Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

1-Algebraic-functions/1.3-Miscellaneous/52-1.3.2-Algebraic-functions

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [1025]. This is test number [52].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.41 (1019)	0.59 (6)
Mathematica	98.44 (1009)	1.56 (16)
Maple	82.34 (844)	17.66 (181)
Fricas	81.76 (838)	18.24 (187)
Giac	50.63 (519)	49.37 (506)
Mupad	44.39 (455)	55.61 (570)
Maxima	35.02 (359)	64.98 (666)
Sympy	31.41 (322)	68.59 (703)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

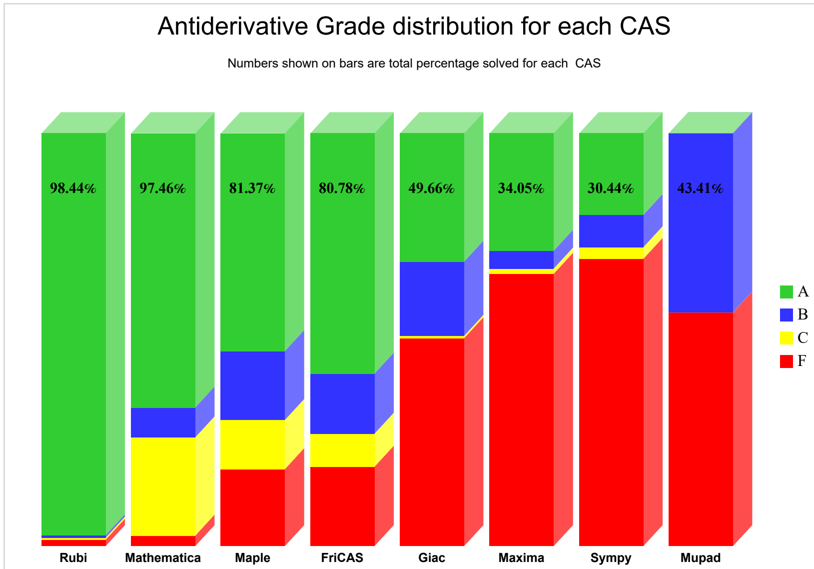
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

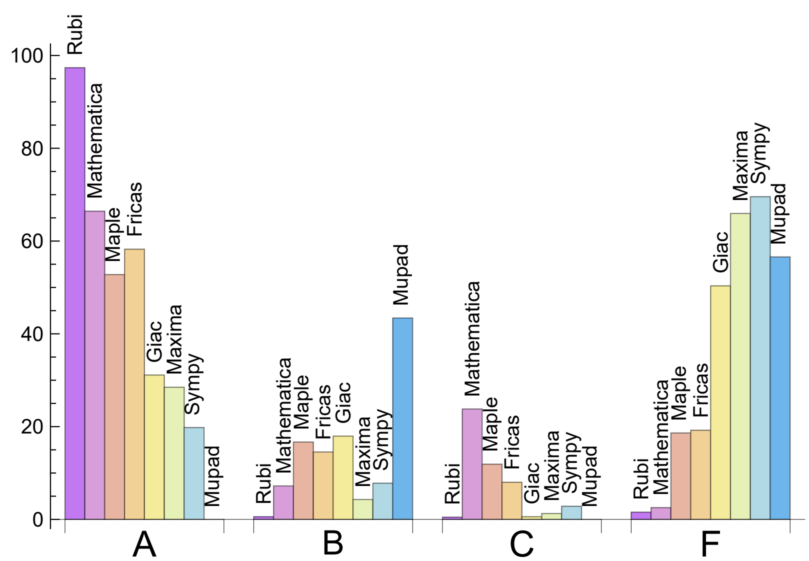
System	% A grade	% B grade	% C grade	% F grade
Rubi	96.878	1.073	0.488	1.561
Mathematica	66.439	7.220	23.805	2.537
Fricas	58.244	14.537	8.000	19.220
Maple	52.780	16.683	11.902	18.634
Giac	31.122	17.951	0.585	50.341
Maxima	28.488	4.293	1.268	65.951
Sympy	19.805	7.805	2.829	69.561
Mupad	0.000	43.415	0.000	56.585

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	6	100.00	0.00	0.00
Mathematica	16	100.00	0.00	0.00
Maple	181	100.00	0.00	0.00
Fricas	187	52.94	40.11	6.95
Giac	506	76.09	10.08	13.83
Mupad	570	0.00	100.00	0.00
Maxima	666	93.39	0.00	6.61
Sympy	703	87.06	11.10	1.85

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.23
Rubi	0.37
Giac	0.57
Fricas	0.63
Maple	1.86
Mathematica	3.48
Sympy	3.69
Mupad	14.17

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	82.99	1.09	38.00	0.90
Rubi	145.18	1.05	85.00	1.00
Giac	220.05	2.02	59.00	1.20
Mupad	229.19	2.71	50.00	1.09
Mathematica	258.12	1.57	81.00	1.00
Fricas	263.45	2.38	81.00	1.28
Maple	381.04	2.73	74.00	1.01
Sympy	956.83	4.60	63.00	1.06

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

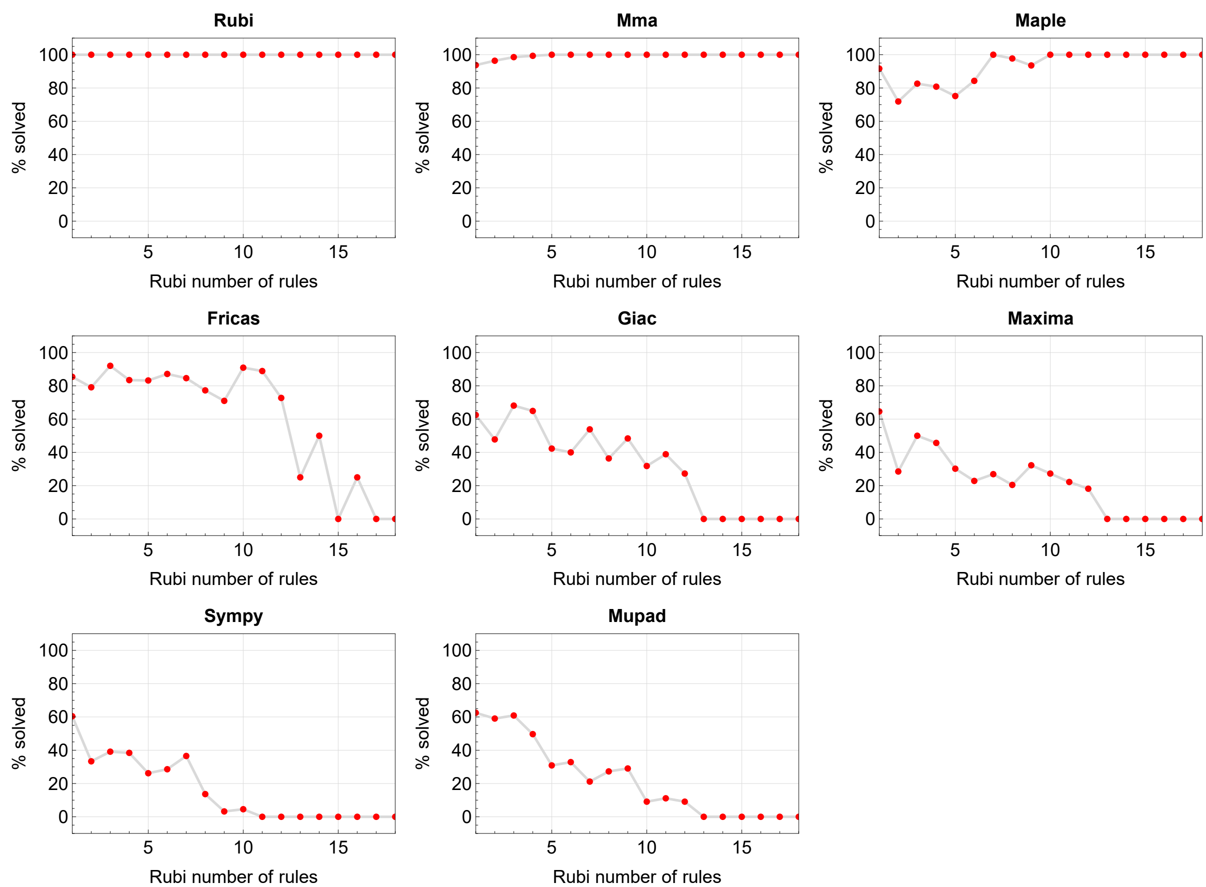


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

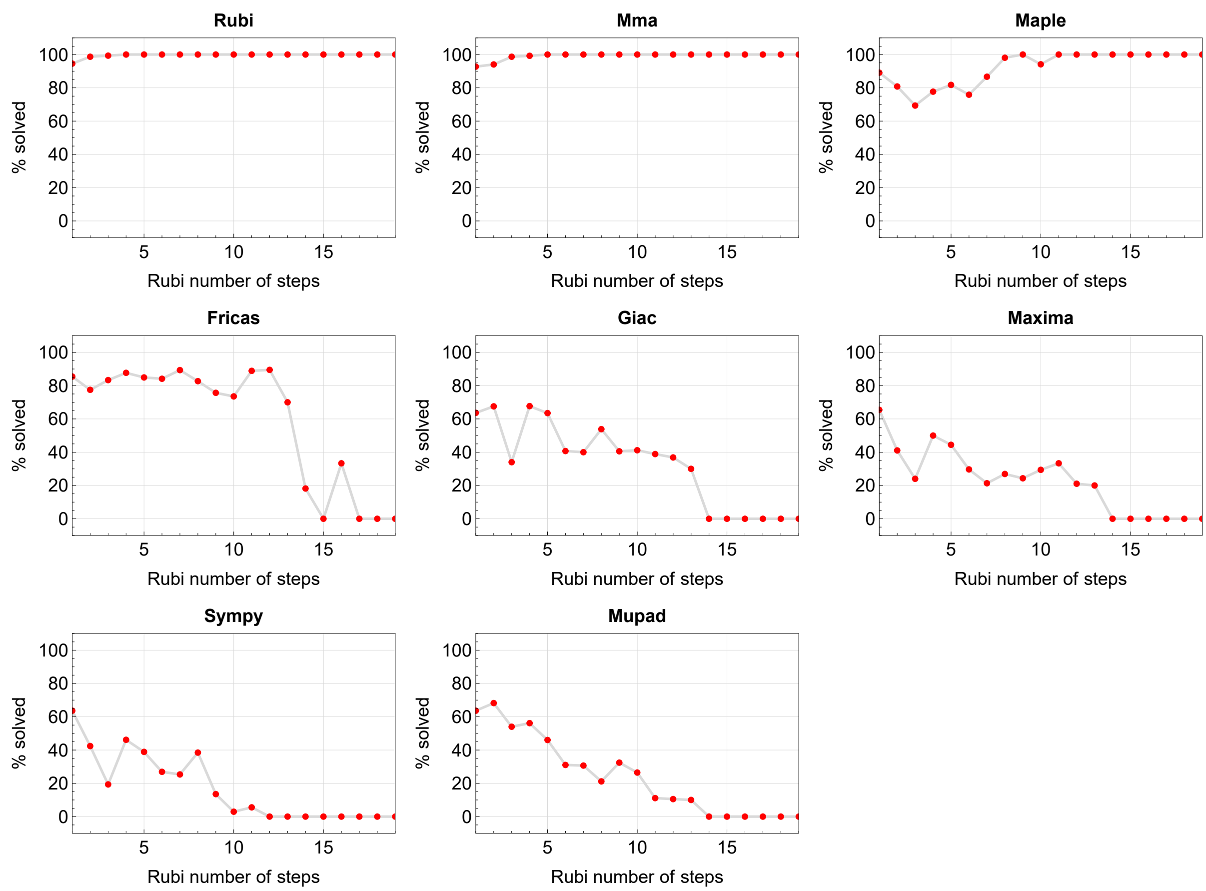


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

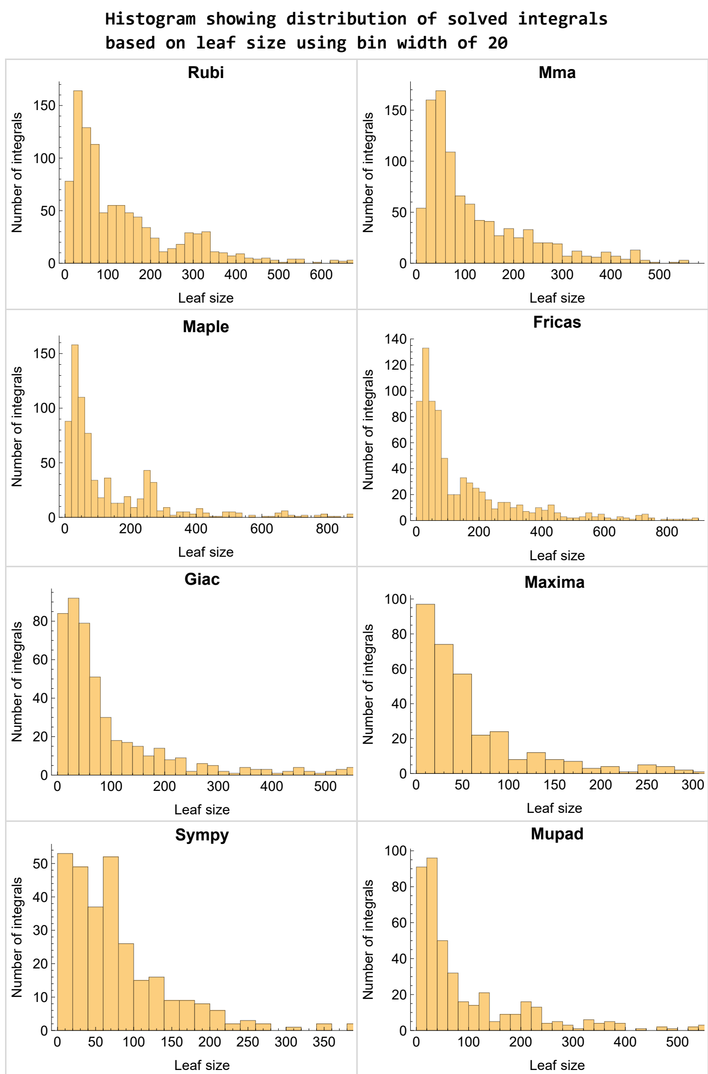


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

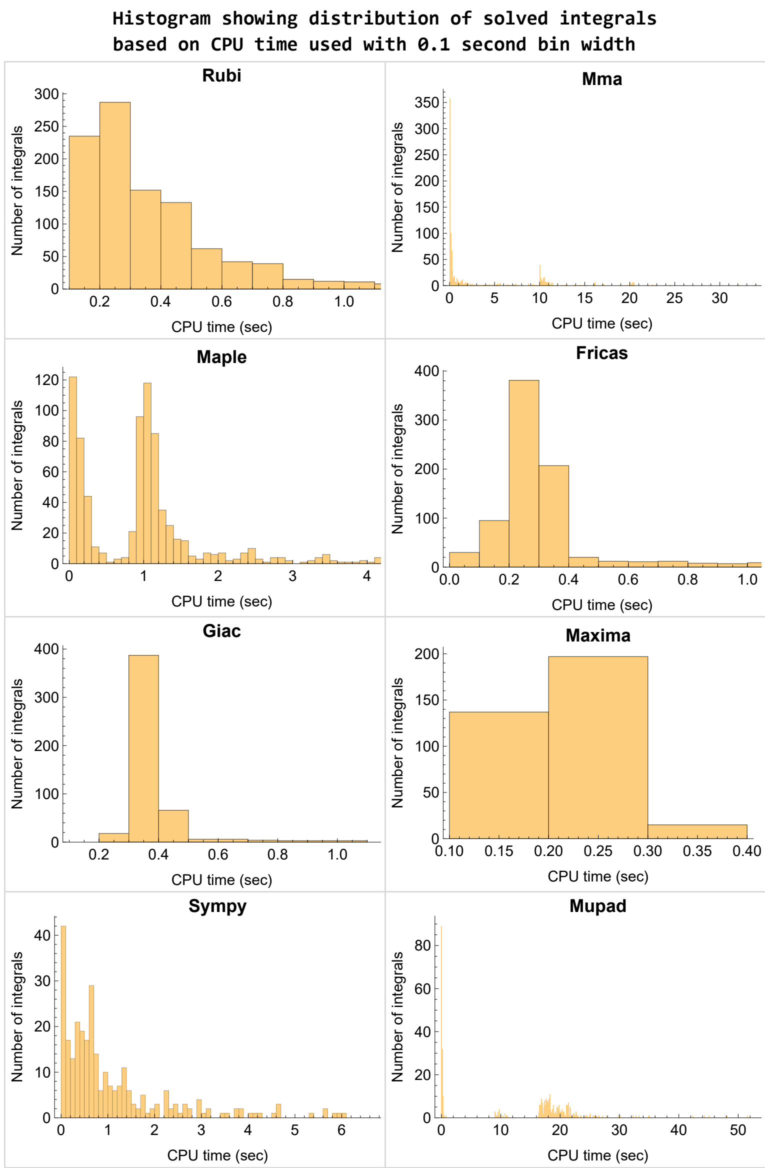


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

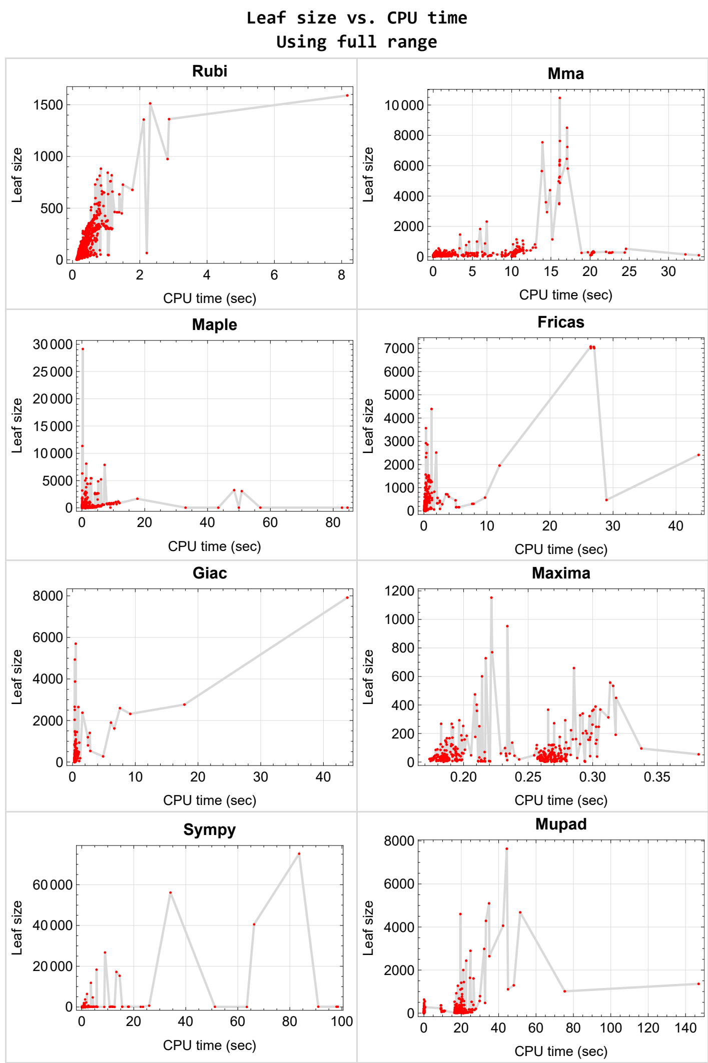


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{901, 902, 903, 904, 905, 906, 907, 908, 909, 910}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {14, 15, 16, 17, 144, 145, 146, 147, 148, 149, 150, 151, 160, 161, 162, 163, 164, 165, 166, 167, 196, 218, 226, 263, 264, 265, 267, 268, 269, 276, 277, 278, 280, 281, 282, 289, 290, 291, 292, 293, 296, 297, 298, 300, 301, 307, 308, 309, 310, 311, 312, 318, 319, 322, 323, 324, 331, 332, 335, 336, 337, 344, 345, 348, 349, 355, 356, 358, 359, 360, 544, 545, 547, 548, 552, 553, 555, 556, 577, 607, 609, 630, 631, 651, 652, 690, 713, 714, 715, 716, 753, 754, 755, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 799, 915, 917, 944, 963, 964}

Mathematica {1, 2, 3, 4, 9, 10, 11, 12, 13, 14, 15, 16, 17, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 69, 70, 71, 72, 73, 83, 84, 85, 86, 109, 110, 111, 112, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 139, 144, 145, 146, 147, 148, 149, 150, 151, 164, 165, 166, 167, 196, 559, 561, 562, 764, 765, 766, 767, 768, 769, 770, 772, 773, 774, 775, 777, 778, 780, 796, 797, 798, 799, 800, 801, 802, 803, 804, 1009}

Maple {170, 557, 558, 559, 560, 561, 580, 774, 775, 777, 778, 780, 781, 782, 784, 785, 786, 787, 789, 790, 791, 792, 793, 794, 795, 796, 800, 803, 838, 997, 998, 1016, 1017, 1024}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

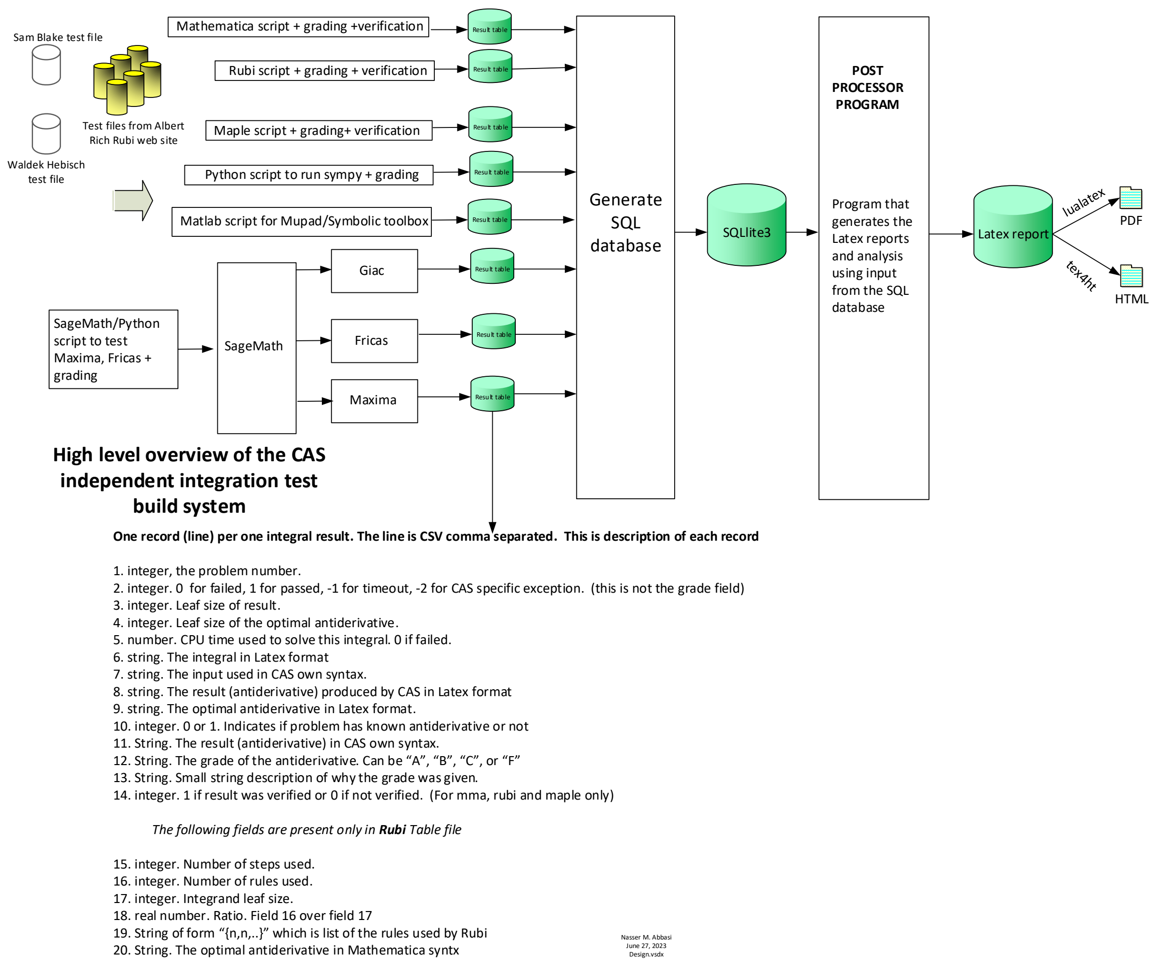
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 613, 614, 615, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1016, 1019, 1020, 1024, 1025 }

B grade { 451, 452, 611, 612, 726, 845, 846, 847, 997, 1015, 1017 }

C grade { 396, 941, 1018, 1021, 1022 }

F normal fail { 197, 616, 617, 995, 996, 1023 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 23, 24, 25, 26, 29, 30, 31, 32, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 74, 75, 76, 77, 78, 79, 80, 81, 82, 101, 102, 103, 104, 105, 106, 107, 108, 113, 114, 115, 116, 117, 118, 119, 120, 172, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 198, 199, 200, 201, 202, 203, 204, 205, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 259, 261, 263, 264, 265, 266, 267, 268, 269, 272, 273, 276, 277, 278, 279, 280, 281, 282, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 304, 305, 307, 308, 309, 310, 311, 312, 318, 319, 320, 321, 322, 323, 324, 327, 328, 331, 332, 333, 334, 335, 336, 337, 344, 345, 346, 347, 348, 349, 352, 353, 355, 356, 357, 358, 359, 360, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 379, 382, 384, 385, 388, 390, 393, 394, 395, 396, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 424, 427, 428, 429, 430, 432, 433, 434, 435, 436, 437, 439, 443, 444, 445, 446, 449, 452, 453, 454, 455, 456, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 528, 529, 530, 531, 534, 535, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 575, 576, 577, 578, 579, 581, 582, 583, 584, 585, 586, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 602, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 733, 734, 735, 736, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 842, 848, 849, 850, 851, 854, 855, 856, 858, 859, 860, 862, 863, 864, 865, 866, 867, 872, 874, 876, 877, 878, 879, 880, 881, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 915, 916, 917, 920, 921, 922, 923, 924, 925, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 974, 975, 976, 977, 979, 980, 982, 983, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 1000, 1001, 1006, 1007, 1008, 1015, 1016, 1017, 1020, 1023, 1024, 1025 }

B grade { 422, 423, 425, 426, 438, 440, 441, 447, 448, 450, 451, 457, 479, 524, 562, 681, 682, 729, 730, 731, 732, 737, 738, 739, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 839, 840, 841, 843, 844, 845, 846, 847, 852, 853, 857, 861, 868, 869, 870, 871, 873, 875, 918, 919, 926, 927, 961, 973, 978, 981, 1009, 1010, 1011, 1012, 1013, 1014 }

C grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 109, 110, 111, 112, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 196, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 255, 256, 257, 258, 260, 262, 270, 271, 274, 275, 283, 284, 285, 286, 287, 288, 302, 303, 306, 313, 314, 315, 316, 317, 325, 326, 329, 330, 338, 339, 340, 341, 342, 343, 350, 351, 354, 361, 362, 363, 364, 365, 378, 380, 381, 383, 386, 387, 389, 391, 392, 397, 431, 442, 523, 525, 526, 527, 532, 533, 536, 537, 574, 580, 601, 603, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 882, 883, 884, 911, 912, 913, 914, 942, 984, 998, 999, 1002, 1003, 1004, 1005, 1018, 1019, 1021, 1022 }

F normal fail { 19, 20, 21, 22, 27, 28, 33, 34, 35, 40, 41, 42, 173, 195, 197, 587 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 10, 11, 12, 13, 14, 15, 16, 17, 53, 54, 55, 57, 58, 59, 66, 68, 84, 85, 86, 93, 127, 128, 129, 130, 137, 138, 144, 145, 146, 147, 148, 149, 150, 151, 154, 158, 160, 161, 162, 163, 164, 165, 166, 167, 176, 196, 200, 201, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 259, 261, 263, 264, 265, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 281, 282, 283, 285, 286, 287, 288, 289, 290, 291, 292, 294, 295, 296, 297, 298, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 312, 313, 314, 315, 316, 317, 318, 319, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 336, 337, 340, 344, 345, 349, 350, 351, 352, 353, 354, 355, 356, 359, 360, 361, 362, 363, 364, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 379, 382, 384, 385, 388, 390, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 423, 425, 426, 427, 428, 429, 430, 431, 433, 438, 439, 440, 441, 444, 445, 446, 448, 450, 451, 455, 456, 474, 475, 498, 504, 523, 524, 525, 530, 531, 538, 539, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 678, 679, 680, 682, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 710, 711, 712, 713, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 727, 728, 729, 731, 733, 734, 735, 736, 738, 741, 742, 743, 744, 748, 749, 750, 751, 752, 756, 757, 758, 762, 763, 805, 806, 807, 808, 809, 810, 813, 814, 815, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 833, 834, 835, 836, 837, 839, 841, 842, 844, 845, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 858, 860, 864, 865, 868, 870, 872, 874, 877, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 893, 894, 895, 896, 897, 898, 899, 900, 915, 916, 917, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 943, 944, 945, 946, 947, 948, 949, 950, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 967, 969, 970, 971, 972, 975, 977, 979, 980, 981, 982, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 1004, 1005, 1006, 1007, 1009, 1010, 1019, 1020, 1023 }

B grade { 9, 52, 56, 64, 65, 67, 73, 74, 75, 83, 91, 92, 94, 95, 100, 125, 126, 135, 136, 139, 174, 175, 178, 179, 180, 182, 183, 184, 260, 262, 266, 279, 284, 299, 310, 320, 321, 322, 333, 334, 335, 338, 339, 341, 342, 343, 346, 347, 348, 357, 358, 365, 408, 413, 422, 424, 432, 442, 443, 447, 449, 452, 454, 457, 458, 459, 473, 476, 477, 478, 487, 528, 529, 534, 535, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 631, 651, 652, 681, 683, 726, 730, 732, 737, 739, 740, 745, 746, 747, 753, 759, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 811, 812, 816, 832, 840, 843, 846, 857, 859, 861, 862, 863, 867, 869, 871, 873, 875, 878, 918, 919, 920, 921, 965, 966, 974, 976, 978, 983, 984, 1015, 1025 }

C grade { 21, 22, 43, 44, 45, 46, 51, 76, 77, 82, 101, 102, 103, 104, 113, 114, 115, 116, 152, 153, 155, 156, 157, 159, 168, 169, 170, 171, 198, 199, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 226, 293, 378, 380, 381, 383, 386, 387, 389, 391, 392, 406, 407, 409, 410, 434, 435, 436, 437, 468, 469, 470, 485, 486, 526, 527, 532, 533, 536, 537, 557, 558, 559, 560, 561, 675, 676, 677, 709, 714, 754, 755, 760, 761, 796, 797, 798, 799, 800, 801, 802, 803, 804, 831, 838, 866, 892, 951, 968, 973, 997, 998, 1002, 1003, 1016, 1017, 1018, 1021, 1022, 1024 }

F normal fail { 5, 6, 7, 8, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 47, 48, 49, 50, 60, 61, 62, 63, 69, 70, 71, 72, 78, 79, 80, 81, 87, 88, 89, 90, 96, 97, 98, 99, 105, 106, 107, 108, 109, 110, 111, 112, 117, 118, 119, 120, 121, 122, 123, 124, 131, 132, 133, 134, 140, 141, 142, 143, 172, 173, 177, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 197, 224, 225, 253, 254, 255, 256, 257, 258, 453, 460, 461, 462, 463, 464, 465, 466, 467, 471, 472, 479, 480, 481, 482, 483, 484, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 562, 563, 564, 587, 588, 589, 590, 591, 592, 593, 594, 653, 654, 655, 656, 657, 876, 911, 912, 913, 914, 942, 995, 996, 999, 1000, 1001, 1008, 1011, 1012, 1013, 1014 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 77, 103, 104, 105, 107, 109, 111, 113, 114, 117, 119, 121, 123, 198, 199, 200, 201, 202, 203, 204, 206, 207, 208, 209, 212, 213, 214, 215, 219, 220, 221, 222, 227, 228, 229, 230, 231, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 349, 350, 351, 352, 353, 354, 355, 356, 359, 361, 362, 363, 364, 365, 366, 369, 370, 371, 372, 373, 376, 377, 379, 382, 384, 385, 390, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 448, 449, 450, 451, 452, 454, 455, 456, 457, 460, 461, 462, 463, 464, 466, 467, 468, 469, 470, 471, 473, 474, 475, 476, 479, 480, 481, 482, 485, 486, 487, 490, 491, 492, 495, 496, 497, 498, 501, 502, 503, 504, 507, 508, 509, 512, 514, 515, 516, 518, 519, 520, 522, 524, 525, 528, 529, 530, 531, 534, 535, 538, 539, 540, 541, 542, 543, 544, 547, 548, 549, 551, 552, 553, 555, 556, 564, 565, 567, 568, 569, 570, 571, 572, 573, 575, 576, 578, 579, 581, 582, 583, 584, 585, 586, 595, 596, 597, 599, 600, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 646, 647, 648, 649, 656, 658, 659, 660, 661, 662, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 685, 686, 691, 692, 693, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 711, 712, 713, 714, 715, 716, 717, 718, 719, 722, 723, 724, 725, 726, 727, 728, 731, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 751, 752, 754, 755, 756, 757, 758, 760, 761, 762, 763, 764, 765, 769, 770, 805, 806, 807, 808, 809, 810, 811, 812, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 835, 836, 837, 838, 848, 849, 850, 851, 852, 853, 854, 856, 858, 860, 862, 867, 872, 874, 877, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 911, 912, 915, 916, 917, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 931, 932, 933, 934, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 960, 961, 962, 963, 964, 965, 966, 967, 969, 970, 971, 972, 975, 977, 979, 980, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 1004, 1005, 1007, 1009, 1010, 1011, 1012, 1013, 1014, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025 }

B grade { 21, 22, 43, 44, 45, 46, 51, 74, 75, 76, 80, 81, 82, 101, 102, 106, 108, 110, 112, 115, 116, 118, 120, 122, 124, 174, 175, 176, 178, 179, 180, 182, 183, 184, 205, 232, 238, 310, 321, 322, 347, 348, 357, 358, 360, 367, 368, 374, 375, 388, 408, 413, 422, 447, 458, 459, 465, 477, 478, 483, 484, 526, 527, 532, 533, 536, 537, 545, 546, 550, 554, 566, 574, 580, 598, 629, 630, 631, 644, 645, 650, 651, 652, 653, 654, 655, 663, 681, 682, 683, 684, 687, 688, 689, 690, 694, 710, 720, 721, 729, 730, 732, 750, 753, 759, 832, 833, 834, 839, 840, 841, 842, 843, 844, 845, 846, 847, 859, 861, 863, 864, 865, 866, 868, 869, 870, 871, 873, 875, 878, 879, 918, 919, 935, 957, 958, 959, 968, 973, 974, 976, 978, 981, 997, 999, 1006, 1015, 1016, 1017 }

C grade { 1, 2, 9, 10, 11, 12, 13, 52, 53, 55, 56, 57, 58, 59, 64, 65, 66, 67, 68, 73, 83, 84, 85, 86, 91, 92, 93, 94, 95, 100, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 152, 153, 154, 155, 156, 157, 158, 159, 168, 169, 170, 171, 378, 380, 381, 383, 387, 389, 391, 392, 397, 577, 601, 766, 767, 768, 771, 772, 773, 813, 855, 857, 998 }

F normal fail { 14, 15, 16, 17, 24, 25, 26, 31, 32, 37, 38, 39, 144, 148, 160, 161, 162, 163, 177, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 210, 224, 225, 255, 256, 257, 386, 453, 488, 489, 493, 494, 499, 500, 505, 506, 510, 511, 513, 517, 521, 562, 563, 587, 588, 589, 590, 591, 592, 593, 594, 657, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 876, 1002, 1003 }

F(-1) timedout fail { 5, 6, 7, 8, 18, 19, 20, 23, 27, 28, 29, 30, 33, 34, 35, 36, 40, 41, 42, 47, 48, 49, 50, 60, 61, 62, 63, 69, 70, 71, 72, 78, 79, 87, 88, 89, 90, 96, 97, 98, 99, 145, 146, 149, 150, 164, 165, 166, 167, 173, 211, 216, 217, 218, 223, 226, 253, 254, 472, 523, 557, 558, 560, 561, 613, 614, 615, 616, 617, 913, 914, 930, 996, 1000, 1001 }

F(-2) exception fail { 3, 4, 54, 147, 151, 172, 258, 393, 394, 395, 396, 559, 1008 }

2.1.5 Maxima

A grade { 174, 175, 176, 179, 180, 227, 228, 229, 230, 231, 233, 234, 235, 236, 244, 246, 247, 248, 249, 250, 251, 252, 253, 254, 290, 293, 318, 319, 321, 322, 331, 332, 333, 334, 335, 336, 344, 345, 347, 348, 355, 356, 357, 358, 359, 370, 371, 372, 373, 374, 375, 376, 379, 384, 385, 396, 398, 399, 400, 419, 420, 421, 422, 423, 424, 425, 443, 444, 445, 446, 447, 448, 449, 450, 454, 455, 456, 530, 531, 538, 539, 544, 545, 546, 552, 553, 554, 565, 567, 568, 569, 570, 571, 572, 573, 575, 576, 577, 578, 579, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 646, 647, 648, 649, 654, 655, 656, 670, 676, 677, 679, 680, 681, 683, 684, 685, 686, 693, 694, 695, 696, 697, 698, 699, 700, 707, 708, 709, 711, 712, 713, 714, 715, 716, 717, 718, 719, 722, 724, 725, 733, 734, 735, 737, 738, 739, 740, 741, 742, 743, 744, 745, 747, 748, 749, 762, 763, 805, 808, 809, 810, 814, 815, 816, 817, 818, 819, 820, 822, 824, 825, 833, 835, 836, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 852, 854, 856, 857, 858, 859, 860, 861, 862, 864, 865, 866, 867, 868, 870, 872, 874, 883, 884, 915, 916, 917, 922, 923, 924, 925, 926, 927, 928, 929, 935, 936, 937, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 952, 953, 954, 959, 960, 961, 963, 964, 971, 972, 975, 977, 979, 980, 981, 982, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 1007, 1020, 1023 }

B grade { 178, 182, 183, 184, 232, 238, 320, 323, 324, 337, 346, 349, 360, 369, 564, 566, 574, 580, 612, 613, 614, 615, 616, 617, 645, 653, 675, 678, 729, 730, 731, 732, 736, 746, 750, 829, 880, 948, 957, 958, 974, 976, 978, 1015 }

C grade { 581, 582, 583, 584, 585, 586, 595, 596, 597, 598, 599, 600, 855 }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 177, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 237, 239, 240, 241, 242, 243, 245, 255, 256, 257, 258, 260, 262, 270, 271, 272, 273, 274, 275, 283, 284, 285, 286, 287, 288, 289, 291, 292, 294, 295, 302, 303, 304, 305, 306, 313, 314, 315, 316, 317, 325, 326, 327, 328, 329, 330, 338, 339, 340, 341, 342, 343, 350, 351, 352, 353, 354, 361, 362, 363, 364, 365, 366, 367, 368, 377, 378, 380, 381, 382, 383, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 451, 452, 453, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 532, 533, 534, 535, 536, 537, 540, 541, 542, 543, 547, 548, 549, 550, 551, 555, 556, 557, 558, 559, 560, 561, 562, 563, 587, 588, 589, 590, 591, 592, 593, 594, 601, 629, 630, 631, 650, 651, 652, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 671, 672, 673, 674, 682, 691, 692, 701, 702, 703, 704, 705, 706, 710, 720, 721, 723, 726, 727, 728, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 806, 807, 813, 821, 823, 826, 827, 828, 830, 831, 834, 837, 838, 850, 851, 853, 863, 869, 871, 873, 875, 876, 877, 878, 879, 881, 882, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 911, 912, 913, 914, 918, 919, 920, 921, 930, 931, 932, 933, 934, 938, 939, 940, 962, 965, 966, 967, 968, 969, 970, 973, 983, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1016, 1017, 1018, 1019, 1021, 1022, 1024, 1025 }

F(-1) timedout fail { }

F(-2) exception fail { 202, 203, 204, 205, 259, 261, 263, 264, 265, 266, 267, 268, 269, 276, 277, 278, 279, 280, 281, 282, 296, 297, 298, 299, 300, 301, 307, 308, 309, 310, 311, 312, 473, 474, 475, 687, 688, 689, 690, 811, 812, 832, 955, 956 }

2.1.6 Giac

A grade { 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 259, 263, 264, 265, 289, 290, 291, 292, 293, 295, 296, 297, 298, 318, 319, 344, 345, 367, 369, 371, 372, 373, 374, 375, 376, 377, 379, 382, 384, 385, 390, 398, 399, 400, 403, 409, 416, 417, 418, 452, 454, 455, 456, 458, 459, 473, 474, 475, 498, 504, 530, 531, 538, 539, 544, 545, 546, 547, 548, 550, 551, 552, 553, 554, 555, 556, 564, 565, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 602, 603, 604, 605, 607, 608, 609, 610, 618, 619, 620, 622, 623, 624, 629, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 670, 675, 676, 677, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 716, 718, 720, 721, 722, 723, 724, 725, 727, 728, 731, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 748, 749, 755, 756, 762, 763, 806, 807, 808, 809, 810, 814, 816, 817, 818, 819, 820, 821, 822, 823, 830, 833, 834, 835, 836, 837, 838, 840, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 865, 872, 874, 877, 882, 883, 891, 893, 894, 895, 896, 897, 898, 899, 900, 915, 916, 917, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 959, 960, 961, 962, 963, 964, 965, 966, 967, 970, 971, 972, 973, 974, 975, 976, 977, 979, 980, 982, 984, 1007, 1020 }

B grade { 174, 175, 176, 178, 179, 180, 183, 184, 227, 261, 267, 268, 269, 300, 301, 320, 322, 323, 324, 331, 332, 333, 346, 348, 349, 355, 356, 366, 368, 388, 401, 402, 404, 405, 406, 407, 408, 410, 411, 412, 413, 414, 415, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 457, 477, 524, 525, 526, 527, 536, 537, 566, 582, 583, 584, 585, 586, 606, 621, 625, 626, 627, 628, 630, 631, 651, 652, 653, 654, 655, 656, 678, 679, 680, 714, 715, 717, 719, 726, 729, 730, 732, 746, 747, 750, 751, 753, 754, 757, 758, 759, 760, 761, 805, 811, 812, 813, 815, 826, 827, 828, 831, 832, 839, 841, 842, 843, 844, 845, 846, 847, 863, 864, 866, 867, 868, 870, 878, 879, 880, 881, 884, 885, 918, 919, 955, 956, 957, 958, 968, 969, 978, 981, 983, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 997, 998, 1006, 1015, 1021, 1022, 1023 }

C grade { 294, 596, 600, 824, 825, 892 }

F normal fail { 1, 9, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 51, 55, 64, 68, 73, 74, 75, 76, 77, 82, 83, 84, 85, 86, 91, 92, 93, 94, 95, 100, 125, 126, 135, 136, 139, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 177, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 255, 256, 257, 258, 260, 262, 270, 271, 272, 273, 274, 275, 283, 284, 285, 286, 287, 288, 302, 303, 304, 305, 306, 313, 314, 315, 316, 317, 325, 326, 327, 328, 329, 330, 335, 336, 337, 338, 339, 340, 341, 342, 343, 350, 351, 352, 353, 354, 359, 360, 361, 362, 363, 364, 365, 378, 380, 381, 383, 386, 387, 389, 391, 392, 393, 394, 395, 396, 397, 453, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 528, 529, 532, 533, 534, 535, 540, 541, 542, 543, 557, 558, 559, 560, 561, 562, 563, 587, 588, 589, 590, 591, 592, 593, 594, 595, 597, 599, 601, 611, 612, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 671, 672, 673, 674, 752, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 829, 869, 871, 873, 875, 876, 886, 887, 888, 889, 890, 911, 912, 913, 914, 941, 942, 995, 996, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1016, 1017, 1018, 1019, 1024, 1025 }

F(-1) timedout fail { 5, 6, 7, 8, 47, 48, 49, 50, 60, 61, 62, 63, 69, 70, 71, 72, 78, 79, 80, 81, 87, 88, 89, 90, 96, 97, 98, 99, 105, 106, 107, 108, 117, 118, 119, 120, 131, 132, 133, 134, 140, 141, 142, 143, 476, 478, 613, 614, 615, 616, 617 }

F(-2) exception fail { 2, 3, 4, 10, 11, 12, 13, 43, 44, 45, 46, 52, 53, 54, 56, 57, 58, 59, 65, 66, 67, 101, 102, 103, 104, 109, 110, 111, 112, 113, 114, 115, 116, 121, 122, 123, 124, 127, 128, 129, 130, 137, 138, 182, 266, 276, 277, 278, 279, 280, 281, 282, 299, 307, 308, 309, 310, 311, 312, 321, 334, 347, 357, 358, 370, 549, 581, 598, 920, 921 }

2.1.7 Mupad

A grade { }

B grade { 14, 15, 16, 17, 43, 44, 45, 46, 47, 48, 49, 50, 51, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 92, 93, 94, 95, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 174, 175, 176, 178, 179, 180, 182, 183, 184, 198, 199, 200, 201, 202, 203, 204, 205, 209, 215, 221, 222, 227, 230, 232, 238, 244, 245, 247, 249, 250, 251, 252, 289, 290, 291, 292, 293, 320, 333, 346, 357, 369, 370, 379, 384, 385, 396, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 455, 456, 498, 504, 516, 518, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 536, 537, 538, 539, 544, 545, 546, 547, 548, 552, 553, 554, 555, 556, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 590, 595, 596, 597, 599, 600, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 628, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 649, 656, 670, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 707, 708, 709, 711, 712, 722, 724, 725, 726, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 750, 751, 762, 763, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 824, 825, 826, 827, 828, 830, 831, 833, 835, 836, 837, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 854, 855, 856, 857, 858, 859, 860, 861, 862, 865, 867, 868, 870, 872, 874, 876, 877, 878, 879, 880, 881, 884, 885, 886, 887, 888, 889, 890, 900, 915, 916, 917, 918, 919, 922, 923, 924, 925, 928, 930, 931, 933, 935, 936, 937, 939, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 957, 958, 959, 960, 963, 964, 965, 966, 967, 968, 969, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 997, 1007, 1015, 1020, 1021, 1022, 1023 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 87, 88, 89, 90, 91, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 172, 173, 177, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 223, 224, 225, 226, 228, 229, 231, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 246, 248, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 371, 372, 373, 374, 375, 376, 377, 378, 380, 381, 382, 383, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 453, 454, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 517, 519, 520, 521, 522, 523, 534, 535, 540, 541, 542, 543, 549, 550, 551, 557, 558, 559, 560, 561, 562, 563, 587, 588, 589, 591, 592, 593, 594, 598, 601, 613, 614, 625, 626, 627, 629, 630, 631, 646, 647, 648, 650, 651, 652, 653, 654, 655, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 671, 672, 673, 674, 701, 702, 703, 704, 705, 706, 710, 713, 714, 715, 716, 717, 718, 719, 720, 721, 723, 727, 728, 749, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 817, 818, 819, 820, 821, 822, 823, 829, 832, 834, 838, 850, 851, 852, 853, 863, 864, 866, 869, 871, 873, 875, 882, 883, 891, 892, 893, 894, 895, 896, 897, 898, 899, 911, 912, 913, 914, 920, 921, 926, 927, 929, 932, 934, 938, 940, 955, 956, 961, 962, 970, 995, 996, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1016, 1017, 1018, 1019, 1024, 1025 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 23, 24, 25, 29, 30, 31, 36, 37, 38, 152, 153, 154, 155, 156, 157, 158, 159, 168, 169, 170, 171, 206, 212, 249, 250, 251, 252, 259, 379, 388, 452, 454, 455, 456, 473, 474, 475, 524, 525, 527, 531, 537, 539, 544, 545, 547, 552, 553, 555, 565, 567, 569, 570, 571, 573, 575, 576, 602, 606, 607, 608, 609, 610, 611, 612, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 632, 633, 634, 635, 636, 639, 640, 641, 643, 646, 647, 648, 649, 658, 659, 662, 663, 664, 665, 668, 669, 670, 675, 676, 677, 678, 679, 680, 681, 682, 684, 686, 687, 689, 691, 692, 693, 694, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 806, 808, 809, 810, 811, 812, 814, 833, 835, 836, 839, 841, 842, 844, 845, 847, 848, 849, 850, 852, 854, 856, 858, 860, 868, 870, 872, 874, 878, 879, 915, 916, 917, 922, 923, 924, 928, 936, 937, 939, 943, 944, 945, 946, 948, 949, 950, 952, 953, 954, 962, 966, 970, 977, 980, 981, 984, 1007, 1015, 1020 }

B grade { 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 197, 244, 245, 398, 399, 403, 408, 412, 413, 416, 417, 418, 422, 443, 447, 469, 470, 486, 487, 498, 504, 526, 528, 529, 530, 532, 533, 534, 535, 536, 538, 546, 554, 564, 566, 603, 604, 605, 642, 683, 685, 695, 714, 762, 763, 829, 837, 877, 933, 955, 956, 957, 958, 959, 963, 964, 967, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 1023 }

C grade { 26, 32, 39, 207, 208, 209, 213, 214, 215, 221, 222, 261, 380, 468, 588, 589, 590, 729, 731, 840, 843, 846, 862, 941, 942, 951, 965, 971, 972 }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 27, 28, 33, 34, 35, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 160, 161, 162, 163, 164, 165, 166, 167, 172, 173, 192, 196, 198, 199, 200, 201, 202, 203, 204, 205, 210, 211, 216, 217, 218, 219, 220, 223, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 246, 247, 248, 253, 254, 255, 256, 257, 258, 260, 262, 289, 290, 291, 294, 295, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 381, 382, 383, 384, 385, 386, 387, 389, 390, 391, 392, 393, 394, 395, 396, 397, 401, 402, 404, 405, 406, 407, 409, 410, 411, 414, 415, 419, 420, 421, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 444, 445, 446, 448, 449, 450, 451, 453, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 471, 476, 477, 478, 479, 480, 481, 482, 483, 484, 488, 489, 490, 491, 492, 493, 494, 496, 497, 499, 500, 502, 503, 505, 506, 509, 516, 523, 542, 543, 548, 549, 550, 551, 556, 557, 558, 559, 560, 561, 562, 563, 568, 572, 574, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 629, 630, 637, 638, 644, 645, 650, 651, 653, 654, 655, 656, 657, 660, 661, 666, 667, 671, 672, 673, 674, 688, 690, 726, 727, 728, 730, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 756, 757, 758, 759, 760, 761, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 807, 813, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 830, 831, 832, 834, 838, 851, 853, 855, 857, 859, 861, 863, 864, 865, 866, 867, 869, 871, 873, 875, 876, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 911, 912, 913, 914, 918, 919, 920, 921, 925, 926, 927, 929, 930, 931, 932, 934, 935, 938, 940, 947, 960, 961, 968, 969, 973, 974, 975, 976, 978, 979, 982, 983, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1016, 1017, 1018, 1019, 1021, 1022, 1024, 1025 }

F(-1) timedout fail { 186, 187, 188, 189, 190, 191, 193, 194, 195, 224, 225, 227, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 292, 293, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 400, 485, 512, 513, 518, 522, 540, 541, 613, 614, 615, 616, 617, 631, 652, 755 }

F(-2) exception fail { 472, 495, 501, 507, 508, 510, 511, 514, 515, 517, 519, 520, 521 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	148	139	0	74	0	0	0
N.S.	1	1.00	1.02	0.96	0.00	0.51	0.00	0.00	0.00
time (sec)	N/A	0.391	20.141	3.109	0.000	0.113	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	148	143	0	79	0	0	0
N.S.	1	1.00	0.92	0.89	0.00	0.49	0.00	0.00	0.00
time (sec)	N/A	0.433	10.098	2.898	0.000	0.121	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	146	143	0	0	0	0	0
N.S.	1	1.00	0.90	0.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.429	20.202	2.870	0.000	0.000	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	150	139	0	0	0	0	0
N.S.	1	1.00	0.96	0.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.421	10.069	2.885	0.000	0.000	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	280	280	164	0	0	0	0	0	0
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.617	10.170	0.000	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	288	288	166	0	0	0	0	0	0
N.S.	1	1.00	0.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.607	10.159	0.000	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	297	167	0	0	0	0	0	0
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.616	10.116	0.000	0.000	0.000	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	293	293	167	0	0	0	0	0	0
N.S.	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.614	10.119	0.000	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	169	495	0	349	0	0	0
N.S.	1	1.00	0.68	1.99	0.00	1.40	0.00	0.00	0.00
time (sec)	N/A	0.521	10.173	1.375	0.000	0.162	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	136	132	0	57	0	0	0
N.S.	1	1.00	0.93	0.90	0.00	0.39	0.00	0.00	0.00
time (sec)	N/A	0.410	20.191	2.095	0.000	0.146	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	136	143	0	66	0	0	0
N.S.	1	1.00	0.83	0.87	0.00	0.40	0.00	0.00	0.00
time (sec)	N/A	0.439	10.100	2.127	0.000	0.122	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	134	132	0	211	0	0	0
N.S.	1	1.00	0.80	0.79	0.00	1.26	0.00	0.00	0.00
time (sec)	N/A	0.421	20.213	2.120	0.000	0.138	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	138	139	0	213	0	0	0
N.S.	1	1.00	0.88	0.89	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.418	10.070	2.074	0.000	0.124	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	329	321	128	123	0	0	0	0	164
N.S.	1	0.98	0.39	0.37	0.00	0.00	0.00	0.00	0.50
time (sec)	N/A	0.938	20.091	1.250	0.000	0.000	0.000	0.000	9.151

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	380	371	128	133	0	0	0	0	180
N.S.	1	0.98	0.34	0.35	0.00	0.00	0.00	0.00	0.47
time (sec)	N/A	0.996	20.086	1.067	0.000	0.000	0.000	0.000	9.149

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	374	374	126	124	0	0	0	0	164
N.S.	1	1.00	0.34	0.33	0.00	0.00	0.00	0.00	0.44
time (sec)	N/A	0.971	20.065	1.245	0.000	0.000	0.000	0.000	0.030

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	340	321	130	133	0	0	0	0	179
N.S.	1	0.94	0.38	0.39	0.00	0.00	0.00	0.00	0.53
time (sec)	N/A	0.914	20.087	1.182	0.000	0.000	0.000	0.000	0.027

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	139	311	0	0	0	0	0	0
N.S.	1	1.00	2.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.280	2.073	0.000	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	186	180	0	0	0	0	0	0	0
N.S.	1	0.97	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.431	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	187	187	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.282	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	B	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	0	3064	0	712	0	0	0
N.S.	1	1.00	0.00	20.84	0.00	4.84	0.00	0.00	0.00
time (sec)	N/A	0.244	0.000	50.888	0.000	3.740	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	C	F	B	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	0	3247	0	720	0	0	0
N.S.	1	1.00	0.00	20.42	0.00	4.53	0.00	0.00	0.00
time (sec)	N/A	0.252	0.000	48.469	0.000	3.519	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	387	373	163	0	0	0	212	0	0
N.S.	1	0.96	0.42	0.00	0.00	0.00	0.55	0.00	0.00
time (sec)	N/A	0.541	6.644	0.000	0.000	0.000	2.638	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	242	234	142	0	0	0	160	0	0
N.S.	1	0.97	0.59	0.00	0.00	0.00	0.66	0.00	0.00
time (sec)	N/A	0.581	6.486	0.000	0.000	0.000	1.754	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	192	186	111	0	0	0	114	0	0
N.S.	1	0.97	0.58	0.00	0.00	0.00	0.59	0.00	0.00
time (sec)	N/A	0.420	6.308	0.000	0.000	0.000	1.383	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	152	75	0	0	0	82	0	0
N.S.	1	0.98	0.48	0.00	0.00	0.00	0.53	0.00	0.00
time (sec)	N/A	0.307	5.627	0.000	0.000	0.000	1.208	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	435	435	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.743	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	818	818	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.104	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	310	310	392	0	0	0	206	0	0
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.66	0.00	0.00
time (sec)	N/A	0.404	10.305	0.000	0.000	0.000	2.269	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	255	255	287	0	0	0	155	0	0
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.61	0.00	0.00
time (sec)	N/A	0.354	10.066	0.000	0.000	0.000	1.908	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	147	201	0	0	0	110	0	0
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.75	0.00	0.00
time (sec)	N/A	0.320	9.639	0.000	0.000	0.000	1.304	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	124	124	163	0	0	0	78	0	0
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.63	0.00	0.00
time (sec)	N/A	0.243	9.204	0.000	0.000	0.000	0.998	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	333	333	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.588	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	761	761	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.080	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1513	1513	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.242	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	306	306	166	0	0	0	204	0	0
N.S.	1	1.00	0.54	0.00	0.00	0.00	0.67	0.00	0.00
time (sec)	N/A	0.407	10.103	0.000	0.000	0.000	2.331	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	187	184	145	0	0	0	153	0	0
N.S.	1	0.98	0.78	0.00	0.00	0.00	0.82	0.00	0.00
time (sec)	N/A	0.448	10.085	0.000	0.000	0.000	1.716	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	141	95	0	0	0	109	0	0
N.S.	1	1.00	0.67	0.00	0.00	0.00	0.77	0.00	0.00
time (sec)	N/A	0.334	10.034	0.000	0.000	0.000	1.324	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	78	0	0	0	78	0	0
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.64	0.00	0.00
time (sec)	N/A	0.244	9.335	0.000	0.000	0.000	1.018	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	332	332	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.561	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	760	760	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.065	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1357	1357	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.041	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	39	108	0	75	0	0	70
N.S.	1	1.00	1.05	2.92	0.00	2.03	0.00	0.00	1.89
time (sec)	N/A	0.245	1.481	4.065	0.000	0.425	0.000	0.000	9.785

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	41	111	0	76	0	0	74
N.S.	1	1.00	1.02	2.78	0.00	1.90	0.00	0.00	1.85
time (sec)	N/A	0.255	1.474	4.126	0.000	0.391	0.000	0.000	9.764

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	39	108	0	238	0	0	62
N.S.	1	1.00	1.03	2.84	0.00	6.26	0.00	0.00	1.63
time (sec)	N/A	0.243	1.484	4.408	0.000	0.399	0.000	0.000	9.533

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	41	112	0	241	0	0	63
N.S.	1	1.00	1.05	2.87	0.00	6.18	0.00	0.00	1.62
time (sec)	N/A	0.244	1.446	4.633	0.000	0.398	0.000	0.000	9.532

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0	106
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	1.68
time (sec)	N/A	0.324	5.392	0.000	0.000	0.000	0.000	0.000	11.152

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	67	0	0	0	0	0	107
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	1.65
time (sec)	N/A	0.339	5.393	0.000	0.000	0.000	0.000	0.000	10.966

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	68	0	0	0	0	0	102
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	1.55
time (sec)	N/A	0.333	5.379	0.000	0.000	0.000	0.000	0.000	10.034

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	68	0	0	0	0	0	103
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	1.56
time (sec)	N/A	0.328	5.392	0.000	0.000	0.000	0.000	0.000	9.762

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	51	889	0	300	0	0	95
N.S.	1	1.00	1.04	18.14	0.00	6.12	0.00	0.00	1.94
time (sec)	N/A	0.267	1.340	0.892	0.000	0.428	0.000	0.000	10.309

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	158	163	336	262	0	157	0	0	0
N.S.	1	1.03	2.13	1.66	0.00	0.99	0.00	0.00	0.00
time (sec)	N/A	0.450	20.428	4.340	0.000	0.163	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	173	176	335	257	0	160	0	0	0
N.S.	1	1.02	1.94	1.49	0.00	0.92	0.00	0.00	0.00
time (sec)	N/A	0.471	20.418	5.645	0.000	0.161	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	176	179	333	266	0	0	0	0	0
N.S.	1	1.02	1.89	1.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.456	20.398	5.348	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	169	174	338	253	0	429	0	0	0
N.S.	1	1.03	2.00	1.50	0.00	2.54	0.00	0.00	0.00
time (sec)	N/A	0.447	20.358	4.912	0.000	0.181	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	159	165	340	264	0	992	0	0	0
N.S.	1	1.04	2.14	1.66	0.00	6.24	0.00	0.00	0.00
time (sec)	N/A	0.458	20.367	1.965	0.000	0.245	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	175	182	340	261	0	1005	0	0	0
N.S.	1	1.04	1.94	1.49	0.00	5.74	0.00	0.00	0.00
time (sec)	N/A	0.483	20.424	1.961	0.000	0.225	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	178	185	338	270	0	999	0	0	0
N.S.	1	1.04	1.90	1.52	0.00	5.61	0.00	0.00	0.00
time (sec)	N/A	0.452	20.418	1.865	0.000	0.237	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	170	176	342	255	0	1002	0	0	0
N.S.	1	1.04	2.01	1.50	0.00	5.89	0.00	0.00	0.00
time (sec)	N/A	0.467	20.387	1.928	0.000	0.224	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	316	316	348	0	0	0	0	0	0
N.S.	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.711	11.037	0.000	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	324	325	399	0	0	0	0	0	0
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.703	10.942	0.000	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	333	334	400	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.694	10.363	0.000	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	329	329	401	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.680	10.848	0.000	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	380	900	0	390	0	0	0
N.S.	1	1.00	1.43	3.40	0.00	1.47	0.00	0.00	0.00
time (sec)	N/A	0.577	11.072	0.960	0.000	0.319	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	207	258	0	82	0	0	0
N.S.	1	1.00	1.43	1.78	0.00	0.57	0.00	0.00	0.00
time (sec)	N/A	0.426	10.323	3.692	0.000	0.130	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	209	253	0	83	0	0	0
N.S.	1	1.00	1.31	1.58	0.00	0.52	0.00	0.00	0.00
time (sec)	N/A	0.462	10.346	3.716	0.000	0.132	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	207	262	0	245	0	0	0
N.S.	1	1.00	1.27	1.61	0.00	1.50	0.00	0.00	0.00
time (sec)	N/A	0.437	10.313	3.579	0.000	0.121	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	209	249	0	248	0	0	0
N.S.	1	1.00	1.34	1.60	0.00	1.59	0.00	0.00	0.00
time (sec)	N/A	0.448	10.274	3.492	0.000	0.124	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	275	275	336	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.623	11.002	0.000	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	283	283	388	0	0	0	0	0	0
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.651	10.788	0.000	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	292	292	389	0	0	0	0	0	0
N.S.	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.653	10.775	0.000	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	288	288	389	0	0	0	0	0	0
N.S.	1	1.00	1.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.681	10.704	0.000	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	372	892	0	350	0	0	0
N.S.	1	1.00	1.51	3.63	0.00	1.42	0.00	0.00	0.00
time (sec)	N/A	0.553	10.949	0.959	0.000	0.168	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	31	44	0	44	0	0	205
N.S.	1	1.00	1.35	1.91	0.00	1.91	0.00	0.00	8.91
time (sec)	N/A	0.212	0.964	1.319	0.000	0.278	0.000	0.000	0.124

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	33	49	0	47	0	0	221
N.S.	1	1.00	1.22	1.81	0.00	1.74	0.00	0.00	8.19
time (sec)	N/A	0.210	0.939	1.366	0.000	0.289	0.000	0.000	0.105

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	21	75	0	40	0	0	205
N.S.	1	1.00	0.84	3.00	0.00	1.60	0.00	0.00	8.20
time (sec)	N/A	0.202	0.951	1.473	0.000	0.288	0.000	0.000	9.072

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	79	0	38	0	0	221
N.S.	1	1.00	0.92	3.16	0.00	1.52	0.00	0.00	8.84
time (sec)	N/A	0.203	0.864	1.617	0.000	0.284	0.000	0.000	9.324

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	50	50	48	0	0	0	0	0	65
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	1.30
time (sec)	N/A	0.280	2.339	0.000	0.000	0.000	0.000	0.000	9.652

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	52	52	50	0	0	0	0	0	67
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	1.29
time (sec)	N/A	0.284	2.289	0.000	0.000	0.000	0.000	0.000	9.586

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	51	0	0	592	0	0	74
N.S.	1	1.00	0.96	0.00	0.00	11.17	0.00	0.00	1.40
time (sec)	N/A	0.283	2.311	0.000	0.000	0.903	0.000	0.000	10.750

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	51	0	0	641	0	0	78
N.S.	1	1.00	0.96	0.00	0.00	12.09	0.00	0.00	1.47
time (sec)	N/A	0.282	2.257	0.000	0.000	0.904	0.000	0.000	10.702

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	44	503	0	294	0	0	67
N.S.	1	1.00	0.96	10.93	0.00	6.39	0.00	0.00	1.46
time (sec)	N/A	0.263	1.241	1.516	0.000	0.405	0.000	0.000	9.705

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	273	246	0	61	0	0	327
N.S.	1	1.00	1.96	1.77	0.00	0.44	0.00	0.00	2.35
time (sec)	N/A	0.365	20.395	0.949	0.000	0.132	0.000	0.000	0.131

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	271	246	0	66	0	0	359
N.S.	1	1.00	1.77	1.61	0.00	0.43	0.00	0.00	2.35
time (sec)	N/A	0.388	20.229	0.960	0.000	0.092	0.000	0.000	9.264

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	269	246	0	55	0	0	327
N.S.	1	1.00	1.72	1.58	0.00	0.35	0.00	0.00	2.10
time (sec)	N/A	0.373	20.224	0.938	0.000	0.118	0.000	0.000	0.067

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	275	246	0	57	0	0	359
N.S.	1	1.00	1.83	1.64	0.00	0.38	0.00	0.00	2.39
time (sec)	N/A	0.400	20.250	0.932	0.000	0.121	0.000	0.000	9.059

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	292	438	0	0	0	0	0	0
N.S.	1	0.98	1.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.604	11.118	0.000	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	304	299	447	0	0	0	0	0	0
N.S.	1	0.98	1.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.593	11.015	0.000	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	313	308	448	0	0	0	0	0	0
N.S.	1	0.98	1.43	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.617	10.955	0.000	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	310	305	441	0	0	0	0	0	0
N.S.	1	0.98	1.42	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.624	11.047	0.000	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	384	521	0	396	0	0	0
N.S.	1	1.00	1.74	2.36	0.00	1.79	0.00	0.00	0.00
time (sec)	N/A	0.537	10.862	1.010	0.000	0.257	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	193	240	0	51	0	0	207
N.S.	1	1.00	1.50	1.86	0.00	0.40	0.00	0.00	1.60
time (sec)	N/A	0.369	10.281	1.036	0.000	0.122	0.000	0.000	0.031

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	195	240	0	54	0	0	224
N.S.	1	1.00	1.34	1.66	0.00	0.37	0.00	0.00	1.54
time (sec)	N/A	0.378	10.214	0.934	0.000	0.113	0.000	0.000	9.058

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	193	240	0	47	0	0	208
N.S.	1	1.00	1.30	1.62	0.00	0.32	0.00	0.00	1.41
time (sec)	N/A	0.368	10.136	1.001	0.000	0.110	0.000	0.000	0.026

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	195	240	0	45	0	0	223
N.S.	1	1.00	1.39	1.71	0.00	0.32	0.00	0.00	1.59
time (sec)	N/A	0.377	10.229	0.947	0.000	0.118	0.000	0.000	0.034

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	260	260	369	0	0	0	0	0	0
N.S.	1	1.00	1.42	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.559	10.652	0.000	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	268	268	371	0	0	0	0	0	0
N.S.	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.575	10.578	0.000	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	277	277	372	0	0	0	0	0	0
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.602	10.470	0.000	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	273	273	372	0	0	0	0	0	0
N.S.	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.610	10.650	0.000	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	295	509	0	356	0	0	0
N.S.	1	1.00	1.46	2.52	0.00	1.76	0.00	0.00	0.00
time (sec)	N/A	0.522	10.553	0.977	0.000	0.150	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	49	130	0	205	0	0	0
N.S.	1	1.00	1.17	3.10	0.00	4.88	0.00	0.00	0.00
time (sec)	N/A	0.264	1.833	3.439	0.000	0.340	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	49	133	0	207	0	0	0
N.S.	1	1.00	1.07	2.89	0.00	4.50	0.00	0.00	0.00
time (sec)	N/A	0.261	1.801	3.220	0.000	0.385	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	47	129	0	50	0	0	0
N.S.	1	1.00	1.07	2.93	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.251	1.812	3.428	0.000	0.339	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	51	134	0	59	0	0	0
N.S.	1	1.00	1.16	3.05	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.243	1.815	3.280	0.000	0.292	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	84	0	0	1240	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	17.97	0.00	0.00	0.00
time (sec)	N/A	0.347	7.404	0.000	0.000	1.068	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	84	0	0	1294	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	18.23	0.00	0.00	0.00
time (sec)	N/A	0.350	7.399	0.000	0.000	1.046	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	85	0	0	1245	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	17.29	0.00	0.00	0.00
time (sec)	N/A	0.339	7.339	0.000	0.000	1.056	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	87	0	0	1303	0	0	0
N.S.	1	1.00	1.21	0.00	0.00	18.10	0.00	0.00	0.00
time (sec)	N/A	0.324	7.289	0.000	0.000	1.093	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	663	0	0	1273	0	0	0
N.S.	1	1.00	9.08	0.00	0.00	17.44	0.00	0.00	0.00
time (sec)	N/A	0.352	10.974	0.000	0.000	0.720	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	648	0	0	1330	0	0	0
N.S.	1	1.00	8.64	0.00	0.00	17.73	0.00	0.00	0.00
time (sec)	N/A	0.364	10.858	0.000	0.000	0.718	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	649	0	0	1278	0	0	0
N.S.	1	1.00	8.54	0.00	0.00	16.82	0.00	0.00	0.00
time (sec)	N/A	0.355	10.903	0.000	0.000	0.680	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	666	0	0	1339	0	0	0
N.S.	1	1.00	8.76	0.00	0.00	17.62	0.00	0.00	0.00
time (sec)	N/A	0.343	10.878	0.000	0.000	0.728	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	49	131	0	50	0	0	0
N.S.	1	1.00	1.17	3.12	0.00	1.19	0.00	0.00	0.00
time (sec)	N/A	0.241	2.018	2.469	0.000	0.282	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	49	135	0	59	0	0	0
N.S.	1	1.00	1.07	2.93	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.255	1.795	2.469	0.000	0.297	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	47	131	0	204	0	0	0
N.S.	1	1.00	1.07	2.98	0.00	4.64	0.00	0.00	0.00
time (sec)	N/A	0.255	1.748	2.528	0.000	0.290	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	51	135	0	206	0	0	0
N.S.	1	1.00	1.16	3.07	0.00	4.68	0.00	0.00	0.00
time (sec)	N/A	0.247	1.795	2.437	0.000	0.336	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	84	0	0	1236	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	17.91	0.00	0.00	0.00
time (sec)	N/A	0.332	7.316	0.000	0.000	1.045	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	71	71	84	0	0	1288	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	18.14	0.00	0.00	0.00
time (sec)	N/A	0.345	7.177	0.000	0.000	1.034	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	85	0	0	1239	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	17.21	0.00	0.00	0.00
time (sec)	N/A	0.343	7.238	0.000	0.000	1.039	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	87	0	0	1299	0	0	0
N.S.	1	1.00	1.21	0.00	0.00	18.04	0.00	0.00	0.00
time (sec)	N/A	0.327	7.183	0.000	0.000	1.071	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	667	0	0	1270	0	0	0
N.S.	1	1.00	9.14	0.00	0.00	17.40	0.00	0.00	0.00
time (sec)	N/A	0.334	10.755	0.000	0.000	0.738	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	649	0	0	1324	0	0	0
N.S.	1	1.00	8.65	0.00	0.00	17.65	0.00	0.00	0.00
time (sec)	N/A	0.351	10.818	0.000	0.000	0.719	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	650	0	0	1273	0	0	0
N.S.	1	1.00	8.55	0.00	0.00	16.75	0.00	0.00	0.00
time (sec)	N/A	0.340	10.845	0.000	0.000	0.731	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	670	0	0	1335	0	0	0
N.S.	1	1.00	8.82	0.00	0.00	17.57	0.00	0.00	0.00
time (sec)	N/A	0.328	10.829	0.000	0.000	0.717	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	269	245	0	55	0	0	0
N.S.	1	1.00	1.86	1.69	0.00	0.38	0.00	0.00	0.00
time (sec)	N/A	0.395	20.393	2.427	0.000	0.106	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	267	245	0	210	0	0	0
N.S.	1	1.00	1.84	1.69	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.430	20.357	2.444	0.000	0.114	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	291	260	0	716	0	0	0
N.S.	1	1.00	1.68	1.50	0.00	4.14	0.00	0.00	0.00
time (sec)	N/A	0.450	20.465	1.587	0.000	0.159	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	187	184	291	264	0	735	0	0	0
N.S.	1	0.98	1.56	1.41	0.00	3.93	0.00	0.00	0.00
time (sec)	N/A	0.490	20.519	1.472	0.000	0.183	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	190	187	289	262	0	723	0	0	0
N.S.	1	0.98	1.52	1.38	0.00	3.81	0.00	0.00	0.00
time (sec)	N/A	0.467	20.511	1.416	0.000	0.175	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	293	258	0	724	0	0	0
N.S.	1	1.00	1.60	1.41	0.00	3.96	0.00	0.00	0.00
time (sec)	N/A	0.475	20.456	1.519	0.000	0.166	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	332	332	456	0	0	7008	0	0	0
N.S.	1	1.00	1.37	0.00	0.00	21.11	0.00	0.00	0.00
time (sec)	N/A	0.745	11.493	0.000	0.000	27.024	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	336	336	466	0	0	7063	0	0	0
N.S.	1	1.00	1.39	0.00	0.00	21.02	0.00	0.00	0.00
time (sec)	N/A	0.776	11.257	0.000	0.000	26.954	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	345	345	467	0	0	7009	0	0	0
N.S.	1	1.00	1.35	0.00	0.00	20.32	0.00	0.00	0.00
time (sec)	N/A	0.775	11.322	0.000	0.000	26.477	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	345	345	459	0	0	7078	0	0	0
N.S.	1	1.00	1.33	0.00	0.00	20.52	0.00	0.00	0.00
time (sec)	N/A	0.727	11.444	0.000	0.000	26.451	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	177	209	255	0	58	0	0	0
N.S.	1	1.30	1.54	1.88	0.00	0.43	0.00	0.00	0.00
time (sec)	N/A	0.444	10.393	2.446	0.000	0.112	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	152	193	232	257	0	69	0	0	0
N.S.	1	1.27	1.53	1.69	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.486	10.536	2.317	0.000	0.110	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	164	187	230	255	0	213	0	0	0
N.S.	1	1.14	1.40	1.55	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.477	11.528	2.401	0.000	0.123	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	179	211	253	0	217	0	0	0
N.S.	1	1.15	1.35	1.62	0.00	1.39	0.00	0.00	0.00
time (sec)	N/A	0.455	10.379	2.373	0.000	0.109	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	147	164	225	253	0	214	0	0	0
N.S.	1	1.12	1.53	1.72	0.00	1.46	0.00	0.00	0.00
time (sec)	N/A	0.465	10.493	2.451	0.000	0.107	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	278	295	445	0	0	1288	0	0	0
N.S.	1	1.06	1.60	0.00	0.00	4.63	0.00	0.00	0.00
time (sec)	N/A	0.695	11.284	0.000	0.000	0.889	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	286	303	454	0	0	1354	0	0	0
N.S.	1	1.06	1.59	0.00	0.00	4.73	0.00	0.00	0.00
time (sec)	N/A	0.705	11.082	0.000	0.000	0.819	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	282	319	455	0	0	1295	0	0	0
N.S.	1	1.13	1.61	0.00	0.00	4.59	0.00	0.00	0.00
time (sec)	N/A	0.700	11.019	0.000	0.000	0.902	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	278	315	448	0	0	1348	0	0	0
N.S.	1	1.13	1.61	0.00	0.00	4.85	0.00	0.00	0.00
time (sec)	N/A	0.713	11.239	0.000	0.000	0.895	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	317	297	214	275	0	0	0	0	0
N.S.	1	0.94	0.68	0.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.147	10.492	1.380	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	329	302	235	264	0	0	0	0	0
N.S.	1	0.92	0.71	0.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.129	10.567	1.397	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	325	300	233	273	0	0	0	0	0
N.S.	1	0.92	0.72	0.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.049	10.540	1.309	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	321	299	233	266	0	0	0	0	0
N.S.	1	0.93	0.73	0.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.091	10.544	1.288	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	358	309	213	275	0	0	0	0	0
N.S.	1	0.86	0.59	0.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.067	10.517	1.233	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	346	301	235	268	0	0	0	0	0
N.S.	1	0.87	0.68	0.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.038	10.565	1.281	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	342	299	233	277	0	0	0	0	0
N.S.	1	0.87	0.68	0.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.971	10.536	1.092	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	362	311	233	266	0	0	0	0	0
N.S.	1	0.86	0.64	0.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.038	10.570	1.212	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	39	90	0	33	56	0	334
N.S.	1	1.00	0.31	0.72	0.00	0.26	0.45	0.00	2.67
time (sec)	N/A	0.283	10.036	1.445	0.000	0.121	2.268	0.000	17.624

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	40	101	0	36	99	0	373
N.S.	1	1.00	0.29	0.73	0.00	0.26	0.71	0.00	2.68
time (sec)	N/A	0.278	10.040	1.352	0.000	0.116	3.743	0.000	18.953

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	58	132	0	52	94	0	334
N.S.	1	1.00	0.41	0.93	0.00	0.37	0.66	0.00	2.35
time (sec)	N/A	0.283	10.035	1.840	0.000	0.113	3.808	0.000	18.892

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	61	94	0	61	61	0	376
N.S.	1	1.00	0.45	0.69	0.00	0.45	0.45	0.00	2.76
time (sec)	N/A	0.295	10.036	1.830	0.000	0.106	2.434	0.000	21.455

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	41	90	0	33	56	0	334
N.S.	1	1.00	0.32	0.71	0.00	0.26	0.44	0.00	2.63
time (sec)	N/A	0.273	10.030	1.067	0.000	0.092	2.268	0.000	19.813

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	42	101	0	36	99	0	373
N.S.	1	1.00	0.30	0.72	0.00	0.26	0.70	0.00	2.65
time (sec)	N/A	0.292	10.026	1.105	0.000	0.102	3.738	0.000	20.839

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	60	132	0	51	94	0	334
N.S.	1	1.00	0.42	0.92	0.00	0.35	0.65	0.00	2.32
time (sec)	N/A	0.287	10.032	1.102	0.000	0.099	3.825	0.000	20.251

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	63	94	0	60	61	0	376
N.S.	1	1.00	0.46	0.68	0.00	0.43	0.44	0.00	2.72
time (sec)	N/A	0.286	10.030	1.080	0.000	0.106	2.406	0.000	21.677

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	328	194	240	0	0	0	0	207
N.S.	1	0.99	0.58	0.72	0.00	0.00	0.00	0.00	0.62
time (sec)	N/A	0.936	10.228	0.958	0.000	0.000	0.000	0.000	0.269

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	378	195	240	0	0	0	0	224
N.S.	1	1.00	0.52	0.64	0.00	0.00	0.00	0.00	0.59
time (sec)	N/A	0.991	10.217	0.924	0.000	0.000	0.000	0.000	0.206

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	373	381	193	240	0	0	0	0	208
N.S.	1	1.02	0.52	0.64	0.00	0.00	0.00	0.00	0.56
time (sec)	N/A	1.024	10.202	0.984	0.000	0.000	0.000	0.000	19.663

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	328	196	240	0	0	0	0	223
N.S.	1	0.96	0.57	0.70	0.00	0.00	0.00	0.00	0.65
time (sec)	N/A	0.954	10.221	0.979	0.000	0.000	0.000	0.000	0.094

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	450	449	211	274	0	0	0	0	356
N.S.	1	1.00	0.47	0.61	0.00	0.00	0.00	0.00	0.79
time (sec)	N/A	1.399	10.446	1.024	0.000	0.000	0.000	0.000	0.147

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	474	460	233	265	0	0	0	0	387
N.S.	1	0.97	0.49	0.56	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	1.385	10.582	1.023	0.000	0.000	0.000	0.000	20.107

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	475	463	231	274	0	0	0	0	355
N.S.	1	0.97	0.49	0.58	0.00	0.00	0.00	0.00	0.75
time (sec)	N/A	1.207	10.527	1.099	0.000	0.000	0.000	0.000	0.108

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	B
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	463	460	213	265	0	0	0	0	388
N.S.	1	0.99	0.46	0.57	0.00	0.00	0.00	0.00	0.84
time (sec)	N/A	1.275	10.443	0.944	0.000	0.000	0.000	0.000	0.112

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	34	53	0	30	42	0	207
N.S.	1	1.00	0.28	0.44	0.00	0.25	0.35	0.00	1.72
time (sec)	N/A	0.259	10.039	0.984	0.000	0.109	1.265	0.000	20.504

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	34	57	0	33	65	0	223
N.S.	1	1.00	0.25	0.43	0.00	0.25	0.49	0.00	1.66
time (sec)	N/A	0.266	10.445	0.930	0.000	0.095	1.412	0.000	20.350

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	137	137	52	93	0	26	60	0	207
N.S.	1	1.00	0.38	0.68	0.00	0.19	0.44	0.00	1.51
time (sec)	N/A	0.265	10.032	0.973	0.000	0.128	1.355	0.000	0.059

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	56	56	0	35	46	0	223
N.S.	1	1.00	0.43	0.43	0.00	0.27	0.35	0.00	1.70
time (sec)	N/A	0.269	10.037	0.979	0.000	0.096	1.400	0.000	19.569

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	159	0	0	0	0	0	0
N.S.	1	1.00	1.67	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.292	1.494	0.000	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	234	227	0	0	0	0	0	0	0
N.S.	1	0.97	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.452	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	133	415	253	490	6397	835	495
N.S.	1	1.00	0.83	2.59	1.58	3.06	39.98	5.22	3.09
time (sec)	N/A	0.336	0.156	0.903	0.200	0.288	2.031	0.314	19.818

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	104	283	184	348	3704	577	363
N.S.	1	1.00	0.83	2.25	1.46	2.76	29.40	4.58	2.88
time (sec)	N/A	0.298	0.096	0.890	0.202	0.336	1.304	0.402	19.170

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	94	167	122	222	1906	361	247
N.S.	1	1.00	1.00	1.78	1.30	2.36	20.28	3.84	2.63
time (sec)	N/A	0.248	0.094	0.878	0.187	0.277	0.814	0.428	19.414

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	99	99	94	0	0	0	675	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	6.82	0.00	0.00
time (sec)	N/A	0.258	0.087	0.000	0.000	0.000	2.560	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	252	1565	601	1565	26746	2660	1410
N.S.	1	1.00	0.86	5.32	2.04	5.32	90.97	9.05	4.80
time (sec)	N/A	0.514	0.252	1.029	0.214	0.285	8.952	0.332	19.988

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	211	893	474	1216	18328	2034	1136
N.S.	1	1.00	0.85	3.60	1.91	4.90	73.90	8.20	4.58
time (sec)	N/A	0.435	0.194	1.000	0.209	0.303	5.693	0.331	20.362

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	172	700	359	893	11851	1477	878
N.S.	1	1.00	0.85	3.45	1.77	4.40	58.38	7.28	4.33
time (sec)	N/A	0.374	0.162	0.954	0.210	0.277	3.529	0.366	20.460

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	209	209	188	0	0	0	4690	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	22.44	0.00	0.00
time (sec)	N/A	0.398	0.168	0.000	0.000	0.000	4.222	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	459	459	402	3780	1153	3564	75191	0	2896
N.S.	1	1.00	0.88	8.24	2.51	7.76	163.81	0.00	6.31
time (sec)	N/A	0.728	0.380	1.348	0.222	0.327	83.592	0.000	24.923

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	396	396	345	2972	953	2919	56151	4934	2436
N.S.	1	1.00	0.87	7.51	2.41	7.37	141.80	12.46	6.15
time (sec)	N/A	0.634	0.308	1.162	0.234	0.324	34.118	0.372	22.664

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	290	2280	770	2313	40536	3874	2001
N.S.	1	1.00	0.86	6.77	2.28	6.86	120.28	11.50	5.94
time (sec)	N/A	0.534	0.292	1.140	0.222	0.331	66.278	0.403	21.092

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	358	358	332	0	0	0	17189	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	48.01	0.00	0.00
time (sec)	N/A	0.561	0.302	0.000	0.000	0.000	13.326	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	324	324	284	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.968	0.558	0.000	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	332	332	292	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.912	0.596	0.000	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	293	293	239	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.686	0.266	0.000	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	253	253	213	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.543	0.202	0.000	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	288	288	237	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.515	0.225	0.000	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	263	263	222	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.433	0.147	0.000	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	300	300	244	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.766	0.295	0.000	0.000	0.000	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	326	326	273	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.768	0.337	0.000	0.000	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	253	253	213	0	0	0	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.723	0.335	0.000	0.000	0.000	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	211	211	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.624	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	1480	1361	877	1126	0	0	0	0	0
N.S.	1	0.92	0.59	0.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.813	6.600	2.473	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	F	F	F	F	B	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	135	0	0	0	0	0	631	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	4.67	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	25.939	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	23	46	0	19	0	0	273
N.S.	1	1.00	1.44	2.88	0.00	1.19	0.00	0.00	17.06
time (sec)	N/A	0.211	1.121	1.616	0.000	0.282	0.000	0.000	0.239

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	23	49	0	28	0	0	292
N.S.	1	1.00	1.15	2.45	0.00	1.40	0.00	0.00	14.60
time (sec)	N/A	0.212	1.113	1.564	0.000	0.269	0.000	0.000	19.534

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	21	26	0	25	0	0	276
N.S.	1	1.00	1.17	1.44	0.00	1.39	0.00	0.00	15.33
time (sec)	N/A	0.197	1.091	1.389	0.000	0.278	0.000	0.000	0.117

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	25	30	0	28	0	0	289
N.S.	1	1.00	1.39	1.67	0.00	1.56	0.00	0.00	16.06
time (sec)	N/A	0.202	1.088	1.328	0.000	0.296	0.000	0.000	0.117

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	37	4397	0	181	0	0	632
N.S.	1	1.00	1.23	146.57	0.00	6.03	0.00	0.00	21.07
time (sec)	N/A	0.226	1.894	1.807	0.000	0.321	0.000	0.000	19.706

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	37	1908	0	191	0	0	677
N.S.	1	1.00	0.97	50.21	0.00	5.03	0.00	0.00	17.82
time (sec)	N/A	0.240	1.975	1.841	0.000	0.310	0.000	0.000	18.721

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	35	4437	0	187	0	0	629
N.S.	1	1.00	0.97	123.25	0.00	5.19	0.00	0.00	17.47
time (sec)	N/A	0.227	1.919	1.686	0.000	0.315	0.000	0.000	0.120

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	39	1888	0	185	0	0	680
N.S.	1	1.00	1.22	59.00	0.00	5.78	0.00	0.00	21.25
time (sec)	N/A	0.231	1.918	1.770	0.000	0.296	0.000	0.000	19.053

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	355	186	261	0	189	175	0	0
N.S.	1	1.00	0.52	0.74	0.00	0.53	0.49	0.00	0.00
time (sec)	N/A	0.471	10.113	1.729	0.000	0.162	2.009	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	326	146	234	0	165	138	0	0
N.S.	1	1.00	0.45	0.72	0.00	0.51	0.42	0.00	0.00
time (sec)	N/A	0.413	8.157	1.500	0.000	0.162	1.806	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	109	119	0	94	88	0	0
N.S.	1	1.00	0.69	0.75	0.00	0.59	0.56	0.00	0.00
time (sec)	N/A	0.268	5.290	1.231	0.000	0.129	1.518	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	89	85	0	41	37	0	37
N.S.	1	1.00	0.85	0.81	0.00	0.39	0.35	0.00	0.35
time (sec)	N/A	0.178	0.087	1.009	0.000	0.078	0.461	0.000	18.066

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	730	660	421	404	0	0	0	0	0
N.S.	1	0.90	0.58	0.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.151	9.226	2.336	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1221	975	394	402	0	0	0	0	0
N.S.	1	0.80	0.32	0.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.770	11.459	1.023	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	157	218	0	152	141	0	0
N.S.	1	1.00	0.53	0.74	0.00	0.52	0.48	0.00	0.00
time (sec)	N/A	0.416	10.111	1.599	0.000	0.137	1.698	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	133	197	0	137	105	0	0
N.S.	1	1.00	0.51	0.75	0.00	0.52	0.40	0.00	0.00
time (sec)	N/A	0.372	10.085	1.058	0.000	0.139	1.420	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	79	96	0	72	61	0	0
N.S.	1	1.00	0.65	0.79	0.00	0.60	0.50	0.00	0.00
time (sec)	N/A	0.264	10.043	1.036	0.000	0.117	1.027	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	74	70	0	29	36	0	37
N.S.	1	1.00	0.84	0.80	0.00	0.33	0.41	0.00	0.42
time (sec)	N/A	0.171	0.038	0.968	0.000	0.083	0.420	0.000	17.623

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	405	431	200	169	0	0	0	0	0
N.S.	1	1.06	0.49	0.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.723	0.213	1.036	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	610	634	448	421	0	0	0	0	0
N.S.	1	1.04	0.73	0.69	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.371	0.811	1.008	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	659	677	614	483	0	0	0	0	0
N.S.	1	1.03	0.93	0.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.781	11.428	1.129	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	295	126	245	0	160	0	0	0
N.S.	1	0.99	0.42	0.82	0.00	0.54	0.00	0.00	0.00
time (sec)	N/A	0.412	10.075	1.282	0.000	0.106	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	268	108	233	0	143	0	0	0
N.S.	1	0.99	0.40	0.86	0.00	0.53	0.00	0.00	0.00
time (sec)	N/A	0.381	10.068	1.109	0.000	0.118	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	59	115	0	76	61	0	57
N.S.	1	1.00	0.52	1.01	0.00	0.67	0.54	0.00	0.50
time (sec)	N/A	0.229	10.040	1.003	0.000	0.098	2.773	0.000	18.042

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	55	94	0	64	36	0	37
N.S.	1	1.00	0.51	0.87	0.00	0.59	0.33	0.00	0.34
time (sec)	N/A	0.192	4.719	0.919	0.000	0.082	0.471	0.000	18.109

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	818	727	455	496	0	0	0	0	0
N.S.	1	0.89	0.56	0.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.533	10.646	0.972	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	349	349	274	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.883	0.427	0.000	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	349	349	274	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.856	0.283	0.000	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1605	1590	455	1153	0	0	0	0	0
N.S.	1	0.99	0.28	0.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	8.073	11.255	1.363	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	87	132	200	119	233	0	355	234
N.S.	1	0.54	0.82	1.24	0.74	1.45	0.00	2.20	1.45
time (sec)	N/A	0.244	0.089	0.041	0.257	0.294	0.000	0.340	18.204

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	72	63	60	136	74	0	72	0
N.S.	1	0.50	0.44	0.42	0.95	0.52	0.00	0.50	0.00
time (sec)	N/A	0.233	0.835	0.169	0.238	0.296	0.000	0.342	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	69	63	60	47	74	0	72	0
N.S.	1	0.48	0.44	0.42	0.33	0.52	0.00	0.50	0.00
time (sec)	N/A	0.225	0.084	0.952	0.255	0.317	0.000	0.305	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	63	63	60	98	74	0	48	50
N.S.	1	0.95	0.95	0.91	1.48	1.12	0.00	0.73	0.76
time (sec)	N/A	0.222	0.729	0.127	0.232	0.304	0.000	0.470	17.832

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	69	63	60	47	74	0	72	0
N.S.	1	0.48	0.44	0.42	0.33	0.52	0.00	0.50	0.00
time (sec)	N/A	0.219	0.078	0.730	0.206	0.272	0.000	0.300	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	29	29	26	60	73	0	25	40
N.S.	1	0.91	0.91	0.81	1.88	2.28	0.00	0.78	1.25
time (sec)	N/A	0.161	0.009	0.136	0.229	0.264	0.000	0.339	18.466

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	65	61	58	43	72	0	46	0
N.S.	1	0.48	0.45	0.43	0.32	0.53	0.00	0.34	0.00
time (sec)	N/A	0.201	0.076	0.483	0.233	0.327	0.000	0.292	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	68	234	59	171	73	0	73	0
N.S.	1	0.49	1.68	0.42	1.23	0.53	0.00	0.53	0.00
time (sec)	N/A	0.216	0.525	0.126	0.191	0.382	0.000	0.305	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	60	62	60	48	72	0	69	0
N.S.	1	0.45	0.46	0.45	0.36	0.54	0.00	0.51	0.00
time (sec)	N/A	0.209	0.101	0.657	0.192	0.246	0.000	0.309	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	67	237	61	176	76	0	91	0
N.S.	1	0.48	1.69	0.44	1.26	0.54	0.00	0.65	0.00
time (sec)	N/A	0.219	0.697	0.138	0.209	0.284	0.000	0.327	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	215	142	155	0	433	0	177	0
N.S.	1	0.85	0.56	0.61	0.00	1.71	0.00	0.70	0.00
time (sec)	N/A	0.312	0.104	2.045	0.000	0.310	0.000	0.463	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	29	29	26	70	87	0	28	62
N.S.	1	0.91	0.91	0.81	2.19	2.72	0.00	0.88	1.94
time (sec)	N/A	0.167	0.012	1.564	0.216	0.278	0.000	0.309	18.395

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	177	123	138	0	402	0	153	0
N.S.	1	0.86	0.59	0.67	0.00	1.94	0.00	0.74	0.00
time (sec)	N/A	0.242	0.066	1.544	0.000	0.297	0.000	0.326	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	143	111	221	0	391	0	185	0
N.S.	1	0.74	0.58	1.15	0.00	2.04	0.00	0.96	0.00
time (sec)	N/A	0.255	0.073	1.498	0.000	0.275	0.000	0.494	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	179	121	141	0	396	0	185	0
N.S.	1	0.86	0.58	0.68	0.00	1.90	0.00	0.89	0.00
time (sec)	N/A	0.268	0.047	2.262	0.000	0.304	0.000	0.326	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	153	118	223	0	411	0	204	0
N.S.	1	0.76	0.58	1.10	0.00	2.03	0.00	1.01	0.00
time (sec)	N/A	0.258	0.055	2.306	0.000	0.283	0.000	0.313	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	80	73	60	0	141	0	71	0
N.S.	1	1.04	0.95	0.78	0.00	1.83	0.00	0.92	0.00
time (sec)	N/A	0.197	0.026	0.070	0.000	0.287	0.000	0.336	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	19	19	42	28	19
N.S.	1	1.00	1.00	0.95	0.90	0.90	2.00	1.33	0.90
time (sec)	N/A	0.157	0.004	0.086	0.193	0.256	0.255	0.334	19.465

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	0	19	46	28	19
N.S.	1	1.00	1.00	0.95	0.00	0.90	2.19	1.33	0.90
time (sec)	N/A	0.148	0.005	0.048	0.000	0.248	0.297	0.333	18.460

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	59	64	80	138	0	59	0
N.S.	1	1.00	0.83	0.90	1.13	1.94	0.00	0.83	0.00
time (sec)	N/A	0.209	0.015	0.064	0.266	0.268	0.000	0.320	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	75	32	37	46	32	0	81	54
N.S.	1	1.56	0.67	0.77	0.96	0.67	0.00	1.69	1.12
time (sec)	N/A	0.194	0.010	0.069	0.197	0.253	0.000	0.343	19.487

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	99	78	81	121	175	0	103	0
N.S.	1	0.95	0.75	0.78	1.16	1.68	0.00	0.99	0.00
time (sec)	N/A	0.227	0.024	0.091	0.269	0.268	0.000	0.445	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	132	63	58	85	75	144	137	109
N.S.	1	0.96	0.46	0.42	0.62	0.54	1.04	0.99	0.79
time (sec)	N/A	0.266	0.056	0.046	0.193	0.296	11.192	0.311	18.884

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	108	52	47	64	63	116	109	88
N.S.	1	1.06	0.51	0.46	0.63	0.62	1.14	1.07	0.86
time (sec)	N/A	0.244	0.038	0.030	0.199	0.285	5.864	0.299	18.945

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	84	41	36	43	51	87	81	67
N.S.	1	1.27	0.62	0.55	0.65	0.77	1.32	1.23	1.02
time (sec)	N/A	0.222	0.036	0.032	0.194	0.267	3.011	0.325	18.251

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	31	31	26	25	37	58	17	28
N.S.	1	0.86	0.86	0.72	0.69	1.03	1.61	0.47	0.78
time (sec)	N/A	0.154	0.006	0.061	0.189	0.299	1.427	0.313	18.868

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	95	96	0	118	0	0	190	0
N.S.	1	0.81	0.82	0.00	1.01	0.00	0.00	1.62	0.00
time (sec)	N/A	0.228	0.084	0.000	0.269	0.000	0.000	0.305	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	100	107	0	138	0	0	212	0
N.S.	1	0.75	0.80	0.00	1.04	0.00	0.00	1.59	0.00
time (sec)	N/A	0.227	0.129	0.000	0.279	0.000	0.000	0.333	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	139	68	0	0	0	0	0	0
N.S.	1	0.91	0.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.235	0.052	0.000	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	119	105	52	0	0	0	0	0	0
N.S.	1	0.88	0.44	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.192	0.009	0.000	0.000	0.000	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	115	109	55	0	0	0	0	0	0
N.S.	1	0.95	0.48	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.210	0.011	0.000	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	139	57	0	0	0	0	0	0
N.S.	1	0.90	0.37	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.231	10.016	0.000	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	75	68	0	80	114	61	0
N.S.	1	1.00	1.06	0.96	0.00	1.13	1.61	0.86	0.00
time (sec)	N/A	0.199	0.071	0.992	0.000	0.314	0.900	0.413	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	53	59	114	0	44	0	0	0
N.S.	1	1.10	1.23	2.38	0.00	0.92	0.00	0.00	0.00
time (sec)	N/A	0.213	0.046	1.531	0.000	0.115	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	55	28	0	43	71	61	0
N.S.	1	1.00	1.72	0.88	0.00	1.34	2.22	1.91	0.00
time (sec)	N/A	0.164	0.012	1.032	0.000	0.336	0.872	0.351	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	18	43	0	9	0	0	0
N.S.	1	1.00	1.50	3.58	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.150	0.013	0.498	0.000	0.113	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	268	198	241	0	541	0	226	0
N.S.	1	1.10	0.81	0.99	0.00	2.22	0.00	0.93	0.00
time (sec)	N/A	0.438	0.437	0.226	0.000	0.373	0.000	0.382	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	185	161	189	0	407	0	169	0
N.S.	1	1.15	1.00	1.17	0.00	2.53	0.00	1.05	0.00
time (sec)	N/A	0.336	0.087	0.123	0.000	0.343	0.000	0.385	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	110	132	154	0	313	0	138	0
N.S.	1	1.07	1.28	1.50	0.00	3.04	0.00	1.34	0.00
time (sec)	N/A	0.240	0.049	0.104	0.000	0.342	0.000	0.395	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	142	147	179	0	865	0	0	0
N.S.	1	1.27	1.31	1.60	0.00	7.72	0.00	0.00	0.00
time (sec)	N/A	0.309	0.041	0.102	0.000	0.479	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	110	137	162	0	333	0	242	0
N.S.	1	0.87	1.08	1.28	0.00	2.62	0.00	1.91	0.00
time (sec)	N/A	0.267	0.057	0.150	0.000	0.482	0.000	0.393	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	184	174	201	0	427	0	549	0
N.S.	1	0.88	0.84	0.97	0.00	2.05	0.00	2.64	0.00
time (sec)	N/A	0.365	0.087	0.159	0.000	0.883	0.000	0.449	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	267	222	257	0	561	0	1001	0
N.S.	1	0.84	0.70	0.81	0.00	1.76	0.00	3.15	0.00
time (sec)	N/A	0.449	1.486	0.192	0.000	2.168	0.000	0.412	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	372	255	490	0	273	0	0	0
N.S.	1	1.04	0.71	1.37	0.00	0.76	0.00	0.00	0.00
time (sec)	N/A	0.539	1.321	5.207	0.000	0.122	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	294	208	350	0	182	0	0	0
N.S.	1	1.11	0.78	1.32	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.418	1.223	4.720	0.000	0.106	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	248	86	184	0	150	0	0	0
N.S.	1	1.28	0.44	0.95	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.332	1.065	1.151	0.000	0.107	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	287	111	192	0	132	0	0	0
N.S.	1	1.20	0.46	0.80	0.00	0.55	0.00	0.00	0.00
time (sec)	N/A	0.396	1.179	4.665	0.000	0.108	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	348	238	378	0	199	0	0	0
N.S.	1	1.08	0.74	1.18	0.00	0.62	0.00	0.00	0.00
time (sec)	N/A	0.500	1.442	5.432	0.000	0.113	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	424	432	302	539	0	289	0	0	0
N.S.	1	1.02	0.71	1.27	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.646	2.786	6.593	0.000	0.114	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	302	222	313	0	553	0	0	0
N.S.	1	1.07	0.79	1.11	0.00	1.96	0.00	0.00	0.00
time (sec)	N/A	0.553	4.541	0.224	0.000	1.287	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	215	177	252	0	417	0	0	0
N.S.	1	1.08	0.89	1.27	0.00	2.10	0.00	0.00	0.00
time (sec)	N/A	0.408	2.846	0.184	0.000	0.681	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	123	136	231	0	328	0	0	0
N.S.	1	0.87	0.96	1.64	0.00	2.33	0.00	0.00	0.00
time (sec)	N/A	0.255	1.894	0.161	0.000	0.462	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	183	187	401	0	1049	0	0	0
N.S.	1	1.21	1.24	2.66	0.00	6.95	0.00	0.00	0.00
time (sec)	N/A	0.366	2.299	0.139	0.000	0.898	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	123	146	239	0	350	0	0	0
N.S.	1	0.75	0.88	1.45	0.00	2.12	0.00	0.00	0.00
time (sec)	N/A	0.289	3.804	0.209	0.000	0.908	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	214	186	270	0	435	0	0	0
N.S.	1	0.84	0.73	1.05	0.00	1.70	0.00	0.00	0.00
time (sec)	N/A	0.414	4.303	0.240	0.000	2.537	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	301	245	333	0	573	0	0	0
N.S.	1	0.82	0.67	0.91	0.00	1.57	0.00	0.00	0.00
time (sec)	N/A	0.537	4.762	0.268	0.000	9.703	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	403	266	775	0	275	0	0	0
N.S.	1	1.03	0.68	1.98	0.00	0.70	0.00	0.00	0.00
time (sec)	N/A	0.631	5.606	9.881	0.000	0.122	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	339	235	734	0	229	0	0	0
N.S.	1	1.09	0.76	2.37	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.477	4.955	8.941	0.000	0.124	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	305	206	527	0	173	0	0	0
N.S.	1	1.16	0.79	2.01	0.00	0.66	0.00	0.00	0.00
time (sec)	N/A	0.398	1.210	3.336	0.000	0.108	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	360	228	670	0	170	0	0	0
N.S.	1	1.17	0.74	2.18	0.00	0.55	0.00	0.00	0.00
time (sec)	N/A	0.518	5.247	8.050	0.000	0.116	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	383	415	253	790	0	214	0	0	0
N.S.	1	1.08	0.66	2.06	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.634	5.625	8.284	0.000	0.117	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	480	489	322	908	0	293	0	0	0
N.S.	1	1.02	0.67	1.89	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.756	6.229	10.053	0.000	0.122	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	57	95	52	0	55	0	18	55
N.S.	1	1.12	1.86	1.02	0.00	1.08	0.00	0.35	1.08
time (sec)	N/A	0.189	0.127	0.410	0.000	0.306	0.000	0.311	17.176

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	76	119	78	76	77	0	30	88
N.S.	1	1.06	1.65	1.08	1.06	1.07	0.00	0.42	1.22
time (sec)	N/A	0.198	0.205	0.442	0.269	0.325	0.000	0.318	0.222

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	59	95	68	0	55	0	22	56
N.S.	1	1.11	1.79	1.28	0.00	1.04	0.00	0.42	1.06
time (sec)	N/A	0.192	0.206	0.830	0.000	0.361	0.000	0.366	16.809

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	108	95	80	0	65	0	57	101
N.S.	1	0.96	0.84	0.71	0.00	0.58	0.00	0.50	0.89
time (sec)	N/A	0.240	0.469	0.869	0.000	0.341	0.000	0.318	16.905

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	117	106	114	121	82	0	47	134
N.S.	1	1.10	1.00	1.08	1.14	0.77	0.00	0.44	1.26
time (sec)	N/A	0.233	0.649	0.777	0.270	0.342	0.000	0.322	0.232

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	49	42	0	32	0	40	0
N.S.	1	1.00	0.94	0.81	0.00	0.62	0.00	0.77	0.00
time (sec)	N/A	0.192	0.031	0.873	0.000	0.303	0.000	0.376	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	65	60	0	42	0	61	0
N.S.	1	1.00	0.96	0.88	0.00	0.62	0.00	0.90	0.00
time (sec)	N/A	0.224	0.043	0.898	0.000	0.342	0.000	0.353	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	274	224	243	0	545	0	235	0
N.S.	1	0.98	0.80	0.86	0.00	1.94	0.00	0.84	0.00
time (sec)	N/A	0.424	1.458	0.161	0.000	0.393	0.000	0.362	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	184	161	191	0	413	0	172	0
N.S.	1	1.09	0.95	1.13	0.00	2.44	0.00	1.02	0.00
time (sec)	N/A	0.309	0.886	0.127	0.000	0.352	0.000	0.366	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	113	136	154	0	313	0	142	0
N.S.	1	1.07	1.28	1.45	0.00	2.95	0.00	1.34	0.00
time (sec)	N/A	0.240	0.377	0.110	0.000	0.322	0.000	0.402	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	142	147	179	0	881	0	0	0
N.S.	1	1.27	1.31	1.60	0.00	7.87	0.00	0.00	0.00
time (sec)	N/A	0.313	0.429	0.092	0.000	0.524	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	113	143	162	0	333	0	238	0
N.S.	1	0.87	1.10	1.25	0.00	2.56	0.00	1.83	0.00
time (sec)	N/A	0.267	0.606	0.145	0.000	0.540	0.000	0.347	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	184	173	200	0	443	0	536	0
N.S.	1	0.84	0.79	0.92	0.00	2.03	0.00	2.46	0.00
time (sec)	N/A	0.325	1.164	0.155	0.000	0.883	0.000	0.393	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	370	258	492	0	276	0	0	0
N.S.	1	0.92	0.64	1.22	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.566	2.517	5.142	0.000	0.123	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	294	212	351	0	185	0	0	0
N.S.	1	0.94	0.68	1.12	0.00	0.59	0.00	0.00	0.00
time (sec)	N/A	0.417	2.018	3.931	0.000	0.120	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	251	86	127	0	130	0	0	0
N.S.	1	1.00	0.34	0.50	0.00	0.52	0.00	0.00	0.00
time (sec)	N/A	0.339	1.038	1.146	0.000	0.110	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	284	111	297	0	156	0	0	0
N.S.	1	0.98	0.38	1.03	0.00	0.54	0.00	0.00	0.00
time (sec)	N/A	0.394	2.082	4.657	0.000	0.112	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	348	238	377	0	206	0	0	0
N.S.	1	0.93	0.63	1.01	0.00	0.55	0.00	0.00	0.00
time (sec)	N/A	0.509	2.415	6.213	0.000	0.109	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	289	247	318	0	781	0	0	0
N.S.	1	0.82	0.70	0.90	0.00	2.21	0.00	0.00	0.00
time (sec)	N/A	0.431	4.435	0.240	0.000	1.251	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	208	179	257	0	585	0	0	0
N.S.	1	1.03	0.89	1.27	0.00	2.90	0.00	0.00	0.00
time (sec)	N/A	0.413	2.612	0.205	0.000	0.702	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	113	138	234	0	443	0	0	0
N.S.	1	0.77	0.95	1.60	0.00	3.03	0.00	0.00	0.00
time (sec)	N/A	0.255	1.860	0.181	0.000	0.540	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F(-2)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	169	189	401	0	1293	0	0	0
N.S.	1	1.11	1.24	2.64	0.00	8.51	0.00	0.00	0.00
time (sec)	N/A	0.371	2.092	0.143	0.000	0.906	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	113	148	242	0	469	0	0	0
N.S.	1	0.66	0.87	1.42	0.00	2.76	0.00	0.00	0.00
time (sec)	N/A	0.300	2.515	0.205	0.000	1.292	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	209	189	274	0	613	0	0	0
N.S.	1	0.82	0.74	1.07	0.00	2.40	0.00	0.00	0.00
time (sec)	N/A	0.417	4.200	0.227	0.000	3.953	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	453	406	271	780	0	425	0	0	0
N.S.	1	0.90	0.60	1.72	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.652	5.549	9.940	0.000	0.121	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	344	219	643	0	305	0	0	0
N.S.	1	0.91	0.58	1.70	0.00	0.81	0.00	0.00	0.00
time (sec)	N/A	0.485	4.909	8.864	0.000	0.116	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	306	203	514	0	251	0	0	0
N.S.	1	0.94	0.62	1.57	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.412	1.317	3.379	0.000	0.107	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	380	360	223	650	0	269	0	0	0
N.S.	1	0.95	0.59	1.71	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.550	5.132	8.116	0.000	0.105	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	444	412	266	866	0	374	0	0	0
N.S.	1	0.93	0.60	1.95	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.634	5.653	8.749	0.000	0.104	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	203	145	237	328	423	0	218	0
N.S.	1	0.94	0.67	1.10	1.52	1.96	0.00	1.01	0.00
time (sec)	N/A	0.417	0.231	0.233	0.290	0.330	0.000	0.394	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	144	107	193	218	325	0	158	0
N.S.	1	1.02	0.76	1.37	1.55	2.30	0.00	1.12	0.00
time (sec)	N/A	0.345	0.141	0.117	0.295	0.315	0.000	0.416	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	66	85	137	126	267	0	123	120
N.S.	1	0.96	1.23	1.99	1.83	3.87	0.00	1.78	1.74
time (sec)	N/A	0.201	0.094	0.155	0.290	0.290	0.000	0.431	17.767

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	104	102	235	159	927	0	0	0
N.S.	1	1.08	1.06	2.45	1.66	9.66	0.00	0.00	0.00
time (sec)	N/A	0.363	0.151	0.100	0.285	0.349	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	109	110	189	156	433	0	281	0
N.S.	1	1.05	1.06	1.82	1.50	4.16	0.00	2.70	0.00
time (sec)	N/A	0.312	0.219	0.145	0.298	0.334	0.000	0.387	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	184	152	239	322	577	0	713	0
N.S.	1	1.06	0.87	1.37	1.85	3.32	0.00	4.10	0.00
time (sec)	N/A	0.388	0.277	0.167	0.298	0.421	0.000	0.426	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	263	216	317	557	755	0	1414	0
N.S.	1	0.99	0.82	1.20	2.10	2.85	0.00	5.34	0.00
time (sec)	N/A	0.476	0.402	0.185	0.314	0.643	0.000	0.421	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	403	294	665	0	238	0	0	0
N.S.	1	1.10	0.80	1.81	0.00	0.65	0.00	0.00	0.00
time (sec)	N/A	0.704	10.064	6.524	0.000	0.094	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	329	243	406	0	166	0	0	0
N.S.	1	1.17	0.86	1.44	0.00	0.59	0.00	0.00	0.00
time (sec)	N/A	0.522	9.204	5.922	0.000	0.109	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	281	98	199	0	148	0	0	0
N.S.	1	1.32	0.46	0.93	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.401	8.869	1.155	0.000	0.098	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	328	136	272	0	196	0	0	0
N.S.	1	1.24	0.51	1.03	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.510	9.541	6.061	0.000	0.098	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	362	394	307	569	0	292	0	0	0
N.S.	1	1.09	0.85	1.57	0.00	0.81	0.00	0.00	0.00
time (sec)	N/A	0.634	10.739	6.997	0.000	0.105	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	466	493	400	778	0	411	0	0	0
N.S.	1	1.06	0.86	1.67	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.775	11.323	7.868	0.000	0.128	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	236	150	295	368	427	0	517	0
N.S.	1	0.95	0.60	1.18	1.48	1.71	0.00	2.08	0.00
time (sec)	N/A	0.485	0.242	0.215	0.306	0.411	0.000	0.802	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	175	113	242	247	335	0	475	0
N.S.	1	1.02	0.66	1.41	1.44	1.95	0.00	2.76	0.00
time (sec)	N/A	0.391	0.171	0.184	0.303	0.346	0.000	0.738	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	90	87	187	156	269	0	381	61
N.S.	1	0.96	0.93	1.99	1.66	2.86	0.00	4.05	0.65
time (sec)	N/A	0.218	0.108	0.209	0.297	0.350	0.000	0.716	18.034

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	140	139	652	201	1073	0	0	0
N.S.	1	1.11	1.10	5.17	1.60	8.52	0.00	0.00	0.00
time (sec)	N/A	0.402	0.213	0.127	0.299	0.402	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	122	124	251	202	404	0	0	0
N.S.	1	0.88	0.90	1.82	1.46	2.93	0.00	0.00	0.00
time (sec)	N/A	0.331	0.207	0.191	0.292	0.393	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	212	153	274	313	557	0	0	0
N.S.	1	1.03	0.75	1.34	1.53	2.72	0.00	0.00	0.00
time (sec)	N/A	0.447	0.228	0.227	0.312	0.621	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	297	226	371	534	733	0	0	0
N.S.	1	1.02	0.77	1.27	1.83	2.51	0.00	0.00	0.00
time (sec)	N/A	0.566	0.565	0.263	0.316	0.884	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	405	434	312	1065	0	233	0	0	0
N.S.	1	1.07	0.77	2.63	0.00	0.58	0.00	0.00	0.00
time (sec)	N/A	0.778	10.628	11.572	0.000	0.108	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	371	256	820	0	204	0	0	0
N.S.	1	1.12	0.77	2.48	0.00	0.62	0.00	0.00	0.00
time (sec)	N/A	0.606	10.419	9.350	0.000	0.094	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	325	229	515	0	160	0	0	0
N.S.	1	1.25	0.88	1.98	0.00	0.62	0.00	0.00	0.00
time (sec)	N/A	0.475	10.415	3.373	0.000	0.108	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	393	270	873	0	251	0	0	0
N.S.	1	1.26	0.87	2.80	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.648	10.447	9.403	0.000	0.109	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	388	440	352	1037	0	284	0	0	0
N.S.	1	1.13	0.91	2.67	0.00	0.73	0.00	0.00	0.00
time (sec)	N/A	0.769	10.923	10.724	0.000	0.121	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	494	521	472	1170	0	393	0	0	0
N.S.	1	1.05	0.96	2.37	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.931	11.228	11.584	0.000	0.106	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	227	148	239	340	425	0	225	0
N.S.	1	1.01	0.66	1.06	1.51	1.89	0.00	1.00	0.00
time (sec)	N/A	0.419	0.233	0.143	0.292	0.304	0.000	0.389	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	146	108	195	223	333	0	166	0
N.S.	1	0.99	0.73	1.32	1.51	2.25	0.00	1.12	0.00
time (sec)	N/A	0.335	0.143	0.115	0.283	0.306	0.000	0.388	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	68	88	134	129	267	0	128	111
N.S.	1	0.94	1.22	1.86	1.79	3.71	0.00	1.78	1.54
time (sec)	N/A	0.204	0.099	1.023	0.302	0.293	0.000	0.370	18.016

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	104	101	313	155	972	0	0	0
N.S.	1	1.08	1.05	3.26	1.61	10.12	0.00	0.00	0.00
time (sec)	N/A	0.331	0.124	0.084	0.303	0.364	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	113	114	197	173	451	0	292	0
N.S.	1	1.05	1.06	1.82	1.60	4.18	0.00	2.70	0.00
time (sec)	N/A	0.308	0.228	0.138	0.298	0.356	0.000	0.405	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	188	149	236	359	593	0	778	0
N.S.	1	1.06	0.84	1.33	2.03	3.35	0.00	4.40	0.00
time (sec)	N/A	0.375	0.259	0.174	0.300	0.400	0.000	0.407	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	443	413	298	664	0	240	0	0	0
N.S.	1	0.93	0.67	1.50	0.00	0.54	0.00	0.00	0.00
time (sec)	N/A	0.701	8.775	8.058	0.000	0.110	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	354	335	248	409	0	168	0	0	0
N.S.	1	0.95	0.70	1.16	0.00	0.47	0.00	0.00	0.00
time (sec)	N/A	0.526	7.620	5.995	0.000	0.101	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	287	107	164	0	127	0	0	0
N.S.	1	1.00	0.37	0.57	0.00	0.44	0.00	0.00	0.00
time (sec)	N/A	0.404	7.333	1.148	0.000	0.102	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	329	151	345	0	239	0	0	0
N.S.	1	0.96	0.44	1.01	0.00	0.70	0.00	0.00	0.00
time (sec)	N/A	0.508	8.738	6.062	0.000	0.101	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	435	402	299	568	0	297	0	0	0
N.S.	1	0.92	0.69	1.31	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.621	9.846	7.003	0.000	0.108	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	254	183	303	389	675	0	663	0
N.S.	1	0.82	0.59	0.98	1.25	2.18	0.00	2.14	0.00
time (sec)	N/A	0.430	0.330	0.227	0.302	0.408	0.000	0.682	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	170	144	248	262	541	0	552	0
N.S.	1	0.91	0.77	1.33	1.40	2.89	0.00	2.95	0.00
time (sec)	N/A	0.420	0.232	0.198	0.300	0.359	0.000	0.610	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	99	114	217	161	395	0	0	61
N.S.	1	0.99	1.14	2.17	1.61	3.95	0.00	0.00	0.61
time (sec)	N/A	0.225	0.129	1.135	0.287	0.343	0.000	0.000	18.323

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F(-2)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	137	140	1015	201	1477	0	0	0
N.S.	1	1.02	1.04	7.57	1.50	11.02	0.00	0.00	0.00
time (sec)	N/A	0.392	0.415	0.129	0.295	0.471	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	112	148	244	247	599	0	0	0
N.S.	1	0.77	1.01	1.67	1.69	4.10	0.00	0.00	0.00
time (sec)	N/A	0.358	0.239	0.211	0.305	0.392	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	199	192	279	450	961	0	0	0
N.S.	1	0.94	0.91	1.32	2.12	4.53	0.00	0.00	0.00
time (sec)	N/A	0.444	0.333	0.248	0.318	0.733	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	482	450	296	879	0	436	0	0	0
N.S.	1	0.93	0.61	1.82	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.815	10.611	11.979	0.000	0.120	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	409	386	253	667	0	326	0	0	0
N.S.	1	0.94	0.62	1.63	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.619	10.409	9.204	0.000	0.115	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	339	240	466	0	327	0	0	0
N.S.	1	0.95	0.67	1.31	0.00	0.92	0.00	0.00	0.00
time (sec)	N/A	0.475	10.434	3.383	0.000	0.118	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	405	261	685	0	432	0	0	0
N.S.	1	0.99	0.64	1.67	0.00	1.05	0.00	0.00	0.00
time (sec)	N/A	0.626	10.513	11.043	0.000	0.108	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	490	470	306	1080	0	601	0	0	0
N.S.	1	0.96	0.62	2.20	0.00	1.23	0.00	0.00	0.00
time (sec)	N/A	0.767	10.788	10.811	0.000	0.121	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	67	59	48	0	169	0	113	0
N.S.	1	0.89	0.79	0.64	0.00	2.25	0.00	1.51	0.00
time (sec)	N/A	0.196	0.018	1.023	0.000	0.349	0.000	0.394	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	44	52	40	0	153	0	68	0
N.S.	1	0.88	1.04	0.80	0.00	3.06	0.00	1.36	0.00
time (sec)	N/A	0.180	0.010	0.957	0.000	0.367	0.000	0.373	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	34	17	0	98	0	58	0
N.S.	1	1.00	1.42	0.71	0.00	4.08	0.00	2.42	0.00
time (sec)	N/A	0.161	0.006	0.955	0.000	0.360	0.000	0.322	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	41	17	0	28	17
N.S.	1	1.00	1.00	0.78	1.78	0.74	0.00	1.22	0.74
time (sec)	N/A	0.142	0.004	0.921	0.298	0.294	0.000	0.335	18.648

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	52	30	25	50	25	0	0	29
N.S.	1	1.06	0.61	0.51	1.02	0.51	0.00	0.00	0.59
time (sec)	N/A	0.162	0.006	0.899	0.283	0.282	0.000	0.000	17.851

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	26	23	18	26	29	0	29	0
N.S.	1	0.70	0.62	0.49	0.70	0.78	0.00	0.78	0.00
time (sec)	N/A	0.150	0.041	2.069	0.271	0.297	0.000	0.306	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	26	23	18	26	29	0	29	0
N.S.	1	0.70	0.62	0.49	0.70	0.78	0.00	0.78	0.00
time (sec)	N/A	0.153	0.001	1.483	0.275	0.340	0.000	0.315	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	37	34	29	40	41	0	42	0
N.S.	1	0.52	0.48	0.41	0.56	0.58	0.00	0.59	0.00
time (sec)	N/A	0.167	0.043	1.492	0.270	0.307	0.000	0.320	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	42	37	42	256	0	48	0
N.S.	1	1.00	0.86	0.76	0.86	5.22	0.00	0.98	0.00
time (sec)	N/A	0.167	0.049	0.934	0.268	0.286	0.000	0.340	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	42	37	42	256	0	48	0
N.S.	1	1.00	0.86	0.76	0.86	5.22	0.00	0.98	0.00
time (sec)	N/A	0.173	0.003	0.938	0.279	0.320	0.000	0.324	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	37	30	26	32	127	0	43	0
N.S.	1	0.84	0.68	0.59	0.73	2.89	0.00	0.98	0.00
time (sec)	N/A	0.156	0.023	1.309	0.279	0.290	0.000	0.343	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	34	42	27	0	42	0	27	0
N.S.	1	0.77	0.95	0.61	0.00	0.95	0.00	0.61	0.00
time (sec)	N/A	0.144	0.034	0.893	0.000	0.267	0.000	0.318	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	77	43	22	0	31	0	0	0
N.S.	1	0.93	0.52	0.27	0.00	0.37	0.00	0.00	0.00
time (sec)	N/A	0.175	10.018	0.977	0.000	0.088	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	19	18	17	19	19
N.S.	1	1.00	1.00	0.86	0.86	0.82	0.77	0.86	0.86
time (sec)	N/A	0.137	0.005	0.893	0.272	0.302	0.123	0.339	16.433

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	163	27	20	0	12	36	0	0
N.S.	1	1.24	0.21	0.15	0.00	0.09	0.27	0.00	0.00
time (sec)	N/A	0.255	10.020	0.925	0.000	0.078	0.532	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	27	22	0	9	0	0	0
N.S.	1	1.00	0.50	0.41	0.00	0.17	0.00	0.00	0.00
time (sec)	N/A	0.157	10.020	0.957	0.000	0.083	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	0	76	0	30	0
N.S.	1	1.00	1.00	0.86	0.00	3.45	0.00	1.36	0.00
time (sec)	N/A	0.148	0.018	0.864	0.000	0.306	0.000	0.329	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	139	27	22	0	30	0	0	0
N.S.	1	0.87	0.17	0.14	0.00	0.19	0.00	0.00	0.00
time (sec)	N/A	0.236	10.017	1.020	0.000	0.089	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	30	0	22	18
N.S.	1	1.00	1.00	0.86	1.10	1.43	0.00	1.05	0.86
time (sec)	N/A	0.133	0.022	0.921	0.278	0.298	0.000	0.332	16.616

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	28	19	0	12	20
N.S.	1	1.00	1.00	0.80	1.12	0.76	0.00	0.48	0.80
time (sec)	N/A	0.134	0.006	0.892	0.288	0.281	0.000	0.319	16.785

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	29	22	0	0	0	0	0
N.S.	1	1.00	0.10	0.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.391	10.023	2.305	0.000	0.000	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	252	29	22	0	19	0	0	0
N.S.	1	0.97	0.11	0.08	0.00	0.07	0.00	0.00	0.00
time (sec)	N/A	0.324	10.018	1.105	0.000	0.088	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	32	15	0	85	14	35	0
N.S.	1	1.00	1.39	0.65	0.00	3.70	0.61	1.52	0.00
time (sec)	N/A	0.167	0.212	1.915	0.000	0.320	0.524	0.322	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	27	22	0	11	0	0	0
N.S.	1	1.00	0.23	0.19	0.00	0.09	0.00	0.00	0.00
time (sec)	N/A	0.224	10.025	1.771	0.000	0.080	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	19	0	68	0	31	0
N.S.	1	1.00	1.00	0.79	0.00	2.83	0.00	1.29	0.00
time (sec)	N/A	0.154	0.004	0.909	0.000	0.299	0.000	0.363	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	310	27	22	0	14	0	0	0
N.S.	1	0.99	0.09	0.07	0.00	0.04	0.00	0.00	0.00
time (sec)	N/A	0.426	10.013	2.239	0.000	0.094	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	271	27	22	0	37	0	0	0
N.S.	1	0.96	0.10	0.08	0.00	0.13	0.00	0.00	0.00
time (sec)	N/A	0.328	10.015	1.194	0.000	0.094	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	36	0	0	0	0	0
N.S.	1	1.00	1.00	0.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.169	0.030	1.086	0.000	0.000	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	62	40	35	0	0	0	0	0
N.S.	1	1.29	0.83	0.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.175	0.028	0.957	0.000	0.000	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	66	44	37	0	0	0	0	0
N.S.	1	1.27	0.85	0.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.177	0.030	1.072	0.000	0.000	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	80	33	30	18	0	0	0	43
N.S.	1	2.35	0.97	0.88	0.53	0.00	0.00	0.00	1.26
time (sec)	N/A	0.205	0.054	0.956	0.243	0.000	0.000	0.000	16.697

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	106	191	0	274	0	0	0
N.S.	1	1.00	0.93	1.68	0.00	2.40	0.00	0.00	0.00
time (sec)	N/A	0.226	2.793	1.036	0.000	0.136	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	17	20	29	20	16
N.S.	1	1.00	1.00	1.06	1.06	1.25	1.81	1.25	1.00
time (sec)	N/A	0.145	0.004	0.064	0.199	0.280	0.399	0.309	17.502

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	32	32	112	32	26
N.S.	1	1.00	1.00	1.04	1.23	1.23	4.31	1.23	1.00
time (sec)	N/A	0.152	0.006	0.714	0.212	0.306	11.021	0.339	16.726

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	37	44	44	0	44	36
N.S.	1	1.00	1.00	1.03	1.22	1.22	0.00	1.22	1.00
time (sec)	N/A	0.157	0.009	49.960	0.239	0.268	0.000	0.324	16.584

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	124	140	90	0	94	0	390	179
N.S.	1	0.84	0.95	0.61	0.00	0.64	0.00	2.65	1.22
time (sec)	N/A	0.314	0.470	0.070	0.000	0.287	0.000	0.356	16.565

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	86	101	66	0	70	0	206	129
N.S.	1	0.91	1.06	0.69	0.00	0.74	0.00	2.17	1.36
time (sec)	N/A	0.269	0.357	0.058	0.000	0.276	0.000	0.351	16.944

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	41	35	40	0	29	136	75	79
N.S.	1	0.87	0.74	0.85	0.00	0.62	2.89	1.60	1.68
time (sec)	N/A	0.238	0.215	0.018	0.000	0.275	0.364	0.342	16.852

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	88	160	73	0	318	0	1015	2983
N.S.	1	0.91	1.65	0.75	0.00	3.28	0.00	10.46	30.75
time (sec)	N/A	0.262	0.504	0.025	0.000	0.317	0.000	0.509	32.295

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	96	187	88	0	399	0	1192	2642
N.S.	1	0.93	1.82	0.85	0.00	3.87	0.00	11.57	25.65
time (sec)	N/A	0.258	0.816	0.034	0.000	0.307	0.000	2.443	35.038

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	206	188	604	0	196	0	797	1358
N.S.	1	0.90	0.82	2.65	0.00	0.86	0.00	3.50	5.96
time (sec)	N/A	0.516	0.526	0.042	0.000	0.290	0.000	0.356	146.996

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	151	133	431	0	149	0	445	1012
N.S.	1	0.92	0.81	2.61	0.00	0.90	0.00	2.70	6.13
time (sec)	N/A	0.389	0.318	0.035	0.000	0.317	0.000	0.341	75.361

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	106	82	184	0	103	388	189	110
N.S.	1	1.68	1.30	2.92	0.00	1.63	6.16	3.00	1.75
time (sec)	N/A	0.292	0.205	0.035	0.000	0.326	0.418	0.310	0.242

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	106	137	258	0	290	0	194	524
N.S.	1	0.80	1.03	1.94	0.00	2.18	0.00	1.46	3.94
time (sec)	N/A	0.400	0.276	0.039	0.000	0.354	0.000	0.422	25.540

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	114	128	274	0	367	0	311	7637
N.S.	1	0.81	0.91	1.94	0.00	2.60	0.00	2.21	54.16
time (sec)	N/A	0.398	0.522	0.038	0.000	0.307	0.000	0.772	44.379

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	285	138	294	0	208	0	1447	529
N.S.	1	0.76	0.37	0.78	0.00	0.55	0.00	3.86	1.41
time (sec)	N/A	0.543	0.751	0.098	0.000	0.271	0.000	0.695	16.436

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	199	93	222	0	159	942	866	385
N.S.	1	0.76	0.36	0.85	0.00	0.61	3.61	3.32	1.48
time (sec)	N/A	0.426	0.586	0.096	0.000	0.276	0.754	0.354	16.830

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	117	55	146	0	106	384	427	252
N.S.	1	1.83	0.86	2.28	0.00	1.66	6.00	6.67	3.94
time (sec)	N/A	0.304	0.483	0.064	0.000	0.286	0.727	0.346	16.640

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	123	244	181	0	516	0	2649	4060
N.S.	1	0.78	1.55	1.15	0.00	3.29	0.00	16.87	25.86
time (sec)	N/A	0.401	0.939	0.025	0.000	0.323	0.000	0.925	42.361

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	175	187	252	0	675	0	2594	4681
N.S.	1	1.08	1.15	1.56	0.00	4.17	0.00	16.01	28.90
time (sec)	N/A	0.446	10.455	0.041	0.000	0.309	0.000	7.563	51.541

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	0	13	63	13	21
N.S.	1	1.00	1.00	0.67	0.00	0.62	3.00	0.62	1.00
time (sec)	N/A	0.146	0.007	0.033	0.000	0.264	0.163	0.325	16.606

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	0	13	63	13	21
N.S.	1	1.00	1.00	0.67	0.00	0.62	3.00	0.62	1.00
time (sec)	N/A	0.144	0.067	0.029	0.000	0.264	0.174	0.296	16.345

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	27	23	16	0	15	51	15	15
N.S.	1	1.17	1.00	0.70	0.00	0.65	2.22	0.65	0.65
time (sec)	N/A	0.196	0.113	0.018	0.000	0.281	0.191	0.317	0.044

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	44	33	31	32	0	77	45
N.S.	1	1.00	1.16	0.87	0.82	0.84	0.00	2.03	1.18
time (sec)	N/A	0.282	0.283	0.913	0.275	0.278	0.000	0.341	16.506

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	67	59	34	51	0	76	563
N.S.	1	1.00	1.40	1.23	0.71	1.06	0.00	1.58	11.73
time (sec)	N/A	0.265	0.250	0.923	0.275	0.288	0.000	0.344	29.813

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	25	24	15	23	0	51	33
N.S.	1	1.00	1.32	1.26	0.79	1.21	0.00	2.68	1.74
time (sec)	N/A	0.230	0.159	0.905	0.269	0.290	0.000	0.341	16.820

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	45	58	17	40	44	48	206
N.S.	1	1.00	2.37	3.05	0.89	2.11	2.32	2.53	10.84
time (sec)	N/A	0.205	0.136	0.968	0.278	0.317	0.904	0.358	21.699

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	72	51	41	41	0	130	122
N.S.	1	1.00	2.25	1.59	1.28	1.28	0.00	4.06	3.81
time (sec)	N/A	0.267	0.146	0.941	0.271	0.342	0.000	0.395	17.759

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	49	50	24	44	0	149	120
N.S.	1	1.00	1.88	1.92	0.92	1.69	0.00	5.73	4.62
time (sec)	N/A	0.255	0.138	0.922	0.264	0.326	0.000	0.313	17.690

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	147	58	54	44	0	235	189
N.S.	1	1.00	4.32	1.71	1.59	1.29	0.00	6.91	5.56
time (sec)	N/A	0.274	0.274	0.921	0.382	0.305	0.000	0.402	19.567

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	124	1832	90	0	122	0	451	179
N.S.	1	0.84	12.46	0.61	0.00	0.83	0.00	3.07	1.22
time (sec)	N/A	0.311	5.983	0.076	0.000	0.314	0.000	0.387	17.613

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	86	113	66	0	92	0	255	129
N.S.	1	0.91	1.19	0.69	0.00	0.97	0.00	2.68	1.36
time (sec)	N/A	0.279	0.866	0.067	0.000	0.312	0.000	0.372	16.540

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	71	40	0	50	0	107	79
N.S.	1	1.00	1.51	0.85	0.00	1.06	0.00	2.28	1.68
time (sec)	N/A	0.210	0.514	0.017	0.000	0.317	0.000	0.337	16.554

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	77	121	73	0	158	0	1093	213
N.S.	1	0.79	1.25	0.75	0.00	1.63	0.00	11.27	2.20
time (sec)	N/A	0.274	0.649	0.020	0.000	0.330	0.000	0.535	18.156

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	96	135	88	0	182	0	1402	1637
N.S.	1	0.93	1.31	0.85	0.00	1.77	0.00	13.61	15.89
time (sec)	N/A	0.258	10.169	0.038	0.000	0.318	0.000	2.749	24.654

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	144	75	120	0	243	0	1895	1610
N.S.	1	0.84	0.44	0.70	0.00	1.42	0.00	11.08	9.42
time (sec)	N/A	0.286	10.091	0.036	0.000	0.369	0.000	6.129	26.553

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	181	182	517	0	479	0	511	1107
N.S.	1	0.93	0.93	2.65	0.00	2.46	0.00	2.62	5.68
time (sec)	N/A	0.504	1.202	0.041	0.000	0.331	0.000	0.795	45.026

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	134	175	187	0	372	0	272	129
N.S.	1	0.94	1.23	1.32	0.00	2.62	0.00	1.92	0.91
time (sec)	N/A	0.412	0.827	0.037	0.000	0.322	0.000	0.835	0.247

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	108	254	266	0	346	0	359	5098
N.S.	1	0.80	1.88	1.97	0.00	2.56	0.00	2.66	37.76
time (sec)	N/A	0.379	1.191	0.046	0.000	0.348	0.000	0.918	34.870

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	111	239	269	0	317	0	438	4285
N.S.	1	0.80	1.73	1.95	0.00	2.30	0.00	3.17	31.05
time (sec)	N/A	0.333	1.144	0.040	0.000	0.381	0.000	0.968	33.209

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	115	109	313	0	126	0	532	787
N.S.	1	0.93	0.89	2.54	0.00	1.02	0.00	4.33	6.40
time (sec)	N/A	0.394	0.814	0.041	0.000	0.318	0.000	2.835	30.043

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	160	153	457	0	182	0	802	1290
N.S.	1	0.92	0.88	2.63	0.00	1.05	0.00	4.61	7.41
time (sec)	N/A	0.442	10.182	0.040	0.000	0.342	0.000	2.460	48.119

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	215	2321	246	0	225	0	932	429
N.S.	1	0.78	8.38	0.89	0.00	0.81	0.00	3.36	1.55
time (sec)	N/A	0.498	6.819	0.101	0.000	0.320	0.000	1.061	17.552

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	129	197	172	0	167	0	480	268
N.S.	1	0.79	1.21	1.06	0.00	1.02	0.00	2.94	1.64
time (sec)	N/A	0.392	1.076	0.062	0.000	0.313	0.000	1.021	16.856

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	121	1004	148	0	321	0	2374	762
N.S.	1	0.78	6.48	0.95	0.00	2.07	0.00	15.32	4.92
time (sec)	N/A	0.405	5.569	0.024	0.000	0.322	0.000	1.576	20.353

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	175	690	237	0	260	0	2318	559
N.S.	1	1.11	4.39	1.51	0.00	1.66	0.00	14.76	3.56
time (sec)	N/A	0.426	10.678	0.036	0.000	0.316	0.000	9.204	21.489

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	213	182	300	0	297	0	2766	287
N.S.	1	1.30	1.11	1.83	0.00	1.81	0.00	16.87	1.75
time (sec)	N/A	0.404	10.213	0.047	0.000	0.334	0.000	17.839	19.715

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	45	63	23	44	48	54	209
N.S.	1	1.00	1.45	2.03	0.74	1.42	1.55	1.74	6.74
time (sec)	N/A	0.237	0.008	0.937	0.270	0.295	1.035	0.297	21.797

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	44	33	31	32	0	77	42
N.S.	1	1.00	1.16	0.87	0.82	0.84	0.00	2.03	1.11
time (sec)	N/A	0.509	0.013	0.946	0.257	0.321	0.000	0.319	17.648

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	68	59	34	51	0	76	381
N.S.	1	1.00	1.42	1.23	0.71	1.06	0.00	1.58	7.94
time (sec)	N/A	0.490	0.017	0.953	0.266	0.321	0.000	0.298	26.488

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	25	26	17	25	0	54	25
N.S.	1	1.00	1.19	1.24	0.81	1.19	0.00	2.57	1.19
time (sec)	N/A	0.345	0.004	0.949	0.272	0.315	0.000	0.287	17.276

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	46	59	20	41	48	49	205
N.S.	1	1.00	2.09	2.68	0.91	1.86	2.18	2.23	9.32
time (sec)	N/A	0.283	0.005	0.945	0.263	0.328	1.061	0.284	17.973

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	72	51	41	41	0	130	122
N.S.	1	1.00	2.25	1.59	1.28	1.28	0.00	4.06	3.81
time (sec)	N/A	0.412	0.008	0.959	0.263	0.293	0.000	0.337	18.142

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	49	50	24	44	0	149	118
N.S.	1	1.00	1.88	1.92	0.92	1.69	0.00	5.73	4.54
time (sec)	N/A	0.418	0.010	1.019	0.263	0.327	0.000	0.346	17.875

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	148	57	51	43	0	233	186
N.S.	1	1.00	4.48	1.73	1.55	1.30	0.00	7.06	5.64
time (sec)	N/A	0.431	0.028	1.000	0.262	0.316	0.000	0.421	19.134

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	69	68	48	0	36	0	123	93
N.S.	1	2.46	2.43	1.71	0.00	1.29	0.00	4.39	3.32
time (sec)	N/A	0.560	0.164	0.952	0.000	0.326	0.000	0.369	19.041

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	79	46	62	0	37	223	41	200
N.S.	1	2.39	1.39	1.88	0.00	1.12	6.76	1.24	6.06
time (sec)	N/A	0.354	0.089	0.919	0.000	0.288	11.609	0.296	27.821

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	116	86	0	0	0	0	0	0
N.S.	1	0.96	0.71	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.270	0.396	0.000	0.000	0.000	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	168	192	341	293	161	459	171	0
N.S.	1	0.96	1.10	1.95	1.67	0.92	2.62	0.98	0.00
time (sec)	N/A	0.286	0.763	0.932	0.197	0.287	0.604	0.372	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	131	113	124	107	114	185	107	210
N.S.	1	0.96	0.83	0.91	0.79	0.84	1.36	0.79	1.54
time (sec)	N/A	0.270	0.559	0.876	0.184	0.365	0.529	0.369	18.794

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	80	75	54	74	54	68	136
N.S.	1	1.00	1.18	1.10	0.79	1.09	0.79	1.00	2.00
time (sec)	N/A	0.197	0.137	0.983	0.191	0.292	1.069	0.361	17.640

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	111	337	651	0	187	0	299	0
N.S.	1	0.95	2.88	5.56	0.00	1.60	0.00	2.56	0.00
time (sec)	N/A	0.275	0.858	0.064	0.000	0.357	0.000	0.501	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	146	228	3182	0	284	0	146	0
N.S.	1	0.97	1.51	21.07	0.00	1.88	0.00	0.97	0.00
time (sec)	N/A	0.284	0.700	0.084	0.000	0.377	0.000	0.458	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	182	278	11352	0	536	0	182	0
N.S.	1	0.94	1.44	58.82	0.00	2.78	0.00	0.94	0.00
time (sec)	N/A	0.313	1.906	0.133	0.000	0.551	0.000	1.222	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	225	219	212	0	0	416	0	0	0
N.S.	1	0.97	0.94	0.00	0.00	1.85	0.00	0.00	0.00
time (sec)	N/A	0.408	1.407	0.000	0.000	0.388	0.000	0.000	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	183	181	170	0	0	337	0	0	0
N.S.	1	0.99	0.93	0.00	0.00	1.84	0.00	0.00	0.00
time (sec)	N/A	0.371	1.236	0.000	0.000	0.366	0.000	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	147	142	0	0	301	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	2.05	0.00	0.00	0.00
time (sec)	N/A	0.315	0.871	0.000	0.000	0.369	0.000	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	147	151	141	0	0	298	0	0	0
N.S.	1	1.03	0.96	0.00	0.00	2.03	0.00	0.00	0.00
time (sec)	N/A	0.278	0.852	0.000	0.000	0.381	0.000	0.000	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	158	160	169	0	0	487	0	0	0
N.S.	1	1.01	1.07	0.00	0.00	3.08	0.00	0.00	0.00
time (sec)	N/A	0.314	1.309	0.000	0.000	0.465	0.000	0.000	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	199	196	209	0	0	812	0	0	0
N.S.	1	0.98	1.05	0.00	0.00	4.08	0.00	0.00	0.00
time (sec)	N/A	0.375	1.371	0.000	0.000	0.657	0.000	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	41	45	41	0	0	26	0	0	0
N.S.	1	1.10	1.00	0.00	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	0.181	0.039	0.000	0.000	0.305	0.000	0.000	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	70	69	0	0	59	0	0	0
N.S.	1	1.01	1.00	0.00	0.00	0.86	0.00	0.00	0.00
time (sec)	N/A	0.228	0.396	0.000	0.000	0.330	0.000	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	35	60	0	34	415	0	0
N.S.	1	1.00	0.78	1.33	0.00	0.76	9.22	0.00	0.00
time (sec)	N/A	0.170	0.057	0.051	0.000	0.338	0.708	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	31	55	0	28	197	0	0
N.S.	1	1.00	0.76	1.34	0.00	0.68	4.80	0.00	0.00
time (sec)	N/A	0.161	0.060	0.052	0.000	0.325	0.657	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	31	64	0	30	197	0	0
N.S.	1	1.00	0.76	1.56	0.00	0.73	4.80	0.00	0.00
time (sec)	N/A	0.164	0.048	0.050	0.000	0.327	0.671	0.000	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	72	0	0	70	0	0	0
N.S.	1	1.00	1.09	0.00	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.209	3.997	0.000	0.000	0.398	0.000	0.000	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	166	168	134	0	0	0	0	0	0
N.S.	1	1.01	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.359	0.969	0.000	0.000	0.000	0.000	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	312	260	809	0	345	1363	397	0
N.S.	1	1.03	0.86	2.67	0.00	1.14	4.50	1.31	0.00
time (sec)	N/A	0.520	1.271	1.261	0.000	0.334	1.136	0.349	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	246	176	300	0	219	490	236	0
N.S.	1	1.04	0.74	1.27	0.00	0.92	2.07	1.00	0.00
time (sec)	N/A	0.438	0.918	1.152	0.000	0.282	0.707	0.354	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	180	123	0	123	197	119	0
N.S.	1	1.00	1.53	1.04	0.00	1.04	1.67	1.01	0.00
time (sec)	N/A	0.252	0.348	1.246	0.000	0.268	0.297	0.303	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	222	266	1263	0	371	0	0	0
N.S.	1	1.03	1.24	5.87	0.00	1.73	0.00	0.00	0.00
time (sec)	N/A	0.401	0.746	0.082	0.000	2.519	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	275	427	6303	0	826	0	1618	0
N.S.	1	1.03	1.61	23.70	0.00	3.11	0.00	6.08	0.00
time (sec)	N/A	0.455	1.448	0.106	0.000	1.667	0.000	6.663	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	339	300	29133	0	1954	0	0	0
N.S.	1	1.03	0.91	88.28	0.00	5.92	0.00	0.00	0.00
time (sec)	N/A	0.502	10.506	0.292	0.000	12.004	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	370	377	985	0	0	923	0	0	0
N.S.	1	1.02	2.66	0.00	0.00	2.49	0.00	0.00	0.00
time (sec)	N/A	0.840	4.555	0.000	0.000	0.447	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	302	309	443	0	0	657	0	0	0
N.S.	1	1.02	1.47	0.00	0.00	2.18	0.00	0.00	0.00
time (sec)	N/A	0.651	2.570	0.000	0.000	0.441	0.000	0.000	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	233	243	327	0	0	692	0	0	0
N.S.	1	1.04	1.40	0.00	0.00	2.97	0.00	0.00	0.00
time (sec)	N/A	0.484	1.361	0.000	0.000	0.432	0.000	0.000	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	244	276	341	0	0	716	0	0	0
N.S.	1	1.13	1.40	0.00	0.00	2.93	0.00	0.00	0.00
time (sec)	N/A	0.438	1.511	0.000	0.000	0.432	0.000	0.000	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	269	281	395	0	0	1456	0	0	0
N.S.	1	1.04	1.47	0.00	0.00	5.41	0.00	0.00	0.00
time (sec)	N/A	0.483	2.408	0.000	0.000	0.785	0.000	0.000	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	335	354	557	0	0	2514	0	0	0
N.S.	1	1.06	1.66	0.00	0.00	7.50	0.00	0.00	0.00
time (sec)	N/A	0.656	3.340	0.000	0.000	1.961	0.000	0.000	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	155	138	216	0	158	0	0	0
N.S.	1	0.95	0.84	1.32	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.288	0.395	0.965	0.000	0.280	0.000	0.000	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	103	92	167	0	78	15302	0	0
N.S.	1	0.95	0.85	1.55	0.00	0.72	141.69	0.00	0.00
time (sec)	N/A	0.247	0.223	0.036	0.000	0.300	14.579	0.000	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	51	43	120	0	32	2236	0	0
N.S.	1	0.98	0.83	2.31	0.00	0.62	43.00	0.00	0.00
time (sec)	N/A	0.189	0.127	0.026	0.000	0.280	1.711	0.000	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	61	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.214	0.159	0.000	0.000	0.000	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	61	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.211	0.166	0.000	0.000	0.000	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	176	167	150	0	0	159	0	0	0
N.S.	1	0.95	0.85	0.00	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.277	0.409	0.000	0.000	0.282	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	116	111	100	0	0	79	0	0	0
N.S.	1	0.96	0.86	0.00	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.233	0.226	0.000	0.000	0.275	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	55	50	0	0	33	0	0	0
N.S.	1	0.98	0.89	0.00	0.00	0.59	0.00	0.00	0.00
time (sec)	N/A	0.183	0.141	0.000	0.000	0.273	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.213	0.157	0.000	0.000	0.000	0.000	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.212	0.163	0.000	0.000	0.000	0.000	0.000	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	187	176	157	0	0	201	0	0	0
N.S.	1	0.94	0.84	0.00	0.00	1.07	0.00	0.00	0.00
time (sec)	N/A	0.302	0.341	0.000	0.000	0.286	0.000	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	124	111	0	0	110	0	0	0
N.S.	1	0.95	0.85	0.00	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.273	0.223	0.000	0.000	0.283	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	72	65	0	0	48	0	0	0
N.S.	1	0.96	0.87	0.00	0.00	0.64	0.00	0.00	0.00
time (sec)	N/A	0.235	0.182	0.000	0.000	0.280	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	0	15	311	15	15
N.S.	1	1.00	1.00	0.94	0.00	0.88	18.29	0.88	0.88
time (sec)	N/A	0.189	0.023	0.854	0.000	0.276	1.509	0.310	16.967

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	61	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.216	0.075	0.000	0.000	0.000	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	61	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.221	0.087	0.000	0.000	0.000	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	201	190	173	0	0	204	0	0	0
N.S.	1	0.95	0.86	0.00	0.00	1.01	0.00	0.00	0.00
time (sec)	N/A	0.288	0.413	0.000	0.000	0.279	0.000	0.000	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	141	134	123	0	0	113	0	0	0
N.S.	1	0.95	0.87	0.00	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.265	0.224	0.000	0.000	0.279	0.000	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	78	73	0	0	51	0	0	0
N.S.	1	0.96	0.90	0.00	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	0.232	0.193	0.000	0.000	0.284	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	18	41	18	18
N.S.	1	1.00	1.00	0.95	0.00	0.90	2.05	0.90	0.90
time (sec)	N/A	0.196	0.025	0.950	0.000	0.270	0.953	0.323	17.024

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.217	0.082	0.000	0.000	0.000	0.000	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.218	0.096	0.000	0.000	0.000	0.000	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	365	327	280	0	0	654	0	0	0
N.S.	1	0.90	0.77	0.00	0.00	1.79	0.00	0.00	0.00
time (sec)	N/A	0.503	4.201	0.000	0.000	0.311	0.000	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	239	217	186	0	0	239	0	0	0
N.S.	1	0.91	0.78	0.00	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.372	1.496	0.000	0.000	0.311	0.000	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	104	89	0	0	80	0	0	0
N.S.	1	0.97	0.83	0.00	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.256	0.997	0.000	0.000	0.317	0.000	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	112	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.372	1.212	0.000	0.000	0.000	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	112	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.359	1.334	0.000	0.000	0.000	0.000	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	104	89	0	0	80	0	0	0
N.S.	1	0.97	0.83	0.00	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.309	0.009	0.000	0.000	0.288	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	112	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.532	0.007	0.000	0.000	0.000	0.000	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	266	228	0	0	377	0	0	0
N.S.	1	0.90	0.77	0.00	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.471	6.541	0.000	0.000	0.298	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	171	156	135	0	0	122	0	0	0
N.S.	1	0.91	0.79	0.00	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.414	2.242	0.000	0.000	0.335	0.000	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	36	0	0	41	0	0	41
N.S.	1	1.00	0.88	0.00	0.00	1.00	0.00	0.00	1.00
time (sec)	N/A	0.342	0.252	0.000	0.000	0.273	0.000	0.000	17.344

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	112	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.387	3.108	0.000	0.000	0.000	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	36	0	0	41	0	0	39
N.S.	1	1.00	0.88	0.00	0.00	1.00	0.00	0.00	0.95
time (sec)	N/A	0.507	0.008	0.000	0.000	0.274	0.000	0.000	16.689

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	327	208	175	0	0	231	0	0	0
N.S.	1	0.64	0.54	0.00	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.677	0.165	0.000	0.000	0.309	0.000	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	76	0	0	117	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	1.26	0.00	0.00	0.00
time (sec)	N/A	0.605	0.034	0.000	0.000	0.290	0.000	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	177	177	152	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.659	0.130	0.000	0.000	0.000	0.000	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	93	76	0	0	117	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	1.26	0.00	0.00	0.00
time (sec)	N/A	0.796	0.021	0.000	0.000	0.282	0.000	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	772	272	0	0	0	0	0
N.S.	1	1.00	4.04	1.42	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.701	4.191	4.541	0.000	0.000	0.000	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	191	74	0	141	70	266	50
N.S.	1	1.00	2.36	0.91	0.00	1.74	0.86	3.28	0.62
time (sec)	N/A	0.246	0.087	0.159	0.000	0.332	0.318	0.403	0.111

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	233	66	0	146	63	282	28
N.S.	1	1.00	3.19	0.90	0.00	2.00	0.86	3.86	0.38
time (sec)	N/A	0.217	0.097	0.148	0.000	0.367	0.334	0.405	17.812

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	87	70	0	153	70	67	54
N.S.	1	1.00	2.29	1.84	0.00	4.03	1.84	1.76	1.42
time (sec)	N/A	0.232	0.047	0.080	0.000	0.325	0.382	0.334	0.221

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	87	72	0	155	63	63	67
N.S.	1	1.00	2.29	1.89	0.00	4.08	1.66	1.66	1.76
time (sec)	N/A	0.225	0.041	0.081	0.000	0.323	0.407	0.329	17.550

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	54	38	78	0	144	109	0	196
N.S.	1	1.42	1.00	2.05	0.00	3.79	2.87	0.00	5.16
time (sec)	N/A	0.268	0.279	0.334	0.000	0.356	15.540	0.000	17.515

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	54	38	72	0	144	99	0	139
N.S.	1	1.42	1.00	1.89	0.00	3.79	2.61	0.00	3.66
time (sec)	N/A	0.257	0.259	0.328	0.000	0.371	15.593	0.000	20.079

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	49	42	42	36	155	78	39	42
N.S.	1	1.17	1.00	1.00	0.86	3.69	1.86	0.93	1.00
time (sec)	N/A	0.189	0.016	0.075	0.271	0.329	0.351	0.421	0.113

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	51	46	42	67	168	75	42	199
N.S.	1	1.16	1.05	0.95	1.52	3.82	1.70	0.95	4.52
time (sec)	N/A	0.187	0.018	0.064	0.262	0.294	0.391	0.398	17.483

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	85	74	0	208	90	0	278
N.S.	1	1.00	2.12	1.85	0.00	5.20	2.25	0.00	6.95
time (sec)	N/A	0.284	0.034	0.222	0.000	0.310	0.651	0.000	17.466

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	85	77	0	213	80	0	30
N.S.	1	1.00	2.12	1.92	0.00	5.32	2.00	0.00	0.75
time (sec)	N/A	0.280	0.035	0.222	0.000	0.296	0.680	0.000	0.155

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	78	42	78	0	146	122	0	0
N.S.	1	1.86	1.00	1.86	0.00	3.48	2.90	0.00	0.00
time (sec)	N/A	0.366	1.017	1.106	0.000	0.287	98.471	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	78	42	74	0	146	112	0	0
N.S.	1	1.86	1.00	1.76	0.00	3.48	2.67	0.00	0.00
time (sec)	N/A	0.349	0.247	1.091	0.000	0.316	97.943	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	86	74	0	153	73	71	233
N.S.	1	1.00	2.15	1.85	0.00	3.82	1.82	1.78	5.82
time (sec)	N/A	0.245	0.033	0.079	0.000	0.300	0.611	0.355	16.921

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	86	74	0	155	66	67	67
N.S.	1	1.00	2.15	1.85	0.00	3.88	1.65	1.68	1.68
time (sec)	N/A	0.256	0.032	0.093	0.000	0.303	0.615	0.340	17.371

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	49	42	42	36	155	78	39	42
N.S.	1	1.17	1.00	1.00	0.86	3.69	1.86	0.93	1.00
time (sec)	N/A	0.186	0.015	0.064	0.266	0.312	0.454	0.363	17.469

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	51	46	42	67	168	75	42	923
N.S.	1	1.16	1.05	0.95	1.52	3.82	1.70	0.95	20.98
time (sec)	N/A	0.191	0.016	0.067	0.268	0.306	0.485	0.362	17.355

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	78	0	146	0	0	0
N.S.	1	1.00	1.00	1.86	0.00	3.48	0.00	0.00	0.00
time (sec)	N/A	0.374	2.304	2.569	0.000	0.314	0.000	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	74	0	146	0	0	0
N.S.	1	1.00	1.00	1.76	0.00	3.48	0.00	0.00	0.00
time (sec)	N/A	0.363	0.415	2.591	0.000	0.312	0.000	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	70	42	84	0	165	0	0	0
N.S.	1	1.67	1.00	2.00	0.00	3.93	0.00	0.00	0.00
time (sec)	N/A	0.391	0.470	3.539	0.000	0.324	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	70	42	78	0	165	0	0	0
N.S.	1	1.67	1.00	1.86	0.00	3.93	0.00	0.00	0.00
time (sec)	N/A	0.393	0.570	3.455	0.000	0.313	0.000	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	109	108	1812	125	233	138	155	167
N.S.	1	0.81	0.81	13.52	0.93	1.74	1.03	1.16	1.25
time (sec)	N/A	0.500	0.163	0.135	0.191	0.292	1.913	0.341	17.478

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	64	66	1699	62	161	85	72	123
N.S.	1	0.93	0.96	24.62	0.90	2.33	1.23	1.04	1.78
time (sec)	N/A	0.407	0.055	0.075	0.194	0.305	1.586	0.329	17.310

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	26	1628	21	105	46	22	45
N.S.	1	1.00	1.13	70.78	0.91	4.57	2.00	0.96	1.96
time (sec)	N/A	0.303	0.024	0.070	0.183	0.290	1.206	0.365	18.109

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	86	69	1697	0	316	139	94	1270
N.S.	1	0.98	0.78	19.28	0.00	3.59	1.58	1.07	14.43
time (sec)	N/A	0.430	0.077	0.078	0.000	0.360	2.932	0.333	18.189

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	184	139	1910	0	530	0	210	4602
N.S.	1	1.22	0.92	12.65	0.00	3.51	0.00	1.39	30.48
time (sec)	N/A	0.558	0.219	0.105	0.000	0.737	0.000	0.334	19.544

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	156	99	1780	0	1168	0	0	0
N.S.	1	1.06	0.67	12.11	0.00	7.95	0.00	0.00	0.00
time (sec)	N/A	0.430	0.258	0.083	0.000	0.395	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	63	1718	0	510	0	107	0
N.S.	1	1.00	0.61	16.68	0.00	4.95	0.00	1.04	0.00
time (sec)	N/A	0.249	0.209	0.076	0.000	0.315	0.000	0.329	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	168	105	1811	0	581	0	211	0
N.S.	1	1.05	0.66	11.32	0.00	3.63	0.00	1.32	0.00
time (sec)	N/A	0.398	0.360	0.086	0.000	0.374	0.000	0.344	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	112	108	332	125	191	146	156	200
N.S.	1	0.80	0.77	2.37	0.89	1.36	1.04	1.11	1.43
time (sec)	N/A	0.497	0.163	0.390	0.182	0.300	2.082	0.317	17.830

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	67	66	246	62	118	92	72	119
N.S.	1	0.92	0.90	3.37	0.85	1.62	1.26	0.99	1.63
time (sec)	N/A	0.409	0.054	0.163	0.184	0.282	1.632	0.331	18.044

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	29	160	22	61	49	23	60
N.S.	1	1.00	1.12	6.15	0.85	2.35	1.88	0.88	2.31
time (sec)	N/A	0.295	0.027	0.147	0.182	0.303	1.226	0.318	17.253

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	90	69	223	0	232	150	94	156
N.S.	1	0.97	0.74	2.40	0.00	2.49	1.61	1.01	1.68
time (sec)	N/A	0.437	0.076	1.111	0.000	0.301	2.973	0.383	17.758

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	187	139	405	0	445	0	211	248
N.S.	1	1.21	0.90	2.63	0.00	2.89	0.00	1.37	1.61
time (sec)	N/A	0.549	0.210	0.970	0.000	0.367	0.000	0.320	18.896

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	311	325	296	994	0	0	0	0	0
N.S.	1	1.05	0.95	3.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.778	10.432	0.992	0.000	0.000	0.000	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	304	307	252	665	0	0	0	0	0
N.S.	1	1.01	0.83	2.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.626	10.205	0.881	0.000	0.000	0.000	0.000	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	300	312	425	665	0	0	0	0	0
N.S.	1	1.04	1.42	2.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.670	10.461	0.863	0.000	0.000	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	319	342	496	1200	0	0	0	0	0
N.S.	1	1.07	1.55	3.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.741	10.523	0.834	0.000	0.000	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	324	337	604	1049	0	0	0	0	0
N.S.	1	1.04	1.86	3.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.715	13.067	0.914	0.000	0.000	0.000	0.000	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	320	0	0	0	0	0	0
N.S.	1	1.00	2.37	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.329	0.550	0.000	0.000	0.000	0.000	0.000	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	167	167	156	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.420	0.407	0.000	0.000	0.000	0.000	0.000	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	B	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	27	27	32	0	61	25	49	26	25
N.S.	1	1.00	1.19	0.00	2.26	0.93	1.81	0.96	0.93
time (sec)	N/A	0.297	0.059	0.000	0.199	0.263	6.000	0.311	16.922

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.143	0.014	0.945	0.276	0.267	0.087	0.322	0.047

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	23	13	10	21	21	26	22	9
N.S.	1	1.77	1.00	0.77	1.62	1.62	2.00	1.69	0.69
time (sec)	N/A	0.158	0.028	0.954	0.276	0.266	0.122	0.325	16.488

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	25	20	19	19	22	20	19
N.S.	1	1.00	0.93	0.74	0.70	0.70	0.81	0.74	0.70
time (sec)	N/A	0.173	0.019	0.957	0.182	0.255	0.089	0.333	16.555

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	36	33	25	24	24	0	24	24
N.S.	1	1.12	1.03	0.78	0.75	0.75	0.00	0.75	0.75
time (sec)	N/A	0.173	0.005	0.954	0.183	0.245	0.000	0.325	0.031

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	27	25	20	19	19	22	19	19
N.S.	1	1.08	1.00	0.80	0.76	0.76	0.88	0.76	0.76
time (sec)	N/A	0.166	0.018	0.947	0.187	0.273	0.092	0.347	0.030

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	18	18	15	14	14	15	15	14
N.S.	1	0.90	0.90	0.75	0.70	0.70	0.75	0.75	0.70
time (sec)	N/A	0.166	0.013	0.951	0.179	0.276	0.062	0.336	0.082

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	71	62	46	45	47	68	45	73
N.S.	1	1.15	1.00	0.74	0.73	0.76	1.10	0.73	1.18
time (sec)	N/A	0.228	0.010	0.941	0.274	0.269	0.196	0.369	0.051

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	81	72	50	49	49	0	49	49
N.S.	1	1.11	0.99	0.68	0.67	0.67	0.00	0.67	0.67
time (sec)	N/A	0.198	0.036	1.035	0.185	0.251	0.000	0.320	0.036

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	140	117	83	82	76	121	82	82
N.S.	1	1.08	0.90	0.64	0.63	0.58	0.93	0.63	0.63
time (sec)	N/A	0.235	0.013	0.988	0.185	0.252	1.125	0.326	0.193

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	205	126	124	272	638	0	139	223
N.S.	1	1.02	0.63	0.62	1.36	3.19	0.00	0.70	1.12
time (sec)	N/A	0.436	0.011	1.033	0.270	0.983	0.000	0.431	0.095

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.130	0.002	1.009	0.275	0.281	0.106	0.346	0.143

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	25	19	16	15	15	17	15	15
N.S.	1	1.32	1.00	0.84	0.79	0.79	0.89	0.79	0.79
time (sec)	N/A	0.161	0.014	0.942	0.197	0.250	0.066	0.329	17.757

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	124	62	66	83	74	0	83	42
N.S.	1	1.15	0.57	0.61	0.77	0.69	0.00	0.77	0.39
time (sec)	N/A	0.306	0.078	0.950	0.275	0.267	0.000	0.350	0.088

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	83	83	61	60	62	0	60	78
N.S.	1	1.09	1.09	0.80	0.79	0.82	0.00	0.79	1.03
time (sec)	N/A	0.196	0.046	1.026	0.274	0.272	0.000	0.341	0.071

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	131	110	76	75	71	0	75	75
N.S.	1	1.10	0.92	0.64	0.63	0.60	0.00	0.63	0.63
time (sec)	N/A	0.216	0.028	1.004	0.194	0.258	0.000	0.351	0.173

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	201	208	127	132	293	547	0	140	208
N.S.	1	1.03	0.63	0.66	1.46	2.72	0.00	0.70	1.03
time (sec)	N/A	0.431	0.007	1.032	0.279	0.940	0.000	0.439	17.040

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	35	36	15	44	0	0	32
N.S.	1	1.00	0.97	1.00	0.42	1.22	0.00	0.00	0.89
time (sec)	N/A	0.179	1.170	0.691	0.214	0.262	0.000	0.000	16.998

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	27	5	35	0	76	23
N.S.	1	1.00	1.00	0.93	0.17	1.21	0.00	2.62	0.79
time (sec)	N/A	0.168	8.770	0.974	0.213	0.285	0.000	0.352	16.946

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	27	27	5	35	0	56	23
N.S.	1	1.00	0.93	0.93	0.17	1.21	0.00	1.93	0.79
time (sec)	N/A	0.159	5.582	0.947	0.215	0.284	0.000	0.351	17.306

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	25	5	33	0	51	21
N.S.	1	1.00	0.96	1.00	0.20	1.32	0.00	2.04	0.84
time (sec)	N/A	0.152	4.391	0.962	0.211	0.266	0.000	0.339	17.040

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	24	5	32	0	42	20
N.S.	1	1.00	1.00	1.00	0.21	1.33	0.00	1.75	0.83
time (sec)	N/A	0.161	1.511	0.987	0.220	0.250	0.000	0.337	16.883

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	26	27	5	35	0	60	23
N.S.	1	1.00	0.90	0.93	0.17	1.21	0.00	2.07	0.79
time (sec)	N/A	0.163	3.142	1.041	0.214	0.251	0.000	0.377	16.739

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.206	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	143	112	0	0	0	117	0	0
N.S.	1	1.04	0.81	0.00	0.00	0.00	0.85	0.00	0.00
time (sec)	N/A	0.268	0.153	0.000	0.000	0.000	11.738	0.000	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	73	0	0	0	73	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.92	0.00	0.00
time (sec)	N/A	0.194	0.102	0.000	0.000	0.000	2.666	0.000	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	50	0	0	0	34	0	51
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.85	0.00	1.28
time (sec)	N/A	0.155	0.043	0.000	0.000	0.000	0.800	0.000	16.892

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	97	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.223	0.136	0.000	0.000	0.000	0.000	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	57	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.189	0.116	0.000	0.000	0.000	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	112	112	99	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.226	0.146	0.000	0.000	0.000	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	185	197	155	0	0	0	0	0	0
N.S.	1	1.06	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.349	0.204	0.000	0.000	0.000	0.000	0.000	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	35	8	44	0	0	29
N.S.	1	1.00	1.00	1.06	0.24	1.33	0.00	0.00	0.88
time (sec)	N/A	0.189	0.073	1.061	0.216	0.267	0.000	0.000	16.812

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	32	31	5	41	0	15	25
N.S.	1	1.00	1.03	1.00	0.16	1.32	0.00	0.48	0.81
time (sec)	N/A	0.178	0.041	1.038	0.220	0.237	0.000	0.381	16.672

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	30	7	41	0	0	27
N.S.	1	1.00	1.00	1.07	0.25	1.46	0.00	0.00	0.96
time (sec)	N/A	0.162	0.045	1.043	0.220	0.264	0.000	0.000	17.076

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	30	4	51	0	0	0
N.S.	1	1.00	1.00	1.07	0.14	1.82	0.00	0.00	0.00
time (sec)	N/A	0.164	0.056	0.935	0.217	0.277	0.000	0.000	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	28	7	41	0	0	22
N.S.	1	1.00	1.00	1.08	0.27	1.58	0.00	0.00	0.85
time (sec)	N/A	0.174	0.048	0.980	0.211	0.254	0.000	0.000	16.821

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	31	5	44	0	5	25
N.S.	1	1.00	1.00	1.00	0.16	1.42	0.00	0.16	0.81
time (sec)	N/A	0.181	0.055	0.988	0.217	0.262	0.000	0.348	17.282

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	406	719	525	623	0	235	0	0	0
N.S.	1	1.77	1.29	1.53	0.00	0.58	0.00	0.00	0.00
time (sec)	N/A	0.820	24.580	2.425	0.000	0.089	0.000	0.000	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	12	11	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.80	0.73	0.73
time (sec)	N/A	0.140	10.023	1.046	0.187	0.243	0.084	0.332	17.831

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	72	14	13	20	31	13	19
N.S.	1	1.00	4.24	0.82	0.76	1.18	1.82	0.76	1.12
time (sec)	N/A	0.142	10.040	1.085	0.196	0.289	0.097	0.373	17.770

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	31	11	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	2.07	0.73	0.73
time (sec)	N/A	0.137	10.026	1.073	0.188	0.270	0.096	0.359	17.503

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	31	11	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	2.07	0.73	0.73
time (sec)	N/A	0.139	10.030	1.037	0.194	0.257	0.093	0.411	17.398

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	23	10
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	1.77	0.77
time (sec)	N/A	0.139	0.019	1.051	0.185	0.244	0.066	0.366	17.099

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	46	45	34	33	32	39	33	32
N.S.	1	1.05	1.02	0.77	0.75	0.73	0.89	0.75	0.73
time (sec)	N/A	0.171	0.023	1.015	0.182	0.247	0.077	0.417	0.115

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	27	19	16	15	15	17	16	15
N.S.	1	1.29	0.90	0.76	0.71	0.71	0.81	0.76	0.71
time (sec)	N/A	0.155	0.003	0.932	0.195	0.250	0.061	0.426	17.303

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	34	27	26	25	27	26	25
N.S.	1	1.00	1.03	0.82	0.79	0.76	0.82	0.79	0.76
time (sec)	N/A	0.168	0.020	0.941	0.194	0.257	0.071	0.345	17.247

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	43	36	35	30	29	42	38	29
N.S.	1	1.30	1.09	1.06	0.91	0.88	1.27	1.15	0.88
time (sec)	N/A	0.179	0.030	0.982	0.186	0.285	0.097	0.340	0.051

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	37	10	12	10	10	14	0	10
N.S.	1	3.70	1.00	1.20	1.00	1.00	1.40	0.00	1.00
time (sec)	N/A	0.225	0.025	1.103	0.185	0.255	0.836	0.000	17.025

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	37	10	12	86	10	24	0	39
N.S.	1	2.18	0.59	0.71	5.06	0.59	1.41	0.00	2.29
time (sec)	N/A	0.193	0.007	1.197	0.201	0.241	12.624	0.000	17.096

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	34	38	97	0	0	0	0
N.S.	1	1.00	0.92	1.03	2.62	0.00	0.00	0.00	0.00
time (sec)	N/A	0.266	3.895	56.810	0.282	0.000	0.000	0.000	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	36	95	0	0	0	0
N.S.	1	1.00	0.91	1.03	2.71	0.00	0.00	0.00	0.00
time (sec)	N/A	0.258	10.182	43.423	0.263	0.000	0.000	0.000	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	31	35	92	0	0	0	148
N.S.	1	1.00	0.91	1.03	2.71	0.00	0.00	0.00	4.35
time (sec)	N/A	0.277	6.960	32.992	0.269	0.000	0.000	0.000	27.137

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	B	F(-1)	F(-1)	F(-1)	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	0	34	38	95	0	0	0	37
N.S.	1	0.00	0.92	1.03	2.57	0.00	0.00	0.00	1.00
time (sec)	N/A	0.000	8.767	84.549	0.278	0.000	0.000	0.000	25.134

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	B	F(-1)	F(-1)	F(-1)	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	0	34	38	95	0	0	0	146
N.S.	1	0.00	0.92	1.03	2.57	0.00	0.00	0.00	3.95
time (sec)	N/A	0.000	9.145	82.797	0.281	0.000	0.000	0.000	26.024

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	171	105	78	151	94	177	151	167
N.S.	1	0.92	0.57	0.42	0.82	0.51	0.96	0.82	0.90
time (sec)	N/A	0.352	0.083	0.980	0.186	0.364	0.581	0.399	17.655

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	129	93	66	112	81	133	127	124
N.S.	1	0.93	0.67	0.48	0.81	0.59	0.96	0.92	0.90
time (sec)	N/A	0.286	0.059	1.046	0.190	0.342	0.559	0.410	0.047

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	85	68	54	72	67	88	131	79
N.S.	1	0.96	0.76	0.61	0.81	0.75	0.99	1.47	0.89
time (sec)	N/A	0.237	0.041	0.981	0.194	0.339	0.557	0.498	0.030

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	47	41	35	35	49	68	82	36
N.S.	1	1.15	1.00	0.85	0.85	1.20	1.66	2.00	0.88
time (sec)	N/A	0.183	0.019	0.927	0.185	0.282	0.095	0.393	0.046

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	71	63	51	70	118	87	59	130
N.S.	1	1.25	1.11	0.89	1.23	2.07	1.53	1.04	2.28
time (sec)	N/A	0.250	0.056	0.895	0.262	0.285	1.390	0.402	0.089

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	66	64	63	73	147	73	80	131
N.S.	1	1.22	1.19	1.17	1.35	2.72	1.35	1.48	2.43
time (sec)	N/A	0.217	0.100	0.967	0.262	0.276	22.673	0.325	0.122

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	91	77	81	113	181	134	105	80
N.S.	1	1.14	0.96	1.01	1.41	2.26	1.68	1.31	1.00
time (sec)	N/A	0.221	0.182	0.983	0.274	0.304	90.954	0.386	0.084

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	326	307	232	385	268	286	359	915	0
N.S.	1	0.94	0.71	1.18	0.82	0.88	1.10	2.81	0.00
time (sec)	N/A	0.404	0.211	0.171	0.191	0.371	0.889	0.417	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	211	147	184	167	184	241	549	0
N.S.	1	0.94	0.66	0.82	0.75	0.82	1.08	2.45	0.00
time (sec)	N/A	0.317	0.127	0.148	0.189	0.315	0.748	0.391	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	126	84	93	93	103	156	279	0
N.S.	1	0.95	0.63	0.70	0.70	0.77	1.17	2.10	0.00
time (sec)	N/A	0.257	0.068	0.142	0.195	0.314	0.669	0.388	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	55	55	41	43	50	70	99	44
N.S.	1	0.98	0.98	0.73	0.77	0.89	1.25	1.77	0.79
time (sec)	N/A	0.181	0.035	0.105	0.183	0.333	0.418	0.344	17.903

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	134	124	154	0	194	0	157	0
N.S.	1	1.16	1.07	1.33	0.00	1.67	0.00	1.35	0.00
time (sec)	N/A	0.367	0.230	0.296	0.000	0.347	0.000	0.443	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	186	144	166	0	1003	0	232	0
N.S.	1	1.36	1.05	1.21	0.00	7.32	0.00	1.69	0.00
time (sec)	N/A	0.341	0.387	0.271	0.000	0.351	0.000	0.413	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	336	217	372	0	2856	0	895	0
N.S.	1	1.50	0.97	1.66	0.00	12.75	0.00	4.00	0.00
time (sec)	N/A	0.519	1.750	0.303	0.000	0.562	0.000	0.612	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	216	235	288	243	228	252	341	317
N.S.	1	0.94	1.02	1.25	1.06	0.99	1.10	1.48	1.38
time (sec)	N/A	0.408	0.199	0.057	0.192	0.289	1.402	0.424	0.070

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	143	151	166	148	138	167	198	184
N.S.	1	0.95	1.00	1.10	0.98	0.91	1.11	1.31	1.22
time (sec)	N/A	0.318	0.113	0.052	0.186	0.282	1.194	0.446	17.676

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	88	85	85	81	71	104	105	89
N.S.	1	0.98	0.94	0.94	0.90	0.79	1.16	1.17	0.99
time (sec)	N/A	0.255	0.062	0.040	0.187	0.285	1.074	0.345	0.057

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	39	40	36	35	33	49	38	33
N.S.	1	0.95	0.98	0.88	0.85	0.80	1.20	0.93	0.80
time (sec)	N/A	0.182	0.026	0.048	0.188	0.276	0.271	0.370	17.445

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	79	62	69	95	125	97	88	181
N.S.	1	0.96	0.76	0.84	1.16	1.52	1.18	1.07	2.21
time (sec)	N/A	0.237	0.050	0.077	0.338	0.298	1.344	0.353	17.828

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	151	118	128	191	283	0	191	220
N.S.	1	1.16	0.91	0.98	1.47	2.18	0.00	1.47	1.69
time (sec)	N/A	0.329	0.174	0.079	0.318	0.339	0.000	0.352	18.515

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	244	198	227	367	534	0	375	1094
N.S.	1	1.20	0.97	1.11	1.80	2.62	0.00	1.84	5.36
time (sec)	N/A	0.438	0.378	0.092	0.301	0.677	0.000	0.361	19.573

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	228	283	277	251	392	267	324	461
N.S.	1	0.95	1.18	1.15	1.05	1.63	1.11	1.35	1.92
time (sec)	N/A	0.424	0.194	0.072	0.212	0.316	5.690	0.339	0.094

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	158	194	161	158	269	189	191	197
N.S.	1	0.95	1.17	0.97	0.95	1.62	1.14	1.15	1.19
time (sec)	N/A	0.334	0.136	0.067	0.201	0.305	4.543	0.423	0.063

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	108	87	90	163	126	102	98
N.S.	1	1.00	1.14	0.92	0.95	1.72	1.33	1.07	1.03
time (sec)	N/A	0.267	0.074	0.060	0.193	0.286	3.497	0.344	0.062

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	44	43	41	43	75	124	44	43
N.S.	1	0.94	0.91	0.87	0.91	1.60	2.64	0.94	0.91
time (sec)	N/A	0.192	0.040	0.062	0.186	0.293	0.561	0.380	17.233

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	152	107	118	176	444	167	174	125
N.S.	1	1.18	0.83	0.91	1.36	3.44	1.29	1.35	0.97
time (sec)	N/A	0.331	0.160	0.078	0.274	0.325	4.639	0.357	17.948

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	219	177	182	367	854	0	311	275
N.S.	1	1.08	0.88	0.90	1.82	4.23	0.00	1.54	1.36
time (sec)	N/A	0.391	0.294	0.111	0.265	0.528	0.000	0.376	0.708

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	336	301	303	659	1252	0	521	1441
N.S.	1	1.10	0.98	0.99	2.15	4.09	0.00	1.70	4.71
time (sec)	N/A	0.527	0.588	0.135	0.285	1.247	0.000	0.394	21.417

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	305	232	384	268	231	357	409	0
N.S.	1	0.94	0.72	1.19	0.83	0.71	1.10	1.26	0.00
time (sec)	N/A	0.416	0.196	0.164	0.183	0.309	0.800	0.383	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	209	147	184	167	140	240	238	0
N.S.	1	0.94	0.66	0.83	0.75	0.63	1.08	1.07	0.00
time (sec)	N/A	0.331	0.119	0.163	0.187	0.340	0.721	0.406	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	124	84	92	93	71	155	115	0
N.S.	1	0.95	0.64	0.70	0.71	0.54	1.18	0.88	0.00
time (sec)	N/A	0.262	0.066	0.118	0.192	0.319	0.629	0.364	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	53	42	41	42	34	68	38	44
N.S.	1	0.98	0.78	0.76	0.78	0.63	1.26	0.70	0.81
time (sec)	N/A	0.188	0.026	0.086	0.186	0.331	0.359	0.406	17.118

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	112	105	92	0	743	0	140	0
N.S.	1	1.15	1.08	0.95	0.00	7.66	0.00	1.44	0.00
time (sec)	N/A	0.311	0.159	0.284	0.000	0.329	0.000	0.381	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	235	170	259	0	2493	0	644	0
N.S.	1	1.44	1.04	1.59	0.00	15.29	0.00	3.95	0.00
time (sec)	N/A	0.393	0.506	0.290	0.000	0.406	0.000	0.618	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	391	249	427	0	4390	0	1303	0
N.S.	1	1.50	0.95	1.64	0.00	16.82	0.00	4.99	0.00
time (sec)	N/A	0.606	1.306	0.418	0.000	1.219	0.000	0.444	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	350	330	555	0	728	1416	0	5699	0
N.S.	1	0.94	1.59	0.00	2.08	4.05	0.00	16.28	0.00
time (sec)	N/A	0.469	0.680	0.000	0.217	0.409	0.000	0.508	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	242	227	284	0	402	712	0	2511	0
N.S.	1	0.94	1.17	0.00	1.66	2.94	0.00	10.38	0.00
time (sec)	N/A	0.358	0.366	0.000	0.210	0.329	0.000	0.358	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	145	137	128	0	187	294	0	806	0
N.S.	1	0.94	0.88	0.00	1.29	2.03	0.00	5.56	0.00
time (sec)	N/A	0.280	0.194	0.000	0.198	0.314	0.000	0.352	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	60	53	0	60	81	0	129	146
N.S.	1	0.97	0.85	0.00	0.97	1.31	0.00	2.08	2.35
time (sec)	N/A	0.196	0.090	0.000	0.200	0.293	0.000	0.568	17.516

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	145	136	0	0	0	0	0	0
N.S.	1	1.04	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.297	0.169	0.000	0.000	0.000	0.000	0.000	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	87	77	70	0	164	134	0	0
N.S.	1	0.94	0.83	0.75	0.00	1.76	1.44	0.00	0.00
time (sec)	N/A	0.207	0.196	1.353	0.000	0.275	18.081	0.000	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	67	61	54	0	130	95	0	0
N.S.	1	0.96	0.87	0.77	0.00	1.86	1.36	0.00	0.00
time (sec)	N/A	0.190	0.136	1.365	0.000	0.290	12.055	0.000	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	47	46	40	0	103	0	0	0
N.S.	1	0.96	0.94	0.82	0.00	2.10	0.00	0.00	0.00
time (sec)	N/A	0.185	0.109	1.281	0.000	0.287	0.000	0.000	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	0	78	0	0	0
N.S.	1	1.00	1.00	0.83	0.00	2.60	0.00	0.00	0.00
time (sec)	N/A	0.173	0.102	1.274	0.000	0.299	0.000	0.000	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	50	52	43	0	164	66	0	0
N.S.	1	0.96	1.00	0.83	0.00	3.15	1.27	0.00	0.00
time (sec)	N/A	0.183	0.146	1.070	0.000	0.283	3.161	0.000	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	67	59	0	262	90	0	0
N.S.	1	1.00	0.89	0.79	0.00	3.49	1.20	0.00	0.00
time (sec)	N/A	0.197	0.184	1.050	0.000	0.310	4.673	0.000	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	97	81	78	0	169	131	0	0
N.S.	1	0.96	0.80	0.77	0.00	1.67	1.30	0.00	0.00
time (sec)	N/A	0.204	0.161	1.406	0.000	0.265	17.815	0.000	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	74	66	60	0	135	94	0	0
N.S.	1	0.97	0.87	0.79	0.00	1.78	1.24	0.00	0.00
time (sec)	N/A	0.198	0.135	1.407	0.000	0.268	11.992	0.000	0.000

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	51	50	44	0	110	0	0	0
N.S.	1	0.96	0.94	0.83	0.00	2.08	0.00	0.00	0.00
time (sec)	N/A	0.181	0.115	1.284	0.000	0.279	0.000	0.000	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	80	0	0	0
N.S.	1	1.00	1.00	0.84	0.00	2.50	0.00	0.00	0.00
time (sec)	N/A	0.175	0.102	1.377	0.000	0.266	0.000	0.000	0.000

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	54	56	47	0	175	66	0	0
N.S.	1	0.96	1.00	0.84	0.00	3.12	1.18	0.00	0.00
time (sec)	N/A	0.189	0.146	1.109	0.000	0.284	4.180	0.000	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	82	70	65	0	277	88	0	0
N.S.	1	1.01	0.86	0.80	0.00	3.42	1.09	0.00	0.00
time (sec)	N/A	0.202	0.207	1.163	0.000	0.293	4.693	0.000	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	32	56	24	21	17
N.S.	1	1.00	1.00	0.78	1.39	2.43	1.04	0.91	0.74
time (sec)	N/A	0.136	0.003	1.077	0.265	0.294	0.517	0.347	16.702

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	0	78	0	0	0
N.S.	1	1.00	1.00	0.83	0.00	2.60	0.00	0.00	0.00
time (sec)	N/A	0.170	0.111	1.415	0.000	0.301	0.000	0.000	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	32	0	116	0	0	0
N.S.	1	1.00	1.00	0.86	0.00	3.14	0.00	0.00	0.00
time (sec)	N/A	0.328	0.184	1.115	0.000	0.282	0.000	0.000	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	0	147	0	0	0
N.S.	1	1.00	1.00	0.89	0.00	3.34	0.00	0.00	0.00
time (sec)	N/A	0.525	0.217	1.152	0.000	0.313	0.000	0.000	0.000

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	46	0	182	0	0	0
N.S.	1	1.00	1.00	0.90	0.00	3.57	0.00	0.00	0.00
time (sec)	N/A	0.809	0.248	1.205	0.000	0.298	0.000	0.000	0.000

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	89	70	63	120	55	80	77	54
N.S.	1	1.17	0.92	0.83	1.58	0.72	1.05	1.01	0.71
time (sec)	N/A	0.217	0.103	1.096	0.266	0.273	51.196	0.361	17.764

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	68	65	58	67	50	60	67	44
N.S.	1	1.13	1.08	0.97	1.12	0.83	1.00	1.12	0.73
time (sec)	N/A	0.199	0.079	0.991	0.269	0.268	23.559	0.336	17.645

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	47	46	53	30	43	39	57	30
N.S.	1	1.07	1.05	1.20	0.68	0.98	0.89	1.30	0.68
time (sec)	N/A	0.175	0.063	0.176	0.264	0.269	10.705	0.337	18.111

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	13	16	12	8	37	14
N.S.	1	1.00	1.00	1.44	1.78	1.33	0.89	4.11	1.56
time (sec)	N/A	0.155	0.024	1.017	0.182	0.261	1.120	0.335	17.763

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	27	24	29	30	28	20	58	25
N.S.	1	1.29	1.14	1.38	1.43	1.33	0.95	2.76	1.19
time (sec)	N/A	0.191	0.029	1.007	0.186	0.279	1.723	0.353	18.035

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	40	32	34	38	38	34	68	28
N.S.	1	1.18	0.94	1.00	1.12	1.12	1.00	2.00	0.82
time (sec)	N/A	0.193	0.043	1.008	0.191	0.274	2.310	0.490	18.170

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	20	23	11	28	8	11	18
N.S.	1	1.00	2.22	2.56	1.22	3.11	0.89	1.22	2.00
time (sec)	N/A	0.145	0.005	0.976	0.259	0.280	1.444	0.344	18.245

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	20	12	0	28	10	13	18
N.S.	1	1.00	2.22	1.33	0.00	3.11	1.11	1.44	2.00
time (sec)	N/A	0.144	0.002	1.000	0.000	0.274	1.397	0.381	18.373

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	20	895	16	67	71	16	26
N.S.	1	1.00	1.11	49.72	0.89	3.72	3.94	0.89	1.44
time (sec)	N/A	0.252	0.019	0.094	0.183	0.277	0.735	0.317	19.438

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	29	16	21	32	19	16	10
N.S.	1	1.00	1.81	1.00	1.31	2.00	1.19	1.00	0.62
time (sec)	N/A	0.229	0.018	1.142	0.184	0.277	0.089	0.330	18.249

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	48	40	27	49	36	94	29	26
N.S.	1	1.09	0.91	0.61	1.11	0.82	2.14	0.66	0.59
time (sec)	N/A	0.183	0.109	1.048	0.260	0.264	0.154	0.345	0.091

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	125	57	49	89	53	133	138	124
N.S.	1	1.03	0.47	0.40	0.74	0.44	1.10	1.14	1.02
time (sec)	N/A	0.314	0.031	1.314	0.269	0.330	0.907	0.342	0.053

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	49	42	0	268	83	41	42
N.S.	1	1.00	0.98	0.84	0.00	5.36	1.66	0.82	0.84
time (sec)	N/A	0.202	0.076	1.081	0.000	0.323	2.703	0.342	18.790

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	54	49	42	0	268	0	41	96
N.S.	1	1.08	0.98	0.84	0.00	5.36	0.00	0.82	1.92
time (sec)	N/A	0.197	0.003	1.522	0.000	0.309	0.000	0.352	19.349

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	62	56	0	382	131	55	56
N.S.	1	1.00	0.91	0.82	0.00	5.62	1.93	0.81	0.82
time (sec)	N/A	0.201	0.092	1.169	0.000	0.326	2.925	0.349	18.383

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	55	62	68	0	382	0	52	50
N.S.	1	0.81	0.91	1.00	0.00	5.62	0.00	0.76	0.74
time (sec)	N/A	0.676	0.057	1.148	0.000	0.320	0.000	0.420	18.896

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	37	26	0	39	32	27	39
N.S.	1	1.00	1.09	0.76	0.00	1.15	0.94	0.79	1.15
time (sec)	N/A	0.179	0.057	0.980	0.000	0.557	0.243	0.333	18.750

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	78	50	41	0	49	48	43	27
N.S.	1	1.05	0.68	0.55	0.00	0.66	0.65	0.58	0.36
time (sec)	N/A	0.210	0.080	0.992	0.000	0.577	0.253	0.369	18.170

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	14	13	16	17	16	13
N.S.	1	1.00	0.95	0.74	0.68	0.84	0.89	0.84	0.68
time (sec)	N/A	0.151	0.011	0.038	0.179	0.284	0.083	0.324	0.030

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	66	52	50	67	96	100	55	162
N.S.	1	1.22	0.96	0.93	1.24	1.78	1.85	1.02	3.00
time (sec)	N/A	0.289	0.111	9.070	0.258	0.298	0.447	0.393	0.104

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	32	12	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	2.29	0.86	0.86
time (sec)	N/A	0.181	0.010	0.102	0.176	0.295	0.650	0.340	17.964

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	63	56	33	46	63	54	50	71
N.S.	1	1.03	0.92	0.54	0.75	1.03	0.89	0.82	1.16
time (sec)	N/A	0.264	0.060	0.253	0.259	0.288	0.683	0.343	0.198

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	43	37	31	30	32	39	30	32
N.S.	1	1.16	1.00	0.84	0.81	0.86	1.05	0.81	0.86
time (sec)	N/A	0.231	0.012	0.237	0.261	0.309	0.578	0.329	18.250

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	63	56	32	45	64	53	49	71
N.S.	1	1.03	0.92	0.52	0.74	1.05	0.87	0.80	1.16
time (sec)	N/A	0.243	0.043	0.225	0.270	0.316	0.619	0.358	17.770

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	33	29	22	21	21	36	22	25
N.S.	1	1.06	0.94	0.71	0.68	0.68	1.16	0.71	0.81
time (sec)	N/A	0.224	0.002	0.136	0.174	0.286	0.696	0.347	0.051

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	67	60	36	51	65	53	54	79
N.S.	1	1.03	0.92	0.55	0.78	1.00	0.82	0.83	1.22
time (sec)	N/A	0.264	0.044	0.239	0.263	0.289	0.649	0.472	18.275

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	69	57	42	0	51	48	45	0
N.S.	1	1.11	0.92	0.68	0.00	0.82	0.77	0.73	0.00
time (sec)	N/A	0.196	0.071	0.044	0.000	0.579	0.290	0.327	0.000

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	82	59	55	0	61	58	55	0
N.S.	1	1.09	0.79	0.73	0.00	0.81	0.77	0.73	0.00
time (sec)	N/A	0.243	0.080	0.048	0.000	0.559	0.326	0.401	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	75	65	48	0	59	51	53	0
N.S.	1	1.10	0.96	0.71	0.00	0.87	0.75	0.78	0.00
time (sec)	N/A	0.232	0.081	0.051	0.000	0.606	0.332	0.363	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	85	75	60	0	73	58	67	0
N.S.	1	1.06	0.94	0.75	0.00	0.91	0.72	0.84	0.00
time (sec)	N/A	0.230	0.083	0.056	0.000	0.523	0.411	0.372	0.000

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	116	83	67	0	101	66	88	0
N.S.	1	1.06	0.76	0.61	0.00	0.93	0.61	0.81	0.00
time (sec)	N/A	0.284	0.200	0.056	0.000	1.205	0.471	0.339	0.000

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	49	43	32	0	47	37	33	0
N.S.	1	1.04	0.91	0.68	0.00	1.00	0.79	0.70	0.00
time (sec)	N/A	0.223	0.060	0.050	0.000	0.515	0.325	0.389	0.000

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	72	63	52	49	48	58	60	102
N.S.	1	1.07	0.94	0.78	0.73	0.72	0.87	0.90	1.52
time (sec)	N/A	0.316	0.095	0.280	0.260	0.283	2.300	0.362	17.319

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	74	67	54	51	59	60	62	118
N.S.	1	1.04	0.94	0.76	0.72	0.83	0.85	0.87	1.66
time (sec)	N/A	0.303	0.066	0.285	0.266	0.289	2.277	0.354	0.034

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	54	32	61	46	23	44	46	43
N.S.	1	1.04	0.62	1.17	0.88	0.44	0.85	0.88	0.83
time (sec)	N/A	0.232	0.042	1.009	0.263	0.260	0.207	0.340	18.593

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	18	31	14	0	37	19	25	0
N.S.	1	0.90	1.55	0.70	0.00	1.85	0.95	1.25	0.00
time (sec)	N/A	0.265	0.063	1.085	0.000	0.452	0.408	0.415	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	45	39	30	29	29	36	30	53
N.S.	1	1.05	0.91	0.70	0.67	0.67	0.84	0.70	1.23
time (sec)	N/A	0.272	0.027	1.089	0.261	0.274	0.418	0.314	18.150

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	124	130	90	89	76	105	89	88
N.S.	1	1.07	1.12	0.78	0.77	0.66	0.91	0.77	0.76
time (sec)	N/A	0.339	0.052	0.028	0.180	0.265	1.103	0.319	0.156

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	61	58	54	53	35	71	82	0
N.S.	1	0.73	0.70	0.65	0.64	0.42	0.86	0.99	0.00
time (sec)	N/A	0.256	0.055	0.365	0.196	0.292	0.403	0.391	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	48	62	17	40	39	216	268	0
N.S.	1	0.75	0.97	0.27	0.62	0.61	3.38	4.19	0.00
time (sec)	N/A	0.252	0.055	0.418	0.198	0.283	1.396	0.386	0.000

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	68	86	59	58	57	65	474	0
N.S.	1	0.83	1.05	0.72	0.71	0.70	0.79	5.78	0.00
time (sec)	N/A	0.280	0.064	0.360	0.195	0.294	0.653	0.412	0.000

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	61	58	54	53	35	71	82	0
N.S.	1	0.73	0.70	0.65	0.64	0.42	0.86	0.99	0.00
time (sec)	N/A	0.256	0.003	0.280	0.194	0.277	0.409	0.351	0.000

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	192	168	121	120	76	165	7916	0
N.S.	1	1.01	0.88	0.64	0.63	0.40	0.87	41.66	0.00
time (sec)	N/A	0.614	0.168	0.779	0.191	0.286	0.651	43.804	0.000

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	235	186	154	153	85	202	271	0
N.S.	1	1.01	0.80	0.66	0.66	0.36	0.87	1.16	0.00
time (sec)	N/A	0.689	0.171	0.967	0.199	0.274	0.730	4.877	0.000

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	162	94	107	106	62	139	859	0
N.S.	1	1.01	0.59	0.67	0.66	0.39	0.87	5.37	0.00
time (sec)	N/A	0.552	0.115	0.438	0.194	0.273	0.631	0.426	0.000

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	31	16	0	35	20	25	0
N.S.	1	1.00	1.55	0.80	0.00	1.75	1.00	1.25	0.00
time (sec)	N/A	0.266	0.002	1.015	0.000	0.446	0.394	0.333	0.000

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	52	59	38	0	85	39	49	0
N.S.	1	1.18	1.34	0.86	0.00	1.93	0.89	1.11	0.00
time (sec)	N/A	0.228	0.071	0.051	0.000	0.655	0.411	0.422	0.000

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	57	51	50	58	45	73	61	50
N.S.	1	1.06	0.94	0.93	1.07	0.83	1.35	1.13	0.93
time (sec)	N/A	0.499	0.048	1.012	0.185	0.263	1.278	0.424	18.262

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	77	64	50	0	84	48	44	0
N.S.	1	1.10	0.91	0.71	0.00	1.20	0.69	0.63	0.00
time (sec)	N/A	0.208	0.093	0.045	0.000	0.843	0.333	0.388	0.000

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	23	19	16	15	15	17	15	15
N.S.	1	1.21	1.00	0.84	0.79	0.79	0.89	0.79	0.79
time (sec)	N/A	0.173	0.014	0.998	0.189	0.253	0.075	0.336	18.260

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	67	50	49	49	71	50	47
N.S.	1	1.00	0.87	0.65	0.64	0.64	0.92	0.65	0.61
time (sec)	N/A	0.227	0.031	1.044	0.186	0.267	0.345	0.361	0.051

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	224	76	289	0	92	0	171	255
N.S.	1	2.80	0.95	3.61	0.00	1.15	0.00	2.14	3.19
time (sec)	N/A	0.536	0.130	1.366	0.000	0.287	0.000	0.826	0.099

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	97	81	130	0	87	0	81	0
N.S.	1	1.09	0.91	1.46	0.00	0.98	0.00	0.91	0.00
time (sec)	N/A	0.299	0.098	1.001	0.000	2.554	0.000	0.538	0.000

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	67	55	68	0	62	0	65	0
N.S.	1	1.10	0.90	1.11	0.00	1.02	0.00	1.07	0.00
time (sec)	N/A	0.636	0.115	1.010	0.000	1.614	0.000	0.634	0.000

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	18	7	27	18	26	14	14
N.S.	1	1.00	2.25	0.88	3.38	2.25	3.25	1.75	1.75
time (sec)	N/A	0.129	0.003	1.071	0.183	0.271	0.540	0.424	0.164

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	18	32	27	27	0	22	12
N.S.	1	1.00	2.25	4.00	3.38	3.38	0.00	2.75	1.50
time (sec)	N/A	0.146	0.004	0.123	0.183	0.271	0.000	0.367	0.066

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	51	27	49	28	63	22	26
N.S.	1	1.00	2.32	1.23	2.23	1.27	2.86	1.00	1.18
time (sec)	N/A	0.136	0.036	0.993	0.184	0.267	0.904	0.323	19.424

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	51	45	51	42	0	35	35
N.S.	1	1.00	2.32	2.05	2.32	1.91	0.00	1.59	1.59
time (sec)	N/A	0.147	0.001	0.109	0.187	0.308	0.000	0.341	18.839

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	66	43	20	39	0	42	138
N.S.	1	1.00	1.83	1.19	0.56	1.08	0.00	1.17	3.83
time (sec)	N/A	0.147	0.119	1.046	0.267	0.260	0.000	0.324	19.795

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	66	56	41	36	0	51	41
N.S.	1	1.00	1.83	1.56	1.14	1.00	0.00	1.42	1.14
time (sec)	N/A	0.164	0.005	0.224	0.259	0.266	0.000	0.343	0.058

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	74	78	60	55	46	0	47	473
N.S.	1	1.07	1.13	0.87	0.80	0.67	0.00	0.68	6.86
time (sec)	N/A	0.165	0.099	1.038	0.185	0.281	0.000	0.347	32.745

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	74	78	70	138	64	0	62	119
N.S.	1	1.07	1.13	1.01	2.00	0.93	0.00	0.90	1.72
time (sec)	N/A	0.184	0.001	0.119	0.187	0.296	0.000	0.339	0.050

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	42	33	13	13	0	20	13
N.S.	1	1.00	2.80	2.20	0.87	0.87	0.00	1.33	0.87
time (sec)	N/A	0.148	0.009	0.174	0.276	0.261	0.000	0.329	0.173

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	51	30	15	15	0	16	15
N.S.	1	1.00	2.83	1.67	0.83	0.83	0.00	0.89	0.83
time (sec)	N/A	0.151	0.030	0.959	0.272	0.265	0.000	0.336	17.865

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	63	85	24	24	0	41	36
N.S.	1	1.00	2.62	3.54	1.00	1.00	0.00	1.71	1.50
time (sec)	N/A	0.187	0.053	1.057	0.277	0.278	0.000	0.379	18.265

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	77	80	59	105	0	61	31
N.S.	1	1.00	1.88	1.95	1.44	2.56	0.00	1.49	0.76
time (sec)	N/A	0.200	0.019	1.129	0.267	0.251	0.000	0.378	0.214

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	48	52	45	37	28	0	36	37
N.S.	1	1.50	1.62	1.41	1.16	0.88	0.00	1.12	1.16
time (sec)	N/A	0.156	0.008	0.181	0.271	0.263	0.000	0.339	0.033

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	57	64	39	43	32	0	29	43
N.S.	1	1.50	1.68	1.03	1.13	0.84	0.00	0.76	1.13
time (sec)	N/A	0.160	0.013	0.177	0.273	0.285	0.000	0.343	0.039

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	58	67	61	49	38	0	36	49
N.S.	1	1.38	1.60	1.45	1.17	0.90	0.00	0.86	1.17
time (sec)	N/A	0.161	0.032	0.154	0.271	0.289	0.000	0.326	17.923

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	60	67	62	70	58	0	40	51
N.S.	1	1.46	1.63	1.51	1.71	1.41	0.00	0.98	1.24
time (sec)	N/A	0.166	0.064	0.136	0.190	0.271	0.000	0.339	0.051

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	97	97	152	118	180	0	109	90
N.S.	1	1.28	1.28	2.00	1.55	2.37	0.00	1.43	1.18
time (sec)	N/A	0.187	0.053	0.091	0.268	0.296	0.000	0.380	0.275

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	75	76	80	54	0	74	57
N.S.	1	1.00	1.53	1.55	1.63	1.10	0.00	1.51	1.16
time (sec)	N/A	0.164	0.090	0.229	0.260	0.273	0.000	0.385	18.228

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	88	84	53	46	0	114	74
N.S.	1	1.00	1.91	1.83	1.15	1.00	0.00	2.48	1.61
time (sec)	N/A	0.179	0.172	0.243	0.269	0.292	0.000	0.338	0.159

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	32	19	17	27	17	0	17	17
N.S.	1	1.60	0.95	0.85	1.35	0.85	0.00	0.85	0.85
time (sec)	N/A	0.173	0.040	1.007	0.187	0.311	0.000	0.300	0.060

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	19	17	16	17	0	17	0
N.S.	1	1.00	1.06	0.94	0.89	0.94	0.00	0.94	0.00
time (sec)	N/A	0.177	0.005	0.985	0.197	0.255	0.000	0.312	0.000

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	59	68	79	103	83	0	129	62
N.S.	1	1.09	1.26	1.46	1.91	1.54	0.00	2.39	1.15
time (sec)	N/A	0.203	0.175	1.018	0.271	0.315	0.000	0.383	18.001

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	19	18	0	17	0	23	15
N.S.	1	1.00	1.73	1.64	0.00	1.55	0.00	2.09	1.36
time (sec)	N/A	0.134	0.025	1.078	0.000	0.268	0.000	0.330	18.254

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	41	40	0	34	0	0	0
N.S.	1	1.00	1.41	1.38	0.00	1.17	0.00	0.00	0.00
time (sec)	N/A	0.173	1.756	1.101	0.000	0.290	0.000	0.000	0.000

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	197	111	359	0	372	0	287	0
N.S.	1	1.09	0.62	1.99	0.00	2.07	0.00	1.59	0.00
time (sec)	N/A	0.481	0.271	1.259	0.000	0.285	0.000	0.394	0.000

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	165	94	104	0	171	0	350	0
N.S.	1	0.96	0.55	0.60	0.00	0.99	0.00	2.03	0.00
time (sec)	N/A	0.349	0.367	0.291	0.000	0.278	0.000	0.374	0.000

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F(-1)	A	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	305	115	129	0	223	0	452	0
N.S.	1	0.99	0.37	0.42	0.00	0.73	0.00	1.47	0.00
time (sec)	N/A	0.538	0.438	0.332	0.000	0.268	0.000	0.380	0.000

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	67	78	71	0	77	0	81	0
N.S.	1	1.03	1.20	1.09	0.00	1.18	0.00	1.25	0.00
time (sec)	N/A	0.215	0.187	0.166	0.000	0.302	0.000	0.340	0.000

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	85	69	61	0	97	0	143	0
N.S.	1	1.02	0.83	0.73	0.00	1.17	0.00	1.72	0.00
time (sec)	N/A	0.212	0.300	0.125	0.000	0.279	0.000	0.337	0.000

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	105	83	69	0	129	0	184	0
N.S.	1	1.04	0.82	0.68	0.00	1.28	0.00	1.82	0.00
time (sec)	N/A	0.216	0.336	0.130	0.000	0.255	0.000	0.338	0.000

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	124	81	370	0	187	0	197	0
N.S.	1	1.15	0.75	3.43	0.00	1.73	0.00	1.82	0.00
time (sec)	N/A	0.324	0.225	1.039	0.000	0.278	0.000	0.329	0.000

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	100	84	110	0	121	0	263	0
N.S.	1	1.15	0.97	1.26	0.00	1.39	0.00	3.02	0.00
time (sec)	N/A	0.247	0.334	0.341	0.000	0.283	0.000	0.335	0.000

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	163	109	130	0	171	0	367	0
N.S.	1	1.09	0.73	0.87	0.00	1.15	0.00	2.46	0.00
time (sec)	N/A	0.304	0.392	0.573	0.000	0.262	0.000	0.328	0.000

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	74	42	35	59	58	182	58	38
N.S.	1	1.76	1.00	0.83	1.40	1.38	4.33	1.38	0.90
time (sec)	N/A	0.439	0.081	2.983	0.233	0.276	0.161	0.308	0.185

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	74	42	35	59	58	182	58	51
N.S.	1	1.76	1.00	0.83	1.40	1.38	4.33	1.38	1.21
time (sec)	N/A	0.450	0.071	1.561	0.236	0.251	0.766	0.320	18.468

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	102	108	278	958	0	90	0	0	0
N.S.	1	1.06	2.73	9.39	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.268	22.479	2.635	0.000	0.082	0.000	0.000	0.000

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	62	66	256	936	0	69	0	0	0
N.S.	1	1.06	4.13	15.10	0.00	1.11	0.00	0.00	0.00
time (sec)	N/A	0.222	22.424	1.999	0.000	0.091	0.000	0.000	0.000

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	156	200	0	16	0	0	0
N.S.	1	1.00	9.18	11.76	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.165	32.153	1.365	0.000	0.071	0.000	0.000	0.000

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	261	932	0	119	0	0	0
N.S.	1	1.00	3.58	12.77	0.00	1.63	0.00	0.00	0.00
time (sec)	N/A	0.228	22.672	2.033	0.000	0.084	0.000	0.000	0.000

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	109	115	298	972	0	195	0	0	0
N.S.	1	1.06	2.73	8.92	0.00	1.79	0.00	0.00	0.00
time (sec)	N/A	0.266	22.849	2.067	0.000	0.081	0.000	0.000	0.000

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	102	108	278	954	0	90	0	0	0
N.S.	1	1.06	2.73	9.35	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.255	24.415	2.223	0.000	0.105	0.000	0.000	0.000

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	62	66	256	932	0	69	0	0	0
N.S.	1	1.06	4.13	15.03	0.00	1.11	0.00	0.00	0.00
time (sec)	N/A	0.217	18.905	1.946	0.000	0.097	0.000	0.000	0.000

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	100	200	0	16	0	0	0
N.S.	1	1.00	5.88	11.76	0.00	0.94	0.00	0.00	0.00
time (sec)	N/A	0.156	33.863	1.416	0.000	0.085	0.000	0.000	0.000

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	298	963	0	119	0	0	0
N.S.	1	1.00	4.08	13.19	0.00	1.63	0.00	0.00	0.00
time (sec)	N/A	0.217	19.717	1.446	0.000	0.088	0.000	0.000	0.000

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	109	115	327	1039	0	195	0	0	0
N.S.	1	1.06	3.00	9.53	0.00	1.79	0.00	0.00	0.00
time (sec)	N/A	0.263	22.028	1.489	0.000	0.102	0.000	0.000	0.000

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	730	843	10468	5229	0	0	0	0	0
N.S.	1	1.15	14.34	7.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.011	16.139	6.079	0.000	0.000	0.000	0.000	0.000

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	622	729	5218	4865	0	0	0	0	0
N.S.	1	1.17	8.39	7.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.652	16.070	5.159	0.000	0.000	0.000	0.000	0.000

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	258	822	1056	0	0	0	0	0
N.S.	1	1.14	3.62	4.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.283	11.488	1.042	0.000	0.000	0.000	0.000	0.000

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	674	777	5276	5024	0	0	0	0	0
N.S.	1	1.15	7.83	7.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.702	16.124	1.033	0.000	0.000	0.000	0.000	0.000

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	663	814	7543	7887	0	0	0	0	0
N.S.	1	1.23	11.38	11.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.768	13.922	7.218	0.000	0.000	0.000	0.000	0.000

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	279	1065	1704	0	0	0	0	0
N.S.	1	1.19	4.53	7.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.299	11.412	1.234	0.000	0.000	0.000	0.000	0.000

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	748	882	7629	8103	0	0	0	0	0
N.S.	1	1.18	10.20	10.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.829	16.144	1.373	0.000	0.000	0.000	0.000	0.000

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	452	539	6287	2655	0	0	0	0	0
N.S.	1	1.19	13.91	5.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.661	16.104	5.237	0.000	0.000	0.000	0.000	0.000

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	397	481	3470	2519	0	0	0	0	0
N.S.	1	1.21	8.74	6.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.521	16.056	4.196	0.000	0.000	0.000	0.000	0.000

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	540	530	0	0	0	0	0
N.S.	1	1.00	3.75	3.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.245	11.033	0.878	0.000	0.000	0.000	0.000	0.000

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	437	509	3526	2601	0	0	0	0	0
N.S.	1	1.16	8.07	5.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.553	16.075	0.928	0.000	0.000	0.000	0.000	0.000

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	517	597	6386	2757	0	0	0	0	0
N.S.	1	1.15	12.35	5.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.647	16.135	0.959	0.000	0.000	0.000	0.000	0.000

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	558	635	7235	2694	0	0	0	0	0
N.S.	1	1.14	12.97	4.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.842	17.097	4.905	0.000	0.000	0.000	0.000	0.000

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	466	543	4389	2551	0	0	0	0	0
N.S.	1	1.17	9.42	5.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.750	14.891	4.755	0.000	0.000	0.000	0.000	0.000

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	173	813	788	0	0	0	0	0
N.S.	1	0.97	4.54	4.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.390	12.964	1.037	0.000	0.000	0.000	0.000	0.000

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	474	540	3593	2616	0	0	0	0	0
N.S.	1	1.14	7.58	5.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.691	14.360	1.122	0.000	0.000	0.000	0.000	0.000

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	591	664	6452	2777	0	0	0	0	0
N.S.	1	1.12	10.92	4.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.834	17.030	1.012	0.000	0.000	0.000	0.000	0.000

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	585	651	8500	2733	0	0	0	0	0
N.S.	1	1.11	14.53	4.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.917	17.053	4.146	0.000	0.000	0.000	0.000	0.000

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	485	555	5647	2582	0	0	0	0	0
N.S.	1	1.14	11.64	5.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.797	13.837	4.107	0.000	0.000	0.000	0.000	0.000

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	388	471	1145	1147	0	0	0	0	0
N.S.	1	1.21	2.95	2.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.680	15.188	1.437	0.000	0.000	0.000	0.000	0.000

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	311	389	2941	2607	0	0	0	0	0
N.S.	1	1.25	9.46	8.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.590	14.505	1.277	0.000	0.000	0.000	0.000	0.000

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD
size	582	653	5812	2780	0	0	0	0	0
N.S.	1	1.12	9.99	4.78	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.889	17.139	1.511	0.000	0.000	0.000	0.000	0.000

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	129	168	927	965	0	0	0	0	0
N.S.	1	1.30	7.19	7.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.401	10.702	1.128	0.000	0.000	0.000	0.000	0.000

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	431	553	4865	4426	0	0	0	0	0
N.S.	1	1.28	11.29	10.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.755	16.151	2.704	0.000	0.000	0.000	0.000	0.000

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	108	115	249	961	0	0	0	0	0
N.S.	1	1.06	2.31	8.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.372	10.382	0.834	0.000	0.000	0.000	0.000	0.000

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	367	400	602	2564	0	0	0	0	0
N.S.	1	1.09	1.64	6.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.703	12.702	0.811	0.000	0.000	0.000	0.000	0.000

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	126	163	1148	1180	0	0	0	0	0
N.S.	1	1.29	9.11	9.37	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.423	10.615	1.158	0.000	0.000	0.000	0.000	0.000

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	434	529	6019	5421	0	0	0	0	0
N.S.	1	1.22	13.87	12.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.759	16.102	2.992	0.000	0.000	0.000	0.000	0.000

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	577	636	6084	5441	0	0	0	0	0
N.S.	1	1.10	10.54	9.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.017	16.098	2.841	0.000	0.000	0.000	0.000	0.000

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	130	158	826	1180	0	0	0	0	0
N.S.	1	1.22	6.35	9.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.400	10.174	0.990	0.000	0.000	0.000	0.000	0.000

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	444	526	4974	5421	0	0	0	0	0
N.S.	1	1.18	11.20	12.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.782	15.989	1.343	0.000	0.000	0.000	0.000	0.000

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	404	76	57	81	0	151	158
N.S.	1	1.00	7.21	1.36	1.02	1.45	0.00	2.70	2.82
time (sec)	N/A	0.362	1.665	0.627	0.263	0.269	0.000	0.487	25.149

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	68	56	0	84	63	89	55
N.S.	1	1.00	1.05	0.86	0.00	1.29	0.97	1.37	0.85
time (sec)	N/A	0.348	0.222	0.213	0.000	0.315	2.633	0.387	0.044

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	101	66	58	0	78	0	80	52
N.S.	1	1.91	1.25	1.09	0.00	1.47	0.00	1.51	0.98
time (sec)	N/A	0.397	0.167	0.139	0.000	0.256	0.000	0.367	0.044

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	12	12	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.86	0.86	0.86
time (sec)	N/A	0.272	0.007	1.100	0.263	0.263	0.064	0.313	20.379

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	66	56	44	45	43	60	43	43
N.S.	1	1.08	0.92	0.72	0.74	0.70	0.98	0.70	0.70
time (sec)	N/A	0.292	0.063	0.086	0.178	0.264	0.648	0.329	21.011

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	121	103	85	85	91	116	84	84
N.S.	1	1.06	0.90	0.75	0.75	0.80	1.02	0.74	0.74
time (sec)	N/A	0.332	0.144	0.122	0.178	0.266	0.916	0.332	0.090

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	60	111	102	0	180	73	92	46
N.S.	1	1.03	1.91	1.76	0.00	3.10	1.26	1.59	0.79
time (sec)	N/A	0.211	0.335	0.115	0.000	0.266	0.956	0.424	21.190

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	60	111	102	0	180	73	92	62
N.S.	1	1.03	1.91	1.76	0.00	3.10	1.26	1.59	1.07
time (sec)	N/A	0.280	0.003	0.069	0.000	0.272	2.218	0.426	21.204

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	55	39	0	125	0	123	67
N.S.	1	1.00	1.04	0.74	0.00	2.36	0.00	2.32	1.26
time (sec)	N/A	0.188	0.510	1.271	0.000	0.263	0.000	0.336	20.636

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	25	18	17	22	27	18	23
N.S.	1	1.00	1.09	0.78	0.74	0.96	1.17	0.78	1.00
time (sec)	N/A	0.161	0.017	1.031	0.183	0.267	0.078	0.315	0.031

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	25	26	17	25	0	54	35
N.S.	1	1.00	1.09	1.13	0.74	1.09	0.00	2.35	1.52
time (sec)	N/A	0.162	0.001	1.048	0.259	0.259	0.000	0.467	21.379

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	54	58	36	37	0	39	180
N.S.	1	1.00	1.64	1.76	1.09	1.12	0.00	1.18	5.45
time (sec)	N/A	0.168	0.061	1.054	0.183	0.300	0.000	0.340	25.922

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	57	35	2	2	0	31	0
N.S.	1	1.00	1.27	0.78	0.04	0.04	0.00	0.69	0.00
time (sec)	N/A	0.316	0.804	1.064	0.264	0.276	0.000	0.324	0.000

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	57	35	2	2	0	31	0
N.S.	1	1.00	1.27	0.78	0.04	0.04	0.00	0.69	0.00
time (sec)	N/A	0.243	0.002	1.056	0.268	0.319	0.000	0.301	0.000

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	57	35	2	2	0	31	0
N.S.	1	1.00	1.27	0.78	0.04	0.04	0.00	0.69	0.00
time (sec)	N/A	0.284	0.001	1.041	0.270	0.273	0.000	0.313	0.000

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	57	20	2	2	0	12	0
N.S.	1	1.00	1.27	0.44	0.04	0.04	0.00	0.27	0.00
time (sec)	N/A	0.332	3.792	1.073	0.263	0.276	0.000	0.308	0.000

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	39	41	0	6	0	6	0
N.S.	1	1.00	0.75	0.79	0.00	0.12	0.00	0.12	0.00
time (sec)	N/A	0.356	1.604	1.064	0.000	0.277	0.000	0.336	0.000

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	39	21	6	6	0	6	0
N.S.	1	1.00	0.75	0.40	0.12	0.12	0.00	0.12	0.00
time (sec)	N/A	0.356	1.547	1.070	0.265	0.264	0.000	0.325	0.000

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	67	40	0	8	0	8	0
N.S.	1	1.00	0.99	0.59	0.00	0.12	0.00	0.12	0.00
time (sec)	N/A	0.422	1.646	0.058	0.000	0.269	0.000	0.310	0.000

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	39	29	27	25	0	54	19
N.S.	1	1.00	1.26	0.94	0.87	0.81	0.00	1.74	0.61
time (sec)	N/A	0.162	0.278	1.112	0.259	0.273	0.000	0.313	0.074

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	23	29	25	25	0	54	19
N.S.	1	1.00	0.74	0.94	0.81	0.81	0.00	1.74	0.61
time (sec)	N/A	0.408	0.247	1.187	0.216	0.265	0.000	0.338	0.037

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	23	29	0	25	0	68	19
N.S.	1	1.00	0.74	0.94	0.00	0.81	0.00	2.19	0.61
time (sec)	N/A	0.292	0.273	0.092	0.000	0.283	0.000	0.369	0.068

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	23	29	0	25	0	68	19
N.S.	1	1.00	0.74	0.94	0.00	0.81	0.00	2.19	0.61
time (sec)	N/A	0.287	0.269	0.091	0.000	0.273	0.000	0.310	0.052

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	23	29	0	25	0	68	19
N.S.	1	1.00	0.74	0.94	0.00	0.81	0.00	2.19	0.61
time (sec)	N/A	0.677	0.278	1.092	0.000	0.290	0.000	0.379	21.082

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	50	38	58	85	44	172	0	0
N.S.	1	1.16	0.88	1.35	1.98	1.02	4.00	0.00	0.00
time (sec)	N/A	0.241	0.133	1.206	0.182	0.304	1.218	0.000	0.000

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	53	45	0	44	0	57	31
N.S.	1	1.00	1.26	1.07	0.00	1.05	0.00	1.36	0.74
time (sec)	N/A	0.294	0.101	1.113	0.000	0.287	0.000	0.340	20.710

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	50	57	0	80	0	133	60
N.S.	1	1.00	0.77	0.88	0.00	1.23	0.00	2.05	0.92
time (sec)	N/A	0.312	0.123	1.117	0.000	0.262	0.000	0.332	0.387

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	206	179	729	0	1174	0	558	0
N.S.	1	1.06	0.92	3.74	0.00	6.02	0.00	2.86	0.00
time (sec)	N/A	0.405	0.933	1.389	0.000	0.299	0.000	0.343	0.000

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	21	34	52	37	34	20
N.S.	1	1.00	1.00	0.75	1.21	1.86	1.32	1.21	0.71
time (sec)	N/A	0.154	0.029	1.543	0.264	0.256	5.908	0.295	21.788

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	21	0	52	0	34	0
N.S.	1	1.00	1.00	0.75	0.00	1.86	0.00	1.21	0.00
time (sec)	N/A	0.229	0.001	1.335	0.000	0.258	0.000	0.307	0.000

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	18	17	18	18
N.S.	1	1.00	1.00	0.86	0.82	0.82	0.77	0.82	0.82
time (sec)	N/A	0.144	0.005	1.122	0.182	0.256	0.065	0.435	21.069

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	18	17	18	18
N.S.	1	1.00	1.00	0.86	0.82	0.82	0.77	0.82	0.82
time (sec)	N/A	0.247	0.001	1.142	0.188	0.273	0.212	0.471	0.032

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	0	11	36	11	11
N.S.	1	1.00	1.00	0.71	0.00	0.65	2.12	0.65	0.65
time (sec)	N/A	0.166	0.031	1.247	0.000	0.281	0.226	0.297	22.172

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	17	17	17	125	0	11	0	11	0
N.S.	1	1.00	1.00	7.35	0.00	0.65	0.00	0.65	0.00
time (sec)	N/A	0.213	0.029	1.148	0.000	0.312	0.000	0.303	0.000

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	23	7	6	19	7	19	6
N.S.	1	1.00	2.30	0.70	0.60	1.90	0.70	1.90	0.60
time (sec)	N/A	0.123	0.022	1.548	0.272	0.287	0.070	0.354	0.011

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	23	34	6	25	49	12	32
N.S.	1	1.00	2.30	3.40	0.60	2.50	4.90	1.20	3.20
time (sec)	N/A	0.122	0.001	1.230	0.275	0.291	0.544	0.330	0.147

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	23	7	6	19	7	19	6
N.S.	1	1.00	2.30	0.70	0.60	1.90	0.70	1.90	0.60
time (sec)	N/A	0.136	0.001	1.091	0.280	0.253	0.519	0.309	0.014

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	23	7	8	29	7	26	6
N.S.	1	1.00	1.92	0.58	0.67	2.42	0.58	2.17	0.50
time (sec)	N/A	0.135	0.050	1.211	0.265	0.259	0.265	0.327	21.282

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	44	31	8	29	39	13	30
N.S.	1	1.00	3.67	2.58	0.67	2.42	3.25	1.08	2.50
time (sec)	N/A	0.138	0.024	1.205	0.262	0.261	0.530	0.327	0.126

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	44	7	8	29	7	26	6
N.S.	1	1.00	3.67	0.58	0.67	2.42	0.58	2.17	0.50
time (sec)	N/A	0.150	0.001	1.204	0.260	0.283	0.569	0.307	21.092

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	12	23	5	8	29	3	24	4
N.S.	1	3.00	5.75	1.25	2.00	7.25	0.75	6.00	1.00
time (sec)	N/A	0.134	0.046	1.264	0.269	0.266	0.271	0.444	21.150

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	B	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	12	44	29	8	29	39	13	33
N.S.	1	3.00	11.00	7.25	2.00	7.25	9.75	3.25	8.25
time (sec)	N/A	0.139	0.024	1.205	0.265	0.280	0.540	0.428	0.090

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	12	44	5	8	29	3	24	4
N.S.	1	3.00	11.00	1.25	2.00	7.25	0.75	6.00	1.00
time (sec)	N/A	0.147	0.001	1.176	0.279	0.304	0.574	0.328	0.064

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	7	8	7	7
N.S.	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	0.64
time (sec)	N/A	0.130	0.001	0.035	0.186	0.249	0.020	0.305	0.027

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	7	8	7	7
N.S.	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	0.64
time (sec)	N/A	0.135	0.000	1.104	0.195	0.283	0.060	0.298	0.027

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	44	24	0	26	7	10	0
N.S.	1	1.00	1.63	0.89	0.00	0.96	0.26	0.37	0.00
time (sec)	N/A	0.145	0.023	1.099	0.000	0.261	0.770	0.360	0.000

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	44	30	0	26	0	10	0
N.S.	1	1.00	1.63	1.11	0.00	0.96	0.00	0.37	0.00
time (sec)	N/A	0.144	0.001	0.160	0.000	0.271	0.000	0.316	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	56	28	14	14	15	26	0
N.S.	1	1.00	2.24	1.12	0.56	0.56	0.60	1.04	0.00
time (sec)	N/A	0.142	0.014	1.108	0.183	0.275	0.656	0.311	0.000

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	29	56	28	0	14	0	21	0
N.S.	1	1.16	2.24	1.12	0.00	0.56	0.00	0.84	0.00
time (sec)	N/A	0.149	0.001	0.114	0.000	0.286	0.000	0.335	0.000

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	9	9
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.82	0.82
time (sec)	N/A	0.118	0.001	1.121	0.184	0.273	0.023	0.313	21.178

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	20	17	12	16	0	15	16
N.S.	1	1.00	1.82	1.55	1.09	1.45	0.00	1.36	1.45
time (sec)	N/A	0.123	0.043	1.108	0.198	0.250	0.000	0.327	21.776

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	7	7	7	7
N.S.	1	1.00	1.00	0.89	0.78	0.78	0.78	0.78	0.78
time (sec)	N/A	0.115	0.001	1.082	0.187	0.249	0.019	0.397	21.409

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	22	22	7	23	0	13	18
N.S.	1	1.00	2.44	2.44	0.78	2.56	0.00	1.44	2.00
time (sec)	N/A	0.118	0.041	1.138	0.195	0.254	0.000	0.316	21.496

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	12	10	9	9
N.S.	1	1.00	1.00	0.77	0.69	0.92	0.77	0.69	0.69
time (sec)	N/A	0.117	0.001	1.199	0.186	0.242	0.022	0.337	0.095

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	22	20	12	19	0	15	20
N.S.	1	1.00	1.69	1.54	0.92	1.46	0.00	1.15	1.54
time (sec)	N/A	0.122	0.036	1.198	0.194	0.240	0.000	0.334	21.238

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	7	8	7	7
N.S.	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	0.64
time (sec)	N/A	0.115	0.001	1.165	0.191	0.238	0.024	0.335	21.306

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	24	22	7	26	0	13	22
N.S.	1	1.00	2.18	2.00	0.64	2.36	0.00	1.18	2.00
time (sec)	N/A	0.120	0.036	1.147	0.195	0.254	0.000	0.297	21.446

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	48	67	41	52	100	39	172
N.S.	1	1.00	1.37	1.91	1.17	1.49	2.86	1.11	4.91
time (sec)	N/A	0.141	0.075	1.144	0.276	0.274	0.950	0.298	24.154

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	49	86	0	96	0	69	0
N.S.	1	1.00	1.40	2.46	0.00	2.74	0.00	1.97	0.00
time (sec)	N/A	0.154	1.678	1.296	0.000	0.279	0.000	0.374	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	69	70	53	74	0	163	0
N.S.	1	1.00	1.60	1.63	1.23	1.72	0.00	3.79	0.00
time (sec)	N/A	0.167	0.168	1.158	0.269	0.255	0.000	0.383	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	59	32	53	63	0	44	37
N.S.	1	1.00	1.69	0.91	1.51	1.80	0.00	1.26	1.06
time (sec)	N/A	0.178	0.112	1.197	0.280	0.245	0.000	0.384	25.777

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	83	134	65	93	0	196	0
N.S.	1	1.00	1.63	2.63	1.27	1.82	0.00	3.84	0.00
time (sec)	N/A	0.185	0.214	1.181	0.270	0.280	0.000	0.435	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	71	99	65	82	0	87	82
N.S.	1	1.00	1.58	2.20	1.44	1.82	0.00	1.93	1.82
time (sec)	N/A	0.200	1.344	1.233	0.275	0.254	0.000	0.340	0.105

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	20	3	2	18	2	17	2
N.S.	1	1.00	10.00	1.50	1.00	9.00	1.00	8.50	1.00
time (sec)	N/A	0.119	0.001	1.148	0.294	0.253	0.062	0.346	0.007

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	32	29	0	27	0	0	0
N.S.	1	1.00	16.00	14.50	0.00	13.50	0.00	0.00	0.00
time (sec)	N/A	0.132	0.542	1.231	0.000	0.250	0.000	0.000	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	16	3	2	14	2	25	2
N.S.	1	1.00	8.00	1.50	1.00	7.00	1.00	12.50	1.00
time (sec)	N/A	0.120	0.011	1.240	0.271	0.270	0.062	0.335	0.029

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	42	29	0	81	0	0	0
N.S.	1	1.00	21.00	14.50	0.00	40.50	0.00	0.00	0.00
time (sec)	N/A	0.133	0.511	1.270	0.000	0.255	0.000	0.000	0.000

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	37	18	17	31	15	17	17
N.S.	1	1.00	1.61	0.78	0.74	1.35	0.65	0.74	0.74
time (sec)	N/A	0.135	0.035	1.269	0.271	0.241	0.079	0.329	0.033

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	50	42	0	60	0	0	0
N.S.	1	1.00	2.17	1.83	0.00	2.61	0.00	0.00	0.00
time (sec)	N/A	0.149	0.610	1.169	0.000	0.246	0.000	0.000	0.000

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	33	16	15	25	15	25	15
N.S.	1	1.00	1.57	0.76	0.71	1.19	0.71	1.19	0.71
time (sec)	N/A	0.129	0.019	1.278	0.280	0.263	0.081	0.304	0.032

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	70	47	0	120	0	0	0
N.S.	1	1.00	3.33	2.24	0.00	5.71	0.00	0.00	0.00
time (sec)	N/A	0.142	0.616	1.118	0.000	0.250	0.000	0.000	0.000

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	49	57	53	0	0	0	0	0	54
N.S.	1	1.16	1.08	0.00	0.00	0.00	0.00	0.00	1.10
time (sec)	N/A	0.182	0.021	0.000	0.000	0.000	0.000	0.000	23.468

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	33	21	0	30	65	30	20
N.S.	1	1.00	1.18	0.75	0.00	1.07	2.32	1.07	0.71
time (sec)	N/A	0.153	0.022	1.140	0.000	0.254	0.189	0.309	0.036

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	39	197	0	84	17	140	105
N.S.	1	1.00	1.05	5.32	0.00	2.27	0.46	3.78	2.84
time (sec)	N/A	0.222	0.040	0.298	0.000	0.243	0.070	0.328	0.153

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	68	33	0	90	27	88	57
N.S.	1	1.00	1.70	0.82	0.00	2.25	0.68	2.20	1.42
time (sec)	N/A	0.224	0.093	0.230	0.000	0.262	0.089	0.384	0.189

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	79	45	94	67	0	117	86
N.S.	1	1.00	1.46	0.83	1.74	1.24	0.00	2.17	1.59
time (sec)	N/A	0.306	0.095	1.125	0.275	0.261	0.000	0.459	21.957

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	78	45	0	67	0	117	86
N.S.	1	1.00	1.30	0.75	0.00	1.12	0.00	1.95	1.43
time (sec)	N/A	0.437	0.079	1.131	0.000	0.240	0.000	0.377	21.712

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	45	35	0	66	0	50	0
N.S.	1	1.00	0.88	0.69	0.00	1.29	0.00	0.98	0.00
time (sec)	N/A	0.288	0.113	0.099	0.000	0.245	0.000	0.314	0.000

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	45	35	54	66	0	50	0
N.S.	1	1.00	0.88	0.69	1.06	1.29	0.00	0.98	0.00
time (sec)	N/A	0.278	0.095	1.193	0.273	0.243	0.000	0.325	0.000

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	45	51	54	64	0	109	56
N.S.	1	1.00	0.88	1.00	1.06	1.25	0.00	2.14	1.10
time (sec)	N/A	0.270	0.003	1.121	0.270	0.255	0.000	0.345	24.313

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	58	69	51	0	64	0	109	56
N.S.	1	1.07	1.28	0.94	0.00	1.19	0.00	2.02	1.04
time (sec)	N/A	0.289	0.105	0.036	0.000	0.260	0.000	0.376	0.063

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	25	23	20	0	20	0	0	20
N.S.	1	0.93	0.85	0.74	0.00	0.74	0.00	0.00	0.74
time (sec)	N/A	0.163	0.013	0.085	0.000	0.247	0.000	0.000	23.576

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	12	13	0	16	0	0	138
N.S.	1	1.00	0.86	0.93	0.00	1.14	0.00	0.00	9.86
time (sec)	N/A	0.205	0.158	1.172	0.000	0.263	0.000	0.000	22.464

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	0	16	0	0	10
N.S.	1	1.00	1.00	0.92	0.00	1.33	0.00	0.00	0.83
time (sec)	N/A	0.198	0.001	1.164	0.000	0.256	0.000	0.000	0.055

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	29	31	0	30	0	0	48
N.S.	1	1.00	0.81	0.86	0.00	0.83	0.00	0.00	1.33
time (sec)	N/A	0.295	0.008	1.158	0.000	0.260	0.000	0.000	22.126

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	29	31	0	30	0	0	43
N.S.	1	1.00	0.88	0.94	0.00	0.91	0.00	0.00	1.30
time (sec)	N/A	0.353	0.002	1.070	0.000	0.266	0.000	0.000	22.947

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	103	0	208	0	92	0
N.S.	1	1.00	1.00	1.47	0.00	2.97	0.00	1.31	0.00
time (sec)	N/A	0.199	0.348	1.108	0.000	0.256	0.000	0.349	0.000

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	62	56	60	0	38	0	46	0
N.S.	1	0.75	0.67	0.72	0.00	0.46	0.00	0.55	0.00
time (sec)	N/A	0.291	0.033	1.695	0.000	0.257	0.000	0.325	0.000

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	44	63	0	36	0	18	0
N.S.	1	1.00	0.94	1.34	0.00	0.77	0.00	0.38	0.00
time (sec)	N/A	0.447	5.268	1.115	0.000	0.289	0.000	0.304	0.000

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	82	70	75	0	74	0	39	0
N.S.	1	0.67	0.57	0.61	0.00	0.60	0.00	0.32	0.00
time (sec)	N/A	0.460	0.031	1.056	0.000	0.251	0.000	0.336	0.000

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	115	76	62	0	117	0	86	0
N.S.	1	0.86	0.57	0.47	0.00	0.88	0.00	0.65	0.00
time (sec)	N/A	0.291	0.255	0.331	0.000	0.258	0.000	0.323	0.000

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	76	56	49	0	83	0	67	0
N.S.	1	0.84	0.62	0.54	0.00	0.92	0.00	0.74	0.00
time (sec)	N/A	0.257	0.137	0.138	0.000	0.255	0.000	0.323	0.000

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	43	53	42	0	75	0	49	0
N.S.	1	0.70	0.87	0.69	0.00	1.23	0.00	0.80	0.00
time (sec)	N/A	0.220	0.063	0.126	0.000	0.266	0.000	0.306	0.000

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	71	82	79	0	142	0	88	0
N.S.	1	0.65	0.75	0.72	0.00	1.30	0.00	0.81	0.00
time (sec)	N/A	0.261	0.106	0.283	0.000	0.265	0.000	0.376	0.000

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	103	108	109	0	205	0	170	0
N.S.	1	0.72	0.75	0.76	0.00	1.42	0.00	1.18	0.00
time (sec)	N/A	0.292	0.266	0.293	0.000	0.288	0.000	0.399	0.000

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	25	25	0	31	0	30	23
N.S.	1	1.00	0.89	0.89	0.00	1.11	0.00	1.07	0.82
time (sec)	N/A	0.279	0.084	1.066	0.000	0.250	0.000	0.300	0.069

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	14	14	14	14
N.S.	1	1.00	1.14	0.86	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.418	0.136	0.042	0.215	0.251	0.287	0.333	20.753

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	24	14	14	14
N.S.	1	1.00	1.14	0.86	1.00	1.71	1.00	1.00	1.00
time (sec)	N/A	0.388	0.194	0.039	0.225	0.252	0.306	0.329	20.188

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	17	19	13	15	15	15	15	15
N.S.	1	1.00	1.12	0.76	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.327	0.004	0.040	0.219	0.254	1.240	0.327	20.414

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	17	17	19	13	15	25	15	15	15
N.S.	1	1.00	1.12	0.76	0.88	1.47	0.88	0.88	0.88
time (sec)	N/A	0.295	0.004	0.037	0.232	0.250	0.529	0.308	20.177

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20
time (sec)	N/A	0.210	0.008	0.012	0.203	0.232	0.182	0.324	20.252

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	12	14	14
N.S.	1	1.00	1.17	1.00	1.17	1.17	1.00	1.17	1.17
time (sec)	N/A	0.212	0.007	0.013	0.178	0.228	0.191	0.302	20.163

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20
time (sec)	N/A	0.173	0.007	0.010	0.184	0.230	0.204	0.325	20.477

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	12	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.14	0.86	1.14	1.14
time (sec)	N/A	0.180	0.008	0.010	0.177	0.243	0.238	0.323	21.677

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	25	12	14	14
N.S.	1	1.00	1.17	1.00	1.17	2.08	1.00	1.17	1.17
time (sec)	N/A	0.173	0.010	0.010	0.176	0.238	0.237	0.306	21.461

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	27	14	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.93	1.00	1.14	1.14
time (sec)	N/A	0.181	0.009	0.013	0.175	0.234	0.280	0.295	19.764

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	47	47	75	0	0	135	0	0	0
N.S.	1	1.00	1.60	0.00	0.00	2.87	0.00	0.00	0.00
time (sec)	N/A	0.246	0.284	0.000	0.000	1.105	0.000	0.000	0.000

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	82	0	0	146	0	0	0
N.S.	1	1.00	1.71	0.00	0.00	3.04	0.00	0.00	0.00
time (sec)	N/A	0.246	0.280	0.000	0.000	1.076	0.000	0.000	0.000

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	169	169	364	0	0	0	0	0	0
N.S.	1	1.00	2.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.413	0.892	0.000	0.000	0.000	0.000	0.000	0.000

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	268	309	1464	0	0	0	0	0	0
N.S.	1	1.15	5.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.501	3.421	0.000	0.000	0.000	0.000	0.000	0.000

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	45	42	28	27	25	37	27	27
N.S.	1	1.10	1.02	0.68	0.66	0.61	0.90	0.66	0.66
time (sec)	N/A	0.270	0.032	1.026	0.264	0.366	3.136	0.316	0.034

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	30	26	21	20	20	24	20	22
N.S.	1	1.15	1.00	0.81	0.77	0.77	0.92	0.77	0.85
time (sec)	N/A	0.216	0.032	0.007	0.268	0.343	0.951	0.330	0.055

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	36	42	31	30	30	39	30	42
N.S.	1	0.86	1.00	0.74	0.71	0.71	0.93	0.71	1.00
time (sec)	N/A	0.258	0.031	1.109	0.265	0.347	4.037	0.337	0.026

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	22	48	50	0	75	0	44	17
N.S.	1	1.10	2.40	2.50	0.00	3.75	0.00	2.20	0.85
time (sec)	N/A	0.158	0.023	1.063	0.000	0.332	0.000	0.321	19.265

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	22	52	61	0	84	0	40	21
N.S.	1	1.10	2.60	3.05	0.00	4.20	0.00	2.00	1.05
time (sec)	N/A	0.159	0.022	1.136	0.000	0.348	0.000	0.320	19.251

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	124	160	247	0	1532	0	0	0
N.S.	1	1.02	1.32	2.04	0.00	12.66	0.00	0.00	0.00
time (sec)	N/A	0.362	0.229	1.104	0.000	0.766	0.000	0.000	0.000

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	184	186	397	0	2411	0	0	0
N.S.	1	1.02	1.03	2.19	0.00	13.32	0.00	0.00	0.00
time (sec)	N/A	0.497	0.457	1.062	0.000	43.573	0.000	0.000	0.000

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	17	16	16	22	11	16
N.S.	1	1.00	1.00	0.65	0.62	0.62	0.85	0.42	0.62
time (sec)	N/A	0.162	6.307	0.023	0.177	0.260	63.480	0.320	19.993

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	28	47	23	22	32	17	22	22
N.S.	1	1.08	1.81	0.88	0.85	1.23	0.65	0.85	0.85
time (sec)	N/A	0.160	0.037	1.443	0.257	0.326	0.380	0.319	19.761

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	13	10	11	11	12	11	9
N.S.	1	1.00	0.87	0.67	0.73	0.73	0.80	0.73	0.60
time (sec)	N/A	0.134	10.009	1.089	0.180	0.260	0.069	0.319	19.903

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	19	16	28	27	0	27	18
N.S.	1	1.00	0.86	0.73	1.27	1.23	0.00	1.23	0.82
time (sec)	N/A	0.141	0.057	1.026	0.179	0.293	0.000	0.322	19.847

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	41	9	9	17	0	17	0
N.S.	1	1.00	3.42	0.75	0.75	1.42	0.00	1.42	0.00
time (sec)	N/A	0.142	0.043	1.288	0.268	0.272	0.000	0.361	0.000

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	43	11	11	17	0	17	0
N.S.	1	1.00	3.58	0.92	0.92	1.42	0.00	1.42	0.00
time (sec)	N/A	0.143	0.035	1.330	0.260	0.300	0.000	0.363	0.000

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	12	11	13	13	10	13	10
N.S.	1	1.00	0.80	0.73	0.87	0.87	0.67	0.87	0.67
time (sec)	N/A	0.135	0.028	1.065	0.180	0.269	0.063	0.302	20.276

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	58	82	47	42	49	0	53	0
N.S.	1	1.07	1.52	0.87	0.78	0.91	0.00	0.98	0.00
time (sec)	N/A	0.219	0.104	1.859	0.266	0.301	0.000	0.300	0.000

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	47	38	0	0	0	45	27
N.S.	1	1.00	0.80	0.64	0.00	0.00	0.00	0.76	0.46
time (sec)	N/A	0.214	0.319	3.477	0.000	0.000	0.000	1.062	20.191

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	64	39	28	0	30	0	33	27
N.S.	1	1.08	0.66	0.47	0.00	0.51	0.00	0.56	0.46
time (sec)	N/A	0.196	0.028	1.047	0.000	0.374	0.000	0.306	19.831

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	109	51	38	0	40	0	51	0
N.S.	1	1.16	0.54	0.40	0.00	0.43	0.00	0.54	0.00
time (sec)	N/A	0.249	0.032	1.061	0.000	0.373	0.000	0.313	0.000

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	20	0	20	29	18	19
N.S.	1	1.00	1.00	1.11	0.00	1.11	1.61	1.00	1.06
time (sec)	N/A	0.157	0.047	1.060	0.000	0.294	0.329	0.334	0.143

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	91	82	47	0	62	0	60	0
N.S.	1	0.85	0.77	0.44	0.00	0.58	0.00	0.56	0.00
time (sec)	N/A	0.220	0.067	0.345	0.000	0.284	0.000	0.332	0.000

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	24	25	24	14	64	0	14	25
N.S.	1	0.96	1.00	0.96	0.56	2.56	0.00	0.56	1.00
time (sec)	N/A	0.157	0.032	2.705	0.179	0.344	0.000	0.325	21.394

Problem 936	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	44	42	34	33	35	49	33	35
N.S.	1	1.05	1.00	0.81	0.79	0.83	1.17	0.79	0.83
time (sec)	N/A	0.195	0.031	0.270	0.268	0.283	0.109	0.301	0.054

Problem 937	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	38	32	26	25	27	37	25	27
N.S.	1	1.19	1.00	0.81	0.78	0.84	1.16	0.78	0.84
time (sec)	N/A	0.181	0.022	0.257	0.274	0.282	0.101	0.334	21.127

Problem 938	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	83	49	53	0	95	0	45	0
N.S.	1	1.09	0.64	0.70	0.00	1.25	0.00	0.59	0.00
time (sec)	N/A	0.191	0.220	0.027	0.000	0.304	0.000	0.327	0.000

Problem 939	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	41	43	0	64	48	79	46
N.S.	1	1.00	0.89	0.93	0.00	1.39	1.04	1.72	1.00
time (sec)	N/A	0.275	0.078	0.115	0.000	0.351	1.419	0.311	20.090

Problem 940	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	18	0	14	0	13	0
N.S.	1	1.00	1.00	0.90	0.00	0.70	0.00	0.65	0.00
time (sec)	N/A	0.133	0.266	1.023	0.000	0.277	0.000	0.301	0.000

Problem 941	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	36	18	20	38	19	68	0	14
N.S.	1	1.03	0.51	0.57	1.09	0.54	1.94	0.00	0.40
time (sec)	N/A	0.160	0.157	1.133	0.303	0.275	0.658	0.000	19.876

Problem 942	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	A	A	C	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	111	0	13	16	122	0	13
N.S.	1	1.00	7.40	0.00	0.87	1.07	8.13	0.00	0.87
time (sec)	N/A	0.153	0.193	0.000	0.234	0.300	5.334	0.000	20.460

Problem 943	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	28	24	37	24	46	37	37
N.S.	1	1.00	0.53	0.45	0.70	0.45	0.87	0.70	0.70
time (sec)	N/A	0.187	0.014	1.089	0.180	0.283	0.496	0.322	0.047

Problem 944	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	22	31	26	25	25	27	25	25
N.S.	1	0.71	1.00	0.84	0.81	0.81	0.87	0.81	0.81
time (sec)	N/A	0.178	0.015	0.029	0.175	0.263	0.129	0.297	0.072

Problem 945	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	9	9
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.82	0.82
time (sec)	N/A	0.133	0.025	1.126	0.181	0.268	0.056	0.300	21.206

Problem 946	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	5	4	4	5	4	4
N.S.	1	1.00	1.00	0.83	0.67	0.67	0.83	0.67	0.67
time (sec)	N/A	0.128	0.009	1.074	0.265	0.303	0.085	0.351	21.159

Problem 947	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	17	14	16	16	0	25	16
N.S.	1	1.00	0.85	0.70	0.80	0.80	0.00	1.25	0.80
time (sec)	N/A	0.140	0.014	1.076	0.265	0.254	0.000	0.310	21.207

Problem 948	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	26	11	12	11	11
N.S.	1	1.00	1.00	0.71	1.53	0.65	0.71	0.65	0.65
time (sec)	N/A	0.141	0.001	0.027	0.188	0.301	0.066	0.301	0.025

Problem 949	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	12	11	11	15	11	12
N.S.	1	1.00	1.00	0.63	0.58	0.58	0.79	0.58	0.63
time (sec)	N/A	0.147	0.002	0.056	0.180	0.279	0.377	0.316	0.025

Problem 950	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	45	42	28	27	25	37	27	27
N.S.	1	1.10	1.02	0.68	0.66	0.61	0.90	0.66	0.66
time (sec)	N/A	0.159	0.024	1.121	0.260	0.280	1.700	0.305	0.027

Problem 951	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	75	86	48	63	65	39	64	73
N.S.	1	1.12	1.28	0.72	0.94	0.97	0.58	0.96	1.09
time (sec)	N/A	0.177	0.043	1.077	0.263	0.270	0.637	0.326	21.538

Problem 952	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	7	8	7	7
N.S.	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	0.64
time (sec)	N/A	0.131	0.001	0.030	0.182	0.279	0.021	0.309	0.003

Problem 953	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	7	8	7	7
N.S.	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	0.64
time (sec)	N/A	0.130	0.001	0.040	0.174	0.278	0.021	0.301	0.026

Problem 954	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	7	8	7	7
N.S.	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	0.64
time (sec)	N/A	0.143	0.000	1.110	0.179	0.288	0.460	0.318	0.028

Problem 955	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	75	49	0	192	162	123	0
N.S.	1	1.00	1.23	0.80	0.00	3.15	2.66	2.02	0.00
time (sec)	N/A	0.186	0.061	1.260	0.000	0.390	1.070	0.309	0.000

Problem 956	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	78	55	0	202	162	131	0
N.S.	1	1.00	1.20	0.85	0.00	3.11	2.49	2.02	0.00
time (sec)	N/A	0.188	0.087	1.176	0.000	0.281	1.133	0.320	0.000

Problem 957	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	23	13	10	21	21	26	22	9
N.S.	1	1.77	1.00	0.77	1.62	1.62	2.00	1.69	0.69
time (sec)	N/A	0.147	0.018	1.048	0.262	0.288	0.125	0.415	0.032

Problem 958	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	23	13	10	21	21	26	22	9
N.S.	1	1.77	1.00	0.77	1.62	1.62	2.00	1.69	0.69
time (sec)	N/A	0.142	0.002	1.071	0.270	0.301	0.158	0.311	0.027

Problem 959	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	77	72	58	77	131	202	80	233
N.S.	1	1.07	1.00	0.81	1.07	1.82	2.81	1.11	3.24
time (sec)	N/A	0.234	0.062	1.194	0.260	0.302	0.699	0.331	20.604

Problem 960	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	23	23	15	22	0	48	22
N.S.	1	1.00	0.62	0.62	0.41	0.59	0.00	1.30	0.59
time (sec)	N/A	0.167	0.012	1.148	0.189	0.251	0.000	0.330	19.204

Problem 961	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	41	9	9	17	0	17	0
N.S.	1	1.00	3.42	0.75	0.75	1.42	0.00	1.42	0.00
time (sec)	N/A	0.146	0.004	1.076	0.261	0.301	0.000	0.331	0.000

Problem 962	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	107	71	67	0	89	56	51	0
N.S.	1	1.13	0.75	0.71	0.00	0.94	0.59	0.54	0.00
time (sec)	N/A	0.245	0.106	0.028	0.000	1.124	0.361	0.318	0.000

Problem 963	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	25	28	24	23	21	184	23	16
N.S.	1	0.71	0.80	0.69	0.66	0.60	5.26	0.66	0.46
time (sec)	N/A	0.160	0.014	0.127	0.174	0.289	0.636	0.287	19.371

Problem 964	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	25	30	28	27	22	265	27	24
N.S.	1	0.68	0.81	0.76	0.73	0.59	7.16	0.73	0.65
time (sec)	N/A	0.167	0.014	0.087	0.183	0.285	0.625	0.310	20.101

Problem 965	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	35	40	48	0	33	32	29	39
N.S.	1	1.21	1.38	1.66	0.00	1.14	1.10	1.00	1.34
time (sec)	N/A	0.164	0.082	1.078	0.000	0.275	1.378	0.332	21.865

Problem 966	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	33	42	48	0	36	42	32	40
N.S.	1	1.32	1.68	1.92	0.00	1.44	1.68	1.28	1.60
time (sec)	N/A	0.163	0.086	1.100	0.000	0.308	1.267	0.326	22.285

Problem 967	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	0	15	56	15	22
N.S.	1	1.00	1.00	0.76	0.00	0.71	2.67	0.71	1.05
time (sec)	N/A	0.167	0.052	1.045	0.000	0.300	0.170	0.470	0.042

Problem 968	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	64	49	57	0	97	0	105	127
N.S.	1	0.98	0.75	0.88	0.00	1.49	0.00	1.62	1.95
time (sec)	N/A	0.220	0.109	0.212	0.000	0.269	0.000	0.323	21.672

Problem 969	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	38	50	28	0	41	0	63	64
N.S.	1	1.23	1.61	0.90	0.00	1.32	0.00	2.03	2.06
time (sec)	N/A	0.205	0.134	0.227	0.000	0.286	0.000	0.323	0.187

Problem 970	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	94	57	46	0	54	58	50	0
N.S.	1	1.15	0.70	0.56	0.00	0.66	0.71	0.61	0.00
time (sec)	N/A	0.214	0.085	1.135	0.000	0.621	0.271	0.335	0.000

Problem 971	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	84	74	56	55	57	155	55	95
N.S.	1	1.14	1.00	0.76	0.74	0.77	2.09	0.74	1.28
time (sec)	N/A	0.337	0.047	1.049	0.279	0.284	2.480	0.336	20.542

Problem 972	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	124	115	81	80	80	221	80	130
N.S.	1	1.08	1.00	0.70	0.70	0.70	1.92	0.70	1.13
time (sec)	N/A	0.397	0.055	1.067	0.266	0.274	2.913	0.348	0.127

Problem 973	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	22	29	0	20	0	4	4
N.S.	1	1.00	5.50	7.25	0.00	5.00	0.00	1.00	1.00
time (sec)	N/A	0.175	0.099	0.142	0.000	0.270	0.000	0.337	0.030

Problem 974	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	39	50	40	0	31	18
N.S.	1	1.00	1.00	1.77	2.27	1.82	0.00	1.41	0.82
time (sec)	N/A	0.160	0.028	0.111	0.180	0.284	0.000	0.333	21.500

Problem 975	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	40	37	26	0	28	20
N.S.	1	1.00	1.00	1.67	1.54	1.08	0.00	1.17	0.83
time (sec)	N/A	0.159	0.020	0.172	0.271	0.291	0.000	0.332	20.532

Problem 976	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	28	34	39	51	40	0	35	24
N.S.	1	1.17	1.42	1.62	2.12	1.67	0.00	1.46	1.00
time (sec)	N/A	0.164	0.050	0.109	0.191	0.308	0.000	0.360	0.031

Problem 977	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	39	38	38	32	38	20
N.S.	1	1.00	1.00	1.62	1.58	1.58	1.33	1.58	0.83
time (sec)	N/A	0.175	0.019	0.092	0.177	0.274	1.000	0.496	0.037

Problem 978	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	51	45	51	42	0	35	35
N.S.	1	1.00	2.32	2.05	2.32	1.91	0.00	1.59	1.59
time (sec)	N/A	0.154	0.035	0.096	0.179	0.278	0.000	0.313	0.040

Problem 979	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	27	52	44	35	25	0	35	23
N.S.	1	0.93	1.79	1.52	1.21	0.86	0.00	1.21	0.79
time (sec)	N/A	0.172	0.009	0.189	0.261	0.279	0.000	0.344	0.041

Problem 980	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	35	49	27	36	35	22	25	26
N.S.	1	1.06	1.48	0.82	1.09	1.06	0.67	0.76	0.79
time (sec)	N/A	0.168	0.066	1.159	0.258	0.295	0.513	0.304	21.653

Problem 981	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	40	7	6	16	5	25	6
N.S.	1	1.00	5.00	0.88	0.75	2.00	0.62	3.12	0.75
time (sec)	N/A	0.143	0.026	1.167	0.262	0.291	0.497	0.313	0.017

Problem 982	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	11	10	11	18	0	18	13
N.S.	1	1.00	0.85	0.77	0.85	1.38	0.00	1.38	1.00
time (sec)	N/A	0.149	0.014	1.084	0.182	0.261	0.000	0.302	22.393

Problem 983	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	24	41	0	33	0	73	17
N.S.	1	1.00	1.09	1.86	0.00	1.50	0.00	3.32	0.77
time (sec)	N/A	0.185	0.039	1.079	0.000	0.273	0.000	0.342	22.370

Problem 984	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	39	30	11	13	15	13	45
N.S.	1	1.00	1.62	1.25	0.46	0.54	0.62	0.54	1.88
time (sec)	N/A	0.152	0.018	1.132	0.265	0.259	0.375	0.295	22.709

Problem 985	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	55	23	27	62	75	59	28
N.S.	1	1.00	1.96	0.82	0.96	2.21	2.68	2.11	1.00
time (sec)	N/A	0.147	0.038	1.122	0.185	0.287	0.329	0.306	21.388

Problem 986	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	55	23	27	62	75	59	28
N.S.	1	1.00	1.96	0.82	0.96	2.21	2.68	2.11	1.00
time (sec)	N/A	0.160	0.001	1.086	0.186	0.295	0.707	0.472	21.322

Problem 987	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	55	29	27	62	75	59	28
N.S.	1	1.00	1.96	1.04	0.96	2.21	2.68	2.11	1.00
time (sec)	N/A	0.158	0.001	0.040	0.184	0.296	0.656	0.312	21.687

Problem 988	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	55	29	27	62	75	59	28
N.S.	1	1.00	1.96	1.04	0.96	2.21	2.68	2.11	1.00
time (sec)	N/A	0.158	0.001	0.037	0.185	0.294	0.705	0.325	22.675

Problem 989	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	55	29	27	62	75	59	28
N.S.	1	1.00	1.96	1.04	0.96	2.21	2.68	2.11	1.00
time (sec)	N/A	0.159	0.001	0.041	0.176	0.278	0.662	0.351	22.632

Problem 990	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	55	29	27	62	75	59	28
N.S.	1	1.00	1.96	1.04	0.96	2.21	2.68	2.11	1.00
time (sec)	N/A	0.157	0.001	0.038	0.178	0.304	0.686	0.362	21.136

Problem 991	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	56	28	36	87	97	76	33
N.S.	1	1.00	1.40	0.70	0.90	2.18	2.42	1.90	0.82
time (sec)	N/A	0.153	0.007	1.189	0.184	0.271	0.334	0.324	21.399

Problem 992	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	56	28	36	87	97	76	33
N.S.	1	1.00	1.40	0.70	0.90	2.18	2.42	1.90	0.82
time (sec)	N/A	0.168	0.001	1.129	0.187	0.282	0.657	0.336	20.973

Problem 993	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	56	28	36	87	97	76	33
N.S.	1	1.00	1.40	0.70	0.90	2.18	2.42	1.90	0.82
time (sec)	N/A	0.161	0.001	1.095	0.181	0.335	0.711	0.301	22.701

Problem 994	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	56	37	36	87	97	76	33
N.S.	1	1.00	1.40	0.92	0.90	2.18	2.42	1.90	0.82
time (sec)	N/A	0.168	0.001	0.045	0.181	0.284	0.691	0.301	23.073

Problem 995	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	A	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	0	110	0	0	68	0	0	0
N.S.	1	0.00	1.75	0.00	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.000	0.501	0.000	0.000	0.639	0.000	0.000	0.000

Problem 996	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	0	91	0	0	0	0	0	0
N.S.	1	0.00	1.38	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.616	0.000	0.000	0.000	0.000	0.000	0.000

Problem 997	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	C	F	B	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	78	319	83	217	0	434	0	218	649
N.S.	1	4.09	1.06	2.78	0.00	5.56	0.00	2.79	8.32
time (sec)	N/A	0.813	0.323	2.327	0.000	0.289	0.000	0.409	20.649

Problem 998	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	126	132	97	430	0	105	0	444	0
N.S.	1	1.05	0.77	3.41	0.00	0.83	0.00	3.52	0.00
time (sec)	N/A	0.333	0.109	1.669	0.000	0.280	0.000	0.376	0.000

Problem 999	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	96	0	0	62	0	0	0
N.S.	1	1.00	4.36	0.00	0.00	2.82	0.00	0.00	0.00
time (sec)	N/A	0.199	0.219	0.000	0.000	0.632	0.000	0.000	0.000

Problem 1000	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	0	0	0	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.263	0.833	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1001	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	41	41	44	0	0	0	0	0	0
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.263	0.797	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1002	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	187	389	1528	0	0	0	0	0
N.S.	1	1.02	2.11	8.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.384	10.356	4.993	0.000	0.000	0.000	0.000	0.000

Problem 1003	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	134	90	1036	0	0	0	0	0
N.S.	1	1.02	0.69	7.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.217	20.049	2.355	0.000	0.000	0.000	0.000	0.000

Problem 1004	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	419	47	0	305	0	0	0
N.S.	1	1.00	7.76	0.87	0.00	5.65	0.00	0.00	0.00
time (sec)	N/A	0.348	12.408	2.702	0.000	7.879	0.000	0.000	0.000

Problem 1005	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	416	46	0	304	0	0	0
N.S.	1	1.00	7.85	0.87	0.00	5.74	0.00	0.00	0.00
time (sec)	N/A	0.361	11.905	2.712	0.000	7.665	0.000	0.000	0.000

Problem 1006	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	83	90	69	0	154	0	164	0
N.S.	1	0.99	1.07	0.82	0.00	1.83	0.00	1.95	0.00
time (sec)	N/A	0.346	0.232	1.487	0.000	0.256	0.000	0.336	0.000

Problem 1007	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	17	23	7	22	26	15	13
N.S.	1	1.00	0.85	1.15	0.35	1.10	1.30	0.75	0.65
time (sec)	N/A	0.147	0.004	0.068	0.294	0.257	0.183	0.299	21.024

Problem 1008	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	60	38	0	0	0	0	0	0
N.S.	1	1.30	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.181	0.026	0.000	0.000	0.000	0.000	0.000	0.000

Problem 1009	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	246	92	0	324	0	0	0
N.S.	1	1.00	2.80	1.05	0.00	3.68	0.00	0.00	0.00
time (sec)	N/A	0.387	0.941	3.960	0.000	2.158	0.000	0.000	0.000

Problem 1010	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	213	96	0	331	0	0	0
N.S.	1	1.00	2.42	1.09	0.00	3.76	0.00	0.00	0.00
time (sec)	N/A	0.441	0.921	3.883	0.000	2.104	0.000	0.000	0.000

Problem 1011	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	46	107	0	0	161	0	0	0
N.S.	1	1.00	2.33	0.00	0.00	3.50	0.00	0.00	0.00
time (sec)	N/A	0.621	5.000	0.000	0.000	5.583	0.000	0.000	0.000

Problem 1012	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	47	114	0	0	161	0	0	0
N.S.	1	1.02	2.48	0.00	0.00	3.50	0.00	0.00	0.00
time (sec)	N/A	0.604	4.440	0.000	0.000	5.158	0.000	0.000	0.000

Problem 1013	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	46	107	0	0	161	0	0	0
N.S.	1	1.00	2.33	0.00	0.00	3.50	0.00	0.00	0.00
time (sec)	N/A	1.037	0.007	0.000	0.000	5.510	0.000	0.000	0.000

Problem 1014	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	47	114	0	0	161	0	0	0
N.S.	1	1.02	2.48	0.00	0.00	3.50	0.00	0.00	0.00
time (sec)	N/A	1.031	0.008	0.000	0.000	5.109	0.000	0.000	0.000

Problem 1015	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	66	27	147	94	96	17	58	132
N.S.	1	3.47	1.42	7.74	4.95	5.05	0.89	3.05	6.95
time (sec)	N/A	2.081	0.266	1.382	0.256	0.243	8.672	0.337	22.531

Problem 1016	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	90	90	180	1673	0	458	0	0	0
N.S.	1	1.00	2.00	18.59	0.00	5.09	0.00	0.00	0.00
time (sec)	N/A	0.208	0.356	17.656	0.000	5.029	0.000	0.000	0.000

Problem 1017	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	103	425	181	692	0	289	0	0	0
N.S.	1	4.13	1.76	6.72	0.00	2.81	0.00	0.00	0.00
time (sec)	N/A	0.957	1.344	10.010	0.000	2.921	0.000	0.000	0.000

Problem 1018	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	175	53	49	0	51	0	0	0
N.S.	1	3.57	1.08	1.00	0.00	1.04	0.00	0.00	0.00
time (sec)	N/A	0.381	0.171	1.869	0.000	0.288	0.000	0.000	0.000

Problem 1019	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	383	66	0	472	0	0	0
N.S.	1	1.00	4.79	0.82	0.00	5.90	0.00	0.00	0.00
time (sec)	N/A	0.522	10.565	3.405	0.000	28.967	0.000	0.000	0.000

Problem 1020	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.131	0.000	1.175	0.184	0.235	0.021	0.316	0.005

Problem 1021	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	C	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	122	129	89	0	73	0	193	549
N.S.	1	2.90	3.07	2.12	0.00	1.74	0.00	4.60	13.07
time (sec)	N/A	0.356	0.074	0.393	0.000	0.262	0.000	0.374	20.258

Problem 1022	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	C	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	149	129	89	0	73	0	193	549
N.S.	1	3.55	3.07	2.12	0.00	1.74	0.00	4.60	13.07
time (sec)	N/A	0.559	0.070	1.071	0.000	0.261	0.000	0.347	0.506

Problem 1023	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	A	B	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	0	31	32	16	31	73	81	31
N.S.	1	0.00	0.91	0.94	0.47	0.91	2.15	2.38	0.91
time (sec)	N/A	0.000	0.092	1.341	0.270	0.266	17.963	0.364	19.805

Problem 1024	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	177	177	182	1597	0	164	0	0	0
N.S.	1	1.00	1.03	9.02	0.00	0.93	0.00	0.00	0.00
time (sec)	N/A	0.296	5.604	1.541	0.000	0.353	0.000	0.000	0.000

Problem 1025	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	139	100	228	0	97	0	0	0
N.S.	1	1.39	1.00	2.28	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	0.254	3.632	2.072	0.000	0.337	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [218] had the largest ratio of [.8947370000000000005]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.00	19	0.211
2	A	6	5	1.00	23	0.217
3	A	6	5	1.00	21	0.238
4	A	5	4	1.00	21	0.190
5	A	5	4	1.00	33	0.121
6	A	6	5	1.00	35	0.143
7	A	6	5	1.00	36	0.139
8	A	5	4	1.00	36	0.111
9	A	5	4	1.00	24	0.167
10	A	6	5	1.00	20	0.250
11	A	6	5	1.00	24	0.208
12	A	6	5	1.00	22	0.227
13	A	6	5	1.00	22	0.227
14	A	10	9	0.98	15	0.600
15	A	9	8	0.98	17	0.471
16	A	9	8	1.00	15	0.533
17	A	10	9	0.94	17	0.529
18	A	1	1	1.00	25	0.040
19	A	3	3	0.97	25	0.120
20	A	1	1	1.00	25	0.040
21	A	1	1	1.00	21	0.048
22	A	1	1	1.00	24	0.042

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	4	4	0.96	19	0.211
24	A	8	8	0.97	19	0.421
25	A	6	6	0.97	19	0.316
26	A	4	4	0.98	17	0.235
27	A	2	2	1.00	19	0.105
28	A	2	2	1.00	19	0.105
29	A	2	2	1.00	19	0.105
30	A	2	2	1.00	19	0.105
31	A	4	4	1.00	19	0.211
32	A	2	2	1.00	17	0.118
33	A	2	2	1.00	19	0.105
34	A	2	2	1.00	19	0.105
35	A	2	2	1.00	19	0.105
36	A	2	2	1.00	19	0.105
37	A	5	5	0.98	19	0.263
38	A	4	4	1.00	19	0.211
39	A	2	2	1.00	17	0.118
40	A	2	2	1.00	19	0.105
41	A	2	2	1.00	19	0.105
42	A	2	2	1.00	19	0.105
43	A	3	2	1.00	28	0.071
44	A	3	2	1.00	32	0.062
45	A	3	2	1.00	30	0.067
46	A	3	2	1.00	30	0.067
47	A	3	2	1.00	53	0.038
48	A	3	2	1.00	55	0.036
49	A	3	2	1.00	56	0.036
50	A	3	2	1.00	56	0.036
51	A	3	2	1.00	30	0.067
52	A	5	4	1.03	24	0.167
53	A	6	5	1.02	28	0.179
54	A	6	5	1.02	26	0.192
55	A	5	4	1.03	26	0.154
56	A	5	4	1.04	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	6	5	1.04	28	0.179
58	A	6	5	1.04	26	0.192
59	A	5	4	1.04	26	0.154
60	A	5	4	1.00	38	0.105
61	A	6	5	1.00	40	0.125
62	A	6	5	1.00	41	0.122
63	A	5	4	1.00	41	0.098
64	A	5	4	1.00	29	0.138
65	A	5	4	1.00	20	0.200
66	A	6	5	1.00	24	0.208
67	A	6	5	1.00	22	0.227
68	A	5	4	1.00	22	0.182
69	A	5	4	1.00	34	0.118
70	A	6	5	1.00	36	0.139
71	A	6	5	1.00	37	0.135
72	A	5	4	1.00	37	0.108
73	A	5	4	1.00	25	0.160
74	A	3	2	1.00	20	0.100
75	A	3	2	1.00	22	0.091
76	A	3	2	1.00	20	0.100
77	A	3	2	1.00	22	0.091
78	A	3	2	1.00	43	0.047
79	A	3	2	1.00	44	0.045
80	A	3	2	1.00	45	0.044
81	A	3	2	1.00	46	0.043
82	A	3	2	1.00	30	0.067
83	A	6	5	1.00	22	0.227
84	A	6	5	1.00	22	0.227
85	A	6	5	1.00	20	0.250
86	A	6	5	1.00	24	0.208
87	A	6	5	0.98	35	0.143
88	A	6	5	0.98	35	0.143
89	A	6	5	0.98	36	0.139
90	A	6	5	0.98	38	0.132

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	5	4	1.00	29	0.138
92	A	6	5	1.00	18	0.278
93	A	6	5	1.00	18	0.278
94	A	6	5	1.00	16	0.312
95	A	6	5	1.00	20	0.250
96	A	6	5	1.00	31	0.161
97	A	6	5	1.00	31	0.161
98	A	6	5	1.00	32	0.156
99	A	6	5	1.00	34	0.147
100	A	5	4	1.00	25	0.160
101	A	3	2	1.00	30	0.067
102	A	3	2	1.00	36	0.056
103	A	3	2	1.00	34	0.059
104	A	3	2	1.00	32	0.062
105	A	3	2	1.00	58	0.034
106	A	3	2	1.00	61	0.033
107	A	3	2	1.00	62	0.032
108	A	3	2	1.00	61	0.033
109	A	3	2	1.00	52	0.038
110	A	3	2	1.00	55	0.036
111	A	3	2	1.00	56	0.036
112	A	3	2	1.00	55	0.036
113	A	3	2	1.00	30	0.067
114	A	3	2	1.00	36	0.056
115	A	3	2	1.00	34	0.059
116	A	3	2	1.00	32	0.062
117	A	3	2	1.00	58	0.034
118	A	3	2	1.00	61	0.033
119	A	3	2	1.00	62	0.032
120	A	3	2	1.00	61	0.033
121	A	3	2	1.00	52	0.038
122	A	3	2	1.00	55	0.036
123	A	3	2	1.00	56	0.036
124	A	3	2	1.00	55	0.036

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	6	5	1.00	23	0.217
126	A	6	5	1.00	25	0.200
127	A	6	5	1.00	25	0.200
128	A	6	5	0.98	29	0.172
129	A	6	5	0.98	27	0.185
130	A	6	5	1.00	27	0.185
131	A	6	5	1.00	42	0.119
132	A	6	5	1.00	44	0.114
133	A	6	5	1.00	45	0.111
134	A	6	5	1.00	45	0.111
135	A	6	5	1.30	21	0.238
136	A	6	5	1.27	25	0.200
137	A	6	5	1.14	23	0.217
138	A	6	5	1.15	23	0.217
139	A	6	5	1.12	23	0.217
140	A	6	5	1.06	38	0.132
141	A	6	5	1.06	40	0.125
142	A	6	5	1.13	41	0.122
143	A	6	5	1.13	41	0.122
144	A	8	7	0.94	25	0.280
145	A	7	6	0.92	29	0.207
146	A	7	6	0.92	27	0.222
147	A	8	7	0.93	27	0.259
148	A	8	7	0.86	27	0.259
149	A	7	6	0.87	31	0.194
150	A	7	6	0.87	29	0.207
151	A	8	7	0.86	29	0.241
152	A	6	5	1.00	21	0.238
153	A	7	6	1.00	25	0.240
154	A	7	6	1.00	23	0.261
155	A	6	5	1.00	23	0.217
156	A	6	5	1.00	23	0.217
157	A	7	6	1.00	27	0.222
158	A	7	6	1.00	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	6	5	1.00	25	0.200
160	A	10	9	0.99	16	0.562
161	A	9	8	1.00	18	0.444
162	A	9	8	1.02	16	0.500
163	A	10	9	0.96	18	0.500
164	A	10	9	1.00	22	0.409
165	A	9	8	0.97	24	0.333
166	A	9	8	0.97	22	0.364
167	A	10	9	0.99	24	0.375
168	A	7	6	1.00	18	0.333
169	A	7	6	1.00	20	0.300
170	A	7	6	1.00	18	0.333
171	A	7	6	1.00	20	0.300
172	A	1	1	1.00	31	0.032
173	A	3	3	0.97	30	0.100
174	A	2	2	1.00	18	0.111
175	A	2	2	1.00	16	0.125
176	A	2	2	1.00	15	0.133
177	A	2	2	1.00	18	0.111
178	A	2	2	1.00	20	0.100
179	A	2	2	1.00	18	0.111
180	A	2	2	1.00	17	0.118
181	A	2	2	1.00	20	0.100
182	A	2	2	1.00	20	0.100
183	A	2	2	1.00	18	0.111
184	A	2	2	1.00	17	0.118
185	A	2	2	1.00	20	0.100
186	A	2	2	1.00	20	0.100
187	A	2	2	1.00	20	0.100
188	A	2	2	1.00	20	0.100
189	A	2	2	1.00	20	0.100
190	A	2	2	1.00	18	0.111
191	A	2	2	1.00	17	0.118
192	A	2	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
193	A	2	2	1.00	20	0.100
194	A	2	2	1.00	22	0.091
195	A	2	2	1.00	20	0.100
196	A	15	14	0.92	19	0.737
197	F	0	0	N/A	0.000	N/A
198	A	3	2	1.00	27	0.074
199	A	3	2	1.00	29	0.069
200	A	3	2	1.00	27	0.074
201	A	3	2	1.00	29	0.069
202	A	3	2	1.00	31	0.065
203	A	3	2	1.00	35	0.057
204	A	3	2	1.00	33	0.061
205	A	3	2	1.00	33	0.061
206	A	2	2	1.00	19	0.105
207	A	2	2	1.00	19	0.105
208	A	2	2	1.00	17	0.118
209	A	2	2	1.00	11	0.182
210	A	17	16	0.90	19	0.842
211	A	13	12	0.80	19	0.632
212	A	2	2	1.00	19	0.105
213	A	2	2	1.00	19	0.105
214	A	2	2	1.00	17	0.118
215	A	1	1	1.00	11	0.091
216	A	9	8	1.06	19	0.421
217	A	16	15	1.04	19	0.789
218	A	18	17	1.03	19	0.895
219	A	7	7	0.99	19	0.368
220	A	6	6	0.99	19	0.316
221	A	4	4	1.00	17	0.235
222	A	2	2	1.00	11	0.182
223	A	18	17	0.89	19	0.895
224	A	2	2	1.00	20	0.100
225	A	2	2	1.00	22	0.091

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
226	A	2	2	0.99	24	0.083
227	A	4	4	0.54	19	0.211
228	A	6	5	0.50	19	0.263
229	A	4	4	0.48	19	0.211
230	A	6	5	0.95	19	0.263
231	A	4	4	0.48	19	0.211
232	A	4	3	0.91	17	0.176
233	A	3	3	0.48	15	0.200
234	A	6	5	0.49	19	0.263
235	A	4	4	0.45	19	0.211
236	A	6	5	0.48	19	0.263
237	A	8	8	0.85	19	0.421
238	A	4	3	0.91	17	0.176
239	A	7	7	0.86	15	0.467
240	A	10	9	0.74	19	0.474
241	A	7	7	0.86	19	0.368
242	A	10	9	0.76	19	0.474
243	A	3	3	1.04	19	0.158
244	A	3	2	1.00	17	0.118
245	A	2	2	1.00	15	0.133
246	A	6	5	1.00	19	0.263
247	A	3	3	1.56	19	0.158
248	A	7	6	0.95	19	0.316
249	A	5	4	0.96	21	0.190
250	A	5	4	1.06	21	0.190
251	A	5	4	1.27	21	0.190
252	A	4	3	0.86	19	0.158
253	A	10	9	0.81	21	0.429
254	A	10	9	0.75	21	0.429
255	A	5	5	0.91	21	0.238
256	A	4	4	0.88	17	0.235
257	A	4	4	0.95	21	0.190
258	A	5	5	0.90	21	0.238
259	A	5	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
260	A	8	8	1.10	17	0.471
261	A	4	3	1.00	15	0.200
262	A	4	4	1.00	17	0.235
263	A	9	8	1.10	26	0.308
264	A	8	7	1.15	26	0.269
265	A	5	4	1.07	24	0.167
266	A	6	5	1.27	26	0.192
267	A	5	4	0.87	26	0.154
268	A	7	6	0.88	26	0.231
269	A	9	8	0.84	26	0.308
270	A	7	7	1.04	26	0.269
271	A	6	6	1.11	26	0.231
272	A	5	5	1.28	22	0.227
273	A	7	7	1.20	26	0.269
274	A	8	8	1.08	26	0.308
275	A	10	10	1.02	26	0.385
276	A	13	12	1.07	26	0.462
277	A	9	8	1.08	26	0.308
278	A	6	5	0.87	24	0.208
279	A	8	7	1.21	26	0.269
280	A	6	5	0.75	26	0.192
281	A	10	9	0.84	26	0.346
282	A	12	11	0.82	26	0.423
283	A	11	11	1.03	26	0.423
284	A	8	8	1.09	26	0.308
285	A	7	7	1.16	22	0.318
286	A	9	9	1.17	26	0.346
287	A	11	11	1.08	26	0.423
288	A	13	13	1.02	26	0.500
289	A	5	4	1.12	21	0.190
290	A	5	4	1.06	23	0.174
291	A	5	4	1.11	23	0.174
292	A	9	8	0.96	23	0.348
293	A	7	6	1.10	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
294	A	6	5	1.00	23	0.217
295	A	6	5	1.00	28	0.179
296	A	9	8	0.98	26	0.308
297	A	7	6	1.09	26	0.231
298	A	5	4	1.07	24	0.167
299	A	6	5	1.27	26	0.192
300	A	5	4	0.87	26	0.154
301	A	7	6	0.84	26	0.231
302	A	8	8	0.92	26	0.308
303	A	6	6	0.94	26	0.231
304	A	5	5	1.00	22	0.227
305	A	7	7	0.98	26	0.269
306	A	10	10	0.93	26	0.385
307	A	10	9	0.82	26	0.346
308	A	12	11	1.03	26	0.423
309	A	6	5	0.77	24	0.208
310	A	7	6	1.11	26	0.231
311	A	6	5	0.66	26	0.192
312	A	12	11	0.82	26	0.423
313	A	10	10	0.90	26	0.385
314	A	7	7	0.91	26	0.269
315	A	7	7	0.94	22	0.318
316	A	11	11	0.95	26	0.423
317	A	12	12	0.93	26	0.462
318	A	11	10	0.94	21	0.476
319	A	9	8	1.02	21	0.381
320	A	6	5	0.96	19	0.263
321	A	9	8	1.08	21	0.381
322	A	6	5	1.05	21	0.238
323	A	10	9	1.06	21	0.429
324	A	11	10	0.99	21	0.476
325	A	10	10	1.10	21	0.476
326	A	7	7	1.17	21	0.333
327	A	6	6	1.32	17	0.353

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
328	A	8	8	1.24	21	0.381
329	A	12	12	1.09	21	0.571
330	A	14	14	1.06	21	0.667
331	A	11	10	0.95	21	0.476
332	A	12	11	1.02	21	0.524
333	A	7	6	0.96	19	0.316
334	A	10	9	1.11	21	0.429
335	A	7	6	0.88	21	0.286
336	A	11	10	1.03	21	0.476
337	A	12	11	1.02	21	0.524
338	A	12	12	1.07	21	0.571
339	A	9	9	1.12	21	0.429
340	A	8	8	1.25	17	0.471
341	A	12	12	1.26	21	0.571
342	A	14	14	1.13	21	0.667
343	A	16	16	1.05	21	0.762
344	A	10	9	1.01	21	0.429
345	A	9	8	0.99	21	0.381
346	A	6	5	0.94	19	0.263
347	A	9	8	1.08	21	0.381
348	A	6	5	1.05	21	0.238
349	A	9	8	1.06	21	0.381
350	A	9	9	0.93	21	0.429
351	A	7	7	0.95	21	0.333
352	A	6	6	1.00	17	0.353
353	A	8	8	0.96	21	0.381
354	A	10	10	0.92	21	0.476
355	A	10	9	0.82	21	0.429
356	A	13	12	0.91	21	0.571
357	A	7	6	0.99	19	0.316
358	A	10	9	1.02	21	0.429
359	A	7	6	0.77	21	0.286
360	A	12	11	0.94	21	0.524
361	A	13	13	0.93	21	0.619

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
362	A	10	10	0.94	21	0.476
363	A	8	8	0.95	17	0.471
364	A	10	10	0.99	21	0.476
365	A	13	13	0.96	21	0.619
366	A	7	6	0.89	19	0.316
367	A	6	5	0.88	19	0.263
368	A	5	4	1.00	19	0.211
369	A	2	2	1.00	19	0.105
370	A	3	3	1.06	19	0.158
371	A	4	4	0.70	22	0.182
372	A	5	5	0.70	19	0.263
373	A	3	3	0.52	22	0.136
374	A	1	1	1.00	33	0.030
375	A	1	1	1.00	30	0.033
376	A	7	6	0.84	19	0.316
377	A	3	3	0.77	19	0.158
378	A	5	4	0.93	19	0.211
379	A	2	2	1.00	19	0.105
380	A	6	5	1.24	17	0.294
381	A	4	3	1.00	19	0.158
382	A	5	4	1.00	19	0.211
383	A	7	6	0.87	19	0.316
384	A	2	2	1.00	19	0.105
385	A	2	2	1.00	19	0.105
386	A	7	6	1.00	19	0.316
387	A	4	4	0.97	19	0.211
388	A	4	3	1.00	17	0.176
389	A	4	3	1.00	19	0.158
390	A	5	4	1.00	19	0.211
391	A	8	7	0.99	19	0.368
392	A	5	5	0.96	19	0.263
393	A	2	2	1.00	21	0.095
394	A	2	2	1.29	19	0.105
395	A	2	2	1.27	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
396	C	1	1	2.35	54	0.019
397	A	2	2	1.00	26	0.077
398	A	2	2	1.00	7	0.286
399	A	2	2	1.00	15	0.133
400	A	2	2	1.00	22	0.091
401	A	3	3	0.84	25	0.120
402	A	3	3	0.91	23	0.130
403	A	2	2	0.87	21	0.095
404	A	5	4	0.91	25	0.160
405	A	5	4	0.93	25	0.160
406	A	2	2	0.90	25	0.080
407	A	2	2	0.92	23	0.087
408	A	2	2	1.68	21	0.095
409	A	2	2	0.80	25	0.080
410	A	2	2	0.81	25	0.080
411	A	2	2	0.76	25	0.080
412	A	2	2	0.76	23	0.087
413	A	2	2	1.83	21	0.095
414	A	2	2	0.78	25	0.080
415	A	2	2	1.08	25	0.080
416	A	3	3	1.00	15	0.200
417	A	3	3	1.00	15	0.200
418	A	2	2	1.17	17	0.118
419	A	2	2	1.00	23	0.087
420	A	2	2	1.00	23	0.087
421	A	2	2	1.00	21	0.095
422	A	2	2	1.00	19	0.105
423	A	2	2	1.00	23	0.087
424	A	2	2	1.00	23	0.087
425	A	2	2	1.00	23	0.087
426	A	3	3	0.84	25	0.120
427	A	3	3	0.91	25	0.120
428	A	2	2	1.00	23	0.087
429	A	2	2	0.79	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
430	A	5	4	0.93	25	0.160
431	A	6	5	0.84	25	0.200
432	A	2	2	0.93	25	0.080
433	A	2	2	0.94	25	0.080
434	A	2	2	0.80	23	0.087
435	A	2	2	0.80	21	0.095
436	A	2	2	0.93	25	0.080
437	A	2	2	0.92	25	0.080
438	A	2	2	0.78	25	0.080
439	A	2	2	0.79	25	0.080
440	A	2	2	0.78	25	0.080
441	A	2	2	1.11	23	0.087
442	A	2	2	1.30	21	0.095
443	A	2	2	1.00	27	0.074
444	A	4	4	1.00	42	0.095
445	A	4	4	1.00	42	0.095
446	A	4	4	1.00	40	0.100
447	A	4	4	1.00	39	0.103
448	A	4	4	1.00	42	0.095
449	A	4	4	1.00	42	0.095
450	A	4	4	1.00	42	0.095
451	B	4	4	2.46	39	0.103
452	B	4	4	2.39	35	0.114
453	A	4	3	0.96	25	0.120
454	A	4	3	0.96	25	0.120
455	A	4	3	0.96	25	0.120
456	A	1	1	1.00	23	0.043
457	A	4	3	0.95	25	0.120
458	A	4	3	0.97	25	0.120
459	A	4	3	0.94	25	0.120
460	A	7	6	0.97	27	0.222
461	A	6	5	0.99	27	0.185
462	A	7	6	1.00	27	0.222
463	A	7	6	1.03	27	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
464	A	7	6	1.01	27	0.222
465	A	6	5	0.98	27	0.185
466	A	5	4	1.10	17	0.235
467	A	4	3	1.01	26	0.115
468	A	1	1	1.00	17	0.059
469	A	1	1	1.00	15	0.067
470	A	1	1	1.00	15	0.067
471	A	1	1	1.00	25	0.040
472	A	4	3	1.01	28	0.107
473	A	4	3	1.03	28	0.107
474	A	4	3	1.04	28	0.107
475	A	1	1	1.00	26	0.038
476	A	4	3	1.03	28	0.107
477	A	4	3	1.03	28	0.107
478	A	4	3	1.03	28	0.107
479	A	7	6	1.02	30	0.200
480	A	6	5	1.02	30	0.167
481	A	7	6	1.04	30	0.200
482	A	7	6	1.13	30	0.200
483	A	7	6	1.04	30	0.200
484	A	7	6	1.06	30	0.200
485	A	4	3	0.95	21	0.143
486	A	4	3	0.95	19	0.158
487	A	4	3	0.98	13	0.231
488	A	3	2	1.00	21	0.095
489	A	3	2	1.00	21	0.095
490	A	4	3	0.95	23	0.130
491	A	4	3	0.96	21	0.143
492	A	4	3	0.98	15	0.200
493	A	3	2	1.00	23	0.087
494	A	3	2	1.00	23	0.087
495	A	4	3	0.94	23	0.130
496	A	4	3	0.95	23	0.130
497	A	4	3	0.96	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
498	A	3	2	1.00	23	0.087
499	A	3	2	1.00	23	0.087
500	A	3	2	1.00	23	0.087
501	A	4	3	0.95	25	0.120
502	A	4	3	0.95	25	0.120
503	A	4	3	0.96	25	0.120
504	A	3	2	1.00	25	0.080
505	A	3	2	1.00	25	0.080
506	A	3	2	1.00	25	0.080
507	A	5	4	0.90	56	0.071
508	A	5	4	0.91	54	0.074
509	A	5	4	0.97	33	0.121
510	A	5	4	1.00	56	0.071
511	A	4	3	1.00	56	0.054
512	A	6	5	0.97	33	0.152
513	A	6	5	1.00	56	0.089
514	A	5	4	0.90	58	0.069
515	A	5	4	0.91	58	0.069
516	A	3	2	1.00	58	0.034
517	A	4	3	1.00	58	0.052
518	A	4	3	1.00	58	0.052
519	A	6	5	0.64	62	0.081
520	A	4	3	1.00	62	0.048
521	A	5	4	1.00	62	0.065
522	A	5	4	1.00	60	0.067
523	A	8	7	1.00	30	0.233
524	A	4	4	1.00	37	0.108
525	A	4	4	1.00	37	0.108
526	A	3	2	1.00	37	0.054
527	A	3	2	1.00	37	0.054
528	A	3	2	1.42	42	0.048
529	A	3	2	1.42	42	0.048
530	A	5	4	1.17	30	0.133
531	A	5	4	1.16	30	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
532	A	3	2	1.00	42	0.048
533	A	3	2	1.00	42	0.048
534	A	3	2	1.86	51	0.039
535	A	3	2	1.86	51	0.039
536	A	3	2	1.00	40	0.050
537	A	3	2	1.00	40	0.050
538	A	5	4	1.17	32	0.125
539	A	5	4	1.16	32	0.125
540	A	3	2	1.00	51	0.039
541	A	3	2	1.00	51	0.039
542	A	3	2	1.67	56	0.036
543	A	3	2	1.67	56	0.036
544	A	5	4	0.81	29	0.138
545	A	6	5	0.93	29	0.172
546	A	4	3	1.00	27	0.111
547	A	8	7	0.98	29	0.241
548	A	7	6	1.22	29	0.207
549	A	10	9	1.06	29	0.310
550	A	6	5	1.00	25	0.200
551	A	10	9	1.05	29	0.310
552	A	5	4	0.80	29	0.138
553	A	6	5	0.92	29	0.172
554	A	4	3	1.00	29	0.103
555	A	8	7	0.97	29	0.241
556	A	7	6	1.21	29	0.207
557	A	14	13	1.05	29	0.448
558	A	13	12	1.01	27	0.444
559	A	14	13	1.04	25	0.520
560	A	14	13	1.07	29	0.448
561	A	14	13	1.04	29	0.448
562	A	4	4	1.00	25	0.160
563	A	4	4	1.00	29	0.138
564	A	4	3	1.00	31	0.097
565	A	4	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
566	A	6	5	1.77	15	0.333
567	A	6	5	1.00	15	0.333
568	A	5	4	1.12	13	0.308
569	A	5	4	1.08	13	0.308
570	A	6	5	0.90	15	0.333
571	A	11	10	1.15	13	0.769
572	A	5	4	1.11	13	0.308
573	A	5	4	1.08	13	0.308
574	A	12	11	1.02	15	0.733
575	A	4	3	1.00	13	0.231
576	A	5	4	1.32	13	0.308
577	A	13	12	1.15	15	0.800
578	A	11	10	1.09	19	0.526
579	A	5	4	1.10	19	0.211
580	A	6	5	1.03	21	0.238
581	A	3	3	1.00	26	0.115
582	A	3	3	1.00	26	0.115
583	A	3	3	1.00	24	0.125
584	A	3	3	1.00	23	0.130
585	A	3	3	1.00	26	0.115
586	A	3	3	1.00	26	0.115
587	A	4	4	1.00	17	0.235
588	A	6	5	1.04	17	0.294
589	A	5	4	1.00	15	0.267
590	A	3	2	1.00	9	0.222
591	A	6	5	1.00	17	0.294
592	A	4	3	1.00	17	0.176
593	A	5	4	1.00	17	0.235
594	A	7	6	1.06	17	0.353
595	A	3	3	1.00	28	0.107
596	A	3	3	1.00	28	0.107
597	A	3	3	1.00	26	0.115
598	A	3	3	1.00	25	0.120
599	A	3	3	1.00	28	0.107

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
600	A	3	3	1.00	28	0.107
601	A	10	9	1.77	21	0.429
602	A	1	1	1.00	17	0.059
603	A	1	1	1.00	21	0.048
604	A	1	1	1.00	17	0.059
605	A	1	1	1.00	19	0.053
606	A	1	1	1.00	21	0.048
607	A	5	4	1.05	25	0.160
608	A	5	4	1.29	17	0.235
609	A	5	4	1.00	27	0.148
610	A	6	5	1.30	25	0.200
611	B	6	5	3.70	22	0.227
612	B	5	4	2.18	29	0.138
613	A	1	1	1.00	176	0.006
614	A	1	1	1.00	174	0.006
615	A	1	1	1.00	164	0.006
616	F	0	0	N/A	0.000	N/A
617	F	0	0	N/A	0.000	N/A
618	A	6	5	0.92	19	0.263
619	A	5	4	0.93	19	0.211
620	A	6	5	0.96	17	0.294
621	A	5	4	1.15	15	0.267
622	A	8	7	1.25	19	0.368
623	A	8	7	1.22	19	0.368
624	A	8	7	1.14	19	0.368
625	A	6	5	0.94	21	0.238
626	A	5	4	0.94	21	0.190
627	A	6	5	0.95	19	0.263
628	A	5	4	0.98	17	0.235
629	A	11	10	1.16	21	0.476
630	A	10	9	1.36	21	0.429
631	A	12	11	1.50	21	0.524
632	A	6	5	0.94	19	0.263

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
633	A	5	4	0.95	19	0.211
634	A	6	5	0.98	17	0.294
635	A	5	4	0.95	15	0.267
636	A	9	8	0.96	19	0.421
637	A	7	6	1.16	19	0.316
638	A	10	9	1.20	19	0.474
639	A	6	5	0.95	19	0.263
640	A	5	4	0.95	19	0.211
641	A	6	5	1.00	17	0.294
642	A	5	4	0.94	15	0.267
643	A	8	7	1.18	19	0.368
644	A	7	6	1.08	19	0.316
645	A	10	9	1.10	19	0.474
646	A	6	5	0.94	21	0.238
647	A	5	4	0.94	21	0.190
648	A	6	5	0.95	19	0.263
649	A	5	4	0.98	17	0.235
650	A	8	7	1.15	21	0.333
651	A	10	9	1.44	21	0.429
652	A	12	11	1.50	21	0.524
653	A	6	5	0.94	19	0.263
654	A	5	4	0.94	19	0.211
655	A	6	5	0.94	17	0.294
656	A	5	4	0.97	15	0.267
657	A	6	5	1.04	19	0.263
658	A	9	8	0.94	17	0.471
659	A	8	7	0.96	17	0.412
660	A	7	6	0.96	17	0.353
661	A	6	5	1.00	17	0.294
662	A	7	6	0.96	17	0.353
663	A	8	7	1.00	17	0.412
664	A	9	8	0.96	19	0.421
665	A	8	7	0.97	19	0.368
666	A	7	6	0.96	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
667	A	6	5	1.00	19	0.263
668	A	7	6	0.96	19	0.316
669	A	8	7	1.01	19	0.368
670	A	3	2	1.00	13	0.154
671	A	6	5	1.00	17	0.294
672	A	7	6	1.00	21	0.286
673	A	8	7	1.00	25	0.280
674	A	9	8	1.00	29	0.276
675	A	10	9	1.17	20	0.450
676	A	9	8	1.13	20	0.400
677	A	8	7	1.07	18	0.389
678	A	3	3	1.00	20	0.150
679	A	6	5	1.29	20	0.250
680	A	6	5	1.18	20	0.250
681	A	2	2	1.00	18	0.111
682	A	2	2	1.00	20	0.100
683	A	4	3	1.00	22	0.136
684	A	4	3	1.00	17	0.176
685	A	4	3	1.09	23	0.130
686	A	4	3	1.03	22	0.136
687	A	1	1	1.00	34	0.029
688	A	6	5	1.08	31	0.161
689	A	1	1	1.00	47	0.021
690	A	9	8	0.81	58	0.138
691	A	5	4	1.00	11	0.364
692	A	7	6	1.05	11	0.545
693	A	2	2	1.00	17	0.118
694	A	5	4	1.22	13	0.308
695	A	3	2	1.00	16	0.125
696	A	5	4	1.03	14	0.286
697	A	6	5	1.16	12	0.417
698	A	5	4	1.03	13	0.308
699	A	5	4	1.06	13	0.308
700	A	5	4	1.03	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
701	A	6	5	1.11	12	0.417
702	A	7	6	1.09	14	0.429
703	A	6	5	1.10	13	0.385
704	A	6	5	1.06	17	0.294
705	A	7	6	1.06	17	0.353
706	A	5	4	1.04	13	0.308
707	A	4	3	1.07	18	0.167
708	A	5	4	1.04	20	0.200
709	A	5	4	1.04	18	0.222
710	A	5	4	0.90	26	0.154
711	A	4	3	1.05	17	0.176
712	A	4	3	1.07	27	0.111
713	A	7	6	0.73	17	0.353
714	A	8	7	0.75	17	0.412
715	A	8	7	0.83	23	0.304
716	A	7	6	0.73	17	0.353
717	A	8	7	1.01	23	0.304
718	A	8	7	1.01	25	0.280
719	A	8	7	1.01	21	0.333
720	A	4	3	1.00	23	0.130
721	A	6	5	1.18	16	0.312
722	A	5	4	1.06	28	0.143
723	A	6	5	1.10	16	0.312
724	A	6	5	1.21	22	0.227
725	A	2	2	1.00	21	0.095
726	B	3	3	2.80	20	0.150
727	A	12	11	1.09	25	0.440
728	A	8	7	1.10	35	0.200
729	A	3	2	1.00	13	0.154
730	A	4	3	1.00	15	0.200
731	A	4	3	1.00	13	0.231
732	A	5	4	1.00	11	0.364
733	A	4	3	1.00	18	0.167
734	A	5	4	1.00	17	0.235

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
735	A	4	4	1.07	18	0.222
736	A	5	5	1.07	17	0.294
737	A	3	2	1.00	16	0.125
738	A	3	2	1.00	21	0.095
739	A	4	3	1.00	26	0.115
740	A	4	3	1.00	25	0.120
741	A	4	3	1.50	12	0.250
742	A	4	3	1.50	15	0.200
743	A	4	3	1.38	15	0.200
744	A	4	3	1.46	15	0.200
745	A	4	3	1.28	17	0.176
746	A	5	4	1.00	15	0.267
747	A	5	4	1.00	21	0.190
748	A	4	3	1.60	22	0.136
749	A	6	5	1.00	20	0.250
750	A	9	8	1.09	20	0.400
751	A	2	2	1.00	15	0.133
752	A	6	5	1.00	21	0.238
753	A	12	11	1.09	18	0.611
754	A	7	6	0.96	18	0.333
755	A	8	7	0.99	18	0.389
756	A	5	4	1.03	16	0.250
757	A	5	4	1.02	16	0.250
758	A	5	4	1.04	16	0.250
759	A	13	12	1.15	18	0.667
760	A	6	5	1.15	18	0.278
761	A	8	7	1.09	18	0.389
762	A	8	7	1.76	35	0.200
763	A	9	8	1.76	28	0.286
764	A	11	10	1.06	23	0.435
765	A	9	8	1.06	23	0.348
766	A	5	4	1.00	23	0.174
767	A	9	8	1.00	23	0.348
768	A	11	10	1.06	23	0.435

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
769	A	11	10	1.06	19	0.526
770	A	9	8	1.06	19	0.421
771	A	5	4	1.00	19	0.211
772	A	9	8	1.00	19	0.421
773	A	11	10	1.06	19	0.526
774	A	10	9	1.15	31	0.290
775	A	8	7	1.17	31	0.226
776	A	3	2	1.14	31	0.065
777	A	8	7	1.15	31	0.226
778	A	7	6	1.23	34	0.176
779	A	3	2	1.19	34	0.059
780	A	7	6	1.18	34	0.176
781	A	11	10	1.19	24	0.417
782	A	9	8	1.21	24	0.333
783	A	4	3	1.00	24	0.125
784	A	9	8	1.16	24	0.333
785	A	11	10	1.15	24	0.417
786	A	17	16	1.14	26	0.615
787	A	14	13	1.17	26	0.500
788	A	8	7	0.97	26	0.269
789	A	12	11	1.14	26	0.423
790	A	15	14	1.12	26	0.538
791	A	19	18	1.11	28	0.643
792	A	16	15	1.14	28	0.536
793	A	13	12	1.21	28	0.429
794	A	12	11	1.25	28	0.393
795	A	17	16	1.12	28	0.571
796	A	5	4	1.30	19	0.211
797	A	14	13	1.28	19	0.684
798	A	5	4	1.06	19	0.211
799	A	14	13	1.09	19	0.684
800	A	5	4	1.29	24	0.167
801	A	14	13	1.22	24	0.542
802	A	18	17	1.10	24	0.708

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
803	A	5	4	1.22	24	0.167
804	A	14	13	1.18	24	0.542
805	A	2	2	1.00	27	0.074
806	A	2	2	1.00	20	0.100
807	A	2	2	1.91	21	0.095
808	A	2	2	1.00	22	0.091
809	A	4	3	1.08	18	0.167
810	A	4	3	1.06	18	0.167
811	A	6	5	1.03	23	0.217
812	A	7	6	1.03	22	0.273
813	A	7	6	1.00	20	0.300
814	A	2	2	1.00	15	0.133
815	A	2	2	1.00	21	0.095
816	A	2	2	1.00	19	0.105
817	A	2	2	1.00	24	0.083
818	A	2	2	1.00	24	0.083
819	A	3	3	1.00	21	0.143
820	A	4	4	1.00	13	0.308
821	A	3	3	1.00	25	0.120
822	A	4	4	1.00	15	0.267
823	A	6	5	1.00	19	0.263
824	A	1	1	1.00	35	0.029
825	A	2	2	1.00	37	0.054
826	A	2	2	1.00	29	0.069
827	A	2	2	1.00	36	0.056
828	A	2	2	1.00	43	0.047
829	A	6	5	1.16	21	0.238
830	A	2	2	1.00	27	0.074
831	A	2	2	1.00	27	0.074
832	A	6	5	1.06	27	0.185
833	A	5	4	1.00	20	0.200
834	A	7	6	1.00	29	0.207
835	A	1	1	1.00	20	0.050
836	A	2	2	1.00	25	0.080

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
837	A	4	3	1.00	28	0.107
838	A	5	4	1.00	26	0.154
839	A	1	1	1.00	11	0.091
840	A	2	2	1.00	19	0.105
841	A	2	2	1.00	15	0.133
842	A	3	2	1.00	14	0.143
843	A	4	3	1.00	17	0.176
844	A	4	3	1.00	13	0.231
845	B	3	2	3.00	14	0.143
846	B	4	3	3.00	17	0.176
847	B	4	3	3.00	13	0.231
848	A	1	1	1.00	9	0.111
849	A	2	2	1.00	15	0.133
850	A	2	2	1.00	13	0.154
851	A	3	3	1.00	19	0.158
852	A	2	2	1.00	11	0.182
853	A	3	3	1.16	17	0.176
854	A	1	1	1.00	9	0.111
855	A	2	2	1.00	19	0.105
856	A	1	1	1.00	7	0.143
857	A	2	2	1.00	21	0.095
858	A	1	1	1.00	9	0.111
859	A	2	2	1.00	19	0.105
860	A	1	1	1.00	7	0.143
861	A	2	2	1.00	21	0.095
862	A	4	3	1.00	17	0.176
863	A	5	4	1.00	30	0.133
864	A	8	7	1.00	20	0.350
865	A	7	6	1.00	20	0.300
866	A	8	7	1.00	23	0.304
867	A	7	6	1.00	25	0.240
868	A	1	1	1.00	11	0.091
869	A	2	2	1.00	21	0.095
870	A	1	1	1.00	9	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
871	A	2	2	1.00	23	0.087
872	A	2	2	1.00	11	0.182
873	A	3	3	1.00	21	0.143
874	A	2	2	1.00	9	0.222
875	A	3	3	1.00	23	0.130
876	A	3	3	1.16	14	0.214
877	A	4	4	1.00	15	0.267
878	A	2	2	1.00	17	0.118
879	A	2	2	1.00	17	0.118
880	A	2	2	1.00	34	0.059
881	A	2	2	1.00	30	0.067
882	A	2	2	1.00	21	0.095
883	A	2	2	1.00	23	0.087
884	A	2	2	1.00	25	0.080
885	A	12	11	1.07	25	0.440
886	A	2	2	0.93	13	0.154
887	A	3	3	1.00	24	0.125
888	A	3	3	1.00	22	0.136
889	A	3	3	1.00	30	0.100
890	A	4	4	1.00	27	0.148
891	A	5	4	1.00	23	0.174
892	A	8	7	0.75	28	0.250
893	A	8	7	1.00	25	0.280
894	A	9	8	0.67	29	0.276
895	A	9	9	0.86	16	0.562
896	A	8	8	0.84	16	0.500
897	A	5	5	0.70	16	0.312
898	A	9	8	0.65	16	0.500
899	A	11	10	0.72	16	0.625
900	A	4	4	1.00	24	0.167
901	N/A	4	0	1.00	14	0.000
902	N/A	4	0	1.00	14	0.000
903	N/A	3	0	1.00	17	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
904	N/A	3	0	1.00	17	0.000
905	N/A	2	0	1.00	10	0.000
906	N/A	2	0	1.00	12	0.000
907	N/A	1	0	1.00	10	0.000
908	N/A	1	0	1.00	14	0.000
909	N/A	1	0	1.00	12	0.000
910	N/A	1	0	1.00	14	0.000
911	A	3	2	1.00	37	0.054
912	A	3	2	1.00	38	0.053
913	A	5	4	1.00	40	0.100
914	A	6	5	1.15	40	0.125
915	A	6	5	1.10	18	0.278
916	A	5	4	1.15	21	0.190
917	A	5	4	0.86	22	0.182
918	A	4	3	1.10	21	0.143
919	A	4	3	1.10	24	0.125
920	A	12	11	1.02	19	0.579
921	A	11	10	1.02	24	0.417
922	A	2	2	1.00	19	0.105
923	A	4	3	1.08	17	0.176
924	A	1	1	1.00	15	0.067
925	A	1	1	1.00	17	0.059
926	A	3	2	1.00	17	0.118
927	A	3	2	1.00	17	0.118
928	A	1	1	1.00	17	0.059
929	A	7	6	1.07	17	0.353
930	A	6	5	1.00	11	0.455
931	A	3	3	1.08	11	0.273
932	A	5	5	1.16	13	0.385
933	A	2	2	1.00	21	0.095
934	A	6	5	0.85	16	0.312
935	A	2	2	0.96	13	0.154
936	A	4	3	1.05	16	0.188

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
937	A	4	3	1.19	12	0.250
938	A	5	4	1.09	18	0.222
939	A	2	2	1.00	17	0.118
940	A	1	1	1.00	17	0.059
941	C	1	1	1.03	27	0.037
942	A	2	2	1.00	37	0.054
943	A	3	3	1.00	17	0.176
944	A	6	5	0.71	17	0.294
945	A	1	1	1.00	15	0.067
946	A	4	3	1.00	14	0.214
947	A	1	1	1.00	17	0.059
948	A	2	2	1.00	13	0.154
949	A	2	2	1.00	15	0.133
950	A	8	7	1.10	15	0.467
951	A	7	6	1.12	15	0.400
952	A	1	1	1.00	9	0.111
953	A	1	1	1.00	9	0.111
954	A	2	2	1.00	19	0.105
955	A	4	3	1.00	15	0.200
956	A	4	3	1.00	16	0.188
957	A	5	4	1.77	15	0.267
958	A	6	5	1.77	15	0.333
959	A	5	4	1.07	25	0.160
960	A	2	2	1.00	11	0.182
961	A	3	2	1.00	17	0.118
962	A	8	7	1.13	22	0.318
963	A	5	4	0.71	13	0.308
964	A	5	4	0.68	15	0.267
965	A	4	3	1.21	19	0.158
966	A	4	3	1.32	21	0.143
967	A	3	3	1.00	17	0.176
968	A	8	7	0.98	19	0.368
969	A	8	7	1.23	19	0.368
970	A	7	6	1.15	17	0.353

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
971	A	5	4	1.14	17	0.235
972	A	7	6	1.08	17	0.353
973	A	4	4	1.00	22	0.182
974	A	6	5	1.00	11	0.455
975	A	6	5	1.00	13	0.385
976	A	6	5	1.17	11	0.455
977	A	6	5	1.00	15	0.333
978	A	5	4	1.00	11	0.364
979	A	6	5	0.93	13	0.385
980	A	5	4	1.06	11	0.364
981	A	4	3	1.00	11	0.273
982	A	3	3	1.00	11	0.273
983	A	7	6	1.00	19	0.316
984	A	2	2	1.00	23	0.087
985	A	3	2	1.00	13	0.154
986	A	4	3	1.00	11	0.273
987	A	4	3	1.00	15	0.200
988	A	4	3	1.00	19	0.158
989	A	4	3	1.00	19	0.158
990	A	4	3	1.00	19	0.158
991	A	3	2	1.00	15	0.133
992	A	4	3	1.00	15	0.200
993	A	4	3	1.00	12	0.250
994	A	4	3	1.00	16	0.188
995	F	0	0	N/A	0.000	N/A
996	F	0	0	N/A	0.000	N/A
997	B	3	3	4.09	31	0.097
998	A	6	5	1.05	25	0.200
999	A	3	2	1.00	27	0.074
1000	A	3	2	1.00	33	0.061
1001	A	3	2	1.00	34	0.059
1002	A	4	3	1.02	51	0.059
1003	A	3	2	1.02	49	0.041

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1004	A	3	2	1.00	43	0.047
1005	A	3	2	1.00	44	0.045
1006	A	10	9	0.99	20	0.450
1007	A	3	3	1.00	15	0.200
1008	A	3	3	1.30	15	0.200
1009	A	1	1	1.00	52	0.019
1010	A	1	1	1.00	57	0.018
1011	A	3	2	1.00	59	0.034
1012	A	3	2	1.02	58	0.034
1013	A	4	3	1.00	58	0.052
1014	A	4	3	1.02	57	0.053
1015	B	2	2	3.47	66	0.030
1016	A	5	4	1.00	31	0.129
1017	B	2	2	4.13	29	0.069
1018	C	7	6	3.57	20	0.300
1019	A	3	2	1.00	46	0.043
1020	A	1	1	1.00	15	0.067
1021	C	2	2	2.90	17	0.118
1022	C	2	2	3.55	33	0.061
1023	F	0	0	N/A	0.000	N/A
1024	A	1	1	1.00	38	0.026
1025	A	4	3	1.39	30	0.100

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

Listing of integrals

3.1 $\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

3.1.1 Optimal result

Integrand size = 19, antiderivative size = 145

$$\int \frac{1}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \frac{2\arctan\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
2/9*arctan((1+2^(1/3)*x)*3^(1/2)/(x^3+1)^(1/2))*3^(1/2)+2/9*2^(1/3)*(1+x)*
EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1
/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3
^(1/2))^2)^(1/2)
```

3.1.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.14 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.02

$$\int \frac{1}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \frac{4i\sqrt{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}},\arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(1+2\ 2^{2/3}-i\sqrt{3}\right)\sqrt{1+x^3}}$$

input
```
Integrate[1/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

output
```
((4*I)*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*Ellipti
cPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (
2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((1 + 2*2^(2/3)
- I*Sqrt[3])*Sqrt[1 + x^3])
```

3.1.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2559, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(x+2^{2/3}\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \ 2559$$

$$\frac{1}{3}\sqrt[3]{2}\int \frac{1}{\sqrt{x^3+1}}dx+\frac{\int \frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{x^3+1}}dx}{3\ 2^{2/3}}$$

$$\Big\downarrow \ 759$$

$$\frac{\int \frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{x^3+1}}dx}{3\ 2^{2/3}}+\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \ 2562$$

$$\frac{2}{3}\int \frac{1}{\frac{3\left(\sqrt[3]{2}x+1\right)^2}{x^3+1}+1}d\frac{\sqrt[3]{2}x+1}{\sqrt{x^3+1}}+$$
$$\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \ 216$$

$$\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+$$
$$\frac{2\arctan\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}$$

input
```
Int[1/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/(3*Sqrt[3]) + (2*2^(1/
3)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellip
ticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1
/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

3.1.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2559
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[2/
(3*c)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[1/(3*c)   Int[(c - 2*d*x)/((c
+ d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*
a*d^3, 0]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.1.4 Maple [A] (verified)

Time = 3.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\Pi\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{2^{\frac{2}{3}}-1},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}+1}\left(2^{\frac{2}{3}}-1\right)}$	139
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\Pi\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{2^{\frac{2}{3}}-1},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}+1}\left(2^{\frac{2}{3}}-1\right)}$	139

input
```
int(1/(2^(2/3)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)/(2^(2/3)-1)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))
^(1/2),(-3/2+1/2*I*3^(1/2))/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*
3^(1/2)))^(1/2))
```

3.1.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.51

$$\int \frac{1}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = -\frac{1}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(5x^3-2^{\frac{2}{3}}(x^5+x^2)+2^{\frac{1}{3}}(7x^4+4x)+2\right)\sqrt{x^3+1}}{6(2x^6+3x^3+1)}\right) + \frac{2}{3}\cdot 2^{\frac{1}{3}}\mathrm{weierstrassPInverse}(0,-4,x)$$

input
```
integrate(1/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/9*sqrt(3)*arctan(-1/6*sqrt(3)*(5*x^3 - 2^(2/3)*(x^5 + x^2) + 2^(1/3)*(7
*x^4 + 4*x) + 2)*sqrt(x^3 + 1)/(2*x^6 + 3*x^3 + 1)) + 2/3*2^(1/3)*weierstr
assPInverse(0, -4, x)
```

3.1.6 Sympy [F]

$$\int \frac{1}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate(1/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

output
```
Integral(1/(sqrt((x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

3.1.7 Maxima [F]

$$\int \frac{1}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate(1/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

3.1.8 Giac [F]

$$\int \frac{1}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate(1/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

3.1.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}\,(x+2^{2/3})}\,dx$$

input
```
int(1/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
```

output
```
int(1/((x^3 + 1)^(1/2)*(x + 2^(2/3))), x)
```

3.2 $\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

3.2.1 Optimal result

Integrand size = 23, antiderivative size = 160

$$\int \frac{1}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}} - \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-2/9*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*3^(1/2)-2/9*2^(1/3)*(1-x
)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^
(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-
x+3^(1/2))^2)^(1/2)
```

3.2.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.92

$$\int \frac{1}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = -\frac{4i\sqrt{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\sqrt{1+x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}},\arcsin\left(\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(1+2\;2^{2/3}-i\sqrt{3}\right)\sqrt{1-x^3}}$$

input
```
Integrate[1/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

output
```
((-4*I)*Sqrt[2]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*El
lipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3
] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((1 + 2*2^(
2/3) - I*Sqrt[3])*Sqrt[1 - x^3])
```

3.2.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2559, 27, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx$$

$$\Big\downarrow \; 2559$$

$$\frac{1}{3}\sqrt[3]{2}\int \frac{1}{\sqrt{1-x^3}}dx+\frac{\int \frac{2^{2/3}\left(\sqrt[3]{2}x+1\right)}{(2^{2/3}-x)\sqrt{1-x^3}}dx}{3\;2^{2/3}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\sqrt[3]{2}\int \frac{1}{\sqrt{1-x^3}}dx+\frac{1}{3}\int \frac{\sqrt[3]{2}x+1}{(2^{2/3}-x)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \; 759$$

$$\frac{1}{3}\int \frac{\sqrt[3]{2}x+1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}dx-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \text{2562}$$

$$-\frac{2}{3}\int \frac{1}{\frac{3\left(1-\sqrt[3]{2}x\right)^2}{1-x^3}+1}d\frac{1-\sqrt[3]{2}x}{\sqrt{1-x^3}}-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}$$

input
```
Int[1/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/(3*Sqrt[3]) - (2*2^(1
/3)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*Elli
pticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(
1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

3.2. $\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

3.2.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2559
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[2/
(3*c)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[1/(3*c)   Int[(c - 2*d*x)/((c
+ d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*
a*d^3, 0]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.2.4 Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\Pi\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{-2^{\frac{2}{3}}-\frac{1}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}\,\left(-2^{\frac{2}{3}}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}$	143
elliptic	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\Pi\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{-2^{\frac{2}{3}}-\frac{1}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}\,\left(-2^{\frac{2}{3}}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}$	143

input
```
int(1/(2^(2/3)-x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-2
^(2/3)-1/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*
3^(1/2))^(1/2),I*3^(1/2)/(-2^(2/3)-1/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2
*I*3^(1/2)))^(1/2))
```

3.2.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.49

$$\int \frac{1}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = -\frac{1}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(5x^3-2^{\frac{2}{3}}(x^5-x^2)-2^{\frac{1}{3}}(7x^4-4x)-2\right)\sqrt{-x^3+1}}{6\,(2x^6-3x^3+1)}\right) - \frac{2}{3}i\cdot 2^{\frac{1}{3}}\text{weierstrassPInverse}(0,4,x)$$

input
```
integrate(1/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/9*sqrt(3)*arctan(-1/6*sqrt(3)*(5*x^3 - 2^(2/3)*(x^5 - x^2) - 2^(1/3)*(7
*x^4 - 4*x) - 2)*sqrt(-x^3 + 1)/(2*x^6 - 3*x^3 + 1)) - 2/3*I*2^(1/3)*weier
strassPInverse(0, 4, x)
```

3.2.6 Sympy [F]

$$\int \frac{1}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = -\int \frac{1}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx$$

input
```
integrate(1/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

output
```
-Integral(1/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

3.2.7 Maxima [F]

$$\int \frac{1}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \int -\frac{1}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate(1/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

3.2.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad
Argument
```

3.2.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx=-\int \frac{1}{\sqrt{1-x^3}\,\left(x-2^{2/3}\right)}\,dx$$

input
```
int(-1/((1 - x^3)^(1/2)*(x - 2^(2/3))),x)
```

output
```
-int(1/((1 - x^3)^(1/2)*(x - 2^(2/3))), x)
```

3.3 $\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

3.3.1 Optimal result

Integrand size = 21, antiderivative size = 163

$$\int \frac{1}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}} - \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
-2/9*arctanh((1-2^(1/3)*x)*3^(1/2)/(x^3-1)^(1/2))*3^(1/2)-2/9*2^(1/3)*(1-x
)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^
(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-
x-3^(1/2))^2)^(1/2)
```

3.3.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = -\frac{4i\sqrt{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\sqrt{1+x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}},\arcsin\left(\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(1+2\ 2^{2/3}-i\sqrt{3}\right)\sqrt{-1+x^3}}$$

input

```
Integrate[1/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

output

```
((-4*I)*Sqrt[2]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*El
lipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3
] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((1 + 2*2^(
2/3) - I*Sqrt[3])*Sqrt[-1 + x^3])
```

3.3.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2559, 27, 760, 2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{ 2559}$$

$$\frac{1}{3}\sqrt[3]{2}\int \frac{1}{\sqrt{x^3-1}}dx+\frac{\int \frac{2^{2/3}\left(\sqrt[3]{2}x+1\right)}{\left(2^{2/3}-x\right)\sqrt{x^3-1}}dx}{3\ 2^{2/3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\sqrt[3]{2}\int \frac{1}{\sqrt{x^3-1}}dx+\frac{1}{3}\int \frac{\sqrt[3]{2}x+1}{\left(2^{2/3}-x\right)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{ 760}$$

$$\frac{1}{3}\int \frac{\sqrt[3]{2}x+1}{\left(2^{2/3}-x\right)\sqrt{x^3-1}}dx-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{2562}$$

$$-\frac{2}{3}\int \frac{1}{1-\frac{3\left(1-\sqrt[3]{2}x\right)^2}{x^3-1}}d\frac{1-\sqrt[3]{2}x}{\sqrt{x^3-1}}-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\text{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}$$

input
```
Int[1/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/(3*Sqrt[3]) - (2*2^
(1/3)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*El
lipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3
^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

3.3. $\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

3.3.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2559
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[2/
(3*c)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[1/(3*c)   Int[(c - 2*d*x)/((c
+ d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*
a*d^3, 0]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.3.4 Maple [A] (verified)

Time = 2.87 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\Pi\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-2^{\frac{2}{3}}+1},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}-1}\,\left(-2^{\frac{2}{3}}+1\right)}$	143
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\Pi\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-2^{\frac{2}{3}}+1},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}-1}\,\left(-2^{\frac{2}{3}}+1\right)}$	143

input
```
int(1/(2^(2/3)-x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)/(-2^(2/3)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2
)))^(1/2),(3/2+1/2*I*3^(1/2))/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I
*3^(1/2)))^(1/2))
```

3.3.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   catd
ef: division by zero
```

3.3.6 Sympy [F]

$$\int \frac{1}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = -\int \frac{1}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

input
```
integrate(1/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

output
```
-Integral(1/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

3.3.7 Maxima [F]

$$\int \frac{1}{(2^{2/3} - x)\sqrt{-1 + x^3}}\, dx = \int -\frac{1}{\sqrt{x^3 - 1}\left(x - 2^{\frac{2}{3}}\right)}\, dx$$

input
```
integrate(1/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

3.3.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(2^{2/3} - x)\sqrt{-1 + x^3}}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[2
]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Argument Value
```

3.3.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2^{2/3} - x)\sqrt{-1 + x^3}}\, dx = -\int \frac{1}{\sqrt{x^3 - 1}\,(x - 2^{2/3})}\, dx$$

input
```
int(-1/((x^3 - 1)^(1/2)*(x - 2^(2/3))),x)
```

output
```
-int(1/((x^3 - 1)^(1/2)*(x - 2^(2/3))), x)
```

3.4 $\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

3.4.1 Optimal result

Integrand size = 21, antiderivative size = 156

$$\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
2/9*arctanh((1+2^(1/3)*x)*3^(1/2)/(-x^3-1)^(1/2))*3^(1/2)+2/9*2^(1/3)*(1+x
)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^
(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1
+x-3^(1/2))^2)^(1/2)
```

3.4.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{4i\sqrt{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}},\arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{-1-x^3}}$$

input
```
Integrate[1/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

output
```
((4*I)*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*Ellipti
cPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (
2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((1 + 2*2^(2/3)
- I*Sqrt[3])*Sqrt[-1 - x^3])
```

3.4.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2559, 760, 2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(x+2^{2/3}\right)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{2559}$$

$$\frac{1}{3}\sqrt[3]{2}\int \frac{1}{\sqrt{-x^3-1}}dx+\frac{\int \frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{-x^3-1}}dx}{3\ 2^{2/3}}$$

$$\Big\downarrow \text{760}$$

$$\frac{\int \frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{-x^3-1}}dx}{3\ 2^{2/3}}+\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{2562}$$

$$\frac{2}{3}\int \frac{1}{1-\frac{3\left(\sqrt[3]{2}x+1\right)^2}{-x^3-1}}d\frac{\sqrt[3]{2}x+1}{\sqrt{-x^3-1}}+$$

$$\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+$$
$$\frac{2\text{arctanh}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}$$

input
```
Int[1/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/(3*Sqrt[3]) + (2*2^(
1/3)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Ell
ipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^
(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

3.4.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2559
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[2/
(3*c)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[1/(3*c)   Int[(c - 2*d*x)/((c
+ d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*
a*d^3, 0]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.4.4 Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\Pi\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{2^{\frac{2}{3}}+\frac{1}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}-1}\,\left(2^{\frac{2}{3}}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}$	139
elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\Pi\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{2^{\frac{2}{3}}+\frac{1}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}-1}\,\left(2^{\frac{2}{3}}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}$	139

input
```
int(1/(2^(2/3)+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(2^
(2/3)+1/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2),I*3^(1/2)/(2^(2/3)+1/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*
3^(1/2)))^(1/2))
```

3.4.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   catd
ef: division by zero
```

3.4.6 Sympy [F]

$$\int \frac{1}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate(1/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

3.4.7 Maxima [F]

$$\int \frac{1}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate(1/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

3.4.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[1
]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Bad Argument Value
```

3.4.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3-1}\,\left(x+2^{2/3}\right)}\,dx$$

input
```
int(1/((- x^3 - 1)^(1/2)*(x + 2^(2/3))),x)
```

output
```
int(1/((- x^3 - 1)^(1/2)*(x + 2^(2/3))), x)
```

3.5 $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

3.5.1 Optimal result

Integrand size = 33, antiderivative size = 280

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \frac{2\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}} + \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/9*arctan(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)*3^(1/2)/(b*x^3+a)^(1/2))/b^
(1/3)*3^(1/2)/a^(1/2)+2/9*2^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x
+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*
6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a
^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(1/3)/b^(1/3)/(b*x^3+a)^(1/2)/(a^(1
/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.5.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.17 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.59

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\frac{2i\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\text{EllipticPi}\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}},\arcsin\left(\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right),\sqrt[3]{-1}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{a+bx^3}}$$

input
```
Integrate[1/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

output
```
((-2*I)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 - (b
^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1
/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1
/3))*a^(1/3))]], (-1)^(1/3)])/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[a + b*x
^3])
```

3.5.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2559, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{2559}$$

$$\frac{\sqrt[3]{2}\int \frac{1}{\sqrt{bx^3+a}}dx}{3\sqrt[3]{a}}+\frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a}\right)\sqrt{bx^3+a}}dx}{3\ 2^{2/3}\sqrt[3]{a}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a}\right)\sqrt{bx^3+a}}dx}{3\ 2^{2/3}\sqrt[3]{a}}+\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$\Big\downarrow$ 2562

$$\frac{2\int \frac{1}{\frac{3\sqrt[3]{a}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{bx^3+a}+1}d\frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{bx^3+a}}}{3\sqrt[3]{b}}+\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$\Big\downarrow$ 216

$$\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

input
```
Int[1/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

3.5. $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}dx$

output
```
(2*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a + b*x^3]]
)/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) + (2*2^(1/3)*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3
)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*Sqrt[a + b*x^3])
```

3.5.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2559
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[2/
(3*c)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[1/(3*c)   Int[(c - 2*d*x)/((c
+ d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*
a*d^3, 0]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.5.4 Maple [F]

$$\int \frac{1}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}+b^{\frac{1}{3}} x\right) \sqrt{b\,x^{3}+a}}dx$$

input
```
int(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output
```
int(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

3.5.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

3.5.6 Sympy [F]

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{1}{\sqrt{a+bx^3}\cdot\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate(1/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

3.5.7 Maxima [F]

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{1}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

3.5.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac
")
```

output
```
Timed out
```

3.5.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{1}{\sqrt{b\,x^3+a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\,dx$$

input
```
int(1/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
```

output
```
int(1/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

3.6 $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

3.6.1 Optimal result

Integrand size = 35, antiderivative size = 288

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

output
```
-2/9*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)*3^(1/2)/(-b*x^3+a)^(1/2))/
b^(1/3)*3^(1/2)/a^(1/2)-2/9*2^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3
)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(
1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(1/3)/b^(1/3)/(-b*x^3+a)^(1/2)
/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.6.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.16 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.58

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \frac{2i\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\text{EllipticPi}\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}},\arcsin\left(\cdots\right.\right.}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{a-bx^3}}$$

input
```
Integrate[1/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
((2*I)*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^
(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/
3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/
3))*a^(1/3))]], (-1)^(1/3)])/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[a - b*x^
3])
```

3.6.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2559, 27, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx$$

$$\Big\downarrow \text{2559}$$

$$\frac{\sqrt[3]{2}\int \frac{1}{\sqrt{a-bx^3}}dx}{3\sqrt[3]{a}}+\frac{\int \frac{2^{2/3}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx}{3\ 2^{2/3}\sqrt[3]{a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[3]{2}\int \frac{1}{\sqrt{a-bx^3}}dx}{3\sqrt[3]{a}}+\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx}{3\sqrt[3]{a}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx}{3\sqrt[3]{a}} - \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\Big\downarrow \text{2562}$$

$$-\frac{2\int \frac{1}{\frac{3\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)^2}{a-bx^3}+1}d\frac{\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{a-bx^3}}}{3\sqrt[3]{b}} - \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}} - \frac{2\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

input
```
Int[1/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

3.6. $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx$

output
```
(-2*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a - b*x^3]
])/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) - (2*2^(1/3)*Sqrt[2 + Sqrt[3]]*(a^(1/3) - b
^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*
a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*
x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/
3)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b
^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

3.6.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2559
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[2/
(3*c)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[1/(3*c)   Int[(c - 2*d*x)/((c
+ d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*
a*d^3, 0]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.6.4 Maple [F]

$$\int \frac{1}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}-b^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3}+a}}dx$$

input
```
int(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

output
```
int(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

3.6.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="fri
cas")
```

output
```
Timed out
```

3.6.6 Sympy [F]

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\int \frac{1}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

input
```
integrate(1/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

output
```
-Integral(1/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x
**3)), x)
```

3.6.7 Maxima [F]

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int -\frac{1}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="max
ima")
```

output
```
-integrate(1/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

3.6.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="gia
c")
```

output
```
Timed out
```

3.6.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{1}{\sqrt{a-b\,x^3}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\,dx$$

input
```
int(1/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
```

output
```
int(1/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

3.7 $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

3.7.1 Optimal result

Integrand size = 36, antiderivative size = 297

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}} - \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

output
```
-2/9*arctanh(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)*3^(1/2)/(b*x^3-a)^(1/2))/
b^(1/3)*3^(1/2)/a^(1/2)-2/9*2^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(
(a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2
)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/a^(1/3)/b^(1/3)/(b*x^3-a)^(1/2)/
(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.7.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.56

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\frac{2i\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\text{EllipticPi}\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}},\arcsin\left(\right.\right.}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{-a+bx^3}}$$

input
```
Integrate[1/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
((2*I)*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^
(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/
3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/
3))*a^(1/3))]], (-1)^(1/3)])/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[-a + b*x
^3])
```

3.7.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2559, 27, 760, 2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{2559}$$

$$\frac{\sqrt[3]{2}\int \frac{1}{\sqrt{bx^3-a}}dx}{3\sqrt[3]{a}}+\frac{\int \frac{2^{2/3}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx}{3\ 2^{2/3}\sqrt[3]{a}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt[3]{2}\int \frac{1}{\sqrt{bx^3-a}}dx}{3\sqrt[3]{a}}+\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx}{3\sqrt[3]{a}}$$

$$\Big\downarrow \; 760$$

$$\frac{\int \frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx}{3\sqrt[3]{a}}-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\Big\downarrow \; 2562$$

$$-\frac{2\int \frac{1}{1-\frac{3\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)^2}{bx^3-a}}d\frac{\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{bx^3-a}}}{3\sqrt[3]{b}}-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\Big\downarrow \; 219$$

$$-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

input
```
Int[1/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

3.7. $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}dx$

output
```
(-2*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[-a + b*x^
3]])/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) - (2*2^(1/3)*Sqrt[2 - Sqrt[3]]*(a^(1/3) -
 b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3]
)*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3
)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(
1/3)*b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3)
 - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

3.7.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2559
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[2/
(3*c)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[1/(3*c)   Int[(c - 2*d*x)/((c
+ d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*
a*d^3, 0]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.7.4 Maple [F]

$$\int \frac{1}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}-b^{\frac{1}{3}} x\right) \sqrt{b\,x^{3}-a}}d x$$

input
```
int(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output
```
int(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

3.7.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

3.7.6 Sympy [F]

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=-\int \frac{1}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

input
```
integrate(1/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

output
```
-Integral(1/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b
*x**3)), x)
```

3.7.7 Maxima [F]

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int -\frac{1}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="maxi
ma")
```

output
```
-integrate(1/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

3.7.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="giac
")
```

output
```
Timed out
```

3.7.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{1}{\sqrt{b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\,dx$$

input
```
int(1/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
```

output
```
int(1/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

3.8 $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

3.8.1 Optimal result

Integrand size = 36, antiderivative size = 293

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}+\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

output
```
2/9*arctanh(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)*3^(1/2)/(-b*x^3-a)^(1/2))/
b^(1/3)*3^(1/2)/a^(1/2)+2/9*2^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(
1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/a^(1/3)/b^(1/3)/(-b*x^3-a)^(1/2)/(-
a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.8.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.57

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\frac{2i\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\text{EllipticPi}\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}},\arcsin\left(\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right),\sqrt[3]{-1}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{-a-bx^3}}$$

input
```
Integrate[1/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
((-2*I)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 - (b
^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1
/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1
/3))*a^(1/3))]], (-1)^(1/3)])/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[-a - b*
x^3])
```

3.8.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2559, 760, 2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$$

$$\Big\downarrow \text{ 2559}$$

$$\frac{\sqrt[3]{2}\int \frac{1}{\sqrt{-bx^3-a}}dx}{3\sqrt[3]{a}}+\frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a}\right)\sqrt{-bx^3-a}}dx}{3\ 2^{2/3}\sqrt[3]{a}}$$

$$\Big\downarrow \text{ 760}$$

$$\frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a}\right)\sqrt{-bx^3-a}}dx}{3\ 2^{2/3}\sqrt[3]{a}}+$$

$$\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$\Big\downarrow$ 2562

$$\frac{2\int \frac{1}{1-\frac{3\sqrt[3]{a}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{-bx^3-a}}d\frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{-bx^3-a}}}{3\sqrt[3]{b}}+$$

$$\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$\Big\downarrow$ 219

$$\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{2\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

input
```
Int[1/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

3.8. $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}dx$

output
```
(2*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-a - b*x^3
]])/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) + (2*2^(1/3)*Sqrt[2 - Sqrt[3]]*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1
/3)*b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

3.8.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2559
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[2/
(3*c)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[1/(3*c)   Int[(c - 2*d*x)/((c
+ d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*
a*d^3, 0]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.8.4 Maple [F]

$$\int \frac{1}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}+b^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3}-a}}dx$$

input
```
int(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

output
```
int(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

3.8.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="fri
cas")
```

output
```
Timed out
```

3.8.6 Sympy [F]

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \int \frac{1}{\sqrt{-a-bx^3}\cdot\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate(1/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

output
```
Integral(1/(sqrt(-a - b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

3.8.7 Maxima [F]

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{1}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(1/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

3.8.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="gia
c")
```

output
```
Timed out
```

3.8.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{1}{\sqrt{-b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\,dx$$

input
```
int(1/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
```

output
```
int(1/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

3.9 $\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

3.9.1 Optimal result

Integrand size = 24, antiderivative size = 249

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \frac{2\arctan\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}c^{3/2}d} + \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(c+2^{2/3}dx)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd\sqrt{\frac{c(c+2^{2/3}dx)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

output
```
2/9*arctan((2*d*x+c)*3^(1/2)*c^(1/2)/(4*d^3*x^3+c^3)^(1/2))/c^(3/2)/d*3^(1
/2)+2/9*2^(1/3)*(c+2^(2/3)*d*x)*EllipticF((2^(2/3)*d*x+c*(1-3^(1/2)))/(2^(
2/3)*d*x+c*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((c^2-2^(
2/3)*c*d*x+2*2^(1/3)*d^2*x^2)/(2^(2/3)*d*x+c*(1+3^(1/2)))^2)^(1/2)*3^(3/4)
/c/d/(4*d^3*x^3+c^3)^(1/2)/(c*(c+2^(2/3)*d*x)/(2^(2/3)*d*x+c*(1+3^(1/2)))^
2)^(1/2)
```

3.9.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.17 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.68

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = -\frac{i2^{5/6}\sqrt{\frac{\sqrt[3]{2}c+2dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{2^{2/3}-\frac{2\sqrt[3]{2}dx}{c}+\frac{4d^2x^2}{c^2}}\ \text{EllipticPi}\left(\frac{i\sqrt[3]{2}\sqrt{3}}{2+\sqrt[3]{-2}},\arcsin\left(\frac{\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}}{\sqrt[6]{2}}\right),\sqrt[3]{-1}\right)}{\left(2+\sqrt[3]{-2}\right)d\sqrt{c^3+4d^3x^3}}$$

input
```
Integrate[1/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

output
```
((-I)*2^(5/6)*Sqrt[(2^(1/3)*c + 2*d*x)/((1 + (-1)^(1/3))*c)]*Sqrt[2^(2/3)
- (2*2^(1/3)*d*x)/c + (4*d^2*x^2)/c^2]*EllipticPi[(I*2^(1/3)*Sqrt[3])/(2 +
 (-2)^(1/3)), ArcSin[Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))
*c)]/2^(1/6)], (-1)^(1/3)])/((2 + (-2)^(1/3))*d*Sqrt[c^3 + 4*d^3*x^3])
```

3.9.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2559, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

$$\Big\downarrow \text{2559}$$

$$\frac{2\int \frac{1}{\sqrt{c^3+4d^3x^3}}dx}{3c}+\frac{\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}dx}{3c}$$

$$\Big\downarrow \text{759}$$

$$\frac{\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}dx}{3c}+\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(c+2^{2/3}dx)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd\sqrt{\frac{c(c+2^{2/3}dx)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

$$\Big\downarrow \text{2562}$$

$$\frac{2\int \frac{1}{\frac{3c(c+2dx)^2}{c^3+4d^3x^3}+1}d\frac{c+2dx}{c\sqrt{c^3+4d^3x^3}}}{3d}+\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(c+2^{2/3}dx)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd\sqrt{\frac{c(c+2^{2/3}dx)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(c+2^{2/3}dx)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd\sqrt{\frac{c(c+2^{2/3}dx)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}+\frac{2\arctan\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}c^{3/2}d}$$

input
```
Int[1/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/(3*Sqrt[3]
*c^(3/2)*d) + (2*2^(1/3)*Sqrt[2 + Sqrt[3]]*(c + 2^(2/3)*d*x)*Sqrt[(c^2 - 2
^(2/3)*c*d*x + 2*2^(1/3)*d^2*x^2)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)^2]*Ellip
ticF[ArcSin[((1 - Sqrt[3])*c + 2^(2/3)*d*x)/((1 + Sqrt[3])*c + 2^(2/3)*d*x
)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*c*d*Sqrt[(c*(c + 2^(2/3)*d*x))/((1 + Sqrt[
3])*c + 2^(2/3)*d*x)^2]*Sqrt[c^3 + 4*d^3*x^3])
```

3.9. $\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

3.9.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2559
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[2/
(3*c)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[1/(3*c)   Int[(c - 2*d*x)/((c
+ d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*
a*d^3, 0]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.9.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(200) = 400$.

Time = 1.38 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.99

method	result
default	$\frac{2\left(\frac{\left(\frac{2^{\frac{1}{3}}}{4}-\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}-\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}\right)\sqrt{\frac{x-\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}}{\frac{\left(\frac{2^{\frac{1}{3}}}{4}-\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}-\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}}}\sqrt{\frac{x+\frac{2^{\frac{1}{3}}c}{2d}}{\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}+\frac{2^{\frac{1}{3}}c}{2d}}}\sqrt{\frac{x-\frac{\left(\frac{2^{\frac{1}{3}}}{4}-i\sqrt{3}\right.}{d}}{\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}-\frac{\left(\right.}{}}}}{d\sqrt{4d^3x^3+c^3}\left(\right.}$
elliptic	$\frac{2\left(\frac{\left(\frac{2^{\frac{1}{3}}}{4}-\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}-\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}\right)\sqrt{\frac{x-\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}}{\frac{\left(\frac{2^{\frac{1}{3}}}{4}-\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}-\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}}}\sqrt{\frac{x+\frac{2^{\frac{1}{3}}c}{2d}}{\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}+\frac{2^{\frac{1}{3}}c}{2d}}}\sqrt{\frac{x-\frac{\left(\frac{2^{\frac{1}{3}}}{4}-i\sqrt{3}\right.}{d}}{\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}-}}}{d\sqrt{4d^3x^3+c^3}\left(\right.}$

input
```
int(1/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^
(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4
*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)*((
x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/
d))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I
*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d
^3*x^3+c^3)^(1/2)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d)*EllipticPi
(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*3^(1/2)*
2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2),((1/4*2^(1/3)
+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4
*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1
/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1
/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/2))
```

3.9. $\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

3.9.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.40

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$
$$= \left[-\frac{\sqrt{3}\sqrt{-c}d^2\log\left(\frac{2\,d^6x^6-36\,cd^5x^5-18\,c^2d^4x^4+28\,c^3d^3x^3+18\,c^4d^2x^2-c^6+\sqrt{3}\left(4\,d^4x^4-10\,cd^3x^3-18\,c^2d^2x^2-8\,c^3dx-c^4\right)\sqrt{4\,d^3x^3+c^3}\sqrt{-}}{d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx+c^6}\right.}{18\,c^2d^3}\right.$$
$$\left.-\frac{\sqrt{3}\sqrt{c}d^2\arctan\left(\frac{\sqrt{3}\sqrt{4\,d^3x^3+c^3}\left(2\,d^3x^3-6\,cd^2x^2-6\,c^2dx-c^3\right)\sqrt{c}}{3\left(8\,cd^4x^4+4\,c^2d^3x^3+2\,c^4dx+c^5\right)}\right)-6\,c\sqrt{d^3}\text{weierstrassPInverse}\left(0,-\frac{c^3}{d^3},x\right)}{9\,c^2d^3}\right]$$

input
```
integrate(1/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

output
```
[-1/18*(sqrt(3)*sqrt(-c)*d^2*log((2*d^6*x^6 - 36*c*d^5*x^5 - 18*c^2*d^4*x^
4 + 28*c^3*d^3*x^3 + 18*c^4*d^2*x^2 - c^6 + sqrt(3)*(4*d^4*x^4 - 10*c*d^3*
x^3 - 18*c^2*d^2*x^2 - 8*c^3*d*x - c^4)*sqrt(4*d^3*x^3 + c^3)*sqrt(-c))/(d
^6*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x^2 +
6*c^5*d*x + c^6)) - 12*c*sqrt(d^3)*weierstrassPInverse(0, -c^3/d^3, x))/(c
^2*d^3), -1/9*(sqrt(3)*sqrt(c)*d^2*arctan(1/3*sqrt(3)*sqrt(4*d^3*x^3 + c^3
)*(2*d^3*x^3 - 6*c*d^2*x^2 - 6*c^2*d*x - c^3)*sqrt(c)/(8*c*d^4*x^4 + 4*c^2
*d^3*x^3 + 2*c^4*d*x + c^5)) - 6*c*sqrt(d^3)*weierstrassPInverse(0, -c^3/d
^3, x))/(c^2*d^3)]
```

3.9.6 Sympy [F]

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

input
```
integrate(1/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

output
```
Integral(1/((c + d*x)*sqrt(c**3 + 4*d**3*x**3)), x)
```

3.9.7 Maxima [F]

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{1}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

3.9.8 Giac [F]

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{1}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

3.9.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{1}{\sqrt{c^3+4\,d^3\,x^3}\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)),x)
```

output
```
int(1/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)), x)
```

3.10 $\int \frac{1}{(1+\sqrt{3}+x)\sqrt{1+x^3}}\,dx$

3.10.1 Optimal result

Integrand size = 20, antiderivative size = 146

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} + \frac{\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
1/3*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2
)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(1/4)/(x^3+1)^(1/2)/((1
+x)/(1+x+3^(1/2))^2)^(1/2)+arctan((1+x)*(3+2*3^(1/2))^(1/2)/(x^3+1)^(1/2))
/(9+6*3^(1/2))^(1/2)
```

3.10.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.19 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{4\sqrt{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}},\arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{1+x^3}}$$

input
```
Integrate[1/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
(-4*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*EllipticPi
[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]
/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((3*I + (1 + 2*I)*Sqrt[
3])*Sqrt[1 + x^3])
```

3.10.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2560, 27, 759, 2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{ 2560}$$

$$\frac{\int \frac{1}{\sqrt{x^3+1}}dx}{2\sqrt{3}}-\frac{\int \frac{6\left(x-\sqrt{3}+1\right)}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx}{12\sqrt{3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1}{\sqrt{x^3+1}}dx}{2\sqrt{3}}-\frac{\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx}{2\sqrt{3}}$$

$\downarrow$ 759

$$\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\int\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx}{2\sqrt{3}}$$

$\downarrow$ 2565

$$\frac{\int\frac{1}{\frac{\left(3+2\sqrt{3}\right)(x+1)^2}{x^3+1}+1}d\frac{x+1}{\sqrt{x^3+1}}}{\sqrt{3}}+\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\downarrow$ 216

$$\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

input
```
Int[1/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/Sqrt[3*(3 + 2*Sqrt[3])
] + (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ell
ipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3
/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

3.10.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2560
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[-6
*a*(d^3/(c*(b*c^3 - 28*a*d^3)))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[1/(c
*(b*c^3 - 28*a*d^3))   Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20
*a*b*c^3*d^3 - 8*a^2*d^6, 0]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

3.10.4 Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\,\Pi\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^{3}+1}}$	132
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\,\Pi\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^{3}+1}}$	132

input
```
int(1/(1+x+3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^
(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3+1)^(1/2)*3^(1/2)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(
1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^
(1/2)))^(1/2))
```

3.10.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.39

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{1}{6}\sqrt{2\sqrt{3}-3}\arctan\left(\frac{\left(\sqrt{3}(x^2-4x-2)-6x-6\right)\sqrt{2\sqrt{3}-3}}{6\sqrt{x^3+1}}\right) + \frac{1}{3}\sqrt{3}\text{weierstrassPInverse}(0,-4,x)$$

input
```
integrate(1/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(2*sqrt(3) - 3)*arctan(1/6*(sqrt(3)*(x^2 - 4*x - 2) - 6*x - 6)*sq
rt(2*sqrt(3) - 3)/sqrt(x^3 + 1)) + 1/3*sqrt(3)*weierstrassPInverse(0, -4,
x)
```

3.10.6 Sympy [F]

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{(x+1)(x^2-x+1)}\left(x+1+\sqrt{3}\right)}\,dx$$

input
```
integrate(1/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

output
```
Integral(1/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

3.10.7 Maxima [F]

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate(1/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

3.10.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[2
]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argument Value
```

3.10.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Hanged}$$

input
```
int(1/((x^3 + 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

output
```
\text{Hanged}
```

3.11 $\int \frac{1}{(1+\sqrt{3}-x)\sqrt{1-x^3}}\,dx$

3.11.1 Optimal result

Integrand size = 24, antiderivative size = 164

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} - \frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-1/3*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/
2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(1/4)/(-x^3+1)^(1/2)/(
(1-x)/(1-x+3^(1/2))^2)^(1/2)-arctan((1-x)*(3+2*3^(1/2))^(1/2)/(-x^3+1)^(1/
2))/(9+6*3^(1/2))^(1/2)
```

3.11.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \frac{4\sqrt{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\sqrt{1+x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}},\arcsin\left(\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{1-x^3}}$$

input
```
Integrate[1/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

output
```
(4*Sqrt[2]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*Ellipti
cPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)
*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((3*I + (1 + 2*I)*Sq
rt[3])*Sqrt[1 - x^3])
```

3.11.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2560, 27, 759, 2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(-x+\sqrt{3}+1\right)\sqrt{1-x^3}}\,dx$$

$$\Big\downarrow \; 2560$$

$$\frac{\int \frac{1}{\sqrt{1-x^3}}dx}{2\sqrt{3}}+\frac{\int -\frac{6\left(-x-\sqrt{3}+1\right)}{\left(-x+\sqrt{3}+1\right)\sqrt{1-x^3}}dx}{12\sqrt{3}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{1}{\sqrt{1-x^3}}dx}{2\sqrt{3}}-\frac{\int \frac{-x-\sqrt{3}+1}{\left(-x+\sqrt{3}+1\right)\sqrt{1-x^3}}dx}{2\sqrt{3}}$$

$$\Big\downarrow \ 759$$

$$-\frac{\int \frac{-x-\sqrt{3}+1}{\left(-x+\sqrt{3}+1\right)\sqrt{1-x^3}}dx}{2\sqrt{3}}-\frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \ 2565$$

$$-\frac{\int \frac{1}{\frac{\left(3+2\sqrt{3}\right)(1-x)^2}{1-x^3}+1}d\frac{1-x}{\sqrt{1-x^3}}}{\sqrt{3}}-\frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \ 216$$

$$-\frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

input
```
Int[1/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

output
```
-(ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]]/Sqrt[3*(3 + 2*Sqrt[3
]])]) - (Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*
EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3
^(3/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

3.11. $\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$

3.11.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2560
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[-6
*a*(d^3/(c*(b*c^3 - 28*a*d^3)))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[1/(c
*(b*c^3 - 28*a*d^3))   Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20
*a*b*c^3*d^3 - 8*a^2*d^6, 0]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

3.11.4 Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\Pi\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{-\frac{3}{2}-\sqrt{3}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}\left(-\frac{3}{2}-\sqrt{3}+\frac{i\sqrt{3}}{2}\right)}$	143
elliptic	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\Pi\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{-\frac{3}{2}-\sqrt{3}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}\left(-\frac{3}{2}-\sqrt{3}+\frac{i\sqrt{3}}{2}\right)}$	143

input
```
int(1/(1-x+3^(1/2))/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-3
/2-3^(1/2)+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*
3^(1/2))^(1/2),I*3^(1/2)/(-3/2-3^(1/2)+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2
*I*3^(1/2)))^(1/2))
```

3.11.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.40

$$\begin{aligned}\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx &= -\frac{1}{6}\sqrt{2\sqrt{3}-3}\arctan\left(\frac{\sqrt{-x^3+1}\left(\sqrt{3}(x^2+4x-2)+6x-6\right)\sqrt{2\sqrt{3}-3}}{6(x^3-1)}\right)\\ &\quad -\frac{1}{3}i\sqrt{3}\text{weierstrassPInverse}(0,4,x)\end{aligned}$$

input
```
integrate(1/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(2*sqrt(3) - 3)*arctan(1/6*sqrt(-x^3 + 1)*(sqrt(3)*(x^2 + 4*x - 2
) + 6*x - 6)*sqrt(2*sqrt(3) - 3)/(x^3 - 1)) - 1/3*I*sqrt(3)*weierstrassPIn
verse(0, 4, x)
```

3.11.6 Sympy [F]

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = -\int \frac{1}{x\sqrt{1-x^3}-\sqrt{3}\sqrt{1-x^3}-\sqrt{1-x^3}}\,dx$$

input
```
integrate(1/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

output
```
-Integral(1/(x*sqrt(1 - x**3) - sqrt(3)*sqrt(1 - x**3) - sqrt(1 - x**3)),
x)
```

3.11.7 Maxima [F]

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \int -\frac{1}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

input
```
integrate(1/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

3.11.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[2
]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argument Value
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \text{Hanged}$$

input
```
int(1/((1 - x^3)^(1/2)*(3^(1/2) - x + 1)),x)
```

output
```
\text{Hanged}
```

3.12 $\int \frac{1}{(1+\sqrt{3}-x)\sqrt{-1+x^3}}\,dx$

3.12.1 Optimal result

Integrand size = 22, antiderivative size = 167

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} - \frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
-1/3*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/
2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(1/4)/(x^3-1)^(1/2)/((
-1+x)/(1-x-3^(1/2))^2)^(1/2)-arctanh((1-x)*(3+2*3^(1/2))^(1/2)/(x^3-1)^(1/
2))/(9+6*3^(1/2))^(1/2)
```

3.12.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \frac{4\sqrt{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\sqrt{1+x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}},\arcsin\left(\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{-1+x^3}}$$

input
```
Integrate[1/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

output
```
(4*Sqrt[2]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*Ellipti
cPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)
*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((3*I + (1 + 2*I)*Sq
rt[3])*Sqrt[-1 + x^3])
```

3.12.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2560, 27, 760, 2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(-x+\sqrt{3}+1\right)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{2560}$$

$$\frac{\int \frac{1}{\sqrt{x^3-1}}dx}{2\sqrt{3}}-\frac{\int \frac{6\left(-x-\sqrt{3}+1\right)}{\left(-x+\sqrt{3}+1\right)\sqrt{x^3-1}}dx}{12\sqrt{3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{\sqrt{x^3-1}}dx}{2\sqrt{3}}-\frac{\int \frac{-x-\sqrt{3}+1}{\left(-x+\sqrt{3}+1\right)\sqrt{x^3-1}}dx}{2\sqrt{3}}$$

$$\Big\downarrow \ 760$$

$$-\frac{\int \frac{-x-\sqrt{3}+1}{\left(-x+\sqrt{3}+1\right)\sqrt{x^3-1}}dx}{2\sqrt{3}}-\frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \ 2565$$

$$-\frac{\int \frac{1}{1-\frac{\left(3+2\sqrt{3}\right)(1-x)^2}{x^3-1}}d\frac{1-x}{\sqrt{x^3-1}}}{\sqrt{3}}-\frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \ 219$$

$$-\frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

input
```
Int[1/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

output
```
-(ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]]/Sqrt[3*(3 + 2*Sqrt
[3])]) - (Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2
]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/
(3^(3/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

3.12. $\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

3.12.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2560
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[-6
*a*(d^3/(c*(b*c^3 - 28*a*d^3)))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[1/(c
*(b*c^3 - 28*a*d^3))   Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20
*a*b*c^3*d^3 - 8*a^2*d^6, 0]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

3.12.4 Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\,\Pi\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^{3}-1}}$	132
elliptic	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\,\Pi\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{x^{3}-1}}$	132

input
```
int(1/(1-x+3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^
(1/2),-1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(
1/2)))^(1/2))
```

3.12.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$$
$$=\frac{1}{12}\sqrt{2\sqrt{3}-3}\log\left(\frac{x^8+16\,x^7+112\,x^6+16\,x^5+112\,x^4-224\,x^3+64\,x^2+4\left(2\,x^6+18\,x^5+42\,x^4+8\,x\right.}{}\right.$$
$$+\frac{1}{3}\sqrt{3}\text{weierstrassPInverse}(0,4,x)$$

input
```
integrate(1/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/12*sqrt(2*sqrt(3) - 3)*log((x^8 + 16*x^7 + 112*x^6 + 16*x^5 + 112*x^4 -
224*x^3 + 64*x^2 + 4*(2*x^6 + 18*x^5 + 42*x^4 + 8*x^3 + sqrt(3)*(x^6 + 12*
x^5 + 18*x^4 + 16*x^3 - 12*x^2 - 8) - 24*x + 8)*sqrt(x^3 - 1)*sqrt(2*sqrt(
3) - 3) + 16*sqrt(3)*(x^7 + 2*x^6 + 6*x^5 - 5*x^4 + 2*x^3 - 6*x^2 + 4*x -
4) - 128*x + 112)/(x^8 - 8*x^7 + 16*x^6 + 16*x^5 - 56*x^4 - 32*x^3 + 64*x^
2 + 64*x + 16)) + 1/3*sqrt(3)*weierstrassPInverse(0, 4, x)
```

3.12.6 Sympy [F]

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = -\int \frac{1}{x\sqrt{x^3-1}-\sqrt{3}\sqrt{x^3-1}-\sqrt{x^3-1}}\,dx$$

input
```
integrate(1/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

output
```
-Integral(1/(x*sqrt(x**3 - 1) - sqrt(3)*sqrt(x**3 - 1) - sqrt(x**3 - 1)),
x)
```

3.12.7 Maxima [F]

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \int -\frac{1}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

input
```
integrate(1/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

3.12.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Ar
gument Va
```

3.12.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Hanged}$$

input
```
int(1/((x^3 - 1)^(1/2)*(3^(1/2) - x + 1)),x)
```

output
```
\text{Hanged}
```

3.13 $\int \frac{1}{(1+\sqrt{3}+x)\sqrt{-1-x^3}}\,dx$

3.13.1 Optimal result

Integrand size = 22, antiderivative size = 157

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} + \frac{\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
1/3*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2
)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(1/4)/(-x^3-1)^(1/2)/((
-1-x)/(1+x-3^(1/2))^2)^(1/2)+arctanh((1+x)*(3+2*3^(1/2))^(1/2)/(-x^3-1)^(1
/2))/(9+6*3^(1/2))^(1/2)
```

3.13.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.07 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = -\frac{4\sqrt{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}},\arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{-1-x^3}}$$

input
```
Integrate[1/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

output
```
(-4*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*EllipticPi
[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]
/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((3*I + (1 + 2*I)*Sqrt[
3])*Sqrt[-1 - x^3])
```

3.13.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2560, 27, 760, 2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(x+\sqrt{3}+1\right)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{2560}$$

$$\frac{\int \frac{1}{\sqrt{-x^3-1}}dx}{2\sqrt{3}}+\frac{\int -\frac{6\left(x-\sqrt{3}+1\right)}{\left(x+\sqrt{3}+1\right)\sqrt{-x^3-1}}dx}{12\sqrt{3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{\sqrt{-x^3-1}}dx}{2\sqrt{3}}-\frac{\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{-x^3-1}}dx}{2\sqrt{3}}$$

$$\Big\downarrow \ 760$$

$$\frac{\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{\int\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{-x^3-1}}dx}{2\sqrt{3}}$$

$$\Big\downarrow \ 2565$$

$$\frac{\int\frac{1}{1-\frac{\left(3+2\sqrt{3}\right)(x+1)^2}{-x^3-1}}d\frac{x+1}{\sqrt{-x^3-1}}}{\sqrt{3}}+\frac{\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \ 219$$

$$\frac{\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

input
```
Int[1/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

output
```
ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]]/Sqrt[3*(3 + 2*Sqrt[3
])] + (Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*E
llipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^
(3/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

3.13.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2560
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[-6
*a*(d^3/(c*(b*c^3 - 28*a*d^3)))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[1/(c
*(b*c^3 - 28*a*d^3))   Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20
*a*b*c^3*d^3 - 8*a^2*d^6, 0]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

3.13.4 Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{2 i \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \Pi\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \frac{i \sqrt{3}}{\frac{3}{2}+\sqrt{3}+\frac{i \sqrt{3}}{2}}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}-1}\, \left(\frac{3}{2}+\sqrt{3}+\frac{i \sqrt{3}}{2}\right)}$	139
elliptic	$-\frac{2 i \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \Pi\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \frac{i \sqrt{3}}{\frac{3}{2}+\sqrt{3}+\frac{i \sqrt{3}}{2}}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}-1}\, \left(\frac{3}{2}+\sqrt{3}+\frac{i \sqrt{3}}{2}\right)}$	139

input
```
int(1/(1+x+3^(1/2))/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(3/
2+3^(1/2)+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2),I*3^(1/2)/(3/2+3^(1/2)+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*
3^(1/2)))^(1/2))
```

3.13.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.36

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$$
$$= \frac{1}{12}\sqrt{2\sqrt{3}-3}\log\left(\frac{x^8-16\,x^7+112\,x^6-16\,x^5+112\,x^4+224\,x^3+64\,x^2-4\left(2\,x^6-18\,x^5+42\,x^4-8\,x\right.}{}\right.$$
$$-\frac{1}{3}i\sqrt{3}\text{weierstrassPInverse}(0,-4,x)$$

input
```
integrate(1/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/12*sqrt(2*sqrt(3) - 3)*log((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112*x^4 +
224*x^3 + 64*x^2 - 4*(2*x^6 - 18*x^5 + 42*x^4 - 8*x^3 + sqrt(3)*(x^6 - 12*
x^5 + 18*x^4 - 16*x^3 - 12*x^2 - 8) + 24*x + 8)*sqrt(-x^3 - 1)*sqrt(2*sqrt
(3) - 3) - 16*sqrt(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*x +
 4) + 128*x + 112)/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64*x
^2 - 64*x + 16)) - 1/3*I*sqrt(3)*weierstrassPInverse(0, -4, x)
```

3.13.6 Sympy [F]

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-(x+1)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

input
```
integrate(1/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

3.13.7 Maxima [F]

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate(1/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

3.13.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Ar
gument Va
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \text{Hanged}$$

input
```
int(1/((- x^3 - 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

output
```
\text{Hanged}
```

3.14 $\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx$

3.14.1 Optimal result

Integrand size = 15, antiderivative size = 329

$$\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx = \frac{(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\arctan\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{2\sqrt{26+15\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{4\sqrt[4]{3}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticPi}\left(97-56\sqrt{3},\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
1/26*(1+x)*arctan(1/2*26^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/((x^2-x+1)/(1
+x+3^(1/2))^2)^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*26^(1/2)/(x^3+1)^(
1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)+4*3^(1/4)*(1+x)*EllipticPi((-1-x+3^(1/2
))/(1+x+3^(1/2)),97-56*3^(1/2),I*3^(1/2)+2*I)*((x^2-x+1)/(1+x+3^(1/2))^2)^
(1/2)/(x^3+1)^(1/2)/(1/2*6^(1/2)-1/2*2^(1/2))/((1+x)/(1+x+3^(1/2))^2)^(1/2
)+2/3*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*((x^2-x+1
)/(1+x+3^(1/2))^2)^(1/2)*(3/2*6^(1/2)+5/2*2^(1/2))*3^(3/4)/(x^3+1)^(1/2)/(
(1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.14.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.39

$$\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx = -\frac{4\sqrt{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{7i+\sqrt{3}},\arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(7i+\sqrt{3}\right)\sqrt{1+x^3}}$$

input
```
Integrate[1/((3 + x)*Sqrt[1 + x^3]),x]
```

output
```
(-4*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*EllipticPi
[(2*Sqrt[3])/(7*I + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*
3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((7*I + Sqrt[3])*Sqrt[1 + x^3])
```

3.14.3 Rubi [A] (warning: unable to verify)

Time = 0.94 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2561, 759, 2567, 25, 2538, 412, 435, 104, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+3)\sqrt{x^3+1}}\,dx$$

$$\Big\downarrow \text{2561}$$

$$\frac{\int \frac{1}{\sqrt{x^3+1}}dx}{2-\sqrt{3}}-\frac{\int \frac{x+\sqrt{3}+1}{(x+3)\sqrt{x^3+1}}dx}{2-\sqrt{3}}$$

$$\Big\downarrow \text{759}$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\int \frac{x+\sqrt{3}+1}{(x+3)\sqrt{x^3+1}}dx}{2-\sqrt{3}}$$

$$\Big\downarrow \text{2567}$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\int -\frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(2-\sqrt{3}\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}+\sqrt{3}+2\right)}d\left(-\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{25}$$

$$\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\int \frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(2-\sqrt{3}\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}+\sqrt{3}+2\right)}d\left(-\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{2538}$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(2-\sqrt{3}\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(7-4\sqrt{3}\right)\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}+4\sqrt{3}+7\right)}d\left(-\right.\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}$$

3.14. $\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx$

$\downarrow$ 412

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-$$

$$\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(2-\sqrt{3}\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(7-4\sqrt{3}\right)\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}+4\sqrt{3}+7\right)}d\left(-\right.\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3}}$$

$\downarrow$ 435

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-$$

$$\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{1}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}\left(\frac{\left(7-4\sqrt{3}\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}+4\sqrt{3}+7\right)}d\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}+\sqrt{7}\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\downarrow$ 104

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-$$

$$\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(2-\sqrt{3}\right)\int \frac{1}{-\frac{52\left(2-\sqrt{3}\right)\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}-8\sqrt{3}}d\frac{\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}+\sqrt{7-4\sqrt{3}}\left(2+\sqrt{3}\right)\text{Ellipti}\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\downarrow$ 217

3.14. $\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}} -$$

$$\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\sqrt{7-4\sqrt{3}}(2+\sqrt{3})\operatorname{EllipticPi}\left(97-56\sqrt{3},\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)+\frac{\sqrt{\frac{1}{26}\left(2-\sqrt{3}\right.}}{}\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

input
```
Int[1/((3 + x)*Sqrt[1 + x^3]),x]
```

output
```
(2*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellip
ticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4
)*(2 - Sqrt[3])*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (4*3^(1
/4)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*((Sqrt[(2 - Sqrt[3])/2
6]*ArcTan[(Sqrt[(13*(2 - Sqrt[3]))/2]*(1 - Sqrt[3] + x))/(3^(1/4)*(1 + Sqr
t[3] + x))])/(4*3^(1/4)) + Sqrt[7 - 4*Sqrt[3]]*(2 + Sqrt[3])*EllipticPi[97
 - 56*Sqrt[3], ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]
]))/(Sqrt[2 - Sqrt[3]]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

3.14.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2561
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[-q/((1 + Sqrt[3])*d - c*q)   Int[1/Sqrt[a + b*x^3], x]
, x] + Simp[d/((1 + Sqrt[3])*d - c*q)   Int[(1 + Sqrt[3] + q*x)/((c + d*x)*
Sqrt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.14. $\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx$

3.14.4 Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.37

method	result	size
default	$\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\Pi\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{3}{4}+\frac{i\sqrt{3}}{4},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}$	123
elliptic	$\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\Pi\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{3}{4}+\frac{i\sqrt{3}}{4},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}$	123

input
```
int(1/(3+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2
))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),-3/4+1/
4*I*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

3.14.5 Fricas [F]

$$\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^3 + 1)/(x^4 + 3*x^3 + x + 3), x)
```

3.14.6 Sympy [F]

$$\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{(x+1)\left(x^2-x+1\right)}\,(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(x**3+1)**(1/2),x)
```

output
```
Integral(1/(sqrt((x + 1)*(x**2 - x + 1))*(x + 3)), x)
```

3.14.7 Maxima [F]

$$\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^3 + 1)*(x + 3)), x)
```

3.14.8 Giac [F]

$$\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3+1}(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^3 + 1)*(x + 3)), x)
```

3.14.9 Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.50

$$\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx = \frac{(3+\sqrt{3}\,1\mathrm{i})\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(-\frac{3}{4}-\frac{\sqrt{3}\,1\mathrm{i}}{4};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{2\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/((x^3 + 1)^(1/2)*(x + 3)),x)
```

output
```
((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(- (3^(1/2)*1i)/4 - 3/4, asin(((x + 1)/
((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
3/2)))/(2*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - (
(3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

3.15 $\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx$

3.15.1 Optimal result

Integrand size = 17, antiderivative size = 380

$$\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx = -\frac{(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{arctanh}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{169}\left(553+304\sqrt{3}\right),\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-1/14*(1-x)*arctanh(1/2*7^(1/2)*((1-x)/(1-x+3^(1/2))^2)^(1/2)/((x^2+x+1)/(
1-x+3^(1/2))^2)^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*7^(1/2)/(-x^3+1)^
(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)+4/13*3^(1/4)*(1-x)*EllipticPi((-1+x+3^
(1/2))/(1-x+3^(1/2)),553/169+304/169*3^(1/2),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1
/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(-x^3+1)^(1/2)/((1-x)/(1-x+3
^(1/2))^2)^(1/2)-2/3*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)
+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/
(4+3^(1/2))/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

3.15.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.34

$$\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx = -\frac{4\sqrt{2}\sqrt{\frac{i(-1+x)}{-3i+\sqrt{3}}}\sqrt{1+x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{5i+\sqrt{3}},\arcsin\left(\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)}{\left(5i+\sqrt{3}\right)\sqrt{1-x^3}}$$

input
```
Integrate[1/((3 + x)*Sqrt[1 - x^3]),x]
```

output
```
(-4*Sqrt[2]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*Elliptic
Pi[(2*Sqrt[3])/(5*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[
2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])]])/((5*I + Sqrt[3])*Sqrt[1 - x^3
])
```

3.15.3 Rubi [A] (warning: unable to verify)

Time = 1.00 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {2561, 759, 2567, 2538, 412, 435, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+3)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{2561}$$

$$\frac{\int \frac{1}{\sqrt{1-x^3}}dx}{4+\sqrt{3}}+\frac{\int \frac{-x+\sqrt{3}+1}{(x+3)\sqrt{1-x^3}}dx}{4+\sqrt{3}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\int \frac{-x+\sqrt{3}+1}{(x+3)\sqrt{1-x^3}}dx}{4+\sqrt{3}}-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$\Big\downarrow$ 2567

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(4+\sqrt{3}\right)\left(-x-\sqrt{3}+1\right)}{-x+\sqrt{3}+1}-\sqrt{3}+4\right)}d\left(-\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$\Big\downarrow$ 2538

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\left(4-\sqrt{3}\right)\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(4+\sqrt{3}\right)^2\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-8\sqrt{3}+19\right)}\right.}{\left(4+\sqrt{3}\right)}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$\Big\downarrow$ 412

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\left(4+\sqrt{3}\right)\int -\frac{-x-\sqrt{3}+1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(4+\sqrt{3}\right)^2\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-8\sqrt{3}\right.}\right.}{\left(4+\sqrt{3}\right.}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$\Big\downarrow$ 435

3.15. $\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\frac{1}{2}\left(4+\sqrt{3}\right)\int \frac{1}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}\left(\frac{\left(4+\sqrt{3}\right)^2\left(-x-\sqrt{3}+1\right)}{-x+\sqrt{3}+1}-8\sqrt{3}+19\right)}d\right.}{\left(4+\sqrt{3}\right)\sqrt{\frac{1}{(-x+}}}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \; 104$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\left(4+\sqrt{3}\right)\int \frac{1}{16\sqrt{3}-\frac{28\left(2-\sqrt{3}\right)\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}}d\frac{\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}-\frac{1}{169}\left(4-\sqrt{3}\right)\right.}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x}}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \; 219$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\frac{\left(4+\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt{7\left(2-\sqrt{3}\right)}\left(-x-\sqrt{3}+1\right)}{2\sqrt[4]{3}\left(-x+\sqrt{3}+1\right)}\right)}{8\sqrt[4]{3}\sqrt{7\left(2-\sqrt{3}\right)}}-\frac{1}{169}\left(4-\sqrt{3}\right)\sqrt{7519+4340\sqrt{3}}\text{Ellip}\right.}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

input
```
Int[1/((3 + x)*Sqrt[1 - x^3]),x]
```

3.15. $\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx$

output
```
(-2*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*Elli
pticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/
4)*(4 + Sqrt[3])*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) + (4*3^(
1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*(((
4 + Sqrt[3])*ArcTanh[(Sqrt[7*(2 - Sqrt[3])]*(1 - Sqrt[3] - x))/(2*3^(1/4)*
(1 + Sqrt[3] - x))])/(8*3^(1/4)*Sqrt[7*(2 - Sqrt[3])]) - ((4 - Sqrt[3])*Sq
rt[7519 + 4340*Sqrt[3]]*EllipticPi[(553 + 304*Sqrt[3])/169, ArcSin[(1 - Sq
rt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/169))/((4 + Sqrt[3])*Sqrt[
(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

3.15.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2561
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[-q/((1 + Sqrt[3])*d - c*q)   Int[1/Sqrt[a + b*x^3], x]
, x] + Simp[d/((1 + Sqrt[3])*d - c*q)   Int[(1 + Sqrt[3] + q*x)/((c + d*x)*
Sqrt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.15.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.35

method	result	size
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\Pi\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{\frac{5}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}\,\left(\frac{5}{2}+\frac{i\sqrt{3}}{2}\right)}$	133
elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\Pi\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{\frac{5}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}\,\left(\frac{5}{2}+\frac{i\sqrt{3}}{2}\right)}$	133

input
```
int(1/(3+x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(5
/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))
^(1/2),I*3^(1/2)/(5/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2
))
```

3.15.5 Fricas [F]

$$\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3+1}(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^3 + 1)/(x^4 + 3*x^3 - x - 3), x)
```

3.15.6 Sympy [F]

$$\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx = \int \frac{1}{\sqrt{-(x-1)(x^2+x+1)}\,(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(-x**3+1)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(x - 1)*(x**2 + x + 1))*(x + 3)), x)
```

3.15.7 Maxima [F]

$$\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3+1}(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^3 + 1)*(x + 3)), x)
```

3.15.8 Giac [F]

$$\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3+1}(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^3 + 1)*(x + 3)), x)
```

3.15.9 Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.47

$$\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx = -\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{8}+\frac{\sqrt{3}\,1\mathrm{i}}{8};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{2\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/((1 - x^3)^(1/2)*(x + 3)),x)
```

output
```
-(((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/
8 + 3/8, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 +
 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(2*(1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*
((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
+ 1) + x^3)^(1/2))
```

3.15. $\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx$

3.16 $\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx$

3.16.1 Optimal result

Integrand size = 15, antiderivative size = 374

$$\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx = -\frac{(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{arctanh}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{2\sqrt{62-35\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{169}\left(553+304\sqrt{3}\right),\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
-2/39*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(5/2*6^(1
/2)-7/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/(
(-1+x)/(1-x-3^(1/2))^2)^(1/2)-1/14*(1-x)*arctanh(1/2*7^(1/2)*((1-x)/(1-x+3
^(1/2))^2)^(1/2)/((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2))*((x^2+x+1)/(1-x+3^(1/2
))^2)^(1/2)*7^(1/2)/(x^3-1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)+4/13*3^(1/
4)*(1-x)*EllipticPi((-1+x+3^(1/2))/(1-x+3^(1/2)),553/169+304/169*3^(1/2),I
*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/
(x^3-1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

3.16.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.06 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.34

$$\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx = -\frac{4\sqrt{2}\sqrt{\frac{i(-1+x)}{-3i+\sqrt{3}}}\sqrt{1+x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{5i+\sqrt{3}},\arcsin\left(\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)}{\left(5i+\sqrt{3}\right)\sqrt{-1+x^3}}$$

input
```
Integrate[1/((3 + x)*Sqrt[-1 + x^3]),x]
```

output
```
(-4*Sqrt[2]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*Elliptic
Pi[(2*Sqrt[3])/(5*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[
2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])])/((5*I + Sqrt[3])*Sqrt[-1 + x^
3])
```

3.16.3 Rubi [A] (warning: unable to verify)

Time = 0.97 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {2561, 760, 2567, 2538, 412, 435, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+3)\sqrt{x^3-1}}\,dx$$

$$\Big\downarrow \text{2561}$$

$$\frac{\int \frac{1}{\sqrt{x^3-1}}dx}{4+\sqrt{3}}+\frac{\int \frac{-x+\sqrt{3}+1}{(x+3)\sqrt{x^3-1}}dx}{4+\sqrt{3}}$$

$$\Big\downarrow \text{760}$$

$$\frac{\int \frac{-x+\sqrt{3}+1}{(x+3)\sqrt{x^3-1}}dx}{4+\sqrt{3}}-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{2567}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(4+\sqrt{3}\right)\left(-x-\sqrt{3}+1\right)}{-x+\sqrt{3}+1}-\sqrt{3}+4\right)}d\left(-\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right.}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{2538}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\left(4-\sqrt{3}\right)\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(4+\sqrt{3}\right)^2\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-8\sqrt{3}+19\right)}\right.}{\left(4+\sqrt{3}\right)\sqrt{}}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{412}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\left(4+\sqrt{3}\right)\int -\frac{-x-\sqrt{3}+1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(4+\sqrt{3}\right)^2\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-8\sqrt{3}\right.}\right.}{\left(4+\sqrt{3}\right.}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{435}$$

3.16. $\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\frac{1}{2}\left(4+\sqrt{3}\right)\int \frac{1}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}\left(\frac{\left(4+\sqrt{3}\right)^2\left(-x-\sqrt{3}+1\right)}{-x+\sqrt{3}+1}-8\sqrt{3}+19\right)}d \dots\right.}{\left(4+\sqrt{3}\right)\sqrt{\frac{1}{\left(-x+ \dots\right.}}\dots}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \; 104$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\left(4+\sqrt{3}\right)\int \frac{1}{16\sqrt{3}-\frac{28\left(2-\sqrt{3}\right)\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}}d\frac{\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}-\frac{1}{169}\left(4-\sqrt{3}\right)\dots\right.}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-\dots}}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \; 219$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\frac{\left(4+\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt{7\left(2-\sqrt{3}\right)}\left(-x-\sqrt{3}+1\right)}{2\sqrt[4]{3}\left(-x+\sqrt{3}+1\right)}\right)}{8\sqrt[4]{3}\sqrt{7\left(2-\sqrt{3}\right)}}-\frac{1}{169}\left(4-\sqrt{3}\right)\sqrt{7519+4340\sqrt{3}}\,\text{Ellip}\dots\right.}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

input
```
Int[1/((3 + x)*Sqrt[-1 + x^3]),x]
```

3.16. $\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx$

output
```
(-2*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*Elli
pticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/
4)*(4 + Sqrt[3])*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) + (4
*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]
*(((4 + Sqrt[3])*ArcTanh[(Sqrt[7*(2 - Sqrt[3])]*(1 - Sqrt[3] - x))/(2*3^(1
/4)*(1 + Sqrt[3] - x))])/(8*3^(1/4)*Sqrt[7*(2 - Sqrt[3])]) - ((4 - Sqrt[3]
)*Sqrt[7519 + 4340*Sqrt[3]]*EllipticPi[(553 + 304*Sqrt[3])/169, ArcSin[(1
- Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/169))/((4 + Sqrt[3])*S
qrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

3.16.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2561
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[-q/((1 + Sqrt[3])*d - c*q)   Int[1/Sqrt[a + b*x^3], x]
, x] + Simp[d/((1 + Sqrt[3])*d - c*q)   Int[(1 + Sqrt[3] + q*x)/((c + d*x)*
Sqrt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.16.4 Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.33

method	result	size
default	$\frac{\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\Pi\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{3}{8}+\frac{i\sqrt{3}}{8},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{2\sqrt{x^{3}-1}}$	124
elliptic	$\frac{\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\Pi\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{3}{8}+\frac{i\sqrt{3}}{8},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{2\sqrt{x^{3}-1}}$	124

input
```
int(1/(3+x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),3/
8+1/8*I*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

3.16.5 Fricas [F]

$$\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3-1}(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^3 - 1)/(x^4 + 3*x^3 - x - 3), x)
```

3.16.6 Sympy [F]

$$\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx = \int \frac{1}{\sqrt{(x-1)\,(x^2+x+1)}\,(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(x**3-1)**(1/2),x)
```

output
```
Integral(1/(sqrt((x - 1)*(x**2 + x + 1))*(x + 3)), x)
```

3.16.7 Maxima [F]

$$\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3-1}(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^3 - 1)*(x + 3)), x)
```

3.16.8 Giac [F]

$$\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx = \int \frac{1}{\sqrt{x^3-1}(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^3 - 1)*(x + 3)), x)
```

3.16.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.44

$$\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx = -\frac{(3+\sqrt{3}\,1\mathrm{i})\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{8}+\frac{\sqrt{3}\,1\mathrm{i}}{8};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{4\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/((x^3 - 1)^(1/2)*(x + 3)),x)
```

output
```
-((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1
/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/8 + 3/8, asin((-(x - 1
)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
- 3/2)))/(4*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.17 $\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx$

3.17.1 Optimal result

Integrand size = 17, antiderivative size = 340

$$\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx = \frac{(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\arctan\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{2(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{4\sqrt[4]{3}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticPi}\left(97-56\sqrt{3},\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
2/3*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*((x^2-x+1)/
(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/(1/2*6^(1/2)-1/2*2^(1/2))/((
-1-x)/(1+x-3^(1/2))^2)^(1/2)+1/26*(1+x)*arctan(1/2*26^(1/2)*((1+x)/(1+x+3^
(1/2))^2)^(1/2)/((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2))*((x^2-x+1)/(1+x+3^(1/2)
)^2)^(1/2)*26^(1/2)/(-x^3-1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)+4*3^(1/4)
*(1+x)*EllipticPi((-1-x+3^(1/2))/(1+x+3^(1/2)),97-56*3^(1/2),I*3^(1/2)+2*I
)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)/(1/2*6^(1/2)-1/2*2^(1/2
))/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.17.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.38

$$\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx = -\frac{4\sqrt{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{7i+\sqrt{3}},\arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(7i+\sqrt{3}\right)\sqrt{-1-x^3}}$$

input
```
Integrate[1/((3 + x)*Sqrt[-1 - x^3]),x]
```

output
```
(-4*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*EllipticPi
[(2*Sqrt[3])/(7*I + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*
3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((7*I + Sqrt[3])*Sqrt[-1 - x^3])
```

3.17.3 Rubi [A] (warning: unable to verify)

Time = 0.91 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {2561, 760, 2567, 25, 2538, 412, 435, 104, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+3)\sqrt{-x^3-1}}\,dx$$

$$\Big\downarrow \; 2561$$

$$\frac{\int \frac{1}{\sqrt{-x^3-1}}dx}{2-\sqrt{3}}-\frac{\int \frac{x+\sqrt{3}+1}{(x+3)\sqrt{-x^3-1}}dx}{2-\sqrt{3}}$$

$$\Big\downarrow \text{760}$$

$$\frac{2(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{\int \frac{x+\sqrt{3}+1}{(x+3)\sqrt{-x^3-1}}dx}{2-\sqrt{3}}$$

$$\Big\downarrow \text{2567}$$

$$\frac{2(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\int -\frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(2-\sqrt{3}\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}+\sqrt{3}+2\right)}d\left(-\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{25}$$

$$\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\int \frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(2-\sqrt{3}\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}+\sqrt{3}+2\right)}d\left(-\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{2538}$$

$$\frac{2(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(2-\sqrt{3}\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(7-4\sqrt{3}\right)\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}+4\sqrt{3}+7\right)}d\left(-\right.\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}$$

3.17. $\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx$

$$\Big\downarrow 412$$

$$\frac{2(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-$$

$$\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(2-\sqrt{3}\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(7-4\sqrt{3}\right)\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}+4\sqrt{3}+7\right)}d\Big(-}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3}}$$

$$\Big\downarrow 435$$

$$\frac{2(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-$$

$$\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\frac{1}{2}\left(2-\sqrt{3}\right)\int \frac{1}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}\left(\frac{\left(7-4\sqrt{3}\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}+4\sqrt{3}+7\right)}d\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}+\sqrt{7}}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow 104$$

$$\frac{2(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-$$

$$\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(2-\sqrt{3}\right)\int \frac{1}{-\frac{52\left(2-\sqrt{3}\right)\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}-8\sqrt{3}}d\frac{\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}+\sqrt{7-4\sqrt{3}}\left(2+\sqrt{3}\right)\text{Ellipti}}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow 217$$

3.17. $\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx$

$$\frac{2(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-$$

$$\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\sqrt{7-4\sqrt{3}}(2+\sqrt{3})\,\text{EllipticPi}\left(97-56\sqrt{3},\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)+\frac{\sqrt{\frac{1}{26}\left(2-\sqrt{3}\right.}}{}\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

input
```
Int[1/((3 + x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + S
qrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[2 - Sqrt[3]
]*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]) - (4*3^(1/4)*(1 + x
)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*((Sqrt[(2 - Sqrt[3])/26]*ArcTan[
(Sqrt[(13*(2 - Sqrt[3]))/2]*(1 - Sqrt[3] + x))/(3^(1/4)*(1 + Sqrt[3] + x))
])/(4*3^(1/4)) + Sqrt[7 - 4*Sqrt[3]]*(2 + Sqrt[3])*EllipticPi[97 - 56*Sqrt
[3], ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]]))/(Sqrt[
2 - Sqrt[3]]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```

3.17.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2561
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[-q/((1 + Sqrt[3])*d - c*q)   Int[1/Sqrt[a + b*x^3], x]
, x] + Simp[d/((1 + Sqrt[3])*d - c*q)   Int[(1 + Sqrt[3] + q*x)/((c + d*x)*
Sqrt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.17. $\int \frac{1}{(3+x)\sqrt{-1-x^3}} \, dx$

3.17.4 Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.39

method	result	size
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\Pi\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{\frac{7}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}-1}\,\left(\frac{7}{2}+\frac{i\sqrt{3}}{2}\right)}$	133
elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\Pi\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\frac{i\sqrt{3}}{\frac{7}{2}+\frac{i\sqrt{3}}{2}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}-1}\,\left(\frac{7}{2}+\frac{i\sqrt{3}}{2}\right)}$	133

input
```
int(1/(3+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(7/
2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^
(1/2),I*3^(1/2)/(7/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

3.17.5 Fricas [F]

$$\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3-1}(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^3 - 1)/(x^4 + 3*x^3 + x + 3), x)
```

3.17.6 Sympy [F]

$$\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-(x+1)(x^2-x+1)}(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(-x**3-1)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 3)), x)
```

3.17.7 Maxima [F]

$$\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3-1}(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^3 - 1)*(x + 3)), x)
```

3.17.8 Giac [F]

$$\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx = \int \frac{1}{\sqrt{-x^3-1}(x+3)}\,dx$$

input
```
integrate(1/(3+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^3 - 1)*(x + 3)), x)
```

3.17.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.53

$$\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx$$
$$= \frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(-\frac{3}{4}-\frac{\sqrt{3}\,1\mathrm{i}}{4};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(1/((- x^3 - 1)^(1/2)*(x + 3)),x)
```

output
```
(((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1
/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*
1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(- (3^(1/2)*1i)/4
 - 3/4, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3
/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/
2 + 1/2))^(1/2))
```

3.18 $\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$

3.18.1 Optimal result

Integrand size = 25, antiderivative size = 139

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{-c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}cd}+\frac{\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}cd}-\frac{3\log\left(d(c-dx)+2^{2/3}d\sqrt[3]{-c^3+d^3x^3}\right)}{4\sqrt[3]{2}cd}$$

output
```
1/8*ln((-d*x+c)*(d*x+c)^2)*2^(2/3)/c/d-3/8*ln(d*(-d*x+c)+2^(2/3)*d*(d^3*x^
3-c^3)^(1/3))*2^(2/3)/c/d+1/4*arctan(1/3*(1-2^(1/3)*(-d*x+c)/(d^3*x^3-c^3)
^(1/3))*3^(1/2))*3^(1/2)*2^(2/3)/c/d
```

3.18.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.07 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.24

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \frac{\sqrt[3]{-\frac{1}{2}}\left(2i\sqrt{3}\text{arctanh}\left(\frac{\sqrt[3]{2}\left(3+i\sqrt{3}\right)c+\sqrt[3]{2}\left(-3-i\sqrt{3}\right)dx+2i\sqrt{3}\sqrt[3]{-c^3+d^3x^3}}{6\sqrt[3]{-c^3+d^3x^3}}\right)+2\log\left(\sqrt{c}\sqrt{d}\left(-c+i\sqrt{3}c+dx-\right.\right.\right.}{}$$

input
```
Integrate[1/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

output
```
((-1/2)^(1/3)*((2*I)*Sqrt[3]*ArcTanh[(2^(1/3)*(3 + I*Sqrt[3])*c + 2^(1/3)*
(-3 - I*Sqrt[3])*d*x + (2*I)*Sqrt[3]*(-c^3 + d^3*x^3)^(1/3))/(6*(-c^3 + d^
3*x^3)^(1/3))] + 2*Log[Sqrt[c]*Sqrt[d]*(-c + I*Sqrt[3]*c + d*x - I*Sqrt[3]
*d*x + 2*2^(2/3)*(-c^3 + d^3*x^3)^(1/3))] - Log[-(c*d*((1 + I*Sqrt[3])*c^2
 + (1 + I*Sqrt[3])*d^2*x^2 - 2*(-2)^(2/3)*d*x*(-c^3 + d^3*x^3)^(1/3) - 4*2
^(1/3)*(-c^3 + d^3*x^3)^(2/3) + 2*c*((-1 - I*Sqrt[3])*d*x + (-2)^(2/3)*(-c
^3 + d^3*x^3)^(1/3)))])))/(4*c*d)
```

3.18.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2574}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)\sqrt[3]{d^3x^3-c^3}}dx$$

$$\Big\downarrow \; 2574$$

$$\frac{\sqrt{3}\arctan\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{d^3x^3-c^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}cd}-\frac{3\log\left(2^{2/3}d\sqrt[3]{d^3x^3-c^3}+d(c-dx)\right)}{4\sqrt[3]{2}cd}+\frac{\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}cd}$$

input
```
Int[1/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

output
```
(Sqrt[3]*ArcTan[(1 - (2^(1/3)*(c - d*x))/(-c^3 + d^3*x^3)^(1/3))/Sqrt[3]])
/(2*2^(1/3)*c*d) + Log[(c - d*x)*(c + d*x)^2]/(4*2^(1/3)*c*d) - (3*Log[d*(
c - d*x) + 2^(2/3)*d*(-c^3 + d^3*x^3)^(1/3)])/(4*2^(1/3)*c*d)
```

3.18.3.1 Defintions of rubi rules used

rule 2574
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

3.18.4 Maple [F]

$$\int \frac{1}{(dx+c)\left(d^{3}x^{3}-c^{3}\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
```

output
```
int(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
```

3.18.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx=\text{Timed out}$$

input
```
integrate(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.18.6 Sympy [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx=\int \frac{1}{\sqrt[3]{(-c+dx)\left(c^2+cdx+d^2x^2\right)}\,(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(d**3*x**3-c**3)**(1/3),x)
```

output
```
Integral(1/(((-c + d*x)*(c**2 + c*d*x + d**2*x**2))**(1/3)*(c + d*x)), x)
```

3.18.7 Maxima [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \int \frac{1}{(d^3x^3-c^3)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

3.18.8 Giac [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \int \frac{1}{(d^3x^3-c^3)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

3.18.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \int \frac{1}{(d^3\,x^3-c^3)^{1/3}\,(c+d\,x)}\,dx$$

input
```
int(1/((d^3*x^3 - c^3)^(1/3)*(c + d*x)),x)
```

output
```
int(1/((d^3*x^3 - c^3)^(1/3)*(c + d*x)), x)
```

3.19 $\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$

3.19.1 Optimal result

Integrand size = 25, antiderivative size = 186

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \frac{\arctan\left(\frac{1+\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}cd} - \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2cd} - \frac{\log(c+dx)}{2cd} - \frac{\log\left(-dx+\sqrt[3]{2c^3+d^3x^3}\right)}{4cd} + \frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{4cd}$$

output
```
-1/2*ln(d*x+c)/c/d-1/4*ln(-d*x+(d^3*x^3+2*c^3)^(1/3))/c/d+3/4*ln(d*(d*x+2*
c)-d*(d^3*x^3+2*c^3)^(1/3))/c/d+1/6*arctan(1/3*(1+2*d*x/(d^3*x^3+2*c^3)^(1
/3))*3^(1/2))/c/d*3^(1/2)-1/2*arctan(1/3*(1+2*(d*x+2*c)/(d^3*x^3+2*c^3)^(1
/3))*3^(1/2))*3^(1/2)/c/d
```

3.19.2 Mathematica [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$$

input
```
Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```

output
```
Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)), x]
```

3.19.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2575, 769, 2576}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$$

$$\Big\downarrow \text{ 2575}$$

$$\frac{\int \frac{1}{\sqrt[3]{2c^3+d^3x^3}}dx}{2c}+\frac{\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}dx}{2c}$$

$$\Big\downarrow \text{ 769}$$

$$\frac{\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}dx}{2c}+\frac{\frac{\arctan\left(\frac{\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d}-\frac{\log\left(\sqrt[3]{2c^3+d^3x^3}-dx\right)}{2d}}{2c}$$

$$\Big\downarrow \text{ 2576}$$

$$\frac{\frac{\arctan\left(\frac{\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d}-\frac{\log\left(\sqrt[3]{2c^3+d^3x^3}-dx\right)}{2d}}{2c}+\frac{-\frac{\sqrt{3}\arctan\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{d}+\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{2d}-\frac{\log(c+dx)}{d}}{2c}$$

input
```
Int[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```

output
```
(ArcTan[(1 + (2*d*x)/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*d) - Log[-
(d*x) + (2*c^3 + d^3*x^3)^(1/3)]/(2*d))/(2*c) + (-((Sqrt[3]*ArcTan[(1 + (2
*(2*c + d*x))/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/d) - Log[c + d*x]/d + (3*
Log[d*(2*c + d*x) - d*(2*c^3 + d^3*x^3)^(1/3)])/(2*d))/(2*c)
```

3.19.3.1 Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 2575
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
1/(2*c)   Int[1/(a + b*x^3)^(1/3), x], x] + Simp[1/(2*c)   Int[(c - d*x)/((
c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[2*b*c^
3 - a*d^3, 0]
```

rule 2576
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[Sqrt[3]*f*(ArcTan[(1 + 2*Rt[b, 3]*((2*c + d*x)/(d*(a + b*
x^3)^(1/3))))/Sqrt[3]]/(Rt[b, 3]*d)), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

3.19.4 Maple [F]

$$\int \frac{1}{(dx+c)\,(d^3x^3+2c^3)^{\frac{1}{3}}}dx$$

input
```
int(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
```

output
```
int(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
```

3.19.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.19.6 Sympy [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \int \frac{1}{(c+dx)\,\sqrt[3]{2c^3+d^3x^3}}\,dx$$

input
```
integrate(1/(d*x+c)/(d**3*x**3+2*c**3)**(1/3),x)
```

output
```
Integral(1/((c + d*x)*(2*c**3 + d**3*x**3)**(1/3)), x)
```

3.19.7 Maxima [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \int \frac{1}{(d^3x^3+2\,c^3)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

3.19.8 Giac [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \int \frac{1}{(d^3x^3+2\,c^3)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \int \frac{1}{\left(2\,c^3+d^3\,x^3\right)^{1/3}\,(c+d\,x)}\,dx$$

input
```
int(1/((2*c^3 + d^3*x^3)^(1/3)*(c + d*x)),x)
```

output
```
int(1/((2*c^3 + d^3*x^3)^(1/3)*(c + d*x)), x)
```

3.20 $\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx$

3.20.1 Optimal result

Integrand size = 25, antiderivative size = 187

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx = -\frac{\arctan\left(\frac{1+\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}c^2d} + \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2c^2d} - \frac{\log(c+dx)}{2c^2d} - \frac{\log\left(dx-\sqrt[3]{2c^3+d^3x^3}\right)}{4c^2d} + \frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{4c^2d}$$

output
```
-1/2*ln(d*x+c)/c^2/d-1/4*ln(d*x-(d^3*x^3+2*c^3)^(1/3))/c^2/d+3/4*ln(d*(d*x
+2*c)-d*(d^3*x^3+2*c^3)^(1/3))/c^2/d-1/6*arctan(1/3*(1+2*d*x/(d^3*x^3+2*c^
3)^(1/3))*3^(1/2))/c^2/d*3^(1/2)+1/2*arctan(1/3*(1+2*(d*x+2*c)/(d^3*x^3+2*
c^3)^(1/3))*3^(1/2))*3^(1/2)/c^2/d
```

3.20.2 Mathematica [F]

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx=\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx$$

input
```
Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)),x]
```

output
```
Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)), x]
```

3.20.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2579}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx$$

$$\Big\downarrow \text{2579}$$

$$-\frac{\arctan\left(\frac{\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2\sqrt{3}c^2d}+\frac{\sqrt{3}\arctan\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2c^2d}-\frac{\log(c+dx)}{2c^2d}-\frac{\log\left(dx-\sqrt[3]{2c^3+d^3x^3}\right)}{4c^2d}+\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{4c^2d}$$

input
```
Int[1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)),x]
```

output
```
-1/2*ArcTan[(1 + (2*d*x)/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*c^2*d)
 + (Sqrt[3]*ArcTan[(1 + (2*(2*c + d*x))/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]])
/(2*c^2*d) - Log[c + d*x]/(2*c^2*d) - Log[d*x - (2*c^3 + d^3*x^3)^(1/3)]/(
4*c^2*d) + (3*Log[d*(2*c + d*x) - d*(2*c^3 + d^3*x^3)^(1/3)])/(4*c^2*d)
```

3.20.3.1 Defintions of rubi rules used

rule 2579
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(2/3)), x_Symbol] :> With[
{q = Rt[b, 3]}, Simp[(-d)*(ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/
(2*Sqrt[3]*q^2*c^2)), x] + (Simp[Sqrt[3]*d*(ArcTan[(1 + 2*q*((2*c + d*x)/(d
*(a + b*x^3)^(1/3))))/Sqrt[3]]/(2*q^2*c^2)), x] - Simp[d*(Log[c + d*x]/(2*q
^2*c^2)), x] - Simp[d*(Log[q*x - (a + b*x^3)^(1/3)]/(4*q^2*c^2)), x] + Simp
[3*d*(Log[q*(2*c + d*x) - d*(a + b*x^3)^(1/3)]/(4*q^2*c^2)), x])] /; FreeQ[
{a, b, c, d}, x] && EqQ[2*b*c^3 - a*d^3, 0]
```

3.20.4 Maple [F]

$$\int \frac{1}{(dx+c)\left(d^3x^3+2c^3\right)^{\frac{2}{3}}}dx$$

input
```
int(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x)
```

output
```
int(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x)
```

3.20.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx=\text{Timed out}$$

input
```
integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.20.6 Sympy [F]

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx=\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(d*x+c)/(d**3*x**3+2*c**3)**(2/3),x)
```

output
```
Integral(1/((c + d*x)*(2*c**3 + d**3*x**3)**(2/3)), x)
```

3.20.7 Maxima [F]

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx=\int \frac{1}{(d^3x^3+2\,c^3)^{\frac{2}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((d^3*x^3 + 2*c^3)^(2/3)*(d*x + c)), x)
```

3.20.8 Giac [F]

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx=\int \frac{1}{(d^3x^3+2\,c^3)^{\frac{2}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((d^3*x^3 + 2*c^3)^(2/3)*(d*x + c)), x)
```

3.20.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx=\int \frac{1}{\left(2\,c^3+d^3\,x^3\right)^{2/3}\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/((2*c^3 + d^3*x^3)^(2/3)*(c + d*x)),x)
```

output
```
int(1/((2*c^3 + d^3*x^3)^(2/3)*(c + d*x)), x)
```

3.21 $\int \frac{1}{\left(1+\sqrt[3]{2}x\right)\left(1+x^3\right)^{2/3}}\,dx$

3.21.1 Optimal result

Integrand size = 21, antiderivative size = 147

$$\int \frac{1}{\left(1+\sqrt[3]{2}x\right)\left(1+x^3\right)^{2/3}}\,dx = -\frac{\arctan\left(\frac{1+\frac{2x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\left(2^{2/3}+x\right)}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{2^{2/3}} - \frac{\log\left(1+\sqrt[3]{2}x\right)}{2^{2/3}}-\frac{\log\left(x-\sqrt[3]{1+x^3}\right)}{2\ 2^{2/3}}+\frac{3\log\left(2+\sqrt[3]{2}x-\sqrt[3]{2}\sqrt[3]{1+x^3}\right)}{2\ 2^{2/3}}$$

output
```
-1/2*ln(1+2^(1/3)*x)*2^(1/3)-1/4*ln(x-(x^3+1)^(1/3))*2^(1/3)+3/4*ln(2+2^(1
/3)*x-2^(1/3)*(x^3+1)^(1/3))*2^(1/3)-1/6*arctan(1/3*(1+2*x/(x^3+1)^(1/3))*
3^(1/2))*2^(1/3)*3^(1/2)+1/2*arctan(1/3*(1+2*(2^(2/3)+x)/(x^3+1)^(1/3))*3^
(1/2))*3^(1/2)*2^(1/3)
```

3.21.2 Mathematica [F]

$$\int \frac{1}{\left(1+\sqrt[3]{2}x\right)(1+x^3)^{2/3}}\,dx = \int \frac{1}{\left(1+\sqrt[3]{2}x\right)(1+x^3)^{2/3}}\,dx$$

input
```
Integrate[1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)),x]
```

output
```
Integrate[1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)), x]
```

3.21.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2579}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt[3]{2}x+1\right)(x^3+1)^{2/3}}\,dx$$

$$\Big\downarrow \ 2579$$

$$-\frac{\arctan\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\sqrt{3}\arctan\left(\frac{\frac{2\left(x+2^{2/3}\right)}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(x-\sqrt[3]{x^3+1}\right)}{2\ 2^{2/3}}+\frac{3\log\left(-\sqrt[3]{2}\sqrt[3]{x^3+1}+\sqrt[3]{2}x+2\right)}{2\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}x+1\right)}{2^{2/3}}$$

input
```
Int[1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)),x]
```

output
```
-(ArcTan[(1 + (2*x)/(1 + x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3])) + (Sqrt[3
]*ArcTan[(1 + (2*(2^(2/3) + x))/(1 + x^3)^(1/3))/Sqrt[3]])/2^(2/3) - Log[1
 + 2^(1/3)*x]/2^(2/3) - Log[x - (1 + x^3)^(1/3)]/(2*2^(2/3)) + (3*Log[2 +
2^(1/3)*x - 2^(1/3)*(1 + x^3)^(1/3)])/(2*2^(2/3))
```

3.21.3.1 Defintions of rubi rules used

rule 2579
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(2/3)), x_Symbol] :> With[
{q = Rt[b, 3]}, Simp[(-d)*(ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/
(2*Sqrt[3]*q^2*c^2)), x] + (Simp[Sqrt[3]*d*(ArcTan[(1 + 2*q*((2*c + d*x)/(d
*(a + b*x^3)^(1/3))))/Sqrt[3]]/(2*q^2*c^2)), x] - Simp[d*(Log[c + d*x]/(2*q
^2*c^2)), x] - Simp[d*(Log[q*x - (a + b*x^3)^(1/3)]/(4*q^2*c^2)), x] + Simp
[3*d*(Log[q*(2*c + d*x) - d*(a + b*x^3)^(1/3)]/(4*q^2*c^2)), x])] /; FreeQ[
{a, b, c, d}, x] && EqQ[2*b*c^3 - a*d^3, 0]
```

3.21.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 50.89 (sec) , antiderivative size = 3064, normalized size of antiderivative = 20.84

method	result	size
trager	Expression too large to display	3064

input
```
int(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-1/6*ln(-(-15559137585059152-1604954020235328*2^(1/3)*x^4+1471207851882384
0*x^3-936223178470608*x^6-12498127505504256*2^(1/3)*(x^3+1)^(1/3)-42798773
87294208*x^5*2^(2/3)+6471910353179844*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z
^2)*(x^3+1)^(2/3)*x^3+1203809884289286*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_
Z^2)*(x^3+1)^(2/3)*x^4-3218487773589102*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+
_Z^2)*(x^3+1)^(1/3)*x^5-72607968203490*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_
Z^2)^2*(x^3+1)^(2/3)*x^4-23004340956706368*2^(1/3)*x-1604954020235328*2^(2
/3)*x^2-1560939140169318*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(x^3+1)
^(1/3)*x^5+3775614346581480*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(x^3
+1)^(2/3)*x^2-3842311729647552*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(
x^3+1)^(1/3)*x^3-10498622607665136*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)
*(x^3+1)^(1/3)*x^4-7759251414704196*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2
)*(x^3+1)^(2/3)*x-3531674097632562*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)
*(x^3+1)^(1/3)*x^2+2613886855325640*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2
)^2*(x^3+1)^(2/3)*x+840505690860402*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2
)^2*(x^3+1)^(1/3)*x^2-11688730639030284*2^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+
_Z^2)*(x^3+1)^(1/3)*x-5504178119120758*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_
Z^2)+7959206999356368*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^4-1039113368969860
8*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x+6434683875648336*RootOf(2^(2/3)+2^(1/3
)*_Z+_Z^2)^2*x^2+6150800763487380*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^5...
```

3.21.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 712 vs. $2(112) = 224$.

Time = 3.74 (sec) , antiderivative size = 712, normalized size of antiderivative = 4.84

$$\int \frac{1}{\left(1+\sqrt[3]{2}x\right)\left(1+x^3\right)^{2/3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*2^(1/3)*arctan(-1/3*(13910019318573948542*sqrt(3)*(44297109310
930172741433829405399636654451725916403400759596345420183*x^16 + 469911753
877577297266687493361266274298219751726156511748796788210304*x^13 - 168603
219036433260440647021325346295645242325246375460547582960409424*x^10 - 197
880630118237657393829295422779262737333028339787658261155833289 3440*x^7 -
144009089168717758142291876308930196860258103687221308438991237030187 2*x^4
 - 2^(2/3)*(522710774531251076129959239776547583493948769228855528192099 99
866413*x^15 + 590674547854548577293285820788340778493299281255213360593997
994805172*x^12 + 306314261222931431619887382966630423064822217690279625339
1978577817900*x^9 + 733104955869757780900835257159703940345796885706673027
7786114959327080*x^6 + 772324480675629044375977054678087297173944475017351
9635544186114816064*x^3 + 291168089878390092195634857418355141558919044601
5106452608070501424800) + 6*2^(1/3)*(126013559962163220933147486791491205 4
3302140685677058235520929344665*x^14 - 555869063001966513924627194919212 67
8478207988900198502271159380897 18*x^11 - 450398920105320599307639536027883
98613179362472930340743623361078 8504*x^8 - 721888705880948261432517052670 3
941062383389438443735539065108798 66584*x^5 - 338668158068684373436309273 06
784946440569136075137850744247292 1774544*x^2) - 623676430454539792290217 01
2355944404253806601409762924332407805 19680*x)*(x^3 + 1)^(2/3) - 1391001931
8573948542*sqrt(3)*(20244151386762728582873176440916642276036913846721 9...
```

3.21.6 Sympy [F]

$$\int \frac{1}{\left(1+\sqrt[3]{2}x\right)\left(1+x^3\right)^{2/3}}\,dx = \int \frac{1}{\left(\left(x+1\right)\left(x^2-x+1\right)\right)^{\frac{2}{3}}\cdot\left(\sqrt[3]{2}x+1\right)}\,dx$$

input
```
integrate(1/(1+2**(1/3)*x)/(x**3+1)**(2/3),x)
```

output
```
Integral(1/(((x + 1)*(x**2 - x + 1))**(2/3)*(2**(1/3)*x + 1)), x)
```

3.21.7 Maxima [F]

$$\int \frac{1}{\left(1+\sqrt[3]{2}x\right)\left(1+x^3\right)^{2/3}}\,dx = \int \frac{1}{\left(x^3+1\right)^{\frac{2}{3}}\left(2^{\frac{1}{3}}x+1\right)}\,dx$$

input
```
integrate(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)), x)
```

3.21.8 Giac [F]

$$\int \frac{1}{\left(1+\sqrt[3]{2}x\right)\left(1+x^3\right)^{2/3}}\,dx = \int \frac{1}{\left(x^3+1\right)^{\frac{2}{3}}\left(2^{\frac{1}{3}}x+1\right)}\,dx$$

input
```
integrate(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)), x)
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1+\sqrt[3]{2}x\right)\left(1+x^3\right)^{2/3}}\,dx = \int \frac{1}{\left(x^3+1\right)^{2/3}\left(2^{1/3}\,x+1\right)}\,dx$$

input
```
int(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)),x)
```

output
```
int(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)), x)
```

3.22 $\int \frac{1}{\left(1-\sqrt[3]{2}x\right)(1-x^3)^{2/3}}\,dx$

3.22.1 Optimal result

Integrand size = 24, antiderivative size = 159

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)(1-x^3)^{2/3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\left(2^{2/3}-2x\right)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}+\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log\left(1-\sqrt[3]{2}x\right)}{2^{2/3}}+\frac{\log\left(-x-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}-\frac{3\log\left(-2+\sqrt[3]{2}x+\sqrt[3]{2}\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

output
```
1/2*ln(1-2^(1/3)*x)*2^(1/3)+1/4*ln(-x-(-x^3+1)^(1/3))*2^(1/3)-3/4*ln(-2+2^
(1/3)*x+2^(1/3)*(-x^3+1)^(1/3))*2^(1/3)+1/6*arctan(1/3*(1-2*x/(-x^3+1)^(1/
3))*3^(1/2))*2^(1/3)*3^(1/2)-1/2*arctan(1/3*(1+(2*2^(2/3)-2*x)/(-x^3+1)^(1
/3))*3^(1/2))*3^(1/2)*2^(1/3)
```

3.22.2 Mathematica [F]

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx=\int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx$$

input
```
Integrate[1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)),x]
```

output
```
Integrate[1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)), x]
```

3.22.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {2579}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx$$

$$\Big\downarrow \text{2579}$$

$$-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\,2^{2/3}-2x}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2^{2/3}}+\frac{\arctan\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(-\sqrt[3]{1-x^3}-x\right)}{2\,2^{2/3}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{1-x^3}+\sqrt[3]{2}x-2\right)}{2\,2^{2/3}}+\frac{\log\left(1-\sqrt[3]{2}x\right)}{2^{2/3}}$$

input
```
Int[1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)),x]
```

output
```
-((Sqrt[3]*ArcTan[(1 + (2*2^(2/3) - 2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/2^(2/3
)) + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log[1
 - 2^(1/3)*x]/2^(2/3) + Log[-x - (1 - x^3)^(1/3)]/(2*2^(2/3)) - (3*Log[-2
+ 2^(1/3)*x + 2^(1/3)*(1 - x^3)^(1/3)])/(2*2^(2/3))
```

3.22.3.1 Defintions of rubi rules used

rule 2579
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(2/3)), x_Symbol] :> With[
{q = Rt[b, 3]}, Simp[(-d)*(ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/
(2*Sqrt[3]*q^2*c^2)), x] + (Simp[Sqrt[3]*d*(ArcTan[(1 + 2*q*((2*c + d*x)/(d
*(a + b*x^3)^(1/3))))/Sqrt[3]]/(2*q^2*c^2)), x] - Simp[d*(Log[c + d*x]/(2*q
^2*c^2)), x] - Simp[d*(Log[q*x - (a + b*x^3)^(1/3)]/(4*q^2*c^2)), x] + Simp
[3*d*(Log[q*(2*c + d*x) - d*(a + b*x^3)^(1/3)]/(4*q^2*c^2)), x])] /; FreeQ[
{a, b, c, d}, x] && EqQ[2*b*c^3 - a*d^3, 0]
```

3.22.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 48.47 (sec) , antiderivative size = 3247, normalized size of antiderivative = 20.42

method	result	size
trager	Expression too large to display	3247

input
```
int(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/6*ln((1604954020235328*2^(1/3)*x^4+14712078518823840*x^3-384231172964755
2*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*(-x^3+1)^(1/3)*x^3+64719103531
79844*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(2/3)*x^3+104986226
07665136*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(1/3)*x^4+261388
6855325640*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(2/3)*x-7759
251414704196*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(2/3)*x-8405
05690860402*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^3+1)^(1/3)*x^2+3
531674097632562*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1)^(1/3)*x^2
+936223178470608*x^6+12498127505504256*2^(1/3)*(-x^3+1)^(1/3)-427987738729
4208*x^5*2^(2/3)-11688730639030284*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)
*(-x^3+1)^(1/3)*x+72607968203490*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)
*(-x^3+1)^(2/3)*x^4-1203809884289286*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/
3)*(-x^3+1)^(2/3)*x^4-1560939140169318*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2
^(1/3)*(-x^3+1)^(1/3)*x^5-3218487773589102*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)
*2^(2/3)*(-x^3+1)^(1/3)*x^5-3775614346581480*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^
2)^2*2^(2/3)*(-x^3+1)^(2/3)*x^2-2884227944870616*RootOf(2^(2/3)+2^(1/3)*_Z
+_Z^2)*(-x^3+1)^(2/3)*x^2-8123294120973864*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)
*(-x^3+1)^(1/3)*x^3+2613886855325640*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(-x
^3+1)^(2/3)*x^3+11138422684341672*2^(1/3)*(-x^3+1)^(2/3)*x^2+3919074648194
292*2^(2/3)*(-x^3+1)^(2/3)*x^3+6964190009986188*RootOf(2^(2/3)+2^(1/3)*...
```

3.22.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 720 vs. $2(124) = 248$.

Time = 3.52 (sec) , antiderivative size = 720, normalized size of antiderivative = 4.53

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)(1-x^3)^{2/3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*2^(1/3)*arctan(1/3*(13910019318573948542*sqrt(3)*(442971093109
3017274143382940539963665445172591640340075959634542018 3*x^16 - 4699117538
7757729726668749336126627429821975172615651174879678821030 4*x^13 - 1686032
190364332604406470213253462956452423252463754605475829604094 24*x^10 + 1978
806301182376573938292954227792627373330283397876582611558332893440*x^7 - 1
4400908916871775814229187630893019686025810368722130843899123703018 72*x^4
+ 2^(2/3)*(5227107745312510761299592397765475834939487692288555281920999 98
66413*x^15 - 590674547854548577293285820788340778493299281255213360593997 9
94805172*x^12 + 306314261222931431619887382966630423064822217690279625339 1
978577817900*x^9 - 733104955869757780900835257159703940345796885706673027 7
786114959327080*x^6 + 7723244806756290443759770546780872971739444750173519
635544186114816064*x^3 - 2911680898783900921956348574183551415589190446015
106452608070501424800) + 6*2^(1/3)*(1260135599621632209331474867914912054 3
3021406856770582355209293446 65*x^14 + 555869063001966513924627194919212678
478207988900198502271159380897 18*x^11 - 450398920105320599307639536027883 9
861317936247293034074362336107885 04*x^8 + 721888705880948261432517052670 39
41062383389438443735539065108798665 84*x^5 - 338668158068684373436309273067
84946440569136075137850744247292177454 4*x^2) + 623676430454539792290217012
35594440425380660140976292433240780519680*x)*(-x^3 + 1)^(2/3) + 1391001931
8573948542*sqrt(3)*(202441513867627285828731764409166422760369138467219...
```

3.22.6 Sympy [F]

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)(1-x^3)^{2/3}}\,dx = -\int \frac{1}{\sqrt[3]{2}x\,(1-x^3)^{\frac{2}{3}}-(1-x^3)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(1-2**(1/3)*x)/(-x**3+1)**(2/3),x)
```

output
```
-Integral(1/(2**(1/3)*x*(1 - x**3)**(2/3) - (1 - x**3)**(2/3)), x)
```

3.22.7 Maxima [F]

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx = \int -\frac{1}{\left(-x^3+1\right)^{\frac{2}{3}}\left(2^{\frac{1}{3}}x-1\right)}\,dx$$

input
```
integrate(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x, algorithm="maxima")
```

output
```
-integrate(1/((-x^3 + 1)^(2/3)*(2^(1/3)*x - 1)), x)
```

3.22.8 Giac [F]

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx = \int -\frac{1}{\left(-x^3+1\right)^{\frac{2}{3}}\left(2^{\frac{1}{3}}x-1\right)}\,dx$$

input
```
integrate(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x, algorithm="giac")
```

output
```
integrate(-1/((-x^3 + 1)^(2/3)*(2^(1/3)*x - 1)), x)
```

3.22.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1-\sqrt[3]{2}x\right)\left(1-x^3\right)^{2/3}}\,dx = -\int \frac{1}{\left(1-x^3\right)^{2/3}\left(2^{1/3}\,x-1\right)}\,dx$$

input
```
int(-1/((1 - x^3)^(2/3)*(2^(1/3)*x - 1)),x)
```

output
```
-int(1/((1 - x^3)^(2/3)*(2^(1/3)*x - 1)), x)
```

3.23 $\int (c+dx)^4 \sqrt[3]{a+bx^3}\,dx$

3.23.1 Optimal result

Integrand size = 19, antiderivative size = 387

$$\begin{aligned}\int (c+dx)^4 \sqrt[3]{a+bx^3}\,dx &= \frac{3ac^2d^2\sqrt[3]{a+bx^3}}{2b}+\frac{ad^4x^2\sqrt[3]{a+bx^3}}{18b}\\ &+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4+5d^4x^5\right)\\ &-\frac{4ac^3d\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}}+\frac{a^2d^4\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{5/3}}\\ &+\frac{ac^4x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}}\\ &+\frac{acd^3x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}}\\ &-\frac{2ac^3d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{2/3}}+\frac{a^2d^4\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{5/3}}\end{aligned}$$

output
```
3/2*a*c^2*d^2*(b*x^3+a)^(1/3)/b+1/18*a*d^4*x^2*(b*x^3+a)^(1/3)/b+1/30*(b*x
^3+a)^(1/3)*(5*d^4*x^5+24*c*d^3*x^4+45*c^2*d^2*x^3+40*c^3*d*x^2+15*c^4*x)+
1/2*a*c^4*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+
a)^(2/3)+1/5*a*c*d^3*x^4*(1+b*x^3/a)^(2/3)*hypergeom([2/3, 4/3],[7/3],-b*x
^3/a)/(b*x^3+a)^(2/3)-2/3*a*c^3*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)+1/
18*a^2*d^4*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(5/3)-4/9*a*c^3*d*arctan(1/3*(1
+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)*3^(1/2)+1/27*a^2*d^4*arctan
(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(5/3)*3^(1/2)
```

3.23.2 Mathematica [A] (verified)

Time = 6.64 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.42

$$\int (c+dx)^4\sqrt[3]{a+bx^3}\,dx$$

$$=\frac{\sqrt[3]{a+bx^3}\left(6bc^4x\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+d(12bc^3-ad^3)\,x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{2}{3},\frac{5}{3}\right.\right.}{6b\sqrt[3]{1+}}$$

input
```
Integrate[(c + d*x)^4*(a + b*x^3)^(1/3),x]
```

output
```
((a + b*x^3)^(1/3)*(6*b*c^4*x*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/
a)] + d*(12*b*c^3 - a*d^3)*x^2*Hypergeometric2F1[-1/3, 2/3, 5/3, -((b*x^3)
/a)] + d^2*((9*c^2 + d^2*x^2)*(a + b*x^3)*(1 + (b*x^3)/a)^(1/3) + 6*b*c*d*
x^4*Hypergeometric2F1[-1/3, 4/3, 7/3, -((b*x^3)/a)])))/(6*b*(1 + (b*x^3)/a
)^(1/3))
```

3.23.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2392, 27, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^3}(c+dx)^4\,dx$$

$$\Big\downarrow \; 2392$$

$$a\int \frac{15c^4+40dxc^3+45d^2x^2c^2+24d^3x^3c+5d^4x^4}{30\left(bx^3+a\right)^{2/3}}dx+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4+5d^4x^5\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{30}a\int \frac{15c^4+40dxc^3+45d^2x^2c^2+24d^3x^3c+5d^4x^4}{\left(bx^3+a\right)^{2/3}}dx+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4+5d^4x^5\right)$$

$$\Big\downarrow \; 2432$$

$$\frac{1}{30}a\int \left(\frac{15c^4}{\left(bx^3+a\right)^{2/3}}+\frac{40dxc^3}{\left(bx^3+a\right)^{2/3}}+\frac{45d^2x^2c^2}{\left(bx^3+a\right)^{2/3}}+\frac{24d^3x^3c}{\left(bx^3+a\right)^{2/3}}+\frac{5d^4x^4}{\left(bx^3+a\right)^{2/3}}\right)dx+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4+5d^4x^5\right)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{30}a\left(-\frac{40c^3d\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}+\frac{10ad^4\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{5/3}}-\frac{20c^3d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}}+\frac{5ad^4}{}\right.$$
$$\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4+5d^4x^5\right)$$

input
```
Int[(c + d*x)^4*(a + b*x^3)^(1/3),x]
```

3.23. $\int (c+dx)^4\sqrt[3]{a+bx^3}\,dx$

output
```
((a + b*x^3)^(1/3)*(15*c^4*x + 40*c^3*d*x^2 + 45*c^2*d^2*x^3 + 24*c*d^3*x^
4 + 5*d^4*x^5))/30 + (a*((45*c^2*d^2*(a + b*x^3)^(1/3))/b + (5*d^4*x^2*(a
+ b*x^3)^(1/3))/(3*b) - (40*c^3*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1
/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)) + (10*a*d^4*ArcTan[(1 + (2*b^(1/3)*x)/(a
+ b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(5/3)) + (15*c^4*x*(1 + (b*x^3)/a)^
(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) +
(6*c*d^3*x^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 4/3, 7/3, -((b*x
^3)/a)])/(a + b*x^3)^(2/3) - (20*c^3*d*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])
/b^(2/3) + (5*a*d^4*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(3*b^(5/3))))/30
```

3.23.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2392
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Simp[a*n*p   Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*
(x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x
] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

3.23.4 Maple [F]

$$\int (dx+c)^4 \left(b\,x^3+a\right)^{\frac{1}{3}}dx$$

input
```
int((d*x+c)^4*(b*x^3+a)^(1/3),x)
```

output
```
int((d*x+c)^4*(b*x^3+a)^(1/3),x)
```

3.23.5 Fricas [F(-1)]

Timed out.

$$\int (c+dx)^4 \sqrt[3]{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^4*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.23.6 Sympy [A] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.55

$$\int (c+dx)^4 \sqrt[3]{a+bx^3}\,dx = \frac{\sqrt[3]{a}c^4x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{4\sqrt[3]{a}c^3dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + \frac{4\sqrt[3]{a}cd^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt[3]{a}d^4x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + 6c^2d^2\left(\begin{cases}\frac{\sqrt[3]{a}x^3}{3} & \text{for } b=0\\ \frac{\left(a+bx^3\right)^{\frac{4}{3}}}{4b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((d*x+c)**4*(b*x**3+a)**(1/3),x)
```

output
```
a**(1/3)*c**4*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(4/3)) + 4*a**(1/3)*c**3*d*x**2*gamma(2/3)*hyper((-1/3, 2/3)
, (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + 4*a**(1/3)*c*d**3*x**
4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma
(7/3)) + a**(1/3)*d**4*x**5*gamma(5/3)*hyper((-1/3, 5/3), (8/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*gamma(8/3)) + 6*c**2*d**2*Piecewise((a**(1/3)*x**3/3,
 Eq(b, 0)), ((a + b*x**3)**(4/3)/(4*b), True))
```

3.23.7 Maxima [F]

$$\int (c+dx)^4\sqrt[3]{a+bx^3}\,dx=\int (bx^3+a)^{\frac{1}{3}}(dx+c)^4\,dx$$

input
```
integrate((d*x+c)^4*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)*(d*x + c)^4, x)
```

3.23.8 Giac [F]

$$\int (c+dx)^4\sqrt[3]{a+bx^3}\,dx=\int (bx^3+a)^{\frac{1}{3}}(dx+c)^4\,dx$$

input
```
integrate((d*x+c)^4*(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*(d*x + c)^4, x)
```

3.23.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4\sqrt[3]{a+bx^3}\,dx=\int (b\,x^3+a)^{1/3}\,(c+d\,x)^4\,dx$$

input
```
int((a + b*x^3)^(1/3)*(c + d*x)^4,x)
```

output
```
int((a + b*x^3)^(1/3)*(c + d*x)^4, x)
```

3.24 $\int (c+dx)^3 \sqrt[3]{a+bx^3}\,dx$

3.24.1 Optimal result

Integrand size = 19, antiderivative size = 242

$$\int (c+dx)^3 \sqrt[3]{a+bx^3}\,dx = \frac{3acd^2\sqrt[3]{a+bx^3}}{4b}+\frac{ad^3x\sqrt[3]{a+bx^3}}{10b} + \frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right) - \frac{ac^2d\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} + \frac{a(5bc^3-ad^3)\,x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{10b\,(a+bx^3)^{2/3}} - \frac{ac^2d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}$$

output
```
3/4*a*c*d^2*(b*x^3+a)^(1/3)/b+1/10*a*d^3*x*(b*x^3+a)^(1/3)/b+1/20*(b*x^3+a
)^(1/3)*(4*d^3*x^4+15*c*d^2*x^3+20*c^2*d*x^2+10*c^3*x)+1/10*a*(-a*d^3+5*b*
c^3)*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/b/(b*x^3+a)^
(2/3)-1/2*a*c^2*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)-1/3*a*c^2*d*arctan
(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)*3^(1/2)
```

3.24.2 Mathematica [A] (verified)

Time = 6.49 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.59

$$\int (c+dx)^3\sqrt[3]{a+bx^3}\,dx$$
$$=\frac{\sqrt[3]{a+bx^3}\left(4bc^3x\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+d\left(6bc^2x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right.\right.}{4b\sqrt[3]{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(c + d*x)^3*(a + b*x^3)^(1/3),x]
```

output
```
((a + b*x^3)^(1/3)*(4*b*c^3*x*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/
a)] + d*(6*b*c^2*x^2*Hypergeometric2F1[-1/3, 2/3, 5/3, -((b*x^3)/a)] + d*(
3*c*(a + b*x^3)*(1 + (b*x^3)/a)^(1/3) + b*d*x^4*Hypergeometric2F1[-1/3, 4/
3, 7/3, -((b*x^3)/a)]))))/(4*b*(1 + (b*x^3)/a)^(1/3))
```

3.24.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2392, 27, 2427, 27, 2425, 793, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^3}(c+dx)^3\,dx$$

$$\Big\downarrow \text{2392}$$

$$a\int \frac{10c^3+20dxc^2+15d^2x^2c+4d^3x^3}{20\left(bx^3+a\right)^{2/3}}dx+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{20}a\int \frac{10c^3+20dxc^2+15d^2x^2c+4d^3x^3}{\left(bx^3+a\right)^{2/3}}dx+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)$$

$$\Big\downarrow \text{2427}$$

$$\frac{1}{20}a\left(\frac{\int \frac{2\left(20bdxc^2+15bd^2x^2c+2\left(5bc^3-ad^3\right)\right)}{\left(bx^3+a\right)^{2/3}}dx}{2b}+\frac{2d^3x\sqrt[3]{a+bx^3}}{b}\right)+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{20}a\left(\frac{\int \frac{20bdxc^2+15bd^2x^2c+2\left(5bc^3-ad^3\right)}{\left(bx^3+a\right)^{2/3}}dx}{b}+\frac{2d^3x\sqrt[3]{a+bx^3}}{b}\right)+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)$$

$$\Big\downarrow \text{ 2425}$$

$$\frac{1}{20}a\left(\frac{\int \frac{20bdxc^2+2\left(5bc^3-ad^3\right)}{\left(bx^3+a\right)^{2/3}}dx+15bcd^2\int \frac{x^2}{\left(bx^3+a\right)^{2/3}}dx}{b}+\frac{2d^3x\sqrt[3]{a+bx^3}}{b}\right)+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)$$

$$\Big\downarrow \text{ 793}$$

$$\frac{1}{20}a\left(\frac{\int \frac{20bdxc^2+2\left(5bc^3-ad^3\right)}{\left(bx^3+a\right)^{2/3}}dx+15cd^2\sqrt[3]{a+bx^3}}{b}+\frac{2d^3x\sqrt[3]{a+bx^3}}{b}\right)+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)$$

$$\Big\downarrow \text{ 2432}$$

$$\frac{1}{20}a\left(\frac{\int \left(\frac{20bdxc^2}{\left(bx^3+a\right)^{2/3}}+\frac{2\left(5bc^3-ad^3\right)}{\left(bx^3+a\right)^{2/3}}\right)dx+15cd^2\sqrt[3]{a+bx^3}}{b}+\frac{2d^3x\sqrt[3]{a+bx^3}}{b}\right)+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)$$

$$\Big\downarrow \text{ 2009}$$

3.24. $\int (c+dx)^3\sqrt[3]{a+bx^3}\,dx$

$$\frac{1}{20}a\left(\frac{-\frac{20\sqrt[3]{b}c^2d\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{2x\left(\frac{bx^3}{a}+1\right)^{2/3}(5bc^3-ad^3)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}-10\sqrt[3]{b}c^2d\log\left(\sqrt[3]{b}x-\right.}{b}\right.$$

$$\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)$$

input
```
Int[(c + d*x)^3*(a + b*x^3)^(1/3),x]
```

output
```
((a + b*x^3)^(1/3)*(10*c^3*x + 20*c^2*d*x^2 + 15*c*d^2*x^3 + 4*d^3*x^4))/2
0 + (a*((2*d^3*x*(a + b*x^3)^(1/3))/b + (15*c*d^2*(a + b*x^3)^(1/3) - (20*
b^(1/3)*c^2*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/Sqrt[
3] + (2*(5*b*c^3 - a*d^3)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2
/3, 4/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) - 10*b^(1/3)*c^2*d*Log[b^(1/3)*x
 - (a + b*x^3)^(1/3)])/b))/20
```

3.24.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2392
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Simp[a*n*p   Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*
(x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x
] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

3.24.4 Maple [F]

$$\int (dx+c)^3 \left(b\,x^3+a\right)^{\frac{1}{3}} dx$$

input
```
int((d*x+c)^3*(b*x^3+a)^(1/3),x)
```

output
```
int((d*x+c)^3*(b*x^3+a)^(1/3),x)
```

3.24.5 Fricas [F]

$$\int (c+dx)^3 \sqrt[3]{a+bx^3}\,dx=\int \left(bx^3+a\right)^{\frac{1}{3}} (dx+c)^3\,dx$$

input
```
integrate((d*x+c)^3*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*(b*x^3 + a)^(1/3), x)
```

3.24.6 Sympy [A] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.66

$$\int (c+dx)^3\sqrt[3]{a+bx^3}\,dx = \frac{\sqrt[3]{a}c^3x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt[3]{a}c^2dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{\Gamma\left(\frac{5}{3}\right)} + \frac{\sqrt[3]{a}d^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + 3cd^2\left(\begin{cases}\frac{\sqrt[3]{a}x^3}{3} & \text{for } b=0\\ \frac{(a+bx^3)^{\frac{4}{3}}}{4b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((d*x+c)**3*(b*x**3+a)**(1/3),x)
```

output
```
a**(1/3)*c**3*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(4/3)) + a**(1/3)*c**2*d*x**2*gamma(2/3)*hyper((-1/3, 2/3),
(5/3,), b*x**3*exp_polar(I*pi)/a)/gamma(5/3) + a**(1/3)*d**3*x**4*gamma(4/
3)*hyper((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + 3
*c*d**2*Piecewise((a**(1/3)*x**3/3, Eq(b, 0)), ((a + b*x**3)**(4/3)/(4*b),
 True))
```

3.24.7 Maxima [F]

$$\int (c+dx)^3\sqrt[3]{a+bx^3}\,dx = \int (bx^3+a)^{\frac{1}{3}}(dx+c)^3\,dx$$

input
```
integrate((d*x+c)^3*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)*(d*x + c)^3, x)
```

3.24.8 Giac [F]

$$\int (c+dx)^3\sqrt[3]{a+bx^3}\,dx=\int (bx^3+a)^{\frac{1}{3}}(dx+c)^3\,dx$$

input
```
integrate((d*x+c)^3*(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*(d*x + c)^3, x)
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3\sqrt[3]{a+bx^3}\,dx=\int \left(b\,x^3+a\right)^{1/3}\,{\left(c+d\,x\right)}^3\,dx$$

input
```
int((a + b*x^3)^(1/3)*(c + d*x)^3,x)
```

output
```
int((a + b*x^3)^(1/3)*(c + d*x)^3, x)
```

3.25 $\int (c+dx)^2 \sqrt[3]{a+bx^3}\,dx$

3.25.1 Optimal result

Integrand size = 19, antiderivative size = 192

$$\int (c+dx)^2 \sqrt[3]{a+bx^3}\,dx = \frac{ad^2\sqrt[3]{a+bx^3}}{4b} + \frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right) - \frac{2acd\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}} + \frac{ac^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}} - \frac{acd\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{2/3}}$$

output
```
1/4*a*d^2*(b*x^3+a)^(1/3)/b+1/12*(b*x^3+a)^(1/3)*(3*d^2*x^3+8*c*d*x^2+6*c^
2*x)+1/2*a*c^2*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b
*x^3+a)^(2/3)-1/3*a*c*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)-2/9*a*c*d*ar
ctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)*3^(1/2)
```

3.25.2 Mathematica [A] (verified)

Time = 6.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.58

$$\int (c+dx)^2\sqrt[3]{a+bx^3}\,dx$$
$$=\frac{\sqrt[3]{a+bx^3}\left(4bc^2x\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+d\left(d(a+bx^3)\sqrt[3]{1+\frac{bx^3}{a}}+4bcx^2\,\text{Hypergeometr}\right.\right.}{4b\sqrt[3]{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(c + d*x)^2*(a + b*x^3)^(1/3),x]
```

output
```
((a + b*x^3)^(1/3)*(4*b*c^2*x*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/
a)] + d*(d*(a + b*x^3)*(1 + (b*x^3)/a)^(1/3) + 4*b*c*x^2*Hypergeometric2F1
[-1/3, 2/3, 5/3, -((b*x^3)/a)])))/(4*b*(1 + (b*x^3)/a)^(1/3))
```

3.25.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2392, 27, 2425, 793, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^3}(c+dx)^2\,dx$$
$$\Big\downarrow \text{ 2392}$$
$$a\int \frac{6c^2+8dxc+3d^2x^2}{12\left(bx^3+a\right)^{2/3}}dx+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)$$
$$\Big\downarrow \text{ 27}$$
$$\frac{1}{12}a\int \frac{6c^2+8dxc+3d^2x^2}{\left(bx^3+a\right)^{2/3}}dx+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)$$
$$\Big\downarrow \text{ 2425}$$
$$\frac{1}{12}a\left(\int \frac{6c^2+8dxc}{\left(bx^3+a\right)^{2/3}}dx+3d^2\int \frac{x^2}{\left(bx^3+a\right)^{2/3}}dx\right)+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)$$

$$\Big\downarrow \text{793}$$

$$\frac{1}{12}a\left(\int \frac{6c^2+8dxc}{(bx^3+a)^{2/3}}dx+\frac{3d^2\sqrt[3]{a+bx^3}}{b}\right)+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)$$

$$\Big\downarrow \text{2432}$$

$$\frac{1}{12}a\left(\int \left(\frac{6c^2}{(bx^3+a)^{2/3}}+\frac{8dxc}{(bx^3+a)^{2/3}}\right)dx+\frac{3d^2\sqrt[3]{a+bx^3}}{b}\right)+ \frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{12}a\left(-\frac{8cd\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{4cd\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}}+\frac{6c^2x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\right.}{(a+bx^3)^{2/3}}\right.$$

$$\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)$$

input
```
Int[(c + d*x)^2*(a + b*x^3)^(1/3),x]
```

output
```
((a + b*x^3)^(1/3)*(6*c^2*x + 8*c*d*x^2 + 3*d^2*x^3))/12 + (a*((3*d^2*(a +
 b*x^3)^(1/3))/b - (8*c*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqr
t[3]])/(Sqrt[3]*b^(2/3)) + (6*c^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F
1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) - (4*c*d*Log[b^(1/3)*x -
 (a + b*x^3)^(1/3)])/b^(2/3)))/12
```

3.25.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2392
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Simp[a*n*p   Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*
(x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x
] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

3.25.4 Maple [F]

$$\int (dx+c)^2 \left(b\,x^3+a\right)^{\frac{1}{3}}dx$$

input
```
int((d*x+c)^2*(b*x^3+a)^(1/3),x)
```

output
```
int((d*x+c)^2*(b*x^3+a)^(1/3),x)
```

3.25.5 Fricas [F]

$$\int (c+dx)^2\sqrt[3]{a+bx^3}\,dx=\int \left(bx^3+a\right)^{\frac{1}{3}}(dx+c)^2\,dx$$

input
```
integrate((d*x+c)^2*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((d^2*x^2 + 2*c*d*x + c^2)*(b*x^3 + a)^(1/3), x)
```

3.25.6 Sympy [A] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.59

$$\int (c+dx)^2\sqrt[3]{a+bx^3}\,dx = \frac{\sqrt[3]{a}c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2\sqrt[3]{a}cdx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)} + d^2\left(\begin{cases}\frac{\sqrt[3]{a}x^3}{3} & \text{for } b=0\\ \frac{(a+bx^3)^{\frac{4}{3}}}{4b} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((d*x+c)**2*(b*x**3+a)**(1/3),x)
```

output
```
a**(1/3)*c**2*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(4/3)) + 2*a**(1/3)*c*d*x**2*gamma(2/3)*hyper((-1/3, 2/3), (
5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + d**2*Piecewise((a**(1/3)
*x**3/3, Eq(b, 0)), ((a + b*x**3)**(4/3)/(4*b), True))
```

3.25.7 Maxima [F]

$$\int (c+dx)^2\sqrt[3]{a+bx^3}\,dx = \int \left(bx^3+a\right)^{\frac{1}{3}}(dx+c)^2\,dx$$

input
```
integrate((d*x+c)^2*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)*(d*x + c)^2, x)
```

3.25.8 Giac [F]

$$\int (c+dx)^2 \sqrt[3]{a+bx^3} \, dx = \int (bx^3+a)^{\frac{1}{3}} (dx+c)^2 \, dx$$

input
```
integrate((d*x+c)^2*(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*(d*x + c)^2, x)
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \sqrt[3]{a+bx^3} \, dx = \int (b\,x^3+a)^{1/3}\,(c+d\,x)^2 \, dx$$

input
```
int((a + b*x^3)^(1/3)*(c + d*x)^2,x)
```

output
```
int((a + b*x^3)^(1/3)*(c + d*x)^2, x)
```

3.26 $\int (c+dx)\sqrt[3]{a+bx^3}\,dx$

3.26.1 Optimal result

Integrand size = 17, antiderivative size = 155

$$\int (c+dx)\sqrt[3]{a+bx^3}\,dx = \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3} - \frac{ad\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}} + \frac{acx\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}} - \frac{ad\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6b^{2/3}}$$

output
```
1/6*(2*d*x^2+3*c*x)*(b*x^3+a)^(1/3)+1/2*a*c*x*(1+b*x^3/a)^(2/3)*hypergeom(
[1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)-1/6*a*d*ln(b^(1/3)*x-(b*x^3+a)^
(1/3))/b^(2/3)-1/9*a*d*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))
/b^(2/3)*3^(1/2)
```

3.26.2 Mathematica [A] (verified)

Time = 5.63 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.48

$$\int (c+dx)\sqrt[3]{a+bx^3}\,dx = \frac{x\sqrt[3]{a+bx^3}\left(2c\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)\right)}{2\sqrt[3]{1+\frac{bx^3}{a}}}$$

input
```
Integrate[(c + d*x)*(a + b*x^3)^(1/3),x]
```

output
```
(x*(a + b*x^3)^(1/3)*(2*c*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)]
+ d*x*Hypergeometric2F1[-1/3, 2/3, 5/3, -((b*x^3)/a)]))/(2*(1 + (b*x^3)/a)
^(1/3))
```

3.26.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2392, 27, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^3}(c+dx)\,dx$$

$$\Big\downarrow \text{2392}$$

$$a\int \frac{3c+2dx}{6\left(bx^3+a\right)^{2/3}}dx+\frac{1}{6}\sqrt[3]{a+bx^3}\left(3cx+2dx^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}a\int \frac{3c+2dx}{\left(bx^3+a\right)^{2/3}}dx+\frac{1}{6}\sqrt[3]{a+bx^3}\left(3cx+2dx^2\right)$$

$$\Big\downarrow \text{2432}$$

$$\frac{1}{6}a\int \left(\frac{3c}{\left(bx^3+a\right)^{2/3}}+\frac{2dx}{\left(bx^3+a\right)^{2/3}}\right)dx+\frac{1}{6}\sqrt[3]{a+bx^3}\left(3cx+2dx^2\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}a\left(-\frac{2d\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}}+\frac{3cx\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\right.}{(a+bx^3)^{2/3}}\right.$$

$$\frac{1}{6}\sqrt[3]{a+bx^3}(3cx+2dx^2)$$

input
```
Int[(c + d*x)*(a + b*x^3)^(1/3),x]
```

output
```
((3*c*x + 2*d*x^2)*(a + b*x^3)^(1/3))/6 + (a*((-2*d*ArcTan[(1 + (2*b^(1/3)
*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)) + (3*c*x*(1 + (b*x^3)/a
)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3)
- (d*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/b^(2/3)))/6
```

3.26.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2392
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Simp[a*n*p   Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*
(x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x
] && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

3.26.4 Maple [F]

$$\int (dx+c)\left(b\,x^{3}+a\right)^{\frac{1}{3}}dx$$

input
```
int((d*x+c)*(b*x^3+a)^(1/3),x)
```

output
```
int((d*x+c)*(b*x^3+a)^(1/3),x)
```

3.26.5 Fricas [F]

$$\int (c+dx)\sqrt[3]{a+bx^3}\,dx=\int \left(bx^{3}+a\right)^{\frac{1}{3}}(dx+c)\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a)^(1/3)*(d*x + c), x)
```

3.26.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.53

$$\int (c+dx)\sqrt[3]{a+bx^3}\,dx=\frac{\sqrt[3]{a}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt[3]{a}dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((d*x+c)*(b*x**3+a)**(1/3),x)
```

output
```
a**(1/3)*c*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(4/3)) + a**(1/3)*d*x**2*gamma(2/3)*hyper((-1/3, 2/3), (5/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3))
```

3.26.7 Maxima [F]

$$\int (c+dx)\sqrt[3]{a+bx^3}\,dx = \int (bx^3+a)^{\frac{1}{3}}(dx+c)\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)*(d*x + c), x)
```

3.26.8 Giac [F]

$$\int (c+dx)\sqrt[3]{a+bx^3}\,dx = \int (bx^3+a)^{\frac{1}{3}}(dx+c)\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)*(d*x + c), x)
```

3.26.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\sqrt[3]{a+bx^3}\,dx = \int (b\,x^3+a)^{1/3}\,(c+d\,x)\,dx$$

input
```
int((a + b*x^3)^(1/3)*(c + d*x),x)
```

output
```
int((a + b*x^3)^(1/3)*(c + d*x), x)
```

3.27 $\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx$

3.27.1 Optimal result

Integrand size = 19, antiderivative size = 435

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx &= \frac{\sqrt[3]{a+bx^3}}{d}+\frac{x\sqrt[3]{a+bx^3}\,\mathrm{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}\\
&+\frac{\sqrt[3]{b}c\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2}-\frac{\sqrt[3]{bc^3-ad^3}\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2}\\
&+\frac{\sqrt[3]{bc^3-ad^3}\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2}\\
&+\frac{\sqrt[3]{bc^3-ad^3}\log\left(c^3+d^3x^3\right)}{3d^2}+\frac{\sqrt[3]{b}c\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d^2}\\
&-\frac{\sqrt[3]{bc^3-ad^3}\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{2d^2}\\
&-\frac{\sqrt[3]{bc^3-ad^3}\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2d^2}\end{aligned}$$

output
```
(b*x^3+a)^(1/3)/d+x*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,1,4/3,-b*x^3/a,-d^3*
x^3/c^3)/c/(1+b*x^3/a)^(1/3)+1/3*(-a*d^3+b*c^3)^(1/3)*ln(d^3*x^3+c^3)/d^2+
1/2*b^(1/3)*c*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/d^2-1/2*(-a*d^3+b*c^3)^(1/3)*l
n((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/d^2-1/2*(-a*d^3+b*c^3)^(1/3)*l
n((-a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^(1/3))/d^2+1/3*b^(1/3)*c*arctan(1/3*(1+
2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/d^2*3^(1/2)-1/3*(-a*d^3+b*c^3)^(1/3)
*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))/d^2*3^
(1/2)+1/3*(-a*d^3+b*c^3)^(1/3)*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3)/(-a*d^3+b
*c^3)^(1/3))*3^(1/2))/d^2*3^(1/2)
```

3.27.2 Mathematica [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx=\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx$$

input
```
Integrate[(a + b*x^3)^(1/3)/(c + d*x),x]
```

output
```
Integrate[(a + b*x^3)^(1/3)/(c + d*x), x]
```

3.27.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2581, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx$$

$$\Big\downarrow \text{2581}$$

$$\int \left(-\frac{cdx\sqrt[3]{a+bx^3}}{c^3+d^3x^3}+\frac{d^2x^2\sqrt[3]{a+bx^3}}{c^3+d^3x^3}+\frac{c^2\sqrt[3]{a+bx^3}}{c^3+d^3x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{\sqrt[3]{bc^3-ad^3}\arctan\left(\frac{\frac{2x\sqrt[3]{bc^3-ad^3}}{c\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d^2}+\frac{\sqrt[3]{bc^3-ad^3}\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2}+\frac{\sqrt[3]{b}c\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d^2}+\frac{\sqrt[3]{bc^3-ad^3}\log\left(c^3+d^3x^3\right)}{3d^2}-\frac{\sqrt[3]{bc^3-ad^3}\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{2d^2}-\frac{\sqrt[3]{bc^3-ad^3}\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2d^2}+\frac{\sqrt[3]{b}c\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d^2}+\frac{\sqrt[3]{a+bx^3}}{d}$$

input
```
Int[(a + b*x^3)^(1/3)/(c + d*x),x]
```

output
```
(a + b*x^3)^(1/3)/d + (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((
b*x^3)/a), -((d^3*x^3)/c^3)])/(c*(1 + (b*x^3)/a)^(1/3)) + (b^(1/3)*c*ArcTa
n[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^2) - ((b*c^3
- a*d^3)^(1/3)*ArcTan[(1 + (2*(b*c^3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3
)))/Sqrt[3]])/(Sqrt[3]*d^2) + ((b*c^3 - a*d^3)^(1/3)*ArcTan[(1 - (2*d*(a +
 b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^2) + ((b*c^3 -
a*d^3)^(1/3)*Log[c^3 + d^3*x^3])/(3*d^2) + (b^(1/3)*c*Log[b^(1/3)*x - (a +
 b*x^3)^(1/3)])/(2*d^2) - ((b*c^3 - a*d^3)^(1/3)*Log[((b*c^3 - a*d^3)^(1/3
)*x)/c - (a + b*x^3)^(1/3)]])/(2*d^2) - ((b*c^3 - a*d^3)^(1/3)*Log[(b*c^3 -
 a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(2*d^2)
```

3.27.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2581
```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

3.27.4 Maple [F]

$$\int \frac{(b\,x^{3}+a)^{\frac{1}{3}}}{dx+c}dx$$

input
```
int((b*x^3+a)^(1/3)/(d*x+c),x)
```

output
```
int((b*x^3+a)^(1/3)/(d*x+c),x)
```

3.27.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x+c),x, algorithm="fricas")
```

output
```
Timed out
```

3.27.6 Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/(d*x+c),x)
```

output
```
Integral((a + b*x**3)**(1/3)/(c + d*x), x)
```

3.27.7 Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{dx+c}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/(d*x + c), x)
```

3.27.8 Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{dx+c}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/(d*x + c), x)
```

3.27.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx = \int \frac{(b\,x^3+a)^{1/3}}{c+d\,x}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(c + d*x),x)
```

output
```
int((a + b*x^3)^(1/3)/(c + d*x), x)
```

3.28 $\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx$

3.28.1 Optimal result

Integrand size = 19, antiderivative size = 818

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx = -\frac{c^2\sqrt[3]{a+bx^3}}{d\,(c^3+d^3x^3)} - \frac{dx^2\sqrt[3]{a+bx^3}}{c^3+d^3x^3} + \frac{x\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^2\sqrt[3]{1+\frac{bx^3}{a}}} - \frac{d^3x^4\sqrt[3]{a+bx^3}\,\text{AppellF1}\left(\frac{4}{3},-\frac{1}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^5\sqrt[3]{1+\frac{bx^3}{a}}} - \frac{\sqrt[3]{b}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2} + \frac{2ad\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c\,(bc^3-ad^3)^{2/3}} + \frac{(3bc^3-2ad^3)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}cd^2\,(bc^3-ad^3)^{2/3}} - \frac{bc^2\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2\,(bc^3-ad^3)^{2/3}} - \frac{bc^2\log(c^3+d^3x^3)}{6d^2\,(bc^3-ad^3)^{2/3}} - \frac{ad\log(c^3+d^3x^3)}{9c\,(bc^3-ad^3)^{2/3}} - \frac{(3bc^3-2ad^3)\log(c^3+d^3x^3)}{18cd^2\,(bc^3-ad^3)^{2/3}} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d^2} + \frac{ad\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{3c\,(bc^3-ad^3)^{2/3}} + \frac{(3bc^3-2ad^3)\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{6cd^2\,(bc^3-ad^3)^{2/3}} + \frac{bc^2\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2d^2\,(bc^3-ad^3)^{2/3}}$$

output
```
-c^2*(b*x^3+a)^(1/3)/d/(d^3*x^3+c^3)-d*x^2*(b*x^3+a)^(1/3)/(d^3*x^3+c^3)+x
*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,2,4/3,-b*x^3/a,-d^3*x^3/c^3)/c^2/(1+b*x
^3/a)^(1/3)-1/2*d^3*x^4*(b*x^3+a)^(1/3)*AppellF1(4/3,-1/3,2,7/3,-b*x^3/a,-
d^3*x^3/c^3)/c^5/(1+b*x^3/a)^(1/3)-1/6*b*c^2*ln(d^3*x^3+c^3)/d^2/(-a*d^3+b
*c^3)^(2/3)-1/9*a*d*ln(d^3*x^3+c^3)/c/(-a*d^3+b*c^3)^(2/3)-1/18*(-2*a*d^3+
3*b*c^3)*ln(d^3*x^3+c^3)/c/d^2/(-a*d^3+b*c^3)^(2/3)-1/2*b^(1/3)*ln(b^(1/3)
*x-(b*x^3+a)^(1/3))/d^2+1/3*a*d*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3
))/c/(-a*d^3+b*c^3)^(2/3)+1/6*(-2*a*d^3+3*b*c^3)*ln((-a*d^3+b*c^3)^(1/3)*x
/c-(b*x^3+a)^(1/3))/c/d^2/(-a*d^3+b*c^3)^(2/3)+1/2*b*c^2*ln((-a*d^3+b*c^3)
^(1/3)+d*(b*x^3+a)^(1/3))/d^2/(-a*d^3+b*c^3)^(2/3)-1/3*b^(1/3)*arctan(1/3*
(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/d^2*3^(1/2)+2/9*a*d*arctan(1/3*(1
+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))/c/(-a*d^3+b*c^3)^(2/
3)*3^(1/2)+1/9*(-2*a*d^3+3*b*c^3)*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c
/(b*x^3+a)^(1/3))*3^(1/2))/c/d^2/(-a*d^3+b*c^3)^(2/3)*3^(1/2)-1/3*b*c^2*ar
ctan(1/3*(1-2*d*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)^(1/3))*3^(1/2))/d^2/(-a*d^3
+b*c^3)^(2/3)*3^(1/2)
```

3.28.2 Mathematica [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx=\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx$$

input
```
Integrate[(a + b*x^3)^(1/3)/(c + d*x)^2,x]
```

output
```
Integrate[(a + b*x^3)^(1/3)/(c + d*x)^2, x]
```

3.28.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 818, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2581, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{2581}$$

$$\int \left(-\frac{2c^3dx\sqrt[3]{a+bx^3}}{(c^3+d^3x^3)^2}-\frac{2cd^3x^3\sqrt[3]{a+bx^3}}{(c^3+d^3x^3)^2}+\frac{d^4x^4\sqrt[3]{a+bx^3}}{(c^3+d^3x^3)^2}+\frac{c^4\sqrt[3]{a+bx^3}}{(c^3+d^3x^3)^2}+\frac{3c^2d^2x^2\sqrt[3]{a+bx^3}}{(c^3+d^3x^3)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^3\sqrt[3]{bx^3+a}\,\text{AppellF1}\left(\frac{4}{3},-\frac{1}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)x^4}{2c^5\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{d\sqrt[3]{bx^3+a}x^2}{c^3+d^3x^3}+\frac{\sqrt[3]{bx^3+a}\,\text{AppellF1}\left(\frac{1}{3},-\frac{1}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)x}{c^2\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{\sqrt[3]{b}\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{bx^3+a}}+1}{\sqrt{3}}\right)}{\sqrt{3}d^2}+\frac{2ad\arctan\left(\frac{\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{bx^3+a}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c\left(bc^3-ad^3\right)^{2/3}}+\frac{\left(3bc^3-2ad^3\right)\arctan\left(\frac{\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{bx^3+a}}+1}{\sqrt{3}}\right)}{3\sqrt{3}cd^2\left(bc^3-ad^3\right)^{2/3}}-\frac{bc^2\arctan\left(\frac{1-\frac{2d\sqrt[3]{bx^3+a}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2\left(bc^3-ad^3\right)^{2/3}}-\frac{ad\log\left(c^3+d^3x^3\right)}{9c\left(bc^3-ad^3\right)^{2/3}}-\frac{\left(3bc^3-2ad^3\right)\log\left(c^3+d^3x^3\right)}{18cd^2\left(bc^3-ad^3\right)^{2/3}}-\frac{bc^2\log\left(c^3+d^3x^3\right)}{6d^2\left(bc^3-ad^3\right)^{2/3}}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}x-\sqrt[3]{bx^3+a}\right)}{2d^2}+\frac{ad\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{bx^3+a}\right)}{3c\left(bc^3-ad^3\right)^{2/3}}+\frac{\left(3bc^3-2ad^3\right)\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{bx^3+a}\right)}{6cd^2\left(bc^3-ad^3\right)^{2/3}}+\frac{bc^2\log\left(\sqrt[3]{bx^3+a}d+\sqrt[3]{bc^3-ad^3}\right)}{2d^2\left(bc^3-ad^3\right)^{2/3}}-\frac{c^2\sqrt[3]{bx^3+a}}{d\left(c^3+d^3x^3\right)}$$

input
```
Int[(a + b*x^3)^(1/3)/(c + d*x)^2,x]
```

3.28. $\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx$

output
```
-((c^2*(a + b*x^3)^(1/3))/(d*(c^3 + d^3*x^3))) - (d*x^2*(a + b*x^3)^(1/3))
/(c^3 + d^3*x^3) + (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 2, 4/3, -((b*x
^3)/a), -((d^3*x^3)/c^3)])/(c^2*(1 + (b*x^3)/a)^(1/3)) - (d^3*x^4*(a + b*x
^3)^(1/3)*AppellF1[4/3, -1/3, 2, 7/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(2*
c^5*(1 + (b*x^3)/a)^(1/3)) - (b^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3
)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^2) + (2*a*d*ArcTan[(1 + (2*(b*c^3 - a*d^3)^(
1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c*(b*c^3 - a*d^3)^(2/3
)) + ((3*b*c^3 - 2*a*d^3)*ArcTan[(1 + (2*(b*c^3 - a*d^3)^(1/3)*x)/(c*(a +
b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c*d^2*(b*c^3 - a*d^3)^(2/3)) - (b*c^2*
ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1/3))/Sqrt[3]])/(Sqrt
[3]*d^2*(b*c^3 - a*d^3)^(2/3)) - (b*c^2*Log[c^3 + d^3*x^3])/(6*d^2*(b*c^3
- a*d^3)^(2/3)) - (a*d*Log[c^3 + d^3*x^3])/(9*c*(b*c^3 - a*d^3)^(2/3)) - (
(3*b*c^3 - 2*a*d^3)*Log[c^3 + d^3*x^3])/(18*c*d^2*(b*c^3 - a*d^3)^(2/3)) -
 (b^(1/3)*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(2*d^2) + (a*d*Log[((b*c^3 -
 a*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(3*c*(b*c^3 - a*d^3)^(2/3)) + ((3
*b*c^3 - 2*a*d^3)*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(6
*c*d^2*(b*c^3 - a*d^3)^(2/3)) + (b*c^2*Log[(b*c^3 - a*d^3)^(1/3) + d*(a +
b*x^3)^(1/3)])/(2*d^2*(b*c^3 - a*d^3)^(2/3))
```

3.28.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2581
```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

3.28.4 Maple [F]

$$\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{(dx+c)^2}dx$$

input
```
int((b*x^3+a)^(1/3)/(d*x+c)^2,x)
```

output
```
int((b*x^3+a)^(1/3)/(d*x+c)^2,x)
```

3.28.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.28.6 Sympy [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx$$

input
```
integrate((b*x**3+a)**(1/3)/(d*x+c)**2,x)
```

output
```
Integral((a + b*x**3)**(1/3)/(c + d*x)**2, x)
```

3.28.7 Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a)^(1/3)/(d*x + c)^2, x)
```

3.28.8 Giac [F]

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx = \int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx+c)^2}\,dx$$

input
```
integrate((b*x^3+a)^(1/3)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a)^(1/3)/(d*x + c)^2, x)
```

3.28.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx = \int \frac{(b\,x^3+a)^{1/3}}{(c+d\,x)^2}\,dx$$

input
```
int((a + b*x^3)^(1/3)/(c + d*x)^2,x)
```

output
```
int((a + b*x^3)^(1/3)/(c + d*x)^2, x)
```

3.29 $\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx$

3.29.1 Optimal result

Integrand size = 19, antiderivative size = 310

$$\begin{aligned}\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx &= \frac{3c^2d^2(a+bx^3)^{2/3}}{b}+\frac{4cd^3x(a+bx^3)^{2/3}}{3b}\\&\quad+\frac{c^4\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{4acd^3\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}}\\&\quad+\frac{2c^3dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}}\\&\quad+\frac{d^4x^5\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{8}{3},-\frac{bx^3}{a}\right)}{5\sqrt[3]{a+bx^3}}\\&\quad-\frac{c^4\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}+\frac{2acd^3\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{3b^{4/3}}\end{aligned}$$

output
```
3*c^2*d^2*(b*x^3+a)^(2/3)/b+4/3*c*d^3*x*(b*x^3+a)^(2/3)/b+2*c^3*d*x^2*(1+b
*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/(b*x^3+a)^(1/3)+1/5*d^4
*x^5*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 5/3],[8/3],-b*x^3/a)/(b*x^3+a)^(1/3
)-1/2*c^4*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)+2/3*a*c*d^3*ln(-b^(1/3)*x
+(b*x^3+a)^(1/3))/b^(4/3)+1/3*c^4*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3
))*3^(1/2))/b^(1/3)*3^(1/2)-4/9*a*c*d^3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a
)^(1/3))*3^(1/2))/b^(4/3)*3^(1/2)
```

3.29.2 Mathematica [A] (verified)

Time = 10.31 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.26

$$\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx$$

$$= \frac{180b^{4/3}c^3dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)+18b^{4/3}d^4x^5\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\right.}{}$$

input
```
Integrate[(c + d*x)^4/(a + b*x^3)^(1/3),x]
```

output
```
(180*b^(4/3)*c^3*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5
/3, -((b*x^3)/a)] + 18*b^(4/3)*d^4*x^5*(1 + (b*x^3)/a)^(1/3)*Hypergeometri
c2F1[1/3, 5/3, 8/3, -((b*x^3)/a)] + 5*c*(54*a*b^(1/3)*c*d^2 + 24*a*b^(1/3)
*d^3*x + 54*b^(4/3)*c*d^2*x^3 + 24*b^(4/3)*d^3*x^4 + 2*Sqrt[3]*(3*b*c^3 -
4*a*d^3)*(a + b*x^3)^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sq
rt[3]] + 2*(-3*b*c^3 + 4*a*d^3)*(a + b*x^3)^(1/3)*Log[1 - (b^(1/3)*x)/(a +
 b*x^3)^(1/3)] + 3*b*c^3*(a + b*x^3)^(1/3)*Log[1 + (b^(2/3)*x^2)/(a + b*x^
3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)] - 4*a*d^3*(a + b*x^3)^(1/3)*Log[
1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)]))/(90
*b^(4/3)*(a + b*x^3)^(1/3))
```

3.29.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx$$

$$\Big\downarrow \text{2432}$$

$$\int \left(\frac{c^4}{\sqrt[3]{a+bx^3}}+\frac{4c^3dx}{\sqrt[3]{a+bx^3}}+\frac{6c^2d^2x^2}{\sqrt[3]{a+bx^3}}+\frac{4cd^3x^3}{\sqrt[3]{a+bx^3}}+\frac{d^4x^4}{\sqrt[3]{a+bx^3}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4acd^3\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}}+\frac{c^4\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{2acd^3\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{3b^{4/3}}-\frac{c^4\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}+\frac{2c^3dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}}+\frac{3c^2d^2\left(a+bx^3\right)^{2/3}}{b}+\frac{4cd^3x\left(a+bx^3\right)^{2/3}}{3b}+\frac{d^4x^5\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{5}{3},\frac{8}{3},-\frac{bx^3}{a}\right)}{5\sqrt[3]{a+bx^3}}$$

input
```
Int[(c + d*x)^4/(a + b*x^3)^(1/3),x]
```

output
```
(3*c^2*d^2*(a + b*x^3)^(2/3))/b + (4*c*d^3*x*(a + b*x^3)^(2/3))/(3*b) + (c
^4*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)
) - (4*a*c*d^3*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*S
qrt[3]*b^(4/3)) + (2*c^3*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3
, 2/3, 5/3, -((b*x^3)/a)])/(a + b*x^3)^(1/3) + (d^4*x^5*(1 + (b*x^3)/a)^(1
/3)*Hypergeometric2F1[1/3, 5/3, 8/3, -((b*x^3)/a)])/(5*(a + b*x^3)^(1/3))
- (c^4*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(2*b^(1/3)) + (2*a*c*d^3*Log
[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(3*b^(4/3))
```

3.29.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

3.29.4 Maple [F]

$$\int \frac{(dx+c)^4}{(b\,x^3+a)^{\frac{1}{3}}}dx$$

input
```
int((d*x+c)^4/(b*x^3+a)^(1/3),x)
```

output
```
int((d*x+c)^4/(b*x^3+a)^(1/3),x)
```

3.29.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((d*x+c)^4/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.29.6 Sympy [A] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.66

$$\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx = 6c^2d^2\left(\begin{cases}\frac{x^3}{3\sqrt[3]{a}} & \text{for } b=0\\ \frac{(a+bx^3)^{\frac{2}{3}}}{2b} & \text{otherwise}\end{cases}\right) + \frac{c^4x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{4c^3dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)} + \frac{4cd^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)} + \frac{d^4x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((d*x+c)**4/(b*x**3+a)**(1/3),x)
```

output
```
6*c**2*d**2*Piecewise((x**3/(3*a**(1/3)), Eq(b, 0)), ((a + b*x**3)**(2/3)/
(2*b), True)) + c**4*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*a**(1/3)*gamma(4/3)) + 4*c**3*d*x**2*gamma(2/3)*hyper((1/3,
 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(5/3)) + 4*c*d**
3*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(1/3)*gamma(7/3)) + d**4*x**5*gamma(5/3)*hyper((1/3, 5/3), (8/3,), b*x**
3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(8/3))
```

3.29.7 Maxima [F]

$$\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(dx+c)^4}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)^4/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/
3))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3
)/x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c^4 + inte
grate((d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x)/(b*x^3 + a)^(1/3
), x)
```

3.29.8 Giac [F]

$$\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(dx+c)^4}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)^4/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((d*x + c)^4/(b*x^3 + a)^(1/3), x)
```

3.29.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(c+d\,x)^4}{(b\,x^3+a)^{1/3}}\,dx$$

input
```
int((c + d*x)^4/(a + b*x^3)^(1/3),x)
```

output
```
int((c + d*x)^4/(a + b*x^3)^(1/3), x)
```

3.30 $\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx$

3.30.1 Optimal result

Integrand size = 19, antiderivative size = 255

$$\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{3cd^2(a+bx^3)^{2/3}}{2b}+\frac{d^3x(a+bx^3)^{2/3}}{3b}+\frac{c^3\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{ad^3\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}}+\frac{3c^2dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}}-\frac{c^3\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}+\frac{ad^3\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6b^{4/3}}$$

output
```
3/2*c*d^2*(b*x^3+a)^(2/3)/b+1/3*d^3*x*(b*x^3+a)^(2/3)/b+3/2*c^2*d*x^2*(1+b
*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/(b*x^3+a)^(1/3)-1/2*c^3
*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)+1/6*a*d^3*ln(-b^(1/3)*x+(b*x^3+a)^
(1/3))/b^(4/3)+1/3*c^3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))
/b^(1/3)*3^(1/2)-1/9*a*d^3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1
/2))/b^(4/3)*3^(1/2)
```

3.30.2 Mathematica [A] (verified)

Time = 10.07 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx = \frac{1}{18}\left(\frac{27c^2dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}}\right.$$

$$+\frac{27\sqrt[3]{b}cd^2(a+bx^3)^{2/3}+6\sqrt[3]{b}d^3x(a+bx^3)^{2/3}+2\sqrt{3}(3bc^3-ad^3)\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)+(-6bc^3+2a}{b}$$

input
```
Integrate[(c + d*x)^3/(a + b*x^3)^(1/3),x]
```

output
```
((27*c^2*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b
*x^3)/a)])/(a + b*x^3)^(1/3) + (27*b^(1/3)*c*d^2*(a + b*x^3)^(2/3) + 6*b^(
1/3)*d^3*x*(a + b*x^3)^(2/3) + 2*Sqrt[3]*(3*b*c^3 - a*d^3)*ArcTan[(1 + (2*
b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]] + (-6*b*c^3 + 2*a*d^3)*Log[1 - (b^(
1/3)*x)/(a + b*x^3)^(1/3)] + 3*b*c^3*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/
3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)] - a*d^3*Log[1 + (b^(2/3)*x^2)/(a + b*x
^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/b^(4/3))/18
```

3.30.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx$$

$$\downarrow \text{2432}$$

$$\int \left(\frac{c^3}{\sqrt[3]{a+bx^3}}+\frac{3c^2dx}{\sqrt[3]{a+bx^3}}+\frac{3cd^2x^2}{\sqrt[3]{a+bx^3}}+\frac{d^3x^3}{\sqrt[3]{a+bx^3}}\right)dx$$

$$\downarrow \text{2009}$$

$$-\frac{ad^3\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}}+\frac{c^3\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{ad^3\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{6b^{4/3}}-\frac{c^3\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}+\frac{3c^2dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}}+\frac{3cd^2\left(a+bx^3\right)^{2/3}}{2b}+\frac{d^3x\left(a+bx^3\right)^{2/3}}{3b}$$

input
```
Int[(c + d*x)^3/(a + b*x^3)^(1/3),x]
```

output
```
(3*c*d^2*(a + b*x^3)^(2/3))/(2*b) + (d^3*x*(a + b*x^3)^(2/3))/(3*b) + (c^3
*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3))
- (a*d^3*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]
*b^(4/3)) + (3*c^2*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3,
 5/3, -((b*x^3)/a)])/(2*(a + b*x^3)^(1/3)) - (c^3*Log[-(b^(1/3)*x) + (a +
b*x^3)^(1/3)])/(2*b^(1/3)) + (a*d^3*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])
/(6*b^(4/3))
```

3.30.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

3.30.4 Maple [F]

$$\int \frac{(dx+c)^3}{(b\,x^3+a)^{\frac{1}{3}}}dx$$

input
```
int((d*x+c)^3/(b*x^3+a)^(1/3),x)
```

output
```
int((d*x+c)^3/(b*x^3+a)^(1/3),x)
```

3.30.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.30.6 Sympy [A] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.61

$$\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx = 3cd^2\left(\begin{cases}\frac{x^3}{3\sqrt[3]{a}} & \text{for}\ b=0\\ \frac{(a+bx^3)^{\frac{2}{3}}}{2b} & \text{otherwise}\end{cases}\right) + \frac{c^3x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{c^2dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)} + \frac{d^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((d*x+c)**3/(b*x**3+a)**(1/3),x)
```

output
```
3*c*d**2*Piecewise((x**3/(3*a**(1/3)), Eq(b, 0)), ((a + b*x**3)**(2/3)/(2*
b), True)) + c**3*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(
I*pi)/a)/(3*a**(1/3)*gamma(4/3)) + c**2*d*x**2*gamma(2/3)*hyper((1/3, 2/3)
, (5/3,), b*x**3*exp_polar(I*pi)/a)/(a**(1/3)*gamma(5/3)) + d**3*x**4*gamm
a(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gam
ma(7/3))
```

3.30.7 Maxima [F]

$$\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(dx+c)^3}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/
3))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3
)/x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c^3 + inte
grate((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x)/(b*x^3 + a)^(1/3), x)
```

3.30.8 Giac [F]

$$\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(dx+c)^3}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(b*x^3 + a)^(1/3), x)
```

3.30.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(c+d\,x)^3}{(b\,x^3+a)^{1/3}}\,dx$$

input
```
int((c + d*x)^3/(a + b*x^3)^(1/3),x)
```

output
```
int((c + d*x)^3/(a + b*x^3)^(1/3), x)
```

3.31 $\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}\,dx$

3.31.1 Optimal result

Integrand size = 19, antiderivative size = 147

$$\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{d^2(a+bx^3)^{2/3}}{2b} + \frac{c^2\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} + \frac{cdx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}} - \frac{c^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}$$

output
```
1/2*d^2*(b*x^3+a)^(2/3)/b+c*d*x^2*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[
5/3],-b*x^3/a)/(b*x^3+a)^(1/3)-1/2*c^2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1
/3)+1/3*c^2*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^
(1/2)
```

3.31.2 Mathematica [A] (verified)

Time = 9.64 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.37

$$\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{d^2(a+bx^3)^{2/3}}{2b} + \frac{c^2\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} + \frac{cdx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}} - \frac{c^2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} + \frac{c^2\log\left(1+\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{6\sqrt[3]{b}}$$

input
```
Integrate[(c + d*x)^2/(a + b*x^3)^(1/3),x]
```

output
```
(d^2*(a + b*x^3)^(2/3))/(2*b) + (c^2*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)
^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) + (c*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hyper
geometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(a + b*x^3)^(1/3) - (c^2*Log[1
- (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*b^(1/3)) + (c^2*Log[1 + (b^(2/3)*x^2)
/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*b^(1/3))
```

3.31.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2425, 793, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}dx$$

$$\Big\downarrow \text{ 2425}$$

$$\int \frac{c^2+2dxc}{\sqrt[3]{bx^3+a}}dx + d^2\int \frac{x^2}{\sqrt[3]{bx^3+a}}dx$$

$$\Big\downarrow \text{ 793}$$

$$\int \frac{c^2+2dxc}{\sqrt[3]{bx^3+a}}dx+\frac{d^2\left(a+bx^3\right)^{2/3}}{2b}$$

$$\Big\downarrow \text{2432}$$

$$\int \left(\frac{c^2}{\sqrt[3]{bx^3+a}}+\frac{2dxc}{\sqrt[3]{bx^3+a}}\right)dx+\frac{d^2\left(a+bx^3\right)^{2/3}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{c^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}+\frac{cdx^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}}+\frac{d^2\left(a+bx^3\right)^{2/3}}{2b}$$

input
```
Int[(c + d*x)^2/(a + b*x^3)^(1/3),x]
```

output
```
(d^2*(a + b*x^3)^(2/3))/(2*b) + (c^2*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)
^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) + (c*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hyper
geometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(a + b*x^3)^(1/3) - (c^2*Log[-(
b^(1/3)*x) + (a + b*x^3)^(1/3)])/(2*b^(1/3))
```

3.31.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

3.31. $\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}\,dx$

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

3.31.4 Maple [F]

$$\int \frac{(dx+c)^2}{(b\,x^3+a)^{\frac{1}{3}}}dx$$

input
```
int((d*x+c)^2/(b*x^3+a)^(1/3),x)
```

output
```
int((d*x+c)^2/(b*x^3+a)^(1/3),x)
```

3.31.5 Fricas [F]

$$\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(dx+c)^2}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((d^2*x^2 + 2*c*d*x + c^2)/(b*x^3 + a)^(1/3), x)
```

3.31.6 Sympy [A] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.75

$$\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}\,dx = d^2\left(\begin{cases} \frac{x^3}{3\sqrt[3]{a}} & \text{for } b=0 \\ \frac{(a+bx^3)^{\frac{2}{3}}}{2b} & \text{otherwise} \end{cases}\right) + \frac{c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{2cdx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((d*x+c)**2/(b*x**3+a)**(1/3),x)
```

output
```
d**2*Piecewise((x**3/(3*a**(1/3)), Eq(b, 0)), ((a + b*x**3)**(2/3)/(2*b),
True)) + c**2*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*a**(1/3)*gamma(4/3)) + 2*c*d*x**2*gamma(2/3)*hyper((1/3, 2/3), (5/
3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(5/3))
```

3.31.7 Maxima [F]

$$\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(dx+c)^2}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/
3))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3
)/x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c^2 + inte
grate((d^2*x^2 + 2*c*d*x)/(b*x^3 + a)^(1/3), x)
```

3.31.8 Giac [F]

$$\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(dx+c)^2}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(b*x^3 + a)^(1/3), x)
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{(c+d\,x)^2}{(b\,x^3+a)^{1/3}}\,dx$$

input
```
int((c + d*x)^2/(a + b*x^3)^(1/3),x)
```

output
```
int((c + d*x)^2/(a + b*x^3)^(1/3), x)
```

3.32 $\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx$

3.32.1 Optimal result

Integrand size = 17, antiderivative size = 124

$$\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx = \frac{c\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} + \frac{dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}} - \frac{c\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}$$

output
```
1/2*d*x^2*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/(b*x^3+a)
^(1/3)-1/2*c*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)+1/3*c*arctan(1/3*(1+2*
b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

3.32.2 Mathematica [A] (verified)

Time = 9.20 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.31

$$\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx = \frac{1}{6}\left(\frac{3dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}} + \frac{c\left(2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)-2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)+\log\left(1+\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)\right)}{\sqrt[3]{b}}\right)$$

input
```
Integrate[(c + d*x)/(a + b*x^3)^(1/3),x]
```

output
```
((3*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)
/a)])/(a + b*x^3)^(1/3) + (c*(2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x
^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[1 + (
b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)]))/b^(1/3))
/6
```

3.32.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx$$

$$\Big\downarrow \text{2432}$$

$$\int \left(\frac{c}{\sqrt[3]{a+bx^3}}+\frac{dx}{\sqrt[3]{a+bx^3}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{c\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}+\frac{dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}}$$

input
```
Int[(c + d*x)/(a + b*x^3)^(1/3),x]
```

output
```
(c*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)
) + (d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3
)/a)])/(2*(a + b*x^3)^(1/3)) - (c*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(
2*b^(1/3))
```

3.32.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

3.32.4 Maple [F]

$$\int \frac{dx+c}{(b\,x^3+a)^{\frac{1}{3}}}dx$$

input
```
int((d*x+c)/(b*x^3+a)^(1/3),x)
```

output
```
int((d*x+c)/(b*x^3+a)^(1/3),x)
```

3.32.5 Fricas [F]

$$\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{dx+c}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((d*x + c)/(b*x^3 + a)^(1/3), x)
```

3.32.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.63

$$\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((d*x+c)/(b*x**3+a)**(1/3),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
1/3)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/3, 2/3), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(1/3)*gamma(5/3))
```

3.32.7 Maxima [F]

$$\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{dx+c}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/
3))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3
)/x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c + d*inte
grate(x/(b*x^3 + a)^(1/3), x)
```

3.32.8 Giac [F]

$$\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{dx+c}{(bx^3+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate((d*x+c)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((d*x + c)/(b*x^3 + a)^(1/3), x)
```

3.32.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx = \int \frac{c+d\,x}{(b\,x^3+a)^{1/3}}\,dx$$

input
```
int((c + d*x)/(a + b*x^3)^(1/3),x)
```

output
```
int((c + d*x)/(a + b*x^3)^(1/3), x)
```

3.33 $\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx$

3.33.1 Optimal result

Integrand size = 19, antiderivative size = 333

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx = -\frac{dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^2\sqrt[3]{a+bx^3}} + \frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{bc^3-ad^3}} - \frac{\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{bc^3-ad^3}} + \frac{\log\left(c^3+d^3x^3\right)}{3\sqrt[3]{bc^3-ad^3}} - \frac{\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{bc^3-ad^3}} - \frac{\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{bc^3-ad^3}}$$

output
```
-1/2*d*x^2*(1+b*x^3/a)^(1/3)*AppellF1(2/3,1/3,1,5/3,-b*x^3/a,-d^3*x^3/c^3)
/c^2/(b*x^3+a)^(1/3)+1/3*ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^(1/3)-1/2*ln((-a*d
^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/(-a*d^3+b*c^3)^(1/3)-1/2*ln((-a*d^3+b
*c^3)^(1/3)+d*(b*x^3+a)^(1/3))/(-a*d^3+b*c^3)^(1/3)+1/3*arctan(1/3*(1+2*(-
a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))/(-a*d^3+b*c^3)^(1/3)*3^(1
/2)-1/3*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)^(1/3))*3^(1/2))/(
-a*d^3+b*c^3)^(1/3)*3^(1/2)
```

3.33.2 Mathematica [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx$$

input
```
Integrate[1/((c + d*x)*(a + b*x^3)^(1/3)),x]
```

output
```
Integrate[1/((c + d*x)*(a + b*x^3)^(1/3)), x]
```

3.33.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2581, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx)}\,dx$$

$$\Big\downarrow \ 2581$$

$$\int \left(-\frac{cdx}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)}+\frac{d^2x^2}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)}+\frac{c^2}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^2\sqrt[3]{a+bx^3}}+\frac{\arctan\left(\frac{\frac{2x\sqrt[3]{bc^3-ad^3}}{c\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{bc^3-ad^3}}-\frac{\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{bc^3-ad^3}}+\frac{\log\left(c^3+d^3x^3\right)}{3\sqrt[3]{bc^3-ad^3}}-\frac{\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{bc^3-ad^3}}-\frac{\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{bc^3-ad^3}}$$

input
```
Int[1/((c + d*x)*(a + b*x^3)^(1/3)),x]
```

output
```
-1/2*(d*x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a),
 -((d^3*x^3)/c^3)])/(c^2*(a + b*x^3)^(1/3)) + ArcTan[(1 + (2*(b*c^3 - a*d^
3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*(b*c^3 - a*d^3)^(1/3)
) - ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1/3))/Sqrt[3]]/(S
qrt[3]*(b*c^3 - a*d^3)^(1/3)) + Log[c^3 + d^3*x^3]/(3*(b*c^3 - a*d^3)^(1/3
)) - Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)]/(2*(b*c^3 - a*d^
3)^(1/3)) - Log[(b*c^3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)]/(2*(b*c^3 - a
*d^3)^(1/3))
```

3.33.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2581
```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

3.33.4 Maple [F]

$$\int \frac{1}{(dx+c)\,(b\,x^3+a)^{\frac{1}{3}}}dx$$

input
```
int(1/(d*x+c)/(b*x^3+a)^(1/3),x)
```

output
```
int(1/(d*x+c)/(b*x^3+a)^(1/3),x)
```

3.33.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.33.6 Sympy [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(b*x**3+a)**(1/3),x)
```

output
```
Integral(1/((a + b*x**3)**(1/3)*(c + d*x)), x)
```

3.33.7 Maxima [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)), x)
```

3.33.8 Giac [F]

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)), x)
```

3.33.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(b\,x^3+a)^{1/3}\,(c+d\,x)}\,dx$$

input
```
int(1/((a + b*x^3)^(1/3)*(c + d*x)),x)
```

output
```
int(1/((a + b*x^3)^(1/3)*(c + d*x)), x)
```

3.34 $\int \frac{1}{(c+dx)^2 \sqrt[3]{a+bx^3}}\,dx$

3.34.1 Optimal result

Integrand size = 19, antiderivative size = 761

$$\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx = \frac{c^2d^2(a+bx^3)^{2/3}}{(bc^3-ad^3)(c^3+d^3x^3)} - \frac{cd^3x(a+bx^3)^{2/3}}{(bc^3-ad^3)(c^3+d^3x^3)} - \frac{dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},2,\frac{5}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^3\sqrt[3]{a+bx^3}} + \frac{d^4x^5\sqrt[3]{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},2,\frac{8}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{5c^6\sqrt[3]{a+bx^3}} + \frac{2ad^3\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c\,(bc^3-ad^3)^{4/3}} + \frac{(3bc^3-2ad^3)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c\,(bc^3-ad^3)^{4/3}} - \frac{bc^2\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,(bc^3-ad^3)^{4/3}} + \frac{bc^2\log(c^3+d^3x^3)}{6(bc^3-ad^3)^{4/3}} + \frac{ad^3\log(c^3+d^3x^3)}{9c\,(bc^3-ad^3)^{4/3}} + \frac{(3bc^3-2ad^3)\log(c^3+d^3x^3)}{18c\,(bc^3-ad^3)^{4/3}} - \frac{ad^3\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{3c\,(bc^3-ad^3)^{4/3}} - \frac{(3bc^3-2ad^3)\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{a+bx^3}\right)}{6c\,(bc^3-ad^3)^{4/3}} - \frac{bc^2\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2(bc^3-ad^3)^{4/3}}$$

output
```
c^2*d^2*(b*x^3+a)^(2/3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)-c*d^3*x*(b*x^3+a)^(2/
3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)-d*x^2*(1+b*x^3/a)^(1/3)*AppellF1(2/3,1/3,2
,5/3,-b*x^3/a,-d^3*x^3/c^3)/c^3/(b*x^3+a)^(1/3)+1/5*d^4*x^5*(1+b*x^3/a)^(1
/3)*AppellF1(5/3,1/3,2,8/3,-b*x^3/a,-d^3*x^3/c^3)/c^6/(b*x^3+a)^(1/3)+1/6*
b*c^2*ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^(4/3)+1/9*a*d^3*ln(d^3*x^3+c^3)/c/(-a
*d^3+b*c^3)^(4/3)+1/18*(-2*a*d^3+3*b*c^3)*ln(d^3*x^3+c^3)/c/(-a*d^3+b*c^3)
^(4/3)-1/3*a*d^3*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c/(-a*d^3+b*
c^3)^(4/3)-1/6*(-2*a*d^3+3*b*c^3)*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1
/3))/c/(-a*d^3+b*c^3)^(4/3)-1/2*b*c^2*ln((-a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^
(1/3))/(-a*d^3+b*c^3)^(4/3)+2/9*a*d^3*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)
*x/c/(b*x^3+a)^(1/3))*3^(1/2))/c/(-a*d^3+b*c^3)^(4/3)*3^(1/2)+1/9*(-2*a*d^
3+3*b*c^3)*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/
2))/c/(-a*d^3+b*c^3)^(4/3)*3^(1/2)-1/3*b*c^2*arctan(1/3*(1-2*d*(b*x^3+a)^(
1/3)/(-a*d^3+b*c^3)^(1/3))*3^(1/2))/(-a*d^3+b*c^3)^(4/3)*3^(1/2)
```

3.34.2 Mathematica [F]

$$\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx$$

input
```
Integrate[1/((c + d*x)^2*(a + b*x^3)^(1/3)),x]
```

output
```
Integrate[1/((c + d*x)^2*(a + b*x^3)^(1/3)), x]
```

3.34.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 761, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2581, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx)^2}\,dx$$

$$\Big\downarrow \text{ 2581}$$

$$\int \left(-\frac{2c^3dx}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^2}-\frac{2cd^3x^3}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^2}+\frac{d^4x^4}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^2}+\frac{c^4}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^2}+\frac{}{\sqrt[3]{}}\right.$$

$\downarrow$ 2009

$$-\frac{dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},2,\frac{5}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^3\sqrt[3]{a+bx^3}}+\frac{d^4x^5\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},2,\frac{8}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{5c^6\sqrt[3]{a+bx^3}}+\frac{2ad^3\arctan\left(\frac{\frac{2x\sqrt[3]{bc^3-ad^3}}{c\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c\,(bc^3-ad^3)^{4/3}}+\frac{(3bc^3-2ad^3)\arctan\left(\frac{\frac{2x\sqrt[3]{bc^3-ad^3}}{c\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c\,(bc^3-ad^3)^{4/3}}-\frac{bc^2\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,(bc^3-ad^3)^{4/3}}-\frac{cd^3x(a+bx^3)^{2/3}}{(c^3+d^3x^3)(bc^3-ad^3)}+\frac{ad^3\log(c^3+d^3x^3)}{9c\,(bc^3-ad^3)^{4/3}}+\frac{(3bc^3-2ad^3)\log(c^3+d^3x^3)}{18c\,(bc^3-ad^3)^{4/3}}-\frac{ad^3\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{3c\,(bc^3-ad^3)^{4/3}}-\frac{(3bc^3-2ad^3)\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{6c\,(bc^3-ad^3)^{4/3}}+\frac{bc^2\log(c^3+d^3x^3)}{6\,(bc^3-ad^3)^{4/3}}-\frac{bc^2\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2\,(bc^3-ad^3)^{4/3}}+\frac{c^2d^2(a+bx^3)^{2/3}}{(c^3+d^3x^3)(bc^3-ad^3)}$$

input
```
Int[1/((c + d*x)^2*(a + b*x^3)^(1/3)),x]
```

3.34. $\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx$

output
```
(c^2*d^2*(a + b*x^3)^(2/3))/((b*c^3 - a*d^3)*(c^3 + d^3*x^3)) - (c*d^3*x*(
a + b*x^3)^(2/3))/((b*c^3 - a*d^3)*(c^3 + d^3*x^3)) - (d*x^2*(1 + (b*x^3)/
a)^(1/3)*AppellF1[2/3, 1/3, 2, 5/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(c^3*
(a + b*x^3)^(1/3)) + (d^4*x^5*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 2,
8/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(5*c^6*(a + b*x^3)^(1/3)) + (2*a*d^3
*ArcTan[(1 + (2*(b*c^3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/
(3*Sqrt[3]*c*(b*c^3 - a*d^3)^(4/3)) + ((3*b*c^3 - 2*a*d^3)*ArcTan[(1 + (2*
(b*c^3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c*(b*
c^3 - a*d^3)^(4/3)) - (b*c^2*ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 -
a*d^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*(b*c^3 - a*d^3)^(4/3)) + (b*c^2*Log[c^3 +
 d^3*x^3])/(6*(b*c^3 - a*d^3)^(4/3)) + (a*d^3*Log[c^3 + d^3*x^3])/(9*c*(b*
c^3 - a*d^3)^(4/3)) + ((3*b*c^3 - 2*a*d^3)*Log[c^3 + d^3*x^3])/(18*c*(b*c^
3 - a*d^3)^(4/3)) - (a*d^3*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a + b*x^3)^(
1/3)])/(3*c*(b*c^3 - a*d^3)^(4/3)) - ((3*b*c^3 - 2*a*d^3)*Log[((b*c^3 - a*
d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(6*c*(b*c^3 - a*d^3)^(4/3)) - (b*c^2
*Log[(b*c^3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(2*(b*c^3 - a*d^3)^(4/3
))
```

3.34.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2581
```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

3.34.4 Maple [F]

$$\int \frac{1}{(dx+c)^2 (b\,x^3+a)^{\frac{1}{3}}}dx$$

input
```
int(1/(d*x+c)^2/(b*x^3+a)^(1/3),x)
```

output
```
int(1/(d*x+c)^2/(b*x^3+a)^(1/3),x)
```

3.34.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.34.6 Sympy [F]

$$\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx)^2}\,dx$$

input
```
integrate(1/(d*x+c)**2/(b*x**3+a)**(1/3),x)
```

output
```
Integral(1/((a + b*x**3)**(1/3)*(c + d*x)**2), x)
```

3.34.7 Maxima [F]

$$\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}\,(dx+c)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)^2), x)
```

3.34.8 Giac [F]

$$\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx+c)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)^2), x)
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(b\,x^3+a)^{1/3}\,(c+d\,x)^2}\,dx$$

input
```
int(1/((a + b*x^3)^(1/3)*(c + d*x)^2),x)
```

output
```
int(1/((a + b*x^3)^(1/3)*(c + d*x)^2), x)
```

3.35 $\int \frac{1}{(c+dx)^3 \sqrt[3]{a+bx^3}} \, dx$

3.35.1 Optimal result

Integrand size = 19, antiderivative size = 1513

$$\int \frac{1}{(c+dx)^3 \sqrt[3]{a+bx^3}} \, dx = \text{Too large to display}$$

output
```
3/2*c^4*d^2*(b*x^3+a)^(2/3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)^2-3/2*c^3*d^3*x*(
b*x^3+a)^(2/3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)^2+4/3*b*c^4*d^2*(b*x^3+a)^(2/3
)/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)-1/3*c*d^2*(-3*a*d^3+b*c^3)*(b*x^3+a)^(2/3
)/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)+1/18*d^3*(-7*a*d^3+3*b*c^3)*x*(b*x^3+a)^(
2/3)/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)-1/18*d^3*(-5*a*d^3+9*b*c^3)*x*(b*x^3+a
)^(2/3)/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)-7/18*d^3*(a*d^3+3*b*c^3)*x*(b*x^3+a
)^(2/3)/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)-3/2*d*x^2*(1+b*x^3/a)^(1/3)*AppellF
1(2/3,1/3,3,5/3,-b*x^3/a,-d^3*x^3/c^3)/c^4/(b*x^3+a)^(1/3)+6/5*d^4*x^5*(1+
b*x^3/a)^(1/3)*AppellF1(5/3,1/3,3,8/3,-b*x^3/a,-d^3*x^3/c^3)/c^7/(b*x^3+a)
^(1/3)+2/9*b^2*c^4*ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^(7/3)+1/27*a^2*d^6*ln(d^
3*x^3+c^3)/c^2/(-a*d^3+b*c^3)^(7/3)-1/18*b*c*(-3*a*d^3+b*c^3)*ln(d^3*x^3+c
^3)/(-a*d^3+b*c^3)^(7/3)+7/54*a*d^3*(-a*d^3+3*b*c^3)*ln(d^3*x^3+c^3)/c^2/(
-a*d^3+b*c^3)^(7/3)+1/54*(5*a^2*d^6-12*a*b*c^3*d^3+9*b^2*c^6)*ln(d^3*x^3+c
^3)/c^2/(-a*d^3+b*c^3)^(7/3)-1/9*a^2*d^6*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^
3+a)^(1/3))/c^2/(-a*d^3+b*c^3)^(7/3)-7/18*a*d^3*(-a*d^3+3*b*c^3)*ln((-a*d^
3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c^2/(-a*d^3+b*c^3)^(7/3)-1/18*(5*a^2*d
^6-12*a*b*c^3*d^3+9*b^2*c^6)*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/
c^2/(-a*d^3+b*c^3)^(7/3)-2/3*b^2*c^4*ln((-a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^(
1/3))/(-a*d^3+b*c^3)^(7/3)+1/6*b*c*(-3*a*d^3+b*c^3)*ln((-a*d^3+b*c^3)^(1/3
)+d*(b*x^3+a)^(1/3))/(-a*d^3+b*c^3)^(7/3)+2/27*a^2*d^6*arctan(1/3*(1+2*...
```

3.35.2 Mathematica [F]

$$\int \frac{1}{(c+dx)^3\sqrt[3]{a+bx^3}}\,dx=\int \frac{1}{(c+dx)^3\sqrt[3]{a+bx^3}}\,dx$$

input
```
Integrate[1/((c + d*x)^3*(a + b*x^3)^(1/3)),x]
```

output
```
Integrate[1/((c + d*x)^3*(a + b*x^3)^(1/3)), x]
```

3.35.3 Rubi [A] (verified)

Time = 2.24 (sec) , antiderivative size = 1513, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2581, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx)^3}\,dx$$

$$\Big\downarrow \text{2581}$$

$$\int \left(-\frac{7c^3d^3x^3}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^3}+\frac{d^6x^6}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^3}-\frac{3cd^5x^5}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^3}+\frac{c^6}{\sqrt[3]{a+bx^3}\,(c^3+d^3x^3)^3}-\frac{}{\sqrt[3]{}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^2\arctan\left(\frac{\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{bx^3+a}}+1}{\sqrt{3}}\right)d^6}{9\sqrt{3}c^2\left(bc^3-ad^3\right)^{7/3}}+\frac{a^2\log\left(c^3+d^3x^3\right)d^6}{27c^2\left(bc^3-ad^3\right)^{7/3}}-$$

$$\frac{a^2\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{bx^3+a}\right)d^6}{9c^2\left(bc^3-ad^3\right)^{7/3}}+\frac{6x^5\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{5}{3},\frac{1}{3},3,\frac{8}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)d^4}{5c^7\sqrt[3]{bx^3+a}}+$$

$$\frac{7a\left(3bc^3-ad^3\right)\arctan\left(\frac{\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{bx^3+a}}+1}{\sqrt{3}}\right)d^3}{9\sqrt{3}c^2\left(bc^3-ad^3\right)^{7/3}}+\frac{7a\left(3bc^3-ad^3\right)\log\left(c^3+d^3x^3\right)d^3}{54c^2\left(bc^3-ad^3\right)^{7/3}}-$$

$$\frac{7a\left(3bc^3-ad^3\right)\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{bx^3+a}\right)d^3}{18c^2\left(bc^3-ad^3\right)^{7/3}}-\frac{7\left(3bc^3+ad^3\right)x\left(bx^3+a\right)^{2/3}d^3}{18\left(bc^3-ad^3\right)^2\left(c^3+d^3x^3\right)}+$$

$$\frac{\left(3bc^3-7ad^3\right)x\left(bx^3+a\right)^{2/3}d^3}{18\left(bc^3-ad^3\right)^2\left(c^3+d^3x^3\right)}-\frac{\left(9bc^3-5ad^3\right)x\left(bx^3+a\right)^{2/3}d^3}{18\left(bc^3-ad^3\right)^2\left(c^3+d^3x^3\right)}-\frac{3c^3x\left(bx^3+a\right)^{2/3}d^3}{2\left(bc^3-ad^3\right)\left(c^3+d^3x^3\right)^2}+$$

$$\frac{4bc^4\left(bx^3+a\right)^{2/3}d^2}{3\left(bc^3-ad^3\right)^2\left(c^3+d^3x^3\right)}-\frac{c\left(bc^3-3ad^3\right)\left(bx^3+a\right)^{2/3}d^2}{3\left(bc^3-ad^3\right)^2\left(c^3+d^3x^3\right)}+\frac{3c^4\left(bx^3+a\right)^{2/3}d^2}{2\left(bc^3-ad^3\right)\left(c^3+d^3x^3\right)^2}-$$

$$\frac{3x^2\sqrt[3]{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},3,\frac{5}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)d}{2c^4\sqrt[3]{bx^3+a}}+$$

$$\frac{\left(9b^2c^6-12abd^3c^3+5a^2d^6\right)\arctan\left(\frac{\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{bx^3+a}}+1}{\sqrt{3}}\right)}{9\sqrt{3}c^2\left(bc^3-ad^3\right)^{7/3}}-\frac{4b^2c^4\arctan\left(\frac{1-\frac{2d\sqrt[3]{bx^3+a}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\left(bc^3-ad^3\right)^{7/3}}+$$

$$\frac{bc\left(bc^3-3ad^3\right)\arctan\left(\frac{1-\frac{2d\sqrt[3]{bx^3+a}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\left(bc^3-ad^3\right)^{7/3}}+\frac{\left(9b^2c^6-12abd^3c^3+5a^2d^6\right)\log\left(c^3+d^3x^3\right)}{54c^2\left(bc^3-ad^3\right)^{7/3}}+$$

$$\frac{2b^2c^4\log\left(c^3+d^3x^3\right)}{9\left(bc^3-ad^3\right)^{7/3}}-\frac{bc\left(bc^3-3ad^3\right)\log\left(c^3+d^3x^3\right)}{18\left(bc^3-ad^3\right)^{7/3}}-$$

$$\frac{\left(9b^2c^6-12abd^3c^3+5a^2d^6\right)\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{bx^3+a}\right)}{18c^2\left(bc^3-ad^3\right)^{7/3}}-$$

$$\frac{2b^2c^4\log\left(\sqrt[3]{bx^3+a}d+\sqrt[3]{bc^3-ad^3}\right)}{3\left(bc^3-ad^3\right)^{7/3}}+\frac{bc\left(bc^3-3ad^3\right)\log\left(\sqrt[3]{bx^3+a}d+\sqrt[3]{bc^3-ad^3}\right)}{6\left(bc^3-ad^3\right)^{7/3}}$$

3.35. $\int \frac{1}{(c+dx)^3\sqrt[3]{a+bx^3}}\,dx$

input
```
Int[1/((c + d*x)^3*(a + b*x^3)^(1/3)),x]
```

output
```
(3*c^4*d^2*(a + b*x^3)^(2/3))/(2*(b*c^3 - a*d^3)*(c^3 + d^3*x^3)^2) - (3*c
^3*d^3*x*(a + b*x^3)^(2/3))/(2*(b*c^3 - a*d^3)*(c^3 + d^3*x^3)^2) + (4*b*c
^4*d^2*(a + b*x^3)^(2/3))/(3*(b*c^3 - a*d^3)^2*(c^3 + d^3*x^3)) - (c*d^2*(
b*c^3 - 3*a*d^3)*(a + b*x^3)^(2/3))/(3*(b*c^3 - a*d^3)^2*(c^3 + d^3*x^3))
+ (d^3*(3*b*c^3 - 7*a*d^3)*x*(a + b*x^3)^(2/3))/(18*(b*c^3 - a*d^3)^2*(c^3
 + d^3*x^3)) - (d^3*(9*b*c^3 - 5*a*d^3)*x*(a + b*x^3)^(2/3))/(18*(b*c^3 -
a*d^3)^2*(c^3 + d^3*x^3)) - (7*d^3*(3*b*c^3 + a*d^3)*x*(a + b*x^3)^(2/3))/
(18*(b*c^3 - a*d^3)^2*(c^3 + d^3*x^3)) - (3*d*x^2*(1 + (b*x^3)/a)^(1/3)*Ap
pellF1[2/3, 1/3, 3, 5/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(2*c^4*(a + b*x^
3)^(1/3)) + (6*d^4*x^5*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 3, 8/3, -(
(b*x^3)/a), -((d^3*x^3)/c^3)])/(5*c^7*(a + b*x^3)^(1/3)) + (2*a^2*d^6*ArcT
an[(1 + (2*(b*c^3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(9*Sq
rt[3]*c^2*(b*c^3 - a*d^3)^(7/3)) + (7*a*d^3*(3*b*c^3 - a*d^3)*ArcTan[(1 +
(2*(b*c^3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[3]*c^
2*(b*c^3 - a*d^3)^(7/3)) + ((9*b^2*c^6 - 12*a*b*c^3*d^3 + 5*a^2*d^6)*ArcTa
n[(1 + (2*(b*c^3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqr
t[3]*c^2*(b*c^3 - a*d^3)^(7/3)) - (4*b^2*c^4*ArcTan[(1 - (2*d*(a + b*x^3)^
(1/3))/(b*c^3 - a*d^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*(b*c^3 - a*d^3)^(7/3))
+ (b*c*(b*c^3 - 3*a*d^3)*ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^
3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*(b*c^3 - a*d^3)^(7/3)) + (2*b^2*c^4*Log[...
```

3.35.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2581
```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

3.35.4 Maple [F]

$$\int \frac{1}{(dx+c)^{3} \left(b\,x^{3}+a\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(d*x+c)^3/(b*x^3+a)^(1/3),x)
```

output
```
int(1/(d*x+c)^3/(b*x^3+a)^(1/3),x)
```

3.35.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^3\sqrt[3]{a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.35.6 Sympy [F]

$$\int \frac{1}{(c+dx)^3\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx)^3}\,dx$$

input
```
integrate(1/(d*x+c)**3/(b*x**3+a)**(1/3),x)
```

output
```
Integral(1/((a + b*x**3)**(1/3)*(c + d*x)**3), x)
```

3.35.7 Maxima [F]

$$\int \frac{1}{(c+dx)^3 \sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx+c)^3}\,dx$$

input
```
integrate(1/(d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)^3), x)
```

3.35.8 Giac [F]

$$\int \frac{1}{(c+dx)^3 \sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx+c)^3}\,dx$$

input
```
integrate(1/(d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)^3), x)
```

3.35.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^3 \sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(b\,x^3+a)^{1/3}\,(c+d\,x)^3}\,dx$$

input
```
int(1/((a + b*x^3)^(1/3)*(c + d*x)^3),x)
```

output
```
int(1/((a + b*x^3)^(1/3)*(c + d*x)^3), x)
```

3.36 $\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}}\,dx$

3.36.1 Optimal result

Integrand size = 19, antiderivative size = 306

$$\begin{aligned}\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}}\,dx &= \frac{6c^2d^2\sqrt[3]{a+bx^3}}{b}+\frac{d^4x^2\sqrt[3]{a+bx^3}}{3b}-\frac{4c^3d\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}\\
&+\frac{2ad^4\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{5/3}}+\frac{c^4x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}\\
&+\frac{cd^3x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}\\
&-\frac{2c^3d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}}+\frac{ad^4\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{5/3}}\end{aligned}$$

output
```
6*c^2*d^2*(b*x^3+a)^(1/3)/b+1/3*d^4*x^2*(b*x^3+a)^(1/3)/b+c^4*x*(1+b*x^3/a
)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)+c*d^3*x^4*(1+
b*x^3/a)^(2/3)*hypergeom([2/3, 4/3],[7/3],-b*x^3/a)/(b*x^3+a)^(2/3)-2*c^3*
d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)+1/3*a*d^4*ln(b^(1/3)*x-(b*x^3+a)^(
1/3))/b^(5/3)-4/3*c^3*d*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2)
)/b^(2/3)*3^(1/2)+2/9*a*d^4*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(
1/2))/b^(5/3)*3^(1/2)
```

3.36.2 Mathematica [A] (verified)

Time = 10.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.54

$$\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}}\,dx = \frac{3bc^4x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+d\left((6bc^3-ad^3)\,x^2\,\text{Hypergeom}\right.}{}$$

input
```
Integrate[(c + d*x)^4/(a + b*x^3)^(2/3),x]
```

output
```
(3*b*c^4*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3
)/a)] + d*((6*b*c^3 - a*d^3)*x^2*Hypergeometric2F1[2/3, 1, 5/3, (b*x^3)/(a
 + b*x^3)] + d*((18*c^2 + d^2*x^2)*(a + b*x^3) + 3*b*c*d*x^4*(1 + (b*x^3)/
a)^(2/3)*Hypergeometric2F1[2/3, 4/3, 7/3, -((b*x^3)/a)])))/(3*b*(a + b*x^3
)^(2/3))
```

3.36.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}}\,dx$$

$$\Big\downarrow \text{ 2432}$$

$$\int \left(\frac{c^4}{(a+bx^3)^{2/3}}+\frac{4c^3dx}{(a+bx^3)^{2/3}}+\frac{6c^2d^2x^2}{(a+bx^3)^{2/3}}+\frac{4cd^3x^3}{(a+bx^3)^{2/3}}+\frac{d^4x^4}{(a+bx^3)^{2/3}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{4c^3d\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}+\frac{2ad^4\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{5/3}}-\frac{2c^3d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}}+\frac{ad^4\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{5/3}}+\frac{c^4x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}+\frac{6c^2d^2\sqrt[3]{a+bx^3}}{b}+\frac{cd^3x^4\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{2}{3},\frac{4}{3},\frac{7}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}+\frac{d^4x^2\sqrt[3]{a+bx^3}}{3b}$$

input
```
Int[(c + d*x)^4/(a + b*x^3)^(2/3),x]
```

output
```
(6*c^2*d^2*(a + b*x^3)^(1/3))/b + (d^4*x^2*(a + b*x^3)^(1/3))/(3*b) - (4*c
^3*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/
3)) + (2*a*d^4*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*S
qrt[3]*b^(5/3)) + (c^4*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3,
 4/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) + (c*d^3*x^4*(1 + (b*x^3)/a)^(2/3)*
Hypergeometric2F1[2/3, 4/3, 7/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) - (2*c^3
*d*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/b^(2/3) + (a*d^4*Log[b^(1/3)*x - (a
 + b*x^3)^(1/3)])/(3*b^(5/3))
```

3.36.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

3.36.4 Maple [F]

$$\int \frac{(dx+c)^4}{(b\,x^3+a)^{\frac{2}{3}}}dx$$

input
```
int((d*x+c)^4/(b*x^3+a)^(2/3),x)
```

output
```
int((d*x+c)^4/(b*x^3+a)^(2/3),x)
```

3.36.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)^4/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.36.6 Sympy [A] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.67

$$\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}}\,dx = 6c^2d^2\left(\begin{cases} \frac{x^3}{3a^{\frac{2}{3}}} & \text{for } b=0 \\ \frac{\sqrt[3]{a+bx^3}}{b} & \text{otherwise} \end{cases}\right) + \frac{c^4x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix} \frac{1}{3},\frac{2}{3} \\ \frac{4}{3} \end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{4c^3dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix} \frac{2}{3},\frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)} + \frac{4cd^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix} \frac{2}{3},\frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)} + \frac{d^4x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix} \frac{2}{3},\frac{5}{3} \\ \frac{8}{3} \end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((d*x+c)**4/(b*x**3+a)**(2/3),x)
```

output
```
6*c**2*d**2*Piecewise((x**3/(3*a**(2/3)), Eq(b, 0)), ((a + b*x**3)**(1/3)/
b, True)) + c**4*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I
*pi)/a)/(3*a**(2/3)*gamma(4/3)) + 4*c**3*d*x**2*gamma(2/3)*hyper((2/3, 2/3
), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(5/3)) + 4*c*d**3*x*
*4*gamma(4/3)*hyper((2/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2
/3)*gamma(7/3)) + d**4*x**5*gamma(5/3)*hyper((2/3, 5/3), (8/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(2/3)*gamma(8/3))
```

3.36.7 Maxima [F]

$$\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}} dx = \int \frac{(dx+c)^4}{(bx^3+a)^{\frac{2}{3}}} dx$$

input
```
integrate((d*x+c)^4/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^4/(b*x^3 + a)^(2/3), x)
```

3.36.8 Giac [F]

$$\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}} dx = \int \frac{(dx+c)^4}{(bx^3+a)^{\frac{2}{3}}} dx$$

input
```
integrate((d*x+c)^4/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((d*x + c)^4/(b*x^3 + a)^(2/3), x)
```

3.36.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}}\,dx = \int \frac{(c+d\,x)^4}{(b\,x^3+a)^{2/3}}\,dx$$

input
```
int((c + d*x)^4/(a + b*x^3)^(2/3),x)
```

output
```
int((c + d*x)^4/(a + b*x^3)^(2/3), x)
```

3.37 $\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}}\,dx$

3.37.1 Optimal result

Integrand size = 19, antiderivative size = 187

$$\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}}\,dx = \frac{3cd^2\sqrt[3]{a+bx^3}}{b}+\frac{d^3x\sqrt[3]{a+bx^3}}{2b}-\frac{\sqrt{3}c^2d\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{(2bc^3-ad^3)\,x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{2b\,(a+bx^3)^{2/3}}-\frac{3c^2d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}$$

output
```
3*c*d^2*(b*x^3+a)^(1/3)/b+1/2*d^3*x*(b*x^3+a)^(1/3)/b+1/2*(-a*d^3+2*b*c^3)
*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/b/(b*x^3+a)^(2/3
)-3/2*c^2*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)-c^2*d*arctan(1/3*(1+2*b^
(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(2/3)
```

3.37.2 Mathematica [A] (verified)

Time = 10.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.78

$$\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}}\,dx = \frac{4bc^3x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+d\left(6bc^2x^2\,\text{Hypergeometric2F1}\right.}{4b\,(}$$

input
```
Integrate[(c + d*x)^3/(a + b*x^3)^(2/3),x]
```

output
```
(4*b*c^3*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3
)/a)] + d*(6*b*c^2*x^2*Hypergeometric2F1[2/3, 1, 5/3, (b*x^3)/(a + b*x^3)]
 + d*(12*c*(a + b*x^3) + b*d*x^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2
/3, 4/3, 7/3, -((b*x^3)/a)])))/(4*b*(a + b*x^3)^(2/3))
```

3.37.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2427, 2425, 793, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}}dx$$

$$\Big\downarrow \text{2427}$$

$$\frac{\int \frac{2bc^3+6bdxc^2+6bd^2x^2c-ad^3}{(bx^3+a)^{2/3}}dx}{2b}+\frac{d^3x\sqrt[3]{a+bx^3}}{2b}$$

$$\Big\downarrow \text{2425}$$

$$\frac{\int \frac{2bc^3+6bdxc^2-ad^3}{(bx^3+a)^{2/3}}dx+6bcd^2\int \frac{x^2}{(bx^3+a)^{2/3}}dx}{2b}+\frac{d^3x\sqrt[3]{a+bx^3}}{2b}$$

$$\Big\downarrow \text{793}$$

$$\frac{\int \frac{2bc^3+6bdxc^2-ad^3}{(bx^3+a)^{2/3}}dx+6cd^2\sqrt[3]{a+bx^3}}{2b}+\frac{d^3x\sqrt[3]{a+bx^3}}{2b}$$

$$\Big\downarrow \text{2432}$$

$$\frac{\int \left(\frac{2b\left(1-\frac{ad^3}{2bc^3}\right)c^3}{(bx^3+a)^{2/3}}+\frac{6bdxc^2}{(bx^3+a)^{2/3}}\right)dx+6cd^2\sqrt[3]{a+bx^3}}{2b}+\frac{d^3x\sqrt[3]{a+bx^3}}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-2\sqrt{3}\sqrt[3]{b}c^2d\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(2bc^3-ad^3)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}-3\sqrt[3]{b}c^2d\log\left(\sqrt[3]{b}x-\sqrt[3]{\ldots}\right.}{2b}$$

$$\frac{d^3x\sqrt[3]{a+bx^3}}{2b}$$

input
```
Int[(c + d*x)^3/(a + b*x^3)^(2/3),x]
```

output
```
(d^3*x*(a + b*x^3)^(1/3))/(2*b) + (6*c*d^2*(a + b*x^3)^(1/3) - 2*Sqrt[3]*b
^(1/3)*c^2*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]] + ((2*b
*c^3 - a*d^3)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((
b*x^3)/a)])/(a + b*x^3)^(2/3) - 3*b^(1/3)*c^2*d*Log[b^(1/3)*x - (a + b*x^3
)^(1/3)])/(2*b)
```

3.37.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2427
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p +
1)/(b*(q + n*p + 1))), x] + Simp[1/(b*(q + n*p + 1))   Int[ExpandToSum[b*(q
 + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^n)^p,
 x], x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ
[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

3.37.4 Maple [F]

$$\int \frac{(dx + c)^3}{(b\,x^3 + a)^{\frac{2}{3}}}dx$$

input
```
int((d*x+c)^3/(b*x^3+a)^(2/3),x)
```

output
```
int((d*x+c)^3/(b*x^3+a)^(2/3),x)
```

3.37.5 Fricas [F]

$$\int \frac{(c + dx)^3}{(a + bx^3)^{2/3}}\,dx = \int \frac{(dx + c)^3}{(bx^3 + a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)/(b*x^3 + a)^(2/3), x)
```

3.37.6 Sympy [A] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.82

$$\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}}\,dx = 3cd^2\left(\begin{cases} \frac{x^3}{3a^{\frac{2}{3}}} & \text{for } b=0 \\ \frac{\sqrt[3]{a+bx^3}}{b} & \text{otherwise} \end{cases}\right) + \frac{c^3x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{2}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{c^2dx^2\Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} \frac{2}{3}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)} + \frac{d^3x^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} \frac{2}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((d*x+c)**3/(b*x**3+a)**(2/3),x)
```

output
```
3*c*d**2*Piecewise((x**3/(3*a**(2/3)), Eq(b, 0)), ((a + b*x**3)**(1/3)/b,
True)) + c**3*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*a**(2/3)*gamma(4/3)) + c**2*d*x**2*gamma(2/3)*hyper((2/3, 2/3), (5
/3,), b*x**3*exp_polar(I*pi)/a)/(a**(2/3)*gamma(5/3)) + d**3*x**4*gamma(4/
3)*hyper((2/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(7
/3))
```

3.37.7 Maxima [F]

$$\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx+c)^3}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^3/(b*x^3 + a)^(2/3), x)
```

3.37.8 Giac [F]

$$\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx+c)^3}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(b*x^3 + a)^(2/3), x)
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}}\,dx = \int \frac{(c+d\,x)^3}{(b\,x^3+a)^{2/3}}\,dx$$

input
```
int((c + d*x)^3/(a + b*x^3)^(2/3),x)
```

output
```
int((c + d*x)^3/(a + b*x^3)^(2/3), x)
```

3.38 $\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}\,dx$

3.38.1 Optimal result

Integrand size = 19, antiderivative size = 141

$$\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}\,dx = \frac{d^2\sqrt[3]{a+bx^3}}{b} - \frac{2cd\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} + \frac{c^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}} - \frac{cd\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}}$$

output
```
d^2*(b*x^3+a)^(1/3)/b+c^2*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-
b*x^3/a)/(b*x^3+a)^(2/3)-c*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)-2/3*c*d
*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)*3^(1/2)
```

3.38.2 Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.67

$$\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}\,dx = \frac{bc^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right) + d\Big(d(a+bx^3) + bcx^2\,\text{Hypergeo}}{b\,(a+bx^3)^{2/3}}$$

input
```
Integrate[(c + d*x)^2/(a + b*x^3)^(2/3),x]
```

output
```
(b*c^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/
a)] + d*(d*(a + b*x^3) + b*c*x^2*Hypergeometric2F1[2/3, 1, 5/3, (b*x^3)/(a
 + b*x^3)]))/(b*(a + b*x^3)^(2/3))
```

3.38.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2425, 793, 2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}dx$$

$$\Big\downarrow \text{2425}$$

$$\int \frac{c^2+2dxc}{(bx^3+a)^{2/3}}dx+d^2\int \frac{x^2}{(bx^3+a)^{2/3}}dx$$

$$\Big\downarrow \text{793}$$

$$\int \frac{c^2+2dxc}{(bx^3+a)^{2/3}}dx+\frac{d^2\sqrt[3]{a+bx^3}}{b}$$

$$\Big\downarrow \text{2432}$$

$$\int \left(\frac{c^2}{(bx^3+a)^{2/3}}+\frac{2dxc}{(bx^3+a)^{2/3}}\right)dx+\frac{d^2\sqrt[3]{a+bx^3}}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2cd\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{cd\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}}+\frac{c^2x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}+\frac{d^2\sqrt[3]{a+bx^3}}{b}$$

input
```
Int[(c + d*x)^2/(a + b*x^3)^(2/3),x]
```

output
```
(d^2*(a + b*x^3)^(1/3))/b - (2*c*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(
1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)) + (c^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeom
etric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) - (c*d*Log[b^(1/3
)*x - (a + b*x^3)^(1/3)])/b^(2/3)
```

3.38.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2425
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Pq, x, n -
 1]   Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq,
x, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[P
q, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

3.38.4 Maple [F]

$$\int \frac{(dx+c)^2}{(b\,x^3+a)^{\frac{2}{3}}}dx$$

input
```
int((d*x+c)^2/(b*x^3+a)^(2/3),x)
```

output
```
int((d*x+c)^2/(b*x^3+a)^(2/3),x)
```

3.38.5 Fricas [F]

$$\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx+c)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((d^2*x^2 + 2*c*d*x + c^2)/(b*x^3 + a)^(2/3), x)
```

3.38.6 Sympy [A] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.77

$$\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}\,dx = d^2\left(\begin{cases}\frac{x^3}{3a^{\frac{2}{3}}} & \text{for } b=0\\ \frac{\sqrt[3]{a+bx^3}}{b} & \text{otherwise}\end{cases}\right) + \frac{c^2x\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{2cdx^2\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((d*x+c)**2/(b*x**3+a)**(2/3),x)
```

output
```
d**2*Piecewise((x**3/(3*a**(2/3)), Eq(b, 0)), ((a + b*x**3)**(1/3)/b, True
)) + c**2*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)
/(3*a**(2/3)*gamma(4/3)) + 2*c*d*x**2*gamma(2/3)*hyper((2/3, 2/3), (5/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(5/3))
```

3.38.7 Maxima [F]

$$\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx+c)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2/(b*x^3 + a)^(2/3), x)
```

3.38.8 Giac [F]

$$\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(dx+c)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(b*x^3 + a)^(2/3), x)
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}\,dx = \int \frac{(c+d\,x)^2}{(b\,x^3+a)^{2/3}}\,dx$$

input
```
int((c + d*x)^2/(a + b*x^3)^(2/3),x)
```

output
```
int((c + d*x)^2/(a + b*x^3)^(2/3), x)
```

3.39 $\int \frac{c+dx}{(a+bx^3)^{2/3}}\,dx$

3.39.1 Optimal result

Integrand size = 17, antiderivative size = 121

$$\int \frac{c+dx}{(a+bx^3)^{2/3}}\,dx = -\frac{d\arctan\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} + \frac{cx\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}} - \frac{d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}$$

output
```
c*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)
-1/2*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)-1/3*d*arctan(1/3*(1+2*b^(1/3)
*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)*3^(1/2)
```

3.39.2 Mathematica [A] (verified)

Time = 9.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.64

$$\int \frac{c+dx}{(a+bx^3)^{2/3}}\,dx = \frac{x\left(2c\left(1+\frac{bx^3}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)+dx\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\right.\right.}{2\left(a+bx^3\right)^{2/3}}$$

input
```
Integrate[(c + d*x)/(a + b*x^3)^(2/3),x]
```

output
```
(x*(2*c*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a
)] + d*x*Hypergeometric2F1[2/3, 1, 5/3, (b*x^3)/(a + b*x^3)]))/(2*(a + b*x
^3)^(2/3))
```

3.39.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a+bx^3)^{2/3}}\,dx$$

$$\Big\downarrow \text{2432}$$

$$\int \left(\frac{c}{(a+bx^3)^{2/3}}+\frac{dx}{(a+bx^3)^{2/3}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d\arctan\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}+\frac{cx\left(\frac{bx^3}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}$$

input
```
Int[(c + d*x)/(a + b*x^3)^(2/3),x]
```

output
```
-((d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/
3))) + (c*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^
3)/a)])/(a + b*x^3)^(2/3) - (d*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(2*b^(2
/3))
```

3.39.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

3.39.4 Maple [F]

$$\int \frac{dx+c}{(b\,x^{3}+a)^{\frac{2}{3}}}dx$$

input
```
int((d*x+c)/(b*x^3+a)^(2/3),x)
```

output
```
int((d*x+c)/(b*x^3+a)^(2/3),x)
```

3.39.5 Fricas [F]

$$\int \frac{c+dx}{(a+bx^3)^{2/3}}\,dx = \int \frac{dx+c}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((d*x + c)/(b*x^3 + a)^(2/3), x)
```

3.39.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.64

$$\int \frac{c+dx}{(a+bx^3)^{2/3}}\,dx = \frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((d*x+c)/(b*x**3+a)**(2/3),x)
```

output
```
c*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
2/3)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 2/3), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(2/3)*gamma(5/3))
```

3.39.7 Maxima [F]

$$\int \frac{c+dx}{(a+bx^3)^{2/3}}\,dx = \int \frac{dx+c}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/(b*x^3 + a)^(2/3), x)
```

3.39.8 Giac [F]

$$\int \frac{c+dx}{(a+bx^3)^{2/3}}\,dx = \int \frac{dx+c}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((d*x+c)/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((d*x + c)/(b*x^3 + a)^(2/3), x)
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{(a+bx^3)^{2/3}}\,dx = \int \frac{c+d\,x}{(b\,x^3+a)^{2/3}}\,dx$$

input
```
int((c + d*x)/(a + b*x^3)^(2/3),x)
```

output
```
int((c + d*x)/(a + b*x^3)^(2/3), x)
```

3.40 $\int \frac{1}{(c+dx)(a+bx^3)^{2/3}}\,dx$

3.40.1 Optimal result

Integrand size = 19, antiderivative size = 332

$$\int \frac{1}{(c+dx)(a+bx^3)^{2/3}}\,dx = \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c\,(a+bx^3)^{2/3}} + \frac{d\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}\,x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,(bc^3-ad^3)^{2/3}} - \frac{d\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,(bc^3-ad^3)^{2/3}} - \frac{d\log\left(c^3+d^3x^3\right)}{3\,(bc^3-ad^3)^{2/3}} + \frac{d\log\left(\frac{\sqrt[3]{bc^3-ad^3}\,x}{c}-\sqrt[3]{a+bx^3}\right)}{2\,(bc^3-ad^3)^{2/3}} + \frac{d\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2\,(bc^3-ad^3)^{2/3}}$$

output
```
x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,1,4/3,-b*x^3/a,-d^3*x^3/c^3)/c/(b*x^3
+a)^(2/3)-1/3*d*ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^(2/3)+1/2*d*ln((-a*d^3+b*c^
3)^(1/3)*x/c-(b*x^3+a)^(1/3))/(-a*d^3+b*c^3)^(2/3)+1/2*d*ln((-a*d^3+b*c^3)
^(1/3)+d*(b*x^3+a)^(1/3))/(-a*d^3+b*c^3)^(2/3)+1/3*d*arctan(1/3*(1+2*(-a*d
^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))/(-a*d^3+b*c^3)^(2/3)*3^(1/2)
-1/3*d*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)^(1/3))*3^(1/2))/(-
a*d^3+b*c^3)^(2/3)*3^(1/2)
```

3.40.2 Mathematica [F]

$$\int \frac{1}{(c+dx)\left(a+bx^3\right)^{2/3}}\,dx=\int \frac{1}{(c+dx)\left(a+bx^3\right)^{2/3}}\,dx$$

input
```
Integrate[1/((c + d*x)*(a + b*x^3)^(2/3)),x]
```

output
```
Integrate[1/((c + d*x)*(a + b*x^3)^(2/3)), x]
```

3.40.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2581, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+bx^3\right)^{2/3}(c+dx)}\,dx$$

$$\Big\downarrow \ 2581$$

$$\int \left(-\frac{cdx}{\left(a+bx^3\right)^{2/3}\left(c^3+d^3x^3\right)}+\frac{d^2x^2}{\left(a+bx^3\right)^{2/3}\left(c^3+d^3x^3\right)}+\frac{c^2}{\left(a+bx^3\right)^{2/3}\left(c^3+d^3x^3\right)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c\left(a+bx^3\right)^{2/3}}+\frac{d\arctan\left(\frac{\frac{2x\sqrt[3]{bc^3-ad^3}}{c\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\left(bc^3-ad^3\right)^{2/3}}-\frac{d\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\left(bc^3-ad^3\right)^{2/3}}-\frac{d\log\left(c^3+d^3x^3\right)}{3\left(bc^3-ad^3\right)^{2/3}}+\frac{d\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{2\left(bc^3-ad^3\right)^{2/3}}+\frac{d\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2\left(bc^3-ad^3\right)^{2/3}}$$

input
```
Int[1/((c + d*x)*(a + b*x^3)^(2/3)),x]
```

output
```
(x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d^3*x
^3)/c^3)])/(c*(a + b*x^3)^(2/3)) + (d*ArcTan[(1 + (2*(b*c^3 - a*d^3)^(1/3)
*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*(b*c^3 - a*d^3)^(2/3)) - (d*
ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1/3))/Sqrt[3]])/(Sqrt
[3]*(b*c^3 - a*d^3)^(2/3)) - (d*Log[c^3 + d^3*x^3])/(3*(b*c^3 - a*d^3)^(2/
3)) + (d*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(2*(b*c^3 -
 a*d^3)^(2/3)) + (d*Log[(b*c^3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(2*(
b*c^3 - a*d^3)^(2/3))
```

3.40.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2581
```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

3.40.4 Maple [F]

$$\int \frac{1}{(dx+c)\,(b\,x^3+a)^{\frac{2}{3}}}dx$$

input
```
int(1/(d*x+c)/(b*x^3+a)^(2/3),x)
```

output
```
int(1/(d*x+c)/(b*x^3+a)^(2/3),x)
```

3.40.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)\left(a+bx^3\right)^{2/3}}\,dx=\text{Timed out}$$

input
```
integrate(1/(d*x+c)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.40.6 Sympy [F]

$$\int \frac{1}{(c+dx)\left(a+bx^3\right)^{2/3}}\,dx=\int \frac{1}{\left(a+bx^3\right)^{\frac{2}{3}}(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(b*x**3+a)**(2/3),x)
```

output
```
Integral(1/((a + b*x**3)**(2/3)*(c + d*x)), x)
```

3.40.7 Maxima [F]

$$\int \frac{1}{(c+dx)\left(a+bx^3\right)^{2/3}}\,dx=\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)), x)
```

3.40.8 Giac [F]

$$\int \frac{1}{(c+dx)(a+bx^3)^{2/3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx+c)}\,dx$$

input
```
integrate(1/(d*x+c)/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)), x)
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)(a+bx^3)^{2/3}}\,dx = \int \frac{1}{(b\,x^3+a)^{2/3}\,(c+d\,x)}\,dx$$

input
```
int(1/((a + b*x^3)^(2/3)*(c + d*x)),x)
```

output
```
int(1/((a + b*x^3)^(2/3)*(c + d*x)), x)
```

3.41 $\int \frac{1}{(c+dx)^2 \left(a+bx^3\right)^{2/3}} \, dx$

3.41.1 Optimal result

Integrand size = 19, antiderivative size = 760

$$
\begin{aligned}
\int \frac{1}{(c+dx)^2\left(a+bx^3\right)^{2/3}}\,dx &= \frac{c^2d^2\sqrt[3]{a+bx^3}}{(bc^3-ad^3)\left(c^3+d^3x^3\right)} \\
&+ \frac{d^4x^2\sqrt[3]{a+bx^3}}{(bc^3-ad^3)\left(c^3+d^3x^3\right)} + \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^2\left(a+bx^3\right)^{2/3}} \\
&- \frac{d^3x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^5\left(a+bx^3\right)^{2/3}} \\
&+ \frac{2ad^4\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}\,x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c\left(bc^3-ad^3\right)^{5/3}} + \frac{2d(3bc^3-ad^3)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}\,x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c\left(bc^3-ad^3\right)^{5/3}} \\
&- \frac{2bc^2d\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\left(bc^3-ad^3\right)^{5/3}} - \frac{bc^2d\log\left(c^3+d^3x^3\right)}{3\left(bc^3-ad^3\right)^{5/3}} - \frac{ad^4\log\left(c^3+d^3x^3\right)}{9c\left(bc^3-ad^3\right)^{5/3}} \\
&- \frac{d(3bc^3-ad^3)\log\left(c^3+d^3x^3\right)}{9c\left(bc^3-ad^3\right)^{5/3}} + \frac{ad^4\log\left(\frac{\sqrt[3]{bc^3-ad^3}\,x}{c}-\sqrt[3]{a+bx^3}\right)}{3c\left(bc^3-ad^3\right)^{5/3}} \\
&+ \frac{d(3bc^3-ad^3)\log\left(\frac{\sqrt[3]{bc^3-ad^3}\,x}{c}-\sqrt[3]{a+bx^3}\right)}{3c\left(bc^3-ad^3\right)^{5/3}} \\
&+ \frac{bc^2d\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{\left(bc^3-ad^3\right)^{5/3}}
\end{aligned}
$$

output
```
c^2*d^2*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)+d^4*x^2*(b*x^3+a)^(1/
3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)+x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,2,4/3
,-b*x^3/a,-d^3*x^3/c^3)/c^2/(b*x^3+a)^(2/3)-1/2*d^3*x^4*(1+b*x^3/a)^(2/3)*
AppellF1(4/3,2/3,2,7/3,-b*x^3/a,-d^3*x^3/c^3)/c^5/(b*x^3+a)^(2/3)-1/3*b*c^
2*d*ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^(5/3)-1/9*a*d^4*ln(d^3*x^3+c^3)/c/(-a*d
^3+b*c^3)^(5/3)-1/9*d*(-a*d^3+3*b*c^3)*ln(d^3*x^3+c^3)/c/(-a*d^3+b*c^3)^(5
/3)+1/3*a*d^4*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c/(-a*d^3+b*c^3
)^(5/3)+1/3*d*(-a*d^3+3*b*c^3)*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3)
)/c/(-a*d^3+b*c^3)^(5/3)+b*c^2*d*ln((-a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^(1/3)
)/(-a*d^3+b*c^3)^(5/3)+2/9*a*d^4*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/
(b*x^3+a)^(1/3))*3^(1/2))/c/(-a*d^3+b*c^3)^(5/3)*3^(1/2)+2/9*d*(-a*d^3+3*b
*c^3)*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))/c
/(-a*d^3+b*c^3)^(5/3)*3^(1/2)-2/3*b*c^2*d*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3
)/(-a*d^3+b*c^3)^(1/3))*3^(1/2))/(-a*d^3+b*c^3)^(5/3)*3^(1/2)
```

3.41.2 Mathematica [F]

$$\int \frac{1}{(c+dx)^2 (a+bx^3)^{2/3}} \, dx = \int \frac{1}{(c+dx)^2 (a+bx^3)^{2/3}} \, dx$$

input
```
Integrate[1/((c + d*x)^2*(a + b*x^3)^(2/3)),x]
```

output
```
Integrate[1/((c + d*x)^2*(a + b*x^3)^(2/3)), x]
```

3.41.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 760, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2581, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{2/3} (c+dx)^2} \, dx$$
$$\Big\downarrow \; 2581$$

$$\int\left(-\frac{2c^3dx}{(a+bx^3)^{2/3}(c^3+d^3x^3)^2}-\frac{2cd^3x^3}{(a+bx^3)^{2/3}(c^3+d^3x^3)^2}+\frac{d^4x^4}{(a+bx^3)^{2/3}(c^3+d^3x^3)^2}+\frac{c^4}{(a+bx^3)^{2/3}(c^3+d^3x\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^3x^4\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},2,\frac{7}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^5(a+bx^3)^{2/3}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},2,\frac{4}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^2(a+bx^3)^{2/3}}+\frac{2d(3bc^3-ad^3)\arctan\left(\frac{\frac{2x\sqrt[3]{bc^3-ad^3}}{c\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c(bc^3-ad^3)^{5/3}}+\frac{2ad^4\arctan\left(\frac{\frac{2x\sqrt[3]{bc^3-ad^3}}{c\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c(bc^3-ad^3)^{5/3}}-\frac{2bc^2d\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}(bc^3-ad^3)^{5/3}}-\frac{d(3bc^3-ad^3)\log(c^3+d^3x^3)}{9c(bc^3-ad^3)^{5/3}}+\frac{d(3bc^3-ad^3)\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{3c(bc^3-ad^3)^{5/3}}-\frac{ad^4\log(c^3+d^3x^3)}{9c(bc^3-ad^3)^{5/3}}+\frac{ad^4\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{3c(bc^3-ad^3)^{5/3}}+\frac{d^4x^2\sqrt[3]{a+bx^3}}{(c^3+d^3x^3)(bc^3-ad^3)}-\frac{bc^2d\log(c^3+d^3x^3)}{3(bc^3-ad^3)^{5/3}}+\frac{bc^2d\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{(bc^3-ad^3)^{5/3}}+\frac{c^2d^2\sqrt[3]{a+bx^3}}{(c^3+d^3x^3)(bc^3-ad^3)}$$

input
```
Int[1/((c + d*x)^2*(a + b*x^3)^(2/3)),x]
```

3.41. $\int \frac{1}{(c+dx)^2(a+bx^3)^{2/3}}\,dx$

output
```
(c^2*d^2*(a + b*x^3)^(1/3))/((b*c^3 - a*d^3)*(c^3 + d^3*x^3)) + (d^4*x^2*(
a + b*x^3)^(1/3))/((b*c^3 - a*d^3)*(c^3 + d^3*x^3)) + (x*(1 + (b*x^3)/a)^(
2/3)*AppellF1[1/3, 2/3, 2, 4/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(c^2*(a +
 b*x^3)^(2/3)) - (d^3*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 2, 7/3,
 -((b*x^3)/a), -((d^3*x^3)/c^3)])/(2*c^5*(a + b*x^3)^(2/3)) + (2*a*d^4*Arc
Tan[(1 + (2*(b*c^3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*S
qrt[3]*c*(b*c^3 - a*d^3)^(5/3)) + (2*d*(3*b*c^3 - a*d^3)*ArcTan[(1 + (2*(b
*c^3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c*(b*c^
3 - a*d^3)^(5/3)) - (2*b*c^2*d*ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3
- a*d^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*(b*c^3 - a*d^3)^(5/3)) - (b*c^2*d*Log[c
^3 + d^3*x^3])/(3*(b*c^3 - a*d^3)^(5/3)) - (a*d^4*Log[c^3 + d^3*x^3])/(9*c
*(b*c^3 - a*d^3)^(5/3)) - (d*(3*b*c^3 - a*d^3)*Log[c^3 + d^3*x^3])/(9*c*(b
*c^3 - a*d^3)^(5/3)) + (a*d^4*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a + b*x^3
)^(1/3)])/(3*c*(b*c^3 - a*d^3)^(5/3)) + (d*(3*b*c^3 - a*d^3)*Log[((b*c^3 -
 a*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(3*c*(b*c^3 - a*d^3)^(5/3)) + (b*
c^2*d*Log[(b*c^3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(b*c^3 - a*d^3)^(5
/3)
```

3.41.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2581
```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

3.41.4 Maple [F]

$$\int \frac{1}{(dx+c)^2 (b\,x^3+a)^{\frac{2}{3}}}dx$$

input
```
int(1/(d*x+c)^2/(b*x^3+a)^(2/3),x)
```

output
```
int(1/(d*x+c)^2/(b*x^3+a)^(2/3),x)
```

3.41.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2 (a+bx^3)^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.41.6 Sympy [F]

$$\int \frac{1}{(c+dx)^2 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{(a+bx^3)^{\frac{2}{3}} (c+dx)^2}\,dx$$

input
```
integrate(1/(d*x+c)**2/(b*x**3+a)**(2/3),x)
```

output
```
Integral(1/((a + b*x**3)**(2/3)*(c + d*x)**2), x)
```

3.41.7 Maxima [F]

$$\int \frac{1}{(c+dx)^2 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}} (dx+c)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)^2), x)
```

3.41.8 Giac [F]

$$\int \frac{1}{(c+dx)^2 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx+c)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)^2), x)
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{(b\,x^3+a)^{2/3}\,(c+d\,x)^2}\,dx$$

input
```
int(1/((a + b*x^3)^(2/3)*(c + d*x)^2),x)
```

output
```
int(1/((a + b*x^3)^(2/3)*(c + d*x)^2), x)
```

3.42 $\int \frac{1}{(c+dx)^3 \left(a+bx^3\right)^{2/3}} \, dx$

3.42.1 Optimal result

Integrand size = 19, antiderivative size = 1357

$$
\begin{aligned}
\int \frac{1}{(c+dx)^3\left(a+bx^3\right)^{2/3}}\,dx &= \frac{3c^4d^2\sqrt[3]{a+bx^3}}{2\left(bc^3-ad^3\right)\left(c^3+d^3x^3\right)^2} \\
&+ \frac{3c^2d^4x^2\sqrt[3]{a+bx^3}}{2\left(bc^3-ad^3\right)\left(c^3+d^3x^3\right)^2} + \frac{5bc^4d^2\sqrt[3]{a+bx^3}}{3\left(bc^3-ad^3\right)^2\left(c^3+d^3x^3\right)} \\
&- \frac{cd^2(bc^3-6ad^3)\sqrt[3]{a+bx^3}}{6\left(bc^3-ad^3\right)^2\left(c^3+d^3x^3\right)} + \frac{d^4(9bc^3-4ad^3)\,x^2\sqrt[3]{a+bx^3}}{6c\left(bc^3-ad^3\right)^2\left(c^3+d^3x^3\right)} \\
&+ \frac{d^4(3bc^3+2ad^3)\,x^2\sqrt[3]{a+bx^3}}{3c\left(bc^3-ad^3\right)^2\left(c^3+d^3x^3\right)} \\
&+ \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{1}{3},\frac{2}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^3\left(a+bx^3\right)^{2/3}} \\
&- \frac{7d^3x^4\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{4}{3},\frac{2}{3},3,\frac{7}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{4c^6\left(a+bx^3\right)^{2/3}} \\
&+ \frac{d^6x^7\left(1+\frac{bx^3}{a}\right)^{2/3}\operatorname{AppellF1}\left(\frac{7}{3},\frac{2}{3},3,\frac{10}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{7c^9\left(a+bx^3\right)^{2/3}} \\
&+ \frac{2ad^4(6bc^3-ad^3)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}\,x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^2\left(bc^3-ad^3\right)^{8/3}} \\
&+ \frac{d(9b^2c^6-6abc^3d^3+2a^2d^6)\arctan\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}\,x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^2\left(bc^3-ad^3\right)^{8/3}} \\
&- \frac{10b^2c^4d\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\left(bc^3-ad^3\right)^{8/3}} \\
&+ \frac{bcd(bc^3-6ad^3)\arctan\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\left(bc^3-ad^3\right)^{8/3}} - \frac{5b^2c^4d\log\left(c^3+d^3x^3\right)}{9\left(bc^3-ad^3\right)^{8/3}} \\
&+ \frac{bcd(bc^3-6ad^3)\log\left(c^3+d^3x^3\right)}{18\left(bc^3-ad^3\right)^{8/3}} - \frac{ad^4(6bc^3-ad^3)\log\left(c^3+d^3x^3\right)}{9c^2\left(bc^3-ad^3\right)^{8/3}} \\
&- \frac{d(9b^2c^6-6abc^3d^3+2a^2d^6)\log\left(c^3+d^3x^3\right)}{18c^2\left(bc^3-ad^3\right)^{8/3}} \\
&\quad ad^4(6bc^3-ad^3)\log\left(\frac{\sqrt[3]{bc^3-ad^3}\,x}{\ldots}-\sqrt[3]{a+bx^3}\right)
\end{aligned}
$$

output
```
3/2*c^4*d^2*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)^2+3/2*c^2*d^4*x^2
*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)^2+5/3*b*c^4*d^2*(b*x^3+a)^(1
/3)/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)-1/6*c*d^2*(-6*a*d^3+b*c^3)*(b*x^3+a)^(1
/3)/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)+1/6*d^4*(-4*a*d^3+9*b*c^3)*x^2*(b*x^3+a
)^(1/3)/c/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)+1/3*d^4*(2*a*d^3+3*b*c^3)*x^2*(b*
x^3+a)^(1/3)/c/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)+x*(1+b*x^3/a)^(2/3)*AppellF1
(1/3,2/3,3,4/3,-b*x^3/a,-d^3*x^3/c^3)/c^3/(b*x^3+a)^(2/3)-7/4*d^3*x^4*(1+b
*x^3/a)^(2/3)*AppellF1(4/3,2/3,3,7/3,-b*x^3/a,-d^3*x^3/c^3)/c^6/(b*x^3+a)^
(2/3)+1/7*d^6*x^7*(1+b*x^3/a)^(2/3)*AppellF1(7/3,2/3,3,10/3,-b*x^3/a,-d^3*
x^3/c^3)/c^9/(b*x^3+a)^(2/3)-5/9*b^2*c^4*d*ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^
(8/3)+1/18*b*c*d*(-6*a*d^3+b*c^3)*ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^(8/3)-1/9
*a*d^4*(-a*d^3+6*b*c^3)*ln(d^3*x^3+c^3)/c^2/(-a*d^3+b*c^3)^(8/3)-1/18*d*(2
*a^2*d^6-6*a*b*c^3*d^3+9*b^2*c^6)*ln(d^3*x^3+c^3)/c^2/(-a*d^3+b*c^3)^(8/3)
+1/3*a*d^4*(-a*d^3+6*b*c^3)*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c
^2/(-a*d^3+b*c^3)^(8/3)+1/6*d*(2*a^2*d^6-6*a*b*c^3*d^3+9*b^2*c^6)*ln((-a*d
^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c^2/(-a*d^3+b*c^3)^(8/3)+5/3*b^2*c^4*
d*ln((-a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^(1/3))/(-a*d^3+b*c^3)^(8/3)-1/6*b*c*
d*(-6*a*d^3+b*c^3)*ln((-a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^(1/3))/(-a*d^3+b*c^
3)^(8/3)+2/9*a*d^4*(-a*d^3+6*b*c^3)*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x
/c/(b*x^3+a)^(1/3))*3^(1/2))/c^2/(-a*d^3+b*c^3)^(8/3)*3^(1/2)+1/9*d*(2*...
```

3.42.2 Mathematica [F]

$$\int \frac{1}{(c+dx)^3 (a+bx^3)^{2/3}}\, dx = \int \frac{1}{(c+dx)^3 (a+bx^3)^{2/3}}\, dx$$

input
```
Integrate[1/((c + d*x)^3*(a + b*x^3)^(2/3)),x]
```

output
```
Integrate[1/((c + d*x)^3*(a + b*x^3)^(2/3)), x]
```

3.42.3 Rubi [A] (verified)

Time = 2.04 (sec) , antiderivative size = 1357, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2581, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx)^3}dx$$

$$\Big\downarrow \text{2581}$$

$$\int \left(-\frac{7c^3d^3x^3}{(a+bx^3)^{2/3}(c^3+d^3x^3)^3}+\frac{d^6x^6}{(a+bx^3)^{2/3}(c^3+d^3x^3)^3}-\frac{3cd^5x^5}{(a+bx^3)^{2/3}(c^3+d^3x^3)^3}+\frac{c^6}{(a+bx^3)^{2/3}(c^3+d^3x$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^6\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{7}{3},\frac{2}{3},3,\frac{10}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)x^7}{7c^9\left(bx^3+a\right)^{2/3}} - \frac{7d^3\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{4}{3},\frac{2}{3},3,\frac{7}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)x^4}{4c^6\left(bx^3+a\right)^{2/3}} + \frac{d^4\left(3bc^3+2ad^3\right)\sqrt[3]{bx^3+a}x^2}{3c\left(bc^3-ad^3\right)^2\left(c^3+d^3x^3\right)} + \frac{d^4\left(9bc^3-4ad^3\right)\sqrt[3]{bx^3+a}x^2}{6c\left(bc^3-ad^3\right)^2\left(c^3+d^3x^3\right)} + \frac{3c^2d^4\sqrt[3]{bx^3+a}x^2}{2\left(bc^3-ad^3\right)\left(c^3+d^3x^3\right)^2} + \frac{\left(\frac{bx^3}{a}+1\right)^{2/3}\text{AppellF1}\left(\frac{1}{3},\frac{2}{3},3,\frac{4}{3},-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)x}{c^3\left(bx^3+a\right)^{2/3}} + \frac{2ad^4\left(6bc^3-ad^3\right)\arctan\left(\frac{\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{bx^3+a}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c^2\left(bc^3-ad^3\right)^{8/3}} + \frac{d\left(9b^2c^6-6abd^3c^3+2a^2d^6\right)\arctan\left(\frac{\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{bx^3+a}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c^2\left(bc^3-ad^3\right)^{8/3}} - \frac{10b^2c^4d\arctan\left(\frac{1-\frac{2d\sqrt[3]{bx^3+a}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\left(bc^3-ad^3\right)^{8/3}} + \frac{bcd\left(bc^3-6ad^3\right)\arctan\left(\frac{1-\frac{2d\sqrt[3]{bx^3+a}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\left(bc^3-ad^3\right)^{8/3}} - \frac{ad^4\left(6bc^3-ad^3\right)\log\left(c^3+d^3x^3\right)}{9c^2\left(bc^3-ad^3\right)^{8/3}} - \frac{d\left(9b^2c^6-6abd^3c^3+2a^2d^6\right)\log\left(c^3+d^3x^3\right)}{18c^2\left(bc^3-ad^3\right)^{8/3}} - \frac{5b^2c^4d\log\left(c^3+d^3x^3\right)}{9\left(bc^3-ad^3\right)^{8/3}} + \frac{bcd\left(bc^3-6ad^3\right)\log\left(c^3+d^3x^3\right)}{18\left(bc^3-ad^3\right)^{8/3}} + \frac{ad^4\left(6bc^3-ad^3\right)\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{bx^3+a}\right)}{3c^2\left(bc^3-ad^3\right)^{8/3}} + \frac{d\left(9b^2c^6-6abd^3c^3+2a^2d^6\right)\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{bx^3+a}\right)}{6c^2\left(bc^3-ad^3\right)^{8/3}} + \frac{5b^2c^4d\log\left(\sqrt[3]{bx^3+a}d+\sqrt[3]{bc^3-ad^3}\right)}{3\left(bc^3-ad^3\right)^{8/3}} - \frac{bcd\left(bc^3-6ad^3\right)\log\left(\sqrt[3]{bx^3+a}d+\sqrt[3]{bc^3-ad^3}\right)}{6\left(bc^3-ad^3\right)^{8/3}} + \frac{5bc^4d^2\sqrt[3]{bx^3+a}}{3\left(bc^3-ad^3\right)^2\left(c^3+d^3x^3\right)} - \frac{cd^2\left(bc^3-6ad^3\right)\sqrt[3]{bx^3+a}}{6\left(bc^3-ad^3\right)^2\left(c^3+d^3x^3\right)} + \frac{3c^4d^2\sqrt[3]{bx^3+a}}{2\left(bc^3-ad^3\right)\left(c^3+d^3x^3\right)^2}$$

input
```
Int[1/((c + d*x)^3*(a + b*x^3)^(2/3)),x]
```

3.42. $\int \frac{1}{(c+dx)^3(a+bx^3)^{2/3}}\,dx$

output
```
(3*c^4*d^2*(a + b*x^3)^(1/3))/(2*(b*c^3 - a*d^3)*(c^3 + d^3*x^3)^2) + (3*c
^2*d^4*x^2*(a + b*x^3)^(1/3))/(2*(b*c^3 - a*d^3)*(c^3 + d^3*x^3)^2) + (5*b
*c^4*d^2*(a + b*x^3)^(1/3))/(3*(b*c^3 - a*d^3)^2*(c^3 + d^3*x^3)) - (c*d^2
*(b*c^3 - 6*a*d^3)*(a + b*x^3)^(1/3))/(6*(b*c^3 - a*d^3)^2*(c^3 + d^3*x^3)
) + (d^4*(9*b*c^3 - 4*a*d^3)*x^2*(a + b*x^3)^(1/3))/(6*c*(b*c^3 - a*d^3)^2
*(c^3 + d^3*x^3)) + (d^4*(3*b*c^3 + 2*a*d^3)*x^2*(a + b*x^3)^(1/3))/(3*c*(
b*c^3 - a*d^3)^2*(c^3 + d^3*x^3)) + (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3,
 2/3, 3, 4/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(c^3*(a + b*x^3)^(2/3)) - (
7*d^3*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 3, 7/3, -((b*x^3)/a), -
((d^3*x^3)/c^3)])/(4*c^6*(a + b*x^3)^(2/3)) + (d^6*x^7*(1 + (b*x^3)/a)^(2/
3)*AppellF1[7/3, 2/3, 3, 10/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(7*c^9*(a
+ b*x^3)^(2/3)) + (2*a*d^4*(6*b*c^3 - a*d^3)*ArcTan[(1 + (2*(b*c^3 - a*d^3
)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c^2*(b*c^3 - a*d^3)
^(8/3)) + (d*(9*b^2*c^6 - 6*a*b*c^3*d^3 + 2*a^2*d^6)*ArcTan[(1 + (2*(b*c^3
 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c^2*(b*c^3
- a*d^3)^(8/3)) - (10*b^2*c^4*d*ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3
 - a*d^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*(b*c^3 - a*d^3)^(8/3)) + (b*c*d*(b*c
^3 - 6*a*d^3)*ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1/3))/S
qrt[3]])/(3*Sqrt[3]*(b*c^3 - a*d^3)^(8/3)) - (5*b^2*c^4*d*Log[c^3 + d^3*x^
3])/(9*(b*c^3 - a*d^3)^(8/3)) + (b*c*d*(b*c^3 - 6*a*d^3)*Log[c^3 + d^3*...
```

3.42.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2581
```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

3.42.4 Maple [F]

$$\int \frac{1}{(dx+c)^3 (b\,x^3+a)^{\frac{2}{3}}} dx$$

input
```
int(1/(d*x+c)^3/(b*x^3+a)^(2/3),x)
```

output
```
int(1/(d*x+c)^3/(b*x^3+a)^(2/3),x)
```

3.42.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^3 (a+bx^3)^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.42.6 Sympy [F]

$$\int \frac{1}{(c+dx)^3 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{(a+bx^3)^{\frac{2}{3}} (c+dx)^3}\,dx$$

input
```
integrate(1/(d*x+c)**3/(b*x**3+a)**(2/3),x)
```

output
```
Integral(1/((a + b*x**3)**(2/3)*(c + d*x)**3), x)
```

3.42.7 Maxima [F]

$$\int \frac{1}{(c+dx)^3 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}} (dx+c)^3}\,dx$$

input
```
integrate(1/(d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)^3), x)
```

3.42.8 Giac [F]

$$\int \frac{1}{(c+dx)^3 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{(bx^3+a)^{\frac{2}{3}}(dx+c)^3}\,dx$$

input
```
integrate(1/(d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)^3), x)
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^3 (a+bx^3)^{2/3}}\,dx = \int \frac{1}{(b\,x^3+a)^{2/3}\,(c+d\,x)^3}\,dx$$

input
```
int(1/((a + b*x^3)^(2/3)*(c + d*x)^3),x)
```

output
```
int(1/((a + b*x^3)^(2/3)*(c + d*x)^3), x)
```

3.43 $\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

3.43.1 Optimal result

Integrand size = 28, antiderivative size = 37

$$\int \frac{2^{2/3}-2x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx=\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{\sqrt{3}}$$

output
```
2/3*2^(2/3)*arctan((1+2^(1/3)*x)*3^(1/2)/(x^3+1)^(1/2))*3^(1/2)
```

3.43.2 Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int \frac{2^{2/3}-2x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx=-\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{1+x^3}}{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}\right)}{\sqrt{3}}$$

input
```
Integrate[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTan[Sqrt[1 + x^3]/(Sqrt[3]*(1 + 2^(1/3)*x))])/Sqrt[3]
```

3.43.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{2562}$$

$$2\ 2^{2/3}\int \frac{1}{\frac{3\left(\sqrt[3]{2}x+1\right)^2}{x^3+1}+1}d\frac{\sqrt[3]{2}x+1}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\ 2^{2/3}\arctan\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

input
```
Int[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

output
```
(2*2^(2/3)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/Sqrt[3]
```

3.43.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.43.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.92

method	result
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)\ln\left(\frac{12\sqrt{x^3+1}\,x+3\,\mathrm{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)2^{\frac{2}{3}}x^2-\mathrm{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)x^3+6\sqrt{x^3+1}\,2^{\frac{2}{3}}+6\,\mathrm{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)2\cdots}{\left(2^{\frac{1}{3}}x+2\right)^3}\cdots\right.}{3}\cdots$
default	$-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{6\,2^{\frac{2}{3}}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}\cdots}{-\frac{3}{2}+\cdots}}\cdots}{\sqrt{x^3+1}\left(\cdots\right.}$
elliptic	$-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{6\,2^{\frac{2}{3}}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}\cdots}{-\frac{3}{2}+\cdots}}\cdots}{\sqrt{x^3+1}\left(\cdots\right.}$

input
```
int((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*RootOf(_Z^2+6*2^(1/3))*ln((12*(x^3+1)^(1/2)*x+3*RootOf(_Z^2+6*2^(1/3)
)*2^(2/3)*x^2-RootOf(_Z^2+6*2^(1/3))*x^3+6*(x^3+1)^(1/2)*2^(2/3)+6*RootOf(
_Z^2+6*2^(1/3))*2^(1/3)*x+2*RootOf(_Z^2+6*2^(1/3)))/(2^(1/3)*x+2)^3)
```

3.43.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(27) = 54$.

Time = 0.43 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.03

$$\int \frac{2^{2/3}-2x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \frac{1}{3}\sqrt{6}2^{\frac{1}{6}}\arctan\left(-\frac{\sqrt{6}2^{\frac{1}{6}}\left(2\,x^5+2\,x^2-2^{\frac{2}{3}}(7\,x^4+4\,x)-2^{\frac{1}{3}}(5\,x^3+2)\right)\sqrt{x^3+1}}{12\,(2\,x^6+3\,x^3+1)}\cdots\right)$$

input
```
integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(6)*2^(1/6)*arctan(-1/12*sqrt(6)*2^(1/6)*(2*x^5 + 2*x^2 - 2^(2/3)*
(7*x^4 + 4*x) - 2^(1/3)*(5*x^3 + 2))*sqrt(x^3 + 1)/(2*x^6 + 3*x^3 + 1))
```

3.43.6 Sympy [F]

$$\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = -\int \left(-\frac{2^{\frac{2}{3}}}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\right)dx - \int \frac{2x}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\,dx$$

input
```
integrate((2**(2/3)-2*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

output
```
-Integral(-2**(2/3)/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x) - Int
egral(2*x/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x)
```

3.43.7 Maxima [F]

$$\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \int -\frac{2\,x-2^{\frac{2}{3}}}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((2*x - 2^(2/3))/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

3.43.8 Giac [F(-2)]

Exception generated.

$$\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Ba
d Argumen
```

3.43.9 Mupad [B] (verification not implemented)

Time = 9.79 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.89

$$\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \frac{2^{2/3}\sqrt{3}\,\ln\left(\frac{\left(\sqrt{3}\,1\mathrm{i}+\sqrt{x^3+1}+2^{1/3}\,\sqrt{3}\,x\,1\mathrm{i}\right)\left(\sqrt{3}\,1\mathrm{i}-\sqrt{x^3+1}+2^{1/3}\,\sqrt{3}\,x\,1\mathrm{i}\right)^3}{\left(x+2^{2/3}\right)^6}\right)\,1\mathrm{i}}{3}$$

input
```
int(-(2*x - 2^(2/3))/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
```

output
```
(2^(2/3)*3^(1/2)*log(((3^(1/2)*1i + (x^3 + 1)^(1/2) + 2^(1/3)*3^(1/2)*x*1i
)*(3^(1/2)*1i - (x^3 + 1)^(1/2) + 2^(1/3)*3^(1/2)*x*1i)^3)/(x + 2^(2/3))^6
)*1i)/3
```

3.44 $\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

3.44.1 Optimal result

Integrand size = 32, antiderivative size = 40

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx=-\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{\sqrt{3}}$$

output
```
-2/3*2^(2/3)*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*3^(1/2)
```

3.44.2 Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx=-\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{1-x^3}}{\sqrt{3}\left(-1+\sqrt[3]{2}x\right)}\right)}{\sqrt{3}}$$

input
```
Integrate[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTan[Sqrt[1 - x^3]/(Sqrt[3]*(-1 + 2^(1/3)*x))])/Sqrt[3]
```

3.44.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+2^{2/3}}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{2562}$$

$$-22^{2/3}\int \frac{1}{\frac{3\left(1-\sqrt[3]{2}x\right)^2}{1-x^3}+1}d\frac{1-\sqrt[3]{2}x}{\sqrt{1-x^3}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\ 2^{2/3}\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{\sqrt{3}}$$

input
```
Int[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/Sqrt[3]
```

3.44.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.44.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.78

method	result
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)\ln\left(\frac{12\sqrt{-x^3+1}\,x+3\operatorname{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)2^{\frac{2}{3}}x^2+\operatorname{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)x^3-6\sqrt{-x^3+1}\,2^{\frac{2}{3}}-6\operatorname{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right.}{\left(2^{\frac{1}{3}}x-2\right)^3}\right.}{3}$
default	$\frac{4i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{2i2^{\frac{2}{3}}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{}}}{}$
elliptic	$\frac{4i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{2i2^{\frac{2}{3}}\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{}}}{}$

input
```
int((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*RootOf(_Z^2+6*2^(1/3))*ln((12*(-x^3+1)^(1/2)*x+3*RootOf(_Z^2+6*2^(1/3
))*2^(2/3)*x^2+RootOf(_Z^2+6*2^(1/3))*x^3-6*(-x^3+1)^(1/2)*2^(2/3)-6*RootO
f(_Z^2+6*2^(1/3))*2^(1/3)*x+2*RootOf(_Z^2+6*2^(1/3)))/(2^(1/3)*x-2)^3)
```

3.44.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(30) = 60.

Time = 0.39 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.90

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx =$$
$$-\frac{1}{3}\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{\sqrt{6}2^{\frac{1}{6}}\left(2\,x^5-2\,x^2+2^{\frac{2}{3}}(7\,x^4-4\,x)-2^{\frac{1}{3}}(5\,x^3-2)\right)\sqrt{-x^3+1}}{12\,(2\,x^6-3\,x^3+1)}\right)$$

input
```
integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(6)*2^(1/6)*arctan(1/12*sqrt(6)*2^(1/6)*(2*x^5 - 2*x^2 + 2^(2/3)*
(7*x^4 - 4*x) - 2^(1/3)*(5*x^3 - 2))*sqrt(-x^3 + 1)/(2*x^6 - 3*x^3 + 1))
```

3.44.6 Sympy [F]

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\int \frac{2^{\frac{2}{3}}}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx - \int \frac{2x}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx$$

input
```
integrate((2**(2/3)+2*x)/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

output
```
-Integral(2**(2/3)/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x) - Inte
gral(2*x/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

3.44.7 Maxima [F]

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = \int -\frac{2\,x+2^{\frac{2}{3}}}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((2*x + 2^(2/3))/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

3.44.8 Giac [F(-2)]

Exception generated.

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad
Argument
```

3.44.9 Mupad [B] (verification not implemented)

Time = 9.76 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.85

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = \frac{2^{2/3}\,\sqrt{3}\,\ln\left(\frac{\left(\sqrt{1-x^3}-\sqrt{3}\,1\mathrm{i}+2^{1/3}\,\sqrt{3}\,x\,1\mathrm{i}\right)\,\left(\sqrt{3}\,1\mathrm{i}+\sqrt{1-x^3}-2^{1/3}\,\sqrt{3}\,x\,1\mathrm{i}\right)^3}{\left(x-2^{2/3}\right)^6}\right)\,1\mathrm{i}}{3}$$

input
```
int(-(2*x + 2^(2/3))/((1 - x^3)^(1/2)*(x - 2^(2/3))),x)
```

output
```
(2^(2/3)*3^(1/2)*log((((1 - x^3)^(1/2) - 3^(1/2)*1i + 2^(1/3)*3^(1/2)*x*1i
)*(3^(1/2)*1i + (1 - x^3)^(1/2) - 2^(1/3)*3^(1/2)*x*1i)^3)/(x - 2^(2/3))^6
)*1i)/3
```

3.45 $\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

3.45.1 Optimal result

Integrand size = 30, antiderivative size = 38

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = -\frac{2\ 2^{2/3}\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{\sqrt{3}}$$

output
```
-2/3*2^(2/3)*arctanh((1-2^(1/3)*x)*3^(1/2)/(x^3-1)^(1/2))*3^(1/2)
```

3.45.2 Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = \frac{2\ 2^{2/3}\operatorname{arctanh}\left(\frac{\sqrt{-1+x^3}}{\sqrt{3}\left(-1+\sqrt[3]{2}x\right)}\right)}{\sqrt{3}}$$

input
```
Integrate[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

output
```
(2*2^(2/3)*ArcTanh[Sqrt[-1 + x^3]/(Sqrt[3]*(-1 + 2^(1/3)*x))])/Sqrt[3]
```

3.45.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+2^{2/3}}{\left(2^{2/3}-x\right)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{ 2562}$$

$$-22^{2/3}\int \frac{1}{1-\frac{3\left(1-\sqrt[3]{2}x\right)^2}{x^3-1}}d\frac{1-\sqrt[3]{2}x}{\sqrt{x^3-1}}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{2\ 2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{\sqrt{3}}$$

input
```
Int[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/Sqrt[3]
```

3.45.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.45.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.41 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.84

method	result
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)\ln\left(\frac{12\sqrt{x^3-1}\,x-3\,2^{\frac{2}{3}}x^2\operatorname{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)-\operatorname{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)x^3-6\sqrt{x^3-1}\,2^{\frac{2}{3}}+6\operatorname{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)2}{\left(2^{\frac{1}{3}}x-2\right)^3}\right.}{3}$
default	$-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{6\,2^{\frac{2}{3}}\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{\sqrt{x^3-}}$
elliptic	$-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{6\,2^{\frac{2}{3}}\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{\sqrt{x^3-}}$

input
```
int((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*RootOf(_Z^2-6*2^(1/3))*ln((12*(x^3-1)^(1/2)*x-3*2^(2/3)*x^2*RootOf(_Z
^2-6*2^(1/3))-RootOf(_Z^2-6*2^(1/3))*x^3-6*(x^3-1)^(1/2)*2^(2/3)+6*RootOf(
_Z^2-6*2^(1/3))*2^(1/3)*x-2*RootOf(_Z^2-6*2^(1/3)))/(2^(1/3)*x-2)^3)
```

3.45.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(28) = 56$.

Time = 0.40 (sec) , antiderivative size = 238, normalized size of antiderivative = 6.26

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = \frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\log\left(\frac{x^{18}+1440\,x^{15}+17400\,x^{12}-21056\,x^9-10368\,x^6+15360\,x^3+}{}\right.$$

input
```
integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(6)*2^(1/6)*log((x^18 + 1440*x^15 + 17400*x^12 - 21056*x^9 - 10368
*x^6 + 15360*x^3 + 2*sqrt(6)*2^(1/6)*(126*x^14 + 2664*x^11 - 4608*x^5 + 23
04*x^2 + 2^(2/3)*(x^16 + 310*x^13 + 2332*x^10 - 2656*x^7 - 256*x^4 + 512*x
) + 2^(1/3)*(17*x^15 + 1058*x^12 + 2528*x^9 - 5408*x^6 + 2560*x^3 - 512))*
sqrt(x^3 - 1) + 24*2^(2/3)*(x^17 + 121*x^14 + 478*x^11 - 1144*x^8 + 608*x^
5 - 64*x^2) + 48*2^(1/3)*(5*x^16 + 176*x^13 + 83*x^10 - 680*x^7 + 544*x^4
- 128*x) - 2048)/(x^18 - 24*x^15 + 240*x^12 - 1280*x^9 + 3840*x^6 - 6144*x
^3 + 4096))
```

3.45.6 Sympy [F]

$$\int \frac{2^{2/3}+2x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = -\int \frac{2^{\frac{2}{3}}}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx - \int \frac{2x}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

input
```
integrate((2**(2/3)+2*x)/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

output
```
-Integral(2**(2/3)/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x) - Inte
gral(2*x/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

3.45.7 Maxima [F]

$$\int \frac{2^{2/3}+2x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \int -\frac{2\,x+2^{\frac{2}{3}}}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((2*x + 2^(2/3))/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

3.45.8 Giac [F(-2)]

Exception generated.

$$\int \frac{2^{2/3}+2x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad
Argument
```

3.45.9 Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.63

$$\int \frac{2^{2/3}+2x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \frac{2^{2/3}\sqrt{3}\ln\left(\frac{\left(\sqrt{x^3-1}-\sqrt{3}+2^{1/3}\sqrt{3}\,x\right)^3\left(\sqrt{3}+\sqrt{x^3-1}-2^{1/3}\sqrt{3}\,x\right)}{(x-2^{2/3})^6}\right)}{3}$$

input
```
int(-(2*x + 2^(2/3))/((x^3 - 1)^(1/2)*(x - 2^(2/3))),x)
```

output
```
(2^(2/3)*3^(1/2)*log((((x^3 - 1)^(1/2) - 3^(1/2) + 2^(1/3)*3^(1/2)*x)^3*(3
^(1/2) + (x^3 - 1)^(1/2) - 2^(1/3)*3^(1/2)*x))/(x - 2^(2/3))^6))/3
```

3.46 $\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

3.46.1 Optimal result

Integrand size = 30, antiderivative size = 39

$$\int \frac{2^{2/3}-2x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx=\frac{2\ 2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{\sqrt{3}}$$

output
```
2/3*2^(2/3)*arctanh((1+2^(1/3)*x)*3^(1/2)/(-x^3-1)^(1/2))*3^(1/2)
```

3.46.2 Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{2^{2/3}-2x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx=\frac{2\ 2^{2/3}\text{arctanh}\left(\frac{\sqrt{-1-x^3}}{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}\right)}{\sqrt{3}}$$

input
```
Integrate[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*2^(2/3)*ArcTanh[Sqrt[-1 - x^3]/(Sqrt[3]*(1 + 2^(1/3)*x))])/Sqrt[3]
```

3.46.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{2562}$$

$$2\ 2^{2/3}\int \frac{1}{1-\frac{3\left(\sqrt[3]{2}x+1\right)^2}{-x^3-1}}d\frac{\sqrt[3]{2}x+1}{\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\ 2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{\sqrt{3}}$$

input
```
Int[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/Sqrt[3]
```

3.46.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.46.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.63 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.87

method	result
trager	$\frac{\text{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)\ln\left(\frac{3\,2^{\frac{2}{3}}x^2\,\text{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)+12\sqrt{-x^3-1}\,x-\text{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)x^3+6\,\text{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)2^{\frac{1}{3}}x+6\sqrt{-x^3-}}{\left(2^{\frac{1}{3}}x+2\right)^3}\right.}{3}$
default	$\frac{4i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2i2^{\frac{2}{3}}\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{}$
elliptic	$\frac{4i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2i2^{\frac{2}{3}}\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{}$

input
```
int((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*RootOf(_Z^2-6*2^(1/3))*ln((3*2^(2/3)*x^2*RootOf(_Z^2-6*2^(1/3))+12*(-x
^3-1)^(1/2)*x-RootOf(_Z^2-6*2^(1/3))*x^3+6*RootOf(_Z^2-6*2^(1/3))*2^(1/3)*
x+6*(-x^3-1)^(1/2)*2^(2/3)+2*RootOf(_Z^2-6*2^(1/3)))/(2^(1/3)*x+2)^3)
```

3.46.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(29) = 58.

Time = 0.40 (sec) , antiderivative size = 241, normalized size of antiderivative = 6.18

$$\int \frac{2^{2/3}-2x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\log\left(\frac{x^{18}-1440\,x^{15}+17400\,x^{12}+21056\,x^9-10368\,x^6-15360\,x^3-}{}\right.$$

input
```
integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(6)*2^(1/6)*log((x^18 - 1440*x^15 + 17400*x^12 + 21056*x^9 - 10368
*x^6 - 15360*x^3 - 2*sqrt(6)*2^(1/6)*(126*x^14 - 2664*x^11 + 4608*x^5 + 23
04*x^2 + 2^(2/3)*(x^16 - 310*x^13 + 2332*x^10 + 2656*x^7 - 256*x^4 - 512*x
) - 2^(1/3)*(17*x^15 - 1058*x^12 + 2528*x^9 + 5408*x^6 + 2560*x^3 + 512))*
sqrt(-x^3 - 1) - 24*2^(2/3)*(x^17 - 121*x^14 + 478*x^11 + 1144*x^8 + 608*x
^5 + 64*x^2) + 48*2^(1/3)*(5*x^16 - 176*x^13 + 83*x^10 + 680*x^7 + 544*x^4
 + 128*x) - 2048)/(x^18 + 24*x^15 + 240*x^12 + 1280*x^9 + 3840*x^6 + 6144*
x^3 + 4096))
```

3.46.6 Sympy [F]

$$\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = -\int\left(-\frac{2^{\frac{2}{3}}}{x\sqrt{-x^3-1}+2^{\frac{2}{3}}\sqrt{-x^3-1}}\right)dx-\int\frac{2x}{x\sqrt{-x^3-1}+2^{\frac{2}{3}}\sqrt{-x^3-1}}\,dx$$

input
```
integrate((2**(2/3)-2*x)/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

output
```
-Integral(-2**(2/3)/(x*sqrt(-x**3 - 1) + 2**(2/3)*sqrt(-x**3 - 1)), x) - I
ntegral(2*x/(x*sqrt(-x**3 - 1) + 2**(2/3)*sqrt(-x**3 - 1)), x)
```

3.46.7 Maxima [F]

$$\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \int -\frac{2\,x-2^{\frac{2}{3}}}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((2*x - 2^(2/3))/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

3.46.8 Giac [F(-2)]

Exception generated.

$$\int \frac{2^{2/3}-2x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Ba
d Argumen
```

3.46.9 Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

$$\int \frac{2^{2/3}-2x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \frac{2^{2/3}\sqrt{3}\ln\left(\frac{\left(\sqrt{3}+\sqrt{-x^3-1}+2^{1/3}\sqrt{3}\,x\right)^3\left(\sqrt{3}-\sqrt{-x^3-1}+2^{1/3}\sqrt{3}\,x\right)}{\left(x+2^{2/3}\right)^6}\right)}{3}$$

input
```
int(-(2*x - 2^(2/3))/((- x^3 - 1)^(1/2)*(x + 2^(2/3))),x)
```

output
```
(2^(2/3)*3^(1/2)*log(((3^(1/2) + (- x^3 - 1)^(1/2) + 2^(1/3)*3^(1/2)*x)^3*
(3^(1/2) - (- x^3 - 1)^(1/2) + 2^(1/3)*3^(1/2)*x))/(x + 2^(2/3))^6))/3
```

3.47 $\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

3.47.1 Optimal result

Integrand size = 53, antiderivative size = 63

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
2/3*2^(2/3)*arctan(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)*3^(1/2)/(b*x^3+a)^(
1/2))/a^(1/6)/b^(1/3)*3^(1/2)
```

3.47.2 Mathematica [A] (verified)

Time = 5.39 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=-\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{a+bx^3}}{\sqrt{3}\left(\sqrt{a}+\sqrt[3]{2}\sqrt[6]{a}\sqrt[3]{b}x\right)}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*S
qrt[a + b*x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTan[Sqrt[a + b*x^3]/(Sqrt[3]*(Sqrt[a] + 2^(1/3)*a^(1/6)*b^(
1/3)*x))])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

3.47.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{2562}$$

$$\frac{2\ 2^{2/3}\sqrt[3]{a}\int \frac{1}{\frac{3\sqrt[3]{a}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{bx^3+a}+1}d\frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{bx^3+a}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\ 2^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a
+ b*x^3]),x]
```

output
```
(2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a +
 b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

3.47.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.47.4 Maple [F]

$$\int \frac{2^{\frac{2}{3}} a^{\frac{1}{3}}-2 b^{\frac{1}{3}} x}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}+b^{\frac{1}{3}} x\right) \sqrt{b \,x^{3}+a}}d x$$

input
```
int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1
/2),x)
```

output
```
int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1
/2),x)
```

3.47.5 Fricas [F(-1)]

Timed out.

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3
+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.47.6 Sympy [F]

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\int \left(-\frac{2^{\frac{2}{3}}\sqrt[3]{a}}{2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\right)dx - \int \frac{2\sqrt[3]{b}x}{2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx$$

input
```
integrate((2**(2/3)*a**(1/3)-2*b**(1/3)*x)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/
(b*x**3+a)**(1/2),x)
```

output
```
-Integral(-2**(2/3)*a**(1/3)/(2**(2/3)*a**(1/3)*sqrt(a + b*x**3) + b**(1/3
)*x*sqrt(a + b*x**3)), x) - Integral(2*b**(1/3)*x/(2**(2/3)*a**(1/3)*sqrt(
a + b*x**3) + b**(1/3)*x*sqrt(a + b*x**3)), x)
```

3.47.7 Maxima [F]

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \int -\frac{2\,b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3
+a)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((2*b^(1/3)*x - 2^(2/3)*a^(1/3))/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2
^(2/3)*a^(1/3))), x)
```

3.47.8 Giac [F(-1)]

Timed out.

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3
+a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.47.9 Mupad [B] (verification not implemented)

Time = 11.15 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.68

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{2^{2/3}\,\sqrt{3}\,\ln\left(\frac{\left(\sqrt{3}\,\sqrt{a}\,1\mathrm{i}-\sqrt{b\,x^3+a}+2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\,1\mathrm{i}\right)^3\,\left(\sqrt{3}\,\sqrt{a}\,1\mathrm{i}+\sqrt{b\,x^3+a}+2^{1/3}\,\sqrt{3}\,a^{1}\right.}{\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)^6}\right.}{3\,a^{1/6}\,b^{1/3}}$$

input
```
int((2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) +
b^(1/3)*x)),x)
```

output
```
(2^(2/3)*3^(1/2)*log(((3^(1/2)*a^(1/2)*1i - (a + b*x^3)^(1/2) + 2^(1/3)*3^
(1/2)*a^(1/6)*b^(1/3)*x*1i)^3*(3^(1/2)*a^(1/2)*1i + (a + b*x^3)^(1/2) + 2^
(1/3)*3^(1/2)*a^(1/6)*b^(1/3)*x*1i))/(2^(2/3)*a^(1/3) + b^(1/3)*x)^6)*1i)/
(3*a^(1/6)*b^(1/3))
```

3.48 $\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

3.48.1 Optimal result

Integrand size = 55, antiderivative size = 65

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=-\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
-2/3*2^(2/3)*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)*3^(1/2)/(-b*x^3+a)
^(1/2))/a^(1/6)/b^(1/3)*3^(1/2)
```

3.48.2 Mathematica [A] (verified)

Time = 5.39 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{a-bx^3}}{\sqrt{3}\left(\sqrt{a}-\sqrt[3]{2}\sqrt[6]{a}\sqrt[3]{b}x\right)}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*S
qrt[a - b*x^3]),x]
```

output
```
(2*2^(2/3)*ArcTan[Sqrt[a - b*x^3]/(Sqrt[3]*(Sqrt[a] - 2^(1/3)*a^(1/6)*b^(1
/3)*x))])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

3.48.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx$$

$$\Big\downarrow \text{2562}$$

$$-\frac{2\ 2^{2/3}\sqrt[3]{a}\int \frac{1}{\frac{3\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)^2}{a-bx^3}+1}d\frac{\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{a-bx^3}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\ 2^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a
- b*x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a
- b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

3.48.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.48.4 Maple [F]

$$\int \frac{2^{\frac{2}{3}} a^{\frac{1}{3}}+2 b^{\frac{1}{3}} x}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}-b^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3}+a}}dx$$

input
```
int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(
1/2),x)
```

output
```
int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(
1/2),x)
```

3.48.5 Fricas [F(-1)]

Timed out.

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^
3+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.48.6 Sympy [F]

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\int \frac{2^{\frac{2}{3}}\sqrt[3]{a}}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx - \int \frac{2\sqrt[3]{b}x}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

input
```
integrate((2**(2/3)*a**(1/3)+2*b**(1/3)*x)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/
(-b*x**3+a)**(1/2),x)
```

output
```
-Integral(2**(2/3)*a**(1/3)/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3
)*x*sqrt(a - b*x**3)), x) - Integral(2*b**(1/3)*x/(-2**(2/3)*a**(1/3)*sqrt
(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x**3)), x)
```

3.48.7 Maxima [F]

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \int -\frac{2\,b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^
3+a)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((2*b^(1/3)*x + 2^(2/3)*a^(1/3))/(sqrt(-b*x^3 + a)*(b^(1/3)*x -
2^(2/3)*a^(1/3))), x)
```

3.48.8 Giac [F(-1)]

Timed out.

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^
3+a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.48.9 Mupad [B] (verification not implemented)

Time = 10.97 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.65

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \frac{2^{2/3}\sqrt{3}\,\ln\left(\frac{\left(\sqrt{a-b\,x^3}-\sqrt{3}\,\sqrt{a}\,1\mathrm{i}+2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\,1\mathrm{i}\right)\left(\sqrt{3}\,\sqrt{a}\,1\mathrm{i}+\sqrt{a-b\,x^3}-2^{1/3}\,\sqrt{3}\,a^{1/}\right.}{\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)^6}\right.}{3\,a^{1/6}\,b^{1/3}}$$

input
```
int((2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) -
b^(1/3)*x)),x)
```

output
```
(2^(2/3)*3^(1/2)*log((((a - b*x^3)^(1/2) - 3^(1/2)*a^(1/2)*1i + 2^(1/3)*3^
(1/2)*a^(1/6)*b^(1/3)*x*1i)*(3^(1/2)*a^(1/2)*1i + (a - b*x^3)^(1/2) - 2^(1
/3)*3^(1/2)*a^(1/6)*b^(1/3)*x*1i)^3)/(2^(2/3)*a^(1/3) - b^(1/3)*x)^6)*1i)/
(3*a^(1/6)*b^(1/3))
```

3.49 $\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

3.49.1 Optimal result

Integrand size = 56, antiderivative size = 66

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=-\frac{2\,2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
-2/3*2^(2/3)*arctanh(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)*3^(1/2)/(b*x^3-a)
^(1/2))/a^(1/6)/b^(1/3)*3^(1/2)
```

3.49.2 Mathematica [A] (verified)

Time = 5.38 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=-\frac{2\,2^{2/3}\text{arctanh}\left(\frac{\sqrt{-a+bx^3}}{\sqrt{3}\left(\sqrt{a}-\sqrt[3]{2}\sqrt[6]{a}\sqrt[3]{b}x\right)}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*S
qrt[-a + b*x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTanh[Sqrt[-a + b*x^3]/(Sqrt[3]*(Sqrt[a] - 2^(1/3)*a^(1/6)*b
^(1/3)*x))])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

3.49.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{2562}$$

$$-\frac{2\ 2^{2/3}\sqrt[3]{a}\int \frac{1}{1-\frac{3\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)^2}{bx^3-a}}d\frac{\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{bx^3-a}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\ 2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a
 + b*x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[-
a + b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

3.49.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.49.4 Maple [F]

$$\int \frac{2^{\frac{2}{3}} a^{\frac{1}{3}} + 2 b^{\frac{1}{3}} x}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}} - b^{\frac{1}{3}} x\right) \sqrt{b\,x^{3} - a}}dx$$

input
```
int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1
/2),x)
```

output
```
int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1
/2),x)
```

3.49.5 Fricas [F(-1)]

Timed out.

$$\int \frac{2^{2/3}\sqrt[3]{a} + 2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a} - \sqrt[3]{b}x\right)\sqrt{-a + bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3
-a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.49.6 Sympy [F]

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\int \frac{2^{\frac{2}{3}}\sqrt[3]{a}}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx - \int \frac{2\sqrt[3]{b}x}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

input
```
integrate((2**(2/3)*a**(1/3)+2*b**(1/3)*x)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/
(b*x**3-a)**(1/2),x)
```

output
```
-Integral(2**(2/3)*a**(1/3)/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/
3)*x*sqrt(-a + b*x**3)), x) - Integral(2*b**(1/3)*x/(-2**(2/3)*a**(1/3)*sq
rt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*x**3)), x)
```

3.49.7 Maxima [F]

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \int -\frac{2\,b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3
-a)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((2*b^(1/3)*x + 2^(2/3)*a^(1/3))/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2
^(2/3)*a^(1/3))), x)
```

3.49.8 Giac [F(-1)]

Timed out.

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3
-a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.49.9 Mupad [B] (verification not implemented)

Time = 10.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.55

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \frac{\sqrt{3}\,4^{1/3}\,\ln\left(\frac{\left(\sqrt{b\,x^3-a}+\sqrt{3}\,\sqrt{a}-2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\right)\left(\sqrt{b\,x^3-a}-\sqrt{3}\,\sqrt{a}+2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1}\right.}{\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)^6}\right.}{3\,a^{1/6}\,b^{1/3}}$$

input
```
int((2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) -
b^(1/3)*x)),x)
```

output
```
(3^(1/2)*4^(1/3)*log((((b*x^3 - a)^(1/2) + 3^(1/2)*a^(1/2) - 2^(1/3)*3^(1/
2)*a^(1/6)*b^(1/3)*x)*((b*x^3 - a)^(1/2) - 3^(1/2)*a^(1/2) + 2^(1/3)*3^(1/
2)*a^(1/6)*b^(1/3)*x)^3)/(2^(2/3)*a^(1/3) - b^(1/3)*x)^6))/(3*a^(1/6)*b^(1
/3))
```

3.50 $\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

3.50.1 Optimal result

Integrand size = 56, antiderivative size = 66

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\frac{2\,2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
2/3*2^(2/3)*arctanh(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)*3^(1/2)/(-b*x^3-a)
^(1/2))/a^(1/6)/b^(1/3)*3^(1/2)
```

3.50.2 Mathematica [A] (verified)

Time = 5.39 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\frac{2\,2^{2/3}\text{arctanh}\left(\frac{\sqrt{-a-bx^3}}{\sqrt{3}\left(\sqrt{a}+\sqrt[3]{2}\sqrt[6]{a}\sqrt[3]{b}x\right)}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*S
qrt[-a - b*x^3]),x]
```

3.50. $\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

output
```
(2*2^(2/3)*ArcTanh[Sqrt[-a - b*x^3]/(Sqrt[3]*(Sqrt[a] + 2^(1/3)*a^(1/6)*b^
(1/3)*x))])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

3.50.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}dx$$

$$\Big\downarrow \text{2562}$$

$$\frac{2\ 2^{2/3}\sqrt[3]{a}\int \frac{1}{1-\frac{3\sqrt[3]{a}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{-bx^3-a}}d\frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{-bx^3-a}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\ 2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a
 - b*x^3]),x]
```

output
```
(2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-a
 - b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

3.50.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.50.4 Maple [F]

$$\int \frac{2^{\frac{2}{3}} a^{\frac{1}{3}}-2 b^{\frac{1}{3}} x}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}+b^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3}-a}}dx$$

input
```
int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(
1/2),x)
```

output
```
int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(
1/2),x)
```

3.50.5 Fricas [F(-1)]

Timed out.

$$\int \frac{2^{2/3} \sqrt[3]{a}-2 \sqrt[3]{b} x}{\left(2^{2/3} \sqrt[3]{a}+\sqrt[3]{b} x\right) \sqrt{-a-b x^3}} \, dx=\text{Timed out}$$

input
```
integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^
3-a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.50.6 Sympy [F]

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\int \left(-\frac{2^{\frac{2}{3}}\sqrt[3]{a}}{2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}x\sqrt{-a-bx^3}}\right)dx - \int \frac{2\sqrt[3]{b}x}{2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}x\sqrt{-a-bx^3}}\,dx$$

input
```
integrate((2**(2/3)*a**(1/3)-2*b**(1/3)*x)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/
(-b*x**3-a)**(1/2),x)
```

output
```
-Integral(-2**(2/3)*a**(1/3)/(2**(2/3)*a**(1/3)*sqrt(-a - b*x**3) + b**(1/
3)*x*sqrt(-a - b*x**3)), x) - Integral(2*b**(1/3)*x/(2**(2/3)*a**(1/3)*sqr
t(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*x**3)), x)
```

3.50.7 Maxima [F]

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \int -\frac{2\,b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^
3-a)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((2*b^(1/3)*x - 2^(2/3)*a^(1/3))/(sqrt(-b*x^3 - a)*(b^(1/3)*x +
2^(2/3)*a^(1/3))), x)
```

3.50.8 Giac [F(-1)]

Timed out.

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^
3-a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.50.9 Mupad [B] (verification not implemented)

Time = 9.76 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.56

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \frac{\sqrt{3}\,4^{1/3}\ln\Bigg(\frac{\left(\sqrt{-b\,x^3-a}+\sqrt{3}\,\sqrt{a}+2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\right)^3\left(\sqrt{3}\,\sqrt{a}-\sqrt{-b\,x^3-a}+2^{1/3}\,\sqrt{3}\,a^{1}\cdots\right.}{\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)^6}\cdots}{3\,a^{1/6}\,b^{1/3}}$$

input
```
int((2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3)
+ b^(1/3)*x)),x)
```

output
```
(3^(1/2)*4^(1/3)*log((((- a - b*x^3)^(1/2) + 3^(1/2)*a^(1/2) + 2^(1/3)*3^(
1/2)*a^(1/6)*b^(1/3)*x)^3*(3^(1/2)*a^(1/2) - (- a - b*x^3)^(1/2) + 2^(1/3)
*3^(1/2)*a^(1/6)*b^(1/3)*x))/(2^(2/3)*a^(1/3) + b^(1/3)*x)^6))/(3*a^(1/6)*
b^(1/3))
```

3.51 $\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

3.51.1 Optimal result

Integrand size = 30, antiderivative size = 49

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \frac{2\arctan\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{\sqrt{3}\sqrt{cd}}$$

output
```
2/3*arctan((2*d*x+c)*3^(1/2)*c^(1/2)/(4*d^3*x^3+c^3)^(1/2))/d*3^(1/2)/c^(1
/2)
```

3.51.2 Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{c^3+4d^3x^3}}{\sqrt{3}\sqrt{c}(c+2dx)}\right)}{\sqrt{3}\sqrt{cd}}$$

input
```
Integrate[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

output
```
(-2*ArcTan[Sqrt[c^3 + 4*d^3*x^3]/(Sqrt[3]*Sqrt[c]*(c + 2*d*x))])/(Sqrt[3]*
Sqrt[c]*d)
```

3.51.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

$$\Big\downarrow \; 2562$$

$$\frac{2c\int \frac{1}{\frac{3c(c+2dx)^2}{c^3+4d^3x^3}+1}d\frac{c+2dx}{c\sqrt{c^3+4d^3x^3}}}{d}$$

$$\Big\downarrow \; 216$$

$$\frac{2\arctan\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{\sqrt{3}\sqrt{c}d}$$

input
```
Int[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/(Sqrt[3]*S
qrt[c]*d)
```

3.51.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.51.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.89 (sec) , antiderivative size = 889, normalized size of antiderivative = 18.14

method	result	size
default	Expression too large to display	889
elliptic	Expression too large to display	889

input
```
int((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(
1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*
I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)*((x
+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d
))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*
3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d^
3*x^3+c^3)^(1/2)*EllipticF(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((
1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))
*c/d))^(1/2),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*
3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)
*c/d))^(1/2))+6*c/d*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+
1/4*I*3^(1/2)*2^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/(
(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3)
)*c/d))^(1/2)*((x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/
d+1/2*2^(1/3)*c/d))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1
/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*
c/d))^(1/2)/(4*d^3*x^3+c^3)^(1/2)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d
+c/d)*EllipticPi(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3
)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/
2),((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*...
```

3.51.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(39) = 78$.

Time = 0.43 (sec) , antiderivative size = 300, normalized size of antiderivative = 6.12

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$
$$= \left[\frac{\sqrt{3}\sqrt{-\frac{1}{c}}\log\left(\frac{2\,d^6x^6-36\,cd^5x^5-18\,c^2d^4x^4+28\,c^3d^3x^3+18\,c^4d^2x^2-c^6-\sqrt{3}(4\,cd^4x^4-10\,c^2d^3x^3-18\,c^3d^2x^2-8\,c^4dx-c^5)\sqrt{4\,d^3x^3+c^3}\sqrt{-\frac{1}{c}}}{d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx+c^6}\right)}{6\,d}, -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{4\,d^3x^3+c^3}(2\,d^3x^3-6\,cd^2x^2-6\,c^2dx-c^3)}{3\,(8\,d^4x^4+4\,cd^3x^3+2\,c^3dx+c^4)\sqrt{c}}\right)}{3\,\sqrt{c}d}\right]$$

input
```
integrate((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*sqrt(3)*sqrt(-1/c)*log((2*d^6*x^6 - 36*c*d^5*x^5 - 18*c^2*d^4*x^4 + 2
8*c^3*d^3*x^3 + 18*c^4*d^2*x^2 - c^6 - sqrt(3)*(4*c*d^4*x^4 - 10*c^2*d^3*x
^3 - 18*c^3*d^2*x^2 - 8*c^4*d*x - c^5)*sqrt(4*d^3*x^3 + c^3)*sqrt(-1/c))/(
d^6*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x^2 +
 6*c^5*d*x + c^6))/d, -1/3*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(4*d^3*x^3 + c^3
)*(2*d^3*x^3 - 6*c*d^2*x^2 - 6*c^2*d*x - c^3)/((8*d^4*x^4 + 4*c*d^3*x^3 +
2*c^3*d*x + c^4)*sqrt(c)))/(sqrt(c)*d)]
```

3.51.6 Sympy [F]

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = -\int \left(-\frac{c}{c\sqrt{c^3+4d^3x^3}+dx\sqrt{c^3+4d^3x^3}}\right)dx - \int \frac{2dx}{c\sqrt{c^3+4d^3x^3}+dx\sqrt{c^3+4d^3x^3}}\,dx$$

input
```
integrate((-2*d*x+c)/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

output
```
-Integral(-c/(c*sqrt(c**3 + 4*d**3*x**3) + d*x*sqrt(c**3 + 4*d**3*x**3)),
x) - Integral(2*d*x/(c*sqrt(c**3 + 4*d**3*x**3) + d*x*sqrt(c**3 + 4*d**3*x
**3)), x)
```

3.51.7 Maxima [F]

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int -\frac{2\,dx-c}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

input
```
integrate((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((2*d*x - c)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

3.51.8 Giac [F]

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int -\frac{2\,dx-c}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

input
```
integrate((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(2*d*x - c)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

3.51.9 Mupad [B] (verification not implemented)

Time = 10.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.94

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \frac{\sqrt{3}\,\ln\left(\frac{\left(-\sqrt{c^3+4\,d^3\,x^3}+\sqrt{3}\,c^{3/2}\,1\mathrm{i}+\sqrt{3}\,\sqrt{c}\,d\,x\,2\mathrm{i}\right)^3\left(\sqrt{c^3+4\,d^3\,x^3}+\sqrt{3}\,c^{3/2}\,1\mathrm{i}+\sqrt{3}\,\sqrt{c}\,d\,x\,2\mathrm{i}\right)}{(c+d\,x)^6}\right)\,1\mathrm{i}}{3\,\sqrt{c}\,d}$$

input
```
int((c - 2*d*x)/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)),x)
```

output
```
(3^(1/2)*log(((3^(1/2)*c^(3/2)*1i - (c^3 + 4*d^3*x^3)^(1/2) + 3^(1/2)*c^(1
/2)*d*x*2i)^3*((c^3 + 4*d^3*x^3)^(1/2) + 3^(1/2)*c^(3/2)*1i + 3^(1/2)*c^(1
/2)*d*x*2i))/(c + d*x)^6)*1i)/(3*c^(1/2)*d)
```

3.52 $\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

3.52.1 Optimal result

Integrand size = 24, antiderivative size = 158

$$\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \frac{2\left(2-3\ 2^{2/3}\right)\arctan\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}} + \frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
2/9*(2-3*2^(2/3))*arctan((1+2^(1/3)*x)*3^(1/2)/(x^3+1)^(1/2))*3^(1/2)+2/9*
(3+2*2^(1/3))*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(
1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)
^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.52.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.43 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.13

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \frac{2\sqrt[6]{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(3\sqrt{-i+\sqrt{3}+2ix}\left(-6-3\sqrt[3]{2}-2i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}+\left(3\sqrt[3]{2}+4i\sqrt{3}\right.\right.\right.}{}$$

input
```
Integrate[(2 + 3*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

output
```
(2*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(3*Sqrt[-I + Sqrt[3] + (2*I)*
x]*(-6 - 3*2^(1/3) - (2*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3] + (3*2^(1/3) + (4*I
)*Sqrt[3] + I*2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I
)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - 4*Sqrt[3]*(-3 + 2^
(1/3))*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3]
)/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[
2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) +
 Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x^3])
```

3.52.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2564, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x+2}{\left(x+2^{2/3}\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{ 2564}$$

$$\frac{1}{3}\left(3+2\sqrt[3]{2}\right)\int \frac{1}{\sqrt{x^3+1}}dx-\frac{1}{3}\left(3-\sqrt[3]{2}\right)\int \frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{ 759}$$

$$\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{1}{3}\left(3-\sqrt[3]{2}\right)\int\frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \ 2562$$

$$\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2}{3}2^{2/3}\left(3-\sqrt[3]{2}\right)\int\frac{1}{\frac{3\left(\sqrt[3]{2}x+1\right)^2}{x^3+1}+1}d\frac{\sqrt[3]{2}x+1}{\sqrt{x^3+1}}$$

$$\Big\downarrow \ 216$$

$$\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\,2^{2/3}\left(3-\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}$$

input
```
Int[(2 + 3*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

output
```
(-2*2^(2/3)*(3 - 2^(1/3))*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])
/(3*Sqrt[3]) + (2*(3 + 2*2^(1/3))*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x +
x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3]
+ x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[
1 + x^3])
```

3.52. $\int \frac{2+3x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx$

3.52.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.52.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(125) = 250$.

Time = 4.34 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.66

method	result
default	$\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(2-3\,2^{\frac{2}{3}}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{\sqrt{x^3+1}}$
elliptic	$\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(2-3\,2^{\frac{2}{3}}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{\sqrt{x^3+1}}$

input
```
int((3*x+2)/(2^(2/3)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
6*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2
+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(2-3*2^(2/3))*(3/2-1/2*I*3^
(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*
I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+
1)^(1/2)/(2^(2/3)-1)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/
2*I*3^(1/2))/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)
)
```

3.52.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \frac{1}{9}\sqrt{3}\sqrt{-12\cdot 2^{\frac{2}{3}}+18\cdot 2^{\frac{1}{3}}+4}\arctan\left(\frac{\sqrt{3}\left(18\,x^5-42\,x^4-10\,x^3+18\,x^2+2^{\frac{2}{3}}(2\right.}{}\right.$$

$$+\frac{2}{3}\left(2\cdot 2^{\frac{1}{3}}+3\right)\text{weierstrassPInverse}(0,-4,x)$$

input
```
integrate((2+3*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/9*sqrt(3)*sqrt(-12*2^(2/3) + 18*2^(1/3) + 4)*arctan(1/300*sqrt(3)*(18*x^
5 - 42*x^4 - 10*x^3 + 18*x^2 + 2^(2/3)*(2*x^5 - 63*x^4 - 15*x^3 + 2*x^2 -
36*x - 6) + 2^(1/3)*(6*x^5 - 14*x^4 - 45*x^3 + 6*x^2 - 8*x - 18) - 24*x -
4)*sqrt(x^3 + 1)*sqrt(-12*2^(2/3) + 18*2^(1/3) + 4)/(2*x^6 + 3*x^3 + 1)) +
 2/3*(2*2^(1/3) + 3)*weierstrassPInverse(0, -4, x)
```

3.52.6 Sympy [F]

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{3x+2}{\sqrt{(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((2+3*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

output
```
Integral((3*x + 2)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

3.52.7 Maxima [F]

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{3\,x+2}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((2+3*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((3*x + 2)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

3.52.8 Giac [F(-2)]

Exception generated.

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2+3*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Ba
d Argumen
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{3\,x+2}{\sqrt{x^3+1}\,\left(x+2^{2/3}\right)}\,dx$$

input
```
int((3*x + 2)/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
```

output
```
int((3*x + 2)/((x^3 + 1)^(1/2)*(x + 2^(2/3))), x)
```

3.53 $\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

3.53.1 Optimal result

Integrand size = 28, antiderivative size = 173

$$\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{2\left(2+3\ 2^{2/3}\right)\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}} + \frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-2/9*(2+3*2^(2/3))*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*3^(1/2)+2/
9*(3-2*2^(1/3))*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)
*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3
+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

3.53.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.42 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.94

$$\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = \frac{2\sqrt[6]{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\left(-3i\sqrt{-i+\sqrt{3}-2ix}\left(-6i-3i\sqrt[3]{2}+2\sqrt{3}-\sqrt[3]{2}\sqrt{3}+\left(-3i\sqrt[3]{2}\right.\right.\right.}{}$$

input
```
Integrate[(2 + 3*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

output
```
(2*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-3*I)*Sqrt[-I + Sqrt[3]
 - (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*
2^(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3
] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 4*Sqrt[3]*
(3 + 2^(1/3))*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*
Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]
/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^
(2/3) + Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x^3])
```

3.53.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2564, 27, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x+2}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{1}{3}\left(3+\sqrt[3]{2}\right)\int \frac{2^{2/3}\left(\sqrt[3]{2}x+1\right)}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}dx-\frac{1}{3}\left(3-2\sqrt[3]{2}\right)\int \frac{1}{\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}2^{2/3}\left(3+\sqrt[3]{2}\right)\int \frac{\sqrt[3]{2}x+1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}dx-\frac{1}{3}\left(3-2\sqrt[3]{2}\right)\int \frac{1}{\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{759}$$

$$\frac{1}{3}2^{2/3}\left(3+\sqrt[3]{2}\right)\int \frac{\sqrt[3]{2}x+1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}dx+\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \text{2562}$$

$$\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{3}2^{2/3}\left(3+\sqrt[3]{2}\right)\int \frac{1}{\frac{3\left(1-\sqrt[3]{2}x\right)^2}{1-x^3}+1}d\frac{1-\sqrt[3]{2}x}{\sqrt{1-x^3}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\ 2^{2/3}\left(3+\sqrt[3]{2}\right)\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}$$

input
```
Int[(2 + 3*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

output
```
(-2*2^(2/3)*(3 + 2^(1/3))*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])
/(3*Sqrt[3]) + (2*(3 - 2*2^(1/3))*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x +
x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3]
- x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[
1 - x^3])
```

3.53. $\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

3.53.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.53.4 Maple [A] (verified)

Time = 5.64 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.49

method	result
default	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-x^{3}+1}}-\frac{2i\left(-2-3\,2^{\frac{2}{3}}\right)\sqrt{3}\,\sqrt{i\left(x+\right.}}{}$
elliptic	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-x^{3}+1}}-\frac{2i\left(-2-3\,2^{\frac{2}{3}}\right)\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}\right.}}{}$

input
```
int((3*x+2)/(2^(2/3)-x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+
1/2*I*3^(1/2)))^(1/2))-2/3*I*(-2-3*2^(2/3))*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2
))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1
/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-2^(2/3)-1/2+1/2*I*3^(1/2))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-2^(2/3)-1
/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

3.53.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.92

$$\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx =$$
$$-\frac{1}{9}\sqrt{3}\sqrt{12\cdot 2^{\frac{2}{3}}+18\cdot 2^{\frac{1}{3}}+4}\arctan\left(\frac{\sqrt{3}\left(18\,x^{5}-42\,x^{4}-10\,x^{3}-18\,x^{2}+2^{\frac{2}{3}}\left(2\,x^{5}+63\,x^{4}+15\,x^{3}-2\,x^{2}\right.\right.}{}\right.$$
$$-\frac{2}{3}\left(2i\cdot 2^{\frac{1}{3}}-3i\right)\text{weierstrassPInverse}(0,4,x)$$

input
```
integrate((2+3*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/9*sqrt(3)*sqrt(12*2^(2/3) + 18*2^(1/3) + 4)*arctan(1/348*sqrt(3)*(18*x^
5 - 42*x^4 - 10*x^3 - 18*x^2 + 2^(2/3)*(2*x^5 + 63*x^4 + 15*x^3 - 2*x^2 -
36*x - 6) - 2^(1/3)*(6*x^5 - 14*x^4 + 45*x^3 - 6*x^2 + 8*x - 18) + 24*x +
4)*sqrt(-x^3 + 1)*sqrt(12*2^(2/3) + 18*2^(1/3) + 4)/(2*x^6 - 3*x^3 + 1)) -
 2/3*(2*I*2^(1/3) - 3*I)*weierstrassPInverse(0, 4, x)
```

3.53.6 Sympy [F]

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = -\int \frac{3x}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx - \int \frac{2}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx$$

input
```
integrate((2+3*x)/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

output
```
-Integral(3*x/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x) - Integral(
2/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

3.53.7 Maxima [F]

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \int -\frac{3x+2}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((2+3*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((3*x + 2)/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

3.53.8 Giac [F(-2)]

Exception generated.

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2+3*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad
Argument
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \int -\frac{3\,x+2}{\sqrt{1-x^3}\,\left(x-2^{2/3}\right)}\,dx$$

input
```
int(-(3*x + 2)/((1 - x^3)^(1/2)*(x - 2^(2/3))),x)
```

output
```
int(-(3*x + 2)/((1 - x^3)^(1/2)*(x - 2^(2/3))), x)
```

3.54 $\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

3.54.1 Optimal result

Integrand size = 26, antiderivative size = 176

$$\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = -\frac{2\left(2+3\ 2^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}} + \frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
-2/9*(2+3*2^(2/3))*arctanh((1-2^(1/3)*x)*3^(1/2)/(x^3-1)^(1/2))*3^(1/2)+2/
9*(3-2*2^(1/3))*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))
*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-
1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

3.54.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.40 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.89

$$\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = \frac{2\sqrt[6]{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\left(-3i\sqrt{-i+\sqrt{3}-2ix}\left(-6i-3i\sqrt[3]{2}+2\sqrt{3}-\sqrt[3]{2}\sqrt{3}+\left(-3i\right.\right.\right.}{}$$

input
```
Integrate[(2 + 3*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

output
```
(2*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-3*I)*Sqrt[-I + Sqrt[3]
 - (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*
2^(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3
] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 4*Sqrt[3]*
(3 + 2^(1/3))*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*
Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]
/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^
(2/3) + Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[-1 + x^3])
```

3.54.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2564, 27, 760, 2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x+2}{\left(2^{2/3}-x\right)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{1}{3}\left(3+\sqrt[3]{2}\right)\int \frac{2^{2/3}\left(\sqrt[3]{2}x+1\right)}{\left(2^{2/3}-x\right)\sqrt{x^3-1}}dx-\frac{1}{3}\left(3-2\sqrt[3]{2}\right)\int \frac{1}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}2^{2/3}\left(3+\sqrt[3]{2}\right)\int \frac{\sqrt[3]{2}x+1}{\left(2^{2/3}-x\right)\sqrt{x^3-1}}dx-\frac{1}{3}\left(3-2\sqrt[3]{2}\right)\int \frac{1}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$\frac{1}{3}2^{2/3}\left(3+\sqrt[3]{2}\right)\int \frac{\sqrt[3]{2}x+1}{\left(2^{2/3}-x\right)\sqrt{x^3-1}}dx+\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{2562}$$

$$\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2}{3}2^{2/3}\left(3+\sqrt[3]{2}\right)\int \frac{1}{1-\frac{3\left(1-\sqrt[3]{2}x\right)^2}{x^3-1}}d\frac{1-\sqrt[3]{2}x}{\sqrt{x^3-1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\ 2^{2/3}\left(3+\sqrt[3]{2}\right)\text{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}$$

input
```
Int[(2 + 3*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

output
```
(-2*2^(2/3)*(3 + 2^(1/3))*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]
])/(3*Sqrt[3]) + (2*(3 - 2*2^(1/3))*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x
+ x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3
] - x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*
Sqrt[-1 + x^3])
```

3.54. $\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

3.54.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.54.4 Maple [A] (verified)

Time = 5.35 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.51

method	result
default	$-\frac{6\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\left(-2-3\,2^{\frac{2}{3}}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+}{}}}{\sqrt{}}$
elliptic	$-\frac{6\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\left(-2-3\,2^{\frac{2}{3}}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+}{}}}{\sqrt{}}$

input
```
int((3*x+2)/(2^(2/3)-x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-6*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/
2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*(-2-3*2^(2/3))*(-3/2-1/2*I*
3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/
2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3
-1)^(1/2)/(-2^(2/3)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+
1/2*I*3^(1/2))/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2
))
```

3.54.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2+3*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   catd
ef: division by zero
```

3.54.6 Sympy [F]

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = -\int \frac{3x}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx - \int \frac{2}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

input
```
integrate((2+3*x)/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

output
```
-Integral(3*x/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x) - Integral(
2/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

3.54.7 Maxima [F]

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \int -\frac{3x+2}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((2+3*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((3*x + 2)/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

3.54.8 Giac [F(-2)]

Exception generated.

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((2+3*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Ba
d Argumen
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx=\int -\frac{3\,x+2}{\sqrt{x^3-1}\,\left(x-2^{2/3}\right)}\,dx$$

input
```
int(-(3*x + 2)/((x^3 - 1)^(1/2)*(x - 2^(2/3))),x)
```

output
```
int(-(3*x + 2)/((x^3 - 1)^(1/2)*(x - 2^(2/3))), x)
```

3.55 $\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

3.55.1 Optimal result

Integrand size = 26, antiderivative size = 169

$$\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{2\left(2-3\ 2^{2/3}\right)\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}} + \frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
2/9*(2-3*2^(2/3))*arctanh((1+2^(1/3)*x)*3^(1/2)/(-x^3-1)^(1/2))*3^(1/2)+2/
9*(3+2*2^(1/3))*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))
*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3
-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

3.55.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.36 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.00

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \frac{2\sqrt[6]{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(3\sqrt{-i+\sqrt{3}+2ix}\left(-6-3\sqrt[3]{2}-2i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}+\left(3\sqrt[3]{2}+4i\right.\right.\right.}{}$$

input
```
Integrate[(2 + 3*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(3*Sqrt[-I + Sqrt[3] + (2*I)*
x]*(-6 - 3*2^(1/3) - (2*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3] + (3*2^(1/3) + (4*I
)*Sqrt[3] + I*2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I
)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - 4*Sqrt[3]*(-3 + 2^
(1/3))*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3]
)/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[
2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) +
 Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[-1 - x^3])
```

3.55.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2564, 760, 2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x+2}{\left(x+2^{2/3}\right)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{ 2564}$$

$$\frac{1}{3}\left(3+2\sqrt[3]{2}\right)\int \frac{1}{\sqrt{-x^3-1}}dx-\frac{1}{3}\left(3-\sqrt[3]{2}\right)\int \frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{ 760}$$

$$\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{1}{3}\left(3-\sqrt[3]{2}\right)\int\frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \; 2562$$

$$\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2}{3}2^{2/3}\left(3-\sqrt[3]{2}\right)\int\frac{1}{1-\frac{3\left(\sqrt[3]{2}x+1\right)^2}{-x^3-1}}d\frac{\sqrt[3]{2}x+1}{\sqrt{-x^3-1}}$$

$$\Big\downarrow \; 219$$

$$\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\,2^{2/3}\left(3-\sqrt[3]{2}\right)\text{arctanh}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}$$

input
```
Int[(2 + 3*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

output
```
(-2*2^(2/3)*(3 - 2^(1/3))*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]
])/(3*Sqrt[3]) + (2*(3 + 2*2^(1/3))*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x
+ x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3
] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*
Sqrt[-1 - x^3])
```

3.55. $\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

3.55.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.55.4 Maple [A] (verified)

Time = 4.91 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.50

method	result
default	$-\frac{2 i \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, F\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{\sqrt{-x^{3}-1}} - \frac{2 i \left(2-3\, 2^{\frac{2}{3}}\right) \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}- \cdots\right.}}{\cdots}$
elliptic	$-\frac{2 i \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, F\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{\sqrt{-x^{3}-1}} - \frac{2 i \left(2-3\, 2^{\frac{2}{3}}\right) \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}- \cdots\right.}}{\cdots}$

input
```
int((3*x+2)/(2^(2/3)+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1
/2*I*3^(1/2)))^(1/2))-2/3*I*(2-3*2^(2/3))*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))
*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2)
)*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(2^(2/3)+1/2+1/2*I*3^(1/2))*EllipticPi(1/3
*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(2^(2/3)+1/2+1/
2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

3.55.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.54

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \frac{1}{18}\sqrt{3}\sqrt{-12\cdot 2^{\frac{2}{3}}+18\cdot 2^{\frac{1}{3}}+4}\log\left(\frac{25\,x^{18}-36000\,x^{15}+435000\,x^{12}+52640\cdots}{\cdots}\right) - \frac{2}{3}\left(2i\cdot 2^{\frac{1}{3}}+3i\right)\text{weierstrassPInverse}(0,-4,x)$$

input
```
integrate((2+3*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/18*sqrt(3)*sqrt(-12*2^(2/3) + 18*2^(1/3) + 4)*log((25*x^18 - 36000*x^15
+ 435000*x^12 + 526400*x^9 - 259200*x^6 - 384000*x^3 + 2*sqrt(3)*(6*x^16 -
 34*x^15 + 1134*x^14 - 1860*x^13 + 2116*x^12 - 23976*x^11 + 13992*x^10 - 5
056*x^9 + 15936*x^7 - 10816*x^6 + 41472*x^5 - 1536*x^4 - 5120*x^3 + 20736*
x^2 + 3*2^(2/3)*(3*x^16 - 17*x^15 + 42*x^14 - 930*x^13 + 1058*x^12 - 888*x
^11 + 6996*x^10 - 2528*x^9 + 7968*x^7 - 5408*x^6 + 1536*x^5 - 768*x^4 - 25
60*x^3 + 768*x^2 - 1536*x - 512) + 2^(1/3)*(2*x^16 - 153*x^15 + 378*x^14 -
 620*x^13 + 9522*x^12 - 7992*x^11 + 4664*x^10 - 22752*x^9 + 5312*x^7 - 486
72*x^6 + 13824*x^5 - 512*x^4 - 23040*x^3 + 6912*x^2 - 1024*x - 4608) - 307
2*x - 1024)*sqrt(-x^3 - 1)*sqrt(-12*2^(2/3) + 18*2^(1/3) + 4) - 600*2^(2/3
)*(x^17 - 121*x^14 + 478*x^11 + 1144*x^8 + 608*x^5 + 64*x^2) + 1200*2^(1/3
)*(5*x^16 - 176*x^13 + 83*x^10 + 680*x^7 + 544*x^4 + 128*x) - 51200)/(x^18
 + 24*x^15 + 240*x^12 + 1280*x^9 + 3840*x^6 + 6144*x^3 + 4096)) - 2/3*(2*I
*2^(1/3) + 3*I)*weierstrassPInverse(0, -4, x)
```

3.55.6 Sympy [F]

$$\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx=\int \frac{3x+2}{\sqrt{-\left(x+1\right)\left(x^2-x+1\right)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((2+3*x)/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

output
```
Integral((3*x + 2)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

3.55.7 Maxima [F]

$$\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx=\int \frac{3\,x+2}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((2+3*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((3*x + 2)/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

3.55.8 Giac [F]

$$\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx=\int \frac{3\,x+2}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((2+3*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((3*x + 2)/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx=\int \frac{3\,x+2}{\sqrt{-x^3-1}\,\left(x+2^{2/3}\right)} \,dx$$

input
```
int((3*x + 2)/((- x^3 - 1)^(1/2)*(x + 2^(2/3))),x)
```

output
```
int((3*x + 2)/((- x^3 - 1)^(1/2)*(x + 2^(2/3))), x)
```

3.56 $\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

3.56.1 Optimal result

Integrand size = 24, antiderivative size = 159

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \frac{2\left(e-2^{2/3}f\right)\arctan\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}e+f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
2/9*(e-2^(2/3)*f)*arctan((1+2^(1/3)*x)*3^(1/2)/(x^3+1)^(1/2))*3^(1/2)+2/9*
(2^(1/3)*e+f)*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(
1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)
^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.56.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.37 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.14

$$\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \frac{2\sqrt[6]{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(f\sqrt{-i+\sqrt{3}+2ix}\left(-6-3\sqrt[3]{2}-2i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}+\left(3\sqrt[3]{2}+4i\sqrt{3}\right.\right.\right.}{}$$

input
```
Integrate[(e + f*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

output
```
(2*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(f*Sqrt[-I + Sqrt[3] + (2*I)*
x]*(-6 - 3*2^(1/3) - (2*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3] + (3*2^(1/3) + (4*I
)*Sqrt[3] + I*2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I
)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - 2*Sqrt[3]*(2^(1/3)
*e - 2*f)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt
[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sq
rt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3
) + Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x^3])
```

3.56.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2564, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(x+2^{2/3}\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{1}{3}\left(\sqrt[3]{2}e+f\right)\int \frac{1}{\sqrt{x^3+1}}dx+\frac{1}{6}\left(\sqrt[3]{2}e-2f\right)\int \frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{759}$$

$$\frac{1}{6}\left(\sqrt[3]{2}e-2f\right)\int \frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{x^3+1}}dx+$$
$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e+f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\downarrow$ 2562

$$\frac{1}{3}2^{2/3}\left(\sqrt[3]{2}e-2f\right)\int \frac{1}{\frac{3\left(\sqrt[3]{2}x+1\right)^2}{x^3+1}+1}d\frac{\sqrt[3]{2}x+1}{\sqrt{x^3+1}}+$$
$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e+f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\downarrow$ 216

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e+f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+$$
$$\frac{2^{2/3}\arctan\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)\left(\sqrt[3]{2}e-2f\right)}{3\sqrt{3}}$$

input
```
Int[(e + f*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

output
```
(2^(2/3)*(2^(1/3)*e - 2*f)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]]
)/(3*Sqrt[3]) + (2*Sqrt[2 + Sqrt[3]]*(2^(1/3)*e + f)*(1 + x)*Sqrt[(1 - x +
 x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3]
 + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt
[1 + x^3])
```

3.56. $\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

3.56.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.56.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(126) = 252$.

Time = 1.96 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.66

method	result
default	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}} + \frac{2\left(e-2^{\frac{2}{3}}f\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\cdots}}{\sqrt{x^3+1}}$
elliptic	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}} + \frac{2\left(e-2^{\frac{2}{3}}f\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\cdots}}{\sqrt{x^3+1}}$

input
```
int((f*x+e)/(2^(2/3)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*f*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^
(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3
/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(e-2^(2/3)*f)*(3/2-1/2*I*
3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/
2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^
3+1)^(1/2)/(2^(2/3)-1)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+
1/2*I*3^(1/2))/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/
2))
```

3.56.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 992, normalized size of antiderivative = 6.24

$$\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
[1/18*sqrt(3)*sqrt(2*2^(2/3)*e*f - 2*2^(1/3)*f^2 - e^2)*log(-((e^3 - 4*f^3
)*x^18 - 1440*(e^3 - 4*f^3)*x^15 + 17400*(e^3 - 4*f^3)*x^12 + 21056*(e^3 -
 4*f^3)*x^9 - 10368*(e^3 - 4*f^3)*x^6 - 15360*(e^3 - 4*f^3)*x^3 - 2048*e^3
 + 8192*f^3 - 4*sqrt(3)*(2*e*f*x^16 - 17*e^2*x^15 + 252*f^2*x^14 - 620*e*f
*x^13 + 1058*e^2*x^12 - 5328*f^2*x^11 + 4664*e*f*x^10 - 2528*e^2*x^9 + 531
2*e*f*x^7 - 5408*e^2*x^6 + 9216*f^2*x^5 - 512*e*f*x^4 - 2560*e^2*x^3 + 460
8*f^2*x^2 - 1024*e*f*x - 512*e^2 + 2^(2/3)*(2*f^2*x^16 - 17*e*f*x^15 + 63*
e^2*x^14 - 620*f^2*x^13 + 1058*e*f*x^12 - 1332*e^2*x^11 + 4664*f^2*x^10 -
2528*e*f*x^9 + 5312*f^2*x^7 - 5408*e*f*x^6 + 2304*e^2*x^5 - 512*f^2*x^4 -
2560*e*f*x^3 + 1152*e^2*x^2 - 1024*f^2*x - 512*e*f) + 2^(1/3)*(e^2*x^16 -
34*f^2*x^15 + 126*e*f*x^14 - 310*e^2*x^13 + 2116*f^2*x^12 - 2664*e*f*x^11
+ 2332*e^2*x^10 - 5056*f^2*x^9 + 2656*e^2*x^7 - 10816*f^2*x^6 + 4608*e*f*x
^5 - 256*e^2*x^4 - 5120*f^2*x^3 + 2304*e*f*x^2 - 512*e^2*x - 1024*f^2))*sq
rt(x^3 + 1)*sqrt(2*2^(2/3)*e*f - 2*2^(1/3)*f^2 - e^2) - 24*2^(2/3)*((e^3 -
 4*f^3)*x^17 - 121*(e^3 - 4*f^3)*x^14 + 478*(e^3 - 4*f^3)*x^11 + 1144*(e^3
 - 4*f^3)*x^8 + 608*(e^3 - 4*f^3)*x^5 + 64*(e^3 - 4*f^3)*x^2) + 48*2^(1/3)
*(5*(e^3 - 4*f^3)*x^16 - 176*(e^3 - 4*f^3)*x^13 + 83*(e^3 - 4*f^3)*x^10 +
680*(e^3 - 4*f^3)*x^7 + 544*(e^3 - 4*f^3)*x^4 + 128*(e^3 - 4*f^3)*x))/(x^1
8 + 24*x^15 + 240*x^12 + 1280*x^9 + 3840*x^6 + 6144*x^3 + 4096)) + 2/3*(2^
(1/3)*e + f)*weierstrassPInverse(0, -4, x), 1/9*sqrt(3)*sqrt(-2*2^(2/3)...
```

3.56.6 Sympy [F]

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx=\int \frac{e+fx}{\sqrt{(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((f*x+e)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

3.56.7 Maxima [F]

$$\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx=\int \frac{fx+e}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((f*x+e)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

3.56.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((f*x+e)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Ba
d Argumen
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx=\int \frac{e+f\,x}{\sqrt{x^3+1}\,\left(x+2^{2/3}\right)}\,dx$$

input
```
int((e + f*x)/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
```

output
```
int((e + f*x)/((x^3 + 1)^(1/2)*(x + 2^(2/3))), x)
```

3.57 $\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

3.57.1 Optimal result

Integrand size = 28, antiderivative size = 175

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = -\frac{2\left(e+2^{2/3}f\right)\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}} - \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}e-f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-2/9*(e+2^(2/3)*f)*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*3^(1/2)-2/
9*(2^(1/3)*e-f)*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)
*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3
+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

3.57.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.42 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.94

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \frac{2\sqrt[6]{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\left(-if\sqrt{-i+\sqrt{3}-2ix}\left(-6i-3i\sqrt[3]{2}+2\sqrt{3}-\sqrt[3]{2}\sqrt{3}+\left(-3i\sqrt[3]{\right.\right.\right.}{}$$

input
```
Integrate[(e + f*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

output
```
(2*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-I)*f*Sqrt[-I + Sqrt[3]
 - (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*
2^(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3
] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 2*Sqrt[3]*
(2^(1/3)*e + 2*f)*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi
[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I
)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I
)*2^(2/3) + Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x^3])
```

3.57.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2564, 27, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{1}{3}\left(\sqrt[3]{2}e-f\right)\int \frac{1}{\sqrt{1-x^3}}dx+\frac{1}{6}\left(\sqrt[3]{2}e+2f\right)\int \frac{2^{2/3}\left(\sqrt[3]{2}x+1\right)}{(2^{2/3}-x)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\sqrt[3]{2}e-f\right)\int \frac{1}{\sqrt{1-x^3}}dx+\frac{\left(\sqrt[3]{2}e+2f\right)\int \frac{\sqrt[3]{2}x+1}{(2^{2/3}-x)\sqrt{1-x^3}}dx}{3\sqrt[3]{2}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\left(\sqrt[3]{2}e+2f\right)\int \frac{\sqrt[3]{2}x+1}{(2^{2/3}-x)\sqrt{1-x^3}}dx}{3\sqrt[3]{2}}-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e-f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \text{2562}$$

$$-\frac{1}{3}2^{2/3}\left(\sqrt[3]{2}e+2f\right)\int \frac{1}{\frac{3\left(1-\sqrt[3]{2}x\right)^2}{1-x^3}+1}d\frac{1-\sqrt[3]{2}x}{\sqrt{1-x^3}}-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e-f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e-f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2^{2/3}\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)\left(\sqrt[3]{2}e+2f\right)}{3\sqrt{3}}$$

input
```
Int[(e + f*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

output
```
-1/3*(2^(2/3)*(2^(1/3)*e + 2*f)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 -
x^3]])/Sqrt[3] - (2*Sqrt[2 + Sqrt[3]]*(2^(1/3)*e - f)*(1 - x)*Sqrt[(1 + x
+ x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3
] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqr
t[1 - x^3])
```

3.57. $\int \frac{e+fx}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx$

3.57.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.57.4 Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.49

method	result
default	$\frac{2 i f \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, F\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}+1}}-\frac{2 i \left(-e-2^{\frac{2}{3}} f\right) \sqrt{3}\, \sqrt{i \left(x+\right.}}{}$
elliptic	$\frac{2 i f \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, F\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}+1}}-\frac{2 i \left(-e-2^{\frac{2}{3}} f\right) \sqrt{3}\, \sqrt{i \left(x+\right.}}{}$

input
```
int((f*x+e)/(2^(2/3)-x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*f*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I
*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-
3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(-e-2^(2/3)*f)*3^(1/2)*(I*(x+1/2-1/2*I*3^
(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*
3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-2^(2/3)-1/2+1/2*I*3^(1/2))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-2^(2/
3)-1/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

3.57.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.22 (sec) , antiderivative size = 1005, normalized size of antiderivative = 5.74

$$\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
[1/18*sqrt(3)*sqrt(-2*2^(2/3)*e*f - 2*2^(1/3)*f^2 - e^2)*log(((e^3 + 4*f^3
)*x^18 + 1440*(e^3 + 4*f^3)*x^15 + 17400*(e^3 + 4*f^3)*x^12 - 21056*(e^3 +
 4*f^3)*x^9 - 10368*(e^3 + 4*f^3)*x^6 + 15360*(e^3 + 4*f^3)*x^3 - 2048*e^3
 - 8192*f^3 - 4*sqrt(3)*(2*e*f*x^16 - 17*e^2*x^15 - 252*f^2*x^14 + 620*e*f
*x^13 - 1058*e^2*x^12 - 5328*f^2*x^11 + 4664*e*f*x^10 - 2528*e^2*x^9 - 531
2*e*f*x^7 + 5408*e^2*x^6 + 9216*f^2*x^5 - 512*e*f*x^4 - 2560*e^2*x^3 - 460
8*f^2*x^2 + 1024*e*f*x + 512*e^2 - 2^(2/3)*(2*f^2*x^16 - 17*e*f*x^15 + 63*
e^2*x^14 + 620*f^2*x^13 - 1058*e*f*x^12 + 1332*e^2*x^11 + 4664*f^2*x^10 -
2528*e*f*x^9 - 5312*f^2*x^7 + 5408*e*f*x^6 - 2304*e^2*x^5 - 512*f^2*x^4 -
2560*e*f*x^3 + 1152*e^2*x^2 + 1024*f^2*x + 512*e*f) - 2^(1/3)*(e^2*x^16 +
34*f^2*x^15 - 126*e*f*x^14 + 310*e^2*x^13 + 2116*f^2*x^12 - 2664*e*f*x^11
+ 2332*e^2*x^10 + 5056*f^2*x^9 - 2656*e^2*x^7 - 10816*f^2*x^6 + 4608*e*f*x
^5 - 256*e^2*x^4 + 5120*f^2*x^3 - 2304*e*f*x^2 + 512*e^2*x - 1024*f^2))*sq
rt(-x^3 + 1)*sqrt(-2*2^(2/3)*e*f - 2*2^(1/3)*f^2 - e^2) + 24*2^(2/3)*((e^3
 + 4*f^3)*x^17 + 121*(e^3 + 4*f^3)*x^14 + 478*(e^3 + 4*f^3)*x^11 - 1144*(e
^3 + 4*f^3)*x^8 + 608*(e^3 + 4*f^3)*x^5 - 64*(e^3 + 4*f^3)*x^2) + 48*2^(1/
3)*(5*(e^3 + 4*f^3)*x^16 + 176*(e^3 + 4*f^3)*x^13 + 83*(e^3 + 4*f^3)*x^10
- 680*(e^3 + 4*f^3)*x^7 + 544*(e^3 + 4*f^3)*x^4 - 128*(e^3 + 4*f^3)*x))/(x
^18 - 24*x^15 + 240*x^12 - 1280*x^9 + 3840*x^6 - 6144*x^3 + 4096)) - 2/3*(
I*2^(1/3)*e - I*f)*weierstrassPInverse(0, 4, x), -1/9*sqrt(3)*sqrt(2*2^...
```

3.57.6 Sympy [F]

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = -\int \frac{e}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx - \int \frac{fx}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx$$

input
```
integrate((f*x+e)/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

output
```
-Integral(e/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x) - Integral(f*
x/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

3.57.7 Maxima [F]

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \int -\frac{fx+e}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((f*x+e)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((f*x + e)/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

3.57.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x+e)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad
Argument
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \int -\frac{e+f\,x}{\sqrt{1-x^3}\,(x-2^{2/3})}\,dx$$

input
```
int(-(e + f*x)/((1 - x^3)^(1/2)*(x - 2^(2/3))),x)
```

output
```
int(-(e + f*x)/((1 - x^3)^(1/2)*(x - 2^(2/3))), x)
```

3.58 $\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

3.58.1 Optimal result

Integrand size = 26, antiderivative size = 178

$$\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx=-\frac{2\left(e+2^{2/3}f\right)\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}e-f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
-2/9*(e+2^(2/3)*f)*arctanh((1-2^(1/3)*x)*3^(1/2)/(x^3-1)^(1/2))*3^(1/2)-2/
9*(2^(1/3)*e-f)*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))
*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-
1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

3.58.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.42 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.90

$$\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = \frac{2\sqrt[6]{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\left(-if\sqrt{-i+\sqrt{3}-2ix}\left(-6i-3i\sqrt[3]{2}+2\sqrt{3}-\sqrt[3]{2}\sqrt{3}+\left(-3i\right.\right.\right.}{}$$

input
```
Integrate[(e + f*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

output
```
(2*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-I)*f*Sqrt[-I + Sqrt[3]
 - (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*
2^(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3
] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 2*Sqrt[3]*
(2^(1/3)*e + 2*f)*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi
[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I
)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I
)*2^(2/3) + Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[-1 + x^3])
```

3.58.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2564, 27, 760, 2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \; 2564$$

$$\frac{1}{3}\left(\sqrt[3]{2}e-f\right)\int \frac{1}{\sqrt{x^3-1}}dx+\frac{1}{6}\left(\sqrt[3]{2}e+2f\right)\int \frac{2^{2/3}\left(\sqrt[3]{2}x+1\right)}{\left(2^{2/3}-x\right)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(\sqrt[3]{2}e-f\right)\int \frac{1}{\sqrt{x^3-1}}dx+\frac{\left(\sqrt[3]{2}e+2f\right)\int \frac{\sqrt[3]{2}x+1}{\left(2^{2/3}-x\right)\sqrt{x^3-1}}dx}{3\sqrt[3]{2}}$$

$$\Big\downarrow \text{760}$$

$$\frac{\left(\sqrt[3]{2}e+2f\right)\int \frac{\sqrt[3]{2}x+1}{(2^{2/3}-x)\sqrt{x^3-1}}dx}{3\sqrt[3]{2}}-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e-f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{2562}$$

$$-\frac{1}{3}2^{2/3}\left(\sqrt[3]{2}e+2f\right)\int \frac{1}{1-\frac{3\left(1-\sqrt[3]{2}x\right)^2}{x^3-1}}d\frac{1-\sqrt[3]{2}x}{\sqrt{x^3-1}}-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e-f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e-f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)\left(\sqrt[3]{2}e+2f\right)}{3\sqrt{3}}$$

input
```
Int[(e + f*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

output
```
-1/3*(2^(2/3)*(2^(1/3)*e + 2*f)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1
+ x^3]])/Sqrt[3] - (2*Sqrt[2 - Sqrt[3]]*(2^(1/3)*e - f)*(1 - x)*Sqrt[(1 +
x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt
[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)
]*Sqrt[-1 + x^3])
```

3.58. $\int \frac{e+fx}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx$

3.58.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.58.4 Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.52

method	result
default	$-\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\left(-e-2^{\frac{2}{3}}f\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{x\cdots}}{\cdots}$
elliptic	$-\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\left(-e-2^{\frac{2}{3}}f\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{x\cdots}}{\cdots}$

input
```
int((f*x+e)/(2^(2/3)-x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*f*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I
*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(
1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((
3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*(-e-2^(2/3)*f)*(-3/2-1/2*
I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-
1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x
^3-1)^(1/2)/(-2^(2/3)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/
2+1/2*I*3^(1/2))/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1
/2))
```

3.58.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.24 (sec) , antiderivative size = 999, normalized size of antiderivative = 5.61

$$\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
[1/18*sqrt(3)*sqrt(2*2^(2/3)*e*f + 2*2^(1/3)*f^2 + e^2)*log(((e^3 + 4*f^3)
*x^18 + 1440*(e^3 + 4*f^3)*x^15 + 17400*(e^3 + 4*f^3)*x^12 - 21056*(e^3 +
4*f^3)*x^9 - 10368*(e^3 + 4*f^3)*x^6 + 15360*(e^3 + 4*f^3)*x^3 - 2048*e^3
- 8192*f^3 - 4*sqrt(3)*(2*e*f*x^16 - 17*e^2*x^15 - 252*f^2*x^14 + 620*e*f*
x^13 - 1058*e^2*x^12 - 5328*f^2*x^11 + 4664*e*f*x^10 - 2528*e^2*x^9 - 5312
*e*f*x^7 + 5408*e^2*x^6 + 9216*f^2*x^5 - 512*e*f*x^4 - 2560*e^2*x^3 - 4608
*f^2*x^2 + 1024*e*f*x + 512*e^2 - 2^(2/3)*(2*f^2*x^16 - 17*e*f*x^15 + 63*e
^2*x^14 + 620*f^2*x^13 - 1058*e*f*x^12 + 1332*e^2*x^11 + 4664*f^2*x^10 - 2
528*e*f*x^9 - 5312*f^2*x^7 + 5408*e*f*x^6 - 2304*e^2*x^5 - 512*f^2*x^4 - 2
560*e*f*x^3 + 1152*e^2*x^2 + 1024*f^2*x + 512*e*f) - 2^(1/3)*(e^2*x^16 + 3
4*f^2*x^15 - 126*e*f*x^14 + 310*e^2*x^13 + 2116*f^2*x^12 - 2664*e*f*x^11 +
 2332*e^2*x^10 + 5056*f^2*x^9 - 2656*e^2*x^7 - 10816*f^2*x^6 + 4608*e*f*x^
5 - 256*e^2*x^4 + 5120*f^2*x^3 - 2304*e*f*x^2 + 512*e^2*x - 1024*f^2))*sqr
t(x^3 - 1)*sqrt(2*2^(2/3)*e*f + 2*2^(1/3)*f^2 + e^2) + 24*2^(2/3)*((e^3 +
4*f^3)*x^17 + 121*(e^3 + 4*f^3)*x^14 + 478*(e^3 + 4*f^3)*x^11 - 1144*(e^3
+ 4*f^3)*x^8 + 608*(e^3 + 4*f^3)*x^5 - 64*(e^3 + 4*f^3)*x^2) + 48*2^(1/3)*
(5*(e^3 + 4*f^3)*x^16 + 176*(e^3 + 4*f^3)*x^13 + 83*(e^3 + 4*f^3)*x^10 - 6
80*(e^3 + 4*f^3)*x^7 + 544*(e^3 + 4*f^3)*x^4 - 128*(e^3 + 4*f^3)*x))/(x^18
 - 24*x^15 + 240*x^12 - 1280*x^9 + 3840*x^6 - 6144*x^3 + 4096)) + 2/3*(2^(
1/3)*e - f)*weierstrassPInverse(0, 4, x), -1/9*sqrt(3)*sqrt(-2*2^(2/3)*...
```

3.58.6 Sympy [F]

$$\int \frac{e+fx}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = -\int \frac{e}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx - \int \frac{fx}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

input
```
integrate((f*x+e)/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

output
```
-Integral(e/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x) - Integral(f*
x/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

3.58.7 Maxima [F]

$$\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx=\int -\frac{fx+e}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((f*x+e)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((f*x + e)/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

3.58.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((f*x+e)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad
Argument
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx=\int -\frac{e+f\,x}{\sqrt{x^3-1}\,\left(x-2^{2/3}\right)}\,dx$$

input
```
int(-(e + f*x)/((x^3 - 1)^(1/2)*(x - 2^(2/3))),x)
```

output
```
int(-(e + f*x)/((x^3 - 1)^(1/2)*(x - 2^(2/3))), x)
```

3.59 $\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

3.59.1 Optimal result

Integrand size = 26, antiderivative size = 170

$$\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{2\left(e-2^{2/3}f\right)\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}e+f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
2/9*(e-2^(2/3)*f)*arctanh((1+2^(1/3)*x)*3^(1/2)/(-x^3-1)^(1/2))*3^(1/2)+2/
9*(2^(1/3)*e+f)*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))
*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3
-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

3.59.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.39 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.01

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \frac{2\sqrt[6]{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(f\sqrt{-i+\sqrt{3}+2ix}\left(-6-3\sqrt[3]{2}-2i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}+\left(3\sqrt[3]{2}+4i\right.\right.\right.}{}$$

input
```
Integrate[(e + f*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(f*Sqrt[-I + Sqrt[3] + (2*I)*
x]*(-6 - 3*2^(1/3) - (2*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3] + (3*2^(1/3) + (4*I
)*Sqrt[3] + I*2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I
)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - 2*Sqrt[3]*(2^(1/3)
*e - 2*f)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt
[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sq
rt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3
) + Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[-1 - x^3])
```

3.59.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2564, 760, 2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(x+2^{2/3}\right)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{1}{3}\left(\sqrt[3]{2}e+f\right)\int \frac{1}{\sqrt{-x^3-1}}dx+\frac{1}{6}\left(\sqrt[3]{2}e-2f\right)\int \frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$\frac{1}{6}\left(\sqrt[3]{2}e-2f\right)\int \frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{-x^3-1}}dx+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e+f\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{2562}$$

$$\frac{1}{3}2^{2/3}\left(\sqrt[3]{2}e-2f\right)\int \frac{1}{1-\frac{3\left(\sqrt[3]{2}x+1\right)^2}{-x^3-1}}d\frac{\sqrt[3]{2}x+1}{\sqrt{-x^3-1}}+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e+f\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e+f\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)\left(\sqrt[3]{2}e-2f\right)}{3\sqrt{3}}$$

input
```
Int[(e + f*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

output
```
(2^(2/3)*(2^(1/3)*e - 2*f)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3
]])/(3*Sqrt[3]) + (2*Sqrt[2 - Sqrt[3]]*(2^(1/3)*e + f)*(1 + x)*Sqrt[(1 - x
 + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[
3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]
*Sqrt[-1 - x^3])
```

3.59. $\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

3.59.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.59.4 Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.50

method	result
default	$-\frac{2 i f \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, F\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}-1}}-\frac{2 i \left(e-2^{\frac{2}{3}} f\right) \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\right.}}{}$
elliptic	$-\frac{2 i f \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, F\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}-1}}-\frac{2 i \left(e-2^{\frac{2}{3}} f\right) \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\right.}}{}$

input
```
int((f*x+e)/(2^(2/3)+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*f*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I
*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3
/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(e-2^(2/3)*f)*3^(1/2)*(I*(x-1/2-1/2*I*3^(1
/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(
1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(2^(2/3)+1/2+1/2*I*3^(1/2))*EllipticPi
(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(2^(2/3)+1/
2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

3.59.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.22 (sec) , antiderivative size = 1002, normalized size of antiderivative = 5.89

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
[1/18*sqrt(3)*sqrt(-2*2^(2/3)*e*f + 2*2^(1/3)*f^2 + e^2)*log(-((e^3 - 4*f^
3)*x^18 - 1440*(e^3 - 4*f^3)*x^15 + 17400*(e^3 - 4*f^3)*x^12 + 21056*(e^3
- 4*f^3)*x^9 - 10368*(e^3 - 4*f^3)*x^6 - 15360*(e^3 - 4*f^3)*x^3 - 2048*e^
3 + 8192*f^3 - 4*sqrt(3)*(2*e*f*x^16 - 17*e^2*x^15 + 252*f^2*x^14 - 620*e*
f*x^13 + 1058*e^2*x^12 - 5328*f^2*x^11 + 4664*e*f*x^10 - 2528*e^2*x^9 + 53
12*e*f*x^7 - 5408*e^2*x^6 + 9216*f^2*x^5 - 512*e*f*x^4 - 2560*e^2*x^3 + 46
08*f^2*x^2 - 1024*e*f*x - 512*e^2 + 2^(2/3)*(2*f^2*x^16 - 17*e*f*x^15 + 63
*e^2*x^14 - 620*f^2*x^13 + 1058*e*f*x^12 - 1332*e^2*x^11 + 4664*f^2*x^10 -
 2528*e*f*x^9 + 5312*f^2*x^7 - 5408*e*f*x^6 + 2304*e^2*x^5 - 512*f^2*x^4 -
 2560*e*f*x^3 + 1152*e^2*x^2 - 1024*f^2*x - 512*e*f) + 2^(1/3)*(e^2*x^16 -
 34*f^2*x^15 + 126*e*f*x^14 - 310*e^2*x^13 + 2116*f^2*x^12 - 2664*e*f*x^11
 + 2332*e^2*x^10 - 5056*f^2*x^9 + 2656*e^2*x^7 - 10816*f^2*x^6 + 4608*e*f*
x^5 - 256*e^2*x^4 - 5120*f^2*x^3 + 2304*e*f*x^2 - 512*e^2*x - 1024*f^2))*s
qrt(-x^3 - 1)*sqrt(-2*2^(2/3)*e*f + 2*2^(1/3)*f^2 + e^2) - 24*2^(2/3)*((e^
3 - 4*f^3)*x^17 - 121*(e^3 - 4*f^3)*x^14 + 478*(e^3 - 4*f^3)*x^11 + 1144*(
e^3 - 4*f^3)*x^8 + 608*(e^3 - 4*f^3)*x^5 + 64*(e^3 - 4*f^3)*x^2) + 48*2^(1
/3)*(5*(e^3 - 4*f^3)*x^16 - 176*(e^3 - 4*f^3)*x^13 + 83*(e^3 - 4*f^3)*x^10
 + 680*(e^3 - 4*f^3)*x^7 + 544*(e^3 - 4*f^3)*x^4 + 128*(e^3 - 4*f^3)*x))/(
x^18 + 24*x^15 + 240*x^12 + 1280*x^9 + 3840*x^6 + 6144*x^3 + 4096)) - 2/3*
(I*2^(1/3)*e + I*f)*weierstrassPInverse(0, -4, x), 1/9*sqrt(3)*sqrt(2*2...
```

3.59.6 Sympy [F]

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \int \frac{e+fx}{\sqrt{-(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((f*x+e)/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

3.59.7 Maxima [F]

$$\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx=\int \frac{fx+e}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate((f*x+e)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

3.59.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((f*x+e)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Ba
d Argumen
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx=\int \frac{e+f\,x}{\sqrt{-x^3-1}\,\left(x+2^{2/3}\right)}\,dx$$

input
```
int((e + f*x)/((- x^3 - 1)^(1/2)*(x + 2^(2/3))),x)
```

output
```
int((e + f*x)/((- x^3 - 1)^(1/2)*(x + 2^(2/3))), x)
```

3.60 $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

3.60.1 Optimal result

Integrand size = 38, antiderivative size = 316

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \frac{2\left(\sqrt[3]{b}e-2^{2/3}\sqrt[3]{a}f\right)\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}\sqrt[3]{b}e+\sqrt[3]{a}f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-\right.}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/9*(b^(1/3)*e-2^(2/3)*a^(1/3)*f)*arctan(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*
x)*3^(1/2)/(b*x^3+a)^(1/2))/b^(2/3)*3^(1/2)/a^(1/2)+2/9*(2^(1/3)*b^(1/3)*e
+a^(1/3)*f)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*(
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)
^(1/2)*3^(3/4)/a^(1/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x
)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.60.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.04 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.10

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\frac{3f\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\operatorname{EllipticF}\left(\right.}{\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}\right.}{}$$

input
```
Integrate[(e + f*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

output
```
(2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-3*f*((-1)^(1/
3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/(
(1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(
1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[(a^(1/3) + (-1)^(2
/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))] + ((-1)^(1/3)*(1 + (-1)^(1/3))*
(-(b^(1/3)*e) + 2^(2/3)*a^(1/3)*f)*Sqrt[3 - (3*b^(1/3)*x)/a^(1/3) + (3*b^(
2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[S
qrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1
/3)])/((-1)^(1/3) + 2^(2/3))))/(3*b^(2/3)*Sqrt[a + b*x^3])
```

3.60.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2564, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \; 2564$$

$$\frac{1}{3}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{1}{6}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a}\right)\sqrt{bx^3+a}}dx$$

$$\Big\downarrow \; 759$$

$$\frac{1}{6}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a}\right)\sqrt{bx^3+a}}dx+ \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 2562$$

$$\frac{2^{2/3}\sqrt[3]{a}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{1}{\frac{3\sqrt[3]{a}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{bx^3+a}+1}d\frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{bx^3+a}}}{3\sqrt[3]{b}}+ \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \; 216$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)}{3\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(e + f*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

3.60. $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}dx$

output
```
(2^(2/3)*((2^(1/3)*e)/a^(1/3) - (2*f)/b^(1/3))*ArcTan[(Sqrt[3]*a^(1/6)*(a^
(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a + b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(1/3))
+ (2*Sqrt[2 + Sqrt[3]]*((2^(1/3)*e)/a^(1/3) + f/b^(1/3))*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(1/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3])
```

3.60.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.60.4 Maple [F]

$$\int \frac{fx+e}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}+b^{\frac{1}{3}} x\right) \sqrt{b\,x^{3}+a}}dx$$

input
```
int((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output
```
int((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

3.60.5 Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

3.60.6 Sympy [F]

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \int \frac{e+fx}{\sqrt{a+bx^3}\cdot\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate((f*x+e)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt(a + b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

3.60.7 Maxima [F]

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{fx+e}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm
="maxima")
```

output
```
integrate((f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

3.60.8 Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm
="giac")
```

output
```
Timed out
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{e+f\,x}{\sqrt{b\,x^3+a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\,dx$$

input
```
int((e + f*x)/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
```

output
```
int((e + f*x)/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

3.61 $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

3.61.1 Optimal result

Integrand size = 40, antiderivative size = 324

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=-\frac{2\left(\sqrt[3]{b}e+2^{2/3}\sqrt[3]{a}f\right)\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}}$$
$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}\sqrt[3]{b}e-\sqrt[3]{a}f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),\ldots\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

output
```
-2/9*(b^(1/3)*e+2^(2/3)*a^(1/3)*f)*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)
*x)*3^(1/2)/(-b*x^3+a)^(1/2))/b^(2/3)*3^(1/2)/a^(1/2)-2/9*(2^(1/3)*b^(1/3)
*e-a^(1/3)*f)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)
))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2
))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)
))^2)^(1/2)*3^(3/4)/a^(1/3)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(
1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.61. $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

3.61.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.94 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.23

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx =$$

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}+2^{2/3}\right)f\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{\sqrt[3]{a}\cdots}{(1\cdots}}\right.\right.\right.}{\left(\sqrt[3]{-\cdots}\right.}$$

input
```
Integrate[(e + f*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
(-2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1)^(1/3) +
2^(2/3))*f*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-
1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^
(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] -
((-1)^(1/3)*(1 + (-1)^(1/3))*(b^(1/3)*e + 2^(2/3)*a^(1/3)*f)*Sqrt[(a^(1/3)
 - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/
a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2
/3)), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/
3))]], (-1)^(1/3)])/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3
) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a - b*x^3])
```

3.61.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2564, 27, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{1}{3}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a-bx^3}}dx+\frac{1}{6}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2^{2/3}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a-bx^3}}dx+\frac{\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx}{3\sqrt[3]{2}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx}{3\sqrt[3]{2}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\Big\downarrow \text{2562}$$

$$-\frac{2^{2/3}\sqrt[3]{a}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{1}{\frac{3\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)^2}{a-bx^3}+1}d\frac{\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{a-bx^3}}}{3\sqrt[3]{b}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right.}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\Big\downarrow \text{216}$$

3.61. $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx$

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\frac{2^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)}{3\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(e + f*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
-1/3*(2^(2/3)*((2^(1/3)*e)/a^(1/3) + (2*f)/b^(1/3))*ArcTan[(Sqrt[3]*a^(1/6
)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a - b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3
)) - (2*Sqrt[2 + Sqrt[3]]*((2^(1/3)*e)/a^(1/3) - f/b^(1/3))*(a^(1/3) - b^(
1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(1/3)
*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^
2]*Sqrt[a - b*x^3])
```

3.61.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.61.4 Maple [F]

$$\int \frac{fx+e}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}-b^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3}+a}}dx$$

input
```
int((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

output
```
int((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

3.61.5 Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorith
m="fricas")
```

output
```
Timed out
```

3.61.6 Sympy [F]

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=-\int \frac{e}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx - \int \frac{fx}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

input
```
integrate((f*x+e)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

output
```
-Integral(e/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x
**3)), x) - Integral(f*x/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x
*sqrt(a - b*x**3)), x)
```

3.61.7 Maxima [F]

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int -\frac{fx+e}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorith
m="maxima")
```

output
```
-integrate((f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

3.61.8 Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorith
m="giac")
```

output
```
Timed out
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{e+f\,x}{\sqrt{a-b\,x^3}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\,dx$$

input
```
int((e + f*x)/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
```

output
```
int((e + f*x)/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

3.62 $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

3.62.1 Optimal result

Integrand size = 41, antiderivative size = 333

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=-\frac{2\left(\sqrt[3]{b}e+2^{2/3}\sqrt[3]{a}f\right)\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}\sqrt[3]{b}e-\sqrt[3]{a}f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-\right.}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

output
```
-2/9*(b^(1/3)*e+2^(2/3)*a^(1/3)*f)*arctanh(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3
)*x)*3^(1/2)/(b*x^3-a)^(1/2))/b^(2/3)*3^(1/2)/a^(1/2)-2/9*(2^(1/3)*b^(1/3)
*e-a^(1/3)*f)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)
))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1
/2*2^(1/2))*3^(3/4)/a^(1/3)/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(
1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.62.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.36 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.20

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx =$$

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}+2^{2/3}\right)f\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{\sqrt[3]{a}\cdots}{(1\cdots}}\right.\right.\right.}{\left(\sqrt[3]{-\cdots}\right.}$$

input
```
Integrate[(e + f*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
(-2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1)^(1/3) +
2^(2/3))*f*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-
1)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^
(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] -
((-1)^(1/3)*(1 + (-1)^(1/3))*(b^(1/3)*e + 2^(2/3)*a^(1/3)*f)*Sqrt[(a^(1/3)
 - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/
a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2
/3)), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/
3))]], (-1)^(1/3)])/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3
) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3])
```

3.62.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {2564, 27, 760, 2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx$$

$\downarrow$ 2564

$$\frac{1}{3}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{bx^3-a}}dx+\frac{1}{6}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2^{2/3}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx$$

$\downarrow$ 27

$$\frac{1}{3}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{bx^3-a}}dx+\frac{\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx}{3\sqrt[3]{2}}$$

$\downarrow$ 760

$$\frac{\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx}{3\sqrt[3]{2}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$\downarrow$ 2562

$$-\frac{2^{2/3}\sqrt[3]{a}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{1}{1-\frac{3\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)^2}{bx^3-a}}d\frac{\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{bx^3-a}}}{3\sqrt[3]{b}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$\downarrow$ 219

3.62. $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}dx$

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\frac{2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)}{3\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(e + f*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
-1/3*(2^(2/3)*((2^(1/3)*e)/a^(1/3) + (2*f)/b^(1/3))*ArcTanh[(Sqrt[3]*a^(1/
6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[-a + b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1
/3)) - (2*Sqrt[2 - Sqrt[3]]*((2^(1/3)*e)/a^(1/3) - f/b^(1/3))*(a^(1/3) - b
^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*
a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*
x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(1/
3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)
*x)^2)]*Sqrt[-a + b*x^3])
```

3.62.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.62.4 Maple [F]

$$\int \frac{fx+e}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}-b^{\frac{1}{3}} x\right) \sqrt{b\,x^{3}-a}}dx$$

input
```
int((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output
```
int((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

3.62.5 Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

3.62.6 Sympy [F]

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\int \frac{e}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx - \int \frac{fx}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

input
```
integrate((f*x+e)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

output
```
-Integral(e/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b
*x**3)), x) - Integral(f*x/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3
)*x*sqrt(-a + b*x**3)), x)
```

3.62.7 Maxima [F]

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \int -\frac{fx+e}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm
="maxima")
```

output
```
-integrate((f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

3.62.8 Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm
="giac")
```

output
```
Timed out
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{e+f\,x}{\sqrt{b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\,dx$$

input
```
int((e + f*x)/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
```

output
```
int((e + f*x)/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

3.63 $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

3.63.1 Optimal result

Integrand size = 41, antiderivative size = 329

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \frac{2\left(\sqrt[3]{b}e-2^{2/3}\sqrt[3]{a}f\right)\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}} + \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}\sqrt[3]{b}e+\sqrt[3]{a}f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-\right.}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

output
```
2/9*(b^(1/3)*e-2^(2/3)*a^(1/3)*f)*arctanh(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)
*x)*3^(1/2)/(-b*x^3-a)^(1/2))/b^(2/3)*3^(1/2)/a^(1/2)+2/9*(2^(1/3)*b^(1/3)
*e+a^(1/3)*f)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)
*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*
2^(1/2))*3^(3/4)/a^(1/3)/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/
3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.63.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.85 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.22

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx =$$

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}+2^{2/3}\right)f\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{\sqrt[3]{a}+\cdots}{\left(1+\cdots\right.}}\right.\right.\right.}{\left(\sqrt[3]{-1}\cdots\right.}$$

input
```
Integrate[(e + f*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
(-2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1)^(1/3) +
2^(2/3))*f*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/3) - (-1
)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1
/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - ((
-1)^(1/3)*(1 + (-1)^(1/3))*(-(b^(1/3)*e) + 2^(2/3)*a^(1/3)*f)*Sqrt[(a^(1/3
) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)
/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(
2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1
/3))]], (-1)^(1/3)])/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/
3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a - b*x^3])
```

3.63.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {2564, 760, 2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}dx$$

$$\Big\downarrow \textbf{2564}$$

$$\frac{1}{3}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{-bx^3-a}}dx+\frac{1}{6}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a}\right)\sqrt{-bx^3-a}}dx$$

$$\Big\downarrow \textbf{760}$$

$$\frac{1}{6}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a}\right)\sqrt{-bx^3-a}}dx+\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\Big\downarrow \textbf{2562}$$

$$\frac{2^{2/3}\sqrt[3]{a}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{1}{1-\frac{3\sqrt[3]{a}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{-bx^3-a}}d\frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{-bx^3-a}}}{3\sqrt[3]{b}}+\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)\left(\frac{\sqrt[3]{2}e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)}{3\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

3.63. $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}dx$

input
```
Int[(e + f*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
(2^(2/3)*((2^(1/3)*e)/a^(1/3) - (2*f)/b^(1/3))*ArcTanh[(Sqrt[3]*a^(1/6)*(a
^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-a - b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(1/3)
) + (2*Sqrt[2 - Sqrt[3]]*((2^(1/3)*e)/a^(1/3) + f/b^(1/3))*(a^(1/3) + b^(1
/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(1/3)*
Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2)]*Sqrt[-a - b*x^3])
```

3.63.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.63.4 Maple [F]

$$\int \frac{fx+e}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}+b^{\frac{1}{3}} x\right)\sqrt{-b\,x^{3}-a}}dx$$

input
```
int((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

output
```
int((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

3.63.5 Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorith
m="fricas")
```

output
```
Timed out
```

3.63.6 Sympy [F]

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{e+fx}{\sqrt{-a-bx^3}\cdot\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate((f*x+e)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt(-a - b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x
)
```

3.63.7 Maxima [F]

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{fx+e}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

3.63.8 Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorith
m="giac")
```

output
```
Timed out
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{e+f\,x}{\sqrt{-b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\,dx$$

input
```
int((e + f*x)/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
```

output
```
int((e + f*x)/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

3.64 $\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

3.64.1 Optimal result

Integrand size = 29, antiderivative size = 265

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \frac{2(de-cf)\arctan\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}c^{3/2}d^2} + \frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}(2de+cf)\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\right.}{3\sqrt[4]{3}cd^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

output
```
2/9*(-c*f+d*e)*arctan((2*d*x+c)*3^(1/2)*c^(1/2)/(4*d^3*x^3+c^3)^(1/2))/c^(
3/2)/d^2*3^(1/2)+1/9*2^(1/3)*(c*f+2*d*e)*(c+2^(2/3)*d*x)*EllipticF((2^(2/3
)*d*x+c*(1-3^(1/2)))/(2^(2/3)*d*x+c*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/
2)+1/2*2^(1/2))*((c^2-2^(2/3)*c*d*x+2*2^(1/3)*d^2*x^2)/(2^(2/3)*d*x+c*(1+3
^(1/2)))^2)^(1/2)*3^(3/4)/c/d^2/(4*d^3*x^3+c^3)^(1/2)/(c*(c+2^(2/3)*d*x)/(
2^(2/3)*d*x+c*(1+3^(1/2)))^2)^(1/2)
```

3.64.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.07 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.43

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

$$= \frac{\sqrt[6]{2}\sqrt{\frac{\sqrt[3]{2}c+2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(-f\sqrt{\frac{\sqrt[3]{-2}c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(\sqrt[3]{-1}\left(2+\sqrt[3]{-2}\right)c-2\left(\sqrt[3]{-1}+2^{2/3}\right)dx\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\cdots}}{\cdots}\right.\right.\right.}{\left(2+\cdots\right.}$$

input
```
Integrate[(e + f*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

output
```
(2^(1/6)*Sqrt[(2^(1/3)*c + 2*d*x)/((1 + (-1)^(1/3))*c)]*(-(f*Sqrt[((-2)^(1
/3)*c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*((-1)^(1/3)*(2 + (-2)^(1/3
))*c - 2*((-1)^(1/3) + 2^(2/3))*d*x)*EllipticF[ArcSin[Sqrt[(2^(1/3)*c + 2*
(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)]) + ((-1)^(1/3)
*2^(2/3)*(1 + (-1)^(1/3))*(-(d*e) + c*f)*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*
x)/((1 + (-1)^(1/3))*c)]*Sqrt[2^(2/3) - (2*2^(1/3)*d*x)/c + (4*d^2*x^2)/c^
2]*EllipticPi[(I*2^(1/3)*Sqrt[3])/(2 + (-2)^(1/3)), ArcSin[Sqrt[(2^(1/3)*c
 + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)])/Sqrt[3])
)/((2 + (-2)^(1/3))*d^2*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/
3))*c)]*Sqrt[c^3 + 4*d^3*x^3])
```

3.64.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2564, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{(cf+2de)\int \frac{1}{\sqrt{c^3+4d^3x^3}}dx}{3cd}+\frac{(de-cf)\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}dx}{3cd}$$

$$\Big\downarrow \text{759}$$

$$\frac{(de-cf)\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}dx}{3cd}+\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}(c+2^{2/3}dx)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}(cf+2de)\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c(c+2^{2/3}dx)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

$$\Big\downarrow \text{2562}$$

$$\frac{2(de-cf)\int \frac{1}{\frac{3c(c+2dx)^2}{c^3+4d^3x^3}+1}d\frac{c+2dx}{c\sqrt{c^3+4d^3x^3}}}{3d^2}+\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}(c+2^{2/3}dx)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}(cf+2de)\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c(c+2^{2/3}dx)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}(c+2^{2/3}dx)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}(cf+2de)\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c(c+2^{2/3}dx)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}+\frac{2\arctan\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)(de-cf)}{3\sqrt{3}c^{3/2}d^2}$$

input
```
Int[(e + f*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

output
```
(2*(d*e - c*f)*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]]
)/(3*Sqrt[3]*c^(3/2)*d^2) + (2^(1/3)*Sqrt[2 + Sqrt[3]]*(2*d*e + c*f)*(c +
2^(2/3)*d*x)*Sqrt[(c^2 - 2^(2/3)*c*d*x + 2*2^(1/3)*d^2*x^2)/((1 + Sqrt[3])
*c + 2^(2/3)*d*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c + 2^(2/3)*d*x)/((1
+ Sqrt[3])*c + 2^(2/3)*d*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*c*d^2*Sqrt[(c*(c
 + 2^(2/3)*d*x))/((1 + Sqrt[3])*c + 2^(2/3)*d*x)^2]*Sqrt[c^3 + 4*d^3*x^3])
```

3.64.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.64.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 899 vs. 2(216) = 432.

Time = 0.96 (sec) , antiderivative size = 900, normalized size of antiderivative = 3.40

method	result	size
default	Expression too large to display	900
elliptic	Expression too large to display	900

input
```
int((f*x+e)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*f/d*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*
2^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1
/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)*
((x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*
c/d))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4
*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4
*d^3*x^3+c^3)^(1/2)*EllipticF(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)
/((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/
3))*c/d))^(1/2),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4
*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1
/3)*c/d))^(1/2))+2*(-c*f+d*e)/d^2*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d
-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2
^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3
^(1/2)*2^(1/3))*c/d))^(1/2)*((x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/
2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(
1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(
1/2)*2^(1/3))*c/d))^(1/2)/(4*d^3*x^3+c^3)^(1/2)/((1/4*2^(1/3)+1/4*I*3^(1/2
)*2^(1/3))*c/d+c/d)*EllipticPi(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d
)/((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1
/3))*c/d))^(1/2),((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-...
```

3.64.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.32 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.47

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$
$$= \left[\frac{\sqrt{3}(d^3e-cd^2f)\sqrt{-c}\log\left(\frac{2\,d^6x^6-36\,cd^5x^5-18\,c^2d^4x^4+28\,c^3d^3x^3+18\,c^4d^2x^2-c^6-\sqrt{3}\left(4\,d^4x^4-10\,cd^3x^3-18\,c^2d^2x^2-8\,c^3dx-c^4\right)\sqrt{4}}{d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx+c^6}\right.}{18\,c^2d^4}\right.$$
$$-\frac{\sqrt{3}(d^3e-cd^2f)\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{4\,d^3x^3+c^3}\left(2\,d^3x^3-6\,cd^2x^2-6\,c^2dx-c^3\right)\sqrt{c}}{3\left(8\,cd^4x^4+4\,c^2d^3x^3+2\,c^4dx+c^5\right)}\right)-3\,\sqrt{d^3}(2\,cde+c^2f)\text{weierstrassPInv}}{9\,c^2d^4}$$

input
```
integrate((f*x+e)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

output
```
[1/18*(sqrt(3)*(d^3*e - c*d^2*f)*sqrt(-c)*log((2*d^6*x^6 - 36*c*d^5*x^5 -
18*c^2*d^4*x^4 + 28*c^3*d^3*x^3 + 18*c^4*d^2*x^2 - c^6 - sqrt(3)*(4*d^4*x^
4 - 10*c*d^3*x^3 - 18*c^2*d^2*x^2 - 8*c^3*d*x - c^4)*sqrt(4*d^3*x^3 + c^3)
*sqrt(-c))/(d^6*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c
^4*d^2*x^2 + 6*c^5*d*x + c^6)) + 6*sqrt(d^3)*(2*c*d*e + c^2*f)*weierstrass
PInverse(0, -c^3/d^3, x))/(c^2*d^4), -1/9*(sqrt(3)*(d^3*e - c*d^2*f)*sqrt(
c)*arctan(1/3*sqrt(3)*sqrt(4*d^3*x^3 + c^3)*(2*d^3*x^3 - 6*c*d^2*x^2 - 6*c
^2*d*x - c^3)*sqrt(c)/(8*c*d^4*x^4 + 4*c^2*d^3*x^3 + 2*c^4*d*x + c^5)) - 3
*sqrt(d^3)*(2*c*d*e + c^2*f)*weierstrassPInverse(0, -c^3/d^3, x))/(c^2*d^4
)]
```

3.64.6 Sympy [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{e+fx}{(c+dx)\,\sqrt{c^3+4d^3x^3}}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

output
```
Integral((e + f*x)/((c + d*x)*sqrt(c**3 + 4*d**3*x**3)), x)
```

3.64.7 Maxima [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{fx+e}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

3.64.8 Giac [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{fx+e}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x + e)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{e+f\,x}{\sqrt{c^3+4\,d^3\,x^3}\,(c+d\,x)}\,dx$$

input
```
int((e + f*x)/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)),x)
```

output
```
int((e + f*x)/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)), x)
```

3.65 $\int \frac{x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

3.65.1 Optimal result

Integrand size = 20, antiderivative size = 145

$$\int \frac{x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
-2/9*2^(2/3)*arctan((1+2^(1/3)*x)*3^(1/2)/(x^3+1)^(1/2))*3^(1/2)+2/9*(1+x)
*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(
1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+
3^(1/2))^2)^(1/2)
```

3.65.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.32 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.43

$$\int \frac{x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{i2^{2}\cdots}{\cdots}\right)}{\sqrt{1+x^3}}$$

input
```
Integrate[x/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))])
 + (I*2^(2/3)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/
3)), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)
^(1/3) + 2^(2/3))))/Sqrt[1 + x^3]
```

3.65.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2564, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(x+2^{2/3}\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{x^3+1}}dx-\frac{1}{3}\int \frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{759}$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{1}{3}\int\frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{2562}$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2}{3}2^{2/3}\int\frac{1}{\frac{3\left(\sqrt[3]{2}x+1\right)^2}{x^3+1}+1}d\frac{\sqrt[3]{2}x+1}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\ 2^{2/3}\arctan\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}$$

input
```
Int[x/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/(3*Sqrt[3]) +
 (2*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Elli
pticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(
1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

3.65. $\int\frac{x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

3.65.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.65.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(114) = 228$.

Time = 3.69 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.78

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\,2^{\frac{2}{3}}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+i}{-\frac{3}{2}+i}}}{\sqrt{x^3+1}\,\left(2^{\frac{2}{3}}\right.}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\,2^{\frac{2}{3}}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+i}{-\frac{3}{2}+i}}}{\sqrt{x^3+1}\,\left(2^{\frac{2}{3}}\right.}$

input
```
int(x/(2^(2/3)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2
+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*2^(2/3)*(3/2-1/2*I*3^(1/2))
*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1
/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/
2)/(2^(2/3)-1)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^
(1/2))/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

3.65.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.57

$$\int \frac{x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = -\frac{1}{9}\sqrt{3}2^{\frac{2}{3}}\arctan\left(-\frac{\sqrt{3}2^{\frac{2}{3}}\left(2x^5+2x^2-2^{\frac{2}{3}}(7x^4+4x)-2^{\frac{1}{3}}(5x^3+2)\right)\sqrt{x^3+1}}{12(2x^6+3x^3+1)}\right) + \frac{2}{3}\,\mathrm{weierstrassPInverse}(0,-4,x)$$

input
```
integrate(x/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/9*sqrt(3)*2^(2/3)*arctan(-1/12*sqrt(3)*2^(2/3)*(2*x^5 + 2*x^2 - 2^(2/3)
*(7*x^4 + 4*x) - 2^(1/3)*(5*x^3 + 2))*sqrt(x^3 + 1)/(2*x^6 + 3*x^3 + 1)) +
 2/3*weierstrassPInverse(0, -4, x)
```

3.65.6 Sympy [F]

$$\int \frac{x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate(x/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

output
```
Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

3.65.7 Maxima [F]

$$\int \frac{x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate(x/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

3.65.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Ba
d Argumen
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx=\int \frac{x}{\sqrt{x^3+1}\,\left(x+2^{2/3}\right)}\,dx$$

input
```
int(x/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
```

output
```
int(x/((x^3 + 1)^(1/2)*(x + 2^(2/3))), x)
```

3.66 $\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

3.66.1 Optimal result

Integrand size = 24, antiderivative size = 160

$$\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx=-\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}+\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-2/9*2^(2/3)*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*3^(1/2)+2/9*(1-x
)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^
(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-
x+3^(1/2))^2)^(1/2)
```

3.66.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.35 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.31

$$\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{i2^{2\cdots}}{\cdots}\right.}{\sqrt{1-x^3}}$$

input
```
Integrate[x/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

output
```
(2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))])
 + (I*2^(2/3)*Sqrt[1 + x + x^2]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/
3)), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)
^(1/3) + 2^(2/3))))/Sqrt[1 - x^3]
```

3.66.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2564, 27, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$$

$$\Big\downarrow \; 2564$$

$$\frac{1}{3}\int \frac{2^{2/3}\left(\sqrt[3]{2}x+1\right)}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}dx-\frac{1}{3}\int \frac{1}{\sqrt{1-x^3}}dx$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}2^{2/3}\int \frac{\sqrt[3]{2}x+1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}dx-\frac{1}{3}\int \frac{1}{\sqrt{1-x^3}}dx$$

$$\Big\downarrow \; 759$$

$$\frac{1}{3}2^{2/3}\int \frac{\sqrt[3]{2}x+1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}dx+\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \; 2562$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{3}2^{2/3}\int \frac{1}{\frac{3\left(1-\sqrt[3]{2}x\right)^2}{1-x^3}+1}d\frac{1-\sqrt[3]{2}x}{\sqrt{1-x^3}}$$

$$\Big\downarrow \; 216$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\ 2^{2/3}\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}$$

input
```
Int[x/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/(3*Sqrt[3]) +
 (2*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*Elli
pticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(
1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

3.66. $\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

3.66.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.66.4 Maple [A] (verified)

Time = 3.72 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.58

method	result
default	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}}+\frac{2i2^{\frac{2}{3}}\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{}}}{}$
elliptic	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}}+\frac{2i2^{\frac{2}{3}}\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{}}}{}$

input
```
int(x/(2^(2/3)-x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/
2+1/2*I*3^(1/2)))^(1/2))+2/3*I*2^(2/3)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*
3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-2^(2/3)-1/2+1/2*I*3^(1/2))*EllipticPi(1/3*
3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-2^(2/3)-1/2+1/
2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

3.66.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.52

$$\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{1}{9}\,\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}2^{\frac{2}{3}}\left(2\,x^{5}-2\,x^{2}+2^{\frac{2}{3}}\left(7\,x^{4}-4\,x\right)-2^{\frac{1}{3}}\left(5\,x^{3}-2\right)\right)\sqrt{-x^{3}+1}}{12\left(2\,x^{6}-3\,x^{3}+1\right)}\right)+\frac{2}{3}i\,\text{weierstrassPInverse}(0,4,x)$$

input
```
integrate(x/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/9*sqrt(3)*2^(2/3)*arctan(1/12*sqrt(3)*2^(2/3)*(2*x^5 - 2*x^2 + 2^(2/3)*
(7*x^4 - 4*x) - 2^(1/3)*(5*x^3 - 2))*sqrt(-x^3 + 1)/(2*x^6 - 3*x^3 + 1)) +
 2/3*I*weierstrassPInverse(0, 4, x)
```

3.66.6 Sympy [F]

$$\int \frac{x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = -\int \frac{x}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx$$

input
```
integrate(x/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

output
```
-Integral(x/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

3.66.7 Maxima [F]

$$\int \frac{x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \int -\frac{x}{\sqrt{-x^3+1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate(x/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

3.66.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad
Argument
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx=-\int \frac{x}{\sqrt{1-x^3}\,\left(x-2^{2/3}\right)}\,dx$$

input
```
int(-x/((1 - x^3)^(1/2)*(x - 2^(2/3))),x)
```

output
```
-int(x/((1 - x^3)^(1/2)*(x - 2^(2/3))), x)
```

3.67 $\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

3.67.1 Optimal result

Integrand size = 22, antiderivative size = 163

$$\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx=-\frac{2\ 2^{2/3}\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}}+\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
-2/9*2^(2/3)*arctanh((1-2^(1/3)*x)*3^(1/2)/(x^3-1)^(1/2))*3^(1/2)+2/9*(1-x
)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^
(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-
x-3^(1/2))^2)^(1/2)
```

3.67.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.31 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.27

$$\int \frac{x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\right.}{\sqrt{-1+x^3}}$$

input
```
Integrate[x/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

output
```
(2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))])
 + (I*2^(2/3)*Sqrt[1 + x + x^2]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/
3)), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)
^(1/3) + 2^(2/3))))/Sqrt[-1 + x^3]
```

3.67.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2564, 27, 760, 2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(2^{2/3}-x)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{1}{3}\int \frac{2^{2/3}\left(\sqrt[3]{2}x+1\right)}{(2^{2/3}-x)\sqrt{x^3-1}}dx-\frac{1}{3}\int \frac{1}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}2^{2/3}\int \frac{\sqrt[3]{2}x+1}{(2^{2/3}-x)\sqrt{x^3-1}}dx-\frac{1}{3}\int \frac{1}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{ 760}$$

$$\frac{1}{3}2^{2/3}\int \frac{\sqrt[3]{2}x+1}{\left(2^{2/3}-x\right)\sqrt{x^3-1}}dx+\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{ 2562}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2}{3}2^{2/3}\int \frac{1}{1-\frac{3\left(1-\sqrt[3]{2}x\right)^2}{x^3-1}}d\frac{1-\sqrt[3]{2}x}{\sqrt{x^3-1}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\ 2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}$$

input
```
Int[x/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/(3*Sqrt[3])
 + (2*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*El
lipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3
^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

3.67. $\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

3.67.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.67.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(127) = 254.

Time = 3.58 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.61

method	result
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{2\,2^{\frac{2}{3}}\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{\sqrt{x^3-}}$
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{2\,2^{\frac{2}{3}}\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{\sqrt{x^3-}}$

input
```
int(x/(2^(2/3)-x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/
2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2*2^(2/3)*(-3/2-1/2*I*3^(1/2)
)*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(
1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/
2)/(-2^(2/3)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3
^(1/2))/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

3.67.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.50

$$\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx=\frac{1}{18}\sqrt{3}2^{\frac{2}{3}}\log\left(\frac{x^{18}+1440\,x^{15}+17400\,x^{12}-21056\,x^9-10368\,x^6+15360\,x^3+}{}\right.$$
$$-\frac{2}{3}\,\mathrm{weierstrassPInverse}(0,4,x)$$

input
```
integrate(x/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/18*sqrt(3)*2^(2/3)*log((x^18 + 1440*x^15 + 17400*x^12 - 21056*x^9 - 1036
8*x^6 + 15360*x^3 + 2*sqrt(3)*2^(2/3)*(126*x^14 + 2664*x^11 - 4608*x^5 + 2
304*x^2 + 2^(2/3)*(x^16 + 310*x^13 + 2332*x^10 - 2656*x^7 - 256*x^4 + 512*
x) + 2^(1/3)*(17*x^15 + 1058*x^12 + 2528*x^9 - 5408*x^6 + 2560*x^3 - 512))
*sqrt(x^3 - 1) + 24*2^(2/3)*(x^17 + 121*x^14 + 478*x^11 - 1144*x^8 + 608*x
^5 - 64*x^2) + 48*2^(1/3)*(5*x^16 + 176*x^13 + 83*x^10 - 680*x^7 + 544*x^4
 - 128*x) - 2048)/(x^18 - 24*x^15 + 240*x^12 - 1280*x^9 + 3840*x^6 - 6144*
x^3 + 4096)) - 2/3*weierstrassPInverse(0, 4, x)
```

3.67.6 Sympy [F]

$$\int \frac{x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = -\int \frac{x}{x\sqrt{x^3-1}-2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

input
```
integrate(x/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

output
```
-Integral(x/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

3.67.7 Maxima [F]

$$\int \frac{x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = \int -\frac{x}{\sqrt{x^3-1}\left(x-2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate(x/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

3.67.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Ba
d Argumen
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = -\int \frac{x}{\sqrt{x^3-1}\,\left(x-2^{2/3}\right)}\,dx$$

input
```
int(-x/((x^3 - 1)^(1/2)*(x - 2^(2/3))),x)
```

output
```
-int(x/((x^3 - 1)^(1/2)*(x - 2^(2/3))), x)
```

3.68 $\int \frac{x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

3.68.1 Optimal result

Integrand size = 22, antiderivative size = 156

$$\int \frac{x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = -\frac{2\,2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
-2/9*2^(2/3)*arctanh((1+2^(1/3)*x)*3^(1/2)/(-x^3-1)^(1/2))*3^(1/2)+2/9*(1+
x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2
^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(
1+x-3^(1/2))^2)^(1/2)
```

3.68.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.27 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.34

$$\int \frac{x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\right.}{\sqrt{-1-x^3}}$$

input
```
Integrate[x/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))])
 + (I*2^(2/3)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/
3)), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)
^(1/3) + 2^(2/3))))/Sqrt[-1 - x^3]
```

3.68.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2564, 760, 2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(x+2^{2/3}\right)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{-x^3-1}}dx-\frac{1}{3}\int \frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{1}{3}\int\frac{2^{2/3}-2x}{\left(x+2^{2/3}\right)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \ 2562$$

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2}{3}2^{2/3}\int\frac{1}{1-\frac{3\left(\sqrt[3]{2}x+1\right)^2}{-x^3-1}}d\frac{\sqrt[3]{2}x+1}{\sqrt{-x^3-1}}$$

$$\Big\downarrow \ 219$$

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\ 2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}$$

input
```
Int[x/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/(3*Sqrt[3])
 + (2*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*El
lipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3
^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

3.68. $\int \frac{x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

3.68.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.68.4 Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.60

method	result
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2i2^{\frac{2}{3}}\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{}}}{}$
elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2i2^{\frac{2}{3}}\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{}}}{}$

input
```
int(x/(2^(2/3)+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2
+1/2*I*3^(1/2)))^(1/2))+2/3*I*2^(2/3)*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(
1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^
(1/2))^(1/2)/(-x^3-1)^(1/2)/(2^(2/3)+1/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(
1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(2^(2/3)+1/2+1/2*I*
3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

3.68.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.59

$$\int \frac{x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \frac{1}{18}\sqrt{3}2^{\frac{2}{3}}\log\left(\frac{x^{18}-1440\,x^{15}+17400\,x^{12}+21056\,x^9-10368\,x^6-15360\,x^3+\cdots}{\cdots}\right) - \frac{2}{3}i\,\text{weierstrassPInverse}(0,-4,x)$$

input
```
integrate(x/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/18*sqrt(3)*2^(2/3)*log((x^18 - 1440*x^15 + 17400*x^12 + 21056*x^9 - 1036
8*x^6 - 15360*x^3 + 2*sqrt(3)*2^(2/3)*(126*x^14 - 2664*x^11 + 4608*x^5 + 2
304*x^2 + 2^(2/3)*(x^16 - 310*x^13 + 2332*x^10 + 2656*x^7 - 256*x^4 - 512*
x) - 2^(1/3)*(17*x^15 - 1058*x^12 + 2528*x^9 + 5408*x^6 + 2560*x^3 + 512))
*sqrt(-x^3 - 1) - 24*2^(2/3)*(x^17 - 121*x^14 + 478*x^11 + 1144*x^8 + 608*
x^5 + 64*x^2) + 48*2^(1/3)*(5*x^16 - 176*x^13 + 83*x^10 + 680*x^7 + 544*x^
4 + 128*x) - 2048)/(x^18 + 24*x^15 + 240*x^12 + 1280*x^9 + 3840*x^6 + 6144
*x^3 + 4096)) - 2/3*I*weierstrassPInverse(0, -4, x)
```

3.68.6 Sympy [F]

$$\int \frac{x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \int \frac{x}{\sqrt{-(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate(x/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

output
```
Integral(x/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

3.68.7 Maxima [F]

$$\int \frac{x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate(x/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

3.68.8 Giac [F]

$$\int \frac{x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3-1}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

input
```
integrate(x/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx=\int \frac{x}{\sqrt{-x^3-1}\,\left(x+2^{2/3}\right)}\,dx$$

input
```
int(x/((- x^3 - 1)^(1/2)*(x + 2^(2/3))),x)
```

output
```
int(x/((- x^3 - 1)^(1/2)*(x + 2^(2/3))), x)
```

3.69 $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

3.69.1 Optimal result

Integrand size = 34, antiderivative size = 275

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-2/9*2^(2/3)*arctan(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)*3^(1/2)/(b*x^3+a)^
(1/2))/a^(1/6)/b^(2/3)*3^(1/2)+2/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*
x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2
*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^
(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.69.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.00 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.22

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\frac{3\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\,\text{EllipticF}\left(\text{ar}\cdots\right.}{\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}\cdots\right.}{\cdots}$$

input
```
Integrate[x/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

output
```
(2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-3*((-1)^(1/3)
*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1
 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/
3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[(a^(1/3) + (-1)^(2/3
)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))] + ((-1)^(1/3)*2^(2/3)*(1 + (-1)^(
1/3))*a^(1/3)*Sqrt[3 - (3*b^(1/3)*x)/a^(1/3) + (3*b^(2/3)*x^2)/a^(2/3)]*El
lipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(
2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/((-1)^(1/3) + 2
^(2/3))))/(3*b^(2/3)*Sqrt[a + b*x^3])
```

3.69.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2564, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \; 2564$$

$$\frac{\int \frac{1}{\sqrt{bx^3+a}}dx}{3\sqrt[3]{b}}-\frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a}\right)\sqrt{bx^3+a}}dx}{3\sqrt[3]{b}}$$

$\downarrow$ 759

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a}\right)\sqrt{bx^3+a}}dx}{3\sqrt[3]{b}}$$

$\downarrow$ 2562

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2\,2^{2/3}\sqrt[3]{a}\int \frac{1}{\frac{3\sqrt[3]{a}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{bx^3+a}+1}d\frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{bx^3+a}}}{3b^{2/3}}$$

$\downarrow$ 216

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

3.69. $\displaystyle\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}dx$

input
```
Int[x/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a
+ b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*
x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/
3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2]*Sqrt[a + b*x^3])
```

3.69.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.69.4 Maple [F]

$$\int \frac{x}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}+b^{\frac{1}{3}} x\right) \sqrt{b\,x^{3}+a}}d x$$

input
```
int(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output
```
int(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

3.69.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

3.69.6 Sympy [F]

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \int \frac{x}{\sqrt{a+bx^3}\cdot\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate(x/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

output
```
Integral(x/(sqrt(a + b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

3.69.7 Maxima [F]

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{x}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

3.69.8 Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac
")
```

output
```
Timed out
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{x}{\sqrt{b\,x^3+a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\,dx$$

input
```
int(x/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
```

output
```
int(x/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

3.70 $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

3.70.1 Optimal result

Integrand size = 36, antiderivative size = 283

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

output

```
-2/9*2^(2/3)*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)*3^(1/2)/(-b*x^3+a)
^(1/2))/a^(1/6)/b^(2/3)*3^(1/2)+2/9*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3
)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(
1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3
)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.70.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.79 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.37

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx =$$

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}+2^{2/3}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{\sqrt[3]{a}-}{(1+}}\right.\right.\right.}{\left(\sqrt[3]{-1}+2\right.}$$

input
```
Integrate[x/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
(-2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1)^(1/3) +
2^(2/3))*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)
^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1
/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - ((
-1)^(1/3)*2^(2/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1
/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)
*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[
(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]
)/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(
1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a - b*x^3])
```

3.70.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2564, 27, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx$$

$$\Big\downarrow \text{ 2564}$$

$$\frac{\int \frac{2^{2/3}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx}{3\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt{a-bx^3}}dx}{3\sqrt[3]{b}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2^{2/3}\int \frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx}{3\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt{a-bx^3}}dx}{3\sqrt[3]{b}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{2^{2/3}\int \frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx}{3\sqrt[3]{b}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\Big\downarrow \text{ 2562}$$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\ 2^{2/3}\sqrt[3]{a}\int \frac{1}{\frac{3\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)^2}{a-bx^3}+1}d\frac{\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{a-bx^3}}}{3b^{2/3}}$$

$$\Big\downarrow \text{ 216}$$

3.70. $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\,2^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

input
```
Int[x/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a
- b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) - b
^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*
a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*
x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/
3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x
)^2]*Sqrt[a - b*x^3])
```

3.70.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.70.4 Maple [F]

$$\int \frac{x}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}-b^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3}+a}}dx$$

input
```
int(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

output
```
int(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

3.70.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="fri
cas")
```

output
```
Timed out
```

3.70.6 Sympy [F]

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\int \frac{x}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

input
```
integrate(x/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

output
```
-Integral(x/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x
**3)), x)
```

3.70.7 Maxima [F]

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \int -\frac{x}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="max
ima")
```

output
```
-integrate(x/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

3.70.8 Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="gia
c")
```

output
```
Timed out
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{x}{\sqrt{a-b\,x^3}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\,dx$$

input
```
int(x/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
```

output
```
int(x/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

3.71 $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

3.71.1 Optimal result

Integrand size = 37, antiderivative size = 292

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\frac{2\,2^{2/3}\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}} + \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

output
```
-2/9*2^(2/3)*arctanh(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)*3^(1/2)/(b*x^3-a)
^(1/2))/a^(1/6)/b^(2/3)*3^(1/2)+2/9*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(
(a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2
)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3
)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.71.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.78 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.33

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx =$$

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}+2^{2/3}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{\sqrt[3]{a}-}{(1+}}\right.\right.\right.}{\left(\sqrt[3]{-1}+2^{2}\right.}$$

input
```
Integrate[x/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
(-2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1)^(1/3) +
2^(2/3))*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)
^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1
/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - ((
-1)^(1/3)*2^(2/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1
/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)
*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[
(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]
)/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(
1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3])
```

3.71.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {2564, 27, 760, 2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \; 2564$$

$$\frac{\int \frac{2^{2/3}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx}{3\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt{bx^3-a}}dx}{3\sqrt[3]{b}}$$

$$\Big\downarrow \; 27$$

$$\frac{2^{2/3}\int \frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx}{3\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt{bx^3-a}}dx}{3\sqrt[3]{b}}$$

$$\Big\downarrow \; 760$$

$$\frac{2^{2/3}\int \frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx}{3\sqrt[3]{b}}+\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\Big\downarrow \; 2562$$

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\,2^{2/3}\sqrt[3]{a}\int \frac{1}{1-\frac{3\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)^2}{bx^3-a}}d\frac{\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{bx^3-a}}}{3b^{2/3}}$$

$$\Big\downarrow \; 219$$

3.71. $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}dx$

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\,2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

input
```
Int[x/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[-
a + b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) -
 b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3]
)*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3
)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(
2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/
3)*x)^2)]*Sqrt[-a + b*x^3])
```

3.71.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.71.4 Maple [F]

$$\int \frac{x}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}-b^{\frac{1}{3}} x\right) \sqrt{b\,x^{3}-a}}dx$$

input
```
int(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output
```
int(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

3.71.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

3.71.6 Sympy [F]

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\int \frac{x}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

input
```
integrate(x/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

output
```
-Integral(x/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b
*x**3)), x)
```

3.71.7 Maxima [F]

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \int -\frac{x}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x-2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="maxi
ma")
```

output
```
-integrate(x/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

3.71.8 Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="giac
")
```

output
```
Timed out
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{x}{\sqrt{b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\,dx$$

input
```
int(x/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
```

output
```
int(x/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

3.72 $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

3.72.1 Optimal result

Integrand size = 37, antiderivative size = 288

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\frac{2\,2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}} + \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

output
```
-2/9*2^(2/3)*arctanh(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)*3^(1/2)/(-b*x^3-a
)^(1/2))/a^(1/6)/b^(2/3)*3^(1/2)+2/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^
(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)
*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.72.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.70 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.35

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx =$$

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}+2^{2/3}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{\sqrt[3]{a}+(\cdots}{(1+\sqrt[3]{\cdots}}}\right.\right.\right.}{\left(\sqrt[3]{-1}+2^{2/\cdots}\right.}$$

input
```
Integrate[x/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
(-2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1)^(1/3) +
2^(2/3))*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^
(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3
) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - ((-1
)^(1/3)*2^(2/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3
)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x
^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a
^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/
Sqrt[3]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/
3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a - b*x^3])
```

3.72.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {2564, 760, 2562, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}dx$$

$$\Big\downarrow \; 2564$$

$$\frac{\int \frac{1}{\sqrt{-bx^3-a}}dx}{3\sqrt[3]{b}}-\frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a}\right)\sqrt{-bx^3-a}}dx}{3\sqrt[3]{b}}$$

$$\Big\downarrow \; 760$$

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2^{2/3}\sqrt[3]{a}\right)\sqrt{-bx^3-a}}dx}{3\sqrt[3]{b}}$$

$$\Big\downarrow \; 2562$$

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{2\ 2^{2/3}\sqrt[3]{a}\int \frac{1}{1-\frac{3\sqrt[3]{a}\left(\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{-bx^3-a}}d\frac{\sqrt[3]{2}\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{-bx^3-a}}}{3b^{2/3}}$$

$$\Big\downarrow \; 219$$

3.72. $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}dx$

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}} - \frac{2\,2^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}b^{2/3}}$$

input
```
Int[x/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
(-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-
a - b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(
2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2)]*Sqrt[-a - b*x^3])
```

3.72.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.72.4 Maple [F]

$$\int \frac{x}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}+b^{\frac{1}{3}} x\right) \sqrt{-b\, x^{3}-a}}dx$$

input
```
int(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

output
```
int(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

3.72.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3} \sqrt[3]{a}+\sqrt[3]{b} x\right) \sqrt{-a-b x^3}}\, dx=\text{Timed out}$$

input
```
integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="fri
cas")
```

output
```
Timed out
```

3.72.6 Sympy [F]

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{x}{\sqrt{-a-bx^3}\cdot\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate(x/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

output
```
Integral(x/(sqrt(-a - b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

3.72.7 Maxima [F]

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{x}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x+2^{\frac{2}{3}}a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

3.72.8 Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="gia
c")
```

output
```
Timed out
```

3.72.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{x}{\sqrt{-b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\,dx$$

input
```
int(x/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
```

output
```
int(x/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

3.73 $\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

3.73.1 Optimal result

Integrand size = 25, antiderivative size = 246

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}\sqrt{c}d^2} + \frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

output
```
-2/9*arctan((2*d*x+c)*3^(1/2)*c^(1/2)/(4*d^3*x^3+c^3)^(1/2))/d^2*3^(1/2)/c
^(1/2)+1/9*2^(1/3)*(c+2^(2/3)*d*x)*EllipticF((2^(2/3)*d*x+c*(1-3^(1/2)))/(
2^(2/3)*d*x+c*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((c^2-
2^(2/3)*c*d*x+2*2^(1/3)*d^2*x^2)/(2^(2/3)*d*x+c*(1+3^(1/2)))^2)^(1/2)*3^(3
/4)/d^2/(4*d^3*x^3+c^3)^(1/2)/(c*(c+2^(2/3)*d*x)/(2^(2/3)*d*x+c*(1+3^(1/2)
))^2)^(1/2)
```

3.73.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.95 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.51

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

$$= \frac{\sqrt[6]{2}\sqrt{\frac{\sqrt[3]{2}c+2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(-\sqrt{\frac{\sqrt[3]{-2}c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(\sqrt[3]{-1}\left(2+\sqrt[3]{-2}\right)c-2\left(\sqrt[3]{-1}+2^{2/3}\right)dx\right)\text{EllipticF}\left(\arcsin\left(\cdots\right.\right.\right.}{\left(2+\sqrt[3]{-2}\cdots\right.}$$

input
```
Integrate[x/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

output
```
(2^(1/6)*Sqrt[(2^(1/3)*c + 2*d*x)/((1 + (-1)^(1/3))*c)]*(-(Sqrt[((-2)^(1/3
)*c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*((-1)^(1/3)*(2 + (-2)^(1/3))
*c - 2*((-1)^(1/3) + 2^(2/3))*d*x)*EllipticF[ArcSin[Sqrt[(2^(1/3)*c + 2*(-
1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)]) + ((-1)^(1/3)*2
^(2/3)*(1 + (-1)^(1/3))*c*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(
1/3))*c)]*Sqrt[2^(2/3) - (2*2^(1/3)*d*x)/c + (4*d^2*x^2)/c^2]*EllipticPi[(
I*2^(1/3)*Sqrt[3])/(2 + (-2)^(1/3)), ArcSin[Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)
*d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)])/Sqrt[3]))/((2 + (-2)^(1
/3))*d^2*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*Sqrt[c^
3 + 4*d^3*x^3])
```

3.73.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2564, 759, 2562, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

$$\Big\downarrow \text{ 2564}$$

$$\frac{\int \frac{1}{\sqrt{c^3+4d^3x^3}}dx}{3d}-\frac{\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}dx}{3d}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}-\frac{\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}dx}{3d}$$

$$\Big\downarrow \text{ 2562}$$

$$\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}-\frac{2c\int \frac{1}{\frac{3c(c+2dx)^2}{c^3+4d^3x^3}+1}d\frac{c+2dx}{c\sqrt{c^3+4d^3x^3}}}{3d^2}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}(c+2^{2/3}dx)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c(c+2^{2/3}dx)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}-\frac{2\arctan\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}\sqrt{c}d^2}$$

input
```
Int[x/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/(3*Sqrt[3
]*Sqrt[c]*d^2) + (2^(1/3)*Sqrt[2 + Sqrt[3]]*(c + 2^(2/3)*d*x)*Sqrt[(c^2 -
2^(2/3)*c*d*x + 2*2^(1/3)*d^2*x^2)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*c + 2^(2/3)*d*x)/((1 + Sqrt[3])*c + 2^(2/3)*d*
x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*d^2*Sqrt[(c*(c + 2^(2/3)*d*x))/((1 + Sqrt
[3])*c + 2^(2/3)*d*x)^2]*Sqrt[c^3 + 4*d^3*x^3])
```

3.73.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2562
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[2*(e/d)   Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))
/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
&& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.73.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 891 vs. $2(197) = 394$.

Time = 0.96 (sec) , antiderivative size = 892, normalized size of antiderivative = 3.63

method	result	size
default	Expression too large to display	892
elliptic	Expression too large to display	892

input
```
int(x/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^
(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4
*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)*((
x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/
d))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I
*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d
^3*x^3+c^3)^(1/2)*EllipticF(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/(
(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3)
)*c/d))^(1/2),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I
*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3
)*c/d))^(1/2))-2*c/d^2*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/
3)+1/4*I*3^(1/2)*2^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d
)/((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1
/3))*c/d))^(1/2)*((x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))
*c/d+1/2*2^(1/3)*c/d))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/
((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3
))*c/d))^(1/2)/(4*d^3*x^3+c^3)^(1/2)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*
c/d+c/d)*EllipticPi(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(
1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^
(1/2),((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/...
```

3.73.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.42

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \left[-\frac{\sqrt{3}\sqrt{-c}d^2\log\left(\frac{2\,d^6x^6-36\,cd^5x^5-18\,c^2d^4x^4+28\,c^3d^3x^3+18\,c^4d^2x^2-c^6-\sqrt{3}(4\,d^4x^4-10\,cd^3x^3-18\,c^2d^2x^2-8\,c^3dx-c^4)\sqrt{4\,d^3x^3+c^3}\sqrt{\cdot}}{d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx+c^6}\right.}{18\,cd^4}\right.$$

input
```
integrate(x/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

output
```
[-1/18*(sqrt(3)*sqrt(-c)*d^2*log((2*d^6*x^6 - 36*c*d^5*x^5 - 18*c^2*d^4*x^
4 + 28*c^3*d^3*x^3 + 18*c^4*d^2*x^2 - c^6 - sqrt(3)*(4*d^4*x^4 - 10*c*d^3*
x^3 - 18*c^2*d^2*x^2 - 8*c^3*d*x - c^4)*sqrt(4*d^3*x^3 + c^3)*sqrt(-c))/(d
^6*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x^2 +
6*c^5*d*x + c^6)) - 6*c*sqrt(d^3)*weierstrassPInverse(0, -c^3/d^3, x))/(c*
d^4), 1/9*(sqrt(3)*sqrt(c)*d^2*arctan(1/3*sqrt(3)*sqrt(4*d^3*x^3 + c^3)*(2
*d^3*x^3 - 6*c*d^2*x^2 - 6*c^2*d*x - c^3)*sqrt(c)/(8*c*d^4*x^4 + 4*c^2*d^3
*x^3 + 2*c^4*d*x + c^5)) + 3*c*sqrt(d^3)*weierstrassPInverse(0, -c^3/d^3,
x))/(c*d^4)]
```

3.73.6 Sympy [F]

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{x}{(c+dx)\,\sqrt{c^3+4d^3x^3}}\,dx$$

input
```
integrate(x/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

output
```
Integral(x/((c + d*x)*sqrt(c**3 + 4*d**3*x**3)), x)
```

3.73.7 Maxima [F]

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{x}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

input
```
integrate(x/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

3.73.8 Giac [F]

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{x}{\sqrt{4\,d^3x^3+c^3}(dx+c)}\,dx$$

input
```
integrate(x/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \int \frac{x}{\sqrt{c^3+4\,d^3\,x^3}\,(c+d\,x)}\,dx$$

input
```
int(x/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)),x)
```

output
```
int(x/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)), x)
```

3.74 $\int \frac{1+x}{(2-x)\sqrt{1+x^3}} dx$

3.74.1 Optimal result

Integrand size = 20, antiderivative size = 23

$$\int \frac{1+x}{(2-x)\sqrt{1+x^3}}\,dx = \frac{2}{3}\operatorname{arctanh}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right)$$

output
```
2/3*arctanh(1/3*(1+x)^2/(x^3+1)^(1/2))
```

3.74.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{1+x}{(2-x)\sqrt{1+x^3}}\,dx = \frac{2}{3}\operatorname{arctanh}\left(\frac{\frac{1}{3}+\frac{2x}{3}+\frac{x^2}{3}}{\sqrt{1+x^3}}\right)$$

input
```
Integrate[(1 + x)/((2 - x)*Sqrt[1 + x^3]),x]
```

output
```
(2*ArcTanh[(1/3 + (2*x)/3 + x^2/3)/Sqrt[1 + x^3]])/3
```

3.74.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{(2-x)\sqrt{x^3+1}}\,dx$$

$$\Big\downarrow \text{2563}$$

$$2\int \frac{1}{9-\frac{(x+1)^4}{x^3+1}}d\frac{(x+1)^2}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2}{3}\text{arctanh}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)$$

input
```
Int[(1 + x)/((2 - x)*Sqrt[1 + x^3]),x]
```

output
```
(2*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/3
```

3.74.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.74.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(17) = 34$.

Time = 1.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

method	result
trager	$-\frac{\ln\left(\frac{-x^3+6\sqrt{x^3+1}\,x-12x^2+6\sqrt{x^3+1}+6x-10}{(x-2)^3}\right)}{3}$
default	$-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+i\sqrt{}}{-\frac{3}{2}+\frac{i\sqrt{}}{2}}}}{\sqrt{x^3+1}}$
elliptic	$-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+i\sqrt{}}{-\frac{3}{2}+\frac{i\sqrt{}}{2}}}}{\sqrt{x^3+1}}$

input
```
int((x+1)/(2-x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln((-x^3+6*(x^3+1)^(1/2)*x-12*x^2+6*(x^3+1)^(1/2)+6*x-10)/(x-2)^3)
```

3.74.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(17) = 34$.

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \frac{1+x}{(2-x)\sqrt{1+x^3}}\,dx = \frac{1}{3}\log\left(\frac{x^3+12\,x^2+6\,\sqrt{x^3+1}(x+1)-6\,x+10}{x^3-6\,x^2+12\,x-8}\right)$$

input
```
integrate((1+x)/(2-x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*log((x^3 + 12*x^2 + 6*sqrt(x^3 + 1)*(x + 1) - 6*x + 10)/(x^3 - 6*x^2 +
 12*x - 8))
```

3.74.6 Sympy [F]

$$\int \frac{1+x}{(2-x)\sqrt{1+x^3}}\,dx = -\int \frac{x}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx - \int \frac{1}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx$$

input
```
integrate((1+x)/(2-x)/(x**3+1)**(1/2),x)
```

output
```
-Integral(x/(x*sqrt(x**3 + 1) - 2*sqrt(x**3 + 1)), x) - Integral(1/(x*sqrt
(x**3 + 1) - 2*sqrt(x**3 + 1)), x)
```

3.74.7 Maxima [F]

$$\int \frac{1+x}{(2-x)\sqrt{1+x^3}}\,dx = \int -\frac{x+1}{\sqrt{x^3+1}(x-2)}\,dx$$

input
```
integrate((1+x)/(2-x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

3.74.8 Giac [F]

$$\int \frac{1+x}{(2-x)\sqrt{1+x^3}}\,dx = \int -\frac{x+1}{\sqrt{x^3+1}(x-2)}\,dx$$

input
```
integrate((1+x)/(2-x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

3.74.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 8.91

$$\int \frac{1+x}{(2-x)\sqrt{1+x^3}}\,dx =$$

$$-\frac{(3+\sqrt{3}\,1\mathrm{i})\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right.\right.}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(-(x + 1)/((x^3 + 1)^(1/2)*(x - 2)),x)
```

output
```
-((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*(ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)
/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6 + 1/2, asin(
((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)
*1i)/2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 -
x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*(
(3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
)^(1/2)
```

3.75 $\int \frac{1-x}{(2+x)\sqrt{1-x^3}}\,dx$

3.75.1 Optimal result

Integrand size = 22, antiderivative size = 27

$$\int \frac{1-x}{(2+x)\sqrt{1-x^3}}\,dx = -\frac{2}{3}\text{arctanh}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)$$

output
```
-2/3*arctanh(1/3*(1-x)^2/(-x^3+1)^(1/2))
```

3.75.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int \frac{1-x}{(2+x)\sqrt{1-x^3}}\,dx = -\frac{2}{3}\text{arctanh}\left(\frac{\frac{1}{3}-\frac{2x}{3}+\frac{x^2}{3}}{\sqrt{1-x^3}}\right)$$

input
```
Integrate[(1 - x)/((2 + x)*Sqrt[1 - x^3]),x]
```

output
```
(-2*ArcTanh[(1/3 - (2*x)/3 + x^2/3)/Sqrt[1 - x^3]])/3
```

3.75.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x}{(x+2)\sqrt{1-x^3}}\,dx$$

$$\Big\downarrow \; 2563$$

$$-2\int \frac{1}{9-\frac{(1-x)^4}{1-x^3}}d\frac{(1-x)^2}{\sqrt{1-x^3}}$$

$$\Big\downarrow \; 219$$

$$-\frac{2}{3}\text{arctanh}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)$$

input
```
Int[(1 - x)/((2 + x)*Sqrt[1 - x^3]),x]
```

output
```
(-2*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/3
```

3.75.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.75.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(21) = 42$.

Time = 1.37 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

method	result
trager	$\frac{\ln\left(-\frac{-x^3+6\sqrt{-x^3+1}\,x+12x^2-6\sqrt{-x^3+1}+6x+10}{(x+2)^3}\right)}{3}$
default	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}} - \frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{}$
elliptic	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}} - \frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{}$

input
```
int((1-x)/(x+2)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(-(-x^3+6*(-x^3+1)^(1/2)*x+12*x^2-6*(-x^3+1)^(1/2)+6*x+10)/(x+2)^3)
```

3.75.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(19) = 38$.

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int \frac{1-x}{(2+x)\sqrt{1-x^3}}\,dx = \frac{1}{3}\log\left(-\frac{x^3-12\,x^2-6\,\sqrt{-x^3+1}(x-1)-6\,x-10}{x^3+6\,x^2+12\,x+8}\right)$$

input
```
integrate((1-x)/(2+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*log(-(x^3 - 12*x^2 - 6*sqrt(-x^3 + 1)*(x - 1) - 6*x - 10)/(x^3 + 6*x^2
 + 12*x + 8))
```

3.75.6 Sympy [F]

$$\int \frac{1-x}{(2+x)\sqrt{1-x^3}}\,dx = -\int \frac{x}{x\sqrt{1-x^3}+2\sqrt{1-x^3}}\,dx - \int \left(-\frac{1}{x\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx$$

input
```
integrate((1-x)/(2+x)/(-x**3+1)**(1/2),x)
```

output
```
-Integral(x/(x*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(-1/(x*sqr
t(1 - x**3) + 2*sqrt(1 - x**3)), x)
```

3.75.7 Maxima [F]

$$\int \frac{1-x}{(2+x)\sqrt{1-x^3}}\,dx = \int -\frac{x-1}{\sqrt{-x^3+1}(x+2)}\,dx$$

input
```
integrate((1-x)/(2+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x - 1)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

3.75.8 Giac [F]

$$\int \frac{1-x}{(2+x)\sqrt{1-x^3}}\,dx = \int -\frac{x-1}{\sqrt{-x^3+1}(x+2)}\,dx$$

input
```
integrate((1-x)/(2+x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x - 1)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

3.75.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 221, normalized size of antiderivative = 8.19

$$\int \frac{1-x}{(2+x)\sqrt{1-x^3}}\,dx$$
$$= \frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\right.\right.\right.}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}\right.}}$$

input
```
int(-(x - 1)/((1 - x^3)^(1/2)*(x + 2)),x)
```

output
```
((3^(1/2)*1i + 3)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1
i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)*(ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i
)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6 + 1/2, asin
((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/
2)*1i)/2 - 3/2)))*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/((1 - x^3)^(1/2
)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.76 $\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx$

3.76.1 Optimal result

Integrand size = 20, antiderivative size = 25

$$\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx = -\frac{2}{3}\arctan\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right)$$

output
```
-2/3*arctan(1/3*(1-x)^2/(x^3-1)^(1/2))
```

3.76.2 Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx = \frac{2}{3}\arctan\left(\frac{3\sqrt{-1+x^3}}{(-1+x)^2}\right)$$

input
```
Integrate[(1 - x)/((2 + x)*Sqrt[-1 + x^3]),x]
```

output
```
(2*ArcTan[(3*Sqrt[-1 + x^3])/(-1 + x)^2])/3
```

3.76.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2563, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x}{(x+2)\sqrt{x^3-1}}\,dx$$

$$\Big\downarrow \text{2563}$$

$$-2\int \frac{1}{\frac{(1-x)^4}{x^3-1}+9}d\frac{(1-x)^2}{\sqrt{x^3-1}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2}{3}\arctan\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)$$

input
```
Int[(1 - x)/((2 + x)*Sqrt[-1 + x^3]),x]
```

output
```
(-2*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/3
```

3.76.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.76.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.47 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.00

method	result
trager	$\frac{\operatorname{RootOf}(\_Z^2+1)\ln\left(-\frac{\operatorname{RootOf}(\_Z^2+1)x^3-12\operatorname{RootOf}(\_Z^2+1)x^2+6\sqrt{x^3-1}\,x-6\operatorname{RootOf}(\_Z^2+1)x-6\sqrt{x^3-1}-10\operatorname{RootOf}(\_Z^2+}{(x+2)^3}\right.}{3}$
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{x}}{\sqrt{}}$
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{x}}{\sqrt{}}$

input
```
int((1-x)/(x+2)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*RootOf(_Z^2+1)*ln(-(RootOf(_Z^2+1)*x^3-12*RootOf(_Z^2+1)*x^2+6*(x^3-1)
^(1/2)*x-6*RootOf(_Z^2+1)*x-6*(x^3-1)^(1/2)-10*RootOf(_Z^2+1))/(x+2)^3)
```

3.76.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(17) = 34.

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx = -\frac{1}{3}\arctan\left(\frac{(x^3-12\,x^2-6\,x-10)\sqrt{x^3-1}}{6\,(x^4-x^3-x+1)}\right)$$

input
```
integrate((1-x)/(2+x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*arctan(1/6*(x^3 - 12*x^2 - 6*x - 10)*sqrt(x^3 - 1)/(x^4 - x^3 - x + 1
))
```

3.76.6 Sympy [F]

$$\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx = -\int \frac{x}{x\sqrt{x^3-1}+2\sqrt{x^3-1}}\,dx - \int \left(-\frac{1}{x\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)\,dx$$

input
```
integrate((1-x)/(2+x)/(x**3-1)**(1/2),x)
```

output
```
-Integral(x/(x*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(-1/(x*sqr
t(x**3 - 1) + 2*sqrt(x**3 - 1)), x)
```

3.76.7 Maxima [F]

$$\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx = \int -\frac{x-1}{\sqrt{x^3-1}(x+2)}\,dx$$

input
```
integrate((1-x)/(2+x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x - 1)/(sqrt(x^3 - 1)*(x + 2)), x)
```

3.76.8 Giac [F]

$$\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx = \int -\frac{x-1}{\sqrt{x^3-1}(x+2)}\,dx$$

input
```
integrate((1-x)/(2+x)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x - 1)/(sqrt(x^3 - 1)*(x + 2)), x)
```

3.76.9 Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 205, normalized size of antiderivative = 8.20

$$\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx$$
$$= \frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{-\frac{x-}{\frac{3}{2}+}}\right.\right.\right.}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(-(x - 1)/((x^3 - 1)^(1/2)*(x + 2)),x)
```

output
```
((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(ellipticF(as
in((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(
1/2)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((-(x - 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))
)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/((((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) +
x^3)^(1/2)
```

3.77 $\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx$

3.77.1 Optimal result

Integrand size = 22, antiderivative size = 25

$$\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx = \frac{2}{3}\arctan\left(\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right)$$

output
```
2/3*arctan(1/3*(1+x)^2/(-x^3-1)^(1/2))
```

3.77.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx = -\frac{2}{3}\arctan\left(\frac{3\sqrt{-1-x^3}}{(1+x)^2}\right)$$

input
```
Integrate[(1 + x)/((2 - x)*Sqrt[-1 - x^3]),x]
```

output
```
(-2*ArcTan[(3*Sqrt[-1 - x^3])/(1 + x)^2])/3
```

3.77.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2563, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{(2-x)\sqrt{-x^3-1}}\,dx$$

$$\Big\downarrow \text{2563}$$

$$2\int \frac{1}{\frac{(x+1)^4}{-x^3-1}+9}d\frac{(x+1)^2}{\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2}{3}\arctan\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)$$

input
```
Int[(1 + x)/((2 - x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/3
```

3.77.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.77.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.62 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.16

method	result
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-\operatorname{RootOf}\left(\_Z^2+1\right)x^3-12\operatorname{RootOf}\left(\_Z^2+1\right)x^2+6\sqrt{-x^3-1}\,x+6\operatorname{RootOf}\left(\_Z^2+1\right)x+6\sqrt{-x^3-1}-10\operatorname{RootOf}\left(\_Z\right.}{(x-2)^3}\right.}{3}$
default	$\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{}{\frac{3}{2}}}}{}$
elliptic	$\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{}{\frac{3}{2}}}}{}$

input
```
int((x+1)/(2-x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*RootOf(_Z^2+1)*ln((-RootOf(_Z^2+1)*x^3-12*RootOf(_Z^2+1)*x^2+6*(-x^3-1
)^(1/2)*x+6*RootOf(_Z^2+1)*x+6*(-x^3-1)^(1/2)-10*RootOf(_Z^2+1))/(x-2)^3)
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx = -\frac{1}{3}\arctan\left(\frac{(x^3+12\,x^2-6\,x+10)\sqrt{-x^3-1}}{6\,(x^4+x^3+x+1)}\right)$$

input
```
integrate((1+x)/(2-x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*arctan(1/6*(x^3 + 12*x^2 - 6*x + 10)*sqrt(-x^3 - 1)/(x^4 + x^3 + x +
1))
```

3.77.6 Sympy [F]

$$\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx = -\int \frac{x}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx - \int \frac{1}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx$$

input
```
integrate((1+x)/(2-x)/(-x**3-1)**(1/2),x)
```

output
```
-Integral(x/(x*sqrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x) - Integral(1/(x*sq
rt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x)
```

3.77.7 Maxima [F]

$$\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx = \int -\frac{x+1}{\sqrt{-x^3-1}(x-2)}\,dx$$

input
```
integrate((1+x)/(2-x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x + 1)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

3.77.8 Giac [F]

$$\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx = \int -\frac{x+1}{\sqrt{-x^3-1}(x-2)}\,dx$$

input
```
integrate((1+x)/(2-x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x + 1)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

3.77.9 Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 221, normalized size of antiderivative = 8.84

$$\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx =$$
$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\right.\right.}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}\right.}}$$

input
```
int(-(x + 1)/((- x^3 - 1)^(1/2)*(x - 2)),x)
```

output
```
-((3^(1/2)*1i + 3)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1
i)/2 - 3/2))^(1/2)*(ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))
, -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i
)/6 + 1/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2
+ 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((
3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/((- x^3 - 1)^(1/2)
*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

3.78 $\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

3.78.1 Optimal result

Integrand size = 43, antiderivative size = 50

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{2\text{arctanh}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
2/3*arctanh(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(b*x^3+a)^(1/2))/a^(1/6)/b^(
1/3)
```

3.78.2 Mathematica [A] (verified)

Time = 2.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{2\text{arctanh}\left(\frac{3\sqrt[6]{a}\sqrt{a+bx^3}}{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),
x]
```

output
```
(2*ArcTanh[(3*a^(1/6)*Sqrt[a + b*x^3])/(a^(1/3) + b^(1/3)*x)^2])/(3*a^(1/6
)*b^(1/3))
```

3.78.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{2563}$$

$$\frac{2\sqrt[3]{a}\int \frac{1}{9-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^4}{\sqrt[3]{a}(bx^3+a)}}d\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\sqrt{bx^3+a}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

output
```
(2*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a + b*x^3])])/(3*a^(1/6
)*b^(1/3))
```

3.78.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.78.4 Maple [F]

$$\int \frac{a^{\frac{1}{3}}+b^{\frac{1}{3}}x}{\left(2a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{b\,x^{3}+a}}dx$$

input
```
int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output
```
int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

3.78.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, alg
orithm="fricas")
```

output
```
Timed out
```

3.78.6 Sympy [F]

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\int \frac{\sqrt[3]{a}}{-2\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx - \int \frac{\sqrt[3]{b}x}{-2\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx$$

input
```
integrate((a**(1/3)+b**(1/3)*x)/(2*a**(1/3)-b**(1/3)*x)/(b*x**3+a)**(1/2),
x)
```

output
```
-Integral(a**(1/3)/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x
**3)), x) - Integral(b**(1/3)*x/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x
*sqrt(a + b*x**3)), x)
```

3.78.7 Maxima [F]

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \int -\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, alg
orithm="maxima")
```

output
```
-integrate((b^(1/3)*x + a^(1/3))/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3)))
, x)
```

3.78.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, alg
orithm="giac")
```

output
```
Timed out
```

3.78.9 Mupad [B] (verification not implemented)

Time = 9.65 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)\left(\sqrt{b\,x^3+a}-\sqrt{a}+2\,a^{1/6}\,b^{1/3}\,x\right)^3}{x^3\,\left(b^{1/3}\,x-2\,a^{1/3}\right)^3}\right)}{3\,a^{1/6}\,b^{1/3}}$$

input
```
int(-(b^(1/3)*x + a^(1/3))/((b^(1/3)*x - 2*a^(1/3))*(a + b*x^3)^(1/2)),x)
```

output
```
log((((a + b*x^3)^(1/2) + a^(1/2))*((a + b*x^3)^(1/2) - a^(1/2) + 2*a^(1/6
)*b^(1/3)*x)^3)/(x^3*(b^(1/3)*x - 2*a^(1/3))^3))/(3*a^(1/6)*b^(1/3))
```

3.79 $\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

3.79.1 Optimal result

Integrand size = 44, antiderivative size = 52

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=-\frac{2\text{arctanh}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
-2/3*arctanh(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(-b*x^3+a)^(1/2))/a^(1/6)/b
^(1/3)
```

3.79.2 Mathematica [A] (verified)

Time = 2.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=-\frac{2\text{arctanh}\left(\frac{3\sqrt[6]{a}\sqrt{a-bx^3}}{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),
x]
```

output
```
(-2*ArcTanh[(3*a^(1/6)*Sqrt[a - b*x^3])/(a^(1/3) - b^(1/3)*x)^2])/(3*a^(1/
6)*b^(1/3))
```

3.79.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx$$

$$\Big\downarrow \text{2563}$$

$$-\frac{2\sqrt[3]{a}\int \frac{1}{9-\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^4}{\sqrt[3]{a}\left(a-bx^3\right)}}d\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{a^{2/3}\sqrt{a-bx^3}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\text{arctanh}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
(-2*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a - b*x^3])])/(3*a^(1/
6)*b^(1/3))
```

3.79.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.79.4 Maple [F]

$$\int \frac{a^{\frac{1}{3}}-b^{\frac{1}{3}}x}{\left(2a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{-b\,x^{3}+a}}dx$$

input
```
int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

output
```
int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

3.79.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, al
gorithm="fricas")
```

output
```
Timed out
```

3.79.6 Sympy [F]

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\int \left(-\frac{\sqrt[3]{a}}{2\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\right)dx - \int \frac{\sqrt[3]{b}x}{2\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

input
```
integrate((a**(1/3)-b**(1/3)*x)/(2*a**(1/3)+b**(1/3)*x)/(-b*x**3+a)**(1/2)
,x)
```

output
```
-Integral(-a**(1/3)/(2*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x
**3)), x) - Integral(b**(1/3)*x/(2*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*
sqrt(a - b*x**3)), x)
```

3.79.7 Maxima [F]

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \int -\frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, al
gorithm="maxima")
```

output
```
-integrate((b^(1/3)*x - a^(1/3))/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3))
), x)
```

3.79.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, al
gorithm="giac")
```

output
```
Timed out
```

3.79.9 Mupad [B] (verification not implemented)

Time = 9.59 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{a-b\,x^3}-\sqrt{a}\right)\,\left(\sqrt{a-b\,x^3}+\sqrt{a}+2\,a^{1/6}\,b^{1/3}\,x\right)^3}{x^3\,\left(b^{1/3}\,x+2\,a^{1/3}\right)^3}\right)}{3\,a^{1/6}\,b^{1/3}}$$

input
```
int(-(b^(1/3)*x - a^(1/3))/((b^(1/3)*x + 2*a^(1/3))*(a - b*x^3)^(1/2)),x)
```

output
```
log((((a - b*x^3)^(1/2) - a^(1/2))*((a - b*x^3)^(1/2) + a^(1/2) + 2*a^(1/6
)*b^(1/3)*x)^3)/(x^3*(b^(1/3)*x + 2*a^(1/3))^3))/(3*a^(1/6)*b^(1/3))
```

3.80 $\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

3.80.1 Optimal result

Integrand size = 45, antiderivative size = 53

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=-\frac{2\arctan\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a+bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
-2/3*arctan(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(b*x^3-a)^(1/2))/a^(1/6)/b^(
1/3)
```

3.80.2 Mathematica [A] (verified)

Time = 2.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\frac{2\arctan\left(\frac{3\sqrt[6]{a}\sqrt{-a+bx^3}}{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3])
,x]
```

output
```
(2*ArcTan[(3*a^(1/6)*Sqrt[-a + b*x^3])/(a^(1/3) - b^(1/3)*x)^2])/(3*a^(1/6
)*b^(1/3))
```

3.80.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.044, Rules used = {2563, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{2563}$$

$$-\frac{2\sqrt[3]{a}\int \frac{1}{\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^4}{\sqrt[3]{a}(bx^3-a)}+9}d\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{a^{2/3}\sqrt{bx^3-a}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\arctan\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{bx^3-a}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
(-2*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a + b*x^3])])/(3*a^(1/
6)*b^(1/3))
```

3.80.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.80.4 Maple [F]

$$\int \frac{a^{\frac{1}{3}}-b^{\frac{1}{3}}x}{\left(2a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{b\,x^{3}-a}}dx$$

input
```
int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output
```
int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

3.80.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. 2(38) = 76.

Time = 0.90 (sec) , antiderivative size = 592, normalized size of antiderivative = 11.17

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$$
$$=\left[\frac{1}{6}\,a^{\frac{1}{3}}\sqrt{-\frac{1}{ab^{\frac{2}{3}}}}\log\left(\frac{b^{6}x^{18}-7800\,ab^{5}x^{15}+535272\,a^{2}b^{4}x^{12}-5147264\,a^{3}b^{3}x^{9}+10516992\,a^{4}b^{2}x^{6}-592281}{}\right.\right.$$

input
```
integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, alg
orithm="fricas")
```

output
```
[1/6*a^(1/3)*sqrt(-1/(a*b^(2/3)))*log((b^6*x^18 - 7800*a*b^5*x^15 + 535272
*a^2*b^4*x^12 - 5147264*a^3*b^3*x^9 + 10516992*a^4*b^2*x^6 - 5922816*a^5*b
*x^3 + 557056*a^6 + 144*(7*b^5*x^16 - 1169*a*b^4*x^13 + 20266*a^2*b^3*x^10
 - 66976*a^3*b^2*x^7 + 58112*a^4*b*x^4 - 10240*a^5*x)*a^(2/3)*b^(1/3) - 72
*(b^5*x^17 - 581*a*b^4*x^14 + 19108*a^2*b^3*x^11 - 106336*a^3*b^2*x^8 + 13
7984*a^4*b*x^5 - 50176*a^5*x^2)*a^(1/3)*b^(2/3) - 12*sqrt(b*x^3 - a)*((b^5
*x^16 - 1568*a*b^4*x^13 + 72520*a^2*b^3*x^10 - 498304*a^3*b^2*x^7 + 625664
*a^4*b*x^4 - 139264*a^5*x)*a^(2/3)*b^(2/3) + 6*(41*a*b^5*x^14 - 4268*a^2*b
^4*x^11 + 52896*a^3*b^3*x^8 - 116480*a^4*b^2*x^5 + 48128*a^5*b*x^2)*a^(1/3
) - (25*a*b^5*x^15 - 7202*a^2*b^4*x^12 + 167392*a^3*b^3*x^9 - 647296*a^4*b
^2*x^6 + 468992*a^5*b*x^3 - 40960*a^6)*b^(1/3))*sqrt(-1/(a*b^(2/3))))/(b^6
*x^18 + 48*a*b^5*x^15 + 960*a^2*b^4*x^12 + 10240*a^3*b^3*x^9 + 61440*a^4*b
^2*x^6 + 196608*a^5*b*x^3 + 262144*a^6)), 1/3*a^(1/3)*sqrt(1/(a*b^(2/3)))*
arctan(1/6*sqrt(b*x^3 - a)*((11*b*x^4 + 16*a*x)*a^(2/3)*b^(2/3) - (b^2*x^5
 - 28*a*b*x^2)*a^(1/3) + (17*a*b*x^3 + 10*a^2)*b^(1/3))*sqrt(1/(a*b^(2/3))
)/(b^2*x^6 - 2*a*b*x^3 + a^2))]
```

3.80.6 Sympy [F]

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\int \left(-\frac{\sqrt[3]{a}}{2\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\right)dx - \int \frac{\sqrt[3]{b}x}{2\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

input
```
integrate((a**(1/3)-b**(1/3)*x)/(2*a**(1/3)+b**(1/3)*x)/(b*x**3-a)**(1/2),
x)
```

output
```
-Integral(-a**(1/3)/(2*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b
*x**3)), x) - Integral(b**(1/3)*x/(2*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)
*x*sqrt(-a + b*x**3)), x)
```

3.80.7 Maxima [F]

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int -\frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, alg
orithm="maxima")
```

output
```
-integrate((b^(1/3)*x - a^(1/3))/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3)))
, x)
```

3.80.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, alg
orithm="giac")
```

output
```
Timed out
```

3.80.9 Mupad [B] (verification not implemented)

Time = 10.75 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\frac{\ln\left(\frac{\left(\sqrt{b\,x^3-a}+\sqrt{a}\,1i\right)\left(\sqrt{a}+2\,a^{1/6}\,b^{1/3}\,x+\sqrt{b\,x^3-a}\,1i\right)^3}{x^3\,\left(b^{1/3}\,x+2\,a^{1/3}\right)^3}\right)\,1i}{3\,a^{1/6}\,b^{1/3}}$$

input
```
int(-(b^(1/3)*x - a^(1/3))/((b^(1/3)*x + 2*a^(1/3))*(b*x^3 - a)^(1/2)),x)
```

output
```
(log((((b*x^3 - a)^(1/2) + a^(1/2)*1i)*((b*x^3 - a)^(1/2)*1i + a^(1/2) + 2
*a^(1/6)*b^(1/3)*x)^3)/(x^3*(b^(1/3)*x + 2*a^(1/3))^3))*1i)/(3*a^(1/6)*b^(
1/3))
```

3.81 $\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

3.81.1 Optimal result

Integrand size = 46, antiderivative size = 53

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\frac{2\arctan\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
2/3*arctan(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(-b*x^3-a)^(1/2))/a^(1/6)/b^(
1/3)
```

3.81.2 Mathematica [A] (verified)

Time = 2.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=-\frac{2\arctan\left(\frac{3\sqrt[6]{a}\sqrt{-a-bx^3}}{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3])
,x]
```

output
```
(-2*ArcTan[(3*a^(1/6)*Sqrt[-a - b*x^3])/(a^(1/3) + b^(1/3)*x)^2])/(3*a^(1/
6)*b^(1/3))
```

3.81.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2563, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}dx$$

$$\Big\downarrow \text{2563}$$

$$\frac{2\sqrt[3]{a}\int \frac{1}{\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^4}{\sqrt[3]{a}\left(-bx^3-a\right)}+9}d\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\sqrt{-bx^3-a}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\arctan\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
(2*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a - b*x^3])])/(3*a^(1/6
)*b^(1/3))
```

3.81.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.81.4 Maple [F]

$$\int \frac{a^{\frac{1}{3}}+b^{\frac{1}{3}} x}{\left(2 a^{\frac{1}{3}}-b^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3}-a}}dx$$

input
```
int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

output
```
int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

3.81.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(37) = 74$.

Time = 0.90 (sec) , antiderivative size = 641, normalized size of antiderivative = 12.09

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$$
$$= \left[\frac{1}{6}\,a^{\frac{1}{3}}\sqrt{-\frac{1}{ab^{\frac{2}{3}}}}\log\left(\frac{b^6x^{18}+7800\,ab^5x^{15}+535272\,a^2b^4x^{12}+5147264\,a^3b^3x^9+10516992\,a^4b^2x^6+592281\cdots}{\cdots}\right.\right.$$
$$-\frac{1}{3}\,a^{\frac{1}{3}}\sqrt{\frac{1}{ab^{\frac{2}{3}}}}\arctan\left(\frac{\left((11\,bx^4-16\,ax)\sqrt{-bx^3-a}a^{\frac{2}{3}}b^{\frac{2}{3}}+(b^2x^5+28\,abx^2)\sqrt{-bx^3-a}a^{\frac{1}{3}}-(17\,abx^3-1\cdots\right.}{6\,(b^2x^6+2\,abx^3+a^2)}\right.$$

input
```
integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, al
gorithm="fricas")
```

output
```
[1/6*a^(1/3)*sqrt(-1/(a*b^(2/3)))*log((b^6*x^18 + 7800*a*b^5*x^15 + 535272
*a^2*b^4*x^12 + 5147264*a^3*b^3*x^9 + 10516992*a^4*b^2*x^6 + 5922816*a^5*b
*x^3 + 557056*a^6 + 144*(7*b^5*x^16 + 1169*a*b^4*x^13 + 20266*a^2*b^3*x^10
 + 66976*a^3*b^2*x^7 + 58112*a^4*b*x^4 + 10240*a^5*x)*a^(2/3)*b^(1/3) + 72
*(b^5*x^17 + 581*a*b^4*x^14 + 19108*a^2*b^3*x^11 + 106336*a^3*b^2*x^8 + 13
7984*a^4*b*x^5 + 50176*a^5*x^2)*a^(1/3)*b^(2/3) + 12*((b^5*x^16 + 1568*a*b
^4*x^13 + 72520*a^2*b^3*x^10 + 498304*a^3*b^2*x^7 + 625664*a^4*b*x^4 + 139
264*a^5*x)*sqrt(-b*x^3 - a)*a^(2/3)*b^(2/3) + 6*(41*a*b^5*x^14 + 4268*a^2*
b^4*x^11 + 52896*a^3*b^3*x^8 + 116480*a^4*b^2*x^5 + 48128*a^5*b*x^2)*sqrt(
-b*x^3 - a)*a^(1/3) + (25*a*b^5*x^15 + 7202*a^2*b^4*x^12 + 167392*a^3*b^3*
x^9 + 647296*a^4*b^2*x^6 + 468992*a^5*b*x^3 + 40960*a^6)*sqrt(-b*x^3 - a)*
b^(1/3))*sqrt(-1/(a*b^(2/3))))/(b^6*x^18 - 48*a*b^5*x^15 + 960*a^2*b^4*x^1
2 - 10240*a^3*b^3*x^9 + 61440*a^4*b^2*x^6 - 196608*a^5*b*x^3 + 262144*a^6)
), -1/3*a^(1/3)*sqrt(1/(a*b^(2/3)))*arctan(1/6*((11*b*x^4 - 16*a*x)*sqrt(-
b*x^3 - a)*a^(2/3)*b^(2/3) + (b^2*x^5 + 28*a*b*x^2)*sqrt(-b*x^3 - a)*a^(1/
3) - (17*a*b*x^3 - 10*a^2)*sqrt(-b*x^3 - a)*b^(1/3))*sqrt(1/(a*b^(2/3)))/(
b^2*x^6 + 2*a*b*x^3 + a^2))]
```

3.81.6 Sympy [F]

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\int \frac{\sqrt[3]{a}}{-2\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}x\sqrt{-a-bx^3}}\,dx - \int \frac{\sqrt[3]{b}x}{-2\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}x\sqrt{-a-bx^3}}\,dx$$

input
```
integrate((a**(1/3)+b**(1/3)*x)/(2*a**(1/3)-b**(1/3)*x)/(-b*x**3-a)**(1/2)
,x)
```

output
```
-Integral(a**(1/3)/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b
*x**3)), x) - Integral(b**(1/3)*x/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3
)*x*sqrt(-a - b*x**3)), x)
```

3.81.7 Maxima [F]

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int -\frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, al
gorithm="maxima")
```

output
```
-integrate((b^(1/3)*x + a^(1/3))/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3))
), x)
```

3.81.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, al
gorithm="giac")
```

output
```
Timed out
```

3.81.9 Mupad [B] (verification not implemented)

Time = 10.70 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\frac{\ln\left(\frac{\left(\sqrt{-b\,x^3-a}-\sqrt{a}\,1\mathrm{i}\right)\,{\left(2\,a^{1/6}\,b^{1/3}\,x-\sqrt{a}+\sqrt{-b\,x^3-a}\,1\mathrm{i}\right)}^3}{x^3\,{\left(b^{1/3}\,x-2\,a^{1/3}\right)}^3}\right)\,1\mathrm{i}}{3\,a^{1/6}\,b^{1/3}}$$

input
```
int(-(b^(1/3)*x + a^(1/3))/((b^(1/3)*x - 2*a^(1/3))*(- a - b*x^3)^(1/2)),x
)
```

output
```
(log((((- a - b*x^3)^(1/2) - a^(1/2)*1i)*((- a - b*x^3)^(1/2)*1i - a^(1/2)
 + 2*a^(1/6)*b^(1/3)*x)^3)/(x^3*(b^(1/3)*x - 2*a^(1/3))^3))*1i)/(3*a^(1/6)
*b^(1/3))
```

3.81. $\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

3.82 $\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$

3.82.1 Optimal result

Integrand size = 30, antiderivative size = 46

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = -\frac{2\text{arctanh}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{3\sqrt{cd}}$$

output
```
-2/3*arctanh(1/3*(-2*d*x+c)^2/c^(1/2)/(-8*d^3*x^3+c^3)^(1/2))/d/c^(1/2)
```

3.82.2 Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = -\frac{2\text{arctanh}\left(\frac{3\sqrt{c}\sqrt{c^3-8d^3x^3}}{(c-2dx)^2}\right)}{3\sqrt{cd}}$$

input
```
Integrate[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

output
```
(-2*ArcTanh[(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])/(c - 2*d*x)^2])/(3*Sqrt[c]*d
)
```

3.82.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$$

$$\Big\downarrow \text{ 2563}$$

$$-\frac{2c\int \frac{1}{9-\frac{(c-2dx)^4}{c\left(c^3-8d^3x^3\right)}}d\frac{(c-2dx)^2}{c^2\sqrt{c^3-8d^3x^3}}}{d}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{2\text{arctanh}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{3\sqrt{c}d}$$

input
```
Int[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

output
```
(-2*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(3*Sqrt[c]*d
)
```

3.82.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

3.82.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 1.52 (sec) , antiderivative size = 503, normalized size of antiderivative = 10.93

method	result
default	$-\frac{4\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}\right)\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}\,F\left(\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}},\sqrt{\frac{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\right.}{\sqrt{-8d^3x^3+c^3}}$
elliptic	$-\frac{4\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}\right)\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}\,F\left(\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}},\sqrt{\frac{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\right.}{\sqrt{-8d^3x^3+c^3}}$

input
```
int((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4*(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^
(1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*
(-1/2+1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*c/
d-1/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)*EllipticF(((
x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2),((1/2*c/d-1/2*(-1
/2-1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2))+4*(1
/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2)
)*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(-1/2
+1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2
*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)*EllipticPi(((x-1/
2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2),2/3*(1/2*c/d-1/2*(-1/
2-1/2*I*3^(1/2))*c/d)/c*d,((1/2*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*c/d
-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2))
```

3.82.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(38) = 76$.

Time = 0.41 (sec) , antiderivative size = 294, normalized size of antiderivative = 6.39

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$$
$$= \left[\frac{\log\left(\frac{8\,d^6x^6-240\,cd^5x^5+408\,c^2d^4x^4+88\,c^3d^3x^3+156\,c^4d^2x^2+12\,c^5dx+17\,c^6-3\,(8\,d^4x^4-52\,cd^3x^3+12\,c^2d^2x^2-4\,c^3dx+5\,c^4)\sqrt{-8\,d^3x^3+c^3}\sqrt{c}}{d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx+c^6}\right)}{6\sqrt{c}d}\right.$$
$$\left. -\frac{\sqrt{-c}\arctan\left(\frac{(4\,d^3x^3-24\,cd^2x^2-6\,c^2dx-5\,c^3)\sqrt{-8\,d^3x^3+c^3}\sqrt{-c}}{3\,(16\,cd^4x^4-8\,c^2d^3x^3-2\,c^4dx+c^5)}\right)}{3\,cd}\right]$$

input
```
integrate((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*log((8*d^6*x^6 - 240*c*d^5*x^5 + 408*c^2*d^4*x^4 + 88*c^3*d^3*x^3 + 1
56*c^4*d^2*x^2 + 12*c^5*d*x + 17*c^6 - 3*(8*d^4*x^4 - 52*c*d^3*x^3 + 12*c^
2*d^2*x^2 - 4*c^3*d*x + 5*c^4)*sqrt(-8*d^3*x^3 + c^3)*sqrt(c))/(d^6*x^6 +
6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x^2 + 6*c^5*d*x
 + c^6))/(sqrt(c)*d), -1/3*sqrt(-c)*arctan(1/3*(4*d^3*x^3 - 24*c*d^2*x^2 -
 6*c^2*d*x - 5*c^3)*sqrt(-8*d^3*x^3 + c^3)*sqrt(-c)/(16*c*d^4*x^4 - 8*c^2*
d^3*x^3 - 2*c^4*d*x + c^5))/(c*d)]
```

3.82.6 Sympy [F]

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = -\int \left(-\frac{c}{c\sqrt{c^3-8d^3x^3}+dx\sqrt{c^3-8d^3x^3}}\right)dx - \int \frac{2dx}{c\sqrt{c^3-8d^3x^3}+dx\sqrt{c^3-8d^3x^3}}\,dx$$

input
```
integrate((-2*d*x+c)/(d*x+c)/(-8*d**3*x**3+c**3)**(1/2),x)
```

output
```
-Integral(-c/(c*sqrt(c**3 - 8*d**3*x**3) + d*x*sqrt(c**3 - 8*d**3*x**3)),
x) - Integral(2*d*x/(c*sqrt(c**3 - 8*d**3*x**3) + d*x*sqrt(c**3 - 8*d**3*x
**3)), x)
```

3.82.7 Maxima [F]

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \int -\frac{2\,dx-c}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}\,dx$$

input
```
integrate((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((2*d*x - c)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

3.82.8 Giac [F]

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \int -\frac{2\,dx-c}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}\,dx$$

input
```
integrate((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(2*d*x - c)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

3.82.9 Mupad [B] (verification not implemented)

Time = 9.71 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.46

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \frac{\ln\left(\frac{\left(\sqrt{c^3-8\,d^3\,x^3}-c^{3/2}\right)\left(\sqrt{c^3-8\,d^3\,x^3}+c^{3/2}+4\,\sqrt{c}\,d\,x\right)^3}{x^3\,(c+d\,x)^3}\right)}{3\,\sqrt{c}\,d}$$

input
```
int((c - 2*d*x)/((c^3 - 8*d^3*x^3)^(1/2)*(c + d*x)),x)
```

output
```
log((((c^3 - 8*d^3*x^3)^(1/2) - c^(3/2))*((c^3 - 8*d^3*x^3)^(1/2) + c^(3/2
) + 4*c^(1/2)*d*x)^3)/(x^3*(c + d*x)^3))/(3*c^(1/2)*d)
```

3.83 $\int \frac{e+fx}{(2-x)\sqrt{1+x^3}} dx$

3.83.1 Optimal result

Integrand size = 22, antiderivative size = 139

$$\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx = \frac{2}{9}(e+2f)\operatorname{arctanh}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right) + \frac{2\sqrt{2+\sqrt{3}}(e-f)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
2/9*(e+2*f)*arctanh(1/3*(1+x)^2/(x^3+1)^(1/2))+2/9*(e-f)*(1+x)*EllipticF((
1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-
x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^
(1/2)
```

3.83.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.40 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.96

$$\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx = \frac{2\sqrt{\frac{2}{3}}\sqrt{-\frac{i(1+x)}{-3i+\sqrt{3}}}\left(-3if\sqrt{i+\sqrt{3}-2ix}\left(-i-\sqrt{3}+\left(-i+\sqrt{3}\right)x\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)\right.}{\left(3i+\sqrt{3}\right)\sqrt{-i+\sqrt{3}+2ix}\ldots}$$

input
```
Integrate[(e + f*x)/((2 - x)*Sqrt[1 + x^3]),x]
```

output
```
(2*Sqrt[2/3]*Sqrt[((-I)*(1 + x))/(-3*I + Sqrt[3])]*((-3*I)*f*Sqrt[I + Sqrt
[3] - (2*I)*x]*(-I - Sqrt[3] + (-I + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[-I
+ Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] + 2
*Sqrt[3]*(e + 2*f)*Sqrt[-I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x + x^2]*Elliptic
Pi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] + (2*I)*x]/(Sqrt[
2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + S
qrt[3] + (2*I)*x]*Sqrt[1 + x^3])
```

3.83.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2564, 27, 759, 2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{(2-x)\sqrt{x^3+1}}\,dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{1}{3}(e-f)\int \frac{1}{\sqrt{x^3+1}}dx+\frac{1}{6}(e+2f)\int \frac{2(x+1)}{(2-x)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}(e-f)\int \frac{1}{\sqrt{x^3+1}}dx+\frac{1}{3}(e+2f)\int \frac{x+1}{(2-x)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \color{blue}{759}$$

$$\frac{1}{3}(e+2f)\int \frac{x+1}{(2-x)\sqrt{x^3+1}}dx+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(e-f)\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \color{blue}{2563}$$

$$\frac{2}{3}(e+2f)\int \frac{1}{9-\frac{(x+1)^4}{x^3+1}}d\frac{(x+1)^2}{\sqrt{x^3+1}}+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(e-f)\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \color{blue}{219}$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(e-f)\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2}{9}\text{arctanh}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)(e+2f)$$

input
```
Int[(e + f*x)/((2 - x)*Sqrt[1 + x^3]),x]
```

output
```
(2*(e + 2*f)*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/9 + (2*Sqrt[2 + Sqrt[3]
]*(e - f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin
[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1
 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

3.83. $\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx$

3.83.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.83.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(114) = 228$.

Time = 0.95 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.77

method	result
default	$-\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2(e+2f)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{}}{3\sqrt{}}$
elliptic	$-\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2(-e-2f)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{3\sqrt{}}$

input
```
int((f*x+e)/(2-x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*f*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3
^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(
1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-
3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(e+2*f)*(3/2-1/2*I*3^(
1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I
*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1
)^(1/2)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),1/2-1/6*I*3^(1/2),((-
3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

3.83.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.44

$$\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx = \frac{1}{9}\,(e+2\,f)\log\left(\frac{x^3+12\,x^2+6\,\sqrt{x^3+1}(x+1)-6\,x+10}{x^3-6\,x^2+12\,x-8}\right) + \frac{2}{3}\,(e-f)\mathrm{weierstrassPInverse}(0,-4,x)$$

input
```
integrate((f*x+e)/(2-x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/9*(e + 2*f)*log((x^3 + 12*x^2 + 6*sqrt(x^3 + 1)*(x + 1) - 6*x + 10)/(x^3
 - 6*x^2 + 12*x - 8)) + 2/3*(e - f)*weierstrassPInverse(0, -4, x)
```

3.83.6 Sympy [F]

$$\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx = -\int \frac{e}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx - \int \frac{fx}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx$$

input
```
integrate((f*x+e)/(2-x)/(x**3+1)**(1/2),x)
```

output
```
-Integral(e/(x*sqrt(x**3 + 1) - 2*sqrt(x**3 + 1)), x) - Integral(f*x/(x*sq
rt(x**3 + 1) - 2*sqrt(x**3 + 1)), x)
```

3.83.7 Maxima [F]

$$\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx = \int -\frac{fx+e}{\sqrt{x^3+1}(x-2)}\,dx$$

input
```
integrate((f*x+e)/(2-x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((f*x + e)/(sqrt(x^3 + 1)*(x - 2)), x)
```

3.83.8 Giac [F]

$$\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx = \int -\frac{fx+e}{\sqrt{x^3+1}(x-2)}\,dx$$

input
```
integrate((f*x+e)/(2-x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(f*x + e)/(sqrt(x^3 + 1)*(x - 2)), x)
```

3.83.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.35

$$\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx$$
$$=\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,(e+2\,f)\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{6};\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$
$$-\frac{2\,f\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

input
```
int(-(e + f*x)/((x^3 + 1)^(1/2)*(x - 2)),x)
```

output
```
(2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/
2))^(1/2)*(e + 2*f)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/
2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/6 + 1/2
, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((
3^(1/2)*1i)/2 - 3/2)))/(3*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (2*
f*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2
))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2
)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*((
(3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

3.84 $\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx$

3.84.1 Optimal result

Integrand size = 22, antiderivative size = 153

$$\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx = -\frac{2}{9}(e-2f)\operatorname{arctanh}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right) - \frac{2\sqrt{2+\sqrt{3}}(e+f)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-2/9*(e-2*f)*arctanh(1/3*(1-x)^2/(-x^3+1)^(1/2))-2/9*(e+f)*(1-x)*EllipticF
((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^
2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^
2)^(1/2)
```

3.84.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.23 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.77

$$\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(-1+x)}{-3i+\sqrt{3}}}\left(3f\sqrt{i+\sqrt{3}+2ix}(-1+i\sqrt{3}+x+i\sqrt{3}x)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-\right.}{(3i+\sqrt{3})\sqrt{-i+\sqrt{3}-}}$$

input
```
Integrate[(e + f*x)/((2 + x)*Sqrt[1 - x^3]),x]
```

output
```
(2*Sqrt[2/3]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(3*f*Sqrt[I + Sqrt[3] + (
2*I)*x]*(-1 + I*Sqrt[3] + x + I*Sqrt[3]*x)*EllipticF[ArcSin[Sqrt[-I + Sqrt
[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] - 2*Sqrt[
3]*(e - 2*f)*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*
Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(
1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + Sqrt[3]
 - (2*I)*x]*Sqrt[1 - x^3])
```

3.84.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {2564, 27, 759, 2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{(x+2)\sqrt{1-x^3}}dx$$
$$\Big\downarrow \text{2564}$$
$$\frac{1}{3}(e+f)\int \frac{1}{\sqrt{1-x^3}}dx+\frac{1}{6}(e-2f)\int \frac{2(1-x)}{(x+2)\sqrt{1-x^3}}dx$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{3}(e+f)\int \frac{1}{\sqrt{1-x^3}}dx+\frac{1}{3}(e-2f)\int \frac{1-x}{(x+2)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \; 759$$

$$\frac{1}{3}(e-2f)\int \frac{1-x}{(x+2)\sqrt{1-x^3}}dx-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(e+f)\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \; 2563$$

$$-\frac{2}{3}(e-2f)\int \frac{1}{9-\frac{(1-x)^4}{1-x^3}}d\frac{(1-x)^2}{\sqrt{1-x^3}}-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(e+f)\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \; 219$$

$$-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(e+f)\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{9}\text{arctanh}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)(e-2f)$$

input
```
Int[(e + f*x)/((2 + x)*Sqrt[1 - x^3]),x]
```

output
```
(-2*(e - 2*f)*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/9 - (2*Sqrt[2 + Sqrt[3
]]*(e + f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSi
n[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(
1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

3.84. $\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx$

3.84.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.84.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.61

method	result
default	$-\frac{2 i f \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, F\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}+1}}-\frac{2 i (e-2 f) \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2} \cdots\right.}}{\cdots}$
elliptic	$-\frac{2 i f \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, F\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}+1}}-\frac{2 i (e-2 f) \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2} \cdots\right.}}{\cdots}$

input
```
int((f*x+e)/(x+2)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*f*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*
I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(
-3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(e-2*f)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))
*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2
))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)),(I
*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

3.84.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.43

$$\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx = -\frac{1}{9}\,(e-2\,f)\log\left(-\frac{x^{3}-12\,x^{2}+6\,\sqrt{-x^{3}+1}(x-1)-6\,x-10}{x^{3}+6\,x^{2}+12\,x+8}\right) - \frac{2}{3}\,(i\,e+i\,f)\text{weierstrassPInverse}(0,4,x)$$

input
```
integrate((f*x+e)/(2+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/9*(e - 2*f)*log(-(x^3 - 12*x^2 + 6*sqrt(-x^3 + 1)*(x - 1) - 6*x - 10)/(
x^3 + 6*x^2 + 12*x + 8)) - 2/3*(I*e + I*f)*weierstrassPInverse(0, 4, x)
```

3.84.6 Sympy [F]

$$\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx = \int \frac{e+fx}{\sqrt{-(x-1)(x^2+x+1)}\,(x+2)}\,dx$$

input
```
integrate((f*x+e)/(2+x)/(-x**3+1)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt(-(x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

3.84.7 Maxima [F]

$$\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx = \int \frac{fx+e}{\sqrt{-x^3+1}(x+2)}\,dx$$

input
```
integrate((f*x+e)/(2+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

3.84.8 Giac [F]

$$\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx = \int \frac{fx+e}{\sqrt{-x^3+1}(x+2)}\,dx$$

input
```
integrate((f*x+e)/(2+x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x + e)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

3.84.9 Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.35

$$\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx = -\frac{2\,f\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} - \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,(e-2\,f)\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\right.}{3\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int((e + f*x)/((1 - x^3)^(1/2)*(x + 2)),x)
```

output
```
- (2*f*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)
/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2
 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x
 - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i
)/2 - 3/2)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/
2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) -
 (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((
3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 +
3/2))^(1/2)*(e - 2*f)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((
3^(1/2)*1i)/6 + 1/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(
1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(1 - x^3)^(1/2)*(((3^(1/2)*1
i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1
i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.85 $\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx$

3.85.1 Optimal result

Integrand size = 20, antiderivative size = 156

$$\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx = -\frac{2}{9}(e-2f)\arctan\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right) - \frac{2\sqrt{2-\sqrt{3}}(e+f)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
-2/9*(e-2*f)*arctan(1/3*(1-x)^2/(x^3-1)^(1/2))-2/9*(e+f)*(1-x)*EllipticF((
1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+
x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)
^(1/2)
```

3.85.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.22 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.72

$$\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(-1+x)}{-3i+\sqrt{3}}}\left(3f\sqrt{i+\sqrt{3}+2ix}(-1+i\sqrt{3}+x+i\sqrt{3}x)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-\right.}{(3i+\sqrt{3})\sqrt{-i+\sqrt{3}-}}$$

input
```
Integrate[(e + f*x)/((2 + x)*Sqrt[-1 + x^3]),x]
```

output
```
(2*Sqrt[2/3]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(3*f*Sqrt[I + Sqrt[3] + (
2*I)*x]*(-1 + I*Sqrt[3] + x + I*Sqrt[3]*x)*EllipticF[ArcSin[Sqrt[-I + Sqrt
[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] - 2*Sqrt[
3]*(e - 2*f)*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*
Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(
1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + Sqrt[3]
 - (2*I)*x]*Sqrt[-1 + x^3])
```

3.85.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2564, 27, 760, 2563, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{(x+2)\sqrt{x^3-1}}\,dx$$
$$\Big\downarrow \text{2564}$$
$$\frac{1}{3}(e+f)\int \frac{1}{\sqrt{x^3-1}}dx+\frac{1}{6}(e-2f)\int \frac{2(1-x)}{(x+2)\sqrt{x^3-1}}dx$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{3}(e+f)\int \frac{1}{\sqrt{x^3-1}}dx+\frac{1}{3}(e-2f)\int \frac{1-x}{(x+2)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$\frac{1}{3}(e-2f)\int \frac{1-x}{(x+2)\sqrt{x^3-1}}dx-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}(e+f)\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{2563}$$

$$-\frac{2}{3}(e-2f)\int \frac{1}{\frac{(1-x)^4}{x^3-1}+9}d\frac{(1-x)^2}{\sqrt{x^3-1}}-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}(e+f)\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}(e+f)\,\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2}{9}\arctan\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)(e-2f)$$

input
```
Int[(e + f*x)/((2 + x)*Sqrt[-1 + x^3]),x]
```

output
```
(-2*(e - 2*f)*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/9 - (2*Sqrt[2 - Sqrt[3
]]*(e + f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSi
n[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-
((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

3.85. $\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx$

3.85.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.85.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.58

method	result
default	$\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2(e-2f)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{3}$
elliptic	$\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2(e-2f)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{3}$

input
```
int((f*x+e)/(x+2)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*f*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3
/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(e-2*f)*(-3/2-1/2*I*3^(1
/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*
3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^
(1/2)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/2+1/6*I*3^(1/2),((3/
2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

3.85.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.35

$$\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx = -\frac{1}{9}\,(e-2\,f)\arctan\left(\frac{(x^3-12\,x^2-6\,x-10)\sqrt{x^3-1}}{6\,(x^4-x^3-x+1)}\right) + \frac{2}{3}\,(e+f)\mathrm{weierstrassPInverse}(0,4,x)$$

input
```
integrate((f*x+e)/(2+x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/9*(e - 2*f)*arctan(1/6*(x^3 - 12*x^2 - 6*x - 10)*sqrt(x^3 - 1)/(x^4 - x
^3 - x + 1)) + 2/3*(e + f)*weierstrassPInverse(0, 4, x)
```

3.85.6 Sympy [F]

$$\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx = \int \frac{e+fx}{\sqrt{(x-1)\,(x^2+x+1)}\,(x+2)}\,dx$$

input
```
integrate((f*x+e)/(2+x)/(x**3-1)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt((x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

3.85.7 Maxima [F]

$$\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx=\int \frac{fx+e}{\sqrt{x^3-1}(x+2)}\,dx$$

input
```
integrate((f*x+e)/(2+x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(x^3 - 1)*(x + 2)), x)
```

3.85.8 Giac [F]

$$\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx=\int \frac{fx+e}{\sqrt{x^3-1}(x+2)}\,dx$$

input
```
integrate((f*x+e)/(2+x)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x + e)/(sqrt(x^3 - 1)*(x + 2)), x)
```

3.85.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.10

$$\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx$$
$$=-\frac{2\,f\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}}$$
$$-\frac{2\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\left(e-2\,f\right)\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}\right)}{3\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}}$$

input
```
int((e + f*x)/((x^3 - 1)^(1/2)*(x + 2)),x)
```

output
```
- (2*f*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2
 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(
-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*
1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((
3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((
3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2) - (2*((3^(1/2)*1i)/2 + 3/2)*(-(x -
(3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 +
 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(e - 2*f)*(-(x - 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((-(x - 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(((3
^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3
^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.85. $\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx$

3.86 $\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx$

3.86.1 Optimal result

Integrand size = 24, antiderivative size = 150

$$\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx = \frac{2}{9}(e+2f)\arctan\left(\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right) + \frac{2\sqrt{2-\sqrt{3}}(e-f)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
2/9*(e+2*f)*arctan(1/3*(1+x)^2/(-x^3-1)^(1/2))+2/9*(e-f)*(1+x)*EllipticF((
1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-
x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2
)^(1/2)
```

3.86.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.25 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.83

$$\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{2}{3}}\sqrt{-\frac{i(1+x)}{-3i+\sqrt{3}}}\left(-3if\sqrt{i+\sqrt{3}-2ix}\left(-i-\sqrt{3}+\left(-i+\sqrt{3}\right)x\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)\ldots}{\left(3i+\sqrt{3}\right)\sqrt{-i+\sqrt{3}\ldots}}$$

input
```
Integrate[(e + f*x)/((2 - x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*Sqrt[2/3]*Sqrt[((-I)*(1 + x))/(-3*I + Sqrt[3])]*((-3*I)*f*Sqrt[I + Sqrt
[3] - (2*I)*x]*(-I - Sqrt[3] + (-I + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[-I
+ Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] + 2
*Sqrt[3]*(e + 2*f)*Sqrt[-I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x + x^2]*Elliptic
Pi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] + (2*I)*x]/(Sqrt[
2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + S
qrt[3] + (2*I)*x]*Sqrt[-1 - x^3])
```

3.86.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2564, 27, 760, 2563, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{(2-x)\sqrt{-x^3-1}}\,dx$$
$$\Big\downarrow \text{ 2564}$$
$$\frac{1}{3}(e-f)\int \frac{1}{\sqrt{-x^3-1}}dx+\frac{1}{6}(e+2f)\int \frac{2(x+1)}{(2-x)\sqrt{-x^3-1}}dx$$
$$\Big\downarrow \text{ 27}$$
$$\frac{1}{3}(e-f)\int \frac{1}{\sqrt{-x^3-1}}dx+\frac{1}{3}(e+2f)\int \frac{x+1}{(2-x)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$\frac{1}{3}(e+2f)\int \frac{x+1}{(2-x)\sqrt{-x^3-1}}dx+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}(e-f)\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{2563}$$

$$\frac{2}{3}(e+2f)\int \frac{1}{\frac{(x+1)^4}{-x^3-1}+9}d\frac{(x+1)^2}{\sqrt{-x^3-1}}+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}(e-f)\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}(e-f)\,\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2}{9}\arctan\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)(e+2f)$$

input
```
Int[(e + f*x)/((2 - x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*(e + 2*f)*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/9 + (2*Sqrt[2 - Sqrt[3]
]*(e - f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin
[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-(
(1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

3.86. $\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx$

3.86.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.86.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.64

method	result
default	$\frac{2if\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}-1}}+\frac{2i(e+2f)\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right.}}{}$
elliptic	$\frac{2if\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}-1}}-\frac{2i(-e-2f)\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right.}}{}$

input
```
int((f*x+e)/(2-x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*f*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/
2+1/2*I*3^(1/2)))^(1/2))+2/3*I*(e+2*f)*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3
^(1/2))^(1/2)/(-x^3-1)^(1/2)/(-3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(
I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)),(I*3
^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

3.86.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.38

$$\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx = -\frac{1}{9}\,(e+2\,f)\arctan\left(\frac{(x^3+12\,x^2-6\,x+10)\sqrt{-x^3-1}}{6\,(x^4+x^3+x+1)}\right) - \frac{2}{3}\,(i\,e-i\,f)\text{weierstrassPInverse}(0,-4,x)$$

input
```
integrate((f*x+e)/(2-x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/9*(e + 2*f)*arctan(1/6*(x^3 + 12*x^2 - 6*x + 10)*sqrt(-x^3 - 1)/(x^4 +
x^3 + x + 1)) - 2/3*(I*e - I*f)*weierstrassPInverse(0, -4, x)
```

3.86.6 Sympy [F]

$$\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx = -\int \frac{e}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx - \int \frac{fx}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx$$

input
```
integrate((f*x+e)/(2-x)/(-x**3-1)**(1/2),x)
```

output
```
-Integral(e/(x*sqrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x) - Integral(f*x/(x*
sqrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x)
```

3.86.7 Maxima [F]

$$\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx = \int -\frac{fx+e}{\sqrt{-x^3-1}(x-2)}\,dx$$

input
```
integrate((f*x+e)/(2-x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((f*x + e)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

3.86.8 Giac [F]

$$\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx = \int -\frac{fx+e}{\sqrt{-x^3-1}(x-2)}\,dx$$

input
```
integrate((f*x+e)/(2-x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(f*x + e)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

3.86.9 Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.39

$$\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx = -\frac{2\,f\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} + \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,(e+2\,f)\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+}{-\frac{3}{2}+}\right.}{3\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(-(e + f*x)/((- x^3 - 1)^(1/2)*(x - 2)),x)
```

output
```
(2*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*(e + 2*f)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(
1/2)*1i)/6 + 1/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)
*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(- x^3 - 1)^(1/2)*(x^3 - x*(((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((
3^(1/2)*1i)/2 + 1/2))^(1/2)) - (2*f*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)
*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1
/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)
*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^
(1/2)*1i)/2 + 1/2))^(1/2))
```

3.87 $\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

3.87.1 Optimal result

Integrand size = 35, antiderivative size = 297

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \frac{2\left(\sqrt[3]{b}e+2\sqrt[3]{a}f\right)\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{9\sqrt{a}b^{2/3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-\right.}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
2/9*(b^(1/3)*e+2*a^(1/3)*f)*arctanh(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(b*x
^3+a)^(1/2))/b^(2/3)/a^(1/2)+2/9*(b^(1/3)*e-a^(1/3)*f)*(a^(1/3)+b^(1/3)*x)
*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(1/3)/b^(2/3)/
(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
))^2)^(1/2)
```

3.87.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.12 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.47

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$$
$$= \frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}f\left(\left(-3-i\sqrt{3}\right)\sqrt[3]{a}+\left(3-i\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}-\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\,\mathrm{EllipticF}\left(\arcsin\left(\right.\right.}{}$$

input
```
Integrate[(e + f*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

output
```
(2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((f*((-3 - I*Sqr
t[3])*a^(1/3) + (3 - I*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt[3])*a^(1/3) -
(I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*EllipticF[ArcSin[Sqrt
[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]],
(1 + I*Sqrt[3])/2])/2 + I*(b^(1/3)*e + 2*a^(1/3)*f)*Sqrt[((-2*I)*a^(1/3) +
 (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)
/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]),
ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a
^(1/3))]], (1 + I*Sqrt[3])/2]))/((-2 + (-1)^(1/3))*b^(2/3)*Sqrt[(a^(1/3) +
 (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

3.87.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2564, 27, 759, 2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \; 2564$$

$$\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{1}{6}\left(\frac{e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3+a}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{bx^3+a}}dx+\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3+a}}dx$$

$$\Big\downarrow \text{759}$$

$$\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3+a}}dx+ \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{2563}$$

$$\frac{2\sqrt[3]{a}\left(\frac{e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{1}{9-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^4}{\sqrt[3]{a}\left(bx^3+a\right)}}d\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\sqrt{bx^3+a}}}{3\sqrt[3]{b}}+ \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{219}$$

3.87. $\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}dx$

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$
$$\frac{2\text{arctanh}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)\left(\frac{e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)}{9\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(e + f*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

output
```
(2*(e/a^(1/3) + (2*f)/b^(1/3))*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*
Sqrt[a + b*x^3])])/(9*a^(1/6)*b^(1/3)) + (2*Sqrt[2 + Sqrt[3]]*(e/a^(1/3) -
 f/b^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sq
rt[3]])/(3*3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

3.87.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.87.4 Maple [F]

$$\int \frac{fx+e}{\left(2a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{b\,x^{3}+a}}dx$$

input
```
int((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output
```
int((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

3.87.5 Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

3.87.6 Sympy [F]

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\int \frac{e}{-2\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx - \int \frac{fx}{-2\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx$$

input
```
integrate((f*x+e)/(2*a**(1/3)-b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

output
```
-Integral(e/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x**3)),
x) - Integral(f*x/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x*
*3)), x)
```

3.87.7 Maxima [F]

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \int -\frac{fx+e}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
-integrate((f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

3.87.8 Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac
")
```

output
```
Timed out
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int -\frac{e+f\,x}{\left(b^{1/3}\,x-2\,a^{1/3}\right)\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int(-(e + f*x)/((b^(1/3)*x - 2*a^(1/3))*(a + b*x^3)^(1/2)),x)
```

output
```
int(-(e + f*x)/((b^(1/3)*x - 2*a^(1/3))*(a + b*x^3)^(1/2)), x)
```

3.88 $\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

3.88.1 Optimal result

Integrand size = 35, antiderivative size = 304

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\frac{2\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{9\sqrt{a}b^{2/3}}$$

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}e+\sqrt[3]{a}f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-\right.}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

output
```
-2/9*(b^(1/3)*e-2*a^(1/3)*f)*arctanh(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(-b
*x^3+a)^(1/2))/b^(2/3)/a^(1/2)-2/9*(b^(1/3)*e+a^(1/3)*f)*(a^(1/3)-b^(1/3)*
x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/
2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+
b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(1/3)/b^(
2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)
```

3.88.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.01 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.47

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$$

$$= \frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\frac{1}{2}if\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\left(\left(-3i+\sqrt{3}\right)\sqrt[3]{a}-\left(3i+\sqrt{3}\right)\sqrt[3]{b}x\right)\text{EllipticF}\left(\text{arcsi}\right.\right.}{}$$

input
```
Integrate[(e + f*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
(2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-1/2*I)*f*Sqrt
[((-I + Sqrt[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1
/3))]*((-3*I + Sqrt[3])*a^(1/3) - (3*I + Sqrt[3])*b^(1/3)*x)*EllipticF[Arc
Sin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*
a^(1/3))]], (1 + I*Sqrt[3])/2] - I*(b^(1/3)*e - 2*a^(1/3)*f)*Sqrt[((-I)*(2
*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1
+ (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(3*I
 + Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((
-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((-2 + (-1)^(1/3))*b^(2/3
)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a
 - b*x^3])
```

3.88.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2564, 27, 759, 2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a-bx^3}}dx+\frac{1}{6}\left(\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{b}x+2\sqrt[3]{a}\right)\sqrt{a-bx^3}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a-bx^3}}dx+\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2\sqrt[3]{a}\right)\sqrt{a-bx^3}}dx$$

$$\Big\downarrow \text{759}$$

$$\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2\sqrt[3]{a}\right)\sqrt{a-bx^3}}dx-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\Big\downarrow \text{2563}$$

$$-\frac{2\sqrt[3]{a}\left(\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{1}{9-\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^4}{\sqrt[3]{a}\left(a-bx^3\right)}}d\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{a^{2/3}\sqrt{a-bx^3}}}{3\sqrt[3]{b}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\Big\downarrow \text{219}$$

3.88. $\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx$

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\frac{2\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)\left(\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)}{9\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(e + f*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
(-2*(e/a^(1/3) - (2*f)/b^(1/3))*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)
*Sqrt[a - b*x^3])])/(9*a^(1/6)*b^(1/3)) - (2*Sqrt[2 + Sqrt[3]]*(e/a^(1/3)
+ f/b^(1/3))*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sq
rt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*S
qrt[3]])/(3*3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqr
t[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

3.88.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.88.4 Maple [F]

$$\int \frac{fx+e}{\left(2a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{-b\,x^{3}+a}}dx$$

input
```
int((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

output
```
int((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

3.88.5 Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="fri
cas")
```

output
```
Timed out
```

3.88.6 Sympy [F]

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$$

input
```
integrate((f*x+e)/(2*a**(1/3)+b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

output
```
Integral((e + f*x)/((2*a**(1/3) + b**(1/3)*x)*sqrt(a - b*x**3)), x)
```

3.88.7 Maxima [F]

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{fx+e}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

3.88.8 Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="gia
c")
```

output
```
Timed out
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{e+f\,x}{\left(b^{1/3}\,x+2\,a^{1/3}\right)\sqrt{a-b\,x^3}}\,dx$$

input
```
int((e + f*x)/((b^(1/3)*x + 2*a^(1/3))*(a - b*x^3)^(1/2)),x)
```

output
```
int((e + f*x)/((b^(1/3)*x + 2*a^(1/3))*(a - b*x^3)^(1/2)), x)
```

3.89 $\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

3.89.1 Optimal result

Integrand size = 36, antiderivative size = 313

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\frac{2\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\arctan\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a+bx^3}}\right)}{9\sqrt{a}b^{2/3}}$$

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}e+\sqrt[3]{a}f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-\right.}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

output
```
-2/9*(b^(1/3)*e-2*a^(1/3)*f)*arctan(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(b*x
^3-a)^(1/2))/b^(2/3)/a^(1/2)-2/9*(b^(1/3)*e+a^(1/3)*f)*(a^(1/3)-b^(1/3)*x)
*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)
)),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(
1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/a^(1/3)/b^(2/
3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^
(1/2)))^2)^(1/2)
```

3.89.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.95 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.43

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$$

$$= \frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\frac{1}{2}if\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\left(\left(-3i+\sqrt{3}\right)\sqrt[3]{a}-\left(3i+\sqrt{3}\right)\sqrt[3]{b}x\right)\text{EllipticF}\left(\text{arcsi}\right.\right.}{}$$

input
```
Integrate[(e + f*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
(2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-1/2*I)*f*Sqrt
[((-I + Sqrt[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1
/3))]*((-3*I + Sqrt[3])*a^(1/3) - (3*I + Sqrt[3])*b^(1/3)*x)*EllipticF[Arc
Sin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*
a^(1/3))]], (1 + I*Sqrt[3])/2] - I*(b^(1/3)*e - 2*a^(1/3)*f)*Sqrt[((-I)*(2
*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1
+ (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(3*I
 + Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((
-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((-2 + (-1)^(1/3))*b^(2/3
)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-
a + b*x^3])
```

3.89.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2564, 27, 760, 2563, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{bx^3-a}}\,dx$$

$$\Big\downarrow \ 2564$$

$$\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{bx^3-a}}dx+\frac{1}{6}\left(\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{b}x+2\sqrt[3]{a}\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \ 27$$

$$\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{bx^3-a}}dx+\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2\sqrt[3]{a}\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \ 760$$

$$\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2\sqrt[3]{a}\right)\sqrt{bx^3-a}}dx-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\Big\downarrow \ 2563$$

$$-\frac{2\sqrt[3]{a}\left(\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{1}{\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^4}{\sqrt[3]{a}\left(bx^3-a\right)}+9}d\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{a^{2/3}\sqrt{bx^3-a}}}{3\sqrt[3]{b}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\Big\downarrow \ 216$$

3.89. $\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\frac{2\arctan\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{bx^3-a}}\right)\left(\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)}{9\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(e + f*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
(-2*(e/a^(1/3) - (2*f)/b^(1/3))*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*
Sqrt[-a + b*x^3])])/(9*a^(1/6)*b^(1/3)) - (2*Sqrt[2 - Sqrt[3]]*(e/a^(1/3)
+ f/b^(1/3))*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sq
rt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*S
qrt[3]])/(3*3^(1/4)*b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - S
qrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

3.89.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.89.4 Maple [F]

$$\int \frac{fx+e}{\left(2a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{b\,x^{3}-a}}dx$$

input
```
int((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output
```
int((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

3.89.5 Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

3.89.6 Sympy [F]

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$$

input
```
integrate((f*x+e)/(2*a**(1/3)+b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

output
```
Integral((e + f*x)/((2*a**(1/3) + b**(1/3)*x)*sqrt(-a + b*x**3)), x)
```

3.89.7 Maxima [F]

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{fx+e}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

3.89.8 Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="giac
")
```

output
```
Timed out
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{e+f\,x}{\left(b^{1/3}\,x+2\,a^{1/3}\right)\,\sqrt{b\,x^3-a}}\,dx$$

input
```
int((e + f*x)/((b^(1/3)*x + 2*a^(1/3))*(b*x^3 - a)^(1/2)),x)
```

output
```
int((e + f*x)/((b^(1/3)*x + 2*a^(1/3))*(b*x^3 - a)^(1/2)), x)
```

3.90 $\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

3.90.1 Optimal result

Integrand size = 38, antiderivative size = 310

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \frac{2\left(\sqrt[3]{b}e+2\sqrt[3]{a}f\right)\arctan\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{9\sqrt{a}b^{2/3}} + \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-\right.}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

output
```
2/9*(b^(1/3)*e+2*a^(1/3)*f)*arctan(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(-b*x
^3-a)^(1/2))/b^(2/3)/a^(1/2)+2/9*(b^(1/3)*e-a^(1/3)*f)*(a^(1/3)+b^(1/3)*x)
*EllipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
,2*I-I*3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3
)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/a^(1/3)/b^(2/3)/
(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/
2)))^2)^(1/2)
```

3.90.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.05 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.42

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$$
$$= \frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}f\left(\left(-3-i\sqrt{3}\right)\sqrt[3]{a}+\left(3-i\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}-\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\operatorname{EllipticF}\left(\arcsin\left(\right.\right.\right.}{}$$

input
```
Integrate[(e + f*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
(2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((f*((-3 - I*Sqr
t[3])*a^(1/3) + (3 - I*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt[3])*a^(1/3) -
(I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*EllipticF[ArcSin[Sqrt
[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]],
(1 + I*Sqrt[3])/2])/2 + I*(b^(1/3)*e + 2*a^(1/3)*f)*Sqrt[((-2*I)*a^(1/3) +
 (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)
/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]),
ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a
^(1/3))]], (1 + I*Sqrt[3])/2]))/((-2 + (-1)^(1/3))*b^(2/3)*Sqrt[(a^(1/3) +
 (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a - b*x^3])
```

3.90.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2564, 27, 760, 2563, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$$

$$\Big\downarrow \text{ 2564}$$

$$\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\int\frac{1}{\sqrt{-bx^3-a}}dx+\frac{1}{6}\left(\frac{e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int\frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-bx^3-a}}dx$$

$$\downarrow 27$$

$$\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\int\frac{1}{\sqrt{-bx^3-a}}dx+\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-bx^3-a}}dx$$

$$\downarrow 760$$

$$\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-bx^3-a}}dx+\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\downarrow 2563$$

$$\frac{2\sqrt[3]{a}\left(\frac{e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int\frac{1}{\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^4}{\sqrt[3]{a}\left(-bx^3-a\right)}+9}d\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\sqrt{-bx^3-a}}}{3\sqrt[3]{b}}+\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\downarrow 216$$

3.90. $\int\frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}dx$

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\frac{2\arctan\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)\left(\frac{e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)}{9\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[(e + f*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
(2*(e/a^(1/3) + (2*f)/b^(1/3))*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*S
qrt[-a - b*x^3])])/(9*a^(1/6)*b^(1/3)) + (2*Sqrt[2 - Sqrt[3]]*(e/a^(1/3) -
 f/b^(1/3))*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sq
rt[3]])/(3*3^(1/4)*b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

3.90.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.90.4 Maple [F]

$$\int \frac{fx+e}{\left(2a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{-b\,x^{3}-a}}dx$$

input
```
int((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

output
```
int((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

3.90.5 Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="fri
cas")
```

output
```
Timed out
```

3.90.6 Sympy [F]

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\int \frac{e}{-2\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}x\sqrt{-a-bx^3}}\,dx - \int \frac{fx}{-2\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}x\sqrt{-a-bx^3}}\,dx$$

input
```
integrate((f*x+e)/(2*a**(1/3)-b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

output
```
-Integral(e/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*x**3))
, x) - Integral(f*x/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a -
b*x**3)), x)
```

3.90.7 Maxima [F]

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \int -\frac{fx+e}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="max
ima")
```

output
```
-integrate((f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

3.90.8 Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="gia
c")
```

output
```
Timed out
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int -\frac{e+f\,x}{\left(b^{1/3}\,x-2\,a^{1/3}\right)\,\sqrt{-b\,x^3-a}}\,dx$$

input
```
int(-(e + f*x)/((b^(1/3)*x - 2*a^(1/3))*(- a - b*x^3)^(1/2)),x)
```

output
```
int(-(e + f*x)/((b^(1/3)*x - 2*a^(1/3))*(- a - b*x^3)^(1/2)), x)
```

3.91 $\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$

3.91.1 Optimal result

Integrand size = 29, antiderivative size = 221

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = -\frac{2(de-cf)\text{arctanh}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9c^{3/2}d^2} - \frac{\sqrt{2+\sqrt{3}}(2de+cf)(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

output
```
-2/9*(-c*f+d*e)*arctanh(1/3*(-2*d*x+c)^2/c^(1/2)/(-8*d^3*x^3+c^3)^(1/2))/c
^(3/2)/d^2-1/9*(c*f+2*d*e)*(-2*d*x+c)*EllipticF((-2*d*x+c*(1-3^(1/2)))/(-2
*d*x+c*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((4*d^2*x^2+2
*c*d*x+c^2)/(-2*d*x+c*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c/d^2/(-8*d^3*x^3+c^3)
^(1/2)/(c*(-2*d*x+c)/(-2*d*x+c*(1+3^(1/2)))^2)^(1/2)
```

3.91.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.86 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.74

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx =$$

$$-\frac{i\sqrt{\frac{c-2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(f\sqrt{\frac{\left(-i+\sqrt{3}\right)c+2\left(i+\sqrt{3}\right)dx}{\left(-3i+\sqrt{3}\right)c}}\left(\left(-3i+\sqrt{3}\right)c-2\left(3i+\sqrt{3}\right)dx\right)\text{EllipticF}\left(\arcsin\left(\sqrt{2}\sqrt{\frac{ic+idx}{3ic-\cdots}}\right.\right.\right.}{2\left(-2+\cdots\right.}$$

input
```
Integrate[(e + f*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

output
```
((-1/2*I)*Sqrt[(c - 2*d*x)/((1 + (-1)^(1/3))*c)]*(f*Sqrt[((-I + Sqrt[3])*c
 + 2*(I + Sqrt[3])*d*x)/((-3*I + Sqrt[3])*c)]*((-3*I + Sqrt[3])*c - 2*(3*I
 + Sqrt[3])*d*x)*EllipticF[ArcSin[Sqrt[2]*Sqrt[(I*c + I*d*x + Sqrt[3]*d*x)
/((3*I)*c - Sqrt[3]*c)]], (1 + I*Sqrt[3])/2] + 4*Sqrt[2]*(d*e - c*f)*Sqrt[
(I*c + I*d*x + Sqrt[3]*d*x)/((3*I)*c - Sqrt[3]*c)]*Sqrt[(c^2 + 2*c*d*x + 4
*d^2*x^2)/c^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[2]*Sqrt
[(I*c + I*d*x + Sqrt[3]*d*x)/((3*I)*c - Sqrt[3]*c)]], (1 + I*Sqrt[3])/2]))
/((-2 + (-1)^(1/3))*d^2*Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*
Sqrt[c^3 - 8*d^3*x^3])
```

3.91.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2564, 759, 2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$$

$$\Big\downarrow \; 2564$$

$$\frac{(cf+2de)\int \frac{1}{\sqrt{c^3-8d^3x^3}}dx}{3cd}+\frac{(de-cf)\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}dx}{3cd}$$

$$\Big\downarrow \; 759$$

$$\frac{(de-cf)\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}dx}{3cd}-\frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}(cf+2de)\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

$$\Big\downarrow \text{2563}$$

$$-\frac{2(de-cf)\int \frac{1}{9-\frac{(c-2dx)^4}{c\left(c^3-8d^3x^3\right)}}d\frac{(c-2dx)^2}{c^2\sqrt{c^3-8d^3x^3}}}{3d^2}-\frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}(cf+2de)\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}(cf+2de)\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}-\frac{2\text{arctanh}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)(de-cf)}{9c^{3/2}d^2}$$

input
```
Int[(e + f*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

output
```
(-2*(d*e - c*f)*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/
(9*c^(3/2)*d^2) - (Sqrt[2 + Sqrt[3]]*(2*d*e + c*f)*(c - 2*d*x)*Sqrt[(c^2 +
 2*c*d*x + 4*d^2*x^2)/((1 + Sqrt[3])*c - 2*d*x)^2]*EllipticF[ArcSin[((1 -
Sqrt[3])*c - 2*d*x)/((1 + Sqrt[3])*c - 2*d*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4
)*c*d^2*Sqrt[(c*(c - 2*d*x))/((1 + Sqrt[3])*c - 2*d*x)^2]*Sqrt[c^3 - 8*d^3
*x^3])
```

3.91. $\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$

3.91.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.91.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 520 vs. 2(192) = 384.

Time = 1.01 (sec) , antiderivative size = 521, normalized size of antiderivative = 2.36

method	result
default	$\frac{2f\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}\right)\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}F\left(\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}},\sqrt{\frac{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\right.}{d\sqrt{-8d^3x^3+c^3}}$
elliptic	$\frac{2f\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}\right)\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}F\left(\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}},\sqrt{\frac{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\right.}{d\sqrt{-8d^3x^3+c^3}}$

input
```
int((f*x+e)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*f/d*(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I
*3^(1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*c/d-1
/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2
*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)*EllipticF
(((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2),((1/2*c/d-1/2*
(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2))+4
/3*(-c*f+d*e)/d*(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d)*((x-1/2*c/d)/(1/2*(
-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/
(1/2*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*(-1/2-1/2*I*3^(1/2))
*c/d)/(1/2*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)
/c*EllipticPi(((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2),2
/3*(1/2*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/c*d,((1/2*c/d-1/2*(-1/2-1/2*I*3^
(1/2))*c/d)/(1/2*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2))
```

3.91.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.26 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.79

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$$
$$= \left[-\frac{3\sqrt{2}\sqrt{-d^3}(2\,cde+c^2f)\text{weierstrassPInverse}\left(0,\frac{c^3}{2\,d^3},x\right)+(d^3e-cd^2f)\sqrt{c}\log\left(\frac{8\,d^6x^6-240\,cd^5x^5+408\,c^2d^4x^4\cdots}{\cdots}\right)}{18\,c^2d^4}\right.$$
$$-\frac{3\sqrt{2}\sqrt{-d^3}(2\,cde+c^2f)\text{weierstrassPInverse}\left(0,\frac{c^3}{2\,d^3},x\right)+2\,(d^3e-cd^2f)\sqrt{-c}\arctan\left(\frac{(4\,d^3x^3-24\,cd^2x^2-6\,c\cdots}{3\,(16\,cd^4x^4-8\cdots}\right)}{18\,c^2d^4}$$

input
```
integrate((f*x+e)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

output
```
[-1/18*(3*sqrt(2)*sqrt(-d^3)*(2*c*d*e + c^2*f)*weierstrassPInverse(0, 1/2*
c^3/d^3, x) + (d^3*e - c*d^2*f)*sqrt(c)*log((8*d^6*x^6 - 240*c*d^5*x^5 + 4
08*c^2*d^4*x^4 + 88*c^3*d^3*x^3 + 156*c^4*d^2*x^2 + 12*c^5*d*x + 17*c^6 +
3*(8*d^4*x^4 - 52*c*d^3*x^3 + 12*c^2*d^2*x^2 - 4*c^3*d*x + 5*c^4)*sqrt(-8*
d^3*x^3 + c^3)*sqrt(c))/(d^6*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d
^3*x^3 + 15*c^4*d^2*x^2 + 6*c^5*d*x + c^6)))/(c^2*d^4), -1/18*(3*sqrt(2)*s
qrt(-d^3)*(2*c*d*e + c^2*f)*weierstrassPInverse(0, 1/2*c^3/d^3, x) + 2*(d^
3*e - c*d^2*f)*sqrt(-c)*arctan(1/3*(4*d^3*x^3 - 24*c*d^2*x^2 - 6*c^2*d*x -
 5*c^3)*sqrt(-8*d^3*x^3 + c^3)*sqrt(-c)/(16*c*d^4*x^4 - 8*c^2*d^3*x^3 - 2*
c^4*d*x + c^5)))/(c^2*d^4)]
```

3.91.6 Sympy [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \int \frac{e+fx}{\sqrt{-\left(-c+2dx\right)\left(c^2+2cdx+4d^2x^2\right)}\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(-8*d**3*x**3+c**3)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt(-(-c + 2*d*x)*(c**2 + 2*c*d*x + 4*d**2*x**2))*(c
+ d*x)), x)
```

3.91.7 Maxima [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \int \frac{fx+e}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

3.91.8 Giac [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \int \frac{fx+e}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x + e)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \int \frac{e+f\,x}{\sqrt{c^3-8\,d^3\,x^3}\,(c+d\,x)}\,dx$$

input
```
int((e + f*x)/((c^3 - 8*d^3*x^3)^(1/2)*(c + d*x)),x)
```

output
```
int((e + f*x)/((c^3 - 8*d^3*x^3)^(1/2)*(c + d*x)), x)
```

3.92 $\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx$

3.92.1 Optimal result

Integrand size = 18, antiderivative size = 129

$$\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx = \frac{4}{9}\operatorname{arctanh}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right) - \frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
4/9*arctanh(1/3*(1+x)^2/(x^3+1)^(1/2))-2/9*(1+x)*EllipticF((1+x-3^(1/2))/(
1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1
/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.92.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.28 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.50

$$\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx = \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{2i\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{3i+\sqrt{3}},\arcsin\left(\right.\right.}{-2+\sqrt[3]{-1}}\right.}{\sqrt{1+x^3}}$$

input
```
Integrate[x/((2 - x)*Sqrt[1 + x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (-
1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((2*I)*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sq
rt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-2 + (-1)^(1/3)))
/Sqrt[1 + x^3]
```

3.92.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2564, 27, 759, 2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(2-x)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{ 2564}$$

$$\frac{1}{3}\int \frac{2(x+1)}{(2-x)\sqrt{x^3+1}}dx-\frac{1}{3}\int \frac{1}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{3}\int \frac{x+1}{(2-x)\sqrt{x^3+1}}dx-\frac{1}{3}\int \frac{1}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{ 759}$$

$$\frac{2}{3}\int \frac{x+1}{(2-x)\sqrt{x^3+1}}dx - \frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{2563}$$

$$\frac{4}{3}\int \frac{1}{9-\frac{(x+1)^4}{x^3+1}}d\frac{(x+1)^2}{\sqrt{x^3+1}} - \frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{4}{9}\text{arctanh}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right) - \frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

input
```
Int[x/((2 - x)*Sqrt[1 + x^3]),x]
```

output
```
(4*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/9 - (2*Sqrt[2 + Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)
/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3]
 + x)^2]*Sqrt[1 + x^3])
```

3.92.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.92.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(104) = 208$.

Time = 1.04 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.86

method	result
default	$-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}}{3\sqrt{x^3+1}}$
elliptic	$-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}}{3\sqrt{x^3+1}}$

input
```
int(x/(2-x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/
2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+4/3*(3/2-1/2*I*3^(1/2))*((x+
1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))
^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*El
lipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),1/2-1/6*I*3^(1/2),((-3/2+1/2*I*
3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

3.92.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.40

$$\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx = \frac{2}{9}\log\left(\frac{x^3+12\,x^2+6\,\sqrt{x^3+1}(x+1)-6\,x+10}{x^3-6\,x^2+12\,x-8}\right) - \frac{2}{3}\,\mathrm{weierstrassPInverse}(0,-4,x)$$

input
```
integrate(x/(2-x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
2/9*log((x^3 + 12*x^2 + 6*sqrt(x^3 + 1)*(x + 1) - 6*x + 10)/(x^3 - 6*x^2 +
 12*x - 8)) - 2/3*weierstrassPInverse(0, -4, x)
```

3.92.6 Sympy [F]

$$\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx = -\int \frac{x}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx$$

input
```
integrate(x/(2-x)/(x**3+1)**(1/2),x)
```

output
```
-Integral(x/(x*sqrt(x**3 + 1) - 2*sqrt(x**3 + 1)), x)
```

3.92.7 Maxima [F]

$$\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx = \int -\frac{x}{\sqrt{x^3+1}(x-2)}\,dx$$

input
```
integrate(x/(2-x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x/(sqrt(x^3 + 1)*(x - 2)), x)
```

3.92.8 Giac [F]

$$\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx = \int -\frac{x}{\sqrt{x^3+1}(x-2)}\,dx$$

input
```
integrate(x/(2-x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-x/(sqrt(x^3 + 1)*(x - 2)), x)
```

3.92.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.60

$$\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx = -\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(3\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-2\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asi}\right.\right.}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right.}}$$

input
```
int(-x/((x^3 + 1)^(1/2)*(x - 2)),x)
```

output
```
-((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)*(3*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - 2*ellipticPi(
(3^(1/2)*1i)/6 + 1/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(
1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(3*(x^3 - x*(((3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
 + 1/2))^(1/2))
```

3.92. $\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx$

3.93 $\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx$

3.93.1 Optimal result

Integrand size = 18, antiderivative size = 145

$$\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx = \frac{4}{9}\text{arctanh}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right) - \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
4/9*arctanh(1/3*(1-x)^2/(-x^3+1)^(1/2))-2/9*(1-x)*EllipticF((1-x-3^(1/2))/
(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(
1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

3.93.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.21 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.34

$$\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx = \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{2i\sqrt{1+x+x^2}\operatorname{EllipticPi}\left(\frac{2\sqrt{3}}{3i+\sqrt{3}},\arcsin\left(\right.\right.}{-2+\sqrt[3]{-1}}\right.}{\sqrt{1-x^3}}$$

input
```
Integrate[x/((2 + x)*Sqrt[1 - x^3]),x]
```

output
```
(2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-
1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((2*I)*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sq
rt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-2 + (-1)^(1/3)))
/Sqrt[1 - x^3]
```

3.93.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2564, 27, 759, 2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+2)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{1-x^3}}dx-\frac{1}{3}\int \frac{2(1-x)}{(x+2)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{1-x^3}}dx-\frac{2}{3}\int \frac{1-x}{(x+2)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{759}$$

$$-\frac{2}{3}\int \frac{1-x}{(x+2)\sqrt{1-x^3}}dx-$$
$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \; 2563$$

$$\frac{4}{3}\int \frac{1}{9-\frac{(1-x)^4}{1-x^3}}d\frac{(1-x)^2}{\sqrt{1-x^3}}-$$
$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \; 219$$

$$\frac{4}{9}\text{arctanh}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)-$$
$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

input
```
Int[x/((2 + x)*Sqrt[1 - x^3]),x]
```

output
```
(4*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/9 - (2*Sqrt[2 + Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)
/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3]
 - x)^2]*Sqrt[1 - x^3])
```

3.93.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.93.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.66

method	result
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}}+\frac{4i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{}}}{}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}}+\frac{4i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{}}}{}$

input
```
int(x/(x+2)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3
/2+1/2*I*3^(1/2)))^(1/2))+4/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^
(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2)
)^(1/2)/(-x^3+1)^(1/2)/(3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/
2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)),(I*3^(1/2)/(
-3/2+1/2*I*3^(1/2)))^(1/2))
```

3.93.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.37

$$\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx = \frac{2}{9}\log\left(-\frac{x^3-12\,x^2+6\,\sqrt{-x^3+1}(x-1)-6\,x-10}{x^3+6\,x^2+12\,x+8}\right) - \frac{2}{3}i\,\mathrm{weierstrassPInverse}(0,4,x)$$

input
```
integrate(x/(2+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
2/9*log(-(x^3 - 12*x^2 + 6*sqrt(-x^3 + 1)*(x - 1) - 6*x - 10)/(x^3 + 6*x^2
 + 12*x + 8)) - 2/3*I*weierstrassPInverse(0, 4, x)
```

3.93.6 Sympy [F]

$$\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx = \int \frac{x}{\sqrt{-(x-1)(x^2+x+1)}\,(x+2)}\,dx$$

input
```
integrate(x/(2+x)/(-x**3+1)**(1/2),x)
```

output
```
Integral(x/(sqrt(-(x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

3.93.7 Maxima [F]

$$\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3+1}(x+2)}\,dx$$

input
```
integrate(x/(2+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(-x^3 + 1)*(x + 2)), x)
```

3.93.8 Giac [F]

$$\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3+1}(x+2)}\,dx$$

input
```
integrate(x/(2+x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(-x^3 + 1)*(x + 2)), x)
```

3.93.9 Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.54

$$\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx = -\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\,\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(3\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-2\,\Pi\right.}{3\,\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right.}}$$

input
```
int(x/((1 - x^3)^(1/2)*(x + 2)),x)
```

output
```
-((3^(1/2)*1i + 3)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*
1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(3*ellipticF(asin((-(x - 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/
2)) - 2*ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((-(x - 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(3*(1 - x^
3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)
/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.93. $\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx$

3.94 $\int \frac{x}{(2+x)\sqrt{-1+x^3}} dx$

3.94.1 Optimal result

Integrand size = 16, antiderivative size = 148

$$\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx = \frac{4}{9}\arctan\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right) - \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
4/9*arctan(1/3*(1-x)^2/(x^3-1)^(1/2))-2/9*(1-x)*EllipticF((1-x+3^(1/2))/(1
-x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/
2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

3.94.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.14 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.30

$$\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{2i\sqrt{1+x+x^2}\operatorname{EllipticPi}\left(\frac{2\sqrt{3}}{3i+\sqrt{3}},\arcsin\left(\right.\right.}{-2+\sqrt[3]{-1}}\right.}{\sqrt{-1+x^3}}$$

input
```
Integrate[x/((2 + x)*Sqrt[-1 + x^3]),x]
```

output
```
(2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-
1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((2*I)*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sq
rt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-2 + (-1)^(1/3)))
/Sqrt[-1 + x^3]
```

3.94.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2564, 27, 760, 2563, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+2)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{ 2564}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{x^3-1}}dx-\frac{1}{3}\int \frac{2(1-x)}{(x+2)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{x^3-1}}dx-\frac{2}{3}\int \frac{1-x}{(x+2)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{ 760}$$

$$-\frac{2}{3}\int \frac{1-x}{(x+2)\sqrt{x^3-1}}dx-$$
$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{2563}$$

$$\frac{4}{3}\int \frac{1}{\frac{(1-x)^4}{x^3-1}+9}d\frac{(1-x)^2}{\sqrt{x^3-1}}-$$
$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{4}{9}\arctan\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)-$$
$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

input
```
Int[x/((2 + x)*Sqrt[-1 + x^3]),x]
```

output
```
(4*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/9 - (2*Sqrt[2 - Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)
/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[
3] - x)^2)]*Sqrt[-1 + x^3])
```

3.94.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.94. $\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx$

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.94.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(118) = 236$.

Time = 1.00 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.62

method	result
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} - \frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}}{\frac{3}{2}+}}}{3\sqrt{x^3}}$
elliptic	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} - \frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}}{\frac{3}{2}+}}}{3\sqrt{x^3}}$

input
```
int(x/(x+2)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-4/3*(-3/2-1/2*I*3^(1/2))*((x-1
)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^
(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*Elli
pticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/2+1/6*I*3^(1/2),((3/2+1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

3.94.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.32

$$\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx = \frac{2}{9}\arctan\left(\frac{(x^3-12\,x^2-6\,x-10)\sqrt{x^3-1}}{6\,(x^4-x^3-x+1)}\right) + \frac{2}{3}\,\mathrm{weierstrassPInverse}(0,4,x)$$

input
```
integrate(x/(2+x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
2/9*arctan(1/6*(x^3 - 12*x^2 - 6*x - 10)*sqrt(x^3 - 1)/(x^4 - x^3 - x + 1)
) + 2/3*weierstrassPInverse(0, 4, x)
```

3.94.6 Sympy [F]

$$\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx = \int \frac{x}{\sqrt{(x-1)\,(x^2+x+1)}\,(x+2)}\,dx$$

input
```
integrate(x/(2+x)/(x**3-1)**(1/2),x)
```

output
```
Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

3.94.7 Maxima [F]

$$\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3-1}(x+2)}\,dx$$

input
```
integrate(x/(2+x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(x^3 - 1)*(x + 2)), x)
```

3.94.8 Giac [F]

$$\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3-1}(x+2)}\,dx$$

input
```
integrate(x/(2+x)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(x^3 - 1)*(x + 2)), x)
```

3.94.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.41

$$\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx = -\frac{(3+\sqrt{3}\,1\mathrm{i})\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(3\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-2\,\Pi\left(\frac{1}{2}+\right.\right.}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\right.}}$$

input
```
int(x/((x^3 - 1)^(1/2)*(x + 2)),x)
```

output
```
-((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1
/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)*(3*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - 2*elliptic
Pi((3^(1/2)*1i)/6 + 1/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -(
(3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(3*(((3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
 + 1) + x^3)^(1/2))
```

3.94. $\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx$

3.95 $\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx$

3.95.1 Optimal result

Integrand size = 20, antiderivative size = 140

$$\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx = \frac{4}{9}\arctan\left(\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right) - \frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
4/9*arctan(1/3*(1+x)^2/(-x^3-1)^(1/2))-2/9*(1+x)*EllipticF((1+x+3^(1/2))/(
1+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1
/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

3.95.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.23 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.39

$$\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx = \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{2i\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{3i+\sqrt{3}},\arcsin\left(\right.\right.}{-2+\sqrt[3]{-1}}\right.}{\sqrt{-1-x^3}}$$

input
```
Integrate[x/((2 - x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (-
1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((2*I)*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sq
rt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-2 + (-1)^(1/3))))
/Sqrt[-1 - x^3]
```

3.95.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2564, 27, 760, 2563, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(2-x)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{1}{3}\int \frac{2(x+1)}{(2-x)\sqrt{-x^3-1}}dx-\frac{1}{3}\int \frac{1}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\int \frac{x+1}{(2-x)\sqrt{-x^3-1}}dx-\frac{1}{3}\int \frac{1}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$\frac{2}{3}\int \frac{x+1}{(2-x)\sqrt{-x^3-1}}dx - \frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \; 2563$$

$$\frac{4}{3}\int \frac{1}{\frac{(x+1)^4}{-x^3-1}+9}d\frac{(x+1)^2}{\sqrt{-x^3-1}} - \frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \; 216$$

$$\frac{4}{9}\arctan\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right) - \frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

input
```
Int[x/((2 - x)*Sqrt[-1 - x^3]),x]
```

output
```
(4*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/9 - (2*Sqrt[2 - Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)
/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[
3] + x)^2)]*Sqrt[-1 - x^3])
```

3.95.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.95.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(114) = 228$.

Time = 0.95 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.71

method	result
default	$\frac{2 i \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, F\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}-1}} + \frac{4 i \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{}{\frac{3}{2}}}}{}$
elliptic	$\frac{2 i \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, F\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}-1}} + \frac{4 i \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{}{\frac{3}{2}}}}{}$

input
```
int(x/(2-x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))+4/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/
2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1
/2)/(-x^3-1)^(1/2)/(-3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1
/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/
2+1/2*I*3^(1/2)))^(1/2))
```

3.95.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.32

$$\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx = -\frac{2}{9}\arctan\left(\frac{(x^3+12\,x^2-6\,x+10)\sqrt{-x^3-1}}{6\,(x^4+x^3+x+1)}\right) + \frac{2}{3}i\,\mathrm{weierstrassPInverse}(0,-4,x)$$

input
```
integrate(x/(2-x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-2/9*arctan(1/6*(x^3 + 12*x^2 - 6*x + 10)*sqrt(-x^3 - 1)/(x^4 + x^3 + x +
1)) + 2/3*I*weierstrassPInverse(0, -4, x)
```

3.95.6 Sympy [F]

$$\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx = -\int \frac{x}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx$$

input
```
integrate(x/(2-x)/(-x**3-1)**(1/2),x)
```

output
```
-Integral(x/(x*sqrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x)
```

3.95.7 Maxima [F]

$$\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx = \int -\frac{x}{\sqrt{-x^3-1}(x-2)}\,dx$$

input
```
integrate(x/(2-x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x/(sqrt(-x^3 - 1)*(x - 2)), x)
```

3.95.8 Giac [F]

$$\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx = \int -\frac{x}{\sqrt{-x^3-1}(x-2)}\,dx$$

input
```
integrate(x/(2-x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-x/(sqrt(-x^3 - 1)*(x - 2)), x)
```

3.95.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.59

$$\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx =$$

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(3\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-2\,\Pi\left(\frac{1}{2}+\right.\right.}{3\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\right.}}$$

input
```
int(-x/((- x^3 - 1)^(1/2)*(x - 2)),x)
```

output
```
-((3^(1/2)*1i + 3)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1
i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2
 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(3*ellipticF(asin(((x + 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))
 - 2*ellipticPi((3^(1/2)*1i)/6 + 1/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(3*(- x^3 - 1)
^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

3.95. $\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx$

3.96 $\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

3.96.1 Optimal result

Integrand size = 31, antiderivative size = 260

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \frac{4\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}} - \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
4/9*arctanh(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(b*x^3+a)^(1/2))/a^(1/6)/b^(
2/3)-2/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^
(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1
/2)*3^(3/4)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.96.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.65 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.42

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$$

$$= \frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(-2+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{\sqrt[3]{a}+(-}{(1+\sqrt[3]{}}}\right.\right.\right.}{\left(-2+\sqrt[3]{-1}\right)b}$$

input
```
Integrate[x/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

output
```
(2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2 + (-1)^(1/3
))*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*
b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-
1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] + (2*(-1)^(1
/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (
-1)^(1/3))*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]
*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)
*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/((-2 + (-
1)^(1/3))*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*
a^(1/3))]*Sqrt[a + b*x^3])
```

3.96.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2564, 27, 759, 2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \ 2564$$

$$\frac{\int \frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3+a}}dx}{3\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt{bx^3+a}}dx}{3\sqrt[3]{b}}$$

$$\Big\downarrow \ 27$$

$$\frac{2\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3+a}}dx}{3\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt{bx^3+a}}dx}{3\sqrt[3]{b}}$$

$$\Big\downarrow \ 759$$

$$\frac{2\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3+a}}dx}{3\sqrt[3]{b}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \ 2563$$

$$\frac{4\sqrt[3]{a}\int \frac{1}{9-\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^4}{\sqrt[3]{a}\left(bx^3+a\right)}}d\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\sqrt{bx^3+a}}}{3b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \ 219$$

3.96. $\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}dx$

$$\frac{4\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

input
```
Int[x/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

output
```
(4*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a + b*x^3])])/(9*a^(1/6
)*b^(2/3)) - (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) +
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

3.96.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.96.4 Maple [F]

$$\int \frac{x}{\left(2a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{b\,x^{3}+a}}dx$$

input
```
int(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

output
```
int(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

3.96.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.96.6 Sympy [F]

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\int \frac{x}{-2\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}x\sqrt{a+bx^3}}\,dx$$

input
```
integrate(x/(2*a**(1/3)-b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

output
```
-Integral(x/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x**3)),
x)
```

3.96.7 Maxima [F]

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \int -\frac{x}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

3.96.8 Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=-\int \frac{x}{\left(b^{1/3}\,x-2\,a^{1/3}\right)\,\sqrt{b\,x^3+a}}\,dx$$

input
```
int(-x/((b^(1/3)*x - 2*a^(1/3))*(a + b*x^3)^(1/2)),x)
```

output
```
-int(x/((b^(1/3)*x - 2*a^(1/3))*(a + b*x^3)^(1/2)), x)
```

3.97 $\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

3.97.1 Optimal result

Integrand size = 31, antiderivative size = 268

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\frac{4\text{arctanh}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

output
```
4/9*arctanh(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(-b*x^3+a)^(1/2))/a^(1/6)/b^
(2/3)-2/9*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*(
(a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2
)^(1/2)*3^(3/4)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.97.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.58 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.38

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$$

$$= \frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(-2+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{\sqrt[3]{a}-}{(1+}}\right.\right.\right.}{\left(-2+\sqrt[3]{-1}\right)}$$

input
```
Integrate[x/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
(2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2 + (-1)^(1/3
))*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(1/3)
*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) -
(-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] + (2*(-1)^
(1/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 +
 (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3
)]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/
3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/((-2 +
(-1)^(1/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3)
)*a^(1/3))]*Sqrt[a - b*x^3])
```

3.97.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2564, 27, 759, 2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx$$

$\Big\downarrow$ 2564

$$\frac{\int \frac{1}{\sqrt{a-bx^3}}dx}{3\sqrt[3]{b}}-\frac{\int \frac{2\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{b}x+2\sqrt[3]{a}\right)\sqrt{a-bx^3}}dx}{3\sqrt[3]{b}}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{1}{\sqrt{a-bx^3}}dx}{3\sqrt[3]{b}}-\frac{2\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2\sqrt[3]{a}\right)\sqrt{a-bx^3}}dx}{3\sqrt[3]{b}}$$

$\Big\downarrow$ 759

$$-\frac{2\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2\sqrt[3]{a}\right)\sqrt{a-bx^3}}dx}{3\sqrt[3]{b}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$\Big\downarrow$ 2563

$$\frac{4\sqrt[3]{a}\int \frac{1}{9-\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^4}{\sqrt[3]{a}\left(a-bx^3\right)}}d\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{a^{2/3}\sqrt{a-bx^3}}}{3b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$\Big\downarrow$ 219

3.97. $$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx$$

$$\frac{4\text{arctanh}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

input
```
Int[x/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
(4*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a - b*x^3])])/(9*a^(1/6
)*b^(2/3)) - (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) -
b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) -
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

3.97.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.97.4 Maple [F]

$$\int \frac{x}{\left(2 a^{\frac{1}{3}}+b^{\frac{1}{3}} x \right) \sqrt{-b \,x^{3}+a}}d x$$

input
```
int(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

output
```
int(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

3.97.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(2 \sqrt[3]{a}+\sqrt[3]{b} x\right) \sqrt{a-b x^3}} \, dx=\text{Timed out}$$

input
```
integrate(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.97.6 Sympy [F]

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$$

input
```
integrate(x/(2*a**(1/3)+b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

output
```
Integral(x/((2*a**(1/3) + b**(1/3)*x)*sqrt(a - b*x**3)), x)
```

3.97.7 Maxima [F]

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{x}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

3.97.8 Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{x}{\left(b^{1/3}\,x+2\,a^{1/3}\right)\,\sqrt{a-b\,x^3}}\,dx$$

input
```
int(x/((b^(1/3)*x + 2*a^(1/3))*(a - b*x^3)^(1/2)),x)
```

output
```
int(x/((b^(1/3)*x + 2*a^(1/3))*(a - b*x^3)^(1/2)), x)
```

3.98 $\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

3.98.1 Optimal result

Integrand size = 32, antiderivative size = 277

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\frac{4\arctan\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a+bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

output
```
4/9*arctan(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(b*x^3-a)^(1/2))/a^(1/6)/b^(2
/3)-2/9*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b
^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b
^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(
1/2))*3^(3/4)/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1
/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.98.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.47 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.34

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$$

$$= \frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(-2+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{\sqrt[3]{a}-}{(1+}}\right.\right.\right.}{\left(-2+\sqrt[3]{-1}\right)b}$$

input
```
Integrate[x/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
(2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2 + (-1)^(1/3
))*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(1/3)
*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) -
(-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] + (2*(-1)^
(1/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 +
 (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3
)]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/
3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/((-2 +
(-1)^(1/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3)
)*a^(1/3))]*Sqrt[-a + b*x^3])
```

3.98.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {2564, 27, 760, 2563, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{ 2564}$$

$$\frac{\int \frac{1}{\sqrt{bx^3-a}}dx}{3\sqrt[3]{b}}-\frac{\int \frac{2\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\sqrt[3]{b}x+2\sqrt[3]{a}\right)\sqrt{bx^3-a}}dx}{3\sqrt[3]{b}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1}{\sqrt{bx^3-a}}dx}{3\sqrt[3]{b}}-\frac{2\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2\sqrt[3]{a}\right)\sqrt{bx^3-a}}dx}{3\sqrt[3]{b}}$$

$$\Big\downarrow \text{ 760}$$

$$-\frac{2\int \frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\sqrt[3]{b}x+2\sqrt[3]{a}\right)\sqrt{bx^3-a}}dx}{3\sqrt[3]{b}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\Big\downarrow \text{ 2563}$$

$$\frac{4\sqrt[3]{a}\int \frac{1}{\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^4}{\sqrt[3]{a}\left(bx^3-a\right)}+9}d\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{a^{2/3}\sqrt{bx^3-a}}}{3b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\Big\downarrow \text{ 216}$$

3.98. $\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}dx$

$$\frac{4\arctan\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{bx^3-a}}\right)}{9\sqrt[6]{a}b^{2/3}} - \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

input
```
Int[x/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
(4*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a + b*x^3])])/(9*a^(1/6
)*b^(2/3)) - (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Elli
pticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) -
b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3)
- b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

3.98.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.98.4 Maple [F]

$$\int \frac{x}{\left(2a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{b\,x^{3}-a}}dx$$

input
```
int(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

output
```
int(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

3.98.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.98.6 Sympy [F]

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$$

input
```
integrate(x/(2*a**(1/3)+b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

output
```
Integral(x/((2*a**(1/3) + b**(1/3)*x)*sqrt(-a + b*x**3)), x)
```

3.98.7 Maxima [F]

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{x}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+2\,a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

3.98.8 Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{x}{\left(b^{1/3}\,x+2\,a^{1/3}\right)\,\sqrt{b\,x^3-a}}\,dx$$

input
```
int(x/((b^(1/3)*x + 2*a^(1/3))*(b*x^3 - a)^(1/2)),x)
```

output
```
int(x/((b^(1/3)*x + 2*a^(1/3))*(b*x^3 - a)^(1/2)), x)
```

3.99 $\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

3.99.1 Optimal result

Integrand size = 34, antiderivative size = 273

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\frac{4\arctan\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

output
```
4/9*arctan(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(-b*x^3-a)^(1/2))/a^(1/6)/b^(
2/3)-2/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^
(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^
(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/
2))*3^(3/4)/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3
)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.99.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.65 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.36

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$$

$$= \frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(-2+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{\sqrt[3]{a}+(-}{\left(1+\sqrt[3]{\right.}}}\right.\right.\right.}{\left(-2+\sqrt[3]{-1}\right)b^2}$$

input
```
Integrate[x/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
(2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2 + (-1)^(1/3
))*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[((-1)^(1/3)*a^(1/3) - (-1)^(2/3)*
b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-
1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] + (2*(-1)^(1
/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (
-1)^(1/3))*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]
*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)
*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/((-2 + (-
1)^(1/3))*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*
a^(1/3))]*Sqrt[-a - b*x^3])
```

3.99.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {2564, 27, 760, 2563, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}dx$$

$$\Big\downarrow \text{2564}$$

$$\frac{\int \frac{2\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-bx^3-a}}dx}{3\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt{-bx^3-a}}dx}{3\sqrt[3]{b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-bx^3-a}}dx}{3\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt{-bx^3-a}}dx}{3\sqrt[3]{b}}$$

$$\Big\downarrow \text{760}$$

$$\frac{2\int \frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-bx^3-a}}dx}{3\sqrt[3]{b}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\Big\downarrow \text{2563}$$

$$\frac{4\sqrt[3]{a}\int \frac{1}{\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^4}{\sqrt[3]{a}\left(-bx^3-a\right)}+9}d\frac{\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{a^{2/3}\sqrt{-bx^3-a}}}{3b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\Big\downarrow \text{216}$$

3.99. $\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}dx$

$$\frac{4\arctan\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

input
```
Int[x/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
(4*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a - b*x^3])])/(9*a^(1/6
)*b^(2/3)) - (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elli
pticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3)
+ b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

3.99.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.99.4 Maple [F]

$$\int \frac{x}{\left(2a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{-b\,x^{3}-a}}dx$$

input
```
int(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

output
```
int(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

3.99.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.99.6 Sympy [F]

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\int \frac{x}{-2\sqrt[3]{a}\sqrt{-a-bx^3}+\sqrt[3]{b}x\sqrt{-a-bx^3}}\,dx$$

input
```
integrate(x/(2*a**(1/3)-b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

output
```
-Integral(x/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*x**3))
, x)
```

3.99.7 Maxima [F]

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \int -\frac{x}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x-2\,a^{\frac{1}{3}}\right)}\,dx$$

input
```
integrate(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

3.99.8 Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=-\int \frac{x}{\left(b^{1/3}\,x-2\,a^{1/3}\right)\,\sqrt{-b\,x^3-a}}\,dx$$

input
```
int(-x/((b^(1/3)*x - 2*a^(1/3))*(- a - b*x^3)^(1/2)),x)
```

output
```
-int(x/((b^(1/3)*x - 2*a^(1/3))*(- a - b*x^3)^(1/2)), x)
```

3.100 $\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$

3.100.1 Optimal result

Integrand size = 25, antiderivative size = 202

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9\sqrt{c}d^2} - \frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

output
```
2/9*arctanh(1/3*(-2*d*x+c)^2/c^(1/2)/(-8*d^3*x^3+c^3)^(1/2))/d^2/c^(1/2)-1
/9*(-2*d*x+c)*EllipticF((-2*d*x+c*(1-3^(1/2)))/(-2*d*x+c*(1+3^(1/2))),I*3^
(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((4*d^2*x^2+2*c*d*x+c^2)/(-2*d*x+c*(1
+3^(1/2)))^2)^(1/2)*3^(3/4)/d^2/(-8*d^3*x^3+c^3)^(1/2)/(c*(-2*d*x+c)/(-2*d
*x+c*(1+3^(1/2)))^2)^(1/2)
```

3.100.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.55 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.46

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$$

$$= \frac{\sqrt{\frac{c-2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(\left(-2+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}c+2dx\right)\sqrt{\frac{\sqrt[3]{-1}\left(c+2\sqrt[3]{-1}dx\right)}{\left(1+\sqrt[3]{-1}\right)c}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\right),\sqrt[3]{\cdots}\right.\right.}{\left(-2+\sqrt[3]{-1}\right)d^2\sqrt{\frac{c-2(-1)^{2/\cdots}}{\left(1+\sqrt[3]{-\cdots}\right.}}}$$

input
```
Integrate[x/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

output
```
(Sqrt[(c - 2*d*x)/((1 + (-1)^(1/3))*c)]*((-2 + (-1)^(1/3))*((-1)^(1/3)*c +
 2*d*x)*Sqrt[((-1)^(1/3)*(c + 2*(-1)^(1/3)*d*x))/((1 + (-1)^(1/3))*c)]*Ell
ipticF[ArcSin[Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]], (-1)^(1/
3)] + (2*(-1)^(1/3)*(1 + (-1)^(1/3))*c*Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (
-1)^(1/3))*c)]*Sqrt[(c^2 + 2*c*d*x + 4*d^2*x^2)/c^2]*EllipticPi[(2*Sqrt[3]
)/(3*I + Sqrt[3]), ArcSin[Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)
]], (-1)^(1/3)])/Sqrt[3]))/((-2 + (-1)^(1/3))*d^2*Sqrt[(c - 2*(-1)^(2/3)*d
*x)/((1 + (-1)^(1/3))*c)]*Sqrt[c^3 - 8*d^3*x^3])
```

3.100.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2564, 759, 2563, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$$

$$\Big\downarrow \ 2564$$

$$\frac{\int \frac{1}{\sqrt{c^3-8d^3x^3}}dx}{3d}-\frac{\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}dx}{3d}$$

$$\Big\downarrow \text{759}$$

$$-\frac{\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}dx}{3d}-\frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

$$\Big\downarrow \text{2563}$$

$$\frac{2c\int \frac{1}{9-\frac{(c-2dx)^4}{c\left(c^3-8d^3x^3\right)}}d\frac{(c-2dx)^2}{c^2\sqrt{c^3-8d^3x^3}}}{3d^2}-\frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9\sqrt{c}d^2}-\frac{\sqrt{2+\sqrt{3}}(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right),-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\sqrt{c^3-8d^3x^3}}$$

input
```
Int[x/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

output
```
(2*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(9*Sqrt[c]*d^
2) - (Sqrt[2 + Sqrt[3]]*(c - 2*d*x)*Sqrt[(c^2 + 2*c*d*x + 4*d^2*x^2)/((1 +
 Sqrt[3])*c - 2*d*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c - 2*d*x)/((1 + S
qrt[3])*c - 2*d*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*d^2*Sqrt[(c*(c - 2*d*x))/
((1 + Sqrt[3])*c - 2*d*x)^2]*Sqrt[c^3 - 8*d^3*x^3])
```

3.100. $\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$

3.100.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2563
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Simp[-2*(e/d)   Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

rule 2564
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[(2*d*e + c*f)/(3*c*d)   Int[1/Sqrt[a + b*x^3], x], x] + Si
mp[(d*e - c*f)/(3*c*d)   Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*
d^3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

3.100.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 508 vs. 2(173) = 346.

Time = 0.98 (sec) , antiderivative size = 509, normalized size of antiderivative = 2.52

method	result
default	$\frac{2\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}\right)\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}F\left(\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}},\sqrt{\frac{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\right.}{d\sqrt{-8d^3x^3+c^3}}$
elliptic	$\frac{2\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}\right)\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}F\left(\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}},\sqrt{\frac{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\right.}{d\sqrt{-8d^3x^3+c^3}}$

input
```
int(x/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3
^(1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2
*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*c
/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)*EllipticF((
(x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2),((1/2*c/d-1/2*(-
1/2-1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2))-4/3
/d*(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^
(1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*
(-1/2+1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*c/
d-1/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)*EllipticPi((
(x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2),2/3*(1/2*c/d-1/2
*(-1/2-1/2*I*3^(1/2))*c/d)/c*d,((1/2*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/
2*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2))
```

3.100.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.76

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$$
$$= \left[\frac{\sqrt{c}d^2\log\left(\frac{8\,d^6x^6-240\,cd^5x^5+408\,c^2d^4x^4+88\,c^3d^3x^3+156\,c^4d^2x^2+12\,c^5dx+17\,c^6+3\,\left(8\,d^4x^4-52\,cd^3x^3+12\,c^2d^2x^2-4\,c^3dx+5\,c^4\right)\sqrt{-8\,d^3x}}{d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx+c^6}\right.}{18\,cd^4}\right.$$

input
```
integrate(x/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

output
```
[1/18*(sqrt(c)*d^2*log((8*d^6*x^6 - 240*c*d^5*x^5 + 408*c^2*d^4*x^4 + 88*c
^3*d^3*x^3 + 156*c^4*d^2*x^2 + 12*c^5*d*x + 17*c^6 + 3*(8*d^4*x^4 - 52*c*d
^3*x^3 + 12*c^2*d^2*x^2 - 4*c^3*d*x + 5*c^4)*sqrt(-8*d^3*x^3 + c^3)*sqrt(c
))/(d^6*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x
^2 + 6*c^5*d*x + c^6)) - 3*sqrt(2)*sqrt(-d^3)*c*weierstrassPInverse(0, 1/2
*c^3/d^3, x))/(c*d^4), 1/18*(2*sqrt(-c)*d^2*arctan(1/3*(4*d^3*x^3 - 24*c*d
^2*x^2 - 6*c^2*d*x - 5*c^3)*sqrt(-8*d^3*x^3 + c^3)*sqrt(-c)/(16*c*d^4*x^4
- 8*c^2*d^3*x^3 - 2*c^4*d*x + c^5)) - 3*sqrt(2)*sqrt(-d^3)*c*weierstrassPI
nverse(0, 1/2*c^3/d^3, x))/(c*d^4)]
```

3.100.6 Sympy [F]

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx=\int \frac{x}{\sqrt{-\left(-c+2dx\right)\left(c^2+2cdx+4d^2x^2\right)}\left(c+dx\right)}\,dx$$

input
```
integrate(x/(d*x+c)/(-8*d**3*x**3+c**3)**(1/2),x)
```

output
```
Integral(x/(sqrt(-(-c + 2*d*x)*(c**2 + 2*c*d*x + 4*d**2*x**2))*(c + d*x)),
 x)
```

3.100.7 Maxima [F]

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx=\int \frac{x}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}\,dx$$

input
```
integrate(x/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

3.100.8 Giac [F]

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \int \frac{x}{\sqrt{-8\,d^3x^3+c^3}(dx+c)}\,dx$$

input
```
integrate(x/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \int \frac{x}{\sqrt{c^3-8\,d^3\,x^3}\,(c+d\,x)}\,dx$$

input
```
int(x/((c^3 - 8*d^3*x^3)^(1/2)*(c + d*x)),x)
```

output
```
int(x/((c^3 - 8*d^3*x^3)^(1/2)*(c + d*x)), x)
```

3.101 $\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

3.101.1 Optimal result

Integrand size = 30, antiderivative size = 42

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx=-\frac{2\text{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

output
```
-2*arctanh((1+x)*(-3+2*3^(1/2))^(1/2)/(x^3+1)^(1/2))/(-3+2*3^(1/2))^(1/2)
```

3.101.2 Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx=-2\sqrt{1+\frac{2}{\sqrt{3}}}\text{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{1+x^3}}{1-x+x^2}\right)$$

input
```
Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
-2*Sqrt[1 + 2/Sqrt[3]]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[1 + x^3])/(1 - x
 + x^2)]
```

3.101.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sqrt{3}+1}{\left(x-\sqrt{3}+1\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{2565}$$

$$-2\int \frac{1}{\frac{\left(3-2\sqrt{3}\right)(x+1)^2}{x^3+1}+1}d\frac{x+1}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

input
```
Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[-3 + 2*Sqr
t[3]]
```

3.101.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

3.101.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.44 (sec) , antiderivative size = 130, normalized size of antiderivative = 3.10

method	result
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)\ln\left(-\frac{6\operatorname{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)x^2+4\operatorname{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)\sqrt{3}\,x^2-4\sqrt{3}\operatorname{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)x \cdots}{\left(x\sqrt{3}+x-2\right)^2}\right)}{6}$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\cdots}{\sqrt{x^3+\cdots}}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\cdots}{\sqrt{x^3+\cdots}}$

input
```
int((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*RootOf(_Z^2-24*3^(1/2)-36)*ln(-(6*RootOf(_Z^2-24*3^(1/2)-36)*x^2+4*Ro
otOf(_Z^2-24*3^(1/2)-36)*3^(1/2)*x^2-4*3^(1/2)*RootOf(_Z^2-24*3^(1/2)-36)*
x+48*(x^3+1)^(1/2)*3^(1/2)+4*RootOf(_Z^2-24*3^(1/2)-36)*3^(1/2)+72*(x^3+1)
^(1/2)+12*RootOf(_Z^2-24*3^(1/2)-36))/(x*3^(1/2)+x-2)^2)
```

3.101.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(32) = 64.

Time = 0.34 (sec) , antiderivative size = 205, normalized size of antiderivative = 4.88

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$$
$$=\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}+3}\log\left(\frac{x^8-16x^7+112x^6-16x^5+112x^4+224x^3+64x^2-4\left(2x^6-18x^5+42x^4-\cdots\right.}{\cdots}\right.$$

input
```
integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*sqrt(2*sqrt(3) + 3)*log((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112
*x^4 + 224*x^3 + 64*x^2 - 4*(2*x^6 - 18*x^5 + 42*x^4 - 8*x^3 - sqrt(3)*(x^
6 - 12*x^5 + 18*x^4 - 16*x^3 - 12*x^2 - 8) + 24*x + 8)*sqrt(x^3 + 1)*sqrt(
2*sqrt(3) + 3) + 16*sqrt(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 +
 4*x + 4) + 128*x + 112)/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3
+ 64*x^2 - 64*x + 16))
```

3.101.6 Sympy [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x+1+\sqrt{3}}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x-\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(x**3+1)**(1/2),x)
```

output
```
Integral((x + 1 + sqrt(3))/(sqrt((x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1)
), x)
```

3.101.7 Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

3.101.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[-1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[-2,4]:[1,0,-3]%%}
,[2]%%%}
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Hanged}$$

input
```
int((x + 3^(1/2) + 1)/((x^3 + 1)^(1/2)*(x - 3^(1/2) + 1)),x)
```

output
```
\text{Hanged}
```

3.102 $\int \frac{1+\sqrt{3}-x}{(1-\sqrt{3}-x)\sqrt{1-x^3}}\,dx$

3.102.1 Optimal result

Integrand size = 36, antiderivative size = 46

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

output
```
2*arctanh((1-x)*(-3+2*3^(1/2))^(1/2)/(-x^3+1)^(1/2))/(-3+2*3^(1/2))^(1/2)
```

3.102.2 Mathematica [A] (verified)

Time = 1.80 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx=2\sqrt{1+\frac{2}{\sqrt{3}}}\text{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{1-x^3}}{1+x+x^2}\right)$$

input
```
Integrate[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

output
```
2*Sqrt[1 + 2/Sqrt[3]]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[1 - x^3])/(1 + x
+ x^2)]
```

3.102.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x+\sqrt{3}+1}{\left(-x-\sqrt{3}+1\right)\sqrt{1-x^3}}dx$$

$$\big\downarrow \text{2565}$$

$$2\int \frac{1}{\frac{\left(3-2\sqrt{3}\right)(1-x)^2}{1-x^3}+1}d\frac{1-x}{\sqrt{1-x^3}}$$

$$\big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{2\sqrt{3}-3}}$$

input
```
Int[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

output
```
(2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[-3 + 2*Sqrt
[3]]
```

3.102.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

3.102.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.89

method	result
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)\ln\left(\frac{6\operatorname{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)x^2+4\operatorname{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)\sqrt{3}\,x^2+4\sqrt{3}\operatorname{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)x-4\cdots}{\left(x\sqrt{3}+x+2\right)^2}\cdots\right.}{6}$
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{4i\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\cdots}{\cdots}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}+\frac{4i\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\cdots}{\cdots}$

input
```
int((1-x+3^(1/2))/(1-x-3^(1/2))/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*RootOf(_Z^2-24*3^(1/2)-36)*ln((6*RootOf(_Z^2-24*3^(1/2)-36)*x^2+4*Roo
tOf(_Z^2-24*3^(1/2)-36)*3^(1/2)*x^2+4*3^(1/2)*RootOf(_Z^2-24*3^(1/2)-36)*x
-48*(-x^3+1)^(1/2)*3^(1/2)+4*RootOf(_Z^2-24*3^(1/2)-36)*3^(1/2)-72*(-x^3+1
)^(1/2)+12*RootOf(_Z^2-24*3^(1/2)-36))/(x*3^(1/2)+x+2)^2)
```

3.102.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(35) = 70.

Time = 0.39 (sec) , antiderivative size = 207, normalized size of antiderivative = 4.50

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$$
$$=\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}+3}\log\left(\frac{x^8+16x^7+112x^6+16x^5+112x^4-224x^3+64x^2+4\left(2x^6+18x^5+42x^4+8\cdots\right.}{\cdots}\right.$$

input
```
integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas"
)
```

output
```
1/6*sqrt(3)*sqrt(2*sqrt(3) + 3)*log((x^8 + 16*x^7 + 112*x^6 + 16*x^5 + 112
*x^4 - 224*x^3 + 64*x^2 + 4*(2*x^6 + 18*x^5 + 42*x^4 + 8*x^3 - sqrt(3)*(x^
6 + 12*x^5 + 18*x^4 + 16*x^3 - 12*x^2 - 8) - 24*x + 8)*sqrt(-x^3 + 1)*sqrt
(2*sqrt(3) + 3) - 16*sqrt(3)*(x^7 + 2*x^6 + 6*x^5 - 5*x^4 + 2*x^3 - 6*x^2
+ 4*x - 4) - 128*x + 112)/(x^8 - 8*x^7 + 16*x^6 + 16*x^5 - 56*x^4 - 32*x^3
 + 64*x^2 + 64*x + 16))
```

3.102.6 Sympy [F]

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \int \frac{x-\sqrt{3}-1}{\sqrt{-\left(x-1\right)\left(x^2+x+1\right)}\left(x-1+\sqrt{3}\right)}\,dx$$

input
```
integrate((1-x+3**(1/2))/(1-x-3**(1/2))/(-x**3+1)**(1/2),x)
```

output
```
Integral((x - sqrt(3) - 1)/(sqrt(-(x - 1)*(x**2 + x + 1))*(x - 1 + sqrt(3)
)), x)
```

3.102.7 Maxima [F]

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \int \frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}\left(x+\sqrt{3}-1\right)}\,dx$$

input
```
integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(x + sqrt(3) - 1)), x)
```

3.102.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[1,1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[-2,4]:[1,0,-3]%%},[
2]%%%} Er
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \text{Hanged}$$

input
```
int(-(3^(1/2) - x + 1)/((1 - x^3)^(1/2)*(x + 3^(1/2) - 1)),x)
```

output
```
\text{Hanged}
```

3.103 $\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

3.103.1 Optimal result

Integrand size = 34, antiderivative size = 44

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

output
```
2*arctan((1-x)*(-3+2*3^(1/2))^(1/2)/(x^3-1)^(1/2))/(-3+2*3^(1/2))^(1/2)
```

3.103.2 Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx=-2\sqrt{1+\frac{2}{\sqrt{3}}}\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{-1+x^3}}{1+x+x^2}\right)$$

input
```
Integrate[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

output
```
-2*Sqrt[1 + 2/Sqrt[3]]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[-1 + x^3])/(1 + x
 + x^2)]
```

3.103.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x+\sqrt{3}+1}{\left(-x-\sqrt{3}+1\right)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{ 2565}$$

$$2\int \frac{1}{1-\frac{\left(3-2\sqrt{3}\right)(1-x)^2}{x^3-1}}d\frac{1-x}{\sqrt{x^3-1}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{2\arctan\left(\frac{\sqrt{2\sqrt{3}-3}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

input
```
Int[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[-3 + 2*Sqrt
[3]]
```

3.103.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

3.103.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.43 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.93

method	result
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)\ln\left(\frac{6\operatorname{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)x^2+4\operatorname{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)\sqrt{3}\,x^2+4\sqrt{3}\operatorname{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)x-48\sqrt{}\cdots}{\left(x\sqrt{3}+x+2\right)^2}\cdots\right.}{6}$
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}\cdots}{\frac{3}{2}+\cdots}}\cdots}{\sqrt{}\cdots}$
elliptic	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}\cdots}{\frac{3}{2}+\cdots}}\cdots}{\sqrt{}\cdots}$

input
```
int((1-x+3^(1/2))/(1-x-3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*RootOf(_Z^2+24*3^(1/2)+36)*ln((6*RootOf(_Z^2+24*3^(1/2)+36)*x^2+4*Root
Of(_Z^2+24*3^(1/2)+36)*3^(1/2)*x^2+4*3^(1/2)*RootOf(_Z^2+24*3^(1/2)+36)*x-
48*(x^3-1)^(1/2)*3^(1/2)+4*RootOf(_Z^2+24*3^(1/2)+36)*3^(1/2)-72*(x^3-1)^(
1/2)+12*RootOf(_Z^2+24*3^(1/2)+36))/(x*3^(1/2)+x+2)^2)
```

3.103.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.14

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$$
$$=\frac{1}{3}\sqrt{3}\sqrt{2\sqrt{3}+3}\arctan\left(\frac{\left(\sqrt{3}(x^2+4x-2)-6x+6\right)\sqrt{2\sqrt{3}+3}}{6\sqrt{x^3-1}}\right)$$

input
```
integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*sqrt(2*sqrt(3) + 3)*arctan(1/6*(sqrt(3)*(x^2 + 4*x - 2) - 6*x
+ 6)*sqrt(2*sqrt(3) + 3)/sqrt(x^3 - 1))
```

3.103.6 Sympy [F]

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \int \frac{x-\sqrt{3}-1}{\sqrt{(x-1)\left(x^2+x+1\right)}\left(x-1+\sqrt{3}\right)}\,dx$$

input
```
integrate((1-x+3**(1/2))/(1-x-3**(1/2))/(x**3-1)**(1/2),x)
```

output
```
Integral((x - sqrt(3) - 1)/(sqrt((x - 1)*(x**2 + x + 1))*(x - 1 + sqrt(3))
), x)
```

3.103.7 Maxima [F]

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \int \frac{x-\sqrt{3}-1}{\sqrt{x^3-1}\left(x+\sqrt{3}-1\right)}\,dx$$

input
```
integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(x + sqrt(3) - 1)), x)
```

3.103.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[1,1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[-2,4]:[1,0,-3]%%},[
2]%%%} Er
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Hanged}$$

input
```
int(-(3^(1/2) - x + 1)/((x^3 - 1)^(1/2)*(x + 3^(1/2) - 1)),x)
```

output
```
\text{Hanged}
```

3.104 $\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$

3.104.1 Optimal result

Integrand size = 32, antiderivative size = 44

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

output
```
-2*arctan((1+x)*(-3+2*3^(1/2))^(1/2)/(-x^3-1)^(1/2))/(-3+2*3^(1/2))^(1/2)
```

3.104.2 Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx=2\sqrt{1+\frac{2}{\sqrt{3}}}\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{-1-x^3}}{1-x+x^2}\right)$$

input
```
Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

output
```
2*Sqrt[1 + 2/Sqrt[3]]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[-1 - x^3])/(1 - x
+ x^2)]
```

3.104.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sqrt{3}+1}{\left(x-\sqrt{3}+1\right)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \; 2565$$

$$-2\int \frac{1}{1-\frac{\left(3-2\sqrt{3}\right)(x+1)^2}{-x^3-1}}d\frac{x+1}{\sqrt{-x^3-1}}$$

$$\Big\downarrow \; 216$$

$$-\frac{2\arctan\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

input
```
Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[-3 + 2*Sqr
t[3]]
```

3.104.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

3.104.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 3.05

method	result
trager	$\frac{\mathrm{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)\ln\left(-\frac{6\,\mathrm{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)x^2+4\,\mathrm{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)\sqrt{3}\,x^2-4\sqrt{3}\,\mathrm{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)x+4}{\left(x\sqrt{3}+x-2\right)^2}\right.}{6}$
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{4i\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{}{\frac{3}{2}}}}{}$
elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{4i\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{}{\frac{3}{2}}}}{}$

input
```
int((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*RootOf(_Z^2+24*3^(1/2)+36)*ln(-(6*RootOf(_Z^2+24*3^(1/2)+36)*x^2+4*Roo
tOf(_Z^2+24*3^(1/2)+36)*3^(1/2)*x^2-4*3^(1/2)*RootOf(_Z^2+24*3^(1/2)+36)*x
+4*RootOf(_Z^2+24*3^(1/2)+36)*3^(1/2)+48*(-x^3-1)^(1/2)*3^(1/2)+12*RootOf(
_Z^2+24*3^(1/2)+36)+72*(-x^3-1)^(1/2))/(x*3^(1/2)+x-2)^2)
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.34

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{1}{3}\sqrt{3}\sqrt{2\sqrt{3}+3}\arctan\left(\frac{\sqrt{-x^3-1}\left(\sqrt{3}(x^2-4x-2)+6x+6\right)\sqrt{2\sqrt{3}+3}}{6\left(x^3+1\right)}\right)$$

input
```
integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas"
)
```

output
```
1/3*sqrt(3)*sqrt(2*sqrt(3) + 3)*arctan(1/6*sqrt(-x^3 - 1)*(sqrt(3)*(x^2 -
4*x - 2) + 6*x + 6)*sqrt(2*sqrt(3) + 3)/(x^3 + 1))
```

3.104.6 Sympy [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \int \frac{x+1+\sqrt{3}}{\sqrt{-\left(x+1\right)\left(x^2-x+1\right)}\left(x-\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(-x**3-1)**(1/2),x)
```

output
```
Integral((x + 1 + sqrt(3))/(sqrt(-(x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1
)), x)
```

3.104.7 Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}\left(x-\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(x - sqrt(3) + 1)), x)
```

3.104.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[-1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[-2,4]:[1,0,-3]%%}
,[2]%%%}
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx=\text{Hanged}$$

input
```
int((x + 3^(1/2) + 1)/((- x^3 - 1)^(1/2)*(x - 3^(1/2) + 1)),x)
```

output
```
\text{Hanged}
```

3.105 $\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

3.105.1 Optimal result

Integrand size = 58, antiderivative size = 69

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
-2*arctanh(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(b*x^3+a)^(1/2
))/a^(1/6)/b^(1/3)/(-3+2*3^(1/2))^(1/2)
```

3.105.2 Mathematica [A] (verified)

Time = 7.40 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.22

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{1+\frac{2}{\sqrt{3}}}\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)*Sqrt[a + b*x^3]),x]
```

3.105. $\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

output
```
(-2*ArcTanh[(Sqrt[1 + 2/Sqrt[3]]*(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2))/(a^(1/6)*Sqrt[a + b*x^3])])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

3.105.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{2565}$$

$$-\frac{2\sqrt[3]{a}\int \frac{1}{\frac{\left(3-2\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{bx^3+a}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{bx^3+a}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*
x)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a +
b*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

3.105.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

3.105.4 Maple [F]

$$\int \frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(1+\sqrt{3}\right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(1-\sqrt{3}\right)\right) \sqrt{b\,x^{3}+a}}dx$$

input
```
int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3
+a)^(1/2),x)
```

output
```
int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3
+a)^(1/2),x)
```

3.105.5 Fricas [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 1240, normalized size of antiderivative = 17.97

$$\int \frac{\left(1+\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b}x\right) \sqrt{a+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/
(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 - 1
840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*
x^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 286
72*a^8 + 4*sqrt(1/3)*sqrt(b*x^3 + a)*((3*b^7*x^22 - 2688*a*b^6*x^19 + 5695
2*a^2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 - 377856*a^5*b^2*
x^7 - 314880*a^6*b*x^4 - 24576*a^7*x - 2*sqrt(3)*(b^7*x^22 - 764*a*b^6*x^1
9 + 16860*a^2*b^5*x^16 - 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 + 104448*
a^5*b^2*x^7 + 90880*a^6*b*x^4 + 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^7
*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 - 24192*a^4*b^4*x^11 - 3974
4*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^
20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5
*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) - 2*(30*a*b^7*x^21
 - 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5
*b^3*x^9 + 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 - sqrt(3)*(17*a
*b^7*x^21 - 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 -
56000*a^5*b^3*x^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*b^(1
/3))*sqrt((2*sqrt(3) + 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 - 846*a*b^6*x^19 +
 4617*a^2*b^5*x^16 + 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^
2*x^7 + 59328*a^6*b*x^4 + 4608*a^7*x - sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^1
9 + 2130*a^2*b^5*x^16 - 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 - 53760*...
```

3.105.6 Sympy [F]

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1-3**(1
/2)))/(b*x**3+a)**(1/2),x)
```

output
```
Integral((a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(a + b*x**3)*(-sq
rt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)), x)
```

3.105.7 Maxima [F]

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(\sqrt{3}+1)}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}(\sqrt{3}-1)\right)}\,dx$$

input
```
integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/
(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/(sqrt(b*x^3 + a)*(b^(1/3)*x
- a^(1/3)*(sqrt(3) - 1))), x)
```

3.105.8 Giac [F(-1)]

Timed out.

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/
(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \text{Hanged}$$

input
```
int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/((a + b*x^3)^(1/2)*(b^(1/3)*x - a^
(1/3)*(3^(1/2) - 1))),x)
```

output
```
\text{Hanged}
```

3.106 $\int \frac{(1+\sqrt{3}) \sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3}) \sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

3.106.1 Optimal result

Integrand size = 61, antiderivative size = 71

$$\int \frac{(1+\sqrt{3}) \sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3}) \sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
2*arctanh(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(-b*x^3+a)^(1/2
))/a^(1/6)/b^(1/3)/(-3+2*3^(1/2))^(1/2)
```

3.106.2 Mathematica [A] (verified)

Time = 7.40 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.18

$$\int \frac{(1+\sqrt{3}) \sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3}) \sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{1+\frac{2}{\sqrt{3}}}\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) - b^
(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
(2*ArcTanh[(Sqrt[1 + 2/Sqrt[3]]*(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
))/(a^(1/6)*Sqrt[a - b*x^3])])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

3.106.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx$$

$$\Big\downarrow \text{2565}$$

$$\frac{2\sqrt[3]{a}\int \frac{1}{\frac{\left(3-2\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{a-bx^3}+1}d\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{a-bx^3}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*
x)*Sqrt[a - b*x^3]),x]
```

output
```
(2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[a - b
*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

3.106.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

3.106.4 Maple [F]

$$\int \frac{-b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(1+\sqrt{3}\right)}{\left(-b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(1-\sqrt{3}\right)\right) \sqrt{-b\,x^{3}+a}}dx$$

input
```
int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*
x^3+a)^(1/2),x)
```

output
```
int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*
x^3+a)^(1/2),x)
```

3.106.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(53) = 106$.

Time = 1.05 (sec) , antiderivative size = 1294, normalized size of antiderivative = 18.23

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Too large to display}$$

input
```
integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))
)/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 + 1
840*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*
x^12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 286
72*a^8 + 32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 - 5472*a^3*b^
4*x^13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 - 4608*a
^7*x - sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 + 4928*a^3
*b^4*x^13 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 + 256
0*a^7*x))*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 13320*a^2*b^
5*x^17 + 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b^2*x^8 - 3
28704*a^6*b*x^5 + 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 + 299*a*b^6*x^20 + 4
260*a^2*b^5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024*a^5*b^2
*x^8 + 93184*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 4*sqrt(1/3)*((3
*b^7*x^22 + 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63
552*a^4*b^3*x^10 + 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 + 24576*a^7*x - 2
*sqrt(3)*(b^7*x^22 + 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 + 19792*a^3*b^4*x
^13 + 42368*a^4*b^3*x^10 - 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 - 7168*a^7
*x))*sqrt(-b*x^3 + a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^
17 + 14472*a^3*b^5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a
^6*b^2*x^5 - 13824*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17
+ 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6...
```

3.106.6 Sympy [F]

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{-\sqrt{3}\sqrt[3]{a}-\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a-bx^3}\left(-\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1-3**
(1/2)))/(-b*x**3+a)**(1/2),x)
```

output
```
Integral((-sqrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)/(sqrt(a - b*x**3)*(-a
**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

3.106.7 Maxima [F]

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

input
```
integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))
)/(-b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/(sqrt(-b*x^3 + a)*(b^(1/3)*x
 + a^(1/3)*(sqrt(3) - 1))), x)
```

3.106.8 Giac [F(-1)]

Timed out.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))
)/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Hanged}$$

input
```
int((b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/((a - b*x^3)^(1/2)*(b^(1/3)*x + a^
(1/3)*(3^(1/2) - 1))),x)
```

output
```
\text{Hanged}
```

3.107 $\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

3.107.1 Optimal result

Integrand size = 62, antiderivative size = 72

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
2*arctan(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(b*x^3-a)^(1/2))
/a^(1/6)/b^(1/3)/(-3+2*3^(1/2))^(1/2)
```

3.107.2 Mathematica [A] (verified)

Time = 7.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.18

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{1+\frac{2}{\sqrt{3}}}\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[6]{a}\sqrt{-a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((((1 - Sqrt[3])*a^(1/3) - b^
(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[1 + 2/Sqrt[3]]*(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)
)/(a^(1/6)*Sqrt[-a + b*x^3])])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

3.107.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{2565}$$

$$\frac{2\sqrt[3]{a}\int \frac{1}{1-\frac{\left(3-2\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{bx^3-a}}d\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{bx^3-a}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\arctan\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*
x)*Sqrt[-a + b*x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[-a + b
*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

3.107.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

3.107.4 Maple [F]

$$\int \frac{-b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(1+\sqrt{3}\right)}{\left(-b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{b\,x^{3}-a}}dx$$

input
```
int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x
^3-a)^(1/2),x)
```

output
```
int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x
^3-a)^(1/2),x)
```

3.107.5 Fricas [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 1245, normalized size of antiderivative = 17.29

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))
)/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 +
1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4
*x^12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28
672*a^8 - 4*sqrt(1/3)*sqrt(b*x^3 - a)*((3*b^7*x^22 + 2688*a*b^6*x^19 + 569
52*a^2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 + 377856*a^5*b^2
*x^7 - 314880*a^6*b*x^4 + 24576*a^7*x - 2*sqrt(3)*(b^7*x^22 + 764*a*b^6*x^
19 + 16860*a^2*b^5*x^16 + 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 - 104448
*a^5*b^2*x^7 + 90880*a^6*b*x^4 - 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^
7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 + 24192*a^4*b^4*x^11 - 397
44*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x
^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^
5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) + 2*(30*a*b^7*x^2
1 + 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^
5*b^3*x^9 - 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(17*
a*b^7*x^21 + 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 -
 56000*a^5*b^3*x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*b^(
1/3))*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 + 846*a*b^6*x^19
 + 4617*a^2*b^5*x^16 - 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 - 98496*a^5*
b^2*x^7 + 59328*a^6*b*x^4 - 4608*a^7*x - sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x
^19 + 2130*a^2*b^5*x^16 + 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 + 5376...
```

3.107.6 Sympy [F]

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{-\sqrt{3}\sqrt[3]{a}-\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a+bx^3}\left(-\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1-3**
(1/2)))/(b*x**3-a)**(1/2),x)
```

output
```
Integral((-sqrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)/(sqrt(-a + b*x**3)*(-
a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

3.107.7 Maxima [F]

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

input
```
integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))
)/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/(sqrt(b*x^3 - a)*(b^(1/3)*x
+ a^(1/3)*(sqrt(3) - 1))), x)
```

3.107.8 Giac [F(-1)]

Timed out.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))
)/(b*x^3-a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Hanged}$$

input
```
int((b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/((b*x^3 - a)^(1/2)*(b^(1/3)*x + a^
(1/3)*(3^(1/2) - 1))),x)
```

output
```
\text{Hanged}
```

3.108 $\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

3.108.1 Optimal result

Integrand size = 61, antiderivative size = 72

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
-2*arctan(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(-b*x^3-a)^(1/2
))/a^(1/6)/b^(1/3)/(-3+2*3^(1/2))^(1/2)
```

3.108.2 Mathematica [A] (verified)

Time = 7.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{1+\frac{2}{\sqrt{3}}}\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[1 + 2/Sqrt[3]]*(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
))/(a^(1/6)*Sqrt[-a - b*x^3])])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

3.108.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1+\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x}{\left(\left(1-\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x\right) \sqrt{-a-b x^3}} \, dx$$

$$\Big\downarrow \textbf{2565}$$

$$-\frac{2 \sqrt[3]{a} \int \frac{1}{1-\frac{\left(3-2 \sqrt{3}\right) \sqrt[3]{a} \left(\sqrt[3]{b} x+\sqrt[3]{a}\right)^2}{-b x^3-a}}d\frac{\sqrt[3]{b} x+\sqrt[3]{a}}{\sqrt[3]{a} \sqrt{-b x^3-a}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \textbf{216}$$

$$-\frac{2 \arctan \left(\frac{\sqrt{2 \sqrt{3}-3} \sqrt[6]{a} \left(\sqrt[3]{a}+\sqrt[3]{b} x\right)}{\sqrt{-a-b x^3}}\right)}{\sqrt{2 \sqrt{3}-3} \sqrt[6]{a} \sqrt[3]{b}}$$

input
```
Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*
x)*Sqrt[-a - b*x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a -
b*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

3.108.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

3.108.4 Maple [F]

$$\int \frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(1+\sqrt{3}\right)}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(1-\sqrt{3}\right)\right) \sqrt{-b\,x^{3}-a}}dx$$

input
```
int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^
3-a)^(1/2),x)
```

output
```
int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^
3-a)^(1/2),x)
```

3.108.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(52) = 104.

Time = 1.09 (sec) , antiderivative size = 1303, normalized size of antiderivative = 18.10

$$\int \frac{\left(1+\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b}x\right) \sqrt{-a-bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/
(-b*x^3-a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 -
1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4
*x^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28
672*a^8 + 32*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 + 5472*a^3*b
^4*x^13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 + 4608*
a^7*x - sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 - 4928*a^
3*b^4*x^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 - 25
60*a^7*x))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x^20 + 13320*a^2*b
^5*x^17 - 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 345024*a^5*b^2*x^8 -
328704*a^6*b*x^5 - 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 - 299*a*b^6*x^20 +
4260*a^2*b^5*x^17 + 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 + 105024*a^5*b^
2*x^8 + 93184*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 4*sqrt(1/3)*((
3*b^7*x^22 - 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 - 93504*a^3*b^4*x^13 - 6
3552*a^4*b^3*x^10 - 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 - 24576*a^7*x -
2*sqrt(3)*(b^7*x^22 - 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 - 19792*a^3*b^4*
x^13 + 42368*a^4*b^3*x^10 + 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 + 7168*a^
7*x))*sqrt(-b*x^3 - a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x
^17 + 14472*a^3*b^5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*
a^6*b^2*x^5 - 13824*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17
 + 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^...
```

3.108.6 Sympy [F]

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1-3**(1
/2)))/(-b*x**3-a)**(1/2),x)
```

output
```
Integral((a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(-a - b*x**3)*(-s
qrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)), x)
```

3.108.7 Maxima [F]

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}+1\right)}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

input
```
integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/
(-b*x^3-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/(sqrt(-b*x^3 - a)*(b^(1/3)*x
 - a^(1/3)*(sqrt(3) - 1))), x)
```

3.108.8 Giac [F(-1)]

Timed out.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/
(-b*x^3-a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Hanged}$$

input
```
int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/((- a - b*x^3)^(1/2)*(b^(1/3)*x -
a^(1/3)*(3^(1/2) - 1))),x)
```

output
```
\text{Hanged}
```

3.109 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx$

3.109.1 Optimal result

Integrand size = 52, antiderivative size = 73

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

output
```
-2*arctanh((1+(b/a)^(1/3)*x)*a^(1/2)*(-3+2*3^(1/2))^(1/2)/(b*x^3+a)^(1/2))
/(b/a)^(1/3)/a^(1/2)/(-3+2*3^(1/2))^(1/2)
```

3.109.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.97 (sec) , antiderivative size = 663, normalized size of antiderivative = 9.08

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx$$
$$= \frac{x\left(12(-3+\sqrt{3})\sqrt[3]{\frac{b}{a}}x\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},\frac{bx^3}{-10a+6\sqrt{3}a}\right)-8\left(\frac{b}{a}\right)^{2/3}x^2\sqrt{3+\frac{3bx^3}{a}}\,\text{AppellF1}\right.}{}$$

input
```
Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqr
t[a + b*x^3]),x]
```

output
```
(x*(12*(-3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2,
 1, 5/3, -((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*
Sqrt[3 + (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), (b*x^3)/(-10*a
+ 6*Sqrt[3]*a)] - (3*(-18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a),
(b*x^3)/(-10*a + 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4
/3, -((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)] + b*x^3*(2*(-5 + 3*Sqrt[3
])*a - b*x^3)*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a),
 (b*x^3)/(-10*a + 6*Sqrt[3]*a)]*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1
, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + 3*b*x^3*(AppellF1[
4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + (5 - 3*
Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt
[3]*a))]))))/(a*(2*(-5 + 3*Sqrt[3])*a - b*x^3)*(8*(-5 + 3*Sqrt[3])*a*Appel
lF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + 3*b
*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]
*a))] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)
/(10*a - 6*Sqrt[3]*a))])))))/(24*(-5 + 3*Sqrt[3])*Sqrt[a + b*x^3])
```

3.109.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1}{\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \text{2565}$$

$$-\frac{2\int \frac{1}{\frac{\left(3-2\sqrt{3}\right)a\left(\sqrt[3]{\frac{b}{a}}x+1\right)^2}{bx^3+a}+1}d\frac{\sqrt[3]{\frac{b}{a}}x+1}{\sqrt{bx^3+a}}}{\sqrt[3]{\frac{b}{a}}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

input
```
Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[a +
b*x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[a + b*
x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

3.109.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

3.109.4 Maple [F]

$$\int \frac{1+\left(\frac{b}{a}\right)^{\frac{1}{3}} x+\sqrt{3}}{\left(1+\left(\frac{b}{a}\right)^{\frac{1}{3}} x-\sqrt{3}\right) \sqrt{b\,x^{3}+a}}dx$$

input
```
int((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x)
```

output
```
int((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x)
```

3.109.5 Fricas [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 1273, normalized size of antiderivative = 17.44

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx=\text{Too large to display}$$

input
```
integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1
/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a*
b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 +
 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8
 + 4*sqrt(1/3)*(486*a*b^7*x^20 - 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14 -
 145152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 - 414720*a^6*b^2*x^5 - 82944*a^7
*b*x^2 + (3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 93504*a^
4*b^4*x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 -
24576*a^8*x - 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^1
6 - 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 + 90880*a
^7*b*x^4 + 7168*a^8*x))*(b/a)^(2/3) - 6*sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*
b^6*x^17 + 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 376
32*a^6*b^2*x^5 + 8192*a^7*b*x^2) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 +
44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b
^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^
6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115
968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 + a
)*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b) - 8*(3*a*b^7*x^23 - 1077*a^2*b^6*x^2
0 + 13320*a^3*b^5*x^17 - 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 - 345024
*a^6*b^2*x^8 - 328704*a^7*b*x^5 - 61440*a^8*x^2 - 2*sqrt(3)*(a*b^7*x^23 -
299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 + 1520*a^4*b^4*x^14 + 26720*a^5*b^...
```

3.109.6 Sympy [F]

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{x\sqrt[3]{\frac{b}{a}}+1+\sqrt{3}}{\sqrt{a+bx^3}\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+(b/a)**(1/3)*x+3**(1/2))/(1+(b/a)**(1/3)*x-3**(1/2))/(b*x**3+
a)**(1/2),x)
```

output
```
Integral((x*(b/a)**(1/3) + 1 + sqrt(3))/(sqrt(a + b*x**3)*(x*(b/a)**(1/3)
- sqrt(3) + 1)), x)
```

3.109.7 Maxima [F]

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}+1}{\sqrt{bx^3+a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1
/2),x, algorithm="maxima")
```

output
```
integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/(sqrt(b*x^3 + a)*(x*(b/a)^(1/3) -
sqrt(3) + 1)), x)
```

3.109.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1
/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(con
st gen &
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}+1}{\sqrt{b\,x^3+a}\,\left(x\left(\frac{b}{a}\right)^{1/3}-\sqrt{3}+1\right)}\,dx$$

input
```
int((3^(1/2) + x*(b/a)^(1/3) + 1)/((a + b*x^3)^(1/2)*(x*(b/a)^(1/3) - 3^(1
/2) + 1)),x)
```

output
```
int((3^(1/2) + x*(b/a)^(1/3) + 1)/((a + b*x^3)^(1/2)*(x*(b/a)^(1/3) - 3^(1
/2) + 1)), x)
```

3.110 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx$

3.110.1 Optimal result

Integrand size = 55, antiderivative size = 75

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

output
```
2*arctanh((1-(b/a)^(1/3)*x)*a^(1/2)*(-3+2*3^(1/2))^(1/2)/(-b*x^3+a)^(1/2))
/(b/a)^(1/3)/a^(1/2)/(-3+2*3^(1/2))^(1/2)
```

3.110.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.86 (sec) , antiderivative size = 648, normalized size of antiderivative = 8.64

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx$$
$$= \frac{x\left(-12(-3+\sqrt{3})\sqrt[3]{\frac{b}{a}}x\sqrt{1-\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)-8\left(\frac{b}{a}\right)^{2/3}x^2\sqrt{3-\frac{3bx^3}{a}}\,\text{AppellF1}\left(\right.\right.}{}$$

input
```
Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqr
t[a - b*x^3]),x]
```

output
```
(x*(-12*(-3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2
, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqr
t[3 - (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqr
t[3]*a)] - (3*(-18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(1
0*a - 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)
/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] - b*x^3*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*S
qrt[1 - (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6
*Sqrt[3]*a)]*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (
b*x^3)/(10*a - 6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)
/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1,
7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)]))))/(a*(2*(-5 + 3*Sqrt[3])*a
 + b*x^3)*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x
^3)/(10*a - 6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a,
 (b*x^3)/(10*a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3
, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)])))))/(24*(-5 + 3*Sqrt[3])*Sqrt[
a - b*x^3])
```

3.110.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1\right)\sqrt{a-bx^3}}dx$$

$$\Big\downarrow \text{2565}$$

$$\frac{2\int \frac{1}{\frac{\left(3-2\sqrt{3}\right)a\left(1-\sqrt[3]{\frac{b}{a}}x\right)^2}{a-bx^3}+1}d\frac{1-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}}{\sqrt[3]{\frac{b}{a}}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

input
```
Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[a -
b*x^3]),x]
```

output
```
(2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[a - b*x
^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

3.110.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

3.110.4 Maple [F]

$$\int \frac{1-\left(\frac{b}{a}\right)^{\frac{1}{3}} x+\sqrt{3}}{\left(1-\left(\frac{b}{a}\right)^{\frac{1}{3}} x-\sqrt{3}\right) \sqrt{-b\,x^{3}+a}}dx$$

input
```
int((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x
)
```

output
```
int((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x
)
```

3.110.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(57) = 114.

Time = 0.72 (sec) , antiderivative size = 1330, normalized size of antiderivative = 17.73

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx=\text{Too large to display}$$

input
```
integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(
1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a*
b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 -
 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8
 - 4*sqrt(1/3)*((3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 9
3504*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b
*x^4 + 24576*a^8*x - 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*
b^5*x^16 + 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 +
90880*a^7*b*x^4 - 7168*a^8*x))*sqrt(-b*x^3 + a)*(b/a)^(2/3) + 2*(30*a*b^7*
x^21 + 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872
*a^5*b^3*x^9 - 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(
17*a*b^7*x^21 + 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^1
2 - 56000*a^5*b^3*x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*
sqrt(-b*x^3 + a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 1447
2*a^3*b^5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^
5 - 13824*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^
3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8
192*a^7*b*x^2))*sqrt(-b*x^3 + a))*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b) + 8*
(3*a*b^7*x^23 + 1077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 + 19200*a^4*b^4*x^1
4 - 111360*a^5*b^3*x^11 + 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 + 61440*a^
8*x^2 - 2*sqrt(3)*(a*b^7*x^23 + 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 - ...
```

3.110.6 Sympy [F]

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx = \int \frac{x\sqrt[3]{\frac{b}{a}}-\sqrt{3}-1}{\sqrt{a-bx^3}\left(x\sqrt[3]{\frac{b}{a}}-1+\sqrt{3}\right)}\,dx$$

input
```
integrate((1-(b/a)**(1/3)*x+3**(1/2))/(1-(b/a)**(1/3)*x-3**(1/2))/(-b*x**3
+a)**(1/2),x)
```

output
```
Integral((x*(b/a)**(1/3) - sqrt(3) - 1)/(sqrt(a - b*x**3)*(x*(b/a)**(1/3)
- 1 + sqrt(3))), x)
```

3.110.7 Maxima [F]

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx = \int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}-1}{\sqrt{-bx^3+a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1\right)}\,dx$$

input
```
integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(
1/2),x, algorithm="maxima")
```

output
```
integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/(sqrt(-b*x^3 + a)*(x*(b/a)^(1/3) +
 sqrt(3) - 1)), x)
```

3.110.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(
1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(con
st gen &
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx=\int -\frac{\sqrt{3}-x\left(\frac{b}{a}\right)^{1/3}+1}{\sqrt{a-b\,x^3}\,\left(\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}-1\right)}\,dx$$

input
```
int(-(3^(1/2) - x*(b/a)^(1/3) + 1)/((a - b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(
1/3) - 1)),x)
```

output
```
int(-(3^(1/2) - x*(b/a)^(1/3) + 1)/((a - b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(
1/3) - 1)), x)
```

3.111 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx$

3.111.1 Optimal result

Integrand size = 56, antiderivative size = 76

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx = \frac{2\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

output
```
2*arctan((1-(b/a)^(1/3)*x)*a^(1/2)*(-3+2*3^(1/2))^(1/2)/(b*x^3-a)^(1/2))/(
b/a)^(1/3)/a^(1/2)/(-3+2*3^(1/2))^(1/2)
```

3.111.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.90 (sec) , antiderivative size = 649, normalized size of antiderivative = 8.54

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx$$
$$= \frac{x\left(-12(-3+\sqrt{3})\sqrt[3]{\frac{b}{a}}x\sqrt{1-\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)-8\left(\frac{b}{a}\right)^{2/3}x^2\sqrt{3-\frac{3bx^3}{a}}\,\text{AppellF1}\left(\right.\right.}{}$$

input
```
Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqr
t[-a + b*x^3]),x]
```

output
```
(x*(-12*(-3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2
, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqr
t[3 - (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqr
t[3]*a)] - (3*(-18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(1
0*a - 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)
/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] - b*x^3*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*S
qrt[1 - (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6
*Sqrt[3]*a)]*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (
b*x^3)/(10*a - 6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)
/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1,
7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)]))))/(a*(2*(-5 + 3*Sqrt[3])*a
 + b*x^3)*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x
^3)/(10*a - 6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a,
 (b*x^3)/(10*a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3
, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)])))))/(24*(-5 + 3*Sqrt[3])*Sqrt[
-a + b*x^3])
```

3.111.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{2565}$$

$$\frac{2\int \frac{1}{1-\frac{\left(3-2\sqrt{3}\right)a\left(1-\sqrt[3]{\frac{b}{a}}x\right)^2}{bx^3-a}}d\frac{1-\sqrt[3]{\frac{b}{a}}x}{\sqrt{bx^3-a}}}{\sqrt[3]{\frac{b}{a}}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\arctan\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

input
```
Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[-a +
b*x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[-a + b*x
^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

3.111.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

3.111.4 Maple [F]

$$\int \frac{1-\left(\frac{b}{a}\right)^{\frac{1}{3}} x+\sqrt{3}}{\left(1-\left(\frac{b}{a}\right)^{\frac{1}{3}} x-\sqrt{3}\right) \sqrt{b\,x^{3}-a}}dx$$

input
```
int((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x)
```

output
```
int((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x)
```

3.111.5 Fricas [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 1278, normalized size of antiderivative = 16.82

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1
/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a
*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12
- 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^
8 - 4*sqrt(1/3)*(486*a*b^7*x^20 + 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14
+ 145152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 + 414720*a^6*b^2*x^5 - 82944*a^
7*b*x^2 + (3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 93504*a
^4*b^4*x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 +
 24576*a^8*x - 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^
16 + 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 + 90880*
a^7*b*x^4 - 7168*a^8*x))*(b/a)^(2/3) - 6*sqrt(3)*(47*a*b^7*x^20 + 2724*a^2
*b^6*x^17 + 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37
632*a^6*b^2*x^5 + 8192*a^7*b*x^2) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 +
 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*
b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b
^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 11
5968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 -
a)*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b) + 8*(3*a*b^7*x^23 + 1077*a^2*b^6*x
^20 + 13320*a^3*b^5*x^17 + 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 + 3450
24*a^6*b^2*x^8 - 328704*a^7*b*x^5 + 61440*a^8*x^2 - 2*sqrt(3)*(a*b^7*x^23
+ 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 - 1520*a^4*b^4*x^14 + 26720*a^5*...
```

3.111.6 Sympy [F]

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{x\sqrt[3]{\frac{b}{a}}-\sqrt{3}-1}{\sqrt{-a+bx^3}\left(x\sqrt[3]{\frac{b}{a}}-1+\sqrt{3}\right)}\,dx$$

input
```
integrate((1-(b/a)**(1/3)*x+3**(1/2))/(1-(b/a)**(1/3)*x-3**(1/2))/(b*x**3-
a)**(1/2),x)
```

output
```
Integral((x*(b/a)**(1/3) - sqrt(3) - 1)/(sqrt(-a + b*x**3)*(x*(b/a)**(1/3)
 - 1 + sqrt(3))), x)
```

3.111.7 Maxima [F]

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}-1}{\sqrt{bx^3-a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1\right)}\,dx$$

input
```
integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1
/2),x, algorithm="maxima")
```

output
```
integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/(sqrt(b*x^3 - a)*(x*(b/a)^(1/3) +
sqrt(3) - 1)), x)
```

3.111.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1
/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(con
st gen &
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx=\int -\frac{\sqrt{3}-x\left(\frac{b}{a}\right)^{1/3}+1}{\sqrt{b\,x^3-a}\,\left(\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}-1\right)}\,dx$$

input
```
int(-(3^(1/2) - x*(b/a)^(1/3) + 1)/((b*x^3 - a)^(1/2)*(3^(1/2) + x*(b/a)^(
1/3) - 1)),x)
```

output
```
int(-(3^(1/2) - x*(b/a)^(1/3) + 1)/((b*x^3 - a)^(1/2)*(3^(1/2) + x*(b/a)^(
1/3) - 1)), x)
```

3.112 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx$

3.112.1 Optimal result

Integrand size = 55, antiderivative size = 76

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

output
```
-2*arctan((1+(b/a)^(1/3)*x)*a^(1/2)*(-3+2*3^(1/2))^(1/2)/(-b*x^3-a)^(1/2))
/(b/a)^(1/3)/a^(1/2)/(-3+2*3^(1/2))^(1/2)
```

3.112.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.88 (sec) , antiderivative size = 666, normalized size of antiderivative = 8.76

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx$$
$$= \frac{x\left(12\left(-3+\sqrt{3}\right)\sqrt[3]{\frac{b}{a}}x\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},\frac{bx^3}{-10a+6\sqrt{3}a}\right)-8\left(\frac{b}{a}\right)^{2/3}x^2\sqrt{3+\frac{3bx^3}{a}}\,\text{AppellF1}\right.}{}$$

input
```
Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqr
t[-a - b*x^3]),x]
```

output
```
(x*(12*(-3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2,
 1, 5/3, -((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*
Sqrt[3 + (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), (b*x^3)/(-10*a
+ 6*Sqrt[3]*a)] - (3*(-18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a),
(b*x^3)/(-10*a + 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4
/3, -((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)] + b*x^3*(2*(-5 + 3*Sqrt[3
])*a - b*x^3)*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a),
 (b*x^3)/(-10*a + 6*Sqrt[3]*a)]*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1
, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + 3*b*x^3*(AppellF1[
4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + (5 - 3*
Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt
[3]*a))]))))/(a*(2*(-5 + 3*Sqrt[3])*a - b*x^3)*(8*(-5 + 3*Sqrt[3])*a*Appel
lF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + 3*b
*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]
*a))] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)
/(10*a - 6*Sqrt[3]*a))])))))/(24*(-5 + 3*Sqrt[3])*Sqrt[-a - b*x^3])
```

3.112.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1}{\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)\sqrt{-a-bx^3}}dx$$

$$\Big\downarrow \text{2565}$$

$$-\frac{2\int \frac{1}{1-\frac{\left(3-2\sqrt{3}\right)a\left(\sqrt[3]{\frac{b}{a}}x+1\right)^2}{-bx^3-a}}d\frac{\sqrt[3]{\frac{b}{a}}x+1}{\sqrt{-bx^3-a}}}{\sqrt[3]{\frac{b}{a}}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\arctan\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

input
```
Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[-a -
 b*x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[-a - b*
x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

3.112.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

3.112.4 Maple [F]

$$\int \frac{1+\left(\frac{b}{a}\right)^{\frac{1}{3}} x+\sqrt{3}}{\left(1+\left(\frac{b}{a}\right)^{\frac{1}{3}} x-\sqrt{3}\right)\sqrt{-b\,x^{3}-a}}dx$$

input
```
int((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x
)
```

output
```
int((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x
)
```

3.112.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(58) = 116.

Time = 0.73 (sec) , antiderivative size = 1339, normalized size of antiderivative = 17.62

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx=\text{Too large to display}$$

input
```
integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(
1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a
*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12
+ 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^
8 + 4*sqrt(1/3)*((3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 -
93504*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*
b*x^4 - 24576*a^8*x - 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3
*b^5*x^16 - 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 +
 90880*a^7*b*x^4 + 7168*a^8*x))*sqrt(-b*x^3 - a)*(b/a)^(2/3) - 2*(30*a*b^7
*x^21 - 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 7987
2*a^5*b^3*x^9 + 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 - sqrt(3)*
(17*a*b^7*x^21 - 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^
12 - 56000*a^5*b^3*x^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))
*sqrt(-b*x^3 - a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 144
72*a^3*b^5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x
^5 - 13824*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a
^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 +
8192*a^7*b*x^2))*sqrt(-b*x^3 - a))*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b) -
8*(3*a*b^7*x^23 - 1077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 - 19200*a^4*b^4*x
^14 - 111360*a^5*b^3*x^11 - 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 - 61440*
a^8*x^2 - 2*sqrt(3)*(a*b^7*x^23 - 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 ...
```

3.112.6 Sympy [F]

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx = \int \frac{x\sqrt[3]{\frac{b}{a}}+1+\sqrt{3}}{\sqrt{-a-bx^3}\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+(b/a)**(1/3)*x+3**(1/2))/(1+(b/a)**(1/3)*x-3**(1/2))/(-b*x**3
-a)**(1/2),x)
```

output
```
Integral((x*(b/a)**(1/3) + 1 + sqrt(3))/(sqrt(-a - b*x**3)*(x*(b/a)**(1/3)
 - sqrt(3) + 1)), x)
```

3.112.7 Maxima [F]

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx = \int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}+1}{\sqrt{-bx^3-a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(
1/2),x, algorithm="maxima")
```

output
```
integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/(sqrt(-b*x^3 - a)*(x*(b/a)^(1/3) -
 sqrt(3) + 1)), x)
```

3.112.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(
1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(con
st gen &
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}+1}{\sqrt{-b\,x^3-a}\,\left(x\left(\frac{b}{a}\right)^{1/3}-\sqrt{3}+1\right)}\,dx$$

input
```
int((3^(1/2) + x*(b/a)^(1/3) + 1)/((- a - b*x^3)^(1/2)*(x*(b/a)^(1/3) - 3^
(1/2) + 1)),x)
```

output
```
int((3^(1/2) + x*(b/a)^(1/3) + 1)/((- a - b*x^3)^(1/2)*(x*(b/a)^(1/3) - 3^
(1/2) + 1)), x)
```

3.113 $\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

3.113.1 Optimal result

Integrand size = 30, antiderivative size = 42

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

output
```
-2*arctan((1+x)*(3+2*3^(1/2))^(1/2)/(x^3+1)^(1/2))/(3+2*3^(1/2))^(1/2)
```

3.113.2 Mathematica [A] (verified)

Time = 2.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx=-2\sqrt{-1+\frac{2}{\sqrt{3}}}\arctan\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{1+x^3}}{1-x+x^2}\right)$$

input
```
Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
-2*Sqrt[-1 + 2/Sqrt[3]]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[1 + x^3])/(1 - x
+ x^2)]
```

3.113.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{2565}$$

$$-2\int \frac{1}{\frac{\left(3+2\sqrt{3}\right)(x+1)^2}{x^3+1}+1}d\frac{x+1}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

input
```
Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[3 + 2*Sqrt[3
]]
```

3.113.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

3.113.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.47 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.12

method	result
trager	$\frac{\operatorname{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)\ln\left(\frac{6\operatorname{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)x^2-4\operatorname{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)\sqrt{3}\,x^2+4\sqrt{3}\operatorname{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)x+48\sqrt{}}{\left(x\sqrt{3}-x+2\right)^2}\right.}{6}$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}}{\sqrt{x^3+}}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}}{\sqrt{x^3+}}$

input
```
int((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*RootOf(_Z^2-36+24*3^(1/2))*ln((6*RootOf(_Z^2-36+24*3^(1/2))*x^2-4*Root
Of(_Z^2-36+24*3^(1/2))*3^(1/2)*x^2+4*3^(1/2)*RootOf(_Z^2-36+24*3^(1/2))*x+
48*(x^3+1)^(1/2)*3^(1/2)-4*RootOf(_Z^2-36+24*3^(1/2))*3^(1/2)-72*(x^3+1)^(
1/2)+12*RootOf(_Z^2-36+24*3^(1/2)))/(x*3^(1/2)-x+2)^2)
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$$
$$=\frac{1}{3}\sqrt{3}\sqrt{2\sqrt{3}-3}\arctan\left(\frac{\left(\sqrt{3}(x^2-4x-2)-6x-6\right)\sqrt{2\sqrt{3}-3}}{6\sqrt{x^3+1}}\right)$$

input
```
integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*sqrt(2*sqrt(3) - 3)*arctan(1/6*(sqrt(3)*(x^2 - 4*x - 2) - 6*x
- 6)*sqrt(2*sqrt(3) - 3)/sqrt(x^3 + 1))
```

3.113.6 Sympy [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

input
```
integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

output
```
Integral((x - sqrt(3) + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))
), x)
```

3.113.7 Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

3.113.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[
2]%%%} Er
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx=\text{Hanged}$$

input
```
int((x - 3^(1/2) + 1)/((x^3 + 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

output
```
\text{Hanged}
```

3.114 $\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$

3.114.1 Optimal result

Integrand size = 36, antiderivative size = 46

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

output
```
2*arctan((1-x)*(3+2*3^(1/2))^(1/2)/(-x^3+1)^(1/2))/(3+2*3^(1/2))^(1/2)
```

3.114.2 Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx=2\sqrt{-1+\frac{2}{\sqrt{3}}}\arctan\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{1-x^3}}{1+x+x^2}\right)$$

input
```
Integrate[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

output
```
2*Sqrt[-1 + 2/Sqrt[3]]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[1 - x^3])/(1 + x +
 x^2)]
```

3.114.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x-\sqrt{3}+1}{\left(-x+\sqrt{3}+1\right)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{2565}$$

$$2\int \frac{1}{\frac{\left(3+2\sqrt{3}\right)(1-x)^2}{1-x^3}+1}d\frac{1-x}{\sqrt{1-x^3}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

input
```
Int[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[3 + 2*Sqrt[3]
]
```

3.114.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

3.114.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.47 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.93

method	result
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)\ln\left(\frac{6\,\mathrm{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)x^2-4\,\mathrm{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)\sqrt{3}\,x^2-4\sqrt{3}\,\mathrm{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)x+4}{\left(x\sqrt{3}-x-2\right)^2}\right.}{6}$
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}-\frac{4i\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{}}{}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}-\frac{4i\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{}}{}$

input
```
int((1-x-3^(1/2))/(1-x+3^(1/2))/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*RootOf(_Z^2-36+24*3^(1/2))*ln((6*RootOf(_Z^2-36+24*3^(1/2))*x^2-4*Roo
tOf(_Z^2-36+24*3^(1/2))*3^(1/2)*x^2-4*3^(1/2)*RootOf(_Z^2-36+24*3^(1/2))*x
+48*(-x^3+1)^(1/2)*3^(1/2)-4*RootOf(_Z^2-36+24*3^(1/2))*3^(1/2)-72*(-x^3+1
)^(1/2)+12*RootOf(_Z^2-36+24*3^(1/2)))/(x*3^(1/2)-x-2)^2)
```

3.114.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \frac{1}{3}\sqrt{3}\sqrt{2\sqrt{3}-3}\arctan\left(\frac{\sqrt{-x^3+1}\left(\sqrt{3}(x^2+4x-2)+6x-6\right)\sqrt{2\sqrt{3}-3}}{6\left(x^3-1\right)}\right)$$

input
```
integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas"
)
```

output
```
1/3*sqrt(3)*sqrt(2*sqrt(3) - 3)*arctan(1/6*sqrt(-x^3 + 1)*(sqrt(3)*(x^2 +
4*x - 2) + 6*x - 6)*sqrt(2*sqrt(3) - 3)/(x^3 - 1))
```

3.114.6 Sympy [F]

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \int \frac{x-1+\sqrt{3}}{\sqrt{-\left(x-1\right)\left(x^2+x+1\right)}\left(x-\sqrt{3}-1\right)}\,dx$$

input
```
integrate((1-x-3**(1/2))/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

output
```
Integral((x - 1 + sqrt(3))/(sqrt(-(x - 1)*(x**2 + x + 1))*(x - sqrt(3) - 1
)), x)
```

3.114.7 Maxima [F]

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \int \frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

input
```
integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

3.114.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[
2]%%%} Er
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx=\text{Hanged}$$

input
```
int(-(x + 3^(1/2) - 1)/((1 - x^3)^(1/2)*(3^(1/2) - x + 1)),x)
```

output
```
\text{Hanged}
```

3.115 $\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

3.115.1 Optimal result

Integrand size = 34, antiderivative size = 44

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

output
```
2*arctanh((1-x)*(3+2*3^(1/2))^(1/2)/(x^3-1)^(1/2))/(3+2*3^(1/2))^(1/2)
```

3.115.2 Mathematica [A] (verified)

Time = 1.75 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx=-2\sqrt{-1+\frac{2}{\sqrt{3}}}\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{-1+x^3}}{1+x+x^2}\right)$$

input
```
Integrate[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

output
```
-2*Sqrt[-1 + 2/Sqrt[3]]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[-1 + x^3])/(1 +
x + x^2)]
```

3.115.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x-\sqrt{3}+1}{\left(-x+\sqrt{3}+1\right)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{ 2565}$$

$$2\int \frac{1}{1-\frac{\left(3+2\sqrt{3}\right)(1-x)^2}{x^3-1}}d\frac{1-x}{\sqrt{x^3-1}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

input
```
Int[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

output
```
(2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[3 + 2*Sqrt[
3]]
```

3.115.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

3.115.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.53 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.98

method	result
trager	$\frac{\mathrm{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)\ln\left(\frac{6\,\mathrm{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)x^2-4\,\mathrm{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)\sqrt{3}\,x^2-4\sqrt{3}\,\mathrm{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)x+48\sqrt{}}{\left(x\sqrt{3}-x-2\right)^2}\right.}{6}$
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}}{\frac{3}{2}+}}}{\sqrt{}}$
elliptic	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}}{\frac{3}{2}+}}}{\sqrt{}}$

input
```
int((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*RootOf(_Z^2-24*3^(1/2)+36)*ln((6*RootOf(_Z^2-24*3^(1/2)+36)*x^2-4*Root
Of(_Z^2-24*3^(1/2)+36)*3^(1/2)*x^2-4*3^(1/2)*RootOf(_Z^2-24*3^(1/2)+36)*x+
48*(x^3-1)^(1/2)*3^(1/2)-4*RootOf(_Z^2-24*3^(1/2)+36)*3^(1/2)-72*(x^3-1)^(
1/2)+12*RootOf(_Z^2-24*3^(1/2)+36))/(x*3^(1/2)-x-2)^2)
```

3.115.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(33) = 66.

Time = 0.29 (sec) , antiderivative size = 204, normalized size of antiderivative = 4.64

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$$
$$=\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}-3}\log\left(\frac{x^8+16\,x^7+112\,x^6+16\,x^5+112\,x^4-224\,x^3+64\,x^2-4\left(2\,x^6+18\,x^5+42\,x^4+8\right.}{}\right.$$

input
```
integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*sqrt(2*sqrt(3) - 3)*log((x^8 + 16*x^7 + 112*x^6 + 16*x^5 + 112
*x^4 - 224*x^3 + 64*x^2 - 4*(2*x^6 + 18*x^5 + 42*x^4 + 8*x^3 + sqrt(3)*(x^
6 + 12*x^5 + 18*x^4 + 16*x^3 - 12*x^2 - 8) - 24*x + 8)*sqrt(x^3 - 1)*sqrt(
2*sqrt(3) - 3) + 16*sqrt(3)*(x^7 + 2*x^6 + 6*x^5 - 5*x^4 + 2*x^3 - 6*x^2 +
 4*x - 4) - 128*x + 112)/(x^8 - 8*x^7 + 16*x^6 + 16*x^5 - 56*x^4 - 32*x^3
+ 64*x^2 + 64*x + 16))
```

3.115.6 Sympy [F]

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \int \frac{x-1+\sqrt{3}}{\sqrt{(x-1)\left(x^2+x+1\right)}\left(x-\sqrt{3}-1\right)}\,dx$$

input
```
integrate((1-x-3**(1/2))/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

output
```
Integral((x - 1 + sqrt(3))/(sqrt((x - 1)*(x**2 + x + 1))*(x - sqrt(3) - 1)
), x)
```

3.115.7 Maxima [F]

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \int \frac{x+\sqrt{3}-1}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

input
```
integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

3.115.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[
2]%%%} Er
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Hanged}$$

input
```
int(-(x + 3^(1/2) - 1)/((x^3 - 1)^(1/2)*(3^(1/2) - x + 1)),x)
```

output
```
\text{Hanged}
```

3.116 $\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$

3.116.1 Optimal result

Integrand size = 32, antiderivative size = 44

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx=-\frac{2\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

output
```
-2*arctanh((1+x)*(3+2*3^(1/2))^(1/2)/(-x^3-1)^(1/2))/(3+2*3^(1/2))^(1/2)
```

3.116.2 Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx=2\sqrt{-1+\frac{2}{\sqrt{3}}}\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{-1-x^3}}{1-x+x^2}\right)$$

input
```
Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

output
```
2*Sqrt[-1 + 2/Sqrt[3]]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[-1 - x^3])/(1 - x
 + x^2)]
```

3.116.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{2565}$$

$$-2\int \frac{1}{1-\frac{\left(3+2\sqrt{3}\right)(x+1)^2}{-x^3-1}}d\frac{x+1}{\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

input
```
Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[3 + 2*Sqrt
[3]]
```

3.116.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

3.116.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.44 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.07

method	result
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)\ln\left(\frac{6\,\mathrm{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)x^2-4\,\mathrm{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)\sqrt{3}\,x^2+4\sqrt{3}\,\mathrm{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)x+4}{\left(x\sqrt{3}-x+2\right)^2}\right.}{6}$
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{4i\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{3}{2}}}{}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{4i\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{3}{2}}}{}$

input
```
int((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*RootOf(_Z^2-24*3^(1/2)+36)*ln((6*RootOf(_Z^2-24*3^(1/2)+36)*x^2-4*Roo
tOf(_Z^2-24*3^(1/2)+36)*3^(1/2)*x^2+4*3^(1/2)*RootOf(_Z^2-24*3^(1/2)+36)*x
+48*(-x^3-1)^(1/2)*3^(1/2)-4*RootOf(_Z^2-24*3^(1/2)+36)*3^(1/2)-72*(-x^3-1
)^(1/2)+12*RootOf(_Z^2-24*3^(1/2)+36))/(x*3^(1/2)-x+2)^2)
```

3.116.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(34) = 68.

Time = 0.34 (sec) , antiderivative size = 206, normalized size of antiderivative = 4.68

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$$
$$=\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}-3}\log\left(\frac{x^8-16x^7+112x^6-16x^5+112x^4+224x^3+64x^2+4\left(2x^6-18x^5+42x^4-\right.}{}\right.$$

input
```
integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas"
)
```

output
```
1/6*sqrt(3)*sqrt(2*sqrt(3) - 3)*log((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112
*x^4 + 224*x^3 + 64*x^2 + 4*(2*x^6 - 18*x^5 + 42*x^4 - 8*x^3 + sqrt(3)*(x^
6 - 12*x^5 + 18*x^4 - 16*x^3 - 12*x^2 - 8) + 24*x + 8)*sqrt(-x^3 - 1)*sqrt
(2*sqrt(3) - 3) - 16*sqrt(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2
+ 4*x + 4) + 128*x + 112)/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3
 + 64*x^2 - 64*x + 16))
```

3.116.6 Sympy [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{-\left(x+1\right)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

input
```
integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

output
```
Integral((x - sqrt(3) + 1)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3)
)), x)
```

3.116.7 Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

3.116.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{[1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[
2]%%%} Er
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \text{Hanged}$$

input
```
int((x - 3^(1/2) + 1)/((- x^3 - 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

output
```
\text{Hanged}
```

3.117 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

3.117.1 Optimal result

Integrand size = 58, antiderivative size = 69

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
-2*arctan(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(3+2*3^(1/2))^(1/2)/(b*x^3+a)^(1/2))
/a^(1/6)/b^(1/3)/(3+2*3^(1/2))^(1/2)
```

3.117.2 Mathematica [A] (verified)

Time = 7.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.22

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{2\arctan\left(\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)*Sqrt[a + b*x^3]),x]
```

3.117. $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

output
```
(2*ArcTan[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(Sqrt[3 + 2*Sqrt[3]]
*a^(1/6)*Sqrt[a + b*x^3])])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

3.117.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b} x\right) \sqrt{a+b x^3}} \, dx$$

$$\Big\downarrow \text{2565}$$

$$-\frac{2 \sqrt[3]{a} \int \frac{1}{\frac{\left(3+2 \sqrt{3}\right) \sqrt[3]{a} \left(\sqrt[3]{b} x+\sqrt[3]{a}\right)^2}{b x^3+a}+1}d\frac{\sqrt[3]{b} x+\sqrt[3]{a}}{\sqrt[3]{a} \sqrt{b x^3+a}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2 \arctan \left(\frac{\sqrt{3+2 \sqrt{3}} \sqrt[6]{a} \left(\sqrt[3]{a}+\sqrt[3]{b} x\right)}{\sqrt{a+b x^3}}\right)}{\sqrt{3+2 \sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}}$$

input
```
Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a + b*
x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

3.117.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

3.117.4 Maple [F]

$$\int \frac{b^{\frac{1}{3}} x + a^{\frac{1}{3}} (1-\sqrt{3})}{\left(b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(1+\sqrt{3}\right)\right) \sqrt{b\,x^{3}+a}}dx$$

input
```
int((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3
+a)^(1/2),x)
```

output
```
int((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3
+a)^(1/2),x)
```

3.117.5 Fricas [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 1236, normalized size of antiderivative = 17.91

$$\int \frac{\left(1-\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}+\sqrt[3]{b}x\right) \sqrt{a+bx^3}}\, dx = \text{Too large to display}$$

input
```
integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/
(b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 -
1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4
*x^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28
672*a^8 + 4*sqrt(1/3)*sqrt(b*x^3 + a)*((3*b^7*x^22 - 2688*a*b^6*x^19 + 569
52*a^2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 - 377856*a^5*b^2
*x^7 - 314880*a^6*b*x^4 - 24576*a^7*x + 2*sqrt(3)*(b^7*x^22 - 764*a*b^6*x^
19 + 16860*a^2*b^5*x^16 - 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 + 104448
*a^5*b^2*x^7 + 90880*a^6*b*x^4 + 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^
7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 - 24192*a^4*b^4*x^11 - 397
44*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x
^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^
5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) - 2*(30*a*b^7*x^2
1 - 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^
5*b^3*x^9 + 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(17*
a*b^7*x^21 - 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 -
 56000*a^5*b^3*x^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*b^(
1/3))*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 - 846*a*b^6*x^19
 + 4617*a^2*b^5*x^16 + 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 + 98496*a^5*
b^2*x^7 + 59328*a^6*b*x^4 + 4608*a^7*x + sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x
^19 + 2130*a^2*b^5*x^16 - 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 - 5376...
```

3.117.6 Sympy [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}\left(\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1+3**(1
/2)))/(b*x**3+a)**(1/2),x)
```

output
```
Integral((-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)/(sqrt(a + b*x**3)*(a*
*(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

3.117. $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

3.117.7 Maxima [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}+1\right)\right)}\,dx$$

input
```
integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/
(b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/(sqrt(b*x^3 + a)*(b^(1/3)*x
+ a^(1/3)*(sqrt(3) + 1))), x)
```

3.117.8 Giac [F(-1)]

Timed out.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/
(b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Hanged}$$

input
```
int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/((a + b*x^3)^(1/2)*(b^(1/3)*x + a^
(1/3)*(3^(1/2) + 1))),x)
```

output
```
\text{Hanged}
```

3.118 $\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

3.118.1 Optimal result

Integrand size = 61, antiderivative size = 71

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
2*arctan(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(3+2*3^(1/2))^(1/2)/(-b*x^3+a)^(1/2))
/a^(1/6)/b^(1/3)/(3+2*3^(1/2))^(1/2)
```

3.118.2 Mathematica [A] (verified)

Time = 7.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.18

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=-\frac{2\arctan\left(\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^
(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
(-2*ArcTan[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(Sqrt[3 + 2*Sqrt[3]
]*a^(1/6)*Sqrt[a - b*x^3])])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

3.118.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b} x}{\left(\left(1+\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b} x\right) \sqrt{a-b x^3}} \, dx$$

$$\Big\downarrow \ 2565$$

$$\frac{2 \sqrt[3]{a} \int \frac{1}{\frac{\left(3+2 \sqrt{3}\right) \sqrt[3]{a} \left(\sqrt[3]{a}-\sqrt[3]{b} x\right)^2}{a-b x^3}+1}d\frac{\sqrt[3]{a}-\sqrt[3]{b} x}{\sqrt[3]{a} \sqrt{a-b x^3}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \ 216$$

$$\frac{2 \arctan \left(\frac{\sqrt{3+2 \sqrt{3}} \sqrt[6]{a} \left(\sqrt[3]{a}-\sqrt[3]{b} x\right)}{\sqrt{a-b x^3}}\right)}{\sqrt{3+2 \sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}}$$

input
```
Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(1/3)*
x)*Sqrt[a - b*x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[a - b*x
^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

3.118.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

3.118.4 Maple [F]

$$\int \frac{-b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1-\sqrt{3})}{\left(-b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(1+\sqrt{3}\right)\right)\sqrt{-b\,x^3+a}}dx$$

input
```
int((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*
x^3+a)^(1/2),x)
```

output
```
int((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*
x^3+a)^(1/2),x)
```

3.118.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(53) = 106.

Time = 1.03 (sec) , antiderivative size = 1288, normalized size of antiderivative = 18.14

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 +
1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4
*x^12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28
672*a^8 + 32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 - 5472*a^3*b
^4*x^13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 - 4608*
a^7*x + sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 + 4928*a^
3*b^4*x^13 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 + 25
60*a^7*x))*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 13320*a^2*b
^5*x^17 + 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b^2*x^8 -
328704*a^6*b*x^5 + 61440*a^7*x^2 + 2*sqrt(3)*(b^7*x^23 + 299*a*b^6*x^20 +
4260*a^2*b^5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024*a^5*b^
2*x^8 + 93184*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 4*sqrt(1/3)*((
3*b^7*x^22 + 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 + 93504*a^3*b^4*x^13 - 6
3552*a^4*b^3*x^10 + 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 + 24576*a^7*x +
2*sqrt(3)*(b^7*x^22 + 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 + 19792*a^3*b^4*
x^13 + 42368*a^4*b^3*x^10 - 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 - 7168*a^
7*x))*sqrt(-b*x^3 + a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x
^17 + 14472*a^3*b^5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*
a^6*b^2*x^5 - 13824*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17
 + 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^...
```

3.118.6 Sympy [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{-\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a-bx^3}\left(-\sqrt{3}\sqrt[3]{a}-\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1+3**
(1/2)))/(-b*x**3+a)**(1/2),x)
```

output
```
Integral((-a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(a - b*x**3)*(-s
qrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)), x)
```

3.118. $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

3.118.7 Maxima [F]

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(\sqrt{3}-1)}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}(\sqrt{3}+1)\right)}\,dx$$

input
```
integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)/(-b*x^3+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/(sqrt(-b*x^3 + a)*(b^(1/3)*x
 - a^(1/3)*(sqrt(3) + 1))), x)
```

3.118.8 Giac [F(-1)]

Timed out.

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \text{Hanged}$$

input
```
int((b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/((a - b*x^3)^(1/2)*(b^(1/3)*x - a^
(1/3)*(3^(1/2) + 1))),x)
```

output
```
\text{Hanged}
```

3.118. $\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

3.119 $\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

3.119.1 Optimal result

Integrand size = 62, antiderivative size = 72

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \frac{2\,\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
2*arctanh(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(3+2*3^(1/2))^(1/2)/(b*x^3-a)^(1/2))
/a^(1/6)/b^(1/3)/(3+2*3^(1/2))^(1/2)
```

3.119.2 Mathematica [A] (verified)

Time = 7.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.18

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = -\frac{2\,\text{arctanh}\left(\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt{-a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^
(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

3.119. $\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

output
```
(-2*ArcTanh[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(Sqrt[3 + 2*Sqrt[3
]]*a^(1/6)*Sqrt[-a + b*x^3])])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

3.119.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{2565}$$

$$\frac{2\sqrt[3]{a}\int \frac{1}{1-\frac{\left(3+2\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{bx^3-a}}d\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{bx^3-a}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(1/3)*
x)*Sqrt[-a + b*x^3]),x]
```

output
```
(2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[-a + b
*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

3.119.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

3.119.4 Maple [F]

$$\int \frac{-b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1-\sqrt{3}\right)}{\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1+\sqrt{3}\right)\right)\sqrt{b\,x^{3}-a}}dx$$

input
```
int((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x
^3-a)^(1/2),x)
```

output
```
int((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x
^3-a)^(1/2),x)
```

3.119.5 Fricas [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 1239, normalized size of antiderivative = 17.21

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Too large to display}$$

input
```
integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 + 1
840*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*
x^12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 286
72*a^8 - 4*sqrt(1/3)*sqrt(b*x^3 - a)*((3*b^7*x^22 + 2688*a*b^6*x^19 + 5695
2*a^2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 + 377856*a^5*b^2*
x^7 - 314880*a^6*b*x^4 + 24576*a^7*x + 2*sqrt(3)*(b^7*x^22 + 764*a*b^6*x^1
9 + 16860*a^2*b^5*x^16 + 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 - 104448*
a^5*b^2*x^7 + 90880*a^6*b*x^4 - 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^7
*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 + 24192*a^4*b^4*x^11 - 3974
4*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^
20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5
*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) + 2*(30*a*b^7*x^21
 + 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5
*b^3*x^9 - 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 + sqrt(3)*(17*a
*b^7*x^21 + 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 -
56000*a^5*b^3*x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*b^(1
/3))*sqrt((2*sqrt(3) - 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 + 846*a*b^6*x^19 +
 4617*a^2*b^5*x^16 - 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^
2*x^7 + 59328*a^6*b*x^4 - 4608*a^7*x + sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^1
9 + 2130*a^2*b^5*x^16 + 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 + 53760*...
```

3.119.6 Sympy [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{-\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a+bx^3}\left(-\sqrt{3}\sqrt[3]{a}-\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1+3**
(1/2)))/(b*x**3-a)**(1/2),x)
```

output
```
Integral((-a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(-a + b*x**3)*(-
sqrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)), x)
```

3.119.7 Maxima [F]

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}(\sqrt{3}-1)}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}(\sqrt{3}+1)\right)}\,dx$$

input
```
integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/(sqrt(b*x^3 - a)*(b^(1/3)*x
- a^(1/3)*(sqrt(3) + 1))), x)
```

3.119.8 Giac [F(-1)]

Timed out.

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)/(b*x^3-a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Hanged}$$

input
```
int((b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/((b*x^3 - a)^(1/2)*(b^(1/3)*x - a^
(1/3)*(3^(1/2) + 1))),x)
```

output
```
\text{Hanged}
```

3.120 $\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

3.120.1 Optimal result

Integrand size = 61, antiderivative size = 72

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

output
```
-2*arctanh(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(3+2*3^(1/2))^(1/2)/(-b*x^3-a)^(1/2
))/a^(1/6)/b^(1/3)/(3+2*3^(1/2))^(1/2)
```

3.120.2 Mathematica [A] (verified)

Time = 7.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
(2*ArcTanh[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(Sqrt[3 + 2*Sqrt[3]
]*a^(1/6)*Sqrt[-a - b*x^3])]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

3.120.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}dx$$

$$\Big\downarrow \text{2565}$$

$$-\frac{2\sqrt[3]{a}\int \frac{1}{1-\frac{\left(3+2\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{-bx^3-a}}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{-bx^3-a}}}{\sqrt[3]{b}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

input
```
Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)*Sqrt[-a - b*x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a -
b*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

3.120.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

3.120.4 Maple [F]

$$\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(1-\sqrt{3})}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(1+\sqrt{3}\right)\right)\sqrt{-b\,x^{3}-a}}dx$$

input
```
int((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^
3-a)^(1/2),x)
```

output
```
int((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^
3-a)^(1/2),x)
```

3.120.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(52) = 104.

Time = 1.07 (sec) , antiderivative size = 1299, normalized size of antiderivative = 18.04

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/
(-b*x^3-a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 - 1
840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*
x^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 286
72*a^8 + 32*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 + 5472*a^3*b^
4*x^13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 + 4608*a
^7*x + sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 - 4928*a^3
*b^4*x^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 - 256
0*a^7*x))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x^20 + 13320*a^2*b^
5*x^17 - 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 345024*a^5*b^2*x^8 - 3
28704*a^6*b*x^5 - 61440*a^7*x^2 + 2*sqrt(3)*(b^7*x^23 - 299*a*b^6*x^20 + 4
260*a^2*b^5*x^17 + 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 + 105024*a^5*b^2
*x^8 + 93184*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 4*sqrt(1/3)*((3
*b^7*x^22 - 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63
552*a^4*b^3*x^10 - 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 - 24576*a^7*x + 2
*sqrt(3)*(b^7*x^22 - 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 - 19792*a^3*b^4*x
^13 + 42368*a^4*b^3*x^10 + 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 + 7168*a^7
*x))*sqrt(-b*x^3 - a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^
17 + 14472*a^3*b^5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a
^6*b^2*x^5 - 13824*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17
+ 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6...
```

3.120.6 Sympy [F]

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{-a-bx^3}\left(\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1+3**(1
/2)))/(-b*x**3-a)**(1/2),x)
```

output
```
Integral((-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)/(sqrt(-a - b*x**3)*(a
**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

3.120.7 Maxima [F]

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}(\sqrt{3}-1)}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(\sqrt{3}+1)\right)}\,dx$$

input
```
integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/
(-b*x^3-a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/(sqrt(-b*x^3 - a)*(b^(1/3)*x
 + a^(1/3)*(sqrt(3) + 1))), x)
```

3.120.8 Giac [F(-1)]

Timed out.

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Timed out}$$

input
```
integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/
(-b*x^3-a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x}{\left((1+\sqrt{3})\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Hanged}$$

input
```
int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/((- a - b*x^3)^(1/2)*(b^(1/3)*x +
a^(1/3)*(3^(1/2) + 1))),x)
```

output
```
\text{Hanged}
```

3.121 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx$

3.121.1 Optimal result

Integrand size = 52, antiderivative size = 73

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

output
```
-2*arctan((1+(b/a)^(1/3)*x)*a^(1/2)*(3+2*3^(1/2))^(1/2)/(b*x^3+a)^(1/2))/(
b/a)^(1/3)/a^(1/2)/(3+2*3^(1/2))^(1/2)
```

3.121.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.76 (sec) , antiderivative size = 667, normalized size of antiderivative = 9.14

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx$$
$$= \frac{x\left(12(3+\sqrt{3})\sqrt[3]{\frac{b}{a}}x\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a+6\sqrt{3}a}\right)-8\left(\frac{b}{a}\right)^{2/3}x^2\sqrt{3+\frac{3bx^3}{a}}\,\text{AppellF1}\left(\right.\right.}{}$$

input
```
Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqr
t[a + b*x^3]),x]
```

output
```
(x*(12*(3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2,
1, 5/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 8*(b/a)^(2/3)*x^2
*Sqrt[3 + (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), -((b*x^3)/(10*
a + 6*Sqrt[3]*a))] - (3*(18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a)
, -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2,
1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - b*x^3*(2*(5 + 3*S
qrt[3])*a + b*x^3)*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3
)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))]*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3,
1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 3*b*x^3*(App
ellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + (
5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a +
6*Sqrt[3]*a))]))))/(a*(2*(5 + 3*Sqrt[3])*a + b*x^3)*(8*(5 + 3*Sqrt[3])*a*A
ppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] -
 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqr
t[3]*a))] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*
x^3)/(10*a + 6*Sqrt[3]*a))])))))/(24*(5 + 3*Sqrt[3])*Sqrt[a + b*x^3])
```

3.121.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1}{\left(x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1\right)\sqrt{a+bx^3}}dx$$

$$\Big\downarrow \ 2565$$

$$-\frac{2\int \frac{1}{\frac{\left(3+2\sqrt{3}\right)a\left(\sqrt[3]{\frac{b}{a}}x+1\right)^2}{bx^3+a}+1}d\frac{\sqrt[3]{\frac{b}{a}}x+1}{\sqrt{bx^3+a}}}{\sqrt[3]{\frac{b}{a}}}$$

$$\Big\downarrow \ 216$$

$$-\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

input
```
Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[a +
b*x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[a + b*x^
3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

3.121.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

3.121.4 Maple [F]

$$\int \frac{1+\left(\frac{b}{a}\right)^{\frac{1}{3}} x -\sqrt{3}}{\left(1+\left(\frac{b}{a}\right)^{\frac{1}{3}} x +\sqrt{3}\right) \sqrt{b \,x^{3}+a}}d x$$

input
```
int((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x)
```

output
```
int((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x)
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 1270, normalized size of antiderivative = 17.40

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx=\text{Too large to display}$$

input
```
integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1
/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a
*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12
+ 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^
8 + 4*sqrt(1/3)*(486*a*b^7*x^20 - 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14
- 145152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 - 414720*a^6*b^2*x^5 - 82944*a^
7*b*x^2 + (3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 93504*a
^4*b^4*x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 -
 24576*a^8*x + 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^
16 - 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 + 90880*
a^7*b*x^4 + 7168*a^8*x))*(b/a)^(2/3) + 6*sqrt(3)*(47*a*b^7*x^20 - 2724*a^2
*b^6*x^17 + 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37
632*a^6*b^2*x^5 + 8192*a^7*b*x^2) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 +
 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*
b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b
^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 11
5968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 +
a)*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b) - 8*(3*a*b^7*x^23 - 1077*a^2*b^6*x
^20 + 13320*a^3*b^5*x^17 - 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 - 3450
24*a^6*b^2*x^8 - 328704*a^7*b*x^5 - 61440*a^8*x^2 + 2*sqrt(3)*(a*b^7*x^23
- 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 + 1520*a^4*b^4*x^14 + 26720*a^5*...
```

3.121.6 Sympy [F]

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx = \int \frac{x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1}{\sqrt{a+bx^3}\left(x\sqrt[3]{\frac{b}{a}}+1+\sqrt{3}\right)}\,dx$$

input
```
integrate((1+(b/a)**(1/3)*x-3**(1/2))/(1+(b/a)**(1/3)*x+3**(1/2))/(b*x**3+
a)**(1/2),x)
```

output
```
Integral((x*(b/a)**(1/3) - sqrt(3) + 1)/(sqrt(a + b*x**3)*(x*(b/a)**(1/3)
+ 1 + sqrt(3))), x)
```

3.121.7 Maxima [F]

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx = \int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}+1}{\sqrt{bx^3+a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1
/2),x, algorithm="maxima")
```

output
```
integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/(sqrt(b*x^3 + a)*(x*(b/a)^(1/3) +
sqrt(3) + 1)), x)
```

3.121.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1
/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(con
st gen &
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{x\left(\frac{b}{a}\right)^{1/3}-\sqrt{3}+1}{\sqrt{b\,x^3+a}\,\left(\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}+1\right)}\,dx$$

input
```
int((x*(b/a)^(1/3) - 3^(1/2) + 1)/((a + b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(1
/3) + 1)),x)
```

output
```
int((x*(b/a)^(1/3) - 3^(1/2) + 1)/((a + b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(1
/3) + 1)), x)
```

3.122 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx$

3.122.1 Optimal result

Integrand size = 55, antiderivative size = 75

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

output

```
2*arctan((1-(b/a)^(1/3)*x)*a^(1/2)*(3+2*3^(1/2))^(1/2)/(-b*x^3+a)^(1/2))/(
b/a)^(1/3)/a^(1/2)/(3+2*3^(1/2))^(1/2)
```

3.122.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.82 (sec) , antiderivative size = 649, normalized size of antiderivative = 8.65

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx$$
$$= \frac{x\left(-12(3+\sqrt{3})\sqrt[3]{\frac{b}{a}}x\sqrt{1-\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right)-8\left(\frac{b}{a}\right)^{2/3}x^2\sqrt{3-\frac{3bx^3}{a}}\operatorname{AppellF1}\left(1,\right.\right.}{}$$

input
```
Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqr
t[a - b*x^3]),x]
```

output
```
(x*(-12*(3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2,
 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqrt
[3 - (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt
[3]*a)] - (3*(18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*
a + 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a
, (b*x^3)/(10*a + 6*Sqrt[3]*a)] + b*x^3*(2*(5 + 3*Sqrt[3])*a - b*x^3)*Sqrt
[1 - (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sq
rt[3]*a)]*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^
3)/(10*a + 6*Sqrt[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a,
(b*x^3)/(10*a + 6*Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3,
 (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)]))))/(a*(2*(5 + 3*Sqrt[3])*a - b*
x^3)*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(1
0*a + 6*Sqrt[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^
3)/(10*a + 6*Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x
^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)])))))/(24*(5 + 3*Sqrt[3])*Sqrt[a - b*x
^3])
```

3.122.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1\right)\sqrt{a-bx^3}}dx$$

$$\Big\downarrow \ 2565$$

$$\frac{2\int \frac{1}{\frac{\left(3+2\sqrt{3}\right)a\left(1-\sqrt[3]{\frac{b}{a}}x\right)^2}{a-bx^3}+1}d\frac{1-\sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}}}{\sqrt[3]{\frac{b}{a}}}$$

$$\Big\downarrow \ 216$$

$$\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

input
```
Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[a -
b*x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[a - b*x^3
]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

3.122.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

3.122.4 Maple [F]

$$\int \frac{1-\left(\frac{b}{a}\right)^{\frac{1}{3}} x-\sqrt{3}}{\left(1-\left(\frac{b}{a}\right)^{\frac{1}{3}} x+\sqrt{3}\right) \sqrt{-b\,x^{3}+a}}dx$$

input
```
int((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x
)
```

output
```
int((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x
)
```

3.122.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(57) = 114.

Time = 0.72 (sec) , antiderivative size = 1324, normalized size of antiderivative = 17.65

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx=\text{Too large to display}$$

input
```
integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(
1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a
*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12
- 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^
8 - 4*sqrt(1/3)*((3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 +
93504*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*
b*x^4 + 24576*a^8*x + 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3
*b^5*x^16 + 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 +
 90880*a^7*b*x^4 - 7168*a^8*x))*sqrt(-b*x^3 + a)*(b/a)^(2/3) + 2*(30*a*b^7
*x^21 + 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 7987
2*a^5*b^3*x^9 - 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 + sqrt(3)*
(17*a*b^7*x^21 + 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^
12 - 56000*a^5*b^3*x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))
*sqrt(-b*x^3 + a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 144
72*a^3*b^5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x
^5 - 13824*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a
^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 +
8192*a^7*b*x^2))*sqrt(-b*x^3 + a))*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b) +
8*(3*a*b^7*x^23 + 1077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 + 19200*a^4*b^4*x
^14 - 111360*a^5*b^3*x^11 + 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 + 61440*
a^8*x^2 + 2*sqrt(3)*(a*b^7*x^23 + 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 ...
```

3.122.6 Sympy [F]

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx=\int \frac{x\sqrt[3]{\frac{b}{a}}-1+\sqrt{3}}{\sqrt{a-bx^3}\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}-1\right)}\,dx$$

input
```
integrate((1-(b/a)**(1/3)*x-3**(1/2))/(1-(b/a)**(1/3)*x+3**(1/2))/(-b*x**3
+a)**(1/2),x)
```

output
```
Integral((x*(b/a)**(1/3) - 1 + sqrt(3))/(sqrt(a - b*x**3)*(x*(b/a)**(1/3)
- sqrt(3) - 1)), x)
```

3.122.7 Maxima [F]

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx = \int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1}{\sqrt{-bx^3+a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}-1\right)}\,dx$$

input
```
integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(
1/2),x, algorithm="maxima")
```

output
```
integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/(sqrt(-b*x^3 + a)*(x*(b/a)^(1/3) -
 sqrt(3) - 1)), x)
```

3.122.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(
1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(con
st gen &
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{a-bx^3}}\,dx=\int -\frac{\sqrt{3}+x\,\left(\frac{b}{a}\right)^{1/3}-1}{\sqrt{a-b\,x^3}\,\left(\sqrt{3}-x\,\left(\frac{b}{a}\right)^{1/3}+1\right)}\,dx$$

input
```
int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/((a - b*x^3)^(1/2)*(3^(1/2) - x*(b/a)^(
1/3) + 1)),x)
```

output
```
int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/((a - b*x^3)^(1/2)*(3^(1/2) - x*(b/a)^(
1/3) + 1)), x)
```

3.123 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx$

3.123.1 Optimal result

Integrand size = 56, antiderivative size = 76

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

output
```
2*arctanh((1-(b/a)^(1/3)*x)*a^(1/2)*(3+2*3^(1/2))^(1/2)/(b*x^3-a)^(1/2))/(
b/a)^(1/3)/a^(1/2)/(3+2*3^(1/2))^(1/2)
```

3.123.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.85 (sec) , antiderivative size = 650, normalized size of antiderivative = 8.55

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx$$
$$= \frac{x\left(-12(3+\sqrt{3})\sqrt[3]{\frac{b}{a}}x\sqrt{1-\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},\frac{bx^3}{a},\frac{bx^3}{10a+6\sqrt{3}a}\right)-8\left(\frac{b}{a}\right)^{2/3}x^2\sqrt{3-\frac{3bx^3}{a}}\,\text{AppellF1}\left(1,\right.\right.}{}$$

input
```
Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqr
t[-a + b*x^3]),x]
```

output
```
(x*(-12*(3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2,
 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqrt
[3 - (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt
[3]*a)] - (3*(18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*
a + 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a
, (b*x^3)/(10*a + 6*Sqrt[3]*a)] + b*x^3*(2*(5 + 3*Sqrt[3])*a - b*x^3)*Sqrt
[1 - (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sq
rt[3]*a)]*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^
3)/(10*a + 6*Sqrt[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a,
(b*x^3)/(10*a + 6*Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3,
 (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)]))))/(a*(2*(5 + 3*Sqrt[3])*a - b*
x^3)*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(1
0*a + 6*Sqrt[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^
3)/(10*a + 6*Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x
^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)])))))/(24*(5 + 3*Sqrt[3])*Sqrt[-a + b*
x^3])
```

3.123.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(-\sqrt[3]{\frac{b}{a}}\right)-\sqrt{3}+1}{\left(x\left(-\sqrt[3]{\frac{b}{a}}\right)+\sqrt{3}+1\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \text{ 2565}$$

$$\frac{2\int \frac{1}{1-\frac{\left(3+2\sqrt{3}\right)a\left(1-\sqrt[3]{\frac{b}{a}}x\right)^2}{bx^3-a}}d\frac{1-\sqrt[3]{\frac{b}{a}}x}{\sqrt{bx^3-a}}}{\sqrt[3]{\frac{b}{a}}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

input
```
Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[-a +
 b*x^3]),x]
```

output
```
(2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[-a + b*x
^3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

3.123. $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}dx$

3.123.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

3.123.4 Maple [F]

$$\int \frac{1-\left(\frac{b}{a}\right)^{\frac{1}{3}} x-\sqrt{3}}{\left(1-\left(\frac{b}{a}\right)^{\frac{1}{3}} x+\sqrt{3}\right) \sqrt{b\,x^{3}-a}}dx$$

input
```
int((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x)
```

output
```
int((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x)
```

3.123.5 Fricas [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 1273, normalized size of antiderivative = 16.75

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1
/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a*
b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 -
 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8
 - 4*sqrt(1/3)*(486*a*b^7*x^20 + 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14 +
 145152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 + 414720*a^6*b^2*x^5 - 82944*a^7
*b*x^2 + (3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 93504*a^
4*b^4*x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 +
24576*a^8*x + 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^1
6 + 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 + 90880*a
^7*b*x^4 - 7168*a^8*x))*(b/a)^(2/3) + 6*sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*
b^6*x^17 + 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 376
32*a^6*b^2*x^5 + 8192*a^7*b*x^2) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 +
44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b
^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^
6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115
968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 - a
)*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b) + 8*(3*a*b^7*x^23 + 1077*a^2*b^6*x^2
0 + 13320*a^3*b^5*x^17 + 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 + 345024
*a^6*b^2*x^8 - 328704*a^7*b*x^5 + 61440*a^8*x^2 + 2*sqrt(3)*(a*b^7*x^23 +
299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 - 1520*a^4*b^4*x^14 + 26720*a^5*b^...
```

3.123.6 Sympy [F]

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx=\int \frac{x\sqrt[3]{\frac{b}{a}}-1+\sqrt{3}}{\sqrt{-a+bx^3}\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}-1\right)}\,dx$$

input
```
integrate((1-(b/a)**(1/3)*x-3**(1/2))/(1-(b/a)**(1/3)*x+3**(1/2))/(b*x**3-
a)**(1/2),x)
```

output
```
Integral((x*(b/a)**(1/3) - 1 + sqrt(3))/(sqrt(-a + b*x**3)*(x*(b/a)**(1/3)
 - sqrt(3) - 1)), x)
```

3.123.7 Maxima [F]

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx = \int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}-1}{\sqrt{bx^3-a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}-1\right)}\,dx$$

input
```
integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1
/2),x, algorithm="maxima")
```

output
```
integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/(sqrt(b*x^3 - a)*(x*(b/a)^(1/3) -
sqrt(3) - 1)), x)
```

3.123.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1
/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(con
st gen &
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx=\int -\frac{\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}-1}{\sqrt{b\,x^3-a}\,\left(\sqrt{3}-x\left(\frac{b}{a}\right)^{1/3}+1\right)}\,dx$$

input
```
int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/((b*x^3 - a)^(1/2)*(3^(1/2) - x*(b/a)^(
1/3) + 1)),x)
```

output
```
int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/((b*x^3 - a)^(1/2)*(3^(1/2) - x*(b/a)^(
1/3) + 1)), x)
```

3.124 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx$

3.124.1 Optimal result

Integrand size = 55, antiderivative size = 76

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx=-\frac{2\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

output
```
-2*arctanh((1+(b/a)^(1/3)*x)*a^(1/2)*(3+2*3^(1/2))^(1/2)/(-b*x^3-a)^(1/2))
/(b/a)^(1/3)/a^(1/2)/(3+2*3^(1/2))^(1/2)
```

3.124.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 10.83 (sec) , antiderivative size = 670, normalized size of antiderivative = 8.82

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx$$
$$= \frac{x\left(12(3+\sqrt{3})\sqrt[3]{\frac{b}{a}}x\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bx^3}{10a+6\sqrt{3}a}\right)-8\left(\frac{b}{a}\right)^{2/3}x^2\sqrt{3+\frac{3bx^3}{a}}\,\text{AppellF1}\left(\right.\right.}{}$$

input
```
Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqr
t[-a - b*x^3]),x]
```

output
```
(x*(12*(3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2,
1, 5/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 8*(b/a)^(2/3)*x^2
*Sqrt[3 + (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), -((b*x^3)/(10*
a + 6*Sqrt[3]*a))] - (3*(18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a)
, -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2,
1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - b*x^3*(2*(5 + 3*S
qrt[3])*a + b*x^3)*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3
)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))]*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3,
1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 3*b*x^3*(App
ellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + (
5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a +
6*Sqrt[3]*a))]))))/(a*(2*(5 + 3*Sqrt[3])*a + b*x^3)*(8*(5 + 3*Sqrt[3])*a*A
ppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] -
 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqr
t[3]*a))] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*
x^3)/(10*a + 6*Sqrt[3]*a))]))))))/(24*(5 + 3*Sqrt[3])*Sqrt[-a - b*x^3])
```

3.124.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1}{\left(x\sqrt[3]{\frac{b}{a}}+\sqrt{3}+1\right)\sqrt{-a-bx^3}}dx$$

$$\Big\downarrow \text{2565}$$

$$-\frac{2\int \frac{1}{1-\frac{\left(3+2\sqrt{3}\right)a\left(\sqrt[3]{\frac{b}{a}}x+1\right)^2}{-bx^3-a}}d\frac{\sqrt[3]{\frac{b}{a}}x+1}{\sqrt{-bx^3-a}}}{\sqrt[3]{\frac{b}{a}}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

input
```
Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[-a -
 b*x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[-a - b*
x^3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

3.124. $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}dx$

3.124.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

3.124.4 Maple [F]

$$\int \frac{1+\left(\frac{b}{a}\right)^{\frac{1}{3}} x -\sqrt{3}}{\left(1+\left(\frac{b}{a}\right)^{\frac{1}{3}} x +\sqrt{3}\right) \sqrt{-b \,x^{3}-a}}d x$$

input
```
int((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x
)
```

output
```
int((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x
)
```

3.124.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(58) = 116.

Time = 0.72 (sec) , antiderivative size = 1335, normalized size of antiderivative = 17.57

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx=\text{Too large to display}$$

input
```
integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(
1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(1/3)*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a*
b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 +
 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8
 + 4*sqrt(1/3)*((3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 9
3504*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b
*x^4 - 24576*a^8*x + 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*
b^5*x^16 - 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 +
90880*a^7*b*x^4 + 7168*a^8*x))*sqrt(-b*x^3 - a)*(b/a)^(2/3) - 2*(30*a*b^7*
x^21 - 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872
*a^5*b^3*x^9 + 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(
17*a*b^7*x^21 - 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^1
2 - 56000*a^5*b^3*x^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*
sqrt(-b*x^3 - a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 1447
2*a^3*b^5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^
5 - 13824*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^
3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8
192*a^7*b*x^2))*sqrt(-b*x^3 - a))*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b) - 8*
(3*a*b^7*x^23 - 1077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 - 19200*a^4*b^4*x^1
4 - 111360*a^5*b^3*x^11 - 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 - 61440*a^
8*x^2 + 2*sqrt(3)*(a*b^7*x^23 - 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 + ...
```

3.124.6 Sympy [F]

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1}{\sqrt{-a-bx^3}\left(x\sqrt[3]{\frac{b}{a}}+1+\sqrt{3}\right)}\,dx$$

input
```
integrate((1+(b/a)**(1/3)*x-3**(1/2))/(1+(b/a)**(1/3)*x+3**(1/2))/(-b*x**3
-a)**(1/2),x)
```

output
```
Integral((x*(b/a)**(1/3) - sqrt(3) + 1)/(sqrt(-a - b*x**3)*(x*(b/a)**(1/3)
 + 1 + sqrt(3))), x)
```

3.124.7 Maxima [F]

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx = \int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\sqrt{3}+1}{\sqrt{-bx^3-a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(
1/2),x, algorithm="maxima")
```

output
```
integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/(sqrt(-b*x^3 - a)*(x*(b/a)^(1/3) +
 sqrt(3) + 1)), x)
```

3.124.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(
1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(con
st gen &
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{x\left(\frac{b}{a}\right)^{1/3}-\sqrt{3}+1}{\sqrt{-b\,x^3-a}\,\left(\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}+1\right)}\,dx$$

input
```
int((x*(b/a)^(1/3) - 3^(1/2) + 1)/((- a - b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^
(1/3) + 1)),x)
```

output
```
int((x*(b/a)^(1/3) - 3^(1/2) + 1)/((- a - b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^
(1/3) + 1)), x)
```

3.125 $\int \frac{1+x}{(1+\sqrt{3}+x)\sqrt{1+x^3}}\,dx$

3.125.1 Optimal result

Integrand size = 23, antiderivative size = 145

$$\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3+2\sqrt{3}}} + \frac{\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
1/3*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2
)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1
+x)/(1+x+3^(1/2))^2)^(1/2)-arctan((1+x)*(3+2*3^(1/2))^(1/2)/(x^3+1)^(1/2))
/(3+2*3^(1/2))^(1/2)
```

3.125.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.39 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.86

$$\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$$
$$= \frac{2\sqrt{6}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(\sqrt{-i+\sqrt{3}+2ix}\left((-2-i)-\sqrt{3}+\left((1+2i)+i\sqrt{3}\right)x\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2}{3i}\right.\right.}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{i+\sqrt{3}}}$$

input
```
Integrate[(1 + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
(2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]
*((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I
+ Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 2*
Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I
+ (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))
], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt
[3] - (2*I)*x]*Sqrt[1 + x^3])
```

3.125.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2566, 27, 759, 2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx$$
$$\Big\downarrow \text{2566}$$
$$\frac{1}{2}\int \frac{1}{\sqrt{x^3+1}}dx+\frac{1}{12}\int \frac{6\left(x-\sqrt{3}+1\right)}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{2}\int \frac{1}{\sqrt{x^3+1}}dx+\frac{1}{2}\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{759}$$

$$\frac{1}{2}\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx+\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{2565}$$

$$\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\int \frac{1}{\frac{\left(3+2\sqrt{3}\right)(x+1)^2}{x^3+1}+1}d\frac{x+1}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

input
```
Int[(1 + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
-(ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/Sqrt[3 + 2*Sqrt[3]])
 + (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Elli
pticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/
4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

3.125.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.125. $\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}dx$

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

3.125.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(119) = 238$.

Time = 2.43 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.69

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\cdots}{\sqrt{x^3+\cdots}}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\cdots}{\sqrt{x^3+\cdots}}$

input
```
int((x+1)/(1+x+3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2
+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*(3/2-1/2*I*3^(1/2))*((x+1)/
(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1
/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*Ellip
ticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),(
(-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

3.125.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.38

$$\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}-3}\arctan\left(\frac{\left(\sqrt{3}(x^2-4x-2)-6x-6\right)\sqrt{2\sqrt{3}-3}}{6\sqrt{x^3+1}}\right) + \text{weierstrassPInverse}(0,-4,x)$$

input
```
integrate((1+x)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*sqrt(2*sqrt(3) - 3)*arctan(1/6*(sqrt(3)*(x^2 - 4*x - 2) - 6*x
- 6)*sqrt(2*sqrt(3) - 3)/sqrt(x^3 + 1)) + weierstrassPInverse(0, -4, x)
```

3.125.6 Sympy [F]

$$\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x+1}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

input
```
integrate((1+x)/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

output
```
Integral((x + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

3.125.7 Maxima [F]

$$\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x+1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+x)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

3.125.8 Giac [F]

$$\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x+1}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+x)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Hanged}$$

input
```
int((x + 1)/((x^3 + 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

output
```
\text{Hanged}
```

3.126 $\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

3.126.1 Optimal result

Integrand size = 25, antiderivative size = 145

$$\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{-3+2\sqrt{3}}} + \frac{\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
1/3*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2
)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1
+x)/(1+x+3^(1/2))^2)^(1/2)-arctanh((1+x)*(-3+2*3^(1/2))^(1/2)/(x^3+1)^(1/2
))/(-3+2*3^(1/2))^(1/2)
```

3.126.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.36 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.84

$$\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{2\sqrt{6}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(\sqrt{-i+\sqrt{3}+2ix}\left((1+2i)-i\sqrt{3}+\left((-2-i)+\sqrt{3}\right)x\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)\right.\right.}{\left(-3+(2+i)\sqrt{3}\right)\sqrt{i+}}$$

input
```
Integrate[(1 + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
(-2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x
]*((1 + 2*I) - I*Sqrt[3] + ((-2 - I) + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I
 + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + (
2*I)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[((2*I)*Sqrt[
3])/(-3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^
(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((-3 + (2 + I)*Sqrt[3])*Sqrt[I + S
qrt[3] - (2*I)*x]*Sqrt[1 + x^3])
```

3.126.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2566, 27, 759, 2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{\left(x-\sqrt{3}+1\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{2566}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x^3+1}}dx+\frac{1}{12}\int \frac{6\left(x+\sqrt{3}+1\right)}{\left(x-\sqrt{3}+1\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x^3+1}}dx+\frac{1}{2}\int \frac{x+\sqrt{3}+1}{\left(x-\sqrt{3}+1\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{759}$$

$$\frac{1}{2}\int \frac{x+\sqrt{3}+1}{\left(x-\sqrt{3}+1\right)\sqrt{x^3+1}}dx+\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{2565}$$

$$\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\int \frac{1}{\frac{\left(3-2\sqrt{3}\right)(x+1)^2}{x^3+1}+1}d\frac{x+1}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

input
```
Int[(1 + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
-(ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/Sqrt[-3 + 2*Sqrt[3
]]) + (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*E
llipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^
(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

3.126.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.126. $\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

3.126.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(119) = 238$.

Time = 2.44 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.69

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\cdots}{\sqrt{x^3\cdots}}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\cdots}{\sqrt{x^3\cdots}}$

input
```
int((x+1)/(1+x-3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2
+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*(3/2-1/2*I*3^(1/2))*((x+1)/
(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1
/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*Ellip
ticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),
((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

3.126.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.45

$$\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$$
$$= \frac{1}{12}\sqrt{3}\sqrt{2\sqrt{3}+3}\log\left(\frac{x^8-16x^7+112x^6-16x^5+112x^4+224x^3+64x^2-4\left(2x^6-18x^5+42x^4-\right.}{}\right.$$
$$+ \text{weierstrassPInverse}(0,-4,x)$$

input
```
integrate((1+x)/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*sqrt(2*sqrt(3) + 3)*log((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 11
2*x^4 + 224*x^3 + 64*x^2 - 4*(2*x^6 - 18*x^5 + 42*x^4 - 8*x^3 - sqrt(3)*(x
^6 - 12*x^5 + 18*x^4 - 16*x^3 - 12*x^2 - 8) + 24*x + 8)*sqrt(x^3 + 1)*sqrt
(2*sqrt(3) + 3) + 16*sqrt(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2
+ 4*x + 4) + 128*x + 112)/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3
 + 64*x^2 - 64*x + 16)) + weierstrassPInverse(0, -4, x)
```

3.126.6 Sympy [F]

$$\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x+1}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x-\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+x)/(1+x-3**(1/2))/(x**3+1)**(1/2),x)
```

output
```
Integral((x + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1)), x)
```

3.126.7 Maxima [F]

$$\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x+1}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+x)/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

3.126.8 Giac [F]

$$\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x+1}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

input
```
integrate((1+x)/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Hanged}$$

input
```
int((x + 1)/((x^3 + 1)^(1/2)*(x - 3^(1/2) + 1)),x)
```

output
```
\text{Hanged}
```

3.127 $\int \frac{e+fx}{(1+\sqrt{3}+x)\sqrt{1+x^3}}\,dx$

3.127.1 Optimal result

Integrand size = 25, antiderivative size = 173

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx=\frac{\left(e-f-\sqrt{3}f\right)\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2+\sqrt{3}}\left(e-\left(1-\sqrt{3}\right)f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
1/3*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(e-f*(1-3^(
1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(1/4)
/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)+arctan((1+x)*(3+2*3^(1/2))^(1
/2)/(x^3+1)^(1/2))*(e-f-f*3^(1/2))/(9+6*3^(1/2))^(1/2)
```

3.127.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.47 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.68

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$$

$$=\frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(3f\sqrt{-i+\sqrt{3}+2ix}\left((-2-i)-\sqrt{3}+\left((1+2i)+i\sqrt{3}\right)x\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)\right.\right.}{(3i+(1+2i)\cdots}$$

input
```
Integrate[(e + f*x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
(2*Sqrt[2/3]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(3*f*Sqrt[-I + Sqrt[3] + (2
*I)*x]*((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*EllipticF[ArcSin[S
qrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])
] + 2*(-(Sqrt[3]*e) + (3 + Sqrt[3])*f)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1
- x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I
 + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(
(3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x^3])
```

3.127.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2566, 27, 759, 2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{2566}$$

$$\frac{\left(e-\left(1-\sqrt{3}\right)f\right)\int \frac{1}{\sqrt{x^3+1}}dx}{2\sqrt{3}}-\frac{1}{12}\left(\frac{e-f}{\sqrt{3}}-f\right)\int \frac{6\left(x-\sqrt{3}+1\right)}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(e-\left(1-\sqrt{3}\right)f\right)\int \frac{1}{\sqrt{x^3+1}}dx}{2\sqrt{3}}-\frac{1}{2}\left(\frac{e-f}{\sqrt{3}}-f\right)\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\left(1-\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{1}{2}\left(\frac{e-f}{\sqrt{3}}-f\right)\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{2565}$$

$$\left(\frac{e-f}{\sqrt{3}}-f\right)\int \frac{1}{\frac{\left(3+2\sqrt{3}\right)(x+1)^2}{x^3+1}+1}d\frac{x+1}{\sqrt{x^3+1}}+$$
$$\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\left(1-\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\big\downarrow$ 216

$$\frac{\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\left(1-\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+$$
$$\frac{\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)\left(\frac{e-f}{\sqrt{3}}-f\right)}{\sqrt{3+2\sqrt{3}}}$$

input
```
Int[(e + f*x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
(((e - f)/Sqrt[3] - f)*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]
)/Sqrt[3 + 2*Sqrt[3]] + (Sqrt[2 + Sqrt[3]]*(e - (1 - Sqrt[3])*f)*(1 + x)*S
qrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] +
x)^2]*Sqrt[1 + x^3])
```

3.127.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.127. $\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

3.127.4 Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.50

method	result
default	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(e-f-f\sqrt{3}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{}$
elliptic	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(e-f-f\sqrt{3}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{}$

input
```
int((f*x+e)/(1+x+3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*f*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^
(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3
/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(e-f-f*3^(1/2))*(3/2-1/
2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/
2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)
/(x^3+1)^(1/2)*3^(1/2)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-
3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/
2))
```

3.127.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 716, normalized size of antiderivative = 4.14

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \Bigg[\frac{1}{3}\left(\sqrt{3}(e-f)+3\,f\right)\text{weierstrassPInverse}(0,-4,x)$$

$$+\frac{1}{12}\sqrt{3\,e^2+6\,ef-2\,\sqrt{3}(e^2+ef+f^2)}\log\Bigg(-\frac{(e^2-2\,ef-2\,f^2)x^8-16\,(e^2-2\,ef-2\,f^2)x^7+112\,(e^2$$

$$-4,x)$$

$$-\frac{1}{6}\sqrt{-3\,e^2-6\,ef+2\,\sqrt{3}(e^2+ef+f^2)}\arctan\Bigg(\frac{(3\,fx^2-6\,(e+f)x+\sqrt{3}((e-f)x^2-2\,(2\,e+f)x-}{6\,((e^2-2\,ef-2}$$

input
```
integrate((f*x+e)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
[1/3*(sqrt(3)*(e - f) + 3*f)*weierstrassPInverse(0, -4, x) + 1/12*sqrt(3*e
^2 + 6*e*f - 2*sqrt(3)*(e^2 + e*f + f^2))*log(-((e^2 - 2*e*f - 2*f^2)*x^8
- 16*(e^2 - 2*e*f - 2*f^2)*x^7 + 112*(e^2 - 2*e*f - 2*f^2)*x^6 - 16*(e^2 -
 2*e*f - 2*f^2)*x^5 + 112*(e^2 - 2*e*f - 2*f^2)*x^4 + 224*(e^2 - 2*e*f - 2
*f^2)*x^3 + 64*(e^2 - 2*e*f - 2*f^2)*x^2 - 4*((2*e + f)*x^6 - 18*(e + f)*x
^5 + 6*(7*e + 2*f)*x^4 - 8*(e + 5*f)*x^3 - 36*f*x^2 + 24*(e - f)*x + sqrt(
3)*((e + f)*x^6 - 6*(2*e + f)*x^5 + 6*(3*e + 4*f)*x^4 - 8*(2*e - f)*x^3 -
12*(e - f)*x^2 + 24*f*x - 8*e + 16*f) + 8*e - 32*f)*sqrt(x^3 + 1)*sqrt(3*e
^2 + 6*e*f - 2*sqrt(3)*(e^2 + e*f + f^2)) + 112*e^2 - 224*e*f - 224*f^2 +
128*(e^2 - 2*e*f - 2*f^2)*x - 16*sqrt(3)*((e^2 - 2*e*f - 2*f^2)*x^7 - 2*(e
^2 - 2*e*f - 2*f^2)*x^6 + 6*(e^2 - 2*e*f - 2*f^2)*x^5 + 5*(e^2 - 2*e*f - 2
*f^2)*x^4 + 2*(e^2 - 2*e*f - 2*f^2)*x^3 + 6*(e^2 - 2*e*f - 2*f^2)*x^2 + 4*
e^2 - 8*e*f - 8*f^2 + 4*(e^2 - 2*e*f - 2*f^2)*x))/(x^8 + 8*x^7 + 16*x^6 -
16*x^5 - 56*x^4 + 32*x^3 + 64*x^2 - 64*x + 16)), 1/3*(sqrt(3)*(e - f) + 3*
f)*weierstrassPInverse(0, -4, x) - 1/6*sqrt(-3*e^2 - 6*e*f + 2*sqrt(3)*(e^
2 + e*f + f^2))*arctan(1/6*(3*f*x^2 - 6*(e + f)*x + sqrt(3)*((e - f)*x^2 -
 2*(2*e + f)*x - 2*e - 4*f) - 6*e)*sqrt(x^3 + 1)*sqrt(-3*e^2 - 6*e*f + 2*s
qrt(3)*(e^2 + e*f + f^2))/((e^2 - 2*e*f - 2*f^2)*x^3 + e^2 - 2*e*f - 2*f^2
))]
```

3.127.6 Sympy [F]

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{e+fx}{\sqrt{(x+1)\,(x^2-x+1)}\,\left(x+1+\sqrt{3}\right)}\,dx$$

input
```
integrate((f*x+e)/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

3.127.7 Maxima [F]

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{fx+e}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate((f*x+e)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

3.127.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x+e)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Ar
gument Va
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Hanged}$$

input
```
int((e + f*x)/((x^3 + 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

output
```
\text{Hanged}
```

3.128 $\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$

3.128.1 Optimal result

Integrand size = 29, antiderivative size = 187

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx=-\frac{\left(e+f+\sqrt{3}f\right)\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\sqrt{2+\sqrt{3}}\left(e+\left(1-\sqrt{3}\right)f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-1/3*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(e+f*(1-3^
(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(1/4
)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)-arctan((1-x)*(3+2*3^(1/2))^
(1/2)/(-x^3+1)^(1/2))*(e+f+f*3^(1/2))/(9+6*3^(1/2))^(1/2)
```

3.128.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.52 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.56

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$$

$$=\frac{2\sqrt{\frac{2}{3}}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\left(-3if\sqrt{-i+\sqrt{3}-2ix}\left(-i\left((2+i)+\sqrt{3}\right)+\left((2-i)+\sqrt{3}\right)x\right)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{i+\sqrt{}}}{\sqrt{2}}\right.\right.\right.}{(3i+(1+2}$$

input
```
Integrate[(e + f*x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

output
```
(2*Sqrt[2/3]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-3*I)*f*Sqrt[-I + Sqr
t[3] - (2*I)*x]*((-I)*((2 + I) + Sqrt[3]) + ((2 - I) + Sqrt[3])*x)*Ellipti
cF[ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I
 + Sqrt[3])] + 2*(Sqrt[3]*e + (3 + Sqrt[3])*f)*Sqrt[I + Sqrt[3] + (2*I)*x]
*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSi
n[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[
3])]))/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x^3
])
```

3.128.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2566, 27, 759, 2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(-x+\sqrt{3}+1\right)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{2566}$$

$$\frac{1}{12}\left(\frac{e+f}{\sqrt{3}}+f\right)\int -\frac{6\left(-x-\sqrt{3}+1\right)}{\left(-x+\sqrt{3}+1\right)\sqrt{1-x^3}}dx-\frac{1}{2}\left(f-\frac{e+f}{\sqrt{3}}\right)\int \frac{1}{\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}\left(f-\frac{e+f}{\sqrt{3}}\right)\int \frac{1}{\sqrt{1-x^3}}dx-\frac{1}{2}\left(\frac{e+f}{\sqrt{3}}+f\right)\int \frac{-x-\sqrt{3}+1}{\left(-x+\sqrt{3}+1\right)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{759}$$

$$\frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(f-\frac{e+f}{\sqrt{3}}\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{1}{2}\left(\frac{e+f}{\sqrt{3}}+f\right)\int \frac{-x-\sqrt{3}+1}{\left(-x+\sqrt{3}+1\right)\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{2565}$$

$$\frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(f-\frac{e+f}{\sqrt{3}}\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\left(\frac{e+f}{\sqrt{3}}+f\right)\int\frac{1}{\frac{\left(3+2\sqrt{3}\right)(1-x)^2}{1-x^3}+1}d\frac{1-x}{\sqrt{1-x^3}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(f-\frac{e+f}{\sqrt{3}}\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)\left(\frac{e+f}{\sqrt{3}}+f\right)}{\sqrt{3+2\sqrt{3}}}$$

input
```
Int[(e + f*x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

output
```
-(((f + (e + f)/Sqrt[3])*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3
]])/Sqrt[3 + 2*Sqrt[3]]) + (Sqrt[2 + Sqrt[3]]*(f - (e + f)/Sqrt[3])*(1 - x
)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] -
x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3]
 - x)^2]*Sqrt[1 - x^3])
```

3.128.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

3.128.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.41

method	result
default	$\frac{2 i f \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, F\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}+1}} -$ $2 i \left(-e-f-f \sqrt{3}\right) \sqrt{3}\, \sqrt{i(\ldots$
elliptic	$\frac{2 i f \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, F\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}+1}} -$ $2 i \left(-e-f-f \sqrt{3}\right) \sqrt{3}\, \sqrt{i(\ldots$

input
```
int((f*x+e)/(1-x+3^(1/2))/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*f*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I
*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-
3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(-e-f-f*3^(1/2))*3^(1/2)*(I*(x+1/2-1/2*I*
3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*
I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-3/2-3^(1/2)+1/2*I*3^(1/2))*Elli
pticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2
-3^(1/2)+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

3.128.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.18 (sec) , antiderivative size = 735, normalized size of antiderivative = 3.93

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \left[-\frac{1}{3}\left(\sqrt{3}(i\,e+i\,f)-3i\,f\right)\text{weierstrassPInverse}(0,4,x)\right.$$
$$+\frac{1}{12}\sqrt{3\,e^2-6\,ef-2\,\sqrt{3}(e^2-ef+f^2)}\log\left(-\frac{(e^2+2\,ef-2\,f^2)x^8+16\,(e^2+2\,ef-2\,f^2)x^7+112\,(e^2\right.$$
$$-\frac{1}{3}\left(\sqrt{3}(i\,e+i\,f)-3i\,f\right)\text{weierstrassPInverse}(0,4,x)$$
$$+\frac{1}{6}\sqrt{-3\,e^2+6\,ef+2\,\sqrt{3}(e^2-ef+f^2)}\arctan\left(\frac{(3\,fx^2-6\,(e-f)x-\sqrt{3}((e+f)x^2+2\,(2\,e-f)x-}{6\,((e^2+2\,ef-2}\right.$$

input
```
integrate((f*x+e)/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
[-1/3*(sqrt(3)*(I*e + I*f) - 3*I*f)*weierstrassPInverse(0, 4, x) + 1/12*sq
rt(3*e^2 - 6*e*f - 2*sqrt(3)*(e^2 - e*f + f^2))*log(-((e^2 + 2*e*f - 2*f^2
)*x^8 + 16*(e^2 + 2*e*f - 2*f^2)*x^7 + 112*(e^2 + 2*e*f - 2*f^2)*x^6 + 16*
(e^2 + 2*e*f - 2*f^2)*x^5 + 112*(e^2 + 2*e*f - 2*f^2)*x^4 - 224*(e^2 + 2*e
*f - 2*f^2)*x^3 + 64*(e^2 + 2*e*f - 2*f^2)*x^2 + 4*((2*e - f)*x^6 + 18*(e
- f)*x^5 + 6*(7*e - 2*f)*x^4 + 8*(e - 5*f)*x^3 + 36*f*x^2 - 24*(e + f)*x +
 sqrt(3)*((e - f)*x^6 + 6*(2*e - f)*x^5 + 6*(3*e - 4*f)*x^4 + 8*(2*e + f)*
x^3 - 12*(e + f)*x^2 + 24*f*x - 8*e - 16*f) + 8*e + 32*f)*sqrt(-x^3 + 1)*s
qrt(3*e^2 - 6*e*f - 2*sqrt(3)*(e^2 - e*f + f^2)) + 112*e^2 + 224*e*f - 224
*f^2 - 128*(e^2 + 2*e*f - 2*f^2)*x + 16*sqrt(3)*((e^2 + 2*e*f - 2*f^2)*x^7
 + 2*(e^2 + 2*e*f - 2*f^2)*x^6 + 6*(e^2 + 2*e*f - 2*f^2)*x^5 - 5*(e^2 + 2*
e*f - 2*f^2)*x^4 + 2*(e^2 + 2*e*f - 2*f^2)*x^3 - 6*(e^2 + 2*e*f - 2*f^2)*x
^2 - 4*e^2 - 8*e*f + 8*f^2 + 4*(e^2 + 2*e*f - 2*f^2)*x))/(x^8 - 8*x^7 + 16
*x^6 + 16*x^5 - 56*x^4 - 32*x^3 + 64*x^2 + 64*x + 16)), -1/3*(sqrt(3)*(I*e
 + I*f) - 3*I*f)*weierstrassPInverse(0, 4, x) + 1/6*sqrt(-3*e^2 + 6*e*f +
2*sqrt(3)*(e^2 - e*f + f^2))*arctan(1/6*(3*f*x^2 - 6*(e - f)*x - sqrt(3)*(
(e + f)*x^2 + 2*(2*e - f)*x - 2*e + 4*f) + 6*e)*sqrt(-x^3 + 1)*sqrt(-3*e^2
 + 6*e*f + 2*sqrt(3)*(e^2 - e*f + f^2))/((e^2 + 2*e*f - 2*f^2)*x^3 - e^2 -
 2*e*f + 2*f^2))]
```

3.128.6 Sympy [F]

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = -\int \frac{e}{x\sqrt{1-x^3}-\sqrt{3}\sqrt{1-x^3}-\sqrt{1-x^3}}\,dx - \int \frac{fx}{x\sqrt{1-x^3}-\sqrt{3}\sqrt{1-x^3}-\sqrt{1-x^3}}\,dx$$

input
```
integrate((f*x+e)/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

output
```
-Integral(e/(x*sqrt(1 - x**3) - sqrt(3)*sqrt(1 - x**3) - sqrt(1 - x**3)),
x) - Integral(f*x/(x*sqrt(1 - x**3) - sqrt(3)*sqrt(1 - x**3) - sqrt(1 - x*
*3)), x)
```

3.128.7 Maxima [F]

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx=\int -\frac{fx+e}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

input
```
integrate((f*x+e)/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((f*x + e)/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

3.128.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((f*x+e)/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Ar
gument Va
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx=\text{Hanged}$$

input
```
int((e + f*x)/((1 - x^3)^(1/2)*(3^(1/2) - x + 1)),x)
```

output
```
\text{Hanged}
```

3.129 $\int \frac{e+fx}{(1+\sqrt{3}-x)\sqrt{-1+x^3}}\,dx$

3.129.1 Optimal result

Integrand size = 27, antiderivative size = 190

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx=-\frac{\left(e+f+\sqrt{3}f\right)\operatorname{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\sqrt{2-\sqrt{3}}\left(e+\left(1-\sqrt{3}\right)f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
-1/3*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(e+f*(1-3^
(1/2)))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(1/4
)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)-arctanh((1-x)*(3+2*3^(1/2))
^(1/2)/(x^3-1)^(1/2))*(e+f+f*3^(1/2))/(9+6*3^(1/2))^(1/2)
```

3.129.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.51 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.52

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$$

$$=\frac{2\sqrt{\frac{2}{3}}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\left(-3if\sqrt{-i+\sqrt{3}-2ix}\left(-i\left((2+i)+\sqrt{3}\right)+\left((2-i)+\sqrt{3}\right)x\right)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{i+\sqrt{}}}{\sqrt{2}}\right.\right.\right.}{\left(3i+(1+2i\right.}$$

input
```
Integrate[(e + f*x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

output
```
(2*Sqrt[2/3]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-3*I)*f*Sqrt[-I + Sqr
t[3] - (2*I)*x]*((-I)*((2 + I) + Sqrt[3]) + ((2 - I) + Sqrt[3])*x)*Ellipti
cF[ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I
 + Sqrt[3])] + 2*(Sqrt[3]*e + (3 + Sqrt[3])*f)*Sqrt[I + Sqrt[3] + (2*I)*x]
*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSi
n[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[
3])]))/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[-1 + x^
3])
```

3.129.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2566, 27, 760, 2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(-x+\sqrt{3}+1\right)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{2566}$$

$$-\frac{1}{2}\left(f-\frac{e+f}{\sqrt{3}}\right)\int \frac{1}{\sqrt{x^3-1}}dx-\frac{1}{12}\left(\frac{e+f}{\sqrt{3}}+f\right)\int \frac{6\left(-x-\sqrt{3}+1\right)}{\left(-x+\sqrt{3}+1\right)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}\left(f-\frac{e+f}{\sqrt{3}}\right)\int \frac{1}{\sqrt{x^3-1}}dx-\frac{1}{2}\left(\frac{e+f}{\sqrt{3}}+f\right)\int \frac{-x-\sqrt{3}+1}{\left(-x+\sqrt{3}+1\right)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$\frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(f-\frac{e+f}{\sqrt{3}}\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{1}{2}\left(\frac{e+f}{\sqrt{3}}+f\right)\int \frac{-x-\sqrt{3}+1}{\left(-x+\sqrt{3}+1\right)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{2565}$$

$$\frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(f-\frac{e+f}{\sqrt{3}}\right)\operatorname{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\left(\frac{e+f}{\sqrt{3}}+f\right)\int\frac{1}{1-\frac{\left(3+2\sqrt{3}\right)(1-x)^2}{x^3-1}}d\frac{1-x}{\sqrt{x^3-1}}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(f-\frac{e+f}{\sqrt{3}}\right)\operatorname{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)\left(\frac{e+f}{\sqrt{3}}+f\right)}{\sqrt{3+2\sqrt{3}}}$$

input
```
Int[(e + f*x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

output
```
-(((f + (e + f)/Sqrt[3])*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x
^3]])/Sqrt[3 + 2*Sqrt[3]]) + (Sqrt[2 - Sqrt[3]]*(f - (e + f)/Sqrt[3])*(1 -
 x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3]
- x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqr
t[3] - x)^2)]*Sqrt[-1 + x^3])
```

3.129.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

3.129.4 Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.38

method	result
default	$-\frac{2 f \left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\, F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} - \frac{2\left(-e-f-f\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{}$
elliptic	$-\frac{2 f \left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\, F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} - \frac{2\left(-e-f-f\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{}$

input
```
int((f*x+e)/(1-x+3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*f*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I
*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(
1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((
3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2/3*(-e-f-f*3^(1/2))*(-3/2-
1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(
3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2
)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3
*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/
2))
```

3.129.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 723, normalized size of antiderivative = 3.81

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \left[\frac{1}{3}\left(\sqrt{3}(e+f)-3f\right)\text{weierstrassPInverse}(0,4,x)\right.$$

$$+\frac{1}{12}\sqrt{-3e^2+6ef+2\sqrt{3}(e^2-ef+f^2)}\log\left(-\frac{(e^2+2ef-2f^2)x^8+16(e^2+2ef-2f^2)x^7+112(e}{}\right.$$

$$+\frac{1}{6}\sqrt{3e^2-6ef-2\sqrt{3}(e^2-ef+f^2)}\arctan\left(\frac{(3fx^2-6(e-f)x-\sqrt{3}((e+f)x^2+2(2e-f)x-2}{6((e^2+2ef-2f^2)}\right.$$

input
```
integrate((f*x+e)/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
[1/3*(sqrt(3)*(e + f) - 3*f)*weierstrassPInverse(0, 4, x) + 1/12*sqrt(-3*e
^2 + 6*e*f + 2*sqrt(3)*(e^2 - e*f + f^2))*log(-((e^2 + 2*e*f - 2*f^2)*x^8
+ 16*(e^2 + 2*e*f - 2*f^2)*x^7 + 112*(e^2 + 2*e*f - 2*f^2)*x^6 + 16*(e^2 +
 2*e*f - 2*f^2)*x^5 + 112*(e^2 + 2*e*f - 2*f^2)*x^4 - 224*(e^2 + 2*e*f - 2
*f^2)*x^3 + 64*(e^2 + 2*e*f - 2*f^2)*x^2 + 4*((2*e - f)*x^6 + 18*(e - f)*x
^5 + 6*(7*e - 2*f)*x^4 + 8*(e - 5*f)*x^3 + 36*f*x^2 - 24*(e + f)*x + sqrt(
3)*((e - f)*x^6 + 6*(2*e - f)*x^5 + 6*(3*e - 4*f)*x^4 + 8*(2*e + f)*x^3 -
12*(e + f)*x^2 + 24*f*x - 8*e - 16*f) + 8*e + 32*f)*sqrt(x^3 - 1)*sqrt(-3*
e^2 + 6*e*f + 2*sqrt(3)*(e^2 - e*f + f^2)) + 112*e^2 + 224*e*f - 224*f^2 -
 128*(e^2 + 2*e*f - 2*f^2)*x + 16*sqrt(3)*((e^2 + 2*e*f - 2*f^2)*x^7 + 2*(
e^2 + 2*e*f - 2*f^2)*x^6 + 6*(e^2 + 2*e*f - 2*f^2)*x^5 - 5*(e^2 + 2*e*f -
2*f^2)*x^4 + 2*(e^2 + 2*e*f - 2*f^2)*x^3 - 6*(e^2 + 2*e*f - 2*f^2)*x^2 - 4
*e^2 - 8*e*f + 8*f^2 + 4*(e^2 + 2*e*f - 2*f^2)*x))/(x^8 - 8*x^7 + 16*x^6 +
 16*x^5 - 56*x^4 - 32*x^3 + 64*x^2 + 64*x + 16)), 1/3*(sqrt(3)*(e + f) - 3
*f)*weierstrassPInverse(0, 4, x) + 1/6*sqrt(3*e^2 - 6*e*f - 2*sqrt(3)*(e^2
 - e*f + f^2))*arctan(1/6*(3*f*x^2 - 6*(e - f)*x - sqrt(3)*((e + f)*x^2 +
2*(2*e - f)*x - 2*e + 4*f) + 6*e)*sqrt(x^3 - 1)*sqrt(3*e^2 - 6*e*f - 2*sqr
t(3)*(e^2 - e*f + f^2))/((e^2 + 2*e*f - 2*f^2)*x^3 - e^2 - 2*e*f + 2*f^2))
]
```

3.129.6 Sympy [F]

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = -\int \frac{e}{x\sqrt{x^3-1}-\sqrt{3}\sqrt{x^3-1}-\sqrt{x^3-1}}\,dx - \int \frac{fx}{x\sqrt{x^3-1}-\sqrt{3}\sqrt{x^3-1}-\sqrt{x^3-1}}\,dx$$

input
```
integrate((f*x+e)/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

output
```
-Integral(e/(x*sqrt(x**3 - 1) - sqrt(3)*sqrt(x**3 - 1) - sqrt(x**3 - 1)),
x) - Integral(f*x/(x*sqrt(x**3 - 1) - sqrt(3)*sqrt(x**3 - 1) - sqrt(x**3 -
 1)), x)
```

3.129.7 Maxima [F]

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \int -\frac{fx+e}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

input
```
integrate((f*x+e)/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((f*x + e)/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

3.129.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x+e)/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Ar
gument Va
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Hanged}$$

input
```
int((e + f*x)/((x^3 - 1)^(1/2)*(3^(1/2) - x + 1)),x)
```

output
```
\text{Hanged}
```

3.130 $\int \frac{e+fx}{(1+\sqrt{3}+x)\sqrt{-1-x^3}}\,dx$

3.130.1 Optimal result

Integrand size = 27, antiderivative size = 183

$$\int \frac{e+fx}{(1+\sqrt{3}+x)\sqrt{-1-x^3}}\,dx=\frac{\left(e-\left(1+\sqrt{3}\right)f\right)\operatorname{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2-\sqrt{3}}\left(e-\left(1-\sqrt{3}\right)f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
1/3*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(e-f*(1-3^(
1/2)))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(1/4)
/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)+arctanh((1+x)*(3+2*3^(1/2))
^(1/2)/(-x^3-1)^(1/2))*(e-f*(1+3^(1/2)))/(9+6*3^(1/2))^(1/2)
```

3.130.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.46 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.60

$$\int \frac{e+fx}{(1+\sqrt{3}+x)\sqrt{-1-x^3}}\,dx$$
$$=\frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(3f\sqrt{-i+\sqrt{3}+2ix}\left((-2-i)-\sqrt{3}+\left((1+2i)+i\sqrt{3}\right)x\right)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)\right.\right.}{(3i+(1+2i)\sqrt{}}$$

input
```
Integrate[(e + f*x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*Sqrt[2/3]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(3*f*Sqrt[-I + Sqrt[3] + (2
*I)*x]*((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*EllipticF[ArcSin[S
qrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])
] + 2*(-(Sqrt[3]*e) + (3 + Sqrt[3])*f)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1
- x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I
 + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(
(3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[-1 - x^3])
```

3.130.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2566, 27, 760, 2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(x+\sqrt{3}+1\right)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{2566}$$

$$\frac{\left(e-\left(1-\sqrt{3}\right)f\right)\int \frac{1}{\sqrt{-x^3-1}}dx}{2\sqrt{3}}+\frac{\left(e-\left(1+\sqrt{3}\right)f\right)\int -\frac{6\left(x-\sqrt{3}+1\right)}{\left(x+\sqrt{3}+1\right)\sqrt{-x^3-1}}dx}{12\sqrt{3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(e-\left(1-\sqrt{3}\right)f\right)\int \frac{1}{\sqrt{-x^3-1}}dx}{2\sqrt{3}}-\frac{\left(e-\left(1+\sqrt{3}\right)f\right)\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{-x^3-1}}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{760}$$

$$\frac{\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\left(1-\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{\left(e-\left(1+\sqrt{3}\right)f\right)\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{-x^3-1}}dx}{2\sqrt{3}}$$

$$\Big\downarrow \text{2565}$$

$$\frac{\left(e-\left(1+\sqrt{3}\right)f\right)\int \frac{1}{1-\frac{\left(3+2\sqrt{3}\right)(x+1)^2}{-x^3-1}}d\frac{x+1}{\sqrt{-x^3-1}}}{\sqrt{3}}+\frac{\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\left(1-\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\left(1-\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)\left(e-\left(1+\sqrt{3}\right)f\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

input
```
Int[(e + f*x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

output
```
((e - (1 + Sqrt[3])*f)*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3
]])/Sqrt[3*(3 + 2*Sqrt[3])] + (Sqrt[2 - Sqrt[3]]*(e - (1 - Sqrt[3])*f)*(1
+ x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3]
 + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(3/4)*Sqrt[-((1 + x)/(1 - Sq
rt[3] + x)^2)]*Sqrt[-1 - x^3])
```

3.130.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.130. $\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}dx$

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

3.130.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.41

method	result
default	$-\frac{2 i f \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, F\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}-1}}-\frac{2 i \left(e-f-f \sqrt{3}\right) \sqrt{3}\, \sqrt{i \left(x-\right.}}{}$
elliptic	$-\frac{2 i f \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, F\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 \sqrt{-x^{3}-1}}-\frac{2 i \left(e-f-f \sqrt{3}\right) \sqrt{3}\, \sqrt{i \left(x-\right.}}{}$

input
```
int((f*x+e)/(1+x+3^(1/2))/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*f*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I
*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3
/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(e-f-f*3^(1/2))*3^(1/2)*(I*(x-1/2-1/2*I*3^
(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3
^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(3/2+3^(1/2)+1/2*I*3^(1/2))*Elliptic
Pi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+3^(1
/2)+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

3.130.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 724, normalized size of antiderivative = 3.96

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \left[-\frac{1}{3}\left(\sqrt{3}(i\,e-i\,f)+3i\,f\right)\text{weierstrassPInverse}(0,-4,x)\right.$$

$$+\frac{1}{12}\sqrt{-3\,e^2-6\,ef+2\,\sqrt{3}(e^2+ef+f^2)}\log\left(-\frac{(e^2-2\,ef-2\,f^2)x^8-16\,(e^2-2\,ef-2\,f^2)x^7+112\,(e\right.$$

$$-\frac{1}{3}\left(\sqrt{3}(i\,e-i\,f)+3i\,f\right)\text{weierstrassPInverse}(0,-4,x)$$

$$-\frac{1}{6}\sqrt{3\,e^2+6\,ef-2\,\sqrt{3}(e^2+ef+f^2)}\arctan\left(\frac{(3\,fx^2-6\,(e+f)x+\sqrt{3}((e-f)x^2-2\,(2\,e+f)x-2\,}{6\,((e^2-2\,ef-2\,f^2}\right.$$

input
```
integrate((f*x+e)/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
[-1/3*(sqrt(3)*(I*e - I*f) + 3*I*f)*weierstrassPInverse(0, -4, x) + 1/12*s
qrt(-3*e^2 - 6*e*f + 2*sqrt(3)*(e^2 + e*f + f^2))*log(-((e^2 - 2*e*f - 2*f
^2)*x^8 - 16*(e^2 - 2*e*f - 2*f^2)*x^7 + 112*(e^2 - 2*e*f - 2*f^2)*x^6 - 1
6*(e^2 - 2*e*f - 2*f^2)*x^5 + 112*(e^2 - 2*e*f - 2*f^2)*x^4 + 224*(e^2 - 2
*e*f - 2*f^2)*x^3 + 64*(e^2 - 2*e*f - 2*f^2)*x^2 - 4*((2*e + f)*x^6 - 18*(
e + f)*x^5 + 6*(7*e + 2*f)*x^4 - 8*(e + 5*f)*x^3 - 36*f*x^2 + 24*(e - f)*x
 + sqrt(3)*((e + f)*x^6 - 6*(2*e + f)*x^5 + 6*(3*e + 4*f)*x^4 - 8*(2*e - f
)*x^3 - 12*(e - f)*x^2 + 24*f*x - 8*e + 16*f) + 8*e - 32*f)*sqrt(-x^3 - 1)
*sqrt(-3*e^2 - 6*e*f + 2*sqrt(3)*(e^2 + e*f + f^2)) + 112*e^2 - 224*e*f -
224*f^2 + 128*(e^2 - 2*e*f - 2*f^2)*x - 16*sqrt(3)*((e^2 - 2*e*f - 2*f^2)*
x^7 - 2*(e^2 - 2*e*f - 2*f^2)*x^6 + 6*(e^2 - 2*e*f - 2*f^2)*x^5 + 5*(e^2 -
 2*e*f - 2*f^2)*x^4 + 2*(e^2 - 2*e*f - 2*f^2)*x^3 + 6*(e^2 - 2*e*f - 2*f^2
)*x^2 + 4*e^2 - 8*e*f - 8*f^2 + 4*(e^2 - 2*e*f - 2*f^2)*x))/(x^8 + 8*x^7 +
 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64*x^2 - 64*x + 16)), -1/3*(sqrt(3)*(
I*e - I*f) + 3*I*f)*weierstrassPInverse(0, -4, x) - 1/6*sqrt(3*e^2 + 6*e*f
 - 2*sqrt(3)*(e^2 + e*f + f^2))*arctan(1/6*(3*f*x^2 - 6*(e + f)*x + sqrt(3
)*((e - f)*x^2 - 2*(2*e + f)*x - 2*e - 4*f) - 6*e)*sqrt(-x^3 - 1)*sqrt(3*e
^2 + 6*e*f - 2*sqrt(3)*(e^2 + e*f + f^2))/((e^2 - 2*e*f - 2*f^2)*x^3 + e^2
 - 2*e*f - 2*f^2))]
```

3.130.6 Sympy [F]

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \int \frac{e+fx}{\sqrt{-\left(x+1\right)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

input
```
integrate((f*x+e)/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

3.130.7 Maxima [F]

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \int \frac{fx+e}{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate((f*x+e)/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

3.130.8 Giac [F(-2)]

Exception generated.

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x+e)/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Ar
gument Va
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \text{Hanged}$$

input
```
int((e + f*x)/((- x^3 - 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

output
```
\text{Hanged}
```

3.131 $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

3.131.1 Optimal result

Integrand size = 42, antiderivative size = 332

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$$

$$= -\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\operatorname{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{a}b^{2/3}}$$

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\ \text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+}\right.\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-arctanh(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(b*x^3+a)^(1/2))
*(b^(1/3)*e-a^(1/3)*f*(1-3^(1/2)))/b^(2/3)/a^(1/2)/(-9+6*3^(1/2))^(1/2)-1/
3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*e-a^(1/3)*f*(1+3^(1/2)))*(1/
2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(1/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^
(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.131.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.49 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.37

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx =$$

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}if\left(\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3i-(1+2i)\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}-\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\,\text{El}\right.}{}$$

input
```
Integrate[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),
x]
```

output
```
(-4*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((I/2)*f*((-3 +
 (2 + I)*Sqrt[3])*a^(1/3) + (3*I - (1 + 2*I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I
 + Sqrt[3])*a^(1/3) - (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]
*EllipticF[ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I +
 Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2] + I*(b^(1/3)*e + (-1 + Sqrt[3])*a
^(1/3)*f)*Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3]
)*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*Elliptic
Pi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (
I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))
/((3 - (2 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1
+ (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

3.131.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.119, Rules used = {2566, 27, 759, 2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow 2566$$

$$\frac{\left(\sqrt[3]{b}e-(1-\sqrt{3})\sqrt[3]{a}f\right)\int \frac{6ab\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)\sqrt{bx^3+a}}dx}{12\sqrt{3}a^{4/3}b^{4/3}}-\frac{\left(\sqrt[3]{b}e-(1+\sqrt{3})\sqrt[3]{a}f\right)\int \frac{1}{\sqrt{bx^3+a}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}$$

$$\Big\downarrow 27$$

$$\frac{\left(\sqrt[3]{b}e-(1-\sqrt{3})\sqrt[3]{a}f\right)\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)\sqrt{bx^3+a}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}-\frac{\left(\sqrt[3]{b}e-(1+\sqrt{3})\sqrt[3]{a}f\right)\int \frac{1}{\sqrt{bx^3+a}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}$$

$$\Big\downarrow 759$$

$$\frac{\left(\sqrt[3]{b}e-(1-\sqrt{3})\sqrt[3]{a}f\right)\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)\sqrt{bx^3+a}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-(1+\sqrt{3})\sqrt[3]{a}f\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow 2565$$

$$-\frac{\left(\sqrt[3]{b}e-(1-\sqrt{3})\sqrt[3]{a}f\right)\int \frac{1}{\frac{\left(3-2\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{bx^3+a}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{bx^3+a}}}{\sqrt{3}b^{2/3}}-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-(1+\sqrt{3})\sqrt[3]{a}f\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow 219$$

3.131. $\displaystyle\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{a}b^{2/3}}$$

input
```
Int[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

output
```
-(((b^(1/3)*e - (1 - Sqrt[3])*a^(1/3)*f)*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(
1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a + b*x^3]])/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sq
rt[a]*b^(2/3))) - (Sqrt[2 + Sqrt[3]]*(b^(1/3)*e - (1 + Sqrt[3])*a^(1/3)*f)
*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/
3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^
(3/4)*a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

3.131.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

3.131.4 Maple [F]

$$\int \frac{fx+e}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{b\,x^{3}+a}}dx$$

input
```
int((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x)
```

output
```
int((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x)
```

3.131.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 27.02 (sec) , antiderivative size = 7008, normalized size of antiderivative = 21.11

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algor
ithm="fricas")
```

output
```
Too large to include
```

3.131.6 Sympy [F]

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{e+fx}{\sqrt{a+bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate((f*x+e)/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3+a)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt(a + b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(1
/3)*x)), x)
```

3.131.7 Maxima [F]

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{fx+e}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

input
```
integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))),
 x)
```

3.131.8 Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algor
ithm="giac")
```

output
```
Timed out
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{e+f\,x}{\sqrt{b\,x^3+a}\,\left(b^{1/3}\,x-a^{1/3}\,\left(\sqrt{3}-1\right)\right)}\,dx$$

input
```
int((e + f*x)/((a + b*x^3)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))),x)
```

output
```
int((e + f*x)/((a + b*x^3)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))), x)
```

3.132 $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

3.132.1 Optimal result

Integrand size = 44, antiderivative size = 336

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \frac{\left(\sqrt[3]{b}e+\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\operatorname{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{a}b^{2/3}} + \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}e+\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-}\right.\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

output
```
arctanh(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(-b*x^3+a)^(1/2))
*(b^(1/3)*e+a^(1/3)*f*(1-3^(1/2)))/b^(2/3)/a^(1/2)/(-9+6*3^(1/2))^(1/2)+1/
3*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)
*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*e+a^(1/3)*f*(1+3^(1/2)))*(
1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(1/3)/b^(2/3)/(-b*x^3+a)^(1/2)
/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.132.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.26 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.39

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx =$$

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}f\left(i\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3-(2-i)\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\,\text{Elli}\right.}{}$$

input
```
Integrate[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),
x]
```

output
```
(-4*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((f*(I*(-3 + (2
 + I)*Sqrt[3])*a^(1/3) + (3 - (2 - I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt
[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Ellipt
icF[ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sq
rt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2])/2 - I*(b^(1/3)*e - (-1 + Sqrt[3])*a
^(1/3)*f)*Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqr
t[3])*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*Elli
pticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3
) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[
3])/2]))/((3 - (2 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)
*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a - b*x^3])
```

3.132.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2566, 27, 759, 2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx$$

$$\Big\downarrow \text{2566}$$

$$-\frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\int -\frac{6ab\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx}{12\sqrt{3}a^{4/3}b^{4/3}}-\frac{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\int \frac{1}{\sqrt{a-bx^3}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}-\frac{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\int \frac{1}{\sqrt{a-bx^3}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}+\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\Big\downarrow \text{2565}$$

$$\frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\int \frac{1}{\frac{\left(3-2\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{a-bx^3}+1}d\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{a-bx^3}}}{\sqrt{3}b^{2/3}}+\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\Big\downarrow \text{219}$$

3.132. $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx$

$$\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{a}b^{2/3}}$$

input
```
Int[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
((b^(1/3)*e + (1 - Sqrt[3])*a^(1/3)*f)*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/
6)*(a^(1/3) - b^(1/3)*x))/Sqrt[a - b*x^3]])/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sqrt
[a]*b^(2/3)) + (Sqrt[2 + Sqrt[3]]*(b^(1/3)*e + (1 + Sqrt[3])*a^(1/3)*f)*(a
^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3)
- b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/
4)*a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(
1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

3.132.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

3.132.4 Maple [F]

$$\int \frac{fx+e}{\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{-b\,x^{3}+a}}dx$$

input
```
int((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x)
```

output
```
int((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x)
```

3.132.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 26.95 (sec) , antiderivative size = 7063, normalized size of antiderivative = 21.02

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, alg
orithm="fricas")
```

output
```
Too large to include
```

3.132.6 Sympy [F]

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$$
$$=-\int \frac{e}{-\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$
$$-\int \frac{fx}{-\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

input
```
integrate((f*x+e)/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3+a)**(1/2),x
)
```

output
```
-Integral(e/(-a**(1/3)*sqrt(a - b*x**3) + sqrt(3)*a**(1/3)*sqrt(a - b*x**3
) + b**(1/3)*x*sqrt(a - b*x**3)), x) - Integral(f*x/(-a**(1/3)*sqrt(a - b*
x**3) + sqrt(3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x**3)),
x)
```

3.132.7 Maxima [F]

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int -\frac{fx+e}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

input
```
integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, alg
orithm="maxima")
```

output
```
-integrate((f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))
), x)
```

3.132.8 Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, alg
orithm="giac")
```

output
```
Timed out
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx=\int -\frac{e+f\,x}{\sqrt{a-b\,x^3}\,\left(b^{1/3}\,x+a^{1/3}\,\left(\sqrt{3}-1\right)\right)}\,dx$$

input
```
int(-(e + f*x)/((a - b*x^3)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))),x)
```

output
```
int(-(e + f*x)/((a - b*x^3)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))), x)
```

3.133 $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

3.133.1 Optimal result

Integrand size = 45, antiderivative size = 345

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \frac{\left(\sqrt[3]{b}e+\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{a}b^{2/3}} + \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}e+\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-}\right.\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

output
```
1/3*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/
3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(b^(1/3)*e+a^(1/3)*f*(1+3^(1/2)))
*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(1/4)/a^(1/3)/b^(2/3)/(b*x^3-a)^(1/2
)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+
arctan(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(b*x^3-a)^(1/2))*(
b^(1/3)*e+a^(1/3)*f*(1-3^(1/2)))/b^(2/3)/a^(1/2)/(-9+6*3^(1/2))^(1/2)
```

3.133.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.32 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.35

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx =$$

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}f\left(i\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3-(2-i)\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\,\mathrm{Elli}\right.}{}$$

input
```
Integrate[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3])
,x]
```

output
```
(-4*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((f*(I*(-3 + (2
 + I)*Sqrt[3])*a^(1/3) + (3 - (2 - I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt
[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Ellipt
icF[ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sq
rt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2])/2 - I*(b^(1/3)*e - (-1 + Sqrt[3])*a
^(1/3)*f)*Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqr
t[3])*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*Elli
pticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3
) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[
3])/2]))/((3 - (2 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)
*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3])
```

3.133.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2566, 27, 760, 2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx$$

$\downarrow$ 2566

$$\frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\int \frac{6ab\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx}{12\sqrt{3}a^{4/3}b^{4/3}}-\frac{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\int \frac{1}{\sqrt{bx^3-a}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}$$

$\downarrow$ 27

$$\frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}-\frac{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\int \frac{1}{\sqrt{bx^3-a}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}$$

$\downarrow$ 760

$$\frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}+\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$\downarrow$ 2565

$$\frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\int \frac{1}{1-\frac{\left(3-2\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{bx^3-a}}d\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{bx^3-a}}}{\sqrt{3}b^{2/3}}+\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$\downarrow$ 216

3.133. $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}dx$

$$\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}$$

$$\frac{\arctan\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{a}b^{2/3}}$$

input
```
Int[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
((b^(1/3)*e + (1 - Sqrt[3])*a^(1/3)*f)*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6
)*(a^(1/3) - b^(1/3)*x))/Sqrt[-a + b*x^3]])/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sqrt
[a]*b^(2/3)) + (Sqrt[2 - Sqrt[3]]*(b^(1/3)*e + (1 + Sqrt[3])*a^(1/3)*f)*(a
^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 -
 Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3)
- b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(3/
4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a
^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

3.133.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

3.133.4 Maple [F]

$$\int \frac{fx+e}{\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{b\,x^{3}-a}}dx$$

input
```
int((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x)
```

output
```
int((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x)
```

3.133.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 26.48 (sec) , antiderivative size = 7009, normalized size of antiderivative = 20.32

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

3.133.6 Sympy [F]

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$$
$$= -\int \frac{e}{-\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$
$$-\int \frac{fx}{-\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

input
```
integrate((f*x+e)/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3-a)**(1/2),x)
```

output
```
-Integral(e/(-a**(1/3)*sqrt(-a + b*x**3) + sqrt(3)*a**(1/3)*sqrt(-a + b*x*
*3) + b**(1/3)*x*sqrt(-a + b*x**3)), x) - Integral(f*x/(-a**(1/3)*sqrt(-a
+ b*x**3) + sqrt(3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*x*
*3)), x)
```

3.133.7 Maxima [F]

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \int -\frac{fx+e}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

input
```
integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algo
rithm="maxima")
```

output
```
-integrate((f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1)))
, x)
```

3.133.8 Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=\text{Hanged}$$

input
```
int(-(e + f*x)/((b*x^3 - a)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))),x)
```

output
```
\text{Hanged}
```

3.134 $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

3.134.1 Optimal result

Integrand size = 45, antiderivative size = 345

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$$

$$= -\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{a}b^{2/3}}$$

$$-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+}\right.\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

output

```
-1/3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3
)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(b^(1/3)*e-a^(1/3)*f*(1+3^(1/2)))*
((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2
)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(1/4)/a^(1/3)/b^(2/3)/(-b*x^3-a)^(1/2)
/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-ar
ctan(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(-b*x^3-a)^(1/2))*(b
^(1/3)*e-a^(1/3)*f*(1-3^(1/2)))/b^(2/3)/a^(1/2)/(-9+6*3^(1/2))^(1/2)
```

3.134.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.44 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.33

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx =$$

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}if\left(\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3i-(1+2i)\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}-\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\,\mathrm{El}}{}$$

input
```
Integrate[(e + f*x)/((((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3])
,x]
```

output
```
(-4*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((I/2)*f*((-3 +
 (2 + I)*Sqrt[3])*a^(1/3) + (3*I - (1 + 2*I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I
 + Sqrt[3])*a^(1/3) - (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]
*EllipticF[ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I +
 Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2] + I*(b^(1/3)*e + (-1 + Sqrt[3])*a
^(1/3)*f)*Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3]
)*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*Elliptic
Pi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (
I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))
/((3 - (2 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1
+ (-1)^(1/3))*a^(1/3))]*Sqrt[-a - b*x^3])
```

3.134.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2566, 27, 760, 2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$$

$$\Big\downarrow 2566$$

$$-\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\int -\frac{6ab\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)\sqrt{-bx^3-a}}dx}{12\sqrt{3}a^{4/3}b^{4/3}}-\frac{\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\int \frac{1}{\sqrt{-bx^3-a}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}$$

$$\Big\downarrow 27$$

$$\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)\sqrt{-bx^3-a}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}-\frac{\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\int \frac{1}{\sqrt{-bx^3-a}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}$$

$$\Big\downarrow 760$$

$$\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)\sqrt{-bx^3-a}}dx}{2\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\Big\downarrow 2565$$

$$-\frac{\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)\int \frac{1}{1-\frac{\left(3-2\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{-bx^3-a}}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{-bx^3-a}}}{\sqrt{3}b^{2/3}}-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\Big\downarrow 216$$

3.134. $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}dx$

$$-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\left(1+\sqrt{3}\right)\sqrt[3]{a}f\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-\right.}{3^{3/4}\sqrt[3]{a}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\frac{\arctan\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)\left(\sqrt[3]{b}e-\left(1-\sqrt{3}\right)\sqrt[3]{a}f\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{a}b^{2/3}}$$

input
```
Int[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
-(((b^(1/3)*e - (1 - Sqrt[3])*a^(1/3)*f)*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1
/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a - b*x^3]])/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sq
rt[a]*b^(2/3))) - (Sqrt[2 - Sqrt[3]]*(b^(1/3)*e - (1 + Sqrt[3])*a^(1/3)*f)
*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((
1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^
(3/4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

3.134.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

3.134.4 Maple [F]

$$\int \frac{fx+e}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{-b\,x^{3}-a}}dx$$

input
```
int((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x)
```

output
```
int((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x)
```

3.134.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 26.45 (sec) , antiderivative size = 7078, normalized size of antiderivative = 20.52

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algo
rithm="fricas")
```

output
```
Too large to include
```

3.134.6 Sympy [F]

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{e+fx}{\sqrt{-a-bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate((f*x+e)/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt(-a - b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(
1/3)*x)), x)
```

3.134.7 Maxima [F]

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{fx+e}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

input
```
integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1)))
, x)
```

3.134.8 Giac [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algo
rithm="giac")
```

output
```
Timed out
```

3.134.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Hanged}$$

input
```
int((e + f*x)/((- a - b*x^3)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))),x)
```

output
```
\text{Hanged}
```

3.135 $\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

3.135.1 Optimal result

Integrand size = 21, antiderivative size = 136

$$\begin{aligned}\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx &= -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{3^{3/4}}\\ &\quad+\frac{\sqrt{2}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}\end{aligned}$$

output
```
-1/3*arctan((1+x)*(3+2*3^(1/2))^(1/2)/(x^3+1)^(1/2))*2^(1/2)*3^(1/4)+1/3*(
1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*((x^2-x+
1)/(1+x+3^(1/2))^2)^(1/2)*3^(1/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1
/2)
```

3.135.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.39 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.54

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{2i\left(1+\sqrt{3}\right)\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{2}{3+(\cdots}\right.}{3+(\cdots}\right.}{\sqrt{1+x^3}}$$

input
```
Integrate[x/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))])
 + ((2*I)*(1 + Sqrt[3])*Sqrt[1 - x + x^2]*EllipticPi[((2*I)*Sqrt[3])/(3 +
(2 + I)*Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^
(1/3)])/(3 + (2 + I)*Sqrt[3])))/Sqrt[1 + x^3]
```

3.135.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2566, 27, 759, 2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}\,dx$$

$$\Big\downarrow \text{ 2566}$$

$$\frac{\left(2-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx}{3-\sqrt{3}}+\frac{\int \frac{6\left(x-\sqrt{3}+1\right)}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx}{6\left(3-\sqrt{3}\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(2-\sqrt{3})\int \frac{1}{\sqrt{x^3+1}}dx}{3-\sqrt{3}}+\frac{\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx}{3-\sqrt{3}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{x^3+1}}dx}{3-\sqrt{3}}+\frac{2(2-\sqrt{3})\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{2565}$$

$$\frac{2(2-\sqrt{3})\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\int \frac{1}{\frac{(3+2\sqrt{3})(x+1)^2}{x^3+1}+1}d\frac{x+1}{\sqrt{x^3+1}}}{3-\sqrt{3}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2(2-\sqrt{3})\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}}\right)}{\left(3-\sqrt{3}\right)\sqrt{3+2\sqrt{3}}}$$

input
```
Int[x/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/((3 - Sqrt[3])*Sq
rt[3 + 2*Sqrt[3]]) + (2*(2 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 -
x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt
[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*(3 - Sqrt[3])*Sqrt[(1 + x)/(1 + Sqrt[
3] + x)^2]*Sqrt[1 + x^3])
```

3.135. $\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

3.135.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

3.135.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(108) = 216.

Time = 2.45 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.88

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(-1-\sqrt{3}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{}}{}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(-1-\sqrt{3}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{}}{}$

input
```
int(x/(1+x+3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2
+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(-1-3^(1/2))*(3/2-1/2*I*3
^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2
*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3
+1)^(1/2)*3^(1/2)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1
/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

3.135.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.43

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{1}{3}\left(\sqrt{3}-3\right)\text{weierstrassPInverse}(0,-4,x)-\frac{1}{6}\cdot 3^{\frac{1}{4}}\sqrt{2}\arctan\left(-\frac{3^{\frac{1}{4}}\sqrt{2}\left(3\,x^2-\sqrt{3}(x^2+2\,x+4)-6\,x\right)}{12\sqrt{x^3+1}}\right)$$

input
```
integrate(x/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(sqrt(3) - 3)*weierstrassPInverse(0, -4, x) - 1/6*3^(1/4)*sqrt(2)*arc
tan(-1/12*3^(1/4)*sqrt(2)*(3*x^2 - sqrt(3)*(x^2 + 2*x + 4) - 6*x)/sqrt(x^3
 + 1))
```

3.135.6 Sympy [F]

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

input
```
integrate(x/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

output
```
Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

3.135.7 Maxima [F]

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate(x/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

3.135.8 Giac [F]

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}\left(x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate(x/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

3.135.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx=\text{Hanged}$$

input
```
int(x/((x^3 + 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

output
```
\text{Hanged}
```

3.136 $\int \frac{x}{(1+\sqrt{3}-x)\sqrt{1-x^3}}\,dx$

3.136.1 Optimal result

Integrand size = 25, antiderivative size = 152

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{3^{3/4}} + \frac{\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-1/3*arctan((1-x)*(3+2*3^(1/2))^(1/2)/(-x^3+1)^(1/2))*2^(1/2)*3^(1/4)+1/3*
(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*((x^2+x
+1)/(1-x+3^(1/2))^2)^(1/2)*3^(1/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^
(1/2)
```

3.136.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.54 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.53

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$$
$$= \frac{2i\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{i\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\left(3i+(1+2i)\sqrt{3}+\left(3+(2+i)\sqrt{3}\right)x\right)\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+2(1+\sqrt{3})\sqrt{}\right.}{\left(3+(2+i)\sqrt{3}\right)\sqrt{1-x^3}}$$

input
```
Integrate[x/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

output
```
((2*I)*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((I*Sqrt[((-1)^(1/3) + (-1)^(2/3)*x)
/(1 + (-1)^(1/3))]*(3*I + (1 + 2*I)*Sqrt[3] + (3 + (2 + I)*Sqrt[3])*x)*Ell
ipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqr
t[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + 2*(1 + Sqrt[3])*Sqrt[1 + x + x^2]
*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(
2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]))/((3 + (2 + I)*Sqrt[3])*Sqrt[1 -
x^3])
```

3.136.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2566, 27, 759, 2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(-x+\sqrt{3}+1\right)\sqrt{1-x^3}}\,dx$$
$$\Big\downarrow \ 2566$$
$$\frac{\int -\frac{6\left(-x-\sqrt{3}+1\right)}{\left(-x+\sqrt{3}+1\right)\sqrt{1-x^3}}dx}{6\left(3-\sqrt{3}\right)}-\frac{\left(2-\sqrt{3}\right)\int \frac{1}{\sqrt{1-x^3}}dx}{3-\sqrt{3}}$$

$$\Big\downarrow 27$$

$$-\frac{\left(2-\sqrt{3}\right)\int \frac{1}{\sqrt{1-x^3}}dx}{3-\sqrt{3}}-\frac{\int \frac{-x-\sqrt{3}+1}{\left(-x+\sqrt{3}+1\right)\sqrt{1-x^3}}dx}{3-\sqrt{3}}$$

$$\Big\downarrow 759$$

$$\frac{2\left(2-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3-\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{\int \frac{-x-\sqrt{3}+1}{\left(-x+\sqrt{3}+1\right)\sqrt{1-x^3}}dx}{3-\sqrt{3}}$$

$$\Big\downarrow 2565$$

$$\frac{2\left(2-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3-\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\int \frac{1}{\frac{\left(3+2\sqrt{3}\right)(1-x)^2}{1-x^3}+1}d\frac{1-x}{\sqrt{1-x^3}}}{3-\sqrt{3}}$$

$$\Big\downarrow 216$$

$$\frac{2\left(2-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3-\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\arctan\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\left(3-\sqrt{3}\right)\sqrt{3+2\sqrt{3}}}$$

input
```
Int[x/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/((3 - Sqrt[3])*Sq
rt[3 + 2*Sqrt[3]]) + (2*(2 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 +
x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt
[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*(3 - Sqrt[3])*Sqrt[(1 - x)/(1 + Sqrt[
3] - x)^2]*Sqrt[1 - x^3])
```

3.136. $\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}dx$

3.136.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

3.136.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(124) = 248$.

Time = 2.32 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.69

method	result
default	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}} - \frac{2i\left(-1-\sqrt{3}\right)\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\right.}}{}$
elliptic	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}} - \frac{2i\left(-1-\sqrt{3}\right)\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\right.}}{}$

input
```
int(x/(1-x+3^(1/2))/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/
2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(-1-3^(1/2))*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2
))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1
/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-3/2-3^(1/2)+1/2*I*3^(1/2))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2-3^(1/
2)+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

3.136.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.45

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{1}{3}\left(i\sqrt{3}-3i\right)\text{weierstrassPInverse}(0,4,x)+\frac{1}{6} \cdot 3^{\frac{1}{4}}\sqrt{2}\arctan\left(-\frac{3^{\frac{1}{4}}\sqrt{2}\sqrt{-x^3+1}\left(3x^2-\sqrt{3}(x^2-2x+4)+6x\right)}{12\left(x^3-1\right)}\right)$$

input
```
integrate(x/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(I*sqrt(3) - 3*I)*weierstrassPInverse(0, 4, x) + 1/6*3^(1/4)*sqrt(2)*
arctan(-1/12*3^(1/4)*sqrt(2)*sqrt(-x^3 + 1)*(3*x^2 - sqrt(3)*(x^2 - 2*x +
4) + 6*x)/(x^3 - 1))
```

3.136.6 Sympy [F]

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = -\int \frac{x}{x\sqrt{1-x^3}-\sqrt{3}\sqrt{1-x^3}-\sqrt{1-x^3}}\,dx$$

input
```
integrate(x/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

output
```
-Integral(x/(x*sqrt(1 - x**3) - sqrt(3)*sqrt(1 - x**3) - sqrt(1 - x**3)),
x)
```

3.136.7 Maxima [F]

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \int -\frac{x}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

input
```
integrate(x/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

3.136.8 Giac [F]

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \int -\frac{x}{\sqrt{-x^3+1}\left(x-\sqrt{3}-1\right)}\,dx$$

input
```
integrate(x/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-x/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \text{Hanged}$$

input
```
int(x/((1 - x^3)^(1/2)*(3^(1/2) - x + 1)),x)
```

output
```
\text{Hanged}
```

3.137 $\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

3.137.1 Optimal result

Integrand size = 23, antiderivative size = 164

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = -\frac{\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}}\right)}{3^{3/4}} + \frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
-1/3*arctanh((1-x)*(3+2*3^(1/2))^(1/2)/(x^3-1)^(1/2))*2^(1/2)*3^(1/4)+2/3*
(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(1/3*6^(1/2)-1/
2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)
/(1-x-3^(1/2))^2)^(1/2)
```

3.137.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 11.53 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.40

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$$
$$= \frac{2i\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{i\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\left(3i+(1+2i)\sqrt{3}+\left(3+(2+i)\sqrt{3}\right)x\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+2(1+\sqrt{3})\sqrt{\right.}}{\left(3+(2+i)\sqrt{3}\right)\sqrt{-1+x^3}}$$

input
```
Integrate[x/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

output
```
((2*I)*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((I*Sqrt[((-1)^(1/3) + (-1)^(2/3)*x)
/(1 + (-1)^(1/3))]*(3*I + (1 + 2*I)*Sqrt[3] + (3 + (2 + I)*Sqrt[3])*x)*Ell
ipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqr
t[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + 2*(1 + Sqrt[3])*Sqrt[1 + x + x^2]
*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(
2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]))/((3 + (2 + I)*Sqrt[3])*Sqrt[-1 +
 x^3])
```

3.137.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2566, 27, 760, 2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(-x+\sqrt{3}+1\right)\sqrt{x^3-1}}\,dx$$
$$\Big\downarrow \; 2566$$
$$-\frac{\left(2-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3-1}}dx}{3-\sqrt{3}}-\frac{\int \frac{6\left(-x-\sqrt{3}+1\right)}{\left(-x+\sqrt{3}+1\right)\sqrt{x^3-1}}dx}{6\left(3-\sqrt{3}\right)}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{\left(2-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3-1}}dx}{3-\sqrt{3}}-\frac{\int \frac{-x-\sqrt{3}+1}{\left(-x+\sqrt{3}+1\right)\sqrt{x^3-1}}dx}{3-\sqrt{3}}$$

$$\Big\downarrow \textbf{760}$$

$$\frac{2\left(2-\sqrt{3}\right)^{3/2}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(3-\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{\int \frac{-x-\sqrt{3}+1}{\left(-x+\sqrt{3}+1\right)\sqrt{x^3-1}}dx}{3-\sqrt{3}}$$

$$\Big\downarrow \textbf{2565}$$

$$\frac{2\left(2-\sqrt{3}\right)^{3/2}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(3-\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\int \frac{1}{1-\frac{\left(3+2\sqrt{3}\right)(1-x)^2}{x^3-1}}d\frac{1-x}{\sqrt{x^3-1}}}{3-\sqrt{3}}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{2\left(2-\sqrt{3}\right)^{3/2}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(3-\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}}\right)}{\left(3-\sqrt{3}\right)\sqrt{3+2\sqrt{3}}}$$

input
```
Int[x/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/((3 - Sqrt[3])*
Sqrt[3 + 2*Sqrt[3]]) + (2*(2 - Sqrt[3])^(3/2)*(1 - x)*Sqrt[(1 + x + x^2)/(
1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)],
 -7 + 4*Sqrt[3]])/(3^(1/4)*(3 - Sqrt[3])*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^
2)]*Sqrt[-1 + x^3])
```

3.137. $\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

3.137.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

3.137.4 Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.55

method	result
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{2\left(-1-\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}}{\frac{3}{2}}}}{}$
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{2\left(-1-\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}}{\frac{3}{2}}}}{}$

input
```
int(x/(1-x+3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/
2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2/3*(-1-3^(1/2))*(-3/2-1/2*I*
3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/
2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3
-1)^(1/2)*3^(1/2)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+
1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

3.137.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.30

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \frac{1}{3}\left(\sqrt{3}-3\right)\text{weierstrassPInverse}(0,4,x)+\frac{1}{12}$$
$$\cdot 3^{\frac{1}{4}}\sqrt{2}\log\left(\frac{x^8+16\,x^7+112\,x^6+16\,x^5+112\,x^4-224\,x^3+2\cdot 3^{\frac{1}{4}}\sqrt{2}\left(x^6+18\,x^5+12\,x^4+40\,x^3-36\,x^2\right.}{}\right.$$

input
```
integrate(x/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(3) - 3)*weierstrassPInverse(0, 4, x) + 1/12*3^(1/4)*sqrt(2)*log(
(x^8 + 16*x^7 + 112*x^6 + 16*x^5 + 112*x^4 - 224*x^3 + 2*3^(1/4)*sqrt(2)*(
x^6 + 18*x^5 + 12*x^4 + 40*x^3 - 36*x^2 + sqrt(3)*(x^6 + 6*x^5 + 24*x^4 -
8*x^3 + 12*x^2 - 24*x + 16) + 24*x - 32)*sqrt(x^3 - 1) + 64*x^2 + 16*sqrt(
3)*(x^7 + 2*x^6 + 6*x^5 - 5*x^4 + 2*x^3 - 6*x^2 + 4*x - 4) - 128*x + 112)/
(x^8 - 8*x^7 + 16*x^6 + 16*x^5 - 56*x^4 - 32*x^3 + 64*x^2 + 64*x + 16))
```

3.137.6 Sympy [F]

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = -\int \frac{x}{x\sqrt{x^3-1}-\sqrt{3}\sqrt{x^3-1}-\sqrt{x^3-1}}\,dx$$

input
```
integrate(x/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

output
```
-Integral(x/(x*sqrt(x**3 - 1) - sqrt(3)*sqrt(x**3 - 1) - sqrt(x**3 - 1)),
x)
```

3.137.7 Maxima [F]

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \int -\frac{x}{\sqrt{x^3-1}\left(x-\sqrt{3}-1\right)}\,dx$$

input
```
integrate(x/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate(x/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

3.137.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Ar
gument Va
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = \text{Hanged}$$

input
```
int(x/((x^3 - 1)^(1/2)*(3^(1/2) - x + 1)),x)
```

output
```
\text{Hanged}
```

3.138 $\int \frac{x}{(1+\sqrt{3}+x)\sqrt{-1-x^3}}\,dx$

3.138.1 Optimal result

Integrand size = 23, antiderivative size = 156

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = -\frac{\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right)}{3^{3/4}} + \frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
-1/3*arctanh((1+x)*(3+2*3^(1/2))^(1/2)/(-x^3-1)^(1/2))*2^(1/2)*3^(1/4)+2/3
*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(1/3*6^(1/2)-1
/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-
x)/(1+x-3^(1/2))^2)^(1/2)
```

3.138.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.38 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.35

$$\int \frac{x}{(1+\sqrt{3}+x)\sqrt{-1-x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{2i\left(1+\sqrt{3}\right)\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{2}{3+(\ldots}\right.}{3+(\ldots}\right.}{\sqrt{-1-x^3}}$$

input
```
Integrate[x/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))])
 + ((2*I)*(1 + Sqrt[3])*Sqrt[1 - x + x^2]*EllipticPi[((2*I)*Sqrt[3])/(3 +
(2 + I)*Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^
(1/3)])/(3 + (2 + I)*Sqrt[3])))/Sqrt[-1 - x^3]
```

3.138.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2566, 27, 760, 2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(x+\sqrt{3}+1\right)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{2566}$$

$$\frac{\left(2-\sqrt{3}\right)\int \frac{1}{\sqrt{-x^3-1}}dx}{3-\sqrt{3}}-\frac{\int -\frac{6\left(x-\sqrt{3}+1\right)}{\left(x+\sqrt{3}+1\right)\sqrt{-x^3-1}}dx}{6\left(3-\sqrt{3}\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(2-\sqrt{3})\int \frac{1}{\sqrt{-x^3-1}}dx}{3-\sqrt{3}}+\frac{\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{-x^3-1}}dx}{3-\sqrt{3}}$$

$$\Big\downarrow \text{760}$$

$$\frac{\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{-x^3-1}}dx}{3-\sqrt{3}}+\frac{2(2-\sqrt{3})^{3/2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(3-\sqrt{3}\right)\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{2565}$$

$$\frac{2(2-\sqrt{3})^{3/2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(3-\sqrt{3}\right)\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\int \frac{1}{1-\frac{\left(3+2\sqrt{3}\right)(x+1)^2}{-x^3-1}}d\frac{x+1}{\sqrt{-x^3-1}}}{3-\sqrt{3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2(2-\sqrt{3})^{3/2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(3-\sqrt{3}\right)\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\text{arctanh}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)}{\left(3-\sqrt{3}\right)\sqrt{3+2\sqrt{3}}}$$

input
```
Int[x/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/((3 - Sqrt[3])*
Sqrt[3 + 2*Sqrt[3]]) + (2*(2 - Sqrt[3])^(3/2)*(1 + x)*Sqrt[(1 - x + x^2)/(
1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)],
 -7 + 4*Sqrt[3]])/(3^(1/4)*(3 - Sqrt[3])*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^
2)]*Sqrt[-1 - x^3])
```

3.138. $\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$

3.138.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

3.138. $\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$

3.138.4 Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.62

method	result
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}-1}}-\frac{2i\left(-1-\sqrt{3}\right)\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\right.}}{}$
elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}-1}}-\frac{2i\left(-1-\sqrt{3}\right)\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\right.}}{}$

input
```
int(x/(1+x+3^(1/2))/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2
+1/2*I*3^(1/2)))^(1/2))-2/3*I*(-1-3^(1/2))*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2)
)*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2
))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(3/2+3^(1/2)+1/2*I*3^(1/2))*EllipticPi(1/
3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+3^(1/2)+1
/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

3.138.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.39

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^{3}}}\,dx=-\frac{1}{3}\left(-i\sqrt{3}+3i\right)\text{weierstrassPInverse}(0,-4,x)+\frac{1}{12}$$
$$\cdot 3^{\frac{1}{4}}\sqrt{2}\log\left(\frac{x^{8}-16\,x^{7}+112\,x^{6}-16\,x^{5}+112\,x^{4}+224\,x^{3}+2\cdot 3^{\frac{1}{4}}\sqrt{2}\left(x^{6}-18\,x^{5}+12\,x^{4}-40\,x^{3}-36\,x^{2}\right.}{}\right.$$

input
```
integrate(x/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(-I*sqrt(3) + 3*I)*weierstrassPInverse(0, -4, x) + 1/12*3^(1/4)*sqrt(
2)*log((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112*x^4 + 224*x^3 + 2*3^(1/4)*sq
rt(2)*(x^6 - 18*x^5 + 12*x^4 - 40*x^3 - 36*x^2 + sqrt(3)*(x^6 - 6*x^5 + 24
*x^4 + 8*x^3 + 12*x^2 + 24*x + 16) - 24*x - 32)*sqrt(-x^3 - 1) + 64*x^2 -
16*sqrt(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*x + 4) + 128*x
 + 112)/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64*x^2 - 64*x +
 16))
```

3.138.6 Sympy [F]

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx=\int \frac{x}{\sqrt{-\left(x+1\right)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

input
```
integrate(x/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

output
```
Integral(x/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

3.138.7 Maxima [F]

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx=\int \frac{x}{\sqrt{-x^3-1}\left(x+\sqrt{3}+1\right)}\,dx$$

input
```
integrate(x/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

3.138.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Ar
gument Va
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \text{Hanged}$$

input
```
int(x/((- x^3 - 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

output
```
\text{Hanged}
```

3.139 $\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

3.139.1 Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{3^{3/4}} + \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output

```
-1/3*arctanh((1+x)*(-3+2*3^(1/2))^(1/2)/(x^3+1)^(1/2))*2^(1/2)*3^(1/4)+2/3
*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/3*6^(1/2)+1
/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)
/(1+x+3^(1/2))^2)^(1/2)
```

3.139.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.49 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.53

$$\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(\frac{\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\left(3-(2+i)\sqrt{3}+\left(-3i+(1+2i)\sqrt{3}\right)x\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}-2\left(-1+\sqrt{3}\right)\cdots\right.}{\left(-3i+(1+2i)\sqrt{3}\right)\sqrt{1+x^3}}$$

input
```
Integrate[x/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((Sqrt[((-1)^(1/3) - (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]*(3 - (2 + I)*Sqrt[3] + (-3*I + (1 + 2*I)*Sqrt[3])*x)*Elliptic
F[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1
+ (-1)^(2/3)*x)/(1 + (-1)^(1/3))] - 2*(-1 + Sqrt[3])*Sqrt[1 - x + x^2]*Ell
ipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3
)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]))/((-3*I + (1 + 2*I)*Sqrt[3])*Sqrt[1
+ x^3])
```

3.139.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2566, 27, 759, 2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(x-\sqrt{3}+1\right)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{2566}$$

$$\frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx}{3+\sqrt{3}}+\frac{\int \frac{6\left(x+\sqrt{3}+1\right)}{\left(x-\sqrt{3}+1\right)\sqrt{x^3+1}}dx}{6\left(3+\sqrt{3}\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{(2+\sqrt{3})\int \frac{1}{\sqrt{x^3+1}}dx}{3+\sqrt{3}}+\frac{\int \frac{x+\sqrt{3}+1}{\left(x-\sqrt{3}+1\right)\sqrt{x^3+1}}dx}{3+\sqrt{3}}$$

$$\Big\downarrow \ 759$$

$$\frac{\int \frac{x+\sqrt{3}+1}{\left(x-\sqrt{3}+1\right)\sqrt{x^3+1}}dx}{3+\sqrt{3}}+\frac{2\left(2+\sqrt{3}\right)^{3/2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \ 2565$$

$$\frac{2\left(2+\sqrt{3}\right)^{3/2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\int \frac{1}{\frac{\left(3-2\sqrt{3}\right)(x+1)^2}{x^3+1}+1}d\frac{x+1}{\sqrt{x^3+1}}}{3+\sqrt{3}}$$

$$\Big\downarrow \ 219$$

$$\frac{2\left(2+\sqrt{3}\right)^{3/2}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{x^3+1}}\right)}{\left(3+\sqrt{3}\right)\sqrt{2\sqrt{3}-3}}$$

input
```
Int[x/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/((3 + Sqrt[3])*
Sqrt[-3 + 2*Sqrt[3]]) + (2*(2 + Sqrt[3])^(3/2)*(1 + x)*Sqrt[(1 - x + x^2)/
(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)]
, -7 - 4*Sqrt[3]])/(3^(1/4)*(3 + Sqrt[3])*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2
]*Sqrt[1 + x^3])
```

3.139. $\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

3.139.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

3.139. $\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

3.139.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(116) = 232.

Time = 2.45 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.72

method	result
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\sqrt{3}-1\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-}{-}}}{}$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(1-\sqrt{3}\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-}{-}}}{}$

input
```
int(x/(1+x-3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2
+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2/3*(3^(1/2)-1)*(3/2-1/2*I*3^
(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*
I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+
1)^(1/2)*3^(1/2)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1
/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

3.139.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.46

$$\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{1}{3}\left(\sqrt{3}+3\right)\text{weierstrassPInverse}(0,-4,x)+\frac{1}{12}$$
$$\cdot 3^{\frac{1}{4}}\sqrt{2}\log\left(\frac{x^8-16\,x^7+112\,x^6-16\,x^5+112\,x^4+224\,x^3+2\cdot 3^{\frac{1}{4}}\sqrt{2}\left(x^6-18\,x^5+12\,x^4-40\,x^3-36\,x^2\right.}{}\right.$$

input
```
integrate(x/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(3) + 3)*weierstrassPInverse(0, -4, x) + 1/12*3^(1/4)*sqrt(2)*log
((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112*x^4 + 224*x^3 + 2*3^(1/4)*sqrt(2)*
(x^6 - 18*x^5 + 12*x^4 - 40*x^3 - 36*x^2 - sqrt(3)*(x^6 - 6*x^5 + 24*x^4 +
 8*x^3 + 12*x^2 + 24*x + 16) - 24*x - 32)*sqrt(x^3 + 1) + 64*x^2 + 16*sqrt
(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*x + 4) + 128*x + 112)
/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64*x^2 - 64*x + 16))
```

3.139.6 Sympy [F]

$$\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{(x+1)\,(x^2-x+1)}\,\left(x-\sqrt{3}+1\right)}\,dx$$

input
```
integrate(x/(1+x-3**(1/2))/(x**3+1)**(1/2),x)
```

output
```
Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1)), x)
```

3.139.7 Maxima [F]

$$\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

input
```
integrate(x/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

3.139.8 Giac [F]

$$\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}\left(x-\sqrt{3}+1\right)}\,dx$$

input
```
integrate(x/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \text{Hanged}$$

input
```
int(x/((x^3 + 1)^(1/2)*(x - 3^(1/2) + 1)),x)
```

output
```
\text{Hanged}
```

3.140 $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

3.140.1 Optimal result

Integrand size = 38, antiderivative size = 278

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=-\frac{\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}}+\frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

output
```
-1/3*arctanh(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(b*x^3+a)^(1
/2))*2^(1/2)*3^(1/4)/a^(1/6)/b^(2/3)+2/3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^
(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I
)*(1/3*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/
3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.140.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.28 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.60

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx =$$

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}i\left(\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3i-(1+2i)\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}-\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\,\text{Elli}\cdots}{\cdots}$$

input
```
Integrate[x/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

output
```
(-4*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((I/2)*((-3 + (
2 + I)*Sqrt[3])*a^(1/3) + (3*I - (1 + 2*I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I +
 Sqrt[3])*a^(1/3) - (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*E
llipticF[ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + S
qrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2] + I*(-1 + Sqrt[3])*a^(1/3)*Sqrt[((-
2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1
 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3
*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/
3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqr
t[3])*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1
/3))]*Sqrt[a + b*x^3])
```

3.140.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2566, 27, 759, 2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$$

$$\Big\downarrow \text{2566}$$

$$\frac{\int \frac{6ab\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)\sqrt{bx^3+a}}dx}{6\left(3+\sqrt{3}\right)ab^{4/3}}+\frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{bx^3+a}}dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{bx^3+a}}dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}+\frac{\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)\sqrt{bx^3+a}}dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$$\Big\downarrow \text{759}$$

$$\frac{\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)\sqrt{bx^3+a}}dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}+\frac{2\left(2+\sqrt{3}\right)^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{2565}$$

$$\frac{2\left(2+\sqrt{3}\right)^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2\sqrt[3]{a}\int \frac{1}{\frac{\left(3-2\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{bx^3+a}+1}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{bx^3+a}}}{\left(3+\sqrt{3}\right)b^{2/3}}$$

$$\Big\downarrow \text{219}$$

3.140. $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}dx$

$$\frac{2(2+\sqrt{3})^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,(3+\sqrt{3})\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{2\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{(3+\sqrt{3})\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}b^{2/3}}$$

input
```
Int[x/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a +
b*x^3]])/((3 + Sqrt[3])*Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(2/3)) + (2*(2 + Sq
rt[3])^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*S
qrt[3]])/(3^(1/4)*(3 + Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x
))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

3.140.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

3.140.4 Maple [F]

$$\int \frac{x}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{b\,x^{3}+a}}dx$$

input
```
int(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x)
```

output
```
int(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x)
```

3.140.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.89 (sec) , antiderivative size = 1288, normalized size of antiderivative = 4.63

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="
fricas")
```

output
```
[1/12*(sqrt(2)*a^(1/3)*b^(4/3)*sqrt(sqrt(3)/a)*log((b^8*x^24 - 1840*a*b^7*
x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 + 214
0160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 - 2
*sqrt(2)*(26*a*b^7*x^21 - 4180*a^2*b^6*x^18 + 39552*a^3*b^5*x^15 + 10432*a
^4*b^4*x^12 + 271744*a^5*b^3*x^9 + 699648*a^6*b^2*x^6 + 284672*a^7*b*x^3 +
 8192*a^8 - (b^7*x^22 - 1160*a*b^6*x^19 + 23232*a^2*b^5*x^16 - 53920*a^3*b
^4*x^13 - 148288*a^4*b^3*x^10 - 586752*a^5*b^2*x^7 - 496640*a^6*b*x^4 - 38
912*a^7*x - sqrt(3)*(b^7*x^22 - 632*a*b^6*x^19 + 14736*a^2*b^5*x^16 - 8416
*a^3*b^4*x^13 + 105920*a^4*b^3*x^10 + 334848*a^5*b^2*x^7 + 286720*a^6*b*x^
4 + 22528*a^7*x))*a^(2/3)*b^(1/3) - 12*(17*a*b^6*x^20 - 1014*a^2*b^5*x^17
+ 2748*a^3*b^4*x^14 - 9632*a^4*b^3*x^11 - 36096*a^5*b^2*x^8 - 53376*a^6*b*
x^5 - 11008*a^7*x^2 - 2*sqrt(3)*(5*a*b^6*x^20 - 285*a^2*b^5*x^17 + 1038*a^
3*b^4*x^14 + 784*a^4*b^3*x^11 + 11424*a^5*b^2*x^8 + 15168*a^6*b*x^5 + 3200
*a^7*x^2))*a^(1/3)*b^(2/3) - 2*sqrt(3)*(7*a*b^7*x^21 - 1250*a^2*b^6*x^18 +
 9984*a^3*b^5*x^15 - 19456*a^4*b^4*x^12 - 82624*a^5*b^3*x^9 - 193920*a^6*b
^2*x^6 - 84992*a^7*b*x^3 - 2048*a^8))*sqrt(b*x^3 + a)*sqrt(sqrt(3)/a) + 32
*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 + 5472*a^3*b^4*x^13 + 43
776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 + 4608*a^7*x - sqrt
(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 - 4928*a^3*b^4*x^13 -
 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 - 2560*a^7*x)...
```

3.140.6 Sympy [F]

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{x}{\sqrt{a+bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate(x/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3+a)**(1/2),x)
```

output
```
Integral(x/(sqrt(a + b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)),
 x)
```

3.140.7 Maxima [F]

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\int \frac{x}{\sqrt{bx^3+a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

input
```
integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))), x)
```

3.140.8 Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="
giac")
```

output
```
Timed out
```

3.140.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\text{Hanged}$$

input
```
int(x/((a + b*x^3)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))),x)
```

output
```
\text{Hanged}
```

3.141 $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

3.141.1 Optimal result

Integrand size = 40, antiderivative size = 286

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = -\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}} + \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

output
```
-1/3*arctanh(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(-b*x^3+a)^(
1/2))*2^(1/2)*3^(1/4)/a^(1/6)/b^(2/3)+2/3*(a^(1/3)-b^(1/3)*x)*EllipticF((-
b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+
2*I)*(1/3*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-
b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(-b*x^3+a)^(1/2)/(
a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

3.141.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.08 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.59

$$\int \frac{x}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx =$$

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}\left(i(-3+(2+i)\sqrt{3})\sqrt[3]{a}+(3-(2-i)\sqrt{3})\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\text{Ellipt}\right.}{}$$

input
```
Integrate[x/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
(-4*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((I*(-3 + (2 +
 I)*Sqrt[3])*a^(1/3) + (3 - (2 - I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt[3
])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Elliptic
F[ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt
[3])*a^(1/3))]], (1 + I*Sqrt[3])/2])/2 + I*(-1 + Sqrt[3])*a^(1/3)*Sqrt[((-
I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]*Sq
rt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])
/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3]
)*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 - (2
- I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/
3))*a^(1/3))]*Sqrt[a - b*x^3])
```

3.141.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2566, 27, 759, 2565, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$$

$$\Big\downarrow \text{2566}$$

$$\frac{\int -\frac{6ab\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx}{6\left(3+\sqrt{3}\right)ab^{4/3}}-\frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{a-bx^3}}dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{a-bx^3}}dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}-\frac{\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$$\Big\downarrow \text{759}$$

$$\frac{2\left(2+\sqrt{3}\right)^{3/2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$$\Big\downarrow \text{2565}$$

$$\frac{2\left(2+\sqrt{3}\right)^{3/2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\sqrt[3]{a}\int \frac{1}{\frac{\left(3-2\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{a-bx^3}+1}d\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{a-bx^3}}}{\left(3+\sqrt{3}\right)b^{2/3}}$$

$$\Big\downarrow \text{219}$$

3.141. $\displaystyle\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}dx$

$$\frac{2(2+\sqrt{3})^{3/2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\text{arctanh}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\left(3+\sqrt{3}\right)\sqrt{2\sqrt{3}-3}\sqrt[6]{a}b^{2/3}}$$

input
```
Int[x/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[a -
b*x^3]])/((3 + Sqrt[3])*Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(2/3)) + (2*(2 + Sq
rt[3])^(3/2)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sq
rt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*S
qrt[3]])/(3^(1/4)*(3 + Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x
))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

3.141.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

3.141.4 Maple [F]

$$\int \frac{x}{\left(-b^{\frac{1}{3}} x+a^{\frac{1}{3}} \left(1-\sqrt{3}\right)\right) \sqrt{-b\,x^{3}+a}}dx$$

input
```
int(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x)
```

output
```
int(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x)
```

3.141.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.82 (sec) , antiderivative size = 1354, normalized size of antiderivative = 4.73

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm
="fricas")
```

output
```
[1/12*(sqrt(2)*a^(1/3)*b^(4/3)*sqrt(sqrt(3)/a)*log((b^8*x^24 + 1840*a*b^7*
x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 - 214
0160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 + 3
2*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 - 5472*a^3*b^4*x^13 + 4
3776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 - 4608*a^7*x - sqr
t(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 + 4928*a^3*b^4*x^13
- 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 + 2560*a^7*x))*
a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 + 1
9200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b^2*x^8 - 328704*a^6*
b*x^5 + 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 + 299*a*b^6*x^20 + 4260*a^2*b^
5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024*a^5*b^2*x^8 + 931
84*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 2*sqrt(2)*((b^7*x^22 + 11
60*a*b^6*x^19 + 23232*a^2*b^5*x^16 + 53920*a^3*b^4*x^13 - 148288*a^4*b^3*x
^10 + 586752*a^5*b^2*x^7 - 496640*a^6*b*x^4 + 38912*a^7*x - sqrt(3)*(b^7*x
^22 + 632*a*b^6*x^19 + 14736*a^2*b^5*x^16 + 8416*a^3*b^4*x^13 + 105920*a^4
*b^3*x^10 - 334848*a^5*b^2*x^7 + 286720*a^6*b*x^4 - 22528*a^7*x))*sqrt(-b*
x^3 + a)*a^(2/3)*b^(1/3) + 12*(17*a*b^6*x^20 + 1014*a^2*b^5*x^17 + 2748*a^
3*b^4*x^14 + 9632*a^4*b^3*x^11 - 36096*a^5*b^2*x^8 + 53376*a^6*b*x^5 - 110
08*a^7*x^2 - 2*sqrt(3)*(5*a*b^6*x^20 + 285*a^2*b^5*x^17 + 1038*a^3*b^4*x^1
4 - 784*a^4*b^3*x^11 + 11424*a^5*b^2*x^8 - 15168*a^6*b*x^5 + 3200*a^7*x...
```

3.141.6 Sympy [F]

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$$
$$= -\int \frac{x}{-\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\,dx$$

input
```
integrate(x/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

output
```
-Integral(x/(-a**(1/3)*sqrt(a - b*x**3) + sqrt(3)*a**(1/3)*sqrt(a - b*x**3
) + b**(1/3)*x*sqrt(a - b*x**3)), x)
```

3.141.7 Maxima [F]

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \int -\frac{x}{\sqrt{-bx^3+a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

input
```
integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm
="maxima")
```

output
```
-integrate(x/(sqrt(-b*x^3 + a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))), x)
```

3.141.8 Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm
="giac")
```

output
```
Timed out
```

3.141.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx = \text{Hanged}$$

input
```
int(-x/((a - b*x^3)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))),x)
```

output
```
\text{Hanged}
```

3.142 $\int \frac{x}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

3.142.1 Optimal result

Integrand size = 41, antiderivative size = 282

$$\int \frac{x}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx=-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}}+\frac{\sqrt{2}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{-a+bx^3}}$$

output
```
-1/3*arctan(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(b*x^3-a)^(1/
2))*2^(1/2)*3^(1/4)/a^(1/6)/b^(2/3)+1/3*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2
))*2^(1/2)*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1
-3^(1/2)))^2)^(1/2)*3^(1/4)/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(
1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.142.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.02 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.61

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx =$$

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}\left(i\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3-(2-i)\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\text{Ellipt}\right.}{}$$

input
```
Integrate[x/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
(-4*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((I*(-3 + (2 +
 I)*Sqrt[3])*a^(1/3) + (3 - (2 - I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt[3
])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Elliptic
F[ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt
[3])*a^(1/3))]], (1 + I*Sqrt[3])/2])/2 + I*(-1 + Sqrt[3])*a^(1/3)*Sqrt[((-
I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]*Sq
rt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])
/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3]
)*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 - (2
- I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/
3))*a^(1/3))]*Sqrt[-a + b*x^3])
```

3.142.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {2566, 27, 760, 2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx$$

$$\Big\downarrow \ 2566$$

$$-\frac{\int \frac{6ab\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx}{6\left(3+\sqrt{3}\right)ab^{4/3}}-\frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{bx^3-a}}dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\left(2+\sqrt{3}\right)\int \frac{1}{\sqrt{bx^3-a}}dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}-\frac{\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$$\Big\downarrow \ 760$$

$$\frac{2\sqrt{2-\sqrt{3}}(2+\sqrt{3})\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{bx^3-a}}dx}{\left(3+\sqrt{3}\right)\sqrt[3]{b}}$$

$$\Big\downarrow \ 2565$$

$$\frac{2\sqrt{2-\sqrt{3}}(2+\sqrt{3})\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\sqrt[3]{a}\int \frac{1}{1-\frac{\left(3-2\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{bx^3-a}}d\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\sqrt[3]{a}\sqrt{bx^3-a}}}{\left(3+\sqrt{3}\right)b^{2/3}}$$

$$\Big\downarrow \ 216$$

3.142. $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}dx$

$$\frac{2\sqrt{2-\sqrt{3}}(2+\sqrt{3})\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\arctan\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{\left(3+\sqrt{3}\right)\sqrt{2\sqrt{3}-3}\sqrt[6]{a}b^{2/3}}$$

input
```
Int[x/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[-a +
b*x^3]])/((3 + Sqrt[3])*Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(2/3)) + (2*Sqrt[2
- Sqrt[3]]*(2 + Sqrt[3])*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[Arc
Sin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x
)], -7 + 4*Sqrt[3]])/(3^(1/4)*(3 + Sqrt[3])*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/
3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

3.142.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

3.142.4 Maple [F]

$$\int \frac{x}{\left(-b^{\frac{1}{3}} x + a^{\frac{1}{3}} \left(1-\sqrt{3}\right)\right) \sqrt{b\,x^{3}-a}}dx$$

input
```
int(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x)
```

output
```
int(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x)
```

3.142.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.90 (sec) , antiderivative size = 1295, normalized size of antiderivative = 4.59

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right) \sqrt[3]{a}-\sqrt[3]{b}x\right) \sqrt{-a+bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm=
"fricas")
```

output
```
[1/12*(sqrt(2)*a^(1/3)*b^(4/3)*sqrt(-sqrt(3)/a)*log((b^8*x^24 + 1840*a*b^7
*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 - 21
40160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 +
32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 - 5472*a^3*b^4*x^13 +
43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 - 4608*a^7*x - sq
rt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 + 4928*a^3*b^4*x^13
 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 + 2560*a^7*x))
*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 +
19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b^2*x^8 - 328704*a^6
*b*x^5 + 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 + 299*a*b^6*x^20 + 4260*a^2*b
^5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024*a^5*b^2*x^8 + 93
184*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 32*sqrt(3)*(35*a*b^7*x^2
1 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39520*a^5*
b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8) + 2*sqrt(b*x^3 -
a)*(sqrt(2)*(b^7*x^22 + 1160*a*b^6*x^19 + 23232*a^2*b^5*x^16 + 53920*a^3*b
^4*x^13 - 148288*a^4*b^3*x^10 + 586752*a^5*b^2*x^7 - 496640*a^6*b*x^4 + 38
912*a^7*x - sqrt(3)*(b^7*x^22 + 632*a*b^6*x^19 + 14736*a^2*b^5*x^16 + 8416
*a^3*b^4*x^13 + 105920*a^4*b^3*x^10 - 334848*a^5*b^2*x^7 + 286720*a^6*b*x^
4 - 22528*a^7*x))*a^(2/3)*b^(1/3)*sqrt(-sqrt(3)/a) + 12*sqrt(2)*(17*a*b^6*
x^20 + 1014*a^2*b^5*x^17 + 2748*a^3*b^4*x^14 + 9632*a^4*b^3*x^11 - 3609...
```

3.142.6 Sympy [F]

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$$
$$= -\int \frac{x}{-\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\,dx$$

input
```
integrate(x/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3-a)**(1/2),x)
```

output
```
-Integral(x/(-a**(1/3)*sqrt(-a + b*x**3) + sqrt(3)*a**(1/3)*sqrt(-a + b*x*
*3) + b**(1/3)*x*sqrt(-a + b*x**3)), x)
```

3.142.7 Maxima [F]

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \int -\frac{x}{\sqrt{bx^3-a}\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

input
```
integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm=
"maxima")
```

output
```
-integrate(x/(sqrt(b*x^3 - a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))), x)
```

3.142.8 Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \text{Timed out}$$

input
```
integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm=
"giac")
```

output
```
Timed out
```

3.142.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx = \text{Hanged}$$

input
```
int(-x/((b*x^3 - a)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))),x)
```

output
```
\text{Hanged}
```

3.143 $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

3.143.1 Optimal result

Integrand size = 41, antiderivative size = 278

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}} + \frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right),-7+4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

output
```
-1/3*arctan(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(-b*x^3-a)^(1
/2))*2^(1/2)*3^(1/4)/a^(1/6)/b^(2/3)+1/3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2)
)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3
^(1/2)))^2)^(1/2)*3^(1/4)/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1
/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

3.143.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.24 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.61

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx =$$

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}i\left(\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3i-(1+2i)\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}-\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\,\mathrm{Elli}}{}$$

input
```
Integrate[x/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
(-4*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((I/2)*((-3 + (
2 + I)*Sqrt[3])*a^(1/3) + (3*I - (1 + 2*I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I +
 Sqrt[3])*a^(1/3) - (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*E
llipticF[ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + S
qrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2] + I*(-1 + Sqrt[3])*a^(1/3)*Sqrt[((-
2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1
 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3
*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/
3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqr
t[3])*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1
/3))]*Sqrt[-a - b*x^3])
```

3.143.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {2566, 27, 760, 2565, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}dx$$

$$\Big\downarrow \; 2566$$

$$\frac{(2+\sqrt{3})\int \frac{1}{\sqrt{-bx^3-a}}dx}{(3+\sqrt{3})\sqrt[3]{b}}-\frac{\int -\frac{6ab\left(\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}\right)}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)\sqrt{-bx^3-a}}dx}{6\left(3+\sqrt{3}\right)ab^{4/3}}$$

$$\Big\downarrow \; 27$$

$$\frac{(2+\sqrt{3})\int \frac{1}{\sqrt{-bx^3-a}}dx}{(3+\sqrt{3})\sqrt[3]{b}}+\frac{\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)\sqrt{-bx^3-a}}dx}{(3+\sqrt{3})\sqrt[3]{b}}$$

$$\Big\downarrow \; 760$$

$$\frac{\int \frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\left(\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}\right)\sqrt{-bx^3-a}}dx}{(3+\sqrt{3})\sqrt[3]{b}}+\frac{2\sqrt{2-\sqrt{3}}(2+\sqrt{3})\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}$$

$$\Big\downarrow \; 2565$$

$$\frac{2\sqrt{2-\sqrt{3}}(2+\sqrt{3})\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{2\sqrt[3]{a}\int \frac{1}{1-\frac{\left(3-2\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)^2}{-bx^3-a}}d\frac{\sqrt[3]{b}x+\sqrt[3]{a}}{\sqrt[3]{a}\sqrt{-bx^3-a}}}{\left(3+\sqrt{3}\right)b^{2/3}}$$

$$\Big\downarrow \; 216$$

3.143. $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}dx$

$$\frac{2\sqrt{2-\sqrt{3}}(2+\sqrt{3})\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(3+\sqrt{3}\right)b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{2\arctan\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\left(3+\sqrt{3}\right)\sqrt{2\sqrt{3}-3}\sqrt[6]{a}b^{2/3}}$$

input
```
Int[x/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a -
b*x^3]])/((3 + Sqrt[3])*Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(2/3)) + (2*Sqrt[2
- Sqrt[3]]*(2 + Sqrt[3])*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Arc
Sin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 + 4*Sqrt[3]])/(3^(1/4)*(3 + Sqrt[3])*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

3.143.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

3.143. $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

rule 2565
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Simp[(1 + k)*(e/d)   S
ubst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x
^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c
^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^
3), 0]
```

rule 2566
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Simp[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3))   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/(c*d*(b*c^3 - 28*a*
d^3))   Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6
 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3)
, 0]
```

3.143.4 Maple [F]

$$\int \frac{x}{\left(b^{\frac{1}{3}} x+a^{\frac{1}{3}} \left(1-\sqrt{3}\right)\right) \sqrt{-b\,x^{3}-a}}dx$$

input
```
int(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x)
```

output
```
int(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x)
```

3.143.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.90 (sec) , antiderivative size = 1348, normalized size of antiderivative = 4.85

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx = \text{Too large to display}$$

input
```
integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm=
"fricas")
```

output
```
[1/12*(sqrt(2)*a^(1/3)*b^(4/3)*sqrt(-sqrt(3)/a)*log((b^8*x^24 - 1840*a*b^7
*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 + 21
40160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 -
4*sqrt(2)*(13*a*b^7*x^21 - 2090*a^2*b^6*x^18 + 19776*a^3*b^5*x^15 + 5216*a
^4*b^4*x^12 + 135872*a^5*b^3*x^9 + 349824*a^6*b^2*x^6 + 142336*a^7*b*x^3 +
 4096*a^8 - sqrt(3)*(7*a*b^7*x^21 - 1250*a^2*b^6*x^18 + 9984*a^3*b^5*x^15
- 19456*a^4*b^4*x^12 - 82624*a^5*b^3*x^9 - 193920*a^6*b^2*x^6 - 84992*a^7*
b*x^3 - 2048*a^8))*sqrt(-b*x^3 - a)*sqrt(-sqrt(3)/a) + 2*(144*b^7*x^22 - 1
3536*a*b^6*x^19 + 73872*a^2*b^5*x^16 + 87552*a^3*b^4*x^13 + 700416*a^4*b^3
*x^10 + 1575936*a^5*b^2*x^7 + 949248*a^6*b*x^4 + 73728*a^7*x + sqrt(2)*(b^
7*x^22 - 1160*a*b^6*x^19 + 23232*a^2*b^5*x^16 - 53920*a^3*b^4*x^13 - 14828
8*a^4*b^3*x^10 - 586752*a^5*b^2*x^7 - 496640*a^6*b*x^4 - 38912*a^7*x - sqr
t(3)*(b^7*x^22 - 632*a*b^6*x^19 + 14736*a^2*b^5*x^16 - 8416*a^3*b^4*x^13 +
 105920*a^4*b^3*x^10 + 334848*a^5*b^2*x^7 + 286720*a^6*b*x^4 + 22528*a^7*x
))*sqrt(-b*x^3 - a)*sqrt(-sqrt(3)/a) - 16*sqrt(3)*(5*b^7*x^22 - 505*a*b^6*
x^19 + 2130*a^2*b^5*x^16 - 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 - 53760*
a^5*b^2*x^7 - 35200*a^6*b*x^4 - 2560*a^7*x))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^
23 - 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 - 19200*a^3*b^4*x^14 - 111360*a^
4*b^3*x^11 - 345024*a^5*b^2*x^8 - 328704*a^6*b*x^5 - 61440*a^7*x^2 - 3*sqr
t(2)*(17*a*b^6*x^20 - 1014*a^2*b^5*x^17 + 2748*a^3*b^4*x^14 - 9632*a^4*...
```

3.143.6 Sympy [F]

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{x}{\sqrt{-a-bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}\,dx$$

input
```
integrate(x/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

output
```
Integral(x/(sqrt(-a - b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x))
, x)
```

3.143.7 Maxima [F]

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\int \frac{x}{\sqrt{-bx^3-a}\left(b^{\frac{1}{3}}x-a^{\frac{1}{3}}\left(\sqrt{3}-1\right)\right)}\,dx$$

input
```
integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))), x)
```

3.143.8 Giac [F(-1)]

Timed out.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm=
"giac")
```

output
```
Timed out
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\text{Hanged}$$

input
```
int(x/((- a - b*x^3)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))),x)
```

output
```
\text{Hanged}
```

3.144 $\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx$

3.144.1 Optimal result

Integrand size = 25, antiderivative size = 317

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = -\frac{\left(c-\left(1+\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\arctan\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticPi}\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\left(c-\left(1-\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
-(1+x)*arctan((c^2+c*d+d^2)^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/(c-d)^(1/2
)/d^(1/2)/((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2))*(c-d*(1+3^(1/2)))*((x^2-x+1)/
(1+x+3^(1/2))^2)^(1/2)/(c-d)^(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/(x^3+1)^(1/
2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-4*3^(1/4)*(1+x)*EllipticPi((-1-x+3^(1/2))
/(1+x+3^(1/2)),(c-d*(1+3^(1/2)))^2/(c-d*(1-3^(1/2)))^2,I*3^(1/2)+2*I)*(1/2
*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(c-d*(1-3^(1/2)))/
(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.144.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.49 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.68

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{i\left(c-\left(1+\sqrt{3}\right)d\right)\sqrt{1-x+x^2}\,\text{EllipticPi}\ldots}{\ldots}\right)}{d\sqrt{1+x^3}}$$

input
```
Integrate[(1 + Sqrt[3] + x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))])
 + (I*(c - (1 + Sqrt[3])*d)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c
+ (-1)^(1/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(
1/3)])/(c + (-1)^(1/3)*d)))/(d*Sqrt[1 + x^3])
```

3.144.3 Rubi [A] (warning: unable to verify)

Time = 1.15 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2567, 25, 2538, 412, 435, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}(c+dx)}\,dx$$
$$\Big\downarrow \text{ 2567}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\int -\frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(c+\sqrt{3}d-d-\frac{\left(c-\sqrt{3}d-d\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}\right)}d\left(-\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\big\downarrow$ 25

$$-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\int \frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(c-\left(1-\sqrt{3}\right)d-\frac{\left(c-\left(1+\sqrt{3}\right)d\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}\right)}d\left(-\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\big\downarrow$ 2538

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(c-\left(1+\sqrt{3}\right)d\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(c-\left(1-\sqrt{3}\right)d\right)^2 \cdots\right.}\right.}{\cdots}$$

$\big\downarrow$ 412

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(c-\left(1+\sqrt{3}\right)d\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(c-\left(1-\sqrt{3}\right)d\right)^2 \cdots\right.}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\big\downarrow$ 435

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\frac{1}{2}\left(c-\left(1+\sqrt{3}\right)d\right)\int \frac{1}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}\left(\left(c-\left(1-\sqrt{3}\right)d\right)^2+\frac{\left(c-\left(1+\sqrt{3}\right)d\right)\cdots}{x+\sqrt{3}\cdots}\right.}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\big\downarrow$ 104

3.144. $\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(c-\left(1+\sqrt{3}\right)d\right)\int \frac{1}{-4\sqrt{3}(c-d)d-\frac{4\left(2-\sqrt{3}\right)\left(c^2+dc+d^2\right)\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}}d\frac{\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+}}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\downarrow$ 218

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\frac{\text{EllipticPi}\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{7-4\sqrt{3}}\left(c-\left(1-\sqrt{3}\right)d\right)}+\frac{\left(c-\left(1+\sqrt{3}\right)d\right)\arctan\left(\frac{\sqrt{2-\sqrt{3}}\left(x-\right.}{\sqrt[4]{3}\sqrt{d}\left(x\right.}\right.}{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{d}\sqrt{c-d}\sqrt{c^2}}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

input
```
Int[(1 + Sqrt[3] + x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

output
```
(4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^
2]*(((c - (1 + Sqrt[3])*d)*ArcTan[(Sqrt[2 - Sqrt[3]]*Sqrt[c^2 + c*d + d^2]
*(1 - Sqrt[3] + x))/(3^(1/4)*Sqrt[c - d]*Sqrt[d]*(1 + Sqrt[3] + x))])/(4*3
^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]) + Elli
pticPi[(c - (1 + Sqrt[3])*d)^2/(c - (1 - Sqrt[3])*d)^2, ArcSin[(1 - Sqrt[3
] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]]/(Sqrt[7 - 4*Sqrt[3]]*(c - (1 -
Sqrt[3])*d))))/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

3.144.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.144. $\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx$

3.144.4 Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.87

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3+1}}+\frac{2\left(d\sqrt{3}-c+d\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{d^2\sqrt{x^3+}}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3+1}}+\frac{2\left(d\sqrt{3}-c+d\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{d^2\sqrt{x^3+}}$

input
```
int((1+x+3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^
(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3
/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(d*3^(1/2)-c+d)/d^2*(3/2-
1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-
3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/
2)/(x^3+1)^(1/2)/(-1+c/d)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3
/2+1/2*I*3^(1/2))/(-1+c/d),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/
2))
```

3.144.5 Fricas [F]

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx=\int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}(dx+c)}\,dx$$

input
```
integrate((1+x+3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^3 + 1)*(x + sqrt(3) + 1)/(d*x^4 + c*x^3 + d*x + c), x)
```

3.144.6 Sympy [F]

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx=\int \frac{x+1+\sqrt{3}}{\sqrt{(x+1)\,(x^2-x+1)}\,(c+dx)}\,dx$$

input
```
integrate((1+x+3**(1/2))/(d*x+c)/(x**3+1)**(1/2),x)
```

output
```
Integral((x + 1 + sqrt(3))/(sqrt((x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

3.144.7 Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx=\int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}(dx+c)}\,dx$$

input
```
integrate((1+x+3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

3.144.8 Giac [F]

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx=\int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}(dx+c)}\,dx$$

input
```
integrate((1+x+3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

3.144.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \text{Hanged}$$

input
```
int((x + 3^(1/2) + 1)/((x^3 + 1)^(1/2)*(c + d*x)),x)
```

output
```
\text{Hanged}
```

3.145 $\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx$

3.145.1 Optimal result

Integrand size = 29, antiderivative size = 329

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = -\frac{\left(c+d+\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2},\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\left(c+d-\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-(1-x)*arctanh((c^2-c*d+d^2)^(1/2)*((1-x)/(1-x+3^(1/2))^2)^(1/2)/d^(1/2)/(
c+d)^(1/2)/((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2))*(c+d+d*3^(1/2))*((x^2+x+1)/(
1-x+3^(1/2))^2)^(1/2)/d^(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/(-x^3+1)^(1/
2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)+4*3^(1/4)*(1-x)*EllipticPi((-1+x+3^(1/2))
/(1-x+3^(1/2)),(c+d+d*3^(1/2))^2/(c+d-d*3^(1/2))^2,I*3^(1/2)+2*I)*(1/2*6^(
1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(c+d-d*3^(1/2))/(-x^3+
1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

3.145.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.57 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.71

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(\sqrt{3}c+\left(3+\sqrt{3}\right)\cdots\right.}{\cdots}\right.}{3d\sqrt{1-x^3}}$$

input
```
Integrate[(1 + Sqrt[3] - x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

output
```
(2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/
(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]
 + ((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c + (3 + Sqrt[3])*d)*Sqrt[1 + x +
 x^2]*EllipticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^
(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c - (-1)^(1/3)*d)))/(3*d*Sqrt[1
 - x^3])
```

3.145.3 Rubi [A] (warning: unable to verify)

Time = 1.13 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2567, 2538, 412, 435, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x+\sqrt{3}+1}{\sqrt{1-x^3}(c+dx)}\,dx$$

$$\Big\downarrow \text{2567}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(c-\sqrt{3}d+d-\frac{\left(c+\sqrt{3}d+d\right)\left(-x-\sqrt{3}+1\right)}{-x+\sqrt{3}+1}\right)}d\left(-\frac{-x-\sqrt{}}{-x+\sqrt{}}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$\downarrow$ 2538

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\left(c-\sqrt{3}d+d\right)\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(c-\sqrt{3}d+d\right)^2-\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(-x+\sqrt{}\right.}\right.}\right.}{}$$

$\downarrow$ 412

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\left(c+\sqrt{3}d+d\right)\int -\frac{-x-\sqrt{3}+1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(c-\sqrt{3}d+d\right)^2-\frac{\left(c+\sqrt{3}d\right.}{\left(\right.}\right.}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$\downarrow$ 435

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\frac{1}{2}\left(c+\sqrt{3}d+d\right)\int \frac{1}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}\left(\left(c-\sqrt{3}d+d\right)^2+\frac{\left(c+\sqrt{3}d+d\right)^2}{-x+}\right.}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$\downarrow$ 104

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\left(c+\sqrt{3}d+d\right)\int \frac{1}{4\sqrt{3}d(c+d)-\frac{4\left(2-\sqrt{3}\right)\left(c^2-dc+d^2\right)\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}}d\frac{\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{}}}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

3.145. $\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx$

$$\Big\downarrow \text{221}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\frac{\left(c+\sqrt{3}d+d\right)\text{arctanh}\left(\frac{\sqrt{2-\sqrt{3}}\left(-x-\sqrt{3}+1\right)\sqrt{c^2-cd+d^2}}{\sqrt[4]{3}\sqrt{d}\left(-x+\sqrt{3}+1\right)\sqrt{c+d}}\right)}{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}}-\frac{\text{EllipticPi}\left(\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(c-\sqrt{3}d+d\right)^2},\arcsin\left(\frac{-}{-}\right.\right.}{\sqrt{7-4\sqrt{3}}\left(c-\sqrt{3}\right.}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

input
```
Int[(1 + Sqrt[3] - x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

output
```
(4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^
2]*(((c + d + Sqrt[3]*d)*ArcTanh[(Sqrt[2 - Sqrt[3]]*Sqrt[c^2 - c*d + d^2]*
(1 - Sqrt[3] - x))/(3^(1/4)*Sqrt[d]*Sqrt[c + d]*(1 + Sqrt[3] - x))])/(4*3^
(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[d]*Sqrt[c + d]*Sqrt[c^2 - c*d + d^2]) - Ellip
ticPi[(c + d + Sqrt[3]*d)^2/(c + d - Sqrt[3]*d)^2, ArcSin[(1 - Sqrt[3] - x
)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]]/(Sqrt[7 - 4*Sqrt[3]]*(c + d - Sqrt[3
]*d))))/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

3.145.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

3.145. $\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx$

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.145.4 Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.80

method	result
default	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3d\sqrt{-x^3+1}}-\frac{2i\left(c+d+d\sqrt{3}\right)\sqrt{3}\,\sqrt{i\left(x+\cdots\right.}}{\cdots}$
elliptic	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3d\sqrt{-x^3+1}}-\frac{2i\left(c+d+d\sqrt{3}\right)\sqrt{3}\,\sqrt{i\left(x+\cdots\right.}}{\cdots}$

input
```
int((1-x+3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I/d*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I
*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-
3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(c+d+d*3^(1/2))/d^2*3^(1/2)*(I*(x+1/2-1/2
*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1
/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+c/d)*Ellip
ticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+
1/2*I*3^(1/2)+c/d),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

3.145.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = \text{Timed out}$$

input
```
integrate((1-x+3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.145.6 Sympy [F]

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = -\int \left(-\frac{\sqrt{3}}{c\sqrt{1-x^3}+dx\sqrt{1-x^3}}\right)dx - \int \frac{x}{c\sqrt{1-x^3}+dx\sqrt{1-x^3}}\,dx - \int \left(-\frac{1}{c\sqrt{1-x^3}+dx\sqrt{1-x^3}}\right)dx$$

input
```
integrate((1-x+3**(1/2))/(d*x+c)/(-x**3+1)**(1/2),x)
```

output
```
-Integral(-sqrt(3)/(c*sqrt(1 - x**3) + d*x*sqrt(1 - x**3)), x) - Integral(
x/(c*sqrt(1 - x**3) + d*x*sqrt(1 - x**3)), x) - Integral(-1/(c*sqrt(1 - x*
*3) + d*x*sqrt(1 - x**3)), x)
```

3.145.7 Maxima [F]

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}(dx+c)}\,dx$$

input
```
integrate((1-x+3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

3.145.8 Giac [F]

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}(dx+c)}\,dx$$

input
```
integrate((1-x+3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = \text{Hanged}$$

input
```
int((3^(1/2) - x + 1)/((1 - x^3)^(1/2)*(c + d*x)),x)
```

output
```
\text{Hanged}
```

3.146 $\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx$

3.146.1 Optimal result

Integrand size = 27, antiderivative size = 325

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = -\frac{\left(c+d+\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2},\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\left(c+d-\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
-(1-x)*arctanh((c^2-c*d+d^2)^(1/2)*((1-x)/(1-x+3^(1/2))^2)^(1/2)/d^(1/2)/(
c+d)^(1/2)/((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2))*(c+d+d*3^(1/2))*((x^2+x+1)/(
1-x+3^(1/2))^2)^(1/2)/d^(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/(x^3-1)^(1/2
)/((1-x)/(1-x+3^(1/2))^2)^(1/2)+4*3^(1/4)*(1-x)*EllipticPi((-1+x+3^(1/2))/
(1-x+3^(1/2)),(c+d+d*3^(1/2))^2/(c+d-d*3^(1/2))^2,I*3^(1/2)+2*I)*(1/2*6^(1
/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(c+d-d*3^(1/2))/(x^3-1)
^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

3.146.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.54 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.72

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(\sqrt{3}c+\left(3+\sqrt{3}\right.\right.}{}\right.}{3d\sqrt{-1+x^3}}$$

input
```
Integrate[(1 + Sqrt[3] - x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

output
```
(2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/
(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]
 + ((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c + (3 + Sqrt[3])*d)*Sqrt[1 + x +
 x^2]*EllipticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^
(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c - (-1)^(1/3)*d)))/(3*d*Sqrt[-
1 + x^3])
```

3.146.3 Rubi [A] (warning: unable to verify)

Time = 1.05 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2567, 2538, 412, 435, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x+\sqrt{3}+1}{\sqrt{x^3-1}(c+dx)}\,dx$$
$$\Big\downarrow \ 2567$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(c-\sqrt{3}d+d-\frac{\left(c+\sqrt{3}d+d\right)\left(-x-\sqrt{3}+1\right)}{-x+\sqrt{3}+1}\right)}d\left(-\frac{-x-\sqrt{}}{-x+\sqrt{}}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$\Big\downarrow$ 2538

$$4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\left(c-\sqrt{3}d+d\right)\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(c-\sqrt{3}d+d\right)^2-\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(-x+\sqrt{}\right.}\right.}\right.$$

$\Big\downarrow$ 412

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\left(c+\sqrt{3}d+d\right)\int -\frac{-x-\sqrt{3}+1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(c-\sqrt{3}d+d\right)^2-\frac{\left(c+\sqrt{3}d\right.}{(}\right.}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$\Big\downarrow$ 435

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\frac{1}{2}\left(c+\sqrt{3}d+d\right)\int \frac{1}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}\left(\left(c-\sqrt{3}d+d\right)^2+\frac{\left(c+\sqrt{3}d+d\right)^2}{-x+}\right.}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$\Big\downarrow$ 104

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\left(c+\sqrt{3}d+d\right)\int \frac{1}{4\sqrt{3}d(c+d)-\frac{4\left(2-\sqrt{3}\right)\left(c^2-dc+d^2\right)\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}}d\frac{\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}\right.}}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

3.146. $\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx$

$$\Big\downarrow \text{221}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\frac{\left(c+\sqrt{3}d+d\right)\text{arctanh}\left(\frac{\sqrt{2-\sqrt{3}}\left(-x-\sqrt{3}+1\right)\sqrt{c^2-cd+d^2}}{\sqrt[4]{3}\sqrt{d}\left(-x+\sqrt{3}+1\right)\sqrt{c+d}}\right)}{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}}-\frac{\text{EllipticPi}\left(\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(c-\sqrt{3}d+d\right)^2},\arcsin\left(\frac{-}{}\right.\right.}{\sqrt{7-4\sqrt{3}}\left(c-\sqrt{3}\right.}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

input
```
Int[(1 + Sqrt[3] - x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

output
```
(4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^
2]*(((c + d + Sqrt[3]*d)*ArcTanh[(Sqrt[2 - Sqrt[3]]*Sqrt[c^2 - c*d + d^2]*
(1 - Sqrt[3] - x))/(3^(1/4)*Sqrt[d]*Sqrt[c + d]*(1 + Sqrt[3] - x))])/(4*3^
(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[d]*Sqrt[c + d]*Sqrt[c^2 - c*d + d^2]) - Ellip
ticPi[(c + d + Sqrt[3]*d)^2/(c + d - Sqrt[3]*d)^2, ArcSin[(1 - Sqrt[3] - x
)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]]/(Sqrt[7 - 4*Sqrt[3]]*(c + d - Sqrt[3
]*d))))/(Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

3.146.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.146.4 Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.84

method	result
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3-1}}+\frac{2\left(c+d+d\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{\cdots}$
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3-1}}+\frac{2\left(c+d+d\sqrt{3}\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\cdots}{\cdots}$

input
```
int((1-x+3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I
*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(
1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((
3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*(c+d+d*3^(1/2))/d^2*(-3/2
-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/
(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/
2)/(x^3-1)^(1/2)/(1+c/d)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/
2+1/2*I*3^(1/2))/(1+c/d),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

3.146.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx=\text{Timed out}$$

input
```
integrate((1-x+3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.146.6 Sympy [F]

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx=-\int\left(-\frac{\sqrt{3}}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\right)dx-\int\frac{x}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\,dx-\int\left(-\frac{1}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\right)dx$$

input
```
integrate((1-x+3**(1/2))/(d*x+c)/(x**3-1)**(1/2),x)
```

output
```
-Integral(-sqrt(3)/(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x) - Integral(
x/(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x) - Integral(-1/(c*sqrt(x**3 -
 1) + d*x*sqrt(x**3 - 1)), x)
```

3.146.7 Maxima [F]

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{x^3-1}(dx+c)}\,dx$$

input
```
integrate((1-x+3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

3.146.8 Giac [F]

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{x^3-1}(dx+c)}\,dx$$

input
```
integrate((1-x+3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

3.146.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = \text{Hanged}$$

input
```
int((3^(1/2) - x + 1)/((x^3 - 1)^(1/2)*(c + d*x)),x)
```

output
```
\text{Hanged}
```

3.147 $\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx$

3.147.1 Optimal result

Integrand size = 27, antiderivative size = 321

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = -\frac{\left(c-\left(1+\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\arctan\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticPi}\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\left(c-\left(1-\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
-(1+x)*arctan((c^2+c*d+d^2)^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/(c-d)^(1/2
)/d^(1/2)/((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2))*(c-d*(1+3^(1/2)))*((x^2-x+1)/
(1+x+3^(1/2))^2)^(1/2)/(c-d)^(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/(-x^3-1)^(1
/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-4*3^(1/4)*(1+x)*EllipticPi((-1-x+3^(1/2)
)/(1+x+3^(1/2)),(c-d*(1+3^(1/2)))^2/(c-d*(1-3^(1/2)))^2,I*3^(1/2)+2*I)*(1/
2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(c-d*(1-3^(1/2)))
/(-x^3-1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.147.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.54 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.73

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(\sqrt{3}c-\left(3+\sqrt{3}\right.\right.}{}\right.}{3d\sqrt{-1-x^3}}$$

input
```
Integrate[(1 + Sqrt[3] + x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/
(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]
 + ((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c - (3 + Sqrt[3])*d)*Sqrt[1 - x +
 x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 + (-1)^(
2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c + (-1)^(1/3)*d)))/(3*d*Sqrt[-1
 - x^3])
```

3.147.3 Rubi [A] (warning: unable to verify)

Time = 1.09 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {2567, 25, 2538, 412, 435, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}(c+dx)}\,dx$$
$$\Big\downarrow \ 2567$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\int -\frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(c+\sqrt{3}d-d-\frac{\left(c-\sqrt{3}d-d\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}\right)}d\left(-\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$\downarrow$ 25

$$-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\int \frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(c-\left(1-\sqrt{3}\right)d-\frac{\left(c-\left(1+\sqrt{3}\right)d\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}\right)}d\left(-\frac{x-\sqrt{}}{x+\sqrt{}}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$\downarrow$ 2538

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(c-\left(1+\sqrt{3}\right)d\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(c-\left(1-\sqrt{3}\right)d\right)^2\right)}\right)}{}$$

$\downarrow$ 412

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(c-\left(1+\sqrt{3}\right)d\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(c-\left(1-\sqrt{3}\right)d\right)^2\right)}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$\downarrow$ 435

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\frac{1}{2}\left(c-\left(1+\sqrt{3}\right)d\right)\int \frac{1}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}\left(\left(c-\left(1-\sqrt{3}\right)d\right)^2+\frac{\left(c-\left(1+\sqrt{3}\right)d\right)}{x+\sqrt{}}\right)}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$\downarrow$ 104

3.147. $\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(c-\left(1+\sqrt{3}\right)d\right)\int \frac{1}{-4\sqrt{3}(c-d)d-\frac{4\left(2-\sqrt{3}\right)\left(c^2+dc+d^2\right)\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}}d\frac{\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+}}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$\downarrow$ 218

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\frac{\text{EllipticPi}\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{7-4\sqrt{3}}\left(c-\left(1-\sqrt{3}\right)d\right)}+\frac{\left(c-\left(1+\sqrt{3}\right)d\right)\arctan\left(\frac{\sqrt{2-\sqrt{3}}(x-\sqrt{}}{\sqrt[4]{3}\sqrt{d}(x}\right.}{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{d}\sqrt{c-d}\sqrt{c^2}}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

input
```
Int[(1 + Sqrt[3] + x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

output
```
(4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^
2]*(((c - (1 + Sqrt[3])*d)*ArcTan[(Sqrt[2 - Sqrt[3]]*Sqrt[c^2 + c*d + d^2]
*(1 - Sqrt[3] + x))/(3^(1/4)*Sqrt[c - d]*Sqrt[d]*(1 + Sqrt[3] + x))])/(4*3
^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]) + Elli
pticPi[(c - (1 + Sqrt[3])*d)^2/(c - (1 - Sqrt[3])*d)^2, ArcSin[(1 - Sqrt[3
] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]]/(Sqrt[7 - 4*Sqrt[3]]*(c - (1 -
Sqrt[3])*d))))/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```

3.147.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.147. $\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx$

3.147.4 Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.83

method	result
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3d\sqrt{-x^{3}-1}}-\frac{2i\left(d\sqrt{3}-c+d\right)\sqrt{3}\,\sqrt{i\left(x-\right.}}{}$
elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3d\sqrt{-x^{3}-1}}-\frac{2i\left(d\sqrt{3}-c+d\right)\sqrt{3}\,\sqrt{i\left(x-\right.}}{}$

input
```
int((1+x+3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I/d*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I
*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3
/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(d*3^(1/2)-c+d)/d^2*3^(1/2)*(I*(x-1/2-1/2*
I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2
*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+c/d)*Elliptic
Pi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*
I*3^(1/2)+c/d),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

3.147.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+x+3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   catd
ef: division by zero
```

3.147.6 Sympy [F]

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \int \frac{x+1+\sqrt{3}}{\sqrt{-(x+1)\,(x^2-x+1)}\,(c+dx)}\,dx$$

input
```
integrate((1+x+3**(1/2))/(d*x+c)/(-x**3-1)**(1/2),x)
```

output
```
Integral((x + 1 + sqrt(3))/(sqrt(-(x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

3.147.7 Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}(dx+c)}\,dx$$

input
```
integrate((1+x+3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

3.147.8 Giac [F]

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}(dx+c)}\,dx$$

input
```
integrate((1+x+3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

3.147.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \text{Hanged}$$

input
```
int((x + 3^(1/2) + 1)/((- x^3 - 1)^(1/2)*(c + d*x)),x)
```

output
```
\text{Hanged}
```

3.148 $\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx$

3.148.1 Optimal result

Integrand size = 27, antiderivative size = 358

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = -\frac{\left(c-\left(1-\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{arctanh}\left(\frac{2\sqrt{2+\sqrt{3}}\sqrt{c^2+cd+d^2}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{7+4\sqrt{3}+\frac{\left(1+\sqrt{3}+x\right)^2}{\left(1-\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
-(1+x)*arctanh(2*(c^2+c*d+d^2)^(1/2)*((-1-x)/(1+x-3^(1/2))^2)^(1/2)*(1/2*6
^(1/2)+1/2*2^(1/2))/(c-d)^(1/2)/d^(1/2)/(7+4*3^(1/2)+(1+x+3^(1/2))^2/(1+x-
3^(1/2))^2)^(1/2))*(c-d*(1-3^(1/2)))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)/(c-
d)^(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/(x^3+1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2
)^(1/2)+4*3^(1/4)*(1+x)*EllipticPi((-1-x-3^(1/2))/(1+x-3^(1/2)),(c-d*(1-3^
(1/2)))^2/(c-d*(1+3^(1/2)))^2,2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x
^2-x+1)/(1+x-3^(1/2))^2)^(1/2)/(-d*3^(1/2)+c-d)/(x^3+1)^(1/2)/((-1-x)/(1+x
-3^(1/2))^2)^(1/2)
```

3.148.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.52 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.59

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{i\left(c+\left(-1+\sqrt{3}\right)d\right)\sqrt{1-x+x^2}\,\text{EllipticP}}{}\right.}{d\sqrt{1+x^3}}$$

input
```
Integrate[(1 - Sqrt[3] + x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))])
 + (I*(c + (-1 + Sqrt[3])*d)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c
 + (-1)^(1/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^
(1/3)])/(c + (-1)^(1/3)*d)))/(d*Sqrt[1 + x^3])
```

3.148.3 Rubi [A] (warning: unable to verify)

Time = 1.07 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.86, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2568, 25, 2538, 412, 435, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}(c+dx)}\,dx$$

$$\Big\downarrow \text{2568}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\int -\frac{1}{\left(c-\sqrt{3}d-d-\frac{\left(c-\left(1-\sqrt{3}\right)d\right)\left(x+\sqrt{3}+1\right)}{x-\sqrt{3}+1}\right)\sqrt{1-\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}}d\left(-\frac{x+\sqrt{3}}{x-\sqrt{3}}\right.}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\downarrow$ 25

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\int \frac{1}{\left(c-\left(1+\sqrt{3}\right)d-\frac{\left(c-\left(1-\sqrt{3}\right)d\right)\left(x+\sqrt{3}+1\right)}{x-\sqrt{3}+1}\right)\sqrt{1-\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}}d\left(-\frac{x+\sqrt{3}+}{x-\sqrt{3}+}\right.}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\downarrow$ 2538

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\left(c-\left(1-\sqrt{3}\right)d\right)\int -\frac{x+\sqrt{3}+1}{\left(x-\sqrt{3}+1\right)\sqrt{1-\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}\left(\left(c-\left(1+\sqrt{3}\right)d\right.\right.}\right.}{}$$

$\downarrow$ 412

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\left(c-\left(1-\sqrt{3}\right)d\right)\int -\frac{x+\sqrt{3}+1}{\left(x-\sqrt{3}+1\right)\sqrt{1-\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}\left(\left(c-\left(1+\sqrt{3}\right)d\right.\right.}\right.}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{}}$$

$\downarrow$ 435

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\frac{1}{2}\left(c-\left(1-\sqrt{3}\right)d\right)\int \frac{1}{\sqrt{\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}+1}\left(\frac{\left(x+\sqrt{3}+1\right)\left(c-\left(1-\sqrt{3}\right)d\right)^2}{x-\sqrt{3}+1}+\left(c-\left(1+\sqrt{3}\right)d\right)^2\right)\sqrt{\frac{\left(x+\sqrt{\right.}}{\left(x-\sqrt{\right.}}}}\right.}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\downarrow$ 104

3.148. $\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\left(c-\left(1-\sqrt{3}\right)d\right)\int \frac{1}{4\sqrt{3}(c-d)d-\frac{4\left(2+\sqrt{3}\right)\left(c^2+dc+d^2\right)\sqrt{\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}}}d\frac{\sqrt{\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+}}\right.}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\downarrow$ 221

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)\text{EllipticPi}\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{7+4\sqrt{3}}\left(c-\sqrt{3}d-d\right)^2}-\frac{\left(c-\left(1-\sqrt{3}\right)d\right)\text{arc}}{4\sqrt[4]{3}\sqrt{}}\right.}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

input
```
Int[(1 - Sqrt[3] + x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

output
```
(-4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)
^2]*(-1/4*((c - (1 - Sqrt[3])*d)*ArcTanh[(Sqrt[2 + Sqrt[3]]*Sqrt[c^2 + c*d
 + d^2]*(1 + Sqrt[3] + x))/(3^(1/4)*Sqrt[c - d]*Sqrt[d]*(1 - Sqrt[3] + x))
])/(3^(1/4)*Sqrt[2 + Sqrt[3]]*Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]) +
 ((c - (1 + Sqrt[3])*d)*EllipticPi[(c - (1 - Sqrt[3])*d)^2/(c - (1 + Sqrt[
3])*d)^2, ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(S
qrt[7 + 4*Sqrt[3]]*(c - d - Sqrt[3]*d)^2)))/(Sqrt[-((1 + x)/(1 - Sqrt[3] +
 x)^2)]*Sqrt[1 + x^3])
```

3.148.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2568
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(-1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
 + Sqrt[3]]*f*(1 - q*x)*(Sqrt[(1 + q*x + q^2*x^2)/(1 - Sqrt[3] - q*x)^2]/(q
*Sqrt[a + b*x^3]*Sqrt[-(1 - q*x)/(1 - Sqrt[3] - q*x)^2]))   Subst[Int[1/(((
1 + Sqrt[3])*d + c*q + ((1 - Sqrt[3])*d + c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 + 4*
Sqrt[3] + x^2]), x], x, (1 + Sqrt[3] - q*x)/(-1 + Sqrt[3] + q*x)], x]] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 - 3*Sq
rt[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.148. $\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx$

3.148.4 Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.77

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3+1}}-\frac{2\left(d\sqrt{3}+c-d\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{d^2\sqrt{x^3+}}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3+1}}-\frac{2\left(d\sqrt{3}+c-d\right)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}}{d^2\sqrt{x^3+}}$

input
```
int((1+x-3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^
(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3
/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*(d*3^(1/2)+c-d)/d^2*(3/2-
1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-
3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/
2)/(x^3+1)^(1/2)/(-1+c/d)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3
/2+1/2*I*3^(1/2))/(-1+c/d),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/
2))
```

3.148.5 Fricas [F]

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}(dx+c)}\,dx$$

input
```
integrate((1+x-3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^3 + 1)*(x - sqrt(3) + 1)/(d*x^4 + c*x^3 + d*x + c), x)
```

3.148.6 Sympy [F]

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{(x+1)\,(x^2-x+1)}\,(c+dx)}\,dx$$

input
```
integrate((1+x-3**(1/2))/(d*x+c)/(x**3+1)**(1/2),x)
```

output
```
Integral((x - sqrt(3) + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

3.148.7 Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}(dx+c)}\,dx$$

input
```
integrate((1+x-3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

3.148.8 Giac [F]

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}(dx+c)}\,dx$$

input
```
integrate((1+x-3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

3.148.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \text{Hanged}$$

input
```
int((x - 3^(1/2) + 1)/((x^3 + 1)^(1/2)*(c + d*x)),x)
```

output
```
\text{Hanged}
```

3.149 $\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx$

3.149.1 Optimal result

Integrand size = 31, antiderivative size = 346

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = -\frac{\left(c+d-\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\arctan\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticPi}\left(\frac{\left(c+d-\sqrt{3}d\right)^2}{\left(c+d+\sqrt{3}d\right)^2},\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output

```
-(1-x)*arctan((c^2-c*d+d^2)^(1/2)*((-1+x)/(1-x-3^(1/2))^2)^(1/2)/d^(1/2)/(
c+d)^(1/2)/((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2))*(c+d-d*3^(1/2))*((x^2+x+1)/(
1-x-3^(1/2))^2)^(1/2)/d^(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/(-x^3+1)^(1/
2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)-4*3^(1/4)*(1-x)*EllipticPi((-1+x-3^(1/2)
)/(1-x-3^(1/2)),(c+d-d*3^(1/2))^2/(c+d+d*3^(1/2))^2,2*I-I*3^(1/2))*(1/2*6^
(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)/(c+d+d*3^(1/2))/(-x^3
+1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

3.149.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.56 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.68

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx$$

$$= \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(\sqrt{3}c+\left(-3+\sqrt{\ldots}\right.\right.}{\ldots}\right.}{3d\sqrt{1-x^3}}$$

input
```
Integrate[(1 - Sqrt[3] - x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

output
```
(2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/
(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]
 + ((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c + (-3 + Sqrt[3])*d)*Sqrt[1 + x
+ x^2]*EllipticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)
^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c - (-1)^(1/3)*d)))/(3*d*Sqrt[
1 - x^3])
```

3.149.3 Rubi [A] (warning: unable to verify)

Time = 1.04 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2568, 2538, 412, 435, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x-\sqrt{3}+1}{\sqrt{1-x^3}(c+dx)}\,dx$$

$$\Big\downarrow \; 2568$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\int\frac{1}{\left(c+\sqrt{3}d+d-\frac{\left(c-\sqrt{3}d+d\right)\left(-x+\sqrt{3}+1\right)}{-x-\sqrt{3}+1}\right)\sqrt{1-\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}}d\left(-\frac{-x\cdots}{-x\cdots}\right.}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$\downarrow$ 2538

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(\left(c+\sqrt{3}d+d\right)\int\frac{1}{\sqrt{1-\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}\left(\left(c+\sqrt{3}d+d\right)^2-\frac{\left(c-\sqrt{3}d+d\cdots\right.}{\left(-x\cdots\right.}\right.}\right.}{\cdots}$$

$\downarrow$ 412

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\left(c-\sqrt{3}d+d\right)\int-\frac{-x+\sqrt{3}+1}{\sqrt{1-\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}\left(\left(c+\sqrt{3}d+d\right)^2-\frac{\left(c-\sqrt{\cdots}\right.}{\cdots}\right.}\right.}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{1-\cdots}}$$

$\downarrow$ 435

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\frac{1}{2}\left(c-\sqrt{3}d+d\right)\int\frac{1}{\sqrt{\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}+1}\left(\frac{\left(-x+\sqrt{3}+1\right)\left(c-\sqrt{3}d+d\right)^2}{-x-\sqrt{3}+1}+\left(c+\sqrt{3}d+d\right)^2\right)\sqrt{\frac{\left(-x+\cdots\right.}{\left(-x-\cdots\right.}}}\right.}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$\downarrow$ 104

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\left(c-\sqrt{3}d+d\right)\int\frac{1}{-4\sqrt{3}d(c+d)-\frac{4\left(2+\sqrt{3}\right)\left(c^2-dc+d^2\right)\sqrt{\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}}}d\frac{\sqrt{\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\cdots}}{\sqrt{\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}+\cdots}}\right.}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

3.149. $\int\frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx$

$$\Big\downarrow 218$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\frac{\text{EllipticPi}\left(\frac{\left(c-\sqrt{3}d+d\right)^2}{\left(c+\sqrt{3}d+d\right)^2},\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{7+4\sqrt{3}}\left(c+\sqrt{3}d+d\right)}-\frac{\left(c-\sqrt{3}d+d\right)\arctan\left(\frac{\sqrt{2+\sqrt{3}}(-\cdots}{\sqrt[4]{3}\sqrt{\cdots}}\right)}{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt{d}\sqrt{c+\cdots}}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

input
```
Int[(1 - Sqrt[3] - x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

output
```
(-4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)
^2]*(-1/4*((c + d - Sqrt[3]*d)*ArcTan[(Sqrt[2 + Sqrt[3]]*Sqrt[c^2 - c*d +
d^2]*(1 + Sqrt[3] - x))/(3^(1/4)*Sqrt[d]*Sqrt[c + d]*(1 - Sqrt[3] - x))])/
(3^(1/4)*Sqrt[2 + Sqrt[3]]*Sqrt[d]*Sqrt[c + d]*Sqrt[c^2 - c*d + d^2]) - El
lipticPi[(c + d - Sqrt[3]*d)^2/(c + d + Sqrt[3]*d)^2, ArcSin[(1 + Sqrt[3]
- x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]]/(Sqrt[7 + 4*Sqrt[3]]*(c + d + Sqr
t[3]*d))))/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[1 - x^3])
```

3.149.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

3.149. $\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx$

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2568
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(-1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
 + Sqrt[3]]*f*(1 - q*x)*(Sqrt[(1 + q*x + q^2*x^2)/(1 - Sqrt[3] - q*x)^2]/(q
*Sqrt[a + b*x^3]*Sqrt[-(1 - q*x)/(1 - Sqrt[3] - q*x)^2]))   Subst[Int[1/(((
1 + Sqrt[3])*d + c*q + ((1 - Sqrt[3])*d + c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 + 4*
Sqrt[3] + x^2]), x], x, (1 + Sqrt[3] - q*x)/(-1 + Sqrt[3] + q*x)], x]] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 - 3*Sq
rt[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.149.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.77

method	result
default	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3d\sqrt{-x^{3}+1}}+\frac{2i\left(d\sqrt{3}-c-d\right)\sqrt{3}\,\sqrt{i\left(x+\right.}}{}$
elliptic	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3d\sqrt{-x^{3}+1}}+\frac{2i\left(d\sqrt{3}-c-d\right)\sqrt{3}\,\sqrt{i\left(x+\right.}}{}$

input
```
int((1-x-3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I/d*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I
*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-
3/2+1/2*I*3^(1/2)))^(1/2))+2/3*I*(d*3^(1/2)-c-d)/d^2*3^(1/2)*(I*(x+1/2-1/2
*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1
/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+c/d)*Ellip
ticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+
1/2*I*3^(1/2)+c/d),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

3.149.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = \text{Timed out}$$

input
```
integrate((1-x-3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.149.6 Sympy [F]

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = -\int \frac{\sqrt{3}}{c\sqrt{1-x^3}+dx\sqrt{1-x^3}}\,dx - \int \frac{x}{c\sqrt{1-x^3}+dx\sqrt{1-x^3}}\,dx - \int \left(-\frac{1}{c\sqrt{1-x^3}+dx\sqrt{1-x^3}}\right)dx$$

input
```
integrate((1-x-3**(1/2))/(d*x+c)/(-x**3+1)**(1/2),x)
```

output
```
-Integral(sqrt(3)/(c*sqrt(1 - x**3) + d*x*sqrt(1 - x**3)), x) - Integral(x
/(c*sqrt(1 - x**3) + d*x*sqrt(1 - x**3)), x) - Integral(-1/(c*sqrt(1 - x**
3) + d*x*sqrt(1 - x**3)), x)
```

3.149.7 Maxima [F]

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}(dx+c)}\,dx$$

input
```
integrate((1-x-3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

3.149.8 Giac [F]

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}(dx+c)}\,dx$$

input
```
integrate((1-x-3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

3.149.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = \text{Hanged}$$

input
```
int(-(x + 3^(1/2) - 1)/((1 - x^3)^(1/2)*(c + d*x)),x)
```

output
```
\text{Hanged}
```

3.150 $\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx$

3.150.1 Optimal result

Integrand size = 29, antiderivative size = 342

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = -\frac{\left(c+d-\sqrt{3}d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\arctan\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticPi}\left(\frac{\left(c+d-\sqrt{3}d\right)^2}{\left(c+d+\sqrt{3}d\right)^2},\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
-(1-x)*arctan((c^2-c*d+d^2)^(1/2)*((-1+x)/(1-x-3^(1/2))^2)^(1/2)/d^(1/2)/(
c+d)^(1/2)/((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2))*(c+d-d*3^(1/2))*((x^2+x+1)/(
1-x-3^(1/2))^2)^(1/2)/d^(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/(x^3-1)^(1/2
)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)-4*3^(1/4)*(1-x)*EllipticPi((-1+x-3^(1/2))
/(1-x-3^(1/2)),(c+d-d*3^(1/2))^2/(c+d+d*3^(1/2))^2,2*I-I*3^(1/2))*(1/2*6^(
1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)/(c+d+d*3^(1/2))/(x^3-1
)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

3.150.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.54 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.68

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(\sqrt{3}c+\left(-3+\sqrt{3}\right)\right.}{}\right.}{3d\sqrt{-1+x^3}}$$

input
```
Integrate[(1 - Sqrt[3] - x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

output
```
(2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/
(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]
 + ((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c + (-3 + Sqrt[3])*d)*Sqrt[1 + x
+ x^2]*EllipticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)
^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c - (-1)^(1/3)*d)))/(3*d*Sqrt[
-1 + x^3])
```

3.150.3 Rubi [A] (warning: unable to verify)

Time = 0.97 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2568, 2538, 412, 435, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x-\sqrt{3}+1}{\sqrt{x^3-1}(c+dx)}\,dx$$
$$\Big\downarrow \text{2568}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\int \frac{1}{\left(c+\sqrt{3}d+d-\frac{\left(c-\sqrt{3}d+d\right)\left(-x+\sqrt{3}+1\right)}{-x-\sqrt{3}+1}\right)\sqrt{1-\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}}d\left(-\frac{-x\cdots}{-x\cdots}\right.}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$\downarrow$ 2538

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(\left(c+\sqrt{3}d+d\right)\int \frac{1}{\sqrt{1-\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}\left(\left(c+\sqrt{3}d+d\right)^2-\frac{\left(c-\sqrt{3}d+d\cdots\right.}{\left(-x\cdots\right.}\right.}\cdots\right.}{\cdots}$$

$\downarrow$ 412

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\left(c-\sqrt{3}d+d\right)\int -\frac{-x+\sqrt{3}+1}{\sqrt{1-\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}\left(\left(c+\sqrt{3}d+d\right)^2-\frac{\left(c-\sqrt{\cdots}\right.}{\cdots}\right.}\cdots\right.}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-\cdots}}$$

$\downarrow$ 435

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\frac{1}{2}\left(c-\sqrt{3}d+d\right)\int \frac{1}{\sqrt{\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}+1}\left(\frac{\left(-x+\sqrt{3}+1\right)\left(c-\sqrt{3}d+d\right)^2}{-x-\sqrt{3}+1}+\left(c+\sqrt{3}d+d\right)^2\right)\sqrt{\frac{\left(-x+\cdots\right.}{\left(-x-\cdots\right.}\cdots}}\cdots\right.}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$\downarrow$ 104

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\left(c-\sqrt{3}d+d\right)\int \frac{1}{-4\sqrt{3}d(c+d)-\frac{4\left(2+\sqrt{3}\right)\left(c^2-dc+d^2\right)\sqrt{\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}}}d\frac{\sqrt{\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\cdots}}{\sqrt{\frac{\left(-x+\sqrt{3}+1\right)^2}{\left(-x-\sqrt{3}+1\right)^2}+\cdots}}\cdots\right.}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

3.150. $\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx$

$$\downarrow 218$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(-\frac{\text{EllipticPi}\left(\frac{\left(c-\sqrt{3}d+d\right)^2}{\left(c+\sqrt{3}d+d\right)^2},\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{7+4\sqrt{3}}\left(c+\sqrt{3}d+d\right)}-\frac{\left(c-\sqrt{3}d+d\right)\arctan\left(\frac{\sqrt{2+\sqrt{3}}(-}{\sqrt[4]{3}\surd}\right.}{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt{d}\sqrt{c+}}\right.}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

input
```
Int[(1 - Sqrt[3] - x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

output
```
(-4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)
^2]*(-1/4*((c + d - Sqrt[3]*d)*ArcTan[(Sqrt[2 + Sqrt[3]]*Sqrt[c^2 - c*d +
d^2]*(1 + Sqrt[3] - x))/(3^(1/4)*Sqrt[d]*Sqrt[c + d]*(1 - Sqrt[3] - x))])/
(3^(1/4)*Sqrt[2 + Sqrt[3]]*Sqrt[d]*Sqrt[c + d]*Sqrt[c^2 - c*d + d^2]) - El
lipticPi[(c + d - Sqrt[3]*d)^2/(c + d + Sqrt[3]*d)^2, ArcSin[(1 + Sqrt[3]
- x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]]/(Sqrt[7 + 4*Sqrt[3]]*(c + d + Sqr
t[3]*d))))/(Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

3.150.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

3.150. $\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx$

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2568
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(-1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
 + Sqrt[3]]*f*(1 - q*x)*(Sqrt[(1 + q*x + q^2*x^2)/(1 - Sqrt[3] - q*x)^2]/(q
*Sqrt[a + b*x^3]*Sqrt[-(1 - q*x)/(1 - Sqrt[3] - q*x)^2]))   Subst[Int[1/(((
1 + Sqrt[3])*d + c*q + ((1 - Sqrt[3])*d + c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 + 4*
Sqrt[3] + x^2]), x], x, (1 + Sqrt[3] - q*x)/(-1 + Sqrt[3] + q*x)], x]] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 - 3*Sq
rt[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.150.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.81

method	result
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3-1}}-\frac{2\left(d\sqrt{3}-c-d\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-}{}}}{}$
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3-1}}-\frac{2\left(d\sqrt{3}-c-d\right)\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-}{}}}{}$

input
```
int((1-x-3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/d*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I
*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(
1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((
3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2*(d*3^(1/2)-c-d)/d^2*(-3/2
-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/
(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/
2)/(x^3-1)^(1/2)/(1+c/d)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/
2+1/2*I*3^(1/2))/(1+c/d),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

3.150.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = \text{Timed out}$$

input
```
integrate((1-x-3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.150.6 Sympy [F]

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = -\int \frac{\sqrt{3}}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\,dx - \int \frac{x}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\,dx - \int \left(-\frac{1}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\right)dx$$

input
```
integrate((1-x-3**(1/2))/(d*x+c)/(x**3-1)**(1/2),x)
```

output
```
-Integral(sqrt(3)/(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x) - Integral(x
/(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x) - Integral(-1/(c*sqrt(x**3 -
1) + d*x*sqrt(x**3 - 1)), x)
```

3.150.7 Maxima [F]

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}(dx+c)}\,dx$$

input
```
integrate((1-x-3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

3.150.8 Giac [F]

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}(dx+c)}\,dx$$

input
```
integrate((1-x-3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

3.150.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = \text{Hanged}$$

input
```
int(-(x + 3^(1/2) - 1)/((x^3 - 1)^(1/2)*(c + d*x)),x)
```

output
```
\text{Hanged}
```

3.151 $\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx$

3.151.1 Optimal result

Integrand size = 29, antiderivative size = 362

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = -\frac{\left(c-\left(1-\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{arctanh}\left(\frac{2\sqrt{2+\sqrt{3}}\sqrt{c^2+cd+d^2}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{7+4\sqrt{3}+\frac{\left(1+\sqrt{3}+x\right)^2}{\left(1-\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
-(1+x)*arctanh(2*(c^2+c*d+d^2)^(1/2)*((-1-x)/(1+x-3^(1/2))^2)^(1/2)*(1/2*6
^(1/2)+1/2*2^(1/2))/(c-d)^(1/2)/d^(1/2)/(7+4*3^(1/2)+(1+x+3^(1/2))^2/(1+x-
3^(1/2))^2)^(1/2))*(c-d*(1-3^(1/2)))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)/(c-
d)^(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^
2)^(1/2)+4*3^(1/4)*(1+x)*EllipticPi((-1-x-3^(1/2))/(1+x-3^(1/2)),(c-d*(1-3
^(1/2)))^2/(c-d*(1+3^(1/2)))^2,2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((
x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)/(-d*3^(1/2)+c-d)/(-x^3-1)^(1/2)/((-1-x)/(1
+x-3^(1/2))^2)^(1/2)
```

3.151.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.57 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.64

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(\sqrt{3}c-\left(-3+\sqrt{\ldots}\right.\right.}{\ldots}\right.}{3d\sqrt{-1-x^3}}$$

input
```
Integrate[(1 - Sqrt[3] + x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/
(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]
 + ((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c - (-3 + Sqrt[3])*d)*Sqrt[1 - x
+ x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 + (-1)^
(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c + (-1)^(1/3)*d)))/(3*d*Sqrt[-
1 - x^3])
```

3.151.3 Rubi [A] (warning: unable to verify)

Time = 1.04 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.86, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2568, 25, 2538, 412, 435, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}(c+dx)}\,dx$$

$$\Big\downarrow \text{2568}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\int -\frac{1}{\left(c-\sqrt{3}d-d-\frac{\left(c-\left(1-\sqrt{3}\right)d\right)\left(x+\sqrt{3}+1\right)}{x-\sqrt{3}+1}\right)\sqrt{1-\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}}d\left(-\frac{x+\sqrt{3}}{x-\sqrt{3}}\right.}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$\downarrow$ 25

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\int \frac{1}{\left(c-\left(1+\sqrt{3}\right)d-\frac{\left(c-\left(1-\sqrt{3}\right)d\right)\left(x+\sqrt{3}+1\right)}{x-\sqrt{3}+1}\right)\sqrt{1-\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}}d\left(-\frac{x+\sqrt{3}+}{x-\sqrt{3}+}\right.}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$\downarrow$ 2538

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\left(c-\left(1-\sqrt{3}\right)d\right)\int -\frac{x+\sqrt{3}+1}{\left(x-\sqrt{3}+1\right)\sqrt{1-\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}\left(\left(c-\left(1+\sqrt{3}\right)d\right.\right.}\right.}{}$$

$\downarrow$ 412

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\left(c-\left(1-\sqrt{3}\right)d\right)\int -\frac{x+\sqrt{3}+1}{\left(x-\sqrt{3}+1\right)\sqrt{1-\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}\left(\left(c-\left(1+\sqrt{3}\right)d\right.\right.}\right.}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{}}$$

$\downarrow$ 435

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\frac{1}{2}\left(c-\left(1-\sqrt{3}\right)d\right)\int \frac{1}{\sqrt{\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}+1}\left(\frac{\left(x+\sqrt{3}+1\right)\left(c-\left(1-\sqrt{3}\right)d\right)^2}{x-\sqrt{3}+1}+\left(c-\left(1+\sqrt{3}\right)d\right)^2\right)\sqrt{\frac{(x+}{(x-}}}\right.}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$\downarrow$ 104

3.151. $\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\left(c-\left(1-\sqrt{3}\right)d\right)\int \frac{1}{4\sqrt{3}(c-d)d-\frac{4\left(2+\sqrt{3}\right)\left(c^2+dc+d^2\right)\sqrt{\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}}}d\frac{\sqrt{\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+}}\right.}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$\downarrow$ 221

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)\text{EllipticPi}\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt{7+4\sqrt{3}}\left(c-\sqrt{3}d-d\right)^2}-\frac{\left(c-\left(1-\sqrt{3}\right)d\right)\text{arc}}{4\sqrt[4]{3}\sqrt{}}\right.}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

input
```
Int[(1 - Sqrt[3] + x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

output
```
(-4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)
^2]*(-1/4*((c - (1 - Sqrt[3])*d)*ArcTanh[(Sqrt[2 + Sqrt[3]]*Sqrt[c^2 + c*d
 + d^2]*(1 + Sqrt[3] + x))/(3^(1/4)*Sqrt[c - d]*Sqrt[d]*(1 - Sqrt[3] + x))
])/(3^(1/4)*Sqrt[2 + Sqrt[3]]*Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]) +
 ((c - (1 + Sqrt[3])*d)*EllipticPi[(c - (1 - Sqrt[3])*d)^2/(c - (1 + Sqrt[
3])*d)^2, ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(S
qrt[7 + 4*Sqrt[3]]*(c - d - Sqrt[3]*d)^2)))/(Sqrt[-((1 + x)/(1 - Sqrt[3] +
 x)^2)]*Sqrt[-1 - x^3])
```

3.151.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2568
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(-1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
 + Sqrt[3]]*f*(1 - q*x)*(Sqrt[(1 + q*x + q^2*x^2)/(1 - Sqrt[3] - q*x)^2]/(q
*Sqrt[a + b*x^3]*Sqrt[-(1 - q*x)/(1 - Sqrt[3] - q*x)^2]))   Subst[Int[1/(((
1 + Sqrt[3])*d + c*q + ((1 - Sqrt[3])*d + c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 + 4*
Sqrt[3] + x^2]), x], x, (1 + Sqrt[3] - q*x)/(-1 + Sqrt[3] + q*x)], x]] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 - 3*Sq
rt[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

3.151. $\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx$

3.151.4 Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.73

method	result
default	$-\frac{2 i \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, F\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 d \sqrt{-x^{3}-1}}+ 2 i \left(d \sqrt{3}+c-d\right) \sqrt{3}\, \sqrt{i \left(x-\right.}$
elliptic	$-\frac{2 i \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, F\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 d \sqrt{-x^{3}-1}}+ 2 i \left(d \sqrt{3}+c-d\right) \sqrt{3}\, \sqrt{i \left(x-\right.}$

input
```
int((1+x-3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I/d*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I
*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3
/2+1/2*I*3^(1/2)))^(1/2))+2/3*I*(d*3^(1/2)+c-d)/d^2*3^(1/2)*(I*(x-1/2-1/2*
I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2
*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+c/d)*Elliptic
Pi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*
I*3^(1/2)+c/d),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

3.151.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+x-3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   catd
ef: division by zero
```

3.151.6 Sympy [F]

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{-(x+1)\,(x^2-x+1)}\,(c+dx)}\,dx$$

input
```
integrate((1+x-3**(1/2))/(d*x+c)/(-x**3-1)**(1/2),x)
```

output
```
Integral((x - sqrt(3) + 1)/(sqrt(-(x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

3.151.7 Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}(dx+c)}\,dx$$

input
```
integrate((1+x-3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

3.151.8 Giac [F]

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}(dx+c)}\,dx$$

input
```
integrate((1+x-3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

3.151.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \text{Hanged}$$

input
```
int((x - 3^(1/2) + 1)/((- x^3 - 1)^(1/2)*(c + d*x)),x)
```

output
```
\text{Hanged}
```

3.152 $\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx$

3.152.1 Optimal result

Integrand size = 21, antiderivative size = 125

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = -\frac{2}{3}\left(1+\sqrt{3}\right)\operatorname{arctanh}\left(\sqrt{1+x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
-2/3*arctanh((x^3+1)^(1/2))*(1+3^(1/2))+2/3*(1+x)*EllipticF((1+x-3^(1/2))/
(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(
1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.152.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.31

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = -\frac{2}{3}\left(1+\sqrt{3}\right)\operatorname{arctanh}\left(\sqrt{1+x^3}\right)+x\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)$$

input
```
Integrate[(1 + Sqrt[3] + x)/(x*Sqrt[1 + x^3]),x]
```

output
```
(-2*(1 + Sqrt[3])*ArcTanh[Sqrt[1 + x^3]])/3 + x*Hypergeometric2F1[1/3, 1/2
, 4/3, -x^3]
```

3.152.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2371, 759, 798, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sqrt{3}+1}{x\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{2371}$$

$$\int \frac{1}{\sqrt{x^3+1}}dx+\left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{759}$$

$$\left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{x^3+1}}dx+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\left(1+\sqrt{3}\right)\int \frac{1}{x^3\sqrt{x^3+1}}dx^3+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{73}$$

$$\frac{2}{3}\left(1+\sqrt{3}\right)\int \frac{1}{x^6-1}d\sqrt{x^3+1}+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \ 220$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2}{3}\left(1+\sqrt{3}\right)\text{arctanh}\left(\sqrt{x^3+1}\right)$$

input
```
Int[(1 + Sqrt[3] + x)/(x*Sqrt[1 + x^3]),x]
```

output
```
(-2*(1 + Sqrt[3])*ArcTanh[Sqrt[1 + x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*(1 + x)
*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x
)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3]
+ x)^2]*Sqrt[1 + x^3])
```

3.152.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

3.152.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.44 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.72

method	result
meijerg	$\frac{-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x))\sqrt{\pi}}{3\sqrt{\pi}}+x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)+\frac{\sqrt{3}\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x))\sqrt{}\right.}{3\sqrt{\pi}}$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\,\operatorname{arctanh}\left(\sqrt{x^3+1}\right)\left(1+\sqrt{3}\right)}{3}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\,\operatorname{arctanh}\left(\sqrt{x^3+1}\right)\left(1+\sqrt{3}\right)}{3}$

input
```
int((1+x+3^(1/2))/x/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/Pi^(1/2)*(-2*Pi^(1/2)*ln(1/2+1/2*(x^3+1)^(1/2))+(-2*ln(2)+3*ln(x))*Pi^
(1/2))+x*hypergeom([1/3,1/2],[4/3],-x^3)+1/3*3^(1/2)/Pi^(1/2)*(-2*Pi^(1/2)
*ln(1/2+1/2*(x^3+1)^(1/2))+(-2*ln(2)+3*ln(x))*Pi^(1/2))
```

3.152.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.26

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = \frac{1}{3}\left(\sqrt{3}+1\right)\log\left(\frac{x^3-2\sqrt{x^3+1}+2}{x^3}\right)+2\,\text{weierstrassPInverse}(0,-4,x)$$

input
```
integrate((1+x+3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(3) + 1)*log((x^3 - 2*sqrt(x^3 + 1) + 2)/x^3) + 2*weierstrassPInv
erse(0, -4, x)
```

3.152.6 Sympy [A] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.45

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{2\sqrt{3}\,\text{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} - \frac{2\,\text{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

input
```
integrate((1+x+3**(1/2))/x/(x**3+1)**(1/2),x)
```

output
```
x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3)
) - 2*sqrt(3)*asinh(x**(-3/2))/3 - 2*asinh(x**(-3/2))/3
```

3.152.7 Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}x}\,dx$$

input
```
integrate((1+x+3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

3.152.8 Giac [F]

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = \int \frac{x+\sqrt{3}+1}{\sqrt{x^3+1}x}\,dx$$

input
```
integrate((1+x+3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

3.152.9 Mupad [B] (verification not implemented)

Time = 17.62 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.67

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = -\frac{2\sqrt{3}\,\mathrm{atanh}\left(\sqrt{x^3+1}\right)}{3} + \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} - \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int((x + 3^(1/2) + 1)/(x*(x^3 + 1)^(1/2)),x)
```

output
```
(2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/
2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/
2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(
((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (2*3^(1/2)*atanh((x^3 + 1)^(1/2)))/3 - (
2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2
))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2
)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x
+ 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)
/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) -
 ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

3.153 $\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx$

3.153.1 Optimal result

Integrand size = 25, antiderivative size = 139

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = -\frac{2}{3}\left(1+\sqrt{3}\right)\operatorname{arctanh}\left(\sqrt{1-x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-2/3*arctanh((-x^3+1)^(1/2))*(1+3^(1/2))+2/3*(1-x)*EllipticF((1-x-3^(1/2))
/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^
(1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

3.153.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.29

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = -\frac{2}{3}\left(1+\sqrt{3}\right)\operatorname{arctanh}\left(\sqrt{1-x^3}\right) - x\,\operatorname{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)$$

input
```
Integrate[(1 + Sqrt[3] - x)/(x*Sqrt[1 - x^3]),x]
```

output
```
(-2*(1 + Sqrt[3])*ArcTanh[Sqrt[1 - x^3]])/3 - x*Hypergeometric2F1[1/3, 1/2
, 4/3, x^3]
```

3.153.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2371, 25, 759, 798, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x+\sqrt{3}+1}{x\sqrt{1-x^3}}dx$$

$$\Big\downarrow \ 2371$$

$$\int -\frac{1}{\sqrt{1-x^3}}dx+\left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{1-x^3}}dx$$

$$\Big\downarrow \ 25$$

$$\left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{1-x^3}}dx-\int \frac{1}{\sqrt{1-x^3}}dx$$

$$\Big\downarrow \ 759$$

$$\left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{1-x^3}}dx+\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \ 798$$

$$\frac{1}{3}\left(1+\sqrt{3}\right)\int \frac{1}{x^3\sqrt{1-x^3}}dx^3+\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \ 73$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{3}\left(1+\sqrt{3}\right)\int\frac{1}{1-x^6}d\sqrt{1-x^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{3}\left(1+\sqrt{3}\right)\operatorname{arctanh}\left(\sqrt{1-x^3}\right)$$

input
```
Int[(1 + Sqrt[3] - x)/(x*Sqrt[1 - x^3]),x]
```

output
```
(-2*(1 + Sqrt[3])*ArcTanh[Sqrt[1 - x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*(1 - x)
*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x
)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3]
- x)^2]*Sqrt[1 - x^3])
```

3.153.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.153. $\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx$

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

3.153.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.73

method	result
meijerg	$\frac{-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x)+i\pi)\sqrt{\pi}}{3\sqrt{\pi}}-x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)+\frac{\sqrt{3}\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)+(-2\ln(2)+3\ln(\right.}{3\sqrt{\pi}}$
default	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}-\frac{2\,\mathrm{arctanh}\left(\sqrt{-x^3+1}\right)\left(1+\right.}{3}$
elliptic	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}-\frac{2\,\mathrm{arctanh}\left(\sqrt{-x^3+1}\right)\left(1+\right.}{3}$

input
```
int((1-x+3^(1/2))/x/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/Pi^(1/2)*(-2*Pi^(1/2)*ln(1/2+1/2*(-x^3+1)^(1/2))+(-2*ln(2)+3*ln(x)+I*P
i)*Pi^(1/2))-x*hypergeom([1/3,1/2],[4/3],x^3)+1/3*3^(1/2)/Pi^(1/2)*(-2*Pi^
(1/2)*ln(1/2+1/2*(-x^3+1)^(1/2))+(-2*ln(2)+3*ln(x)+I*Pi)*Pi^(1/2))
```

3.153.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.26

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = \frac{1}{3}\left(\sqrt{3}+1\right)\log\left(-\frac{x^3+2\sqrt{-x^3+1}-2}{x^3}\right)+2i\,\text{weierstrassPInverse}(0,4,x)$$

input
```
integrate((1-x+3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(3) + 1)*log(-(x^3 + 2*sqrt(-x^3 + 1) - 2)/x^3) + 2*I*weierstrass
PInverse(0, 4, x)
```

3.153.6 Sympy [A] (verification not implemented)

Time = 3.74 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.71

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = -\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\begin{cases}-\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}+\sqrt{3}\left(\begin{cases}-\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((1-x+3**(1/2))/x/(-x**3+1)**(1/2),x)
```

output
```
-x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4
/3)) + Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (2*I*asin(x**(-
3/2))/3, True)) + sqrt(3)*Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) >
1), (2*I*asin(x**(-3/2))/3, True))
```

3.153.7 Maxima [F]

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx=\int -\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}x}\,dx$$

input
```
integrate((1-x+3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

3.153.8 Giac [F]

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx=\int -\frac{x-\sqrt{3}-1}{\sqrt{-x^3+1}x}\,dx$$

input
```
integrate((1-x+3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

3.153.9 Mupad [B] (verification not implemented)

Time = 18.95 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.68

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx=\frac{\sqrt{3}\,\ln\left(\frac{\left(\sqrt{1-x^3}-1\right)^3\left(\sqrt{1-x^3}+1\right)}{x^6}\right)}{3}+\frac{\sqrt{x^3-1}\left(\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}\cdots}{-\frac{3}{2}+\cdots}}\cdots}{\sqrt{x\cdots}}\cdots\right)}{\sqrt{1-x^3}}$$

input
```
int((3^(1/2) - x + 1)/(x*(1 - x^3)^(1/2)),x)
```

output
```
(3^(1/2)*log((((1 - x^3)^(1/2) - 1)^3*((1 - x^3)^(1/2) + 1))/x^6))/3 + ((x
^3 - 1)^(1/2)*((2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/
2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2) - (2*((3^(1/2)*1i)/2 + 3
/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1
/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(
1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(
1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)))/(1 - x^3)^(1/2)
```

3.153. $\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx$

3.154 $\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx$

3.154.1 Optimal result

Integrand size = 23, antiderivative size = 142

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \frac{2}{3}\left(1+\sqrt{3}\right)\arctan\left(\sqrt{-1+x^3}\right) + \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
2/3*arctan((x^3-1)^(1/2))*(1+3^(1/2))+2/3*(1-x)*EllipticF((1-x+3^(1/2))/(1
-x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/
2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

3.154.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.41

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \frac{2}{3}\left(1+\sqrt{3}\right)\arctan\left(\sqrt{-1+x^3}\right) - \frac{x\sqrt{1-x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)}{\sqrt{-1+x^3}}$$

input
```
Integrate[(1 + Sqrt[3] - x)/(x*Sqrt[-1 + x^3]),x]
```

output
```
(2*(1 + Sqrt[3])*ArcTan[Sqrt[-1 + x^3]])/3 - (x*Sqrt[1 - x^3]*Hypergeometr
ic2F1[1/3, 1/2, 4/3, x^3])/Sqrt[-1 + x^3]
```

3.154.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2371, 25, 760, 798, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x+\sqrt{3}+1}{x\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{2371}$$

$$\int -\frac{1}{\sqrt{x^3-1}}dx+\left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{25}$$

$$\left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{x^3-1}}dx-\int \frac{1}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$\left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{x^3-1}}dx+\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\left(1+\sqrt{3}\right)\int \frac{1}{x^3\sqrt{x^3-1}}dx^3+\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2}{3}\left(1+\sqrt{3}\right)\int \frac{1}{x^6+1}d\sqrt{x^3-1}+}{} \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+ \frac{2}{3}\left(1+\sqrt{3}\right)\arctan\left(\sqrt{x^3-1}\right)$$

input
```
Int[(1 + Sqrt[3] - x)/(x*Sqrt[-1 + x^3]),x]
```

output
```
(2*(1 + Sqrt[3])*ArcTan[Sqrt[-1 + x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)
/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3]
 - x)^2)]*Sqrt[-1 + x^3])
```

3.154.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.154. $\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx$

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

3.154.4 Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.93

method	result
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\arctan\left(\sqrt{x^3-1}\right)\left(1+\sqrt{3}\right)}{3}$
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\arctan\left(\sqrt{x^3-1}\right)\left(1+\sqrt{3}\right)}{3}$
meijerg	$\frac{\sqrt{-\operatorname{signum}(x^3-1)}\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x)+i\pi)\sqrt{\pi}\right)}{3\sqrt{\pi}\,\sqrt{\operatorname{signum}(x^3-1)}}-\frac{\sqrt{-\operatorname{signum}(x^3-1)}\,x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)}{\sqrt{\operatorname{signum}(x^3-1)}}+\sqrt{3}\,\sqrt{-\mathrm{si}$

input
```
int((1-x+3^(1/2))/x/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/
2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2/3*arctan((x^3-1)^(1/2))*(1+
3^(1/2))
```

3.154.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.37

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \frac{1}{3}\sqrt{2\sqrt{3}+4}\arctan\left(-\frac{(x^3-\sqrt{3}(x^3-2)-2)\sqrt{2\sqrt{3}+4}}{4\sqrt{x^3-1}}\right) - 2\,\text{weierstrassPInverse}(0,4,x)$$

input
```
integrate((1-x+3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(2*sqrt(3) + 4)*arctan(-1/4*(x^3 - sqrt(3)*(x^3 - 2) - 2)*sqrt(2*s
qrt(3) + 4)/sqrt(x^3 - 1)) - 2*weierstrassPInverse(0, 4, x)
```

3.154.6 Sympy [A] (verification not implemented)

Time = 3.81 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.66

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)} + \begin{cases}\frac{2i\,\text{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ -\frac{2\,\text{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases} + \sqrt{3}\left(\begin{cases}\frac{2i\,\text{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ -\frac{2\,\text{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((1-x+3**(1/2))/x/(x**3-1)**(1/2),x)
```

output
```
I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) + Piecewise(
(2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**(-3/2))/3, True)) +
 sqrt(3)*Piecewise((2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**
(-3/2))/3, True))
```

3.154.7 Maxima [F]

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{x^3-1}x}\,dx$$

input
```
integrate((1-x+3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x - sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

3.154.8 Giac [F]

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \int -\frac{x-\sqrt{3}-1}{\sqrt{x^3-1}x}\,dx$$

input
```
integrate((1-x+3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x - sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

3.154.9 Mupad [B] (verification not implemented)

Time = 18.89 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.35

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{3}\,\mathrm{atan}\left(\sqrt{x^3-1}\right)}{3} + \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}} - \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

input
```
int((3^(1/2) - x + 1)/(x*(x^3 - 1)^(1/2)),x)
```

output
```
(2*3^(1/2)*atan((x^3 - 1)^(1/2)))/3 + (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^
(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/
2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*e
llipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 +
 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1
/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2) -
 (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 -
3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x
 - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(
(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2
)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(
1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)
```

3.154. $\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx$

3.155 $\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx$

3.155.1 Optimal result

Integrand size = 23, antiderivative size = 136

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \frac{2}{3}\left(1+\sqrt{3}\right)\arctan\left(\sqrt{-1-x^3}\right) + \frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
2/3*arctan((-x^3-1)^(1/2))*(1+3^(1/2))+2/3*(1+x)*EllipticF((1+x+3^(1/2))/(
1+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1
/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

3.155.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.45

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \frac{2}{3}\left(1+\sqrt{3}\right)\arctan\left(\sqrt{-1-x^3}\right) + \frac{x\sqrt{1+x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)}{\sqrt{-1-x^3}}$$

input
```
Integrate[(1 + Sqrt[3] + x)/(x*Sqrt[-1 - x^3]),x]
```

output
```
(2*(1 + Sqrt[3])*ArcTan[Sqrt[-1 - x^3]])/3 + (x*Sqrt[1 + x^3]*Hypergeometr
ic2F1[1/3, 1/2, 4/3, -x^3])/Sqrt[-1 - x^3]
```

3.155.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2371, 760, 798, 73, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sqrt{3}+1}{x\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{ 2371}$$

$$\int \frac{1}{\sqrt{-x^3-1}}dx+\left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{ 760}$$

$$\left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{-x^3-1}}dx+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{3}\left(1+\sqrt{3}\right)\int \frac{1}{x^3\sqrt{-x^3-1}}dx^3+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2}{3}\left(1+\sqrt{3}\right)\int\frac{1}{-x^6-1}d\sqrt{-x^3-1}$$

$$\Big\downarrow \text{217}$$

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2}{3}\left(1+\sqrt{3}\right)\arctan\left(\sqrt{-x^3-1}\right)$$

input
```
Int[(1 + Sqrt[3] + x)/(x*Sqrt[-1 - x^3]),x]
```

output
```
(2*(1 + Sqrt[3])*ArcTan[Sqrt[-1 - x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)
/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3]
 + x)^2)]*Sqrt[-1 - x^3])
```

3.155.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

3.155.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.83 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.69

method	result
meijerg	$-\frac{i\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x))\sqrt{\pi}\right)}{3\sqrt{\pi}}-ix\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)-\frac{i\sqrt{3}\,\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+(-2\ln(2)+}{3\sqrt{\pi}}$
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2\arctan\left(\sqrt{-x^3-1}\right)(1+\sqrt{3}}{3}$
elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2\arctan\left(\sqrt{-x^3-1}\right)(1+\sqrt{3}}{3}$

input
```
int((1+x+3^(1/2))/x/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I/Pi^(1/2)*(-2*Pi^(1/2)*ln(1/2+1/2*(x^3+1)^(1/2))+(-2*ln(2)+3*ln(x))*
Pi^(1/2))-I*x*hypergeom([1/3,1/2],[4/3],-x^3)-1/3*I*3^(1/2)/Pi^(1/2)*(-2*P
i^(1/2)*ln(1/2+1/2*(x^3+1)^(1/2))+(-2*ln(2)+3*ln(x))*Pi^(1/2))
```

3.155.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.45

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \frac{1}{3}\sqrt{2\sqrt{3}+4}\arctan\left(-\frac{(x^3-\sqrt{3}(x^3+2)+2)\sqrt{-x^3-1}\sqrt{2\sqrt{3}+4}}{4(x^3+1)}\right) - 2i\,\mathrm{weierstrassPInverse}(0,-4,x)$$

input
```
integrate((1+x+3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(2*sqrt(3) + 4)*arctan(-1/4*(x^3 - sqrt(3)*(x^3 + 2) + 2)*sqrt(-x^
3 - 1)*sqrt(2*sqrt(3) + 4)/(x^3 + 1)) - 2*I*weierstrassPInverse(0, -4, x)
```

3.155.6 Sympy [A] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.45

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = -\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2i\,\mathrm{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} + \frac{2\sqrt{3}i\,\mathrm{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

input
```
integrate((1+x+3**(1/2))/x/(-x**3-1)**(1/2),x)
```

output
```
-I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4
/3)) + 2*I*asinh(x**(-3/2))/3 + 2*sqrt(3)*I*asinh(x**(-3/2))/3
```

3.155.7 Maxima [F]

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx=\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}x}\,dx$$

input
```
integrate((1+x+3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

3.155.8 Giac [F]

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx=\int \frac{x+\sqrt{3}+1}{\sqrt{-x^3-1}x}\,dx$$

input
```
integrate((1+x+3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

3.155.9 Mupad [B] (verification not implemented)

Time = 21.45 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.76

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx=\frac{\sqrt{3}\,\ln\left(\frac{\left(\sqrt{-x^3-1}-\mathrm{i}\right)\left(\sqrt{-x^3-1}+1\mathrm{i}\right)^3}{x^6}\right)\,1\mathrm{i}}{3}+\frac{\sqrt{x^3+1}\left(\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\cdots}{\sqrt{x^3+\left(-\left(\cdots\right.\right.}}\cdots\right)}{\sqrt{-x^3-1}}$$

input
```
int((x + 3^(1/2) + 1)/(x*(- x^3 - 1)^(1/2)),x)
```

output
```
(3^(1/2)*log((((- x^3 - 1)^(1/2) - 1i)*((- x^3 - 1)^(1/2) + 1i)^3)/x^6)*1i
)/3 + ((x^3 + 1)^(1/2)*((2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1
/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(
((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((
x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1
i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1)
 - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (2*((3^(1/2)*1i)
/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x +
1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^(1/2)*
1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^
3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/
2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)))/(- x^3 - 1)^(1/2)
```

3.155. $\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx$

3.156 $\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx$

3.156.1 Optimal result

Integrand size = 23, antiderivative size = 127

$$\begin{aligned}\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx &= -\frac{2}{3}\left(1-\sqrt{3}\right)\operatorname{arctanh}\left(\sqrt{1+x^3}\right)\\ &+\frac{2\sqrt{2+\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}\end{aligned}$$

output
```
-2/3*arctanh((x^3+1)^(1/2))*(1-3^(1/2))+2/3*(1+x)*EllipticF((1+x-3^(1/2))/
(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(
1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.156.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.32

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx=-\frac{2}{3}\left(1-\sqrt{3}\right)\operatorname{arctanh}\left(\sqrt{1+x^3}\right)+x\operatorname{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)$$

input
```
Integrate[(1 - Sqrt[3] + x)/(x*Sqrt[1 + x^3]),x]
```

output
```
(-2*(1 - Sqrt[3])*ArcTanh[Sqrt[1 + x^3]])/3 + x*Hypergeometric2F1[1/3, 1/2
, 4/3, -x^3]
```

3.156.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2371, 759, 798, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-\sqrt{3}+1}{x\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{2371}$$

$$\int \frac{1}{\sqrt{x^3+1}}dx+\left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{759}$$

$$\left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{x^3+1}}dx+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\left(1-\sqrt{3}\right)\int \frac{1}{x^3\sqrt{x^3+1}}dx^3+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2}{3}\left(1-\sqrt{3}\right)\int \frac{1}{x^6-1}d\sqrt{x^3+1}+}{}$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\downarrow$ 220

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-$$
$$\frac{2}{3}\left(1-\sqrt{3}\right)\text{arctanh}\left(\sqrt{x^3+1}\right)$$

input
```
Int[(1 - Sqrt[3] + x)/(x*Sqrt[1 + x^3]),x]
```

output
```
(-2*(1 - Sqrt[3])*ArcTanh[Sqrt[1 + x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*(1 + x)
*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x
)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3]
+ x)^2]*Sqrt[1 + x^3])
```

3.156.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

3.156. $\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx$

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

3.156.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.71

method	result
meijerg	$\frac{-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x))\sqrt{\pi}}{3\sqrt{\pi}}+x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)-\frac{\sqrt{3}\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x))\right.}{3\sqrt{\pi}}$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(\sqrt{3}-1\right)\operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{3}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\operatorname{arctanh}\left(\sqrt{x^3+1}\right)\left(1-\sqrt{3}\right)}{3}$

input
```
int((1+x-3^(1/2))/x/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/Pi^(1/2)*(-2*Pi^(1/2)*ln(1/2+1/2*(x^3+1)^(1/2))+(-2*ln(2)+3*ln(x))*Pi^
(1/2))+x*hypergeom([1/3,1/2],[4/3],-x^3)-1/3*3^(1/2)/Pi^(1/2)*(-2*Pi^(1/2)
*ln(1/2+1/2*(x^3+1)^(1/2))+(-2*ln(2)+3*ln(x))*Pi^(1/2))
```

3.156.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.26

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = \frac{1}{3}\left(\sqrt{3}-1\right)\log\left(\frac{x^3+2\sqrt{x^3+1}+2}{x^3}\right)+2\,\text{weierstrassPInverse}(0,-4,x)$$

input
```
integrate((1+x-3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(3) - 1)*log((x^3 + 2*sqrt(x^3 + 1) + 2)/x^3) + 2*weierstrassPInv
erse(0, -4, x)
```

3.156.6 Sympy [A] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.44

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = \frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{2\,\text{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} + \frac{2\sqrt{3}\,\text{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

input
```
integrate((1+x-3**(1/2))/x/(x**3+1)**(1/2),x)
```

output
```
x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3)
) - 2*asinh(x**(-3/2))/3 + 2*sqrt(3)*asinh(x**(-3/2))/3
```

3.156.7 Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}x}\,dx$$

input
```
integrate((1+x-3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

3.156.8 Giac [F]

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx=\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}x}\,dx$$

input
```
integrate((1+x-3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

3.156.9 Mupad [B] (verification not implemented)

Time = 19.81 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.63

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx=\frac{2\sqrt{3}\,\mathrm{atanh}\left(\sqrt{x^3+1}\right)}{3}+\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int((x - 3^(1/2) + 1)/(x*(x^3 + 1)^(1/2)),x)
```

output
```
(2*3^(1/2)*atanh((x^3 + 1)^(1/2)))/3 + (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^
(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2
+ 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*el
lipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3
/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i
)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (
2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2
))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2
)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x
+ 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)
/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) -
 ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

3.157 $\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx$

3.157.1 Optimal result

Integrand size = 27, antiderivative size = 141

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = -\frac{2}{3}\left(1-\sqrt{3}\right)\operatorname{arctanh}\left(\sqrt{1-x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-2/3*arctanh((-x^3+1)^(1/2))*(1-3^(1/2))+2/3*(1-x)*EllipticF((1-x-3^(1/2))
/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^
(1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

3.157.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.30

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = -\frac{2}{3}\left(1-\sqrt{3}\right)\operatorname{arctanh}\left(\sqrt{1-x^3}\right) - x\,\operatorname{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)$$

input
```
Integrate[(1 - Sqrt[3] - x)/(x*Sqrt[1 - x^3]),x]
```

output
```
(-2*(1 - Sqrt[3])*ArcTanh[Sqrt[1 - x^3]])/3 - x*Hypergeometric2F1[1/3, 1/2
, 4/3, x^3]
```

3.157.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2371, 25, 759, 798, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x-\sqrt{3}+1}{x\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{2371}$$

$$\int -\frac{1}{\sqrt{1-x^3}}dx+\left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{25}$$

$$\left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{1-x^3}}dx-\int \frac{1}{\sqrt{1-x^3}}dx$$

$$\Big\downarrow \text{759}$$

$$\left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{1-x^3}}dx+\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\left(1-\sqrt{3}\right)\int \frac{1}{x^3\sqrt{1-x^3}}dx^3+\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{3}\left(1-\sqrt{3}\right)\int\frac{1}{1-x^6}d\sqrt{1-x^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{3}\left(1-\sqrt{3}\right)\text{arctanh}\left(\sqrt{1-x^3}\right)$$

input
```
Int[(1 - Sqrt[3] - x)/(x*Sqrt[1 - x^3]),x]
```

output
```
(-2*(1 - Sqrt[3])*ArcTanh[Sqrt[1 - x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*(1 - x)
*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x
)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3]
- x)^2]*Sqrt[1 - x^3])
```

3.157.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

3.157.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.72

method	result
meijerg	$\frac{-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x)+i\pi)\sqrt{\pi}}{3\sqrt{\pi}} - x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right) - \frac{\sqrt{3}\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)+(-2\ln(2)+3\ln(\right.}{3\sqrt{\pi}}$
default	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}} + \frac{2\left(\sqrt{3}-1\right)\operatorname{arctanh}\left(\sqrt{-x^3\right.}{3}$
elliptic	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}} - \frac{2\operatorname{arctanh}\left(\sqrt{-x^3+1}\right)\left(1-\right.}{3}$

input
```
int((1-x-3^(1/2))/x/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/Pi^(1/2)*(-2*Pi^(1/2)*ln(1/2+1/2*(-x^3+1)^(1/2))+(-2*ln(2)+3*ln(x)+I*P
i)*Pi^(1/2))-x*hypergeom([1/3,1/2],[4/3],x^3)-1/3*3^(1/2)/Pi^(1/2)*(-2*Pi^
(1/2)*ln(1/2+1/2*(-x^3+1)^(1/2))+(-2*ln(2)+3*ln(x)+I*Pi)*Pi^(1/2))
```

3.157.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.26

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = \frac{1}{3}\left(\sqrt{3}-1\right)\log\left(-\frac{x^3-2\sqrt{-x^3+1}-2}{x^3}\right)+2i\,\text{weierstrassPInverse}(0,4,x)$$

input
```
integrate((1-x-3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(3) - 1)*log(-(x^3 - 2*sqrt(-x^3 + 1) - 2)/x^3) + 2*I*weierstrass
PInverse(0, 4, x)
```

3.157.6 Sympy [A] (verification not implemented)

Time = 3.74 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.70

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = -\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\right|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\sqrt{3}\left(\begin{cases}-\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}$$

input
```
integrate((1-x-3**(1/2))/x/(-x**3+1)**(1/2),x)
```

output
```
-x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4
/3)) - sqrt(3)*Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (2*I*as
in(x**(-3/2))/3, True)) + Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) >
1), (2*I*asin(x**(-3/2))/3, True))
```

3.157.7 Maxima [F]

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}x}\,dx$$

input
```
integrate((1-x-3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

3.157.8 Giac [F]

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{-x^3+1}x}\,dx$$

input
```
integrate((1-x-3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

3.157.9 Mupad [B] (verification not implemented)

Time = 20.84 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.65

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = \frac{\sqrt{3}\,\ln\left(\frac{\left(\sqrt{1-x^3}-1\right)\left(\sqrt{1-x^3}+1\right)^3}{x^6}\right)}{3} + \frac{\sqrt{x^3-1}\left(\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}} - \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}\cdots}{-\frac{3}{2}+\cdots}}\cdots}{\sqrt{x\cdots}}\cdots}{\sqrt{1-x^3}}$$

input
```
int(-(x + 3^(1/2) - 1)/(x*(1 - x^3)^(1/2)),x)
```

output
```
(3^(1/2)*log((((1 - x^3)^(1/2) - 1)*((1 - x^3)^(1/2) + 1)^3)/x^6))/3 + ((x
^3 - 1)^(1/2)*((2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/
2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2) - (2*((3^(1/2)*1i)/2 + 3
/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1
/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(
1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(
1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)))/(1 - x^3)^(1/2)
```

3.157. $\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx$

3.158 $\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx$

3.158.1 Optimal result

Integrand size = 25, antiderivative size = 144

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \frac{2}{3}\left(1-\sqrt{3}\right)\arctan\left(\sqrt{-1+x^3}\right) + \frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
2/3*arctan((x^3-1)^(1/2))*(1-3^(1/2))+2/3*(1-x)*EllipticF((1-x+3^(1/2))/(1
-x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/
2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

3.158.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.42

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \frac{2}{3}\left(1-\sqrt{3}\right)\arctan\left(\sqrt{-1+x^3}\right) - \frac{x\sqrt{1-x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)}{\sqrt{-1+x^3}}$$

input
```
Integrate[(1 - Sqrt[3] - x)/(x*Sqrt[-1 + x^3]),x]
```

output
```
(2*(1 - Sqrt[3])*ArcTan[Sqrt[-1 + x^3]])/3 - (x*Sqrt[1 - x^3]*Hypergeometr
ic2F1[1/3, 1/2, 4/3, x^3])/Sqrt[-1 + x^3]
```

3.158.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2371, 25, 760, 798, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x-\sqrt{3}+1}{x\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{2371}$$

$$\int -\frac{1}{\sqrt{x^3-1}}dx+\left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{25}$$

$$\left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{x^3-1}}dx-\int \frac{1}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$\left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{x^3-1}}dx+\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}\left(1-\sqrt{3}\right)\int \frac{1}{x^3\sqrt{x^3-1}}dx^3+\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2}{3}\left(1-\sqrt{3}\right)\int \frac{1}{x^6+1}d\sqrt{x^3-1}+}{}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \; 216$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+$$
$$\frac{2}{3}\left(1-\sqrt{3}\right)\arctan\left(\sqrt{x^3-1}\right)$$

input
```
Int[(1 - Sqrt[3] - x)/(x*Sqrt[-1 + x^3]),x]
```

output
```
(2*(1 - Sqrt[3])*ArcTan[Sqrt[-1 + x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)
/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3]
 - x)^2)]*Sqrt[-1 + x^3])
```

3.158.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.158. $\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx$

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

3.158.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.92

method	result
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}-1}}-\frac{2\left(\sqrt{3}-1\right)\arctan\left(\sqrt{x^{3}-1}\right)}{3}$
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}-1}}+\frac{2\arctan\left(\sqrt{x^{3}-1}\right)\left(1-\sqrt{3}\right)}{3}$
meijerg	$\frac{\sqrt{-\operatorname{signum}\left(x^{3}-1\right)}\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^{3}+1}}{2}\right)+\left(-2\ln\left(2\right)+3\ln\left(x\right)+i\pi\right)\sqrt{\pi}\right)}{3\sqrt{\pi}\,\sqrt{\operatorname{signum}\left(x^{3}-1\right)}}-\frac{\sqrt{-\operatorname{signum}\left(x^{3}-1\right)}\,x\,{}_{2}F_{1}\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^{3}\right)}{\sqrt{\operatorname{signum}\left(x^{3}-1\right)}}-\sqrt{3}\,\sqrt{-\mathrm{si}}$

input
```
int((1-x-3^(1/2))/x/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/
2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2/3*(3^(1/2)-1)*arctan((x^3-1
)^(1/2))
```

3.158.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.35

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = -\frac{1}{3}\sqrt{-2\sqrt{3}+4}\arctan\left(\frac{(x^3+\sqrt{3}(x^3-2)-2)\sqrt{-2\sqrt{3}+4}}{4\sqrt{x^3-1}}\right) - 2\,\text{weierstrassPInverse}(0,4,x)$$

input
```
integrate((1-x-3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(-2*sqrt(3) + 4)*arctan(1/4*(x^3 + sqrt(3)*(x^3 - 2) - 2)*sqrt(-2
*sqrt(3) + 4)/sqrt(x^3 - 1)) - 2*weierstrassPInverse(0, 4, x)
```

3.158.6 Sympy [A] (verification not implemented)

Time = 3.83 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.65

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)} - \sqrt{3}\left(\begin{cases}\frac{2i\,\text{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ -\frac{2\,\text{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right) + \begin{cases}\frac{2i\,\text{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ -\frac{2\,\text{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}$$

input
```
integrate((1-x-3**(1/2))/x/(x**3-1)**(1/2),x)
```

output
```
I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) - sqrt(3)*Pi
ecewise((2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**(-3/2))/3,
True)) + Piecewise((2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**
(-3/2))/3, True))
```

3.158.7 Maxima [F]

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}x}\,dx$$

input
```
integrate((1-x-3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x + sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

3.158.8 Giac [F]

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \int -\frac{x+\sqrt{3}-1}{\sqrt{x^3-1}x}\,dx$$

input
```
integrate((1-x-3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x + sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

3.158.9 Mupad [B] (verification not implemented)

Time = 20.25 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.32

$$\begin{aligned}\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx &= -\frac{2\sqrt{3}\,\mathrm{atan}\left(\sqrt{x^3-1}\right)}{3}\\
&+\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}\\
&-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}\end{aligned}$$

input
```
int(-(x + 3^(1/2) - 1)/(x*(x^3 - 1)^(1/2)),x)
```

output
```
(2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3
/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x
- 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1
/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2) - (2*3^(1/2)*atan((x^3 - 1)^(1/2)))/3 -
 (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 -
3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x
 - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(
(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2
)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(
1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)
```

3.158. $\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx$

3.159 $\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx$

3.159.1 Optimal result

Integrand size = 25, antiderivative size = 138

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \frac{2}{3}\left(1-\sqrt{3}\right)\arctan\left(\sqrt{-1-x^3}\right) + \frac{2\sqrt{2-\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
2/3*arctan((-x^3-1)^(1/2))*(1-3^(1/2))+2/3*(1+x)*EllipticF((1+x+3^(1/2))/(
1+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1
/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

3.159.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.46

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \frac{2}{3}\left(1-\sqrt{3}\right)\arctan\left(\sqrt{-1-x^3}\right) + \frac{x\sqrt{1+x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)}{\sqrt{-1-x^3}}$$

input
```
Integrate[(1 - Sqrt[3] + x)/(x*Sqrt[-1 - x^3]),x]
```

output
```
(2*(1 - Sqrt[3])*ArcTan[Sqrt[-1 - x^3]])/3 + (x*Sqrt[1 + x^3]*Hypergeometr
ic2F1[1/3, 1/2, 4/3, -x^3])/Sqrt[-1 - x^3]
```

3.159.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2371, 760, 798, 73, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-\sqrt{3}+1}{x\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{ 2371}$$

$$\int \frac{1}{\sqrt{-x^3-1}}dx+\left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{ 760}$$

$$\left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{-x^3-1}}dx+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{3}\left(1-\sqrt{3}\right)\int \frac{1}{x^3\sqrt{-x^3-1}}dx^3+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2}{3}\left(1-\sqrt{3}\right)\int\frac{1}{-x^6-1}d\sqrt{-x^3-1}$$

$$\Big\downarrow \text{217}$$

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2}{3}\left(1-\sqrt{3}\right)\arctan\left(\sqrt{-x^3-1}\right)$$

input
```
Int[(1 - Sqrt[3] + x)/(x*Sqrt[-1 - x^3]),x]
```

output
```
(2*(1 - Sqrt[3])*ArcTan[Sqrt[-1 - x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)
/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3]
 + x)^2)]*Sqrt[-1 - x^3])
```

3.159.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

3.159.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.68

method	result
meijerg	$-\frac{i\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x))\sqrt{\pi}\right)}{3\sqrt{\pi}}-ix\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)+\frac{i\sqrt{3}\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+(-2\ln(2)+\cdots\right)}{3\sqrt{\pi}}$
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2\left(\sqrt{3}-1\right)\arctan\left(\sqrt{-x^3-1}\cdots\right)}{3}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2\arctan\left(\sqrt{-x^3-1}\right)\left(1-\sqrt{\cdots}\right)}{3}$

input
```
int((1+x-3^(1/2))/x/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I/Pi^(1/2)*(-2*Pi^(1/2)*ln(1/2+1/2*(x^3+1)^(1/2))+(-2*ln(2)+3*ln(x))*
Pi^(1/2))-I*x*hypergeom([1/3,1/2],[4/3],-x^3)+1/3*I*3^(1/2)/Pi^(1/2)*(-2*P
i^(1/2)*ln(1/2+1/2*(x^3+1)^(1/2))+(-2*ln(2)+3*ln(x))*Pi^(1/2))
```

3.159.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.43

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = -\frac{1}{3}\sqrt{-2\sqrt{3}+4}\arctan\left(\frac{\left(x^3+\sqrt{3}(x^3+2)+2\right)\sqrt{-x^3-1}\sqrt{-2\sqrt{3}+4}}{4\,(x^3+1)}\right) - 2i\,\text{weierstrassPInverse}(0,-4,x)$$

input
```
integrate((1+x-3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(-2*sqrt(3) + 4)*arctan(1/4*(x^3 + sqrt(3)*(x^3 + 2) + 2)*sqrt(-x
^3 - 1)*sqrt(-2*sqrt(3) + 4)/(x^3 + 1)) - 2*I*weierstrassPInverse(0, -4, x
)
```

3.159.6 Sympy [A] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.44

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = -\frac{ix\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{2\sqrt{3}i\,\text{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} + \frac{2i\,\text{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

input
```
integrate((1+x-3**(1/2))/x/(-x**3-1)**(1/2),x)
```

output
```
-I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4
/3)) - 2*sqrt(3)*I*asinh(x**(-3/2))/3 + 2*I*asinh(x**(-3/2))/3
```

3.159.7 Maxima [F]

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}x}\,dx$$

input
```
integrate((1+x-3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

3.159.8 Giac [F]

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \int \frac{x-\sqrt{3}+1}{\sqrt{-x^3-1}x}\,dx$$

input
```
integrate((1+x-3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

3.159.9 Mupad [B] (verification not implemented)

Time = 21.68 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.72

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \frac{\sqrt{3}\,\ln\left(\frac{\left(\sqrt{-x^3-1}-\mathrm{i}\right)^3\left(\sqrt{-x^3-1}+1\mathrm{i}\right)}{x^6}\right)\,1\mathrm{i}}{3} + \frac{\sqrt{x^3+1}\left(\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} - \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\cdots}{\sqrt{x^3+\left(-\left(\cdots\right.\right.}}\cdots\right)}{\sqrt{-x^3-1}}$$

input
```
int((x - 3^(1/2) + 1)/(x*(- x^3 - 1)^(1/2)),x)
```

output
```
(3^(1/2)*log((((- x^3 - 1)^(1/2) - 1i)^3*((- x^3 - 1)^(1/2) + 1i))/x^6)*1i
)/3 + ((x^3 + 1)^(1/2)*((2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1
/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(
((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((
x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1
i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1)
 - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (2*((3^(1/2)*1i)
/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x +
1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^(1/2)*
1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^
3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/
2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)))/(- x^3 - 1)^(1/2)
```

3.159. $\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx$

3.160 $\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx$

3.160.1 Optimal result

Integrand size = 16, antiderivative size = 332

$$\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx = -\frac{3(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\arctan\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{2\sqrt{2\left(97+56\sqrt{3}\right)}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{12\sqrt[4]{3}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticPi}\left(97-56\sqrt{3},\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
-3/26*(1+x)*arctan(1/2*26^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/((x^2-x+1)/(
1+x+3^(1/2))^2)^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*26^(1/2)/(x^3+1)^
(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-12*3^(1/4)*(1+x)*EllipticPi((-1-x+3^(1
/2))/(1+x+3^(1/2)),97-56*3^(1/2),I*3^(1/2)+2*I)*((x^2-x+1)/(1+x+3^(1/2))^2
)^(1/2)/(x^3+1)^(1/2)/(1/2*6^(1/2)-1/2*2^(1/2))/((1+x)/(1+x+3^(1/2))^2)^(1
/2)-2/3*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*((x^2-x
+1)/(1+x+3^(1/2))^2)^(1/2)*(7*2^(1/2)+4*6^(1/2))*3^(3/4)/(x^3+1)^(1/2)/((1
+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.160.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.23 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.58

$$\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx = \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{3i\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{i\sqrt{3}}{3+\sqrt[3]{-1}},\text{ar}\ldots\right.}{3+\sqrt[3]{-\ldots}}\right.}{\sqrt{1+x^3}}$$

input
```
Integrate[x/((3 + x)*Sqrt[1 + x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))])
 + ((3*I)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/(3 + (-1)^(1/3)), ArcSi
n[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(3 + (-1)^(1/3)
)))/Sqrt[1 + x^3]
```

3.160.3 Rubi [A] (warning: unable to verify)

Time = 0.94 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {2569, 759, 2567, 25, 2538, 412, 435, 104, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+3)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{ 2569}$$

$$\frac{3\int \frac{x+\sqrt{3}+1}{(x+3)\sqrt{x^3+1}}dx}{2-\sqrt{3}}-\frac{\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx}{2-\sqrt{3}}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{3\int \frac{x+\sqrt{3}+1}{(x+3)\sqrt{x^3+1}}dx}{2-\sqrt{3}}-\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{ 2567}$$

$$\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\int -\frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(2-\sqrt{3}\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}+\sqrt{3}+2\right)}d\left(-\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\int \frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(2-\sqrt{3}\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}+\sqrt{3}+2\right)}d\left(-\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\downarrow$ 2538

$$\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(2-\sqrt{3}\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(7-4\sqrt{3}\right)\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}+4\sqrt{3}+7\right)}d\left(\right.\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)}}}$$

$$\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\downarrow$ 412

$$\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(2-\sqrt{3}\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(7-4\sqrt{3}\right)\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}+4\sqrt{3}+7\right)}d\left(\right.\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3}}$$

$$\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\downarrow$ 435

3.160. $\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx$

$$\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\frac{1}{2}\left(2-\sqrt{3}\right)\int\frac{1}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}\left(\frac{\left(7-4\sqrt{3}\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}+4\sqrt{3}+7\right)}d\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}+\sqrt{}\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\downarrow$ 104

$$\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(2-\sqrt{3}\right)\int\frac{1}{-\frac{52\left(2-\sqrt{3}\right)\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}-8\sqrt{3}}d\frac{\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}+\sqrt{7-4\sqrt{3}}\left(2+\sqrt{3}\right)\text{Ellipt}\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$\downarrow$ 217

$$\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\sqrt{7-4\sqrt{3}}\left(2+\sqrt{3}\right)\text{EllipticPi}\left(97-56\sqrt{3},\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)+\frac{\sqrt{\frac{1}{26}\left(2-\sqrt{}\right.}}{}\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(2-\sqrt{3}\right)\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

input
```
Int[x/((3 + x)*Sqrt[1 + x^3]),x]
```

3.160. $\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx$

output
```
(-2*(1 + Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3
] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sq
rt[3]])/(3^(1/4)*(2 - Sqrt[3])*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 +
x^3]) + (12*3^(1/4)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*((Sqrt
[(2 - Sqrt[3])/26]*ArcTan[(Sqrt[(13*(2 - Sqrt[3]))/2]*(1 - Sqrt[3] + x))/(
3^(1/4)*(1 + Sqrt[3] + x))])/(4*3^(1/4)) + Sqrt[7 - 4*Sqrt[3]]*(2 + Sqrt[3
])*EllipticPi[97 - 56*Sqrt[3], ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)]
, -7 - 4*Sqrt[3]]))/(Sqrt[2 - Sqrt[3]]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*S
qrt[1 + x^3])
```

3.160.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2569
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Simp[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/((1 + Sqrt[
3])*d - c*q)   Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

3.160.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.72

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{3\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}}{\sqrt{x^3+1}}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{3\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}}{\sqrt{x^3+1}}$

input
```
int(x/(3+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2
+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-3*(3/2-1/2*I*3^(1/2))*((x+1)/
(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1
/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*Ellip
ticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),-3/4+1/4*I*3^(1/2),((-3/2+1/2*I*3^
(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

3.160.5 Fricas [F]

$$\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^3 + 1)*x/(x^4 + 3*x^3 + x + 3), x)
```

3.160.6 Sympy [F]

$$\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{(x+1)\,(x^2-x+1)}\,(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(x**3+1)**(1/2),x)
```

output
```
Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x + 3)), x)
```

3.160.7 Maxima [F]

$$\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(x^3 + 1)*(x + 3)), x)
```

3.160.8 Giac [F]

$$\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3+1}(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(x^3 + 1)*(x + 3)), x)
```

3.160.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.62

$$\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx = \frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(2\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-3\,\Pi\left(-\frac{3}{4}-\frac{\sqrt{3}\,1\mathrm{i}}{4};\mathrm{asi}\right.\right.}{2\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int(x/((x^3 + 1)^(1/2)*(x + 3)),x)
```

output
```
((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*(2*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - 3*ellipticPi(-
 (3^(1/2)*1i)/4 - 3/4, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^
(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(2*(x^3 - x*(((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/
2 + 1/2))^(1/2))
```

3.160. $\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx$

3.161 $\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx$

3.161.1 Optimal result

Integrand size = 18, antiderivative size = 377

$$\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx = \frac{3(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{arctanh}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{2\sqrt{2\left(37+20\sqrt{3}\right)}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{169}\left(553+304\sqrt{3}\right),\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
3/14*(1-x)*arctanh(1/2*7^(1/2)*((1-x)/(1-x+3^(1/2))^2)^(1/2)/((x^2+x+1)/(1
-x+3^(1/2))^2)^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*7^(1/2)/(-x^3+1)^(
1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)-12/13*3^(1/4)*(1-x)*EllipticPi((-1+x+3^
(1/2))/(1-x+3^(1/2)),553/169+304/169*3^(1/2),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1
/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(-x^3+1)^(1/2)/((1-x)/(1-x+3
^(1/2))^2)^(1/2)-2/39*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2
)+2*I)*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*(5*2^(1/2)+2*6^(1/2))*3^(3/4)/(-x
^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

3.161.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.22 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.52

$$\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx = \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{3i\sqrt{1+x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{5i+\sqrt{3}},\arcsin\left(\right.\right.}{-3+\sqrt[3]{-1}}\right.}{\sqrt{1-x^3}}$$

input
```
Integrate[x/((3 + x)*Sqrt[1 - x^3]),x]
```

output
```
(2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-
1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((3*I)*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(5*I + Sqrt[3]), ArcSin[Sq
rt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-3 + (-1)^(1/3))))
/Sqrt[1 - x^3]
```

3.161.3 Rubi [A] (warning: unable to verify)

Time = 0.99 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2569, 759, 2567, 2538, 412, 435, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+3)\sqrt{1-x^3}}\,dx$$

$$\Big\downarrow \text{ 2569}$$

$$\frac{\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{1-x^3}}dx}{4+\sqrt{3}}-\frac{3\int \frac{-x+\sqrt{3}+1}{(x+3)\sqrt{1-x^3}}dx}{4+\sqrt{3}}$$

$$\Big\downarrow \text{ 759}$$

$$-\frac{3\int \frac{-x+\sqrt{3}+1}{(x+3)\sqrt{1-x^3}}dx}{4+\sqrt{3}}-\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$\big\downarrow$ 2567

$$-\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(4+\sqrt{3}\right)\left(-x-\sqrt{3}+1\right)}{-x+\sqrt{3}+1}-\sqrt{3}+4\right)}d\left(-\frac{-x-\sqrt{3}}{-x+\sqrt{3}}\right.}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$\big\downarrow$ 2538

$$-\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\left(4-\sqrt{3}\right)\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(4+\sqrt{3}\right)^2\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-8\sqrt{3}+\right.}\right.}{\left(4+\sqrt{3}\right.}$$

$$\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$\big\downarrow$ 412

$$-\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\left(4+\sqrt{3}\right)\int -\frac{-x-\sqrt{3}+1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(4+\sqrt{3}\right)^2\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-8\right.}\right.}{\left(4+\right.}$$

$$\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

3.161. $\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx$

$$\Big\downarrow \ 435$$

$$-\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\frac{1}{2}\left(4+\sqrt{3}\right)\int \frac{1}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}\left(\frac{\left(4+\sqrt{3}\right)^2\left(-x-\sqrt{3}+1\right)}{-x+\sqrt{3}+1}-8\sqrt{3}+19\right.}\right.}{\left(4+\sqrt{3}\right)\sqrt{\left(-\right.}}$$

$$\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \ 104$$

$$-\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\left(4+\sqrt{3}\right)\int \frac{1}{16\sqrt{3}-\frac{28\left(2-\sqrt{3}\right)\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}}d\frac{\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}-\frac{1}{169}\left(4-\right.\right.}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-}}$$

$$\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \ 219$$

$$-\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\frac{\left(4+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{\sqrt{7\left(2-\sqrt{3}\right)}\left(-x-\sqrt{3}+1\right)}{2\sqrt[4]{3}\left(-x+\sqrt{3}+1\right)}\right)}{8\sqrt[4]{3}\sqrt{7\left(2-\sqrt{3}\right)}}-\frac{1}{169}\left(4-\sqrt{3}\right)\sqrt{7519+4340\sqrt{3}}\,\mathrm{El}\right.}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\frac{2\left(1+\sqrt{3}\right)\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

3.161. $\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx$

input
```
Int[x/((3 + x)*Sqrt[1 - x^3]),x]
```

output
```
(-2*(1 + Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3
] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sq
rt[3]])/(3^(1/4)*(4 + Sqrt[3])*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 -
x^3]) - (12*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt
[3] - x)^2]*(((4 + Sqrt[3])*ArcTanh[(Sqrt[7*(2 - Sqrt[3])]*(1 - Sqrt[3] -
x))/(2*3^(1/4)*(1 + Sqrt[3] - x))])/(8*3^(1/4)*Sqrt[7*(2 - Sqrt[3])]) - ((
4 - Sqrt[3])*Sqrt[7519 + 4340*Sqrt[3]]*EllipticPi[(553 + 304*Sqrt[3])/169,
 ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/169))/((4 +
 Sqrt[3])*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

3.161.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2569
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Simp[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/((1 + Sqrt[
3])*d - c*q)   Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

3.161. $\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx$

3.161.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.64

method	result
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}}+\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{}}}{}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}+1}}+\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{}}}{}$

input
```
int(x/(3+x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3
/2+1/2*I*3^(1/2)))^(1/2))+2*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1
/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^
(1/2)/(-x^3+1)^(1/2)/(5/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-
1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(5/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3
/2+1/2*I*3^(1/2)))^(1/2))
```

3.161.5 Fricas [F]

$$\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3+1}(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^3 + 1)*x/(x^4 + 3*x^3 - x - 3), x)
```

3.161.6 Sympy [F]

$$\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx = \int \frac{x}{\sqrt{-(x-1)(x^2+x+1)}\,(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(-x**3+1)**(1/2),x)
```

output
```
Integral(x/(sqrt(-(x - 1)*(x**2 + x + 1))*(x + 3)), x)
```

3.161.7 Maxima [F]

$$\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3+1}(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(-x^3 + 1)*(x + 3)), x)
```

3.161.8 Giac [F]

$$\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3+1}(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(-x^3 + 1)*(x + 3)), x)
```

3.161.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.59

$$\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx = -\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(4\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-3\,\Pi\right.}{4\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right.}}$$

input
```
int(x/((1 - x^3)^(1/2)*(x + 3)),x)
```

output
```
-((3^(1/2)*1i + 3)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*
1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(4*ellipticF(asin((-(x - 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/
2)) - 3*ellipticPi((3^(1/2)*1i)/8 + 3/8, asin((-(x - 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(4*(1 - x^
3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)
/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.162 $\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx$

3.162.1 Optimal result

Integrand size = 16, antiderivative size = 373

$$\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx = \frac{3(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{arctanh}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{2\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticPi}\left(\frac{1}{169}\left(553+304\sqrt{3}\right),\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
-2/3*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((
x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(4+3^(1/2))/(x^3-1)^(1/2)/((-1+x)/
(1-x-3^(1/2))^2)^(1/2)+3/14*(1-x)*arctanh(1/2*7^(1/2)*((1-x)/(1-x+3^(1/2))
^2)^(1/2)/((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(
1/2)*7^(1/2)/(x^3-1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)-12/13*3^(1/4)*(1-
x)*EllipticPi((-1+x+3^(1/2))/(1-x+3^(1/2)),553/169+304/169*3^(1/2),I*3^(1/
2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(x^3-1
)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

3.162.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.20 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.52

$$\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx = \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{3i\sqrt{1+x+x^2}\,\text{EllipticPi}\left(\frac{2\sqrt{3}}{5i+\sqrt{3}},\arcsin\left(\right.\right.}{-3+\sqrt[3]{-1}}\right.}{\sqrt{-1+x^3}}$$

input
```
Integrate[x/((3 + x)*Sqrt[-1 + x^3]),x]
```

output
```
(2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-
1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((3*I)*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(5*I + Sqrt[3]), ArcSin[Sq
rt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-3 + (-1)^(1/3))))
/Sqrt[-1 + x^3]
```

3.162.3 Rubi [A] (warning: unable to verify)

Time = 1.02 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2569, 760, 2567, 2538, 412, 435, 104, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+3)\sqrt{x^3-1}}\,dx$$

$$\Big\downarrow \text{2569}$$

$$\frac{\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{x^3-1}}dx}{4+\sqrt{3}}-\frac{3\int \frac{-x+\sqrt{3}+1}{(x+3)\sqrt{x^3-1}}dx}{4+\sqrt{3}}$$

$$\Big\downarrow \text{760}$$

$$-\frac{3\int \frac{-x+\sqrt{3}+1}{(x+3)\sqrt{x^3-1}}dx}{4+\sqrt{3}}-\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$\downarrow$ 2567

$$-\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(4+\sqrt{3}\right)\left(-x-\sqrt{3}+1\right)}{-x+\sqrt{3}+1}-\sqrt{3}+4\right)}d\left(-\frac{-x-\sqrt{3}}{-x+\sqrt{3}}\right.}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$
$$\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$\downarrow$ 2538

$$-\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\left(4-\sqrt{3}\right)\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(4+\sqrt{3}\right)^2\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-8\sqrt{3}+\right.}\right.}{\left(4+\sqrt{3}\right.}$$
$$\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$\downarrow$ 412

$$-\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\left(4+\sqrt{3}\right)\int -\frac{-x-\sqrt{3}+1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(4+\sqrt{3}\right)^2\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-8\right.}\right.}{\left(4+\right.}$$
$$\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

3.162. $\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx$

$\Big\downarrow$ 435

$$-\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\frac{1}{2}\left(4+\sqrt{3}\right)\int \frac{1}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}\left(\frac{\left(4+\sqrt{3}\right)^2\left(-x-\sqrt{3}+1\right)}{-x+\sqrt{3}+1}-8\sqrt{3}+19\right.}}{\left(4+\sqrt{3}\right)\sqrt{\frac{}{(-}}}$$
$$\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$\Big\downarrow$ 104

$$-\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(-\left(4+\sqrt{3}\right)\int \frac{1}{16\sqrt{3}-\frac{28\left(2-\sqrt{3}\right)\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}}d\frac{\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}-\frac{1}{169}\left(4-\right.\right.}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3}}$$
$$\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$\Big\downarrow$ 219

$$-\frac{12\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\frac{\left(4+\sqrt{3}\right)\text{arctanh}\left(\frac{\sqrt{7\left(2-\sqrt{3}\right)}\left(-x-\sqrt{3}+1\right)}{2\sqrt[4]{3}\left(-x+\sqrt{3}+1\right)}\right)}{8\sqrt[4]{3}\sqrt{7\left(2-\sqrt{3}\right)}}-\frac{1}{169}\left(4-\sqrt{3}\right)\sqrt{7519+4340\sqrt{3}}\text{El}\right.}{\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$
$$\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

3.162. $\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx$

input
```
Int[x/((3 + x)*Sqrt[-1 + x^3]),x]
```

output
```
(-2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3
] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sq
rt[3]])/(3^(1/4)*(4 + Sqrt[3])*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-
1 + x^3]) - (12*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 +
Sqrt[3] - x)^2]*(((4 + Sqrt[3])*ArcTanh[(Sqrt[7*(2 - Sqrt[3])]*(1 - Sqrt[3
] - x))/(2*3^(1/4)*(1 + Sqrt[3] - x))])/(8*3^(1/4)*Sqrt[7*(2 - Sqrt[3])])
- ((4 - Sqrt[3])*Sqrt[7519 + 4340*Sqrt[3]]*EllipticPi[(553 + 304*Sqrt[3])/
169, ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/169))/(
(4 + Sqrt[3])*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

3.162.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2569
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Simp[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/((1 + Sqrt[
3])*d - c*q)   Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

3.162.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.64

method	result
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{3\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}}{\frac{3}{2}}}}{2\sqrt{x^3}}$
elliptic	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{3\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}}{\frac{3}{2}}}}{2\sqrt{x^3}}$

input
```
int(x/(3+x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-3/2*(-3/2-1/2*I*3^(1/2))*((x-1
)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^
(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*Elli
pticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),3/8+1/8*I*3^(1/2),((3/2+1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

3.162.5 Fricas [F]

$$\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3-1}(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^3 - 1)*x/(x^4 + 3*x^3 - x - 3), x)
```

3.162.6 Sympy [F]

$$\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx = \int \frac{x}{\sqrt{(x-1)\,(x^2+x+1)}\,(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(x**3-1)**(1/2),x)
```

output
```
Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x + 3)), x)
```

3.162.7 Maxima [F]

$$\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3-1}(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(x^3 - 1)*(x + 3)), x)
```

3.162.8 Giac [F]

$$\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx = \int \frac{x}{\sqrt{x^3-1}(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(x^3 - 1)*(x + 3)), x)
```

3.162.9 Mupad [B] (verification not implemented)

Time = 19.66 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.56

$$\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx = -\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(4\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-3\,\Pi\left(\frac{3}{8}+\frac{\sqrt{3}}{8}\right.\right.}{4\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\sqrt{}\right.}}$$

input
```
int(x/((x^3 - 1)^(1/2)*(x + 3)),x)
```

output
```
-((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1
/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)*(4*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - 3*elliptic
Pi((3^(1/2)*1i)/8 + 3/8, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -(
(3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(4*(((3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
 + 1) + x^3)^(1/2))
```

3.162. $\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx$

3.163 $\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx$

3.163.1 Optimal result

Integrand size = 18, antiderivative size = 341

$$\begin{aligned}\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx &= -\frac{3(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\arctan\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} \\ &\quad - \frac{2\sqrt{14+8\sqrt{3}}(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} \\ &\quad + \frac{12\sqrt[4]{3}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticPi}\left(97-56\sqrt{3},\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}\end{aligned}$$

output
```
-3/26*(1+x)*arctan(1/2*26^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/((x^2-x+1)/(
1+x+3^(1/2))^2)^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*26^(1/2)/(-x^3-1)
^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-12*3^(1/4)*(1+x)*EllipticPi((-1-x+3^(
1/2))/(1+x+3^(1/2)),97-56*3^(1/2),I*3^(1/2)+2*I)*((x^2-x+1)/(1+x+3^(1/2))^
2)^(1/2)/(-x^3-1)^(1/2)/(1/2*6^(1/2)-1/2*2^(1/2))/((1+x)/(1+x+3^(1/2))^2)^
(1/2)-2/3*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*((x^2
-x+1)/(1+x-3^(1/2))^2)^(1/2)*(2*2^(1/2)+6^(1/2))*3^(3/4)/(-x^3-1)^(1/2)/((
-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

3.163.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.22 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.57

$$\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx = \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{3i\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{i\sqrt{3}}{3+\sqrt[3]{-1}},\text{ar}\ldots\right.}{3+\sqrt[3]{-\ldots}}\right.}{\sqrt{-1-x^3}}$$

input
```
Integrate[x/((3 + x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))])
 + ((3*I)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/(3 + (-1)^(1/3)), ArcSi
n[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(3 + (-1)^(1/3)
)))/Sqrt[-1 - x^3]
```

3.163.3 Rubi [A] (warning: unable to verify)

Time = 0.95 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2569, 760, 2567, 25, 2538, 412, 435, 104, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+3)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{2569}$$

$$\frac{3\int \frac{x+\sqrt{3}+1}{(x+3)\sqrt{-x^3-1}}dx}{2-\sqrt{3}}-\frac{\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{-x^3-1}}dx}{2-\sqrt{3}}$$

$$\Big\downarrow \text{760}$$

$$\frac{3\int \frac{x+\sqrt{3}+1}{(x+3)\sqrt{-x^3-1}}dx}{2-\sqrt{3}}-\frac{2\left(1+\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{2567}$$

$$\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\int -\frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(2-\sqrt{3}\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}+\sqrt{3}+2\right)}d\left(-\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\left(1+\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\int \frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(2-\sqrt{3}\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}+\sqrt{3}+2\right)}d\left(-\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\left(1+\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{2538}$$

$$\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(2-\sqrt{3}\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(7-4\sqrt{3}\right)\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}+4\sqrt{3}+7\right)}d\left(\right.\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}$$

$$\frac{2\left(1+\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{412}$$

$$\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\left(2-\sqrt{3}\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(-\frac{\left(7-4\sqrt{3}\right)\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}+4\sqrt{3}+7\right)}d\left(\right.\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^{}}}$$

$$\frac{2\left(1+\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{435}$$

3.163. $\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx$

$$\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\frac{1}{2}(2-\sqrt{3})\int \frac{1}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}\left(\frac{\left(7-4\sqrt{3}\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}+4\sqrt{3}+7\right)}d\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}+\sqrt{}\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\frac{2\left(1+\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$\Big\downarrow$ 104

$$\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left((2-\sqrt{3})\int \frac{1}{-\frac{52\left(2-\sqrt{3}\right)\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}-8\sqrt{3}}d\frac{\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}+\sqrt{7-4\sqrt{3}}(2+\sqrt{3})\,\text{Ellipt}\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\frac{2\left(1+\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$\Big\downarrow$ 217

$$\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\sqrt{7-4\sqrt{3}}(2+\sqrt{3})\,\text{EllipticPi}\left(97-56\sqrt{3},\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)+\frac{\sqrt{\frac{1}{26}\left(2-\sqrt{}\right.}}{}\right.}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\frac{2\left(1+\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

input
```
Int[x/((3 + x)*Sqrt[-1 - x^3]),x]
```

3.163. $\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx$

output
```
(-2*(1 + Sqrt[3])*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Elliptic
F[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*S
qrt[2 - Sqrt[3]]*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]) + (1
2*3^(1/4)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*((Sqrt[(2 - Sqrt
[3])/26]*ArcTan[(Sqrt[(13*(2 - Sqrt[3]))/2]*(1 - Sqrt[3] + x))/(3^(1/4)*(1
 + Sqrt[3] + x))])/(4*3^(1/4)) + Sqrt[7 - 4*Sqrt[3]]*(2 + Sqrt[3])*Ellipti
cPi[97 - 56*Sqrt[3], ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*S
qrt[3]]))/(Sqrt[2 - Sqrt[3]]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x
^3])
```

3.163.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

3.163. $\int \frac{x}{(3+x)\sqrt{-1-x^3}} \, dx$

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2569
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Simp[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/((1 + Sqrt[
3])*d - c*q)   Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

3.163. $\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx$

3.163.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.70

method	result
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}-1}}+\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{}}{}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^{3}-1}}+\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{}}{}$

input
```
int(x/(3+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2
+1/2*I*3^(1/2)))^(1/2))+2*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2
)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/
2)/(-x^3-1)^(1/2)/(7/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2
*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(7/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1
/2*I*3^(1/2)))^(1/2))
```

3.163.5 Fricas [F]

$$\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3-1}(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^3 - 1)*x/(x^4 + 3*x^3 + x + 3), x)
```

3.163.6 Sympy [F]

$$\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx = \int \frac{x}{\sqrt{-(x+1)(x^2-x+1)}\,(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(-x**3-1)**(1/2),x)
```

output
```
Integral(x/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 3)), x)
```

3.163.7 Maxima [F]

$$\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3-1}(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(-x^3 - 1)*(x + 3)), x)
```

3.163.8 Giac [F]

$$\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx = \int \frac{x}{\sqrt{-x^3-1}(x+3)}\,dx$$

input
```
integrate(x/(3+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(-x^3 - 1)*(x + 3)), x)
```

3.163.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.65

$$\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx$$
$$= \frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(2\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-3\,\Pi\left(-\frac{3}{4}-\right.\right.}{2\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}\right.}}$$

input
```
int(x/((- x^3 - 1)^(1/2)*(x + 3)),x)
```

output
```
((3^(1/2)*1i + 3)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i
)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2
- x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(2*ellipticF(asin(((x + 1)/((3^(1
/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))
- 3*ellipticPi(- (3^(1/2)*1i)/4 - 3/4, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(2*(- x^3 - 1
)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3
^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

3.164 $\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx$

3.164.1 Optimal result

Integrand size = 22, antiderivative size = 450

$$\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx = \frac{(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\arctan\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$
$$+\frac{2\sqrt{2+\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$
$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticPi}\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\left(c^2-2cd-2d^2\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
(-c*f+d*e)*(1+x)*arctan((c^2+c*d+d^2)^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/
(c-d)^(1/2)/d^(1/2)/((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2))*((x^2-x+1)/(1+x+3^(
1/2))^2)^(1/2)/(c-d)^(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/(x^3+1)^(1/2)/(((1+x
)/(1+x+3^(1/2))^2)^(1/2)+4*3^(1/4)*(-c*f+d*e)*(1+x)*EllipticPi((-1-x+3^(1/
2))/(1+x+3^(1/2)),(c-d*(1+3^(1/2)))^2/(c-d*(1-3^(1/2)))^2,I*3^(1/2)+2*I)*(
1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(c^2-2*c*d-2*d^
2)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)+2/3*(1+x)*EllipticF((1+x-3^
(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(e-f-f*3^(1/2))*(1/2*6^(1/2)+1/2*2^(1/
2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(-d*3^(1/2)+c-d)/(x^3+1)^(1/
2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.164.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.45 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.47

$$\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{f\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{i(-de+cf)\sqrt{1-x+x^2}\operatorname{EllipticPi}\left(-\dots\right)}{c-\dots}\right.}{d\sqrt{1+x^3}}$$

input
```
Integrate[(e + f*x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((f*((-1)^(1/3) - x)*Sqrt[((-1)^(1/3)
- (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)
/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))
]) + (I*(-(d*e) + c*f)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-1
)^(1/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]
)/(c + (-1)^(1/3)*d)))/(d*Sqrt[1 + x^3])
```

3.164.3 Rubi [A] (warning: unable to verify)

Time = 1.40 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {2569, 759, 2567, 25, 2538, 412, 435, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\sqrt{x^3+1}(c+dx)} \, dx$$

$$\Big\downarrow \text{ 2569}$$

$$\frac{\left(e-\left(1+\sqrt{3}\right) f\right) \int \frac{1}{\sqrt{x^3+1}}dx}{c-\left(1+\sqrt{3}\right) d}-\frac{(de-cf) \int \frac{x+\sqrt{3}+1}{(c+dx)\sqrt{x^3+1}}dx}{c-\left(1+\sqrt{3}\right) d}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right) f\right) \text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1+\sqrt{3}\right) d\right)}-\frac{(de-cf) \int \frac{x+\sqrt{3}+1}{(c+dx)\sqrt{x^3+1}}dx}{c-\left(1+\sqrt{3}\right) d}$$

$$\Big\downarrow \text{ 2567}$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right) f\right) \text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1+\sqrt{3}\right) d\right)}-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf) \int -\frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(c+\sqrt{3}d-d-\frac{\left(c-\sqrt{3}d-d\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}\right)}d\Bigg(}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1+\sqrt{3}\right) d\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\int \frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(c-\left(1-\sqrt{3}\right)d-\frac{\left(c-\left(1+\sqrt{3}\right)d\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}\right)}d}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1+\sqrt{3}\right)d\right)}$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1+\sqrt{3}\right)d\right)}$$

$\downarrow$ 2538

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1+\sqrt{3}\right)d\right)}-$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\left(\left(c-\left(1+\sqrt{3}\right)d\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(c-\right.\right.}\right.}{}$$

$\downarrow$ 412

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1+\sqrt{3}\right)d\right)}-$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\left(\left(c-\left(1+\sqrt{3}\right)d\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(c-\right.\right.}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\right.}$$

$\downarrow$ 435

3.164. $\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1+\sqrt{3}\right)d\right)}-$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\left(\frac{1}{2}\left(c-\left(1+\sqrt{3}\right)d\right)\int\frac{1}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}\left(\left(c-\left(1-\sqrt{3}\right)d\right)^2+\cdots\right.}\cdots\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1+\sqrt{3}\cdots\right.\right.}$$

$$\big\downarrow \; 104$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1+\sqrt{3}\right)d\right)}-$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\left(\left(c-\left(1+\sqrt{3}\right)d\right)\int\frac{1}{-4\sqrt{3}(c-d)d-\frac{4\left(2-\sqrt{3}\right)\left(c^2+dc+d^2\right)\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}}d\frac{\sqrt{\frac{x-\cdots}{x+\cdots}}}{\sqrt{\frac{\left(x-\sqrt{3}\cdots\right.}{\left(x+\sqrt{3}\cdots\right.}}}\cdots\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1+\sqrt{3}\right)d\right)}$$

$$\big\downarrow \; 218$$

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1+\sqrt{3}\right)d\right)}-$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\left(\frac{\text{EllipticPi}\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{7-4\sqrt{3}}\left(c-\left(1-\sqrt{3}\right)d\right)}+\frac{\left(c-\left(1+\sqrt{3}\right)d\right)\arctan\left(\cdots\right.}{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\cdots}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1+\sqrt{3}\right)d\right)}$$

input
```
Int[(e + f*x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

3.164. $\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx$

output
```
(2*Sqrt[2 + Sqrt[3]]*(e - (1 + Sqrt[3])*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 +
 Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
 - 4*Sqrt[3]])/(3^(1/4)*(c - (1 + Sqrt[3])*d)*Sqrt[(1 + x)/(1 + Sqrt[3] +
x)^2]*Sqrt[1 + x^3]) - (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(d*e - c*f)*(1 + x)*Sq
rt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*(((c - (1 + Sqrt[3])*d)*ArcTan[(Sqrt
[2 - Sqrt[3]]*Sqrt[c^2 + c*d + d^2]*(1 - Sqrt[3] + x))/(3^(1/4)*Sqrt[c - d
]*Sqrt[d]*(1 + Sqrt[3] + x))])/(4*3^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[c - d]*Sq
rt[d]*Sqrt[c^2 + c*d + d^2]) + EllipticPi[(c - (1 + Sqrt[3])*d)^2/(c - (1
- Sqrt[3])*d)^2, ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[
3]]/(Sqrt[7 - 4*Sqrt[3]]*(c - (1 - Sqrt[3])*d))))/((c - (1 + Sqrt[3])*d)*S
qrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

3.164.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2569
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Simp[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/((1 + Sqrt[
3])*d - c*q)   Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

3.164. $\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx$

3.164.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.61

method	result
default	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3+1}}+\frac{2(-cf+ed)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{}}{d^2\sqrt{x^3+}}$
elliptic	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3+1}}-\frac{2(cf-ed)\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{}}{d^2\sqrt{x^3+}}$

input
```
int((f*x+e)/(d*x+c)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*f/d*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*
3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^
(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((
-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(-c*f+d*e)/d^2*(3/2-1/2
*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/
(x^3+1)^(1/2)/(-1+c/d)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+
1/2*I*3^(1/2))/(-1+c/d),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

3.164.5 Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)/(d*x+c)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.164.6 Sympy [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx = \int \frac{e+fx}{\sqrt{(x+1)\left(x^2-x+1\right)}\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(x**3+1)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt((x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

3.164.7 Maxima [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx = \int \frac{fx+e}{\sqrt{x^3+1}(dx+c)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

3.164.8 Giac [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx = \int \frac{fx+e}{\sqrt{x^3+1}(dx+c)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x + e)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

3.164.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.79

$$\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx$$
$$= \frac{2f\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{d\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$
$$-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,(cf-de)\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{c}{d}-1};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{d^2\left(\frac{c}{d}-1\right)\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

input
```
int((e + f*x)/((x^3 + 1)^(1/2)*(c + d*x)),x)
```

output
```
(2*f*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 -
3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x +
1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2
 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(d*(x^3
- x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3
^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*(c*f - d*e)*((x + 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3
/2))^(1/2)*ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(c/d - 1), asin(((x + 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2
)))/(d^2*(c/d - 1)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
 + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

3.165 $\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx$

3.165.1 Optimal result

Integrand size = 24, antiderivative size = 474

$$\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx = -\frac{(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$-\frac{2\sqrt{2+\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2},\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\left(c^2+2cd-2d^2\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-(-c*f+d*e)*(1-x)*arctanh((c^2-c*d+d^2)^(1/2)*((1-x)/(1-x+3^(1/2))^2)^(1/2
)/d^(1/2)/(c+d)^(1/2)/((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2))*((x^2+x+1)/(1-x+3
^(1/2))^2)^(1/2)/d^(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/(-x^3+1)^(1/2)/((
1-x)/(1-x+3^(1/2))^2)^(1/2)+4*3^(1/4)*(-c*f+d*e)*(1-x)*EllipticPi((-1+x+3^
(1/2))/(1-x+3^(1/2)),(c+d+d*3^(1/2))^2/(c+d-d*3^(1/2))^2,I*3^(1/2)+2*I)*(1
/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(c^2+2*c*d-2*d^2
)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)-2/3*(1-x)*EllipticF((1-x-3^
(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(e+f+f*3^(1/2))*(1/2*6^(1/2)+1/2*2^(1/
2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(c+d+d*3^(1/2))/(-x^3+1)^(1/
2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

3.165.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.58 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.49

$$\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx = \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{3f\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\operatorname{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\sqrt{3}\left(1+\sqrt[3]{-1}\right)(-de+cf)\sqrt{1}}{}\right.}{3d\sqrt{1-x^3}}$$

input
```
Integrate[(e + f*x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

output
```
(2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((3*f*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3)
+ (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)
/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))
] + ((-1)^(1/3)*Sqrt[3]*(1 + (-1)^(1/3))*(-(d*e) + c*f)*Sqrt[1 + x + x^2]*
EllipticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3)*
x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-c + (-1)^(1/3)*d)))/(3*d*Sqrt[1 - x^
3])
```

3.165.3 Rubi [A] (warning: unable to verify)

Time = 1.39 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2569, 759, 2567, 2538, 412, 435, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\sqrt{1-x^3}(c+dx)}\,dx$$

$$\Big\downarrow \text{2569}$$

$$\frac{\left(e+\sqrt{3}f+f\right)\int \frac{1}{\sqrt{1-x^3}}dx}{c+\sqrt{3}d+d}+\frac{(de-cf)\int \frac{-x+\sqrt{3}+1}{(c+dx)\sqrt{1-x^3}}dx}{c+\sqrt{3}d+d}$$

$$\Big\downarrow \text{759}$$

$$\frac{(de-cf)\int \frac{-x+\sqrt{3}+1}{(c+dx)\sqrt{1-x^3}}dx}{c+\sqrt{3}d+d}-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)}$$

$$\Big\downarrow \text{2567}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(c-\sqrt{3}d+d-\frac{\left(c+\sqrt{3}d+d\right)\left(-x-\sqrt{3}+1\right)}{-x+\sqrt{3}+1}\right)}}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)}-\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)}$$

$$\Big\downarrow \text{2538}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\left(\left(c-\sqrt{3}d+d\right)\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(c-\sqrt{3}d+d\right)^2-\right.}\right.}{}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)}$$

$\downarrow$ 412

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\left(-\left(c+\sqrt{3}d+d\right)\int -\frac{-x-\sqrt{3}+1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(c-\sqrt{3}d+d\right.\right.}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\,(c}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)}$$

$\downarrow$ 435

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\left(-\frac{1}{2}\left(c+\sqrt{3}d+d\right)\int \frac{1}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}\left(\left(c-\sqrt{3}d+d\right)^2+\right.}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d\right.}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)}$$

$\downarrow$ 104

3.165. $\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\left(-\left(c+\sqrt{3}d+d\right)\int \frac{1}{4\sqrt{3}d(c+d)-\frac{4\left(2-\sqrt{3}\right)\left(c^2-dc+d^2\right)\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}}d\frac{\sqrt{\ldots}}{\sqrt{\frac{\left(-x-\ldots\right)}{\left(-x+\ldots\right)}}}\ldots\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)}$$

$$\Big\downarrow \; 221$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\left(\frac{\left(c+\sqrt{3}d+d\right)\text{arctanh}\left(\frac{\sqrt{2-\sqrt{3}}\left(-x-\sqrt{3}+1\right)\sqrt{c^2-cd+d^2}}{\sqrt[4]{3}\sqrt{d}\left(-x+\sqrt{3}+1\right)\sqrt{c+d}}\right)}{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}}-\frac{\text{EllipticPi}\left(\frac{\left(c+\sqrt{3}d+\ldots\right)}{\left(c-\sqrt{3}d+\ldots\right)}\ldots\right)}{\sqrt{7-\ldots}}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)}$$

$$\frac{2\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)}$$

input
```
Int[(e + f*x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

output
```
(-2*Sqrt[2 + Sqrt[3]]*(e + f + Sqrt[3]*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 +
Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7
- 4*Sqrt[3]])/(3^(1/4)*(c + d + Sqrt[3]*d)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^
2]*Sqrt[1 - x^3]) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(d*e - c*f)*(1 - x)*Sqrt[
(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*(((c + d + Sqrt[3]*d)*ArcTanh[(Sqrt[2 -
 Sqrt[3]]*Sqrt[c^2 - c*d + d^2]*(1 - Sqrt[3] - x))/(3^(1/4)*Sqrt[d]*Sqrt[c
 + d]*(1 + Sqrt[3] - x))])/(4*3^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[d]*Sqrt[c + d
]*Sqrt[c^2 - c*d + d^2]) - EllipticPi[(c + d + Sqrt[3]*d)^2/(c + d - Sqrt[
3]*d)^2, ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]]/(Sqr
t[7 - 4*Sqrt[3]]*(c + d - Sqrt[3]*d))))/((c + d + Sqrt[3]*d)*Sqrt[(1 - x)/
(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

3.165. $\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx$

3.165.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2569
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Simp[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/((1 + Sqrt[
3])*d - c*q)   Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

3.165.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.56

method	result
default	$-\frac{2 i f \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, F\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 d \sqrt{-x^{3}+1}}-\frac{2 i (-c f+e d) \sqrt{3}\, \sqrt{i \left(x+\right.}}{}$
elliptic	$-\frac{2 i f \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, F\left(\frac{\sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}}{3}, \sqrt{\frac{i \sqrt{3}}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\right)}{3 d \sqrt{-x^{3}+1}}+\frac{2 i (c f-e d) \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}\right.}}{}$

input
```
int((f*x+e)/(d*x+c)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*f/d*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/
2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2
)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)
/(-3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(-c*f+d*e)/d^2*3^(1/2)*(I*(x+1/2-1/2*I
*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2
*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+c/d)*Ellipti
cPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/
2*I*3^(1/2)+c/d),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

3.165.5 Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.165.6 Sympy [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx = \int \frac{e+fx}{\sqrt{-(x-1)(x^2+x+1)}\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(-x**3+1)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt(-(x - 1)*(x**2 + x + 1))*(c + d*x)), x)
```

3.165.7 Maxima [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx=\int \frac{fx+e}{\sqrt{-x^3+1}(dx+c)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

3.165.8 Giac [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx=\int \frac{fx+e}{\sqrt{-x^3+1}(dx+c)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x + e)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

3.165.9 Mupad [B] (verification not implemented)

Time = 20.11 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.82

$$\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx=$$
$$-\frac{2f\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{d\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$
$$+\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,(cf-de)\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{c}{d}+1};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle| \cdots\right.}{d^2\sqrt{1-x^3}\left(\frac{c}{d}+1\right)\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right.}}$$

input
```
int((e + f*x)/((1 - x^3)^(1/2)*(c + d*x)),x)
```

output
```
(2*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3
^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3
/2))^(1/2)*(c*f - d*e)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(
((3^(1/2)*1i)/2 + 3/2)/(c/d + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(
1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(d^2*(1 - x^3)^(1/
2)*(c/d + 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)
*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (2*f*((3^(1/2)*1
i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 -
 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(
x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i
)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(d*(1
 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2
)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.165. $\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx$

3.166 $\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx$

3.166.1 Optimal result

Integrand size = 22, antiderivative size = 475

$$\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx = -\frac{(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{arctanh}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$
$$-\frac{2\sqrt{2-\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$
$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(c+d+\sqrt{3}d\right)^2}{\left(c+d-\sqrt{3}d\right)^2},\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{(c^2+2cd-2d^2)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
-2/3*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(e+f+f*3^(
1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/
(c+d+d*3^(1/2))/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)-(-c*f+d*e)*(1
-x)*arctanh((c^2-c*d+d^2)^(1/2)*((1-x)/(1-x+3^(1/2))^2)^(1/2)/d^(1/2)/(c+d
)^(1/2)/((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/
2)/d^(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/(x^3-1)^(1/2)/((1-x)/(1-x+3^(1/
2))^2)^(1/2)+4*3^(1/4)*(-c*f+d*e)*(1-x)*EllipticPi((-1+x+3^(1/2))/(1-x+3^(
1/2)),(c+d+d*3^(1/2))^2/(c+d-d*3^(1/2))^2,I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*
2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(c^2+2*c*d-2*d^2)/(x^3-1)^(1/2)
/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

3.166.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.53 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.49

$$\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{3f\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\sqrt{3}\left(1+\sqrt[3]{-1}\right)(-de+cf)\sqrt{1}}{}\right.}{3d\sqrt{-1+x^3}}$$

input
```
Integrate[(e + f*x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

output
```
(2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((3*f*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3)
+ (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)
/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))
] + ((-1)^(1/3)*Sqrt[3]*(1 + (-1)^(1/3))*(-(d*e) + c*f)*Sqrt[1 + x + x^2]*
EllipticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3)*
x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-c + (-1)^(1/3)*d)))/(3*d*Sqrt[-1 + x
^3])
```

3.166.3 Rubi [A] (warning: unable to verify)

Time = 1.21 (sec) , antiderivative size = 463, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2569, 760, 2567, 2538, 412, 435, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\sqrt{x^3-1}(c+dx)}dx$$

$$\Big\downarrow \text{2569}$$

$$\frac{\left(e+\sqrt{3}f+f\right)\int \frac{1}{\sqrt{x^3-1}}dx}{c+\sqrt{3}d+d}+\frac{(de-cf)\int \frac{-x+\sqrt{3}+1}{(c+dx)\sqrt{x^3-1}}dx}{c+\sqrt{3}d+d}$$

$$\Big\downarrow \text{760}$$

$$\frac{(de-cf)\int \frac{-x+\sqrt{3}+1}{(c+dx)\sqrt{x^3-1}}dx}{c+\sqrt{3}d+d}-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d+d\right)}$$

$$\Big\downarrow \text{2567}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(c-\sqrt{3}d+d-\frac{\left(c+\sqrt{3}d+d\right)\left(-x-\sqrt{3}+1\right)}{-x+\sqrt{3}+1}\right)}}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d+d\right)}-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d+d\right)}$$

$$\Big\downarrow \text{2538}$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\left(\left(c-\sqrt{3}d+d\right)\int \frac{1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(c-\sqrt{3}d+d\right)^2-\right.}\right.}{}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d+d\right)}$$

$\downarrow$ 412

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\left(-\left(c+\sqrt{3}d+d\right)\int -\frac{-x-\sqrt{3}+1}{\sqrt{1-\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(c-\sqrt{3}d+d\right.\right.}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c\right.}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d+d\right)}$$

$\downarrow$ 435

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\left(-\frac{1}{2}\left(c+\sqrt{3}d+d\right)\int \frac{1}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}\left(\left(c-\sqrt{3}d+d\right)^2+\right.}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d\right.}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d+d\right)}$$

$\downarrow$ 104

3.166. $\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\left(-(c+\sqrt{3}d+d)\int \frac{1}{4\sqrt{3}d(c+d)-\frac{4\left(2-\sqrt{3}\right)\left(c^2-dc+d^2\right)\sqrt{\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(-x-\sqrt{3}+1\right)^2}{\left(-x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}}d\frac{\sqrt{\dots}}{\sqrt{\frac{\left(-x-\dots\right)}{\left(-x+\dots\right)}}}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d+d\right)}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d+d\right)}$$

$$\Big\downarrow \; 221$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\left(\frac{\left(c+\sqrt{3}d+d\right)\text{arctanh}\left(\frac{\sqrt{2-\sqrt{3}}\left(-x-\sqrt{3}+1\right)\sqrt{c^2-cd+d^2}}{\sqrt[4]{3}\sqrt{d}\left(-x+\sqrt{3}+1\right)\sqrt{c+d}}\right)}{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}}-\frac{\text{EllipticPi}\left(\frac{\left(c+\sqrt{3}d+\dots\right)}{\left(c-\sqrt{3}d+\dots\right)}\dots\right)}{\sqrt{7-\dots}}\right.}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d+d\right)}$$

$$\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d+d\right)}$$

input
```
Int[(e + f*x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

output
```
(-2*Sqrt[2 - Sqrt[3]]*(e + f + Sqrt[3]*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 -
Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7
+ 4*Sqrt[3]])/(3^(1/4)*(c + d + Sqrt[3]*d)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x
)^2)]*Sqrt[-1 + x^3]) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(d*e - c*f)*(1 - x)*S
qrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*(((c + d + Sqrt[3]*d)*ArcTanh[(Sqrt
[2 - Sqrt[3]]*Sqrt[c^2 - c*d + d^2]*(1 - Sqrt[3] - x))/(3^(1/4)*Sqrt[d]*Sq
rt[c + d]*(1 + Sqrt[3] - x))])/(4*3^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[d]*Sqrt[c
 + d]*Sqrt[c^2 - c*d + d^2]) - EllipticPi[(c + d + Sqrt[3]*d)^2/(c + d - S
qrt[3]*d)^2, ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]]/
(Sqrt[7 - 4*Sqrt[3]]*(c + d - Sqrt[3]*d))))/((c + d + Sqrt[3]*d)*Sqrt[(1 -
 x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

3.166. $\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx$

3.166.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2569
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Simp[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/((1 + Sqrt[
3])*d - c*q)   Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

3.166.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.58

method	result
default	$\frac{2 f \left(-\frac{3}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{\frac{x -1}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x +\frac{1}{2}-\frac{i \sqrt{3}}{2}}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x +\frac{1}{2}+\frac{i \sqrt{3}}{2}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, F\left(\sqrt{\frac{x -1}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}, \sqrt{\frac{\frac{3}{2}+\frac{i \sqrt{3}}{2}}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\right)}{d \sqrt{x^{3}-1}}+\frac{2 \left(-c f +e d \right) \left(-\frac{3}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{\frac{x -1}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x +\frac{1}{2}-}{\frac{3}{2}-i}}}{d^{2}}$
elliptic	$\frac{2 f \left(-\frac{3}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{\frac{x -1}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x +\frac{1}{2}-\frac{i \sqrt{3}}{2}}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x +\frac{1}{2}+\frac{i \sqrt{3}}{2}}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, F\left(\sqrt{\frac{x -1}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}, \sqrt{\frac{\frac{3}{2}+\frac{i \sqrt{3}}{2}}{\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\right)}{d \sqrt{x^{3}-1}}-\frac{2 \left(c f -e d \right) \left(-\frac{3}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{\frac{x -1}{-\frac{3}{2}-\frac{i \sqrt{3}}{2}}}\, \sqrt{\frac{x +\frac{1}{2}-i}{\frac{3}{2}-i\sqrt{3}}}}{d^{2}}$

input
```
int((f*x+e)/(d*x+c)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*f/d*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*
I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^
(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(
(3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*(-c*f+d*e)/d^2*(-3/2-1/2
*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(
x^3-1)^(1/2)/(1+c/d)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/
2*I*3^(1/2))/(1+c/d),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

3.166.5 Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)/(d*x+c)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.166.6 Sympy [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx = \int \frac{e+fx}{\sqrt{(x-1)\left(x^2+x+1\right)}\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(x**3-1)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt((x - 1)*(x**2 + x + 1))*(c + d*x)), x)
```

3.166.7 Maxima [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx = \int \frac{fx+e}{\sqrt{x^3-1}(dx+c)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

3.166.8 Giac [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx = \int \frac{fx+e}{\sqrt{x^3-1}(dx+c)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x + e)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

3.166.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.75

$$\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx$$
$$= -\frac{2f\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{d\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$
$$+\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,(cf-de)\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{c}{d}+1};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{d^2\left(\frac{c}{d}+1\right)\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int((e + f*x)/((x^3 - 1)^(1/2)*(c + d*x)),x)
```

output
```
(2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3
/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(c*f
- d*e)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2
+ 3/2)/(c/d + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2
)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(d^2*(c/d + 1)*(((3^(1/2)*1i)/2 -
1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
1/2) + 1) + x^3)^(1/2)) - (2*f*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/
2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/
2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(a
sin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^
(1/2)*1i)/2 - 3/2)))/(d*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x
*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.167 $\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx$

3.167.1 Optimal result

Integrand size = 24, antiderivative size = 463

$$\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx = \frac{(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\arctan\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$+\frac{2\sqrt{2-\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticPi}\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\left(c^2-2cd-2d^2\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
2/3*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(e-f-f*3^(1
/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(
-d*3^(1/2)+c-d)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)+(-c*f+d*e)*(
1+x)*arctan((c^2+c*d+d^2)^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/(c-d)^(1/2)/
d^(1/2)/((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/
2)/(c-d)^(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/(-x^3-1)^(1/2)/((1+x)/(1+x+3^(1
/2))^2)^(1/2)+4*3^(1/4)*(-c*f+d*e)*(1+x)*EllipticPi((-1-x+3^(1/2))/(1+x+3^
(1/2)),(c-d*(1+3^(1/2)))^2/(c-d*(1-3^(1/2)))^2,I*3^(1/2)+2*I)*(1/2*6^(1/2)
+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(c^2-2*c*d-2*d^2)/(-x^3-1)
^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.167.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.44 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.46

$$\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx$$
$$= \frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{f\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right),\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{i(-de+cf)\sqrt{1-x+x^2}\,\text{EllipticPi}\left(\frac{}{c+\cdots}\right.}{c-\cdots}\right.}{d\sqrt{-1-x^3}}$$

input
```
Integrate[(e + f*x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

output
```
(2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((f*((-1)^(1/3) - x)*Sqrt[((-1)^(1/3)
- (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)
/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))
]) + (I*(-(d*e) + c*f)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-1
)^(1/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]
)/(c + (-1)^(1/3)*d)))/(d*Sqrt[-1 - x^3])
```

3.167.3 Rubi [A] (warning: unable to verify)

Time = 1.28 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2569, 760, 2567, 25, 2538, 412, 435, 104, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{\sqrt{-x^3-1}(c+dx)}dx$$

$$\Big\downarrow \; 2569$$

$$\frac{\left(e-\left(1+\sqrt{3}\right)f\right)\int \frac{1}{\sqrt{-x^3-1}}dx}{c-\left(1+\sqrt{3}\right)d}-\frac{(de-cf)\int \frac{x+\sqrt{3}+1}{(c+dx)\sqrt{-x^3-1}}dx}{c-\left(1+\sqrt{3}\right)d}$$

$$\Big\downarrow \; 760$$

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1+\sqrt{3}\right)d\right)}-\frac{(de-cf)\int \frac{x+\sqrt{3}+1}{(c+dx)\sqrt{-x^3-1}}dx}{c-\left(1+\sqrt{3}\right)d}$$

$$\Big\downarrow \; 2567$$

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1+\sqrt{3}\right)d\right)}-$$
$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\int -\frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(c+\sqrt{3}d-d-\frac{\left(c-\sqrt{3}d-d\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}\right)}d\Big(}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1+\sqrt{3}\right)d\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\int \frac{1}{\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(c-\left(1-\sqrt{3}\right)d-\frac{\left(c-\left(1+\sqrt{3}\right)d\right)\left(x-\sqrt{3}+1\right)}{x+\sqrt{3}+1}\right)}d}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1+\sqrt{3}\right)d\right)}$$

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1+\sqrt{3}\right)d\right)}$$

$\downarrow$ 2538

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1+\sqrt{3}\right)d\right)}-$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\left(\left(c-\left(1+\sqrt{3}\right)d\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(c-\right.\right.}\right.}{}$$

$\downarrow$ 412

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1+\sqrt{3}\right)d\right)}-$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\left(\left(c-\left(1+\sqrt{3}\right)d\right)\int -\frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\sqrt{1-\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\left(\left(c-\right.\right.}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\right.}$$

$\downarrow$ 435

3.167. $\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx$

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\operatorname{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1+\sqrt{3}\right)d\right)}-$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\left(\frac{1}{2}\left(c-\left(1+\sqrt{3}\right)d\right)\int\frac{1}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}\left(\left(c-\left(1-\sqrt{3}\right)d\right)^2+\right.}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1+\sqrt{\right.\right.}}$$

$$\Big\downarrow \; 104$$

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\operatorname{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1+\sqrt{3}\right)d\right)}-$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\left(\left(c-\left(1+\sqrt{3}\right)d\right)\int\frac{1}{-4\sqrt{3}(c-d)d-\frac{4\left(2-\sqrt{3}\right)\left(c^2+dc+d^2\right)\sqrt{\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}+1}}{\sqrt{\frac{\left(x-\sqrt{3}+1\right)^2}{\left(x+\sqrt{3}+1\right)^2}-4\sqrt{3}+7}}}d\frac{\sqrt{\frac{x-}{x+}}}{\sqrt{\frac{(x-\sqrt{3}}{(x+\sqrt{3}}}}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1+\sqrt{3}\right)d\right)}$$

$$\Big\downarrow \; 218$$

$$\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\left(1+\sqrt{3}\right)f\right)\operatorname{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1+\sqrt{3}\right)d\right)}-$$

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\left(\frac{\operatorname{EllipticPi}\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2},\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt{7-4\sqrt{3}}\left(c-\left(1-\sqrt{3}\right)d\right)}+\frac{\left(c-\left(1+\sqrt{3}\right)d\right)\arctan\left(\right.}{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}}\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1+\sqrt{3}\right)d\right)}$$

input
```
Int[(e + f*x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

3.167. $\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx$

output
```
(2*Sqrt[2 - Sqrt[3]]*(e - (1 + Sqrt[3])*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 -
 Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7
 + 4*Sqrt[3]])/(3^(1/4)*(c - (1 + Sqrt[3])*d)*Sqrt[-((1 + x)/(1 - Sqrt[3]
+ x)^2)]*Sqrt[-1 - x^3]) - (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(d*e - c*f)*(1 + x
)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*(((c - (1 + Sqrt[3])*d)*ArcTan[(
Sqrt[2 - Sqrt[3]]*Sqrt[c^2 + c*d + d^2]*(1 - Sqrt[3] + x))/(3^(1/4)*Sqrt[c
 - d]*Sqrt[d]*(1 + Sqrt[3] + x))])/(4*3^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[c - d
]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]) + EllipticPi[(c - (1 + Sqrt[3])*d)^2/(c -
 (1 - Sqrt[3])*d)^2, ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*S
qrt[3]]/(Sqrt[7 - 4*Sqrt[3]]*(c - (1 - Sqrt[3])*d))))/((c - (1 + Sqrt[3])*
d)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```

3.167.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2569
```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Simp[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q)   Int[1/Sqrt[a + b*x^3], x], x] + Simp[(d*e - c*f)/((1 + Sqrt[
3])*d - c*q)   Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

3.167. $\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx$

3.167.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.57

method	result
default	$-\frac{2 i f \sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3d\sqrt{-x^{3}-1}}-\frac{2i(-cf+ed)\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}\right.}}{}$
elliptic	$-\frac{2 i f \sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3d\sqrt{-x^{3}-1}}+\frac{2i(cf-ed)\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}\right.}}{}$

input
```
int((f*x+e)/(d*x+c)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*f/d*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2
*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)
*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/
(3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(-c*f+d*e)/d^2*3^(1/2)*(I*(x-1/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*
3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+c/d)*EllipticPi(
1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3
^(1/2)+c/d),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

3.167.5 Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx=\text{Timed out}$$

input
```
integrate((f*x+e)/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.167.6 Sympy [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx = \int \frac{e+fx}{\sqrt{-(x+1)(x^2-x+1)}\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(-x**3-1)**(1/2),x)
```

output
```
Integral((e + f*x)/(sqrt(-(x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

3.167.7 Maxima [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx = \int \frac{fx+e}{\sqrt{-x^3-1}(dx+c)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

3.167.8 Giac [F]

$$\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx = \int \frac{fx+e}{\sqrt{-x^3-1}(dx+c)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x + e)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

3.167.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.84

$$\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx$$
$$= \frac{2\,f\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{d\,\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$
$$- \frac{2\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(c\,f-d\,e\right)\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{c}{d}-1};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2} \cdots}{-\frac{3}{2} \cdots}\right)}{d^2\,\sqrt{-x^3-1}\,\left(\frac{c}{d}-1\right)\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int((e + f*x)/((- x^3 - 1)^(1/2)*(c + d*x)),x)
```

output
```
(2*f*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((
3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1
/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
 3/2)))/(d*(- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i
)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) -
(2*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*(c*f - d*e)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-(
(3^(1/2)*1i)/2 + 3/2)/(c/d - 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(d^2*(- x^3 - 1)^(1/
2)*(c/d - 1)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1)
- ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

3.168 $\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx$

3.168.1 Optimal result

Integrand size = 18, antiderivative size = 120

$$\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx = -\frac{2}{3}e\,\text{arctanh}\left(\sqrt{1+x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}f(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
-2/3*e*arctanh((x^3+1)^(1/2))+2/3*f*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(
1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)
^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.168.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.28

$$\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx = -\frac{2}{3}e\,\text{arctanh}\left(\sqrt{1+x^3}\right) + fx\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)$$

input
```
Integrate[(e + f*x)/(x*Sqrt[1 + x^3]),x]
```

output
```
(-2*e*ArcTanh[Sqrt[1 + x^3]])/3 + f*x*Hypergeometric2F1[1/3, 1/2, 4/3, -x^
3]
```

3.168.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2371, 27, 759, 798, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{x\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{2371}$$

$$e\int \frac{1}{x\sqrt{x^3+1}}dx+\int \frac{f}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{27}$$

$$e\int \frac{1}{x\sqrt{x^3+1}}dx+f\int \frac{1}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{759}$$

$$e\int \frac{1}{x\sqrt{x^3+1}}dx+\frac{2\sqrt{2+\sqrt{3}}f(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}e\int \frac{1}{x^3\sqrt{x^3+1}}dx^3+\frac{2\sqrt{2+\sqrt{3}}f(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\Big\downarrow \text{73}$$

$$\frac{2}{3}e\int \frac{1}{x^6-1}d\sqrt{x^3+1}+\frac{2\sqrt{2+\sqrt{3}}f(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

$$\downarrow 220$$

$$\frac{2\sqrt{2+\sqrt{3}}f(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2}{3}e\text{arctanh}\left(\sqrt{x^3+1}\right)$$

input
```
Int[(e + f*x)/(x*Sqrt[1 + x^3]),x]
```

output
```
(-2*e*ArcTanh[Sqrt[1 + x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*f*(1 + x)*Sqrt[(1 -
 x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqr
t[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sq
rt[1 + x^3])
```

3.168.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

3.168.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.98 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.44

method	result	size
meijerg	$f x {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)+\frac{e\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+\left(-2\ln\left(2\right)+3\ln\left(x\right)\right)\sqrt{\pi}\right)}{3\sqrt{\pi}}$	53
default	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2e\,\operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{3}$	129
elliptic	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2e\,\operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{3}$	129

input
```
int((f*x+e)/x/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
f*x*hypergeom([1/3,1/2],[4/3],-x^3)+1/3*e/Pi^(1/2)*(-2*Pi^(1/2)*ln(1/2+1/2
*(x^3+1)^(1/2))+(-2*ln(2)+3*ln(x))*Pi^(1/2))
```

3.168.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.25

$$\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx = \frac{1}{3}\,e\log\left(\frac{x^3-2\sqrt{x^3+1}+2}{x^3}\right) + 2\,f\text{weierstrassPInverse}(0,-4,x)$$

input
```
integrate((f*x+e)/x/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*e*log((x^3 - 2*sqrt(x^3 + 1) + 2)/x^3) + 2*f*weierstrassPInverse(0, -4
, x)
```

3.168.6 Sympy [A] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.35

$$\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx = -\frac{2e\,\text{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} + \frac{fx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((f*x+e)/x/(x**3+1)**(1/2),x)
```

output
```
-2*e*asinh(x**(-3/2))/3 + f*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*ex
p_polar(I*pi))/(3*gamma(4/3))
```

3.168.7 Maxima [F]

$$\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx = \int \frac{fx+e}{\sqrt{x^3+1}x}\,dx$$

input
```
integrate((f*x+e)/x/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(x^3 + 1)*x), x)
```

3.168.8 Giac [F]

$$\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx=\int \frac{fx+e}{\sqrt{x^3+1}x}\,dx$$

input
```
integrate((f*x+e)/x/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x + e)/(sqrt(x^3 + 1)*x), x)
```

3.168.9 Mupad [B] (verification not implemented)

Time = 20.50 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.72

$$\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx = \frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(f\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-e\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}}{2}}\right.\right.}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

input
```
int((e + f*x)/(x*(x^3 + 1)^(1/2)),x)
```

output
```
((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*(f*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i
)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - e*ellipticPi((3^(1/2)*1i)/2 + 3/2, as
in(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1
/2)*1i)/2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2
 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1
/2))^(1/2)
```

3.169 $\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx$

3.169.1 Optimal result

Integrand size = 20, antiderivative size = 134

$$\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx = -\frac{2}{3}e\,\text{arctanh}\left(\sqrt{1-x^3}\right) - \frac{2\sqrt{2+\sqrt{3}}f(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

output
```
-2/3*e*arctanh((-x^3+1)^(1/2))-2/3*f*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^
(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2
)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

3.169.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.44 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.25

$$\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx = -\frac{2}{3}e\,\text{arctanh}\left(\sqrt{1-x^3}\right) + fx\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)$$

input
```
Integrate[(e + f*x)/(x*Sqrt[1 - x^3]),x]
```

output
```
(-2*e*ArcTanh[Sqrt[1 - x^3]])/3 + f*x*Hypergeometric2F1[1/3, 1/2, 4/3, x^3
]
```

3.169.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2371, 27, 759, 798, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{x\sqrt{1-x^3}}dx$$

$$\Big\downarrow \ 2371$$

$$e\int \frac{1}{x\sqrt{1-x^3}}dx+\int \frac{f}{\sqrt{1-x^3}}dx$$

$$\Big\downarrow \ 27$$

$$e\int \frac{1}{x\sqrt{1-x^3}}dx+f\int \frac{1}{\sqrt{1-x^3}}dx$$

$$\Big\downarrow \ 759$$

$$e\int \frac{1}{x\sqrt{1-x^3}}dx-\frac{2\sqrt{2+\sqrt{3}}f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \ 798$$

$$\frac{1}{3}e\int \frac{1}{x^3\sqrt{1-x^3}}dx^3-\frac{2\sqrt{2+\sqrt{3}}f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \ 73$$

$$-\frac{2}{3}e\int \frac{1}{1-x^6}d\sqrt{1-x^3}-$$
$$\frac{2\sqrt{2+\sqrt{3}}f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\sqrt{2+\sqrt{3}}f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-$$
$$\frac{2}{3}e\text{arctanh}\left(\sqrt{1-x^3}\right)$$

input
```
Int[(e + f*x)/(x*Sqrt[1 - x^3]),x]
```

output
```
(-2*e*ArcTanh[Sqrt[1 - x^3]])/3 - (2*Sqrt[2 + Sqrt[3]]*f*(1 - x)*Sqrt[(1 +
 x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqr
t[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sq
rt[1 - x^3])
```

3.169.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

3.169.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.93 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.43

method	result
meijerg	$f x\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)+\frac{e\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x)+i\pi)\sqrt{\pi}\right)}{3\sqrt{\pi}}$
default	$-\frac{2 i f\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}-\frac{2e\,\operatorname{arctanh}\left(\sqrt{-x^3+1}\right)}{3}$
elliptic	$-\frac{2 i f\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}-\frac{2e\,\operatorname{arctanh}\left(\sqrt{-x^3+1}\right)}{3}$

input
```
int((f*x+e)/x/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
f*x*hypergeom([1/3,1/2],[4/3],x^3)+1/3*e/Pi^(1/2)*(-2*Pi^(1/2)*ln(1/2+1/2*
(-x^3+1)^(1/2))+(-2*ln(2)+3*ln(x)+I*Pi)*Pi^(1/2))
```

3.169.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.25

$$\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx = \frac{1}{3}\,e\log\left(-\frac{x^3+2\sqrt{-x^3+1}-2}{x^3}\right) - 2i\,f\text{weierstrassPInverse}(0,4,x)$$

input
```
integrate((f*x+e)/x/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*e*log(-(x^3 + 2*sqrt(-x^3 + 1) - 2)/x^3) - 2*I*f*weierstrassPInverse(0
, 4, x)
```

3.169.6 Sympy [A] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.49

$$\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx = e\left(\begin{cases} -\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|} > 1 \\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise} \end{cases}\right) + \frac{fx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((f*x+e)/x/(-x**3+1)**(1/2),x)
```

output
```
e*Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (2*I*asin(x**(-3/2))
/3, True)) + f*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*p
i))/(3*gamma(4/3))
```

3.169.7 Maxima [F]

$$\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx = \int \frac{fx+e}{\sqrt{-x^3+1}x}\,dx$$

input
```
integrate((f*x+e)/x/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(-x^3 + 1)*x), x)
```

3.169.8 Giac [F]

$$\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx = \int \frac{fx+e}{\sqrt{-x^3+1}x}\,dx$$

input
```
integrate((f*x+e)/x/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x + e)/(sqrt(-x^3 + 1)*x), x)
```

3.169.9 Mupad [B] (verification not implemented)

Time = 20.35 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.66

$$\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx = -\frac{\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\left(f\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)+e\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2};\mathrm{asin}\left(\sqrt{-\frac{}{\frac{3}{2}}}\right.\right.\right.}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

input
```
int((e + f*x)/(x*(1 - x^3)^(1/2)),x)
```

output
```
-((x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(f*ellipticF(
asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3
^(1/2)*1i)/2 - 3/2)) + e*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/(
(3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3
/2)))*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(3^(1/2) - 3i)*1i)/((1 - x^3
)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/
2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.170 $\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx$

3.170.1 Optimal result

Integrand size = 18, antiderivative size = 137

$$\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx = \frac{2}{3}e\arctan\left(\sqrt{-1+x^3}\right) - \frac{2\sqrt{2-\sqrt{3}}f(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

output
```
2/3*e*arctan((x^3-1)^(1/2))-2/3*f*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/
2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(
1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

3.170.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.38

$$\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx = \frac{2}{3}e\arctan\left(\sqrt{-1+x^3}\right) + \frac{fx\sqrt{1-x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},x^3\right)}{\sqrt{-1+x^3}}$$

input
```
Integrate[(e + f*x)/(x*Sqrt[-1 + x^3]),x]
```

output
```
(2*e*ArcTan[Sqrt[-1 + x^3]])/3 + (f*x*Sqrt[1 - x^3]*Hypergeometric2F1[1/3,
 1/2, 4/3, x^3])/Sqrt[-1 + x^3]
```

3.170.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2371, 27, 760, 798, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{x\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{2371}$$

$$e\int \frac{1}{x\sqrt{x^3-1}}dx+\int \frac{f}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{27}$$

$$e\int \frac{1}{x\sqrt{x^3-1}}dx+f\int \frac{1}{\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$e\int \frac{1}{x\sqrt{x^3-1}}dx-\frac{2\sqrt{2-\sqrt{3}}f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}e\int \frac{1}{x^3\sqrt{x^3-1}}dx^3-\frac{2\sqrt{2-\sqrt{3}}f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2}{3}e\int \frac{1}{x^6+1}d\sqrt{x^3-1}-2\sqrt{2-\sqrt{3}}f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{2}{3}e\arctan\left(\sqrt{x^3-1}\right)-2\sqrt{2-\sqrt{3}}f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

input
```
Int[(e + f*x)/(x*Sqrt[-1 + x^3]),x]
```

output
```
(2*e*ArcTan[Sqrt[-1 + x^3]])/3 - (2*Sqrt[2 - Sqrt[3]]*f*(1 - x)*Sqrt[(1 +
x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt
[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*
Sqrt[-1 + x^3])
```

3.170.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

3.170.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.97 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.68

method	result	size
meijerg	$\frac{f\sqrt{-\operatorname{signum}(x^3-1)}\, x\, {}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};x^3\right)}{\sqrt{\operatorname{signum}(x^3-1)}}+\frac{e\sqrt{-\operatorname{signum}(x^3-1)}\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x)+i\pi)\sqrt{\pi}\right)}{3\sqrt{\pi}\,\sqrt{\operatorname{signum}(x^3-1)}}$	93
default	$\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2e\arctan\left(\sqrt{x^3-1}\right)}{3}$	129
elliptic	$\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,F\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2e\arctan\left(\sqrt{x^3-1}\right)}{3}$	129

input
```
int((f*x+e)/x/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
f/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*x*hypergeom([1/3,1/2],[4/3],x
^3)+1/3*e/Pi^(1/2)/signum(x^3-1)^(1/2)*(-signum(x^3-1))^(1/2)*(-2*Pi^(1/2)
*ln(1/2+1/2*(-x^3+1)^(1/2))+(-2*ln(2)+3*ln(x)+I*Pi)*Pi^(1/2))
```

3.170.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.19

$$\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx = \frac{1}{3}\,e\arctan\left(\frac{x^3-2}{2\sqrt{x^3-1}}\right) + 2\,f\text{weierstrassPInverse}(0,4,x)$$

input
```
integrate((f*x+e)/x/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*e*arctan(1/2*(x^3 - 2)/sqrt(x^3 - 1)) + 2*f*weierstrassPInverse(0, 4,
x)
```

3.170.6 Sympy [A] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.44

$$\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx = e\left(\begin{cases} \frac{2i\,\text{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1 \\ -\frac{2\,\text{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise} \end{cases}\right) - \frac{ifx\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((f*x+e)/x/(x**3-1)**(1/2),x)
```

output
```
e*Piecewise((2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**(-3/2))
/3, True)) - I*f*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3
))
```

3.170.7 Maxima [F]

$$\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx = \int \frac{fx+e}{\sqrt{x^3-1}x}\,dx$$

input
```
integrate((f*x+e)/x/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(x^3 - 1)*x), x)
```

3.170.8 Giac [F]

$$\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx = \int \frac{fx+e}{\sqrt{x^3-1}x}\,dx$$

input
```
integrate((f*x+e)/x/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x + e)/(sqrt(x^3 - 1)*x), x)
```

3.170.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.51

$$\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx = -\frac{\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\left(f\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)+e\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\right.\right.}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\sqrt{}\right.}}$$

input
```
int((e + f*x)/(x*(x^3 - 1)^(1/2)),x)
```

output
```
-((-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2
)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(f*ellipticF(asin((-(x - 1)/(
(3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3
/2)) + e*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*(-(x - 1)/
((3^(1/2)*1i)/2 + 3/2))^(1/2)*(3^(1/2) - 3i)*1i)/(((3^(1/2)*1i)/2 - 1/2)*(
(3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) +
 1) + x^3)^(1/2)
```

3.171 $\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx$

3.171.1 Optimal result

Integrand size = 20, antiderivative size = 131

$$\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx = \frac{2}{3}e\arctan\left(\sqrt{-1-x^3}\right) + \frac{2\sqrt{2-\sqrt{3}}f(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

output
```
2/3*e*arctan((-x^3-1)^(1/2))+2/3*f*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1
/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^
(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

3.171.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.43

$$\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx = \frac{2}{3}e\arctan\left(\sqrt{-1-x^3}\right) + \frac{fx\sqrt{1+x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-x^3\right)}{\sqrt{-1-x^3}}$$

input
```
Integrate[(e + f*x)/(x*Sqrt[-1 - x^3]),x]
```

output
```
(2*e*ArcTan[Sqrt[-1 - x^3]])/3 + (f*x*Sqrt[1 + x^3]*Hypergeometric2F1[1/3,
 1/2, 4/3, -x^3])/Sqrt[-1 - x^3]
```

3.171.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2371, 27, 760, 798, 73, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{x\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{2371}$$

$$e\int \frac{1}{x\sqrt{-x^3-1}}dx+\int \frac{f}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{27}$$

$$e\int \frac{1}{x\sqrt{-x^3-1}}dx+f\int \frac{1}{\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{760}$$

$$e\int \frac{1}{x\sqrt{-x^3-1}}dx+\frac{2\sqrt{2-\sqrt{3}}f(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}e\int \frac{1}{x^3\sqrt{-x^3-1}}dx^3+\frac{2\sqrt{2-\sqrt{3}}f(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\sqrt{2-\sqrt{3}}f(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2}{3}e\int\frac{1}{-x^6-1}d\sqrt{-x^3-1}$$

$$\Big\downarrow \text{217}$$

$$\frac{2\sqrt{2-\sqrt{3}}f(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2}{3}e\arctan\left(\sqrt{-x^3-1}\right)$$

input
```
Int[(e + f*x)/(x*Sqrt[-1 - x^3]),x]
```

output
```
(2*e*ArcTan[Sqrt[-1 - x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*f*(1 + x)*Sqrt[(1 -
x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt
[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*
Sqrt[-1 - x^3])
```

3.171.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2371
```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Simp[Coeff[Pq,
x, 0]   Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
 x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IG
tQ[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

3.171.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.98 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.43

method	result	s
meijerg	$-ifx\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-x^3\right)-\frac{ie\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+\left(-2\ln(2)+3\ln(x)\right)\sqrt{\pi}\right)}{3\sqrt{\pi}}$	5
default	$-\frac{2if\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2e\arctan\left(\sqrt{-x^3-1}\right)}{3}$	1
elliptic	$-\frac{2if\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}+\frac{2e\arctan\left(\sqrt{-x^3-1}\right)}{3}$	1

input
```
int((f*x+e)/x/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*f*x*hypergeom([1/3,1/2],[4/3],-x^3)-1/3*I*e/Pi^(1/2)*(-2*Pi^(1/2)*ln(1/
2+1/2*(x^3+1)^(1/2))+(-2*ln(2)+3*ln(x))*Pi^(1/2))
```

3.171.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.27

$$\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx = \frac{1}{3}\,e\arctan\left(\frac{(x^3+2)\sqrt{-x^3-1}}{2\,(x^3+1)}\right) - 2i\,f\text{weierstrassPInverse}(0,-4,x)$$

input
```
integrate((f*x+e)/x/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
1/3*e*arctan(1/2*(x^3 + 2)*sqrt(-x^3 - 1)/(x^3 + 1)) - 2*I*f*weierstrassPI
nverse(0, -4, x)
```

3.171.6 Sympy [A] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.35

$$\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx = \frac{2ie\,\text{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} - \frac{ifx\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((f*x+e)/x/(-x**3-1)**(1/2),x)
```

output
```
2*I*e*asinh(x**(-3/2))/3 - I*f*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3
*exp_polar(I*pi))/(3*gamma(4/3))
```

3.171.7 Maxima [F]

$$\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx = \int \frac{fx+e}{\sqrt{-x^3-1}x}\,dx$$

input
```
integrate((f*x+e)/x/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/(sqrt(-x^3 - 1)*x), x)
```

3.171.8 Giac [F]

$$\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx = \int \frac{fx+e}{\sqrt{-x^3-1}x}\,dx$$

input
```
integrate((f*x+e)/x/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate((f*x + e)/(sqrt(-x^3 - 1)*x), x)
```

3.171.9 Mupad [B] (verification not implemented)

Time = 19.57 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.70

$$\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx$$
$$= \frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(f\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-e\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\right|\right.}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}\right.}}$$

input
```
int((e + f*x)/(x*(- x^3 - 1)^(1/2)),x)
```

output
```
((3^(1/2)*1i + 3)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i
)/2 - 3/2))^(1/2)*(f*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - e*ellipticPi((3^(1/2)
*1i)/2 + 3/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)
/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*
(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/((- x^3 - 1)^(1
/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/
2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

3.172 $\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$

3.172.1 Optimal result

Integrand size = 31, antiderivative size = 95

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1+\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{d} - \frac{\log(c+dx)}{d} + \frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{2d}$$

output
```
-ln(d*x+c)/d+3/2*ln(d*(d*x+2*c)-d*(d^3*x^3+2*c^3)^(1/3))/d-arctan(1/3*(1+2
*(d*x+2*c)/(d^3*x^3+2*c^3)^(1/3))*3^(1/2))*3^(1/2)/d
```

3.172.2 Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.67

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{2c^3+d^3x^3}}{4c+2dx+\sqrt[3]{2c^3+d^3x^3}}\right)}{d} + \frac{\log\left(-2c-dx+\sqrt[3]{2c^3+d^3x^3}\right)}{d} - \frac{\log\left(4c^2+4cdx+d^2x^2+(2c+dx)\sqrt[3]{2c^3+d^3x^3}+(2c^3+d^3x^3)^{2/3}\right)}{2d}$$

input
```
Integrate[(c - d*x)/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```

output
```
(Sqrt[3]*ArcTan[(Sqrt[3]*(2*c^3 + d^3*x^3)^(1/3))/(4*c + 2*d*x + (2*c^3 +
d^3*x^3)^(1/3))])/d + Log[-2*c - d*x + (2*c^3 + d^3*x^3)^(1/3)]/d - Log[4*
c^2 + 4*c*d*x + d^2*x^2 + (2*c + d*x)*(2*c^3 + d^3*x^3)^(1/3) + (2*c^3 + d
^3*x^3)^(2/3)]/(2*d)
```

3.172.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {2576}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}dx$$

$$\Big\downarrow \text{2576}$$

$$-\frac{\sqrt{3}\arctan\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{d}+\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{2d}-\frac{\log(c+dx)}{d}$$

input
```
Int[(c - d*x)/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```

output
```
-((Sqrt[3]*ArcTan[(1 + (2*(2*c + d*x))/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/
d) - Log[c + d*x]/d + (3*Log[d*(2*c + d*x) - d*(2*c^3 + d^3*x^3)^(1/3)])/(
2*d)
```

3.172.3.1 Defintions of rubi rules used

rule 2576
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[Sqrt[3]*f*(ArcTan[(1 + 2*Rt[b, 3]*((2*c + d*x)/(d*(a + b*
x^3)^(1/3))))/Sqrt[3]]/(Rt[b, 3]*d)), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

3.172.4 Maple [F]

$$\int \frac{-dx+c}{(dx+c)\left(d^3x^3+2c^3\right)^{\frac{1}{3}}}dx$$

input
```
int((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
```

output
```
int((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
```

3.172.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (residue poly has multiple non-linear fac
tors)
```

3.172.6 Sympy [F]

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = -\int \left(-\frac{c}{c\sqrt[3]{2c^3+d^3x^3}+dx\sqrt[3]{2c^3+d^3x^3}}\right)dx - \int \frac{dx}{c\sqrt[3]{2c^3+d^3x^3}+dx\sqrt[3]{2c^3+d^3x^3}}\,dx$$

input
```
integrate((-d*x+c)/(d*x+c)/(d**3*x**3+2*c**3)**(1/3),x)
```

output
```
-Integral(-c/(c*(2*c**3 + d**3*x**3)**(1/3) + d*x*(2*c**3 + d**3*x**3)**(1
/3)), x) - Integral(d*x/(c*(2*c**3 + d**3*x**3)**(1/3) + d*x*(2*c**3 + d**
3*x**3)**(1/3)), x)
```

3.172.7 Maxima [F]

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \int -\frac{dx-c}{\left(d^3x^3+2\,c^3\right)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="maxima")
```

output
```
-integrate((d*x - c)/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

3.172.8 Giac [F]

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \int -\frac{dx-c}{\left(d^3x^3+2\,c^3\right)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="giac")
```

output
```
integrate(-(d*x - c)/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

3.172.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \int \frac{c-d\,x}{\left(2\,c^3+d^3\,x^3\right)^{1/3}\,(c+d\,x)}\,dx$$

input
```
int((c - d*x)/((2*c^3 + d^3*x^3)^(1/3)*(c + d*x)),x)
```

output
```
int((c - d*x)/((2*c^3 + d^3*x^3)^(1/3)*(c + d*x)), x)
```

3.173 $\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$

3.173.1 Optimal result

Integrand size = 30, antiderivative size = 234

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \frac{f\arctan\left(\frac{1+\frac{2dx}{\sqrt[3]{-c^3+d^3x^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2} + \frac{\sqrt{3}(de-cf)\arctan\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{-c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}cd^2} + \frac{(de-cf)\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}cd^2} - \frac{f\log\left(-dx+\sqrt[3]{-c^3+d^3x^3}\right)}{2d^2} - \frac{3(de-cf)\log\left(d(c-dx)+2^{2/3}d\sqrt[3]{-c^3+d^3x^3}\right)}{4\sqrt[3]{2}cd^2}$$

output
```
1/8*(-c*f+d*e)*ln((-d*x+c)*(d*x+c)^2)*2^(2/3)/c/d^2-1/2*f*ln(-d*x+(d^3*x^3
-c^3)^(1/3))/d^2-3/8*(-c*f+d*e)*ln(d*(-d*x+c)+2^(2/3)*d*(d^3*x^3-c^3)^(1/3
))*2^(2/3)/c/d^2+1/3*f*arctan(1/3*(1+2*d*x/(d^3*x^3-c^3)^(1/3))*3^(1/2))/d
^2*3^(1/2)+1/4*(-c*f+d*e)*arctan(1/3*(1-2^(1/3)*(-d*x+c)/(d^3*x^3-c^3)^(1/
3))*3^(1/2))*3^(1/2)*2^(2/3)/c/d^2
```

3.173.2 Mathematica [F]

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx=\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$$

input
```
Integrate[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

output
```
Integrate[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)), x]
```

3.173.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2577, 769, 2574}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{d^3x^3-c^3}}dx$$

$$\Big\downarrow \text{2577}$$

$$\frac{(de-cf)\int \frac{1}{(c+dx)\sqrt[3]{d^3x^3-c^3}}dx}{d}+\frac{f\int \frac{1}{\sqrt[3]{d^3x^3-c^3}}dx}{d}$$

$$\Big\downarrow \text{769}$$

$$\frac{(de-cf)\int \frac{1}{(c+dx)\sqrt[3]{d^3x^3-c^3}}dx}{d}+\frac{f\left(\frac{\arctan\left(\frac{\frac{2dx}{\sqrt[3]{d^3x^3-c^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d}-\frac{\log\left(\sqrt[3]{d^3x^3-c^3}-dx\right)}{2d}\right)}{d}$$

$$\Big\downarrow \text{2574}$$

$$\frac{(de-cf)\left(\frac{\sqrt{3}\arctan\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{d^3x^3-c^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}cd}-\frac{3\log\left(2^{2/3}d\sqrt[3]{d^3x^3-c^3}+d(c-dx)\right)}{4\sqrt[3]{2}cd}+\frac{\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}cd}\right)}{d}+\frac{f\left(\frac{\arctan\left(\frac{\frac{2dx}{\sqrt[3]{d^3x^3-c^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d}-\frac{\log\left(\sqrt[3]{d^3x^3-c^3}-dx\right)}{2d}\right)}{d}$$

input
```
Int[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

output
```
(f*(ArcTan[(1 + (2*d*x)/(-c^3 + d^3*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*d) - Log
[-(d*x) + (-c^3 + d^3*x^3)^(1/3)]/(2*d)))/d + ((d*e - c*f)*((Sqrt[3]*ArcTa
n[(1 - (2^(1/3)*(c - d*x))/(-c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/(2*2^(1/3)*c*
d) + Log[(c - d*x)*(c + d*x)^2]/(4*2^(1/3)*c*d) - (3*Log[d*(c - d*x) + 2^(
2/3)*d*(-c^3 + d^3*x^3)^(1/3)])/(4*2^(1/3)*c*d)))/d
```

3.173.3.1 Defintions of rubi rules used

rule 769
```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

rule 2574
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

3.173. $\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$

rule 2577
```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Simp[f/d   Int[1/(a + b*x^3)^(1/3), x], x] + Simp[(d*e - c*f
)/d   Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e,
f}, x]
```

3.173.4 Maple [F]

$$\int \frac{fx+e}{(dx+c)\left(d^3x^3-c^3\right)^{\frac{1}{3}}}dx$$

input
```
int((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
```

output
```
int((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
```

3.173.5 Fricas [F(-1)]

Timed out.

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.173.6 Sympy [F]

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \int \frac{e+fx}{\sqrt[3]{(-c+dx)\left(c^2+cdx+d^2x^2\right)}\,(c+dx)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(d**3*x**3-c**3)**(1/3),x)
```

output
```
Integral((e + f*x)/(((-c + d*x)*(c**2 + c*d*x + d**2*x**2))**(1/3)*(c + d*
x)), x)
```

3.173.7 Maxima [F]

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \int \frac{fx+e}{(d^3x^3-c^3)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="maxima")
```

output
```
integrate((f*x + e)/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

3.173.8 Giac [F]

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \int \frac{fx+e}{(d^3x^3-c^3)^{\frac{1}{3}}(dx+c)}\,dx$$

input
```
integrate((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="giac")
```

output
```
integrate((f*x + e)/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

3.173.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \int \frac{e+f\,x}{(d^3\,x^3-c^3)^{1/3}\,(c+d\,x)}\,dx$$

input
```
int((e + f*x)/((d^3*x^3 - c^3)^(1/3)*(c + d*x)),x)
```

output
```
int((e + f*x)/((d^3*x^3 - c^3)^(1/3)*(c + d*x)), x)
```

3.174 $\int x^2(a+bx)^n \left(c+dx^3\right) dx$

3.174.1 Optimal result

Integrand size = 18, antiderivative size = 160

$$\int x^2(a+bx)^n \left(c+dx^3\right) dx = \frac{a^2(b^3c-a^3d)(a+bx)^{1+n}}{b^6(1+n)} - \frac{a(2b^3c-5a^3d)(a+bx)^{2+n}}{b^6(2+n)} + \frac{(b^3c-10a^3d)(a+bx)^{3+n}}{b^6(3+n)} + \frac{10a^2d(a+bx)^{4+n}}{b^6(4+n)} - \frac{5ad(a+bx)^{5+n}}{b^6(5+n)} + \frac{d(a+bx)^{6+n}}{b^6(6+n)}$$

output
```
a^2*(-a^3*d+b^3*c)*(b*x+a)^(1+n)/b^6/(1+n)-a*(-5*a^3*d+2*b^3*c)*(b*x+a)^(2
+n)/b^6/(2+n)+(-10*a^3*d+b^3*c)*(b*x+a)^(3+n)/b^6/(3+n)+10*a^2*d*(b*x+a)^(
4+n)/b^6/(4+n)-5*a*d*(b*x+a)^(5+n)/b^6/(5+n)+d*(b*x+a)^(6+n)/b^6/(6+n)
```

3.174.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.83

$$\int x^2(a+bx)^n \left(c+dx^3\right) dx = \frac{(a+bx)^{1+n}\left(\frac{a^2b^3c-a^5d}{1+n} + \frac{a(-2b^3c+5a^3d)(a+bx)}{2+n} + \frac{(b^3c-10a^3d)(a+bx)^2}{3+n} + \frac{10a^2d(a+bx)^3}{4+n} - \frac{5ad(a+bx)^4}{5+n} + \frac{d(a+bx)^5}{6+n}\right)}{b^6}$$

input
```
Integrate[x^2*(a + b*x)^n*(c + d*x^3),x]
```

output
```
((a + b*x)^(1 + n)*((a^2*b^3*c - a^5*d)/(1 + n) + (a*(-2*b^3*c + 5*a^3*d)*
(a + b*x))/(2 + n) + ((b^3*c - 10*a^3*d)*(a + b*x)^2)/(3 + n) + (10*a^2*d*
(a + b*x)^3)/(4 + n) - (5*a*d*(a + b*x)^4)/(5 + n) + (d*(a + b*x)^5)/(6 +
n)))/b^6
```

3.174.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(c+dx^3\right) (a+bx)^n \, dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{a\left(5a^3d-2b^3c\right)(a+bx)^{n+1}}{b^5}+\frac{\left(b^3c-10a^3d\right)(a+bx)^{n+2}}{b^5}+\frac{10a^2d(a+bx)^{n+3}}{b^5}+\frac{\left(a^2b^3c-a^5d\right)(a+bx)^n}{b^5}-\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\left(2b^3c-5a^3d\right)(a+bx)^{n+2}}{b^6(n+2)}+\frac{\left(b^3c-10a^3d\right)(a+bx)^{n+3}}{b^6(n+3)}+\frac{10a^2d(a+bx)^{n+4}}{b^6(n+4)}+\frac{a^2\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^6(n+1)}-\frac{5ad(a+bx)^{n+5}}{b^6(n+5)}+\frac{d(a+bx)^{n+6}}{b^6(n+6)}$$

input
```
Int[x^2*(a + b*x)^n*(c + d*x^3),x]
```

output
```
(a^2*(b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^6*(1 + n)) - (a*(2*b^3*c - 5*a^
3*d)*(a + b*x)^(2 + n))/(b^6*(2 + n)) + ((b^3*c - 10*a^3*d)*(a + b*x)^(3 +
 n))/(b^6*(3 + n)) + (10*a^2*d*(a + b*x)^(4 + n))/(b^6*(4 + n)) - (5*a*d*(
a + b*x)^(5 + n))/(b^6*(5 + n)) + (d*(a + b*x)^(6 + n))/(b^6*(6 + n))
```

3.174.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

3.174.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(160) = 320$.

Time = 0.90 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.59

method	result
norman	$\frac{d\,x^{6} e^{n \ln(bx+a)}}{6+n}+\frac{(b^{3} c\,n^{3}+15 b^{3} c\,n^{2}+20 a^{3} d n+74 b^{3} c n+120 b^{3} c) x^{3} e^{n \ln(bx+a)}}{b^{3} (n^{4}+18 n^{3}+119 n^{2}+342 n+360)}+\frac{a d n\,x^{5} e^{n \ln(bx+a)}}{b (n^{2}+11 n+30)}-\frac{2 a^{3} (-b^{3} c\,n^{3}-15 b^{3} c}{b^{6} (n^{6}+21 n^{5}+17}$
gosper	$-\frac{(bx+a)^{1+n} (-b^{5} d\,n^{5} x^{5}-15 b^{5} d\,n^{4} x^{5}+5 a\,b^{4} d\,n^{4} x^{4}-85 b^{5} d\,n^{3} x^{5}+50 a\,b^{4} d\,n^{3} x^{4}-b^{5} c\,n^{5} x^{2}-225 b^{5} d\,n^{2} x^{5}-20 a^{2} b^{3} d\,n^{3} x^{3}+175}{}$
risch	$-\frac{(-b^{6} d\,n^{5} x^{6}-a\,b^{5} d\,n^{5} x^{5}-15 b^{6} d\,n^{4} x^{6}-10 a\,b^{5} d\,n^{4} x^{5}-85 b^{6} d\,n^{3} x^{6}+5 a^{2} b^{4} d\,n^{4} x^{4}-35 a\,b^{5} d\,n^{3} x^{5}-b^{6} c\,n^{5} x^{3}-225 b^{6} d\,n^{2} x^{6}+30}{}$
parallelrisch	$\frac{121 x^{3} (bx+a)^{n} a\,b^{6} c\,n^{3}+16 x^{2} (bx+a)^{n} a^{2} b^{5} c\,n^{4}-30 x^{4} (bx+a)^{n} a^{3} b^{4} d n+60 x^{3} (bx+a)^{n} a^{4} b^{3} d\,n^{2}+372 x^{3} (bx+a)^{n} a\,b^{6} c\,n^{2}+89 x^{2} (b}{}$

input
```
int(x^2*(b*x+a)^n*(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
d/(6+n)*x^6*exp(n*ln(b*x+a))+(b^3*c*n^3+15*b^3*c*n^2+20*a^3*d*n+74*b^3*c*n
+120*b^3*c)/b^3/(n^4+18*n^3+119*n^2+342*n+360)*x^3*exp(n*ln(b*x+a))+a*d*n/
b/(n^2+11*n+30)*x^5*exp(n*ln(b*x+a))-2*a^3*(-b^3*c*n^3-15*b^3*c*n^2-74*b^3
*c*n+60*a^3*d-120*b^3*c)/b^6/(n^6+21*n^5+175*n^4+735*n^3+1624*n^2+1764*n+7
20)*exp(n*ln(b*x+a))+2/b^5*n*a^2*(-b^3*c*n^3-15*b^3*c*n^2-74*b^3*c*n+60*a^
3*d-120*b^3*c)/(n^6+21*n^5+175*n^4+735*n^3+1624*n^2+1764*n+720)*x*exp(n*ln
(b*x+a))-5*n*d*a^2/b^2/(n^3+15*n^2+74*n+120)*x^4*exp(n*ln(b*x+a))-(-b^3*c*
n^3-15*b^3*c*n^2-74*b^3*c*n+60*a^3*d-120*b^3*c)*a/b^4*n/(n^5+20*n^4+155*n^
3+580*n^2+1044*n+720)*x^2*exp(n*ln(b*x+a))
```

3.174.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(160) = 320$.

Time = 0.29 (sec) , antiderivative size = 490, normalized size of antiderivative = 3.06

$$\int x^2(a+bx)^n\left(c+dx^3\right)\,dx = \frac{(2\,a^3b^3cn^3+30\,a^3b^3cn^2+148\,a^3b^3cn+240\,a^3b^3c-120\,a^6d+(b^6dn^5+15\,b^6dn^4+85\,b^6dn^3+225\,b^6dn^2+}{}$$

input
```
integrate(x^2*(b*x+a)^n*(d*x^3+c),x, algorithm="fricas")
```

output
```
(2*a^3*b^3*c*n^3 + 30*a^3*b^3*c*n^2 + 148*a^3*b^3*c*n + 240*a^3*b^3*c - 12
0*a^6*d + (b^6*d*n^5 + 15*b^6*d*n^4 + 85*b^6*d*n^3 + 225*b^6*d*n^2 + 274*b
^6*d*n + 120*b^6*d)*x^6 + (a*b^5*d*n^5 + 10*a*b^5*d*n^4 + 35*a*b^5*d*n^3 +
 50*a*b^5*d*n^2 + 24*a*b^5*d*n)*x^5 - 5*(a^2*b^4*d*n^4 + 6*a^2*b^4*d*n^3 +
 11*a^2*b^4*d*n^2 + 6*a^2*b^4*d*n)*x^4 + (b^6*c*n^5 + 18*b^6*c*n^4 + 240*b
^6*c + (121*b^6*c + 20*a^3*b^3*d)*n^3 + 12*(31*b^6*c + 5*a^3*b^3*d)*n^2 +
4*(127*b^6*c + 10*a^3*b^3*d)*n)*x^3 + (a*b^5*c*n^5 + 16*a*b^5*c*n^4 + 89*a
*b^5*c*n^3 + 2*(97*a*b^5*c - 30*a^4*b^2*d)*n^2 + 60*(2*a*b^5*c - a^4*b^2*d
)*n)*x^2 - 2*(a^2*b^4*c*n^4 + 15*a^2*b^4*c*n^3 + 74*a^2*b^4*c*n^2 + 60*(2*
a^2*b^4*c - a^5*b*d)*n)*x)*(b*x + a)^n/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4
 + 735*b^6*n^3 + 1624*b^6*n^2 + 1764*b^6*n + 720*b^6)
```

3.174.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6397 vs. $2(144) = 288$.

Time = 2.03 (sec) , antiderivative size = 6397, normalized size of antiderivative = 39.98

$$\int x^2(a+bx)^n\left(c+dx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(b*x+a)**n*(d*x**3+c),x)
```

output
```
Piecewise((a**n*(c*x**3/3 + d*x**6/6), Eq(b, 0)), (60*a**5*d*log(a/b + x)/
(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3
+ 300*a*b**10*x**4 + 60*b**11*x**5) + 137*a**5*d/(60*a**5*b**6 + 300*a**4*
b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b
**11*x**5) + 300*a**4*b*d*x*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x +
 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**
5) + 625*a**4*b*d*x/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 +
 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 600*a**3*b**2*d*
x**2*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 6
00*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 1100*a**3*b**2*d*x
**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x
**3 + 300*a*b**10*x**4 + 60*b**11*x**5) - 2*a**2*b**3*c/(60*a**5*b**6 + 30
0*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4
 + 60*b**11*x**5) + 600*a**2*b**3*d*x**3*log(a/b + x)/(60*a**5*b**6 + 300*
a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 +
 60*b**11*x**5) + 900*a**2*b**3*d*x**3/(60*a**5*b**6 + 300*a**4*b**7*x + 6
00*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5)
 - 10*a*b**4*c*x/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 60
0*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*d*x**4*l
og(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*...
```

3.174.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.58

$$\int x^2(a+bx)^n \left(c+dx^3\right) \, dx$$
$$= \frac{((n^2+3n+2)b^3x^3+(n^2+n)ab^2x^2-2a^2bnx+2a^3)(bx+a)^n c}{(n^3+6n^2+11n+6)b^3} + \frac{((n^5+15n^4+85n^3+225n^2+274n+120)b^6x^6+(n^5+10n^4+35n^3+50n^2+24n)ab^5x^5-5(n^4+}{(n^6+21n^5+175n^4+73}$$

input
```
integrate(x^2*(b*x+a)^n*(d*x^3+c),x, algorithm="maxima")
```

output
```
((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
 + a)^n*c/((n^3 + 6*n^2 + 11*n + 6)*b^3) + ((n^5 + 15*n^4 + 85*n^3 + 225*n
^2 + 274*n + 120)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5*
x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*
a^3*b^3*x^3 - 60*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a
)^n*d/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6)
```

3.174.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 835 vs. $2(160) = 320$.

Time = 0.31 (sec) , antiderivative size = 835, normalized size of antiderivative = 5.22

$$\int x^2(a+bx)^n\left(c+dx^3\right)\,dx$$
$$= \frac{(bx+a)^n b^6 dn^5 x^6 + (bx+a)^n ab^5 dn^5 x^5 + 15\,(bx+a)^n b^6 dn^4 x^6 + 10\,(bx+a)^n ab^5 dn^4 x^5 + 85\,(bx+a)^n b^6 dn}{}$$

input
```
integrate(x^2*(b*x+a)^n*(d*x^3+c),x, algorithm="giac")
```

output
```
((b*x + a)^n*b^6*d*n^5*x^6 + (b*x + a)^n*a*b^5*d*n^5*x^5 + 15*(b*x + a)^n*
b^6*d*n^4*x^6 + 10*(b*x + a)^n*a*b^5*d*n^4*x^5 + 85*(b*x + a)^n*b^6*d*n^3*
x^6 + (b*x + a)^n*b^6*c*n^5*x^3 - 5*(b*x + a)^n*a^2*b^4*d*n^4*x^4 + 35*(b*
x + a)^n*a*b^5*d*n^3*x^5 + 225*(b*x + a)^n*b^6*d*n^2*x^6 + (b*x + a)^n*a*b
^5*c*n^5*x^2 + 18*(b*x + a)^n*b^6*c*n^4*x^3 - 30*(b*x + a)^n*a^2*b^4*d*n^3
*x^4 + 50*(b*x + a)^n*a*b^5*d*n^2*x^5 + 274*(b*x + a)^n*b^6*d*n*x^6 + 16*(
b*x + a)^n*a*b^5*c*n^4*x^2 + 121*(b*x + a)^n*b^6*c*n^3*x^3 + 20*(b*x + a)^
n*a^3*b^3*d*n^3*x^3 - 55*(b*x + a)^n*a^2*b^4*d*n^2*x^4 + 24*(b*x + a)^n*a*
b^5*d*n*x^5 + 120*(b*x + a)^n*b^6*d*x^6 - 2*(b*x + a)^n*a^2*b^4*c*n^4*x +
89*(b*x + a)^n*a*b^5*c*n^3*x^2 + 372*(b*x + a)^n*b^6*c*n^2*x^3 + 60*(b*x +
 a)^n*a^3*b^3*d*n^2*x^3 - 30*(b*x + a)^n*a^2*b^4*d*n*x^4 - 30*(b*x + a)^n*
a^2*b^4*c*n^3*x + 194*(b*x + a)^n*a*b^5*c*n^2*x^2 - 60*(b*x + a)^n*a^4*b^2
*d*n^2*x^2 + 508*(b*x + a)^n*b^6*c*n*x^3 + 40*(b*x + a)^n*a^3*b^3*d*n*x^3
+ 2*(b*x + a)^n*a^3*b^3*c*n^3 - 148*(b*x + a)^n*a^2*b^4*c*n^2*x + 120*(b*x
 + a)^n*a*b^5*c*n*x^2 - 60*(b*x + a)^n*a^4*b^2*d*n*x^2 + 240*(b*x + a)^n*b
^6*c*x^3 + 30*(b*x + a)^n*a^3*b^3*c*n^2 - 240*(b*x + a)^n*a^2*b^4*c*n*x +
120*(b*x + a)^n*a^5*b*d*n*x + 148*(b*x + a)^n*a^3*b^3*c*n + 240*(b*x + a)^
n*a^3*b^3*c - 120*(b*x + a)^n*a^6*d)/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 +
 735*b^6*n^3 + 1624*b^6*n^2 + 1764*b^6*n + 720*b^6)
```

3.174.9 Mupad [B] (verification not implemented)

Time = 19.82 (sec) , antiderivative size = 495, normalized size of antiderivative = 3.09

$$\int x^2(a+bx)^n\left(c+dx^3\right)\,dx = (a+b\,x)^n\left(\frac{d\,x^6\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)}{n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720} + \frac{2\,a^3\,(-60\,d\,a^3+c\,b^3\,n^3+15\,c\,b^3\,n^2+74\,c\,b^3\,n+120\,c\,b^3)}{b^6\,(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)} + \frac{x^3\,(n^2+3\,n+2)\,(20\,d\,a^3\,n+c\,b^3\,n^3+15\,c\,b^3\,n^2+74\,c\,b^3\,n+120\,c\,b^3)}{b^3\,(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)} - \frac{2\,a^2\,n\,x\,(-60\,d\,a^3+c\,b^3\,n^3+15\,c\,b^3\,n^2+74\,c\,b^3\,n+120\,c\,b^3)}{b^5\,(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)} + \frac{a\,d\,n\,x^5\,(n^4+10\,n^3+35\,n^2+50\,n+24)}{b\,(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)} + \frac{a\,n\,x^2\,(n+1)\,(-60\,d\,a^3+c\,b^3\,n^3+15\,c\,b^3\,n^2+74\,c\,b^3\,n+120\,c\,b^3)}{b^4\,(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)} - \frac{5\,a^2\,d\,n\,x^4\,(n^3+6\,n^2+11\,n+6)}{b^2\,(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)}\right)$$

input
```
int(x^2*(c + d*x^3)*(a + b*x)^n,x)
```

output
```
(a + b*x)^n*((d*x^6*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(1764
*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720) + (2*a^3*(120*b^3*
c - 60*a^3*d + 15*b^3*c*n^2 + b^3*c*n^3 + 74*b^3*c*n))/(b^6*(1764*n + 1624
*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (x^3*(3*n + n^2 + 2)*(12
0*b^3*c + 15*b^3*c*n^2 + b^3*c*n^3 + 20*a^3*d*n + 74*b^3*c*n))/(b^3*(1764*
n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) - (2*a^2*n*x*(120*
b^3*c - 60*a^3*d + 15*b^3*c*n^2 + b^3*c*n^3 + 74*b^3*c*n))/(b^5*(1764*n +
1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (a*d*n*x^5*(50*n + 3
5*n^2 + 10*n^3 + n^4 + 24))/(b*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21
*n^5 + n^6 + 720)) + (a*n*x^2*(n + 1)*(120*b^3*c - 60*a^3*d + 15*b^3*c*n^2
 + b^3*c*n^3 + 74*b^3*c*n))/(b^4*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 +
21*n^5 + n^6 + 720)) - (5*a^2*d*n*x^4*(11*n + 6*n^2 + n^3 + 6))/(b^2*(1764
*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)))
```

3.175 $\int x(a+bx)^n (c+dx^3) \, dx$

3.175.1 Optimal result

Integrand size = 16, antiderivative size = 126

$$\int x(a+bx)^n (c+dx^3) \, dx = -\frac{a(b^3c-a^3d)(a+bx)^{1+n}}{b^5(1+n)} + \frac{(b^3c-4a^3d)(a+bx)^{2+n}}{b^5(2+n)} + \frac{6a^2d(a+bx)^{3+n}}{b^5(3+n)} - \frac{4ad(a+bx)^{4+n}}{b^5(4+n)} + \frac{d(a+bx)^{5+n}}{b^5(5+n)}$$

output
```
-a*(-a^3*d+b^3*c)*(b*x+a)^(1+n)/b^5/(1+n)+(-4*a^3*d+b^3*c)*(b*x+a)^(2+n)/b
^5/(2+n)+6*a^2*d*(b*x+a)^(3+n)/b^5/(3+n)-4*a*d*(b*x+a)^(4+n)/b^5/(4+n)+d*(
b*x+a)^(5+n)/b^5/(5+n)
```

3.175.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

$$\int x(a+bx)^n (c+dx^3) \, dx = \frac{(a+bx)^{1+n}\left(\frac{a(-b^3c+a^3d)}{1+n} + \frac{(b^3c-4a^3d)(a+bx)}{2+n} + \frac{6a^2d(a+bx)^2}{3+n} - \frac{4ad(a+bx)^3}{4+n} + \frac{d(a+bx)^4}{5+n}\right)}{b^5}$$

input
```
Integrate[x*(a + b*x)^n*(c + d*x^3),x]
```

output
```
((a + b*x)^(1 + n)*((a*(-(b^3*c) + a^3*d))/(1 + n) + ((b^3*c - 4*a^3*d)*(a
 + b*x))/(2 + n) + (6*a^2*d*(a + b*x)^2)/(3 + n) - (4*a*d*(a + b*x)^3)/(4
+ n) + (d*(a + b*x)^4)/(5 + n)))/b^5
```

3.175.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(c+dx^3)(a+bx)^n\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{a(a^3d-b^3c)(a+bx)^n}{b^4}+\frac{(b^3c-4a^3d)(a+bx)^{n+1}}{b^4}+\frac{6a^2d(a+bx)^{n+2}}{b^4}-\frac{4ad(a+bx)^{n+3}}{b^4}+\frac{d(a+bx)^{n+4}}{b^4}\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a(b^3c-a^3d)(a+bx)^{n+1}}{b^5(n+1)}+\frac{(b^3c-4a^3d)(a+bx)^{n+2}}{b^5(n+2)}+\frac{6a^2d(a+bx)^{n+3}}{b^5(n+3)}-\frac{4ad(a+bx)^{n+4}}{b^5(n+4)}+\frac{d(a+bx)^{n+5}}{b^5(n+5)}$$

input
```
Int[x*(a + b*x)^n*(c + d*x^3),x]
```

output
```
-((a*(b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^5*(1 + n))) + ((b^3*c - 4*a^3*d
)*(a + b*x)^(2 + n))/(b^5*(2 + n)) + (6*a^2*d*(a + b*x)^(3 + n))/(b^5*(3 +
 n)) - (4*a*d*(a + b*x)^(4 + n))/(b^5*(4 + n)) + (d*(a + b*x)^(5 + n))/(b^
5*(5 + n))
```

3.175.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

3.175.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(126) = 252$.

Time = 0.89 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.25

method	result
gosper	$\frac{(bx+a)^{1+n}(b^4dn^4x^4+10b^4dn^3x^4-4ab^3dn^3x^3+35b^4dn^2x^4-24ab^3dn^2x^3+b^4cn^4x+50b^4dnx^4+12a^2b^2dn^2x^2-44ab^3dnx^3}{b}$
norman	$\frac{dx^5e^{n\ln(bx+a)}}{5+n}+\frac{a^2(-b^3cn^3-12b^3cn^2-47b^3cn+24a^3d-60b^3c)e^{n\ln(bx+a)}}{b^5(n^5+15n^4+85n^3+225n^2+274n+120)}+\frac{(b^3cn^3+12b^3cn^2+12a^3dn+47b^3cn+60b^3c)}{b^3(n^4+14n^3+71n^2+154n+120}$
risch	$\frac{(b^5dn^4x^5+ab^4dn^4x^4+10b^5dn^3x^5+6ab^4dn^3x^4+35b^5dn^2x^5-4a^2b^3dn^3x^3+11ab^4dn^2x^4+b^5cn^4x^2+50b^5dnx^5-12a^2b^3dn^2}{}$
parallelrisch	$\frac{x^5(bx+a)^nb^5dn^4+10x^5(bx+a)^nb^5dn^3+35x^5(bx+a)^nb^5dn^2+50x^5(bx+a)^nb^5dn+x^2(bx+a)^nb^5cn^4+13x^2(bx+a)^nb^5cn^3+59}{}$

input
```
int(x*(b*x+a)^n*(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
1/b^5*(b*x+a)^(1+n)/(n^5+15*n^4+85*n^3+225*n^2+274*n+120)*(b^4*d*n^4*x^4+1
0*b^4*d*n^3*x^4-4*a*b^3*d*n^3*x^3+35*b^4*d*n^2*x^4-24*a*b^3*d*n^2*x^3+b^4*
c*n^4*x+50*b^4*d*n*x^4+12*a^2*b^2*d*n^2*x^2-44*a*b^3*d*n*x^3+13*b^4*c*n^3*
x+24*b^4*d*x^4+36*a^2*b^2*d*n*x^2-a*b^3*c*n^3-24*a*b^3*d*x^3+59*b^4*c*n^2*
x-24*a^3*b*d*n*x+24*a^2*b^2*d*x^2-12*a*b^3*c*n^2+107*b^4*c*n*x-24*a^3*b*d*
x-47*a*b^3*c*n+60*b^4*c*x+24*a^4*d-60*a*b^3*c)
```

3.175.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(126) = 252$.

Time = 0.34 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.76

$$\int x(a+bx)^n\left(c+dx^3\right)\,dx =$$
$$-\frac{(a^2b^3cn^3+12\,a^2b^3cn^2+47\,a^2b^3cn+60\,a^2b^3c-24\,a^5d-(b^5dn^4+10\,b^5dn^3+35\,b^5dn^2+50\,b^5dn+24\,b}{}$$

input
```
integrate(x*(b*x+a)^n*(d*x^3+c),x, algorithm="fricas")
```

output
```
-(a^2*b^3*c*n^3 + 12*a^2*b^3*c*n^2 + 47*a^2*b^3*c*n + 60*a^2*b^3*c - 24*a^
5*d - (b^5*d*n^4 + 10*b^5*d*n^3 + 35*b^5*d*n^2 + 50*b^5*d*n + 24*b^5*d)*x^
5 - (a*b^4*d*n^4 + 6*a*b^4*d*n^3 + 11*a*b^4*d*n^2 + 6*a*b^4*d*n)*x^4 + 4*(
a^2*b^3*d*n^3 + 3*a^2*b^3*d*n^2 + 2*a^2*b^3*d*n)*x^3 - (b^5*c*n^4 + 13*b^5
*c*n^3 + 60*b^5*c + (59*b^5*c + 12*a^3*b^2*d)*n^2 + (107*b^5*c + 12*a^3*b^
2*d)*n)*x^2 - (a*b^4*c*n^4 + 12*a*b^4*c*n^3 + 47*a*b^4*c*n^2 + 12*(5*a*b^4
*c - 2*a^4*b*d)*n)*x)*(b*x + a)^n/(b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225
*b^5*n^2 + 274*b^5*n + 120*b^5)
```

3.175.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3704 vs. $2(112) = 224$.

Time = 1.30 (sec) , antiderivative size = 3704, normalized size of antiderivative = 29.40

$$\int x(a+bx)^n \left(c+dx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate(x*(b*x+a)**n*(d*x**3+c),x)
```

output
```
Piecewise((a**n*(c*x**2/2 + d*x**5/5), Eq(b, 0)), (12*a**4*d*log(a/b + x)/
(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b
**9*x**4) + 25*a**4*d/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 +
 48*a*b**8*x**3 + 12*b**9*x**4) + 48*a**3*b*d*x*log(a/b + x)/(12*a**4*b**5
 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 8
8*a**3*b*d*x/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**
8*x**3 + 12*b**9*x**4) + 72*a**2*b**2*d*x**2*log(a/b + x)/(12*a**4*b**5 +
48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 108*
a**2*b**2*d*x**2/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a
*b**8*x**3 + 12*b**9*x**4) - a*b**3*c/(12*a**4*b**5 + 48*a**3*b**6*x + 72*
a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 48*a*b**3*d*x**3*log(a/b
 + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3
+ 12*b**9*x**4) + 48*a*b**3*d*x**3/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**
2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) - 4*b**4*c*x/(12*a**4*b**5 +
48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 12*b
**4*d*x**4*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2
 + 48*a*b**8*x**3 + 12*b**9*x**4), Eq(n, -5)), (-24*a**4*d*log(a/b + x)/(6
*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 44*a**4*d/(6
*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 72*a**3*b*d*
x*log(a/b + x)/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*...
```

3.175.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.46

$$\int x(a+bx)^n \left(c+dx^3\right) dx = \frac{(b^2(n+1)x^2+abnx-a^2)(bx+a)^n c}{(n^2+3n+2)b^2} + \frac{((n^4+10n^3+35n^2+50n+24)b^5x^5+(n^4+6n^3+11n^2+6n)ab^4x^4-4(n^3+3n^2+2n)a^2b^3x^3+1}{(n^5+15n^4+85n^3+225n^2+274n+120)b^5}$$

input
```
integrate(x*(b*x+a)^n*(d*x^3+c),x, algorithm="maxima")
```

output
```
(b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c/((n^2 + 3*n + 2)*b^2) + ((
n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*
a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 -
 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 +
274*n + 120)*b^5)
```

3.175.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. 2(126) = 252.

Time = 0.40 (sec) , antiderivative size = 577, normalized size of antiderivative = 4.58

$$\int x(a+bx)^n \left(c+dx^3\right) dx = \frac{(bx+a)^n b^5 dn^4 x^5 + (bx+a)^n ab^4 dn^4 x^4 + 10(bx+a)^n b^5 dn^3 x^5 + 6(bx+a)^n ab^4 dn^3 x^4 + 35(bx+a)^n b^5 dn^2}{}$$

input
```
integrate(x*(b*x+a)^n*(d*x^3+c),x, algorithm="giac")
```

output
```
((b*x + a)^n*b^5*d*n^4*x^5 + (b*x + a)^n*a*b^4*d*n^4*x^4 + 10*(b*x + a)^n*
b^5*d*n^3*x^5 + 6*(b*x + a)^n*a*b^4*d*n^3*x^4 + 35*(b*x + a)^n*b^5*d*n^2*x
^5 + (b*x + a)^n*b^5*c*n^4*x^2 - 4*(b*x + a)^n*a^2*b^3*d*n^3*x^3 + 11*(b*x
 + a)^n*a*b^4*d*n^2*x^4 + 50*(b*x + a)^n*b^5*d*n*x^5 + (b*x + a)^n*a*b^4*c
*n^4*x + 13*(b*x + a)^n*b^5*c*n^3*x^2 - 12*(b*x + a)^n*a^2*b^3*d*n^2*x^3 +
 6*(b*x + a)^n*a*b^4*d*n*x^4 + 24*(b*x + a)^n*b^5*d*x^5 + 12*(b*x + a)^n*a
*b^4*c*n^3*x + 59*(b*x + a)^n*b^5*c*n^2*x^2 + 12*(b*x + a)^n*a^3*b^2*d*n^2
*x^2 - 8*(b*x + a)^n*a^2*b^3*d*n*x^3 - (b*x + a)^n*a^2*b^3*c*n^3 + 47*(b*x
 + a)^n*a*b^4*c*n^2*x + 107*(b*x + a)^n*b^5*c*n*x^2 + 12*(b*x + a)^n*a^3*b
^2*d*n*x^2 - 12*(b*x + a)^n*a^2*b^3*c*n^2 + 60*(b*x + a)^n*a*b^4*c*n*x - 2
4*(b*x + a)^n*a^4*b*d*n*x + 60*(b*x + a)^n*b^5*c*x^2 - 47*(b*x + a)^n*a^2*
b^3*c*n - 60*(b*x + a)^n*a^2*b^3*c + 24*(b*x + a)^n*a^5*d)/(b^5*n^5 + 15*b
^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

3.175.9 Mupad [B] (verification not implemented)

Time = 19.17 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.88

$$\begin{aligned}
&\int x(a+bx)^n\left(c+dx^3\right)\,dx \\
&= (a+b\,x)^n\left(\frac{d\,x^5\,(n^4+10\,n^3+35\,n^2+50\,n+24)}{n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120}\right. \\
&\quad -\frac{a^2\,(-24\,d\,a^3+c\,b^3\,n^3+12\,c\,b^3\,n^2+47\,c\,b^3\,n+60\,c\,b^3)}{b^5\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)} \\
&\quad +\frac{x^2\,(n+1)\,\left(12\,d\,a^3\,n+c\,b^3\,n^3+12\,c\,b^3\,n^2+47\,c\,b^3\,n+60\,c\,b^3\right)}{b^3\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)} \\
&\quad +\frac{a\,n\,x\,(-24\,d\,a^3+c\,b^3\,n^3+12\,c\,b^3\,n^2+47\,c\,b^3\,n+60\,c\,b^3)}{b^4\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)} \\
&\quad +\frac{a\,d\,n\,x^4\,(n^3+6\,n^2+11\,n+6)}{b\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)} \\
&\quad \left.-\frac{4\,a^2\,d\,n\,x^3\,(n^2+3\,n+2)}{b^2\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}\right)
\end{aligned}$$

input
```
int(x*(c + d*x^3)*(a + b*x)^n,x)
```

output
```
(a + b*x)^n*((d*x^5*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(274*n + 225*n^2
+ 85*n^3 + 15*n^4 + n^5 + 120) - (a^2*(60*b^3*c - 24*a^3*d + 12*b^3*c*n^2
+ b^3*c*n^3 + 47*b^3*c*n))/(b^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 +
 120)) + (x^2*(n + 1)*(60*b^3*c + 12*b^3*c*n^2 + b^3*c*n^3 + 12*a^3*d*n +
47*b^3*c*n))/(b^3*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*n*
x*(60*b^3*c - 24*a^3*d + 12*b^3*c*n^2 + b^3*c*n^3 + 47*b^3*c*n))/(b^4*(274
*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*d*n*x^4*(11*n + 6*n^2 +
n^3 + 6))/(b*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (4*a^2*d*n
*x^3*(3*n + n^2 + 2))/(b^2*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)
))
```

3.175. $\int x(a+bx)^n (c+dx^3) \, dx$

3.176 $\int (a+bx)^n (c+dx^3) \, dx$

3.176.1 Optimal result

Integrand size = 15, antiderivative size = 94

$$\int (a+bx)^n (c+dx^3) \, dx = \frac{(b^3c-a^3d)(a+bx)^{1+n}}{b^4(1+n)} + \frac{3a^2d(a+bx)^{2+n}}{b^4(2+n)} - \frac{3ad(a+bx)^{3+n}}{b^4(3+n)} + \frac{d(a+bx)^{4+n}}{b^4(4+n)}$$

output
```
(-a^3*d+b^3*c)*(b*x+a)^(1+n)/b^4/(1+n)+3*a^2*d*(b*x+a)^(2+n)/b^4/(2+n)-3*a
*d*(b*x+a)^(3+n)/b^4/(3+n)+d*(b*x+a)^(4+n)/b^4/(4+n)
```

3.176.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int (a+bx)^n (c+dx^3) \, dx = \frac{(b^3c-a^3d)(a+bx)^{1+n}}{b^4(1+n)} + \frac{3a^2d(a+bx)^{2+n}}{b^4(2+n)} - \frac{3ad(a+bx)^{3+n}}{b^4(3+n)} + \frac{d(a+bx)^{4+n}}{b^4(4+n)}$$

input
```
Integrate[(a + b*x)^n*(c + d*x^3),x]
```

output
```
((b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^4*(1 + n)) + (3*a^2*d*(a + b*x)^(2
+ n))/(b^4*(2 + n)) - (3*a*d*(a + b*x)^(3 + n))/(b^4*(3 + n)) + (d*(a + b*
x)^(4 + n))/(b^4*(4 + n))
```

3.176.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx^3)(a+bx)^n\,dx$$

$$\Big\downarrow \ 2389$$

$$\int \left(\frac{(b^3c-a^3d)(a+bx)^n}{b^3}+\frac{3a^2d(a+bx)^{n+1}}{b^3}-\frac{3ad(a+bx)^{n+2}}{b^3}+\frac{d(a+bx)^{n+3}}{b^3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{(b^3c-a^3d)(a+bx)^{n+1}}{b^4(n+1)}+\frac{3a^2d(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad(a+bx)^{n+3}}{b^4(n+3)}+\frac{d(a+bx)^{n+4}}{b^4(n+4)}$$

input
```
Int[(a + b*x)^n*(c + d*x^3),x]
```

output
```
((b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^4*(1 + n)) + (3*a^2*d*(a + b*x)^(2
+ n))/(b^4*(2 + n)) - (3*a*d*(a + b*x)^(3 + n))/(b^4*(3 + n)) + (d*(a + b*
x)^(4 + n))/(b^4*(4 + n))
```

3.176.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.176.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.78

method	result
gosper	$-\frac{(bx+a)^{1+n}(-b^3dn^3x^3-6b^3dn^2x^3+3ab^2dn^2x^2-11b^3dnx^3+9ab^2dnx^2-b^3cn^3-6x^3db^3-6a^2bdnx+6adx^2b^2-9b^3cn^2-6}{b^4(n^4+10n^3+35n^2+50n+24)}$
risch	$-\frac{(-b^4dn^3x^4-ab^3dn^3x^3-6b^4dn^2x^4-3ab^3dn^2x^3-11b^4dnx^4+3a^2b^2dn^2x^2-2ab^3dnx^3-b^4cn^3x-6dx^4b^4+3a^2b^2dnx^2-a}{(3+n)(4+n)(2+n)(1+n)b^4}$
norman	$\frac{dx^4e^{n\ln(bx+a)}}{4+n}+\frac{(b^3cn^3+9b^3cn^2+6a^3dn+26b^3cn+24b^3c)xe^{n\ln(bx+a)}}{b^3(n^4+10n^3+35n^2+50n+24)}+\frac{ndax^3e^{n\ln(bx+a)}}{b(n^2+7n+12)}-\frac{a(-b^3cn^3-9b^3cn^2-26b}{b^4(n^4+10n^3}$
parallelrisch	$\frac{x^4(bx+a)^nab^4dn^3+6x^4(bx+a)^nab^4dn^2+x^3(bx+a)^na^2b^3dn^3+11x^4(bx+a)^nab^4dn+3x^3(bx+a)^na^2b^3dn^2+6x^4(bx+a)^nab^4}{}$

input
```
int((b*x+a)^n*(d*x^3+c),x,method=_RETURNVERBOSE)
```

output
```
-1/b^4*(b*x+a)^(1+n)/(n^4+10*n^3+35*n^2+50*n+24)*(-b^3*d*n^3*x^3-6*b^3*d*n
^2*x^3+3*a*b^2*d*n^2*x^2-11*b^3*d*n*x^3+9*a*b^2*d*n*x^2-b^3*c*n^3-6*b^3*d*
x^3-6*a^2*b*d*n*x+6*a*b^2*d*x^2-9*b^3*c*n^2-6*a^2*b*d*x-26*b^3*c*n+6*a^3*d
-24*b^3*c)
```

3.176.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(94) = 188.

Time = 0.28 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.36

$$\int (a+bx)^n\,(c+dx^3)\;dx$$
$$=\frac{(ab^3cn^3+9\,ab^3cn^2+26\,ab^3cn+24\,ab^3c-6\,a^4d+(b^4dn^3+6\,b^4dn^2+11\,b^4dn+6\,b^4d)x^4+(ab^3dn^3+3\,ab}{b^4n^4+10\,b^4n^3+35}$$

input
```
integrate((b*x+a)^n*(d*x^3+c),x, algorithm="fricas")
```

output
```
(a*b^3*c*n^3 + 9*a*b^3*c*n^2 + 26*a*b^3*c*n + 24*a*b^3*c - 6*a^4*d + (b^4*
d*n^3 + 6*b^4*d*n^2 + 11*b^4*d*n + 6*b^4*d)*x^4 + (a*b^3*d*n^3 + 3*a*b^3*d
*n^2 + 2*a*b^3*d*n)*x^3 - 3*(a^2*b^2*d*n^2 + a^2*b^2*d*n)*x^2 + (b^4*c*n^3
 + 9*b^4*c*n^2 + 24*b^4*c + 2*(13*b^4*c + 3*a^3*b*d)*n)*x)*(b*x + a)^n/(b^
4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

3.176.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1906 vs. 2(83) = 166.

Time = 0.81 (sec) , antiderivative size = 1906, normalized size of antiderivative = 20.28

$$\int (a+bx)^n \left(c+dx^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**n*(d*x**3+c),x)
```

output
```
Piecewise((a**n*(c*x + d*x**4/4), Eq(b, 0)), (6*a**3*d*log(a/b + x)/(6*a**
3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 11*a**3*d/(6*a**
3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*d*x*lo
g(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) +
 27*a**2*b*d*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**
3) + 18*a*b**2*d*x**2*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b*
*6*x**2 + 6*b**7*x**3) + 18*a*b**2*d*x**2/(6*a**3*b**4 + 18*a**2*b**5*x +
18*a*b**6*x**2 + 6*b**7*x**3) - 2*b**3*c/(6*a**3*b**4 + 18*a**2*b**5*x + 1
8*a*b**6*x**2 + 6*b**7*x**3) + 6*b**3*d*x**3*log(a/b + x)/(6*a**3*b**4 + 1
8*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3), Eq(n, -4)), (-6*a**3*d*log(
a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 9*a**3*d/(2*a**2*b**4
+ 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*d*x*log(a/b + x)/(2*a**2*b**4 + 4*
a*b**5*x + 2*b**6*x**2) - 12*a**2*b*d*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6
*x**2) - 6*a*b**2*d*x**2*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x
**2) - b**3*c/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*d*x**3/(2*
a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2), Eq(n, -3)), (6*a**3*d*log(a/b + x)/
(2*a*b**4 + 2*b**5*x) + 6*a**3*d/(2*a*b**4 + 2*b**5*x) + 6*a**2*b*d*x*log(
a/b + x)/(2*a*b**4 + 2*b**5*x) - 3*a*b**2*d*x**2/(2*a*b**4 + 2*b**5*x) - 2
*b**3*c/(2*a*b**4 + 2*b**5*x) + b**3*d*x**3/(2*a*b**4 + 2*b**5*x), Eq(n, -
2)), (-a**3*d*log(a/b + x)/b**4 + a**2*d*x/b**3 - a*d*x**2/(2*b**2) + c...
```

3.176.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.30

$$\int (a+bx)^n \left(c+dx^3\right) \, dx = \frac{(bx+a)^{n+1}c}{b(n+1)} + \frac{((n^3+6\,n^2+11\,n+6)b^4x^4 + (n^3+3\,n^2+2\,n)ab^3x^3 - 3\,(n^2+n)a^2b^2x^2 + 6\,a^3bnx - 6\,a^4)(bx+a)^n d}{(n^4+10\,n^3+35\,n^2+50\,n+24)b^4}$$

input
```
integrate((b*x+a)^n*(d*x^3+c),x, algorithm="maxima")
```

output
```
(b*x + a)^(n + 1)*c/(b*(n + 1)) + ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3
 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)
*(b*x + a)^n*d/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4)
```

3.176.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(94) = 188$.

Time = 0.43 (sec) , antiderivative size = 361, normalized size of antiderivative = 3.84

$$\int (a+bx)^n \left(c+dx^3\right) \, dx$$
$$= \frac{(bx+a)^n b^4 d n^3 x^4 + (bx+a)^n a b^3 d n^3 x^3 + 6\,(bx+a)^n b^4 d n^2 x^4 + 3\,(bx+a)^n a b^3 d n^2 x^3 + 11\,(bx+a)^n b^4 d n x^4}{}$$

input
```
integrate((b*x+a)^n*(d*x^3+c),x, algorithm="giac")
```

output
```
((b*x + a)^n*b^4*d*n^3*x^4 + (b*x + a)^n*a*b^3*d*n^3*x^3 + 6*(b*x + a)^n*b
^4*d*n^2*x^4 + 3*(b*x + a)^n*a*b^3*d*n^2*x^3 + 11*(b*x + a)^n*b^4*d*n*x^4
+ (b*x + a)^n*b^4*c*n^3*x - 3*(b*x + a)^n*a^2*b^2*d*n^2*x^2 + 2*(b*x + a)^
n*a*b^3*d*n*x^3 + 6*(b*x + a)^n*b^4*d*x^4 + (b*x + a)^n*a*b^3*c*n^3 + 9*(b
*x + a)^n*b^4*c*n^2*x - 3*(b*x + a)^n*a^2*b^2*d*n*x^2 + 9*(b*x + a)^n*a*b^
3*c*n^2 + 26*(b*x + a)^n*b^4*c*n*x + 6*(b*x + a)^n*a^3*b*d*n*x + 26*(b*x +
 a)^n*a*b^3*c*n + 24*(b*x + a)^n*b^4*c*x + 24*(b*x + a)^n*a*b^3*c - 6*(b*x
 + a)^n*a^4*d)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

3.176.9 Mupad [B] (verification not implemented)

Time = 19.41 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.63

$$\int (a+bx)^n \left(c+dx^3\right) \, dx = (a+b\,x)^n \left(\frac{x\,(6\,d\,a^3\,b\,n+c\,b^4\,n^3+9\,c\,b^4\,n^2+26\,c\,b^4\,n+24\,c\,b^4)}{b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)} + \frac{a\,(-6\,d\,a^3+c\,b^3\,n^3+9\,c\,b^3\,n^2+26\,c\,b^3\,n+24\,c\,b^3)}{b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)} + \frac{d\,x^4\,(n^3+6\,n^2+11\,n+6)}{n^4+10\,n^3+35\,n^2+50\,n+24} - \frac{3\,a^2\,d\,n\,x^2\,(n+1)}{b^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)} + \frac{a\,d\,n\,x^3\,(n^2+3\,n+2)}{b\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)$$

input
```
int((c + d*x^3)*(a + b*x)^n,x)
```

output
```
(a + b*x)^n*((x*(24*b^4*c + 9*b^4*c*n^2 + b^4*c*n^3 + 26*b^4*c*n + 6*a^3*b
*d*n))/(b^4*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*(24*b^3*c - 6*a^3*d
+ 9*b^3*c*n^2 + b^3*c*n^3 + 26*b^3*c*n))/(b^4*(50*n + 35*n^2 + 10*n^3 + n^
4 + 24)) + (d*x^4*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 + 10*n^3 + n^4
+ 24) - (3*a^2*d*n*x^2*(n + 1))/(b^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))
+ (a*d*n*x^3*(3*n + n^2 + 2))/(b*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)))
```

3.177 $\int \frac{(a+bx)^n(c+dx^3)}{x}\,dx$

3.177.1 Optimal result

Integrand size = 18, antiderivative size = 99

$$\int \frac{(a+bx)^n\left(c+dx^3\right)}{x}\,dx=\frac{a^2d(a+bx)^{1+n}}{b^3(1+n)}-\frac{2ad(a+bx)^{2+n}}{b^3(2+n)}+\frac{d(a+bx)^{3+n}}{b^3(3+n)}-\frac{c(a+bx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{bx}{a}\right)}{a(1+n)}$$

output
```
a^2*d*(b*x+a)^(1+n)/b^3/(1+n)-2*a*d*(b*x+a)^(2+n)/b^3/(2+n)+d*(b*x+a)^(3+n
)/b^3/(3+n)-c*(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

3.177.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx)^n\left(c+dx^3\right)}{x}\,dx=\frac{(a+bx)^{1+n}\left(ad\left(2a^2-2ab(1+n)x+b^2\left(2+3n+n^2\right)x^2\right)-b^3c\left(6+5n+n^2\right)\text{Hypergeometric2F1}\left(1,1+\right.\right.}{ab^3(1+n)(2+n)(3+n)}$$

input
```
Integrate[((a + b*x)^n*(c + d*x^3))/x,x]
```

output
```
((a + b*x)^(1 + n)*(a*d*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2
) - b^3*c*(6 + 5*n + n^2)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])
)/(a*b^3*(1 + n)*(2 + n)*(3 + n))
```

3.177.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)(a+bx)^n}{x} \, dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{a^2d(a+bx)^n}{b^2}-\frac{2ad(a+bx)^{n+1}}{b^2}+\frac{d(a+bx)^{n+2}}{b^2}+\frac{c(a+bx)^n}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2d(a+bx)^{n+1}}{b^3(n+1)}-\frac{2ad(a+bx)^{n+2}}{b^3(n+2)}+\frac{d(a+bx)^{n+3}}{b^3(n+3)}-\frac{c(a+bx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{bx}{a}+1\right)}{a(n+1)}$$

input
```
Int[((a + b*x)^n*(c + d*x^3))/x,x]
```

output
```
(a^2*d*(a + b*x)^(1 + n))/(b^3*(1 + n)) - (2*a*d*(a + b*x)^(2 + n))/(b^3*(
2 + n)) + (d*(a + b*x)^(3 + n))/(b^3*(3 + n)) - (c*(a + b*x)^(1 + n)*Hyper
geometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n))
```

3.177.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

3.177.4 Maple [F]

$$\int \frac{(bx+a)^n (x^3 d + c)}{x}dx$$

input
```
int((b*x+a)^n*(d*x^3+c)/x,x)
```

output
```
int((b*x+a)^n*(d*x^3+c)/x,x)
```

3.177.5 Fricas [F]

$$\int \frac{(a+bx)^n (c+dx^3)}{x}\,dx = \int \frac{(dx^3+c)(bx+a)^n}{x}\,dx$$

input
```
integrate((b*x+a)^n*(d*x^3+c)/x,x, algorithm="fricas")
```

output
```
integral((d*x^3 + c)*(b*x + a)^n/x, x)
```

3.177.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(83) = 166$.

Time = 2.56 (sec) , antiderivative size = 675, normalized size of antiderivative = 6.82

$$\int \frac{(a+bx)^n (c+dx^3)}{x}\,dx$$
$$= d \left(\begin{cases} \frac{a^n x^3}{3} \\ \frac{2a^2 \log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{3a^2}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{4abx \log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{4abx}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{2b^2x^2 \log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} \\ -\frac{2a^2 \log\left(\frac{a}{b}+x\right)}{ab^3+b^4x} - \frac{2a^2}{ab^3+b^4x} - \frac{2abx \log\left(\frac{a}{b}+x\right)}{ab^3+b^4x} + \frac{b^2x^2}{ab^3+b^4x} \\ \frac{a^2 \log\left(\frac{a}{b}+x\right)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b} \\ \frac{2a^3(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} - \frac{2a^2bnx(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{ab^2n^2x^2(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{ab^2nx^2(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{b^3n^2x^3(a+b}{b^3n^3+6b^3n^2+11} \end{cases} \right.$$
$$- \frac{b^{n+1} cn \left(\frac{a}{b}+x\right)^{n+1} \Phi\left(1+\frac{bx}{a}, 1, n+1\right) \Gamma(n+1)}{a\Gamma(n+2)}$$
$$- \frac{b^{n+1} c \left(\frac{a}{b}+x\right)^{n+1} \Phi\left(1+\frac{bx}{a}, 1, n+1\right) \Gamma(n+1)}{a\Gamma(n+2)}$$

input
```
integrate((b*x+a)**n*(d*x**3+c)/x,x)
```

output
```
d*Piecewise((a**n*x**3/3, Eq(b, 0)), (2*a**2*log(a/b + x)/(2*a**2*b**3 + 4
*a*b**4*x + 2*b**5*x**2) + 3*a**2/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2)
 + 4*a*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*a*b*x
/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*log(a/b + x)/(2*a*
*2*b**3 + 4*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a**2*log(a/b + x)/(a*
b**3 + b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/b + x)/(a*b**3 +
 b**4*x) + b**2*x**2/(a*b**3 + b**4*x), Eq(n, -2)), (a**2*log(a/b + x)/b**
3 - a*x/b**2 + x**2/(2*b), Eq(n, -1)), (2*a**3*(a + b*x)**n/(b**3*n**3 + 6
*b**3*n**2 + 11*b**3*n + 6*b**3) - 2*a**2*b*n*x*(a + b*x)**n/(b**3*n**3 +
6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n**2*x**2*(a + b*x)**n/(b**3*n*
*3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n*x**2*(a + b*x)**n/(b**3*
n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + b**3*n**2*x**3*(a + b*x)**n/(b*
*3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 3*b**3*n*x**3*(a + b*x)**n/(
b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 2*b**3*x**3*(a + b*x)**n/(
b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3), True)) - b**(n + 1)*c*n*(a/
b + x)**(n + 1)*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2)
) - b**(n + 1)*c*(a/b + x)**(n + 1)*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n
+ 1)/(a*gamma(n + 2))
```

3.177.7 Maxima [F]

$$\int \frac{(a+bx)^n\,(c+dx^3)}{x}\,dx = \int \frac{(dx^3+c)(bx+a)^n}{x}\,dx$$

input
```
integrate((b*x+a)^n*(d*x^3+c)/x,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)*(b*x + a)^n/x, x)
```

3.177.8 Giac [F]

$$\int \frac{(a+bx)^n (c+dx^3)}{x} \, dx = \int \frac{(dx^3+c)(bx+a)^n}{x} \, dx$$

input
```
integrate((b*x+a)^n*(d*x^3+c)/x,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)*(b*x + a)^n/x, x)
```

3.177.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^n (c+dx^3)}{x} \, dx = \int \frac{(d\,x^3+c)\,(a+b\,x)^n}{x} \, dx$$

input
```
int(((c + d*x^3)*(a + b*x)^n)/x,x)
```

output
```
int(((c + d*x^3)*(a + b*x)^n)/x, x)
```

3.178 $\int x^2(a+bx)^n (c+dx^3)^2 \, dx$

3.178.1 Optimal result

Integrand size = 20, antiderivative size = 294

$$\begin{aligned}\int x^2(a+bx)^n (c+dx^3)^2 \, dx = {} & \frac{a^2(b^3c-a^3d)^2 (a+bx)^{1+n}}{b^9(1+n)} \\ & - \frac{2a(b^3c-4a^3d)(b^3c-a^3d)(a+bx)^{2+n}}{b^9(2+n)} \\ & + \frac{(b^6c^2-20a^3b^3cd+28a^6d^2)(a+bx)^{3+n}}{b^9(3+n)} \\ & + \frac{4a^2d(5b^3c-14a^3d)(a+bx)^{4+n}}{b^9(4+n)} \\ & - \frac{10ad(b^3c-7a^3d)(a+bx)^{5+n}}{b^9(5+n)} \\ & + \frac{2d(b^3c-28a^3d)(a+bx)^{6+n}}{b^9(6+n)} + \frac{28a^2d^2(a+bx)^{7+n}}{b^9(7+n)} \\ & - \frac{8ad^2(a+bx)^{8+n}}{b^9(8+n)} + \frac{d^2(a+bx)^{9+n}}{b^9(9+n)}\end{aligned}$$

output

```
a^2*(-a^3*d+b^3*c)^2*(b*x+a)^(1+n)/b^9/(1+n)-2*a*(-4*a^3*d+b^3*c)*(-a^3*d+
b^3*c)*(b*x+a)^(2+n)/b^9/(2+n)+(28*a^6*d^2-20*a^3*b^3*c*d+b^6*c^2)*(b*x+a)
^(3+n)/b^9/(3+n)+4*a^2*d*(-14*a^3*d+5*b^3*c)*(b*x+a)^(4+n)/b^9/(4+n)-10*a*
d*(-7*a^3*d+b^3*c)*(b*x+a)^(5+n)/b^9/(5+n)+2*d*(-28*a^3*d+b^3*c)*(b*x+a)^(
6+n)/b^9/(6+n)+28*a^2*d^2*(b*x+a)^(7+n)/b^9/(7+n)-8*a*d^2*(b*x+a)^(8+n)/b^
9/(8+n)+d^2*(b*x+a)^(9+n)/b^9/(9+n)
```

3.178.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.86

$$\int x^2(a+bx)^n \left(c+dx^3\right)^2 dx$$
$$= \frac{(a+bx)^{1+n}\left(\frac{(ab^3c-a^4d)^2}{1+n}-\frac{2a(b^3c-4a^3d)(b^3c-a^3d)(a+bx)}{2+n}+\frac{(b^6c^2-20a^3b^3cd+28a^6d^2)(a+bx)^2}{3+n}+\frac{4a^2d(5b^3c-14a^3d)(a+bx)^3}{4+n}+\right.}{b^9}$$

input
```
Integrate[x^2*(a + b*x)^n*(c + d*x^3)^2,x]
```

output
```
((a + b*x)^(1 + n)*((a*b^3*c - a^4*d)^2/(1 + n) - (2*a*(b^3*c - 4*a^3*d)*(
b^3*c - a^3*d)*(a + b*x))/(2 + n) + ((b^6*c^2 - 20*a^3*b^3*c*d + 28*a^6*d^
2)*(a + b*x)^2)/(3 + n) + (4*a^2*d*(5*b^3*c - 14*a^3*d)*(a + b*x)^3)/(4 +
n) + (10*a*d*(-(b^3*c) + 7*a^3*d)*(a + b*x)^4)/(5 + n) + (2*d*(b^3*c - 28*
a^3*d)*(a + b*x)^5)/(6 + n) + (28*a^2*d^2*(a + b*x)^6)/(7 + n) - (8*a*d^2*
(a + b*x)^7)/(8 + n) + (d^2*(a + b*x)^8)/(9 + n)))/b^9
```

3.178.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(c+dx^3\right)^2 (a+bx)^n \, dx$$
$$\Big\downarrow \text{2123}$$
$$\int \left(\frac{(ab^3c-a^4d)^2 (a+bx)^n}{b^8}+\frac{10ad(7a^3d-b^3c)(a+bx)^{n+4}}{b^8}+\frac{2d(b^3c-28a^3d)(a+bx)^{n+5}}{b^8}+\frac{28a^2d^2(a+bx)^{n}}{b^8}\right.$$
$$\Big\downarrow \text{2009}$$

$$-\frac{2a(b^3c-4a^3d)\,(b^3c-a^3d)\,(a+bx)^{n+2}}{b^9(n+2)}-\frac{10ad(b^3c-7a^3d)\,(a+bx)^{n+5}}{b^9(n+5)}+\frac{2d(b^3c-28a^3d)\,(a+bx)^{n+6}}{b^9(n+6)}+\frac{28a^2d^2(a+bx)^{n+7}}{b^9(n+7)}+\frac{(28a^6d^2-20a^3b^3cd+b^6c^2)\,(a+bx)^{n+3}}{b^9(n+3)}+\frac{a^2(b^3c-a^3d)^2\,(a+bx)^{n+1}}{b^9(n+1)}+\frac{4a^2d(5b^3c-14a^3d)\,(a+bx)^{n+4}}{b^9(n+4)}-\frac{8ad^2(a+bx)^{n+8}}{b^9(n+8)}+\frac{d^2(a+bx)^{n+9}}{b^9(n+9)}$$

input
```
Int[x^2*(a + b*x)^n*(c + d*x^3)^2,x]
```

output
```
(a^2*(b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^9*(1 + n)) - (2*a*(b^3*c - 4*
a^3*d)*(b^3*c - a^3*d)*(a + b*x)^(2 + n))/(b^9*(2 + n)) + ((b^6*c^2 - 20*a
^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^(3 + n))/(b^9*(3 + n)) + (4*a^2*d*(5*b^
3*c - 14*a^3*d)*(a + b*x)^(4 + n))/(b^9*(4 + n)) - (10*a*d*(b^3*c - 7*a^3*
d)*(a + b*x)^(5 + n))/(b^9*(5 + n)) + (2*d*(b^3*c - 28*a^3*d)*(a + b*x)^(6
 + n))/(b^9*(6 + n)) + (28*a^2*d^2*(a + b*x)^(7 + n))/(b^9*(7 + n)) - (8*a
*d^2*(a + b*x)^(8 + n))/(b^9*(8 + n)) + (d^2*(a + b*x)^(9 + n))/(b^9*(9 +
n))
```

3.178.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

3.178.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1564 vs. 2(294) = 588.

Time = 1.03 (sec) , antiderivative size = 1565, normalized size of antiderivative = 5.32

method	result	size
gosper	Expression too large to display	1565
risch	Expression too large to display	1799
parallelrisch	Expression too large to display	2710

input
```
int(x^2*(b*x+a)^n*(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^9*(b*x+a)^(1+n)/(n^9+45*n^8+870*n^7+9450*n^6+63273*n^5+269325*n^4+7236
80*n^3+1172700*n^2+1026576*n+362880)*(b^8*d^2*n^8*x^8+36*b^8*d^2*n^7*x^8-8
*a*b^7*d^2*n^7*x^7+546*b^8*d^2*n^6*x^8-224*a*b^7*d^2*n^6*x^7+2*b^8*c*d*n^8
*x^5+4536*b^8*d^2*n^5*x^8+56*a^2*b^6*d^2*n^6*x^6-2576*a*b^7*d^2*n^5*x^7+78
*b^8*c*d*n^7*x^5+22449*b^8*d^2*n^4*x^8+1176*a^2*b^6*d^2*n^5*x^6-10*a*b^7*c
*d*n^7*x^4-15680*a*b^7*d^2*n^4*x^7+1272*b^8*c*d*n^6*x^5+67284*b^8*d^2*n^3*
x^8-336*a^3*b^5*d^2*n^5*x^5+9800*a^2*b^6*d^2*n^4*x^6-340*a*b^7*c*d*n^6*x^4
-54152*a*b^7*d^2*n^3*x^7+b^8*c^2*n^8*x^2+11268*b^8*c*d*n^5*x^5+118124*b^8*
d^2*n^2*x^8-5040*a^3*b^5*d^2*n^4*x^5+40*a^2*b^6*c*d*n^6*x^3+41160*a^2*b^6*
d^2*n^3*x^6-4660*a*b^7*c*d*n^5*x^4-105056*a*b^7*d^2*n^2*x^7+42*b^8*c^2*n^7
*x^2+58938*b^8*c*d*n^4*x^5+109584*b^8*d^2*n*x^8+1680*a^4*b^4*d^2*n^4*x^4-2
8560*a^3*b^5*d^2*n^3*x^5+1200*a^2*b^6*c*d*n^5*x^3+90944*a^2*b^6*d^2*n^2*x^
6-2*a*b^7*c^2*n^7*x-33040*a*b^7*c*d*n^4*x^4-104544*a*b^7*d^2*n*x^7+744*b^8
*c^2*n^6*x^2+185022*b^8*c*d*n^3*x^5+40320*b^8*d^2*x^8+16800*a^4*b^4*d^2*n^
3*x^4-120*a^3*b^5*c*d*n^5*x^2-75600*a^3*b^5*d^2*n^2*x^5+13840*a^2*b^6*c*d*
n^4*x^3+98784*a^2*b^6*d^2*n*x^6-80*a*b^7*c^2*n^6*x-129490*a*b^7*c*d*n^3*x^
4-40320*a*b^7*d^2*x^7+7218*b^8*c^2*n^5*x^2+337228*b^8*c*d*n^2*x^5-6720*a^5
*b^3*d^2*n^3*x^3+58800*a^4*b^4*d^2*n^2*x^4-3240*a^3*b^5*c*d*n^4*x^2-92064*
a^3*b^5*d^2*n*x^5+2*a^2*b^6*c^2*n^6+76800*a^2*b^6*c*d*n^3*x^3+40320*a^2*b^
6*d^2*x^6-1328*a*b^7*c^2*n^5*x-277660*a*b^7*c*d*n^2*x^4+41619*b^8*c^2*n...
```

3.178.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1565 vs. $2(294) = 588$.

Time = 0.29 (sec) , antiderivative size = 1565, normalized size of antiderivative = 5.32

$$\int x^2(a+bx)^n\left(c+dx^3\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
(2*a^3*b^6*c^2*n^6 + 78*a^3*b^6*c^2*n^5 + 1250*a^3*b^6*c^2*n^4 + 120960*a^
3*b^6*c^2 - 120960*a^6*b^3*c*d + 40320*a^9*d^2 + (b^9*d^2*n^8 + 36*b^9*d^2
*n^7 + 546*b^9*d^2*n^6 + 4536*b^9*d^2*n^5 + 22449*b^9*d^2*n^4 + 67284*b^9*
d^2*n^3 + 118124*b^9*d^2*n^2 + 109584*b^9*d^2*n + 40320*b^9*d^2)*x^9 + (a*
b^8*d^2*n^8 + 28*a*b^8*d^2*n^7 + 322*a*b^8*d^2*n^6 + 1960*a*b^8*d^2*n^5 +
6769*a*b^8*d^2*n^4 + 13132*a*b^8*d^2*n^3 + 13068*a*b^8*d^2*n^2 + 5040*a*b^
8*d^2*n)*x^8 - 8*(a^2*b^7*d^2*n^7 + 21*a^2*b^7*d^2*n^6 + 175*a^2*b^7*d^2*n
^5 + 735*a^2*b^7*d^2*n^4 + 1624*a^2*b^7*d^2*n^3 + 1764*a^2*b^7*d^2*n^2 + 7
20*a^2*b^7*d^2*n)*x^7 + 2*(b^9*c*d*n^8 + 39*b^9*c*d*n^7 + 60480*b^9*c*d +
4*(159*b^9*c*d + 7*a^3*b^6*d^2)*n^6 + 6*(939*b^9*c*d + 70*a^3*b^6*d^2)*n^5
 + (29469*b^9*c*d + 2380*a^3*b^6*d^2)*n^4 + 9*(10279*b^9*c*d + 700*a^3*b^6
*d^2)*n^3 + 2*(84307*b^9*c*d + 3836*a^3*b^6*d^2)*n^2 + 24*(6709*b^9*c*d +
140*a^3*b^6*d^2)*n)*x^6 + 2*(a*b^8*c*d*n^8 + 34*a*b^8*c*d*n^7 + 466*a*b^8*
c*d*n^6 + 56*(59*a*b^8*c*d - 3*a^4*b^5*d^2)*n^5 + (12949*a*b^8*c*d - 1680*
a^4*b^5*d^2)*n^4 + 2*(13883*a*b^8*c*d - 2940*a^4*b^5*d^2)*n^3 + 24*(1241*a
*b^8*c*d - 350*a^4*b^5*d^2)*n^2 + 4032*(3*a*b^8*c*d - a^4*b^5*d^2)*n)*x^5
- 10*(a^2*b^7*c*d*n^7 + 30*a^2*b^7*c*d*n^6 + 346*a^2*b^7*c*d*n^5 + 24*(80*
a^2*b^7*c*d - 7*a^5*b^4*d^2)*n^4 + (5269*a^2*b^7*c*d - 1008*a^5*b^4*d^2)*n
^3 + 6*(1115*a^2*b^7*c*d - 308*a^5*b^4*d^2)*n^2 + 1008*(3*a^2*b^7*c*d - a^
5*b^4*d^2)*n)*x^4 + 30*(351*a^3*b^6*c^2 - 8*a^6*b^3*c*d)*n^3 + (b^9*c^2...
```

3.178.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26746 vs. $2(275) = 550$.

Time = 8.95 (sec) , antiderivative size = 26746, normalized size of antiderivative = 90.97

$$\int x^2(a+bx)^n \left(c+dx^3\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate(x**2*(b*x+a)**n*(d*x**3+c)**2,x)
```

output
```
Piecewise((a**n*(c**2*x**3/3 + c*d*x**6/3 + d**2*x**9/9), Eq(b, 0)), (840*
a**8*d**2*log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**
11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14
*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 2283
*a**8*d**2/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47
040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 2352
0*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 6720*a**7*b*d**2
*x*log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2
 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 +
 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 17424*a**7*
b*d**2*x/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 4704
0*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*
a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 23520*a**6*b**2*d*
*2*x**2*log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11
*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x
**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 57624*
a**6*b**2*d**2*x**2/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*
x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x*
*5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) - 10*a**5
*b**3*c*d/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + ...
```

3.178.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(294) = 588$.

Time = 0.21 (sec) , antiderivative size = 601, normalized size of antiderivative = 2.04

$$\int x^2(a+bx)^n\left(c+dx^3\right)^2\,dx$$
$$= \frac{((n^2+3\,n+2)b^3x^3+(n^2+n)ab^2x^2-2\,a^2bnx+2\,a^3)(bx+a)^nc^2}{(n^3+6\,n^2+11\,n+6)b^3}$$
$$+ \frac{2\,((n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)b^6x^6+(n^5+10\,n^4+35\,n^3+50\,n^2+24\,n)ab^5x^5-5\,(n^4}{(n^6+21\,n^5+175\,n^4+7}$$
$$+ \frac{((n^8+36\,n^7+546\,n^6+4536\,n^5+22449\,n^4+67284\,n^3+118124\,n^2+109584\,n+40320)b^9x^9+(n^8+}{}$$

input
```
integrate(x^2*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
 + a)^n*c^2/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 2*((n^5 + 15*n^4 + 85*n^3 + 2
25*n^2 + 274*n + 120)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*
b^5*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2
*n)*a^3*b^3*x^3 - 60*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x
 + a)^n*c*d/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*
b^6) + ((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 11812
4*n^2 + 109584*n + 40320)*b^9*x^9 + (n^8 + 28*n^7 + 322*n^6 + 1960*n^5 + 6
769*n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a*b^8*x^8 - 8*(n^7 + 21*n^6 + 17
5*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a^2*b^7*x^7 + 56*(n^6 + 15*
n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^3*b^6*x^6 - 336*(n^5 + 10*n^4
+ 35*n^3 + 50*n^2 + 24*n)*a^4*b^5*x^5 + 1680*(n^4 + 6*n^3 + 11*n^2 + 6*n)*
a^5*b^4*x^4 - 6720*(n^3 + 3*n^2 + 2*n)*a^6*b^3*x^3 + 20160*(n^2 + n)*a^7*b
^2*x^2 - 40320*a^8*b*n*x + 40320*a^9)*(b*x + a)^n*d^2/((n^9 + 45*n^8 + 870
*n^7 + 9450*n^6 + 63273*n^5 + 269325*n^4 + 723680*n^3 + 1172700*n^2 + 1026
576*n + 362880)*b^9)
```

3.178.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2660 vs. 2(294) = 588.

Time = 0.33 (sec) , antiderivative size = 2660, normalized size of antiderivative = 9.05

$$\int x^2(a+bx)^n \left(c+dx^3\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="giac")
```

output
```
((b*x + a)^n*b^9*d^2*n^8*x^9 + (b*x + a)^n*a*b^8*d^2*n^8*x^8 + 36*(b*x + a
)^n*b^9*d^2*n^7*x^9 + 28*(b*x + a)^n*a*b^8*d^2*n^7*x^8 + 546*(b*x + a)^n*b
^9*d^2*n^6*x^9 + 2*(b*x + a)^n*b^9*c*d*n^8*x^6 - 8*(b*x + a)^n*a^2*b^7*d^2
*n^7*x^7 + 322*(b*x + a)^n*a*b^8*d^2*n^6*x^8 + 4536*(b*x + a)^n*b^9*d^2*n^
5*x^9 + 2*(b*x + a)^n*a*b^8*c*d*n^8*x^5 + 78*(b*x + a)^n*b^9*c*d*n^7*x^6 -
 168*(b*x + a)^n*a^2*b^7*d^2*n^6*x^7 + 1960*(b*x + a)^n*a*b^8*d^2*n^5*x^8
+ 22449*(b*x + a)^n*b^9*d^2*n^4*x^9 + 68*(b*x + a)^n*a*b^8*c*d*n^7*x^5 + 1
272*(b*x + a)^n*b^9*c*d*n^6*x^6 + 56*(b*x + a)^n*a^3*b^6*d^2*n^6*x^6 - 140
0*(b*x + a)^n*a^2*b^7*d^2*n^5*x^7 + 6769*(b*x + a)^n*a*b^8*d^2*n^4*x^8 + 6
7284*(b*x + a)^n*b^9*d^2*n^3*x^9 + (b*x + a)^n*b^9*c^2*n^8*x^3 - 10*(b*x +
 a)^n*a^2*b^7*c*d*n^7*x^4 + 932*(b*x + a)^n*a*b^8*c*d*n^6*x^5 + 11268*(b*x
 + a)^n*b^9*c*d*n^5*x^6 + 840*(b*x + a)^n*a^3*b^6*d^2*n^5*x^6 - 5880*(b*x
+ a)^n*a^2*b^7*d^2*n^4*x^7 + 13132*(b*x + a)^n*a*b^8*d^2*n^3*x^8 + 118124*
(b*x + a)^n*b^9*d^2*n^2*x^9 + (b*x + a)^n*a*b^8*c^2*n^8*x^2 + 42*(b*x + a)
^n*b^9*c^2*n^7*x^3 - 300*(b*x + a)^n*a^2*b^7*c*d*n^6*x^4 + 6608*(b*x + a)^
n*a*b^8*c*d*n^5*x^5 - 336*(b*x + a)^n*a^4*b^5*d^2*n^5*x^5 + 58938*(b*x + a
)^n*b^9*c*d*n^4*x^6 + 4760*(b*x + a)^n*a^3*b^6*d^2*n^4*x^6 - 12992*(b*x +
a)^n*a^2*b^7*d^2*n^3*x^7 + 13068*(b*x + a)^n*a*b^8*d^2*n^2*x^8 + 109584*(b
*x + a)^n*b^9*d^2*n*x^9 + 40*(b*x + a)^n*a*b^8*c^2*n^7*x^2 + 744*(b*x + a)
^n*b^9*c^2*n^6*x^3 + 40*(b*x + a)^n*a^3*b^6*c*d*n^6*x^3 - 3460*(b*x + a...
```

3.178.9 Mupad [B] (verification not implemented)

Time = 19.99 (sec) , antiderivative size = 1410, normalized size of antiderivative = 4.80

$$\int x^2(a+bx)^n\left(c+dx^3\right)^2\,dx=\text{Too large to display}$$

input
```
int(x^2*(c + d*x^3)^2*(a + b*x)^n,x)
```

output
```
(d^2*x^9*(a + b*x)^n*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536
*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))/(1026576*n + 1172700*n^2 + 723680*
n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)
 + (2*a^3*(a + b*x)^n*(20160*a^6*d^2 + 60480*b^6*c^2 + 60216*b^6*c^2*n + 2
4574*b^6*c^2*n^2 + 5265*b^6*c^2*n^3 + 625*b^6*c^2*n^4 + 39*b^6*c^2*n^5 + b
^6*c^2*n^6 - 60480*a^3*b^3*c*d - 22920*a^3*b^3*c*d*n - 2880*a^3*b^3*c*d*n^
2 - 120*a^3*b^3*c*d*n^3))/(b^9*(1026576*n + 1172700*n^2 + 723680*n^3 + 269
325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) + (x^3*
(a + b*x)^n*(3*n + n^2 + 2)*(60480*b^6*c^2 - 6720*a^6*d^2*n + 60216*b^6*c^
2*n + 24574*b^6*c^2*n^2 + 5265*b^6*c^2*n^3 + 625*b^6*c^2*n^4 + 39*b^6*c^2*
n^5 + b^6*c^2*n^6 + 20160*a^3*b^3*c*d*n + 7640*a^3*b^3*c*d*n^2 + 960*a^3*b
^3*c*d*n^3 + 40*a^3*b^3*c*d*n^4))/(b^6*(1026576*n + 1172700*n^2 + 723680*n
^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880))
 + (2*d*x^6*(a + b*x)^n*(504*b^3*c + 24*b^3*c*n^2 + b^3*c*n^3 + 28*a^3*d*n
 + 191*b^3*c*n)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(b^3*(102
6576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 87
0*n^7 + 45*n^8 + n^9 + 362880)) - (2*a^2*n*x*(a + b*x)^n*(20160*a^6*d^2 +
60480*b^6*c^2 + 60216*b^6*c^2*n + 24574*b^6*c^2*n^2 + 5265*b^6*c^2*n^3 + 6
25*b^6*c^2*n^4 + 39*b^6*c^2*n^5 + b^6*c^2*n^6 - 60480*a^3*b^3*c*d - 22920*
a^3*b^3*c*d*n - 2880*a^3*b^3*c*d*n^2 - 120*a^3*b^3*c*d*n^3))/(b^8*(1026...
```

3.178. $\int x^2(a+bx)^n \left(c+dx^3\right)^2 \, dx$

3.179 $\int x(a+bx)^n \left(c+dx^3\right)^2 \, dx$

3.179.1 Optimal result

Integrand size = 18, antiderivative size = 248

$$\begin{aligned}\int x(a+bx)^n \left(c+dx^3\right)^2 \, dx = & -\frac{a(b^3c-a^3d)^2 (a+bx)^{1+n}}{b^8(1+n)} \\ & + \frac{(b^3c-7a^3d)(b^3c-a^3d)(a+bx)^{2+n}}{b^8(2+n)} \\ & + \frac{3a^2d(4b^3c-7a^3d)(a+bx)^{3+n}}{b^8(3+n)} \\ & - \frac{ad(8b^3c-35a^3d)(a+bx)^{4+n}}{b^8(4+n)} \\ & + \frac{d(2b^3c-35a^3d)(a+bx)^{5+n}}{b^8(5+n)} + \frac{21a^2d^2(a+bx)^{6+n}}{b^8(6+n)} \\ & - \frac{7ad^2(a+bx)^{7+n}}{b^8(7+n)} + \frac{d^2(a+bx)^{8+n}}{b^8(8+n)}\end{aligned}$$

output
```
-a*(-a^3*d+b^3*c)^2*(b*x+a)^(1+n)/b^8/(1+n)+(-7*a^3*d+b^3*c)*(-a^3*d+b^3*c
)*(b*x+a)^(2+n)/b^8/(2+n)+3*a^2*d*(-7*a^3*d+4*b^3*c)*(b*x+a)^(3+n)/b^8/(3+
n)-a*d*(-35*a^3*d+8*b^3*c)*(b*x+a)^(4+n)/b^8/(4+n)+d*(-35*a^3*d+2*b^3*c)*(
b*x+a)^(5+n)/b^8/(5+n)+21*a^2*d^2*(b*x+a)^(6+n)/b^8/(6+n)-7*a*d^2*(b*x+a)^
(7+n)/b^8/(7+n)+d^2*(b*x+a)^(8+n)/b^8/(8+n)
```

3.179.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.85

$$\int x(a+bx)^n \left(c+dx^3\right)^2 \, dx = \frac{(a+bx)^{1+n}\left(-\frac{a(b^3c-a^3d)^2}{1+n}+\frac{(b^3c-7a^3d)(b^3c-a^3d)(a+bx)}{2+n}+\frac{3a^2d(4b^3c-7a^3d)(a+bx)^2}{3+n}+\frac{ad(-8b^3c+35a^3d)(a+bx)^3}{4+n}+\frac{d(2b^3c-\cdots}{}\right.}{b^8}$$

input
```
Integrate[x*(a + b*x)^n*(c + d*x^3)^2,x]
```

output
```
((a + b*x)^(1 + n)*(-((a*(b^3*c - a^3*d)^2)/(1 + n)) + ((b^3*c - 7*a^3*d)*
(b^3*c - a^3*d)*(a + b*x))/(2 + n) + (3*a^2*d*(4*b^3*c - 7*a^3*d)*(a + b*x
)^2)/(3 + n) + (a*d*(-8*b^3*c + 35*a^3*d)*(a + b*x)^3)/(4 + n) + (d*(2*b^3
*c - 35*a^3*d)*(a + b*x)^4)/(5 + n) + (21*a^2*d^2*(a + b*x)^5)/(6 + n) - (
7*a*d^2*(a + b*x)^6)/(7 + n) + (d^2*(a + b*x)^7)/(8 + n)))/b^8
```

3.179.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(c+dx^3\right)^2 (a+bx)^n \, dx$$

$$\Big\downarrow \; 2123$$

$$\int \left(-\frac{a\left(a^3d-b^3c\right)^2 (a+bx)^n}{b^7}+\frac{\left(b^3c-7a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+1}}{b^7}+\frac{ad\left(35a^3d-8b^3c\right)(a+bx)^{n+3}}{b^7}+\frac{d(2b^3c\cdots}{}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{a\left(b^3c-a^3d\right)^2 (a+bx)^{n+1}}{b^8(n+1)}+\frac{\left(b^3c-7a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+2}}{b^8(n+2)}-\frac{ad\left(8b^3c-35a^3d\right)(a+bx)^{n+4}}{b^8(n+4)}+\frac{d\left(2b^3c-35a^3d\right)(a+bx)^{n+5}}{b^8(n+5)}+\frac{21a^2d^2(a+bx)^{n+6}}{b^8(n+6)}+\frac{3a^2d\left(4b^3c-7a^3d\right)(a+bx)^{n+3}}{b^8(n+3)}-\frac{7ad^2(a+bx)^{n+7}}{b^8(n+7)}+\frac{d^2(a+bx)^{n+8}}{b^8(n+8)}$$

input
```
Int[x*(a + b*x)^n*(c + d*x^3)^2,x]
```

output
```
-((a*(b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^8*(1 + n))) + ((b^3*c - 7*a^3
*d)*(b^3*c - a^3*d)*(a + b*x)^(2 + n))/(b^8*(2 + n)) + (3*a^2*d*(4*b^3*c -
 7*a^3*d)*(a + b*x)^(3 + n))/(b^8*(3 + n)) - (a*d*(8*b^3*c - 35*a^3*d)*(a
+ b*x)^(4 + n))/(b^8*(4 + n)) + (d*(2*b^3*c - 35*a^3*d)*(a + b*x)^(5 + n))
/(b^8*(5 + n)) + (21*a^2*d^2*(a + b*x)^(6 + n))/(b^8*(6 + n)) - (7*a*d^2*(
a + b*x)^(7 + n))/(b^8*(7 + n)) + (d^2*(a + b*x)^(8 + n))/(b^8*(8 + n))
```

3.179.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

3.179.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 892 vs. $2(248) = 496$.

Time = 1.00 (sec) , antiderivative size = 893, normalized size of antiderivative = 3.60

method	result
norman	$\frac{d^2 x^8 e^{n \ln(bx+a)}}{8+n} + \frac{na\left(b^6c^2n^6+33b^6c^2n^5+445b^6c^2n^4-48a^3b^3cd\,n^3+3135b^6c^2n^3-1008a^3b^3cd\,n^2+12154b^6c^2n^2-7008a^3b^3c\right.}{b^7\left(n^8+36n^7+546n^6+4536n^5+22449n^4+67284n^3+118124\right.}$
gosper	Expression too large to display
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int(x*(b*x+a)^n*(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
d^2/(8+n)*x^8*exp(n*ln(b*x+a))+1/b^7*n*a*(b^6*c^2*n^6+33*b^6*c^2*n^5+445*b
^6*c^2*n^4-48*a^3*b^3*c*d*n^3+3135*b^6*c^2*n^3-1008*a^3*b^3*c*d*n^2+12154*
b^6*c^2*n^2-7008*a^3*b^3*c*d*n+24552*b^6*c^2*n+5040*a^6*d^2-16128*a^3*b^3*
c*d+20160*b^6*c^2)/(n^8+36*n^7+546*n^6+4536*n^5+22449*n^4+67284*n^3+118124
*n^2+109584*n+40320)*x*exp(n*ln(b*x+a))+d^2*a*n/b/(n^2+15*n+56)*x^7*exp(n*
ln(b*x+a))-a^2*(b^6*c^2*n^6+33*b^6*c^2*n^5+445*b^6*c^2*n^4-48*a^3*b^3*c*d*
n^3+3135*b^6*c^2*n^3-1008*a^3*b^3*c*d*n^2+12154*b^6*c^2*n^2-7008*a^3*b^3*c
*d*n+24552*b^6*c^2*n+5040*a^6*d^2-16128*a^3*b^3*c*d+20160*b^6*c^2)/b^8/(n^
8+36*n^7+546*n^6+4536*n^5+22449*n^4+67284*n^3+118124*n^2+109584*n+40320)*e
xp(n*ln(b*x+a))-(-b^6*c^2*n^6-33*b^6*c^2*n^5-24*a^3*b^3*c*d*n^4-445*b^6*c^
2*n^4-504*a^3*b^3*c*d*n^3-3135*b^6*c^2*n^3-3504*a^3*b^3*c*d*n^2-12154*b^6*
c^2*n^2+2520*a^6*d^2*n-8064*a^3*b^3*c*d*n-24552*b^6*c^2*n-20160*b^6*c^2)/b
^6/(n^7+35*n^6+511*n^5+4025*n^4+18424*n^3+48860*n^2+69264*n+40320)*x^2*exp
(n*ln(b*x+a))+2*d*(b^3*c*n^3+21*b^3*c*n^2+21*a^3*d*n+146*b^3*c*n+336*b^3*c
)/b^3/(n^4+26*n^3+251*n^2+1066*n+1680)*x^5*exp(n*ln(b*x+a))-7*n*a^2*d^2/b^
2/(n^3+21*n^2+146*n+336)*x^6*exp(n*ln(b*x+a))-2*n*a*d*(-b^3*c*n^3-21*b^3*c
*n^2-146*b^3*c*n+105*a^3*d-336*b^3*c)/b^4/(n^5+30*n^4+355*n^3+2070*n^2+594
4*n+6720)*x^4*exp(n*ln(b*x+a))+8*(-b^3*c*n^3-21*b^3*c*n^2-146*b^3*c*n+105*
a^3*d-336*b^3*c)*a^2/b^5*d*n/(n^6+33*n^5+445*n^4+3135*n^3+12154*n^2+24552*
n+20160)*x^3*exp(n*ln(b*x+a))
```

3.179.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1216 vs. $2(248) = 496$.

Time = 0.30 (sec) , antiderivative size = 1216, normalized size of antiderivative = 4.90

$$\int x(a+bx)^n \left(c+dx^3\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate(x*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
-(a^2*b^6*c^2*n^6 + 33*a^2*b^6*c^2*n^5 + 445*a^2*b^6*c^2*n^4 + 20160*a^2*b
^6*c^2 - 16128*a^5*b^3*c*d + 5040*a^8*d^2 - (b^8*d^2*n^7 + 28*b^8*d^2*n^6
+ 322*b^8*d^2*n^5 + 1960*b^8*d^2*n^4 + 6769*b^8*d^2*n^3 + 13132*b^8*d^2*n^
2 + 13068*b^8*d^2*n + 5040*b^8*d^2)*x^8 - (a*b^7*d^2*n^7 + 21*a*b^7*d^2*n^
6 + 175*a*b^7*d^2*n^5 + 735*a*b^7*d^2*n^4 + 1624*a*b^7*d^2*n^3 + 1764*a*b^
7*d^2*n^2 + 720*a*b^7*d^2*n)*x^7 + 7*(a^2*b^6*d^2*n^6 + 15*a^2*b^6*d^2*n^5
 + 85*a^2*b^6*d^2*n^4 + 225*a^2*b^6*d^2*n^3 + 274*a^2*b^6*d^2*n^2 + 120*a^
2*b^6*d^2*n)*x^6 - 2*(b^8*c*d*n^7 + 31*b^8*c*d*n^6 + 8064*b^8*c*d + (391*b
^8*c*d + 21*a^3*b^5*d^2)*n^5 + (2581*b^8*c*d + 210*a^3*b^5*d^2)*n^4 + (954
4*b^8*c*d + 735*a^3*b^5*d^2)*n^3 + 2*(9782*b^8*c*d + 525*a^3*b^5*d^2)*n^2
+ 72*(282*b^8*c*d + 7*a^3*b^5*d^2)*n)*x^5 - 2*(a*b^7*c*d*n^7 + 27*a*b^7*c*
d*n^6 + 283*a*b^7*c*d*n^5 + 21*(69*a*b^7*c*d - 5*a^4*b^4*d^2)*n^4 + 2*(187
4*a*b^7*c*d - 315*a^4*b^4*d^2)*n^3 + 3*(1524*a*b^7*c*d - 385*a^4*b^4*d^2)*
n^2 + 126*(16*a*b^7*c*d - 5*a^4*b^4*d^2)*n)*x^4 + 3*(1045*a^2*b^6*c^2 - 16
*a^5*b^3*c*d)*n^3 + 8*(a^2*b^6*c*d*n^6 + 24*a^2*b^6*c*d*n^5 + 211*a^2*b^6*
c*d*n^4 + 3*(272*a^2*b^6*c*d - 35*a^5*b^3*d^2)*n^3 + 5*(260*a^2*b^6*c*d -
63*a^5*b^3*d^2)*n^2 + 42*(16*a^2*b^6*c*d - 5*a^5*b^3*d^2)*n)*x^3 + 2*(6077
*a^2*b^6*c^2 - 504*a^5*b^3*c*d)*n^2 - (b^8*c^2*n^7 + 34*b^8*c^2*n^6 + 2016
0*b^8*c^2 + 2*(239*b^8*c^2 + 12*a^3*b^5*c*d)*n^5 + 4*(895*b^8*c^2 + 132*a^
3*b^5*c*d)*n^4 + (15289*b^8*c^2 + 4008*a^3*b^5*c*d)*n^3 + 2*(18353*b^8*...
```

3.179.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18328 vs. 2(228) = 456.

Time = 5.69 (sec) , antiderivative size = 18328, normalized size of antiderivative = 73.90

$$\int x(a+bx)^n \left(c+dx^3\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate(x*(b*x+a)**n*(d*x**3+c)**2,x)
```

output
```
Piecewise((a**n*(c**2*x**2/2 + 2*c*d*x**5/5 + d**2*x**8/8), Eq(b, 0)), (42
0*a**7*d**2*log(a/b + x)/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**
10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*
x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 1089*a**7*d**2/(420*a**7*b**8
 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700
*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x*
*7) + 2940*a**6*b*d**2*x*log(a/b + x)/(420*a**7*b**8 + 2940*a**6*b**9*x +
8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 882
0*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 7203*a**6*b*d**2
*x/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b
**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x*
*6 + 420*b**15*x**7) + 8820*a**5*b**2*d**2*x**2*log(a/b + x)/(420*a**7*b**
8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 1470
0*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x
**7) + 20139*a**5*b**2*d**2*x**2/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*
a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**
2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) - 8*a**4*b**3*c*d/(420*
a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**
3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420
*b**15*x**7) + 14700*a**4*b**3*d**2*x**3*log(a/b + x)/(420*a**7*b**8 + ...
```

3.179.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.91

$$\int x(a+bx)^n \left(c+dx^3\right)^2 \, dx = \frac{(b^2(n+1)x^2 + abnx - a^2)(bx+a)^n c^2}{(n^2+3n+2)b^2} + \frac{2\left((n^4+10n^3+35n^2+50n+24)b^5x^5 + (n^4+6n^3+11n^2+6n)ab^4x^4 - 4(n^3+3n^2+2n)a^2b^3x^3 + \right.}{(n^5+15n^4+85n^3+225n^2+274n+120)b^5} + \frac{\left((n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040)b^8x^8 + (n^7+21n^6+175n^5+\right.}{}$$

input
```
integrate(x*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
(b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^2/((n^2 + 3*n + 2)*b^2) +
2*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6
*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x
^2 - 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*c*d/((n^5 + 15*n^4 + 85*n^3 + 225*
n^2 + 274*n + 120)*b^5) + ((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769*n^3 +
 13132*n^2 + 13068*n + 5040)*b^8*x^8 + (n^7 + 21*n^6 + 175*n^5 + 735*n^4 +
 1624*n^3 + 1764*n^2 + 720*n)*a*b^7*x^7 - 7*(n^6 + 15*n^5 + 85*n^4 + 225*n
^3 + 274*n^2 + 120*n)*a^2*b^6*x^6 + 42*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 2
4*n)*a^3*b^5*x^5 - 210*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^4*b^4*x^4 + 840*(n^3
 + 3*n^2 + 2*n)*a^5*b^3*x^3 - 2520*(n^2 + n)*a^6*b^2*x^2 + 5040*a^7*b*n*x
- 5040*a^8)*(b*x + a)^n*d^2/((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^
4 + 67284*n^3 + 118124*n^2 + 109584*n + 40320)*b^8)
```

3.179.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2034 vs. 2(248) = 496.

Time = 0.33 (sec) , antiderivative size = 2034, normalized size of antiderivative = 8.20

$$\int x(a+bx)^n \left(c+dx^3\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate(x*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="giac")
```

output
```
((b*x + a)^n*b^8*d^2*n^7*x^8 + (b*x + a)^n*a*b^7*d^2*n^7*x^7 + 28*(b*x + a
)^n*b^8*d^2*n^6*x^8 + 21*(b*x + a)^n*a*b^7*d^2*n^6*x^7 + 322*(b*x + a)^n*b
^8*d^2*n^5*x^8 + 2*(b*x + a)^n*b^8*c*d*n^7*x^5 - 7*(b*x + a)^n*a^2*b^6*d^2
*n^6*x^6 + 175*(b*x + a)^n*a*b^7*d^2*n^5*x^7 + 1960*(b*x + a)^n*b^8*d^2*n^
4*x^8 + 2*(b*x + a)^n*a*b^7*c*d*n^7*x^4 + 62*(b*x + a)^n*b^8*c*d*n^6*x^5 -
 105*(b*x + a)^n*a^2*b^6*d^2*n^5*x^6 + 735*(b*x + a)^n*a*b^7*d^2*n^4*x^7 +
 6769*(b*x + a)^n*b^8*d^2*n^3*x^8 + 54*(b*x + a)^n*a*b^7*c*d*n^6*x^4 + 782
*(b*x + a)^n*b^8*c*d*n^5*x^5 + 42*(b*x + a)^n*a^3*b^5*d^2*n^5*x^5 - 595*(b
*x + a)^n*a^2*b^6*d^2*n^4*x^6 + 1624*(b*x + a)^n*a*b^7*d^2*n^3*x^7 + 13132
*(b*x + a)^n*b^8*d^2*n^2*x^8 + (b*x + a)^n*b^8*c^2*n^7*x^2 - 8*(b*x + a)^n
*a^2*b^6*c*d*n^6*x^3 + 566*(b*x + a)^n*a*b^7*c*d*n^5*x^4 + 5162*(b*x + a)^
n*b^8*c*d*n^4*x^5 + 420*(b*x + a)^n*a^3*b^5*d^2*n^4*x^5 - 1575*(b*x + a)^n
*a^2*b^6*d^2*n^3*x^6 + 1764*(b*x + a)^n*a*b^7*d^2*n^2*x^7 + 13068*(b*x + a
)^n*b^8*d^2*n*x^8 + (b*x + a)^n*a*b^7*c^2*n^7*x + 34*(b*x + a)^n*b^8*c^2*n
^6*x^2 - 192*(b*x + a)^n*a^2*b^6*c*d*n^5*x^3 + 2898*(b*x + a)^n*a*b^7*c*d*
n^4*x^4 - 210*(b*x + a)^n*a^4*b^4*d^2*n^4*x^4 + 19088*(b*x + a)^n*b^8*c*d*
n^3*x^5 + 1470*(b*x + a)^n*a^3*b^5*d^2*n^3*x^5 - 1918*(b*x + a)^n*a^2*b^6*
d^2*n^2*x^6 + 720*(b*x + a)^n*a*b^7*d^2*n*x^7 + 5040*(b*x + a)^n*b^8*d^2*x
^8 + 33*(b*x + a)^n*a*b^7*c^2*n^6*x + 478*(b*x + a)^n*b^8*c^2*n^5*x^2 + 24
*(b*x + a)^n*a^3*b^5*c*d*n^5*x^2 - 1688*(b*x + a)^n*a^2*b^6*c*d*n^4*x^3...
```

3.179. $\int x(a+bx)^n \left(c+dx^3\right)^2 \, dx$

3.179.9 Mupad [B] (verification not implemented)

Time = 20.36 (sec) , antiderivative size = 1136, normalized size of antiderivative = 4.58

$$\int x(a+bx)^n\left(c+dx^3\right)^2\,dx$$
$$= \frac{d^2\,x^8\,(a+b\,x)^n\,(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040)}{n^8+36\,n^7+546\,n^6+4536\,n^5+22449\,n^4+67284\,n^3+118124\,n^2+109584\,n+40320}$$
$$- \frac{a^2\,(a+b\,x)^n\,(5040\,a^6\,d^2-48\,a^3\,b^3\,c\,d\,n^3-1008\,a^3\,b^3\,c\,d\,n^2-7008\,a^3\,b^3\,c\,d\,n-16128\,a^3\,b^3\,c\,d+b^6\,c^2\,n^6}{b^8\,(n^8+36\,n^7+546\,n^6+4536\,n^5+22449\,n^4+6728}$$
$$+ \frac{x^2\,(n+1)\,(a+b\,x)^n\,(-2520\,a^6\,d^2\,n+24\,a^3\,b^3\,c\,d\,n^4+504\,a^3\,b^3\,c\,d\,n^3+3504\,a^3\,b^3\,c\,d\,n^2+8064\,a^3\,b^3\,c\,d}{b^6\,(n^8+36\,n^7+546\,n^6+4536\,n^5+22449\,n^4+}$$
$$+ \frac{a\,n\,x\,(a+b\,x)^n\,(5040\,a^6\,d^2-48\,a^3\,b^3\,c\,d\,n^3-1008\,a^3\,b^3\,c\,d\,n^2-7008\,a^3\,b^3\,c\,d\,n-16128\,a^3\,b^3\,c\,d+b^6\,c^2}{b^7\,(n^8+36\,n^7+546\,n^6+4536\,n^5+22449\,n^4+672}$$
$$+ \frac{2\,d\,x^5\,(a+b\,x)^n\,(n^4+10\,n^3+35\,n^2+50\,n+24)\,(21\,d\,a^3\,n+c\,b^3\,n^3+21\,c\,b^3\,n^2+146\,c\,b^3\,n+336\,c\,b^3)}{b^3\,(n^8+36\,n^7+546\,n^6+4536\,n^5+22449\,n^4+67284\,n^3+118124\,n^2+109584\,n+40320)}$$
$$+ \frac{a\,d^2\,n\,x^7\,(a+b\,x)^n\,(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)}{b\,(n^8+36\,n^7+546\,n^6+4536\,n^5+22449\,n^4+67284\,n^3+118124\,n^2+109584\,n+40320)}$$
$$- \frac{7\,a^2\,d^2\,n\,x^6\,(a+b\,x)^n\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)}{b^2\,(n^8+36\,n^7+546\,n^6+4536\,n^5+22449\,n^4+67284\,n^3+118124\,n^2+109584\,n+40320)}$$
$$+ \frac{2\,a\,d\,n\,x^4\,(a+b\,x)^n\,(n^3+6\,n^2+11\,n+6)\,(-105\,d\,a^3+c\,b^3\,n^3+21\,c\,b^3\,n^2+146\,c\,b^3\,n+336\,c\,b^3)}{b^4\,(n^8+36\,n^7+546\,n^6+4536\,n^5+22449\,n^4+67284\,n^3+118124\,n^2+109584\,n+40320)}$$
$$- \frac{8\,a^2\,d\,n\,x^3\,(a+b\,x)^n\,(n^2+3\,n+2)\,(-105\,d\,a^3+c\,b^3\,n^3+21\,c\,b^3\,n^2+146\,c\,b^3\,n+336\,c\,b^3)}{b^5\,(n^8+36\,n^7+546\,n^6+4536\,n^5+22449\,n^4+67284\,n^3+118124\,n^2+109584\,n+40320)}$$

input

```
int(x*(c + d*x^3)^2*(a + b*x)^n,x)
```

output
```
(d^2*x^8*(a + b*x)^n*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5
+ 28*n^6 + n^7 + 5040))/(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4
536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320) - (a^2*(a + b*x)^n*(5040*a^6*d^2
 + 20160*b^6*c^2 + 24552*b^6*c^2*n + 12154*b^6*c^2*n^2 + 3135*b^6*c^2*n^3
+ 445*b^6*c^2*n^4 + 33*b^6*c^2*n^5 + b^6*c^2*n^6 - 16128*a^3*b^3*c*d - 700
8*a^3*b^3*c*d*n - 1008*a^3*b^3*c*d*n^2 - 48*a^3*b^3*c*d*n^3))/(b^8*(109584
*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^
8 + 40320)) + (x^2*(n + 1)*(a + b*x)^n*(20160*b^6*c^2 - 2520*a^6*d^2*n + 2
4552*b^6*c^2*n + 12154*b^6*c^2*n^2 + 3135*b^6*c^2*n^3 + 445*b^6*c^2*n^4 +
33*b^6*c^2*n^5 + b^6*c^2*n^6 + 8064*a^3*b^3*c*d*n + 3504*a^3*b^3*c*d*n^2 +
 504*a^3*b^3*c*d*n^3 + 24*a^3*b^3*c*d*n^4))/(b^6*(109584*n + 118124*n^2 +
67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (a*n
*x*(a + b*x)^n*(5040*a^6*d^2 + 20160*b^6*c^2 + 24552*b^6*c^2*n + 12154*b^6
*c^2*n^2 + 3135*b^6*c^2*n^3 + 445*b^6*c^2*n^4 + 33*b^6*c^2*n^5 + b^6*c^2*n
^6 - 16128*a^3*b^3*c*d - 7008*a^3*b^3*c*d*n - 1008*a^3*b^3*c*d*n^2 - 48*a^
3*b^3*c*d*n^3))/(b^7*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536
*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (2*d*x^5*(a + b*x)^n*(50*n + 35*
n^2 + 10*n^3 + n^4 + 24)*(336*b^3*c + 21*b^3*c*n^2 + b^3*c*n^3 + 21*a^3*d*
n + 146*b^3*c*n))/(b^3*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 45
36*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (a*d^2*n*x^7*(a + b*x)^n*(1...
```

3.179. $\int x(a+bx)^n \left(c+dx^3\right)^2 \, dx$

3.180 $\int (a+bx)^n \left(c+dx^3\right)^2 \, dx$

3.180.1 Optimal result

Integrand size = 17, antiderivative size = 203

$$\int (a+bx)^n \left(c+dx^3\right)^2 \, dx = \frac{\left(b^3c-a^3d\right)^2 (a+bx)^{1+n}}{b^7(1+n)}+\frac{6a^2d(b^3c-a^3d)\,(a+bx)^{2+n}}{b^7(2+n)} -\frac{3ad(2b^3c-5a^3d)\,(a+bx)^{3+n}}{b^7(3+n)}+\frac{2d(b^3c-10a^3d)\,(a+bx)^{4+n}}{b^7(4+n)} +\frac{15a^2d^2(a+bx)^{5+n}}{b^7(5+n)}-\frac{6ad^2(a+bx)^{6+n}}{b^7(6+n)}+\frac{d^2(a+bx)^{7+n}}{b^7(7+n)}$$

output
```
(-a^3*d+b^3*c)^2*(b*x+a)^(1+n)/b^7/(1+n)+6*a^2*d*(-a^3*d+b^3*c)*(b*x+a)^(2
+n)/b^7/(2+n)-3*a*d*(-5*a^3*d+2*b^3*c)*(b*x+a)^(3+n)/b^7/(3+n)+2*d*(-10*a^
3*d+b^3*c)*(b*x+a)^(4+n)/b^7/(4+n)+15*a^2*d^2*(b*x+a)^(5+n)/b^7/(5+n)-6*a*
d^2*(b*x+a)^(6+n)/b^7/(6+n)+d^2*(b*x+a)^(7+n)/b^7/(7+n)
```

3.180.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.85

$$\int (a+bx)^n \left(c+dx^3\right)^2 \, dx = \frac{(a+bx)^{1+n}\left(\frac{\left(b^3c-a^3d\right)^2}{1+n}+\frac{6a^2d\left(b^3c-a^3d\right)(a+bx)}{2+n}+\frac{3ad\left(-2b^3c+5a^3d\right)(a+bx)^2}{3+n}+\frac{2d\left(b^3c-10a^3d\right)(a+bx)^3}{4+n}+\frac{15a^2d^2(a+bx)^4}{5+n}-6\right.}{b^7}$$

input
```
Integrate[(a + b*x)^n*(c + d*x^3)^2,x]
```

output
```
((a + b*x)^(1 + n)*((b^3*c - a^3*d)^2/(1 + n) + (6*a^2*d*(b^3*c - a^3*d)*(
a + b*x))/(2 + n) + (3*a*d*(-2*b^3*c + 5*a^3*d)*(a + b*x)^2)/(3 + n) + (2*
d*(b^3*c - 10*a^3*d)*(a + b*x)^3)/(4 + n) + (15*a^2*d^2*(a + b*x)^4)/(5 +
n) - (6*a*d^2*(a + b*x)^5)/(6 + n) + (d^2*(a + b*x)^6)/(7 + n)))/b^7
```

3.180.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+dx^3\right)^2 (a+bx)^n \, dx$$

$$\Big\downarrow \text{2389}$$

$$\int \left(\frac{\left(b^3c-a^3d\right)^2 (a+bx)^n}{b^6}+\frac{3ad\left(5a^3d-2b^3c\right)(a+bx)^{n+2}}{b^6}+\frac{2d\left(b^3c-10a^3d\right)(a+bx)^{n+3}}{b^6}+\frac{15a^2d^2(a+bx)^{n+}}{b^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(b^3c-a^3d\right)^2 (a+bx)^{n+1}}{b^7(n+1)}-\frac{3ad\left(2b^3c-5a^3d\right)(a+bx)^{n+3}}{b^7(n+3)}+\frac{2d\left(b^3c-10a^3d\right)(a+bx)^{n+4}}{b^7(n+4)}+\frac{15a^2d^2(a+bx)^{n+5}}{b^7(n+5)}+\frac{6a^2d\left(b^3c-a^3d\right)(a+bx)^{n+2}}{b^7(n+2)}-\frac{6ad^2(a+bx)^{n+6}}{b^7(n+6)}+\frac{d^2(a+bx)^{n+7}}{b^7(n+7)}$$

input
```
Int[(a + b*x)^n*(c + d*x^3)^2,x]
```

output
```
((b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^7*(1 + n)) + (6*a^2*d*(b^3*c - a^
3*d)*(a + b*x)^(2 + n))/(b^7*(2 + n)) - (3*a*d*(2*b^3*c - 5*a^3*d)*(a + b*
x)^(3 + n))/(b^7*(3 + n)) + (2*d*(b^3*c - 10*a^3*d)*(a + b*x)^(4 + n))/(b^
7*(4 + n)) + (15*a^2*d^2*(a + b*x)^(5 + n))/(b^7*(5 + n)) - (6*a*d^2*(a +
b*x)^(6 + n))/(b^7*(6 + n)) + (d^2*(a + b*x)^(7 + n))/(b^7*(7 + n))
```

3.180.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.180.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(203) = 406$.

Time = 0.95 (sec) , antiderivative size = 700, normalized size of antiderivative = 3.45

method	result
norman	$\frac{d^2x^7\mathrm{e}^{n\ln(bx+a)}}{7+n}+\frac{a\left(b^6c^2n^6+27b^6c^2n^5+295b^6c^2n^4-12a^3b^3cd\,n^3+1665b^6c^2n^3-216a^3b^3cd\,n^2+5104b^6c^2n^2-1284a^3b^3cdn+\right.}{b^7\left(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068\right.}$
gosper	$\frac{(bx+a)^{1+n}\left(b^6d^2n^6x^6+21b^6d^2n^5x^6-6a\,b^5d^2n^5x^5+175b^6d^2n^4x^6-90a\,b^5d^2n^4x^5+2b^6cd\,n^6x^3+735b^6d^2n^3x^6+30a^2b^4d^2n^4x^4-\right.}{}$
risch	$\frac{\left(b^7d^2n^6x^7+a\,b^6d^2n^6x^6+21b^7d^2n^5x^7+15a\,b^6d^2n^5x^6+175b^7d^2n^4x^7-6a^2b^5d^2n^5x^5+85a\,b^6d^2n^4x^6+2b^7cd\,n^6x^4+735b^7d^2n^3x\right.}{}$
parallelrisch	Expression too large to display

input
```
int((b*x+a)^n*(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

output
```
d^2/(7+n)*x^7*exp(n*ln(b*x+a))+a*(b^6*c^2*n^6+27*b^6*c^2*n^5+295*b^6*c^2*n
^4-12*a^3*b^3*c*d*n^3+1665*b^6*c^2*n^3-216*a^3*b^3*c*d*n^2+5104*b^6*c^2*n^
2-1284*a^3*b^3*c*d*n+8028*b^6*c^2*n+720*a^6*d^2-2520*a^3*b^3*c*d+5040*b^6*
c^2)/b^7/(n^7+28*n^6+322*n^5+1960*n^4+6769*n^3+13132*n^2+13068*n+5040)*exp
(n*ln(b*x+a))+d^2*a*n/b/(n^2+13*n+42)*x^6*exp(n*ln(b*x+a))-(-b^6*c^2*n^6-2
7*b^6*c^2*n^5-12*a^3*b^3*c*d*n^4-295*b^6*c^2*n^4-216*a^3*b^3*c*d*n^3-1665*
b^6*c^2*n^3-1284*a^3*b^3*c*d*n^2-5104*b^6*c^2*n^2+720*a^6*d^2*n-2520*a^3*b
^3*c*d*n-8028*b^6*c^2*n-5040*b^6*c^2)/b^6/(n^7+28*n^6+322*n^5+1960*n^4+676
9*n^3+13132*n^2+13068*n+5040)*x*exp(n*ln(b*x+a))+2*(b^3*c*n^3+18*b^3*c*n^2
+15*a^3*d*n+107*b^3*c*n+210*b^3*c)*d/b^3/(n^4+22*n^3+179*n^2+638*n+840)*x^
4*exp(n*ln(b*x+a))-6*n*a^2*d^2/b^2/(n^3+18*n^2+107*n+210)*x^5*exp(n*ln(b*x
+a))-2*n*a*d*(-b^3*c*n^3-18*b^3*c*n^2-107*b^3*c*n+60*a^3*d-210*b^3*c)/b^4/
(n^5+25*n^4+245*n^3+1175*n^2+2754*n+2520)*x^3*exp(n*ln(b*x+a))+6*(-b^3*c*n
^3-18*b^3*c*n^2-107*b^3*c*n+60*a^3*d-210*b^3*c)*d*a^2/b^5*n/(n^6+27*n^5+29
5*n^4+1665*n^3+5104*n^2+8028*n+5040)*x^2*exp(n*ln(b*x+a))
```

3.180.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 893 vs. $2(203) = 406$.

Time = 0.28 (sec) , antiderivative size = 893, normalized size of antiderivative = 4.40

$$\int (a+bx)^n \left(c+dx^3\right)^2 \, dx$$
$$= \frac{(ab^6c^2n^6 + 27\, ab^6c^2n^5 + 295\, ab^6c^2n^4 + 5040\, ab^6c^2 - 2520\, a^4b^3cd + 720\, a^7d^2 + (b^7d^2n^6 + 21\, b^7d^2n^5 + 175\, b}{}$$

input
```
integrate((b*x+a)^n*(d*x^3+c)^2,x, algorithm="fricas")
```

output
```
(a*b^6*c^2*n^6 + 27*a*b^6*c^2*n^5 + 295*a*b^6*c^2*n^4 + 5040*a*b^6*c^2 - 2
520*a^4*b^3*c*d + 720*a^7*d^2 + (b^7*d^2*n^6 + 21*b^7*d^2*n^5 + 175*b^7*d^
2*n^4 + 735*b^7*d^2*n^3 + 1624*b^7*d^2*n^2 + 1764*b^7*d^2*n + 720*b^7*d^2)
*x^7 + (a*b^6*d^2*n^6 + 15*a*b^6*d^2*n^5 + 85*a*b^6*d^2*n^4 + 225*a*b^6*d^
2*n^3 + 274*a*b^6*d^2*n^2 + 120*a*b^6*d^2*n)*x^6 - 6*(a^2*b^5*d^2*n^5 + 10
*a^2*b^5*d^2*n^4 + 35*a^2*b^5*d^2*n^3 + 50*a^2*b^5*d^2*n^2 + 24*a^2*b^5*d^
2*n)*x^5 + 2*(b^7*c*d*n^6 + 24*b^7*c*d*n^5 + 1260*b^7*c*d + (226*b^7*c*d +
 15*a^3*b^4*d^2)*n^4 + 6*(176*b^7*c*d + 15*a^3*b^4*d^2)*n^3 + 5*(509*b^7*c
*d + 33*a^3*b^4*d^2)*n^2 + 18*(164*b^7*c*d + 5*a^3*b^4*d^2)*n)*x^4 + 3*(55
5*a*b^6*c^2 - 4*a^4*b^3*c*d)*n^3 + 2*(a*b^6*c*d*n^6 + 21*a*b^6*c*d*n^5 + 1
63*a*b^6*c*d*n^4 + 3*(189*a*b^6*c*d - 20*a^4*b^3*d^2)*n^3 + 4*(211*a*b^6*c
*d - 45*a^4*b^3*d^2)*n^2 + 60*(7*a*b^6*c*d - 2*a^4*b^3*d^2)*n)*x^3 + 8*(63
8*a*b^6*c^2 - 27*a^4*b^3*c*d)*n^2 - 6*(a^2*b^5*c*d*n^5 + 19*a^2*b^5*c*d*n^
4 + 125*a^2*b^5*c*d*n^3 + (317*a^2*b^5*c*d - 60*a^5*b^2*d^2)*n^2 + 30*(7*a
^2*b^5*c*d - 2*a^5*b^2*d^2)*n)*x^2 + 12*(669*a*b^6*c^2 - 107*a^4*b^3*c*d)*
n + (b^7*c^2*n^6 + 27*b^7*c^2*n^5 + 5040*b^7*c^2 + (295*b^7*c^2 + 12*a^3*b
^4*c*d)*n^4 + 9*(185*b^7*c^2 + 24*a^3*b^4*c*d)*n^3 + 4*(1276*b^7*c^2 + 321
*a^3*b^4*c*d)*n^2 + 36*(223*b^7*c^2 + 70*a^3*b^4*c*d - 20*a^6*b*d^2)*n)*x)
*(b*x + a)^n/(b^7*n^7 + 28*b^7*n^6 + 322*b^7*n^5 + 1960*b^7*n^4 + 6769*b^7
*n^3 + 13132*b^7*n^2 + 13068*b^7*n + 5040*b^7)
```

3.180.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11851 vs. $2(187) = 374$.

Time = 3.53 (sec) , antiderivative size = 11851, normalized size of antiderivative = 58.38

$$\int (a+bx)^n \left(c+dx^3\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**n*(d*x**3+c)**2,x)
```

output
```
Piecewise((a**n*(c**2*x + c*d*x**4/2 + d**2*x**7/7), Eq(b, 0)), (60*a**6*d
**2*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 12
00*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**
6) + 147*a**6*d**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 +
1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x
**6) + 360*a**5*b*d**2*x*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 90
0*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**1
2*x**5 + 60*b**13*x**6) + 822*a**5*b*d**2*x/(60*a**6*b**7 + 360*a**5*b**8*
x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*
a*b**12*x**5 + 60*b**13*x**6) + 900*a**4*b**2*d**2*x**2*log(a/b + x)/(60*a
**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 9
00*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1875*a**4*b**2*d*
*2*x**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b
**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 2*a*
*3*b**3*c*d/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a*
*3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) +
1200*a**3*b**3*d**2*x**3*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 90
0*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**1
2*x**5 + 60*b**13*x**6) + 2200*a**3*b**3*d**2*x**3/(60*a**6*b**7 + 360*a**
5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x...
```

3.180.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.77

$$\int (a+bx)^n \left(c+dx^3\right)^2 \, dx = \frac{(bx+a)^{n+1}c^2}{b(n+1)}$$
$$+ \frac{2\left((n^3+6\,n^2+11\,n+6)b^4x^4+(n^3+3\,n^2+2\,n)ab^3x^3-3\,(n^2+n)a^2b^2x^2+6\,a^3bnx-6\,a^4\right)(bx+a)^n c}{(n^4+10\,n^3+35\,n^2+50\,n+24)b^4}$$
$$+ \frac{\left((n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)b^7x^7+(n^6+15\,n^5+85\,n^4+225\,n^3+274\,n^2\right.}{}$$

input
```
integrate((b*x+a)^n*(d*x^3+c)^2,x, algorithm="maxima")
```

output
```
(b*x + a)^(n + 1)*c^2/(b*(n + 1)) + 2*((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 +
(n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*
a^4)*(b*x + a)^n*c*d/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4) + ((n^6 + 2
1*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 + 15*n
^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4 + 35*
n^3 + 50*n^2 + 24*n)*a^2*b^5*x^5 + 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^3*b^4
*x^4 - 120*(n^3 + 3*n^2 + 2*n)*a^4*b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x^2 - 7
20*a^6*b*n*x + 720*a^7)*(b*x + a)^n*d^2/((n^7 + 28*n^6 + 322*n^5 + 1960*n^
4 + 6769*n^3 + 13132*n^2 + 13068*n + 5040)*b^7)
```

3.180.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1477 vs. $2(203) = 406$.

Time = 0.37 (sec) , antiderivative size = 1477, normalized size of antiderivative = 7.28

$$\int (a+bx)^n \left(c+dx^3\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^n*(d*x^3+c)^2,x, algorithm="giac")
```

output
```
((b*x + a)^n*b^7*d^2*n^6*x^7 + (b*x + a)^n*a*b^6*d^2*n^6*x^6 + 21*(b*x + a
)^n*b^7*d^2*n^5*x^7 + 15*(b*x + a)^n*a*b^6*d^2*n^5*x^6 + 175*(b*x + a)^n*b
^7*d^2*n^4*x^7 + 2*(b*x + a)^n*b^7*c*d*n^6*x^4 - 6*(b*x + a)^n*a^2*b^5*d^2
*n^5*x^5 + 85*(b*x + a)^n*a*b^6*d^2*n^4*x^6 + 735*(b*x + a)^n*b^7*d^2*n^3*
x^7 + 2*(b*x + a)^n*a*b^6*c*d*n^6*x^3 + 48*(b*x + a)^n*b^7*c*d*n^5*x^4 - 6
0*(b*x + a)^n*a^2*b^5*d^2*n^4*x^5 + 225*(b*x + a)^n*a*b^6*d^2*n^3*x^6 + 16
24*(b*x + a)^n*b^7*d^2*n^2*x^7 + 42*(b*x + a)^n*a*b^6*c*d*n^5*x^3 + 452*(b
*x + a)^n*b^7*c*d*n^4*x^4 + 30*(b*x + a)^n*a^3*b^4*d^2*n^4*x^4 - 210*(b*x
+ a)^n*a^2*b^5*d^2*n^3*x^5 + 274*(b*x + a)^n*a*b^6*d^2*n^2*x^6 + 1764*(b*x
 + a)^n*b^7*d^2*n*x^7 + (b*x + a)^n*b^7*c^2*n^6*x - 6*(b*x + a)^n*a^2*b^5*
c*d*n^5*x^2 + 326*(b*x + a)^n*a*b^6*c*d*n^4*x^3 + 2112*(b*x + a)^n*b^7*c*d
*n^3*x^4 + 180*(b*x + a)^n*a^3*b^4*d^2*n^3*x^4 - 300*(b*x + a)^n*a^2*b^5*d
^2*n^2*x^5 + 120*(b*x + a)^n*a*b^6*d^2*n*x^6 + 720*(b*x + a)^n*b^7*d^2*x^7
 + (b*x + a)^n*a*b^6*c^2*n^6 + 27*(b*x + a)^n*b^7*c^2*n^5*x - 114*(b*x + a
)^n*a^2*b^5*c*d*n^4*x^2 + 1134*(b*x + a)^n*a*b^6*c*d*n^3*x^3 - 120*(b*x +
a)^n*a^4*b^3*d^2*n^3*x^3 + 5090*(b*x + a)^n*b^7*c*d*n^2*x^4 + 330*(b*x + a
)^n*a^3*b^4*d^2*n^2*x^4 - 144*(b*x + a)^n*a^2*b^5*d^2*n*x^5 + 27*(b*x + a)
^n*a*b^6*c^2*n^5 + 295*(b*x + a)^n*b^7*c^2*n^4*x + 12*(b*x + a)^n*a^3*b^4*
c*d*n^4*x - 750*(b*x + a)^n*a^2*b^5*c*d*n^3*x^2 + 1688*(b*x + a)^n*a*b^6*c
*d*n^2*x^3 - 360*(b*x + a)^n*a^4*b^3*d^2*n^2*x^3 + 5904*(b*x + a)^n*b^7...
```

3.180. $\int (a+bx)^n \left(c+dx^3\right)^2 \, dx$

3.180.9 Mupad [B] (verification not implemented)

Time = 20.46 (sec) , antiderivative size = 878, normalized size of antiderivative = 4.33

$$\int (a+bx)^n \left(c+dx^3\right)^2 dx$$

$$= \frac{a\,(a+b\,x)^n\,(720\,a^6\,d^2-12\,a^3\,b^3\,c\,d\,n^3-216\,a^3\,b^3\,c\,d\,n^2-1284\,a^3\,b^3\,c\,d\,n-2520\,a^3\,b^3\,c\,d+b^6\,c^2\,n^6+27\,b^6}{b^7\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+1313}$$

$$+\frac{d^2\,x^7\,(a+b\,x)^n\,(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720)}{n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040}$$

$$+\frac{x\,(a+b\,x)^n\,(-720\,a^6\,b\,d^2\,n+12\,a^3\,b^4\,c\,d\,n^4+216\,a^3\,b^4\,c\,d\,n^3+1284\,a^3\,b^4\,c\,d\,n^2+2520\,a^3\,b^4\,c\,d\,n+b^7\,c^2}{b^7\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+}$$

$$+\frac{2\,d\,x^4\,(a+b\,x)^n\,(n^3+6\,n^2+11\,n+6)\,\left(15\,d\,a^3\,n+c\,b^3\,n^3+18\,c\,b^3\,n^2+107\,c\,b^3\,n+210\,c\,b^3\right)}{b^3\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040\right)}$$

$$+\frac{a\,d^2\,n\,x^6\,(a+b\,x)^n\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)}{b\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040\right)}$$

$$-\frac{6\,a^2\,d^2\,n\,x^5\,(a+b\,x)^n\,(n^4+10\,n^3+35\,n^2+50\,n+24)}{b^2\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040\right)}$$

$$+\frac{2\,a\,d\,n\,x^3\,(a+b\,x)^n\,(n^2+3\,n+2)\,\left(-60\,d\,a^3+c\,b^3\,n^3+18\,c\,b^3\,n^2+107\,c\,b^3\,n+210\,c\,b^3\right)}{b^4\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040\right)}$$

$$-\frac{6\,a^2\,d\,n\,x^2\,(n+1)\,(a+b\,x)^n\,\left(-60\,d\,a^3+c\,b^3\,n^3+18\,c\,b^3\,n^2+107\,c\,b^3\,n+210\,c\,b^3\right)}{b^5\,\left(n^7+28\,n^6+322\,n^5+1960\,n^4+6769\,n^3+13132\,n^2+13068\,n+5040\right)}$$

input
```
int((c + d*x^3)^2*(a + b*x)^n,x)
```

output
```
(a*(a + b*x)^n*(720*a^6*d^2 + 5040*b^6*c^2 + 8028*b^6*c^2*n + 5104*b^6*c^2
*n^2 + 1665*b^6*c^2*n^3 + 295*b^6*c^2*n^4 + 27*b^6*c^2*n^5 + b^6*c^2*n^6 -
 2520*a^3*b^3*c*d - 1284*a^3*b^3*c*d*n - 216*a^3*b^3*c*d*n^2 - 12*a^3*b^3*
c*d*n^3))/(b^7*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n
^6 + n^7 + 5040)) + (d^2*x^7*(a + b*x)^n*(1764*n + 1624*n^2 + 735*n^3 + 17
5*n^4 + 21*n^5 + n^6 + 720))/(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 +
322*n^5 + 28*n^6 + n^7 + 5040) + (x*(a + b*x)^n*(5040*b^7*c^2 + 8028*b^7*c
^2*n + 5104*b^7*c^2*n^2 + 1665*b^7*c^2*n^3 + 295*b^7*c^2*n^4 + 27*b^7*c^2*
n^5 + b^7*c^2*n^6 - 720*a^6*b*d^2*n + 2520*a^3*b^4*c*d*n + 1284*a^3*b^4*c*
d*n^2 + 216*a^3*b^4*c*d*n^3 + 12*a^3*b^4*c*d*n^4))/(b^7*(13068*n + 13132*n
^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (2*d*x^4*(a +
 b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(210*b^3*c + 18*b^3*c*n^2 + b^3*c*n^3 + 1
5*a^3*d*n + 107*b^3*c*n))/(b^3*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4
+ 322*n^5 + 28*n^6 + n^7 + 5040)) + (a*d^2*n*x^6*(a + b*x)^n*(274*n + 225*
n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(b*(13068*n + 13132*n^2 + 6769*n^3 + 1
960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) - (6*a^2*d^2*n*x^5*(a + b*x)^n*(
50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(b^2*(13068*n + 13132*n^2 + 6769*n^3 +
 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (2*a*d*n*x^3*(a + b*x)^n*(3*
n + n^2 + 2)*(210*b^3*c - 60*a^3*d + 18*b^3*c*n^2 + b^3*c*n^3 + 107*b^3*c*
n))/(b^4*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 ...
```

3.180. $\int (a+bx)^n \left(c+dx^3\right)^2 \, dx$

3.181 $\int \frac{(a+bx)^n(c+dx^3)^2}{x}\,dx$

3.181.1 Optimal result

Integrand size = 20, antiderivative size = 209

$$\int \frac{(a+bx)^n(c+dx^3)^2}{x}\,dx = \frac{a^2d(2b^3c-a^3d)(a+bx)^{1+n}}{b^6(1+n)} - \frac{ad(4b^3c-5a^3d)(a+bx)^{2+n}}{b^6(2+n)} + \frac{2d(b^3c-5a^3d)(a+bx)^{3+n}}{b^6(3+n)} + \frac{10a^2d^2(a+bx)^{4+n}}{b^6(4+n)} - \frac{5ad^2(a+bx)^{5+n}}{b^6(5+n)} + \frac{d^2(a+bx)^{6+n}}{b^6(6+n)} - \frac{c^2(a+bx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{bx}{a}\right)}{a(1+n)}$$

output
```
a^2*d*(-a^3*d+2*b^3*c)*(b*x+a)^(1+n)/b^6/(1+n)-a*d*(-5*a^3*d+4*b^3*c)*(b*x
+a)^(2+n)/b^6/(2+n)+2*d*(-5*a^3*d+b^3*c)*(b*x+a)^(3+n)/b^6/(3+n)+10*a^2*d^
2*(b*x+a)^(4+n)/b^6/(4+n)-5*a*d^2*(b*x+a)^(5+n)/b^6/(5+n)+d^2*(b*x+a)^(6+n
)/b^6/(6+n)-c^2*(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

3.181.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^n \left(c+dx^3\right)^2}{x} \, dx = (a+bx)^{1+n} \left(\frac{a^2d(2b^3c-a^3d)}{b^6(1+n)} + \frac{ad(-4b^3c+5a^3d)\,(a+bx)}{b^6(2+n)} + \frac{2d(b^3c-5a^3d)\,(a+bx)^2}{b^6(3+n)} + \frac{10a^2d^2(a+bx)^3}{b^6(4+n)} - \frac{5ad^2(a+bx)^4}{b^6(5+n)} + \frac{d^2(a+bx)^5}{b^6(6+n)} - \frac{c^2\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+bx}{a}\right)}{a+an}\right)$$

input
```
Integrate[((a + b*x)^n*(c + d*x^3)^2)/x,x]
```

output
```
(a + b*x)^(1 + n)*((a^2*d*(2*b^3*c - a^3*d))/(b^6*(1 + n)) + (a*d*(-4*b^3*
c + 5*a^3*d)*(a + b*x))/(b^6*(2 + n)) + (2*d*(b^3*c - 5*a^3*d)*(a + b*x)^2
)/(b^6*(3 + n)) + (10*a^2*d^2*(a + b*x)^3)/(b^6*(4 + n)) - (5*a*d^2*(a + b
*x)^4)/(b^6*(5 + n)) + (d^2*(a + b*x)^5)/(b^6*(6 + n)) - (c^2*Hypergeometr
ic2F1[1, 1 + n, 2 + n, (a + b*x)/a])/(a + a*n))
```

3.181.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c+dx^3\right)^2 (a+bx)^n}{x} \, dx$$

$$\Big\downarrow \; 2123$$

$$\int \left(\frac{ad\left(5a^3d-4b^3c\right)(a+bx)^{n+1}}{b^5} + \frac{2d\left(b^3c-5a^3d\right)(a+bx)^{n+2}}{b^5} + \frac{10a^2d^2(a+bx)^{n+3}}{b^5} - \frac{a^2d\left(a^3d-2b^3c\right)(a+bx}{b^5}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{ad(4b^3c-5a^3d)\,(a+bx)^{n+2}}{b^6(n+2)}+\frac{2d(b^3c-5a^3d)\,(a+bx)^{n+3}}{b^6(n+3)}+\frac{10a^2d^2(a+bx)^{n+4}}{b^6(n+4)}+\frac{a^2d(2b^3c-a^3d)\,(a+bx)^{n+1}}{b^6(n+1)}-\frac{5ad^2(a+bx)^{n+5}}{b^6(n+5)}+\frac{d^2(a+bx)^{n+6}}{b^6(n+6)}-\frac{c^2(a+bx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{bx}{a}+1\right)}{a(n+1)}$$

input
```
Int[((a + b*x)^n*(c + d*x^3)^2)/x,x]
```

output
```
(a^2*d*(2*b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^6*(1 + n)) - (a*d*(4*b^3*c
- 5*a^3*d)*(a + b*x)^(2 + n))/(b^6*(2 + n)) + (2*d*(b^3*c - 5*a^3*d)*(a +
b*x)^(3 + n))/(b^6*(3 + n)) + (10*a^2*d^2*(a + b*x)^(4 + n))/(b^6*(4 + n))
 - (5*a*d^2*(a + b*x)^(5 + n))/(b^6*(5 + n)) + (d^2*(a + b*x)^(6 + n))/(b^
6*(6 + n)) - (c^2*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 +
 (b*x)/a])/(a*(1 + n))
```

3.181.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

3.181.4 Maple [F]

$$\int \frac{(bx+a)^n\,(x^3d+c)^2}{x}dx$$

input
```
int((b*x+a)^n*(d*x^3+c)^2/x,x)
```

output
```
int((b*x+a)^n*(d*x^3+c)^2/x,x)
```

3.181.5 Fricas [F]

$$\int \frac{(a+bx)^n (c+dx^3)^2}{x}\,dx = \int \frac{(dx^3+c)^2(bx+a)^n}{x}\,dx$$

input
```
integrate((b*x+a)^n*(d*x^3+c)^2/x,x, algorithm="fricas")
```

output
```
integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x + a)^n/x, x)
```

3.181.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4007 vs. $2(187) = 374$.

Time = 4.22 (sec) , antiderivative size = 4690, normalized size of antiderivative = 22.44

$$\int \frac{(a+bx)^n (c+dx^3)^2}{x}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**n*(d*x**3+c)**2/x,x)
```

output
```
2*c*d*Piecewise((a**n*x**3/3, Eq(b, 0)), (2*a**2*log(a/b + x)/(2*a**2*b**3
 + 4*a*b**4*x + 2*b**5*x**2) + 3*a**2/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x
**2) + 4*a*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*a
*b*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*log(a/b + x)/(
2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a**2*log(a/b + x)
/(a*b**3 + b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/b + x)/(a*b*
*3 + b**4*x) + b**2*x**2/(a*b**3 + b**4*x), Eq(n, -2)), (a**2*log(a/b + x)
/b**3 - a*x/b**2 + x**2/(2*b), Eq(n, -1)), (2*a**3*(a + b*x)**n/(b**3*n**3
 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) - 2*a**2*b*n*x*(a + b*x)**n/(b**3*n**
3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n**2*x**2*(a + b*x)**n/(b**
3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n*x**2*(a + b*x)**n/(b
**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + b**3*n**2*x**3*(a + b*x)**n
/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 3*b**3*n*x**3*(a + b*x)*
*n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 2*b**3*x**3*(a + b*x)*
*n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3), True)) + d**2*Piecewise
((a**n*x**6/6, Eq(b, 0)), (60*a**5*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b
**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b*
*11*x**5) + 137*a**5/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2
+ 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a**4*b*x*lo
g(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a...
```

3.181.7 Maxima [F]

$$\int \frac{(a+bx)^n \left(c+dx^3\right)^2}{x}\,dx = \int \frac{\left(dx^3+c\right)^2 (bx+a)^n}{x}\,dx$$

input
```
integrate((b*x+a)^n*(d*x^3+c)^2/x,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^2*(b*x + a)^n/x, x)
```

3.181.8 Giac [F]

$$\int \frac{(a+bx)^n \left(c+dx^3\right)^2}{x}\,dx = \int \frac{\left(dx^3+c\right)^2 (bx+a)^n}{x}\,dx$$

input
```
integrate((b*x+a)^n*(d*x^3+c)^2/x,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^2*(b*x + a)^n/x, x)
```

3.181.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^n \left(c+dx^3\right)^2}{x}\,dx = \int \frac{\left(d\,x^3+c\right)^2 \left(a+b\,x\right)^n}{x}\,dx$$

input
```
int(((c + d*x^3)^2*(a + b*x)^n)/x,x)
```

output
```
int(((c + d*x^3)^2*(a + b*x)^n)/x, x)
```

3.182 $\int x^2(a+bx)^n \left(c+dx^3\right)^3 \, dx$

3.182.1 Optimal result

Integrand size = 20, antiderivative size = 459

$$\begin{aligned}\int x^2(a+bx)^n \left(c+dx^3\right)^3 \, dx &= \frac{a^2(b^3c-a^3d)^3 (a+bx)^{1+n}}{b^{12}(1+n)} \\ &- \frac{a(2b^3c-11a^3d)\left(b^3c-a^3d\right)^2 (a+bx)^{2+n}}{b^{12}(2+n)} \\ &+ \frac{(b^3c-a^3d)\left(b^6c^2-29a^3b^3cd+55a^6d^2\right)(a+bx)^{3+n}}{b^{12}(3+n)} \\ &+ \frac{3a^2d(10b^6c^2-56a^3b^3cd+55a^6d^2)\left(a+bx\right)^{4+n}}{b^{12}(4+n)} \\ &- \frac{15ad(b^6c^2-14a^3b^3cd+22a^6d^2)\left(a+bx\right)^{5+n}}{b^{12}(5+n)} \\ &+ \frac{3d(b^6c^2-56a^3b^3cd+154a^6d^2)\left(a+bx\right)^{6+n}}{b^{12}(6+n)} \\ &+ \frac{42a^2d^2(2b^3c-11a^3d)\left(a+bx\right)^{7+n}}{b^{12}(7+n)} \\ &- \frac{6ad^2(4b^3c-55a^3d)\left(a+bx\right)^{8+n}}{b^{12}(8+n)} \\ &+ \frac{3d^2(b^3c-55a^3d)\left(a+bx\right)^{9+n}}{b^{12}(9+n)} + \frac{55a^2d^3(a+bx)^{10+n}}{b^{12}(10+n)} \\ &- \frac{11ad^3(a+bx)^{11+n}}{b^{12}(11+n)} + \frac{d^3(a+bx)^{12+n}}{b^{12}(12+n)}\end{aligned}$$

output
```
a^2*(-a^3*d+b^3*c)^3*(b*x+a)^(1+n)/b^12/(1+n)-a*(-11*a^3*d+2*b^3*c)*(-a^3*
d+b^3*c)^2*(b*x+a)^(2+n)/b^12/(2+n)+(-a^3*d+b^3*c)*(55*a^6*d^2-29*a^3*b^3*
c*d+b^6*c^2)*(b*x+a)^(3+n)/b^12/(3+n)+3*a^2*d*(55*a^6*d^2-56*a^3*b^3*c*d+1
0*b^6*c^2)*(b*x+a)^(4+n)/b^12/(4+n)-15*a*d*(22*a^6*d^2-14*a^3*b^3*c*d+b^6*
c^2)*(b*x+a)^(5+n)/b^12/(5+n)+3*d*(154*a^6*d^2-56*a^3*b^3*c*d+b^6*c^2)*(b*
x+a)^(6+n)/b^12/(6+n)+42*a^2*d^2*(-11*a^3*d+2*b^3*c)*(b*x+a)^(7+n)/b^12/(7
+n)-6*a*d^2*(-55*a^3*d+4*b^3*c)*(b*x+a)^(8+n)/b^12/(8+n)+3*d^2*(-55*a^3*d+
b^3*c)*(b*x+a)^(9+n)/b^12/(9+n)+55*a^2*d^3*(b*x+a)^(10+n)/b^12/(10+n)-11*a
*d^3*(b*x+a)^(11+n)/b^12/(11+n)+d^3*(b*x+a)^(12+n)/b^12/(12+n)
```

3.182.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.88

$$\int x^2(a+bx)^n \left(c+dx^3\right)^3 dx = \frac{(a+bx)^{1+n}\left(\frac{a^2\left(b^3c-a^3d\right)^3}{1+n}+\frac{a\left(b^3c-a^3d\right)^2\left(-2b^3c+11a^3d\right)(a+bx)}{2+n}+\frac{\left(b^3c-a^3d\right)\left(b^6c^2-29a^3b^3cd+55a^6d^2\right)(a+bx)^2}{3+n}+\frac{3a^2d(10b^6c^2-5}{}\right.}{}$$

input
```
Integrate[x^2*(a + b*x)^n*(c + d*x^3)^3,x]
```

output
```
((a + b*x)^(1 + n)*((a^2*(b^3*c - a^3*d)^3)/(1 + n) + (a*(b^3*c - a^3*d)^2
*(-2*b^3*c + 11*a^3*d)*(a + b*x))/(2 + n) + ((b^3*c - a^3*d)*(b^6*c^2 - 29
*a^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^2)/(3 + n) + (3*a^2*d*(10*b^6*c^2 - 5
6*a^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^3)/(4 + n) - (15*a*d*(b^6*c^2 - 14*a
^3*b^3*c*d + 22*a^6*d^2)*(a + b*x)^4)/(5 + n) + (3*d*(b^6*c^2 - 56*a^3*b^3
*c*d + 154*a^6*d^2)*(a + b*x)^5)/(6 + n) + (42*a^2*d^2*(2*b^3*c - 11*a^3*d
)*(a + b*x)^6)/(7 + n) + (6*a*d^2*(-4*b^3*c + 55*a^3*d)*(a + b*x)^7)/(8 +
n) + (3*d^2*(b^3*c - 55*a^3*d)*(a + b*x)^8)/(9 + n) + (55*a^2*d^3*(a + b*x
)^9)/(10 + n) - (11*a*d^3*(a + b*x)^10)/(11 + n) + (d^3*(a + b*x)^11)/(12
+ n)))/b^12
```

3.182.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(c+dx^3\right)^3(a+bx)^n\,dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{6ad^2\left(55a^3d-4b^3c\right)(a+bx)^{n+7}}{b^{11}}+\frac{3d^2\left(b^3c-55a^3d\right)(a+bx)^{n+8}}{b^{11}}+\frac{a\left(a^3d-b^3c\right)^2\left(11a^3d-2b^3c\right)(a+bx)^{n+}}{b^{11}}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6ad^2\left(4b^3c-55a^3d\right)(a+bx)^{n+8}}{b^{12}(n+8)}+\frac{3d^2\left(b^3c-55a^3d\right)(a+bx)^{n+9}}{b^{12}(n+9)}-\frac{a\left(2b^3c-11a^3d\right)\left(b^3c-a^3d\right)^2(a+bx)^{n+2}}{b^{12}(n+2)}+\frac{55a^2d^3(a+bx)^{n+10}}{b^{12}(n+10)}+\frac{\left(b^3c-a^3d\right)\left(55a^6d^2-29a^3b^3cd+b^6c^2\right)(a+bx)^{n+3}}{b^{12}(n+3)}-\frac{15ad\left(22a^6d^2-14a^3b^3cd+b^6c^2\right)(a+bx)^{n+5}}{b^{12}(n+5)}+\frac{3d\left(154a^6d^2-56a^3b^3cd+b^6c^2\right)(a+bx)^{n+6}}{b^{12}(n+6)}+\frac{42a^2d^2\left(2b^3c-11a^3d\right)(a+bx)^{n+7}}{b^{12}(n+7)}+\frac{a^2\left(b^3c-a^3d\right)^3(a+bx)^{n+1}}{b^{12}(n+1)}+\frac{3a^2d\left(55a^6d^2-56a^3b^3cd+10b^6c^2\right)(a+bx)^{n+4}}{b^{12}(n+4)}-\frac{11ad^3(a+bx)^{n+11}}{b^{12}(n+11)}+\frac{d^3(a+bx)^{n+12}}{b^{12}(n+12)}$$

input
```
Int[x^2*(a + b*x)^n*(c + d*x^3)^3,x]
```

output
```
(a^2*(b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^12*(1 + n)) - (a*(2*b^3*c - 1
1*a^3*d)*(b^3*c - a^3*d)^2*(a + b*x)^(2 + n))/(b^12*(2 + n)) + ((b^3*c - a
^3*d)*(b^6*c^2 - 29*a^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^(3 + n))/(b^12*(3
+ n)) + (3*a^2*d*(10*b^6*c^2 - 56*a^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^(4 +
 n))/(b^12*(4 + n)) - (15*a*d*(b^6*c^2 - 14*a^3*b^3*c*d + 22*a^6*d^2)*(a +
 b*x)^(5 + n))/(b^12*(5 + n)) + (3*d*(b^6*c^2 - 56*a^3*b^3*c*d + 154*a^6*d
^2)*(a + b*x)^(6 + n))/(b^12*(6 + n)) + (42*a^2*d^2*(2*b^3*c - 11*a^3*d)*(
a + b*x)^(7 + n))/(b^12*(7 + n)) - (6*a*d^2*(4*b^3*c - 55*a^3*d)*(a + b*x)
^(8 + n))/(b^12*(8 + n)) + (3*d^2*(b^3*c - 55*a^3*d)*(a + b*x)^(9 + n))/(b
^12*(9 + n)) + (55*a^2*d^3*(a + b*x)^(10 + n))/(b^12*(10 + n)) - (11*a*d^3
*(a + b*x)^(11 + n))/(b^12*(11 + n)) + (d^3*(a + b*x)^(12 + n))/(b^12*(12
+ n))
```

3.182.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

3.182.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3779 vs. $2(459) = 918$.

Time = 1.35 (sec) , antiderivative size = 3780, normalized size of antiderivative = 8.24

method	result	size
gosper	Expression too large to display	3780
risch	Expression too large to display	4231
parallelrisch	Expression too large to display	6192

input
```
int(x^2*(b*x+a)^n*(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/b^12*(b*x+a)^(1+n)/(n^12+78*n^11+2717*n^10+55770*n^9+749463*n^8+6926634
*n^7+44990231*n^6+206070150*n^5+657206836*n^4+1414014888*n^3+1931559552*n^
2+1486442880*n+479001600)*(-b^11*d^3*n^11*x^11-66*b^11*d^3*n^10*x^11+11*a*
b^10*d^3*n^10*x^10-1925*b^11*d^3*n^9*x^11+605*a*b^10*d^3*n^9*x^10-3*b^11*c
*d^2*n^11*x^8-32670*b^11*d^3*n^8*x^11-110*a^2*b^9*d^3*n^9*x^9+14520*a*b^10
*d^3*n^8*x^10-207*b^11*c*d^2*n^10*x^8-357423*b^11*d^3*n^7*x^11-4950*a^2*b^
9*d^3*n^8*x^9+24*a*b^10*c*d^2*n^10*x^7+199650*a*b^10*d^3*n^7*x^10-6288*b^1
1*c*d^2*n^9*x^8-2637558*b^11*d^3*n^6*x^11+990*a^3*b^8*d^3*n^8*x^8-95700*a^
2*b^9*d^3*n^7*x^9+1464*a*b^10*c*d^2*n^9*x^7+1735503*a*b^10*d^3*n^6*x^10-3*
b^11*c^2*d*n^11*x^5-110718*b^11*c*d^2*n^8*x^8-13339535*b^11*d^3*n^5*x^11+3
5640*a^3*b^8*d^3*n^7*x^8-168*a^2*b^9*c*d^2*n^9*x^6-1039500*a^2*b^9*d^3*n^6
*x^9+38592*a*b^10*c*d^2*n^8*x^7+9922605*a*b^10*d^3*n^5*x^10-216*b^11*c^2*d
*n^10*x^5-1251927*b^11*c*d^2*n^7*x^8-45995730*b^11*d^3*n^4*x^11-7920*a^4*b
^7*d^3*n^7*x^7+540540*a^3*b^8*d^3*n^6*x^8-9072*a^2*b^9*c*d^2*n^8*x^6-69600
30*a^2*b^9*d^3*n^5*x^9+15*a*b^10*c^2*d*n^10*x^4+577008*a*b^10*c*d^2*n^7*x^
7+37586230*a*b^10*d^3*n^4*x^10-6855*b^11*c^2*d*n^9*x^5-9512559*b^11*c*d^2*
n^6*x^8-105258076*b^11*d^3*n^3*x^11-221760*a^4*b^7*d^3*n^6*x^7+1008*a^3*b^
8*c*d^2*n^8*x^5+4490640*a^3*b^8*d^3*n^5*x^8-206640*a^2*b^9*c*d^2*n^7*x^6-2
9625750*a^2*b^9*d^3*n^4*x^9+1005*a*b^10*c^2*d*n^9*x^4+5399352*a*b^10*c*d^2
*n^6*x^7+92504500*a*b^10*d^3*n^3*x^10-b^11*c^3*n^11*x^2-126180*b^11*c^2...
```

3.182.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3564 vs. $2(459) = 918$.

Time = 0.33 (sec) , antiderivative size = 3564, normalized size of antiderivative = 7.76

$$\int x^2(a+bx)^n\left(c+dx^3\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
(2*a^3*b^9*c^3*n^9 + 144*a^3*b^9*c^3*n^8 + 4548*a^3*b^9*c^3*n^7 + 15966720
0*a^3*b^9*c^3 - 239500800*a^6*b^6*c^2*d + 159667200*a^9*b^3*c*d^2 - 399168
00*a^12*d^3 + (b^12*d^3*n^11 + 66*b^12*d^3*n^10 + 1925*b^12*d^3*n^9 + 3267
0*b^12*d^3*n^8 + 357423*b^12*d^3*n^7 + 2637558*b^12*d^3*n^6 + 13339535*b^1
2*d^3*n^5 + 45995730*b^12*d^3*n^4 + 105258076*b^12*d^3*n^3 + 150917976*b^1
2*d^3*n^2 + 120543840*b^12*d^3*n + 39916800*b^12*d^3)*x^12 + (a*b^11*d^3*n
^11 + 55*a*b^11*d^3*n^10 + 1320*a*b^11*d^3*n^9 + 18150*a*b^11*d^3*n^8 + 15
7773*a*b^11*d^3*n^7 + 902055*a*b^11*d^3*n^6 + 3416930*a*b^11*d^3*n^5 + 840
9500*a*b^11*d^3*n^4 + 12753576*a*b^11*d^3*n^3 + 10628640*a*b^11*d^3*n^2 +
3628800*a*b^11*d^3*n)*x^11 - 11*(a^2*b^10*d^3*n^10 + 45*a^2*b^10*d^3*n^9 +
 870*a^2*b^10*d^3*n^8 + 9450*a^2*b^10*d^3*n^7 + 63273*a^2*b^10*d^3*n^6 + 2
69325*a^2*b^10*d^3*n^5 + 723680*a^2*b^10*d^3*n^4 + 1172700*a^2*b^10*d^3*n^
3 + 1026576*a^2*b^10*d^3*n^2 + 362880*a^2*b^10*d^3*n)*x^10 + (3*b^12*c*d^2
*n^11 + 207*b^12*c*d^2*n^10 + 159667200*b^12*c*d^2 + 2*(3144*b^12*c*d^2 +
55*a^3*b^9*d^3)*n^9 + 18*(6151*b^12*c*d^2 + 220*a^3*b^9*d^3)*n^8 + 3*(4173
09*b^12*c*d^2 + 20020*a^3*b^9*d^3)*n^7 + 567*(16777*b^12*c*d^2 + 880*a^3*b
^9*d^3)*n^6 + 6*(8226277*b^12*c*d^2 + 411565*a^3*b^9*d^3)*n^5 + 36*(483309
7*b^12*c*d^2 + 205590*a^3*b^9*d^3)*n^4 + 40*(10142427*b^12*c*d^2 + 324841*
a^3*b^9*d^3)*n^3 + 288*(2051288*b^12*c*d^2 + 41855*a^3*b^9*d^3)*n^2 + 5760
*(82941*b^12*c*d^2 + 770*a^3*b^9*d^3)*n)*x^9 + 3*(a*b^11*c*d^2*n^11 + 6...
```

3.182.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75191 vs. 2(439) = 878.

Time = 83.59 (sec) , antiderivative size = 75191, normalized size of antiderivative = 163.81

$$\int x^2(a+bx)^n \left(c+dx^3\right)^3 dx = \text{Too large to display}$$

input
```
integrate(x**2*(b*x+a)**n*(d*x**3+c)**3,x)
```

output
```
Piecewise((a**n*(c**3*x**3/3 + c**2*d*x**6/2 + c*d**2*x**9/3 + d**3*x**12/
12), Eq(b, 0)), (27720*a**11*d**3*log(a/b + x)/(27720*a**11*b**12 + 304920
*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 91476
00*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 +
 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x*
*9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) + 83711*a**11*d**3/(27720*a
**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8
*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 1280664
0*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 15
24600*a**2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) + 304920
*a**10*b*d**3*x*log(a/b + x)/(27720*a**11*b**12 + 304920*a**10*b**13*x + 1
524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4
 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a**4*b**1
9*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*a*b**2
2*x**10 + 27720*b**23*x**11) + 893101*a**10*b*d**3*x/(27720*a**11*b**12 +
304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 +
 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*
x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b*
*21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) + 1524600*a**9*b**2*d
**3*x**2*log(a/b + x)/(27720*a**11*b**12 + 304920*a**10*b**13*x + 15246...
```

3.182.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1153 vs. $2(459) = 918$.

Time = 0.22 (sec) , antiderivative size = 1153, normalized size of antiderivative = 2.51

$$\int x^2(a+bx)^n \left(c+dx^3\right)^3 dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
 + a)^n*c^3/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 3*((n^5 + 15*n^4 + 85*n^3 + 2
25*n^2 + 274*n + 120)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*
b^5*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2
*n)*a^3*b^3*x^3 - 60*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x
 + a)^n*c^2*d/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720
)*b^6) + 3*((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 1
18124*n^2 + 109584*n + 40320)*b^9*x^9 + (n^8 + 28*n^7 + 322*n^6 + 1960*n^5
 + 6769*n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a*b^8*x^8 - 8*(n^7 + 21*n^6
+ 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a^2*b^7*x^7 + 56*(n^6 +
 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^3*b^6*x^6 - 336*(n^5 + 10*
n^4 + 35*n^3 + 50*n^2 + 24*n)*a^4*b^5*x^5 + 1680*(n^4 + 6*n^3 + 11*n^2 + 6
*n)*a^5*b^4*x^4 - 6720*(n^3 + 3*n^2 + 2*n)*a^6*b^3*x^3 + 20160*(n^2 + n)*a
^7*b^2*x^2 - 40320*a^8*b*n*x + 40320*a^9)*(b*x + a)^n*c*d^2/((n^9 + 45*n^8
 + 870*n^7 + 9450*n^6 + 63273*n^5 + 269325*n^4 + 723680*n^3 + 1172700*n^2
+ 1026576*n + 362880)*b^9) + ((n^11 + 66*n^10 + 1925*n^9 + 32670*n^8 + 357
423*n^7 + 2637558*n^6 + 13339535*n^5 + 45995730*n^4 + 105258076*n^3 + 1509
17976*n^2 + 120543840*n + 39916800)*b^12*x^12 + (n^11 + 55*n^10 + 1320*n^9
 + 18150*n^8 + 157773*n^7 + 902055*n^6 + 3416930*n^5 + 8409500*n^4 + 12753
576*n^3 + 10628640*n^2 + 3628800*n)*a*b^11*x^11 - 11*(n^10 + 45*n^9 + 8...
```

3.182.8 Giac [F(-2)]

Exception generated.

$$\int x^2(a+bx)^n\left(c+dx^3\right)^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Polynomial exponent overflow. Error
: Bad Argument Value
```

3.182.9 Mupad [B] (verification not implemented)

Time = 24.92 (sec) , antiderivative size = 2896, normalized size of antiderivative = 6.31

$$\int x^2(a+bx)^n \left(c+dx^3\right)^3 dx = \text{Too large to display}$$

input
```
int(x^2*(c + d*x^3)^3*(a + b*x)^n,x)
```

output
```
(2*a^3*(a + b*x)^n*(79833600*b^9*c^3 - 19958400*a^9*d^3 + 101378880*b^9*c^
3*n + 56231712*b^9*c^3*n^2 + 17893196*b^9*c^3*n^3 + 3602088*b^9*c^3*n^4 +
476049*b^9*c^3*n^5 + 41328*b^9*c^3*n^6 + 2274*b^9*c^3*n^7 + 72*b^9*c^3*n^8
 + b^9*c^3*n^9 - 119750400*a^3*b^6*c^2*d + 79833600*a^6*b^3*c*d^2 - 782222
40*a^3*b^6*c^2*d*n + 21893760*a^6*b^3*c*d^2*n - 21141720*a^3*b^6*c^2*d*n^2
 + 1995840*a^6*b^3*c*d^2*n^2 - 3026700*a^3*b^6*c^2*d*n^3 + 60480*a^6*b^3*c
*d^2*n^3 - 242100*a^3*b^6*c^2*d*n^4 - 10260*a^3*b^6*c^2*d*n^5 - 180*a^3*b^
6*c^2*d*n^6))/(b^12*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 6572
06836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 5577
0*n^9 + 2717*n^10 + 78*n^11 + n^12 + 479001600)) + (d^3*x^12*(a + b*x)^n*(
120543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5
+ 2637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916
800))/(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 20
6070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n
^10 + 78*n^11 + n^12 + 479001600) + (x^3*(a + b*x)^n*(3*n + n^2 + 2)*(7983
3600*b^9*c^3 + 6652800*a^9*d^3*n + 101378880*b^9*c^3*n + 56231712*b^9*c^3*
n^2 + 17893196*b^9*c^3*n^3 + 3602088*b^9*c^3*n^4 + 476049*b^9*c^3*n^5 + 41
328*b^9*c^3*n^6 + 2274*b^9*c^3*n^7 + 72*b^9*c^3*n^8 + b^9*c^3*n^9 + 399168
00*a^3*b^6*c^2*d*n - 26611200*a^6*b^3*c*d^2*n + 26074080*a^3*b^6*c^2*d*n^2
 - 7297920*a^6*b^3*c*d^2*n^2 + 7047240*a^3*b^6*c^2*d*n^3 - 665280*a^6*b...
```

3.183 $\int x(a+bx)^n \left(c+dx^3\right)^3 \, dx$

3.183.1 Optimal result

Integrand size = 18, antiderivative size = 396

$$\begin{aligned}\int x(a+bx)^n \left(c+dx^3\right)^3 \, dx = &-\frac{a(b^3c-a^3d)^3 (a+bx)^{1+n}}{b^{11}(1+n)} \\ &+\frac{(b^3c-10a^3d)(b^3c-a^3d)^2 (a+bx)^{2+n}}{b^{11}(2+n)} \\ &+\frac{9a^2d(2b^3c-5a^3d)(b^3c-a^3d)(a+bx)^{3+n}}{b^{11}(3+n)} \\ &-\frac{3ad(4b^6c^2-35a^3b^3cd+40a^6d^2)(a+bx)^{4+n}}{b^{11}(4+n)} \\ &+\frac{3d(b^6c^2-35a^3b^3cd+70a^6d^2)(a+bx)^{5+n}}{b^{11}(5+n)} \\ &+\frac{63a^2d^2(b^3c-4a^3d)(a+bx)^{6+n}}{b^{11}(6+n)} \\ &-\frac{21ad^2(b^3c-10a^3d)(a+bx)^{7+n}}{b^{11}(7+n)} \\ &+\frac{3d^2(b^3c-40a^3d)(a+bx)^{8+n}}{b^{11}(8+n)}+\frac{45a^2d^3(a+bx)^{9+n}}{b^{11}(9+n)} \\ &-\frac{10ad^3(a+bx)^{10+n}}{b^{11}(10+n)}+\frac{d^3(a+bx)^{11+n}}{b^{11}(11+n)}\end{aligned}$$

output
```
-a*(-a^3*d+b^3*c)^3*(b*x+a)^(1+n)/b^11/(1+n)+(-10*a^3*d+b^3*c)*(-a^3*d+b^3
*c)^2*(b*x+a)^(2+n)/b^11/(2+n)+9*a^2*d*(-5*a^3*d+2*b^3*c)*(-a^3*d+b^3*c)*(
b*x+a)^(3+n)/b^11/(3+n)-3*a*d*(40*a^6*d^2-35*a^3*b^3*c*d+4*b^6*c^2)*(b*x+a
)^(4+n)/b^11/(4+n)+3*d*(70*a^6*d^2-35*a^3*b^3*c*d+b^6*c^2)*(b*x+a)^(5+n)/b
^11/(5+n)+63*a^2*d^2*(-4*a^3*d+b^3*c)*(b*x+a)^(6+n)/b^11/(6+n)-21*a*d^2*(-
10*a^3*d+b^3*c)*(b*x+a)^(7+n)/b^11/(7+n)+3*d^2*(-40*a^3*d+b^3*c)*(b*x+a)^(
8+n)/b^11/(8+n)+45*a^2*d^3*(b*x+a)^(9+n)/b^11/(9+n)-10*a*d^3*(b*x+a)^(10+n
)/b^11/(10+n)+d^3*(b*x+a)^(11+n)/b^11/(11+n)
```

3.183.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.87

$$\int x(a+bx)^n \left(c+dx^3\right)^3 dx$$
$$= \frac{(a+bx)^{1+n}\left(\frac{a\left(-b^3c+a^3d\right)^3}{1+n}+\frac{\left(b^3c-10a^3d\right)\left(b^3c-a^3d\right)^2(a+bx)}{2+n}+\frac{9a^2d\left(-b^3c+a^3d\right)\left(-2b^3c+5a^3d\right)(a+bx)^2}{3+n}-\frac{3ad\left(4b^6c^2-35a^3b^3cd+4\right.}{4+n}\right.}{}$$

input
```
Integrate[x*(a + b*x)^n*(c + d*x^3)^3,x]
```

output
```
((a + b*x)^(1 + n)*((a*(-(b^3*c) + a^3*d)^3)/(1 + n) + ((b^3*c - 10*a^3*d)
*(b^3*c - a^3*d)^2*(a + b*x))/(2 + n) + (9*a^2*d*(-(b^3*c) + a^3*d)*(-2*b^
3*c + 5*a^3*d)*(a + b*x)^2)/(3 + n) - (3*a*d*(4*b^6*c^2 - 35*a^3*b^3*c*d +
 40*a^6*d^2)*(a + b*x)^3)/(4 + n) + (3*d*(b^6*c^2 - 35*a^3*b^3*c*d + 70*a^
6*d^2)*(a + b*x)^4)/(5 + n) + (63*a^2*d^2*(b^3*c - 4*a^3*d)*(a + b*x)^5)/(
6 + n) + (21*a*d^2*(-(b^3*c) + 10*a^3*d)*(a + b*x)^6)/(7 + n) + (3*d^2*(b^
3*c - 40*a^3*d)*(a + b*x)^7)/(8 + n) + (45*a^2*d^3*(a + b*x)^8)/(9 + n) -
(10*a*d^3*(a + b*x)^9)/(10 + n) + (d^3*(a + b*x)^10)/(11 + n)))/b^11
```

3.183.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(c+dx^3\right)^3(a+bx)^n\,dx$$

$$\Big\downarrow \ 2123$$

$$\int \left(\frac{21ad^2\left(10a^3d-b^3c\right)(a+bx)^{n+6}}{b^{10}}+\frac{3d^2\left(b^3c-40a^3d\right)(a+bx)^{n+7}}{b^{10}}+\frac{a\left(a^3d-b^3c\right)^3(a+bx)^n}{b^{10}}+\frac{\left(b^3c-10a^3d\right)}{}\right.$$

$$\Big\downarrow \ 2009$$

$$-\frac{21ad^2\left(b^3c-10a^3d\right)(a+bx)^{n+7}}{b^{11}(n+7)}+\frac{3d^2\left(b^3c-40a^3d\right)(a+bx)^{n+8}}{b^{11}(n+8)}-\frac{a\left(b^3c-a^3d\right)^3(a+bx)^{n+1}}{b^{11}(n+1)}+\frac{\left(b^3c-10a^3d\right)\left(b^3c-a^3d\right)^2(a+bx)^{n+2}}{b^{11}(n+2)}+\frac{45a^2d^3(a+bx)^{n+9}}{b^{11}(n+9)}-\frac{3ad\left(40a^6d^2-35a^3b^3cd+4b^6c^2\right)(a+bx)^{n+4}}{b^{11}(n+4)}+\frac{3d\left(70a^6d^2-35a^3b^3cd+b^6c^2\right)(a+bx)^{n+5}}{b^{11}(n+5)}+\frac{63a^2d^2\left(b^3c-4a^3d\right)(a+bx)^{n+6}}{b^{11}(n+6)}+\frac{9a^2d\left(2b^3c-5a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+3}}{b^{11}(n+3)}-\frac{10ad^3(a+bx)^{n+10}}{b^{11}(n+10)}+\frac{d^3(a+bx)^{n+11}}{b^{11}(n+11)}$$

input
```
Int[x*(a + b*x)^n*(c + d*x^3)^3,x]
```

output
```
-((a*(b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^11*(1 + n))) + ((b^3*c - 10*a
^3*d)*(b^3*c - a^3*d)^2*(a + b*x)^(2 + n))/(b^11*(2 + n)) + (9*a^2*d*(2*b^
3*c - 5*a^3*d)*(b^3*c - a^3*d)*(a + b*x)^(3 + n))/(b^11*(3 + n)) - (3*a*d*
(4*b^6*c^2 - 35*a^3*b^3*c*d + 40*a^6*d^2)*(a + b*x)^(4 + n))/(b^11*(4 + n)
) + (3*d*(b^6*c^2 - 35*a^3*b^3*c*d + 70*a^6*d^2)*(a + b*x)^(5 + n))/(b^11*
(5 + n)) + (63*a^2*d^2*(b^3*c - 4*a^3*d)*(a + b*x)^(6 + n))/(b^11*(6 + n))
 - (21*a*d^2*(b^3*c - 10*a^3*d)*(a + b*x)^(7 + n))/(b^11*(7 + n)) + (3*d^2
*(b^3*c - 40*a^3*d)*(a + b*x)^(8 + n))/(b^11*(8 + n)) + (45*a^2*d^3*(a + b
*x)^(9 + n))/(b^11*(9 + n)) - (10*a*d^3*(a + b*x)^(10 + n))/(b^11*(10 + n)
) + (d^3*(a + b*x)^(11 + n))/(b^11*(11 + n))
```

3.183. $\int x(a+bx)^n\left(c+dx^3\right)^3\,dx$

3.183.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

3.183.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2971 vs. 2(396) = 792.

Time = 1.16 (sec) , antiderivative size = 2972, normalized size of antiderivative = 7.51

method	result	size
gosper	Expression too large to display	2972
risch	Expression too large to display	3409
parallelrisch	Expression too large to display	4900

input
```
int(x*(b*x+a)^n*(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^11*(b*x+a)^(1+n)/(n^11+66*n^10+1925*n^9+32670*n^8+357423*n^7+2637558*n
^6+13339535*n^5+45995730*n^4+105258076*n^3+150917976*n^2+120543840*n+39916
800)*(b^10*d^3*n^10*x^10+55*b^10*d^3*n^9*x^10-10*a*b^9*d^3*n^9*x^9+1320*b^
10*d^3*n^8*x^10-450*a*b^9*d^3*n^8*x^9+3*b^10*c*d^2*n^10*x^7+18150*b^10*d^3
*n^7*x^10+90*a^2*b^8*d^3*n^8*x^8-8700*a*b^9*d^3*n^7*x^9+174*b^10*c*d^2*n^9
*x^7+157773*b^10*d^3*n^6*x^10+3240*a^2*b^8*d^3*n^7*x^8-21*a*b^9*c*d^2*n^9*
x^6-94500*a*b^9*d^3*n^6*x^9+4383*b^10*c*d^2*n^8*x^7+902055*b^10*d^3*n^5*x^
10-720*a^3*b^7*d^3*n^7*x^7+49140*a^2*b^8*d^3*n^6*x^8-1071*a*b^9*c*d^2*n^8*
x^6-632730*a*b^9*d^3*n^5*x^9+3*b^10*c^2*d*n^10*x^4+62946*b^10*c*d^2*n^7*x^
7+3416930*b^10*d^3*n^4*x^10-20160*a^3*b^7*d^3*n^6*x^7+126*a^2*b^8*c*d^2*n^
8*x^5+408240*a^2*b^8*d^3*n^5*x^8-23184*a*b^9*c*d^2*n^7*x^6-2693250*a*b^9*d
^3*n^4*x^9+183*b^10*c^2*d*n^9*x^4+568701*b^10*c*d^2*n^6*x^7+8409500*b^10*d
^3*n^3*x^10+5040*a^4*b^6*d^3*n^6*x^6-231840*a^3*b^7*d^3*n^5*x^7+5670*a^2*b
^8*c*d^2*n^7*x^5+2020410*a^2*b^8*d^3*n^4*x^8-12*a*b^9*c^2*d*n^9*x^3-278334
*a*b^9*c*d^2*n^6*x^6-7236800*a*b^9*d^3*n^3*x^9+4860*b^10*c^2*d*n^8*x^4+336
3066*b^10*c*d^2*n^5*x^7+12753576*b^10*d^3*n^2*x^10+105840*a^4*b^6*d^3*n^5*
x^6-630*a^3*b^7*c*d^2*n^7*x^4-1411200*a^3*b^7*d^3*n^4*x^7+105084*a^2*b^8*c
*d^2*n^6*x^5+6055560*a^2*b^8*d^3*n^3*x^8-684*a*b^9*c^2*d*n^8*x^3-2032569*a
*b^9*c*d^2*n^5*x^6-11727000*a*b^9*d^3*n^2*x^9+b^10*c^3*n^10*x+73710*b^10*c
^2*d*n^7*x^4+13114077*b^10*c*d^2*n^4*x^7+10628640*b^10*d^3*n*x^10-30240...
```

3.183.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2919 vs. $2(396) = 792$.

Time = 0.32 (sec) , antiderivative size = 2919, normalized size of antiderivative = 7.37

$$\int x(a+bx)^n\left(c+dx^3\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(x*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
-(a^2*b^9*c^3*n^9 + 63*a^2*b^9*c^3*n^8 + 1734*a^2*b^9*c^3*n^7 + 19958400*a
^2*b^9*c^3 - 23950080*a^5*b^6*c^2*d + 14968800*a^8*b^3*c*d^2 - 3628800*a^1
1*d^3 - (b^11*d^3*n^10 + 55*b^11*d^3*n^9 + 1320*b^11*d^3*n^8 + 18150*b^11*
d^3*n^7 + 157773*b^11*d^3*n^6 + 902055*b^11*d^3*n^5 + 3416930*b^11*d^3*n^4
 + 8409500*b^11*d^3*n^3 + 12753576*b^11*d^3*n^2 + 10628640*b^11*d^3*n + 36
28800*b^11*d^3)*x^11 - (a*b^10*d^3*n^10 + 45*a*b^10*d^3*n^9 + 870*a*b^10*d
^3*n^8 + 9450*a*b^10*d^3*n^7 + 63273*a*b^10*d^3*n^6 + 269325*a*b^10*d^3*n^
5 + 723680*a*b^10*d^3*n^4 + 1172700*a*b^10*d^3*n^3 + 1026576*a*b^10*d^3*n^
2 + 362880*a*b^10*d^3*n)*x^10 + 10*(a^2*b^9*d^3*n^9 + 36*a^2*b^9*d^3*n^8 +
 546*a^2*b^9*d^3*n^7 + 4536*a^2*b^9*d^3*n^6 + 22449*a^2*b^9*d^3*n^5 + 6728
4*a^2*b^9*d^3*n^4 + 118124*a^2*b^9*d^3*n^3 + 109584*a^2*b^9*d^3*n^2 + 4032
0*a^2*b^9*d^3*n)*x^9 - 3*(b^11*c*d^2*n^10 + 58*b^11*c*d^2*n^9 + 4989600*b^
11*c*d^2 + 3*(487*b^11*c*d^2 + 10*a^3*b^8*d^3)*n^8 + 6*(3497*b^11*c*d^2 +
140*a^3*b^8*d^3)*n^7 + 21*(9027*b^11*c*d^2 + 460*a^3*b^8*d^3)*n^6 + 294*(3
813*b^11*c*d^2 + 200*a^3*b^8*d^3)*n^5 + (4371359*b^11*c*d^2 + 203070*a^3*b
^8*d^3)*n^4 + 2*(5512429*b^11*c*d^2 + 196980*a^3*b^8*d^3)*n^3 + 36*(473867
*b^11*c*d^2 + 10890*a^3*b^8*d^3)*n^2 + 360*(40123*b^11*c*d^2 + 420*a^3*b^8
*d^3)*n)*x^8 - 3*(a*b^10*c*d^2*n^10 + 51*a*b^10*c*d^2*n^9 + 1104*a*b^10*c*
d^2*n^8 + 6*(2209*a*b^10*c*d^2 - 40*a^4*b^7*d^3)*n^7 + 21*(4609*a*b^10*c*d
^2 - 240*a^4*b^7*d^3)*n^6 + 21*(21119*a*b^10*c*d^2 - 2000*a^4*b^7*d^3)*...
```

3.183.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56151 vs. 2(374) = 748.

Time = 34.12 (sec) , antiderivative size = 56151, normalized size of antiderivative = 141.80

$$\int x(a+bx)^n\left(c+dx^3\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(x*(b*x+a)**n*(d*x**3+c)**3,x)
```

output
```
Piecewise((a**n*(c**3*x**2/2 + 3*c**2*d*x**5/5 + 3*c*d**2*x**8/8 + d**3*x*
*11/11), Eq(b, 0)), (2520*a**10*d**3*log(a/b + x)/(2520*a**10*b**11 + 2520
0*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*
a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400
*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**2
1*x**10) + 7381*a**10*d**3/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400
*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 63504
0*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 1134
00*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 25200*a**9*b
*d**3*x*log(a/b + x)/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*
b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5
*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**
2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 71290*a**9*b*d**3*
x/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400
*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 52920
0*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 2520
0*a*b**20*x**9 + 2520*b**21*x**10) + 113400*a**8*b**2*d**3*x**2*log(a/b +
x)/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 30240
0*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 5292
00*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + ...
```

3.183.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 953 vs. $2(396) = 792$.

Time = 0.23 (sec) , antiderivative size = 953, normalized size of antiderivative = 2.41

$$\int x(a+bx)^n \left(c+dx^3\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(x*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
(b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^3/((n^2 + 3*n + 2)*b^2) +
3*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6
*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x
^2 - 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*c^2*d/((n^5 + 15*n^4 + 85*n^3 + 22
5*n^2 + 274*n + 120)*b^5) + 3*((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769*n
^3 + 13132*n^2 + 13068*n + 5040)*b^8*x^8 + (n^7 + 21*n^6 + 175*n^5 + 735*n
^4 + 1624*n^3 + 1764*n^2 + 720*n)*a*b^7*x^7 - 7*(n^6 + 15*n^5 + 85*n^4 + 2
25*n^3 + 274*n^2 + 120*n)*a^2*b^6*x^6 + 42*(n^5 + 10*n^4 + 35*n^3 + 50*n^2
 + 24*n)*a^3*b^5*x^5 - 210*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^4*b^4*x^4 + 840*
(n^3 + 3*n^2 + 2*n)*a^5*b^3*x^3 - 2520*(n^2 + n)*a^6*b^2*x^2 + 5040*a^7*b*
n*x - 5040*a^8)*(b*x + a)^n*c*d^2/((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22
449*n^4 + 67284*n^3 + 118124*n^2 + 109584*n + 40320)*b^8) + ((n^10 + 55*n^
9 + 1320*n^8 + 18150*n^7 + 157773*n^6 + 902055*n^5 + 3416930*n^4 + 8409500
*n^3 + 12753576*n^2 + 10628640*n + 3628800)*b^11*x^11 + (n^10 + 45*n^9 + 8
70*n^8 + 9450*n^7 + 63273*n^6 + 269325*n^5 + 723680*n^4 + 1172700*n^3 + 10
26576*n^2 + 362880*n)*a*b^10*x^10 - 10*(n^9 + 36*n^8 + 546*n^7 + 4536*n^6
+ 22449*n^5 + 67284*n^4 + 118124*n^3 + 109584*n^2 + 40320*n)*a^2*b^9*x^9 +
 90*(n^8 + 28*n^7 + 322*n^6 + 1960*n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2
+ 5040*n)*a^3*b^8*x^8 - 720*(n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 +
 1764*n^2 + 720*n)*a^4*b^7*x^7 + 5040*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 ...
```

3.183.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4934 vs. 2(396) = 792.

Time = 0.37 (sec) , antiderivative size = 4934, normalized size of antiderivative = 12.46

$$\int x(a+bx)^n \left(c+dx^3\right)^3 dx = \text{Too large to display}$$

input
```
integrate(x*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="giac")
```

output
```
((b*x + a)^n*b^11*d^3*n^10*x^11 + (b*x + a)^n*a*b^10*d^3*n^10*x^10 + 55*(b
*x + a)^n*b^11*d^3*n^9*x^11 + 45*(b*x + a)^n*a*b^10*d^3*n^9*x^10 + 1320*(b
*x + a)^n*b^11*d^3*n^8*x^11 + 3*(b*x + a)^n*b^11*c*d^2*n^10*x^8 - 10*(b*x
+ a)^n*a^2*b^9*d^3*n^9*x^9 + 870*(b*x + a)^n*a*b^10*d^3*n^8*x^10 + 18150*(
b*x + a)^n*b^11*d^3*n^7*x^11 + 3*(b*x + a)^n*a*b^10*c*d^2*n^10*x^7 + 174*(
b*x + a)^n*b^11*c*d^2*n^9*x^8 - 360*(b*x + a)^n*a^2*b^9*d^3*n^8*x^9 + 9450
*(b*x + a)^n*a*b^10*d^3*n^7*x^10 + 157773*(b*x + a)^n*b^11*d^3*n^6*x^11 +
153*(b*x + a)^n*a*b^10*c*d^2*n^9*x^7 + 4383*(b*x + a)^n*b^11*c*d^2*n^8*x^8
 + 90*(b*x + a)^n*a^3*b^8*d^3*n^8*x^8 - 5460*(b*x + a)^n*a^2*b^9*d^3*n^7*x
^9 + 63273*(b*x + a)^n*a*b^10*d^3*n^6*x^10 + 902055*(b*x + a)^n*b^11*d^3*n
^5*x^11 + 3*(b*x + a)^n*b^11*c^2*d*n^10*x^5 - 21*(b*x + a)^n*a^2*b^9*c*d^2
*n^9*x^6 + 3312*(b*x + a)^n*a*b^10*c*d^2*n^8*x^7 + 62946*(b*x + a)^n*b^11*
c*d^2*n^7*x^8 + 2520*(b*x + a)^n*a^3*b^8*d^3*n^7*x^8 - 45360*(b*x + a)^n*a
^2*b^9*d^3*n^6*x^9 + 269325*(b*x + a)^n*a*b^10*d^3*n^5*x^10 + 3416930*(b*x
 + a)^n*b^11*d^3*n^4*x^11 + 3*(b*x + a)^n*a*b^10*c^2*d*n^10*x^4 + 183*(b*x
 + a)^n*b^11*c^2*d*n^9*x^5 - 945*(b*x + a)^n*a^2*b^9*c*d^2*n^8*x^6 + 39762
*(b*x + a)^n*a*b^10*c*d^2*n^7*x^7 - 720*(b*x + a)^n*a^4*b^7*d^3*n^7*x^7 +
568701*(b*x + a)^n*b^11*c*d^2*n^6*x^8 + 28980*(b*x + a)^n*a^3*b^8*d^3*n^6*
x^8 - 224490*(b*x + a)^n*a^2*b^9*d^3*n^5*x^9 + 723680*(b*x + a)^n*a*b^10*d
^3*n^4*x^10 + 8409500*(b*x + a)^n*b^11*d^3*n^3*x^11 + 171*(b*x + a)^n*a...
```

3.183.9 Mupad [B] (verification not implemented)

Time = 22.66 (sec) , antiderivative size = 2436, normalized size of antiderivative = 6.15

$$\int x(a+bx)^n\left(c+dx^3\right)^3\,dx=\text{Too large to display}$$

input
```
int(x*(c + d*x^3)^3*(a + b*x)^n,x)
```

output
```
(d^3*x^11*(a + b*x)^n*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n
^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628
800))/(120543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 133395
35*n^5 + 2637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11
+ 39916800) - (a^2*(a + b*x)^n*(19958400*b^9*c^3 - 3628800*a^9*d^3 + 30334
320*b^9*c^3*n + 19978308*b^9*c^3*n^2 + 7494416*b^9*c^3*n^3 + 1767087*b^9*c
^3*n^4 + 271929*b^9*c^3*n^5 + 27342*b^9*c^3*n^6 + 1734*b^9*c^3*n^7 + 63*b^
9*c^3*n^8 + b^9*c^3*n^9 - 23950080*a^3*b^6*c^2*d + 14968800*a^6*b^3*c*d^2
- 17640288*a^3*b^6*c^2*d*n + 4520880*a^6*b^3*c*d^2*n - 5365728*a^3*b^6*c^2
*d*n^2 + 453600*a^6*b^3*c*d^2*n^2 - 862920*a^3*b^6*c^2*d*n^3 + 15120*a^6*b
^3*c*d^2*n^3 - 77400*a^3*b^6*c^2*d*n^4 - 3672*a^3*b^6*c^2*d*n^5 - 72*a^3*b
^6*c^2*d*n^6))/(b^11*(120543840*n + 150917976*n^2 + 105258076*n^3 + 459957
30*n^4 + 13339535*n^5 + 2637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 +
66*n^10 + n^11 + 39916800)) + (x^2*(n + 1)*(a + b*x)^n*(19958400*b^9*c^3 +
 1814400*a^9*d^3*n + 30334320*b^9*c^3*n + 19978308*b^9*c^3*n^2 + 7494416*b
^9*c^3*n^3 + 1767087*b^9*c^3*n^4 + 271929*b^9*c^3*n^5 + 27342*b^9*c^3*n^6
+ 1734*b^9*c^3*n^7 + 63*b^9*c^3*n^8 + b^9*c^3*n^9 + 11975040*a^3*b^6*c^2*d
*n - 7484400*a^6*b^3*c*d^2*n + 8820144*a^3*b^6*c^2*d*n^2 - 2260440*a^6*b^3
*c*d^2*n^2 + 2682864*a^3*b^6*c^2*d*n^3 - 226800*a^6*b^3*c*d^2*n^3 + 431460
*a^3*b^6*c^2*d*n^4 - 7560*a^6*b^3*c*d^2*n^4 + 38700*a^3*b^6*c^2*d*n^5 +...
```

3.183. $\int x(a+bx)^n \left(c+dx^3\right)^3 \, dx$

3.184 $\int (a+bx)^n \left(c+dx^3\right)^3 \, dx$

3.184.1 Optimal result

Integrand size = 17, antiderivative size = 337

$$\begin{aligned}\int (a+bx)^n \left(c+dx^3\right)^3 \, dx = & \frac{\left(b^3c-a^3d\right)^3 (a+bx)^{1+n}}{b^{10}(1+n)}+\frac{9a^2d\left(b^3c-a^3d\right)^2 (a+bx)^{2+n}}{b^{10}(2+n)} \\ & -\frac{9ad\left(b^3c-4a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{3+n}}{b^{10}(3+n)} \\ & +\frac{3d\left(b^6c^2-20a^3b^3cd+28a^6d^2\right)(a+bx)^{4+n}}{b^{10}(4+n)} \\ & +\frac{9a^2d^2\left(5b^3c-14a^3d\right)(a+bx)^{5+n}}{b^{10}(5+n)} \\ & -\frac{18ad^2\left(b^3c-7a^3d\right)(a+bx)^{6+n}}{b^{10}(6+n)} \\ & +\frac{3d^2\left(b^3c-28a^3d\right)(a+bx)^{7+n}}{b^{10}(7+n)}+\frac{36a^2d^3(a+bx)^{8+n}}{b^{10}(8+n)} \\ & -\frac{9ad^3(a+bx)^{9+n}}{b^{10}(9+n)}+\frac{d^3(a+bx)^{10+n}}{b^{10}(10+n)}\end{aligned}$$

output
```
(-a^3*d+b^3*c)^3*(b*x+a)^(1+n)/b^10/(1+n)+9*a^2*d*(-a^3*d+b^3*c)^2*(b*x+a)
^(2+n)/b^10/(2+n)-9*a*d*(-4*a^3*d+b^3*c)*(-a^3*d+b^3*c)*(b*x+a)^(3+n)/b^10
/(3+n)+3*d*(28*a^6*d^2-20*a^3*b^3*c*d+b^6*c^2)*(b*x+a)^(4+n)/b^10/(4+n)+9*
a^2*d^2*(-14*a^3*d+5*b^3*c)*(b*x+a)^(5+n)/b^10/(5+n)-18*a*d^2*(-7*a^3*d+b^
3*c)*(b*x+a)^(6+n)/b^10/(6+n)+3*d^2*(-28*a^3*d+b^3*c)*(b*x+a)^(7+n)/b^10/(
7+n)+36*a^2*d^3*(b*x+a)^(8+n)/b^10/(8+n)-9*a*d^3*(b*x+a)^(9+n)/b^10/(9+n)+
d^3*(b*x+a)^(10+n)/b^10/(10+n)
```

3.184.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.86

$$\int (a+bx)^n \left(c+dx^3\right)^3 \, dx$$
$$= \frac{(a+bx)^{1+n} \left(\frac{\left(b^3c-a^3d\right)^3}{1+n} + \frac{9d\left(ab^3c-a^4d\right)^2(a+bx)}{2+n} - \frac{9ad\left(b^3c-4a^3d\right)\left(b^3c-a^3d\right)(a+bx)^2}{3+n} + \frac{3d\left(b^6c^2-20a^3b^3cd+28a^6d^2\right)(a+bx)^3}{4+n} + \cdots\right)}{b \cdots}$$

input
```
Integrate[(a + b*x)^n*(c + d*x^3)^3,x]
```

output
```
((a + b*x)^(1 + n)*((b^3*c - a^3*d)^3/(1 + n) + (9*d*(a*b^3*c - a^4*d)^2*(
a + b*x))/(2 + n) - (9*a*d*(b^3*c - 4*a^3*d)*(b^3*c - a^3*d)*(a + b*x)^2)/
(3 + n) + (3*d*(b^6*c^2 - 20*a^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^3)/(4 + n
) + (9*a^2*d^2*(5*b^3*c - 14*a^3*d)*(a + b*x)^4)/(5 + n) + (18*a*d^2*(-(b^
3*c) + 7*a^3*d)*(a + b*x)^5)/(6 + n) + (3*d^2*(b^3*c - 28*a^3*d)*(a + b*x)
^6)/(7 + n) + (36*a^2*d^3*(a + b*x)^7)/(8 + n) - (9*a*d^3*(a + b*x)^8)/(9
+ n) + (d^3*(a + b*x)^9)/(10 + n)))/b^10
```

3.184.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c+dx^3\right)^3 (a+bx)^n \, dx$$
$$\Big\downarrow \; 2389$$
$$\int \left(\frac{9d\left(ab^3c-a^4d\right)^2(a+bx)^{n+1}}{b^9} + \frac{18ad^2\left(7a^3d-b^3c\right)(a+bx)^{n+5}}{b^9} + \frac{3d^2\left(b^3c-28a^3d\right)(a+bx)^{n+6}}{b^9} + \frac{\left(b^3c-a^3\cdots\right.}{\cdots}\right.$$
$$\Big\downarrow \; 2009$$

$$-\frac{18ad^2(b^3c-7a^3d)(a+bx)^{n+6}}{b^{10}(n+6)}+\frac{3d^2(b^3c-28a^3d)(a+bx)^{n+7}}{b^{10}(n+7)}+\frac{(b^3c-a^3d)^3(a+bx)^{n+1}}{b^{10}(n+1)}-\frac{9ad(b^3c-4a^3d)(b^3c-a^3d)(a+bx)^{n+3}}{b^{10}(n+3)}+\frac{36a^2d^3(a+bx)^{n+8}}{b^{10}(n+8)}+\frac{3d(28a^6d^2-20a^3b^3cd+b^6c^2)(a+bx)^{n+4}}{b^{10}(n+4)}+\frac{9a^2d^2(5b^3c-14a^3d)(a+bx)^{n+5}}{b^{10}(n+5)}+\frac{9a^2d(b^3c-a^3d)^2(a+bx)^{n+2}}{b^{10}(n+2)}-\frac{9ad^3(a+bx)^{n+9}}{b^{10}(n+9)}+\frac{d^3(a+bx)^{n+10}}{b^{10}(n+10)}$$

input
```
Int[(a + b*x)^n*(c + d*x^3)^3,x]
```

output
```
((b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^10*(1 + n)) + (9*a^2*d*(b^3*c - a
^3*d)^2*(a + b*x)^(2 + n))/(b^10*(2 + n)) - (9*a*d*(b^3*c - 4*a^3*d)*(b^3*
c - a^3*d)*(a + b*x)^(3 + n))/(b^10*(3 + n)) + (3*d*(b^6*c^2 - 20*a^3*b^3*
c*d + 28*a^6*d^2)*(a + b*x)^(4 + n))/(b^10*(4 + n)) + (9*a^2*d^2*(5*b^3*c
- 14*a^3*d)*(a + b*x)^(5 + n))/(b^10*(5 + n)) - (18*a*d^2*(b^3*c - 7*a^3*d
)*(a + b*x)^(6 + n))/(b^10*(6 + n)) + (3*d^2*(b^3*c - 28*a^3*d)*(a + b*x)^
(7 + n))/(b^10*(7 + n)) + (36*a^2*d^3*(a + b*x)^(8 + n))/(b^10*(8 + n)) -
(9*a*d^3*(a + b*x)^(9 + n))/(b^10*(9 + n)) + (d^3*(a + b*x)^(10 + n))/(b^1
0*(10 + n))
```

3.184.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.184.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2279 vs. $2(337) = 674$.

Time = 1.14 (sec) , antiderivative size = 2280, normalized size of antiderivative = 6.77

method	result	size
gosper	Expression too large to display	2280
risch	Expression too large to display	2665
parallelrisch	Expression too large to display	3960

input
```
int((b*x+a)^n*(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/b^10*(b*x+a)^(1+n)/(n^10+55*n^9+1320*n^8+18150*n^7+157773*n^6+902055*n^
5+3416930*n^4+8409500*n^3+12753576*n^2+10628640*n+3628800)*(-b^9*d^3*n^9*x
^9-45*b^9*d^3*n^8*x^9+9*a*b^8*d^3*n^8*x^8-870*b^9*d^3*n^7*x^9+324*a*b^8*d^
3*n^7*x^8-3*b^9*c*d^2*n^9*x^6-9450*b^9*d^3*n^6*x^9-72*a^2*b^7*d^3*n^7*x^7+
4914*a*b^8*d^3*n^6*x^8-144*b^9*c*d^2*n^8*x^6-63273*b^9*d^3*n^5*x^9-2016*a^
2*b^7*d^3*n^6*x^7+18*a*b^8*c*d^2*n^8*x^5+40824*a*b^8*d^3*n^5*x^8-2952*b^9*
c*d^2*n^7*x^6-269325*b^9*d^3*n^4*x^9+504*a^3*b^6*d^3*n^6*x^6-23184*a^2*b^7
*d^3*n^5*x^7+756*a*b^8*c*d^2*n^7*x^5+202041*a*b^8*d^3*n^4*x^8-3*b^9*c^2*d*
n^9*x^3-33786*b^9*c*d^2*n^6*x^6-723680*b^9*d^3*n^3*x^9+10584*a^3*b^6*d^3*n
^5*x^6-90*a^2*b^7*c*d^2*n^7*x^4-141120*a^2*b^7*d^3*n^4*x^7+13176*a*b^8*c*d
^2*n^6*x^5+605556*a*b^8*d^3*n^3*x^8-153*b^9*c^2*d*n^8*x^3-236817*b^9*c*d^2
*n^5*x^6-1172700*b^9*d^3*n^2*x^9-3024*a^4*b^5*d^3*n^5*x^5+88200*a^3*b^6*d^
3*n^4*x^6-3330*a^2*b^7*c*d^2*n^6*x^4-487368*a^2*b^7*d^3*n^3*x^7+9*a*b^8*c^
2*d*n^8*x^2+123660*a*b^8*c*d^2*n^5*x^5+1063116*a*b^8*d^3*n^2*x^8-3348*b^9*
c^2*d*n^7*x^3-1048446*b^9*c*d^2*n^4*x^6-1026576*b^9*d^3*n*x^9-45360*a^4*b^
5*d^3*n^4*x^5+360*a^3*b^6*c*d^2*n^6*x^3+370440*a^3*b^6*d^3*n^3*x^6-49230*a
^2*b^7*c*d^2*n^5*x^4-945504*a^2*b^7*d^3*n^2*x^7+432*a*b^8*c^2*d*n^7*x^2+67
8942*a*b^8*c*d^2*n^4*x^5+986256*a*b^8*d^3*n*x^8-b^9*c^3*n^9-41058*b^9*c^2*
d*n^6*x^3-2911668*b^9*c*d^2*n^3*x^6-362880*b^9*d^3*x^9+15120*a^5*b^4*d^3*n
^4*x^4-257040*a^4*b^5*d^3*n^3*x^5+11880*a^3*b^6*c*d^2*n^5*x^3+818496*a^...
```

3.184.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2313 vs. $2(337) = 674$.

Time = 0.33 (sec) , antiderivative size = 2313, normalized size of antiderivative = 6.86

$$\int (a+bx)^n \left(c+dx^3\right)^3 dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^n*(d*x^3+c)^3,x, algorithm="fricas")
```

output
```
(a*b^9*c^3*n^9 + 54*a*b^9*c^3*n^8 + 1266*a*b^9*c^3*n^7 + 3628800*a*b^9*c^3
 - 2721600*a^4*b^6*c^2*d + 1555200*a^7*b^3*c*d^2 - 362880*a^10*d^3 + (b^10
*d^3*n^9 + 45*b^10*d^3*n^8 + 870*b^10*d^3*n^7 + 9450*b^10*d^3*n^6 + 63273*
b^10*d^3*n^5 + 269325*b^10*d^3*n^4 + 723680*b^10*d^3*n^3 + 1172700*b^10*d^
3*n^2 + 1026576*b^10*d^3*n + 362880*b^10*d^3)*x^10 + (a*b^9*d^3*n^9 + 36*a
*b^9*d^3*n^8 + 546*a*b^9*d^3*n^7 + 4536*a*b^9*d^3*n^6 + 22449*a*b^9*d^3*n^
5 + 67284*a*b^9*d^3*n^4 + 118124*a*b^9*d^3*n^3 + 109584*a*b^9*d^3*n^2 + 40
320*a*b^9*d^3*n)*x^9 - 9*(a^2*b^8*d^3*n^8 + 28*a^2*b^8*d^3*n^7 + 322*a^2*b
^8*d^3*n^6 + 1960*a^2*b^8*d^3*n^5 + 6769*a^2*b^8*d^3*n^4 + 13132*a^2*b^8*d
^3*n^3 + 13068*a^2*b^8*d^3*n^2 + 5040*a^2*b^8*d^3*n)*x^8 + 3*(b^10*c*d^2*n
^9 + 48*b^10*c*d^2*n^8 + 518400*b^10*c*d^2 + 24*(41*b^10*c*d^2 + a^3*b^7*d
^3)*n^7 + 6*(1877*b^10*c*d^2 + 84*a^3*b^7*d^3)*n^6 + 21*(3759*b^10*c*d^2 +
 200*a^3*b^7*d^3)*n^5 + 42*(8321*b^10*c*d^2 + 420*a^3*b^7*d^3)*n^4 + 4*(24
2639*b^10*c*d^2 + 9744*a^3*b^7*d^3)*n^3 + 72*(22439*b^10*c*d^2 + 588*a^3*b
^7*d^3)*n^2 + 1440*(1003*b^10*c*d^2 + 12*a^3*b^7*d^3)*n)*x^7 + 18*(938*a*b
^9*c^3 - a^4*b^6*c^2*d)*n^6 + 3*(a*b^9*c*d^2*n^9 + 42*a*b^9*c*d^2*n^8 + 73
2*a*b^9*c*d^2*n^7 + 6*(1145*a*b^9*c*d^2 - 28*a^4*b^6*d^3)*n^6 + 9*(4191*a*
b^9*c*d^2 - 280*a^4*b^6*d^3)*n^5 + 24*(5132*a*b^9*c*d^2 - 595*a^4*b^6*d^3)
*n^4 + 4*(57887*a*b^9*c*d^2 - 9450*a^4*b^6*d^3)*n^3 + 48*(4715*a*b^9*c*d^2
 - 959*a^4*b^6*d^3)*n^2 + 2880*(30*a*b^9*c*d^2 - 7*a^4*b^6*d^3)*n)*x^6 ...
```

3.184.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40536 vs. $2(316) = 632$.

Time = 66.28 (sec) , antiderivative size = 40536, normalized size of antiderivative = 120.28

$$\int (a+bx)^n \left(c+dx^3\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**n*(d*x**3+c)**3,x)
```

output
```
Piecewise((a**n*(c**3*x + 3*c**2*d*x**4/4 + 3*c*d**2*x**7/7 + d**3*x**10/1
0), Eq(b, 0)), (2520*a**9*d**3*log(a/b + x)/(2520*a**9*b**10 + 22680*a**8*
b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**
14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**
17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + 7129*a**9*d**3/(2520*a**
9*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x
**3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*
x**6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + 226
80*a**8*b*d**3*x*log(a/b + x)/(2520*a**9*b**10 + 22680*a**8*b**11*x + 9072
0*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 3175
20*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 2268
0*a*b**18*x**8 + 2520*b**19*x**9) + 61641*a**8*b*d**3*x/(2520*a**9*b**10 +
 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317
520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90
720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + 90720*a**7*b
**2*d**3*x**2*log(a/b + x)/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90720*a
**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*
a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a
*b**18*x**8 + 2520*b**19*x**9) + 235224*a**7*b**2*d**3*x**2/(2520*a**9*b**
10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**...
```

3.184.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(337) = 674$.

Time = 0.22 (sec) , antiderivative size = 770, normalized size of antiderivative = 2.28

$$\int (a+bx)^n (c+dx^3)^3 \, dx = \frac{(bx+a)^{n+1}c^3}{b(n+1)}$$
$$+ \frac{3\left((n^3+6n^2+11n+6)b^4x^4+(n^3+3n^2+2n)ab^3x^3-3(n^2+n)a^2b^2x^2+6a^3bnx-6a^4\right)(bx+a)^n c^2}{(n^4+10n^3+35n^2+50n+24)b^4}$$
$$+ \frac{3\left((n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)b^7x^7+(n^6+15n^5+85n^4+225n^3+274n\right.}{}$$
$$+ \frac{\left((n^9+45n^8+870n^7+9450n^6+63273n^5+269325n^4+723680n^3+1172700n^2+1026576n+3628\right.}{}$$

input
```
integrate((b*x+a)^n*(d*x^3+c)^3,x, algorithm="maxima")
```

output
```
(b*x + a)^(n + 1)*c^3/(b*(n + 1)) + 3*((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 +
(n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*
a^4)*(b*x + a)^n*c^2*d/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4) + 3*((n^6
 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 +
15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4 +
 35*n^3 + 50*n^2 + 24*n)*a^2*b^5*x^5 + 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^3
*b^4*x^4 - 120*(n^3 + 3*n^2 + 2*n)*a^4*b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x^2
 - 720*a^6*b*n*x + 720*a^7)*(b*x + a)^n*c*d^2/((n^7 + 28*n^6 + 322*n^5 + 1
960*n^4 + 6769*n^3 + 13132*n^2 + 13068*n + 5040)*b^7) + ((n^9 + 45*n^8 + 8
70*n^7 + 9450*n^6 + 63273*n^5 + 269325*n^4 + 723680*n^3 + 1172700*n^2 + 10
26576*n + 362880)*b^10*x^10 + (n^9 + 36*n^8 + 546*n^7 + 4536*n^6 + 22449*n
^5 + 67284*n^4 + 118124*n^3 + 109584*n^2 + 40320*n)*a*b^9*x^9 - 9*(n^8 + 2
8*n^7 + 322*n^6 + 1960*n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a^
2*b^8*x^8 + 72*(n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 7
20*n)*a^3*b^7*x^7 - 504*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n
)*a^4*b^6*x^6 + 3024*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^5*b^5*x^5 -
 15120*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^6*b^4*x^4 + 60480*(n^3 + 3*n^2 + 2*n
)*a^7*b^3*x^3 - 181440*(n^2 + n)*a^8*b^2*x^2 + 362880*a^9*b*n*x - 362880*a
^10)*(b*x + a)^n*d^3/((n^10 + 55*n^9 + 1320*n^8 + 18150*n^7 + 157773*n^6 +
 902055*n^5 + 3416930*n^4 + 8409500*n^3 + 12753576*n^2 + 10628640*n + 3...
```

3.184.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3874 vs. $2(337) = 674$.

Time = 0.40 (sec) , antiderivative size = 3874, normalized size of antiderivative = 11.50

$$\int (a+bx)^n \left(c+dx^3\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((b*x+a)^n*(d*x^3+c)^3,x, algorithm="giac")
```

output
```
((b*x + a)^n*b^10*d^3*n^9*x^10 + (b*x + a)^n*a*b^9*d^3*n^9*x^9 + 45*(b*x +
 a)^n*b^10*d^3*n^8*x^10 + 36*(b*x + a)^n*a*b^9*d^3*n^8*x^9 + 870*(b*x + a)
^n*b^10*d^3*n^7*x^10 + 3*(b*x + a)^n*b^10*c*d^2*n^9*x^7 - 9*(b*x + a)^n*a^
2*b^8*d^3*n^8*x^8 + 546*(b*x + a)^n*a*b^9*d^3*n^7*x^9 + 9450*(b*x + a)^n*b
^10*d^3*n^6*x^10 + 3*(b*x + a)^n*a*b^9*c*d^2*n^9*x^6 + 144*(b*x + a)^n*b^1
0*c*d^2*n^8*x^7 - 252*(b*x + a)^n*a^2*b^8*d^3*n^7*x^8 + 4536*(b*x + a)^n*a
*b^9*d^3*n^6*x^9 + 63273*(b*x + a)^n*b^10*d^3*n^5*x^10 + 126*(b*x + a)^n*a
*b^9*c*d^2*n^8*x^6 + 2952*(b*x + a)^n*b^10*c*d^2*n^7*x^7 + 72*(b*x + a)^n*
a^3*b^7*d^3*n^7*x^7 - 2898*(b*x + a)^n*a^2*b^8*d^3*n^6*x^8 + 22449*(b*x +
a)^n*a*b^9*d^3*n^5*x^9 + 269325*(b*x + a)^n*b^10*d^3*n^4*x^10 + 3*(b*x + a
)^n*b^10*c^2*d*n^9*x^4 - 18*(b*x + a)^n*a^2*b^8*c*d^2*n^8*x^5 + 2196*(b*x
+ a)^n*a*b^9*c*d^2*n^7*x^6 + 33786*(b*x + a)^n*b^10*c*d^2*n^6*x^7 + 1512*(
b*x + a)^n*a^3*b^7*d^3*n^6*x^7 - 17640*(b*x + a)^n*a^2*b^8*d^3*n^5*x^8 + 6
7284*(b*x + a)^n*a*b^9*d^3*n^4*x^9 + 723680*(b*x + a)^n*b^10*d^3*n^3*x^10
+ 3*(b*x + a)^n*a*b^9*c^2*d*n^9*x^3 + 153*(b*x + a)^n*b^10*c^2*d*n^8*x^4 -
 666*(b*x + a)^n*a^2*b^8*c*d^2*n^7*x^5 + 20610*(b*x + a)^n*a*b^9*c*d^2*n^6
*x^6 - 504*(b*x + a)^n*a^4*b^6*d^3*n^6*x^6 + 236817*(b*x + a)^n*b^10*c*d^2
*n^5*x^7 + 12600*(b*x + a)^n*a^3*b^7*d^3*n^5*x^7 - 60921*(b*x + a)^n*a^2*b
^8*d^3*n^4*x^8 + 118124*(b*x + a)^n*a*b^9*d^3*n^3*x^9 + 1172700*(b*x + a)^
n*b^10*d^3*n^2*x^10 + 144*(b*x + a)^n*a*b^9*c^2*d*n^8*x^3 + 3348*(b*x +...
```

3.184.9 Mupad [B] (verification not implemented)

Time = 21.09 (sec) , antiderivative size = 2001, normalized size of antiderivative = 5.94

$$\int (a+bx)^n \left(c+dx^3\right)^3 \, dx = \text{Too large to display}$$

input
```
int((c + d*x^3)^3*(a + b*x)^n,x)
```

output
```
((a + b*x)^n*(3628800*a*b^9*c^3 - 362880*a^10*d^3 - 2721600*a^4*b^6*c^2*d
+ 1555200*a^7*b^3*c*d^2 + 5753736*a*b^9*c^3*n^2 + 2655764*a*b^9*c^3*n^3 +
761166*a*b^9*c^3*n^4 + 140889*a*b^9*c^3*n^5 + 16884*a*b^9*c^3*n^6 + 1266*a
*b^9*c^3*n^7 + 54*a*b^9*c^3*n^8 + a*b^9*c^3*n^9 + 6999840*a*b^9*c^3*n - 23
01480*a^4*b^6*c^2*d*n + 522720*a^7*b^3*c*d^2*n - 801432*a^4*b^6*c^2*d*n^2
+ 58320*a^7*b^3*c*d^2*n^2 - 147150*a^4*b^6*c^2*d*n^3 + 2160*a^7*b^3*c*d^2*
n^3 - 15030*a^4*b^6*c^2*d*n^4 - 810*a^4*b^6*c^2*d*n^5 - 18*a^4*b^6*c^2*d*n
^6))/(b^10*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055
*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) + (x*
(a + b*x)^n*(3628800*b^10*c^3 + 6999840*b^10*c^3*n + 5753736*b^10*c^3*n^2
+ 2655764*b^10*c^3*n^3 + 761166*b^10*c^3*n^4 + 140889*b^10*c^3*n^5 + 16884
*b^10*c^3*n^6 + 1266*b^10*c^3*n^7 + 54*b^10*c^3*n^8 + b^10*c^3*n^9 + 36288
0*a^9*b*d^3*n + 2721600*a^3*b^7*c^2*d*n - 1555200*a^6*b^4*c*d^2*n + 230148
0*a^3*b^7*c^2*d*n^2 - 522720*a^6*b^4*c*d^2*n^2 + 801432*a^3*b^7*c^2*d*n^3
- 58320*a^6*b^4*c*d^2*n^3 + 147150*a^3*b^7*c^2*d*n^4 - 2160*a^6*b^4*c*d^2*
n^4 + 15030*a^3*b^7*c^2*d*n^5 + 810*a^3*b^7*c^2*d*n^6 + 18*a^3*b^7*c^2*d*n
^7))/(b^10*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055
*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) + (d^
3*x^10*(a + b*x)^n*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63
273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880))/(10628640*n + 12...
```

3.184. $\int (a+bx)^n \left(c+dx^3\right)^3 \, dx$

3.185 $\int \frac{(a+bx)^n(c+dx^3)^3}{x} dx$

3.185.1 Optimal result

Integrand size = 20, antiderivative size = 358

$$\begin{aligned}\int \frac{(a+bx)^n(c+dx^3)^3}{x} dx = {} & \frac{a^2d(3b^6c^2-3a^3b^3cd+a^6d^2)(a+bx)^{1+n}}{b^9(1+n)} \\ & -\frac{ad(6b^6c^2-15a^3b^3cd+8a^6d^2)(a+bx)^{2+n}}{b^9(2+n)} \\ & +\frac{d(3b^6c^2-30a^3b^3cd+28a^6d^2)(a+bx)^{3+n}}{b^9(3+n)} \\ & +\frac{2a^2d^2(15b^3c-28a^3d)(a+bx)^{4+n}}{b^9(4+n)} \\ & -\frac{5ad^2(3b^3c-14a^3d)(a+bx)^{5+n}}{b^9(5+n)} \\ & +\frac{d^2(3b^3c-56a^3d)(a+bx)^{6+n}}{b^9(6+n)}+\frac{28a^2d^3(a+bx)^{7+n}}{b^9(7+n)} \\ & -\frac{8ad^3(a+bx)^{8+n}}{b^9(8+n)}+\frac{d^3(a+bx)^{9+n}}{b^9(9+n)} \\ & -\frac{c^3(a+bx)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{bx}{a}\right)}{a(1+n)}\end{aligned}$$

output
```
a^2*d*(a^6*d^2-3*a^3*b^3*c*d+3*b^6*c^2)*(b*x+a)^(1+n)/b^9/(1+n)-a*d*(8*a^6
*d^2-15*a^3*b^3*c*d+6*b^6*c^2)*(b*x+a)^(2+n)/b^9/(2+n)+d*(28*a^6*d^2-30*a^
3*b^3*c*d+3*b^6*c^2)*(b*x+a)^(3+n)/b^9/(3+n)+2*a^2*d^2*(-28*a^3*d+15*b^3*c
)*(b*x+a)^(4+n)/b^9/(4+n)-5*a*d^2*(-14*a^3*d+3*b^3*c)*(b*x+a)^(5+n)/b^9/(5
+n)+d^2*(-56*a^3*d+3*b^3*c)*(b*x+a)^(6+n)/b^9/(6+n)+28*a^2*d^3*(b*x+a)^(7+
n)/b^9/(7+n)-8*a*d^3*(b*x+a)^(8+n)/b^9/(8+n)+d^3*(b*x+a)^(9+n)/b^9/(9+n)-c
^3*(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

3.185.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^n (c+dx^3)^3}{x}\,dx = (a+bx)^{1+n}\left(\frac{a^2d(3b^6c^2-3a^3b^3cd+a^6d^2)}{b^9(1+n)} - \frac{ad(6b^6c^2-15a^3b^3cd+8a^6d^2)(a+bx)}{b^9(2+n)} + \frac{d(3b^6c^2-30a^3b^3cd+28a^6d^2)(a+bx)^2}{b^9(3+n)} + \frac{2a^2d^2(15b^3c-28a^3d)(a+bx)^3}{b^9(4+n)} + \frac{5ad^2(-3b^3c+14a^3d)(a+bx)^4}{b^9(5+n)} + \frac{d^2(3b^3c-56a^3d)(a+bx)^5}{b^9(6+n)} + \frac{28a^2d^3(a+bx)^6}{b^9(7+n)} - \frac{8ad^3(a+bx)^7}{b^9(8+n)} + \frac{d^3(a+bx)^8}{b^9(9+n)} - \frac{c^3\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{a+bx}{a}\right)}{a+an}\right)$$

input
```
Integrate[((a + b*x)^n*(c + d*x^3)^3)/x,x]
```

output
```
(a + b*x)^(1 + n)*((a^2*d*(3*b^6*c^2 - 3*a^3*b^3*c*d + a^6*d^2))/(b^9*(1 +
 n)) - (a*d*(6*b^6*c^2 - 15*a^3*b^3*c*d + 8*a^6*d^2)*(a + b*x))/(b^9*(2 +
n)) + (d*(3*b^6*c^2 - 30*a^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^2)/(b^9*(3 +
n)) + (2*a^2*d^2*(15*b^3*c - 28*a^3*d)*(a + b*x)^3)/(b^9*(4 + n)) + (5*a*d
^2*(-3*b^3*c + 14*a^3*d)*(a + b*x)^4)/(b^9*(5 + n)) + (d^2*(3*b^3*c - 56*a
^3*d)*(a + b*x)^5)/(b^9*(6 + n)) + (28*a^2*d^3*(a + b*x)^6)/(b^9*(7 + n))
- (8*a*d^3*(a + b*x)^7)/(b^9*(8 + n)) + (d^3*(a + b*x)^8)/(b^9*(9 + n)) -
(c^3*Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*x)/a])/(a + a*n))
```

3.185.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx^3)^3 (a+bx)^n}{x} dx$$

$$\Big\downarrow \text{2123}$$

$$\int \left(\frac{5ad^2(14a^3d-3b^3c)(a+bx)^{n+4}}{b^8}+\frac{d^2(3b^3c-56a^3d)(a+bx)^{n+5}}{b^8}+\frac{28a^2d^3(a+bx)^{n+6}}{b^8}-\frac{ad(8a^6d^2-15a^3b^3\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{5ad^2(3b^3c-14a^3d)(a+bx)^{n+5}}{b^9(n+5)}+\frac{d^2(3b^3c-56a^3d)(a+bx)^{n+6}}{b^9(n+6)}+\frac{28a^2d^3(a+bx)^{n+7}}{b^9(n+7)}-\frac{ad(8a^6d^2-15a^3b^3cd+6b^6c^2)(a+bx)^{n+2}}{b^9(n+2)}+\frac{d(28a^6d^2-30a^3b^3cd+3b^6c^2)(a+bx)^{n+3}}{b^9(n+3)}+\frac{2a^2d^2(15b^3c-28a^3d)(a+bx)^{n+4}}{b^9(n+4)}+\frac{a^2d(a^6d^2-3a^3b^3cd+3b^6c^2)(a+bx)^{n+1}}{b^9(n+1)}-\frac{8ad^3(a+bx)^{n+8}}{b^9(n+8)}+\frac{d^3(a+bx)^{n+9}}{b^9(n+9)}-\frac{c^3(a+bx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{bx}{a}+1\right)}{a(n+1)}$$

input
```
Int[((a + b*x)^n*(c + d*x^3)^3)/x,x]
```

output
```
(a^2*d*(3*b^6*c^2 - 3*a^3*b^3*c*d + a^6*d^2)*(a + b*x)^(1 + n))/(b^9*(1 +
n)) - (a*d*(6*b^6*c^2 - 15*a^3*b^3*c*d + 8*a^6*d^2)*(a + b*x)^(2 + n))/(b^
9*(2 + n)) + (d*(3*b^6*c^2 - 30*a^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^(3 + n
))/(b^9*(3 + n)) + (2*a^2*d^2*(15*b^3*c - 28*a^3*d)*(a + b*x)^(4 + n))/(b^
9*(4 + n)) - (5*a*d^2*(3*b^3*c - 14*a^3*d)*(a + b*x)^(5 + n))/(b^9*(5 + n)
) + (d^2*(3*b^3*c - 56*a^3*d)*(a + b*x)^(6 + n))/(b^9*(6 + n)) + (28*a^2*d
^3*(a + b*x)^(7 + n))/(b^9*(7 + n)) - (8*a*d^3*(a + b*x)^(8 + n))/(b^9*(8
+ n)) + (d^3*(a + b*x)^(9 + n))/(b^9*(9 + n)) - (c^3*(a + b*x)^(1 + n)*Hyp
ergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n))
```

3.185.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

3.185.4 Maple [F]

$$\int \frac{(bx+a)^n\,(x^3d+c)^3}{x}dx$$

input
```
int((b*x+a)^n*(d*x^3+c)^3/x,x)
```

output
```
int((b*x+a)^n*(d*x^3+c)^3/x,x)
```

3.185.5 Fricas [F]

$$\int \frac{(a+bx)^n\,(c+dx^3)^3}{x}\,dx = \int \frac{(dx^3+c)^3(bx+a)^n}{x}\,dx$$

input
```
integrate((b*x+a)^n*(d*x^3+c)^3/x,x, algorithm="fricas")
```

output
```
integral((d^3*x^9 + 3*c*d^2*x^6 + 3*c^2*d*x^3 + c^3)*(b*x + a)^n/x, x)
```

3.185.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12492 vs. 2(338) = 676.

Time = 13.33 (sec) , antiderivative size = 17189, normalized size of antiderivative = 48.01

$$\int \frac{(a+bx)^n\,(c+dx^3)^3}{x}\,dx = \text{Too large to display}$$

input
```
integrate((b*x+a)**n*(d*x**3+c)**3/x,x)
```

output
```
3*c**2*d*Piecewise((a**n*x**3/3, Eq(b, 0)), (2*a**2*log(a/b + x)/(2*a**2*b
**3 + 4*a*b**4*x + 2*b**5*x**2) + 3*a**2/(2*a**2*b**3 + 4*a*b**4*x + 2*b**
5*x**2) + 4*a*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) +
4*a*b*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*log(a/b + x
)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a**2*log(a/b +
 x)/(a*b**3 + b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/b + x)/(a
*b**3 + b**4*x) + b**2*x**2/(a*b**3 + b**4*x), Eq(n, -2)), (a**2*log(a/b +
 x)/b**3 - a*x/b**2 + x**2/(2*b), Eq(n, -1)), (2*a**3*(a + b*x)**n/(b**3*n
**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) - 2*a**2*b*n*x*(a + b*x)**n/(b**3*
n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n**2*x**2*(a + b*x)**n/(
b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n*x**2*(a + b*x)**n
/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + b**3*n**2*x**3*(a + b*x)
**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 3*b**3*n*x**3*(a + b*
x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 2*b**3*x**3*(a + b*
x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3), True)) + 3*c*d**2*Pi
ecewise((a**n*x**6/6, Eq(b, 0)), (60*a**5*log(a/b + x)/(60*a**5*b**6 + 300
*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4
+ 60*b**11*x**5) + 137*a**5/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**
8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a**4
*b*x*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 ...
```

3.185.7 Maxima [F]

$$\int \frac{(a+bx)^n (c+dx^3)^3}{x} dx = \int \frac{(dx^3+c)^3(bx+a)^n}{x} dx$$

input
```
integrate((b*x+a)^n*(d*x^3+c)^3/x,x, algorithm="maxima")
```

output
```
integrate((d*x^3 + c)^3*(b*x + a)^n/x, x)
```

3.185.8 Giac [F]

$$\int \frac{(a+bx)^n (c+dx^3)^3}{x} \, dx = \int \frac{(dx^3+c)^3 (bx+a)^n}{x} \, dx$$

input
```
integrate((b*x+a)^n*(d*x^3+c)^3/x,x, algorithm="giac")
```

output
```
integrate((d*x^3 + c)^3*(b*x + a)^n/x, x)
```

3.185.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^n (c+dx^3)^3}{x} \, dx = \int \frac{(d\,x^3+c)^3 (a+b\,x)^n}{x} \, dx$$

input
```
int(((c + d*x^3)^3*(a + b*x)^n)/x,x)
```

output
```
int(((c + d*x^3)^3*(a + b*x)^n)/x, x)
```

3.186 $\int \frac{x^5(e+fx)^n}{a+bx^3} dx$

3.186.1 Optimal result

Integrand size = 20, antiderivative size = 324

$$\begin{aligned}\int \frac{x^5(e+fx)^n}{a+bx^3} dx =& \frac{e^2(e+fx)^{1+n}}{bf^3(1+n)} - \frac{2e(e+fx)^{2+n}}{bf^3(2+n)} + \frac{(e+fx)^{3+n}}{bf^3(3+n)} \\ &+ \frac{a(e+fx)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{5/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} \\ &+ \frac{a(e+fx)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3b^{5/3}\left(\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f\right)(1+n)} \\ &+ \frac{a(e+fx)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3b^{5/3}\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)(1+n)}\end{aligned}$$

output
```
e^2*(f*x+e)^(1+n)/b/f^3/(1+n)-2*e*(f*x+e)^(2+n)/b/f^3/(2+n)+(f*x+e)^(3+n)/
b/f^3/(3+n)+1/3*a*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(
b^(1/3)*e-a^(1/3)*f))/b^(5/3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*a*(f*x+e)^(1
+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e+(-1)^(1/3)*a^(1/3)
*f))/b^(5/3)/(b^(1/3)*e+(-1)^(1/3)*a^(1/3)*f)/(1+n)+1/3*a*(f*x+e)^(1+n)*hy
pergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-(-1)^(2/3)*a^(1/3)*f))/b
^(5/3)/(b^(1/3)*e-(-1)^(2/3)*a^(1/3)*f)/(1+n)
```

3.186.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.88

$$\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx = \frac{(e+fx)^{1+n}\left(\frac{3b^{2/3}e^2}{f^3(1+n)}-\frac{6b^{2/3}e(e+fx)}{f^3(2+n)}+\frac{3b^{2/3}(e+fx)^2}{f^3(3+n)}+\frac{a\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)}+\frac{a\,\text{Hypergeometric}}{\left(\right.}\right.}{3b^{5/3}}$$

input
```
Integrate[(x^5*(e + f*x)^n)/(a + b*x^3),x]
```

output
```
((e + f*x)^(1 + n)*((3*b^(2/3)*e^2)/(f^3*(1 + n)) - (6*b^(2/3)*e*(e + f*x)
)/(f^3*(2 + n)) + (3*b^(2/3)*(e + f*x)^2)/(f^3*(3 + n)) + (a*Hypergeometri
c2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/((b^(1
/3)*e - a^(1/3)*f)*(1 + n)) + (a*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/
3)*(e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)])/((b^(1/3)*e + (-1)^(1/3
)*a^(1/3)*f)*(1 + n)) + (a*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e
+ f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/((b^(1/3)*e - (-1)^(2/3)*a^(1
/3)*f)*(1 + n))))/(3*b^(5/3))
```

3.186.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx$$

$$\Big\downarrow \; 7276$$

$$\int \left(-\frac{ax^2(e+fx)^n}{b\left(a+bx^3\right)}+\frac{e^2(e+fx)^n}{bf^2}-\frac{2e(e+fx)^{n+1}}{bf^2}+\frac{(e+fx)^{n+2}}{bf^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{5/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{a(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3b^{5/3}(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}+\frac{a(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3b^{5/3}(n+1)\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)}+\frac{e^2(e+fx)^{n+1}}{bf^3(n+1)}-\frac{2e(e+fx)^{n+2}}{bf^3(n+2)}+\frac{(e+fx)^{n+3}}{bf^3(n+3)}$$

input
```
Int[(x^5*(e + f*x)^n)/(a + b*x^3),x]
```

output
```
(e^2*(e + f*x)^(1 + n))/(b*f^3*(1 + n)) - (2*e*(e + f*x)^(2 + n))/(b*f^3*(
2 + n)) + (e + f*x)^(3 + n)/(b*f^3*(3 + n)) + (a*(e + f*x)^(1 + n)*Hyperge
ometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/
(3*b^(5/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) + (a*(e + f*x)^(1 + n)*Hyperge
ometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a^
(1/3)*f)])/(3*b^(5/3)*(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)*(1 + n)) + (a*(e
+ f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(
1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/(3*b^(5/3)*(b^(1/3)*e - (-1)^(2/3)*a^(1/3
)*f)*(1 + n))
```

3.186.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.186.4 Maple [F]

$$\int \frac{x^5 (fx+e)^n}{b\,x^3+a}dx$$

input
```
int(x^5*(f*x+e)^n/(b*x^3+a),x)
```

output
```
int(x^5*(f*x+e)^n/(b*x^3+a),x)
```

3.186.5 Fricas [F]

$$\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^5}{bx^3+a}\,dx$$

input
```
integrate(x^5*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n*x^5/(b*x^3 + a), x)
```

3.186.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(f*x+e)**n/(b*x**3+a),x)
```

output
```
Timed out
```

3.186.7 Maxima [F]

$$\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx=\int \frac{(fx+e)^n x^5}{bx^3+a}\,dx$$

input
```
integrate(x^5*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n*x^5/(b*x^3 + a), x)
```

3.186.8 Giac [F]

$$\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx=\int \frac{(fx+e)^n x^5}{bx^3+a}\,dx$$

input
```
integrate(x^5*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n*x^5/(b*x^3 + a), x)
```

3.186.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx=\int \frac{x^5\,(e+f\,x)^n}{b\,x^3+a}\,dx$$

input
```
int((x^5*(e + f*x)^n)/(a + b*x^3),x)
```

output
```
int((x^5*(e + f*x)^n)/(a + b*x^3), x)
```

3.187 $\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx$

3.187.1 Optimal result

Integrand size = 20, antiderivative size = 332

$$\begin{aligned}
\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx &= -\frac{e(e+fx)^{1+n}}{bf^2(1+n)}+\frac{(e+fx)^{2+n}}{bf^2(2+n)} \\
&\quad -\frac{a^{2/3}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} \\
&\quad +\frac{\sqrt[3]{-1}a^{2/3}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} \\
&\quad +\frac{(-1)^{2/3}a^{2/3}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3b^{4/3}\left(\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f\right)(1+n)}
\end{aligned}$$

output
```
-e*(f*x+e)^(1+n)/b/f^2/(1+n)+(f*x+e)^(2+n)/b/f^2/(2+n)-1/3*a^(2/3)*(f*x+e)
^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-a^(1/3)*f))/b^(
4/3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*(-1)^(1/3)*a^(2/3)*(f*x+e)^(1+n)*hype
rgeom([1, 1+n],[2+n],(-1)^(2/3)*b^(1/3)*(f*x+e)/((-1)^(2/3)*b^(1/3)*e-a^(1
/3)*f))/b^(4/3)/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*(-1)^(2/3)*a^(2
/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(1/3)*b^(1/3)*(f*x+e)/((-1
)^(1/3)*b^(1/3)*e+a^(1/3)*f))/b^(4/3)/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f)/(1+
n)
```

3.187.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.88

$$\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx = \frac{(e+fx)^{1+n}\left(-\frac{3\sqrt[3]{b}e}{f^2(1+n)}+\frac{3\sqrt[3]{b}(e+fx)}{f^2(2+n)}-\frac{a^{2/3}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)}+\frac{\sqrt[3]{-1}a^{2/3}\,\text{Hypergeometric2F}}{\left((-1)^{2/3}\sqrt[3]{}\right.}\right.}{3b^{4/3}}$$

input
```
Integrate[(x^4*(e + f*x)^n)/(a + b*x^3),x]
```

output
```
((e + f*x)^(1 + n)*((-3*b^(1/3)*e)/(f^2*(1 + n)) + (3*b^(1/3)*(e + f*x))/(
f^2*(2 + n)) - (a^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f
*x))/(b^(1/3)*e - a^(1/3)*f)])/((b^(1/3)*e - a^(1/3)*f)*(1 + n)) + ((-1)^(
1/3)*a^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f
*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(((-1)^(2/3)*b^(1/3)*e - a^(1/3)
*f)*(1 + n)) + ((-1)^(2/3)*a^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1
)^(1/3)*b^(1/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(((-1)^(1/
3)*b^(1/3)*e + a^(1/3)*f)*(1 + n))))/(3*b^(4/3))
```

3.187.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(e+fx)^n}{a+bx^3}dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(-\frac{ax(e+fx)^n}{b(a+bx^3)}-\frac{e(e+fx)^n}{bf}+\frac{(e+fx)^{n+1}}{bf}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{2/3}(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{\sqrt[3]{-1}a^{2/3}(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{(-1)^{2/3}a^{2/3}(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3b^{4/3}(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}-\frac{e(e+fx)^{n+1}}{bf^2(n+1)}+\frac{(e+fx)^{n+2}}{bf^2(n+2)}$$

input
```
Int[(x^4*(e + f*x)^n)/(a + b*x^3),x]
```

output
```
-((e*(e + f*x)^(1 + n))/(b*f^2*(1 + n))) + (e + f*x)^(2 + n)/(b*f^2*(2 + n
)) - (a^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3
)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*b^(4/3)*(b^(1/3)*e - a^(1/3)*f)*
(1 + n)) + ((-1)^(1/3)*a^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 +
n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f
)])/(3*b^(4/3)*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) + ((-1)^(2/3)*a
^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^
(1/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*b^(4/3)*((-1)^(1/
3)*b^(1/3)*e + a^(1/3)*f)*(1 + n))
```

3.187.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.187.4 Maple [F]

$$\int \frac{x^4(fx+e)^n}{b\,x^3+a}dx$$

input
```
int(x^4*(f*x+e)^n/(b*x^3+a),x)
```

output
```
int(x^4*(f*x+e)^n/(b*x^3+a),x)
```

3.187.5 Fricas [F]

$$\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^4}{bx^3+a}\,dx$$

input
```
integrate(x^4*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n*x^4/(b*x^3 + a), x)
```

3.187.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(f*x+e)**n/(b*x**3+a),x)
```

output
```
Timed out
```

3.187.7 Maxima [F]

$$\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^4}{bx^3+a}\,dx$$

input
```
integrate(x^4*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n*x^4/(b*x^3 + a), x)
```

3.187.8 Giac [F]

$$\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^4}{bx^3+a}\,dx$$

input
```
integrate(x^4*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n*x^4/(b*x^3 + a), x)
```

3.187.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx = \int \frac{x^4\,(e+f\,x)^n}{b\,x^3+a}\,dx$$

input
```
int((x^4*(e + f*x)^n)/(a + b*x^3),x)
```

output
```
int((x^4*(e + f*x)^n)/(a + b*x^3), x)
```

3.188 $\int \frac{x^3(e+fx)^n}{a+bx^3} \, dx$

3.188.1 Optimal result

Integrand size = 20, antiderivative size = 293

$$\int \frac{x^3(e+fx)^n}{a+bx^3} \, dx = \frac{(e+fx)^{1+n}}{bf(1+n)} + \frac{\sqrt[3]{a}(e+fx)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} + \frac{\sqrt[3]{a}(e+fx)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} - \frac{\sqrt[3]{a}(e+fx)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3b\left(\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f\right)(1+n)}$$

output
```
(f*x+e)^(1+n)/b/f/(1+n)+1/3*a^(1/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n]
,b^(1/3)*(f*x+e)/(b^(1/3)*e-a^(1/3)*f))/b/(b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*
a^(1/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(2/3)*b^(1/3)*(f*x+e)/
((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f))/b/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f)/(1+n)
-1/3*a^(1/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(1/3)*b^(1/3)*(f*
x+e)/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f))/b/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f)/
(1+n)
```

3.188.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.82

$$\int \frac{x^3(e+fx)^n}{a+bx^3} \, dx$$
$$= \frac{(e+fx)^{1+n}\left(\frac{3}{f}+\frac{\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f}+\frac{\sqrt[3]{a}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right.}{3b(1+n)}$$

input
```
Integrate[(x^3*(e + f*x)^n)/(a + b*x^3),x]
```

output
```
((e + f*x)^(1 + n)*(3/f + (a^(1/3)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(
1/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(b^(1/3)*e - a^(1/3)*f) + (a^(1/
3)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)
^(2/3)*b^(1/3)*e - a^(1/3)*f)])/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f) - (a^(1
/3)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((-1
)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)))/(3*b
*(1 + n))
```

3.188.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(e+fx)^n}{a+bx^3} \, dx$$
$$\Big\downarrow \text{ 7276}$$
$$\int \left(\frac{(e+fx)^n}{b}-\frac{a(e+fx)^n}{b(a+bx^3)}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt[3]{a}(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{\sqrt[3]{a}(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{\sqrt[3]{a}(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3b(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}+\frac{(e+fx)^{n+1}}{bf(n+1)}$$

input
```
Int[(x^3*(e + f*x)^n)/(a + b*x^3),x]
```

output
```
(e + f*x)^(1 + n)/(b*f*(1 + n)) + (a^(1/3)*(e + f*x)^(1 + n)*Hypergeometri
c2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*b*(
b^(1/3)*e - a^(1/3)*f)*(1 + n)) + (a^(1/3)*(e + f*x)^(1 + n)*Hypergeometri
c2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e
 - a^(1/3)*f)])/(3*b*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) - (a^(1/3
)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)
*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*b*((-1)^(1/3)*b^(1/3)*
e + a^(1/3)*f)*(1 + n))
```

3.188.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.188.4 Maple [F]

$$\int \frac{x^3(fx+e)^n}{b\,x^3+a}dx$$

input
```
int(x^3*(f*x+e)^n/(b*x^3+a),x)
```

output
```
int(x^3*(f*x+e)^n/(b*x^3+a),x)
```

3.188.5 Fricas [F]

$$\int \frac{x^3(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^3}{bx^3+a}\,dx$$

input
```
integrate(x^3*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n*x^3/(b*x^3 + a), x)
```

3.188.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(e+fx)^n}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(f*x+e)**n/(b*x**3+a),x)
```

output
```
Timed out
```

3.188.7 Maxima [F]

$$\int \frac{x^3(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^3}{bx^3+a}\,dx$$

input
```
integrate(x^3*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n*x^3/(b*x^3 + a), x)
```

3.188.8 Giac [F]

$$\int \frac{x^3(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^3}{bx^3+a}\,dx$$

input
```
integrate(x^3*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n*x^3/(b*x^3 + a), x)
```

3.188.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(e+fx)^n}{a+bx^3}\,dx = \int \frac{x^3\,(e+f\,x)^n}{b\,x^3+a}\,dx$$

input
```
int((x^3*(e + f*x)^n)/(a + b*x^3),x)
```

output
```
int((x^3*(e + f*x)^n)/(a + b*x^3), x)
```

3.189 $\int \frac{x^2(e+fx)^n}{a+bx^3} dx$

3.189.1 Optimal result

Integrand size = 20, antiderivative size = 253

$$\int \frac{x^2(e+fx)^n}{a+bx^3} dx = -\frac{(e+fx)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{2/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} - \frac{(e+fx)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3b^{2/3}\left(\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f\right)(1+n)} - \frac{(e+fx)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3b^{2/3}\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)(1+n)}$$

output
```
-1/3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-a^(
1/3)*f))/b^(2/3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)-1/3*(f*x+e)^(1+n)*hypergeom([
1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e+(-1)^(1/3)*a^(1/3)*f))/b^(2/3)/(b
^(1/3)*e+(-1)^(1/3)*a^(1/3)*f)/(1+n)-1/3*(f*x+e)^(1+n)*hypergeom([1, 1+n],
[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-(-1)^(2/3)*a^(1/3)*f))/b^(2/3)/(b^(1/3)*e
-(-1)^(2/3)*a^(1/3)*f)/(1+n)
```

3.189.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.84

$$\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx = \frac{(e+fx)^{1+n}\left(-\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f}-\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}-\text{Hype}\cdots\right)}{3b^{2/3}(1+n)}$$

input
```
Integrate[(x^2*(e + f*x)^n)/(a + b*x^3),x]
```

output
```
((e + f*x)^(1 + n)*(-(Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x
))/(b^(1/3)*e - a^(1/3)*f)]/(b^(1/3)*e - a^(1/3)*f)) - Hypergeometric2F1[1
, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)]/(b
^(1/3)*e + (-1)^(1/3)*a^(1/3)*f) - Hypergeometric2F1[1, 1 + n, 2 + n, (b^(
1/3)*(e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)]/(b^(1/3)*e - (-1)^(2/3
)*a^(1/3)*f)))/(3*b^(2/3)*(1 + n))
```

3.189.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx$$

$$\Big\downarrow \text{ 7276}$$

$$\int \left(\frac{(e+fx)^n}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{(e+fx)^n}{3b^{2/3}\left(\sqrt[3]{b}x-\sqrt[3]{-1}\sqrt[3]{a}\right)}+\frac{(e+fx)^n}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{2/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3b^{2/3}(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}-\frac{(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3b^{2/3}(n+1)\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)}$$

input
```
Int[(x^2*(e + f*x)^n)/(a + b*x^3),x]
```

output
```
-1/3*((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f
*x))/(b^(1/3)*e - a^(1/3)*f)])/(b^(2/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) -
 ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))
/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)])/(3*b^(2/3)*(b^(1/3)*e + (-1)^(1/3)*a
^(1/3)*f)*(1 + n)) - ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n,
 (b^(1/3)*(e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/(3*b^(2/3)*(b^(1
/3)*e - (-1)^(2/3)*a^(1/3)*f)*(1 + n))
```

3.189.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.189.4 Maple [F]

$$\int \frac{x^2(fx+e)^n}{b\,x^3+a}dx$$

input
```
int(x^2*(f*x+e)^n/(b*x^3+a),x)
```

output
```
int(x^2*(f*x+e)^n/(b*x^3+a),x)
```

3.189.5 Fricas [F]

$$\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^2}{bx^3+a}\,dx$$

input
```
integrate(x^2*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n*x^2/(b*x^3 + a), x)
```

3.189.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(f*x+e)**n/(b*x**3+a),x)
```

output
```
Timed out
```

3.189.7 Maxima [F]

$$\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^2}{bx^3+a}\,dx$$

input
```
integrate(x^2*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n*x^2/(b*x^3 + a), x)
```

3.189.8 Giac [F]

$$\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^2}{bx^3+a}\,dx$$

input
```
integrate(x^2*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n*x^2/(b*x^3 + a), x)
```

3.189.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx = \int \frac{x^2\,(e+f\,x)^n}{b\,x^3+a}\,dx$$

input
```
int((x^2*(e + f*x)^n)/(a + b*x^3),x)
```

output
```
int((x^2*(e + f*x)^n)/(a + b*x^3), x)
```

3.190 $\int \frac{x(e+fx)^n}{a+bx^3} dx$

3.190.1 Optimal result

Integrand size = 18, antiderivative size = 288

$$\int \frac{x(e+fx)^n}{a+bx^3} dx = \frac{(e+fx)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} - \frac{\sqrt[3]{-1}(e+fx)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} - \frac{(-1)^{2/3}(e+fx)^{1+n} \text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f\right)(1+n)}$$

output
```
1/3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-a^(1
/3)*f))/a^(1/3)/b^(1/3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)-1/3*(-1)^(1/3)*(f*x+e)
^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(2/3)*b^(1/3)*(f*x+e)/((-1)^(2/3)*b^(
1/3)*e-a^(1/3)*f))/a^(1/3)/b^(1/3)/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f)/(1+n)-
1/3*(-1)^(2/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(1/3)*b^(1/3)*(
f*x+e)/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f))/a^(1/3)/b^(1/3)/((-1)^(1/3)*b^(1/
3)*e+a^(1/3)*f)/(1+n)
```

3.190.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.82

$$\int \frac{x(e+fx)^n}{a+bx^3}\,dx = \frac{(e+fx)^{1+n}\left(\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f}-\frac{\sqrt[3]{-1}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}-\right.}{3\sqrt[3]{a}\sqrt[3]{b}(1+n)}$$

input
```
Integrate[(x*(e + f*x)^n)/(a + b*x^3),x]
```

output
```
((e + f*x)^(1 + n)*(Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))
/(b^(1/3)*e - a^(1/3)*f)]/(b^(1/3)*e - a^(1/3)*f) - ((-1)^(1/3)*Hypergeome
tric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3
)*e - a^(1/3)*f)])/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f) - ((-1)^(2/3)*Hyperg
eometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((-1)^(1/3)*b^
(1/3)*e + a^(1/3)*f)])/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)))/(3*a^(1/3)*b^(
1/3)*(1 + n))
```

3.190.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(e+fx)^n}{a+bx^3}\,dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(-\frac{(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{(-1)^{2/3}(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}+\frac{\sqrt[3]{-1}(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{\sqrt[3]{-1}(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{(-1)^{2/3}(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3\sqrt[3]{a}\sqrt[3]{b}(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}$$

input
```
Int[(x*(e + f*x)^n)/(a + b*x^3),x]
```

output
```
((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/
(b^(1/3)*e - a^(1/3)*f)])/(3*a^(1/3)*b^(1/3)*(b^(1/3)*e - a^(1/3)*f)*(1 +
n)) - ((-1)^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-
1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(3*a^(1/3
)*b^(1/3)*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) - ((-1)^(2/3)*(e + f
*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*
x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*a^(1/3)*b^(1/3)*((-1)^(1/3)*b^
(1/3)*e + a^(1/3)*f)*(1 + n))
```

3.190.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.190.4 Maple [F]

$$\int \frac{x(fx+e)^{n}}{b\,x^{3}+a}dx$$

input
```
int(x*(f*x+e)^n/(b*x^3+a),x)
```

output
```
int(x*(f*x+e)^n/(b*x^3+a),x)
```

3.190.5 Fricas [F]

$$\int \frac{x(e+fx)^n}{a+bx^3}\,dx=\int \frac{(fx+e)^{n}x}{bx^{3}+a}\,dx$$

input
```
integrate(x*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n*x/(b*x^3 + a), x)
```

3.190.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(e+fx)^n}{a+bx^3}\,dx=\text{Timed out}$$

input
```
integrate(x*(f*x+e)**n/(b*x**3+a),x)
```

output
```
Timed out
```

3.190.7 Maxima [F]

$$\int \frac{x(e+fx)^n}{a+bx^3} \, dx = \int \frac{(fx+e)^n x}{bx^3+a} \, dx$$

input
```
integrate(x*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n*x/(b*x^3 + a), x)
```

3.190.8 Giac [F]

$$\int \frac{x(e+fx)^n}{a+bx^3} \, dx = \int \frac{(fx+e)^n x}{bx^3+a} \, dx$$

input
```
integrate(x*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n*x/(b*x^3 + a), x)
```

3.190.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(e+fx)^n}{a+bx^3} \, dx = \int \frac{x\,(e+f\,x)^n}{b\,x^3+a} \, dx$$

input
```
int((x*(e + f*x)^n)/(a + b*x^3),x)
```

output
```
int((x*(e + f*x)^n)/(a + b*x^3), x)
```

3.191 $\int \frac{(e+fx)^n}{a+bx^3} dx$

3.191.1 Optimal result

Integrand size = 17, antiderivative size = 263

$$\int \frac{(e+fx)^n}{a+bx^3}\,dx = -\frac{(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} - \frac{(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} + \frac{(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3a^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f\right)(1+n)}$$

output
```
-1/3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-a^(
1/3)*f))/a^(2/3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)-1/3*(f*x+e)^(1+n)*hypergeom([
1, 1+n],[2+n],(-1)^(2/3)*b^(1/3)*(f*x+e)/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f))
/a^(2/3)/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*(f*x+e)^(1+n)*hypergeo
m([1, 1+n],[2+n],(-1)^(1/3)*b^(1/3)*(f*x+e)/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*
f))/a^(2/3)/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f)/(1+n)
```

3.191.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.84

$$\int \frac{(e+fx)^n}{a+bx^3}\,dx = \frac{(e+fx)^{1+n}\left(-\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f}-\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}+\text{Hyp}\ldots}{3a^{2/3}(1+n)}$$

input
```
Integrate[(e + f*x)^n/(a + b*x^3),x]
```

output
```
((e + f*x)^(1 + n)*(-(Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x
))/(b^(1/3)*e - a^(1/3)*f)]/(b^(1/3)*e - a^(1/3)*f)) - Hypergeometric2F1[1
, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(
1/3)*f)]/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f) + Hypergeometric2F1[1, 1 + n,
2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)]/
((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)))/(3*a^(2/3)*(1 + n))
```

3.191.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^n}{a+bx^3}\,dx$$

$$\Big\downarrow \; 7276$$

$$\int \left(-\frac{(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{(e+fx)^n}{3a^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{b}x-\sqrt[3]{a}\right)}-\frac{(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3a^{2/3}(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}$$

input
```
Int[(e + f*x)^n/(a + b*x^3),x]
```

output
```
-1/3*((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f
*x))/(b^(1/3)*e - a^(1/3)*f)])/(a^(2/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) -
 ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)
*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(3*a^(2/3)*((-1)^(2/3)*b^
(1/3)*e - a^(1/3)*f)*(1 + n)) + ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1
+ n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)
*f)])/(3*a^(2/3)*((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)*(1 + n))
```

3.191.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.191.4 Maple [F]

$$\int \frac{(fx+e)^n}{b\,x^3+a}dx$$

input
```
int((f*x+e)^n/(b*x^3+a),x)
```

output
```
int((f*x+e)^n/(b*x^3+a),x)
```

3.191.5 Fricas [F]

$$\int \frac{(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n}{bx^3+a}\,dx$$

input
```
integrate((f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n/(b*x^3 + a), x)
```

3.191.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**n/(b*x**3+a),x)
```

output
```
Timed out
```

3.191.7 Maxima [F]

$$\int \frac{(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n}{bx^3+a}\,dx$$

input
```
integrate((f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n/(b*x^3 + a), x)
```

3.191.8 Giac [F]

$$\int \frac{(e+fx)^n}{a+bx^3}\,dx=\int \frac{(fx+e)^n}{bx^3+a}\,dx$$

input
```
integrate((f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n/(b*x^3 + a), x)
```

3.191.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{a+bx^3}\,dx=\int \frac{(e+f\,x)^n}{b\,x^3+a}\,dx$$

input
```
int((e + f*x)^n/(a + b*x^3),x)
```

output
```
int((e + f*x)^n/(a + b*x^3), x)
```

3.192 $\int \frac{(e+fx)^n}{x(a+bx^3)}\,dx$

3.192.1 Optimal result

Integrand size = 20, antiderivative size = 300

$$\begin{aligned}\int \frac{(e+fx)^n}{x(a+bx^3)}\,dx &= \frac{\sqrt[3]{b}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)} \\ &+ \frac{\sqrt[3]{b}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3a\left(\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f\right)(1+n)} \\ &+ \frac{\sqrt[3]{b}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3a\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)(1+n)} \\ &- \frac{(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,1+\frac{fx}{e}\right)}{ae(1+n)}\end{aligned}$$

output
```
1/3*b^(1/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3
)*e-a^(1/3)*f))/a/(b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*b^(1/3)*(f*x+e)^(1+n)*hy
pergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e+(-1)^(1/3)*a^(1/3)*f))/a
/(b^(1/3)*e+(-1)^(1/3)*a^(1/3)*f)/(1+n)+1/3*b^(1/3)*(f*x+e)^(1+n)*hypergeo
m([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-(-1)^(2/3)*a^(1/3)*f))/a/(b^(1
/3)*e-(-1)^(2/3)*a^(1/3)*f)/(1+n)-(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],1
+f*x/e)/a/e/(1+n)
```

3.192.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.81

$$\int \frac{(e+fx)^n}{x(a+bx^3)}\,dx$$
$$= \frac{(e+fx)^{1+n}\left(\frac{\sqrt[3]{b}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f}+\frac{\sqrt[3]{b}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}-\right.}{3a(1+n)}$$

input
```
Integrate[(e + f*x)^n/(x*(a + b*x^3)),x]
```

output
```
((e + f*x)^(1 + n)*((b^(1/3)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(
e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(b^(1/3)*e - a^(1/3)*f) + (b^(1/3)*Hyp
ergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e + (-1)^(1/3
)*a^(1/3)*f)])/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f) + (b^(1/3)*Hypergeometri
c2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*
f)])/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f) - (3*Hypergeometric2F1[1, 1 + n, 2
 + n, 1 + (f*x)/e])/e))/(3*a*(1 + n))
```

3.192.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^n}{x(a+bx^3)}\,dx$$
$$\Big\downarrow \; 7276$$
$$\int \left(\frac{(e+fx)^n}{ax}-\frac{bx^2(e+fx)^n}{a(a+bx^3)}\right)dx$$
$$\Big\downarrow \; 2009$$

$$\frac{\sqrt[3]{b}(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{\sqrt[3]{b}(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e+\sqrt[3]{-1}\sqrt[3]{a}f}\right)}{3a(n+1)\left(\sqrt[3]{-1}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}+\frac{\sqrt[3]{b}(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f}\right)}{3a(n+1)\left(\sqrt[3]{b}e-(-1)^{2/3}\sqrt[3]{a}f\right)}-\frac{(e+fx)^{n+1}\,\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{fx}{e}+1\right)}{ae(n+1)}$$

input
```
Int[(e + f*x)^n/(x*(a + b*x^3)),x]
```

output
```
(b^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e
+ f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*a*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) +
(b^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e
+ f*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)])/(3*a*(b^(1/3)*e + (-1)^(1/3)*
a^(1/3)*f)*(1 + n)) + (b^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 +
n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/(3*a*(b
^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)*(1 + n)) - ((e + f*x)^(1 + n)*Hypergeomet
ric2F1[1, 1 + n, 2 + n, 1 + (f*x)/e])/(a*e*(1 + n))
```

3.192.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.192.4 Maple [F]

$$\int \frac{(fx+e)^n}{x\,(b\,x^3+a)}dx$$

input
```
int((f*x+e)^n/x/(b*x^3+a),x)
```

output
```
int((f*x+e)^n/x/(b*x^3+a),x)
```

3.192.5 Fricas [F]

$$\int \frac{(e+fx)^n}{x\,(a+bx^3)}\,dx = \int \frac{(fx+e)^n}{(bx^3+a)x}\,dx$$

input
```
integrate((f*x+e)^n/x/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n/(b*x^4 + a*x), x)
```

3.192.6 Sympy [F]

$$\int \frac{(e+fx)^n}{x\,(a+bx^3)}\,dx = \int \frac{(e+fx)^n}{x\,(a+bx^3)}\,dx$$

input
```
integrate((f*x+e)**n/x/(b*x**3+a),x)
```

output
```
Integral((e + f*x)**n/(x*(a + b*x**3)), x)
```

3.192.7 Maxima [F]

$$\int \frac{(e+fx)^n}{x(a+bx^3)} \, dx = \int \frac{(fx+e)^n}{(bx^3+a)x} \, dx$$

input
```
integrate((f*x+e)^n/x/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n/((b*x^3 + a)*x), x)
```

3.192.8 Giac [F]

$$\int \frac{(e+fx)^n}{x(a+bx^3)} \, dx = \int \frac{(fx+e)^n}{(bx^3+a)x} \, dx$$

input
```
integrate((f*x+e)^n/x/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n/((b*x^3 + a)*x), x)
```

3.192.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{x(a+bx^3)} \, dx = \int \frac{(e+f\,x)^n}{x\,(b\,x^3+a)} \, dx$$

input
```
int((e + f*x)^n/(x*(a + b*x^3)),x)
```

output
```
int((e + f*x)^n/(x*(a + b*x^3)), x)
```

3.193 $\int \frac{(e+fx)^n}{x^2(a+bx^3)}\,dx$

3.193.1 Optimal result

Integrand size = 20, antiderivative size = 326

$$\begin{aligned}\int \frac{(e+fx)^n}{x^2(a+bx^3)}\,dx &= -\frac{b^{2/3}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{4/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)}\\
&\quad+\frac{\sqrt[3]{-1}b^{2/3}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{4/3}\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)}\\
&\quad+\frac{(-1)^{2/3}b^{2/3}(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3a^{4/3}\left(\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f\right)(1+n)}\\
&\quad+\frac{f(e+fx)^{1+n}\,\text{Hypergeometric2F1}\left(2,1+n,2+n,1+\frac{fx}{e}\right)}{ae^2(1+n)}\end{aligned}$$

output
```
-1/3*b^(2/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/
3)*e-a^(1/3)*f))/a^(4/3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*(-1)^(1/3)*b^(2/3
)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(2/3)*b^(1/3)*(f*x+e)/((-1)^
(2/3)*b^(1/3)*e-a^(1/3)*f))/a^(4/3)/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f)/(1+n)
+1/3*(-1)^(2/3)*b^(2/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(1/3)*
b^(1/3)*(f*x+e)/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f))/a^(4/3)/((-1)^(1/3)*b^(1
/3)*e+a^(1/3)*f)/(1+n)+f*(f*x+e)^(1+n)*hypergeom([2, 1+n],[2+n],1+f*x/e)/a
/e^2/(1+n)
```

3.193.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.84

$$\int \frac{(e+fx)^n}{x^2(a+bx^3)}\,dx$$
$$= \frac{(e+fx)^{1+n}\left(-\frac{b^{2/3}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\sqrt[3]{b}e-\sqrt[3]{a}f}+\frac{\sqrt[3]{-1}b^{2/3}\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{(-1)^{2/3}\sqrt[3]{b}(e+\cdots}{(-1)^{2/3}\sqrt[3]{b}e-\cdots}\right.}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\cdots\right.}{3a^{4/3}(1\cdots}$$

input
```
Integrate[(e + f*x)^n/(x^2*(a + b*x^3)),x]
```

output
```
((e + f*x)^(1 + n)*(-((b^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)
*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(b^(1/3)*e - a^(1/3)*f)) + ((-1)^(1/
3)*b^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x
))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)
 + ((-1)^(2/3)*b^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1
/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/((-1)^(1/3)*b^(1/3)*e
+ a^(1/3)*f) + (3*a^(1/3)*f*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (f*x)/e
])/e^2))/(3*a^(4/3)*(1 + n))
```

3.193.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^n}{x^2\left(a+bx^3\right)}dx$$

$$\Big\downarrow \text{ 7276}$$

$$\int \left(\frac{(e+fx)^n}{ax^2}-\frac{bx(e+fx)^n}{a\left(a+bx^3\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b^{2/3}(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{4/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{\sqrt[3]{-1}b^{2/3}(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{4/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{(-1)^{2/3}b^{2/3}(e+fx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3a^{4/3}(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}+\frac{f(e+fx)^{n+1}\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{fx}{e}+1\right)}{ae^2(n+1)}$$

input
```
Int[(e + f*x)^n/(x^2*(a + b*x^3)),x]
```

output
```
-1/3*(b^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3
)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(a^(4/3)*(b^(1/3)*e - a^(1/3)*f)*(1
 + n)) + ((-1)^(1/3)*b^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n,
 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)]
)/(3*a^(4/3)*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) + ((-1)^(2/3)*b^(
2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1
/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*a^(4/3)*((-1)^(1/3)
*b^(1/3)*e + a^(1/3)*f)*(1 + n)) + (f*(e + f*x)^(1 + n)*Hypergeometric2F1[
2, 1 + n, 2 + n, 1 + (f*x)/e])/(a*e^2*(1 + n))
```

3.193.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.193.4 Maple [F]

$$\int \frac{(fx+e)^n}{x^2\,(b\,x^3+a)}dx$$

input
```
int((f*x+e)^n/x^2/(b*x^3+a),x)
```

output
```
int((f*x+e)^n/x^2/(b*x^3+a),x)
```

3.193.5 Fricas [F]

$$\int \frac{(e+fx)^n}{x^2\,(a+bx^3)}\,dx = \int \frac{(fx+e)^n}{(bx^3+a)x^2}\,dx$$

input
```
integrate((f*x+e)^n/x^2/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n/(b*x^5 + a*x^2), x)
```

3.193.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{x^2\,(a+bx^3)}\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)**n/x**2/(b*x**3+a),x)
```

output
```
Timed out
```

3.193.7 Maxima [F]

$$\int \frac{(e+fx)^n}{x^2(a+bx^3)}\,dx = \int \frac{(fx+e)^n}{(bx^3+a)x^2}\,dx$$

input
```
integrate((f*x+e)^n/x^2/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n/((b*x^3 + a)*x^2), x)
```

3.193.8 Giac [F]

$$\int \frac{(e+fx)^n}{x^2(a+bx^3)}\,dx = \int \frac{(fx+e)^n}{(bx^3+a)x^2}\,dx$$

input
```
integrate((f*x+e)^n/x^2/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n/((b*x^3 + a)*x^2), x)
```

3.193.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx)^n}{x^2(a+bx^3)}\,dx = \int \frac{(e+f\,x)^n}{x^2\,(b\,x^3+a)}\,dx$$

input
```
int((e + f*x)^n/(x^2*(a + b*x^3)),x)
```

output
```
int((e + f*x)^n/(x^2*(a + b*x^3)), x)
```

3.194 $\int \frac{x^2(c+dx)^{1+n}}{a+bx^3}\,dx$

3.194.1 Optimal result

Integrand size = 22, antiderivative size = 253

$$\int \frac{x^2(c+dx)^{1+n}}{a+bx^3}\,dx = -\frac{(c+dx)^{2+n}\,\text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{2/3}\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)(2+n)} - \frac{(c+dx)^{2+n}\,\text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3b^{2/3}\left(\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d\right)(2+n)} - \frac{(c+dx)^{2+n}\,\text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b^{2/3}\left(\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d\right)(2+n)}$$

output
```
-1/3*(d*x+c)^(2+n)*hypergeom([1, 2+n],[3+n],b^(1/3)*(d*x+c)/(b^(1/3)*c-a^(
1/3)*d))/b^(2/3)/(b^(1/3)*c-a^(1/3)*d)/(2+n)-1/3*(d*x+c)^(2+n)*hypergeom([
1, 2+n],[3+n],b^(1/3)*(d*x+c)/(b^(1/3)*c+(-1)^(1/3)*a^(1/3)*d))/b^(2/3)/(b
^(1/3)*c+(-1)^(1/3)*a^(1/3)*d)/(2+n)-1/3*(d*x+c)^(2+n)*hypergeom([1, 2+n],
[3+n],b^(1/3)*(d*x+c)/(b^(1/3)*c-(-1)^(2/3)*a^(1/3)*d))/b^(2/3)/(b^(1/3)*c
-(-1)^(2/3)*a^(1/3)*d)/(2+n)
```

3.194.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.84

$$\int \frac{x^2(c+dx)^{1+n}}{a+bx^3}\,dx = \frac{(c+dx)^{2+n}\left(-\frac{\text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{\sqrt[3]{b}c-\sqrt[3]{a}d}-\frac{\text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}-\text{Hype}\ldots\right.}{3b^{2/3}(2+n)}$$

input
```
Integrate[(x^2*(c + d*x)^(1 + n))/(a + b*x^3),x]
```

output
```
((c + d*x)^(2 + n)*(-(Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/3)*(c + d*x
))/(b^(1/3)*c - a^(1/3)*d)]/(b^(1/3)*c - a^(1/3)*d)) - Hypergeometric2F1[1
, 2 + n, 3 + n, (b^(1/3)*(c + d*x))/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)]/(b
^(1/3)*c + (-1)^(1/3)*a^(1/3)*d) - Hypergeometric2F1[1, 2 + n, 3 + n, (b^(
1/3)*(c + d*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d)]/(b^(1/3)*c - (-1)^(2/3
)*a^(1/3)*d)))/(3*b^(2/3)*(2 + n))
```

3.194.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx)^{n+1}}{a+bx^3}\,dx$$

$$\Big\downarrow \text{ 7276}$$

$$\int \left(\frac{(c+dx)^{n+1}}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{(c+dx)^{n+1}}{3b^{2/3}\left(\sqrt[3]{b}x-\sqrt[3]{-1}\sqrt[3]{a}\right)}+\frac{(c+dx)^{n+1}}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{(c+dx)^{n+2}\,\text{Hypergeometric2F1}\left(1,n+2,n+3,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-\sqrt[3]{a}d}\right)}{3b^{2/3}(n+2)\left(\sqrt[3]{b}c-\sqrt[3]{a}d\right)}-\frac{(c+dx)^{n+2}\,\text{Hypergeometric2F1}\left(1,n+2,n+3,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c+\sqrt[3]{-1}\sqrt[3]{a}d}\right)}{3b^{2/3}(n+2)\left(\sqrt[3]{-1}\sqrt[3]{a}d+\sqrt[3]{b}c\right)}-\frac{(c+dx)^{n+2}\,\text{Hypergeometric2F1}\left(1,n+2,n+3,\frac{\sqrt[3]{b}(c+dx)}{\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d}\right)}{3b^{2/3}(n+2)\left(\sqrt[3]{b}c-(-1)^{2/3}\sqrt[3]{a}d\right)}$$

input
```
Int[(x^2*(c + d*x)^(1 + n))/(a + b*x^3),x]
```

output
```
-1/3*((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/3)*(c + d
*x))/(b^(1/3)*c - a^(1/3)*d)])/(b^(2/3)*(b^(1/3)*c - a^(1/3)*d)*(2 + n)) -
 ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/3)*(c + d*x))
/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)])/(3*b^(2/3)*(b^(1/3)*c + (-1)^(1/3)*a
^(1/3)*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n,
 (b^(1/3)*(c + d*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d)])/(3*b^(2/3)*(b^(1
/3)*c - (-1)^(2/3)*a^(1/3)*d)*(2 + n))
```

3.194.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.194.4 Maple [F]

$$\int \frac{x^2(dx+c)^{1+n}}{b\,x^3+a}dx$$

input
```
int(x^2*(d*x+c)^(1+n)/(b*x^3+a),x)
```

output
```
int(x^2*(d*x+c)^(1+n)/(b*x^3+a),x)
```

3.194.5 Fricas [F]

$$\int \frac{x^2(c+dx)^{1+n}}{a+bx^3}\,dx = \int \frac{(dx+c)^{n+1}x^2}{bx^3+a}\,dx$$

input
```
integrate(x^2*(d*x+c)^(1+n)/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^(n + 1)*x^2/(b*x^3 + a), x)
```

3.194.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx)^{1+n}}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(d*x+c)**(1+n)/(b*x**3+a),x)
```

output
```
Timed out
```

3.194.7 Maxima [F]

$$\int \frac{x^2(c+dx)^{1+n}}{a+bx^3}\,dx = \int \frac{(dx+c)^{n+1}x^2}{bx^3+a}\,dx$$

input
```
integrate(x^2*(d*x+c)^(1+n)/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(n + 1)*x^2/(b*x^3 + a), x)
```

3.194.8 Giac [F]

$$\int \frac{x^2(c+dx)^{1+n}}{a+bx^3}\,dx = \int \frac{(dx+c)^{n+1}x^2}{bx^3+a}\,dx$$

input
```
integrate(x^2*(d*x+c)^(1+n)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(n + 1)*x^2/(b*x^3 + a), x)
```

3.194.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx)^{1+n}}{a+bx^3}\,dx = \int \frac{x^2\,(c+d\,x)^{n+1}}{b\,x^3+a}\,dx$$

input
```
int((x^2*(c + d*x)^(n + 1))/(a + b*x^3),x)
```

output
```
int((x^2*(c + d*x)^(n + 1))/(a + b*x^3), x)
```

3.195 $\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx$

3.195.1 Optimal result

Integrand size = 20, antiderivative size = 211

$$\begin{aligned}\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx &= \frac{x^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\operatorname{AppellF1}\left(1+m,-n,1,2+m,-\frac{fx}{e},-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(1+m)}\\ &+\frac{x^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\operatorname{AppellF1}\left(1+m,-n,1,2+m,-\frac{fx}{e},\frac{\sqrt[3]{-1}\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(1+m)}\\ &+\frac{x^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\operatorname{AppellF1}\left(1+m,-n,1,2+m,-\frac{fx}{e},-\frac{(-1)^{2/3}\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(1+m)}\end{aligned}$$

output
```
1/3*x^(1+m)*(f*x+e)^n*AppellF1(1+m,1,-n,2+m,-b^(1/3)*x/a^(1/3),-f*x/e)/a/(
1+m)/((1+f*x/e)^n)+1/3*x^(1+m)*(f*x+e)^n*AppellF1(1+m,1,-n,2+m,(-1)^(1/3)*
b^(1/3)*x/a^(1/3),-f*x/e)/a/(1+m)/((1+f*x/e)^n)+1/3*x^(1+m)*(f*x+e)^n*Appe
llF1(1+m,-n,1,2+m,-f*x/e,-(-1)^(2/3)*b^(1/3)*x/a^(1/3))/a/(1+m)/((1+f*x/e)
^n)
```

3.195.2 Mathematica [F]

$$\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx=\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx$$

input
```
Integrate[(x^m*(e + f*x)^n)/(a + b*x^3),x]
```

output
```
Integrate[(x^m*(e + f*x)^n)/(a + b*x^3), x]
```

3.195.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(-\frac{x^m(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{x^m(e+fx)^n}{3a^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{b}x-\sqrt[3]{a}\right)}-\frac{x^m(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{fx}{e},-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(m+1)}+\frac{x^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{fx}{e},\frac{\sqrt[3]{-1}\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(m+1)}+\frac{x^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}\text{AppellF1}\left(m+1,-n,1,m+2,-\frac{fx}{e},-\frac{(-1)^{2/3}\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{3a(m+1)}$$

input
```
Int[(x^m*(e + f*x)^n)/(a + b*x^3),x]
```

output
```
(x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -((b^(1/3
)*x)/a^(1/3))])/(3*a*(1 + m)*(1 + (f*x)/e)^n) + (x^(1 + m)*(e + f*x)^n*App
ellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), ((-1)^(1/3)*b^(1/3)*x)/a^(1/3)])/(3
*a*(1 + m)*(1 + (f*x)/e)^n) + (x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1
, 2 + m, -((f*x)/e), -(((-1)^(2/3)*b^(1/3)*x)/a^(1/3))])/(3*a*(1 + m)*(1 +
 (f*x)/e)^n)
```

3.195.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.195.4 Maple [F]

$$\int \frac{x^m (fx+e)^n}{b\,x^3+a}dx$$

input
```
int(x^m*(f*x+e)^n/(b*x^3+a),x)
```

output
```
int(x^m*(f*x+e)^n/(b*x^3+a),x)
```

3.195.5 Fricas [F]

$$\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx = \int \frac{(fx+e)^n x^m}{bx^3+a}\,dx$$

input
```
integrate(x^m*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

output
```
integral((f*x + e)^n*x^m/(b*x^3 + a), x)
```

3.195.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m (e + fx)^n}{a + bx^3} \, dx = \text{Timed out}$$

input
```
integrate(x**m*(f*x+e)**n/(b*x**3+a),x)
```

output
```
Timed out
```

3.195.7 Maxima [F]

$$\int \frac{x^m (e + fx)^n}{a + bx^3} \, dx = \int \frac{(fx + e)^n x^m}{bx^3 + a} \, dx$$

input
```
integrate(x^m*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate((f*x + e)^n*x^m/(b*x^3 + a), x)
```

3.195.8 Giac [F]

$$\int \frac{x^m (e + fx)^n}{a + bx^3} \, dx = \int \frac{(fx + e)^n x^m}{bx^3 + a} \, dx$$

input
```
integrate(x^m*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate((f*x + e)^n*x^m/(b*x^3 + a), x)
```

3.195.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx=\int \frac{x^m\,{\left(e+f\,x\right)}^n}{b\,x^3+a}\,dx$$

input
```
int((x^m*(e + f*x)^n)/(a + b*x^3),x)
```

output
```
int((x^m*(e + f*x)^n)/(a + b*x^3), x)
```

3.196 $\int \frac{\sqrt{c+dx^3}}{a+bx} \, dx$

3.196.1 Optimal result

Integrand size = 19, antiderivative size = 1480

$$\int \frac{\sqrt{c+dx^3}}{a+bx} \, dx = \text{Too large to display}$$

output
```
2/3*(d*x^3+c)^(1/2)/b-2*a*d^(1/3)*(d*x^3+c)^(1/2)/b^2/(d^(1/3)*x+c^(1/3)*(
1+3^(1/2)))+3^(1/4)*a*c^(1/3)*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/
3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(
1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)
*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^2/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(
1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/3*a*d^(1/3)*(c^(1/3)+d^
(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3
^(1/2))),I*3^(1/2)+2*I)*(a*d^(1/3)+b*c^(1/3)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2
*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3
^(1/2)))^2)^(1/2)*3^(3/4)/b^3/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)
/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-2/3*(-a^3*d+b^3*c)*(c^(1/3)+d^(1
/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(
1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*
x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^3/(-a*d^
(1/3)+b*c^(1/3)*(1+3^(1/2)))/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/
(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-c^(1/6)*(c^(1/3)+d^(1/3)*x)*arcta
nh(1/3*(b^2*c^(2/3)+a*b*c^(1/3)*d^(1/3)+a^2*d^(2/3))^(1/2)*(1/2*6^(1/2)-1/
2*2^(1/2))*(1-(d^(1/3)*x+c^(1/3)*(1-3^(1/2)))^2/(d^(1/3)*x+c^(1/3)*(1+3^(1
/2)))^2)^(1/2)*3^(3/4)/c^(1/6)/b^(1/2)/(b*c^(1/3)-a*d^(1/3))^(1/2)/(7-4*3^
(1/2)+(d^(1/3)*x+c^(1/3)*(1-3^(1/2)))^2/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))...
```

3.196.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.60 (sec) , antiderivative size = 877, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx$$

$$= \frac{2\left(c+dx^3+\frac{3\sqrt{2}a\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{-1}\sqrt[3]{c}-\sqrt[3]{d}x\right)\sqrt{\frac{\sqrt[3]{-1}\sqrt[3]{c}-(-1)^{2/3}\sqrt[3]{d}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{c}}}\sqrt{\frac{i\left(1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{3i+\sqrt{3}}}\left((-1+(-1)^{2/3})E\left(\arcsin\left(\sqrt{-\frac{(-1)^{2/3}\cdots}{\cdots}}\right.\right.\right.}{b\sqrt{\frac{\sqrt[3]{c}+(-1)^{2/3}\cdots}{\left(1+\sqrt[3]{-1}\cdots\right.}}}\right.}{\cdots}$$

input
```
Integrate[Sqrt[c + d*x^3]/(a + b*x),x]
```

output
```
(2*(c + d*x^3 + (3*Sqrt[2]*a*c^(1/3)*d^(1/3)*((-1)^(1/3)*c^(1/3) - d^(1/3)
*x)*Sqrt[((-1)^(1/3)*c^(1/3) - (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1/3))*c^(
1/3))]*Sqrt[(I*(1 + (d^(1/3)*x)/c^(1/3)))/(3*I + Sqrt[3])]*((-1 + (-1)^(2/
3))*EllipticE[ArcSin[Sqrt[-(((-1)^(2/3)*((-1)^(2/3)*c^(1/3) + d^(1/3)*x))/
((1 + (-1)^(1/3))*c^(1/3)))]], (-1)^(1/3)/(-1 + (-1)^(1/3))] + EllipticF[A
rcSin[Sqrt[-(((-1)^(2/3)*((-1)^(2/3)*c^(1/3) + d^(1/3)*x))/((1 + (-1)^(1/3
))*c^(1/3)))]], (-1)^(1/3)/(-1 + (-1)^(1/3))]))/(b*Sqrt[(c^(1/3) + (-1)^(2
/3)*d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/3))]) - (3*a^2*d^(2/3)*((-1)^(1/3)*c
^(1/3) - d^(1/3)*x)*Sqrt[(c^(1/3) + d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/3))]
*Sqrt[((-1)^(1/3)*c^(1/3) - (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/3
))]*EllipticF[ArcSin[Sqrt[(c^(1/3) + (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1/3
))*c^(1/3))]], (-1)^(1/3)])/(b^2*Sqrt[(c^(1/3) + (-1)^(2/3)*d^(1/3)*x)/((1
 + (-1)^(1/3))*c^(1/3))]) - ((3*I)*b*c^(4/3)*Sqrt[(c^(1/3) + d^(1/3)*x)/((
1 + (-1)^(1/3))*c^(1/3))]*Sqrt[1 - (d^(1/3)*x)/c^(1/3) + (d^(2/3)*x^2)/c^(
2/3)]*EllipticPi[(I*Sqrt[3]*b*c^(1/3))/((-1)^(1/3)*b*c^(1/3) + a*d^(1/3)),
 ArcSin[Sqrt[(c^(1/3) + (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/3))]]
, (-1)^(1/3)])/((-1)^(1/3)*b*c^(1/3) + a*d^(1/3)) + ((-1)^(1/3)*Sqrt[3]*(1
 + (-1)^(1/3))*a^3*c^(1/3)*d*Sqrt[(c^(1/3) + d^(1/3)*x)/((1 + (-1)^(1/3))*
c^(1/3))]*Sqrt[1 - (d^(1/3)*x)/c^(1/3) + (d^(2/3)*x^2)/c^(2/3)]*EllipticPi
[(I*Sqrt[3]*b*c^(1/3))/((-1)^(1/3)*b*c^(1/3) + a*d^(1/3)), ArcSin[Sqrt[...
```

3.196.3 Rubi [A] (warning: unable to verify)

Time = 2.81 (sec) , antiderivative size = 1361, normalized size of antiderivative = 0.92, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {2573, 793, 2417, 759, 2416, 2561, 27, 759, 2567, 2538, 412, 435, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx^3}}{a+bx}dx$$

$$\Big\downarrow \text{2573}$$

$$\left(c-\frac{a^3d}{b^3}\right)\int \frac{1}{(a+bx)\sqrt{dx^3+c}}dx+\frac{ad\int \frac{a-bx}{\sqrt{dx^3+c}}dx}{b^3}+\frac{d\int \frac{x^2}{\sqrt{dx^3+c}}dx}{b}$$

$$\Big\downarrow \text{793}$$

$$\left(c-\frac{a^3d}{b^3}\right)\int \frac{1}{(a+bx)\sqrt{dx^3+c}}dx+\frac{ad\int \frac{a-bx}{\sqrt{dx^3+c}}dx}{b^3}+\frac{2\sqrt{c+dx^3}}{3b}$$

$$\Big\downarrow \text{2417}$$

$$\left(c-\frac{a^3d}{b^3}\right)\int \frac{1}{(a+bx)\sqrt{dx^3+c}}dx+\frac{ad\left(\left(a+\frac{\left(1-\sqrt{3}\right)b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\int \frac{1}{\sqrt{dx^3+c}}dx-\frac{b\int \frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt{dx^3+c}}dx}{\sqrt[3]{d}}\right)}{b^3}+\frac{2\sqrt{c+dx^3}}{3b}$$

$$\Big\downarrow \text{759}$$

$$\left(c-\frac{a^3d}{b^3}\right)\int \frac{1}{(a+bx)\sqrt{dx^3+c}}dx+$$
$$\frac{ad\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\left(a+\frac{\left(1-\sqrt{3}\right)b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{b\int \frac{\sqrt[3]{d}x\cdots}{\cdots}}{\cdots}\cdots\right)}{b^3}$$
$$\frac{2\sqrt{c+dx^3}}{3b}$$

$$\Big\downarrow \text{2416}$$

$$\left(c-\frac{a^3 d}{b^3}\right)\int \frac{1}{(a+bx)\sqrt{dx^3+c}}dx+$$

$$\frac{ad\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\left(a+\frac{\left(1-\sqrt{3}\right)b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{b\left(\frac{\ldots}{\sqrt[3]{d}(\ldots}\right.}{\ldots}\right.}{b^3}$$

$$\frac{2\sqrt{c+dx^3}}{3b}$$

$$\Big\downarrow \text{ 2561}$$

$$\left(c-\frac{a^3 d}{b^3}\right)\left(\frac{b\int \frac{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{c}(a+bx)\sqrt{dx^3+c}}dx}{-\frac{a\sqrt[3]{d}}{\sqrt[3]{c}}+\sqrt{3}b+b}-\frac{\sqrt[3]{d}\int \frac{1}{\sqrt{dx^3+c}}dx}{\left(1+\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{d}}\right)+$$

$$\frac{ad\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\left(a+\frac{\left(1-\sqrt{3}\right)b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{b\left(\frac{\ldots}{\sqrt[3]{d}(\ldots}\right.}{\ldots}\right.}{b^3}$$

$$\frac{2\sqrt{c+dx^3}}{3b}$$

$$\Big\downarrow \text{ 27}$$

3.196. $\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx$

$$\left(c-\frac{a^3 d}{b^3}\right)\left(\frac{b\int \frac{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}{(a+bx)\sqrt{dx^3+c}}dx}{\sqrt[3]{c}\left(-\frac{a\sqrt[3]{d}}{\sqrt[3]{c}}+\sqrt{3}b+b\right)}-\frac{\sqrt[3]{d}\int \frac{1}{\sqrt{dx^3+c}}dx}{\left(1+\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{d}}\right)+$$

$$\frac{ad\left(\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\left(a+\frac{\left(1-\sqrt{3}\right)b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{b\left(\sqrt[3]{d}\left(\cdots\right.\right.}{\cdots}\right.}{b^3 \cdots}$$

$$\frac{2\sqrt{c+dx^3}}{3b}$$

$$\Big\downarrow \; 759$$

$$\frac{ad\left(\frac{2\sqrt{2+\sqrt{3}}\left(a+\frac{\left(1-\sqrt{3}\right)b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\sqrt{dx^3+c}}-\frac{b\left(\sqrt[3]{d}\left(\cdots\right.\right.}{\cdots}\right.}{b^3 \cdots}$$

$$\left(c-\frac{a^3 d}{b^3}\right)\left(\frac{b\int \frac{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}{(a+bx)\sqrt{dx^3+c}}dx}{\sqrt[3]{c}\left(-\frac{\sqrt[3]{d}a}{\sqrt[3]{c}}+\sqrt{3}b+b\right)}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x \cdots}{\sqrt[3]{d}x \cdots}\right.\right.}{\sqrt[4]{3}\left(\left(1+\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{d}\right)\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\cdots}\right.$$

$$\frac{2\sqrt{dx^3+c}}{3b}$$

3.196. $\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx$

$$\Big\downarrow \text{2567}$$

$$\frac{ad\left(\frac{2\sqrt{2+\sqrt{3}}\left(a+\frac{(1-\sqrt{3})b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}{\left(\sqrt[3]{d}x+(1+\sqrt{3})\sqrt[3]{c}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+(1-\sqrt{3})\sqrt[3]{c}}{\sqrt[3]{d}x+(1+\sqrt{3})\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\left(\sqrt[3]{d}x+(1+\sqrt{3})\sqrt[3]{c}\right)^2}}\sqrt{dx^3+c}}-\frac{b\left(\frac{}{\sqrt[3]{d}(}\right.}{}\right.}{b^3}$$

$$\left(c-\frac{a^3d}{b^3}\right)\left(\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{c^{2/3}\left(\frac{d^{2/3}x^2}{c^{2/3}}-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}+1\right)}{\left(\sqrt[3]{d}x+(1+\sqrt{3})\sqrt[3]{c}\right)^2}}\int\frac{}{\sqrt{1-\frac{\left(\sqrt[3]{d}x+(1-\sqrt{3})\sqrt[3]{c}\right)^2}{\left(\sqrt[3]{d}x+(1+\sqrt{3})\sqrt[3]{c}\right)^2}}\sqrt{\frac{\left(\sqrt[3]{d}x+(1-\sqrt{3})\sqrt[3]{c}\right)^2}{\left(\sqrt[3]{d}x+(1+\sqrt{3})\sqrt[3]{c}\right)^2}-}}}{\sqrt[3]{c}\left(-\frac{\sqrt[3]{d}a}{\sqrt[3]{c}}+\sqrt{3}b+b\right)\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{d}x\right.}{\left(\sqrt[3]{d}x+(1+\right.}}}\right.$$

$$\frac{2\sqrt{dx^3+c}}{3b}$$

$$\Big\downarrow \text{2538}$$

3.196. $\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx$

$$\frac{ad\left(\frac{2\sqrt{2+\sqrt{3}}\left(a+\frac{(1-\sqrt{3})b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}{\left(\sqrt[3]{d}x+(1+\sqrt{3})\sqrt[3]{c}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+(1-\sqrt{3})\sqrt[3]{c}}{\sqrt[3]{d}x+(1+\sqrt{3})\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\left(\sqrt[3]{d}x+(1+\sqrt{3})\sqrt[3]{c}\right)^2}}\sqrt{dx^3+c}}-\frac{b\left(\frac{\ldots}{\sqrt[3]{d}(\ldots}\right.}{\ldots}\right)}{b^3 \ldots}$$

$$\left(c-\frac{a^3d}{b^3}\right)\left(\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{c^{2/3}\left(\frac{d^{2/3}x^2}{c^{2/3}}-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}+1\right)}{\left(\sqrt[3]{d}x+(1+\sqrt{3})\sqrt[3]{c}\right)^2}}\left(\left(-\frac{\sqrt[3]{d}a}{\sqrt[3]{c}}-\sqrt{3}b+b\right)\int\frac{\ldots}{\sqrt{1-\frac{\left(\sqrt[3]{d}x+(1-\sqrt{3})\sqrt[3]{c}\right)\ldots}{\left(\sqrt[3]{d}x+(1+\sqrt{3})\sqrt[3]{c}\right)\ldots}}\ldots}\right.}{\ldots}\right.$$

$$\frac{2\sqrt{dx^3+c}}{3b}$$

$\downarrow$ 412

3.196. $\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx$

$$\frac{ad\left(\frac{2\sqrt{2+\sqrt{3}}\left(a+\frac{\left(1-\sqrt{3}\right)b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\sqrt{dx^3+c}}-\frac{b\left(\ldots\right.}{\sqrt[3]{d}\left(\ldots\right.}\right.}{b^3}$$

$$\left(c-\frac{a^3d}{b^3}\right)\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{c^{2/3}\left(\frac{d^{2/3}x^2}{c^{2/3}}-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}+1\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\left(-\frac{\left(-\frac{\sqrt[3]{d}a}{\sqrt[3]{c}}-\sqrt{3}b+b\right)\text{EllipticPi}\left(\frac{\left(\left(1+\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{d}\right)\ldots}{\left(\left(1-\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{d}\right)\ldots}\right.}{\sqrt{7-4\sqrt{3}}\left(\left(1-\sqrt{3}\right)b\ldots\right.}\right.}{\ldots}$$

$$\frac{2\sqrt{dx^3+c}}{3b}$$

$\downarrow$ 435

3.196. $\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx$

$$ad\left(\frac{2\sqrt{2+\sqrt{3}}\left(a+\frac{\left(1-\sqrt{3}\right)b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\sqrt{dx^3+c}}-\frac{b\left(\frac{}{\sqrt[3]{d}(}\right.}{}\right.$$

$$b^3$$

$$\left(c-\frac{a^3d}{b^3}\right)\left(\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{c^{2/3}\left(\frac{d^{2/3}x^2}{c^{2/3}}-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}+1\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\left(-\frac{\left(-\frac{\sqrt[3]{d}a}{\sqrt[3]{c}}-\sqrt{3}b+b\right)\text{EllipticPi}\left(\frac{\left(\left(1+\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{d}\right)}{\left(\left(1-\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{d}\right)}\right.}{\sqrt{7-4\sqrt{3}}\left(\left(1-\sqrt{3}\right)b\right.}\right.}{}\right.$$

$$\frac{2\sqrt{dx^3+c}}{3b}$$

$$\downarrow 104$$

3.196. $\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx$

$$\frac{ad\left(\frac{2\sqrt{2+\sqrt{3}}\left(a+\frac{\left(1-\sqrt{3}\right)b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\sqrt{dx^3+c}}-\frac{b\left(\ldots\right.}{\sqrt[3]{d}\left(\ldots\right.}\right)}{b^3}$$

$$\left(c-\frac{a^3d}{b^3}\right)\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{c^{2/3}\left(\frac{d^{2/3}x^2}{c^{2/3}}-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}+1\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\left(-\frac{\left(-\frac{\sqrt[3]{d}a}{\sqrt[3]{c}}-\sqrt{3}b+b\right)\text{EllipticPi}\left(\frac{\left(\left(1+\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{d}\right)\ldots}{\left(\left(1-\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{d}\right)\ldots}\ldots\right.}{\sqrt{7-4\sqrt{3}}\left(\left(1-\sqrt{3}\right)b\ldots\right.}\right.\right.}{\ldots}$$

$$\frac{2\sqrt{dx^3+c}}{3b}$$

$\downarrow$ 221

3.196. $\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx$

$$\frac{ad\left(\frac{2\sqrt{2+\sqrt{3}}\left(a+\frac{(1-\sqrt{3})b\sqrt[3]{c}}{\sqrt[3]{d}}\right)\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}{\left(\sqrt[3]{d}x+(1+\sqrt{3})\sqrt[3]{c}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[3]{d}x+(1-\sqrt{3})\sqrt[3]{c}}{\sqrt[3]{d}x+(1+\sqrt{3})\sqrt[3]{c}}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{d}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\left(\sqrt[3]{d}x+(1+\sqrt{3})\sqrt[3]{c}\right)^2}}\sqrt{dx^3+c}}-\frac{b\left(\cdots\right.}{\sqrt[3]{d}(\cdots}\right.}{b^3\cdots}$$

$$\left(c-\frac{a^3d}{b^3}\right)\left(\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}b\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{c^{2/3}\left(\frac{d^{2/3}x^2}{c^{2/3}}-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}+1\right)}{\left(\sqrt[3]{d}x+(1+\sqrt{3})\sqrt[3]{c}\right)^2}}\left(\frac{\sqrt{c}\left(-\frac{\sqrt[3]{d}a}{\sqrt[3]{c}}+\sqrt{3}b+b\right)\text{arctanh}\left(\frac{\sqrt{2-\sqrt{3}}\sqrt{d^{2/3}a^2+b\sqrt[3]{c}\cdots}}{\sqrt[4]{3}\sqrt{b}\sqrt[6]{c}\sqrt{b\sqrt[3]{\cdots}}}\right.}{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{b}\sqrt{b\sqrt[3]{c}-a\sqrt[3]{d}}\sqrt{d^{2/3}a^2\cdots}}\right.}{\sqrt[3]{c}\left(-\frac{\sqrt[3]{d}a}{\sqrt[3]{c}}+\sqrt{3}b+\cdots\right.}\right.$$

$$\frac{2\sqrt{dx^3+c}}{3b}$$

input
```
Int[Sqrt[c + d*x^3]/(a + b*x),x]
```

3.196. $\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx$

output
```
(2*Sqrt[c + d*x^3])/(3*b) + (a*d*(-((b*((2*Sqrt[c + d*x^3])/(d^(1/3)*((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1
/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d
^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(1/3)*
Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2
]*Sqrt[c + d*x^3])))/d^(1/3)) + (2*Sqrt[2 + Sqrt[3]]*(a + ((1 - Sqrt[3])*b
*c^(1/3))/d^(1/3))*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x
 + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((
1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7
 - 4*Sqrt[3]])/(3^(1/4)*d^(1/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])))/b^3 + (c - (a^3*d)/b^
3)*((-2*Sqrt[2 + Sqrt[3]]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^
(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[Ar
cSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)], -7 - 4*Sqrt[3]])/(3^(1/4)*((1 + Sqrt[3])*b*c^(1/3) - a*d^(1/3))*Sqrt[
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqr
t[c + d*x^3]) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*b*(c^(1/3) + d^(1/3)*x)*Sqrt[
(c^(2/3)*(1 - (d^(1/3)*x)/c^(1/3) + (d^(2/3)*x^2)/c^(2/3)))/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)^2]*((Sqrt[c]*(b + Sqrt[3]*b - (a*d^(1/3))/c^(1/3)...
```

3.196.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2417
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(c*r - (1 - Sqrt[3])*d*s)/r
  Int[1/Sqrt[a + b*x^3], x], x] + Simp[d/r   Int[((1 - Sqrt[3])*s + r*x)/Sq
rt[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 -
2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.196. $\int \frac{\sqrt{c+dx^3}}{a+bx} \, dx$

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 2561
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Simp[-q/((1 + Sqrt[3])*d - c*q)   Int[1/Sqrt[a + b*x^3], x]
, x] + Simp[d/((1 + Sqrt[3])*d - c*q)   Int[(1 + Sqrt[3] + q*x)/((c + d*x)*
Sqrt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0]
```

rule 2567
```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Simp[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2]))   Subst[Int[1/(((1
- Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sq
rt[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt
[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

rule 2573
```
Int[Sqrt[(a_) + (b_.)*(x_)^3]/((c_) + (d_.)*(x_)), x_Symbol] :> Simp[b/d
Int[x^2/Sqrt[a + b*x^3], x], x] + (-Simp[(b*c^3 - a*d^3)/d^3   Int[1/((c +
d*x)*Sqrt[a + b*x^3]), x], x] + Simp[b*(c/d^3)   Int[(c - d*x)/Sqrt[a + b*x
^3], x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[b*c^3 - a*d^3, 0]
```

3.196.4 Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 1126, normalized size of antiderivative = 0.76

method	result	size
default	Expression too large to display	1126
elliptic	Expression too large to display	1126
risch	Expression too large to display	1137

input
```
int((d*x^3+c)^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/3*(d*x^3+c)^(1/2)/b-2/3*I*a^2/b^3*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^
(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*
3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^
(1/3))^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1
/2)/d*(-c*d^2)^(1/3)))^(1/2))+2/3*I*a/b^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3)
)^(1/2)*((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c
*d^2)^(1/3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(
1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1
/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*
d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)
,(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^
2)^(1/3)))^(1/2))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c
*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2
),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d
^2)^(1/3)))^(1/2)))+2/3*I*(a^3*d-b^3*c)/b^4*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x
+1/2/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2...
```

3.196.5 Fricas [F]

$$\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx=\int \frac{\sqrt{dx^3+c}}{bx+a}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/(b*x+a),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x^3 + c)/(b*x + a), x)
```

3.196.6 Sympy [F]

$$\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx = \int \frac{\sqrt{c+dx^3}}{a+bx}\,dx$$

input
```
integrate((d*x**3+c)**(1/2)/(b*x+a),x)
```

output
```
Integral(sqrt(c + d*x**3)/(a + b*x), x)
```

3.196.7 Maxima [F]

$$\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx = \int \frac{\sqrt{dx^3+c}}{bx+a}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/(b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^3 + c)/(b*x + a), x)
```

3.196.8 Giac [F]

$$\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx = \int \frac{\sqrt{dx^3+c}}{bx+a}\,dx$$

input
```
integrate((d*x^3+c)^(1/2)/(b*x+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x^3 + c)/(b*x + a), x)
```

3.196.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx=\int \frac{\sqrt{d\,x^3+c}}{a+b\,x}\,dx$$

input
```
int((c + d*x^3)^(1/2)/(a + b*x),x)
```

output
```
int((c + d*x^3)^(1/2)/(a + b*x), x)
```

3.197 $\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx$

3.197.1 Optimal result

Integrand size = 21, antiderivative size = 135

$$\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx = \frac{(d^3+e^3x^3)^p\left(1+\frac{2(d+ex)}{\left(-3+i\sqrt{3}\right)d}\right)^{-p}\left(1-\frac{2(d+ex)}{\left(3+i\sqrt{3}\right)d}\right)^{-p}\text{AppellF1}\left(p,-p,-p,1+p,-\frac{2(d+ex)}{\left(-3+i\sqrt{3}\right)d},\frac{2(d+ex)}{\left(3+i\sqrt{3}\right)d}\right)}{ep}$$

output
```
(e^3*x^3+d^3)^p*AppellF1(p,-p,-p,p+1,-2*(e*x+d)/d/(-3+I*3^(1/2)),2*(e*x+d)
/d/(3+I*3^(1/2)))/e/p/((1+2*(e*x+d)/d/(-3+I*3^(1/2)))^p)/((1-2*(e*x+d)/d/(
3+I*3^(1/2)))^p)
```

3.197.2 Mathematica [F]

$$\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx = \int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx$$

input
```
Integrate[(d^3 + e^3*x^3)^p/(d + e*x),x]
```

output
```
Integrate[(d^3 + e^3*x^3)^p/(d + e*x), x]
```

3.197.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{(d^3+e^3x^3)^p}{d+ex}dx$$

input
```
Int[(d^3 + e^3*x^3)^p/(d + e*x),x]
```

output
```
$Aborted
```

3.197.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.197.4 Maple [F]

$$\int \frac{(e^3x^3+d^3)^p}{ex+d}dx$$

input
```
int((e^3*x^3+d^3)^p/(e*x+d),x)
```

output
```
int((e^3*x^3+d^3)^p/(e*x+d),x)
```

3.197.5 Fricas [F]

$$\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx = \int \frac{(e^3x^3+d^3)^p}{ex+d}\,dx$$

input
```
integrate((e^3*x^3+d^3)^p/(e*x+d),x, algorithm="fricas")
```

output
```
integral((e^3*x^3 + d^3)^p/(e*x + d), x)
```

3.197.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(102) = 204$.

Time = 25.94 (sec) , antiderivative size = 631, normalized size of antiderivative = 4.67

$$\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx$$
$$= \frac{0^p \log\left(1+\frac{e^3x^3}{d^3}\right)\Gamma\left(-\frac{2}{3}\right)\Gamma\left(-\frac{1}{3}\right)\Gamma\left(\frac{4}{3}\right)\Gamma\left(\frac{5}{3}\right)}{4\pi^2 e}$$
$$+ \frac{0^p e^{\frac{i\pi}{3}} \log\left(1-\frac{exe^{\frac{i\pi}{3}}}{d}\right)\Gamma\left(-\frac{1}{3}\right)\Gamma\left(\frac{1}{3}\right)\Gamma^2\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}{6\pi^2 e\Gamma\left(\frac{5}{3}\right)}$$
$$+ \frac{0^p e^{\frac{2i\pi}{3}} \log\left(1-\frac{exe^{\frac{i\pi}{3}}}{d}\right)\Gamma^3\left(\frac{1}{3}\right)\Gamma^2\left(\frac{2}{3}\right)}{12\pi^2 e\Gamma\left(\frac{4}{3}\right)} - \frac{0^p \log\left(1-\frac{exe^{i\pi}}{d}\right)\Gamma\left(-\frac{1}{3}\right)\Gamma\left(\frac{1}{3}\right)\Gamma^2\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}{6\pi^2 e\Gamma\left(\frac{5}{3}\right)}$$
$$+ \frac{0^p \log\left(1-\frac{exe^{i\pi}}{d}\right)\Gamma^3\left(\frac{1}{3}\right)\Gamma^2\left(\frac{2}{3}\right)}{12\pi^2 e\Gamma\left(\frac{4}{3}\right)} + \frac{0^p e^{-\frac{2i\pi}{3}} \log\left(1-\frac{exe^{\frac{5i\pi}{3}}}{d}\right)\Gamma^3\left(\frac{1}{3}\right)\Gamma^2\left(\frac{2}{3}\right)}{12\pi^2 e\Gamma\left(\frac{4}{3}\right)}$$
$$+ \frac{0^p e^{-\frac{i\pi}{3}} \log\left(1-\frac{exe^{\frac{5i\pi}{3}}}{d}\right)\Gamma\left(-\frac{1}{3}\right)\Gamma\left(\frac{1}{3}\right)\Gamma^2\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}{6\pi^2 e\Gamma\left(\frac{5}{3}\right)}$$
$$- \frac{d^2 e^{3p-3} p x^{3p-2}\Gamma\left(-\frac{2}{3}\right)\Gamma\left(-\frac{1}{3}\right)\Gamma\left(\frac{4}{3}\right)\Gamma\left(\frac{5}{3}\right)\Gamma(p)\Gamma\left(\frac{2}{3}-p\right){}_2F_1\left(\left.\begin{matrix}1-p,\frac{2}{3}-p\\ \frac{5}{3}-p\end{matrix}\right|\frac{d^3e^{i\pi}}{e^3x^3}\right)}{4\pi^2\Gamma\left(\frac{5}{3}-p\right)\Gamma(p+1)}$$
$$- \frac{d e^{3p-2} p x^{3p-1}\Gamma\left(-\frac{1}{3}\right)\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)\Gamma(p)\Gamma\left(\frac{1}{3}-p\right){}_2F_1\left(\left.\begin{matrix}1-p,\frac{1}{3}-p\\ \frac{4}{3}-p\end{matrix}\right|\frac{d^3e^{i\pi}}{e^3x^3}\right)}{4\pi^2\Gamma\left(\frac{4}{3}-p\right)\Gamma(p+1)}$$
$$- \frac{d^{3p} e^2 x^3\Gamma^2\left(\frac{1}{3}\right)\Gamma^2\left(\frac{2}{3}\right)\Gamma(p)\Gamma(1-p){}_3F_2\left(\left.\begin{matrix}2,1,1-p\\ 2,2\end{matrix}\right|\frac{e^3x^3e^{i\pi}}{d^3}\right)}{4\pi^2 d^3\Gamma(-p)\Gamma(p+1)}$$

input
```
integrate((e**3*x**3+d**3)**p/(e*x+d),x)
```

3.197. $\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx$

output
```
0**p*log(1 + e**3*x**3/d**3)*gamma(-2/3)*gamma(-1/3)*gamma(4/3)*gamma(5/3)
/(4*pi**2*e) + 0**p*exp(I*pi/3)*log(1 - e*x*exp_polar(I*pi/3)/d)*gamma(-1/
3)*gamma(1/3)*gamma(2/3)**2*gamma(4/3)/(6*pi**2*e*gamma(5/3)) + 0**p*exp(2
*I*pi/3)*log(1 - e*x*exp_polar(I*pi/3)/d)*gamma(1/3)**3*gamma(2/3)**2/(12*
pi**2*e*gamma(4/3)) - 0**p*log(1 - e*x*exp_polar(I*pi)/d)*gamma(-1/3)*gamm
a(1/3)*gamma(2/3)**2*gamma(4/3)/(6*pi**2*e*gamma(5/3)) + 0**p*log(1 - e*x*
exp_polar(I*pi)/d)*gamma(1/3)**3*gamma(2/3)**2/(12*pi**2*e*gamma(4/3)) + 0
**p*exp(-2*I*pi/3)*log(1 - e*x*exp_polar(5*I*pi/3)/d)*gamma(1/3)**3*gamma(
2/3)**2/(12*pi**2*e*gamma(4/3)) + 0**p*exp(-I*pi/3)*log(1 - e*x*exp_polar(
5*I*pi/3)/d)*gamma(-1/3)*gamma(1/3)*gamma(2/3)**2*gamma(4/3)/(6*pi**2*e*ga
mma(5/3)) - d**2*e**(3*p - 3)*p*x**(3*p - 2)*gamma(-2/3)*gamma(-1/3)*gamma
(4/3)*gamma(5/3)*gamma(p)*gamma(2/3 - p)*hyper((1 - p, 2/3 - p), (5/3 - p,
), d**3*exp_polar(I*pi)/(e**3*x**3))/(4*pi**2*gamma(5/3 - p)*gamma(p + 1))
 - d*e**(3*p - 2)*p*x**(3*p - 1)*gamma(-1/3)*gamma(1/3)*gamma(2/3)*gamma(4
/3)*gamma(p)*gamma(1/3 - p)*hyper((1 - p, 1/3 - p), (4/3 - p,), d**3*exp_p
olar(I*pi)/(e**3*x**3))/(4*pi**2*gamma(4/3 - p)*gamma(p + 1)) - d**(3*p)*e
**2*x**3*gamma(1/3)**2*gamma(2/3)**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1
- p), (2, 2), e**3*x**3*exp_polar(I*pi)/d**3)/(4*pi**2*d**3*gamma(-p)*gamm
a(p + 1))
```

3.197.7 Maxima [F]

$$\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx = \int \frac{(e^3x^3+d^3)^p}{ex+d}\,dx$$

input
```
integrate((e^3*x^3+d^3)^p/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((e^3*x^3 + d^3)^p/(e*x + d), x)
```

3.197.8 Giac [F]

$$\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx = \int \frac{(e^3x^3+d^3)^p}{ex+d}\,dx$$

input
```
integrate((e^3*x^3+d^3)^p/(e*x+d),x, algorithm="giac")
```

output
```
integrate((e^3*x^3 + d^3)^p/(e*x + d), x)
```

3.197.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d^3+e^3x^3)^p}{d+ex}\,dx=\int \frac{(d^3+e^3\,x^3)^p}{d+e\,x}\,dx$$

input
```
int((d^3 + e^3*x^3)^p/(d + e*x),x)
```

output
```
int((d^3 + e^3*x^3)^p/(d + e*x), x)
```

3.198 $\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx$

3.198.1 Optimal result

Integrand size = 27, antiderivative size = 16

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx = 2\arctan\left(\frac{1+x}{\sqrt{1+x^3}}\right)$$

output
```
2*arctan((1+x)/(x^3+1)^(1/2))
```

3.198.2 Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx = 2\arctan\left(\frac{\sqrt{1+x^3}}{1-x+x^2}\right)$$

input
```
Integrate[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[1 + x^3]),x]
```

output
```
2*ArcTan[Sqrt[1 + x^3]/(1 - x + x^2)]
```

3.198.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2571, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x^2-2x+2}{(x^2+2)\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{2571}$$

$$2\int \frac{1}{\frac{(x+1)^2}{x^3+1}+1}d\frac{x+1}{\sqrt{x^3+1}}$$

$$\Big\downarrow \text{216}$$

$$2\arctan\left(\frac{x+1}{\sqrt{x^3+1}}\right)$$

input
```
Int[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[1 + x^3]),x]
```

output
```
2*ArcTan[(1 + x)/Sqrt[1 + x^3]]
```

3.198.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2571
```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> Simp[-g/e   Subst[Int[1/(1 + a*x^2), x], x, (1
+ 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ
[b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

3.198.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.62 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.88

method	result	size
trager	$\operatorname{RootOf}\left(\_Z^{2}+1\right) \ln \left(\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right) x^{2}-2 \operatorname{RootOf}\left(\_Z^{2}+1\right) x+2 \sqrt{x^{3}+1}}{x^{2}+2}\right)$	46
default	Expression too large to display	1640
elliptic	Expression too large to display	1845

input
```
int((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
RootOf(_Z^2+1)*ln((RootOf(_Z^2+1)*x^2-2*RootOf(_Z^2+1)*x+2*(x^3+1)^(1/2))/
(x^2+2))
```

3.198.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx = -\arctan\left(\frac{x^2-2\,x}{2\,\sqrt{x^3+1}}\right)$$

input
```
integrate((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-arctan(1/2*(x^2 - 2*x)/sqrt(x^3 + 1))
```

3.198.6 Sympy [F]

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx = -\int \frac{2x}{x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx - \int \frac{x^2}{x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx - \int \left(-\frac{2}{x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\right)dx$$

input
```
integrate((-x**2-2*x+2)/(x**2+2)/(x**3+1)**(1/2),x)
```

output
```
-Integral(2*x/(x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x) - Integral(x**2
/(x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x) - Integral(-2/(x**2*sqrt(x**
3 + 1) + 2*sqrt(x**3 + 1)), x)
```

3.198.7 Maxima [F]

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx = \int -\frac{x^2+2\,x-2}{\sqrt{x^3+1}(x^2+2)}\,dx$$

input
```
integrate((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(x^2 + 2)), x)
```

3.198.8 Giac [F]

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx = \int -\frac{x^2+2\,x-2}{\sqrt{x^3+1}(x^2+2)}\,dx$$

input
```
integrate((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(x^2 + 2)), x)
```

3.198.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 273, normalized size of antiderivative = 17.06

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx = \frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{1+\sqrt{2}\,1\mathrm{i}};\mathrm{asin}\left(\sqrt{\right.\right.\right.}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\right.}}$$

input
```
int(-(2*x + x^2 - 2)/((x^2 + 2)*(x^3 + 1)^(1/2)),x)
```

output
```
((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i +
1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/
((3^(1/2)*1i)/2 - 3/2)) - ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + ellipticPi(-((3^
(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i - 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(x^3 - x*(((3^(
1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3
^(1/2)*1i)/2 + 1/2))^(1/2)
```

3.198. $\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx$

3.199 $\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx$

3.199.1 Optimal result

Integrand size = 29, antiderivative size = 20

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx = -2\arctan\left(\frac{1-x}{\sqrt{1-x^3}}\right)$$

output
```
-2*arctan((1-x)/(-x^3+1)^(1/2))
```

3.199.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx = -2\arctan\left(\frac{\sqrt{1-x^3}}{1+x+x^2}\right)$$

input
```
Integrate[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[1 - x^3]),x]
```

output
```
-2*ArcTan[Sqrt[1 - x^3]/(1 + x + x^2)]
```

3.199.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2571, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x^2+2x+2}{(x^2+2)\sqrt{1-x^3}}\,dx$$

$$\Big\downarrow \; 2571$$

$$-2\int \frac{1}{\frac{(1-x)^2}{1-x^3}+1}d\frac{1-x}{\sqrt{1-x^3}}$$

$$\Big\downarrow \; 216$$

$$-2\arctan\left(\frac{1-x}{\sqrt{1-x^3}}\right)$$

input
```
Int[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[1 - x^3]),x]
```

output
```
-2*ArcTan[(1 - x)/Sqrt[1 - x^3]]
```

3.199.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2571
```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> Simp[-g/e   Subst[Int[1/(1 + a*x^2), x], x, (1
+ 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ
[b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

3.199.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.56 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.45

method	result
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)x^2+2\operatorname{RootOf}\left(\_Z^2+1\right)x-2\sqrt{-x^3+1}}{x^2+2}\right)$
default	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}-\frac{2i\sqrt{3}\sqrt{i\sqrt{3}x+\frac{i\sqrt{3}}{2}+\frac{3}{2}}\sqrt{\cdots}}{\cdots}$
elliptic	$\frac{2i\sqrt{3}\sqrt{i\sqrt{3}x+\frac{i\sqrt{3}}{2}+\frac{3}{2}}\sqrt{\frac{x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}-\frac{1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\sqrt{3}x-\frac{i\sqrt{3}}{2}+\frac{3}{2}}\,F\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}}-\frac{2\sqrt{2}\sqrt{3}\sqrt{i\sqrt{3}x+\cdots}}{\cdots}$

input
```
int((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
RootOf(_Z^2+1)*ln(-(RootOf(_Z^2+1)*x^2+2*RootOf(_Z^2+1)*x-2*(-x^3+1)^(1/2)
)/(x^2+2))
```

3.199.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx = -\arctan\left(\frac{\sqrt{-x^3+1}(x^2+2\,x)}{2\,(x^3-1)}\right)$$

input
```
integrate((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-arctan(1/2*sqrt(-x^3 + 1)*(x^2 + 2*x)/(x^3 - 1))
```

3.199.6 Sympy [F]

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx = -\int \left(-\frac{2x}{x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx - \int \frac{x^2}{x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\,dx - \int \left(-\frac{2}{x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx$$

input
```
integrate((-x**2+2*x+2)/(x**2+2)/(-x**3+1)**(1/2),x)
```

output
```
-Integral(-2*x/(x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(x**
2/(x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(-2/(x**2*sqrt(1
- x**3) + 2*sqrt(1 - x**3)), x)
```

3.199.7 Maxima [F]

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx = \int -\frac{x^2-2x-2}{\sqrt{-x^3+1}(x^2+2)}\,dx$$

input
```
integrate((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(x^2 + 2)), x)
```

3.199.8 Giac [F]

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx = \int -\frac{x^2-2x-2}{\sqrt{-x^3+1}(x^2+2)}\,dx$$

input
```
integrate((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(x^2 + 2)), x)
```

3.199.9 Mupad [B] (verification not implemented)

Time = 19.53 (sec) , antiderivative size = 292, normalized size of antiderivative = 14.60

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx = -\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3-1}\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(-F\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(\right.\right.}{\sqrt{1-x^3}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right.}}$$

input
```
int((2*x - x^2 + 2)/((x^2 + 2)*(1 - x^3)^(1/2)),x)
```

output
```
-((3^(1/2)*1i + 3)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*
1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 +
3/2)/(2^(1/2)*1i + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3
^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticF(asin((-(x - 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2
)) + ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i - 1), asin((-(x - 1)/(
(3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3
/2))))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x
*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.200 $\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx$

3.200.1 Optimal result

Integrand size = 27, antiderivative size = 18

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx = -2\text{arctanh}\left(\frac{1-x}{\sqrt{-1+x^3}}\right)$$

output
```
-2*arctanh((1-x)/(x^3-1)^(1/2))
```

3.200.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx = 2\text{arctanh}\left(\frac{\sqrt{-1+x^3}}{1+x+x^2}\right)$$

input
```
Integrate[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[-1 + x^3]),x]
```

output
```
2*ArcTanh[Sqrt[-1 + x^3]/(1 + x + x^2)]
```

3.200.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2571, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x^2+2x+2}{(x^2+2)\sqrt{x^3-1}}dx$$

$$\Big\downarrow \text{2571}$$

$$-2\int \frac{1}{1-\frac{(1-x)^2}{x^3-1}}d\frac{1-x}{\sqrt{x^3-1}}$$

$$\Big\downarrow \text{219}$$

$$-2\text{arctanh}\left(\frac{1-x}{\sqrt{x^3-1}}\right)$$

input
```
Int[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[-1 + x^3]),x]
```

output
```
-2*ArcTanh[(1 - x)/Sqrt[-1 + x^3]]
```

3.200.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2571
```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> Simp[-g/e   Subst[Int[1/(1 + a*x^2), x], x, (1
+ 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ
[b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

3.200.4 Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

method	result	size
trager	$\ln\left(\frac{x^2+2\sqrt{x^3-1}+2x}{x^2+2}\right)$	26
default	Expression too large to display	1656
elliptic	Expression too large to display	1865

input
```
int((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln((x^2+2*(x^3-1)^(1/2)+2*x)/(x^2+2))
```

3.200.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx = \log\left(\frac{x^2+2\,x+2\,\sqrt{x^3-1}}{x^2+2}\right)$$

input
```
integrate((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
log((x^2 + 2*x + 2*sqrt(x^3 - 1))/(x^2 + 2))
```

3.200.6 Sympy [F]

$$\begin{aligned}\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx = & -\int\left(-\frac{2x}{x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx \\ & -\int\frac{x^2}{x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\,dx \\ & -\int\left(-\frac{2}{x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx\end{aligned}$$

input
```
integrate((-x**2+2*x+2)/(x**2+2)/(x**3-1)**(1/2),x)
```

output
```
-Integral(-2*x/(x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(x**
2/(x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(-2/(x**2*sqrt(x*
*3 - 1) + 2*sqrt(x**3 - 1)), x)
```

3.200.7 Maxima [F]

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx = \int -\frac{x^2-2\,x-2}{\sqrt{x^3-1}(x^2+2)}\,dx$$

input
```
integrate((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 + 2)), x)
```

3.200.8 Giac [F]

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx = \int -\frac{x^2-2\,x-2}{\sqrt{x^3-1}(x^2+2)}\,dx$$

input
```
integrate((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 + 2)), x)
```

3.200.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 276, normalized size of antiderivative = 15.33

$$\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx =$$

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{1+\sqrt{2}\,1\mathrm{i}};\right.\right.}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+}}$$

input
```
int((2*x - x^2 + 2)/((x^2 + 2)*(x^3 - 1)^(1/2)),x)
```

output
```
-((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1
/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i
 + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3
/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3
/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + ellipticPi(
-((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i - 1), asin((-(x - 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(((3^(1/2
)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2
)*1i)/2 + 1/2) + 1) + x^3)^(1/2)
```

3.200. $\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx$

3.201 $\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx$

3.201.1 Optimal result

Integrand size = 29, antiderivative size = 18

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx = 2\text{arctanh}\left(\frac{1+x}{\sqrt{-1-x^3}}\right)$$

output
```
2*arctanh((1+x)/(-x^3-1)^(1/2))
```

3.201.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx = -2\text{arctanh}\left(\frac{\sqrt{-1-x^3}}{1-x+x^2}\right)$$

input
```
Integrate[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[-1 - x^3]),x]
```

output
```
-2*ArcTanh[Sqrt[-1 - x^3]/(1 - x + x^2)]
```

3.201.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2571, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x^2-2x+2}{(x^2+2)\sqrt{-x^3-1}}dx$$

$$\Big\downarrow \text{2571}$$

$$2\int \frac{1}{1-\frac{(x+1)^2}{-x^3-1}}d\frac{x+1}{\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{219}$$

$$2\text{arctanh}\left(\frac{x+1}{\sqrt{-x^3-1}}\right)$$

input
```
Int[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[-1 - x^3]),x]
```

output
```
2*ArcTanh[(1 + x)/Sqrt[-1 - x^3]]
```

3.201.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2571
```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> Simp[-g/e   Subst[Int[1/(1 + a*x^2), x], x, (1
+ 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ
[b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

3.201.4 Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

method	result
trager	$-\ln\left(\frac{x^2+2\sqrt{-x^3-1}-2x}{x^2+2}\right)$
default	$\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2\sqrt{2}\,\sqrt{3}\,\sqrt{i\sqrt{3}\,x-\frac{i\sqrt{3}}{2}+\frac{3}{2}}\,\sqrt{\cdots}}{\cdots}$
elliptic	$\frac{2i\sqrt{3}\,\sqrt{i\sqrt{3}\,x-\frac{i\sqrt{3}}{2}+\frac{3}{2}}\,\sqrt{\frac{x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}+\frac{1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\sqrt{3}\,x+\frac{i\sqrt{3}}{2}+\frac{3}{2}}\,F\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1}}-\frac{2\sqrt{2}\,\sqrt{3}\,\sqrt{i\sqrt{3}\,x-\frac{i\sqrt{3}}{2}+\cdots}}{\cdots}$

input
```
int((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-ln((x^2+2*(-x^3-1)^(1/2)-2*x)/(x^2+2))
```

3.201.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.56

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx = \log\left(-\frac{x^2-2\,x-2\,\sqrt{-x^3-1}}{x^2+2}\right)$$

input
```
integrate((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
log(-(x^2 - 2*x - 2*sqrt(-x^3 - 1))/(x^2 + 2))
```

3.201.6 Sympy [F]

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx = -\int \frac{2x}{x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx - \int \frac{x^2}{x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx - \int \left(-\frac{2}{x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\right)dx$$

input
```
integrate((-x**2-2*x+2)/(x**2+2)/(-x**3-1)**(1/2),x)
```

output
```
-Integral(2*x/(x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x) - Integral(x*
*2/(x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x) - Integral(-2/(x**2*sqrt
(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x)
```

3.201.7 Maxima [F]

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx = \int -\frac{x^2+2\,x-2}{\sqrt{-x^3-1}(x^2+2)}\,dx$$

input
```
integrate((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(x^2 + 2)), x)
```

3.201.8 Giac [F]

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx = \int -\frac{x^2+2\,x-2}{\sqrt{-x^3-1}(x^2+2)}\,dx$$

input
```
integrate((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(x^2 + 2)), x)
```

3.201.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 289, normalized size of antiderivative = 16.06

$$\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx$$
$$=\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{1+\sqrt{2}\,1\mathrm{i}}\right.\right.}{\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x}}$$

input
```
int(-(2*x + x^2 - 2)/((x^2 + 2)*(- x^3 - 1)^(1/2)),x)
```

output
```
((3^(1/2)*1i + 3)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i
)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2
- x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 + 3/2
)/(2^(1/2)*1i + 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/
2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticF(asin(((x + 1)/((3^(1/2
)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) +
ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i - 1), asin(((x + 1)/((3^(1/
2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))
/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2
) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

3.202 $\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx$

3.202.1 Optimal result

Integrand size = 31, antiderivative size = 30

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{1+d}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{1+d}}$$

output
```
2*arctan((1+x)*(1+d)^(1/2)/(x^3+1)^(1/2))/(1+d)^(1/2)
```

3.202.2 Mathematica [A] (verified)

Time = 1.89 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx=\frac{2\arctan\left(\frac{\sqrt{1+d}\sqrt{1+x^3}}{1-x+x^2}\right)}{\sqrt{1+d}}$$

input
```
Integrate[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[1 + x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[1 + d]*Sqrt[1 + x^3])/(1 - x + x^2)])/Sqrt[1 + d]
```

3.202.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2570, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x^2-2x+2}{\sqrt{x^3+1}\,(dx+d+x^2+2)}dx$$
$$\Big\downarrow \text{2570}$$
$$-4\int \frac{1}{-\frac{2(d+1)(x+1)^2}{x^3+1}-2}d\frac{x+1}{\sqrt{x^3+1}}$$
$$\Big\downarrow \text{217}$$
$$\frac{2\arctan\left(\frac{\sqrt{d+1}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{d+1}}$$

input
```
Int[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[1 + x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[1 + d]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[1 + d]
```

3.202.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 2570
```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[-2*g*h   Subst[Int[1/(2*e*h -
 (b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Fre
eQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8
*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

3.202.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 1.81 (sec) , antiderivative size = 4397, normalized size of antiderivative = 146.57

method	result	size
default	Expression too large to display	4397
elliptic	Expression too large to display	4602

input
```
int((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/
2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-3/2/(d^2-4*d-8)^(1/2)*(1/(3/
2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-
1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/
2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(
1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((x+
1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-
8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d^2-1/2*I/(d^
2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-
3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3
/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(
1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-
1+1/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))
^(1/2))*d^2*3^(1/2)+3/2*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1
/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+
1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(...
```

3.202.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 181, normalized size of antiderivative = 6.03

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx$$
$$=\left[-\frac{\sqrt{-d-1}\log\left(-\frac{2\,(3\,d+4)x^3-x^4-(d^2+2\,d+4)x^2-d^2+4\,\sqrt{x^3+1}((d+2)x-x^2+d)\sqrt{-d-1}-2\,(d^2+2\,d)x+4\,d+4}{2\,dx^3+x^4+(d^2+2\,d+4)x^2+d^2+2\,(d^2+2\,d)x+4\,d+4}\right)}{2\,(d+1)},\right.$$
$$\left.-\frac{\arctan\left(-\frac{\sqrt{x^3+1}((d+2)x-x^2+d)\sqrt{d+1}}{2\,((d+1)x^3+d+1)}\right)}{\sqrt{d+1}}\right]$$

input
```
integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-d - 1)*log(-(2*(3*d + 4)*x^3 - x^4 - (d^2 + 2*d + 4)*x^2 - d^2
 + 4*sqrt(x^3 + 1)*((d + 2)*x - x^2 + d)*sqrt(-d - 1) - 2*(d^2 + 2*d)*x +
4*d + 4)/(2*d*x^3 + x^4 + (d^2 + 2*d + 4)*x^2 + d^2 + 2*(d^2 + 2*d)*x + 4*
d + 4))/(d + 1), -arctan(-1/2*sqrt(x^3 + 1)*((d + 2)*x - x^2 + d)*sqrt(d +
 1)/((d + 1)*x^3 + d + 1))/sqrt(d + 1)]
```

3.202.6 Sympy [F]

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx$$
$$=-\int \frac{2x}{dx\sqrt{x^3+1}+d\sqrt{x^3+1}+x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx$$
$$-\int \frac{x^2}{dx\sqrt{x^3+1}+d\sqrt{x^3+1}+x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx$$
$$-\int \left(-\frac{2}{dx\sqrt{x^3+1}+d\sqrt{x^3+1}+x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\right)dx$$

input
```
integrate((-x**2-2*x+2)/(d*x+x**2+d+2)/(x**3+1)**(1/2),x)
```

output
```
-Integral(2*x/(d*x*sqrt(x**3 + 1) + d*sqrt(x**3 + 1) + x**2*sqrt(x**3 + 1)
 + 2*sqrt(x**3 + 1)), x) - Integral(x**2/(d*x*sqrt(x**3 + 1) + d*sqrt(x**3
 + 1) + x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x) - Integral(-2/(d*x*sqr
t(x**3 + 1) + d*sqrt(x**3 + 1) + x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)),
x)
```

3.202.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d^2-4*(d+2)>0)', see `assume?` f
or more de
```

3.202.8 Giac [F]

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx = \int -\frac{x^2+2\,x-2}{\sqrt{x^3+1}(dx+x^2+d+2)}\,dx$$

input
```
integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(d*x + x^2 + d + 2)), x)
```

3.202.9 Mupad [B] (verification not implemented)

Time = 19.71 (sec) , antiderivative size = 632, normalized size of antiderivative = 21.07

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx = \text{Too large to display}$$

input
```
int(-(2*x + x^2 - 2)/((x^3 + 1)^(1/2)*(d + d*x + x^2 + 2)),x)
```

output
```
- (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 -
3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x +
1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2
 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x
*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1
/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/
2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3
/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellip
ticPi(((3^(1/2)*1i)/2 + 3/2)/((d^2 - 4*d - 8)^(1/2)/2 - d/2 + 1), asin(((x
 + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i
)/2 - 3/2))*(d - (d - 2)*(d/2 - (d^2 - 4*d - 8)^(1/2)/2) + 4))/((x^3 - x*(
((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/2 + 1/2))^(1/2)*(d^2 - 4*d - 8)^(1/2)*((d^2 - 4*d - 8)^(1/
2)/2 - d/2 + 1)) - (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/(
(3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(
1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3^(1/2)*
1i)/2 + 3/2)/(d/2 + (d^2 - 4*d - 8)^(1/2)/2 - 1), asin(((x + 1)/((3^(1/2)*
1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d -
 (d - 2)*(d/2 + (d^2 - 4*d - 8)^(1/2)/2) + 4))/((x^3 - x*(((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1...
```

3.202. $\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx$

3.203 $\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx$

3.203.1 Optimal result

Integrand size = 35, antiderivative size = 38

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx=-\frac{2\arctan\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{1-d}}$$

output
```
-2*arctan((1-x)*(1-d)^(1/2)/(-x^3+1)^(1/2))/(1-d)^(1/2)
```

3.203.2 Mathematica [A] (verified)

Time = 1.98 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx=-\frac{2\text{arctanh}\left(\frac{\sqrt{-1+d}\sqrt{1-x^3}}{1+x+x^2}\right)}{\sqrt{-1+d}}$$

input
```
Integrate[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[1 - x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[-1 + d]*Sqrt[1 - x^3])/(1 + x + x^2)])/Sqrt[-1 + d]
```

3.203.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {2570, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x^2+2x+2}{\sqrt{1-x^3}\,(dx-d+x^2+2)}\,dx$$

$$\Big\downarrow \, 2570$$

$$4\int \frac{1}{-\frac{2(1-d)(1-x)^2}{1-x^3}-2}d\frac{1-x}{\sqrt{1-x^3}}$$

$$\Big\downarrow \, 217$$

$$-\frac{2\arctan\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{1-d}}$$

input
```
Int[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[1 - x^3]),x]
```

output
```
(-2*ArcTan[(Sqrt[1 - d]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[1 - d]
```

3.203.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 2570
```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[-2*g*h   Subst[Int[1/(2*e*h -
 (b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Fre
eQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8
*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

3.203.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 1.84 (sec) , antiderivative size = 1908, normalized size of antiderivative = 50.21

method	result	size
default	Expression too large to display	1908
elliptic	Expression too large to display	1919

input
```
int((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/
2+1/2*I*3^(1/2)))^(1/2))+1/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*
I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/
2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/
2)+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(
1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))*d^2-1/3*I*3^(1/2)*(I*3^(1/2)
*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2
)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*
I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/
2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4
*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))*d+4/3*I/(d^2+4*d-8)^(
1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))
*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-
x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1
/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3
^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2)
)*d-2/3*I*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*...
```

3.203.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 191, normalized size of antiderivative = 5.03

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx = \left[\frac{\log\left(-\frac{2\,(3\,d-4)x^3-x^4-(d^2-2\,d+4)x^2-4\,\sqrt{-x^3+1}\,((d-2)x-x^2-d)\,\sqrt{d-1}-d^2+2\,(d^2-2\,d)x-4\,d+4}{2\,dx^3+x^4+(d^2-2\,d+4)x^2+d^2-2\,(d^2-2\,d)x-4\,d+4}\right)}{2\,\sqrt{d-1}}, -\frac{\sqrt{-d+1}\arctan\left(-\frac{\sqrt{-x^3+1}\,((d-2)x-x^2-d)\,\sqrt{-d+1}}{2\,((d-1)x^3-d+1)}\right)}{d-1}\right]$$

input
```
integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-(2*(3*d - 4)*x^3 - x^4 - (d^2 - 2*d + 4)*x^2 - 4*sqrt(-x^3 + 1)*
((d - 2)*x - x^2 - d)*sqrt(d - 1) - d^2 + 2*(d^2 - 2*d)*x - 4*d + 4)/(2*d*
x^3 + x^4 + (d^2 - 2*d + 4)*x^2 + d^2 - 2*(d^2 - 2*d)*x - 4*d + 4))/sqrt(d
 - 1), -sqrt(-d + 1)*arctan(-1/2*sqrt(-x^3 + 1)*((d - 2)*x - x^2 - d)*sqrt
(-d + 1)/((d - 1)*x^3 - d + 1))/(d - 1)]
```

3.203.6 Sympy [F]

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx = -\int \left(-\frac{2x}{dx\sqrt{1-x^3}-d\sqrt{1-x^3}+x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx - \int \frac{x^2}{dx\sqrt{1-x^3}-d\sqrt{1-x^3}+x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\,dx - \int \left(-\frac{2}{dx\sqrt{1-x^3}-d\sqrt{1-x^3}+x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx$$

input
```
integrate((-x**2+2*x+2)/(d*x+x**2-d+2)/(-x**3+1)**(1/2),x)
```

output
```
-Integral(-2*x/(d*x*sqrt(1 - x**3) - d*sqrt(1 - x**3) + x**2*sqrt(1 - x**3
) + 2*sqrt(1 - x**3)), x) - Integral(x**2/(d*x*sqrt(1 - x**3) - d*sqrt(1 -
 x**3) + x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(-2/(d*x*sq
rt(1 - x**3) - d*sqrt(1 - x**3) + x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)),
 x)
```

3.203.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d^2-4*(2-d)>0)', see `assume?` f
or more de
```

3.203.8 Giac [F]

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx = \int -\frac{x^2-2\,x-2}{\sqrt{-x^3+1}(dx+x^2-d+2)}\,dx$$

input
```
integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(d*x + x^2 - d + 2)), x)
```

3.203.9 Mupad [B] (verification not implemented)

Time = 18.72 (sec) , antiderivative size = 677, normalized size of antiderivative = 17.82

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx = \text{Too large to display}$$

input
```
int((2*x - x^2 + 2)/((1 - x^3)^(1/2)*(d*x - d + x^2 + 2)),x)
```

output
```
(2*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3
^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3
/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1
)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
- 3/2)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) -
 x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) + (2*
((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1
/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2
 + 3/2)/(d/2 - (4*d + d^2 - 8)^(1/2)/2 + 1), asin((-(x - 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d + (d
+ 2)*(d/2 - (4*d + d^2 - 8)^(1/2)/2) - 4))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)
/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)
/2 + 1/2) + 1) + x^3)^(1/2)*(d/2 - (4*d + d^2 - 8)^(1/2)/2 + 1)*(4*d + d^2
 - 8)^(1/2)) - (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1
i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*elliptic
Pi(((3^(1/2)*1i)/2 + 3/2)/(d/2 + (4*d + d^2 - 8)^(1/2)/2 + 1), asin((-(x -
 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/
2 - 3/2))*(d + (d + 2)*(d/2 + (4*d + d^2 - 8)^(1/2)/2) - 4))/((1 - x^3)...
```

3.203. $\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}}\,dx$

3.204 $\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}} dx$

3.204.1 Optimal result

Integrand size = 33, antiderivative size = 36

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}} dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{1-d}}$$

output
```
-2*arctanh((1-x)*(1-d)^(1/2)/(x^3-1)^(1/2))/(1-d)^(1/2)
```

3.204.2 Mathematica [A] (verified)

Time = 1.92 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}} dx = \frac{2\arctan\left(\frac{\sqrt{-1+d}\sqrt{-1+x^3}}{1+x+x^2}\right)}{\sqrt{-1+d}}$$

input
```
Integrate[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[-1 + x^3]),x]
```

output
```
(2*ArcTan[(Sqrt[-1 + d]*Sqrt[-1 + x^3])/(1 + x + x^2)])/Sqrt[-1 + d]
```

3.204.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2570, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x^2+2x+2}{\sqrt{x^3-1}\,(dx-d+x^2+2)}dx$$

$$\Big\downarrow \; 2570$$

$$4\int \frac{1}{\frac{2(1-d)(1-x)^2}{x^3-1}-2}d\frac{1-x}{\sqrt{x^3-1}}$$

$$\Big\downarrow \; 220$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{1-d}}$$

input
```
Int[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[-1 + x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[1 - d]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[1 - d]
```

3.204.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 2570
```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[-2*g*h   Subst[Int[1/(2*e*h -
 (b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Fre
eQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8
*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

3.204.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 1.69 (sec) , antiderivative size = 4437, normalized size of antiderivative = 123.25

method	result	size
default	Expression too large to display	4437
elliptic	Expression too large to display	4646

input
```
int((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/
2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+3/2/(d^2+4*d-8)^(1/2)*(1/(-3/
2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+
1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1
/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))
^(1/2)/(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((x-1)/(-3
/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2
)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d^2+4*I/(d^2+4*d-8)^(1
/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*
3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2
)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2
))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi
(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d+1/2*(d^2+
4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-3/2
*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(
1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(
1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*
3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi...
```

3.204.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 187, normalized size of antiderivative = 5.19

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx$$
$$=\left[-\frac{\sqrt{-d+1}\log\left(-\frac{2\,(3\,d-4)x^3-x^4-(d^2-2\,d+4)x^2-d^2+4\,\sqrt{x^3-1}((d-2)x-x^2-d)\sqrt{-d+1}+2\,(d^2-2\,d)x-4\,d+4}{2\,dx^3+x^4+(d^2-2\,d+4)x^2+d^2-2\,(d^2-2\,d)x-4\,d+4}\right)}{2\,(d-1)},\right.$$
$$\left.-\frac{\arctan\left(-\frac{\sqrt{x^3-1}((d-2)x-x^2-d)\sqrt{d-1}}{2\,((d-1)x^3-d+1)}\right)}{\sqrt{d-1}}\right]$$

input
```
integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-d + 1)*log(-(2*(3*d - 4)*x^3 - x^4 - (d^2 - 2*d + 4)*x^2 - d^2
 + 4*sqrt(x^3 - 1)*((d - 2)*x - x^2 - d)*sqrt(-d + 1) + 2*(d^2 - 2*d)*x -
4*d + 4)/(2*d*x^3 + x^4 + (d^2 - 2*d + 4)*x^2 + d^2 - 2*(d^2 - 2*d)*x - 4*
d + 4))/(d - 1), -arctan(-1/2*sqrt(x^3 - 1)*((d - 2)*x - x^2 - d)*sqrt(d -
 1)/((d - 1)*x^3 - d + 1))/sqrt(d - 1)]
```

3.204.6 Sympy [F]

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx$$
$$=-\int\left(-\frac{2x}{dx\sqrt{x^3-1}-d\sqrt{x^3-1}+x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx$$
$$-\int\frac{x^2}{dx\sqrt{x^3-1}-d\sqrt{x^3-1}+x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\,dx$$
$$-\int\left(-\frac{2}{dx\sqrt{x^3-1}-d\sqrt{x^3-1}+x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx$$

input
```
integrate((-x**2+2*x+2)/(d*x+x**2-d+2)/(x**3-1)**(1/2),x)
```

output
```
-Integral(-2*x/(d*x*sqrt(x**3 - 1) - d*sqrt(x**3 - 1) + x**2*sqrt(x**3 - 1
) + 2*sqrt(x**3 - 1)), x) - Integral(x**2/(d*x*sqrt(x**3 - 1) - d*sqrt(x**
3 - 1) + x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(-2/(d*x*sq
rt(x**3 - 1) - d*sqrt(x**3 - 1) + x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)),
 x)
```

3.204.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d^2-4*(2-d)>0)', see `assume?` f
or more de
```

3.204.8 Giac [F]

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx = \int -\frac{x^2-2x-2}{\sqrt{x^3-1}(dx+x^2-d+2)}\,dx$$

input
```
integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(d*x + x^2 - d + 2)), x)
```

3.204.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 629, normalized size of antiderivative = 17.47

$$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx = \text{Too large to display}$$

input
```
int((2*x - x^2 + 2)/((x^3 - 1)^(1/2)*(d*x - d + x^2 + 2)),x)
```

output
```
(2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3
/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x
- 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1
/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2) + (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(
1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2
)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*el
lipticPi(((3^(1/2)*1i)/2 + 3/2)/(d/2 - (4*d + d^2 - 8)^(1/2)/2 + 1), asin(
(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2
)*1i)/2 - 3/2))*(d + (d + 2)*(d/2 - (4*d + d^2 - 8)^(1/2)/2) - 4))/((((3^(
1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(
1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)*(d/2 - (4*d + d^2 - 8)^(1/2)/2 + 1)*(4
*d + d^2 - 8)^(1/2)) - (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1
/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i
)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(
1/2)*1i)/2 + 3/2)/(d/2 + (4*d + d^2 - 8)^(1/2)/2 + 1), asin((-(x - 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)
)*(d + (d + 2)*(d/2 + (4*d + d^2 - 8)^(1/2)/2) - 4))/((((3^(1/2)*1i)/2 - 1
/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 ...
```

3.204. $\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx$

3.205 $\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx$

3.205.1 Optimal result

Integrand size = 33, antiderivative size = 32

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{1+d}(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{1+d}}$$

output
```
2*arctanh((1+x)*(1+d)^(1/2)/(-x^3-1)^(1/2))/(1+d)^(1/2)
```

3.205.2 Mathematica [A] (verified)

Time = 1.92 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx=-\frac{2\text{arctanh}\left(\frac{\sqrt{1+d}\sqrt{-1-x^3}}{1-x+x^2}\right)}{\sqrt{1+d}}$$

input
```
Integrate[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[-1 - x^3]),x]
```

output
```
(-2*ArcTanh[(Sqrt[1 + d]*Sqrt[-1 - x^3])/(1 - x + x^2)])/Sqrt[1 + d]
```

3.205.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2570, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x^2-2x+2}{\sqrt{-x^3-1}\,(dx+d+x^2+2)}dx$$

$$\Big\downarrow \text{ 2570}$$

$$-4\int \frac{1}{\frac{2(d+1)(x+1)^2}{-x^3-1}-2}d\frac{x+1}{\sqrt{-x^3-1}}$$

$$\Big\downarrow \text{ 220}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{d+1}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{d+1}}$$

input
```
Int[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[-1 - x^3]),x]
```

output
```
(2*ArcTanh[(Sqrt[1 + d]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[1 + d]
```

3.205.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 2570
```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Simp[-2*g*h   Subst[Int[1/(2*e*h -
 (b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Fre
eQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8
*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

3.205.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 1.77 (sec) , antiderivative size = 1888, normalized size of antiderivative = 59.00

method	result	size
default	Expression too large to display	1888
elliptic	Expression too large to display	1897

input
```
int((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))+1/3*I/(d^2-4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*
3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(
-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/
2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))
*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2)),
(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d^2-1/3*I*3^(1/2)*(I*3^(1/2)*x-1/2*
I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)
*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+
1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2
))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2)
),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d-4/3*I/(d^2-4*d-8)^(1/2)*3^(1/2)
*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2
*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(
1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(
x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2
*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d+2/3*I*3^(1/2)
*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+...
```

3.205.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(26) = 52$.

Time = 0.30 (sec) , antiderivative size = 185, normalized size of antiderivative = 5.78

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx$$
$$= \left[\frac{\log\left(-\frac{2\,(3\,d+4)x^3-x^4-(d^2+2\,d+4)x^2-4\,\sqrt{-x^3-1}\,((d+2)x-x^2+d)\,\sqrt{d+1}-d^2-2\,(d^2+2\,d)x+4\,d+4}{2\,dx^3+x^4+(d^2+2\,d+4)x^2+d^2+2\,(d^2+2\,d)x+4\,d+4}\right)}{2\,\sqrt{d+1}},\right.$$
$$\left.-\frac{\sqrt{-d-1}\arctan\left(-\frac{\sqrt{-x^3-1}\,((d+2)x-x^2+d)\,\sqrt{-d-1}}{2\,((d+1)x^3+d+1)}\right)}{d+1}\right]$$

input
```
integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-(2*(3*d + 4)*x^3 - x^4 - (d^2 + 2*d + 4)*x^2 - 4*sqrt(-x^3 - 1)*
((d + 2)*x - x^2 + d)*sqrt(d + 1) - d^2 - 2*(d^2 + 2*d)*x + 4*d + 4)/(2*d*
x^3 + x^4 + (d^2 + 2*d + 4)*x^2 + d^2 + 2*(d^2 + 2*d)*x + 4*d + 4))/sqrt(d
 + 1), -sqrt(-d - 1)*arctan(-1/2*sqrt(-x^3 - 1)*((d + 2)*x - x^2 + d)*sqrt
(-d - 1)/((d + 1)*x^3 + d + 1))/(d + 1)]
```

3.205.6 Sympy [F]

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx$$
$$= -\int \frac{2x}{dx\sqrt{-x^3-1}+d\sqrt{-x^3-1}+x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx$$
$$-\int \frac{x^2}{dx\sqrt{-x^3-1}+d\sqrt{-x^3-1}+x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx$$
$$-\int \left(-\frac{2}{dx\sqrt{-x^3-1}+d\sqrt{-x^3-1}+x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\right)dx$$

input
```
integrate((-x**2-2*x+2)/(d*x+x**2+d+2)/(-x**3-1)**(1/2),x)
```

output
```
-Integral(2*x/(d*x*sqrt(-x**3 - 1) + d*sqrt(-x**3 - 1) + x**2*sqrt(-x**3 -
 1) + 2*sqrt(-x**3 - 1)), x) - Integral(x**2/(d*x*sqrt(-x**3 - 1) + d*sqrt
(-x**3 - 1) + x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x) - Integral(-2/
(d*x*sqrt(-x**3 - 1) + d*sqrt(-x**3 - 1) + x**2*sqrt(-x**3 - 1) + 2*sqrt(-
x**3 - 1)), x)
```

3.205.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d^2-4*(d+2)>0)', see `assume?` f
or more de
```

3.205.8 Giac [F]

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx = \int -\frac{x^2+2\,x-2}{\sqrt{-x^3-1}(dx+x^2+d+2)}\,dx$$

input
```
integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(d*x + x^2 + d + 2)), x)
```

3.205.9 Mupad [B] (verification not implemented)

Time = 19.05 (sec) , antiderivative size = 680, normalized size of antiderivative = 21.25

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx = \text{Too large to display}$$

input
```
int(-(2*x + x^2 - 2)/((- x^3 - 1)^(1/2)*(d + d*x + x^2 + 2)),x)
```

output
```
- (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((
3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1
/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/
2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (2
*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1
/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*
1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2
+ 3/2)/((d^2 - 4*d - 8)^(1/2)/2 - d/2 + 1), asin(((x + 1)/((3^(1/2)*1i)/2
+ 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d - (d -
2)*(d/2 - (d^2 - 4*d - 8)^(1/2)/2) + 4))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((
3^(1/2)*1i)/2 + 1/2))^(1/2)*(d^2 - 4*d - 8)^(1/2)*((d^2 - 4*d - 8)^(1/2)/2
 - d/2 + 1)) - (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i
)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi
(-((3^(1/2)*1i)/2 + 3/2)/(d/2 + (d^2 - 4*d - 8)^(1/2)/2 - 1), asin(((x + 1
)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
- 3/2))*(d - (d - 2)*(d/2 + (d^2 - 4*d - 8)^(1/2)/2) + 4))/((- x^3 - 1)...
```

3.206 $\int (d+ex)^3\sqrt{a+cx^4}\,dx$

3.206.1 Optimal result

Integrand size = 19, antiderivative size = 355

$$\begin{aligned}\int (d+ex)^3\sqrt{a+cx^4}\,dx &= \frac{3}{4}d^2ex^2\sqrt{a+cx^4}+\frac{6ade^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{1}{15}dx\left(5d^2+9e^2x^2\right)\sqrt{a+cx^4}+\frac{e^3\left(a+cx^4\right)^{3/2}}{6c}\\
&+\frac{3ad^2e\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}}-\frac{6a^{5/4}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}}\\
&+\frac{a^{3/4}d\left(5\sqrt{c}d^2+9\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}\end{aligned}$$

output
```
1/6*e^3*(c*x^4+a)^(3/2)/c+3/4*a*d^2*e*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))
/c^(1/2)+3/4*d^2*e*x^2*(c*x^4+a)^(1/2)+1/15*d*x*(9*e^2*x^2+5*d^2)*(c*x^4+a
)^(1/2)+6/5*a*d*e^2*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-6/5*a^
(5/4)*d*e^2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4
)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(
1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4
+a)^(1/2)+1/15*a^(3/4)*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*
arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*
2^(1/2))*(9*e^2*a^(1/2)+5*d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a
^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)
```

3.206.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.52

$$\int (d+ex)^3\sqrt{a+cx^4}\,dx$$
$$= \frac{\sqrt{a+cx^4}\left(2ae^3\sqrt{1+\frac{cx^4}{a}}+9cd^2ex^2\sqrt{1+\frac{cx^4}{a}}+2ce^3x^4\sqrt{1+\frac{cx^4}{a}}+9\sqrt{a}\sqrt{c}d^2e\,\text{arcsinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)+12cd^3x\,\text{Hy}\right.}{12c\sqrt{1+\frac{cx^4}{a}}}$$

input
```
Integrate[(d + e*x)^3*Sqrt[a + c*x^4],x]
```

output
```
(Sqrt[a + c*x^4]*(2*a*e^3*Sqrt[1 + (c*x^4)/a] + 9*c*d^2*e*x^2*Sqrt[1 + (c*
x^4)/a] + 2*c*e^3*x^4*Sqrt[1 + (c*x^4)/a] + 9*Sqrt[a]*Sqrt[c]*d^2*e*ArcSin
h[(Sqrt[c]*x^2)/Sqrt[a]] + 12*c*d^3*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -(
(c*x^4)/a)] + 12*c*d*e^2*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^4)/a
)]))/(12*c*Sqrt[1 + (c*x^4)/a])
```

3.206.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+cx^4}(d+ex)^3\,dx$$
$$\Big\downarrow \text{ 2424}$$
$$\int \left(\sqrt{a+cx^4}\left(d^3+3de^2x^2\right)+x\sqrt{a+cx^4}\left(3d^2e+e^3x^2\right)\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{a^{3/4}d(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(9\sqrt{a}e^2+5\sqrt{c}d^2)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}-\frac{6a^{5/4}de^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}}+\frac{3ad^2e\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}}+\frac{1}{15}dx\sqrt{a+cx^4}(5d^2+9e^2x^2)+\frac{3}{4}d^2ex^2\sqrt{a+cx^4}+\frac{6ade^2x\sqrt{a+cx^4}}{5\sqrt{c}(\sqrt{a}+\sqrt{c}x^2)}+\frac{e^3(a+cx^4)^{3/2}}{6c}$$

input
```
Int[(d + e*x)^3*Sqrt[a + c*x^4],x]
```

output
```
(3*d^2*e*x^2*Sqrt[a + c*x^4])/4 + (6*a*d*e^2*x*Sqrt[a + c*x^4])/(5*Sqrt[c]
*(Sqrt[a] + Sqrt[c]*x^2)) + (d*x*(5*d^2 + 9*e^2*x^2)*Sqrt[a + c*x^4])/15 +
 (e^3*(a + c*x^4)^(3/2))/(6*c) + (3*a*d^2*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a +
 c*x^4]])/(4*Sqrt[c]) - (6*a^(5/4)*d*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
 c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)]
, 1/2])/(5*c^(3/4)*Sqrt[a + c*x^4]) + (a^(3/4)*d*(5*Sqrt[c]*d^2 + 9*Sqrt[a
]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(15*c^(3/4)*Sqrt[a + c*x^4
])
```

3.206.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.206.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.73 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.74

method	result
risch	$\frac{(10e^3x^4c+36d\,e^2x^3c+45c\,d^2e\,x^2+20d^3cx+10a\,e^3)\sqrt{c\,x^4+a}}{60c}+\frac{da\left(\frac{20d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}+\frac{36ie^2\sqrt{a}\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\ldots}{\ldots}\right)}{3\ldots}$
default	$d^3\left(\frac{x\sqrt{c\,x^4+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}\right)+\frac{e^3(c\,x^4+a)^{\frac{3}{2}}}{6c}+3d\,e^2\left(\frac{x^3\sqrt{c\,x^4+a}}{5}+\frac{2ia^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\ldots}{\ldots}\right.$
elliptic	$\frac{e^3x^4\sqrt{c\,x^4+a}}{6}+\frac{3d\,e^2x^3\sqrt{c\,x^4+a}}{5}+\frac{3d^2e\,x^2\sqrt{c\,x^4+a}}{4}+\frac{d^3x\sqrt{c\,x^4+a}}{3}+\frac{e^3a\sqrt{c\,x^4+a}}{6c}+\frac{2a\,d^3\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\ldots}\right.}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}$

input
```
int((e*x+d)^3*(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/60*(10*c*e^3*x^4+36*c*d*e^2*x^3+45*c*d^2*e*x^2+20*c*d^3*x+10*a*e^3)/c*(c
*x^4+a)^(1/2)+1/30*d*a*(20*d^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1
/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x
*(I/a^(1/2)*c^(1/2))^(1/2),I)+36*I*e^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(
1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(
1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1
/2)*c^(1/2))^(1/2),I))+45/2*e*d*ln(x^2*c^(1/2)+(c*x^4+a)^(1/2))/c^(1/2))
```

3.206.5 Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.53

$$\int (d+ex)^3\sqrt{a+cx^4}\,dx$$
$$=\frac{144\,a\sqrt{c}de^2x\left(-\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1)+45\,a\sqrt{c}d^2ex\log\left(-2\,cx^4-2\sqrt{cx^4+a}\sqrt{c}x^2-a\right)+16\,(5\,c\ldots}{\ldots}$$

input
```
integrate((e*x+d)^3*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/120*(144*a*sqrt(c)*d*e^2*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x
), -1) + 45*a*sqrt(c)*d^2*e*x*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2
 - a) + 16*(5*c*d^3 - 9*a*d*e^2)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin(
(-a/c)^(1/4)/x), -1) + 2*(10*c*e^3*x^5 + 36*c*d*e^2*x^4 + 45*c*d^2*e*x^3 +
 20*c*d^3*x^2 + 10*a*e^3*x + 72*a*d*e^2)*sqrt(c*x^4 + a))/(c*x)
```

3.206.6 Sympy [A] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.49

$$\int (d+ex)^3\sqrt{a+cx^4}\,dx = \frac{\sqrt{a}d^3x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{3\sqrt{a}d^2ex^2\sqrt{1+\frac{cx^4}{a}}}{4} + \frac{3\sqrt{a}de^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{3ad^2e\operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{c}} + e^3\left(\begin{cases}\frac{\sqrt{a}x^4}{4} & \text{for } c=0\\ \frac{(a+cx^4)^{\frac{3}{2}}}{6c} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((e*x+d)**3*(c*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*d**3*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi
)/a)/(4*gamma(5/4)) + 3*sqrt(a)*d**2*e*x**2*sqrt(1 + c*x**4/a)/4 + 3*sqrt(
a)*d*e**2*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi
)/a)/(4*gamma(7/4)) + 3*a*d**2*e*asinh(sqrt(c)*x**2/sqrt(a))/(4*sqrt(c)) +
 e**3*Piecewise((sqrt(a)*x**4/4, Eq(c, 0)), ((a + c*x**4)**(3/2)/(6*c), Tr
ue))
```

3.206.7 Maxima [F]

$$\int (d+ex)^3\sqrt{a+cx^4}\,dx = \int \sqrt{cx^4+a}(ex+d)^3\,dx$$

input
```
integrate((e*x+d)^3*(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + a)*(e*x + d)^3, x)
```

3.206.8 Giac [F]

$$\int (d+ex)^3\sqrt{a+cx^4}\,dx = \int \sqrt{cx^4+a}(ex+d)^3\,dx$$

input
```
integrate((e*x+d)^3*(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + a)*(e*x + d)^3, x)
```

3.206.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3\sqrt{a+cx^4}\,dx = \int \sqrt{c\,x^4+a}\,(d+e\,x)^3\,dx$$

input
```
int((a + c*x^4)^(1/2)*(d + e*x)^3,x)
```

output
```
int((a + c*x^4)^(1/2)*(d + e*x)^3, x)
```

3.207 $\int (d+ex)^2\sqrt{a+cx^4}\,dx$

3.207.1 Optimal result

Integrand size = 19, antiderivative size = 326

$$\begin{aligned}\int (d+ex)^2\sqrt{a+cx^4}\,dx &= \frac{1}{2}dex^2\sqrt{a+cx^4}+\frac{2ae^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{1}{15}x\left(5d^2+3e^2x^2\right)\sqrt{a+cx^4}\\&\quad+\frac{ade\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}-\frac{2a^{5/4}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}}\\&\quad+\frac{a^{3/4}\left(5\sqrt{c}d^2+3\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}\end{aligned}$$

output
```
1/2*a*d*e*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/c^(1/2)+1/2*d*e*x^2*(c*x^4+
a)^(1/2)+1/15*x*(3*e^2*x^2+5*d^2)*(c*x^4+a)^(1/2)+2/5*a*e^2*x*(c*x^4+a)^(1
/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-2/5*a^(5/4)*e^2*(cos(2*arctan(c^(1/4)*x/
a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan
(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2
)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)+1/15*a^(3/4)*(cos(2*arctan
(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(s
in(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(3*e^2*a^(1/2)+5*d^2*c^(1/2))
*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(
c*x^4+a)^(1/2)
```

3.207.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.16 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.45

$$\int (d+ex)^2\sqrt{a+cx^4}\,dx$$
$$=\frac{\sqrt{a+cx^4}\left(6\sqrt{c}d^2x\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{cx^4}{a}\right)+e\left(3d\left(\sqrt{c}x^2\sqrt{1+\frac{cx^4}{a}}+\sqrt{a}\text{arcsinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)\right)\right.\right.}{6\sqrt{c}\sqrt{1+\frac{cx^4}{a}}}$$

input
```
Integrate[(d + e*x)^2*Sqrt[a + c*x^4],x]
```

output
```
(Sqrt[a + c*x^4]*(6*Sqrt[c]*d^2*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x
^4)/a)] + e*(3*d*(Sqrt[c]*x^2*Sqrt[1 + (c*x^4)/a] + Sqrt[a]*ArcSinh[(Sqrt[
c]*x^2)/Sqrt[a]]) + 2*Sqrt[c]*e*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c
*x^4)/a)])))/(6*Sqrt[c]*Sqrt[1 + (c*x^4)/a])
```

3.207.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+cx^4}(d+ex)^2\,dx$$
$$\Big\downarrow \ 2424$$
$$\int \left(\sqrt{a+cx^4}(d^2+e^2x^2)+2dex\sqrt{a+cx^4}\right)dx$$
$$\Big\downarrow \ 2009$$

$$\frac{a^{3/4}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(3\sqrt{a}e^2+5\sqrt{c}d^2)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}} - \frac{2a^{5/4}e^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}} + \frac{ade\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}} + \frac{1}{15}x\sqrt{a+cx^4}(5d^2+3e^2x^2) + \frac{1}{2}dex^2\sqrt{a+cx^4} + \frac{2ae^2x\sqrt{a+cx^4}}{5\sqrt{c}\,(\sqrt{a}+\sqrt{c}x^2)}$$

input
```
Int[(d + e*x)^2*Sqrt[a + c*x^4],x]
```

output
```
(d*e*x^2*Sqrt[a + c*x^4])/2 + (2*a*e^2*x*Sqrt[a + c*x^4])/(5*Sqrt[c]*(Sqrt
[a] + Sqrt[c]*x^2)) + (x*(5*d^2 + 3*e^2*x^2)*Sqrt[a + c*x^4])/15 + (a*d*e*
ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt[c]) - (2*a^(5/4)*e^2*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2
*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(3/4)*Sqrt[a + c*x^4]) + (a^(3/4)
*(5*Sqrt[c]*d^2 + 3*Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/
(15*c^(3/4)*Sqrt[a + c*x^4])
```

3.207.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.207.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.50 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.72

method	result
risch	$\frac{x(6e^2x^2+15edx+10d^2)\sqrt{cx^4+a}}{30}+\frac{a\left(\frac{10d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{cx^4+a}}+\frac{6ie^2\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(\cdots\right)\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{cx^4+a}\,\sqrt{c}}\cdots\right)}{15}$
default	$d^2\left(\frac{x\sqrt{cx^4+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{cx^4+a}}\right)+e^2\left(\frac{x^3\sqrt{cx^4+a}}{5}+\frac{2ia^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\cdots}\right)\cdots\right)}{5\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{cx^4+a}\cdots}\cdots\right)$
elliptic	$\frac{e^2x^3\sqrt{cx^4+a}}{5}+\frac{dex^2\sqrt{cx^4+a}}{2}+\frac{d^2x\sqrt{cx^4+a}}{3}+\frac{2ad^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{cx^4+a}}+\frac{eda\ln\left(2x^2\sqrt{c}+2\sqrt{cx^4+a}\right)}{2\sqrt{c}}\cdots$

input
```
int((e*x+d)^2*(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/30*x*(6*e^2*x^2+15*d*e*x+10*d^2)*(c*x^4+a)^(1/2)+1/15*a*(10*d^2/(I/a^(1/
2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2
)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+6*I*e^2*a
^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1
/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(
1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+15/2*e*d*ln(x^2*c
^(1/2)+(c*x^4+a)^(1/2))/c^(1/2))
```

3.207.5 Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.51

$$\int (d+ex)^2\sqrt{a+cx^4}\,dx$$

$$=\frac{24\,a\sqrt{c}e^2x\left(-\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\mid -1)+15\,a\sqrt{c}dex\log\left(-2\,cx^4-2\,\sqrt{cx^4+a}\sqrt{c}x^2-a\right)+8\,(5\,cd^2-\cdots}{60\,cx}$$

input
```
integrate((e*x+d)^2*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/60*(24*a*sqrt(c)*e^2*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x), -
1) + 15*a*sqrt(c)*d*e*x*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a)
+ 8*(5*c*d^2 - 3*a*e^2)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1
/4)/x), -1) + 2*(6*c*e^2*x^4 + 15*c*d*e*x^3 + 10*c*d^2*x^2 + 12*a*e^2)*sqr
t(c*x^4 + a))/(c*x)
```

3.207.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.81 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.42

$$\int (d+ex)^2\sqrt{a+cx^4}\,dx = \frac{\sqrt{a}d^2x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{\sqrt{a}dex^2\sqrt{1+\frac{cx^4}{a}}}{2} + \frac{\sqrt{a}e^2x^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{ade\operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}}$$

input
```
integrate((e*x+d)**2*(c*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*d**2*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi
)/a)/(4*gamma(5/4)) + sqrt(a)*d*e*x**2*sqrt(1 + c*x**4/a)/2 + sqrt(a)*e**2
*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*g
amma(7/4)) + a*d*e*asinh(sqrt(c)*x**2/sqrt(a))/(2*sqrt(c))
```

3.207.7 Maxima [F]

$$\int (d+ex)^2\sqrt{a+cx^4}\,dx = \int \sqrt{cx^4+a}(ex+d)^2\,dx$$

input
```
integrate((e*x+d)^2*(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + a)*(e*x + d)^2, x)
```

3.207.8 Giac [F]

$$\int (d+ex)^2\sqrt{a+cx^4}\,dx=\int \sqrt{cx^4+a}(ex+d)^2\,dx$$

input
```
integrate((e*x+d)^2*(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + a)*(e*x + d)^2, x)
```

3.207.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2\sqrt{a+cx^4}\,dx=\int \sqrt{c\,x^4+a}\,(d+e\,x)^2\,dx$$

input
```
int((a + c*x^4)^(1/2)*(d + e*x)^2,x)
```

output
```
int((a + c*x^4)^(1/2)*(d + e*x)^2, x)
```

3.208 $\int (d+ex)\sqrt{a+cx^4}\,dx$

3.208.1 Optimal result

Integrand size = 17, antiderivative size = 158

$$\int (d+ex)\sqrt{a+cx^4}\,dx = \frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{4}ex^2\sqrt{a+cx^4}+\frac{ae\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}}+\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}$$

output

```
1/4*a*e*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/c^(1/2)+1/3*d*x*(c*x^4+a)^(1/
2)+1/4*e*x^2*(c*x^4+a)^(1/2)+1/3*a^(3/4)*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)
))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4
)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c
^(1/2))^2)^(1/2)/c^(1/4)/(c*x^4+a)^(1/2)
```

3.208.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.69

$$\int (d+ex)\sqrt{a+cx^4}\,dx$$
$$=\frac{\sqrt{a+cx^4}\left(\sqrt{c}ex^2\sqrt{1+\frac{cx^4}{a}}+\sqrt{a}e\operatorname{arcsinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)+4\sqrt{c}dx\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{cx^4}{a}\right)\right)}{4\sqrt{c}\sqrt{1+\frac{cx^4}{a}}}$$

input
```
Integrate[(d + e*x)*Sqrt[a + c*x^4],x]
```

output
```
(Sqrt[a + c*x^4]*(Sqrt[c]*e*x^2*Sqrt[1 + (c*x^4)/a] + Sqrt[a]*e*ArcSinh[(S
qrt[c]*x^2)/Sqrt[a]] + 4*Sqrt[c]*d*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((
c*x^4)/a)]))/(4*Sqrt[c]*Sqrt[1 + (c*x^4)/a])
```

3.208.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+cx^4}(d+ex)\,dx$$
$$\Big\downarrow \text{2424}$$
$$\int \left(d\sqrt{a+cx^4}+ex\sqrt{a+cx^4}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{ae\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}}+\frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{4}ex^2\sqrt{a+cx^4}$$

input
```
Int[(d + e*x)*Sqrt[a + c*x^4],x]
```

output
```
(d*x*Sqrt[a + c*x^4])/3 + (e*x^2*Sqrt[a + c*x^4])/4 + (a*e*ArcTanh[(Sqrt[c
]*x^2)/Sqrt[a + c*x^4]])/(4*Sqrt[c]) + (a^(3/4)*d*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], 1/2])/(3*c^(1/4)*Sqrt[a + c*x^4])
```

3.208.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.208.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.23 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{x(3ex+4d)\sqrt{c\,x^{4}+a}}{12}+\frac{2ad\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{ae\ln\left(x^{2}\sqrt{c}+\sqrt{c\,x^{4}+a}\right)}{4\sqrt{c}}$	119
default	$d\left(\frac{x\sqrt{c\,x^{4}+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}\right)+e\left(\frac{x^{2}\sqrt{c\,x^{4}+a}}{4}+\frac{a\ln\left(x^{2}\sqrt{c}+\sqrt{c\,x^{4}+a}\right)}{4\sqrt{c}}\right)$	129
elliptic	$\frac{e\,x^{2}\sqrt{c\,x^{4}+a}}{4}+\frac{dx\sqrt{c\,x^{4}+a}}{3}+\frac{2ad\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{ae\ln\left(2x^{2}\sqrt{c}+2\sqrt{c\,x^{4}+a}\right)}{4\sqrt{c}}$	130

input
```
int((e*x+d)*(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*x*(3*e*x+4*d)*(c*x^4+a)^(1/2)+2/3*a*d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/
a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)
*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/4*a*e*ln(x^2*c^(1/2)+(c*x^4+a)
^(1/2))/c^(1/2)
```

3.208.5 Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.59

$$\int (d+ex)\sqrt{a+cx^4}\,dx$$
$$= \frac{16\,c^{\frac{3}{2}}d\left(-\frac{a}{c}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+3\,a\sqrt{c}e\log\left(-2\,cx^4-2\,\sqrt{cx^4+a}\sqrt{c}x^2-a\right)+2\,\sqrt{cx^4+a}(3\,cex^2}{24\,c}$$

input
```
integrate((e*x+d)*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/24*(16*c^(3/2)*d*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) + 3
*a*sqrt(c)*e*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a) + 2*sqrt(c*
x^4 + a)*(3*c*e*x^2 + 4*c*d*x))/c
```

3.208.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.52 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.56

$$\int (d+ex)\sqrt{a+cx^4}\,dx = \frac{\sqrt{a}dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{a}ex^2\sqrt{1+\frac{cx^4}{a}}}{4}+\frac{ae\,\mathrm{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{c}}$$

input
```
integrate((e*x+d)*(c*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*d*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)/a
)/(4*gamma(5/4)) + sqrt(a)*e*x**2*sqrt(1 + c*x**4/a)/4 + a*e*asinh(sqrt(c)
*x**2/sqrt(a))/(4*sqrt(c))
```

3.208.7 Maxima [F]

$$\int (d+ex)\sqrt{a+cx^4}\,dx = \int \sqrt{cx^4+a}(ex+d)\,dx$$

input
```
integrate((e*x+d)*(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + a)*(e*x + d), x)
```

3.208.8 Giac [F]

$$\int (d+ex)\sqrt{a+cx^4}\,dx = \int \sqrt{cx^4+a}(ex+d)\,dx$$

input
```
integrate((e*x+d)*(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + a)*(e*x + d), x)
```

3.208.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)\sqrt{a+cx^4}\,dx = \int \sqrt{c\,x^4+a}\,(d+e\,x)\,dx$$

input
```
int((a + c*x^4)^(1/2)*(d + e*x),x)
```

output
```
int((a + c*x^4)^(1/2)*(d + e*x), x)
```

3.209 $\int \sqrt{a+cx^4}\,dx$

3.209.1 Optimal result

Integrand size = 11, antiderivative size = 105

$$\int \sqrt{a+cx^4}\,dx=\frac{1}{3}x\sqrt{a+cx^4}+\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}$$

output
```
1/3*x*(c*x^4+a)^(1/2)+1/3*a^(3/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/
2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/
4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2
)^(1/2)/c^(1/4)/(c*x^4+a)^(1/2)
```

3.209.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.85

$$\int \sqrt{a+cx^4}\,dx=\frac{x(a+cx^4)-\frac{2ia\sqrt{1+\frac{cx^4}{a}}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right),-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}}{3\sqrt{a+cx^4}}$$

input
```
Integrate[Sqrt[a + c*x^4],x]
```

output
```
(x*(a + c*x^4) - ((2*I)*a*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*
Sqrt[c])/Sqrt[a]]*x], -1])/Sqrt[(I*Sqrt[c])/Sqrt[a]])/(3*Sqrt[a + c*x^4])
```

3.209.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {748, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+cx^4}\,dx$$

$$\Big\downarrow \text{ 748}$$

$$\frac{2}{3}a\int \frac{1}{\sqrt{cx^4+a}}dx+\frac{1}{3}x\sqrt{a+cx^4}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{1}{3}x\sqrt{a+cx^4}$$

input
```
Int[Sqrt[a + c*x^4],x]
```

output
```
(x*Sqrt[a + c*x^4])/3 + (a^(3/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/
(3*c^(1/4)*Sqrt[a + c*x^4])
```

3.209.3.1 Defintions of rubi rules used

rule 748
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Simp[a*n*(p/(n*p + 1))   Int[(a + b*x^n)^(p - 1), x], x] /; Fre
eQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || LtQ[Denominat
or[p + 1/n], Denominator[p]])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.209.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{x\sqrt{c\,x^4+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	85
risch	$\frac{x\sqrt{c\,x^4+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	85
elliptic	$\frac{x\sqrt{c\,x^4+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	85

input
```
int((c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(c*x^4+a)^(1/2)+2/3*a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)
*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I
/a^(1/2)*c^(1/2))^(1/2),I)
```

3.209.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.39

$$\int \sqrt{a+cx^4}\,dx = \frac{2}{3}\sqrt{c}\left(-\frac{a}{c}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)|-1\right)+\frac{1}{3}\sqrt{cx^4+ax}$$

input
```
integrate((c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(c)*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) + 1/3*sqrt
(c*x^4 + a)*x
```

3.209.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.35

$$\int \sqrt{a+cx^4}\,dx = \frac{\sqrt{a}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((c*x**4+a)**(1/2),x)
```

output
```
sqrt(a)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)/a)/
(4*gamma(5/4))
```

3.209.7 Maxima [F]

$$\int \sqrt{a+cx^4}\,dx = \int \sqrt{cx^4+a}\,dx$$

input
```
integrate((c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + a), x)
```

3.209.8 Giac [F]

$$\int \sqrt{a+cx^4}\,dx = \int \sqrt{cx^4+a}\,dx$$

input
```
integrate((c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + a), x)
```

3.209.9 Mupad [B] (verification not implemented)

Time = 18.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.35

$$\int \sqrt{a+cx^4}\,dx = \frac{x\,\sqrt{c\,x^4+a}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{1}{4};\ \frac{5}{4};\ -\frac{c\,x^4}{a}\right)}{\sqrt{\frac{c\,x^4}{a}+1}}$$

input
```
int((a + c*x^4)^(1/2),x)
```

output
```
(x*(a + c*x^4)^(1/2)*hypergeom([-1/2, 1/4], 5/4, -(c*x^4)/a))/((c*x^4)/a +
 1)^(1/2)
```

3.210 $\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx$

3.210.1 Optimal result

Integrand size = 19, antiderivative size = 730

$$\begin{aligned}\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx &= \frac{\sqrt{a+cx^4}}{2e}-\frac{\sqrt{c}dx\sqrt{a+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt{-cd^4-ae^4}\arctan\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2e^3}\\
&+\frac{\sqrt{c}d^2\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2e^3}-\frac{\sqrt{cd^4+ae^4}\operatorname{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{2e^3}\\
&+\frac{\sqrt[4]{a}\sqrt[4]{c}d(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{e^2\sqrt{a+cx^4}}\\
&-\frac{\sqrt[4]{a}\sqrt[4]{c}d\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2e^4\sqrt{a+cx^4}}\\
&+\frac{\sqrt[4]{c}d(cd^4+ae^4)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}e^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}\\
&-\frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)(cd^4+ae^4)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}de^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}\end{aligned}$$

output
```
1/2*d^2*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))*c^(1/2)/e^3-1/2*arctan(x*(-a*
e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2))*(-a*e^4-c*d^4)^(1/2)/e^3-1/2*arctanh
((c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/2))*(a*e^4+c*d^4)^(1/2
)/e^3+1/2*(c*x^4+a)^(1/2)/e-d*x*c^(1/2)*(c*x^4+a)^(1/2)/e^2/(a^(1/2)+x^2*c
^(1/2))+a^(1/4)*c^(1/4)*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2
*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2
*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/
e^2/(c*x^4+a)^(1/2)+1/2*c^(1/4)*d*(a*e^4+c*d^4)*(cos(2*arctan(c^(1/4)*x/a^
(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c
^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+
x^2*c^(1/2))^2)^(1/2)/a^(1/4)/e^4/(e^2*a^(1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2
)-1/4*(a*e^4+c*d^4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/4*(e^2
*a^(1/2)+d^2*c^(1/2))^2/d^2/e^2/a^(1/2)/c^(1/2),1/2*2^(1/2))*(-e^2*a^(1/2)
+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1
/2)/a^(1/4)/c^(1/4)/d/e^4/(e^2*a^(1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)-1/2*a^
(1/4)*c^(1/4)*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^
(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*
(a^(1/2)+x^2*c^(1/2))*(e^2+d^2*c^(1/2)/a^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^
(1/2))^2)^(1/2)/e^4/(c*x^4+a)^(1/2)
```

3.210.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.23 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx$$
$$= \frac{-2\sqrt{a}c^{3/4}d^2e^2\sqrt{1+\frac{cx^4}{a}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+2c^{3/4}d^2\left(i\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{1+\frac{cx^4}{a}}\operatorname{EllipticF}\left(i\operatorname{arcsi}\right.}{}$$

input
```
Integrate[Sqrt[a + c*x^4]/(d + e*x),x]
```

output
```
(-2*Sqrt[a]*c^(3/4)*d^2*e^2*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(
I*Sqrt[c])/Sqrt[a]]*x], -1] + 2*c^(3/4)*d^2*(I*Sqrt[c]*d^2 + Sqrt[a]*e^2)*
Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]
+ Sqrt[(I*Sqrt[c])/Sqrt[a]]*(-2*(-1)^(1/4)*a^(1/4)*(c*d^4 + a*e^4)*Sqrt[1
+ (c*x^4)/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*
c^(1/4)*x)/a^(1/4)], -1] + c^(1/4)*d*e*(e^2*(a + c*x^4) - 2*Sqrt[-(c*d^4)
- a*e^4]*Sqrt[a + c*x^4]*ArcTan[(Sqrt[c]*(d^2 - e^2*x^2) + e^2*Sqrt[a + c*
x^4])/Sqrt[-(c*d^4) - a*e^4]] - Sqrt[c]*d^2*Sqrt[a + c*x^4]*Log[-(Sqrt[c]*
x^2) + Sqrt[a + c*x^4]])))/(2*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^(1/4)*d*e^4*Sqrt
[a + c*x^4])
```

3.210.3 Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 660, normalized size of antiderivative = 0.90, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.842, Rules used = {2267, 1524, 27, 1512, 27, 761, 1510, 1577, 493, 25, 719, 224, 219, 488, 219, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^4}}{d+ex}dx$$

$$\Big\downarrow \text{2267}$$

$$d\int \frac{\sqrt{cx^4+a}}{d^2-e^2x^2}dx-e\int \frac{x\sqrt{cx^4+a}}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{1524}$$

$$d\left(\frac{\left(ae^4+cd^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\int \frac{\sqrt{c}\left(\sqrt{c}d^2-\sqrt{a}e^2+\sqrt{c}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)x^2\right)}{\sqrt{cx^4+a}}dx}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right)-e\int \frac{x\sqrt{cx^4+a}}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{27}$$

$$d\left(\frac{\left(ae^4+cd^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\sqrt{c}\int \frac{\sqrt{c}d^2-\sqrt{a}e^2+\sqrt{c}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)x^2}{\sqrt{cx^4+a}}dx}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right)-e\int \frac{x\sqrt{cx^4+a}}{d^2-e^2x^2}dx$$

$$\Big\downarrow \text{1512}$$

$$d\left(\frac{\left(ae^4+cd^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\sqrt{c}\left(2\sqrt{c}d^2\int \frac{1}{\sqrt{cx^4+a}}dx-\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx\right)}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right)-e\int \frac{x\sqrt{cx^4+a}}{d^2-e^2x^2}dx$$

$$\downarrow \text{27}$$

$$d\left(\frac{\left(ae^4+cd^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\sqrt{c}\left(2\sqrt{c}d^2\int \frac{1}{\sqrt{cx^4+a}}dx-\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{a}}\right)}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right)-e\int \frac{x\sqrt{cx^4+a}}{d^2-e^2x^2}dx$$

$$\downarrow \text{761}$$

$$d\left(\frac{\left(ae^4+cd^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\sqrt{c}\left(\frac{\sqrt[4]{c}d^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}}-\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\int}{\sqrt{a}}\right.}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right.$$

$$e\int \frac{x\sqrt{cx^4+a}}{d^2-e^2x^2}dx$$

$$\downarrow \text{1510}$$

3.210. $\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx$

$$d\left(\frac{(ae^4+cd^4)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\sqrt{c}\left(\frac{\sqrt[4]{c}d^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}}-\frac{(\sqrt{a}e^2+\sqrt{c}d^2)(\cdots}{\cdots}\right.}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\cdots}\right.$$

$$e\int \frac{x\sqrt{cx^4+a}}{d^2-e^2x^2}dx$$

$\big\downarrow$ 1577

$$d\left(\frac{(ae^4+cd^4)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\sqrt{c}\left(\frac{\sqrt[4]{c}d^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}}-\frac{(\sqrt{a}e^2+\sqrt{c}d^2)(\cdots}{\cdots}\right.}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\cdots}\right.$$

$$\frac{1}{2}e\int \frac{\sqrt{cx^4+a}}{d^2-e^2x^2}dx^2$$

$\big\downarrow$ 493

3.210. $\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx$

$$d\left(\frac{\left(ae^4+cd^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\sqrt{c}\left(\frac{\sqrt[4]{c}d^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}}-\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\left(\cdots\right.}{\cdots}\cdots\right.}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\cdots}\cdots\right.$$

$$\frac{1}{2}e\left(-\frac{\int -\frac{ae^2+cd^2x^2}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx^2}{e^2}-\frac{\sqrt{a+cx^4}}{e^2}\right)$$

$$\Big\downarrow \; 25$$

$$d\left(\frac{\left(ae^4+cd^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\sqrt{c}\left(\frac{\sqrt[4]{c}d^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}}-\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\left(\cdots\right.}{\cdots}\cdots\right.}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\cdots}\cdots\right.$$

$$\frac{1}{2}e\left(\frac{\int \frac{ae^2+cd^2x^2}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx^2}{e^2}-\frac{\sqrt{a+cx^4}}{e^2}\right)$$

$$\Big\downarrow \; 719$$

3.210. $\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx$

$$d\left(\frac{\left(ae^4+cd^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\sqrt{c}\left(\frac{\sqrt[4]{c}d^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}}-\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\left(\cdots\right.}{\cdots}\right.}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\cdots}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{\left(ae^4+cd^4\right)\int\frac{1}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx^2}{e^2}-\frac{cd^2\int\frac{1}{\sqrt{cx^4+a}}dx^2}{e^2}}{e^2}-\frac{\sqrt{a+cx^4}}{e^2}\right)$$

$$\Big\downarrow \; 224$$

$$d\left(\frac{\left(ae^4+cd^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\sqrt{c}\left(\frac{\sqrt[4]{c}d^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}}-\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\left(\cdots\right.}{\cdots}\right.}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\cdots}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{\left(ae^4+cd^4\right)\int\frac{1}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx^2}{e^2}-\frac{cd^2\int\frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+a}}}{e^2}}{e^2}-\frac{\sqrt{a+cx^4}}{e^2}\right)$$

$$\Big\downarrow \; 219$$

3.210. $\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx$

$$d\left(\frac{\left(ae^4+cd^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\sqrt{c}\left(\frac{\sqrt[4]{c}d^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}}-\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\left(\right.}{}\right.}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right.$$

$$\frac{1}{2}e\left(\frac{\frac{\left(ae^4+cd^4\right)\int\frac{1}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx^2}{e^2}-\frac{\sqrt{c}d^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{e^2}}{e^2}-\frac{\sqrt{a+cx^4}}{e^2}\right)$$

$\Big\downarrow$ 488

$$d\left(\frac{\left(ae^4+cd^4\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\sqrt{c}\left(\frac{\sqrt[4]{c}d^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}}-\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\left(\right.}{}\right.}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right.$$

$$\frac{1}{2}e\left(\frac{-\frac{\left(ae^4+cd^4\right)\int\frac{1}{cd^4+ae^4-x^4}d\frac{-ae^2-cd^2x^2}{\sqrt{cx^4+a}}}{e^2}-\frac{\sqrt{c}d^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{e^2}}{e^2}-\frac{\sqrt{a+cx^4}}{e^2}\right)$$

$\Big\downarrow$ 219

3.210. $\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx$

$$d\left(\frac{(ae^4+cd^4)\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}-\frac{\sqrt{c}\left(\frac{\sqrt[4]{c}d^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{a+cx^4}}-\frac{(\sqrt{a}e^2+\sqrt{c}d^2)\left(\right.}{}\right.}{e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right.$$

$$\frac{1}{2}e\left(\frac{-\frac{\sqrt{c}d^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{e^2}-\frac{\sqrt{ae^4+cd^4}\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{e^2}}{e^2}-\frac{\sqrt{a+cx^4}}{e^2}\right)$$

$$\downarrow 2223$$

$$d\left(\frac{(ae^4+cd^4)\left(\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)\text{EllipticPi}\left(\frac{(\sqrt{c}d^2+\sqrt{a}e^2)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{(\sqrt{a}e^2+\sqrt{c}d^2)\text{arctanh}\left(\frac{x\sqrt{}}{de}\right.}{2de\sqrt{ae^4+cd^4}}\right.}{\sqrt{a}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}\right.$$

$$\frac{1}{2}e\left(\frac{-\frac{\sqrt{c}d^2\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{e^2}-\frac{\sqrt{ae^4+cd^4}\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{e^2}}{e^2}-\frac{\sqrt{a+cx^4}}{e^2}\right)$$

input
```
Int[Sqrt[a + c*x^4]/(d + e*x),x]
```

3.210. $\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx$

output
```
-1/2*(e*(-(Sqrt[a + c*x^4]/e^2) + (-((Sqrt[c]*d^2*ArcTanh[(Sqrt[c]*x^2)/Sq
rt[a + c*x^4]])/e^2) - (Sqrt[c*d^4 + a*e^4]*ArcTanh[(-(a*e^2) - c*d^2*x^2)
/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])])/e^2)/e^2)) + d*(-((Sqrt[c]*(-(((S
qrt[c]*d^2 + Sqrt[a]*e^2)*(-((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2))
+ (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^
4])))/Sqrt[a]) + (c^(1/4)*d^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sq
rt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(a^
(1/4)*Sqrt[a + c*x^4])))/(e^2*((Sqrt[c]*d^2)/Sqrt[a] + e^2))) + ((c*d^4 +
a*e^4)*(((Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTanh[(Sqrt[c*d^4 + a*e^4]*x)/(d*e*
Sqrt[a + c*x^4])])/(2*d*e*Sqrt[c*d^4 + a*e^4]) + ((Sqrt[a]/d^2 - Sqrt[c]/e
^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*El
lipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcT
an[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4])))/(Sqrt
[a]*e^2*((Sqrt[c]*d^2)/Sqrt[a] + e^2))
```

3.210.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 1524
```
Int[Sqrt[(a_) + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2), x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(c*d^2 + a*e^2)/(e*(e - d*q))   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x] - Simp[1/(e*(e - d*q))   Int[(c*d + a*e*q - (
c*e - a*d*q^3)*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && N
eQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

3.210. $\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx$

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

rule 2267
```
Int[((a_) + (c_.)*(x_)^4)^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[d
Int[(a + c*x^4)^p/(d^2 - e^2*x^2), x], x] - Simp[e   Int[x*((a + c*x^4)^p/(
d^2 - e^2*x^2)), x], x] /; FreeQ[{a, c, d, e}, x] && IntegerQ[p + 1/2]
```

3.210.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.34 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.55

method	result
default	$\frac{\sqrt{c\,x^4+a}}{2e}-\frac{d^3 c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e^4\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{\sqrt{c}\,d^2\ln\left(2x^2\sqrt{c}+2\sqrt{c\,x^4+a}\right)}{2e^3}-\frac{i\sqrt{c}\,d\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(F\right.}{e^2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c}}$
elliptic	$\frac{\sqrt{c\,x^4+a}}{2e}-\frac{d^3 c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e^4\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{\sqrt{c}\,d^2\ln\left(2x^2\sqrt{c}+2\sqrt{c\,x^4+a}\right)}{2e^3}-\frac{i\sqrt{c}\,d\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(F\right.}{e^2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c}}$
risch	$\frac{\sqrt{c\,x^4+a}}{2e}-\frac{\left(-e^4 a-d^4 c\right)\left(-\frac{\operatorname{arctanh}\left(\frac{\frac{2c\,x^2 d^2}{e^2}+2a}{2\sqrt{\frac{c\,d^4}{e^4}+a}\,\sqrt{c\,x^4+a}}\right)}{2\sqrt{\frac{c\,d^4}{e^4}+a}}+\frac{e\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\Pi\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},-\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,d\sqrt{c\,x^4+a}}\right)}{e^4}+\frac{cd\left(\frac{d^2\sqrt{1-\frac{i\sqrt{c}}{}}}{}\right.}{e}$

input
```
int((c*x^4+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*(c*x^4+a)^(1/2)/e-d^3*c/e^4/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(
1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(
x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/2*c^(1/2)*d^2/e^3*ln(2*x^2*c^(1/2)+2*(c*x
^4+a)^(1/2))-I*c^(1/2)/e^2*d*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2
)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*(Elli
pticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2)
,I))+(a*e^4+c*d^4)/e^5*(-1/2/(c/e^4*d^4+a)^(1/2)*arctanh(1/2*(2*c*x^2/e^2*
d^2+2*a)/(c/e^4*d^4+a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)*
e/d*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4
+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)*e^2/d^
2,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

3.210.5 Fricas [F]

$$\int \frac{\sqrt{a + cx^4}}{d + ex} dx = \int \frac{\sqrt{cx^4 + a}}{ex + d} dx$$

input
```
integrate((c*x^4+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^4 + a)/(e*x + d), x)
```

3.210.6 Sympy [F]

$$\int \frac{\sqrt{a + cx^4}}{d + ex} dx = \int \frac{\sqrt{a + cx^4}}{d + ex} dx$$

input
```
integrate((c*x**4+a)**(1/2)/(e*x+d),x)
```

output
```
Integral(sqrt(a + c*x**4)/(d + e*x), x)
```

3.210.7 Maxima [F]

$$\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx = \int \frac{\sqrt{cx^4+a}}{ex+d}\,dx$$

input
```
integrate((c*x^4+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + a)/(e*x + d), x)
```

3.210.8 Giac [F]

$$\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx = \int \frac{\sqrt{cx^4+a}}{ex+d}\,dx$$

input
```
integrate((c*x^4+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + a)/(e*x + d), x)
```

3.210.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx = \int \frac{\sqrt{c\,x^4+a}}{d+e\,x}\,dx$$

input
```
int((a + c*x^4)^(1/2)/(d + e*x),x)
```

output
```
int((a + c*x^4)^(1/2)/(d + e*x), x)
```

3.211 $\int \frac{\sqrt{a+cx^4}}{(d+ex)^2} \, dx$

3.211.1 Optimal result

Integrand size = 19, antiderivative size = 1221

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^2} \, dx$$
$$= \frac{2\sqrt{c}x\sqrt{a+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{d\sqrt{a+cx^4}}{e\left(d^2-e^2x^2\right)} + \frac{x\sqrt{a+cx^4}}{d^2-e^2x^2} + \frac{\sqrt{-cd^4-ae^4}\arctan\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2de^3}$$
$$- \frac{\left(cd^4-ae^4\right)\arctan\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2de^3\sqrt{-cd^4-ae^4}} - \frac{\sqrt{c}d\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{e^3} + \frac{cd^3\text{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{e^3\sqrt{cd^4+ae^4}}$$
$$- \frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{e^2\sqrt{a+cx^4}}$$
$$+ \frac{3\sqrt[4]{a}\sqrt[4]{c}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4e^4\sqrt{a+cx^4}}$$
$$- \frac{\sqrt[4]{c}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}e^4\sqrt{a+cx^4}}$$
$$+ \frac{\sqrt[4]{c}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}e^4\sqrt{a+cx^4}}$$
$$- \frac{\sqrt[4]{c}\left(cd^4+ae^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}e^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}$$
$$+ \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}d^2e^4\sqrt{a+cx^4}}$$
$$+ \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(cd^4+ae^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}d^2e^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}$$

output
```
-d*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))*c^(1/2)/e^3-1/2*(-a*e^4+c*d^4)*arc
tan(x*(-a*e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2))/d/e^3/(-a*e^4-c*d^4)^(1/2)
+1/2*arctan(x*(-a*e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2))*(-a*e^4-c*d^4)^(1/
2)/d/e^3+c*d^3*arctanh((c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/
2))/e^3/(a*e^4+c*d^4)^(1/2)-d*(c*x^4+a)^(1/2)/e/(-e^2*x^2+d^2)+x*(c*x^4+a)
^(1/2)/(-e^2*x^2+d^2)+2*x*c^(1/2)*(c*x^4+a)^(1/2)/e^2/(a^(1/2)+x^2*c^(1/2)
)-2*a^(1/4)*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/
2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/e^2/(c
*x^4+a)^(1/2)-1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2
*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2
*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(
1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/e^4/(c*x^4+a)^(1/2)+1/4*(cos(2*arctan(c
^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(si
n(2*arctan(c^(1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2/d^2/e^2/a^(
1/2)/c^(1/2),1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))^2*(a^(1/2)+x^2*c^(1/2
))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d^2/e^4/(c*x^
4+a)^(1/2)-1/2*c^(1/4)*(a*e^4+c*d^4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^
(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^
(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1...
```

3.211.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.46 (sec) , antiderivative size = 394, normalized size of antiderivative = 0.32

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx$$

$$= \frac{-\frac{e^3(a+cx^4)}{d+ex} - \frac{2cd^3e\sqrt{a+cx^4}\arctan\left(\frac{\sqrt{c}\left(d^2-e^2x^2\right)+e^2\sqrt{a+cx^4}}{\sqrt{-cd^4-ae^4}}\right)}{\sqrt{-cd^4-ae^4}} - 2ia\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}e^2\sqrt{1+\frac{cx^4}{a}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right) - \cdots}{\cdots}$$

input
```
Integrate[Sqrt[a + c*x^4]/(d + e*x)^2,x]
```

output
```
(-((e^3*(a + c*x^4))/(d + e*x)) - (2*c*d^3*e*Sqrt[a + c*x^4]*ArcTan[(Sqrt[
c]*(d^2 - e^2*x^2) + e^2*Sqrt[a + c*x^4])/Sqrt[-(c*d^4) - a*e^4]])/Sqrt[-(
c*d^4) - a*e^4] - (2*I)*a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*e^2*Sqrt[1 + (c*x^4)/a
]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - (2*Sqrt[c]*(I*Sq
rt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*S
qrt[c])/Sqrt[a]]*x], -1])/Sqrt[(I*Sqrt[c])/Sqrt[a]] + 2*(-1)^(1/4)*a^(1/4)
*c^(3/4)*d^2*Sqrt[1 + (c*x^4)/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2),
 ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4)], -1] + Sqrt[c]*d*e*Sqrt[a + c*x^4]
*Log[-(Sqrt[c]*x^2) + Sqrt[a + c*x^4]])/(e^4*Sqrt[a + c*x^4])
```

3.211.3 Rubi [A] (verified)

Time = 2.77 (sec) , antiderivative size = 975, normalized size of antiderivative = 0.80, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {2584, 2255, 27, 1577, 492, 605, 224, 219, 488, 219, 2259, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}dx$$

$$\Big\downarrow \text{2584}$$

$$\int \frac{\sqrt{a+cx^4}\left(d^2-2dex+e^2x^2\right)}{\left(d^2-e^2x^2\right)^2}dx$$

$$\Big\downarrow \text{2255}$$

$$\int -\frac{2dex\sqrt{cx^4+a}}{\left(d^2-e^2x^2\right)^2}dx+\int \frac{\left(d^2+e^2x^2\right)\sqrt{cx^4+a}}{\left(d^2-e^2x^2\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\left(d^2+e^2x^2\right)\sqrt{cx^4+a}}{\left(d^2-e^2x^2\right)^2}dx-2de\int \frac{x\sqrt{cx^4+a}}{\left(d^2-e^2x^2\right)^2}dx$$

$$\Big\downarrow \text{1577}$$

$$\int \frac{\left(d^2+e^2x^2\right)\sqrt{cx^4+a}}{\left(d^2-e^2x^2\right)^2}dx-de\int \frac{\sqrt{cx^4+a}}{\left(d^2-e^2x^2\right)^2}dx^2$$

$$\Big\downarrow \text{492}$$

$$\int \frac{(d^2+e^2x^2)\sqrt{cx^4+a}}{(d^2-e^2x^2)^2}dx-de\left(\frac{\sqrt{a+cx^4}}{e^2(d^2-e^2x^2)}-\frac{c\int \frac{x^2}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx^2}{e^2}\right)$$

$$\Big\downarrow \text{605}$$

$$\int \frac{(d^2+e^2x^2)\sqrt{cx^4+a}}{(d^2-e^2x^2)^2}dx-de\left(\frac{\sqrt{a+cx^4}}{e^2(d^2-e^2x^2)}-\frac{c\left(\frac{d^2\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx^2}{e^2}-\frac{\int \frac{1}{\sqrt{cx^4+a}}dx^2}{e^2}\right)}{e^2}\right)$$

$$\Big\downarrow \text{224}$$

$$\int \frac{(d^2+e^2x^2)\sqrt{cx^4+a}}{(d^2-e^2x^2)^2}dx-de\left(\frac{\sqrt{a+cx^4}}{e^2(d^2-e^2x^2)}-\frac{c\left(\frac{d^2\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx^2}{e^2}-\frac{\int \frac{1}{1-cx^4}d\frac{x^2}{\sqrt{cx^4+a}}}{e^2}\right)}{e^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\int \frac{(d^2+e^2x^2)\sqrt{cx^4+a}}{(d^2-e^2x^2)^2}dx-$$
$$de\left(\frac{\sqrt{a+cx^4}}{e^2(d^2-e^2x^2)}-\frac{c\left(\frac{d^2\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx^2}{e^2}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}e^2}\right)}{e^2}\right)$$

$$\Big\downarrow \text{488}$$

$$\int \frac{(d^2+e^2x^2)\sqrt{cx^4+a}}{(d^2-e^2x^2)^2}dx-$$
$$de\left(\frac{\sqrt{a+cx^4}}{e^2(d^2-e^2x^2)}-\frac{c\left(-\frac{d^2\int \frac{1}{cd^4+ae^4-x^4}d\frac{-ae^2-cd^2x^2}{\sqrt{cx^4+a}}}{e^2}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}e^2}\right)}{e^2}\right)$$

$$\Big\downarrow \text{219}$$

3.211. $\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx$

$$\int \frac{(d^2+e^2x^2)\sqrt{cx^4+a}}{(d^2-e^2x^2)^2}dx - de\left(\frac{\sqrt{a+cx^4}}{e^2(d^2-e^2x^2)} - \frac{c\left(-\frac{d^2\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{e^2\sqrt{ae^4+cd^4}} - \frac{\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}e^2}\right)}{e^2}\right)$$

$$\Big\downarrow \text{2259}$$

$$\int \left(-\frac{4cd^4}{e^4(d^2-e^2x^2)\sqrt{cx^4+a}} + \frac{3cd^2}{e^4\sqrt{cx^4+a}} + \frac{cx^2}{e^2\sqrt{cx^4+a}} + \frac{cd^4+ae^4}{2e^4(ex-d)^2\sqrt{cx^4+a}} + \frac{cd^4+ae^4}{2e^4(d+ex)^2\sqrt{cx^4+a}}\right) - de\left(\frac{\sqrt{a+cx^4}}{e^2(d^2-e^2x^2)} - \frac{c\left(-\frac{d^2\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{e^2\sqrt{ae^4+cd^4}} - \frac{\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}e^2}\right)}{e^2}\right)$$

$$\Big\downarrow \text{2009}$$

3.211. $\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx$

$$-\frac{2c^{5/4}\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)d^4}{\sqrt[4]{a}e^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{cx^4+a}}-\frac{c\,\text{arctanh}\left(\frac{\sqrt{cd^4+ae^4}x}{de\sqrt{cx^4+a}}\right)d^3}{e^3\sqrt{cd^4+ae^4}}+\frac{3c^{3/4}\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)d^2}{2\sqrt[4]{a}e^4\sqrt{cx^4+a}}+\frac{c^{3/4}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)d^2}{2\sqrt[4]{a}e^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{cx^4+a}}-e\left(\frac{\sqrt{cx^4+a}}{e^2\left(d^2-e^2x^2\right)}-\frac{c\left(-\frac{\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{cx^4+a}}\right)d^2}{e^2\sqrt{cd^4+ae^4}}-\frac{\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{cx^4+a}}\right)}{\sqrt{c}e^2}\right)}{e^2}\right)d-\frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{e^2\sqrt{cx^4+a}}+\frac{\sqrt[4]{c}\left(cd^4+ae^4\right)\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}e^4\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{cx^4+a}}+\frac{\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{c}x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2e^2\sqrt{cx^4+a}}+\frac{\sqrt{cx^4+a}}{2e(d-ex)}-\frac{\sqrt{cx^4+a}}{2e(d+ex)}+\frac{2\sqrt{c}x\sqrt{cx^4+a}}{e^2\left(\sqrt{c}x^2+\sqrt{a}\right)}$$

input
```
Int[Sqrt[a + c*x^4]/(d + e*x)^2,x]
```

3.211. $\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx$

output
```
Sqrt[a + c*x^4]/(2*e*(d - e*x)) - Sqrt[a + c*x^4]/(2*e*(d + e*x)) + (2*Sqr
t[c]*x*Sqrt[a + c*x^4])/(e^2*(Sqrt[a] + Sqrt[c]*x^2)) - (c*d^3*ArcTanh[(Sq
rt[c*d^4 + a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(e^3*Sqrt[c*d^4 + a*e^4]) - d
*e*(Sqrt[a + c*x^4]/(e^2*(d^2 - e^2*x^2)) - (c*(-(ArcTanh[(Sqrt[c]*x^2)/Sq
rt[a + c*x^4]]/(Sqrt[c]*e^2)) - (d^2*ArcTanh[(-(a*e^2) - c*d^2*x^2)/(Sqrt[
c*d^4 + a*e^4]*Sqrt[a + c*x^4])])/(e^2*Sqrt[c*d^4 + a*e^4])))/e^2) - (2*a^
(1/4)*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*
x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(e^2*Sqrt[a + c*x^4
]) + (3*c^(3/4)*d^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*e^
4*Sqrt[a + c*x^4]) + (a^(1/4)*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*
x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1
/2])/(2*e^2*Sqrt[a + c*x^4]) - (2*c^(5/4)*d^4*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt
[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(
1/4)], 1/2])/(a^(1/4)*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) + (
c^(1/4)*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4
)*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) + (c^(3/4)*d^2*(Sqrt[c]
*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[...
```

3.211.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 605
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[1/d   Int[x^(m - 1)*(a + b*x^2)^p, x], x] - Simp[c/d   Int[x^(m - 1
)*((a + b*x^2)^p/(c + d*x)), x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m,
 0] && LtQ[-1, p, 0]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2255
```
Int[(Pr_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Module[{r = Expon[Pr, x], k}, Int[Sum[Coeff[Pr, x, 2*k]*x^(2*k), {k, 0,
 r/2}]*(d + e*x^2)^q*(a + c*x^4)^p, x] + Int[x*Sum[Coeff[Pr, x, 2*k + 1]*x^
(2*k), {k, 0, (r - 1)/2}]*(d + e*x^2)^q*(a + c*x^4)^p, x]] /; FreeQ[{a, c,
d, e, p, q}, x] && PolyQ[Pr, x] &&  !PolyQ[Pr, x^2]
```

rule 2259
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(d + e*x^2)^q*(a + c*x^4)^(p
+ 1/2), x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && IntegerQ[p + 1/
2] && IntegerQ[q]
```

rule 2584
```
Int[((c_) + (d_.)*(x_)^(n_.))^(q_)*((a_) + (b_.)*(x_)^(nn_.))^(p_), x_Symbo
l] :> Int[ExpandToSum[(c - d*x^n)^(-q), x]*((a + b*x^nn)^p/(c^2 - d^2*x^(2*
n))^(-q)), x] /; FreeQ[{a, b, c, d, n, nn, p}, x] &&  !IntegerQ[p] && ILtQ[
q, 0] && IGtQ[Log[2, nn/n], 0]
```

3.211. $\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx$

3.211.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.33

method	result
default	$-\frac{\sqrt{c\,x^4+a}}{e(ex+d)}+\frac{2c\,d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e^4\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}-\frac{d\sqrt{c}\,\ln\left(2x^2\sqrt{c}+2\sqrt{c\,x^4+a}\right)}{e^3}+\frac{2i\sqrt{c}\,\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}}{e^2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}$
elliptic	$-\frac{\sqrt{c\,x^4+a}}{e(ex+d)}+\frac{2c\,d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e^4\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}-\frac{d\sqrt{c}\,\ln\left(2x^2\sqrt{c}+2\sqrt{c\,x^4+a}\right)}{e^3}+\frac{2i\sqrt{c}\,\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}}{e^2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}$

input
```
int((c*x^4+a)^(1/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/e*(c*x^4+a)^(1/2)/(e*x+d)+2*c*d^2/e^4/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^
(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*E
llipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-d*c^(1/2)/e^3*ln(2*x^2*c^(1/2)+2*(
c*x^4+a)^(1/2))+2*I*c^(1/2)/e^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(
1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*(E
llipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1
/2),I))-2*d^3*c/e^5*(-1/2/(c/e^4*d^4+a)^(1/2)*arctanh(1/2*(2*c*x^2/e^2*d^2
+2*a)/(c/e^4*d^4+a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)*e/d
*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)
^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)*e^2/d^2,(
-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

3.211.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x^4+a)^(1/2)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.211.6 Sympy [F]

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx = \int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx$$

input
```
integrate((c*x**4+a)**(1/2)/(e*x+d)**2,x)
```

output
```
Integral(sqrt(a + c*x**4)/(d + e*x)**2, x)
```

3.211.7 Maxima [F]

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx = \int \frac{\sqrt{cx^4+a}}{(ex+d)^2}\,dx$$

input
```
integrate((c*x^4+a)^(1/2)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^4 + a)/(e*x + d)^2, x)
```

3.211.8 Giac [F]

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx = \int \frac{\sqrt{cx^4+a}}{(ex+d)^2}\,dx$$

input
```
integrate((c*x^4+a)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^4 + a)/(e*x + d)^2, x)
```

3.211.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx = \int \frac{\sqrt{c\,x^4+a}}{(d+e\,x)^2}\,dx$$

input
```
int((a + c*x^4)^(1/2)/(d + e*x)^2,x)
```

output
```
int((a + c*x^4)^(1/2)/(d + e*x)^2, x)
```

3.212 $\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx$

3.212.1 Optimal result

Integrand size = 19, antiderivative size = 295

$$\begin{aligned}\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx &= \frac{e^3\sqrt{a+cx^4}}{2c}+\frac{3de^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{3d^2e\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}\\ &\quad-\frac{3\sqrt[4]{a}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}\\ &\quad+\frac{d\left(\sqrt{c}d^2+3\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+cx^4}}\end{aligned}$$

output
```
3/2*d^2*e*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/c^(1/2)+1/2*e^3*(c*x^4+a)^(
1/2)/c+3*d*e^2*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-3*a^(1/4)*d
*e^2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(
1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^
2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/
2)+1/2*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x
/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(3*e^2*
a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2)
)^2)^(1/2)/a^(1/4)/c^(3/4)/(c*x^4+a)^(1/2)
```

3.212.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.11 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.53

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx = \frac{e^3\sqrt{a+cx^4}}{2c} + \frac{3d^2e\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}} + \frac{d^3x\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)}{\sqrt{a+cx^4}} + \frac{de^2x^3\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^4}{a}\right)}{\sqrt{a+cx^4}}$$

input
```
Integrate[(d + e*x)^3/Sqrt[a + c*x^4],x]
```

output
```
(e^3*Sqrt[a + c*x^4])/(2*c) + (3*d^2*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^
4]])/(2*Sqrt[c]) + (d^3*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2,
5/4, -((c*x^4)/a)])/Sqrt[a + c*x^4] + (d*e^2*x^3*Sqrt[1 + (c*x^4)/a]*Hyper
geometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)])/Sqrt[a + c*x^4]
```

3.212.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}dx$$

$$\Big\downarrow \text{2424}$$

$$\int \left(\frac{d^3+3de^2x^2}{\sqrt{a+cx^4}}+\frac{x(3d^2e+e^3x^2)}{\sqrt{a+cx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(3\sqrt{a}e^2+\sqrt{c}d^2)\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}c^{3/4}\sqrt{a+cx^4}} - \frac{3\sqrt[4]{a}de^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{3d^2e\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}} + \frac{3de^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{e^3\sqrt{a+cx^4}}{2c}$$

input
```
Int[(d + e*x)^3/Sqrt[a + c*x^4],x]
```

output
```
(e^3*Sqrt[a + c*x^4])/(2*c) + (3*d*e^2*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a
] + Sqrt[c]*x^2)) + (3*d^2*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sq
rt[c]) - (3*a^(1/4)*d*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4
)*Sqrt[a + c*x^4]) + (d*(Sqrt[c]*d^2 + 3*Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x
^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4
)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(3/4)*Sqrt[a + c*x^4])
```

3.212.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.212.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.74

method	result
default	$\frac{d^{3} \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}}+\frac{e^{3} \sqrt{c\,x^{4}+a}}{2 c}+\frac{3 i d\,e^{2} \sqrt{a}\, \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \left(F\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)-E\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)\right)}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}\, \sqrt{c}}$
risch	$\frac{e^{3} \sqrt{c\,x^{4}+a}}{2 c}+d \left(\frac{d^{2} \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}}+\frac{3 i e^{2} \sqrt{a}\, \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \left(F\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)-E\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)\right.}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}\, \sqrt{c}}\right.$
elliptic	$\frac{e^{3} \sqrt{c\,x^{4}+a}}{2 c}+\frac{d^{3} \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, F\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}}+\frac{3 d^{2} e \ln\left(2 x^{2} \sqrt{c}+2 \sqrt{c\,x^{4}+a}\right)}{2 \sqrt{c}}+\frac{3 i d\,e^{2} \sqrt{a}\, \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, (}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}}}$

input
```
int((e*x+d)^3/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
d^3/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)
*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),
I)+1/2*e^3*(c*x^4+a)^(1/2)/c+3*I*d*e^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(
1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(
1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1
/2)*c^(1/2))^(1/2),I))+3/2*d^2*e*ln(x^2*c^(1/2)+(c*x^4+a)^(1/2))/c^(1/2)
```

3.212.5 Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.52

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx$$
$$= \frac{12\,a\sqrt{c}de^2x\left(-\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\,|-1)+3\,a\sqrt{c}d^2ex\log\left(-2\,cx^4-2\,\sqrt{cx^4+a}\sqrt{c}x^2-a\right)+4\,(cd^3-3}{4\,acx}$$

input
```
integrate((e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(12*a*sqrt(c)*d*e^2*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x),
-1) + 3*a*sqrt(c)*d^2*e*x*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a
) + 4*(c*d^3 - 3*a*d*e^2)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^
(1/4)/x), -1) + 2*(a*e^3*x + 6*a*d*e^2)*sqrt(c*x^4 + a))/(a*c*x)
```

3.212.6 Sympy [A] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.48

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx = e^3\left(\begin{cases} \frac{x^4}{4\sqrt{a}} & \text{for } c=0 \\ \frac{\sqrt{a+cx^4}}{2c} & \text{otherwise} \end{cases}\right) + \frac{3d^2e\operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}} + \frac{d^3x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{3de^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x+d)**3/(c*x**4+a)**(1/2),x)
```

output
```
e**3*Piecewise((x**4/(4*sqrt(a)), Eq(c, 0)), (sqrt(a + c*x**4)/(2*c), True
)) + 3*d**2*e*asinh(sqrt(c)*x**2/sqrt(a))/(2*sqrt(c)) + d**3*x*gamma(1/4)*
hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
 + 3*d*e**2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_polar(I*p
i)/a)/(4*sqrt(a)*gamma(7/4))
```

3.212.7 Maxima [F]

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx = \int \frac{(ex+d)^3}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3/sqrt(c*x^4 + a), x)
```

3.212.8 Giac [F]

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx = \int \frac{(ex+d)^3}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^3/sqrt(c*x^4 + a), x)
```

3.212.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx = \int \frac{(d+e\,x)^3}{\sqrt{c\,x^4+a}}\,dx$$

input
```
int((d + e*x)^3/(a + c*x^4)^(1/2),x)
```

output
```
int((d + e*x)^3/(a + c*x^4)^(1/2), x)
```

3.213 $\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx$

3.213.1 Optimal result

Integrand size = 19, antiderivative size = 263

$$\begin{aligned}
&\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx \\
&= \frac{e^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{de\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}} \\
&\quad - \frac{\sqrt[4]{a}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} \\
&\quad + \frac{\sqrt[4]{a}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}
\end{aligned}$$

output
```
d*e*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/c^(1/2)+e^2*x*(c*x^4+a)^(1/2)/c^(
1/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*e^2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2
)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/
a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/
2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)+1/2*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a
^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(
c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e^2+d^2*c^(1/2)/a^
(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)
```

3.213.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.51

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx = \frac{de\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}} + \frac{d^2x\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)}{\sqrt{a+cx^4}} + \frac{e^2x^3\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{cx^4}{a}\right)}{3\sqrt{a+cx^4}}$$

input
```
Integrate[(d + e*x)^2/Sqrt[a + c*x^4],x]
```

output
```
(d*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/Sqrt[c] + (d^2*x*Sqrt[1 + (c*
x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)])/Sqrt[a + c*x^4] +
(e^2*x^3*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)
])/(3*Sqrt[a + c*x^4])
```

3.213.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx$$

$$\Big\downarrow \text{2424}$$

$$\int \left(\frac{d^2+e^2x^2}{\sqrt{a+cx^4}}+\frac{2dex}{\sqrt{a+cx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt[4]{a}(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}} - \frac{\sqrt[4]{a}e^2(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}} + \frac{de\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}} + \frac{e^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

input
```
Int[(d + e*x)^2/Sqrt[a + c*x^4],x]
```

output
```
(e^2*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) + (d*e*ArcTanh[(
Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/Sqrt[c] - (a^(1/4)*e^2*(Sqrt[a] + Sqrt[c]*x
^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4
)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (a^(1/4)*((Sqrt[c]*d^2)/S
qrt[a] + e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*
x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a +
 c*x^4])
```

3.213.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

3.213.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.75

method	result
default	$\frac{d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{ie^2\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{c}}+\frac{ed\ln\left(x^2\sqrt{c}+\sqrt{\right.}}{\sqrt{c}}$
elliptic	$\frac{d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{ed\ln\left(2x^2\sqrt{c}+2\sqrt{c\,x^4+a}\right)}{\sqrt{c}}+\frac{ie^2\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\right.\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{c}}$

input
```
int((e*x+d)^2/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
d^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)
*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),
I)+I*e^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)
*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a
^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+e*d*ln(
x^2*c^(1/2)+(c*x^4+a)^(1/2))/c^(1/2)
```

3.213.5 Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.52

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx$$
$$= \frac{2\,a\sqrt{c}e^2x\left(-\frac{a}{c}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\mid-1)+a\sqrt{c}dex\log\left(-2\,cx^4-2\,\sqrt{cx^4+a}\sqrt{c}x^2-a\right)+2\left(cd^2-ae^2\right)\sqrt{}}{2\,acx}$$

input
```
integrate((e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*a*sqrt(c)*e^2*x*(-a/c)^(3/4)*elliptic_e(arcsin((-a/c)^(1/4)/x), -1)
 + a*sqrt(c)*d*e*x*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a) + 2*(
c*d^2 - a*e^2)*sqrt(c)*x*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -
1) + 2*sqrt(c*x^4 + a)*a*e^2)/(a*c*x)
```

3.213.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.40

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx = \frac{de\operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{c}} + \frac{d^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)} + \frac{e^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((e*x+d)**2/(c*x**4+a)**(1/2),x)
```

output
```
d*e*asinh(sqrt(c)*x**2/sqrt(a))/sqrt(c) + d**2*x*gamma(1/4)*hyper((1/4, 1/
2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + e**2*x**3*g
amma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*g
amma(7/4))
```

3.213.7 Maxima [F]

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx = \int \frac{(ex+d)^2}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2/sqrt(c*x^4 + a), x)
```

3.213.8 Giac [F]

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx = \int \frac{(ex+d)^2}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2/sqrt(c*x^4 + a), x)
```

3.213.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx = \int \frac{{(d+e\,x)}^2}{\sqrt{c\,x^4+a}}\,dx$$

input
```
int((d + e*x)^2/(a + c*x^4)^(1/2),x)
```

output
```
int((d + e*x)^2/(a + c*x^4)^(1/2), x)
```

3.214 $\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx$

3.214.1 Optimal result

Integrand size = 17, antiderivative size = 121

$$\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx=\frac{e\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}+\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}$$

output
```
1/2*e*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/c^(1/2)+1/2*d*(cos(2*arctan(c^(
1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2
*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/
(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/(c*x^4+a)^(1/2)
```

3.214.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.65

$$\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx=\frac{e\operatorname{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}+\frac{dx\sqrt{1+\frac{cx^4}{a}}\operatorname{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)}{\sqrt{a+cx^4}}$$

input
```
Integrate[(d + e*x)/Sqrt[a + c*x^4],x]
```

output
```
(e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt[c]) + (d*x*Sqrt[1 + (c*
x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)])/Sqrt[a + c*x^4]
```

3.214.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx$$

$$\Big\downarrow \text{2424}$$

$$\int \left(\frac{d}{\sqrt{a+cx^4}}+\frac{ex}{\sqrt{a+cx^4}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{e\,\text{arctanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

input
```
Int[(d + e*x)/Sqrt[a + c*x^4],x]
```

output
```
(e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt[c]) + (d*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(
c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4])
```

3.214.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

3.214.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{d\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{e\ln\left(x^2\sqrt{c}+\sqrt{c\,x^4+a}\right)}{2\sqrt{c}}$	96
elliptic	$\frac{d\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{e\ln\left(2x^2\sqrt{c}+2\sqrt{c\,x^4+a}\right)}{2\sqrt{c}}$	99

input
```
int((e*x+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c
^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
+1/2*e*ln(x^2*c^(1/2)+(c*x^4+a)^(1/2))/c^(1/2)
```

3.214.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.60

$$\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx$$
$$=\frac{4\,c^{\frac{3}{2}}d\left(-\frac{a}{c}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x}\right)\mid -1)+a\sqrt{c}e\log\left(-2\,cx^4-2\,\sqrt{cx^4+a}\sqrt{c}x^2-a\right)}{4\,ac}$$

input
```
integrate((e*x+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
1/4*(4*c^(3/2)*d*(-a/c)^(3/4)*elliptic_f(arcsin((-a/c)^(1/4)/x), -1) + a*s
qrt(c)*e*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a))/(a*c)
```

3.214.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.50

$$\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx = \frac{e\operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}} + \frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((e*x+d)/(c*x**4+a)**(1/2),x)
```

output
```
e*asinh(sqrt(c)*x**2/sqrt(a))/(2*sqrt(c)) + d*x*gamma(1/4)*hyper((1/4, 1/2
), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

3.214.7 Maxima [F]

$$\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx = \int \frac{ex+d}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((e*x+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)/sqrt(c*x^4 + a), x)
```

3.214.8 Giac [F]

$$\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx = \int \frac{ex+d}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate((e*x+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)/sqrt(c*x^4 + a), x)
```

3.214.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx=\int \frac{d+e\,x}{\sqrt{c\,x^4+a}}\,dx$$

input
```
int((d + e*x)/(a + c*x^4)^(1/2),x)
```

output
```
int((d + e*x)/(a + c*x^4)^(1/2), x)
```

3.215 $\int \frac{1}{\sqrt{a+cx^4}}\,dx$

3.215.1 Optimal result

Integrand size = 11, antiderivative size = 88

$$\int \frac{1}{\sqrt{a+cx^4}}\,dx=\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}$$

output
```
1/2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1
/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2
*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/(c*x^4
+a)^(1/2)
```

3.215.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{a+cx^4}}\,dx=-\frac{i\sqrt{1+\frac{cx^4}{a}}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right),-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{a+cx^4}}$$

input
```
Integrate[1/Sqrt[a + c*x^4],x]
```

output
```
((-I)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x]
, -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*Sqrt[a + c*x^4])
```

3.215.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^4}}\,dx$$

$$\Big\downarrow \text{761}$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}$$

input
```
Int[1/Sqrt[a + c*x^4],x]
```

output
```
((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellip
ticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^
4])
```

3.215.3.1 Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.215.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	70
elliptic	$\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	70

input
```
int(1/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c
^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

3.215.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.33

$$\int \frac{1}{\sqrt{a+cx^4}}\,dx = -\frac{\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right)\middle|-1\right)}{c}$$

input
```
integrate(1/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(a)*(-c/a)^(3/4)*elliptic_f(arcsin(x*(-c/a)^(1/4)), -1)/c
```

3.215.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.41

$$\int \frac{1}{\sqrt{a+cx^4}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(c*x**4+a)**(1/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a
)*gamma(5/4))
```

3.215.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate(1/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(c*x^4 + a), x)
```

3.215.8 Giac [F]

$$\int \frac{1}{\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}}\,dx$$

input
```
integrate(1/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*x^4 + a), x)
```

3.215.9 Mupad [B] (verification not implemented)

Time = 17.62 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sqrt{a+cx^4}}\,dx = \frac{x\,\sqrt{\frac{c\,x^4}{a}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\;\frac{5}{4};\;-\frac{c\,x^4}{a}\right)}{\sqrt{c\,x^4+a}}$$

input
```
int(1/(a + c*x^4)^(1/2),x)
```

output
```
(x*((c*x^4)/a + 1)^(1/2)*hypergeom([1/4, 1/2], 5/4, -(c*x^4)/a))/(a + c*x^
4)^(1/2)
```

3.216 $\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx$

3.216.1 Optimal result

Integrand size = 19, antiderivative size = 405

$$\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx = \frac{e\arctan\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-cd^4-ae^4}} - \frac{e\text{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{2\sqrt{cd^4+ae^4}} + \frac{\sqrt[4]{c}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}} - \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}}$$

output
```
1/2*e*arctan(x*(-a*e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2))/(-a*e^4-c*d^4)^(1
/2)-1/2*e*arctanh((c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/2))/(
a*e^4+c*d^4)^(1/2)+1/2*c^(1/4)*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2
)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4
))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)
^(1/2)/a^(1/4)/(e^2*a^(1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)-1/4*(cos(2*arctan
(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(
sin(2*arctan(c^(1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2/d^2/e^2/a
^(1/2)/c^(1/2),1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2
))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d/(e^2*a^(1/2
)+d^2*c^(1/2))/(c*x^4+a)^(1/2)
```

3.216.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.49

$$\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx$$
$$= \frac{\sqrt{1+\frac{cx^4}{a}}\left(-2\sqrt[4]{-1}\sqrt[4]{a}\sqrt{1+\frac{cd^4}{ae^4}}e\,\text{EllipticPi}\left(\frac{i\sqrt{a}e^2}{\sqrt{c}d^2},\arcsin\left(\frac{(-1)^{3/4}\sqrt[4]{c}x}{\sqrt[4]{a}}\right),-1\right)+\sqrt[4]{c}d\log\left(\frac{-d^2+}{cd^2x^2+ae^2\left(1+\sqrt{\right.}\right.}\right.}{2\sqrt[4]{c}d\sqrt{1+\frac{cd^4}{ae^4}}e\sqrt{a+cx^4}}$$

input
```
Integrate[1/((d + e*x)*Sqrt[a + c*x^4]),x]
```

output
```
(Sqrt[1 + (c*x^4)/a]*(-2*(-1)^(1/4)*a^(1/4)*Sqrt[1 + (c*d^4)/(a*e^4)]*e*El
lipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1
/4)], -1] + c^(1/4)*d*Log[(-d^2 + e^2*x^2)/(c*d^2*x^2 + a*e^2*(1 + Sqrt[1
+ (c*d^4)/(a*e^4)]*Sqrt[1 + (c*x^4)/a]))]))/(2*c^(1/4)*d*Sqrt[1 + (c*d^4)/
(a*e^4)]*e*Sqrt[a + c*x^4])
```

3.216.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2263, 1541, 27, 761, 1577, 488, 219, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^4}(d+ex)}\,dx$$
$$\Big\downarrow \text{2263}$$
$$d\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx-e\int \frac{x}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx$$
$$\Big\downarrow \text{1541}$$
$$d\left(\frac{\sqrt{c}\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\sqrt{a}e^2\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}\right)-e\int \frac{x}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx$$

$$\Big\downarrow \ 27$$

$$d\left(\frac{\sqrt{c}\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{e^2\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}\right)-e\int \frac{x}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx$$

$$\Big\downarrow \ 761$$

$$d\left(\frac{e^2\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}\right)-e\int \frac{x}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx$$

$$\Big\downarrow \ 1577$$

$$d\left(\frac{e^2\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}\right)-\frac{1}{2}e\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx^2$$

$$\Big\downarrow \ 488$$

$$d\left(\frac{e^2\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}\right)+\frac{1}{2}e\int \frac{1}{cd^4+ae^4-x^4}d\frac{-ae^2-cd^2x^2}{\sqrt{cx^4+a}}$$

$$\Big\downarrow \ 219$$

$$d\left(\frac{e^2\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}\right)+\frac{e\,\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}$$

$$\Big\downarrow \ 2223$$

3.216. $\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx$

$$d\left(\frac{e^2\left(\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)\text{EllipticPi}\left(\frac{(\sqrt{c}d^2+\sqrt{a}e^2)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{(\sqrt{a}e^2+\sqrt{c}d^2)\text{arctanh}\left(\frac{x\sqrt{ae^4+cd^4}}{de\sqrt{a+cx^4}}\right)}{2de\sqrt{ae^4+cd^4}}\right)}{\sqrt{a}e^2+\sqrt{c}d^2}\right.$$

$$\frac{e\,\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}$$

input
```
Int[1/((d + e*x)*Sqrt[a + c*x^4]),x]
```

output
```
(e*ArcTanh[(-(a*e^2) - c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])])/
(2*Sqrt[c*d^4 + a*e^4]) + d*((c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*
x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1
/2])/(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) + (e^2*(((Sqr
t[c]*d^2 + Sqrt[a]*e^2)*ArcTanh[(Sqrt[c*d^4 + a*e^4]*x)/(d*e*Sqrt[a + c*x^
4])])/(2*d*e*Sqrt[c*d^4 + a*e^4]) + ((Sqrt[a]/d^2 - Sqrt[c]/e^2)*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqr
t[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x
)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4])))/(Sqrt[c]*d^2 + Sqr
t[a]*e^2))
```

3.216.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1541
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4
], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   Int[(1 + q*x^2)/((d + e*
x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

rule 2263
```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[d
  Int[1/((d^2 - e^2*x^2)*Sqrt[a + c*x^4]), x], x] - Simp[e   Int[x/((d^2 -
e^2*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x]
```

3.216. $\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx$

3.216.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.42

method	result	size
default	$\frac{-\frac{\operatorname{arctanh}\left(\frac{\frac{2c\,x^2 d^2}{e^2}+2a}{2\sqrt{\frac{c\,d^4}{e^4}+a}\,\sqrt{c\,x^4+a}}\right)}{2\sqrt{\frac{c\,d^4}{e^4}+a}}+\frac{e\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\Pi\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},-\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,d\sqrt{c\,x^4+a}}}{e}$	169
elliptic	$\frac{-\frac{\operatorname{arctanh}\left(\frac{\frac{2c\,x^2 d^2}{e^2}+2a}{2\sqrt{\frac{c\,d^4}{e^4}+a}\,\sqrt{c\,x^4+a}}\right)}{2\sqrt{\frac{c\,d^4}{e^4}+a}}+\frac{e\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\Pi\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},-\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,d\sqrt{c\,x^4+a}}}{e}$	169

input
```
int(1/(e*x+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/e*(-1/2/(c/e^4*d^4+a)^(1/2)*arctanh(1/2*(2*c*x^2/e^2*d^2+2*a)/(c/e^4*d^4
+a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)*e/d*(1-I/a^(1/2)*c^
(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticP
i(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)*e^2/d^2,(-I/a^(1/2)*c^(1/
2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

3.216.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.216.6 Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{a+cx^4}\,(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x**4+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + c*x**4)*(d + e*x)), x)
```

3.216.7 Maxima [F]

$$\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + a)*(e*x + d)), x)
```

3.216.8 Giac [F]

$$\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + a)*(e*x + d)), x)
```

3.216.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{c\,x^4+a}\,(d+e\,x)}\,dx$$

input
```
int(1/((a + c*x^4)^(1/2)*(d + e*x)),x)
```

output
```
int(1/((a + c*x^4)^(1/2)*(d + e*x)), x)
```

3.217 $\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx$

3.217.1 Optimal result

Integrand size = 19, antiderivative size = 610

$$\begin{aligned}\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx &= -\frac{e^3\sqrt{a+cx^4}}{(cd^4+ae^4)(d+ex)}+\frac{\sqrt{c}e^2x\sqrt{a+cx^4}}{(cd^4+ae^4)\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&-\frac{cd^3e\arctan\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{(-cd^4-ae^4)^{3/2}}-\frac{cd^3e\,\mathrm{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{(cd^4+ae^4)^{3/2}} \\
&-\frac{\sqrt[4]{a}\sqrt[4]{c}e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{(cd^4+ae^4)\sqrt{a+cx^4}} \\
&+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{a+cx^4}} \\
&-\frac{c^{3/4}d^2\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\mathrm{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)(cd^4+ae^4)\sqrt{a+cx^4}}\end{aligned}$$

output
```
-c*d^3*e*arctan(x*(-a*e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2))/(-a*e^4-c*d^4)
^(3/2)-c*d^3*e*arctanh((c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/
2))/(a*e^4+c*d^4)^(3/2)-e^3*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)/(e*x+d)+e^2*x*c^
(1/2)*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*c^(1/4)*
e^2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1
/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2
*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/(a*e^4+c*d^4)/(c*x^4+a
)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/
2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4
)/(e^2*a^(1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)-1/2*c^(3/4)*d^2*(cos(2*arctan(
c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(s
in(2*arctan(c^(1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2/d^2/e^2/a^
(1/2)/c^(1/2),1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2)
)*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(a*e^4+c*d^4)/(e^2*a^(
1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)
```

3.217.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 448, normalized size of antiderivative = 0.73

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx =$$

$$-\frac{\sqrt{a}\sqrt{c}e^2\sqrt{-cd^4-ae^4}(d+ex)\sqrt{1+\frac{cx^4}{a}}E\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+i\sqrt{c}\left(\sqrt{c}d^2+i\sqrt{a}e^2\right)\sqrt{-cd^4-a}}{}$$

input
```
Integrate[1/((d + e*x)^2*Sqrt[a + c*x^4]),x]
```

output
```
-((Sqrt[a]*Sqrt[c]*e^2*Sqrt[-(c*d^4) - a*e^4]*(d + e*x)*Sqrt[1 + (c*x^4)/a
]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + I*Sqrt[c]*(Sqrt[
c]*d^2 + I*Sqrt[a]*e^2)*Sqrt[-(c*d^4) - a*e^4]*(d + e*x)*Sqrt[1 + (c*x^4)/
a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - Sqrt[(I*Sqrt[c]
)/Sqrt[a]]*(e^3*Sqrt[-(c*d^4) - a*e^4]*(a + c*x^4) - 2*c*d^3*e*(d + e*x)*S
qrt[a + c*x^4]*ArcTan[(Sqrt[c]*(d^2 - e^2*x^2) + e^2*Sqrt[a + c*x^4])/Sqrt
[-(c*d^4) - a*e^4]] + 2*(-1)^(1/4)*a^(1/4)*c^(3/4)*d^2*Sqrt[-(c*d^4) - a*e
^4]*(d + e*x)*Sqrt[1 + (c*x^4)/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2)
, ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4)], -1]))/(Sqrt[(I*Sqrt[c])/Sqrt[a]]
*(-(c*d^4) - a*e^4)^(3/2)*(d + e*x)*Sqrt[a + c*x^4]))
```

3.217.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 634, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.789$, Rules used = {2265, 25, 2280, 27, 1577, 488, 219, 2233, 25, 27, 1510, 2227, 27, 761, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^4}(d+ex)^2} \, dx$$

$$\Big\downarrow \text{2265}$$

$$-\frac{c\int -\frac{d^3-exd^2+e^2x^2d+e^3x^3}{(d+ex)\sqrt{cx^4+a}}dx}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{c\int \frac{d^3-exd^2+e^2x^2d+e^3x^3}{(d+ex)\sqrt{cx^4+a}}dx}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \text{2280}$$

$$\frac{c\left(\int \frac{d^4+2e^2x^2d^2-e^4x^4}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx+\int -\frac{2d^3ex}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\left(\int \frac{d^4+2e^2x^2d^2-e^4x^4}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx-2d^3e\int \frac{x}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \text{1577}$$

$$\frac{c\left(\int \frac{d^4+2e^2x^2d^2-e^4x^4}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx-d^3e\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx^2\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \text{488}$$

$$\frac{c\left(\int \frac{d^4+2e^2x^2d^2-e^4x^4}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx+d^3e\int \frac{1}{cd^4+ae^4-x^4}d\frac{-ae^2-cd^2x^2}{\sqrt{cx^4+a}}\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{c\left(\int \frac{d^4+2e^2x^2d^2-e^4x^4}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx+\frac{d^3e\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{\sqrt{ae^4+cd^4}}\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \text{2233}$$

$$\frac{c\left(-\frac{\int -\frac{\sqrt{c}e^2\left(\left(\sqrt{c}d^2+\sqrt{a}e^2\right)d^2+e^2\left(\sqrt{c}d^2-\sqrt{a}e^2\right)x^2\right)}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{ce^2}-\frac{\sqrt{a}e^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}}+\frac{d^3e\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{\sqrt{ae^4+cd^4}}\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{c\left(\frac{\int \frac{\sqrt{c}e^2\left(\left(\sqrt{c}d^2+\sqrt{a}e^2\right)d^2+e^2\left(\sqrt{c}d^2-\sqrt{a}e^2\right)x^2\right)}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{ce^2}-\frac{\sqrt{a}e^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}}+\frac{d^3e\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{\sqrt{ae^4+cd^4}}\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\left(\frac{\int \frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)d^2+e^2\left(\sqrt{c}d^2-\sqrt{a}e^2\right)x^2}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{\sqrt{c}}-\frac{e^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}+\frac{d^3e\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{\sqrt{ae^4+cd^4}}\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \text{1510}$$

3.217. $\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx$

$$\frac{c\left(\frac{\int \frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)d^2+e^2\left(\sqrt{c}d^2-\sqrt{a}e^2\right)x^2}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{\sqrt{c}}-\frac{e^2\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\sqrt{a+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}+\frac{d^3e\,\text{arctanh}\left(\cdots\right.}{\sqrt{ae^4\cdots}}\right.}{ae^4+cd^4}$$

$$\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

$\big\downarrow$ 2227

$$\frac{c\left(\frac{\frac{2\sqrt{a}\sqrt{c}d^4e^2\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\left(ae^4+cd^4\right)\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}}{\sqrt{c}}-\frac{e^2\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\sqrt{a+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}\cdots\right.}{ae^4+cd^4}$$

$$\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

$\big\downarrow$ 27

$$\frac{c\left(\frac{\frac{2\sqrt{c}d^4e^2\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\left(ae^4+cd^4\right)\int \frac{1}{\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}}{\sqrt{c}}-\frac{e^2\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\sqrt{a+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right)}{\sqrt{c}}+\frac{d^3\cdots}{\cdots}\right.}{ae^4+cd^4}$$

$$\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

$\big\downarrow$ 761

3.217. $\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx$

$$\frac{c\left(\frac{\frac{2\sqrt{c}d^4e^2\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(ae^4+cd^4\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}}{\sqrt{c}}-\frac{e^2\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\cdots}{\sqrt[4]{c}\sqrt{a+c\cdots}}\cdots\right)}{\cdots}\cdots\right)}{ae^4+cd^4\cdots}$$

$$\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

$\downarrow$ 2223

$$\frac{c\left(\frac{\frac{2\sqrt{c}d^4e^2\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\text{arctanh}\left(\frac{x\sqrt{ae^4+cd^4}}{de\sqrt{a+cx^4}}\right)}{2de\sqrt{ae^4+cd^4}}\right)}{\sqrt{a}e^2+\sqrt{c}d^2}+\frac{\left(\cdots\right.}{\cdots}\cdots}{\sqrt{c}}\cdots\right)}{\cdots}$$

$$\frac{e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}$$

input
```
Int[1/((d + e*x)^2*Sqrt[a + c*x^4]),x]
```

3.217. $\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx$

output
```
-((e^3*Sqrt[a + c*x^4])/((c*d^4 + a*e^4)*(d + e*x))) + (c*((d^3*e*ArcTanh[
(-(a*e^2) - c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])])/Sqrt[c*d^4
+ a*e^4] - (e^2*(-((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellip
ticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^4])))/Sqrt
[c] + (((c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4
)*c^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) + (2*Sqrt[c]*d^4*e^
2*(((Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTanh[(Sqrt[c*d^4 + a*e^4]*x)/(d*e*Sqrt[
a + c*x^4])])/(2*d*e*Sqrt[c*d^4 + a*e^4]) + ((Sqrt[a]/d^2 - Sqrt[c]/e^2)*(
Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c
^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4])))/(Sqrt[c]*d
^2 + Sqrt[a]*e^2))/Sqrt[c]))/(c*d^4 + a*e^4)
```

3.217.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

rule 2227
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q) - a*B*(e + d*q)
)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4], x], x] + Simp[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x]
, x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
 && NeQ[c*A^2 - a*B^2, 0]
```

rule 2233
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Sim
p[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x
^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2,
2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

3.217. $\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx$

rule 2265
```
Int[((d_) + (e_.)*(x_))^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[e
^3*(d + e*x)^(q + 1)*(Sqrt[a + c*x^4]/((q + 1)*(c*d^4 + a*e^4))), x] + Simp
[c/((q + 1)*(c*d^4 + a*e^4))   Int[((d + e*x)^(q + 1)/Sqrt[a + c*x^4])*Simp
[d^3*(q + 1) - d^2*e*(q + 1)*x + d*e^2*(q + 1)*x^2 - e^3*(q + 3)*x^3, x], x
], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^4 + a*e^4, 0] && ILtQ[q, -1]
```

rule 2280
```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Wit
h[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff
[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a
+ c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt
[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px
, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

3.217.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.69

method	result
default	$-\frac{e^{3}\sqrt{c\,x^{4}+a}}{\left(e^{4}a+d^{4}c\right)\left(ex+d\right)}-\frac{c\,d^{2}\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\left(e^{4}a+d^{4}c\right)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{i e^{2}\sqrt{c}\,\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\right.\right.}{\left(e^{4}a+d^{4}c\right)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$
elliptic	$-\frac{e^{3}\sqrt{c\,x^{4}+a}}{\left(e^{4}a+d^{4}c\right)\left(ex+d\right)}-\frac{c\,d^{2}\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\left(e^{4}a+d^{4}c\right)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{i e^{2}\sqrt{c}\,\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\right.\right.}{\left(e^{4}a+d^{4}c\right)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$

input
```
int(1/(e*x+d)^2/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-e^3*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)/(e*x+d)-c*d^2/(a*e^4+c*d^4)/(I/a^(1/2)*
c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(
1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+I*e^2*c^(1/2
)/(a*e^4+c*d^4)*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2
)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*(EllipticF(x*(I/a^
(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+2*d^3*c/
(a*e^4+c*d^4)/e*(-1/2/(c/e^4*d^4+a)^(1/2)*arctanh(1/2*(2*c*x^2/e^2*d^2+2*a
)/(c/e^4*d^4+a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)*e/d*(1-
I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/
2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)*e^2/d^2,(-I/a
^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

3.217.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.217.6 Sympy [F]

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{a+cx^4}\,(d+ex)^2}\,dx$$

input
```
integrate(1/(e*x+d)**2/(c*x**4+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + c*x**4)*(d + e*x)**2), x)
```

3.217.7 Maxima [F]

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex+d)^2}\,dx$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^2), x)
```

3.217.8 Giac [F]

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex+d)^2}\,dx$$

input
```
integrate(1/(e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^2), x)
```

3.217.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{c\,x^4+a}\,(d+e\,x)^2}\,dx$$

input
```
int(1/((a + c*x^4)^(1/2)*(d + e*x)^2),x)
```

output
```
int(1/((a + c*x^4)^(1/2)*(d + e*x)^2), x)
```

3.218 $\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx$

3.218.1 Optimal result

Integrand size = 19, antiderivative size = 659

$$\begin{aligned}
&\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx \\
&= -\frac{e^3\sqrt{a+cx^4}}{2(cd^4+ae^4)(d+ex)^2}-\frac{3cd^3e^3\sqrt{a+cx^4}}{(cd^4+ae^4)^2(d+ex)}+\frac{3c^{3/2}d^3e^2x\sqrt{a+cx^4}}{(cd^4+ae^4)^2\left(\sqrt{a}+\sqrt{c}x^2\right)} \\
&\quad+\frac{3cd^2e(cd^4-ae^4)\arctan\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{2(-cd^4-ae^4)^{5/2}}-\frac{3cd^2e(cd^4-ae^4)\operatorname{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{2(cd^4+ae^4)^{5/2}} \\
&\quad-\frac{3\sqrt[4]{a}c^{5/4}d^3e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{(cd^4+ae^4)^2\sqrt{a+cx^4}} \\
&\quad+\frac{c^{3/4}d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}(cd^4+ae^4)\sqrt{a+cx^4}} \\
&\quad-\frac{3c^{3/4}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\operatorname{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}(cd^4+ae^4)^2\sqrt{a+cx^4}}
\end{aligned}$$

output
```
3/2*c*d^2*e*(-a*e^4+c*d^4)*arctan(x*(-a*e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/
2))/(-a*e^4-c*d^4)^(5/2)-3/2*c*d^2*e*(-a*e^4+c*d^4)*arctanh((c*d^2*x^2+a*e
^2)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/2))/(a*e^4+c*d^4)^(5/2)-1/2*e^3*(c*x^
4+a)^(1/2)/(a*e^4+c*d^4)/(e*x+d)^2-3*c*d^3*e^3*(c*x^4+a)^(1/2)/(a*e^4+c*d^
4)^2/(e*x+d)+3*c^(3/2)*d^3*e^2*x*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)^2/(a^(1/2)+
x^2*c^(1/2))-3*a^(1/4)*c^(5/4)*d^3*e^2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2
)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/
a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/
2))^2)^(1/2)/(a*e^4+c*d^4)^2/(c*x^4+a)^(1/2)+1/2*c^(3/4)*d*(cos(2*arctan(c
^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin
(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a
)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(a*e^4+c*d^4)/(c*x^4+a)^(1/2)-3/4
*c^(3/4)*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)
*x/a^(1/4)))*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+
d^2*c^(1/2))^2/d^2/e^2/a^(1/2)/c^(1/2),1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1
/2))^2*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(
1/4)/(a*e^4+c*d^4)^2/(c*x^4+a)^(1/2)
```

3.218.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.43 (sec) , antiderivative size = 614, normalized size of antiderivative = 0.93

$$\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx =$$

$$-\frac{\frac{cd^4e^3\sqrt{a+cx^4}}{(d+ex)^2}+\frac{ae^7\sqrt{a+cx^4}}{(d+ex)^2}+\frac{6cd^3e^3\sqrt{a+cx^4}}{d+ex}-\frac{6c^2d^6e\arctan\left(\frac{\sqrt{c}\left(d^2-e^2x^2\right)+e^2\sqrt{a+cx^4}}{\sqrt{-cd^4-ae^4}}\right)}{\sqrt{-cd^4-ae^4}}+\frac{6acd^2e^5\arctan\left(\frac{\sqrt{c}\left(d^2-e^2x^2\right)+e^2\sqrt{a+}}{\sqrt{-cd^4-ae^4}}\right)}{\sqrt{-cd^4-ae^4}}}{}$$

input
```
Integrate[1/((d + e*x)^3*Sqrt[a + c*x^4]),x]
```

output
```
-1/2*((c*d^4*e^3*Sqrt[a + c*x^4])/(d + e*x)^2 + (a*e^7*Sqrt[a + c*x^4])/(d
 + e*x)^2 + (6*c*d^3*e^3*Sqrt[a + c*x^4])/(d + e*x) - (6*c^2*d^6*e*ArcTan[
(Sqrt[c]*(d^2 - e^2*x^2) + e^2*Sqrt[a + c*x^4])/Sqrt[-(c*d^4) - a*e^4]])/S
qrt[-(c*d^4) - a*e^4] + (6*a*c*d^2*e^5*ArcTan[(Sqrt[c]*(d^2 - e^2*x^2) + e
^2*Sqrt[a + c*x^4])/Sqrt[-(c*d^4) - a*e^4]])/Sqrt[-(c*d^4) - a*e^4] + ((6*
I)*a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*d^3*e^2*Sqrt[1 + (c*x^4)/a]*EllipticE[I*A
rcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/Sqrt[a + c*x^4] + ((2*I)*c*d*(-2
*c*d^4 - (3*I)*Sqrt[a]*Sqrt[c]*d^2*e^2 + a*e^4)*Sqrt[1 + (c*x^4)/a]*Ellipt
icF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]
]*Sqrt[a + c*x^4]) + (6*(-1)^(1/4)*a^(1/4)*c^(7/4)*d^5*Sqrt[1 + (c*x^4)/a]
*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a
^(1/4)], -1])/Sqrt[a + c*x^4] - (6*(-1)^(1/4)*a^(5/4)*c^(3/4)*d*e^4*Sqrt[1
 + (c*x^4)/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)
*c^(1/4)*x)/a^(1/4)], -1])/Sqrt[a + c*x^4])/(c*d^4 + a*e^4)^2
```

3.218.3 Rubi [A] (warning: unable to verify)

Time = 1.78 (sec) , antiderivative size = 677, normalized size of antiderivative = 1.03, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.895$, Rules used = {2265, 27, 2277, 25, 2280, 27, 1577, 488, 219, 2233, 25, 27, 1510, 2227, 27, 761, 2223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^4}(d+ex)^3}dx$$

$$\Big\downarrow \text{2265}$$

$$-\frac{c\int -\frac{2\left(d^3-exd^2+e^2x^2d\right)}{(d+ex)^2\sqrt{cx^4+a}}dx}{2\left(ae^4+cd^4\right)}-\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\int \frac{d^3-exd^2+e^2x^2d}{(d+ex)^2\sqrt{cx^4+a}}dx}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \text{2277}$$

$$\frac{c\left(-\frac{\int -\frac{3ce^2x^2d^4+3ce^3x^3d^3+\left(cd^4-2ae^4\right)d^2-e\left(2cd^4-ae^4\right)xd}{(d+ex)\sqrt{cx^4+a}}dx}{ae^4+cd^4}-\frac{3d^3e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)}\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$$\Big\downarrow 25$$

$$\frac{c\left(\frac{\int \frac{3ce^2x^2d^4+3ce^3x^3d^3+\left(cd^4-2ae^4\right)d^2-e\left(2cd^4-ae^4\right)xd}{(d+ex)\sqrt{cx^4+a}}dx}{ae^4+cd^4}-\frac{3d^3e^3\sqrt{a+cx^4}}{(d+ex)(ae^4+cd^4)}\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$$\Big\downarrow 2280$$

$$\frac{c\left(\frac{\int \frac{\left(-e\left(cd^4-2ae^4\right)d^2-e\left(2cd^4-ae^4\right)d^2\right)x}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx+\int \frac{-3cd^3e^4x^4+\left(3ce^2d^5+e^2\left(2cd^4-ae^4\right)d\right)x^2+d^3\left(cd^4-2ae^4\right)}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{ae^4+cd^4}-\frac{3d^3e^3\sqrt{a+cx^4}}{(d+ex)(ae^4+cd^4)}\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$$\Big\downarrow 27$$

$$\frac{c\left(\frac{\int \frac{-3cd^3e^4x^4+\left(3ce^2d^5+e^2\left(2cd^4-ae^4\right)d\right)x^2+d^3\left(cd^4-2ae^4\right)}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx-3d^2e\left(cd^4-ae^4\right)\int \frac{x}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{ae^4+cd^4}-\frac{3d^3e^3\sqrt{a+cx^4}}{(d+ex)(ae^4+cd^4)}\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$$\Big\downarrow 1577$$

$$\frac{c\left(\frac{\int \frac{-3cd^3e^4x^4+\left(3ce^2d^5+e^2\left(2cd^4-ae^4\right)d\right)x^2+d^3\left(cd^4-2ae^4\right)}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx-\frac{3}{2}d^2e\left(cd^4-ae^4\right)\int \frac{1}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx^2}{ae^4+cd^4}-\frac{3d^3e^3\sqrt{a+cx^4}}{(d+ex)(ae^4+cd^4)}\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$$\Big\downarrow 488$$

$$\frac{c\left(\frac{\frac{3}{2}d^2e\left(cd^4-ae^4\right)\int \frac{1}{cd^4+ae^4-x^4}d\frac{-ae^2-cd^2x^2}{\sqrt{cx^4+a}}+\int \frac{-3cd^3e^4x^4+\left(3ce^2d^5+e^2\left(2cd^4-ae^4\right)d\right)x^2+d^3\left(cd^4-2ae^4\right)}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{ae^4+cd^4}-\frac{3d^3e^3\sqrt{a+cx^4}}{(d+ex)(ae^4+cd^4)}\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$$\Big\downarrow 219$$

3.218. $\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx$

$$\frac{c\left(\frac{\int \frac{-3cd^3e^4x^4+\left(3ce^2d^5+e^2\left(2cd^4-ae^4\right)d\right)x^2+d^3\left(cd^4-2ae^4\right)}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx+\frac{3d^2e\left(cd^4-ae^4\right)\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}}{ae^4+cd^4}-\frac{3d^3e^3\sqrt{a+cx^4}}{(d+ex)(ae^4+cd^4)}\right)}{ae^4+cd^4}-\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$\big\downarrow$ 2233

$$\frac{c\left(\frac{-3\sqrt{a}\sqrt{c}d^3e^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx-\frac{\int -\frac{cde^2\left(\left(cd^4+3\sqrt{a}\sqrt{c}e^2d^2-2ae^4\right)d^2+e^2\left(2cd^4-3\sqrt{a}\sqrt{c}e^2d^2-ae^4\right)x^2\right)}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{ce^2}+\frac{3d^2e\left(cd^4-ae^4\right)\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4 \cdots}}\right)}{2\sqrt{ae^4+cd^4}}}{ae^4+cd^4}\cdots\right)}{ae^4+cd^4}\cdots$$

$$\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$\big\downarrow$ 25

$$\frac{c\left(\frac{-3\sqrt{a}\sqrt{c}d^3e^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx+\frac{\int \frac{cde^2\left(\left(cd^4+3\sqrt{a}\sqrt{c}e^2d^2-2ae^4\right)d^2+e^2\left(2cd^4-3\sqrt{a}\sqrt{c}e^2d^2-ae^4\right)x^2\right)}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{ce^2}+\frac{3d^2e\left(cd^4-ae^4\right)\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+\cdots}}\right)}{2\sqrt{ae^4+cd^4}}}{ae^4+cd^4}\cdots\right)}{ae^4+cd^4}\cdots$$

$$\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$\big\downarrow$ 27

$$\frac{c\left(\frac{-3\sqrt{c}d^3e^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx+d\int \frac{\left(cd^4+3\sqrt{a}\sqrt{c}e^2d^2-2ae^4\right)d^2+e^2\left(2cd^4-3\sqrt{a}\sqrt{c}e^2d^2-ae^4\right)x^2}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx+\frac{3d^2e\left(cd^4-ae^4\right)\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}}{ae^4+cd^4}-\cdots\right)}{ae^4+cd^4}\cdots$$

$$\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$\big\downarrow$ 1510

3.218. $\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx$

$$\frac{c\left(\frac{d\int \frac{\left(cd^4+3\sqrt{a}\sqrt{c}e^2d^2-2ae^4\right)d^2+e^2\left(2cd^4-3\sqrt{a}\sqrt{c}e^2d^2-ae^4\right)x^2}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx-3\sqrt{c}d^3e^2\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\sqrt{a+cx^4}}{\sqrt{a}+\sqrt{c}x^2}\right.}{ae^4+cd^4}\right.}{ae^4+cd^4}$$

$$\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$\downarrow$ 2227

$$\frac{c\left(\frac{d\left(\left(ae^4+cd^4\right)\int \frac{1}{\sqrt{cx^4+a}}dx+3\sqrt{a}d^2e^2\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx\right)-3\sqrt{c}d^3e^2\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\right.}{\sqrt[4]{c}\sqrt{a+cx^4}}\right.}{ae^4+cd^4}\right.}{ae^4+cd^4}$$

$$\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$\downarrow$ 27

$$\frac{c\left(\frac{d\left(\left(ae^4+cd^4\right)\int \frac{1}{\sqrt{cx^4+a}}dx+3d^2e^2\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx\right)-3\sqrt{c}d^3e^2\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right.}{\sqrt[4]{c}\sqrt{a+cx^4}}\right.}{ae^4+cd^4}\right.}{ae^4+cd^4}$$

$$\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$\downarrow$ 761

3.218. $\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx$

$$c\left(\frac{d\left(3d^2e^2\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\int \frac{\sqrt{c}x^2+\sqrt{a}}{\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(ae^4+cd^4\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}\right)-3\sqrt{c}d^3e^2\left(\frac{\sqrt[4]{a}\left(\sqrt{a}-\right.}{}\right.}{ae^4+cd^4}\right.$$

$$\overline{\qquad ae^4+cd^4}$$

$$\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

$$\Big\downarrow \; 2223$$

$$c\left(\frac{d\left(3d^2e^2\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\text{arctanh}\left(\frac{x\sqrt{}}{de}\right.}{2de\sqrt{ae^4+cd^4}}\right.\right.}{}\right.$$

$$\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}$$

input
```
Int[1/((d + e*x)^3*Sqrt[a + c*x^4]),x]
```

output
```
-1/2*(e^3*Sqrt[a + c*x^4])/((c*d^4 + a*e^4)*(d + e*x)^2) + (c*((-3*d^3*e^3
*Sqrt[a + c*x^4])/((c*d^4 + a*e^4)*(d + e*x)) + ((3*d^2*e*(c*d^4 - a*e^4)*
ArcTanh[(-(a*e^2) - c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])])/(2*
Sqrt[c*d^4 + a*e^4]) - 3*Sqrt[c]*d^3*e^2*(-((x*Sqrt[a + c*x^4])/(Sqrt[a] +
 Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4
)*Sqrt[a + c*x^4])) + d*(((c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
+ c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4]) + 3*d^2*e^2*(Sqrt[c]*d^2 - Sq
rt[a]*e^2)*(((Sqrt[c]*d^2 + Sqrt[a]*e^2)*ArcTanh[(Sqrt[c*d^4 + a*e^4]*x)/(
d*e*Sqrt[a + c*x^4])])/(2*d*e*Sqrt[c*d^4 + a*e^4]) + ((Sqrt[a]/d^2 - Sqrt[
c]/e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*
ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4]))))/
(c*d^4 + a*e^4)))/(c*d^4 + a*e^4)
```

3.218. $\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx$

3.218.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

rule 2227
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q) - a*B*(e + d*q)
)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4], x], x] + Simp[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x]
, x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
 && NeQ[c*A^2 - a*B^2, 0]
```

rule 2233
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Sim
p[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x
^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2,
2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2265
```
Int[((d_) + (e_.)*(x_))^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[e
^3*(d + e*x)^(q + 1)*(Sqrt[a + c*x^4]/((q + 1)*(c*d^4 + a*e^4))), x] + Simp
[c/((q + 1)*(c*d^4 + a*e^4))   Int[((d + e*x)^(q + 1)/Sqrt[a + c*x^4])*Simp
[d^3*(q + 1) - d^2*e*(q + 1)*x + d*e^2*(q + 1)*x^2 - e^3*(q + 3)*x^3, x], x
], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^4 + a*e^4, 0] && ILtQ[q, -1]
```

3.218. $\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx$

rule 2277
```
Int[((Px_)*((d_) + (e_.)*(x_))^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :
> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D =
Coeff[Px, x, 3]}, Simp[(-(d^3*D - C*d^2*e + B*d*e^2 - A*e^3))*(d + e*x)^(q
+ 1)*(Sqrt[a + c*x^4]/((q + 1)*(c*d^4 + a*e^4))), x] + Simp[1/((q + 1)*(c*d
^4 + a*e^4))   Int[((d + e*x)^(q + 1)/Sqrt[a + c*x^4])*Simp[(q + 1)*(a*e*(d
^2*D - C*d*e + B*e^2) + A*d*(c*d^2)) - (e*(q + 1)*(A*c*d^2 + a*e*(d*D - C*e
)) - B*d*(c*d^2*(q + 1)))*x + (q + 1)*(D*e*(a*e^2) + c*d*(C*d^2 - e*(B*d -
A*e)))*x^2 + c*(q + 3)*(d^3*D - C*d^2*e + B*d*e^2 - A*e^3)*x^3, x], x], x]]
 /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px, x], 3] && NeQ[c
*d^4 + a*e^4, 0] && LtQ[q, -1]
```

rule 2280
```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Wit
h[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff
[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a
+ c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt
[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px
, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

3.218.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.73

method	result
default	$-\frac{e^3\sqrt{c\,x^4+a}}{2(e^4a+d^4c)(ex+d)^2}-\frac{3c\,d^3e^3\sqrt{c\,x^4+a}}{(e^4a+d^4c)^2(ex+d)}+\frac{dc(e^4a-2d^4c)\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{(e^4a+d^4c)^2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{3ic^{\frac{3}{2}}d^3e^2\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}}{\sqrt{\cdot}}}}{}$
elliptic	$-\frac{e^3\sqrt{c\,x^4+a}}{2(e^4a+d^4c)(ex+d)^2}-\frac{3c\,d^3e^3\sqrt{c\,x^4+a}}{(e^4a+d^4c)^2(ex+d)}+\frac{dc(e^4a-2d^4c)\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{(e^4a+d^4c)^2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{3ic^{\frac{3}{2}}d^3e^2\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}}{\sqrt{\cdot}}}}{}$

input
```
int(1/(e*x+d)^3/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*e^3*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)/(e*x+d)^2-3*c*d^3*e^3*(c*x^4+a)^(1/
2)/(a*e^4+c*d^4)^2/(e*x+d)+d*c*(a*e^4-2*c*d^4)/(a*e^4+c*d^4)^2/(I/a^(1/2)*
c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(
1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+3*I*c^(3/2)*
d^3*e^2/(a*e^4+c*d^4)^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(
1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*(EllipticF
(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))-
3*c*d^2*(a*e^4-c*d^4)/(a*e^4+c*d^4)^2/e*(-1/2/(c/e^4*d^4+a)^(1/2)*arctanh(
1/2*(2*c*x^2/e^2*d^2+2*a)/(c/e^4*d^4+a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2
)*c^(1/2))^(1/2)*e/d*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*
x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/
2)/c^(1/2)*e^2/d^2,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

3.218.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.218.6 Sympy [F]

$$\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{a+cx^4}\,(d+ex)^3}\,dx$$

input
```
integrate(1/(e*x+d)**3/(c*x**4+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + c*x**4)*(d + e*x)**3), x)
```

3.218.7 Maxima [F]

$$\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex+d)^3}\,dx$$

input
```
integrate(1/(e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^3), x)
```

3.218.8 Giac [F]

$$\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{cx^4+a}(ex+d)^3}\,dx$$

input
```
integrate(1/(e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^4 + a)*(e*x + d)^3), x)
```

3.218.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx = \int \frac{1}{\sqrt{c\,x^4+a}\,(d+e\,x)^3}\,dx$$

input
```
int(1/((a + c*x^4)^(1/2)*(d + e*x)^3),x)
```

output
```
int(1/((a + c*x^4)^(1/2)*(d + e*x)^3), x)
```

3.219 $\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx$

3.219.1 Optimal result

Integrand size = 19, antiderivative size = 298

$$\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx = -\frac{3de^2x\sqrt{a+cx^4}}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{2ac\sqrt{a+cx^4}} + \frac{3de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}} + \frac{d\left(\sqrt{c}d^2-3\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}}$$

output
```
1/2*(-a*e^3+c*x*(3*d*e^2*x^2+3*d^2*e*x+d^3))/a/c/(c*x^4+a)^(1/2)-3/2*d*e^2
*x*(c*x^4+a)^(1/2)/a/c^(1/2)/(a^(1/2)+x^2*c^(1/2))+3/2*d*e^2*(cos(2*arctan
(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(s
in(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4
+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/c^(3/4)/(c*x^4+a)^(1/2)+1/4*d*(
cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))
*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-3*e^2*a^(1/2)+d
^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2
)/a^(5/4)/c^(3/4)/(c*x^4+a)^(1/2)
```

3.219.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.42

$$\int \frac{(d+ex)^3}{\left(a+cx^4\right)^{3/2}}\,dx = \frac{-ae^3+cd^3x+3cd^2ex^2+cd^3x\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)+2cde^2x^3}{2ac\sqrt{a+cx^4}}$$

input
```
Integrate[(d + e*x)^3/(a + c*x^4)^(3/2),x]
```

output
```
(-(a*e^3) + c*d^3*x + 3*c*d^2*e*x^2 + c*d^3*x*Sqrt[1 + (c*x^4)/a]*Hypergeo
metric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)] + 2*c*d*e^2*x^3*Sqrt[1 + (c*x^4)/a]
*Hypergeometric2F1[3/4, 3/2, 7/4, -((c*x^4)/a)])/(2*a*c*Sqrt[a + c*x^4])
```

3.219.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2393, 25, 27, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{\left(a+cx^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{2393}$$

$$-\frac{\int -\frac{d\left(d^2-3e^2x^2\right)}{\sqrt{cx^4+a}}dx}{2a}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{2ac\sqrt{a+cx^4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{d\left(d^2-3e^2x^2\right)}{\sqrt{cx^4+a}}dx}{2a}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{2ac\sqrt{a+cx^4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\int \frac{d^2-3e^2x^2}{\sqrt{cx^4+a}}dx}{2a}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{2ac\sqrt{a+cx^4}}$$

$$\Big\downarrow \text{1512}$$

$$\frac{d\left(\left(d^2-\frac{3\sqrt{a}e^2}{\sqrt{c}}\right)\int\frac{1}{\sqrt{cx^4+a}}dx+\frac{3\sqrt{a}e^2\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)}{2a}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{2ac\sqrt{a+cx^4}}$$

$\Big\downarrow$ 27

$$\frac{d\left(\left(d^2-\frac{3\sqrt{a}e^2}{\sqrt{c}}\right)\int\frac{1}{\sqrt{cx^4+a}}dx+\frac{3e^2\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}\right)}{2a}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{2ac\sqrt{a+cx^4}}$$

$\Big\downarrow$ 761

$$\frac{d\left(\frac{3e^2\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(d^2-\frac{3\sqrt{a}e^2}{\sqrt{c}}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}\right)}{2a}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{2ac\sqrt{a+cx^4}}$$

$\Big\downarrow$ 1510

$$\frac{d\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(d^2-\frac{3\sqrt{a}e^2}{\sqrt{c}}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{3e^2\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}\right.}{\sqrt{c}}\right.}{2a}-\frac{ae^3-cx\left(d^3+3d^2ex+3de^2x^2\right)}{2ac\sqrt{a+cx^4}}$$

input
```
Int[(d + e*x)^3/(a + c*x^4)^(3/2),x]
```

output
```
-1/2*(a*e^3 - c*x*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(a*c*Sqrt[a + c*x^4]) +
 (d*((3*e^2*(-((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE
[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^4])))/Sqrt[c]
+ ((d^2 - (3*Sqrt[a]*e^2)/Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2]
)/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4])))/(2*a)
```

3.219. $\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx$

3.219.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 2393
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   In
t[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(
p + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n
, 0] && LtQ[p, -1]
```

3.219. $\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx$

3.219.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.28 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.82

method	result
elliptic	$-\frac{2c\left(-\frac{3d e^2 x^3}{4ac}-\frac{3d^2 e\, x^2}{4ca}-\frac{d^3 x}{4ac}+\frac{e^3}{4c^2}\right)}{\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{d^3\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}-\frac{3id\,e^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\right.\right.}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{}}$
default	$d^3\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}\right)-\frac{e^3}{2c\sqrt{c\,x^4+a}}+3d\,e^2\left(\frac{x^3}{2a\sqrt{\left(x^4+\frac{a}{c}\right)c}}-\frac{i\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1}}{}\right.$

input
```
int((e*x+d)^3/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(-3/4/a*d*e^2/c*x^3-3/4*d^2*e/c/a*x^2-1/4*d^3/a/c*x+1/4*e^3/c^2)/((x^
4+a/c)*c)^(1/2)+1/2*d^3/a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x
^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a
^(1/2)*c^(1/2))^(1/2),I)-3/2*I/a^(1/2)*d*e^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-
I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/
2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2
)*c^(1/2))^(1/2),I))
```

3.219.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.54

$$\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx = \frac{3\left(cde^2x^4+ade^2\right)\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right)\mid -1\right)-\left(\left(cd^3+3\,cde^2\right)x^4+ad^3+3\,a\right.}{2\left(ac^2x\right.}$$

input
```
integrate((e*x+d)^3/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*(3*(c*d*e^2*x^4 + a*d*e^2)*sqrt(a)*(-c/a)^(3/4)*elliptic_e(arcsin(x*(-
c/a)^(1/4)), -1) - ((c*d^3 + 3*c*d*e^2)*x^4 + a*d^3 + 3*a*d*e^2)*sqrt(a)*(
-c/a)^(3/4)*elliptic_f(arcsin(x*(-c/a)^(1/4)), -1) + (3*c*d*e^2*x^3 + 3*c*
d^2*e*x^2 + c*d^3*x - a*e^3)*sqrt(c*x^4 + a))/(a*c^2*x^4 + a^2*c)
```

3.219.6 Sympy [F]

$$\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx = \int \frac{(d+ex)^3}{(a+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**3/(c*x**4+a)**(3/2),x)
```

output
```
Integral((d + e*x)**3/(a + c*x**4)**(3/2), x)
```

3.219.7 Maxima [F]

$$\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx = \int \frac{(ex+d)^3}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^3/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^3/(c*x^4 + a)^(3/2), x)
```

3.219.8 Giac [F]

$$\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx = \int \frac{(ex+d)^3}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^3/(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^3/(c*x^4 + a)^(3/2), x)
```

3.219.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx = \int \frac{(d+e\,x)^3}{(c\,x^4+a)^{3/2}}\,dx$$

input
```
int((d + e*x)^3/(a + c*x^4)^(3/2),x)
```

output
```
int((d + e*x)^3/(a + c*x^4)^(3/2), x)
```

3.220 $\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}} dx$

3.220.1 Optimal result

Integrand size = 19, antiderivative size = 270

$$\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}} dx = \frac{x(d+ex)^2}{2a\sqrt{a+cx^4}} - \frac{e^2x\sqrt{a+cx^4}}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}} + \frac{\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}}$$

output
```
1/2*x*(e*x+d)^2/a/(c*x^4+a)^(1/2)-1/2*e^2*x*(c*x^4+a)^(1/2)/a/c^(1/2)/(a^(
1/2)+x^2*c^(1/2))+1/2*e^2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2
*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2
*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/
a^(3/4)/c^(3/4)/(c*x^4+a)^(1/2)+1/4*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(
1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(
1/4))),1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x
^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(5/4)/c^(3/4)/(c*x^4+a)^(1/2)
```

3.220.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.40

$$\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}}\,dx = \frac{x\left(3d(d+2ex)+3d^2\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)+2e^2x^2\sqrt{1+\frac{cx^4}{a}}\,\text{H}}{6a\sqrt{a+cx^4}}$$

input
```
Integrate[(d + e*x)^2/(a + c*x^4)^(3/2),x]
```

output
```
(x*(3*d*(d + 2*e*x) + 3*d^2*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2
, 5/4, -((c*x^4)/a)] + 2*e^2*x^2*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[3/4
, 3/2, 7/4, -((c*x^4)/a)]))/(6*a*Sqrt[a + c*x^4])
```

3.220.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2394, 25, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \; 2394$$

$$\frac{x(d+ex)^2}{2a\sqrt{a+cx^4}}-\frac{\int -\frac{d^2-e^2x^2}{\sqrt{cx^4+a}}dx}{2a}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{d^2-e^2x^2}{\sqrt{cx^4+a}}dx}{2a}+\frac{x(d+ex)^2}{2a\sqrt{a+cx^4}}$$

$$\Big\downarrow \; 1512$$

$$\frac{\left(d^2-\frac{\sqrt{a}e^2}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+a}}dx+\frac{\sqrt{a}e^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx}{\sqrt{c}}}{2a}+\frac{x(d+ex)^2}{2a\sqrt{a+cx^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\left(d^2-\frac{\sqrt{a}e^2}{\sqrt{c}}\right)\int \frac{1}{\sqrt{cx^4+a}}dx+\frac{e^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}}{2a}+\frac{x(d+ex)^2}{2a\sqrt{a+cx^4}}$$

$$\Big\downarrow \; 761$$

$$\frac{\frac{e^2\int \frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx}{\sqrt{c}}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(d^2-\frac{\sqrt{a}e^2}{\sqrt{c}}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}}{2a}+\frac{x(d+ex)^2}{2a\sqrt{a+cx^4}}$$

$$\Big\downarrow \; 1510$$

$$\frac{\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(d^2-\frac{\sqrt{a}e^2}{\sqrt{c}}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{e^2\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{c}\sqrt{a+cx^4}}-\frac{x\sqrt{a}}{\sqrt{a}+}\right.}{\sqrt{c}}}{2a}$$

$$\frac{x(d+ex)^2}{2a\sqrt{a+cx^4}}$$

input
```
Int[(d + e*x)^2/(a + c*x^4)^(3/2),x]
```

output
```
(x*(d + e*x)^2)/(2*a*Sqrt[a + c*x^4]) + ((e^2*(-((x*Sqrt[a + c*x^4])/(Sqrt
[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(S
qrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c
^(1/4)*Sqrt[a + c*x^4])))/Sqrt[c] + ((d^2 - (Sqrt[a]*e^2)/Sqrt[c])*(Sqrt[a
] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4]))/(2*
a)
```

3.220.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

3.220.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.86

method	result
elliptic	$-\frac{2c\left(-\frac{e^2x^3}{4ac}-\frac{ed\,x^2}{2ca}-\frac{d^2x}{4ac}\right)}{\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}-\frac{ie^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-E\left(x\sqrt{\cdots}\right.\right.}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{c}}$
default	$d^2\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}\right)+e^2\left(\frac{x^3}{2a\sqrt{\left(x^4+\frac{a}{c}\right)c}}-\frac{i\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(F\left(x\sqrt{\cdots}\right.\right.}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4\cdots}}\right.$

input
```
int((e*x+d)^2/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(-1/4/a*e^2/c*x^3-1/2/c*e*d/a*x^2-1/4/a*d^2/c*x)/((x^4+a/c)*c)^(1/2)+
1/2/a*d^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a
^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^
(1/2),I)-1/2*I/a^(1/2)*e^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(Ellipt
icF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I
))
```

3.220.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.53

$$\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}}\,dx = \frac{(ce^2x^4+ae^2)\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}}E\left(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right)|-1\right) - ((cd^2+ce^2)x^4+ad^2+ae^2)\sqrt{a}(-}{2\left(ac^2x^4+a^2c\right)}$$

input
```
integrate((e*x+d)^2/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((c*e^2*x^4 + a*e^2)*sqrt(a)*(-c/a)^(3/4)*elliptic_e(arcsin(x*(-c/a)^(
1/4)), -1) - ((c*d^2 + c*e^2)*x^4 + a*d^2 + a*e^2)*sqrt(a)*(-c/a)^(3/4)*el
liptic_f(arcsin(x*(-c/a)^(1/4)), -1) + (c*e^2*x^3 + 2*c*d*e*x^2 + c*d^2*x)
*sqrt(c*x^4 + a))/(a*c^2*x^4 + a^2*c)
```

3.220.6 Sympy [F]

$$\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}}\,dx = \int \frac{(d+ex)^2}{(a+cx^4)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**2/(c*x**4+a)**(3/2),x)
```

output
```
Integral((d + e*x)**2/(a + c*x**4)**(3/2), x)
```

3.220.7 Maxima [F]

$$\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}}\,dx=\int \frac{(ex+d)^2}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^2/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2/(c*x^4 + a)^(3/2), x)
```

3.220.8 Giac [F]

$$\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}}\,dx=\int \frac{(ex+d)^2}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^2/(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2/(c*x^4 + a)^(3/2), x)
```

3.220.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}}\,dx=\int \frac{(d+e\,x)^2}{(c\,x^4+a)^{3/2}}\,dx$$

input
```
int((d + e*x)^2/(a + c*x^4)^(3/2),x)
```

output
```
int((d + e*x)^2/(a + c*x^4)^(3/2), x)
```

3.221 $\int \frac{d+ex}{(a+cx^4)^{3/2}} \, dx$

3.221.1 Optimal result

Integrand size = 17, antiderivative size = 114

$$\int \frac{d+ex}{(a+cx^4)^{3/2}} \, dx = \frac{x(d+ex)}{2a\sqrt{a+cx^4}} + \frac{d(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}}$$

output
```
1/2*x*(e*x+d)/a/(c*x^4+a)^(1/2)+1/4*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)
^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a
^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2
))^2)^(1/2)/a^(5/4)/c^(1/4)/(c*x^4+a)^(1/2)
```

3.221.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.52

$$\int \frac{d+ex}{(a+cx^4)^{3/2}} \, dx = \frac{x\left(d+ex+d\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)\right)}{2a\sqrt{a+cx^4}}$$

input
```
Integrate[(d + e*x)/(a + c*x^4)^(3/2),x]
```

output
```
(x*(d + e*x + d*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*
x^4)/a)]))/(2*a*Sqrt[a + c*x^4])
```

3.221.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2394, 25, 27, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(a+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2394}$$

$$\frac{x(d+ex)}{2a\sqrt{a+cx^4}}-\frac{\int -\frac{d}{\sqrt{cx^4+a}}dx}{2a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{d}{\sqrt{cx^4+a}}dx}{2a}+\frac{x(d+ex)}{2a\sqrt{a+cx^4}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{d\int \frac{1}{\sqrt{cx^4+a}}dx}{2a}+\frac{x(d+ex)}{2a\sqrt{a+cx^4}}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{x(d+ex)}{2a\sqrt{a+cx^4}}$$

input
```
Int[(d + e*x)/(a + c*x^4)^(3/2),x]
```

output
```
(x*(d + e*x))/(2*a*Sqrt[a + c*x^4]) + (d*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
 c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)]
, 1/2])/(4*a^(5/4)*c^(1/4)*Sqrt[a + c*x^4])
```

3.221.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 2394
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Simp[1/(a*n*(p + 1))   Int[ExpandToSum[n
*(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x
] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

3.221.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01

method	result	size
default	$d\left(\frac{x}{2a\sqrt{\left(x^{4}+\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}\right)+\frac{e\,x^{2}}{2\sqrt{c\,x^{4}+a}\,a}$	115
elliptic	$-\frac{2c\left(-\frac{e\,x^{2}}{4ca}-\frac{dx}{4ac}\right)}{\sqrt{\left(x^{4}+\frac{a}{c}\right)c}}+\frac{d\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	115

input
```
int((e*x+d)/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
d*(1/2*x/a/((x^4+a/c)*c)^(1/2)+1/2/a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2
)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ellip
ticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+1/2*e/(c*x^4+a)^(1/2)*x^2/a
```

3.221.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.67

$$\int \frac{d+ex}{(a+cx^4)^{3/2}}\,dx = -\frac{(cdx^4+ad)\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}} F\left(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right) | -1\right) - \sqrt{cx^4+a}(cex^2+cdx)}{2\left(ac^2x^4+a^2c\right)}$$

input
```
integrate((e*x+d)/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*((c*d*x^4 + a*d)*sqrt(a)*(-c/a)^(3/4)*elliptic_f(arcsin(x*(-c/a)^(1/4
)), -1) - sqrt(c*x^4 + a)*(c*e*x^2 + c*d*x))/(a*c^2*x^4 + a^2*c)
```

3.221.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.77 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.54

$$\int \frac{d+ex}{(a+cx^4)^{3/2}}\,dx = \frac{dx\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{4} \end{matrix}\middle| \frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)} + \frac{ex^2}{2a^{\frac{3}{2}}\sqrt{1+\frac{cx^4}{a}}}$$

input
```
integrate((e*x+d)/(c*x**4+a)**(3/2),x)
```

output
```
d*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*a**(
3/2)*gamma(5/4)) + e*x**2/(2*a**(3/2)*sqrt(1 + c*x**4/a))
```

3.221.7 Maxima [F]

$$\int \frac{d+ex}{(a+cx^4)^{3/2}}\,dx = \int \frac{ex+d}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)/(c*x^4 + a)^(3/2), x)
```

3.221.8 Giac [F]

$$\int \frac{d+ex}{(a+cx^4)^{3/2}}\,dx = \int \frac{ex+d}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)/(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)/(c*x^4 + a)^(3/2), x)
```

3.221.9 Mupad [B] (verification not implemented)

Time = 18.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.50

$$\int \frac{d+ex}{(a+cx^4)^{3/2}}\,dx = \frac{e\,x^2}{2\,a\,\sqrt{c\,x^4+a}} + \frac{d\,x\,\left(\frac{c\,x^4}{a}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{c\,x^4}{a}\right)}{(c\,x^4+a)^{3/2}}$$

input
```
int((d + e*x)/(a + c*x^4)^(3/2),x)
```

output
```
(e*x^2)/(2*a*(a + c*x^4)^(1/2)) + (d*x*((c*x^4)/a + 1)^(3/2)*hypergeom([1/
4, 3/2], 5/4, -(c*x^4)/a))/(a + c*x^4)^(3/2)
```

3.222 $\int \frac{1}{(a+cx^4)^{3/2}}\,dx$

3.222.1 Optimal result

Integrand size = 11, antiderivative size = 108

$$\int \frac{1}{(a+cx^4)^{3/2}}\,dx=\frac{x}{2a\sqrt{a+cx^4}}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}}$$

output
```
1/2*x/a/(c*x^4+a)^(1/2)+1/4*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos
(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1
/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2
)/a^(5/4)/c^(1/4)/(c*x^4+a)^(1/2)
```

3.222.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.72 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.51

$$\int \frac{1}{(a+cx^4)^{3/2}}\,dx=\frac{x+x\sqrt{1+\frac{cx^4}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{cx^4}{a}\right)}{2a\sqrt{a+cx^4}}$$

input
```
Integrate[(a + c*x^4)^(-3/2),x]
```

output
```
(x + x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)])
/(2*a*Sqrt[a + c*x^4])
```

3.222.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {749, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^4)^{3/2}}\,dx$$

$$\Big\downarrow \; 749$$

$$\frac{\int \frac{1}{\sqrt{cx^4+a}}dx}{2a}+\frac{x}{2a\sqrt{a+cx^4}}$$

$$\Big\downarrow \; 761$$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\sqrt{a+cx^4}}+\frac{x}{2a\sqrt{a+cx^4}}$$

input
```
Int[(a + c*x^4)^(-3/2),x]
```

output
```
x/(2*a*Sqrt[a + c*x^4]) + ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[
a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(
5/4)*c^(1/4)*Sqrt[a + c*x^4])
```

3.222.3.1 Defintions of rubi rules used

rule 749
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Simp[(n*(p + 1) + 1)/(a*n*(p + 1))   Int[(a + b*x^
n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Inte
gerQ[2*p] || Denominator[p + 1/n] < Denominator[p])
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.222.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{x}{2a\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	94
elliptic	$\frac{x}{2a\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	94

input
```
int(1/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x/a/((x^4+a/c)*c)^(1/2)+1/2/a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c
^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Elliptic
F(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

3.222.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.59

$$\int \frac{1}{\left(a+cx^4\right)^{3/2}}\,dx = -\frac{\left(cx^4+a\right)\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}}F\left(\arcsin\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}}\right)\mid -1\right)-\sqrt{cx^4+a}\,cx}{2\left(ac^2x^4+a^2c\right)}$$

input
```
integrate(1/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*((c*x^4 + a)*sqrt(a)*(-c/a)^(3/4)*elliptic_f(arcsin(x*(-c/a)^(1/4)),
-1) - sqrt(c*x^4 + a)*c*x)/(a*c^2*x^4 + a^2*c)
```

3.222.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.33

$$\int \frac{1}{(a+cx^4)^{3/2}}\,dx = \frac{x\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{4} \end{matrix}\middle| \frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(c*x**4+a)**(3/2),x)
```

output
```
x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*a**(3/
2)*gamma(5/4))
```

3.222.7 Maxima [F]

$$\int \frac{1}{(a+cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^4 + a)^(-3/2), x)
```

3.222.8 Giac [F]

$$\int \frac{1}{(a+cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^4 + a)^(-3/2), x)
```

3.222.9 Mupad [B] (verification not implemented)

Time = 18.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.34

$$\int \frac{1}{(a+cx^4)^{3/2}}\,dx = \frac{x\left(\frac{c\,x^4}{a}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{c\,x^4}{a}\right)}{(c\,x^4+a)^{3/2}}$$

input
```
int(1/(a + c*x^4)^(3/2),x)
```

output
```
(x*((c*x^4)/a + 1)^(3/2)*hypergeom([1/4, 3/2], 5/4, -(c*x^4)/a))/(a + c*x^
4)^(3/2)
```

3.223 $\int \frac{1}{(d+ex)(a+cx^4)^{3/2}} dx$

3.223.1 Optimal result

Integrand size = 19, antiderivative size = 818

$$\begin{aligned}
\int \frac{1}{(d+ex)(a+cx^4)^{3/2}}\,dx &= \frac{e(ae^2-cd^2x^2)}{2a(cd^4+ae^4)\sqrt{a+cx^4}}+\frac{cdx(d^2+e^2x^2)}{2a(cd^4+ae^4)\sqrt{a+cx^4}} \\
&- \frac{\sqrt{c}de^2x\sqrt{a+cx^4}}{2a(cd^4+ae^4)\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{e^5\arctan\left(\frac{\sqrt{-cd^4-ae^4}x}{de\sqrt{a+cx^4}}\right)}{2(-cd^4-ae^4)^{3/2}}-\frac{e^5\text{arctanh}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{2(cd^4+ae^4)^{3/2}} \\
&+ \frac{\sqrt[4]{c}de^2\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}(cd^4+ae^4)\sqrt{a+cx^4}} \\
&+ \frac{\sqrt[4]{c}d\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4a^{5/4}(cd^4+ae^4)\sqrt{a+cx^4}} \\
&+ \frac{\sqrt[4]{c}de^4\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\left(\sqrt{c}d^2+\sqrt{a}e^2\right)(cd^4+ae^4)\sqrt{a+cx^4}} \\
&- \frac{e^4\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2},2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}d\left(\sqrt{c}d^2+\sqrt{a}e^2\right)(cd^4+ae^4)\sqrt{a+cx^4}}
\end{aligned}$$

output
```
-1/2*e^5*arctan(x*(-a*e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2))/(-a*e^4-c*d^4)
^(3/2)-1/2*e^5*arctanh((c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/
2))/(a*e^4+c*d^4)^(3/2)+1/2*e*(-c*d^2*x^2+a*e^2)/a/(a*e^4+c*d^4)/(c*x^4+a)
^(1/2)+1/2*c*d*x*(e^2*x^2+d^2)/a/(a*e^4+c*d^4)/(c*x^4+a)^(1/2)-1/2*d*e^2*x
*c^(1/2)*(c*x^4+a)^(1/2)/a/(a*e^4+c*d^4)/(a^(1/2)+x^2*c^(1/2))+1/2*c^(1/4)
*d*e^2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a
^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+
x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(a*e^4+c*d^
4)/(c*x^4+a)^(1/2)+1/4*c^(1/4)*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2
)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4
))),1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+
a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(5/4)/(a*e^4+c*d^4)/(c*x^4+a)^(1/2)+1/
2*c^(1/4)*d*e^4*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^
(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*
(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(a
*e^4+c*d^4)/(e^2*a^(1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)-1/4*e^4*(cos(2*arcta
n(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi
(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2/d^2/e^2/
a^(1/2)/c^(1/2),1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/
2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d/(a*e^4+...
```

3.223.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.65 (sec) , antiderivative size = 455, normalized size of antiderivative = 0.56

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^{3/2}}\,dx =$$

$$-\frac{-\sqrt{a}c^{3/4}d^2e^2\sqrt{-cd^4-ae^4}\sqrt{1+\frac{cx^4}{a}}E\left(i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)+c^{3/4}d^2\left(-i\sqrt{c}d^2+\sqrt{a}e^2\right)\sqrt{-cd^4-ae^4}\sqrt{\ldots}}{\ldots}$$

input
```
Integrate[1/((d + e*x)*(a + c*x^4)^(3/2)),x]
```

output
```
-1/2*(-(Sqrt[a]*c^(3/4)*d^2*e^2*Sqrt[-(c*d^4) - a*e^4]*Sqrt[1 + (c*x^4)/a]
*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]) + c^(3/4)*d^2*((-I
)*Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[-(c*d^4) - a*e^4]*Sqrt[1 + (c*x^4)/a]*El
lipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + Sqrt[(I*Sqrt[c])/Sqr
t[a]]*(c^(1/4)*d*(Sqrt[-(c*d^4) - a*e^4]*(a*e^3 + c*d*x*(d^2 - d*e*x + e^2
*x^2)) + 2*a*e^5*Sqrt[a + c*x^4]*ArcTan[(Sqrt[c]*(d^2 - e^2*x^2) + e^2*Sqr
t[a + c*x^4])/Sqrt[-(c*d^4) - a*e^4]]) - 2*(-1)^(1/4)*a^(5/4)*e^4*Sqrt[-(c
*d^4) - a*e^4]*Sqrt[1 + (c*x^4)/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2
), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4)], -1]))/(a*Sqrt[(I*Sqrt[c])/Sqrt[
a]]*c^(1/4)*d*(-(c*d^4) - a*e^4)^(3/2)*Sqrt[a + c*x^4])
```

3.223.3 Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 727, normalized size of antiderivative = 0.89, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.895, Rules used = {2267, 1548, 27, 1577, 496, 25, 27, 488, 219, 2223, 2397, 25, 27, 1512, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+cx^4\right)^{3/2}(d+ex)}dx$$

$$\Big\downarrow \ 2267$$

$$d\int \frac{1}{\left(d^2-e^2x^2\right)\left(cx^4+a\right)^{3/2}}dx-e\int \frac{x}{\left(d^2-e^2x^2\right)\left(cx^4+a\right)^{3/2}}dx$$

$$\Big\downarrow \ 1548$$

$$d\left(\frac{\int \frac{\frac{c^{3/2}e^4x^4}{\sqrt{a}}+ce^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)x^2+\sqrt{c}\left(\frac{cd^4}{\sqrt{a}}+\sqrt{c}e^2d^2+\sqrt{a}e^4\right)}{\left(cx^4+a\right)^{3/2}}dx}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\left(ae^4+cd^4\right)}+\frac{e^6\int \frac{\sqrt{c}x^2+\sqrt{a}}{\sqrt{a}\left(d^2-e^2x^2\right)\sqrt{cx^4+a}}dx}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\left(ae^4+cd^4\right)}\right)-e\int \frac{x}{\left(d^2-e^2x^2\right)\left(cx^4+a\right)^{3/2}}dx$$

$$\Big\downarrow \ 27$$

$$d\left(\frac{\int \frac{\frac{c^{3/2}e^4x^4}{\sqrt{a}}+ce^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)x^2+\sqrt{c}\left(\frac{cd^4}{\sqrt{a}}+\sqrt{c}e^2d^2+\sqrt{a}e^4\right)}{(cx^4+a)^{3/2}}dx}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}+\frac{e^6\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}\right)-e\int \frac{x}{(d^2-e^2x^2)(cx^4+a)^{3/2}}dx$$

$$\Big\downarrow \text{1577}$$

$$d\left(\frac{\int \frac{\frac{c^{3/2}e^4x^4}{\sqrt{a}}+ce^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)x^2+\sqrt{c}\left(\frac{cd^4}{\sqrt{a}}+\sqrt{c}e^2d^2+\sqrt{a}e^4\right)}{(cx^4+a)^{3/2}}dx}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}+\frac{e^6\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}\right)-\frac{1}{2}e\int \frac{1}{(d^2-e^2x^2)(cx^4+a)^{3/2}}dx^2$$

$$\Big\downarrow \text{496}$$

$$d\left(\frac{\int \frac{\frac{c^{3/2}e^4x^4}{\sqrt{a}}+ce^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)x^2+\sqrt{c}\left(\frac{cd^4}{\sqrt{a}}+\sqrt{c}e^2d^2+\sqrt{a}e^4\right)}{(cx^4+a)^{3/2}}dx}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}+\frac{e^6\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}\right)-\frac{1}{2}e\left(-\frac{\int -\frac{ae^4}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx^2}{a(ae^4+cd^4)}-\frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}(ae^4+cd^4)}\right)$$

$$\Big\downarrow \text{25}$$

$$d\left(\frac{\int \frac{\frac{c^{3/2}e^4x^4}{\sqrt{a}}+ce^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)x^2+\sqrt{c}\left(\frac{cd^4}{\sqrt{a}}+\sqrt{c}e^2d^2+\sqrt{a}e^4\right)}{(cx^4+a)^{3/2}}dx}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}+\frac{e^6\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}\right)-\frac{1}{2}e\left(\frac{\int \frac{ae^4}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx^2}{a(ae^4+cd^4)}-\frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}(ae^4+cd^4)}\right)$$

$$\Big\downarrow \text{27}$$

$$d\left(\frac{\int \frac{\frac{c^{3/2}e^4x^4}{\sqrt{a}}+ce^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)x^2+\sqrt{c}\left(\frac{cd^4}{\sqrt{a}}+\sqrt{c}e^2d^2+\sqrt{a}e^4\right)}{(cx^4+a)^{3/2}}dx}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}+\frac{e^6\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}\right)-\frac{1}{2}e\left(\frac{e^4\int \frac{1}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx^2}{ae^4+cd^4}-\frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}(ae^4+cd^4)}\right)$$

3.223. $\int \frac{1}{(d+ex)(a+cx^4)^{3/2}}\,dx$

$$\Big\downarrow \text{488}$$

$$d\left(\frac{\int \frac{\frac{c^{3/2}e^4x^4}{\sqrt{a}}+ce^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)x^2+\sqrt{c}\left(\frac{cd^4}{\sqrt{a}}+\sqrt{c}e^2d^2+\sqrt{a}e^4\right)}{(cx^4+a)^{3/2}}dx}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}+\frac{e^6\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}\right)-\frac{1}{2}e\left(-\frac{e^4\int \frac{1}{cd^4+ae^4-x^4}d\frac{-ae^2-cd^2x^2}{\sqrt{cx^4+a}}}{ae^4+cd^4}-\frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}\,(ae^4+cd^4)}\right)$$

$$\Big\downarrow \text{219}$$

$$d\left(\frac{\int \frac{\frac{c^{3/2}e^4x^4}{\sqrt{a}}+ce^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)x^2+\sqrt{c}\left(\frac{cd^4}{\sqrt{a}}+\sqrt{c}e^2d^2+\sqrt{a}e^4\right)}{(cx^4+a)^{3/2}}dx}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}+\frac{e^6\int \frac{\sqrt{c}x^2+\sqrt{a}}{(d^2-e^2x^2)\sqrt{cx^4+a}}dx}{\sqrt{a}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}\right)-\frac{1}{2}e\left(-\frac{e^4\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{(ae^4+cd^4)^{3/2}}-\frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}\,(ae^4+cd^4)}\right)$$

$$\Big\downarrow \text{2223}$$

$$d\left(\frac{\int \frac{\frac{c^{3/2}e^4x^4}{\sqrt{a}}+ce^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)x^2+\sqrt{c}\left(\frac{cd^4}{\sqrt{a}}+\sqrt{c}e^2d^2+\sqrt{a}e^4\right)}{(cx^4+a)^{3/2}}dx}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}+\frac{e^6\left(\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)\text{EllipticPi}\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2}\right.}{4\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}\right.}{\sqrt{a}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right.}\right.$$

$$-\frac{1}{2}e\left(-\frac{e^4\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{(ae^4+cd^4)^{3/2}}-\frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}\,(ae^4+cd^4)}\right)$$

$$\Big\downarrow \text{2397}$$

3.223. $\int \frac{1}{(d+ex)(a+cx^4)^{3/2}}\,dx$

$$d\left(\frac{\frac{cx(d^2+e^2x^2)\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}{2a\sqrt{a+cx^4}}-\frac{\int -\frac{c^{3/2}\left(cd^4+\sqrt{a}\sqrt{c}e^2d^2+2ae^4-\sqrt{a}\sqrt{c}e^2\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)x^2\right)}{\sqrt{a}\sqrt{cx^4+a}}dx}{2ac}}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}+\frac{e^6\left(\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)\text{El}}{4\sqrt[4]{a}}\right.}{}\right.$$

$$\frac{1}{2}e\left(-\frac{e^4\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{(ae^4+cd^4)^{3/2}}-\frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}(ae^4+cd^4)}\right)$$

$$\downarrow 25$$

$$d\left(\frac{\frac{\int \frac{c^{3/2}\left(cd^4+\sqrt{a}\sqrt{c}e^2d^2+2ae^4-\sqrt{c}e^2\left(\sqrt{c}d^2+\sqrt{a}e^2\right)x^2\right)}{\sqrt{a}\sqrt{cx^4+a}}dx}{2ac}+\frac{cx(d^2+e^2x^2)\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}{2a\sqrt{a+cx^4}}}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}+\frac{e^6\left(\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)\text{Ellipti}}{4\sqrt[4]{a}\sqrt[4]{c}}\right.}{}\right.$$

$$\frac{1}{2}e\left(-\frac{e^4\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{(ae^4+cd^4)^{3/2}}-\frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}(ae^4+cd^4)}\right)$$

$$\downarrow 27$$

$$d\left(\frac{\frac{\sqrt{c}\int \frac{cd^4+\sqrt{a}\sqrt{c}e^2d^2+2ae^4-\sqrt{c}e^2\left(\sqrt{c}d^2+\sqrt{a}e^2\right)x^2}{\sqrt{cx^4+a}}dx}{2a^{3/2}}+\frac{cx(d^2+e^2x^2)\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}{2a\sqrt{a+cx^4}}}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)(ae^4+cd^4)}+\frac{e^6\left(\frac{(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(\frac{\sqrt{a}}{d^2}-\frac{\sqrt{c}}{e^2}\right)\text{EllipticP}}{4\sqrt[4]{a}\sqrt[4]{c}\sqrt{a}}\right.}{\sqrt{}}\right.$$

$$\frac{1}{2}e\left(-\frac{e^4\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{(ae^4+cd^4)^{3/2}}-\frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}(ae^4+cd^4)}\right)$$

$$\downarrow 1512$$

3.223. $\int \frac{1}{(d+ex)(a+cx^4)^{3/2}}\,dx$

$$d\left(\frac{\frac{\sqrt{c}\left((ae^4+cd^4)\int\frac{1}{\sqrt{cx^4+a}}dx+\sqrt{a}e^2\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{a}\sqrt{cx^4+a}}dx\right)}{2a^{3/2}}+\frac{cx\left(d^2+e^2x^2\right)\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}{2a\sqrt{a+cx^4}}}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\left(ae^4+cd^4\right)}+\frac{e^6\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}}{d^2}\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

$$\frac{1}{2}e\left(-\frac{e^4\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{\left(ae^4+cd^4\right)^{3/2}}-\frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}\left(ae^4+cd^4\right)}\right)$$

$$\Big\downarrow \ 27$$

$$d\left(\frac{\frac{\sqrt{c}\left((ae^4+cd^4)\int\frac{1}{\sqrt{cx^4+a}}dx+e^2\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx\right)}{2a^{3/2}}+\frac{cx\left(d^2+e^2x^2\right)\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}{2a\sqrt{a+cx^4}}}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\left(ae^4+cd^4\right)}+\frac{e^6\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{a}}{d^2}-\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

$$\frac{1}{2}e\left(-\frac{e^4\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{\left(ae^4+cd^4\right)^{3/2}}-\frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}\left(ae^4+cd^4\right)}\right)$$

$$\Big\downarrow \ 761$$

$$d\left(\frac{\frac{\sqrt{c}\left(e^2\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\int\frac{\sqrt{a}-\sqrt{c}x^2}{\sqrt{cx^4+a}}dx+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(ae^4+cd^4\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}\right)}{2a^{3/2}}+\frac{cx\left(d^2+e^2x^2\right)\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)}{2a\sqrt{a+cx^4}}}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\left(ae^4+cd^4\right)}+\cdots\right.$$

$$\frac{1}{2}e\left(-\frac{e^4\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{\left(ae^4+cd^4\right)^{3/2}}-\frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}\left(ae^4+cd^4\right)}\right)$$

$$\Big\downarrow \ 1510$$

3.223. $\int\frac{1}{(d+ex)(a+cx^4)^{3/2}}\,dx$

$$d\left(\frac{\frac{\sqrt{c}\left(\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(ae^4+cd^4\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}+e^2\left(\sqrt{a}e^2+\sqrt{c}d^2\right)\left(\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{c}\ldots}{\sqrt[4]{\ldots}}\right.\right.}{\sqrt[4]{c}\sqrt{a+cx^4}}\right.\right.}{2a^{3/2}}}{\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)\left(ae^4+cd^4\right)}\right.$$

$$\frac{1}{2}e\left(-\frac{e^4\text{arctanh}\left(\frac{-ae^2-cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{\left(ae^4+cd^4\right)^{3/2}}-\frac{ae^2-cd^2x^2}{a\sqrt{a+cx^4}\left(ae^4+cd^4\right)}\right)$$

input
```
Int[1/((d + e*x)*(a + c*x^4)^(3/2)),x]
```

output
```
-1/2*(e*(-((a*e^2 - c*d^2*x^2)/(a*(c*d^4 + a*e^4)*Sqrt[a + c*x^4])) - (e^4
*ArcTanh[(-(a*e^2) - c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])])/(c
*d^4 + a*e^4)^(3/2))) + d*(((c*((Sqrt[c]*d^2)/Sqrt[a] + e^2)*x*(d^2 + e^2*
x^2))/(2*a*Sqrt[a + c*x^4]) + (Sqrt[c]*(e^2*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(-
((x*Sqrt[a + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(Sqrt[a] + Sqrt[c
]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(
1/4)*x)/a^(1/4)], 1/2])/(c^(1/4)*Sqrt[a + c*x^4])) + ((c*d^4 + a*e^4)*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[
2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4])))
/(2*a^(3/2)))/(((Sqrt[c]*d^2)/Sqrt[a] + e^2)*(c*d^4 + a*e^4)) + (e^6*(((Sq
rt[c]*d^2 + Sqrt[a]*e^2)*ArcTanh[(Sqrt[c*d^4 + a*e^4]*x)/(d*e*Sqrt[a + c*x
^4])])/(2*d*e*Sqrt[c*d^4 + a*e^4]) + ((Sqrt[a]/d^2 - Sqrt[c]/e^2)*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sq
rt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*
x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4])))/(Sqrt[a]*((Sqrt[c
]*d^2)/Sqrt[a] + e^2)*(c*d^4 + a*e^4)))
```

3.223.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 1512
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + c*x^4], x], x] - Simp[e/q
 Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c
, d, e}, x] && PosQ[c/a]
```

3.223. $\int \frac{1}{(d+ex)(a+cx^4)^{3/2}}\,dx$

rule 1548
```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Simp[-(
c*d^2 + a*e^2)^(p + 1/2)/(e^(2*p)*(Rt[c/a, 2]*d - e))   Int[(1 + Rt[c/a, 2]
*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] + Simp[(c*d^2 + a*e^2)^(p + 1/2
)/(Rt[c/a, 2]*d - e)   Int[(a + c*x^4)^p*ExpandToSum[((Rt[c/a, 2]*d - e)*(c
*d^2 + a*e^2)^(-p - 1/2) + ((1 + Rt[c/a, 2]*x^2)*(a + c*x^4)^(-p - 1/2))/e^
(2*p))/(d + e*x^2), x], x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e
^2, 0] && ILtQ[p + 1/2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 2223
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTanh[Rt[(-c)*
(d/e) - a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[(-c)*(d/e) - a*(e/d), 2]
)), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^
2)]/(4*d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*Arc
Tan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[c*(d/e) + a*(e
/d)]
```

rule 2267
```
Int[((a_) + (c_.)*(x_)^4)^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[d
Int[(a + c*x^4)^p/(d^2 - e^2*x^2), x], x] - Simp[e   Int[x*((a + c*x^4)^p/(
d^2 - e^2*x^2)), x], x] /; FreeQ[{a, c, d, e}, x] && IntegerQ[p + 1/2]
```

rule 2397
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, S
imp[(-x)*R*((a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]
 + Simp[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))   Int[(a + b*x^n)^(p + 1)*
ExpandToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x]] /; GeQ[q,
n]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

3.223. $\int \frac{1}{(d+ex)(a+cx^4)^{3/2}}\,dx$

3.223.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 496, normalized size of antiderivative = 0.61

method	result
default	$-\frac{2c\left(-\frac{d\,e^{2}x^{3}}{4a\left(e^{4}a+d^{4}c\right)}+\frac{d^{2}e\,x^{2}}{4a\left(e^{4}a+d^{4}c\right)}-\frac{d^{3}x}{4a\left(e^{4}a+d^{4}c\right)}-\frac{e^{3}}{4\left(e^{4}a+d^{4}c\right)c}\right)}{\sqrt{\left(x^{4}+\frac{a}{c}\right)c}}+\frac{d^{3}c\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a(e^{4}a+d^{4}c)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}-\frac{i\sqrt{c}\,d\,e^{2}\sqrt{1\cdots}}{\cdots}$
elliptic	$-\frac{2c\left(-\frac{d\,e^{2}x^{3}}{4a\left(e^{4}a+d^{4}c\right)}+\frac{d^{2}e\,x^{2}}{4a\left(e^{4}a+d^{4}c\right)}-\frac{d^{3}x}{4a\left(e^{4}a+d^{4}c\right)}-\frac{e^{3}}{4\left(e^{4}a+d^{4}c\right)c}\right)}{\sqrt{\left(x^{4}+\frac{a}{c}\right)c}}+\frac{d^{3}c\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,F\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a(e^{4}a+d^{4}c)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}-\frac{i\sqrt{c}\,d\,e^{2}\sqrt{1\cdots}}{\cdots}$

input
```
int(1/(e*x+d)/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c*(-1/4/a*d*e^2/(a*e^4+c*d^4)*x^3+1/4/a*d^2*e/(a*e^4+c*d^4)*x^2-1/4*d^3
/a/(a*e^4+c*d^4)*x-1/4*e^3/(a*e^4+c*d^4)/c)/((x^4+a/c)*c)^(1/2)+1/2*d^3/a*
c/(a*e^4+c*d^4)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*
(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(
1/2))^(1/2),I)-1/2*I*c^(1/2)*d/a^(1/2)*e^2/(a*e^4+c*d^4)/(I/a^(1/2)*c^(1/2
))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(
c*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^
(1/2)*c^(1/2))^(1/2),I))+e^3/(a*e^4+c*d^4)*(-1/2/(c/e^4*d^4+a)^(1/2)*arcta
nh(1/2*(2*c*x^2/e^2*d^2+2*a)/(c/e^4*d^4+a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(
1/2)*c^(1/2))^(1/2)*e/d*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/
2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^
(1/2)/c^(1/2)*e^2/d^2,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)
))
```

3.223.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(a+cx^4)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.223.6 Sympy [F]

$$\int \frac{1}{(d+ex)(a+cx^4)^{3/2}}\,dx = \int \frac{1}{(a+cx^4)^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x**4+a)**(3/2),x)
```

output
```
Integral(1/((a + c*x**4)**(3/2)*(d + e*x)), x)
```

3.223.7 Maxima [F]

$$\int \frac{1}{(d+ex)(a+cx^4)^{3/2}}\,dx = \int \frac{1}{(cx^4+a)^{\frac{3}{2}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^4 + a)^(3/2)*(e*x + d)), x)
```

3.223.8 Giac [F]

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^{3/2}}\,dx = \int \frac{1}{(cx^4+a)^{\frac{3}{2}}(ex+d)}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x^4+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^4 + a)^(3/2)*(e*x + d)), x)
```

3.223.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^{3/2}}\,dx = \int \frac{1}{\left(c\,x^4+a\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

input
```
int(1/((a + c*x^4)^(3/2)*(d + e*x)),x)
```

output
```
int(1/((a + c*x^4)^(3/2)*(d + e*x)), x)
```

3.224 $\int \frac{x^3(c+dx)^n}{a+bx^4} dx$

3.224.1 Optimal result

Integrand size = 20, antiderivative size = 349

$$\int \frac{x^3(c+dx)^n}{a+bx^4} dx = -\frac{(c+dx)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d\right)(1+n)} - \frac{(c+dx)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d\right)(1+n)} - \frac{(c+dx)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c-\sqrt[4]{-a}d\right)(1+n)} - \frac{(c+dx)^{1+n}\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c+\sqrt[4]{-a}d\right)(1+n)}$$

output
```
-1/4*(d*x+c)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/4)*(d*x+c)/(b^(1/4)*c-(-a
)^(1/4)*d))/b^(3/4)/(b^(1/4)*c-(-a)^(1/4)*d)/(1+n)-1/4*(d*x+c)^(1+n)*hyper
geom([1, 1+n],[2+n],b^(1/4)*(d*x+c)/(b^(1/4)*c+(-a)^(1/4)*d))/b^(3/4)/(b^(
1/4)*c+(-a)^(1/4)*d)/(1+n)-1/4*(d*x+c)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1
/4)*(d*x+c)/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2)))/b^(3/4)/(1+n)/(b^(1/4)*c-d*
(-(-a)^(1/2))^(1/2))-1/4*(d*x+c)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/4)*(d
*x+c)/(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))/b^(3/4)/(1+n)/(b^(1/4)*c+d*(-(-a)
^(1/2))^(1/2))
```

3.224.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.79

$$\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx = \frac{(c+dx)^{1+n}\left(-\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}-\frac{\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}\right)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}-\frac{\text{Hyperg}}{}\right.}{4b^{3/4}(1+n)}$$

input
```
Integrate[(x^3*(c + d*x)^n)/(a + b*x^4),x]
```

output
```
((c + d*x)^(1 + n)*(-(Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)*(c + d*x
))/(b^(1/4)*c - (-a)^(1/4)*d)]/(b^(1/4)*c - (-a)^(1/4)*d)) - Hypergeometri
c2F1[1, 1 + n, 2 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c - I*(-a)^(1/4)*d)]/(b
^(1/4)*c - I*(-a)^(1/4)*d) - Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)*(
c + d*x))/(b^(1/4)*c + I*(-a)^(1/4)*d)]/(b^(1/4)*c + I*(-a)^(1/4)*d) - Hyp
ergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4
)*d)]/(b^(1/4)*c + (-a)^(1/4)*d)))/(4*b^(3/4)*(1 + n))
```

3.224.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx$$
$$\Big\downarrow \text{7276}$$
$$\int \left(\frac{x(c+dx)^n}{2\left(bx^2-\sqrt{-a}\sqrt{b}\right)}+\frac{x(c+dx)^n}{2\left(\sqrt{-a}\sqrt{b}+bx^2\right)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{(c+dx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}(n+1)\left(\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d\right)}-\frac{(c+dx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}(n+1)\left(\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c\right)}-\frac{(c+dx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{3/4}(n+1)\left(\sqrt[4]{b}c-\sqrt[4]{-a}d\right)}-\frac{(c+dx)^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^{3/4}(n+1)\left(\sqrt[4]{-a}d+\sqrt[4]{b}c\right)}$$

input
```
Int[(x^3*(c + d*x)^n)/(a + b*x^4),x]
```

output
```
-1/4*((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)*(c + d
*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)])/(b^(3/4)*(b^(1/4)*c - Sqrt[-Sqrt[-a
]]*d)*(1 + n)) - ((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^
(1/4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)])/(4*b^(3/4)*(b^(1/4)*c +
 Sqrt[-Sqrt[-a]]*d)*(1 + n)) - ((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 +
 n, 2 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)])/(4*b^(3/4)*(b^
(1/4)*c - (-a)^(1/4)*d)*(1 + n)) - ((c + d*x)^(1 + n)*Hypergeometric2F1[1,
 1 + n, 2 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*b^(3/4)
*(b^(1/4)*c + (-a)^(1/4)*d)*(1 + n))
```

3.224.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.224. $\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx$

3.224.4 Maple [F]

$$\int \frac{x^3 (dx + c)^n}{b\,x^4 + a}dx$$

input
```
int(x^3*(d*x+c)^n/(b*x^4+a),x)
```

output
```
int(x^3*(d*x+c)^n/(b*x^4+a),x)
```

3.224.5 Fricas [F]

$$\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx = \int \frac{(dx+c)^n x^3}{bx^4+a}\,dx$$

input
```
integrate(x^3*(d*x+c)^n/(b*x^4+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^n*x^3/(b*x^4 + a), x)
```

3.224.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(d*x+c)**n/(b*x**4+a),x)
```

output
```
Timed out
```

3.224.7 Maxima [F]

$$\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx = \int \frac{(dx+c)^n x^3}{bx^4+a}\,dx$$

input
```
integrate(x^3*(d*x+c)^n/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^n*x^3/(b*x^4 + a), x)
```

3.224.8 Giac [F]

$$\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx = \int \frac{(dx+c)^n x^3}{bx^4+a}\,dx$$

input
```
integrate(x^3*(d*x+c)^n/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^n*x^3/(b*x^4 + a), x)
```

3.224.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx = \int \frac{x^3\,(c+d\,x)^n}{b\,x^4+a}\,dx$$

input
```
int((x^3*(c + d*x)^n)/(a + b*x^4),x)
```

output
```
int((x^3*(c + d*x)^n)/(a + b*x^4), x)
```

3.225 $\int \frac{x^3(c+dx)^{1+n}}{a+bx^4} dx$

3.225.1 Optimal result

Integrand size = 22, antiderivative size = 349

$$\int \frac{x^3(c+dx)^{1+n}}{a+bx^4} dx = -\frac{(c+dx)^{2+n} \text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d\right)(2+n)} - \frac{(c+dx)^{2+n} \text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d\right)(2+n)} - \frac{(c+dx)^{2+n} \text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c-\sqrt[4]{-a}d\right)(2+n)} - \frac{(c+dx)^{2+n} \text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^{3/4}\left(\sqrt[4]{b}c+\sqrt[4]{-a}d\right)(2+n)}$$

output
```
-1/4*(d*x+c)^(2+n)*hypergeom([1, 2+n],[3+n],b^(1/4)*(d*x+c)/(b^(1/4)*c-(-a
)^(1/4)*d))/b^(3/4)/(b^(1/4)*c-(-a)^(1/4)*d)/(2+n)-1/4*(d*x+c)^(2+n)*hyper
geom([1, 2+n],[3+n],b^(1/4)*(d*x+c)/(b^(1/4)*c+(-a)^(1/4)*d))/b^(3/4)/(b^(
1/4)*c+(-a)^(1/4)*d)/(2+n)-1/4*(d*x+c)^(2+n)*hypergeom([1, 2+n],[3+n],b^(1
/4)*(d*x+c)/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2)))/b^(3/4)/(2+n)/(b^(1/4)*c-d*
(-(-a)^(1/2))^(1/2))-1/4*(d*x+c)^(2+n)*hypergeom([1, 2+n],[3+n],b^(1/4)*(d
*x+c)/(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))/b^(3/4)/(2+n)/(b^(1/4)*c+d*(-(-a)
^(1/2))^(1/2))
```

3.225.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.79

$$\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx$$

$$= \frac{(c+dx)^{2+n}\left(-\frac{\text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}-\frac{\text{Hypergeometric2F1}\left(1,2+n,3+n,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}\right)}{\sqrt[4]{b}c-i\sqrt[4]{-a}d}-\frac{\text{Hyperg}}{}\right.}{4b^{3/4}(2+n)}$$

input
```
Integrate[(x^3*(c + d*x)^(1 + n))/(a + b*x^4),x]
```

output
```
((c + d*x)^(2 + n)*(-(Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/4)*(c + d*x
))/(b^(1/4)*c - (-a)^(1/4)*d)]/(b^(1/4)*c - (-a)^(1/4)*d)) - Hypergeometri
c2F1[1, 2 + n, 3 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c - I*(-a)^(1/4)*d)]/(b
^(1/4)*c - I*(-a)^(1/4)*d) - Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/4)*(
c + d*x))/(b^(1/4)*c + I*(-a)^(1/4)*d)]/(b^(1/4)*c + I*(-a)^(1/4)*d) - Hyp
ergeometric2F1[1, 2 + n, 3 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4
)*d)]/(b^(1/4)*c + (-a)^(1/4)*d)))/(4*b^(3/4)*(2 + n))
```

3.225.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c+dx)^{n+1}}{a+bx^4}\,dx$$

$$\Big\downarrow \; 7276$$

$$\int \left(\frac{x(c+dx)^{n+1}}{2\left(bx^2-\sqrt{-a}\sqrt{b}\right)}+\frac{x(c+dx)^{n+1}}{2\left(\sqrt{-a}\sqrt{b}+bx^2\right)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{(c+dx)^{n+2}\,\text{Hypergeometric2F1}\left(1,n+2,n+3,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}(n+2)\left(\sqrt[4]{b}c-\sqrt{-\sqrt{-a}}d\right)}-\frac{(c+dx)^{n+2}\,\text{Hypergeometric2F1}\left(1,n+2,n+3,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt{-\sqrt{-a}}d}\right)}{4b^{3/4}(n+2)\left(\sqrt{-\sqrt{-a}}d+\sqrt[4]{b}c\right)}-\frac{(c+dx)^{n+2}\,\text{Hypergeometric2F1}\left(1,n+2,n+3,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c-\sqrt[4]{-a}d}\right)}{4b^{3/4}(n+2)\left(\sqrt[4]{b}c-\sqrt[4]{-a}d\right)}-\frac{(c+dx)^{n+2}\,\text{Hypergeometric2F1}\left(1,n+2,n+3,\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{b}c+\sqrt[4]{-a}d}\right)}{4b^{3/4}(n+2)\left(\sqrt[4]{-a}d+\sqrt[4]{b}c\right)}$$

input
```
Int[(x^3*(c + d*x)^(1 + n))/(a + b*x^4),x]
```

output
```
-1/4*((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/4)*(c + d
*x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)])/(b^(3/4)*(b^(1/4)*c - Sqrt[-Sqrt[-a
]]*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^
(1/4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)])/(4*b^(3/4)*(b^(1/4)*c +
 Sqrt[-Sqrt[-a]]*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 +
 n, 3 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)])/(4*b^(3/4)*(b^
(1/4)*c - (-a)^(1/4)*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1,
 2 + n, 3 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*b^(3/4)
*(b^(1/4)*c + (-a)^(1/4)*d)*(2 + n))
```

3.225.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.225. $\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx$

3.225.4 Maple [F]

$$\int \frac{x^3 (dx+c)^{1+n}}{b\,x^4+a}dx$$

input
```
int(x^3*(d*x+c)^(1+n)/(b*x^4+a),x)
```

output
```
int(x^3*(d*x+c)^(1+n)/(b*x^4+a),x)
```

3.225.5 Fricas [F]

$$\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx = \int \frac{(dx+c)^{n+1}x^3}{bx^4+a}\,dx$$

input
```
integrate(x^3*(d*x+c)^(1+n)/(b*x^4+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^(n + 1)*x^3/(b*x^4 + a), x)
```

3.225.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(d*x+c)**(1+n)/(b*x**4+a),x)
```

output
```
Timed out
```

3.225.7 Maxima [F]

$$\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx = \int \frac{(dx+c)^{n+1}x^3}{bx^4+a}\,dx$$

input
```
integrate(x^3*(d*x+c)^(1+n)/(b*x^4+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(n + 1)*x^3/(b*x^4 + a), x)
```

3.225.8 Giac [F]

$$\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx = \int \frac{(dx+c)^{n+1}x^3}{bx^4+a}\,dx$$

input
```
integrate(x^3*(d*x+c)^(1+n)/(b*x^4+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(n + 1)*x^3/(b*x^4 + a), x)
```

3.225.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx = \int \frac{x^3\,(c+d\,x)^{n+1}}{b\,x^4+a}\,dx$$

input
```
int((x^3*(c + d*x)^(n + 1))/(a + b*x^4),x)
```

output
```
int((x^3*(c + d*x)^(n + 1))/(a + b*x^4), x)
```

3.226 $\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx$

3.226.1 Optimal result

Integrand size = 24, antiderivative size = 1605

$$\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx = \text{Too large to display}$$

output
```
1/2*b^(1/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1
/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a
^(1/2)+x^2*b^(1/2))*(d-(-4*c*e+d^2)^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2)
)^2)^(1/2)/a^(1/4)/(-4*c*e+d^2)^(1/2)/(2*e^2*a^(1/2)+b^(1/2)*(d^2-2*c*e-d*
(-4*c*e+d^2)^(1/2)))/(b*x^4+a)^(1/2)+1/2*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)
))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1/
4)*x/a^(1/4))),1/4*(2*e^2*a^(1/2)+b^(1/2)*(d^2-2*c*e-d*(-4*c*e+d^2)^(1/2))
)^2/e^2/a^(1/2)/b^(1/2)/(d-(-4*c*e+d^2)^(1/2))^2,1/2*2^(1/2))*(a^(1/2)+x^2
*b^(1/2))*(2*e^2*a^(1/2)-b^(1/2)*(d^2-2*c*e-d*(-4*c*e+d^2)^(1/2)))*((b*x^4
+a)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/(d-(-4*c*e+d^2)^(1/2))/
(-4*c*e+d^2)^(1/2)/(2*e^2*a^(1/2)+b^(1/2)*(d^2-2*c*e-d*(-4*c*e+d^2)^(1/2))
)/(b*x^4+a)^(1/2)-1/2*b^(1/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4)
)),1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2))*(d+(-4*c*e+d^2)^(1/2))*((b*x^4+a)/(a
^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/(-4*c*e+d^2)^(1/2)/(2*e^2*a^(1/2)+b^(
1/2)*(d^2-2*c*e+d*(-4*c*e+d^2)^(1/2)))/(b*x^4+a)^(1/2)-1/2*e*(cos(2*arctan
(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(
sin(2*arctan(b^(1/4)*x/a^(1/4))),1/4*(2*e^2*a^(1/2)+b^(1/2)*(d^2-2*c*e+d*(
-4*c*e+d^2)^(1/2)))^2/e^2/a^(1/2)/b^(1/2)/(d+(-4*c*e+d^2)^(1/2))^2,1/2*2^(
1/2))*(a^(1/2)+x^2*b^(1/2))*(2*e^2*a^(1/2)-b^(1/2)*(d^2-2*c*e+d*(-4*c*e...
```

3.226.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 11.25 (sec) , antiderivative size = 455, normalized size of antiderivative = 0.28

$$\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx =$$

$$-\frac{i\sqrt{1+\frac{bx^4}{a}}\left(\left(-d^2+\sqrt{d^4-4cd^2e}\right)\text{EllipticPi}\left(\frac{2i\sqrt{a}e^2}{\sqrt{b}\left(d^2-2ce+\sqrt{d^4-4cd^2e}\right)},i\text{arcsinh}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x\right),-1\right)+\left(d^2+\sqrt{\right.\right.}{2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}c\sqrt{d^4-4cd^2e}\sqrt{a+bx}}$$

$$+\sqrt{b}d\text{RootSum}\Bigg[a^2e^2-2a\sqrt{b}d^2\#1+4a\sqrt{b}ce\#1+4bc^2\#1^2-2ae^2\#1^2+2\sqrt{b}d^2\#1^3$$

$$-4\sqrt{b}ce\#1^3$$

$$+e^2\#1^4\&,\frac{\log\left(-\sqrt{b}x^2+\sqrt{a+bx^4}-\#1\right)\#1}{-a\sqrt{b}d^2+2a\sqrt{b}ce+4bc^2\#1-2ae^2\#1+3\sqrt{b}d^2\#1^2-6\sqrt{b}ce\#1^2+2e^2\#1^3}\&\Bigg]$$

input
```
Integrate[1/((c + d*x + e*x^2)*Sqrt[a + b*x^4]),x]
```

output
```
((-1/2*I)*Sqrt[1 + (b*x^4)/a]*((-d^2 + Sqrt[d^4 - 4*c*d^2*e])*EllipticPi[(
(2*I)*Sqrt[a]*e^2)/(Sqrt[b]*(d^2 - 2*c*e + Sqrt[d^4 - 4*c*d^2*e])), I*ArcS
inh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] + (d^2 + Sqrt[d^4 - 4*c*d^2*e])*Elli
pticPi[((-2*I)*Sqrt[a]*e^2)/(Sqrt[b]*(-d^2 + 2*c*e + Sqrt[d^4 - 4*c*d^2*e]
)), I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1]))/(Sqrt[(I*Sqrt[b])/Sqrt[a
]]*c*Sqrt[d^4 - 4*c*d^2*e]*Sqrt[a + b*x^4]) + Sqrt[b]*d*RootSum[a^2*e^2 -
2*a*Sqrt[b]*d^2*#1 + 4*a*Sqrt[b]*c*e*#1 + 4*b*c^2*#1^2 - 2*a*e^2*#1^2 + 2*
Sqrt[b]*d^2*#1^3 - 4*Sqrt[b]*c*e*#1^3 + e^2*#1^4 & , (Log[-(Sqrt[b]*x^2) +
 Sqrt[a + b*x^4] - #1]*#1)/(-(a*Sqrt[b]*d^2) + 2*a*Sqrt[b]*c*e + 4*b*c^2*#
1 - 2*a*e^2*#1 + 3*Sqrt[b]*d^2*#1^2 - 6*Sqrt[b]*c*e*#1^2 + 2*e^2*#1^3) & ]
```

3.226.3 Rubi [A] (warning: unable to verify)

Time = 8.07 (sec) , antiderivative size = 1590, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx+ex^2)}\,dx$$

$$\Big\downarrow \text{ 7279}$$

$$\int \left(\frac{2e}{\sqrt{a+bx^4}\sqrt{d^2-4ce}\left(-\sqrt{d^2-4ce}+d+2ex\right)}-\frac{2e}{\sqrt{a+bx^4}\sqrt{d^2-4ce}\left(\sqrt{d^2-4ce}+d+2ex\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt{bd^4-4bced^2-b\sqrt{d^2-4ce}(d^2-2ce)d+2ae^4+2bc^2e^2}x}{e\left(d-\sqrt{d^2-4ce}\right)\sqrt{bx^4+a}}\right)e^2}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{2ae^4+b\left(d^4-\sqrt{d^2-4ce}d^3-4ced^2+2ce\sqrt{d^2-4ce}d+2c^2e^2\right)}}-\frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt{bd^4-4bced^2+b\sqrt{d^2-4ce}(d^2-2ce)d+2ae^4+2bc^2e^2}x}{e\left(d+\sqrt{d^2-4ce}\right)\sqrt{bx^4+a}}\right)e^2}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{2ae^4+b\left(d^4+\sqrt{d^2-4ce}d^3-4ced^2-2ce\sqrt{d^2-4ce}d+2c^2e^2\right)}}-\frac{\text{arctanh}\left(\frac{4ae^2+b\left(d-\sqrt{d^2-4ce}\right)^2x^2}{2\sqrt{2}\sqrt{bd^4-4bced^2-b\sqrt{d^2-4ce}(d^2-2ce)d+2ae^4+2bc^2e^2}\sqrt{bx^4+a}}\right)e^2}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{bd^4-4bced^2-b\sqrt{d^2-4ce}\left(d^2-2ce\right)d+2ae^4+2bc^2e^2}}+\frac{\text{arctanh}\left(\frac{4ae^2+b\left(d+\sqrt{d^2-4ce}\right)^2x^2}{2\sqrt{2}\sqrt{bd^4-4bced^2+b\sqrt{d^2-4ce}(d^2-2ce)d+2ae^4+2bc^2e^2}\sqrt{bx^4+a}}\right)e^2}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{bd^4-4bced^2+b\sqrt{d^2-4ce}\left(d^2-2ce\right)d+2ae^4+2bc^2e^2}}+\frac{\sqrt[4]{b}\left(d-\sqrt{d^2-4ce}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)e}{2\sqrt[4]{a}\sqrt{d^2-4ce}\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2-\sqrt{d^2-4ce}d-2ce\right)\right)\sqrt{bx^4+a}}-\frac{\sqrt[4]{b}\left(d+\sqrt{d^2-4ce}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)e}{2\sqrt[4]{a}\sqrt{d^2-4ce}\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2+\sqrt{d^2-4ce}d-2ce\right)\right)\sqrt{bx^4+a}}+\frac{\left(2\sqrt{a}e^2-\sqrt{b}\left(d^2-\sqrt{d^2-4ce}d-2ce\right)\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{EllipticPi}\left(\frac{\sqrt{a}\left(4e^2+\frac{\sqrt{b}\left(d-\sqrt{d^2-4ce}\right)^2}{\sqrt{a}}\right)^2}{16\sqrt{b}e^2\left(d-\sqrt{d^2-4ce}\right)^2},2\,\text{arct}\right.}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{d^2-4ce}\left(d-\sqrt{d^2-4ce}\right)\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2-\sqrt{d^2-4ce}d-2ce\right)\right)\sqrt{bx^4+a}}$$

$$\frac{\left(2\sqrt{a}e^2-\sqrt{b}\left(d^2+\sqrt{d^2-4ce}d-2ce\right)\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\text{EllipticPi}\left(\frac{\sqrt{a}\left(4e^2+\frac{\sqrt{b}\left(d+\sqrt{d^2-4ce}\right)^2}{\sqrt{a}}\right)^2}{16\sqrt{b}e^2\left(d+\sqrt{d^2-4ce}\right)^2},2\,\text{arct}\right.}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{d^2-4ce}\left(d+\sqrt{d^2-4ce}\right)\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2+\sqrt{d^2-4ce}d-2ce\right)\right)\sqrt{bx^4+a}}$$

input
```
Int[1/((c + d*x + e*x^2)*Sqrt[a + b*x^4]),x]
```

3.226. $\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx$

output
```
(e^2*ArcTanh[(Sqrt[2]*Sqrt[b*d^4 - 4*b*c*d^2*e + 2*b*c^2*e^2 + 2*a*e^4 - b
*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]*x)/(e*(d - Sqrt[d^2 - 4*c*e])*Sqrt[a +
 b*x^4])])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[2*a*e^4 + b*(d^4 - 4*c*d^2*e +
2*c^2*e^2 - d^3*Sqrt[d^2 - 4*c*e] + 2*c*d*e*Sqrt[d^2 - 4*c*e])]) - (e^2*Ar
cTanh[(Sqrt[2]*Sqrt[b*d^4 - 4*b*c*d^2*e + 2*b*c^2*e^2 + 2*a*e^4 + b*d*Sqrt
[d^2 - 4*c*e]*(d^2 - 2*c*e)]*x)/(e*(d + Sqrt[d^2 - 4*c*e])*Sqrt[a + b*x^4]
)])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[2*a*e^4 + b*(d^4 - 4*c*d^2*e + 2*c^2*e
^2 + d^3*Sqrt[d^2 - 4*c*e] - 2*c*d*e*Sqrt[d^2 - 4*c*e])]) - (e^2*ArcTanh[(
4*a*e^2 + b*(d - Sqrt[d^2 - 4*c*e])^2*x^2)/(2*Sqrt[2]*Sqrt[b*d^4 - 4*b*c*d
^2*e + 2*b*c^2*e^2 + 2*a*e^4 - b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]*Sqrt[a
 + b*x^4])])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[b*d^4 - 4*b*c*d^2*e + 2*b*c^2
*e^2 + 2*a*e^4 - b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]) + (e^2*ArcTanh[(4*a
*e^2 + b*(d + Sqrt[d^2 - 4*c*e])^2*x^2)/(2*Sqrt[2]*Sqrt[b*d^4 - 4*b*c*d^2*
e + 2*b*c^2*e^2 + 2*a*e^4 + b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]*Sqrt[a +
b*x^4])])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[b*d^4 - 4*b*c*d^2*e + 2*b*c^2*e^
2 + 2*a*e^4 + b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]) + (b^(1/4)*e*(d - Sqrt
[d^2 - 4*c*e])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]
*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*Sqrt[d^
2 - 4*c*e]*(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e - d*Sqrt[d^2 - 4*c*e]))*S
qrt[a + b*x^4]) - (b^(1/4)*e*(d + Sqrt[d^2 - 4*c*e])*(Sqrt[a] + Sqrt[b]...
```

3.226.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.226.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.36 (sec) , antiderivative size = 1153, normalized size of antiderivative = 0.72

method	result	size
default	Expression too large to display	1153
elliptic	Expression too large to display	1153

input
```
int(1/(e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(-4*c*e+d^2)^(1/2)/(1/2*b/e^4*d^4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*
b/e^3*c*d^2+b/e^3*c*d*(-4*c*e+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)*arctanh(1/2/(1
/2*b/e^4*d^4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*c*d^2+b/e^3*c*d*(-4*
c*e+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*b*x^2/e^2*d^2-1/2/(1/2*b
/e^4*d^4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*c*d^2+b/e^3*c*d*(-4*c*e+
d^2)^(1/2)+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*b*x^2/e^2*d*(-4*c*e+d^2)^(1/
2)-1/(1/2*b/e^4*d^4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*c*d^2+b/e^3*c
*d*(-4*c*e+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*b*x^2/e*c+1/(1/2*
b/e^4*d^4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*c*d^2+b/e^3*c*d*(-4*c*e
+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*a)-2/(-4*c*e+d^2)^(1/2)/(I/
a^(1/2)*b^(1/2))^(1/2)*e/(-d+(-4*c*e+d^2)^(1/2))*(1-I/a^(1/2)*b^(1/2)*x^2)
^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(
1/2)*b^(1/2))^(1/2),-4*I*a^(1/2)/b^(1/2)*e^2/(-d+(-4*c*e+d^2)^(1/2))^2,(-I
/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))+1/2/(-4*c*e+d^2)^(1/2)/
(1/2*b/e^4*d^4+1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*c*d^2-b/e^3*c*d*(-
4*c*e+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)*arctanh(1/2/(1/2*b/e^4*d^4+1/2*b/e^4*d
^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*c*d^2-b/e^3*c*d*(-4*c*e+d^2)^(1/2)+b/e^2*c^2
+a)^(1/2)/(b*x^4+a)^(1/2)*b*x^2/e^2*d^2+1/2/(1/2*b/e^4*d^4+1/2*b/e^4*d^3*(
-4*c*e+d^2)^(1/2)-2*b/e^3*c*d^2-b/e^3*c*d*(-4*c*e+d^2)^(1/2)+b/e^2*c^2+a)^
(1/2)/(b*x^4+a)^(1/2)*b*x^2/e^2*d*(-4*c*e+d^2)^(1/2)-1/(1/2*b/e^4*d^4+1...
```

3.226.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx=\text{Timed out}$$

input
```
integrate(1/(e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.226.6 Sympy [F]

$$\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx = \int \frac{1}{\sqrt{a+bx^4}\,(c+dx+ex^2)}\,dx$$

input
```
integrate(1/(e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**4)*(c + d*x + e*x**2)), x)
```

3.226.7 Maxima [F]

$$\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+a}(ex^2+dx+c)}\,dx$$

input
```
integrate(1/(e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^4 + a)*(e*x^2 + d*x + c)), x)
```

3.226.8 Giac [F]

$$\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx = \int \frac{1}{\sqrt{bx^4+a}(ex^2+dx+c)}\,dx$$

input
```
integrate(1/(e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^4 + a)*(e*x^2 + d*x + c)), x)
```

3.226.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx = \int \frac{1}{\sqrt{b\,x^4+a}\,\left(e\,x^2+d\,x+c\right)}\,dx$$

input
```
int(1/((a + b*x^4)^(1/2)*(c + d*x + e*x^2)),x)
```

output
```
int(1/((a + b*x^4)^(1/2)*(c + d*x + e*x^2)), x)
```

3.227 $\int x^m \left(c(a+bx^2)^2\right)^{3/2}\,dx$

3.227.1 Optimal result

Integrand size = 19, antiderivative size = 161

$$\int x^m \left(c(a+bx^2)^2\right)^{3/2}\,dx = \frac{a^3cx^{1+m}\sqrt{c\,(a+bx^2)^2}}{(1+m)\,(a+bx^2)} + \frac{3a^2bcx^{3+m}\sqrt{c\,(a+bx^2)^2}}{(3+m)\,(a+bx^2)} + \frac{3ab^2cx^{5+m}\sqrt{c\,(a+bx^2)^2}}{(5+m)\,(a+bx^2)} + \frac{b^3cx^{7+m}\sqrt{c\,(a+bx^2)^2}}{(7+m)\,(a+bx^2)}$$

output
```
a^3*c*x^(1+m)*(c*(b*x^2+a)^2)^(1/2)/(1+m)/(b*x^2+a)+3*a^2*b*c*x^(3+m)*(c*(
b*x^2+a)^2)^(1/2)/(3+m)/(b*x^2+a)+3*a*b^2*c*x^(5+m)*(c*(b*x^2+a)^2)^(1/2)/
(5+m)/(b*x^2+a)+b^3*c*x^(7+m)*(c*(b*x^2+a)^2)^(1/2)/(7+m)/(b*x^2+a)
```

3.227.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.82

$$\int x^m \left(c(a+bx^2)^2\right)^{3/2}\,dx = \frac{x^{1+m}\left(c(a+bx^2)^2\right)^{3/2}\left(a^3(105+71m+15m^2+m^3)+3a^2b(35+47m+13m^2+m^3)\,x^2+\right.}{(1+m)(3+m)(5+m)(7+m)\,(}$$

input
```
Integrate[x^m*(c*(a + b*x^2)^2)^(3/2),x]
```

output
```
(x^(1 + m)*(c*(a + b*x^2)^2)^(3/2)*(a^3*(105 + 71*m + 15*m^2 + m^3) + 3*a^
2*b*(35 + 47*m + 13*m^2 + m^3)*x^2 + 3*a*b^2*(21 + 31*m + 11*m^2 + m^3)*x^
4 + b^3*(15 + 23*m + 9*m^2 + m^3)*x^6))/((1 + m)*(3 + m)*(5 + m)*(7 + m)*(
a + b*x^2)^3)
```

3.227.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.54, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2045, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m\left(c\left(a+bx^2\right)^2\right)^{3/2}dx$$
$$\Big\downarrow \ 2045$$
$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^2}\int \frac{x^m\left(bx^2+a\right)^3}{a^3}dx}{a+bx^2}$$
$$\Big\downarrow \ 27$$
$$\frac{c\sqrt{c\left(a+bx^2\right)^2}\int x^m\left(bx^2+a\right)^3dx}{a+bx^2}$$
$$\Big\downarrow \ 244$$
$$\frac{c\sqrt{c\left(a+bx^2\right)^2}\int \left(a^3x^m+3a^2bx^{m+2}+3ab^2x^{m+4}+b^3x^{m+6}\right)dx}{a+bx^2}$$
$$\Big\downarrow \ 2009$$
$$\frac{c\sqrt{c\left(a+bx^2\right)^2}\left(\frac{a^3x^{m+1}}{m+1}+\frac{3a^2bx^{m+3}}{m+3}+\frac{3ab^2x^{m+5}}{m+5}+\frac{b^3x^{m+7}}{m+7}\right)}{a+bx^2}$$

input
```
Int[x^m*(c*(a + b*x^2)^2)^(3/2),x]
```

output
```
(c*Sqrt[c*(a + b*x^2)^2]*((a^3*x^(1 + m))/(1 + m) + (3*a^2*b*x^(3 + m))/(3
 + m) + (3*a*b^2*x^(5 + m))/(5 + m) + (b^3*x^(7 + m))/(7 + m)))/(a + b*x^2
)
```

3.227.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.227.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.24

method	result
gosper	$\frac{x^{1+m} \left(c \left(b\,x^{2}+a \right)^{2}\right)^{\frac{3}{2}} \left(b^{3} m^{3} x^{6}+9 b^{3} m^{2} x^{6}+3 a\,b^{2} m^{3} x^{4}+23 m\,x^{6} b^{3}+33 a\,b^{2} m^{2} x^{4}+15 b^{3} x^{6}+3 a^{2} b\,m^{3} x^{2}+93 m\,x^{4} b^{2} a+39 a^{2} b\,m^{2} x^{2}+63 b\right.}{\left(1+m \right) \left(3+m \right) \left(5+m \right) \left(7+m \right) \left(b\,x^{2}+a \right)^{3}}$
risch	$\frac{c \sqrt{c \left(b\,x^{2}+a \right)^{2}}\, \left(b^{3} m^{3} x^{6}+9 b^{3} m^{2} x^{6}+3 a\,b^{2} m^{3} x^{4}+23 m\,x^{6} b^{3}+33 a\,b^{2} m^{2} x^{4}+15 b^{3} x^{6}+3 a^{2} b\,m^{3} x^{2}+93 m\,x^{4} b^{2} a+39 a^{2} b\,m^{2} x^{2}+63 b^{2} x^{4} a+\right.}{\left(b\,x^{2}+a \right) \left(7+m \right) \left(5+m \right) \left(3+m \right) \left(1+m \right)}$

input
```
int(x^m*(c*(b*x^2+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x^(1+m)/(1+m)/(3+m)/(5+m)/(7+m)/(b*x^2+a)^3*(c*(b*x^2+a)^2)^(3/2)*(b^3*m^3
*x^6+9*b^3*m^2*x^6+3*a*b^2*m^3*x^4+23*b^3*m*x^6+33*a*b^2*m^2*x^4+15*b^3*x^
6+3*a^2*b*m^3*x^2+93*a*b^2*m*x^4+39*a^2*b*m^2*x^2+63*a*b^2*x^4+a^3*m^3+141
*a^2*b*m*x^2+15*a^3*m^2+105*a^2*b*x^2+71*a^3*m+105*a^3)
```

3.227.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.45

$$\int x^m \left(c(a+bx^2)^2\right)^{3/2} dx = \frac{((b^3cm^3 + 9\,b^3cm^2 + 23\,b^3cm + 15\,b^3c)x^7 + 3\,(ab^2cm^3 + 11\,ab^2cm^2 + 31\,ab^2cm + 21\,ab^2c)x}{am^4 + 16\,am^3 + 86\,am^2 + (b}$$

input
```
integrate(x^m*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

output
```
((b^3*c*m^3 + 9*b^3*c*m^2 + 23*b^3*c*m + 15*b^3*c)*x^7 + 3*(a*b^2*c*m^3 +
11*a*b^2*c*m^2 + 31*a*b^2*c*m + 21*a*b^2*c)*x^5 + 3*(a^2*b*c*m^3 + 13*a^2*
b*c*m^2 + 47*a^2*b*c*m + 35*a^2*b*c)*x^3 + (a^3*c*m^3 + 15*a^3*c*m^2 + 71*
a^3*c*m + 105*a^3*c)*x)*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)*x^m/(a*m^4 +
 16*a*m^3 + 86*a*m^2 + (b*m^4 + 16*b*m^3 + 86*b*m^2 + 176*b*m + 105*b)*x^2
 + 176*a*m + 105*a)
```

3.227.6 Sympy [F(-1)]

Timed out.

$$\int x^m \left(c(a+bx^2)^2\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate(x**m*(c*(b*x**2+a)**2)**(3/2),x)
```

output
```
Timed out
```

3.227.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.74

$$\int x^m \left(c(a+bx^2)^2\right)^{3/2} dx = \frac{\left((m^3 + 9\,m^2 + 23\,m + 15)b^3c^{\frac{3}{2}}x^7 + 3\,(m^3 + 11\,m^2 + 31\,m + 21)ab^2c^{\frac{3}{2}}x^5 + 3\,(m^3 + 13\,m^2 +}{m^4 + 16\,m^3 + 86\,m^2 + 176\,m +}$$

input
```
integrate(x^m*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

output
```
((m^3 + 9*m^2 + 23*m + 15)*b^3*c^(3/2)*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)*
a*b^2*c^(3/2)*x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*a^2*b*c^(3/2)*x^3 + (m^3
+ 15*m^2 + 71*m + 105)*a^3*c^(3/2)*x)*x^m/(m^4 + 16*m^3 + 86*m^2 + 176*m +
 105)
```

3.227.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(153) = 306$.

Time = 0.34 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.20

$$\int x^m\left(c(a+bx^2)^2\right)^{3/2}\,dx = \frac{(b^3m^3x^7x^m\mathrm{sgn}(bx^2+a) + 9\,b^3m^2x^7x^m\mathrm{sgn}(bx^2+a) + 3\,ab^2m^3x^5x^m\mathrm{sgn}(bx^2+a) + 23\,b^3mx}{}$$

input
```
integrate(x^m*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

output
```
(b^3*m^3*x^7*x^m*sgn(b*x^2 + a) + 9*b^3*m^2*x^7*x^m*sgn(b*x^2 + a) + 3*a*b
^2*m^3*x^5*x^m*sgn(b*x^2 + a) + 23*b^3*m*x^7*x^m*sgn(b*x^2 + a) + 33*a*b^2
*m^2*x^5*x^m*sgn(b*x^2 + a) + 15*b^3*x^7*x^m*sgn(b*x^2 + a) + 3*a^2*b*m^3*
x^3*x^m*sgn(b*x^2 + a) + 93*a*b^2*m*x^5*x^m*sgn(b*x^2 + a) + 39*a^2*b*m^2*
x^3*x^m*sgn(b*x^2 + a) + 63*a*b^2*x^5*x^m*sgn(b*x^2 + a) + a^3*m^3*x*x^m*s
gn(b*x^2 + a) + 141*a^2*b*m*x^3*x^m*sgn(b*x^2 + a) + 15*a^3*m^2*x*x^m*sgn(
b*x^2 + a) + 105*a^2*b*x^3*x^m*sgn(b*x^2 + a) + 71*a^3*m*x*x^m*sgn(b*x^2 +
 a) + 105*a^3*x*x^m*sgn(b*x^2 + a))*c^(3/2)/(m^4 + 16*m^3 + 86*m^2 + 176*m
 + 105)
```

3.227.9 Mupad [B] (verification not implemented)

Time = 18.20 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.45

$$\int x^m\left(c(a+bx^2)^2\right)^{3/2}\,dx = \frac{x^m\left(\frac{3\,a^2\,c\,x^3\,\sqrt{c\,(b\,x^2+a)^2}\,(m^3+13\,m^2+47\,m+35)}{m^4+16\,m^3+86\,m^2+176\,m+105} + \frac{b^2\,c\,x^7\,\sqrt{c\,(b\,x^2+a)^2}\,(m^3+9\,m^2+23\,m+15)}{m^4+16\,m^3+86\,m^2+176\,m+105} + \frac{3\,a\,b\,c\,x^5\,\sqrt{c\,(b\,x^2}{m^4+16\,m}\right.}{\frac{a}{b}+x^2}$$

input
```
int(x^m*(c*(a + b*x^2)^2)^(3/2),x)
```

output
```
(x^m*((3*a^2*c*x^3*(c*(a + b*x^2)^2)^(1/2)*(47*m + 13*m^2 + m^3 + 35))/(17
6*m + 86*m^2 + 16*m^3 + m^4 + 105) + (b^2*c*x^7*(c*(a + b*x^2)^2)^(1/2)*(2
3*m + 9*m^2 + m^3 + 15))/(176*m + 86*m^2 + 16*m^3 + m^4 + 105) + (3*a*b*c*
x^5*(c*(a + b*x^2)^2)^(1/2)*(31*m + 11*m^2 + m^3 + 21))/(176*m + 86*m^2 +
16*m^3 + m^4 + 105) + (a^3*c*x*(c*(a + b*x^2)^2)^(1/2)*(71*m + 15*m^2 + m^
3 + 105))/(b*(176*m + 86*m^2 + 16*m^3 + m^4 + 105))))/(a/b + x^2)
```

3.227. $\int x^m \left(c(a+bx^2)^2\right)^{3/2} dx$

3.228 $\int x^5 \left(c(a+bx^2)^2\right)^{3/2} dx$

3.228.1 Optimal result

Integrand size = 19, antiderivative size = 143

$$\int x^5 \left(c(a+bx^2)^2\right)^{3/2} dx = \frac{a^3cx^6\sqrt{c\,(a+bx^2)^2}}{6\,(a+bx^2)} + \frac{3a^2bcx^8\sqrt{c\,(a+bx^2)^2}}{8\,(a+bx^2)} + \frac{3ab^2cx^{10}\sqrt{c\,(a+bx^2)^2}}{10\,(a+bx^2)} + \frac{b^3cx^{12}\sqrt{c\,(a+bx^2)^2}}{12\,(a+bx^2)}$$

output
```
1/6*a^3*c*x^6*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+3/8*a^2*b*c*x^8*(c*(b*x^2+a)
^2)^(1/2)/(b*x^2+a)+3/10*a*b^2*c*x^10*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/12
*b^3*c*x^12*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

3.228.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.44

$$\int x^5 \left(c(a+bx^2)^2\right)^{3/2} dx = \frac{x^6\left(c(a+bx^2)^2\right)^{3/2}(20a^3+45a^2bx^2+36ab^2x^4+10b^3x^6)}{120\,(a+bx^2)^3}$$

input
```
Integrate[x^5*(c*(a + b*x^2)^2)^(3/2),x]
```

output
```
(x^6*(c*(a + b*x^2)^2)^(3/2)*(20*a^3 + 45*a^2*b*x^2 + 36*a*b^2*x^4 + 10*b^
3*x^6))/(120*(a + b*x^2)^3)
```

3.228.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.50, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2045, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(c(a+bx^2)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{2045}$$

$$\frac{a^3c\sqrt{c\,(a+bx^2)^2}\int \frac{x^5\,(bx^2+a)^3}{a^3}dx}{a+bx^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\sqrt{c\,(a+bx^2)^2}\int x^5\,(bx^2+a)^3\,dx}{a+bx^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{c\sqrt{c\,(a+bx^2)^2}\int x^4\,(bx^2+a)^3\,dx^2}{2\,(a+bx^2)}$$

$$\Big\downarrow \text{49}$$

$$\frac{c\sqrt{c\,(a+bx^2)^2}\int \left(b^3x^{10}+3ab^2x^8+3a^2bx^6+a^3x^4\right)dx^2}{2\,(a+bx^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\left(\frac{a^3x^6}{3}+\frac{3}{4}a^2bx^8+\frac{3}{5}ab^2x^{10}+\frac{b^3x^{12}}{6}\right)\sqrt{c\,(a+bx^2)^2}}{2\,(a+bx^2)}$$

input
```
Int[x^5*(c*(a + b*x^2)^2)^(3/2),x]
```

output
```
(c*Sqrt[c*(a + b*x^2)^2]*((a^3*x^6)/3 + (3*a^2*b*x^8)/4 + (3*a*b^2*x^10)/5
 + (b^3*x^12)/6))/(2*(a + b*x^2))
```

3.228.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.228.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.42

method	result	size
gosper	$\frac{x^{6}\left(10 b^{3} x^{6}+36 b^{2} x^{4} a+45 a^{2} b \,x^{2}+20 a^{3}\right) \left(c \left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{120 \left(b \,x^{2}+a \right)^{3}}$	60
default	$\frac{x^{6}\left(10 b^{3} x^{6}+36 b^{2} x^{4} a+45 a^{2} b \,x^{2}+20 a^{3}\right) \left(c \left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{120 \left(b \,x^{2}+a \right)^{3}}$	60
pseudoelliptic	$\frac{x^{6}\left(10 b^{3} x^{6}+36 b^{2} x^{4} a+45 a^{2} b \,x^{2}+20 a^{3}\right) c \sqrt{c \left(b \,x^{2}+a \right)^{2}}}{120 b \,x^{2}+120 a}$	63
trager	$\frac{c \,x^{6}\left(10 b^{3} x^{6}+36 b^{2} x^{4} a+45 a^{2} b \,x^{2}+20 a^{3}\right) \sqrt{b^{2} c \,x^{4}+2 a b c \,x^{2}+c \,a^{2}}}{120 b \,x^{2}+120 a}$	72
risch	$\frac{a^{3} c \,x^{6} \sqrt{c \left(b \,x^{2}+a \right)^{2}}}{6 b \,x^{2}+6 a}+\frac{3 a^{2} b c \,x^{8} \sqrt{c \left(b \,x^{2}+a \right)^{2}}}{8 \left(b \,x^{2}+a \right)}+\frac{3 a \,b^{2} c \,x^{10} \sqrt{c \left(b \,x^{2}+a \right)^{2}}}{10 \left(b \,x^{2}+a \right)}+\frac{b^{3} c \,x^{12} \sqrt{c \left(b \,x^{2}+a \right)^{2}}}{12 b \,x^{2}+12 a}$	128

input
```
int(x^5*(c*(b*x^2+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/120*x^6*(10*b^3*x^6+36*a*b^2*x^4+45*a^2*b*x^2+20*a^3)*(c*(b*x^2+a)^2)^(3
/2)/(b*x^2+a)^3
```

3.228.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.52

$$\int x^5\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\frac{\left(10\,b^3cx^{12}+36\,ab^2cx^{10}+45\,a^2bcx^8+20\,a^3cx^6\right)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{120\,\left(bx^2+a\right)}$$

input
```
integrate(x^5*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/120*(10*b^3*c*x^12 + 36*a*b^2*c*x^10 + 45*a^2*b*c*x^8 + 20*a^3*c*x^6)*sq
rt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

3.228.6 Sympy [F]

$$\int x^5\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\int x^5\left(c\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**5*(c*(b*x**2+a)**2)**(3/2),x)
```

output
```
Integral(x**5*(c*(a + b*x**2)**2)**(3/2), x)
```

3.228.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.95

$$\int x^5\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\frac{\left(b^2cx^4+2\,abcx^2+a^2c\right)^{\frac{3}{2}}a^2x^2}{8\,b^2}+\frac{\left(b^2cx^4+2\,abcx^2+a^2c\right)^{\frac{3}{2}}a^3}{8\,b^3}+\frac{\left(b^2cx^4+2\,abcx^2+a^2c\right)^{\frac{5}{2}}x^2}{12\,b^2c}-\frac{7\left(b^2cx^4+2\,abcx^2+a^2c\right)^{\frac{5}{2}}a}{60\,b^3c}$$

input
```
integrate(x^5*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/8*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3/2)*a^2*x^2/b^2 + 1/8*(b^2*c*x^4 +
 2*a*b*c*x^2 + a^2*c)^(3/2)*a^3/b^3 + 1/12*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*
c)^(5/2)*x^2/(b^2*c) - 7/60*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(5/2)*a/(b^3
*c)
```

3.228.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.50

$$\int x^5\left(c(a+bx^2)^2\right)^{3/2}\,dx = \frac{1}{120}\left(10\,b^3x^{12}\mathrm{sgn}\left(bx^2+a\right)+36\,ab^2x^{10}\mathrm{sgn}\left(bx^2+a\right)+45\,a^2bx^8\mathrm{sgn}\left(bx^2+a\right)+20\,a^3x^6\mathrm{sgn}(\right.$$

input
```
integrate(x^5*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

output
```
1/120*(10*b^3*x^12*sgn(b*x^2 + a) + 36*a*b^2*x^10*sgn(b*x^2 + a) + 45*a^2*
b*x^8*sgn(b*x^2 + a) + 20*a^3*x^6*sgn(b*x^2 + a))*c^(3/2)
```

3.228.9 Mupad [F(-1)]

Timed out.

$$\int x^5\left(c(a+bx^2)^2\right)^{3/2}\,dx = \int x^5\,\left(c\,{\left(b\,x^2+a\right)}^2\right)^{3/2}\,dx$$

input
```
int(x^5*(c*(a + b*x^2)^2)^(3/2),x)
```

output
```
int(x^5*(c*(a + b*x^2)^2)^(3/2), x)
```

3.229 $\int x^4 \left(c(a+bx^2)^2\right)^{3/2} dx$

3.229.1 Optimal result

Integrand size = 19, antiderivative size = 143

$$\int x^4 \left(c(a+bx^2)^2\right)^{3/2} dx = \frac{a^3cx^5\sqrt{c(a+bx^2)^2}}{5(a+bx^2)} + \frac{3a^2bcx^7\sqrt{c(a+bx^2)^2}}{7(a+bx^2)} + \frac{ab^2cx^9\sqrt{c(a+bx^2)^2}}{3(a+bx^2)} + \frac{b^3cx^{11}\sqrt{c(a+bx^2)^2}}{11(a+bx^2)}$$

output
```
1/5*a^3*c*x^5*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+3/7*a^2*b*c*x^7*(c*(b*x^2+a)
^2)^(1/2)/(b*x^2+a)+1/3*a*b^2*c*x^9*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/11*b
^3*c*x^11*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

3.229.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.44

$$\int x^4 \left(c(a+bx^2)^2\right)^{3/2} dx = \frac{x^5\left(c(a+bx^2)^2\right)^{3/2}(231a^3+495a^2bx^2+385ab^2x^4+105b^3x^6)}{1155(a+bx^2)^3}$$

input
```
Integrate[x^4*(c*(a + b*x^2)^2)^(3/2),x]
```

output
```
(x^5*(c*(a + b*x^2)^2)^(3/2)*(231*a^3 + 495*a^2*b*x^2 + 385*a*b^2*x^4 + 10
5*b^3*x^6))/(1155*(a + b*x^2)^3)
```

3.229.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.48, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2045, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(c \left(a+b x^2\right)^2\right)^{3/2} \, dx$$

$$\Big\downarrow \text{2045}$$

$$\frac{a^3 c \sqrt{c \left(a+b x^2\right)^2} \int \frac{x^4 \left(b x^2+a\right)^3}{a^3}dx}{a+b x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{c \sqrt{c \left(a+b x^2\right)^2} \int x^4 \left(b x^2+a\right)^3dx}{a+b x^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{c \sqrt{c \left(a+b x^2\right)^2} \int \left(b^3 x^{10}+3 a b^2 x^8+3 a^2 b x^6+a^3 x^4\right)dx}{a+b x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c \left(\frac{a^3 x^5}{5}+\frac{3}{7} a^2 b x^7+\frac{1}{3} a b^2 x^9+\frac{b^3 x^{11}}{11}\right) \sqrt{c \left(a+b x^2\right)^2}}{a+b x^2}$$

input
```
Int[x^4*(c*(a + b*x^2)^2)^(3/2),x]
```

output
```
(c*Sqrt[c*(a + b*x^2)^2]*((a^3*x^5)/5 + (3*a^2*b*x^7)/7 + (a*b^2*x^9)/3 +
(b^3*x^11)/11))/(a + b*x^2)
```

3.229.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.229.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.42

method	result	size
gosper	$\frac{x^{5}\left(105 b^{3} x^{6}+385 b^{2} x^{4} a+495 a^{2} b\,x^{2}+231 a^{3}\right)\left(c\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{1155\left(b\,x^{2}+a\right)^{3}}$	60
default	$\frac{x^{5}\left(105 b^{3} x^{6}+385 b^{2} x^{4} a+495 a^{2} b\,x^{2}+231 a^{3}\right)\left(c\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{1155\left(b\,x^{2}+a\right)^{3}}$	60
trager	$\frac{c\,x^{5}\left(105 b^{3} x^{6}+385 b^{2} x^{4} a+495 a^{2} b\,x^{2}+231 a^{3}\right)\sqrt{b^{2} c\,x^{4}+2 a b c\,x^{2}+c\,a^{2}}}{1155 b\,x^{2}+1155 a}$	72
risch	$\frac{a^{3} c\,x^{5}\sqrt{c\left(b\,x^{2}+a\right)^{2}}}{5 b\,x^{2}+5 a}+\frac{3 a^{2} b c\,x^{7}\sqrt{c\left(b\,x^{2}+a\right)^{2}}}{7\left(b\,x^{2}+a\right)}+\frac{a\,b^{2} c\,x^{9}\sqrt{c\left(b\,x^{2}+a\right)^{2}}}{3 b\,x^{2}+3 a}+\frac{b^{3} c\,x^{11}\sqrt{c\left(b\,x^{2}+a\right)^{2}}}{11 b\,x^{2}+11 a}$	128

input
```
int(x^4*(c*(b*x^2+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/1155*x^5*(105*b^3*x^6+385*a*b^2*x^4+495*a^2*b*x^2+231*a^3)*(c*(b*x^2+a)^
2)^(3/2)/(b*x^2+a)^3
```

3.229.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.52

$$\int x^4 \left(c(a+bx^2)^2\right)^{3/2} dx = \frac{(105\,b^3cx^{11} + 385\,ab^2cx^9 + 495\,a^2bcx^7 + 231\,a^3cx^5)\sqrt{b^2cx^4 + 2\,abcx^2 + a^2c}}{1155\,(bx^2 + a)}$$

input
```
integrate(x^4*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/1155*(105*b^3*c*x^11 + 385*a*b^2*c*x^9 + 495*a^2*b*c*x^7 + 231*a^3*c*x^5
)*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

3.229.6 Sympy [F]

$$\int x^4 \left(c(a+bx^2)^2\right)^{3/2} dx = \int x^4 \left(c(a+bx^2)^2\right)^{\frac{3}{2}} dx$$

input
```
integrate(x**4*(c*(b*x**2+a)**2)**(3/2),x)
```

output
```
Integral(x**4*(c*(a + b*x**2)**2)**(3/2), x)
```

3.229.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.33

$$\int x^4 \left(c(a+bx^2)^2\right)^{3/2} dx = \frac{1}{11}\,b^3c^{\frac{3}{2}}x^{11} + \frac{1}{3}\,ab^2c^{\frac{3}{2}}x^9 + \frac{3}{7}\,a^2bc^{\frac{3}{2}}x^7 + \frac{1}{5}\,a^3c^{\frac{3}{2}}x^5$$

input
```
integrate(x^4*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/11*b^3*c^(3/2)*x^11 + 1/3*a*b^2*c^(3/2)*x^9 + 3/7*a^2*b*c^(3/2)*x^7 + 1/
5*a^3*c^(3/2)*x^5
```

3.229.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.50

$$\int x^4\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\frac{1}{1155}\left(105\,b^3x^{11}\mathrm{sgn}\left(bx^2+a\right)+385\,ab^2x^9\mathrm{sgn}\left(bx^2+a\right)+495\,a^2bx^7\mathrm{sgn}\left(bx^2+a\right)+231\,a^3x^5$$

input
```
integrate(x^4*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

output
```
1/1155*(105*b^3*x^11*sgn(b*x^2 + a) + 385*a*b^2*x^9*sgn(b*x^2 + a) + 495*a
^2*b*x^7*sgn(b*x^2 + a) + 231*a^3*x^5*sgn(b*x^2 + a))*c^(3/2)
```

3.229.9 Mupad [F(-1)]

Timed out.

$$\int x^4\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\int x^4\,\left(c\,{\left(b\,x^2+a\right)}^2\right)^{3/2}\,dx$$

input
```
int(x^4*(c*(a + b*x^2)^2)^(3/2),x)
```

output
```
int(x^4*(c*(a + b*x^2)^2)^(3/2), x)
```

3.230 $\int x^3 \left(c(a+bx^2)^2\right)^{3/2} dx$

3.230.1 Optimal result

Integrand size = 19, antiderivative size = 66

$$\int x^3 \left(c(a+bx^2)^2\right)^{3/2} dx = -\frac{ac(a+bx^2)^3\sqrt{c(a+bx^2)^2}}{8b^2} + \frac{c(a+bx^2)^4\sqrt{c(a+bx^2)^2}}{10b^2}$$

output
```
-1/8*a*c*(b*x^2+a)^3*(c*(b*x^2+a)^2)^(1/2)/b^2+1/10*c*(b*x^2+a)^4*(c*(b*x^
2+a)^2)^(1/2)/b^2
```

3.230.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int x^3 \left(c(a+bx^2)^2\right)^{3/2} dx = \frac{x^4\left(c(a+bx^2)^2\right)^{3/2}(10a^3+20a^2bx^2+15ab^2x^4+4b^3x^6)}{40(a+bx^2)^3}$$

input
```
Integrate[x^3*(c*(a + b*x^2)^2)^(3/2),x]
```

output
```
(x^4*(c*(a + b*x^2)^2)^(3/2)*(10*a^3 + 20*a^2*b*x^2 + 15*a*b^2*x^4 + 4*b^3
*x^6))/(40*(a + b*x^2)^3)
```

3.230.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2045, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(c(a+bx^2)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{ 2045}$$

$$\frac{a^3 c\sqrt{c\left(a+bx^2\right)^2}\int \frac{x^3\left(bx^2+a\right)^3}{a^3}dx}{a+bx^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c\sqrt{c\left(a+bx^2\right)^2}\int x^3\left(bx^2+a\right)^3dx}{a+bx^2}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{c\sqrt{c\left(a+bx^2\right)^2}\int x^2\left(bx^2+a\right)^3dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{c\sqrt{c\left(a+bx^2\right)^2}\int \left(\frac{\left(bx^2+a\right)^4}{b}-\frac{a\left(bx^2+a\right)^3}{b}\right)dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c\left(\frac{\left(a+bx^2\right)^5}{5b^2}-\frac{a\left(a+bx^2\right)^4}{4b^2}\right)\sqrt{c\left(a+bx^2\right)^2}}{2\left(a+bx^2\right)}$$

input
```
Int[x^3*(c*(a + b*x^2)^2)^(3/2),x]
```

output
```
(c*Sqrt[c*(a + b*x^2)^2]*(-1/4*(a*(a + b*x^2)^4)/b^2 + (a + b*x^2)^5/(5*b^
2)))/(2*(a + b*x^2))
```

3.230.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.230.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.91

method	result	size
gosper	$\frac{x^{4}\left(4b^{3}x^{6}+15b^{2}x^{4}a+20a^{2}b\,x^{2}+10a^{3}\right)\left(c\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{40\left(b\,x^{2}+a\right)^{3}}$	60
default	$\frac{x^{4}\left(4b^{3}x^{6}+15b^{2}x^{4}a+20a^{2}b\,x^{2}+10a^{3}\right)\left(c\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{40\left(b\,x^{2}+a\right)^{3}}$	60
pseudoelliptic	$\frac{x^{4}\left(4b^{3}x^{6}+15b^{2}x^{4}a+20a^{2}b\,x^{2}+10a^{3}\right)c\sqrt{c\left(b\,x^{2}+a\right)^{2}}}{40b\,x^{2}+40a}$	63
trager	$\frac{c\,x^{4}\left(4b^{3}x^{6}+15b^{2}x^{4}a+20a^{2}b\,x^{2}+10a^{3}\right)\sqrt{b^{2}c\,x^{4}+2abc\,x^{2}+c\,a^{2}}}{40b\,x^{2}+40a}$	72
risch	$\frac{c\sqrt{c\left(b\,x^{2}+a\right)^{2}}\,b^{3}x^{10}}{10b\,x^{2}+10a}+\frac{3c\sqrt{c\left(b\,x^{2}+a\right)^{2}}\,a\,b^{2}x^{8}}{8\left(b\,x^{2}+a\right)}+\frac{c\sqrt{c\left(b\,x^{2}+a\right)^{2}}\,a^{2}b\,x^{6}}{2b\,x^{2}+2a}+\frac{c\sqrt{c\left(b\,x^{2}+a\right)^{2}}\,a^{3}x^{4}}{4b\,x^{2}+4a}$	128

input
```
int(x^3*(c*(b*x^2+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/40*x^4*(4*b^3*x^6+15*a*b^2*x^4+20*a^2*b*x^2+10*a^3)*(c*(b*x^2+a)^2)^(3/2
)/(b*x^2+a)^3
```

3.230.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.12

$$\int x^3\left(c(a+bx^2)^2\right)^{3/2}\,dx = \frac{(4\,b^3cx^{10}+15\,ab^2cx^8+20\,a^2bcx^6+10\,a^3cx^4)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{40\,(bx^2+a)}$$

input
```
integrate(x^3*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/40*(4*b^3*c*x^10 + 15*a*b^2*c*x^8 + 20*a^2*b*c*x^6 + 10*a^3*c*x^4)*sqrt(
b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

3.230.6 Sympy [F]

$$\int x^3\left(c(a+bx^2)^2\right)^{3/2}\,dx = \int x^3\left(c(a+bx^2)^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**3*(c*(b*x**2+a)**2)**(3/2),x)
```

output
```
Integral(x**3*(c*(a + b*x**2)**2)**(3/2), x)
```

3.230.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.48

$$\int x^3\left(c(a+bx^2)^2\right)^{3/2}\,dx = -\frac{(b^2cx^4+2\,abcx^2+a^2c)^{\frac{3}{2}}ax^2}{8\,b} - \frac{(b^2cx^4+2\,abcx^2+a^2c)^{\frac{3}{2}}a^2}{8\,b^2} + \frac{(b^2cx^4+2\,abcx^2+a^2c)^{\frac{5}{2}}}{10\,b^2c}$$

input
```
integrate(x^3*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

output
```
-1/8*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3/2)*a*x^2/b - 1/8*(b^2*c*x^4 + 2*
a*b*c*x^2 + a^2*c)^(3/2)*a^2/b^2 + 1/10*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^
(5/2)/(b^2*c)
```

3.230.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.73

$$\int x^3 \left(c \left(a+b x^2\right)^2\right)^{3/2} \, dx = \frac{1}{40} \left(4 \, b^3 x^{10} + 15 \, a b^2 x^8 + 20 \, a^2 b x^6 + 10 \, a^3 x^4\right) c^{\frac{3}{2}} \mathrm{sgn}\left(b x^2 + a\right)$$

input
```
integrate(x^3*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

output
```
1/40*(4*b^3*x^10 + 15*a*b^2*x^8 + 20*a^2*b*x^6 + 10*a^3*x^4)*c^(3/2)*sgn(b
*x^2 + a)
```

3.230.9 Mupad [B] (verification not implemented)

Time = 17.83 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int x^3 \left(c \left(a+b x^2\right)^2\right)^{3/2} \, dx = \frac{\left(-a^2 + 3 \, a \, b \, x^2 + 4 \, b^2 \, x^4\right) \, \left(c \, a^2 + 2 \, c \, a \, b \, x^2 + c \, b^2 \, x^4\right)^{3/2}}{40 \, b^2}$$

input
```
int(x^3*(c*(a + b*x^2)^2)^(3/2),x)
```

output
```
((4*b^2*x^4 - a^2 + 3*a*b*x^2)*(a^2*c + b^2*c*x^4 + 2*a*b*c*x^2)^(3/2))/(4
0*b^2)
```

3.231 $\int x^2 \left(c \left(a+b x^2\right)^2\right)^{3/2} \, dx$

3.231.1 Optimal result

Integrand size = 19, antiderivative size = 143

$$\int x^2 \left(c \left(a+b x^2\right)^2\right)^{3/2} \, dx = \frac{a^3 c x^3 \sqrt{c \left(a+b x^2\right)^2}}{3 \left(a+b x^2\right)} + \frac{3 a^2 b c x^5 \sqrt{c \left(a+b x^2\right)^2}}{5 \left(a+b x^2\right)} + \frac{3 a b^2 c x^7 \sqrt{c \left(a+b x^2\right)^2}}{7 \left(a+b x^2\right)} + \frac{b^3 c x^9 \sqrt{c \left(a+b x^2\right)^2}}{9 \left(a+b x^2\right)}$$

output
```
1/3*a^3*c*x^3*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+3/5*a^2*b*c*x^5*(c*(b*x^2+a)
^2)^(1/2)/(b*x^2+a)+3/7*a*b^2*c*x^7*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/9*b^
3*c*x^9*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

3.231.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.44

$$\int x^2 \left(c \left(a+b x^2\right)^2\right)^{3/2} \, dx = \frac{x^3 \left(c \left(a+b x^2\right)^2\right)^{3/2} \left(105 a^3+189 a^2 b x^2+135 a b^2 x^4+35 b^3 x^6\right)}{315 \left(a+b x^2\right)^3}$$

input
```
Integrate[x^2*(c*(a + b*x^2)^2)^(3/2),x]
```

output
```
(x^3*(c*(a + b*x^2)^2)^(3/2)*(105*a^3 + 189*a^2*b*x^2 + 135*a*b^2*x^4 + 35
*b^3*x^6))/(315*(a + b*x^2)^3)
```

3.231.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.48, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2045, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(c(a+bx^2)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{2045}$$

$$\frac{a^3 c\sqrt{c\left(a+bx^2\right)^2}\int \frac{x^2 \left(bx^2+a\right)^3}{a^3}dx}{a+bx^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\sqrt{c\left(a+bx^2\right)^2}\int x^2 \left(bx^2+a\right)^3 dx}{a+bx^2}$$

$$\Big\downarrow \text{244}$$

$$\frac{c\sqrt{c\left(a+bx^2\right)^2}\int \left(b^3x^8+3ab^2x^6+3a^2bx^4+a^3x^2\right)dx}{a+bx^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\left(\frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{7}ab^2x^7+\frac{b^3x^9}{9}\right)\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}$$

input
```
Int[x^2*(c*(a + b*x^2)^2)^(3/2),x]
```

output
```
(c*Sqrt[c*(a + b*x^2)^2]*((a^3*x^3)/3 + (3*a^2*b*x^5)/5 + (3*a*b^2*x^7)/7
+ (b^3*x^9)/9))/(a + b*x^2)
```

3.231.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.231.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.42

method	result	size
gosper	$\frac{x^{3}\left(35 b^{3} x^{6}+135 b^{2} x^{4} a+189 a^{2} b \,x^{2}+105 a^{3}\right) \left(c \left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{315 \left(b \,x^{2}+a \right)^{3}}$	60
default	$\frac{x^{3}\left(35 b^{3} x^{6}+135 b^{2} x^{4} a+189 a^{2} b \,x^{2}+105 a^{3}\right) \left(c \left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{315 \left(b \,x^{2}+a \right)^{3}}$	60
trager	$\frac{c \,x^{3} \left(35 b^{3} x^{6}+135 b^{2} x^{4} a+189 a^{2} b \,x^{2}+105 a^{3}\right) \sqrt{b^{2} c \,x^{4}+2 a b c \,x^{2}+c \,a^{2}}}{315 b \,x^{2}+315 a}$	72
risch	$\frac{a^{3} c \,x^{3} \sqrt{c \left(b \,x^{2}+a \right)^{2}}}{3 b \,x^{2}+3 a}+\frac{3 a^{2} b c \,x^{5} \sqrt{c \left(b \,x^{2}+a \right)^{2}}}{5 \left(b \,x^{2}+a \right)}+\frac{3 a \,b^{2} c \,x^{7} \sqrt{c \left(b \,x^{2}+a \right)^{2}}}{7 \left(b \,x^{2}+a \right)}+\frac{b^{3} c \,x^{9} \sqrt{c \left(b \,x^{2}+a \right)^{2}}}{9 b \,x^{2}+9 a}$	128

input
```
int(x^2*(c*(b*x^2+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*x^3*(35*b^3*x^6+135*a*b^2*x^4+189*a^2*b*x^2+105*a^3)*(c*(b*x^2+a)^2)
^(3/2)/(b*x^2+a)^3
```

3.231.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.52

$$\int x^2\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\frac{\left(35\,b^3cx^9+135\,ab^2cx^7+189\,a^2bcx^5+105\,a^3cx^3\right)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{315\,(bx^2+a)}$$

input
```
integrate(x^2*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/315*(35*b^3*c*x^9 + 135*a*b^2*c*x^7 + 189*a^2*b*c*x^5 + 105*a^3*c*x^3)*s
qrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

3.231.6 Sympy [F]

$$\int x^2\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\int x^2\left(c\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**2*(c*(b*x**2+a)**2)**(3/2),x)
```

output
```
Integral(x**2*(c*(a + b*x**2)**2)**(3/2), x)
```

3.231.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.33

$$\int x^2\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\frac{1}{9}\,b^3c^{\frac{3}{2}}x^9+\frac{3}{7}\,ab^2c^{\frac{3}{2}}x^7+\frac{3}{5}\,a^2bc^{\frac{3}{2}}x^5+\frac{1}{3}\,a^3c^{\frac{3}{2}}x^3$$

input
```
integrate(x^2*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/9*b^3*c^(3/2)*x^9 + 3/7*a*b^2*c^(3/2)*x^7 + 3/5*a^2*b*c^(3/2)*x^5 + 1/3*
a^3*c^(3/2)*x^3
```

3.231.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.50

$$\int x^2\left(c(a+bx^2)^2\right)^{3/2}\,dx=\frac{1}{315}\left(35\,b^3x^9\mathrm{sgn}\left(bx^2+a\right)+135\,ab^2x^7\mathrm{sgn}\left(bx^2+a\right)+189\,a^2bx^5\mathrm{sgn}\left(bx^2+a\right)+105\,a^3x^3\mathrm{sgn}$$

input
```
integrate(x^2*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

output
```
1/315*(35*b^3*x^9*sgn(b*x^2 + a) + 135*a*b^2*x^7*sgn(b*x^2 + a) + 189*a^2*
b*x^5*sgn(b*x^2 + a) + 105*a^3*x^3*sgn(b*x^2 + a))*c^(3/2)
```

3.231.9 Mupad [F(-1)]

Timed out.

$$\int x^2\left(c(a+bx^2)^2\right)^{3/2}\,dx=\int x^2\,\left(c\,{\left(b\,x^2+a\right)}^2\right)^{3/2}\,dx$$

input
```
int(x^2*(c*(a + b*x^2)^2)^(3/2),x)
```

output
```
int(x^2*(c*(a + b*x^2)^2)^(3/2), x)
```

3.232 $\int x\left(c(a+bx^2)^2\right)^{3/2}\,dx$

3.232.1 Optimal result

Integrand size = 17, antiderivative size = 32

$$\int x\left(c(a+bx^2)^2\right)^{3/2}\,dx=\frac{c(a+bx^2)^3\sqrt{c\left(a+bx^2\right)^2}}{8b}$$

output
```
1/8*c*(b*x^2+a)^3*(c*(b*x^2+a)^2)^(1/2)/b
```

3.232.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int x\left(c(a+bx^2)^2\right)^{3/2}\,dx=\frac{(a+bx^2)\left(c(a+bx^2)^2\right)^{3/2}}{8b}$$

input
```
Integrate[x*(c*(a + b*x^2)^2)^(3/2),x]
```

output
```
((a + b*x^2)*(c*(a + b*x^2)^2)^(3/2))/(8*b)
```

3.232.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2024, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(c\left(a+bx^2\right)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{2024}$$

$$\frac{\int \left(c\left(bx^2+a\right)^2\right)^{3/2}d\left(bx^2+a\right)}{2b}$$

$$\Big\downarrow \text{20}$$

$$\frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}\int \left(bx^2+a\right)^3d\left(bx^2+a\right)}{2b\left(a+bx^2\right)^3}$$

$$\Big\downarrow \text{15}$$

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^2\right)^{3/2}}{8b}$$

input
```
Int[x*(c*(a + b*x^2)^2)^(3/2),x]
```

output
```
((a + b*x^2)*(c*(a + b*x^2)^2)^(3/2))/(8*b)
```

3.232.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] && !IntegerQ[p]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

3.232.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\left(c\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}\left(b\,x^2+a\right)}{8b}$	26
risch	$\frac{c\left(b\,x^2+a\right)^3\sqrt{c\left(b\,x^2+a\right)^2}}{8b}$	29
gosper	$\frac{x^2\left(b^3x^6+4b^2x^4a+6a^2b\,x^2+4a^3\right)\left(c\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{8\left(b\,x^2+a\right)^3}$	59
pseudoelliptic	$\frac{x^2\left(b^3x^6+4b^2x^4a+6a^2b\,x^2+4a^3\right)c\sqrt{c\left(b\,x^2+a\right)^2}}{8b\,x^2+8a}$	62
trager	$\frac{c\,x^2\left(b^3x^6+4b^2x^4a+6a^2b\,x^2+4a^3\right)\sqrt{b^2c\,x^4+2abc\,x^2+c\,a^2}}{8b\,x^2+8a}$	71

input
```
int(x*(c*(b*x^2+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(c*(b*x^2+a)^2)^(3/2)*(b*x^2+a)/b
```

3.232.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(28) = 56$.

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.28

$$\int x\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\frac{\left(b^3cx^8+4\,ab^2cx^6+6\,a^2bcx^4+4\,a^3cx^2\right)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{8\,\left(bx^2+a\right)}$$

input
```
integrate(x*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/8*(b^3*c*x^8 + 4*a*b^2*c*x^6 + 6*a^2*b*c*x^4 + 4*a^3*c*x^2)*sqrt(b^2*c*x
^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

3.232.6 Sympy [F]

$$\int x\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\int x\left(c\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(c*(b*x**2+a)**2)**(3/2),x)
```

output
```
Integral(x*(c*(a + b*x**2)**2)**(3/2), x)
```

3.232.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(28) = 56.

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.88

$$\int x\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\frac{1}{8}\left(b^2cx^4+2\,abcx^2+a^2c\right)^{\frac{3}{2}}x^2+\frac{\left(b^2cx^4+2\,abcx^2+a^2c\right)^{\frac{3}{2}}a}{8\,b}$$

input
```
integrate(x*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/8*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3/2)*x^2 + 1/8*(b^2*c*x^4 + 2*a*b*c
*x^2 + a^2*c)^(3/2)*a/b
```

3.232.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int x\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\frac{\left(bx^2+a\right)^4c^{\frac{3}{2}}\mathrm{sgn}\left(bx^2+a\right)}{8\,b}$$

input
```
integrate(x*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

output
```
1/8*(b*x^2 + a)^4*c^(3/2)*sgn(b*x^2 + a)/b
```

3.232.9 Mupad [B] (verification not implemented)

Time = 18.47 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int x\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\frac{\left(b^2\,x^2+a\,b\right)\,\left(c\,a^2+2\,c\,a\,b\,x^2+c\,b^2\,x^4\right)^{3/2}}{8\,b^2}$$

input
```
int(x*(c*(a + b*x^2)^2)^(3/2),x)
```

output
```
((a*b + b^2*x^2)*(a^2*c + b^2*c*x^4 + 2*a*b*c*x^2)^(3/2))/(8*b^2)
```

3.233 $\int \left(c(a+bx^2)^2\right)^{3/2}\,dx$

3.233.1 Optimal result

Integrand size = 15, antiderivative size = 135

$$\int \left(c(a+bx^2)^2\right)^{3/2}\,dx=\frac{a^3cx\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{a^2bcx^3\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{3ab^2cx^5\sqrt{c\left(a+bx^2\right)^2}}{5\left(a+bx^2\right)}+\frac{b^3cx^7\sqrt{c\left(a+bx^2\right)^2}}{7\left(a+bx^2\right)}$$

output
```
a^3*c*x*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+a^2*b*c*x^3*(c*(b*x^2+a)^2)^(1/2)/
(b*x^2+a)+3/5*a*b^2*c*x^5*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/7*b^3*c*x^7*(c
*(b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

3.233.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.45

$$\int \left(c(a+bx^2)^2\right)^{3/2}\,dx=\frac{x\left(c(a+bx^2)^2\right)^{3/2}\left(35a^3+35a^2bx^2+21ab^2x^4+5b^3x^6\right)}{35\left(a+bx^2\right)^3}$$

input
```
Integrate[(c*(a + b*x^2)^2)^(3/2),x]
```

output
```
(x*(c*(a + b*x^2)^2)^(3/2)*(35*a^3 + 35*a^2*b*x^2 + 21*a*b^2*x^4 + 5*b^3*x
^6))/(35*(a + b*x^2)^3)
```

3.233.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.48, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2045, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c\left(a+bx^2\right)^2\right)^{3/2} dx$$

$$\Big\downarrow \ 2045$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^2}\int \left(\frac{bx^2}{a}+1\right)^3dx}{a+bx^2}$$

$$\Big\downarrow \ 210$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^2}\int \left(\frac{b^3x^6}{a^3}+\frac{3b^2x^4}{a^2}+\frac{3bx^2}{a}+1\right)dx}{a+bx^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{a^3c\left(\frac{b^3x^7}{7a^3}+\frac{3b^2x^5}{5a^2}+\frac{bx^3}{a}+x\right)\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}$$

input
```
Int[(c*(a + b*x^2)^2)^(3/2),x]
```

output
```
(a^3*c*Sqrt[c*(a + b*x^2)^2]*(x + (b*x^3)/a + (3*b^2*x^5)/(5*a^2) + (b^3*x
^7)/(7*a^3)))/(a + b*x^2)
```

3.233.3.1 Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.233.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.43

method	result	size
gosper	$\frac{x\left(5b^{3}x^{6}+21b^{2}x^{4}a+35a^{2}b\,x^{2}+35a^{3}\right)\left(c\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{35\left(b\,x^{2}+a\right)^{3}}$	58
default	$\frac{x\left(5b^{3}x^{6}+21b^{2}x^{4}a+35a^{2}b\,x^{2}+35a^{3}\right)\left(c\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{35\left(b\,x^{2}+a\right)^{3}}$	58
trager	$\frac{cx\left(5b^{3}x^{6}+21b^{2}x^{4}a+35a^{2}b\,x^{2}+35a^{3}\right)\sqrt{b^{2}c\,x^{4}+2abc\,x^{2}+c\,a^{2}}}{35b\,x^{2}+35a}$	70
risch	$\frac{a^{3}cx\sqrt{c\left(b\,x^{2}+a\right)^{2}}}{b\,x^{2}+a}+\frac{a^{2}bc\,x^{3}\sqrt{c\left(b\,x^{2}+a\right)^{2}}}{b\,x^{2}+a}+\frac{3a\,b^{2}c\,x^{5}\sqrt{c\left(b\,x^{2}+a\right)^{2}}}{5\left(b\,x^{2}+a\right)}+\frac{b^{3}c\,x^{7}\sqrt{c\left(b\,x^{2}+a\right)^{2}}}{7b\,x^{2}+7a}$	124

input
```
int((c*(b*x^2+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*x*(5*b^3*x^6+21*a*b^2*x^4+35*a^2*b*x^2+35*a^3)*(c*(b*x^2+a)^2)^(3/2)/
(b*x^2+a)^3
```

3.233.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.53

$$\int \left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\frac{(5\,b^3cx^7+21\,ab^2cx^5+35\,a^2bcx^3+35\,a^3cx)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{35\,(bx^2+a)}$$

input
```
integrate((c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/35*(5*b^3*c*x^7 + 21*a*b^2*c*x^5 + 35*a^2*b*c*x^3 + 35*a^3*c*x)*sqrt(b^2
*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

3.233.6 Sympy [F]

$$\int \left(c\left(a+bx^2\right)^2\right)^{3/2} dx = \int \left(c\left(a+bx^2\right)^2\right)^{\frac{3}{2}} dx$$

input
```
integrate((c*(b*x**2+a)**2)**(3/2),x)
```

output
```
Integral((c*(a + b*x**2)**2)**(3/2), x)
```

3.233.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.32

$$\int \left(c\left(a+bx^2\right)^2\right)^{3/2} dx = \frac{1}{7}b^3c^{\frac{3}{2}}x^7 + \frac{3}{5}ab^2c^{\frac{3}{2}}x^5 + a^2bc^{\frac{3}{2}}x^3 + a^3c^{\frac{3}{2}}x$$

input
```
integrate((c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/7*b^3*c^(3/2)*x^7 + 3/5*a*b^2*c^(3/2)*x^5 + a^2*b*c^(3/2)*x^3 + a^3*c^(3
/2)*x
```

3.233.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.34

$$\int \left(c\left(a+bx^2\right)^2\right)^{3/2} dx = \frac{1}{35}\left(5\,b^3x^7 + 21\,ab^2x^5 + 35\,a^2bx^3 + 35\,a^3x\right)c^{\frac{3}{2}}\text{sgn}\left(bx^2+a\right)$$

input
```
integrate((c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

output
```
1/35*(5*b^3*x^7 + 21*a*b^2*x^5 + 35*a^2*b*x^3 + 35*a^3*x)*c^(3/2)*sgn(b*x^
2 + a)
```

3.233.9 Mupad [F(-1)]

Timed out.

$$\int \left(c(a+bx^2)^2\right)^{3/2} dx = \int \left(c\left(b\,x^2+a\right)^2\right)^{3/2} dx$$

input
```
int((c*(a + b*x^2)^2)^(3/2),x)
```

output
```
int((c*(a + b*x^2)^2)^(3/2), x)
```

3.234 $\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x}\,dx$

3.234.1 Optimal result

Integrand size = 19, antiderivative size = 139

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x}\,dx = \frac{3a^2bcx^2\sqrt{c\left(a+bx^2\right)^2}}{2\left(a+bx^2\right)} + \frac{3ab^2cx^4\sqrt{c\left(a+bx^2\right)^2}}{4\left(a+bx^2\right)} + \frac{b^3cx^6\sqrt{c\left(a+bx^2\right)^2}}{6\left(a+bx^2\right)} + \frac{a^3c\sqrt{c\left(a+bx^2\right)^2}\log(x)}{a+bx^2}$$

output
```
3/2*a^2*b*c*x^2*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+3/4*a*b^2*c*x^4*(c*(b*x^2+
a)^2)^(1/2)/(b*x^2+a)+1/6*b^3*c*x^6*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+a^3*c*
ln(x)*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

3.234.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.68

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x}\,dx = \frac{c\left(b\left(2b^2x^4\left(-\sqrt{b^2}cx^2+\sqrt{c\left(a+bx^2\right)^2}\right)+abx^2\left(-9\sqrt{b^2}cx^2+7\sqrt{c\left(a+bx^2\right)^2}\right)\right.\right.}{}$$

input
```
Integrate[(c*(a + b*x^2)^2)^(3/2)/x,x]
```

output
```
(c*(b*(2*b^2*x^4*(-(Sqrt[b^2*c]*x^2) + Sqrt[c*(a + b*x^2)^2]) + a*b*x^2*(-
9*Sqrt[b^2*c]*x^2 + 7*Sqrt[c*(a + b*x^2)^2]) + a^2*(-18*Sqrt[b^2*c]*x^2 +
11*Sqrt[c*(a + b*x^2)^2])) + 12*a^3*b*Sqrt[c]*ArcTanh[(Sqrt[b^2*c]*x^2 - S
qrt[c*(a + b*x^2)^2])/(a*Sqrt[c])] - 6*a^3*Sqrt[b^2*c]*Log[x^2*(a*b*c + b^
2*c*x^2 - Sqrt[b^2*c]*Sqrt[c*(a + b*x^2)^2])]))/(24*b)
```

3.234.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.49, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2045, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x}\,dx$$

$$\Big\downarrow \ 2045$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^2}\int \frac{\left(bx^2+a\right)^3}{a^3x}dx}{a+bx^2}$$

$$\Big\downarrow \ 27$$

$$\frac{c\sqrt{c\left(a+bx^2\right)^2}\int \frac{\left(bx^2+a\right)^3}{x}dx}{a+bx^2}$$

$$\Big\downarrow \ 243$$

$$\frac{c\sqrt{c\left(a+bx^2\right)^2}\int \frac{\left(bx^2+a\right)^3}{x^2}dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \ 49$$

$$\frac{c\sqrt{c\left(a+bx^2\right)^2}\int \left(b^3x^4+3ab^2x^2+3a^2b+\frac{a^3}{x^2}\right)dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \ 2009$$

$$\frac{c\sqrt{c\left(a+bx^2\right)^2}\left(a^3\log\left(x^2\right)+3a^2bx^2+\frac{3}{2}ab^2x^4+\frac{b^3x^6}{3}\right)}{2\left(a+bx^2\right)}$$

input
```
Int[(c*(a + b*x^2)^2)^(3/2)/x,x]
```

output
```
(c*Sqrt[c*(a + b*x^2)^2]*(3*a^2*b*x^2 + (3*a*b^2*x^4)/2 + (b^3*x^6)/3 + a^
3*Log[x^2]))/(2*(a + b*x^2))
```

3.234.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.234.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.42

method	result	size
default	$\frac{\left(c\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}\left(2b^{3}x^{6}+9b^{2}x^{4}a+18a^{2}b\,x^{2}+12a^{3}\ln\left(x\right)\right)}{12\left(b\,x^{2}+a\right)^{3}}$	59
pseudoelliptic	$\frac{c\sqrt{c\left(b\,x^{2}+a\right)^{2}}\,\left(2b^{3}x^{6}+9b^{2}x^{4}a+6a^{3}\ln\left(x^{2}\right)+18a^{2}b\,x^{2}\right)}{12b\,x^{2}+12a}$	64
risch	$\frac{c\sqrt{c\left(b\,x^{2}+a\right)^{2}}\,b\left(\frac{1}{6}b^{2}x^{6}+\frac{3}{4}a b\,x^{4}+\frac{3}{2}a^{2}x^{2}\right)}{b\,x^{2}+a}+\frac{a^{3}c\ln\left(x\right)\sqrt{c\left(b\,x^{2}+a\right)^{2}}}{b\,x^{2}+a}$	80

input
```
int((c*(b*x^2+a)^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/12*(c*(b*x^2+a)^2)^(3/2)*(2*b^3*x^6+9*b^2*x^4*a+18*a^2*b*x^2+12*a^3*ln(x
))/(b*x^2+a)^3
```

3.234.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.53

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x}\,dx = \frac{(2\,b^3cx^6+9\,ab^2cx^4+18\,a^2bcx^2+12\,a^3c\log{(x)})\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{12\,(bx^2+a)}$$

input
```
integrate((c*(b*x^2+a)^2)^(3/2)/x,x, algorithm="fricas")
```

output
```
1/12*(2*b^3*c*x^6 + 9*a*b^2*c*x^4 + 18*a^2*b*c*x^2 + 12*a^3*c*log(x))*sqrt
(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

3.234.6 Sympy [F]

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x}\,dx = \int \frac{\left(c(a+bx^2)^2\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((c*(b*x**2+a)**2)**(3/2)/x,x)
```

output
```
Integral((c*(a + b*x**2)**2)**(3/2)/x, x)
```

3.234.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.23

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x}\,dx = \frac{1}{2}\,(-1)^{2\,b^2cx^2+2\,abc}\,a^3c^{\frac{3}{2}}\log\left(2\,b^2cx^2+2\,abc\right) - \frac{1}{2}\,(-1)^{2\,abcx^2+2\,a^2c}\,a^3c^{\frac{3}{2}}\log\left(2\,abc+\frac{2\,a^2c}{x^2}\right) + \frac{1}{4}\,\sqrt{b^2cx^4+2\,abcx^2+a^2c}abcx^2 + \frac{3}{4}\,\sqrt{b^2cx^4+2\,abcx^2+a^2c}a^2c + \frac{1}{6}\,\left(b^2cx^4+2\,abcx^2+a^2c\right)^{\frac{3}{2}}$$

input
```
integrate((c*(b*x^2+a)^2)^(3/2)/x,x, algorithm="maxima")
```

output
```
1/2*(-1)^(2*b^2*c*x^2 + 2*a*b*c)*a^3*c^(3/2)*log(2*b^2*c*x^2 + 2*a*b*c) -
1/2*(-1)^(2*a*b*c*x^2 + 2*a^2*c)*a^3*c^(3/2)*log(2*a*b*c + 2*a^2*c/x^2) +
1/4*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)*a*b*c*x^2 + 3/4*sqrt(b^2*c*x^4 +
 2*a*b*c*x^2 + a^2*c)*a^2*c + 1/6*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3/2)
```

3.234.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.53

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x}\,dx = \frac{1}{12}\,\left(2\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+9\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+18\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+6\,a^3\log\left(\right.\right.$$

input
```
integrate((c*(b*x^2+a)^2)^(3/2)/x,x, algorithm="giac")
```

output
```
1/12*(2*b^3*x^6*sgn(b*x^2 + a) + 9*a*b^2*x^4*sgn(b*x^2 + a) + 18*a^2*b*x^2
*sgn(b*x^2 + a) + 6*a^3*log(x^2)*sgn(b*x^2 + a))*c^(3/2)
```

3.234.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x}\,dx = \int \frac{\left(c\,(b\,x^2+a)^2\right)^{3/2}}{x}\,dx$$

input
```
int((c*(a + b*x^2)^2)^(3/2)/x,x)
```

output
```
int((c*(a + b*x^2)^2)^(3/2)/x, x)
```

3.235 $\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x^2}\,dx$

3.235.1 Optimal result

Integrand size = 19, antiderivative size = 134

$$\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x^2}\,dx=-\frac{a^3c\sqrt{c\left(a+bx^2\right)^2}}{x\left(a+bx^2\right)}+\frac{3a^2bcx\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{ab^2cx^3\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{b^3cx^5\sqrt{c\left(a+bx^2\right)^2}}{5\left(a+bx^2\right)}$$

output
```
-a^3*c*(c*(b*x^2+a)^2)^(1/2)/x/(b*x^2+a)+3*a^2*b*c*x*(c*(b*x^2+a)^2)^(1/2)
/(b*x^2+a)+a*b^2*c*x^3*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/5*b^3*c*x^5*(c*(b
*x^2+a)^2)^(1/2)/(b*x^2+a)
```

3.235.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.46

$$\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x^2}\,dx=\frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}\left(-5a^3+15a^2bx^2+5ab^2x^4+b^3x^6\right)}{5x\left(a+bx^2\right)^3}$$

input
```
Integrate[(c*(a + b*x^2)^2)^(3/2)/x^2,x]
```

output
```
((c*(a + b*x^2)^2)^(3/2)*(-5*a^3 + 15*a^2*b*x^2 + 5*a*b^2*x^4 + b^3*x^6))/
(5*x*(a + b*x^2)^3)
```

3.235.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.45, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2045, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \ 2045$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^2}\int \frac{\left(bx^2+a\right)^3}{a^3x^2}dx}{a+bx^2}$$

$$\Big\downarrow \ 27$$

$$\frac{c\sqrt{c\left(a+bx^2\right)^2}\int \frac{\left(bx^2+a\right)^3}{x^2}dx}{a+bx^2}$$

$$\Big\downarrow \ 244$$

$$\frac{c\sqrt{c\left(a+bx^2\right)^2}\int \left(b^3x^4+3ab^2x^2+3a^2b+\frac{a^3}{x^2}\right)dx}{a+bx^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{c\left(-\frac{a^3}{x}+3a^2bx+ab^2x^3+\frac{b^3x^5}{5}\right)\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}$$

input
```
Int[(c*(a + b*x^2)^2)^(3/2)/x^2,x]
```

output
```
(c*Sqrt[c*(a + b*x^2)^2]*(-(a^3/x) + 3*a^2*b*x + a*b^2*x^3 + (b^3*x^5)/5))
/(a + b*x^2)
```

3.235.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.235.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.45

method	result	size
gosper	$-\frac{\left(-b^{3} x^{6}-5 b^{2} x^{4} a-15 a^{2} b \,x^{2}+5 a^{3}\right) \left(c \left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{5 x \left(b \,x^{2}+a \right)^{3}}$	60
default	$-\frac{\left(-b^{3} x^{6}-5 b^{2} x^{4} a-15 a^{2} b \,x^{2}+5 a^{3}\right) \left(c \left(b \,x^{2}+a \right)^{2}\right)^{\frac{3}{2}}}{5 x \left(b \,x^{2}+a \right)^{3}}$	60
risch	$\frac{c \sqrt{c \left(b \,x^{2}+a \right)^{2}}\, b \left(\frac{1}{5} b^{2} x^{5}+a b \,x^{3}+3 a^{2} x \right)}{b \,x^{2}+a}-\frac{a^{3} c \sqrt{c \left(b \,x^{2}+a \right)^{2}}}{x \left(b \,x^{2}+a \right)}$	79
trager	$\frac{c \left(b^{3} x^{5}+b^{3} x^{4}+5 a \,b^{2} x^{3}+b^{3} x^{3}+5 a \,b^{2} x^{2}+b^{3} x^{2}+15 a^{2} b x +5 b^{2} a x +b^{3} x +5 a^{3}\right) \left(x -1\right) \sqrt{b^{2} c \,x^{4}+2 a b c \,x^{2}+c \,a^{2}}}{5 x \left(b \,x^{2}+a \right)}$	114

input
```
int((c*(b*x^2+a)^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(-b^3*x^6-5*a*b^2*x^4-15*a^2*b*x^2+5*a^3)*(c*(b*x^2+a)^2)^(3/2)/x/(b*
x^2+a)^3
```

3.235.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.54

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^2}\,dx = \frac{(b^3cx^6+5\,ab^2cx^4+15\,a^2bcx^2-5\,a^3c)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{5\,(bx^3+ax)}$$

input
```
integrate((c*(b*x^2+a)^2)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
1/5*(b^3*c*x^6 + 5*a*b^2*c*x^4 + 15*a^2*b*c*x^2 - 5*a^3*c)*sqrt(b^2*c*x^4
+ 2*a*b*c*x^2 + a^2*c)/(b*x^3 + a*x)
```

3.235.6 Sympy [F]

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^2}\,dx = \int \frac{\left(c(a+bx^2)^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*(b*x**2+a)**2)**(3/2)/x**2,x)
```

output
```
Integral((c*(a + b*x**2)**2)**(3/2)/x**2, x)
```

3.235.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.36

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^2}\,dx = \frac{b^3c^{\frac{3}{2}}x^6+5\,ab^2c^{\frac{3}{2}}x^4+15\,a^2bc^{\frac{3}{2}}x^2-5\,a^3c^{\frac{3}{2}}}{5\,x}$$

input
```
integrate((c*(b*x^2+a)^2)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
1/5*(b^3*c^(3/2)*x^6 + 5*a*b^2*c^(3/2)*x^4 + 15*a^2*b*c^(3/2)*x^2 - 5*a^3*
c^(3/2))/x
```

3.235.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.51

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^2}\,dx = \frac{1}{5}\left(b^3x^5\mathrm{sgn}\left(bx^2+a\right)+5\,ab^2x^3\mathrm{sgn}\left(bx^2+a\right)+15\,a^2bx\mathrm{sgn}\left(bx^2+a\right)-\frac{5\,a^3\mathrm{sgn}(bx^2}{x}\right.$$

input
```
integrate((c*(b*x^2+a)^2)^(3/2)/x^2,x, algorithm="giac")
```

output
```
1/5*(b^3*x^5*sgn(b*x^2 + a) + 5*a*b^2*x^3*sgn(b*x^2 + a) + 15*a^2*b*x*sgn(
b*x^2 + a) - 5*a^3*sgn(b*x^2 + a)/x)*c^(3/2)
```

3.235.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^2}\,dx = \int \frac{\left(c\left(b\,x^2+a\right)^2\right)^{3/2}}{x^2}\,dx$$

input
```
int((c*(a + b*x^2)^2)^(3/2)/x^2,x)
```

output
```
int((c*(a + b*x^2)^2)^(3/2)/x^2, x)
```

3.236 $\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x^3}\,dx$

3.236.1 Optimal result

Integrand size = 19, antiderivative size = 140

$$\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x^3}\,dx = -\frac{a^3c\sqrt{c\left(a+bx^2\right)^2}}{2x^2\left(a+bx^2\right)}+\frac{3ab^2cx^2\sqrt{c\left(a+bx^2\right)^2}}{2\left(a+bx^2\right)}+\frac{b^3cx^4\sqrt{c\left(a+bx^2\right)^2}}{4\left(a+bx^2\right)}+\frac{3a^2bc\sqrt{c\left(a+bx^2\right)^2}\log(x)}{a+bx^2}$$

output

```
-1/2*a^3*c*(c*(b*x^2+a)^2)^(1/2)/x^2/(b*x^2+a)+3/2*a*b^2*c*x^2*(c*(b*x^2+a
)^2)^(1/2)/(b*x^2+a)+1/4*b^3*c*x^4*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+3*a^2*b
*c*ln(x)*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

3.236.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.69

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^3}\,dx = \frac{1}{16}c\left(\frac{(8a^3+21a^2bx^2-24ab^2x^4-4b^3x^6)\left(abc+b^2cx^2-\sqrt{b^2c}\sqrt{c\,(a+bx^2)^2}\right)}{x^2\left(a\sqrt{b^2c}+b\sqrt{b^2c}x^2-b\sqrt{c\,(a+bx^2)^2}\right)} + 24a^2b\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{b^2c}x^2-\sqrt{c\,(a+bx^2)^2}}{a\sqrt{c}}\right) - 12a^2\sqrt{b^2c}\log\left(x^2\left(abc+b^2cx^2-\sqrt{b^2c}\sqrt{c\,(a+bx^2)^2}\right)\right)\right)$$

input
```
Integrate[(c*(a + b*x^2)^2)^(3/2)/x^3,x]
```

output
```
(c*(((8*a^3 + 21*a^2*b*x^2 - 24*a*b^2*x^4 - 4*b^3*x^6)*(a*b*c + b^2*c*x^2
- Sqrt[b^2*c]*Sqrt[c*(a + b*x^2)^2]))/(x^2*(a*Sqrt[b^2*c] + b*Sqrt[b^2*c]*
x^2 - b*Sqrt[c*(a + b*x^2)^2])) + 24*a^2*b*Sqrt[c]*ArcTanh[(Sqrt[b^2*c]*x^
2 - Sqrt[c*(a + b*x^2)^2])/(a*Sqrt[c])] - 12*a^2*Sqrt[b^2*c]*Log[x^2*(a*b*
c + b^2*c*x^2 - Sqrt[b^2*c]*Sqrt[c*(a + b*x^2)^2])]))/16
```

3.236.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.48, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2045, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^3}\,dx$$

$$\Big\downarrow \text{2045}$$

$$\frac{a^3c\sqrt{c\,(a+bx^2)^2}\int \frac{(bx^2+a)^3}{a^3x^3}dx}{a+bx^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\sqrt{c\left(a+bx^2\right)^2}\int\frac{\left(bx^2+a\right)^3}{x^3}dx}{a+bx^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{c\sqrt{c\left(a+bx^2\right)^2}\int\frac{\left(bx^2+a\right)^3}{x^4}dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{49}$$

$$\frac{c\sqrt{c\left(a+bx^2\right)^2}\int\left(\frac{a^3}{x^4}+\frac{3ba^2}{x^2}+3b^2a+b^3x^2\right)dx^2}{2\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\sqrt{c\left(a+bx^2\right)^2}\left(-\frac{a^3}{x^2}+3a^2b\log\left(x^2\right)+3ab^2x^2+\frac{b^3x^4}{2}\right)}{2\left(a+bx^2\right)}$$

input
```
Int[(c*(a + b*x^2)^2)^(3/2)/x^3,x]
```

output
```
(c*Sqrt[c*(a + b*x^2)^2]*(-(a^3/x^2) + 3*a*b^2*x^2 + (b^3*x^4)/2 + 3*a^2*b
*Log[x^2]))/(2*(a + b*x^2))
```

3.236.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.236. $\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x^3}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.236.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.44

method	result	size
default	$\frac{\left(c\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}\left(b^{3}x^{6}+6b^{2}x^{4}a+12a^{2}b\ln\left(x\right)x^{2}-2a^{3}\right)}{4x^{2}\left(b\,x^{2}+a\right)^{3}}$	61
pseudoelliptic	$-\frac{c\left(-\frac{b^{3}x^{6}}{2}-3b^{2}x^{4}a-3a^{2}b\ln\left(x^{2}\right)x^{2}+a^{3}\right)\sqrt{c\left(b\,x^{2}+a\right)^{2}}}{2\left(b\,x^{2}+a\right)x^{2}}$	63
risch	$\frac{c\sqrt{c\left(b\,x^{2}+a\right)^{2}}\,b\left(b\,x^{2}+3a\right)^{2}}{4b\,x^{2}+4a}-\frac{a^{3}c\sqrt{c\left(b\,x^{2}+a\right)^{2}}}{2x^{2}\left(b\,x^{2}+a\right)}+\frac{3a^{2}bc\ln\left(x\right)\sqrt{c\left(b\,x^{2}+a\right)^{2}}}{b\,x^{2}+a}$	101

input
```
int((c*(b*x^2+a)^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(c*(b*x^2+a)^2)^(3/2)*(b^3*x^6+6*b^2*x^4*a+12*a^2*b*ln(x)*x^2-2*a^3)/x
^2/(b*x^2+a)^3
```

3.236.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.54

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^3}\,dx=\frac{\left(b^3cx^6+6\,ab^2cx^4+12\,a^2bcx^2\log\left(x\right)-2\,a^3c\right)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{4\left(bx^4+ax^2\right)}$$

input
```
integrate((c*(b*x^2+a)^2)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
1/4*(b^3*c*x^6 + 6*a*b^2*c*x^4 + 12*a^2*b*c*x^2*log(x) - 2*a^3*c)*sqrt(b^2
*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^4 + a*x^2)
```

3.236.6 Sympy [F]

$$\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x^3}\,dx=\int \frac{\left(c\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((c*(b*x**2+a)**2)**(3/2)/x**3,x)
```

output
```
Integral((c*(a + b*x**2)**2)**(3/2)/x**3, x)
```

3.236.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.26

$$\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x^3}\,dx=\frac{3}{2}\left(-1\right)^{2\,b^2cx^2+2\,abc}a^2bc^{\frac{3}{2}}\log\left(2\,b^2cx^2+2\,abc\right)-\frac{3}{2}\left(-1\right)^{2\,abcx^2+2\,a^2c}a^2bc^{\frac{3}{2}}\log\left(2\,abc+\frac{2\,a^2c}{x^2}\right)+\frac{3}{4}\sqrt{b^2cx^4+2\,abcx^2+a^2c}b^2cx^2+\frac{9}{4}\sqrt{b^2cx^4+2\,abcx^2+a^2c}abc-\frac{\left(b^2cx^4+2\,abcx^2+a^2c\right)^{\frac{3}{2}}}{2\,x^2}$$

input
```
integrate((c*(b*x^2+a)^2)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
3/2*(-1)^(2*b^2*c*x^2 + 2*a*b*c)*a^2*b*c^(3/2)*log(2*b^2*c*x^2 + 2*a*b*c)
- 3/2*(-1)^(2*a*b*c*x^2 + 2*a^2*c)*a^2*b*c^(3/2)*log(2*a*b*c + 2*a^2*c/x^2
) + 3/4*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)*b^2*c*x^2 + 9/4*sqrt(b^2*c*x
^4 + 2*a*b*c*x^2 + a^2*c)*a*b*c - 1/2*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3
/2)/x^2
```

3.236.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.65

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^3}\,dx = \frac{1}{4}\left(b^3x^4\mathrm{sgn}\left(bx^2+a\right)+6\,ab^2x^2\mathrm{sgn}\left(bx^2+a\right)+6\,a^2b\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)-\frac{2\,(3\,a}{}\right.$$

input
```
integrate((c*(b*x^2+a)^2)^(3/2)/x^3,x, algorithm="giac")
```

output
```
1/4*(b^3*x^4*sgn(b*x^2 + a) + 6*a*b^2*x^2*sgn(b*x^2 + a) + 6*a^2*b*log(x^2
)*sgn(b*x^2 + a) - 2*(3*a^2*b*x^2*sgn(b*x^2 + a) + a^3*sgn(b*x^2 + a))/x^2
)*c^(3/2)
```

3.236.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^3}\,dx = \int \frac{\left(c\,{\left(b\,x^2+a\right)}^2\right)^{3/2}}{x^3}\,dx$$

input
```
int((c*(a + b*x^2)^2)^(3/2)/x^3,x)
```

output
```
int((c*(a + b*x^2)^2)^(3/2)/x^3, x)
```

3.237 $\int x^2 \left(c(a+bx^2)^3\right)^{3/2} dx$

3.237.1 Optimal result

Integrand size = 19, antiderivative size = 253

$$\begin{aligned}\int x^2 \left(c(a+bx^2)^3\right)^{3/2} dx &= \frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3} \\ &+ \frac{21a^5cx\sqrt{c\left(a+bx^2\right)^3}}{1024b\left(a+bx^2\right)} + \frac{21a^4cx^3\sqrt{c\left(a+bx^2\right)^3}}{512\left(a+bx^2\right)} \\ &+ \frac{21}{320}a^2cx^3\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{3}{40}acx^3\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} \\ &+ \frac{1}{12}cx^3\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} - \frac{21a^{9/2}c\sqrt{c\left(a+bx^2\right)^3}\operatorname{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{1024b^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/2}}\end{aligned}$$

output
```
7/128*a^3*c*x^3*(c*(b*x^2+a)^3)^(1/2)+21/1024*a^5*c*x*(c*(b*x^2+a)^3)^(1/2
)/b/(b*x^2+a)+21/512*a^4*c*x^3*(c*(b*x^2+a)^3)^(1/2)/(b*x^2+a)+21/320*a^2*
c*x^3*(b*x^2+a)*(c*(b*x^2+a)^3)^(1/2)+3/40*a*c*x^3*(b*x^2+a)^2*(c*(b*x^2+a
)^3)^(1/2)+1/12*c*x^3*(b*x^2+a)^3*(c*(b*x^2+a)^3)^(1/2)-21/1024*a^(9/2)*c*
arcsinh(x*b^(1/2)/a^(1/2))*(c*(b*x^2+a)^3)^(1/2)/b^(3/2)/(1+b*x^2/a)^(3/2)
```

3.237.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.56

$$\int x^2\left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx=\frac{\left(c(a+bx^2)^3\right)^{3/2}\left(\sqrt{b}x\sqrt{a+bx^2}(315a^5+4910a^4bx^2+11432a^3b^2x^4+12144a^2b^3x^6+6272\right.}{15360b^{3/2}\left(a+bx^2\right)^{9/2}}$$

input
```
Integrate[x^2*(c*(a + b*x^2)^3)^(3/2),x]
```

output
```
((c*(a + b*x^2)^3)^(3/2)*(Sqrt[b]*x*Sqrt[a + b*x^2]*(315*a^5 + 4910*a^4*b*
x^2 + 11432*a^3*b^2*x^4 + 12144*a^2*b^3*x^6 + 6272*a*b^4*x^8 + 1280*b^5*x^
10) + 630*a^6*ArcTanh[(Sqrt[b]*x)/(Sqrt[a] - Sqrt[a + b*x^2])]))/(15360*b^
(3/2)*(a + b*x^2)^(9/2))
```

3.237.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.85, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2045, 248, 248, 248, 248, 248, 262, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(c\left(a+bx^2\right)^3\right)^{3/2}dx$$

$$\Big\downarrow \text{ 2045}$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\int x^2\left(\frac{bx^2}{a}+1\right)^{9/2}dx}{\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \text{ 248}$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\left(\frac{3}{4}\int x^2\left(\frac{bx^2}{a}+1\right)^{7/2}dx+\frac{1}{12}x^3\left(\frac{bx^2}{a}+1\right)^{9/2}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \text{ 248}$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\left(\frac{3}{4}\left(\frac{7}{10}\int x^2\left(\frac{bx^2}{a}+1\right)^{5/2}dx+\frac{1}{10}x^3\left(\frac{bx^2}{a}+1\right)^{7/2}\right)+\frac{1}{12}x^3\left(\frac{bx^2}{a}+1\right)^{9/2}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$\big\downarrow$ 248

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\left(\frac{3}{4}\left(\frac{7}{10}\left(\frac{5}{8}\int x^2\left(\frac{bx^2}{a}+1\right)^{3/2}dx+\frac{1}{8}x^3\left(\frac{bx^2}{a}+1\right)^{5/2}\right)+\frac{1}{10}x^3\left(\frac{bx^2}{a}+1\right)^{7/2}\right)+\frac{1}{12}x^3\left(\frac{bx^2}{a}+1\right)^{9/2}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$\big\downarrow$ 248

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\left(\frac{3}{4}\left(\frac{7}{10}\left(\frac{5}{8}\left(\frac{1}{2}\int x^2\sqrt{\frac{bx^2}{a}+1}dx+\frac{1}{6}x^3\left(\frac{bx^2}{a}+1\right)^{3/2}\right)+\frac{1}{8}x^3\left(\frac{bx^2}{a}+1\right)^{5/2}\right)+\frac{1}{10}x^3\left(\frac{bx^2}{a}+1\right)^{7/2}\right)\right.\right.}{\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$\big\downarrow$ 248

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\left(\frac{3}{4}\left(\frac{7}{10}\left(\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\int\frac{x^2}{\sqrt{\frac{bx^2}{a}+1}}dx+\frac{1}{4}x^3\sqrt{\frac{bx^2}{a}+1}\right)+\frac{1}{6}x^3\left(\frac{bx^2}{a}+1\right)^{3/2}\right)+\frac{1}{8}x^3\left(\frac{bx^2}{a}+1\right)^{5/2}\right)+\frac{1}{10}\right.\right.\right.}{\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$\big\downarrow$ 262

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\left(\frac{3}{4}\left(\frac{7}{10}\left(\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{ax\sqrt{\frac{bx^2}{a}+1}}{2b}-\frac{a\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}}dx}{2b}\right)+\frac{1}{4}x^3\sqrt{\frac{bx^2}{a}+1}\right)+\frac{1}{6}x^3\left(\frac{bx^2}{a}+1\right)^{3/2}\right)+\frac{1}{8}x^3\left(\right.\right.\right.\right.\right.}{\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$\big\downarrow$ 222

$$\frac{a^3c\left(\frac{3}{4}\left(\frac{7}{10}\left(\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{ax\sqrt{\frac{bx^2}{a}+1}}{2b}-\frac{a^{3/2}\text{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{3/2}}\right)+\frac{1}{4}x^3\sqrt{\frac{bx^2}{a}+1}\right)+\frac{1}{6}x^3\left(\frac{bx^2}{a}+1\right)^{3/2}\right)+\frac{1}{8}x^3\left(\frac{bx^2}{a}+1\right)^{5/2}\right.\right.\right.\right.}{\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

input

```
Int[x^2*(c*(a + b*x^2)^3)^(3/2),x]
```

3.237. $\int x^2\left(c(a+bx^2)^3\right)^{3/2}dx$

output
```
(a^3*c*Sqrt[c*(a + b*x^2)^3]*((x^3*(1 + (b*x^2)/a)^(9/2))/12 + (3*((x^3*(1
 + (b*x^2)/a)^(7/2))/10 + (7*((x^3*(1 + (b*x^2)/a)^(5/2))/8 + (5*((x^3*(1
+ (b*x^2)/a)^(3/2))/6 + ((x^3*Sqrt[1 + (b*x^2)/a]])/4 + ((a*x*Sqrt[1 + (b*x
^2)/a]])/(2*b) - (a^(3/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(3/2)))/4)/2))
/8))/10))/4))/(1 + (b*x^2)/a)^(3/2)
```

3.237.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.237.4 Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.61

method	result
risch	$\frac{x\left(1280x^{10}b^5+6272a\,x^8b^4+12144a^2x^6b^3+11432a^3x^4b^2+4910x^2a^4b+315a^5\right)c\sqrt{c\left(b\,x^2+a\right)^3}}{15360\left(b\,x^2+a\right)b}-\frac{21a^6\ln\left(\frac{bcx}{\sqrt{bc}}+\sqrt{bc\,x^2+ac}\right)c\sqrt{c(b\,x^2}}{1024b\sqrt{bc}\,(b\,x^2+a)^2}$
default	$-\frac{\left(c\left(b\,x^2+a\right)^3\right)^{\frac{3}{2}}\left(-1280x^7\left(bc\,x^2+ac\right)^{\frac{5}{2}}b^3\sqrt{bc}-3712\sqrt{bc}\left(bc\,x^2+ac\right)^{\frac{5}{2}}a\,b^2x^5-3440\sqrt{bc}\left(bc\,x^2+ac\right)^{\frac{5}{2}}a^2b\,x^3+315\ln\left(\frac{bcx+\sqrt{bc\,x^2+a}}{\sqrt{bc}}\right.\right.}{15360b\left(b\,x^2+a\right)^3\left(c\left(b\,x^2+a\right)\right)^{\frac{3}{2}}c\sqrt{bc}}$

input
```
int(x^2*(c*(b*x^2+a)^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/15360*x*(1280*b^5*x^10+6272*a*b^4*x^8+12144*a^2*b^3*x^6+11432*a^3*b^2*x^
4+4910*a^4*b*x^2+315*a^5)/(b*x^2+a)/b*c*(c*(b*x^2+a)^3)^(1/2)-21/1024*a^6/
b*ln(b*c*x/(b*c)^(1/2)+(b*c*x^2+a*c)^(1/2))/(b*c)^(1/2)*c/(b*x^2+a)^2*(c*(
b*x^2+a)^3)^(1/2)*(c*(b*x^2+a))^(1/2)
```

3.237.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.71

$$\int x^2\Big(c\big(a+bx^2\big)^3\Big)^{3/2}\,dx = \left[\frac{315\,(a^6bcx^2+a^7c)\sqrt{\frac{c}{b}}\log\left(-\frac{2\,b^2cx^4+3\,abcx^2+a^2c-2\,\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}bx\sqrt{\frac{c}{b}}}{bx^2+a}\right)+2\,(1280\right.$$

input
```
integrate(x^2*(c*(b*x^2+a)^3)^(3/2),x, algorithm="fricas")
```

output
```
[1/30720*(315*(a^6*b*c*x^2 + a^7*c)*sqrt(c/b)*log(-(2*b^2*c*x^4 + 3*a*b*c*
x^2 + a^2*c - 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*b*
x*sqrt(c/b))/(b*x^2 + a)) + 2*(1280*b^5*c*x^11 + 6272*a*b^4*c*x^9 + 12144*
a^2*b^3*c*x^7 + 11432*a^3*b^2*c*x^5 + 4910*a^4*b*c*x^3 + 315*a^5*c*x)*sqrt
(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b^2*x^2 + a*b), 1/15
360*(315*(a^6*b*c*x^2 + a^7*c)*sqrt(-c/b)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*
c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*b*x*sqrt(-c/b)/(b^2*c*x^4 + 2*a*b*c*x^2 + a
^2*c)) + (1280*b^5*c*x^11 + 6272*a*b^4*c*x^9 + 12144*a^2*b^3*c*x^7 + 11432
*a^3*b^2*c*x^5 + 4910*a^4*b*c*x^3 + 315*a^5*c*x)*sqrt(b^3*c*x^6 + 3*a*b^2*
c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b^2*x^2 + a*b)]
```

3.237.6 Sympy [F]

$$\int x^2 \left(c \left(a+b x^2\right)^3\right)^{3/2} dx=\int x^2 \left(c \left(a+b x^2\right)^3\right)^{\frac{3}{2}} dx$$

input
```
integrate(x**2*(c*(b*x**2+a)**3)**(3/2),x)
```

output
```
Integral(x**2*(c*(a + b*x**2)**3)**(3/2), x)
```

3.237.7 Maxima [F]

$$\int x^2 \left(c \left(a+b x^2\right)^3\right)^{3/2} dx=\int \left(\left(b x^2+a\right)^3 c\right)^{\frac{3}{2}} x^2 dx$$

input
```
integrate(x^2*(c*(b*x^2+a)^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate(((b*x^2 + a)^3*c)^(3/2)*x^2, x)
```

3.237.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.70

$$\int x^2 \left(c \left(a+b x^2\right)^3\right)^{3/2} dx=\frac{1}{15360} \left(\frac{315 a^6 c \log \left(\left|-\sqrt{bc}x+\sqrt{bcx^2+ac}\right|\right) \operatorname{sgn}(bx^2+a)}{\sqrt{bc}b}+\left(\frac{315 a^5 \operatorname{sgn}(bx^2+a)}{b}+2 (2\right.\right.$$

input
```
integrate(x^2*(c*(b*x^2+a)^3)^(3/2),x, algorithm="giac")
```

output
```
1/15360*(315*a^6*c*log(abs(-sqrt(b*c)*x + sqrt(b*c*x^2 + a*c)))*sgn(b*x^2
+ a)/(sqrt(b*c)*b) + (315*a^5*sgn(b*x^2 + a)/b + 2*(2455*a^4*sgn(b*x^2 + a
) + 4*(1429*a^3*b*sgn(b*x^2 + a) + 2*(759*a^2*b^2*sgn(b*x^2 + a) + 8*(10*b
^4*x^2*sgn(b*x^2 + a) + 49*a*b^3*sgn(b*x^2 + a))*x^2)*x^2)*x^2)*x^2)*sqrt(
b*c*x^2 + a*c)*x)*c
```

3.237.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \left(c\left(a+bx^2\right)^3\right)^{3/2} dx = \int x^2 \left(c\left(b\,x^2+a\right)^3\right)^{3/2} dx$$

input
```
int(x^2*(c*(a + b*x^2)^3)^(3/2),x)
```

output
```
int(x^2*(c*(a + b*x^2)^3)^(3/2), x)
```

3.238 $\int x\left(c(a+bx^2)^3\right)^{3/2}\,dx$

3.238.1 Optimal result

Integrand size = 17, antiderivative size = 32

$$\int x\left(c(a+bx^2)^3\right)^{3/2}\,dx=\frac{c(a+bx^2)^4\sqrt{c\,(a+bx^2)^3}}{11b}$$

output
```
1/11*c*(b*x^2+a)^4*(c*(b*x^2+a)^3)^(1/2)/b
```

3.238.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int x\left(c(a+bx^2)^3\right)^{3/2}\,dx=\frac{(a+bx^2)\left(c(a+bx^2)^3\right)^{3/2}}{11b}$$

input
```
Integrate[x*(c*(a + b*x^2)^3)^(3/2),x]
```

output
```
((a + b*x^2)*(c*(a + b*x^2)^3)^(3/2))/(11*b)
```

3.238.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2024, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(c\left(a+bx^2\right)^3\right)^{3/2}dx$$

$$\Big\downarrow \text{ 2024}$$

$$\frac{\int \left(c\left(bx^2+a\right)^3\right)^{3/2}d\left(bx^2+a\right)}{2b}$$

$$\Big\downarrow \text{ 20}$$

$$\frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}\int \left(bx^2+a\right)^{9/2}d\left(bx^2+a\right)}{2b\left(a+bx^2\right)^{9/2}}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^3\right)^{3/2}}{11b}$$

input
```
Int[x*(c*(a + b*x^2)^3)^(3/2),x]
```

output
```
((a + b*x^2)*(c*(a + b*x^2)^3)^(3/2))/(11*b)
```

3.238.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

3.238.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

method	result
gosper	$\frac{(b\,x^{2}+a)\left(c\,(b\,x^{2}+a)^{3}\right)^{\frac{3}{2}}}{11b}$
risch	$\frac{c\sqrt{c\,(b\,x^{2}+a)^{3}}\,(x^{10}b^{5}+5a\,x^{8}b^{4}+10a^{2}x^{6}b^{3}+10a^{3}x^{4}b^{2}+5x^{2}a^{4}b+a^{5})}{11(b\,x^{2}+a)b}$
trager	$\frac{c(b^{4}x^{8}+4a\,b^{3}x^{6}+6a^{2}b^{2}x^{4}+4a^{3}b\,x^{2}+a^{4})\sqrt{b^{3}c\,x^{6}+3a\,b^{2}c\,x^{4}+3a^{2}bc\,x^{2}+a^{3}c}}{11b}$
default	$-\frac{\left(c\,(b\,x^{2}+a)^{3}\right)^{\frac{3}{2}}\left(-5x^{6}(bc\,x^{2}+ac)^{\frac{5}{2}}b^{3}-15(bc\,x^{2}+ac)^{\frac{5}{2}}a\,b^{2}x^{4}-15(bc\,x^{2}+ac)^{\frac{5}{2}}a^{2}b\,x^{2}+6(bc\,x^{2}+ac)^{\frac{5}{2}}a^{3}-11a^{3}\left(c\,(b\,x^{2}+a)\right)^{\frac{5}{2}}\right)}{55b(b\,x^{2}+a)^{3}(c\,(b\,x^{2}+a))^{\frac{3}{2}}c}$

input
```
int(x*(c*(b*x^2+a)^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/11*(b*x^2+a)/b*(c*(b*x^2+a)^3)^(3/2)
```

3.238.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(28) = 56$.

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.72

$$\int x\left(c(a+bx^2)^3\right)^{3/2}\,dx = \frac{(b^4cx^8+4\,ab^3cx^6+6\,a^2b^2cx^4+4\,a^3bcx^2+a^4c)\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}}{11\,b}$$

input
```
integrate(x*(c*(b*x^2+a)^3)^(3/2),x, algorithm="fricas")
```

output
```
1/11*(b^4*c*x^8 + 4*a*b^3*c*x^6 + 6*a^2*b^2*c*x^4 + 4*a^3*b*c*x^2 + a^4*c)
*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)/b
```

3.238.6 Sympy [F]

$$\int x\left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx=\int x\left(c\left(a+bx^2\right)^3\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(c*(b*x**2+a)**3)**(3/2),x)
```

output
```
Integral(x*(c*(a + b*x**2)**3)**(3/2), x)
```

3.238.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(28) = 56$.

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.19

$$\int x\left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx=\frac{\left(b^4c^{\frac{3}{2}}x^8+4\,ab^3c^{\frac{3}{2}}x^6+6\,a^2b^2c^{\frac{3}{2}}x^4+4\,a^3bc^{\frac{3}{2}}x^2+a^4c^{\frac{3}{2}}\right)\left(bx^2+a\right)^{\frac{3}{2}}}{11\,b}$$

input
```
integrate(x*(c*(b*x^2+a)^3)^(3/2),x, algorithm="maxima")
```

output
```
1/11*(b^4*c^(3/2)*x^8 + 4*a*b^3*c^(3/2)*x^6 + 6*a^2*b^2*c^(3/2)*x^4 + 4*a^
3*b*c^(3/2)*x^2 + a^4*c^(3/2))*(b*x^2 + a)^(3/2)/b
```

3.238.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int x\left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx=\frac{\left(bcx^2+ac\right)^{\frac{11}{2}}\operatorname{sgn}\left(bx^2+a\right)}{11\,bc^4}$$

input
```
integrate(x*(c*(b*x^2+a)^3)^(3/2),x, algorithm="giac")
```

output
```
1/11*(b*c*x^2 + a*c)^(11/2)*sgn(b*x^2 + a)/(b*c^4)
```

3.238.9 Mupad [B] (verification not implemented)

Time = 18.39 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.94

$$\int x\left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx=\sqrt{c\left(b\,x^2+a\right)^3}\,\left(\frac{a^4\,c}{11\,b}+\frac{4\,a^3\,c\,x^2}{11}+\frac{b^3\,c\,x^8}{11}+\frac{6\,a^2\,b\,c\,x^4}{11}+\frac{4\,a\,b^2\,c\,x^6}{11}\right)$$

input
```
int(x*(c*(a + b*x^2)^3)^(3/2),x)
```

output
```
(c*(a + b*x^2)^3)^(1/2)*((a^4*c)/(11*b) + (4*a^3*c*x^2)/11 + (b^3*c*x^8)/1
1 + (6*a^2*b*c*x^4)/11 + (4*a*b^2*c*x^6)/11)
```

3.239 $\int \left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx$

3.239.1 Optimal result

Integrand size = 15, antiderivative size = 207

$$\int \left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx=\frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3}+\frac{63a^4cx\sqrt{c\left(a+bx^2\right)^3}}{256\left(a+bx^2\right)}+\frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{9}{80}acx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{10}cx\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}+\frac{63a^{7/2}c\sqrt{c\left(a+bx^2\right)^3}\operatorname{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

output
```
21/128*a^3*c*x*(c*(b*x^2+a)^3)^(1/2)+63/256*a^4*c*x*(c*(b*x^2+a)^3)^(1/2)/
(b*x^2+a)+21/160*a^2*c*x*(b*x^2+a)*(c*(b*x^2+a)^3)^(1/2)+9/80*a*c*x*(b*x^2
+a)^2*(c*(b*x^2+a)^3)^(1/2)+1/10*c*x*(b*x^2+a)^3*(c*(b*x^2+a)^3)^(1/2)+63/
256*a^(7/2)*c*arcsinh(x*b^(1/2)/a^(1/2))*(c*(b*x^2+a)^3)^(1/2)/(1+b*x^2/a)
^(3/2)/b^(1/2)
```

3.239.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.59

$$\int \left(c\left(a+bx^2\right)^3\right)^{3/2} dx = \frac{\left(c(a+bx^2)^3\right)^{3/2}\left(\sqrt{b}x\sqrt{a+bx^2}(965a^4+1490a^3bx^2+1368a^2b^2x^4+656ab^3x^6+128b^4x^8)\right.}{1280\sqrt{b}\left(a+bx^2\right)^{9/2}}$$

input
```
Integrate[(c*(a + b*x^2)^3)^(3/2),x]
```

output
```
((c*(a + b*x^2)^3)^(3/2)*(Sqrt[b]*x*Sqrt[a + b*x^2]*(965*a^4 + 1490*a^3*b*
x^2 + 1368*a^2*b^2*x^4 + 656*a*b^3*x^6 + 128*b^4*x^8) - 315*a^5*Log[-(Sqrt
[b]*x) + Sqrt[a + b*x^2]]))/(1280*Sqrt[b]*(a + b*x^2)^(9/2))
```

3.239.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {2045, 211, 211, 211, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c\left(a+bx^2\right)^3\right)^{3/2} dx$$

$$\Big\downarrow \; 2045$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\int \left(\frac{bx^2}{a}+1\right)^{9/2}dx}{\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \; 211$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\left(\frac{9}{10}\int \left(\frac{bx^2}{a}+1\right)^{7/2}dx+\frac{1}{10}x\left(\frac{bx^2}{a}+1\right)^{9/2}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \; 211$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\left(\frac{9}{10}\left(\frac{7}{8}\int\left(\frac{bx^2}{a}+1\right)^{5/2}dx+\frac{1}{8}x\left(\frac{bx^2}{a}+1\right)^{7/2}\right)+\frac{1}{10}x\left(\frac{bx^2}{a}+1\right)^{9/2}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \; 211$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\int\left(\frac{bx^2}{a}+1\right)^{3/2}dx+\frac{1}{6}x\left(\frac{bx^2}{a}+1\right)^{5/2}\right)+\frac{1}{8}x\left(\frac{bx^2}{a}+1\right)^{7/2}\right)+\frac{1}{10}x\left(\frac{bx^2}{a}+1\right)^{9/2}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \; 211$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int\sqrt{\frac{bx^2}{a}+1}dx+\frac{1}{4}x\left(\frac{bx^2}{a}+1\right)^{3/2}\right)+\frac{1}{6}x\left(\frac{bx^2}{a}+1\right)^{5/2}\right)+\frac{1}{8}x\left(\frac{bx^2}{a}+1\right)^{7/2}\right)+\frac{1}{10}x}{\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \; 211$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}}dx+\frac{1}{2}x\sqrt{\frac{bx^2}{a}+1}\right)+\frac{1}{4}x\left(\frac{bx^2}{a}+1\right)^{3/2}\right)+\frac{1}{6}x\left(\frac{bx^2}{a}+1\right)^{5/2}\right)+\frac{1}{8}x\left(b\right.\right.\right.}{\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \; 222$$

$$\frac{a^3c\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\sqrt{a}\operatorname{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{\frac{bx^2}{a}+1}\right)+\frac{1}{4}x\left(\frac{bx^2}{a}+1\right)^{3/2}\right)+\frac{1}{6}x\left(\frac{bx^2}{a}+1\right)^{5/2}\right)+\frac{1}{8}x\left(\frac{bx^2}{a}+1\right)^{7/2}\right)\right.\right.}{\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

input
```
Int[(c*(a + b*x^2)^3)^(3/2),x]
```

output
```
(a^3*c*Sqrt[c*(a + b*x^2)^3]*((x*(1 + (b*x^2)/a)^(9/2))/10 + (9*((x*(1 + (
b*x^2)/a)^(7/2))/8 + (7*((x*(1 + (b*x^2)/a)^(5/2))/6 + (5*((x*(1 + (b*x^2)
/a)^(3/2))/4 + (3*((x*Sqrt[1 + (b*x^2)/a])/2 + (Sqrt[a]*ArcSinh[(Sqrt[b]*x
)/Sqrt[a]])/(2*Sqrt[b])))/4))/6))/8))/10))/(1 + (b*x^2)/a)^(3/2)
```

3.239. $\int\left(c\left(a+bx^2\right)^3\right)^{3/2}dx$

3.239.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.239.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.67

method	result
risch	$\frac{x\left(128 b^{4} x^{8}+656 a\,b^{3} x^{6}+1368 a^{2} b^{2} x^{4}+1490 a^{3} b\,x^{2}+965 a^{4}\right) c\sqrt{c\left(b\,x^{2}+a\right)^{3}}}{1280 b\,x^{2}+1280 a}+\frac{63 a^{5} \ln\left(\frac{b c x}{\sqrt{b c}}+\sqrt{b c\,x^{2}+a c}\right) c\sqrt{c\left(b\,x^{2}+a\right)^{3}}\,\sqrt{c\left(b\,x^{2}+a\right)}}{256 \sqrt{b c}\,\left(b\,x^{2}+a\right)^{2}}$
default	$\frac{\left(c\left(b\,x^{2}+a\right)^{3}\right)^{\frac{3}{2}} \left(128 x^{5} \left(b c\,x^{2}+a c\right)^{\frac{5}{2}} b^{2} \sqrt{b c}+400 \left(b c\,x^{2}+a c\right)^{\frac{5}{2}} \sqrt{b c}\,a b\,x^{3}+440 \left(b c\,x^{2}+a c\right)^{\frac{5}{2}} \sqrt{b c}\,a^{2} x+210 \left(b c\,x^{2}+a c\right)^{\frac{3}{2}} \sqrt{b c}\,a^{3} c x+315\sqrt{}\right.}{1280 \left(b\,x^{2}+a\right)^{3} \left(c\left(b\,x^{2}+a\right)\right)^{\frac{3}{2}} c\sqrt{b c}}$

input
```
int((c*(b*x^2+a)^3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/1280*x*(128*b^4*x^8+656*a*b^3*x^6+1368*a^2*b^2*x^4+1490*a^3*b*x^2+965*a^
4)/(b*x^2+a)*c*(c*(b*x^2+a)^3)^(1/2)+63/256*a^5*ln(b*c*x/(b*c)^(1/2)+(b*c*
x^2+a*c)^(1/2))/(b*c)^(1/2)*c/(b*x^2+a)^2*(c*(b*x^2+a)^3)^(1/2)*(c*(b*x^2+
a))^(1/2)
```

3.239.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.94

$$\int \left(c(a+bx^2)^3\right)^{3/2}\,dx = \left[\frac{315\,(a^5bcx^2+a^6c)\sqrt{\frac{c}{b}}\log\left(-\frac{2\,b^2cx^4+3\,abcx^2+a^2c+2\,\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\,bx\sqrt{\frac{c}{b}}}{bx^2+a}\right)+2\,(128\,b\cdots}{256\cdots}\right.$$

$$-\frac{315\,(a^5bcx^2+a^6c)\sqrt{-\frac{c}{b}}\arctan\left(\frac{\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\,bx\sqrt{-\frac{c}{b}}}{b^2cx^4+2\,abcx^2+a^2c}\right)-(128\,b^4cx^9+656\,ab^3cx^7+1368\,a^2b^2c\cdots}{1280\,(bx^2+a)}$$

input
```
integrate((c*(b*x^2+a)^3)^(3/2),x, algorithm="fricas")
```

output
```
[1/2560*(315*(a^5*b*c*x^2 + a^6*c)*sqrt(c/b)*log(-(2*b^2*c*x^4 + 3*a*b*c*x
^2 + a^2*c + 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*b*x
*sqrt(c/b))/(b*x^2 + a)) + 2*(128*b^4*c*x^9 + 656*a*b^3*c*x^7 + 1368*a^2*b
^2*c*x^5 + 1490*a^3*b*c*x^3 + 965*a^4*c*x)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4
+ 3*a^2*b*c*x^2 + a^3*c))/(b*x^2 + a), -1/1280*(315*(a^5*b*c*x^2 + a^6*c)*
sqrt(-c/b)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*
b*x*sqrt(-c/b)/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) - (128*b^4*c*x^9 + 656*a
*b^3*c*x^7 + 1368*a^2*b^2*c*x^5 + 1490*a^3*b*c*x^3 + 965*a^4*c*x)*sqrt(b^3
*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^2 + a)]
```

3.239.6 Sympy [F]

$$\int \left(c(a+bx^2)^3\right)^{3/2}\,dx = \int \left(c(a+bx^2)^3\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*(b*x**2+a)**3)**(3/2),x)
```

output
```
Integral((c*(a + b*x**2)**3)**(3/2), x)
```

3.239.7 Maxima [F]

$$\int \left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx=\int \left(\left(bx^2+a\right)^3c\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*(b*x^2+a)^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate(((b*x^2 + a)^3*c)^(3/2), x)
```

3.239.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.74

$$\int \left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx=$$
$$-\frac{1}{1280}\left(\frac{315\,a^5c\log\left(\left|-\sqrt{bc}x+\sqrt{bcx^2+ac}\right|\right)\operatorname{sgn}(bx^2+a)}{\sqrt{bc}}-\left(965\,a^4\operatorname{sgn}\left(bx^2+a\right)+2\left(745\,a^3b\operatorname{sgn}\left(bx^2+\right.\right.\right.\right.$$

input
```
integrate((c*(b*x^2+a)^3)^(3/2),x, algorithm="giac")
```

output
```
-1/1280*(315*a^5*c*log(abs(-sqrt(b*c)*x + sqrt(b*c*x^2 + a*c)))*sgn(b*x^2
+ a)/sqrt(b*c) - (965*a^4*sgn(b*x^2 + a) + 2*(745*a^3*b*sgn(b*x^2 + a) + 4
*(171*a^2*b^2*sgn(b*x^2 + a) + 2*(8*b^4*x^2*sgn(b*x^2 + a) + 41*a*b^3*sgn(
b*x^2 + a))*x^2)*x^2)*x^2)*sqrt(b*c*x^2 + a*c)*x)*c
```

3.239.9 Mupad [F(-1)]

Timed out.

$$\int \left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx=\int \left(c\left(b\,x^2+a\right)^3\right)^{3/2}\,dx$$

input
```
int((c*(a + b*x^2)^3)^(3/2),x)
```

output
```
int((c*(a + b*x^2)^3)^(3/2), x)
```

3.240 $\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x} \, dx$

3.240.1 Optimal result

Integrand size = 19, antiderivative size = 192

$$\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x} \, dx = \frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3}+\frac{a^4c\sqrt{c\left(a+bx^2\right)^3}}{a+bx^2} + \frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{7}ac\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{9}c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}-\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\operatorname{arctanh}\left(\sqrt{1+\frac{bx^2}{a}}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

output

```
1/3*a^3*c*(c*(b*x^2+a)^3)^(1/2)+a^4*c*(c*(b*x^2+a)^3)^(1/2)/(b*x^2+a)+1/5*
a^2*c*(b*x^2+a)*(c*(b*x^2+a)^3)^(1/2)+1/7*a*c*(b*x^2+a)^2*(c*(b*x^2+a)^3)^
(1/2)+1/9*c*(b*x^2+a)^3*(c*(b*x^2+a)^3)^(1/2)-a^3*c*arctanh((1+b*x^2/a)^(1
/2))*(c*(b*x^2+a)^3)^(1/2)/(1+b*x^2/a)^(3/2)
```

3.240.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.58

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x}\,dx = \frac{\left(c(a+bx^2)^3\right)^{3/2}\left(\sqrt{a+bx^2}(563a^4+506a^3bx^2+408a^2b^2x^4+185ab^3x^6+35b^4x^8\right)}{315\,(a+bx^2)^{9/2}}$$

input
```
Integrate[(c*(a + b*x^2)^3)^(3/2)/x,x]
```

output
```
((c*(a + b*x^2)^3)^(3/2)*(Sqrt[a + b*x^2]*(563*a^4 + 506*a^3*b*x^2 + 408*a
^2*b^2*x^4 + 185*a*b^3*x^6 + 35*b^4*x^8) - 315*a^(9/2)*ArcTanh[Sqrt[a + b*
x^2]/Sqrt[a]]))/(315*(a + b*x^2)^(9/2))
```

3.240.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.74, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {2045, 243, 60, 60, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x}dx$$

$$\Big\downarrow \; 2045$$

$$\frac{a^3c\sqrt{c\,(a+bx^2)^3}\int \frac{\left(\frac{bx^2}{a}+1\right)^{9/2}}{x}dx}{\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \; 243$$

$$\frac{a^3c\sqrt{c\,(a+bx^2)^3}\int \frac{\left(\frac{bx^2}{a}+1\right)^{9/2}}{x^2}dx^2}{2\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \; 60$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\left(\int \frac{\left(\frac{bx^2}{a}+1\right)^{7/2}}{x^2}dx^2+\frac{2}{9}\left(\frac{bx^2}{a}+1\right)^{9/2}\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \; 60$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\left(\int \frac{\left(\frac{bx^2}{a}+1\right)^{5/2}}{x^2}dx^2+\frac{2}{9}\left(\frac{bx^2}{a}+1\right)^{9/2}+\frac{2}{7}\left(\frac{bx^2}{a}+1\right)^{7/2}\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \; 60$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\left(\int \frac{\left(\frac{bx^2}{a}+1\right)^{3/2}}{x^2}dx^2+\frac{2}{9}\left(\frac{bx^2}{a}+1\right)^{9/2}+\frac{2}{7}\left(\frac{bx^2}{a}+1\right)^{7/2}+\frac{2}{5}\left(\frac{bx^2}{a}+1\right)^{5/2}\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \; 60$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\left(\int \frac{\sqrt{\frac{bx^2}{a}+1}}{x^2}dx^2+\frac{2}{9}\left(\frac{bx^2}{a}+1\right)^{9/2}+\frac{2}{7}\left(\frac{bx^2}{a}+1\right)^{7/2}+\frac{2}{5}\left(\frac{bx^2}{a}+1\right)^{5/2}+\frac{2}{3}\left(\frac{bx^2}{a}+1\right)^{3/2}\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \; 60$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\left(\int \frac{1}{x^2\sqrt{\frac{bx^2}{a}+1}}dx^2+\frac{2}{9}\left(\frac{bx^2}{a}+1\right)^{9/2}+\frac{2}{7}\left(\frac{bx^2}{a}+1\right)^{7/2}+\frac{2}{5}\left(\frac{bx^2}{a}+1\right)^{5/2}+\frac{2}{3}\left(\frac{bx^2}{a}+1\right)^{3/2}+2\sqrt{\frac{bx^2}{a}}\right.}{2\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \; 73$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\left(\frac{2a\int \frac{1}{\frac{ax^4}{b}-\frac{a}{b}}d\sqrt{\frac{bx^2}{a}+1}}{b}+\frac{2}{9}\left(\frac{bx^2}{a}+1\right)^{9/2}+\frac{2}{7}\left(\frac{bx^2}{a}+1\right)^{7/2}+\frac{2}{5}\left(\frac{bx^2}{a}+1\right)^{5/2}+\frac{2}{3}\left(\frac{bx^2}{a}+1\right)^{3/2}+2\sqrt{\right.}{2\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \; 221$$

3.240. $\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x}\,dx$

$$\frac{a^3c\left(-2\text{arctanh}\left(\sqrt{\frac{bx^2}{a}+1}\right)+\frac{2}{9}\left(\frac{bx^2}{a}+1\right)^{9/2}+\frac{2}{7}\left(\frac{bx^2}{a}+1\right)^{7/2}+\frac{2}{5}\left(\frac{bx^2}{a}+1\right)^{5/2}+\frac{2}{3}\left(\frac{bx^2}{a}+1\right)^{3/2}+2\sqrt{\frac{bx^2}{a}+1}\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

input
```
Int[(c*(a + b*x^2)^3)^(3/2)/x,x]
```

output
```
(a^3*c*Sqrt[c*(a + b*x^2)^3]*(2*Sqrt[1 + (b*x^2)/a] + (2*(1 + (b*x^2)/a)^(
3/2))/3 + (2*(1 + (b*x^2)/a)^(5/2))/5 + (2*(1 + (b*x^2)/a)^(7/2))/7 + (2*(
1 + (b*x^2)/a)^(9/2))/9 - 2*ArcTanh[Sqrt[1 + (b*x^2)/a]]))/(2*(1 + (b*x^2)
/a)^(3/2))
```

3.240.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.240. $\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x}\,dx$

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.240.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.15

method	result
default	$-\frac{\left(c\left(b\,x^{2}+a\right)^{3}\right)^{\frac{3}{2}} \left(-35\sqrt{ac}\,\left(bc\,x^{2}+ac\right)^{\frac{5}{2}} b^{2} x^{4}-115\sqrt{ac}\,\left(bc\,x^{2}+ac\right)^{\frac{5}{2}} ab\,x^{2}+315\ln\left(\frac{2ac+2\sqrt{ac}\,\sqrt{bc\,x^{2}+ac}}{x}\right) a^{5} c^{3}+46\sqrt{ac}\,\left(bc\,x^{2}+ac\right)^{\frac{5}{2}}\right.}{315\left(b\,x^{2}+a\right)^{3} \left(c\left(b\,x^{2}+a\right)\right)^{\frac{3}{2}} c\sqrt{ac}}$

input
```
int((c*(b*x^2+a)^3)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/315*(c*(b*x^2+a)^3)^(3/2)*(-35*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*b^2*x^4-
115*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*a*b*x^2+315*ln(2*((a*c)^(1/2)*(b*c*x^2
+a*c)^(1/2)+a*c)/x)*a^5*c^3+46*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*a^2-105*(a*
c)^(1/2)*(b*c*x^2+a*c)^(3/2)*a^3*c-315*(a*c)^(1/2)*(b*c*x^2+a*c)^(1/2)*a^4
*c^2-189*a^2*(c*(b*x^2+a))^(5/2)*(a*c)^(1/2))/(b*x^2+a)^3/(c*(b*x^2+a))^(3
/2)/c/(a*c)^(1/2)
```

3.240.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 391, normalized size of antiderivative = 2.04

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x}\,dx = \left[\frac{315\left(a^4bcx^2+a^5c\right)\sqrt{ac}\log\left(-\frac{b^2cx^4+3\,abcx^2+2\,a^2c-2\,\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\sqrt{ac}}{bx^4+ax^2}\right)+}{}\right.$$

input
```
integrate((c*(b*x^2+a)^3)^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/630*(315*(a^4*b*c*x^2 + a^5*c)*sqrt(a*c)*log(-(b^2*c*x^4 + 3*a*b*c*x^2
+ 2*a^2*c - 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sqrt
(a*c))/(b*x^4 + a*x^2)) + 2*(35*b^4*c*x^8 + 185*a*b^3*c*x^6 + 408*a^2*b^2*
c*x^4 + 506*a^3*b*c*x^2 + 563*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^
2*b*c*x^2 + a^3*c))/(b*x^2 + a), 1/315*(315*(a^4*b*c*x^2 + a^5*c)*sqrt(-a*
c)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sqrt(-a*
c)/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) + (35*b^4*c*x^8 + 185*a*b^3*c*x^6 +
408*a^2*b^2*c*x^4 + 506*a^3*b*c*x^2 + 563*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*
c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^2 + a)]
```

3.240.6 Sympy [F]

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x}\,dx = \int \frac{\left(c(a+bx^2)^3\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((c*(b*x**2+a)**3)**(3/2)/x,x)
```

output
```
Integral((c*(a + b*x**2)**3)**(3/2)/x, x)
```

3.240.7 Maxima [F]

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x}\,dx = \int \frac{\left((bx^2+a)^3c\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((c*(b*x^2+a)^3)^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate(((b*x^2 + a)^3*c)^(3/2)/x, x)
```

3.240.8 Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.96

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x}\,dx = \frac{1}{315}\left(\frac{315\,a^5\arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)\operatorname{sgn}(bx^2+a)}{\sqrt{-ac}} + \frac{315\sqrt{bcx^2+ac}a^4c^{44}\operatorname{sgn}(bx^2+a)\right.$$

input
```
integrate((c*(b*x^2+a)^3)^(3/2)/x,x, algorithm="giac")
```

output
```
1/315*(315*a^5*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))*sgn(b*x^2 + a)/sqrt(
-a*c) + (315*sqrt(b*c*x^2 + a*c)*a^4*c^44*sgn(b*x^2 + a) + 105*(b*c*x^2 +
a*c)^(3/2)*a^3*c^43*sgn(b*x^2 + a) + 63*(b*c*x^2 + a*c)^(5/2)*a^2*c^42*sgn
(b*x^2 + a) + 45*(b*c*x^2 + a*c)^(7/2)*a*c^41*sgn(b*x^2 + a) + 35*(b*c*x^2
 + a*c)^(9/2)*c^40*sgn(b*x^2 + a))/c^45)*c^2
```

3.240.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x}\,dx = \int \frac{\left(c\,{\left(b\,x^2+a\right)}^3\right)^{3/2}}{x}\,dx$$

input
```
int((c*(a + b*x^2)^3)^(3/2)/x,x)
```

output
```
int((c*(a + b*x^2)^3)^(3/2)/x, x)
```

3.241 $\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x^2} dx$

3.241.1 Optimal result

Integrand size = 19, antiderivative size = 208

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^2} dx = \frac{105}{64}a^2bcx\sqrt{c(a+bx^2)^3} + \frac{315a^3bcx\sqrt{c(a+bx^2)^3}}{128(a+bx^2)} + \frac{21}{16}abcx(a+bx^2)\sqrt{c(a+bx^2)^3} + \frac{9}{8}bcx(a+bx^2)^2\sqrt{c(a+bx^2)^3} - \frac{c(a+bx^2)^3\sqrt{c(a+bx^2)^3}}{x} + \frac{315a^{5/2}\sqrt{b}c\sqrt{c(a+bx^2)^3}\operatorname{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

output
```
105/64*a^2*b*c*x*(c*(b*x^2+a)^3)^(1/2)+315/128*a^3*b*c*x*(c*(b*x^2+a)^3)^(
1/2)/(b*x^2+a)+21/16*a*b*c*x*(b*x^2+a)*(c*(b*x^2+a)^3)^(1/2)+9/8*b*c*x*(b*
x^2+a)^2*(c*(b*x^2+a)^3)^(1/2)-c*(b*x^2+a)^3*(c*(b*x^2+a)^3)^(1/2)/x+315/1
28*a^(5/2)*c*arcsinh(x*b^(1/2)/a^(1/2))*b^(1/2)*(c*(b*x^2+a)^3)^(1/2)/(1+b
*x^2/a)^(3/2)
```

3.241.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.58

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^2}\,dx =$$

$$-\frac{\left(c(a+bx^2)^3\right)^{3/2}\left(\sqrt{a+bx^2}(128a^4-325a^3bx^2-210a^2b^2x^4-88ab^3x^6-16b^4x^8)+315a^4\sqrt{b}x\log\left(-\sqrt{b}x\right.\right.}{128x\,(a+bx^2)^{9/2}}$$

input
```
Integrate[(c*(a + b*x^2)^3)^(3/2)/x^2,x]
```

output
```
-1/128*((c*(a + b*x^2)^3)^(3/2)*(Sqrt[a + b*x^2]*(128*a^4 - 325*a^3*b*x^2
- 210*a^2*b^2*x^4 - 88*a*b^3*x^6 - 16*b^4*x^8) + 315*a^4*Sqrt[b]*x*Log[-(S
qrt[b]*x) + Sqrt[a + b*x^2]]))/(x*(a + b*x^2)^(9/2))
```

3.241.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.86, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2045, 247, 211, 211, 211, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{ 2045}$$

$$\frac{a^3c\sqrt{c\,(a+bx^2)^3}\int \frac{\left(\frac{bx^2}{a}+1\right)^{9/2}}{x^2}dx}{\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \text{ 247}$$

$$\frac{a^3c\sqrt{c\,(a+bx^2)^3}\left(\frac{9b\int \left(\frac{bx^2}{a}+1\right)^{7/2}dx}{a}-\frac{\left(\frac{bx^2}{a}+1\right)^{9/2}}{x}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \; 211$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\left(\frac{9b\left(\frac{7}{8}\int\left(\frac{bx^2}{a}+1\right)^{5/2}dx+\frac{1}{8}x\left(\frac{bx^2}{a}+1\right)^{7/2}\right)}{a}-\frac{\left(\frac{bx^2}{a}+1\right)^{9/2}}{x}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \; 211$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\left(\frac{9b\left(\frac{7}{8}\left(\frac{5}{6}\int\left(\frac{bx^2}{a}+1\right)^{3/2}dx+\frac{1}{6}x\left(\frac{bx^2}{a}+1\right)^{5/2}\right)+\frac{1}{8}x\left(\frac{bx^2}{a}+1\right)^{7/2}\right)}{a}-\frac{\left(\frac{bx^2}{a}+1\right)^{9/2}}{x}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \; 211$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\left(\frac{9b\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int\sqrt{\frac{bx^2}{a}+1}dx+\frac{1}{4}x\left(\frac{bx^2}{a}+1\right)^{3/2}\right)+\frac{1}{6}x\left(\frac{bx^2}{a}+1\right)^{5/2}\right)+\frac{1}{8}x\left(\frac{bx^2}{a}+1\right)^{7/2}\right)}{a}-\frac{\left(\frac{bx^2}{a}+1\right)^{9/2}}{x}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \; 211$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\left(\frac{9b\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int\frac{1}{\sqrt{\frac{bx^2}{a}+1}}dx+\frac{1}{2}x\sqrt{\frac{bx^2}{a}+1}\right)+\frac{1}{4}x\left(\frac{bx^2}{a}+1\right)^{3/2}\right)+\frac{1}{6}x\left(\frac{bx^2}{a}+1\right)^{5/2}\right)+\frac{1}{8}x\left(\frac{bx^2}{a}+1\right)^{7/2}\right)}{a}-\frac{\left(\frac{bx^2}{a}+1\right)\cdots}{x}\right.}{\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \; 222$$

$$\frac{a^3c\left(\frac{9b\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\sqrt{a}\,\text{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{\frac{bx^2}{a}+1}\right)+\frac{1}{4}x\left(\frac{bx^2}{a}+1\right)^{3/2}\right)+\frac{1}{6}x\left(\frac{bx^2}{a}+1\right)^{5/2}\right)+\frac{1}{8}x\left(\frac{bx^2}{a}+1\right)^{7/2}\right)}{a}-\frac{\left(\frac{bx^2}{a}+1\right)^{9/2}}{x}\right)\sqrt{c(a\cdots}}{\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

input
```
Int[(c*(a + b*x^2)^3)^(3/2)/x^2,x]
```

3.241. $\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x^2}\,dx$

output
```
(a^3*c*Sqrt[c*(a + b*x^2)^3]*(-((1 + (b*x^2)/a)^(9/2)/x) + (9*b*((x*(1 + (
b*x^2)/a)^(7/2))/8 + (7*((x*(1 + (b*x^2)/a)^(5/2))/6 + (5*((x*(1 + (b*x^2)
/a)^(3/2))/4 + (3*((x*Sqrt[1 + (b*x^2)/a])/2 + (Sqrt[a]*ArcSinh[(Sqrt[b]*x
)/Sqrt[a]])/(2*Sqrt[b])))/4))/6))/8))/a))/(1 + (b*x^2)/a)^(3/2)
```

3.241.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.241.4 Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{\left(-16 b^{4} x^{8}-88 a\, b^{3} x^{6}-210 a^{2} b^{2} x^{4}-325 a^{3} b\, x^{2}+128 a^{4}\right) c \sqrt{c \left(b\, x^{2}+a\right)^{3}}}{128 \left(b\, x^{2}+a \right) x}+\frac{315 b\, a^{4} \ln \left(\frac{b c x}{\sqrt{b c}}+\sqrt{b c\, x^{2}+a c}\right) c \sqrt{c \left(b\, x^{2}+a\right)^{3}}\, \sqrt{c \left(b\, x^{2}+a\right)}}{128 \sqrt{b c}\, \left(b\, x^{2}+a\right)^{2}}$
default	$-\frac{\left(c \left(b\, x^{2}+a\right)^{3}\right)^{\frac{3}{2}} \left(-16 \left(b c\, x^{2}+a c\right)^{\frac{5}{2}} \sqrt{b c}\, b^{2} x^{4}-56 \left(b c\, x^{2}+a c\right)^{\frac{5}{2}} \sqrt{b c}\, a b\, x^{2}-210 \left(b c\, x^{2}+a c\right)^{\frac{3}{2}} \sqrt{b c}\, a^{2} b c\, x^{2}-315 \sqrt{b c\, x^{2}+a c}\, \sqrt{b c}\, a^{3} b\, c^{2} x^{2}}{128 \left(b\, x^{2}+a\right)^{3} \left(c \left(b\, x^{2}+a\right)\right)^{\frac{3}{2}} c \sqrt{b c}\, x}$

input
```
int((c*(b*x^2+a)^3)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

3.241. $\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x^2}\,dx$

output
```
-1/128/(b*x^2+a)*(-16*b^4*x^8-88*a*b^3*x^6-210*a^2*b^2*x^4-325*a^3*b*x^2+1
28*a^4)/x*c*(c*(b*x^2+a)^3)^(1/2)+315/128*b*a^4*ln(b*c*x/(b*c)^(1/2)+(b*c*
x^2+a*c)^(1/2))/(b*c)^(1/2)*c/(b*x^2+a)^2*(c*(b*x^2+a)^3)^(1/2)*(c*(b*x^2+
a))^(1/2)
```

3.241.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.90

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^2}\,dx = \left[\frac{315\,(a^4bcx^3+a^5cx)\sqrt{bc}\log\left(-\frac{2\,b^2cx^4+3\,abcx^2+a^2c+2\,\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\sqrt{bc}x}{bx^2+a}\right)}{}\right.$$

$$-\frac{315\,(a^4bcx^3+a^5cx)\sqrt{-bc}\arctan\left(\frac{\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\sqrt{-bc}x}{b^2cx^4+2\,abcx^2+a^2c}\right)-(16\,b^4cx^8+88\,ab^3cx^6+210\,a^2b^2cx^4}{128\,(bx^3+ax)}$$

input
```
integrate((c*(b*x^2+a)^3)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
[1/256*(315*(a^4*b*c*x^3 + a^5*c*x)*sqrt(b*c)*log(-(2*b^2*c*x^4 + 3*a*b*c*
x^2 + a^2*c + 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sq
rt(b*c)*x)/(b*x^2 + a)) + 2*(16*b^4*c*x^8 + 88*a*b^3*c*x^6 + 210*a^2*b^2*c
*x^4 + 325*a^3*b*c*x^2 - 128*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2
*b*c*x^2 + a^3*c))/(b*x^3 + a*x), -1/128*(315*(a^4*b*c*x^3 + a^5*c*x)*sqrt
(-b*c)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sqrt
(-b*c)*x/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) - (16*b^4*c*x^8 + 88*a*b^3*c*x
^6 + 210*a^2*b^2*c*x^4 + 325*a^3*b*c*x^2 - 128*a^4*c)*sqrt(b^3*c*x^6 + 3*a
*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^3 + a*x)]
```

3.241.6 Sympy [F]

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^2}\,dx = \int \frac{\left(c(a+bx^2)^3\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*(b*x**2+a)**3)**(3/2)/x**2,x)
```

output
```
Integral((c*(a + b*x**2)**3)**(3/2)/x**2, x)
```

3.241.7 Maxima [F]

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^2}\,dx=\int \frac{\left((bx^2+a)^3c\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*(b*x^2+a)^3)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(((b*x^2 + a)^3*c)^(3/2)/x^2, x)
```

3.241.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.89

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^2}\,dx=\frac{1}{256}\left(\frac{512\sqrt{bc}a^5c\text{sgn}(bx^2+a)}{\left(\sqrt{bc}x-\sqrt{bcx^2+ac}\right)^2-ac}-315\sqrt{bc}a^4\log\left(\left(\sqrt{bc}x-\sqrt{bcx^2+ac}\right)^2\right)\text{sg}\right.$$

input
```
integrate((c*(b*x^2+a)^3)^(3/2)/x^2,x, algorithm="giac")
```

output
```
1/256*(512*sqrt(b*c)*a^5*c*sgn(b*x^2 + a)/((sqrt(b*c)*x - sqrt(b*c*x^2 + a
*c))^2 - a*c) - 315*sqrt(b*c)*a^4*log((sqrt(b*c)*x - sqrt(b*c*x^2 + a*c))^
2)*sgn(b*x^2 + a) + 2*(325*a^3*b*sgn(b*x^2 + a) + 2*(105*a^2*b^2*sgn(b*x^2
 + a) + 4*(2*b^4*x^2*sgn(b*x^2 + a) + 11*a*b^3*sgn(b*x^2 + a))*x^2)*x^2)*s
qrt(b*c*x^2 + a*c)*x)*c
```

3.241.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^2}\,dx=\int \frac{\left(c\,(b\,x^2+a)^3\right)^{3/2}}{x^2}\,dx$$

input
```
int((c*(a + b*x^2)^3)^(3/2)/x^2,x)
```

output
```
int((c*(a + b*x^2)^3)^(3/2)/x^2, x)
```

3.242 $\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x^3}\,dx$

3.242.1 Optimal result

Integrand size = 19, antiderivative size = 202

$$\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x^3}\,dx = \frac{3}{2}a^2bc\sqrt{c\left(a+bx^2\right)^3}+\frac{9a^3bc\sqrt{c\left(a+bx^2\right)^3}}{2\left(a+bx^2\right)} + \frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{9}{14}bc\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} - \frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}}{2x^2}-\frac{9a^2bc\sqrt{c\left(a+bx^2\right)^3}\,\text{arctanh}\left(\sqrt{1+\frac{bx^2}{a}}\right)}{2\left(1+\frac{bx^2}{a}\right)^{3/2}}$$

output
```
3/2*a^2*b*c*(c*(b*x^2+a)^3)^(1/2)+9/2*a^3*b*c*(c*(b*x^2+a)^3)^(1/2)/(b*x^2
+a)+9/10*a*b*c*(b*x^2+a)*(c*(b*x^2+a)^3)^(1/2)+9/14*b*c*(b*x^2+a)^2*(c*(b*
x^2+a)^3)^(1/2)-1/2*c*(b*x^2+a)^3*(c*(b*x^2+a)^3)^(1/2)/x^2-9/2*a^2*b*c*ar
ctanh((1+b*x^2/a)^(1/2))*(c*(b*x^2+a)^3)^(1/2)/(1+b*x^2/a)^(3/2)
```

3.242.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.58

$$\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x^3}\,dx = -\frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}\left(\sqrt{a+bx^2}\left(35a^4-388a^3bx^2-156a^2b^2x^4-58ab^3x^6-10b^4x^8\right)+315a^{7/2}bx^2\text{arctanh}\left(\frac{\sqrt{a}}{\cdots}\right)\right)}{70x^2\left(a+bx^2\right)^{9/2}}$$

input
```
Integrate[(c*(a + b*x^2)^3)^(3/2)/x^3,x]
```

output
```
-1/70*((c*(a + b*x^2)^3)^(3/2)*(Sqrt[a + b*x^2]*(35*a^4 - 388*a^3*b*x^2 -
156*a^2*b^2*x^4 - 58*a*b^3*x^6 - 10*b^4*x^8) + 315*a^(7/2)*b*x^2*ArcTanh[S
qrt[a + b*x^2]/Sqrt[a]]))/(x^2*(a + b*x^2)^(9/2))
```

3.242.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.76, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {2045, 243, 51, 60, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \; 2045$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\int \frac{\left(\frac{bx^2}{a}+1\right)^{9/2}}{x^3}dx}{\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \; 243$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\int \frac{\left(\frac{bx^2}{a}+1\right)^{9/2}}{x^4}dx^2}{2\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \; 51$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\left(\frac{9b\int\frac{\left(\frac{bx^2}{a}+1\right)^{7/2}}{x^2}dx^2}{2a}-\frac{\left(\frac{bx^2}{a}+1\right)^{9/2}}{x^2}\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\left(\frac{9b\left(\int\frac{\left(\frac{bx^2}{a}+1\right)^{5/2}}{x^2}dx^2+\frac{2}{7}\left(\frac{bx^2}{a}+1\right)^{7/2}\right)}{2a}-\frac{\left(\frac{bx^2}{a}+1\right)^{9/2}}{x^2}\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\left(\frac{9b\left(\int\frac{\left(\frac{bx^2}{a}+1\right)^{3/2}}{x^2}dx^2+\frac{2}{7}\left(\frac{bx^2}{a}+1\right)^{7/2}+\frac{2}{5}\left(\frac{bx^2}{a}+1\right)^{5/2}\right)}{2a}-\frac{\left(\frac{bx^2}{a}+1\right)^{9/2}}{x^2}\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\left(\frac{9b\left(\int\frac{\sqrt{\frac{bx^2}{a}+1}}{x^2}dx^2+\frac{2}{7}\left(\frac{bx^2}{a}+1\right)^{7/2}+\frac{2}{5}\left(\frac{bx^2}{a}+1\right)^{5/2}+\frac{2}{3}\left(\frac{bx^2}{a}+1\right)^{3/2}\right)}{2a}-\frac{\left(\frac{bx^2}{a}+1\right)^{9/2}}{x^2}\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\left(\frac{9b\left(\int\frac{1}{x^2\sqrt{\frac{bx^2}{a}+1}}dx^2+\frac{2}{7}\left(\frac{bx^2}{a}+1\right)^{7/2}+\frac{2}{5}\left(\frac{bx^2}{a}+1\right)^{5/2}+\frac{2}{3}\left(\frac{bx^2}{a}+1\right)^{3/2}+2\sqrt{\frac{bx^2}{a}+1}\right)}{2a}-\frac{\left(\frac{bx^2}{a}+1\right)^{9/2}}{x^2}\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \text{ 73}$$

3.242. $\int\frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x^3}dx$

$$\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\left(\frac{9b\left(\frac{2a\int\frac{1}{\frac{ax^4}{b}-\frac{a}{b}}d\sqrt{\frac{bx^2}{a}+1}}{b}+\frac{2}{7}\left(\frac{bx^2}{a}+1\right)^{7/2}+\frac{2}{5}\left(\frac{bx^2}{a}+1\right)^{5/2}+\frac{2}{3}\left(\frac{bx^2}{a}+1\right)^{3/2}+2\sqrt{\frac{bx^2}{a}+1}\right)}{2a}-\frac{\left(\frac{bx^2}{a}+1\right)^{9/2}}{x^2}\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{a^3c\left(\frac{9b\left(-2\text{arctanh}\left(\sqrt{\frac{bx^2}{a}+1}\right)+\frac{2}{7}\left(\frac{bx^2}{a}+1\right)^{7/2}+\frac{2}{5}\left(\frac{bx^2}{a}+1\right)^{5/2}+\frac{2}{3}\left(\frac{bx^2}{a}+1\right)^{3/2}+2\sqrt{\frac{bx^2}{a}+1}\right)}{2a}-\frac{\left(\frac{bx^2}{a}+1\right)^{9/2}}{x^2}\right)\sqrt{c\left(a+bx^2\right)^3}}{2\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

input
```
Int[(c*(a + b*x^2)^3)^(3/2)/x^3,x]
```

output
```
(a^3*c*Sqrt[c*(a + b*x^2)^3]*(-((1 + (b*x^2)/a)^(9/2)/x^2) + (9*b*(2*Sqrt[
1 + (b*x^2)/a] + (2*(1 + (b*x^2)/a)^(3/2))/3 + (2*(1 + (b*x^2)/a)^(5/2))/5
 + (2*(1 + (b*x^2)/a)^(7/2))/7 - 2*ArcTanh[Sqrt[1 + (b*x^2)/a]]))/(2*a)))/
(2*(1 + (b*x^2)/a)^(3/2))
```

3.242.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.242.4 Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.10

method	result
risch	$-\frac{a^4 c\sqrt{c(b\,x^2+a)^3}}{2(b\,x^2+a)x^2}+\frac{\left(\frac{b^4x^6\sqrt{bc\,x^2+ac}}{7c}+\frac{29b^3a\,x^4\sqrt{bc\,x^2+ac}}{35c}+\frac{78b^2a^2x^2\sqrt{bc\,x^2+ac}}{35c}-\frac{156b\,a^3\sqrt{bc\,x^2+ac}}{35c}-\frac{9b\,a^4\ln\left(\frac{2ac+2\sqrt{ac}\,\sqrt{bc\,x^2+ac}}{x}\right.}{2\sqrt{ac}}\right.}{(b\,x^2+a)^2}$
default	$\frac{\left(c(b\,x^2+a)^3\right)^{\frac{3}{2}}\left(10\sqrt{ac}\,(bc\,x^2+ac)^{\frac{5}{2}}b^2x^4-315\ln\left(\frac{2ac+2\sqrt{ac}\,\sqrt{bc\,x^2+ac}}{x}\right)a^4bc^3x^2+42ba\left(c(b\,x^2+a)\right)^{\frac{5}{2}}x^2\sqrt{ac}-4\sqrt{ac}\,(bc\,x^2+ac)^{\frac{5}{2}}ab\right.}{70(b\,x^2+a)^3\left(c(b\,x^2+a)\right)^{\frac{3}{2}}c\,x^2\sqrt{ac}}$

input
```
int((c*(b*x^2+a)^3)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^4/(b*x^2+a)/x^2*c*(c*(b*x^2+a)^3)^(1/2)+(1/7*b^4*x^6/c*(b*c*x^2+a*c
)^(1/2)+29/35*b^3*a*x^4/c*(b*c*x^2+a*c)^(1/2)+78/35*b^2*a^2*x^2/c*(b*c*x^2
+a*c)^(1/2)-156/35*b*a^3/c*(b*c*x^2+a*c)^(1/2)-9/2*b*a^4/(a*c)^(1/2)*ln((2
*a*c+2*(a*c)^(1/2)*(b*c*x^2+a*c)^(1/2))/x)+10*b*a^3/c*(c*(b*x^2+a))^(1/2))
*c/(b*x^2+a)^2*(c*(b*x^2+a)^3)^(1/2)*(c*(b*x^2+a))^(1/2)
```

3.242. $\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^3}\,dx$

3.242.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.03

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^3}\,dx = \left[\frac{315\,(a^3b^2cx^4+a^4bcx^2)\sqrt{ac}\log\left(-\frac{b^2cx^4+3\,abcx^2+2\,a^2c-2\,\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\sqrt{ac}}{bx^4+ax^2}\right.}{}\right.$$

input
```
integrate((c*(b*x^2+a)^3)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
[1/140*(315*(a^3*b^2*c*x^4 + a^4*b*c*x^2)*sqrt(a*c)*log(-(b^2*c*x^4 + 3*a*
b*c*x^2 + 2*a^2*c - 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3
*c)*sqrt(a*c))/(b*x^4 + a*x^2)) + 2*(10*b^4*c*x^8 + 58*a*b^3*c*x^6 + 156*a
^2*b^2*c*x^4 + 388*a^3*b*c*x^2 - 35*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4
+ 3*a^2*b*c*x^2 + a^3*c))/(b*x^4 + a*x^2), 1/70*(315*(a^3*b^2*c*x^4 + a^4*
b*c*x^2)*sqrt(-a*c)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2
+ a^3*c)*sqrt(-a*c)/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) + (10*b^4*c*x^8 + 5
8*a*b^3*c*x^6 + 156*a^2*b^2*c*x^4 + 388*a^3*b*c*x^2 - 35*a^4*c)*sqrt(b^3*c
*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^4 + a*x^2)]
```

3.242.6 Sympy [F]

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^3}\,dx = \int \frac{\left(c(a+bx^2)^3\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((c*(b*x**2+a)**3)**(3/2)/x**3,x)
```

output
```
Integral((c*(a + b*x**2)**3)**(3/2)/x**3, x)
```

3.242.7 Maxima [F]

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^3}\,dx = \int \frac{\left((bx^2+a)^3c\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((c*(b*x^2+a)^3)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(((b*x^2 + a)^3*c)^(3/2)/x^3, x)
```

3.242.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.01

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^3}\,dx = \frac{\left(\frac{315\,a^4b^2c\arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)\mathrm{sgn}(bx^2+a)}{\sqrt{-ac}} - \frac{35\,\sqrt{bcx^2+ac}a^4b\mathrm{sgn}(bx^2+a)}{x^2} + \frac{2\left(140\,\sqrt{bcx^2+ac}a^3b^2c^{21}\mathrm{sgn}\right.}{}\right.}{}$$

input
```
integrate((c*(b*x^2+a)^3)^(3/2)/x^3,x, algorithm="giac")
```

output
```
1/70*(315*a^4*b^2*c*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))*sgn(b*x^2 + a)/
sqrt(-a*c) - 35*sqrt(b*c*x^2 + a*c)*a^4*b*sgn(b*x^2 + a)/x^2 + 2*(140*sqrt
(b*c*x^2 + a*c)*a^3*b^2*c^21*sgn(b*x^2 + a) + 35*(b*c*x^2 + a*c)^(3/2)*a^2
*b^2*c^20*sgn(b*x^2 + a) + 14*(b*c*x^2 + a*c)^(5/2)*a*b^2*c^19*sgn(b*x^2 +
 a) + 5*(b*c*x^2 + a*c)^(7/2)*b^2*c^18*sgn(b*x^2 + a))/c^21)*c/b
```

3.242.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^3}\,dx = \int \frac{\left(c\,(b\,x^2+a)^3\right)^{3/2}}{x^3}\,dx$$

input
```
int((c*(a + b*x^2)^3)^(3/2)/x^3,x)
```

output
```
int((c*(a + b*x^2)^3)^(3/2)/x^3, x)
```

3.243 $\int x^2\left(\frac{c}{a+bx^2}\right)^{3/2}\,dx$

3.243.1 Optimal result

Integrand size = 19, antiderivative size = 77

$$\int x^2\left(\frac{c}{a+bx^2}\right)^{3/2}\,dx=-\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b}+\frac{\sqrt{a}c\sqrt{\frac{c}{a+bx^2}}\sqrt{1+\frac{bx^2}{a}}\text{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

output
```
-c*x*(c/(b*x^2+a))^(1/2)/b+c*arcsinh(x*b^(1/2)/a^(1/2))*a^(1/2)*(c/(b*x^2+
a))^(1/2)*(1+b*x^2/a)^(1/2)/b^(3/2)
```

3.243.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int x^2\left(\frac{c}{a+bx^2}\right)^{3/2}\,dx=-\frac{c\sqrt{\frac{c}{a+bx^2}}\left(\sqrt{b}x+2\sqrt{a+bx^2}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}-\sqrt{a+bx^2}}\right)\right)}{b^{3/2}}$$

input
```
Integrate[x^2*(c/(a + b*x^2))^(3/2),x]
```

output
```
-((c*Sqrt[c/(a + b*x^2)]*(Sqrt[b]*x + 2*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x
)/(Sqrt[a] - Sqrt[a + b*x^2])]))/b^(3/2))
```

3.243.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2045, 252, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(\frac{c}{a+bx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{ 2045}$$

$$\frac{c\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{c}{a+bx^2}}\int \frac{x^2}{\left(\frac{bx^2}{a}+1\right)^{3/2}}dx}{a}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{c\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{c}{a+bx^2}}\left(\frac{a\int \frac{1}{\sqrt{\frac{bx^2}{a}+1}}dx}{b}-\frac{ax}{b\sqrt{\frac{bx^2}{a}+1}}\right)}{a}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{c\sqrt{\frac{bx^2}{a}+1}\left(\frac{a^{3/2}\text{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}-\frac{ax}{b\sqrt{\frac{bx^2}{a}+1}}\right)\sqrt{\frac{c}{a+bx^2}}}{a}$$

input
```
Int[x^2*(c/(a + b*x^2))^(3/2),x]
```

output
```
(c*Sqrt[c/(a + b*x^2)]*Sqrt[1 + (b*x^2)/a]*(-((a*x)/(b*Sqrt[1 + (b*x^2)/a]
)) + (a^(3/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)))/a
```

3.243.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.243.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\left(\frac{c}{b\,x^{2}+a}\right)^{\frac{3}{2}} \left(b\,x^{2}+a\right) \left(x\,b^{\frac{3}{2}}-\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right) b\sqrt{b\,x^{2}+a}\right)}{b^{\frac{5}{2}}}$	60

input
```
int(x^2*(c/(b*x^2+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(c/(b*x^2+a))^(3/2)*(b*x^2+a)*(x*b^(3/2)-ln(b^(1/2)*x+(b*x^2+a)^(1/2))*b*
(b*x^2+a)^(1/2))/b^(5/2)
```

3.243.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.83

$$\int x^2 \left(\frac{c}{a+bx^2}\right)^{3/2} dx = \left[-\frac{2\,cx\sqrt{\frac{c}{bx^2+a}} - c\sqrt{\frac{c}{b}}\log\left(-2\,bcx^2 - ac - 2\,(b^2x^3 + abx)\sqrt{\frac{c}{bx^2+a}}\sqrt{\frac{c}{b}}\right)}{2\,b}, -\frac{cx\sqrt{\frac{c}{bx^2+a}} + c\sqrt{-\frac{c}{b}}\arctan\left(\frac{bx\sqrt{\frac{c}{bx^2+a}}\sqrt{-\frac{c}{b}}}{c}\right)}{b}\right]$$

input
```
integrate(x^2*(c/(b*x^2+a))^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*(2*c*x*sqrt(c/(b*x^2 + a)) - c*sqrt(c/b)*log(-2*b*c*x^2 - a*c - 2*(b
^2*x^3 + a*b*x)*sqrt(c/(b*x^2 + a))*sqrt(c/b)))/b, -(c*x*sqrt(c/(b*x^2 + a
)) + c*sqrt(-c/b)*arctan(b*x*sqrt(c/(b*x^2 + a))*sqrt(-c/b)/c))/b]
```

3.243.6 Sympy [F]

$$\int x^2 \left(\frac{c}{a+bx^2}\right)^{3/2} dx = \int x^2 \left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}} dx$$

input
```
integrate(x**2*(c/(b*x**2+a))**(3/2),x)
```

output
```
Integral(x**2*(c/(a + b*x**2))**(3/2), x)
```

3.243.7 Maxima [F]

$$\int x^2 \left(\frac{c}{a+bx^2}\right)^{3/2} dx = \int x^2 \left(\frac{c}{bx^2+a}\right)^{\frac{3}{2}} dx$$

input
```
integrate(x^2*(c/(b*x^2+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2*(c/(b*x^2 + a))^(3/2), x)
```

3.243.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int x^2 \left(\frac{c}{a+bx^2}\right)^{3/2} dx = -\left(\frac{cx\mathrm{sgn}(bx^2+a)}{\sqrt{bcx^2+ac}b} + \frac{c\log\left(\left|-\sqrt{bc}x+\sqrt{bcx^2+ac}\right|\right)\mathrm{sgn}(bx^2+a)}{\sqrt{bc}b}\right)c$$

input
```
integrate(x^2*(c/(b*x^2+a))^(3/2),x, algorithm="giac")
```

output
```
-(c*x*sgn(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*b) + c*log(abs(-sqrt(b*c)*x + sq
rt(b*c*x^2 + a*c)))*sgn(b*x^2 + a)/(sqrt(b*c)*b))*c
```

3.243.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \left(\frac{c}{a+bx^2}\right)^{3/2} dx = \int x^2 \left(\frac{c}{b\,x^2+a}\right)^{3/2} dx$$

input
```
int(x^2*(c/(a + b*x^2))^(3/2),x)
```

output
```
int(x^2*(c/(a + b*x^2))^(3/2), x)
```

3.244 $\int x\left(\frac{c}{a+bx^2}\right)^{3/2} dx$

3.244.1 Optimal result

Integrand size = 17, antiderivative size = 21

$$\int x\left(\frac{c}{a+bx^2}\right)^{3/2} dx = -\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

output
```
-c*(c/(b*x^2+a))^(1/2)/b
```

3.244.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x\left(\frac{c}{a+bx^2}\right)^{3/2} dx = -\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

input
```
Integrate[x*(c/(a + b*x^2))^(3/2),x]
```

output
```
-((c*Sqrt[c/(a + b*x^2)])/b)
```

3.244.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2024, 19}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(\frac{c}{a+bx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{2024}$$

$$\frac{\int \left(\frac{c}{bx^2+a}\right)^{3/2}d(bx^2+a)}{2b}$$

$$\Big\downarrow \text{19}$$

$$-\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

input
```
Int[x*(c/(a + b*x^2))^(3/2),x]
```

output
```
-((c*Sqrt[c/(a + b*x^2)])/b)
```

3.244.3.1 Defintions of rubi rules used

rule 19
```
Int[((a_.)/(x_))^(p_), x_Symbol] :> Simp[(-a)*((a/x)^(p - 1)/(p - 1)), x] /
; FreeQ[{a, p}, x] &&  !IntegerQ[p]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

3.244.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

method	result	size
trager	$-\frac{c\sqrt{\frac{c}{b\,x^{2}+a}}}{b}$	20
gosper	$-\frac{\left(b\,x^{2}+a\right)\left(\frac{c}{b\,x^{2}+a}\right)^{\frac{3}{2}}}{b}$	26
derivativedivides	$-\frac{\left(b\,x^{2}+a\right)\left(\frac{c}{b\,x^{2}+a}\right)^{\frac{3}{2}}}{b}$	26
default	$-\frac{\left(b\,x^{2}+a\right)\left(\frac{c}{b\,x^{2}+a}\right)^{\frac{3}{2}}}{b}$	26

input
```
int(x*(c/(b*x^2+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-c*(c/(b*x^2+a))^(1/2)/b
```

3.244.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x\left(\frac{c}{a+bx^2}\right)^{3/2}\,dx=-\frac{c\sqrt{\frac{c}{bx^2+a}}}{b}$$

input
```
integrate(x*(c/(b*x^2+a))^(3/2),x, algorithm="fricas")
```

output
```
-c*sqrt(c/(b*x^2 + a))/b
```

3.244.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(15) = 30.

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

$$\int x\left(\frac{c}{a+bx^2}\right)^{3/2}\,dx=\begin{cases}-\frac{a\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}}{b}-x^2\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}} & \text{for } b\neq 0\\ \frac{x^2\left(\frac{c}{a}\right)^{\frac{3}{2}}}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*(c/(b*x**2+a))**(3/2),x)
```

output
```
Piecewise((-a*(c/(a + b*x**2))**(3/2)/b - x**2*(c/(a + b*x**2))**(3/2), Ne
(b, 0)), (x**2*(c/a)**(3/2)/2, True))
```

3.244.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x\left(\frac{c}{a+bx^2}\right)^{3/2}\,dx=-\frac{c\sqrt{\frac{c}{bx^2+a}}}{b}$$

input
```
integrate(x*(c/(b*x^2+a))^(3/2),x, algorithm="maxima")
```

output
```
-c*sqrt(c/(b*x^2 + a))/b
```

3.244.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int x\left(\frac{c}{a+bx^2}\right)^{3/2}\,dx=-\frac{c^2\operatorname{sgn}(bx^2+a)}{\sqrt{bcx^2+ac}b}$$

input
```
integrate(x*(c/(b*x^2+a))^(3/2),x, algorithm="giac")
```

output
```
-c^2*sgn(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*b)
```

3.244.9 Mupad [B] (verification not implemented)

Time = 19.47 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x\left(\frac{c}{a+bx^2}\right)^{3/2}\,dx=-\frac{c\,\sqrt{\frac{c}{b\,x^2+a}}}{b}$$

input
```
int(x*(c/(a + b*x^2))^(3/2),x)
```

output
```
-(c*(c/(a + b*x^2))^(1/2))/b
```

3.245 $\int \left(\frac{c}{a+bx^2}\right)^{3/2} dx$

3.245.1 Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \left(\frac{c}{a+bx^2}\right)^{3/2} dx = \frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}$$

output
```
c*x*(c/(b*x^2+a))^(1/2)/a
```

3.245.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(\frac{c}{a+bx^2}\right)^{3/2} dx = \frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}$$

input
```
Integrate[(c/(a + b*x^2))^(3/2),x]
```

output
```
(c*x*Sqrt[c/(a + b*x^2)])/a
```

3.245.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2045, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{c}{a+bx^2}\right)^{3/2} dx$$

$$\Big\downarrow \ 2045$$

$$\frac{c\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{c}{a+bx^2}}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/2}}dx}{a}$$

$$\Big\downarrow \ 208$$

$$\frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}$$

input
```
Int[(c/(a + b*x^2))^(3/2),x]
```

output
```
(c*x*Sqrt[c/(a + b*x^2)])/a
```

3.245.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.245.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

method	result	size
trager	$\frac{cx\sqrt{\frac{c}{b\,x^{2}+a}}}{a}$	20
gosper	$\frac{\left(b\,x^{2}+a\right)x\left(\frac{c}{b\,x^{2}+a}\right)^{\frac{3}{2}}}{a}$	26
default	$\frac{\left(b\,x^{2}+a\right)x\left(\frac{c}{b\,x^{2}+a}\right)^{\frac{3}{2}}}{a}$	26

input
```
int((c/(b*x^2+a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
c*x*(c/(b*x^2+a))^(1/2)/a
```

3.245.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \left(\frac{c}{a+bx^2}\right)^{3/2} dx = \frac{cx\sqrt{\frac{c}{bx^2+a}}}{a}$$

input
```
integrate((c/(b*x^2+a))^(3/2),x, algorithm="fricas")
```

output
```
c*x*sqrt(c/(b*x^2 + a))/a
```

3.245.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(15) = 30.

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.19

$$\int \left(\frac{c}{a+bx^2}\right)^{3/2} dx = \begin{cases} x\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}} + \frac{bx^3\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}}{a} & \text{for}\ a \neq 0 \\ -\frac{x\left(\frac{c}{bx^2}\right)^{\frac{3}{2}}}{2} & \text{otherwise} \end{cases}$$

input
```
integrate((c/(b*x**2+a))**(3/2),x)
```

output
```
Piecewise((x*(c/(a + b*x**2))**(3/2) + b*x**3*(c/(a + b*x**2))**(3/2)/a, N
e(a, 0)), (-x*(c/(b*x**2))**(3/2)/2, True))
```

3.245.7 Maxima [F]

$$\int \left(\frac{c}{a+bx^2}\right)^{3/2} dx = \int \left(\frac{c}{bx^2+a}\right)^{\frac{3}{2}} dx$$

input
```
integrate((c/(b*x^2+a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c/(b*x^2 + a))^(3/2), x)
```

3.245.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int \left(\frac{c}{a+bx^2}\right)^{3/2} dx = \frac{c^2 x \mathrm{sgn}(bx^2+a)}{\sqrt{bcx^2+ac}a}$$

input
```
integrate((c/(b*x^2+a))^(3/2),x, algorithm="giac")
```

output
```
c^2*x*sgn(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*a)
```

3.245.9 Mupad [B] (verification not implemented)

Time = 18.46 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \left(\frac{c}{a+bx^2}\right)^{3/2} dx = \frac{c\,x\,\sqrt{\frac{c}{b\,x^2+a}}}{a}$$

input
```
int((c/(a + b*x^2))^(3/2),x)
```

output
```
(c*x*(c/(a + b*x^2))^(1/2))/a
```

3.246 $\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x} dx$

3.246.1 Optimal result

Integrand size = 19, antiderivative size = 71

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x} dx = \frac{c\sqrt{\frac{c}{a+bx^2}}}{a} - \frac{c\sqrt{\frac{c}{a+bx^2}}\sqrt{1+\frac{bx^2}{a}}\text{arctanh}\left(\sqrt{1+\frac{bx^2}{a}}\right)}{a}$$

output
```
c*(c/(b*x^2+a))^(1/2)/a-c*arctanh((1+b*x^2/a)^(1/2))*(c/(b*x^2+a))^(1/2)*(
1+b*x^2/a)^(1/2)/a
```

3.246.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x} dx = \frac{c\sqrt{\frac{c}{a+bx^2}}\left(\sqrt{a} - \sqrt{a+bx^2}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{a^{3/2}}$$

input
```
Integrate[(c/(a + b*x^2))^(3/2)/x,x]
```

output
```
(c*Sqrt[c/(a + b*x^2)]*(Sqrt[a] - Sqrt[a + b*x^2]*ArcTanh[Sqrt[a + b*x^2]/
Sqrt[a]]))/a^(3/2)
```

3.246.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2045, 243, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{ 2045}$$

$$\frac{c\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{c}{a+bx^2}}\int \frac{1}{x\left(\frac{bx^2}{a}+1\right)^{3/2}}dx}{a}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{c\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{c}{a+bx^2}}\int \frac{1}{x^2\left(\frac{bx^2}{a}+1\right)^{3/2}}dx^2}{2a}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{c\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{c}{a+bx^2}}\left(\int \frac{1}{x^2\sqrt{\frac{bx^2}{a}+1}}dx^2+\frac{2}{\sqrt{\frac{bx^2}{a}+1}}\right)}{2a}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{c\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{c}{a+bx^2}}\left(\frac{2a\int \frac{1}{\frac{ax^4}{b}-\frac{a}{b}}d\sqrt{\frac{bx^2}{a}+1}}{b}+\frac{2}{\sqrt{\frac{bx^2}{a}+1}}\right)}{2a}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{c\sqrt{\frac{bx^2}{a}+1}\left(\frac{2}{\sqrt{\frac{bx^2}{a}+1}}-2\text{arctanh}\left(\sqrt{\frac{bx^2}{a}+1}\right)\right)\sqrt{\frac{c}{a+bx^2}}}{2a}$$

input
```
Int[(c/(a + b*x^2))^(3/2)/x,x]
```

output
```
(c*Sqrt[c/(a + b*x^2)]*Sqrt[1 + (b*x^2)/a]*(2/Sqrt[1 + (b*x^2)/a] - 2*ArcT
anh[Sqrt[1 + (b*x^2)/a]]))/(2*a)
```

3.246. $\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x}dx$

3.246.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.246.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{\left(\frac{c}{b\,x^{2}+a}\right)^{\frac{3}{2}} \left(b\,x^{2}+a\right) \left(\ln\left(\frac{2 a+2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right) a \sqrt{b\,x^{2}+a}-a^{\frac{3}{2}}\right)}{a^{\frac{5}{2}}}$	64

input
```
int((c/(b*x^2+a))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-(c/(b*x^2+a))^(3/2)*(b*x^2+a)*(ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*a*(b*x
^2+a)^(1/2)-a^(3/2))/a^(5/2)
```

3.246.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.94

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x}\,dx = \left[\frac{c\sqrt{\frac{c}{a}}\log\left(-\frac{bcx^2+2\,ac-2\,(abx^2+a^2)\sqrt{\frac{c}{bx^2+a}}\sqrt{\frac{c}{a}}}{x^2}\right)+2\,c\sqrt{\frac{c}{bx^2+a}}}{2\,a}, \frac{c\sqrt{-\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{bx^2+a}}\sqrt{-\frac{c}{a}}}{c}\right)}{a}\right.$$

input
```
integrate((c/(b*x^2+a))^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/2*(c*sqrt(c/a)*log(-(b*c*x^2 + 2*a*c - 2*(a*b*x^2 + a^2)*sqrt(c/(b*x^2
+ a))*sqrt(c/a))/x^2) + 2*c*sqrt(c/(b*x^2 + a)))/a, (c*sqrt(-c/a)*arctan(a
*sqrt(c/(b*x^2 + a))*sqrt(-c/a)/c) + c*sqrt(c/(b*x^2 + a)))/a]
```

3.246.6 Sympy [F]

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x}\,dx = \int \frac{\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((c/(b*x**2+a))**(3/2)/x,x)
```

output
```
Integral((c/(a + b*x**2))**(3/2)/x, x)
```

3.246.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x}\,dx = \frac{1}{2}\,c\left(\frac{c\log\left(\frac{a\sqrt{\frac{c}{bx^2+a}}-\sqrt{ac}}{a\sqrt{\frac{c}{bx^2+a}}+\sqrt{ac}}\right)}{\sqrt{ac}a}+\frac{2\sqrt{\frac{c}{bx^2+a}}}{a}\right)$$

input
```
integrate((c/(b*x^2+a))^(3/2)/x,x, algorithm="maxima")
```

output
```
1/2*c*(c*log((a*sqrt(c/(b*x^2 + a)) - sqrt(a*c))/(a*sqrt(c/(b*x^2 + a)) +
sqrt(a*c)))/(sqrt(a*c)*a) + 2*sqrt(c/(b*x^2 + a))/a)
```

3.246.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x}\,dx = c\left(\frac{c\arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}a}+\frac{c}{\sqrt{bcx^2+ac}a}\right)\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate((c/(b*x^2+a))^(3/2)/x,x, algorithm="giac")
```

output
```
c*(c*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a) + c/(sqrt(b*c*x
^2 + a*c)*a))*sgn(b*x^2 + a)
```

3.246.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x}\,dx = \int \frac{\left(\frac{c}{b\,x^2+a}\right)^{3/2}}{x}\,dx$$

input
```
int((c/(a + b*x^2))^(3/2)/x,x)
```

output
```
int((c/(a + b*x^2))^(3/2)/x, x)
```

3.247 $\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2} \, dx$

3.247.1 Optimal result

Integrand size = 19, antiderivative size = 48

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2} \, dx = -\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax} - \frac{2bcx\sqrt{\frac{c}{a+bx^2}}}{a^2}$$

output
```
-c*(c/(b*x^2+a))^(1/2)/a/x-2*b*c*x*(c/(b*x^2+a))^(1/2)/a^2
```

3.247.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.67

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2} \, dx = -\frac{c\sqrt{\frac{c}{a+bx^2}}(a+2bx^2)}{a^2x}$$

input
```
Integrate[(c/(a + b*x^2))^(3/2)/x^2,x]
```

output
```
-((c*Sqrt[c/(a + b*x^2)]*(a + 2*b*x^2))/(a^2*x))
```

3.247.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.56, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2045, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2} \, dx$$

$$\Big\downarrow \ 2045$$

$$\frac{c\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{c}{a+bx^2}}\int \frac{1}{x^2\left(\frac{bx^2}{a}+1\right)^{3/2}}dx}{a}$$

$$\Big\downarrow \ 245$$

$$\frac{c\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{c}{a+bx^2}}\left(-\frac{2b\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/2}}dx}{a}-\frac{1}{x\sqrt{\frac{bx^2}{a}+1}}\right)}{a}$$

$$\Big\downarrow \ 208$$

$$\frac{c\sqrt{\frac{bx^2}{a}+1}\left(-\frac{2bx}{a\sqrt{\frac{bx^2}{a}+1}}-\frac{1}{x\sqrt{\frac{bx^2}{a}+1}}\right)\sqrt{\frac{c}{a+bx^2}}}{a}$$

input
```
Int[(c/(a + b*x^2))^(3/2)/x^2,x]
```

output
```
(c*Sqrt[c/(a + b*x^2)]*Sqrt[1 + (b*x^2)/a]*(-(1/(x*Sqrt[1 + (b*x^2)/a])) -
 (2*b*x)/(a*Sqrt[1 + (b*x^2)/a])))/a
```

3.247.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.247.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)\left(2b\,x^{2}+a\right)\left(\frac{c}{b\,x^{2}+a}\right)^{\frac{3}{2}}}{a^{2}x}$	37
default	$-\frac{\left(b\,x^{2}+a\right)\left(2b\,x^{2}+a\right)\left(\frac{c}{b\,x^{2}+a}\right)^{\frac{3}{2}}}{a^{2}x}$	37
trager	$-\frac{\left(ac+bc\right)\left(2b\,x^{2}+a\right)\sqrt{\frac{c}{b\,x^{2}+a}}}{a^{2}\left(a+b\right)x}$	42
risch	$-\frac{\left(b\,x^{2}+a\right)c\sqrt{\frac{c}{b\,x^{2}+a}}}{a^{2}x}-\frac{bcx\sqrt{\frac{c}{b\,x^{2}+a}}}{a^{2}}$	52

input
```
int((c/(b*x^2+a))^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(b*x^2+a)*(2*b*x^2+a)*(c/(b*x^2+a))^(3/2)/a^2/x
```

3.247.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.67

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2}\,dx = -\frac{(2\,bcx^2+ac)\sqrt{\frac{c}{bx^2+a}}}{a^2x}$$

input
```
integrate((c/(b*x^2+a))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
-(2*b*c*x^2 + a*c)*sqrt(c/(b*x^2 + a))/(a^2*x)
```

3.247.6 Sympy [F]

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2}\,dx = \int \frac{\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c/(b*x**2+a))**(3/2)/x**2,x)
```

output
```
Integral((c/(a + b*x**2))**(3/2)/x**2, x)
```

3.247.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2}\,dx = -\frac{2\,b^2c^{\frac{3}{2}}x^4+3\,abc^{\frac{3}{2}}x^2+a^2c^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}a^2x}$$

input
```
integrate((c/(b*x^2+a))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
-(2*b^2*c^(3/2)*x^4 + 3*a*b*c^(3/2)*x^2 + a^2*c^(3/2))/((b*x^2 + a)^(3/2)*
a^2*x)
```

3.247.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.69

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2}\,dx = -\left(\frac{bcx\mathrm{sgn}(bx^2+a)}{\sqrt{bcx^2+ac}a^2} - \frac{2\sqrt{bc}c\mathrm{sgn}(bx^2+a)}{\left(\left(\sqrt{bc}x-\sqrt{bcx^2+ac}\right)^2-ac\right)a}\right)c$$

input
```
integrate((c/(b*x^2+a))^(3/2)/x^2,x, algorithm="giac")
```

output
```
-(b*c*x*sgn(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*a^2) - 2*sqrt(b*c)*c*sgn(b*x^2
 + a)/(((sqrt(b*c)*x - sqrt(b*c*x^2 + a*c))^2 - a*c)*a))*c
```

3.247.9 Mupad [B] (verification not implemented)

Time = 19.49 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2}\,dx = -\frac{\left(\frac{b\,c}{a}+\frac{2\,b^2\,c\,x^2}{a^2}\right)\,\sqrt{\frac{c}{b\,x^2+a}}\,\left(\frac{a}{b}+x^2\right)}{b\,x^3+a\,x}$$

input
```
int((c/(a + b*x^2))^(3/2)/x^2,x)
```

output
```
-(((b*c)/a + (2*b^2*c*x^2)/a^2)*(c/(a + b*x^2))^(1/2)*(a/b + x^2))/(a*x +
b*x^3)
```

3.248 $\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3}\,dx$

3.248.1 Optimal result

Integrand size = 19, antiderivative size = 104

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3}\,dx=-\frac{3bc\sqrt{\frac{c}{a+bx^2}}}{2a^2}-\frac{c\sqrt{\frac{c}{a+bx^2}}}{2ax^2}+\frac{3bc\sqrt{\frac{c}{a+bx^2}}\sqrt{1+\frac{bx^2}{a}}\,\text{arctanh}\left(\sqrt{1+\frac{bx^2}{a}}\right)}{2a^2}$$

output
```
-3/2*b*c*(c/(b*x^2+a))^(1/2)/a^2-1/2*c*(c/(b*x^2+a))^(1/2)/a/x^2+3/2*b*c*a
rctanh((1+b*x^2/a)^(1/2))*(c/(b*x^2+a))^(1/2)*(1+b*x^2/a)^(1/2)/a^2
```

3.248.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.75

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3}\,dx=-\frac{c\sqrt{\frac{c}{a+bx^2}}\left(\sqrt{a}(a+3bx^2)-3bx^2\sqrt{a+bx^2}\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{2a^{5/2}x^2}$$

input
```
Integrate[(c/(a + b*x^2))^(3/2)/x^3,x]
```

output
```
-1/2*(c*Sqrt[c/(a + b*x^2)]*(Sqrt[a]*(a + 3*b*x^2) - 3*b*x^2*Sqrt[a + b*x^
2]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]))/(a^(5/2)*x^2)
```

3.248.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2045, 243, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3}\,dx$$

$$\Big\downarrow \text{ 2045}$$

$$\frac{c\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{c}{a+bx^2}}\int \frac{1}{x^3\left(\frac{bx^2}{a}+1\right)^{3/2}}dx}{a}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{c\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{c}{a+bx^2}}\int \frac{1}{x^4\left(\frac{bx^2}{a}+1\right)^{3/2}}dx^2}{2a}$$

$$\Big\downarrow \text{ 52}$$

$$\frac{c\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{c}{a+bx^2}}\left(-\frac{3b\int \frac{1}{x^2\left(\frac{bx^2}{a}+1\right)^{3/2}}dx^2}{2a}-\frac{1}{x^2\sqrt{\frac{bx^2}{a}+1}}\right)}{2a}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{c\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{c}{a+bx^2}}\left(-\frac{3b\left(\int \frac{1}{x^2\sqrt{\frac{bx^2}{a}+1}}dx^2+\frac{2}{\sqrt{\frac{bx^2}{a}+1}}\right)}{2a}-\frac{1}{x^2\sqrt{\frac{bx^2}{a}+1}}\right)}{2a}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{c\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{c}{a+bx^2}}\left(-\frac{3b\left(\frac{2a\int \frac{1}{\frac{ax^4}{b}-\frac{a}{b}}d\sqrt{\frac{bx^2}{a}+1}}{b}+\frac{2}{\sqrt{\frac{bx^2}{a}+1}}\right)}{2a}-\frac{1}{x^2\sqrt{\frac{bx^2}{a}+1}}\right)}{2a}$$

$$\downarrow 221$$

$$\frac{c\sqrt{\frac{bx^2}{a}+1}\left(-\frac{3b\left(\frac{2}{\sqrt{\frac{bx^2}{a}+1}}-2\text{arctanh}\left(\sqrt{\frac{bx^2}{a}+1}\right)\right)}{2a}-\frac{1}{x^2\sqrt{\frac{bx^2}{a}+1}}\right)\sqrt{\frac{c}{a+bx^2}}}{2a}$$

input
```
Int[(c/(a + b*x^2))^(3/2)/x^3,x]
```

output
```
(c*Sqrt[c/(a + b*x^2)]*Sqrt[1 + (b*x^2)/a]*(-(1/(x^2*Sqrt[1 + (b*x^2)/a]))
 - (3*b*(2/Sqrt[1 + (b*x^2)/a] - 2*ArcTanh[Sqrt[1 + (b*x^2)/a]]))/(2*a)))/
(2*a)
```

3.248.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.248. $\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3}\,dx$

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.248.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.78

method	result
default	$\frac{\left(\frac{c}{b\,x^{2}+a}\right)^{\frac{3}{2}} \left(b\,x^{2}+a\right) \left(3 \sqrt{b\,x^{2}+a}\,\ln\left(\frac{2 a+2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right) a b\,x^{2}-3 a^{\frac{3}{2}} b\,x^{2}-a^{\frac{5}{2}}\right)}{2 a^{\frac{7}{2}} x^{2}}$
risch	$-\frac{\left(b\,x^{2}+a\right) c \sqrt{\frac{c}{b\,x^{2}+a}}}{2 a^{2} x^{2}}-\frac{b \left(-\frac{3 \ln\left(\frac{2 a c+2 \sqrt{a c}\,\sqrt{b c\,x^{2}+a c}}{x}\right)}{\sqrt{a c}}-\frac{\sqrt{b c \left(x+\frac{\sqrt{-a b}}{b}\right)^{2}-2 c \sqrt{-a b}\,\left(x+\frac{\sqrt{-a b}}{b}\right)}}{c \sqrt{-a b}\,\left(x+\frac{\sqrt{-a b}}{b}\right)}+\frac{\sqrt{b c \left(x-\frac{\sqrt{-a b}}{b}\right)^{2}+2 c \sqrt{-a b}\,\left(x-\cdots\right.}}{c \sqrt{-a b}\,\left(x-\frac{\sqrt{-a b}}{b}\right)}\cdots\right.}{2 a^{2}}$

input
```
int((c/(b*x^2+a))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(c/(b*x^2+a))^(3/2)*(b*x^2+a)*(3*(b*x^2+a)^(1/2)*ln(2*(a^(1/2)*(b*x^2+
a)^(1/2)+a)/x)*a*b*x^2-3*a^(3/2)*b*x^2-a^(5/2))/a^(7/2)/x^2
```

3.248.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.68

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3}\,dx = \left[\frac{3\,bcx^2\sqrt{\frac{c}{a}}\log\left(-\frac{bcx^2+2\,ac+2\,(abx^2+a^2)\sqrt{\frac{c}{bx^2+a}}\sqrt{\frac{c}{a}}}{x^2}\right)-2\,(3\,bcx^2+ac)\sqrt{\frac{c}{bx^2+a}}}{4\,a^2x^2},\right.$$
$$\left.-\frac{3\,bcx^2\sqrt{-\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{bx^2+a}}\sqrt{-\frac{c}{a}}}{c}\right)+(3\,bcx^2+ac)\sqrt{\frac{c}{bx^2+a}}}{2\,a^2x^2}\right]$$

input
```
integrate((c/(b*x^2+a))^(3/2)/x^3,x, algorithm="fricas")
```

output
```
[1/4*(3*b*c*x^2*sqrt(c/a)*log(-(b*c*x^2 + 2*a*c + 2*(a*b*x^2 + a^2)*sqrt(c
/(b*x^2 + a))*sqrt(c/a))/x^2) - 2*(3*b*c*x^2 + a*c)*sqrt(c/(b*x^2 + a)))/(
a^2*x^2), -1/2*(3*b*c*x^2*sqrt(-c/a)*arctan(a*sqrt(c/(b*x^2 + a))*sqrt(-c/
a)/c) + (3*b*c*x^2 + a*c)*sqrt(c/(b*x^2 + a)))/(a^2*x^2)]
```

3.248.6 Sympy [F]

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3}\,dx = \int \frac{\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((c/(b*x**2+a))**(3/2)/x**3,x)
```

output
```
Integral((c/(a + b*x**2))**(3/2)/x**3, x)
```

3.248.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.16

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3}\,dx = -\frac{1}{4}\,bc\left(\frac{2\,c\sqrt{\frac{c}{bx^2+a}}}{a^2c-\frac{a^3c}{bx^2+a}}+\frac{3\,c\log\left(\frac{a\sqrt{\frac{c}{bx^2+a}}-\sqrt{ac}}{a\sqrt{\frac{c}{bx^2+a}}+\sqrt{ac}}\right)}{\sqrt{ac}a^2}+\frac{4\sqrt{\frac{c}{bx^2+a}}}{a^2}\right)$$

input
```
integrate((c/(b*x^2+a))^(3/2)/x^3,x, algorithm="maxima")
```

output
```
-1/4*b*c*(2*c*sqrt(c/(b*x^2 + a))/(a^2*c - a^3*c/(b*x^2 + a)) + 3*c*log((a
*sqrt(c/(b*x^2 + a)) - sqrt(a*c))/(a*sqrt(c/(b*x^2 + a)) + sqrt(a*c)))/(sq
rt(a*c)*a^2) + 4*sqrt(c/(b*x^2 + a))/a^2)
```

3.248.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.99

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3}\,dx = -\frac{1}{2}\,c\left(\frac{3\,bc\arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}a^2}+\frac{2\,abc^2-3\,(bcx^2+ac)bc}{\left(\sqrt{bcx^2+ac}ac-(bcx^2+ac)^{\frac{3}{2}}\right)a^2}\right)\mathrm{sgn}\left(bx^2+a\right)$$

input
```
integrate((c/(b*x^2+a))^(3/2)/x^3,x, algorithm="giac")
```

output
```
-1/2*c*(3*b*c*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a^2) + (2
*a*b*c^2 - 3*(b*c*x^2 + a*c)*b*c)/((sqrt(b*c*x^2 + a*c)*a*c - (b*c*x^2 + a
*c)^(3/2))*a^2))*sgn(b*x^2 + a)
```

3.248.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3}\,dx = \int \frac{\left(\frac{c}{b\,x^2+a}\right)^{3/2}}{x^3}\,dx$$

input
```
int((c/(a + b*x^2))^(3/2)/x^3,x)
```

output
```
int((c/(a + b*x^2))^(3/2)/x^3, x)
```

3.249 $\int x^7 \left(c\sqrt{a+bx^2}\right)^{3/2} dx$

3.249.1 Optimal result

Integrand size = 21, antiderivative size = 138

$$\int x^7 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = -\frac{2a^3\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)}{7b^4} + \frac{6a^2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)^2}{11b^4} - \frac{2a\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)^3}{5b^4} + \frac{2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)^4}{19b^4}$$

output
```
-2/7*a^3*(b*x^2+a)*(c*(b*x^2+a)^(1/2))^(3/2)/b^4+6/11*a^2*(b*x^2+a)^2*(c*(
b*x^2+a)^(1/2))^(3/2)/b^4-2/5*a*(b*x^2+a)^3*(c*(b*x^2+a)^(1/2))^(3/2)/b^4+
2/19*(b*x^2+a)^4*(c*(b*x^2+a)^(1/2))^(3/2)/b^4
```

3.249.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.46

$$\int x^7 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \frac{2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)\left(-128a^3+224a^2bx^2-308ab^2x^4+385b^3x^6\right)}{7315b^4}$$

input
```
Integrate[x^7*(c*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
(2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2)*(-128*a^3 + 224*a^2*b*x^2 - 308*a
*b^2*x^4 + 385*b^3*x^6))/(7315*b^4)
```

3.249.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2045, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7\left(c\sqrt{a+bx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{2045}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\int x^7\left(\frac{bx^2}{a}+1\right)^{3/4}dx}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\int x^6\left(\frac{bx^2}{a}+1\right)^{3/4}dx^2}{2\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \text{53}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\int\left(\frac{a^3\left(\frac{bx^2}{a}+1\right)^{15/4}}{b^3}-\frac{3a^3\left(\frac{bx^2}{a}+1\right)^{11/4}}{b^3}+\frac{3a^3\left(\frac{bx^2}{a}+1\right)^{7/4}}{b^3}-\frac{a^3\left(\frac{bx^2}{a}+1\right)^{3/4}}{b^3}\right)dx^2}{2\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{4a^4\left(\frac{bx^2}{a}+1\right)^{19/4}}{19b^4}-\frac{4a^4\left(\frac{bx^2}{a}+1\right)^{15/4}}{5b^4}+\frac{12a^4\left(\frac{bx^2}{a}+1\right)^{11/4}}{11b^4}-\frac{4a^4\left(\frac{bx^2}{a}+1\right)^{7/4}}{7b^4}\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{2\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

input
```
Int[x^7*(c*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
((c*Sqrt[a + b*x^2])^(3/2)*((-4*a^4*(1 + (b*x^2)/a)^(7/4))/(7*b^4) + (12*a
^4*(1 + (b*x^2)/a)^(11/4))/(11*b^4) - (4*a^4*(1 + (b*x^2)/a)^(15/4))/(5*b^
4) + (4*a^4*(1 + (b*x^2)/a)^(19/4))/(19*b^4)))/(2*(1 + (b*x^2)/a)^(3/4))
```

3.249.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.249.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.42

method	result	size
gosper	$-\frac{2\left(b\,x^{2}+a\right)\left(-385 b^{3} x^{6}+308 b^{2} x^{4} a-224 a^{2} b\,x^{2}+128 a^{3}\right)\left(c\sqrt{b\,x^{2}+a}\right)^{\frac{3}{2}}}{7315 b^{4}}$	58

input
```
int(x^7*(c*(b*x^2+a)^(1/2))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7315*(b*x^2+a)*(-385*b^3*x^6+308*a*b^2*x^4-224*a^2*b*x^2+128*a^3)*(c*(b
*x^2+a)^(1/2))^(3/2)/b^4
```

3.249.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.54

$$\int x^7\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2\,(385\,b^4cx^8+77\,ab^3cx^6-84\,a^2b^2cx^4+96\,a^3bcx^2-128\,a^4c)\sqrt{bx^2+a}\sqrt{\sqrt{bx^2+}}}{7315\,b^4}$$

input
```
integrate(x^7*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
2/7315*(385*b^4*c*x^8 + 77*a*b^3*c*x^6 - 84*a^2*b^2*c*x^4 + 96*a^3*b*c*x^2
 - 128*a^4*c)*sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)/b^4
```

3.249.6 Sympy [A] (verification not implemented)

Time = 11.19 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.04

$$\int x^7\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\begin{cases}-\frac{256a^4\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{7315b^4}+\frac{192a^3x^2\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{7315b^3}-\frac{24a^2x^4\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{1045b^2}+\frac{2ax^6\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{95b}+\frac{2x^{}}{} \\ \frac{x^8\left(\sqrt{a}c\right)^{\frac{3}{2}}}{8}\end{cases}$$

input
```
integrate(x**7*(c*(b*x**2+a)**(1/2))**(3/2),x)
```

output
```
Piecewise((-256*a**4*(c*sqrt(a + b*x**2))**(3/2)/(7315*b**4) + 192*a**3*x*
*2*(c*sqrt(a + b*x**2))**(3/2)/(7315*b**3) - 24*a**2*x**4*(c*sqrt(a + b*x*
*2))**(3/2)/(1045*b**2) + 2*a*x**6*(c*sqrt(a + b*x**2))**(3/2)/(95*b) + 2*
x**8*(c*sqrt(a + b*x**2))**(3/2)/19, Ne(b, 0)), (x**8*(sqrt(a)*c)**(3/2)/8
, True))
```

3.249.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.62

$$\int x^7\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=$$
$$-\frac{2\left(1045\left(\sqrt{bx^2+a}c\right)^{\frac{7}{2}}a^3c^6-1995\left(\sqrt{bx^2+a}c\right)^{\frac{11}{2}}a^2c^4+1463\left(\sqrt{bx^2+a}c\right)^{\frac{15}{2}}ac^2-385\left(\sqrt{bx^2+a}c\right)^{\frac{19}{2}}\right)}{7315\,b^4c^8}$$

input
```
integrate(x^7*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
-2/7315*(1045*(sqrt(b*x^2 + a)*c)^(7/2)*a^3*c^6 - 1995*(sqrt(b*x^2 + a)*c)
^(11/2)*a^2*c^4 + 1463*(sqrt(b*x^2 + a)*c)^(15/2)*a*c^2 - 385*(sqrt(b*x^2
+ a)*c)^(19/2))/(b^4*c^8)
```

3.249.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.99

$$\int x^7\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx = \frac{2\,c^{\frac{3}{2}}\left(\frac{19\left(77\,(bx^2+a)^{\frac{15}{4}}-315\,(bx^2+a)^{\frac{11}{4}}a+495\,(bx^2+a)^{\frac{7}{4}}a^2-385\,(bx^2+a)^{\frac{3}{4}}a^3\right)a}{b^3}+\frac{1155\,(bx^2+a)^{\frac{19}{4}}-5}{}\right.}{21945\,b}$$

input
```
integrate(x^7*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
2/21945*c^(3/2)*(19*(77*(b*x^2 + a)^(15/4) - 315*(b*x^2 + a)^(11/4)*a + 49
5*(b*x^2 + a)^(7/4)*a^2 - 385*(b*x^2 + a)^(3/4)*a^3)*a/b^3 + (1155*(b*x^2
+ a)^(19/4) - 5852*(b*x^2 + a)^(15/4)*a + 11970*(b*x^2 + a)^(11/4)*a^2 - 1
2540*(b*x^2 + a)^(7/4)*a^3 + 7315*(b*x^2 + a)^(3/4)*a^4)/b^3)/b
```

3.249.9 Mupad [B] (verification not implemented)

Time = 18.88 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.79

$$\int x^7\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx = \sqrt{c\,\sqrt{b\,x^2+a}}\left(\frac{2\,c\,x^8\,\sqrt{b\,x^2+a}}{19}-\frac{256\,a^4\,c\,\sqrt{b\,x^2+a}}{7315\,b^4}+\frac{2\,a\,c\,x^6\,\sqrt{b\,x^2+a}}{95\,b}-\frac{24\,a^2\,c\,x^4\,\sqrt{b\,x^2+a}}{1045\,b^2}+\frac{192\,a^3\,c\,x^2\,\sqrt{b\,x^2+a}}{7315\,b^3}\right)$$

input
```
int(x^7*(c*(a + b*x^2)^(1/2))^(3/2),x)
```

output
```
(c*(a + b*x^2)^(1/2))^(1/2)*((2*c*x^8*(a + b*x^2)^(1/2))/19 - (256*a^4*c*(
a + b*x^2)^(1/2))/(7315*b^4) + (2*a*c*x^6*(a + b*x^2)^(1/2))/(95*b) - (24*
a^2*c*x^4*(a + b*x^2)^(1/2))/(1045*b^2) + (192*a^3*c*x^2*(a + b*x^2)^(1/2)
)/(7315*b^3))
```

3.250 $\int x^5 \left(c\sqrt{a+bx^2}\right)^{3/2} dx$

3.250.1 Optimal result

Integrand size = 21, antiderivative size = 102

$$\int x^5 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \frac{2a^2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)}{7b^3} - \frac{4a\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)^2}{11b^3} + \frac{2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)^3}{15b^3}$$

output
```
2/7*a^2*(b*x^2+a)*(c*(b*x^2+a)^(1/2))^(3/2)/b^3-4/11*a*(b*x^2+a)^2*(c*(b*x
^2+a)^(1/2))^(3/2)/b^3+2/15*(b*x^2+a)^3*(c*(b*x^2+a)^(1/2))^(3/2)/b^3
```

3.250.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.51

$$\int x^5 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \frac{2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)\left(32a^2-56abx^2+77b^2x^4\right)}{1155b^3}$$

input
```
Integrate[x^5*(c*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
(2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2)*(32*a^2 - 56*a*b*x^2 + 77*b^2*x^4
))/(1155*b^3)
```

3.250.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2045, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(c\sqrt{a+bx^2}\right)^{3/2} dx$$

$$\Big\downarrow \text{2045}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2} \int x^5 \left(\frac{bx^2}{a}+1\right)^{3/4} dx}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2} \int x^4 \left(\frac{bx^2}{a}+1\right)^{3/4} dx^2}{2\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \text{53}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2} \int \left(\frac{a^2\left(\frac{bx^2}{a}+1\right)^{11/4}}{b^2}-\frac{2a^2\left(\frac{bx^2}{a}+1\right)^{7/4}}{b^2}+\frac{a^2\left(\frac{bx^2}{a}+1\right)^{3/4}}{b^2}\right) dx^2}{2\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{4a^3\left(\frac{bx^2}{a}+1\right)^{15/4}}{15b^3}-\frac{8a^3\left(\frac{bx^2}{a}+1\right)^{11/4}}{11b^3}+\frac{4a^3\left(\frac{bx^2}{a}+1\right)^{7/4}}{7b^3}\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{2\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

input
```
Int[x^5*(c*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
((c*Sqrt[a + b*x^2])^(3/2)*((4*a^3*(1 + (b*x^2)/a)^(7/4))/(7*b^3) - (8*a^3
*(1 + (b*x^2)/a)^(11/4))/(11*b^3) + (4*a^3*(1 + (b*x^2)/a)^(15/4))/(15*b^3
)))/(2*(1 + (b*x^2)/a)^(3/4))
```

3.250.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.250.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.46

method	result	size
gosper	$\frac{2(b\,x^{2}+a)\left(77b^{2}x^{4}-56ab\,x^{2}+32a^{2}\right)\left(c\sqrt{b\,x^{2}+a}\right)^{\frac{3}{2}}}{1155b^{3}}$	47

input
```
int(x^5*(c*(b*x^2+a)^(1/2))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/1155*(b*x^2+a)*(77*b^2*x^4-56*a*b*x^2+32*a^2)*(c*(b*x^2+a)^(1/2))^(3/2)/
b^3
```

3.250.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

$$\int x^5\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2\,(77\,b^3cx^6+21\,ab^2cx^4-24\,a^2bcx^2+32\,a^3c)\sqrt{bx^2+a}\sqrt{\sqrt{bx^2+a}c}}{1155\,b^3}$$

input
```
integrate(x^5*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
2/1155*(77*b^3*c*x^6 + 21*a*b^2*c*x^4 - 24*a^2*b*c*x^2 + 32*a^3*c)*sqrt(b*
x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)/b^3
```

3.250.6 Sympy [A] (verification not implemented)

Time = 5.86 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.14

$$\int x^5\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\begin{cases}\frac{64a^3\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{1155b^3}-\frac{16a^2x^2\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{385b^2}+\frac{2ax^4\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{55b}+\frac{2x^6\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{15} & \text{for } b\neq 0\\ \frac{x^6\left(\sqrt{a}c\right)^{\frac{3}{2}}}{6} & \text{otherwise}\end{cases}$$

input
```
integrate(x**5*(c*(b*x**2+a)**(1/2))**(3/2),x)
```

output
```
Piecewise((64*a**3*(c*sqrt(a + b*x**2))**(3/2)/(1155*b**3) - 16*a**2*x**2*
(c*sqrt(a + b*x**2))**(3/2)/(385*b**2) + 2*a*x**4*(c*sqrt(a + b*x**2))**(3
/2)/(55*b) + 2*x**6*(c*sqrt(a + b*x**2))**(3/2)/15, Ne(b, 0)), (x**6*(sqrt
(a)*c)**(3/2)/6, True))
```

3.250.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

$$\int x^5\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2\left(165\left(\sqrt{bx^2+a}c\right)^{\frac{7}{2}}a^2c^4-210\left(\sqrt{bx^2+a}c\right)^{\frac{11}{2}}ac^2+77\left(\sqrt{bx^2+a}c\right)^{\frac{15}{2}}\right)}{1155\,b^3c^6}$$

input
```
integrate(x^5*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
2/1155*(165*(sqrt(b*x^2 + a)*c)^(7/2)*a^2*c^4 - 210*(sqrt(b*x^2 + a)*c)^(1
1/2)*a*c^2 + 77*(sqrt(b*x^2 + a)*c)^(15/2))/(b^3*c^6)
```

3.250.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.07

$$\int x^5 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \frac{2\,c^{\frac{3}{2}}\left(\frac{5\left(21\,(bx^2+a)^{\frac{11}{4}}-66\,(bx^2+a)^{\frac{7}{4}}a+77\,(bx^2+a)^{\frac{3}{4}}a^2\right)a}{b^2}+\frac{77\,(bx^2+a)^{\frac{15}{4}}-315\,(bx^2+a)^{\frac{11}{4}}a+495\,(bx^2}{b^2}\right.}{1155\,b}$$

input
```
integrate(x^5*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
2/1155*c^(3/2)*(5*(21*(b*x^2 + a)^(11/4) - 66*(b*x^2 + a)^(7/4)*a + 77*(b*
x^2 + a)^(3/4)*a^2)*a/b^2 + (77*(b*x^2 + a)^(15/4) - 315*(b*x^2 + a)^(11/4
)*a + 495*(b*x^2 + a)^(7/4)*a^2 - 385*(b*x^2 + a)^(3/4)*a^3)/b^2)/b
```

3.250.9 Mupad [B] (verification not implemented)

Time = 18.94 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.86

$$\int x^5 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \sqrt{c\,\sqrt{b\,x^2+a}}\left(\frac{2\,c\,x^6\,\sqrt{b\,x^2+a}}{15}\right.$$
$$\left.+\frac{64\,a^3\,c\,\sqrt{b\,x^2+a}}{1155\,b^3}+\frac{2\,a\,c\,x^4\,\sqrt{b\,x^2+a}}{55\,b}-\frac{16\,a^2\,c\,x^2\,\sqrt{b\,x^2+a}}{385\,b^2}\right)$$

input
```
int(x^5*(c*(a + b*x^2)^(1/2))^(3/2),x)
```

output
```
(c*(a + b*x^2)^(1/2))^(1/2)*((2*c*x^6*(a + b*x^2)^(1/2))/15 + (64*a^3*c*(a
 + b*x^2)^(1/2))/(1155*b^3) + (2*a*c*x^4*(a + b*x^2)^(1/2))/(55*b) - (16*a
^2*c*x^2*(a + b*x^2)^(1/2))/(385*b^2))
```

3.251 $\int x^3 \left(c\sqrt{a+bx^2}\right)^{3/2} dx$

3.251.1 Optimal result

Integrand size = 21, antiderivative size = 66

$$\int x^3 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = -\frac{2a\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)}{7b^2} + \frac{2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)^2}{11b^2}$$

output
```
-2/7*a*(b*x^2+a)*(c*(b*x^2+a)^(1/2))^(3/2)/b^2+2/11*(b*x^2+a)^2*(c*(b*x^2+
a)^(1/2))^(3/2)/b^2
```

3.251.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.62

$$\int x^3 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \frac{2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)(-4a+7bx^2)}{77b^2}$$

input
```
Integrate[x^3*(c*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
(2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2)*(-4*a + 7*b*x^2))/(77*b^2)
```

3.251.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.27, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2045, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(c\sqrt{a+bx^2}\right)^{3/2} dx$$

$$\Big\downarrow \text{2045}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2} \int x^3 \left(\frac{bx^2}{a}+1\right)^{3/4} dx}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2} \int x^2 \left(\frac{bx^2}{a}+1\right)^{3/4} dx^2}{2\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \text{53}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2} \int \left(\frac{a\left(\frac{bx^2}{a}+1\right)^{7/4}}{b}-\frac{a\left(\frac{bx^2}{a}+1\right)^{3/4}}{b}\right) dx^2}{2\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{4a^2\left(\frac{bx^2}{a}+1\right)^{11/4}}{11b^2}-\frac{4a^2\left(\frac{bx^2}{a}+1\right)^{7/4}}{7b^2}\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{2\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

input
```
Int[x^3*(c*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
((c*Sqrt[a + b*x^2])^(3/2)*((-4*a^2*(1 + (b*x^2)/a)^(7/4))/(7*b^2) + (4*a^
2*(1 + (b*x^2)/a)^(11/4))/(11*b^2)))/(2*(1 + (b*x^2)/a)^(3/4))
```

3.251.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.251.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.55

method	result	size
gosper	$-\frac{2\left(b\,x^{2}+a\right)\left(-7b\,x^{2}+4a\right)\left(c\sqrt{b\,x^{2}+a}\right)^{\frac{3}{2}}}{77b^{2}}$	36

input
```
int(x^3*(c*(b*x^2+a)^(1/2))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/77*(b*x^2+a)*(-7*b*x^2+4*a)*(c*(b*x^2+a)^(1/2))^(3/2)/b^2
```

3.251.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.77

$$\int x^3\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2\,(7\,b^2cx^4+3\,abcx^2-4\,a^2c)\sqrt{bx^2+a}\sqrt{\sqrt{bx^2+a}c}}{77\,b^2}$$

input
```
integrate(x^3*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
2/77*(7*b^2*c*x^4 + 3*a*b*c*x^2 - 4*a^2*c)*sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2
 + a)*c)/b^2
```

3.251.6 Sympy [A] (verification not implemented)

Time = 3.01 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.32

$$\int x^3\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\begin{cases}-\frac{8a^2\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{77b^2}+\frac{6ax^2\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{77b}+\frac{2x^4\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{11} & \text{for } b\neq 0\\ \frac{x^4\left(\sqrt{a}c\right)^{\frac{3}{2}}}{4} & \text{otherwise}\end{cases}$$

input
```
integrate(x**3*(c*(b*x**2+a)**(1/2))**(3/2),x)
```

output
```
Piecewise((-8*a**2*(c*sqrt(a + b*x**2))**(3/2)/(77*b**2) + 6*a*x**2*(c*sqr
t(a + b*x**2))**(3/2)/(77*b) + 2*x**4*(c*sqrt(a + b*x**2))**(3/2)/11, Ne(b
, 0)), (x**4*(sqrt(a)*c)**(3/2)/4, True))
```

3.251.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.65

$$\int x^3\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=-\frac{2\left(11\left(\sqrt{bx^2+a}c\right)^{\frac{7}{2}}ac^2-7\left(\sqrt{bx^2+a}c\right)^{\frac{11}{2}}\right)}{77\,b^2c^4}$$

input
```
integrate(x^3*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
-2/77*(11*(sqrt(b*x^2 + a)*c)^(7/2)*a*c^2 - 7*(sqrt(b*x^2 + a)*c)^(11/2))/
(b^2*c^4)
```

3.251.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23

$$\int x^3 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \frac{2\left(\frac{11\left(3\,(bx^2+a)^{\frac{7}{4}}-7\,(bx^2+a)^{\frac{3}{4}}a\right)a}{b}+\frac{21\,(bx^2+a)^{\frac{11}{4}}-66\,(bx^2+a)^{\frac{7}{4}}a+77\,(bx^2+a)^{\frac{3}{4}}a^2}{b}\right)c^{\frac{3}{2}}}{231\,b}$$

input
```
integrate(x^3*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
2/231*(11*(3*(b*x^2 + a)^(7/4) - 7*(b*x^2 + a)^(3/4)*a)*a/b + (21*(b*x^2 +
 a)^(11/4) - 66*(b*x^2 + a)^(7/4)*a + 77*(b*x^2 + a)^(3/4)*a^2)/b)*c^(3/2)
/b
```

3.251.9 Mupad [B] (verification not implemented)

Time = 18.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int x^3 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \sqrt{c\,\sqrt{b\,x^2+a}}\left(\frac{2\,c\,x^4\,\sqrt{b\,x^2+a}}{11} - \frac{8\,a^2\,c\,\sqrt{b\,x^2+a}}{77\,b^2}+\frac{6\,a\,c\,x^2\,\sqrt{b\,x^2+a}}{77\,b}\right)$$

input
```
int(x^3*(c*(a + b*x^2)^(1/2))^(3/2),x)
```

output
```
(c*(a + b*x^2)^(1/2))^(1/2)*((2*c*x^4*(a + b*x^2)^(1/2))/11 - (8*a^2*c*(a
+ b*x^2)^(1/2))/(77*b^2) + (6*a*c*x^2*(a + b*x^2)^(1/2))/(77*b))
```

3.252 $\int x\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx$

3.252.1 Optimal result

Integrand size = 19, antiderivative size = 36

$$\int x\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2c\sqrt{c\sqrt{a+bx^2}}\,(a+bx^2)^{3/2}}{7b}$$

output
```
2/7*c*(b*x^2+a)^(3/2)*(c*(b*x^2+a)^(1/2))^(1/2)/b
```

3.252.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int x\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)}{7b}$$

input
```
Integrate[x*(c*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
(2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2))/(7*b)
```

3.252.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2024, 20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(c\sqrt{a+bx^2}\right)^{3/2}dx$$
$$\Big\downarrow \text{2024}$$
$$\frac{\int \left(c\sqrt{bx^2+a}\right)^{3/2}d(bx^2+a)}{2b}$$
$$\Big\downarrow \text{20}$$
$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\int (bx^2+a)^{3/4}d(bx^2+a)}{2b\,(a+bx^2)^{3/4}}$$
$$\Big\downarrow \text{15}$$
$$\frac{2(a+bx^2)\left(c\sqrt{a+bx^2}\right)^{3/2}}{7b}$$

input
```
Int[x*(c*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
(2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2))/(7*b)
```

3.252.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

3.252.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{2(b\,x^2+a)\left(c\sqrt{b\,x^2+a}\right)^{\frac{3}{2}}}{7b}$	26
derivativedivides	$\frac{2(b\,x^2+a)\left(c\sqrt{b\,x^2+a}\right)^{\frac{3}{2}}}{7b}$	26
default	$\frac{2(b\,x^2+a)\left(c\sqrt{b\,x^2+a}\right)^{\frac{3}{2}}}{7b}$	26

input
```
int(x*(c*(b*x^2+a)^(1/2))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*(b*x^2+a)*(c*(b*x^2+a)^(1/2))^(3/2)/b
```

3.252.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int x\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx = \frac{2\,(bcx^2+ac)\sqrt{bx^2+a}\sqrt{\sqrt{bx^2+a}c}}{7\,b}$$

input
```
integrate(x*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
2/7*(b*c*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)/b
```

3.252.6 Sympy [A] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.61

$$\int x\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\begin{cases}\frac{2a\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{7b}+\frac{2x^2\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{7} & \text{for } b\neq 0\\ \frac{x^2\left(\sqrt{a}c\right)^{\frac{3}{2}}}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*(c*(b*x**2+a)**(1/2))**(3/2),x)
```

output
```
Piecewise((2*a*(c*sqrt(a + b*x**2))**(3/2)/(7*b) + 2*x**2*(c*sqrt(a + b*x*
*2))**(3/2)/7, Ne(b, 0)), (x**2*(sqrt(a)*c)**(3/2)/2, True))
```

3.252.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int x\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2\,(bx^2+a)\left(\sqrt{bx^2+a}c\right)^{\frac{3}{2}}}{7\,b}$$

input
```
integrate(x*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
2/7*(b*x^2 + a)*(sqrt(b*x^2 + a)*c)^(3/2)/b
```

3.252.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.47

$$\int x\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2\,(bx^2+a)^{\frac{7}{4}}c^{\frac{3}{2}}}{7\,b}$$

input
```
integrate(x*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
2/7*(b*x^2 + a)^(7/4)*c^(3/2)/b
```

3.252.9 Mupad [B] (verification not implemented)

Time = 18.87 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int x\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{2\,c\,{\left(b\,x^2+a\right)}^{3/2}\,\sqrt{c\,\sqrt{b\,x^2+a}}}{7\,b}$$

input
```
int(x*(c*(a + b*x^2)^(1/2))^(3/2),x)
```

output
```
(2*c*(a + b*x^2)^(3/2)*(c*(a + b*x^2)^(1/2))^(1/2))/(7*b)
```

3.253 $\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x} \, dx$

3.253.1 Optimal result

Integrand size = 21, antiderivative size = 117

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x} \, dx = \frac{2}{3}\left(c\sqrt{a+bx^2}\right)^{3/2} + \frac{\left(c\sqrt{a+bx^2}\right)^{3/2} \arctan\left(\sqrt[4]{1+\frac{bx^2}{a}}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/4}} - \frac{\left(c\sqrt{a+bx^2}\right)^{3/2} \operatorname{arctanh}\left(\sqrt[4]{1+\frac{bx^2}{a}}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

output
```
2/3*(c*(b*x^2+a)^(1/2))^(3/2)+arctan((1+b*x^2/a)^(1/4))*(c*(b*x^2+a)^(1/2)
)^(3/2)/(1+b*x^2/a)^(3/4)-arctanh((1+b*x^2/a)^(1/4))*(c*(b*x^2+a)^(1/2))^(
3/2)/(1+b*x^2/a)^(3/4)
```

3.253.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.82

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x} \, dx = \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(2(a+bx^2)^{3/4} + 3a^{3/4}\arctan\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right) - 3a^{3/4}\operatorname{arctanh}\left(\frac{\sqrt[4]{a+}}{\sqrt[4]{}}\right.\right.}{3\left(a+bx^2\right)^{3/4}}$$

input
```
Integrate[(c*Sqrt[a + b*x^2])^(3/2)/x,x]
```

output
```
((c*Sqrt[a + b*x^2])^(3/2)*(2*(a + b*x^2)^(3/4) + 3*a^(3/4)*ArcTan[(a + b*
x^2)^(1/4)/a^(1/4)] - 3*a^(3/4)*ArcTanh[(a + b*x^2)^(1/4)/a^(1/4)]))/(3*(a
 + b*x^2)^(3/4))
```

3.253.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.81, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2045, 243, 60, 73, 25, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{2045}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\int \frac{\left(\frac{bx^2}{a}+1\right)^{3/4}}{x}dx}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\int \frac{\left(\frac{bx^2}{a}+1\right)^{3/4}}{x^2}dx^2}{2\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \text{60}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(\int \frac{1}{x^2\sqrt[4]{\frac{bx^2}{a}+1}}dx^2+\frac{4}{3}\left(\frac{bx^2}{a}+1\right)^{3/4}\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(\frac{4a\int -\frac{bx^4}{a\left(1-x^8\right)}d\sqrt[4]{\frac{bx^2}{a}+1}}{b}+\frac{4}{3}\left(\frac{bx^2}{a}+1\right)^{3/4}\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow 25$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(\frac{4}{3}\left(\frac{bx^2}{a}+1\right)^{3/4}-\frac{4a\int\frac{bx^4}{a(1-x^8)}d\sqrt[4]{\frac{bx^2}{a}+1}}{b}\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow 27$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(\frac{4}{3}\left(\frac{bx^2}{a}+1\right)^{3/4}-4\int\frac{x^4}{1-x^8}d\sqrt[4]{\frac{bx^2}{a}+1}\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow 827$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(\frac{4}{3}\left(\frac{bx^2}{a}+1\right)^{3/4}-4\left(\frac{1}{2}\int\frac{1}{1-x^4}d\sqrt[4]{\frac{bx^2}{a}+1}-\frac{1}{2}\int\frac{1}{x^4+1}d\sqrt[4]{\frac{bx^2}{a}+1}\right)\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow 216$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(\frac{4}{3}\left(\frac{bx^2}{a}+1\right)^{3/4}-4\left(\frac{1}{2}\int\frac{1}{1-x^4}d\sqrt[4]{\frac{bx^2}{a}+1}-\frac{1}{2}\arctan\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)\right)\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow 219$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(\frac{4}{3}\left(\frac{bx^2}{a}+1\right)^{3/4}-4\left(\frac{1}{2}\text{arctanh}\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)-\frac{1}{2}\arctan\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)\right)\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

input
```
Int[(c*Sqrt[a + b*x^2])^(3/2)/x,x]
```

output
```
((c*Sqrt[a + b*x^2])^(3/2)*((4*(1 + (b*x^2)/a)^(3/4))/3 - 4*(-1/2*ArcTan[(
1 + (b*x^2)/a)^(1/4)] + ArcTanh[(1 + (b*x^2)/a)^(1/4)]/2)))/(2*(1 + (b*x^2
)/a)^(3/4))
```

3.253. $\int\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx$

3.253.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

3.253. $\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx$

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.253.4 Maple [F]

$$\int \frac{\left(c\sqrt{b\,x^{2}+a}\right)^{\frac{3}{2}}}{x}dx$$

input
```
int((c*(b*x^2+a)^(1/2))^(3/2)/x,x)
```

output
```
int((c*(b*x^2+a)^(1/2))^(3/2)/x,x)
```

3.253.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx = \text{Timed out}$$

input
```
integrate((c*(b*x^2+a)^(1/2))^(3/2)/x,x, algorithm="fricas")
```

output
```
Timed out
```

3.253.6 Sympy [F]

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx = \int \frac{\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((c*(b*x**2+a)**(1/2))**(3/2)/x,x)
```

output
```
Integral((c*sqrt(a + b*x**2))**(3/2)/x, x)
```

3.253.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.01

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx = \frac{3\,ac^4\left(\frac{2\,\arctan\left(\frac{\sqrt{\sqrt{bx^2+a}c}}{\left(ac^2\right)^{\frac{1}{4}}}\right)}{\left(ac^2\right)^{\frac{1}{4}}} + \frac{\log\left(\frac{\sqrt{\sqrt{bx^2+a}c}-\left(ac^2\right)^{\frac{1}{4}}}{\sqrt{\sqrt{bx^2+a}c}+\left(ac^2\right)^{\frac{1}{4}}}\right)}{\left(ac^2\right)^{\frac{1}{4}}}\right) + 4\left(\sqrt{bx^2+a}c\right)^{\frac{3}{2}}c^2}{6\,c^2}$$

input
```
integrate((c*(b*x^2+a)^(1/2))^(3/2)/x,x, algorithm="maxima")
```

output
```
1/6*(3*a*c^4*(2*arctan(sqrt(sqrt(b*x^2 + a)*c)/(a*c^2)^(1/4))/(a*c^2)^(1/4
) + log((sqrt(sqrt(b*x^2 + a)*c) - (a*c^2)^(1/4))/(sqrt(sqrt(b*x^2 + a)*c)
 + (a*c^2)^(1/4)))/(a*c^2)^(1/4)) + 4*(sqrt(b*x^2 + a)*c)^(3/2)*c^2)/c^2
```

3.253.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.62

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx =$$

$$-\frac{1}{12}\left(6\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(bx^2+a\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)+6\sqrt{2}(-a)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(bx\right.\right.}{2\left(-a\right)^{\frac{1}{4}}}\right.\right.$$

input
```
integrate((c*(b*x^2+a)^(1/2))^(3/2)/x,x, algorithm="giac")
```

output
```
-1/12*(6*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*
x^2 + a)^(1/4))/(-a)^(1/4)) + 6*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(-a)^(1/4) - 2*(b*x^2 + a)^(1/4))/(-a)^(1/4)) - 3*sqrt(2)*(-a)^(3/4)
*log(sqrt(2)*(b*x^2 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^2 + a) + sqrt(-a)) +
3*sqrt(2)*(-a)^(3/4)*log(-sqrt(2)*(b*x^2 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^
2 + a) + sqrt(-a)) - 8*(b*x^2 + a)^(3/4))*c^(3/2)
```

3.253.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx=\int \frac{{\left(c\,\sqrt{b\,x^2+a}\right)}^{3/2}}{x}\,dx$$

input
```
int((c*(a + b*x^2)^(1/2))^(3/2)/x,x)
```

output
```
int((c*(a + b*x^2)^(1/2))^(3/2)/x, x)
```

3.254 $\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx$

3.254.1 Optimal result

Integrand size = 21, antiderivative size = 133

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx = -\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{2x^2} + \frac{3b\left(c\sqrt{a+bx^2}\right)^{3/2}\arctan\left(\sqrt[4]{1+\frac{bx^2}{a}}\right)}{4a\left(1+\frac{bx^2}{a}\right)^{3/4}} - \frac{3b\left(c\sqrt{a+bx^2}\right)^{3/2}\operatorname{arctanh}\left(\sqrt[4]{1+\frac{bx^2}{a}}\right)}{4a\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

output
```
-1/2*(c*(b*x^2+a)^(1/2))^(3/2)/x^2+3/4*b*arctan((1+b*x^2/a)^(1/4))*(c*(b*x
^2+a)^(1/2))^(3/2)/a/(1+b*x^2/a)^(3/4)-3/4*b*arctanh((1+b*x^2/a)^(1/4))*(c
*(b*x^2+a)^(1/2))^(3/2)/a/(1+b*x^2/a)^(3/4)
```

3.254.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.80

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx = -\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(2\sqrt[4]{a}(a+bx^2)^{3/4} - 3bx^2\arctan\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right) + 3bx^2\operatorname{arctanh}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\right)}{4\sqrt[4]{a}x^2\left(a+bx^2\right)^{3/4}}$$

input
```
Integrate[(c*Sqrt[a + b*x^2])^(3/2)/x^3,x]
```

output
```
-1/4*((c*Sqrt[a + b*x^2])^(3/2)*(2*a^(1/4)*(a + b*x^2)^(3/4) - 3*b*x^2*Arc
Tan[(a + b*x^2)^(1/4)/a^(1/4)] + 3*b*x^2*ArcTanh[(a + b*x^2)^(1/4)/a^(1/4)
]))/(a^(1/4)*x^2*(a + b*x^2)^(3/4))
```

3.254.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.75, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2045, 243, 51, 73, 25, 27, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{2045}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\int \frac{\left(\frac{bx^2}{a}+1\right)^{3/4}}{x^3}dx}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\int \frac{\left(\frac{bx^2}{a}+1\right)^{3/4}}{x^4}dx^2}{2\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \text{51}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(\frac{3b\int \frac{1}{x^2\sqrt[4]{\frac{bx^2}{a}+1}}dx^2}{4a}-\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}}{x^2}\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(3\int -\frac{bx^4}{a(1-x^8)}d\sqrt[4]{\frac{bx^2}{a}+1}-\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}}{x^2}\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(-3\int \frac{bx^4}{a(1-x^8)}d\sqrt[4]{\frac{bx^2}{a}+1}-\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}}{x^2}\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(-\frac{3b\int \frac{x^4}{1-x^8}d\sqrt[4]{\frac{bx^2}{a}+1}}{a}-\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}}{x^2}\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \text{827}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(-\frac{3b\left(\frac{1}{2}\int \frac{1}{1-x^4}d\sqrt[4]{\frac{bx^2}{a}+1}-\frac{1}{2}\int \frac{1}{x^4+1}d\sqrt[4]{\frac{bx^2}{a}+1}\right)}{a}-\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}}{x^2}\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(-\frac{3b\left(\frac{1}{2}\int \frac{1}{1-x^4}d\sqrt[4]{\frac{bx^2}{a}+1}-\frac{1}{2}\arctan\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)\right)}{a}-\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}}{x^2}\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(-\frac{3b\left(\frac{1}{2}\text{arctanh}\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)-\frac{1}{2}\arctan\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)\right)}{a}-\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}}{x^2}\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

3.254. $\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx$

input
```
Int[(c*Sqrt[a + b*x^2])^(3/2)/x^3,x]
```

output
```
((c*Sqrt[a + b*x^2])^(3/2)*(-((1 + (b*x^2)/a)^(3/4)/x^2) - (3*b*(-1/2*ArcT
an[(1 + (b*x^2)/a)^(1/4)] + ArcTanh[(1 + (b*x^2)/a)^(1/4)]/2))/a))/(2*(1 +
 (b*x^2)/a)^(3/4))
```

3.254.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.254.4 Maple [F]

$$\int \frac{\left(c\sqrt{b\,x^{2}+a}\right)^{\frac{3}{2}}}{x^{3}}d x$$

input
```
int((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x)
```

output
```
int((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x)
```

3.254.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.254.6 Sympy [F]

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx = \int \frac{\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((c*(b*x**2+a)**(1/2))**(3/2)/x**3,x)
```

output
```
Integral((c*sqrt(a + b*x**2))**(3/2)/x**3, x)
```

3.254.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.04

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx = \frac{\left(3\,c^4\left(\frac{2\,\arctan\left(\frac{\sqrt{\sqrt{bx^2+a}c}}{(ac^2)^{\frac{1}{4}}}\right)}{(ac^2)^{\frac{1}{4}}} + \frac{\log\left(\frac{\sqrt{\sqrt{bx^2+a}c}-(ac^2)^{\frac{1}{4}}}{\sqrt{\sqrt{bx^2+a}c}+(ac^2)^{\frac{1}{4}}}\right)}{(ac^2)^{\frac{1}{4}}}\right) - \frac{4\left(\sqrt{bx^2+a}c\right)^{\frac{3}{2}}c^4}{(bx^2+a)c^2-ac^2}\right)b}{8\,c^2}$$

input
```
integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x, algorithm="maxima")
```

output
```
1/8*(3*c^4*(2*arctan(sqrt(sqrt(b*x^2 + a)*c)/(a*c^2)^(1/4))/(a*c^2)^(1/4)
+ log((sqrt(sqrt(b*x^2 + a)*c) - (a*c^2)^(1/4))/(sqrt(sqrt(b*x^2 + a)*c) +
 (a*c^2)^(1/4)))/(a*c^2)^(1/4)) - 4*(sqrt(b*x^2 + a)*c)^(3/2)*c^4/((b*x^2
+ a)*c^2 - a*c^2))*b/c^2
```

3.254.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.59

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx = \frac{\left(\frac{6\sqrt{2}b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}+2\left(bx^2+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{1}{4}}} + \frac{6\sqrt{2}b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}(-a)^{\frac{1}{4}}-2\left(bx^2+a\right)^{\frac{1}{4}}\right)}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{1}{4}}} + \frac{3\sqrt{2}(-\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x, algorithm="giac")
```

output
```
1/16*(6*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^2 + a)
^(1/4))/(-a)^(1/4))/(-a)^(1/4) + 6*sqrt(2)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2
)*(-a)^(1/4) - 2*(b*x^2 + a)^(1/4))/(-a)^(1/4))/(-a)^(1/4) + 3*sqrt(2)*(-a
)^(3/4)*b^2*log(sqrt(2)*(b*x^2 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^2 + a) + s
qrt(-a))/a + 3*sqrt(2)*b^2*log(-sqrt(2)*(b*x^2 + a)^(1/4)*(-a)^(1/4) + sqr
t(b*x^2 + a) + sqrt(-a))/(-a)^(1/4) - 8*(b*x^2 + a)^(3/4)*b/x^2)*c^(3/2)/b
```

3.254.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx = \int \frac{\left(c\,\sqrt{b\,x^2+a}\right)^{3/2}}{x^3}\,dx$$

input
```
int((c*(a + b*x^2)^(1/2))^(3/2)/x^3,x)
```

output
```
int((c*(a + b*x^2)^(1/2))^(3/2)/x^3, x)
```

3.255 $\int x^2 \left(c\sqrt{a+bx^2}\right)^{3/2} dx$

3.255.1 Optimal result

Integrand size = 21, antiderivative size = 152

$$\int x^2 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \frac{2ax\left(c\sqrt{a+bx^2}\right)^{3/2}}{15b} + \frac{2}{9}x^3\left(c\sqrt{a+bx^2}\right)^{3/2} - \frac{4a^2x\left(c\sqrt{a+bx^2}\right)^{3/2}}{15b\left(a+bx^2\right)} + \frac{4a^{3/2}\left(c\sqrt{a+bx^2}\right)^{3/2} E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

output
```
2/15*a*x*(c*(b*x^2+a)^(1/2))^(3/2)/b+2/9*x^3*(c*(b*x^2+a)^(1/2))^(3/2)-4/1
5*a^2*x*(c*(b*x^2+a)^(1/2))^(3/2)/b/(b*x^2+a)+4/15*a^(3/2)*(cos(1/2*arctan
(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE
(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*(c*(b*x^2+a)^(1/2))^(3/2)/b^(
3/2)/(1+b*x^2/a)^(3/4)
```

3.255.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.45

$$\int x^2 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \frac{2x\left(c\sqrt{a+bx^2}\right)^{3/2}\left(a+bx^2-\frac{a\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\right)}{9b}$$

input
```
Integrate[x^2*(c*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
(2*x*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2 - (a*Hypergeometric2F1[-3/4, 1/2
, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^(3/4)))/(9*b)
```

3.255.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2045, 248, 262, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(c\sqrt{a+bx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{2045}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\int x^2\left(\frac{bx^2}{a}+1\right)^{3/4}dx}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \text{248}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(\frac{1}{3}\int \frac{x^2}{\sqrt[4]{\frac{bx^2}{a}+1}}dx+\frac{2}{9}x^3\left(\frac{bx^2}{a}+1\right)^{3/4}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \text{262}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(\frac{1}{3}\left(\frac{2ax\left(\frac{bx^2}{a}+1\right)^{3/4}}{5b}-\frac{2a\int \frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{5b}\right)+\frac{2}{9}x^3\left(\frac{bx^2}{a}+1\right)^{3/4}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \text{225}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(\frac{1}{3}\left(\frac{2ax\left(\frac{bx^2}{a}+1\right)^{3/4}}{5b}-\frac{2a\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int\frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{5b}\right)+\frac{2}{9}x^3\left(\frac{bx^2}{a}+1\right)^{3/4}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \ 212$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(\frac{1}{3}\left(\frac{2ax\left(\frac{bx^2}{a}+1\right)^{3/4}}{5b}-\frac{2a\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{5b}\right)+\frac{2}{9}x^3\left(\frac{bx^2}{a}+1\right)^{3/4}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

input
```
Int[x^2*(c*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
((c*Sqrt[a + b*x^2])^(3/2)*((2*x^3*(1 + (b*x^2)/a)^(3/4))/9 + ((2*a*x*(1 +
 (b*x^2)/a)^(3/4))/(5*b) - (2*a*((2*x)/(1 + (b*x^2)/a)^(1/4) - (2*Sqrt[a]*
EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/Sqrt[b]))/(5*b))/3))/(1 + (b*
x^2)/a)^(3/4)
```

3.255.3.1 Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

3.255. $\int x^2\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx$

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.255.4 Maple [F]

$$\int x^{2} \left(c \sqrt{b \,x^{2}+a}\right)^{\frac{3}{2}}d x$$

input
```
int(x^2*(c*(b*x^2+a)^(1/2))^(3/2),x)
```

output
```
int(x^2*(c*(b*x^2+a)^(1/2))^(3/2),x)
```

3.255.5 Fricas [F]

$$\int x^2 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \int \left(\sqrt{bx^2+ac}\right)^{\frac{3}{2}} x^2 \, dx$$

input
```
integrate(x^2*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)*c*x^2, x)
```

3.255.6 Sympy [F]

$$\int x^2 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \int x^2 \left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}} dx$$

input
```
integrate(x**2*(c*(b*x**2+a)**(1/2))**(3/2),x)
```

output
```
Integral(x**2*(c*sqrt(a + b*x**2))**(3/2), x)
```

3.255.7 Maxima [F]

$$\int x^2 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \int \left(\sqrt{bx^2+a}c\right)^{\frac{3}{2}} x^2 dx$$

input
```
integrate(x^2*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
integrate((sqrt(b*x^2 + a)*c)^(3/2)*x^2, x)
```

3.255.8 Giac [F]

$$\int x^2 \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \int \left(\sqrt{bx^2+a}c\right)^{\frac{3}{2}} x^2 dx$$

input
```
integrate(x^2*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
integrate((sqrt(b*x^2 + a)*c)^(3/2)*x^2, x)
```

3.255.9 Mupad [F(-1)]

Timed out.

$$\int x^2\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\int x^2\,\left(c\,\sqrt{b\,x^2+a}\right)^{3/2}\,dx$$

input
```
int(x^2*(c*(a + b*x^2)^(1/2))^(3/2),x)
```

output
```
int(x^2*(c*(a + b*x^2)^(1/2))^(3/2), x)
```

3.256 $\int \left(c\sqrt{a+bx^2}\right)^{3/2} dx$

3.256.1 Optimal result

Integrand size = 17, antiderivative size = 119

$$\int \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \frac{2}{5}x\left(c\sqrt{a+bx^2}\right)^{3/2}+\frac{6ax\left(c\sqrt{a+bx^2}\right)^{3/2}}{5\left(a+bx^2\right)} - \frac{6\sqrt{a}\left(c\sqrt{a+bx^2}\right)^{3/2}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

output
```
2/5*x*(c*(b*x^2+a)^(1/2))^(3/2)+6/5*a*x*(c*(b*x^2+a)^(1/2))^(3/2)/(b*x^2+a
)-6/5*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2
)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*
(c*(b*x^2+a)^(1/2))^(3/2)/(1+b*x^2/a)^(3/4)/b^(1/2)
```

3.256.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.44

$$\int \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \frac{x\left(c\sqrt{a+bx^2}\right)^{3/2}\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

input
```
Integrate[(c*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
(x*(c*Sqrt[a + b*x^2])^(3/2)*Hypergeometric2F1[-3/4, 1/2, 3/2, -((b*x^2)/a
)])/(1 + (b*x^2)/a)^(3/4)
```

3.256.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2045, 211, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(c\sqrt{a+bx^2}\right)^{3/2} dx$$

$$\Big\downarrow \; 2045$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2} \int \left(\frac{bx^2}{a}+1\right)^{3/4} dx}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \; 211$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2} \left(\frac{3}{5}\int \frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx+\frac{2}{5}x\left(\frac{bx^2}{a}+1\right)^{3/4}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \; 225$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2} \left(\frac{3}{5}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)+\frac{2}{5}x\left(\frac{bx^2}{a}+1\right)^{3/4}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \; 212$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(\frac{3}{5}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)+\frac{2}{5}x\left(\frac{bx^2}{a}+1\right)^{3/4}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

input
```
Int[(c*Sqrt[a + b*x^2])^(3/2),x]
```

output
```
((c*Sqrt[a + b*x^2])^(3/2)*((2*x*(1 + (b*x^2)/a)^(3/4))/5 + (3*((2*x)/(1 +
 (b*x^2)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2]
)/Sqrt[b]))/5))/(1 + (b*x^2)/a)^(3/4)
```

3.256.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.256.4 Maple [F]

$$\int \left(c\sqrt{b\,x^{2}+a}\right)^{\frac{3}{2}}dx$$

input
```
int((c*(b*x^2+a)^(1/2))^(3/2),x)
```

output
```
int((c*(b*x^2+a)^(1/2))^(3/2),x)
```

3.256.5 Fricas [F]

$$\int \left(c\sqrt{a+bx^2}\right)^{3/2}\,dx = \int \left(\sqrt{bx^2+a}c\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)*c, x)
```

3.256.6 Sympy [F]

$$\int \left(c\sqrt{a+bx^2}\right)^{3/2}\,dx = \int \left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*(b*x**2+a)**(1/2))**(3/2),x)
```

output
```
Integral((c*sqrt(a + b*x**2))**(3/2), x)
```

3.256.7 Maxima [F]

$$\int \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \int \left(\sqrt{bx^2+a}c\right)^{\frac{3}{2}} dx$$

input
```
integrate((c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
integrate((sqrt(b*x^2 + a)*c)^(3/2), x)
```

3.256.8 Giac [F]

$$\int \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \int \left(\sqrt{bx^2+a}c\right)^{\frac{3}{2}} dx$$

input
```
integrate((c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
integrate((sqrt(b*x^2 + a)*c)^(3/2), x)
```

3.256.9 Mupad [F(-1)]

Timed out.

$$\int \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \int \left(c\,\sqrt{b\,x^2+a}\right)^{3/2} dx$$

input
```
int((c*(a + b*x^2)^(1/2))^(3/2),x)
```

output
```
int((c*(a + b*x^2)^(1/2))^(3/2), x)
```

3.257 $\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx$

3.257.1 Optimal result

Integrand size = 21, antiderivative size = 115

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx = -\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}+\frac{3bx\left(c\sqrt{a+bx^2}\right)^{3/2}}{a+bx^2} - \frac{3\sqrt{b}\left(c\sqrt{a+bx^2}\right)^{3/2}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

output
```
-(c*(b*x^2+a)^(1/2))^(3/2)/x+3*b*x*(c*(b*x^2+a)^(1/2))^(3/2)/(b*x^2+a)-3*(
cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/
2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)*(c*(b*x
^2+a)^(1/2))^(3/2)/(1+b*x^2/a)^(3/4)/a^(1/2)
```

3.257.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.48

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx = -\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{2},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

input
```
Integrate[(c*Sqrt[a + b*x^2])^(3/2)/x^2,x]
```

output
```
-(((c*Sqrt[a + b*x^2])^(3/2)*Hypergeometric2F1[-3/4, -1/2, 1/2, -((b*x^2)/
a)])/(x*(1 + (b*x^2)/a)^(3/4)))
```

3.257.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2045, 247, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{2045}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\int \frac{\left(\frac{bx^2}{a}+1\right)^{3/4}}{x^2}dx}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \text{247}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(\frac{3b\int \frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{2a}-\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}}{x}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \text{225}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(\frac{3b\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{2a}-\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}}{x}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\downarrow \text{212}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(\frac{3b\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{2a}-\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}}{x}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

input
```
Int[(c*Sqrt[a + b*x^2])^(3/2)/x^2,x]
```

output
```
((c*Sqrt[a + b*x^2])^(3/2)*(-((1 + (b*x^2)/a)^(3/4)/x) + (3*b*((2*x)/(1 +
(b*x^2)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])
/Sqrt[b]))/(2*a)))/(1 + (b*x^2)/a)^(3/4)
```

3.257.3.1 Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.257. $\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx$

3.257.4 Maple [F]

$$\int \frac{\left(c\sqrt{b\,x^{2}+a}\right)^{\frac{3}{2}}}{x^{2}}dx$$

input
```
int((c*(b*x^2+a)^(1/2))^(3/2)/x^2,x)
```

output
```
int((c*(b*x^2+a)^(1/2))^(3/2)/x^2,x)
```

3.257.5 Fricas [F]

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx = \int \frac{\left(\sqrt{bx^2+a}c\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)*c/x^2, x)
```

3.257.6 Sympy [F]

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx = \int \frac{\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*(b*x**2+a)**(1/2))**(3/2)/x**2,x)
```

output
```
Integral((c*sqrt(a + b*x**2))**(3/2)/x**2, x)
```

3.257.7 Maxima [F]

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx=\int \frac{\left(\sqrt{bx^2+a}c\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((sqrt(b*x^2 + a)*c)^(3/2)/x^2, x)
```

3.257.8 Giac [F]

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx=\int \frac{\left(\sqrt{bx^2+a}c\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((sqrt(b*x^2 + a)*c)^(3/2)/x^2, x)
```

3.257.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx=\int \frac{\left(c\sqrt{b\,x^2+a}\right)^{3/2}}{x^2}\,dx$$

input
```
int((c*(a + b*x^2)^(1/2))^(3/2)/x^2,x)
```

output
```
int((c*(a + b*x^2)^(1/2))^(3/2)/x^2, x)
```

3.258 $\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx$

3.258.1 Optimal result

Integrand size = 21, antiderivative size = 154

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx = -\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{3x^3} - \frac{b\left(c\sqrt{a+bx^2}\right)^{3/2}}{2ax} + \frac{b^2x\left(c\sqrt{a+bx^2}\right)^{3/2}}{2a\left(a+bx^2\right)} - \frac{b^{3/2}\left(c\sqrt{a+bx^2}\right)^{3/2} E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

output
```
-1/3*(c*(b*x^2+a)^(1/2))^(3/2)/x^3-1/2*b*(c*(b*x^2+a)^(1/2))^(3/2)/a/x+1/2
*b^2*x*(c*(b*x^2+a)^(1/2))^(3/2)/a/(b*x^2+a)-1/2*b^(3/2)*(cos(1/2*arctan(x
*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(s
in(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*(c*(b*x^2+a)^(1/2))^(3/2)/a^(3/
2)/(1+b*x^2/a)^(3/4)
```

3.258.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.37

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx = -\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{4},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

input
```
Integrate[(c*Sqrt[a + b*x^2])^(3/2)/x^4,x]
```

output
```
-1/3*((c*Sqrt[a + b*x^2])^(3/2)*Hypergeometric2F1[-3/2, -3/4, -1/2, -((b*x
^2)/a)])/(x^3*(1 + (b*x^2)/a)^(3/4))
```

3.258.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2045, 247, 264, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{2045}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\int \frac{\left(\frac{bx^2}{a}+1\right)^{3/4}}{x^4}dx}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \text{247}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(\frac{b\int \frac{1}{x^2\sqrt[4]{\frac{bx^2}{a}+1}}dx}{2a}-\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}}{3x^3}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \text{264}$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(\frac{b\left(\frac{b\int\frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{2a}-\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}}{x}\right)}{2a}-\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}}{3x^3}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \; 225$$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(\frac{b\left(\frac{b\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int\frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{2a}-\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}}{x}\right)}{2a}-\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}}{3x^3}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

$$\Big\downarrow \; 212$$

3.258. $\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx$

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(\frac{b\left(\frac{b\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{2a}-\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}}{x}\right)}{2a}-\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}}{3x^3}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

input
```
Int[(c*Sqrt[a + b*x^2])^(3/2)/x^4,x]
```

output
```
((c*Sqrt[a + b*x^2])^(3/2)*(-1/3*(1 + (b*x^2)/a)^(3/4)/x^3 + (b*(-((1 + (b
*x^2)/a)^(3/4)/x) + (b*((2*x)/(1 + (b*x^2)/a)^(1/4) - (2*Sqrt[a]*EllipticE
[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/Sqrt[b]))/(2*a)))/(2*a)))/(1 + (b*x^2)
/a)^(3/4)
```

3.258.3.1 Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.258.4 Maple [F]

$$\int \frac{\left(c\sqrt{b\,x^{2}+a}\right)^{\frac{3}{2}}}{x^{4}}dx$$

input
```
int((c*(b*x^2+a)^(1/2))^(3/2)/x^4,x)
```

output
```
int((c*(b*x^2+a)^(1/2))^(3/2)/x^4,x)
```

3.258.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^4,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   Shou
ldn not  happen
```

3.258.6 Sympy [F]

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c*(b*x**2+a)**(1/2))**(3/2)/x**4,x)
```

output
```
Integral((c*sqrt(a + b*x**2))**(3/2)/x**4, x)
```

3.258.7 Maxima [F]

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(\sqrt{bx^2+a}c\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((sqrt(b*x^2 + a)*c)^(3/2)/x^4, x)
```

3.258.8 Giac [F]

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(\sqrt{bx^2+a}c\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^4,x, algorithm="giac")
```

output
```
integrate((sqrt(b*x^2 + a)*c)^(3/2)/x^4, x)
```

3.258.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(c\,\sqrt{b\,x^2+a}\right)^{3/2}}{x^4}\,dx$$

input
```
int((c*(a + b*x^2)^(1/2))^(3/2)/x^4,x)
```

output
```
int((c*(a + b*x^2)^(1/2))^(3/2)/x^4, x)
```

3.259 $\int \sqrt{(b-x)(-a+x)}\,dx$

3.259.1 Optimal result

Integrand size = 15, antiderivative size = 71

$$\int \sqrt{(b-x)(-a+x)}\,dx = -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2} - \frac{1}{8}(a-b)^2\arctan\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)$$

output
```
-1/8*(a-b)^2*arctan(1/2*(a+b-2*x)/(-a*b+(a+b)*x-x^2)^(1/2))-1/4*(a+b-2*x)*
(-a*b+(a+b)*x-x^2)^(1/2)
```

3.259.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \sqrt{(b-x)(-a+x)}\,dx = \frac{1}{4}\sqrt{(a-x)(-b+x)}\left(-a-b+2x+\frac{(a-b)^2\arctan\left(\frac{\sqrt{-a+x}}{\sqrt{b-x}}\right)}{\sqrt{b-x}\sqrt{-a+x}}\right)$$

input
```
Integrate[Sqrt[(b - x)*(-a + x)],x]
```

output
```
(Sqrt[(a - x)*(-b + x)]*(-a - b + 2*x + ((a - b)^2*ArcTan[Sqrt[-a + x]/Sqr
t[b - x]])/(Sqrt[b - x]*Sqrt[-a + x])))/4
```

3.259.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2048, 1087, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{(x-a)(b-x)}\,dx$$

$$\Big\downarrow \text{2048}$$

$$\int \sqrt{x(a+b)-ab-x^2}dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{8}(a-b)^2\int \frac{1}{\sqrt{-x^2+(a+b)x-ab}}dx-\frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2}$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{4}(a-b)^2\int \frac{1}{-\frac{(a+b-2x)^2}{-x^2+(a+b)x-ab}-4}d\frac{a+b-2x}{\sqrt{-x^2+(a+b)x-ab}}-\frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2}$$

$$\Big\downarrow \text{217}$$

$$-\frac{1}{8}(a-b)^2\arctan\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)-\frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2}$$

input
```
Int[Sqrt[(b - x)*(-a + x)],x]
```

output
```
-1/4*((a + b - 2*x)*Sqrt[-(a*b) + (a + b)*x - x^2]) - ((a - b)^2*ArcTan[(a
 + b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])])/8
```

3.259.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

3.259.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{(a+b-2x)\sqrt{-ab+(a+b)x-x^2}}{4}-\frac{\left(4ab-(a+b)^2\right)\arctan\left(\frac{x-\frac{a}{2}-\frac{b}{2}}{\sqrt{-ab+(a+b)x-x^2}}\right)}{8}$	68
risch	$\frac{(b-x)(a-x)(a+b-2x)}{4\sqrt{-(-b+x)(-a+x)}}-\left(\frac{1}{4}ab-\frac{1}{8}a^2-\frac{1}{8}b^2\right)\arctan\left(\frac{x-\frac{a}{2}-\frac{b}{2}}{\sqrt{-ab+(a+b)x-x^2}}\right)$	78

input
```
int(((b-x)*(-a+x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(a+b-2*x)*(-a*b+(a+b)*x-x^2)^(1/2)-1/8*(4*a*b-(a+b)^2)*arctan((x-1/2*
a-1/2*b)/(-a*b+(a+b)*x-x^2)^(1/2))
```

3.259.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13

$$\int \sqrt{(b-x)(-a+x)}\,dx = -\frac{1}{8}\left(a^2-2\,ab+b^2\right)\arctan\left(-\frac{\sqrt{-ab+(a+b)x-x^2}(a+b-2\,x)}{2\,(ab-(a+b)x+x^2)}\right) - \frac{1}{4}\sqrt{-ab+(a+b)x-x^2}(a+b-2\,x)$$

input
```
integrate(((b-x)*(-a+x))^(1/2),x, algorithm="fricas")
```

output
```
-1/8*(a^2 - 2*a*b + b^2)*arctan(-1/2*sqrt(-a*b + (a + b)*x - x^2)*(a + b -
 2*x)/(a*b - (a + b)*x + x^2)) - 1/4*sqrt(-a*b + (a + b)*x - x^2)*(a + b -
 2*x)
```

3.259.6 Sympy [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.61

$$\int \sqrt{(b-x)(-a+x)}\,dx = \left(-\frac{ab}{2} + \frac{\left(\frac{a}{4}+\frac{b}{4}\right)(a+b)}{2}\right)\left(\begin{cases} -i\log\left(a+b-2x+2i\sqrt{-ab-x^2+x\,(a+b)}\right) & \text{for } ab-\frac{(a+b)^2}{4}\neq 0 \\ \frac{\left(-\frac{a}{2}-\frac{b}{2}+x\right)\log\left(-\frac{a}{2}-\frac{b}{2}+x\right)}{\sqrt{-\left(-\frac{a}{2}-\frac{b}{2}+x\right)^2}} & \text{otherwise} \end{cases}\right) + \left(-\frac{a}{4}-\frac{b}{4}+\frac{x}{2}\right)\sqrt{-ab-x^2+x\,(a+b)}$$

input
```
integrate(((b-x)*(-a+x))**(1/2),x)
```

output
```
(-a*b/2 + (a/4 + b/4)*(a + b)/2)*Piecewise((-I*log(a + b - 2*x + 2*I*sqrt(
-a*b - x**2 + x*(a + b))), Ne(a*b - (a + b)**2/4, 0)), ((-a/2 - b/2 + x)*l
og(-a/2 - b/2 + x)/sqrt(-(-a/2 - b/2 + x)**2), True)) + (-a/4 - b/4 + x/2)
*sqrt(-a*b - x**2 + x*(a + b))
```

3.259.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{(b-x)(-a+x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(((b-x)*(-a+x))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

3.259.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int \sqrt{(b-x)(-a+x)}\,dx = \frac{1}{8}\left(a^2-2\,ab+b^2\right)\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\operatorname{sgn}(-a+b) - \frac{1}{4}\sqrt{-ab+ax+bx-x^2}(a+b-2\,x)$$

input
```
integrate(((b-x)*(-a+x))^(1/2),x, algorithm="giac")
```

output
```
1/8*(a^2 - 2*a*b + b^2)*arcsin((a + b - 2*x)/(a - b))*sgn(-a + b) - 1/4*sq
rt(-a*b + a*x + b*x - x^2)*(a + b - 2*x)
```

3.259.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{(b-x)(-a+x)}\,dx = \int \sqrt{-\left(a-x\right)\,\left(b-x\right)}\,dx$$

input
```
int((-(a - x)*(b - x))^(1/2),x)
```

output
```
int((-(a - x)*(b - x))^(1/2), x)
```

3.260 $\int \sqrt{(1-x^2)(3+x^2)}\,dx$

3.260.1 Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \sqrt{(1-x^2)(3+x^2)}\,dx = \frac{1}{3}x\sqrt{3-2x^2-x^4}-\frac{2E\left(\arcsin(x)\left|-\frac{1}{3}\right.\right)}{\sqrt{3}}+\frac{4\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)}{\sqrt{3}}$$

output
```
-2/3*EllipticE(x,1/3*I*3^(1/2))*3^(1/2)+4/3*EllipticF(x,1/3*I*3^(1/2))*3^(
1/2)+1/3*x*(-x^4-2*x^2+3)^(1/2)
```

3.260.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

$$\int \sqrt{(1-x^2)(3+x^2)}\,dx = \frac{1}{3}\left(x\sqrt{3-2x^2-x^4}-2iE\left(i\text{arcsinh}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)-4i\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{3}}\right),-3\right)\right)$$

input
```
Integrate[Sqrt[(1 - x^2)*(3 + x^2)],x]
```

output
```
(x*Sqrt[3 - 2*x^2 - x^4] - (2*I)*EllipticE[I*ArcSinh[x/Sqrt[3]], -3] - (4*
I)*EllipticF[I*ArcSinh[x/Sqrt[3]], -3])/3
```

3.260.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {2048, 1404, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{(1-x^2)(x^2+3)}\,dx$$

$$\Big\downarrow \text{ 2048}$$

$$\int \sqrt{-x^4-2x^2+3}dx$$

$$\Big\downarrow \text{ 1404}$$

$$\frac{1}{3}\int \frac{2(3-x^2)}{\sqrt{-x^4-2x^2+3}}dx+\frac{1}{3}\sqrt{-x^4-2x^2+3}x$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{3}\int \frac{3-x^2}{\sqrt{-x^4-2x^2+3}}dx+\frac{1}{3}\sqrt{-x^4-2x^2+3}x$$

$$\Big\downarrow \text{ 1494}$$

$$\frac{4}{3}\int \frac{3-x^2}{2\sqrt{1-x^2}\sqrt{x^2+3}}dx+\frac{1}{3}\sqrt{-x^4-2x^2+3}x$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{3}\int \frac{3-x^2}{\sqrt{1-x^2}\sqrt{x^2+3}}dx+\frac{1}{3}\sqrt{-x^4-2x^2+3}x$$

$$\Big\downarrow \text{ 399}$$

$$\frac{2}{3}\left(6\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2+3}}dx-\int \frac{\sqrt{x^2+3}}{\sqrt{1-x^2}}dx\right)+\frac{1}{3}\sqrt{-x^4-2x^2+3}x$$

$$\Big\downarrow \text{ 321}$$

$$\frac{2}{3}\left(2\sqrt{3}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)-\int \frac{\sqrt{x^2+3}}{\sqrt{1-x^2}}dx\right)+\frac{1}{3}\sqrt{-x^4-2x^2+3}x$$

$$\Big\downarrow \text{ 327}$$

$$\frac{2}{3}\left(2\sqrt{3}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)-\sqrt{3}E\left(\arcsin(x)\left|-\frac{1}{3}\right.\right)\right)+\frac{1}{3}\sqrt{-x^4-2x^2+3}x$$

input
```
Int[Sqrt[(1 - x^2)*(3 + x^2)],x]
```

output
```
(x*Sqrt[3 - 2*x^2 - x^4])/3 + (2*(-(Sqrt[3]*EllipticE[ArcSin[x], -1/3]) +
2*Sqrt[3]*EllipticF[ArcSin[x], -1/3]))/3
```

3.260.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

3.260.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(44) = 88$.

Time = 1.53 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.38

method	result	size
default	$\frac{x\sqrt{-x^4-2x^2+3}}{3}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,F\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\left(F\left(x,\frac{i\sqrt{3}}{3}\right)-E\left(x,\frac{i\sqrt{3}}{3}\right)\right)}{3\sqrt{-x^4-2x^2+3}}$	114
elliptic	$\frac{x\sqrt{-x^4-2x^2+3}}{3}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,F\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\left(F\left(x,\frac{i\sqrt{3}}{3}\right)-E\left(x,\frac{i\sqrt{3}}{3}\right)\right)}{3\sqrt{-x^4-2x^2+3}}$	114
risch	$-\frac{x\left(x^2-1\right)\left(x^2+3\right)}{3\sqrt{-\left(x^2-1\right)\left(x^2+3\right)}}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,F\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\left(F\left(x,\frac{i\sqrt{3}}{3}\right)-E\left(x,\frac{i\sqrt{3}}{3}\right)\right)}{3\sqrt{-x^4-2x^2+3}}$	124

input
```
int(((-x^2+1)*(x^2+3))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(-x^4-2*x^2+3)^(1/2)+2/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+
3)^(1/2)*EllipticF(x,1/3*I*3^(1/2))+2/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x
^4-2*x^2+3)^(1/2)*(EllipticF(x,1/3*I*3^(1/2))-EllipticE(x,1/3*I*3^(1/2)))
```

3.260.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \sqrt{(1-x^2)\,(3+x^2)}\,dx = \frac{2i\,xE(\arcsin\left(\frac{1}{x}\right)\,|-3)+4i\,xF(\arcsin\left(\frac{1}{x}\right)\,|-3)+\sqrt{-x^4-2\,x^2+3}(x^2+2)}{3\,x}$$

input
```
integrate(((-x^2+1)*(x^2+3))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*I*x*elliptic_e(arcsin(1/x), -3) + 4*I*x*elliptic_f(arcsin(1/x), -3)
 + sqrt(-x^4 - 2*x^2 + 3)*(x^2 + 2))/x
```

3.260.6 Sympy [F]

$$\int \sqrt{(1-x^2)\,(3+x^2)}\,dx = \int \sqrt{(1-x^2)\,(x^2+3)}\,dx$$

input
```
integrate(((-x**2+1)*(x**2+3))**(1/2),x)
```

output
```
Integral(sqrt((1 - x**2)*(x**2 + 3)), x)
```

3.260.7 Maxima [F]

$$\int \sqrt{(1-x^2)\,(3+x^2)}\,dx = \int \sqrt{-(x^2+3)(x^2-1)}\,dx$$

input
```
integrate(((-x^2+1)*(x^2+3))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-(x^2 + 3)*(x^2 - 1)), x)
```

3.260.8 Giac [F]

$$\int \sqrt{(1-x^2)\,(3+x^2)}\,dx = \int \sqrt{-(x^2+3)(x^2-1)}\,dx$$

input
```
integrate(((-x^2+1)*(x^2+3))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-(x^2 + 3)*(x^2 - 1)), x)
```

3.260.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{(1-x^2)(3+x^2)}\,dx = \int \sqrt{-(x^2-1)\,(x^2+3)}\,dx$$

input
```
int((-(x^2 - 1)*(x^2 + 3))^(1/2),x)
```

output
```
int((-(x^2 - 1)*(x^2 + 3))^(1/2), x)
```

3.261 $\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx$

3.261.1 Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = -\arctan\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)$$

output
```
-arctan(1/2*(a+b-2*x)/(-a*b+(a+b)*x-x^2)^(1/2))
```

3.261.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.72

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = \frac{2\sqrt{b-x}\sqrt{-a+x}\arctan\left(\frac{\sqrt{-a+x}}{\sqrt{b-x}}\right)}{\sqrt{(a-x)(-b+x)}}$$

input
```
Integrate[1/Sqrt[(b - x)*(-a + x)],x]
```

output
```
(2*Sqrt[b - x]*Sqrt[-a + x]*ArcTan[Sqrt[-a + x]/Sqrt[b - x]])/Sqrt[(a - x)
*(-b + x)]
```

3.261.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2048, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{(x-a)(b-x)}} \, dx$$

$$\Big\downarrow \; 2048$$

$$\int \frac{1}{\sqrt{x(a+b)-ab-x^2}}dx$$

$$\Big\downarrow \; 1092$$

$$2\int \frac{1}{-\frac{(a+b-2x)^2}{-x^2+(a+b)x-ab}-4}d\frac{a+b-2x}{\sqrt{-x^2+(a+b)x-ab}}$$

$$\Big\downarrow \; 217$$

$$-\arctan\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

input
```
Int[1/Sqrt[(b - x)*(-a + x)],x]
```

output
```
-ArcTan[(a + b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])]
```

3.261.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

3.261.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

method	result	size
default	$\arctan\left(\frac{x-\frac{a}{2}-\frac{b}{2}}{\sqrt{-ab+(a+b)x-x^2}}\right)$	28

input
```
int(1/((b-x)*(-a+x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan((x-1/2*a-1/2*b)/(-a*b+(a+b)*x-x^2)^(1/2))
```

3.261.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.34

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = -\arctan\left(-\frac{\sqrt{-ab+(a+b)x-x^2}(a+b-2x)}{2(ab-(a+b)x+x^2)}\right)$$

input
```
integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="fricas")
```

output
```
-arctan(-1/2*sqrt(-a*b + (a + b)*x - x^2)*(a + b - 2*x)/(a*b - (a + b)*x +
 x^2))
```

3.261.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.22

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = \begin{cases} -i\log\left(a+b-2x+2i\sqrt{-ab-x^2+x\,(a+b)}\right) & \text{for } ab-\frac{(a+b)^2}{4}\neq 0 \\ \frac{\left(-\frac{a}{2}-\frac{b}{2}+x\right)\log\left(-\frac{a}{2}-\frac{b}{2}+x\right)}{\sqrt{-\left(-\frac{a}{2}-\frac{b}{2}+x\right)^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/((b-x)*(-a+x))**(1/2),x)
```

output
```
Piecewise((-I*log(a + b - 2*x + 2*I*sqrt(-a*b - x**2 + x*(a + b))), Ne(a*b
 - (a + b)**2/4, 0)), ((-a/2 - b/2 + x)*log(-a/2 - b/2 + x)/sqrt(-(-a/2 -
b/2 + x)**2), True))
```

3.261.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more
details)Is
```

3.261.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(28) = 56.

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.91

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = \frac{1}{8}\,(a^2-2\,ab+b^2)\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\operatorname{sgn}(-a+b) - \frac{1}{4}\sqrt{-ab+ax+bx-x^2}(a+b-2\,x)$$

input
```
integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="giac")
```

output
```
1/8*(a^2 - 2*a*b + b^2)*arcsin((a + b - 2*x)/(a - b))*sgn(-a + b) - 1/4*sq
rt(-a*b + a*x + b*x - x^2)*(a + b - 2*x)
```

3.261.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = \int \frac{1}{\sqrt{-(a-x)\,(b-x)}}\,dx$$

input
```
int(1/(-(a - x)*(b - x))^(1/2),x)
```

output
```
int(1/(-(a - x)*(b - x))^(1/2), x)
```

3.262 $\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}}\,dx$

3.262.1 Optimal result

Integrand size = 17, antiderivative size = 12

$$\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}}\,dx = \frac{\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)}{\sqrt{3}}$$

output
```
1/3*EllipticF(x,1/3*I*3^(1/2))*3^(1/2)
```

3.262.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}}\,dx = -i\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{3}}\right),-3\right)$$

input
```
Integrate[1/Sqrt[(1 - x^2)*(3 + x^2)],x]
```

output
```
(-I)*EllipticF[I*ArcSinh[x/Sqrt[3]], -3]
```

3.262.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2048, 1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{(1-x^2)(x^2+3)}}dx$$
$$\Big\downarrow \text{ 2048}$$
$$\int \frac{1}{\sqrt{-x^4-2x^2+3}}dx$$
$$\Big\downarrow \text{ 1408}$$
$$2\int \frac{1}{2\sqrt{1-x^2}\sqrt{x^2+3}}dx$$
$$\Big\downarrow \text{ 27}$$
$$\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2+3}}dx$$
$$\Big\downarrow \text{ 321}$$
$$\frac{\text{EllipticF}\left(\arcsin(x),-\frac{1}{3}\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[(1 - x^2)*(3 + x^2)],x]
```

output
```
EllipticF[ArcSin[x], -1/3]/Sqrt[3]
```

3.262.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

3.262.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(13) = 26$.

Time = 0.50 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.58

method	result	size
default	$\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,F\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}$	43
elliptic	$\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,F\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}$	43

input
```
int(1/((-x^2+1)*(x^2+3))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*EllipticF(x,1/3*I*
3^(1/2))
```

3.262.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}}\,dx = \frac{1}{3}\sqrt{3}F\left(\arcsin(x)\,\middle|\,-\frac{1}{3}\right)$$

input
```
integrate(1/((-x^2+1)*(x^2+3))^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*elliptic_f(arcsin(x), -1/3)
```

3.262.6 Sympy [F]

$$\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}}\,dx = \int \frac{1}{\sqrt{(1-x^2)(x^2+3)}}\,dx$$

input
```
integrate(1/((-x**2+1)*(x**2+3))**(1/2),x)
```

output
```
Integral(1/sqrt((1 - x**2)*(x**2 + 3)), x)
```

3.262.7 Maxima [F]

$$\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}}\,dx = \int \frac{1}{\sqrt{-(x^2+3)(x^2-1)}}\,dx$$

input
```
integrate(1/((-x^2+1)*(x^2+3))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-(x^2 + 3)*(x^2 - 1)), x)
```

3.262.8 Giac [F]

$$\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}}\,dx = \int \frac{1}{\sqrt{-(x^2+3)(x^2-1)}}\,dx$$

input
```
integrate(1/((-x^2+1)*(x^2+3))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-(x^2 + 3)*(x^2 - 1)), x)
```

3.262.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}}\,dx = \int \frac{1}{\sqrt{-(x^2-1)(x^2+3)}}\,dx$$

input
```
int(1/(-(x^2 - 1)*(x^2 + 3))^(1/2),x)
```

output
```
int(1/(-(x^2 - 1)*(x^2 + 3))^(1/2), x)
```

3.263 $\int x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

3.263.1 Optimal result

Integrand size = 26, antiderivative size = 244

$$\int x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx=\frac{(11b^2c^2-2abcd-a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{16b^2d^3}-\frac{(3bc+ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{8bd^3}+\frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}(c+dx^2)^3}{6bd^2e}-\frac{(bc-ad)(5b^2c^2+2abcd+a^2d^2)\sqrt{e}\,\mathrm{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{5/2}d^{7/2}}$$

output
```
1/6*(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)^3/b/d^2/e-1/16*(-a*d+b*c)*(a^2
*d^2+2*a*b*c*d+5*b^2*c^2)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^
(1/2)/e^(1/2))*e^(1/2)/b^(5/2)/d^(7/2)+1/16*(-a^2*d^2-2*a*b*c*d+11*b^2*c^2
)*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^2/d^3-1/8*(a*d+3*b*c)*(d*x^2+c
)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b/d^3
```

3.263.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.81

$$\int x^5\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx$$
$$=\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-b\sqrt{d}(c+dx^2)\left(3a^2d^2-2abd(-2c+dx^2)+b^2(-15c^2+10cdx^2-8d^2x^4)\right)-\frac{3(bc-ad)^{3/2}(5b^2c^2+\cdots}{\cdots}\right.}{48b^3d^{7/2}}$$

input
```
Integrate[x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(b*Sqrt[d]*(c + d*x^2)*(3*a^2*d^2 - 2
*a*b*d*(-2*c + d*x^2) + b^2*(-15*c^2 + 10*c*d*x^2 - 8*d^2*x^4))) - (3*(b*c
 - a*d)^(3/2)*(5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[(b*(c + d*x^2))/(b*c
- a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/Sqrt[a + b*x^2
]))/(48*b^3*d^(7/2))
```

3.263.3 Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2053, 2052, 366, 27, 360, 27, 298, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx$$
$$\Big\downarrow \text{2053}$$
$$\frac{1}{2}\int x^4\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}dx^2$$
$$\Big\downarrow \text{2052}$$
$$e(bc-ad)\int\frac{x^4\left(ae-cx^4\right)^2}{\left(be-dx^4\right)^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$
$$\Big\downarrow \text{366}$$

$$e(bc-ad)\left(\frac{ex^6(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\int \frac{3ex^4\left(2bc^2dx^4+\left(b^2c^2-2abdc-a^2d^2\right)e\right)}{\left(be-dx^4\right)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{6bd^2e}\right)$$

$$\Big\downarrow \; 27$$

$$e(bc-ad)\left(\frac{ex^6(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\int \frac{x^4\left(2bc^2dx^4+\left(b^2c^2-2abdc-a^2d^2\right)e\right)}{\left(be-dx^4\right)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{2bd^2}\right)$$

$$\Big\downarrow \; 360$$

$$e(bc-ad)\left(\frac{ex^6(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\frac{e(bc-ad)(ad+3bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4d\left(be-dx^4\right)^2}-\frac{\int \frac{d\left(8bc^2dx^4+(bc-ad)(3bc+ad)e\right)}{\left(be-dx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{4d^2}}{2bd^2}\right)$$

$$\Big\downarrow \; 27$$

$$e(bc-ad)\left(\frac{ex^6(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\frac{e(bc-ad)(ad+3bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4d\left(be-dx^4\right)^2}-\frac{\int \frac{8bc^2dx^4+(bc-ad)(3bc+ad)e}{\left(be-dx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{4d}}{2bd^2}\right)$$

$$\Big\downarrow \; 298$$

$$e(bc-ad)\left(\frac{ex^6(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\frac{e(bc-ad)(ad+3bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4d\left(be-dx^4\right)^2}-\frac{\frac{\left(-a^2d^2-2abcd+11b^2c^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2b\left(be-dx^4\right)}-\frac{\left(a^2d^2+2abcd+5b^2c^2\right)\int \frac{1}{be-dx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{2b}}{4d}}{2bd^2}\right)$$

$$\Big\downarrow \; 221$$

3.263. $\int x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

$$e(bc-ad)\left(\frac{ex^6(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\frac{e(bc-ad)(ad+3bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4d(be-dx^4)^2}-\frac{\frac{\left(-a^2d^2-2abcd+11b^2c^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2b\left(be-dx^4\right)}-\frac{\left(a^2d^2+2abcd+5b^2c^2\right)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a}{c-}}}{\sqrt{b}\sqrt{}}\right)}{2b^{3/2}\sqrt{d}\sqrt{e}}}{4d}}{2bd^2}\right.$$

input
```
Int[x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(b*c - a*d)*e*(((b*c - a*d)^2*e*x^6)/(6*b*d^2*(b*e - d*x^4)^3) - (((b*c -
a*d)*(3*b*c + a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*d*(b*e - d*x^4)
^2) - (((11*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2
)])/(2*b*(b*e - d*x^4)) - ((5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt
[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2*b^(3/2)*Sqrt
[d]*Sqrt[e]))/(4*d))/(2*b*d^2))
```

3.263.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.263.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.99

method	result
risch	$-\frac{\left(-8b^2d^2x^4-2ab\,d^2x^2+10b^2cd\,x^2+3a^2d^2+4abcd-15b^2c^2\right)\left(d\,x^2+c\right)\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}{48b^2d^3}+\frac{\left(a^3d^3+a^2bc\,d^2+3d\,c^2b^2a-5b^3c^3\right)\ln\left(\frac{1}{2}eda+\right.}{}$
default	$\frac{\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,\left(d\,x^2+c\right)\left(-12\sqrt{bd}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,ab\,d^2x^2-36\sqrt{bd}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,b^2cd\,x^2+3\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+a}}{}\right.\right.}{}$

3.263. $\int x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

input
```
int(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*(-8*b^2*d^2*x^4-2*a*b*d^2*x^2+10*b^2*c*d*x^2+3*a^2*d^2+4*a*b*c*d-15*
b^2*c^2)*(d*x^2+c)/b^2/d^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/32*(a^3*d^3+a^2
*b*c*d^2+3*a*b^2*c^2*d-5*b^3*c^3)/b^2/d^3*ln((1/2*e*d*a+1/2*e*b*c+b*d*e*x^
2)/(b*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)*
(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)/(b*x^2+a)
```

3.263.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 541, normalized size of antiderivative = 2.22

$$\int x^5\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx$$

$$= \left[-\frac{3\left(5\,b^3c^3-3\,ab^2c^2d-a^2bcd^2-a^3d^3\right)\sqrt{\frac{e}{bd}}\log\left(8\,b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6\,abcd+a^2d^2\right)\right.}{}\right.$$

input
```
integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/192*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt(e/(b*d
))*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*
d + a^2*d^2)*e + 4*(2*b^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*d^2 +
 a*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) - 4*(8*b^2
*d^3*x^6 + 15*b^2*c^3 - 4*a*b*c^2*d - 3*a^2*c*d^2 - 2*(b^2*c*d^2 - a*b*d^3
)*x^4 + (5*b^2*c^2*d - 2*a*b*c*d^2 - 3*a^2*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/
(d*x^2 + c)))/(b^2*d^3), 1/96*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2
- a^3*d^3)*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2
 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d))/(b*e*x^2 + a*e)) + 2*(8*b^2*d^3*x^6 +
15*b^2*c^3 - 4*a*b*c^2*d - 3*a^2*c*d^2 - 2*(b^2*c*d^2 - a*b*d^3)*x^4 + (5*
b^2*c^2*d - 2*a*b*c*d^2 - 3*a^2*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)
))/(b^2*d^3)]
```

3.263.6 Sympy [F(-1)]

Timed out.

$$\int x^5\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx=\text{Timed out}$$

input
```
integrate(x**5*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

3.263.7 Maxima [F(-2)]

Exception generated.

$$\int x^5\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.263.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.93

$$\int x^5\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx$$
$$=\frac{1}{96}\left(2\sqrt{bdex^4+bcex^2+adex^2+ace}\left(2x^2\left(\frac{4x^2}{d}-\frac{5b^2cd-abd^2}{b^2d^3}\right)+\frac{15b^2c^2-4abcd-3a^2d^2}{b^2d^3}\right)+\frac{3(5\cdots}{\cdots}\right.$$
$$+c)$$

input
```
integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
1/96*(2*sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e)*(2*x^2*(4*x^2/d -
(5*b^2*c*d - a*b*d^2)/(b^2*d^3)) + (15*b^2*c^2 - 4*a*b*c*d - 3*a^2*d^2)/(b
^2*d^3)) + 3*(5*b^3*c^3*e - 3*a*b^2*c^2*d*e - a^2*b*c*d^2*e - a^3*d^3*e)*l
og(abs(-b*c*e - a*d*e - 2*sqrt(b*d*e)*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 +
b*c*e*x^2 + a*d*e*x^2 + a*c*e))))/(sqrt(b*d*e)*b^2*d^3))*sgn(d*x^2 + c)
```

3.263.9 Mupad [F(-1)]

Timed out.

$$\int x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx = \int x^5\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

input
```
int(x^5*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

output
```
int(x^5*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.264 $\int x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

3.264.1 Optimal result

Integrand size = 26, antiderivative size = 161

$$\int x^3\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx=-\frac{(5bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}{8bd^2}+\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)^2}{4d^2}+\frac{(bc-ad)(3bc+ad)\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{3/2}d^{5/2}}$$

output
```
1/8*(-a*d+b*c)*(a*d+3*b*c)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b
^(1/2)/e^(1/2))*e^(1/2)/b^(3/2)/d^(5/2)-1/8*(-a*d+5*b*c)*(d*x^2+c)*(e*(b*x
^2+a)/(d*x^2+c))^(1/2)/b/d^2+1/4*(d*x^2+c)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)
/d^2
```

3.264.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00

$$\int x^3\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx$$
$$=\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{b}\sqrt{d}\sqrt{a+bx^2}\left(c+dx^2\right)\left(-3bc+ad+2bdx^2\right)+\left(3b^2c^2-2abcd-a^2d^2\right)\sqrt{c+dx^2}\,\text{arctanh}\left(\frac{\sqrt{d}\cdots}{\sqrt{b}\cdots}\right.\right.}{8b^{3/2}d^{5/2}\sqrt{a+bx^2}}$$

input
```
Integrate[x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b]*Sqrt[d]*Sqrt[a + b*x^2]*(c + d
*x^2)*(-3*b*c + a*d + 2*b*d*x^2) + (3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*Sqrt[
c + d*x^2]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]))/
(8*b^(3/2)*d^(5/2)*Sqrt[a + b*x^2])
```

3.264.3 Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2053, 2052, 25, 360, 25, 298, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int x^2\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}dx^2$$

$$\Big\downarrow \text{2052}$$

$$e(bc-ad)\int -\frac{x^4\left(ae-cx^4\right)}{\left(be-dx^4\right)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$-\left(e(bc-ad)\int \frac{x^4\left(ae-cx^4\right)}{\left(be-dx^4\right)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}\right)$$

$$\Big\downarrow \text{360}$$

$$e(bc-ad)\left(\frac{\int -\frac{4cdx^4+(bc-ad)e}{\left(be-dx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{4d^2}+\frac{e(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4d^2\left(be-dx^4\right)^2}\right)$$

$$\Big\downarrow \text{25}$$

$$e(bc-ad)\left(\frac{e(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4d^2\left(be-dx^4\right)^2}-\frac{\int \frac{4cdx^4+(bc-ad)e}{\left(be-dx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{4d^2}\right)$$

$$\Big\downarrow \; 298$$

$$e(bc-ad)\left(\frac{e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^2\,(be-dx^4)^2}-\frac{\frac{(5bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2b(be-dx^4)}-\frac{(ad+3bc)\int\frac{1}{be-dx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{2b}}{4d^2}\right)$$

$$\Big\downarrow \; 221$$

$$e(bc-ad)\left(\frac{e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^2\,(be-dx^4)^2}-\frac{\frac{(5bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2b(be-dx^4)}-\frac{(ad+3bc)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{3/2}\sqrt{d}\sqrt{e}}}{4d^2}\right)$$

input
```
Int[x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(b*c - a*d)*e*(((b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*d^2*(b
*e - d*x^4)^2) - (((5*b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(2*b*(
b*e - d*x^4)) - ((3*b*c + a*d)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c +
d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2*b^(3/2)*Sqrt[d]*Sqrt[e]))/(4*d^2))
```

3.264.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

3.264. $\int x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.264.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.17

method	result
risch	$\frac{\left(2 b d \,x^{2}+d a -3 b c \right) \left(d \,x^{2}+c \right) \sqrt{\frac{e \left(b \,x^{2}+a \right)}{d \,x^{2}+c}}}{8 b \,d^{2}}-\frac{\left(a^{2} d^{2}+2 a b c d -3 b^{2} c^{2}\right) \ln \left(\frac{\frac{1}{2} e d a +\frac{1}{2} e b c +b d e \,x^{2}}{\sqrt{b d e}}+\sqrt{b d e \,x^{4}+\left(e d a +e b c \right) x^{2}+a c e}\right) \sqrt{\frac{e \left(b \,x^{2}+a \right.}{d \,x^{2}+c}}}{16 b \,d^{2} \sqrt{b d e}\, \left(b \,x^{2}+a \right)}$
default	$\frac{\sqrt{\frac{e \left(b \,x^{2}+a \right)}{d \,x^{2}+c}}\, \left(d \,x^{2}+c \right) \left(4 \sqrt{b d}\, \sqrt{b d \,x^{4}+a d \,x^{2}+b c \,x^{2}+a c}\, b d \,x^{2}-\ln \left(\frac{2 b d \,x^{2}+2 \sqrt{b d \,x^{4}+a d \,x^{2}+b c \,x^{2}+a c}\, \sqrt{b d}+d a +b c}{2 \sqrt{b d}}\right) a^{2} d^{2}-2 \ln \left(\frac{2 b d \,x^{2}+2 \sqrt{}\right.\right.}{}$

input
```
int(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*b*d*x^2+a*d-3*b*c)*(d*x^2+c)/b/d^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/
16*(a^2*d^2+2*a*b*c*d-3*b^2*c^2)/b/d^2*ln((1/2*e*d*a+1/2*e*b*c+b*d*e*x^2)/
(b*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)*(e*
(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)/(b*x^2+a)
```

3.264.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 407, normalized size of antiderivative = 2.53

$$\int x^3\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx$$
$$=\left[-\frac{\left(3\,b^2c^2-2\,abcd-a^2d^2\right)\sqrt{\frac{e}{bd}}\log\left(8\,b^2d^2ex^4+8\,(b^2cd+abd^2)ex^2+(b^2c^2+6\,abcd+a^2d^2)e-4\,(2\,b^2d^3x\right.}{}\right.$$
$$-\frac{\left(3\,b^2c^2-2\,abcd-a^2d^2\right)\sqrt{-\frac{e}{bd}}\arctan\left(\frac{\left(2\,bdx^2+bc+ad\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{e}{bd}}}{2\,(bex^2+ae)}\right)-2\,\left(2\,bd^2x^4-3\,bc^2+acd-(bcd-\right.}{16\,bd^2}$$

input
```
integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(e/(b*d))*log(8*b^2*d^2*e*x^
4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e - 4*(2
*b^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*d^2 + a*b*d^3)*x^2)*sqrt((
b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) - 4*(2*b*d^2*x^4 - 3*b*c^2 + a*
c*d - (b*c*d - a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*d^2), -1/
16*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2
 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d))/(b*e*x^2 +
a*e)) - 2*(2*b*d^2*x^4 - 3*b*c^2 + a*c*d - (b*c*d - a*d^2)*x^2)*sqrt((b*e*
x^2 + a*e)/(d*x^2 + c)))/(b*d^2)]
```

3.264.6 Sympy [F(-1)]

Timed out.

$$\int x^3\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx=\text{Timed out}$$

input
```
integrate(x**3*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

3.264.7 Maxima [F(-2)]

Exception generated.

$$\int x^3\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.264.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.05

$$\int x^3\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx$$
$$= \frac{1}{16}\left(2\sqrt{bdex^4+bcex^2+adex^2+ace}\left(\frac{2\,x^2}{d}-\frac{3\,bc-ad}{bd^2}\right)-\frac{(3\,b^2c^2e-2\,abcde-a^2d^2e)\log\left(\left|-bce-ad\right.\right.}{}\right.$$
$$+\,c)$$

input
```
integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
1/16*(2*sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e)*(2*x^2/d - (3*b*c
- a*d)/(b*d^2)) - (3*b^2*c^2*e - 2*a*b*c*d*e - a^2*d^2*e)*log(abs(-b*c*e -
 a*d*e - 2*sqrt(b*d*e)*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d
*e*x^2 + a*c*e))))/(sqrt(b*d*e)*b*d^2))*sgn(d*x^2 + c)
```

3.264.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx = \int x^3\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

input
```
int(x^3*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

output
```
int(x^3*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.265 $\int x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

3.265.1 Optimal result

Integrand size = 24, antiderivative size = 103

$$\int x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx=\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{2d}-\frac{(bc-ad)\sqrt{e}\,\mathrm{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2\sqrt{b}d^{3/2}}$$

output
```
-1/2*(-a*d+b*c)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1
/2))*e^(1/2)/d^(3/2)/b^(1/2)+1/2*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d
```

3.265.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.28

$$\int x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$$
$$=\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{b}\sqrt{d}\sqrt{a+bx^2}(c+dx^2)-(bc-ad)\sqrt{c+dx^2}\,\mathrm{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)\right)}{2\sqrt{b}d^{3/2}\sqrt{a+bx^2}}$$

input
```
Integrate[x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b]*Sqrt[d]*Sqrt[a + b*x^2]*(c + d
*x^2) - (b*c - a*d)*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqr
t[b]*Sqrt[c + d*x^2])]))/(2*Sqrt[b]*d^(3/2)*Sqrt[a + b*x^2])
```

3.265.3 Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2053, 2051, 252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}dx$$

$$\Big\downarrow \; 2053$$

$$\frac{1}{2}\int \sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}dx^2$$

$$\Big\downarrow \; 2051$$

$$e(bc-ad)\int \frac{x^4}{\left(be-dx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$

$$\Big\downarrow \; 252$$

$$e(bc-ad)\left(\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2d\left(be-dx^4\right)}-\frac{\int \frac{1}{be-dx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{2d}\right)$$

$$\Big\downarrow \; 221$$

$$e(bc-ad)\left(\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2d\left(be-dx^4\right)}-\frac{\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2\sqrt{b}d^{3/2}\sqrt{e}}\right)$$

input
```
Int[x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(b*c - a*d)*e*(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(2*d*(b*e - d*x^4)) - Arc
Tanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])]/(2*Sqr
t[b]*d^(3/2)*Sqrt[e]))
```

3.265.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2051
```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)   Subst[Int[
x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
 x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x]
&& FractionQ[p] && IntegerQ[1/n]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.265.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.50

method	result
risch	$\frac{(d\,x^2+c)\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}{2d}+\frac{(da-bc)\ln\left(\frac{\frac{1}{2}eda+\frac{1}{2}ebc+bde\,x^2}{\sqrt{bde}}+\sqrt{bde\,x^4+(eda+ebc)x^2+ace}\right)\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,\sqrt{(d\,x^2+c)e(b\,x^2+a)}}{4d\sqrt{bde}\,(b\,x^2+a)}$
default	$\frac{\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,(d\,x^2+c)\left(a\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+da+bc}{2\sqrt{bd}}\right)d-b\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+da+bc}{2\sqrt{bd}}\right)c+2\sqrt{}\right.}{4\sqrt{(d\,x^2+c)(b\,x^2+a)}\,d\sqrt{bd}}$

input
```
int(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d+1/4*(a*d-b*c)/d*ln((1/2*e*d*
a+1/2*e*b*c+b*d*e*x^2)/(b*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(
1/2))/(b*d*e)^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^
(1/2)/(b*x^2+a)
```

3.265.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 313, normalized size of antiderivative = 3.04

$$\int x\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx$$

$$= \left[-\frac{(bc-ad)\sqrt{\frac{e}{bd}}\log\left(8\,b^2d^2ex^4+8\,(b^2cd+abd^2)ex^2+(b^2c^2+6\,abcd+a^2d^2)e+4\,(2\,b^2d^3x^4+b^2c^2d+ab\right.}{8\,d}\right.$$

input
```
integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*((b*c - a*d)*sqrt(e/(b*d))*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^
2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b^2*d^3*x^4 + b^2*c^2*
d + a*b*c*d^2 + (3*b^2*c*d^2 + a*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 +
 c))*sqrt(e/(b*d))) - 4*(d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/d,
1/4*((b*c - a*d)*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b
*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d))/(b*e*x^2 + a*e)) + 2*(d*x^2 + c)
*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/d]
```

3.265.6 Sympy [F(-1)]

Timed out.

$$\int x\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

3.265.7 Maxima [F(-2)]

Exception generated.

$$\int x\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.265.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.34

$$\int x\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx$$
$$= \frac{1}{4}\left(\frac{2\sqrt{bdex^4+bcex^2+adex^2+ace}}{d} + \frac{(bce-ade)\sqrt{bde}\log\left(\left|-2\left(\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+\right.\right.\right.}{bd^2e}\right.$$
$$\left. + c\right)$$

input
```
integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
1/4*(2*sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e)/d + (b*c*e - a*d*e)
*sqrt(b*d*e)*log(abs(-2*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*
d*e*x^2 + a*c*e))*b*d - sqrt(b*d*e)*b*c - sqrt(b*d*e)*a*d))/(b*d^2*e))*sgn
(d*x^2 + c)
```

3.265.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx=\int x\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

input
```
int(x*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

output
```
int(x*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.266 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx$

3.266.1 Optimal result

Integrand size = 26, antiderivative size = 112

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx=-\frac{\sqrt{a}\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{c}}+\frac{\sqrt{b}\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{d}}$$

output
```
-arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2))*a^(1/2)*e^
(1/2)/c^(1/2)+arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2
))*b^(1/2)*e^(1/2)/d^(1/2)
```

3.266.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx$$
$$=\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\left(-\sqrt{a}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)+\sqrt{b}\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)\right)}{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}}$$

input
```
Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x,x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(-(Sqrt[a]*Sqrt[d]*ArcT
anh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]) + Sqrt[b]*Sqrt[c
]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]))/(Sqrt[c]*
Sqrt[d]*Sqrt[a + b*x^2])
```

3.266.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2053, 2052, 25, 383, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{x^2}dx^2$$

$$\Big\downarrow \text{2052}$$

$$e(bc-ad)\int -\frac{x^4}{(ae-cx^4)(be-dx^4)}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$-\left(e(bc-ad)\int \frac{x^4}{(ae-cx^4)(be-dx^4)}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}\right)$$

$$\Big\downarrow \text{383}$$

$$e(bc-ad)\left(\frac{b\int \frac{1}{be-dx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{bc-ad}-\frac{a\int \frac{1}{ae-cx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{bc-ad}\right)$$

$$\Big\downarrow \text{221}$$

$$e(bc-ad)\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{d}\sqrt{e}(bc-ad)}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{c}\sqrt{e}(bc-ad)}\right)$$

input
```
Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x,x]
```

output
```
(b*c - a*d)*e*(-((Sqrt[a]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2
)])/(Sqrt[a]*Sqrt[e])])/(Sqrt[c]*(b*c - a*d)*Sqrt[e])) + (Sqrt[b]*ArcTanh[
(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(Sqrt[d]*(
b*c - a*d)*Sqrt[e]))
```

3.266.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 383
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[(-a)*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(a + b*x^2), x], x]
 + Simp[c*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; Free
Q[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.266. $\int \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{x}\,dx$

3.266.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(84) = 168$.

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.60

method	result
default	$-\frac{\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}\,\left(d\,x^{2}+c\right)\left(a\ln\left(\frac{a d\,x^{2}+b c\,x^{2}+2\sqrt{a c}\,\sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+a c}+2 a c}{x^{2}}\right)\sqrt{b d}-\sqrt{a c}\,\ln\left(\frac{2 b d\,x^{2}+2\sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+a c}\,\sqrt{b d}+d\cdots}{2\sqrt{b d}}\right.\right.}{2\sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\,\sqrt{b d}\,\sqrt{a c}}$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*(a*ln((a*d*x^2+b*c*x^2+2*(a*c
)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*(b*d)^(1/2)-(a*c)^
(1/2)*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+
d*a+b*c)/(b*d)^(1/2))*b)/((d*x^2+c)*(b*x^2+a))^(1/2)/(b*d)^(1/2)/(a*c)^(1/
2)
```

3.266.5 Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 865, normalized size of antiderivative = 7.72

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx = \left[\frac{1}{4}\sqrt{\frac{be}{d}}\log\left(8\,b^2d^2ex^4 + 8\,(b^2cd + abd^2)ex^2 + (b^2c^2 + 6\,abcd + a^2d^2)e + 4\,(2\,bd^3x^4 + bc^2d + acd^2 + (3\,bcd^2 + ad^3)x^2)\sqrt{\frac{be}{d}}\sqrt{\frac{bex^2+ae}{dx^2+c}}\right)\right.$$

$$+\frac{1}{4}\sqrt{\frac{ae}{c}}\log\left(\frac{(b^2c^2 + 6\,abcd + a^2d^2)ex^4 + 8\,a^2c^2e + 8\,(abc^2 + a^2cd)ex^2 - 4\,((bc^2d + acd^2)x^4 + 2\,ac^3 + (}{x^4}\right.$$

$$-\frac{1}{2}\sqrt{-\frac{be}{d}}\arctan\left(\frac{(2\,bdx^2 + bc + ad)\sqrt{-\frac{be}{d}}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\,(b^2ex^2 + abe)}\right)$$

$$+\frac{1}{4}\sqrt{\frac{ae}{c}}\log\left(\frac{(b^2c^2 + 6\,abcd + a^2d^2)ex^4 + 8\,a^2c^2e + 8\,(abc^2 + a^2cd)ex^2 - 4\,((bc^2d + acd^2)x^4 + 2\,ac^3 + (}{x^4}\right.$$

$$+\frac{1}{4}\sqrt{\frac{be}{d}}\log\left(8\,b^2d^2ex^4 + 8\,(b^2cd + abd^2)ex^2 + (b^2c^2 + 6\,abcd + a^2d^2)e\right.$$

$$\left.+4\,(2\,bd^3x^4 + bc^2d + acd^2 + (3\,bcd^2 + ad^3)x^2)\sqrt{\frac{be}{d}}\sqrt{\frac{bex^2+ae}{dx^2+c}}\right), \frac{1}{2}\sqrt{-\frac{ae}{c}}\arctan\left(\frac{((bc+ad)x^2 + 2\,a}{2\,(abex^2}\right.$$

$$\left.-\frac{1}{2}\sqrt{-\frac{be}{d}}\arctan\left(\frac{(2\,bdx^2 + bc + ad)\sqrt{-\frac{be}{d}}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\,(b^2ex^2 + abe)}\right)\right]$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/4*sqrt(b*e/d)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*
c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*d^2 + (3*b*c
*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) + 1/4*sq
rt(a*e/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*
b*c^2 + a^2*c*d)*e*x^2 - 4*((b*c^2*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3 + 3
*a*c^2*d)*x^2)*sqrt(a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/x^4), -1/2*s
qrt(-b*e/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*e/d)*sqrt((b*e*x^2
+ a*e)/(d*x^2 + c))/(b^2*e*x^2 + a*b*e)) + 1/4*sqrt(a*e/c)*log(((b^2*c^2 +
 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 -
4*((b*c^2*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3 + 3*a*c^2*d)*x^2)*sqrt(a*e/c
)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/x^4), 1/2*sqrt(-a*e/c)*arctan(1/2*((b
*c + a*d)*x^2 + 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b
*e*x^2 + a^2*e)) + 1/4*sqrt(b*e/d)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*
d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d
+ a*c*d^2 + (3*b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x
^2 + c))), 1/2*sqrt(-a*e/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(-a*e
/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b*e*x^2 + a^2*e)) - 1/2*sqrt(-b*e
/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e)/(
d*x^2 + c))/(b^2*e*x^2 + a*b*e))]
```

3.266.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x,x)
```

output
```
Timed out
```

3.266.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.266.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.266.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx = \int \frac{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}{x}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x,x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x, x)
```

3.267 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx$

3.267.1 Optimal result

Integrand size = 26, antiderivative size = 127

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx = \frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} - \frac{(bc-ad)\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2\sqrt{a}c^{3/2}}$$

output
```
-1/2*(-a*d+b*c)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1
/2))*e^(1/2)/c^(3/2)/a^(1/2)+1/2*(-a*d+b*c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/
c/(a-c*(b*x^2+a)/(d*x^2+c))
```

3.267.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx = -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{a}\sqrt{c}\sqrt{a+bx^2}(c+dx^2)+(bc-ad)x^2\sqrt{c+dx^2}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)\right)}{2\sqrt{a}c^{3/2}x^2\sqrt{a+bx^2}}$$

input
```
Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^3,x]
```

output
```
-1/2*(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2]*(
c + d*x^2) + (b*c - a*d)*x^2*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x
^2])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(Sqrt[a]*c^(3/2)*x^2*Sqrt[a + b*x^2])
```

3.267.3 Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2053, 2052, 252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3} \, dx$$

$$\Big\downarrow \text{ 2053}$$

$$\frac{1}{2}\int \frac{\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{x^4}dx^2$$

$$\Big\downarrow \text{ 2052}$$

$$e(bc-ad)\int \frac{x^4}{(ae-cx^4)^2}d\sqrt{\frac{e\,(bx^2+a)}{dx^2+c}}$$

$$\Big\downarrow \text{ 252}$$

$$e(bc-ad)\left(\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c\,(ae-cx^4)}-\frac{\int \frac{1}{ae-cx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{2c}\right)$$

$$\Big\downarrow \text{ 221}$$

$$e(bc-ad)\left(\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c\,(ae-cx^4)}-\frac{\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2\sqrt{a}c^{3/2}\sqrt{e}}\right)$$

input
```
Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^3,x]
```

output
```
(b*c - a*d)*e*(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(2*c*(a*e - c*x^4)) - Arc
Tanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])]/(2*Sqr
t[a]*c^(3/2)*Sqrt[e]))
```

3.267.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.267.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.28

method	result
risch	$-\frac{(d x^2+c)\sqrt{\frac{e(b x^2+a)}{d x^2+c}}}{2c x^2}+\frac{(da-bc)\ln\left(\frac{2ace+(eda+ebc)x^2+2\sqrt{ace}\sqrt{bde x^4+(eda+ebc)x^2+ace}}{x^2}\right)\sqrt{\frac{e(b x^2+a)}{d x^2+c}}\sqrt{(d x^2+c)e(b x^2+a)}}{4c\sqrt{ace}\,(b x^2+a)}$
default	$-\frac{\sqrt{\frac{e(b x^2+a)}{d x^2+c}}\,(d x^2+c)\left(-2bd\sqrt{bd x^4+ad x^2+bc x^2+ac}\,x^4\sqrt{ac}-a^2\ln\left(\frac{ad x^2+bc x^2+2\sqrt{ac}\sqrt{bd x^4+ad x^2+bc x^2+ac}+2ac}{x^2}\right)dc x^2+c^2\ln\left(\right.\right.}{}$

3.267. $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/c*(d*x^2+c)/x^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/4*(a*d-b*c)/c/(a*c*e)
^(1/2)*ln((2*a*c*e+(a*d*e+b*c*e)*x^2+2*(a*c*e)^(1/2)*(b*d*e*x^4+(a*d*e+b*c
*e)*x^2+a*c*e)^(1/2))/x^2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x
^2+a))^(1/2)/(b*x^2+a)
```

3.267.5 Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.62

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx$$
$$= \left[-\frac{(bc-ad)x^2\sqrt{\frac{e}{ac}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)ex^4+8\,a^2c^2e+8\,(abc^2+a^2cd)ex^2+4\,(2\,a^2c^3+(abc^2d+a^2cd^2)x^4+(abc^3+3\,a^2c^2d)x^2)\sqrt{\frac{be}{c}}}{x^4}\right.}{8\,cx^2}\right.$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[-1/8*((b*c - a*d)*x^2*sqrt(e/(a*c))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*
e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 + 4*(2*a^2*c^3 + (a*b*c^
2*d + a^2*c*d^2)*x^4 + (a*b*c^3 + 3*a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(
d*x^2 + c))*sqrt(e/(a*c)))/x^4) + 4*(d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^
2 + c)))/(c*x^2), 1/4*((b*c - a*d)*x^2*sqrt(-e/(a*c))*arctan(1/2*((b*c + a
*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(a*c))/(b*e*x^2
 + a*e)) - 2*(d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c*x^2)]
```

3.267.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**3,x)
```

output
```
Timed out
```

3.267.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.267.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(107) = 214.

Time = 0.39 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.91

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx$$
$$= \frac{1}{2}\left(\frac{(bce-ade)\arctan\left(-\frac{\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+ace}}{\sqrt{-ace}}\right)}{\sqrt{-ace}c} - \frac{\left(\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+ace}\right)bc}{\left(ace-\left(\sqrt{bde}x^2 \cdots\right.\right.} \cdots + c)\right.$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x, algorithm="giac")
```

output
```
1/2*((b*c*e - a*d*e)*arctan(-(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2
 + a*d*e*x^2 + a*c*e))/sqrt(-a*c*e))/(sqrt(-a*c*e)*c) - ((sqrt(b*d*e)*x^2
- sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))*b*c*e + (sqrt(b*d*e)*x^
2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))*a*d*e + 2*sqrt(b*d*e)
*a*c*e)/((a*c*e - (sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^
2 + a*c*e))^2)*c))*sgn(d*x^2 + c)
```

3.267.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx = \int \frac{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}{x^3}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^3,x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^3, x)
```

3.268 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx$

3.268.1 Optimal result

Integrand size = 26, antiderivative size = 208

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx = -\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2} + \frac{(bc-5ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} + \frac{(bc-ad)(bc+3ad)\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{3/2}c^{5/2}}$$

output
```
1/8*(-a*d+b*c)*(3*a*d+b*c)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a
^(1/2)/e^(1/2))*e^(1/2)/a^(3/2)/c^(5/2)-1/4*(-a*d+b*c)^2*(e*(b*x^2+a)/(d*x
^2+c))^(1/2)/c^2/(a-c*(b*x^2+a)/(d*x^2+c))^2+1/8*(-5*a*d+b*c)*(-a*d+b*c)*(
e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^2/(a-c*(b*x^2+a)/(d*x^2+c))
```

3.268.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx$$
$$= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\left(\sqrt{a}\sqrt{c}\sqrt{a+bx^2}\sqrt{c+dx^2}(-2ac-bcx^2+3adx^2)+(b^2c^2+2abcd-3a^2d^2)\,x^4\text{arctanh}\right.}{8a^{3/2}c^{5/2}x^4\sqrt{a+bx^2}}$$

input
```
Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^5,x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(Sqrt[a]*Sqrt[c]*Sqrt[a
 + b*x^2]*Sqrt[c + d*x^2]*(-2*a*c - b*c*x^2 + 3*a*d*x^2) + (b^2*c^2 + 2*a*
b*c*d - 3*a^2*d^2)*x^4*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c +
 d*x^2])]))/(8*a^(3/2)*c^(5/2)*x^4*Sqrt[a + b*x^2])
```

3.268.3 Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2053, 2052, 25, 360, 298, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx$$

$$\Big\downarrow \ 2053$$

$$\frac{1}{2}\int \frac{\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{x^6}\,dx^2$$

$$\Big\downarrow \ 2052$$

$$e(bc-ad)\int -\frac{x^4(be-dx^4)}{(ae-cx^4)^3}\,d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}$$

$$\Big\downarrow \ 25$$

$$-\left(e(bc-ad)\int \frac{x^4\left(be-dx^4\right)}{\left(ae-cx^4\right)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}\right)$$

$$\Big\downarrow \; 360$$

$$e(bc-ad)\left(\frac{\int \frac{(bc-ad)e-4cdx^4}{\left(ae-cx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{4c^2}-\frac{e(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^2\left(ae-cx^4\right)^2}\right)$$

$$\Big\downarrow \; 298$$

$$e(bc-ad)\left(\frac{\frac{(3ad+bc)\int \frac{1}{ae-cx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{2a}+\frac{(bc-5ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2a\left(ae-cx^4\right)}}{4c^2}-\frac{e(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^2\left(ae-cx^4\right)^2}\right)$$

$$\Big\downarrow \; 221$$

$$e(bc-ad)\left(\frac{\frac{(3ad+bc)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{3/2}\sqrt{c}\sqrt{e}}+\frac{(bc-5ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2a\left(ae-cx^4\right)}}{4c^2}-\frac{e(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^2\left(ae-cx^4\right)^2}\right)$$

input
```
Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^5,x]
```

output
```
(b*c - a*d)*e*(-1/4*((b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c^2
*(a*e - c*x^4)^2) + (((b*c - 5*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(2*
a*(a*e - c*x^4)) + ((b*c + 3*a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c
 + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(2*a^(3/2)*Sqrt[c]*Sqrt[e]))/(4*c^2))
```

3.268.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.268. $\int \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{x^5}\,dx$

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.268.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{(d\,x^2+c)\left(-3ad\,x^2+bc\,x^2+2ac\right)\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}{8c^2x^4a}-\frac{\left(3a^2d^2-2abcd-b^2c^2\right)\ln\left(\frac{2ace+(eda+ebc)x^2+2\sqrt{ace}\,\sqrt{bde\,x^4+(eda+ebc)x^2+ace}}{x^2}\right)\sqrt{}}{16a\,c^2\sqrt{ace}\,(b\,x^2+a)}$
default	$-\frac{\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,(d\,x^2+c)\left(10b\,d^2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,x^6a\sqrt{ac}+2b^2d\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,x^6c\sqrt{ac}+3a^3\ln\left(\frac{ad\,x^2+bc\,x^2+2\sqrt{a}}{}\right.\right.}{}$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

3.268. $\int \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{x^5}\,dx$

output
```
-1/8*(d*x^2+c)*(-3*a*d*x^2+b*c*x^2+2*a*c)/c^2/x^4/a*(e*(b*x^2+a)/(d*x^2+c)
)^(1/2)-1/16*(3*a^2*d^2-2*a*b*c*d-b^2*c^2)/a/c^2/(a*c*e)^(1/2)*ln((2*a*c*e
+(a*d*e+b*c*e)*x^2+2*(a*c*e)^(1/2)*(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/
2))/x^2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)/(b*x^
2+a)
```

3.268.5 Fricas [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 427, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx$$

$$= \Bigg[-\frac{(b^2c^2+2\,abcd-3\,a^2d^2)x^4\sqrt{\frac{e}{ac}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)ex^4+8\,a^2c^2e+8\,(abc^2+a^2cd)ex^2-4\,(2\,a^2c^3+(abc^2d+a^2cd^2)x^4+(ab}{x^4}\right.}{32\,ac^2x^4}$$

$$-\frac{(b^2c^2+2\,abcd-3\,a^2d^2)x^4\sqrt{-\frac{e}{ac}}\arctan\left(\frac{((bc+ad)x^2+2\,ac)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{e}{ac}}}{2\,(bex^2+ae)}\right)+2\,((bcd-3\,ad^2)x^4+2\,ac^2+}{16\,ac^2x^4}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x, algorithm="fricas")
```

output
```
[-1/32*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*x^4*sqrt(e/(a*c))*log(((b^2*c^2
+ 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 -
 4*(2*a^2*c^3 + (a*b*c^2*d + a^2*c*d^2)*x^4 + (a*b*c^3 + 3*a^2*c^2*d)*x^2)
*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(a*c)))/x^4) + 4*((b*c*d - 3*a*d
^2)*x^4 + 2*a*c^2 + (b*c^2 - a*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))
)/(a*c^2*x^4), -1/16*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*x^4*sqrt(-e/(a*c))
*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sq
rt(-e/(a*c))/(b*e*x^2 + a*e)) + 2*((b*c*d - 3*a*d^2)*x^4 + 2*a*c^2 + (b*c^
2 - a*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c^2*x^4)]
```

3.268.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**5,x)
```

output
```
Timed out
```

3.268.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.268.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 549 vs. 2(184) = 368.

Time = 0.45 (sec) , antiderivative size = 549, normalized size of antiderivative = 2.64

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx =$$

$$-\frac{1}{8}\left(\frac{(b^2c^2e + 2\,abcde - 3\,a^2d^2e)\arctan\left(-\frac{\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+ace}}{\sqrt{-ace}}\right)}{\sqrt{-ace}ac^2} - \frac{\left(\sqrt{bde}x^2 - \sqrt{bdex^4 + bcex^2\right.}{}\right.$$

$$+ c)$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x, algorithm="giac")
```

output
```
-1/8*((b^2*c^2*e + 2*a*b*c*d*e - 3*a^2*d^2*e)*arctan(-(sqrt(b*d*e)*x^2 - s
qrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))/sqrt(-a*c*e))/(sqrt(-a*c*e
)*a*c^2) - ((sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*
c*e))*a*b^2*c^3*e^2 + 10*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a
*d*e*x^2 + a*c*e))*a^2*b*c^2*d*e^2 + 5*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 +
 b*c*e*x^2 + a*d*e*x^2 + a*c*e))*a^3*c*d^2*e^2 + 8*sqrt(b*d*e)*a^3*c^2*d*e
^2 + (sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^3
*b^2*c^2*e + 2*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 +
 a*c*e))^3*a*b*c*d*e - 3*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a
*d*e*x^2 + a*c*e))^3*a^2*d^2*e + 8*sqrt(b*d*e)*(sqrt(b*d*e)*x^2 - sqrt(b*d
*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^2*a*b*c^2*e)/((a*c*e - (sqrt(b*d*
e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^2)^2*a*c^2))*sgn
(d*x^2 + c)
```

3.268.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx = \int \frac{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}{x^5}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^5,x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^5, x)
```

3.269 $\int \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{x^7} dx$

3.269.1 Optimal result

Integrand size = 26, antiderivative size = 318

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7} dx = \frac{(bc-ad)^2(bc+3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)\left(b^2c^2+2abcd-11a^2d^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16a^2c^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} + \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} - \frac{(bc-ad)\left(b^2c^2+2abcd+5a^2d^2\right)\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{16a^{5/2}c^{7/2}}$$

output
```
1/6*(-a*d+b*c)^3*e^2*(e*(b*x^2+a)/(d*x^2+c))^(3/2)/a/c^2/(a*e-c*e*(b*x^2+a
)/(d*x^2+c))^3-1/16*(-a*d+b*c)*(5*a^2*d^2+2*a*b*c*d+b^2*c^2)*arctanh(c^(1/
2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2))*e^(1/2)/a^(5/2)/c^(7/2)+
1/8*(-a*d+b*c)^2*(3*a*d+b*c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^3/(a-c*(b*x
^2+a)/(d*x^2+c))^2-1/16*(-a*d+b*c)*(-11*a^2*d^2+2*a*b*c*d+b^2*c^2)*(e*(b*x
^2+a)/(d*x^2+c))^(1/2)/a^2/c^3/(a-c*(b*x^2+a)/(d*x^2+c))
```

3.269. $\int \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{x^7} dx$

3.269.2 Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx$$
$$= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\left(\sqrt{a}\sqrt{c}\sqrt{a+bx^2}\sqrt{c+dx^2}(3b^2c^2x^4-2abcx^2(c-2dx^2)+a^2(-8c^2+10cdx^2-15d^2x^4}{48a^{5/2}c^{7/2}x^6\sqrt{a+bx^2}}$$

input
```
Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^7,x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(Sqrt[a]*Sqrt[c]*Sqrt[a
 + b*x^2]*Sqrt[c + d*x^2]*(3*b^2*c^2*x^4 - 2*a*b*c*x^2*(c - 2*d*x^2) + a^2
*(-8*c^2 + 10*c*d*x^2 - 15*d^2*x^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*
c*d^2 - 5*a^3*d^3)*x^6*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c +
 d*x^2])]))/(48*a^(5/2)*c^(7/2)*x^6*Sqrt[a + b*x^2])
```

3.269.3 Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.84, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2053, 2052, 366, 27, 360, 27, 298, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{x^8}dx^2$$

$$\Big\downarrow \text{2052}$$

$$e(bc-ad)\int \frac{x^4\left(be-dx^4\right)^2}{\left(ae-cx^4\right)^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$

$$\Big\downarrow \text{366}$$

$$e(bc-ad)\left(\frac{ex^6(bc-ad)^2}{6ac^2\left(ae-cx^4\right)^3}-\frac{\int -\frac{3ex^4\left(\left(b^2c^2+2abdc-a^2d^2\right)e-2acd^2x^4\right)}{\left(ae-cx^4\right)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{6ac^2e}\right)$$

$$\Big\downarrow \; 27$$

$$e(bc-ad)\left(\frac{\int \frac{x^4\left(\left(b^2c^2+2abdc-a^2d^2\right)e-2acd^2x^4\right)}{\left(ae-cx^4\right)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{2ac^2}+\frac{ex^6(bc-ad)^2}{6ac^2\left(ae-cx^4\right)^3}\right)$$

$$\Big\downarrow \; 360$$

$$e(bc-ad)\left(\frac{\frac{e(bc-ad)(3ad+bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c(ae-cx^4)^2}-\frac{\int \frac{c\left((bc-ad)(bc+3ad)e-8acd^2x^4\right)}{\left(ae-cx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{4c^2}}{2ac^2}+\frac{ex^6(bc-ad)^2}{6ac^2\left(ae-cx^4\right)^3}\right)$$

$$\Big\downarrow \; 27$$

$$e(bc-ad)\left(\frac{\frac{e(bc-ad)(3ad+bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c(ae-cx^4)^2}-\frac{\int \frac{(bc-ad)(bc+3ad)e-8acd^2x^4}{\left(ae-cx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{4c}}{2ac^2}+\frac{ex^6(bc-ad)^2}{6ac^2\left(ae-cx^4\right)^3}\right)$$

$$\Big\downarrow \; 298$$

$$e(bc-ad)\left(\frac{\frac{e(bc-ad)(3ad+bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c(ae-cx^4)^2}-\frac{\frac{\left(5a^2d^2+2abcd+b^2c^2\right)\int \frac{1}{ae-cx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{2a}+\frac{\left(-11a^2d^2+2abcd+b^2c^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2a\left(ae-cx^4\right)}}{4c}}{2ac^2}+\frac{ex^6(bc-ad)^2}{6ac^2\left(ae-cx^4\right)^3}\right)$$

$$\Big\downarrow \; 221$$

3.269. $\displaystyle\int \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{x^7}dx$

$$e(bc-ad)\left(\frac{\frac{e(bc-ad)(3ad+bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c(ae-cx^4)^2}-\frac{\frac{(-11a^2d^2+2abcd+b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2a(ae-cx^4)}+\frac{(5a^2d^2+2abcd+b^2c^2)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{3/2}\sqrt{c}\sqrt{e}}}{4c}}{2ac^2}+\frac{ex^6(bc-}{6ac^2(ae-}\right.$$

input
```
Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^7,x]
```

output
```
(b*c - a*d)*e*(((b*c - a*d)^2*e*x^6)/(6*a*c^2*(a*e - c*x^4)^3) + (((b*c -
a*d)*(b*c + 3*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*c*(a*e - c*x^4)
^2) - (((b^2*c^2 + 2*a*b*c*d - 11*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2
)])/(2*a*(a*e - c*x^4)) + ((b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt
[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(2*a^(3/2)*Sqrt
[c]*Sqrt[e]))/(4*c))/(2*a*c^2))
```

3.269.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

3.269. $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx$

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.269.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{\left(d\,x^{2}+c\right)\left(15a^{2}d^{2}x^{4}-4bdac\,x^{4}-3b^{2}c^{2}x^{4}-10a^{2}cd\,x^{2}+2ab\,c^{2}x^{2}+8a^{2}c^{2}\right)\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}{48c^{3}x^{6}a^{2}}+\frac{\left(5a^{3}d^{3}-3a^{2}bc\,d^{2}-d\,c^{2}b^{2}a-b^{3}c^{3}\right)\ln\left(\frac{2ac\cdots}{}\right.}{}$
default	$-\frac{\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}\left(d\,x^{2}+c\right)\left(-66\sqrt{ac}\,\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,a^{2}b\,d^{3}x^{8}-24\sqrt{ac}\,\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,a\,b^{2}c\,d^{2}x^{8}-6\sqrt{ac}\,\sqrt{bd\,x^{4}+a\cdots}\right.}{}$

3.269. $\int \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{x^7}\,dx$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(d*x^2+c)*(15*a^2*d^2*x^4-4*a*b*c*d*x^4-3*b^2*c^2*x^4-10*a^2*c*d*x^2
+2*a*b*c^2*x^2+8*a^2*c^2)/c^3/x^6/a^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/32*(
5*a^3*d^3-3*a^2*b*c*d^2-a*b^2*c^2*d-b^3*c^3)/a^2/c^3/(a*c*e)^(1/2)*ln((2*a
*c*e+(a*d*e+b*c*e)*x^2+2*(a*c*e)^(1/2)*(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)
^(1/2))/x^2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)/(
b*x^2+a)
```

3.269.5 Fricas [A] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.76

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx$$

$$= \left[-\frac{3\,(b^3c^3 + ab^2c^2d + 3\,a^2bcd^2 - 5\,a^3d^3)x^6\sqrt{\frac{e}{ac}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)ex^4+8\,a^2c^2e+8\,(abc^2+a^2cd)ex^2+4\,(2\,a^2c^3+(abc^2\cdots}{x^4}\right.}{\cdots}\right.$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x, algorithm="fricas")
```

output
```
[-1/192*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*x^6*sqrt(e/
(a*c))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c
^2 + a^2*c*d)*e*x^2 + 4*(2*a^2*c^3 + (a*b*c^2*d + a^2*c*d^2)*x^4 + (a*b*c^
3 + 3*a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(a*c)))/x^4
) - 4*((3*b^2*c^2*d + 4*a*b*c*d^2 - 15*a^2*d^3)*x^6 - 8*a^2*c^3 + (3*b^2*c
^3 + 2*a*b*c^2*d - 5*a^2*c*d^2)*x^4 - 2*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b
*e*x^2 + a*e)/(d*x^2 + c)))/(a^2*c^3*x^6), 1/96*(3*(b^3*c^3 + a*b^2*c^2*d
+ 3*a^2*b*c*d^2 - 5*a^3*d^3)*x^6*sqrt(-e/(a*c))*arctan(1/2*((b*c + a*d)*x^
2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(a*c))/(b*e*x^2 + a*e
)) + 2*((3*b^2*c^2*d + 4*a*b*c*d^2 - 15*a^2*d^3)*x^6 - 8*a^2*c^3 + (3*b^2*
c^3 + 2*a*b*c^2*d - 5*a^2*c*d^2)*x^4 - 2*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((
b*e*x^2 + a*e)/(d*x^2 + c)))/(a^2*c^3*x^6)]
```

3.269.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**7,x)
```

output
```
Timed out
```

3.269.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.269.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. 2(290) = 580.

Time = 0.41 (sec) , antiderivative size = 1001, normalized size of antiderivative = 3.15

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx$$

$$= \frac{1}{48}\left(\frac{3\left(b^3c^3e + ab^2c^2de + 3a^2bcd^2e - 5a^3d^3e\right)\arctan\left(-\frac{\sqrt{bde}x^2 - \sqrt{bdex^4 + bcex^2 + adex^2 + ace}}{\sqrt{-ace}}\right)}{\sqrt{-ace}a^2c^3} - \frac{3\left(\sqrt{bde}x^2 - \sqrt{\right.}}{}\right.$$

$$+ c)$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x, algorithm="giac")
```

output
```
1/48*(3*(b^3*c^3*e + a*b^2*c^2*d*e + 3*a^2*b*c*d^2*e - 5*a^3*d^3*e)*arctan
(-(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))/sqrt
(-a*c*e))/(sqrt(-a*c*e)*a^2*c^3) - (3*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 +
b*c*e*x^2 + a*d*e*x^2 + a*c*e))*a^2*b^3*c^5*e^3 + 51*(sqrt(b*d*e)*x^2 - sq
rt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))*a^3*b^2*c^4*d*e^3 + 105*(sq
rt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))*a^4*b*c^3
*d^2*e^3 + 33*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 +
a*c*e))*a^5*c^2*d^3*e^3 + 16*sqrt(b*d*e)*a^4*b*c^4*d*e^3 + 48*sqrt(b*d*e)*
a^5*c^3*d^2*e^3 + 8*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*
x^2 + a*c*e))^3*a*b^3*c^4*e^2 + 72*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c
*e*x^2 + a*d*e*x^2 + a*c*e))^3*a^2*b^2*c^3*d*e^2 + 24*(sqrt(b*d*e)*x^2 - s
qrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^3*a^3*b*c^2*d^2*e^2 - 40*(
sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^3*a^4*c
*d^3*e^2 + 48*sqrt(b*d*e)*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 +
a*d*e*x^2 + a*c*e))^2*a^2*b^2*c^4*e^2 + 144*sqrt(b*d*e)*(sqrt(b*d*e)*x^2 -
 sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^2*a^3*b*c^3*d*e^2 - 3*(s
qrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^5*b^3*c^
3*e - 3*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e)
)^5*a*b^2*c^2*d*e - 9*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*
e*x^2 + a*c*e))^5*a^2*b*c*d^2*e + 15*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 ...
```

3.269.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx = \int \frac{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}{x^7}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^7,x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^7, x)
```

3.270 $\int x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

3.270.1 Optimal result

Integrand size = 26, antiderivative size = 357

$$\int x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx=\frac{(8b^2c^2-3abcd-2a^2d^2)\,x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15b^2d^2}-\frac{(4bc-ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{15bd^2}+\frac{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5d}-\frac{\sqrt{c}(8b^2c^2-3abcd-2a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{c^{3/2}(4bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15bd^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

output
```
1/15*(-2*a^2*d^2-3*a*b*c*d+8*b^2*c^2)*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^2/
d^2-1/15*(-a*d+4*b*c)*x*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b/d^2+1/5*
x^3*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d+1/15*c^(3/2)*(-a*d+4*b*c)*(1
/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2
/c)^(1/2),(1-b*c/a/d)^(1/2))*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b/d^(5/2)/(c*(b
*x^2+a)/a/(d*x^2+c))^(1/2)-1/15*(-2*a^2*d^2-3*a*b*c*d+8*b^2*c^2)*(1/(1+d*x
^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/
2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^2/d^(5/2)/(c
*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

3.270.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.32 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.71

$$\int x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$$
$$=\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)(-4bc+ad+3bdx^2)+ic(-8b^2c^2+3abcd+2a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\right.}{15b\sqrt{\frac{b}{a}}}$$

input
```
Integrate[x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*
(-4*b*c + a*d + 3*b*d*x^2) + I*c*(-8*b^2*c^2 + 3*a*b*c*d + 2*a^2*d^2)*Sqrt
[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d
)/(b*c)] - I*c*(-8*b^2*c^2 + 7*a*b*c*d + a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(15*b*Sqr
t[b/a]*d^3*(a + b*x^2))
```

3.270.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2058, 380, 444, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$$
$$\Big\downarrow \; 2058$$
$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\int \frac{x^4\sqrt{bx^2+a}}{\sqrt{dx^2+c}}dx}{\sqrt{a+bx^2}}$$
$$\Big\downarrow \; 380$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\int \frac{x^2\left((4bc-ad)x^2+3ac\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5d}\right)}{\sqrt{a+bx^2}}$$

$\downarrow$ 444

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-ad)}{3bd}-\frac{\int \frac{\left(8b^2c^2-3abdc-2a^2d^2\right)x^2+ac(4bc-ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}}{5d}\right)}{\sqrt{a+bx^2}}$$

$\downarrow$ 406

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-ad)}{3bd}-\frac{\left(-2a^2d^2-3abcd+8b^2c^2\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(4bc-ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{}}}{3bd}}{5d}\right.}{\sqrt{a+bx^2}}$$

$\downarrow$ 320

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-ad)}{3bd}-\frac{\left(-2a^2d^2-3abcd+8b^2c^2\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(4bc-ad)\,\mathrm{E}}{\sqrt{d}\sqrt{c+}}}{3bd}}{5d}\right.}{\sqrt{a+bx^2}}$$

$\downarrow$ 388

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-ad)}{3bd}-\frac{\left(-2a^2d^2-3abcd+8b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+}}{}}{3bd}}{5d}\right.}{\sqrt{a+bx^2}}$$

$\downarrow$ 313

3.270. $\int x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-ad)}{3bd}-\frac{(-2a^2d^2-3abcd+8b^2c^2)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\ldots\right.}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\ldots\right.}{3bd}}{5d}\ldots\right.}{\sqrt{a+bx^2}}$$

input
```
Int[x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*((x^3*Sqrt[a + b*x^2]*S
qrt[c + d*x^2])/(5*d) - (((4*b*c - a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
/(3*b*d) - ((8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*((x*Sqrt[a + b*x^2])/(b*Sq
rt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqr
t[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*
Sqrt[c + d*x^2])) + (c^(3/2)*(4*b*c - a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTa
n[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a
*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b*d))/(5*d)))/Sqrt[a + b*x^2]
```

3.270.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.270. $\int x^4 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

3.270.4 Maple [A] (verified)

Time = 5.21 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.37

method	result
risch	$\frac{x(3bd\,x^2+ad-4bc)(d\,x^2+c)\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{15d^2b} - \left(\frac{a^2cd\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)}{\sqrt{-\frac{b}{a}}\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}} - \frac{4abc^2\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}}\right.}{\sqrt{-\frac{b}{a}}\sqrt{bde\,x^4+ade\,x^2+bce}}\right.$
default	$\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}\,(d\,x^2+c)\left(3\sqrt{-\frac{b}{a}}\,b^2d^3x^7+4\sqrt{-\frac{b}{a}}\,ab\,d^3x^5-\sqrt{-\frac{b}{a}}\,b^2c\,d^2x^5+\sqrt{-\frac{b}{a}}\,a^2d^3x^3-4\sqrt{-\frac{b}{a}}\,b^2c^2d\,x^3+\sqrt{\frac{b\,x^2+a}{a}}\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\right.\right.$

input
```
int(x^4*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(3*b*d*x^2+a*d-4*b*c)*(d*x^2+c)/d^2/b*(e*(b*x^2+a)/(d*x^2+c))^(1/2)
-1/15/b/d^2*(a^2*c*d/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b
*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*
d*e+b*c*e)/c/b/e)^(1/2))-4*a*b*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d
*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)
^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-2*(2*a^2*d^2+3*a*b*c*d-8*b^2*c^2)*a
*c*e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x
^2+b*c*e*x^2+a*c*e)^(1/2)/(e*d*a+e*b*c+e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1
/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*
c*e)/c/b/e)^(1/2))))*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))
^(1/2)/(b*x^2+a)
```

3.270.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.76

$$\int x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx =$$

$$-\frac{(8\,b^2c^3-3\,abc^2d-2\,a^2cd^2)\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(8\,b^2c^3-3\,abc^2d-a^2d^3-2\,(a^2-2\,ab}{}$$

input
```
integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/15*((8*b^2*c^3 - 3*a*b*c^2*d - 2*a^2*c*d^2)*sqrt(b*e/d)*x*sqrt(-c/d)*el
liptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (8*b^2*c^3 - 3*a*b*c^2*d - a^2
*d^3 - 2*(a^2 - 2*a*b)*c*d^2)*sqrt(b*e/d)*x*sqrt(-c/d)*elliptic_f(arcsin(s
qrt(-c/d)/x), a*d/(b*c)) - (3*b^2*d^3*x^6 + 8*b^2*c^3 - 3*a*b*c^2*d - 2*a^
2*c*d^2 - (b^2*c*d^2 - a*b*d^3)*x^4 + 2*(2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3
)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d^3*x)
```

3.270.6 Sympy [F(-1)]

Timed out.

$$\int x^4\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

3.270.7 Maxima [F]

$$\int x^4\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx = \int \sqrt{\frac{(bx^2+a)e}{dx^2+c}}x^4\,dx$$

input
```
integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^4, x)
```

3.270.8 Giac [F]

$$\int x^4\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx = \int \sqrt{\frac{(bx^2+a)e}{dx^2+c}}x^4\,dx$$

input
```
integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^4, x)
```

3.270.9 Mupad [F(-1)]

Timed out.

$$\int x^4 \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx = \int x^4\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

input
```
int(x^4*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

output
```
int(x^4*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.271 $\int x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

3.271.1 Optimal result

Integrand size = 26, antiderivative size = 266

$$\int x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx = -\frac{(2bc-ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3bd}+\frac{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3d}+\frac{\sqrt{c}(2bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3bd^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{c^{3/2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

output
```
-1/3*(-a*d+2*b*c)*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b/d+1/3*x*(d*x^2+c)*(e*(
b*x^2+a)/(d*x^2+c))^(1/2)/d-1/3*c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^
(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(e*
(b*x^2+a)/(d*x^2+c))^(1/2)/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)+1/3*(-a
*d+2*b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1
/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(
1/2)/b/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

3.271.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.78

$$\int x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$$
$$=\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)-ic(-2bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+2ic(-\right.}{3\sqrt{\frac{b}{a}}d^2\left(a+bx^2\right)}$$

input
```
Integrate[x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)
- I*c*(-2*b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2
)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/
(3*Sqrt[b/a]*d^2*(a + b*x^2))
```

3.271.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2058, 380, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$$
$$\Big\downarrow \; 2058$$
$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\int\frac{x^2\sqrt{bx^2+a}}{\sqrt{dx^2+c}}dx}{\sqrt{a+bx^2}}$$
$$\Big\downarrow \; 380$$
$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{\int\frac{(2bc-ad)x^2+ac}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{406}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{ac\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(2bc-ad)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{(2bc-ad)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3d}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{388}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3d}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{313}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\ldots\right.}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right.}{3d}\right.}{\sqrt{a+bx^2}}$$

input
```
Int[x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

3.271. $\int x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*((x*Sqrt[a + b*x^2]*Sqr
t[c + d*x^2])/(3*d) - ((2*b*c - a*d)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^
2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*
x^2])) + (c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d
*x^2]))/(3*d)))/Sqrt[a + b*x^2]
```

3.271.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.271. $\int x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.271.4 Maple [A] (verified)

Time = 4.72 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.32

method	result
risch	$\frac{x(d\,x^2+c)\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{3d}-\frac{\left(\frac{ac\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}}+\frac{2(ad-2bc)ace\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+e}{cbe}}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}\,(edc}\right.}{3d(b\,x^2+a)}$
default	$\frac{\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}\,(d\,x^2+c)\left(\sqrt{-\frac{b}{a}}\,b\,d^2x^5+\sqrt{-\frac{b}{a}}\,a\,d^2x^3+\sqrt{-\frac{b}{a}}\,bcd\,x^3-2ac\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d+2\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\right.}{3\sqrt{(d\,x^2+c)(b\,x^2+a)}\,d^2\sqrt{-\frac{b}{a}}\,\sqrt{}}$

input
```
int(x^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d-1/3/d*(a*c/(-b/a)^(1/2)*(1
+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^
(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))+2*(a*d-2*b*
c)*a*c*e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d
*e*x^2+b*c*e*x^2+a*c*e)^(1/2)/(e*d*a+e*b*c+e*(a*d-b*c))*(EllipticF(x*(-b/a
)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*
e+b*c*e)/c/b/e)^(1/2))))*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2
+a))^(1/2)/(b*x^2+a)
```

3.271.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.68

$$\int x^2\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx$$
$$=\frac{(2\,bc^2-acd)\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(2\,bc^2-acd+ad^2)\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})+}{3\,bd^2x}$$

input
```
integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*((2*b*c^2 - a*c*d)*sqrt(b*e/d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/
d)/x), a*d/(b*c)) - (2*b*c^2 - a*c*d + a*d^2)*sqrt(b*e/d)*x*sqrt(-c/d)*ell
iptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (b*d^2*x^4 - 2*b*c^2 + a*c*d -
(b*c*d - a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*d^2*x)
```

3.271.6 Sympy [F(-1)]

Timed out.

$$\int x^2\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx=\text{Timed out}$$

input
```
integrate(x**2*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

3.271.7 Maxima [F]

$$\int x^2\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx=\int\sqrt{\frac{(bx^2+a)e}{dx^2+c}}x^2\,dx$$

input
```
integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^2, x)
```

3.271.8 Giac [F]

$$\int x^2\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx=\int\sqrt{\frac{(bx^2+a)e}{dx^2+c}}x^2\,dx$$

input
```
integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^2, x)
```

3.271.9 Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx=\int x^2\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

input
```
int(x^2*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

output
```
int(x^2*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.272 $\int \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

3.272.1 Optimal result

Integrand size = 22, antiderivative size = 194

$$\int \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx = x\sqrt{\frac{e(a+bx^2)}{c+dx^2}} - \frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

output
```
x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*El
lipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*
(b*x^2+a)/(d*x^2+c))^(1/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)+(1/(1+d
*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(
1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^(1/2)/(c*(
b*x^2+a)/a/(d*x^2+c))^(1/2)
```

3.272.2 Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.44

$$\int \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx = \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{\frac{c+dx^2}{c}}E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}}$$

input
```
Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sq
rt[-(d/c)]*x], (b*c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a])
```

3.272.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.28, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2058, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{324}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(a\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+b\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(b\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{388}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{313}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{\sqrt{a+bx^2}}$$

input
```
Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(b*((x*Sqrt[a + b*x^2])
/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*
x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x
^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[
d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*
x^2))]*Sqrt[c + d*x^2])))/Sqrt[a + b*x^2]
```

3.272.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.272.4 Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}\,\left(d\,x^{2}+c\right)\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,\left(aF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d-bcF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)+bcE\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\right)}{\sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\,\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,d}$	184

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^
(1/2)*(a*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d-b*c*EllipticF(x*(-b/a
)^(1/2),(a*d/b/c)^(1/2))+b*c*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)))/((
d*x^2+c)*(b*x^2+a))^(1/2)/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
/d
```

3.272.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.77

$$\int\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx =$$
$$-\frac{bc^2\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(bc^2+ad^2)\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(bcdx^2+bc^2)\sqrt{}}{bcdx}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
-(b*c^2*sqrt(b*e/d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c
)) - (b*c^2 + a*d^2)*sqrt(b*e/d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)
/x), a*d/(b*c)) - (b*c*d*x^2 + b*c^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(
b*c*d*x)
```

3.272.6 Sympy [F(-1)]

Timed out.

$$\int \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

3.272.7 Maxima [F]

$$\int \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx = \int \sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

3.272.8 Giac [F]

$$\int \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx = \int \sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

3.272.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx = \int \sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.273 $\int \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{x^2}\,dx$

3.273.1 Optimal result

Integrand size = 26, antiderivative size = 239

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2}\,dx = \frac{dx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{cx} - \frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{b\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

output
```
d*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c-(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2
)/c/x+b*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2
)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/
2)/a/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^
2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2)
)*d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^
(1/2)
```

3.273.2 Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2} dx = \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)\left(-\frac{1}{x}+\frac{b\sqrt{1+\frac{bx^2}{a}}E\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}(a+bx^2)\sqrt{1+\frac{dx^2}{c}}}\right)}{c}$$

input
```
Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^2,x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)*(-x^(-1) + (b*Sqrt[1 + (b*x
^2)/a]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*d)/(b*c)])/(Sqrt[-(b/a)]*(a +
b*x^2)*Sqrt[1 + (d*x^2)/c])))/c
```

3.273.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2058, 377, 27, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2} dx$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\int \frac{\sqrt{bx^2+a}}{x^2\sqrt{dx^2+c}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 377}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{\int \frac{b\sqrt{dx^2+c}}{\sqrt{bx^2+a}}dx}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{b\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}dx}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}\right)}{\sqrt{a+bx^2}}$$

$\downarrow$ 324

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{b\left(c\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+d\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}\right)}{\sqrt{a+bx^2}}$$

$\downarrow$ 320

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{b\left(d\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}\right)}{\sqrt{a+bx^2}}$$

$\downarrow$ 388

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{b\left(d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}\right)}{\sqrt{a+bx^2}}$$

$\downarrow$ 313

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}\right)}{\sqrt{a+bx^2}}$$

input
```
Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^2,x]
```

3.273. $\int\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{x^2}\,dx$

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(-((Sqrt[a + b*x^2]*Sqr
t[c + d*x^2])/(c*x)) + (b*(d*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (S
qrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a
*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) +
 (c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)
/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
))/c))/Sqrt[a + b*x^2]
```

3.273.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

3.273. $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2}\,dx$

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.273.4 Maple [A] (verified)

Time = 4.66 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.80

method	result
default	$-\frac{\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}\,(d\,x^{2}+c)\left(\sqrt{-\frac{b}{a}}\,bd\,x^{4}-bc\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,xE\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)+\sqrt{-\frac{b}{a}}\,ad\,x^{2}+\sqrt{-\frac{b}{a}}\,bc\,x^{2}+\sqrt{-\frac{b}{a}}\,ac\right)}{\sqrt{(d\,x^{2}+c)(b\,x^{2}+a)}\,cx\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}}$
risch	$-\frac{(d\,x^{2}+c)\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}{cx}+\frac{b\left(\frac{c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^{4}+ade\,x^{2}+bce\,x^{2}+ace}}-\frac{2dace\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)-\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^{4}+ade\,x^{2}+bce\,x^{2}+ace}\,(eda+e}\right.}{c(b\,x^{2}+a)}$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*((-b/a)^(1/2)*b*d*x^4-b*c*((b*x^2
+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*x*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2
))+(-b/a)^(1/2)*a*d*x^2+(-b/a)^(1/2)*b*c*x^2+(-b/a)^(1/2)*a*c)/((d*x^2+c)*
(b*x^2+a))^(1/2)/c/x/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

3.273.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2}\,dx$$
$$=\frac{bd\sqrt{\frac{ace}{d^2}}x\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-(a+b)d\sqrt{\frac{ace}{d^2}}x\sqrt{-\frac{b}{a}}F(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-(adx^2+ac)\sqrt{\frac{be}{c}}}{acx}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
(b*d*sqrt(a*c*e/d^2)*x*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*
c)) - (a + b)*d*sqrt(a*c*e/d^2)*x*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a
)), a*d/(b*c)) - (a*d*x^2 + a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c*x
)
```

3.273.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**2,x)
```

output
```
Timed out
```

3.273.7 Maxima [F]

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2}\,dx = \int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^2}\,dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^2, x)
```

3.273.8 Giac [F]

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2}\,dx = \int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^2}\,dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^2, x)
```

3.273.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2}\,dx = \int \frac{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}{x^2}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^2,x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^2, x)
```

3.274 $\int \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{x^4}\,dx$

3.274.1 Optimal result

Integrand size = 26, antiderivative size = 321

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4}\,dx = \frac{d(bc-2ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3ac^2} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3cx^3} - \frac{(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3ac^2x} - \frac{\sqrt{d}(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ac^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{b\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

output
```
1/3*d*(-2*a*d+b*c)*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^2-1/3*(d*x^2+c)*(e*
(b*x^2+a)/(d*x^2+c))^(1/2)/c/x^3-1/3*(-2*a*d+b*c)*(d*x^2+c)*(e*(b*x^2+a)/(
d*x^2+c))^(1/2)/a/c^2/x-1/3*(-2*a*d+b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)
^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^
(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1
/2)-1/3*b*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1
/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(
1/2)/a/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

3.274.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.44 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4}\,dx = -\frac{\sqrt{\frac{b}{a}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}(a+bx^2)(c+dx^2)(bcx^2+a(c-2dx^2))-ibc(-bc+2ad)x^3\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(\right.\right.}{3bc^2x^3\left(a+b\right.}$$

input
```
Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^4,x]
```

output
```
-1/3*(Sqrt[b/a]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*(a + b*x^2)*(
c + d*x^2)*(b*c*x^2 + a*(c - 2*d*x^2)) - I*b*c*(-(b*c) + 2*a*d)*x^3*Sqrt[1
 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/
(b*c)] + I*b*c*(-(b*c) + a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*
EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(b*c^2*x^3*(a + b*x^2))
```

3.274.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2058, 377, 445, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\int \frac{\sqrt{bx^2+a}}{x^4\sqrt{dx^2+c}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{377}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{\int \frac{-bdx^2+bc-2ad}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 445$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{-\frac{\int \frac{bd\left(ac-(bc-2ad)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{acx}}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{-\frac{bd\int \frac{ac-(bc-2ad)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{acx}}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 406$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{-\frac{bd\left(ac\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-(bc-2ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{acx}}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 320$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{-\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-(bc-2ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{acx}}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 388$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{-\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{acx}}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}\right)}{\sqrt{a+bx^2}}$$

3.274. $\displaystyle \int \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{x^4}\,dx$

$\Big\downarrow$ 313

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{-\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{ac}-\frac{\sqrt{a+bx}\cdots}{\cdots}}{3c}\cdots\right)}{\sqrt{a+bx^2}}$$

input
```
Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^4,x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(-1/3*(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2])/(c*x^3) + (-(((b*c - 2*a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^
2])/(a*c*x)) - (b*d*(-((b*c - 2*a*d)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^
2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*
x^2]))) + (c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
d*x^2])))/(a*c))/(3*c)))/Sqrt[a + b*x^2]
```

3.274.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

3.274. $\int \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{x^4}\,dx$

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.274. $\int \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{x^4}\,dx$

3.274.4 Maple [A] (verified)

Time = 5.43 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.18

method	result
risch	$-\frac{(dx^2+c)(-2adx^2+bcx^2+ac)\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{3c^2x^3a}-\frac{db\left(\frac{ac\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)}{\sqrt{-\frac{b}{a}}\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}}-\frac{2(2ad-bc)ace\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx}{c}}}{\sqrt{-\frac{b}{a}}\sqrt{bde}}\right.}{3}$
default	$-\frac{\sqrt{\frac{e(bx^2+a)}{dx^2+c}}\,(dx^2+c)\left(-2\sqrt{-\frac{b}{a}}\,abd^2x^6+\sqrt{-\frac{b}{a}}\,b^2cdx^6-bd\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)x^3ac+\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\,F\left(x\right.\right.}{}$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(d*x^2+c)*(-2*a*d*x^2+b*c*x^2+a*c)/c^2/x^3/a*(e*(b*x^2+a)/(d*x^2+c))^
(1/2)-1/3*d/a*b/c^2*(a*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2
)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1
+(a*d*e+b*c*e)/c/b/e)^(1/2))-2*(2*a*d-b*c)*a*c*e/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)/(e*d
*a+e*b*c+e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(
1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))))*(e*(b*x^2
+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)/(b*x^2+a)
```

3.274.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4}\,dx$$
$$=\frac{(b^2cd-2\,abd^2)\sqrt{\frac{ace}{d^2}}x^3\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-(b^2cd-(a^2+2\,ab)d^2)\sqrt{\frac{ace}{d^2}}x^3\sqrt{-\frac{b}{a}}F(\arcsin\left(x\sqrt{\right.\right.}{3\,a^2c^2x^3}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^4,x, algorithm="fricas")
```

output
```
1/3*((b^2*c*d - 2*a*b*d^2)*sqrt(a*c*e/d^2)*x^3*sqrt(-b/a)*elliptic_e(arcsi
n(x*sqrt(-b/a)), a*d/(b*c)) - (b^2*c*d - (a^2 + 2*a*b)*d^2)*sqrt(a*c*e/d^2
)*x^3*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((a*b*c*d -
 2*a^2*d^2)*x^4 + a^2*c^2 + (a*b*c^2 - a^2*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/
(d*x^2 + c)))/(a^2*c^2*x^3)
```

3.274.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**4,x)
```

output
```
Timed out
```

3.274.7 Maxima [F]

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4}\,dx = \int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^4}\,dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^4, x)
```

3.274.8 Giac [F]

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4}\,dx = \int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^4}\,dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^4, x)
```

3.274.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4}\,dx = \int \frac{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}{x^4}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^4,x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^4, x)
```

3.275 $\int \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{x^6}\,dx$

3.275.1 Optimal result

Integrand size = 26, antiderivative size = 424

$$\begin{aligned}\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6}\,dx =& -\frac{d(2b^2c^2+3abcd-8a^2d^2)\,x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15a^2c^3} \\ & -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5cx^5}-\frac{(bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{15ac^2x^3} \\ & +\frac{(2b^2c^2+3abcd-8a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{15a^2c^3x} \\ & +\frac{\sqrt{d}(2b^2c^2+3abcd-8a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15a^2c^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} \\ & -\frac{b\sqrt{d}(bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15a^2c^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\end{aligned}$$

output
```
-1/15*d*(-8*a^2*d^2+3*a*b*c*d+2*b^2*c^2)*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a
^2/c^3-1/5*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c/x^5-1/15*(-4*a*d+b*c)
*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^2/x^3+1/15*(-8*a^2*d^2+3*a*b*
c*d+2*b^2*c^2)*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^2/c^3/x+1/15*(-8*
a^2*d^2+3*a*b*c*d+2*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellip
ticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(e*(b*
x^2+a)/(d*x^2+c))^(1/2)/a^2/c^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)-1/15*b
*(-4*a*d+b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/
c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(e*(b*x^2+a)/(d*x^2+c
))^(1/2)/a^2/c^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

3.275.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.79 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6}\,dx =$$
$$-\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}(a+bx^2)\,(c+dx^2)\,(-2b^2c^2x^4+abcx^2(c-3dx^2)+a^2(3c^2-4cdx^2+8d^2x^4))+ibc(-2b\right.}{}$$

input
```
Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^6,x]
```

output
```
-1/15*(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2
)*(-2*b^2*c^2*x^4 + a*b*c*x^2*(c - 3*d*x^2) + a^2*(3*c^2 - 4*c*d*x^2 + 8*d
^2*x^4)) + I*b*c*(-2*b^2*c^2 - 3*a*b*c*d + 8*a^2*d^2)*x^5*Sqrt[1 + (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (
2*I)*b*c*(-(b^2*c^2) - a*b*c*d + 2*a^2*d^2)*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1
 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(a^2*Sqrt[b
/a]*c^3*x^5*(a + b*x^2))
```

3.275.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2058, 377, 445, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6} \, dx$$

$$\Big\downarrow \; 2058$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}} \int \frac{\sqrt{bx^2+a}}{x^6\sqrt{dx^2+c}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 377$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{\int \frac{-3bdx^2+bc-4ad}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{5cx^5}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 445$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{-\frac{\int \frac{2b^2c^2+3abdc-8a^2d^2+bd(bc-4ad)x^2}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-4ad)}{3acx^3}}{5c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{5cx^5}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 445$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{-\frac{-\frac{\int -\frac{bd\left(\left(2b^2c^2+3abdc-8a^2d^2\right)x^2+ac(bc-4ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2c}{a}-\frac{8ad^2}{c}+3bd\right)}{x}}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-4ad)}{3acx^3}}{5c}- \dots\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{-\frac{\frac{\int \frac{bd\left(\left(2b^2c^2+3abdc-8a^2d^2\right)x^2+ac(bc-4ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2c}{a}-\frac{8ad^2}{c}+3bd\right)}{x}}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-4ad)}{3acx^3}}{5c}-\sqrt{a\dots}\right)}{\sqrt{a+bx^2}}$$

3.275. $\int \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{x^6} \, dx$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{\frac{bd\int\frac{(2b^2c^2+3abdc-8a^2d^2)x^2+ac(bc-4ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2c}{a}-\frac{8ad^2}{c}+3bd\right)}{x}}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-4ad)}{3acx^3}}{5c}-\sqrt{a+\cdots}\right.}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 406$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{\frac{bd\left((-8a^2d^2+3abcd+2b^2c^2)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(bc-4ad)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2c}{a}-\frac{8ad^2}{c}+3bd\cdots\right)}{x}}{3ac}\cdots}{5c}\right.}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 320$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{\frac{bd\left((-8a^2d^2+3abcd+2b^2c^2)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(bc-4ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+\cdots}}{\cdots}}{3ac}\cdots}{5c}\right.}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 388$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{\frac{bd\left((-8a^2d^2+3abcd+2b^2c^2)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(bc-4ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{ac}-\cdots}{3ac}\cdots}{5c}\right.}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 313$$

3.275. $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6}\,dx$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{\frac{bd\left(\left(-8a^2d^2+3abcd+2b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(bc-4ad)\,\text{EllipticF}\left(\arctan\left(\cdots\right.\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\cdots}{ac}}{3ac}}{5c}\cdots\right.}{\sqrt{a+bx^2}}$$

input
```
Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^6,x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(-1/5*(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2])/(c*x^5) + (-1/3*((b*c - 4*a*d)*Sqrt[a + b*x^2]*Sqrt[c + d
*x^2])/(a*c*x^3) - (-((((2*b^2*c)/a + 3*b*d - (8*a*d^2)/c)*Sqrt[a + b*x^2]
*Sqrt[c + d*x^2])/x) + (b*d*((2*b^2*c^2 + 3*a*b*c*d - 8*a^2*d^2)*((x*Sqrt[
a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTa
n[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/
(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(b*c - 4*a*d)*Sqrt[a + b*x^2
]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(a*c))/(3*a*c))/(5*c)))
/Sqrt[a + b*x^2]
```

3.275.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

3.275. $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6}\,dx$

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.275. $\int \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{x^6}\,dx$

3.275.4 Maple [A] (verified)

Time = 6.59 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.27

method	result
risch	$-\frac{(d\,x^2+c)\left(8a^2d^2x^4-3bdac\,x^4-2b^2c^2x^4-4a^2cd\,x^2+ab\,c^2x^2+3a^2c^2\right)\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}{15c^3x^5a^2}+\frac{bd\left(\frac{4a^2cd\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\cdots}\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace\cdots}}\right.}{\cdots}$
default	$-\frac{\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,(d\,x^2+c)\left(8\sqrt{-\frac{b}{a}}\,a^2b\,d^3x^8-3\sqrt{-\frac{b}{a}}\,a\,b^2c\,d^2x^8-2\sqrt{-\frac{b}{a}}\,b^3c^2d\,x^8+4\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a^2bc\,d^2x^5-\cdots\right.}{\cdots}$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(d*x^2+c)*(8*a^2*d^2*x^4-3*a*b*c*d*x^4-2*b^2*c^2*x^4-4*a^2*c*d*x^2+a
*b*c^2*x^2+3*a^2*c^2)/c^3/x^5/a^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/15/a^2*b
*d/c^3*(4*a^2*c*d/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*
e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e
+b*c*e)/c/b/e)^(1/2))-a*b*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)
^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2
),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-2*(8*a^2*d^2-3*a*b*c*d-2*b^2*c^2)*a*c*e/
(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*
c*e*x^2+a*c*e)^(1/2)/(e*d*a+e*b*c+e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1/2),(
-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/
c/b/e)^(1/2))))*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2
)/(b*x^2+a)
```

3.275.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6}\,dx = -\frac{\left(2\,b^3c^2d+3\,ab^2cd^2-8\,a^2bd^3\right)\sqrt{\frac{ace}{d^2}}x^5\sqrt{-\frac{b}{a}}E\left(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc}\right)-\left(2\,b^3c^2d+(a^2b+3\,ab^2)cd^2-4\,(a\cdots\right.}{\cdots}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^6,x, algorithm="fricas")
```

output
```
-1/15*((2*b^3*c^2*d + 3*a*b^2*c*d^2 - 8*a^2*b*d^3)*sqrt(a*c*e/d^2)*x^5*sqr
t(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (2*b^3*c^2*d + (a^2*
b + 3*a*b^2)*c*d^2 - 4*(a^3 + 2*a^2*b)*d^3)*sqrt(a*c*e/d^2)*x^5*sqrt(-b/a)
*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((2*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - 8*a^3*d^3)*x^6 - 3*a^3*c^3 + 2*(a*b^2*c^3 + a^2*b*c^2*d - 2*a^3*c*d
^2)*x^4 - (a^2*b*c^3 - a^3*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/
(a^3*c^3*x^5)
```

3.275.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**6,x)
```

output
```
Timed out
```

3.275.7 Maxima [F]

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6}\,dx = \int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^6}\,dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^6,x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^6, x)
```

3.275.8 Giac [F]

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6}\,dx = \int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^6}\,dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^6,x, algorithm="giac")
```

output
```
integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^6, x)
```

3.275.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6}\,dx = \int \frac{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}{x^6}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^6,x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^6, x)
```

3.276 $\int x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

3.276.1 Optimal result

Integrand size = 26, antiderivative size = 282

$$\int x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \frac{c^2(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d^4} + \frac{(79b^2c^2-50abcd-5a^2d^2)\,e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{48bd^4} - \frac{(11bc+ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{24d^4} + \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}(c+dx^2)^3}{6bd^2e} - \frac{(bc-ad)\,(35b^2c^2-10abcd-a^2d^2)\,e^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{3/2}d^{9/2}}$$

output
```
1/6*(e*(b*x^2+a)/(d*x^2+c))^(5/2)*(d*x^2+c)^3/b/d^2/e-1/16*(-a*d+b*c)*(-a^
2*d^2-10*a*b*c*d+35*b^2*c^2)*e^(3/2)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c
))^(1/2)/b^(1/2)/e^(1/2))/b^(3/2)/d^(9/2)+c^2*(-a*d+b*c)*e*(e*(b*x^2+a)/(d
*x^2+c))^(1/2)/d^4+1/48*(-5*a^2*d^2-50*a*b*c*d+79*b^2*c^2)*e*(d*x^2+c)*(e*
(b*x^2+a)/(d*x^2+c))^(1/2)/b/d^4-1/24*(a*d+11*b*c)*e*(d*x^2+c)^2*(e*(b*x^2
+a)/(d*x^2+c))^(1/2)/d^4
```

3.276.2 Mathematica [A] (verified)

Time = 4.54 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.79

$$\int x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = \frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(b\sqrt{d}(3a^2d^2(c+dx^2)+2abd(-50c^2-19cdx^2+7d^2x^4)+b^2(105c^3\right.}{}$$

input
```
Integrate[x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(b*Sqrt[d]*(3*a^2*d^2*(c + d*x^2) + 2
*a*b*d*(-50*c^2 - 19*c*d*x^2 + 7*d^2*x^4) + b^2*(105*c^3 + 35*c^2*d*x^2 -
14*c*d^2*x^4 + 8*d^3*x^6)) - (3*(b*c - a*d)^(3/2)*(35*b^2*c^2 - 10*a*b*c*d
 - a^2*d^2)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*
x^2])/Sqrt[b*c - a*d]])/Sqrt[a + b*x^2]))/(48*b^2*d^(9/2))
```

3.276.3 Rubi [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2053, 2052, 366, 25, 25, 27, 360, 25, 1471, 27, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int x^4\left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}dx^2$$

$$\Big\downarrow \text{2052}$$

$$e(bc-ad)\int \frac{x^8(ae-cx^4)^2}{(be-dx^4)^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}$$

$$\Big\downarrow \text{366}$$

$$e(bc-ad)\left(\frac{ex^{10}(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\int -\frac{x^8\left(-6bc^2dex^4+6a^2d^2e^2-5(bce-ade)^2\right)}{\left(be-dx^4\right)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{6bd^2e}\right)$$

$$\Big\downarrow \; 25$$

$$e(bc-ad)\left(\frac{\int -\frac{ex^8\left(6bc^2dx^4+\left(5b^2c^2-10abdc-a^2d^2\right)e\right)}{\left(be-dx^4\right)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{6bd^2e}+\frac{ex^{10}(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}\right)$$

$$\Big\downarrow \; 25$$

$$e(bc-ad)\left(\frac{ex^{10}(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\int \frac{ex^8\left(6bc^2dx^4+\left(5b^2c^2-10abdc-a^2d^2\right)e\right)}{\left(be-dx^4\right)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{6bd^2e}\right)$$

$$\Big\downarrow \; 27$$

$$e(bc-ad)\left(\frac{ex^{10}(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\int \frac{x^8\left(6bc^2dx^4+\left(5b^2c^2-10abdc-a^2d^2\right)e\right)}{\left(be-dx^4\right)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{6bd^2}\right)$$

$$\Big\downarrow \; 360$$

$$e(bc-ad)\left(\frac{ex^{10}(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\frac{\int -\frac{24bc^2d^3x^8+4d^2(bc-ad)(11bc+ad)ex^4+bd(bc-ad)(11bc+ad)e^2}{\left(be-dx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{4d^3}+\frac{be^2(bc-ad)(ad+11bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4d^2\left(be-dx^4\right)^2}}{6bd^2}\right)$$

$$\Big\downarrow \; 25$$

$$e(bc-ad)\left(\frac{ex^{10}(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\frac{be^2(bc-ad)(ad+11bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4d^2\left(be-dx^4\right)^2}-\frac{\int \frac{24bc^2d^3x^8+4d^2(bc-ad)(11bc+ad)ex^4+bd(bc-ad)(11bc+ad)e^2}{\left(be-dx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{4d^3}}{6bd^2}\right)$$

$$\Big\downarrow \; 1471$$

3.276. $\int x^5\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}dx$

$$e(bc-ad)\left(\frac{ex^{10}(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\frac{be^2(bc-ad)(ad+11bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4d^2\left(be-dx^4\right)^2}-\frac{\frac{de\left(-5a^2d^2-50abcd+79b^2c^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2\left(be-dx^4\right)}-\frac{\int\frac{3bde\left(16bc^2dx^4+\left(19b^2c^2-10abdc-\right.\right.}{be-dx^4}}{2be}}{4d^3}}{6bd^2}\right.$$

$\downarrow$ 27

$$e(bc-ad)\left(\frac{ex^{10}(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\frac{be^2(bc-ad)(ad+11bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4d^2\left(be-dx^4\right)^2}-\frac{\frac{de\left(-5a^2d^2-50abcd+79b^2c^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2\left(be-dx^4\right)}-\frac{3}{2}d\int\frac{16bc^2dx^4+\left(19b^2c^2-10abdc-a\right.}{be-dx^4}}{4d^3}}{6bd^2}\right.$$

$\downarrow$ 299

$$e(bc-ad)\left(\frac{ex^{10}(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\frac{be^2(bc-ad)(ad+11bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4d^2\left(be-dx^4\right)^2}-\frac{\frac{de\left(-5a^2d^2-50abcd+79b^2c^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2\left(be-dx^4\right)}-\frac{3}{2}d\left(e\left(-a^2d^2-10abcd+35b^2c^2\right)\int\right.}{4d^3}}{6bd^2}\right.$$

$\downarrow$ 221

$$e(bc-ad)\left(\frac{ex^{10}(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\frac{be^2(bc-ad)(ad+11bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4d^2\left(be-dx^4\right)^2}-\frac{\frac{de\left(-5a^2d^2-50abcd+79b^2c^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2\left(be-dx^4\right)}-\frac{3}{2}d\left(\frac{\sqrt{e}\left(-a^2d^2-10abcd+35b^2c^2\right)\mathrm{a}}{\sqrt{b}}\right.}{4d^3}}{6bd^2}\right.$$

3.276. $\int x^5\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}dx$

input
```
Int[x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(b*c - a*d)*e*(((b*c - a*d)^2*e*x^10)/(6*b*d^2*(b*e - d*x^4)^3) - ((b*(b*c
 - a*d)*(11*b*c + a*d)*e^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*d^2*(b*e
- d*x^4)^2) - ((d*(79*b^2*c^2 - 50*a*b*c*d - 5*a^2*d^2)*e*Sqrt[(e*(a + b*x
^2))/(c + d*x^2)])/(2*(b*e - d*x^4)) - (3*d*(-16*b*c^2*Sqrt[(e*(a + b*x^2)
)/(c + d*x^2)] + ((35*b^2*c^2 - 10*a*b*c*d - a^2*d^2)*Sqrt[e]*ArcTanh[(Sqr
t[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(Sqrt[b]*Sqrt[
d]))/2)/(4*d^3))/(6*b*d^2))
```

3.276.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.276.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.11

method	result
risch	$\frac{\left(8b^2d^2x^4+14ab\,d^2x^2-22b^2cd\,x^2+3a^2d^2-52abcd+57b^2c^2\right)\left(d\,x^2+c\right)e\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}{48b\,d^4}-\frac{\left(\frac{\left(a^2d^2+10abcd-35b^2c^2\right)(ad-bc)\ln\left(\frac{\frac{1}{2}eda+\frac{1}{2}eb\cdots}{\sqrt{bd\cdots}}\right)}{2\sqrt{bde}}\cdots\right.}{\cdots}$
default	Expression too large to display

input
```
int(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/b*(8*b^2*d^2*x^4+14*a*b*d^2*x^2-22*b^2*c*d*x^2+3*a^2*d^2-52*a*b*c*d+5
7*b^2*c^2)*(d*x^2+c)/d^4*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/16/b/d^4*(1/2*(
a^2*d^2+10*a*b*c*d-35*b^2*c^2)*(a*d-b*c)*ln((1/2*e*d*a+1/2*e*b*c+b*d*e*x^2
)/(b*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)+1
6*c^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)*b*(b*x^2+a)/(a*d-b*c)/(b*d*e*x^4+a*d*e*x
^2+b*c*e*x^2+a*c*e)^(1/2))*e/(b*x^2+a)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x
^2+c)*e*(b*x^2+a))^(1/2)
```

3.276.5 Fricas [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.96

$$\int x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \left[\frac{3\,(35\,b^3c^3 - 45\,ab^2c^2d + 9\,a^2bcd^2 + a^3d^3)e\sqrt{\frac{e}{bd}}\log\left(8\,b^2d^2ex^4 + 8\,(b^2cd + abd^2\right.}{}\right.$$

input
```
integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/192*(3*(35*b^3*c^3 - 45*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e*sqrt(e
/(b*d))*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a
*b*c*d + a^2*d^2)*e - 4*(2*b^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*
d^2 + a*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) + 4*(
8*b^2*d^3*e*x^6 - 14*(b^2*c*d^2 - a*b*d^3)*e*x^4 + (35*b^2*c^2*d - 38*a*b*
c*d^2 + 3*a^2*d^3)*e*x^2 + (105*b^2*c^3 - 100*a*b*c^2*d + 3*a^2*c*d^2)*e)*
sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*d^4), 1/96*(3*(35*b^3*c^3 - 45*a*b^2
*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2 +
 b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d))/(b*e*x^2 + a*
e)) + 2*(8*b^2*d^3*e*x^6 - 14*(b^2*c*d^2 - a*b*d^3)*e*x^4 + (35*b^2*c^2*d
- 38*a*b*c*d^2 + 3*a^2*d^3)*e*x^2 + (105*b^2*c^3 - 100*a*b*c^2*d + 3*a^2*c
*d^2)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*d^4)]
```

3.276.6 Sympy [F(-1)]

Timed out.

$$\int x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate(x**5*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

3.276.7 Maxima [F(-2)]

Exception generated.

$$\int x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.276.8 Giac [F(-2)]

Exception generated.

$$\int x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{2,[0,5,0]%%%},[2,0,0,0]%%%}+%%%{%%{[%%%{-4,[0,4,0]%%%}
,0]:[1,0,
```

3.276.9 Mupad [F(-1)]

Timed out.

$$\int x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx=\int x^5\,\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}\,dx$$

input
```
int(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

output
```
int(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.277 $\int x^3 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

3.277.1 Optimal result

Integrand size = 26, antiderivative size = 199

$$\int x^3 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = -\frac{c(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d^3} - \frac{(9bc-5ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{8d^3} + \frac{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{4d^3} + \frac{3(bc-ad)(5bc-ad)e^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8\sqrt{b}d^{7/2}}$$

output
```
3/8*(-a*d+b*c)*(-a*d+5*b*c)*e^(3/2)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c)
)^(1/2)/b^(1/2)/e^(1/2))/d^(7/2)/b^(1/2)-c*(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^
2+c))^(1/2)/d^3-1/8*(-5*a*d+9*b*c)*e*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/
2)/d^3+1/4*b*e*(d*x^2+c)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^3
```

3.277.2 Mathematica [A] (verified)

Time = 2.85 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.89

$$\int x^3 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{b}\sqrt{d}\sqrt{a+bx^2}(ad(13c+5dx^2)+b(-15c^2-5cdx^2+2d^2x^4))+3(\right.}{8\sqrt{b}d^{7/2}\sqrt{a+bx^2}}$$

input
```
Integrate[x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b]*Sqrt[d]*Sqrt[a + b*x^2]*(a*d
*(13*c + 5*d*x^2) + b*(-15*c^2 - 5*c*d*x^2 + 2*d^2*x^4)) + 3*(5*b^2*c^2 -
6*a*b*c*d + a^2*d^2)*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/(Sq
rt[d]*Sqrt[a + b*x^2])]))/(8*Sqrt[b]*d^(7/2)*Sqrt[a + b*x^2])
```

3.277.3 Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2053, 2052, 25, 360, 1471, 27, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int x^2\left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}dx^2$$

$$\Big\downarrow \text{2052}$$

$$e(bc-ad)\int -\frac{x^8(ae-cx^4)}{(be-dx^4)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$-\left(e(bc-ad)\int \frac{x^8(ae-cx^4)}{(be-dx^4)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}\right)$$

$$\Big\downarrow \text{360}$$

$$e(bc-ad)\left(\frac{be^2(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^3(be-dx^4)^2}-\frac{\int \frac{4cd^2x^8+4d(bc-ad)ex^4+b(bc-ad)e^2}{(be-dx^4)^2}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{4d^3}\right)$$

$$\Big\downarrow \text{1471}$$

$$e(bc-ad)\left(\frac{be^2(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^3(be-dx^4)^2}-\frac{\frac{e(9bc-5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2(be-dx^4)}-\frac{\int \frac{be\left(8cdx^4+(7bc-3ad)e\right)}{be-dx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{2be}}{4d^3}\right)$$

$\Big\downarrow$ 27

$$e(bc-ad)\left(\frac{be^2(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^3\left(be-dx^4\right)^2}-\frac{\frac{e(9bc-5ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2(be-dx^4)}-\frac{1}{2}\int\frac{8cdx^4+(7bc-3ad)e}{be-dx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{4d^3}\right)$$

$\Big\downarrow$ 299

$$e(bc-ad)\left(\frac{be^2(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^3\left(be-dx^4\right)^2}-\frac{\frac{1}{2}\left(8c\sqrt{\frac{e(a+bx^2)}{c+dx^2}}-3e(5bc-ad)\int\frac{1}{be-dx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}\right)+\frac{e(9bc-5ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2(be-dx^4)}}{4d^3}\right)$$

$\Big\downarrow$ 221

$$e(bc-ad)\left(\frac{be^2(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^3\left(be-dx^4\right)^2}-\frac{\frac{1}{2}\left(8c\sqrt{\frac{e(a+bx^2)}{c+dx^2}}-\frac{3\sqrt{e}(5bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{b}\sqrt{d}}\right)+\frac{e(9bc-5ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2(be-dx^4)}}{4d^3}\right)$$

input
```
Int[x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(b*c - a*d)*e*((b*(b*c - a*d)*e^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*d^
3*(b*e - d*x^4)^2) - (((9*b*c - 5*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]
)/(2*(b*e - d*x^4)) + (8*c*Sqrt[(e*(a + b*x^2))/(c + d*x^2)] - (3*(5*b*c -
 a*d)*Sqrt[e]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]
*Sqrt[e])])/(Sqrt[b]*Sqrt[d]))/2)/(4*d^3))
```

3.277. $\int x^3\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}dx$

3.277.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

3.277. $\int x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx$

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.277.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.27

method	result
risch	$\frac{\left(2bd\,x^{2}+5ad-7bc\right)\left(d\,x^{2}+c\right)e\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}{8d^{3}}+\frac{\left(\frac{3(ad-5bc)(ad-bc)\ln\left(\frac{\frac{1}{2}eda+\frac{1}{2}ebc+bde\,x^{2}}{\sqrt{bde}}+\sqrt{bde\,x^{4}+(eda+ebc)x^{2}+ace}\right)}{2\sqrt{bde}}+\frac{8c\left(a^{2}d^{2}-2a\right.}{(ad-bc)\sqrt{bde\,x}}\right.}{8d^{3}(b\,x^{2}+a)}$
default	$\frac{\left(4\sqrt{bd}\,\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,b\,d^{2}x^{4}+3\ln\left(\frac{2bd\,x^{2}+2\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)a^{2}d^{3}x^{2}-18\ln\left(\frac{2bd\,x^{2}+2\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}}}{2\sqrt{bd}}\right.\right.}{}$

input
```
int(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*b*d*x^2+5*a*d-7*b*c)*(d*x^2+c)/d^3*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)+
1/8/d^3*(3/2*(a*d-5*b*c)*(a*d-b*c)*ln((1/2*e*d*a+1/2*e*b*c+b*d*e*x^2)/(b*d
*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)+8*c*(a^
2*d^2-2*a*b*c*d+b^2*c^2)*(b*x^2+a)/(a*d-b*c)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^
2+a*c*e)^(1/2))*e/(b*x^2+a)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*
x^2+a))^(1/2)
```

3.277.5 Fricas [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.10

$$\int x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx = \left[\frac{3\,(5\,b^2c^2-6\,abcd+a^2d^2)e\sqrt{\frac{e}{bd}}\log\left(8\,b^2d^2ex^4+8\,(b^2cd+abd^2)ex^2+(b^2c^2+\cdots\right.}{\cdots}\right.$$

$$-\frac{3\,(5\,b^2c^2-6\,abcd+a^2d^2)e\sqrt{-\frac{e}{bd}}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{e}{bd}}}{2\,(bex^2+ae)}\right)-2\,(2\,bd^2ex^4-5\,(bcd-ad^2)ex^2-\cdots}{16\,d^3}$$

input
```
integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/32*(3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*e*sqrt(e/(b*d))*log(8*b^2*d^2*e
*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4
*(2*b^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*d^2 + a*b*d^3)*x^2)*sqr
t((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) + 4*(2*b*d^2*e*x^4 - 5*(b*c*
d - a*d^2)*e*x^2 - (15*b*c^2 - 13*a*c*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 +
c)))/d^3, -1/16*(3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*e*sqrt(-e/(b*d))*arct
an(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(
b*d))/(b*e*x^2 + a*e)) - 2*(2*b*d^2*e*x^4 - 5*(b*c*d - a*d^2)*e*x^2 - (15*
b*c^2 - 13*a*c*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/d^3]
```

3.277.6 Sympy [F(-1)]

Timed out.

$$\int x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

3.277.7 Maxima [F(-2)]

Exception generated.

$$\int x^3 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.277.8 Giac [F(-2)]

Exception generated.

$$\int x^3 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{2,[0,4,0]%%%},[2,0,0,0]%%%}+%%%{%%{[%%%{-4,[0,3,0]%%%}
,0]:[1,0,
```

3.277.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \int x^3 \left(\frac{e(b\,x^2+a)}{d\,x^2+c}\right)^{3/2} dx$$

input
```
int(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

output
```
int(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.278 $\int x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx$

3.278.1 Optimal result

Integrand size = 24, antiderivative size = 141

$$\int x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx=\frac{3(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2d^2}+\frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}(c+dx^2)}{2d}-\frac{3\sqrt{b}(bc-ad)e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2d^{5/2}}$$

output
```
1/2*(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)/d-3/2*(-a*d+b*c)*e^(3/2)*arcta
nh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2))*b^(1/2)/d^(5/2)+
3/2*(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^2
```

3.278.2 Mathematica [A] (verified)

Time = 1.89 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.96

$$\int x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx=\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{d}\sqrt{a+bx^2}(3bc-2ad+bdx^2)-3\sqrt{b}(bc-ad)\sqrt{c+dx^2}\operatorname{arctanh}\left(\right.\right.}{2d^{5/2}\sqrt{a+bx^2}}$$

input
```
Integrate[x*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[d]*Sqrt[a + b*x^2]*(3*b*c - 2*a
*d + b*d*x^2) - 3*Sqrt[b]*(b*c - a*d)*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[d]*Sqr
t[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]))/(2*d^(5/2)*Sqrt[a + b*x^2])
```

3.278.3 Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2053, 2051, 252, 262, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx$$

$$\Big\downarrow \; 2053$$

$$\frac{1}{2}\int \left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}dx^2$$

$$\Big\downarrow \; 2051$$

$$e(bc-ad)\int \frac{x^8}{(be-dx^4)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$

$$\Big\downarrow \; 252$$

$$e(bc-ad)\left(\frac{x^6}{2d\left(be-dx^4\right)}-\frac{3\int \frac{x^4}{be-dx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{2d}\right)$$

$$\Big\downarrow \; 262$$

$$e(bc-ad)\left(\frac{x^6}{2d\left(be-dx^4\right)}-\frac{3\left(\frac{be\int \frac{1}{be-dx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{d}-\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{d}\right)}{2d}\right)$$

$$\Big\downarrow \; 221$$

$$e(bc-ad)\left(\frac{x^6}{2d\left(be-dx^4\right)}-\frac{3\left(\frac{\sqrt{b}\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{d^{3/2}}-\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{d}\right)}{2d}\right)$$

input
```
Int[x*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(b*c - a*d)*e*(x^6/(2*d*(b*e - d*x^4)) - (3*(-(Sqrt[(e*(a + b*x^2))/(c + d
*x^2)]/d) + (Sqrt[b]*Sqrt[e]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*
x^2)])/(Sqrt[b]*Sqrt[e])])/d^(3/2)))/(2*d))
```

3.278.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2051
```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)   Subst[Int[
x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
 x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x]
&& FractionQ[p] && IntegerQ[1/n]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.278.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.64

method	result
risch	$\frac{(d\,x^2+c)be\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}{2d^2}+\frac{\left(\frac{3b(ad-bc)\ln\left(\frac{\frac{1}{2}eda+\frac{1}{2}ebc+bde\,x^2}{\sqrt{bde}}+\sqrt{bde\,x^4+(eda+ebc)x^2+ace}\right)}{2\sqrt{bde}}-\frac{\left(2a^2d^2-4abcd+2b^2c^2\right)\left(b\,x^2+a\right)}{(ad-bc)\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}}\right)e\sqrt{}}{2d^2(b\,x^2+a)}$
default	$-\frac{\left(-3\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)ab\,d^2x^2+3\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)b^2cd\,x^2-2\sqrt{bd}\,\sqrt{}\right.}{}$

input
```
int(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d^2*(d*x^2+c)*b*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/2/d^2*(3/2*b*(a*d-b*
c)*ln((1/2*e*d*a+1/2*e*b*c+b*d*e*x^2)/(b*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*
e)*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)-(2*a^2*d^2-4*a*b*c*d+2*b^2*c^2)*(b*x^2+
a)/(a*d-b*c)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2))*e/(b*x^2+a)*(e*(
b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)
```

3.278.5 Fricas [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.33

$$\int x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \left[-\frac{3\,(bc-ad)\sqrt{\frac{be}{d}}e\log\left(8\,b^2d^2ex^4+8\,(b^2cd+abd^2)ex^2+(b^2c^2+6\,abcd+a^2d^2)\right.}{}\right.$$

input
```
integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(3*(b*c - a*d)*sqrt(b*e/d)*e*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*
d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d
+ a*c*d^2 + (3*b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x
^2 + c))) - 4*(b*d*e*x^2 + (3*b*c - 2*a*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2
+ c)))/d^2, 1/4*(3*(b*c - a*d)*sqrt(-b*e/d)*e*arctan(1/2*(2*b*d*x^2 + b*c
+ a*d)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*e*x^2 + a*b*e))
 + 2*(b*d*e*x^2 + (3*b*c - 2*a*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/d^
2]
```

3.278.6 Sympy [F(-1)]

Timed out.

$$\int x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate(x*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

3.278.7 Maxima [F(-2)]

Exception generated.

$$\int x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx=\text{Exception raised: ValueError}$$

input
```
integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.278.8 Giac [F(-2)]

Exception generated.

$$\int x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{2,[0,3,0]%%%},[2,0,0,0]%%%}+%%%{%%{[%%%{-4,[0,2,0]%%%}
,0]:[1,0,
```

3.278.9 Mupad [F(-1)]

Timed out.

$$\int x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx=\int x\left(\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}dx$$

input
```
int(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

output
```
int(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.279 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx$

3.279.1 Optimal result

Integrand size = 26, antiderivative size = 151

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx = -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} - \frac{a^{3/2}e^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{c^{3/2}} + \frac{b^{3/2}e^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{d^{3/2}}$$

output
```
-a^(3/2)*e^(3/2)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(
1/2))/c^(3/2)+b^(3/2)*e^(3/2)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2
)/b^(1/2)/e^(1/2))/d^(3/2)-(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c/d
```

3.279.2 Mathematica [A] (verified)

Time = 2.30 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.24

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx = \frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-a^{3/2}d^{3/2}\sqrt{c+dx^2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)+\sqrt{c}\left(\sqrt{d}(-bc+ad)\sqrt{a+bx^2}+\right.\right.}{c^{3/2}d^{3/2}\sqrt{a+bx^2}}$$

input
```
Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x,x]
```

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(a^(3/2)*d^(3/2)*Sqrt[c + d*x^2]*Ar
cTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]) + Sqrt[c]*(Sqr
t[d]*(-(b*c) + a*d)*Sqrt[a + b*x^2] + b^(3/2)*c*Sqrt[c + d*x^2]*ArcTanh[(S
qrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])))/(c^(3/2)*d^(3/2)*Sqr
t[a + b*x^2])
```

3.279.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2053, 2052, 25, 381, 27, 397, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{\left(\frac{e\left(bx^2+a\right)}{dx^2+c}\right)^{3/2}}{x^2}dx^2$$

$$\Big\downarrow \text{2052}$$

$$e(bc-ad)\int -\frac{x^8}{\left(ae-cx^4\right)\left(be-dx^4\right)}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$-\left(e(bc-ad)\int \frac{x^8}{\left(ae-cx^4\right)\left(be-dx^4\right)}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}\right)$$

$$\Big\downarrow \text{381}$$

$$e(bc-ad)\left(\frac{\int \frac{e\left(abe-(bc+ad)x^4\right)}{\left(ae-cx^4\right)\left(be-dx^4\right)}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{cd}-\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{cd}\right)$$

$$\Big\downarrow \text{27}$$

$$e(bc-ad)\left(\frac{e\int \frac{abe-(bc+ad)x^4}{(ae-cx^4)(be-dx^4)}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{cd}-\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}\right)$$

$$\Big\downarrow \text{397}$$

$$e(bc-ad)\left(\frac{e\left(\frac{b^2c\int \frac{1}{be-dx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{bc-ad}-\frac{a^2d\int \frac{1}{ae-cx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{bc-ad}\right)}{cd}-\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}\right)$$

$$\Big\downarrow \text{221}$$

$$e(bc-ad)\left(\frac{e\left(\frac{b^{3/2}c\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{d}\sqrt{e}(bc-ad)}-\frac{a^{3/2}d\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{c}\sqrt{e}(bc-ad)}\right)}{cd}-\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}\right)$$

input

```
Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x,x]
```

output

```
(b*c - a*d)*e*(-(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(c*d)) + (e*(-((a^(3/2)
*d*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])
/(Sqrt[c]*(b*c - a*d)*Sqrt[e])) + (b^(3/2)*c*ArcTanh[(Sqrt[d]*Sqrt[(e*(a +
 b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(Sqrt[d]*(b*c - a*d)*Sqrt[e])))
/(c*d))
```

3.279. $\int \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{x}\,dx$

3.279.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.279. $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx$

3.279.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(121) = 242$.

Time = 0.14 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.66

method	result
default	$\frac{\left(-\sqrt{bd}\,\ln\left(\frac{ad\,x^2+bc\,x^2+2\sqrt{ac}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}+2ac}{x^2}\right)a^2d^2x^2+\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)\sqrt{ac}\,b^2cd\,x^2-\sqrt{}\right.}{}$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)+2*a*c)/x^2)*a^2*d^2*x^2+ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)*b^2*c*d*
x^2-(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)+2*a*c)/x^2)*a^2*c*d+ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)*b^2*c^2+2*(b*d
)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*(a*c)^(1/2)*a*d-2*(b*d)^(1/2)*((d*x^2+
c)*(b*x^2+a))^(1/2)*(a*c)^(1/2)*b*c)/c/d*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))
^(3/2)/(a*c)^(1/2)/(b*d)^(1/2)/(b*x^2+a)/((d*x^2+c)*(b*x^2+a))^(1/2)
```

3.279.5 Fricas [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 1049, normalized size of antiderivative = 6.95

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx = \left[\frac{bc\sqrt{\frac{be}{d}}e\log\left(8\,b^2d^2ex^4+8\,(b^2cd+abd^2)ex^2+(b^2c^2+6\,abcd+a^2d^2)e+4\,(2\,bd^3x^4+\right.}{}\right.$$

$$-\frac{2\,bc\sqrt{-\frac{be}{d}}e\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{-\frac{be}{d}}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\,(b^2ex^2+abe)}\right)-ad\sqrt{\frac{ae}{c}}e\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)ex^4+8\,a^2c^2e+8\,(abc^2+a^2cd)ex^2-}{}\right.}{4\,cd}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/4*(b*c*sqrt(b*e/d)*e*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2
+ (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*d^2 +
 (3*b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)) +
 a*d*sqrt(a*e/c)*e*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*
e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b*c^2*d + a*c*d^2)*x^4 + 2*a*c^3 + (
b*c^3 + 3*a*c^2*d)*x^2)*sqrt(a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/x^4
) - 4*(b*c - a*d)*e*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c*d), -1/4*(2*b*c*
sqrt(-b*e/d)*e*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*e/d)*sqrt((b*e*x
^2 + a*e)/(d*x^2 + c))/(b^2*e*x^2 + a*b*e)) - a*d*sqrt(a*e/c)*e*log(((b^2*
c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x
^2 - 4*((b*c^2*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3 + 3*a*c^2*d)*x^2)*sqrt(
a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/x^4) + 4*(b*c - a*d)*e*sqrt((b*e
*x^2 + a*e)/(d*x^2 + c)))/(c*d), 1/4*(2*a*d*sqrt(-a*e/c)*e*arctan(1/2*((b*
c + a*d)*x^2 + 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b*
e*x^2 + a^2*e)) + b*c*sqrt(b*e/d)*e*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b
*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d
 + a*c*d^2 + (3*b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*
x^2 + c))) - 4*(b*c - a*d)*e*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c*d), 1/2
*(a*d*sqrt(-a*e/c)*e*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(-a*e/c)*sqr
t((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b*e*x^2 + a^2*e)) - b*c*sqrt(-b*e/d)*...
```

3.279.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x,x)
```

output
```
Timed out
```

3.279.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.279.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.279.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx = \int \frac{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}{x}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x,x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x, x)
```

3.280 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx$

3.280.1 Optimal result

Integrand size = 26, antiderivative size = 165

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx=\frac{3(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c^2}+\frac{(bc-ad)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}-\frac{3\sqrt{a}(bc-ad)e^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2c^{5/2}}$$

output
```
1/2*(-a*d+b*c)*(e*(b*x^2+a)/(d*x^2+c))^(3/2)/c/(a-c*(b*x^2+a)/(d*x^2+c))-3
/2*(-a*d+b*c)*e^(3/2)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2
)/e^(1/2))*a^(1/2)/c^(5/2)+3/2*(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/
c^2
```

3.280.2 Mathematica [A] (verified)

Time = 3.80 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.88

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{c}\sqrt{a+bx^2}(2bcx^2-a(c+3dx^2))-3\sqrt{a}(bc-ad)x^2\sqrt{c+dx^2}\operatorname{arctanh}\left(\right.\right.}{2c^{5/2}x^2\sqrt{a+bx^2}}$$

input
```
Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3,x]
```

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[c]*Sqrt[a + b*x^2]*(2*b*c*x^2 -
 a*(c + 3*d*x^2)) - 3*Sqrt[a]*(b*c - a*d)*x^2*Sqrt[c + d*x^2]*ArcTanh[(Sqr
t[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(2*c^(5/2)*x^2*Sqrt[a +
 b*x^2])
```

3.280.3 Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2053, 2052, 252, 262, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \ 2053$$

$$\frac{1}{2}\int \frac{\left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}}{x^4}dx^2$$

$$\Big\downarrow \ 2052$$

$$e(bc-ad)\int \frac{x^8}{(ae-cx^4)^2}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}$$

$$\Big\downarrow \ 252$$

$$e(bc-ad)\left(\frac{x^6}{2c(ae-cx^4)}-\frac{3\int \frac{x^4}{ae-cx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{2c}\right)$$

$$\Big\downarrow \ 262$$

$$e(bc-ad)\left(\frac{x^6}{2c\,(ae-cx^4)}-\frac{3\left(\frac{ae\int \frac{1}{ae-cx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{c}-\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c}\right)}{2c}\right)$$

$$\Big\downarrow \ 221$$

$$e(bc-ad)\left(\frac{x^6}{2c\,(ae-cx^4)}-\frac{3\left(\frac{\sqrt{a}\sqrt{e}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{c^{3/2}}-\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c}\right)}{2c}\right)$$

input
```
Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3,x]
```

output
```
(b*c - a*d)*e*(x^6/(2*c*(a*e - c*x^4)) - (3*(-(Sqrt[(e*(a + b*x^2))/(c + d
*x^2)]/c) + (Sqrt[a]*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*
x^2)])/(Sqrt[a]*Sqrt[e])])/c^(3/2)))/(2*c))
```

3.280.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.280.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.45

method	result
risch	$-\frac{a\left(d\,x^{2}+c\right)e\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}{2c^{2}x^{2}}-\frac{\left(-\frac{3a\left(ad-bc\right)\ln\left(\frac{2ace+\left(eda+ebc\right)x^{2}+2\sqrt{ace}\,\sqrt{bde\,x^{4}+\left(eda+ebc\right)x^{2}+ace}}{x^{2}}\right)}{2\sqrt{ace}}-\frac{\left(-2a^{2}d^{2}+4abcd-2b^{2}c^{2}\right)\left(b\,x^{2}\right.}{\left(ad-bc\right)\sqrt{bde\,x^{4}+ade\,x^{2}+bce\,x^{2}}}\right.}{2c^{2}\left(b\,x^{2}+a\right)}$
default	$-\frac{\left(-2\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{ac}\,b\,d^{2}x^{6}-3\ln\left(\frac{ad\,x^{2}+bc\,x^{2}+2\sqrt{ac}\,\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}+2ac}{x^{2}}\right)a^{2}c\,d^{2}x^{4}+3\ln\left(\frac{ad\,x^{2}+bc\,x^{2}+2\sqrt{ac}}{}\right.\right.}{}$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/c^2*(d*x^2+c)/x^2*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/2/c^2*(-3/2*a*(
a*d-b*c)/(a*c*e)^(1/2)*ln((2*a*c*e+(a*d*e+b*c*e)*x^2+2*(a*c*e)^(1/2)*(b*d*
e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/x^2)-(-2*a^2*d^2+4*a*b*c*d-2*b^2*c^2
)*(b*x^2+a)/(a*d-b*c)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2))*e/(b*x^
2+a)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)
```

3.280. $\int \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{x^3}\,dx$

3.280.5 Fricas [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.12

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \left[-\frac{3\,(bc-ad)\sqrt{\frac{ae}{c}}ex^2\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)ex^4+8\,a^2c^2e+8\,(abc^2+a^2cd)ex^2+4\,((bc^2d+acd^2)x^4+2\,a}{x^4}\right.}{8\,c^2x^2}\right.$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x, algorithm="fricas")
```

output
```
[-1/8*(3*(b*c - a*d)*sqrt(a*e/c)*e*x^2*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2
)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 + 4*((b*c^2*d + a*c*d^
2)*x^4 + 2*a*c^3 + (b*c^3 + 3*a*c^2*d)*x^2)*sqrt(a*e/c)*sqrt((b*e*x^2 + a*
e)/(d*x^2 + c)))/x^4) - 4*((2*b*c - 3*a*d)*e*x^2 - a*c*e)*sqrt((b*e*x^2 +
a*e)/(d*x^2 + c)))/(c^2*x^2), 1/4*(3*(b*c - a*d)*sqrt(-a*e/c)*e*x^2*arctan
(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 +
c))/(a*b*e*x^2 + a^2*e)) + 2*((2*b*c - 3*a*d)*e*x^2 - a*c*e)*sqrt((b*e*x^2
 + a*e)/(d*x^2 + c)))/(c^2*x^2)]
```

3.280.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**3,x)
```

output
```
Timed out
```

3.280.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.280.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{2,[0,1,0]%%%},[6,0,0]%%%}+%%%{%%{[-4,0]:[1,0,%%%{-1,[1
,1,1]%%%}
```

3.280.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \int \frac{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}{x^3}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3,x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3, x)
```

3.281 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx$

3.281.1 Optimal result

Integrand size = 26, antiderivative size = 256

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx = -\frac{d(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^3} - \frac{a(bc-ad)^2e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} + \frac{(5bc-9ad)(bc-ad)e^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} - \frac{3(bc-5ad)(bc-ad)e^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8\sqrt{a}c^{7/2}}$$

output
```
-3/8*(-5*a*d+b*c)*(-a*d+b*c)*e^(3/2)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c
))^(1/2)/a^(1/2)/e^(1/2))/c^(7/2)/a^(1/2)-d*(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x
^2+c))^(1/2)/c^3-1/4*a*(-a*d+b*c)^2*e^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^3/
(a*e-c*e*(b*x^2+a)/(d*x^2+c))^2+1/8*(-9*a*d+5*b*c)*(-a*d+b*c)*e^2*(e*(b*x^
2+a)/(d*x^2+c))^(1/2)/c^3/(a*e-c*e*(b*x^2+a)/(d*x^2+c))
```

3.281.2 Mathematica [A] (verified)

Time = 4.30 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.73

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx =$$
$$-\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{a}\sqrt{c}\sqrt{a+bx^2}(bcx^2(5c+13dx^2)+a(2c^2-5cdx^2-15d^2x^4))+3(b^2c^2-6abcd+5a^2d^2)\,x^4\sqrt{\right.}{8\sqrt{a}c^{7/2}x^4\sqrt{a+bx^2}}$$

input
```
Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5,x]
```

output
```
-1/8*(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2]
*(b*c*x^2*(5*c + 13*d*x^2) + a*(2*c^2 - 5*c*d*x^2 - 15*d^2*x^4)) + 3*(b^2*
c^2 - 6*a*b*c*d + 5*a^2*d^2)*x^4*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a +
 b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(Sqrt[a]*c^(7/2)*x^4*Sqrt[a + b*x^2]
)
```

3.281.3 Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.84, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {2053, 2052, 25, 360, 25, 1471, 27, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{\left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}}{x^6}dx^2$$

$$\Big\downarrow \text{2052}$$

$$e(bc-ad)\int -\frac{x^8(be-dx^4)}{(ae-cx^4)^3}d\sqrt{\frac{e\,(bx^2+a)}{dx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$-\left(e(bc-ad)\int \frac{x^8\left(be-dx^4\right)}{\left(ae-cx^4\right)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}\right)$$

$$\Big\downarrow \ 360$$

$$e(bc-ad)\left(-\frac{\int -\frac{-4c^2dx^8+4c(bc-ad)ex^4+a(bc-ad)e^2}{\left(ae-cx^4\right)^2}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{4c^3}-\frac{ae^2(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae-cx^4\right)^2}\right)$$

$$\Big\downarrow \ 25$$

$$e(bc-ad)\left(\frac{\int \frac{-4c^2dx^8+4c(bc-ad)ex^4+a(bc-ad)e^2}{\left(ae-cx^4\right)^2}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{4c^3}-\frac{ae^2(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae-cx^4\right)^2}\right)$$

$$\Big\downarrow \ 1471$$

$$e(bc-ad)\left(\frac{\frac{e(5bc-9ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2(ae-cx^4)}-\frac{\int \frac{ae\left((3bc-7ad)e-8cdx^4\right)}{ae-cx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{2ae}}{4c^3}-\frac{ae^2(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae-cx^4\right)^2}\right)$$

$$\Big\downarrow \ 27$$

$$e(bc-ad)\left(\frac{\frac{e(5bc-9ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2(ae-cx^4)}-\frac{1}{2}\int \frac{(3bc-7ad)e-8cdx^4}{ae-cx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{4c^3}-\frac{ae^2(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae-cx^4\right)^2}\right)$$

$$\Big\downarrow \ 299$$

$$e(bc-ad)\left(\frac{\frac{1}{2}\left(-3e(bc-5ad)\int \frac{1}{ae-cx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}-8d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)+\frac{e(5bc-9ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2(ae-cx^4)}}{4c^3}-\frac{ae^2(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae-cx^4\right)^2}\right)$$

$$\Big\downarrow \ 221$$

3.281. $\displaystyle\int \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{x^5}dx$

$$e(bc-ad)\left(\frac{\frac{1}{2}\left(-\frac{3\sqrt{e}(bc-5ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\sqrt{c}}-8d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)+\frac{e(5bc-9ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2(ae-cx^4)}}{4c^3}-\frac{ae^2(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae-cx^4\right)^2}\right)$$

input
```
Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5,x]
```

output
```
(b*c - a*d)*e*(-1/4*(a*(b*c - a*d)*e^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/
(c^3*(a*e - c*x^4)^2) + (((5*b*c - 9*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^
2)])/(2*(a*e - c*x^4)) + (-8*d*Sqrt[(e*(a + b*x^2))/(c + d*x^2)] - (3*(b*c
 - 5*a*d)*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqr
t[a]*Sqrt[e])])/(Sqrt[a]*Sqrt[c]))/2)/(4*c^3))
```

3.281.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

3.281. $\int \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{x^5}\,dx$

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.281.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.05

method	result
risch	$-\frac{(dx^2+c)(-7adx^2+5bcx^2+2ac)e\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{8c^3x^4}+\frac{\left(-\frac{(15a^2d^2-18abcd+3b^2c^2)\ln\left(\frac{2ace+(eda+ebc)x^2+2\sqrt{ace}\sqrt{bdex^4+(eda+ebc)x^2+\cdots}}{x^2}\right)}{2\sqrt{ace}}\cdots\right.}{8c^3(b\cdots}$
default	Expression too large to display

3.281. $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(d*x^2+c)*(-7*a*d*x^2+5*b*c*x^2+2*a*c)/c^3/x^4*e*(e*(b*x^2+a)/(d*x^2+
c))^(1/2)+1/8/c^3*(-1/2*(15*a^2*d^2-18*a*b*c*d+3*b^2*c^2)/(a*c*e)^(1/2)*ln
((2*a*c*e+(a*d*e+b*c*e)*x^2+2*(a*c*e)^(1/2)*(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a
*c*e)^(1/2))/x^2)+8*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(b*x^2+a)/(a*d-b*c)/(b*d
*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2))*e/(b*x^2+a)*(e*(b*x^2+a)/(d*x^2+c
))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)
```

3.281.5 Fricas [A] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.70

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \left[\frac{3\,(b^2c^2-6\,abcd+5\,a^2d^2)ex^4\sqrt{\frac{e}{ac}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)ex^4+8\,a^2c^2e+8\,(abc^2+a^2cd)ex^2-4\,(2\,a}{x^{}}\right.}{}\right.$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x, algorithm="fricas")
```

output
```
[1/32*(3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*e*x^4*sqrt(e/(a*c))*log(((b^2*c
^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^
2 - 4*(2*a^2*c^3 + (a*b*c^2*d + a^2*c*d^2)*x^4 + (a*b*c^3 + 3*a^2*c^2*d)*x
^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(a*c)))/x^4) - 4*((13*b*c*d -
 15*a*d^2)*e*x^4 + 2*a*c^2*e + 5*(b*c^2 - a*c*d)*e*x^2)*sqrt((b*e*x^2 + a*
e)/(d*x^2 + c)))/(c^3*x^4), 1/16*(3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*e*x^
4*sqrt(-e/(a*c))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)
/(d*x^2 + c))*sqrt(-e/(a*c))/(b*e*x^2 + a*e)) - 2*((13*b*c*d - 15*a*d^2)*e
*x^4 + 2*a*c^2*e + 5*(b*c^2 - a*c*d)*e*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 +
c)))/(c^3*x^4)]
```

3.281.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**5,x)
```

output
```
Timed out
```

3.281.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.281.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{2,[4,1,4]%%%},[2,7,0]%%%}+%%%{%%%{-8,[3,2,4]%%%},[2,6,
1]%%%}+%%
```

3.281.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \int \frac{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}{x^5}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5,x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5, x)
```

3.282 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx$

3.282.1 Optimal result

Integrand size = 26, antiderivative size = 366

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \frac{d^2(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^4} + \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)^2(bc+11ad)e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24c^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)\left(5b^2c^2+50abcd-79a^2d^2\right)e^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{48ac^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} + \frac{(bc-ad)\left(b^2c^2+10abcd-35a^2d^2\right)e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{16a^{3/2}c^{9/2}}$$

output
```
1/6*(-a*d+b*c)^3*e^2*(e*(b*x^2+a)/(d*x^2+c))^(5/2)/a/c^2/(a*e-c*e*(b*x^2+a
)/(d*x^2+c))^3+1/16*(-a*d+b*c)*(-35*a^2*d^2+10*a*b*c*d+b^2*c^2)*e^(3/2)*ar
ctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2))/a^(3/2)/c^(9/
2)+d^2*(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^4+1/24*(-a*d+b*c)^2*(1
1*a*d+b*c)*e^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^4/(a*e-c*e*(b*x^2+a)/(d*x^2
+c))^2-1/48*(-a*d+b*c)*(-79*a^2*d^2+50*a*b*c*d+5*b^2*c^2)*e^2*(e*(b*x^2+a)
/(d*x^2+c))^(1/2)/a/c^4/(a*e-c*e*(b*x^2+a)/(d*x^2+c))
```

3.282.2 Mathematica [A] (verified)

Time = 4.76 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.67

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\sqrt{a}\sqrt{c}\sqrt{a+bx^2}(3b^2c^2x^4(c+dx^2)+2abcx^2(7c^2-19cdx^2-50d^2x^4)+a\right.}{}$$

input
```
Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7,x]
```

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2]*(3
*b^2*c^2*x^4*(c + d*x^2) + 2*a*b*c*x^2*(7*c^2 - 19*c*d*x^2 - 50*d^2*x^4) +
 a^2*(8*c^3 - 14*c^2*d*x^2 + 35*c*d^2*x^4 + 105*d^3*x^6))) + 3*(b^3*c^3 +
9*a*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*x^6*Sqrt[c + d*x^2]*ArcTanh[(
Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(48*a^(3/2)*c^(9/2)*
x^6*Sqrt[a + b*x^2])
```

3.282.3 Rubi [A] (warning: unable to verify)

Time = 0.54 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.82, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {2053, 2052, 366, 25, 27, 360, 25, 1471, 27, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}dx$$

$$\Big\downarrow \text{ 2053}$$

$$\frac{1}{2}\int \frac{\left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}}{x^8}dx^2$$

$$\Big\downarrow \text{ 2052}$$

$$e(bc-ad)\int \frac{x^8(be-dx^4)^2}{(ae-cx^4)^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}$$

$$\Big\downarrow \text{ 366}$$

$$e(bc-ad)\left(\frac{ex^{10}(bc-ad)^2}{6ac^2\left(ae-cx^4\right)^3}-\frac{\int -\frac{x^8\left(-6acd^2ex^4+6b^2c^2e^2-5(bce-ade)^2\right)}{\left(ae-cx^4\right)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{6ac^2e}\right)$$

$\downarrow$ 25

$$e(bc-ad)\left(\frac{\int \frac{ex^8\left(\left(b^2c^2+10abdc-5a^2d^2\right)e-6acd^2x^4\right)}{\left(ae-cx^4\right)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{6ac^2e}+\frac{ex^{10}(bc-ad)^2}{6ac^2\left(ae-cx^4\right)^3}\right)$$

$\downarrow$ 27

$$e(bc-ad)\left(\frac{\int \frac{x^8\left(\left(b^2c^2+10abdc-5a^2d^2\right)e-6acd^2x^4\right)}{\left(ae-cx^4\right)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{6ac^2}+\frac{ex^{10}(bc-ad)^2}{6ac^2\left(ae-cx^4\right)^3}\right)$$

$\downarrow$ 360

$$e(bc-ad)\left(\frac{\frac{\int -\frac{-24ac^3d^2x^8+4c^2(bc-ad)(bc+11ad)ex^4+ac(bc-ad)(bc+11ad)e^2}{\left(ae-cx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{4c^3}+\frac{ae^2(bc-ad)(11ad+bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^2\left(ae-cx^4\right)^2}}{6ac^2}+\frac{ex^{10}(bc-ad)^2}{6ac^2\left(ae-cx^4\right)^3}\right)$$

$\downarrow$ 25

$$e(bc-ad)\left(\frac{\frac{ae^2(bc-ad)(11ad+bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^2\left(ae-cx^4\right)^2}-\frac{\int \frac{-24ac^3d^2x^8+4c^2(bc-ad)(bc+11ad)ex^4+ac(bc-ad)(bc+11ad)e^2}{\left(ae-cx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{4c^3}}{6ac^2}+\frac{ex^{10}(bc-ad)^2}{6ac^2\left(ae-cx^4\right)^3}\right)$$

$\downarrow$ 1471

$$e(bc-ad)\left(\frac{\frac{ae^2(bc-ad)(11ad+bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^2\left(ae-cx^4\right)^2}-\frac{\frac{ce\left(-79a^2d^2+50abcd+5b^2c^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2\left(ae-cx^4\right)}-\frac{\int \frac{3ace\left(\left(b^2c^2+10abdc-19a^2d^2\right)e-16acd^2x^4\right)}{ae-cx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{2ae}}{4c^3}}{6ac^2}+\right.$$

$\downarrow$ 27

3.282. $\displaystyle\int \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{x^7}\,dx$

$$e(bc-$$

$$ad)\left(\frac{\frac{ae^2(bc-ad)(11ad+bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2(ae-cx^4)^2}-\frac{\frac{ce(-79a^2d^2+50abcd+5b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2(ae-cx^4)}-\frac{3}{2}c\int\frac{(b^2c^2+10abdc-19a^2d^2)e-16acd^2x^4}{ae-cx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{4c^3}}{6ac^2}+\frac{}{6}\right.$$

$$\Big\downarrow \; 299$$

$$e(bc-$$

$$ad)\left(\frac{\frac{ae^2(bc-ad)(11ad+bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2(ae-cx^4)^2}-\frac{\frac{ce(-79a^2d^2+50abcd+5b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2(ae-cx^4)}-\frac{3}{2}c\left(e(-35a^2d^2+10abcd+b^2c^2)\int\frac{1}{ae-cx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}+16\right.}{4c^3}}{6ac^2}\right.$$

$$\Big\downarrow \; 221$$

$$e(bc-$$

$$ad)\left(\frac{\frac{ae^2(bc-ad)(11ad+bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2(ae-cx^4)^2}-\frac{\frac{ce(-79a^2d^2+50abcd+5b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2(ae-cx^4)}-\frac{3}{2}c\left(\frac{\sqrt{e}(-35a^2d^2+10abcd+b^2c^2)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\sqrt{c}}\right.}{4c^3}}{6ac^2}\right.$$

input
```
Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7,x]
```

3.282. $\int\frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx$

output
```
(b*c - a*d)*e*(((b*c - a*d)^2*e*x^10)/(6*a*c^2*(a*e - c*x^4)^3) + ((a*(b*c
 - a*d)*(b*c + 11*a*d)*e^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*c^2*(a*e
- c*x^4)^2) - ((c*(5*b^2*c^2 + 50*a*b*c*d - 79*a^2*d^2)*e*Sqrt[(e*(a + b*x
^2))/(c + d*x^2)])/(2*(a*e - c*x^4)) - (3*c*(16*a*d^2*Sqrt[(e*(a + b*x^2))
/(c + d*x^2)] + ((b^2*c^2 + 10*a*b*c*d - 35*a^2*d^2)*Sqrt[e]*ArcTanh[(Sqrt
[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(Sqrt[a]*Sqrt[c
])))/2)/(4*c^3))/(6*a*c^2))
```

3.282.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

3.282. $\int \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{x^7}\,dx$

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.282.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{(d\,x^2+c)\left(57a^2d^2x^4-52bdac\,x^4+3b^2c^2x^4-22a^2cd\,x^2+14ab\,c^2x^2+8a^2c^2\right)e\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}{48c^4x^6a}-\frac{\left(-\frac{\left(35a^3d^3-45a^2bc\,d^2+9d\,c^2b^2a+b^3c^3\right)}{}\right.}{}$
default	Expression too large to display

3.282. $\int \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{x^7}\,dx$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(d*x^2+c)*(57*a^2*d^2*x^4-52*a*b*c*d*x^4+3*b^2*c^2*x^4-22*a^2*c*d*x^
2+14*a*b*c^2*x^2+8*a^2*c^2)/c^4/x^6/a*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/16
/c^4/a*(-1/2*(35*a^3*d^3-45*a^2*b*c*d^2+9*a*b^2*c^2*d+b^3*c^3)/(a*c*e)^(1/
2)*ln((2*a*c*e+(a*d*e+b*c*e)*x^2+2*(a*c*e)^(1/2)*(b*d*e*x^4+(a*d*e+b*c*e)*
x^2+a*c*e)^(1/2))/x^2)+16*a*d^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(b*x^2+a)/(a*d
-b*c)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2))*e/(b*x^2+a)*(e*(b*x^2+a
)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)
```

3.282.5 Fricas [A] (verification not implemented)

Time = 9.70 (sec) , antiderivative size = 573, normalized size of antiderivative = 1.57

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \left[\frac{3\,(b^3c^3 + 9\,ab^2c^2d - 45\,a^2bcd^2 + 35\,a^3d^3)ex^6\sqrt{\frac{e}{ac}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)ex^4+8\,a^2c^2e+8\,(ab}{}\right.}{}\right.$$

$$-\frac{3\,(b^3c^3 + 9\,ab^2c^2d - 45\,a^2bcd^2 + 35\,a^3d^3)ex^6\sqrt{-\frac{e}{ac}}\arctan\left(\frac{((bc+ad)x^2+2\,ac)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{e}{ac}}}{2\,(bex^2+ae)}\right) + 2\,((3\,b^2c^2d - 1}{96\,ac}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x, algorithm="fricas")
```

output
```
[1/192*(3*(b^3*c^3 + 9*a*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*e*x^6*sq
rt(e/(a*c))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(
a*b*c^2 + a^2*c*d)*e*x^2 + 4*(2*a^2*c^3 + (a*b*c^2*d + a^2*c*d^2)*x^4 + (a
*b*c^3 + 3*a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(a*c))
)/x^4) - 4*((3*b^2*c^2*d - 100*a*b*c*d^2 + 105*a^2*d^3)*e*x^6 + 8*a^2*c^3*
e + (3*b^2*c^3 - 38*a*b*c^2*d + 35*a^2*c*d^2)*e*x^4 + 14*(a*b*c^3 - a^2*c^
2*d)*e*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c^4*x^6), -1/96*(3*(b^3*
c^3 + 9*a*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*e*x^6*sqrt(-e/(a*c))*ar
ctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(
-e/(a*c))/(b*e*x^2 + a*e)) + 2*((3*b^2*c^2*d - 100*a*b*c*d^2 + 105*a^2*d^3
)*e*x^6 + 8*a^2*c^3*e + (3*b^2*c^3 - 38*a*b*c^2*d + 35*a^2*c*d^2)*e*x^4 +
14*(a*b*c^3 - a^2*c^2*d)*e*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c^4*
x^6)]
```

3.282.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**7,x)
```

output
```
Timed out
```

3.282.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.282.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{2,[5,1,5]%%%},[2,9,0]%%%}+%%%{%%%{-10,[4,2,5]%%%},[2,8
,1]%%%}+%
```

3.282.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \int \frac{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}{x^7}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7,x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7, x)
```

3.283 $\int x^4 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

3.283.1 Optimal result

Integrand size = 26, antiderivative size = 391

$$\int x^4 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = -\frac{\left(16ac-\frac{16bc^2}{d}-\frac{a^2d}{b}\right) ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5d^2} - \frac{ex^3(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d} - \frac{(8bc-7ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5d^3} + \frac{6bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5d^2} - \frac{\sqrt{c}(16b^2c^2-16abcd+a^2d^2)\, e\sqrt{\frac{e(a+bx^2)}{c+dx^2}} E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5bd^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{c^{3/2}(8bc-7ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\, \text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right), 1-\frac{bc}{ad}\right)}{5d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

output
```
-1/5*(16*a*c-16*b*c^2/d-a^2*d/b)*e*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^2-e*x
^3*(b*x^2+a)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d-1/5*(-7*a*d+8*b*c)*e*x*(d*x^2
+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^3+6/5*b*e*x^3*(d*x^2+c)*(e*(b*x^2+a)/(
d*x^2+c))^(1/2)/d^2+1/5*c^(3/2)*(-7*a*d+8*b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1+
d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(
1/2))*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2
)-1/5*(a^2*d^2-16*a*b*c*d+16*b^2*c^2)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^
(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(
1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b/d^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/
2)
```

3.283.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.61 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.68

$$\int x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx=\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}dx(a+bx^2)\left(ad(7c+2dx^2)+b(-8c^2-2cdx^2+d^2x^4)\right)-ic(16\right.}{}$$

input
```
Integrate[x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(a + b*x^2)*(a*d*(7*c
+ 2*d*x^2) + b*(-8*c^2 - 2*c*d*x^2 + d^2*x^4)) - I*c*(16*b^2*c^2 - 16*a*b*
c*d + a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh
[Sqrt[b/a]*x], (a*d)/(b*c)] + (8*I)*c*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*Sq
rt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)]))/(5*Sqrt[b/a]*d^4*(a + b*x^2))
```

3.283.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {2058, 369, 27, 443, 25, 444, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\int\frac{x^4(bx^2+a)^{3/2}}{(dx^2+c)^{3/2}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{369}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{\int\frac{3x^2\sqrt{bx^2+a}(2bx^2+a)}{\sqrt{dx^2+c}}dx}{d}-\frac{x^3(a+bx^2)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{3\int\frac{x^2\sqrt{bx^2+a}(2bx^2+a)}{\sqrt{dx^2+c}}dx}{d}-\frac{x^3(a+bx^2)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{443}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{3\left(\frac{\int-\frac{x^2(b(8bc-7ad)x^2+a(6bc-5ad))}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5d}+\frac{2bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}\right)}{d}-\frac{x^3(a+bx^2)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{3\left(\frac{2bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\int\frac{x^2(b(8bc-7ad)x^2+a(6bc-5ad))}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5d}\right)}{d}-\frac{x^3(a+bx^2)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{444}$$

3.283. $\int x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{3\left(\frac{2bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(8bc-7ad)}{3d}-\frac{\int\frac{b\left(\left(16b^2c^2-16abdc+a^2d^2\right)x^2+ac(8bc-7ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}}{5d}\right)}{d}-\frac{x^3\left(a+bx^2\right.}{d\sqrt{c+d}}\right.}{\sqrt{a+bx^2}}$$

$\downarrow$ 27

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{3\left(\frac{2bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(8bc-7ad)}{3d}-\frac{\int\frac{\left(16b^2c^2-16abdc+a^2d^2\right)x^2+ac(8bc-7ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}}{5d}\right)}{d}-\frac{x^3\left(a+bx^2\right)^{3/}}{d\sqrt{c+dx^2}}\right.}{\sqrt{a+bx^2}}$$

$\downarrow$ 406

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{3\left(\frac{2bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(8bc-7ad)}{3d}-\frac{\left(a^2d^2-16abcd+16b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(8bc-7ad)\int\frac{}{\sqrt{bx^2}}}{3d}}{5d}\right.}{d}\right.}{\sqrt{a+bx^2}}$$

$\downarrow$ 320

3.283. $\int x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{3\left(\frac{2bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(8bc-7ad)}{3d}-\frac{\left(a^2d^2-16abcd+16b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(8bc-}{\sqrt{d}}}{3d}}{5d}\right.}{d}\right.}{\sqrt{a+bx^2}}$$

$\downarrow$ 388

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{3\left(\frac{2bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(8bc-7ad)}{3d}-\frac{\left(a^2d^2-16abcd+16b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{}}{}}{3d}}{5d}\right.}{d}\right.}{\sqrt{a+bx^2}}$$

$\downarrow$ 313

3.283. $\int x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{3\left(\frac{2bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(8bc-7ad)}{3d}-\frac{(a^2d^2-16abcd+16b^2c^2)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right.}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2}{a(c+dx^2}}}\right.}{3}}{5d}\right.}{d}\right.}{\sqrt{a+bx^2}}$$

input
```
Int[x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(-((x^3*(a + b*x^2)^(
3/2))/(d*Sqrt[c + d*x^2])) + (3*((2*b*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
/(5*d) - (((8*b*c - 7*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*d) - ((16
*b^2*c^2 - 16*a*b*c*d + a^2*d^2)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2])
- (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c
)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]
)) + (c^(3/2)*(8*b*c - 7*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)
/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))
]*Sqrt[c + d*x^2]))/(3*d))/(5*d)))/d))/Sqrt[a + b*x^2]
```

3.283.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.283. $\int x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx$

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

3.283. $\int x^4 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.283.4 Maple [A] (verified)

Time = 9.88 (sec) , antiderivative size = 775, normalized size of antiderivative = 1.98

method	result
risch	$\frac{x\left(bd\,x^2+2ad-3bc\right)\left(d\,x^2+c\right)e\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}{5d^3}+\frac{\left(-\frac{2\left(a^2d^2-11abcd+11b^2c^2\right)ace\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)-E\left(x\sqrt{-\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}\,(eda+ebc+e(ad-bc))}}{}$
default	$\left(\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{\frac{3}{2}}\left(d\,x^2+c\right)\left(\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,b^2d^3x^7+3\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,ab\,d^3x^5-2\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,b^2c\,d\right.$

input
```
int(x^4*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*x*(b*d*x^2+2*a*d-3*b*c)*(d*x^2+c)/d^3*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)+
1/5/d^3*(-2*(a^2*d^2-11*a*b*c*d+11*b^2*c^2)*a*c*e/(-b/a)^(1/2)*(1+b*x^2/a)
^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)/(e*
d*a+e*b*c+e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^
(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2)))-c*(7*a^2*
d^2-13*a*b*c*d+5*b^2*c^2)/d/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(
1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),
(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))+5*c^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d*((b*d*
e*x^2+a*d*e)/c/(a*d-b*c)*x/e/((x^2+c/d)*(b*d*e*x^2+a*d*e))^(1/2)+(1/c-d*a/
c/(a*d-b*c))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4
+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*
e)/c/b/e)^(1/2))+2*b*d/(a*d-b*c)*a*e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c
*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)/(e*d*a+e*b*c+e*(
a*d-b*c))*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-Ellipt
icE(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2)))))*e/(b*x^2+a)*(e*(b*x^
2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)
```

3.283.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.70

$$\int x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx =$$

$$-\frac{(16\,b^2c^3-16\,abc^2d+a^2cd^2)\sqrt{\frac{be}{d}}ex\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(16\,b^2c^3-16\,abc^2d-7\,a^2d^3+(a^2+8\,a}{}$$

input
```
integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/5*((16*b^2*c^3 - 16*a*b*c^2*d + a^2*c*d^2)*sqrt(b*e/d)*e*x*sqrt(-c/d)*e
lliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (16*b^2*c^3 - 16*a*b*c^2*d -
7*a^2*d^3 + (a^2 + 8*a*b)*c*d^2)*sqrt(b*e/d)*e*x*sqrt(-c/d)*elliptic_f(arc
sin(sqrt(-c/d)/x), a*d/(b*c)) - (b^2*d^3*e*x^6 - 2*(b^2*c*d^2 - a*b*d^3)*e
*x^4 + (8*b^2*c^2*d - 9*a*b*c*d^2 + a^2*d^3)*e*x^2 + (16*b^2*c^3 - 16*a*b*
c^2*d + a^2*c*d^2)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*d^4*x)
```

3.283.6 Sympy [F(-1)]

Timed out.

$$\int x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(x**4*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

3.283.7 Maxima [F]

$$\int x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx=\int\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}x^4\,dx$$

input
```
integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^4, x)
```

3.283.8 Giac [F]

$$\int x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx=\int\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}x^4\,dx$$

input
```
integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^4, x)
```

3.283.9 Mupad [F(-1)]

Timed out.

$$\int x^4 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \int x^4 \left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2} dx$$

input
```
int(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

output
```
int(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.284 $\int x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx$

3.284.1 Optimal result

Integrand size = 26, antiderivative size = 310

$$\int x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx=-\frac{(8bc-7ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3d^2}-\frac{ex(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d}+\frac{4bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3d^2}+\frac{\sqrt{c}(8bc-7ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}(4bc-3ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

output

```
-1/3*(-7*a*d+8*b*c)*e*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^2-e*x*(b*x^2+a)*(e
*(b*x^2+a)/(d*x^2+c))^(1/2)/d+4/3*b*e*x*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^
(1/2)/d^2+1/3*(-7*a*d+8*b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ell
ipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*(
b*x^2+a)/(d*x^2+c))^(1/2)/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)-1/3*(-3*
a*d+4*b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c
^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*(b*x^2+a)/(d*x^2+c)
)^(1/2)/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

3.284.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.96 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.76

$$\int x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx =$$

$$-\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}dx(a+bx^2)\left(3ad-b(4c+dx^2)\right)+ibc(-8bc+7ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right.\right.\right.}{3\sqrt{\frac{b}{a}}d^3\left(a+bx^2\right)}$$

input
```
Integrate[x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
-1/3*(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(a + b*x^2)*(3*a*
d - b*(4*c + d*x^2)) + I*b*c*(-8*b*c + 7*a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*(8*b^2*c^2
- 11*a*b*c*d + 3*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elliptic
F[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(Sqrt[b/a]*d^3*(a + b*x^2))
```

3.284.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2058, 369, 403, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\int\frac{x^2(bx^2+a)^{3/2}}{(dx^2+c)^{3/2}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{369}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{\int \frac{\sqrt{bx^2+a}(4bx^2+a)}{\sqrt{dx^2+c}}dx}{d}-\frac{x(a+bx^2)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{403}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{\frac{\int -\frac{b(8bc-7ad)x^2+a(4bc-3ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{4bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}}{d}-\frac{x(a+bx^2)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{\frac{4bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{\int \frac{b(8bc-7ad)x^2+a(4bc-3ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}}{d}-\frac{x(a+bx^2)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{406}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{\frac{4bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{a(4bc-3ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+b(8bc-7ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}}{d}-\frac{x(a+bx^2)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{320}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{\frac{4bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{b(8bc-7ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}(4bc-3ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3d}}{d}-\frac{x(a+bx}{d\sqrt{c+}}\right.}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{388}$$

3.284. $\int x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{\frac{4bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{b(8bc-7ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(4bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3d}}{d}\right.}{\sqrt{a+bx^2}}$$

$\downarrow$ 313

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{\frac{4bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{\frac{\sqrt{c}\sqrt{a+bx^2}(4bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+b(8bc-7ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\right.\right.}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c}{a}}}\right.}{3d}}{d}\right.}{\sqrt{a+bx^2}}$$

input
```
Int[x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(-((x*(a + b*x^2)^(3/
2))/(d*Sqrt[c + d*x^2])) + ((4*b*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*d)
- (b*(8*b*c - 7*a*d)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*S
qrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b
*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c
]*(4*b*c - 3*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d
*x^2]))/(3*d))/d))/Sqrt[a + b*x^2]
```

3.284. $\int x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx$

3.284.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.284. $\int x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.284.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 733 vs. $2(350) = 700$.

Time = 8.94 (sec) , antiderivative size = 734, normalized size of antiderivative = 2.37

method	result
default	$\frac{\left(\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}\right)^{\frac{3}{2}}\left(d\,x^{2}+c\right)\left(\sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\,\sqrt{-\frac{b}{a}}\,b^{2}d^{2}x^{5}+\sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\,\sqrt{-\frac{b}{a}}\,a b\,d^{2}x^{3}+\sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\,\sqrt{-\frac{b}{a}}\,b^{2}c d\,x^{3}\right.}{}$
risch	$\frac{bex\left(d\,x^{2}+c\right)\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}{3d^{2}}+\frac{\left(-\frac{3c\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\left(\frac{\left(bde\,x^{2}+eda\right)x}{c(ad-bc)e\sqrt{\left(x^{2}+\frac{c}{d}\right)\left(bde\,x^{2}+eda\right)}}+\frac{\left(\frac{1}{c}-\frac{da}{c(ad-bc)}\right)\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^{4}+ade\,x^{2}+bce\,x^{2}}}\right.}{}\right.}{}$

input
```
int(x^2*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)*(((d*x^2+c)*(b*x^2+a))^(1/2)*(
-b/a)^(1/2)*b^2*d^2*x^5+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a*b*d^2*x
^3+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*b^2*c*d*x^3-3*(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*d^2*x^3+3*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*(-b/a)^(1/2)*b^2*c*d*x^3+3*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)
/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^
2*d^2-11*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/
2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d+8*((d*x^2+c)*(b*x^2+a
))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),
(a*d/b/c)^(1/2))*b^2*c^2+7*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)
*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d-8*(
(d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipti
cE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b
/a)^(1/2)*a*b*c*d*x-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a^2
*d^2*x+3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*c*d*x)/(b*x^
2+a)^2/d^3/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

3.284.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.74

$$\int x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = \frac{(8\,b^2c^3-7\,abc^2d)\sqrt{\frac{be}{d}}ex\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(8\,b^2c^3-7\,abc^2d+4\,abc$$

input
```
integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*((8*b^2*c^3 - 7*a*b*c^2*d)*sqrt(b*e/d)*e*x*sqrt(-c/d)*elliptic_e(arcsi
n(sqrt(-c/d)/x), a*d/(b*c)) - (8*b^2*c^3 - 7*a*b*c^2*d + 4*a*b*c*d^2 - 3*a
^2*d^3)*sqrt(b*e/d)*e*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b
*c)) + (b^2*c*d^2*e*x^4 - 4*(b^2*c^2*d - a*b*c*d^2)*e*x^2 - (8*b^2*c^3 - 7
*a*b*c^2*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*c*d^3*x)
```

3.284.6 Sympy [F(-1)]

Timed out.

$$\int x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = \text{Timed out}$$

input
```
integrate(x**2*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

3.284.7 Maxima [F]

$$\int x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = \int\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^2, x)
```

3.284.8 Giac [F]

$$\int x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx=\int\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^2, x)
```

3.284.9 Mupad [F(-1)]

Timed out.

$$\int x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx=\int x^2\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}\,dx$$

input
```
int(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

output
```
int(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.285 $\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

3.285.1 Optimal result

Integrand size = 22, antiderivative size = 262

$$\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = -\frac{(bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}+\frac{(2bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} - \frac{(2bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{b\sqrt{c}e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

output
```
-(-a*d+b*c)*e*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c/d+(-a*d+2*b*c)*e*x*(e*(b*x
^2+a)/(d*x^2+c))^(1/2)/c/d-(-a*d+2*b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c
)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(
e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^(3/2)/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/
2)+b*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)
/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2
)/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

3.285.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.79

$$\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(ibc(-2bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+(-bc+\right.}{\sqrt{\frac{b}{a}}cd^2}$$

input
```
Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(I*b*c*(-2*b*c + a*d)*Sqrt[1 + (b*x^2
)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] +
(-(b*c) + a*d)*(Sqrt[b/a]*d*x*(a + b*x^2) - (2*I)*b*c*Sqrt[1 + (b*x^2)/a]*
Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])))/(Sqr
t[b/a]*c*d^2*(a + b*x^2))
```

3.285.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2058, 315, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)^{3/2}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{315}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{\int \frac{b((2bc-ad)x^2+ac)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}-\frac{x\sqrt{a+bx^2}(bc-ad)}{cd\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{b\int \frac{(2bc-ad)x^2+ac}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}-\frac{x\sqrt{a+bx^2}(bc-ad)}{cd\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 406$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{b\left(ac\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(2bc-ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{cd}-\frac{x\sqrt{a+bx^2}(bc-ad)}{cd\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 320$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{b\left((2bc-ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{cd}-\frac{x\sqrt{a+bx^2}(bc-ad)}{cd\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 388$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{b\left((2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{cd}-\frac{x\sqrt{a+bx^2}(bc-ad)}{cd\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 313$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{cd}-x\sqrt{\ }\right.}{\sqrt{a+bx^2}}$$

3.285. $\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

input
```
Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(-(((b*c - a*d)*x*Sqr
t[a + b*x^2])/(c*d*Sqrt[c + d*x^2])) + (b*((2*b*c - a*d)*((x*Sqrt[a + b*x^
2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[
d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c +
d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sq
rt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c +
 d*x^2))]*Sqrt[c + d*x^2])))/(c*d)))/Sqrt[a + b*x^2]
```

3.285.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.285.4 Maple [A] (verified)

Time = 3.34 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.01

method	result
default	$\frac{\left(\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}\right)^{\frac{3}{2}}(d\,x^{2}+c)\left(\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{-\frac{b}{a}}\,ab\,d^{2}x^{3}-\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{-\frac{b}{a}}\,b^{2}cd\,x^{3}+2\sqrt{(d\,x^{2}+c)(b\,x^{2}+a)}\,\sqrt{}\right.}{}$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)*((b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*(-b/a)^(1/2)*a*b*d^2*x^3-(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/
2)*b^2*c*d*x^3+2*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c
)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d-2*((d*x^2+c)*
(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a
)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2-((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)
^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c
*d+2*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*E
llipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2+(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*(-b/a)^(1/2)*a^2*d^2*x-(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/
a)^(1/2)*a*b*c*d*x)/(b*x^2+a)^2/d^2/c/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)
```

3.285.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.66

$$\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx =$$
$$-\frac{(2\,bc^2-acd)\sqrt{\frac{be}{d}}ex\sqrt{-\frac{c}{d}}E\left(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc}\right)-(2\,bc^2-acd+ad^2)\sqrt{\frac{be}{d}}ex\sqrt{-\frac{c}{d}}F\left(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc}\right)}{cd^2x}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
-((2*b*c^2 - a*c*d)*sqrt(b*e/d)*e*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d
)/x), a*d/(b*c)) - (2*b*c^2 - a*c*d + a*d^2)*sqrt(b*e/d)*e*x*sqrt(-c/d)*el
liptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (b*c*d*e*x^2 + (2*b*c^2 - a*c*
d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c*d^2*x)
```

3.285.6 Sympy [F(-1)]

Timed out.

$$\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

3.285.7 Maxima [F]

$$\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \int \left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

3.285.8 Giac [F]

$$\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \int \left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

3.285.9 Mupad [F(-1)]

Timed out.

$$\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \int \left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2} dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.286 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx$

3.286.1 Optimal result

Integrand size = 26, antiderivative size = 307

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx = -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx} - \frac{(bc-2ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^2} + \frac{(bc-2ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{c^2dx} + \frac{(bc-2ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{c^{3/2}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

output
```
-(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c/d/x-(-2*a*d+b*c)*e*x*(e*(b*x
^2+a)/(d*x^2+c))^(1/2)/c^2+(-2*a*d+b*c)*e*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c)
)^(1/2)/c^2/d/x+(-2*a*d+b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ell
ipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(e*(b*x^2+a)
/(d*x^2+c))^(1/2)/c^(3/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)+b*e*(1/(
1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c
)^(1/2),(1-b*c/a/d)^(1/2))*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^(1/2)/d^(1/2)/(
c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

3.286.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.25 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.74

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx =$$
$$-\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}d(a+bx^2)\,(ac-bcx^2+2adx^2)+ibc(-bc+2ad)x\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)\right.\right.}{\sqrt{\frac{b}{a}}c^2dx\,(a+bx^2)}$$

input
```
Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^2,x]
```

output
```
-((e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*(a + b*x^2)*(a*c - b*c
*x^2 + 2*a*d*x^2) + I*b*c*(-(b*c) + 2*a*d)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
(d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(-(b*c)
+ a*d)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[
b/a]*x], (a*d)/(b*c)]))/(Sqrt[b/a]*c^2*d*x*(a + b*x^2)))
```

3.286.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {2058, 370, 27, 445, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx$$

$$\Big\downarrow \ 2058$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\int \frac{(bx^2+a)^{3/2}}{x^2(dx^2+c)^{3/2}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 370$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{\int \frac{a\left(-bdx^2+bc-2ad\right)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}-\frac{\sqrt{a+bx^2}(bc-ad)}{cdx\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{a\int \frac{-bdx^2+bc-2ad}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}-\frac{\sqrt{a+bx^2}(bc-ad)}{cdx\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 445}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{a\left(-\frac{\int \frac{bd\left(ac-(bc-2ad)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{acx}\right)}{cd}-\frac{\sqrt{a+bx^2}(bc-ad)}{cdx\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{a\left(-\frac{bd\int \frac{ac-(bc-2ad)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{acx}\right)}{cd}-\frac{\sqrt{a+bx^2}(bc-ad)}{cdx\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 406}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{a\left(-\frac{bd\left(ac\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-(bc-2ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{acx}\right)}{cd}-\frac{\sqrt{a+bx^2}(bc-ad)}{cdx\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 320}$$

3.286. $\displaystyle \int \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{x^2}\,dx$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{a\left(-\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-(bc-2ad)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{acx}\right)}{cd}-\right.}{\sqrt{a+bx^2}}$$

↓ 388

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{a\left(-\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(}{acx}\right.}{cd}\right.}{\sqrt{a+bx^2}}$$

↓ 313

3.286. $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{a\left(-\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{ac}-\right.}{cd}\right.}{\sqrt{a+bx^2}}$$

input
```
Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^2,x]
```

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(-(((b*c - a*d)*Sqrt[
a + b*x^2])/(c*d*x*Sqrt[c + d*x^2])) - (a*(-(((b*c - 2*a*d)*Sqrt[a + b*x^2
]*Sqrt[c + d*x^2])/(a*c*x)) - (b*d*(-((b*c - 2*a*d)*((x*Sqrt[a + b*x^2])/(
b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)
/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2
))]*Sqrt[c + d*x^2]))) + (c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d
]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x
^2))]*Sqrt[c + d*x^2])))/(a*c)))/(c*d)))/Sqrt[a + b*x^2]
```

3.286.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

3.286. $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx$

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.286. $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx$

3.286.4 Maple [A] (verified)

Time = 8.05 (sec) , antiderivative size = 670, normalized size of antiderivative = 2.18

method	result
default	$-\frac{\left(\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}\right)^{\frac{3}{2}}\left(d\,x^{2}+c\right)\left(\sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\,\sqrt{-\frac{b}{a}}\,a b\,d^{2}x^{4}+\sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+a c}\,\sqrt{-\frac{b}{a}}\,a b\,d^{2}x^{4}-\sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+a c}\right.}{}$
risch	$-\frac{a\left(d\,x^{2}+c\right)e\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}{c^{2}x}+\frac{\left(\frac{b^{2}c^{2}\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)}{d\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^{4}+ade\,x^{2}+bce\,x^{2}+ace}}-\frac{2d\,a^{2}bce\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+}{cb}}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^{4}+ade\,x^{2}+bce\,x^{2}+ace}}\right.}{}$

input

```
int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output

```
-(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)*(((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/
a)^(1/2)*a*b*d^2*x^4+(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*
d^2*x^4-(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b^2*c*d*x^4+((d*x
^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x
*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d*x-((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x
^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2
))*b^2*c^2*x-2*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/
c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d*x+((d*x^2+c)*(b
*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^
(1/2),(a*d/b/c)^(1/2))*b^2*c^2*x+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*
a^2*d^2*x^2+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a*b*c*d*x^2+(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a^2*d^2*x^2-(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*c*d*x^2+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^
(1/2)*a^2*c*d)/(b*x^2+a)^2/c^2/x/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)/d
```

3.286.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.55

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx =$$
$$-\frac{(b^2c-2\,abd)\sqrt{\frac{ace}{d^2}}ex\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-(b^2c-(a^2+2\,ab)d)\sqrt{\frac{ace}{d^2}}ex\sqrt{-\frac{b}{a}}F(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)}{ac^2x}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
-((b^2*c - 2*a*b*d)*sqrt(a*c*e/d^2)*e*x*sqrt(-b/a)*elliptic_e(arcsin(x*sqr
t(-b/a)), a*d/(b*c)) - (b^2*c - (a^2 + 2*a*b)*d)*sqrt(a*c*e/d^2)*e*x*sqrt(
-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) + (a^2*c*e - (a*b*c - 2*
a^2*d)*e*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c^2*x)
```

3.286.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**2,x)
```

output
```
Timed out
```

3.286.7 Maxima [F]

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx = \int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^2, x)
```

3.286.8 Giac [F]

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx = \int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^2, x)
```

3.286.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx = \int \frac{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}{x^2}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^2,x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^2, x)
```

3.287 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx$

3.287.1 Optimal result

Integrand size = 26, antiderivative size = 383

$$\begin{aligned}\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx =& -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3}+\frac{d(7bc-8ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3c^3}\\&+\frac{(3bc-4ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3c^2dx^3}-\frac{(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3c^3x}\\&-\frac{\sqrt{d}(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\\&+\frac{b(3bc-4ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3ac^{3/2}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\end{aligned}$$

output

```
-(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c/d/x^3+1/3*d*(-8*a*d+7*b*c)*e
*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^3+1/3*(-4*a*d+3*b*c)*e*(d*x^2+c)*(e*(b*
x^2+a)/(d*x^2+c))^(1/2)/c^2/d/x^3-1/3*(-8*a*d+7*b*c)*e*(d*x^2+c)*(e*(b*x^2
+a)/(d*x^2+c))^(1/2)/c^3/x+1/3*b*(-4*a*d+3*b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1
+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^
(1/2))*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^(3/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x
^2+c))^(1/2)-1/3*(-8*a*d+7*b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*
EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(
e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

3.287.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.63 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.66

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}(a+bx^2)\left(-bcx^2(4c+7dx^2)+a(-c^2+4cdx^2+8d^2x^4)\right)+ibc(-7bc\right.}{}$$

input
```
Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^4,x]
```

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*(a + b*x^2)*(-(b*c*x^2*(4*
c + 7*d*x^2)) + a*(-c^2 + 4*c*d*x^2 + 8*d^2*x^4)) + I*b*c*(-7*b*c + 8*a*d)
*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]
*x], (a*d)/(b*c)] - (4*I)*b*c*(-(b*c) + a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[
1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*Sqrt[b/
a]*c^3*x^3*(a + b*x^2))
```

3.287.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.08, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {2058, 370, 445, 27, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx$$

$$\Big\downarrow \ 2058$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\int \frac{(bx^2+a)^{3/2}}{x^4(dx^2+c)^{3/2}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 370$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{\int \frac{b(2bc-3ad)x^2+a(3bc-4ad)}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}-\frac{\sqrt{a+bx^2}(bc-ad)}{cdx^3\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\downarrow 445$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{\int \frac{ad\left(b(3bc-4ad)x^2+a(7bc-8ad)\right)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-4ad)}{3cx^3}}{cd}-\frac{\sqrt{a+bx^2}(bc-ad)}{cdx^3\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\downarrow 27$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{d\int \frac{b(3bc-4ad)x^2+a(7bc-8ad)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-4ad)}{3cx^3}}{cd}-\frac{\sqrt{a+bx^2}(bc-ad)}{cdx^3\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\downarrow 445$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{d\left(-\frac{\int -\frac{ab\left(d(7bc-8ad)x^2+c(3bc-4ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-8ad)}{cx}\right)}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-4ad)}{3cx^3}}{cd}-\frac{\sqrt{a+bx^2}(bc-ad)}{cdx^3\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\downarrow 25$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{d\left(\frac{\int \frac{ab\left(d(7bc-8ad)x^2+c(3bc-4ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-8ad)}{cx}\right)}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-4ad)}{3cx^3}}{cd}-\frac{\sqrt{a+bx^2}(bc-ad)}{cdx^3\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\downarrow 27$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{d\left(\frac{b\int \frac{d(7bc-8ad)x^2+c(3bc-4ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-8ad)}{cx}\right)}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-4ad)}{3cx^3}}{cd}-\frac{\sqrt{a+bx^2}(bc-ad)}{cdx^3\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\downarrow 406$$

3.287. $\displaystyle\int \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{x^4}\,dx$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{d\left(\frac{b\left(c(3bc-4ad)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+d(7bc-8ad)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-8ad)}{cx}\right)}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c}}{3}}{cd}\right.}{\sqrt{a+bx^2}}$$

$\downarrow$ 320

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{d\left(\frac{b\left(d(7bc-8ad)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(3bc-4ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(7b}{cx}\right.}{3c}}{cd}\right.}{\sqrt{a+bx^2}}$$

$\downarrow$ 388

3.287. $\displaystyle\int\frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{x^4}\,dx$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{d\left(\frac{b\left(d(7bc-8ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(3bc-4ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{c}-\frac{\sqrt{a+bx^2}\cdots}{\cdots}\right)}{3c}}{cd}\cdots\right)}{\sqrt{a+bx^2}}$$

$\downarrow$ 313

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{d\left(\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(3bc-4ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+d(7bc-8ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\cdots\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\cdots\right)\cdots\right)}{c}\cdots\right)}{3c}}{cd}\cdots\right)}{\sqrt{a+bx^2}}$$

input
```
Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^4,x]
```

3.287. $\int\frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx$

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(-(((b*c - a*d)*Sqrt[
a + b*x^2])/(c*d*x^3*Sqrt[c + d*x^2])) - (-1/3*((3*b*c - 4*a*d)*Sqrt[a + b
*x^2]*Sqrt[c + d*x^2])/(c*x^3) - (d*(-(((7*b*c - 8*a*d)*Sqrt[a + b*x^2]*Sq
rt[c + d*x^2])/(c*x)) + (b*(d*(7*b*c - 8*a*d)*((x*Sqrt[a + b*x^2])/(b*Sqrt
[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sq
rt[c + d*x^2])) + (c^(3/2)*(3*b*c - 4*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTa
n[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/
(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/c))/(3*c))/(c*d)))/Sqrt[a + b*x^2]
```

3.287.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

3.287. $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx$

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.287.4 Maple [A] (verified)

Time = 8.28 (sec) , antiderivative size = 790, normalized size of antiderivative = 2.06

method	result
default	$-\frac{\left(\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{\frac{3}{2}}\left(d\,x^2+c\right)\left(-5\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{-\frac{b}{a}}\,ab\,d^2x^6+4\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{-\frac{b}{a}}\,b^2cd\,x^6-3\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\cdots\right.}{\cdots}$
risch	$-\frac{\left(d\,x^2+c\right)\left(-5ad\,x^2+4bc\,x^2+ac\right)e\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}{3c^3x^3}-\frac{\left(\frac{abcd\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}}-\frac{10a^2b\,d^2ce\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\cdots}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\cdots}}\cdots\right.}{\cdots}$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

3.287. $\int \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{x^4}\,dx$

output
```
-1/3*(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)*(-5*((d*x^2+c)*(b*x^2+a))^(1/
2)*(-b/a)^(1/2)*a*b*d^2*x^6+4*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*b^2
*c*d*x^6-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*d^2*x^6+3*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b^2*c*d*x^6-4*((d*x^2+c)*
(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a
)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d*x^3+4*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2
+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))
*b^2*c^2*x^3+8*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/
c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d*x^3-7*((d*x^2+c
)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b
/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2*x^3-5*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a
)^(1/2)*a^2*d^2*x^4+4*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*b^2*c^2*x^4
-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a^2*d^2*x^4+3*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*c*d*x^4-4*((d*x^2+c)*(b*x^2+a
))^(1/2)*(-b/a)^(1/2)*a^2*c*d*x^2+5*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/
2)*a*b*c^2*x^2+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a^2*c^2)/(b*x^2+a)
^2/c^3/x^3/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

3.287.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.56

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \frac{(7\,b^2cd - 8\,abd^2)\sqrt{\frac{ace}{d^2}}ex^3\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}) - ((3\,ab + 7\,b^2)cd - 4\,(a^2 + 2}{}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^4,x, algorithm="fricas")
```

output
```
1/3*((7*b^2*c*d - 8*a*b*d^2)*sqrt(a*c*e/d^2)*e*x^3*sqrt(-b/a)*elliptic_e(a
rcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((3*a*b + 7*b^2)*c*d - 4*(a^2 + 2*a*b)*d
^2)*sqrt(a*c*e/d^2)*e*x^3*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/
(b*c)) - ((7*a*b*c*d - 8*a^2*d^2)*e*x^4 + a^2*c^2*e + 4*(a*b*c^2 - a^2*c*d
)*e*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c^3*x^3)
```

3.287.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**4,x)
```

output
```
Timed out
```

3.287.7 Maxima [F]

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^4, x)
```

3.287.8 Giac [F]

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^4,x, algorithm="giac")
```

output
```
integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^4, x)
```

3.287.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}{x^4}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^4,x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^4, x)
```

3.288 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx$

3.288.1 Optimal result

Integrand size = 26, antiderivative size = 480

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx = -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} + \frac{d(b^2c^2-16abcd+16a^2d^2)\,ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5ac^4} + \frac{(5bc-6ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5c^2dx^5} - \frac{(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5c^3x^3} - \frac{(b^2c^2-16abcd+16a^2d^2)\,e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{5ac^4x} - \frac{\sqrt{d}(b^2c^2-16abcd+16a^2d^2)\,e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5ac^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{b\sqrt{d}(7bc-8ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{5ac^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

output
```
-(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c/d/x^5+1/5*d*(16*a^2*d^2-16*a
*b*c*d+b^2*c^2)*e*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^4+1/5*(-6*a*d+5*b*c)
*e*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^2/d/x^5-1/5*(-8*a*d+7*b*c)*e*
(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^3/x^3-1/5*(16*a^2*d^2-16*a*b*c*d
+b^2*c^2)*e*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^4/x-1/5*(16*a^2*d^
2-16*a*b*c*d+b^2*c^2)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(
x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(e*(b*x^2+a
)/(d*x^2+c))^(1/2)/a/c^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)-1/5*b*(-8*a*d
+7*b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1
/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(
1/2)/a/c^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

3.288.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.23 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.67

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx =$$

$$-\frac{\sqrt{\frac{b}{a}}e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}(a+bx^2)\left(b^2c^2x^4(c+dx^2)+abcx^2(2c^2-9cdx^2-16d^2x^4)+a^2(c^3-2c^2dx^2+8cd^2x^4\right.\right.}{}$$

input
```
Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^6,x]
```

output
```
-1/5*(Sqrt[b/a]*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*(a + b*x^2)
*(b^2*c^2*x^4*(c + d*x^2) + a*b*c*x^2*(2*c^2 - 9*c*d*x^2 - 16*d^2*x^4) + a
^2*(c^3 - 2*c^2*d*x^2 + 8*c*d^2*x^4 + 16*d^3*x^6)) + I*b*c*(b^2*c^2 - 16*a
*b*c*d + 16*a^2*d^2)*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE
[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(b^2*c^2 - 9*a*b*c*d + 8*a^2
*d^2)*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt
[b/a]*x], (a*d)/(b*c)]))/(b*c^4*x^5*(a + b*x^2))
```

3.288.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2058, 370, 445, 27, 445, 25, 27, 445, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}dx$$

$$\Big\downarrow \ 2058$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\int \frac{(bx^2+a)^{3/2}}{x^6(dx^2+c)^{3/2}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 370$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{\int \frac{b(4bc-5ad)x^2+a(5bc-6ad)}{x^6\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}-\frac{\sqrt{a+bx^2}(bc-ad)}{cdx^5\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 445$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{\int \frac{3ad\left(b(5bc-6ad)x^2+a(7bc-8ad)\right)}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(5bc-6ad)}{5cx^5}}{cd}-\frac{\sqrt{a+bx^2}(bc-ad)}{cdx^5\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 27$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{3d\int \frac{b(5bc-6ad)x^2+a(7bc-8ad)}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(5bc-6ad)}{5cx^5}}{cd}-\frac{\sqrt{a+bx^2}(bc-ad)}{cdx^5\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 445$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{3d\left(-\frac{\int -\frac{a\left(b^2c^2-16abdc+16a^2d^2-bd(7bc-8ad)x^2\right)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-8ad)}{3cx^3}\right)}{5c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(5bc-6ad)}{5cx^5}}{cd}-\cdots\right)}{\sqrt{a+bx^2}}$$

$$\downarrow 25$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{3d\left(\frac{\int \frac{a\left(b^2c^2-16abdc+16a^2d^2-bd(7bc-8ad)x^2\right)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-8ad)}{3cx^3}\right)}{5c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(5bc-6ad)}{5cx^5}}{cd}-\frac{\sqrt{a+\cdots}}{cd\cdots}\right)}{\sqrt{a+bx^2}}$$

$$\downarrow 27$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{3d\left(\frac{\int \frac{b^2c^2-16abdc+16a^2d^2-bd(7bc-8ad)x^2}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-8ad)}{3cx^3}\right)}{5c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(5bc-6ad)}{5cx^5}}{cd}-\frac{\sqrt{a+bx\cdots}}{cdx^5\cdots}\right)}{\sqrt{a+bx^2}}$$

$$\downarrow 445$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{3d\left(\frac{-\frac{\int \frac{bd\left(ac(7bc-8ad)-\left(b^2c^2-16abdc+16a^2d^2\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{b^2c}{a}+\frac{16ad^2}{c}-16bd\right)}{x}}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-\cdots}{3cx^3}\cdots\right)}{5c}\cdots}{cd}\cdots\right)}{\sqrt{a+bx^2}}$$

$$\downarrow 27$$

3.288. $\displaystyle\int \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{x^6}\,dx$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{3d\left(\frac{-\frac{bd\int\frac{ac(7bc-8ad)-\left(b^2c^2-16abdc+16a^2d^2\right)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{b^2c}{a}+\frac{16ad^2}{c}-16bd\right)}{x}}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-8\cdots}{3cx^3}\cdots\right)\cdots}{5c}\cdots}{cd}\cdots\right)}{\sqrt{a+bx^2}\cdots}$$

$\big\downarrow$ 406

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{3d\left(\frac{-\frac{bd\left(ac(7bc-8ad)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\left(16a^2d^2-16abcd+b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{b^2c}{a}+\cdots\right)}{x}\cdots\right)\cdots}{5c}\cdots}{cd}\cdots\right)}{\sqrt{a+bx^2}\cdots}$$

$\big\downarrow$ 320

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{3d\left(\frac{-\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}(7bc-8ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(16a^2d^2-16abcd+b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}}{3c}-\frac{\sqrt{\cdots}}{\cdots}\cdots\right)\cdots}{5c}\cdots}{cd}\cdots\right)}{\sqrt{a+bx^2}\cdots}$$

$\big\downarrow$ 388

3.288. $\displaystyle\int\frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{x^6}\,dx$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{3d\left(-\frac{\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}(7bc-8ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\left(16a^2d^2-16abcd+b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/}}}{b}\right.\cdots}{ac}}{3c}\cdots}{5c}\cdots}{cd}\cdots}{\sqrt{a+bx^2}\cdots}$$

$\downarrow$ 313

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{3d\left(-\frac{\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}(7bc-8ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\left(16a^2d^2-16abcd+b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\cdots\right.}{b\sqrt{d}\sqrt{c+\cdots}}\right.\cdots}{ac}}{3c}\cdots}{5c}\cdots}{\sqrt{a+\cdots}}$$

input

```
Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^6,x]
```

3.288. $\displaystyle\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx$

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(-(((b*c - a*d)*Sqrt[
a + b*x^2])/(c*d*x^5*Sqrt[c + d*x^2])) - (-1/5*((5*b*c - 6*a*d)*Sqrt[a + b
*x^2]*Sqrt[c + d*x^2])/(c*x^5) - (3*d*(-1/3*((7*b*c - 8*a*d)*Sqrt[a + b*x^
2]*Sqrt[c + d*x^2])/(c*x^3) + (-((((b^2*c)/a - 16*b*d + (16*a*d^2)/c)*Sqrt
[a + b*x^2]*Sqrt[c + d*x^2])/x) - (b*d*(-((b^2*c^2 - 16*a*b*c*d + 16*a^2*d
^2)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (c^(3/2)*(7*b*c - 8*a*d
)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(a*c))/
(3*c)))/(5*c))/(c*d)))/Sqrt[a + b*x^2]
```

3.288.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

3.288. $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx$

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.288.4 Maple [A] (verified)

Time = 10.05 (sec) , antiderivative size = 908, normalized size of antiderivative = 1.89

method	result
risch	$-\frac{(d\,x^2+c)\left(11a^2d^2x^4-11bdac\,x^4+b^2c^2x^4-3a^2cd\,x^2+2ab\,c^2x^2+a^2c^2\right)e\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}{5c^4x^5a}+\frac{d\left(-\frac{2b^2a\,c^2\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x^2+bce\,x^2}}\right.}{}$
default	Expression too large to display

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(d*x^2+c)*(11*a^2*d^2*x^4-11*a*b*c*d*x^4+b^2*c^2*x^4-3*a^2*c*d*x^2+2*
a*b*c^2*x^2+a^2*c^2)/c^4/x^5/a*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/5/c^4/a*d
*(-2*b^2*a*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x
^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*
c*e)/c/b/e)^(1/2))+3*a^2*b*c*d/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2
)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/
2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-2*(11*a^2*b*d^2-11*a*b^2*c*d+b^3*c^2)*a
*c*e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x
^2+b*c*e*x^2+a*c*e)^(1/2)/(e*d*a+e*b*c+e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1
/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*
c*e)/c/b/e)^(1/2)))-5*(a^2*d^2-2*a*b*c*d+b^2*c^2)*a*c*((b*d*e*x^2+a*d*e)/c
/(a*d-b*c)*x/e/((x^2+c/d)*(b*d*e*x^2+a*d*e))^(1/2)+(1/c-d*a/c/(a*d-b*c))/(
-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c
*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2
))+2*b*d/(a*d-b*c)*a*e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/
(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)/(e*d*a+e*b*c+e*(a*d-b*c))*(Ell
ipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-EllipticE(x*(-b/a)^(
1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2)))))*e/(b*x^2+a)*(e*(b*x^2+a)/(d*x^2+c)
)^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)
```

3.288.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.61

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx = \frac{(b^3c^2d - 16\,ab^2cd^2 + 16\,a^2bd^3)\sqrt{\frac{ace}{d^2}}ex^5\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}) - (b^3c^2d - (7\,a^2$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^6,x, algorithm="fricas")
```

output
```
1/5*((b^3*c^2*d - 16*a*b^2*c*d^2 + 16*a^2*b*d^3)*sqrt(a*c*e/d^2)*e*x^5*sqr
t(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (b^3*c^2*d - (7*a^2*
b + 16*a*b^2)*c*d^2 + 8*(a^3 + 2*a^2*b)*d^3)*sqrt(a*c*e/d^2)*e*x^5*sqrt(-b
/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((a*b^2*c^2*d - 16*a^2*b
*c*d^2 + 16*a^3*d^3)*e*x^6 + a^3*c^3*e + (a*b^2*c^3 - 9*a^2*b*c^2*d + 8*a^
3*c*d^2)*e*x^4 + 2*(a^2*b*c^3 - a^3*c^2*d)*e*x^2)*sqrt((b*e*x^2 + a*e)/(d*
x^2 + c)))/(a^2*c^4*x^5)
```

3.288.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**6,x)
```

output
```
Timed out
```

3.288.7 Maxima [F]

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx = \int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^6,x, algorithm="maxima")
```

output
```
integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^6, x)
```

3.288.8 Giac [F]

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx = \int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^6,x, algorithm="giac")
```

output
```
integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^6, x)
```

3.288.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6} \, dx = \int \frac{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}{x^6} \, dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^6,x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^6, x)
```

3.289 $\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx$

3.289.1 Optimal result

Integrand size = 21, antiderivative size = 51

$$\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx=\frac{1}{2}\sqrt{\frac{1-x^2}{1+x^2}}(1+x^2)-\arctan\left(\sqrt{\frac{1-x^2}{1+x^2}}\right)$$

output
```
-arctan(((-x^2+1)/(x^2+1))^(1/2))+1/2*(x^2+1)*((-x^2+1)/(x^2+1))^(1/2)
```

3.289.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.86

$$\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx=\frac{\sqrt{\frac{1-x^2}{1+x^2}}\left(\sqrt{1-x^2}(1+x^2)+4\sqrt{1+x^2}\arctan\left(\frac{\sqrt{1-x^2}}{\sqrt{2}-\sqrt{1+x^2}}\right)\right)}{2\sqrt{1-x^2}}$$

input
```
Integrate[x*Sqrt[(1 - x^2)/(1 + x^2)],x]
```

output
```
(Sqrt[(1 - x^2)/(1 + x^2)]*(Sqrt[1 - x^2]*(1 + x^2) + 4*Sqrt[1 + x^2]*ArcT
an[Sqrt[1 - x^2]/(Sqrt[2] - Sqrt[1 + x^2])]))/(2*Sqrt[1 - x^2])
```

3.289.3 Rubi [A] (warning: unable to verify)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2053, 2051, 252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\frac{1-x^2}{x^2+1}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \sqrt{\frac{1-x^2}{x^2+1}}dx^2$$

$$\Big\downarrow \text{2051}$$

$$-2\int \frac{x^4}{(x^4+1)^2}d\sqrt{\frac{1-x^2}{x^2+1}}$$

$$\Big\downarrow \text{252}$$

$$-2\left(\frac{1}{2}\int \frac{1}{x^4+1}d\sqrt{\frac{1-x^2}{x^2+1}}-\frac{\sqrt{\frac{1-x^2}{x^2+1}}}{2(x^4+1)}\right)$$

$$\Big\downarrow \text{216}$$

$$-2\left(\frac{1}{2}\arctan\left(\sqrt{\frac{1-x^2}{x^2+1}}\right)-\frac{\sqrt{\frac{1-x^2}{x^2+1}}}{2(x^4+1)}\right)$$

input
```
Int[x*Sqrt[(1 - x^2)/(1 + x^2)],x]
```

output
```
-2*(-1/2*Sqrt[(1 - x^2)/(1 + x^2)]/(1 + x^4) + ArcTan[Sqrt[(1 - x^2)/(1 +
x^2)]]/2)
```

3.289.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2051
```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)   Subst[Int[
x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
 x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x]
&& FractionQ[p] && IntegerQ[1/n]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.289.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{\sqrt{-\frac{x^2-1}{x^2+1}}\,(x^2+1)\left(\arcsin(x^2)+\sqrt{-x^4+1}\right)}{2\sqrt{-(x^2+1)(x^2-1)}}$	52
risch	$\frac{(x^2+1)\sqrt{-\frac{x^2-1}{x^2+1}}}{2}-\frac{\arcsin(x^2)\sqrt{-\frac{x^2-1}{x^2+1}}\,\sqrt{-(x^2+1)(x^2-1)}}{2(x^2-1)}$	68
trager	$\left(\frac{x^2}{2}+\frac{1}{2}\right)\sqrt{-\frac{x^2-1}{x^2+1}}+\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{x^2-1}{x^2+1}}\,x^2+\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{x^2-1}{x^2+1}}+x^2\right)}{2}$	88

input
```
int(x*((-x^2+1)/(x^2+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-(x^2-1)/(x^2+1))^(1/2)*(x^2+1)*(arcsin(x^2)+(-x^4+1)^(1/2))/(-(x^2+1
)*(x^2-1))^(1/2)
```

3.289.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx = \frac{1}{2}\left(x^2+1\right)\sqrt{-\frac{x^2-1}{x^2+1}} - \arctan\left(\frac{(x^2+1)\sqrt{-\frac{x^2-1}{x^2+1}}-1}{x^2}\right)$$

input
```
integrate(x*((-x^2+1)/(x^2+1))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(x^2 + 1)*sqrt(-(x^2 - 1)/(x^2 + 1)) - arctan(((x^2 + 1)*sqrt(-(x^2 -
1)/(x^2 + 1)) - 1)/x^2)
```

3.289.6 Sympy [F]

$$\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx = \int x\sqrt{-\frac{(x-1)(x+1)}{x^2+1}}\,dx$$

input
```
integrate(x*((-x**2+1)/(x**2+1))**(1/2),x)
```

output
```
Integral(x*sqrt(-(x - 1)*(x + 1)/(x**2 + 1)), x)
```

3.289.7 Maxima [F]

$$\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx = \int x\sqrt{-\frac{x^2-1}{x^2+1}}\,dx$$

input
```
integrate(x*((-x^2+1)/(x^2+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*sqrt(-(x^2 - 1)/(x^2 + 1)), x)
```

3.289.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.35

$$\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx = \frac{1}{2}\sqrt{-x^4+1} + \frac{1}{2}\arcsin\left(x^2\right)$$

input
```
integrate(x*((-x^2+1)/(x^2+1))^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^4 + 1) + 1/2*arcsin(x^2)
```

3.289.9 Mupad [B] (verification not implemented)

Time = 17.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.08

$$\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx = -\mathrm{atan}\left(\sqrt{-\frac{x^2-1}{x^2+1}}\right) - \frac{\sqrt{-\frac{x^2-1}{x^2+1}}}{\frac{x^2-1}{x^2+1}-1}$$

input
```
int(x*(-(x^2 - 1)/(x^2 + 1))^(1/2),x)
```

output
```
- atan((-(x^2 - 1)/(x^2 + 1))^(1/2)) - (-(x^2 - 1)/(x^2 + 1))^(1/2)/((x^2
- 1)/(x^2 + 1) - 1)
```

3.290 $\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx$

3.290.1 Optimal result

Integrand size = 23, antiderivative size = 72

$$\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx=\frac{1}{10}\sqrt{\frac{5-7x^2}{7+5x^2}}\left(7+5x^2\right)-\frac{37\arctan\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^2}{7+5x^2}}\right)}{5\sqrt{35}}$$

output
```
-37/175*arctan(1/7*35^(1/2)*((-7*x^2+5)/(5*x^2+7))^(1/2))*35^(1/2)+1/10*(5
*x^2+7)*((-7*x^2+5)/(5*x^2+7))^(1/2)
```

3.290.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.65

$$\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx=\frac{\sqrt{\frac{5-7x^2}{7+5x^2}}\left(35\sqrt{5-7x^2}(7+5x^2)+148\sqrt{35}\sqrt{7+5x^2}\arctan\left(\frac{\sqrt{5}\sqrt{5-7x^2}}{\sqrt{74}-\sqrt{7}\sqrt{7+5x^2}}\right)\right)}{350\sqrt{5-7x^2}}$$

input
```
Integrate[x*Sqrt[(5 - 7*x^2)/(7 + 5*x^2)],x]
```

output
```
(Sqrt[(5 - 7*x^2)/(7 + 5*x^2)]*(35*Sqrt[5 - 7*x^2]*(7 + 5*x^2) + 148*Sqrt[
35]*Sqrt[7 + 5*x^2]*ArcTan[(Sqrt[5]*Sqrt[5 - 7*x^2])/(Sqrt[74] - Sqrt[7]*S
qrt[7 + 5*x^2])]))/(350*Sqrt[5 - 7*x^2])
```

3.290.3 Rubi [A] (warning: unable to verify)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2053, 2051, 252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\frac{5-7x^2}{5x^2+7}}\,dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \sqrt{\frac{5-7x^2}{5x^2+7}}dx^2$$

$$\Big\downarrow \text{2051}$$

$$-74\int \frac{x^4}{(5x^4+7)^2}d\sqrt{\frac{5-7x^2}{5x^2+7}}$$

$$\Big\downarrow \text{252}$$

$$-74\left(\frac{1}{10}\int \frac{1}{5x^4+7}d\sqrt{\frac{5-7x^2}{5x^2+7}}-\frac{\sqrt{\frac{5-7x^2}{5x^2+7}}}{10\,(5x^4+7)}\right)$$

$$\Big\downarrow \text{216}$$

$$-74\left(\frac{\arctan\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^2}{5x^2+7}}\right)}{10\sqrt{35}}-\frac{\sqrt{\frac{5-7x^2}{5x^2+7}}}{10\,(5x^4+7)}\right)$$

input
```
Int[x*Sqrt[(5 - 7*x^2)/(7 + 5*x^2)],x]
```

output
```
-74*(-1/10*Sqrt[(5 - 7*x^2)/(7 + 5*x^2)]/(7 + 5*x^4) + ArcTan[Sqrt[5/7]*Sq
rt[(5 - 7*x^2)/(7 + 5*x^2)]]/(10*Sqrt[35]))
```

3.290.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2051
```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)   Subst[Int[
x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
 x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x]
&& FractionQ[p] && IntegerQ[1/n]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x))/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.290.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08

method	result
default	$\frac{\sqrt{-\frac{7x^2-5}{5x^2+7}}\,(5x^2+7)\left(37\sqrt{35}\,\arcsin\left(\frac{35x^2}{37}+\frac{12}{37}\right)+35\sqrt{-35x^4-24x^2+35}\right)}{350\sqrt{-(7x^2-5)(5x^2+7)}}$
risch	$\frac{(5x^2+7)\sqrt{-\frac{7x^2-5}{5x^2+7}}}{10}-\frac{37\sqrt{35}\,\arcsin\left(\frac{35x^2}{37}+\frac{12}{37}\right)\sqrt{-\frac{7x^2-5}{5x^2+7}}\,\sqrt{-(7x^2-5)(5x^2+7)}}{350(7x^2-5)}$
trager	$7\left(\frac{x^2}{14}+\frac{1}{10}\right)\sqrt{-\frac{7x^2-5}{5x^2+7}}-\frac{37\,\mathrm{RootOf}(\_Z^2+35)\ln(35\,\mathrm{RootOf}(\_Z^2+35)x^2+175\sqrt{-\frac{7x^2-5}{5x^2+7}}\,x^2+12\,\mathrm{RootOf}(\_Z^2+35)+}{350}$

input
```
int(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/350*(-(7*x^2-5)/(5*x^2+7))^(1/2)*(5*x^2+7)*(37*35^(1/2)*arcsin(35/37*x^2
+12/37)+35*(-35*x^4-24*x^2+35)^(1/2))/(-(7*x^2-5)*(5*x^2+7))^(1/2)
```

3.290.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

$$\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx$$
$$=\frac{1}{10}\left(5\,x^2+7\right)\sqrt{-\frac{7\,x^2-5}{5\,x^2+7}}-\frac{37}{350}\,\sqrt{35}\arctan\left(\frac{\sqrt{35}(35\,x^2+12)\sqrt{-\frac{7\,x^2-5}{5\,x^2+7}}}{35\,(7\,x^2-5)}\right)$$

input
```
integrate(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x, algorithm="fricas")
```

output
```
1/10*(5*x^2 + 7)*sqrt(-(7*x^2 - 5)/(5*x^2 + 7)) - 37/350*sqrt(35)*arctan(1
/35*sqrt(35)*(35*x^2 + 12)*sqrt(-(7*x^2 - 5)/(5*x^2 + 7))/(7*x^2 - 5))
```

3.290.6 Sympy [F]

$$\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx=\int x\sqrt{-\frac{7x^2-5}{5x^2+7}}\,dx$$

input
```
integrate(x*((-7*x**2+5)/(5*x**2+7))**(1/2),x)
```

output
```
Integral(x*sqrt(-(7*x**2 - 5)/(5*x**2 + 7)), x)
```

3.290.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06

$$\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx=-\frac{37}{175}\,\sqrt{35}\arctan\left(\frac{1}{7}\,\sqrt{35}\sqrt{-\frac{7\,x^2-5}{5\,x^2+7}}\right)-\frac{37\,\sqrt{-\frac{7\,x^2-5}{5\,x^2+7}}}{5\left(\frac{5\,(7\,x^2-5)}{5\,x^2+7}-7\right)}$$

input
```
integrate(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x, algorithm="maxima")
```

output
```
-37/175*sqrt(35)*arctan(1/7*sqrt(35)*sqrt(-(7*x^2 - 5)/(5*x^2 + 7))) - 37/
5*sqrt(-(7*x^2 - 5)/(5*x^2 + 7))/(5*(7*x^2 - 5)/(5*x^2 + 7) - 7)
```

3.290.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.42

$$\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx = \frac{37}{350}\sqrt{35}\arcsin\left(\frac{35}{37}x^2+\frac{12}{37}\right)+\frac{1}{10}\sqrt{-35\,x^4-24\,x^2+35}$$

input
```
integrate(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x, algorithm="giac")
```

output
```
37/350*sqrt(35)*arcsin(35/37*x^2 + 12/37) + 1/10*sqrt(-35*x^4 - 24*x^2 + 3
5)
```

3.290.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.22

$$\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx = -\frac{37\sqrt{35}\,\mathrm{atan}\left(\frac{\sqrt{5}\,\sqrt{7}\,\sqrt{-\frac{7\,x^2-5}{5\,x^2+7}}}{7}\right)}{175}-\frac{37\sqrt{5}\,\sqrt{7}\,\sqrt{35}\,\sqrt{-\frac{7\,x^2-5}{5\,x^2+7}}}{1225\,\left(\frac{5\,x^2-\frac{25}{7}}{5\,x^2+7}-1\right)}$$

input
```
int(x*(-(7*x^2 - 5)/(5*x^2 + 7))^(1/2),x)
```

output
```
- (37*35^(1/2)*atan((5^(1/2)*7^(1/2)*(-(7*x^2 - 5)/(5*x^2 + 7))^(1/2))/7))
/175 - (37*5^(1/2)*7^(1/2)*35^(1/2)*(-(7*x^2 - 5)/(5*x^2 + 7))^(1/2))/(122
5*((5*x^2 - 25/7)/(5*x^2 + 7) - 1))
```

3.291 $\int x^2\sqrt{\frac{1-x^3}{1+x^3}}\,dx$

3.291.1 Optimal result

Integrand size = 23, antiderivative size = 53

$$\int x^2\sqrt{\frac{1-x^3}{1+x^3}}\,dx=\frac{1}{3}\sqrt{\frac{1-x^3}{1+x^3}}(1+x^3)-\frac{2}{3}\arctan\left(\sqrt{\frac{1-x^3}{1+x^3}}\right)$$

output
```
-2/3*arctan(((-x^3+1)/(x^3+1))^(1/2))+1/3*(x^3+1)*((-x^3+1)/(x^3+1))^(1/2)
```

3.291.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.79

$$\int x^2\sqrt{\frac{1-x^3}{1+x^3}}\,dx=\frac{\sqrt{\frac{1-x^3}{1+x^3}}\left(\sqrt{1-x^3}(1+x^3)+4\sqrt{1+x^3}\arctan\left(\frac{\sqrt{1-x^3}}{\sqrt{2}-\sqrt{1+x^3}}\right)\right)}{3\sqrt{1-x^3}}$$

input
```
Integrate[x^2*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

output
```
(Sqrt[(1 - x^3)/(1 + x^3)]*(Sqrt[1 - x^3]*(1 + x^3) + 4*Sqrt[1 + x^3]*ArcT
an[Sqrt[1 - x^3]/(Sqrt[2] - Sqrt[1 + x^3])]))/(3*Sqrt[1 - x^3])
```

3.291.3 Rubi [A] (warning: unable to verify)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2053, 2051, 252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{\frac{1-x^3}{x^3+1}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{3}\int \sqrt{\frac{1-x^3}{x^3+1}}dx^3$$

$$\Big\downarrow \text{2051}$$

$$-\frac{4}{3}\int \frac{x^6}{(x^6+1)^2}d\sqrt{\frac{1-x^3}{x^3+1}}$$

$$\Big\downarrow \text{252}$$

$$-\frac{4}{3}\left(\frac{1}{2}\int \frac{1}{x^6+1}d\sqrt{\frac{1-x^3}{x^3+1}}-\frac{\sqrt{\frac{1-x^3}{x^3+1}}}{2(x^6+1)}\right)$$

$$\Big\downarrow \text{216}$$

$$-\frac{4}{3}\left(\frac{1}{2}\arctan\left(\sqrt{\frac{1-x^3}{x^3+1}}\right)-\frac{\sqrt{\frac{1-x^3}{x^3+1}}}{2(x^6+1)}\right)$$

input
```
Int[x^2*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

output
```
(-4*(-1/2*Sqrt[(1 - x^3)/(1 + x^3)]/(1 + x^6) + ArcTan[Sqrt[(1 - x^3)/(1 +
x^3)]]/2))/3
```

3.291.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2051
```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)   Subst[Int[
x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
 x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x]
&& FractionQ[p] && IntegerQ[1/n]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.291.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.28

method	result	size
risch	$\frac{(x^3+1)\sqrt{-\frac{x^3-1}{x^3+1}}}{3}-\frac{\arcsin(x^3)\sqrt{-\frac{x^3-1}{x^3+1}}\,\sqrt{-(x^3+1)(x^3-1)}}{3(x^3-1)}$	68
trager	$\left(\frac{x^3}{3}+\frac{1}{3}\right)\sqrt{-\frac{x^3-1}{x^3+1}}+\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{x^3-1}{x^3+1}}\,x^3+x^3+\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{x^3-1}{x^3+1}}\right)}{3}$	88

input
```
int(x^2*((-x^3+1)/(x^3+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(x^3+1)*(-(x^3-1)/(x^3+1))^(1/2)-1/3*arcsin(x^3)*(-(x^3-1)/(x^3+1))^(1
/2)*(-(x^3+1)*(x^3-1))^(1/2)/(x^3-1)
```

3.291.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int x^2\sqrt{\frac{1-x^3}{1+x^3}}\,dx = \frac{1}{3}\left(x^3+1\right)\sqrt{-\frac{x^3-1}{x^3+1}} - \frac{2}{3}\arctan\left(\frac{\left(x^3+1\right)\sqrt{-\frac{x^3-1}{x^3+1}}-1}{x^3}\right)$$

input
```
integrate(x^2*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(x^3 + 1)*sqrt(-(x^3 - 1)/(x^3 + 1)) - 2/3*arctan(((x^3 + 1)*sqrt(-(x^
3 - 1)/(x^3 + 1)) - 1)/x^3)
```

3.291.6 Sympy [F]

$$\int x^2\sqrt{\frac{1-x^3}{1+x^3}}\,dx = \int x^2\sqrt{-\frac{(x-1)\left(x^2+x+1\right)}{x^3+1}}\,dx$$

input
```
integrate(x**2*((-x**3+1)/(x**3+1))**(1/2),x)
```

output
```
Integral(x**2*sqrt(-(x - 1)*(x**2 + x + 1)/(x**3 + 1)), x)
```

3.291.7 Maxima [F]

$$\int x^2\sqrt{\frac{1-x^3}{1+x^3}}\,dx = \int x^2\sqrt{-\frac{x^3-1}{x^3+1}}\,dx$$

input
```
integrate(x^2*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*sqrt(-(x^3 - 1)/(x^3 + 1)), x)
```

3.291.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.42

$$\int x^2\sqrt{\frac{1-x^3}{1+x^3}}\,dx = \frac{1}{3}\left(\sqrt{-x^6+1}+\arcsin\left(x^3\right)\right)\operatorname{sgn}\left(x^3+1\right)$$

input
```
integrate(x^2*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="giac")
```

output
```
1/3*(sqrt(-x^6 + 1) + arcsin(x^3))*sgn(x^3 + 1)
```

3.291.9 Mupad [B] (verification not implemented)

Time = 16.81 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int x^2\sqrt{\frac{1-x^3}{1+x^3}}\,dx = -\frac{2\,\mathrm{atan}\left(\sqrt{-\frac{x^3-1}{x^3+1}}\right)}{3}-\frac{2\,\sqrt{-\frac{x^3-1}{x^3+1}}}{\frac{3\,(x^3-1)}{x^3+1}-3}$$

input
```
int(x^2*(-(x^3 - 1)/(x^3 + 1))^(1/2),x)
```

output
```
- (2*atan((-(x^3 - 1)/(x^3 + 1))^(1/2)))/3 - (2*(-(x^3 - 1)/(x^3 + 1))^(1/
2))/((3*(x^3 - 1))/(x^3 + 1) - 3)
```

3.292 $\int x^8 \sqrt{\frac{1-x^3}{1+x^3}} \, dx$

3.292.1 Optimal result

Integrand size = 23, antiderivative size = 113

$$\int x^8 \sqrt{\frac{1-x^3}{1+x^3}} \, dx = \frac{1}{2}\sqrt{\frac{1-x^3}{1+x^3}}(1+x^3) - \frac{1}{6}\sqrt{\frac{1-x^3}{1+x^3}}(1+x^3)^2 - \frac{1}{9}\left(\frac{1-x^3}{1+x^3}\right)^{3/2}(1+x^3)^3 - \frac{1}{3}\arctan\left(\sqrt{\frac{1-x^3}{1+x^3}}\right)$$

output
```
-1/9*((-x^3+1)/(x^3+1))^(3/2)*(x^3+1)^3-1/3*arctan(((-x^3+1)/(x^3+1))^(1/2
))+1/2*(x^3+1)*((-x^3+1)/(x^3+1))^(1/2)-1/6*(x^3+1)^2*((-x^3+1)/(x^3+1))^(
1/2)
```

3.292.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.84

$$\int x^8 \sqrt{\frac{1-x^3}{1+x^3}} \, dx = \frac{\sqrt{\frac{1-x^3}{1+x^3}}\left(\sqrt{1-x^3}(4+x^3-x^6+2x^9) - 6\sqrt{1+x^3}\arctan\left(\frac{\sqrt{1-x^3}}{\sqrt{1+x^3}}\right)\right)}{18\sqrt{1-x^3}}$$

input
```
Integrate[x^8*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

output
```
(Sqrt[(1 - x^3)/(1 + x^3)]*(Sqrt[1 - x^3]*(4 + x^3 - x^6 + 2*x^9) - 6*Sqrt
[1 + x^3]*ArcTan[Sqrt[1 - x^3]/Sqrt[1 + x^3]]))/(18*Sqrt[1 - x^3])
```

3.292.3 Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2053, 2052, 366, 27, 360, 27, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8\sqrt{\frac{1-x^3}{x^3+1}}dx$$

$$\Big\downarrow \; 2053$$

$$\frac{1}{3}\int x^6\sqrt{\frac{1-x^3}{x^3+1}}dx^3$$

$$\Big\downarrow \; 2052$$

$$-\frac{4}{3}\int \frac{x^6\left(1-x^6\right)^2}{\left(x^6+1\right)^4}d\sqrt{\frac{1-x^3}{x^3+1}}$$

$$\Big\downarrow \; 366$$

$$-\frac{4}{3}\left(\frac{2x^9}{3\left(x^6+1\right)^3}-\frac{1}{6}\int \frac{6x^6\left(1-x^6\right)}{\left(x^6+1\right)^3}d\sqrt{\frac{1-x^3}{x^3+1}}\right)$$

$$\Big\downarrow \; 27$$

$$-\frac{4}{3}\left(\frac{2x^9}{3\left(x^6+1\right)^3}-\int \frac{x^6\left(1-x^6\right)}{\left(x^6+1\right)^3}d\sqrt{\frac{1-x^3}{x^3+1}}\right)$$

$$\Big\downarrow \; 360$$

$$-\frac{4}{3}\left(\frac{1}{4}\int -\frac{2\left(1-2x^6\right)}{\left(x^6+1\right)^2}d\sqrt{\frac{1-x^3}{x^3+1}}+\frac{2x^9}{3\left(x^6+1\right)^3}+\frac{\sqrt{\frac{1-x^3}{x^3+1}}}{2\left(x^6+1\right)^2}\right)$$

$$\Big\downarrow \; 27$$

$$-\frac{4}{3}\left(-\frac{1}{2}\int \frac{1-2x^6}{\left(x^6+1\right)^2}d\sqrt{\frac{1-x^3}{x^3+1}}+\frac{2x^9}{3\left(x^6+1\right)^3}+\frac{\sqrt{\frac{1-x^3}{x^3+1}}}{2\left(x^6+1\right)^2}\right)$$

$$\Big\downarrow \; 298$$

$$-\frac{4}{3}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^6+1}d\sqrt{\frac{1-x^3}{x^3+1}}-\frac{3\sqrt{\frac{1-x^3}{x^3+1}}}{2\left(x^6+1\right)}\right)+\frac{2x^9}{3\left(x^6+1\right)^3}+\frac{\sqrt{\frac{1-x^3}{x^3+1}}}{2\left(x^6+1\right)^2}\right)$$

$$\Big\downarrow \text{216}$$

$$-\frac{4}{3}\left(\frac{1}{2}\left(\frac{1}{2}\arctan\left(\sqrt{\frac{1-x^3}{x^3+1}}\right)-\frac{3\sqrt{\frac{1-x^3}{x^3+1}}}{2\left(x^6+1\right)}\right)+\frac{2x^9}{3\left(x^6+1\right)^3}+\frac{\sqrt{\frac{1-x^3}{x^3+1}}}{2\left(x^6+1\right)^2}\right)$$

input
```
Int[x^8*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

output
```
(-4*((2*x^9)/(3*(1 + x^6)^3) + Sqrt[(1 - x^3)/(1 + x^3)]/(2*(1 + x^6)^2) +
 ((-3*Sqrt[(1 - x^3)/(1 + x^3)])/(2*(1 + x^6)) + ArcTan[Sqrt[(1 - x^3)/(1
+ x^3)]]/2)/2))/3
```

3.292.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.292.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.71

method	result
risch	$\frac{(x^3+1)(2x^6-3x^3+4)\sqrt{-\frac{x^3-1}{x^3+1}}}{18}-\frac{\arcsin(x^3)\sqrt{-\frac{x^3-1}{x^3+1}}\sqrt{-(x^3+1)(x^3-1)}}{6(x^3-1)}$
trager	$\frac{(x^3+1)(2x^6-3x^3+4)\sqrt{-\frac{x^3-1}{x^3+1}}}{18}+\frac{\operatorname{RootOf}(\_Z^2+1)\ln\left(\operatorname{RootOf}(\_Z^2+1)\sqrt{-\frac{x^3-1}{x^3+1}}\,x^3+x^3+\operatorname{RootOf}(\_Z^2+1)\sqrt{-\frac{x^3-1}{x^3+1}}\right)}{6}$

input
```
int(x^8*((-x^3+1)/(x^3+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/18*(x^3+1)*(2*x^6-3*x^3+4)*(-(x^3-1)/(x^3+1))^(1/2)-1/6*arcsin(x^3)*(-(x
^3-1)/(x^3+1))^(1/2)*(-(x^3+1)*(x^3-1))^(1/2)/(x^3-1)
```

3.292.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.58

$$\int x^8\sqrt{\frac{1-x^3}{1+x^3}}\,dx=\frac{1}{18}\left(2\,x^9-x^6+x^3+4\right)\sqrt{-\frac{x^3-1}{x^3+1}}-\frac{1}{3}\arctan\left(\frac{\left(x^3+1\right)\sqrt{-\frac{x^3-1}{x^3+1}}-1}{x^3}\right)$$

input
```
integrate(x^8*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="fricas")
```

output
```
1/18*(2*x^9 - x^6 + x^3 + 4)*sqrt(-(x^3 - 1)/(x^3 + 1)) - 1/3*arctan(((x^3
 + 1)*sqrt(-(x^3 - 1)/(x^3 + 1)) - 1)/x^3)
```

3.292.6 Sympy [F(-1)]

Timed out.

$$\int x^8\sqrt{\frac{1-x^3}{1+x^3}}\,dx=\text{Timed out}$$

input
```
integrate(x**8*((-x**3+1)/(x**3+1))**(1/2),x)
```

output
```
Timed out
```

3.292.7 Maxima [F]

$$\int x^8\sqrt{\frac{1-x^3}{1+x^3}}\,dx=\int x^8\sqrt{-\frac{x^3-1}{x^3+1}}\,dx$$

input
```
integrate(x^8*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^8*sqrt(-(x^3 - 1)/(x^3 + 1)), x)
```

3.292.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.50

$$\int x^8\sqrt{\frac{1-x^3}{1+x^3}}\,dx = \frac{1}{6}\arcsin\left(x^3\right)\operatorname{sgn}\left(x^3+1\right) + \frac{1}{18}\sqrt{-x^6+1}\left(\left(2\,x^3\operatorname{sgn}\left(x^3+1\right)-3\operatorname{sgn}\left(x^3+1\right)\right)x^3+4\operatorname{sgn}\left(x^3+1\right)\right)$$

input
```
integrate(x^8*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="giac")
```

output
```
1/6*arcsin(x^3)*sgn(x^3 + 1) + 1/18*sqrt(-x^6 + 1)*((2*x^3*sgn(x^3 + 1) -
3*sgn(x^3 + 1))*x^3 + 4*sgn(x^3 + 1))
```

3.292.9 Mupad [B] (verification not implemented)

Time = 16.90 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int x^8\sqrt{\frac{1-x^3}{1+x^3}}\,dx = \frac{2\sqrt{-\frac{x^3-1}{x^3+1}}}{9} - \frac{\operatorname{atan}\left(\sqrt{-\frac{x^3-1}{x^3+1}}\right)}{3} + \frac{x^3\sqrt{-\frac{x^3-1}{x^3+1}}}{18} - \frac{x^6\sqrt{-\frac{x^3-1}{x^3+1}}}{18} + \frac{x^9\sqrt{-\frac{x^3-1}{x^3+1}}}{9}$$

input
```
int(x^8*(-(x^3 - 1)/(x^3 + 1))^(1/2),x)
```

output
```
(2*(-(x^3 - 1)/(x^3 + 1))^(1/2))/9 - atan((-(x^3 - 1)/(x^3 + 1))^(1/2))/3
+ (x^3*(-(x^3 - 1)/(x^3 + 1))^(1/2))/18 - (x^6*(-(x^3 - 1)/(x^3 + 1))^(1/2
))/18 + (x^9*(-(x^3 - 1)/(x^3 + 1))^(1/2))/9
```

3.293 $\int x^9 \sqrt{\frac{5-7x^5}{7+5x^5}}\,dx$

3.293.1 Optimal result

Integrand size = 25, antiderivative size = 106

$$\int x^9 \sqrt{\frac{5-7x^5}{7+5x^5}}\,dx = -\frac{27}{350}\sqrt{\frac{5-7x^5}{7+5x^5}}\left(7+5x^5\right)+\frac{1}{250}\sqrt{\frac{5-7x^5}{7+5x^5}}\left(7+5x^5\right)^2 + \frac{2257\arctan\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^5}{7+5x^5}}\right)}{875\sqrt{35}}$$

output
```
2257/30625*arctan(1/7*35^(1/2)*((-7*x^5+5)/(5*x^5+7))^(1/2))*35^(1/2)-27/3
50*(5*x^5+7)*((-7*x^5+5)/(5*x^5+7))^(1/2)+1/250*(5*x^5+7)^2*((-7*x^5+5)/(5
*x^5+7))^(1/2)
```

3.293.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$\int x^9 \sqrt{\frac{5-7x^5}{7+5x^5}}\,dx = \frac{\sqrt{\frac{5-7x^5}{7+5x^5}}\left(35\sqrt{5-7x^5}\left(-602-185x^5+175x^{10}\right)+4514\sqrt{35}\sqrt{7+5x^5}\arctan\left(\frac{\sqrt{\frac{25}{7}-5x^5}}{\sqrt{7+5x^5}}\right)\right)}{61250\sqrt{5-7x^5}}$$

input
```
Integrate[x^9*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)],x]
```

output
```
(Sqrt[(5 - 7*x^5)/(7 + 5*x^5)]*(35*Sqrt[5 - 7*x^5]*(-602 - 185*x^5 + 175*x
^10) + 4514*Sqrt[35]*Sqrt[7 + 5*x^5]*ArcTan[Sqrt[25/7 - 5*x^5]/Sqrt[7 + 5*
x^5]]))/(61250*Sqrt[5 - 7*x^5])
```

3.293.3 Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2053, 2052, 360, 27, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9\sqrt{\frac{5-7x^5}{5x^5+7}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{5}\int x^5\sqrt{\frac{5-7x^5}{5x^5+7}}dx^5$$

$$\Big\downarrow \text{2052}$$

$$-\frac{148}{5}\int \frac{x^{10}(5-7x^{10})}{(5x^{10}+7)^3}d\sqrt{\frac{5-7x^5}{5x^5+7}}$$

$$\Big\downarrow \text{360}$$

$$-\frac{148}{5}\left(-\frac{1}{100}\int -\frac{2(37-70x^{10})}{(5x^{10}+7)^2}d\sqrt{\frac{5-7x^5}{5x^5+7}}-\frac{37\sqrt{\frac{5-7x^5}{5x^5+7}}}{50(5x^{10}+7)^2}\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{148}{5}\left(\frac{1}{50}\int \frac{37-70x^{10}}{(5x^{10}+7)^2}d\sqrt{\frac{5-7x^5}{5x^5+7}}-\frac{37\sqrt{\frac{5-7x^5}{5x^5+7}}}{50(5x^{10}+7)^2}\right)$$

$$\Big\downarrow \text{298}$$

$$-\frac{148}{5}\left(\frac{1}{50}\left(\frac{135\sqrt{\frac{5-7x^5}{5x^5+7}}}{14(5x^{10}+7)}-\frac{61}{14}\int \frac{1}{5x^{10}+7}d\sqrt{\frac{5-7x^5}{5x^5+7}}\right)-\frac{37\sqrt{\frac{5-7x^5}{5x^5+7}}}{50(5x^{10}+7)^2}\right)$$

$$\Big\downarrow \text{216}$$

$$-\frac{148}{5}\left(\frac{1}{50}\left(\frac{135\sqrt{\frac{5-7x^5}{5x^5+7}}}{14\,(5x^{10}+7)}-\frac{61\arctan\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^5}{5x^5+7}}\right)}{14\sqrt{35}}\right)-\frac{37\sqrt{\frac{5-7x^5}{5x^5+7}}}{50\,(5x^{10}+7)^2}\right)$$

input
```
Int[x^9*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)],x]
```

output
```
(-148*((-37*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)])/(50*(7 + 5*x^10)^2) + ((135*Sqr
t[(5 - 7*x^5)/(7 + 5*x^5)])/(14*(7 + 5*x^10)) - (61*ArcTan[Sqrt[5/7]*Sqrt[
(5 - 7*x^5)/(7 + 5*x^5)]])/(14*Sqrt[35]))/50))/5
```

3.293.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

3.293. $\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx$

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.293.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.78 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.08

method	result
trager	$\frac{(5x^5+7)(35x^5-86)\sqrt{-\frac{7x^5-5}{5x^5+7}}}{1750}+\frac{2257\,\mathrm{RootOf}\left(\_Z^2+35\right)\ln\left(175\sqrt{-\frac{7x^5-5}{5x^5+7}}\,x^5+35\,\mathrm{RootOf}\left(\_Z^2+35\right)x^5+245\sqrt{-\frac{7x^5-5}{5x^5+7}}+12\,\mathrm{R}\right.}{61250}$
risch	$\frac{(5x^5+7)(35x^5-86)\sqrt{-\frac{7x^5-5}{5x^5+7}}}{1750}+\frac{2257\,\mathrm{RootOf}\left(\_Z^2+35\right)\ln\left(-35\,\mathrm{RootOf}\left(\_Z^2+35\right)x^5+35\sqrt{-35x^{10}-24x^5+35}-12\,\mathrm{RootOf}\left(\_\right.\right.}{61250(7x^5-5)}$

input
```
int(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/1750*(5*x^5+7)*(35*x^5-86)*(-(7*x^5-5)/(5*x^5+7))^(1/2)+2257/61250*RootO
f(_Z^2+35)*ln(175*(-(7*x^5-5)/(5*x^5+7))^(1/2)*x^5+35*RootOf(_Z^2+35)*x^5+
245*(-(7*x^5-5)/(5*x^5+7))^(1/2)+12*RootOf(_Z^2+35))
```

3.293.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.77

$$\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx=\frac{1}{1750}\left(175\,x^{10}-185\,x^5-602\right)\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}+\frac{2257}{61250}\sqrt{35}\arctan\left(\frac{\sqrt{35}(35\,x^5+12)\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}}{35\,(7\,x^5-5)}\right)$$

input
```
integrate(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x, algorithm="fricas")
```

output
```
1/1750*(175*x^10 - 185*x^5 - 602)*sqrt(-(7*x^5 - 5)/(5*x^5 + 7)) + 2257/61
250*sqrt(35)*arctan(1/35*sqrt(35)*(35*x^5 + 12)*sqrt(-(7*x^5 - 5)/(5*x^5 +
 7))/(7*x^5 - 5))
```

3.293.6 Sympy [F(-1)]

Timed out.

$$\int x^9 \sqrt{\frac{5-7x^5}{7+5x^5}}\,dx = \text{Timed out}$$

input
```
integrate(x**9*((-7*x**5+5)/(5*x**5+7))**(1/2),x)
```

output
```
Timed out
```

3.293.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.14

$$\int x^9 \sqrt{\frac{5-7x^5}{7+5x^5}}\,dx = \frac{2257}{30625}\sqrt{35}\arctan\left(\frac{1}{7}\sqrt{35}\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}\right) - \frac{37\left(675\left(-\frac{7\,x^5-5}{5\,x^5+7}\right)^{\frac{3}{2}}+427\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}\right)}{875\left(\frac{25\,(7\,x^5-5)^2}{(5\,x^5+7)^2}-\frac{70\,(7\,x^5-5)}{5\,x^5+7}+49\right)}$$

input
```
integrate(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x, algorithm="maxima")
```

output
```
2257/30625*sqrt(35)*arctan(1/7*sqrt(35)*sqrt(-(7*x^5 - 5)/(5*x^5 + 7))) -
37/875*(675*(-(7*x^5 - 5)/(5*x^5 + 7))^(3/2) + 427*sqrt(-(7*x^5 - 5)/(5*x^
5 + 7)))/(25*(7*x^5 - 5)^2/(5*x^5 + 7)^2 - 70*(7*x^5 - 5)/(5*x^5 + 7) + 49
)
```

3.293.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.44

$$\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx = \frac{1}{61250}\left(35\sqrt{-35\,x^{10}-24\,x^5+35}(35\,x^5-86)-2257\sqrt{35}\arcsin\left(\frac{35}{37}\,x^5+\frac{12}{37}\right)\right)\mathrm{sgn}(5\,x^5+7)$$

input
```
integrate(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x, algorithm="giac")
```

output
```
1/61250*(35*sqrt(-35*x^10 - 24*x^5 + 35)*(35*x^5 - 86) - 2257*sqrt(35)*arc
sin(35/37*x^5 + 12/37))*sgn(5*x^5 + 7)
```

3.293.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.26

$$\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx = \frac{2257\sqrt{35}\,\mathrm{atan}\left(\frac{\sqrt{5}\sqrt{7}\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}}{7}\right)}{30625} - \frac{43\sqrt{5}\sqrt{7}\sqrt{35}\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}}{4375} - \frac{37\sqrt{5}\sqrt{7}\sqrt{35}\,x^5\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}}{12250} + \frac{\sqrt{5}\sqrt{7}\sqrt{35}\,x^{10}\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}}{350}$$

input
```
int(x^9*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2),x)
```

output
```
(2257*35^(1/2)*atan((5^(1/2)*7^(1/2)*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2))/7))
/30625 - (43*5^(1/2)*7^(1/2)*35^(1/2)*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2))/43
75 - (37*5^(1/2)*7^(1/2)*35^(1/2)*x^5*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2))/12
250 + (5^(1/2)*7^(1/2)*35^(1/2)*x^10*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2))/350
```

3.294 $\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2} \, dx$

3.294.1 Optimal result

Integrand size = 23, antiderivative size = 52

$$\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2} \, dx = \frac{\sqrt{-\frac{x^2}{1-x^2}}\sqrt{-1+x^2}\arctan\left(\frac{\sqrt{-1+x^2}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

output
```
1/2*arctan(1/2*(x^2-1)^(1/2)*2^(1/2))*(-x^2/(-x^2+1))^(1/2)*(x^2-1)^(1/2)/
x*2^(1/2)
```

3.294.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2} \, dx = \frac{\sqrt{\frac{x^2}{-1+x^2}}\sqrt{-1+x^2}\arctan\left(\frac{\sqrt{-1+x^2}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

input
```
Integrate[Sqrt[x^2/(-1 + x^2)]/(1 + x^2),x]
```

output
```
(Sqrt[x^2/(-1 + x^2)]*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]/Sqrt[2]])/(Sqrt
[2]*x)
```

3.294.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {2058, 34, 353, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{x^2}{x^2-1}}}{x^2+1} \, dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{-\frac{x^2}{1-x^2}}\sqrt{x^2-1}\int \frac{\sqrt{x^2}}{\sqrt{x^2-1}(x^2+1)}dx}{\sqrt{x^2}}$$

$$\Big\downarrow \text{34}$$

$$\frac{\sqrt{-\frac{x^2}{1-x^2}}\sqrt{x^2-1}\int \frac{x}{\sqrt{x^2-1}(x^2+1)}dx}{x}$$

$$\Big\downarrow \text{353}$$

$$\frac{\sqrt{-\frac{x^2}{1-x^2}}\sqrt{x^2-1}\int \frac{1}{\sqrt{x^2-1}(x^2+1)}dx^2}{2x}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt{-\frac{x^2}{1-x^2}}\sqrt{x^2-1}\int \frac{1}{x^4+2}d\sqrt{x^2-1}}{x}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{-\frac{x^2}{1-x^2}}\sqrt{x^2-1}\arctan\left(\frac{\sqrt{x^2-1}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

input
```
Int[Sqrt[x^2/(-1 + x^2)]/(1 + x^2),x]
```

output
```
(Sqrt[-(x^2/(1 - x^2))]*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]/Sqrt[2]])/(Sq
rt[2]*x)
```

3.294. $\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2} \, dx$

3.294.3.1 Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.294.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\sqrt{\frac{x^2}{x^2-1}}\,\sqrt{x^2-1}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{x^2-1}\,\sqrt{2}}{2}\right)}{2x}$	42
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{-x^3\operatorname{RootOf}\left(\_Z^2+2\right)+4x^2\sqrt{\frac{x^2}{x^2-1}}+3\operatorname{RootOf}\left(\_Z^2+2\right)x-4\sqrt{\frac{x^2}{x^2-1}}}{x\left(x^2+1\right)}\right)}{4}$	75

3.294. $\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2}\,dx$

input
```
int((x^2/(x^2-1))^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x^2/(x^2-1))^(1/2)/x*(x^2-1)^(1/2)*2^(1/2)*arctan(1/2*(x^2-1)^(1/2)*2
^(1/2))
```

3.294.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}(x^2-1)\sqrt{\frac{x^2}{x^2-1}}}{2\,x}\right)$$

input
```
integrate((x^2/(x^2-1))^(1/2)/(x^2+1),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 - 1)*sqrt(x^2/(x^2 - 1))/x)
```

3.294.6 Sympy [F]

$$\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{x^2}{x^2-1}}}{x^2+1}\,dx$$

input
```
integrate((x**2/(x**2-1))**(1/2)/(x**2+1),x)
```

output
```
Integral(sqrt(x**2/(x**2 - 1))/(x**2 + 1), x)
```

3.294.7 Maxima [F]

$$\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{x^2}{x^2-1}}}{x^2+1}\,dx$$

input
```
integrate((x^2/(x^2-1))^(1/2)/(x^2+1),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2/(x^2 - 1))/(x^2 + 1), x)
```

3.294.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x^2-1}\right)\operatorname{sgn}\left(x^2-1\right)\operatorname{sgn}(x) + \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}i\sqrt{2}\right)\operatorname{sgn}(x)$$

input
```
integrate((x^2/(x^2-1))^(1/2)/(x^2+1),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x^2 - 1))*sgn(x^2 - 1)*sgn(x) + 1/2*sq
rt(2)*arctan(1/2*I*sqrt(2))*sgn(x)
```

3.294.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{x^2}{x^2-1}}}{x^2+1}\,dx$$

input
```
int((x^2/(x^2 - 1))^(1/2)/(x^2 + 1),x)
```

output
```
int((x^2/(x^2 - 1))^(1/2)/(x^2 + 1), x)
```

3.295 $\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx$

3.295.1 Optimal result

Integrand size = 28, antiderivative size = 68

$$\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx=\frac{\sqrt{-\frac{x^2}{1-a-(1+a)x^2}}\sqrt{-1+a+(1+a)x^2}\arctan\left(\frac{\sqrt{-1+a+(1+a)x^2}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

output
```
1/2*arctan(1/2*(-1+a+(1+a)*x^2)^(1/2)*2^(1/2))*(-x^2/(1-a-(1+a)*x^2))^(1/2
)*(-1+a+(1+a)*x^2)^(1/2)/x*2^(1/2)
```

3.295.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx=\frac{\sqrt{-1+a+x^2+ax^2}\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}\arctan\left(\frac{\sqrt{-1+a+(1+a)x^2}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

input
```
Integrate[Sqrt[x^2/(-1 + a + (1 + a)*x^2)]/(1 + x^2),x]
```

output
```
(Sqrt[-1 + a + x^2 + a*x^2]*Sqrt[x^2/(-1 + a + (1 + a)*x^2)]*ArcTan[Sqrt[-
1 + a + (1 + a)*x^2]/Sqrt[2]])/(Sqrt[2]*x)
```

3.295.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2058, 34, 353, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{x^2}{(a+1)x^2+a-1}}}{x^2+1} \, dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{-\frac{x^2}{-((a+1)x^2)-a+1}}\sqrt{(a+1)x^2+a-1}\int \frac{\sqrt{x^2}}{(x^2+1)\sqrt{(a+1)x^2+a-1}}dx}{\sqrt{x^2}}$$

$$\Big\downarrow \text{34}$$

$$\frac{\sqrt{-\frac{x^2}{-((a+1)x^2)-a+1}}\sqrt{(a+1)x^2+a-1}\int \frac{x}{(x^2+1)\sqrt{(a+1)x^2+a-1}}dx}{x}$$

$$\Big\downarrow \text{353}$$

$$\frac{\sqrt{-\frac{x^2}{-((a+1)x^2)-a+1}}\sqrt{(a+1)x^2+a-1}\int \frac{1}{(x^2+1)\sqrt{(a+1)x^2+a-1}}dx^2}{2x}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt{-\frac{x^2}{-((a+1)x^2)-a+1}}\sqrt{(a+1)x^2+a-1}\int \frac{1}{\frac{x^4}{a+1}+\frac{2}{a+1}}d\sqrt{(a+1)x^2+a-1}}{(a+1)x}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{-\frac{x^2}{-((a+1)x^2)-a+1}}\sqrt{(a+1)x^2+a-1}\arctan\left(\frac{\sqrt{(a+1)x^2+a-1}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

input
```
Int[Sqrt[x^2/(-1 + a + (1 + a)*x^2)]/(1 + x^2),x]
```

output
```
(Sqrt[-(x^2/(1 - a - (1 + a)*x^2))]*Sqrt[-1 + a + (1 + a)*x^2]*ArcTan[Sqrt
[-1 + a + (1 + a)*x^2]/Sqrt[2]])/(Sqrt[2]*x)
```

3.295. $\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2} \, dx$

3.295.3.1 Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.295.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\sqrt{\frac{x^2}{a\,x^2+x^2+a-1}}\,\sqrt{a\,x^2+x^2+a-1}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{a\,x^2+x^2+a-1}\,\sqrt{2}}{2}\right)}{2x}$	60

input
```
int((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x^2/(a*x^2+x^2+a-1))^(1/2)/x*(a*x^2+x^2+a-1)^(1/2)*2^(1/2)*arctan(1/2
*(a*x^2+x^2+a-1)^(1/2)*2^(1/2))
```

3.295. $\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx$

3.295.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx=\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}((a+1)x^2+a-3)\sqrt{\frac{x^2}{(a+1)x^2+a-1}}}{4\,x}\right)$$

input
```
integrate((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*arctan(1/4*sqrt(2)*((a + 1)*x^2 + a - 3)*sqrt(x^2/((a + 1)*x^2
 + a - 1))/x)
```

3.295.6 Sympy [F]

$$\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx=\int \frac{\sqrt{\frac{x^2}{ax^2+a+x^2-1}}}{x^2+1}\,dx$$

input
```
integrate((x**2/(-1+a+(1+a)*x**2))**(1/2)/(x**2+1),x)
```

output
```
Integral(sqrt(x**2/(a*x**2 + a + x**2 - 1))/(x**2 + 1), x)
```

3.295.7 Maxima [F]

$$\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx=\int \frac{\sqrt{\frac{x^2}{(a+1)x^2+a-1}}}{x^2+1}\,dx$$

input
```
integrate((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2/((a + 1)*x^2 + a - 1))/(x^2 + 1), x)
```

3.295.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{ax^2+x^2+a-1}\right)\operatorname{sgn}\left(ax^2+x^2+a-1\right)\operatorname{sgn}(x) - \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{a-1}\right)\operatorname{sgn}(a-1)\operatorname{sgn}(x)$$

input
```
integrate((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*x^2 + x^2 + a - 1))*sgn(a*x^2 + x^2
+ a - 1)*sgn(x) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a - 1))*sgn(a - 1)*s
gn(x)
```

3.295.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{x^2}{(a+1)\,x^2+a-1}}}{x^2+1}\,dx$$

input
```
int((x^2/(a + x^2*(a + 1) - 1))^(1/2)/(x^2 + 1),x)
```

output
```
int((x^2/(a + x^2*(a + 1) - 1))^(1/2)/(x^2 + 1), x)
```

3.296 $\int \frac{x^5}{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}\,dx$

3.296.1 Optimal result

Integrand size = 26, antiderivative size = 281

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{(b^2c^2+2abcd+5a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{16b^3d^2e} - \frac{(3bc+5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{24b^2d^2e} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}{6bd(bc-ad)e} + \frac{(bc-ad)(b^2c^2+2abcd+5a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{7/2}d^{5/2}\sqrt{e}}$$

output
```
1/16*(-a*d+b*c)*(5*a^2*d^2+2*a*b*c*d+b^2*c^2)*arctanh(d^(1/2)*(e*(b*x^2+a)
/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2))/b^(7/2)/d^(5/2)/e^(1/2)+1/16*(5*a^2*d^2
+2*a*b*c*d+b^2*c^2)*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^3/d^2/e-1/24
*(5*a*d+3*b*c)*(d*x^2+c)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^2/d^2/e-1/6*(d*
x^2+c)^3*(a-c*(b*x^2+a)/(d*x^2+c))*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b/d/(-a*d
+b*c)/e
```

3.296.2 Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.80

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$
$$= \frac{\sqrt{a+bx^2}\left(\sqrt{d}\sqrt{a+bx^2}\sqrt{\frac{b(c+dx^2)}{bc-ad}}(15a^2d^2-2abd(2c+5dx^2)+b^2(-3c^2+2cdx^2+8d^2x^4))+3\sqrt{bc-ad}(\right.}{48b^3d^{5/2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{\frac{b(c+dx^2)}{bc-ad}}}$$

input
```
Integrate[x^5/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[a + b*x^2]*(Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)
]*(15*a^2*d^2 - 2*a*b*d*(2*c + 5*d*x^2) + b^2*(-3*c^2 + 2*c*d*x^2 + 8*d^2*
x^4)) + 3*Sqrt[b*c - a*d]*(b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*ArcSinh[(Sqrt[
d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]]))/(48*b^3*d^(5/2)*Sqrt[(e*(a + b*x^2)
)/(c + d*x^2)]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)])
```

3.296.3 Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2053, 2052, 315, 25, 27, 298, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$
$$\Big\downarrow \; 2053$$
$$\frac{1}{2}\int \frac{x^4}{\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}dx^2$$
$$\Big\downarrow \; 2052$$
$$e(bc-ad)\int \frac{\left(ae-cx^4\right)^2}{\left(be-dx^4\right)^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$

$$\Big\downarrow \text{315}$$

$$e(bc-ad)\left(-\frac{\int -\frac{e(a(bc+5ad)e-3c(bc+ad)x^4)}{(be-dx^4)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{6bde}-\frac{(bc-ad)\left(ae-cx^4\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{6bd\left(be-dx^4\right)^3}\right)$$

$$\Big\downarrow \text{25}$$

$$e(bc-ad)\left(\frac{\int \frac{e(a(bc+5ad)e-3c(bc+ad)x^4)}{(be-dx^4)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{6bde}-\frac{(bc-ad)\left(ae-cx^4\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{6bd\left(be-dx^4\right)^3}\right)$$

$$\Big\downarrow \text{27}$$

$$e(bc-ad)\left(\frac{\int \frac{a(bc+5ad)e-3c(bc+ad)x^4}{(be-dx^4)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{6bd}-\frac{(bc-ad)\left(ae-cx^4\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{6bd\left(be-dx^4\right)^3}\right)$$

$$\Big\downarrow \text{298}$$

$$e(bc-ad)\left(\frac{\frac{3}{4}\left(\frac{5a^2d}{b}+2ac+\frac{bc^2}{d}\right)\int \frac{1}{(be-dx^4)^2}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}-\frac{(bc-ad)(5ad+3bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4bd(be-dx^4)^2}}{6bd}-\frac{(bc-ad)\left(ae-cx^4\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{6bd\left(be-dx^4\right)^3}\right)$$

$$\Big\downarrow \text{215}$$

$$e(bc-ad)\left(\frac{\frac{3}{4}\left(\frac{5a^2d}{b}+2ac+\frac{bc^2}{d}\right)\left(\frac{\int \frac{1}{be-dx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{2be}+\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2be(be-dx^4)}\right)-\frac{(bc-ad)(5ad+3bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4bd(be-dx^4)^2}}{6bd}-\frac{(bc-ad)\left(ae-cx^4\right)}{6bd\left(be-dx\right.}\right.$$

$$\Big\downarrow \text{221}$$

3.296. $\int \frac{x^5}{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}\,dx$

$$e(bc-ad)\left(\frac{\frac{3}{4}\left(\frac{5a^2d}{b}+2ac+\frac{bc^2}{d}\right)\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{3/2}\sqrt{d}e^{3/2}}+\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2be(be-dx^4)}\right)-\frac{(bc-ad)(5ad+3bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4bd(be-dx^4)^2}}{6bd}-\frac{(bc-ad)(ae-}{6bd(be}\right.$$

input
```
Int[x^5/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(b*c - a*d)*e*(-1/6*((b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(a*e -
c*x^4))/(b*d*(b*e - d*x^4)^3) + (-1/4*((b*c - a*d)*(3*b*c + 5*a*d)*Sqrt[(e
*(a + b*x^2))/(c + d*x^2)])/(b*d*(b*e - d*x^4)^2) + (3*(2*a*c + (b*c^2)/d
+ (5*a^2*d)/b)*(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(2*b*e*(b*e - d*x^4)) +
ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])]/(2*
b^(3/2)*Sqrt[d]*e^(3/2))))/4)/(6*b*d))
```

3.296.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.296.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.86

method	result
risch	$\frac{\left(8b^2d^2x^4-10ab\,d^2x^2+2b^2cd\,x^2+15a^2d^2-4abcd-3b^2c^2\right)\left(b\,x^2+a\right)}{48b^3d^2\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}-\frac{\left(5a^3d^3-3a^2bc\,d^2-d\,c^2b^2a-b^3c^3\right)\ln\left(\frac{\frac{1}{2}eda+\frac{1}{2}ebc+bde\,x^2}{\sqrt{bde}}+\sqrt{b\cdots}\right.}{32b^3d^2\sqrt{bde}\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\cdots}$
default	$\frac{\left(b\,x^2+a\right)\left(-36\sqrt{bd}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,ab\,d^2x^2-12\sqrt{bd}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,b^2cd\,x^2-15\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+\cdots}}{2\sqrt{bd}}\cdots\right.\right.}{\cdots}$

input
```
int(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

3.296. $\int \frac{x^5}{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}\,dx$

output
```
1/48*(8*b^2*d^2*x^4-10*a*b*d^2*x^2+2*b^2*c*d*x^2+15*a^2*d^2-4*a*b*c*d-3*b^
2*c^2)*(b*x^2+a)/b^3/d^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/32*(5*a^3*d^3-3*a
^2*b*c*d^2-a*b^2*c^2*d-b^3*c^3)/b^3/d^2*ln((1/2*e*d*a+1/2*e*b*c+b*d*e*x^2)
/(b*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)/(e
*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)/(d*x^2+c)
```

3.296.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.94

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$

$$= \Bigg[-\frac{3\,(b^3c^3 + ab^2c^2d + 3\,a^2bcd^2 - 5\,a^3d^3)\sqrt{bde}\log\Big(8\,b^2d^2ex^4 + 8\,(b^2cd + abd^2)ex^2 + (b^2c^2 + 6\,abcd + a^2d^2}{}$$

$$-\frac{3\,(b^3c^3 + ab^2c^2d + 3\,a^2bcd^2 - 5\,a^3d^3)\sqrt{-bde}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{-bde}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\,(b^2dex^2+abde)}\right) - 2\,(8\,b^3d^4x^6 - 3\,b^3c^3}{96\,b^4d^3e}$$

input
```
integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/192*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*sqrt(b*d*e)
*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d
+ a^2*d^2)*e - 4*(2*b*d^2*x^4 + b*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqr
t(b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) - 4*(8*b^3*d^4*x^6 - 3*b^3*c^3
*d - 4*a*b^2*c^2*d^2 + 15*a^2*b*c*d^3 + 10*(b^3*c*d^3 - a*b^2*d^4)*x^4 - (
b^3*c^2*d^2 + 14*a*b^2*c*d^3 - 15*a^2*b*d^4)*x^2)*sqrt((b*e*x^2 + a*e)/(d*
x^2 + c)))/(b^4*d^3*e), -1/96*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 -
5*a^3*d^3)*sqrt(-b*d*e)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d*e)*sq
rt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*d*e*x^2 + a*b*d*e)) - 2*(8*b^3*d^4*x^
6 - 3*b^3*c^3*d - 4*a*b^2*c^2*d^2 + 15*a^2*b*c*d^3 + 10*(b^3*c*d^3 - a*b^2
*d^4)*x^4 - (b^3*c^2*d^2 + 14*a*b^2*c*d^3 - 15*a^2*b*d^4)*x^2)*sqrt((b*e*x
^2 + a*e)/(d*x^2 + c)))/(b^4*d^3*e)]
```

3.296.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

input
```
integrate(x**5/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

3.296.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.296.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.84

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$
$$= \frac{2\sqrt{bdex^4+bcex^2+adex^2+ace}\left(2x^2\left(\frac{4x^2}{be}+\frac{b^2cde-5abd^2e}{b^3d^2e^2}\right)-\frac{3b^2c^2e+4abcde-15a^2d^2e}{b^3d^2e^2}\right)-\frac{3(b^3c^3+ab^2c^2d+3a^2bcd^2-}{}}{96\,\mathrm{sgn}\left(dx^2+c\right)}$$

input
```
integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
1/96*(2*sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e)*(2*x^2*(4*x^2/(b*e
) + (b^2*c*d*e - 5*a*b*d^2*e)/(b^3*d^2*e^2)) - (3*b^2*c^2*e + 4*a*b*c*d*e
- 15*a^2*d^2*e)/(b^3*d^2*e^2)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2
- 5*a^3*d^3)*log(abs(-b*c*e - a*d*e - 2*sqrt(b*d*e)*(sqrt(b*d*e)*x^2 - sqr
t(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))))/(sqrt(b*d*e)*b^3*d^2))/sgn
(d*x^2 + c)
```

3.296.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{x^5}{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

input
```
int(x^5/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

output
```
int(x^5/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.297 $\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

3.297.1 Optimal result

Integrand size = 26, antiderivative size = 169

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = -\frac{(bc+3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{8b^2de}+\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{4bde}-\frac{(bc-ad)(bc+3ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{5/2}d^{3/2}\sqrt{e}}$$

output
```
-1/8*(-a*d+b*c)*(3*a*d+b*c)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/
b^(1/2)/e^(1/2))/b^(5/2)/d^(3/2)/e^(1/2)-1/8*(3*a*d+b*c)*(d*x^2+c)*(e*(b*x
^2+a)/(d*x^2+c))^(1/2)/b^2/d/e+1/4*(d*x^2+c)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/
2)/b/d/e
```

3.297.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{b}\sqrt{d}(a+bx^2)\sqrt{c+dx^2}(-3ad+b(c+2dx^2))-(b^2c^2+2abcd-3a^2d^2)\sqrt{a+bx^2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8b^{5/2}d^{3/2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

input
```
Integrate[x^3/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[b]*Sqrt[d]*(a + b*x^2)*Sqrt[c + d*x^2]*(-3*a*d + b*(c + 2*d*x^2)) -
(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[d]*Sqrt[a
+ b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(8*b^(5/2)*d^(3/2)*Sqrt[(e*(a + b*x^
2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

3.297.3 Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2053, 2052, 25, 298, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}dx$$

$$\Big\downarrow \text{ 2053}$$

$$\frac{1}{2}\int \frac{x^2}{\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}dx^2$$

$$\Big\downarrow \text{ 2052}$$

$$e(bc-ad)\int -\frac{ae-cx^4}{(be-dx^4)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}$$

$$\Big\downarrow \text{ 25}$$

$$-\left(e(bc-ad)\int \frac{ae-cx^4}{(be-dx^4)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}\right)$$

$\big\downarrow$ 298

$$e(bc-ad)\left(\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4bd(be-dx^4)^2}-\frac{(3ad+bc)\int \frac{1}{(be-dx^4)^2}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{4bd}\right)$$

$\big\downarrow$ 215

$$e(bc-ad)\left(\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4bd(be-dx^4)^2}-\frac{(3ad+bc)\left(\frac{\int \frac{1}{be-dx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{2be}+\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2be(be-dx^4)}\right)}{4bd}\right)$$

$\big\downarrow$ 221

$$e(bc-ad)\left(\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4bd(be-dx^4)^2}-\frac{(3ad+bc)\left(\frac{\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{3/2}\sqrt{d}e^{3/2}}+\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2be(be-dx^4)}\right)}{4bd}\right)$$

input
```
Int[x^3/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(b*c - a*d)*e*(((b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*b*d*(b*e
 - d*x^4)^2) - ((b*c + 3*a*d)*(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(2*b*e*(b
*e - d*x^4)) + ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b
]*Sqrt[e])]/(2*b^(3/2)*Sqrt[d]*e^(3/2))))/(4*b*d))
```

3.297. $\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}dx$

3.297.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.297. $\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

3.297.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{(-2bd\,x^2+3ad-bc)(b\,x^2+a)}{8b^2d\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}+\frac{(3a^2d^2-2abcd-b^2c^2)\ln\left(\frac{\frac{1}{2}eda+\frac{1}{2}ebc+bde\,x^2}{\sqrt{bde}}+\sqrt{bde\,x^4+(eda+ebc)x^2+ace}\right)\sqrt{(d\,x^2+c)e(b\,x^2+}}{16b^2d\sqrt{bde}\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}(d\,x^2+c)}$
default	$-\frac{(b\,x^2+a)\left(-4\sqrt{bd}\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,bd\,x^2-3\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)a^2d^2+2\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+}}{}\right.\right.}{16\sqrt{\frac{e(}{}}}$

input
```
int(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-2*b*d*x^2+3*a*d-b*c)*(b*x^2+a)/b^2/d/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+
1/16*(3*a^2*d^2-2*a*b*c*d-b^2*c^2)/b^2/d*ln((1/2*e*d*a+1/2*e*b*c+b*d*e*x^2
)/(b*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)/(
e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)/(d*x^2+c)
```

3.297.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.44

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$

$$= \left[-\frac{(b^2c^2+2\,abcd-3\,a^2d^2)\sqrt{bde}\log\left(8\,b^2d^2ex^4+8\,(b^2cd+abd^2)ex^2+(b^2c^2+6\,abcd+a^2d^2)e+4\,(2\,bd^2x\right.}{}\right.$$

input
```
integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*sqrt(b*d*e)*log(8*b^2*d^2*e*x^4
+ 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b
*d^2*x^4 + b*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqrt(b*d*e)*sqrt((b*e*x^
2 + a*e)/(d*x^2 + c))) - 4*(2*b^2*d^3*x^4 + b^2*c^2*d - 3*a*b*c*d^2 + 3*(b
^2*c*d^2 - a*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*d^2*e), 1
/16*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*sqrt(-b*d*e)*arctan(1/2*(2*b*d*x^2
+ b*c + a*d)*sqrt(-b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*d*e*x^2 +
 a*b*d*e)) + 2*(2*b^2*d^3*x^4 + b^2*c^2*d - 3*a*b*c*d^2 + 3*(b^2*c*d^2 - a
*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*d^2*e)]
```

3.297.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

input
```
integrate(x**3/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

3.297.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.297.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{2\sqrt{bdex^4+bcex^2+adex^2+ace}\left(\frac{2x^2}{be}+\frac{bce-3ade}{b^2de^2}\right)+\frac{(b^2c^2+2abcd-3a^2d^2)\log\left(\left|-bce-ade-2\sqrt{bde}\left(\sqrt{bde}x^2-\sqrt{bdex^4+bcex}\right.\right.\right.}{\sqrt{bde}b^2d}}{16\,\mathrm{sgn}\left(dx^2+c\right)}$$

input
```
integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
1/16*(2*sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e)*(2*x^2/(b*e) + (b*
c*e - 3*a*d*e)/(b^2*d*e^2)) + (b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*log(abs(-b
*c*e - a*d*e - 2*sqrt(b*d*e)*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2
 + a*d*e*x^2 + a*c*e))))/(sqrt(b*d*e)*b^2*d))/sgn(d*x^2 + c)
```

3.297.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{x^3}{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

input
```
int(x^3/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

output
```
int(x^3/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.298 $\int \frac{x}{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}\,dx$

3.298.1 Optimal result

Integrand size = 24, antiderivative size = 106

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx=\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{2be}+\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{3/2}\sqrt{d}\sqrt{e}}$$

output
```
1/2*(-a*d+b*c)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/
2))/b^(3/2)/d^(1/2)/e^(1/2)+1/2*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b/
e
```

3.298.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.28

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{b}\sqrt{d}(a+bx^2)\left(c+dx^2\right)+(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2b^{3/2}\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}$$

input
```
Integrate[x/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[b]*Sqrt[d]*(a + b*x^2)*(c + d*x^2) + (b*c - a*d)*Sqrt[a + b*x^2]*Sqr
t[c + d*x^2]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])
/(2*b^(3/2)*Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

3.298.3 Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2053, 2051, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}dx^2$$

$$\Big\downarrow \text{2051}$$

$$e(bc-ad)\int \frac{1}{(be-dx^4)^2}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}$$

$$\Big\downarrow \text{215}$$

$$e(bc-ad)\left(\frac{\int \frac{1}{be-dx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{2be}+\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2be(be-dx^4)}\right)$$

$$\Big\downarrow \text{221}$$

$$e(bc-ad)\left(\frac{\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{3/2}\sqrt{d}e^{3/2}}+\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2be(be-dx^4)}\right)$$

input
```
Int[x/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(b*c - a*d)*e*(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(2*b*e*(b*e - d*x^4)) + A
rcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])]/(2*b
^(3/2)*Sqrt[d]*e^(3/2)))
```

3.298.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2051
```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)   Subst[Int[
x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
 x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x]
&& FractionQ[p] && IntegerQ[1/n]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.298.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.45

method	result
risch	$\frac{b\,x^{2}+a}{2b\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}-\frac{\left(ad-bc\right)\ln\left(\frac{\frac{1}{2}eda+\frac{1}{2}ebc+bde\,x^{2}}{\sqrt{bde}}+\sqrt{bde\,x^{4}+\left(eda+ebc\right)x^{2}+ace}\right)\sqrt{\left(d\,x^{2}+c\right)e\left(b\,x^{2}+a\right)}}{4b\sqrt{bde}\,\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}\,\left(d\,x^{2}+c\right)}$
default	$-\frac{\left(b\,x^{2}+a\right)\left(d\ln\left(\frac{2bd\,x^{2}+2\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)a-c\ln\left(\frac{2bd\,x^{2}+2\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)b-2\sqrt{bd\,x^{4}+ad\,x^{2}}}{4\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}\,\sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\,b\sqrt{bd}}$

3.298. $\int \frac{x}{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}\,dx$

input
```
int(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*(b*x^2+a)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/4*(a*d-b*c)/b*ln((1/2*e*d*
a+1/2*e*b*c+b*d*e*x^2)/(b*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(
1/2))/(b*d*e)^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^
(1/2)/(d*x^2+c)
```

3.298.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.95

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$
$$= \left[-\frac{\sqrt{bde}(bc-ad)\log\left(8\,b^2d^2ex^4+8\,(b^2cd+abd^2)ex^2+(b^2c^2+6\,abcd+a^2d^2)e-4\,(2\,bd^2x^4+bc^2+acd\right.}{8\,b^2de}\right.$$
$$\left.-\frac{\sqrt{-bde}(bc-ad)\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{-bde}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\,(b^2dex^2+abde)}\right)-2\,(bd^2x^2+bcd)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{4\,b^2de}\right]$$

input
```
integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(b*d*e)*(b*c - a*d)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)
*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e - 4*(2*b*d^2*x^4 + b*c^2 + a*c*
d + (3*b*c*d + a*d^2)*x^2)*sqrt(b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))
- 4*(b*d^2*x^2 + b*c*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d*e), -1/4
*(sqrt(-b*d*e)*(b*c - a*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d*e)
*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*d*e*x^2 + a*b*d*e)) - 2*(b*d^2*x^2
 + b*c*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d*e)]
```

3.298.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

input
```
integrate(x/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

3.298.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.298.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.34

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\frac{2\sqrt{bdex^4+bcex^2+adex^2+ace}}{be} - \frac{\sqrt{bde}(bc-ad)\log\left(\left|-2\left(\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+ace}\right)bd-\sqrt{bde}bc-\sqrt{bde}ad\right|\right)}{b^2de}}{4\,\mathrm{sgn}\left(dx^2+c\right)}$$

input
```
integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
1/4*(2*sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e)/(b*e) - sqrt(b*d*e)
*(b*c - a*d)*log(abs(-2*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*
d*e*x^2 + a*c*e))*b*d - sqrt(b*d*e)*b*c - sqrt(b*d*e)*a*d))/(b^2*d*e))/sgn
(d*x^2 + c)
```

3.298.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{x}{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

input
```
int(x/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

output
```
int(x/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.299 $\int \frac{1}{x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}\,dx$

3.299.1 Optimal result

Integrand size = 26, antiderivative size = 112

$$\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx=-\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\sqrt{e}}+\frac{\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{b}\sqrt{e}}$$

output
```
-arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2))*c^(1/2)/a^
(1/2)/e^(1/2)+arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2
))*d^(1/2)/b^(1/2)/e^(1/2)
```

3.299.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.31

$$\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx=\frac{\sqrt{a+bx^2}\left(-\sqrt{b}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)+\sqrt{a}\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)\right)}{\sqrt{a}\sqrt{b}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

input
```
Integrate[1/(x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

output
```
(Sqrt[a + b*x^2]*(-(Sqrt[b]*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqr
t[a]*Sqrt[c + d*x^2])]) + Sqrt[a]*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2]
)/(Sqrt[b]*Sqrt[c + d*x^2])]))/(Sqrt[a]*Sqrt[b]*Sqrt[(e*(a + b*x^2))/(c +
d*x^2)]*Sqrt[c + d*x^2])
```

3.299.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2053, 2052, 25, 303, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}dx^2$$

$$\Big\downarrow \text{2052}$$

$$e(bc-ad)\int -\frac{1}{(ae-cx^4)(be-dx^4)}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$-\left(e(bc-ad)\int \frac{1}{(ae-cx^4)(be-dx^4)}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}\right)$$

$$\Big\downarrow \text{303}$$

$$e(bc-ad)\left(\frac{d\int \frac{1}{be-dx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{e(bc-ad)}-\frac{c\int \frac{1}{ae-cx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{e(bc-ad)}\right)$$

$$\Big\downarrow \text{221}$$

$$e(bc-ad)\left(\frac{\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{b}e^{3/2}(bc-ad)}-\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}e^{3/2}(bc-ad)}\right)$$

input
```
Int[1/(x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

output
```
(b*c - a*d)*e*(-((Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2
)])/(Sqrt[a]*Sqrt[e])])/(Sqrt[a]*(b*c - a*d)*e^(3/2))) + (Sqrt[d]*ArcTanh[
(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(Sqrt[b]*(
b*c - a*d)*e^(3/2)))
```

3.299.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.299. $\int \frac{1}{x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}\,dx$

3.299.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(84) = 168$.

Time = 0.09 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.60

method	result	s
default	$-\frac{(b x^2+a)\left(c \ln\left(\frac{a d x^2+b c x^2+2\sqrt{ac}\,\sqrt{b d x^4+a d x^2+b c x^2+a c}+2 a c}{x^2}\right)\sqrt{bd}-\ln\left(\frac{2 b d x^2+2\sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{bd}+a d+b c}{2\sqrt{bd}}\right)\sqrt{ac}\, d\right)}{2\sqrt{\frac{e\left(b x^2+a\right)}{d x^2+c}}\,\sqrt{(d x^2+c)(b x^2+a)}\,\sqrt{bd}\,\sqrt{ac}}$	1

input
```
int(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x^2+a)*(c*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)+2*a*c)/x^2)*(b*d)^(1/2)-ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)*d)/(e*(b
*x^2+a)/(d*x^2+c))^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/(b*d)^(1/2)/(a*c)^(1/
2)
```

3.299.5 Fricas [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 881, normalized size of antiderivative = 7.87

$$\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \left[\frac{1}{4}\sqrt{\frac{d}{be}}\log\left(8\,b^2d^2x^4 + b^2c^2 + 6\,abcd + a^2d^2 + 8\left(b^2cd + abd^2\right)x^2 + 4\left(2\,b^2d^2x^4 + b^2c^2 + abcd + \left(3\,b^2cd + abd^2\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{\frac{d}{be}}\right) + \frac{1}{4}\sqrt{\frac{c}{ae}}\log\left(\frac{(b^2c^2 + 6\,abcd + a^2d^2)x^4 + 8\,a^2c^2 + 8\,(abc^2 + a^2cd)x^2 - 4\,((abcd + a^2d^2)x^4 + 2\,a^2c^2 + (ab}{x^4}\right. - \frac{1}{2}\sqrt{-\frac{d}{be}}\arctan\left(\frac{(2\,bdx^2 + bc + ad)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{d}{be}}}{2\,(bdx^2 + ad)}\right) + \frac{1}{4}\sqrt{\frac{c}{ae}}\log\left(\frac{(b^2c^2 + 6\,abcd + a^2d^2)x^4 + 8\,a^2c^2 + 8\,(abc^2 + a^2cd)x^2 - 4\,((abcd + a^2d^2)x^4 + 2\,a^2c^2 + (ab}{x^4}\right. + \frac{1}{4}\sqrt{\frac{d}{be}}\log\left(8\,b^2d^2x^4 + b^2c^2 + 6\,abcd + a^2d^2 + 8\left(b^2cd + abd^2\right)x^2 + 4\left(2\,b^2d^2x^4 + b^2c^2 + abcd + \left(3\,b^2cd + abd^2\right)x^2\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{\frac{d}{be}}\right), \frac{1}{2}\sqrt{-\frac{c}{ae}}\arctan\left(\frac{((bc+ad)x^2 + 2}{2\,(bc}\right. - \frac{1}{2}\sqrt{-\frac{d}{be}}\arctan\left(\frac{(2\,bdx^2 + bc + ad)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{d}{be}}}{2\,(bdx^2 + ad)}\right)\right]$$

input
```
integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*sqrt(d/(b*e))*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(
b^2*c*d + a*b*d^2)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c^2 + a*b*c*d + (3*b^2*c*d
 + a*b*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(d/(b*e))) + 1/4*sq
rt(c/(a*e))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*
c^2 + a^2*c*d)*x^2 - 4*((a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3
*a^2*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(c/(a*e)))/x^4), -1/2
*sqrt(-d/(b*e))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d
*x^2 + c))*sqrt(-d/(b*e))/(b*d*x^2 + a*d)) + 1/4*sqrt(c/(a*e))*log(((b^2*c
^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*
((a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt((b*
e*x^2 + a*e)/(d*x^2 + c))*sqrt(c/(a*e)))/x^4), 1/2*sqrt(-c/(a*e))*arctan(1
/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-c/(a*
e))/(b*c*x^2 + a*c)) + 1/4*sqrt(d/(b*e))*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a
*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c^2
+ a*b*c*d + (3*b^2*c*d + a*b*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*s
qrt(d/(b*e))), 1/2*sqrt(-c/(a*e))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqr
t((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-c/(a*e))/(b*c*x^2 + a*c)) - 1/2*sqrt(
-d/(b*e))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 +
 c))*sqrt(-d/(b*e))/(b*d*x^2 + a*d))]
```

3.299.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

3.299.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.299.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.299.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{x\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

input
```
int(1/(x*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
```

output
```
int(1/(x*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

3.300 $\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

3.300.1 Optimal result

Integrand size = 26, antiderivative size = 130

$$\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2a\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} + \frac{(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{3/2}\sqrt{c}\sqrt{e}}$$

output
```
1/2*(-a*d+b*c)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/
2))/a^(3/2)/c^(1/2)/e^(1/2)+1/2*(-a*d+b*c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a
/(a*e-c*e*(b*x^2+a)/(d*x^2+c))
```

3.300.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{-\sqrt{a}\sqrt{c}(a+bx^2)(c+dx^2)+(bc-ad)x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}\sqrt{c}x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}$$

input
```
Integrate[1/(x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

output
```
(-(Sqrt[a]*Sqrt[c]*(a + b*x^2)*(c + d*x^2)) + (b*c - a*d)*x^2*Sqrt[a + b*x
^2]*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*
x^2])])/(2*a^(3/2)*Sqrt[c]*x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^
2))
```

3.300.3 Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2053, 2052, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}dx$$

$$\Big\downarrow \text{ 2053}$$

$$\frac{1}{2}\int \frac{1}{x^4\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}dx^2$$

$$\Big\downarrow \text{ 2052}$$

$$e(bc-ad)\int \frac{1}{(cx^4-ae)^2}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}$$

$$\Big\downarrow \text{ 215}$$

$$e(bc-ad)\left(\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2ae(ae-cx^4)}-\frac{\int \frac{1}{cx^4-ae}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{2ae}\right)$$

$$\Big\downarrow \text{ 221}$$

$$e(bc-ad)\left(\frac{\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{3/2}\sqrt{c}e^{3/2}}+\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2ae(ae-cx^4)}\right)$$

input
```
Int[1/(x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

output
```
(b*c - a*d)*e*(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(2*a*e*(a*e - c*x^4)) + A
rcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])]/(2*a
^(3/2)*Sqrt[c]*e^(3/2)))
```

3.300.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.300.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.25

method	result
risch	$-\frac{b\,x^{2}+a}{2a\,x^{2}\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}-\frac{(ad-bc)\ln\left(\frac{2ace+(eda+ebc)x^{2}+2\sqrt{ace}\,\sqrt{bde\,x^{4}+(eda+ebc)x^{2}+ace}}{x^{2}}\right)\sqrt{(d\,x^{2}+c)e(b\,x^{2}+a)}}{4a\sqrt{ace}\,\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}\,(d\,x^{2}+c)}$
default	$-\frac{(b\,x^{2}+a)\left(-2bd\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,x^{4}\sqrt{ac}+a^{2}\ln\left(\frac{ad\,x^{2}+bc\,x^{2}+2\sqrt{ac}\,\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}+2ac}{x^{2}}\right)dc\,x^{2}-c^{2}\ln\left(\frac{ad\,x^{2}+bc\,x^{2}+\cdots}{}\right.}{4\sqrt{\frac{e\left(b\,x^{2}+\cdots\right)}{d\,x^{2}+\cdots}}}$

input
```
int(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a*(b*x^2+a)/x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/4*(a*d-b*c)/a/(a*c*e)
^(1/2)*ln((2*a*c*e+(a*d*e+b*c*e)*x^2+2*(a*c*e)^(1/2)*(b*d*e*x^4+(a*d*e+b*c
*e)*x^2+a*c*e)^(1/2))/x^2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x
^2+a))^(1/2)/(d*x^2+c)
```

3.300.5 Fricas [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.56

$$\int \frac{1}{x^{3}\sqrt{\frac{e(a+bx^{2})}{c+dx^{2}}}}\,dx$$

$$= \left[-\frac{\sqrt{ace}(bc-ad)x^{2}\log\left(\frac{(b^{2}c^{2}+6\,abcd+a^{2}d^{2})ex^{4}+8\,a^{2}c^{2}e+8\,(abc^{2}+a^{2}cd)ex^{2}-4\,((bcd+ad^{2})x^{4}+2\,ac^{2}+(bc^{2}+3\,acd)x^{2})\sqrt{ace}\sqrt{\frac{bex^{2}\cdots}{dx^{2}\cdots}}}{x^{4}}\right.}{8\,a^{2}cex^{2}}\right.$$

$$\left.-\frac{\sqrt{-ace}(bc-ad)x^{2}\arctan\left(\frac{\sqrt{-ace}((bc+ad)x^{2}+2\,ac)\sqrt{\frac{bex^{2}+ae}{dx^{2}+c}}}{2\,(abcex^{2}+a^{2}ce)}\right)+2\,(acdx^{2}+ac^{2})\sqrt{\frac{bex^{2}+ae}{dx^{2}+c}}}{4\,a^{2}cex^{2}}\right]$$

input
```
integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(a*c*e)*(b*c - a*d)*x^2*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*
x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b*c*d + a*d^2)*x^4 +
 2*a*c^2 + (b*c^2 + 3*a*c*d)*x^2)*sqrt(a*c*e)*sqrt((b*e*x^2 + a*e)/(d*x^2
+ c)))/x^4) + 4*(a*c*d*x^2 + a*c^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^
2*c*e*x^2), -1/4*(sqrt(-a*c*e)*(b*c - a*d)*x^2*arctan(1/2*sqrt(-a*c*e)*((b
*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b*c*e*x^2 + a^
2*c*e)) + 2*(a*c*d*x^2 + a*c^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^2*c*
e*x^2)]
```

3.300.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**3/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

3.300.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.300.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(110) = 220$.

Time = 0.35 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.83

$$\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = -\frac{\frac{(bc-ad)\arctan\left(-\frac{\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+ace}}{\sqrt{-ace}}\right)}{\sqrt{-acea}}+\frac{\left(\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+ace}\right)bc+\left(\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+a\right.}{\left(ace-\left(\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+ace}\right)^2\right)a}}{2\operatorname{sgn}(dx^2+c)}$$

input
```
integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
-1/2*((b*c - a*d)*arctan(-(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 +
a*d*e*x^2 + a*c*e))/sqrt(-a*c*e))/(sqrt(-a*c*e)*a) + ((sqrt(b*d*e)*x^2 - s
qrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))*b*c + (sqrt(b*d*e)*x^2 - s
qrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))*a*d + 2*sqrt(b*d*e)*a*c)/(
(a*c*e - (sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e
))^2)*a))/sgn(d*x^2 + c)
```

3.300.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{x^3\,\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

input
```
int(1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
```

output
```
int(1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

3.301 $\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

3.301.1 Optimal result

Integrand size = 26, antiderivative size = 218

$$\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx=-\frac{(bc-ad)^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2}-\frac{(bc-ad)(3bc+ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^2c\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}-\frac{(bc-ad)(3bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{5/2}c^{3/2}\sqrt{e}}$$

output
```
-1/8*(-a*d+b*c)*(a*d+3*b*c)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/
a^(1/2)/e^(1/2))/a^(5/2)/c^(3/2)/e^(1/2)-1/4*(-a*d+b*c)^2*e*(e*(b*x^2+a)/(
d*x^2+c))^(1/2)/a/c/(a*e-c*e*(b*x^2+a)/(d*x^2+c))^2-1/8*(-a*d+b*c)*(a*d+3*
b*c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^2/c/(a*e-c*e*(b*x^2+a)/(d*x^2+c))
```

3.301.2 Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$
$$= \frac{\sqrt{a}\sqrt{c}(a+bx^2)\sqrt{c+dx^2}(3bcx^2-a(2c+dx^2))-(3b^2c^2-2abcd-a^2d^2)\,x^4\sqrt{a+bx^2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}c^{3/2}x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

input
```
Integrate[1/(x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

output
```
(Sqrt[a]*Sqrt[c]*(a + b*x^2)*Sqrt[c + d*x^2]*(3*b*c*x^2 - a*(2*c + d*x^2))
 - (3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*x^4*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[c]*
Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(5/2)*c^(3/2)*x^4*Sqrt[(
e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

3.301.3 Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2053, 2052, 25, 298, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}dx$$

$$\Big\downarrow \text{ 2053}$$

$$\frac{1}{2}\int \frac{1}{x^6\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}dx^2$$

$$\Big\downarrow \text{ 2052}$$

$$e(bc-ad)\int -\frac{be-dx^4}{(ae-cx^4)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$

$$\Big\downarrow \text{ 25}$$

$$-\left(e(bc-ad)\int \frac{be-dx^4}{(ae-cx^4)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}\right)$$

$\downarrow$ 298

$$e(bc-ad)\left(-\frac{(ad+3bc)\int \frac{1}{(ae-cx^4)^2}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{4ac}-\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac\left(ae-cx^4\right)^2}\right)$$

$\downarrow$ 215

$$e(bc-ad)\left(-\frac{(ad+3bc)\left(\frac{\int \frac{1}{ae-cx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{2ae}+\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2ae(ae-cx^4)}\right)}{4ac}-\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac\left(ae-cx^4\right)^2}\right)$$

$\downarrow$ 221

$$e(bc-ad)\left(-\frac{(ad+3bc)\left(\frac{\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{3/2}\sqrt{c}e^{3/2}}+\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2ae(ae-cx^4)}\right)}{4ac}-\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac\left(ae-cx^4\right)^2}\right)$$

input
```
Int[1/(x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

output
```
(b*c - a*d)*e*(-1/4*((b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(a*c*(
a*e - c*x^4)^2) - ((3*b*c + a*d)*(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(2*a*e
*(a*e - c*x^4)) + ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqr
t[a]*Sqrt[e])]/(2*a^(3/2)*Sqrt[c]*e^(3/2))))/(4*a*c))
```

3.301. $\int \frac{1}{x^5\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}\,dx$

3.301.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.301. $\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

3.301.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.92

method	result
risch	$-\frac{(b\,x^2+a)\,(a d\,x^2-3 b c\,x^2+2 a c)}{8 a^2 x^4 c \sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}+\frac{(a^2 d^2+2 a b c d-3 b^2 c^2)\ln\left(\frac{2 a c e+(e d a+e b c)x^2+2\sqrt{a c e}\,\sqrt{b d e\,x^4+(e d a+e b c)x^2+a c e}}{x^2}\right)\sqrt{(d\,x^2+c)e(b\,x^2}}{16 c\,a^2\sqrt{a c e}\,\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,(d\,x^2+c)}$
default	$-\frac{(b\,x^2+a)\left(2 b\,d^2\sqrt{b d\,x^4+a d\,x^2+b c\,x^2+a c}\,x^6 a\sqrt{a c}+10 b^2 d\sqrt{b d\,x^4+a d\,x^2+b c\,x^2+a c}\,x^6 c\sqrt{a c}-a^3\ln\left(\frac{a d\,x^2+b c\,x^2+2\sqrt{a c}\,\sqrt{b d\,x^4+a d\,x^2}}{x^2}\right.\right.}{}$

input
```
int(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(b*x^2+a)*(a*d*x^2-3*b*c*x^2+2*a*c)/a^2/x^4/c/(e*(b*x^2+a)/(d*x^2+c))
^(1/2)+1/16*(a^2*d^2+2*a*b*c*d-3*b^2*c^2)/c/a^2/(a*c*e)^(1/2)*ln((2*a*c*e+
(a*d*e+b*c*e)*x^2+2*(a*c*e)^(1/2)*(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2
))/x^2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)/(d*x^2
+c)
```

3.301.5 Fricas [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.03

$$\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$

$$=\left[-\frac{(3\,b^2c^2-2\,abcd-a^2d^2)\sqrt{ace}x^4\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)ex^4+8\,a^2c^2e+8\,(abc^2+a^2cd)ex^2+4\,((bcd+ad^2)x^4+2\,ac^2+(bc^2+3\,}{x^4}\right.}{32\,a^3c^2ex^4}\right.$$

input
```
integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(a*c*e)*x^4*log(((b^2*c^2 +
6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 + 4
*((b*c*d + a*d^2)*x^4 + 2*a*c^2 + (b*c^2 + 3*a*c*d)*x^2)*sqrt(a*c*e)*sqrt(
(b*e*x^2 + a*e)/(d*x^2 + c)))/x^4) + 4*(2*a^2*c^3 - (3*a*b*c^2*d - a^2*c*d
^2)*x^4 - 3*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/
(a^3*c^2*e*x^4), 1/16*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(-a*c*e)*x^4*
arctan(1/2*sqrt(-a*c*e)*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*
x^2 + c))/(a*b*c*e*x^2 + a^2*c*e)) - 2*(2*a^2*c^3 - (3*a*b*c^2*d - a^2*c*d
^2)*x^4 - 3*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/
(a^3*c^2*e*x^4)]
```

3.301.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**5/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

3.301.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.301.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. $2(194) = 388$.

Time = 0.39 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.46

$$\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$

$$= \frac{\frac{(3b^2c^2-2abcd-a^2d^2)\arctan\left(-\frac{\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+ace}}{\sqrt{-ace}}\right)}{\sqrt{-ace}a^2c} + \frac{5\left(\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+ace}\right)ab^2c^3e+10\left(\sqrt{bde}x^2-\sqrt{bdex}\right.}{}}{}$$

input
```
integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
1/8*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(-(sqrt(b*d*e)*x^2 - sqrt(b*d
*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))/sqrt(-a*c*e))/(sqrt(-a*c*e)*a^2*c
) + (5*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))
*a*b^2*c^3*e + 10*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^
2 + a*c*e))*a^2*b*c^2*d*e + (sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2
+ a*d*e*x^2 + a*c*e))*a^3*c*d^2*e + 8*sqrt(b*d*e)*a^2*b*c^3*e - 3*(sqrt(b*
d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^3*b^2*c^2 + 2*
(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^3*a*b*
c*d + (sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^
3*a^2*d^2 + 8*sqrt(b*d*e)*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 +
a*d*e*x^2 + a*c*e))^2*a^2*c*d)/((a*c*e - (sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4
 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^2)^2*a^2*c))/sgn(d*x^2 + c)
```

3.301.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{x^5\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

input
```
int(1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
```

output
```
int(1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

3.302 $\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

3.302.1 Optimal result

Integrand size = 26, antiderivative size = 403

$$\begin{aligned}\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = {} & \frac{(bc-4ad)x(a+bx^2)}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{x^3(a+bx^2)}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{(2b^2c^2+3abcd-8a^2d^2)\,x(a+bx^2)}{15b^3d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} \\ & +\frac{\sqrt{c}(2b^2c^2+3abcd-8a^2d^2)\,(a+bx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^3d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} \\ & -\frac{c^{3/2}(bc-4ad)\,(a+bx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15b^2d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}\end{aligned}$$

output
```
1/15*(-4*a*d+b*c)*x*(b*x^2+a)/b^2/d/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/5*x^3*
(b*x^2+a)/b/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/15*(-8*a^2*d^2+3*a*b*c*d+2*b^2
*c^2)*x*(b*x^2+a)/b^3/d/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/15*c^(3/
2)*(-4*a*d+b*c)*(b*x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Elliptic
F(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/b^2/d^(3/2)/(d*x^
2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/15*(-
8*a^2*d^2+3*a*b*c*d+2*b^2*c^2)*(b*x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)
^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^
(1/2)/b^3/d^(3/2)/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(
d*x^2+c))^(1/2)
```

3.302.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.52 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.64

$$\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$
$$= \frac{-\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(4ad-b(c+3dx^2)\right)-ic(-2b^2c^2-3abcd+8a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\Big(i\mathrm{arc}}{15a^2\left(\frac{b}{a}\right)^{5/2}d^2\sqrt{\frac{e(a+}{c+}}}$$

input
```
Integrate[x^4/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(-(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(4*a*d - b*(c + 3*d*x^2))) - I*c*
(-2*b^2*c^2 - 3*a*b*c*d + 8*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/
c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*c*(-(b^2*c^2) -
a*b*c*d + 2*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*a^2*(b/a)^(5/2)*d^2*Sqrt[(e*(a + b*
x^2))/(c + d*x^2)]*(c + d*x^2))
```

3.302.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2058, 380, 444, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}dx$$
$$\Big\downarrow \text{ 2058}$$
$$\frac{\sqrt{a+bx^2}\int \frac{x^4\sqrt{dx^2+c}}{\sqrt{bx^2+a}}dx}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$
$$\Big\downarrow \text{ 380}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5b}-\frac{\int\frac{x^2\left(3ac-(bc-4ad)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5b}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{444}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5b}-\frac{-\frac{\int-\frac{\left(2b^2c^2+3abdc-8a^2d^2\right)x^2+ac(bc-4ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}-\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-4ad)}{3bd}}{5b}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5b}-\frac{\frac{\int\frac{\left(2b^2c^2+3abdc-8a^2d^2\right)x^2+ac(bc-4ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}-\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-4ad)}{3bd}}{5b}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{406}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5b}-\frac{\frac{\left(-8a^2d^2+3abcd+2b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(bc-4ad)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}-\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-4ad)}{3bd}}{5b}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5b}-\frac{\frac{\left(-8a^2d^2+3abcd+2b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(bc-4ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3bd}-\frac{x\sqrt{a+bx^2}}{}\cdots}{5b}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{388}$$

3.302. $\int\frac{x^4}{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}dx$

$$\frac{\sqrt{a+bx^2}\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5b}-\frac{\frac{\left(-8a^2d^2+3abcd+2b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(bc-4ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3bd}}{5b}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\downarrow$ 313

$$\frac{\sqrt{a+bx^2}\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5b}-\frac{\frac{\left(-8a^2d^2+3abcd+2b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(bc-4ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3bd}}{5b}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

input
```
Int[x^4/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[a + b*x^2]*((x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*b) - (-1/3*((b*
c - 4*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(b*d) + ((2*b^2*c^2 + 3*a*b*
c*d - 8*a^2*d^2)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[
a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqr
t[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(b
*c - 4*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*
c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
)/(3*b*d))/(5*b)))/(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

3.302. $\int \frac{x^4}{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}dx$

3.302.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.302. $\int \frac{x^4}{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}\,dx$

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.302.4 Maple [A] (verified)

Time = 5.14 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.22

method	result
risch	$-\frac{x\left(-3bd\,x^{2}+4ad-bc\right)\left(b\,x^{2}+a\right)}{15d\,b^{2}\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}+\frac{\left(\frac{4a^{2}cd\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^{4}+ade\,x^{2}+bce\,x^{2}+ace}}-\frac{ab\,c^{2}\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\cdots}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^{4}+ade\,x^{2}+bce\,x^{2}+ace}}\cdots\right.}{\cdots}$
default	$-\frac{\left(b\,x^{2}+a\right)\left(-3\sqrt{-\frac{b}{a}}\,b^{2}d^{3}x^{7}+\sqrt{-\frac{b}{a}}\,ab\,d^{3}x^{5}-4\sqrt{-\frac{b}{a}}\,b^{2}c\,d^{2}x^{5}+4\sqrt{-\frac{b}{a}}\,a^{2}d^{3}x^{3}-\sqrt{-\frac{b}{a}}\,b^{2}c^{2}d\,x^{3}+4\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\cdots\right)\cdots\right.}{\cdots}$

input
```
int(x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*x*(-3*b*d*x^2+4*a*d-b*c)*(b*x^2+a)/d/b^2/(e*(b*x^2+a)/(d*x^2+c))^(1/
2)+1/15/d/b^2*(4*a^2*c*d/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2
)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1
+(a*d*e+b*c*e)/c/b/e)^(1/2))-a*b*c^2/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c
*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/
a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-2*(8*a^2*d^2-3*a*b*c*d-2*b^2*c^2)
*a*c*e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e
*x^2+b*c*e*x^2+a*c*e)^(1/2)/(e*d*a+e*b*c+e*(a*d-b*c))*(EllipticF(x*(-b/a)^
(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*e+
b*c*e)/c/b/e)^(1/2))))/(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a
))^(1/2)/(d*x^2+c)
```

3.302.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.68

$$\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$

$$= \frac{(2\,b^2c^3+3\,abc^2d-8\,a^2cd^2)\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(2\,b^2c^3+3\,abc^2d-4\,a^2d^3-(8\,a^2-ab)cd}{}$$

input
```
integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
1/15*((2*b^2*c^3 + 3*a*b*c^2*d - 8*a^2*c*d^2)*sqrt(b*e/d)*x*sqrt(-c/d)*ell
iptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (2*b^2*c^3 + 3*a*b*c^2*d - 4*a^
2*d^3 - (8*a^2 - a*b)*c*d^2)*sqrt(b*e/d)*x*sqrt(-c/d)*elliptic_f(arcsin(sq
rt(-c/d)/x), a*d/(b*c)) + (3*b^2*d^3*x^6 - 2*b^2*c^3 - 3*a*b*c^2*d + 8*a^2
*c*d^2 + 4*(b^2*c*d^2 - a*b*d^3)*x^4 - (b^2*c^2*d + 7*a*b*c*d^2 - 8*a^2*d^
3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*d^2*e*x)
```

3.302.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

input
```
integrate(x**4/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

3.302.7 Maxima [F]

$$\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{x^4}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

input
```
integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

3.302.8 Giac [F]

$$\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{x^4}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

input
```
integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

3.302.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{x^4}{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

input
```
int(x^4/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

output
```
int(x^4/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.303 $\int \frac{x^2}{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} dx$

3.303.1 Optimal result

Integrand size = 26, antiderivative size = 312

$$\begin{aligned}\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx &= \frac{x(a+bx^2)}{3b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(bc-2ad)x(a+bx^2)}{3b^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} \\ &\quad - \frac{\sqrt{c}(bc-2ad)\,(a+bx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} \\ &\quad - \frac{c^{3/2}(a+bx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3b\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}\end{aligned}$$

output
```
1/3*x*(b*x^2+a)/b/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/3*(-2*a*d+b*c)*x*(b*x^2+
a)/b^2/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*c^(3/2)*(b*x^2+a)*(1/(1
+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)
^(1/2),(1-b*c/a/d)^(1/2))/b/(d*x^2+c)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1
/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*(-2*a*d+b*c)*(b*x^2+a)*(1/(1+d*x^2/c
))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(
1-b*c/a/d)^(1/2))*c^(1/2)/b^2/(d*x^2+c)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^
(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

3.303.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.02 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.68

$$\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)+ic(-bc+2ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-ic(-bc+ad)\sqrt{1}}{3b\sqrt{\frac{b}{a}}d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}$$

input
```
Integrate[x^2/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) + I*c*(-(b*c) + 2*a*d)*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
 - I*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*
ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*b*Sqrt[b/a]*d*Sqrt[(e*(a + b*x^2))/
(c + d*x^2)]*(c + d*x^2))
```

3.303.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2058, 380, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{a+bx^2}\int \frac{x^2\sqrt{dx^2+c}}{\sqrt{bx^2+a}}dx}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{ 380}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}-\frac{\int\frac{ac-(bc-2ad)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{406}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}-\frac{ac\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-(bc-2ad)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-(bc-2ad)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{388}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)}{3b}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{313}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{3b}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

3.303. $\int\frac{x^2}{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}dx$

input
```
Int[x^2/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[a + b*x^2]*((x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*b) - (-((b*c - 2*
a*d)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*E
llipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c
*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (c^(3/2)*Sqrt[a + b*x^
2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b)))/(Sqrt[(e*(a + b
*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

3.303.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.303.4 Maple [A] (verified)

Time = 3.93 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.12

method	result
risch	$\frac{x(b\,x^2+a)}{3b\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}-\frac{\left(\frac{ac\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}}-\frac{2(2ad-bc)ace\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)-E\left(\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}\,(eda+ebc+e(}\right.}{3b\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}\,(d\,x^2+c)}$
default	$\frac{(b\,x^2+a)\left(\sqrt{-\frac{b}{a}}\,b\,d^2x^5+\sqrt{-\frac{b}{a}}\,a\,d^2x^3+\sqrt{-\frac{b}{a}}\,bcd\,x^3+ac\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right.\right.}{3\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}\,\sqrt{(d\,x^2+c)(b\,x^2+a)}\,b\sqrt{-\frac{b}{a}}\,\sqrt{ba}}$

input
```
int(x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(b*x^2+a)/b/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3/b*(a*c/(-b/a)^(1/2)*(1
+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^
(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-2*(2*a*d-b*
c)*a*c*e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d
*e*x^2+b*c*e*x^2+a*c*e)^(1/2)/(e*d*a+e*b*c+e*(a*d-b*c))*(EllipticF(x*(-b/a
)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*
e+b*c*e)/c/b/e)^(1/2))))/(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2
+a))^(1/2)/(d*x^2+c)
```

3.303.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.59

$$\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx =$$

$$-\frac{(bc^2-2\,acd)\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - (bc^2-2\,acd-ad^2)\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}}{3\,b^2dex}$$

input
```
integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/3*((b*c^2 - 2*a*c*d)*sqrt(b*e/d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c
/d)/x), a*d/(b*c)) - (b*c^2 - 2*a*c*d - a*d^2)*sqrt(b*e/d)*x*sqrt(-c/d)*el
liptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (b*d^2*x^4 + b*c^2 - 2*a*c*d +
 2*(b*c*d - a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d*e*x)
```

3.303.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

3.303.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{x^2}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

input
```
integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

3.303.8 Giac [F]

$$\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{x^2}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

input
```
integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

3.303.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{x^2}{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

input
```
int(x^2/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

output
```
int(x^2/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.304 $\int \frac{1}{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}\,dx$

3.304.1 Optimal result

Integrand size = 22, antiderivative size = 252

$$\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{dx(a+bx^2)}{b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} - \frac{\sqrt{c}\sqrt{d}(a+bx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} + \frac{c^{3/2}(a+bx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}$$

output
```
d*x*(b*x^2+a)/b/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+c^(3/2)*(b*x^2+a)*
(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x
^2/c)^(1/2),(1-b*c/a/d)^(1/2))/a/(d*x^2+c)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c
))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-(b*x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+
d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(
1/2))*c^(1/2)*d^(1/2)/b/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^
2+a)/(d*x^2+c))^(1/2)
```

3.304.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.34

$$\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{\frac{a+bx^2}{a}}E\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{\frac{c+dx^2}{c}}}$$

input
```
Integrate[1/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[(a + b*x^2)/a]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*d)/(b*c)])/(Sqrt
[-(b/a)]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(c + d*x^2)/c])
```

3.304.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2058, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}dx$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{a+bx^2}\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}dx}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{ 324}$$

$$\frac{\sqrt{a+bx^2}\left(c\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+d\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{\sqrt{a+bx^2}\left(d\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \ 388$$

$$\frac{\sqrt{a+bx^2}\left(d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \ 313$$

$$\frac{\sqrt{a+bx^2}\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

input
```
Int[1/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[a + b*x^2]*(d*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sq
rt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*
Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)
*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/
(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(Sqrt[
(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

3.304.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.304.4 Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.50

method	result	size
default	$\frac{(b\,x^2+a)c\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,E\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)}{\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}}$	127

input
```
int(1/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(b*x^2+a)*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/
2),(a*d/b/c)^(1/2))/(e*(b*x^2+a)/(d*x^2+c))^(1/2)/((d*x^2+c)*(b*x^2+a))^(1
/2)/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

3.304.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = -\frac{c\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}E\left(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}\right) - (c+d)\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}F\left(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}\right) - (dx^2+c)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{bex}$$

input
```
integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
-(c*sqrt(b*e/d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) -
 (c + d)*sqrt(b*e/d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*
c)) - (d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*e*x)
```

3.304.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

3.304.7 Maxima [F]

$$\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

input
```
integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

3.304.8 Giac [F]

$$\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

input
```
integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

3.304.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

input
```
int(1/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

output
```
int(1/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.305 $\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

3.305.1 Optimal result

Integrand size = 26, antiderivative size = 289

$$\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = -\frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{dx(a+bx^2)}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} - \frac{\sqrt{c}\sqrt{d}(a+bx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} + \frac{\sqrt{c}\sqrt{d}(a+bx^2)\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}$$

output
```
(-b*x^2-a)/a/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+d*x*(b*x^2+a)/a/(d*x^2+c)/(e*
(b*x^2+a)/(d*x^2+c))^(1/2)-(b*x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/
2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2
)*d^(1/2)/a/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+
c))^(1/2)+(b*x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^
(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)/a/(d*x^
2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

3.305.2 Mathematica [A] (verified)

Time = 2.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.38

$$\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{(a+bx^2)\left(-\frac{1}{x}+\frac{d\sqrt{1+\frac{dx^2}{c}}E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{1+\frac{bx^2}{a}}(c+dx^2)}\right)}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

input
```
Integrate[1/(x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

output
```
((a + b*x^2)*(-x^(-1) + (d*Sqrt[1 + (d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c
)]*x], (b*c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[1 + (b*x^2)/a]*(c + d*x^2))))/(a*S
qrt[(e*(a + b*x^2))/(c + d*x^2)])
```

3.305.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2058, 377, 27, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{a+bx^2}\int \frac{\sqrt{dx^2+c}}{x^2\sqrt{bx^2+a}}dx}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{377}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{\int \frac{d\sqrt{bx^2+a}}{\sqrt{dx^2+c}}dx}{a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{ax}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{d\int\frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}dx}{a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{ax}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{ 324}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{d\left(a\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+b\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{ax}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{d\left(b\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{ax}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{ 388}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{d\left(b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{ax}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{ 313}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{ax}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

3.305. $\int\frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}dx$

input
```
Int[1/(x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

output
```
(Sqrt[a + b*x^2]*(-((Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*x)) + (d*(b*((x*S
qrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^
2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*Sqrt[a + b*x^2]*Elliptic
F[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x
^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/a))/(Sqrt[(e*(a + b*x^2))/(c + d*
x^2)]*Sqrt[c + d*x^2])
```

3.305.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

3.305. $\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.305.4 Maple [A] (verified)

Time = 4.66 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.03

method	result
default	$-\frac{\left(b\,x^{2}+a\right)\left(\sqrt{-\frac{b}{a}}\,bd\,x^{4}-\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)adx+bc\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,xF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)-bc\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+}{c}}}{\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}\,\sqrt{(d\,x^{2}+c)(b\,x^{2}+a)}\,ax\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}}$
risch	$-\frac{b\,x^{2}+a}{ax\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}+\frac{d\left(\frac{a\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^{4}+ade\,x^{2}+bce\,x^{2}+ace}}-\frac{2bace\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)-E\left(x\sqrt{-}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^{4}+ade\,x^{2}+bce\,x^{2}+ace}\,(eda+ebc+e(}\right.}{a\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}\,(d\,x^{2}+c)}$

input
```
int(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*x^2+a)*((-b/a)^(1/2)*b*d*x^4-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*E
llipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*d*x+b*c*((b*x^2+a)/a)^(1/2)*((d
*x^2+c)/c)^(1/2)*x*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))-b*c*((b*x^2+a
)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*x*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))
+(-b/a)^(1/2)*a*d*x^2+(-b/a)^(1/2)*b*c*x^2+(-b/a)^(1/2)*a*c)/(e*(b*x^2+a)/
(d*x^2+c))^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/a/x/(-b/a)^(1/2)/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)
```

3.305.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$
$$= \frac{b^2cd\sqrt{\frac{ace}{d^2}}x\sqrt{-\frac{b}{a}}E\left(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc}\right) - (b^2cd+a^2d^2)\sqrt{\frac{ace}{d^2}}x\sqrt{-\frac{b}{a}}F\left(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc}\right) - (abcdx^2}{a^2bcex}$$

input
```
integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
(b^2*c*d*sqrt(a*c*e/d^2)*x*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d
/(b*c)) - (b^2*c*d + a^2*d^2)*sqrt(a*c*e/d^2)*x*sqrt(-b/a)*elliptic_f(arcs
in(x*sqrt(-b/a)), a*d/(b*c)) - (a*b*c*d*x^2 + a*b*c^2)*sqrt((b*e*x^2 + a*e
)/(d*x^2 + c)))/(a^2*b*c*e*x)
```

3.305.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**2/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

3.305.7 Maxima [F]

$$\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}x^2}\,dx$$

input
```
integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^2), x)
```

3.305.8 Giac [F]

$$\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}x^2}\,dx$$

input
```
integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^2), x)
```

3.305.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{x^2\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

input
```
int(1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
```

output
```
int(1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

3.306 $\int \frac{1}{x^4\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}\,dx$

3.306.1 Optimal result

Integrand size = 26, antiderivative size = 375

$$\begin{aligned}\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx &= \frac{-a-bx^2}{3ax^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{(2bc-ad)\,(a+bx^2)}{3a^2cx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{d(2bc-ad)x(a+bx^2)}{3a^2c\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}\\ &+\frac{\sqrt{d}(2bc-ad)\,(a+bx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}\\ &-\frac{b\sqrt{c}\sqrt{d}(a+bx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}\end{aligned}$$

output
```
1/3*(-b*x^2-a)/a/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/3*(-a*d+2*b*c)*(b*x^2
+a)/a^2/c/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*d*(-a*d+2*b*c)*x*(b*x^2+a)/a
^2/c/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/3*(-a*d+2*b*c)*(b*x^2+a)*(1
/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2
/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)/a^2/(d*x^2+c)/c^(1/2)/(c*(b*x^2+a)/a/
(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*b*(b*x^2+a)*(1/(1+d*x^2
/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2)
,(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)/a^2/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c)
)^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

3.306.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.42 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$
$$= \frac{-\sqrt{\frac{b}{a}}(a+bx^2)(c+dx^2)(-2bcx^2+a(c+dx^2)) - ibc(-2bc+ad)x^3\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right.\right.}{3a^2\sqrt{\frac{b}{a}}cx^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx}$$

input
```
Integrate[1/(x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

output
```
(-(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(-2*b*c*x^2 + a*(c + d*x^2))) - I*b*c
*(-2*b*c + a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*Ar
cSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*b*c*(-(b*c) + a*d)*x^3*Sqrt[1 + (
b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c
)])/(3*a^2*Sqrt[b/a]*c*x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

3.306.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2058, 377, 25, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$
$$\Big\downarrow \text{ 2058}$$
$$\frac{\sqrt{a+bx^2}\int \frac{\sqrt{dx^2+c}}{x^4\sqrt{bx^2+a}}dx}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$
$$\Big\downarrow \text{ 377}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{\int -\frac{bdx^2+2bc-ad}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ax^3}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{a+bx^2}\left(-\frac{\int \frac{bdx^2+2bc-ad}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ax^3}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{445}$$

$$\frac{\sqrt{a+bx^2}\left(-\frac{-\frac{\int -\frac{bd\left((2bc-ad)x^2+ac\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{acx}}{3a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ax^3}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{a+bx^2}\left(-\frac{\frac{\int \frac{bd\left((2bc-ad)x^2+ac\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{acx}}{3a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ax^3}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a+bx^2}\left(-\frac{\frac{bd\int \frac{(2bc-ad)x^2+ac}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{acx}}{3a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ax^3}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{406}$$

$$\frac{\sqrt{a+bx^2}\left(-\frac{\frac{bd\left(ac\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(2bc-ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{acx}}{3a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ax^3}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{320}$$

3.306. $\int \frac{1}{x^4\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}\,dx$

$$\frac{\sqrt{a+bx^2}\left(-\frac{\frac{bd\left((2bc-ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{acx}}{3a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ax^3}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\downarrow$ 388

$$\frac{\sqrt{a+bx^2}\left(-\frac{\frac{bd\left((2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{acx}}{3a}-\frac{\sqrt{a+bx}}{3}\cdots}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\downarrow$ 313

$$\frac{\sqrt{a+bx^2}\left(-\frac{\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{ac}}{3a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc}{acx}\cdots}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

input
```
Int[1/(x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

3.306. $\int \frac{1}{x^4\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}\,dx$

output
```
(Sqrt[a + b*x^2]*(-1/3*(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*x^3) - (-(((2*
b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (b*d*((2*b*c - a*d)
*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Ellip
ticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*Sqrt[a + b*x^2]*El
lipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(a*c))/(3*a)))/(Sqrt[(e*(a
+ b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

3.306.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

3.306. $\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.306.4 Maple [A] (verified)

Time = 6.21 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.01

method	result
risch	$-\frac{\left(b\,x^{2}+a\right)\left(a d\,x^{2}-2 b c\,x^{2}+a c\right)}{3 a^{2} x^{3} c\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}-\frac{b d\left(\frac{a c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^{4}+ade\,x^{2}+bce\,x^{2}+ace}}+\frac{2(ad-2bc)ace\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^{4}+ade\,x^{2}+bce}}\right.}{3 c\,a^{2}\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}\,\left(d\,x^{2}+c\right)}$
default	$-\frac{\left(b\,x^{2}+a\right)\left(\sqrt{-\frac{b}{a}}\,a b\,d^{2} x^{6}-2\sqrt{-\frac{b}{a}}\,b^{2} c d\,x^{6}+2 b d\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{a d}{b c}}\right)x^{3} a c-2\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{a d}{b c}}\right)\right.}{3\sqrt{\frac{e\left(b\,x^{2}\right.}{d\,x^{2}+}}}$

input
```
int(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^2+a)*(a*d*x^2-2*b*c*x^2+a*c)/a^2/x^3/c/(e*(b*x^2+a)/(d*x^2+c))^(
1/2)-1/3*b*d/c/a^2*(a*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)
/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+
(a*d*e+b*c*e)/c/b/e)^(1/2))+2*(a*d-2*b*c)*a*c*e/(-b/a)^(1/2)*(1+b*x^2/a)^(
1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)/(e*d*
a+e*b*c+e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1
/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))))/(e*(b*x^2+
a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)/(d*x^2+c)
```

3.306.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = -\frac{(2\,b^2cd-abd^2)\sqrt{\frac{ace}{d^2}}x^3\sqrt{-\frac{b}{a}}E\left(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc}\right)-(2\,b^2cd+(a^2-ab)d^2)\sqrt{\frac{ace}{d^2}}x^3\sqrt{-\frac{b}{a}}F\left(\arcsin\left(x\right.\right.}{3\,a^3cex^3}$$

input
```
integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/3*((2*b^2*c*d - a*b*d^2)*sqrt(a*c*e/d^2)*x^3*sqrt(-b/a)*elliptic_e(arcs
in(x*sqrt(-b/a)), a*d/(b*c)) - (2*b^2*c*d + (a^2 - a*b)*d^2)*sqrt(a*c*e/d^
2)*x^3*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((2*a*b*c*
d - a^2*d^2)*x^4 - a^2*c^2 + 2*(a*b*c^2 - a^2*c*d)*x^2)*sqrt((b*e*x^2 + a*
e)/(d*x^2 + c)))/(a^3*c*e*x^3)
```

3.306.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**4/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

3.306.7 Maxima [F]

$$\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}x^4}\,dx$$

input
```
integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^4), x)
```

3.306.8 Giac [F]

$$\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}x^4}\,dx$$

input
```
integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^4), x)
```

3.306.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{x^4\,\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

input
```
int(1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
```

output
```
int(1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

3.307 $\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

3.307.1 Optimal result

Integrand size = 26, antiderivative size = 354

$$\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = -\frac{(b^2c^2+5ad(2bc-7ad))\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{16b^4de^2} - \frac{(b^2c^2+5ad(2bc-7ad))\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{24b^3d(bc-ad)e^2} - \frac{a^2(c+dx^2)^3}{b(bc-ad)^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(b^2c^2-2abcd+7a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^3}{6b^2d(bc-ad)^2e^2} - \frac{(bc-ad)\,(b^2c^2+5ad(2bc-7ad))\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{9/2}d^{3/2}e^{3/2}}$$

output
```
-1/16*(-a*d+b*c)*(b^2*c^2+5*a*d*(-7*a*d+2*b*c))*arctanh(d^(1/2)*(e*(b*x^2+
a)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2))/b^(9/2)/d^(3/2)/e^(3/2)-a^2*(d*x^2+c)
^3/b/(-a*d+b*c)^2/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/16*(b^2*c^2+5*a*d*(-7*
a*d+2*b*c))*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^4/d/e^2-1/24*(b^2*c^
2+5*a*d*(-7*a*d+2*b*c))*(d*x^2+c)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^3/d/(-
a*d+b*c)/e^2+1/6*(7*a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x^2+c)^3*(e*(b*x^2+a)/(d
*x^2+c))^(1/2)/b^2/d/(-a*d+b*c)^2/e^2
```

3.307.2 Mathematica [A] (verified)

Time = 4.43 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.70

$$\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{d}\sqrt{\frac{b(c+dx^2)}{bc-ad}}(105a^3d^2+5a^2bd(-20c+7dx^2)+ab^2(3c^2-38cdx^2-14d^2x^4)+b^3x^2(3c}{48b^4d^{3/2}e\sqrt{}}$$

input
```
Integrate[x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[d]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*(105*a^3*d^2 + 5*a^2*b*d*(-20*c
 + 7*d*x^2) + a*b^2*(3*c^2 - 38*c*d*x^2 - 14*d^2*x^4) + b^3*x^2*(3*c^2 + 1
4*c*d*x^2 + 8*d^2*x^4)) - 3*Sqrt[b*c - a*d]*(b^2*c^2 + 10*a*b*c*d - 35*a^2
*d^2)*Sqrt[a + b*x^2]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/
(48*b^4*d^(3/2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(b*(c + d*x^2))/(
b*c - a*d)])
```

3.307.3 Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.82, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {2053, 2052, 365, 25, 27, 298, 215, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \ 2053$$

$$\frac{1}{2}\int \frac{x^4}{\left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}}dx^2$$

$$\Big\downarrow \ 2052$$

$$e(bc-ad)\int \frac{\left(ae-cx^4\right)^2}{x^4\left(be-dx^4\right)^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$

$$\Big\downarrow \ 365$$

$$e(bc-ad)\left(\frac{\int -\frac{e\left(a(2bc-7ad)e-bc^2x^4\right)}{(be-dx^4)^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{be}-\frac{a^2e}{bx^2\left(be-dx^4\right)^3}\right)$$

$\downarrow$ 25

$$e(bc-ad)\left(-\frac{\int \frac{e\left(a(2bc-7ad)e-bc^2x^4\right)}{(be-dx^4)^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{be}-\frac{a^2e}{bx^2\left(be-dx^4\right)^3}\right)$$

$\downarrow$ 27

$$e(bc-ad)\left(-\frac{\int \frac{a(2bc-7ad)e-bc^2x^4}{(be-dx^4)^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{b}-\frac{a^2e}{bx^2\left(be-dx^4\right)^3}\right)$$

$\downarrow$ 298

$$e(bc-ad)\left(-\frac{\frac{1}{6}\left(\frac{5a(2bc-7ad)}{b}+\frac{bc^2}{d}\right)\int \frac{1}{(be-dx^4)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}-\frac{\left(\frac{bc^2}{d}-\frac{a(2bc-7ad)}{b}\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{6(be-dx^4)^3}}{b}-\frac{a^2e}{bx^2\left(be-dx^4\right)^3}\right)$$

$\downarrow$ 215

$$e(bc-ad)\left(-\frac{\frac{1}{6}\left(\frac{5a(2bc-7ad)}{b}+\frac{bc^2}{d}\right)\left(\frac{3\int \frac{1}{\left(be-dx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{4be}+\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4be(be-dx^4)^2}\right)-\frac{\left(\frac{bc^2}{d}-\frac{a(2bc-7ad)}{b}\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{6(be-dx^4)^3}}{b}-\frac{a^2e}{bx^2\left(be-dx^4\right.}\right.$$

$\downarrow$ 215

3.307. $\int \frac{x^5}{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

$$e(bc-ad)\left(-\frac{\frac{1}{6}\left(\frac{5a(2bc-7ad)}{b}+\frac{bc^2}{d}\right)\left(\frac{3\left(\frac{\int \frac{1}{be-dx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{2be}+\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2be(be-dx^4)}\right)}{4be}+\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4be(be-dx^4)^2}\right)-\frac{\left(\frac{bc^2}{d}-\frac{a(2bc-7ad)}{b}\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{6(be-dx^4)^3}}{b}-\frac{}{bx}\right.$$

$\Big\downarrow$ 221

$$e(bc-ad)\left(-\frac{a^2e}{bx^2(be-dx^4)^3}-\frac{\frac{1}{6}\left(\frac{5a(2bc-7ad)}{b}+\frac{bc^2}{d}\right)\left(\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{3/2}\sqrt{d}e^{3/2}}+\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2be(be-dx^4)}\right)}{4be}+\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4be(be-dx^4)^2}\right)-\frac{\left(\frac{bc^2}{d}-\right.}{}}{b}\right.$$

3.307. $\displaystyle\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

input
```
Int[x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(b*c - a*d)*e*(-((a^2*e)/(b*x^2*(b*e - d*x^4)^3)) - (-1/6*(((b*c^2)/d - (a
*(2*b*c - 7*a*d))/b)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(b*e - d*x^4)^3 +
(((b*c^2)/d + (5*a*(2*b*c - 7*a*d))/b)*(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/
(4*b*e*(b*e - d*x^4)^2) + (3*(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(2*b*e*(b*
e - d*x^4)) + ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]
*Sqrt[e])]/(2*b^(3/2)*Sqrt[d]*e^(3/2))))/(4*b*e)))/6)/b)
```

3.307.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.307.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.90

method	result
risch	$\frac{\left(8b^2d^2x^4-22ab\,d^2x^2+14b^2cd\,x^2+57a^2d^2-52abcd+3b^2c^2\right)\left(b\,x^2+a\right)}{48d\,b^4e\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}-\frac{\left(\frac{\left(35a^2d^2-10abcd-b^2c^2\right)\left(ad-bc\right)\ln\left(\frac{\frac{1}{2}eda+\frac{1}{2}ebc+bde\,x^2}{\sqrt{bde}}+\sqrt{bd\cdots}\right.}{2\sqrt{bde}}\right.}{16d\cdots}$
default	Expression too large to display

input
```
int(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/d*(8*b^2*d^2*x^4-22*a*b*d^2*x^2+14*b^2*c*d*x^2+57*a^2*d^2-52*a*b*c*d+
3*b^2*c^2)*(b*x^2+a)/b^4/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/16/d/b^4*(1/2*(
35*a^2*d^2-10*a*b*c*d-b^2*c^2)*(a*d-b*c)*ln((1/2*e*d*a+1/2*e*b*c+b*d*e*x^2
)/(b*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)-1
6*a^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)*d*(d*x^2+c)/(a*d-b*c)/(b*d*e*x^4+a*d*e*x
^2+b*c*e*x^2+a*c*e)^(1/2))/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b
*x^2+a))^(1/2)/(d*x^2+c)
```

3.307.5 Fricas [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 781, normalized size of antiderivative = 2.21

$$\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \left[\frac{3\left(ab^3c^3+9\,a^2b^2c^2d-45\,a^3bcd^2+35\,a^4d^3+\left(b^4c^3+9\,ab^3c^2d-45\,a^2b^2cd^2+35\,a^3bd^3\right.\right.}{}\right.$$

input
```
integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/192*(3*(a*b^3*c^3 + 9*a^2*b^2*c^2*d - 45*a^3*b*c*d^2 + 35*a^4*d^3 + (b^
4*c^3 + 9*a*b^3*c^2*d - 45*a^2*b^2*c*d^2 + 35*a^3*b*d^3)*x^2)*sqrt(b*d*e)*
log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d +
 a^2*d^2)*e - 4*(2*b*d^2*x^4 + b*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqrt
(b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) + 4*(8*b^4*d^4*x^8 + 3*a*b^3*c^
3*d - 100*a^2*b^2*c^2*d^2 + 105*a^3*b*c*d^3 + 2*(11*b^4*c*d^3 - 7*a*b^3*d^
4)*x^6 + (17*b^4*c^2*d^2 - 52*a*b^3*c*d^3 + 35*a^2*b^2*d^4)*x^4 + (3*b^4*c
^3*d - 35*a*b^3*c^2*d^2 - 65*a^2*b^2*c*d^3 + 105*a^3*b*d^4)*x^2)*sqrt((b*e
*x^2 + a*e)/(d*x^2 + c)))/(b^6*d^2*e^2*x^2 + a*b^5*d^2*e^2), 1/96*(3*(a*b^
3*c^3 + 9*a^2*b^2*c^2*d - 45*a^3*b*c*d^2 + 35*a^4*d^3 + (b^4*c^3 + 9*a*b^3
*c^2*d - 45*a^2*b^2*c*d^2 + 35*a^3*b*d^3)*x^2)*sqrt(-b*d*e)*arctan(1/2*(2*
b*d*x^2 + b*c + a*d)*sqrt(-b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*d
*e*x^2 + a*b*d*e)) + 2*(8*b^4*d^4*x^8 + 3*a*b^3*c^3*d - 100*a^2*b^2*c^2*d^
2 + 105*a^3*b*c*d^3 + 2*(11*b^4*c*d^3 - 7*a*b^3*d^4)*x^6 + (17*b^4*c^2*d^2
 - 52*a*b^3*c*d^3 + 35*a^2*b^2*d^4)*x^4 + (3*b^4*c^3*d - 35*a*b^3*c^2*d^2
- 65*a^2*b^2*c*d^3 + 105*a^3*b*d^4)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))
)/(b^6*d^2*e^2*x^2 + a*b^5*d^2*e^2)]
```

3.307.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**5/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

3.307.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.307.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{2,[1,0,0]%%%},[2,1,0]%%%}+%%%{%%{[-4,0]:[1,0,%%%{-1,[1
,1,1]%%%}
```

3.307.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^5}{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

output
```
int(x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.308 $\int \frac{x^3}{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

3.308.1 Optimal result

Integrand size = 26, antiderivative size = 202

$$\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx=\frac{a(bc-ad)}{b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{(3bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{8b^3e^2}+\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{4b^2e^2}+\frac{3(bc-5ad)(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{7/2}\sqrt{d}e^{3/2}}$$

output
```
3/8*(-5*a*d+b*c)*(-a*d+b*c)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/
b^(1/2)/e^(1/2))/b^(7/2)/e^(3/2)/d^(1/2)+a*(-a*d+b*c)/b^3/e/(e*(b*x^2+a)/(
d*x^2+c))^(1/2)+1/8*(-7*a*d+3*b*c)*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)
/b^3/e^2+1/4*(d*x^2+c)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^2/e^2
```

3.308.2 Mathematica [A] (verified)

Time = 2.61 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b}\sqrt{d}\sqrt{c+dx^2}(-15a^2d+ab(13c-5dx^2)+b^2x^2(5c+2dx^2))+3(b^2c^2-6abcd+5a^2}{8b^{7/2}\sqrt{d}e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

input
```
Integrate[x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[b]*Sqrt[d]*Sqrt[c + d*x^2]*(-15*a^2*d + a*b*(13*c - 5*d*x^2) + b^2*x
^2*(5*c + 2*d*x^2)) + 3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*Sqrt[a + b*x^2]*
ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(8*b^(7/2)*S
qrt[d]*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

3.308.3 Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {2053, 2052, 25, 361, 25, 27, 361, 25, 27, 359, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \ 2053$$

$$\frac{1}{2}\int \frac{x^2}{\left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}}dx^2$$

$$\Big\downarrow \ 2052$$

$$e(bc-ad)\int -\frac{ae-cx^4}{x^4\left(be-dx^4\right)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$

$$\Big\downarrow \ 25$$

$$-\left(e(bc-ad)\int \frac{ae-cx^4}{x^4\left(be-dx^4\right)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}\right)$$

$$\Big\downarrow \text{361}$$

$$e(bc-ad)\left(\frac{1}{4}\int -\frac{4abe-3(bc-ad)x^4}{b^2ex^4\left(be-dx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}+\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4b^2e\left(be-dx^4\right)^2}\right)$$

$$\Big\downarrow \text{25}$$

$$e(bc-ad)\left(\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4b^2e\left(be-dx^4\right)^2}-\frac{1}{4}\int \frac{4abe-3(bc-ad)x^4}{b^2ex^4\left(be-dx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}\right)$$

$$\Big\downarrow \text{27}$$

$$e(bc-ad)\left(\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4b^2e\left(be-dx^4\right)^2}-\frac{\int \frac{4abe-3(bc-ad)x^4}{x^4(be-dx^4)^2}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{4b^2e}\right)$$

$$\Big\downarrow \text{361}$$

$$e(bc-ad)\left(\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4b^2e\left(be-dx^4\right)^2}-\frac{-\frac{1}{2}\int -\frac{8ae-\left(3c-\frac{7ad}{b}\right)x^4}{ex^4(be-dx^4)}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}-\frac{(3bc-7ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2be(be-dx^4)}}{4b^2e}\right)$$

$$\Big\downarrow \text{25}$$

$$e(bc-ad)\left(\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4b^2e\left(be-dx^4\right)^2}-\frac{\frac{1}{2}\int \frac{8ae-\left(3c-\frac{7ad}{b}\right)x^4}{ex^4(be-dx^4)}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}-\frac{(3bc-7ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2be(be-dx^4)}}{4b^2e}\right)$$

$$\Big\downarrow \text{27}$$

$$e(bc-ad)\left(\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4b^2e\left(be-dx^4\right)^2}-\frac{\frac{\int \frac{8ae-\left(3c-\frac{7ad}{b}\right)x^4}{x^4\left(be-dx^4\right)}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{2e}-\frac{(3bc-7ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2be(be-dx^4)}}{4b^2e}\right)$$

$$\Big\downarrow \text{359}$$

$$e(bc-ad)\left(\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4b^2e\left(be-dx^4\right)^2}-\frac{\frac{-\frac{3(bc-5ad)\int \frac{1}{be-dx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{b}-\frac{8a}{bx^2}}{2e}-\frac{(3bc-7ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2be(be-dx^4)}}{4b^2e}\right)$$

$$\Big\downarrow \text{221}$$

3.308. $\int \frac{x^3}{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}dx$

$$e(bc-ad)\left(\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4b^2e\left(be-dx^4\right)^2}-\frac{\frac{-\frac{3(bc-5ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{b^{3/2}\sqrt{d}\sqrt{e}}-\frac{8a}{bx^2}}{2e}-\frac{(3bc-7ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2be(be-dx^4)}}{4b^2e}\right)$$

input
```
Int[x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(b*c - a*d)*e*(((b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*b^2*e*(b
*e - d*x^4)^2) - (-1/2*((3*b*c - 7*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])
/(b*e*(b*e - d*x^4)) + ((-8*a)/(b*x^2) - (3*(b*c - 5*a*d)*ArcTanh[(Sqrt[d]
*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(b^(3/2)*Sqrt[d]*S
qrt[e]))/(2*e))/(4*b^2*e))
```

3.308.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.308. $\int \frac{x^3}{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.308.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.27

method	result
risch	$-\frac{\left(-2 b d x^{2}+7 a d-5 b c\right)\left(b x^{2}+a\right)}{8 b^{3} e \sqrt{\frac{e\left(b x^{2}+a\right)}{d x^{2}+c}}}+\frac{\left(\frac{3(5 a d-b c)(a d-b c) \ln \left(\frac{\frac{1}{2} e d a+\frac{1}{2} e b c+b d e x^{2}}{\sqrt{b d e}}+\sqrt{b d e x^{4}+(e d a+e b c) x^{2}+a c e}\right)}{2 \sqrt{b d e}}-\frac{8 a\left(a^{2} d^{2}-2 a b c d+b^{2} c^{2}\right)}{(a d-b c) \sqrt{b d e x^{4}+a d e x^{2}+\cdots}}\cdots\right)}{8 b^{3} e \sqrt{\frac{e\left(b x^{2}+a\right)}{d x^{2}+c}}\left(d x^{2}+c\right)}$
default	$-\frac{\left(-4 \sqrt{b d} \sqrt{b d x^{4}+a d x^{2}+b c x^{2}+a c}\, b^{2} d x^{4}-15 \ln \left(\frac{2 b d x^{2}+2 \sqrt{b d x^{4}+a d x^{2}+b c x^{2}+a c} \sqrt{b d}+a d+b c}{2 \sqrt{b d}}\right) a^{2} b d^{2} x^{2}+18 \ln \left(\frac{2 b d x^{2}+2 \sqrt{b d x^{4}+a d \cdots}}{\cdots}\right)\cdots\right)}{\cdots}$

input
```
int(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-2*b*d*x^2+7*a*d-5*b*c)*(b*x^2+a)/b^3/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2
)+1/8/b^3*(3/2*(5*a*d-b*c)*(a*d-b*c)*ln((1/2*e*d*a+1/2*e*b*c+b*d*e*x^2)/(b
*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)-8*a*(
a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x^2+c)/(a*d-b*c)/(b*d*e*x^4+a*d*e*x^2+b*c*e*
x^2+a*c*e)^(1/2))/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^
(1/2)/(d*x^2+c)
```

3.308.5 Fricas [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 585, normalized size of antiderivative = 2.90

$$\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \left[\frac{3\,(ab^2c^2 - 6\,a^2bcd + 5\,a^3d^2 + (b^3c^2 - 6\,ab^2cd + 5\,a^2bd^2)x^2)\sqrt{bde}\log\left(8\,b^2d^2ex^4 + 8\,(b\right.}{}\right.$$

$$-\frac{3\,(ab^2c^2 - 6\,a^2bcd + 5\,a^3d^2 + (b^3c^2 - 6\,ab^2cd + 5\,a^2bd^2)x^2)\sqrt{-bde}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{-bde}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\,(b^2dex^2+abde)}\right) -}{16\,(b^5de^2x^2}$$

input
```
integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/32*(3*(a*b^2*c^2 - 6*a^2*b*c*d + 5*a^3*d^2 + (b^3*c^2 - 6*a*b^2*c*d + 5
*a^2*b*d^2)*x^2)*sqrt(b*d*e)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e
*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^2*x^4 + b*c^2 + a*c*d
+ (3*b*c*d + a*d^2)*x^2)*sqrt(b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) +
4*(2*b^3*d^3*x^6 + 13*a*b^2*c^2*d - 15*a^2*b*c*d^2 + (7*b^3*c*d^2 - 5*a*b^
2*d^3)*x^4 + (5*b^3*c^2*d + 8*a*b^2*c*d^2 - 15*a^2*b*d^3)*x^2)*sqrt((b*e*x
^2 + a*e)/(d*x^2 + c)))/(b^5*d*e^2*x^2 + a*b^4*d*e^2), -1/16*(3*(a*b^2*c^2
 - 6*a^2*b*c*d + 5*a^3*d^2 + (b^3*c^2 - 6*a*b^2*c*d + 5*a^2*b*d^2)*x^2)*sq
rt(-b*d*e)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d*e)*sqrt((b*e*x^2 +
 a*e)/(d*x^2 + c))/(b^2*d*e*x^2 + a*b*d*e)) - 2*(2*b^3*d^3*x^6 + 13*a*b^2*
c^2*d - 15*a^2*b*c*d^2 + (7*b^3*c*d^2 - 5*a*b^2*d^3)*x^4 + (5*b^3*c^2*d +
8*a*b^2*c*d^2 - 15*a^2*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^5
*d*e^2*x^2 + a*b^4*d*e^2)]
```

3.308.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**3/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

3.308.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.308.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{2,[1,0,0]%%%},[2,1,0]%%%}+%%%{%%{[-4,0]:[1,0,%%%{-1,[1
,1,1]%%%}
```

3.308.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^3}{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

output
```
int(x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.309 $\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

3.309.1 Optimal result

Integrand size = 24, antiderivative size = 146

$$\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx=-\frac{3(bc-ad)}{2b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{c+dx^2}{2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{3\sqrt{d}(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{5/2}e^{3/2}}$$

output
```
3/2*(-a*d+b*c)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/
2))*d^(1/2)/b^(5/2)/e^(3/2)-3/2*(-a*d+b*c)/b^2/e/(e*(b*x^2+a)/(d*x^2+c))^(
1/2)+1/2*(d*x^2+c)/b/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

3.309.2 Mathematica [A] (verified)

Time = 1.86 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.95

$$\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx=\frac{\sqrt{b}\sqrt{c+dx^2}(-2bc+3ad+bdx^2)+3\sqrt{d}(bc-ad)\sqrt{a+bx^2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2b^{5/2}e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

input
```
Integrate[x/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[b]*Sqrt[c + d*x^2]*(-2*b*c + 3*a*d + b*d*x^2) + 3*Sqrt[d]*(b*c - a*d
)*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^
2])])/(2*b^(5/2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

3.309.3 Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2053, 2051, 253, 264, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{1}{\left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{2051}$$

$$e(bc-ad)\int \frac{1}{x^4\left(be-dx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$

$$\Big\downarrow \text{253}$$

$$e(bc-ad)\left(\frac{3\int \frac{1}{x^4(be-dx^4)}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{2be}+\frac{1}{2bex^2\left(be-dx^4\right)}\right)$$

$$\Big\downarrow \text{264}$$

$$e(bc-ad)\left(\frac{3\left(\frac{d\int \frac{1}{be-dx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{be}-\frac{1}{bex^2}\right)}{2be}+\frac{1}{2bex^2\left(be-dx^4\right)}\right)$$

$$\Big\downarrow \text{221}$$

$$e(bc-ad)\left(\frac{3\left(\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{b^{3/2}e^{3/2}}-\frac{1}{bex^2}\right)}{2be}+\frac{1}{2bex^2\left(be-dx^4\right)}\right)$$

input
```
Int[x/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(b*c - a*d)*e*(1/(2*b*e*x^2*(b*e - d*x^4)) + (3*(-(1/(b*e*x^2)) + (Sqrt[d]
*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(
b^(3/2)*e^(3/2))))/(2*b*e))
```

3.309.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2051
```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)   Subst[Int[
x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
 x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x]
&& FractionQ[p] && IntegerQ[1/n]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.309.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.60

method	result
risch	$\frac{(b x^2+a) d}{2b^2 e\sqrt{\frac{e(b x^2+a)}{d x^2+c}}}-\frac{\left(\frac{3d(ad-bc)\ln\left(\frac{\frac{1}{2}eda+\frac{1}{2}ebc+bde\,x^2}{\sqrt{bde}}+\sqrt{bde\,x^4+(eda+ebc)x^2+ace}\right)}{2\sqrt{bde}}+\frac{\left(-2a^2d^2+4abcd-2b^2c^2\right)\left(d x^2+c\right)}{(ad-bc)\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}}\right)\sqrt{(d x^2+\cdots}}{2b^2 e\sqrt{\frac{e(b x^2+a)}{d x^2+c}}\,(d x^2+c)}$
default	$-\frac{\left(3\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)ab\,d^2x^2-3\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)b^2cd\,x^2-2\sqrt{bd}\,\sqrt{bd\cdots}}{\cdots}$

input
```
int(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^2*(b*x^2+a)*d/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/2/b^2*(3/2*d*(a*d-b*
c)*ln((1/2*e*d*a+1/2*e*b*c+b*d*e*x^2)/(b*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*
e)*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)+(-2*a^2*d^2+4*a*b*c*d-2*b^2*c^2)*(d*x^2
+c)/(a*d-b*c)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2))/e/(e*(b*x^2+a)/
(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)/(d*x^2+c)
```

3.309.5 Fricas [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 443, normalized size of antiderivative = 3.03

$$\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \left[-\frac{3\left((b^2c-abd)ex^2+(abc-a^2d)e\right)\sqrt{\frac{d}{be}}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\,(b^2\right.}{}\right.$$

$$-\frac{3\left((b^2c-abd)ex^2+(abc-a^2d)e\right)\sqrt{-\frac{d}{be}}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{d}{be}}}{2\,(bdx^2+ad)}\right)-2\left(bd^2x^4-2\,bc^2+3\,acd-\right.}{4\,(b^3e^2x^2+ab^2e^2)}$$

input
```
integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(3*((b^2*c - a*b*d)*e*x^2 + (a*b*c - a^2*d)*e)*sqrt(d/(b*e))*log(8*b
^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 - 4
*(2*b^2*d^2*x^4 + b^2*c^2 + a*b*c*d + (3*b^2*c*d + a*b*d^2)*x^2)*sqrt((b*e
*x^2 + a*e)/(d*x^2 + c))*sqrt(d/(b*e))) - 4*(b*d^2*x^4 - 2*b*c^2 + 3*a*c*d
 - (b*c*d - 3*a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*e^2*x^2
+ a*b^2*e^2), -1/4*(3*((b^2*c - a*b*d)*e*x^2 + (a*b*c - a^2*d)*e)*sqrt(-d/
(b*e))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)
)*sqrt(-d/(b*e))/(b*d*x^2 + a*d)) - 2*(b*d^2*x^4 - 2*b*c^2 + 3*a*c*d - (b*
c*d - 3*a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*e^2*x^2 + a*b^
2*e^2)]
```

3.309.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

3.309.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.309.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{2,[1,0,0]%%%},[2,1,0]%%%}+%%%{%%{[-4,0]:[1,0,%%%{-1,[1
,1,1]%%%}
```

3.309.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x}{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(x/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

output
```
int(x/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.310 $\int \frac{1}{x\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

3.310.1 Optimal result

Integrand size = 26, antiderivative size = 152

$$\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx=\frac{bc-ad}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}e^{3/2}}+\frac{d^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{b^{3/2}e^{3/2}}$$

output
```
-c^(3/2)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2))/a^
(3/2)/e^(3/2)+d^(3/2)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2
)/e^(1/2))/b^(3/2)/e^(3/2)+(-a*d+b*c)/a/b/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

3.310.2 Mathematica [A] (verified)

Time = 2.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.24

$$\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx=\frac{-b^{3/2}c^{3/2}\sqrt{a+bx^2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)+\sqrt{a}\left(\sqrt{b}(bc-ad)\sqrt{c+dx^2}+ad^{3/2}\sqrt{a+bx^2}\right.}{a^{3/2}b^{3/2}e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

input
```
Integrate[1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

output
```
(-(b^(3/2)*c^(3/2)*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt
[a]*Sqrt[c + d*x^2])]) + Sqrt[a]*(Sqrt[b]*(b*c - a*d)*Sqrt[c + d*x^2] + a*
d^(3/2)*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c
+ d*x^2])]))/(a^(3/2)*b^(3/2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c +
 d*x^2])
```

3.310.3 Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2053, 2052, 25, 382, 397, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{1}{x^2\left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{2052}$$

$$e(bc-ad)\int -\frac{1}{x^4\left(ae-cx^4\right)\left(be-dx^4\right)}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$-\left(e(bc-ad)\int \frac{1}{x^4\left(ae-cx^4\right)\left(be-dx^4\right)}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}\right)$$

$$\Big\downarrow \text{382}$$

$$e(bc-ad)\left(\frac{1}{abe^2x^2}-\frac{\int \frac{(bc+ad)e-cdx^4}{(ae-cx^4)(be-dx^4)}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{abe^2}\right)$$

$$\Big\downarrow \text{397}$$

$$e(bc-ad)\left(\frac{1}{abe^2x^2}-\frac{\frac{bc^2\int\frac{1}{ae-cx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{bc-ad}-\frac{ad^2\int\frac{1}{be-dx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{bc-ad}}{abe^2}\right)$$

$$\Big\downarrow \text{221}$$

$$e(bc-ad)\left(\frac{1}{abe^2x^2}-\frac{\frac{bc^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\sqrt{e}(bc-ad)}-\frac{ad^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{b}\sqrt{e}(bc-ad)}}{abe^2}\right)$$

input
```
Int[1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

output
```
(b*c - a*d)*e*(1/(a*b*e^2*x^2) - ((b*c^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a +
 b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(Sqrt[a]*(b*c - a*d)*Sqrt[e]) -
 (a*d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*S
qrt[e])])/(Sqrt[b]*(b*c - a*d)*Sqrt[e]))/(a*b*e^2))
```

3.310.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

3.310. $\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.310.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(122) = 244$.

Time = 0.14 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.64

method	result
default	$-\frac{\left(\sqrt{bd}\,\ln\left(\frac{a d\,x^{2}+b c\,x^{2}+2\sqrt{a c}\,\sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+a c}+2 a c}{x^{2}}\right) b^{2} c^{2} x^{2}-\ln\left(\frac{2 b d\,x^{2}+2\sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+a c}\,\sqrt{b d}+a d+b c}{2\sqrt{b d}}\right)\sqrt{a c}\,a b\,d^{2} x^{2}+\sqrt{}}{}$

input
```
int(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((b*d)^(1/2)*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)+2*a*c)/x^2)*b^2*c^2*x^2-ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)*a*b*d^2*
x^2+(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)+2*a*c)/x^2)*a*b*c^2-ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)*a^2*d^2+2*(b*d
)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*(a*c)^(1/2)*a*d-2*(b*d)^(1/2)*((d*x^2+
c)*(b*x^2+a))^(1/2)*(a*c)^(1/2)*b*c)/a*(b*x^2+a)/b/(a*c)^(1/2)/(b*d)^(1/2)
/((d*x^2+c)*(b*x^2+a))^(1/2)/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(3/2)
```

3.310.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(122) = 244$.

Time = 0.91 (sec) , antiderivative size = 1293, normalized size of antiderivative = 8.51

$$\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((a*b*d*e*x^2 + a^2*d*e)*sqrt(d/(b*e))*log(8*b^2*d^2*x^4 + b^2*c^2 +
6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c
^2 + a*b*c*d + (3*b^2*c*d + a*b*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)
)*sqrt(d/(b*e))) + (b^2*c*e*x^2 + a*b*c*e)*sqrt(c/(a*e))*log(((b^2*c^2 + 6
*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((a*b*
c*d + a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt((b*e*x^2
+ a*e)/(d*x^2 + c))*sqrt(c/(a*e)))/x^4) + 4*(b*c^2 - a*c*d + (b*c*d - a*d^
2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*b^2*e^2*x^2 + a^2*b*e^2), -1
/4*(2*(a*b*d*e*x^2 + a^2*d*e)*sqrt(-d/(b*e))*arctan(1/2*(2*b*d*x^2 + b*c +
 a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-d/(b*e))/(b*d*x^2 + a*d)) -
(b^2*c*e*x^2 + a*b*c*e)*sqrt(c/(a*e))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)
*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((a*b*c*d + a^2*d^2)*x^4
+ 2*a^2*c^2 + (a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))
*sqrt(c/(a*e)))/x^4) - 4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*sqrt((b*e*x
^2 + a*e)/(d*x^2 + c)))/(a*b^2*e^2*x^2 + a^2*b*e^2), 1/4*(2*(b^2*c*e*x^2 +
 a*b*c*e)*sqrt(-c/(a*e))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^
2 + a*e)/(d*x^2 + c))*sqrt(-c/(a*e))/(b*c*x^2 + a*c)) + (a*b*d*e*x^2 + a^2
*d*e)*sqrt(d/(b*e))*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*
(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c^2 + a*b*c*d + (3*b^2*c*
d + a*b*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(d/(b*e))) + 4*...
```

3.310.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

3.310.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.310.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{2,[1,2,2]%%%},[2,1,3,0]%%%}+%%%{%%%{-4,[2,1,2]%%%},[2,
1,2,1]%%%
```

3.310.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{x\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
```

output
```
int(1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)
```

3.311 $\int \frac{1}{x^3\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

3.311.1 Optimal result

Integrand size = 26, antiderivative size = 170

$$\int \frac{1}{x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx=-\frac{3(bc-ad)}{2a^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{bc-ad}{2a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}+\frac{3\sqrt{c}(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{5/2}e^{3/2}}$$

output
```
3/2*(-a*d+b*c)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/
2))*c^(1/2)/a^(5/2)/e^(3/2)-3/2*(-a*d+b*c)/a^2/e/(e*(b*x^2+a)/(d*x^2+c))^(
1/2)+1/2*(-a*d+b*c)/a/(a*e-c*e*(b*x^2+a)/(d*x^2+c))/(e*(b*x^2+a)/(d*x^2+c)
)^(1/2)
```

3.311.2 Mathematica [A] (verified)

Time = 2.52 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\, dx = \frac{-\sqrt{a}\sqrt{c+dx^2}(3bcx^2 + a(c-2dx^2)) + 3\sqrt{c}(bc-ad)x^2\sqrt{a+bx^2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{5/2}ex^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

input
```
Integrate[1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

output
```
(-(Sqrt[a]*Sqrt[c + d*x^2]*(3*b*c*x^2 + a*(c - 2*d*x^2))) + 3*Sqrt[c]*(b*c
 - a*d)*x^2*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqr
t[c + d*x^2])])/(2*a^(5/2)*e*x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c
+ d*x^2])
```

3.311.3 Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.66, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2053, 2052, 253, 264, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2053}$$

$$\frac{1}{2}\int \frac{1}{x^4 \left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{ 2052}$$

$$e(bc-ad)\int \frac{1}{x^4\left(ae-cx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$

$$\Big\downarrow \text{ 253}$$

$$e(bc-ad)\left(\frac{3\int \frac{1}{x^4(ae-cx^4)}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{2ae}+\frac{1}{2aex^2\left(ae-cx^4\right)}\right)$$

$$\Big\downarrow \text{264}$$

$$e(bc-ad)\left(\frac{3\left(\frac{c\int \frac{1}{ae-cx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{ae}-\frac{1}{aex^2}\right)}{2ae}+\frac{1}{2aex^2\,(ae-cx^4)}\right)$$

$$\Big\downarrow \text{221}$$

$$e(bc-ad)\left(\frac{3\left(\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}e^{3/2}}-\frac{1}{aex^2}\right)}{2ae}+\frac{1}{2aex^2\,(ae-cx^4)}\right)$$

input
```
Int[1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

output
```
(b*c - a*d)*e*(1/(2*a*e*x^2*(a*e - c*x^4)) + (3*(-(1/(a*e*x^2)) + (Sqrt[c]
*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(
a^(3/2)*e^(3/2))))/(2*a*e))
```

3.311.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

3.311. $\int \frac{1}{x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.311.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.42

method	result
risch	$-\frac{c\left(b\,x^2+a\right)}{2a^2x^2e\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}+\frac{\left(-\frac{3c(ad-bc)\ln\left(\frac{2ace+(eda+ebc)x^2+2\sqrt{ace}\,\sqrt{bde\,x^4+(eda+ebc)x^2+ace}}{x^2}\right)}{2\sqrt{ace}}+\frac{\left(2a^2d^2-4abcd+2b^2c^2\right)\left(d\,x^2+c\right)}{(ad-bc)\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ac}}\right.}{2a^2e\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,\left(d\,x^2+c\right)}$
default	$\frac{\left(2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{ac}\,b^2d\,x^6-3\ln\left(\frac{ad\,x^2+bc\,x^2+2\sqrt{ac}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}+2ac}{x^2}\right)a^2bcd\,x^4+3\ln\left(\frac{ad\,x^2+bc\,x^2+2\sqrt{ac}\,\sqrt{bd\,x}}{x}\right.\right.}{}$

input
```
int(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2*c*(b*x^2+a)/x^2/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/2/a^2*(-3/2*c*(
a*d-b*c)/(a*c*e)^(1/2)*ln((2*a*c*e+(a*d*e+b*c*e)*x^2+2*(a*c*e)^(1/2)*(b*d*
e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/x^2)+(2*a^2*d^2-4*a*b*c*d+2*b^2*c^2)
*(d*x^2+c)/(a*d-b*c)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2))/e/(e*(b*
x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)/(d*x^2+c)
```

3.311.5 Fricas [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.76

$$\int \frac{1}{x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \left[-\frac{3\left((b^2c-abd)ex^4+(abc-a^2d)ex^2\right)\sqrt{\frac{c}{ae}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^4+8\,a^2c^2+8\,(abc^2+a^2c}{}\right.}{}\right.$$

$$-\frac{3\left((b^2c-abd)ex^4+(abc-a^2d)ex^2\right)\sqrt{-\frac{c}{ae}}\arctan\left(\frac{((bc+ad)x^2+2\,ac)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{c}{ae}}}{2\,(bcx^2+ac)}\right)+2\left((3\,bcd-2\,ad^2)x^4+\right.}{4\,(a^2be^2x^4+a^3e^2x^2)}$$

input
```
integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(3*((b^2*c - a*b*d)*e*x^4 + (a*b*c - a^2*d)*e*x^2)*sqrt(c/(a*e))*log
(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*
x^2 - 4*((a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*c*d)*x^2)*
sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(c/(a*e)))/x^4) + 4*((3*b*c*d - 2*a*
d^2)*x^4 + a*c^2 + (3*b*c^2 - a*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)
))/(a^2*b*e^2*x^4 + a^3*e^2*x^2), -1/4*(3*((b^2*c - a*b*d)*e*x^4 + (a*b*c
- a^2*d)*e*x^2)*sqrt(-c/(a*e))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((
b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-c/(a*e))/(b*c*x^2 + a*c)) + 2*((3*b*c*d
- 2*a*d^2)*x^4 + a*c^2 + (3*b*c^2 - a*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^
2 + c)))/(a^2*b*e^2*x^4 + a^3*e^2*x^2)]
```

3.311.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**3/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

3.311.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.311.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{2,[1,0,0]%%%},[6,1,0,0]%%%}+%%%{%%{[-4,0]:[1,0,%%%{-1,
[1,1,1]%%
```

3.311.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{x^3 \left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
```

output
```
int(1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)
```

3.312 $\int \frac{1}{x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

3.312.1 Optimal result

Integrand size = 26, antiderivative size = 255

$$\int \frac{1}{x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{b(bc-ad)}{a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2}-\frac{(7bc-3ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^3\left(ae^2-\frac{ce^2(a+bx^2)}{c+dx^2}\right)}-\frac{3(bc-ad)(5bc-ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{7/2}\sqrt{c}e^{3/2}}$$

output
```
-3/8*(-a*d+b*c)*(-a*d+5*b*c)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)
/a^(1/2)/e^(1/2))/a^(7/2)/e^(3/2)/c^(1/2)+b*(-a*d+b*c)/a^3/e/(e*(b*x^2+a)/
(d*x^2+c))^(1/2)-1/4*(-a*d+b*c)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^2/(a*e-c
*e*(b*x^2+a)/(d*x^2+c))^2-1/8*(-3*a*d+7*b*c)*(-a*d+b*c)*(e*(b*x^2+a)/(d*x^
2+c))^(1/2)/a^3/(a*e^2-c*e^2*(b*x^2+a)/(d*x^2+c))
```

3.312.2 Mathematica [A] (verified)

Time = 4.20 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{a}\sqrt{c}\sqrt{c+dx^2}(15b^2cx^4+abx^2(5c-13dx^2)-a^2(2c+5dx^2))-3(5b^2c^2-6abcd+}{8a^{7/2}\sqrt{c}ex^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

input
```
Integrate[1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

output
```
(Sqrt[a]*Sqrt[c]*Sqrt[c + d*x^2]*(15*b^2*c*x^4 + a*b*x^2*(5*c - 13*d*x^2)
- a^2*(2*c + 5*d*x^2)) - 3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x^4*Sqrt[a +
b*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^
(7/2)*Sqrt[c]*e*x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

3.312.3 Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.82, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {2053, 2052, 25, 361, 25, 27, 361, 25, 27, 359, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \; 2053$$

$$\frac{1}{2}\int \frac{1}{x^6 \left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}}dx^2$$

$$\Big\downarrow \; 2052$$

$$e(bc-ad)\int -\frac{be-dx^4}{x^4(ae-cx^4)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}$$

$$\Big\downarrow \; 25$$

$$-\left(e(bc-ad)\int \frac{be-dx^4}{x^4(ae-cx^4)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}\right)$$

$$\Big\downarrow 361$$

$$e(bc-ad)\left(\frac{1}{4}\int -\frac{3(bc-ad)x^4+4abe}{a^2ex^4\left(ae-cx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}-\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2e\left(ae-cx^4\right)^2}\right)$$

$$\Big\downarrow 25$$

$$e(bc-ad)\left(-\frac{1}{4}\int \frac{3(bc-ad)x^4+4abe}{a^2ex^4\left(ae-cx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}-\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2e\left(ae-cx^4\right)^2}\right)$$

$$\Big\downarrow 27$$

$$e(bc-ad)\left(-\frac{\int \frac{3(bc-ad)x^4+4abe}{x^4(ae-cx^4)^2}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{4a^2e}-\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2e\left(ae-cx^4\right)^2}\right)$$

$$\Big\downarrow 361$$

$$e(bc-ad)\left(-\frac{\frac{(7bc-3ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2ae(ae-cx^4)}-\frac{1}{2}\int -\frac{\left(\frac{7bc}{a}-3d\right)x^4+8be}{ex^4(ae-cx^4)}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{4a^2e}-\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2e\left(ae-cx^4\right)^2}\right)$$

$$\Big\downarrow 25$$

$$e(bc-ad)\left(-\frac{\frac{1}{2}\int \frac{\left(\frac{7bc}{a}-3d\right)x^4+8be}{ex^4(ae-cx^4)}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}+\frac{(7bc-3ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2ae(ae-cx^4)}}{4a^2e}-\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2e\left(ae-cx^4\right)^2}\right)$$

$$\Big\downarrow 27$$

$$e(bc-ad)\left(-\frac{\frac{\int \frac{\left(\frac{7bc}{a}-3d\right)x^4+8be}{x^4\left(ae-cx^4\right)}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{2e}+\frac{(7bc-3ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2ae(ae-cx^4)}}{4a^2e}-\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2e\left(ae-cx^4\right)^2}\right)$$

$$\Big\downarrow 359$$

$$e(bc-ad)\left(-\frac{\frac{\frac{3(5bc-ad)\int \frac{1}{ae-cx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{a}-\frac{8b}{ax^2}}{2e}+\frac{(7bc-3ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2ae(ae-cx^4)}}{4a^2e}-\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2e\left(ae-cx^4\right)^2}\right)$$

$$\Big\downarrow 221$$

3.312. $\displaystyle\int \frac{1}{x^5\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

$$e(bc-ad)\left(-\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2e\left(ae-cx^4\right)^2}-\frac{\frac{\frac{3(5bc-ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}\sqrt{c}\sqrt{e}}-\frac{8b}{ax^2}}{2e}+\frac{(7bc-3ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2ae(ae-cx^4)}}{4a^2e}\right)$$

input
```
Int[1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

output
```
(b*c - a*d)*e*(-1/4*((b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(a^2*e
*(a*e - c*x^4)^2) - (((7*b*c - 3*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(
2*a*e*(a*e - c*x^4)) + ((-8*b)/(a*x^2) + (3*(5*b*c - a*d)*ArcTanh[(Sqrt[c]
*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(a^(3/2)*Sqrt[c]*S
qrt[e]))/(2*e))/(4*a^2*e))
```

3.312.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.312.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{\left(b\,x^{2}+a\right)\left(5ad\,x^{2}-7bc\,x^{2}+2ac\right)}{8a^{3}x^{4}e\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}+\frac{\left(\frac{\left(-3a^{2}d^{2}+18abcd-15b^{2}c^{2}\right)\ln\left(\frac{2ace+(eda+ebc)x^{2}+2\sqrt{ace}\,\sqrt{bde\,x^{4}+(eda+ebc)x^{2}+ace}}{x^{2}}\right)}{2\sqrt{ace}}-\frac{8b\left(a^{2}c\right.}{(ad-bc)\sqrt{}}\right.}{8a^{3}e\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}\,(d\,x^{2}+c)}$
default	Expression too large to display

input
```
int(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(b*x^2+a)*(5*a*d*x^2-7*b*c*x^2+2*a*c)/a^3/x^4/e/(e*(b*x^2+a)/(d*x^2+c
))^(1/2)+1/8/a^3*(1/2*(-3*a^2*d^2+18*a*b*c*d-15*b^2*c^2)/(a*c*e)^(1/2)*ln(
(2*a*c*e+(a*d*e+b*c*e)*x^2+2*(a*c*e)^(1/2)*(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*
c*e)^(1/2))/x^2)-8*b*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x^2+c)/(a*d-b*c)/(b*d*
e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2))/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((
d*x^2+c)*e*(b*x^2+a))^(1/2)/(d*x^2+c)
```

3.312.5 Fricas [A] (verification not implemented)

Time = 3.95 (sec) , antiderivative size = 613, normalized size of antiderivative = 2.40

$$\int \frac{1}{x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \left[\frac{3\left((5\,b^3c^2 - 6\,ab^2cd + a^2bd^2)x^6 + (5\,ab^2c^2 - 6\,a^2bcd + a^3d^2)x^4\right)\sqrt{ace}\log\left(\frac{(b^2c^2+6\,a\right.}{}\right.}{}\right.$$

input
```
integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/32*(3*((5*b^3*c^2 - 6*a*b^2*c*d + a^2*b*d^2)*x^6 + (5*a*b^2*c^2 - 6*a^2
*b*c*d + a^3*d^2)*x^4)*sqrt(a*c*e)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*
x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b*c*d + a*d^2)*x^4 +
 2*a*c^2 + (b*c^2 + 3*a*c*d)*x^2)*sqrt(a*c*e)*sqrt((b*e*x^2 + a*e)/(d*x^2
+ c)))/x^4) + 4*((15*a*b^2*c^2*d - 13*a^2*b*c*d^2)*x^6 - 2*a^3*c^3 + (15*a
*b^2*c^3 - 8*a^2*b*c^2*d - 5*a^3*c*d^2)*x^4 + (5*a^2*b*c^3 - 7*a^3*c^2*d)*
x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^4*b*c*e^2*x^6 + a^5*c*e^2*x^4),
 1/16*(3*((5*b^3*c^2 - 6*a*b^2*c*d + a^2*b*d^2)*x^6 + (5*a*b^2*c^2 - 6*a^2
*b*c*d + a^3*d^2)*x^4)*sqrt(-a*c*e)*arctan(1/2*sqrt(-a*c*e)*((b*c + a*d)*x
^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b*c*e*x^2 + a^2*c*e)) + 2
*((15*a*b^2*c^2*d - 13*a^2*b*c*d^2)*x^6 - 2*a^3*c^3 + (15*a*b^2*c^3 - 8*a^
2*b*c^2*d - 5*a^3*c*d^2)*x^4 + (5*a^2*b*c^3 - 7*a^3*c^2*d)*x^2)*sqrt((b*e*
x^2 + a*e)/(d*x^2 + c)))/(a^4*b*c*e^2*x^6 + a^5*c*e^2*x^4)]
```

3.312.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**5/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

3.312.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.312.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{2,[1,4,4]%%%},[2,1,7,0]%%%}+%%%{%%%{-8,[2,3,4]%%%},[2,
1,6,1]%%%
```

3.312.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{x^5\,{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)}^{3/2}}\,dx$$

input
```
int(1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
```

output
```
int(1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)
```

3.313 $\int \frac{x^4}{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

3.313.1 Optimal result

Integrand size = 26, antiderivative size = 453

$$\begin{aligned}\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx &= \frac{(7bc-8ad)x(a+bx^2)}{5b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{6dx^3(a+bx^2)}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\\
&+\frac{(b^2c^2-16abcd+16a^2d^2)\,x(a+bx^2)}{5b^4e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}-\frac{x^3(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\\
&-\frac{\sqrt{c}(b^2c^2-16abcd+16a^2d^2)\,(a+bx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5b^4\sqrt{d}e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}\\
&-\frac{c^{3/2}(7bc-8ad)\,(a+bx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{5b^3\sqrt{d}e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}\end{aligned}$$

output
```
1/5*(-8*a*d+7*b*c)*x*(b*x^2+a)/b^3/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+6/5*d*x
^3*(b*x^2+a)/b^2/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/5*(16*a^2*d^2-16*a*b*c*
d+b^2*c^2)*x*(b*x^2+a)/b^4/e/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-x^3*(
d*x^2+c)/b/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/5*c^(3/2)*(-8*a*d+7*b*c)*(b*x
^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/
(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/b^3/e/(d*x^2+c)/d^(1/2)/(c*(b*x^2+a)/
a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/5*(16*a^2*d^2-16*a*b*c*
d+b^2*c^2)*(b*x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d
^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)/b^4/e/(d*x^2+c
)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

3.313.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.55 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.60

$$\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}dx(c+dx^2)\left(-8a^2d+ab(7c-2dx^2)+b^2x^2(2c+dx^2)\right)-ic(b^2c^2-16\right.}{}$$

input
```
Integrate[x^4/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(c + d*x^2)*(-8*a^2*d +
a*b*(7*c - 2*d*x^2) + b^2*x^2*(2*c + d*x^2)) - I*c*(b^2*c^2 - 16*a*b*c*d +
 16*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[S
qrt[b/a]*x], (a*d)/(b*c)] + I*c*(b^2*c^2 - 9*a*b*c*d + 8*a^2*d^2)*Sqrt[1 +
 (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b
*c)]))/(5*b^3*Sqrt[b/a]*d*e^2*(a + b*x^2))
```

3.313.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 406, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2058, 369, 27, 443, 444, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{a+bx^2}\int \frac{x^4\left(dx^2+c\right)^{3/2}}{\left(bx^2+a\right)^{3/2}}dx}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{ 369}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{\int \frac{3x^2\sqrt{dx^2+c}\left(2dx^2+c\right)}{\sqrt{bx^2+a}}dx}{b}-\frac{x^3\left(c+dx^2\right)^{3/2}}{b\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{3\int \frac{x^2\sqrt{dx^2+c}\left(2dx^2+c\right)}{\sqrt{bx^2+a}}dx}{b}-\frac{x^3\left(c+dx^2\right)^{3/2}}{b\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{ 443}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{3\left(\frac{\int \frac{x^2\left(d(7bc-8ad)x^2+c(5bc-6ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5b}+\frac{2dx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5b}\right)}{b}-\frac{x^3\left(c+dx^2\right)^{3/2}}{b\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{ 444}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{3\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-8ad)}{3b}-\frac{\int \frac{d\left(ac(7bc-8ad)-\left(b^2c^2-16abdc+16a^2d^2\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}}{5b}+\frac{2dx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5b}\right)}{b}-\frac{x^3\left(c+dx^2\right)^{3/2}}{b\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{ 27}$$

3.313. $\int \frac{x^4}{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}dx$

$$\frac{\sqrt{a+bx^2}\left(\frac{3\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-8ad)}{3b}-\frac{\frac{\int \frac{ac(7bc-8ad)-\left(b^2c^2-16abdc+16a^2d^2\right)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}}{5b}+\frac{2dx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5b}\right)}{b}-\frac{x^3\left(c+dx^2\right)^{3/2}}{b\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\downarrow$ 406

$$\frac{\sqrt{a+bx^2}\left(\frac{3\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-8ad)}{3b}-\frac{\frac{ac(7bc-8ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\left(16a^2d^2-16abcd+b^2c^2\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}}{5b}+\frac{2dx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5b}\right.}{b}\right.}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\downarrow$ 320

$$\frac{\sqrt{a+bx^2}\left(\frac{3\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-8ad)}{3b}-\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(7bc-8ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(16a^2d^2-16abcd+b^2c^2\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}}{5b}+\right.}{b}\right.}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\downarrow$ 388

3.313. $\int \frac{x^4}{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}dx$

$$\frac{\sqrt{a+bx^2}\left(\frac{3\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-8ad)}{3b}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}(7bc-8ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(16a^2d^2-16abcd+b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2}}{\left(dx^2+c\right.}}{b}\right.}{3b}}{5b}\right.}{b}}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\Big\downarrow$ 313

$$\frac{\sqrt{a+bx^2}\left(\frac{3\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-8ad)}{3b}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}(7bc-8ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(16a^2d^2-16abcd+b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}}{b\sqrt{d}\sqrt{}}\right.}{3b}}{5b}\right.}{b}}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

input
```
Int[x^4/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

3.313. $\int \frac{x^4}{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

output
```
(Sqrt[a + b*x^2]*(-((x^3*(c + d*x^2)^(3/2))/(b*Sqrt[a + b*x^2])) + (3*((2*
d*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*b) + (((7*b*c - 8*a*d)*x*Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2])/(3*b) - (-((b^2*c^2 - 16*a*b*c*d + 16*a^2*d^2)*(
(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Ellipti
cE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (c^(3/2)*(7*b*c - 8*a*d)*Sqr
t[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqr
t[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b))/(5*b))
)/b))/(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

3.313.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.313. $\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.313.4 Maple [A] (verified)

Time = 9.94 (sec) , antiderivative size = 780, normalized size of antiderivative = 1.72

method	result
risch	$-\frac{x(-bd\,x^2+3ad-2bc)(b\,x^2+a)}{5b^3e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}+\frac{\left(-\frac{2(11a^2d^2-11abcd+b^2c^2)ace\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)-E\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{e}{}}\right.\right.}{\sqrt{-\frac{b}{a}}\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}\,(eda+ebc+e(ad-bc))}\right.}{}$
default	$-\frac{(b\,x^2+a)\left(-\sqrt{(d\,x^2+c)(b\,x^2+a)}\sqrt{-\frac{b}{a}}\,b^2d^3x^7+2\sqrt{(d\,x^2+c)(b\,x^2+a)}\sqrt{-\frac{b}{a}}\,ab\,d^3x^5-3\sqrt{(d\,x^2+c)(b\,x^2+a)}\sqrt{-\frac{b}{a}}\,b^2c\,d^2x^5+5\sqrt{bd}\right.}{}$

input
```
int(x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/b^3*x*(-b*d*x^2+3*a*d-2*b*c)*(b*x^2+a)/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2
)+1/5/b^3*(-2*(11*a^2*d^2-11*a*b*c*d+b^2*c^2)*a*c*e/(-b/a)^(1/2)*(1+b*x^2/
a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)/(
e*d*a+e*b*c+e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e
)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2)))-a*(5*a^
2*d^2-13*a*b*c*d+7*b^2*c^2)/b/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)
^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2
),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))+5*a^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b*(-(b
*d*e*x^2+b*c*e)/a/(a*d-b*c)*x/e/((x^2+a/b)*(b*d*e*x^2+b*c*e))^(1/2)+(1/a+b
*c/a/(a*d-b*c))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+1/c*d*x^2)^(1/2)/(b*d*e*
x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b
*c*e)/c/b/e)^(1/2))-2*d*b/(a*d-b*c)*c*e/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+
1/c*d*x^2)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)/(e*d*a+e*b*c+
e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-Ell
ipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))))))/e/(e*(b*x^2+a)/(d
*x^2+c))^(1/2)*((d*x^2+c)*e*(b*x^2+a))^(1/2)/(d*x^2+c)
```

3.313.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 425, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx =$$

$$-\frac{((b^3c^3-16\,ab^2c^2d+16\,a^2bcd^2)x^3+(ab^2c^3-16\,a^2bc^2d+16\,a^3cd^2)x)\sqrt{\frac{be}{d}}\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(}{}$$

input
```
integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/5*(((b^3*c^3 - 16*a*b^2*c^2*d + 16*a^2*b*c*d^2)*x^3 + (a*b^2*c^3 - 16*a
^2*b*c^2*d + 16*a^3*c*d^2)*x)*sqrt(b*e/d)*sqrt(-c/d)*elliptic_e(arcsin(sqr
t(-c/d)/x), a*d/(b*c)) - ((b^3*c^3 - 16*a*b^2*c^2*d + 8*a^2*b*d^3 + (16*a^
2*b - 7*a*b^2)*c*d^2)*x^3 + (a*b^2*c^3 - 16*a^2*b*c^2*d + 8*a^3*d^3 + (16*
a^3 - 7*a^2*b)*c*d^2)*x)*sqrt(b*e/d)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/
d)/x), a*d/(b*c)) - (b^3*d^3*x^8 + (3*b^3*c*d^2 - 2*a*b^2*d^3)*x^6 + a*b^2
*c^3 - 16*a^2*b*c^2*d + 16*a^3*c*d^2 + (3*b^3*c^2*d - 11*a*b^2*c*d^2 + 8*a
^2*b*d^3)*x^4 + (b^3*c^3 - 8*a*b^2*c^2*d - 8*a^2*b*c*d^2 + 16*a^3*d^3)*x^2
)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^5*d*e^2*x^3 + a*b^4*d*e^2*x)
```

3.313.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**4/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

3.313.7 Maxima [F]

$$\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^4}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

3.313.8 Giac [F]

$$\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^4}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^4/((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

3.313.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^4}{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(x^4/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

output
```
int(x^4/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.314 $\int \frac{x^2}{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

3.314.1 Optimal result

Integrand size = 26, antiderivative size = 378

$$\begin{aligned}\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx &= \frac{4dx(a+bx^2)}{3b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{d(7bc-8ad)x(a+bx^2)}{3b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} \\ &\quad -\frac{x(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\sqrt{d}(7bc-8ad)\,(a+bx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^3e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} \\ &\quad +\frac{c^{3/2}(3bc-4ad)\,(a+bx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3ab^2\sqrt{d}e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}\end{aligned}$$

output
```
4/3*d*x*(b*x^2+a)/b^2/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/3*d*(-8*a*d+7*b*c)
*x*(b*x^2+a)/b^3/e/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-x*(d*x^2+c)/b/e
/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/3*c^(3/2)*(-4*a*d+3*b*c)*(b*x^2+a)*(1/(1+
d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^
(1/2),(1-b*c/a/d)^(1/2))/a/b^2/e/(d*x^2+c)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c
))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*(-8*a*d+7*b*c)*(b*x^2+a)*(1/(1+
d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^
(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)/b^3/e/(d*x^2+c)/(c*(b*x^2+a)/a/(d
*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

3.314.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.91 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.58

$$\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}x(c+dx^2)\left(-3bc+4ad+bdx^2\right)+ic(-7bc+8ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\right.}{3a^2\left(\frac{b}{a}\right.}$$

input
```
Integrate[x^2/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*x*(c + d*x^2)*(-3*b*c + 4*a*
d + b*d*x^2) + I*c*(-7*b*c + 8*a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (4*I)*c*(-(b*c) + a*d)*
Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x],
(a*d)/(b*c)]))/(3*a^2*(b/a)^(5/2)*e^2*(a + b*x^2))
```

3.314.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2058, 369, 403, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 2058$$

$$\frac{\sqrt{a+bx^2}\int \frac{x^2\left(dx^2+c\right)^{3/2}}{\left(bx^2+a\right)^{3/2}}dx}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \; 369$$

$$\frac{\sqrt{a+bx^2}\left(\frac{\int \frac{\sqrt{dx^2+c}\left(4dx^2+c\right)}{\sqrt{bx^2+a}}dx}{b}-\frac{x\left(c+dx^2\right)^{3/2}}{b\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{ 403}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{\frac{\int \frac{d(7bc-8ad)x^2+c(3bc-4ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}+\frac{4dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}}{b}-\frac{x\left(c+dx^2\right)^{3/2}}{b\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{ 406}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{\frac{c(3bc-4ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+d(7bc-8ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}+\frac{4dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}}{b}-\frac{x\left(c+dx^2\right)^{3/2}}{b\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{\frac{d(7bc-8ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(3bc-4ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3b}+\frac{4dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}}{b}-\frac{x\left(c+dx^2\right)^{3/2}}{b\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{ 388}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{\frac{d(7bc-8ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(3bc-4ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3b}+\frac{4dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}}{b}-\frac{x\left(c+dx^2\right)^{3/2}}{b\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{ 313}$$

3.314. $\displaystyle\int \frac{x^2}{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

$$\frac{\sqrt{a+bx^2}\left(\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3bc-4ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d(7bc-8ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{3b}+\frac{4dx\sqrt{a+bx^2}\sqrt{c+\cdots}}{3b}}{b}\cdots\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}$$

input
```
Int[x^2/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[a + b*x^2]*(-((x*(c + d*x^2)^(3/2))/(b*Sqrt[a + b*x^2])) + ((4*d*x*S
qrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*b) + (d*(7*b*c - 8*a*d)*((x*Sqrt[a + b*
x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqr
t[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(3*b*c - 4*a*d)*Sqrt[a + b*x^2]*El
lipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b))/b))/(e*Sqrt[(e*(a +
 b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

3.314.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.314. $\int \frac{x^2}{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

3.314.4 Maple [A] (verified)

Time = 8.86 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.70

method	result
default	$\frac{(b\,x^2+a)\left(\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,b\,d^2x^5+\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,a\,d^2x^3+\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,bcd\,x^3+3\sqrt{bd\,x^4+ad\,x^2+}\right.}{}$
risch	$\frac{dx(b\,x^2+a)}{3b^2e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}-\frac{\left(\frac{3a\left(a^2d^2-2abcd+b^2c^2\right)\left(-\frac{(bde\,x^2+ebc)x}{a(ad-bc)e\sqrt{\left(x^2+\frac{a}{b}\right)(bde\,x^2+ebc)}}+\frac{\left(\frac{1}{a}+\frac{bc}{a(ad-bc)}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+}\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}}\right.}{b}\right.}{}$

input
```
int(x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*x^2+a)*(((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*b*d^2*x^5+((d*x^2+
c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a*d^2*x^3+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b
/a)^(1/2)*b*c*d*x^3+3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*d
^2*x^3-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b*c*d*x^3+4*((d*
x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(
x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*d-4*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)
)*b*c^2-8*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1
/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*d+7*((d*x^2+c)*(b*x^2+a)
)^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2))*b*c^2+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a*c*d*x+3*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*c*d*x-3*(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b*c^2*x)/b^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2
)/(d*x^2+c)^2/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

3.314.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx =$$

$$-\frac{((7\,b^2c^2-8\,abcd)x^3+(7\,abc^2-8\,a^2cd)x)\sqrt{\frac{be}{d}}\sqrt{-\frac{c}{d}}E\left(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc}\right)-((7\,b^2c^2-4\,abd^2-(8\,ab-3}{}$$

input
```
integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(((7*b^2*c^2 - 8*a*b*c*d)*x^3 + (7*a*b*c^2 - 8*a^2*c*d)*x)*sqrt(b*e/d
)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((7*b^2*c^2 - 4
*a*b*d^2 - (8*a*b - 3*b^2)*c*d)*x^3 + (7*a*b*c^2 - 4*a^2*d^2 - (8*a^2 - 3*
a*b)*c*d)*x)*sqrt(b*e/d)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(
b*c)) - (b^2*d^2*x^6 + (5*b^2*c*d - 4*a*b*d^2)*x^4 + 7*a*b*c^2 - 8*a^2*c*d
 + (4*b^2*c^2 + 3*a*b*c*d - 8*a^2*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 +
c)))/(b^4*e^2*x^3 + a*b^3*e^2*x)
```

3.314.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

3.314.7 Maxima [F]

$$\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^2}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

3.314.8 Giac [F]

$$\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^2}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

3.314.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^2}{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(x^2/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

output
```
int(x^2/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.315 $\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

3.315.1 Optimal result

Integrand size = 22, antiderivative size = 327

$$\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{(bc-ad)x}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{d(bc-2ad)x(a+bx^2)}{ab^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} + \frac{\sqrt{c}\sqrt{d}(bc-2ad)\,(a+bx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab^2e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} + \frac{c^{3/2}\sqrt{d}(a+bx^2)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{abe\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}$$

output
```
(-a*d+b*c)*x/a/b/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-d*(-2*a*d+b*c)*x*(b*x^2+a
)/a/b^2/e/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+c^(3/2)*(b*x^2+a)*(1/(1+
d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^
(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)/a/b/e/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))
^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+(-2*a*d+b*c)*(b*x^2+a)*(1/(1+d*x^2/c)
)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1
-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)/a/b^2/e/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c
))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

3.315.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.32 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.62

$$\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-ic(-bc+2ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)\mid\frac{ad}{bc}\right)+(bc-ad)\left(\right.\right.}{a^2\left(\frac{b}{a}\right)^{3/2}e^2(a+}$$

input
```
Integrate[((e*(a + b*x^2))/(c + d*x^2))^(-3/2),x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*((-I)*c*(-(b*c) + 2*a*d)*Sqrt[1 + (b*x^
2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] +
 (b*c - a*d)*(Sqrt[b/a]*x*(c + d*x^2) - I*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (
d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])))/(a^2*(b/a)^(3/
2)*e^2*(a + b*x^2))
```

3.315.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {2058, 315, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \; 2058$$

$$\frac{\sqrt{a+bx^2}\int \frac{(dx^2+c)^{3/2}}{(bx^2+a)^{3/2}}dx}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \; 315$$

$$\frac{\sqrt{a+bx^2}\left(\frac{\int \frac{d\left(ac-(bc-2ad)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{ab\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a+bx^2}\left(\frac{d\int \frac{ac-(bc-2ad)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{ab\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \; 406$$

$$\frac{\sqrt{a+bx^2}\left(\frac{d\left(ac\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-(bc-2ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{ab\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \; 320$$

$$\frac{\sqrt{a+bx^2}\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-(bc-2ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{ab\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \; 388$$

$$\frac{\sqrt{a+bx^2}\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)\right)}{ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{ab\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \; 313$$

3.315. $\int \frac{1}{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

$$\frac{\sqrt{a+bx^2}\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{ab}+\frac{x\sqrt{c+dx^2}(bc-ad}{ab\sqrt{a+bx^2}}\right.}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

input
```
Int[((e*(a + b*x^2))/(c + d*x^2))^(-3/2),x]
```

output
```
(Sqrt[a + b*x^2]*(((b*c - a*d)*x*Sqrt[c + d*x^2])/(a*b*Sqrt[a + b*x^2]) +
(d*(-((b*c - 2*a*d)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sq
rt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*
Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (c^(3/2
)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(a*b)))
/(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

3.315.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

3.315. $\int \frac{1}{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.315.4 Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.57

method	result
default	$-\frac{(b x^2+a)\left(\sqrt{bd x^4+ad x^2+bc x^2+ac}\,\sqrt{-\frac{b}{a}}\,a d^2x^3-\sqrt{bd x^4+ad x^2+bc x^2+ac}\,\sqrt{-\frac{b}{a}}\,bcd x^3+\sqrt{(d x^2+c)(b x^2+a)}\,\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{d x^2+c}{c}}\right.}{}$

input
```
int(1/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*x^2+a)/b*((b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*d^2*x^3-(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b*c*d*x^3+((d*x^2+c)*(b*x^
2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/
2),(a*d/b/c)^(1/2))*a*c*d-((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*
((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2-2*((d*
x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(
x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*d+((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+
a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*
b*c^2+(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*c*d*x-(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b*c^2*x)/(e*(b*x^2+a)/(d*x^2+c))^(3/
2)/(d*x^2+c)^2/a/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

3.315.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{((b^2c^2-2\,abcd)x^3+(abc^2-2\,a^2cd)x)\sqrt{\frac{be}{d}}\sqrt{-\frac{c}{d}}E\left(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc}\right)-((b^2c^2-2\,a}{}$$

input
```
integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
(((b^2*c^2 - 2*a*b*c*d)*x^3 + (a*b*c^2 - 2*a^2*c*d)*x)*sqrt(b*e/d)*sqrt(-c
/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((b^2*c^2 - 2*a*b*c*d -
a*b*d^2)*x^3 + (a*b*c^2 - 2*a^2*c*d - a^2*d^2)*x)*sqrt(b*e/d)*sqrt(-c/d)*e
lliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (a*b*d^2*x^4 + 2*a^2*d^2*x^2
- a*b*c^2 + 2*a^2*c*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*b^3*e^2*x^3 +
 a^2*b^2*e^2*x)
```

3.315.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

3.315.7 Maxima [F]

$$\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(((b*x^2 + a)*e/(d*x^2 + c))^(-3/2), x)
```

3.315.8 Giac [F]

$$\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(((b*x^2 + a)*e/(d*x^2 + c))^(-3/2), x)
```

3.315.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(1/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

output
```
int(1/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.316 $\int \frac{1}{x^2\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

3.316.1 Optimal result

Integrand size = 26, antiderivative size = 380

$$\begin{aligned}\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx &= \frac{bc-ad}{abex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{(2bc-ad)\left(a+bx^2\right)}{a^2bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\\ &+\frac{d(2bc-ad)x(a+bx^2)}{a^2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}-\frac{\sqrt{c}\sqrt{d}(2bc-ad)\left(a+bx^2\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a^2be\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}\\ &+\frac{c^{3/2}\sqrt{d}(a+bx^2)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a^2e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}\end{aligned}$$

output
```
(-a*d+b*c)/a/b/e/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-(-a*d+2*b*c)*(b*x^2+a)/a^
2/b/e/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+d*(-a*d+2*b*c)*x*(b*x^2+a)/a^2/b/e/(
d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+c^(3/2)*(b*x^2+a)*(1/(1+d*x^2/c))^(
1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*
c/a/d)^(1/2))*d^(1/2)/a^2/e/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(
b*x^2+a)/(d*x^2+c))^(1/2)-(-a*d+2*b*c)*(b*x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+
d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(
1/2))*c^(1/2)*d^(1/2)/a^2/b/e/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e
*(b*x^2+a)/(d*x^2+c))^(1/2)
```

3.316.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.13 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\sqrt{\frac{b}{a}}(c+dx^2)\,(ac+2bcx^2-adx^2)+ic(-2bc+ad)x\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\cdots}\right.}{a^2\cdots}$$

input
```
Integrate[1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(Sqrt[b/a]*(c + d*x^2)*(a*c + 2*b*c*x
^2 - a*d*x^2)) + I*c*(-2*b*c + a*d)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)
/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)*c*(-(b*c) + a*d
)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*
x], (a*d)/(b*c)]))/(a^2*Sqrt[b/a]*e^2*x*(a + b*x^2))
```

3.316.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {2058, 370, 25, 27, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{a+bx^2}\int \frac{(dx^2+c)^{3/2}}{x^2(bx^2+a)^{3/2}}dx}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{370}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{\sqrt{c+dx^2}(bc-ad)}{abx\sqrt{a+bx^2}}-\frac{\int -\frac{c\left(bdx^2+2bc-ad\right)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ab}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{a+bx^2}\left(\frac{\int \frac{c\left(bdx^2+2bc-ad\right)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{abx\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a+bx^2}\left(\frac{c\int \frac{bdx^2+2bc-ad}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{abx\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \; 445$$

$$\frac{\sqrt{a+bx^2}\left(\frac{c\left(-\frac{\int -\frac{bd\left((2bc-ad)x^2+ac\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{acx}\right)}{ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{abx\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{a+bx^2}\left(\frac{c\left(\frac{\int \frac{bd\left((2bc-ad)x^2+ac\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{acx}\right)}{ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{abx\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a+bx^2}\left(\frac{c\left(\frac{bd\int \frac{(2bc-ad)x^2+ac}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{acx}\right)}{ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{abx\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

3.316. $\int \frac{1}{x^2\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

$$\Big\downarrow \text{406}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{c\left(\frac{bd\left(ac\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(2bc-ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{acx}\right)}{ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{abx\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{c\left(\frac{bd\left((2bc-ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{acx}\right)}{ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{abx\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{388}$$

3.316. $\displaystyle\int \frac{1}{x^2\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

$$\frac{\sqrt{a+bx^2}\left(\frac{c\left(\frac{bd\left((2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{acx}\right)}{ab}+\frac{\sqrt{c+}}{ab}\cdots\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\downarrow$ 313

$$\frac{\sqrt{a+bx^2}\left(\frac{c\left(\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2b}{acx}\cdots\right)}{ab}\cdots\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

input
```
Int[1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

3.316. $\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

output
```
(Sqrt[a + b*x^2]*(((b*c - a*d)*Sqrt[c + d*x^2])/(a*b*x*Sqrt[a + b*x^2]) +
(c*(-(((2*b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (b*d*((2*
b*c - a*d)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*
x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*S
qrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*Sqrt[a +
 b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*
Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(a*c)))/(a*b)))/(
e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

3.316.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.316.4 Maple [A] (verified)

Time = 8.12 (sec) , antiderivative size = 650, normalized size of antiderivative = 1.71

method	result
default	$-\frac{(bx^2+a)\left(\sqrt{(dx^2+c)(bx^2+a)}\sqrt{-\frac{b}{a}}\,bcdx^4-\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{-\frac{b}{a}}\,ad^2x^4+\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{-\frac{b}{a}}\,bcdx^4-2\sqrt{(\cdots}\right.}{}$
risch	$-\frac{c(bx^2+a)}{a^2xe\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}+\frac{\left(\frac{a^2d^2\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)}{b\sqrt{-\frac{b}{a}}\sqrt{bdex^4+adex^2+bcex^2+ace}}-\frac{2bc^2dae\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)-\cdots\right.}{\sqrt{-\frac{b}{a}}\sqrt{bdex^4+adex^2+bcex^2+ace}\,(eda+e\cdots}\right.}{}$

input
```
int(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*x^2+a)*(((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*b*c*d*x^4-(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*d^2*x^4+(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*(-b/a)^(1/2)*b*c*d*x^4-2*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/
a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c
*d*x+2*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)
*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2*x+((d*x^2+c)*(b*x^2+a))^(
1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d
/b/c)^(1/2))*a*c*d*x-2*c^2*b*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellip
ticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x*((d*x^2+c)*(b*x^2+a))^(1/2)+((d*x^2
+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a*c*d*x^2+((d*x^2+c)*(b*x^2+a))^(1/2)*(-
b/a)^(1/2)*b*c^2*x^2-(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*c*
d*x^2+(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b*c^2*x^2+((d*x^2+c
)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a*c^2)/(e*(b*x^2+a)/(d*x^2+c))^(3/2)/(d*x^
2+c)^2/a^2/x/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

3.316.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{((2\,b^3cd-ab^2d^2)x^3+(2\,ab^2cd-a^2bd^2)x)\sqrt{\frac{ace}{d^2}}\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-((2}{}$$

input
```
integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
(((2*b^3*c*d - a*b^2*d^2)*x^3 + (2*a*b^2*c*d - a^2*b*d^2)*x)*sqrt(a*c*e/d^
2)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((2*b^3*c*d +
(a^2*b - a*b^2)*d^2)*x^3 + (2*a*b^2*c*d + (a^3 - a^2*b)*d^2)*x)*sqrt(a*c*e
/d^2)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (2*a*b^2*c^
2*x^2 + a^2*b*c^2 + (2*a*b^2*c*d - a^2*b*d^2)*x^4)*sqrt((b*e*x^2 + a*e)/(d
*x^2 + c)))/(a^3*b^2*e^2*x^3 + a^4*b*e^2*x)
```

3.316.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**2/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

3.316.7 Maxima [F]

$$\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^2), x)
```

3.316.8 Giac [F]

$$\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^2), x)
```

3.316.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} \, dx = \int \frac{1}{x^2 \left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}} \, dx$$

input
```
int(1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
```

output
```
int(1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)
```

3.317 $\int \frac{1}{x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

3.317.1 Optimal result

Integrand size = 26, antiderivative size = 444

$$\begin{aligned}\int \frac{1}{x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx &= \frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(4bc-3ad)\,(a+bx^2)}{3a^2bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} \\ &+ \frac{(8bc-7ad)\,(a+bx^2)}{3a^3ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{d(8bc-7ad)x(a+bx^2)}{3a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} \\ &+ \frac{\sqrt{c}\sqrt{d}(8bc-7ad)\,(a+bx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^3e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} \\ &- \frac{\sqrt{c}\sqrt{d}(4bc-3ad)\,(a+bx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^3e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}\end{aligned}$$

output
```
(-a*d+b*c)/a/b/e/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*(-3*a*d+4*b*c)*(b*x
^2+a)/a^2/b/e/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/3*(-7*a*d+8*b*c)*(b*x^2+
a)/a^3/e/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*d*(-7*a*d+8*b*c)*x*(b*x^2+a)/
a^3/e/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/3*(-7*a*d+8*b*c)*(b*x^2+a)
*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*
x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)/a^3/e/(d*x^2+c)/(c*(b*x^2+
a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*(-3*a*d+4*b*c)*(b*
x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)
/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)/a^3/e/(d*x^2+c)/(c*(
b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

3.317.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.65 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\sqrt{\frac{b}{a}}(c+dx^2)\left(-8b^2cx^4+a^2(c+4dx^2)+ab(-4cx^2+7dx^4)\right)-ibc(-8\right.}{}$$

input
```
Integrate[1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(Sqrt[b/a]*(c + d*x^2)*(-8*b^2*c*x^4
+ a^2*(c + 4*d*x^2) + a*b*(-4*c*x^2 + 7*d*x^4))) - I*b*c*(-8*b*c + 7*a*d)*
x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*
x], (a*d)/(b*c)] - I*(8*b^2*c^2 - 11*a*b*c*d + 3*a^2*d^2)*x^3*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
))/(3*a^3*Sqrt[b/a]*e^2*x^3*(a + b*x^2))
```

3.317.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 412, normalized size of antiderivative = 0.93, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2058, 370, 25, 445, 27, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$$

$\Big\downarrow$ 2058

$$\frac{\sqrt{a+bx^2}\int \frac{(dx^2+c)^{3/2}}{x^4(bx^2+a)^{3/2}}dx}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\Big\downarrow$ 370

$$\frac{\sqrt{a+bx^2}\left(\frac{\sqrt{c+dx^2}(bc-ad)}{abx^3\sqrt{a+bx^2}}-\frac{\int -\frac{d(3bc-2ad)x^2+c(4bc-3ad)}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ab}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{a+bx^2}\left(\frac{\int \frac{d(3bc-2ad)x^2+c(4bc-3ad)}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{abx^3\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\Big\downarrow$ 445

$$\frac{\sqrt{a+bx^2}\left(\frac{-\frac{\int \frac{bc\left(d(4bc-3ad)x^2+c(8bc-7ad)\right)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-3ad)}{3ax^3}}{ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{abx^3\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{a+bx^2}\left(\frac{-\frac{b\int \frac{d(4bc-3ad)x^2+c(8bc-7ad)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-3ad)}{3ax^3}}{ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{abx^3\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\Big\downarrow$ 445

3.317. $\int \frac{1}{x^4 \left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

$$\frac{\sqrt{a+bx^2}\left(-\frac{-\frac{b\left(-\frac{\int -\frac{cd\left(b(8bc-7ad)x^2+a(4bc-3ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(8bc-7ad)}{ax}\right)}{3a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-3ad)}{3ax^3}}{ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{abx^3\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\downarrow$ 25

$$\frac{\sqrt{a+bx^2}\left(-\frac{-\frac{b\left(\frac{\int \frac{cd\left(b(8bc-7ad)x^2+a(4bc-3ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(8bc-7ad)}{ax}\right)}{3a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-3ad)}{3ax^3}}{ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{abx^3\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\downarrow$ 27

$$\frac{\sqrt{a+bx^2}\left(-\frac{-\frac{b\left(\frac{d\int \frac{b(8bc-7ad)x^2+a(4bc-3ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(8bc-7ad)}{ax}\right)}{3a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-3ad)}{3ax^3}}{ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{abx^3\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\downarrow$ 406

$$\frac{\sqrt{a+bx^2}\left(-\frac{-\frac{b\left(\frac{d\left(a(4bc-3ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+b(8bc-7ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(8bc-7ad)}{ax}\right)}{3a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-3ad)}{3ax^3}}{ab}+\right.}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\downarrow$ 320

3.317. $\int \frac{1}{x^4\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

$$\frac{\sqrt{a+bx^2}\left(-\frac{-\frac{b\left(\frac{d\left(b(8bc-7ad)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}(4bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(8bc-7ad)}{ax}\right)}{3a}-\frac{\sqrt{a+bx^2}}{}}{ab}\right.}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\downarrow$ 388

$$\frac{\sqrt{a+bx^2}\left(-\frac{-\frac{b\left(\frac{d\left(b(8bc-7ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(4bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(8bc-7ad)}{ax}\right)}{3a}}{ab}\right.}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\downarrow$ 313

3.317. $\displaystyle\int \frac{1}{x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

$$\frac{\sqrt{a+bx^2}\left(-\frac{-\frac{b\left(\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(4bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+b(8bc-7ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{a}-\frac{\sqrt{a+bx^2}}{}\right)}{3a}}{ab}\right.}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

input
```
Int[1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

output
```
(Sqrt[a + b*x^2]*(((b*c - a*d)*Sqrt[c + d*x^2])/(a*b*x^3*Sqrt[a + b*x^2])
+ (-1/3*((4*b*c - 3*a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*x^3) - (b*(-(
((8*b*c - 7*a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*x)) + (d*(b*(8*b*c -
7*a*d)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]
*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(4*b*c - 3*a
*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)
])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/a))/(
3*a))/(a*b)))/(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

3.317.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.317. $\int \frac{1}{x^4\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.317. $\int \frac{1}{x^4 \left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.317.4 Maple [A] (verified)

Time = 8.75 (sec) , antiderivative size = 866, normalized size of antiderivative = 1.95

method	result
default	$-\frac{(b x^2+a)\left(4\sqrt{(d x^2+c)(b x^2+a)}\,\sqrt{-\frac{b}{a}}\,a b d^2 x^6-5\sqrt{(d x^2+c)(b x^2+a)}\,\sqrt{-\frac{b}{a}}\,b^2 c d x^6+3\sqrt{b d x^4+a d x^2+b c x^2+a c}\,\sqrt{-\frac{b}{a}}\,a b d^2 x^6-3\sqrt{}\right.}{}$
risch	$-\frac{(b x^2+a)(4ad x^2-5bc x^2+ac)}{3a^3x^3e\sqrt{\frac{e(b x^2+a)}{d x^2+c}}}+\frac{\left(-\frac{abcd\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{eda+ebc}{cbe}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bde x^4+ade x^2+bce x^2+ace}}-\frac{8a^2b d^2ce\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde x^4+ade x^2+bce}}\right.}{}$

input
```
int(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^2+a)*(4*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a*b*d^2*x^6-5*(
(d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*b^2*c*d*x^6+3*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*d^2*x^6-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)*(-b/a)^(1/2)*b^2*c*d*x^6-3*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^
(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*d^
2*x^3+11*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/
2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d*x^3-8*((d*x^2+c)*(b*x
^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1
/2),(a*d/b/c)^(1/2))*b^2*c^2*x^3-7*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/
a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b
*c*d*x^3+8*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(
1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2*x^3+4*((d*x^2+c)*(b
*x^2+a))^(1/2)*(-b/a)^(1/2)*a^2*d^2*x^4-5*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/
a)^(1/2)*b^2*c^2*x^4+3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*
b*c*d*x^4-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b^2*c^2*x^4+5
*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a^2*c*d*x^2-4*((d*x^2+c)*(b*x^2+
a))^(1/2)*(-b/a)^(1/2)*a*b*c^2*x^2+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2
)*a^2*c^2)/(e*(b*x^2+a)/(d*x^2+c))^(3/2)/(d*x^2+c)^2/a^3/x^3/(-b/a)^(1/2)/
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

3.317.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^4 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\, dx =$$

$$-\frac{((8\,b^4c^2d-7\,ab^3cd^2)x^5+(8\,ab^3c^2d-7\,a^2b^2cd^2)x^3)\sqrt{\frac{ace}{d^2}}\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-((8\,b^4c^2d-3\,a^3b}{}$$

input
```
integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(((8*b^4*c^2*d - 7*a*b^3*c*d^2)*x^5 + (8*a*b^3*c^2*d - 7*a^2*b^2*c*d^
2)*x^3)*sqrt(a*c*e/d^2)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b
*c)) - ((8*b^4*c^2*d - 3*a^3*b*d^3 + (4*a^2*b^2 - 7*a*b^3)*c*d^2)*x^5 + (8
*a*b^3*c^2*d - 3*a^4*d^3 + (4*a^3*b - 7*a^2*b^2)*c*d^2)*x^3)*sqrt(a*c*e/d^
2)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) + (a^3*b*c^3 - (
8*a*b^3*c^2*d - 7*a^2*b^2*c*d^2)*x^6 - (8*a*b^3*c^3 - 3*a^2*b^2*c^2*d - 4*
a^3*b*c*d^2)*x^4 - (4*a^2*b^2*c^3 - 5*a^3*b*c^2*d)*x^2)*sqrt((b*e*x^2 + a*
e)/(d*x^2 + c)))/(a^4*b^2*c*e^2*x^5 + a^5*b*c*e^2*x^3)
```

3.317.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\, dx = \text{Timed out}$$

input
```
integrate(1/x**4/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

3.317.7 Maxima [F]

$$\int \frac{1}{x^4 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} x^4}\,dx$$

input
```
integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^4), x)
```

3.317.8 Giac [F]

$$\int \frac{1}{x^4 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} x^4}\,dx$$

input
```
integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^4), x)
```

3.317.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{x^4 \left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
```

output
```
int(1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)
```

3.318 $\int x^5 \sqrt{a+\frac{b}{c+dx^2}}\,dx$

3.318.1 Optimal result

Integrand size = 21, antiderivative size = 216

$$\int x^5 \sqrt{a+\frac{b}{c+dx^2}}\,dx = -\frac{(b^2+4abc-8a^2c^2)\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{16a^2d^3} - \frac{(b+4ac)\,(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8ad^3} + \frac{(c+dx^2)^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}{6ad^3} + \frac{b(b^2+4abc+8a^2c^2)\,\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{5/2}d^3}$$

output
```
1/6*(d*x^2+c)^3*((a*d*x^2+a*c+b)/(d*x^2+c))^(3/2)/a/d^3+1/16*b*(8*a^2*c^2+
4*a*b*c+b^2)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/a^(5/2)/d^
3-1/16*(-8*a^2*c^2+4*a*b*c+b^2)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2
)/a^2/d^3-1/8*(4*a*c+b)*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^
3
```

3.318.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.67

$$\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx$$

$$= \frac{\sqrt{a}(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}(-3b^2+2ab(-5c+dx^2)+8a^2(c^2-cdx^2+d^2x^4))+3b(b^2+4abc+8a^2c^2)\,\text{arcta}}{48a^{5/2}d^3}$$

input
```
Integrate[x^5*Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-3*b^2 + 2*a*b
*(-5*c + d*x^2) + 8*a^2*(c^2 - c*d*x^2 + d^2*x^4)) + 3*b*(b^2 + 4*a*b*c +
8*a^2*c^2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(48*a^(
5/2)*d^3)
```

3.318.3 Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.94, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2057, 2053, 2052, 27, 366, 27, 360, 25, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx$$

$$\Big\downarrow \text{2057}$$

$$\int x^5\sqrt{\frac{ac+adx^2+b}{c+dx^2}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int x^4\sqrt{\frac{adx^2+b+ac}{dx^2+c}}dx^2$$

$$\Big\downarrow \text{2052}$$

$$-bd\int \frac{x^4\left(-cx^4+b+ac\right)^2}{d^4\left(a-x^4\right)^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \ 27$$

$$-\frac{b\int \frac{x^4(-cx^4+b+ac)^2}{(a-x^4)^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{d^3}$$

$$\Big\downarrow \ 366$$

$$-\frac{b\left(\frac{b^2x^6}{6a(a-x^4)^3}-\frac{\int \frac{3x^4\left(2ac^2x^4+b^2-2(b+ac)^2\right)}{\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{6a}\right)}{d^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{b\left(\frac{b^2x^6}{6a(a-x^4)^3}-\frac{\int \frac{x^4\left(2ac^2x^4+b^2-2(b+ac)^2\right)}{\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2a}\right)}{d^3}$$

$$\Big\downarrow \ 360$$

$$-\frac{b\left(\frac{b^2x^6}{6a(a-x^4)^3}-\frac{-\frac{1}{4}\int -\frac{b(b+4ac)-8ac^2x^4}{\left(a-x^4\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}-\frac{b(4ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4\left(a-x^4\right)^2}}{2a}\right)}{d^3}$$

$$\Big\downarrow \ 25$$

$$-\frac{b\left(\frac{b^2x^6}{6a(a-x^4)^3}-\frac{\frac{1}{4}\int \frac{b(b+4ac)-8ac^2x^4}{\left(a-x^4\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}-\frac{b(4ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4\left(a-x^4\right)^2}}{2a}\right)}{d^3}$$

$$\Big\downarrow \ 298$$

$$-\frac{b\left(\frac{b^2x^6}{6a(a-x^4)^3}-\frac{\frac{1}{4}\left(\frac{\left(8a^2c^2+4abc+b^2\right)\int \frac{1}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2a}+\frac{\left(-8a^2c^2+4abc+b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2a\left(a-x^4\right)}\right)-\frac{b(4ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4\left(a-x^4\right)^2}}{2a}\right)}{d^3}$$

$$\Big\downarrow \ 219$$

3.318. $\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx$

$$-\frac{b\left(\frac{b^2x^6}{6a(a-x^4)^3}-\frac{\frac{1}{4}\left(\frac{(-8a^2c^2+4abc+b^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2a(a-x^4)}+\frac{(8a^2c^2+4abc+b^2)\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}}\right)-\frac{b(4ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4(a-x^4)^2}}{2a}\right)}{d^3}$$

input
```
Int[x^5*Sqrt[a + b/(c + d*x^2)],x]
```

output
```
-((b*((b^2*x^6)/(6*a*(a - x^4)^3) - (-1/4*(b*(b + 4*a*c)*Sqrt[(b + a*c + a
*d*x^2)/(c + d*x^2)])/(a - x^4)^2 + (((b^2 + 4*a*b*c - 8*a^2*c^2)*Sqrt[(b
+ a*c + a*d*x^2)/(c + d*x^2)])/(2*a*(a - x^4)) + ((b^2 + 4*a*b*c + 8*a^2*c
^2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(2*a^(3/2)))/4
)/(2*a)))/d^3)
```

3.318.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

3.318. $\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx$

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

3.318. $\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx$

3.318.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.10

method	result
risch	$\frac{(8a^2d^2x^4-8a^2cd\,x^2+2abd\,x^2+8a^2c^2-10abc-3b^2)(d\,x^2+c)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{48d^3a^2}+\frac{b(8a^2c^2+4abc+b^2)\ln\left(\frac{acd+\frac{1}{2}bd+a\,d^2x^2}{\sqrt{a\,d^2}}+\sqrt{a\,c^2+bc+}\right.}{32d^2a^2\sqrt{}}$
default	$\frac{\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,(d\,x^2+c)\left(-48\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}\,a^2cd\,x^2-12\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}\,abd\,x^2+2\right.}{}$

input
```
int(x^5*(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/d^3*(8*a^2*d^2*x^4-8*a^2*c*d*x^2+2*a*b*d*x^2+8*a^2*c^2-10*a*b*c-3*b^2
)*(d*x^2+c)/a^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/32*b/d^2*(8*a^2*c^2+4*
a*b*c+b^2)/a^2*ln((a*c*d+1/2*b*d+a*d^2*x^2)/(a*d^2)^(1/2)+(a*c^2+b*c+(2*a*
c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/(a*d^2)^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(
1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(a*d*x^2+a*c+b)
```

3.318.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.96

$$\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx$$

$$=\left[\frac{3\,(8\,a^2bc^2+4\,ab^2c+b^3)\sqrt{a}\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2+4\,(2\,ad^2x^4+(4\,ac+\right.}{}\right.$$

$$-\frac{3\,(8\,a^2bc^2+4\,ab^2c+b^3)\sqrt{-a}\arctan\left(\frac{(2\,adx^2+2\,ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2dx^2+a^2c+ab)}\right)-2\,(8\,a^3d^3x^6+2\,a^2bd^2x^4+8\,a^3c^3}{96\,a^3d^3}$$

input
```
integrate(x^5*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/192*(3*(8*a^2*b*c^2 + 4*a*b^2*c + b^3)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^
2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c
+ b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
 + 4*(8*a^3*d^3*x^6 + 2*a^2*b*d^2*x^4 + 8*a^3*c^3 - 10*a^2*b*c^2 - 3*a*b^2
*c - (8*a^2*b*c + 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(
a^3*d^3), -1/96*(3*(8*a^2*b*c^2 + 4*a*b^2*c + b^3)*sqrt(-a)*arctan(1/2*(2*
a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d
*x^2 + a^2*c + a*b)) - 2*(8*a^3*d^3*x^6 + 2*a^2*b*d^2*x^4 + 8*a^3*c^3 - 10
*a^2*b*c^2 - 3*a*b^2*c - (8*a^2*b*c + 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c
+ b)/(d*x^2 + c)))/(a^3*d^3)]
```

3.318.6 Sympy [F]

$$\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int x^5\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,dx$$

input
```
integrate(x**5*(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(x**5*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

3.318.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.52

$$\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx=$$
$$-\frac{3\,(8\,a^2bc^2-4\,ab^2c-b^3)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-8\,(6\,a^3bc^2-ab^3)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+3\,(8\,a^4bc^2+4\,a^3b^2c+a^2b^3)\sqrt{\cdots}}{48\left(a^5d^3-\frac{3\,(adx^2+ac+b)a^4d^3}{dx^2+c}+\frac{3\,(adx^2+ac+b)^2a^3d^3}{(dx^2+c)^2}-\frac{(adx^2+ac+b)^3a^2d^3}{(dx^2+c)^3}\right)}$$
$$-\frac{(8\,a^2c^2+4\,abc+b^2)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{32\,a^{\frac{5}{2}}d^3}$$

input
```
integrate(x^5*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/48*(3*(8*a^2*b*c^2 - 4*a*b^2*c - b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))
^(5/2) - 8*(6*a^3*b*c^2 - a*b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) +
 3*(8*a^4*b*c^2 + 4*a^3*b^2*c + a^2*b^3)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
 c)))/(a^5*d^3 - 3*(a*d*x^2 + a*c + b)*a^4*d^3/(d*x^2 + c) + 3*(a*d*x^2 +
a*c + b)^2*a^3*d^3/(d*x^2 + c)^2 - (a*d*x^2 + a*c + b)^3*a^2*d^3/(d*x^2 +
c)^3) - 1/32*(8*a^2*c^2 + 4*a*b*c + b^2)*b*log(-(sqrt(a) - sqrt((a*d*x^2 +
 a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))
/(a^(5/2)*d^3)
```

3.318.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.01

$$\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx = \frac{1}{96}\left(2\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(2\,x^2\left(\frac{4\,x^2}{d}-\frac{4\,a^2cd^3-abd^3}{a^2d^5}\right)+\frac{8\,a^2c^2d^2-10\,abcd^2-3\,b^2d^2}{a^2d^5}\right.\right.$$
$$+c)$$

input
```
integrate(x^5*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
1/96*(2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2*(4*x^
2/d - (4*a^2*c*d^3 - a*b*d^3)/(a^2*d^5)) + (8*a^2*c^2*d^2 - 10*a*b*c*d^2 -
 3*b^2*d^2)/(a^2*d^5)) - 3*(8*a^2*b*c^2 + 4*a*b^2*c + b^3)*log(abs(2*a*c*d
 + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b
*c))*sqrt(a)*abs(d) + b*d))/(a^(5/2)*d^2*abs(d)))*sgn(d*x^2 + c)
```

3.318.9 Mupad [F(-1)]

Timed out.

$$\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx = \int x^5\,\sqrt{a+\frac{b}{d\,x^2+c}}\,dx$$

input
```
int(x^5*(a + b/(c + d*x^2))^(1/2),x)
```

output
```
int(x^5*(a + b/(c + d*x^2))^(1/2), x)
```

3.319 $\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx$

3.319.1 Optimal result

Integrand size = 21, antiderivative size = 141

$$\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx=\frac{(b-4ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8ad^2}+\frac{\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4d^2}-\frac{b(b+4ac)\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{3/2}d^2}$$

output
```
-1/8*b*(4*a*c+b)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/a^(3/2
)/d^2+1/8*(-4*a*c+b)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^2+1/4
*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2
```

3.319.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.76

$$\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx=\frac{\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(b-2ac+2adx^2\right)}{8ad^2}-\frac{b(b+4ac)\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{3/2}d^2}$$

input
```
Integrate[x^3*Sqrt[a + b/(c + d*x^2)],x]
```

output
```
((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b - 2*a*c + 2*a*d*x^2)
)/(8*a*d^2) - (b*(b + 4*a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]
/Sqrt[a]])/(8*a^(3/2)*d^2)
```

3.319.3 Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2057, 2053, 2052, 25, 27, 360, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+\frac{b}{c+dx^2}}dx$$

$$\Big\downarrow \text{2057}$$

$$\int x^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int x^2\sqrt{\frac{adx^2+b+ac}{dx^2+c}}dx^2$$

$$\Big\downarrow \text{2052}$$

$$-bd\int -\frac{x^4\left(-cx^4+b+ac\right)}{d^3\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$bd\int \frac{x^4\left(-cx^4+b+ac\right)}{d^3\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{x^4(-cx^4+b+ac)}{(a-x^4)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{d^2}$$

$$\Big\downarrow \text{360}$$

$$\frac{b\left(\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4(a-x^4)^2}-\frac{1}{4}\int\frac{b-4cx^4}{(a-x^4)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}\right)}{d^2}$$

$$\Big\downarrow \textbf{298}$$

$$\frac{b\left(\frac{1}{4}\left(-\frac{(4ac+b)\int\frac{1}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2a}-\frac{(b-4ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2a(a-x^4)}\right)+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4(a-x^4)^2}\right)}{d^2}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{b\left(\frac{1}{4}\left(-\frac{(4ac+b)\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{(b-4ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2a(a-x^4)}\right)+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4(a-x^4)^2}\right)}{d^2}$$

input
```
Int[x^3*Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(b*((b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*(a - x^4)^2) + (-1/2*((b
- 4*a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(a*(a - x^4)) - ((b + 4*a*
c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(2*a^(3/2)))/4)
)/d^2
```

3.319.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.319. $\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx$

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

3.319.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.37

method	result
risch	$-\frac{\left(-2 a d \,x^{2}+2 a c-b\right) \left(d \,x^{2}+c\right) \sqrt{\frac{a d \,x^{2}+a c+b}{d \,x^{2}+c}}}{8 d^{2} a}-\frac{b \left(4 a c+b\right) \ln \left(\frac{a c d+\frac{1}{2} b d+a \,d^{2} x^{2}}{\sqrt{a \,d^{2}}}+\sqrt{a \,c^{2}+b c+\left(2 a c d+b d\right) x^{2}+a \,d^{2} x^{4}}\right) \sqrt{\frac{a d \,x^{2}+a c+b}{d \,x^{2}+c}}\, \sqrt{}}{16 d a \sqrt{a \,d^{2}}\, \left(a d \,x^{2}+a c+b\right)}$
default	$-\frac{\sqrt{\frac{a d \,x^{2}+a c+b}{d \,x^{2}+c}}\, \left(d \,x^{2}+c\right) \left(-4 \sqrt{a \,d^{2} x^{4}+2 a c d \,x^{2}+b d \,x^{2}+a \,c^{2}+b c}\, \sqrt{a \,d^{2}}\, a d \,x^{2}+4 \ln \left(\frac{2 a \,d^{2} x^{2}+2 a c d+2 \sqrt{a \,d^{2} x^{4}+2 a c d \,x^{2}+b d \,x^{2}+a \,c^{2}+b c}\, \sqrt{}}{2 \sqrt{a \,d^{2}}}\right.\right.}{}$

input
```
int(x^3*(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/d^2*(-2*a*d*x^2+2*a*c-b)*(d*x^2+c)/a*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2
)-1/16*b/d*(4*a*c+b)/a*ln((a*c*d+1/2*b*d+a*d^2*x^2)/(a*d^2)^(1/2)+(a*c^2+b
*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/(a*d^2)^(1/2)*((a*d*x^2+a*c+b)/(d*x
^2+c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(a*d*x^2+a*c+b)
```

3.319.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.30

$$\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx$$

$$= \left[\frac{(4\,abc+b^2)\sqrt{a}\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2-4\,(2\,ad^2x^4+(4\,ac+b)dx^2+2\,c\right.}{32\,a^2d^2}\right.$$

input
```
integrate(x^3*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/32*((4*a*b*c + b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c
+ a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^
2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^2*d^2*x^4
 + a*b*d*x^2 - 2*a^2*c^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(
a^2*d^2), 1/16*((4*a*b*c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b
)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)
) + 2*(2*a^2*d^2*x^4 + a*b*d*x^2 - 2*a^2*c^2 + a*b*c)*sqrt((a*d*x^2 + a*c
+ b)/(d*x^2 + c)))/(a^2*d^2)]
```

3.319.6 Sympy [F]

$$\int x^3 \sqrt{a + \frac{b}{c + dx^2}}\, dx = \int x^3 \sqrt{\frac{ac + adx^2 + b}{c + dx^2}}\, dx$$

input
```
integrate(x**3*(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(x**3*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

3.319.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.55

$$\int x^3 \sqrt{a + \frac{b}{c + dx^2}}\, dx = -\frac{(4\,abc - b^2)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}} - (4\,a^2bc + ab^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^3d^2 - \frac{2\,(adx^2+ac+b)a^2d^2}{dx^2+c} + \frac{(adx^2+ac+b)^2ad^2}{(dx^2+c)^2}\right)} + \frac{(4\,ac + b)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{16\,a^{\frac{3}{2}}d^2}$$

input
```
integrate(x^3*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/8*((4*a*b*c - b^2)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*c
 + a*b^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^3*d^2 - 2*(a*d*x^2 + a
*c + b)*a^2*d^2/(d*x^2 + c) + (a*d*x^2 + a*c + b)^2*a*d^2/(d*x^2 + c)^2) +
 1/16*(4*a*c + b)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(3/2)*d^2)
```

3.319.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.12

$$\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx$$
$$=\frac{1}{16}\left(2\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(\frac{2\,x^2}{d}-\frac{2\,acd-bd}{ad^3}\right)+\frac{(4\,abc+b^2)\log\left(\left|2\,acd+2\left(\sqrt{ad^2}x^2\right.\right.\right.}{}\right.$$
$$\left.+c\right)$$

input
```
integrate(x^3*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
1/16*(2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2/d - (
2*a*c*d - b*d)/(a*d^3)) + (4*a*b*c + b^2)*log(abs(2*a*c*d + 2*(sqrt(a*d^2)
*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*sqrt(a)*abs(
d) + b*d))/(a^(3/2)*d*abs(d)))*sgn(d*x^2 + c)
```

3.319.9 Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int x^3\,\sqrt{a+\frac{b}{d\,x^2+c}}\,dx$$

input
```
int(x^3*(a + b/(c + d*x^2))^(1/2),x)
```

output
```
int(x^3*(a + b/(c + d*x^2))^(1/2), x)
```

3.320 $\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx$

3.320.1 Optimal result

Integrand size = 19, antiderivative size = 69

$$\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx=\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a}d}$$

output
```
1/2*b*arctanh((a+b/(d*x^2+c))^(1/2)/a^(1/2))/d/a^(1/2)+1/2*(d*x^2+c)*(a+b/
(d*x^2+c))^(1/2)/d
```

3.320.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.23

$$\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx=\frac{(c+dx^2)\sqrt{\frac{b+a(c+dx^2)}{c+dx^2}}}{2d}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+a(c+dx^2)}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a}d}$$

input
```
Integrate[x*Sqrt[a + b/(c + d*x^2)],x]
```

output
```
((c + d*x^2)*Sqrt[(b + a*(c + d*x^2))/(c + d*x^2)])/(2*d) + (b*ArcTanh[Sqr
t[(b + a*(c + d*x^2))/(c + d*x^2)]/Sqrt[a]])/(2*Sqrt[a]*d)
```

3.320.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2024, 773, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx$$

$$\Big\downarrow \text{2024}$$

$$\frac{\int \sqrt{a+\frac{b}{dx^2+c}}d(dx^2+c)}{2d}$$

$$\Big\downarrow \text{773}$$

$$-\frac{\int (dx^2+c)^2\sqrt{a+\frac{b}{dx^2+c}}d\frac{1}{dx^2+c}}{2d}$$

$$\Big\downarrow \text{51}$$

$$-\frac{\frac{1}{2}b\int \frac{dx^2+c}{\sqrt{a+\frac{b}{dx^2+c}}}d\frac{1}{dx^2+c}-(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\int \frac{1}{\frac{1}{b(dx^2+c)^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{dx^2+c}}-(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2d}$$

$$\Big\downarrow \text{221}$$

$$-\frac{(c+dx^2)\left(-\sqrt{a+\frac{b}{c+dx^2}}\right)-\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}}}{2d}$$

input
```
Int[x*Sqrt[a + b/(c + d*x^2)],x]
```

output
```
-1/2*(-((c + d*x^2)*Sqrt[a + b/(c + d*x^2)]) - (b*ArcTanh[Sqrt[a + b/(c +
d*x^2)]/Sqrt[a]])/Sqrt[a])/d
```

3.320.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

3.320.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(57) = 114$.

Time = 0.16 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.99

method	result
derivativedivides	$\frac{\sqrt{\frac{a\left(d\,x^{2}+c\right)+b}{d\,x^{2}+c}}\,(d\,x^{2}+c)\left(2\sqrt{a(d\,x^{2}+c)^{2}+b(d\,x^{2}+c)}\,\sqrt{a}+b\ln\left(\frac{2\sqrt{a\left(d\,x^{2}+c\right)^{2}+b\left(d\,x^{2}+c\right)}\,\sqrt{a}+2a\left(d\,x^{2}+c\right)+b}{2\sqrt{a}}\right)\right)}{4d\sqrt{(a(d\,x^{2}+c)+b)(d\,x^{2}+c)}\,\sqrt{a}}$
risch	$\frac{(d\,x^{2}+c)\sqrt{\frac{a d\,x^{2}+a c+b}{d\,x^{2}+c}}}{2d}+\frac{b\ln\left(\frac{a c d+\frac{1}{2}b d+a\,d^{2}x^{2}}{\sqrt{a\,d^{2}}}+\sqrt{a\,c^{2}+b c+(2a c d+b d)x^{2}+a\,d^{2}x^{4}}\right)\sqrt{\frac{a d\,x^{2}+a c+b}{d\,x^{2}+c}}\,\sqrt{(a d\,x^{2}+a c+b)(d}}{4\sqrt{a\,d^{2}}\,(a d\,x^{2}+a c+b)}$
default	$\frac{\sqrt{\frac{a d\,x^{2}+a c+b}{d\,x^{2}+c}}\,(d\,x^{2}+c)\left(b\ln\left(\frac{2a\,d^{2}x^{2}+2a c d+2\sqrt{a\,d^{2}x^{4}+2a c d\,x^{2}+b d\,x^{2}+a\,c^{2}+b c}\,\sqrt{a\,d^{2}}+b d}{2\sqrt{a\,d^{2}}}\right)d+2\sqrt{a\,d^{2}x^{4}+2a c d\,x^{2}+b d\,x^{2}+}\right.}{4\sqrt{(a d\,x^{2}+a c+b)(d\,x^{2}+c)}\,\sqrt{a\,d^{2}}\,d}$

input
```
int(x*(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*((a*(d*x^2+c)+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(2*(a*(d*x^2+c)^2+b*(d*x
^2+c))^(1/2)*a^(1/2)+b*ln(1/2*(2*(a*(d*x^2+c)^2+b*(d*x^2+c))^(1/2)*a^(1/2)
+2*a*(d*x^2+c)+b)/a^(1/2)))/((a*(d*x^2+c)+b)*(d*x^2+c))^(1/2)/a^(1/2)
```

3.320.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.87

$$\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx$$

$$=\left[\frac{\sqrt{a}b\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2+4\,(2\,ad^2x^4+(4\,ac+b)dx^2+2\,ac^2+bc)\sqrt{\right.}{8\,ad}\right.$$

$$\left.-\frac{\sqrt{-a}b\arctan\left(\frac{(2\,adx^2+2\,ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2dx^2+a^2c+ab)}\right)-2\,(adx^2+ac)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{4\,ad}\right]$$

input
```
integrate(x*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(a)*b*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 +
8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a
)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a*d*x^2 + a*c)*sqrt((a*d*x^2
 + a*c + b)/(d*x^2 + c)))/(a*d), -1/4*(sqrt(-a)*b*arctan(1/2*(2*a*d*x^2 +
2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2
*c + a*b)) - 2*(a*d*x^2 + a*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a*d
)]
```

3.320.6 Sympy [F]

$$\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int x\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,dx$$

input
```
integrate(x*(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(x*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

3.320.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(57) = 114.

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.83

$$\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx=-\frac{b\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(ad-\frac{(adx^2+ac+b)d}{dx^2+c}\right)}-\frac{b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{4\sqrt{a}d}$$

input
```
integrate(x*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/2*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*d - (a*d*x^2 + a*c + b)*d/
(d*x^2 + c)) - 1/4*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))
)/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(sqrt(a)*d)
```

3.320.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(57) = 114$.

Time = 0.43 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.78

$$\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx =$$
$$-\frac{1}{4}\left(\frac{b\log\left(\left|2\,acd+2\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)\sqrt{a}|d|+bd\right|\right)}{\sqrt{a}|d|}-\frac{2\sqrt{ad^2x^4+2\,a}}{}\right.$$
$$\left.+c\right)$$

input
```
integrate(x*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
-1/4*(b*log(abs(2*a*c*d + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^
2 + b*d*x^2 + a*c^2 + b*c))*sqrt(a)*abs(d) + b*d))/(sqrt(a)*abs(d)) - 2*sq
rt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)/d)*sgn(d*x^2 + c)
```

3.320.9 Mupad [B] (verification not implemented)

Time = 17.77 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.74

$$\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx$$
$$=\frac{\sqrt{\frac{b\,(d\,x^2+c)+a\,(d\,x^2+c)^2}{(d\,x^2+c)^2}}\,(d\,x^2+c)\left(\frac{b\,\ln\left(\frac{\frac{b}{2}+a\,\left(d\,x^2+c\right)+\sqrt{a}\,\sqrt{b\,\left(d\,x^2+c\right)+a\,\left(d\,x^2+c\right)^2}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b\,(d\,x^2+c)+a\,(d\,x^2+c)^2}}+2\right)}{4\,d}$$

input
```
int(x*(a + b/(c + d*x^2))^(1/2),x)
```

output
```
(((b*(c + d*x^2) + a*(c + d*x^2)^2)/(c + d*x^2)^2)^(1/2)*(c + d*x^2)*((b*l
og((b/2 + a*(c + d*x^2) + a^(1/2)*(b*(c + d*x^2) + a*(c + d*x^2)^2)^(1/2))
/a^(1/2)))/(a^(1/2)*(b*(c + d*x^2) + a*(c + d*x^2)^2)^(1/2)) + 2))/(4*d)
```

3.321 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx$

3.321.1 Optimal result

Integrand size = 21, antiderivative size = 96

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx=\sqrt{a}\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)-\frac{\sqrt{b+ac}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{\sqrt{c}}$$

output
```
arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))*a^(1/2)-arctanh(c^(1/2)
*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))*(a*c+b)^(1/2)/c^(1/2)
```

3.321.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx=-\frac{\sqrt{-b-ac}\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{\sqrt{c}}+\sqrt{a}\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)$$

input
```
Integrate[Sqrt[a + b/(c + d*x^2)]/x,x]
```

output
```
-((Sqrt[-b - a*c]*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/S
qrt[-b - a*c]])/Sqrt[c]) + Sqrt[a]*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d
*x^2)]/Sqrt[a]]
```

3.321.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2057, 2053, 2052, 25, 27, 383, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x} \, dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{x^2}dx^2$$

$$\Big\downarrow \text{2052}$$

$$-bd\int -\frac{x^4}{d\left(a-x^4\right)\left(-cx^4+b+ac\right)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$bd\int \frac{x^4}{d\left(a-x^4\right)\left(-cx^4+b+ac\right)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{27}$$

$$b\int \frac{x^4}{\left(a-x^4\right)\left(-cx^4+b+ac\right)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{383}$$

$$b\left(\frac{a\int \frac{1}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{b}-\frac{(ac+b)\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{b}\right)$$

$$\Big\downarrow \text{219}$$

$$b\left(\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{b}-\frac{(ac+b)\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{b}\right)$$

$$\Big\downarrow \text{221}$$

$$b\left(\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{b}-\frac{\sqrt{ac+b}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{b\sqrt{c}}\right)$$

input
```
Int[Sqrt[a + b/(c + d*x^2)]/x,x]
```

output
```
b*((Sqrt[a]*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/b - (S
qrt[b + a*c]*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[
b + a*c]])/(b*Sqrt[c]))
```

3.321.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.321. $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx$

rule 383
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[(-a)*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(a + b*x^2), x], x]
 + Simp[c*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; Free
Q[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

3.321.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(80) = 160$.

Time = 0.10 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.45

method	result
default	$-\frac{\sqrt{\frac{a d\,x^{2}+a c+b}{d\,x^{2}+c}}\,\left(d\,x^{2}+c\right)\left(-\ln\left(\frac{2 a\,d^{2} x^{2}+2 a c d+2 \sqrt{a\,d^{2} x^{4}+2 a c d\,x^{2}+b d\,x^{2}+a\,c^{2}+b c}\,\sqrt{a\,d^{2}}+b d}{2 \sqrt{a\,d^{2}}}\right) a c d+\sqrt{a\,c^{2}+b c}\,\ln\left(\frac{2 a c d\,x^{2}+b d\,x^{2}+2 a\,c^{2}+}{}\right.\right.}{2 \sqrt{\left(a d\,x^{2}+a c+b\right)\left(d\,x^{2}+c\right)}\,c \sqrt{a\,d^{2}}}$

input
```
int((a+b/(d*x^2+c))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-ln(1/2*(2*a*d^2*x^2+2*a
*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/
(a*d^2)^(1/2))*a*c*d+(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*(
a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^
2)*(a*d^2)^(1/2))/((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/c/(a*d^2)^(1/2)
```

3.321.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(80) = 160$.

Time = 0.35 (sec) , antiderivative size = 927, normalized size of antiderivative = 9.66

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx = \left[\frac{1}{4}\sqrt{a}\log\left(8a^2d^2x^4 + 8a^2c^2 + 8\left(2a^2c+ab\right)dx^2 + 8abc + b^2 + 4\left(2ad^2x^4 + (4ac+b)dx^2 + 2ac^2 + bc\right)\sqrt{a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right) + \frac{1}{4}\sqrt{\frac{ac+b}{c}}\log\left(\frac{(8a^2c^2+8abc+b^2)d^2x^4 + 8a^2c^4 + 16abc^3 + 8b^2c^2 + 8(2a^2c^3+3abc^2+b^2c)dx^2 - 4}{x^4}\right.\right.$$

$$-\frac{1}{2}\sqrt{-a}\arctan\left(\frac{(2adx^2+2ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2(a^2dx^2+a^2c+ab)}\right)$$

$$+\frac{1}{4}\sqrt{\frac{ac+b}{c}}\log\left(\frac{(8a^2c^2+8abc+b^2)d^2x^4 + 8a^2c^4 + 16abc^3 + 8b^2c^2 + 8(2a^2c^3+3abc^2+b^2c)dx^2 - 4}{x^4}\right.$$

$$+\frac{1}{4}\sqrt{a}\log\left(8a^2d^2x^4 + 8a^2c^2 + 8\left(2a^2c+ab\right)dx^2 + 8abc + b^2 + 4\left(2ad^2x^4 + (4ac+b)dx^2 + 2ac^2 + bc\right)\sqrt{a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right),$$

$$-\frac{1}{2}\sqrt{-a}\arctan\left(\frac{(2adx^2+2ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2(a^2dx^2+a^2c+ab)}\right)$$

$$\left.+\frac{1}{2}\sqrt{-\frac{ac+b}{c}}\arctan\left(\frac{((2ac+b)dx^2+2ac^2+2bc)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\sqrt{-\frac{ac+b}{c}}}{2(a^2c^2+(a^2c+ab)dx^2+2abc+b^2)}\right)\right]$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/4*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a
*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*s
qrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 1/4*sqrt((a*c + b)/c)*log(((8*a^2*
c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a
^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c^2 + b*c)*d^2*x^4 + 2*a*c^4 +
 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
)*sqrt((a*c + b)/c))/x^4), -1/2*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b
)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)
) + 1/4*sqrt((a*c + b)/c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2
*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 -
4*((2*a*c^2 + b*c)*d^2*x^4 + 2*a*c^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2
)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt((a*c + b)/c))/x^4), 1/2*sqrt(
-(a*c + b)/c)*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x
^2 + a*c + b)/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(a^2*c^2 + (a^2*c + a*b)*d*x
^2 + 2*a*b*c + b^2)) + 1/4*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^
2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*
a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))), -1/2*sqrt(-a)
*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x
^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) + 1/2*sqrt(-(a*c + b)/c)*arctan(1/2*((
2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c...
```

3.321.6 Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx = \int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(1/2)/x,x)
```

output
```
Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x, x)
```

3.321.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.66

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx = \frac{(ac+b)\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{2\sqrt{(ac+b)c}} - \frac{1}{2}\sqrt{a}\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x,x, algorithm="maxima")
```

output
```
1/2*(a*c + b)*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b
)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/sqrt(
(a*c + b)*c) - 1/2*sqrt(a)*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2
 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))
```

3.321.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.321.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx = \int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x}\,dx$$

input
```
int((a + b/(c + d*x^2))^(1/2)/x,x)
```

output
```
int((a + b/(c + d*x^2))^(1/2)/x, x)
```

3.322 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}\,dx$

3.322.1 Optimal result

Integrand size = 21, antiderivative size = 104

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}\,dx=-\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{2cx^2}+\frac{bd\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{2c^{3/2}\sqrt{b+ac}}$$

output
```
1/2*b*d*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))/c
^(3/2)/(a*c+b)^(1/2)-1/2*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c/x^2
```

3.322.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}\,dx=-\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{2cx^2}-\frac{bd\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{2c^{3/2}\sqrt{-b-ac}}$$

input
```
Integrate[Sqrt[a + b/(c + d*x^2)]/x^3,x]
```

output
```
-1/2*((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(c*x^2) - (b*d*Ar
cTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(2*c
^(3/2)*Sqrt[-b - a*c])
```

3.322.3 Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2057, 2053, 2052, 252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^3}dx$$

$$\Big\downarrow \text{ 2053}$$

$$\frac{1}{2}\int \frac{\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{x^4}dx^2$$

$$\Big\downarrow \text{ 2052}$$

$$-bd\int \frac{x^4}{(-cx^4+b+ac)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{ 252}$$

$$-bd\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2c\left(ac+b-cx^4\right)}-\frac{\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2c}\right)$$

$$\Big\downarrow \text{ 221}$$

$$-bd\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2c\left(ac+b-cx^4\right)}-\frac{\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2c^{3/2}\sqrt{ac+b}}\right)$$

input
```
Int[Sqrt[a + b/(c + d*x^2)]/x^3,x]
```

output
```
-(b*d*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/(2*c*(b + a*c - c*x^4)) - Arc
Tanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]]/(2*c^(
3/2)*Sqrt[b + a*c])))
```

3.322.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

3.322. $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}\,dx$

3.322.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(88) = 176.

Time = 0.14 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.82

method	result
risch	$-\frac{(d\,x^2+c)\sqrt{\frac{a d\,x^2+a c+b}{d\,x^2+c}}}{2c\,x^2}+\frac{b d\ln\left(\frac{2a\,c^2+2bc+(2acd+bd)x^2+2\sqrt{a\,c^2+bc}\,\sqrt{a\,c^2+bc+(2acd+bd)x^2+a\,d^2x^4}}{x^2}\right)\sqrt{\frac{a d\,x^2+a c+b}{d\,x^2+c}}\,\sqrt{(a d\,x^2+ac+}}{4c\sqrt{a\,c^2+bc}\,(a d\,x^2+ac+b)}$
default	$-\frac{\sqrt{\frac{a d\,x^2+ac+b}{d\,x^2+c}}\,(d\,x^2+c)\left(-2a\,d^2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,x^4\sqrt{a\,c^2+bc}-\ln\left(\frac{2acd\,x^2+bd\,x^2+2a\,c^2+2\sqrt{a\,c^2+bc}\,\sqrt{a\,d^2x^4+2acd}}{x^2}\right.\right.}{}$

input
```
int((a+b/(d*x^2+c))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c/x^2+1/4*b*d/c/(a*c^2+b*
c)^(1/2)*ln((2*a*c^2+2*b*c+(2*a*c*d+b*d)*x^2+2*(a*c^2+b*c)^(1/2)*(a*c^2+b*
c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/x^2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/
2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(a*d*x^2+a*c+b)
```

3.322.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(88) = 176.

Time = 0.33 (sec) , antiderivative size = 433, normalized size of antiderivative = 4.16

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}\,dx$$

$$= \left[\frac{\sqrt{ac^2+bc}\,bdx^2\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,(2\,a^2c^3+3\,abc^2+b^2c)dx^2+4\,((2\,ac+b)d^2x^4+2\,ac^3+(4\,ac^2+3\,b}{x^4}\right.}{8\,(ac^3+bc^2)x^2}\right.$$

$$-\frac{\sqrt{-ac^2-bc}\,bdx^2\arctan\left(\frac{((2\,ac+b)dx^2+2\,ac^2+2\,bc)\sqrt{-ac^2-bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2c^3+2\,abc^2+(a^2c^2+abc)dx^2+b^2c)}\right)+2\,(ac^3+(ac^2+bc)dx^2+bc^2)\sqrt{\frac{ad}{}}}{4\,(ac^3+bc^2)x^2}$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/8*(sqrt(a*c^2 + b*c)*b*d*x^2*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 +
 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*
x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2
)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4) - 4*(a*c^3
 + (a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a
*c^3 + b*c^2)*x^2), -1/4*(sqrt(-a*c^2 - b*c)*b*d*x^2*arctan(1/2*((2*a*c +
b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d
*x^2 + c))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(a
*c^3 + (a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
/((a*c^3 + b*c^2)*x^2)]
```

3.322.6 Sympy [F]

$$\int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x^3}\,dx = \int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^3}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(1/2)/x**3,x)
```

output
```
Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**3, x)
```

3.322.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x^3}\,dx = -\frac{bd\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(ac^2+bc-\frac{(adx^2+ac+b)c^2}{dx^2+c}\right)} - \frac{bd\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{4\sqrt{(ac+b)c}c}$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-1/2*b*d*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*c^2 + b*c - (a*d*x^2 + a
*c + b)*c^2/(d*x^2 + c)) - 1/4*b*d*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2
+ c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt
((a*c + b)*c)))/(sqrt((a*c + b)*c)*c)
```

3.322.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(88) = 176$.

Time = 0.39 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.70

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}\,dx = -\frac{1}{2}\left(\frac{bd\arctan\left(-\frac{\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{\sqrt{-ac^2-bc}}\right)}{\sqrt{-ac^2-bc}\,c}+\frac{2\,a^{\frac{3}{2}}c^2|d|+2\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2\right.}{\left(ac^2-\left(\sqrt{ad^2}\right.\right.}\right.$$
$$+c\Big)$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^3,x, algorithm="giac")
```

output
```
-1/2*(b*d*arctan(-(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^
2 + a*c^2 + b*c))/sqrt(-a*c^2 - b*c))/(sqrt(-a*c^2 - b*c)*c) + (2*a^(3/2)*
c^2*abs(d) + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 +
 a*c^2 + b*c))*a*c*d + 2*sqrt(a)*b*c*abs(d) + (sqrt(a*d^2)*x^2 - sqrt(a*d^
2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*b*d)/((a*c^2 - (sqrt(a*d^2)*
x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2 + b*c)*c))*
sgn(d*x^2 + c)
```

3.322.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}\,dx = \int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x^3}\,dx$$

input
```
int((a + b/(c + d*x^2))^(1/2)/x^3,x)
```

output
```
int((a + b/(c + d*x^2))^(1/2)/x^3, x)
```

3.323 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx$

3.323.1 Optimal result

Integrand size = 21, antiderivative size = 174

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx=\frac{(5b+4ac)d\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8c^2(b+ac)x^2}-\frac{\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4c^2x^4}-\frac{b(3b+4ac)d^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{8c^{5/2}(b+ac)^{3/2}}$$

output
```
-1/8*b*(4*a*c+3*b)*d^2*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(
a*c+b)^(1/2))/c^(5/2)/(a*c+b)^(3/2)+1/8*(4*a*c+5*b)*d*(d*x^2+c)*((a*d*x^2+
a*c+b)/(d*x^2+c))^(1/2)/c^2/(a*c+b)/x^2-1/4*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(
d*x^2+c))^(1/2)/c^2/x^4
```

3.323.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx=-\frac{\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(b\left(2c-3dx^2\right)+2ac\left(c-dx^2\right)\right)}{8c^2(b+ac)x^4}-\frac{b(3b+4ac)d^2\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{8c^{5/2}(-b-ac)^{3/2}}$$

input
```
Integrate[Sqrt[a + b/(c + d*x^2)]/x^5,x]
```

output
```
-1/8*((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b*(2*c - 3*d*x^2)
 + 2*a*c*(c - d*x^2)))/(c^2*(b + a*c)*x^4) - (b*(3*b + 4*a*c)*d^2*ArcTan[(
Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(8*c^(5/2)
*(-b - a*c)^(3/2))
```

3.323.3 Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2057, 2053, 2052, 25, 27, 360, 25, 298, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5} dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^5}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{x^6}dx^2$$

$$\Big\downarrow \text{2052}$$

$$-bd\int -\frac{dx^4(a-x^4)}{(-cx^4+b+ac)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$bd\int \frac{dx^4(a-x^4)}{(-cx^4+b+ac)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{27}$$

$$bd^2\int \frac{x^4(a-x^4)}{(-cx^4+b+ac)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{360}$$

$$bd^2\left(-\frac{\int -\frac{4cx^4+b}{(-cx^4+b+ac)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{4c^2}-\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^2\left(ac+b-cx^4\right)^2}\right)$$

$$\Big\downarrow \ 25$$

$$bd^2\left(\frac{\int \frac{4cx^4+b}{(-cx^4+b+ac)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{4c^2}-\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^2\left(ac+b-cx^4\right)^2}\right)$$

$$\Big\downarrow \ 298$$

$$bd^2\left(\frac{\frac{(4ac+5b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b)(ac+b-cx^4)}-\frac{(4ac+3b)\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2(ac+b)}}{4c^2}-\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^2\left(ac+b-cx^4\right)^2}\right)$$

$$\Big\downarrow \ 221$$

$$bd^2\left(\frac{\frac{(4ac+5b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b)(ac+b-cx^4)}-\frac{(4ac+3b)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2\sqrt{c}(ac+b)^{3/2}}}{4c^2}-\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^2\left(ac+b-cx^4\right)^2}\right)$$

input
```
Int[Sqrt[a + b/(c + d*x^2)]/x^5,x]
```

output
```
b*d^2*(-1/4*(b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(c^2*(b + a*c - c*x^
4)^2) + (((5*b + 4*a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(2*(b + a*c
)*(b + a*c - c*x^4)) - ((3*b + 4*a*c)*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d
*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/(2*Sqrt[c]*(b + a*c)^(3/2)))/(4*c^2))
```

3.323.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.323. $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

3.323. $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5} \, dx$

3.323.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.37

method	result
risch	$-\frac{(d\,x^2+c)\left(-2acd\,x^2-3bd\,x^2+2a\,c^2+2bc\right)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{8c^2x^4(ac+b)}-\frac{d^2b(4ac+3b)\ln\left(\frac{2a\,c^2+2bc+(2acd+bd)x^2+2\sqrt{a\,c^2+bc}\,\sqrt{a\,c^2+bc+(2acd+bd}}{x^2}\right.}{16c^2(ac+b)\sqrt{a\,c^2+bc}\,(ad\,x}$
default	$-\frac{\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,(d\,x^2+c)\left(12a^2d^3\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,x^6c\left(a\,c^2+bc\right)^{\frac{3}{2}}+4\ln\left(\frac{2acd\,x^2+bd\,x^2+2a\,c^2+2\sqrt{a\,c^2+bc}\,\sqrt{a\,d^2x^4+}}{x^2}\right.\right.}{}$

input
```
int((a+b/(d*x^2+c))^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(d*x^2+c)*(-2*a*c*d*x^2-3*b*d*x^2+2*a*c^2+2*b*c)/c^2/x^4/(a*c+b)*((a*
d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/16*d^2*b*(4*a*c+3*b)/c^2/(a*c+b)/(a*c^2+b*
c)^(1/2)*ln((2*a*c^2+2*b*c+(2*a*c*d+b*d)*x^2+2*(a*c^2+b*c)^(1/2)*(a*c^2+b*
c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/x^2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/
2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(a*d*x^2+a*c+b)
```

3.323.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 577, normalized size of antiderivative = 3.32

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx$$
$$= \left[\frac{(4\,abc+3\,b^2)\sqrt{ac^2+bc}d^2x^4\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,(2\,a^2c^3+3\,abc^2+b^2c)dx^2-4\,((2\,ac+b)d^2x^4+}{x^4}\right.}{}\right.$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^5,x, algorithm="fricas")
```

output
```
[1/32*((4*a*b*c + 3*b^2)*sqrt(a*c^2 + b*c)*d^2*x^4*log(((8*a^2*c^2 + 8*a*b
*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*
a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b
*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c
)))/x^4) - 4*(2*a^2*c^5 - (2*a^2*c^3 + 5*a*b*c^2 + 3*b^2*c)*d^2*x^4 + 4*a*
b*c^4 + 2*b^2*c^3 - (a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d
*x^2 + c)))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*x^4), 1/16*((4*a*b*c + 3*b^2)
*sqrt(-a*c^2 - b*c)*d^2*x^4*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*
c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2*a
*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) - 2*(2*a^2*c^5 - (2*a^2*c^3 + 5
*a*b*c^2 + 3*b^2*c)*d^2*x^4 + 4*a*b*c^4 + 2*b^2*c^3 - (a*b*c^3 + b^2*c^2)*
d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^2*c^5 + 2*a*b*c^4 + b^2*
c^3)*x^4)]
```

3.323.6 Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx = \int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^5}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(1/2)/x**5,x)
```

output
```
Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**5, x)
```

3.323.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(154) = 308$.

Time = 0.30 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.85

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx = \frac{(4\,abc+3\,b^2)d^2\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{16\,(ac^3+bc^2)\sqrt{(ac+b)c}} - \frac{(4\,abc^2+5\,b^2c)d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}} - (4\,a^2bc^2+7\,ab^2c+3\,b^3)d^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^3c^5+3\,a^2bc^4+3\,ab^2c^3+b^3c^2+\frac{(ac^5+bc^4)(adx^2+ac+b)^2}{(dx^2+c)^2}-\frac{2\,(a^2c^5+2\,abc^4+b^2c^3)(adx^2+ac+b)}{dx^2+c}\right)}$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^5,x, algorithm="maxima")
```

output
```
1/16*(4*a*b*c + 3*b^2)*d^2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) -
sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c +
b)*c)))/((a*c^3 + b*c^2)*sqrt((a*c + b)*c)) - 1/8*((4*a*b*c^2 + 5*b^2*c)*d
^2*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*c^2 + 7*a*b^2*c + 3*
b^3)*d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^3*c^5 + 3*a^2*b*c^4 + 3
*a*b^2*c^3 + b^3*c^2 + (a*c^5 + b*c^4)*(a*d*x^2 + a*c + b)^2/(d*x^2 + c)^2
 - 2*(a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*(a*d*x^2 + a*c + b)/(d*x^2 + c))
```

3.323.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 713 vs. $2(154) = 308$.

Time = 0.43 (sec) , antiderivative size = 713, normalized size of antiderivative = 4.10

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx$$

$$= \frac{1}{8}\left(\frac{(4\,abcd^2+3\,b^2d^2)\arctan\left(-\frac{\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{\sqrt{-ac^2-bc}}\right)}{(ac^3+bc^2)\sqrt{-ac^2-bc}}+\frac{8\,a^{\frac{7}{2}}c^5d|d|+16\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4}\right.}{}\right.$$

$$+c)$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^5,x, algorithm="giac")
```

output
```
1/8*((4*a*b*c*d^2 + 3*b^2*d^2)*arctan(-(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 +
 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))/sqrt(-a*c^2 - b*c))/((a*c^3 + b*c^2
)*sqrt(-a*c^2 - b*c)) + (8*a^(7/2)*c^5*d*abs(d) + 16*(sqrt(a*d^2)*x^2 - sq
rt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3*c^4*d^2 + 8*(sqrt
(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(
5/2)*c^3*d*abs(d) + 24*a^(5/2)*b*c^4*d*abs(d) + 36*(sqrt(a*d^2)*x^2 - sqrt
(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b*c^3*d^2 + 8*(sqrt
(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(
3/2)*b*c^2*d*abs(d) + 24*a^(3/2)*b^2*c^3*d*abs(d) - 4*(sqrt(a*d^2)*x^2 - s
qrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a*b*c*d^2 + 25*(sq
rt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*b
^2*c^2*d^2 + 8*sqrt(a)*b^3*c^2*d*abs(d) - 3*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*
x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*b^2*d^2 + 5*(sqrt(a*d^2)*x^2
 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*b^3*c*d^2)/((a*c
^3 + b*c^2)*(a*c^2 - (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d
*x^2 + a*c^2 + b*c))^2 + b*c)^2))*sgn(d*x^2 + c)
```

3.323.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx = \int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x^5}\,dx$$

input
```
int((a + b/(c + d*x^2))^(1/2)/x^5,x)
```

output
```
int((a + b/(c + d*x^2))^(1/2)/x^5, x)
```

3.324 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7} dx$

3.324.1 Optimal result

Integrand size = 21, antiderivative size = 265

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7} dx = -\frac{(11b^2+20abc+8a^2c^2)\,d^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{16c^3(b+ac)^2x^2} + \frac{(3b+4ac)d(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8c^3(b+ac)x^4} - \frac{(c+dx^2)^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}{6c^2(b+ac)x^6} + \frac{b(5b^2+12abc+8a^2c^2)\,d^3\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{16c^{7/2}(b+ac)^{5/2}}$$

output
```
-1/6*(d*x^2+c)^3*((a*d*x^2+a*c+b)/(d*x^2+c))^(3/2)/c^2/(a*c+b)/x^6+1/16*b*
(8*a^2*c^2+12*a*b*c+5*b^2)*d^3*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))
^(1/2)/(a*c+b)^(1/2))/c^(7/2)/(a*c+b)^(5/2)-1/16*(8*a^2*c^2+20*a*b*c+11*b^
2)*d^2*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^3/(a*c+b)^2/x^2+1/8*(
4*a*c+3*b)*d*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^3/(a*c+b)/x^4
```

3.324.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}\,dx =$$

$$-\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}(8a^2c^2(c^2-cdx^2+d^2x^4)+2abc(8c^2-9cdx^2+13d^2x^4)+b^2(8c^2-10cdx^2+15d^2}{48c^3(b+ac)^2x^6}$$

$$-\frac{b(5b^2+12abc+8a^2c^2)\,d^3\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{16c^{7/2}(-b-ac)^{5/2}}$$

input
```
Integrate[Sqrt[a + b/(c + d*x^2)]/x^7,x]
```

output
```
-1/48*((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(8*a^2*c^2*(c^2 -
 c*d*x^2 + d^2*x^4) + 2*a*b*c*(8*c^2 - 9*c*d*x^2 + 13*d^2*x^4) + b^2*(8*c^
2 - 10*c*d*x^2 + 15*d^2*x^4)))/(c^3*(b + a*c)^2*x^6) - (b*(5*b^2 + 12*a*b*
c + 8*a^2*c^2)*d^3*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/
Sqrt[-b - a*c]])/(16*c^(7/2)*(-b - a*c)^(5/2))
```

3.324.3 Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2057, 2053, 2052, 27, 366, 27, 360, 27, 298, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}\,dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^7}dx$$

$$\Big\downarrow \text{ 2053}$$

$$\frac{1}{2}\int \frac{\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{x^8}dx^2$$

$$\Big\downarrow \ 2052$$

$$-bd\int \frac{d^2x^4\left(a-x^4\right)^2}{\left(-cx^4+b+ac\right)^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \ 27$$

$$-bd^3\int \frac{x^4\left(a-x^4\right)^2}{\left(-cx^4+b+ac\right)^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \ 366$$

$$-bd^3\left(\frac{b^2x^6}{6c^2(ac+b)\left(ac+b-cx^4\right)^3}-\frac{\int \frac{3x^4\left(2c(b+ac)x^4+b^2-2a^2c^2\right)}{\left(-cx^4+b+ac\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{6c^2(ac+b)}\right)$$

$$\Big\downarrow \ 27$$

$$-bd^3\left(\frac{b^2x^6}{6c^2(ac+b)\left(ac+b-cx^4\right)^3}-\frac{\int \frac{x^4\left(2c(b+ac)x^4+b^2-2a^2c^2\right)}{\left(-cx^4+b+ac\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2c^2(ac+b)}\right)$$

$$\Big\downarrow \ 360$$

$$-bd^3\left(\frac{b^2x^6}{6c^2(ac+b)\left(ac+b-cx^4\right)^3}-\frac{\frac{b(4ac+3b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c\left(ac+b-cx^4\right)^2}-\frac{\int \frac{c\left(8c(b+ac)x^4+b(3b+4ac)\right)}{\left(-cx^4+b+ac\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{4c^2}}{2c^2(ac+b)}\right)$$

$$\Big\downarrow \ 27$$

$$-bd^3\left(\frac{b^2x^6}{6c^2(ac+b)\left(ac+b-cx^4\right)^3}-\frac{\frac{b(4ac+3b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c\left(ac+b-cx^4\right)^2}-\frac{\int \frac{8c(b+ac)x^4+b(3b+4ac)}{\left(-cx^4+b+ac\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{4c}}{2c^2(ac+b)}\right)$$

$$\Big\downarrow \ 298$$

3.324. $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}dx$

$$-bd^3\left(\frac{b^2x^6}{6c^2(ac+b)(ac+b-cx^4)^3}-\frac{\frac{b(4ac+3b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c(ac+b-cx^4)^2}-\frac{\frac{\left(8a^2c^2+20abc+11b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b)\left(ac+b-cx^4\right)}-\frac{\left(8a^2c^2+12abc+5b^2\right)\int\frac{1}{-cx^4+b+ac}d}{2(ac+b)}}{4c}}{2c^2(ac+b)}\right.$$

$$\Big\downarrow \text{221}$$

$$-bd^3\left(\frac{b^2x^6}{6c^2(ac+b)(ac+b-cx^4)^3}-\frac{\frac{b(4ac+3b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c(ac+b-cx^4)^2}-\frac{\frac{\left(8a^2c^2+20abc+11b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b)\left(ac+b-cx^4\right)}-\frac{\left(8a^2c^2+12abc+5b^2\right)\text{arctanh}\left(\frac{\sqrt{c}}{}\right.}{2\sqrt{c}(ac+b)^{3/2}}}{4c}}{2c^2(ac+b)}\right.$$

input
```
Int[Sqrt[a + b/(c + d*x^2)]/x^7,x]
```

output
```
-(b*d^3*((b^2*x^6)/(6*c^2*(b + a*c)*(b + a*c - c*x^4)^3) - ((b*(3*b + 4*a*
c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*c*(b + a*c - c*x^4)^2) - (((1
1*b^2 + 20*a*b*c + 8*a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(2*(b
 + a*c)*(b + a*c - c*x^4)) - ((5*b^2 + 12*a*b*c + 8*a^2*c^2)*ArcTanh[(Sqrt
[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/(2*Sqrt[c]*(b +
 a*c)^(3/2)))/(4*c))/(2*c^2*(b + a*c))))
```

3.324.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

3.324. $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}\,dx$

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

3.324. $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7} \, dx$

3.324.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.20

method	result
risch	$-\frac{(d\,x^2+c)\left(8a^2c^2d^2x^4+26ac\,d^2b\,x^4-8a^2c^3d\,x^2+15b^2d^2x^4-18ab\,c^2d\,x^2+8a^2c^4-10b^2cd\,x^2+16ab\,c^3+8b^2c^2\right)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{48c^3x^6(ac+b)^2}+\frac{d^3b(}{}$
default	Expression too large to display

input
```
int((a+b/(d*x^2+c))^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(d*x^2+c)*(8*a^2*c^2*d^2*x^4+26*a*b*c*d^2*x^4-8*a^2*c^3*d*x^2+15*b^2
*d^2*x^4-18*a*b*c^2*d*x^2+8*a^2*c^4-10*b^2*c*d*x^2+16*a*b*c^3+8*b^2*c^2)/c
^3/x^6/(a*c+b)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/32*d^3*b*(8*a^2*c^2+1
2*a*b*c+5*b^2)/(a*c+b)^2/c^3/(a*c^2+b*c)^(1/2)*ln((2*a*c^2+2*b*c+(2*a*c*d+
b*d)*x^2+2*(a*c^2+b*c)^(1/2)*(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2)
)/x^2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)
/(a*d*x^2+a*c+b)
```

3.324.5 Fricas [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 755, normalized size of antiderivative = 2.85

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}\,dx$$

$$= \left[\frac{3\,(8\,a^2bc^2+12\,ab^2c+5\,b^3)\sqrt{ac^2+bc}d^3x^6\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,(2\,a^2c^3+3\,abc^2+b^2c)dx^2\cdots}{x^4}\right.}{\cdots}\right.$$

$$-\frac{3\,(8\,a^2bc^2+12\,ab^2c+5\,b^3)\sqrt{-ac^2-bc}d^3x^6\arctan\left(\frac{((2\,ac+b)dx^2+2\,ac^2+2\,bc)\sqrt{-ac^2-bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2c^3+2\,abc^2+(a^2c^2+abc)dx^2+b^2c)}\right)+2\,(8\,a^3c\cdots}{\cdots}$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^7,x, algorithm="fricas")
```

output
```
[1/192*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*sqrt(a*c^2 + b*c)*d^3*x^6*log
(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2
 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*
c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 +
 a*c + b)/(d*x^2 + c)))/x^4) - 4*(8*a^3*c^7 + (8*a^3*c^4 + 34*a^2*b*c^3 +
41*a*b^2*c^2 + 15*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 + 24*a*b^2*c^5 + 8*b^3*c^4
 + (8*a^2*b*c^4 + 13*a*b^2*c^3 + 5*b^3*c^2)*d^2*x^4 - 2*(a^2*b*c^5 + 2*a*b
^2*c^4 + b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^7
+ 3*a^2*b*c^6 + 3*a*b^2*c^5 + b^3*c^4)*x^6), -1/96*(3*(8*a^2*b*c^2 + 12*a*
b^2*c + 5*b^3)*sqrt(-a*c^2 - b*c)*d^3*x^6*arctan(1/2*((2*a*c + b)*d*x^2 +
2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/
(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(8*a^3*c^7 +
(8*a^3*c^4 + 34*a^2*b*c^3 + 41*a*b^2*c^2 + 15*b^3*c)*d^3*x^6 + 24*a^2*b*c^
6 + 24*a*b^2*c^5 + 8*b^3*c^4 + (8*a^2*b*c^4 + 13*a*b^2*c^3 + 5*b^3*c^2)*d^
2*x^4 - 2*(a^2*b*c^5 + 2*a*b^2*c^4 + b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c)))/((a^3*c^7 + 3*a^2*b*c^6 + 3*a*b^2*c^5 + b^3*c^4)*x^6)]
```

3.324.6 Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}\,dx=\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^7}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(1/2)/x**7,x)
```

output
```
Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**7, x)
```

3.324.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 557 vs. 2(241) = 482.

Time = 0.31 (sec) , antiderivative size = 557, normalized size of antiderivative = 2.10

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}\,dx=-\frac{(8\,a^2bc^2+12\,ab^2c+5\,b^3)d^3\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{32\,(a^2c^5+2\,abc^4+b^2c^3)\sqrt{(ac+b)c}} - \frac{3\,(8\,a^2bc^4+20\,ab^2c^3+11\,b^3c^2)d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-8\,(6\,a^3bc^4+18\,a^2b^2c^3+17\,ab^3c^2+5\,b^4c)d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right.}{48\left(a^5c^8+5\,a^4bc^7+10\,a^3b^2c^6+10\,a^2b^3c^5+5\,ab^4c^4+b^5c^3-\frac{(a^2c^8+2\,abc^7+b^2c^6)(adx^2+ac+b)^3}{(dx^2+c)^3}+\frac{3\,(a^3c^8+3\,a^2bc^7}{}\right.}$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^7,x, algorithm="maxima")
```

output
```
-1/32*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*d^3*log((c*sqrt((a*d*x^2 + a*c +
b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c)) + sqrt((a*c + b)*c)))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*sqrt((a*c + b)*
c)) - 1/48*(3*(8*a^2*b*c^4 + 20*a*b^2*c^3 + 11*b^3*c^2)*d^3*((a*d*x^2 + a*
c + b)/(d*x^2 + c))^(5/2) - 8*(6*a^3*b*c^4 + 18*a^2*b^2*c^3 + 17*a*b^3*c^2
 + 5*b^4*c)*d^3*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) + 3*(8*a^4*b*c^4 +
 28*a^3*b^2*c^3 + 37*a^2*b^3*c^2 + 22*a*b^4*c + 5*b^5)*d^3*sqrt((a*d*x^2 +
 a*c + b)/(d*x^2 + c)))/(a^5*c^8 + 5*a^4*b*c^7 + 10*a^3*b^2*c^6 + 10*a^2*b
^3*c^5 + 5*a*b^4*c^4 + b^5*c^3 - (a^2*c^8 + 2*a*b*c^7 + b^2*c^6)*(a*d*x^2
+ a*c + b)^3/(d*x^2 + c)^3 + 3*(a^3*c^8 + 3*a^2*b*c^7 + 3*a*b^2*c^6 + b^3*
c^5)*(a*d*x^2 + a*c + b)^2/(d*x^2 + c)^2 - 3*(a^4*c^8 + 4*a^3*b*c^7 + 6*a^
2*b^2*c^6 + 4*a*b^3*c^5 + b^4*c^4)*(a*d*x^2 + a*c + b)/(d*x^2 + c))
```

3.324.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1414 vs. $2(241) = 482$.

Time = 0.42 (sec) , antiderivative size = 1414, normalized size of antiderivative = 5.34

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}\,dx = \text{Too large to display}$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^7,x, algorithm="giac")
```

output
```
-1/48*(3*(8*a^2*b*c^2*d^3 + 12*a*b^2*c*d^3 + 5*b^3*d^3)*arctan(-(sqrt(a*d^
2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))/sqrt(-a*c^
2 - b*c))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*sqrt(-a*c^2 - b*c)) + (64*a^(11
/2)*c^8*d^2*abs(d) + 192*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 +
 b*d*x^2 + a*c^2 + b*c))*a^5*c^7*d^3 + 192*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x
^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(9/2)*c^6*d^2*abs(d) + 304*
a^(9/2)*b*c^7*d^2*abs(d) + 64*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*
x^2 + b*d*x^2 + a*c^2 + b*c))^3*a^4*c^5*d^3 + 744*(sqrt(a*d^2)*x^2 - sqrt(
a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^4*b*c^6*d^3 + 528*(sqr
t(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^
(7/2)*b*c^5*d^2*abs(d) + 576*a^(7/2)*b^2*c^6*d^2*abs(d) + 64*(sqrt(a*d^2)*
x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a^3*b*c^4*d
^3 + 1116*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^
2 + b*c))*a^3*b^2*c^5*d^3 + 480*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*
d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(5/2)*b^2*c^4*d^2*abs(d) + 544*a^(5/2)
*b^3*c^5*d^2*abs(d) + 24*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 +
 b*d*x^2 + a*c^2 + b*c))^5*a^2*b*c^2*d^3 - 96*(sqrt(a*d^2)*x^2 - sqrt(a*d^
2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a^2*b^2*c^3*d^3 + 801*(sqr
t(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*
b^3*c^4*d^3 + 144*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d...
```

3.324.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}\,dx = \int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x^7}\,dx$$

input
```
int((a + b/(c + d*x^2))^(1/2)/x^7,x)
```

output
```
int((a + b/(c + d*x^2))^(1/2)/x^7, x)
```

3.325 $\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx$

3.325.1 Optimal result

Integrand size = 21, antiderivative size = 368

$$\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx=-\frac{(2b^2+7abc-3a^2c^2)\,x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15a^2d^2}+\frac{(b-3ac)x(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15ad^2}+\frac{x^3(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5d}+\frac{\sqrt{c}(2b^2+7abc-3a^2c^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15a^2d^{5/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}-\frac{c^{3/2}(b-3ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{15ad^{5/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

output
```
-1/15*(-3*a^2*c^2+7*a*b*c+2*b^2)*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^2/d
^2+1/15*(-3*a*c+b)*x*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^2+1/5
*x^3*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d-1/15*c^(3/2)*(-3*a*c+b)
*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*
x^2/c)^(1/2),(b/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^(5/2
)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+1/15*(-3*a^2*c^2+7*a*b*c+2*b
^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1
+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/
2)/a^2/d^(5/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

3.325.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.06 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.80

$$\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx$$
$$=\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(a\sqrt{\frac{d}{c}}x(c+dx^2)\left(b^2-2ab(c-2dx^2)-3a^2(c^2-d^2x^4)\right)+i(2b^3+9ab^2c+4a^2bc^2-3a^3c^3)\sqrt{}\right.}{15a}$$

input
```
Integrate[x^4*Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*Sqrt[d/c]*x*(c + d*x^2)*(b^2 - 2
*a*b*(c - 2*d*x^2) - 3*a^2*(c^2 - d^2*x^4)) + I*(2*b^3 + 9*a*b^2*c + 4*a^2
*b*c^2 - 3*a^3*c^3)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c
]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] - (2*I)*b*(b^2 + 4*a*
b*c + 3*a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*E
llipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(15*a^2*c^2*(d/c)^(5/2
)*(b + a*(c + d*x^2)))
```

3.325.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2057, 2058, 380, 444, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx$$
$$\Big\downarrow \text{2057}$$
$$\int x^4\sqrt{\frac{ac+adx^2+b}{c+dx^2}}dx$$
$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\int \frac{x^4\sqrt{adx^2+b+ac}}{\sqrt{dx^2+c}}dx}{\sqrt{ac+adx^2+b}}$$

$\Big\downarrow$ 380

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5d}-\frac{\int \frac{x^2\left(3c(b+ac)-(b-3ac)dx^2\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{5d}\right)}{\sqrt{ac+adx^2+b}}$$

$\Big\downarrow$ 444

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5d}-\frac{-\frac{\int -\frac{d\left(\left(2b^2+7acb-3a^2c^2\right)dx^2+c(b-3ac)(b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3ad^2}-\frac{x(b-3ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}}{5d}\right)}{\sqrt{ac+adx^2+b}}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5d}-\frac{\frac{\int \frac{d\left(\left(2b^2+7acb-3a^2c^2\right)dx^2+c(b-3ac)(b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3ad^2}-\frac{x(b-3ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}}{5d}\right)}{\sqrt{ac+adx^2+b}}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5d}-\frac{\frac{\int \frac{\left(2b^2+7acb-3a^2c^2\right)dx^2+c(b-3ac)(b+ac)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3ad}-\frac{x(b-3ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}}{5d}\right)}{\sqrt{ac+adx^2+b}}$$

$\Big\downarrow$ 406

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5d}-\frac{\frac{d\left(-3a^2c^2+7abc+2b^2\right)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+c(b-3ac)(ac+b)\int \frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3ad}}{5d}\right.}{\sqrt{ac+adx^2+b}}$$

$\Big\downarrow$ 320

3.325. $\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5d}-\frac{\frac{d\left(-3a^2c^2+7abc+2b^2\right)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{c^{3/2}(b-3ac)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\ldots\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\ldots\right)}{(ac+b)\left(c+dx^2\ldots\right)}}}}{3ad}}{5d}\ldots\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 388

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5d}-\frac{\frac{d\left(-3a^2c^2+7abc+2b^2\right)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)+\frac{c^{3/2}(b-3ac)\sqrt{ac+adx^2+b}\,\text{E}\ldots}{\sqrt{d}\sqrt{c+dx^2}\ldots}}{3ad}}{5d}\ldots\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 313

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5d}-\frac{\frac{d\left(-3a^2c^2+7abc+2b^2\right)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}(b\ldots}{\ldots}}{3ad}}{5d}\ldots\right)}{\sqrt{ac+adx^2+b}}$$

input
```
Int[x^4*Sqrt[a + b/(c + d*x^2)],x]
```

3.325. $\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx$

output
```
(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*((x^3*Sqrt[c + d*x^
2]*Sqrt[b + a*c + a*d*x^2])/(5*d) - (-1/3*((b - 3*a*c)*x*Sqrt[c + d*x^2]*S
qrt[b + a*c + a*d*x^2])/(a*d) + ((2*b^2 + 7*a*b*c - 3*a^2*c^2)*d*((x*Sqrt[
b + a*c + a*d*x^2])/(a*d*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^
2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c
+ d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])) + (c^(3/2
)*(b - 3*a*c)*Sqrt[b + a*c + a*d*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]
], b/(b + a*c)])/(Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b
 + a*c)*(c + d*x^2))]))/(3*a*d))/(5*d)))/Sqrt[b + a*c + a*d*x^2]
```

3.325.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

3.325. $\int x^4 \sqrt{a+\frac{b}{c+dx^2}}\,dx$

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.325.4 Maple [A] (verified)

Time = 6.52 (sec) , antiderivative size = 665, normalized size of antiderivative = 1.81

method	result
default	$-\frac{\left(-3\sqrt{-\frac{ad}{ac+b}}\,a^2d^3x^7-3\sqrt{-\frac{ad}{ac+b}}\,a^2c\,d^2x^5-4\sqrt{-\frac{ad}{ac+b}}\,ab\,d^2x^5+3\sqrt{-\frac{ad}{ac+b}}\,a^2c^2d\,x^3-2\sqrt{-\frac{ad}{ac+b}}\,abcd\,x^3-3\sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\,\sqrt{\frac{d\,x^2\cdots}{c}}\cdots\right.}{\cdots}$
risch	$-\frac{x\left(-3ad\,x^2+3ac-b\right)\left(d\,x^2+c\right)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{15d^2a}+\frac{\left(\frac{3a^2c^3\sqrt{1+\frac{ad\,x^2}{ac+b}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{-1+\frac{2acd+bd}{dca}}\right)}{\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}}-\frac{b^2c\sqrt{1+\frac{ad\,x^2}{ac+b}}\,\sqrt{1+\frac{d\,x^2}{c}}\cdots}{\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a\,d^2x^4\cdots}}\cdots\right.}{\cdots}$

3.325. $\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx$

input
```
int(x^4*(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-3*(-a*d/(a*c+b))^(1/2)*a^2*d^3*x^7-3*(-a*d/(a*c+b))^(1/2)*a^2*c*d^
2*x^5-4*(-a*d/(a*c+b))^(1/2)*a*b*d^2*x^5+3*(-a*d/(a*c+b))^(1/2)*a^2*c^2*d*
x^3-2*(-a*d/(a*c+b))^(1/2)*a*b*c*d*x^3-3*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*(
(d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a
^2*c^3+3*(-a*d/(a*c+b))^(1/2)*a^2*c^3*x-(-a*d/(a*c+b))^(1/2)*b^2*d*x^3-9*(
(a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+
b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c^2+7*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*(
(d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a
*b*c^2+2*(-a*d/(a*c+b))^(1/2)*a*b*c^2*x-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((
d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b^
2*c+2*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*
d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b^2*c-(-a*d/(a*c+b))^(1/2)*b^2*c*x)*
(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2/(a*d^2*x^4+2*a*c*d*x^2+b*d
*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/a/((a*d*x^2+a*c+b)*(d*x^2+c))^(
1/2)
```

3.325.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.65

$$\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx =$$

$$-\frac{(3\,a^2c^3-7\,abc^2-2\,b^2c)\sqrt{a}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ac+b}{ac})-(3\,a^2c^3-7\,abc^2-2\,b^2c+(3\,a^2c^2+2\,abc}{}$$

input
```
integrate(x^4*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/15*((3*a^2*c^3 - 7*a*b*c^2 - 2*b^2*c)*sqrt(a)*x*sqrt(-c/d)*elliptic_e(a
rcsin(sqrt(-c/d)/x), (a*c + b)/(a*c)) - (3*a^2*c^3 - 7*a*b*c^2 - 2*b^2*c +
 (3*a^2*c^2 + 2*a*b*c - b^2)*d)*sqrt(a)*x*sqrt(-c/d)*elliptic_f(arcsin(sqr
t(-c/d)/x), (a*c + b)/(a*c)) - (3*a^2*d^3*x^6 + a*b*d^2*x^4 + 3*a^2*c^3 -
7*a*b*c^2 - 2*(3*a*b*c + b^2)*d*x^2 - 2*b^2*c)*sqrt((a*d*x^2 + a*c + b)/(d
*x^2 + c)))/(a^2*d^3*x)
```

3.325.6 Sympy [F]

$$\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int x^4\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,dx$$

input
```
integrate(x**4*(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(x**4*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

3.325.7 Maxima [F]

$$\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int\sqrt{a+\frac{b}{dx^2+c}}x^4\,dx$$

input
```
integrate(x^4*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x^2 + c))*x^4, x)
```

3.325.8 Giac [F]

$$\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int\sqrt{a+\frac{b}{dx^2+c}}x^4\,dx$$

input
```
integrate(x^4*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/(d*x^2 + c))*x^4, x)
```

3.325.9 Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int x^4\,\sqrt{a+\frac{b}{d\,x^2+c}}\,dx$$

input
```
int(x^4*(a + b/(c + d*x^2))^(1/2),x)
```

output
```
int(x^4*(a + b/(c + d*x^2))^(1/2), x)
```

3.326 $\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx$

3.326.1 Optimal result

Integrand size = 21, antiderivative size = 282

$$\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx=\frac{(b-ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3ad}+\frac{x(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3d}-\frac{\sqrt{c}(b-ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3ad^{3/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}-\frac{c^{3/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

output
```
1/3*(-a*c+b)*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d+1/3*x*(d*x^2+c)*((a*d
*x^2+a*c+b)/(d*x^2+c))^(1/2)/d-1/3*c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/
c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*
((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^(3/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^
2+c))^(1/2)-1/3*(-a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE
(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*((a*d*x^2+
a*c+b)/(d*x^2+c))^(1/2)/a/d^(3/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1
/2)
```

3.326.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.20 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.86

$$\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx$$
$$= \frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(a\sqrt{\frac{d}{c}}x(c+dx^2)\left(b+a(c+dx^2)\right)+i(-b^2+a^2c^2)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)|\right.\right.}{3ad\sqrt{\frac{d}{c}}\left(b+a\left(c+dx^2\right)\right)}$$

input
```
Integrate[x^2*Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*Sqrt[d/c]*x*(c + d*x^2)*(b + a*(
c + d*x^2)) + I*(-b^2 + a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[
1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] + I*b*(b
 + a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[
I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(3*a*d*Sqrt[d/c]*(b + a*(c + d*
x^2)))
```

3.326.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2057, 2058, 380, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx$$
$$\Big\downarrow \; 2057$$
$$\int x^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}dx$$
$$\Big\downarrow \; 2058$$
$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\int\frac{x^2\sqrt{adx^2+b+ac}}{\sqrt{dx^2+c}}dx}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 380

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3d}-\frac{\int\frac{c(b+ac)-(b-ac)dx^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3d}\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 406

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3d}-\frac{c(ac+b)\int\frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx-d(b-ac)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3d}\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 320

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3d}-\frac{\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-d(b-ac)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3d}\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 388

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3d}-\frac{\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-d(b-ac)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b\cdots}}{\left(dx^2+c\right)\cdots}}{ad}\cdots\right.}{3d}\cdots\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 313

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3d}-\frac{\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-d(b-ac)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2\cdots}}{ad^{3/2}\sqrt{c\cdots}}\cdots\right.}{3d}\cdots\right.}{\sqrt{ac+adx^2+b}}$$

3.326. $\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx$

input
```
Int[x^2*Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*((x*Sqrt[c + d*x^2]
*Sqrt[b + a*c + a*d*x^2])/(3*d) - (-((b - a*c)*d*((x*Sqrt[b + a*c + a*d*x^
2])/(a*d*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c
*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]))) + (c^(3/2)*Sqrt[b + a*c
+ a*d*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d]*S
qrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]))/(3*
d)))/Sqrt[b + a*c + a*d*x^2]
```

3.326.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.326.4 Maple [A] (verified)

Time = 5.92 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.44

method	result
default	$\frac{\left(\sqrt{-\frac{ad}{ac+b}}\, a\, d^2 x^5+2\sqrt{-\frac{ad}{ac+b}}\, acd\, x^3+\sqrt{-\frac{ad}{ac+b}}\, bd\, x^3-\sqrt{\frac{ad\, x^2+ac+b}{ac+b}}\, \sqrt{\frac{d\, x^2+c}{c}}\, E\left(x\sqrt{-\frac{ad}{ac+b}}, \sqrt{\frac{ac+b}{ac}}\right) a\, c^2+\sqrt{-\frac{ad}{ac+b}}\, a\, c^2 x-2\sqrt{\frac{ac}{}}\right.}{3d\sqrt{a\, d^2 x^4+2acd\, x^2+bd\, x^2+a\, c^2}}$
risch	$\frac{x\left(d\, x^2+c\right)\sqrt{\frac{ad\, x^2+ac+b}{d\, x^2+c}}}{3d}-\frac{\left(\frac{a\, c^2\sqrt{1+\frac{ad\, x^2}{ac+b}}\, \sqrt{1+\frac{d\, x^2}{c}}\, F\left(x\sqrt{-\frac{ad}{ac+b}}, \sqrt{-1+\frac{2acd+bd}{dca}}\right)}{\sqrt{-\frac{ad}{ac+b}}\, \sqrt{a\, d^2 x^4+2acd\, x^2+bd\, x^2+a\, c^2+bc}}+\frac{bc\sqrt{1+\frac{ad\, x^2}{ac+b}}\, \sqrt{1+\frac{d\, x^2}{c}}\, F\left(x\sqrt{-\frac{ad}{ac+b}}, \sqrt{-1+\frac{2acd}{dc}}\right)}{\sqrt{-\frac{ad}{ac+b}}\, \sqrt{a\, d^2 x^4+2acd\, x^2+bd\, x^2+a\, c^2+bc}}\right.}{}$

input
```
int(x^2*(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((-a*d/(a*c+b))^(1/2)*a*d^2*x^5+2*(-a*d/(a*c+b))^(1/2)*a*c*d*x^3+(-a*d
/(a*c+b))^(1/2)*b*d*x^3-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2
)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*c^2+(-a*d/(a*c+b
))^(1/2)*a*c^2*x-2*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*Ell
ipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*c+((a*d*x^2+a*c+b)/(a
*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)
/a/c)^(1/2))*b*c+(-a*d/(a*c+b))^(1/2)*b*c*x)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d
*x^2+c))^(1/2)/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*
c+b))^(1/2)/((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)
```

3.326.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.59

$$\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx$$
$$=\frac{(ac^2-bc)\sqrt{a}x\sqrt{-\frac{c}{d}}E\left(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ac+b}{ac}\right)-(ac^2-bc+(ac+b)d)\sqrt{a}x\sqrt{-\frac{c}{d}}F\left(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ac+b}{ac}\right)}{3\,ad^2x}$$

input
```
integrate(x^2*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*((a*c^2 - b*c)*sqrt(a)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), (
a*c + b)/(a*c)) - (a*c^2 - b*c + (a*c + b)*d)*sqrt(a)*x*sqrt(-c/d)*ellipti
c_f(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c)) + (a*d^2*x^4 + b*d*x^2 - a*c^2
+ b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a*d^2*x)
```

3.326.6 Sympy [F]

$$\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int x^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,dx$$

input
```
integrate(x**2*(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(x**2*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

3.326.7 Maxima [F]

$$\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx=\int\sqrt{a+\frac{b}{dx^2+c}}x^2\,dx$$

input
```
integrate(x^2*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x^2 + c))*x^2, x)
```

3.326.8 Giac [F]

$$\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx = \int \sqrt{a+\frac{b}{dx^2+c}}x^2\,dx$$

input
```
integrate(x^2*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/(d*x^2 + c))*x^2, x)
```

3.326.9 Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx = \int x^2\,\sqrt{a+\frac{b}{d\,x^2+c}}\,dx$$

input
```
int(x^2*(a + b/(c + d*x^2))^(1/2),x)
```

output
```
int(x^2*(a + b/(c + d*x^2))^(1/2), x)
```

3.327 $\int \sqrt{a+\frac{b}{c+dx^2}}\,dx$

3.327.1 Optimal result

Integrand size = 17, antiderivative size = 213

$$\int \sqrt{a+\frac{b}{c+dx^2}}\,dx = x\sqrt{\frac{b+ac+adx^2}{c+dx^2}} - \frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

output
```
x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2
)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)
*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x
^2+c))^(1/2)+(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c
^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*((a*d*x^2+a*c+b)/(d*x^
2+c))^(1/2)/d^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

3.327.2 Mathematica [A] (verified)

Time = 8.87 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.46

$$\int \sqrt{a+\frac{b}{c+dx^2}}\,dx=\frac{\sqrt{\frac{c+dx^2}{c}}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|\frac{ac}{b+ac}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{b+ac+adx^2}{b+ac}}}$$

input
```
Integrate[Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(Sqrt[(c + d*x^2)/c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticE[ArcSi
n[Sqrt[-(d/c)]*x], (a*c)/(b + a*c)])/(Sqrt[-(d/c)]*Sqrt[(b + a*c + a*d*x^2
)/(b + a*c)])
```

3.327.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.32, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2057, 2058, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+\frac{b}{c+dx^2}}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \sqrt{\frac{ac+adx^2+b}{c+dx^2}}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\int \frac{\sqrt{adx^2+b+ac}}{\sqrt{dx^2+c}}dx}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{324}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left((ac+b)\int \frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+ad\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(ad\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{\sqrt{c}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{388}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(ad\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{(dx^2+c)^{3/2}}dx}{ad}\right)+\frac{\sqrt{c}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{313}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(ad\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}\right)+\frac{\sqrt{c}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}\right.}{\sqrt{ac+adx^2+b}}$$

input
```
Int[Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*d*((x*Sqrt[b + a
*c + a*d*x^2])/(a*d*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x
^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])) + (Sqrt[c]*Sqr
t[b + a*c + a*d*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/
(Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^
2))])))/Sqrt[b + a*c + a*d*x^2]
```

3.327. $\int\sqrt{a+\frac{b}{c+dx^2}}\,dx$

3.327.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.327.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{\left(acE\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)+F\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)b\right)\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\,(d\,x^2+c)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{-\frac{ad}{ac+b}}\,\sqrt{(ad\,x^2+ac+b)(d\,x^2+c)}}$	199

input
```
int((a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a*c*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))+EllipticF(x*(-a
*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b)*((d*x^2+c)/c)^(1/2)*((a*d*x^2+a*
c+b)/(a*c+b))^(1/2)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4
+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/((a*d*x^2+a*c+b
)*(d*x^2+c))^(1/2)
```

3.327.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.69

$$\int \sqrt{a+\frac{b}{c+dx^2}}\,dx =$$

$$-\frac{a^{\frac{3}{2}}c^2x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ac+b}{ac}) - (ac^2+(ac+b)d)\sqrt{a}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ac+b}{ac}) - (acdx^2 +}{acdx}$$

input
```
integrate((a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
-(a^(3/2)*c^2*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c
)) - (a*c^2 + (a*c + b)*d)*sqrt(a)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/
d)/x), (a*c + b)/(a*c)) - (a*c*d*x^2 + a*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*
x^2 + c)))/(a*c*d*x)
```

3.327.6 Sympy [F]

$$\int \sqrt{a+\frac{b}{c+dx^2}}\,dx=\int \sqrt{a+\frac{b}{c+dx^2}}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b/(c + d*x**2)), x)
```

3.327.7 Maxima [F]

$$\int \sqrt{a+\frac{b}{c+dx^2}}\,dx=\int \sqrt{a+\frac{b}{dx^2+c}}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x^2 + c)), x)
```

3.327.8 Giac [F]

$$\int \sqrt{a+\frac{b}{c+dx^2}}\,dx=\int \sqrt{a+\frac{b}{dx^2+c}}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/(d*x^2 + c)), x)
```

3.327.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{b}{c+dx^2}}\,dx=\int \sqrt{a+\frac{b}{d\,x^2+c}}\,dx$$

input
```
int((a + b/(c + d*x^2))^(1/2),x)
```

output
```
int((a + b/(c + d*x^2))^(1/2), x)
```

3.328 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2} dx$

3.328.1 Optimal result

Integrand size = 21, antiderivative size = 265

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2} dx = \frac{dx\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c} - \frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx} - \frac{\sqrt{d}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{c}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{a\sqrt{c}\sqrt{d}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

output

```
d*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c-(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+
c))^(1/2)/c/x-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/
c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*d^(1/2)*((a*d*x^2+a*c+b)/(d*x
^2+c))^(1/2)/c^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+a*(1/(1+d
*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(
1/2),(b/(a*c+b))^(1/2))*c^(1/2)*d^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/
(a*c+b)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

3.328.2 Mathematica [A] (verified)

Time = 9.54 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\,dx = \sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(-\frac{1}{x}-\frac{dx}{c}+\frac{ad\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\left(\arcsin\left(\sqrt{-\frac{ad}{b+ac}}x\right)|1+\frac{b}{ac}\right)}{\sqrt{-\frac{ad}{b+ac}}\left(b+a\left(c+dx^2\right)\right)}\right)$$

input
```
Integrate[Sqrt[a + b/(c + d*x^2)]/x^2,x]
```

output
```
Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-x^(-1) - (d*x)/c + (a*d*Sqrt[(b +
a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[ArcSin[Sqrt[-((a*d
)/(b + a*c))]*x], 1 + b/(a*c)])/(Sqrt[-((a*d)/(b + a*c))]*(b + a*(c + d*x^
2))))
```

3.328.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2057, 2058, 377, 27, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\,dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^2}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\int \frac{\sqrt{adx^2+b+ac}}{x^2\sqrt{dx^2+c}}dx}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{377}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\int \frac{ad\sqrt{dx^2+c}}{\sqrt{adx^2+b+ac}}dx}{c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{ad\int \frac{\sqrt{dx^2+c}}{\sqrt{adx^2+b+ac}}dx}{c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{324}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{ad\left(c\int \frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+d\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{ad\left(d\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}(ac+b)\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{388}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{ad\left(d\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)+\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}(ac+b)\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{c}-\frac{\sqrt{c+dx^2}\sqrt{ac+\cdots}}{cx}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{313}$$

3.328. $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\,dx$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{ad\left(\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}(ac+b)\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}+d\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}\right)\right)}{c}\right.}{\sqrt{ac+adx^2+b}}$$

input
```
Int[Sqrt[a + b/(c + d*x^2)]/x^2,x]
```

output
```
(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-((Sqrt[c + d*x^2]
*Sqrt[b + a*c + a*d*x^2])/(c*x)) + (a*d*(d*((x*Sqrt[b + a*c + a*d*x^2])/(a
*d*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcTan[(S
qrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(b +
a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])) + (c^(3/2)*Sqrt[b + a*c + a*d*x
^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/((b + a*c)*Sqrt[d
]*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])))
/c))/Sqrt[b + a*c + a*d*x^2]
```

3.328.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

3.328. $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\,dx$

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.328.4 Maple [A] (verified)

Time = 6.06 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.03

method	result
default	$-\frac{\left(\sqrt{-\frac{ad}{ac+b}}\, a\, d^{2} x^{4}-adc\sqrt{\frac{ad\,x^{2}+ac+b}{ac+b}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\, xE\left(x\sqrt{-\frac{ad}{ac+b}}, \sqrt{\frac{ac+b}{ac}}\right)+2\sqrt{-\frac{ad}{ac+b}}\, acd\,x^{2}+\sqrt{-\frac{ad}{ac+b}}\, bd\,x^{2}+\sqrt{-\frac{ad}{ac+b}}\, a\,c^{2}+\sqrt{-\cdots}\right.}{\sqrt{a\,d^{2} x^{4}+2acd\,x^{2}+bd\,x^{2}+a\,c^{2}+bc}\,\sqrt{-\frac{ad}{ac+b}}\, xc\sqrt{\left(ad\,x^{2}+ac+b\right)\left(d\,x^{2}+c\right)}}$
risch	$-\frac{\left(d\,x^{2}+c\right)\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}}{cx}+\frac{ad\left(\frac{c\sqrt{1+\frac{ad\,x^{2}}{ac+b}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\, F\left(x\sqrt{-\frac{ad}{ac+b}}, \sqrt{-1+\frac{2acd+bd}{dca}}\right)}{\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a\,d^{2} x^{4}+2acd\,x^{2}+bd\,x^{2}+a\,c^{2}+bc}}-\frac{2d\left(a\,c^{2}+bc\right)\sqrt{1+\frac{ad\,x^{2}}{ac+b}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{-\frac{ad}{ac+b}}\cdots\right.\right.}{\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a\,d^{2}x^{4}+2acd\cdots}}\right.}{c(ad\,x^{2}+ac+\cdots}$

3.328. $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\, dx$

input
```
int((a+b/(d*x^2+c))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-((-a*d/(a*c+b))^(1/2)*a*d^2*x^4-a*d*c*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d
*x^2+c)/c)^(1/2)*x*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))+2
*(-a*d/(a*c+b))^(1/2)*a*c*d*x^2+(-a*d/(a*c+b))^(1/2)*b*d*x^2+(-a*d/(a*c+b)
)^(1/2)*a*c^2+(-a*d/(a*c+b))^(1/2)*b*c)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+
c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(
1/2)/x/c/((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)
```

3.328.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\,dx$$
$$= \frac{a\sqrt{-\frac{ad}{ac+b}}d^2x\sqrt{\frac{ac^2+bc}{d^2}}E(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right) \mid \frac{ac+b}{ac}) - (ad^2+(ac+b)d)\sqrt{-\frac{ad}{ac+b}}x\sqrt{\frac{ac^2+bc}{d^2}}F(\arcsin\left(\sqrt{-\frac{}{a}}\right.}{(ac^2+bc)x}$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
(a*sqrt(-a*d/(a*c + b))*d^2*x*sqrt((a*c^2 + b*c)/d^2)*elliptic_e(arcsin(sq
rt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - (a*d^2 + (a*c + b)*d)*sqrt(-a*d/
(a*c + b))*x*sqrt((a*c^2 + b*c)/d^2)*elliptic_f(arcsin(sqrt(-a*d/(a*c + b)
)*x), (a*c + b)/(a*c)) - ((a*c + b)*d*x^2 + a*c^2 + b*c)*sqrt((a*d*x^2 + a
*c + b)/(d*x^2 + c)))/((a*c^2 + b*c)*x)
```

3.328.6 Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\,dx = \int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^2}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(1/2)/x**2,x)
```

output
```
Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**2, x)
```

3.328.7 Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\,dx = \int \frac{\sqrt{a+\frac{b}{dx^2+c}}}{x^2}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x^2 + c))/x^2, x)
```

3.328.8 Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\,dx = \int \frac{\sqrt{a+\frac{b}{dx^2+c}}}{x^2}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/(d*x^2 + c))/x^2, x)
```

3.328.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\,dx = \int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x^2}\,dx$$

input
```
int((a + b/(c + d*x^2))^(1/2)/x^2,x)
```

output
```
int((a + b/(c + d*x^2))^(1/2)/x^2, x)
```

3.329 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4} dx$

3.329.1 Optimal result

Integrand size = 21, antiderivative size = 362

$$\begin{aligned}\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4} dx = &-\frac{(2b+ac)d^2x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2(b+ac)} - \frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3cx^3} \\ &+ \frac{(2b+ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2(b+ac)x} \\ &+ \frac{(2b+ac)d^{3/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3c^{3/2}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} \\ &- \frac{ad^{3/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3\sqrt{c}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\end{aligned}$$

output
```
-1/3*(a*c+2*b)*d^2*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^2/(a*c+b)-1/3*(d*
x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c/x^3+1/3*(a*c+2*b)*d*(d*x^2+c)*(
(a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^2/(a*c+b)/x+1/3*(a*c+2*b)*d^(3/2)*(1/(1
+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)
^(1/2),(b/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^(3/2)/(a*c+b
)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)-1/3*a*d^(3/2)*(1/(1+d*x^2/c)
)^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b
/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)/c^(1/2)/(c*(a*d
*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

3.329.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.74 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx =$$

$$-\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{d}{c}}(c+dx^2)\left(b^2(c-2dx^2)+a^2c(c^2-d^2x^4)+2ab(c^2-cdx^2-d^2x^4)\right)-i(2b^2+3abc+a\right.}{3c}$$

input
```
Integrate[Sqrt[a + b/(c + d*x^2)]/x^4,x]
```

output
```
-1/3*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[d/c]*(c + d*x^2)*(b^2*(c
 - 2*d*x^2) + a^2*c*(c^2 - d^2*x^4) + 2*a*b*(c^2 - c*d*x^2 - d^2*x^4)) - I
*(2*b^2 + 3*a*b*c + a^2*c^2)*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*S
qrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] + (2
*I)*b*(b + a*c)*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^
2)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(c^2*(b + a*c)*
Sqrt[d/c]*x^3*(b + a*(c + d*x^2)))
```

3.329.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {2057, 2058, 377, 25, 27, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^4}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\int \frac{\sqrt{adx^2+b+ac}}{x^4\sqrt{dx^2+c}}dx}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{377}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\int -\frac{d\left(adx^2+2b+ac\right)}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{\int \frac{d\left(adx^2+2b+ac\right)}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\int \frac{adx^2+2b+ac}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{445}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(-\frac{\int -\frac{ad\left((2b+ac)dx^2+c(b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{(ac+2b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{3c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(\frac{\int \frac{ad\left((2b+ac)dx^2+c(b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{(ac+2b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{3c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{27}$$

3.329. $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(\frac{ad\int\frac{(2b+ac)dx^2+c(b+ac)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{(ac+2b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{3c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{ 406}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(\frac{ad\left(c(ac+b)\int\frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+d(ac+2b)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{c(ac+b)}-\frac{(ac+2b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{3c}-\ldots\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(\frac{ad\left(d(ac+2b)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{c(ac+b)}-\frac{(ac+2b)\sqrt{c+dx^2}\sqrt{ac-\ldots}}{cx(ac+b)}\ldots\right)}{3c}\ldots\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{ 388}$$

3.329. $\int\frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(\frac{ad\left(d(ac+2b)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{(dx^2+c)^{3/2}}dx}{ad}\right)+\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}\right)}{c(ac+b)}-\frac{(ac+2b}{}\right)}{3c}\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 313

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(\frac{ad\left(\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}+d(ac+2b)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.\right.}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+}{(ac+b)(c+dx}}}\right.\right.}{c(ac+b)}\right)}{3c}\right.}{\sqrt{ac+adx^2+b}}$$

input

```
Int[Sqrt[a + b/(c + d*x^2)]/x^4,x]
```

3.329. $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx$

output
```
(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-1/3*(Sqrt[c + d*x
^2]*Sqrt[b + a*c + a*d*x^2])/(c*x^3) - (d*(-(((2*b + a*c)*Sqrt[c + d*x^2]*
Sqrt[b + a*c + a*d*x^2])/(c*(b + a*c)*x)) + (a*d*((2*b + a*c)*d*((x*Sqrt[b
 + a*c + a*d*x^2])/(a*d*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2
]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c +
 d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])) + (c^(3/2)
*Sqrt[b + a*c + a*d*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c
)])/(Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c +
d*x^2))])))/(c*(b + a*c))))/(3*c)))/Sqrt[b + a*c + a*d*x^2]
```

3.329.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

3.329. $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4} \, dx$

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.329.4 Maple [A] (verified)

Time = 7.00 (sec) , antiderivative size = 569, normalized size of antiderivative = 1.57

method	result
risch	$-\frac{\left(d\,x^{2}+c\right)\left(-acd\,x^{2}-2bd\,x^{2}+a\,c^{2}+bc\right)\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}}{3c^{2}x^{3}\left(ac+b\right)}-\frac{a\,d^{2}\left(\frac{a\,c^{2}\sqrt{1+\frac{ad\,x^{2}}{ac+b}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{-1+\frac{2acd+bd}{dca}}\right)}{\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a\,d^{2}x^{4}+2acd\,x^{2}+bd\,x^{2}+a\,c^{2}+bc}}+\frac{bc\sqrt{1+\frac{ad\,x^{2}}{ac+b}}}{\sqrt{-\frac{ad}{ac+b}}}\right.}{}$
default	$-\frac{\left(-\sqrt{-\frac{ad}{ac+b}}\,a^{2}c\,d^{3}x^{6}-2\sqrt{-\frac{ad}{ac+b}}\,ab\,d^{3}x^{6}+\sqrt{\frac{ad\,x^{2}+ac+b}{ac+b}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,E\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)a^{2}c^{2}d^{2}x^{3}-\sqrt{-\frac{ad}{ac+b}}\,a^{2}c^{2}d^{2}x^{4}-\sqrt{\frac{ad}{}}\right.}{}$

3.329. $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx$

input
```
int((a+b/(d*x^2+c))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(d*x^2+c)*(-a*c*d*x^2-2*b*d*x^2+a*c^2+b*c)/c^2/x^3/(a*c+b)*((a*d*x^2+
a*c+b)/(d*x^2+c))^(1/2)-1/3*a*d^2/c^2/(a*c+b)*(a*c^2/(-a*d/(a*c+b))^(1/2)*
(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x
^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c
/a)^(1/2))+b*c/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2
)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/
(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-2*(a*c*d+2*b*d)*(a*c^2+b*c)
/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2
*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(2*a*c*d+2*b*d)*(EllipticF(x*(-a
*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-EllipticE(x*(-a*d/(a*c+b
))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/
2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(a*d*x^2+a*c+b)
```

3.329.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx =$$

$$-\frac{(a^2c+2\,ab)\sqrt{-\frac{ad}{ac+b}}d^3x^3\sqrt{\frac{ac^2+bc}{d^2}}E(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right) \mid \frac{ac+b}{ac}) - ((a^2c+2\,ab)d^3 + (a^2c^2+2\,abc+b^2)d}{}$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-1/3*((a^2*c + 2*a*b)*sqrt(-a*d/(a*c + b))*d^3*x^3*sqrt((a*c^2 + b*c)/d^2)
*elliptic_e(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - ((a^2*c + 2
*a*b)*d^3 + (a^2*c^2 + 2*a*b*c + b^2)*d^2)*sqrt(-a*d/(a*c + b))*x^3*sqrt((
a*c^2 + b*c)/d^2)*elliptic_f(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*
c)) - ((a^2*c^2 + 3*a*b*c + 2*b^2)*d^2*x^4 - a^2*c^4 - 2*a*b*c^3 - b^2*c^2
 + (a*b*c^2 + b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^2*c
^4 + 2*a*b*c^3 + b^2*c^2)*x^3)
```

3.329.6 Sympy [F]

$$\int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x^4} \, dx = \int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^4} \, dx$$

input
```
integrate((a+b/(d*x**2+c))**(1/2)/x**4,x)
```

output
```
Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**4, x)
```

3.329.7 Maxima [F]

$$\int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x^4} \, dx = \int \frac{\sqrt{a + \frac{b}{dx^2+c}}}{x^4} \, dx$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x^2 + c))/x^4, x)
```

3.329.8 Giac [F]

$$\int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x^4} \, dx = \int \frac{\sqrt{a + \frac{b}{dx^2+c}}}{x^4} \, dx$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/(d*x^2 + c))/x^4, x)
```

3.329.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx=\int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x^4}\,dx$$

input
```
int((a + b/(c + d*x^2))^(1/2)/x^4,x)
```

output
```
int((a + b/(c + d*x^2))^(1/2)/x^4, x)
```

3.330 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx$

3.330.1 Optimal result

Integrand size = 21, antiderivative size = 466

$$\begin{aligned}\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx =& \frac{(8b^2+13abc+3a^2c^2)\,d^3x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15c^3(b+ac)^2} \\ &- \frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5cx^5} + \frac{(4b+3ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15c^2(b+ac)x^3} \\ &- \frac{(8b^2+13abc+3a^2c^2)\,d^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15c^3(b+ac)^2x} \\ &- \frac{(8b^2+13abc+3a^2c^2)\,d^{5/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15c^{5/2}(b+ac)^2\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} \\ &+ \frac{a(4b+3ac)d^{5/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{15c^{3/2}(b+ac)^2\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\end{aligned}$$

output
```
1/15*(3*a^2*c^2+13*a*b*c+8*b^2)*d^3*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^
3/(a*c+b)^2-1/5*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c/x^5+1/15*(3*
a*c+4*b)*d*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^2/(a*c+b)/x^3-1/1
5*(3*a^2*c^2+13*a*b*c+8*b^2)*d^2*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/
2)/c^3/(a*c+b)^2/x-1/15*(3*a^2*c^2+13*a*b*c+8*b^2)*d^(5/2)*(1/(1+d*x^2/c))
^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/
(a*c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^(5/2)/(a*c+b)^2/(c*(a*
d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+1/15*a*(3*a*c+4*b)*d^(5/2)*(1/(1+d*x
^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/
2),(b/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^(3/2)/(a*c+b)^2/
(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

3.330.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.32 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx =$$

$$-\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{d}{c}}(c+dx^2)\left(b^3(3c^2-4cdx^2+8d^2x^4)+3a^3c^2(c^3+d^3x^6)+ab^2(9c^3-8c^2dx^2+17cd^2x^4\right.\right.}{}$$

input
```
Integrate[Sqrt[a + b/(c + d*x^2)]/x^6,x]
```

output
```
-1/15*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[d/c]*(c + d*x^2)*(b^3*(
3*c^2 - 4*c*d*x^2 + 8*d^2*x^4) + 3*a^3*c^2*(c^3 + d^3*x^6) + a*b^2*(9*c^3
- 8*c^2*d*x^2 + 17*c*d^2*x^4 + 8*d^3*x^6) + a^2*b*c*(9*c^3 - 4*c^2*d*x^2 +
 9*c*d^2*x^4 + 13*d^3*x^6)) + I*(8*b^3 + 21*a*b^2*c + 16*a^2*b*c^2 + 3*a^3
*c^3)*d^3*x^5*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*Elli
pticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] - I*b*(8*b^2 + 17*a*b*c + 9
*a^2*c^2)*d^3*x^5*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*
EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(c^3*(b + a*c)^2*Sqrt
[d/c]*x^5*(b + a*(c + d*x^2)))
```

3.330.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {2057, 2058, 377, 25, 27, 445, 27, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6} \, dx$$

$$\Big\downarrow \ 2057$$

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^6}dx$$

$$\Big\downarrow \ 2058$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\int \frac{\sqrt{adx^2+b+ac}}{x^6\sqrt{dx^2+c}}dx}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \ 377$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\int -\frac{d\left(3adx^2+4b+3ac\right)}{x^4\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{5c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5cx^5}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{\int \frac{d\left(3adx^2+4b+3ac\right)}{x^4\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{5c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5cx^5}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\int \frac{3adx^2+4b+3ac}{x^4\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{5c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5cx^5}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \ 445$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(-\frac{\int \frac{d\left(8b^2+13acb+3a^2c^2+a(4b+3ac)dx^2\right)}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3c(ac+b)}-\frac{(3ac+4b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3(ac+b)}\right)}{5c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5cx^5}\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 27

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(-\frac{d\int \frac{8b^2+13acb+3a^2c^2+a(4b+3ac)dx^2}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3c(ac+b)}-\frac{(3ac+4b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3(ac+b)}\right)}{5c}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5cx^5}\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 445

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(-\frac{d\left(-\frac{\int -\frac{ad\left(\left(8b^2+13acb+3a^2c^2\right)dx^2+c(b+ac)(4b+3ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{\left(3a^2c^2+13abc+8b^2\right)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{3c(ac+b)}-\frac{(3ac+4b}{}\right.}{5c}\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 25

3.330. $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(-\frac{d\left(\frac{\int \frac{ad\left(\left(8b^2+13acb+3a^2c^2\right)dx^2+c(b+ac)(4b+3ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{\left(3a^2c^2+13abc+8b^2\right)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{3c(ac+b)}-\frac{(3ac+4b)\sqrt{c\cdots}}{3c\cdots}\right)}{5c}\cdots\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 27

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(-\frac{d\left(\frac{ad\int \frac{\left(8b^2+13acb+3a^2c^2\right)dx^2+c(b+ac)(4b+3ac)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{\left(3a^2c^2+13abc+8b^2\right)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{3c(ac+b)}-\frac{(3ac+4b)\sqrt{c+\cdots}}{3cx^3\cdots}\right)}{5c}\cdots\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 406

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(-\frac{d\left(\frac{ad\left(d\left(3a^2c^2+13abc+8b^2\right)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+c(ac+b)(3ac+4b)\int \frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{c(ac+b)}-\frac{\left(3a^2c^2+13a\cdots\right.}{\cdots}\right)}{3c(ac+b)}\cdots\right)}{5c}\cdots\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 320

3.330. $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(d-\frac{d\left(ad\left(d\left(3a^2c^2+13abc+8b^2\right)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{c^{3/2}(3ac+4b)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right.}{c(ac+b)}}{3c(ac+b)}}{5c}}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 388

3.330. $\int\frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(d-\frac{d\left(\frac{ad\left(d\left(3a^2c^2+13abc+8b^2\right)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)+\frac{c^{3/2}(3ac+4b)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\cdots\right.\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}\cdots}\cdots\right.}{c(ac+b)}\cdots\right.}{3c(ac+b)\cdots}\cdots\right.}{5c\cdots}\cdots\right.}{\sqrt{ac+adx^2+b}\cdots}$$

$\downarrow$ 313

3.330. $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{d\left(-\frac{d\left(\frac{d\left(ad\left(d\left(3a^2c^2+13abc+8b^2\right)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}(3ac+4b)\sqrt{ac+adx\cdots}}{\sqrt{d}\sqrt{c+\cdots}}\cdots}{c(ac+b)}\cdots\right)}{3c(ac+b)}\cdots\right)}{5\cdots}\cdots\right)}{\sqrt{ac+ad\cdots}}$$

input `Int[Sqrt[a + b/(c + d*x^2)]/x^6,x]`

3.330. $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx$

output
```
(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-1/5*(Sqrt[c + d*x
^2]*Sqrt[b + a*c + a*d*x^2])/(c*x^5) - (d*(-1/3*((4*b + 3*a*c)*Sqrt[c + d*
x^2]*Sqrt[b + a*c + a*d*x^2])/(c*(b + a*c)*x^3) - (d*(-(((8*b^2 + 13*a*b*c
 + 3*a^2*c^2)*Sqrt[c + d*x^2]*Sqrt[b + a*c + a*d*x^2])/(c*(b + a*c)*x)) +
(a*d*((8*b^2 + 13*a*b*c + 3*a^2*c^2)*d*((x*Sqrt[b + a*c + a*d*x^2])/(a*d*S
qrt[c + d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcTan[(Sqrt[
d]*x)/Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c
+ a*d*x^2))/((b + a*c)*(c + d*x^2))])) + (c^(3/2)*(4*b + 3*a*c)*Sqrt[b + a
*c + a*d*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d
]*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])))
/(c*(b + a*c))))/(3*c*(b + a*c))))/(5*c)))/Sqrt[b + a*c + a*d*x^2]
```

3.330.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

3.330. $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6} dx$

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.330.4 Maple [A] (verified)

Time = 7.87 (sec) , antiderivative size = 778, normalized size of antiderivative = 1.67

method	result
risch	$-\frac{\left(d\,x^{2}+c\right)\left(3a^{2}c^{2}d^{2}x^{4}+13ac\,d^{2}b\,x^{4}-3a^{2}c^{3}d\,x^{2}+8b^{2}d^{2}x^{4}-7ab\,c^{2}d\,x^{2}+3a^{2}c^{4}-4b^{2}cd\,x^{2}+6ab\,c^{3}+3b^{2}c^{2}\right)\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}}{15c^{3}x^{5}\left(ac+b\right)^{2}}+\frac{a\,d^{3}\left(\frac{3a^{2}\cdots}{\cdots}\right.}{\cdots}$
default	$-\frac{\left(3\sqrt{-\frac{ad}{ac+b}}\,a^{3}c^{2}d^{4}x^{8}+13\sqrt{-\frac{ad}{ac+b}}\,a^{2}bc\,d^{4}x^{8}-3\sqrt{\frac{ad\,x^{2}+ac+b}{ac+b}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,E\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)a^{3}c^{3}d^{3}x^{5}+3\sqrt{-\frac{ad}{ac+b}}\,a^{3}c^{3}d^{3}x^{6}-\cdots\right.}{\cdots}$

3.330. $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx$

input
```
int((a+b/(d*x^2+c))^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(d*x^2+c)*(3*a^2*c^2*d^2*x^4+13*a*b*c*d^2*x^4-3*a^2*c^3*d*x^2+8*b^2*
d^2*x^4-7*a*b*c^2*d*x^2+3*a^2*c^4-4*b^2*c*d*x^2+6*a*b*c^3+3*b^2*c^2)/c^3/x
^5/(a*c+b)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/15*a*d^3/(a*c+b)^2/c^3*(3
*a^2*c^3/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2
)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b
))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))+4*b^2*c/(-a*d/(a*c+b))^(1/2)*(1+a
*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a
*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^
(1/2))+7*a*b*c^2/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x
^2)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*
d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-2*(3*a^2*c^2*d+13*a*b*c*d
+8*b^2*d)*(a*c^2+b*c)/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/
c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(2*a*c*d+2*
b*d)*(EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-Ell
ipticE(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))))*((a*d*x^2+
a*c+b)/(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(a*d*x^2+a*c+b)
```

3.330.5 Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx$$
$$= \frac{(3\,a^3c^2+13\,a^2bc+8\,ab^2)\sqrt{-\frac{ad}{ac+b}}d^4x^5\sqrt{\frac{ac^2+bc}{d^2}}E(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right) \mid \frac{ac+b}{ac}) - ((3\,a^3c^2+13\,a^2bc+8\,ab^2)c}{}$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^6,x, algorithm="fricas")
```

output
```
1/15*((3*a^3*c^2 + 13*a^2*b*c + 8*a*b^2)*sqrt(-a*d/(a*c + b))*d^4*x^5*sqrt
((a*c^2 + b*c)/d^2)*elliptic_e(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(
a*c)) - ((3*a^3*c^2 + 13*a^2*b*c + 8*a*b^2)*d^4 + (3*a^3*c^3 + 10*a^2*b*c^
2 + 11*a*b^2*c + 4*b^3)*d^3)*sqrt(-a*d/(a*c + b))*x^5*sqrt((a*c^2 + b*c)/d
^2)*elliptic_f(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - ((3*a^3*
c^3 + 16*a^2*b*c^2 + 21*a*b^2*c + 8*b^3)*d^3*x^6 + 3*a^3*c^6 + 9*a^2*b*c^5
 + 9*a*b^2*c^4 + 2*(3*a^2*b*c^3 + 5*a*b^2*c^2 + 2*b^3*c)*d^2*x^4 + 3*b^3*c
^3 - (a^2*b*c^4 + 2*a*b^2*c^3 + b^3*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(
d*x^2 + c)))/((a^3*c^6 + 3*a^2*b*c^5 + 3*a*b^2*c^4 + b^3*c^3)*x^5)
```

3.330.6 Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx=\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^6}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(1/2)/x**6,x)
```

output
```
Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**6, x)
```

3.330.7 Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx=\int \frac{\sqrt{a+\frac{b}{dx^2+c}}}{x^6}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^6,x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x^2 + c))/x^6, x)
```

3.330.8 Giac [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx = \int \frac{\sqrt{a+\frac{b}{dx^2+c}}}{x^6}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(1/2)/x^6,x, algorithm="giac")
```

output
```
integrate(sqrt(a + b/(d*x^2 + c))/x^6, x)
```

3.330.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx = \int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x^6}\,dx$$

input
```
int((a + b/(c + d*x^2))^(1/2)/x^6,x)
```

output
```
int((a + b/(c + d*x^2))^(1/2)/x^6, x)
```

3.331 $\int x^5 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

3.331.1 Optimal result

Integrand size = 21, antiderivative size = 249

$$\int x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = -\frac{bc^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d^3} - \frac{(5b^2+60abc-24a^2c^2)(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{48ad^3} - \frac{(b+12ac)(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{24d^3} + \frac{(c+dx^2)^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{5/2}}{6ad^3} - \frac{b(b^2+12abc-24a^2c^2)\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{3/2}d^3}$$

output
```
1/6*(d*x^2+c)^3*((a*d*x^2+a*c+b)/(d*x^2+c))^(5/2)/a/d^3-1/16*b*(-24*a^2*c^
2+12*a*b*c+b^2)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/a^(3/2)
/d^3-b*c^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^3-1/48*(-24*a^2*c^2+60*a*b*
c+5*b^2)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^3-1/24*(12*a*c+b)
*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^3
```

3.331.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.60

$$\int x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx = \frac{\sqrt{a}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}(3b^2(c+dx^2)-2ab(47c^2+16cdx^2-7d^2x^4)+8a^2(c^3+d^3x^6))-3b(b^2+}{48a^{3/2}d^3}$$

input
```
Integrate[x^5*(a + b/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[a]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(3*b^2*(c + d*x^2) - 2*a*b*
(47*c^2 + 16*c*d*x^2 - 7*d^2*x^4) + 8*a^2*(c^3 + d^3*x^6)) - 3*b*(b^2 + 12
*a*b*c - 24*a^2*c^2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]
])/(48*a^(3/2)*d^3)
```

3.331.3 Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2057, 2053, 2052, 27, 366, 360, 1471, 27, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int x^5\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{ 2053}$$

$$\frac{1}{2}\int x^4\left(\frac{adx^2+b+ac}{dx^2+c}\right)^{3/2}dx^2$$

$$\Big\downarrow \text{ 2052}$$

$$-bd\int \frac{x^8\left(-cx^4+b+ac\right)^2}{d^4\left(a-x^4\right)^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$\Big\downarrow$ 27

$$-\frac{b\int \frac{x^8\left(-cx^4+b+ac\right)^2}{\left(a-x^4\right)^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{d^3}$$

$\Big\downarrow$ 366

$$-\frac{b\left(\frac{b^2x^{10}}{6a\left(a-x^4\right)^3}-\frac{\int \frac{x^8\left(6ac^2x^4+5b^2-6(b+ac)^2\right)}{\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{6a}\right)}{d^3}$$

$\Big\downarrow$ 360

$$-\frac{b\left(\frac{b^2x^{10}}{6a\left(a-x^4\right)^3}-\frac{\frac{1}{4}\int \frac{-24ac^2x^8+4b(b+12ac)x^4+ab(b+12ac)}{\left(a-x^4\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}-\frac{ab(12ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4\left(a-x^4\right)^2}}{6a}\right)}{d^3}$$

$\Big\downarrow$ 1471

$$-\frac{b\left(\frac{b^2x^{10}}{6a\left(a-x^4\right)^3}-\frac{\frac{1}{4}\left(\frac{\left(-24a^2c^2+60abc+5b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2\left(a-x^4\right)}-\frac{\int \frac{3a\left(-16ac^2x^4+b^2-8a^2c^2+12abc\right)}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2a}\right)-\frac{ab(12ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4\left(a-x^4\right)^2}}{6a}\right)}{d^3}$$

$\Big\downarrow$ 27

$$-\frac{b\left(\frac{b^2x^{10}}{6a\left(a-x^4\right)^3}-\frac{\frac{1}{4}\left(\frac{\left(-24a^2c^2+60abc+5b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2\left(a-x^4\right)}-\frac{3}{2}\int \frac{-16ac^2x^4+b^2-8a^2c^2+12abc}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}\right)-\frac{ab(12ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4\left(a-x^4\right)^2}}{6a}\right)}{d^3}$$

$\Big\downarrow$ 299

3.331. $\int x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx$

$$-\frac{b\left(\frac{b^2x^{10}}{6a(a-x^4)^3}-\frac{\frac{1}{4}\left(\frac{(-24a^2c^2+60abc+5b^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(a-x^4)}-\frac{3}{2}\left((-24a^2c^2+12abc+b^2)\int\frac{1}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}+16ac^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\right)\right)-\frac{ab(12ac-}{4}}{6a}\right.}{d^3}$$

$\downarrow$ 219

$$-\frac{b\left(\frac{b^2x^{10}}{6a(a-x^4)^3}-\frac{\frac{1}{4}\left(\frac{(-24a^2c^2+60abc+5b^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(a-x^4)}-\frac{3}{2}\left(\frac{(-24a^2c^2+12abc+b^2)\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}}+16ac^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\right)\right)-\frac{ab(1}{}}{6a}\right.}{d^3}$$

input
```
Int[x^5*(a + b/(c + d*x^2))^(3/2),x]
```

output
```
-((b*((b^2*x^10)/(6*a*(a - x^4)^3) - (-1/4*(a*b*(b + 12*a*c)*Sqrt[(b + a*c
 + a*d*x^2)/(c + d*x^2)])/(a - x^4)^2 + (((5*b^2 + 60*a*b*c - 24*a^2*c^2)*
Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(2*(a - x^4)) - (3*(16*a*c^2*Sqrt[(
b + a*c + a*d*x^2)/(c + d*x^2)] + ((b^2 + 12*a*b*c - 24*a^2*c^2)*ArcTanh[S
qrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/Sqrt[a]))/2)/4)/(6*a)))/d^3
)
```

3.331.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.331. $\int x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx$

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

3.331. $\int x^5 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

3.331.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.18

method	result
risch	$\frac{\left(8a^2d^2x^4-8a^2cd\,x^2+14abd\,x^2+8a^2c^2-46abc+3b^2\right)\left(d\,x^2+c\right)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{48a\,d^3}+\frac{b\left(-\frac{16a\,c^2\left(ad\,x^2+ac+b\right)}{d\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}}+\frac{\left(24a^2c^2-12a\right.}{}\right.}{}$
default	Expression too large to display

input
```
int(x^5*(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/a/d^3*(8*a^2*d^2*x^4-8*a^2*c*d*x^2+14*a*b*d*x^2+8*a^2*c^2-46*a*b*c+3*
b^2)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/16*b/a/d^2*(-16*a*c^2*(
a*d*x^2+a*c+b)/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+1/2*(24*a
^2*c^2-12*a*b*c-b^2)*ln((a*c*d+1/2*b*d+a*d^2*x^2)/(a*d^2)^(1/2)+(a*c^2+b*c
+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/(a*d^2)^(1/2))*((a*d*x^2+a*c+b)/(d*x^
2+c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(a*d*x^2+a*c+b)
```

3.331.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.71

$$\int x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\left[\frac{3\,(24\,a^2bc^2-12\,ab^2c-b^3)\sqrt{a}\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2+\right.}{}\right.$$

$$-\frac{3\,(24\,a^2bc^2-12\,ab^2c-b^3)\sqrt{-a}\arctan\left(\frac{(2\,adx^2+2\,ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2dx^2+a^2c+ab)}\right)-2\,(8\,a^3d^3x^6+14\,a^2bd^2x^4+8\,a^3c^3}{96\,a^2d^3}$$

input
```
integrate(x^5*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/192*(3*(24*a^2*b*c^2 - 12*a*b^2*c - b^3)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*
a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*
c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
)) + 4*(8*a^3*d^3*x^6 + 14*a^2*b*d^2*x^4 + 8*a^3*c^3 - 94*a^2*b*c^2 + 3*a*
b^2*c - (32*a^2*b*c - 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
))/(a^2*d^3), -1/96*(3*(24*a^2*b*c^2 - 12*a*b^2*c - b^3)*sqrt(-a)*arctan(1
/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/
(a^2*d*x^2 + a^2*c + a*b)) - 2*(8*a^3*d^3*x^6 + 14*a^2*b*d^2*x^4 + 8*a^3*c
^3 - 94*a^2*b*c^2 + 3*a*b^2*c - (32*a^2*b*c - 3*a*b^2)*d*x^2)*sqrt((a*d*x^
2 + a*c + b)/(d*x^2 + c)))/(a^2*d^3)]
```

3.331.6 Sympy [F]

$$\int x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\int x^5\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}dx$$

input
```
integrate(x**5*(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(x**5*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

3.331.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.48

$$\int x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=-\frac{bc^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{d^3}-\frac{3\left(8\,a^2bc^2-20\,ab^2c+b^3\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-8\left(6\,a^3bc^2-12\,a^2b^2c-ab^3\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+3\left(8\,a^4bc^2-12\,a^3b^2c\right.}{48\left(a^4d^3-\frac{3\,(adx^2+ac+b)a^3d^3}{dx^2+c}+\frac{3\,(adx^2+ac+b)^2a^2d^3}{(dx^2+c)^2}-\frac{(adx^2+ac+b)^3ad^3}{(dx^2+c)^3}\right)}-\frac{(24\,a^2c^2-12\,abc-b^2)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{32\,a^{\frac{3}{2}}d^3}$$

input
```
integrate(x^5*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
-b*c^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/d^3 - 1/48*(3*(8*a^2*b*c^2 -
20*a*b^2*c + b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - 8*(6*a^3*b*c^2
 - 12*a^2*b^2*c - a*b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) + 3*(8*a^
4*b*c^2 - 12*a^3*b^2*c - a^2*b^3)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(
a^4*d^3 - 3*(a*d*x^2 + a*c + b)*a^3*d^3/(d*x^2 + c) + 3*(a*d*x^2 + a*c + b
)^2*a^2*d^3/(d*x^2 + c)^2 - (a*d*x^2 + a*c + b)^3*a*d^3/(d*x^2 + c)^3) - 1
/32*(24*a^2*c^2 - 12*a*b*c - b^2)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c +
b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(3/
2)*d^3)
```

3.331.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. 2(229) = 458.

Time = 0.80 (sec) , antiderivative size = 517, normalized size of antiderivative = 2.08

$$\int x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{1}{48}\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(2\left(\frac{4\,ax^2\mathrm{sgn}(dx^2+c)}{d}-\frac{4\,a^3cd^6\mathrm{sgn}(dx^2+c)-}{a^2}\right.\right.$$
$$-\frac{(24\,a^2bc^2\mathrm{sgn}(dx^2+c)-12\,ab^2c\mathrm{sgn}(dx^2+c)-b^3\mathrm{sgn}(dx^2+c))\log\left(\left|2\,a^2c^3d+6\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2}\right.\right.\right.}{}$$

input
```
integrate(x^5*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
1/48*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*(4*a*x^2*sgn
(d*x^2 + c)/d - (4*a^3*c*d^6*sgn(d*x^2 + c) - 7*a^2*b*d^6*sgn(d*x^2 + c))/
(a^2*d^8))*x^2 + (8*a^3*c^2*d^5*sgn(d*x^2 + c) - 46*a^2*b*c*d^5*sgn(d*x^2
+ c) + 3*a*b^2*d^5*sgn(d*x^2 + c))/(a^2*d^8)) - 1/96*(24*a^2*b*c^2*sgn(d*x
^2 + c) - 12*a*b^2*c*sgn(d*x^2 + c) - b^3*sgn(d*x^2 + c))*log(abs(2*a^2*c^
3*d + 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2
+ b*c))*a^(3/2)*c^2*abs(d) + 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d
*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a*c*d + a*b*c^2*d + 2*(sqrt(a*d^2)*x^2 -
sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*sqrt(a)*abs(d) +
2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)
)*sqrt(a)*b*c*abs(d) + (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b
*d*x^2 + a*c^2 + b*c))^2*b*d))/(a^(3/2)*d^2*abs(d))
```

3.331.9 Mupad [F(-1)]

Timed out.

$$\int x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\int x^5\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}dx$$

input
```
int(x^5*(a + b/(c + d*x^2))^(3/2),x)
```

output
```
int(x^5*(a + b/(c + d*x^2))^(3/2), x)
```

3.332 $\int x^3 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

3.332.1 Optimal result

Integrand size = 21, antiderivative size = 172

$$\int x^3 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \frac{bc\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d^2} + \frac{(5b-4ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8d^2} + \frac{a(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4d^2} + \frac{3b(b-4ac)\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{8\sqrt{a}d^2}$$

output
```
3/8*b*(-4*a*c+b)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/d^2/a^
(1/2)+b*c*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2+1/8*(-4*a*c+5*b)*(d*x^2+c)
*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2+1/4*a*(d*x^2+c)^2*((a*d*x^2+a*c+b)/
(d*x^2+c))^(1/2)/d^2
```

3.332.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.66

$$\int x^3 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(13bc-2ac^2+5bdx^2+2ad^2x^4\right)}{8d^2} - \frac{3b(-b+4ac)\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{8\sqrt{a}d^2}$$

input
```
Integrate[x^3*(a + b/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(13*b*c - 2*a*c^2 + 5*b*d*x^2 + 2*a
*d^2*x^4))/(8*d^2) - (3*b*(-b + 4*a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c
 + d*x^2)]/Sqrt[a]])/(8*Sqrt[a]*d^2)
```

3.332.3 Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {2057, 2053, 2052, 25, 27, 360, 25, 1471, 27, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{2057}$$

$$\int x^3\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int x^2\left(\frac{adx^2+b+ac}{dx^2+c}\right)^{3/2}dx^2$$

$$\Big\downarrow \text{2052}$$

$$-bd\int -\frac{x^8\left(-cx^4+b+ac\right)}{d^3\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$bd\int \frac{x^8\left(-cx^4+b+ac\right)}{d^3\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{x^8\left(-cx^4+b+ac\right)}{\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{d^2}$$

$$\Big\downarrow \text{360}$$

$$\frac{b\left(\frac{1}{4}\int -\frac{-4cx^8+4bx^4+ab}{(a-x^4)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}+\frac{ab\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4(a-x^4)^2}\right)}{d^2}$$

$$\Big\downarrow \; 25$$

$$\frac{b\left(\frac{ab\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4(a-x^4)^2}-\frac{1}{4}\int \frac{-4cx^8+4bx^4+ab}{(a-x^4)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}\right)}{d^2}$$

$$\Big\downarrow \; 1471$$

$$\frac{b\left(\frac{1}{4}\left(\frac{\int \frac{a\left(-8cx^4+3b-4ac\right)}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2a}-\frac{(5b-4ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(a-x^4)}\right)+\frac{ab\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4(a-x^4)^2}\right)}{d^2}$$

$$\Big\downarrow \; 27$$

$$\frac{b\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{-8cx^4+3b-4ac}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}-\frac{(5b-4ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(a-x^4)}\right)+\frac{ab\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4(a-x^4)^2}\right)}{d^2}$$

$$\Big\downarrow \; 299$$

$$\frac{b\left(\frac{1}{4}\left(\frac{1}{2}\left(3(b-4ac)\int \frac{1}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}+8c\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\right)-\frac{(5b-4ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(a-x^4)}\right)+\frac{ab\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4(a-x^4)^2}\right)}{d^2}$$

$$\Big\downarrow \; 219$$

$$\frac{b\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{3(b-4ac)\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}}+8c\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\right)-\frac{(5b-4ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(a-x^4)}\right)+\frac{ab\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4(a-x^4)^2}\right)}{d^2}$$

input
```
Int[x^3*(a + b/(c + d*x^2))^(3/2),x]
```

3.332. $\int x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx$

output
```
(b*((a*b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*(a - x^4)^2) + (-1/2*((
5*b - 4*a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(a - x^4) + (8*c*Sqrt[
(b + a*c + a*d*x^2)/(c + d*x^2)] + (3*(b - 4*a*c)*ArcTanh[Sqrt[(b + a*c +
a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/Sqrt[a])/2)/4))/d^2
```

3.332.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

3.332. $\int x^3 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

3.332.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.41

method	result
risch	$-\frac{\left(-2ad\,x^2+2ac-5b\right)\left(d\,x^2+c\right)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{8d^2}-\frac{b\left(\frac{(12ac-3b)\ln\left(\frac{acd+\frac{1}{2}bd+a\,d^2x^2}{\sqrt{a\,d^2}}+\sqrt{a\,c^2+bc+(2acd+bd)x^2+a\,d^2x^4}\right)}{2\sqrt{a\,d^2}}-\frac{8c\left(ad\right.}{d\sqrt{a\,d^2x^4+2ac}}\right.}{8d(ad\,x^2+ac+b)}$
default	$\frac{\left(4\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}\,a\,d^2x^4-12\ln\left(\frac{2a\,d^2x^2+2acd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}+bd}{2\sqrt{a\,d^2}}\right)abc\,d^2x^2+3\ln\left(\right.\right.}{}$

input
```
int(x^3*(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/d^2*(-2*a*d*x^2+2*a*c-5*b)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2
)-1/8*b/d*(1/2*(12*a*c-3*b)*ln((a*c*d+1/2*b*d+a*d^2*x^2)/(a*d^2)^(1/2)+(a*
c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/(a*d^2)^(1/2)-8*c*(a*d*x^2+a*c
+b)/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))*((a*d*x^2+a*c+b)/(d
*x^2+c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(a*d*x^2+a*c+b)
```

3.332.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.95

$$\int x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=\left[\frac{3\,(4\,abc-b^2)\sqrt{a}\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2-4\,(2\,ad^2x^4+\right.}{}\right.$$

input
```
integrate(x^3*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/32*(3*(4*a*b*c - b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*
c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*
c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^2*d^2*x
^4 + 5*a*b*d*x^2 - 2*a^2*c^2 + 13*a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
 c)))/(a*d^2), 1/16*(3*(4*a*b*c - b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*
a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c
 + a*b)) + 2*(2*a^2*d^2*x^4 + 5*a*b*d*x^2 - 2*a^2*c^2 + 13*a*b*c)*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c)))/(a*d^2)]
```

3.332.6 Sympy [F]

$$\int x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=\int x^3\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**3*(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(x**3*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

3.332.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.44

$$\int x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{bc\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{d^2}+\frac{3\,(4\,ac-b)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{16\,\sqrt{a}d^2}-\frac{(4\,abc-5\,b^2)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-(4\,a^2bc-3\,ab^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^2d^2-\frac{2\,(adx^2+ac+b)ad^2}{dx^2+c}+\frac{(adx^2+ac+b)^2d^2}{(dx^2+c)^2}\right)}$$

input
```
integrate(x^3*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
b*c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/d^2 + 3/16*(4*a*c - b)*b*log(-(s
qrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 +
 a*c + b)/(d*x^2 + c))))/(sqrt(a)*d^2) - 1/8*((4*a*b*c - 5*b^2)*((a*d*x^2
+ a*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*c - 3*a*b^2)*sqrt((a*d*x^2 + a*c
+ b)/(d*x^2 + c)))/(a^2*d^2 - 2*(a*d*x^2 + a*c + b)*a*d^2/(d*x^2 + c) + (a
*d*x^2 + a*c + b)^2*d^2/(d*x^2 + c)^2)
```

3.332.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(156) = 312$.

Time = 0.74 (sec) , antiderivative size = 475, normalized size of antiderivative = 2.76

$$\int x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{1}{8}\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(\frac{2\,ax^2\mathrm{sgn}(dx^2+c)}{d}-\frac{2\,a^2cd^2\mathrm{sgn}(dx^2+c)-5\,a\cdots}{ad^4}\right.$$
$$+\frac{(4\,abc\mathrm{sgn}(dx^2+c)-b^2\mathrm{sgn}(dx^2+c))\log\left(\left|2\,a^2c^3d+6\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)a\cdots\right.\right.}{\cdots}$$
$$+\frac{(4\,abcd^2|d|\mathrm{sgn}(dx^2+c)-b^2d^2|d|\mathrm{sgn}(dx^2+c))\log(96)}{8\,\sqrt{a}d^5}$$

input
```
integrate(x^3*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
1/8*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*a*x^2*sgn(d*x
^2 + c)/d - (2*a^2*c*d^2*sgn(d*x^2 + c) - 5*a*b*d^2*sgn(d*x^2 + c))/(a*d^4
)) + 1/16*(4*a*b*c*sgn(d*x^2 + c) - b^2*sgn(d*x^2 + c))*log(abs(2*a^2*c^3*
d + 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 +
b*c))*a^(3/2)*c^2*abs(d) + 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x
^2 + b*d*x^2 + a*c^2 + b*c))^2*a*c*d + a*b*c^2*d + 2*(sqrt(a*d^2)*x^2 - sq
rt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*sqrt(a)*abs(d) + 2*
(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*
sqrt(a)*b*c*abs(d) + (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d
*x^2 + a*c^2 + b*c))^2*b*d))/(sqrt(a)*d*abs(d)) + 1/8*(4*a*b*c*d^2*abs(d)*
sgn(d*x^2 + c) - b^2*d^2*abs(d)*sgn(d*x^2 + c))*log(96)/(sqrt(a)*d^5)
```

3.332.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a + \frac{b}{c + dx^2}\right)^{3/2} dx = \int x^3 \left(a + \frac{b}{d\,x^2 + c}\right)^{3/2} dx$$

input
```
int(x^3*(a + b/(c + d*x^2))^(3/2),x)
```

output
```
int(x^3*(a + b/(c + d*x^2))^(3/2), x)
```

3.333 $\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx$

3.333.1 Optimal result

Integrand size = 19, antiderivative size = 94

$$\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=-\frac{3b\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{(c+dx^2)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}+\frac{3\sqrt{a}b\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2d}$$

output
```
1/2*(d*x^2+c)*(a+b/(d*x^2+c))^(3/2)/d+3/2*b*arctanh((a+b/(d*x^2+c))^(1/2)/
a^(1/2))*a^(1/2)/d-3/2*b*(a+b/(d*x^2+c))^(1/2)/d
```

3.333.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.93

$$\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(-2b+a(c+dx^2)\right)+3\sqrt{a}b\,\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{2d}$$

input
```
Integrate[x*(a + b/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*b + a*(c + d*x^2)) + 3*Sqrt[a]*
b*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(2*d)
```

3.333.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2024, 773, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{2024}$$

$$\frac{\int \left(a+\frac{b}{dx^2+c}\right)^{3/2}d(dx^2+c)}{2d}$$

$$\Big\downarrow \text{773}$$

$$-\frac{\int \left(dx^2+c\right)^2\left(a+\frac{b}{dx^2+c}\right)^{3/2}d\frac{1}{dx^2+c}}{2d}$$

$$\Big\downarrow \text{51}$$

$$-\frac{\frac{3}{2}b\int \left(dx^2+c\right)\sqrt{a+\frac{b}{dx^2+c}}d\frac{1}{dx^2+c}-\left(c+dx^2\right)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}$$

$$\Big\downarrow \text{60}$$

$$-\frac{\frac{3}{2}b\left(a\int \frac{dx^2+c}{\sqrt{a+\frac{b}{dx^2+c}}}d\frac{1}{dx^2+c}+2\sqrt{a+\frac{b}{c+dx^2}}\right)-\left(c+dx^2\right)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{3}{2}b\left(\frac{2a\int \frac{1}{\frac{1}{b\left(dx^2+c\right)^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{dx^2+c}}}{b}+2\sqrt{a+\frac{b}{c+dx^2}}\right)-\left(c+dx^2\right)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{3}{2}b\left(2\sqrt{a+\frac{b}{c+dx^2}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)\right)-(c+dx^2)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}$$

input
```
Int[x*(a + b/(c + d*x^2))^(3/2),x]
```

output
```
-1/2*(-((c + d*x^2)*(a + b/(c + d*x^2))^(3/2)) + (3*b*(2*Sqrt[a + b/(c + d
*x^2)] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]]))/2)/d
```

3.333.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

3.333. $\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx$

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

3.333.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(78) = 156$.

Time = 0.21 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.99

method	result
derivativedivides	$\frac{\sqrt{\frac{a(dx^2+c)+b}{dx^2+c}}\left(6a^{\frac{3}{2}}\sqrt{a(dx^2+c)^2+b(dx^2+c)}\,(dx^2+c)^2+3\ln\left(\frac{2\sqrt{a(dx^2+c)^2+b(dx^2+c)}\,\sqrt{a}+2a(dx^2+c)+b}{2\sqrt{a}}\right)ab(dx^2+\cdots\right.}{4d(dx^2+c)\sqrt{(a(dx^2+c)+b)(dx^2+c)}\,\sqrt{a}}$
risch	$\frac{(dx^2+c)a\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2d}+\frac{b\left(\frac{3a\ln\left(\frac{acd+\frac{1}{2}bd+ad^2x^2}{\sqrt{ad^2}}+\sqrt{ac^2+bc+(2acd+bd)x^2+ad^2x^4}\right)}{2\sqrt{ad^2}}-\frac{2(adx^2+ac+b)}{d\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2\cdots}}\right.}{2adx^2+2ac+2b}$
default	$\left(3\ln\left(\frac{2ad^2x^2+2acd+2\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\,\sqrt{ad^2}+bd}{2\sqrt{ad^2}}\right)abd^2x^2+2\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\,\sqrt{ad^2}\,ad\cdots\right.$

input
```
int(x*(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*((a*(d*x^2+c)+b)/(d*x^2+c))^(1/2)*(6*a^(3/2)*(a*(d*x^2+c)^2+b*(d*x^2
+c))^(1/2)*(d*x^2+c)^2+3*ln(1/2*(2*(a*(d*x^2+c)^2+b*(d*x^2+c))^(1/2)*a^(1/
2)+2*a*(d*x^2+c)+b)/a^(1/2))*a*b*(d*x^2+c)^2-4*(a*(d*x^2+c)^2+b*(d*x^2+c))
^(3/2)*a^(1/2))/(d*x^2+c)/((a*(d*x^2+c)+b)*(d*x^2+c))^(1/2)/a^(1/2)
```

3.333.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.86

$$\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx = \left[\frac{3\sqrt{a}b\log\left(8a^2d^2x^4+8a^2c^2+8(2a^2c+ab)dx^2+8abc+b^2+4(2ad^2x^4+(4ac+b)\right.}{8d}\right.$$

$$\left.-\frac{3\sqrt{-a}b\arctan\left(\frac{(2adx^2+2ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2(a^2dx^2+a^2c+ab)}\right)-2(adx^2+ac-2b)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{4d}\right]$$

input
```
integrate(x*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*sqrt(a)*b*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2
+ 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt
(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a*d*x^2 + a*c - 2*b)*sqrt(
(a*d*x^2 + a*c + b)/(d*x^2 + c)))/d, -1/4*(3*sqrt(-a)*b*arctan(1/2*(2*a*d*
x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2
 + a^2*c + a*b)) - 2*(a*d*x^2 + a*c - 2*b)*sqrt((a*d*x^2 + a*c + b)/(d*x^2
 + c)))/d]
```

3.333.6 Sympy [F]

$$\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx = \int x\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}dx$$

input
```
integrate(x*(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(x*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

3.333.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.66

$$\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=-\frac{ab\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(ad-\frac{(adx^2+ac+b)d}{dx^2+c}\right)}-\frac{3\sqrt{a}b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{4\,d}-\frac{b\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{d}$$

input
```
integrate(x*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/2*a*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*d - (a*d*x^2 + a*c + b)*
d/(d*x^2 + c)) - 3/4*sqrt(a)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d
*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/d - b*sqrt(
(a*d*x^2 + a*c + b)/(d*x^2 + c))/d
```

3.333.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. 2(78) = 156.

Time = 0.72 (sec) , antiderivative size = 381, normalized size of antiderivative = 4.05

$$\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=-\frac{\sqrt{a}b|d|\log(24)\,\mathrm{sgn}(dx^2+c)}{2\,d^2}$$
$$-\frac{\sqrt{a}b\log\left(\left|2\,a^2c^3d+6\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)a^{\frac{3}{2}}c^2|d|+6\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+\right.\right.}{}$$
$$+\frac{\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}a\,\mathrm{sgn}(dx^2+c)}{2\,d}$$

input
```
integrate(x*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(a)*b*abs(d)*log(24)*sgn(d*x^2 + c)/d^2 - 1/4*sqrt(a)*b*log(abs(2
*a^2*c^3*d + 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 +
 a*c^2 + b*c))*a^(3/2)*c^2*abs(d) + 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 +
2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a*c*d + a*b*c^2*d + 2*(sqrt(a*d^2)
*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*sqrt(a)*ab
s(d) + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2
 + b*c))*sqrt(a)*b*c*abs(d) + (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*
x^2 + b*d*x^2 + a*c^2 + b*c))^2*b*d))*sgn(d*x^2 + c)/abs(d) + 1/2*sqrt(a*d
^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*a*sgn(d*x^2 + c)/d
```

3.333.9 Mupad [B] (verification not implemented)

Time = 18.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.65

$$\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=-\frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}\left(d\,x^2+c\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{a\,(d\,x^2+c)}{b}\right)}{d\,\left(\frac{a\,(d\,x^2+c)}{b}+1\right)^{3/2}}$$

input
```
int(x*(a + b/(c + d*x^2))^(3/2),x)
```

output
```
-((a + b/(c + d*x^2))^(3/2)*(c + d*x^2)*hypergeom([-3/2, -1/2], 1/2, -(a*(
c + d*x^2))/b))/(d*((a*(c + d*x^2))/b + 1)^(3/2))
```

3.334 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x} dx$

3.334.1 Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x} dx = \frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c} + a^{3/2}\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right) - \frac{(b+ac)^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{c^{3/2}}$$

output
```
a^(3/2)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))-(a*c+b)^(3/2)*a
rctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))/c^(3/2)+b*
((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c
```

3.334.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x} dx = \frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c} - \frac{(-b-ac)^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{-b-ac}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{b+ac}\right)}{c^{3/2}} + a^{3/2}\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b/(c + d*x^2))^(3/2)/x,x]
```

output
```
(b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/c - ((-b - a*c)^(3/2)*ArcTan[(Sq
rt[c]*Sqrt[-b - a*c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(b + a*c)])/c^
(3/2) + a^(3/2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]]
```

3.334.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2057, 2053, 2052, 25, 27, 381, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{\left(\frac{adx^2+b+ac}{dx^2+c}\right)^{3/2}}{x^2}dx^2$$

$$\Big\downarrow \text{2052}$$

$$-bd\int -\frac{x^8}{d\left(a-x^4\right)\left(-cx^4+b+ac\right)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$bd\int \frac{x^8}{d\left(a-x^4\right)\left(-cx^4+b+ac\right)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{27}$$

$$b\int \frac{x^8}{\left(a-x^4\right)\left(-cx^4+b+ac\right)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{381}$$

$$b\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}-\frac{\int \frac{a(b+ac)-(b+2ac)x^4}{(a-x^4)(-cx^4+b+ac)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{c}\right)$$

$$\Big\downarrow \text{397}$$

$$b\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}-\frac{\frac{(ac+b)^2\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{b}-\frac{a^2c\int \frac{1}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{b}}{c}\right)$$

$$\Big\downarrow \text{219}$$

$$b\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}-\frac{\frac{(ac+b)^2\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{b}-\frac{a^{3/2}c\,\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{b}}{c}\right)$$

$$\Big\downarrow \text{221}$$

$$b\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}-\frac{\frac{(ac+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{b\sqrt{c}}-\frac{a^{3/2}c\,\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{b}}{c}\right)$$

input
```
Int[(a + b/(c + d*x^2))^(3/2)/x,x]
```

output
```
b*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/c - (-((a^(3/2)*c*ArcTanh[Sqrt[(b
 + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/b) + ((b + a*c)^(3/2)*ArcTanh[(Sq
rt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/(b*Sqrt[c]))/
c)
```

3.334.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.334. $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x}\,dx$

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

3.334. $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x} \, dx$

3.334.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(108) = 216$.

Time = 0.13 (sec) , antiderivative size = 652, normalized size of antiderivative = 5.17

method	result
default	$\frac{\left(\ln\left(\frac{2a\,d^2x^2+2acd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}+bd}{2\sqrt{a\,d^2}}\right)a^2c^2d^2x^2-\sqrt{a\,d^2}\,\sqrt{a\,c^2+bc}\,\ln\left(\frac{2acd\,x^2+bd\,x^2+2a\,c^2+2\sqrt{a\,c^2+bc}\,\sqrt{a\,d^2}}{x^2}\right.\right.}{}$

input
```
int((a+b/(d*x^2+c))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*
c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*c^2*d^2*x^2-(a*d^2)^(1/2)*(
a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*(a*c^2+b*c)^(1/2)*(a*d^
2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a*c*d*x^2-(a*d^2)^(
1/2)*(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*(a*c^2+b*c)^(1/2)
*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*b*d*x^2+ln(1/
2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(
a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*c^3*d-(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)
*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^
2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a*c^2-(a*d^2)^(1/2)*(a*c^2+b*c)^(1/
2)*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*
x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*b*c+2*((a*d*x^2+a*c+b)*(d*x^2+c))
^(1/2)*(a*d^2)^(1/2)*b*c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2)^(1/2)/
c^2/((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)
```

3.334.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 1073, normalized size of antiderivative = 8.52

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x}\,dx = \left[\frac{a^{\frac{3}{2}}c\log\left(8a^2d^2x^4+8a^2c^2+8(2a^2c+ab)dx^2+8abc+b^2+4(2ad^2x^4+(4ac+b\right.}{}\right.$$

$$-\frac{2\sqrt{-a}ac\arctan\left(\frac{(2adx^2+2ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2(a^2dx^2+a^2c+ab)}\right)-(ac+b)\sqrt{\frac{ac+b}{c}}\log\left(\frac{(8a^2c^2+8abc+b^2)d^2x^4+8a^2c^4+16abc^3+8b^2c^}{}\right.}{4c}$$

$$-\frac{\sqrt{-a}ac\arctan\left(\frac{(2adx^2+2ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2(a^2dx^2+a^2c+ab)}\right)-(ac+b)\sqrt{-\frac{ac+b}{c}}\arctan\left(\frac{((2ac+b)dx^2+2ac^2+2bc)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\sqrt{}}{2(a^2c^2+(a^2c+ab)dx^2+2abc+b^2)}\right.}{2c}$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/4*(a^(3/2)*c*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 +
8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a
)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + (a*c + b)*sqrt((a*c + b)/c)*log
(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2
 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c^2 + b*c)*d^2*x^4 +
2*a*c^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d
*x^2 + c))*sqrt((a*c + b)/c))/x^4) + 4*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
 c)))/c, -1/4*(2*sqrt(-a)*a*c*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*
sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - (a*c +
b)*sqrt((a*c + b)/c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4
+ 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2
*a*c^2 + b*c)*d^2*x^4 + 2*a*c^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqr
t((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt((a*c + b)/c))/x^4) - 4*b*sqrt((a*d
*x^2 + a*c + b)/(d*x^2 + c)))/c, 1/4*(a^(3/2)*c*log(8*a^2*d^2*x^4 + 8*a^2*
c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c +
b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) +
 2*(a*c + b)*sqrt(-(a*c + b)/c)*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 +
2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(a^2*c^2 +
 (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2)) + 4*b*sqrt((a*d*x^2 + a*c + b)/(d*x
^2 + c)))/c, -1/2*(sqrt(-a)*a*c*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt...
```

3.334.6 Sympy [F]

$$\int \frac{\left(a + \frac{b}{c+dx^2}\right)^{3/2}}{x}\,dx = \int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(3/2)/x,x)
```

output
```
Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x, x)
```

3.334.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.60

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x}\,dx = -\frac{1}{2}\,a^{\frac{3}{2}}\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right) + \frac{b\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{c} + \frac{(a^2c^2+2\,abc+b^2)\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{2\,\sqrt{(ac+b)c}c}$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x,x, algorithm="maxima")
```

output
```
-1/2*a^(3/2)*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(
a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))) + b*sqrt((a*d*x^2 + a*c + b)/
(d*x^2 + c))/c + 1/2*(a^2*c^2 + 2*a*b*c + b^2)*log((c*sqrt((a*d*x^2 + a*c
+ b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2
+ c)) + sqrt((a*c + b)*c)))/(sqrt((a*c + b)*c)*c)
```

3.334.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.334.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x}\,dx=\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x}\,dx$$

input
```
int((a + b/(c + d*x^2))^(3/2)/x,x)
```

output
```
int((a + b/(c + d*x^2))^(3/2)/x, x)
```

3.335 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3} dx$

3.335.1 Optimal result

Integrand size = 21, antiderivative size = 138

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3} dx = -\frac{3bd\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{2c^2} - \frac{(c+dx^2)\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}{2cx^2} + \frac{3b\sqrt{b+ac}\,d\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{2c^{5/2}}$$

output
```
-1/2*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(3/2)/c/x^2+3/2*b*d*arctanh(c^(
1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))*(a*c+b)^(1/2)/c^(5/2
)-3/2*b*d*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^2
```

3.335.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3} dx = -\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(ac(c+dx^2)+b(c+3dx^2)\right)}{2c^2x^2} + \frac{3b\sqrt{-b-ac}\,d\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{2c^{5/2}}$$

input
```
Integrate[(a + b/(c + d*x^2))^(3/2)/x^3,x]
```

output
```
-1/2*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*c*(c + d*x^2) + b*(c + 3*d*
x^2)))/(c^2*x^2) + (3*b*Sqrt[-b - a*c]*d*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a
*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(2*c^(5/2))
```

3.335.3 Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2057, 2053, 2052, 252, 262, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{ 2053}$$

$$\frac{1}{2}\int \frac{\left(\frac{adx^2+b+ac}{dx^2+c}\right)^{3/2}}{x^4}dx^2$$

$$\Big\downarrow \text{ 2052}$$

$$-bd\int \frac{x^8}{\left(-cx^4+b+ac\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{ 252}$$

$$-bd\left(\frac{x^6}{2c\left(ac+b-cx^4\right)}-\frac{3\int \frac{x^4}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2c}\right)$$

$$\Big\downarrow \text{ 262}$$

$$-bd\left(\frac{x^6}{2c\left(ac+b-cx^4\right)}-\frac{3\left(\frac{(ac+b)\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{c}-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}\right)}{2c}\right)$$

$$\Big\downarrow \text{221}$$

$$-bd\left(\frac{x^6}{2c\left(ac+b-cx^4\right)}-\frac{3\left(\frac{\sqrt{ac+b}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{c^{3/2}}-\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}\right)}{2c}\right)$$

input
```
Int[(a + b/(c + d*x^2))^(3/2)/x^3,x]
```

output
```
-(b*d*(x^6/(2*c*(b + a*c - c*x^4)) - (3*(-(Sqrt[(b + a*c + a*d*x^2)/(c + d
*x^2)]/c) + (Sqrt[b + a*c]*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c +
d*x^2)])/Sqrt[b + a*c]])/c^(3/2)))/(2*c)))
```

3.335.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

3.335. $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3}\,dx$

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

3.335.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(118) = 236$.

Time = 0.19 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.82

method	result
risch	$-\frac{(ac+b)(dx^2+c)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2c^2x^2}-\frac{bd\left(-\frac{(3ac+3b)\ln\left(\frac{2ac^2+2bc+(2acd+bd)x^2+2\sqrt{ac^2+bc}\sqrt{ac^2+bc+(2acd+bd)x^2+ad^2x^4}}{x^2}\right)}{2\sqrt{ac^2+bc}}+\frac{\cdots}{\sqrt{ad^2x^4 \cdots}} \cdots\right)}{2c^2(adx^2+ac+b)}$
default	$-\frac{\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\,\Big(-2\sqrt{ac^2+bc}\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\,ad^3x^6-3\ln\Big(\frac{2acdx^2+bdx^2+2ac^2+2\sqrt{ac^2+bc}\sqrt{ad^2x^4+2acdx^2+bdx^2 \cdots}}{x^2} \cdots}{\cdots}$

input
```
int((a+b/(d*x^2+c))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

3.335. $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3}\,dx$

output
```
-1/2*(a*c+b)/c^2*(d*x^2+c)/x^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/2*b*d/c
^2*(-1/2*(3*a*c+3*b)/(a*c^2+b*c)^(1/2)*ln((2*a*c^2+2*b*c+(2*a*c*d+b*d)*x^2
+2*(a*c^2+b*c)^(1/2)*(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/x^2)+2
*(a*d*x^2+a*c+b)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))*((a*d*x^
2+a*c+b)/(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(a*d*x^2+a*c+b
)
```

3.335.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 404, normalized size of antiderivative = 2.93

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \left[\frac{3\,bdx^2\sqrt{\frac{ac+b}{c}}\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,(2\,a^2c^3+3\,abc^2+b^2c)dx^2+4\,((2\,ac^2\cdots}{x^4}\right.}{\cdots}\right.$$

$$-\frac{3\,bdx^2\sqrt{-\frac{ac+b}{c}}\arctan\left(\frac{((2\,ac+b)dx^2+2\,ac^2+2\,bc)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\sqrt{-\frac{ac+b}{c}}}{2\,(a^2c^2+(a^2c+ab)dx^2+2\,abc+b^2)}\right)+2\,((ac+3\,b)dx^2+ac^2+bc)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{4\,c^2x^2}$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^3,x, algorithm="fricas")
```

output
```
[1/8*(3*b*d*x^2*sqrt((a*c + b)/c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4
 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*
d*x^2 + 4*((2*a*c^2 + b*c)*d^2*x^4 + 2*a*c^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^
2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt((a*c + b)/c))/x^4) -
4*((a*c + 3*b)*d*x^2 + a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
/(c^2*x^2), -1/4*(3*b*d*x^2*sqrt(-(a*c + b)/c)*arctan(1/2*((2*a*c + b)*d*x
^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-(a*c + b
)/c)/(a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2)) + 2*((a*c + 3*b)*d*x
^2 + a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(c^2*x^2)]
```

3.335.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(3/2)/x**3,x)
```

output
```
Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**3, x)
```

3.335.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.46

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3}\,dx = -\frac{(abc+b^2)d\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(ac^3+bc^2-\frac{(adx^2+ac+b)c^3}{dx^2+c}\right)} - \frac{bd\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{c^2} - \frac{3\,(abc+b^2)d\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{4\sqrt{(ac+b)c}c^2}$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^3,x, algorithm="maxima")
```

output
```
-1/2*(a*b*c + b^2)*d*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*c^3 + b*c^2
- (a*d*x^2 + a*c + b)*c^3/(d*x^2 + c)) - b*d*sqrt((a*d*x^2 + a*c + b)/(d*x
^2 + c))/c^2 - 3/4*(a*b*c + b^2)*d*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2
+ c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt
((a*c + b)*c)))/(sqrt((a*c + b)*c)*c^2)
```

3.335.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^3,x, algorithm="giac")
```

output
```
undef
```

3.335.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3}\,dx=\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^3}\,dx$$

input
```
int((a + b/(c + d*x^2))^(3/2)/x^3,x)
```

output
```
int((a + b/(c + d*x^2))^(3/2)/x^3, x)
```

3.336 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx$

3.336.1 Optimal result

Integrand size = 21, antiderivative size = 205

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \frac{bd^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c^3} + \frac{(9b+4ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8c^3x^2} - \frac{(b+ac)\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4c^3x^4} - \frac{3b(5b+4ac)d^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{8c^{7/2}\sqrt{b+ac}}$$

output
```
-3/8*b*(4*a*c+5*b)*d^2*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(
a*c+b)^(1/2))/c^(7/2)/(a*c+b)^(1/2)+b*d^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2
)/c^3+1/8*(4*a*c+9*b)*d*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^3/x^
2-1/4*(a*c+b)*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^3/x^4
```

3.336.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.75

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(-2bc^2-2ac^3+5bcdx^2+15bd^2x^4+2acd^2x^4\right)}{8c^3x^4} + \frac{3b(5b+4ac)d^2\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{8c^{7/2}\sqrt{-b-ac}}$$

input
```
Integrate[(a + b/(c + d*x^2))^(3/2)/x^5,x]
```

output
```
(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*b*c^2 - 2*a*c^3 + 5*b*c*d*x^2 +
 15*b*d^2*x^4 + 2*a*c*d^2*x^4))/(8*c^3*x^4) + (3*b*(5*b + 4*a*c)*d^2*ArcTa
n[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(8*c^(7
/2)*Sqrt[-b - a*c])
```

3.336.3 Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2057, 2053, 2052, 25, 27, 360, 1471, 27, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}{x^5}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{\left(\frac{adx^2+b+ac}{dx^2+c}\right)^{3/2}}{x^6}dx^2$$

$$\Big\downarrow \text{2052}$$

$$-bd\int -\frac{dx^8\left(a-x^4\right)}{\left(-cx^4+b+ac\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$bd\int \frac{dx^8\left(a-x^4\right)}{\left(-cx^4+b+ac\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{27}$$

$$bd^2\int \frac{x^8\left(a-x^4\right)}{\left(-cx^4+b+ac\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{360}$$

$$bd^2\left(\frac{\int \frac{4c^2x^8+4bcx^4+b(b+ac)}{(-cx^4+b+ac)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{4c^3}-\frac{b(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^3\left(ac+b-cx^4\right)^2}\right)$$

$$\Big\downarrow \text{1471}$$

$$bd^2\left(\frac{\frac{(4ac+9b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b-cx^4)}-\frac{\int \frac{(b+ac)\left(8cx^4+7b+4ac\right)}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2(ac+b)}}{4c^3}-\frac{b(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^3\left(ac+b-cx^4\right)^2}\right)$$

$$\Big\downarrow \text{27}$$

$$bd^2\left(\frac{\frac{(4ac+9b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b-cx^4)}-\frac{1}{2}\int \frac{8cx^4+7b+4ac}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{4c^3}-\frac{b(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^3\left(ac+b-cx^4\right)^2}\right)$$

$$\Big\downarrow \text{299}$$

$$bd^2\left(\frac{\frac{1}{2}\left(8\sqrt{\frac{ac+adx^2+b}{c+dx^2}}-3(4ac+5b)\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}\right)+\frac{(4ac+9b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b-cx^4)}}{4c^3}-\frac{b(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^3\left(ac+b-cx^4\right)^2}\right)$$

$$\Big\downarrow \text{221}$$

$$bd^2\left(\frac{\frac{1}{2}\left(8\sqrt{\frac{ac+adx^2+b}{c+dx^2}}-\frac{3(4ac+5b)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{\sqrt{c}\sqrt{ac+b}}\right)+\frac{(4ac+9b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b-cx^4)}}{4c^3}-\frac{b(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^3\left(ac+b-cx^4\right)^2}\right)$$

input
```
Int[(a + b/(c + d*x^2))^(3/2)/x^5,x]
```

3.336. $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx$

output
```
b*d^2*(-1/4*(b*(b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(c^3*(b +
a*c - c*x^4)^2) + (((9*b + 4*a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(
2*(b + a*c - c*x^4)) + (8*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)] - (3*(5*b
+ 4*a*c)*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b +
a*c]])/(Sqrt[c]*Sqrt[b + a*c]))/2)/(4*c^3))
```

3.336.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

3.336. $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5} dx$

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

3.336.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.34

method	result
risch	$-\frac{(d\,x^2+c)\left(-2acd\,x^2-7bd\,x^2+2a\,c^2+2bc\right)\sqrt{\frac{a d\,x^2+ac+b}{d\,x^2+c}}}{8c^3x^4}+\frac{b\,d^2\left(-\frac{(12ac+15b)\ln\left(\frac{2a\,c^2+2bc+(2acd+bd)x^2+2\sqrt{a\,c^2+bc}\,\sqrt{a\,c^2+bc+(2ac}}{x^2}\right.}{2\sqrt{a\,c^2+bc}}\right.}{}$
default	Expression too large to display

input
```
int((a+b/(d*x^2+c))^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(d*x^2+c)*(-2*a*c*d*x^2-7*b*d*x^2+2*a*c^2+2*b*c)/c^3/x^4*((a*d*x^2+a*
c+b)/(d*x^2+c))^(1/2)+1/8*b*d^2/c^3*(-1/2*(12*a*c+15*b)/(a*c^2+b*c)^(1/2)*
ln((2*a*c^2+2*b*c+(2*a*c*d+b*d)*x^2+2*(a*c^2+b*c)^(1/2)*(a*c^2+b*c+(2*a*c*
d+b*d)*x^2+a*d^2*x^4)^(1/2))/x^2)+8*(a*d*x^2+a*c+b)/(a*d^2*x^4+2*a*c*d*x^2
+b*d*x^2+a*c^2+b*c)^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*((a*d*x^2+a*c
+b)*(d*x^2+c))^(1/2)/(a*d*x^2+a*c+b)
```

3.336.5 Fricas [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 557, normalized size of antiderivative = 2.72

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \left[\frac{3\,(4\,abc+5\,b^2)\sqrt{ac^2+bcd}d^2x^4\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)\,d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,(2\,a^2c^3+3\,ab}{}\right.}{}\right.$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^5,x, algorithm="fricas")
```

output
```
[1/32*(3*(4*a*b*c + 5*b^2)*sqrt(a*c^2 + b*c)*d^2*x^4*log(((8*a^2*c^2 + 8*a
*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 +
3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3
*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
 c)))/x^4) - 4*(2*a^2*c^5 - (2*a^2*c^3 + 17*a*b*c^2 + 15*b^2*c)*d^2*x^4 +
4*a*b*c^4 + 2*b^2*c^3 - 5*(a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c)))/((a*c^5 + b*c^4)*x^4), 1/16*(3*(4*a*b*c + 5*b^2)*sqrt(-a
*c^2 - b*c)*d^2*x^4*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(
-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2*a*b*c^2 +
 (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) - 2*(2*a^2*c^5 - (2*a^2*c^3 + 17*a*b*c^
2 + 15*b^2*c)*d^2*x^4 + 4*a*b*c^4 + 2*b^2*c^3 - 5*(a*b*c^3 + b^2*c^2)*d*x^
2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a*c^5 + b*c^4)*x^4)]
```

3.336.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(3/2)/x**5,x)
```

output
```
Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**5, x)
```

3.336.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.53

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \frac{bd^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{c^3} + \frac{3\,(4\,abc+5\,b^2)d^2\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{16\,\sqrt{(ac+b)c}c^3} - \frac{(4\,abc^2+9\,b^2c)d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-(4\,a^2bc^2+11\,ab^2c+7\,b^3)d^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^2c^5+2\,abc^4+b^2c^3+\frac{(adx^2+ac+b)^2c^5}{(dx^2+c)^2}-\frac{2\,(ac^5+bc^4)(adx^2+ac+b)}{dx^2+c}\right)}$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^5,x, algorithm="maxima")
```

output
```
b*d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/c^3 + 3/16*(4*a*b*c + 5*b^2)*d
^2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sq
rt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/(sqrt((a*c + b)*
c)*c^3) - 1/8*((4*a*b*c^2 + 9*b^2*c)*d^2*((a*d*x^2 + a*c + b)/(d*x^2 + c))
^(3/2) - (4*a^2*b*c^2 + 11*a*b^2*c + 7*b^3)*d^2*sqrt((a*d*x^2 + a*c + b)/(
d*x^2 + c)))/(a^2*c^5 + 2*a*b*c^4 + b^2*c^3 + (a*d*x^2 + a*c + b)^2*c^5/(d
*x^2 + c)^2 - 2*(a*c^5 + b*c^4)*(a*d*x^2 + a*c + b)/(d*x^2 + c))
```

3.336.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^5,x, algorithm="giac")
```

output
```
undef
```

3.336.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx=\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^5}\,dx$$

input
```
int((a + b/(c + d*x^2))^(3/2)/x^5,x)
```

output
```
int((a + b/(c + d*x^2))^(3/2)/x^5, x)
```

3.337 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx$

3.337.1 Optimal result

Integrand size = 21, antiderivative size = 292

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx = -\frac{bd^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c^4} - \frac{(79b^2+108abc+24a^2c^2)\,d^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{48c^4(b+ac)x^2} + \frac{(11b+12ac)d(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{24c^4x^4} - \frac{(c+dx^2)^3\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{5/2}}{6c^2(b+ac)x^6} + \frac{b(35b^2+60abc+24a^2c^2)\,d^3\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{16c^{9/2}(b+ac)^{3/2}}$$

output
```
-1/6*(d*x^2+c)^3*((a*d*x^2+a*c+b)/(d*x^2+c))^(5/2)/c^2/(a*c+b)/x^6+1/16*b*
(24*a^2*c^2+60*a*b*c+35*b^2)*d^3*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c
))^(1/2)/(a*c+b)^(1/2))/c^(9/2)/(a*c+b)^(3/2)-b*d^3*((a*d*x^2+a*c+b)/(d*x^
2+c))^(1/2)/c^4-1/48*(24*a^2*c^2+108*a*b*c+79*b^2)*d^2*(d*x^2+c)*((a*d*x^2
+a*c+b)/(d*x^2+c))^(1/2)/c^4/(a*c+b)/x^2+1/24*(12*a*c+11*b)*d*(d*x^2+c)^2*
((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^4/x^4
```

3.337.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.77

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx = -\frac{\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(8a^2c^2\left(c^3+d^3x^6\right)+2abc\left(8c^3-7c^2dx^2+16cd^2x^4+55d^3x^6\right)+b^2\left(8c^3-14c^2dx^2+35cd^2x^4+105d^3x^6\right.\right.}{(b+ac)x^6}}{48c^{9/2}}$$

input
```
Integrate[(a + b/(c + d*x^2))^(3/2)/x^7,x]
```

output
```
(-((Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(8*a^2*c^2*(c^3 + d^3*x^
6) + 2*a*b*c*(8*c^3 - 7*c^2*d*x^2 + 16*c*d^2*x^4 + 55*d^3*x^6) + b^2*(8*c^
3 - 14*c^2*d*x^2 + 35*c*d^2*x^4 + 105*d^3*x^6)))/((b + a*c)*x^6)) + (3*b*(
35*b^2 + 60*a*b*c + 24*a^2*c^2)*d^3*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^
2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(-b - a*c)^(3/2))/(48*c^(9/2))
```

3.337.3 Rubi [A] (warning: unable to verify)

Time = 0.57 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {2057, 2053, 2052, 27, 366, 360, 25, 1471, 27, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}{x^7}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{\left(\frac{adx^2+b+ac}{dx^2+c}\right)^{3/2}}{x^8}dx^2$$

$$\Big\downarrow \text{2052}$$

$$-bd\int \frac{d^2x^8\left(a-x^4\right)^2}{\left(-cx^4+b+ac\right)^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \; 27$$

$$-bd^3\int \frac{x^8\left(a-x^4\right)^2}{\left(-cx^4+b+ac\right)^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \; 366$$

$$-bd^3\left(\frac{b^2x^{10}}{6c^2(ac+b)\left(ac+b-cx^4\right)^3}-\frac{\int \frac{x^8\left(6c(b+ac)x^4+5b^2-6a^2c^2\right)}{\left(-cx^4+b+ac\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{6c^2(ac+b)}\right)$$

$$\Big\downarrow \; 360$$

$$-bd^3\left(\frac{b^2x^{10}}{6c^2(ac+b)\left(ac+b-cx^4\right)^3}-\frac{\frac{\int -\frac{24c^3(b+ac)x^8+4bc^2(11b+12ac)x^4+bc(b+ac)(11b+12ac)}{\left(-cx^4+b+ac\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{4c^3}+\frac{b(ac+b)(12ac+11b)\sqrt{\frac{a\cdots}{}}}{4c^2(ac+b-cx^4)\cdots}}{6c^2(ac+b)}\cdots\right.$$

$$\Big\downarrow \; 25$$

$$-bd^3\left(\frac{b^2x^{10}}{6c^2(ac+b)\left(ac+b-cx^4\right)^3}-\frac{\frac{b(ac+b)(12ac+11b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^2(ac+b-cx^4)^2}-\frac{\int \frac{24c^3(b+ac)x^8+4bc^2(11b+12ac)x^4+bc(b+ac)(11b+12ac)}{\left(-cx^4+b+ac\right)^2}d\sqrt{\frac{adx\cdots}{d\cdots}}}{4c^3}}{6c^2(ac+b)}\cdots\right.$$

$$\Big\downarrow \; 1471$$

$$-bd^3\left(\frac{b^2x^{10}}{6c^2(ac+b)\left(ac+b-cx^4\right)^3}-\frac{\frac{b(ac+b)(12ac+11b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^2(ac+b-cx^4)^2}-\frac{\frac{c\left(24a^2c^2+108abc+79b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2\left(ac+b-cx^4\right)}-\frac{\int \frac{3c(b+ac)\left(16c(b+ac)x\cdots\right.}{-cx^4\cdots}}{\cdots}}{4c^3}}{6c^2(ac+b)}\cdots\right.$$

$$\Big\downarrow \; 27$$

3.337. $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx$

$$-bd^3\left(\frac{b^2x^{10}}{6c^2(ac+b)\left(ac+b-cx^4\right)^3}-\frac{\frac{b(ac+b)(12ac+11b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^2(ac+b-cx^4)^2}-\frac{\frac{c\left(24a^2c^2+108abc+79b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2\left(ac+b-cx^4\right)}-\frac{3}{2}c\int\frac{16c(b+ac)x^4+19b^2}{-cx^4+}}{4c^3}}{6c^2(ac+b)}\right.$$

$\downarrow$ 299

$$-bd^3\left(\frac{b^2x^{10}}{6c^2(ac+b)\left(ac+b-cx^4\right)^3}-\frac{\frac{b(ac+b)(12ac+11b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^2(ac+b-cx^4)^2}-\frac{\frac{c\left(24a^2c^2+108abc+79b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2\left(ac+b-cx^4\right)}-\frac{3}{2}c\left(\left(24a^2c^2+60abc+\right.\right.}{4}}{6c^2(ac+b)}\right.$$

$\downarrow$ 221

$$-bd^3\left(\frac{b^2x^{10}}{6c^2(ac+b)\left(ac+b-cx^4\right)^3}-\frac{\frac{b(ac+b)(12ac+11b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^2(ac+b-cx^4)^2}-\frac{\frac{c\left(24a^2c^2+108abc+79b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2\left(ac+b-cx^4\right)}-\frac{3}{2}c\left(\frac{\left(24a^2c^2+60abc+3\right.}{}\right.}{4}}{6c^2(ac+b)}\right.$$

input

```
Int[(a + b/(c + d*x^2))^(3/2)/x^7,x]
```

output

```
-(b*d^3*((b^2*x^10)/(6*c^2*(b + a*c)*(b + a*c - c*x^4)^3) - ((b*(b + a*c)*
(11*b + 12*a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*c^2*(b + a*c - c
*x^4)^2) - ((c*(79*b^2 + 108*a*b*c + 24*a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/
(c + d*x^2)])/(2*(b + a*c - c*x^4)) - (3*c*(-16*(b + a*c)*Sqrt[(b + a*c +
a*d*x^2)/(c + d*x^2)] + ((35*b^2 + 60*a*b*c + 24*a^2*c^2)*ArcTanh[(Sqrt[c]
*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/(Sqrt[c]*Sqrt[b +
a*c])))/2)/(4*c^3))/(6*c^2*(b + a*c)))
```

3.337. $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx$

3.337.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

3.337. $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx$

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

3.337.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.27

method	result
risch	$-\frac{(d\,x^2+c)\left(8a^2c^2d^2x^4+62ac\,d^2b\,x^4-8a^2c^3d\,x^2+57b^2d^2x^4-30ab\,c^2d\,x^2+8a^2c^4-22b^2cd\,x^2+16ab\,c^3+8b^2c^2\right)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{48c^4x^6(ac+b)}-\frac{d^3b}{}$
default	Expression too large to display

input
```
int((a+b/(d*x^2+c))^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(d*x^2+c)*(8*a^2*c^2*d^2*x^4+62*a*b*c*d^2*x^4-8*a^2*c^3*d*x^2+57*b^2
*d^2*x^4-30*a*b*c^2*d*x^2+8*a^2*c^4-22*b^2*c*d*x^2+16*a*b*c^3+8*b^2*c^2)/c
^4/x^6/(a*c+b)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/16*d^3*b/c^4/(a*c+b)*(-
1/2*(24*a^2*c^2+60*a*b*c+35*b^2)/(a*c^2+b*c)^(1/2)*ln((2*a*c^2+2*b*c+(2*a*
c*d+b*d)*x^2+2*(a*c^2+b*c)^(1/2)*(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(
1/2))/x^2)+16*(a*c+b)*(a*d*x^2+a*c+b)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2
+b*c)^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))
^(1/2)/(a*d*x^2+a*c+b)
```

3.337.5 Fricas [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 733, normalized size of antiderivative = 2.51

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \left[\frac{3\left(24\,a^2bc^2+60\,ab^2c+35\,b^3\right)\sqrt{ac^2+bc}\,d^3x^6\log\left(\frac{\left(8\,a^2c^2+8\,abc+b^2\right)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b}{}\right.}{}\right.$$

$$-\frac{3\left(24\,a^2bc^2+60\,ab^2c+35\,b^3\right)\sqrt{-ac^2-bc}\,d^3x^6\arctan\left(\frac{\left((2\,ac+b)dx^2+2\,ac^2+2\,bc\right)\sqrt{-ac^2-bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2c^3+2\,abc^2+\left(a^2c^2+abc\right)dx^2+b^2c\right)}\right)+2\left(8\,a^3c^7\right.}{}$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^7,x, algorithm="fricas")
```

output
```
[1/192*(3*(24*a^2*b*c^2 + 60*a*b^2*c + 35*b^3)*sqrt(a*c^2 + b*c)*d^3*x^6*l
og(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c
^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*
a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2
 + a*c + b)/(d*x^2 + c)))/x^4) - 4*(8*a^3*c^7 + (8*a^3*c^4 + 118*a^2*b*c^3
 + 215*a*b^2*c^2 + 105*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 + 24*a*b^2*c^5 + 8*b^
3*c^4 + (32*a^2*b*c^4 + 67*a*b^2*c^3 + 35*b^3*c^2)*d^2*x^4 - 14*(a^2*b*c^5
 + 2*a*b^2*c^4 + b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((
a^2*c^7 + 2*a*b*c^6 + b^2*c^5)*x^6), -1/96*(3*(24*a^2*b*c^2 + 60*a*b^2*c +
 35*b^3)*sqrt(-a*c^2 - b*c)*d^3*x^6*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^
2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c
^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(8*a^3*c^7 + (8*a^3
*c^4 + 118*a^2*b*c^3 + 215*a*b^2*c^2 + 105*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 +
 24*a*b^2*c^5 + 8*b^3*c^4 + (32*a^2*b*c^4 + 67*a*b^2*c^3 + 35*b^3*c^2)*d^2
*x^4 - 14*(a^2*b*c^5 + 2*a*b^2*c^4 + b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c)))/((a^2*c^7 + 2*a*b*c^6 + b^2*c^5)*x^6)]
```

3.337.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(3/2)/x**7,x)
```

output
```
Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**7, x)
```

3.337.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. 2(266) = 532.

Time = 0.32 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.83

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx =$$

$$-\frac{(24\,a^2bc^2+60\,ab^2c+35\,b^3)d^3\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{32\,(ac^5+bc^4)\sqrt{(ac+b)c}} - \frac{bd^3\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{c^4}$$

$$-\frac{3\,(8\,a^2bc^4+36\,ab^2c^3+29\,b^3c^2)d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}} - 8\,(6\,a^3bc^4+30\,a^2b^2c^3+41\,ab^3c^2+17\,b^4c)d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}}{48\left(a^4c^8+4\,a^3bc^7+6\,a^2b^2c^6+4\,ab^3c^5+b^4c^4-\frac{(ac^8+bc^7)(adx^2+ac+b)^3}{(dx^2+c)^3}+\frac{3\,(a^2c^8+2\,abc^7+b^2c^6)}{(dx^2+c)}\right.}$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^7,x, algorithm="maxima")
```

output
```
-1/32*(24*a^2*b*c^2 + 60*a*b^2*c + 35*b^3)*d^3*log((c*sqrt((a*d*x^2 + a*c
+ b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2
+ c)) + sqrt((a*c + b)*c)))/((a*c^5 + b*c^4)*sqrt((a*c + b)*c)) - b*d^3*sq
rt((a*d*x^2 + a*c + b)/(d*x^2 + c))/c^4 - 1/48*(3*(8*a^2*b*c^4 + 36*a*b^2*
c^3 + 29*b^3*c^2)*d^3*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - 8*(6*a^3*b
*c^4 + 30*a^2*b^2*c^3 + 41*a*b^3*c^2 + 17*b^4*c)*d^3*((a*d*x^2 + a*c + b)/
(d*x^2 + c))^(3/2) + 3*(8*a^4*b*c^4 + 44*a^3*b^2*c^3 + 83*a^2*b^3*c^2 + 66
*a*b^4*c + 19*b^5)*d^3*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^8 + 4
*a^3*b*c^7 + 6*a^2*b^2*c^6 + 4*a*b^3*c^5 + b^4*c^4 - (a*c^8 + b*c^7)*(a*d*
x^2 + a*c + b)^3/(d*x^2 + c)^3 + 3*(a^2*c^8 + 2*a*b*c^7 + b^2*c^6)*(a*d*x^
2 + a*c + b)^2/(d*x^2 + c)^2 - 3*(a^3*c^8 + 3*a^2*b*c^7 + 3*a*b^2*c^6 + b^
3*c^5)*(a*d*x^2 + a*c + b)/(d*x^2 + c))
```

3.337.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^7}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^7,x, algorithm="giac")
```

output
```
undef
```

3.337.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx=\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^7}\,dx$$

input
```
int((a + b/(c + d*x^2))^(3/2)/x^7,x)
```

output
```
int((a + b/(c + d*x^2))^(3/2)/x^7, x)
```

3.338 $\int x^4 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

3.338.1 Optimal result

Integrand size = 21, antiderivative size = 405

$$\begin{aligned}\int x^4 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx &= \frac{(b^2-14abc+a^2c^2)\,x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5ad^2} \\ &+ \frac{(7b-ac)x(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5d^2} \\ &+ \frac{6ax^3(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5d} - \frac{x^3(b+ac+adx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d} \\ &- \frac{\sqrt{c}(b^2-14abc+a^2c^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5ad^{5/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} \\ &- \frac{c^{3/2}(7b-ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{5d^{5/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\end{aligned}$$

output
```
1/5*(a^2*c^2-14*a*b*c+b^2)*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^2+1/5*(
-a*c+7*b)*x*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2+6/5*a*x^3*(d*x
^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d-x^3*(a*d*x^2+a*c+b)*((a*d*x^2+a*
c+b)/(d*x^2+c))^(1/2)/d-1/5*c^(3/2)*(-a*c+7*b)*(1/(1+d*x^2/c))^(1/2)*(1+d*
x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/
2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^(5/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(
d*x^2+c))^(1/2)-1/5*(a^2*c^2-14*a*b*c+b^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/
c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*
c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^(5/2)/(c*(a*d*x^2+a*c+b)/(a*
c+b)/(d*x^2+c))^(1/2)
```

3.338.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.63 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.77

$$\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(a\sqrt{\frac{d}{c}}x\left(-a^2(c-dx^2)\left(c+dx^2\right)^2+b^2(7c+2dx^2)+3ab(2c^2+3cdx^2+d^2x^4\right.\right.}{}$$

input
```
Integrate[x^4*(a + b/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*Sqrt[d/c]*x*(-(a^2*(c - d*x^2)*(
c + d*x^2)^2) + b^2*(7*c + 2*d*x^2) + 3*a*b*(2*c^2 + 3*c*d*x^2 + d^2*x^4))
 - I*(b^3 - 13*a*b^2*c - 13*a^2*b*c^2 + a^3*c^3)*Sqrt[(b + a*c + a*d*x^2)/
(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b
+ a*c)] + I*b*(b^2 - 6*a*b*c - 7*a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*
c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]
))/(5*a*c^2*(d/c)^(5/2)*(b + a*(c + d*x^2)))
```

3.338.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {2057, 2058, 369, 27, 443, 27, 444, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{2057}$$

$$\int x^4\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\int \frac{x^4(adx^2+b+ac)^{3/2}}{(dx^2+c)^{3/2}}dx}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{369}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\int \frac{3x^2\sqrt{adx^2+b+ac}(2adx^2+b+ac)}{\sqrt{dx^2+c}}dx}{d}-\frac{x^3(ac+adx^2+b)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{3\int \frac{x^2\sqrt{adx^2+b+ac}(2adx^2+b+ac)}{\sqrt{dx^2+c}}dx}{d}-\frac{x^3(ac+adx^2+b)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{443}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{3\left(\frac{\int \frac{dx^2(a(7b-ac)dx^2+(5b-ac)(b+ac))}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{5d}+\frac{2}{5}ax^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}\right)}{d}-\frac{x^3(ac+adx^2+b)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{3\left(\frac{1}{5}\int\frac{x^2\left(a(7b-ac)dx^2+(5b-ac)(b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{2}{5}ax^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}\right)}{d}-\frac{x^3\left(ac+adx^2+b\right)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 444

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{3\left(\frac{1}{5}\left(\frac{x(7b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3d}-\frac{\int\frac{ad\left(c(7b-ac)(b+ac)-\left(b^2-14acb+a^2c^2\right)dx^2\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3ad^2}\right)+\frac{2}{5}ax^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}\right.}{d}\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 27

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{3\left(\frac{1}{5}\left(\frac{x(7b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3d}-\frac{\int\frac{c(7b-ac)(b+ac)-\left(b^2-14acb+a^2c^2\right)dx^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3d}\right)+\frac{2}{5}ax^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}\right)}{d}-\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 406

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{3\left(\frac{1}{5}\left(\frac{x(7b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3d}-\frac{c(7b-ac)(ac+b)\int\frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx-d\left(a^2c^2-14abc+b^2\right)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+}}\right.}{3d}\right.}{d}\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 320

3.338. $\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{3\left(\frac{1}{5}\left(\frac{x(7b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3d}-\frac{\frac{c^{3/2}(7b-ac)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-d\left(a^2c^2-14abc+b^2\right)\int \cdots}{3d}\right.\right.}{d}\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 388

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{3\left(\frac{1}{5}\left(\frac{x(7b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3d}-\frac{\frac{c^{3/2}(7b-ac)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-d\left(a^2c^2-14abc+b^2\right)\left(\frac{x}{\cdots}\right.}{3d}\right.\right.}{d}\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 313

3.338. $\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{3\left(\frac{1}{5}\left(\frac{x(7b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3d}-\frac{\frac{c^{3/2}(7b-ac)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-d\left(a^2c^2-14abc+b^2\right)\left(\frac{x}{\cdots}\right.}{3d}\right.\right.}{d}\right.}{\sqrt{ac+adx^2+b}}$$

input
```
Int[x^4*(a + b/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-((x^3*(b + a*c +
a*d*x^2)^(3/2))/(d*Sqrt[c + d*x^2])) + (3*((2*a*x^3*Sqrt[c + d*x^2]*Sqrt[b
 + a*c + a*d*x^2])/5 + (((7*b - a*c)*x*Sqrt[c + d*x^2]*Sqrt[b + a*c + a*d*
x^2])/(3*d) - (-((b^2 - 14*a*b*c + a^2*c^2)*d*((x*Sqrt[b + a*c + a*d*x^2])
/(a*d*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcTan
[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(b
 + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]))) + (c^(3/2)*(7*b - a*c)*Sqrt
[b + a*c + a*d*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(
Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2
))]))/(3*d))/5))/d))/Sqrt[b + a*c + a*d*x^2]
```

3.338.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] && !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

3.338. $\int x^4 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.338.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1064 vs. $2(441) = 882$.

Time = 11.57 (sec) , antiderivative size = 1065, normalized size of antiderivative = 2.63

method	result	size
risch	Expression too large to display	1065
default	Expression too large to display	1101

input
```
int(x^4*(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/d^2*x*(-a*d*x^2+a*c-2*b)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+
1/5/d^2*(a^2*c^3/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x
^2)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*
d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-7*b^2*c/(-a*d/(a*c+b))^(1
/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b
*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)
/d/c/a)^(1/2))-a*b*c^2/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1
/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(
x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))+5*b^2*c^2*((a*d^2*x
^2+a*c*d+b*d)/c/b*x/d/((x^2+c/d)*(a*d^2*x^2+a*c*d+b*d))^(1/2)+(1/c-(a*c*d+
b*d)/c/b/d)/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(
1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*
c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))+2*a*d/b/c*(a*c^2+b*c)/(-a*d/(a
*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*
c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(2*a*c*d+2*b*d)*(EllipticF(x*(-a*d/(a*c+b
))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-EllipticE(x*(-a*d/(a*c+b))^(1/2),
(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))))-2*(a^2*c^2*d-9*a*b*c*d+b^2*d)*(a*c^2+b*c
)/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^
2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(2*a*c*d+2*b*d)*(EllipticF(x*(-
a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-EllipticE(x*(-a*d/(a...
```

3.338.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.58

$$\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2} dx =$$

$$-\frac{(a^2c^3-14\,abc^2+b^2c)\sqrt{a}x\sqrt{-\frac{c}{d}}E\left(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ac+b}{ac}\right)-(a^2c^3-14\,abc^2+b^2c+(a^2c^2-6\,abc-7\,b^2)d)\cdots}{5\cdots}$$

input
```
integrate(x^4*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/5*((a^2*c^3 - 14*a*b*c^2 + b^2*c)*sqrt(a)*x*sqrt(-c/d)*elliptic_e(arcsi
n(sqrt(-c/d)/x), (a*c + b)/(a*c)) - (a^2*c^3 - 14*a*b*c^2 + b^2*c + (a^2*c
^2 - 6*a*b*c - 7*b^2)*d)*sqrt(a)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)
/x), (a*c + b)/(a*c)) - (a^2*d^3*x^6 + 2*a*b*d^2*x^4 + a^2*c^3 - 14*a*b*c^
2 - (7*a*b*c - b^2)*d*x^2 + b^2*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/
(a*d^3*x)
```

3.338.6 Sympy [F]

$$\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=\int x^4\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**4*(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(x**4*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

3.338.7 Maxima [F]

$$\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=\int\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^4\,dx$$

input
```
integrate(x^4*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^2 + c))^(3/2)*x^4, x)
```

3.338.8 Giac [F]

$$\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=\int\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^4\,dx$$

input
```
integrate(x^4*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^2 + c))^(3/2)*x^4, x)
```

3.338.9 Mupad [F(-1)]

Timed out.

$$\int x^4 \left(a + \frac{b}{c + dx^2}\right)^{3/2} dx = \int x^4 \left(a + \frac{b}{d\,x^2 + c}\right)^{3/2} dx$$

input
```
int(x^4*(a + b/(c + d*x^2))^(3/2),x)
```

output
```
int(x^4*(a + b/(c + d*x^2))^(3/2), x)
```

3.339 $\int x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx$

3.339.1 Optimal result

Integrand size = 21, antiderivative size = 331

$$\int x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{(7b-ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3d}+\frac{4ax(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3d}-\frac{x(b+ac+adx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d}-\frac{\sqrt{c}(7b-ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}+\frac{\sqrt{c}(3b-ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

output
```
1/3*(-a*c+7*b)*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d+4/3*a*x*(d*x^2+c)*((a
*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d-x*(a*d*x^2+a*c+b)*((a*d*x^2+a*c+b)/(d*x^2
+c))^(1/2)/d-1/3*(-a*c+7*b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellipt
icE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*((a*d*x
^2+a*c+b)/(d*x^2+c))^(1/2)/d^(3/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(
1/2)+1/3*(-a*c+3*b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^
(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*((a*d*x^2+a*c+b
)/(d*x^2+c))^(1/2)/d^(3/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

3.339.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.42 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.77

$$\int x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{d}{c}}x\left(-3b^2-2ab(c+dx^2)+a^2(c+dx^2)^2\right)+i(-7b^2-6abc+a^2c^2)\sqrt{\frac{b+\cdots}{}}\right.}{}$$

input
```
Integrate[x^2*(a + b/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[d/c]*x*(-3*b^2 - 2*a*b*(c + d
*x^2) + a^2*(c + d*x^2)^2) + I*(-7*b^2 - 6*a*b*c + a^2*c^2)*Sqrt[(b + a*c
+ a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x]
, (a*c)/(b + a*c)] + (4*I)*b*(b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]
*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/
(3*d*Sqrt[d/c]*(b + a*(c + d*x^2)))
```

3.339.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2057, 2058, 369, 403, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int x^2\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\int \frac{x^2\left(adx^2+b+ac\right)^{3/2}}{\left(dx^2+c\right)^{3/2}}dx}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 369

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\int \frac{\sqrt{adx^2+b+ac}\left(4adx^2+b+ac\right)}{\sqrt{dx^2+c}}dx}{d}-\frac{x\left(ac+adx^2+b\right)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 403

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\frac{\int \frac{d\left(a(7b-ac)dx^2+(3b-ac)(b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3d}+\frac{4}{3}ax\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{d}-\frac{x\left(ac+adx^2+b\right)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 27

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\frac{1}{3}\int \frac{a(7b-ac)dx^2+(3b-ac)(b+ac)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{4}{3}ax\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{d}-\frac{x\left(ac+adx^2+b\right)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 406

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\frac{1}{3}\left((3b-ac)(ac+b)\int \frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+ad(7b-ac)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)+\frac{4}{3}ax\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{d}-\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 320

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\frac{1}{3}\left(ad(7b-ac)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{\sqrt{c}(3b-ac)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)+\frac{4}{3}ax\sqrt{c+dx^2}\sqrt{ac}}{d}\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 388

3.339. $\int x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\frac{1}{3}\left(ad(7b-ac)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{(dx^2+c)^{3/2}}dx}{ad}\right)+\frac{\sqrt{c}(3b-ac)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}\right)+\frac{4}{3}ax}{d}\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 313

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\frac{1}{3}\left(ad(7b-ac)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}\right)+\frac{\sqrt{c}(3b-ac)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}\right)}{d}\right.}{\sqrt{ac+adx^2+b}}$$

input
```
Int[x^2*(a + b/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-((x*(b + a*c + a*
d*x^2)^(3/2))/(d*Sqrt[c + d*x^2])) + ((4*a*x*Sqrt[c + d*x^2]*Sqrt[b + a*c
+ a*d*x^2])/3 + (a*(7*b - a*c)*d*((x*Sqrt[b + a*c + a*d*x^2])/(a*d*Sqrt[c
+ d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/
Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*
x^2))/((b + a*c)*(c + d*x^2))])) + (Sqrt[c]*(3*b - a*c)*Sqrt[b + a*c + a*d
*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d]*Sqrt[c
 + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]))/3)/d))/S
qrt[b + a*c + a*d*x^2]
```

3.339. $\int x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx$

3.339.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

3.339. $\int x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.339.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 819 vs. $2(371) = 742$.

Time = 9.35 (sec) , antiderivative size = 820, normalized size of antiderivative = 2.48

method	result
default	$\frac{\left(\sqrt{(ad\,x^2+ac+b)(d\,x^2+c)}\,\sqrt{-\frac{ad}{ac+b}}\,a^2d^2x^5+2\sqrt{(ad\,x^2+ac+b)(d\,x^2+c)}\,\sqrt{-\frac{ad}{ac+b}}\,a^2cd\,x^3+\sqrt{(ad\,x^2+ac+b)(d\,x^2+c)}\,\sqrt{-\frac{ad}{ac+b}}\,abd\,x}{}$
risch	Expression too large to display

input
```
int(x^2*(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*d^2*x^5+2*
((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c*d*x^3+((a*d*x
^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*d*x^3-((a*d*x^2+a*c+b)
*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
pticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*c^2-3*(a*d^2*x^4+2*a
*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*d*x^3+((a*d*x^2
+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c^2*x-5*((a*d*x^2+a*c+b)
*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
pticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c+7*((a*d*x^2+a*c+b)
*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
pticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c+((a*d*x^2+a*c+b)*(
d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*c*x+3*((a*d*x^2+a*c+b)*(d*x^2+c))
^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a
*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b^2-3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^
2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*c*x-3*(a*d^2*x^4+2*a*c*d*x^2+b
*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*b^2*x)*((a*d*x^2+a*c+b)/(d*x^
2+c))^(1/2)/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b
))^(1/2)/(a*d*x^2+a*c+b)
```

3.339.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.62

$$\int x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{(a^2c^3-7\,abc^2)\sqrt{a}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ac+b}{ac})-(a^2c^3-7\,abc^2+(a^2c^2-2\,abc-3\,b}{3\,a}$$

input
```
integrate(x^2*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*((a^2*c^3 - 7*a*b*c^2)*sqrt(a)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/
d)/x), (a*c + b)/(a*c)) - (a^2*c^3 - 7*a*b*c^2 + (a^2*c^2 - 2*a*b*c - 3*b^
2)*d)*sqrt(a)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c
)) + (a^2*c*d^2*x^4 + 4*a*b*c*d*x^2 - a^2*c^3 + 7*a*b*c^2)*sqrt((a*d*x^2 +
 a*c + b)/(d*x^2 + c)))/(a*c*d^2*x)
```

3.339.6 Sympy [F]

$$\int x^2 \left(a + \frac{b}{c + dx^2}\right)^{3/2} dx = \int x^2 \left(\frac{ac + adx^2 + b}{c + dx^2}\right)^{\frac{3}{2}} dx$$

input
```
integrate(x**2*(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(x**2*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

3.339.7 Maxima [F]

$$\int x^2 \left(a + \frac{b}{c + dx^2}\right)^{3/2} dx = \int \left(a + \frac{b}{dx^2 + c}\right)^{\frac{3}{2}} x^2 \, dx$$

input
```
integrate(x^2*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^2 + c))^(3/2)*x^2, x)
```

3.339.8 Giac [F]

$$\int x^2 \left(a + \frac{b}{c + dx^2}\right)^{3/2} dx = \int \left(a + \frac{b}{dx^2 + c}\right)^{\frac{3}{2}} x^2 \, dx$$

input
```
integrate(x^2*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^2 + c))^(3/2)*x^2, x)
```

3.339.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \int x^2 \left(a+\frac{b}{d x^2+c}\right)^{3/2} dx$$

input
```
int(x^2*(a + b/(c + d*x^2))^(3/2),x)
```

output
```
int(x^2*(a + b/(c + d*x^2))^(3/2), x)
```

3.340 $\int \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

3.340.1 Optimal result

Integrand size = 17, antiderivative size = 260

$$\int \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \frac{bx\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c}-\frac{(b-ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c}+\frac{(b-ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}+\frac{a\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

output
```
b*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c-(-a*c+b)*x*((a*d*x^2+a*c+b)/(d*x^2
+c))^(1/2)/c+(-a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*
d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x
^2+c))^(1/2)/c^(1/2)/d^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+a
*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*
x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/
d^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

3.340.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.42 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.88

$$\int \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \frac{\sqrt{\frac{d}{c}}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(b\sqrt{\frac{d}{c}}x(b+a(c+dx^2))+i(b^2-a^2c^2)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\right.\right.}{d\left(b+a\left(c\right.\right.}$$

input

```
Integrate[(a + b/(c + d*x^2))^(3/2),x]
```

output

```
(Sqrt[d/c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b*Sqrt[d/c]*x*(b + a*(c
+ d*x^2)) + I*(b^2 - a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 +
 (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] - I*b*(b +
a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*A
rcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(d*(b + a*(c + d*x^2)))
```

3.340.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.25, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {2057, 2058, 315, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2} dx$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\int \frac{(adx^2+b+ac)^{3/2}}{(dx^2+c)^{3/2}}dx}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{ 315}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\int \frac{ad\left(c(b+ac)-(b-ac)dx^2\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{cd}+\frac{bx\sqrt{ac+adx^2+b}}{c\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 27

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{a\int \frac{c(b+ac)-(b-ac)dx^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c}+\frac{bx\sqrt{ac+adx^2+b}}{c\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 406

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{a\left(c(ac+b)\int \frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx-d(b-ac)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{c}+\frac{bx\sqrt{ac+adx^2+b}}{c\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 320

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{a\left(\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-d(b-ac)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{c}+\frac{bx\sqrt{ac+adx^2+b}}{c\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 388

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{a\left(\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-d(b-ac)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)\right)}{c}+\frac{bx\sqrt{ac+a\cdots}}{c\sqrt{c+\cdots}}\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 313

3.340. $\int \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{a\left(\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-d(b-ac)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right.\right.}{c}\right.}{\sqrt{ac+adx^2+b}}$$

input
```
Int[(a + b/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*((b*x*Sqrt[b + a*c
+ a*d*x^2])/(c*Sqrt[c + d*x^2]) + (a*(-((b - a*c)*d*((x*Sqrt[b + a*c + a*d
*x^2])/(a*d*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[
ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt
[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]))) + (c^(3/2)*Sqrt[b + a
*c + a*d*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d
]*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])))
/c))/Sqrt[b + a*c + a*d*x^2]
```

3.340.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.340.4 Maple [A] (verified)

Time = 3.37 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.98

method	result
default	$\frac{\left(\sqrt{(ad\,x^2+ac+b)(d\,x^2+c)}\,\sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\,\sqrt{\frac{d\,x^2+c}{c}}\,E\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)a^2c^2+\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{-\frac{ad}{ac+b}}\,abd\,x^{3}\right.}{}$

input
```
int((a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*c^2
+(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*
d*x^3+2*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*
((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*
a*b*c-((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((
d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*
b*c+(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*a
*b*c*x+(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2
)*b^2*x)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+
a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/c/(a*d*x^2+a*c+b)
```

3.340.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.62

$$\int \left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx =$$
$$-\frac{(ac^2-bc)\sqrt{a}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ac+b}{ac}) - (ac^2-bc+(ac+b)d)\sqrt{a}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ac+b}{ac})}{cdx}$$

input
```
integrate((a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
-((a*c^2 - b*c)*sqrt(a)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), (a*c
 + b)/(a*c)) - (a*c^2 - b*c + (a*c + b)*d)*sqrt(a)*x*sqrt(-c/d)*elliptic_f
(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c)) - (a*c*d*x^2 + a*c^2 - b*c)*sqrt((
a*d*x^2 + a*c + b)/(d*x^2 + c)))/(c*d*x)
```

3.340.6 Sympy [F]

$$\int \left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx = \int \left(a+\frac{b}{c+dx^2}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral((a + b/(c + d*x**2))**(3/2), x)
```

3.340.7 Maxima [F]

$$\int \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \int \left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^2 + c))^(3/2), x)
```

3.340.8 Giac [F]

$$\int \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \int \left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^2 + c))^(3/2), x)
```

3.340.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \int \left(a+\frac{b}{d\,x^2+c}\right)^{3/2} dx$$

input
```
int((a + b/(c + d*x^2))^(3/2),x)
```

output
```
int((a + b/(c + d*x^2))^(3/2), x)
```

3.341 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2} dx$

3.341.1 Optimal result

Integrand size = 21, antiderivative size = 312

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2} dx = \frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx} + \frac{(2b+ac)dx\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c^2} - \frac{(2b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c^2x} - \frac{(2b+ac)\sqrt{d}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{c^{3/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{a\sqrt{d}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{c}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

output
```
b*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c/x+(a*c+2*b)*d*x*((a*d*x^2+a*c+b)/(d*
x^2+c))^(1/2)/c^2-(a*c+2*b)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^
2/x-(a*c+2*b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/
c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*d^(1/2)*((a*d*x^2+a*c+b)/(d*x
^2+c))^(1/2)/c^(3/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+a*(1/(1+d
*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(
1/2),(b/(a*c+b))^(1/2))*d^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^(1/2)/
(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

3.341.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.45 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx = -\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{d}{c}}\left(2ab(c+dx^2)^2+a^2c(c+dx^2)^2+b^2(c+2dx^2)\right)+i(2b^2+3abc+a^2c^2)\,dx\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{\right.}{c^2\sqrt{\frac{d}{c}}x\,(b+}$$

input
```
Integrate[(a + b/(c + d*x^2))^(3/2)/x^2,x]
```

output
```
-((Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[d/c]*(2*a*b*(c + d*x^2)^2 +
 a^2*c*(c + d*x^2)^2 + b^2*(c + 2*d*x^2)) + I*(2*b^2 + 3*a*b*c + a^2*c^2)*
d*x*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*Ar
cSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] - (2*I)*b*(b + a*c)*d*x*Sqrt[(b + a*c
 + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x
], (a*c)/(b + a*c)]))/(c^2*Sqrt[d/c]*x*(b + a*(c + d*x^2))))
```

3.341.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.26, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {2057, 2058, 370, 25, 27, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\int \frac{\left(adx^2+b+ac\right)^{3/2}}{x^2\left(dx^2+c\right)^{3/2}}dx}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{370}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{b\sqrt{ac+adx^2+b}}{cx\sqrt{c+dx^2}}-\frac{\int -\frac{(b+ac)d\left(adx^2+2b+ac\right)}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{cd}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\int \frac{(b+ac)d\left(adx^2+2b+ac\right)}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{cd}+\frac{b\sqrt{ac+adx^2+b}}{cx\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{(ac+b)\int \frac{adx^2+2b+ac}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c}+\frac{b\sqrt{ac+adx^2+b}}{cx\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{445}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{(ac+b)\left(-\frac{\int -\frac{ad\left((2b+ac)dx^2+c(b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{(ac+2b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{c}+\frac{b\sqrt{ac+adx^2+b}}{cx\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{(ac+b)\left(\frac{\int \frac{ad\left((2b+ac)dx^2+c(b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{(ac+2b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{c}+\frac{b\sqrt{ac+adx^2+b}}{cx\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{27}$$

3.341. $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}dx$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{(ac+b)\left(\frac{ad\int \frac{(2b+ac)dx^2+c(b+ac)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{(ac+2b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{c}+\frac{b\sqrt{ac+adx^2+b}}{cx\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 406

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{(ac+b)\left(\frac{ad\left(c(ac+b)\int\frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+d(ac+2b)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{c(ac+b)}-\frac{(ac+2b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{c}\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 320

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{(ac+b)\left(\frac{ad\left(d(ac+2b)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{c(ac+b)}-\frac{(ac+2b)\sqrt{c+dx^2}\sqrt{\ldots}}{cx(ac+\ldots}\right.}{c}\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 388

3.341. $\displaystyle\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{(ac+b)\left(\frac{ad\left(d(ac+2b)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{(dx^2+c)^{3/2}}dx}{ad}\right)+\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}\right)}{c(ac+b)}-\frac{(ac\cdots}{\cdots}\right)}{c}\cdots\right)}{\sqrt{ac+adx^2+b}}$$

$\big\downarrow$ 313

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{(ac+b)\left(\frac{ad\left(\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}+d(ac+2b)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\left(\frac{\sqrt{d}\cdots}{\sqrt{\cdots}}\right)\cdots\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(ac+ad\cdots)}{(ac+b)(c\cdots)}}}\cdots\right)\right)}{c(ac+b)}\cdots\right)}{c}\cdots\right)}{\sqrt{ac+adx^2+b}}$$

input
```
Int[(a + b/(c + d*x^2))^(3/2)/x^2,x]
```

3.341. $\displaystyle\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx$

output
```
(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*((b*Sqrt[b + a*c +
a*d*x^2])/(c*x*Sqrt[c + d*x^2]) + ((b + a*c)*(-(((2*b + a*c)*Sqrt[c + d*x^
2]*Sqrt[b + a*c + a*d*x^2])/(c*(b + a*c)*x)) + (a*d*((2*b + a*c)*d*((x*Sqr
t[b + a*c + a*d*x^2])/(a*d*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*
x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[
c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])) + (c^(3
/2)*Sqrt[b + a*c + a*d*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b +
a*c)])/(Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c
 + d*x^2))])))/(c*(b + a*c))))/c))/Sqrt[b + a*c + a*d*x^2]
```

3.341.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

3.341. $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx$

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.341.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 872 vs. $2(360) = 720$.

Time = 9.40 (sec) , antiderivative size = 873, normalized size of antiderivative = 2.80

method	result
default	$-\frac{\left(\sqrt{(a d x^2+ac+b)(d x^2+c)}\,\sqrt{-\frac{ad}{ac+b}}\,a^2 c d^2 x^4+\sqrt{(a d x^2+ac+b)(d x^2+c)}\,\sqrt{-\frac{ad}{ac+b}}\,a b d^2 x^4-\sqrt{(a d x^2+ac+b)(d x^2+c)}\,\sqrt{\frac{a d x^2+ac}{ac+b}} \cdots\right.}{\cdots}$
risch	$-\frac{(ac+b)(d x^2+c)\sqrt{\frac{a d x^2+ac+b}{d x^2+c}}}{c^2 x}+\frac{d\left(\frac{a^2 c^2\sqrt{1+\frac{a d x^2}{ac+b}}\,\sqrt{1+\frac{d x^2}{c}}\,F\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{-1+\frac{2acd+bd}{dca}}\right)}{\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a d^2 x^4+2acd x^2+bd x^2+a c^2+bc}}-\frac{2a^2 cd\left(a c^2+bc\right)\sqrt{1+\frac{a d x^2}{ac+b}}\,\sqrt{1+\frac{d x^2}{c}}\,\left(\cdots\right.}{\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a \cdots}}\right.}{\cdots}$

3.341. $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx$

input
```
int((a+b/(d*x^2+c))^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c*d^2*x^4+((a
*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*d^2*x^4-((a*d*x^2+
a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2
)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*c^2*d*x+(a*d^2
*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*d^2*x^4
+2*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c^2*d*x^2+((
a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)
/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c*d*x-
2*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^
2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c*
d*x+3*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*c*d*x^2+(
a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*c*
d*x^2+((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c^3+((a*d
*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*b^2*d*x^2+(a*d^2*x^4+2*a
*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*b^2*d*x^2+2*((a*d*x
^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*c^2+((a*d*x^2+a*c+b)*(
d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*b^2*c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1
/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/x
/c^2/(a*d*x^2+a*c+b)
```

3.341.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx = \frac{(a^2c+2\,ab)\sqrt{-\frac{ad}{ac+b}}d^2x\sqrt{\frac{ac^2+bc}{d^2}}E(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right) \mid \frac{ac+b}{ac}) - ((a^2c+2\,ab)d^2+(a^2$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
((a^2*c + 2*a*b)*sqrt(-a*d/(a*c + b))*d^2*x*sqrt((a*c^2 + b*c)/d^2)*ellipt
ic_e(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - ((a^2*c + 2*a*b)*d
^2 + (a^2*c^2 + 2*a*b*c + b^2)*d)*sqrt(-a*d/(a*c + b))*x*sqrt((a*c^2 + b*c
)/d^2)*elliptic_f(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - (a^2*
c^3 + 2*a*b*c^2 + (a^2*c^2 + 3*a*b*c + 2*b^2)*d*x^2 + b^2*c)*sqrt((a*d*x^2
 + a*c + b)/(d*x^2 + c)))/((a*c^3 + b*c^2)*x)
```

3.341.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx=\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(3/2)/x**2,x)
```

output
```
Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**2, x)
```

3.341.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^2 + c))^(3/2)/x^2, x)
```

3.341.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^2 + c))^(3/2)/x^2, x)
```

3.341.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx=\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^2}\,dx$$

input
```
int((a + b/(c + d*x^2))^(3/2)/x^2,x)
```

output
```
int((a + b/(c + d*x^2))^(3/2)/x^2, x)
```

3.342 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx$

3.342.1 Optimal result

Integrand size = 21, antiderivative size = 388

$$\begin{aligned}\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx &= \frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx^3}-\frac{(8b+ac)d^2x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^3}\\ &-\frac{(4b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2x^3}+\frac{(8b+ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^3x}\\ &+\frac{(8b+ac)d^{3/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3c^{5/2}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\\ &-\frac{a(4b+ac)d^{3/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3c^{3/2}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\end{aligned}$$

output
```
b*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c/x^3-1/3*(a*c+8*b)*d^2*x*((a*d*x^2+a*
c+b)/(d*x^2+c))^(1/2)/c^3-1/3*(a*c+4*b)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+
c))^(1/2)/c^2/x^3+1/3*(a*c+8*b)*d*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1
/2)/c^3/x+1/3*(a*c+8*b)*d^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*El
lipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*((a*d*x^2+a
*c+b)/(d*x^2+c))^(1/2)/c^(5/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
-1/3*a*(a*c+4*b)*d^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF
(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)/(
d*x^2+c))^(1/2)/c^(3/2)/(a*c+b)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2
)
```

3.342.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.92 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(-b^2c^2-2abc^3-a^2c^4+4b^2cdx^2+3abc^2dx^2-a^2c^3dx^2+8b^2d^2x^4+13a\right.}{}$$

input
```
Integrate[(a + b/(c + d*x^2))^(3/2)/x^4,x]
```

output
```
(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-(b^2*c^2) - 2*a*b*c^3 - a^2*c^4 +
 4*b^2*c*d*x^2 + 3*a*b*c^2*d*x^2 - a^2*c^3*d*x^2 + 8*b^2*d^2*x^4 + 13*a*b*
c*d^2*x^4 + a^2*c^2*d^2*x^4 + 8*a*b*d^3*x^6 + a^2*c*d^3*x^6 + I*c*(8*b^2 +
 9*a*b*c + a^2*c^2)*d*Sqrt[d/c]*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sq
rt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] - I*b
*c*(8*b + 5*a*c)*d*Sqrt[d/c]*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[
1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(3*c^3
*x^3*(b + a*(c + d*x^2)))
```

3.342.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.13, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {2057, 2058, 370, 25, 27, 445, 27, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\int \frac{(adx^2+b+ac)^{3/2}}{x^4(dx^2+c)^{3/2}}dx}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{370}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{b\sqrt{ac+adx^2+b}}{cx^3\sqrt{c+dx^2}}-\frac{\int -\frac{d\left(a(3b+ac)dx^2+(b+ac)(4b+ac)\right)}{x^4\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{cd}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\int \frac{d\left(a(3b+ac)dx^2+(b+ac)(4b+ac)\right)}{x^4\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{cd}+\frac{b\sqrt{ac+adx^2+b}}{cx^3\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\int \frac{a(3b+ac)dx^2+(b+ac)(4b+ac)}{x^4\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c}+\frac{b\sqrt{ac+adx^2+b}}{cx^3\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{445}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{-\frac{\int \frac{(b+ac)d\left(a(4b+ac)dx^2+(b+ac)(8b+ac)\right)}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3c(ac+b)}-\frac{(ac+4b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3}}{c}+\frac{b\sqrt{ac+adx^2+b}}{cx^3\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{-\frac{d\int \frac{a(4b+ac)dx^2+(b+ac)(8b+ac)}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3c}-\frac{(ac+4b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3}}{c}+\frac{b\sqrt{ac+adx^2+b}}{cx^3\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{445}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{-\frac{d\left(-\frac{\int -\frac{a(b+ac)d\left((8b+ac)dx^2+c(4b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{(ac+8b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx}\right)}{3c}-\frac{(ac+4b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3}}{c}+\frac{b\sqrt{ac-}}{cx^3\sqrt{}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{25}$$

3.342. $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{-\frac{d\left(\frac{\int \frac{a(b+ac)d\left((8b+ac)dx^2+c(4b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{(ac+8b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx}\right)}{3c}-\frac{(ac+4b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3}}{c}+\frac{b\sqrt{ac+ac}}{cx^3\sqrt{c+}}\right.}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{-\frac{d\left(\frac{ad\int \frac{(8b+ac)dx^2+c(4b+ac)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c}-\frac{(ac+8b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx}\right)}{3c}-\frac{(ac+4b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3}}{c}+\frac{b\sqrt{ac+adx^2+b}}{cx^3\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \; 406$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{-\frac{d\left(\frac{ad\left(c(ac+4b)\int \frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+d(ac+8b)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{c}-\frac{(ac+8b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx}\right)}{3c}-\frac{(ac}{}}{c}\right.}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \; 320$$

3.342. $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{-\frac{d\left(\frac{ad\left(d(ac+8b)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{c^{3/2}(ac+4b)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}(ac+b)\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{c}-\frac{(ac+8b)\sqrt{c+dx}}{c}\right.}{3c}}{c}\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 388

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{-\frac{d\left(\frac{ad\left(d(ac+8b)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)+\frac{c^{3/2}(ac+4b)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}(ac+b)\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{c}-\right.}{3c}}{c}\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 313

3.342. $\int\frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{-\frac{d\left(\frac{ad\left(\frac{c^{3/2}(ac+4b)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}(ac+b)\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+d(ac+8b)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\left(\cdots\right.\right.}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac\cdots\right.}{(ac+b\cdots}}}\right.\right.}{c}\cdots\right.}{3c}}{c}\right.\cdots}{\sqrt{ac+adx^2+b}}$$

input
```
Int[(a + b/(c + d*x^2))^(3/2)/x^4,x]
```

output
```
(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*((b*Sqrt[b + a*c +
a*d*x^2])/(c*x^3*Sqrt[c + d*x^2]) + (-1/3*((4*b + a*c)*Sqrt[c + d*x^2]*Sqr
t[b + a*c + a*d*x^2])/(c*x^3) - (d*(-(((8*b + a*c)*Sqrt[c + d*x^2]*Sqrt[b
+ a*c + a*d*x^2])/(c*x)) + (a*d*((8*b + a*c)*d*((x*Sqrt[b + a*c + a*d*x^2]
)/(a*d*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcTa
n[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(
b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])) + (c^(3/2)*(4*b + a*c)*Sqrt
[b + a*c + a*d*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(
(b + a*c)*Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*
(c + d*x^2))])))/c))/(3*c))/c))/Sqrt[b + a*c + a*d*x^2]
```

3.342.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.342. $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx$

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

3.342. $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx$

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.342.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1036 vs. $2(424) = 848$.

Time = 10.72 (sec) , antiderivative size = 1037, normalized size of antiderivative = 2.67

method	result	size
default	Expression too large to display	1037
risch	Expression too large to display	1122

input
```
int((a+b/(d*x^2+c))^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c*d^3*x^
6-5*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*d^3*x^6+((a
*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/
c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*c^2*d^2
*x^3-3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2
)*a*b*d^3*x^6-((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c
^2*d^2*x^4-4*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(
1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1
/2))*a*b*c*d^2*x^3+8*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a
*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)
/a/c)^(1/2))*a*b*c*d^2*x^3-10*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c
+b))^(1/2)*a*b*c*d^2*x^4-3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)
*(-a*d/(a*c+b))^(1/2)*a*b*c*d^2*x^4+((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*
d/(a*c+b))^(1/2)*a^2*c^3*d*x^2-5*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(
a*c+b))^(1/2)*b^2*d^2*x^4-3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2
)*(-a*d/(a*c+b))^(1/2)*b^2*d^2*x^4-3*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a
*d/(a*c+b))^(1/2)*a*b*c^2*d*x^2+((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a
*c+b))^(1/2)*a^2*c^4-4*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1
/2)*b^2*c*d*x^2+2*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a
*b*c^3+((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*b^2*c^2)*...
```

3.342.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.73

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx =$$

$$-\frac{(a^2c+8\,ab)\sqrt{-\frac{ad}{ac+b}}d^3x^3\sqrt{\frac{ac^2+bc}{d^2}}E(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right)\mid\frac{ac+b}{ac})-((a^2c+8\,ab)d^3+(a^2c^2+5\,abc+4\,b^2)d^2)}{}$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^4,x, algorithm="fricas")
```

output
```
-1/3*((a^2*c + 8*a*b)*sqrt(-a*d/(a*c + b))*d^3*x^3*sqrt((a*c^2 + b*c)/d^2)
*elliptic_e(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - ((a^2*c + 8
*a*b)*d^3 + (a^2*c^2 + 5*a*b*c + 4*b^2)*d^2)*sqrt(-a*d/(a*c + b))*x^3*sqrt
((a*c^2 + b*c)/d^2)*elliptic_f(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(
a*c)) - ((a^2*c^2 + 9*a*b*c + 8*b^2)*d^2*x^4 - a^2*c^4 - 2*a*b*c^3 - b^2*c
^2 + 4*(a*b*c^2 + b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a
*c^4 + b*c^3)*x^3)
```

3.342.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx=\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(3/2)/x**4,x)
```

output
```
Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**4, x)
```

3.342.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^2 + c))^(3/2)/x^4, x)
```

3.342.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^4,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^2 + c))^(3/2)/x^4, x)
```

3.342.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx=\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^4}\,dx$$

input
```
int((a + b/(c + d*x^2))^(3/2)/x^4,x)
```

output
```
int((a + b/(c + d*x^2))^(3/2)/x^4, x)
```

3.343 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx$

3.343.1 Optimal result

Integrand size = 21, antiderivative size = 494

$$\begin{aligned}\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx &= \frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx^5}+\frac{(16b^2+16abc+a^2c^2)\,d^3x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^4(b+ac)}\\
&-\frac{(6b+ac)\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^2x^5}+\frac{(8b+ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^3x^3}\\
&-\frac{(16b^2+16abc+a^2c^2)\,d^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^4(b+ac)x}\\
&-\frac{(16b^2+16abc+a^2c^2)\,d^{5/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5c^{7/2}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\\
&+\frac{a(8b+ac)d^{5/2}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{5c^{5/2}(b+ac)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\end{aligned}$$

output
```
b*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c/x^5+1/5*(a^2*c^2+16*a*b*c+16*b^2)*d^
3*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^4/(a*c+b)-1/5*(a*c+6*b)*(d*x^2+c)*
((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^2/x^5+1/5*(a*c+8*b)*d*(d*x^2+c)*((a*d*
x^2+a*c+b)/(d*x^2+c))^(1/2)/c^3/x^3-1/5*(a^2*c^2+16*a*b*c+16*b^2)*d^2*(d*x
^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^4/(a*c+b)/x-1/5*(a^2*c^2+16*a*b*
c+16*b^2)*d^(5/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1
/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c
))^(1/2)/c^(7/2)/(a*c+b)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+1/5*a
*(a*c+8*b)*d^(5/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(
1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+
c))^(1/2)/c^(5/2)/(a*c+b)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

3.343.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.23 (sec) , antiderivative size = 472, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx =$$

$$-\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(b^3c^3+3ab^2c^4+3a^2bc^5+a^3c^6-2b^3c^2dx^2-3ab^2c^3dx^2+a^3c^5dx^2+8b^3cd^2x^4+13ab^2c^2d^2x^4\right.}{}$$

input
```
Integrate[(a + b/(c + d*x^2))^(3/2)/x^6,x]
```

output
```
-1/5*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b^3*c^3 + 3*a*b^2*c^4 + 3*a^2
*b*c^5 + a^3*c^6 - 2*b^3*c^2*d*x^2 - 3*a*b^2*c^3*d*x^2 + a^3*c^5*d*x^2 + 8
*b^3*c*d^2*x^4 + 13*a*b^2*c^2*d^2*x^4 + 5*a^2*b*c^3*d^2*x^4 + 16*b^3*d^3*x
^6 + 40*a*b^2*c*d^3*x^6 + 24*a^2*b*c^2*d^3*x^6 + a^3*c^3*d^3*x^6 + 16*a*b^
2*d^4*x^8 + 16*a^2*b*c*d^4*x^8 + a^3*c^2*d^4*x^8 + I*c*(16*b^3 + 32*a*b^2*
c + 17*a^2*b*c^2 + a^3*c^3)*d^2*Sqrt[d/c]*x^5*Sqrt[(b + a*c + a*d*x^2)/(b
+ a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a
*c)] - (8*I)*b*c*(2*b^2 + 3*a*b*c + a^2*c^2)*d^2*Sqrt[d/c]*x^5*Sqrt[(b + a
*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]
*x], (a*c)/(b + a*c)]))/(c^4*(b + a*c)*x^5*(b + a*(c + d*x^2)))
```

3.343.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.05, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.762, Rules used = {2057, 2058, 370, 25, 27, 445, 27, 445, 27, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}{x^6}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\int \frac{\left(adx^2+b+ac\right)^{3/2}}{x^6\left(dx^2+c\right)^{3/2}}dx}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{370}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{b\sqrt{ac+adx^2+b}}{cx^5\sqrt{c+dx^2}}-\frac{\int -\frac{d\left(a(5b+ac)dx^2+(b+ac)(6b+ac)\right)}{x^6\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{cd}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\int \frac{d\left(a(5b+ac)dx^2+(b+ac)(6b+ac)\right)}{x^6\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{cd}+\frac{b\sqrt{ac+adx^2+b}}{cx^5\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{\int \frac{a(5b+ac)dx^2+(b+ac)(6b+ac)}{x^6\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c}+\frac{b\sqrt{ac+adx^2+b}}{cx^5\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$$\Big\downarrow \text{445}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{-\frac{\int \frac{3(b+ac)d\left(a(6b+ac)dx^2+(b+ac)(8b+ac)\right)}{x^4\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{5c(ac+b)}-\frac{(ac+6b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5cx^5}}{c}+\frac{b\sqrt{ac+adx^2+b}}{cx^5\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 27

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{-\frac{3d\int \frac{a(6b+ac)dx^2+(b+ac)(8b+ac)}{x^4\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{5c}-\frac{(ac+6b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5cx^5}}{c}+\frac{b\sqrt{ac+adx^2+b}}{cx^5\sqrt{c+dx^2}}\right)}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 445

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{-\frac{3d\left(-\frac{\int \frac{(b+ac)d\left(16b^2+16acb+a^2c^2+a(8b+ac)dx^2\right)}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3c(ac+b)}-\frac{(ac+8b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3}\right)}{5c}-\frac{(ac+6b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5cx^5}}{c}\cdots\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 27

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{-\frac{3d\left(-\frac{d\int \frac{16b^2+16acb+a^2c^2+a(8b+ac)dx^2}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3c}-\frac{(ac+8b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3cx^3}\right)}{5c}-\frac{(ac+6b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5cx^5}}{c}+\frac{b\sqrt{ac\cdots}}{cx^5\cdots}\right.}{\sqrt{ac+adx^2+b}}$$

$\downarrow$ 445

3.343. $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{-\frac{3d\left(-\frac{d\left(-\frac{\int -\frac{ad\left(\left(16b^2+16acb+a^2c^2\right)dx^2+c(b+ac)(8b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{\left(a^2c^2+16abc+16b^2\right)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{3c}-\frac{(ac+8b)\sqrt{\ldots}}{\ldots}\right)}{5c}}{c}\ldots\right)}{\sqrt{ac+adx^2+b}}$$

$\Big\downarrow$ 25

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{-\frac{3d\left(-\frac{d\left(\frac{\int \frac{ad\left(\left(16b^2+16acb+a^2c^2\right)dx^2+c(b+ac)(8b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{\left(a^2c^2+16abc+16b^2\right)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{3c}-\frac{(ac+8b)\sqrt{c+\ldots}}{\ldots}\right)}{5c}}{c}\ldots\right)}{\sqrt{ac+adx^2+b}}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{-\frac{3d\left(-\frac{d\left(\frac{ad\int \frac{\left(16b^2+16acb+a^2c^2\right)dx^2+c(b+ac)(8b+ac)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{\left(a^2c^2+16abc+16b^2\right)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{3c}-\frac{(ac+8b)\sqrt{c+dx\ldots}}{3c\ldots}\right)}{5c}}{c}\ldots\right)}{\sqrt{ac+adx^2+b}}$$

3.343. $\displaystyle\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx$

$\downarrow$ 406

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(\frac{-\frac{3d\left(-\frac{d\left(\frac{ad\left(d\left(a^2c^2+16abc+16b^2\right)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+c(ac+b)(ac+8b)\int\frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{c(ac+b)}-\left(a^2c^2+16abc\ldots\right.}{3c}\ldots\right)}{5c}\ldots}{c}\ldots\right)}{\sqrt{ac+adx^2+b}\ldots}$$

$\downarrow$ 320

3.343. $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{3d-\frac{d\left(\frac{ad\left(d\left(a^2c^2+16abc+16b^2\right)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{c^{3/2}(ac+8b)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}}{c(ac+b)}}{3c}}{5c}}{c}\right.}{\sqrt{ac+adx^2+}}$$

$\downarrow$ 388

3.343. $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{3d\left(-\frac{d\left(\frac{ad\left(d\left(a^2c^2+16abc+16b^2\right)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)+\frac{c^{3/2}(ac+8b)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\right.\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right.}{c(ac+b)}\right)}{3c}\right)}{5c}\right.}{\sqrt{ac+ad}}$$

$\downarrow$ 313

3.343. $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\left(-\frac{3d\left(-\frac{d\left(\frac{ad\left(d\left(a^2c^2+16abc+16b^2\right)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}(ac+8b)\sqrt{ac+adx^2\cdots}}{\sqrt{d}\sqrt{c+\cdots}}\cdots\right)}{c(ac+b)}\cdots\right)}{3c}\cdots\right)}{5c}\cdots\right)}{\sqrt{\cdots}}$$

input
```
Int[(a + b/(c + d*x^2))^(3/2)/x^6,x]
```

3.343. $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx$

output
```
(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*((b*Sqrt[b + a*c +
a*d*x^2])/(c*x^5*Sqrt[c + d*x^2]) + (-1/5*((6*b + a*c)*Sqrt[c + d*x^2]*Sqr
t[b + a*c + a*d*x^2])/(c*x^5) - (3*d*(-1/3*((8*b + a*c)*Sqrt[c + d*x^2]*Sq
rt[b + a*c + a*d*x^2])/(c*x^3) - (d*(-(((16*b^2 + 16*a*b*c + a^2*c^2)*Sqrt
[c + d*x^2]*Sqrt[b + a*c + a*d*x^2])/(c*(b + a*c)*x)) + (a*d*((16*b^2 + 16
*a*b*c + a^2*c^2)*d*((x*Sqrt[b + a*c + a*d*x^2])/(a*d*Sqrt[c + d*x^2]) - (
Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(
b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a
*c)*(c + d*x^2))])) + (c^(3/2)*(8*b + a*c)*Sqrt[b + a*c + a*d*x^2]*Ellipti
cF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d]*Sqrt[c + d*x^2]*Sqr
t[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])))/(c*(b + a*c))))/(3*c
)))/(5*c))/c))/Sqrt[b + a*c + a*d*x^2]
```

3.343.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

3.343. $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx$

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.343.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1169 vs. $2(526) = 1052$.

Time = 11.58 (sec) , antiderivative size = 1170, normalized size of antiderivative = 2.37

method	result	size
risch	Expression too large to display	1170
default	Expression too large to display	1666

input
```
int((a+b/(d*x^2+c))^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(d*x^2+c)*(a^2*c^2*d^2*x^4+11*a*b*c*d^2*x^4-a^2*c^3*d*x^2+11*b^2*d^2*
x^4-4*a*b*c^2*d*x^2+a^2*c^4-3*b^2*c*d*x^2+2*a*b*c^3+b^2*c^2)/c^4/x^5/(a*c+
b)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/5/c^4*d^3/(a*c+b)*(c^3*a^3/(-a*d/(a
*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*
c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a
*c*d+b*d)/d/c/a)^(1/2))+4*a^2*b*c^2/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^
2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/
2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))+3*a*b^
2*c/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*
d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1
/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-5*b^2*c*(a*c+b)*((a*d^2*x^2+a*c*d+b*d)
/c/b*x/d/((x^2+c/d)*(a*d^2*x^2+a*c*d+b*d))^(1/2)+(1/c-(a*c*d+b*d)/c/b/d)/(
-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x
^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),(
-1+(2*a*c*d+b*d)/d/c/a)^(1/2))+2*a*d/b/c*(a*c^2+b*c)/(-a*d/(a*c+b))^(1/2)*
(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x
^2+a*c^2+b*c)^(1/2)/(2*a*c*d+2*b*d)*(EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+
(2*a*c*d+b*d)/d/c/a)^(1/2))-EllipticE(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+
b*d)/d/c/a)^(1/2))))-2*(a^3*c^2*d+11*a^2*b*c*d+11*a*b^2*d)*(a*c^2+b*c)/(-a
*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*...
```

3.343.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 393, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx = \frac{(a^3c^2+16\,a^2bc+16\,ab^2)\sqrt{-\frac{ad}{ac+b}}d^4x^5\sqrt{\frac{ac^2+bc}{d^2}}E(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right) \mid \frac{ac+b}{ac}) - ((a^3c^2$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^6,x, algorithm="fricas")
```

output
```
1/5*((a^3*c^2 + 16*a^2*b*c + 16*a*b^2)*sqrt(-a*d/(a*c + b))*d^4*x^5*sqrt((
a*c^2 + b*c)/d^2)*elliptic_e(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*
c)) - ((a^3*c^2 + 16*a^2*b*c + 16*a*b^2)*d^4 + (a^3*c^3 + 10*a^2*b*c^2 + 1
7*a*b^2*c + 8*b^3)*d^3)*sqrt(-a*d/(a*c + b))*x^5*sqrt((a*c^2 + b*c)/d^2)*e
lliptic_f(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - ((a^3*c^3 + 1
7*a^2*b*c^2 + 32*a*b^2*c + 16*b^3)*d^3*x^6 + a^3*c^6 + 3*a^2*b*c^5 + 3*a*b
^2*c^4 + (7*a^2*b*c^3 + 15*a*b^2*c^2 + 8*b^3*c)*d^2*x^4 + b^3*c^3 - 2*(a^2
*b*c^4 + 2*a*b^2*c^3 + b^3*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c
)))/((a^2*c^6 + 2*a*b*c^5 + b^2*c^4)*x^5)
```

3.343.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx=\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((a+b/(d*x**2+c))**(3/2)/x**6,x)
```

output
```
Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**6, x)
```

3.343.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^6,x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^2 + c))^(3/2)/x^6, x)
```

3.343.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx=\int \frac{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((a+b/(d*x^2+c))^(3/2)/x^6,x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^2 + c))^(3/2)/x^6, x)
```

3.343.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx=\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^6}\,dx$$

input
```
int((a + b/(c + d*x^2))^(3/2)/x^6,x)
```

output
```
int((a + b/(c + d*x^2))^(3/2)/x^6, x)
```

3.344 $\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

3.344.1 Optimal result

Integrand size = 21, antiderivative size = 225

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{(5b^2+12abc+8a^2c^2)\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{16a^3d^3} - \frac{(5b+8ac)\,(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{24a^2d^3} + \frac{x^2(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{6ad^2} - \frac{b(5b^2+12abc+8a^2c^2)\,\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{7/2}d^3}$$

output
```
-1/16*b*(8*a^2*c^2+12*a*b*c+5*b^2)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/
2)/a^(1/2))/a^(7/2)/d^3+1/16*(8*a^2*c^2+12*a*b*c+5*b^2)*(d*x^2+c)*((a*d*x^
2+a*c+b)/(d*x^2+c))^(1/2)/a^3/d^3-1/24*(8*a*c+5*b)*(d*x^2+c)^2*((a*d*x^2+a
*c+b)/(d*x^2+c))^(1/2)/a^2/d^3+1/6*x^2*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2
+c))^(1/2)/a/d^2
```

3.344.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.66

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

$$= \frac{\sqrt{a}(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}(15b^2+2ab(13c-5dx^2)+8a^2(c^2-cdx^2+d^2x^4))-3b(5b^2+12abc+8a^2c^2)\,\mathrm{arc}}{48a^{7/2}d^3}$$

input
```
Integrate[x^5/Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(15*b^2 + 2*a*b
*(13*c - 5*d*x^2) + 8*a^2*(c^2 - c*d*x^2 + d^2*x^4)) - 3*b*(5*b^2 + 12*a*b
*c + 8*a^2*c^2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(4
8*a^(7/2)*d^3)
```

3.344.3 Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2057, 2053, 2052, 27, 315, 25, 298, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

$$\Big\downarrow \; 2057$$

$$\int \frac{x^5}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}dx$$

$$\Big\downarrow \; 2053$$

$$\frac{1}{2}\int \frac{x^4}{\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}dx^2$$

$$\Big\downarrow \; 2052$$

$$-bd\int \frac{\left(-cx^4+b+ac\right)^2}{d^4\left(a-x^4\right)^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$\Big\downarrow$ 27

$$-\frac{b\int \frac{\left(-cx^4+b+ac\right)^2}{\left(a-x^4\right)^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{d^3}$$

$\Big\downarrow$ 315

$$-\frac{b\left(\frac{b\left(ac+b-cx^4\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{6a\left(a-x^4\right)^3}-\frac{\int -\frac{(b+ac)(5b+6ac)-3c(b+2ac)x^4}{\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{6a}\right)}{d^3}$$

$\Big\downarrow$ 25

$$-\frac{b\left(\frac{\int \frac{(b+ac)(5b+6ac)-3c(b+2ac)x^4}{\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{6a}+\frac{b\left(ac+b-cx^4\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{6a\left(a-x^4\right)^3}\right)}{d^3}$$

$\Big\downarrow$ 298

$$-\frac{b\left(\frac{\frac{3\left(8a^2c^2+12abc+5b^2\right)\int \frac{1}{\left(a-x^4\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{4a}+\frac{b(8ac+5b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a\left(a-x^4\right)^2}}{6a}+\frac{b\left(ac+b-cx^4\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{6a\left(a-x^4\right)^3}\right)}{d^3}$$

$\Big\downarrow$ 215

$$-\frac{b\left(\frac{\frac{3\left(8a^2c^2+12abc+5b^2\right)\left(\frac{\int \frac{1}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2a}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2a\left(a-x^4\right)}\right)}{4a}+\frac{b(8ac+5b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a\left(a-x^4\right)^2}}{6a}+\frac{b\left(ac+b-cx^4\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{6a\left(a-x^4\right)^3}\right)}{d^3}$$

$\Big\downarrow$ 219

3.344. $\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

$$-\frac{b\left(\frac{\frac{3\left(8a^2c^2+12abc+5b^2\right)\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2a\left(a-x^4\right)}\right)}{4a}+\frac{b(8ac+5b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a\left(a-x^4\right)^2}}{6a}+\frac{b\left(ac+b-cx^4\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{6a(a-x^4)^3}\right)}{d^3}$$

input
```
Int[x^5/Sqrt[a + b/(c + d*x^2)],x]
```

output
```
-((b*((b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b + a*c - c*x^4))/(6*a*(a
- x^4)^3) + ((b*(5*b + 8*a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*a*
(a - x^4)^2) + (3*(5*b^2 + 12*a*b*c + 8*a^2*c^2)*(Sqrt[(b + a*c + a*d*x^2)
/(c + d*x^2)]/(2*a*(a - x^4)) + ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^
2)]/Sqrt[a]]/(2*a^(3/2))))/(4*a))/(6*a)))/d^3)
```

3.344.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

3.344.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.06

method	result
risch	$\frac{\left(8a^{2}d^{2}x^{4}-8a^{2}cd\,x^{2}-10abd\,x^{2}+8a^{2}c^{2}+26abc+15b^{2}\right)\left(ad\,x^{2}+ac+b\right)}{48d^{3}a^{3}\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}}-\frac{b\left(8a^{2}c^{2}+12abc+5b^{2}\right)\ln\left(\frac{acd+\frac{1}{2}bd+a\,d^{2}x^{2}}{\sqrt{a\,d^{2}}}+\sqrt{a\,c^{2}+bc+(2a\cdots}\right.}{32d^{2}a^{3}\sqrt{a\,d^{2}}\,\sqrt{\frac{ad\,x^{2}+ac+\cdots}{d\,x^{2}+c}}}$
default	$\frac{\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}\,\left(d\,x^{2}+c\right)\left(-48\sqrt{a\,d^{2}x^{4}+2acd\,x^{2}+bd\,x^{2}+a\,c^{2}+bc}\,\sqrt{a\,d^{2}}\,a^{2}cd\,x^{2}-36\sqrt{a\,d^{2}x^{4}+2acd\,x^{2}+bd\,x^{2}+a\,c^{2}+bc}\,\sqrt{a\,d^{2}}\,abd\,x^{2}-2\cdots\right.}{}$

input
```
int(x^5/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/d^3*(8*a^2*d^2*x^4-8*a^2*c*d*x^2-10*a*b*d*x^2+8*a^2*c^2+26*a*b*c+15*b
^2)*(a*d*x^2+a*c+b)/a^3/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/32*b/d^2*(8*a^
2*c^2+12*a*b*c+5*b^2)/a^3*ln((a*c*d+1/2*b*d+a*d^2*x^2)/(a*d^2)^(1/2)+(a*c^
2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/(a*d^2)^(1/2)/((a*d*x^2+a*c+b)/(
d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(d*x^2+c)
```

3.344.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.89

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

$$= \left[\frac{3\,(8\,a^2bc^2+12\,ab^2c+5\,b^3)\sqrt{a}\log\Big(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2-4\,(2\,ad^2x^4+(4\,a\right.$$

input
```
integrate(x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/192*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*sqrt(a)*log(8*a^2*d^2*x^4 + 8
*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a
*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c
))) + 4*(8*a^3*d^3*x^6 - 10*a^2*b*d^2*x^4 + 8*a^3*c^3 + 26*a^2*b*c^2 + 15*
a*b^2*c + (16*a^2*b*c + 15*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
 c)))/(a^4*d^3), 1/96*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*sqrt(-a)*arcta
n(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c
))/(a^2*d*x^2 + a^2*c + a*b)) + 2*(8*a^3*d^3*x^6 - 10*a^2*b*d^2*x^4 + 8*a^
3*c^3 + 26*a^2*b*c^2 + 15*a*b^2*c + (16*a^2*b*c + 15*a*b^2)*d*x^2)*sqrt((a
*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*d^3)]
```

3.344.6 Sympy [F]

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{x^5}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

input
```
integrate(x**5/(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(x**5/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

3.344.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.51

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx =$$

$$-\frac{3\left(8a^2bc^2+12ab^2c+5b^3\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-8\left(6a^3bc^2+12a^2b^2c+5ab^3\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+3\left(8a^4bc^2+20\right.}{48\left(a^6d^3-\frac{3\left(adx^2+ac+b\right)a^5d^3}{dx^2+c}+\frac{3\left(adx^2+ac+b\right)^2a^4d^3}{\left(dx^2+c\right)^2}-\frac{\left(adx^2+ac+b\right)^3a^3d^3}{\left(dx^2+c\right)^3}\right)}$$

$$+\frac{\left(8a^2c^2+12abc+5b^2\right)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{32a^{\frac{7}{2}}d^3}$$

input
```
integrate(x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/48*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*((a*d*x^2 + a*c + b)/(d*x^2 +
c))^(5/2) - 8*(6*a^3*b*c^2 + 12*a^2*b^2*c + 5*a*b^3)*((a*d*x^2 + a*c + b)/
(d*x^2 + c))^(3/2) + 3*(8*a^4*b*c^2 + 20*a^3*b^2*c + 11*a^2*b^3)*sqrt((a*d
*x^2 + a*c + b)/(d*x^2 + c)))/(a^6*d^3 - 3*(a*d*x^2 + a*c + b)*a^5*d^3/(d*
x^2 + c) + 3*(a*d*x^2 + a*c + b)^2*a^4*d^3/(d*x^2 + c)^2 - (a*d*x^2 + a*c
+ b)^3*a^3*d^3/(d*x^2 + c)^3) + 1/32*(8*a^2*c^2 + 12*a*b*c + 5*b^2)*b*log(
-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^
2 + a*c + b)/(d*x^2 + c))))/(a^(7/2)*d^3)
```

3.344.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

$$= \frac{2\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(2\,x^2\left(\frac{4\,x^2}{ad}-\frac{4\,a^2cd^3+5\,abd^3}{a^3d^5}\right)+\frac{8\,a^2c^2d^2+26\,abcd^2+15\,b^2d^2}{a^3d^5}\right)+\frac{3\,(8\,a^2bc^2+12\,ab^2}{}}{96\,\mathrm{sgn}\left(dx^2+c\right)}$$

input
```
integrate(x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
1/96*(2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2*(4*x^
2/(a*d) - (4*a^2*c*d^3 + 5*a*b*d^3)/(a^3*d^5)) + (8*a^2*c^2*d^2 + 26*a*b*c
*d^2 + 15*b^2*d^2)/(a^3*d^5)) + 3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*log(a
bs(2*a*c*d + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 +
 a*c^2 + b*c))*sqrt(a)*abs(d) + b*d))/(a^(7/2)*d^2*abs(d)))/sgn(d*x^2 + c)
```

3.344.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{x^5}{\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

input
```
int(x^5/(a + b/(c + d*x^2))^(1/2),x)
```

output
```
int(x^5/(a + b/(c + d*x^2))^(1/2), x)
```

3.345 $\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

3.345.1 Optimal result

Integrand size = 21, antiderivative size = 148

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=-\frac{(3b+4ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8a^2d^2}+\frac{\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4ad^2}+\frac{b(3b+4ac)\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{5/2}d^2}$$

output
```
1/8*b*(4*a*c+3*b)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/a^(5/
2)/d^2-1/8*(4*a*c+3*b)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^2/d^2
+1/4*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^2
```

3.345.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\frac{\sqrt{a}\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(-3b-2ac+2adx^2\right)+b(3b+4ac)\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{5/2}d^2}$$

input
```
Integrate[x^3/Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-3*b - 2*a*c +
 2*a*d*x^2) + b*(3*b + 4*a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)
]/Sqrt[a]])/(8*a^(5/2)*d^2)
```

3.345.3 Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2057, 2053, 2052, 25, 27, 298, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{x^3}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{x^2}{\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}dx^2$$

$$\Big\downarrow \text{2052}$$

$$-bd\int -\frac{-cx^4+b+ac}{d^3\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$bd\int \frac{-cx^4+b+ac}{d^3\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{-cx^4+b+ac}{\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{d^2}$$

$$\Big\downarrow \text{298}$$

$$\frac{b\left(\frac{(4ac+3b)\int \frac{1}{\left(a-x^4\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{4a}+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a(a-x^4)^2}\right)}{d^2}$$

$$\Big\downarrow \; 215$$

$$\frac{b\left(\frac{(4ac+3b)\left(\frac{\int \frac{1}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2a}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2a\left(a-x^4\right)}\right)}{4a}+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a(a-x^4)^2}\right)}{d^2}$$

$$\Big\downarrow \; 219$$

$$\frac{b\left(\frac{(4ac+3b)\left(\frac{\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2a\left(a-x^4\right)}\right)}{4a}+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a(a-x^4)^2}\right)}{d^2}$$

input
```
Int[x^3/Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(b*((b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*a*(a - x^4)^2) + ((3*b +
4*a*c)*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/(2*a*(a - x^4)) + ArcTanh[Sq
rt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]]/(2*a^(3/2))))/(4*a)))/d^2
```

3.345.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.345. $\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

3.345. $\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

3.345.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.32

method	result
risch	$-\frac{(-2ad\,x^2+2ac+3b)\,(ad\,x^2+ac+b)}{8d^2a^2\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}+\frac{b(4ac+3b)\ln\left(\frac{acd+\frac{1}{2}bd+a\,d^2x^2}{\sqrt{a\,d^2}}+\sqrt{a\,c^2+bc+(2acd+bd)x^2+a\,d^2x^4}\right)\sqrt{(ad\,x^2+ac+b)(d\,x^2+}}{16d\,a^2\sqrt{a\,d^2}\,\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,(d\,x^2+c)}$
default	$-\frac{\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,(d\,x^2+c)\left(-4\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}\,ad\,x^2-4\ln\left(\frac{2a\,d^2x^2+2acd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}}{2\sqrt{a\,d^2}}\right.\right.}{}$

input
```
int(x^3/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/d^2*(-2*a*d*x^2+2*a*c+3*b)*(a*d*x^2+a*c+b)/a^2/((a*d*x^2+a*c+b)/(d*x^
2+c))^(1/2)+1/16*b/d*(4*a*c+3*b)/a^2*ln((a*c*d+1/2*b*d+a*d^2*x^2)/(a*d^2)^
(1/2)+(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/(a*d^2)^(1/2)/((a*d*x
^2+a*c+b)/(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(d*x^2+c)
```

3.345.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.25

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

$$= \left[\frac{(4\,abc+3\,b^2)\sqrt{a}\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2+4\,(2\,ad^2x^4+(4\,ac+b)dx^2+2\right.}{32\,a^3d^2}\right.$$

$$-\frac{(4\,abc+3\,b^2)\sqrt{-a}\arctan\left(\frac{(2\,adx^2+2\,ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2dx^2+a^2c+ab)}\right)-2\,(2\,a^2d^2x^4-3\,abdx^2-2\,a^2c^2-3\,abc)\sqrt{\frac{ad}{}}}{16\,a^3d^2}$$

input
```
integrate(x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/32*((4*a*b*c + 3*b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*
c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*
c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^2*d^2*x
^4 - 3*a*b*d*x^2 - 2*a^2*c^2 - 3*a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c)))/(a^3*d^2), -1/16*((4*a*b*c + 3*b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 +
2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2
*c + a*b)) - 2*(2*a^2*d^2*x^4 - 3*a*b*d*x^2 - 2*a^2*c^2 - 3*a*b*c)*sqrt((a
*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^3*d^2)]
```

3.345.6 Sympy [F]

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{x^3}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

input
```
integrate(x**3/(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(x**3/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

3.345.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.51

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=-\frac{(4\,abc+3\,b^2)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-(4\,a^2bc+5\,ab^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^4d^2-\frac{2\,(adx^2+ac+b)a^3d^2}{dx^2+c}+\frac{(adx^2+ac+b)^2a^2d^2}{(dx^2+c)^2}\right)}-\frac{(4\,ac+3\,b)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{16\,a^{\frac{5}{2}}d^2}$$

input
```
integrate(x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/8*((4*a*b*c + 3*b^2)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b
*c + 5*a*b^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*d^2 - 2*(a*d*x^2
 + a*c + b)*a^3*d^2/(d*x^2 + c) + (a*d*x^2 + a*c + b)^2*a^2*d^2/(d*x^2 + c
)^2) - 1/16*(4*a*c + 3*b)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^
2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(5/2)*d^2)
```

3.345.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.12

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$
$$= \frac{2\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(\frac{2\,x^2}{ad}-\frac{2\,acd+3\,bd}{a^2d^3}\right)-\frac{(4\,abc+3\,b^2)\log\left(\left|2\,acd+2\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+\cdots}\right.\right.\right.}{a^{\frac{5}{2}}d|d|}}{16\,\mathrm{sgn}\left(dx^2+c\right)}$$

input
```
integrate(x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
1/16*(2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2/(a*d)
 - (2*a*c*d + 3*b*d)/(a^2*d^3)) - (4*a*b*c + 3*b^2)*log(abs(2*a*c*d + 2*(s
qrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*sq
rt(a)*abs(d) + b*d))/(a^(5/2)*d*abs(d)))/sgn(d*x^2 + c)
```

3.345.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{x^3}{\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

input
```
int(x^3/(a + b/(c + d*x^2))^(1/2),x)
```

output
```
int(x^3/(a + b/(c + d*x^2))^(1/2), x)
```

3.346 $\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

3.346.1 Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2ad}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}$$

output
```
-1/2*b*arctanh((a+b/(d*x^2+c))^(1/2)/a^(1/2))/a^(3/2)/d+1/2*(d*x^2+c)*(a+b
/(d*x^2+c))^(1/2)/a/d
```

3.346.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.22

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\frac{(c+dx^2)\sqrt{\frac{b+a(c+dx^2)}{c+dx^2}}}{2ad}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+a(c+dx^2)}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}$$

input
```
Integrate[x/Sqrt[a + b/(c + d*x^2)],x]
```

output
```
((c + d*x^2)*Sqrt[(b + a*(c + d*x^2))/(c + d*x^2)])/(2*a*d) - (b*ArcTanh[S
qrt[(b + a*(c + d*x^2))/(c + d*x^2)]/Sqrt[a]])/(2*a^(3/2)*d)
```

3.346.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2024, 773, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{ 2024}$$

$$\frac{\int \frac{1}{\sqrt{a+\frac{b}{dx^2+c}}}d\left(dx^2+c\right)}{2d}$$

$$\Big\downarrow \text{ 773}$$

$$-\frac{\int \frac{\left(dx^2+c\right)^2}{\sqrt{a+\frac{b}{dx^2+c}}}d\frac{1}{dx^2+c}}{2d}$$

$$\Big\downarrow \text{ 52}$$

$$-\frac{-\frac{b\int \frac{dx^2+c}{\sqrt{a+\frac{b}{dx^2+c}}}d\frac{1}{dx^2+c}}{2a}-\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{a}}{2d}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{-\frac{\int \frac{1}{\frac{1}{b\left(dx^2+c\right)^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{dx^2+c}}}{a}-\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{a}}{2d}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{\frac{b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{a}}{2d}$$

input
```
Int[x/Sqrt[a + b/(c + d*x^2)],x]
```

output
```
-1/2*(-(((c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/a) + (b*ArcTanh[Sqrt[a + b/(
c + d*x^2)]/Sqrt[a]])/a^(3/2))/d
```

3.346.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

3.346.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(60) = 120$.

Time = 1.02 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.86

method	result
derivativedivides	$\frac{\sqrt{\frac{a\left(d\,x^2+c\right)+b}{d\,x^2+c}}\,\left(d\,x^2+c\right)\left(2\sqrt{\left(a\left(d\,x^2+c\right)+b\right)\left(d\,x^2+c\right)}\,\sqrt{a}-b\ln\left(\frac{2\sqrt{\left(a\left(d\,x^2+c\right)+b\right)\left(d\,x^2+c\right)}\,\sqrt{a}+2a\left(d\,x^2+c\right)+b}{2\sqrt{a}}\right)\right)}{4d\sqrt{\left(a\left(d\,x^2+c\right)+b\right)\left(d\,x^2+c\right)}\,a^{\frac{3}{2}}}$
risch	$\frac{a\,d\,x^2+ac+b}{2da\sqrt{\frac{a\,d\,x^2+ac+b}{d\,x^2+c}}}-\frac{\ln\left(\frac{acd+\frac{1}{2}bd+a\,d^2x^2}{\sqrt{a\,d^2}}+\sqrt{a\,c^2+bc+(2acd+bd)x^2+a\,d^2x^4}\right)b\sqrt{(ad\,x^2+ac+b)(d\,x^2+c)}}{4a\sqrt{a\,d^2}\,\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,(d\,x^2+c)}$
default	$\frac{\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,(d\,x^2+c)\left(-b\ln\left(\frac{2a\,d^2x^2+2acd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,\sqrt{a\,d^2}+bd}{2\sqrt{a\,d^2}}\right)d+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2}\right.}{4\sqrt{(ad\,x^2+ac+b)(d\,x^2+c)}\,ad\sqrt{a\,d^2}}$

input
```
int(x/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*((a*(d*x^2+c)+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(2*((a*(d*x^2+c)+b)*(d*x
^2+c))^(1/2)*a^(1/2)-b*ln(1/2*(2*((a*(d*x^2+c)+b)*(d*x^2+c))^(1/2)*a^(1/2)
+2*a*(d*x^2+c)+b)/a^(1/2)))/((a*(d*x^2+c)+b)*(d*x^2+c))^(1/2)/a^(3/2)
```

3.346.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.71

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

$$= \left[\frac{\sqrt{a}b\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2-4\,(2\,ad^2x^4+(4\,ac+b)dx^2+2\,ac^2+bc)\sqrt{a}\right.}{8\,a^2d}\right.$$

input
```
integrate(x/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(a)*b*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 +
8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a
)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a*d*x^2 + a*c)*sqrt((a*d*x^2
 + a*c + b)/(d*x^2 + c)))/(a^2*d), 1/4*(sqrt(-a)*b*arctan(1/2*(2*a*d*x^2 +
 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^
2*c + a*b)) + 2*(a*d*x^2 + a*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^
2*d)]
```

3.346.6 Sympy [F]

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{x}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

input
```
integrate(x/(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(x/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

3.346.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(60) = 120$.

Time = 0.30 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.79

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = -\frac{b\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2d-\frac{(adx^2+ac+b)ad}{dx^2+c}\right)} + \frac{b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{4\,a^{\frac{3}{2}}d}$$

input
```
integrate(x/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/2*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d - (a*d*x^2 + a*c + b)*
a*d/(d*x^2 + c)) + 1/4*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
 c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(3/2)*d)
```

3.346.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(60) = 120$.

Time = 0.37 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.78

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\frac{b\log\left(\left|2\,acd+2\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)\sqrt{a}|d|+bd\right|\right)}{a^{\frac{3}{2}}|d|}+\frac{2\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{ad}}{4\,\mathrm{sgn}\left(dx^2+c\right)}$$

input
```
integrate(x/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
1/4*(b*log(abs(2*a*c*d + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2
 + b*d*x^2 + a*c^2 + b*c))*sqrt(a)*abs(d) + b*d))/(a^(3/2)*abs(d)) + 2*sqr
t(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)/(a*d))/sgn(d*x^2 + c)
```

3.346.9 Mupad [B] (verification not implemented)

Time = 18.02 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.54

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{\frac{a\,(d\,x^2+c)}{b}+1}\,(d\,x^2+c)\,\left(\frac{3\,\sqrt{b}\,\sqrt{b+a\,(d\,x^2+c)}}{2\,a\,(d\,x^2+c)}+\frac{b^{3/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\,\sqrt{d\,x^2+c}\,1\mathrm{i}}{\sqrt{b}}\right)\,3\mathrm{i}}{2\,a^{3/2}\,{(d\,x^2+c)}^{3/2}}\right)}{3\,d\,\sqrt{a+\frac{b}{d\,x^2+c}}}$$

input
```
int(x/(a + b/(c + d*x^2))^(1/2),x)
```

output
```
(((a*(c + d*x^2))/b + 1)^(1/2)*(c + d*x^2)*((b^(3/2)*asin((a^(1/2)*(c + d*
x^2)^(1/2)*1i)/b^(1/2))*3i)/(2*a^(3/2)*(c + d*x^2)^(3/2)) + (3*b^(1/2)*(b
+ a*(c + d*x^2))^(1/2))/(2*a*(c + d*x^2))))/(3*d*(a + b/(c + d*x^2))^(1/2)
)
```

3.347 $\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

3.347.1 Optimal result

Integrand size = 21, antiderivative size = 96

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{\sqrt{b+ac}}$$

output
```
arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/a^(1/2)-arctanh(c^(1/2)
*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))*c^(1/2)/(a*c+b)^(1/2)
```

3.347.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\frac{\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{\sqrt{-b-ac}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[1/(x*Sqrt[a + b/(c + d*x^2)]),x]
```

output
```
(Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b -
a*c]])/Sqrt[-b - a*c] + ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt
[a]]/Sqrt[a]
```

3.347.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2057, 2053, 2052, 25, 27, 303, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \frac{1}{x\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{ 2053}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}dx^2$$

$$\Big\downarrow \text{ 2052}$$

$$-bd\int -\frac{1}{d(a-x^4)(-cx^4+b+ac)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{ 25}$$

$$bd\int \frac{1}{d(a-x^4)(-cx^4+b+ac)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{ 27}$$

$$b\int \frac{1}{(a-x^4)(-cx^4+b+ac)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{ 303}$$

$$b\left(\frac{\int \frac{1}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{b}-\frac{c\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{b}\right)$$

$$\Big\downarrow \text{219}$$

$$b\left(\frac{\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{c\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{b}\right)$$

$$\Big\downarrow \text{221}$$

$$b\left(\frac{\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{b\sqrt{ac+b}}\right)$$

input
```
Int[1/(x*Sqrt[a + b/(c + d*x^2)]),x]
```

output
```
b*(ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]]/(Sqrt[a]*b) - (S
qrt[c]*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*
c]])/(b*Sqrt[b + a*c]))
```

3.347.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.347. $\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

3.347.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(80) = 160$.

Time = 0.08 (sec) , antiderivative size = 313, normalized size of antiderivative = 3.26

method	result
default	$-\frac{\sqrt{\frac{a d\,x^{2}+a c+b}{d\,x^{2}+c}}\,\left(d\,x^{2}+c\right)\left(-\ln\left(\frac{2 a\,d^{2} x^{2}+2 a c d+2 \sqrt{a\,d^{2} x^{4}+2 a c d\,x^{2}+b d\,x^{2}+a\,c^{2}+b c}\,\sqrt{a\,d^{2}}+b d}{2 \sqrt{a\,d^{2}}}\right) a c d+\sqrt{a\,c^{2}+b c}\,\ln\left(\frac{2 a c d\,x^{2}+b d\,x^{2}+2 a\,c^{2}+}{}\right.}{2 \sqrt{\left(a d\,x^{2}+a c+b\right)\left(d\,x^{2}+\right.}}$

input
```
int(1/x/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-ln(1/2*(2*a*d^2*x^2+2*a
*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/
(a*d^2)^(1/2))*a*c*d+(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*(
a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^
2)*(a*d^2)^(1/2)-b*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*
d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*d)/((a*d*x^2+a*c+
b)*(d*x^2+c))^(1/2)/(a*c+b)/(a*d^2)^(1/2)
```

3.347.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(80) = 160$.

Time = 0.36 (sec) , antiderivative size = 972, normalized size of antiderivative = 10.12

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

$$= \left[\frac{a\sqrt{\frac{c}{ac+b}}\log\left(\frac{(8a^2c^2+8abc+b^2)d^2x^4+8a^2c^4+16abc^3+8b^2c^2+8(2a^2c^3+3abc^2+b^2c)dx^2-4((2a^2c^2+3abc+b^2)d^2x^4+2a^2c^4+4abc^3+}{x^4}\right.}{}\right.$$

input
```
integrate(1/x/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(a*sqrt(c/(a*c + b))*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2
*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 -
4*((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4 + 2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2
 + (4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^
2 + c))*sqrt(c/(a*c + b)))/x^4) + sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 +
8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x
^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/a, 1/4
*(a*sqrt(c/(a*c + b))*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4
 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((
2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4 + 2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 + (
4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c))*sqrt(c/(a*c + b)))/x^4) - 2*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b
)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)
))/a, 1/4*(2*a*sqrt(-c/(a*c + b))*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2
+ 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-c/(a*c + b))/(a*c*d*x
^2 + a*c^2 + b*c)) + sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c +
a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2
+ b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/a, 1/2*(a*sqrt(-c/(
a*c + b))*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 +
 a*c + b)/(d*x^2 + c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 + a*c^2 + b*c)) - ...
```

3.347.6 Sympy [F]

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{1}{x\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

input
```
integrate(1/x/(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(1/(x*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

3.347.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.61

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\frac{c\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{2\sqrt{(ac+b)c}}-\frac{\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{2\sqrt{a}}$$

input
```
integrate(1/x/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
1/2*c*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c
*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/sqrt((a*c + b
)*c) - 1/2*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a)
 + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/sqrt(a)
```

3.347.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.347.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{1}{x\,\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

input
```
int(1/(x*(a + b/(c + d*x^2))^(1/2)),x)
```

output
```
int(1/(x*(a + b/(c + d*x^2))^(1/2)), x)
```

3.348 $\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

3.348.1 Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx=-\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{2(b+ac)x^2}-\frac{bd\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{2\sqrt{c}(b+ac)^{3/2}}$$

output
```
-1/2*b*d*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))/
(a*c+b)^(3/2)/c^(1/2)-1/2*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c
+b)/x^2
```

3.348.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\frac{1}{2}\left(-\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{(b+ac)x^2}-\frac{bd\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{\sqrt{c}(-b-ac)^{3/2}}\right)$$

input
```
Integrate[1/(x^3*Sqrt[a + b/(c + d*x^2)]),x]
```

output
```
(-(((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/((b + a*c)*x^2)) -
(b*d*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]
])/(Sqrt[c]*(-b - a*c)^(3/2)))/2
```

3.348.3 Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2057, 2053, 2052, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{1}{x^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{1}{x^4\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}dx^2$$

$$\Big\downarrow \text{2052}$$

$$-bd\int \frac{1}{(cx^4-b-ac)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{215}$$

$$-bd\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b)\left(ac+b-cx^4\right)}-\frac{\int \frac{1}{cx^4-b-ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2(ac+b)}\right)$$

$$\Big\downarrow \text{221}$$

$$-bd\left(\frac{\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2\sqrt{c}(ac+b)^{3/2}}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b)\left(ac+b-cx^4\right)}\right)$$

input
```
Int[1/(x^3*Sqrt[a + b/(c + d*x^2)]),x]
```

output
```
-(b*d*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/(2*(b + a*c)*(b + a*c - c*x^4
)) + ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]
]/(2*Sqrt[c]*(b + a*c)^(3/2))))
```

3.348.3.1 Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

3.348. $\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

3.348.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(92) = 184.

Time = 0.14 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.82

method	result
risch	$-\frac{a d x^2+a c+b}{2(ac+b)x^2\sqrt{\frac{a d x^2+ac+b}{d x^2+c}}}-\frac{b d \ln\left(\frac{2a c^2+2bc+(2acd+bd)x^2+2\sqrt{a c^2+bc}\,\sqrt{a c^2+bc+(2acd+bd)x^2+a d^2x^4}}{x^2}\right)\sqrt{(a d x^2+ac+b)(d x^2+c)}}{4(ac+b)\sqrt{a c^2+bc}\,\sqrt{\frac{a d x^2+ac+b}{d x^2+c}}\,(d x^2+c)}$
default	$-\frac{\sqrt{\frac{a d x^2+ac+b}{d x^2+c}}\,(d x^2+c)\left(-2a d^2\sqrt{a d^2x^4+2acd x^2+bd x^2+a c^2+bc}\,x^4\sqrt{a c^2+bc}+\ln\left(\frac{2acd x^2+bd x^2+2a c^2+2\sqrt{a c^2+bc}\,\sqrt{a d^2x^4+2acd}}{x^2}\right.\right.}{}$

input
```
int(1/x^3/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a*c+b)*(a*d*x^2+a*c+b)/x^2/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/4*b*d
/(a*c+b)/(a*c^2+b*c)^(1/2)*ln((2*a*c^2+2*b*c+(2*a*c*d+b*d)*x^2+2*(a*c^2+b*
c)^(1/2)*(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/x^2)/((a*d*x^2+a*c
+b)/(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(d*x^2+c)
```

3.348.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(92) = 184.

Time = 0.36 (sec) , antiderivative size = 451, normalized size of antiderivative = 4.18

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

$$= \left[\frac{\sqrt{ac^2+bc}bdx^2\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,(2\,a^2c^3+3\,abc^2+b^2c)dx^2-4\,((2\,ac+b)d^2x^4+2\,ac^3+(4\,ac^2+3\,b}{x^4}\right.}{8\,(a^2c^3+2\,abc^2+b^2c)x^2}\right.$$

input
```
integrate(1/x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(a*c^2 + b*c)*b*d*x^2*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 +
 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*
x^2 - 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2
)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4) - 4*(a*c^3
 + (a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a
^2*c^3 + 2*a*b*c^2 + b^2*c)*x^2), 1/4*(sqrt(-a*c^2 - b*c)*b*d*x^2*arctan(1
/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2
+ a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b
^2*c)) - 2*(a*c^3 + (a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/
(d*x^2 + c)))/((a^2*c^3 + 2*a*b*c^2 + b^2*c)*x^2)]
```

3.348.6 Sympy [F]

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{x^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

input
```
integrate(1/x**3/(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

3.348.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.60

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx = -\frac{bd\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2c^2+2\,abc+b^2-\frac{(adx^2+ac+b)(ac^2+bc)}{dx^2+c}\right)} + \frac{bd\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{4\sqrt{(ac+b)c}(ac+b)}$$

input
```
integrate(1/x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/2*b*d*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^2 + 2*a*b*c + b^2 -
(a*d*x^2 + a*c + b)*(a*c^2 + b*c)/(d*x^2 + c)) + 1/4*b*d*log((c*sqrt((a*d*
x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c +
b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/(sqrt((a*c + b)*c)*(a*c + b))
```

3.348.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(92) = 184$.

Time = 0.40 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.70

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\frac{bd\arctan\left(-\frac{\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{\sqrt{-ac^2-bc}}\right)}{\sqrt{-ac^2-bc}(ac+b)} - \frac{2\,a^{\frac{3}{2}}c^2|d|+2\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)acd+2\sqrt{a}bc|d|+\left(\sqrt{ad^2}x^2-\right.}{\left(ac^2-\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)^2+bc\right.}}{2\,\mathrm{sgn}\left(dx^2+c\right)}$$

input
```
integrate(1/x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
1/2*(b*d*arctan(-(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2
 + a*c^2 + b*c))/sqrt(-a*c^2 - b*c))/(sqrt(-a*c^2 - b*c)*(a*c + b)) - (2*a
^(3/2)*c^2*abs(d) + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*
d*x^2 + a*c^2 + b*c))*a*c*d + 2*sqrt(a)*b*c*abs(d) + (sqrt(a*d^2)*x^2 - sq
rt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*b*d)/((a*c^2 - (sqrt(
a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2 + b*
c)*(a*c + b)))/sgn(d*x^2 + c)
```

3.348.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{x^3\,\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

input
```
int(1/(x^3*(a + b/(c + d*x^2))^(1/2)),x)
```

output
```
int(1/(x^3*(a + b/(c + d*x^2))^(1/2)), x)
```

3.349 $\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

3.349.1 Optimal result

Integrand size = 21, antiderivative size = 177

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{(b+4ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8c(b+ac)^2x^2} - \frac{(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4c(b+ac)x^4} + \frac{b(b+4ac)d^2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{8c^{3/2}(b+ac)^{5/2}}$$

output
```
1/8*b*(4*a*c+b)*d^2*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c
+b)^(1/2))/c^(3/2)/(a*c+b)^(5/2)+1/8*(4*a*c+b)*d*(d*x^2+c)*((a*d*x^2+a*c+b
)/(d*x^2+c))^(1/2)/c/(a*c+b)^2/x^2-1/4*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2
+c))^(1/2)/c/(a*c+b)/x^4
```

3.349.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx = -\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,(2ac(c-dx^2)+b(2c+dx^2))}{8c(b+ac)^2x^4} - \frac{b(b+4ac)d^2\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{8c^{3/2}(-b-ac)^{5/2}}$$

input
```
Integrate[1/(x^5*Sqrt[a + b/(c + d*x^2)]),x]
```

output
```
-1/8*((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(2*a*c*(c - d*x^2)
 + b*(2*c + d*x^2)))/(c*(b + a*c)^2*x^4) - (b*(b + 4*a*c)*d^2*ArcTan[(Sqrt
[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(8*c^(3/2)*(-b
 - a*c)^(5/2))
```

3.349.3 Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2057, 2053, 2052, 25, 27, 298, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{1}{x^5\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{1}{x^6\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}dx^2$$

$$\Big\downarrow \text{2052}$$

$$-bd\int -\frac{d\left(a-x^4\right)}{\left(-cx^4+b+ac\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$\Big\downarrow$ 25

$$bd\int \frac{d\left(a-x^4\right)}{\left(-cx^4+b+ac\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$\Big\downarrow$ 27

$$bd^2\int \frac{a-x^4}{\left(-cx^4+b+ac\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$\Big\downarrow$ 298

$$bd^2\left(\frac{(4ac+b)\int \frac{1}{\left(-cx^4+b+ac\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{4c(ac+b)}-\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c(ac+b)\left(ac+b-cx^4\right)^2}\right)$$

$\Big\downarrow$ 215

$$bd^2\left(\frac{(4ac+b)\left(\frac{\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2(ac+b)}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b)(ac+b-cx^4)}\right)}{4c(ac+b)}-\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c(ac+b)\left(ac+b-cx^4\right)^2}\right)$$

$\Big\downarrow$ 221

$$bd^2\left(\frac{(4ac+b)\left(\frac{\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2\sqrt{c}(ac+b)^{3/2}}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b)(ac+b-cx^4)}\right)}{4c(ac+b)}-\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c(ac+b)\left(ac+b-cx^4\right)^2}\right)$$

input
```
Int[1/(x^5*Sqrt[a + b/(c + d*x^2)]),x]
```

output
```
b*d^2*(-1/4*(b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(c*(b + a*c)*(b + a*
c - c*x^4)^2) + ((b + 4*a*c)*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/(2*(b
+ a*c)*(b + a*c - c*x^4)) + ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c +
 d*x^2)])/Sqrt[b + a*c]]/(2*Sqrt[c]*(b + a*c)^(3/2))))/(4*c*(b + a*c)))
```

3.349. $\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

3.349.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

3.349. $\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

3.349.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.33

method	result
risch	$-\frac{(a d\,x^{2}+a c+b)\left(-2 a c d\,x^{2}+b d\,x^{2}+2 a\,c^{2}+2 b c\right)}{8 (a c+b)^{2} x^{4} c \sqrt{\frac{a d\,x^{2}+a c+b}{d\,x^{2}+c}}}+\frac{d^{2} b (4 a c+b) \ln\left(\frac{2 a\,c^{2}+2 b c+(2 a c d+b d) x^{2}+2 \sqrt{a\,c^{2}+b c}\,\sqrt{a\,c^{2}+b c+(2 a c d+b d) x^{2}+a\,d^{2} x^{4}}}{x^{2}}\right.}{16 (a c+b)^{2} c \sqrt{a\,c^{2}+b c}\,\sqrt{\frac{a d\,x^{2}+a c+b}{d\,x^{2}+c}}\,(d\,x^{2}+c)}$
default	$-\frac{\sqrt{\frac{a d\,x^{2}+a c+b}{d\,x^{2}+c}}\,(d\,x^{2}+c)\left(12 a^{2} d^{3} \sqrt{a\,d^{2} x^{4}+2 a c d\,x^{2}+b d\,x^{2}+a\,c^{2}+b c}\,x^{6} c (a\,c^{2}+b c)^{\frac{3}{2}}-4 \ln\left(\frac{2 a c d\,x^{2}+b d\,x^{2}+2 a\,c^{2}+2 \sqrt{a\,c^{2}+b c}\,\sqrt{a\,d^{2} x^{4}+}}{x^{2}}\right.\right.}{}$

input
```
int(1/x^5/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(a*d*x^2+a*c+b)*(-2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c)/(a*c+b)^2/x^4/c/
((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/16*d^2*b*(4*a*c+b)/(a*c+b)^2/c/(a*c^2+
b*c)^(1/2)*ln((2*a*c^2+2*b*c+(2*a*c*d+b*d)*x^2+2*(a*c^2+b*c)^(1/2)*(a*c^2+
b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/x^2)/((a*d*x^2+a*c+b)/(d*x^2+c))^(
1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(d*x^2+c)
```

3.349.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 593, normalized size of antiderivative = 3.35

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

$$= \left[\frac{(4\,abc+b^2)\sqrt{ac^2+bc}d^2x^4\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,(2\,a^2c^3+3\,abc^2+b^2c)dx^2+4\,((2\,ac+b)d^2x^4+2}{x^4}\right.}{32\,(a^3}\right.$$

$$-\frac{(4\,abc+b^2)\sqrt{-ac^2-bc}d^2x^4\arctan\left(\frac{((2\,ac+b)dx^2+2\,ac^2+2\,bc)\sqrt{-ac^2-bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2c^3+2\,abc^2+(a^2c^2+abc)dx^2+b^2c)}\right)+2\,(2\,a^2c^5-(2\,a^2c^3+a}{16\,(a^3c^5+3\,a^2bc^4+3\,ab^2c^3+b^3c^2)x^4}$$

input
```
integrate(1/x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/32*((4*a*b*c + b^2)*sqrt(a*c^2 + b*c)*d^2*x^4*log(((8*a^2*c^2 + 8*a*b*c
 + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*
b*c^2 + b^2*c)*d*x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c
)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))
)/x^4) - 4*(2*a^2*c^5 - (2*a^2*c^3 + a*b*c^2 - b^2*c)*d^2*x^4 + 4*a*b*c^4
+ 2*b^2*c^3 + 3*(a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2
 + c)))/((a^3*c^5 + 3*a^2*b*c^4 + 3*a*b^2*c^3 + b^3*c^2)*x^4), -1/16*((4*a
*b*c + b^2)*sqrt(-a*c^2 - b*c)*d^2*x^4*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a
*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^
2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(2*a^2*c^5 - (2*
a^2*c^3 + a*b*c^2 - b^2*c)*d^2*x^4 + 4*a*b*c^4 + 2*b^2*c^3 + 3*(a*b*c^3 +
b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^5 + 3*a^2*b
*c^4 + 3*a*b^2*c^3 + b^3*c^2)*x^4)]
```

3.349.6 Sympy [F]

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{1}{x^5\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

input
```
integrate(1/x**5/(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(1/(x**5*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

3.349.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(157) = 314$.

Time = 0.30 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.03

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx=-\frac{(4\,abc+b^2)d^2\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{16\,(a^2c^3+2\,abc^2+b^2c)\sqrt{(ac+b)c}}-\frac{(4\,abc^2+b^2c)d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-(4\,a^2bc^2+3\,ab^2c-b^3)d^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^4c^5+4\,a^3bc^4+6\,a^2b^2c^3+4\,ab^3c^2+b^4c+\frac{(a^2c^5+2\,abc^4+b^2c^3)(adx^2+ac+b)^2}{(dx^2+c)^2}-\frac{2\,(a^3c^5+3\,a^2bc^4+3\,ab^2c^3+b^3c^2)(adx^2}{dx^2+c}\right.}$$

input
```
integrate(1/x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
-1/16*(4*a*b*c + b^2)*d^2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - s
qrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b
)*c)))/((a^2*c^3 + 2*a*b*c^2 + b^2*c)*sqrt((a*c + b)*c)) - 1/8*((4*a*b*c^2
 + b^2*c)*d^2*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*c^2 + 3*a
*b^2*c - b^3)*d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^5 + 4*a^3*
b*c^4 + 6*a^2*b^2*c^3 + 4*a*b^3*c^2 + b^4*c + (a^2*c^5 + 2*a*b*c^4 + b^2*c
^3)*(a*d*x^2 + a*c + b)^2/(d*x^2 + c)^2 - 2*(a^3*c^5 + 3*a^2*b*c^4 + 3*a*b
^2*c^3 + b^3*c^2)*(a*d*x^2 + a*c + b)/(d*x^2 + c))
```

3.349.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 778 vs. $2(157) = 314$.

Time = 0.41 (sec) , antiderivative size = 778, normalized size of antiderivative = 4.40

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx =$$

$$-\frac{\frac{\left(4\,abcd^2+b^2d^2\right)\arctan\left(-\frac{\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{\sqrt{-ac^2-bc}}\right)}{\left(a^2c^3+2\,abc^2+b^2c\right)\sqrt{-ac^2-bc}}-\frac{8\,a^{\frac{7}{2}}c^5d|d|+16\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)a^3c^4d^2-}{}}{}$$

input
```
integrate(1/x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
-1/8*((4*a*b*c*d^2 + b^2*d^2)*arctan(-(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 +
2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))/sqrt(-a*c^2 - b*c))/((a^2*c^3 + 2*a*
b*c^2 + b^2*c)*sqrt(-a*c^2 - b*c)) - (8*a^(7/2)*c^5*d*abs(d) + 16*(sqrt(a*
d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3*c^4*
d^2 + 8*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2
+ b*c))^2*a^(5/2)*c^3*d*abs(d) + 16*a^(5/2)*b*c^4*d*abs(d) + 28*(sqrt(a*d^
2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b*c^3*
d^2 + 16*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2
 + b*c))^2*a^(3/2)*b*c^2*d*abs(d) + 8*a^(3/2)*b^2*c^3*d*abs(d) + 4*(sqrt(a
*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a*b*c
*d^2 + 13*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^
2 + b*c))*a*b^2*c^2*d^2 + 8*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^
2 + b*d*x^2 + a*c^2 + b*c))^2*sqrt(a)*b^2*c*d*abs(d) + (sqrt(a*d^2)*x^2 -
sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*b^2*d^2 + (sqrt(a
*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*b^3*c*d
^2)/((a^2*c^3 + 2*a*b*c^2 + b^2*c)*(a*c^2 - (sqrt(a*d^2)*x^2 - sqrt(a*d^2*
x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2 + b*c)^2))/sgn(d*x^2 + c)
```

3.349.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{1}{x^5\,\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

input
```
int(1/(x^5*(a + b/(c + d*x^2))^(1/2)),x)
```

output
```
int(1/(x^5*(a + b/(c + d*x^2))^(1/2)), x)
```

3.350 $\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

3.350.1 Optimal result

Integrand size = 21, antiderivative size = 443

$$\begin{aligned}\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx =& -\frac{(4b+3ac)x(b+ac+adx^2)}{15a^2d^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{x^3(b+ac+adx^2)}{5ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\\ &+\frac{(8b^2+13abc+3a^2c^2)\,x(b+ac+adx^2)}{15a^3d^2\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\\ &-\frac{\sqrt{c}(8b^2+13abc+3a^2c^2)\,(b+ac+adx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15a^3d^{5/2}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\\ &+\frac{c^{3/2}(4b+3ac)\,(b+ac+adx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{15a^2d^{5/2}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\end{aligned}$$

output
```
-1/15*(3*a*c+4*b)*x*(a*d*x^2+a*c+b)/a^2/d^2/((a*d*x^2+a*c+b)/(d*x^2+c))^(1
/2)+1/5*x^3*(a*d*x^2+a*c+b)/a/d/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/15*(3*
a^2*c^2+13*a*b*c+8*b^2)*x*(a*d*x^2+a*c+b)/a^3/d^2/(d*x^2+c)/((a*d*x^2+a*c+
b)/(d*x^2+c))^(1/2)+1/15*c^(3/2)*(3*a*c+4*b)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c
))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(
b/(a*c+b))^(1/2))/a^2/d^(5/2)/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/
(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)-1/15*(3*a^2*c^2+13*a*b*c+8*b^2
)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1
/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)/a^3/d^(5/2)/(d*x^
2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c
))^(1/2)
```

3.350.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.78 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = -\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(a\sqrt{\frac{d}{c}}x(c+dx^2)\left(4b^2+ab(7c+dx^2)+3a^2(c^2-d^2x^4)\right)+i(8b^3+21ab^2c+16a^2bc^2+3a^3c\right.}{}$$

input
```
Integrate[x^4/Sqrt[a + b/(c + d*x^2)],x]
```

output
```
-1/15*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*Sqrt[d/c]*x*(c + d*x^2)*(4
*b^2 + a*b*(7*c + d*x^2) + 3*a^2*(c^2 - d^2*x^4)) + I*(8*b^3 + 21*a*b^2*c
+ 16*a^2*b*c^2 + 3*a^3*c^3)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (
d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] - I*b*(8*b^2
+ 17*a*b*c + 9*a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^
2)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(a^3*c^2*(d/c)^
(5/2)*(b + a*(c + d*x^2)))
```

3.350.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 413, normalized size of antiderivative = 0.93, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2057, 2058, 380, 444, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \frac{x^4}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{ac+adx^2+b}\int \frac{x^4\sqrt{dx^2+c}}{\sqrt{adx^2+b+ac}}dx}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{ 380}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5ad}-\frac{\int \frac{x^2\left((4b+3ac)dx^2+3c(b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{5ad}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{ 444}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5ad}-\frac{\frac{x(3ac+4b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{\int \frac{d\left(\left(8b^2+13acb+3a^2c^2\right)dx^2+c(b+ac)(4b+3ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3ad^2}}{5ad}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5ad}-\frac{\frac{x(3ac+4b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{\int \frac{\left(8b^2+13acb+3a^2c^2\right)dx^2+c(b+ac)(4b+3ac)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3ad}}{5ad}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \; 406$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5ad}-\frac{\frac{x(3ac+4b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{d\left(3a^2c^2+13abc+8b^2\right)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+c(ac+b)(3ac+4b)}{3ad}}{5ad}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \; 320$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5ad}-\frac{\frac{x(3ac+4b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{d\left(3a^2c^2+13abc+8b^2\right)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{c^{3/2}(3ac+4b)\sqrt{}}{}}{3ad}}{5ad}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \; 388$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5ad}-\frac{\frac{x(3ac+4b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{d\left(3a^2c^2+13abc+8b^2\right)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)+\frac{c^{3}}{}}{3ad}}{5ad}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \; 313$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5ad}-\frac{\frac{x(3ac+4b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{d\left(3a^2c^2+13abc+8b^2\right)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\right.}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(ac+}{(ac+}}}\right.}{}}{5ad}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

3.350. $\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

input
```
Int[x^4/Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(Sqrt[b + a*c + a*d*x^2]*((x^3*Sqrt[c + d*x^2]*Sqrt[b + a*c + a*d*x^2])/(5
*a*d) - (((4*b + 3*a*c)*x*Sqrt[c + d*x^2]*Sqrt[b + a*c + a*d*x^2])/(3*a*d)
 - ((8*b^2 + 13*a*b*c + 3*a^2*c^2)*d*((x*Sqrt[b + a*c + a*d*x^2])/(a*d*Sqr
t[c + d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcTan[(Sqrt[d]
*x)/Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c +
a*d*x^2))/((b + a*c)*(c + d*x^2))])) + (c^(3/2)*(4*b + 3*a*c)*Sqrt[b + a*c
 + a*d*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d]*
Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]))/(3
*a*d))/(5*a*d)))/(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])
```

3.350.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.350. $\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

3.350.4 Maple [A] (verified)

Time = 8.06 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.50

method	result
default	$-\frac{\left(-3\sqrt{-\frac{ad}{ac+b}}\,a^2d^3x^7-3\sqrt{-\frac{ad}{ac+b}}\,a^2c\,d^2x^5+\sqrt{-\frac{ad}{ac+b}}\,ab\,d^2x^5+3\sqrt{-\frac{ad}{ac+b}}\,a^2c^2d\,x^3+8\sqrt{-\frac{ad}{ac+b}}\,abcd\,x^3-3\sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\,\sqrt{\frac{d\,x^2+}{c}}\right.}{}$
risch	$-\frac{x(-3ad\,x^2+3ac+4b)\,(ad\,x^2+ac+b)}{15d^2a^2\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}+\frac{\left(\frac{3a^2c^3\sqrt{1+\frac{ad\,x^2}{ac+b}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{-1+\frac{2acd+bd}{dca}}\right)}{\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}}+\frac{4b^2c\sqrt{1+\frac{ad\,x^2}{ac+b}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\right.}{\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a\,d^2x^4+2acd\,x}}\right.}{}$

input
```
int(x^4/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-3*(-a*d/(a*c+b))^(1/2)*a^2*d^3*x^7-3*(-a*d/(a*c+b))^(1/2)*a^2*c*d^
2*x^5+(-a*d/(a*c+b))^(1/2)*a*b*d^2*x^5+3*(-a*d/(a*c+b))^(1/2)*a^2*c^2*d*x^
3+8*(-a*d/(a*c+b))^(1/2)*a*b*c*d*x^3-3*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d
*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2
*c^3+3*(-a*d/(a*c+b))^(1/2)*a^2*c^3*x+4*(-a*d/(a*c+b))^(1/2)*b^2*d*x^3+6*(
(a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+
b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c^2-13*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*
((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*
a*b*c^2+7*(-a*d/(a*c+b))^(1/2)*a*b*c^2*x+4*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)
*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))
*b^2*c-8*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(
-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b^2*c+4*(-a*d/(a*c+b))^(1/2)*b^2*
c*x)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^2/d^2/(a*d^2*x^4+2*a*c*
d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/((a*d*x^2+a*c+b)*(d*x^
2+c))^(1/2)
```

3.350.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.54

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx =$$

$$-\frac{(3\,a^2c^3+13\,abc^2+8\,b^2c)\sqrt{a}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ac+b}{ac})-(3\,a^2c^3+13\,abc^2+8\,b^2c+(3\,a^2c^2+7\,ab}{}$$

input
```
integrate(x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/15*((3*a^2*c^3 + 13*a*b*c^2 + 8*b^2*c)*sqrt(a)*x*sqrt(-c/d)*elliptic_e(
arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c)) - (3*a^2*c^3 + 13*a*b*c^2 + 8*b^2*c
 + (3*a^2*c^2 + 7*a*b*c + 4*b^2)*d)*sqrt(a)*x*sqrt(-c/d)*elliptic_f(arcsin
(sqrt(-c/d)/x), (a*c + b)/(a*c)) - (3*a^2*d^3*x^6 - 4*a*b*d^2*x^4 + 3*a^2*
c^3 + 13*a*b*c^2 + (9*a*b*c + 8*b^2)*d*x^2 + 8*b^2*c)*sqrt((a*d*x^2 + a*c
+ b)/(d*x^2 + c)))/(a^3*d^3*x)
```

3.350.6 Sympy [F]

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{x^4}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

input
```
integrate(x**4/(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(x**4/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

3.350.7 Maxima [F]

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{x^4}{\sqrt{a+\frac{b}{dx^2+c}}}\,dx$$

input
```
integrate(x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt(a + b/(d*x^2 + c)), x)
```

3.350.8 Giac [F]

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{x^4}{\sqrt{a+\frac{b}{dx^2+c}}}\,dx$$

input
```
integrate(x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(a + b/(d*x^2 + c)), x)
```

3.350.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{x^4}{\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

input
```
int(x^4/(a + b/(c + d*x^2))^(1/2),x)
```

output
```
int(x^4/(a + b/(c + d*x^2))^(1/2), x)
```

3.351 $\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

3.351.1 Optimal result

Integrand size = 21, antiderivative size = 354

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx &= \frac{x(b+ac+adx^2)}{3ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-\frac{(2b+ac)x(b+ac+adx^2)}{3a^2d\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\\ &+\frac{\sqrt{c}(2b+ac)\,(b+ac+adx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3a^2d^{3/2}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\\ &-\frac{c^{3/2}(b+ac+adx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3ad^{3/2}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\end{aligned}$$

output
```
1/3*x*(a*d*x^2+a*c+b)/a/d/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/3*(a*c+2*b)*
x*(a*d*x^2+a*c+b)/a^2/d/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/3*c^
(3/2)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*
d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/a/d^(3/2)/(d*x^2+c)/(
(a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/
2)+1/3*(a*c+2*b)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*E
llipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)/a^
2/d^(3/2)/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(
a*c+b)/(d*x^2+c))^(1/2)
```

3.351.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.62 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$
$$= \frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(a\sqrt{\frac{d}{c}}x(c+dx^2)\left(b+a(c+dx^2)\right)+i(2b^2+3abc+a^2c^2)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\right.\right.\right.}{3a^2d\sqrt{\frac{d}{c}}\left(b+a\left(c+dx^2\right.\right.}$$

input
```
Integrate[x^2/Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*Sqrt[d/c]*x*(c + d*x^2)*(b + a*(
c + d*x^2)) + I*(2*b^2 + 3*a*b*c + a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b +
a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c
)] - (2*I)*b*(b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2
)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(3*a^2*d*Sqrt[d/
c]*(b + a*(c + d*x^2)))
```

3.351.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 335, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2057, 2058, 380, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$
$$\Big\downarrow \ 2057$$
$$\int \frac{x^2}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}dx$$
$$\Big\downarrow \ 2058$$

$$\frac{\sqrt{ac+adx^2+b}\int \frac{x^2\sqrt{dx^2+c}}{\sqrt{adx^2+b+ac}}dx}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{ 380}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{\int \frac{(2b+ac)dx^2+c(b+ac)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3ad}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{ 406}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{c(ac+b)\int \frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+d(ac+2b)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3ad}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{d(ac+2b)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}}{3ad}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{ 388}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{d(ac+2b)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)+\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}}{3ad}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{ 313}$$

3.351. $\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}dx$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+d(ac+2b)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\right.}{ad^{3/2}\sqrt{c+dx^2}}\right)}{3ad}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

input
```
Int[x^2/Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(Sqrt[b + a*c + a*d*x^2]*((x*Sqrt[c + d*x^2]*Sqrt[b + a*c + a*d*x^2])/(3*a
*d) - ((2*b + a*c)*d*((x*Sqrt[b + a*c + a*d*x^2])/(a*d*Sqrt[c + d*x^2]) -
(Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/
(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b +
a*c)*(c + d*x^2))])) + (c^(3/2)*Sqrt[b + a*c + a*d*x^2]*EllipticF[ArcTan[(
Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[(c*(b + a
*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]))/(3*a*d)))/(Sqrt[c + d*x^2]*Sqrt[
(b + a*c + a*d*x^2)/(c + d*x^2)])
```

3.351.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.351. $\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

3.351.4 Maple [A] (verified)

Time = 6.00 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.16

method	result
default	$\frac{\left(\sqrt{-\frac{ad}{ac+b}}\, a\, d^{2} x^{5}+2 \sqrt{-\frac{ad}{ac+b}}\, a c d\, x^{3}+\sqrt{-\frac{ad}{ac+b}}\, b d\, x^{3}-\sqrt{\frac{a d\, x^{2}+a c+b}{a c+b}}\, \sqrt{\frac{d\, x^{2}+c}{c}}\, E\left(x \sqrt{-\frac{ad}{ac+b}}, \sqrt{\frac{ac+b}{ac}}\right) a\, c^{2}+\sqrt{-\frac{ad}{ac+b}}\, a\, c^{2} x+\sqrt{\frac{a d\, x^{2}}{}}\right.}{3 d \sqrt{a\, d^{2} x^{4}+2 a c d\, x^{2}+b d\, x^{2}+a\, c^{2}+}}$
risch	$\frac{x\left(a d\, x^{2}+a c+b\right)}{3 a d \sqrt{\frac{a d\, x^{2}+a c+b}{d\, x^{2}+c}}}-\frac{\left(\frac{a\, c^{2} \sqrt{1+\frac{a d\, x^{2}}{a c+b}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, F\left(x \sqrt{-\frac{ad}{ac+b}}, \sqrt{-1+\frac{2 a c d+b d}{d c a}}\right)}{\sqrt{-\frac{ad}{ac+b}}\, \sqrt{a\, d^{2} x^{4}+2 a c d\, x^{2}+b d\, x^{2}+a\, c^{2}+b c}}+\frac{b c \sqrt{1+\frac{a d\, x^{2}}{a c+b}}\, \sqrt{1+\frac{d\, x^{2}}{c}}\, F\left(x \sqrt{-\frac{ad}{ac+b}}, \sqrt{-1+\frac{2 a c d+b d}{d c a}}\right)}{\sqrt{-\frac{ad}{ac+b}}\, \sqrt{a\, d^{2} x^{4}+2 a c d\, x^{2}+b d\, x^{2}+a\, c^{2}+b c}}-\right.}{3 d a}$

input
```
int(x^2/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((-a*d/(a*c+b))^(1/2)*a*d^2*x^5+2*(-a*d/(a*c+b))^(1/2)*a*c*d*x^3+(-a*d
/(a*c+b))^(1/2)*b*d*x^3-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2
)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*c^2+(-a*d/(a*c+b
))^(1/2)*a*c^2*x+((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*Ellip
ticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*c-2*((a*d*x^2+a*c+b)/(a
*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)
/a/c)^(1/2))*b*c+(-a*d/(a*c+b))^(1/2)*b*c*x)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d
*x^2+c))^(1/2)/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*
c+b))^(1/2)/a/((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)
```

3.351.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.47

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\, dx$$
$$= \frac{(ac^2+2\,bc)\sqrt{a}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ac+b}{ac}) - (ac^2+2\,bc+(ac+b)d)\sqrt{a}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ac-}{a}}{3\,a^2d^2x}$$

input
```
integrate(x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*((a*c^2 + 2*b*c)*sqrt(a)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x),
 (a*c + b)/(a*c)) - (a*c^2 + 2*b*c + (a*c + b)*d)*sqrt(a)*x*sqrt(-c/d)*ell
iptic_f(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c)) + (a*d^2*x^4 - 2*b*d*x^2 -
a*c^2 - 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d^2*x)
```

3.351.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{x^2}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

input
```
integrate(x**2/(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(x**2/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

3.351.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{x^2}{\sqrt{a+\frac{b}{dx^2+c}}}\,dx$$

input
```
integrate(x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(a + b/(d*x^2 + c)), x)
```

3.351.8 Giac [F]

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{x^2}{\sqrt{a+\frac{b}{dx^2+c}}}\,dx$$

input
```
integrate(x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(a + b/(d*x^2 + c)), x)
```

3.351.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{x^2}{\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

input
```
int(x^2/(a + b/(c + d*x^2))^(1/2),x)
```

output
```
int(x^2/(a + b/(c + d*x^2))^(1/2), x)
```

3.352 $\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

3.352.1 Optimal result

Integrand size = 17, antiderivative size = 286

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{x(b+ac+adx^2)}{a(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{\sqrt{c}(b+ac+adx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{a\sqrt{d}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{c^{3/2}(b+ac+adx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{(b+ac)\sqrt{d}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

output
```
x*(a*d*x^2+a*c+b)/a/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+c^(3/2)*(a
*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/
c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/(a*c+b)/(d*x^2+c)/d^(1/2)/((a
*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
-(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/
2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)/a/(d*x^2+c)/d^(1/2
)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^
(1/2)
```

3.352.2 Mathematica [A] (verified)

Time = 7.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.37

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\frac{\sqrt{\frac{b+ac+adx^2}{b+ac}}E\left(\arcsin\left(\sqrt{-\frac{ad}{b+ac}}x\right)|1+\frac{b}{ac}\right)}{\sqrt{-\frac{ad}{b+ac}}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{1+\frac{dx^2}{c}}}$$

input
```
Integrate[1/Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*EllipticE[ArcSin[Sqrt[-((a*d)/(b + a*
c))]*x], 1 + b/(a*c)])/(Sqrt[-((a*d)/(b + a*c))]*Sqrt[(b + a*c + a*d*x^2)/
(c + d*x^2)]*Sqrt[1 + (d*x^2)/c])
```

3.352.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2057, 2058, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}dx$$

$$\Big\downarrow \; 2057$$

$$\int \frac{1}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}dx$$

$$\Big\downarrow \; 2058$$

$$\frac{\sqrt{ac+adx^2+b}\int \frac{\sqrt{dx^2+c}}{\sqrt{adx^2+b+ac}}dx}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \; 324$$

$$\frac{\sqrt{ac+adx^2+b}\left(c\int \frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+d\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{ac+adx^2+b}\left(d\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}(ac+b)\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{388}$$

$$\frac{\sqrt{ac+adx^2+b}\left(d\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)+\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}(ac+b)\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{313}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}(ac+b)\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

input
```
Int[1/Sqrt[a + b/(c + d*x^2)],x]
```

output
```
(Sqrt[b + a*c + a*d*x^2]*(d*((x*Sqrt[b + a*c + a*d*x^2])/(a*d*Sqrt[c + d*x
^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))
/((b + a*c)*(c + d*x^2))])) + (c^(3/2)*Sqrt[b + a*c + a*d*x^2]*EllipticF[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/((b + a*c)*Sqrt[d]*Sqrt[c + d*x^
2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])))/(Sqrt[c + d*x^
2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])
```

3.352. $\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

3.352.3.1 Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.352. $\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

3.352.4 Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{E\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)\sqrt{\frac{d\,x^{2}+c}{c}}\,\sqrt{\frac{ad\,x^{2}+ac+b}{ac+b}}\,c\left(d\,x^{2}+c\right)\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}}{\sqrt{a\,d^{2}x^{4}+2acd\,x^{2}+bd\,x^{2}+a\,c^{2}+bc}\,\sqrt{-\frac{ad}{ac+b}}\,\sqrt{\left(ad\,x^{2}+ac+b\right)\left(d\,x^{2}+c\right)}}$	164

input
```
int(1/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)/c)^(1/2)*
((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*c*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1
/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/(
(a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)
```

3.352.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx =$$

$$-\frac{\sqrt{a}cx\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ac+b}{ac}) - \sqrt{a}(c+d)x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ac+b}{ac}) - (dx^2+c)\sqrt{\frac{adx^2+ac-}{dx^2+c}}}{adx}$$

input
```
integrate(1/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(a)*c*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c))
 - sqrt(a)*(c + d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), (a*c + b)
/(a*c)) - (d*x^2 + c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a*d*x)
```

3.352.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$

input
```
integrate(1/(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b/(c + d*x**2)), x)
```

3.352.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{dx^2+c}}}\,dx$$

input
```
integrate(1/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a + b/(d*x^2 + c)), x)
```

3.352.8 Giac [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{dx^2+c}}}\,dx$$

input
```
integrate(1/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(a + b/(d*x^2 + c)), x)
```

3.352.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

input
```
int(1/(a + b/(c + d*x^2))^(1/2),x)
```

output
```
int(1/(a + b/(c + d*x^2))^(1/2), x)
```

3.353 $\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

3.353.1 Optimal result

Integrand size = 21, antiderivative size = 343

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx = -\frac{b+ac+adx^2}{(b+ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{dx(b+ac+adx^2)}{(b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{\sqrt{c}\sqrt{d}(b+ac+adx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{(b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{\sqrt{c}\sqrt{d}(b+ac+adx^2)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{(b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

output
```
(-a*d*x^2-a*c-b)/(a*c+b)/x/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+d*x*(a*d*x^2+
a*c+b)/(a*c+b)/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-(a*d*x^2+a*c+b)
*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*
x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*d^(1/2)/(a*c+b)/(d*x^2+c)/((a*d*x^
2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+(a*d
*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^
(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*d^(1/2)/(a*c+b)/(d*x^2+
c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))
^(1/2)
```

3.353.2 Mathematica [A] (verified)

Time = 8.74 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$
$$= -\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{(b+ac)x} + \frac{d\sqrt{\frac{c+dx^2}{c}}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|\frac{ac}{b+ac}\right)}{(b+ac)\sqrt{-\frac{d}{c}}\sqrt{\frac{b+ac+adx^2}{b+ac}}}$$

input
```
Integrate[1/(x^2*Sqrt[a + b/(c + d*x^2)]),x]
```

output
```
-(((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/((b + a*c)*x)) + (d*
Sqrt[(c + d*x^2)/c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticE[ArcSin
[Sqrt[-(d/c)]*x], (a*c)/(b + a*c)])/((b + a*c)*Sqrt[-(d/c)]*Sqrt[(b + a*c
+ a*d*x^2)/(b + a*c)])
```

3.353.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {2057, 2058, 377, 27, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$
$$\Big\downarrow \text{2057}$$
$$\int \frac{1}{x^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}dx$$
$$\Big\downarrow \text{2058}$$
$$\frac{\sqrt{ac+adx^2+b}\int \frac{\sqrt{dx^2+c}}{x^2\sqrt{adx^2+b+ac}}dx}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$
$$\Big\downarrow \text{377}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{\int \frac{d\sqrt{adx^2+b+ac}}{\sqrt{dx^2+c}}dx}{ac+b}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{x(ac+b)}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{d\int \frac{\sqrt{adx^2+b+ac}}{\sqrt{dx^2+c}}dx}{ac+b}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{x(ac+b)}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{324}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{d\left((ac+b)\int \frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+ad\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{ac+b}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{x(ac+b)}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{d\left(ad\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{\sqrt{c}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{ac+b}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{x(ac+b)}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{388}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{d\left(ad\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)+\frac{\sqrt{c}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{ac+b}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{x(ac+b)}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{313}$$

3.353. $\displaystyle\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{d\left(ad\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)+\frac{\sqrt{c}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{ac+b}-\sqrt{c+}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

input
```
Int[1/(x^2*Sqrt[a + b/(c + d*x^2)]),x]
```

output
```
(Sqrt[b + a*c + a*d*x^2]*(-((Sqrt[c + d*x^2]*Sqrt[b + a*c + a*d*x^2])/((b
+ a*c)*x)) + (d*(a*d*((x*Sqrt[b + a*c + a*d*x^2])/(a*d*Sqrt[c + d*x^2]) -
(Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/
(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b +
a*c)*(c + d*x^2))])) + (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticF[ArcTan[(
Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[(c*(b + a
*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])))/(b + a*c)))/(Sqrt[c + d*x^2]*Sq
rt[(b + a*c + a*d*x^2)/(c + d*x^2)])
```

3.353.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

3.353. $\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.353. $\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

3.353.4 Maple [A] (verified)

Time = 6.06 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.01

method	result
default	$-\frac{\left(\sqrt{-\frac{ad}{ac+b}}\, a\, d^2x^4-adc\sqrt{\frac{a d\, x^2+ac+b}{ac+b}}\, \sqrt{\frac{d\, x^2+c}{c}}\, xE\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)+2\sqrt{-\frac{ad}{ac+b}}\, acd\, x^2-\sqrt{\frac{ad\, x^2+ac+b}{ac+b}}\, \sqrt{\frac{d\, x^2+c}{c}}\, F\left(x\sqrt{-\frac{a}{ac}}\right.\right.}{\sqrt{a\, d^2x^4+2acd\, x^2+bd\, x^2+a\, c^2+bc}\, \sqrt{-\frac{ad}{ac+b}}\, x(ac+b)\sqrt{(ad\, x}}$
risch	$-\frac{ad\, x^2+ac+b}{(ac+b)x\sqrt{\frac{ad\, x^2+ac+b}{d\, x^2+c}}}+\frac{d\left(\frac{b\sqrt{1+\frac{ad\, x^2}{ac+b}}\, \sqrt{1+\frac{d\, x^2}{c}}\, F\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{-1+\frac{2acd+bd}{dca}}\right)}{\sqrt{-\frac{ad}{ac+b}}\, \sqrt{a\, d^2x^4+2acd\, x^2+bd\, x^2+a\, c^2+bc}}+\frac{ac\sqrt{1+\frac{ad\, x^2}{ac+b}}\, \sqrt{1+\frac{d\, x^2}{c}}\, F\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{-1+\frac{2acd+}{dca}}\right.}{\sqrt{-\frac{ad}{ac+b}}\, \sqrt{a\, d^2x^4+2acd\, x^2+bd\, x^2+a\, c^2+bc}}\right.}{(a}$

input
```
int(1/x^2/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((-a*d/(a*c+b))^(1/2)*a*d^2*x^4-a*d*c*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d
*x^2+c)/c)^(1/2)*x*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))+2
*(-a*d/(a*c+b))^(1/2)*a*c*d*x^2-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)
/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*d*x+(-a*
d/(a*c+b))^(1/2)*b*d*x^2+(-a*d/(a*c+b))^(1/2)*a*c^2+(-a*d/(a*c+b))^(1/2)*b
*c)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d
*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/x/(a*c+b)/((a*d*x^2+a*c+b)*(d*x
^2+c))^(1/2)
```

3.353.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx$$
$$=\frac{a^2c\sqrt{-\frac{ad}{ac+b}}d^2x\sqrt{\frac{ac^2+bc}{d^2}}E(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right)\mid\frac{ac+b}{ac})-(a^2cd^2+(a^2c^2+2\,abc+b^2)d)\sqrt{-\frac{ad}{ac+b}}x\sqrt{\frac{ac^2+bc}{d^2}}F}{(a^3c^3+2\,a^2bc^2+ab^2c)x}$$

input
```
integrate(1/x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
(a^2*c*sqrt(-a*d/(a*c + b))*d^2*x*sqrt((a*c^2 + b*c)/d^2)*elliptic_e(arcsi
n(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - (a^2*c*d^2 + (a^2*c^2 + 2*a*
b*c + b^2)*d)*sqrt(-a*d/(a*c + b))*x*sqrt((a*c^2 + b*c)/d^2)*elliptic_f(ar
csin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - (a^2*c^3 + a*b*c^2 + (a^2
*c^2 + a*b*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^3 + 2*
a^2*b*c^2 + a*b^2*c)*x)
```

3.353.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{1}{x^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

input
```
integrate(1/x**2/(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

3.353.7 Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{dx^2+c}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/(d*x^2 + c))*x^2), x)
```

3.353.8 Giac [F]

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{dx^2+c}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a + b/(d*x^2 + c))*x^2), x)
```

3.353.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{1}{x^2\,\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

input
```
int(1/(x^2*(a + b/(c + d*x^2))^(1/2)),x)
```

output
```
int(1/(x^2*(a + b/(c + d*x^2))^(1/2)), x)
```

3.354 $\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

3.354.1 Optimal result

Integrand size = 21, antiderivative size = 435

$$\begin{aligned}\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx &= \frac{-b-ac-adx^2}{3(b+ac)x^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-\frac{(b-ac)d(b+ac+adx^2)}{3c(b+ac)^2x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\\ &\quad+\frac{(b-ac)d^2x(b+ac+adx^2)}{3c(b+ac)^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\\ &\quad-\frac{(b-ac)d^{3/2}(b+ac+adx^2)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3\sqrt{c}(b+ac)^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\\ &\quad-\frac{a\sqrt{c}d^{3/2}(b+ac+adx^2)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3(b+ac)^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\end{aligned}$$

output
```
1/3*(-a*d*x^2-a*c-b)/(a*c+b)/x^3/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/3*(-a
*c+b)*d*(a*d*x^2+a*c+b)/c/(a*c+b)^2/x/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/
3*(-a*c+b)*d^2*x*(a*d*x^2+a*c+b)/c/(a*c+b)^2/(d*x^2+c)/((a*d*x^2+a*c+b)/(d
*x^2+c))^(1/2)-1/3*(-a*c+b)*d^(3/2)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*
(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b)
)^(1/2))/(a*c+b)^2/(d*x^2+c)/c^(1/2)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*
(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)-1/3*a*d^(3/2)*(a*d*x^2+a*c+b)*(1/
(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/
c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)/(a*c+b)^2/(d*x^2+c)/((a*d*x^2+a*c+b)/(
d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

3.354.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.85 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx = -\frac{\sqrt{\frac{d}{c}}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{d}{c}}(c+dx^2)\left(b^2(c+dx^2)+a^2c(c^2-d^2x^4)+ab(2c^2+cdx^2+d^2x^4)\right)+i(b^2-a^2c^2)\,d^2\right.}{3(b}$$

input
```
Integrate[1/(x^4*Sqrt[a + b/(c + d*x^2)]),x]
```

output
```
-1/3*(Sqrt[d/c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[d/c]*(c + d*x^
2)*(b^2*(c + d*x^2) + a^2*c*(c^2 - d^2*x^4) + a*b*(2*c^2 + c*d*x^2 + d^2*x
^4)) + I*(b^2 - a^2*c^2)*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[
1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] - I*b*(b
 + a*c)*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*El
lipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/((b + a*c)^2*d*x^3*(b +
 a*(c + d*x^2)))
```

3.354.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2057, 2058, 377, 27, 445, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{1}{x^4\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{ac+adx^2+b}\int \frac{\sqrt{dx^2+c}}{x^4\sqrt{adx^2+b+ac}}dx}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \; 377$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{\int \frac{d\left(-adx^2+b-ac\right)}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3(ac+b)}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3x^3(ac+b)}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{d\int \frac{-adx^2+b-ac}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3(ac+b)}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3x^3(ac+b)}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \; 445$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{d\left(-\frac{\int \frac{ad\left(c(b+ac)-(b-ac)dx^2\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{(b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{3(ac+b)}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3x^3(ac+b)}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{d\left(-\frac{ad\int \frac{c(b+ac)-(b-ac)dx^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{(b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{3(ac+b)}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3x^3(ac+b)}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \; 406$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{d\left(-\frac{ad\left(c(ac+b)\int \frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx-d(b-ac)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{c(ac+b)}-\frac{(b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{3(ac+b)}-\frac{\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3x^3(ac+b)}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \; 320$$

3.354. $\displaystyle\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{d\left(-\frac{ad\left(\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-d(b-ac)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{c(ac+b)}-\frac{(b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{3(ac+b)}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\downarrow$ 388

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{d\left(-\frac{ad\left(\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-d(b-ac)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)\right)}{c(ac+b)}-\frac{(b-ac)\sqrt{c+dx^2}\sqrt{\cdots}}{cx(ac+\cdots}\right)}{3(ac+b)}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\downarrow$ 313

3.354. $\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{d\left(-\frac{ad\left(\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}-d(b-ac)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}\right)\right.}{c(ac+b)}}{3(ac+b)}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

input
```
Int[1/(x^4*Sqrt[a + b/(c + d*x^2)]),x]
```

output
```
(Sqrt[b + a*c + a*d*x^2]*(-1/3*(Sqrt[c + d*x^2]*Sqrt[b + a*c + a*d*x^2])/(
(b + a*c)*x^3) + (d*(-(((b - a*c)*Sqrt[c + d*x^2]*Sqrt[b + a*c + a*d*x^2])
/(c*(b + a*c)*x)) - (a*d*(-((b - a*c)*d*((x*Sqrt[b + a*c + a*d*x^2])/(a*d*
Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcTan[(Sqrt
[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c
 + a*d*x^2))/((b + a*c)*(c + d*x^2))]))) + (c^(3/2)*Sqrt[b + a*c + a*d*x^2
]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d]*Sqrt[c + d
*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])))/(c*(b + a*c
))))/(3*(b + a*c))))/(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)
])
```

3.354. $\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

3.354.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.354. $\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.354.4 Maple [A] (verified)

Time = 7.00 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.31

method	result
risch	$-\frac{(a d\,x^{2}+a c+b)\left(-a c d\,x^{2}+b d\,x^{2}+a\,c^{2}+b c\right)}{3(a c+b)^{2} x^{3} c\sqrt{\frac{a d\,x^{2}+a c+b}{d\,x^{2}+c}}}-\frac{d^{2} a\left(\frac{a\,c^{2}\sqrt{1+\frac{a d\,x^{2}}{a c+b}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{a d}{a c+b}},\sqrt{-1+\frac{2 a c d+b d}{d c a}}\right)}{\sqrt{-\frac{a d}{a c+b}}\,\sqrt{a\,d^{2} x^{4}+2 a c d\,x^{2}+b d\,x^{2}+a\,c^{2}+b c}}+\frac{b c\sqrt{1+\frac{a d\,x^{2}}{a c+b}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F}{\sqrt{-\frac{a d}{a c+b}}\,\sqrt{a\,d^{2} x^{4}}}\right.}{}$
default	$-\frac{\left(-\sqrt{-\frac{a d}{a c+b}}\,a^{2} c\,d^{3} x^{6}+\sqrt{-\frac{a d}{a c+b}}\,a b\,d^{3} x^{6}+\sqrt{\frac{a d\,x^{2}+a c+b}{a c+b}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,E\left(x\sqrt{-\frac{a d}{a c+b}},\sqrt{\frac{a c+b}{a c}}\right) a^{2} c^{2} d^{2} x^{3}-\sqrt{-\frac{a d}{a c+b}}\,a^{2} c^{2} d^{2} x^{4}+2\sqrt{\frac{a d}{}}\right.}{}$

input
```
int(1/x^4/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a*d*x^2+a*c+b)*(-a*c*d*x^2+b*d*x^2+a*c^2+b*c)/(a*c+b)^2/x^3/c/((a*d*
x^2+a*c+b)/(d*x^2+c))^(1/2)-1/3*d^2*a/(a*c+b)^2/c*(a*c^2/(-a*d/(a*c+b))^(1
/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b
*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)
/d/c/a)^(1/2))+b*c/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d
*x^2)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-
a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-2*(a*c*d-b*d)*(a*c^2+b*
c)/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d
^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(2*a*c*d+2*b*d)*(EllipticF(x*(
-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-EllipticE(x*(-a*d/(a*c
+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))))/((a*d*x^2+a*c+b)/(d*x^2+c))^(
1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(d*x^2+c)
```

3.354.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx = -\frac{(a^2c-ab)\sqrt{-\frac{ad}{ac+b}}d^3x^3\sqrt{\frac{ac^2+bc}{d^2}}E\left(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right)\mid\frac{ac+b}{ac}\right)-((a^2c-ab)d^3+(a^2c^2+2\,abc+b^2)d^2)\sqrt{\ldots}}{3\,(a^3c^4+\ldots}$$

input
```
integrate(1/x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/3*((a^2*c - a*b)*sqrt(-a*d/(a*c + b))*d^3*x^3*sqrt((a*c^2 + b*c)/d^2)*e
lliptic_e(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - ((a^2*c - a*b
)*d^3 + (a^2*c^2 + 2*a*b*c + b^2)*d^2)*sqrt(-a*d/(a*c + b))*x^3*sqrt((a*c^
2 + b*c)/d^2)*elliptic_f(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c))
- ((a^2*c^2 - b^2)*d^2*x^4 - a^2*c^4 - 2*a*b*c^3 - b^2*c^2 - 2*(a*b*c^2 +
b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^4 + 3*a^2*b*c
^3 + 3*a*b^2*c^2 + b^3*c)*x^3)
```

3.354.6 Sympy [F]

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{1}{x^4\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

input
```
integrate(1/x**4/(a+b/(d*x**2+c))**(1/2),x)
```

output
```
Integral(1/(x**4*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

3.354.7 Maxima [F]

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{dx^2+c}}\,x^4}\,dx$$

input
```
integrate(1/x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a + b/(d*x^2 + c))*x^4), x)
```

3.354.8 Giac [F]

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{dx^2+c}}\,x^4}\,dx$$

input
```
integrate(1/x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a + b/(d*x^2 + c))*x^4), x)
```

3.354.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \int \frac{1}{x^4\,\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

input
```
int(1/(x^4*(a + b/(c + d*x^2))^(1/2)),x)
```

output
```
int(1/(x^4*(a + b/(c + d*x^2))^(1/2)), x)
```

3.355 $\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}} dx$

3.355.1 Optimal result

Integrand size = 21, antiderivative size = 310

$$\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}} dx = -\frac{(b+ac)^2(c+dx^2)^3}{ab^2d^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(35b^2+60abc+24a^2c^2)(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{16a^4d^3} - \frac{(35b^2+60abc+24a^2c^2)(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{24a^3bd^3} + \frac{(7b^2+12abc+6a^2c^2)(c+dx^2)^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{6a^2b^2d^3} - \frac{b(35b^2+60abc+24a^2c^2)\,\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{9/2}d^3}$$

output
```
-1/16*b*(24*a^2*c^2+60*a*b*c+35*b^2)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(
1/2)/a^(1/2))/a^(9/2)/d^3-(a*c+b)^2*(d*x^2+c)^3/a/b^2/d^3/((a*d*x^2+a*c+b)
/(d*x^2+c))^(1/2)+1/16*(24*a^2*c^2+60*a*b*c+35*b^2)*(d*x^2+c)*((a*d*x^2+a*
c+b)/(d*x^2+c))^(1/2)/a^4/d^3-1/24*(24*a^2*c^2+60*a*b*c+35*b^2)*(d*x^2+c)^
2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^3/b/d^3+1/6*(6*a^2*c^2+12*a*b*c+7*b^
2)*(d*x^2+c)^3*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^2/b^2/d^3
```

3.355.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.59

$$\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\frac{\sqrt{a}(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(105b^3+5ab^2(43c+7dx^2)+2a^2b(59c^2+16cdx^2-7d^2x^4)+8a^3(c^3+d^3x^6)\right)}{b+a(c+dx^2)} - 3b(35b^2+}{48a^{9/2}d^3}$$

input
```
Integrate[x^5/(a + b/(c + d*x^2))^(3/2),x]
```

output
```
((Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(105*b^3 + 5*a
*b^2*(43*c + 7*d*x^2) + 2*a^2*b*(59*c^2 + 16*c*d*x^2 - 7*d^2*x^4) + 8*a^3*
(c^3 + d^3*x^6)))/(b + a*(c + d*x^2)) - 3*b*(35*b^2 + 60*a*b*c + 24*a^2*c^
2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(48*a^(9/2)*d^3
)
```

3.355.3 Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.82, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2057, 2053, 2052, 27, 365, 298, 215, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{x^5}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{x^4}{\left(\frac{adx^2+b+ac}{dx^2+c}\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{2052}$$

$$-bd\int \frac{\left(-cx^4+b+ac\right)^2}{d^4x^4\left(a-x^4\right)^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \ 27$$

$$-\frac{b\int \frac{\left(-cx^4+b+ac\right)^2}{x^4\left(a-x^4\right)^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{d^3}$$

$$\Big\downarrow \ 365$$

$$-\frac{b\left(\frac{\int \frac{ac^2x^4+(b+ac)(7b+5ac)}{\left(a-x^4\right)^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{a}-\frac{(ac+b)^2}{ax^2\left(a-x^4\right)^3}\right)}{d^3}$$

$$\Big\downarrow \ 298$$

$$-\frac{b\left(\frac{\frac{\left(24a^2c^2+60abc+35b^2\right)\int \frac{1}{\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{6a}+\frac{\left(6a^2c^2+12abc+7b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{6a\left(a-x^4\right)^3}}{a}-\frac{(ac+b)^2}{ax^2\left(a-x^4\right)^3}\right)}{d^3}$$

$$\Big\downarrow \ 215$$

$$-\frac{b\left(\frac{\frac{\left(24a^2c^2+60abc+35b^2\right)\left(\frac{3\int \frac{1}{\left(a-x^4\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{4a}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a\left(a-x^4\right)^2}\right)}{6a}+\frac{\left(6a^2c^2+12abc+7b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{6a\left(a-x^4\right)^3}}{a}-\frac{(ac+b)^2}{ax^2\left(a-x^4\right)^3}\right)}{d^3}$$

$$\Big\downarrow \ 215$$

3.355. $\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

$$-\frac{b\left(\frac{\frac{\left(24a^2c^2+60abc+35b^2\right)\left(\frac{3\left(\frac{\int \frac{1}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2a}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2a\left(a-x^4\right)}\right)}{4a}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a\left(a-x^4\right)^2}\right)}{6a}+\frac{\left(6a^2c^2+12abc+7b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{6a\left(a-x^4\right)^3}}{a}-\frac{(ac+b)^2}{ax^2(a-x^4)^3}\right)}{d^3}$$

$$\Big\downarrow \; 219$$

$$-\frac{b\left(\frac{\frac{\left(6a^2c^2+12abc+7b^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{6a\left(a-x^4\right)^3}+\frac{\left(24a^2c^2+60abc+35b^2\right)\left(\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2a\left(a-x^4\right)}\right)}{4a}+\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a\left(a-x^4\right)^2}\right)}{6a}}{a}-\frac{(ac+b)^2}{ax^2(a-x^4)^3}\right)}{d^3}$$

input

```
Int[x^5/(a + b/(c + d*x^2))^(3/2),x]
```

3.355. $\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

output
```
-((b*(-((b + a*c)^2/(a*x^2*(a - x^4)^3)) + (((7*b^2 + 12*a*b*c + 6*a^2*c^2
)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(6*a*(a - x^4)^3) + ((35*b^2 + 60
*a*b*c + 24*a^2*c^2)*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/(4*a*(a - x^4)
^2) + (3*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/(2*a*(a - x^4)) + ArcTanh[
Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]]/(2*a^(3/2))))/(4*a)))/(6*a)
)/a))/d^3)
```

3.355.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

3.355. $\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

3.355.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.98

method	result
risch	$\frac{\left(8a^2d^2x^4-8a^2cd\,x^2-22abd\,x^2+8a^2c^2+62abc+57b^2\right)\left(ad\,x^2+ac+b\right)}{48d^3a^4\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}-\frac{b\left(\frac{\left(24a^2c^2+60abc+35b^2\right)\ln\left(\frac{acd+\frac{1}{2}bd+a\,d^2x^2}{\sqrt{a\,d^2}}+\sqrt{a\,c^2+bc+(2ac\ldots}\right.}{2\sqrt{a\,d^2}}\ldots\right.}{16a^4\ldots}$
default	Expression too large to display

input
```
int(x^5/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/d^3*(8*a^2*d^2*x^4-8*a^2*c*d*x^2-22*a*b*d*x^2+8*a^2*c^2+62*a*b*c+57*b
^2)*(a*d*x^2+a*c+b)/a^4/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/16*b/a^4/d^2*(
1/2*(24*a^2*c^2+60*a*b*c+35*b^2)*ln((a*c*d+1/2*b*d+a*d^2*x^2)/(a*d^2)^(1/2
)+(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/(a*d^2)^(1/2)-16*(a^2*c^2
+2*a*b*c+b^2)*(d*x^2+c)/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))
/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(d*x^
2+c)
```

3.355.5 Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 675, normalized size of antiderivative = 2.18

$$\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \left[\frac{3\,(24\,a^3bc^3+84\,a^2b^2c^2+95\,ab^3c+35\,b^4+(24\,a^3bc^2+60\,a^2b^2c+35\,ab^3)dx^2)\sqrt{a}\log\right.$$

input
```
integrate(x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/192*(3*(24*a^3*b*c^3 + 84*a^2*b^2*c^2 + 95*a*b^3*c + 35*b^4 + (24*a^3*b
*c^2 + 60*a^2*b^2*c + 35*a*b^3)*d*x^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c
^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b
)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) +
4*(8*a^4*d^4*x^8 + 2*(4*a^4*c - 7*a^3*b)*d^3*x^6 + 8*a^4*c^4 + 118*a^3*b*c
^3 + (18*a^3*b*c + 35*a^2*b^2)*d^2*x^4 + 215*a^2*b^2*c^2 + 105*a*b^3*c + (
8*a^4*c^3 + 150*a^3*b*c^2 + 250*a^2*b^2*c + 105*a*b^3)*d*x^2)*sqrt((a*d*x^
2 + a*c + b)/(d*x^2 + c)))/(a^6*d^4*x^2 + (a^6*c + a^5*b)*d^3), 1/96*(3*(2
4*a^3*b*c^3 + 84*a^2*b^2*c^2 + 95*a*b^3*c + 35*b^4 + (24*a^3*b*c^2 + 60*a^
2*b^2*c + 35*a*b^3)*d*x^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqr
t(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) + 2
*(8*a^4*d^4*x^8 + 2*(4*a^4*c - 7*a^3*b)*d^3*x^6 + 8*a^4*c^4 + 118*a^3*b*c^
3 + (18*a^3*b*c + 35*a^2*b^2)*d^2*x^4 + 215*a^2*b^2*c^2 + 105*a*b^3*c + (8
*a^4*c^3 + 150*a^3*b*c^2 + 250*a^2*b^2*c + 105*a*b^3)*d*x^2)*sqrt((a*d*x^2
 + a*c + b)/(d*x^2 + c)))/(a^6*d^4*x^2 + (a^6*c + a^5*b)*d^3)]
```

3.355.6 Sympy [F]

$$\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^5}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5/(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(x**5/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

3.355.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.25

$$\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{48\,a^5bc^2+96\,a^4b^2c+48\,a^3b^3-\frac{3\,(24\,a^2bc^2+60\,ab^2c+35\,b^3)\,(adx^2+ac+b)^3}{(dx^2+c)^3}+\frac{8\,(24\,a^3bc^2+60\,a^2b^2c+35}{(dx^2+}}{48\left(a^7d^3\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-3\,a^6d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+3\,a^5d^3\left(\frac{adx^2+a}{dx^2+}\right.\right.} + \frac{(24\,a^2c^2+60\,abc+35\,b^2)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{32\,a^{\frac{9}{2}}d^3}$$

input
```
integrate(x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
1/48*(48*a^5*b*c^2 + 96*a^4*b^2*c + 48*a^3*b^3 - 3*(24*a^2*b*c^2 + 60*a*b^
2*c + 35*b^3)*(a*d*x^2 + a*c + b)^3/(d*x^2 + c)^3 + 8*(24*a^3*b*c^2 + 60*a
^2*b^2*c + 35*a*b^3)*(a*d*x^2 + a*c + b)^2/(d*x^2 + c)^2 - 3*(56*a^4*b*c^2
 + 132*a^3*b^2*c + 77*a^2*b^3)*(a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^7*d^3*s
qrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - 3*a^6*d^3*((a*d*x^2 + a*c + b)/(d*x
^2 + c))^(3/2) + 3*a^5*d^3*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - a^4*d
^3*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(7/2)) + 1/32*(24*a^2*c^2 + 60*a*b*c
+ 35*b^2)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a
) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(9/2)*d^3)
```

3.355.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 663 vs. $2(288) = 576$.

Time = 0.68 (sec) , antiderivative size = 663, normalized size of antiderivative = 2.14

$$\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{1}{48}\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(2\,x^2\left(\frac{4\,x^2}{a^2d\mathrm{sgn}\,(dx^2+c)}-\frac{4\,a^{11}cd^6\mathrm{sgn}(dx^2-}{}\right.\right.$$
$$+\frac{(24\,a^2bc^2+60\,ab^2c+35\,b^3)\log\left(\left|2\,a^3c^3d+6\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)a^{\frac{5}{2}}c^2|d|+6\right.\right.}{}$$
$$+\frac{\left(24\,a^{\frac{13}{2}}bc^2d^3|d|\mathrm{sgn}(dx^2+c)+60\,a^{\frac{11}{2}}b^2cd^3|d|\mathrm{sgn}(dx^2+c)+35\,a^{\frac{9}{2}}b^3d^3|d|\mathrm{sgn}(dx^2+c)\right)\log\left(|a|\right)}{96\,a^9d^7}$$

input
```
integrate(x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
1/48*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2*(4*x^2/(
a^2*d*sgn(d*x^2 + c)) - (4*a^11*c*d^6*sgn(d*x^2 + c) + 11*a^10*b*d^6*sgn(d
*x^2 + c))/(a^13*d^8)) + (8*a^11*c^2*d^5*sgn(d*x^2 + c) + 62*a^10*b*c*d^5*
sgn(d*x^2 + c) + 57*a^9*b^2*d^5*sgn(d*x^2 + c))/(a^13*d^8)) + 1/96*(24*a^2
*b*c^2 + 60*a*b^2*c + 35*b^3)*log(abs(2*a^3*c^3*d + 6*(sqrt(a*d^2)*x^2 - s
qrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^(5/2)*c^2*abs(d) +
 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c
))^2*a^2*c*d + 5*a^2*b*c^2*d + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c
*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a^(3/2)*abs(d) + 10*(sqrt(a*d^2)*x^2 -
sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^(3/2)*b*c*abs(d)
+ 5*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*
c))^2*a*b*d + 4*a*b^2*c*d + 4*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*
x^2 + b*d*x^2 + a*c^2 + b*c))*sqrt(a)*b^2*abs(d) + b^3*d))/(a^(9/2)*d^2*ab
s(d)*sgn(d*x^2 + c)) + 1/96*(24*a^(13/2)*b*c^2*d^3*abs(d)*sgn(d*x^2 + c) +
 60*a^(11/2)*b^2*c*d^3*abs(d)*sgn(d*x^2 + c) + 35*a^(9/2)*b^3*d^3*abs(d)*s
gn(d*x^2 + c))*log(abs(a))/(a^9*d^7)
```

3.355.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{x^5}{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(x^5/(a + b/(c + d*x^2))^(3/2),x)
```

output
```
int(x^5/(a + b/(c + d*x^2))^(3/2), x)
```

3.356 $\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

3.356.1 Optimal result

Integrand size = 21, antiderivative size = 187

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{b(b+ac)}{a^3d^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(7b+4ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8a^3d^2} + \frac{\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4a^2d^2} + \frac{3b(5b+4ac)\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{7/2}d^2}$$

output
```
3/8*b*(4*a*c+5*b)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/a^(7/
2)/d^2-b*(a*c+b)/a^3/d^2/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/8*(4*a*c+7*b)
*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^3/d^2+1/4*(d*x^2+c)^2*((a*d
*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^2/d^2
```

3.356.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{-\frac{\sqrt{a}\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(15b^2+ab\left(17c+5dx^2\right)+2a^2\left(c^2-d^2x^4\right)\right)}{b+a\left(c+dx^2\right)} + 3b(5b+4ac)\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right.}{8a^{7/2}d^2}$$

input
```
Integrate[x^3/(a + b/(c + d*x^2))^(3/2),x]
```

output
```
(-((Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(15*b^2 + a*
b*(17*c + 5*d*x^2) + 2*a^2*(c^2 - d^2*x^4)))/(b + a*(c + d*x^2))) + 3*b*(5
*b + 4*a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(8*a^(
7/2)*d^2)
```

3.356.3 Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.91, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {2057, 2053, 2052, 25, 27, 361, 25, 27, 361, 25, 359, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{x^3}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{x^2}{\left(\frac{adx^2+b+ac}{dx^2+c}\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{2052}$$

$$-bd\int -\frac{-cx^4+b+ac}{d^3x^4\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$bd\int \frac{-cx^4+b+ac}{d^3x^4\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{-cx^4+b+ac}{x^4\left(a-x^4\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{d^2}$$

$$\Big\downarrow \text{361}$$

$$\frac{b\left(\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a^2(a-x^4)^2}-\frac{1}{4}\int -\frac{3bx^4+4a(b+ac)}{a^2x^4(a-x^4)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}\right)}{d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{1}{4}\int \frac{3bx^4+4a(b+ac)}{a^2x^4(a-x^4)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a^2(a-x^4)^2}\right)}{d^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\left(\frac{\int \frac{3bx^4+4a(b+ac)}{x^4\left(a-x^4\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{4a^2}+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a^2(a-x^4)^2}\right)}{d^2}$$

$$\Big\downarrow \text{361}$$

$$\frac{b\left(\frac{\frac{(4ac+7b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2a\left(a-x^4\right)}-\frac{1}{2}\int -\frac{\left(\frac{7b}{a}+4c\right)x^4+8(b+ac)}{x^4\left(a-x^4\right)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{4a^2}+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a^2(a-x^4)^2}\right)}{d^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\left(\frac{\frac{1}{2}\int \frac{\left(\frac{7b}{a}+4c\right)x^4+8(b+ac)}{x^4\left(a-x^4\right)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}+\frac{(4ac+7b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2a\left(a-x^4\right)}}{4a^2}+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a^2(a-x^4)^2}\right)}{d^2}$$

$$\Big\downarrow \text{359}$$

$$\frac{b\left(\frac{\frac{1}{2}\left(\frac{3(4ac+5b)\int \frac{1}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{a}-\frac{8(ac+b)}{ax^2}\right)+\frac{(4ac+7b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2a\left(a-x^4\right)}}{4a^2}+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a^2(a-x^4)^2}\right)}{d^2}$$

$$\Big\downarrow \text{219}$$

3.356. $\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

$$\frac{b\left(\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a^2(a-x^4)^2}+\frac{\frac{1}{2}\left(\frac{3(4ac+5b)\operatorname{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{8(ac+b)}{ax^2}\right)+\frac{(4ac+7b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2a\left(a-x^4\right)}}{4a^2}\right)}{d^2}$$

input
```
Int[x^3/(a + b/(c + d*x^2))^(3/2),x]
```

output
```
(b*((b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*a^2*(a - x^4)^2) + (((7*b
 + 4*a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(2*a*(a - x^4)) + ((-8*(b
 + a*c))/(a*x^2) + (3*(5*b + 4*a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c +
d*x^2)]/Sqrt[a]])/a^(3/2))/2)/(4*a^2)))/d^2
```

3.356.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.356. $\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

3.356.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.33

method	result
risch	$-\frac{\left(-2ad\,x^{2}+2ac+7b\right)\left(ad\,x^{2}+ac+b\right)}{8d^{2}a^{3}\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}}+\frac{b\left(\frac{\left(12ac+15b\right)\ln\left(\frac{acd+\frac{1}{2}bd+a\,d^{2}x^{2}}{\sqrt{a\,d^{2}}}+\sqrt{a\,c^{2}+bc+\left(2acd+bd\right)x^{2}+a\,d^{2}x^{4}}\right)}{2\sqrt{a\,d^{2}}}-\frac{8\left(ac+b\right)\left(d\,x^{2}+\right.}{d\sqrt{a\,d^{2}x^{4}+2acd\,x^{2}+bd\,}}\right.}{8a^{3}d\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}\left(d\,x^{2}+c\right)}$
default	$-\frac{\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}\left(d\,x^{2}+c\right)\left(-4\sqrt{a\,d^{2}x^{4}+2acd\,x^{2}+bd\,x^{2}+a\,c^{2}+bc}\,\sqrt{a\,d^{2}}\,a^{2}d^{2}x^{4}-12\ln\left(\frac{2a\,d^{2}x^{2}+2acd+2\sqrt{a\,d^{2}x^{4}+2acd\,x^{2}+bd\,x^{2}+a\,c^{2}+}}{2\sqrt{a\,d^{2}}}\right.\right.}{}$

input
```
int(x^3/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

3.356. $\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

output
```
-1/8/d^2*(-2*a*d*x^2+2*a*c+7*b)*(a*d*x^2+a*c+b)/a^3/((a*d*x^2+a*c+b)/(d*x^
2+c))^(1/2)+1/8*b/a^3/d*(1/2*(12*a*c+15*b)*ln((a*c*d+1/2*b*d+a*d^2*x^2)/(a
*d^2)^(1/2)+(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/(a*d^2)^(1/2)-8
*(a*c+b)*(d*x^2+c)/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/((a*
d*x^2+a*c+b)/(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(d*x^2+c)
```

3.356.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 541, normalized size of antiderivative = 2.89

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \left[\frac{3\left(4a^2bc^2+9ab^2c+(4a^2bc+5ab^2)dx^2+5b^3\right)\sqrt{a}\log\left(8a^2d^2x^4+8a^2c^2+8(2a^2c\right.}{}\right.$$

$$-\frac{3\left(4a^2bc^2+9ab^2c+(4a^2bc+5ab^2)dx^2+5b^3\right)\sqrt{-a}\arctan\left(\frac{(2adx^2+2ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2(a^2dx^2+a^2c+ab)}\right)-2\left(2a^3d^3x^6\right.}{16\left(a^5d^3x^2+(a^5c\right.}$$

input
```
integrate(x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/32*(3*(4*a^2*b*c^2 + 9*a*b^2*c + (4*a^2*b*c + 5*a*b^2)*d*x^2 + 5*b^3)*s
qrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c +
 b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a
*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^3*d^3*x^6 + (2*a^3*c - 5*a^2*b)*d
^2*x^4 - 2*a^3*c^3 - 17*a^2*b*c^2 - 15*a*b^2*c - (2*a^3*c^2 + 22*a^2*b*c +
 15*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^5*d^3*x^2 + (a
^5*c + a^4*b)*d^2), -1/16*(3*(4*a^2*b*c^2 + 9*a*b^2*c + (4*a^2*b*c + 5*a*b
^2)*d*x^2 + 5*b^3)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sq
rt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - 2*(2*a^3*
d^3*x^6 + (2*a^3*c - 5*a^2*b)*d^2*x^4 - 2*a^3*c^3 - 17*a^2*b*c^2 - 15*a*b^
2*c - (2*a^3*c^2 + 22*a^2*b*c + 15*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/
(d*x^2 + c)))/(a^5*d^3*x^2 + (a^5*c + a^4*b)*d^2)]
```

3.356.6 Sympy [F]

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^3}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(x**3/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

3.356.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.40

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{8\,a^3bc+8\,a^2b^2+\frac{3\,(adx^2+ac+b)^2(4\,abc+5\,b^2)}{(dx^2+c)^2}-\frac{5\,(4\,a^2bc+5\,ab^2)(adx^2+ac+b)}{dx^2+c}}{8\left(a^5d^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-2\,a^4d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+a^3d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}\right)} - \frac{3\,(4\,ac+5\,b)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{16\,a^{\frac{7}{2}}d^2}$$

input
```
integrate(x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
-1/8*(8*a^3*b*c + 8*a^2*b^2 + 3*(a*d*x^2 + a*c + b)^2*(4*a*b*c + 5*b^2)/(d
*x^2 + c)^2 - 5*(4*a^2*b*c + 5*a*b^2)*(a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^
5*d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - 2*a^4*d^2*((a*d*x^2 + a*c +
b)/(d*x^2 + c))^(3/2) + a^3*d^2*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2)) -
 3/16*(4*a*c + 5*b)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(7/2)*d^2)
```

3.356.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 552 vs. $2(169) = 338$.

Time = 0.61 (sec) , antiderivative size = 552, normalized size of antiderivative = 2.95

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{1}{8}\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(\frac{2\,x^2}{a^2d\operatorname{sgn}\left(dx^2+c\right)}-\frac{2\,a^6cd^2+7\,a^5bd^2}{a^8d^4\operatorname{sgn}\left(dx^2+c\right)}\right)$$
$$-\frac{\left(4\,abc+5\,b^2\right)\log\left(\left|2\,a^3c^3d+6\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)a^{\frac{5}{2}}c^2|d|+6\left(\sqrt{ad^2}x^2-\right.\right.\right.}{}$$
$$-\frac{\left(4\,a^{\frac{9}{2}}bcd^2|d|\operatorname{sgn}(dx^2+c)+5\,a^{\frac{7}{2}}b^2d^2|d|\operatorname{sgn}(dx^2+c)\right)\log\left(|a|\right)}{16\,a^7d^5}$$

input
```
integrate(x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
1/8*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2/(a^2*d*sg
n(d*x^2 + c)) - (2*a^6*c*d^2 + 7*a^5*b*d^2)/(a^8*d^4*sgn(d*x^2 + c))) - 1/
16*(4*a*b*c + 5*b^2)*log(abs(2*a^3*c^3*d + 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2
*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^(5/2)*c^2*abs(d) + 6*(sqrt(
a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^2*
c*d + 5*a^2*b*c^2*d + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 +
b*d*x^2 + a*c^2 + b*c))^3*a^(3/2)*abs(d) + 10*(sqrt(a*d^2)*x^2 - sqrt(a*d^
2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^(3/2)*b*c*abs(d) + 5*(sqrt
(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a*b
*d + 4*a*b^2*c*d + 4*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d
*x^2 + a*c^2 + b*c))*sqrt(a)*b^2*abs(d) + b^3*d))/(a^(7/2)*d*abs(d)*sgn(d*
x^2 + c)) - 1/16*(4*a^(9/2)*b*c*d^2*abs(d)*sgn(d*x^2 + c) + 5*a^(7/2)*b^2*
d^2*abs(d)*sgn(d*x^2 + c))*log(abs(a))/(a^7*d^5)
```

3.356.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^3}{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(x^3/(a + b/(c + d*x^2))^(3/2),x)
```

output
```
int(x^3/(a + b/(c + d*x^2))^(3/2), x)
```

3.357 $\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

3.357.1 Optimal result

Integrand size = 19, antiderivative size = 100

$$\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\frac{3b}{2a^2d\sqrt{a+\frac{b}{c+dx^2}}}+\frac{c+dx^2}{2ad\sqrt{a+\frac{b}{c+dx^2}}}-\frac{3b\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{5/2}d}$$

output
```
-3/2*b*arctanh((a+b/(d*x^2+c))^(1/2)/a^(1/2))/a^(5/2)/d+3/2*b/a^2/d/(a+b/(
d*x^2+c))^(1/2)+1/2*(d*x^2+c)/a/d/(a+b/(d*x^2+c))^(1/2)
```

3.357.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.14

$$\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\frac{(c+dx^2)\sqrt{\frac{b+a(c+dx^2)}{c+dx^2}}(3b+a(c+dx^2))}{2a^2d\,(b+a\,(c+dx^2))}-\frac{3b\text{arctanh}\left(\frac{\sqrt{\frac{b+a\left(c+dx^2\right)}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{5/2}d}$$

input
```
Integrate[x/(a + b/(c + d*x^2))^(3/2),x]
```

output
```
((c + d*x^2)*Sqrt[(b + a*(c + d*x^2))/(c + d*x^2)]*(3*b + a*(c + d*x^2)))/
(2*a^2*d*(b + a*(c + d*x^2))) - (3*b*ArcTanh[Sqrt[(b + a*(c + d*x^2))/(c +
 d*x^2)]/Sqrt[a]])/(2*a^(5/2)*d)
```

3.357.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2024, 773, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2024}$$

$$\frac{\int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{3/2}}d(dx^2+c)}{2d}$$

$$\Big\downarrow \text{773}$$

$$-\frac{\int \frac{(dx^2+c)^2}{\left(a+\frac{b}{dx^2+c}\right)^{3/2}}d\frac{1}{dx^2+c}}{2d}$$

$$\Big\downarrow \text{52}$$

$$-\frac{-\frac{3b\int \frac{dx^2+c}{\left(a+\frac{b}{dx^2+c}\right)^{3/2}}d\frac{1}{dx^2+c}}{2a}-\frac{c+dx^2}{a\sqrt{a+\frac{b}{c+dx^2}}}}{2d}$$

$$\Big\downarrow \text{61}$$

$$-\frac{-\frac{3b\left(\frac{\int \frac{dx^2+c}{\sqrt{a+\frac{b}{dx^2+c}}}d\frac{1}{dx^2+c}}{a}+\frac{2}{a\sqrt{a+\frac{b}{c+dx^2}}}\right)}{2a}-\frac{c+dx^2}{a\sqrt{a+\frac{b}{c+dx^2}}}}{2d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{-\frac{3b\left(\frac{2\int \frac{1}{\frac{1}{b(dx^2+c)^2}-\frac{a}{b}}d\sqrt{a+\frac{b}{dx^2+c}}}{ab}+\frac{2}{a\sqrt{a+\frac{b}{c+dx^2}}}\right)}{2a}-\frac{c+dx^2}{a\sqrt{a+\frac{b}{c+dx^2}}}}{2d}$$

$$\Big\downarrow \text{221}$$

$$-\frac{-\frac{3b\left(\frac{2}{a\sqrt{a+\frac{b}{c+dx^2}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}-\frac{c+dx^2}{a\sqrt{a+\frac{b}{c+dx^2}}}}{2d}$$

input
```
Int[x/(a + b/(c + d*x^2))^(3/2),x]
```

output
```
-1/2*(-((c + d*x^2)/(a*Sqrt[a + b/(c + d*x^2)])) - (3*b*(2/(a*Sqrt[a + b/(
c + d*x^2)]) - (2*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/a^(3/2)))/(2*a
))/d
```

3.357.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 2024
```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Simp[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q])   Subst[
Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D
[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &
& PolyQ[Qr, x]
```

3.357.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(84) = 168$.

Time = 1.14 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.17

method	result
risch	$\frac{a d x^2+a c+b}{2 d a^2 \sqrt{\frac{a d x^2+a c+b}{d x^2+c}}}-\frac{b\left(\frac{3 \ln\left(\frac{a c d+\frac{1}{2} b d+a d^2 x^2}{\sqrt{a d^2}}+\sqrt{a c^2+b c+(2 a c d+b d) x^2+a d^2 x^4}\right)}{2 \sqrt{a d^2}}-\frac{2\left(d x^2+c\right)}{d \sqrt{a d^2 x^4+2 a c d x^2+b d x^2+a c^2+b c}}\right) \sqrt{}}{2 a^2 \sqrt{\frac{a d x^2+a c+b}{d x^2+c}}\,(d x^2+c)}$
derivativedivides	$-\frac{\sqrt{\frac{a\left(d x^2+c\right)+b}{d x^2+c}}\,(d x^2+c)\left(-6 \sqrt{(a(d x^2+c)+b)(d x^2+c)}\, a^{\frac{5}{2}}\,(d x^2+c)^2+3 \ln\left(\frac{2 \sqrt{\left(a\left(d x^2+c\right)+b\right)\left(d x^2+c\right)}\,\sqrt{a}+2 a\left(d x^2+\right.}{2 \sqrt{a}}\right.\right.}{}$
default	$\frac{\sqrt{\frac{a d x^2+a c+b}{d x^2+c}}\,(d x^2+c)\left(-3 \ln\left(\frac{2 a d^2 x^2+2 a c d+2 \sqrt{a d^2 x^4+2 a c d x^2+b d x^2+a c^2+b c}\,\sqrt{a d^2}+b d}{2 \sqrt{a d^2}}\right) a b d^2 x^2+2 \sqrt{a d^2 x^4+2 a c d x^2}\right.}{}$

input
```
int(x/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d/a^2*(a*d*x^2+a*c+b)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/2*b/a^2*(3/2
*ln((a*c*d+1/2*b*d+a*d^2*x^2)/(a*d^2)^(1/2)+(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a
*d^2*x^4)^(1/2))/(a*d^2)^(1/2)-2*(d*x^2+c)/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^
2+a*c^2+b*c)^(1/2))/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)*(d*
x^2+c))^(1/2)/(d*x^2+c)
```

3.357.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(84) = 168$.

Time = 0.34 (sec) , antiderivative size = 395, normalized size of antiderivative = 3.95

$$\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \left[\frac{3\left(abdx^2+abc+b^2\right)\sqrt{a}\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\left(2\,a^2c+ab\right)dx^2+8\,abc+b^2-4\right.}{}\right.$$

input
```
integrate(x/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*(a*b*d*x^2 + a*b*c + b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 +
8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x
^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a^
2*d^2*x^4 + a^2*c^2 + (2*a^2*c + 3*a*b)*d*x^2 + 3*a*b*c)*sqrt((a*d*x^2 + a
*c + b)/(d*x^2 + c)))/(a^4*d^2*x^2 + (a^4*c + a^3*b)*d), 1/4*(3*(a*b*d*x^2
 + a*b*c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt(
(a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) + 2*(a^2*d^2*x
^4 + a^2*c^2 + (2*a^2*c + 3*a*b)*d*x^2 + 3*a*b*c)*sqrt((a*d*x^2 + a*c + b)
/(d*x^2 + c)))/(a^4*d^2*x^2 + (a^4*c + a^3*b)*d)]
```

3.357.6 Sympy [F]

$$\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(x/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

3.357.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.61

$$\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{2\,ab-\frac{3\,(adx^2+ac+b)b}{dx^2+c}}{2\left(a^3d\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-a^2d\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}\right)}+\frac{3\,b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{4\,a^{\frac{5}{2}}d}$$

input
```
integrate(x/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
1/2*(2*a*b - 3*(a*d*x^2 + a*c + b)*b/(d*x^2 + c))/(a^3*d*sqrt((a*d*x^2 + a
*c + b)/(d*x^2 + c)) - a^2*d*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2)) + 3/
4*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt
((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(5/2)*d)
```

3.357.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.357.9 Mupad [B] (verification not implemented)

Time = 18.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.61

$$\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\left(\frac{a\,(d\,x^2+c)}{b}+1\right)^{3/2}(d\,x^2+c)\;{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{5}{2};\;\frac{7}{2};\;-\frac{a\,(d\,x^2+c)}{b}\right)}{5\,d\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}$$

input
```
int(x/(a + b/(c + d*x^2))^(3/2),x)
```

output
```
(((a*(c + d*x^2))/b + 1)^(3/2)*(c + d*x^2)*hypergeom([3/2, 5/2], 7/2, -(a*
(c + d*x^2))/b))/(5*d*(a + b/(c + d*x^2))^(3/2))
```

3.357. $\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

3.358 $\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

3.358.1 Optimal result

Integrand size = 21, antiderivative size = 134

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{b}{a(b+ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{(b+ac)^{3/2}}$$

output
```
arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/a^(3/2)-c^(3/2)*arctanh
(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))/(a*c+b)^(3/2)-b/
a/(a*c+b)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)
```

3.358.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{b}{a(b+ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{c^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{(-b-ac)^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[1/(x*(a + b/(c + d*x^2))^(3/2)),x]
```

output
```
-(b/(a*(b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])) - (c^(3/2)*ArcTan
[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(-b - a*
c)^(3/2) + ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]]/a^(3/2)
```

3.358.3 Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2057, 2053, 2052, 25, 27, 382, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \left(a+\frac{b}{c+dx^2}\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{1}{x \left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2} \int \frac{1}{x^2 \left(\frac{adx^2+b+ac}{dx^2+c}\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{2052}$$

$$-bd \int -\frac{1}{dx^4 (a-x^4) (-cx^4+b+ac)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$bd \int \frac{1}{dx^4 (a-x^4) (-cx^4+b+ac)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{27}$$

$$b \int \frac{1}{x^4 (a-x^4) (-cx^4+b+ac)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{382}$$

$$b\left(\frac{\int \frac{-cx^4+b+2ac}{(a-x^4)(-cx^4+b+ac)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{a(ac+b)}-\frac{1}{ax^2(ac+b)}\right)$$

$\Big\downarrow$ 397

$$b\left(\frac{\frac{(ac+b)\int \frac{1}{a-x^4}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{b}-\frac{ac^2\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{b}}{a(ac+b)}-\frac{1}{ax^2(ac+b)}\right)$$

$\Big\downarrow$ 219

$$b\left(\frac{\frac{(ac+b)\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{ac^2\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{b}}{a(ac+b)}-\frac{1}{ax^2(ac+b)}\right)$$

$\Big\downarrow$ 221

$$b\left(\frac{\frac{(ac+b)\text{arctanh}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{ac^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{b\sqrt{ac+b}}}{a(ac+b)}-\frac{1}{ax^2(ac+b)}\right)$$

input
```
Int[1/(x*(a + b/(c + d*x^2))^(3/2)),x]
```

output
```
b*(-(1/(a*(b + a*c)*x^2)) + (((b + a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(
c + d*x^2)]/Sqrt[a]])/(Sqrt[a]*b) - (a*c^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(b +
a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/(b*Sqrt[b + a*c]))/(a*(b + a*
c)))
```

3.358.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.358. $\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

3.358. $\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

3.358.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1014 vs. 2(116) = 232.

Time = 0.13 (sec) , antiderivative size = 1015, normalized size of antiderivative = 7.57

method	result	size
default	Expression too large to display	1015

input
```
int(1/x/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/a*(-ln(1/2*(2*a*d^2*x^2+2
*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d
)/(a*d^2)^(1/2))*a^3*c^2*d^2*x^2-2*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^
4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a
^2*b*c*d^2*x^2+(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*(a*c^2+
b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*(a*
d^2)^(1/2)*a^2*c*d*x^2-ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^
2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^3*c^3*d-ln(
1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)
*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b^2*d^2*x^2-3*ln(1/2*(2*a*d^2*x^2+2*a
*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/
(a*d^2)^(1/2))*a^2*b*c^2*d+(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*a*c
^2+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b
*c)/x^2)*(a*d^2)^(1/2)*a^2*c^2-3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+
2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b
^2*c*d+(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*(a*c^2+b*c)^(1/
2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*(a*d^2)^(1/
2)*a*b*c+2*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(a*d^2)^(1/2)*a*b*c-ln(1/2*(2
*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^
2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^3*d+2*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*...
```

3.358.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(116) = 232.

Time = 0.47 (sec) , antiderivative size = 1477, normalized size of antiderivative = 11.02

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2)*sqrt(a)*log(8*a^2*d^
2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x
^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(
d*x^2 + c))) + (a^3*c*d*x^2 + a^3*c^2 + a^2*b*c)*sqrt(c/(a*c + b))*log(((8
*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8
*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a^2*c^2 + 3*a*b*c + b^2)*d^
2*x^4 + 2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 + (4*a^2*c^3 + 7*a*b*c^2 + 3*b^2
*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(c/(a*c + b)))/x^4) -
 4*(a*b*d*x^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^2 + 2
*a^3*b*c + a^2*b^2 + (a^4*c + a^3*b)*d*x^2), -1/4*(2*(a^2*c^2 + (a^2*c + a
*b)*d*x^2 + 2*a*b*c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqr
t(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - (
a^3*c*d*x^2 + a^3*c^2 + a^2*b*c)*sqrt(c/(a*c + b))*log(((8*a^2*c^2 + 8*a*b
*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*
a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4 + 2*a^2*c^
4 + 4*a*b*c^3 + 2*b^2*c^2 + (4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x^2)*sqrt(
(a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(c/(a*c + b)))/x^4) + 4*(a*b*d*x^2 +
a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^2 + 2*a^3*b*c + a^2*b
^2 + (a^4*c + a^3*b)*d*x^2), 1/4*(2*(a^3*c*d*x^2 + a^3*c^2 + a^2*b*c)*sqrt
(-c/(a*c + b))*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a...
```

3.358.6 Sympy [F]

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{x\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(1/(x*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

3.358.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.50

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{c^2\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{2\sqrt{(ac+b)c}(ac+b)} - \frac{b}{(a^2c+ab)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}} - \frac{\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{2\,a^{\frac{3}{2}}}$$

input
```
integrate(1/x/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
1/2*c^2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/
(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/(sqrt((a*c
+ b)*c)*(a*c + b)) - b/((a^2*c + a*b)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
)) - 1/2*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) +
 sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/a^(3/2)
```

3.358.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.358.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{x\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(1/(x*(a + b/(c + d*x^2))^(3/2)),x)
```

output
```
int(1/(x*(a + b/(c + d*x^2))^(3/2)), x)
```

3.359 $\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

3.359.1 Optimal result

Integrand size = 21, antiderivative size = 146

$$\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\frac{3bd}{2(b+ac)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-\frac{c+dx^2}{2(b+ac)x^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-\frac{3b\sqrt{c}d\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{2(b+ac)^{5/2}}$$

output
```
-3/2*b*d*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))*
c^(1/2)/(a*c+b)^(5/2)+3/2*b*d/(a*c+b)^2/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+
1/2*(-d*x^2-c)/(a*c+b)/x^2/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)
```

3.359.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=-\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,(b(c-2dx^2)+ac(c+dx^2))}{2(b+ac)^2x^2\left(b+a\left(c+dx^2\right)\right)}+\frac{3b\sqrt{c}d\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{2(-b-ac)^{5/2}}$$

input
```
Integrate[1/(x^3*(a + b/(c + d*x^2))^(3/2)),x]
```

output
```
-1/2*((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b*(c - 2*d*x^2) +
 a*c*(c + d*x^2)))/((b + a*c)^2*x^2*(b + a*(c + d*x^2))) + (3*b*Sqrt[c]*d*
ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(2
*(-b - a*c)^(5/2))
```

3.359.3 Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.77, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2057, 2053, 2052, 253, 264, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+\frac{b}{c+dx^2}\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \frac{1}{x^3 \left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2053}$$

$$\frac{1}{2} \int \frac{1}{x^4 \left(\frac{adx^2+b+ac}{dx^2+c}\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{ 2052}$$

$$-bd \int \frac{1}{x^4 \left(-cx^4+b+ac\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\Big\downarrow \text{ 253}$$

$$-bd \left(\frac{3 \int \frac{1}{x^4 (-cx^4+b+ac)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{2(ac+b)}+\frac{1}{2x^2(ac+b)\left(ac+b-cx^4\right)}\right)$$

$$\Big\downarrow \text{ 264}$$

$$-bd\left(\frac{3\left(\frac{c\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{ac+b}-\frac{1}{x^2(ac+b)}\right)}{2(ac+b)}+\frac{1}{2x^2(ac+b)\left(ac+b-cx^4\right)}\right)$$

$$\Big\downarrow \; 221$$

$$-bd\left(\frac{3\left(\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{(ac+b)^{3/2}}-\frac{1}{x^2(ac+b)}\right)}{2(ac+b)}+\frac{1}{2x^2(ac+b)\left(ac+b-cx^4\right)}\right)$$

input
```
Int[1/(x^3*(a + b/(c + d*x^2))^(3/2)),x]
```

output
```
-(b*d*(1/(2*(b + a*c)*x^2*(b + a*c - c*x^4)) + (3*(-(1/((b + a*c)*x^2)) +
(Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b +
a*c]])/(b + a*c)^(3/2)))/(2*(b + a*c))))
```

3.359.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

3.359. $\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

3.359.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.67

method	result
risch	$-\frac{c\left(a d\,x^{2}+a c+b\right)}{2\left(a c+b\right)^{2} x^{2} \sqrt{\frac{a d\,x^{2}+a c+b}{d\,x^{2}+c}}}+\frac{d b \left(-\frac{3 c \ln\left(\frac{2 a\,c^{2}+2 b c+\left(2 a c d+b d\right) x^{2}+2 \sqrt{a\,c^{2}+b c}\,\sqrt{a\,c^{2}+b c+\left(2 a c d+b d\right) x^{2}+a\,d^{2} x^{4}}}{x^{2}}\right)}{2 \sqrt{a\,c^{2}+b c}}+\frac{2 d\,x^{2}+2\cdots}{\sqrt{a\,d^{2} x^{4}+2 a c d\,x^{2}+b\cdots}}\cdots\right.}{2\left(a c+b\right)^{2} \sqrt{\frac{a d\,x^{2}+a c+b}{d\,x^{2}+c}}\,\left(d\,x^{2}+c\right)\cdots}$
default	Expression too large to display

input
```
int(1/x^3/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a*c+b)^2*c*(a*d*x^2+a*c+b)/x^2/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/2
*d*b/(a*c+b)^2*(-3/2*c/(a*c^2+b*c)^(1/2)*ln((2*a*c^2+2*b*c+(2*a*c*d+b*d)*x
^2+2*(a*c^2+b*c)^(1/2)*(a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/x^2)
+2*(d*x^2+c)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/((a*d*x^2+a*
c+b)/(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(d*x^2+c)
```

3.359.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 599, normalized size of antiderivative = 4.10

$$\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \left[\frac{3\,(abd^2x^4+(abc+b^2)dx^2)\sqrt{\frac{c}{ac+b}}\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)\,d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,(2\,a\cdots}{\cdots}\right.}{\cdots}\right.$$

input
```
integrate(1/x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(3*(a*b*d^2*x^4 + (a*b*c + b^2)*d*x^2)*sqrt(c/(a*c + b))*log(((8*a^2*
c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a
^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4
 + 2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 + (4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d
*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(c/(a*c + b)))/x^4) - 4*((
a*c - 2*b)*d^2*x^4 + a*c^3 + (2*a*c^2 - b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2
+ a*c + b)/(d*x^2 + c)))/((a^3*c^2 + 2*a^2*b*c + a*b^2)*d*x^4 + (a^3*c^3 +
 3*a^2*b*c^2 + 3*a*b^2*c + b^3)*x^2), 1/4*(3*(a*b*d^2*x^4 + (a*b*c + b^2)*
d*x^2)*sqrt(-c/(a*c + b))*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)
*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 + a*c
^2 + b*c)) - 2*((a*c - 2*b)*d^2*x^4 + a*c^3 + (2*a*c^2 - b*c)*d*x^2 + b*c^
2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^2 + 2*a^2*b*c + a*b^2)*d
*x^4 + (a^3*c^3 + 3*a^2*b*c^2 + 3*a*b^2*c + b^3)*x^2)]
```

3.359.6 Sympy [F]

$$\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{x^3\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**3/(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(1/(x**3*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

3.359.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.69

$$\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{3\,bcd\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{4\,(a^2c^2+2\,abc+b^2)\sqrt{(ac+b)c}} + \frac{\frac{3\,(adx^2+ac+b)bcd}{dx^2+c}-2\,(abc+b^2)d}{2\left((a^2c^3+2\,abc^2+b^2c)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-(a^3c^3+3\,a^2bc^2+3\,ab^2c+b^3)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)}$$

input
```
integrate(1/x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
3/4*b*c*d*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c)
)/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/((a^2*c^2
 + 2*a*b*c + b^2)*sqrt((a*c + b)*c)) + 1/2*(3*(a*d*x^2 + a*c + b)*b*c*d/(d
*x^2 + c) - 2*(a*b*c + b^2)*d)/((a^2*c^3 + 2*a*b*c^2 + b^2*c)*((a*d*x^2 +
a*c + b)/(d*x^2 + c))^(3/2) - (a^3*c^3 + 3*a^2*b*c^2 + 3*a*b^2*c + b^3)*sq
rt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
```

3.359.8 Giac [F]

$$\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(1/x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
undef
```

3.359.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3 \left(a + \frac{b}{c+dx^2}\right)^{3/2}} \, dx = \int \frac{1}{x^3 \left(a + \frac{b}{d\,x^2+c}\right)^{3/2}} \, dx$$

input
```
int(1/(x^3*(a + b/(c + d*x^2))^(3/2)),x)
```

output
```
int(1/(x^3*(a + b/(c + d*x^2))^(3/2)), x)
```

3.360 $\int \frac{1}{x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

3.360.1 Optimal result

Integrand size = 21, antiderivative size = 212

$$\int \frac{1}{x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=-\frac{abd^2}{(b+ac)^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-\frac{(3b-4ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8(b+ac)^3x^2}-\frac{(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4(b+ac)^2x^4}-\frac{3b(b-4ac)d^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{8\sqrt{c}(b+ac)^{7/2}}$$

output
```
-3/8*b*(-4*a*c+b)*d^2*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a
*c+b)^(1/2))/(a*c+b)^(7/2)/c^(1/2)-a*b*d^2/(a*c+b)^3/((a*d*x^2+a*c+b)/(d*x
^2+c))^(1/2)-1/8*(-4*a*c+3*b)*d*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2
)/(a*c+b)^3/x^2-1/4*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^
2/x^4
```

3.360.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(b^2(2c+5dx^2)+2a^2c(c^2-d^2x^4)+ab(4c^2+5cdx^2+13d^2x^4)\right)}{8(b+ac)^3x^4\left(b+a\left(c+dx^2\right)\right)} - \frac{3b(b-4ac)d^2\arctan\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{8\sqrt{c}(-b-ac)^{7/2}}$$

input
```
Integrate[1/(x^5*(a + b/(c + d*x^2))^(3/2)),x]
```

output
```
-1/8*((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b^2*(2*c + 5*d*x^
2) + 2*a^2*c*(c^2 - d^2*x^4) + a*b*(4*c^2 + 5*c*d*x^2 + 13*d^2*x^4)))/((b
+ a*c)^3*x^4*(b + a*(c + d*x^2))) - (3*b*(b - 4*a*c)*d^2*ArcTan[(Sqrt[c]*S
qrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(8*Sqrt[c]*(-b - a*
c)^(7/2))
```

3.360.3 Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.94, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {2057, 2053, 2052, 25, 27, 361, 25, 361, 25, 359, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{1}{x^5\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{1}{x^6\left(\frac{adx^2+b+ac}{dx^2+c}\right)^{3/2}}dx^2$$

$$\downarrow 2052$$

$$-bd\int -\frac{d\left(a-x^4\right)}{x^4\left(-cx^4+b+ac\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\downarrow 25$$

$$bd\int \frac{d\left(a-x^4\right)}{x^4\left(-cx^4+b+ac\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\downarrow 27$$

$$bd^2\int \frac{a-x^4}{x^4\left(-cx^4+b+ac\right)^3}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}$$

$$\downarrow 361$$

$$bd^2\left(-\frac{1}{4}\int -\frac{\frac{4a}{b+ac}-\frac{3bx^4}{(b+ac)^2}}{x^4\left(-cx^4+b+ac\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}-\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4(ac+b)^2\left(ac+b-cx^4\right)^2}\right)$$

$$\downarrow 25$$

$$bd^2\left(\frac{1}{4}\int \frac{\frac{4a}{b+ac}-\frac{3bx^4}{(b+ac)^2}}{x^4\left(-cx^4+b+ac\right)^2}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}-\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4(ac+b)^2\left(ac+b-cx^4\right)^2}\right)$$

$$\downarrow 361$$

$$bd^2\left(\frac{1}{4}\left(-\frac{1}{2}\int -\frac{\frac{8a}{(b+ac)^2}-\frac{(3b-4ac)x^4}{(b+ac)^3}}{x^4\left(-cx^4+b+ac\right)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}-\frac{(3b-4ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b)^3\left(ac+b-cx^4\right)}\right)-\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4(ac+b)^2\left(ac+b-cx^4\right)}\right.$$

$$\downarrow 25$$

$$bd^2\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{\frac{8a}{(b+ac)^2}-\frac{(3b-4ac)x^4}{(b+ac)^3}}{x^4\left(-cx^4+b+ac\right)}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}-\frac{(3b-4ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b)^3\left(ac+b-cx^4\right)}\right)-\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4(ac+b)^2\left(ac+b-cx^4\right)^2}\right)$$

$$\downarrow 359$$

$$bd^2\left(\frac{1}{4}\left(\frac{1}{2}\left(-\frac{3(b-4ac)\int \frac{1}{-cx^4+b+ac}d\sqrt{\frac{adx^2+b+ac}{dx^2+c}}}{(ac+b)^3}-\frac{8a}{x^2(ac+b)^3}\right)-\frac{(3b-4ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b)^3\left(ac+b-cx^4\right)}\right)-\frac{b\sqrt{\frac{ac\ldots}{\ldots}}}{4(ac+b)^2(a\ldots}\right.$$

3.360. $\int \frac{1}{x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

$$\Big\downarrow 221$$

$$bd^2\left(\frac{1}{4}\left(\frac{1}{2}\left(-\frac{3(b-4ac)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{\sqrt{c}(ac+b)^{7/2}}-\frac{8a}{x^2(ac+b)^3}\right)-\frac{(3b-4ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2(ac+b)^3\left(ac+b-cx^4\right)}\right)-\frac{b\sqrt{\frac{ac+}{c}}}{4(ac+b)^2\,(a}$$

input
```
Int[1/(x^5*(a + b/(c + d*x^2))^(3/2)),x]
```

output
```
b*d^2*(-1/4*(b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/((b + a*c)^2*(b + a*
c - c*x^4)^2) + (-1/2*((3*b - 4*a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]
)/((b + a*c)^3*(b + a*c - c*x^4)) + ((-8*a)/((b + a*c)^3*x^2) - (3*(b - 4*
a*c)*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]
])/(Sqrt[c]*(b + a*c)^(7/2)))/2)/4)
```

3.360.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

3.360.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.32

method	result
risch	$-\frac{\left(a d\,x^{2}+a c+b\right)\left(-2 a c d\,x^{2}+5 b d\,x^{2}+2 a\,c^{2}+2 b c\right)}{8\left(a c+b\right)^{3} x^{4} \sqrt{\frac{a d\,x^{2}+a c+b}{d\,x^{2}+c}}}-\frac{d^{2} b\left(-\frac{\left(12 a c-3 b\right)\ln\left(\frac{2 a\,c^{2}+2 b c+\left(2 a c d+b d\right)x^{2}+2\sqrt{a\,c^{2}+b c}\,\sqrt{a\,c^{2}+b c+\left(2 a c d+b d\right)x^{2}+ \cdots}}{x^{2}}\right)}{2\sqrt{a\,c^{2}+b c}} \cdots\right)}{8\left(a c+b\right)^{3}\sqrt{\frac{a d\,x^{2}+a c+ \cdots}{d\,x^{2}+c}}}$
default	Expression too large to display

input
```
int(1/x^5/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(a*d*x^2+a*c+b)*(-2*a*c*d*x^2+5*b*d*x^2+2*a*c^2+2*b*c)/(a*c+b)^3/x^4/
((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/8*d^2*b/(a*c+b)^3*(-1/2*(12*a*c-3*b)/(
a*c^2+b*c)^(1/2)*ln((2*a*c^2+2*b*c+(2*a*c*d+b*d)*x^2+2*(a*c^2+b*c)^(1/2)*(
a*c^2+b*c+(2*a*c*d+b*d)*x^2+a*d^2*x^4)^(1/2))/x^2)+8*a*(d*x^2+c)/(a*d^2*x^
4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*
((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(d*x^2+c)
```

3.360.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(192) = 384$.

Time = 0.73 (sec) , antiderivative size = 961, normalized size of antiderivative = 4.53

$$\int \frac{1}{x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \left[\frac{3\left((4\,a^2bc-ab^2)d^3x^6+(4\,a^2bc^2+3\,ab^2c-b^3)d^2x^4\right)\sqrt{ac^2+bc}\log\left(\frac{(8\,a^2c^2+8\,abc+b\cdots}{\cdots}\right.}{\cdots}\right.$$

$$-\frac{3\left((4\,a^2bc-ab^2)d^3x^6+(4\,a^2bc^2+3\,ab^2c-b^3)d^2x^4\right)\sqrt{-ac^2-bc}\arctan\left(\frac{((2\,ac+b)dx^2+2\,ac^2+2\,bc)\sqrt{-ac^2-bc}\sqrt{\frac{adx^2\cdots}{dx\cdots}}}{2\,(a^2c^3+2\,abc^2+(a^2c^2+abc)dx^2+b^2c)\cdots}\right.}{16\left((a^5c^5+4\,a^4bc\cdots\right.}$$

input
```
integrate(1/x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/32*(3*((4*a^2*b*c - a*b^2)*d^3*x^6 + (4*a^2*b*c^2 + 3*a*b^2*c - b^3)*d^
2*x^4)*sqrt(a*c^2 + b*c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*
c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 + 4
*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(
a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4) + 4*((2*a^3*c^3 -
 11*a^2*b*c^2 - 13*a*b^2*c)*d^3*x^6 - 2*a^3*c^6 - 6*a^2*b*c^5 - 6*a*b^2*c^
4 + (2*a^3*c^4 - 16*a^2*b*c^3 - 23*a*b^2*c^2 - 5*b^3*c)*d^2*x^4 - 2*b^3*c^
3 - (2*a^3*c^5 + 11*a^2*b*c^4 + 16*a*b^2*c^3 + 7*b^3*c^2)*d*x^2)*sqrt((a*d
*x^2 + a*c + b)/(d*x^2 + c)))/((a^5*c^5 + 4*a^4*b*c^4 + 6*a^3*b^2*c^3 + 4*
a^2*b^3*c^2 + a*b^4*c)*d*x^6 + (a^5*c^6 + 5*a^4*b*c^5 + 10*a^3*b^2*c^4 + 1
0*a^2*b^3*c^3 + 5*a*b^4*c^2 + b^5*c)*x^4), -1/16*(3*((4*a^2*b*c - a*b^2)*d
^3*x^6 + (4*a^2*b*c^2 + 3*a*b^2*c - b^3)*d^2*x^4)*sqrt(-a*c^2 - b*c)*arcta
n(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x
^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2
+ b^2*c)) - 2*((2*a^3*c^3 - 11*a^2*b*c^2 - 13*a*b^2*c)*d^3*x^6 - 2*a^3*c^6
 - 6*a^2*b*c^5 - 6*a*b^2*c^4 + (2*a^3*c^4 - 16*a^2*b*c^3 - 23*a*b^2*c^2 -
5*b^3*c)*d^2*x^4 - 2*b^3*c^3 - (2*a^3*c^5 + 11*a^2*b*c^4 + 16*a*b^2*c^3 +
7*b^3*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^5*c^5 + 4*a^4
*b*c^4 + 6*a^3*b^2*c^3 + 4*a^2*b^3*c^2 + a*b^4*c)*d*x^6 + (a^5*c^6 + 5*a^4
*b*c^5 + 10*a^3*b^2*c^4 + 10*a^2*b^3*c^3 + 5*a*b^4*c^2 + b^5*c)*x^4)]
```

3.360.6 Sympy [F]

$$\int \frac{1}{x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{x^5\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**5/(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(1/(x**5*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

3.360.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(192) = 384$.

Time = 0.32 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.12

$$\int \frac{1}{x^5 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{3\,(4\,abc-b^2)d^2\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{16\,(a^3c^3+3\,a^2bc^2+3\,ab^2c+b^3)\sqrt{(ac+b)c}}$$

$$-\frac{8\,(a^3bc^2+2\,a^2b^2c+ab^3)d^2+\frac{3\,(4\,abc^2-b^2c)\left(adx^2+ac+b\right)^2d^2}{(dx^2+c)^2}-\frac{5\,(4}{}}{8\left((a^3c^5+3\,a^2bc^4+3\,ab^2c^3+b^3c^2)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-2\,(a^4c^5+4\,a^3bc^4+6\,a^2b^2c^3+4\,ab^3c^2+b^4c)\left(\frac{adx^2+ac+b}{dx^2+c}\right.\right.}$$

input
```
integrate(1/x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
-3/16*(4*a*b*c - b^2)*d^2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - s
qrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b
)*c)))/((a^3*c^3 + 3*a^2*b*c^2 + 3*a*b^2*c + b^3)*sqrt((a*c + b)*c)) - 1/8
*(8*(a^3*b*c^2 + 2*a^2*b^2*c + a*b^3)*d^2 + 3*(4*a*b*c^2 - b^2*c)*(a*d*x^2
 + a*c + b)^2*d^2/(d*x^2 + c)^2 - 5*(4*a^2*b*c^2 + 3*a*b^2*c - b^3)*(a*d*x
^2 + a*c + b)*d^2/(d*x^2 + c))/((a^3*c^5 + 3*a^2*b*c^4 + 3*a*b^2*c^3 + b^3
*c^2)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - 2*(a^4*c^5 + 4*a^3*b*c^4 +
 6*a^2*b^2*c^3 + 4*a*b^3*c^2 + b^4*c)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3
/2) + (a^5*c^5 + 5*a^4*b*c^4 + 10*a^3*b^2*c^3 + 10*a^2*b^3*c^2 + 5*a*b^4*c
 + b^5)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
```

3.360.8 Giac [F]

$$\int \frac{1}{x^5 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^5}\,dx$$

input
```
integrate(1/x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
undef
```

3.360.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{x^5 \left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(1/(x^5*(a + b/(c + d*x^2))^(3/2)),x)
```

output
```
int(1/(x^5*(a + b/(c + d*x^2))^(3/2)), x)
```

3.361 $\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

3.361.1 Optimal result

Integrand size = 21, antiderivative size = 482

$$\begin{aligned}\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx &= -\frac{x^3(c+dx^2)}{ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-\frac{(8b+ac)x(b+ac+adx^2)}{5a^3d^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\\
&+\frac{6x^3(b+ac+adx^2)}{5a^2d\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{(16b^2+16abc+a^2c^2)\,x(b+ac+adx^2)}{5a^4d^2\,(c+dx^2)\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\\
&-\frac{\sqrt{c}(16b^2+16abc+a^2c^2)\,(b+ac+adx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5a^4d^{5/2}\,(c+dx^2)\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\\
&+\frac{c^{3/2}(8b+ac)\,(b+ac+adx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{5a^3d^{5/2}\,(c+dx^2)\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\end{aligned}$$

output
```
-x^3*(d*x^2+c)/a/d/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/5*(a*c+8*b)*x*(a*d*
x^2+a*c+b)/a^3/d^2/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+6/5*x^3*(a*d*x^2+a*c+
b)/a^2/d/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/5*(a^2*c^2+16*a*b*c+16*b^2)*x
*(a*d*x^2+a*c+b)/a^4/d^2/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/5*c
^(3/2)*(a*c+8*b)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*E
llipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/a^3/d^(5/2
)/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(
d*x^2+c))^(1/2)-1/5*(a^2*c^2+16*a*b*c+16*b^2)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/
c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),
(b/(a*c+b))^(1/2))*c^(1/2)/a^4/d^(5/2)/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c
))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

3.361.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.61 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.61

$$\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx =$$

$$-\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(a\sqrt{\frac{d}{c}}x(c+dx^2)\left(8b^2+ab(9c+2dx^2)+a^2(c^2-d^2x^4)\right)+i(16b^3+32ab^2c+17a^2bc^2+a^3c^3)\right.}{5a}$$

input
```
Integrate[x^4/(a + b/(c + d*x^2))^(3/2),x]
```

output
```
-1/5*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*Sqrt[d/c]*x*(c + d*x^2)*(8*
b^2 + a*b*(9*c + 2*d*x^2) + a^2*(c^2 - d^2*x^4)) + I*(16*b^3 + 32*a*b^2*c
+ 17*a^2*b*c^2 + a^3*c^3)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*
x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] - (8*I)*b*(2*b^
2 + 3*a*b*c + a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2
)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(a^4*c^2*(d/c)^(
5/2)*(b + a*(c + d*x^2)))
```

3.361.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 450, normalized size of antiderivative = 0.93, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {2057, 2058, 369, 27, 443, 25, 27, 444, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{x^4}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{ac+adx^2+b}\int \frac{x^4\left(dx^2+c\right)^{3/2}}{\left(adx^2+b+ac\right)^{3/2}}dx}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{369}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{\int \frac{3x^2\sqrt{dx^2+c}\left(2dx^2+c\right)}{\sqrt{adx^2+b+ac}}dx}{ad}-\frac{x^3\left(c+dx^2\right)^{3/2}}{ad\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{3\int \frac{x^2\sqrt{dx^2+c}\left(2dx^2+c\right)}{\sqrt{adx^2+b+ac}}dx}{ad}-\frac{x^3\left(c+dx^2\right)^{3/2}}{ad\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{443}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{3\left(\frac{\int -\frac{dx^2\left((8b+ac)dx^2+c(6b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{5ad}+\frac{2x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5a}\right)}{ad}-\frac{x^3\left(c+dx^2\right)^{3/2}}{ad\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{3\left(\frac{2x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5a}-\frac{\int \frac{dx^2\left((8b+ac)dx^2+c(6b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{5ad}\right)}{ad}-\frac{x^3\left(c+dx^2\right)^{3/2}}{ad\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{3\left(\frac{2x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5a}-\frac{\int \frac{x^2\left((8b+ac)dx^2+c(6b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{5a}\right)}{ad}-\frac{x^3\left(c+dx^2\right)^{3/2}}{ad\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \ 444$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{3\left(\frac{2x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5a}-\frac{\frac{x(ac+8b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{\int \frac{d\left(\left(16b^2+16acb+a^2c^2\right)dx^2+c(b+ac)(8b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3ad^2}}{5a}\right)}{ad}-\frac{x^3\left(c+dx^2\right)^{3/2}}{ad\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{3\left(\frac{2x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5a}-\frac{\frac{x(ac+8b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{\int \frac{\left(16b^2+16acb+a^2c^2\right)dx^2+c(b+ac)(8b+ac)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3ad}}{5a}\right)}{ad}-\frac{x^3\left(c+dx^2\right)^{3/2}}{ad\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \ 406$$

3.361. $\displaystyle\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{3\left(\frac{2x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5a}-\frac{\frac{x(ac+8b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{d\left(a^2c^2+16abc+16b^2\right)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+c(ac+b)(ac+8b)\int}{3ad}}{5a}}{ad}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\downarrow$ 320

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{3\left(\frac{2x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5a}-\frac{\frac{x(ac+8b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{d\left(a^2c^2+16abc+16b^2\right)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{c^{3/2}(ac+8b)\sqrt{ac}}{\sqrt{c}}}{3ad}}{5a}}{ad}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\downarrow$ 388

3.361. $\int\frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{3\left(\frac{2x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5a}-\frac{x(ac+8b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{\frac{d\left(a^2c^2+16abc+16b^2\right)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)+\frac{c^{3}\cdots}{\cdots}}{3ad}}{5a}\cdots\right)}{ad}\cdots\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\cdots}$$

$\downarrow$ 313

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{3\left(\frac{2x^3\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{5a}-\frac{x(ac+8b)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3ad}-\frac{d\left(a^2c^2+16abc+16b^2\right)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\left(\cdots\right)\cdots\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac\cdots\right)}{(ac+b\cdots)}}}\cdots\right)}{5a}\cdots\right)}{ad}\cdots\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\cdots}$$

input
```
Int[x^4/(a + b/(c + d*x^2))^(3/2),x]
```

3.361. $\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

output
```
(Sqrt[b + a*c + a*d*x^2]*(-((x^3*(c + d*x^2)^(3/2))/(a*d*Sqrt[b + a*c + a*
d*x^2])) + (3*((2*x^3*Sqrt[c + d*x^2]*Sqrt[b + a*c + a*d*x^2])/(5*a) - (((
8*b + a*c)*x*Sqrt[c + d*x^2]*Sqrt[b + a*c + a*d*x^2])/(3*a*d) - ((16*b^2 +
 16*a*b*c + a^2*c^2)*d*((x*Sqrt[b + a*c + a*d*x^2])/(a*d*Sqrt[c + d*x^2])
- (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b
+ a*c)*(c + d*x^2))])) + (c^(3/2)*(8*b + a*c)*Sqrt[b + a*c + a*d*x^2]*Elli
pticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d]*Sqrt[c + d*x^2]*
Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]))/(3*a*d))/(5*a)))/(
a*d)))/(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])
```

3.361.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

3.361. $\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.361. $\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

3.361.4 Maple [A] (verified)

Time = 11.98 (sec) , antiderivative size = 879, normalized size of antiderivative = 1.82

method	result
risch	$-\frac{x\left(-a d\,x^{2}+a c+3 b\right)\left(a d\,x^{2}+a c+b\right)}{5 d^{2} a^{3} \sqrt{\frac{a d\,x^{2}+a c+b}{d\,x^{2}+c}}}+\frac{\left(-\frac{2 d\left(a^{2} c^{2}+11 a b c+11 b^{2}\right)\left(a\,c^{2}+b c\right)\sqrt{1+\frac{a d\,x^{2}}{a c+b}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{-\frac{a d}{a c+b}},\sqrt{-1+\frac{2 a c d+b d}{d c a}}\right)-E\left(x\sqrt{\right.\right.}{\sqrt{-\frac{a d}{a c+b}}\,\sqrt{a\,d^{2} x^{4}+2 a c d\,x^{2}+b d\,x^{2}+a\,c^{2}+b c}\,\left(2 a c d+2 b d\right)}}{}$
default	Expression too large to display

input
```
int(x^4/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/d^2*x*(-a*d*x^2+a*c+3*b)*(a*d*x^2+a*c+b)/a^3/((a*d*x^2+a*c+b)/(d*x^2+
c))^(1/2)+1/5/a^3/d^2*(-2*d*(a^2*c^2+11*a*b*c+11*b^2)*(a*c^2+b*c)/(-a*d/(a
*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x^4+2*a*
c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(2*a*c*d+2*b*d)*(EllipticF(x*(-a*d/(a*c+b
))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-EllipticE(x*(-a*d/(a*c+b))^(1/2),
(-1+(2*a*c*d+b*d)/d/c/a)^(1/2)))+(a^3*c^3+4*a^2*b*c^2-2*a*b^2*c-5*b^3)/a/(
-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d^2*x
^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),(
-1+(2*a*c*d+b*d)/d/c/a)^(1/2))+5*b^2*(a^2*c^2+2*a*b*c+b^2)/a*(-(a*d^2*x^2+
a*c*d)/(a*c+b)/b*x/d/((x^2+(a*c+b)/a/d)*(a*d^2*x^2+a*c*d))^(1/2)+(1/(a*c+b
)+a*c/(a*c+b)/b)/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x
^2)^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*EllipticF(x*(-a*
d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-2/b*a*d/(a*c+b)*(a*c^2+b*
c)/(-a*d/(a*c+b))^(1/2)*(1+a*d/(a*c+b)*x^2)^(1/2)*(1+1/c*d*x^2)^(1/2)/(a*d
^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(2*a*c*d+2*b*d)*(EllipticF(x*(
-a*d/(a*c+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2))-EllipticE(x*(-a*d/(a*c
+b))^(1/2),(-1+(2*a*c*d+b*d)/d/c/a)^(1/2)))))/((a*d*x^2+a*c+b)/(d*x^2+c))^
(1/2)*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)/(d*x^2+c)
```

3.361.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 436, normalized size of antiderivative = 0.90

$$\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx =$$
$$-\frac{((a^3c^3+16\,a^2bc^2+16\,ab^2c)dx^3+(a^3c^4+17\,a^2bc^3+32\,ab^2c^2+16\,b^3c)x)\sqrt{a}\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ac+b}{ac}}{}$$

input
```
integrate(x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/5*(((a^3*c^3 + 16*a^2*b*c^2 + 16*a*b^2*c)*d*x^3 + (a^3*c^4 + 17*a^2*b*c
^3 + 32*a*b^2*c^2 + 16*b^3*c)*x)*sqrt(a)*sqrt(-c/d)*elliptic_e(arcsin(sqrt
(-c/d)/x), (a*c + b)/(a*c)) - (((a^3*c^2 + 9*a^2*b*c + 8*a*b^2)*d^2 + (a^3
*c^3 + 16*a^2*b*c^2 + 16*a*b^2*c)*d)*x^3 + (a^3*c^4 + 17*a^2*b*c^3 + 32*a*
b^2*c^2 + 16*b^3*c + (a^3*c^3 + 10*a^2*b*c^2 + 17*a*b^2*c + 8*b^3)*d)*x)*s
qrt(a)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c)) - (a^3
*d^4*x^8 + (a^3*c - 2*a^2*b)*d^3*x^6 + a^3*c^4 + (5*a^2*b*c + 8*a*b^2)*d^2
*x^4 + 17*a^2*b*c^3 + 32*a*b^2*c^2 + 16*b^3*c + (a^3*c^3 + 24*a^2*b*c^2 +
40*a*b^2*c + 16*b^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^5*d^
4*x^3 + (a^5*c + a^4*b)*d^3*x)
```

3.361.6 Sympy [F]

$$\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^4}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4/(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(x**4/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

3.361.7 Maxima [F]

$$\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{x^4}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/(a + b/(d*x^2 + c))^(3/2), x)
```

3.361.8 Giac [F]

$$\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{x^4}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^4/(a + b/(d*x^2 + c))^(3/2), x)
```

3.361.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{x^4}{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(x^4/(a + b/(c + d*x^2))^(3/2),x)
```

output
```
int(x^4/(a + b/(c + d*x^2))^(3/2), x)
```

3.362 $\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

3.362.1 Optimal result

Integrand size = 21, antiderivative size = 409

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{x(c+dx^2)}{ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{4x(b+ac+adx^2)}{3a^2d\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-\frac{(8b+ac)x(b+ac+adx^2)}{3a^3d\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{\sqrt{c}(8b+ac)\left(b+ac+adx^2\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3a^3d^{3/2}\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}-\frac{c^{3/2}(4b+ac)\left(b+ac+adx^2\right)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3a^2(b+ac)d^{3/2}\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

output
```
-x*(d*x^2+c)/a/d/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+4/3*x*(a*d*x^2+a*c+b)/a
^2/d/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/3*(a*c+8*b)*x*(a*d*x^2+a*c+b)/a^3
/d/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/3*c^(3/2)*(a*c+4*b)*(a*d*
x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(
1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/a^2/(a*c+b)/d^(3/2)/(d*x^2+c)/((
a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2
)+1/3*(a*c+8*b)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*El
lipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)/a^3
/d^(3/2)/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a
*c+b)/(d*x^2+c))^(1/2)
```

3.362.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.41 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.62

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(a\sqrt{\frac{d}{c}}x(c+dx^2)\left(4b+a(c+dx^2)\right)+i(8b^2+9abc+a^2c^2)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{\cdots}\right.}{3a\cdots}$$

input
```
Integrate[x^2/(a + b/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*Sqrt[d/c]*x*(c + d*x^2)*(4*b + a
*(c + d*x^2)) + I*(8*b^2 + 9*a*b*c + a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b
+ a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a
*c)] - I*b*(8*b + 5*a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x
^2)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(3*a^3*d*Sqrt[
d/c]*(b + a*(c + d*x^2)))
```

3.362.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {2057, 2058, 369, 403, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \frac{x^2}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{ac+adx^2+b}\int \frac{x^2\left(dx^2+c\right)^{3/2}}{\left(adx^2+b+ac\right)^{3/2}}dx}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{369}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{\int \frac{\sqrt{dx^2+c}\left(4dx^2+c\right)}{\sqrt{adx^2+b+ac}}dx}{ad}-\frac{x\left(c+dx^2\right)^{3/2}}{ad\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{403}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{\frac{\int -\frac{d\left((8b+ac)dx^2+c(4b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3ad}+\frac{4x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3a}}{ad}-\frac{x\left(c+dx^2\right)^{3/2}}{ad\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{\frac{4x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3a}-\frac{\int \frac{d\left((8b+ac)dx^2+c(4b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3ad}}{ad}-\frac{x\left(c+dx^2\right)^{3/2}}{ad\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{\frac{4x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3a}-\frac{\int \frac{(8b+ac)dx^2+c(4b+ac)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3a}}{ad}-\frac{x\left(c+dx^2\right)^{3/2}}{ad\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{406}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{\frac{4x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3a}-\frac{c(ac+4b)\int \frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+d(ac+8b)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3a}}{ad}-\frac{x\left(c+dx^2\right)^{3/2}}{ad\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{320}$$

3.362. $\displaystyle\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}dx$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{\frac{4x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3a}-\frac{d(ac+8b)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{c^{3/2}(ac+4b)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}(ac+b)\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}}{3a}}{ad}-\frac{x(\ldots}{ad\sqrt{\ldots}}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\big\downarrow$ 388

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{\frac{4x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3a}-\frac{d(ac+8b)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{(dx^2+c)^{3/2}}dx}{ad}\right)+\frac{c^{3/2}(ac+4b)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+\ldots}\right)}{\sqrt{d}(ac+b)\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}}{3a}}{ad}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\big\downarrow$ 313

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{\frac{4x\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3a}-\frac{\frac{c^{3/2}(ac+4b)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}(ac+b)\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}+d(ac+8b)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}\ldots}{ad^{3/2}\sqrt{c+d\ldots}}\right.}{3a}}{ad}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

input
```
Int[x^2/(a + b/(c + d*x^2))^(3/2),x]
```

3.362. $\int\frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

output
```
(Sqrt[b + a*c + a*d*x^2]*(-((x*(c + d*x^2)^(3/2))/(a*d*Sqrt[b + a*c + a*d*
x^2])) + ((4*x*Sqrt[c + d*x^2]*Sqrt[b + a*c + a*d*x^2])/(3*a) - ((8*b + a*
c)*d*((x*Sqrt[b + a*c + a*d*x^2])/(a*d*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[b
+ a*c + a*d*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*d
^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2)
)])) + (c^(3/2)*(4*b + a*c)*Sqrt[b + a*c + a*d*x^2]*EllipticF[ArcTan[(Sqrt
[d]*x)/Sqrt[c]], b/(b + a*c)])/((b + a*c)*Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[(c*
(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]))/(3*a))/(a*d)))/(Sqrt[c + d
*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])
```

3.362.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

3.362. $\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.362.4 Maple [A] (verified)

Time = 9.20 (sec) , antiderivative size = 667, normalized size of antiderivative = 1.63

method	result
default	$\frac{\left(\sqrt{(ad\,x^2+ac+b)(d\,x^2+c)}\,\sqrt{-\frac{ad}{ac+b}}\,a\,d^2x^5+2\sqrt{(ad\,x^2+ac+b)(d\,x^2+c)}\,\sqrt{-\frac{ad}{ac+b}}\,acd\,x^3+\sqrt{(ad\,x^2+ac+b)(d\,x^2+c)}\,\sqrt{-\frac{ad}{ac+b}}\,bd\,x^3-}{}$
risch	$\frac{x(ad\,x^2+ac+b)}{3a^2d\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}-\frac{\left(-\frac{2d(ac+5b)\left(a\,c^2+bc\right)\sqrt{1+\frac{ad\,x^2}{ac+b}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(F\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{-1+\frac{2acd+bd}{dca}}\right)-E\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{-1+\frac{2acd+bd}{dca}}\right)\right)}{\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+a\,c^2+bc}\,(2acd+2bd)}+\right.}{}$

input
```
int(x^2/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*d^2*x^5+2*((
a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*c*d*x^3+((a*d*x^2+a
*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*b*d*x^3-((a*d*x^2+a*c+b)*(d*x^
2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(
x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*c^2+3*(-a*d/(a*c+b))^(1/2)*(
a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*b*d*x^3+((a*d*x^2+a*c+b)*(d
*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*c^2*x+4*((a*d*x^2+a*c+b)*(d*x^2+c))^
(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*
d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*c-8*((a*d*x^2+a*c+b)*(d*x^2+c))^(1
/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/
(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*c+((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*
(-a*d/(a*c+b))^(1/2)*b*c*x+3*(-a*d/(a*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b
*d*x^2+a*c^2+b*c)^(1/2)*b*c*x)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^2/d/(a*
d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/(a*d*x^2
+a*c+b)
```

3.362.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{((a^2c^2+8\,abc)dx^3+(a^2c^3+9\,abc^2+8\,b^2c)x)\sqrt{a}\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ac+b}{ac})-(((}{}$$

input
```
integrate(x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(((a^2*c^2 + 8*a*b*c)*d*x^3 + (a^2*c^3 + 9*a*b*c^2 + 8*b^2*c)*x)*sqrt(
a)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c)) - (((a^2*c
 + 4*a*b)*d^2 + (a^2*c^2 + 8*a*b*c)*d)*x^3 + (a^2*c^3 + 9*a*b*c^2 + 8*b^2*
c + (a^2*c^2 + 5*a*b*c + 4*b^2)*d)*x)*sqrt(a)*sqrt(-c/d)*elliptic_f(arcsin
(sqrt(-c/d)/x), (a*c + b)/(a*c)) + (a^2*d^3*x^6 + (a^2*c - 4*a*b)*d^2*x^4
- a^2*c^3 - 9*a*b*c^2 - (a^2*c^2 + 13*a*b*c + 8*b^2)*d*x^2 - 8*b^2*c)*sqrt
((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*d^3*x^3 + (a^4*c + a^3*b)*d^2*x)
```

3.362.6 Sympy [F]

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^2}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(x**2/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

3.362.7 Maxima [F]

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^2}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(a + b/(d*x^2 + c))^(3/2), x)
```

3.362.8 Giac [F]

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^2}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/(a + b/(d*x^2 + c))^(3/2), x)
```

3.362.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^2}{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(x^2/(a + b/(c + d*x^2))^(3/2),x)
```

output
```
int(x^2/(a + b/(c + d*x^2))^(3/2), x)
```

3.363 $\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}} dx$

3.363.1 Optimal result

Integrand size = 17, antiderivative size = 356

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}} dx = -\frac{bx}{a(b+ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} + \frac{(2b+ac)x(b+ac+adx^2)}{a^2(b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{\sqrt{c}(2b+ac)\left(b+ac+adx^2\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{a^2(b+ac)\sqrt{d}\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} + \frac{c^{3/2}(b+ac+adx^2)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{a(b+ac)\sqrt{d}\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}$$

output
```
-b*x/a/(a*c+b)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+(a*c+2*b)*x*(a*d*x^2+a*c+
b)/a^2/(a*c+b)/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+c^(3/2)*(a*d*x^
2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/
2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/a/(a*c+b)/(d*x^2+c)/d^(1/2)/((a*d*
x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)-(a
*c+2*b)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(
x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)/a^2/(a*c+b)
/(d*x^2+c)/d^(1/2)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a
*c+b)/(d*x^2+c))^(1/2)
```

3.363.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.43 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.67

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx =$$

$$-\frac{i\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(-iab\sqrt{\frac{d}{c}}x(c+dx^2)+(2b^2+3abc+a^2c^2)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)|\frac{ac}{b+ac}\right)-\right.}{a^2(b+ac)\sqrt{\frac{d}{c}}\left(b+a\left(c+dx^2\right)\right)}$$

input
```
Integrate[(a + b/(c + d*x^2))^(-3/2),x]
```

output
```
((-I)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*((-I)*a*b*Sqrt[d/c]*x*(c + d*x
^2) + (2*b^2 + 3*a*b*c + a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt
[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)] - 2*b*(
b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF
[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(a^2*(b + a*c)*Sqrt[d/c]*(b +
a*(c + d*x^2)))
```

3.363.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {2057, 2058, 315, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \frac{1}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{ac+adx^2+b}\int \frac{\left(dx^2+c\right)^{3/2}}{\left(adx^2+b+ac\right)^{3/2}}dx}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{315}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{\int \frac{d\left((2b+ac)dx^2+c(b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{ad(ac+b)}-\frac{bx\sqrt{c+dx^2}}{a(ac+b)\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{\int \frac{(2b+ac)dx^2+c(b+ac)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{a(ac+b)}-\frac{bx\sqrt{c+dx^2}}{a(ac+b)\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{406}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{c(ac+b)\int \frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+d(ac+2b)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{a(ac+b)}-\frac{bx\sqrt{c+dx^2}}{a(ac+b)\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{d(ac+2b)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}}{a(ac+b)}-\frac{bx\sqrt{c+dx^2}}{a(ac+b)\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{388}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{d(ac+2b)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)+\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}}{a(ac+b)}-\frac{bx\sqrt{c+dx^2}}{a(ac+b)\sqrt{ac+adx^2+}}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

3.363. $\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

$$\Big\downarrow \text{313}$$

$$\frac{\sqrt{ac+adx^2+b}\left(\frac{\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}+d(ac+2b)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}\right)}{a(ac+b)}-\frac{}{a}\cdots\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

input
```
Int[(a + b/(c + d*x^2))^(-3/2),x]
```

output
```
(Sqrt[b + a*c + a*d*x^2]*(-((b*x*Sqrt[c + d*x^2])/(a*(b + a*c)*Sqrt[b + a*
c + a*d*x^2])) + ((2*b + a*c)*d*((x*Sqrt[b + a*c + a*d*x^2])/(a*d*Sqrt[c +
 d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/S
qrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x
^2))/((b + a*c)*(c + d*x^2))])) + (c^(3/2)*Sqrt[b + a*c + a*d*x^2]*Ellipti
cF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d]*Sqrt[c + d*x^2]*Sqr
t[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]))/(a*(b + a*c))))/(Sqrt
[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])
```

3.363.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

3.363. $\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.363. $\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}} \, dx$

3.363.4 Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.31

method	result
default	$-\frac{\left(-\sqrt{(a d x^2+ac+b)(d x^2+c)}\,\sqrt{\frac{a d x^2+ac+b}{ac+b}}\,\sqrt{\frac{d x^2+c}{c}}\,E\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)a c^2+\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a d^2 x^4+2acd x^2+bd x^2+a c^2+bc}\,ba$

input
```
int(1/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x
^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*c^2
+(-a*d/(a*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*b*d*
x^3+((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*
x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*c-
2*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^
2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*c+(-
a*d/(a*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*b*c*x)/
a*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/(a*c+b)/(a*d*x^2+a*c+b)
```

3.363.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx =$$

$$-\frac{((a^2c^2+2\,abc)dx^3+(a^2c^3+3\,abc^2+2\,b^2c)x)\sqrt{a}\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ac+b}{ac})-(((a^2c+ab)d^2+(a^2c^2}{}$$

input
```
integrate(1/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
-(((a^2*c^2 + 2*a*b*c)*d*x^3 + (a^2*c^3 + 3*a*b*c^2 + 2*b^2*c)*x)*sqrt(a)*
sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), (a*c + b)/(a*c)) - (((a^2*c +
a*b)*d^2 + (a^2*c^2 + 2*a*b*c)*d)*x^3 + (a^2*c^3 + 3*a*b*c^2 + 2*b^2*c + (
a^2*c^2 + 2*a*b*c + b^2)*d)*x)*sqrt(a)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-
c/d)/x), (a*c + b)/(a*c)) - ((a^2*c + a*b)*d^2*x^4 + a^2*c^3 + 3*a*b*c^2 +
 2*(a^2*c^2 + 2*a*b*c + b^2)*d*x^2 + 2*b^2*c)*sqrt((a*d*x^2 + a*c + b)/(d*
x^2 + c)))/((a^4*c + a^3*b)*d^2*x^3 + (a^4*c^2 + 2*a^3*b*c + a^2*b^2)*d*x)
```

3.363.6 Sympy [F]

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral((a + b/(c + d*x**2))**(-3/2), x)
```

3.363.7 Maxima [F]

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a + b/(d*x^2 + c))^(-3/2), x)
```

3.363.8 Giac [F]

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((a + b/(d*x^2 + c))^(-3/2), x)
```

3.363.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(1/(a + b/(c + d*x^2))^(3/2),x)
```

output
```
int(1/(a + b/(c + d*x^2))^(3/2), x)
```

3.364 $\int \frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

3.364.1 Optimal result

Integrand size = 21, antiderivative size = 410

$$\begin{aligned}\int \frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx =& -\frac{b}{a(b+ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} \\ &+ \frac{(b-ac)\left(b+ac+adx^2\right)}{a(b+ac)^2x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} - \frac{(b-ac)dx(b+ac+adx^2)}{a(b+ac)^2\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}} \\ &+ \frac{\sqrt{c}(b-ac)\sqrt{d}(b+ac+adx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{a(b+ac)^2\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}} \\ &+ \frac{c^{3/2}\sqrt{d}(b+ac+adx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{(b+ac)^2\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\end{aligned}$$

output
```
-b/a/(a*c+b)/x/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+(-a*c+b)*(a*d*x^2+a*c+b)/
a/(a*c+b)^2/x/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-(-a*c+b)*d*x*(a*d*x^2+a*c+
b)/a/(a*c+b)^2/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+c^(3/2)*(a*d*x^
2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/
2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*d^(1/2)/(a*c+b)^2/(d*x^2+c)/((a*d*
x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+(-
a*c+b)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x
*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*d^(1/2)/a/(a
*c+b)^2/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*
c+b)/(d*x^2+c))^(1/2)
```

3.364.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.51 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^2 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = -\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(a\sqrt{\frac{d}{c}}(c+dx^2)\left(b(c-dx^2)+ac(c+dx^2)\right)+i(-b^2+a^2c^2)\,dx\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\Big(i\text{arcsin}\right.}{a(b+ac)^2\sqrt{\frac{d}{c}}x\,(b+a\,(c}$$

input
```
Integrate[1/(x^2*(a + b/(c + d*x^2))^(3/2)),x]
```

output
```
-((Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*Sqrt[d/c]*(c + d*x^2)*(b*(c -
d*x^2) + a*c*(c + d*x^2)) + I*(-b^2 + a^2*c^2)*d*x*Sqrt[(b + a*c + a*d*x^2
)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(
b + a*c)] + I*b*(b + a*c)*d*x*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 +
 (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/(a*(b + a
*c)^2*Sqrt[d/c]*x*(b + a*(c + d*x^2))))
```

3.364.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 405, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2057, 2058, 370, 27, 445, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2057}$$

$$\int \frac{1}{x^2 \left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{ac+adx^2+b}\int \frac{\left(dx^2+c\right)^{3/2}}{x^2\left(adx^2+b+ac\right)^{3/2}}dx}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\big\downarrow$ 370

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{\int \frac{cd\left(-adx^2+b-ac\right)}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{ad(ac+b)}-\frac{b\sqrt{c+dx^2}}{ax(ac+b)\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\big\downarrow$ 27

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{c\int \frac{-adx^2+b-ac}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{a(ac+b)}-\frac{b\sqrt{c+dx^2}}{ax(ac+b)\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\big\downarrow$ 445

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{c\left(-\frac{\int \frac{ad\left(c(b+ac)-(b-ac)dx^2\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{(b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{a(ac+b)}-\frac{b\sqrt{c+dx^2}}{ax(ac+b)\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\big\downarrow$ 27

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{c\left(-\frac{ad\int \frac{c(b+ac)-(b-ac)dx^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{(b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{a(ac+b)}-\frac{b\sqrt{c+dx^2}}{ax(ac+b)\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\big\downarrow$ 406

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{c\left(-\frac{ad\left(c(ac+b)\int \frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx-d(b-ac)\int \frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{c(ac+b)}-\frac{(b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\right)}{a(ac+b)}-\frac{b\cdots}{ax(ac+b\cdots}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\big\downarrow$ 320

3.364. $\displaystyle\int \frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{c\left(-\frac{ad\left(\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-d(b-ac)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{c(ac+b)}-\frac{(b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{cx(ac+b)}\cdots\right)}{a(ac+b)}\cdots\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\cdots}$$

$\downarrow$ 388

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{c\left(-\frac{ad\left(\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-d(b-ac)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)\right)}{c(ac+b)}-\frac{(b-ac)\sqrt{c+dx^2}\cdots}{cx(a\cdots}\cdots\right)}{a(ac+b)}\cdots\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\cdots}$$

$\downarrow$ 313

3.364. $\displaystyle\int\frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{c\left(-\frac{ad\left(\frac{c^{3/2}\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}-d(b-ac)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}\right.\right.}{c(ac+b)}}{a(ac+b)}}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

input
```
Int[1/(x^2*(a + b/(c + d*x^2))^(3/2)),x]
```

output
```
(Sqrt[b + a*c + a*d*x^2]*(-((b*Sqrt[c + d*x^2])/(a*(b + a*c)*x*Sqrt[b + a*
c + a*d*x^2])) - (c*(-(((b - a*c)*Sqrt[c + d*x^2]*Sqrt[b + a*c + a*d*x^2])
/(c*(b + a*c)*x)) - (a*d*(-((b - a*c)*d*((x*Sqrt[b + a*c + a*d*x^2])/(a*d*
Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcTan[(Sqrt
[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c
 + a*d*x^2))/((b + a*c)*(c + d*x^2))]))) + (c^(3/2)*Sqrt[b + a*c + a*d*x^2
]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d]*Sqrt[c + d
*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])))/(c*(b + a*c
))))/(a*(b + a*c))))/(Sqrt[c + d*x^2]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)
])
```

3.364. $\int \frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

3.364.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

3.364. $\int \frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.364.4 Maple [A] (verified)

Time = 11.04 (sec) , antiderivative size = 685, normalized size of antiderivative = 1.67

method	result
default	$-\frac{\left(\sqrt{(ad\,x^2+ac+b)(d\,x^2+c)}\,\sqrt{-\frac{ad}{ac+b}}\,ac\,d^2x^4-a\,c^2d\sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\,\sqrt{\frac{d\,x^2+c}{c}}\,E\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)x\sqrt{(ad\,x^2+ac+b)(d\,x^2+c)}-\sqrt{}\right.}{}$
risch	Expression too large to display

input
```
int(1/x^2/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*c*d^2*x^4-a*c^2
*d*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(
a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*x*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)-(a*
d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*b*d^2*x^
4+2*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*c^2*d*x^2-2*(
(a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c
)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*c*d*x+(
(a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c
)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*c*d*x+(
(a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*b*c*d*x^2-(a*d^2*x^4
+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*b*c*d*x^2+((a*d
*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*c^3+((a*d*x^2+a*c+b)*(
d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*b*c^2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1
/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/x
/(a*c+b)^2/(a*d*x^2+a*c+b)
```

3.364.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{((a^3c-a^2b)d^3x^3+(a^3c^2-ab^2)d^2x)\sqrt{-\frac{ad}{ac+b}}\sqrt{\frac{ac^2+bc}{d^2}}E(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right) \mid \frac{ac+b}{ac})}{}$$

input
```
integrate(1/x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
(((a^3*c - a^2*b)*d^3*x^3 + (a^3*c^2 - a*b^2)*d^2*x)*sqrt(-a*d/(a*c + b))*
sqrt((a*c^2 + b*c)/d^2)*elliptic_e(arcsin(sqrt(-a*d/(a*c + b))*x), (a*c +
b)/(a*c)) - (((a^3*c - a^2*b)*d^3 + (a^3*c^2 + 2*a^2*b*c + a*b^2)*d^2)*x^3
 + ((a^3*c^2 - a*b^2)*d^2 + (a^3*c^3 + 3*a^2*b*c^2 + 3*a*b^2*c + b^3)*d)*x
)*sqrt(-a*d/(a*c + b))*sqrt((a*c^2 + b*c)/d^2)*elliptic_f(arcsin(sqrt(-a*d
/(a*c + b))*x), (a*c + b)/(a*c)) - (a^3*c^4 + (a^3*c^2 - a*b^2)*d^2*x^4 +
2*a^2*b*c^3 + a*b^2*c^2 + 2*(a^3*c^3 + a^2*b*c^2)*d*x^2)*sqrt((a*d*x^2 + a
*c + b)/(d*x^2 + c)))/((a^5*c^3 + 3*a^4*b*c^2 + 3*a^3*b^2*c + a^2*b^3)*d*x
^3 + (a^5*c^4 + 4*a^4*b*c^3 + 6*a^3*b^2*c^2 + 4*a^2*b^3*c + a*b^4)*x)
```

3.364.6 Sympy [F]

$$\int \frac{1}{x^2 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{x^2 \left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(1/(x**2*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

3.364.7 Maxima [F]

$$\int \frac{1}{x^2 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a + b/(d*x^2 + c))^(3/2)*x^2), x)
```

3.364.8 Giac [F]

$$\int \frac{1}{x^2 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a + b/(d*x^2 + c))^(3/2)*x^2), x)
```

3.364.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{x^2\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(1/(x^2*(a + b/(c + d*x^2))^(3/2)),x)
```

output
```
int(1/(x^2*(a + b/(c + d*x^2))^(3/2)), x)
```

3.365 $\int \frac{1}{x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

3.365.1 Optimal result

Integrand size = 21, antiderivative size = 490

$$\begin{aligned}\int \frac{1}{x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx =& -\frac{b}{a(b+ac)x^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{(3b-ac)\left(b+ac+adx^2\right)}{3a(b+ac)^2x^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\\
&-\frac{(7b-ac)d(b+ac+adx^2)}{3(b+ac)^3x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{(7b-ac)d^2x(b+ac+adx^2)}{3(b+ac)^3\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}\\
&-\frac{\sqrt{c}(7b-ac)d^{3/2}(b+ac+adx^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3(b+ac)^3\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\\
&+\frac{\sqrt{c}(3b-ac)d^{3/2}(b+ac+adx^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{3(b+ac)^3\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}\end{aligned}$$

output
```
-b/a/(a*c+b)/x^3/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/3*(-a*c+3*b)*(a*d*x^2
+a*c+b)/a/(a*c+b)^2/x^3/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/3*(-a*c+7*b)*d
*(a*d*x^2+a*c+b)/(a*c+b)^3/x/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/3*(-a*c+7
*b)*d^2*x*(a*d*x^2+a*c+b)/(a*c+b)^3/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^
(1/2)-1/3*(-a*c+7*b)*d^(3/2)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^
2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2)
)*c^(1/2)/(a*c+b)^3/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^
2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+1/3*(-a*c+3*b)*d^(3/2)*(a*d*x^2+a*c+b)*(
1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^
2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)/(a*c+b)^3/(d*x^2+c)/((a*d*x^2+a*c+b)
/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

3.365.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.79 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx =$$
$$-\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{d}{c}}(c+dx^2)\left(b^2(c+4dx^2)+a^2c(c^2-d^2x^4)+ab(2c^2+4cdx^2+7d^2x^4)\right)+i(7b^2+6abc-a^2\right.}{3(b}$$

input
```
Integrate[1/(x^4*(a + b/(c + d*x^2))^(3/2)),x]
```

output
```
-1/3*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[d/c]*(c + d*x^2)*(b^2*(c
 + 4*d*x^2) + a^2*c*(c^2 - d^2*x^4) + a*b*(2*c^2 + 4*c*d*x^2 + 7*d^2*x^4))
 + I*(7*b^2 + 6*a*b*c - a^2*c^2)*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c
)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]
- (4*I)*b*(b + a*c)*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (
d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (a*c)/(b + a*c)]))/((b + a*c)^
3*Sqrt[d/c]*x^3*(b + a*(c + d*x^2)))
```

3.365.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 470, normalized size of antiderivative = 0.96, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {2057, 2058, 370, 27, 445, 27, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2057}$$

$$\int \frac{1}{x^4 \left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{ac+adx^2+b}\int \frac{\left(dx^2+c\right)^{3/2}}{x^4\left(adx^2+b+ac\right)^{3/2}}dx}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{370}$$

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{\int \frac{d\left((2b-ac)dx^2+c(3b-ac)\right)}{x^4\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{ad(ac+b)}-\frac{b\sqrt{c+dx^2}}{ax^3(ac+b)\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{\int \frac{(2b-ac)dx^2+c(3b-ac)}{x^4\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{a(ac+b)}-\frac{b\sqrt{c+dx^2}}{ax^3(ac+b)\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{445}$$

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{-\frac{\int \frac{acd\left((3b-ac)dx^2+c(7b-ac)\right)}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3c(ac+b)}-\frac{(3b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3x^3(ac+b)}}{a(ac+b)}-\frac{b\sqrt{c+dx^2}}{ax^3(ac+b)\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{-\frac{ad\int \frac{(3b-ac)dx^2+c(7b-ac)}{x^2\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{3(ac+b)}-\frac{(3b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3x^3(ac+b)}}{a(ac+b)}-\frac{b\sqrt{c+dx^2}}{ax^3(ac+b)\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{445}$$

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{-\frac{ad\left(-\frac{\int -\frac{cd\left(a(7b-ac)dx^2+(3b-ac)(b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{(7b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{x(ac+b)}\right)}{3(ac+b)}-\frac{(3b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3x^3(ac+b)}}{a(ac+b)}-\frac{b\sqrt{c+dx^2}}{ax^3(ac+b)\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{-\frac{ad\left(\frac{\int \frac{cd\left(a(7b-ac)dx^2+(3b-ac)(b+ac)\right)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{c(ac+b)}-\frac{(7b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{x(ac+b)}\right)}{3(ac+b)}-\frac{(3b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3x^3(ac+b)}}{a(ac+b)}-\frac{b\sqrt{c+dx^2}}{ax^3(ac+b)\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{-\frac{ad\left(\frac{d\int \frac{a(7b-ac)dx^2+(3b-ac)(b+ac)}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx}{ac+b}-\frac{(7b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{x(ac+b)}\right)}{3(ac+b)}-\frac{(3b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{3x^3(ac+b)}}{a(ac+b)}-\frac{b\sqrt{c+dx^2}}{ax^3(ac+b)\sqrt{ac+adx^2+b}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$$\Big\downarrow \text{406}$$

3.365. $\int \frac{1}{x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{-\frac{ad\left(\frac{d\left((3b-ac)(ac+b)\int\frac{1}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+ad(7b-ac)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx\right)}{ac+b}-\frac{(7b-ac)\sqrt{c+dx^2}\sqrt{ac+adx^2+b}}{x(ac+b)}\right)}{3(ac+b)}-\frac{(3}{}}{a(ac+b)}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\downarrow$ 320

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{-\frac{ad\left(\frac{d\left(ad(7b-ac)\int\frac{x^2}{\sqrt{dx^2+c}\sqrt{adx^2+b+ac}}dx+\frac{\sqrt{c}(3b-ac)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{ac+b}-\frac{(7b-ac)\sqrt{c+dx^2}\sqrt{ac}}{x(ac+b)}\right.}{3(ac+b)}}{a(ac+b)}\right.}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

$\downarrow$ 388

3.365. $\int\frac{1}{x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{-\frac{ad\left(\frac{d\left(ad(7b-ac)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{adx^2+b+ac}}{\left(dx^2+c\right)^{3/2}}dx}{ad}\right)+\frac{\sqrt{c}(3b-ac)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)}{ac+b}-\frac{(7b-a\cdots}{\cdots}\right)}{3(ac+b)}\cdots}{a(ac+b)}\cdots}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\cdots}$$

$\downarrow$ 313

$$\frac{\sqrt{ac+adx^2+b}\left(-\frac{-\frac{ad\left(\frac{d\left(ad(7b-ac)\left(\frac{x\sqrt{ac+adx^2+b}}{ad\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{ac+adx^2+b}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{ad^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\right)+\frac{\sqrt{c}(3b-ac)\sqrt{ac+adx^2+b}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}\cdots}{\sqrt{c}\cdots}\right)\cdots\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}\cdots\right)}{ac+b}\cdots\right)}{3(ac+b)}\cdots}{a(ac+b)}\cdots}{\sqrt{c+dx^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\cdots}$$

input
```
Int[1/(x^4*(a + b/(c + d*x^2))^(3/2)),x]
```

3.365. $\int \frac{1}{x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

output
```
(Sqrt[b + a*c + a*d*x^2]*(-((b*Sqrt[c + d*x^2])/(a*(b + a*c)*x^3*Sqrt[b +
a*c + a*d*x^2])) - (-1/3*((3*b - a*c)*Sqrt[c + d*x^2]*Sqrt[b + a*c + a*d*x
^2])/((b + a*c)*x^3) - (a*d*(-(((7*b - a*c)*Sqrt[c + d*x^2]*Sqrt[b + a*c +
 a*d*x^2])/((b + a*c)*x)) + (d*(a*(7*b - a*c)*d*((x*Sqrt[b + a*c + a*d*x^2
])/(a*d*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[b + a*c + a*d*x^2]*EllipticE[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*d^(3/2)*Sqrt[c + d*x^2]*Sqrt[(c*
(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])) + (Sqrt[c]*(3*b - a*c)*Sqr
t[b + a*c + a*d*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/
(Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^
2))])))/(b + a*c)))/(3*(b + a*c)))/(a*(b + a*c))))/(Sqrt[c + d*x^2]*Sqrt[(
b + a*c + a*d*x^2)/(c + d*x^2)])
```

3.365.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

3.365. $\int \frac{1}{x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2057
```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

3.365.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1079 vs. $2(526) = 1052$.

Time = 10.81 (sec) , antiderivative size = 1080, normalized size of antiderivative = 2.20

method	result	size
default	Expression too large to display	1080
risch	Expression too large to display	1142

input
```
int(1/x^4/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c*d^3*x^
6+4*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*d^3*x^6+((a
*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/
c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*c^2*d^2
*x^3+3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2
)*a*b*d^3*x^6-((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c
^2*d^2*x^4+5*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(
1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1
/2))*a*b*c*d^2*x^3-7*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+b)/(a
*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)
/a/c)^(1/2))*a*b*c*d^2*x^3+8*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+
b))^(1/2)*a*b*c*d^2*x^4-3*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*((a*d*x^2+a*c+
b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a
*c+b)/a/c)^(1/2))*b^2*d^2*x^3+3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^
(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*c*d^2*x^4+((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)
*(-a*d/(a*c+b))^(1/2)*a^2*c^3*d*x^2+4*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-
a*d/(a*c+b))^(1/2)*b^2*d^2*x^4+6*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(
a*c+b))^(1/2)*a*b*c^2*d*x^2+((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b
))^(1/2)*a^2*c^4+5*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*
b^2*c*d*x^2+2*((a*d*x^2+a*c+b)*(d*x^2+c))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*...
```

3.365.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^4 \left(a+\frac{b}{c+dx^2}\right)^{3/2}} \, dx =$$

$$-\frac{((a^4c^2 - 7\,a^3bc)d^4x^5 + (a^4c^3 - 6\,a^3bc^2 - 7\,a^2b^2c)d^3x^3)\sqrt{-\frac{ad}{ac+b}}\sqrt{\frac{ac^2+bc}{d^2}}E(\arcsin\left(\sqrt{-\frac{ad}{ac+b}}x\right) \mid \frac{ac+b}{ac}) - (((\cdots}{\cdots}$$

input
```
integrate(1/x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(((a^4*c^2 - 7*a^3*b*c)*d^4*x^5 + (a^4*c^3 - 6*a^3*b*c^2 - 7*a^2*b^2*
c)*d^3*x^3)*sqrt(-a*d/(a*c + b))*sqrt((a*c^2 + b*c)/d^2)*elliptic_e(arcsin
(sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) - (((a^4*c^2 - 7*a^3*b*c)*d^4 +
 (a^4*c^3 - a^3*b*c^2 - 5*a^2*b^2*c - 3*a*b^3)*d^3)*x^5 + ((a^4*c^3 - 6*a^
3*b*c^2 - 7*a^2*b^2*c)*d^3 + (a^4*c^4 - 6*a^2*b^2*c^2 - 8*a*b^3*c - 3*b^4)
*d^2)*x^3)*sqrt(-a*d/(a*c + b))*sqrt((a*c^2 + b*c)/d^2)*elliptic_f(arcsin(
sqrt(-a*d/(a*c + b))*x), (a*c + b)/(a*c)) + (a^4*c^6 - (a^4*c^3 - 6*a^3*b*
c^2 - 7*a^2*b^2*c)*d^3*x^6 + 3*a^3*b*c^5 + 3*a^2*b^2*c^4 + a*b^3*c^3 - (a^
4*c^4 - 10*a^3*b*c^3 - 15*a^2*b^2*c^2 - 4*a*b^3*c)*d^2*x^4 + (a^4*c^5 + 7*
a^3*b*c^4 + 11*a^2*b^2*c^3 + 5*a*b^3*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/
(d*x^2 + c)))/((a^6*c^5 + 4*a^5*b*c^4 + 6*a^4*b^2*c^3 + 4*a^3*b^3*c^2 + a^
2*b^4*c)*d*x^5 + (a^6*c^6 + 5*a^5*b*c^5 + 10*a^4*b^2*c^4 + 10*a^3*b^3*c^3
+ 5*a^2*b^4*c^2 + a*b^5*c)*x^3)
```

3.365.6 Sympy [F]

$$\int \frac{1}{x^4 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{x^4 \left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**4/(a+b/(d*x**2+c))**(3/2),x)
```

output
```
Integral(1/(x**4*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

3.365.7 Maxima [F]

$$\int \frac{1}{x^4 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{b}{dx^2+c}\right)^{\frac{3}{2}} x^4}\,dx$$

input
```
integrate(1/x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a + b/(d*x^2 + c))^(3/2)*x^4), x)
```

3.365.8 Giac [F]

$$\int \frac{1}{x^4 \left(a + \frac{b}{c+dx^2}\right)^{3/2}} \, dx = \int \frac{1}{\left(a + \frac{b}{dx^2+c}\right)^{\frac{3}{2}} x^4} \, dx$$

input
```
integrate(1/x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a + b/(d*x^2 + c))^(3/2)*x^4), x)
```

3.365.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(a + \frac{b}{c+dx^2}\right)^{3/2}} \, dx = \int \frac{1}{x^4 \left(a + \frac{b}{d\,x^2+c}\right)^{3/2}} \, dx$$

input
```
int(1/(x^4*(a + b/(c + d*x^2))^(3/2)),x)
```

output
```
int(1/(x^4*(a + b/(c + d*x^2))^(3/2)), x)
```

3.366 $\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx$

3.366.1 Optimal result

Integrand size = 19, antiderivative size = 75

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx=-\frac{3\sqrt{ax^{23}}\sqrt{1+x^5}}{20x^9}+\frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4}+\frac{3\sqrt{ax^{23}}\operatorname{arcsinh}\left(x^{5/2}\right)}{20x^{23/2}}$$

output
```
3/20*arcsinh(x^(5/2))*(a*x^23)^(1/2)/x^(23/2)-3/20*(a*x^23)^(1/2)*(x^5+1)^
(1/2)/x^9+1/10*(a*x^23)^(1/2)*(x^5+1)^(1/2)/x^4
```

3.366.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx=\frac{\sqrt{ax^{23}}\left(x^{5/2}\sqrt{1+x^5}(-3+2x^5)+3\log\left(x^{5/2}+\sqrt{1+x^5}\right)\right)}{20x^{23/2}}$$

input
```
Integrate[Sqrt[a*x^23]/Sqrt[1 + x^5],x]
```

output
```
(Sqrt[a*x^23]*(x^(5/2)*Sqrt[1 + x^5]*(-3 + 2*x^5) + 3*Log[x^(5/2) + Sqrt[1
 + x^5]]))/(20*x^(23/2))
```

3.366.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {34, 843, 843, 851, 807, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{x^5+1}}dx$$

$$\Big\downarrow \text{34}$$

$$\frac{\sqrt{ax^{23}}\int \frac{x^{23/2}}{\sqrt{x^5+1}}dx}{x^{23/2}}$$

$$\Big\downarrow \text{843}$$

$$\frac{\sqrt{ax^{23}}\left(\frac{1}{10}x^{15/2}\sqrt{x^5+1}-\frac{3}{4}\int \frac{x^{13/2}}{\sqrt{x^5+1}}dx\right)}{x^{23/2}}$$

$$\Big\downarrow \text{843}$$

$$\frac{\sqrt{ax^{23}}\left(\frac{1}{10}x^{15/2}\sqrt{x^5+1}-\frac{3}{4}\left(\frac{1}{5}x^{5/2}\sqrt{x^5+1}-\frac{1}{2}\int \frac{x^{3/2}}{\sqrt{x^5+1}}dx\right)\right)}{x^{23/2}}$$

$$\Big\downarrow \text{851}$$

$$\frac{\sqrt{ax^{23}}\left(\frac{1}{10}x^{15/2}\sqrt{x^5+1}-\frac{3}{4}\left(\frac{1}{5}x^{5/2}\sqrt{x^5+1}-\int \frac{x^2}{\sqrt{x^5+1}}d\sqrt{x}\right)\right)}{x^{23/2}}$$

$$\Big\downarrow \text{807}$$

$$\frac{\sqrt{ax^{23}}\left(\frac{1}{10}x^{15/2}\sqrt{x^5+1}-\frac{3}{4}\left(\frac{1}{5}x^{5/2}\sqrt{x^5+1}-\frac{1}{5}\int \frac{1}{\sqrt{x+1}}dx^{5/2}\right)\right)}{x^{23/2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{\sqrt{ax^{23}}\left(\frac{1}{10}x^{15/2}\sqrt{x^5+1}-\frac{3}{4}\left(\frac{1}{5}x^{5/2}\sqrt{x^5+1}-\frac{1}{5}\text{arcsinh}\left(x^{5/2}\right)\right)\right)}{x^{23/2}}$$

input
```
Int[Sqrt[a*x^23]/Sqrt[1 + x^5],x]
```

output
```
(Sqrt[a*x^23]*((x^(15/2)*Sqrt[1 + x^5])/10 - (3*((x^(5/2)*Sqrt[1 + x^5])/5
 - ArcSinh[x^(5/2)]/5))/4))/x^(23/2)
```

3.366.3.1 Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

3.366.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.64

method	result	size
meijerg	$\frac{\sqrt{a\,x^{23}}\left(-\frac{\sqrt{\pi}\,x^{\frac{5}{2}}\left(-10x^{5}+15\right)\sqrt{x^{5}+1}}{20}+\frac{3\sqrt{\pi}\,\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)}{4}\right)}{5x^{\frac{23}{2}}\sqrt{\pi}}$	48
risch	$\frac{(2x^{5}-3)\sqrt{x^{5}+1}\,\sqrt{a\,x^{23}}}{20x^{9}}+\frac{3\,\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)\sqrt{a\,x^{23}}\,\sqrt{ax(x^{5}+1)}}{20\sqrt{a}\,x^{12}\sqrt{x^{5}+1}}$	64

input
```
int((a*x^23)^(1/2)/(x^5+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*(a*x^23)^(1/2)/x^(23/2)/Pi^(1/2)*(-1/20*Pi^(1/2)*x^(5/2)*(-10*x^5+15)*
(x^5+1)^(1/2)+3/4*Pi^(1/2)*arcsinh(x^(5/2)))
```

3.366.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.25

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx = \left[\frac{3\sqrt{a}x^9\log\left(-\frac{8ax^{19}+8ax^{14}+ax^9+4\sqrt{ax^{23}}(2x^5+1)\sqrt{x^5+1}\sqrt{a}}{x^9}\right)+4\sqrt{ax^{23}}(2x^5-3)\sqrt{x^5+1}}{80x^9}, -\frac{3\sqrt{-a}x^9\arctan\left(\frac{\sqrt{ax^{23}}(2x^5+1)\sqrt{x^5+1}\sqrt{-a}}{2(ax^{19}+ax^{14})}\right)-2\sqrt{ax^{23}}(2x^5-3)\sqrt{x^5+1}}{40x^9}\right]$$

input
```
integrate((a*x^23)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

output
```
[1/80*(3*sqrt(a)*x^9*log(-(8*a*x^19 + 8*a*x^14 + a*x^9 + 4*sqrt(a*x^23)*(2
*x^5 + 1)*sqrt(x^5 + 1)*sqrt(a))/x^9) + 4*sqrt(a*x^23)*(2*x^5 - 3)*sqrt(x^
5 + 1))/x^9, -1/40*(3*sqrt(-a)*x^9*arctan(1/2*sqrt(a*x^23)*(2*x^5 + 1)*sqr
t(x^5 + 1)*sqrt(-a)/(a*x^19 + a*x^14)) - 2*sqrt(a*x^23)*(2*x^5 - 3)*sqrt(x
^5 + 1))/x^9]
```

3.366.6 Sympy [F]

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{ax^{23}}}{\sqrt{(x+1)(x^4-x^3+x^2-x+1)}}\,dx$$

input
```
integrate((a*x**23)**(1/2)/(x**5+1)**(1/2),x)
```

output
```
Integral(sqrt(a*x**23)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

3.366.7 Maxima [F]

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{ax^{23}}}{\sqrt{x^5+1}}\,dx$$

input
```
integrate((a*x^23)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^23)/sqrt(x^5 + 1), x)
```

3.366.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(55) = 110$.

Time = 0.39 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx = \frac{\sqrt{a^6x^5+a^6}\sqrt{ax}a^6x^2\left(\frac{2x^5}{a^8}-\frac{3}{a^8}\right)\mathrm{sgn}(x)}{20\,|a|} - \frac{3\left(\frac{a^{\frac{5}{2}}\log\left(-\sqrt{ax}a^{\frac{5}{2}}x^2+\sqrt{a^6x^5+a^6}\right)\mathrm{sgn}(x)}{|a|}-\frac{a^{\frac{5}{2}}\log(a^2|a|)\mathrm{sgn}(x)}{|a|}\right)a^3}{20\,|a|^4}$$

input
```
integrate((a*x^23)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

output
```
1/20*sqrt(a^6*x^5 + a^6)*sqrt(a*x)*a^6*x^2*(2*x^5/a^8 - 3/a^8)*sgn(x)/abs(
a) - 3/20*(a^(5/2)*log(-sqrt(a*x)*a^(5/2)*x^2 + sqrt(a^6*x^5 + a^6))*sgn(x
)/abs(a) - a^(5/2)*log(a^2*abs(a))*sgn(x)/abs(a))*a^3/abs(a)^4
```

3.366.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{a\,x^{23}}}{\sqrt{x^5+1}}\,dx$$

input
```
int((a*x^23)^(1/2)/(x^5 + 1)^(1/2),x)
```

output
```
int((a*x^23)^(1/2)/(x^5 + 1)^(1/2), x)
```

3.367 $\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx$

3.367.1 Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx=\frac{\sqrt{ax^{13}}\sqrt{1+x^5}}{5x^4}-\frac{\sqrt{ax^{13}}\operatorname{arcsinh}\left(x^{5/2}\right)}{5x^{13/2}}$$

output
```
-1/5*arcsinh(x^(5/2))*(a*x^13)^(1/2)/x^(13/2)+1/5*(a*x^13)^(1/2)*(x^5+1)^(
1/2)/x^4
```

3.367.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx=\frac{\sqrt{ax^{13}}\left(x^{5/2}\sqrt{1+x^5}-\log\left(x^{5/2}+\sqrt{1+x^5}\right)\right)}{5x^{13/2}}$$

input
```
Integrate[Sqrt[a*x^13]/Sqrt[1 + x^5],x]
```

output
```
(Sqrt[a*x^13]*(x^(5/2)*Sqrt[1 + x^5] - Log[x^(5/2) + Sqrt[1 + x^5]]))/(5*x
^(13/2))
```

3.367.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {34, 843, 851, 807, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{x^5+1}}dx$$
$$\Big\downarrow \text{ 34}$$
$$\frac{\sqrt{ax^{13}}\int \frac{x^{13/2}}{\sqrt{x^5+1}}dx}{x^{13/2}}$$
$$\Big\downarrow \text{ 843}$$
$$\frac{\sqrt{ax^{13}}\left(\frac{1}{5}x^{5/2}\sqrt{x^5+1}-\frac{1}{2}\int \frac{x^{3/2}}{\sqrt{x^5+1}}dx\right)}{x^{13/2}}$$
$$\Big\downarrow \text{ 851}$$
$$\frac{\sqrt{ax^{13}}\left(\frac{1}{5}x^{5/2}\sqrt{x^5+1}-\int \frac{x^2}{\sqrt{x^5+1}}d\sqrt{x}\right)}{x^{13/2}}$$
$$\Big\downarrow \text{ 807}$$
$$\frac{\sqrt{ax^{13}}\left(\frac{1}{5}x^{5/2}\sqrt{x^5+1}-\frac{1}{5}\int \frac{1}{\sqrt{x+1}}dx^{5/2}\right)}{x^{13/2}}$$
$$\Big\downarrow \text{ 222}$$
$$\frac{\sqrt{ax^{13}}\left(\frac{1}{5}x^{5/2}\sqrt{x^5+1}-\frac{1}{5}\text{arcsinh}\left(x^{5/2}\right)\right)}{x^{13/2}}$$

input
```
Int[Sqrt[a*x^13]/Sqrt[1 + x^5],x]
```

output
```
(Sqrt[a*x^13]*((x^(5/2)*Sqrt[1 + x^5])/5 - ArcSinh[x^(5/2)]/5))/x^(13/2)
```

3.367.3.1 Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

3.367.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

method	result	size
meijerg	$\frac{\sqrt{a\,x^{13}}\,\left(\sqrt{\pi}\,x^{\frac{5}{2}}\,\sqrt{x^{5}+1}-\sqrt{\pi}\,\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)\right)}{5x^{\frac{13}{2}}\,\sqrt{\pi}}$	40
risch	$\frac{\sqrt{a\,x^{13}}\,\sqrt{x^{5}+1}}{5x^{4}}-\frac{\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)\sqrt{a\,x^{13}}\,\sqrt{a x \left(x^{5}+1\right)}}{5\sqrt{a}\,x^{7}\,\sqrt{x^{5}+1}}$	57

input
```
int((a*x^13)^(1/2)/(x^5+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*(a*x^13)^(1/2)/x^(13/2)/Pi^(1/2)*(Pi^(1/2)*x^(5/2)*(x^5+1)^(1/2)-Pi^(1
/2)*arcsinh(x^(5/2)))
```

3.367.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(36) = 72.

Time = 0.37 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.06

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx$$
$$= \left[\frac{\sqrt{a}x^4\log\left(-\frac{8\,ax^{14}+8\,ax^9+ax^4-4\,\sqrt{ax^{13}}(2\,x^5+1)\sqrt{x^5+1}\sqrt{a}}{x^4}\right)+4\,\sqrt{ax^{13}}\sqrt{x^5+1}}{20\,x^4}, \frac{\sqrt{-a}x^4\arctan\left(\frac{\sqrt{ax^{13}}(2\,x^5+1)\sqrt{x^5+}}{2\,(ax^{14}+ax^9)}\right.}{10\,x^4}\right.$$

input
```
integrate((a*x^13)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

output
```
[1/20*(sqrt(a)*x^4*log(-(8*a*x^14 + 8*a*x^9 + a*x^4 - 4*sqrt(a*x^13)*(2*x^
5 + 1)*sqrt(x^5 + 1)*sqrt(a))/x^4) + 4*sqrt(a*x^13)*sqrt(x^5 + 1))/x^4, 1/
10*(sqrt(-a)*x^4*arctan(1/2*sqrt(a*x^13)*(2*x^5 + 1)*sqrt(x^5 + 1)*sqrt(-a
)/(a*x^14 + a*x^9)) + 2*sqrt(a*x^13)*sqrt(x^5 + 1))/x^4]
```

3.367.6 Sympy [F]

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{ax^{13}}}{\sqrt{(x+1)\,(x^4-x^3+x^2-x+1)}}\,dx$$

input
```
integrate((a*x**13)**(1/2)/(x**5+1)**(1/2),x)
```

output
```
Integral(sqrt(a*x**13)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

3.367.7 Maxima [F]

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{ax^{13}}}{\sqrt{x^5+1}}\,dx$$

input
```
integrate((a*x^13)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^13)/sqrt(x^5 + 1), x)
```

3.367.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx = \frac{a^{\frac{11}{2}}\log\left(-\sqrt{ax}a^{\frac{5}{2}}x^2+\sqrt{a^6x^5+a^6}\right)}{5\,|a|^5} + \frac{\sqrt{a^6x^5+a^6}\sqrt{ax}x^2}{5\,a^2|a|}$$

input
```
integrate((a*x^13)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

output
```
1/5*a^(11/2)*log(-sqrt(a*x)*a^(5/2)*x^2 + sqrt(a^6*x^5 + a^6))/abs(a)^5 +
1/5*sqrt(a^6*x^5 + a^6)*sqrt(a*x)*x^2/(a^2*abs(a))
```

3.367.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{a\,x^{13}}}{\sqrt{x^5+1}}\,dx$$

input
```
int((a*x^13)^(1/2)/(x^5 + 1)^(1/2),x)
```

output
```
int((a*x^13)^(1/2)/(x^5 + 1)^(1/2), x)
```

3.368 $\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx$

3.368.1 Optimal result

Integrand size = 19, antiderivative size = 24

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx = \frac{2\sqrt{ax^3}\,\text{arcsinh}\left(x^{5/2}\right)}{5x^{3/2}}$$

output
```
2/5*arcsinh(x^(5/2))*(a*x^3)^(1/2)/x^(3/2)
```

3.368.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx = \frac{2\sqrt{ax^3}\log\left(x^{5/2}+\sqrt{1+x^5}\right)}{5x^{3/2}}$$

input
```
Integrate[Sqrt[a*x^3]/Sqrt[1 + x^5],x]
```

output
```
(2*Sqrt[a*x^3]*Log[x^(5/2) + Sqrt[1 + x^5]])/(5*x^(3/2))
```

3.368.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {34, 851, 807, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^3}}{\sqrt{x^5+1}}dx$$

$$\Big\downarrow \text{34}$$

$$\frac{\sqrt{ax^3}\int \frac{x^{3/2}}{\sqrt{x^5+1}}dx}{x^{3/2}}$$

$$\Big\downarrow \text{851}$$

$$\frac{2\sqrt{ax^3}\int \frac{x^2}{\sqrt{x^5+1}}d\sqrt{x}}{x^{3/2}}$$

$$\Big\downarrow \text{807}$$

$$\frac{2\sqrt{ax^3}\int \frac{1}{\sqrt{x+1}}dx^{5/2}}{5x^{3/2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{2\sqrt{ax^3}\text{arcsinh}\left(x^{5/2}\right)}{5x^{3/2}}$$

input
```
Int[Sqrt[a*x^3]/Sqrt[1 + x^5],x]
```

output
```
(2*Sqrt[a*x^3]*ArcSinh[x^(5/2)])/(5*x^(3/2))
```

3.368.3.1 Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

3.368.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

method	result	size
meijerg	$\frac{2\,\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)\sqrt{a\,x^{3}}}{5x^{\frac{3}{2}}}$	17

input
```
int((a*x^3)^(1/2)/(x^5+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*arcsinh(x^(5/2))*(a*x^3)^(1/2)/x^(3/2)
```

3.368.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(16) = 32.

Time = 0.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 4.08

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx = \left[\frac{1}{10}\,\sqrt{a}\log\left(-8\,ax^{10}-8\,ax^5-4\left(2\,x^6+x\right)\sqrt{x^5+1}\sqrt{ax^3}\sqrt{a}-a\right),\right.$$
$$\left.-\frac{1}{5}\,\sqrt{-a}\arctan\left(\frac{\left(2\,x^5+1\right)\sqrt{x^5+1}\sqrt{ax^3}\sqrt{-a}}{2\left(ax^9+ax^4\right)}\right)\right]$$

input
```
integrate((a*x^3)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

output
```
[1/10*sqrt(a)*log(-8*a*x^10 - 8*a*x^5 - 4*(2*x^6 + x)*sqrt(x^5 + 1)*sqrt(a
*x^3)*sqrt(a) - a), -1/5*sqrt(-a)*arctan(1/2*(2*x^5 + 1)*sqrt(x^5 + 1)*sqr
t(a*x^3)*sqrt(-a)/(a*x^9 + a*x^4))]
```

3.368.6 Sympy [F]

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{ax^3}}{\sqrt{(x+1)\,(x^4-x^3+x^2-x+1)}}\,dx$$

input
```
integrate((a*x**3)**(1/2)/(x**5+1)**(1/2),x)
```

output
```
Integral(sqrt(a*x**3)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

3.368.7 Maxima [F]

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{ax^3}}{\sqrt{x^5+1}}\,dx$$

input
```
integrate((a*x^3)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^3)/sqrt(x^5 + 1), x)
```

3.368.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(16) = 32$.

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.42

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx = -\frac{2\,a^{\frac{3}{2}}\log\left(-\sqrt{ax}a^{\frac{5}{2}}x^2+\sqrt{a^6x^5+a^6}\right)\operatorname{sgn}(x)}{5\,|a|} + \frac{2\,a^{\frac{3}{2}}\log\left(a^2|a|\right)\operatorname{sgn}(x)}{5\,|a|}$$

input
```
integrate((a*x^3)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

output
```
-2/5*a^(3/2)*log(-sqrt(a*x)*a^(5/2)*x^2 + sqrt(a^6*x^5 + a^6))*sgn(x)/abs(
a) + 2/5*a^(3/2)*log(a^2*abs(a))*sgn(x)/abs(a)
```

3.368.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{a\,x^3}}{\sqrt{x^5+1}}\,dx$$

input

```
int((a*x^3)^(1/2)/(x^5 + 1)^(1/2),x)
```

output

```
int((a*x^3)^(1/2)/(x^5 + 1)^(1/2), x)
```

3.369 $\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx$

3.369.1 Optimal result

Integrand size = 19, antiderivative size = 23

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx=-\frac{2}{5}\sqrt{\frac{a}{x^7}}x\sqrt{1+x^5}$$

output
```
-2/5*x*(a/x^7)^(1/2)*(x^5+1)^(1/2)
```

3.369.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx=-\frac{2}{5}\sqrt{\frac{a}{x^7}}x\sqrt{1+x^5}$$

input
```
Integrate[Sqrt[a/x^7]/Sqrt[1 + x^5],x]
```

output
```
(-2*Sqrt[a/x^7]*x*Sqrt[1 + x^5])/5
```

3.369.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {34, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{x^5+1}}dx$$

$$\Big\downarrow \text{ 34}$$

$$x^{7/2}\sqrt{\frac{a}{x^7}}\int \frac{1}{x^{7/2}\sqrt{x^5+1}}dx$$

$$\Big\downarrow \text{ 796}$$

$$-\frac{2}{5}x\sqrt{x^5+1}\sqrt{\frac{a}{x^7}}$$

input
```
Int[Sqrt[a/x^7]/Sqrt[1 + x^5],x]
```

output
```
(-2*Sqrt[a/x^7]*x*Sqrt[1 + x^5])/5
```

3.369.3.1 Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

3.369.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
meijerg	$-\frac{2x\sqrt{\frac{a}{x^7}}\,\sqrt{x^5+1}}{5}$	18
risch	$-\frac{2x\sqrt{\frac{a}{x^7}}\,\sqrt{x^5+1}}{5}$	18
gosper	$-\frac{2x(x+1)(x^4-x^3+x^2-x+1)\sqrt{\frac{a}{x^7}}}{5\sqrt{x^5+1}}$	37

input
```
int((a/x^7)^(1/2)/(x^5+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*x*(a/x^7)^(1/2)*(x^5+1)^(1/2)
```

3.369.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx = -\frac{2}{5}\,\sqrt{x^5+1}x\sqrt{\frac{a}{x^7}}$$

input
```
integrate((a/x^7)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

output
```
-2/5*sqrt(x^5 + 1)*x*sqrt(a/x^7)
```

3.369.6 Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{(x+1)\,(x^4-x^3+x^2-x+1)}}\,dx$$

input
```
integrate((a/x**7)**(1/2)/(x**5+1)**(1/2),x)
```

output
```
Integral(sqrt(a/x**7)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

3.369.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(17) = 34$.

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx = -\frac{2\left(\sqrt{a}x^6+\sqrt{a}x\right)}{5\sqrt{x^4-x^3+x^2-x+1}\sqrt{x+1}x^{\frac{7}{2}}}$$

input
```
integrate((a/x^7)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

output
```
-2/5*(sqrt(a)*x^6 + sqrt(a)*x)/(sqrt(x^4 - x^3 + x^2 - x + 1)*sqrt(x + 1)*
x^(7/2))
```

3.369.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx = -\frac{2a^4\left(\frac{\sqrt{a+\frac{a}{x^5}}}{a^3}-\frac{1}{a^{\frac{5}{2}}}\right)}{5|a|}$$

input
```
integrate((a/x^7)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

output
```
-2/5*a^4*(sqrt(a + a/x^5)/a^3 - 1/a^(5/2))/abs(a)
```

3.369.9 Mupad [B] (verification not implemented)

Time = 18.65 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx = -\frac{2x\sqrt{x^5+1}\sqrt{\frac{a}{x^7}}}{5}$$

input
```
int((a/x^7)^(1/2)/(x^5 + 1)^(1/2),x)
```

output
```
-(2*x*(x^5 + 1)^(1/2)*(a/x^7)^(1/2))/5
```

3.370 $\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}}\,dx$

3.370.1 Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}}\,dx=-\frac{2}{15}\sqrt{\frac{a}{x^{17}}}x\sqrt{1+x^5}+\frac{4}{15}\sqrt{\frac{a}{x^{17}}}x^6\sqrt{1+x^5}$$

output
```
-2/15*x*(a/x^17)^(1/2)*(x^5+1)^(1/2)+4/15*x^6*(a/x^17)^(1/2)*(x^5+1)^(1/2)
```

3.370.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}}\,dx=\frac{2}{15}\sqrt{\frac{a}{x^{17}}}x\sqrt{1+x^5}\left(-1+2x^5\right)$$

input
```
Integrate[Sqrt[a/x^17]/Sqrt[1 + x^5],x]
```

output
```
(2*Sqrt[a/x^17]*x*Sqrt[1 + x^5]*(-1 + 2*x^5))/15
```

3.370.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {34, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{x^5+1}}dx$$

$$\Big\downarrow \text{34}$$

$$x^{17/2}\sqrt{\frac{a}{x^{17}}}\int \frac{1}{x^{17/2}\sqrt{x^5+1}}dx$$

$$\Big\downarrow \text{803}$$

$$x^{17/2}\sqrt{\frac{a}{x^{17}}}\left(-\frac{2}{3}\int \frac{1}{x^{7/2}\sqrt{x^5+1}}dx-\frac{2\sqrt{x^5+1}}{15x^{15/2}}\right)$$

$$\Big\downarrow \text{796}$$

$$x^{17/2}\left(\frac{4\sqrt{x^5+1}}{15x^{5/2}}-\frac{2\sqrt{x^5+1}}{15x^{15/2}}\right)\sqrt{\frac{a}{x^{17}}}$$

input
```
Int[Sqrt[a/x^17]/Sqrt[1 + x^5],x]
```

output
```
Sqrt[a/x^17]*x^(17/2)*((-2*Sqrt[1 + x^5])/(15*x^(15/2)) + (4*Sqrt[1 + x^5]
)/(15*x^(5/2)))
```

3.370.3.1 Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

3.370.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.51

method	result	size
meijerg	$-\frac{2\sqrt{\frac{a}{x^{17}}}\,x\left(-2x^{5}+1\right)\sqrt{x^{5}+1}}{15}$	25
risch	$\frac{2\sqrt{\frac{a}{x^{17}}}\,x\left(2x^{10}+x^{5}-1\right)}{15\sqrt{x^{5}+1}}$	28
gosper	$\frac{2x(x+1)\left(x^{4}-x^{3}+x^{2}-x+1\right)\left(2x^{5}-1\right)\sqrt{\frac{a}{x^{17}}}}{15\sqrt{x^{5}+1}}$	44

input
```
int((a/x^17)^(1/2)/(x^5+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(a/x^17)^(1/2)*x*(-2*x^5+1)*(x^5+1)^(1/2)
```

3.370.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}}\,dx = \frac{2}{15}\left(2\,x^{6}-x\right)\sqrt{x^{5}+1}\sqrt{\frac{a}{x^{17}}}$$

input
```
integrate((a/x^17)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(2*x^6 - x)*sqrt(x^5 + 1)*sqrt(a/x^17)
```

3.370.6 Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}}\,dx = \int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{(x+1)\,(x^4-x^3+x^2-x+1)}}\,dx$$

input
```
integrate((a/x**17)**(1/2)/(x**5+1)**(1/2),x)
```

output
```
Integral(sqrt(a/x**17)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

3.370.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}}\,dx = \frac{2\left(2\sqrt{a}x^{11}+\sqrt{a}x^6-\sqrt{a}x\right)}{15\sqrt{x^4-x^3+x^2-x+1}\sqrt{x+1}x^{\frac{17}{2}}}$$

input
```
integrate((a/x^17)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(2*sqrt(a)*x^11 + sqrt(a)*x^6 - sqrt(a)*x)/(sqrt(x^4 - x^3 + x^2 - x
+ 1)*sqrt(x + 1)*x^(17/2))
```

3.370.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a/x^17)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.370.9 Mupad [B] (verification not implemented)

Time = 17.85 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}}\,dx = \frac{\sqrt{\frac{a}{x^{17}}}\left(\frac{4\,x^{11}}{15}+\frac{2\,x^6}{15}-\frac{2\,x}{15}\right)}{\sqrt{x^5+1}}$$

input
```
int((a/x^17)^(1/2)/(x^5 + 1)^(1/2),x)
```

output
```
((a/x^17)^(1/2)*((2*x^6)/15 - (2*x)/15 + (4*x^11)/15))/(x^5 + 1)^(1/2)
```

3.371 $\int \frac{\sqrt{ax^6}}{x(1-x^4)} dx$

3.371.1 Optimal result

Integrand size = 22, antiderivative size = 37

$$\int \frac{\sqrt{ax^6}}{x(1-x^4)} dx = -\frac{\sqrt{ax^6}\arctan(x)}{2x^3} + \frac{\sqrt{ax^6}\text{arctanh}(x)}{2x^3}$$

output
```
-1/2*arctan(x)*(a*x^6)^(1/2)/x^3+1/2*arctanh(x)*(a*x^6)^(1/2)/x^3
```

3.371.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{ax^6}}{x(1-x^4)} dx = \frac{\sqrt{ax^6}(-\arctan(x) + \text{arctanh}(x))}{2x^3}$$

input
```
Integrate[Sqrt[a*x^6]/(x*(1 - x^4)),x]
```

output
```
(Sqrt[a*x^6]*(-ArcTan[x] + ArcTanh[x]))/(2*x^3)
```

3.371.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {30, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^6}}{x(1-x^4)}dx$$

$$\Big\downarrow \text{30}$$

$$\frac{\sqrt{ax^6}\int \frac{x^2}{1-x^4}dx}{x^3}$$

$$\Big\downarrow \text{827}$$

$$\frac{\sqrt{ax^6}\left(\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{1}{2}\int \frac{1}{x^2+1}dx\right)}{x^3}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{ax^6}\left(\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{\arctan(x)}{2}\right)}{x^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{ax^6}\left(\frac{\text{arctanh}(x)}{2}-\frac{\arctan(x)}{2}\right)}{x^3}$$

input
```
Int[Sqrt[a*x^6]/(x*(1 - x^4)),x]
```

output
```
(Sqrt[a*x^6]*(-1/2*ArcTan[x] + ArcTanh[x]/2))/x^3
```

3.371.3.1 Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

3.371.4 Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.49

method	result	size
pseudoelliptic	$\frac{\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a\,x^{4}}}{\sqrt{a}}\right)}{2}$	18
default	$-\frac{\sqrt{a\,x^{6}}\,(\ln(x-1)-\ln(x+1)+2\arctan(x))}{4x^{3}}$	28
meijerg	$-\frac{\sqrt{a\,x^{6}}\left(\ln\left(1-(x^{4})^{\frac{1}{4}}\right)-\ln\left(1+(x^{4})^{\frac{1}{4}}\right)+2\arctan\left((x^{4})^{\frac{1}{4}}\right)\right)}{4(x^{4})^{\frac{3}{4}}}$	44
risch	$-\frac{\sqrt{a\,x^{6}}\,\ln(x-1)}{4x^{3}}-\frac{i\sqrt{a\,x^{6}}\,\ln(x+i)}{4x^{3}}+\frac{i\sqrt{a\,x^{6}}\,\ln(x-i)}{4x^{3}}+\frac{\sqrt{a\,x^{6}}\,\ln(x+1)}{4x^{3}}$	70

input
```
int((a*x^6)^(1/2)/x/(-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*a^(1/2)*arctanh((a*x^4)^(1/2)/a^(1/2))
```

3.371.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{ax^6}}{x(1-x^4)}\,dx = -\frac{\sqrt{ax^6}\left(2\arctan(x) - \log\left(\frac{x+1}{x-1}\right)\right)}{4x^3}$$

input
```
integrate((a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="fricas")
```

output
```
-1/4*sqrt(a*x^6)*(2*arctan(x) - log((x + 1)/(x - 1)))/x^3
```

3.371.6 Sympy [F]

$$\int \frac{\sqrt{ax^6}}{x(1-x^4)}\,dx = -\int \frac{\sqrt{ax^6}}{x^5-x}\,dx$$

input
```
integrate((a*x**6)**(1/2)/x/(-x**4+1),x)
```

output
```
-Integral(sqrt(a*x**6)/(x**5 - x), x)
```

3.371.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{ax^6}}{x(1-x^4)}\,dx = -\frac{1}{2}\sqrt{a}\arctan(x) + \frac{1}{4}\sqrt{a}\log(x+1) - \frac{1}{4}\sqrt{a}\log(x-1)$$

input
```
integrate((a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="maxima")
```

output
```
-1/2*sqrt(a)*arctan(x) + 1/4*sqrt(a)*log(x + 1) - 1/4*sqrt(a)*log(x - 1)
```

3.371.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{ax^6}}{x(1-x^4)}\,dx = -\frac{1}{4}\left(2\arctan\left(x\right)\operatorname{sgn}(x) - \log\left(|x+1|\right)\operatorname{sgn}(x) + \log\left(|x-1|\right)\operatorname{sgn}(x)\right)\sqrt{a}$$

input
```
integrate((a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="giac")
```

output
```
-1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x)
)*sqrt(a)
```

3.371.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^6}}{x(1-x^4)}\,dx = -\int \frac{\sqrt{a\,x^6}}{x\,(x^4-1)}\,dx$$

input
```
int(-(a*x^6)^(1/2)/(x*(x^4 - 1)),x)
```

output
```
-int((a*x^6)^(1/2)/(x*(x^4 - 1)), x)
```

3.372 $\int \frac{\sqrt{ax^6}}{x-x^5} \, dx$

3.372.1 Optimal result

Integrand size = 19, antiderivative size = 37

$$\int \frac{\sqrt{ax^6}}{x-x^5} \, dx = -\frac{\sqrt{ax^6}\arctan(x)}{2x^3} + \frac{\sqrt{ax^6}\text{arctanh}(x)}{2x^3}$$

output
```
-1/2*arctan(x)*(a*x^6)^(1/2)/x^3+1/2*arctanh(x)*(a*x^6)^(1/2)/x^3
```

3.372.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{ax^6}}{x-x^5} \, dx = \frac{\sqrt{ax^6}(-\arctan(x) + \text{arctanh}(x))}{2x^3}$$

input
```
Integrate[Sqrt[a*x^6]/(x - x^5),x]
```

output
```
(Sqrt[a*x^6]*(-ArcTan[x] + ArcTanh[x]))/(2*x^3)
```

3.372.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {34, 9, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^6}}{x-x^5}\,dx$$
$$\Big\downarrow \text{34}$$
$$\frac{\sqrt{ax^6}\int \frac{x^3}{x-x^5}dx}{x^3}$$
$$\Big\downarrow \text{9}$$
$$\frac{\sqrt{ax^6}\int \frac{x^2}{1-x^4}dx}{x^3}$$
$$\Big\downarrow \text{827}$$
$$\frac{\sqrt{ax^6}\left(\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{1}{2}\int \frac{1}{x^2+1}dx\right)}{x^3}$$
$$\Big\downarrow \text{216}$$
$$\frac{\sqrt{ax^6}\left(\frac{1}{2}\int \frac{1}{1-x^2}dx-\frac{\arctan(x)}{2}\right)}{x^3}$$
$$\Big\downarrow \text{219}$$
$$\frac{\sqrt{ax^6}\left(\frac{\text{arctanh}(x)}{2}-\frac{\arctan(x)}{2}\right)}{x^3}$$

input
```
Int[Sqrt[a*x^6]/(x - x^5),x]
```

output
```
(Sqrt[a*x^6]*(-1/2*ArcTan[x] + ArcTanh[x]/2))/x^3
```

3.372.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

3.372.4 Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.49

method	result	size
pseudoelliptic	$\frac{\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a\,x^{4}}}{\sqrt{a}}\right)}{2}$	18
default	$-\frac{\sqrt{a\,x^{6}}\,(\ln(x-1)-\ln(x+1)+2\arctan(x))}{4x^{3}}$	28
meijerg	$-\frac{\sqrt{a\,x^{6}}\left(\ln\left(1-(x^{4})^{\frac{1}{4}}\right)-\ln\left(1+(x^{4})^{\frac{1}{4}}\right)+2\arctan\left((x^{4})^{\frac{1}{4}}\right)\right)}{4(x^{4})^{\frac{3}{4}}}$	44
risch	$-\frac{\sqrt{a\,x^{6}}\,\ln(x-1)}{4x^{3}}-\frac{i\sqrt{a\,x^{6}}\,\ln(x+i)}{4x^{3}}+\frac{i\sqrt{a\,x^{6}}\,\ln(x-i)}{4x^{3}}+\frac{\sqrt{a\,x^{6}}\,\ln(x+1)}{4x^{3}}$	70

input
```
int((a*x^6)^(1/2)/(-x^5+x),x,method=_RETURNVERBOSE)
```

output
```
1/2*a^(1/2)*arctanh((a*x^4)^(1/2)/a^(1/2))
```

3.372.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{ax^6}}{x-x^5}\,dx = -\frac{\sqrt{ax^6}\left(2\arctan(x)-\log\left(\frac{x+1}{x-1}\right)\right)}{4\,x^3}$$

input
```
integrate((a*x^6)^(1/2)/(-x^5+x),x, algorithm="fricas")
```

output
```
-1/4*sqrt(a*x^6)*(2*arctan(x) - log((x + 1)/(x - 1)))/x^3
```

3.372.6 Sympy [F]

$$\int \frac{\sqrt{ax^6}}{x-x^5}\,dx = -\int \frac{\sqrt{ax^6}}{x^5-x}\,dx$$

input
```
integrate((a*x**6)**(1/2)/(-x**5+x),x)
```

output
```
-Integral(sqrt(a*x**6)/(x**5 - x), x)
```

3.372.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{ax^6}}{x-x^5}\,dx = -\frac{1}{2}\sqrt{a}\arctan(x)+\frac{1}{4}\sqrt{a}\log(x+1)-\frac{1}{4}\sqrt{a}\log(x-1)$$

input
```
integrate((a*x^6)^(1/2)/(-x^5+x),x, algorithm="maxima")
```

output
```
-1/2*sqrt(a)*arctan(x) + 1/4*sqrt(a)*log(x + 1) - 1/4*sqrt(a)*log(x - 1)
```

3.372.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{ax^6}}{x-x^5}\,dx = -\frac{1}{4}\left(2\arctan\left(x\right)\operatorname{sgn}(x) - \log\left(|x+1|\right)\operatorname{sgn}(x) + \log\left(|x-1|\right)\operatorname{sgn}(x)\right)\sqrt{a}$$

input
```
integrate((a*x^6)^(1/2)/(-x^5+x),x, algorithm="giac")
```

output
```
-1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x)
)*sqrt(a)
```

3.372.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^6}}{x-x^5}\,dx = \int \frac{\sqrt{a\,x^6}}{x-x^5}\,dx$$

input
```
int((a*x^6)^(1/2)/(x - x^5),x)
```

output
```
int((a*x^6)^(1/2)/(x - x^5), x)
```

3.373 $\int \frac{(ax^6)^{3/2}}{x(1-x^4)}\,dx$

3.373.1 Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{(ax^6)^{3/2}}{x(1-x^4)}\,dx = -\frac{a\sqrt{ax^6}}{x^2}-\frac{1}{5}ax^2\sqrt{ax^6}+\frac{a\sqrt{ax^6}\arctan(x)}{2x^3}+\frac{a\sqrt{ax^6}\operatorname{arctanh}(x)}{2x^3}$$

output
```
-a*(a*x^6)^(1/2)/x^2-1/5*a*x^2*(a*x^6)^(1/2)+1/2*a*arctan(x)*(a*x^6)^(1/2)
/x^3+1/2*a*arctanh(x)*(a*x^6)^(1/2)/x^3
```

3.373.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.48

$$\int \frac{(ax^6)^{3/2}}{x(1-x^4)}\,dx = -\frac{a\sqrt{ax^6}(2x(5+x^4)-5\arctan(x)-5\operatorname{arctanh}(x))}{10x^3}$$

input
```
Integrate[(a*x^6)^(3/2)/(x*(1 - x^4)),x]
```

output
```
-1/10*(a*Sqrt[a*x^6]*(2*x*(5 + x^4) - 5*ArcTan[x] - 5*ArcTanh[x]))/x^3
```

3.373.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.52, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {30, 831, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^6)^{3/2}}{x(1-x^4)}dx$$

$$\Big\downarrow \; 30$$

$$\frac{a\sqrt{ax^6}\int \frac{x^8}{1-x^4}dx}{x^3}$$

$$\Big\downarrow \; 831$$

$$\frac{a\sqrt{ax^6}\int \left(-x^4+\frac{1}{1-x^4}-1\right)dx}{x^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{a\sqrt{ax^6}\left(\frac{\arctan(x)}{2}+\frac{\text{arctanh}(x)}{2}-\frac{x^5}{5}-x\right)}{x^3}$$

input
```
Int[(a*x^6)^(3/2)/(x*(1 - x^4)),x]
```

output
```
(a*Sqrt[a*x^6]*(-x - x^5/5 + ArcTan[x]/2 + ArcTanh[x]/2))/x^3
```

3.373.3.1 Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 831
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.373.4 Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.41

method	result	size
pseudoelliptic	$-\frac{a\left(-\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a\,x^{4}}}{\sqrt{a}}\right)+\sqrt{a\,x^{4}}\right)}{2}$	29
default	$-\frac{\left(a\,x^{6}\right)^{\frac{3}{2}}\left(4x^{5}+5\ln(x-1)-5\ln(x+1)-10\arctan(x)+20x\right)}{20x^{9}}$	38
meijerg	$-\frac{\left(a\,x^{6}\right)^{\frac{3}{2}}(-1)^{\frac{3}{4}}\left(-\frac{4x(-1)^{\frac{1}{4}}\left(9x^{4}+45\right)}{45}-\frac{x(-1)^{\frac{1}{4}}\left(\ln\left(1-\left(x^{4}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(x^{4}\right)^{\frac{1}{4}}\right)-2\arctan\left(\left(x^{4}\right)^{\frac{1}{4}}\right)\right)}{\left(x^{4}\right)^{\frac{1}{4}}}\right)}{4x^{9}}$	70
risch	$-\frac{a\,x^{2}\sqrt{a\,x^{6}}}{5}-\frac{a\sqrt{a\,x^{6}}}{x^{2}}+\frac{a\sqrt{a\,x^{6}}\,\ln(x+1)}{4x^{3}}-\frac{a\sqrt{a\,x^{6}}\,\ln(x-1)}{4x^{3}}-\frac{ia\sqrt{a\,x^{6}}\,\ln(x-i)}{4x^{3}}+\frac{ia\sqrt{a\,x^{6}}\,\ln(x+i)}{4x^{3}}$	100

input
```
int((a*x^6)^(3/2)/x/(-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*(-a^(1/2)*arctanh((a*x^4)^(1/2)/a^(1/2))+(a*x^4)^(1/2))
```

3.373.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.58

$$\int \frac{(ax^6)^{3/2}}{x(1-x^4)}\,dx = -\frac{\sqrt{ax^6}\left(4\,ax^5+20\,ax-10\,a\arctan\left(x\right)-5\,a\log\left(\frac{x+1}{x-1}\right)\right)}{20\,x^3}$$

input
```
integrate((a*x^6)^(3/2)/x/(-x^4+1),x, algorithm="fricas")
```

output
```
-1/20*sqrt(a*x^6)*(4*a*x^5 + 20*a*x - 10*a*arctan(x) - 5*a*log((x + 1)/(x
 - 1)))/x^3
```

3.373.6 Sympy [F]

$$\int \frac{(ax^6)^{3/2}}{x(1-x^4)}\,dx = -\int \frac{(ax^6)^{\frac{3}{2}}}{x^5-x}\,dx$$

input
```
integrate((a*x**6)**(3/2)/x/(-x**4+1),x)
```

output
```
-Integral((a*x**6)**(3/2)/(x**5 - x), x)
```

3.373.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

$$\int \frac{(ax^6)^{3/2}}{x(1-x^4)}\,dx = -\frac{1}{5}\,a^{\frac{3}{2}}x^5 - a^{\frac{3}{2}}x + \frac{1}{2}\,a^{\frac{3}{2}}\arctan(x) + \frac{1}{4}\,a^{\frac{3}{2}}\log(x+1) - \frac{1}{4}\,a^{\frac{3}{2}}\log(x-1)$$

input
```
integrate((a*x^6)^(3/2)/x/(-x^4+1),x, algorithm="maxima")
```

output
```
-1/5*a^(3/2)*x^5 - a^(3/2)*x + 1/2*a^(3/2)*arctan(x) + 1/4*a^(3/2)*log(x +
 1) - 1/4*a^(3/2)*log(x - 1)
```

3.373.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.59

$$\int \frac{(ax^6)^{3/2}}{x(1-x^4)}\,dx = -\frac{1}{20}\left(4\,x^5\mathrm{sgn}(x) + 20\,x\mathrm{sgn}(x) - 10\,\arctan(x)\,\mathrm{sgn}(x) - 5\,\log(|x+1|)\,\mathrm{sgn}(x) + 5\,\log(|x-1|)\,\mathrm{sgn}(x)\right)a^{\frac{3}{2}}$$

input
```
integrate((a*x^6)^(3/2)/x/(-x^4+1),x, algorithm="giac")
```

output
```
-1/20*(4*x^5*sgn(x) + 20*x*sgn(x) - 10*arctan(x)*sgn(x) - 5*log(abs(x + 1)
)*sgn(x) + 5*log(abs(x - 1))*sgn(x))*a^(3/2)
```

3.373.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ax^6)^{3/2}}{x(1-x^4)}\,dx = -\int \frac{(a\,x^6)^{3/2}}{x\,(x^4-1)}\,dx$$

input
```
int(-(a*x^6)^(3/2)/(x*(x^4 - 1)),x)
```

output
```
-int((a*x^6)^(3/2)/(x*(x^4 - 1)), x)
```

3.374 $\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x(1-x^4)}\right) dx$

3.374.1 Optimal result

Integrand size = 33, antiderivative size = 49

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x(1-x^4)}\right) dx=\frac{\arctan(x)}{2}+\frac{\sqrt{ax^6}\arctan(x)}{2x^3}+\frac{\operatorname{arctanh}(x)}{2}-\frac{\sqrt{ax^6}\operatorname{arctanh}(x)}{2x^3}$$

output
```
1/2*arctan(x)+1/2*arctanh(x)+1/2*arctan(x)*(a*x^6)^(1/2)/x^3-1/2*arctanh(x
)*(a*x^6)^(1/2)/x^3
```

3.374.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x(1-x^4)}\right) dx=\frac{\left(x^3+\sqrt{ax^6}\right)\arctan(x)+\left(x^3-\sqrt{ax^6}\right)\operatorname{arctanh}(x)}{2x^3}$$

input
```
Integrate[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x*(1 - x^4)),x]
```

output
```
((x^3 + Sqrt[a*x^6])*ArcTan[x] + (x^3 - Sqrt[a*x^6])*ArcTanh[x])/(2*x^3)
```

3.374.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x\,(1-x^4)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{ax^6}\arctan(x)}{2x^3}-\frac{\sqrt{ax^6}\text{arctanh}(x)}{2x^3}+\frac{\arctan(x)}{2}+\frac{\text{arctanh}(x)}{2}$$

input
```
Int[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x*(1 - x^4)),x]
```

output
```
ArcTan[x]/2 + (Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[a*x^6
]*ArcTanh[x])/(2*x^3)
```

3.374.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.374.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

method	result	siz
default	$\frac{\arctan(x)}{2}+\frac{\text{arctanh}(x)}{2}+\frac{\sqrt{a\,x^{6}}\,(\ln(x-1)-\ln(x+1)+2\arctan(x))}{4x^{3}}$	37
meijerg	$-\frac{x\left(\ln\left(1-(x^{4})^{\frac{1}{4}}\right)-\ln\left(1+(x^{4})^{\frac{1}{4}}\right)-2\arctan\left((x^{4})^{\frac{1}{4}}\right)\right)}{4(x^{4})^{\frac{1}{4}}}+\frac{\sqrt{a\,x^{6}}\,\left(\ln\left(1-(x^{4})^{\frac{1}{4}}\right)-\ln\left(1+(x^{4})^{\frac{1}{4}}\right)+2\arctan\left((x^{4})^{\frac{1}{4}}\right)\right)}{4(x^{4})^{\frac{3}{4}}}$	82
risch	$\frac{i\ln(x+i)x^{3}-i\ln(x-i)x^{3}-\ln(x-1)x^{3}+\ln(x+1)x^{3}+i\sqrt{a\,x^{6}}\,\ln(x+i)-i\sqrt{a\,x^{6}}\,\ln(x-i)+\sqrt{a\,x^{6}}\,\ln(x-1)-\sqrt{a\,x^{6}}\,\ln(x+1)}{4x^{3}}$	10

input
```
int(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(x)+1/2*arctanh(x)+1/4*(a*x^6)^(1/2)*(ln(x-1)-ln(x+1)+2*arctan(x
))/x^3
```

3.374.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(37) = 74.

Time = 0.29 (sec) , antiderivative size = 256, normalized size of antiderivative = 5.22

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x(1-x^4)}\right)dx$$
$$=\left[\frac{x^3\sqrt{-\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}\log\left(\frac{(a-1)x^4-(a-1)x^2-2\left(x^3-\sqrt{ax^6}\right)\sqrt{-\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}}{x^4+x^2}\right)+x^3\log(x+1)-x^3\log(x-1)-}{4\,x^3}\right.$$

input
```
integrate(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="fricas")
```

output
```
[1/4*(x^3*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*log(((a - 1)*x^4 - (a -
 1)*x^2 - 2*(x^3 - sqrt(a*x^6))*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3))/
(x^4 + x^2)) + x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) -
 log(x - 1)))/x^3, 1/4*(2*x^3*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*arct
an(-(x^3 - sqrt(a*x^6))*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)/((a - 1)*x
^2)) + x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - log(x -
 1)))/x^3]
```

3.374.6 Sympy [F]

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x(1-x^4)}\right)dx=-\int \frac{x}{x^5-x}\,dx-\int \left(-\frac{\sqrt{ax^6}}{x^5-x}\right)dx$$

input
```
integrate(1/(-x**4+1)-(a*x**6)**(1/2)/x/(-x**4+1),x)
```

output
```
-Integral(x/(x**5 - x), x) - Integral(-sqrt(a*x**6)/(x**5 - x), x)
```

3.374.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x(1-x^4)}\right)\,dx=\frac{1}{2}\sqrt{a}\arctan(x)-\frac{1}{4}\sqrt{a}\log(x+1)+\frac{1}{4}\sqrt{a}\log(x-1)+\frac{1}{2}\arctan(x)+\frac{1}{4}\log(x+1)-\frac{1}{4}\log(x-1)$$

input
```
integrate(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="maxima")
```

output
```
1/2*sqrt(a)*arctan(x) - 1/4*sqrt(a)*log(x + 1) + 1/4*sqrt(a)*log(x - 1) +
1/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

3.374.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x(1-x^4)}\right)\,dx=\frac{1}{4}\left(2\arctan(x)\,\mathrm{sgn}(x)-\log(|x+1|)\,\mathrm{sgn}(x)+\log(|x-1|)\,\mathrm{sgn}(x)\right)\sqrt{a}+\frac{1}{2}\arctan(x)+\frac{1}{4}\log(|x+1|)-\frac{1}{4}\log(|x-1|)$$

input
```
integrate(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="giac")
```

output
```
1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x))
*sqrt(a) + 1/2*arctan(x) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

3.374.9 Mupad [F(-1)]

Timed out.

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x\,(1-x^4)}\right)\,dx=\int \frac{\sqrt{a\,x^6}}{x\,\left(x^4-1\right)}-\frac{1}{x^4-1}\,dx$$

input
```
int((a*x^6)^(1/2)/(x*(x^4 - 1)) - 1/(x^4 - 1),x)
```

output
```
int((a*x^6)^(1/2)/(x*(x^4 - 1)) - 1/(x^4 - 1), x)
```

3.375 $\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right)\,dx$

3.375.1 Optimal result

Integrand size = 30, antiderivative size = 49

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right)\,dx=\frac{\arctan(x)}{2}+\frac{\sqrt{ax^6}\arctan(x)}{2x^3}+\frac{\text{arctanh}(x)}{2}-\frac{\sqrt{ax^6}\text{arctanh}(x)}{2x^3}$$

output
```
1/2*arctan(x)+1/2*arctanh(x)+1/2*arctan(x)*(a*x^6)^(1/2)/x^3-1/2*arctanh(x
)*(a*x^6)^(1/2)/x^3
```

3.375.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right)\,dx=\frac{\left(x^3+\sqrt{ax^6}\right)\arctan(x)+\left(x^3-\sqrt{ax^6}\right)\text{arctanh}(x)}{2x^3}$$

input
```
Integrate[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x - x^5),x]
```

output
```
((x^3 + Sqrt[a*x^6])*ArcTan[x] + (x^3 - Sqrt[a*x^6])*ArcTanh[x])/(2*x^3)
```

3.375.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{ax^6}\arctan(x)}{2x^3}-\frac{\sqrt{ax^6}\text{arctanh}(x)}{2x^3}+\frac{\arctan(x)}{2}+\frac{\text{arctanh}(x)}{2}$$

input
```
Int[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x - x^5),x]
```

output
```
ArcTan[x]/2 + (Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[a*x^6
]*ArcTanh[x])/(2*x^3)
```

3.375.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.375.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

method	result	siz
default	$\frac{\arctan(x)}{2}+\frac{\text{arctanh}(x)}{2}+\frac{\sqrt{a\,x^6}\,(\ln(x-1)-\ln(x+1)+2\arctan(x))}{4x^3}$	37
meijerg	$-\frac{x\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)-2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{1}{4}}}+\frac{\sqrt{a\,x^6}\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)+2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{3}{4}}}$	82
risch	$\frac{i\ln(x+i)x^3-i\ln(x-i)x^3-\ln(x-1)x^3+\ln(x+1)x^3+i\sqrt{a\,x^6}\,\ln(x+i)-i\sqrt{a\,x^6}\,\ln(x-i)+\sqrt{a\,x^6}\,\ln(x-1)-\sqrt{a\,x^6}\,\ln(x+1)}{4x^3}$	10

input
```
int(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(x)+1/2*arctanh(x)+1/4*(a*x^6)^(1/2)*(ln(x-1)-ln(x+1)+2*arctan(x
))/x^3
```

3.375.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(37) = 74$.

Time = 0.32 (sec) , antiderivative size = 256, normalized size of antiderivative = 5.22

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right) dx$$
$$= \left[\frac{x^3\sqrt{-\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}\log\left(\frac{(a-1)x^4-(a-1)x^2-2\left(x^3-\sqrt{ax^6}\right)\sqrt{-\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}}{x^4+x^2}\right)+x^3\log(x+1)-x^3\log(x-1)-}{4\,x^3}\right.$$

input
```
integrate(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x, algorithm="fricas")
```

output
```
[1/4*(x^3*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*log(((a - 1)*x^4 - (a -
 1)*x^2 - 2*(x^3 - sqrt(a*x^6))*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3))/
(x^4 + x^2)) + x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) -
 log(x - 1)))/x^3, 1/4*(2*x^3*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*arct
an(-(x^3 - sqrt(a*x^6))*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)/((a - 1)*x
^2)) + x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - log(x -
 1)))/x^3]
```

3.375.6 Sympy [F]

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right) dx = -\int \frac{x}{x^5-x}\,dx - \int \left(-\frac{\sqrt{ax^6}}{x^5-x}\right) dx$$

input
```
integrate(1/(-x**4+1)-(a*x**6)**(1/2)/(-x**5+x),x)
```

output
```
-Integral(x/(x**5 - x), x) - Integral(-sqrt(a*x**6)/(x**5 - x), x)
```

3.375.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right)\,dx = \frac{1}{2}\sqrt{a}\arctan\left(x\right)-\frac{1}{4}\sqrt{a}\log\left(x+1\right)+\frac{1}{4}\sqrt{a}\log\left(x-1\right)+\frac{1}{2}\arctan\left(x\right)+\frac{1}{4}\log\left(x+1\right)-\frac{1}{4}\log\left(x-1\right)$$

input
```
integrate(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x, algorithm="maxima")
```

output
```
1/2*sqrt(a)*arctan(x) - 1/4*sqrt(a)*log(x + 1) + 1/4*sqrt(a)*log(x - 1) +
1/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

3.375.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right)\,dx = \frac{1}{4}\left(2\arctan\left(x\right)\operatorname{sgn}(x)-\log\left(|x+1|\right)\operatorname{sgn}(x)+\log\left(|x-1|\right)\operatorname{sgn}(x)\right)\sqrt{a}+\frac{1}{2}\arctan\left(x\right)+\frac{1}{4}\log\left(|x+1|\right)-\frac{1}{4}\log\left(|x-1|\right)$$

input
```
integrate(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x, algorithm="giac")
```

output
```
1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x))
*sqrt(a) + 1/2*arctan(x) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

3.375.9 Mupad [F(-1)]

Timed out.

$$\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right)\,dx=\int -\frac{1}{x^4-1}-\frac{\sqrt{a\,x^6}}{x-x^5}\,dx$$

input
```
int(- 1/(x^4 - 1) - (a*x^6)^(1/2)/(x - x^5),x)
```

output
```
int(- 1/(x^4 - 1) - (a*x^6)^(1/2)/(x - x^5), x)
```

3.376 $\int \frac{\sqrt{a x^3}}{x-x^3} \, dx$

3.376.1 Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \frac{\sqrt{a x^3}}{x-x^3} \, dx = -\frac{\sqrt{a x^3} \arctan \left(\sqrt{x}\right)}{x^{3/2}}+\frac{\sqrt{a x^3} \operatorname{arctanh}\left(\sqrt{x}\right)}{x^{3/2}}$$

output
```
-arctan(x^(1/2))*(a*x^3)^(1/2)/x^(3/2)+arctanh(x^(1/2))*(a*x^3)^(1/2)/x^(3
/2)
```

3.376.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{a x^3}}{x-x^3} \, dx = \frac{\sqrt{a x^3} \left(-\arctan \left(\sqrt{x}\right)+\operatorname{arctanh}\left(\sqrt{x}\right)\right)}{x^{3/2}}$$

input
```
Integrate[Sqrt[a*x^3]/(x - x^3),x]
```

output
```
(Sqrt[a*x^3]*(-ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]))/x^(3/2)
```

3.376.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {34, 9, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^3}}{x-x^3}dx$$

$$\Big\downarrow \text{34}$$

$$\frac{\sqrt{ax^3}\int \frac{x^{3/2}}{x-x^3}dx}{x^{3/2}}$$

$$\Big\downarrow \text{9}$$

$$\frac{\sqrt{ax^3}\int \frac{\sqrt{x}}{1-x^2}dx}{x^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2\sqrt{ax^3}\int \frac{x}{1-x^2}d\sqrt{x}}{x^{3/2}}$$

$$\Big\downarrow \text{827}$$

$$\frac{2\sqrt{ax^3}\left(\frac{1}{2}\int \frac{1}{1-x}d\sqrt{x}-\frac{1}{2}\int \frac{1}{x+1}d\sqrt{x}\right)}{x^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\sqrt{ax^3}\left(\frac{1}{2}\int \frac{1}{1-x}d\sqrt{x}-\frac{\arctan(\sqrt{x})}{2}\right)}{x^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\sqrt{ax^3}\left(\frac{\text{arctanh}(\sqrt{x})}{2}-\frac{\arctan(\sqrt{x})}{2}\right)}{x^{3/2}}$$

input
```
Int[Sqrt[a*x^3]/(x - x^3),x]
```

output
```
(2*Sqrt[a*x^3]*(-1/2*ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]/2))/x^(3/2)
```

3.376.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

3.376. $\int \frac{\sqrt{ax^3}}{x-x^3} \, dx$

3.376.4 Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

method	result	size
pseudoelliptic	$\left(\operatorname{arctanh}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)+\arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)\right)\sqrt{a}$	26
default	$\frac{\sqrt{a\,x^{3}}\,\sqrt{a}\,\left(\operatorname{arctanh}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)-\arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)\right)}{x\sqrt{ax}}$	43
meijerg	$-\frac{\sqrt{a\,x^{3}}\,\left(\ln\left(1-\left(x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(x^{2}\right)^{\frac{1}{4}}\right)+2\arctan\left(\left(x^{2}\right)^{\frac{1}{4}}\right)\right)}{2\left(x^{2}\right)^{\frac{3}{4}}}$	44

input
```
int((a*x^3)^(1/2)/(-x^3+x),x,method=_RETURNVERBOSE)
```

output
```
(arctanh((a*x)^(1/2)/a^(1/2))+arctan((a*x)^(1/2)/a^(1/2)))*a^(1/2)
```

3.376.5 Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(32) = 64.

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.89

$$\int \frac{\sqrt{ax^3}}{x-x^3}\,dx = \left[-\sqrt{a}\arctan\left(\frac{\sqrt{ax^3}}{\sqrt{a}x}\right)+\frac{1}{2}\sqrt{a}\log\left(\frac{ax^2+ax+2\sqrt{ax^3}\sqrt{a}}{x^2-x}\right),\right.$$
$$\left.-\sqrt{-a}\arctan\left(\frac{\sqrt{ax^3}\sqrt{-a}}{ax}\right)+\frac{1}{2}\sqrt{-a}\log\left(\frac{ax^2-ax-2\sqrt{ax^3}\sqrt{-a}}{x^2+x}\right)\right]$$

input
```
integrate((a*x^3)^(1/2)/(-x^3+x),x, algorithm="fricas")
```

output
```
[-sqrt(a)*arctan(sqrt(a*x^3)/(sqrt(a)*x)) + 1/2*sqrt(a)*log((a*x^2 + a*x +
 2*sqrt(a*x^3)*sqrt(a))/(x^2 - x)), -sqrt(-a)*arctan(sqrt(a*x^3)*sqrt(-a)/
(a*x)) + 1/2*sqrt(-a)*log((a*x^2 - a*x - 2*sqrt(a*x^3)*sqrt(-a))/(x^2 + x)
)]
```

3.376.6 Sympy [F]

$$\int \frac{\sqrt{ax^3}}{x-x^3}\,dx = -\int \frac{\sqrt{ax^3}}{x^3-x}\,dx$$

input
```
integrate((a*x**3)**(1/2)/(-x**3+x),x)
```

output
```
-Integral(sqrt(a*x**3)/(x**3 - x), x)
```

3.376.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{ax^3}}{x-x^3}\,dx = -\sqrt{a}\arctan\left(\sqrt{x}\right) + \frac{1}{2}\sqrt{a}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\sqrt{a}\log\left(\sqrt{x}-1\right)$$

input
```
integrate((a*x^3)^(1/2)/(-x^3+x),x, algorithm="maxima")
```

output
```
-sqrt(a)*arctan(sqrt(x)) + 1/2*sqrt(a)*log(sqrt(x) + 1) - 1/2*sqrt(a)*log(
sqrt(x) - 1)
```

3.376.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{ax^3}}{x-x^3}\,dx = -\frac{\left(\frac{a^2\arctan\left(\frac{\sqrt{ax}}{\sqrt{-a}}\right)}{\sqrt{-a}} + a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)\right)\operatorname{sgn}(x)}{a}$$

input
```
integrate((a*x^3)^(1/2)/(-x^3+x),x, algorithm="giac")
```

output
```
-(a^2*arctan(sqrt(a*x)/sqrt(-a))/sqrt(-a) + a^(3/2)*arctan(sqrt(a*x)/sqrt(
a)))*sgn(x)/a
```

3.376.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^3}}{x-x^3}\,dx=\int \frac{\sqrt{a\,x^3}}{x-x^3}\,dx$$

input
```
int((a*x^3)^(1/2)/(x - x^3),x)
```

output
```
int((a*x^3)^(1/2)/(x - x^3), x)
```

3.377 $\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx$

3.377.1 Optimal result

Integrand size = 19, antiderivative size = 44

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx=\frac{\sqrt{ax^4}\sqrt{1+x^2}}{2x}-\frac{\sqrt{ax^4}\operatorname{arcsinh}(x)}{2x^2}$$

output
```
-1/2*arcsinh(x)*(a*x^4)^(1/2)/x^2+1/2*(a*x^4)^(1/2)*(x^2+1)^(1/2)/x
```

3.377.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx=\frac{\sqrt{ax^4}\left(x\sqrt{1+x^2}+\log\left(-x+\sqrt{1+x^2}\right)\right)}{2x^2}$$

input
```
Integrate[Sqrt[a*x^4]/Sqrt[1 + x^2],x]
```

output
```
(Sqrt[a*x^4]*(x*Sqrt[1 + x^2] + Log[-x + Sqrt[1 + x^2]]))/(2*x^2)
```

3.377.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {34, 262, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^4}}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{ 34}$$

$$\frac{\sqrt{ax^4}\int \frac{x^2}{\sqrt{x^2+1}}dx}{x^2}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{\sqrt{ax^4}\left(\frac{1}{2}x\sqrt{x^2+1}-\frac{1}{2}\int \frac{1}{\sqrt{x^2+1}}dx\right)}{x^2}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{\sqrt{ax^4}\left(\frac{1}{2}x\sqrt{x^2+1}-\frac{\text{arcsinh}(x)}{2}\right)}{x^2}$$

input
```
Int[Sqrt[a*x^4]/Sqrt[1 + x^2],x]
```

output
```
(Sqrt[a*x^4]*((x*Sqrt[1 + x^2])/2 - ArcSinh[x]/2))/x^2
```

3.377.3.1 Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

3.377.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{\sqrt{a\,x^{4}}\,\left(x\sqrt{x^{2}+1}-\operatorname{arcsinh}\left(x\right)\right)}{2x^{2}}$	27
meijerg	$\frac{\sqrt{a\,x^{4}}\,\left(\sqrt{\pi}\,x\sqrt{x^{2}+1}-\sqrt{\pi}\,\operatorname{arcsinh}\left(x\right)\right)}{2x^{2}\sqrt{\pi}}$	36
risch	$\frac{\sqrt{a\,x^{4}}\,\sqrt{x^{2}+1}}{2x}-\frac{\ln\left(x\sqrt{a}+\sqrt{a\,x^{2}+a}\right)\sqrt{a\,x^{4}}\,\sqrt{\left(x^{2}+1\right)a}}{2\sqrt{a}\,x^{2}\sqrt{x^{2}+1}}$	68

input
```
int((a*x^4)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x^4)^(1/2)*(x*(x^2+1)^(1/2)-arcsinh(x))/x^2
```

3.377.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{ax^4}\sqrt{x^2+1}x+\sqrt{ax^4}\log\left(-x+\sqrt{x^2+1}\right)}{2\,x^2}$$

input
```
integrate((a*x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(a*x^4)*sqrt(x^2 + 1)*x + sqrt(a*x^4)*log(-x + sqrt(x^2 + 1)))/x^
2
```

3.377.6 Sympy [F]

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{ax^4}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a*x**4)**(1/2)/(x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(a*x**4)/sqrt(x**2 + 1), x)
```

3.377.7 Maxima [F]

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{ax^4}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a*x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^4)/sqrt(x^2 + 1), x)
```

3.377.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx = \frac{1}{2}\left(\sqrt{x^2+1}x + \log\left(-x+\sqrt{x^2+1}\right)\right)\sqrt{a}$$

input
```
integrate((a*x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(x^2 + 1)*x + log(-x + sqrt(x^2 + 1)))*sqrt(a)
```

3.377.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{a\,x^4}}{\sqrt{x^2+1}}\,dx$$

input
```
int((a*x^4)^(1/2)/(x^2 + 1)^(1/2),x)
```

output
```
int((a*x^4)^(1/2)/(x^2 + 1)^(1/2), x)
```

3.378 $\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx$

3.378.1 Optimal result

Integrand size = 19, antiderivative size = 83

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx = \frac{2\sqrt{ax^3}\sqrt{1+x^2}}{3x} - \frac{\sqrt{ax^3}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{x}\right),\frac{1}{2}\right)}{3x^{3/2}\sqrt{1+x^2}}$$

output
```
2/3*(a*x^3)^(1/2)*(x^2+1)^(1/2)/x-1/3*(1+x)*(cos(2*arctan(x^(1/2)))^2)^(1/
2)/cos(2*arctan(x^(1/2)))*EllipticF(sin(2*arctan(x^(1/2))),1/2*2^(1/2))*(a
*x^3)^(1/2)*((x^2+1)/(1+x)^2)^(1/2)/x^(3/2)/(x^2+1)^(1/2)
```

3.378.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx = \frac{2\sqrt{ax^3}\left(\sqrt{1+x^2} - \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-x^2\right)\right)}{3x}$$

input
```
Integrate[Sqrt[a*x^3]/Sqrt[1 + x^2],x]
```

output
```
(2*Sqrt[a*x^3]*(Sqrt[1 + x^2] - Hypergeometric2F1[1/4, 1/2, 5/4, -x^2]))/(
3*x)
```

3.378.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {34, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^3}}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{34}$$

$$\frac{\sqrt{ax^3}\int \frac{x^{3/2}}{\sqrt{x^2+1}}dx}{x^{3/2}}$$

$$\Big\downarrow \text{262}$$

$$\frac{\sqrt{ax^3}\left(\frac{2}{3}\sqrt{x}\sqrt{x^2+1}-\frac{1}{3}\int \frac{1}{\sqrt{x}\sqrt{x^2+1}}dx\right)}{x^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\sqrt{ax^3}\left(\frac{2}{3}\sqrt{x}\sqrt{x^2+1}-\frac{2}{3}\int \frac{1}{\sqrt{x^2+1}}d\sqrt{x}\right)}{x^{3/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{\sqrt{ax^3}\left(\frac{2}{3}\sqrt{x}\sqrt{x^2+1}-\frac{(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt{x}\right),\frac{1}{2}\right)}{3\sqrt{x^2+1}}\right)}{x^{3/2}}$$

input
```
Int[Sqrt[a*x^3]/Sqrt[1 + x^2],x]
```

output
```
(Sqrt[a*x^3]*((2*Sqrt[x]*Sqrt[1 + x^2])/3 - ((1 + x)*Sqrt[(1 + x^2)/(1 + x
)^2]*EllipticF[2*ArcTan[Sqrt[x]], 1/2])/(3*Sqrt[1 + x^2])))/x^(3/2)
```

3.378.3.1 Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.378.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.27

method	result	size
meijerg	$\frac{2\sqrt{a\,x^{3}}\,x\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};-x^{2}\right)}{5}$	22
default	$-\frac{\sqrt{a\,x^{3}}\,\left(i\sqrt{-i(x+i)}\,\sqrt{2}\,\sqrt{-i(-x+i)}\,\sqrt{ix}\,F\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)-2x^{3}-2x\right)}{3x^{2}\sqrt{x^{2}+1}}$	76
risch	$\frac{2\sqrt{a\,x^{3}}\,\sqrt{x^{2}+1}}{3x}-\frac{i\sqrt{-i(x+i)}\,\sqrt{2}\,\sqrt{i(x-i)}\,\sqrt{ix}\,F\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)\sqrt{a\,x^{3}}\,\sqrt{ax(x^{2}+1)}}{3\sqrt{a\,x^{3}+ax}\,x^{2}\sqrt{x^{2}+1}}$	104

input
```
int((a*x^3)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(a*x^3)^(1/2)*x*hypergeom([1/2,5/4],[9/4],-x^2)
```

3.378.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx = -\frac{2\left(\sqrt{a}x\text{weierstrassPInverse}(-4,0,x)-\sqrt{ax^3}\sqrt{x^2+1}\right)}{3\,x}$$

input
```
integrate((a*x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(a)*x*weierstrassPInverse(-4, 0, x) - sqrt(a*x^3)*sqrt(x^2 + 1))
/x
```

3.378.6 Sympy [F]

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{ax^3}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a*x**3)**(1/2)/(x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(a*x**3)/sqrt(x**2 + 1), x)
```

3.378.7 Maxima [F]

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{ax^3}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a*x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^3)/sqrt(x^2 + 1), x)
```

3.378.8 Giac [F]

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{ax^3}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a*x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x^3)/sqrt(x^2 + 1), x)
```

3.378.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{a\,x^3}}{\sqrt{x^2+1}}\,dx$$

input
```
int((a*x^3)^(1/2)/(x^2 + 1)^(1/2),x)
```

output
```
int((a*x^3)^(1/2)/(x^2 + 1)^(1/2), x)
```

3.379 $\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}}\,dx$

3.379.1 Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{ax^2}\sqrt{1+x^2}}{x}$$

output
```
(a*x^2)^(1/2)*(x^2+1)^(1/2)/x
```

3.379.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{ax^2}\sqrt{1+x^2}}{x}$$

input
```
Integrate[Sqrt[a*x^2]/Sqrt[1 + x^2],x]
```

output
```
(Sqrt[a*x^2]*Sqrt[1 + x^2])/x
```

3.379.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {34, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2}}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \; 34$$

$$\frac{\sqrt{ax^2}\int \frac{x}{\sqrt{x^2+1}}dx}{x}$$

$$\Big\downarrow \; 241$$

$$\frac{\sqrt{x^2+1}\sqrt{ax^2}}{x}$$

input
```
Int[Sqrt[a*x^2]/Sqrt[1 + x^2],x]
```

output
```
(Sqrt[a*x^2]*Sqrt[1 + x^2])/x
```

3.379.3.1 Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

3.379.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
gosper	$\frac{\sqrt{a\,x^{2}}\,\sqrt{x^{2}+1}}{x}$	19
default	$\frac{\sqrt{a\,x^{2}}\,\sqrt{x^{2}+1}}{x}$	19
risch	$\frac{\sqrt{a\,x^{2}}\,\sqrt{x^{2}+1}}{x}$	19
meijerg	$\frac{\sqrt{a\,x^{2}}\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{x^{2}+1}\right)}{2x\sqrt{\pi}}$	34

input
```
int((a*x^2)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x^2)^(1/2)*(x^2+1)^(1/2)/x
```

3.379.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{ax^2}\sqrt{x^2+1}}{x}$$

input
```
integrate((a*x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(a*x^2)*sqrt(x^2 + 1)/x
```

3.379.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{ax^2}\sqrt{x^2+1}}{x}$$

input
```
integrate((a*x**2)**(1/2)/(x**2+1)**(1/2),x)
```

output
```
sqrt(a*x**2)*sqrt(x**2 + 1)/x
```

3.379.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{a}x^2 + \sqrt{a}}{\sqrt{x^2+1}}$$

input
```
integrate((a*x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
(sqrt(a)*x^2 + sqrt(a))/sqrt(x^2 + 1)
```

3.379.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}}\,dx = \left(\sqrt{x^2+1}\mathrm{sgn}(x) - \mathrm{sgn}(x)\right)\sqrt{a}$$

input
```
integrate((a*x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
(sqrt(x^2 + 1)*sgn(x) - sgn(x))*sqrt(a)
```

3.379.9 Mupad [B] (verification not implemented)

Time = 16.43 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{a}\,\sqrt{x^2+1}\,\sqrt{x^2}}{x}$$

input
```
int((a*x^2)^(1/2)/(x^2 + 1)^(1/2),x)
```

output
```
(a^(1/2)*(x^2 + 1)^(1/2)*(x^2)^(1/2))/x
```

3.380 $\int \frac{\sqrt{ax}}{\sqrt{1+x^2}} dx$

3.380.1 Optimal result

Integrand size = 17, antiderivative size = 131

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^2}}\,dx = \frac{2\sqrt{ax}\sqrt{1+x^2}}{1+x} - \frac{2\sqrt{a}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}E\left(2\arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)|\frac{1}{2}\right)}{\sqrt{1+x^2}} + \frac{\sqrt{a}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right),\frac{1}{2}\right)}{\sqrt{1+x^2}}$$

output
```
2*(a*x)^(1/2)*(x^2+1)^(1/2)/(1+x)-2*(1+x)*(cos(2*arctan((a*x)^(1/2)/a^(1/2
)))^2)^(1/2)/cos(2*arctan((a*x)^(1/2)/a^(1/2)))*EllipticE(sin(2*arctan((a*
x)^(1/2)/a^(1/2))),1/2*2^(1/2))*a^(1/2)*((x^2+1)/(1+x)^2)^(1/2)/(x^2+1)^(1
/2)+(1+x)*(cos(2*arctan((a*x)^(1/2)/a^(1/2)))^2)^(1/2)/cos(2*arctan((a*x)^
(1/2)/a^(1/2)))*EllipticF(sin(2*arctan((a*x)^(1/2)/a^(1/2))),1/2*2^(1/2))*
a^(1/2)*((x^2+1)/(1+x)^2)^(1/2)/(x^2+1)^(1/2)
```

3.380.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^2}}\,dx = \frac{2}{3}x\sqrt{ax}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-x^2\right)$$

input
```
Integrate[Sqrt[a*x]/Sqrt[1 + x^2],x]
```

output
```
(2*x*Sqrt[a*x]*Hypergeometric2F1[1/2, 3/4, 7/4, -x^2])/3
```

3.380.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax}}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2\int \frac{ax}{\sqrt{x^2+1}}d\sqrt{ax}}{a}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{2\left(a\int \frac{1}{\sqrt{x^2+1}}d\sqrt{ax}-a\int \frac{a-ax}{a\sqrt{x^2+1}}d\sqrt{ax}\right)}{a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2\left(a\int \frac{1}{\sqrt{x^2+1}}d\sqrt{ax}-\int \frac{a-ax}{\sqrt{x^2+1}}d\sqrt{ax}\right)}{a}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{2\left(\frac{\sqrt{a}(ax+a)\sqrt{\frac{a^2x^2+a^2}{(ax+a)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right),\frac{1}{2}\right)}{2\sqrt{x^2+1}}-\int \frac{a-ax}{\sqrt{x^2+1}}d\sqrt{ax}\right)}{a}$$

$$\Big\downarrow \text{ 1510}$$

$$\frac{2\left(\frac{\sqrt{a}(ax+a)\sqrt{\frac{a^2x^2+a^2}{(ax+a)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right),\frac{1}{2}\right)}{2\sqrt{x^2+1}}-\frac{\sqrt{a}(ax+a)\sqrt{\frac{a^2x^2+a^2}{(ax+a)^2}}E\left(2\arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}+\frac{a^2\sqrt{x^2+1}\sqrt{ax}}{ax+a}\right)}{a}$$

input
```
Int[Sqrt[a*x]/Sqrt[1 + x^2],x]
```

output
```
(2*((a^2*Sqrt[a*x]*Sqrt[1 + x^2])/(a + a*x) - (Sqrt[a]*(a + a*x)*Sqrt[(a^2
 + a^2*x^2)/(a + a*x)^2]*EllipticE[2*ArcTan[Sqrt[a*x]/Sqrt[a]], 1/2])/Sqrt
[1 + x^2] + (Sqrt[a]*(a + a*x)*Sqrt[(a^2 + a^2*x^2)/(a + a*x)^2]*EllipticF
[2*ArcTan[Sqrt[a*x]/Sqrt[a]], 1/2])/(2*Sqrt[1 + x^2])))/a
```

3.380.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.380.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.92 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.15

method	result	size
meijerg	$\frac{2\sqrt{ax}\, x\, {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-x^2\right)}{3}$	20
default	$\frac{\sqrt{ax}\,\sqrt{-i(x+i)}\,\sqrt{2}\,\sqrt{-i(-x+i)}\,\sqrt{ix}\,\left(2E\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)-F\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)\right)}{\sqrt{x^2+1}\,x}$	81
elliptic	$\frac{i\sqrt{ax}\,\sqrt{ax(x^2+1)}\,\sqrt{-i(x+i)}\,\sqrt{2}\,\sqrt{i(x-i)}\,\sqrt{ix}\,\left(-2iE\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)+iF\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)\right)}{\sqrt{x^2+1}\,x\sqrt{a\,x^3+ax}}$	104

input
```
int((a*x)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(a*x)^(1/2)*x*hypergeom([1/2,3/4],[7/4],-x^2)
```

3.380.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.09

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^2}}\,dx = -2\sqrt{a}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,x))$$

input
```
integrate((a*x)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, x))
```

3.380.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.27

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{a}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|x^2e^{i\pi}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((a*x)**(1/2)/(x**2+1)**(1/2),x)
```

output
```
sqrt(a)*x**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**2*exp_polar(I*pi)
)/(2*gamma(7/4))
```

3.380.7 Maxima [F]

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{ax}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a*x)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x)/sqrt(x^2 + 1), x)
```

3.380.8 Giac [F]

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{ax}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a*x)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x)/sqrt(x^2 + 1), x)
```

3.380.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{a\,x}}{\sqrt{x^2+1}}\,dx$$

input
```
int((a*x)^(1/2)/(x^2 + 1)^(1/2),x)
```

output
```
int((a*x)^(1/2)/(x^2 + 1)^(1/2), x)
```

3.381 $\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}} dx$

3.381.1 Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}} dx = \frac{\sqrt{\frac{a}{x}}\sqrt{x}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{x}\right),\frac{1}{2}\right)}{\sqrt{1+x^2}}$$

output
```
(1+x)*(cos(2*arctan(x^(1/2)))^2)^(1/2)/cos(2*arctan(x^(1/2)))*EllipticF(si
n(2*arctan(x^(1/2))),1/2*2^(1/2))*(a/x)^(1/2)*x^(1/2)*((x^2+1)/(1+x)^2)^(1
/2)/(x^2+1)^(1/2)
```

3.381.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}} dx = 2\sqrt{\frac{a}{x}}x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-x^2\right)$$

input
```
Integrate[Sqrt[a/x]/Sqrt[1 + x^2],x]
```

output
```
2*Sqrt[a/x]*x*Hypergeometric2F1[1/4, 1/2, 5/4, -x^2]
```

3.381.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {34, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{ 34}$$

$$\sqrt{x}\sqrt{\frac{a}{x}}\int \frac{1}{\sqrt{x}\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{ 266}$$

$$2\sqrt{x}\sqrt{\frac{a}{x}}\int \frac{1}{\sqrt{x^2+1}}d\sqrt{x}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{\sqrt{x}(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\sqrt{\frac{a}{x}}\text{EllipticF}\left(2\arctan\left(\sqrt{x}\right),\frac{1}{2}\right)}{\sqrt{x^2+1}}$$

input
```
Int[Sqrt[a/x]/Sqrt[1 + x^2],x]
```

output
```
(Sqrt[a/x]*Sqrt[x]*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sq
rt[x]], 1/2])/Sqrt[1 + x^2]
```

3.381.3.1 Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.381.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.96 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.41

method	result	size
meijerg	$2\sqrt{\frac{a}{x}}\, x\, {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-x^2\right)$	22
default	$\frac{i\sqrt{\frac{a}{x}}\,\sqrt{-i(x+i)}\,\sqrt{2}\,\sqrt{-i(-x+i)}\,\sqrt{ix}\,F\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)}{\sqrt{x^2+1}}$	62

input
```
int((a/x)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a/x)^(1/2)*x*hypergeom([1/4,1/2],[5/4],-x^2)
```

3.381.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}}\, dx = 2\sqrt{a}\text{weierstrassPInverse}(-4,0,x)$$

input
```
integrate((a/x)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(a)*weierstrassPInverse(-4, 0, x)
```

3.381.6 Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a/x)**(1/2)/(x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(a/x)/sqrt(x**2 + 1), x)
```

3.381.7 Maxima [F]

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a/x)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a/x)/sqrt(x^2 + 1), x)
```

3.381.8 Giac [F]

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a/x)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a/x)/sqrt(x^2 + 1), x)
```

3.381.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}}\,dx=\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}}\,dx$$

input
```
int((a/x)^(1/2)/(x^2 + 1)^(1/2),x)
```

output
```
int((a/x)^(1/2)/(x^2 + 1)^(1/2), x)
```

3.382 $\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx$

3.382.1 Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx = -\sqrt{\frac{a}{x^2}}x\operatorname{arctanh}\left(\sqrt{1+x^2}\right)$$

output
```
-x*arctanh((x^2+1)^(1/2))*(a/x^2)^(1/2)
```

3.382.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx = -\sqrt{\frac{a}{x^2}}x\operatorname{arctanh}\left(\sqrt{1+x^2}\right)$$

input
```
Integrate[Sqrt[a/x^2]/Sqrt[1 + x^2],x]
```

output
```
-(Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^2]])
```

3.382.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {34, 243, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{ 34}$$

$$x\sqrt{\frac{a}{x^2}}\int \frac{1}{x\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}x\sqrt{\frac{a}{x^2}}\int \frac{1}{x^2\sqrt{x^2+1}}dx^2$$

$$\Big\downarrow \text{ 73}$$

$$x\sqrt{\frac{a}{x^2}}\int \frac{1}{x^4-1}d\sqrt{x^2+1}$$

$$\Big\downarrow \text{ 220}$$

$$x\left(-\sqrt{\frac{a}{x^2}}\right)\text{arctanh}\left(\sqrt{x^2+1}\right)$$

input
```
Int[Sqrt[a/x^2]/Sqrt[1 + x^2],x]
```

output
```
-(Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^2]])
```

3.382.3.1 Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.382.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
default	$-\sqrt{\frac{a}{x^2}}\, x \,\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)$	19
meijerg	$\frac{\sqrt{\frac{a}{x^2}}\, x \left(-2\sqrt{\pi}\, \ln\left(\frac{1}{2}+\frac{\sqrt{x^2+1}}{2}\right)+\left(-2\ln(2)+2\ln(x)\right)\sqrt{\pi}\right)}{2\sqrt{\pi}}$	45

input
```
int((a/x^2)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(a/x^2)^(1/2)*x*arctanh(1/(x^2+1)^(1/2))
```

3.382.5 Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(18) = 36.

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.45

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\, dx = \left[x\sqrt{\frac{a}{x^2}}\log\left(\frac{\sqrt{x^2+1}-1}{x}\right),\, 2\sqrt{-a}\arctan\left(-\frac{\sqrt{-a}x^2\sqrt{\frac{a}{x^2}}-\sqrt{x^2+1}\sqrt{-a}x\sqrt{\frac{a}{x^2}}}{a}\right)\right]$$

input
```
integrate((a/x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
[x*sqrt(a/x^2)*log((sqrt(x^2 + 1) - 1)/x), 2*sqrt(-a)*arctan(-(sqrt(-a)*x^
2*sqrt(a/x^2) - sqrt(x^2 + 1)*sqrt(-a)*x*sqrt(a/x^2))/a)]
```

3.382.6 Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a/x**2)**(1/2)/(x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(a/x**2)/sqrt(x**2 + 1), x)
```

3.382.7 Maxima [F]

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a/x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a/x^2)/sqrt(x^2 + 1), x)
```

3.382.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx = -\frac{1}{2}\sqrt{a}\left(\log\left(\sqrt{x^2+1}+1\right)-\log\left(\sqrt{x^2+1}-1\right)\right)\mathrm{sgn}(x)$$

input
```
integrate((a/x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(a)*(log(sqrt(x^2 + 1) + 1) - log(sqrt(x^2 + 1) - 1))*sgn(x)
```

3.382.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^2+1}}\,dx$$

input
```
int((a/x^2)^(1/2)/(x^2 + 1)^(1/2),x)
```

output
```
int((a/x^2)^(1/2)/(x^2 + 1)^(1/2), x)
```

3.383 $\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx$

3.383.1 Optimal result

Integrand size = 19, antiderivative size = 159

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx = -2\sqrt{\frac{a}{x^3}}x\sqrt{1+x^2}+\frac{2\sqrt{\frac{a}{x^3}}x^2\sqrt{1+x^2}}{1+x} - \frac{2\sqrt{\frac{a}{x^3}}x^{3/2}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}E\left(2\arctan\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{1+x^2}} + \frac{\sqrt{\frac{a}{x^3}}x^{3/2}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{x}\right),\frac{1}{2}\right)}{\sqrt{1+x^2}}$$

output
```
-2*x*(a/x^3)^(1/2)*(x^2+1)^(1/2)+2*x^2*(a/x^3)^(1/2)*(x^2+1)^(1/2)/(1+x)-2
*x^(3/2)*(1+x)*(cos(2*arctan(x^(1/2)))^2)^(1/2)/cos(2*arctan(x^(1/2)))*Ell
ipticE(sin(2*arctan(x^(1/2))),1/2*2^(1/2))*(a/x^3)^(1/2)*((x^2+1)/(1+x)^2)
^(1/2)/(x^2+1)^(1/2)+x^(3/2)*(1+x)*(cos(2*arctan(x^(1/2)))^2)^(1/2)/cos(2*
arctan(x^(1/2)))*EllipticF(sin(2*arctan(x^(1/2))),1/2*2^(1/2))*(a/x^3)^(1/
2)*((x^2+1)/(1+x)^2)^(1/2)/(x^2+1)^(1/2)
```

3.383.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx = -2\sqrt{\frac{a}{x^3}}x \,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-x^2\right)$$

input
```
Integrate[Sqrt[a/x^3]/Sqrt[1 + x^2],x]
```

output
```
-2*Sqrt[a/x^3]*x*Hypergeometric2F1[-1/4, 1/2, 3/4, -x^2]
```

3.383.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {34, 264, 266, 834, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{34}$$

$$x^{3/2}\sqrt{\frac{a}{x^3}}\int \frac{1}{x^{3/2}\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{264}$$

$$x^{3/2}\sqrt{\frac{a}{x^3}}\left(\int \frac{\sqrt{x}}{\sqrt{x^2+1}}dx-\frac{2\sqrt{x^2+1}}{\sqrt{x}}\right)$$

$$\Big\downarrow \text{266}$$

$$x^{3/2}\sqrt{\frac{a}{x^3}}\left(2\int \frac{x}{\sqrt{x^2+1}}d\sqrt{x}-\frac{2\sqrt{x^2+1}}{\sqrt{x}}\right)$$

$$\Big\downarrow \text{834}$$

$$x^{3/2}\sqrt{\frac{a}{x^3}}\left(2\left(\int \frac{1}{\sqrt{x^2+1}}d\sqrt{x}-\int \frac{1-x}{\sqrt{x^2+1}}d\sqrt{x}\right)-\frac{2\sqrt{x^2+1}}{\sqrt{x}}\right)$$

$$\Big\downarrow \text{761}$$

$$x^{3/2}\sqrt{\frac{a}{x^3}}\left(2\left(\frac{(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{x}\right),\frac{1}{2}\right)}{2\sqrt{x^2+1}}-\int\frac{1-x}{\sqrt{x^2+1}}d\sqrt{x}\right)-\frac{2\sqrt{x^2+1}}{\sqrt{x}}\right)$$

$$\Big\downarrow \text{1510}$$

$$x^{3/2}\sqrt{\frac{a}{x^3}}\left(2\left(\frac{(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt{x}\right),\frac{1}{2}\right)}{2\sqrt{x^2+1}}-\frac{(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}E\left(2\arctan\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}+\frac{\sqrt{x}\sqrt{x^2+1}}{x+1}\right.\right.$$

input
```
Int[Sqrt[a/x^3]/Sqrt[1 + x^2],x]
```

output
```
Sqrt[a/x^3]*x^(3/2)*((-2*Sqrt[1 + x^2])/Sqrt[x] + 2*((Sqrt[x]*Sqrt[1 + x^2
])/(1 + x) - ((1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*EllipticE[2*ArcTan[Sqrt[x]
], 1/2])/Sqrt[1 + x^2] + ((1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*EllipticF[2*Ar
cTan[Sqrt[x]], 1/2])/(2*Sqrt[1 + x^2])))
```

3.383.3.1 Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

3.383. $\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx$

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

3.383.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.14

method	result	siz
meijerg	$-2\sqrt{\frac{a}{x^3}}\, x\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-x^2\right)$	22
default	$\frac{\sqrt{\frac{a}{x^3}}\, x\left(2\sqrt{-i(x+i)}\,\sqrt{2}\,\sqrt{-i(-x+i)}\,\sqrt{ix}\, E\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)-\sqrt{-i(x+i)}\,\sqrt{2}\,\sqrt{-i(-x+i)}\,\sqrt{ix}\, F\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)-2x^2-2\right)}{\sqrt{x^2+1}}$	11
risch	$-2x\sqrt{\frac{a}{x^3}}\,\sqrt{x^2+1}+\frac{i\sqrt{-i(x+i)}\,\sqrt{2}\,\sqrt{i(x-i)}\,\sqrt{ix}\left(-2iE\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)+iF\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)\right)\sqrt{\frac{a}{x^3}}\, x\sqrt{ax(x^2+1)}}{\sqrt{a\,x^3+ax}\,\sqrt{x^2+1}}$	12

input
```
int((a/x^3)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(a/x^3)^(1/2)*x*hypergeom([-1/4,1/2],[3/4],-x^2)
```

3.383.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx$$
$$= -2\sqrt{x^2+1}x\sqrt{\frac{a}{x^3}}-2\sqrt{a}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,x))$$

input
```
integrate((a/x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(x^2 + 1)*x*sqrt(a/x^3) - 2*sqrt(a)*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, x))
```

3.383.6 Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a/x**3)**(1/2)/(x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(a/x**3)/sqrt(x**2 + 1), x)
```

3.383.7 Maxima [F]

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a/x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a/x^3)/sqrt(x^2 + 1), x)
```

3.383.8 Giac [F]

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a/x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a/x^3)/sqrt(x^2 + 1), x)
```

3.383.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}\,dx$$

input
```
int((a/x^3)^(1/2)/(x^2 + 1)^(1/2),x)
```

output
```
int((a/x^3)^(1/2)/(x^2 + 1)^(1/2), x)
```

3.384 $\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx$

3.384.1 Optimal result

Integrand size = 19, antiderivative size = 21

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx = -\sqrt{\frac{a}{x^4}}x\sqrt{1+x^2}$$

output
```
-x*(a/x^4)^(1/2)*(x^2+1)^(1/2)
```

3.384.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx = -\sqrt{\frac{a}{x^4}}x\sqrt{1+x^2}$$

input
```
Integrate[Sqrt[a/x^4]/Sqrt[1 + x^2],x]
```

output
```
-(Sqrt[a/x^4]*x*Sqrt[1 + x^2])
```

3.384.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {34, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{34}$$

$$x^2\sqrt{\frac{a}{x^4}}\int \frac{1}{x^2\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{242}$$

$$x\sqrt{x^2+1}\left(-\sqrt{\frac{a}{x^4}}\right)$$

input
```
Int[Sqrt[a/x^4]/Sqrt[1 + x^2],x]
```

output
```
-(Sqrt[a/x^4]*x*Sqrt[1 + x^2])
```

3.384.3.1 Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

3.384.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$-x\sqrt{\frac{a}{x^4}}\,\sqrt{x^2+1}$	18
default	$-x\sqrt{\frac{a}{x^4}}\,\sqrt{x^2+1}$	18
meijerg	$-x\sqrt{\frac{a}{x^4}}\,\sqrt{x^2+1}$	18
risch	$-x\sqrt{\frac{a}{x^4}}\,\sqrt{x^2+1}$	18

input
```
int((a/x^4)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-x*(a/x^4)^(1/2)*(x^2+1)^(1/2)
```

3.384.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx = -x^2\sqrt{\frac{a}{x^4}} - \sqrt{x^2+1}x\sqrt{\frac{a}{x^4}}$$

input
```
integrate((a/x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-x^2*sqrt(a/x^4) - sqrt(x^2 + 1)*x*sqrt(a/x^4)
```

3.384.6 Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((a/x**4)**(1/2)/(x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(a/x**4)/sqrt(x**2 + 1), x)
```

3.384.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx = -\frac{\sqrt{a}x^2+\sqrt{a}}{\sqrt{x^2+1}x}$$

input
```
integrate((a/x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-(sqrt(a)*x^2 + sqrt(a))/(sqrt(x^2 + 1)*x)
```

3.384.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx = \frac{2\sqrt{a}}{\left(x-\sqrt{x^2+1}\right)^2-1}$$

input
```
integrate((a/x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(a)/((x - sqrt(x^2 + 1))^2 - 1)
```

3.384.9 Mupad [B] (verification not implemented)

Time = 16.62 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx = -\sqrt{a}\,x\,\sqrt{x^2+1}\,\sqrt{\frac{1}{x^4}}$$

input
```
int((a/x^4)^(1/2)/(x^2 + 1)^(1/2),x)
```

output
```
-a^(1/2)*x*(x^2 + 1)^(1/2)*(1/x^4)^(1/2)
```

3.385 $\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx$

3.385.1 Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{ax^4}\sqrt{1+x^3}}{3x^2}$$

output
```
2/3*(a*x^4)^(1/2)*(x^3+1)^(1/2)/x^2
```

3.385.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{ax^4}\sqrt{1+x^3}}{3x^2}$$

input
```
Integrate[Sqrt[a*x^4]/Sqrt[1 + x^3],x]
```

output
```
(2*Sqrt[a*x^4]*Sqrt[1 + x^3])/(3*x^2)
```

3.385.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {34, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^4}}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{ 34}$$

$$\frac{\sqrt{ax^4}\int \frac{x^2}{\sqrt{x^3+1}}dx}{x^2}$$

$$\Big\downarrow \text{ 793}$$

$$\frac{2\sqrt{x^3+1}\sqrt{ax^4}}{3x^2}$$

input
```
Int[Sqrt[a*x^4]/Sqrt[1 + x^3],x]
```

output
```
(2*Sqrt[a*x^4]*Sqrt[1 + x^3])/(3*x^2)
```

3.385.3.1 Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

3.385.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{2\sqrt{a\,x^4}\,\sqrt{x^3+1}}{3x^2}$	20
risch	$\frac{2\sqrt{a\,x^4}\,\sqrt{x^3+1}}{3x^2}$	20
gosper	$\frac{2(x+1)\left(x^2-x+1\right)\sqrt{a\,x^4}}{3x^2\sqrt{x^3+1}}$	31
meijerg	$\frac{\sqrt{a\,x^4}\,\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{x^3+1}\right)}{3x^2\sqrt{\pi}}$	34

input
```
int((a*x^4)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(a*x^4)^(1/2)*(x^3+1)^(1/2)/x^2
```

3.385.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx = \frac{2\,\sqrt{ax^4}\sqrt{x^3+1}}{3\,x^2}$$

input
```
integrate((a*x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(a*x^4)*sqrt(x^3 + 1)/x^2
```

3.385.6 Sympy [F]

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{ax^4}}{\sqrt{(x+1)\,(x^2-x+1)}}\,dx$$

input
```
integrate((a*x**4)**(1/2)/(x**3+1)**(1/2),x)
```

output
```
Integral(sqrt(a*x**4)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

3.385.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx = \frac{2\left(\sqrt{a}x^3+\sqrt{a}\right)}{3\sqrt{x^2-x+1}\sqrt{x+1}}$$

input
```
integrate((a*x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(sqrt(a)*x^3 + sqrt(a))/(sqrt(x^2 - x + 1)*sqrt(x + 1))
```

3.385.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx = \frac{2}{3}\sqrt{x^3+1}\sqrt{a}$$

input
```
integrate((a*x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
2/3*sqrt(x^3 + 1)*sqrt(a)
```

3.385.9 Mupad [B] (verification not implemented)

Time = 16.79 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{a}\sqrt{x^3+1}\sqrt{x^4}}{3x^2}$$

input
```
int((a*x^4)^(1/2)/(x^3 + 1)^(1/2),x)
```

output
```
(2*a^(1/2)*(x^3 + 1)^(1/2)*(x^4)^(1/2))/(3*x^2)
```

3.386 $\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx$

3.386.1 Optimal result

Integrand size = 19, antiderivative size = 292

$$\begin{aligned}\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx &= \frac{\left(1+\sqrt{3}\right)\sqrt{ax^3}\sqrt{1+x^3}}{x\left(1+\left(1+\sqrt{3}\right)x\right)} \\ &\quad - \frac{\sqrt[4]{3}\sqrt{ax^3}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}E\left(\arccos\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{x\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{1+x^3}} \\ &\quad - \frac{\left(1-\sqrt{3}\right)\sqrt{ax^3}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}x\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{1+x^3}}\end{aligned}$$

output
```
(1+3^(1/2))*(a*x^3)^(1/2)*(x^3+1)^(1/2)/x/(1+x*(1+3^(1/2)))-3^(1/4)*(1+x)*
((1+x*(1-3^(1/2)))^2/(1+x*(1+3^(1/2)))^2)^(1/2)/(1+x*(1-3^(1/2)))*(1+x*(1+
3^(1/2)))*EllipticE((1-(1+x*(1-3^(1/2)))^2/(1+x*(1+3^(1/2)))^2)^(1/2),1/4*
6^(1/2)+1/4*2^(1/2))*(a*x^3)^(1/2)*((x^2-x+1)/(1+x*(1+3^(1/2)))^2)^(1/2)/x
/(x^3+1)^(1/2)/(x*(1+x)/(1+x*(1+3^(1/2)))^2)^(1/2)-1/6*(1+x)*((1+x*(1-3^(1
/2)))^2/(1+x*(1+3^(1/2)))^2)^(1/2)/(1+x*(1-3^(1/2)))*(1+x*(1+3^(1/2)))*Ell
ipticF((1-(1+x*(1-3^(1/2)))^2/(1+x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2
^(1/2))*(1-3^(1/2))*(a*x^3)^(1/2)*((x^2-x+1)/(1+x*(1+3^(1/2)))^2)^(1/2)*3^
(3/4)/x/(x^3+1)^(1/2)/(x*(1+x)/(1+x*(1+3^(1/2)))^2)^(1/2)
```

3.386.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.10

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx = \frac{2}{5}x\sqrt{ax^3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-x^3\right)$$

input
```
Integrate[Sqrt[a*x^3]/Sqrt[1 + x^3],x]
```

output
```
(2*x*Sqrt[a*x^3]*Hypergeometric2F1[1/2, 5/6, 11/6, -x^3])/5
```

3.386.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {34, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^3}}{\sqrt{x^3+1}}\,dx$$

$$\Big\downarrow \; 34$$

$$\frac{\sqrt{ax^3}\int \frac{x^{3/2}}{\sqrt{x^3+1}}dx}{x^{3/2}}$$

$$\Big\downarrow \; 851$$

$$\frac{2\sqrt{ax^3}\int \frac{x^2}{\sqrt{x^3+1}}d\sqrt{x}}{x^{3/2}}$$

$$\Big\downarrow \; 837$$

$$\frac{2\sqrt{ax^3}\left(-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}d\sqrt{x}-\frac{1}{2}\int -\frac{2x^2-\sqrt{3}+1}{\sqrt{x^3+1}}d\sqrt{x}\right)}{x^{3/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{2\sqrt{ax^3}\left(\frac{1}{2}\int \frac{2x^2-\sqrt{3}+1}{\sqrt{x^3+1}}d\sqrt{x}-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}d\sqrt{x}\right)}{x^{3/2}}$$

$$\Big\downarrow \; 766$$

$$\frac{2\sqrt{ax^3}\left(\frac{1}{2}\int \frac{2x^2-\sqrt{3}+1}{\sqrt{x^3+1}}d\sqrt{x}-\frac{\left(1-\sqrt{3}\right)\sqrt{x}(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}\right)}{x^{3/2}}$$

$$\Big\downarrow \; 2420$$

$$\frac{2\sqrt{ax^3}\left(\frac{1}{2}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{x}\sqrt{x^3+1}}{\left(1+\sqrt{3}\right)x+1}-\frac{\sqrt[4]{3}\sqrt{x}(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}\right)-\frac{\left(1-\sqrt{3}\right)\sqrt{x}(x+1)\sqrt{\frac{x^2-\cdots}{\left(\left(1+\sqrt{3}\cdots\right.\right.}}}{4\sqrt[4]{\cdots}}\right)}{x^{3/2}}$$

input
```
Int[Sqrt[a*x^3]/Sqrt[1 + x^3],x]
```

output
```
(2*Sqrt[a*x^3]*((((1 + Sqrt[3])*Sqrt[x]*Sqrt[1 + x^3])/(1 + (1 + Sqrt[3])*
x) - (3^(1/4)*Sqrt[x]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*
EllipticE[ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3
])/4])/(Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3]))/2 - ((1
- Sqrt[3])*Sqrt[x]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*Ell
ipticF[ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/
4])/(4*3^(1/4)*Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3])))/
x^(3/2)
```

3.386.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

3.386. $\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx$

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

3.386.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 2.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.08

method	result	size
meijerg	$\frac{2\sqrt{a\,x^{3}}\,x\,{}_{2}F_{1}\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-x^{3}\right)}{5}$	22
default	Expression too large to display	1521

input
```
int((a*x^3)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(a*x^3)^(1/2)*x*hypergeom([1/2,5/6],[11/6],-x^3)
```

3.386.5 Fricas [F]

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx=\int \frac{\sqrt{ax^3}}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((a*x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*x^3)/sqrt(x^3 + 1), x)
```

3.386.6 Sympy [F]

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx=\int \frac{\sqrt{ax^3}}{\sqrt{(x+1)\,(x^2-x+1)}}\,dx$$

input
```
integrate((a*x**3)**(1/2)/(x**3+1)**(1/2),x)
```

output
```
Integral(sqrt(a*x**3)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

3.386.7 Maxima [F]

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx=\int \frac{\sqrt{ax^3}}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((a*x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^3)/sqrt(x^3 + 1), x)
```

3.386.8 Giac [F]

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{ax^3}}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((a*x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x^3)/sqrt(x^3 + 1), x)
```

3.386.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{a\,x^3}}{\sqrt{x^3+1}}\,dx$$

input
```
int((a*x^3)^(1/2)/(x^3 + 1)^(1/2),x)
```

output
```
int((a*x^3)^(1/2)/(x^3 + 1)^(1/2), x)
```

3.387 $\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}}\,dx$

3.387.1 Optimal result

Integrand size = 19, antiderivative size = 260

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{ax^2}\sqrt{1+x^3}}{x\left(1+\sqrt{3}+x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{ax^2}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{x\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{2\sqrt{2}\sqrt{ax^2}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}x\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
2*(a*x^2)^(1/2)*(x^3+1)^(1/2)/x/(1+x+3^(1/2))+2/3*(1+x)*EllipticF((1+x-3^(
1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*(a*x^2)^(1/2)*((x^2-x+1)/(1+x+3
^(1/2))^2)^(1/2)*3^(3/4)/x/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-3^(
1/4)*(1+x)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(a*x^2)^(1
/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/x/(x^3+1)^
(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.387.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.11

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}}\,dx = \frac{1}{2}x\sqrt{ax^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-x^3\right)$$

input
```
Integrate[Sqrt[a*x^2]/Sqrt[1 + x^3],x]
```

output
```
(x*Sqrt[a*x^2]*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3])/2
```

3.387.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {34, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2}}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{ 34}$$

$$\frac{\sqrt{ax^2}\int \frac{x}{\sqrt{x^3+1}}dx}{x}$$

$$\Big\downarrow \text{ 832}$$

$$\frac{\sqrt{ax^2}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx\right)}{x}$$

$$\Big\downarrow \text{ 759}$$

$$\frac{\sqrt{ax^2}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right)}{x}$$

$$\Big\downarrow \text{ 2416}$$

$$\frac{\sqrt{ax^2}\left(-\frac{2\left(1-\sqrt{3}\right)\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,E\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\right.}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}}{x}$$

input
```
Int[Sqrt[a*x^2]/Sqrt[1 + x^3],x]
```

output
```
(Sqrt[a*x^2]*((2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x) - (3^(1/4)*Sqrt[2 - Sqrt
[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 -
 Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt
[3] + x)^2]*Sqrt[1 + x^3]) - (2*(1 - Sqrt[3])*Sqrt[2 + Sqrt[3]]*(1 + x)*Sq
rt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(
1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x
)^2]*Sqrt[1 + x^3])))/x
```

3.387.3.1 Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.387.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.08

method	result
meijerg	$\frac{\sqrt{a\,x^{2}}\,x\,{}_{2}F_{1}\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^{3}\right)}{2}$
default	$\frac{\sqrt{a\,x^{2}}\,\left(i\sqrt{3}-3\right)\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{3+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\,\left(iE\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{3+i\sqrt{3}}}\right)\sqrt{3}-iF\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}},\sqrt{-\frac{i\sqrt{3}-3}{3+i\sqrt{3}}}\right)\sqrt{3}+3E\left(\sqrt{-\right.\right.}{2x\sqrt{x^{3}+1}}$

input
```
int((a*x^2)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(a*x^2)^(1/2)*x*hypergeom([1/2,2/3],[5/3],-x^3)
```

3.387.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.07

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}}\,dx = -\frac{2\sqrt{ax^2}\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x))}{x}$$

input
```
integrate((a*x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(a*x^2)*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x))/x
```

3.387.6 Sympy [F]

$$\int \frac{\sqrt{a x^2}}{\sqrt{1+x^3}}\, dx = \int \frac{\sqrt{a x^2}}{\sqrt{(x+1)\left(x^2-x+1\right)}}\, dx$$

input
```
integrate((a*x**2)**(1/2)/(x**3+1)**(1/2),x)
```

output
```
Integral(sqrt(a*x**2)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

3.387.7 Maxima [F]

$$\int \frac{\sqrt{a x^2}}{\sqrt{1+x^3}}\, dx = \int \frac{\sqrt{a x^2}}{\sqrt{x^3+1}}\, dx$$

input
```
integrate((a*x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^2)/sqrt(x^3 + 1), x)
```

3.387.8 Giac [F]

$$\int \frac{\sqrt{a x^2}}{\sqrt{1+x^3}}\, dx = \int \frac{\sqrt{a x^2}}{\sqrt{x^3+1}}\, dx$$

input
```
integrate((a*x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x^2)/sqrt(x^3 + 1), x)
```

3.387.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{a\,x^2}}{\sqrt{x^3+1}}\,dx$$

input
```
int((a*x^2)^(1/2)/(x^3 + 1)^(1/2),x)
```

output
```
int((a*x^2)^(1/2)/(x^3 + 1)^(1/2), x)
```

3.388 $\int \frac{\sqrt{ax}}{\sqrt{1+x^3}} dx$

3.388.1 Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^3}} dx = \frac{2}{3}\sqrt{a}\operatorname{arcsinh}\left(\frac{(ax)^{3/2}}{a^{3/2}}\right)$$

output
```
2/3*arcsinh((a*x)^(3/2)/a^(3/2))*a^(1/2)
```

3.388.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^3}} dx = \frac{2\sqrt{ax}\log\left(x^{3/2}+\sqrt{1+x^3}\right)}{3\sqrt{x}}$$

input
```
Integrate[Sqrt[a*x]/Sqrt[1 + x^3],x]
```

output
```
(2*Sqrt[a*x]*Log[x^(3/2) + Sqrt[1 + x^3]])/(3*Sqrt[x])
```

3.388.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {851, 807, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax}}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{ 851}$$

$$\frac{2\int \frac{ax}{\sqrt{x^3+1}}d\sqrt{ax}}{a}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{2\int \frac{1}{\sqrt{\frac{x}{a^2}+1}}d(ax)^{3/2}}{3a}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{2}{3}\sqrt{a}\text{arcsinh}\left(\frac{(ax)^{3/2}}{a^{3/2}}\right)$$

input
```
Int[Sqrt[a*x]/Sqrt[1 + x^3],x]
```

output
```
(2*Sqrt[a]*ArcSinh[(a*x)^(3/2)/a^(3/2)])/3
```

3.388.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

3.388.4 Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

method	result
meijerg	$\frac{2\sqrt{ax}\,\operatorname{arcsinh}\left(x^{\frac{3}{2}}\right)}{3\sqrt{x}}$
default	$\frac{2\sqrt{ax}\,\sqrt{x^3+1}\,\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{x\left(x^3+1\right)a}}{x^2\sqrt{a}}\right)}{3\sqrt{x(x^3+1)a}}$
elliptic	$-\frac{2\sqrt{ax}\,\sqrt{x(x^3+1)a}\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(x+1)}}\,(x+1)^2\sqrt{-\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(x+1)}}\,\sqrt{-\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(x+1)}}\left(-F\left(\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(x+1)}},\sqrt{-\right.\right.\right.}{\sqrt{x^3+1}\,x\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{ax(x+1)\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x-\right.}}$

input
```
int((a*x)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(a*x)^(1/2)/x^(1/2)*arcsinh(x^(3/2))
```

3.388.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(15) = 30.

Time = 0.32 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.70

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx = \left[\frac{1}{6}\sqrt{a}\log\left(-8\,ax^6-8\,ax^3-4\left(2\,x^4+x\right)\sqrt{x^3+1}\sqrt{ax}\sqrt{a}-a\right),\ -\frac{1}{3}\sqrt{-a}\arctan\left(\frac{2\sqrt{x^3+1}\sqrt{ax}\sqrt{-ax}}{2\,ax^3+a}\right)\right]$$

input
```
integrate((a*x)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
[1/6*sqrt(a)*log(-8*a*x^6 - 8*a*x^3 - 4*(2*x^4 + x)*sqrt(x^3 + 1)*sqrt(a*x
)*sqrt(a) - a), -1/3*sqrt(-a)*arctan(2*sqrt(x^3 + 1)*sqrt(a*x)*sqrt(-a)*x/
(2*a*x^3 + a))]
```

3.388.6 Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx = \frac{2\sqrt{a}\,\mathrm{asinh}\left(x^{\frac{3}{2}}\right)}{3}$$

input
```
integrate((a*x)**(1/2)/(x**3+1)**(1/2),x)
```

output
```
2*sqrt(a)*asinh(x**(3/2))/3
```

3.388.7 Maxima [F]

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{ax}}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((a*x)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x)/sqrt(x^3 + 1), x)
```

3.388.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(15) = 30.

Time = 0.32 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx = -\frac{2\,a^{\frac{5}{2}}\log\left(-\sqrt{ax}a^{\frac{3}{2}}x+\sqrt{a^4x^3+a^4}\right)}{3\,|a|^2}$$

input
```
integrate((a*x)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
-2/3*a^(5/2)*log(-sqrt(a*x)*a^(3/2)*x + sqrt(a^4*x^3 + a^4))/abs(a)^2
```

3.388.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{a\,x}}{\sqrt{x^3+1}}\,dx$$

input
```
int((a*x)^(1/2)/(x^3 + 1)^(1/2),x)
```

output
```
int((a*x)^(1/2)/(x^3 + 1)^(1/2), x)
```

3.389 $\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}} dx$

3.389.1 Optimal result

Integrand size = 19, antiderivative size = 116

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}} dx = \frac{\sqrt{\frac{a}{x}}x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right),\frac{1}{4}(2+\sqrt{3})\right)}{\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{1+x^3}}$$

output
```
1/3*x*(1+x)*((1+x*(1-3^(1/2)))^2/(1+x*(1+3^(1/2)))^2)^(1/2)/(1+x*(1-3^(1/2
)))*(1+x*(1+3^(1/2)))*EllipticF((1-(1+x*(1-3^(1/2)))^2/(1+x*(1+3^(1/2)))^2
)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a/x)^(1/2)*((x^2-x+1)/(1+x*(1+3^(1/2)))^
2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/(x*(1+x)/(1+x*(1+3^(1/2)))^2)^(1/2)
```

3.389.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.23

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}} dx = 2\sqrt{\frac{a}{x}}x \operatorname{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-x^3\right)$$

input
```
Integrate[Sqrt[a/x]/Sqrt[1 + x^3],x]
```

output
```
2*Sqrt[a/x]*x*Hypergeometric2F1[1/6, 1/2, 7/6, -x^3]
```

3.389.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {34, 851, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{34}$$

$$\sqrt{x}\sqrt{\frac{a}{x}}\int \frac{1}{\sqrt{x}\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{851}$$

$$2\sqrt{x}\sqrt{\frac{a}{x}}\int \frac{1}{\sqrt{x^3+1}}d\sqrt{x}$$

$$\Big\downarrow \text{766}$$

$$\frac{x(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{\frac{a}{x}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}$$

input
```
Int[Sqrt[a/x]/Sqrt[1 + x^3],x]
```

output
```
(Sqrt[a/x]*x*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticF
[ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(3
^(1/4)*Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3])
```

3.389.3.1 Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

3.389.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.19

method	result
meijerg	$2\sqrt{\frac{a}{x}}\, x\, {}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-x^{3}\right)$
default	$\frac{4\sqrt{\frac{a}{x}}\, x\sqrt{x^{3}+1}\,\left(1+i\sqrt{3}\right)\sqrt{\frac{\left(3+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)\left(x+1\right)}}\,\left(x+1\right)^{2}\sqrt{\frac{i\sqrt{3}+2x-1}{\left(-1+i\sqrt{3}\right)\left(x+1\right)}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{\left(1+i\sqrt{3}\right)\left(x+1\right)}}\, F\left(\sqrt{\frac{\left(3+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)\left(x+1\right)}},\sqrt{\frac{\left(i\sqrt{3}-3\right)\left(1+i\sqrt{3}\right)}{\left(-1+i\sqrt{3}\right)\left(3+i\sqrt{3}\right)}}\right)}{\sqrt{\left(x^{3}+1\right)x}\,\left(3+i\sqrt{3}\right)\sqrt{-x\left(x+1\right)\left(i\sqrt{3}+2x-1\right)\left(i\sqrt{3}-2x+1\right)}}$

input
```
int((a/x)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a/x)^(1/2)*x*hypergeom([1/6,1/2],[7/6],-x^3)
```

3.389.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.09

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx = -2\,\sqrt{a}\text{weierstrassPInverse}\left(0,-4,\frac{1}{x}\right)$$

input
```
integrate((a/x)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(a)*weierstrassPInverse(0, -4, 1/x)
```

3.389.6 Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x}}}{\sqrt{(x+1)\,(x^2-x+1)}}\,dx$$

input
```
integrate((a/x)**(1/2)/(x**3+1)**(1/2),x)
```

output
```
Integral(sqrt(a/x)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

3.389.7 Maxima [F]

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((a/x)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a/x)/sqrt(x^3 + 1), x)
```

3.389.8 Giac [F]

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((a/x)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a/x)/sqrt(x^3 + 1), x)
```

3.389.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^3+1}}\,dx$$

input
```
int((a/x)^(1/2)/(x^3 + 1)^(1/2),x)
```

output
```
int((a/x)^(1/2)/(x^3 + 1)^(1/2), x)
```

3.390 $\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\,dx$

3.390.1 Optimal result

Integrand size = 19, antiderivative size = 24

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\,dx = -\frac{2}{3}\sqrt{\frac{a}{x^2}}x\operatorname{arctanh}\left(\sqrt{1+x^3}\right)$$

output
```
-2/3*x*arctanh((x^3+1)^(1/2))*(a/x^2)^(1/2)
```

3.390.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\,dx = -\frac{2}{3}\sqrt{\frac{a}{x^2}}x\operatorname{arctanh}\left(\sqrt{1+x^3}\right)$$

input
```
Integrate[Sqrt[a/x^2]/Sqrt[1 + x^3],x]
```

output
```
(-2*Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^3]])/3
```

3.390.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {34, 798, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{34}$$

$$x\sqrt{\frac{a}{x^2}}\int \frac{1}{x\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}x\sqrt{\frac{a}{x^2}}\int \frac{1}{x^3\sqrt{x^3+1}}dx^3$$

$$\Big\downarrow \text{73}$$

$$\frac{2}{3}x\sqrt{\frac{a}{x^2}}\int \frac{1}{x^6-1}d\sqrt{x^3+1}$$

$$\Big\downarrow \text{220}$$

$$-\frac{2}{3}x\sqrt{\frac{a}{x^2}}\text{arctanh}\left(\sqrt{x^3+1}\right)$$

input
```
Int[Sqrt[a/x^2]/Sqrt[1 + x^3],x]
```

output
```
(-2*Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^3]])/3
```

3.390.3.1 Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.390.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{2x\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)\sqrt{\frac{a}{x^2}}}{3}$	19
meijerg	$\frac{\sqrt{\frac{a}{x^2}}\,x\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x))\sqrt{\pi}\right)}{3\sqrt{\pi}}$	45

input
```
int((a/x^2)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*x*arctanh((x^3+1)^(1/2))*(a/x^2)^(1/2)
```

3.390.5 Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(18) = 36.

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.83

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\,dx = \left[\frac{1}{3}\,x\sqrt{\frac{a}{x^2}}\log\left(\frac{x^3-2\sqrt{x^3+1}+2}{x^3}\right), \frac{2}{3}\,\sqrt{-a}\,\arctan\left(\frac{\sqrt{x^3+1}\sqrt{-a}x\sqrt{\frac{a}{x^2}}}{ax^3+a}\right)\right]$$

input
```
integrate((a/x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
[1/3*x*sqrt(a/x^2)*log((x^3 - 2*sqrt(x^3 + 1) + 2)/x^3), 2/3*sqrt(-a)*arct
an(sqrt(x^3 + 1)*sqrt(-a)*x*sqrt(a/x^2)/(a*x^3 + a))]
```

3.390.6 Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{(x+1)\,(x^2-x+1)}}\,dx$$

input
```
integrate((a/x**2)**(1/2)/(x**3+1)**(1/2),x)
```

output
```
Integral(sqrt(a/x**2)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

3.390.7 Maxima [F]

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((a/x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a/x^2)/sqrt(x^3 + 1), x)
```

3.390.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\,dx = -\frac{1}{3}\sqrt{a}\left(\log\left(\sqrt{x^3+1}+1\right) - \log\left(\left|\sqrt{x^3+1}-1\right|\right)\right)\mathrm{sgn}(x)$$

input
```
integrate((a/x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
-1/3*sqrt(a)*(log(sqrt(x^3 + 1) + 1) - log(abs(sqrt(x^3 + 1) - 1)))*sgn(x)
```

3.390.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^3+1}}\,dx$$

input
```
int((a/x^2)^(1/2)/(x^3 + 1)^(1/2),x)
```

output
```
int((a/x^2)^(1/2)/(x^3 + 1)^(1/2), x)
```

3.391 $\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx$

3.391.1 Optimal result

Integrand size = 19, antiderivative size = 312

$$\begin{aligned}
\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx
&= -2\sqrt{\frac{a}{x^3}}x\sqrt{1+x^3}+\frac{2\left(1+\sqrt{3}\right)\sqrt{\frac{a}{x^3}}x^2\sqrt{1+x^3}}{1+\left(1+\sqrt{3}\right)x} \\
&\quad -\frac{2\sqrt[4]{3}\sqrt{\frac{a}{x^3}}x^2(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}E\left(\arccos\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{1+x^3}} \\
&\quad -\frac{\left(1-\sqrt{3}\right)\sqrt{\frac{a}{x^3}}x^2(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{1+x^3}}
\end{aligned}$$

output
```
-2*x*(a/x^3)^(1/2)*(x^3+1)^(1/2)+2*x^2*(1+3^(1/2))*(a/x^3)^(1/2)*(x^3+1)^(
1/2)/(1+x*(1+3^(1/2)))-2*3^(1/4)*x^2*(1+x)*((1+x*(1-3^(1/2)))^2/(1+x*(1+3^
(1/2)))^2)^(1/2)/(1+x*(1-3^(1/2)))*(1+x*(1+3^(1/2)))*EllipticE((1-(1+x*(1-
3^(1/2)))^2/(1+x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a/x^3)^(1
/2)*((x^2-x+1)/(1+x*(1+3^(1/2)))^2)^(1/2)/(x^3+1)^(1/2)/(x*(1+x)/(1+x*(1+3
^(1/2)))^2)^(1/2)-1/3*x^2*(1+x)*((1+x*(1-3^(1/2)))^2/(1+x*(1+3^(1/2)))^2)^
(1/2)/(1+x*(1-3^(1/2)))*(1+x*(1+3^(1/2)))*EllipticF((1-(1+x*(1-3^(1/2)))^2
/(1+x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(a/x^3)^(
1/2)*((x^2-x+1)/(1+x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/(x*(1+x)/
(1+x*(1+3^(1/2)))^2)^(1/2)
```

3.391.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.09

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx = -2\sqrt{\frac{a}{x^3}}x\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},-x^3\right)$$

input
```
Integrate[Sqrt[a/x^3]/Sqrt[1 + x^3],x]
```

output
```
-2*Sqrt[a/x^3]*x*Hypergeometric2F1[-1/6, 1/2, 5/6, -x^3]
```

3.391.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {34, 847, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{ 34}$$

$$x^{3/2}\sqrt{\frac{a}{x^3}}\int \frac{1}{x^{3/2}\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{847}$$

$$x^{3/2}\sqrt{\frac{a}{x^3}}\left(2\int \frac{x^{3/2}}{\sqrt{x^3+1}}dx-\frac{2\sqrt{x^3+1}}{\sqrt{x}}\right)$$

$$\Big\downarrow \text{851}$$

$$x^{3/2}\sqrt{\frac{a}{x^3}}\left(4\int \frac{x^2}{\sqrt{x^3+1}}d\sqrt{x}-\frac{2\sqrt{x^3+1}}{\sqrt{x}}\right)$$

$$\Big\downarrow \text{837}$$

$$x^{3/2}\sqrt{\frac{a}{x^3}}\left(4\left(-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}d\sqrt{x}-\frac{1}{2}\int -\frac{2x^2-\sqrt{3}+1}{\sqrt{x^3+1}}d\sqrt{x}\right)-\frac{2\sqrt{x^3+1}}{\sqrt{x}}\right)$$

$$\Big\downarrow \text{25}$$

$$x^{3/2}\sqrt{\frac{a}{x^3}}\left(4\left(\frac{1}{2}\int \frac{2x^2-\sqrt{3}+1}{\sqrt{x^3+1}}d\sqrt{x}-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}d\sqrt{x}\right)-\frac{2\sqrt{x^3+1}}{\sqrt{x}}\right)$$

$$\Big\downarrow \text{766}$$

$$x^{3/2}\sqrt{\frac{a}{x^3}}\left(4\left(\frac{1}{2}\int \frac{2x^2-\sqrt{3}+1}{\sqrt{x^3+1}}d\sqrt{x}-\frac{\left(1-\sqrt{3}\right)\sqrt{x}(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right),\frac{1}{4}\right.}{4\sqrt[4]{3}\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}\right.\right.$$

$$\Big\downarrow \text{2420}$$

$$x^{3/2}\sqrt{\frac{a}{x^3}}\left(4\left(\frac{1}{2}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{x}\sqrt{x^3+1}}{\left(1+\sqrt{3}\right)x+1}-\frac{\sqrt[4]{3}\sqrt{x}(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}\right.\right.\right.$$

input
```
Int[Sqrt[a/x^3]/Sqrt[1 + x^3],x]
```

3.391. $\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx$

output
```
Sqrt[a/x^3]*x^(3/2)*((-2*Sqrt[1 + x^3])/Sqrt[x] + 4*((((1 + Sqrt[3])*Sqrt[
x]*Sqrt[1 + x^3])/(1 + (1 + Sqrt[3])*x) - (3^(1/4)*Sqrt[x]*(1 + x)*Sqrt[(1
 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticE[ArcCos[(1 + (1 - Sqrt[3])*x
)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(Sqrt[(x*(1 + x))/(1 + (1 + Sq
rt[3])*x)^2]*Sqrt[1 + x^3]))/2 - ((1 - Sqrt[3])*Sqrt[x]*(1 + x)*Sqrt[(1 -
x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticF[ArcCos[(1 + (1 - Sqrt[3])*x)/(
1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*Sqrt[(x*(1 + x))/(1 +
(1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3])))
```

3.391.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

3.391. $\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx$

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

3.391.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 2.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.07

method	result
meijerg	$-2\sqrt{\frac{a}{x^3}}\,x\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};-x^3\right)$
risch	$-2x\sqrt{\frac{a}{x^3}}\,\sqrt{x^3+1}+\frac{2\left(x\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(x+1)}}\,(x+1)^2\sqrt{-\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(x+1)}}\,\sqrt{-\frac{x-\cdots}{\left(\frac{1}{2}+i\cdots\right)}}\cdots\right.}{\cdots}$
default	Expression too large to display

input
```
int((a/x^3)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(a/x^3)^(1/2)*x*hypergeom([-1/6,1/2],[5/6],-x^3)
```

3.391.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.04

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx = 2\sqrt{a}\text{weierstrassZeta}\left(0,-4,\text{weierstrassPInverse}\left(0,-4,\frac{1}{x}\right)\right)$$

input
```
integrate((a/x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(a)*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, 1/x))
```

3.391.6 Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{(x+1)\,(x^2-x+1)}}\,dx$$

input
```
integrate((a/x**3)**(1/2)/(x**3+1)**(1/2),x)
```

output
```
Integral(sqrt(a/x**3)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

3.391.7 Maxima [F]

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((a/x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a/x^3)/sqrt(x^3 + 1), x)
```

3.391.8 Giac [F]

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((a/x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a/x^3)/sqrt(x^3 + 1), x)
```

3.391.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^3+1}}\,dx$$

input
```
int((a/x^3)^(1/2)/(x^3 + 1)^(1/2),x)
```

output
```
int((a/x^3)^(1/2)/(x^3 + 1)^(1/2), x)
```

3.392 $\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx$

3.392.1 Optimal result

Integrand size = 19, antiderivative size = 281

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx = -\sqrt{\frac{a}{x^4}}x\sqrt{1+x^3}+\frac{\sqrt{\frac{a}{x^4}}x^2\sqrt{1+x^3}}{1+\sqrt{3}+x} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{\frac{a}{x^4}}x^2(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}E\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{\sqrt{2}\sqrt{\frac{a}{x^4}}x^2(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

output
```
-x*(a/x^4)^(1/2)*(x^3+1)^(1/2)+x^2*(a/x^4)^(1/2)*(x^3+1)^(1/2)/(1+x+3^(1/2
))+1/3*x^2*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1
/2)*(a/x^4)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/
((1+x)/(1+x+3^(1/2))^2)^(1/2)-1/2*3^(1/4)*x^2*(1+x)*EllipticE((1+x-3^(1/2)
)/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(a/x^4)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x
^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

3.392.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.10

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx = -\sqrt{\frac{a}{x^4}}\,x\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},-x^3\right)$$

input
```
Integrate[Sqrt[a/x^4]/Sqrt[1 + x^3],x]
```

output
```
-(Sqrt[a/x^4]*x*Hypergeometric2F1[-1/3, 1/2, 2/3, -x^3])
```

3.392.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {34, 847, 832, 759, 2416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{34}$$

$$x^2\sqrt{\frac{a}{x^4}}\int \frac{1}{x^2\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{847}$$

$$x^2\sqrt{\frac{a}{x^4}}\left(\frac{1}{2}\int \frac{x}{\sqrt{x^3+1}}dx-\frac{\sqrt{x^3+1}}{x}\right)$$

$$\Big\downarrow \text{832}$$

$$x^2\sqrt{\frac{a}{x^4}}\left(\frac{1}{2}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}dx\right)-\frac{\sqrt{x^3+1}}{x}\right)$$

$$\Big\downarrow \text{759}$$

$$x^2\sqrt{\frac{a}{x^4}}\left(\frac{1}{2}\left(\int \frac{x-\sqrt{3}+1}{\sqrt{x^3+1}}dx-\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}\right.\right.$$

$$\Big\downarrow \text{2416}$$

$$x^2\sqrt{\frac{a}{x^4}}\left(\frac{1}{2}\left(-\frac{2(1-\sqrt{3})\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}}{}\right.\right.$$

input
```
Int[Sqrt[a/x^4]/Sqrt[1 + x^3],x]
```

output
```
Sqrt[a/x^4]*x^2*(-(Sqrt[1 + x^3]/x) + ((2*Sqrt[1 + x^3])/(1 + Sqrt[3] + x)
 - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)
^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]]
)/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (2*(1 - Sqrt[3])*Sqr
t[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[A
rcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt
[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]))/2)
```

3.392.3.1 Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 759
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

rule 832
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 - Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && PosQ[a]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 2416
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

3.392.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 1.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.08

method	result
meijerg	$-\sqrt{\frac{a}{x^4}}\, x\, {}_2F_1\left(-\frac{1}{3}, \frac{1}{2}; \frac{2}{3}; -x^3\right)$
risch	$-x\sqrt{\frac{a}{x^4}}\,\sqrt{x^3+1} - \frac{i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\left(\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)E\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}}{3}, \sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}} \cdots\right.\right.}{3\sqrt{a\,x^3+a}\,\sqrt{x^3+1}\cdots}$
default	$\frac{\sqrt{\frac{a}{x^4}}\, x\left(i\sqrt{3}\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\, F\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}, \sqrt{-\frac{i\sqrt{3}-3}{3+i\sqrt{3}}}\right)x - 6\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}}\sqrt{\frac{i\sqrt{3}-2x+1}{3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}+2x-1}{i\sqrt{3}-3}}\, E\left(\sqrt{-\frac{2(x+1)}{i\sqrt{3}-3}} \cdots\right.\right.}{2\sqrt{x^3+1}}$

input
```
int((a/x^4)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(a/x^4)^(1/2)*x*hypergeom([-1/3,1/2],[2/3],-x^3)
```

3.392.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.13

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx = -x^2\sqrt{\frac{a}{x^4}}\text{weierstrassZeta}(0,-4,\text{weierstrassPInverse}(0,-4,x)) - \sqrt{x^3+1}x\sqrt{\frac{a}{x^4}}$$

input
```
integrate((a/x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
-x^2*sqrt(a/x^4)*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x)) - s
qrt(x^3 + 1)*x*sqrt(a/x^4)
```

3.392.6 Sympy [F]

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{(x+1)(x^2-x+1)}}\,dx$$

input
```
integrate((a/x**4)**(1/2)/(x**3+1)**(1/2),x)
```

output
```
Integral(sqrt(a/x**4)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

3.392.7 Maxima [F]

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((a/x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a/x^4)/sqrt(x^3 + 1), x)
```

3.392.8 Giac [F]

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^3+1}}\,dx$$

input
```
integrate((a/x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a/x^4)/sqrt(x^3 + 1), x)
```

3.392.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx = \int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^3+1}}\,dx$$

input
```
int((a/x^4)^(1/2)/(x^3 + 1)^(1/2),x)
```

output
```
int((a/x^4)^(1/2)/(x^3 + 1)^(1/2), x)
```

3.393 $\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx$

3.393.1 Optimal result

Integrand size = 21, antiderivative size = 37

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx=\frac{x\sqrt{ax^{2n}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+\frac{1}{n},2+\frac{1}{n},-x^n\right)}{1+n}$$

output
```
x*hypergeom([1/2, 1+1/n],[2+1/n],-x^n)*(a*x^(2*n))^(1/2)/(1+n)
```

3.393.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx=\frac{x\sqrt{ax^{2n}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+\frac{1}{n},2+\frac{1}{n},-x^n\right)}{1+n}$$

input
```
Integrate[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n],x]
```

output
```
(x*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, 1 + n^(-1), 2 + n^(-1), -x^n])/(
1 + n)
```

3.393.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {34, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{x^n+1}}dx$$
$$\Big\downarrow \text{34}$$
$$x^{-n}\sqrt{ax^{2n}}\int \frac{x^n}{\sqrt{x^n+1}}dx$$
$$\Big\downarrow \text{888}$$
$$\frac{x\sqrt{ax^{2n}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+\frac{1}{n},2+\frac{1}{n},-x^n\right)}{n+1}$$

input
```
Int[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n],x]
```

output
```
(x*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, 1 + n^(-1), 2 + n^(-1), -x^n])/(
1 + n)
```

3.393.3.1 Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

3.393.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

method	result	size
meijerg	$\frac{x{}_{2}F_{1}\left(\frac{1}{2},1+\frac{1}{n};2+\frac{1}{n};-x^{n}\right)\sqrt{x^{2n}a}}{1+n}$	36

input
```
int((x^(2*n)*a)^(1/2)/(1+x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*hypergeom([1/2,1+1/n],[2+1/n],-x^n)*(x^(2*n)*a)^(1/2)/(1+n)
```

3.393.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.393.6 Sympy [F]

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\,dx$$

input
```
integrate((a*x**(2*n))**(1/2)/(1+x**n)**(1/2),x)
```

output
```
Integral(sqrt(a*x**(2*n))/sqrt(x**n + 1), x)
```

3.393.7 Maxima [F]

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{ax^{2\,n}}}{\sqrt{x^n+1}}\,dx$$

input
```
integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1), x)
```

3.393.8 Giac [F]

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{ax^{2\,n}}}{\sqrt{x^n+1}}\,dx$$

input
```
integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1), x)
```

3.393.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{a\,x^{2\,n}}}{\sqrt{x^n+1}}\,dx$$

input
```
int((a*x^(2*n))^(1/2)/(x^n + 1)^(1/2),x)
```

output
```
int((a*x^(2*n))^(1/2)/(x^n + 1)^(1/2), x)
```

3.394 $\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}}\,dx$

3.394.1 Optimal result

Integrand size = 19, antiderivative size = 48

$$\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}}\,dx=\frac{2x\sqrt{ax^n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right),\frac{1}{2}\left(3+\frac{2}{n}\right),-x^n\right)}{2+n}$$

output
```
2*x*hypergeom([1/2, 1/2+1/n],[3/2+1/n],-x^n)*(a*x^n)^(1/2)/(2+n)
```

3.394.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}}\,dx=\frac{2x\sqrt{ax^n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}+\frac{1}{n},\frac{3}{2}+\frac{1}{n},-x^n\right)}{2+n}$$

input
```
Integrate[Sqrt[a*x^n]/Sqrt[1 + x^n],x]
```

output
```
(2*x*Sqrt[a*x^n]*Hypergeometric2F1[1/2, 1/2 + n^(-1), 3/2 + n^(-1), -x^n])
/(2 + n)
```

3.394.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.29, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {34, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^n}}{\sqrt{x^n+1}}dx$$

$$\Big\downarrow \text{ 34}$$

$$x^{-n/2}\sqrt{ax^n}\int \frac{x^{n/2}}{\sqrt{x^n+1}}dx$$

$$\Big\downarrow \text{ 888}$$

$$\frac{2x^{\frac{n+2}{2}-\frac{n}{2}}\sqrt{ax^n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right),\frac{1}{2}\left(3+\frac{2}{n}\right),-x^n\right)}{n+2}$$

input
```
Int[Sqrt[a*x^n]/Sqrt[1 + x^n],x]
```

output
```
(2*x^(-1/2*n + (2 + n)/2)*Sqrt[a*x^n]*Hypergeometric2F1[1/2, (1 + 2/n)/2,
(3 + 2/n)/2, -x^n])/(2 + n)
```

3.394.3.1 Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

3.394.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.73

method	result	size
meijerg	$\frac{2x{}_2F_1\left(\frac{1}{2},\frac{1}{2}+\frac{1}{n};\frac{3}{2}+\frac{1}{n};-x^n\right)\sqrt{a\,x^n}}{2+n}$	35

input
```
int((a*x^n)^(1/2)/(1+x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x*hypergeom([1/2,1/2+1/n],[3/2+1/n],-x^n)*(a*x^n)^(1/2)/(2+n)
```

3.394.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x^n)^(1/2)/(1+x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.394.6 Sympy [F]

$$\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{ax^n}}{\sqrt{x^n+1}}\,dx$$

input
```
integrate((a*x**n)**(1/2)/(1+x**n)**(1/2),x)
```

output
```
Integral(sqrt(a*x**n)/sqrt(x**n + 1), x)
```

3.394.7 Maxima [F]

$$\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{ax^n}}{\sqrt{x^n+1}}\,dx$$

input
```
integrate((a*x^n)^(1/2)/(1+x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^n)/sqrt(x^n + 1), x)
```

3.394.8 Giac [F]

$$\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{ax^n}}{\sqrt{x^n+1}}\,dx$$

input
```
integrate((a*x^n)^(1/2)/(1+x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x^n)/sqrt(x^n + 1), x)
```

3.394.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{a\,x^n}}{\sqrt{x^n+1}}\,dx$$

input
```
int((a*x^n)^(1/2)/(x^n + 1)^(1/2),x)
```

output
```
int((a*x^n)^(1/2)/(x^n + 1)^(1/2), x)
```

3.395 $\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx$

3.395.1 Optimal result

Integrand size = 23, antiderivative size = 52

$$\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx = \frac{4x\sqrt{ax^{n/2}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}\left(1+\frac{4}{n}\right),\frac{1}{4}\left(5+\frac{4}{n}\right),-x^n\right)}{4+n}$$

output
```
4*x*hypergeom([1/2, 1/4+1/n],[5/4+1/n],-x^n)*(a*x^(1/2*n))^(1/2)/(4+n)
```

3.395.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx = \frac{4x\sqrt{ax^{n/2}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}+\frac{1}{n},\frac{5}{4}+\frac{1}{n},-x^n\right)}{4+n}$$

input
```
Integrate[Sqrt[a*x^(n/2)]/Sqrt[1 + x^n],x]
```

output
```
(4*x*Sqrt[a*x^(n/2)]*Hypergeometric2F1[1/2, 1/4 + n^(-1), 5/4 + n^(-1), -x
^n])/(4 + n)
```

3.395.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {34, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^{n/2}}}{\sqrt{x^n+1}}\,dx$$

$$\Big\downarrow \text{ 34}$$

$$x^{-n/4}\sqrt{ax^{n/2}}\int \frac{x^{n/4}}{\sqrt{x^n+1}}dx$$

$$\Big\downarrow \text{ 888}$$

$$\frac{4x^{\frac{n+4}{4}-\frac{n}{4}}\sqrt{ax^{n/2}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}\left(1+\frac{4}{n}\right),\frac{1}{4}\left(5+\frac{4}{n}\right),-x^n\right)}{n+4}$$

input
```
Int[Sqrt[a*x^(n/2)]/Sqrt[1 + x^n],x]
```

output
```
(4*x^(-1/4*n + (4 + n)/4)*Sqrt[a*x^(n/2)]*Hypergeometric2F1[1/2, (1 + 4/n)
/4, (5 + 4/n)/4, -x^n])/(4 + n)
```

3.395.3.1 Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

3.395.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.71

method	result	size
meijerg	$\frac{4x\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}+\frac{1}{n};\frac{5}{4}+\frac{1}{n};-x^n\right)\sqrt{a\,x^{\frac{n}{2}}}}{4+n}$	37

input
```
int((a*x^(1/2*n))^(1/2)/(1+x^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4*x*hypergeom([1/2,1/4+1/n],[5/4+1/n],-x^n)*(a*x^(1/2*n))^(1/2)/(4+n)
```

3.395.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x^(1/2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.395.6 Sympy [F]

$$\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{ax^{\frac{n}{2}}}}{\sqrt{x^n+1}}\,dx$$

input
```
integrate((a*x**(1/2*n))**(1/2)/(1+x**n)**(1/2),x)
```

output
```
Integral(sqrt(a*x**(n/2))/sqrt(x**n + 1), x)
```

3.395.7 Maxima [F]

$$\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{ax^{\frac{1}{2}n}}}{\sqrt{x^n+1}}\,dx$$

input
```
integrate((a*x^(1/2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^(1/2*n))/sqrt(x^n + 1), x)
```

3.395.8 Giac [F]

$$\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{ax^{\frac{1}{2}n}}}{\sqrt{x^n+1}}\,dx$$

input
```
integrate((a*x^(1/2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x^(1/2*n))/sqrt(x^n + 1), x)
```

3.395.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx = \int \frac{\sqrt{a\,x^{n/2}}}{\sqrt{x^n+1}}\,dx$$

input
```
int((a*x^(n/2))^(1/2)/(x^n + 1)^(1/2),x)
```

output
```
int((a*x^(n/2))^(1/2)/(x^n + 1)^(1/2), x)
```

3.396 $\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right)\,dx$

3.396.1 Optimal result

Integrand size = 54, antiderivative size = 34

$$\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right)\,dx=\frac{2x^{1-n}\sqrt{ax^{2n}}\sqrt{1+x^n}}{2+n}$$

output
```
2*x^(1-n)*(a*x^(2*n))^(1/2)*(1+x^n)^(1/2)/(2+n)
```

3.396.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right)\,dx=\frac{2ax^{1+n}\sqrt{1+x^n}}{(2+n)\sqrt{ax^{2n}}}$$

input
```
Integrate[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n] + (2*Sqrt[a*x^(2*n)])/((2 + n)*x^n
*Sqrt[1 + x^n]),x]
```

output
```
(2*a*x^(1 + n)*Sqrt[1 + x^n])/((2 + n)*Sqrt[a*x^(2*n)])
```

3.396.3 Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.35, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{2x^{-n}\sqrt{ax^{2n}}}{(n+2)\sqrt{x^n+1}}+\frac{\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2x^{1-n}\sqrt{ax^{2n}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{n},1+\frac{1}{n},-x^n\right)}{n+2}+\frac{x\sqrt{ax^{2n}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+\frac{1}{n},2+\frac{1}{n},-x^n\right)}{n+1}$$

input
```
Int[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n] + (2*Sqrt[a*x^(2*n)])/((2 + n)*x^n*Sqrt[
1 + x^n]),x]
```

output
```
(x*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, 1 + n^(-1), 2 + n^(-1), -x^n])/(
1 + n) + (2*x^(1 - n)*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, n^(-1), 1 + n
^(-1), -x^n])/(2 + n)
```

3.396.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.396.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{2x\sqrt{1+x^{n}}\,\sqrt{x^{2n}a}\,x^{-n}}{2+n}$	30
meijerg	$\frac{x\,{}_2F_1\left(\frac{1}{2},1+\frac{1}{n};2+\frac{1}{n};-x^{n}\right)\sqrt{x^{2n}a}}{1+n}+\frac{2\sqrt{x^{2n}a}\,x^{1-n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{n};1+\frac{1}{n};-x^{n}\right)}{2+n}$	77

input
```
int((x^(2*n)*a)^(1/2)/(1+x^n)^(1/2)+2*(x^(2*n)*a)^(1/2)/(2+n)/(x^n)/(1+x^n
)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x*(1+x^n)^(1/2)/(2+n)*((x^n)^2*a)^(1/2)/(x^n)
```

3.396.5 Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right)dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2)+2*(a*x^(2*n))^(1/2)/(2+n)/(x^n)/
(1+x^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.396.6 Sympy [F]

$$\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right)dx = \frac{\int \frac{2\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\,dx+\int \frac{n\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\,dx+\int \frac{2x^{-n}\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\,dx}{n+2}$$

input
```
integrate((a*x**(2*n))**(1/2)/(1+x**n)**(1/2)+2*(a*x**(2*n))**(1/2)/(2+n)/
(x**n)/(1+x**n)**(1/2),x)
```

output
```
(Integral(2*sqrt(a*x**(2*n))/sqrt(x**n + 1), x) + Integral(n*sqrt(a*x**(2*
n))/sqrt(x**n + 1), x) + Integral(2*sqrt(a*x**(2*n))/(x**n*sqrt(x**n + 1))
, x))/(n + 2)
```

3.396.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.53

$$\int\left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right)dx=\frac{2\sqrt{a}\sqrt{x^n+1}x}{n+2}$$

input
```
integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2)+2*(a*x^(2*n))^(1/2)/(2+n)/(x^n)/
(1+x^n)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(a)*sqrt(x^n + 1)*x/(n + 2)
```

3.396.8 Giac [F]

$$\int\left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right)dx=\int\frac{\sqrt{ax^{2n}}}{\sqrt{x^n+1}}+\frac{2\sqrt{ax^{2n}}}{(n+2)\sqrt{x^n+1}x^n}\,dx$$

input
```
integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2)+2*(a*x^(2*n))^(1/2)/(2+n)/(x^n)/
(1+x^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1) + 2*sqrt(a*x^(2*n))/((n + 2)*sqrt(
x^n + 1)*x^n), x)
```

3.396.9 Mupad [B] (verification not implemented)

Time = 16.70 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int\left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right)dx=\frac{\sqrt{a\,x^{2\,n}}\,\left(\frac{2\,x}{n+2}+\frac{2\,x^{n+1}}{n+2}\right)}{x^n\,\sqrt{x^n+1}}$$

input
```
int((a*x^(2*n))^(1/2)/(x^n + 1)^(1/2) + (2*(a*x^(2*n))^(1/2))/(x^n*(x^n +
1)^(1/2)*(n + 2)),x)
```

output
```
((a*x^(2*n))^(1/2)*((2*x)/(n + 2) + (2*x^(n + 1))/(n + 2)))/(x^n*(x^n + 1)
^(1/2))
```

3.397 $\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx$

3.397.1 Optimal result

Integrand size = 26, antiderivative size = 114

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx = \frac{2\sqrt{-e^2+df}\sqrt{ax}\sqrt{\frac{e(e+fx)}{e^2-df}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{-e^2+df}}\right)|1-\frac{e^2}{df}\right)}{e\sqrt{f}\sqrt{-\frac{ex}{d}}\sqrt{e+fx}}$$

output
```
2*EllipticE(f^(1/2)*(e*x+d)^(1/2)/(d*f-e^2)^(1/2),(1-e^2/d/f)^(1/2))*(d*f-
e^2)^(1/2)*(a*x)^(1/2)*(e*(f*x+e)/(-d*f+e^2))^(1/2)/e/f^(1/2)/(-e*x/d)^(1/
2)/(f*x+e)^(1/2)
```

3.397.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.79 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx = -\frac{2ie\sqrt{ax}\sqrt{1+\frac{fx}{e}}\left(E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{ex}{d}}\right)|\frac{df}{e^2}\right)-\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{ex}{d}}\right),\frac{df}{e^2}\right)\right)}{f\sqrt{\frac{ex}{d+ex}}\sqrt{d+ex}\sqrt{e+fx}}$$

input
```
Integrate[Sqrt[a*x]/(Sqrt[d + e*x]*Sqrt[e + f*x]),x]
```

output
```
((-2*I)*e*Sqrt[a*x]*Sqrt[1 + (f*x)/e]*(EllipticE[I*ArcSinh[Sqrt[(e*x)/d]],
 (d*f)/e^2] - EllipticF[I*ArcSinh[Sqrt[(e*x)/d]], (d*f)/e^2]))/(f*Sqrt[(e*
x)/(d + e*x)]*Sqrt[d + e*x]*Sqrt[e + f*x])
```

3.397.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {124, 123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx$$

$$\Big\downarrow \; 124$$

$$\frac{\sqrt{ax}\sqrt{\frac{e(e+fx)}{e^2-df}}\int \frac{\sqrt{-\frac{ex}{d}}}{\sqrt{d+ex}\sqrt{\frac{e^2}{e^2-df}+\frac{fxe}{e^2-df}}}dx}{\sqrt{-\frac{ex}{d}}\sqrt{e+fx}}$$

$$\Big\downarrow \; 123$$

$$\frac{2\sqrt{ax}\sqrt{df-e^2}\sqrt{\frac{e(e+fx)}{e^2-df}}E\left(\arcsin\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{df-e^2}}\right)|1-\frac{e^2}{df}\right)}{e\sqrt{f}\sqrt{-\frac{ex}{d}}\sqrt{e+fx}}$$

input
```
Int[Sqrt[a*x]/(Sqrt[d + e*x]*Sqrt[e + f*x]),x]
```

output
```
(2*Sqrt[-e^2 + d*f]*Sqrt[a*x]*Sqrt[(e*(e + f*x))/(e^2 - d*f)]*EllipticE[Ar
cSin[(Sqrt[f]*Sqrt[d + e*x])/Sqrt[-e^2 + d*f]], 1 - e^2/(d*f)])/(e*Sqrt[f]
*Sqrt[-((e*x)/d)]*Sqrt[e + f*x])
```

3.397.3.1 Defintions of rubi rules used

rule 123
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a + b*x]
/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; FreeQ[{a,
b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !L
tQ[-(b*c - a*d)/d, 0] && !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c - a*d
), 0] && GtQ[d/(d*e - c*f), 0] && !LtQ[(b*c - a*d)/b, 0])
```

rule 124
```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_] :> Simp[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt[c + d
*x]*Sqrt[b*((e + f*x)/(b*e - a*f))]))   Int[Sqrt[b*(e/(b*e - a*f)) + b*f*(x
/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d))]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !(GtQ[b/(b*c - a*d), 0] && Gt
Q[b/(b*e - a*f), 0]) && !LtQ[-(b*c - a*d)/d, 0]
```

3.397.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.68

method	result
default	$-\frac{2\left(dF\left(\sqrt{\frac{fx+e}{e}},\sqrt{-\frac{e^2}{df-e^2}}\right)f-E\left(\sqrt{\frac{fx+e}{e}},\sqrt{-\frac{e^2}{df-e^2}}\right)df+E\left(\sqrt{\frac{fx+e}{e}},\sqrt{-\frac{e^2}{df-e^2}}\right)e^2\right)\sqrt{-\frac{fx}{e}}\,\sqrt{\frac{(ex+d)f}{df-e^2}}\,\sqrt{\frac{fx+e}{e}}\,\sqrt{ax}\,\sqrt{ex+d}\,\sqrt{}}{f^2x(ef\,x^2+dfx+e^2x+ed)}$
elliptic	$\frac{2\sqrt{ax}\,\sqrt{(ex+d)(fx+e)ax}\,e\sqrt{\frac{\left(x+\frac{e}{f}\right)f}{e}}\,\sqrt{\frac{x+\frac{d}{e}}{-\frac{e}{f}+\frac{d}{e}}}\,\sqrt{-\frac{fx}{e}}\left(\left(-\frac{e}{f}+\frac{d}{e}\right)E\left(\sqrt{\frac{\left(x+\frac{e}{f}\right)f}{e}},\sqrt{-\frac{e}{f\left(-\frac{e}{f}+\frac{d}{e}\right)}}\right)-\frac{dF\left(\sqrt{\frac{\left(x+\frac{e}{f}\right)f}{e}},\sqrt{-\frac{e}{f\left(-\frac{e}{f}+\frac{d}{e}\right.}}\right.}{e}\right.}{\sqrt{ex+d}\,\sqrt{fx+e}\,xf\,\sqrt{aef\,x^3+adf\,x^2+a\,e^2x^2+edax}}$

input
```
int((a*x)^(1/2)/(e*x+d)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(d*EllipticF(((f*x+e)/e)^(1/2),(-e^2/(d*f-e^2))^(1/2))*f-EllipticE(((f*
x+e)/e)^(1/2),(-e^2/(d*f-e^2))^(1/2))*d*f+EllipticE(((f*x+e)/e)^(1/2),(-e^
2/(d*f-e^2))^(1/2))*e^2)*(-f*x/e)^(1/2)*((e*x+d)*f/(d*f-e^2))^(1/2)*((f*x+
e)/e)^(1/2)*(a*x)^(1/2)*(e*x+d)^(1/2)*(f*x+e)^(1/2)/f^2/x/(e*f*x^2+d*f*x+e
^2*x+d*e)
```

3.397.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.40

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx = -\frac{2\left(3\sqrt{aef}\,ef\,\text{weierstrassZeta}\left(\frac{4\left(e^4-de^2f+d^2f^2\right)}{3\,e^2f^2},-\frac{4\left(2\,e^6-3\,de^4f-3\,d^2e^2f^2+2\,d^3f^3\right)}{27\,e^3f^3},\text{weierstrassPInverse}\left(\frac{4\left(e^4-de\right.}{3\,e}\right.\right.\right.}{}$$

input
```
integrate((a*x)^(1/2)/(e*x+d)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(3*sqrt(a*e*f)*e*f*weierstrassZeta(4/3*(e^4 - d*e^2*f + d^2*f^2)/(e^2
*f^2), -4/27*(2*e^6 - 3*d*e^4*f - 3*d^2*e^2*f^2 + 2*d^3*f^3)/(e^3*f^3), we
ierstrassPInverse(4/3*(e^4 - d*e^2*f + d^2*f^2)/(e^2*f^2), -4/27*(2*e^6 -
3*d*e^4*f - 3*d^2*e^2*f^2 + 2*d^3*f^3)/(e^3*f^3), 1/3*(3*e*f*x + e^2 + d*f
)/(e*f))) + sqrt(a*e*f)*(e^2 + d*f)*weierstrassPInverse(4/3*(e^4 - d*e^2*f
 + d^2*f^2)/(e^2*f^2), -4/27*(2*e^6 - 3*d*e^4*f - 3*d^2*e^2*f^2 + 2*d^3*f^
3)/(e^3*f^3), 1/3*(3*e*f*x + e^2 + d*f)/(e*f)))/(e^2*f^2)
```

3.397.6 Sympy [F]

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx$$

input
```
integrate((a*x)**(1/2)/(e*x+d)**(1/2)/(f*x+e)**(1/2),x)
```

output
```
Integral(sqrt(a*x)/(sqrt(d + e*x)*sqrt(e + f*x)), x)
```

3.397.7 Maxima [F]

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{ax}}{\sqrt{ex+d}\sqrt{fx+e}}\,dx$$

input
```
integrate((a*x)^(1/2)/(e*x+d)^(1/2)/(f*x+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x)/(sqrt(e*x + d)*sqrt(f*x + e)), x)
```

3.397.8 Giac [F]

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{ax}}{\sqrt{ex+d}\sqrt{fx+e}}\,dx$$

input
```
integrate((a*x)^(1/2)/(e*x+d)^(1/2)/(f*x+e)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x)/(sqrt(e*x + d)*sqrt(f*x + e)), x)
```

3.397.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx = \int \frac{\sqrt{a\,x}}{\sqrt{e+f\,x}\,\sqrt{d+e\,x}}\,dx$$

input
```
int((a*x)^(1/2)/((e + f*x)^(1/2)*(d + e*x)^(1/2)),x)
```

output
```
int((a*x)^(1/2)/((e + f*x)^(1/2)*(d + e*x)^(1/2)), x)
```

3.398 $\int (ax^m)^r \, dx$

3.398.1 Optimal result

Integrand size = 7, antiderivative size = 16

$$\int (ax^m)^r \, dx = \frac{x(ax^m)^r}{1+mr}$$

output
```
x*(a*x^m)^r/(m*r+1)
```

3.398.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (ax^m)^r \, dx = \frac{x(ax^m)^r}{1+mr}$$

input
```
Integrate[(a*x^m)^r,x]
```

output
```
(x*(a*x^m)^r)/(1 + m*r)
```

3.398.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {20, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax^m)^r \, dx$$

$$\Big\downarrow \text{ 20}$$

$$x^{-mr}(ax^m)^r \int x^{mr}dx$$

$$\Big\downarrow \text{ 15}$$

$$\frac{x(ax^m)^r}{mr+1}$$

input
```
Int[(a*x^m)^r,x]
```

output
```
(x*(a*x^m)^r)/(1 + m*r)
```

3.398.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

3.398.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{x(x^m a)^r}{mr+1}$	17
parallelrisch	$\frac{x(x^m a)^r}{mr+1}$	17
norman	$\frac{x\,\mathrm{e}^{r\ln\left(\mathrm{e}^{m\ln(x)}a\right)}}{mr+1}$	21

input
```
int((x^m*a)^r,x,method=_RETURNVERBOSE)
```

output
```
x*(x^m*a)^r/(m*r+1)
```

3.398.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int (ax^m)^r \,dx = \frac{xe^{(mr\log(x)+r\log(a))}}{mr+1}$$

input
```
integrate((a*x^m)^r,x, algorithm="fricas")
```

output
```
x*e^(m*r*log(x) + r*log(a))/(m*r + 1)
```

3.398.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 0.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int (ax^m)^r \,dx = \begin{cases} \frac{x(ax^m)^r}{mr+1} & \text{for } m \neq -\frac{1}{r} \\ x\left(ax^{-\frac{1}{r}}\right)^r \log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((a*x**m)**r,x)
```

output
```
Piecewise((x*(a*x**m)**r/(m*r + 1), Ne(m, -1/r)), (x*(a/x**(1/r))**r*log(x
), True))
```

3.398.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (ax^m)^r \, dx = \frac{a^r x (x^m)^r}{mr + 1}$$

input
```
integrate((a*x^m)^r,x, algorithm="maxima")
```

output
```
a^r*x*(x^m)^r/(m*r + 1)
```

3.398.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int (ax^m)^r \, dx = \frac{xe^{(mr \log(x) + r \log(a))}}{mr + 1}$$

input
```
integrate((a*x^m)^r,x, algorithm="giac")
```

output
```
x*e^(m*r*log(x) + r*log(a))/(m*r + 1)
```

3.398.9 Mupad [B] (verification not implemented)

Time = 17.50 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (ax^m)^r \, dx = \frac{x \, (a \, x^m)^r}{m \, r + 1}$$

input
```
int((a*x^m)^r,x)
```

output
```
(x*(a*x^m)^r)/(m*r + 1)
```

3.399 $\int (ax^m)^r (bx^n)^s \, dx$

3.399.1 Optimal result

Integrand size = 15, antiderivative size = 26

$$\int (ax^m)^r (bx^n)^s \, dx = \frac{x(ax^m)^r (bx^n)^s}{1 + mr + ns}$$

output
```
x*(a*x^m)^r*(b*x^n)^s/(m*r+n*s+1)
```

3.399.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (ax^m)^r (bx^n)^s \, dx = \frac{x(ax^m)^r (bx^n)^s}{1 + mr + ns}$$

input
```
Integrate[(a*x^m)^r*(b*x^n)^s,x]
```

output
```
(x*(a*x^m)^r*(b*x^n)^s)/(1 + m*r + n*s)
```

3.399.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {33, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax^m)^r (bx^n)^s \, dx$$

$$\Big\downarrow \; 33$$

$$(ax^m)^r (bx^n)^s x^{-mr-ns} \int x^{mr+ns} dx$$

$$\Big\downarrow \; 15$$

$$\frac{x(ax^m)^r (bx^n)^s}{mr+ns+1}$$

input
```
Int[(a*x^m)^r*(b*x^n)^s,x]
```

output
```
(x*(a*x^m)^r*(b*x^n)^s)/(1 + m*r + n*s)
```

3.399.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 33
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_.)*((b_.)*(x_)^(n_))^(q_.), x_Symbol] :> Sim
p[a^IntPart[p]*b^IntPart[q]*(a*x^m)^FracPart[p]*((b*x^n)^FracPart[q]/x^(m*F
racPart[p] + n*FracPart[q]))   Int[u*x^(m*p + n*q), x], x] /; FreeQ[{a, b,
m, n, p, q}, x]
```

3.399.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

method	result
gosper	$\frac{x(x^m a)^r (b x^n)^s}{mr+ns+1}$
parallelrisch	$\frac{x(x^m a)^r (b x^n)^s}{mr+ns+1}$
risch	$\frac{b^s (x^n)^s (x^m)^r a^r x\, \mathrm{e}^{\frac{i\pi\left(-\operatorname{csgn}(ib\,x^n)^3 s+\operatorname{csgn}(ib\,x^n)^2 \operatorname{csgn}(ib)s+\operatorname{csgn}(ib\,x^n)^2 \operatorname{csgn}(ix^n)s-\operatorname{csgn}(ib\,x^n)\operatorname{csgn}(ib)\operatorname{csgn}(ix^n)s+\operatorname{csgn}(ix^m)\operatorname{csg}}{2}}}{mr+ns+1}$

input
```
int((x^m*a)^r*(b*x^n)^s,x,method=_RETURNVERBOSE)
```

output
```
x*(x^m*a)^r*(b*x^n)^s/(m*r+n*s+1)
```

3.399.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int (ax^m)^r (bx^n)^s \, dx = \frac{xe^{(mr\log(x)+ns\log(x)+r\log(a)+s\log(b))}}{mr+ns+1}$$

input
```
integrate((a*x^m)^r*(b*x^n)^s,x, algorithm="fricas")
```

output
```
x*e^(m*r*log(x) + n*s*log(x) + r*log(a) + s*log(b))/(m*r + n*s + 1)
```

3.399.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(22) = 44.

Time = 11.02 (sec) , antiderivative size = 112, normalized size of antiderivative = 4.31

$$\int (ax^m)^r (bx^n)^s \, dx = \begin{cases} \frac{x(ax^m)^r(bx^n)^s}{mr+ns+1} & \text{for } m \neq -\frac{ns+1}{r} \\ \begin{cases} \frac{x\left(ax^{-\frac{ns}{r}-\frac{1}{r}}\right)^r (bx^n)^s}{ns+r\left(-\frac{ns}{r}-\frac{1}{r}\right)+1} & \text{for } ns+r\left(-\frac{ns}{r}-\frac{1}{r}\right) \neq -1 \\ x\left(ax^{-\frac{ns}{r}-\frac{1}{r}}\right)^r (bx^n)^s \log(x) & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate((a*x**m)**r*(b*x**n)**s,x)
```

output
```
Piecewise((x*(a*x**m)**r*(b*x**n)**s/(m*r + n*s + 1), Ne(m, -(n*s + 1)/r))
, (Piecewise((x*(a*x**(-n*s/r - 1/r))**r*(b*x**n)**s/(n*s + r*(-n*s/r - 1/
r) + 1), Ne(n*s + r*(-n*s/r - 1/r), -1)), (x*(a*x**(-n*s/r - 1/r))**r*(b*x
**n)**s*log(x), True)), True))
```

3.399.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int (ax^m)^r (bx^n)^s \, dx = \frac{a^r b^s x e^{(r \log(x^m) + s \log(x^n))}}{mr + ns + 1}$$

input
```
integrate((a*x^m)^r*(b*x^n)^s,x, algorithm="maxima")
```

output
```
a^r*b^s*x*e^(r*log(x^m) + s*log(x^n))/(m*r + n*s + 1)
```

3.399.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int (ax^m)^r (bx^n)^s \, dx = \frac{x e^{(mr \log(x) + ns \log(x) + r \log(a) + s \log(b))}}{mr + ns + 1}$$

input
```
integrate((a*x^m)^r*(b*x^n)^s,x, algorithm="giac")
```

output
```
x*e^(m*r*log(x) + n*s*log(x) + r*log(a) + s*log(b))/(m*r + n*s + 1)
```

3.399.9 Mupad [B] (verification not implemented)

Time = 16.73 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (ax^m)^r (bx^n)^s \, dx = \frac{x\,(a\,x^m)^r\,(b\,x^n)^s}{m\,r+n\,s+1}$$

input
```
int((a*x^m)^r*(b*x^n)^s,x)
```

output
```
(x*(a*x^m)^r*(b*x^n)^s)/(m*r + n*s + 1)
```

3.400 $\int (ax^m)^r (bx^n)^s (cx^p)^t \, dx$

3.400.1 Optimal result

Integrand size = 22, antiderivative size = 36

$$\int (ax^m)^r (bx^n)^s (cx^p)^t \, dx = \frac{x(ax^m)^r (bx^n)^s (cx^p)^t}{1 + mr + ns + pt}$$

output
```
x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t/(m*r+n*s+p*t+1)
```

3.400.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int (ax^m)^r (bx^n)^s (cx^p)^t \, dx = \frac{x(ax^m)^r (bx^n)^s (cx^p)^t}{1 + mr + ns + pt}$$

input
```
Integrate[(a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x]
```

output
```
(x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t)/(1 + m*r + n*s + p*t)
```

3.400.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {32, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax^m)^r (bx^n)^s (cx^p)^t \, dx$$

$$\Big\downarrow \; 32$$

$$(ax^m)^r (bx^n)^s (cx^p)^t x^{-mr-ns-pt} \int x^{mr+ns+pt} dx$$

$$\Big\downarrow \; 15$$

$$\frac{x(ax^m)^r (bx^n)^s (cx^p)^t}{mr+ns+pt+1}$$

input
```
Int[(a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x]
```

output
```
(x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t)/(1 + m*r + n*s + p*t)
```

3.400.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 32
```
Int[(u_.)*((c_.)*(x_)^(k_))^(r_.)*((a_.)*(x_)^(m_))^(p_.)*((b_.)*(x_)^(n_))
^(q_.), x_Symbol] :> Simp[(a*x^m)^p*(b*x^n)^q*((c*x^k)^r/x^(m*p + n*q + k*r
))   Int[u*x^(m*p + n*q + k*r), x], x] /; FreeQ[{a, b, c, m, n, k, p, q, r}
, x]
```

3.400.4 Maple [A] (verified)

Time = 49.96 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

method	result
gosper	$\frac{x(x^{m}a)^{r}(b\,x^{n})^{s}(c\,x^{p})^{t}}{mr+ns+pt+1}$
parallelrisch	$\frac{x(x^{m}a)^{r}(b\,x^{n})^{s}(c\,x^{p})^{t}}{mr+ns+pt+1}$
risch	$(x^{p})^{t}c^{t}(x^{n})^{s}b^{s}(x^{m})^{r}a^{r}x\,\mathrm{e}^{\frac{i\pi\left(-\operatorname{csgn}(ic\,x^{p})^{3}t+\operatorname{csgn}(ic\,x^{p})^{2}\operatorname{csgn}(ic)t+\operatorname{csgn}(ic\,x^{p})^{2}\operatorname{csgn}(ix^{p})t-\operatorname{csgn}(ic\,x^{p})\operatorname{csgn}(ic)\operatorname{csgn}(ix^{p})t+\operatorname{csgn}(ix^{r}\right.}{}}$

input
```
int((x^m*a)^r*(b*x^n)^s*(c*x^p)^t,x,method=_RETURNVERBOSE)
```

output
```
x*(x^m*a)^r*(b*x^n)^s*(c*x^p)^t/(m*r+n*s+p*t+1)
```

3.400.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int (ax^m)^r (bx^n)^s (cx^p)^t \, dx = \frac{xe^{(mr\log(x)+ns\log(x)+pt\log(x)+r\log(a)+s\log(b)+t\log(c))}}{mr+ns+pt+1}$$

input
```
integrate((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x, algorithm="fricas")
```

output
```
x*e^(m*r*log(x) + n*s*log(x) + p*t*log(x) + r*log(a) + s*log(b) + t*log(c)
)/(m*r + n*s + p*t + 1)
```

3.400.6 Sympy [F(-1)]

Timed out.

$$\int (ax^m)^r (bx^n)^s (cx^p)^t \, dx = \text{Timed out}$$

input
```
integrate((a*x**m)**r*(b*x**n)**s*(c*x**p)**t,x)
```

output
```
Timed out
```

3.400.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int (ax^m)^r (bx^n)^s (cx^p)^t \, dx = \frac{a^r b^s c^t x e^{(r \log(x^m)+s \log(x^n)+t \log(x^p))}}{mr+ns+pt+1}$$

input
```
integrate((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x, algorithm="maxima")
```

output
```
a^r*b^s*c^t*x*e^(r*log(x^m) + s*log(x^n) + t*log(x^p))/(m*r + n*s + p*t +
1)
```

3.400.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int (ax^m)^r (bx^n)^s (cx^p)^t \, dx = \frac{x e^{(mr \log(x)+ns \log(x)+pt \log(x)+r \log(a)+s \log(b)+t \log(c))}}{mr+ns+pt+1}$$

input
```
integrate((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x, algorithm="giac")
```

output
```
x*e^(m*r*log(x) + n*s*log(x) + p*t*log(x) + r*log(a) + s*log(b) + t*log(c)
)/(m*r + n*s + p*t + 1)
```

3.400.9 Mupad [B] (verification not implemented)

Time = 16.58 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int (ax^m)^r (bx^n)^s (cx^p)^t \, dx = \frac{x \left(a\,x^m\right)^r \left(b\,x^n\right)^s \left(c\,x^p\right)^t}{m\,r+n\,s+p\,t+1}$$

input
```
int((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x)
```

output
```
(x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t)/(m*r + n*s + p*t + 1)
```

3.401 $\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}} dx$

3.401.1 Optimal result

Integrand size = 25, antiderivative size = 147

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}} dx = \frac{2a^2(a+bx)^{3/2}}{3b^3(a-c)} - \frac{4a(a+bx)^{5/2}}{5b^3(a-c)} + \frac{2(a+bx)^{7/2}}{7b^3(a-c)} - \frac{2c^2(c+bx)^{3/2}}{3b^3(a-c)} + \frac{4c(c+bx)^{5/2}}{5b^3(a-c)} - \frac{2(c+bx)^{7/2}}{7b^3(a-c)}$$

output
```
2/3*a^2*(b*x+a)^(3/2)/b^3/(a-c)-4/5*a*(b*x+a)^(5/2)/b^3/(a-c)+2/7*(b*x+a)^
(7/2)/b^3/(a-c)-2/3*c^2*(b*x+c)^(3/2)/b^3/(a-c)+4/5*c*(b*x+c)^(5/2)/b^3/(a
-c)-2/7*(b*x+c)^(7/2)/b^3/(a-c)
```

3.401.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}} dx = \frac{2\left(8a^3\sqrt{a+bx} - 4a^2bx\sqrt{a+bx} + 3ab^2x^2\sqrt{a+bx} - 8c^3\sqrt{c+bx} + 4bc^2x\sqrt{c+bx} - 3b^2cx^2\sqrt{c+bx} + 15b^3\right.}{105b^3(a-c)}$$

input
```
Integrate[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

output
```
(2*(8*a^3*Sqrt[a + b*x] - 4*a^2*b*x*Sqrt[a + b*x] + 3*a*b^2*x^2*Sqrt[a + b
*x] - 8*c^3*Sqrt[c + b*x] + 4*b*c^2*x*Sqrt[c + b*x] - 3*b^2*c*x^2*Sqrt[c +
 b*x] + 15*b^3*x^3*(Sqrt[a + b*x] - Sqrt[c + b*x])))/(105*b^3*(a - c))
```

3.401.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.84, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2529, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{bx+c}}dx$$

$$\Big\downarrow \text{2529}$$

$$\frac{\int x^2\sqrt{a+bx}dx}{a-c}-\frac{\int x^2\sqrt{c+bx}dx}{a-c}$$

$$\Big\downarrow \text{53}$$

$$\frac{\int \left(\frac{(a+bx)^{5/2}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{a^2\sqrt{a+bx}}{b^2}\right)dx}{a-c}-\frac{\int \left(\frac{(c+bx)^{5/2}}{b^2}-\frac{2c(c+bx)^{3/2}}{b^2}+\frac{c^2\sqrt{c+bx}}{b^2}\right)dx}{a-c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2a^2(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}}{a-c}-\frac{\frac{2c^2(bx+c)^{3/2}}{3b^3}+\frac{2(bx+c)^{7/2}}{7b^3}-\frac{4c(bx+c)^{5/2}}{5b^3}}{a-c}$$

input
```
Int[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

output
```
((2*a^2*(a + b*x)^(3/2))/(3*b^3) - (4*a*(a + b*x)^(5/2))/(5*b^3) + (2*(a +
 b*x)^(7/2))/(7*b^3))/(a - c) - ((2*c^2*(c + b*x)^(3/2))/(3*b^3) - (4*c*(c
 + b*x)^(5/2))/(5*b^3) + (2*(c + b*x)^(7/2))/(7*b^3))/(a - c)
```

3.401.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2529
```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Simp[-d/(e*(b*c - a*d))   Int[u*Sqrt[a + b*x], x], x] + Simp[
b/(f*(b*c - a*d))   Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

3.401.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{4(bx+a)^{\frac{5}{2}}a}{5}+\frac{2(bx+a)^{\frac{3}{2}}a^{2}}{3}}{(a-c)b^{3}}-\frac{2\left(\frac{(bx+c)^{\frac{7}{2}}}{7}-\frac{2c(bx+c)^{\frac{5}{2}}}{5}+\frac{c^{2}(bx+c)^{\frac{3}{2}}}{3}\right)}{(a-c)b^{3}}$	90

input
```
int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/(a-c)/b^3*(1/7*(b*x+a)^(7/2)-2/5*(b*x+a)^(5/2)*a+1/3*(b*x+a)^(3/2)*a^2)-
2/(a-c)/b^3*(1/7*(b*x+c)^(7/2)-2/5*c*(b*x+c)^(5/2)+1/3*c^2*(b*x+c)^(3/2))
```

3.401.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.64

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx$$
$$=\frac{2\left((15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3)\sqrt{bx+a}-(15\,b^3x^3+3\,b^2cx^2-4\,bc^2x+8\,c^3)\sqrt{bx+c}\right)}{105\,(ab^3-b^3c)}$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
2/105*((15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a) - (15*
b^3*x^3 + 3*b^2*c*x^2 - 4*b*c^2*x + 8*c^3)*sqrt(b*x + c))/(a*b^3 - b^3*c)
```

3.401.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \int \frac{x^2}{\sqrt{a+bx}+\sqrt{bx+c}}\,dx$$

input
```
integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

output
```
Integral(x**2/(sqrt(a + b*x) + sqrt(b*x + c)), x)
```

3.401.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \int \frac{x^2}{\sqrt{bx+a}+\sqrt{bx+c}}\,dx$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c)), x)
```

3.401.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. 2(123) = 246.

Time = 0.36 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.65

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = -\frac{2}{105}\left(\left(3\,(bx+a)\left(\frac{5\,(a^2b^9-2\,ab^9c+b^9c^2)(bx+a)}{a^3b^{12}-3\,a^2b^{12}c+3\,ab^{12}c^2-b^{12}c^3}-\frac{15\,a^3b^9-31\,a^2b^9c+17\,ab^9c^2-b^9c^3}{a^3b^{12}-3\,a^2b^{12}c+3\,ab^{12}c^2-b^{12}c^3}\right)+\frac{45\,a^4}{} \right.\right. + \frac{2\left(15\,(bx+a)^{\frac{7}{2}}-42\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2\right)}{105\,(ab^3-b^3c)}$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

output
```
-2/105*((3*(b*x + a)*(5*(a^2*b^9 - 2*a*b^9*c + b^9*c^2)*(b*x + a)/(a^3*b^1
2 - 3*a^2*b^12*c + 3*a*b^12*c^2 - b^12*c^3) - (15*a^3*b^9 - 31*a^2*b^9*c +
 17*a*b^9*c^2 - b^9*c^3)/(a^3*b^12 - 3*a^2*b^12*c + 3*a*b^12*c^2 - b^12*c^
3)) + (45*a^4*b^9 - 96*a^3*b^9*c + 53*a^2*b^9*c^2 + 2*a*b^9*c^3 - 4*b^9*c^
4)/(a^3*b^12 - 3*a^2*b^12*c + 3*a*b^12*c^2 - b^12*c^3))*(b*x + a) - (15*a^
5*b^9 - 33*a^4*b^9*c + 17*a^3*b^9*c^2 - 3*a^2*b^9*c^3 + 12*a*b^9*c^4 - 8*b
^9*c^5)/(a^3*b^12 - 3*a^2*b^12*c + 3*a*b^12*c^2 - b^12*c^3))*sqrt(b*x + c)
 + 2/105*(15*(b*x + a)^(7/2) - 42*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a
^2)/(a*b^3 - b^3*c)
```

3.401.9 Mupad [B] (verification not implemented)

Time = 16.57 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \frac{2\,x^3\,\sqrt{a+b\,x}}{7\,\left(a-c\right)}-\frac{2\,x^3\,\sqrt{c+b\,x}}{7\,\left(a-c\right)}+\frac{16\,a^3\,\sqrt{a+b\,x}}{105\,b^3\,\left(a-c\right)}-\frac{16\,c^3\,\sqrt{c+b\,x}}{105\,b^3\,\left(a-c\right)}+\frac{2\,a\,x^2\,\sqrt{a+b\,x}}{35\,b\,\left(a-c\right)}-\frac{8\,a^2\,x\,\sqrt{a+b\,x}}{105\,b^2\,\left(a-c\right)}-\frac{2\,c\,x^2\,\sqrt{c+b\,x}}{35\,b\,\left(a-c\right)}+\frac{8\,c^2\,x\,\sqrt{c+b\,x}}{105\,b^2\,\left(a-c\right)}$$

input
```
int(x^2/((a + b*x)^(1/2) + (c + b*x)^(1/2)),x)
```

output
```
(2*x^3*(a + b*x)^(1/2))/(7*(a - c)) - (2*x^3*(c + b*x)^(1/2))/(7*(a - c))
+ (16*a^3*(a + b*x)^(1/2))/(105*b^3*(a - c)) - (16*c^3*(c + b*x)^(1/2))/(1
05*b^3*(a - c)) + (2*a*x^2*(a + b*x)^(1/2))/(35*b*(a - c)) - (8*a^2*x*(a +
 b*x)^(1/2))/(105*b^2*(a - c)) - (2*c*x^2*(c + b*x)^(1/2))/(35*b*(a - c))
+ (8*c^2*x*(c + b*x)^(1/2))/(105*b^2*(a - c))
```

3.402 $\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx$

3.402.1 Optimal result

Integrand size = 23, antiderivative size = 95

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx=-\frac{2a(a+bx)^{3/2}}{3b^2(a-c)}+\frac{2(a+bx)^{5/2}}{5b^2(a-c)}+\frac{2c(c+bx)^{3/2}}{3b^2(a-c)}-\frac{2(c+bx)^{5/2}}{5b^2(a-c)}$$

output
```
-2/3*a*(b*x+a)^(3/2)/b^2/(a-c)+2/5*(b*x+a)^(5/2)/b^2/(a-c)+2/3*c*(b*x+c)^(
3/2)/b^2/(a-c)-2/5*(b*x+c)^(5/2)/b^2/(a-c)
```

3.402.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06

$$\begin{aligned}&\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx\\&=-\frac{2\sqrt{a+bx}(2a^2+ac-3c^2-a(c+bx)+6c(c+bx)-3(c+bx)^2)}{15b^2(a-c)}\\&\quad+\frac{2\left(5c(c+bx)^{3/2}-3(c+bx)^{5/2}\right)}{15b^2(a-c)}\end{aligned}$$

input
```
Integrate[x/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

output
```
(-2*Sqrt[a + b*x]*(2*a^2 + a*c - 3*c^2 - a*(c + b*x) + 6*c*(c + b*x) - 3*(
c + b*x)^2))/(15*b^2*(a - c)) + (2*(5*c*(c + b*x)^(3/2) - 3*(c + b*x)^(5/2
)))/(15*b^2*(a - c))
```

3.402.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2529, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{bx+c}}dx$$

$$\Big\downarrow \text{ 2529}$$

$$\frac{\int x\sqrt{a+bx}dx}{a-c}-\frac{\int x\sqrt{c+bx}dx}{a-c}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{\int \left(\frac{(a+bx)^{3/2}}{b}-\frac{a\sqrt{a+bx}}{b}\right)dx}{a-c}-\frac{\int \left(\frac{(c+bx)^{3/2}}{b}-\frac{c\sqrt{c+bx}}{b}\right)dx}{a-c}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{2(a+bx)^{5/2}}{5b^2}-\frac{2a(a+bx)^{3/2}}{3b^2}}{a-c}-\frac{\frac{2(bx+c)^{5/2}}{5b^2}-\frac{2c(bx+c)^{3/2}}{3b^2}}{a-c}$$

input
```
Int[x/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

output
```
((-2*a*(a + b*x)^(3/2))/(3*b^2) + (2*(a + b*x)^(5/2))/(5*b^2))/(a - c) - (
(-2*c*(c + b*x)^(3/2))/(3*b^2) + (2*(c + b*x)^(5/2))/(5*b^2))/(a - c)
```

3.402.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2529
```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Simp[-d/(e*(b*c - a*d))   Int[u*Sqrt[a + b*x], x], x] + Simp[
b/(f*(b*c - a*d))   Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

3.402.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{2(bx+a)^{\frac{3}{2}}a}{3}}{(a-c)b^{2}}-\frac{2\left(\frac{(bx+c)^{\frac{5}{2}}}{5}-\frac{c(bx+c)^{\frac{3}{2}}}{3}\right)}{(a-c)b^{2}}$	66

input
```
int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/(a-c)/b^2*(1/5*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a)-2/(a-c)/b^2*(1/5*(b*x+
c)^(5/2)-1/3*c*(b*x+c)^(3/2))
```

3.402.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx=\frac{2\left((3\,b^2x^2+abx-2\,a^2)\sqrt{bx+a}-(3\,b^2x^2+bcx-2\,c^2)\sqrt{bx+c}\right)}{15\,(ab^2-b^2c)}$$

input
```
integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
2/15*((3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a) - (3*b^2*x^2 + b*c*x - 2*c
^2)*sqrt(b*x + c))/(a*b^2 - b^2*c)
```

3.402.6 Sympy [F]

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx=\int \frac{x}{\sqrt{a+bx}+\sqrt{bx+c}}\,dx$$

input
```
integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

output
```
Integral(x/(sqrt(a + b*x) + sqrt(b*x + c)), x)
```

3.402.7 Maxima [F]

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx=\int \frac{x}{\sqrt{bx+a}+\sqrt{bx+c}}\,dx$$

input
```
integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(b*x + a) + sqrt(b*x + c)), x)
```

3.402.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(79) = 158.

Time = 0.35 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.17

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx=-\frac{2\left(\left(\left((bx+a)\left(\frac{3\,(ab^2-b^2c)(bx+a)}{a^2b^3-2\,ab^3c+b^3c^2}-\frac{6\,a^2b^2-7\,ab^2c+b^2c^2}{a^2b^3-2\,ab^3c+b^3c^2}\right)+\frac{3\,a^3b^2-4\,a^2b^2c-ab^2c^2+2\,b^2c^3}{a^2b^3-2\,ab^3c+b^3c^2}\right)\sqrt{bx+c}-\frac{3\,(bx+a)^{\frac{5}{2}}-5\,(bx+a)}{ab-bc}\right.}{15\,b}$$

input
```
integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

output
```
-2/15*(((b*x + a)*(3*(a*b^2 - b^2*c)*(b*x + a)/(a^2*b^3 - 2*a*b^3*c + b^3*
c^2) - (6*a^2*b^2 - 7*a*b^2*c + b^2*c^2)/(a^2*b^3 - 2*a*b^3*c + b^3*c^2))
+ (3*a^3*b^2 - 4*a^2*b^2*c - a*b^2*c^2 + 2*b^2*c^3)/(a^2*b^3 - 2*a*b^3*c +
 b^3*c^2))*sqrt(b*x + c) - (3*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a)/(a*b
- b*c))/b
```

3.402.9 Mupad [B] (verification not implemented)

Time = 16.94 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.36

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \frac{2\,x^2\,\sqrt{a+b\,x}}{5\,\left(a-c\right)}-\frac{2\,x^2\,\sqrt{c+b\,x}}{5\,\left(a-c\right)}-\frac{4\,a^2\,\sqrt{a+b\,x}}{15\,b^2\,\left(a-c\right)}+\frac{4\,c^2\,\sqrt{c+b\,x}}{15\,b^2\,\left(a-c\right)}+\frac{2\,a\,x\,\sqrt{a+b\,x}}{15\,b\,\left(a-c\right)}-\frac{2\,c\,x\,\sqrt{c+b\,x}}{15\,b\,\left(a-c\right)}$$

input
```
int(x/((a + b*x)^(1/2) + (c + b*x)^(1/2)),x)
```

output
```
(2*x^2*(a + b*x)^(1/2))/(5*(a - c)) - (2*x^2*(c + b*x)^(1/2))/(5*(a - c))
- (4*a^2*(a + b*x)^(1/2))/(15*b^2*(a - c)) + (4*c^2*(c + b*x)^(1/2))/(15*b
^2*(a - c)) + (2*a*x*(a + b*x)^(1/2))/(15*b*(a - c)) - (2*c*x*(c + b*x)^(1
/2))/(15*b*(a - c))
```

3.403 $\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx$

3.403.1 Optimal result

Integrand size = 21, antiderivative size = 47

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx=\frac{2(a+bx)^{3/2}}{3b(a-c)}-\frac{2(c+bx)^{3/2}}{3b(a-c)}$$

output
```
2/3*(b*x+a)^(3/2)/b/(a-c)-2/3*(b*x+c)^(3/2)/b/(a-c)
```

3.403.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx=\frac{2\left((a+bx)^{3/2}-(c+bx)^{3/2}\right)}{3b(a-c)}$$

input
```
Integrate[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-1),x]
```

output
```
(2*((a + b*x)^(3/2) - (c + b*x)^(3/2)))/(3*b*(a - c))
```

3.403.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {7240, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{bx+c}}dx$$

$$\Big\downarrow \text{7240}$$

$$\frac{\int \left(\sqrt{a+bx}-\sqrt{c+bx}\right)dx}{a-c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2(a+bx)^{3/2}}{3b}-\frac{2(bx+c)^{3/2}}{3b}}{a-c}$$

input
```
Int[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-1),x]
```

output
```
((2*(a + b*x)^(3/2))/(3*b) - (2*(c + b*x)^(3/2))/(3*b))/(a - c)
```

3.403.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7240
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(a*e^2 - c*f^2)^m   Int[ExpandIntegran
d[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c
, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

3.403.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{2(bx+a)^{\frac{3}{2}}}{3b(a-c)}-\frac{2(bx+c)^{\frac{3}{2}}}{3b(a-c)}$	40

input
```
int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/3*(b*x+a)^(3/2)/b/(a-c)-2/3*(b*x+c)^(3/2)/b/(a-c)
```

3.403.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx=\frac{2\left((bx+a)^{\frac{3}{2}}-(bx+c)^{\frac{3}{2}}\right)}{3\,(ab-bc)}$$

input
```
integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
2/3*((b*x + a)^(3/2) - (b*x + c)^(3/2))/(a*b - b*c)
```

3.403.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(32) = 64.

Time = 0.36 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.89

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx =\begin{cases}\frac{2a}{3b\sqrt{a+bx}+3b\sqrt{bx+c}}+\frac{4bx}{3b\sqrt{a+bx}+3b\sqrt{bx+c}}+\frac{2c}{3b\sqrt{a+bx}+3b\sqrt{bx+c}}+\frac{2\sqrt{a+bx}\sqrt{bx+c}}{3b\sqrt{a+bx}+3b\sqrt{bx+c}} & \text{for } b\neq 0\\ \frac{x}{\sqrt{a}+\sqrt{c}} & \text{otherwise}\end{cases}$$

input
```
integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

output
```
Piecewise((2*a/(3*b*sqrt(a + b*x) + 3*b*sqrt(b*x + c)) + 4*b*x/(3*b*sqrt(a
 + b*x) + 3*b*sqrt(b*x + c)) + 2*c/(3*b*sqrt(a + b*x) + 3*b*sqrt(b*x + c))
 + 2*sqrt(a + b*x)*sqrt(b*x + c)/(3*b*sqrt(a + b*x) + 3*b*sqrt(b*x + c)),
Ne(b, 0)), (x/(sqrt(a) + sqrt(c)), True))
```

3.403.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \int \frac{1}{\sqrt{bx+a}+\sqrt{bx+c}}\,dx$$

input
```
integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x + a) + sqrt(b*x + c)), x)
```

3.403.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = -\frac{2}{3}\sqrt{bx+c}\left(\frac{(bx+a)b}{ab^2-b^2c}-\frac{ab-bc}{ab^2-b^2c}\right)+\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,(ab-bc)}$$

input
```
integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

output
```
-2/3*sqrt(b*x + c)*((b*x + a)*b/(a*b^2 - b^2*c) - (a*b - b*c)/(a*b^2 - b^2
*c)) + 2/3*(b*x + a)^(3/2)/(a*b - b*c)
```

3.403.9 Mupad [B] (verification not implemented)

Time = 16.85 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.68

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = \frac{2\,x\,\sqrt{a+b\,x}}{3\,(a-c)}-\frac{2\,x\,\sqrt{c+b\,x}}{3\,(a-c)}+\frac{2\,a\,\sqrt{a+b\,x}}{3\,b\,(a-c)}-\frac{2\,c\,\sqrt{c+b\,x}}{3\,b\,(a-c)}$$

input
```
int(1/((a + b*x)^(1/2) + (c + b*x)^(1/2)),x)
```

output
```
(2*x*(a + b*x)^(1/2))/(3*(a - c)) - (2*x*(c + b*x)^(1/2))/(3*(a - c)) + (2
*a*(a + b*x)^(1/2))/(3*b*(a - c)) - (2*c*(c + b*x)^(1/2))/(3*b*(a - c))
```

3.404 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx$

3.404.1 Optimal result

Integrand size = 25, antiderivative size = 97

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx=\frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{c+bx}}{a-c}-\frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a-c}+\frac{2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)}{a-c}$$

output
```
-2*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)/(a-c)+2*arctanh((b*x+c)^(1/2)/c^
(1/2))*c^(1/2)/(a-c)+2*(b*x+a)^(1/2)/(a-c)-2*(b*x+c)^(1/2)/(a-c)
```

3.404.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.65

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx$$
$$=\frac{2\left(\sqrt{a+bx}-\sqrt{c+bx}-\sqrt{-\left(\sqrt{a}-\sqrt{c}\right)^2}\arctan\left(\frac{\sqrt{a+bx}-\sqrt{c+bx}}{\sqrt{-\left(\sqrt{a}-\sqrt{c}\right)^2}}\right)-\sqrt{-\left(\sqrt{a}+\sqrt{c}\right)^2}\arctan\left(\frac{\sqrt{a+bx}-\sqrt{c+}}{\sqrt{-(\sqrt{a}+\sqrt{c}}}\right.\right.}{a-c}$$

input
```
Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

output
```
(2*(Sqrt[a + b*x] - Sqrt[c + b*x] - Sqrt[-(Sqrt[a] - Sqrt[c])^2]*ArcTan[(S
qrt[a + b*x] - Sqrt[c + b*x])/Sqrt[-(Sqrt[a] - Sqrt[c])^2]] - Sqrt[-(Sqrt[
a] + Sqrt[c])^2]*ArcTan[(Sqrt[a + b*x] - Sqrt[c + b*x])/Sqrt[-(Sqrt[a] + S
qrt[c])^2]]))/(a - c)
```

3.404.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2529, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}dx$$

$$\Big\downarrow \ 2529$$

$$\frac{\int \frac{\sqrt{a+bx}}{x}dx}{a-c}-\frac{\int \frac{\sqrt{c+bx}}{x}dx}{a-c}$$

$$\Big\downarrow \ 60$$

$$\frac{a\int \frac{1}{x\sqrt{a+bx}}dx+2\sqrt{a+bx}}{a-c}-\frac{c\int \frac{1}{x\sqrt{c+bx}}dx+2\sqrt{bx+c}}{a-c}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{2a\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}+2\sqrt{a+bx}}{a-c}-\frac{\frac{2c\int \frac{1}{\frac{c+bx}{b}-\frac{c}{b}}d\sqrt{c+bx}}{b}+2\sqrt{bx+c}}{a-c}$$

$$\Big\downarrow \ 221$$

$$\frac{2\sqrt{a+bx}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a-c}-\frac{2\sqrt{bx+c}-2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{a-c}$$

input
```
Int[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

output
```
(2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(a - c) - (2*
Sqrt[c + b*x] - 2*Sqrt[c]*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/(a - c)
```

3.404.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2529
```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Simp[-d/(e*(b*c - a*d))   Int[u*Sqrt[a + b*x], x], x] + Simp[
b/(f*(b*c - a*d))   Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

3.404.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{2\sqrt{bx+a}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a-c}-\frac{2\sqrt{bx+c}-2\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{a-c}$	73

input
```
int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/(a-c)*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-1/(a-c)
*(2*(b*x+c)^(1/2)-2*c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1/2)))
```

3.404.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.28

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx$$
$$= \left[-\frac{\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\sqrt{c}\log\left(\frac{bx-2\sqrt{bx+c}\sqrt{c}+2c}{x}\right)-2\sqrt{bx+a}+2\sqrt{bx+c}}{a-c}, -\frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{bx+c}\sqrt{-c}}{c}\right)+\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}+2\sqrt{bx+c}}{a-c}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+}}{a}\right.}{}\right.$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
[-(sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(c)*log((b*x
 - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) - 2*sqrt(b*x + a) + 2*sqrt(b*x + c))/
(a - c), -(2*sqrt(-c)*arctan(sqrt(b*x + c)*sqrt(-c)/c) + sqrt(a)*log((b*x
+ 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x + a) + 2*sqrt(b*x + c))/(
a - c), (2*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(c)*log((b*x -
2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) + 2*sqrt(b*x + a) - 2*sqrt(b*x + c))/(a
- c), 2*(sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(-c)*arctan(sqrt(
b*x + c)*sqrt(-c)/c) + sqrt(b*x + a) - sqrt(b*x + c))/(a - c)]
```

3.404.6 Sympy [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = \int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}\,dx$$

input
```
integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

output
```
Integral(1/(x*(sqrt(a + b*x) + sqrt(b*x + c))), x)
```

3.404.7 Maxima [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx=\int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{bx+c}\right)}\,dx$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))), x)
```

3.404.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1015 vs. $2(81)=162$.

Time = 0.51 (sec) , antiderivative size = 1015, normalized size of antiderivative = 10.46

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx=\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}(a-c)}$$

$$-\frac{2\left(a^4c-a^3c^2-a^2c^3+ac^4+2\left(ac^2+\sqrt{ac}c^2\right)(a-c)^2\mathrm{sgn}(-a+c)-2\left(ac^2+\sqrt{ac}ac\right)(a-c)^2+\left(a^2c^2-\right.}{}$$

$$+\frac{2\left(a^4c-a^3c^2-a^2c^3+ac^4-2\left(ac^2-\sqrt{ac}c^2\right)(a-c)^2\mathrm{sgn}(-a+c)-2\left(ac^2+\sqrt{ac}ac\right)(a-c)^2+\left(a^2c^2-\right.}{}$$

$$+\frac{2\sqrt{bx+a}}{a-c}-\frac{2\sqrt{bx+c}}{a-c}$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

output
```
2*a*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*(a - c)) - 2*(a^4*c - a^3*c^2
 - a^2*c^3 + a*c^4 + 2*(a*c^2 + sqrt(a*c)*c^2)*(a - c)^2*sgn(-a + c) - 2*(
a*c^2 + sqrt(a*c)*a*c)*(a - c)^2 + (a^2*c^2 - 2*a*c^3 + c^4 + (a^2*c - 2*a
*c^2 + c^3)*sqrt(a*c))*abs(-a + c)*sgn(-a + c) - (a^3*c - 2*a^2*c^2 + a*c^
3 + (a^2*c - 2*a*c^2 + c^3)*sqrt(a*c))*abs(-a + c) - (a^4*c - a^3*c^2 - a^
2*c^3 + a*c^4 - (a^3*c - a^2*c^2 - a*c^3 + c^4)*sqrt(a*c))*sgn(-a + c) + (
a^4 - a^3*c - a^2*c^2 + a*c^3)*sqrt(a*c))*arctan(-(sqrt(b*x + a) - sqrt(b*
x + c))/sqrt(-(a^2 - c^2 + sqrt((a^2 - c^2)^2 - (a^3 - 3*a^2*c + 3*a*c^2 -
 c^3)*(a - c)))/(a - c)))/((sqrt(-a)*a^4 - a^4*sqrt(-c) - 4*sqrt(-a)*a^3*c
 + 4*a^3*sqrt(-c)*c + 6*sqrt(-a)*a^2*c^2 - 6*a^2*sqrt(-c)*c^2 - 4*sqrt(-a)
*a*c^3 + 4*a*sqrt(-c)*c^3 + sqrt(-a)*c^4 - sqrt(-c)*c^4)*abs(-a + c)) + 2*
(a^4*c - a^3*c^2 - a^2*c^3 + a*c^4 - 2*(a*c^2 - sqrt(a*c)*c^2)*(a - c)^2*s
gn(-a + c) - 2*(a*c^2 + sqrt(a*c)*a*c)*(a - c)^2 + (a^2*c^2 - 2*a*c^3 + c^
4 - (a^2*c - 2*a*c^2 + c^3)*sqrt(a*c))*abs(-a + c)*sgn(-a + c) + (a^3*c -
2*a^2*c^2 + a*c^3 + (a^2*c - 2*a*c^2 + c^3)*sqrt(a*c))*abs(-a + c) + (a^4*
c - a^3*c^2 - a^2*c^3 + a*c^4 + (a^3*c - a^2*c^2 - a*c^3 + c^4)*sqrt(a*c))
*sgn(-a + c) + (a^4 - a^3*c - a^2*c^2 + a*c^3)*sqrt(a*c))*arctan(-(sqrt(b*
x + a) - sqrt(b*x + c))/sqrt(-(a^2 - c^2 - sqrt((a^2 - c^2)^2 - (a^3 - 3*a
^2*c + 3*a*c^2 - c^3)*(a - c)))/(a - c)))/((sqrt(-a)*a^4 - a^4*sqrt(-c) -
4*sqrt(-a)*a^3*c + 4*a^3*sqrt(-c)*c + 6*sqrt(-a)*a^2*c^2 - 6*a^2*sqrt(-...
```

3.404.9 Mupad [B] (verification not implemented)

Time = 32.29 (sec) , antiderivative size = 2983, normalized size of antiderivative = 30.75

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(x*((a + b*x)^(1/2) + (c + b*x)^(1/2))),x)
```

output
```
(atan((a^2*c^(5/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^3*c^(3/2)*(a*c
^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^(7/2)*c*(a*c^3 + a^3*c - 2*a^2*c^2)^(
1/2)*2i + a^(5/2)*c^2*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a*c^3*(a + b*
x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^3*c*(a + b*x)^(1/2)*(a*c
^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(3/2)*c^(5/2)*(a + b*x)^(1/2)*(a*c^3
+ a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(5/2)*c^(3/2)*(a + b*x)^(1/2)*(a*c^3 + a
^3*c - 2*a^2*c^2)^(1/2)*2i - a^2*c^2*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^
2*c^2)^(1/2)*4i - a^(3/2)*c^(5/2)*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c
^2)^(1/2)*2i - a^(5/2)*c^(3/2)*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)
^(1/2)*2i)/(2*a^5*c^(3/2) - 4*a^4*c^(5/2) + 2*a^(5/2)*c^4 + 2*a^3*c^(7/2)
- 4*a^(7/2)*c^3 + 2*a^(9/2)*c^2 - 2*a^2*c^4*(a + b*x)^(1/2) + 4*a^3*c^3*(a
 + b*x)^(1/2) - 2*a^4*c^2*(a + b*x)^(1/2) - 2*a^(3/2)*c^(9/2)*(a + b*x)^(1
/2) + 2*a^(5/2)*c^(7/2)*(a + b*x)^(1/2) + 2*a^(7/2)*c^(5/2)*(a + b*x)^(1/2
) - 2*a^(9/2)*c^(3/2)*(a + b*x)^(1/2) + 2*a^2*c^4*(c + b*x)^(1/2) - 4*a^3*
c^3*(c + b*x)^(1/2) + 2*a^4*c^2*(c + b*x)^(1/2)))*(a + b*x)^(1/2)*(a*c^3 +
 a^3*c - 2*a^2*c^2)^(1/2)*2i - 4*a^(3/2)*c - 8*a*c^(3/2) + atan((a^2*c^(5/
2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^3*c^(3/2)*(a*c^3 + a^3*c - 2*a
^2*c^2)^(1/2)*2i - a^(7/2)*c*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(5/2
)*c^2*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a*c^3*(a + b*x)^(1/2)*(a*c^3
+ a^3*c - 2*a^2*c^2)^(1/2)*2i + a^3*c*(a + b*x)^(1/2)*(a*c^3 + a^3*c - ...
```

3.404. $\int \frac{1}{x(\sqrt{a+bx}+\sqrt{c+bx})}\,dx$

3.405 $\int \frac{1}{x^2(\sqrt{a+bx}+\sqrt{c+bx})} dx$

3.405.1 Optimal result

Integrand size = 25, antiderivative size = 103

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)} dx = -\frac{\sqrt{a+bx}}{(a-c)x}+\frac{\sqrt{c+bx}}{(a-c)x} - \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(a-c)}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)}{(a-c)\sqrt{c}}$$

output
```
-b*arctanh((b*x+a)^(1/2)/a^(1/2))/(a-c)/a^(1/2)+b*arctanh((b*x+c)^(1/2)/c^
(1/2))/(a-c)/c^(1/2)-(b*x+a)^(1/2)/(a-c)/x+(b*x+c)^(1/2)/(a-c)/x
```

3.405.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)} dx = \frac{\sqrt{a}\sqrt{c}(-\sqrt{a+bx}+\sqrt{c+bx})+b\sqrt{-\left(\sqrt{a}-\sqrt{c}\right)^2}x\arctan\left(\frac{\sqrt{a+bx}-\sqrt{c+bx}}{\sqrt{-\left(\sqrt{a}-\sqrt{c}\right)^2}}\right)-b\sqrt{-\left(\sqrt{a}+\sqrt{c}\right)^2}x\arctan}{\sqrt{a}(a-c)\sqrt{c}x}$$

input
```
Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

output
```
(Sqrt[a]*Sqrt[c]*(-Sqrt[a + b*x] + Sqrt[c + b*x]) + b*Sqrt[-(Sqrt[a] - Sqr
t[c])^2]*x*ArcTan[(Sqrt[a + b*x] - Sqrt[c + b*x])/Sqrt[-(Sqrt[a] - Sqrt[c]
)^2]] - b*Sqrt[-(Sqrt[a] + Sqrt[c])^2]*x*ArcTan[(Sqrt[a + b*x] - Sqrt[c +
b*x])/Sqrt[-(Sqrt[a] + Sqrt[c])^2]])/(Sqrt[a]*(a - c)*Sqrt[c]*x)
```

3.405.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2529, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}dx$$

$$\Big\downarrow \text{ 2529}$$

$$\frac{\int \frac{\sqrt{a+bx}}{x^2}dx}{a-c}-\frac{\int \frac{\sqrt{c+bx}}{x^2}dx}{a-c}$$

$$\Big\downarrow \text{ 51}$$

$$\frac{\frac{1}{2}b\int \frac{1}{x\sqrt{a+bx}}dx-\frac{\sqrt{a+bx}}{x}}{a-c}-\frac{\frac{1}{2}b\int \frac{1}{x\sqrt{c+bx}}dx-\frac{\sqrt{bx+c}}{x}}{a-c}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}-\frac{\sqrt{a+bx}}{x}}{a-c}-\frac{\int \frac{1}{\frac{c+bx}{b}-\frac{c}{b}}d\sqrt{c+bx}-\frac{\sqrt{bx+c}}{x}}{a-c}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{-\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx}}{x}}{a-c}-\frac{-\frac{b\text{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{bx+c}}{x}}{a-c}$$

input
```
Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

output
```
(-(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a])/(a - c)
- (-(Sqrt[c + b*x]/x) - (b*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/Sqrt[c])/(a - c
)
```

3.405.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2529
```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Simp[-d/(e*(b*c - a*d))   Int[u*Sqrt[a + b*x], x], x] + Simp[
b/(f*(b*c - a*d))   Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

3.405.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{2b\left(-\frac{\sqrt{bx+a}}{2xb}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{a-c}-\frac{2b\left(-\frac{\sqrt{bx+c}}{2xb}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)}{a-c}$	88

input
```
int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/(a-c)*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/
2))-2/(a-c)*b*(-1/2*(b*x+c)^(1/2)/x/b-1/2/c^(1/2)*arctanh((b*x+c)^(1/2)/c^
(1/2)))
```

3.405.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 399, normalized size of antiderivative = 3.87

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = \left[-\frac{\sqrt{a}bcx\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+ab\sqrt{c}x\log\left(\frac{bx-2\sqrt{bx+c}\sqrt{c}+2c}{x}\right)+2\sqrt{bx+a}ac-2\sqrt{bx+c}ac}{2(a^2c-ac^2)x},\right.$$
$$-\frac{2ab\sqrt{-c}x\arctan\left(\frac{\sqrt{bx+c}\sqrt{-c}}{c}\right)+\sqrt{a}bcx\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}ac-2\sqrt{bx+c}ac}{2(a^2c-ac^2)x}, \frac{2\sqrt{-abc}}{}$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(a)*b*c*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + a*b*sq
rt(c)*x*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) + 2*sqrt(b*x + a)*a*c
 - 2*sqrt(b*x + c)*a*c)/((a^2*c - a*c^2)*x), -1/2*(2*a*b*sqrt(-c)*x*arctan
(sqrt(b*x + c)*sqrt(-c)/c) + sqrt(a)*b*c*x*log((b*x + 2*sqrt(b*x + a)*sqrt
(a) + 2*a)/x) + 2*sqrt(b*x + a)*a*c - 2*sqrt(b*x + c)*a*c)/((a^2*c - a*c^2
)*x), 1/2*(2*sqrt(-a)*b*c*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - a*b*sqrt(c)
*x*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) - 2*sqrt(b*x + a)*a*c + 2*
sqrt(b*x + c)*a*c)/((a^2*c - a*c^2)*x), (sqrt(-a)*b*c*x*arctan(sqrt(b*x +
a)*sqrt(-a)/a) - a*b*sqrt(-c)*x*arctan(sqrt(b*x + c)*sqrt(-c)/c) - sqrt(b*
x + a)*a*c + sqrt(b*x + c)*a*c)/((a^2*c - a*c^2)*x)]
```

3.405.6 Sympy [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = \int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}\,dx$$

input
```
integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

output
```
Integral(1/(x**2*(sqrt(a + b*x) + sqrt(b*x + c))), x)
```

3.405.7 Maxima [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx=\int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{bx+c}\right)}\,dx$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))), x)
```

3.405.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1192 vs. $2(87) = 174$.

Time = 2.44 (sec) , antiderivative size = 1192, normalized size of antiderivative = 11.57

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx=\text{Too large to display}$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

output
```
b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*(a - c)) + (2*(a*c^2 - sqrt(a*c
)*c^2)*(a - c)^2*b*sgn(2*a - 2*c) + 2*(a*c^2 + sqrt(a*c)*a*c)*(a - c)^2*b
+ (a^2*c^2 - 2*a*c^3 + c^4 - (a^2*c - 2*a*c^2 + c^3)*sqrt(a*c))*b*abs(a -
c)*sgn(2*a - 2*c) + (a^3*c - 2*a^2*c^2 + a*c^3 - (a^2*c - 2*a*c^2 + c^3)*s
qrt(a*c))*b*abs(a - c) - (a^4*c - a^3*c^2 - a^2*c^3 + a*c^4 + (a^3*c - a^2
*c^2 - a*c^3 + c^4)*sqrt(a*c))*b*sgn(2*a - 2*c) - (a^4*c - a^3*c^2 - a^2*c
^3 + a*c^4 + (a^4 - a^3*c - a^2*c^2 + a*c^3)*sqrt(a*c))*b)*arctan(-(sqrt(b
*x + a) - sqrt(b*x + c))/sqrt(-(a^2 - c^2 + sqrt((a^2 - c^2)^2 - (a^3 - 3*
a^2*c + 3*a*c^2 - c^3)*(a - c)))/(a - c)))/((sqrt(-a)*a^4*c - a^4*sqrt(-c)
*c - 4*sqrt(-a)*a^3*c^2 + 4*a^3*sqrt(-c)*c^2 + 6*sqrt(-a)*a^2*c^3 - 6*a^2*
sqrt(-c)*c^3 - 4*sqrt(-a)*a*c^4 + 4*a*sqrt(-c)*c^4 + sqrt(-a)*c^5 - sqrt(-
c)*c^5)*abs(a - c)) - (2*(a*c^2 - sqrt(a*c)*c^2)*(a - c)^2*b*sgn(2*a - 2*c
) - 2*(a*c^2 + sqrt(a*c)*a*c)*(a - c)^2*b + (a^2*c^2 - 2*a*c^3 + c^4 + (a^
2*c - 2*a*c^2 + c^3)*sqrt(a*c))*b*abs(a - c)*sgn(2*a - 2*c) - (a^3*c - 2*a
^2*c^2 + a*c^3 - (a^2*c - 2*a*c^2 + c^3)*sqrt(a*c))*b*abs(a - c) - (a^4*c
- a^3*c^2 - a^2*c^3 + a*c^4 + (a^3*c - a^2*c^2 - a*c^3 + c^4)*sqrt(a*c))*b
*sgn(2*a - 2*c) + (a^4*c - a^3*c^2 - a^2*c^3 + a*c^4 + (a^4 - a^3*c - a^2*
c^2 + a*c^3)*sqrt(a*c))*b)*arctan(-(sqrt(b*x + a) - sqrt(b*x + c))/sqrt(-(
a^2 - c^2 - sqrt((a^2 - c^2)^2 - (a^3 - 3*a^2*c + 3*a*c^2 - c^3)*(a - c)))
/(a - c)))/((sqrt(-a)*a^4*c - a^4*sqrt(-c)*c - 4*sqrt(-a)*a^3*c^2 + 4*a...
```

3.405.9 Mupad [B] (verification not implemented)

Time = 35.04 (sec) , antiderivative size = 2642, normalized size of antiderivative = 25.65

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(x^2*((a + b*x)^(1/2) + (c + b*x)^(1/2))),x)
```

output
```
(b*atan(((b*(a*c^(1/2) + a^(1/2)*c)*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^
(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((a^3*b*c^(7/2)
- a^(7/2)*b*c^3 - a^2*b*c^(9/2) + a^(9/2)*b*c^2)/(a^3*c^5 - 2*a^4*c^4 + a^
5*c^3) + (((a + b*x)^(1/2) - a^(1/2))*(2*a^(3/2)*b*c^5 - 2*a^5*b*c^(3/2) +
 2*a^4*b*c^(5/2) - 2*a^(5/2)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c
^5 - 2*a^4*c^4 + a^5*c^3)) - (b*(a*c^(1/2) + a^(1/2)*c)*((a^(5/2)*c^(11/2)
 - a^(7/2)*c^(9/2) - a^(9/2)*c^(7/2) + a^(11/2)*c^(5/2))/(a^3*c^5 - 2*a^4*
c^4 + a^5*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^6 - 12*a^3*c^5 + 16
*a^4*c^4 - 12*a^5*c^3 + 4*a^6*c^2))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^
5 - 2*a^4*c^4 + a^5*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2
*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2))/(2*(2*a^2*c^3 - 2*a^3*c^
2 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2))))*1i)/(2*(2*a^2*c^3 - 2*a^3*c^2 + a
^(3/2)*c^(7/2) - a^(7/2)*c^(3/2))) + (b*(a*c^(1/2) + a^(1/2)*c)*((a^(1/2)*
c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1
/2)))^(1/2)*((a^3*b*c^(7/2) - a^(7/2)*b*c^3 - a^2*b*c^(9/2) + a^(9/2)*b*c^
2)/(a^3*c^5 - 2*a^4*c^4 + a^5*c^3) + (((a + b*x)^(1/2) - a^(1/2))*(2*a^(3/
2)*b*c^5 - 2*a^5*b*c^(3/2) + 2*a^4*b*c^(5/2) - 2*a^(5/2)*b*c^4))/(2*((c +
b*x)^(1/2) - c^(1/2))*(a^3*c^5 - 2*a^4*c^4 + a^5*c^3)) + (b*(a*c^(1/2) + a
^(1/2)*c)*((a^(5/2)*c^(11/2) - a^(7/2)*c^(9/2) - a^(9/2)*c^(7/2) + a^(11/2
)*c^(5/2))/(a^3*c^5 - 2*a^4*c^4 + a^5*c^3) - (((a + b*x)^(1/2) - a^(1/2...
```

3.406 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

3.406.1 Optimal result

Integrand size = 25, antiderivative size = 228

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}-\frac{(4ac-5(a+c)^2)\sqrt{a+bx}\sqrt{c+bx}}{32b^3(a-c)} + \frac{(4ac-5(a+c)^2)(a+bx)^{3/2}\sqrt{c+bx}}{16b^3(a-c)^2} + \frac{5(a+c)(a+bx)^{3/2}(c+bx)^{3/2}}{12b^3(a-c)^2}-\frac{x(a+bx)^{3/2}(c+bx)^{3/2}}{2b^2(a-c)^2} - \frac{(4ac-5(a+c)^2)\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{32b^3}$$

output
```
1/3*(a+c)*x^3/(a-c)^2+1/2*b*x^4/(a-c)^2+5/12*(a+c)*(b*x+a)^(3/2)*(b*x+c)^(
3/2)/b^3/(a-c)^2-1/2*x*(b*x+a)^(3/2)*(b*x+c)^(3/2)/b^2/(a-c)^2-1/32*(4*a*c
-5*(a+c)^2)*arctanh((b*x+a)^(1/2)/(b*x+c)^(1/2))/b^3+1/16*(4*a*c-5*(a+c)^2
)*(b*x+a)^(3/2)*(b*x+c)^(1/2)/b^3/(a-c)^2-1/32*(4*a*c-5*(a+c)^2)*(b*x+a)^(
1/2)*(b*x+c)^(1/2)/b^3/(a-c)
```

3.406.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = -\frac{\sqrt{a+bx}\sqrt{c+bx}(15a^3+15c^3-10bc^2x+8b^2cx^2+48b^3x^3-a^2(7c+10bx)+a(-7c^2+4bcx+8b^2x^2))}{96b^3(a}$$

input
```
Integrate[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

output
```
-1/96*(Sqrt[a + b*x]*Sqrt[c + b*x]*(15*a^3 + 15*c^3 - 10*b*c^2*x + 8*b^2*c
*x^2 + 48*b^3*x^3 - a^2*(7*c + 10*b*x) + a*(-7*c^2 + 4*b*c*x + 8*b^2*x^2))
 - 16*(-c^4 + 2*b^3*c*x^3 + 3*b^4*x^4 + 2*a*(c^3 + b^3*x^3)) + 3*(a - c)^2
*(5*a^2 + 6*a*c + 5*c^2)*Log[Sqrt[a + b*x] - Sqrt[c + b*x]])/(b^3*(a - c)^
2)
```

3.406.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.90, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7240, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}dx$$

$$\Big\downarrow \text{7240}$$

$$\frac{\int \left(2bx^3+(a+c)x^2-2\sqrt{a+bx}\sqrt{c+bx}x^2\right)dx}{(a-c)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{(a-c)^2(4ac-5(a+c)^2)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{32b^3}+\frac{5(a+c)(a+bx)^{3/2}(bx+c)^{3/2}}{12b^3}+\frac{(4ac-5(a+c)^2)(a+bx)^{3/2}\sqrt{bx+c}}{16b^3}-\frac{(a-c)(4ac-5(a+c)^2)\sqrt{a+}}{32b^3}}{(a-c)^2}$$

input
```
Int[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

output
```
(((a + c)*x^3)/3 + (b*x^4)/2 - ((a - c)*(4*a*c - 5*(a + c)^2)*Sqrt[a + b*x
]*Sqrt[c + b*x])/(32*b^3) + ((4*a*c - 5*(a + c)^2)*(a + b*x)^(3/2)*Sqrt[c
+ b*x])/(16*b^3) + (5*(a + c)*(a + b*x)^(3/2)*(c + b*x)^(3/2))/(12*b^3) -
(x*(a + b*x)^(3/2)*(c + b*x)^(3/2))/(2*b^2) - ((a - c)^2*(4*a*c - 5*(a + c
)^2)*ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(32*b^3))/(a - c)^2
```

3.406.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7240
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(a*e^2 - c*f^2)^m   Int[ExpandIntegran
d[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c
, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

3.406.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 604, normalized size of antiderivative = 2.65

method	result
default	$\frac{a\,x^{3}}{3(a-c)^{2}}+\frac{c\,x^{3}}{3(a-c)^{2}}+\frac{b\,x^{4}}{2(a-c)^{2}}-\frac{\sqrt{bx+a}\,\sqrt{bx+c}\,\left(96\ \mathrm{csgn}(b)x^{3}b^{3}\sqrt{b^{2}x^{2}+abx+bcx+ac}+16\ \mathrm{csgn}(b)x^{2}a\,b^{2}\sqrt{b^{2}x^{2}+abx+bcx+ac}\right.}{}$

input
```
int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/(a-c)^2*a*x^3+1/3/(a-c)^2*c*x^3+1/2*b*x^4/(a-c)^2-1/192/(a-c)^2*(b*x+a
)^(1/2)*(b*x+c)^(1/2)*(96*csgn(b)*x^3*b^3*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+
16*csgn(b)*x^2*a*b^2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+16*csgn(b)*x^2*b^2*c*
(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)-20*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)
*x*a^2*b+8*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*x*a*b*c-20*(b^2*x^2+a*b
*x+b*c*x+a*c)^(1/2)*csgn(b)*x*b*c^2+30*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csg
n(b)*a^3-14*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*a^2*c-14*(b^2*x^2+a*b*
x+b*c*x+a*c)^(1/2)*csgn(b)*a*c^2+30*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b
)*c^3-15*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c)*csgn
(b))*a^4+12*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c)*c
sgn(b))*a^3*c+6*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+
c)*csgn(b))*a^2*c^2+12*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2
*b*x+a+c)*csgn(b))*a*c^3-15*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn
(b)+2*b*x+a+c)*csgn(b))*c^4)*csgn(b)/b^3/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)
```

3.406.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$$
$$= \frac{96\,b^4x^4 + 64\,(ab^3 + b^3c)x^3 - 2\,(48\,b^3x^3 + 15\,a^3 - 7\,a^2c - 7\,ac^2 + 15\,c^3 + 8\,(ab^2 + b^2c)x^2 - 2\,(5\,a^2b - 2\,abc}{192\,(a^2b^3 -}$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/192*(96*b^4*x^4 + 64*(a*b^3 + b^3*c)*x^3 - 2*(48*b^3*x^3 + 15*a^3 - 7*a^
2*c - 7*a*c^2 + 15*c^3 + 8*(a*b^2 + b^2*c)*x^2 - 2*(5*a^2*b - 2*a*b*c + 5*
b*c^2)*x)*sqrt(b*x + a)*sqrt(b*x + c) - 3*(5*a^4 - 4*a^3*c - 2*a^2*c^2 - 4
*a*c^3 + 5*c^4)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c))/(a^2*
b^3 - 2*a*b^3*c + b^3*c^2)
```

3.406.6 Sympy [F]

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx=\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$

input
```
integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

output
```
Integral(x**2/(sqrt(a + b*x) + sqrt(b*x + c))**2, x)
```

3.406.7 Maxima [F]

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx=\int \frac{x^2}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c))^2, x)
```

3.406.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 797 vs. $2(194) = 388$.

Time = 0.36 (sec) , antiderivative size = 797, normalized size of antiderivative = 3.50

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = -\frac{1}{96}\left(2\left(4\,(bx+a)\left(\frac{6\,(a^5b^9-5\,a^4b^9c+10\,a^3b^9c^2-10\,a^2b^9c^3+5\,ab^9c^4-b^9c^5)(bx+a)}{a^7b^{12}-7\,a^6b^{12}c+21\,a^5b^{12}c^2-35\,a^4b^{12}c^3+35\,a^3b^{12}c^4-21\,a^2b^{12}c^5+7\,ab^{12}c^6-b^{12}\cdots}\right.\right.\right.$$
$$+\frac{3\,(bx+a)^4-10\,(bx+a)^3a+12\,(bx+a)^2a^2-6\,(bx+a)a^3+2\,(bx+a)^3c-6\,(bx+a)^2ac+6\,(bx+a)\cdots}{6\,(a^2b^3-2\,ab^3c+b^3c^2)}$$
$$-\frac{(5\,a^2+6\,ac+5\,c^2)\log\left(\left|-\sqrt{bx+a}+\sqrt{bx+c}\right|\right)}{32\,b^3}$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
-1/96*(2*(4*(b*x + a)*(6*(a^5*b^9 - 5*a^4*b^9*c + 10*a^3*b^9*c^2 - 10*a^2*
b^9*c^3 + 5*a*b^9*c^4 - b^9*c^5)*(b*x + a)/(a^7*b^12 - 7*a^6*b^12*c + 21*a
^5*b^12*c^2 - 35*a^4*b^12*c^3 + 35*a^3*b^12*c^4 - 21*a^2*b^12*c^5 + 7*a*b^
12*c^6 - b^12*c^7) - (17*a^6*b^9 - 86*a^5*b^9*c + 175*a^4*b^9*c^2 - 180*a^
3*b^9*c^3 + 95*a^2*b^9*c^4 - 22*a*b^9*c^5 + b^9*c^6)/(a^7*b^12 - 7*a^6*b^1
2*c + 21*a^5*b^12*c^2 - 35*a^4*b^12*c^3 + 35*a^3*b^12*c^4 - 21*a^2*b^12*c^
5 + 7*a*b^12*c^6 - b^12*c^7)) + (59*a^7*b^9 - 301*a^6*b^9*c + 615*a^5*b^9*
c^2 - 625*a^4*b^9*c^3 + 305*a^3*b^9*c^4 - 39*a^2*b^9*c^5 - 19*a*b^9*c^6 +
5*b^9*c^7)/(a^7*b^12 - 7*a^6*b^12*c + 21*a^5*b^12*c^2 - 35*a^4*b^12*c^3 +
35*a^3*b^12*c^4 - 21*a^2*b^12*c^5 + 7*a*b^12*c^6 - b^12*c^7))*(b*x + a) -
3*(5*a^8*b^9 - 24*a^7*b^9*c + 44*a^6*b^9*c^2 - 40*a^5*b^9*c^3 + 30*a^4*b^9
*c^4 - 40*a^3*b^9*c^5 + 44*a^2*b^9*c^6 - 24*a*b^9*c^7 + 5*b^9*c^8)/(a^7*b^
12 - 7*a^6*b^12*c + 21*a^5*b^12*c^2 - 35*a^4*b^12*c^3 + 35*a^3*b^12*c^4 -
21*a^2*b^12*c^5 + 7*a*b^12*c^6 - b^12*c^7))*sqrt(b*x + a)*sqrt(b*x + c) +
1/6*(3*(b*x + a)^4 - 10*(b*x + a)^3*a + 12*(b*x + a)^2*a^2 - 6*(b*x + a)*a
^3 + 2*(b*x + a)^3*c - 6*(b*x + a)^2*a*c + 6*(b*x + a)*a^2*c)/(a^2*b^3 - 2
*a*b^3*c + b^3*c^2) - 1/32*(5*a^2 + 6*a*c + 5*c^2)*log(abs(-sqrt(b*x + a)
+ sqrt(b*x + c)))/b^3
```

3.406.9 Mupad [B] (verification not implemented)

Time = 147.00 (sec) , antiderivative size = 1358, normalized size of antiderivative = 5.96

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \text{Too large to display}$$

input
```
int(x^2/((a + b*x)^(1/2) + (c + b*x)^(1/2))^2,x)
```

output
```
(x^3*(a + c))/(3*(a - c)^2) - ((((a + b*x)^(1/2) - a^(1/2))^15*((3*a*c)/8
+ (5*a^2)/16 + (5*c^2)/16))/(b^3*((c + b*x)^(1/2) - c^(1/2))^15) + (((a +
b*x)^(1/2) - a^(1/2))^3*((23*a*c^3)/12 + (23*a^3*c)/12 - (115*a^4)/48 - (1
15*c^4)/48 + (349*a^2*c^2)/8))/(((c + b*x)^(1/2) - c^(1/2))^3*(a^2*b^3 + b
^3*c^2 - 2*a*b^3*c)) + (((a + b*x)^(1/2) - a^(1/2))^13*((23*a*c^3)/12 + (2
3*a^3*c)/12 - (115*a^4)/48 - (115*c^4)/48 + (349*a^2*c^2)/8))/(((c + b*x)^
(1/2) - c^(1/2))^13*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) + (((a + b*x)^(1/2) -
 a^(1/2))^5*((3917*a*c^3)/12 + (3917*a^3*c)/12 + (383*a^4)/48 + (383*c^4)/
48 + (7279*a^2*c^2)/8))/(((c + b*x)^(1/2) - c^(1/2))^5*(a^2*b^3 + b^3*c^2
- 2*a*b^3*c)) + (((a + b*x)^(1/2) - a^(1/2))^11*((3917*a*c^3)/12 + (3917*a
^3*c)/12 + (383*a^4)/48 + (383*c^4)/48 + (7279*a^2*c^2)/8))/(((c + b*x)^(1
/2) - c^(1/2))^11*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) + (((a + b*x)^(1/2) - a
^(1/2))^7*((17567*a*c^3)/12 + (17567*a^3*c)/12 + (2789*a^4)/48 + (2789*c^4
)/48 + (28213*a^2*c^2)/8))/(((c + b*x)^(1/2) - c^(1/2))^7*(a^2*b^3 + b^3*c
^2 - 2*a*b^3*c)) + (((a + b*x)^(1/2) - a^(1/2))^9*((17567*a*c^3)/12 + (175
67*a^3*c)/12 + (2789*a^4)/48 + (2789*c^4)/48 + (28213*a^2*c^2)/8))/(((c +
b*x)^(1/2) - c^(1/2))^9*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) + (((a + b*x)^(1/
2) - a^(1/2))*((3*a*c)/8 + (5*a^2)/16 + (5*c^2)/16))/(b^3*((c + b*x)^(1/2)
 - c^(1/2))) - (a^(1/2)*c^(1/2)*(192*a*c^2 + 192*a^2*c)*((a + b*x)^(1/2) -
 a^(1/2))^4)/(((c + b*x)^(1/2) - c^(1/2))^4*(a^2*b^3 + b^3*c^2 - 2*a*b^...
```

3.406. $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

3.407 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

3.407.1 Optimal result

Integrand size = 23, antiderivative size = 165

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\sqrt{c+bx}}{4b^2(a-c)} + \frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2} - \frac{2(a+bx)^{3/2}(c+bx)^{3/2}}{3b^2(a-c)^2}-\frac{(a+c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{4b^2}$$

output
```
1/2*(a+c)*x^2/(a-c)^2+2/3*b*x^3/(a-c)^2-2/3*(b*x+a)^(3/2)*(b*x+c)^(3/2)/b^
2/(a-c)^2-1/4*(a+c)*arctanh((b*x+a)^(1/2)/(b*x+c)^(1/2))/b^2+1/2*(a+c)*(b*
x+a)^(3/2)*(b*x+c)^(1/2)/b^2/(a-c)^2-1/4*(a+c)*(b*x+a)^(1/2)*(b*x+c)^(1/2)
/b^2/(a-c)
```

3.407.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.81

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{\frac{2(c+bx)(-3ac+c^2+3abx-bcx+4b^2x^2)}{(a-c)^2}+\frac{\sqrt{a+bx}\sqrt{c+bx}(3a^2+3c^2-2bcx-8b^2x^2-2a(c+bx))}{(a-c)^2}+3(a+c)\log\left(\sqrt{a+bx}-\sqrt{c+bx}\right.}{12b^2}$$

input
```
Integrate[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

output
```
((2*(c + b*x)*(-3*a*c + c^2 + 3*a*b*x - b*c*x + 4*b^2*x^2))/(a - c)^2 + (S
qrt[a + b*x]*Sqrt[c + b*x]*(3*a^2 + 3*c^2 - 2*b*c*x - 8*b^2*x^2 - 2*a*(c +
 b*x)))/(a - c)^2 + 3*(a + c)*Log[Sqrt[a + b*x] - Sqrt[c + b*x]])/(12*b^2)
```

3.407.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.92, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {7240, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}dx$$

$$\Big\downarrow \text{ 7240}$$

$$\frac{\int \left(2bx^2+(a+c)x-2\sqrt{a+bx}\sqrt{c+bx}x\right)dx}{(a-c)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{(a^2-c^2)\sqrt{a+bx}\sqrt{bx+c}}{4b^2}-\frac{(a-c)^2(a+c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{4b^2}-\frac{2(a+bx)^{3/2}(bx+c)^{3/2}}{3b^2}+\frac{(a+c)(a+bx)^{3/2}\sqrt{bx+c}}{2b^2}+\frac{1}{2}x^2(a+c)+\frac{2bx^3}{3}}{(a-c)^2}$$

input
```
Int[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

output
```
(((a + c)*x^2)/2 + (2*b*x^3)/3 - ((a^2 - c^2)*Sqrt[a + b*x]*Sqrt[c + b*x])
/(4*b^2) + ((a + c)*(a + b*x)^(3/2)*Sqrt[c + b*x])/(2*b^2) - (2*(a + b*x)^
(3/2)*(c + b*x)^(3/2))/(3*b^2) - ((a - c)^2*(a + c)*ArcTanh[Sqrt[a + b*x]/
Sqrt[c + b*x]])/(4*b^2))/(a - c)^2
```

3.407.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7240
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(a*e^2 - c*f^2)^m   Int[ExpandIntegran
d[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c
, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

3.407.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.61

method	result
default	$\frac{x^2 a}{2(a-c)^2}+\frac{x^2 c}{2(a-c)^2}+\frac{2b\,x^3}{3(a-c)^2}-\frac{\sqrt{bx+a}\,\sqrt{bx+c}\left(16\ \mathrm{csgn}(b)x^2b^2\sqrt{b^2x^2+abx+bcx+ac}+4\ \mathrm{csgn}(b)\sqrt{b^2x^2+abx+bcx+ac}\,xab+4\right.}{}$

input
```
int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/(a-c)^2*a+1/2*x^2/(a-c)^2*c+2/3*b*x^3/(a-c)^2-1/24/(a-c)^2*(b*x+a)
^(1/2)*(b*x+c)^(1/2)*(16*csgn(b)*x^2*b^2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+4
*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*x*a*b+4*csgn(b)*(b^2*x^2+a*b*x+b*
c*x+a*c)^(1/2)*x*b*c-6*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*a^2+4*csgn(
b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*a*c-6*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)
^(1/2)*c^2+3*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c)*
csgn(b))*a^3-3*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c
)*csgn(b))*a^2*c-3*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x
+a+c)*csgn(b))*a*c^2+3*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2
*b*x+a+c)*csgn(b))*c^3)*csgn(b)/b^2/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)
```

3.407.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.90

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$$
$$= \frac{16\,b^3x^3 + 12\,(ab^2+b^2c)x^2 - 2\,(8\,b^2x^2 - 3\,a^2 + 2\,ac - 3\,c^2 + 2\,(ab+bc)x)\sqrt{bx+a}\sqrt{bx+c} + 3\,(a^3 - a^2c -}{24\,(a^2b^2 - 2\,ab^2c + b^2c^2)}$$

input
```
integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/24*(16*b^3*x^3 + 12*(a*b^2 + b^2*c)*x^2 - 2*(8*b^2*x^2 - 3*a^2 + 2*a*c -
 3*c^2 + 2*(a*b + b*c)*x)*sqrt(b*x + a)*sqrt(b*x + c) + 3*(a^3 - a^2*c - a
*c^2 + c^3)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c))/(a^2*b^2
- 2*a*b^2*c + b^2*c^2)
```

3.407.6 Sympy [F]

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \int \frac{x}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$

input
```
integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

output
```
Integral(x/(sqrt(a + b*x) + sqrt(b*x + c))**2, x)
```

3.407.7 Maxima [F]

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \int \frac{x}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

input
```
integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(b*x + a) + sqrt(b*x + c))^2, x)
```

3.407.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(137) = 274$.

Time = 0.34 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.70

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx =$$

$$-\frac{\left(2\,(bx+a)\left(\frac{4\,(a^3b^2-3\,a^2b^2c+3\,ab^2c^2-b^2c^3)(bx+a)}{a^5b^3-5\,a^4b^3c+10\,a^3b^3c^2-10\,a^2b^3c^3+5\,ab^3c^4-b^3c^5}-\frac{7\,a^4b^2-22\,a^3b^2c+24\,a^2b^2c^2-10\,ab^2c^3+b^2c^4}{a^5b^3-5\,a^4b^3c+10\,a^3b^3c^2-10\,a^2b^3c^3+5\,ab^3c^4-b^3c^5}\right)+\frac{3\,(a^5b^2-\cdots}{a^5b^3-5\,a\cdots}\right.}{\cdots}$$

input
```
integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
-1/12*((2*(b*x + a)*(4*(a^3*b^2 - 3*a^2*b^2*c + 3*a*b^2*c^2 - b^2*c^3)*(b*
x + a)/(a^5*b^3 - 5*a^4*b^3*c + 10*a^3*b^3*c^2 - 10*a^2*b^3*c^3 + 5*a*b^3*
c^4 - b^3*c^5) - (7*a^4*b^2 - 22*a^3*b^2*c + 24*a^2*b^2*c^2 - 10*a*b^2*c^3
 + b^2*c^4)/(a^5*b^3 - 5*a^4*b^3*c + 10*a^3*b^3*c^2 - 10*a^2*b^3*c^3 + 5*a
*b^3*c^4 - b^3*c^5)) + 3*(a^5*b^2 - 3*a^4*b^2*c + 2*a^3*b^2*c^2 + 2*a^2*b^
2*c^3 - 3*a*b^2*c^4 + b^2*c^5)/(a^5*b^3 - 5*a^4*b^3*c + 10*a^3*b^3*c^2 - 1
0*a^2*b^3*c^3 + 5*a*b^3*c^4 - b^3*c^5))*sqrt(b*x + a)*sqrt(b*x + c) - 3*(a
 + c)*log(abs(-sqrt(b*x + a) + sqrt(b*x + c)))/b - 2*(4*(b*x + a)^3 - 9*(b
*x + a)^2*a + 6*(b*x + a)*a^2 + 3*(b*x + a)^2*c - 6*(b*x + a)*a*c)/(a^2*b
- 2*a*b*c + b*c^2))/b
```

3.407.9 Mupad [B] (verification not implemented)

Time = 75.36 (sec) , antiderivative size = 1012, normalized size of antiderivative = 6.13

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$$

$$=\frac{\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)\left(\frac{a}{2}+\frac{c}{2}\right)}{b^2\left(\sqrt{c+b\,x}-\sqrt{c}\right)}+\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^{11}\left(\frac{a}{2}+\frac{c}{2}\right)}{b^2\left(\sqrt{c+b\,x}-\sqrt{c}\right)^{11}}-\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3\left(\frac{17\,a^3}{6}+\frac{101\,a^2\,c}{2}+\frac{101\,a\,c^2}{2}+\frac{17\,c^3}{6}\right)}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^3\left(a^2\,b^2-2\,a\,b^2\,c+b^2\,c^2\right)}-\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^9\left(\frac{17\,a^3}{6}+\frac{101\,a^2\cdots}{2}\right.}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^9\left(a^2\,b^2-2\cdots\right.}}{\cdots}$$

$$-\frac{\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{c+b\,x}-\sqrt{c}}\right)\,(a+c)}{2\,b^2}+\frac{x^2\,(a+c)}{2\,(a-c)^2}+\frac{2\,b\,x^3}{3\,(a-c)^2}$$

input
```
int(x/((a + b*x)^(1/2) + (c + b*x)^(1/2))^2,x)
```

output
```
((((a + b*x)^(1/2) - a^(1/2))*(a/2 + c/2))/(b^2*((c + b*x)^(1/2) - c^(1/2)
)) + (((a + b*x)^(1/2) - a^(1/2))^11*(a/2 + c/2))/(b^2*((c + b*x)^(1/2) -
c^(1/2))^11) - (((a + b*x)^(1/2) - a^(1/2))^3*((101*a*c^2)/2 + (101*a^2*c)
/2 + (17*a^3)/6 + (17*c^3)/6))/(((c + b*x)^(1/2) - c^(1/2))^3*(a^2*b^2 + b
^2*c^2 - 2*a*b^2*c)) - (((a + b*x)^(1/2) - a^(1/2))^9*((101*a*c^2)/2 + (10
1*a^2*c)/2 + (17*a^3)/6 + (17*c^3)/6))/(((c + b*x)^(1/2) - c^(1/2))^9*(a^2
*b^2 + b^2*c^2 - 2*a*b^2*c)) - (((a + b*x)^(1/2) - a^(1/2))^5*(269*a*c^2 +
 269*a^2*c + 19*a^3 + 19*c^3))/(((c + b*x)^(1/2) - c^(1/2))^5*(a^2*b^2 + b
^2*c^2 - 2*a*b^2*c)) - (((a + b*x)^(1/2) - a^(1/2))^7*(269*a*c^2 + 269*a^2
*c + 19*a^3 + 19*c^3))/(((c + b*x)^(1/2) - c^(1/2))^7*(a^2*b^2 + b^2*c^2 -
 2*a*b^2*c)) + (16*a^(3/2)*c^(3/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(((c + b
*x)^(1/2) - c^(1/2))^2*(a^2*b^2 + b^2*c^2 - 2*a*b^2*c)) + (16*a^(3/2)*c^(3
/2)*((a + b*x)^(1/2) - a^(1/2))^10)/(((c + b*x)^(1/2) - c^(1/2))^10*(a^2*b
^2 + b^2*c^2 - 2*a*b^2*c)) + (a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^
4*(192*a*c + 64*a^2 + 64*c^2))/(((c + b*x)^(1/2) - c^(1/2))^4*(a^2*b^2 + b
^2*c^2 - 2*a*b^2*c)) + (a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^8*(192
*a*c + 64*a^2 + 64*c^2))/(((c + b*x)^(1/2) - c^(1/2))^8*(a^2*b^2 + b^2*c^2
 - 2*a*b^2*c)) + (a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^6*((1312*a*c
)/3 + 128*a^2 + 128*c^2))/(((c + b*x)^(1/2) - c^(1/2))^6*(a^2*b^2 + b^2*c^
2 - 2*a*b^2*c)))/((15*((a + b*x)^(1/2) - a^(1/2))^4)/((c + b*x)^(1/2) -...
```

3.407. $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

3.408 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2} dx$

3.408.1 Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2} dx = \frac{(a-c)^2}{8b\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^4}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{2b}$$

output
```
1/2*arctanh((b*x+a)^(1/2)/(b*x+c)^(1/2))/b+1/8*(a-c)^2/b/((b*x+a)^(1/2)+(b
*x+c)^(1/2))^4
```

3.408.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.30

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2} dx = -\frac{-\frac{2(a+bx)(c+bx)}{(a-c)^2}+\frac{\sqrt{a+bx}\sqrt{c+bx}(a+c+2bx)}{(a-c)^2}+\log\left(\sqrt{a+bx}-\sqrt{c+bx}\right)}{2b}$$

input
```
Integrate[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-2),x]
```

output
```
-1/2*((-2*(a + b*x)*(c + b*x))/(a - c)^2 + (Sqrt[a + b*x]*Sqrt[c + b*x]*(a
 + c + 2*b*x))/(a - c)^2 + Log[Sqrt[a + b*x] - Sqrt[c + b*x]])/b
```

3.408.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.68, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {7240, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}dx$$

$$\Big\downarrow \text{7240}$$

$$\frac{\int \left(a+c+2bx-2\sqrt{a+bx}\sqrt{c+bx}\right)dx}{(a-c)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(a-c)^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{2b}+\frac{(a-c)\sqrt{a+bx}\sqrt{bx+c}}{2b}-\frac{(a+bx)^{3/2}\sqrt{bx+c}}{b}+x(a+c)+bx^2}{(a-c)^2}$$

input
```
Int[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-2),x]
```

output
```
((a + c)*x + b*x^2 + ((a - c)*Sqrt[a + b*x]*Sqrt[c + b*x])/(2*b) - ((a + b
*x)^(3/2)*Sqrt[c + b*x])/b + ((a - c)^2*ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x
]])/(2*b))/(a - c)^2
```

3.408.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7240
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(a*e^2 - c*f^2)^m   Int[ExpandIntegran
d[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c
, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

3.408.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(51) = 102$.

Time = 0.04 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.92

method	result
default	$\frac{xa}{(a-c)^2}+\frac{xc}{(a-c)^2}+\frac{x^2b}{(a-c)^2}-\frac{2\left(\frac{\sqrt{bx+a}\,(bx+c)^{\frac{3}{2}}}{2b}-\frac{(-ab+bc)\left(\frac{\sqrt{bx+c}\,\sqrt{bx+a}}{b}-\frac{(ab-bc)\sqrt{(bx+a)(bx+c)}\,\ln\left(\frac{\frac{1}{2}ab+\frac{1}{2}bc+b^2x}{\sqrt{b^2}}+\sqrt{b^2x^2+(}\right.}{2b\sqrt{bx+c}\,\sqrt{bx+a}\,\sqrt{b^2}}\right.}{4b}\right.}{(a-c)^2}$

input
```
int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
x/(a-c)^2*a+x/(a-c)^2*c+x^2/(a-c)^2*b-2/(a-c)^2*(1/2/b*(b*x+a)^(1/2)*(b*x+
c)^(3/2)-1/4*(-a*b+b*c)/b*(1/b*(b*x+c)^(1/2)*(b*x+a)^(1/2)-1/2*(a*b-b*c)/b
*((b*x+a)*(b*x+c))^(1/2)/(b*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*b+1/2*b*c+b
^2*x)/(b^2)^(1/2)+(b^2*x^2+(a*b+b*c)*x+a*c)^(1/2))/(b^2)^(1/2)))
```

3.408.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(51) = 102$.

Time = 0.33 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.63

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$$
$$= \frac{4\,b^2x^2 - 2\,(2\,bx + a + c)\sqrt{bx+a}\sqrt{bx+c} + 4\,(ab+bc)x - (a^2 - 2\,ac + c^2)\log\left(-2\,bx + 2\,\sqrt{bx+a}\sqrt{bx+\right.}{4\,(a^2b - 2\,abc + bc^2)}$$

input
```
integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/4*(4*b^2*x^2 - 2*(2*b*x + a + c)*sqrt(b*x + a)*sqrt(b*x + c) + 4*(a*b +
b*c)*x - (a^2 - 2*a*c + c^2)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) -
a - c))/(a^2*b - 2*a*b*c + b*c^2)
```

3.408.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(48) = 96$.

Time = 0.42 (sec) , antiderivative size = 388, normalized size of antiderivative = 6.16

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \begin{cases} \frac{2a\log\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\sqrt{bx+c}} + \frac{a}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\sqrt{bx+c}} + \frac{4bx\log\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\sqrt{bx+c}} + \frac{2bx}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\sqrt{}} \\ \frac{x}{\left(\sqrt{a}+\sqrt{c}\right)^2} \end{cases}$$

input
```
integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

output
```
Piecewise((2*a*log(sqrt(a + b*x) + sqrt(b*x + c))/(4*a*b + 8*b**2*x + 4*b*
c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + a/(4*a*b + 8*b**2*x + 4*b*c + 8*b*s
qrt(a + b*x)*sqrt(b*x + c)) + 4*b*x*log(sqrt(a + b*x) + sqrt(b*x + c))/(4*
a*b + 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + 2*b*x/(4*a*b +
 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + 2*c*log(sqrt(a + b*
x) + sqrt(b*x + c))/(4*a*b + 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x
 + c)) + c/(4*a*b + 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) +
4*sqrt(a + b*x)*sqrt(b*x + c)*log(sqrt(a + b*x) + sqrt(b*x + c))/(4*a*b +
8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)), Ne(b, 0)), (x/(sqrt(a
) + sqrt(c))**2, True))
```

3.408.7 Maxima [F]

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \int \frac{1}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

input
```
integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate((sqrt(b*x + a) + sqrt(b*x + c))^(-2), x)
```

3.408.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(51) = 102$.

Time = 0.31 (sec) , antiderivative size = 189, normalized size of antiderivative = 3.00

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = -\frac{1}{2}\sqrt{bx+a}\sqrt{bx+c}\left(\frac{2\,(ab-bc)(bx+a)}{a^3b^2-3\,a^2b^2c+3\,ab^2c^2-b^2c^3}-\frac{a^2b-2\,abc+bc^2}{a^3b^2-3\,a^2b^2c+3\,ab^2c^2-b^2c^3}\right) + \frac{(bx+a)^2-(bx+a)a+(bx+a)c}{a^2b-2\,abc+bc^2} - \frac{\log\left(\left|-\sqrt{bx+a}+\sqrt{bx+c}\right|\right)}{2\,b}$$

input
```
integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
-1/2*sqrt(b*x + a)*sqrt(b*x + c)*(2*(a*b - b*c)*(b*x + a)/(a^3*b^2 - 3*a^2
*b^2*c + 3*a*b^2*c^2 - b^2*c^3) - (a^2*b - 2*a*b*c + b*c^2)/(a^3*b^2 - 3*a
^2*b^2*c + 3*a*b^2*c^2 - b^2*c^3)) + ((b*x + a)^2 - (b*x + a)*a + (b*x + a
)*c)/(a^2*b - 2*a*b*c + b*c^2) - 1/2*log(abs(-sqrt(b*x + a) + sqrt(b*x + c
)))/b
```

3.408.9 Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.75

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{b\,x^2}{(a-c)^2}+\frac{x\,(a+c)}{(a-c)^2} + \frac{\ln\left(a+c+2\,\sqrt{a+b\,x}\,\sqrt{c+b\,x}+2\,b\,x\right)\,\left(a\,b-b\,c\right)^2}{4\,b^3\,(a-c)^2} - \frac{2\,\sqrt{a+b\,x}\,\sqrt{c+b\,x}\,\left(\frac{x}{2}+\frac{a\,b+b\,c}{4\,b^2}\right)}{(a-c)^2}$$

input
```
int(1/((a + b*x)^(1/2) + (c + b*x)^(1/2))^2,x)
```

output
```
(b*x^2)/(a - c)^2 + (x*(a + c))/(a - c)^2 + (log(a + c + 2*(a + b*x)^(1/2)
*(c + b*x)^(1/2) + 2*b*x)*(a*b - b*c)^2)/(4*b^3*(a - c)^2) - (2*(a + b*x)^
(1/2)*(c + b*x)^(1/2)*(x/2 + (a*b + b*c)/(4*b^2)))/(a - c)^2
```

3.409 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

3.409.1 Optimal result

Integrand size = 25, antiderivative size = 133

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2}-\frac{2(a+c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{(a-c)^2}+\frac{4\sqrt{a}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{c+bx}}\right)}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}$$

output
```
2*b*x/(a-c)^2-2*(a+c)*arctanh((b*x+a)^(1/2)/(b*x+c)^(1/2))/(a-c)^2+(a+c)*l
n(x)/(a-c)^2+4*arctanh(c^(1/2)*(b*x+a)^(1/2)/a^(1/2)/(b*x+c)^(1/2))*a^(1/2
)*c^(1/2)/(a-c)^2-2*(b*x+a)^(1/2)*(b*x+c)^(1/2)/(a-c)^2
```

3.409.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.03

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{\log\left(\sqrt{a}\sqrt{c}+bx-\sqrt{a+bx}\sqrt{c+bx}\right)}{\left(\sqrt{a}+\sqrt{c}\right)^2} + \frac{2\left(c+bx-\sqrt{a+bx}\sqrt{c+bx}\right)+\left(\sqrt{a}+\sqrt{c}\right)^2\log\left(\sqrt{a}\sqrt{c}-bx+\sqrt{a+bx}\sqrt{c+bx}\right)}{(a-c)^2}$$

input
```
Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

output
```
Log[Sqrt[a]*Sqrt[c] + b*x - Sqrt[a + b*x]*Sqrt[c + b*x]]/(Sqrt[a] + Sqrt[c
])^2 + (2*(c + b*x - Sqrt[a + b*x]*Sqrt[c + b*x]) + (Sqrt[a] + Sqrt[c])^2*
Log[Sqrt[a]*Sqrt[c] - b*x + Sqrt[a + b*x]*Sqrt[c + b*x]])/(a - c)^2
```

3.409.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.80, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7240, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}dx$$

$$\Big\downarrow \text{7240}$$

$$\frac{\int \left(2b-\frac{2\sqrt{a+bx}\sqrt{c+bx}}{x}+\frac{a+c}{x}\right)dx}{(a-c)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-2(a+c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)+4\sqrt{a}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{bx+c}}\right)-2\sqrt{a+bx}\sqrt{bx+c}+(a+c)\log(x)+2bx}{(a-c)^2}$$

input
```
Int[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

output
```
(2*b*x - 2*Sqrt[a + b*x]*Sqrt[c + b*x] - 2*(a + c)*ArcTanh[Sqrt[a + b*x]/S
qrt[c + b*x]] + 4*Sqrt[a]*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[a]
*Sqrt[c + b*x])] + (a + c)*Log[x])/(a - c)^2
```

3.409.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7240
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(a*e^2 - c*f^2)^m   Int[ExpandIntegran
d[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c
, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

3.409.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.94

method	result
default	$\frac{a\ln(x)}{(a-c)^2}+\frac{c\ln(x)}{(a-c)^2}+\frac{2bx}{(a-c)^2}+\frac{\sqrt{bx+a}\,\sqrt{bx+c}\left(2\ \mathrm{csgn}(b)\ln\left(\frac{abx+bcx+2\sqrt{ac}\,\sqrt{b^2x^2+abx+bcx+ac}+2ac}{x}\right)ac-2\ \mathrm{csgn}(b)\sqrt{b^2x^2+abx}\right.}{}$

input
```
int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/(a-c)^2*a*ln(x)+1/(a-c)^2*c*ln(x)+2*b*x/(a-c)^2+1/(a-c)^2*(b*x+a)^(1/2)*
(b*x+c)^(1/2)*(2*csgn(b)*ln((a*b*x+b*c*x+2*(a*c)^(1/2)*(b^2*x^2+a*b*x+b*c*
x+a*c)^(1/2)+2*a*c)/x)*a*c-2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*(a*c)
^(1/2)-ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a+c)*csgn(b
))*(a*c)^(1/2)*a-ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)+2*b*x+a
+c)*csgn(b))*(a*c)^(1/2)*c)*csgn(b)/(a*c)^(1/2)/(b^2*x^2+a*b*x+b*c*x+a*c)^
(1/2)
```

3.409.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.18

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$$
$$=\left[\frac{2\,bx+(a+c)\log\left(-2\,bx+2\,\sqrt{bx+a}\sqrt{bx+c}-a-c\right)+(a+c)\log\left(x\right)+2\,\sqrt{ac}\log\left(\frac{2\,a^2c+2\,ac^2+2\,(2\,ac+\sqrt{}}{}\right.}{a^2-2\,ac+c^2}\right.$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
[(2*b*x + (a + c)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c) + (a
 + c)*log(x) + 2*sqrt(a*c)*log((2*a^2*c + 2*a*c^2 + 2*(2*a*c + sqrt(a*c)*(
a + c))*sqrt(b*x + a)*sqrt(b*x + c) + (a^2*b + 2*a*b*c + b*c^2)*x + 2*(2*a
*c + (a*b + b*c)*x)*sqrt(a*c))/x) - 2*sqrt(b*x + a)*sqrt(b*x + c))/(a^2 -
2*a*c + c^2), (2*b*x + (a + c)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c)
- a - c) + (a + c)*log(x) - 4*sqrt(-a*c)*arctan(-(sqrt(-a*c)*b*x - sqrt(-a
*c)*sqrt(b*x + a)*sqrt(b*x + c))/(a*c)) - 2*sqrt(b*x + a)*sqrt(b*x + c))/(
a^2 - 2*a*c + c^2)]
```

3.409.6 Sympy [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$

input
```
integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

output
```
Integral(1/(x*(sqrt(a + b*x) + sqrt(b*x + c))**2), x)
```

3.409.7 Maxima [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))^2), x)
```

3.409.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.46

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{4\,ac\arctan\left(\frac{\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2-a-c}{2\sqrt{-ac}}\right)}{(a^2-2\,ac+c^2)\sqrt{-ac}} - \frac{2\,(a^2-2\,ac+c^2)\sqrt{bx+a}\sqrt{bx+c}}{a^4-4\,a^3c+6\,a^2c^2-4\,ac^3+c^4} + \frac{(a+c)\log\left(\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2\right)}{a^2-2\,ac+c^2} + \frac{(a+c)\log\left(|bx|\right)}{a^2-2\,ac+c^2} + \frac{2\,(bx+a)}{a^2-2\,ac+c^2}$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
4*a*c*arctan(1/2*((sqrt(b*x + a) - sqrt(b*x + c))^2 - a - c)/sqrt(-a*c))/(
(a^2 - 2*a*c + c^2)*sqrt(-a*c)) - 2*(a^2 - 2*a*c + c^2)*sqrt(b*x + a)*sqrt
(b*x + c)/(a^4 - 4*a^3*c + 6*a^2*c^2 - 4*a*c^3 + c^4) + (a + c)*log((sqrt(
b*x + a) - sqrt(b*x + c))^2)/(a^2 - 2*a*c + c^2) + (a + c)*log(abs(b*x))/(
a^2 - 2*a*c + c^2) + 2*(b*x + a)/(a^2 - 2*a*c + c^2)
```

3.409.9 Mupad [B] (verification not implemented)

Time = 25.54 (sec) , antiderivative size = 524, normalized size of antiderivative = 3.94

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{2\,b\,x}{(a-c)^2} - \ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{c+b\,x}-\sqrt{c}}+1\right)\left(\frac{4\,c}{(a-c)^2}+\frac{2}{a-c}\right) - \frac{\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3(4\,a+4\,c)}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^3(a^2-2\,a\,c+c^2)} + \frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)(4\,a+4\,c)}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)(a^2-2\,a\,c+c^2)} - \frac{16\sqrt{a}\sqrt{c}\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^2(a^2-2\,a\,c+c^2)}}{\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^4} - \frac{2\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^2}+1} + \frac{2\,\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{c+b\,x}-\sqrt{c}}-1\right)(a+c)}{(a-c)^2} + \frac{\ln(x)\,(a+c)}{a^2-2\,a\,c+c^2} + \frac{2\sqrt{a}\sqrt{c}\,\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{c+b\,x}-\sqrt{c}}\right)}{(a-c)^2} - \frac{2\sqrt{a}\sqrt{c}\,\ln\left(\frac{a\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{c+b\,x}-\sqrt{c}} - \sqrt{a}\sqrt{c} + \frac{c\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{c+b\,x}-\sqrt{c}} - \frac{\sqrt{a}\sqrt{c}\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)^2}\right)}{a^2-2\,a\,c+c^2}$$

input
```
int(1/(x*((a + b*x)^(1/2) + (c + b*x)^(1/2))^2),x)
```

output
```
(2*b*x)/(a - c)^2 - log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(
1/2)) + 1)*((4*c)/(a - c)^2 + 2/(a - c)) - ((((a + b*x)^(1/2) - a^(1/2))^3
*(4*a + 4*c))/(((c + b*x)^(1/2) - c^(1/2))^3*(a^2 - 2*a*c + c^2)) + (((a +
 b*x)^(1/2) - a^(1/2))*(4*a + 4*c))/(((c + b*x)^(1/2) - c^(1/2))*(a^2 - 2*
a*c + c^2)) - (16*a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(((c + b*
x)^(1/2) - c^(1/2))^2*(a^2 - 2*a*c + c^2)))/((((a + b*x)^(1/2) - a^(1/2))^4
/((c + b*x)^(1/2) - c^(1/2))^4 - (2*((a + b*x)^(1/2) - a^(1/2))^2)/((c + b
*x)^(1/2) - c^(1/2))^2 + 1) + (2*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x
)^(1/2) - c^(1/2)) - 1)*(a + c))/(a - c)^2 + (log(x)*(a + c))/(a^2 - 2*a*c
 + c^2) + (2*a^(1/2)*c^(1/2)*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1
/2) - c^(1/2))))/(a - c)^2 - (2*a^(1/2)*c^(1/2)*log((a*((a + b*x)^(1/2) -
a^(1/2)))/((c + b*x)^(1/2) - c^(1/2)) - a^(1/2)*c^(1/2) + (c*((a + b*x)^(1
/2) - a^(1/2)))/((c + b*x)^(1/2) - c^(1/2)) - (a^(1/2)*c^(1/2)*((a + b*x)^
(1/2) - a^(1/2))^2)/((c + b*x)^(1/2) - c^(1/2))^2))/(a^2 - 2*a*c + c^2)
```

3.409. $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

3.410 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

3.410.1 Optimal result

Integrand size = 25, antiderivative size = 141

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = -\frac{a+c}{(a-c)^2x}+\frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2x}-\frac{4b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{(a-c)^2}+\frac{2b(a+c)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{c+bx}}\right)}{\sqrt{a}(a-c)^2\sqrt{c}}+\frac{2b\log(x)}{(a-c)^2}$$

output
```
(-a-c)/(a-c)^2/x-4*b*arctanh((b*x+a)^(1/2)/(b*x+c)^(1/2))/(a-c)^2+2*b*ln(x
)/(a-c)^2+2*b*(a+c)*arctanh(c^(1/2)*(b*x+a)^(1/2)/a^(1/2)/(b*x+c)^(1/2))/(
a-c)^2/a^(1/2)/c^(1/2)+2*(b*x+a)^(1/2)*(b*x+c)^(1/2)/(a-c)^2/x
```

3.410.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{\frac{2b(a+c)\text{arctanh}\left(\frac{-bx+\sqrt{a+bx}\sqrt{c+bx}}{\sqrt{a}\sqrt{c}}\right)}{\sqrt{a}\sqrt{c}}-\frac{a+c-2bx-2\sqrt{a+bx}\sqrt{c+bx}-2bx\log\left(bx\left(a+c+2bx-2\sqrt{a+bx}\sqrt{c+bx}\right)\right)}{x}}{(a-c)^2}$$

input
```
Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

output
```
((2*b*(a + c)*ArcTanh[(-(b*x) + Sqrt[a + b*x]*Sqrt[c + b*x])/(Sqrt[a]*Sqrt
[c])])/(Sqrt[a]*Sqrt[c]) - (a + c - 2*b*x - 2*Sqrt[a + b*x]*Sqrt[c + b*x]
- 2*b*x*Log[b*x*(a + c + 2*b*x - 2*Sqrt[a + b*x]*Sqrt[c + b*x])])/x)/(a -
c)^2
```

3.410.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.81, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7240, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}dx$$

$$\Big\downarrow \text{ 7240}$$

$$\frac{\int \left(\frac{2b}{x}-\frac{2\sqrt{a+bx}\sqrt{c+bx}}{x^2}+\frac{a+c}{x^2}\right)dx}{(a-c)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{2b(a+c)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{bx+c}}\right)}{\sqrt{a}\sqrt{c}}-4b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)+\frac{2\sqrt{a+bx}\sqrt{bx+c}}{x}-\frac{a+c}{x}+2b\log(x)}{(a-c)^2}$$

input
```
Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

output
```
(-((a + c)/x) + (2*Sqrt[a + b*x]*Sqrt[c + b*x])/x - 4*b*ArcTanh[Sqrt[a + b
*x]/Sqrt[c + b*x]] + (2*b*(a + c)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[a]
*Sqrt[c + b*x])])/(Sqrt[a]*Sqrt[c]) + 2*b*Log[x])/(a - c)^2
```

3.410.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7240
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(a*e^2 - c*f^2)^m   Int[ExpandIntegran
d[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c
, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

3.410.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.94

method	result
default	$-\frac{a}{x(a-c)^2}-\frac{c}{x(a-c)^2}+\frac{2b\ln(x)}{(a-c)^2}+\frac{\sqrt{bx+a}\,\sqrt{bx+c}\left(\operatorname{csgn}(b)\ln\left(\frac{abx+bcx+2\sqrt{ac}\,\sqrt{b^2x^2+abx+bcx+ac}+2ac}{x}\right)xab+\operatorname{csgn}(b)\ln\left(\frac{abx+\cdots}{}\right.\right.}{}$

input
```
int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x/(a-c)^2*a-1/x/(a-c)^2*c+2*b*ln(x)/(a-c)^2+1/(a-c)^2*(b*x+a)^(1/2)*(b*
x+c)^(1/2)*(csgn(b)*ln((a*b*x+b*c*x+2*(a*c)^(1/2)*(b^2*x^2+a*b*x+b*c*x+a*c
)^(1/2)+2*a*c)/x)*x*a*b+csgn(b)*ln((a*b*x+b*c*x+2*(a*c)^(1/2)*(b^2*x^2+a*b
*x+b*c*x+a*c)^(1/2)+2*a*c)/x)*x*b*c-2*ln(1/2*(2*(b^2*x^2+a*b*x+b*c*x+a*c)^
(1/2)*csgn(b)+2*b*x+a+c)*csgn(b))*x*b*(a*c)^(1/2)+2*csgn(b)*(b^2*x^2+a*b*x
+b*c*x+a*c)^(1/2)*(a*c)^(1/2))*csgn(b)/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)/x/(
a*c)^(1/2)
```

3.410.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.60

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$$

$$=\left[\frac{2\,abcx\log\left(-2\,bx+2\,\sqrt{bx+a}\sqrt{bx+c}-a-c\right)+2\,abcx\log\left(x\right)+2\,abcx+(ab+bc)\sqrt{ac}x\log\left(\frac{2\,a^2c+2\,ac\cdots}{}\right.}{(a^3c-2\,a^2c^2\cdots}\right.$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
[(2*a*b*c*x*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c) + 2*a*b*c*
x*log(x) + 2*a*b*c*x + (a*b + b*c)*sqrt(a*c)*x*log((2*a^2*c + 2*a*c^2 + 2*
(2*a*c + sqrt(a*c)*(a + c))*sqrt(b*x + a)*sqrt(b*x + c) + (a^2*b + 2*a*b*c
 + b*c^2)*x + 2*(2*a*c + (a*b + b*c)*x)*sqrt(a*c))/x) + 2*sqrt(b*x + a)*sq
rt(b*x + c)*a*c - a^2*c - a*c^2)/((a^3*c - 2*a^2*c^2 + a*c^3)*x), (2*a*b*c
*x*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c) + 2*a*b*c*x*log(x)
+ 2*a*b*c*x - 2*(a*b + b*c)*sqrt(-a*c)*x*arctan(-(sqrt(-a*c)*b*x - sqrt(-a
*c)*sqrt(b*x + a)*sqrt(b*x + c))/(a*c)) + 2*sqrt(b*x + a)*sqrt(b*x + c)*a*
c - a^2*c - a*c^2)/((a^3*c - 2*a^2*c^2 + a*c^3)*x)]
```

3.410.6 Sympy [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$

input
```
integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

output
```
Integral(1/(x**2*(sqrt(a + b*x) + sqrt(b*x + c))**2), x)
```

3.410.7 Maxima [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^2}\,dx$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))^2), x)
```

3.410.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(121) = 242$.

Time = 0.77 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.21

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{2\,b\log\left(\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2\right)}{a^2-2\,ac+c^2} + \frac{2\,b\log\left(|bx|\right)}{a^2-2\,ac+c^2} + \frac{2\,(ab+bc)\arctan\left(\frac{\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2-a-c}{2\sqrt{-ac}}\right)}{(a^2-2\,ac+c^2)\sqrt{-ac}} - \frac{4\left(ab\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2+bc\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2-a^2b+2\,abc-bc^2\right)}{\left(\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^4-2\,a\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2-2\,c\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2+a^2-2\,ac+c^2\right)(a^2} - \frac{2\,(bx+a)b-ab+bc}{(a^2-2\,ac+c^2)bx}$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
2*b*log((sqrt(b*x + a) - sqrt(b*x + c))^2)/(a^2 - 2*a*c + c^2) + 2*b*log(a
bs(b*x))/(a^2 - 2*a*c + c^2) + 2*(a*b + b*c)*arctan(1/2*((sqrt(b*x + a) -
sqrt(b*x + c))^2 - a - c)/sqrt(-a*c))/((a^2 - 2*a*c + c^2)*sqrt(-a*c)) - 4
*(a*b*(sqrt(b*x + a) - sqrt(b*x + c))^2 + b*c*(sqrt(b*x + a) - sqrt(b*x +
c))^2 - a^2*b + 2*a*b*c - b*c^2)/(((sqrt(b*x + a) - sqrt(b*x + c))^4 - 2*a
*(sqrt(b*x + a) - sqrt(b*x + c))^2 - 2*c*(sqrt(b*x + a) - sqrt(b*x + c))^2
 + a^2 - 2*a*c + c^2)*(a^2 - 2*a*c + c^2)) - (2*(b*x + a)*b - a*b + b*c)/(
(a^2 - 2*a*c + c^2)*b*x)
```

3.410.9 Mupad [B] (verification not implemented)

Time = 44.38 (sec) , antiderivative size = 7637, normalized size of antiderivative = 54.16

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x^2*((a + b*x)^(1/2) + (c + b*x)^(1/2))^2),x)
```

output
```
(2*b*log(x))/(a^2 - 2*a*c + c^2) - ((((a + b*x)^(1/2) - a^(1/2))^2*((a^2*b
)/2 + (b*c^2)/2 - (3*a*b*c)/2))/(((c + b*x)^(1/2) - c^(1/2))^2*(a*c^3 + a^
3*c - 2*a^2*c^2)) - b/(2*(a^2 - 2*a*c + c^2)) + (a^(1/2)*c^(1/2)*((a*b)/2
+ (b*c)/2)*((a + b*x)^(1/2) - a^(1/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a*c^
3 + a^3*c - 2*a^2*c^2)))/(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c
^(1/2)) + ((a + b*x)^(1/2) - a^(1/2))^3/((c + b*x)^(1/2) - c^(1/2))^3 - ((
a + c)*((a + b*x)^(1/2) - a^(1/2))^2)/(a^(1/2)*c^(1/2)*((c + b*x)^(1/2) -
c^(1/2))^2)) + (b*atan(((b*((4*(4*a^4*b^3*c^12 + 8*a^5*b^3*c^11 - 32*a^6*b
^3*c^10 - 8*a^7*b^3*c^9 + 56*a^8*b^3*c^8 - 8*a^9*b^3*c^7 - 32*a^10*b^3*c^6
 + 8*a^11*b^3*c^5 + 4*a^12*b^3*c^4))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13
- 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 +
a^15*c^7) + (4*b*((4*b*((4*(16*a^6*b*c^14 - 4*a^5*b*c^15 + 12*a^7*b*c^13 -
 192*a^8*b*c^12 + 504*a^9*b*c^11 - 672*a^10*b*c^10 + 504*a^11*b*c^9 - 192*
a^12*b*c^8 + 12*a^13*b*c^7 + 16*a^14*b*c^6 - 4*a^15*b*c^5))/(a^7*c^15 - 8*
a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a
^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*((4*(a^(9/2)*c^(35/2) - 8*a^(11/2)
*c^(33/2) + 27*a^(13/2)*c^(31/2) - 49*a^(15/2)*c^(29/2) + 50*a^(17/2)*c^(2
7/2) - 27*a^(19/2)*c^(25/2) + 6*a^(21/2)*c^(23/2) + 6*a^(23/2)*c^(21/2) -
27*a^(25/2)*c^(19/2) + 50*a^(27/2)*c^(17/2) - 49*a^(29/2)*c^(15/2) + 27*a^
(31/2)*c^(13/2) - 8*a^(33/2)*c^(11/2) + a^(35/2)*c^(9/2)))/(a^7*c^15 - ...
```

3.410. $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

3.411 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

3.411.1 Optimal result

Integrand size = 25, antiderivative size = 375

$$\begin{aligned}\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = &-\frac{8a^3(a+bx)^{3/2}}{3b^3(a-c)^3}+\frac{2a^2(a+3c)(a+bx)^{3/2}}{3b^3(a-c)^3}\\ &+\frac{24a^2(a+bx)^{5/2}}{5b^3(a-c)^3}-\frac{4a(a+3c)(a+bx)^{5/2}}{5b^3(a-c)^3}\\ &-\frac{24a(a+bx)^{7/2}}{7b^3(a-c)^3}+\frac{2(a+3c)(a+bx)^{7/2}}{7b^3(a-c)^3}+\frac{8(a+bx)^{9/2}}{9b^3(a-c)^3}\\ &+\frac{8c^3(c+bx)^{3/2}}{3b^3(a-c)^3}-\frac{2c^2(3a+c)(c+bx)^{3/2}}{3b^3(a-c)^3}\\ &-\frac{24c^2(c+bx)^{5/2}}{5b^3(a-c)^3}+\frac{4c(3a+c)(c+bx)^{5/2}}{5b^3(a-c)^3}\\ &+\frac{24c(c+bx)^{7/2}}{7b^3(a-c)^3}-\frac{2(3a+c)(c+bx)^{7/2}}{7b^3(a-c)^3}-\frac{8(c+bx)^{9/2}}{9b^3(a-c)^3}\end{aligned}$$

output
```
-8/3*a^3*(b*x+a)^(3/2)/b^3/(a-c)^3+2/3*a^2*(a+3*c)*(b*x+a)^(3/2)/b^3/(a-c)
^3+24/5*a^2*(b*x+a)^(5/2)/b^3/(a-c)^3-4/5*a*(a+3*c)*(b*x+a)^(5/2)/b^3/(a-c
)^3-24/7*a*(b*x+a)^(7/2)/b^3/(a-c)^3+2/7*(a+3*c)*(b*x+a)^(7/2)/b^3/(a-c)^3
+8/9*(b*x+a)^(9/2)/b^3/(a-c)^3+8/3*c^3*(b*x+c)^(3/2)/b^3/(a-c)^3-2/3*c^2*(
3*a+c)*(b*x+c)^(3/2)/b^3/(a-c)^3-24/5*c^2*(b*x+c)^(5/2)/b^3/(a-c)^3+4/5*c*
(3*a+c)*(b*x+c)^(5/2)/b^3/(a-c)^3+24/7*c*(b*x+c)^(7/2)/b^3/(a-c)^3-2/7*(3*
a+c)*(b*x+c)^(7/2)/b^3/(a-c)^3-8/9*(b*x+c)^(9/2)/b^3/(a-c)^3
```

3.411.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.37

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$$
$$= \frac{2\left((a+bx)^{3/2}\left(-40a^3+12a^2(6c+5bx)-3abx(36c+25bx)+5b^2x^2(27c+28bx)\right)+(c+bx)^{3/2}\left(-9a(8c^2\right.\right.}{315b^3(a-c)^3}$$

input
```
Integrate[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

output
```
(2*((a + b*x)^(3/2)*(-40*a^3 + 12*a^2*(6*c + 5*b*x) - 3*a*b*x*(36*c + 25*b
*x) + 5*b^2*x^2*(27*c + 28*b*x)) + (c + b*x)^(3/2)*(-9*a*(8*c^2 - 12*b*c*x
 + 15*b^2*x^2) + 5*(8*c^3 - 12*b*c^2*x + 15*b^2*c*x^2 - 28*b^3*x^3))))/(31
5*b^3*(a - c)^3)
```

3.411.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.76, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7240, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}\,dx$$
$$\Big\downarrow \text{7240}$$
$$\frac{\int \left(4b\sqrt{a+bx}x^3-4b\sqrt{c+bx}x^3+(a+3c)\sqrt{a+bx}x^2-(3a+c)\sqrt{c+bx}x^2\right)dx}{(a-c)^3}$$
$$\Big\downarrow \text{2009}$$
$$\frac{-\frac{8a^3(a+bx)^{3/2}}{3b^3}+\frac{2a^2(a+3c)(a+bx)^{3/2}}{3b^3}+\frac{24a^2(a+bx)^{5/2}}{5b^3}-\frac{2c^2(3a+c)(bx+c)^{3/2}}{3b^3}+\frac{2(a+3c)(a+bx)^{7/2}}{7b^3}-\frac{4a(a+3c)(a+bx)^{5/2}}{5b^3}-\frac{2(3a+c}{}}{(a-c}$$

input
```
Int[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

output
```
((-8*a^3*(a + b*x)^(3/2))/(3*b^3) + (2*a^2*(a + 3*c)*(a + b*x)^(3/2))/(3*b
^3) + (24*a^2*(a + b*x)^(5/2))/(5*b^3) - (4*a*(a + 3*c)*(a + b*x)^(5/2))/(
5*b^3) - (24*a*(a + b*x)^(7/2))/(7*b^3) + (2*(a + 3*c)*(a + b*x)^(7/2))/(7
*b^3) + (8*(a + b*x)^(9/2))/(9*b^3) + (8*c^3*(c + b*x)^(3/2))/(3*b^3) - (2
*c^2*(3*a + c)*(c + b*x)^(3/2))/(3*b^3) - (24*c^2*(c + b*x)^(5/2))/(5*b^3)
 + (4*c*(3*a + c)*(c + b*x)^(5/2))/(5*b^3) + (24*c*(c + b*x)^(7/2))/(7*b^3
) - (2*(3*a + c)*(c + b*x)^(7/2))/(7*b^3) - (8*(c + b*x)^(9/2))/(9*b^3))/(
a - c)^3
```

3.411.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7240
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(a*e^2 - c*f^2)^m   Int[ExpandIntegran
d[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c
, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

3.411.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.78

method	result
default	$\frac{2a\left(\frac{(bx+a)^{\frac{7}{2}}}{7}-\frac{2(bx+a)^{\frac{5}{2}}a}{5}+\frac{(bx+a)^{\frac{3}{2}}a^2}{3}\right)}{(a-c)^3b^3}+\frac{6c\left(\frac{(bx+a)^{\frac{7}{2}}}{7}-\frac{2(bx+a)^{\frac{5}{2}}a}{5}+\frac{(bx+a)^{\frac{3}{2}}a^2}{3}\right)}{(a-c)^3b^3}-\frac{6a\left(\frac{(bx+c)^{\frac{7}{2}}}{7}-\frac{2c(bx+c)^{\frac{5}{2}}}{5}+\frac{c^2(bx+c)^{\frac{3}{2}}}{3}\right)}{(a-c)^3b^3}$

input
```
int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
2/(a-c)^3*a/b^3*(1/7*(b*x+a)^(7/2)-2/5*(b*x+a)^(5/2)*a+1/3*(b*x+a)^(3/2)*a
^2)+6/(a-c)^3*c/b^3*(1/7*(b*x+a)^(7/2)-2/5*(b*x+a)^(5/2)*a+1/3*(b*x+a)^(3/
2)*a^2)-6/(a-c)^3*a/b^3*(1/7*(b*x+c)^(7/2)-2/5*c*(b*x+c)^(5/2)+1/3*c^2*(b*
x+c)^(3/2))-2/(a-c)^3*c/b^3*(1/7*(b*x+c)^(7/2)-2/5*c*(b*x+c)^(5/2)+1/3*c^2
*(b*x+c)^(3/2))+8/(a-c)^3/b^3*(1/9*(b*x+a)^(9/2)-3/7*a*(b*x+a)^(7/2)+3/5*a
^2*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a^3)-8/(a-c)^3/b^3*(1/9*(b*x+c)^(9/2)-3
/7*c*(b*x+c)^(7/2)+3/5*c^2*(b*x+c)^(5/2)-1/3*c^3*(b*x+c)^(3/2))
```

3.411.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.55

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{2\left(\left(140\,b^4x^4-40\,a^4+72\,a^3c+5\,(13\,ab^3+27\,b^3c)x^3-3\,(5\,a^2b^2-9\,ab^2c)x^2+4\,(5\,a^3b-9\,a^2bc)x\right)\sqrt{bx+c}}{315\,(a^3b^3-3\,a^2b^3}$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

output
```
2/315*((140*b^4*x^4 - 40*a^4 + 72*a^3*c + 5*(13*a*b^3 + 27*b^3*c)*x^3 - 3*
(5*a^2*b^2 - 9*a*b^2*c)*x^2 + 4*(5*a^3*b - 9*a^2*b*c)*x)*sqrt(b*x + a) - (
140*b^4*x^4 + 72*a*c^3 - 40*c^4 + 5*(27*a*b^3 + 13*b^3*c)*x^3 + 3*(9*a*b^2
*c - 5*b^2*c^2)*x^2 - 4*(9*a*b*c^2 - 5*b*c^3)*x)*sqrt(b*x + c))/(a^3*b^3 -
 3*a^2*b^3*c + 3*a*b^3*c^2 - b^3*c^3)
```

3.411.6 Sympy [F]

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}\,dx$$

input
```
integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

output
```
Integral(x**2/(sqrt(a + b*x) + sqrt(b*x + c))**3, x)
```

3.411.7 Maxima [F]

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \int \frac{x^2}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}\,dx$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c))^3, x)
```

3.411.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1447 vs. $2(319) = 638$.

Time = 0.69 (sec) , antiderivative size = 1447, normalized size of antiderivative = 3.86

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

output
```
-2/315*(((5*(b*x + a)*(28*(a^9*b^12 - 9*a^8*b^12*c + 36*a^7*b^12*c^2 - 84*
a^6*b^12*c^3 + 126*a^5*b^12*c^4 - 126*a^4*b^12*c^5 + 84*a^3*b^12*c^6 - 36*
a^2*b^12*c^7 + 9*a*b^12*c^8 - b^12*c^9)*(b*x + a)/(a^12*b^15 - 12*a^11*b^1
5*c + 66*a^10*b^15*c^2 - 220*a^9*b^15*c^3 + 495*a^8*b^15*c^4 - 792*a^7*b^1
5*c^5 + 924*a^6*b^15*c^6 - 792*a^5*b^15*c^7 + 495*a^4*b^15*c^8 - 220*a^3*b
^15*c^9 + 66*a^2*b^15*c^10 - 12*a*b^15*c^11 + b^15*c^12) - (85*a^10*b^12 -
 778*a^9*b^12*c + 3177*a^8*b^12*c^2 - 7608*a^7*b^12*c^3 + 11802*a^6*b^12*c
^4 - 12348*a^5*b^12*c^5 + 8778*a^4*b^12*c^6 - 4152*a^3*b^12*c^7 + 1233*a^2
*b^12*c^8 - 202*a*b^12*c^9 + 13*b^12*c^10)/(a^12*b^15 - 12*a^11*b^15*c + 6
6*a^10*b^15*c^2 - 220*a^9*b^15*c^3 + 495*a^8*b^15*c^4 - 792*a^7*b^15*c^5 +
 924*a^6*b^15*c^6 - 792*a^5*b^15*c^7 + 495*a^4*b^15*c^8 - 220*a^3*b^15*c^9
 + 66*a^2*b^15*c^10 - 12*a*b^15*c^11 + b^15*c^12)) + 3*(145*a^11*b^12 - 13
61*a^10*b^12*c + 5719*a^9*b^12*c^2 - 14151*a^8*b^12*c^3 + 22794*a^7*b^12*c
^4 - 24906*a^6*b^12*c^5 + 18606*a^5*b^12*c^6 - 9294*a^4*b^12*c^7 + 2901*a^
3*b^12*c^8 - 469*a^2*b^12*c^9 + 11*a*b^12*c^10 + 5*b^12*c^11)/(a^12*b^15 -
 12*a^11*b^15*c + 66*a^10*b^15*c^2 - 220*a^9*b^15*c^3 + 495*a^8*b^15*c^4 -
 792*a^7*b^15*c^5 + 924*a^6*b^15*c^6 - 792*a^5*b^15*c^7 + 495*a^4*b^15*c^8
 - 220*a^3*b^15*c^9 + 66*a^2*b^15*c^10 - 12*a*b^15*c^11 + b^15*c^12))*(b*x
 + a) - (155*a^12*b^12 - 1536*a^11*b^12*c + 6855*a^10*b^12*c^2 - 18170*a^9
*b^12*c^3 + 31770*a^8*b^12*c^4 - 38520*a^7*b^12*c^5 + 33222*a^6*b^12*c^...
```

3.411.9 Mupad [B] (verification not implemented)

Time = 16.44 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.41

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{x^3\left(\frac{64\,b\,c}{9\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)\sqrt{c+b\,x}}{7\,b} - \frac{x^3\left(\frac{64\,a\,b}{9\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)\sqrt{a+b\,x}}{7\,b} - \frac{8\,c^2\left(\frac{2\,c\,(3\,a+c)}{(a-c)^3}+\frac{6\,c\left(\frac{64\,b\,c}{9\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)}{7\,b}\right)\sqrt{c+b\,x}}{15\,b^3} - \frac{x^2\left(\frac{2\,c\,(3\,a+c)}{(a-c)^3}+\frac{6\,c\left(\frac{64\,b\,c}{9\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)}{7\,b}\right)\sqrt{c+b\,x}}{5\,b} + \frac{8\,a^2\left(\frac{2\,(a^2+3\,c\,a)}{(a-c)^3}+\frac{6\,a\left(\frac{64\,a\,b}{9\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)}{7\,b}\right)\sqrt{a+b\,x}}{15\,b^3} + \frac{x^2\left(\frac{2\,(a^2+3\,c\,a)}{(a-c)^3}+\frac{6\,a\left(\frac{64\,a\,b}{9\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)}{7\,b}\right)\sqrt{a+b\,x}}{5\,b} + \frac{8\,b\,x^4\sqrt{a+b\,x}}{9\,(a-c)^3} - \frac{8\,b\,x^4\sqrt{c+b\,x}}{9\,(a-c)^3} + \frac{4\,c\,x\left(\frac{2\,c\,(3\,a+c)}{(a-c)^3}+\frac{6\,c\left(\frac{64\,b\,c}{9\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)}{7\,b}\right)\sqrt{c+b\,x}}{15\,b^2} - \frac{4\,a\,x\left(\frac{2\,(a^2+3\,c\,a)}{(a-c)^3}+\frac{6\,a\left(\frac{64\,a\,b}{9\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)}{7\,b}\right)\sqrt{a+b\,x}}{15\,b^2}$$

input
```
int(x^2/((a + b*x)^(1/2) + (c + b*x)^(1/2))^3,x)
```

output
```
(x^3*((64*b*c)/(9*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3)*(c + b*x)^(1/2
))/(7*b) - (x^3*((64*a*b)/(9*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3)*(a
+ b*x)^(1/2))/(7*b) - (8*c^2*((2*c*(3*a + c))/(a - c)^3 + (6*c*((64*b*c)/(
9*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(7*b))*(c + b*x)^(1/2))/(15*b
^3) - (x^2*((2*c*(3*a + c))/(a - c)^3 + (6*c*((64*b*c)/(9*(a - c)^3) - (2*
b*(3*a + 5*c))/(a - c)^3))/(7*b))*(c + b*x)^(1/2))/(5*b) + (8*a^2*((2*(3*a
*c + a^2))/(a - c)^3 + (6*a*((64*a*b)/(9*(a - c)^3) - (2*b*(5*a + 3*c))/(a
 - c)^3))/(7*b))*(a + b*x)^(1/2))/(15*b^3) + (x^2*((2*(3*a*c + a^2))/(a -
c)^3 + (6*a*((64*a*b)/(9*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3))/(7*b))
*(a + b*x)^(1/2))/(5*b) + (8*b*x^4*(a + b*x)^(1/2))/(9*(a - c)^3) - (8*b*x
^4*(c + b*x)^(1/2))/(9*(a - c)^3) + (4*c*x*((2*c*(3*a + c))/(a - c)^3 + (6
*c*((64*b*c)/(9*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(7*b))*(c + b*x
)^(1/2))/(15*b^2) - (4*a*x*((2*(3*a*c + a^2))/(a - c)^3 + (6*a*((64*a*b)/(
9*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3))/(7*b))*(a + b*x)^(1/2))/(15*b
^2)
```

3.411. $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

3.412 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

3.412.1 Optimal result

Integrand size = 23, antiderivative size = 261

$$\begin{aligned}\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx &= \frac{8a^2(a+bx)^{3/2}}{3b^2(a-c)^3}-\frac{2a(a+3c)(a+bx)^{3/2}}{3b^2(a-c)^3}\\&\quad-\frac{16a(a+bx)^{5/2}}{5b^2(a-c)^3}+\frac{2(a+3c)(a+bx)^{5/2}}{5b^2(a-c)^3}+\frac{8(a+bx)^{7/2}}{7b^2(a-c)^3}\\&\quad-\frac{8c^2(c+bx)^{3/2}}{3b^2(a-c)^3}+\frac{2c(3a+c)(c+bx)^{3/2}}{3b^2(a-c)^3}\\&\quad+\frac{16c(c+bx)^{5/2}}{5b^2(a-c)^3}-\frac{2(3a+c)(c+bx)^{5/2}}{5b^2(a-c)^3}-\frac{8(c+bx)^{7/2}}{7b^2(a-c)^3}\end{aligned}$$

output
```
8/3*a^2*(b*x+a)^(3/2)/b^2/(a-c)^3-2/3*a*(a+3*c)*(b*x+a)^(3/2)/b^2/(a-c)^3-
16/5*a*(b*x+a)^(5/2)/b^2/(a-c)^3+2/5*(a+3*c)*(b*x+a)^(5/2)/b^2/(a-c)^3+8/7
*(b*x+a)^(7/2)/b^2/(a-c)^3-8/3*c^2*(b*x+c)^(3/2)/b^2/(a-c)^3+2/3*c*(3*a+c)
*(b*x+c)^(3/2)/b^2/(a-c)^3+16/5*c*(b*x+c)^(5/2)/b^2/(a-c)^3-2/5*(3*a+c)*(b
*x+c)^(5/2)/b^2/(a-c)^3-8/7*(b*x+c)^(7/2)/b^2/(a-c)^3
```

3.412.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.36

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{2\left((c+bx)^{3/2}\left(-6c^2+9bcx-20b^2x^2+7a(2c-3bx)\right)+(a+bx)^{3/2}\left(6a^2-a(14c+9bx)+bx(21c+20bx\right.\right.}{35b^2(a-c)^3}$$

input
```
Integrate[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

output
```
(2*((c + b*x)^(3/2)*(-6*c^2 + 9*b*c*x - 20*b^2*x^2 + 7*a*(2*c - 3*b*x)) +
(a + b*x)^(3/2)*(6*a^2 - a*(14*c + 9*b*x) + b*x*(21*c + 20*b*x))))/(35*b^2
*(a - c)^3)
```

3.412.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.76, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {7240, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}\,dx$$

$$\Big\downarrow \text{7240}$$

$$\frac{\int \left(4b\sqrt{a+bx}x^2-4b\sqrt{c+bx}x^2+(a+3c)\sqrt{a+bx}x-(3a+c)\sqrt{c+bx}x\right)dx}{(a-c)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{8a^2(a+bx)^{3/2}}{3b^2}+\frac{2(a+3c)(a+bx)^{5/2}}{5b^2}-\frac{2a(a+3c)(a+bx)^{3/2}}{3b^2}-\frac{2(3a+c)(bx+c)^{5/2}}{5b^2}+\frac{2c(3a+c)(bx+c)^{3/2}}{3b^2}+\frac{8(a+bx)^{7/2}}{7b^2}-\frac{16a(a+bx)^{5/2}}{5b^2}-}{(a-c)^3}$$

input
```
Int[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

output
```
((8*a^2*(a + b*x)^(3/2))/(3*b^2) - (2*a*(a + 3*c)*(a + b*x)^(3/2))/(3*b^2)
 - (16*a*(a + b*x)^(5/2))/(5*b^2) + (2*(a + 3*c)*(a + b*x)^(5/2))/(5*b^2)
+ (8*(a + b*x)^(7/2))/(7*b^2) - (8*c^2*(c + b*x)^(3/2))/(3*b^2) + (2*c*(3*
a + c)*(c + b*x)^(3/2))/(3*b^2) + (16*c*(c + b*x)^(5/2))/(5*b^2) - (2*(3*a
 + c)*(c + b*x)^(5/2))/(5*b^2) - (8*(c + b*x)^(7/2))/(7*b^2))/(a - c)^3
```

3.412.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7240
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(a*e^2 - c*f^2)^m   Int[ExpandIntegran
d[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c
, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

3.412.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.85

method	result
default	$\frac{2a\left(\frac{(bx+a)^{\frac{5}{2}}}{5}-\frac{(bx+a)^{\frac{3}{2}}a}{3}\right)}{(a-c)^{3}b^{2}}+\frac{6c\left(\frac{(bx+a)^{\frac{5}{2}}}{5}-\frac{(bx+a)^{\frac{3}{2}}a}{3}\right)}{(a-c)^{3}b^{2}}-\frac{6a\left(\frac{(bx+c)^{\frac{5}{2}}}{5}-\frac{c(bx+c)^{\frac{3}{2}}}{3}\right)}{(a-c)^{3}b^{2}}-\frac{2c\left(\frac{(bx+c)^{\frac{5}{2}}}{5}-\frac{c(bx+c)^{\frac{3}{2}}}{3}\right)}{(a-c)^{3}b^{2}}+\frac{8(bx+a)}{7}$

input
```
int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
2/(a-c)^3*a/b^2*(1/5*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a)+6/(a-c)^3*c/b^2*(1
/5*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a)-6/(a-c)^3*a/b^2*(1/5*(b*x+c)^(5/2)-1
/3*c*(b*x+c)^(3/2))-2/(a-c)^3*c/b^2*(1/5*(b*x+c)^(5/2)-1/3*c*(b*x+c)^(3/2)
)+8/(a-c)^3/b^2*(1/7*(b*x+a)^(7/2)-2/5*(b*x+a)^(5/2)*a+1/3*(b*x+a)^(3/2)*a
^2)-8/(a-c)^3/b^2*(1/7*(b*x+c)^(7/2)-2/5*c*(b*x+c)^(5/2)+1/3*c^2*(b*x+c)^(
3/2))
```

3.412.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.61

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$$
$$= \frac{2\left((20\,b^3x^3+6\,a^3-14\,a^2c+(11\,ab^2+21\,b^2c)x^2-(3\,a^2b-7\,abc)x)\sqrt{bx+a}-(20\,b^3x^3-14\,ac^2+6\,c^3+\right.}{35\left(a^3b^2-3\,a^2b^2c+3\,ab^2c^2-b^2c^3\right)}$$

input
```
integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

output
```
2/35*((20*b^3*x^3 + 6*a^3 - 14*a^2*c + (11*a*b^2 + 21*b^2*c)*x^2 - (3*a^2*
b - 7*a*b*c)*x)*sqrt(b*x + a) - (20*b^3*x^3 - 14*a*c^2 + 6*c^3 + (21*a*b^2
 + 11*b^2*c)*x^2 + (7*a*b*c - 3*b*c^2)*x)*sqrt(b*x + c))/(a^3*b^2 - 3*a^2*
b^2*c + 3*a*b^2*c^2 - b^2*c^3)
```

3.412.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 942 vs. 2(240) = 480.

Time = 0.75 (sec) , antiderivative size = 942, normalized size of antiderivative = 3.61

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$$
$$= \begin{cases} \frac{12a^2}{35ab^2\sqrt{a+bx}+105ab^2\sqrt{bx+c}+140b^3x\sqrt{a+bx}+140b^3x\sqrt{bx+c}+105b^2c\sqrt{a+bx}+35b^2c\sqrt{bx+c}} + \frac{54}{35ab^2\sqrt{a+bx}+105ab^2\sqrt{bx+c}+140b^3x\sqrt{a+bx}} \\ \frac{x^2}{2\left(\sqrt{a}+\sqrt{c}\right)^3} \end{cases}$$

input
```
integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

output
```
Piecewise((12*a**2/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 1
40*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b
*x) + 35*b**2*c*sqrt(b*x + c)) + 54*a*b*x/(35*a*b**2*sqrt(a + b*x) + 105*a
*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c)
+ 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 44*a*c/(35*a*b**2*
sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*
b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c))
 + 36*a*sqrt(a + b*x)*sqrt(b*x + c)/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*
sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105*
b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 40*b**2*x**2/(35*a*b**2*
sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*
b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c))
 + 54*b*c*x/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3
*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 3
5*b**2*c*sqrt(b*x + c)) + 30*b*x*sqrt(a + b*x)*sqrt(b*x + c)/(35*a*b**2*sq
rt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b*
*3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) +
 12*c**2/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*
sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b
**2*c*sqrt(b*x + c)) + 36*c*sqrt(a + b*x)*sqrt(b*x + c)/(35*a*b**2*sqrt...
```

3.412.7 Maxima [F]

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \int \frac{x}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}\,dx$$

input
```
integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(b*x + a) + sqrt(b*x + c))^3, x)
```

3.412.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(221) = 442$.

Time = 0.35 (sec) , antiderivative size = 866, normalized size of antiderivative = 3.32

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx =$$

$$-\frac{2\left(\left(\left((bx+a)\left(\frac{20\left(a^6b^3-6a^5b^3c+15a^4b^3c^2-20a^3b^3c^3+15a^2b^3c^4-6ab^3c^5+b^3c^6\right)(bx+a)}{a^9b^4-9a^8b^4c+36a^7b^4c^2-84a^6b^4c^3+126a^5b^4c^4-126a^4b^4c^5+84a^3b^4c^6-36a^2b^4c^7+9ab^4c^8-b^4c^9}-\frac{39a^7b^3-2}{a^9b^4-9a^8b^4c+}\right.\right.\right.\right.}{}$$

input
```
integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

output
```
-2/35*((((b*x + a)*(20*(a^6*b^3 - 6*a^5*b^3*c + 15*a^4*b^3*c^2 - 20*a^3*b^
3*c^3 + 15*a^2*b^3*c^4 - 6*a*b^3*c^5 + b^3*c^6)*(b*x + a)/(a^9*b^4 - 9*a^8
*b^4*c + 36*a^7*b^4*c^2 - 84*a^6*b^4*c^3 + 126*a^5*b^4*c^4 - 126*a^4*b^4*c
^5 + 84*a^3*b^4*c^6 - 36*a^2*b^4*c^7 + 9*a*b^4*c^8 - b^4*c^9) - (39*a^7*b^
3 - 245*a^6*b^3*c + 651*a^5*b^3*c^2 - 945*a^4*b^3*c^3 + 805*a^3*b^3*c^4 -
399*a^2*b^3*c^5 + 105*a*b^3*c^6 - 11*b^3*c^7)/(a^9*b^4 - 9*a^8*b^4*c + 36*
a^7*b^4*c^2 - 84*a^6*b^4*c^3 + 126*a^5*b^4*c^4 - 126*a^4*b^4*c^5 + 84*a^3*
b^4*c^6 - 36*a^2*b^4*c^7 + 9*a*b^4*c^8 - b^4*c^9)) + 3*(6*a^8*b^3 - 41*a^7
*b^3*c + 119*a^6*b^3*c^2 - 189*a^5*b^3*c^3 + 175*a^4*b^3*c^4 - 91*a^3*b^3*
c^5 + 21*a^2*b^3*c^6 + a*b^3*c^7 - b^3*c^8)/(a^9*b^4 - 9*a^8*b^4*c + 36*a^
7*b^4*c^2 - 84*a^6*b^4*c^3 + 126*a^5*b^4*c^4 - 126*a^4*b^4*c^5 + 84*a^3*b^
4*c^6 - 36*a^2*b^4*c^7 + 9*a*b^4*c^8 - b^4*c^9))*(b*x + a) + (a^9*b^3 - 2*
a^8*b^3*c - 20*a^7*b^3*c^2 + 112*a^6*b^3*c^3 - 266*a^5*b^3*c^4 + 364*a^4*b
^3*c^5 - 308*a^3*b^3*c^6 + 160*a^2*b^3*c^7 - 47*a*b^3*c^8 + 6*b^3*c^9)/(a^
9*b^4 - 9*a^8*b^4*c + 36*a^7*b^4*c^2 - 84*a^6*b^4*c^3 + 126*a^5*b^4*c^4 -
126*a^4*b^4*c^5 + 84*a^3*b^4*c^6 - 36*a^2*b^4*c^7 + 9*a*b^4*c^8 - b^4*c^9)
)*sqrt(b*x + c) - (20*(b*x + a)^(7/2) - 49*(b*x + a)^(5/2)*a + 35*(b*x + a
)^(3/2)*a^2 + 21*(b*x + a)^(5/2)*c - 35*(b*x + a)^(3/2)*a*c)/(a^3*b - 3*a^
2*b*c + 3*a*b*c^2 - b*c^3))/b
```

3.412.9 Mupad [B] (verification not implemented)

Time = 16.83 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.48

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{x^2\left(\frac{48\,b\,c}{7\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)\sqrt{c+b\,x}}{5\,b} - \frac{x^2\left(\frac{48\,a\,b}{7\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)\sqrt{a+b\,x}}{5\,b} - \frac{2\,a\left(\frac{2\,a\,(a+3\,c)}{(a-c)^3}+\frac{4\,a\left(\frac{48\,a\,b}{7\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)}{5\,b}\right)\sqrt{a+b\,x}}{3\,b^2} + \frac{8\,b\,x^3\sqrt{a+b\,x}}{7\,(a-c)^3} + \frac{2\,c\left(\frac{2\,c\,(3\,a+c)}{(a-c)^3}+\frac{4\,c\left(\frac{48\,b\,c}{7\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)}{5\,b}\right)\sqrt{c+b\,x}}{3\,b^2} - \frac{8\,b\,x^3\sqrt{c+b\,x}}{7\,(a-c)^3} + \frac{x\left(\frac{2\,a\,(a+3\,c)}{(a-c)^3}+\frac{4\,a\left(\frac{48\,a\,b}{7\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)}{5\,b}\right)\sqrt{a+b\,x}}{3\,b} - \frac{x\left(\frac{2\,c\,(3\,a+c)}{(a-c)^3}+\frac{4\,c\left(\frac{48\,b\,c}{7\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)}{5\,b}\right)\sqrt{c+b\,x}}{3\,b}$$

input
```
int(x/((a + b*x)^(1/2) + (c + b*x)^(1/2))^3,x)
```

output
```
(x^2*((48*b*c)/(7*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3)*(c + b*x)^(1/2
))/(5*b) - (x^2*((48*a*b)/(7*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3)*(a
+ b*x)^(1/2))/(5*b) - (2*a*((2*a*(a + 3*c))/(a - c)^3 + (4*a*((48*a*b)/(7*
(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3))/(5*b))*(a + b*x)^(1/2))/(3*b^2)
 + (8*b*x^3*(a + b*x)^(1/2))/(7*(a - c)^3) + (2*c*((2*c*(3*a + c))/(a - c)
^3 + (4*c*((48*b*c)/(7*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(5*b))*(
c + b*x)^(1/2))/(3*b^2) - (8*b*x^3*(c + b*x)^(1/2))/(7*(a - c)^3) + (x*((2
*a*(a + 3*c))/(a - c)^3 + (4*a*((48*a*b)/(7*(a - c)^3) - (2*b*(5*a + 3*c))
/(a - c)^3))/(5*b))*(a + b*x)^(1/2))/(3*b) - (x*((2*c*(3*a + c))/(a - c)^3
 + (4*c*((48*b*c)/(7*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(5*b))*(c
+ b*x)^(1/2))/(3*b)
```

3.413 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3} \, dx$

3.413.1 Optimal result

Integrand size = 21, antiderivative size = 64

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3} \, dx = \frac{(a-c)^2}{10b\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^5} - \frac{1}{2b\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}$$

output
```
1/10*(a-c)^2/b/((b*x+a)^(1/2)+(b*x+c)^(1/2))^5-1/2/b/((b*x+a)^(1/2)+(b*x+c
)^(1/2))
```

3.413.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3} \, dx = \frac{2\left((-5a+c-4bx)(c+bx)^{3/2}+(a+bx)^{3/2}(-a+5c+4bx)\right)}{5b(a-c)^3}$$

input
```
Integrate[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-3),x]
```

output
```
(2*((-5*a + c - 4*b*x)*(c + b*x)^(3/2) + (a + b*x)^(3/2)*(-a + 5*c + 4*b*x
)))/(5*b*(a - c)^3)
```

3.413.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.83, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {7240, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}dx$$

$$\Big\downarrow \text{7240}$$

$$\frac{\int \left(-\sqrt{c+bx}(3a+c)+(a+3c)\sqrt{a+bx}+4bx\sqrt{a+bx}-4bx\sqrt{c+bx}\right)dx}{(a-c)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2(a+3c)(a+bx)^{3/2}}{3b}-\frac{2(3a+c)(bx+c)^{3/2}}{3b}+\frac{8(a+bx)^{5/2}}{5b}-\frac{8a(a+bx)^{3/2}}{3b}-\frac{8(bx+c)^{5/2}}{5b}+\frac{8c(bx+c)^{3/2}}{3b}}{(a-c)^3}$$

input
```
Int[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-3),x]
```

output
```
((-8*a*(a + b*x)^(3/2))/(3*b) + (2*(a + 3*c)*(a + b*x)^(3/2))/(3*b) + (8*(
a + b*x)^(5/2))/(5*b) + (8*c*(c + b*x)^(3/2))/(3*b) - (2*(3*a + c)*(c + b*
x)^(3/2))/(3*b) - (8*(c + b*x)^(5/2))/(5*b))/(a - c)^3
```

3.413.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7240
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(a*e^2 - c*f^2)^m   Int[ExpandIntegran
d[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c
, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

3.413.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(52) = 104.

Time = 0.06 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.28

method	result	size
default	$\frac{2a(bx+a)^{\frac{3}{2}}}{3(a-c)^3 b}+\frac{2c(bx+a)^{\frac{3}{2}}}{(a-c)^3 b}-\frac{2a(bx+c)^{\frac{3}{2}}}{(a-c)^3 b}-\frac{2c(bx+c)^{\frac{3}{2}}}{3(a-c)^3 b}+\frac{\frac{8(bx+a)^{\frac{5}{2}}}{5}-\frac{8(bx+a)^{\frac{3}{2}}a}{3}}{(a-c)^3 b}-\frac{8\left(\frac{(bx+c)^{\frac{5}{2}}}{5}-\frac{c(bx+c)^{\frac{3}{2}}}{3}\right)}{(a-c)^3 b}$	146

input
```
int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
2/3/(a-c)^3*a*(b*x+a)^(3/2)/b+2/(a-c)^3*c*(b*x+a)^(3/2)/b-2/(a-c)^3*a*(b*x
+c)^(3/2)/b-2/3/(a-c)^3*c*(b*x+c)^(3/2)/b+8/(a-c)^3/b*(1/5*(b*x+a)^(5/2)-1
/3*(b*x+a)^(3/2)*a)-8/(a-c)^3/b*(1/5*(b*x+c)^(5/2)-1/3*c*(b*x+c)^(3/2))
```

3.413.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(52) = 104.

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.66

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$$
$$=\frac{2\left((4\,b^2x^2-a^2+5\,ac+(3\,ab+5\,bc)x)\sqrt{bx+a}-(4\,b^2x^2+5\,ac-c^2+(5\,ab+3\,bc)x)\sqrt{bx+c}\right)}{5\left(a^3b-3\,a^2bc+3\,abc^2-bc^3\right)}$$

input
```
integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

output
```
2/5*((4*b^2*x^2 - a^2 + 5*a*c + (3*a*b + 5*b*c)*x)*sqrt(b*x + a) - (4*b^2*
x^2 + 5*a*c - c^2 + (5*a*b + 3*b*c)*x)*sqrt(b*x + c))/(a^3*b - 3*a^2*b*c +
 3*a*b*c^2 - b*c^3)
```

3.413.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(48) = 96$.

Time = 0.73 (sec) , antiderivative size = 384, normalized size of antiderivative = 6.00

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$$
$$= \begin{cases} -\frac{2a}{5ab\sqrt{a+bx}+15ab\sqrt{bx+c}+20b^2x\sqrt{a+bx}+20b^2x\sqrt{bx+c}+15bc\sqrt{a+bx}+5bc\sqrt{bx+c}} - \frac{4bx}{5ab\sqrt{a+bx}+15ab\sqrt{bx+c}+20b^2x\sqrt{a+bx}+20b^2x\sqrt{bx+c} \\ \frac{x}{\left(\sqrt{a}+\sqrt{c}\right)^3} \end{cases}$$

input
```
integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

output
```
Piecewise((-2*a/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*x + c) + 20*b**2*x*sq
rt(a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a + b*x) + 5*b*c*sqrt(
b*x + c)) - 4*b*x/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*x + c) + 20*b**2*x*
sqrt(a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a + b*x) + 5*b*c*sqr
t(b*x + c)) - 2*c/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*x + c) + 20*b**2*x*
sqrt(a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a + b*x) + 5*b*c*sqr
t(b*x + c)) - 6*sqrt(a + b*x)*sqrt(b*x + c)/(5*a*b*sqrt(a + b*x) + 15*a*b*
sqrt(b*x + c) + 20*b**2*x*sqrt(a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c
*sqrt(a + b*x) + 5*b*c*sqrt(b*x + c)), Ne(b, 0)), (x/(sqrt(a) + sqrt(c))**
3, True))
```

3.413.7 Maxima [F]

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \int \frac{1}{\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}\,dx$$

input
```
integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate((sqrt(b*x + a) + sqrt(b*x + c))^(-3), x)
```

3.413.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(52) = 104$.

Time = 0.35 (sec) , antiderivative size = 427, normalized size of antiderivative = 6.67

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = -\frac{2}{5}\left((bx+a)\left(\frac{4\,(a^3b^2-3\,a^2b^2c+3\,ab^2c^2-b^2c^3)(bx+a)}{a^6b^3-6\,a^5b^3c+15\,a^4b^3c^2-20\,a^3b^3c^3+15\,a^2b^3c^4-6\,ab^3c^5+b^3c^6}-\frac{3\,(a^4b^2-}{a^6b^3-6\,a^5b^3c+15}\right.\right.$$
$$+\frac{2\left(4\,(bx+a)^{\frac{5}{2}}-5\,(bx+a)^{\frac{3}{2}}a+5\,(bx+a)^{\frac{3}{2}}c\right)}{5\,(a^3b-3\,a^2bc+3\,abc^2-bc^3)}$$

input
```
integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

output
```
-2/5*((b*x + a)*(4*(a^3*b^2 - 3*a^2*b^2*c + 3*a*b^2*c^2 - b^2*c^3)*(b*x +
a)/(a^6*b^3 - 6*a^5*b^3*c + 15*a^4*b^3*c^2 - 20*a^3*b^3*c^3 + 15*a^2*b^3*c
^4 - 6*a*b^3*c^5 + b^3*c^6) - 3*(a^4*b^2 - 4*a^3*b^2*c + 6*a^2*b^2*c^2 - 4
*a*b^2*c^3 + b^2*c^4)/(a^6*b^3 - 6*a^5*b^3*c + 15*a^4*b^3*c^2 - 20*a^3*b^3
*c^3 + 15*a^2*b^3*c^4 - 6*a*b^3*c^5 + b^3*c^6)) - (a^5*b^2 - 5*a^4*b^2*c +
 10*a^3*b^2*c^2 - 10*a^2*b^2*c^3 + 5*a*b^2*c^4 - b^2*c^5)/(a^6*b^3 - 6*a^5
*b^3*c + 15*a^4*b^3*c^2 - 20*a^3*b^3*c^3 + 15*a^2*b^3*c^4 - 6*a*b^3*c^5 +
b^3*c^6))*sqrt(b*x + c) + 2/5*(4*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a + 5
*(b*x + a)^(3/2)*c)/(a^3*b - 3*a^2*b*c + 3*a*b*c^2 - b*c^3)
```

3.413.9 Mupad [B] (verification not implemented)

Time = 16.64 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.94

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{\left(\frac{2\,(a^2+3\,c\,a)}{(a-c)^3}+\frac{2\,a\,\left(\frac{32\,a\,b}{5\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)}{3\,b}\right)\sqrt{a+b\,x}}{b} - \frac{\left(\frac{2\,c\,(3\,a+c)}{(a-c)^3}+\frac{2\,c\,\left(\frac{32\,b\,c}{5\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)}{3\,b}\right)\sqrt{c+b\,x}}{b} + \frac{8\,b\,x^2\,\sqrt{a+b\,x}}{5\,(a-c)^3} - \frac{8\,b\,x^2\,\sqrt{c+b\,x}}{5\,(a-c)^3} - \frac{x\,\left(\frac{32\,a\,b}{5\,(a-c)^3}-\frac{2\,b\,(5\,a+3\,c)}{(a-c)^3}\right)\sqrt{a+b\,x}}{3\,b} + \frac{x\,\left(\frac{32\,b\,c}{5\,(a-c)^3}-\frac{2\,b\,(3\,a+5\,c)}{(a-c)^3}\right)\sqrt{c+b\,x}}{3\,b}$$

input
```
int(1/((a + b*x)^(1/2) + (c + b*x)^(1/2))^3,x)
```

output
```
(((2*(3*a*c + a^2))/(a - c)^3 + (2*a*((32*a*b)/(5*(a - c)^3) - (2*b*(5*a +
 3*c))/(a - c)^3))/(3*b))*(a + b*x)^(1/2))/b - (((2*c*(3*a + c))/(a - c)^3
 + (2*c*((32*b*c)/(5*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(3*b))*(c
+ b*x)^(1/2))/b + (8*b*x^2*(a + b*x)^(1/2))/(5*(a - c)^3) - (8*b*x^2*(c +
b*x)^(1/2))/(5*(a - c)^3) - (x*((32*a*b)/(5*(a - c)^3) - (2*b*(5*a + 3*c))
/(a - c)^3)*(a + b*x)^(1/2))/(3*b) + (x*((32*b*c)/(5*(a - c)^3) - (2*b*(3*
a + 5*c))/(a - c)^3)*(c + b*x)^(1/2))/(3*b)
```

3.414 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

3.414.1 Optimal result

Integrand size = 25, antiderivative size = 157

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3}+\frac{8(a+bx)^{3/2}}{3(a-c)^3}-\frac{2(3a+c)\sqrt{c+bx}}{(a-c)^3}-\frac{8(c+bx)^{3/2}}{3(a-c)^3}-\frac{2\sqrt{a}(a+3c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(a-c)^3}+\frac{2\sqrt{c}(3a+c)\text{arctanh}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)}{(a-c)^3}$$

output
```
8/3*(b*x+a)^(3/2)/(a-c)^3-8/3*(b*x+c)^(3/2)/(a-c)^3-2*(a+3*c)*arctanh((b*x
+a)^(1/2)/a^(1/2))*a^(1/2)/(a-c)^3+2*(3*a+c)*arctanh((b*x+c)^(1/2)/c^(1/2)
)*c^(1/2)/(a-c)^3+2*(a+3*c)*(b*x+a)^(1/2)/(a-c)^3-2*(3*a+c)*(b*x+c)^(1/2)/
(a-c)^3
```

3.414.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.55

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{2}{3}\left(-\frac{\sqrt{c+bx}(9a+7c+4bx)}{(a-c)^3}+\frac{\sqrt{a+bx}(7a+9c+4bx)}{(a-c)^3} - \frac{3(\sqrt{a}-\sqrt{c})\arctan\left(\frac{-\sqrt{a+bx}+\sqrt{c+bx}}{\sqrt{-(\sqrt{a}-\sqrt{c})^2}}\right)}{\sqrt{-\left(\sqrt{a}-\sqrt{c}\right)^2}\left(\sqrt{a}+\sqrt{c}\right)^3} - \frac{3(\sqrt{a}+\sqrt{c})\arctan\left(\frac{-\sqrt{a+bx}+\sqrt{c+bx}}{\sqrt{-(\sqrt{a}+\sqrt{c})^2}}\right)}{\left(\sqrt{a}-\sqrt{c}\right)^3\sqrt{-\left(\sqrt{a}+\sqrt{c}\right)^2}}\right)$$

input
```
Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

output
```
(2*(-((Sqrt[c + b*x]*(9*a + 7*c + 4*b*x))/(a - c)^3) + (Sqrt[a + b*x]*(7*a
 + 9*c + 4*b*x))/(a - c)^3 - (3*(Sqrt[a] - Sqrt[c])*ArcTan[(-Sqrt[a + b*x]
 + Sqrt[c + b*x])/Sqrt[-(Sqrt[a] - Sqrt[c])^2]])/(Sqrt[-(Sqrt[a] - Sqrt[c]
)^2]*(Sqrt[a] + Sqrt[c])^3) - (3*(Sqrt[a] + Sqrt[c])*ArcTan[(-Sqrt[a + b*x
] + Sqrt[c + b*x])/Sqrt[-(Sqrt[a] + Sqrt[c])^2]])/((Sqrt[a] - Sqrt[c])^3*S
qrt[-(Sqrt[a] + Sqrt[c])^2])))/3
```

3.414.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.78, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7240, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}\,dx$$

$$\Big\downarrow \text{7240}$$

$$\frac{\int \left(4\sqrt{a+bx}b-4\sqrt{c+bx}b+\frac{(a+3c)\sqrt{a+bx}}{x}-\frac{(3a+c)\sqrt{c+bx}}{x}\right)dx}{(a-c)^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-2\sqrt{a}(a+3c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+2\sqrt{c}(3a+c)\text{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)+2(a+3c)\sqrt{a+bx}-2(3a+c)\sqrt{bx+c}+\frac{8}{3}(a+}{(a-c)^3}$$

input
```
Int[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

output
```
(2*(a + 3*c)*Sqrt[a + b*x] + (8*(a + b*x)^(3/2))/3 - 2*(3*a + c)*Sqrt[c +
b*x] - (8*(c + b*x)^(3/2))/3 - 2*Sqrt[a]*(a + 3*c)*ArcTanh[Sqrt[a + b*x]/S
qrt[a]] + 2*Sqrt[c]*(3*a + c)*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/(a - c)^3
```

3.414.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7240
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(a*e^2 - c*f^2)^m   Int[ExpandIntegran
d[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c
, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

3.414.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.15

method	result
default	$\frac{a\left(2\sqrt{bx+a}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{(a-c)^3}+\frac{8(bx+a)^{\frac{3}{2}}}{3(a-c)^3}-\frac{8(bx+c)^{\frac{3}{2}}}{3(a-c)^3}+\frac{3c\left(2\sqrt{bx+a}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{(a-c)^3}-\frac{3a\left(2\sqrt{bx+c}-2\sqrt{\right.}{(a}$

input
```
int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
1/(a-c)^3*a*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))+8/3
*(b*x+a)^(3/2)/(a-c)^3-8/3*(b*x+c)^(3/2)/(a-c)^3+3/(a-c)^3*c*(2*(b*x+a)^(1
/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-3/(a-c)^3*a*(2*(b*x+c)^(1/2)
-2*c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1/2)))-1/(a-c)^3*c*(2*(b*x+c)^(1/2)-2*
c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1/2)))
```

3.414.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 516, normalized size of antiderivative = 3.29

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$$
$$= \left[-\frac{3\,(a+3\,c)\sqrt{a}\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+3\,(3\,a+c)\sqrt{c}\log\left(\frac{bx-2\,\sqrt{bx+c}\sqrt{c}+2\,c}{x}\right)-2\,(4\,bx+7\,a+9\,c)\sqrt{bx+\cdots}}{3\,(a^3-3\,a^2c+3\,ac^2-c^3)}\right.$$
$$-\frac{6\,(3\,a+c)\sqrt{-c}\arctan\left(\frac{\sqrt{bx+c}\sqrt{-c}}{c}\right)+3\,(a+3\,c)\sqrt{a}\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\,(4\,bx+7\,a+9\,c)\sqrt{bx+\cdots}}{3\,(a^3-3\,a^2c+3\,ac^2-c^3)}$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

output
```
[-1/3*(3*(a + 3*c)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) +
3*(3*a + c)*sqrt(c)*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) - 2*(4*b*
x + 7*a + 9*c)*sqrt(b*x + a) + 2*(4*b*x + 9*a + 7*c)*sqrt(b*x + c))/(a^3 -
 3*a^2*c + 3*a*c^2 - c^3), -1/3*(6*(3*a + c)*sqrt(-c)*arctan(sqrt(b*x + c)
*sqrt(-c)/c) + 3*(a + 3*c)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*
a)/x) - 2*(4*b*x + 7*a + 9*c)*sqrt(b*x + a) + 2*(4*b*x + 9*a + 7*c)*sqrt(b
*x + c))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3), 1/3*(6*sqrt(-a)*(a + 3*c)*arctan
(sqrt(b*x + a)*sqrt(-a)/a) - 3*(3*a + c)*sqrt(c)*log((b*x - 2*sqrt(b*x + c
)*sqrt(c) + 2*c)/x) + 2*(4*b*x + 7*a + 9*c)*sqrt(b*x + a) - 2*(4*b*x + 9*a
 + 7*c)*sqrt(b*x + c))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3), 2/3*(3*sqrt(-a)*(a
 + 3*c)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - 3*(3*a + c)*sqrt(-c)*arctan(sqr
t(b*x + c)*sqrt(-c)/c) + (4*b*x + 7*a + 9*c)*sqrt(b*x + a) - (4*b*x + 9*a
+ 7*c)*sqrt(b*x + c))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3)]
```

3.414.6 Sympy [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx=\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}\,dx$$

input
```
integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

output
```
Integral(1/(x*(sqrt(a + b*x) + sqrt(b*x + c))**3), x)
```

3.414.7 Maxima [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx=\int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}\,dx$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))^3), x)
```

3.414.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2649 vs. 2(133) = 266.

Time = 0.93 (sec) , antiderivative size = 2649, normalized size of antiderivative = 16.87

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

output
```
-2/3*sqrt(b*x + c)*(4*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)*(b*x + a)/(a^6 - 6*a
^5*c + 15*a^4*c^2 - 20*a^3*c^3 + 15*a^2*c^4 - 6*a*c^5 + c^6) + (5*a^4 - 8*
a^3*c - 6*a^2*c^2 + 16*a*c^3 - 7*c^4)/(a^6 - 6*a^5*c + 15*a^4*c^2 - 20*a^3
*c^3 + 15*a^2*c^4 - 6*a*c^5 + c^6)) + 2*(a^2 + 3*a*c)*arctan(sqrt(b*x + a)
/sqrt(-a))/((a^3 - 3*a^2*c + 3*a*c^2 - c^3)*sqrt(-a)) + 2/3*(4*(b*x + a)^(
3/2)*a^6 + 3*sqrt(b*x + a)*a^7 - 24*(b*x + a)^(3/2)*a^5*c - 9*sqrt(b*x + a
)*a^6*c + 60*(b*x + a)^(3/2)*a^4*c^2 - 9*sqrt(b*x + a)*a^5*c^2 - 80*(b*x +
 a)^(3/2)*a^3*c^3 + 75*sqrt(b*x + a)*a^4*c^3 + 60*(b*x + a)^(3/2)*a^2*c^4
- 135*sqrt(b*x + a)*a^3*c^4 - 24*(b*x + a)^(3/2)*a*c^5 + 117*sqrt(b*x + a)
*a^2*c^5 + 4*(b*x + a)^(3/2)*c^6 - 51*sqrt(b*x + a)*a*c^6 + 9*sqrt(b*x + a
)*c^7)/(a^9 - 9*a^8*c + 36*a^7*c^2 - 84*a^6*c^3 + 126*a^5*c^4 - 126*a^4*c^
5 + 84*a^3*c^6 - 36*a^2*c^7 + 9*a*c^8 - c^9) + 2*(3*a^9*c - 14*a^8*c^2 + 2
2*a^7*c^3 - 6*a^6*c^4 - 20*a^5*c^5 + 22*a^4*c^6 - 6*a^3*c^7 - 2*a^2*c^8 +
a*c^9 - 2*(3*a^2*c^2 + a*c^3 - (3*a*c^2 + c^3)*sqrt(a*c))*(a^3 - 3*a^2*c +
 3*a*c^2 - c^3)^2*sgn(a^3 - 3*a^2*c + 3*a*c^2 - c^3) - 2*(3*a^2*c^2 + a*c^
3 + (3*a^2*c + a*c^2)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2 + (3*a^
5*c^2 - 11*a^4*c^3 + 14*a^3*c^4 - 6*a^2*c^5 - a*c^6 + c^7 - (3*a^5*c - 11*
a^4*c^2 + 14*a^3*c^3 - 6*a^2*c^4 - a*c^5 + c^6)*sqrt(a*c))*abs(-a^3 + 3*a^
2*c - 3*a*c^2 + c^3)*sgn(a^3 - 3*a^2*c + 3*a*c^2 - c^3) + (3*a^6*c - 11*a^
5*c^2 + 14*a^4*c^3 - 6*a^3*c^4 - a^2*c^5 + a*c^6 + (3*a^5*c - 11*a^4*c^...
```

3.414.9 Mupad [B] (verification not implemented)

Time = 42.36 (sec) , antiderivative size = 4060, normalized size of antiderivative = 25.86

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(x*((a + b*x)^(1/2) + (c + b*x)^(1/2))^3),x)
```

output
```
((((a^(1/2)*(16*a + 16*c))/(3*a*c^2 - 3*a^2*c + a^3 - c^3) + (c^(1/2)*(16*
a + 16*c))/(3*a*c^2 - 3*a^2*c + a^3 - c^3))*((a + b*x)^(1/2) - a^(1/2)))/(
(c + b*x)^(1/2) - c^(1/2)) + (((a^(1/2)*(12*a + 20*c))/(3*a*c^2 - 3*a^2*c
+ a^3 - c^3) + (c^(1/2)*(20*a + 12*c))/(3*a*c^2 - 3*a^2*c + a^3 - c^3))*((
a + b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1/2) - c^(1/2))^2 + (a^(1/2)*((28
*a)/3 + 12*c))/(3*a*c^2 - 3*a^2*c + a^3 - c^3) + (c^(1/2)*(12*a + (28*c)/3
))/(3*a*c^2 - 3*a^2*c + a^3 - c^3))/((3*((a + b*x)^(1/2) - a^(1/2)))/((c +
 b*x)^(1/2) - c^(1/2)) + (3*((a + b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1/2
) - c^(1/2))^2 + ((a + b*x)^(1/2) - a^(1/2))^3/((c + b*x)^(1/2) - c^(1/2))
^3 + 1) + (log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(a
*(a^(1/2) + 3*c^(1/2)) + c*(3*a^(1/2) + c^(1/2))))/(3*a*c^2 - 3*a^2*c + a^
3 - c^3) + (atan(((((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a
^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((6*a*c^(11/2) - 6*a^(11/2)*c + 2
*a^(3/2)*c^5 - 2*a^5*c^(3/2) + 12*a^3*c^(7/2) - 12*a^(7/2)*c^3 - 16*a^2*c^
(9/2) + 16*a^(9/2)*c^2)/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 - 20*a^4*c
^4 + 15*a^5*c^3 - 6*a^6*c^2) + (((a^(1/2)*c^(15/2) - 5*a^(3/2)*c^(13/2) +
9*a^(5/2)*c^(11/2) - 5*a^(7/2)*c^(9/2) - 5*a^(9/2)*c^(7/2) + 9*a^(11/2)*c^
(5/2) - 5*a^(13/2)*c^(3/2) + a^(15/2)*c^(1/2))/(a*c^7 + a^7*c - 6*a^2*c^6
+ 15*a^3*c^5 - 20*a^4*c^4 + 15*a^5*c^3 - 6*a^6*c^2) - (2*((a + b*x)^(1/2)
- a^(1/2))*(a*c^9 + a^9*c - 7*a^2*c^8 + 22*a^3*c^7 - 41*a^4*c^6 + 50*a^...
```

3.414. $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

3.415 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

3.415.1 Optimal result

Integrand size = 25, antiderivative size = 162

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{(a-c)^3x}-\frac{8b\sqrt{c+bx}}{(a-c)^3} + \frac{(3a+c)\sqrt{c+bx}}{(a-c)^3x}-\frac{3b(3a+c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(a-c)^3} - \frac{3b(a+3c)\text{arctanh}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)}{\sqrt{c}(-a+c)^3}$$

output
```
-3*b*(3*a+c)*arctanh((b*x+a)^(1/2)/a^(1/2))/(a-c)^3/a^(1/2)-3*b*(a+3*c)*ar
ctanh((b*x+c)^(1/2)/c^(1/2))/(-a+c)^3/c^(1/2)+8*b*(b*x+a)^(1/2)/(a-c)^3-(a
+3*c)*(b*x+a)^(1/2)/(a-c)^3/x-8*b*(b*x+c)^(1/2)/(a-c)^3+(3*a+c)*(b*x+c)^(1
/2)/(a-c)^3/x
```

3.415.2 Mathematica [A] (verified)

Time = 10.46 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$$

$$= \frac{b\left(8\sqrt{a+bx}-8\sqrt{c+bx}-8\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+8\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)-\frac{(a+3c)\left(a+bx+bx\sqrt{1+\frac{bx}{a}}\text{arctanh}\left(\sqrt{\right.\right.}}{bx\sqrt{a+bx}}\right.}{(a-c)^3}$$

input
```
Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

output
```
(b*(8*Sqrt[a + b*x] - 8*Sqrt[c + b*x] - 8*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sq
rt[a]] + 8*Sqrt[c]*ArcTanh[Sqrt[c + b*x]/Sqrt[c]] - ((a + 3*c)*(a + b*x +
b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]]))/(b*x*Sqrt[a + b*x]) + (
(3*a + c)*(c + b*x + b*x*Sqrt[1 + (b*x)/c]*ArcTanh[Sqrt[1 + (b*x)/c]]))/(b
*x*Sqrt[c + b*x])))/(a - c)^3
```

3.415.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7240, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}dx$$

$$\Big\downarrow \text{7240}$$

$$\frac{\int \left(\frac{4\sqrt{a+bx}b}{x}-\frac{4\sqrt{c+bx}b}{x}+\frac{(a+3c)\sqrt{a+bx}}{x^2}-\frac{(3a+c)\sqrt{c+bx}}{x^2}\right)dx}{(a-c)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b(a+3c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{b(3a+c)\text{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{\sqrt{c}}-8\sqrt{a}b\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{(a+3c)\sqrt{a+bx}}{x}+\frac{(3a+c)\sqrt{bx+c}}{x}+8b\sqrt{a+}}{(a-c)^3}$$

input
```
Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

output
```
(8*b*Sqrt[a + b*x] - ((a + 3*c)*Sqrt[a + b*x])/x - 8*b*Sqrt[c + b*x] + ((3
*a + c)*Sqrt[c + b*x])/x - 8*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]] - (b
*(a + 3*c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a] + 8*b*Sqrt[c]*ArcTanh[S
qrt[c + b*x]/Sqrt[c]] + (b*(3*a + c)*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/Sqrt[
c])/(a - c)^3
```

3.415.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7240
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(a*e^2 - c*f^2)^m   Int[ExpandIntegran
d[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c
, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

3.415.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.56

method	result
default	$\frac{2ab\left(-\frac{\sqrt{bx+a}}{2xb}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{(a-c)^3}+\frac{6cb\left(-\frac{\sqrt{bx+a}}{2xb}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{(a-c)^3}-\frac{6ab\left(-\frac{\sqrt{bx+c}}{2xb}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)}{(a-c)^3}-\frac{2cb\left(-\frac{\sqrt{bx}}{2x}\right.}{}$

input
```
int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
2/(a-c)^3*a*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a
^(1/2))+6/(a-c)^3*c*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a^
(1/2))/a^(1/2))-6/(a-c)^3*a*b*(-1/2*(b*x+c)^(1/2)/x/b-1/2/c^(1/2)*arctanh(
(b*x+c)^(1/2)/c^(1/2)))-2/(a-c)^3*c*b*(-1/2*(b*x+c)^(1/2)/x/b-1/2/c^(1/2)*
arctanh((b*x+c)^(1/2)/c^(1/2)))+4/(a-c)^3*b*(2*(b*x+a)^(1/2)-2*a^(1/2)*arc
tanh((b*x+a)^(1/2)/a^(1/2)))-4/(a-c)^3*b*(2*(b*x+c)^(1/2)-2*c^(1/2)*arctan
h((b*x+c)^(1/2)/c^(1/2)))
```

3.415.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 675, normalized size of antiderivative = 4.17

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$$

$$= \left[-\frac{3\,(3\,abc+bc^2)\sqrt{a}x\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+3\,(a^2b+3\,abc)\sqrt{c}x\log\left(\frac{bx-2\,\sqrt{bx+c}\sqrt{c}+2\,c}{x}\right)-2\,(8\,abcx-a^2}{2\,(a^4c-3\,a^3c^2+3\,a^2c^3-ac^4)x}\right.$$

$$-\frac{6\,(a^2b+3\,abc)\sqrt{-c}x\arctan\left(\frac{\sqrt{bx+c}\sqrt{-c}}{c}\right)+3\,(3\,abc+bc^2)\sqrt{a}x\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)-2\,(8\,abcx-a^2c}{2\,(a^4c-3\,a^3c^2+3\,a^2c^3-ac^4)x}$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

output
```
[-1/2*(3*(3*a*b*c + b*c^2)*sqrt(a)*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) +
2*a)/x) + 3*(a^2*b + 3*a*b*c)*sqrt(c)*x*log((b*x - 2*sqrt(b*x + c)*sqrt(c)
 + 2*c)/x) - 2*(8*a*b*c*x - a^2*c - 3*a*c^2)*sqrt(b*x + a) + 2*(8*a*b*c*x
- 3*a^2*c - a*c^2)*sqrt(b*x + c))/((a^4*c - 3*a^3*c^2 + 3*a^2*c^3 - a*c^4)
*x), -1/2*(6*(a^2*b + 3*a*b*c)*sqrt(-c)*x*arctan(sqrt(b*x + c)*sqrt(-c)/c)
 + 3*(3*a*b*c + b*c^2)*sqrt(a)*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)
/x) - 2*(8*a*b*c*x - a^2*c - 3*a*c^2)*sqrt(b*x + a) + 2*(8*a*b*c*x - 3*a^2
*c - a*c^2)*sqrt(b*x + c))/((a^4*c - 3*a^3*c^2 + 3*a^2*c^3 - a*c^4)*x), 1/
2*(6*(3*a*b*c + b*c^2)*sqrt(-a)*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - 3*(a^
2*b + 3*a*b*c)*sqrt(c)*x*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) + 2*
(8*a*b*c*x - a^2*c - 3*a*c^2)*sqrt(b*x + a) - 2*(8*a*b*c*x - 3*a^2*c - a*c
^2)*sqrt(b*x + c))/((a^4*c - 3*a^3*c^2 + 3*a^2*c^3 - a*c^4)*x), (3*(3*a*b*
c + b*c^2)*sqrt(-a)*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - 3*(a^2*b + 3*a*b*
c)*sqrt(-c)*x*arctan(sqrt(b*x + c)*sqrt(-c)/c) + (8*a*b*c*x - a^2*c - 3*a*
c^2)*sqrt(b*x + a) - (8*a*b*c*x - 3*a^2*c - a*c^2)*sqrt(b*x + c))/((a^4*c
- 3*a^3*c^2 + 3*a^2*c^3 - a*c^4)*x)]
```

3.415.6 Sympy [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx=\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}\,dx$$

input
```
integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

output
```
Integral(1/(x**2*(sqrt(a + b*x) + sqrt(b*x + c))**3), x)
```

3.415.7 Maxima [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx=\int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{bx+c}\right)^3}\,dx$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))^3), x)
```

3.415.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2594 vs. 2(142) = 284.

Time = 7.56 (sec) , antiderivative size = 2594, normalized size of antiderivative = 16.01

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

output
```
8*sqrt(b*x + a)*b/(a^3 - 3*a^2*c + 3*a*c^2 - c^3) - 8*sqrt(b*x + c)*b/(a^3
 - 3*a^2*c + 3*a*c^2 - c^3) + 3*(3*a*b + b*c)*arctan(sqrt(b*x + a)/sqrt(-a
))/((a^3 - 3*a^2*c + 3*a*c^2 - c^3)*sqrt(-a)) - 3*(2*(a^2*c^2 + 3*a*c^3 +
(a*c^2 + 3*c^3)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2*b*sgn(-2*a^3
+ 6*a^2*c - 6*a*c^2 + 2*c^3) - 2*(a^2*c^2 + 3*a*c^3 + (a^2*c + 3*a*c^2)*sq
rt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2*b + (a^5*c^2 - a^4*c^3 - 6*a^3*
c^4 + 14*a^2*c^5 - 11*a*c^6 + 3*c^7 + (a^5*c - a^4*c^2 - 6*a^3*c^3 + 14*a^
2*c^4 - 11*a*c^5 + 3*c^6)*sqrt(a*c))*b*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3)
*sgn(-2*a^3 + 6*a^2*c - 6*a*c^2 + 2*c^3) - (a^6*c - a^5*c^2 - 6*a^4*c^3 +
14*a^3*c^4 - 11*a^2*c^5 + 3*a*c^6 + (a^5*c - a^4*c^2 - 6*a^3*c^3 + 14*a^2*
c^4 - 11*a*c^5 + 3*c^6)*sqrt(a*c))*b*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3) -
 (a^9*c - 2*a^8*c^2 - 6*a^7*c^3 + 22*a^6*c^4 - 20*a^5*c^5 - 6*a^4*c^6 + 22
*a^3*c^7 - 14*a^2*c^8 + 3*a*c^9 + (a^8*c - 2*a^7*c^2 - 6*a^6*c^3 + 22*a^5*
c^4 - 20*a^4*c^5 - 6*a^3*c^6 + 22*a^2*c^7 - 14*a*c^8 + 3*c^9)*sqrt(a*c))*b
*sgn(-2*a^3 + 6*a^2*c - 6*a*c^2 + 2*c^3) + (a^9*c - 2*a^8*c^2 - 6*a^7*c^3
+ 22*a^6*c^4 - 20*a^5*c^5 - 6*a^4*c^6 + 22*a^3*c^7 - 14*a^2*c^8 + 3*a*c^9
+ (a^9 - 2*a^8*c - 6*a^7*c^2 + 22*a^6*c^3 - 20*a^5*c^4 - 6*a^4*c^5 + 22*a^
3*c^6 - 14*a^2*c^7 + 3*a*c^8)*sqrt(a*c))*b)*arctan(-(sqrt(b*x + a) - sqrt(
b*x + c))/sqrt(-(a^4 - 2*a^3*c + 2*a*c^3 - c^4 + sqrt((a^4 - 2*a^3*c + 2*a
*c^3 - c^4)^2 - (a^5 - 5*a^4*c + 10*a^3*c^2 - 10*a^2*c^3 + 5*a*c^4 - c^...
```

3.415.9 Mupad [B] (verification not implemented)

Time = 51.54 (sec) , antiderivative size = 4681, normalized size of antiderivative = 28.90

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(x^2*((a + b*x)^(1/2) + (c + b*x)^(1/2))^3),x)
```

output
```
(b*atan(((b*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*
c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((9*a^6*b*c^(7/2) - 9*a^(7/2)*b*c^6 - 24
*a^5*b*c^(9/2) + 24*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/2)*b*c^4
- 3*a^2*b*c^(15/2) + 3*a^(15/2)*b*c^2)/(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 -
 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3) + (((a + b*x)^(1/2) - a^(1
/2))*(6*a^(3/2)*b*c^8 - 6*a^8*b*c^(3/2) + 36*a^6*b*c^(7/2) - 36*a^(7/2)*b*
c^6 - 48*a^5*b*c^(9/2) + 48*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/2
)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7
 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)) - (3*b*((a^(5/2)*c^(19/
2) - 5*a^(7/2)*c^(17/2) + 9*a^(9/2)*c^(15/2) - 5*a^(11/2)*c^(13/2) - 5*a^(
13/2)*c^(11/2) + 9*a^(15/2)*c^(9/2) - 5*a^(17/2)*c^(7/2) + a^(19/2)*c^(5/2
))/(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4
 + a^9*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^10 - 28*a^3*c^9 + 88*a
^4*c^8 - 164*a^5*c^7 + 200*a^6*c^6 - 164*a^7*c^5 + 88*a^8*c^4 - 28*a^9*c^3
 + 4*a^10*c^2))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a
^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)))*((a^(1/2)*c^(3/2
) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^
(1/2)*(a*c^(7/2) + a^(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a^2*c^(5/
2) + 2*a^(5/2)*c^2))/(2*(a^2*c^7 - 5*a^3*c^6 + 10*a^4*c^5 - 10*a^5*c^4 + 5
*a^6*c^3 - a^7*c^2)))*(a*c^(7/2) + a^(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2...
```

3.415. $\int \frac{1}{x^2(\sqrt{a+bx}+\sqrt{c+bx})^3}\,dx$

3.416 $\int \frac{1}{\sqrt{x}+\sqrt{1+x}} \, dx$

3.416.1 Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}} \, dx = -\frac{2x^{3/2}}{3}+\frac{2}{3}(1+x)^{3/2}$$

output
```
-2/3*x^(3/2)+2/3*(1+x)^(3/2)
```

3.416.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}} \, dx = -\frac{2x^{3/2}}{3}+\frac{2}{3}(1+x)^{3/2}$$

input
```
Integrate[(Sqrt[x] + Sqrt[1 + x])^(-1),x]
```

output
```
(-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3
```

3.416.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2531, 15, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}+\sqrt{x+1}}dx$$

$$\Big\downarrow \text{2531}$$

$$\int \sqrt{x+1}dx-\int \sqrt{x}dx$$

$$\Big\downarrow \text{15}$$

$$\int \sqrt{x+1}dx-\frac{2x^{3/2}}{3}$$

$$\Big\downarrow \text{17}$$

$$\frac{2}{3}(x+1)^{3/2}-\frac{2x^{3/2}}{3}$$

input
```
Int[(Sqrt[x] + Sqrt[1 + x])^(-1),x]
```

output
```
(-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3
```

3.416.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2531
```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> Simp[-b/(a*d)   Int[u*x^n, x], x] + Simp[1/(a*c)   Int[u*Sqrt[a + b*
x^(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 -
 d^2, 0]
```

3.416.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
default	$-\frac{2x^{\frac{3}{2}}}{3}+\frac{2(x+1)^{\frac{3}{2}}}{3}$	14
meijerg	$-\frac{\frac{4\sqrt{\pi}\,x^{\frac{3}{2}}}{3}-\frac{2\sqrt{\pi}\,x^{\frac{3}{2}}\left(2+\frac{2}{x}\right)\sqrt{1+\frac{1}{x}}}{3}}{2\sqrt{\pi}}$	37

input
```
int(1/(x^(1/2)+(x+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-2/3*x^(3/2)+2/3*(x+1)^(3/2)
```

3.416.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx=\frac{2}{3}\,(x+1)^{\frac{3}{2}}-\frac{2}{3}\,x^{\frac{3}{2}}$$

input
```
integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
```

output
```
2/3*(x + 1)^(3/2) - 2/3*x^(3/2)
```

3.416.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(17) = 34$.

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.00

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx = \frac{2\sqrt{x}\sqrt{x+1}}{3\sqrt{x}+3\sqrt{x+1}} + \frac{4x}{3\sqrt{x}+3\sqrt{x+1}} + \frac{2}{3\sqrt{x}+3\sqrt{x+1}}$$

input
```
integrate(1/(x**(1/2)+(1+x)**(1/2)),x)
```

output
```
2*sqrt(x)*sqrt(x + 1)/(3*sqrt(x) + 3*sqrt(x + 1)) + 4*x/(3*sqrt(x) + 3*sqr
t(x + 1)) + 2/(3*sqrt(x) + 3*sqrt(x + 1))
```

3.416.7 Maxima [F]

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx = \int \frac{1}{\sqrt{x+1}+\sqrt{x}}\,dx$$

input
```
integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x + 1) + sqrt(x)), x)
```

3.416.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx = \frac{2}{3}\,(x+1)^{\frac{3}{2}} - \frac{2}{3}\,x^{\frac{3}{2}}$$

input
```
integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
```

output
```
2/3*(x + 1)^(3/2) - 2/3*x^(3/2)
```

3.416.9 Mupad [B] (verification not implemented)

Time = 16.61 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx=\frac{2\,x\,\sqrt{x+1}}{3}+\frac{2\,\sqrt{x+1}}{3}-\frac{2\,x^{3/2}}{3}$$

input
```
int(1/((x + 1)^(1/2) + x^(1/2)),x)
```

output
```
(2*x*(x + 1)^(1/2))/3 + (2*(x + 1)^(1/2))/3 - (2*x^(3/2))/3
```

3.417 $\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx$

3.417.1 Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx = -\frac{2}{3}(-1+x)^{3/2}+\frac{2x^{3/2}}{3}$$

output
```
-2/3*(-1+x)^(3/2)+2/3*x^(3/2)
```

3.417.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx = -\frac{2}{3}(-1+x)^{3/2}+\frac{2x^{3/2}}{3}$$

input
```
Integrate[(Sqrt[-1 + x] + Sqrt[x])^(-1),x]
```

output
```
(-2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3
```

3.417.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2531, 15, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x-1}+\sqrt{x}}dx$$

$$\Big\downarrow \text{2531}$$

$$\int \sqrt{x}dx-\int \sqrt{x-1}dx$$

$$\Big\downarrow \text{15}$$

$$\frac{2x^{3/2}}{3}-\int \sqrt{x-1}dx$$

$$\Big\downarrow \text{17}$$

$$\frac{2x^{3/2}}{3}-\frac{2}{3}(x-1)^{3/2}$$

input
```
Int[(Sqrt[-1 + x] + Sqrt[x])^(-1),x]
```

output
```
(-2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3
```

3.417.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2531
```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> Simp[-b/(a*d)   Int[u*x^n, x], x] + Simp[1/(a*c)   Int[u*Sqrt[a + b*
x^(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 -
 d^2, 0]
```

3.417.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
default	$-\frac{2(x-1)^{\frac{3}{2}}}{3}+\frac{2x^{\frac{3}{2}}}{3}$	14
meijerg	$-\frac{i\left(\frac{4i\sqrt{\pi}\,x^{\frac{3}{2}}}{3}-\frac{2i\sqrt{\pi}\,x^{\frac{3}{2}}\left(2-\frac{2}{x}\right)\sqrt{1-\frac{1}{x}}}{3}\right)}{2\sqrt{\pi}}$	42

input
```
int(1/((x-1)^(1/2)+x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(x-1)^(3/2)+2/3*x^(3/2)
```

3.417.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx = -\frac{2}{3}\,(x-1)^{\frac{3}{2}}+\frac{2}{3}\,x^{\frac{3}{2}}$$

input
```
integrate(1/((-1+x)^(1/2)+x^(1/2)),x, algorithm="fricas")
```

output
```
-2/3*(x - 1)^(3/2) + 2/3*x^(3/2)
```

3.417.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(17) = 34.

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.00

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx = \frac{2\sqrt{x}\sqrt{x-1}}{3\sqrt{x}+3\sqrt{x-1}} + \frac{4x}{3\sqrt{x}+3\sqrt{x-1}} - \frac{2}{3\sqrt{x}+3\sqrt{x-1}}$$

input
```
integrate(1/((-1+x)**(1/2)+x**(1/2)),x)
```

output
```
2*sqrt(x)*sqrt(x - 1)/(3*sqrt(x) + 3*sqrt(x - 1)) + 4*x/(3*sqrt(x) + 3*sqr
t(x - 1)) - 2/(3*sqrt(x) + 3*sqrt(x - 1))
```

3.417.7 Maxima [F]

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx = \int \frac{1}{\sqrt{x-1}+\sqrt{x}}\,dx$$

input
```
integrate(1/((-1+x)^(1/2)+x^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x - 1) + sqrt(x)), x)
```

3.417.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx = -\frac{2}{3}\,(x-1)^{\frac{3}{2}} + \frac{2}{3}\,x^{\frac{3}{2}}$$

input
```
integrate(1/((-1+x)^(1/2)+x^(1/2)),x, algorithm="giac")
```

output
```
-2/3*(x - 1)^(3/2) + 2/3*x^(3/2)
```

3.417.9 Mupad [B] (verification not implemented)

Time = 16.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx=\frac{2\,\sqrt{x-1}}{3}-\frac{2\,x\,\sqrt{x-1}}{3}+\frac{2\,x^{3/2}}{3}$$

input
```
int(1/((x - 1)^(1/2) + x^(1/2)),x)
```

output
```
(2*(x - 1)^(1/2))/3 - (2*x*(x - 1)^(1/2))/3 + (2*x^(3/2))/3
```

3.418 $\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}} \, dx$

3.418.1 Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}} \, dx = -\frac{1}{3}(-1+x)^{3/2}+\frac{1}{3}(1+x)^{3/2}$$

output
```
-1/3*(-1+x)^(3/2)+1/3*(1+x)^(3/2)
```

3.418.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}} \, dx = -\frac{1}{3}(-1+x)^{3/2}+\frac{1}{3}(1+x)^{3/2}$$

input
```
Integrate[(Sqrt[-1 + x] + Sqrt[1 + x])^(-1),x]
```

output
```
-1/3*(-1 + x)^(3/2) + (1 + x)^(3/2)/3
```

3.418.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7240, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x-1}+\sqrt{x+1}}\,dx$$

$$\Big\downarrow \text{7240}$$

$$-\frac{1}{2}\int \left(\sqrt{x-1}-\sqrt{x+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2}{3}(x+1)^{3/2}-\frac{2}{3}(x-1)^{3/2}\right)$$

input
```
Int[(Sqrt[-1 + x] + Sqrt[1 + x])^(-1),x]
```

output
```
((-2*(-1 + x)^(3/2))/3 + (2*(1 + x)^(3/2))/3)/2
```

3.418.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7240
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(a*e^2 - c*f^2)^m   Int[ExpandIntegran
d[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c
, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

3.418.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{(x-1)^{\frac{3}{2}}}{3}+\frac{(x+1)^{\frac{3}{2}}}{3}$	16

input
```
int(1/((x-1)^(1/2)+(x+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(x-1)^(3/2)+1/3*(x+1)^(3/2)
```

3.418.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx=\frac{1}{3}\,(x+1)^{\frac{3}{2}}-\frac{1}{3}\,(x-1)^{\frac{3}{2}}$$

input
```
integrate(1/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
```

output
```
1/3*(x + 1)^(3/2) - 1/3*(x - 1)^(3/2)
```

3.418.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(15) = 30.

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.22

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx=\frac{4x}{3\sqrt{x-1}+3\sqrt{x+1}}+\frac{2\sqrt{x-1}\sqrt{x+1}}{3\sqrt{x-1}+3\sqrt{x+1}}$$

input
```
integrate(1/((-1+x)**(1/2)+(1+x)**(1/2)),x)
```

output
```
4*x/(3*sqrt(x - 1) + 3*sqrt(x + 1)) + 2*sqrt(x - 1)*sqrt(x + 1)/(3*sqrt(x
- 1) + 3*sqrt(x + 1))
```

3.418.7 Maxima [F]

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = \int \frac{1}{\sqrt{x+1}+\sqrt{x-1}}\,dx$$

input
```
integrate(1/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x + 1) + sqrt(x - 1)), x)
```

3.418.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = \frac{1}{3}\,(x+1)^{\frac{3}{2}} - \frac{1}{3}\,(x-1)^{\frac{3}{2}}$$

input
```
integrate(1/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
```

output
```
1/3*(x + 1)^(3/2) - 1/3*(x - 1)^(3/2)
```

3.418.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = \frac{(x+1)^{3/2}}{3} - \frac{(x-1)^{3/2}}{3}$$

input
```
int(1/((x - 1)^(1/2) + (x + 1)^(1/2)),x)
```

output
```
(x + 1)^(3/2)/3 - (x - 1)^(3/2)/3
```

3.419 $\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx$

3.419.1 Optimal result

Integrand size = 23, antiderivative size = 38

$$\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx=\frac{x^4}{2}-\frac{2}{3}(1-x^2)^{3/2}+\frac{2}{5}(1-x^2)^{5/2}$$

output
```
1/2*x^4-2/3*(-x^2+1)^(3/2)+2/5*(-x^2+1)^(5/2)
```

3.419.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx=\frac{1}{30}(-1+x^2)\left(15+8\sqrt{1-x^2}+3x^2\left(5+4\sqrt{1-x^2}\right)\right)$$

input
```
Integrate[x^3*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

output
```
((-1 + x^2)*(15 + 8*Sqrt[1 - x^2] + 3*x^2*(5 + 4*Sqrt[1 - x^2])))/30
```

3.419.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(\sqrt{1-x}+\sqrt{x+1}\right)^2 dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(2x^3+2\sqrt{1-x^2}x^3\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^4}{2}+\frac{2}{5}\left(1-x^2\right)^{5/2}-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

input
```
Int[x^3*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

output
```
x^4/2 - (2*(1 - x^2)^(3/2))/3 + (2*(1 - x^2)^(5/2))/5
```

3.419.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.419.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{x^4}{2}+\frac{2\sqrt{1-x}\,\sqrt{x+1}\,(x^2-1)\,(3x^2+2)}{15}$	33

input
```
int(x^3*((1-x)^(1/2)+(x+1)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^4+2/15*(1-x)^(1/2)*(x+1)^(1/2)*(x^2-1)*(3*x^2+2)
```

3.419.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\frac{1}{2}\,x^4+\frac{2}{15}\,(3\,x^4-x^2-2)\sqrt{x+1}\sqrt{-x+1}$$

input
```
integrate(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/2*x^4 + 2/15*(3*x^4 - x^2 - 2)*sqrt(x + 1)*sqrt(-x + 1)
```

3.419.6 Sympy [F]

$$\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\int x^3\left(\sqrt{1-x}+\sqrt{x+1}\right)^2\,dx$$

input
```
integrate(x**3*((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

output
```
Integral(x**3*(sqrt(1 - x) + sqrt(x + 1))**2, x)
```

3.419.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\frac{1}{2}\,x^4-\frac{2}{5}\,\left(-x^2+1\right)^{\frac{3}{2}}x^2-\frac{4}{15}\,\left(-x^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

output
```
1/2*x^4 - 2/5*(-x^2 + 1)^(3/2)*x^2 - 4/15*(-x^2 + 1)^(3/2)
```

3.419.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(28) = 56.

Time = 0.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.03

$$\begin{aligned}&\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx\\&=\frac{1}{2}\,x^4+\frac{1}{60}\,((2\,(3\,(4\,x-17)(x+1)+133)(x+1)-295)(x+1)+195)\sqrt{x+1}\sqrt{-x+1}\\&\quad+\frac{1}{12}\,((2\,(3\,x-10)(x+1)+43)(x+1)-39)\sqrt{x+1}\sqrt{-x+1}\end{aligned}$$

input
```
integrate(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

output
```
1/2*x^4 + 1/60*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) + 1
95)*sqrt(x + 1)*sqrt(-x + 1) + 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) -
 39)*sqrt(x + 1)*sqrt(-x + 1)
```

3.419.9 Mupad [B] (verification not implemented)

Time = 16.51 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\frac{x^4}{2}-\frac{\sqrt{1-x}\,\left(-\frac{2\,x^5}{5}-\frac{2\,x^4}{5}+\frac{2\,x^3}{15}+\frac{2\,x^2}{15}+\frac{4\,x}{15}+\frac{4}{15}\right)}{\sqrt{x+1}}$$

input
```
int(x^3*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

output
```
x^4/2 - ((1 - x)^(1/2)*((4*x)/15 + (2*x^2)/15 + (2*x^3)/15 - (2*x^4)/5 - (
2*x^5)/5 + 4/15))/(x + 1)^(1/2)
```

3.419. $\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx$

3.420 $\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx$

3.420.1 Optimal result

Integrand size = 23, antiderivative size = 48

$$\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\frac{2x^3}{3}-\frac{1}{4}x\sqrt{1-x^2}+\frac{1}{2}x^3\sqrt{1-x^2}+\frac{\arcsin(x)}{4}$$

output
```
2/3*x^3+1/4*arcsin(x)-1/4*x*(-x^2+1)^(1/2)+1/2*x^3*(-x^2+1)^(1/2)
```

3.420.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.40

$$\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\frac{1}{12}\left(8-3x\sqrt{1-x^2}+x^3\left(8+6\sqrt{1-x^2}\right)+12\arctan\left(\frac{-\sqrt{2}+\sqrt{1+x}}{\sqrt{1-x}}\right)\right)$$

input
```
Integrate[x^2*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

output
```
(8 - 3*x*Sqrt[1 - x^2] + x^3*(8 + 6*Sqrt[1 - x^2]) + 12*ArcTan[(-Sqrt[2] +
 Sqrt[1 + x])/Sqrt[1 - x]])/12
```

3.420.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(\sqrt{1-x}+\sqrt{x+1}\right)^2\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(2\sqrt{1-x^2}x^2+2x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arcsin(x)}{4}+\frac{2x^3}{3}-\frac{1}{4}\sqrt{1-x^2}x+\frac{1}{2}\sqrt{1-x^2}x^3$$

input
```
Int[x^2*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

output
```
(2*x^3)/3 - (x*Sqrt[1 - x^2])/4 + (x^3*Sqrt[1 - x^2])/2 + ArcSin[x]/4
```

3.420.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.420.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

method	result	size
default	$\frac{2x^3}{3}+\frac{\sqrt{1-x}\,\sqrt{x+1}\,\left(2x^3\sqrt{-x^2+1}-x\sqrt{-x^2+1}+\arcsin\left(x\right)\right)}{4\sqrt{-x^2+1}}$	59

input
```
int(x^2*((1-x)^(1/2)+(x+1)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*x^3+1/4*(1-x)^(1/2)*(x+1)^(1/2)*(2*x^3*(-x^2+1)^(1/2)-x*(-x^2+1)^(1/2)
+arcsin(x))/(-x^2+1)^(1/2)
```

3.420.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\frac{2}{3}\,x^3+\frac{1}{4}\,(2\,x^3-x)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{2}\,\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

output
```
2/3*x^3 + 1/4*(2*x^3 - x)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*arctan((sqrt(x +
1)*sqrt(-x + 1) - 1)/x)
```

3.420.6 Sympy [F]

$$\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\int x^2\left(\sqrt{1-x}+\sqrt{x+1}\right)^2\,dx$$

input
```
integrate(x**2*((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

output
```
Integral(x**2*(sqrt(1 - x) + sqrt(x + 1))**2, x)
```

3.420.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.71

$$\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\frac{2}{3}\,x^3-\frac{1}{2}\,(-x^2+1)^{\frac{3}{2}}x+\frac{1}{4}\,\sqrt{-x^2+1}x+\frac{1}{4}\,\arcsin{(x)}$$

input
```
integrate(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

output
```
2/3*x^3 - 1/2*(-x^2 + 1)^(3/2)*x + 1/4*sqrt(-x^2 + 1)*x + 1/4*arcsin(x)
```

3.420.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(36) = 72.

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.58

$$\begin{aligned}&\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx\\&=\frac{2}{3}\,x^3+\frac{1}{12}\,((2\,(3\,x-10)(x+1)+43)(x+1)-39)\sqrt{x+1}\sqrt{-x+1}\\&\quad+\frac{1}{3}\,((2\,x-5)(x+1)+9)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{2}\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{x+1}\right)\end{aligned}$$

input
```
integrate(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

output
```
2/3*x^3 + 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt
(-x + 1) + 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*arcs
in(1/2*sqrt(2)*sqrt(x + 1))
```

3.420.9 Mupad [B] (verification not implemented)

Time = 29.81 (sec) , antiderivative size = 563, normalized size of antiderivative = 11.73

$$\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx$$
$$=\frac{\frac{4\left(\sqrt{1-x}-1\right)}{\sqrt{x+1}-1}-\frac{28\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{28\left(\sqrt{1-x}-1\right)^5}{\left(\sqrt{x+1}-1\right)^5}-\frac{4\left(\sqrt{1-x}-1\right)^7}{\left(\sqrt{x+1}-1\right)^7}}{\frac{4\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{6\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{4\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{\left(\sqrt{1-x}-1\right)^8}{\left(\sqrt{x+1}-1\right)^8}+1}-\operatorname{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)$$
$$-\frac{\frac{3\left(\sqrt{1-x}-1\right)}{\sqrt{x+1}-1}+\frac{23\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}-\frac{333\left(\sqrt{1-x}-1\right)^5}{\left(\sqrt{x+1}-1\right)^5}+\frac{671\left(\sqrt{1-x}-1\right)^7}{\left(\sqrt{x+1}-1\right)^7}-\frac{671\left(\sqrt{1-x}-1\right)^9}{\left(\sqrt{x+1}-1\right)^9}+\frac{333\left(\sqrt{1-x}-1\right)^{11}}{\left(\sqrt{x+1}-1\right)^{11}}-\frac{23\left(\sqrt{1-x}-1\right)^{13}}{\left(\sqrt{x+1}-1\right)^{13}}}{\frac{8\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{28\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{56\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{70\left(\sqrt{1-x}-1\right)^8}{\left(\sqrt{x+1}-1\right)^8}+\frac{56\left(\sqrt{1-x}-1\right)^{10}}{\left(\sqrt{x+1}-1\right)^{10}}+\frac{28\left(\sqrt{1-x}-1\right)^{12}}{\left(\sqrt{x+1}-1\right)^{12}}+\frac{8\left(\sqrt{1-x}-1\right)^{14}}{\left(\sqrt{x+1}-1\right)^{14}}+\cdots}$$
$$+\frac{2x^3}{3}$$

input
```
int(x^2*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

output
```
((4*((1 - x)^(1/2) - 1))/((x + 1)^(1/2) - 1) - (28*((1 - x)^(1/2) - 1)^3)/
((x + 1)^(1/2) - 1)^3 + (28*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 -
 (4*((1 - x)^(1/2) - 1)^7)/((x + 1)^(1/2) - 1)^7)/((4*((1 - x)^(1/2) - 1)^
2)/((x + 1)^(1/2) - 1)^2 + (6*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4
 + (4*((1 - x)^(1/2) - 1)^6)/((x + 1)^(1/2) - 1)^6 + ((1 - x)^(1/2) - 1)^8
/((x + 1)^(1/2) - 1)^8 + 1) - atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)
) - ((3*((1 - x)^(1/2) - 1))/((x + 1)^(1/2) - 1) + (23*((1 - x)^(1/2) - 1)
^3)/((x + 1)^(1/2) - 1)^3 - (333*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1
)^5 + (671*((1 - x)^(1/2) - 1)^7)/((x + 1)^(1/2) - 1)^7 - (671*((1 - x)^(1
/2) - 1)^9)/((x + 1)^(1/2) - 1)^9 + (333*((1 - x)^(1/2) - 1)^11)/((x + 1)^
(1/2) - 1)^11 - (23*((1 - x)^(1/2) - 1)^13)/((x + 1)^(1/2) - 1)^13 - (3*((
1 - x)^(1/2) - 1)^15)/((x + 1)^(1/2) - 1)^15)/((8*((1 - x)^(1/2) - 1)^2)/(
(x + 1)^(1/2) - 1)^2 + (28*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 +
(56*((1 - x)^(1/2) - 1)^6)/((x + 1)^(1/2) - 1)^6 + (70*((1 - x)^(1/2) - 1)
^8)/((x + 1)^(1/2) - 1)^8 + (56*((1 - x)^(1/2) - 1)^10)/((x + 1)^(1/2) - 1
)^10 + (28*((1 - x)^(1/2) - 1)^12)/((x + 1)^(1/2) - 1)^12 + (8*((1 - x)^(1
/2) - 1)^14)/((x + 1)^(1/2) - 1)^14 + ((1 - x)^(1/2) - 1)^16/((x + 1)^(1/2
) - 1)^16 + 1) + (2*x^3)/3
```

3.421 $\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx$

3.421.1 Optimal result

Integrand size = 21, antiderivative size = 19

$$\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=x^2-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

output
```
x^2-2/3*(-x^2+1)^(3/2)
```

3.421.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\frac{1}{3}(-1+x)(1+x)\left(3+2\sqrt{1-x^2}\right)$$

input
```
Integrate[x*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

output
```
((-1 + x)*(1 + x)*(3 + 2*Sqrt[1 - x^2]))/3
```

3.421.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(\sqrt{1-x}+\sqrt{x+1}\right)^2 dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(2\sqrt{1-x^2}x+2x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x^2-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

input
```
Int[x*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

output
```
x^2 - (2*(1 - x^2)^(3/2))/3
```

3.421.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.421.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

method	result	size
default	$x^{2}+\frac{2\sqrt{1-x}\,\sqrt{x+1}\,\left(x^{2}-1\right)}{3}$	24

input
```
int(x*((1-x)^(1/2)+(x+1)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
x^2+2/3*(1-x)^(1/2)*(x+1)^(1/2)*(x^2-1)
```

3.421.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=x^2+\frac{2}{3}\left(x^2-1\right)\sqrt{x+1}\sqrt{-x+1}$$

input
```
integrate(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

output
```
x^2 + 2/3*(x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)
```

3.421.6 Sympy [F]

$$\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx=\int x\left(\sqrt{1-x}+\sqrt{x+1}\right)^2\,dx$$

input
```
integrate(x*((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

output
```
Integral(x*(sqrt(1 - x) + sqrt(x + 1))**2, x)
```

3.421.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx = x^2 - \frac{2}{3}\left(-x^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

output
```
x^2 - 2/3*(-x^2 + 1)^(3/2)
```

3.421.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(15) = 30.

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.68

$$\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx = (x+1)^2 + \frac{1}{3}\left((2\,x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1} + \sqrt{x+1}(x-2)\sqrt{-x+1} - 2\,x - 2$$

input
```
integrate(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

output
```
(x + 1)^2 + 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + sqrt(x
+ 1)*(x - 2)*sqrt(-x + 1) - 2*x - 2
```

3.421.9 Mupad [B] (verification not implemented)

Time = 16.82 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx = x^2 - \frac{\sqrt{1-x}\left(-\frac{2\,x^3}{3}-\frac{2\,x^2}{3}+\frac{2\,x}{3}+\frac{2}{3}\right)}{\sqrt{x+1}}$$

input
```
int(x*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

output
```
x^2 - ((1 - x)^(1/2)*((2*x)/3 - (2*x^2)/3 - (2*x^3)/3 + 2/3))/(x + 1)^(1/2
)
```

3.422 $\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx$

3.422.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx = 2x+x\sqrt{1-x^2}+\arcsin(x)$$

output
```
2*x+arcsin(x)+x*(-x^2+1)^(1/2)
```

3.422.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 45 vs. 2(19) = 38.

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.37

$$\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx = 2+x\left(2+\sqrt{1-x^2}\right)+4\arctan\left(\frac{-\sqrt{2}+\sqrt{1+x}}{\sqrt{1-x}}\right)$$

input
```
Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

output
```
2 + x*(2 + Sqrt[1 - x^2]) + 4*ArcTan[(-Sqrt[2] + Sqrt[1 + x])/Sqrt[1 - x]]
```

3.422.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sqrt{1-x}+\sqrt{x+1}\right)^2 \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(2\sqrt{1-x^2}+2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\arcsin(x)+\sqrt{1-x^2}x+2x$$

input
```
Int[(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

output
```
2*x + x*Sqrt[1 - x^2] + ArcSin[x]
```

3.422.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.422.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(17) = 34.

Time = 0.97 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.05

method	result	size
default	$2x+\sqrt{1-x}\,(x+1)^{\frac{3}{2}}-\sqrt{1-x}\,\sqrt{x+1}+\frac{\sqrt{(1-x)(x+1)}\,\arcsin(x)}{\sqrt{x+1}\,\sqrt{1-x}}$	58

input
```
int(((1-x)^(1/2)+(x+1)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
2*x+(1-x)^(1/2)*(x+1)^(3/2)-(1-x)^(1/2)*(x+1)^(1/2)+((1-x)*(x+1))^(1/2)/(x
+1)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

3.422.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(17) = 34$.

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.11

$$\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx = \sqrt{x+1}x\sqrt{-x+1}+2\,x-2\,\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate(((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

output
```
sqrt(x + 1)*x*sqrt(-x + 1) + 2*x - 2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)
/x)
```

3.422.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(15) = 30$.

Time = 0.90 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.32

$$\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx = 2x+4\sqrt{1-x}\left(\frac{(x+1)^{\frac{3}{2}}}{4}-\frac{\sqrt{x+1}}{4}\right)+2\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)$$

input
```
integrate(((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

output
```
2*x + 4*sqrt(1 - x)*((x + 1)**(3/2)/4 - sqrt(x + 1)/4) + 2*asin(sqrt(2)*sq
rt(x + 1)/2)
```

3.422.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx = \sqrt{-x^2+1}x + 2x + \arcsin(x)$$

input
```
integrate(((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

output
```
sqrt(-x^2 + 1)*x + 2*x + arcsin(x)
```

3.422.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(17) = 34.

Time = 0.36 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.53

$$\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx = \sqrt{x+1}(x-2)\sqrt{-x+1} + 2x + 2\sqrt{x+1}\sqrt{-x+1} + 2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right) + 2$$

input
```
integrate(((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

output
```
sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + 2*x + 2*sqrt(x + 1)*sqrt(-x + 1) + 2*ar
csin(1/2*sqrt(2)*sqrt(x + 1)) + 2
```

3.422.9 Mupad [B] (verification not implemented)

Time = 21.70 (sec) , antiderivative size = 206, normalized size of antiderivative = 10.84

$$\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx = 2x - 4\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right) - \frac{\frac{4\left(\sqrt{1-x}-1\right)}{\sqrt{x+1}-1} - \frac{28\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3} + \frac{28\left(\sqrt{1-x}-1\right)^5}{\left(\sqrt{x+1}-1\right)^5} - \frac{4\left(\sqrt{1-x}-1\right)^7}{\left(\sqrt{x+1}-1\right)^7}}{\frac{4\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2} + \frac{6\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4} + \frac{4\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6} + \frac{\left(\sqrt{1-x}-1\right)^8}{\left(\sqrt{x+1}-1\right)^8} + 1}$$

input
```
int(((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

output
```
2*x - 4*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - ((4*((1 - x)^(1/2)
 - 1))/((x + 1)^(1/2) - 1) - (28*((1 - x)^(1/2) - 1)^3)/((x + 1)^(1/2) - 1
)^3 + (28*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (4*((1 - x)^(1/2)
 - 1)^7)/((x + 1)^(1/2) - 1)^7)/((4*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2)
- 1)^2 + (6*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (4*((1 - x)^(1/
2) - 1)^6)/((x + 1)^(1/2) - 1)^6 + ((1 - x)^(1/2) - 1)^8/((x + 1)^(1/2) -
1)^8 + 1)
```

3.422. $\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 \, dx$

3.423 $\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx$

3.423.1 Optimal result

Integrand size = 23, antiderivative size = 32

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx = 2\sqrt{1-x^2}-2\text{arctanh}\left(\sqrt{1-x^2}\right)+2\log(x)$$

output
```
-2*arctanh((-x^2+1)^(1/2))+2*ln(x)+2*(-x^2+1)^(1/2)
```

3.423.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 72 vs. $2(32) = 64$.

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.25

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx = 2\left(\sqrt{1-x^2}+2\log\left(\sqrt{2}-\sqrt{1+x}\right)+2\log\left(\sqrt{1-x}-\sqrt{1+x}\right)-2\log\left(-2+\sqrt{2}\sqrt{1+x}\right)\right)$$

input
```
Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2/x,x]
```

output
```
2*(Sqrt[1 - x^2] + 2*Log[Sqrt[2] - Sqrt[1 + x]] + 2*Log[Sqrt[1 - x] - Sqrt
[1 + x]] - 2*Log[-2 + Sqrt[2]*Sqrt[1 + x]])
```

3.423.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x} \, dx$$

$$\Big\downarrow \; 7293$$

$$\int \left(\frac{2\sqrt{1-x^2}}{x}+\frac{2}{x}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-2\text{arctanh}\left(\sqrt{1-x^2}\right)+2\sqrt{1-x^2}+2\log(x)$$

input
```
Int[(Sqrt[1 - x] + Sqrt[1 + x])^2/x,x]
```

output
```
2*Sqrt[1 - x^2] - 2*ArcTanh[Sqrt[1 - x^2]] + 2*Log[x]
```

3.423.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.423.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.59

method	result	size
default	$2\ln\left(x\right)+\frac{2\sqrt{1-x}\,\sqrt{x+1}\,\left(\sqrt{-x^{2}+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)\right)}{\sqrt{-x^{2}+1}}$	51

input
```
int(((1-x)^(1/2)+(x+1)^(1/2))^2/x,x,method=_RETURNVERBOSE)
```

output
```
2*ln(x)+2*(1-x)^(1/2)*(x+1)^(1/2)/(-x^2+1)^(1/2)*((-x^2+1)^(1/2)-arctanh(1
/(-x^2+1)^(1/2)))
```

3.423.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx = 2\,\sqrt{x+1}\sqrt{-x+1}+2\,\log\left(x\right)+2\,\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x, algorithm="fricas")
```

output
```
2*sqrt(x + 1)*sqrt(-x + 1) + 2*log(x) + 2*log((sqrt(x + 1)*sqrt(-x + 1) -
1)/x)
```

3.423.6 Sympy [F]

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx = \int \frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x}\,dx$$

input
```
integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x,x)
```

output
```
Integral((sqrt(1 - x) + sqrt(x + 1))**2/x, x)
```

3.423.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx = 2\sqrt{-x^2+1} + 2\log(x) - 2\log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

input
```
integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x, algorithm="maxima")
```

output
```
2*sqrt(-x^2 + 1) + 2*log(x) - 2*log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

3.423.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(28) = 56.

Time = 0.40 (sec) , antiderivative size = 130, normalized size of antiderivative = 4.06

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx = 2\sqrt{x+1}\sqrt{-x+1} + 2\log\left(\sqrt{x+1}+1\right) + 2\log\left(\left|\sqrt{x+1}-1\right|\right) - 2\log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} + \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}} + 2\right|\right) + 2\log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} + \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}} - 2\right|\right)$$

input
```
integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x, algorithm="giac")
```

output
```
2*sqrt(x + 1)*sqrt(-x + 1) + 2*log(sqrt(x + 1) + 1) + 2*log(abs(sqrt(x + 1
) - 1)) - 2*log(abs(-(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + sqrt(x + 1)/(s
qrt(2) - sqrt(-x + 1)) + 2)) + 2*log(abs(-(sqrt(2) - sqrt(-x + 1))/sqrt(x
+ 1) + sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) - 2))
```

3.423.9 Mupad [B] (verification not implemented)

Time = 17.76 (sec) , antiderivative size = 122, normalized size of antiderivative = 3.81

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx = 2\ln\left(\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-1\right)-2\ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)+2\ln\left(x\right)+\frac{16\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2\left(\frac{2\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+1\right)}$$

input
```
int(((x + 1)^(1/2) + (1 - x)^(1/2))^2/x,x)
```

output
```
2*log(((1 - x)^(1/2) - 1)^2/((x + 1)^(1/2) - 1)^2 - 1) - 2*log(((1 - x)^(1
/2) - 1)/((x + 1)^(1/2) - 1)) + 2*log(x) + (16*((1 - x)^(1/2) - 1)^2)/(((x
 + 1)^(1/2) - 1)^2*((2*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)^2 + ((1
- x)^(1/2) - 1)^4/((x + 1)^(1/2) - 1)^4 + 1))
```

3.424 $\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2} \, dx$

3.424.1 Optimal result

Integrand size = 23, antiderivative size = 26

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2} \, dx=-\frac{2}{x}-\frac{2\sqrt{1-x^2}}{x}-2\arcsin(x)$$

output
```
-2/x-2*arcsin(x)-2*(-x^2+1)^(1/2)/x
```

3.424.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2} \, dx=-\frac{2\left(1+\sqrt{1-x^2}-4x\arctan\left(\frac{\sqrt{1+x}}{\sqrt{2}-\sqrt{1-x}}\right)\right)}{x}$$

input
```
Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^2,x]
```

output
```
(-2*(1 + Sqrt[1 - x^2] - 4*x*ArcTan[Sqrt[1 + x]/(Sqrt[2] - Sqrt[1 - x])]))
/x
```

3.424.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x^2} \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{2\sqrt{1-x^2}}{x^2}+\frac{2}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-2\arcsin(x)-\frac{2\sqrt{1-x^2}}{x}-\frac{2}{x}$$

input
```
Int[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^2,x]
```

output
```
-2/x - (2*Sqrt[1 - x^2])/x - 2*ArcSin[x]
```

3.424.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.424.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(24) = 48$.

Time = 0.92 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

method	result	size
default	$-\frac{2}{x}+\frac{2\left(-\arcsin(x)x-\sqrt{-x^2+1}\right)\sqrt{1-x}\,\sqrt{x+1}}{x\sqrt{-x^2+1}}$	50

input
```
int(((1-x)^(1/2)+(x+1)^(1/2))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-2/x+2*(-arcsin(x)*x-(-x^2+1)^(1/2))*(1-x)^(1/2)*(x+1)^(1/2)/x/(-x^2+1)^(1
/2)
```

3.424.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.69

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2}\,dx = \frac{2\left(2\,x\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)-\sqrt{x+1}\sqrt{-x+1}-1\right)}{x}$$

input
```
integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x, algorithm="fricas")
```

output
```
2*(2*x*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - sqrt(x + 1)*sqrt(-x + 1)
 - 1)/x
```

3.424.6 Sympy [F]

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2}\,dx = \int \frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x^2}\,dx$$

input
```
integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x**2,x)
```

output
```
Integral((sqrt(1 - x) + sqrt(x + 1))**2/x**2, x)
```

3.424.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2}\,dx = -\frac{2\sqrt{-x^2+1}}{x}-\frac{2}{x}-2\arcsin(x)$$

input
```
integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x, algorithm="maxima")
```

output
```
-2*sqrt(-x^2 + 1)/x - 2/x - 2*arcsin(x)
```

3.424.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(24) = 48.

Time = 0.31 (sec) , antiderivative size = 149, normalized size of antiderivative = 5.73

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2}\,dx = -2\pi-\frac{8\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)}{\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^2-4}-\frac{2}{x}-4\arctan\left(\frac{\sqrt{x+1}\left(\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}-1\right)}{2\left(\sqrt{2}-\sqrt{-x+1}\right)}\right)$$

input
```
integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x, algorithm="giac")
```

output
```
-2*pi - 8*((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - s
qrt(-x + 1)))/(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2
) - sqrt(-x + 1)))^2 - 4) - 2/x - 4*arctan(1/2*sqrt(x + 1)*((sqrt(2) - sqr
t(-x + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1)))
```

3.424.9 Mupad [B] (verification not implemented)

Time = 17.69 (sec) , antiderivative size = 120, normalized size of antiderivative = 4.62

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2}\,dx = 8\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-\frac{\frac{5\left(\sqrt{1-x}-1\right)^2}{2\left(\sqrt{x+1}-1\right)^2}-\frac{1}{2}}{\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}-\frac{\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}}-\frac{\sqrt{1-x}-1}{2\left(\sqrt{x+1}-1\right)}-\frac{2}{x}$$

input
```
int(((x + 1)^(1/2) + (1 - x)^(1/2))^2/x^2,x)
```

output
```
8*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - ((5*((1 - x)^(1/2) - 1)^
2)/(2*((x + 1)^(1/2) - 1)^2) - 1/2)/(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) -
1) - ((1 - x)^(1/2) - 1)^3/((x + 1)^(1/2) - 1)^3) - ((1 - x)^(1/2) - 1)/(2
*((x + 1)^(1/2) - 1)) - 2/x
```

3.425 $\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx$

3.425.1 Optimal result

Integrand size = 23, antiderivative size = 34

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx=-\frac{1}{x^2}-\frac{\sqrt{1-x^2}}{x^2}+\operatorname{arctanh}\left(\sqrt{1-x^2}\right)$$

output
```
-1/x^2+arctanh((-x^2+1)^(1/2))-(-x^2+1)^(1/2)/x^2
```

3.425.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 147 vs. 2(34) = 68.

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 4.32

$$\begin{aligned}&\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx\\&=2\operatorname{arctanh}\left(\frac{2-\sqrt{2}+2\sqrt{1-x}+2\sqrt{1+x}-\sqrt{2}\sqrt{1+x}}{-2+\sqrt{2}+\sqrt{2}\sqrt{1+x}}\right)+\log\left(\sqrt{2}-\sqrt{1+x}\right)\\&\quad-\frac{1+\sqrt{1-x^2}+x^2\log\left(-2-\sqrt{2}+\sqrt{1-x}+\sqrt{1+x}+\sqrt{2}\sqrt{1+x}\right)}{x^2}\end{aligned}$$

input
```
Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^3,x]
```

output
```
2*ArcTanh[(2 - Sqrt[2] + 2*Sqrt[1 - x] + 2*Sqrt[1 + x] - Sqrt[2]*Sqrt[1 +
x])/(-2 + Sqrt[2] + Sqrt[2]*Sqrt[1 + x])] + Log[Sqrt[2] - Sqrt[1 + x]] - (
1 + Sqrt[1 - x^2] + x^2*Log[-2 - Sqrt[2] + Sqrt[1 - x] + Sqrt[1 + x] + Sqr
t[2]*Sqrt[1 + x]])/x^2
```

3.425.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x^3} \, dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{2}{x^3}+\frac{2\sqrt{1-x^2}}{x^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\text{arctanh}\left(\sqrt{1-x^2}\right)-\frac{\sqrt{1-x^2}}{x^2}-\frac{1}{x^2}$$

input
```
Int[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^3,x]
```

output
```
-x^(-2) - Sqrt[1 - x^2]/x^2 + ArcTanh[Sqrt[1 - x^2]]
```

3.425.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.425.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.71

method	result	size
default	$-\frac{1}{x^{2}}+\frac{\sqrt{1-x}\,\sqrt{x+1}\,\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)x^{2}-\sqrt{-x^{2}+1}\right)}{x^{2}\sqrt{-x^{2}+1}}$	58

input
```
int(((1-x)^(1/2)+(x+1)^(1/2))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/x^2+(1-x)^(1/2)*(x+1)^(1/2)*(arctanh(1/(-x^2+1)^(1/2))*x^2-(-x^2+1)^(1/
2))/x^2/(-x^2+1)^(1/2)
```

3.425.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx=-\frac{x^{2}\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+\sqrt{x+1}\sqrt{-x+1}+1}{x^{2}}$$

input
```
integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3,x, algorithm="fricas")
```

output
```
-(x^2*log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + sqrt(x + 1)*sqrt(-x + 1) + 1
)/x^2
```

3.425.6 Sympy [F]

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx=\int \frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^{2}}{x^{3}}\,dx$$

input
```
integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x**3,x)
```

output
```
Integral((sqrt(1 - x) + sqrt(x + 1))**2/x**3, x)
```

3.425.7 Maxima [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.59

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx = -\sqrt{-x^2+1} - \frac{\left(-x^2+1\right)^{\frac{3}{2}}}{x^2} - \frac{1}{x^2} + \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

input
```
integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3,x, algorithm="maxima")
```

output
```
-sqrt(-x^2 + 1) - (-x^2 + 1)^(3/2)/x^2 - 1/x^2 + log(2*sqrt(-x^2 + 1)/abs(
x) + 2/abs(x))
```

3.425.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(30) = 60.

Time = 0.40 (sec) , antiderivative size = 235, normalized size of antiderivative = 6.91

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx = \frac{4\left(\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} - \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^3 + \frac{4\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}} - \frac{4\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)}{\left(\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} - \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^2 - 4\right)^2} - \frac{1}{x^2} + \log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} + \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}} + 2\right|\right) - \log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} + \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}} - 2\right|\right)$$

input
```
integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3,x, algorithm="giac")
```

output
```
4*(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - sqrt(-x
+ 1)))^3 + 4*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 4*sqrt(x + 1)/(sqrt(2)
 - sqrt(-x + 1)))/(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sq
rt(2) - sqrt(-x + 1)))^2 - 4)^2 - 1/x^2 + log(abs(-(sqrt(2) - sqrt(-x + 1)
)/sqrt(x + 1) + sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) + 2)) - log(abs(-(sqr
t(2) - sqrt(-x + 1))/sqrt(x + 1) + sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) -
2))
```

3.425.9 Mupad [B] (verification not implemented)

Time = 19.57 (sec) , antiderivative size = 189, normalized size of antiderivative = 5.56

$$\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx = \ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right) - \ln\left(\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-1\right) + \frac{\left(\sqrt{1-x}-1\right)^2}{16\left(\sqrt{x+1}-1\right)^2} - \frac{\frac{\left(\sqrt{1-x}-1\right)^2}{8\left(\sqrt{x+1}-1\right)^2}+\frac{15\left(\sqrt{1-x}-1\right)^4}{16\left(\sqrt{x+1}-1\right)^4}-\frac{1}{16}}{\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-\frac{2\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}} - \frac{1}{x^2}$$

input
```
int(((x + 1)^(1/2) + (1 - x)^(1/2))^2/x^3,x)
```

output
```
log(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - log(((1 - x)^(1/2) - 1)^2/(
(x + 1)^(1/2) - 1)^2 - 1) + ((1 - x)^(1/2) - 1)^2/(16*((x + 1)^(1/2) - 1)^
2) - (((1 - x)^(1/2) - 1)^2/(8*((x + 1)^(1/2) - 1)^2) + (15*((1 - x)^(1/2)
 - 1)^4)/(16*((x + 1)^(1/2) - 1)^4) - 1/16)/(((1 - x)^(1/2) - 1)^2/((x + 1
)^(1/2) - 1)^2 - (2*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + ((1 - x
)^(1/2) - 1)^6/((x + 1)^(1/2) - 1)^6) - 1/x^2
```

3.426 $\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

3.426.1 Optimal result

Integrand size = 25, antiderivative size = 147

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{2a^2(a+bx)^{3/2}}{3b^3(b-c)} - \frac{4a(a+bx)^{5/2}}{5b^3(b-c)} + \frac{2(a+bx)^{7/2}}{7b^3(b-c)} - \frac{2a^2(a+cx)^{3/2}}{3(b-c)c^3} + \frac{4a(a+cx)^{5/2}}{5(b-c)c^3} - \frac{2(a+cx)^{7/2}}{7(b-c)c^3}$$

output
```
2/3*a^2*(b*x+a)^(3/2)/b^3/(b-c)-4/5*a*(b*x+a)^(5/2)/b^3/(b-c)+2/7*(b*x+a)^
(7/2)/b^3/(b-c)-2/3*a^2*(c*x+a)^(3/2)/(b-c)/c^3+4/5*a*(c*x+a)^(5/2)/(b-c)/
c^3-2/7*(c*x+a)^(7/2)/(b-c)/c^3
```

3.426.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1832 vs. 2(147) = 294.

Time = 5.98 (sec) , antiderivative size = 1832, normalized size of antiderivative = 12.46

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \text{Too large to display}$$

input
```
Integrate[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

output
```
(-2*a^3*(b - c)^2*(a + c*x)*(15*b^3*Sqrt[a - (a*b)/c]*c^6*x^5*(b*x - Sqrt[
a + b*x]*Sqrt[a + c*x]) + 3*a*b^2*c^5*x^4*(109*b*Sqrt[a - (a*b)/c]*Sqrt[a
+ b*x]*Sqrt[a + c*x] - 120*Sqrt[a - (a*b)/c]*c*Sqrt[a + b*x]*Sqrt[a + c*x]
 + 7*b*c*x*(18*Sqrt[a - (a*b)/c] - 5*Sqrt[a + b*x] + 5*Sqrt[a + c*x]) - 5*
b^2*x*(22*Sqrt[a - (a*b)/c] - 7*Sqrt[a + b*x] + 7*Sqrt[a + c*x])) + a^6*(b
^3*c*(-378*Sqrt[a - (a*b)/c] + 966*Sqrt[a + b*x] - 200*Sqrt[a + c*x]) + b^
4*(15*Sqrt[a - (a*b)/c] - 105*Sqrt[a + b*x] + 8*Sqrt[a + c*x]) + 32*c^4*(3
5*Sqrt[a - (a*b)/c] - 35*Sqrt[a + b*x] + 16*Sqrt[a + c*x]) - 4*b*c^3*(595*
Sqrt[a - (a*b)/c] - 735*Sqrt[a + b*x] + 288*Sqrt[a + c*x]) + b^2*c^2*(1631
*Sqrt[a - (a*b)/c] - 2681*Sqrt[a + b*x] + 832*Sqrt[a + c*x])) + a^3*c^3*x^
2*(-960*Sqrt[a - (a*b)/c]*c^3*Sqrt[a + b*x]*Sqrt[a + c*x] + b^4*x*(-960*Sq
rt[a - (a*b)/c] + 945*Sqrt[a + b*x] - 791*Sqrt[a + c*x]) + 20*b*c^2*(132*S
qrt[a - (a*b)/c]*Sqrt[a + b*x]*Sqrt[a + c*x] + c*x*(119*Sqrt[a - (a*b)/c]
- 91*Sqrt[a + b*x] + 84*Sqrt[a + c*x])) + b^2*(-2376*Sqrt[a - (a*b)/c]*c*S
qrt[a + b*x]*Sqrt[a + c*x] - 14*c^2*x*(379*Sqrt[a - (a*b)/c] - 319*Sqrt[a
+ b*x] + 288*Sqrt[a + c*x])) + b^3*(693*Sqrt[a - (a*b)/c]*Sqrt[a + b*x]*Sq
rt[a + c*x] + 7*c*x*(558*Sqrt[a - (a*b)/c] - 513*Sqrt[a + b*x] + 449*Sqrt[
a + c*x]))) + a^2*b*c^4*x^3*(-1200*Sqrt[a - (a*b)/c]*c^2*Sqrt[a + b*x]*Sqr
t[a + c*x] + b^3*x*(855*Sqrt[a - (a*b)/c] - 630*Sqrt[a + b*x] + 609*Sqrt[a
 + c*x]) + b^2*(-785*Sqrt[a - (a*b)/c]*Sqrt[a + b*x]*Sqrt[a + c*x] - 21...
```

3.426.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.84, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2528, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

$$\Big\downarrow \ 2528$$

$$\frac{\int x^2\sqrt{a+bx}dx}{b-c}-\frac{\int x^2\sqrt{a+cx}dx}{b-c}$$

$$\Big\downarrow \ 53$$

$$\frac{\int \left(\frac{(a+bx)^{5/2}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{a^2\sqrt{a+bx}}{b^2}\right)dx}{b-c}-\frac{\int \left(\frac{(a+cx)^{5/2}}{c^2}-\frac{2a(a+cx)^{3/2}}{c^2}+\frac{a^2\sqrt{a+cx}}{c^2}\right)dx}{b-c}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{2a^2(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}}{b-c}-\frac{\frac{2a^2(a+cx)^{3/2}}{3c^3}+\frac{2(a+cx)^{7/2}}{7c^3}-\frac{4a(a+cx)^{5/2}}{5c^3}}{b-c}$$

input
```
Int[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

output
```
((2*a^2*(a + b*x)^(3/2))/(3*b^3) - (4*a*(a + b*x)^(5/2))/(5*b^3) + (2*(a +
 b*x)^(7/2))/(7*b^3))/(b - c) - ((2*a^2*(a + c*x)^(3/2))/(3*c^3) - (4*a*(a
 + c*x)^(5/2))/(5*c^3) + (2*(a + c*x)^(7/2))/(7*c^3))/(b - c)
```

3.426.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2528
```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Simp[c/(e*(b*c - a*d))   Int[(u*Sqrt[a + b*x])/x, x], x] - Si
mp[a/(f*(b*c - a*d))   Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

3.426.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{4(bx+a)^{\frac{5}{2}}a}{5}+\frac{2(bx+a)^{\frac{3}{2}}a^2}{3}}{(b-c)b^3}-\frac{2\left(\frac{(cx+a)^{\frac{7}{2}}}{7}-\frac{2(cx+a)^{\frac{5}{2}}a}{5}+\frac{(cx+a)^{\frac{3}{2}}a^2}{3}\right)}{(b-c)c^3}$	90

input
```
int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/(b-c)/b^3*(1/7*(b*x+a)^(7/2)-2/5*(b*x+a)^(5/2)*a+1/3*(b*x+a)^(3/2)*a^2)-
2/(b-c)/c^3*(1/7*(c*x+a)^(7/2)-2/5*(c*x+a)^(5/2)*a+1/3*(c*x+a)^(3/2)*a^2)
```

3.426.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$
$$= \frac{2\left(\left(15\,b^3c^3x^3+3\,ab^2c^3x^2-4\,a^2bc^3x+8\,a^3c^3\right)\sqrt{bx+a}-\left(15\,b^3c^3x^3+3\,ab^3c^2x^2-4\,a^2b^3cx+8\,a^3b^3\right)\sqrt{cx+}\right.}{105\left(b^4c^3-b^3c^4\right)}$$

input
```
integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

output
```
2/105*((15*b^3*c^3*x^3 + 3*a*b^2*c^3*x^2 - 4*a^2*b*c^3*x + 8*a^3*c^3)*sqrt
(b*x + a) - (15*b^3*c^3*x^3 + 3*a*b^3*c^2*x^2 - 4*a^2*b^3*c*x + 8*a^3*b^3)
*sqrt(c*x + a))/(b^4*c^3 - b^3*c^4)
```

3.426.6 Sympy [F]

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

input
```
integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

output
```
Integral(x**3/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

3.426.7 Maxima [F]

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \int \frac{x^3}{\sqrt{bx+a}+\sqrt{cx+a}}\,dx$$

input
```
integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

3.426.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(123) = 246$.

Time = 0.39 (sec) , antiderivative size = 451, normalized size of antiderivative = 3.07

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx =$$
$$-\frac{2}{105}\sqrt{ab^2+(bx+a)bc-abc}\left(\left(3\,(bx+a)\left(\frac{5\,(b^{17}c^5|b|-2\,b^{16}c^6|b|+b^{15}c^7|b|)(bx+a)}{b^{23}c^5-3\,b^{22}c^6+3\,b^{21}c^7-b^{20}c^8}+\frac{ab^{18}c^4|b|-17}{b^{23}c^5}\right.\right.\right.$$
$$+\frac{2\left(15\,(bx+a)^{\frac{7}{2}}-42\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2\right)}{105\,(b^4-b^3c)}$$

input
```
integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

output
```
-2/105*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*((3*(b*x + a)*(5*(b^17*c^5*abs(
b) - 2*b^16*c^6*abs(b) + b^15*c^7*abs(b))*(b*x + a)/(b^23*c^5 - 3*b^22*c^6
 + 3*b^21*c^7 - b^20*c^8) + (a*b^18*c^4*abs(b) - 17*a*b^17*c^5*abs(b) + 31
*a*b^16*c^6*abs(b) - 15*a*b^15*c^7*abs(b))/(b^23*c^5 - 3*b^22*c^6 + 3*b^21
*c^7 - b^20*c^8)) - (4*a^2*b^19*c^3*abs(b) - 2*a^2*b^18*c^4*abs(b) - 53*a^
2*b^17*c^5*abs(b) + 96*a^2*b^16*c^6*abs(b) - 45*a^2*b^15*c^7*abs(b))/(b^23
*c^5 - 3*b^22*c^6 + 3*b^21*c^7 - b^20*c^8))*(b*x + a) + (8*a^3*b^20*c^2*ab
s(b) - 12*a^3*b^19*c^3*abs(b) + 3*a^3*b^18*c^4*abs(b) - 17*a^3*b^17*c^5*ab
s(b) + 33*a^3*b^16*c^6*abs(b) - 15*a^3*b^15*c^7*abs(b))/(b^23*c^5 - 3*b^22
*c^6 + 3*b^21*c^7 - b^20*c^8)) + 2/105*(15*(b*x + a)^(7/2) - 42*(b*x + a)^
(5/2)*a + 35*(b*x + a)^(3/2)*a^2)/(b^4 - b^3*c)
```

3.426.9 Mupad [B] (verification not implemented)

Time = 17.61 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.22

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{2\,x^3\,\sqrt{a+b\,x}}{7\,(b-c)}-\frac{2\,x^3\,\sqrt{a+c\,x}}{7\,(b-c)}+\frac{16\,a^3\,\sqrt{a+b\,x}}{105\,b^3\,(b-c)}-\frac{16\,a^3\,\sqrt{a+c\,x}}{105\,c^3\,(b-c)}+\frac{2\,a\,x^2\,\sqrt{a+b\,x}}{35\,b\,(b-c)}-\frac{8\,a^2\,x\,\sqrt{a+b\,x}}{105\,b^2\,(b-c)}-\frac{2\,a\,x^2\,\sqrt{a+c\,x}}{35\,c\,(b-c)}+\frac{8\,a^2\,x\,\sqrt{a+c\,x}}{105\,c^2\,(b-c)}$$

input
```
int(x^3/((a + b*x)^(1/2) + (a + c*x)^(1/2)),x)
```

output
```
(2*x^3*(a + b*x)^(1/2))/(7*(b - c)) - (2*x^3*(a + c*x)^(1/2))/(7*(b - c))
+ (16*a^3*(a + b*x)^(1/2))/(105*b^3*(b - c)) - (16*a^3*(a + c*x)^(1/2))/(1
05*c^3*(b - c)) + (2*a*x^2*(a + b*x)^(1/2))/(35*b*(b - c)) - (8*a^2*x*(a +
 b*x)^(1/2))/(105*b^2*(b - c)) - (2*a*x^2*(a + c*x)^(1/2))/(35*c*(b - c))
+ (8*a^2*x*(a + c*x)^(1/2))/(105*c^2*(b - c))
```

3.426. $\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

3.427 $\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}} \, dx$

3.427.1 Optimal result

Integrand size = 25, antiderivative size = 95

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}} \, dx = -\frac{2a(a+bx)^{3/2}}{3b^2(b-c)}+\frac{2(a+bx)^{5/2}}{5b^2(b-c)}+\frac{2a(a+cx)^{3/2}}{3(b-c)c^2}-\frac{2(a+cx)^{5/2}}{5(b-c)c^2}$$

output
```
-2/3*a*(b*x+a)^(3/2)/b^2/(b-c)+2/5*(b*x+a)^(5/2)/b^2/(b-c)+2/3*a*(c*x+a)^(
3/2)/(b-c)/c^2-2/5*(c*x+a)^(5/2)/(b-c)/c^2
```

3.427.2 Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.19

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}} \, dx = \frac{6b^2c^2x^2\left(\sqrt{a+bx}-\sqrt{a+cx}\right)+2abcx\left(c\sqrt{a+bx}-b\sqrt{a+cx}\right)+a^2\left(-4c^2\sqrt{a+bx}+4b^2\sqrt{a+cx}\right)}{15b^2(b-c)c^2}$$

input
```
Integrate[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

output
```
(6*b^2*c^2*x^2*(Sqrt[a + b*x] - Sqrt[a + c*x]) + 2*a*b*c*x*(c*Sqrt[a + b*x
] - b*Sqrt[a + c*x]) + a^2*(-4*c^2*Sqrt[a + b*x] + 4*b^2*Sqrt[a + c*x]))/(
15*b^2*(b - c)*c^2)
```

3.427.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2528, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

$$\Big\downarrow \text{ 2528}$$

$$\frac{\int x\sqrt{a+bx}dx}{b-c}-\frac{\int x\sqrt{a+cx}dx}{b-c}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{\int \left(\frac{(a+bx)^{3/2}}{b}-\frac{a\sqrt{a+bx}}{b}\right)dx}{b-c}-\frac{\int \left(\frac{(a+cx)^{3/2}}{c}-\frac{a\sqrt{a+cx}}{c}\right)dx}{b-c}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{2(a+bx)^{5/2}}{5b^2}-\frac{2a(a+bx)^{3/2}}{3b^2}}{b-c}-\frac{\frac{2(a+cx)^{5/2}}{5c^2}-\frac{2a(a+cx)^{3/2}}{3c^2}}{b-c}$$

input
```
Int[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

output
```
((-2*a*(a + b*x)^(3/2))/(3*b^2) + (2*(a + b*x)^(5/2))/(5*b^2))/(b - c) - (
(-2*a*(a + c*x)^(3/2))/(3*c^2) + (2*(a + c*x)^(5/2))/(5*c^2))/(b - c)
```

3.427.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2528
```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Simp[c/(e*(b*c - a*d))   Int[(u*Sqrt[a + b*x])/x, x], x] - Si
mp[a/(f*(b*c - a*d))   Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

3.427.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{2(bx+a)^{\frac{3}{2}}a}{3}}{(b-c)b^2}-\frac{2\left(\frac{(cx+a)^{\frac{5}{2}}}{5}-\frac{(cx+a)^{\frac{3}{2}}a}{3}\right)}{(b-c)c^2}$	66

input
```
int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/(b-c)/b^2*(1/5*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a)-2/(b-c)/c^2*(1/5*(c*x+
a)^(5/2)-1/3*(c*x+a)^(3/2)*a)
```

3.427.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{2\left((3\,b^2c^2x^2+abc^2x-2\,a^2c^2)\sqrt{bx+a}-(3\,b^2c^2x^2+ab^2cx-2\,a^2b^2)\sqrt{cx+a}\right)}{15\,(b^3c^2-b^2c^3)}$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

output
```
2/15*((3*b^2*c^2*x^2 + a*b*c^2*x - 2*a^2*c^2)*sqrt(b*x + a) - (3*b^2*c^2*x
^2 + a*b^2*c*x - 2*a^2*b^2)*sqrt(c*x + a))/(b^3*c^2 - b^2*c^3)
```

3.427.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx=\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

input
```
integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

output
```
Integral(x**2/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

3.427.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx=\int \frac{x^2}{\sqrt{bx+a}+\sqrt{cx+a}}\,dx$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

3.427.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(79) = 158.

Time = 0.37 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.68

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx= -\frac{2}{15}\sqrt{ab^2+(bx+a)bc-abc}\left((bx+a)\left(\frac{3\,(b^9c^3|b|-b^8c^4|b|)(bx+a)}{b^{14}c^3-2\,b^{13}c^4+b^{12}c^5}+\frac{ab^{10}c^2|b|-7\,ab^9c^3|b|+6\,ab^8c^4|b\right.\right.}{b^{14}c^3-2\,b^{13}c^4+b^{12}c^5}$$
$$+\frac{2\left(3\,(bx+a)^{\frac{5}{2}}-5\,(bx+a)^{\frac{3}{2}}a\right)}{15\,(b^3-b^2c)}$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

output
```
-2/15*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*((b*x + a)*(3*(b^9*c^3*abs(b) -
b^8*c^4*abs(b))*(b*x + a)/(b^14*c^3 - 2*b^13*c^4 + b^12*c^5) + (a*b^10*c^2
*abs(b) - 7*a*b^9*c^3*abs(b) + 6*a*b^8*c^4*abs(b))/(b^14*c^3 - 2*b^13*c^4
+ b^12*c^5)) - (2*a^2*b^11*c*abs(b) - a^2*b^10*c^2*abs(b) - 4*a^2*b^9*c^3*
abs(b) + 3*a^2*b^8*c^4*abs(b))/(b^14*c^3 - 2*b^13*c^4 + b^12*c^5)) + 2/15*
(3*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a)/(b^3 - b^2*c)
```

3.427.9 Mupad [B] (verification not implemented)

Time = 16.54 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.36

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{2\,x^2\,\sqrt{a+b\,x}}{5\,\left(b-c\right)}-\frac{2\,x^2\,\sqrt{a+c\,x}}{5\,\left(b-c\right)}-\frac{4\,a^2\,\sqrt{a+b\,x}}{15\,b^2\,\left(b-c\right)}+\frac{4\,a^2\,\sqrt{a+c\,x}}{15\,c^2\,\left(b-c\right)}+\frac{2\,a\,x\,\sqrt{a+b\,x}}{15\,b\,\left(b-c\right)}-\frac{2\,a\,x\,\sqrt{a+c\,x}}{15\,c\,\left(b-c\right)}$$

input
```
int(x^2/((a + b*x)^(1/2) + (a + c*x)^(1/2)),x)
```

output
```
(2*x^2*(a + b*x)^(1/2))/(5*(b - c)) - (2*x^2*(a + c*x)^(1/2))/(5*(b - c))
- (4*a^2*(a + b*x)^(1/2))/(15*b^2*(b - c)) + (4*a^2*(a + c*x)^(1/2))/(15*c
^2*(b - c)) + (2*a*x*(a + b*x)^(1/2))/(15*b*(b - c)) - (2*a*x*(a + c*x)^(1
/2))/(15*c*(b - c))
```

3.428 $\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

3.428.1 Optimal result

Integrand size = 23, antiderivative size = 47

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{2(a+bx)^{3/2}}{3b(b-c)} - \frac{2(a+cx)^{3/2}}{3(b-c)c}$$

output
```
2/3*(b*x+a)^(3/2)/b/(b-c)-2/3*(c*x+a)^(3/2)/(b-c)/c
```

3.428.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.51

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{2ac\sqrt{a+bx}+2bcx\sqrt{a+bx}-2ab\sqrt{a+cx}-2bcx\sqrt{a+cx}}{3b^2c-3bc^2}$$

input
```
Integrate[x/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

output
```
(2*a*c*Sqrt[a + b*x] + 2*b*c*x*Sqrt[a + b*x] - 2*a*b*Sqrt[a + c*x] - 2*b*c
*x*Sqrt[a + c*x])/(3*b^2*c - 3*b*c^2)
```

3.428.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2528, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}dx$$

$$\Big\downarrow \text{ 2528}$$

$$\frac{\int \sqrt{a+bx}dx}{b-c}-\frac{\int \sqrt{a+cx}dx}{b-c}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{2(a+bx)^{3/2}}{3b(b-c)}-\frac{2(a+cx)^{3/2}}{3c(b-c)}$$

input
```
Int[x/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

output
```
(2*(a + b*x)^(3/2))/(3*b*(b - c)) - (2*(a + c*x)^(3/2))/(3*(b - c)*c)
```

3.428.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2528
```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Simp[c/(e*(b*c - a*d))   Int[(u*Sqrt[a + b*x])/x, x], x] - Si
mp[a/(f*(b*c - a*d))   Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

3.428.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{2(bx+a)^{\frac{3}{2}}}{3b(b-c)}-\frac{2(cx+a)^{\frac{3}{2}}}{3(b-c)c}$	40

input
```
int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/3*(b*x+a)^(3/2)/b/(b-c)-2/3*(c*x+a)^(3/2)/(b-c)/c
```

3.428.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{2\left((bcx+ac)\sqrt{bx+a}-(bcx+ab)\sqrt{cx+a}\right)}{3\left(b^2c-bc^2\right)}$$

input
```
integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

output
```
2/3*((b*c*x + a*c)*sqrt(b*x + a) - (b*c*x + a*b)*sqrt(c*x + a))/(b^2*c - b
*c^2)
```

3.428.6 Sympy [F]

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

input
```
integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

output
```
Integral(x/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

3.428.7 Maxima [F]

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx=\int \frac{x}{\sqrt{bx+a}+\sqrt{cx+a}}\,dx$$

input
```
integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

3.428.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(39) = 78.

Time = 0.34 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.28

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = -\frac{2\left(\left(\frac{(bx+a)b^2c|b|}{b^5c-b^4c^2}+\frac{ab^3|b|-ab^2c|b|}{b^5c-b^4c^2}\right)\sqrt{ab^2+(bx+a)bc-abc}-\frac{(bx+a)^{\frac{3}{2}}}{b-c}\right)}{3\,b}$$

input
```
integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

output
```
-2/3*(((b*x + a)*b^2*c*abs(b)/(b^5*c - b^4*c^2) + (a*b^3*abs(b) - a*b^2*c*
abs(b))/(b^5*c - b^4*c^2))*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c) - (b*x + a)
^(3/2)/(b - c))/b
```

3.428.9 Mupad [B] (verification not implemented)

Time = 16.55 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.68

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx=\frac{2\,x\,\sqrt{a+b\,x}}{3\,(b-c)}-\frac{2\,x\,\sqrt{a+c\,x}}{3\,(b-c)}+\frac{2\,a\,\sqrt{a+b\,x}}{3\,b\,(b-c)}-\frac{2\,a\,\sqrt{a+c\,x}}{3\,c\,(b-c)}$$

input
```
int(x/((a + b*x)^(1/2) + (a + c*x)^(1/2)),x)
```

output
```
(2*x*(a + b*x)^(1/2))/(3*(b - c)) - (2*x*(a + c*x)^(1/2))/(3*(b - c)) + (2
*a*(a + b*x)^(1/2))/(3*b*(b - c)) - (2*a*(a + c*x)^(1/2))/(3*c*(b - c))
```

3.429 $\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

3.429.1 Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx=\frac{2\sqrt{a+bx}}{b-c}-\frac{2\sqrt{a+cx}}{b-c}-\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{b-c}+\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{b-c}$$

output
```
-2*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)/(b-c)+2*arctanh((c*x+a)^(1/2)/a^
(1/2))*a^(1/2)/(b-c)+2*(b*x+a)^(1/2)/(b-c)-2*(c*x+a)^(1/2)/(b-c)
```

3.429.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx=\frac{2\left(\sqrt{a+bx}-\sqrt{a+cx}\right)}{b-c}-\frac{4\sqrt{a-\frac{ab}{c}}\sqrt{c}\arctan\left(\frac{\sqrt{b-c}\sqrt{a+cx}}{\sqrt{c}\left(-\sqrt{a-\frac{ab}{c}}+\sqrt{a+bx}+\sqrt{a+cx}\right)}\right)}{(b-c)^{3/2}}$$

input
```
Integrate[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-1),x]
```

output
```
(2*(Sqrt[a + b*x] - Sqrt[a + c*x]))/(b - c) - (4*Sqrt[a - (a*b)/c]*Sqrt[c]
*ArcTan[(Sqrt[b - c]*Sqrt[a + c*x])/(Sqrt[c]*(-Sqrt[a - (a*b)/c] + Sqrt[a
+ b*x] + Sqrt[a + c*x]))])/(b - c)^(3/2)
```

3.429.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {7241, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}dx$$

$$\Big\downarrow \text{7241}$$

$$\frac{\int \left(\frac{\sqrt{a+bx}}{x}-\frac{\sqrt{a+cx}}{x}\right)dx}{b-c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)+2\sqrt{a+bx}-2\sqrt{a+cx}}{b-c}$$

input
```
Int[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-1),x]
```

output
```
(2*Sqrt[a + b*x] - 2*Sqrt[a + c*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[
a]] + 2*Sqrt[a]*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(b - c)
```

3.429.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7241
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(b*e^2 - d*f^2)^m   Int[ExpandIntegran
d[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

3.429.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{2\sqrt{bx+a}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b-c}-\frac{2\sqrt{cx+a}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{b-c}$	73

input
```
int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/(b-c)*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-1/(b-c)
*(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))
```

3.429.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.63

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$
$$=\left[-\frac{\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\sqrt{a}\log\left(\frac{cx-2\sqrt{cx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}+2\sqrt{cx+a}}{b-c},\ \frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{b}}{}\right.\right.}{}\right.$$

input
```
integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

output
```
[-(sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(a)*log((c*x
 - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x + a) + 2*sqrt(c*x + a))/
(b - c), 2*(sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(-a)*arctan(sq
rt(c*x + a)*sqrt(-a)/a) + sqrt(b*x + a) - sqrt(c*x + a))/(b - c)]
```

3.429.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx=\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

input
```
integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

output
```
Integral(1/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

3.429.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \int \frac{1}{\sqrt{bx+a}+\sqrt{cx+a}}\,dx$$

input
```
integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

3.429.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1093 vs. $2(81) = 162$.

Time = 0.54 (sec) , antiderivative size = 1093, normalized size of antiderivative = 11.27

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \text{Too large to display}$$

input
```
integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

output
```
-2*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*abs(b)/(b^3 - b^2*c) + 2*a*arctan(s
qrt(b*x + a)/sqrt(-a))/(sqrt(-a)*(b - c)) + 2*sqrt(b*x + a)/(b - c) - 2*(2
*(a*b^3*c - a*b^2*c^2)*(a*b^2 - a*b*c)^2*sqrt(-a)*abs(b)*sgn(b - c) + 2*(a
*b^3 - a*b^2*c)*(a*b^2 - a*b*c)^2*sqrt(-a*b*c)*abs(b) + (a^2*b^5 - 3*a^2*b
^4*c + 3*a^2*b^3*c^2 - a^2*b^2*c^3)*sqrt(-a*b*c)*abs(a*b^2 - a*b*c)*abs(b)
*sgn(b - c) + (a^2*b^6 - 3*a^2*b^5*c + 3*a^2*b^4*c^2 - a^2*b^3*c^3)*sqrt(-
a)*abs(a*b^2 - a*b*c)*abs(b) + (a^3*b^7*c - 2*a^3*b^6*c^2 + 2*a^3*b^4*c^4
- a^3*b^3*c^5)*sqrt(-a)*abs(b)*sgn(b - c) + (a^3*b^7 - 2*a^3*b^6*c + 2*a^3
*b^4*c^3 - a^3*b^3*c^4)*sqrt(-a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x +
 a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a*b^3 - a*b*c^2 + sqrt((
a*b^3 - a*b*c^2)^2 - (a^2*b^5 - 3*a^2*b^4*c + 3*a^2*b^3*c^2 - a^2*b^2*c^3)
*(b - c)))/(b - c)))/((b^8 - 5*b^7*c + 10*b^6*c^2 - 10*b^5*c^3 + 5*b^4*c^4
 - b^3*c^5)*a^2*abs(a*b^2 - a*b*c)) + 2*(2*(a*b^3*c - a*b^2*c^2)*(a*b^2 -
a*b*c)^2*sqrt(-a)*abs(b)*sgn(b - c) + 2*(a*b^3 - a*b^2*c)*(a*b^2 - a*b*c)^
2*sqrt(-a*b*c)*abs(b) + (a^2*b^5 - 3*a^2*b^4*c + 3*a^2*b^3*c^2 - a^2*b^2*c
^3)*sqrt(-a*b*c)*abs(a*b^2 - a*b*c)*abs(b)*sgn(b - c) + (a^2*b^6 - 3*a^2*b
^5*c + 3*a^2*b^4*c^2 - a^2*b^3*c^3)*sqrt(-a)*abs(a*b^2 - a*b*c)*abs(b) + (
a^3*b^7*c - 2*a^3*b^6*c^2 + 2*a^3*b^4*c^4 - a^3*b^3*c^5)*sqrt(-a)*abs(b)*s
gn(b - c) + (a^3*b^7 - 2*a^3*b^6*c + 2*a^3*b^4*c^3 - a^3*b^3*c^4)*sqrt(-a*
b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*...
```

3.429.9 Mupad [B] (verification not implemented)

Time = 18.16 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.20

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = -\frac{2\sqrt{a}\,c\left(\frac{2\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}+\frac{\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}\right)-2\sqrt{a}\,b\left(\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)-\frac{2\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}+4\right)}{(b-c)\left(b-\frac{c\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}\right)}$$

input
```
int(1/((a + b*x)^(1/2) + (a + c*x)^(1/2)),x)
```

output
```
-(2*a^(1/2)*c*((2*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2))
 + (log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*((a + b*x
)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2) - 2*a^(1/2)*b*(log(((
a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2))) - (2*((a + b*x)^(1/
2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) + 4))/((b - c)*(b - (c*((a + b*
x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2))
```

3.430 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$

3.430.1 Optimal result

Integrand size = 25, antiderivative size = 103

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = -\frac{\sqrt{a+bx}}{(b-c)x}+\frac{\sqrt{a+cx}}{(b-c)x} - \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)}+\frac{c\operatorname{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)}$$

output
```
-b*arctanh((b*x+a)^(1/2)/a^(1/2))/(b-c)/a^(1/2)+c*arctanh((c*x+a)^(1/2)/a^
(1/2))/(b-c)/a^(1/2)-(b*x+a)^(1/2)/(b-c)/x+(c*x+a)^(1/2)/(b-c)/x
```

3.430.2 Mathematica [A] (verified)

Time = 10.17 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.31

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \frac{-\frac{a}{\sqrt{a+bx}}-\frac{bx}{\sqrt{a+bx}}+\frac{a}{\sqrt{a+cx}}+\frac{cx}{\sqrt{a+cx}}-\frac{bx\sqrt{1+\frac{bx}{a}}\operatorname{arctanh}\left(\sqrt{1+\frac{bx}{a}}\right)}{\sqrt{a+bx}}+\frac{cx\sqrt{1+\frac{cx}{a}}\operatorname{arctanh}\left(\sqrt{1+\frac{cx}{a}}\right)}{\sqrt{a+cx}}}{bx-cx}$$

input
```
Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

output
```
(-(a/Sqrt[a + b*x]) - (b*x)/Sqrt[a + b*x] + a/Sqrt[a + c*x] + (c*x)/Sqrt[a
 + c*x] - (b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]])/Sqrt[a + b*x]
 + (c*x*Sqrt[1 + (c*x)/a]*ArcTanh[Sqrt[1 + (c*x)/a]])/Sqrt[a + c*x])/(b*x
- c*x)
```

3.430.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2528, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}dx$$

$$\Big\downarrow \text{2528}$$

$$\frac{\int \frac{\sqrt{a+bx}}{x^2}dx}{b-c}-\frac{\int \frac{\sqrt{a+cx}}{x^2}dx}{b-c}$$

$$\Big\downarrow \text{51}$$

$$\frac{\frac{1}{2}b\int \frac{1}{x\sqrt{a+bx}}dx-\frac{\sqrt{a+bx}}{x}}{b-c}-\frac{\frac{1}{2}c\int \frac{1}{x\sqrt{a+cx}}dx-\frac{\sqrt{a+cx}}{x}}{b-c}$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}-\frac{\sqrt{a+bx}}{x}}{b-c}-\frac{\int \frac{1}{\frac{a+cx}{c}-\frac{a}{c}}d\sqrt{a+cx}-\frac{\sqrt{a+cx}}{x}}{b-c}$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx}}{x}}{b-c}-\frac{-\frac{c\,\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+cx}}{x}}{b-c}$$

input
```
Int[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

output
```
(-(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a])/(b - c)
- (-(Sqrt[a + c*x]/x) - (c*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/Sqrt[a])/(b - c
)
```

3.430.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2528
```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Simp[c/(e*(b*c - a*d))   Int[(u*Sqrt[a + b*x])/x, x], x] - Si
mp[a/(f*(b*c - a*d))   Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

3.430.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{2b\left(-\frac{\sqrt{bx+a}}{2xb}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{b-c}-\frac{2c\left(-\frac{\sqrt{cx+a}}{2cx}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{b-c}$	88

input
```
int(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/(b-c)*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/
2))-2/(b-c)*c*(-1/2*(c*x+a)^(1/2)/c/x-1/2/a^(1/2)*arctanh((c*x+a)^(1/2)/a^
(1/2)))
```

3.430.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.77

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \left[-\frac{\sqrt{a}bx\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\sqrt{a}cx\log\left(\frac{cx-2\sqrt{cx+a}\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}a-2\sqrt{cx+a}a}{2(ab-ac)x}, \frac{\sqrt{-a}bx\,\mathrm{arcta}}{}\right.$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(a)*b*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(a)*
c*x*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a)*a - 2*s
qrt(c*x + a)*a)/((a*b - a*c)*x), (sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-
a)/a) - sqrt(-a)*c*x*arctan(sqrt(c*x + a)*sqrt(-a)/a) - sqrt(b*x + a)*a +
sqrt(c*x + a)*a)/((a*b - a*c)*x)]
```

3.430.6 Sympy [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$$

input
```
integrate(1/x/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

output
```
Integral(1/(x*(sqrt(a + b*x) + sqrt(a + c*x))), x)
```

3.430.7 Maxima [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{cx+a}\right)}\,dx$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(x*(sqrt(b*x + a) + sqrt(c*x + a))), x)
```

3.430.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1402 vs. $2(87) = 174$.

Time = 2.75 (sec) , antiderivative size = 1402, normalized size of antiderivative = 13.61

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

output
```
b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*(b - c)) - 2*((sqrt(b*c)*sqrt(b
*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a*b^2*c*abs(b) - (sqrt(b*c)
*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a*b*c^2*abs(b) + (sq
rt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*c*abs(b))/(
(a^2*b^4 - 2*a^2*b^3*c + a^2*b^2*c^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a
*b^2 + (b*x + a)*b*c - a*b*c))^2*a*b^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt
(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*b*c + (sqrt(b*c)*sqrt(b*x + a) - sqrt
(a*b^2 + (b*x + a)*b*c - a*b*c))^4)*(b - c)) - sqrt(b*x + a)/((b - c)*x) +
 (2*(a*b^3*c^2 - a*b^2*c^3)*(a*b^2 - a*b*c)^2*sqrt(-a)*abs(b)*sgn(-2*b + 2
*c) + 2*(a*b^3*c - a*b^2*c^2)*(a*b^2 - a*b*c)^2*sqrt(-a*b*c)*abs(b) + (a^2
*b^5*c - 3*a^2*b^4*c^2 + 3*a^2*b^3*c^3 - a^2*b^2*c^4)*sqrt(-a*b*c)*abs(-a*
b^2 + a*b*c)*abs(b)*sgn(-2*b + 2*c) + (a^2*b^6*c - 3*a^2*b^5*c^2 + 3*a^2*b
^4*c^3 - a^2*b^3*c^4)*sqrt(-a)*abs(-a*b^2 + a*b*c)*abs(b) + (a^3*b^7*c^2 -
 2*a^3*b^6*c^3 + 2*a^3*b^4*c^5 - a^3*b^3*c^6)*sqrt(-a)*abs(b)*sgn(-2*b + 2
*c) + (a^3*b^7*c - 2*a^3*b^6*c^2 + 2*a^3*b^4*c^4 - a^3*b^3*c^5)*sqrt(-a*b*
c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c -
 a*b*c))/sqrt(-(a*b^3 - a*b*c^2 + sqrt((a*b^3 - a*b*c^2)^2 - (a^2*b^5 - 3*
a^2*b^4*c + 3*a^2*b^3*c^2 - a^2*b^2*c^3)*(b - c)))/(b - c)))/((b^8 - 5*b^7
*c + 10*b^6*c^2 - 10*b^5*c^3 + 5*b^4*c^4 - b^3*c^5)*a^3*abs(-a*b^2 + a*b*c
)) - (2*(a*b^3*c^2 - a*b^2*c^3)*(a*b^2 - a*b*c)^2*sqrt(-a)*abs(b)*sgn(-...
```

3.430.9 Mupad [B] (verification not implemented)

Time = 24.65 (sec) , antiderivative size = 1637, normalized size of antiderivative = 15.89

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(x*((a + b*x)^(1/2) + (a + c*x)^(1/2))),x)
```

output
```
(2*a*b - 2*a*c + a*c*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^
(1/2))) - 2*a^(1/2)*b*(a + c*x)^(1/2) + 2*a^(1/2)*c*(a + b*x)^(1/2) + a*b*
atan((b^3*(a + b*x)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2
)*1i - a^(1/2)*c^3*1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*
(a + b*x)^(1/2) - b^3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3
- a^(1/2)*b*c^2 + b*c^2*(a + c*x)^(1/2)))*2i - a*c*atan((b^3*(a + b*x)^(1/
2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1i - a^(1/2)*c^3*1i +
 a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a + b*x)^(1/2) - b^3*(
a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(1/2)*b*c^2 + b*c^2
*(a + c*x)^(1/2)))*2i + a*b*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/
2) - a^(1/2))) + b*atan((b^3*(a + b*x)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i +
 c^3*(a + b*x)^(1/2)*1i - a^(1/2)*c^3*1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c
*x)^(1/2)*1i)/(b^3*(a + b*x)^(1/2) - b^3*(a + c*x)^(1/2) - c^3*(a + b*x)^(
1/2) + a^(1/2)*c^3 - a^(1/2)*b*c^2 + b*c^2*(a + c*x)^(1/2)))*(a + b*x)^(1/
2)*(a + c*x)^(1/2)*2i - c*atan((b^3*(a + b*x)^(1/2)*1i - b^3*(a + c*x)^(1/
2)*1i + c^3*(a + b*x)^(1/2)*1i - a^(1/2)*c^3*1i + a^(1/2)*b*c^2*1i - b*c^2
*(a + c*x)^(1/2)*1i)/(b^3*(a + b*x)^(1/2) - b^3*(a + c*x)^(1/2) - c^3*(a +
 b*x)^(1/2) + a^(1/2)*c^3 - a^(1/2)*b*c^2 + b*c^2*(a + c*x)^(1/2)))*(a + b
*x)^(1/2)*(a + c*x)^(1/2)*2i + b*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x
)^(1/2) - a^(1/2)))*(a + b*x)^(1/2)*(a + c*x)^(1/2) + c*log(((a + b*x)^...
```

3.430. $\int \frac{1}{x(\sqrt{a+bx}+\sqrt{a+cx})}\,dx$

3.431 $\int \frac{1}{x^2(\sqrt{a+bx}+\sqrt{a+cx})}\,dx$

3.431.1 Optimal result

Integrand size = 25, antiderivative size = 171

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = -\frac{\sqrt{a+bx}}{2(b-c)x^2}-\frac{b\sqrt{a+bx}}{4a(b-c)x}+\frac{\sqrt{a+cx}}{2(b-c)x^2}+\frac{c\sqrt{a+cx}}{4a(b-c)x}+\frac{b^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}-\frac{c^2\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}$$

output
```
1/4*b^2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)/(b-c)-1/4*c^2*arctanh((c*x+
a)^(1/2)/a^(1/2))/a^(3/2)/(b-c)-1/2*(b*x+a)^(1/2)/(b-c)/x^2-1/4*b*(b*x+a)^
(1/2)/a/(b-c)/x+1/2*(c*x+a)^(1/2)/(b-c)/x^2+1/4*c*(c*x+a)^(1/2)/a/(b-c)/x
```

3.431.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \frac{-2b^2(a+bx)^{3/2}\,\text{Hypergeometric2F1}\left(\frac{3}{2},3,\frac{5}{2},1+\frac{bx}{a}\right)+2c^2(a+cx)^{3/2}\,\text{Hypergeometric2F1}\left(\frac{3}{2},3,\frac{5}{2},1+\frac{cx}{a}\right.}{3a^3(b-c)}$$

input
```
Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

output
```
(-2*b^2*(a + b*x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x)/a] + 2*c^
2*(a + c*x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (c*x)/a])/(3*a^3*(b -
 c))
```

3.431.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2528, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}dx$$

$$\Big\downarrow \text{ 2528}$$

$$\frac{\int \frac{\sqrt{a+bx}}{x^3}dx}{b-c}-\frac{\int \frac{\sqrt{a+cx}}{x^3}dx}{b-c}$$

$$\Big\downarrow \text{ 51}$$

$$\frac{\frac{1}{4}b\int \frac{1}{x^2\sqrt{a+bx}}dx-\frac{\sqrt{a+bx}}{2x^2}}{b-c}-\frac{\frac{1}{4}c\int \frac{1}{x^2\sqrt{a+cx}}dx-\frac{\sqrt{a+cx}}{2x^2}}{b-c}$$

$$\Big\downarrow \text{ 52}$$

$$\frac{\frac{1}{4}b\left(-\frac{b\int \frac{1}{x\sqrt{a+bx}}dx}{2a}-\frac{\sqrt{a+bx}}{ax}\right)-\frac{\sqrt{a+bx}}{2x^2}}{b-c}-\frac{\frac{1}{4}c\left(-\frac{c\int \frac{1}{x\sqrt{a+cx}}dx}{2a}-\frac{\sqrt{a+cx}}{ax}\right)-\frac{\sqrt{a+cx}}{2x^2}}{b-c}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\frac{1}{4}b\left(-\frac{\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{a}-\frac{\sqrt{a+bx}}{ax}\right)-\frac{\sqrt{a+bx}}{2x^2}}{b-c}-\frac{\frac{1}{4}c\left(-\frac{\int \frac{1}{\frac{a+cx}{c}-\frac{a}{c}}d\sqrt{a+cx}}{a}-\frac{\sqrt{a+cx}}{ax}\right)-\frac{\sqrt{a+cx}}{2x^2}}{b-c}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{1}{4}b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}\right)-\frac{\sqrt{a+bx}}{2x^2}}{b-c}-\frac{\frac{1}{4}c\left(\frac{c\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+cx}}{ax}\right)-\frac{\sqrt{a+cx}}{2x^2}}{b-c}$$

input
```
Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

3.431. $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}dx$

output
```
(-1/2*Sqrt[a + b*x]/x^2 + (b*(-(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a +
 b*x]/Sqrt[a]])/a^(3/2)))/4)/(b - c) - (-1/2*Sqrt[a + c*x]/x^2 + (c*(-(Sqr
t[a + c*x]/(a*x)) + (c*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/a^(3/2)))/4)/(b - c
)
```

3.431.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2528
```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Simp[c/(e*(b*c - a*d))   Int[(u*Sqrt[a + b*x])/x, x], x] - Si
mp[a/(f*(b*c - a*d))   Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

3.431.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{2b^2\left(\frac{-\frac{(bx+a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{bx+a}}{8}}{x^2b^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)}{b-c}-\frac{2c^2\left(\frac{-\frac{(cx+a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{cx+a}}{8}}{c^2x^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)}{b-c}$	120

input
```
int(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/(b-c)*b^2*((-1/8/a*(b*x+a)^(3/2)-1/8*(b*x+a)^(1/2))/x^2/b^2+1/8/a^(3/2)*
arctanh((b*x+a)^(1/2)/a^(1/2)))-2/(b-c)*c^2*((-1/8/a*(c*x+a)^(3/2)-1/8*(c*
x+a)^(1/2))/c^2/x^2+1/8/a^(3/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))
```

3.431.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$$
$$= \left[-\frac{\sqrt{a}b^2x^2\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\sqrt{a}c^2x^2\log\left(\frac{cx+2\sqrt{cx+a}\sqrt{a}+2a}{x}\right)+2\left(abx+2a^2\right)\sqrt{bx+a}-2\left(acx+2\right.}{8\left(a^2b-a^2c\right)x^2}\right.$$
$$-\frac{\sqrt{-a}b^2x^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\sqrt{-a}c^2x^2\arctan\left(\frac{\sqrt{cx+a}\sqrt{-a}}{a}\right)+\left(abx+2a^2\right)\sqrt{bx+a}-\left(acx+2a^2\right)\sqrt{c}}{4\left(a^2b-a^2c\right)x^2}$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt
(a)*c^2*x^2*log((c*x + 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) + 2*(a*b*x + 2*a^
2)*sqrt(b*x + a) - 2*(a*c*x + 2*a^2)*sqrt(c*x + a))/((a^2*b - a^2*c)*x^2),
 -1/4*(sqrt(-a)*b^2*x^2*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(-a)*c^2*x^
2*arctan(sqrt(c*x + a)*sqrt(-a)/a) + (a*b*x + 2*a^2)*sqrt(b*x + a) - (a*c*
x + 2*a^2)*sqrt(c*x + a))/((a^2*b - a^2*c)*x^2)]
```

3.431.6 Sympy [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx=\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$$

input
```
integrate(1/x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

output
```
Integral(1/(x**2*(sqrt(a + b*x) + sqrt(a + c*x))), x)
```

3.431.7 Maxima [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx=\int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{cx+a}\right)}\,dx$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(x^2*(sqrt(b*x + a) + sqrt(c*x + a))), x)
```

3.431.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1895 vs. $2(139) = 278$.

Time = 6.13 (sec) , antiderivative size = 1895, normalized size of antiderivative = 11.08

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx=\text{Too large to display}$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

output
```
-1/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/((a*b - a*c)*sqrt(-a)) - 1/2*((sqr
t(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a^3*b^6*c^2*ab
s(b) - 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a
^3*b^5*c^3*abs(b) + 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*
c - a*b*c))*a^3*b^4*c^4*abs(b) - (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (
b*x + a)*b*c - a*b*c))*a^3*b^3*c^5*abs(b) + 7*(sqrt(b*c)*sqrt(b*x + a) - s
qrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*a^2*b^4*c^2*abs(b) - 10*(sqrt(b*c)*s
qrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*a^2*b^3*c^3*abs(b) +
 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*a^2*b
^2*c^4*abs(b) + 7*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c -
a*b*c))^5*a*b^2*c^2*abs(b) - 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*
x + a)*b*c - a*b*c))^5*a*b*c^3*abs(b) + (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*
b^2 + (b*x + a)*b*c - a*b*c))^7*c^2*abs(b))/((a^2*b^4 - 2*a^2*b^3*c + a^2*
b^2*c^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)
)^2*a*b^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*
c))^2*a*b*c + (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*
c))^4)^2*(a*b - a*c)) - 1/4*((b*x + a)^(3/2)*b^2 + sqrt(b*x + a)*a*b^2)/((
a*b - a*c)*b^2*x^2) - 1/4*(2*(a*b^3*c^3 - a*b^2*c^4)*(a^2*b^2 - a^2*b*c)^2
*sqrt(-a)*abs(b)*sgn(8*a*b - 8*a*c) + 2*(a*b^3*c^2 - a*b^2*c^3)*(a^2*b^2 -
 a^2*b*c)^2*sqrt(-a*b*c)*abs(b) + (a^3*b^5*c^2 - 3*a^3*b^4*c^3 + 3*a^3*...
```

3.431.9 Mupad [B] (verification not implemented)

Time = 26.55 (sec) , antiderivative size = 1610, normalized size of antiderivative = 9.42

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(x^2*((a + b*x)^(1/2) + (a + c*x)^(1/2))),x)
```

output
```
((a^(3/2)*b^3)/(16*(a^3*c^2 - a^3*b*c)) + (a^(3/2)*((a + b*x)^(1/2) - a^(1
/2))^2*((b*c^2)/4 - (7*b^2*c)/16 + b^3/4))/((a^3*c^2 - a^3*b*c)*((a + c*x)
^(1/2) - a^(1/2))^2) - (a^(3/2)*((b^2*c)/16 + b^3/16)*((a + b*x)^(1/2) - a
^(1/2)))/((a^3*c^2 - a^3*b*c)*((a + c*x)^(1/2) - a^(1/2))) + ((b^2/8 - c^2
/8)*((a + b*x)^(1/2) - a^(1/2))^3)/(a^(3/2)*c*((a + c*x)^(1/2) - a^(1/2))^
3))/(((a + b*x)^(1/2) - a^(1/2))^4/((a + c*x)^(1/2) - a^(1/2))^4 - ((b + c
)*((a + b*x)^(1/2) - a^(1/2))^3)/(c*((a + c*x)^(1/2) - a^(1/2))^3) + (b*((
a + b*x)^(1/2) - a^(1/2))^2)/(c*((a + c*x)^(1/2) - a^(1/2))^2)) - (((c*(b
+ c))/(4*a^(3/2)*(b - c)) - (c*(b^2 - c^2))/(4*a^(3/2)*(b - c)^2))*((a + b
*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (log(((a + b*x)^(1/2)
- a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*(a^(3/2)*b^2 + a^(3/2)*c^2))/(8*a^
3*b - 8*a^3*c) + (atan((((b + c)*(((b + c)*((64*a^6*b^3 - 64*a^6*b*c^2)/(6
4*(a^6*c^3 - a^6*b*c^2)) - (((a + b*x)^(1/2) - a^(1/2))*(64*a^6*b^3 - 64*a
^6*c^3 + 128*a^6*b*c^2 - 128*a^6*b^2*c))/(32*(a^6*c^3 - a^6*b*c^2)*((a + c
*x)^(1/2) - a^(1/2)))))/(8*a^3) - (16*a^3*b^4 + 16*a^3*b*c^3)/(64*(a^6*c^3
 - a^6*b*c^2)) + ((8*a^3*b^4 + 8*a^3*c^4)*((a + b*x)^(1/2) - a^(1/2)))/(32
*(a^6*c^3 - a^6*b*c^2)*((a + c*x)^(1/2) - a^(1/2))))*1i)/(8*a^3) - ((b + c
)*((16*a^3*b^4 + 16*a^3*b*c^3)/(64*(a^6*c^3 - a^6*b*c^2)) + ((b + c)*((64*
a^6*b^3 - 64*a^6*b*c^2)/(64*(a^6*c^3 - a^6*b*c^2)) - (((a + b*x)^(1/2) - a
^(1/2))*(64*a^6*b^3 - 64*a^6*c^3 + 128*a^6*b*c^2 - 128*a^6*b^2*c))/(32*...
```

3.431. $\int \frac{1}{x^2(\sqrt{a+bx}+\sqrt{a+cx})}\,dx$

3.432 $\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

3.432.1 Optimal result

Integrand size = 25, antiderivative size = 195

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a^2(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4b^2(b-c)c^2} + \frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c}-\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3b(b-c)^2c} - \frac{a^3(b+c)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{4b^{5/2}c^{5/2}}$$

output
```
a*x^2/(b-c)^2+1/3*(b+c)*x^3/(b-c)^2-2/3*(b*x+a)^(3/2)*(c*x+a)^(3/2)/b/(b-c
)^2/c-1/4*a^3*(b+c)*arctanh(c^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(c*x+a)^(1/2))/b
^(5/2)/c^(5/2)+1/2*a*(b+c)*(b*x+a)^(3/2)*(c*x+a)^(1/2)/b^2/(b-c)^2/c+1/4*a
^2*(b+c)*(b*x+a)^(1/2)*(c*x+a)^(1/2)/b^2/(b-c)/c^2
```

3.432.2 Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

$$= \frac{\frac{c\sqrt{a+bx}\sqrt{a+cx}\left(a^2\left(3b^2-2bc+3c^2\right)-2abc(b+c)x-8b^2c^2x^2\right)}{b^2(b-c)^2}+\frac{4\left(a^3(b-2c)+3ac^3x^2+c^3(b+c)x^3\right)}{(b-c)^2}+\frac{6a^3\sqrt{c}(b+c)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{a+cx}}{\sqrt{c}\left(\sqrt{a-\frac{ab}{c}}-\sqrt{a+\right.}\right.}\right.}{b^{5/2}}}{12c^3}$$

input
```
Integrate[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

output
```
((c*Sqrt[a + b*x]*Sqrt[a + c*x]*(a^2*(3*b^2 - 2*b*c + 3*c^2) - 2*a*b*c*(b
+ c)*x - 8*b^2*c^2*x^2))/(b^2*(b - c)^2) + (4*(a^3*(b - 2*c) + 3*a*c^3*x^2
 + c^3*(b + c)*x^3))/(b - c)^2 + (6*a^3*Sqrt[c]*(b + c)*ArcTanh[(Sqrt[b]*S
qrt[a + c*x])/(Sqrt[c]*(Sqrt[a - (a*b)/c] - Sqrt[a + b*x]))])/b^(5/2))/(12
*c^3)
```

3.432.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7241, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2} \, dx$$

$$\Big\downarrow \; 7241$$

$$\frac{\int \left((b+c)x^2+2ax-2\sqrt{a+bx}\sqrt{a+cx}x\right)dx}{(b-c)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{a^3(b-c)^2(b+c)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{4b^{5/2}c^{5/2}}+\frac{a^2(b^2-c^2)\sqrt{a+bx}\sqrt{a+cx}}{4b^2c^2}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2c}-\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3bc}+ax^2+\frac{1}{3}x^3(b}{(b-c)^2}$$

input
```
Int[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

output
```
(a*x^2 + ((b + c)*x^3)/3 + (a^2*(b^2 - c^2)*Sqrt[a + b*x]*Sqrt[a + c*x])/(
4*b^2*c^2) + (a*(b + c)*(a + b*x)^(3/2)*Sqrt[a + c*x])/(2*b^2*c) - (2*(a +
 b*x)^(3/2)*(a + c*x)^(3/2))/(3*b*c) - (a^3*(b - c)^2*(b + c)*ArcTanh[(Sqr
t[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(4*b^(5/2)*c^(5/2)))/(b - c)
^2
```

3.432.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7241
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(b*e^2 - d*f^2)^m   Int[ExpandIntegran
d[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

3.432.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(161) = 322$.

Time = 0.04 (sec) , antiderivative size = 517, normalized size of antiderivative = 2.65

method	result
default	$\frac{b\,x^3}{3(b-c)^2}+\frac{c\,x^3}{3(b-c)^2}+\frac{a\,x^2}{(b-c)^2}-\frac{\sqrt{bx+a}\,\sqrt{cx+a}\,\left(16x^2b^2c^2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}+3\ln\left(\frac{2bcx+2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}+ab+}{2\sqrt{bc}}\right.\right.}{}$

input
```
int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/(b-c)^2*b*x^3+1/3/(b-c)^2*c*x^3+a*x^2/(b-c)^2-1/24/(b-c)^2*(b*x+a)^(1/
2)*(c*x+a)^(1/2)*(16*x^2*b^2*c^2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/
2)+3*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c
)/(b*c)^(1/2))*a^3*b^3-3*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)
*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*a^3*b^2*c-3*ln(1/2*(2*b*c*x+2*(b*c*x^2+
a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*a^3*b*c^2+3*ln(1/
2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1
/2))*a^3*c^3+4*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)*x*a*b^2*c+4*(b*
c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)*x*a*b*c^2-6*(b*c*x^2+a*b*x+a*c*x+
a^2)^(1/2)*(b*c)^(1/2)*a^2*b^2+4*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/
2)*a^2*b*c-6*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)*a^2*c^2)/(b*c*x^2
+a*b*x+a*c*x+a^2)^(1/2)/b^2/c^2/(b*c)^(1/2)
```

3.432.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 479, normalized size of antiderivative = 2.46

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$
$$= \left[\frac{24\,ab^3c^3x^2 + 8\,(b^4c^3 + b^3c^4)x^3 + 3\,(a^3b^3 - a^3b^2c - a^3bc^2 + a^3c^3)\sqrt{bc}\log\left(ab^2 + 2\,abc + ac^2 + 2\left(2\,bc - \sqrt{}\right.\right.}{}\right.$$

input
```
integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

output
```
[1/24*(24*a*b^3*c^3*x^2 + 8*(b^4*c^3 + b^3*c^4)*x^3 + 3*(a^3*b^3 - a^3*b^2
*c - a^3*b*c^2 + a^3*c^3)*sqrt(b*c)*log(a*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*c
 - sqrt(b*c)*(b + c))*sqrt(b*x + a)*sqrt(c*x + a) + 2*(b^2*c + b*c^2)*x -
2*(2*b*c*x + a*b + a*c)*sqrt(b*c)) - 2*(8*b^3*c^3*x^2 - 3*a^2*b^3*c + 2*a^
2*b^2*c^2 - 3*a^2*b*c^3 + 2*(a*b^3*c^2 + a*b^2*c^3)*x)*sqrt(b*x + a)*sqrt(
c*x + a))/(b^5*c^3 - 2*b^4*c^4 + b^3*c^5), 1/12*(12*a*b^3*c^3*x^2 + 4*(b^4
*c^3 + b^3*c^4)*x^3 + 3*(a^3*b^3 - a^3*b^2*c - a^3*b*c^2 + a^3*c^3)*sqrt(-
b*c)*arctan((sqrt(-b*c)*sqrt(b*x + a)*sqrt(c*x + a) - sqrt(-b*c)*a)/(b*c*x
)) - (8*b^3*c^3*x^2 - 3*a^2*b^3*c + 2*a^2*b^2*c^2 - 3*a^2*b*c^3 + 2*(a*b^3
*c^2 + a*b^2*c^3)*x)*sqrt(b*x + a)*sqrt(c*x + a))/(b^5*c^3 - 2*b^4*c^4 + b
^3*c^5)]
```

3.432.6 Sympy [F]

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

input
```
integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

output
```
Integral(x**3/(sqrt(a + b*x) + sqrt(a + c*x))**2, x)
```

3.432.7 Maxima [F]

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \int \frac{x^3}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

input
```
integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a))^2, x)
```

3.432.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(161) = 322$.

Time = 0.80 (sec) , antiderivative size = 511, normalized size of antiderivative = 2.62

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = -\frac{1}{12}\sqrt{ab^2+(bx+a)bc-abc}\left(2\,(bx+a)\left(\frac{4\,(b^{11}c^4|b|-3\,b^{10}c^5|b|+3\,b^9c^6|b|-b^8c^7|b|)(bx+a)}{b^{17}c^4-5\,b^{16}c^5+10\,b^{15}c^6-10\,b^{14}c^7+5\,b^{13}c^8-b^{12}c^9}+\frac{ab^{12}c^{3}\cdots}{b\cdots}\right.\right.$$
$$+\frac{(bx+a)^3b-3\,(bx+a)a^2b+(bx+a)^3c-3\,(bx+a)^2ac+3\,(bx+a)a^2c}{3\,(b^5-2\,b^4c+b^3c^2)}$$
$$+\frac{(a^3b|b|+a^3c|b|)\log\left(\left|-\sqrt{bc}\sqrt{bx+a}+\sqrt{ab^2+(bx+a)bc-abc}\right|\right)}{4\,\sqrt{bc}b^3c^2}$$

input
```
integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

output
```
-1/12*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*(2*(b*x + a)*(4*(b^11*c^4*abs(b)
 - 3*b^10*c^5*abs(b) + 3*b^9*c^6*abs(b) - b^8*c^7*abs(b))*(b*x + a)/(b^17*
c^4 - 5*b^16*c^5 + 10*b^15*c^6 - 10*b^14*c^7 + 5*b^13*c^8 - b^12*c^9) + (a
*b^12*c^3*abs(b) - 10*a*b^11*c^4*abs(b) + 24*a*b^10*c^5*abs(b) - 22*a*b^9*
c^6*abs(b) + 7*a*b^8*c^7*abs(b))/(b^17*c^4 - 5*b^16*c^5 + 10*b^15*c^6 - 10
*b^14*c^7 + 5*b^13*c^8 - b^12*c^9)) - 3*(a^2*b^13*c^2*abs(b) - 3*a^2*b^12*
c^3*abs(b) + 2*a^2*b^11*c^4*abs(b) + 2*a^2*b^10*c^5*abs(b) - 3*a^2*b^9*c^6
*abs(b) + a^2*b^8*c^7*abs(b))/(b^17*c^4 - 5*b^16*c^5 + 10*b^15*c^6 - 10*b^
14*c^7 + 5*b^13*c^8 - b^12*c^9))*sqrt(b*x + a) + 1/3*((b*x + a)^3*b - 3*(b
*x + a)*a^2*b + (b*x + a)^3*c - 3*(b*x + a)^2*a*c + 3*(b*x + a)*a^2*c)/(b^
5 - 2*b^4*c + b^3*c^2) + 1/4*(a^3*b*abs(b) + a^3*c*abs(b))*log(abs(-sqrt(b
*c)*sqrt(b*x + a) + sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)))/(sqrt(b*c)*b^3*c
^2)
```

3.432.9 Mupad [B] (verification not implemented)

Time = 45.03 (sec) , antiderivative size = 1107, normalized size of antiderivative = 5.68

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$
$$= \frac{\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^6\,\left(128\,a^3\,b^3\,c+\frac{1312\,a^3\,b^2\,c^2}{3}+128\,a^3\,b\,c^3\right)}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^6}-\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^7\,\left(19\,a^3\,b^3\,c+269\,a^3\,b^2\,c^2+269\,a^3\,b\,c^3+19\,a^3\,c^4\right)}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^7}-\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^5}{}}{}$$
$$+\frac{x^3\,(b+c)}{3\,(b-c)^2}+\frac{a\,x^2}{(b-c)^2}-\frac{a^3\,\mathrm{atanh}\left(\frac{\sqrt{c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{b}\,\left(\sqrt{a+c\,x}-\sqrt{a}\right)}\right)\,(b+c)}{2\,b^{5/2}\,c^{5/2}}$$

input
```
int(x^3/((a + b*x)^(1/2) + (a + c*x)^(1/2))^2,x)
```

output
```
((((a + b*x)^(1/2) - a^(1/2))^6*(128*a^3*b*c^3 + 128*a^3*b^3*c + (1312*a^3
*b^2*c^2)/3))/((a + c*x)^(1/2) - a^(1/2))^6 - (((a + b*x)^(1/2) - a^(1/2))
^7*(19*a^3*c^4 + 269*a^3*b*c^3 + 19*a^3*b^3*c + 269*a^3*b^2*c^2))/((a + c*
x)^(1/2) - a^(1/2))^7 - (((a + b*x)^(1/2) - a^(1/2))^5*(19*a^3*b^4 + 19*a^
3*b*c^3 + 269*a^3*b^3*c + 269*a^3*b^2*c^2))/((a + c*x)^(1/2) - a^(1/2))^5
+ (((a + b*x)^(1/2) - a^(1/2))^4*(64*a^3*b^4 + 192*a^3*b^3*c + 64*a^3*b^2*
c^2))/((a + c*x)^(1/2) - a^(1/2))^4 + (((a + b*x)^(1/2) - a^(1/2))^8*(64*a
^3*c^4 + 192*a^3*b*c^3 + 64*a^3*b^2*c^2))/((a + c*x)^(1/2) - a^(1/2))^8 +
(16*a^3*b^4*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 +
 (16*a^3*c^4*((a + b*x)^(1/2) - a^(1/2))^10)/((a + c*x)^(1/2) - a^(1/2))^1
0 + (((a + b*x)^(1/2) - a^(1/2))^11*(a^3*c^6 - a^3*b*c^5 - a^3*b^2*c^4 + a
^3*b^3*c^3))/(2*b^2*((a + c*x)^(1/2) - a^(1/2))^11) - (((a + b*x)^(1/2) -
a^(1/2))^3*(17*a^3*b^5 + 303*a^3*b^4*c + 17*a^3*b^2*c^3 + 303*a^3*b^3*c^2)
)/(6*c*((a + c*x)^(1/2) - a^(1/2))^3) - (((a + b*x)^(1/2) - a^(1/2))^9*(17
*a^3*c^5 + 303*a^3*b*c^4 + 303*a^3*b^2*c^3 + 17*a^3*b^3*c^2))/(6*b*((a + c
*x)^(1/2) - a^(1/2))^9) + ((a^3*b + a^3*c)*((a + b*x)^(1/2) - a^(1/2))*(b^
5 - 2*b^4*c + b^3*c^2))/(2*c^2*((a + c*x)^(1/2) - a^(1/2))))/(b^8 - 2*b^7*
c + b^6*c^2 + (((a + b*x)^(1/2) - a^(1/2))^12*(c^8 - 2*b*c^7 + b^2*c^6))/(
(a + c*x)^(1/2) - a^(1/2))^12 - (((a + b*x)^(1/2) - a^(1/2))^2*(6*b^7*c +
6*b^5*c^3 - 12*b^6*c^2))/((a + c*x)^(1/2) - a^(1/2))^2 - (((a + b*x)^(1...
```

3.433 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

3.433.1 Optimal result

Integrand size = 25, antiderivative size = 142

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{a\sqrt{a+bx}\sqrt{a+cx}}{2b(b-c)c} - \frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+\frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{2b^{3/2}c^{3/2}}$$

output
```
2*a*x/(b-c)^2+1/2*(b+c)*x^2/(b-c)^2+1/2*a^2*arctanh(c^(1/2)*(b*x+a)^(1/2)/
b^(1/2)/(c*x+a)^(1/2))/b^(3/2)/c^(3/2)-(b*x+a)^(3/2)*(c*x+a)^(1/2)/b/(b-c)
^2-1/2*a*(b*x+a)^(1/2)*(c*x+a)^(1/2)/b/(b-c)/c
```

3.433.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.23

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = -\frac{a^2b(b-3c)-bc^2x\left(bx+cx-2\sqrt{a+bx}\sqrt{a+cx}\right)+ac\left(-4bcx+b\sqrt{a+bx}\sqrt{a+cx}+c\sqrt{a+bx}\sqrt{a+c}\right.}{2b(b-c)^2c^2} - \frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{a+cx}}{\sqrt{c}\left(\sqrt{a-\frac{ab}{c}}-\sqrt{a+bx}\right)}\right)}{b^{3/2}c^{3/2}}$$

input
```
Integrate[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

output
```
-1/2*(a^2*b*(b - 3*c) - b*c^2*x*(b*x + c*x - 2*Sqrt[a + b*x]*Sqrt[a + c*x]
) + a*c*(-4*b*c*x + b*Sqrt[a + b*x]*Sqrt[a + c*x] + c*Sqrt[a + b*x]*Sqrt[a
 + c*x]))/(b*(b - c)^2*c^2) - (a^2*ArcTanh[(Sqrt[b]*Sqrt[a + c*x])/(Sqrt[c
]*(Sqrt[a - (a*b)/c] - Sqrt[a + b*x]))])/(b^(3/2)*c^(3/2))
```

3.433.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7241, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2} \, dx$$

$$\Big\downarrow \text{ 7241}$$

$$\frac{\int \left(2a+(b+c)x-2\sqrt{a+bx}\sqrt{a+cx}\right)dx}{(b-c)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{a^2(b-c)^2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{2b^{3/2}c^{3/2}}-\frac{a(b-c)\sqrt{a+bx}\sqrt{a+cx}}{2bc}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b}+2ax+\frac{1}{2}x^2(b+c)}{(b-c)^2}$$

input
```
Int[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

output
```
(2*a*x + ((b + c)*x^2)/2 - (a*(b - c)*Sqrt[a + b*x]*Sqrt[a + c*x])/(2*b*c)
 - ((a + b*x)^(3/2)*Sqrt[a + c*x])/b + (a^2*(b - c)^2*ArcTanh[(Sqrt[c]*Sqr
t[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(2*b^(3/2)*c^(3/2)))/(b - c)^2
```

3.433.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7241
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(b*e^2 - d*f^2)^m   Int[ExpandIntegran
d[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

3.433.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.32

method	result
default	$\frac{x^2 b}{2(b-c)^2}+\frac{x^2 c}{2(b-c)^2}+\frac{2ax}{(b-c)^2}-\frac{2\left(\frac{\sqrt{bx+a}\,(cx+a)^{\frac{3}{2}}}{2c}-\frac{(ab-ac)\left(\frac{\sqrt{cx+a}\,\sqrt{bx+a}}{b}-\frac{(-ab+ac)\sqrt{(bx+a)(cx+a)}\,\ln\left(\frac{\frac{1}{2}ab+\frac{1}{2}ac+bcx}{\sqrt{bc}}+\sqrt{bc\,x^2}\right.}{2b\sqrt{cx+a}\,\sqrt{bx+a}\,\sqrt{bc}}\right.}{4c}\right.}{(b-c)^2}$

input
```
int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/(b-c)^2*b+1/2*x^2/(b-c)^2*c+2*a*x/(b-c)^2-2/(b-c)^2*(1/2/c*(b*x+a)
^(1/2)*(c*x+a)^(3/2)-1/4*(a*b-a*c)/c*(1/b*(c*x+a)^(1/2)*(b*x+a)^(1/2)-1/2*
(-a*b+a*c)/b*((b*x+a)*(c*x+a))^(1/2)/(c*x+a)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a
*b+1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+(a*b+a*c)*x+a^2)^(1/2))/(b*c)^(1/2)
))
```

3.433.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.62

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$
$$= \left[\frac{8\,ab^2c^2x+2\,(b^3c^2+b^2c^3)x^2+(a^2b^2-2\,a^2bc+a^2c^2)\sqrt{bc}\log\left(ab^2+2\,abc+ac^2+2\left(2\,bc+\sqrt{bc}(b+c)\right)\right.}{4\,(b^4c^2-2\right.$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

output
```
[1/4*(8*a*b^2*c^2*x + 2*(b^3*c^2 + b^2*c^3)*x^2 + (a^2*b^2 - 2*a^2*b*c + a
^2*c^2)*sqrt(b*c)*log(a*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*c + sqrt(b*c)*(b +
c))*sqrt(b*x + a)*sqrt(c*x + a) + 2*(b^2*c + b*c^2)*x + 2*(2*b*c*x + a*b +
 a*c)*sqrt(b*c)) - 2*(2*b^2*c^2*x + a*b^2*c + a*b*c^2)*sqrt(b*x + a)*sqrt(
c*x + a))/(b^4*c^2 - 2*b^3*c^3 + b^2*c^4), 1/2*(4*a*b^2*c^2*x + (b^3*c^2 +
 b^2*c^3)*x^2 - (a^2*b^2 - 2*a^2*b*c + a^2*c^2)*sqrt(-b*c)*arctan((sqrt(-b
*c)*sqrt(b*x + a)*sqrt(c*x + a) - sqrt(-b*c)*a)/(b*c*x)) - (2*b^2*c^2*x +
a*b^2*c + a*b*c^2)*sqrt(b*x + a)*sqrt(c*x + a))/(b^4*c^2 - 2*b^3*c^3 + b^2
*c^4)]
```

3.433.6 Sympy [F]

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx=\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

input
```
integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

output
```
Integral(x**2/(sqrt(a + b*x) + sqrt(a + c*x))**2, x)
```

3.433.7 Maxima [F]

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx=\int \frac{x^2}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a))^2, x)
```

3.433.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(116) = 232$.

Time = 0.83 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.92

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = -\frac{1}{2}\sqrt{ab^2+(bx+a)bc-abc}\sqrt{bx+a}\left(\frac{2\,(b^4c^2|b|-b^3c^3|b|)(bx+a)}{b^9c^2-3\,b^8c^3+3\,b^7c^4-b^6c^5}+\frac{ab^5c|b|-2\,ab^4c^2|b|+ab^3c^3|b|}{b^9c^2-3\,b^8c^3+3\,b^7c^4-b^6c^5}\right) - \frac{a^2|b|\log\left(\left|-\sqrt{bc}\sqrt{bx+a}+\sqrt{ab^2+(bx+a)bc-abc}\right|\right)}{2\sqrt{bc}b^2c} + \frac{(bx+a)^2b+2\,(bx+a)ab+(bx+a)^2c-2\,(bx+a)ac}{2\,(b^4-2\,b^3c+b^2c^2)}$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

output
```
-1/2*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*sqrt(b*x + a)*(2*(b^4*c^2*abs(b)
- b^3*c^3*abs(b))*(b*x + a)/(b^9*c^2 - 3*b^8*c^3 + 3*b^7*c^4 - b^6*c^5) +
(a*b^5*c*abs(b) - 2*a*b^4*c^2*abs(b) + a*b^3*c^3*abs(b))/(b^9*c^2 - 3*b^8*
c^3 + 3*b^7*c^4 - b^6*c^5)) - 1/2*a^2*abs(b)*log(abs(-sqrt(b*c)*sqrt(b*x +
 a) + sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)))/(sqrt(b*c)*b^2*c) + 1/2*((b*x
+ a)^2*b + 2*(b*x + a)*a*b + (b*x + a)^2*c - 2*(b*x + a)*a*c)/(b^4 - 2*b^3
*c + b^2*c^2)
```

3.433.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{2\,a\,x}{{\left(b-c\right)}^2}+\frac{x^2\,\left(b+c\right)}{2\,{\left(b-c\right)}^2}-\frac{2\,\left(\frac{x}{2}+\frac{a\,b+a\,c}{4\,b\,c}\right)\,\sqrt{a+b\,x}\,\sqrt{a+c\,x}}{{\left(b-c\right)}^2} + \frac{\ln\left(a\,b+a\,c+2\,b\,c\,x+2\,\sqrt{b}\,\sqrt{c}\,\sqrt{a+b\,x}\,\sqrt{a+c\,x}\right)\,{\left(a\,b-a\,c\right)}^2}{4\,b^{3/2}\,c^{3/2}\,{\left(b-c\right)}^2}$$

input
```
int(x^2/((a + b*x)^(1/2) + (a + c*x)^(1/2))^2,x)
```

output
```
(2*a*x)/(b - c)^2 + (x^2*(b + c))/(2*(b - c)^2) - (2*(x/2 + (a*b + a*c)/(4
*b*c))*(a + b*x)^(1/2)*(a + c*x)^(1/2))/(b - c)^2 + (log(a*b + a*c + 2*b*c
*x + 2*b^(1/2)*c^(1/2)*(a + b*x)^(1/2)*(a + c*x)^(1/2))*(a*b - a*c)^2)/(4*
b^(3/2)*c^(3/2)*(b - c)^2)
```

3.433. $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

3.434 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

3.434.1 Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{(b+c)x}{(b-c)^2}-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2}+\frac{4a\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{2a(b+c)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{\sqrt{b}(b-c)^2\sqrt{c}}+\frac{2a\log(x)}{(b-c)^2}$$

output
```
(b+c)*x/(b-c)^2+4*a*arctanh((b*x+a)^(1/2)/(c*x+a)^(1/2))/(b-c)^2+2*a*ln(x)
/(b-c)^2-2*a*(b+c)*arctanh(c^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(c*x+a)^(1/2))/(b
-c)^2/b^(1/2)/c^(1/2)-2*(b*x+a)^(1/2)*(c*x+a)^(1/2)/(b-c)^2
```

3.434.2 Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.88

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{4a\sqrt{c}(b+c)\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{a+cx}}{\sqrt{c}\left(\sqrt{a-\frac{ab}{c}}-\sqrt{a+bx}\right)}\right)+\sqrt{b}\left(a(b+c)+c\left(bx+cx-2\sqrt{a+bx}\sqrt{a+cx}\right)+8acarctan\right.}{\sqrt{b}(b-c)^2c}$$

input
```
Integrate[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

output
```
(4*a*Sqrt[c]*(b + c)*ArcTanh[(Sqrt[b]*Sqrt[a + c*x])/(Sqrt[c]*(Sqrt[a - (a
*b)/c] - Sqrt[a + b*x]))] + Sqrt[b]*(a*(b + c) + c*(b*x + c*x - 2*Sqrt[a +
 b*x]*Sqrt[a + c*x]) + 8*a*c*ArcTanh[(-(a*b) - b*c*x + c*Sqrt[a + c*x]*(Sq
rt[a - (a*b)/c] - Sqrt[a + b*x]))/(a*(b - 2*c) - b*c*x + 2*Sqrt[a - (a*b)/
c]*c*Sqrt[a + b*x] + Sqrt[a - (a*b)/c]*c*Sqrt[a + c*x] - c*Sqrt[a + b*x]*S
qrt[a + c*x])]))/(Sqrt[b]*(b - c)^2*c)
```

3.434.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.80, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {7241, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}dx$$

$$\Big\downarrow \text{ 7241}$$

$$\frac{\int \left(\frac{2a}{x}+b+c-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{x}\right)dx}{(b-c)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{4a\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)-\frac{2a(b+c)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{\sqrt{b}\sqrt{c}}-2\sqrt{a+bx}\sqrt{a+cx}+2a\log(x)+x(b+c)}{(b-c)^2}$$

input
```
Int[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

output
```
((b + c)*x - 2*Sqrt[a + b*x]*Sqrt[a + c*x] + 4*a*ArcTanh[Sqrt[a + b*x]/Sqr
t[a + c*x]] - (2*a*(b + c)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a
 + c*x])])/(Sqrt[b]*Sqrt[c]) + 2*a*Log[x])/(b - c)^2
```

3.434.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7241
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(b*e^2 - d*f^2)^m   Int[ExpandIntegran
d[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

3.434.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.97

method	result
default	$\frac{xb}{(b-c)^2}+\frac{xc}{(b-c)^2}+\frac{2a\ln(x)}{(b-c)^2}-\frac{\sqrt{bx+a}\,\sqrt{cx+a}\left(\operatorname{csgn}(a)\ln\left(\frac{2bcx+2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}+ab+ac}{2\sqrt{bc}}\right)ab+\operatorname{csgn}(a)\ln\left(\frac{2bcx+2\sqrt{bc\,x}}{}\right.\right.}{}$

input
```
int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
x/(b-c)^2*b+x/(b-c)^2*c+2*a*ln(x)/(b-c)^2-1/(b-c)^2*(b*x+a)^(1/2)*(c*x+a)^
(1/2)*(csgn(a)*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/
2)+a*b+a*c)/(b*c)^(1/2))*a*b+csgn(a)*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*
x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*a*c+2*csgn(a)*(b*c)^(1/2)*(
b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)-2*ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)
^(1/2)+b*x+c*x+2*a)/x)*(b*c)^(1/2)*a)*csgn(a)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1
/2)/(b*c)^(1/2)
```

3.434.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.56

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$
$$=\left[\frac{2\,abc\log\left(x\right)-2\,abc\log\left(-\frac{(b+c)x-2\,\sqrt{bx+a}\sqrt{cx+a}+2\,a}{x}\right)-2\,\sqrt{bx+a}\sqrt{cx+a}\,abc+(ab+ac)\sqrt{bc}\log\left(ab^2+2\right.}{b^3c-}\right.$$

input
```
integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

output
```
[(2*a*b*c*log(x) - 2*a*b*c*log(-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a)
 + 2*a)/x) - 2*sqrt(b*x + a)*sqrt(c*x + a)*b*c + (a*b + a*c)*sqrt(b*c)*log
(a*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*c - sqrt(b*c)*(b + c))*sqrt(b*x + a)*sqr
t(c*x + a) + 2*(b^2*c + b*c^2)*x - 2*(2*b*c*x + a*b + a*c)*sqrt(b*c)) + (b
^2*c + b*c^2)*x)/(b^3*c - 2*b^2*c^2 + b*c^3), (2*a*b*c*log(x) - 2*a*b*c*lo
g(-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a) + 2*a)/x) - 2*sqrt(b*x + a)*
sqrt(c*x + a)*b*c + 2*(a*b + a*c)*sqrt(-b*c)*arctan((sqrt(-b*c)*sqrt(b*x +
 a)*sqrt(c*x + a) - sqrt(-b*c)*a)/(b*c*x)) + (b^2*c + b*c^2)*x)/(b^3*c - 2
*b^2*c^2 + b*c^3)]
```

3.434.6 Sympy [F]

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

input
```
integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

output
```
Integral(x/(sqrt(a + b*x) + sqrt(a + c*x))**2, x)
```

3.434.7 Maxima [F]

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \int \frac{x}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

input
```
integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(b*x + a) + sqrt(c*x + a))^2, x)
```

3.434.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(115) = 230$.

Time = 0.92 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.66

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{\frac{a(b+c)|b|\log\left(\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2\right)}{(b^2-2bc+c^2)\sqrt{bc}}+\frac{2a|b|\log\left(\left|\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2-\left(b^2+bc+2\sqrt{bc}b\right)a\right|\right)}{b^2-2bc+c^2}-\frac{2a|b|\log}{}}{}$$

input
```
integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

output
```
(a*(b + c)*abs(b)*log((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*
c - a*b*c))^2)/((b^2 - 2*b*c + c^2)*sqrt(b*c)) + 2*a*abs(b)*log(abs((sqrt(
b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2 - (b^2 + b*c +
 2*sqrt(b*c)*b)*a))/(b^2 - 2*b*c + c^2) - 2*a*abs(b)*log(abs((sqrt(b*c)*sq
rt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2 - (b^2 + b*c - 2*sqrt
(b*c)*b)*a))/(b^2 - 2*b*c + c^2) + 2*a*b*log(abs(b*x))/(b^2 - 2*b*c + c^2)
 - 2*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*(b^2*abs(b) - 2*b*c*abs(b) + c^2*
abs(b))*sqrt(b*x + a)/(b^5 - 4*b^4*c + 6*b^3*c^2 - 4*b^2*c^3 + b*c^4) + ((
b*x + a)*b + (b*x + a)*c)/(b^2 - 2*b*c + c^2))/b
```

3.434.9 Mupad [B] (verification not implemented)

Time = 34.87 (sec) , antiderivative size = 5098, normalized size of antiderivative = 37.76

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \text{Too large to display}$$

input
```
int(x/((a + b*x)^(1/2) + (a + c*x)^(1/2))^2,x)
```

output
```
(2*a*log(x))/(b^2 - 2*b*c + c^2) - (((4*a*c^2 + 4*a*b*c)*((a + b*x)^(1/2)
- a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3 + ((4*a*b^2 + 4*a*b*c)*((a + b
*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (16*a*b*c*((a + b*x)^(
1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2)/(b^4 - 2*b^3*c + b^2*c^2
 - (((a + b*x)^(1/2) - a^(1/2))^2*(2*b*c^3 + 2*b^3*c - 4*b^2*c^2))/((a + c
*x)^(1/2) - a^(1/2))^2 + (((a + b*x)^(1/2) - a^(1/2))^4*(c^4 - 2*b*c^3 + b
^2*c^2))/((a + c*x)^(1/2) - a^(1/2))^4) - (2*a*log((((a + b*x)^(1/2) - (a
+ c*x)^(1/2))*(b - (c*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1
/2))))/((a + c*x)^(1/2) - a^(1/2))))/(b^2 - 2*b*c + c^2) + (2*a*log(((a +
b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2))))/(b - c)^2 + (x*(b + c)
)/(b - c)^2 + (a*atan(((a*(b*c)^(1/2)*(b + c)*((2*((a + b*x)^(1/2) - a^(1/
2))*(32*a^3*b^2*c^10 - 64*a^3*b^3*c^9 + 8*a^3*b^4*c^8 + 240*a^3*b^5*c^7 +
8*a^3*b^6*c^6 - 64*a^3*b^7*c^5 + 32*a^3*b^8*c^4))/(((a + c*x)^(1/2) - a^(1
/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) - (4*(4*a^3*b^4*c^8 + 44
*a^3*b^5*c^7 + 44*a^3*b^6*c^6 + 4*a^3*b^7*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 +
 c^4 + 6*b^2*c^2) + (2*a*(b*c)^(1/2)*(b + c)*((4*(4*a^2*b^3*c^11 + 2*a^2*b
^4*c^10 - 18*a^2*b^5*c^9 + 12*a^2*b^6*c^8 + 12*a^2*b^7*c^7 - 18*a^2*b^8*c^
6 + 2*a^2*b^9*c^5 + 4*a^2*b^10*c^4))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^
2*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*(16*a^2*b^2*c^12 - 32*a^2*b^3*c^11
 + 36*a^2*b^4*c^10 - 64*a^2*b^5*c^9 + 88*a^2*b^6*c^8 - 64*a^2*b^7*c^7 +...
```

3.434. $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

3.435 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2} \, dx$

3.435.1 Optimal result

Integrand size = 21, antiderivative size = 138

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2} \, dx=-\frac{2a}{(b-c)^2x}+\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2x}+\frac{2(b+c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{4\sqrt{b}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{(b-c)^2}+\frac{(b+c)\log(x)}{(b-c)^2}$$

output
```
-2*a/(b-c)^2/x+2*(b+c)*arctanh((b*x+a)^(1/2)/(c*x+a)^(1/2))/(b-c)^2+(b+c)*
ln(x)/(b-c)^2-4*arctanh(c^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(c*x+a)^(1/2))*b^(1/
2)*c^(1/2)/(b-c)^2+2*(b*x+a)^(1/2)*(c*x+a)^(1/2)/(b-c)^2/x
```

3.435.2 Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.73

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2} \, dx = \frac{-2a-2cx+2\sqrt{a+bx}\sqrt{a+cx}+8\sqrt{b}\sqrt{c}x\text{arctanh}\left(\frac{\sqrt{b}\sqrt{a+cx}}{\sqrt{c}\left(\sqrt{a-\frac{ab}{c}}-\sqrt{a+bx}\right)}\right)+4(b+c)x\text{arctanh}\left(\frac{}{a(b-2c)-bcx-}\right.}{(b-c)^2x}$$

input
```
Integrate[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-2),x]
```

output
```
(-2*a - 2*c*x + 2*Sqrt[a + b*x]*Sqrt[a + c*x] + 8*Sqrt[b]*Sqrt[c]*x*ArcTan
h[(Sqrt[b]*Sqrt[a + c*x])/(Sqrt[c]*(Sqrt[a - (a*b)/c] - Sqrt[a + b*x]))] +
 4*(b + c)*x*ArcTanh[(-(a*b) - b*c*x + c*Sqrt[a + c*x]*(Sqrt[a - (a*b)/c]
- Sqrt[a + b*x]))/(a*(b - 2*c) - b*c*x + 2*Sqrt[a - (a*b)/c]*c*Sqrt[a + b*
x] + Sqrt[a - (a*b)/c]*c*Sqrt[a + c*x] - c*Sqrt[a + b*x]*Sqrt[a + c*x])])/
((b - c)^2*x)
```

3.435.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.80, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {7241, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}dx$$

$$\Big\downarrow \text{ 7241}$$

$$\frac{\int \left(\frac{2a}{x^2}-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{x^2}+\frac{b+c}{x}\right)dx}{(b-c)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2(b+c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)-4\sqrt{b}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)+\frac{2\sqrt{a+bx}\sqrt{a+cx}}{x}-\frac{2a}{x}+(b+c)\log(x)}{(b-c)^2}$$

input
```
Int[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-2),x]
```

output
```
((-2*a)/x + (2*Sqrt[a + b*x]*Sqrt[a + c*x])/x + 2*(b + c)*ArcTanh[Sqrt[a +
 b*x]/Sqrt[a + c*x]] - 4*Sqrt[b]*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(
Sqrt[b]*Sqrt[a + c*x])] + (b + c)*Log[x])/(b - c)^2
```

3.435.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7241
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(b*e^2 - d*f^2)^m   Int[ExpandIntegran
d[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

3.435.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.95

method	result
default	$\frac{b\ln(x)}{(b-c)^2}+\frac{c\ln(x)}{(b-c)^2}-\frac{2a}{(b-c)^2x}+\frac{\sqrt{bx+a}\,\sqrt{cx+a}\left(-2\,\mathrm{csgn}(a)\ln\left(\frac{2bcx+2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}+ab+ac}{2\sqrt{bc}}\right)xbc+\ln\left(\frac{a\left(2\,\mathrm{csgn}(a)\sqrt{bc}\right.}{}\right.\right.}{}$

input
```
int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/(b-c)^2*b*ln(x)+1/(b-c)^2*c*ln(x)-2*a/(b-c)^2/x+1/(b-c)^2*(b*x+a)^(1/2)*
(c*x+a)^(1/2)*(-2*csgn(a)*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2
)*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*x*b*c+ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a
*c*x+a^2)^(1/2)+b*x+c*x+2*a)/x)*x*b*(b*c)^(1/2)+ln(a*(2*csgn(a)*(b*c*x^2+a
*b*x+a*c*x+a^2)^(1/2)+b*x+c*x+2*a)/x)*x*c*(b*c)^(1/2)+2*csgn(a)*(b*c)^(1/2
)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2))*csgn(a)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)
/x/(b*c)^(1/2)
```

3.435.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.30

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

$$=\left[\frac{2\,(b+c)x\log(x)-2\,(b+c)x\log\left(-\frac{(b+c)x-2\,\sqrt{bx+a}\sqrt{cx+a}+2\,a}{x}\right)+4\,\sqrt{bc}x\log\left(ab^2+2\,abc+ac^2+2\,\left(2\,bc-\right.\right.}{2\,(b^2}\right.$$

input
```
integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

output
```
[1/2*(2*(b + c)*x*log(x) - 2*(b + c)*x*log(-((b + c)*x - 2*sqrt(b*x + a)*s
qrt(c*x + a) + 2*a)/x) + 4*sqrt(b*c)*x*log(a*b^2 + 2*a*b*c + a*c^2 + 2*(2*
b*c - sqrt(b*c)*(b + c))*sqrt(b*x + a)*sqrt(c*x + a) + 2*(b^2*c + b*c^2)*x
 - 2*(2*b*c*x + a*b + a*c)*sqrt(b*c)) + (b + c)*x + 4*sqrt(b*x + a)*sqrt(c
*x + a) - 4*a)/((b^2 - 2*b*c + c^2)*x), 1/2*(2*(b + c)*x*log(x) - 2*(b + c
)*x*log(-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a) + 2*a)/x) + 8*sqrt(-b*
c)*x*arctan((sqrt(-b*c)*sqrt(b*x + a)*sqrt(c*x + a) - sqrt(-b*c)*a)/(b*c*x
)) + (b + c)*x + 4*sqrt(b*x + a)*sqrt(c*x + a) - 4*a)/((b^2 - 2*b*c + c^2)
*x)]
```

3.435.6 Sympy [F]

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx=\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

input
```
integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

output
```
Integral((sqrt(a + b*x) + sqrt(a + c*x))**(-2), x)
```

3.435.7 Maxima [F]

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx=\int \frac{1}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

input
```
integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate((sqrt(b*x + a) + sqrt(c*x + a))^(-2), x)
```

3.435.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(118) = 236$.

Time = 0.97 (sec) , antiderivative size = 438, normalized size of antiderivative = 3.17

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{2\sqrt{bc}|b|\log\left(\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2\right)}{b^3-2\,b^2c+bc^2}$$
$$+\frac{2\sqrt{bc}(b+c)|b|\arctan\left(-\frac{ab^2+abc-\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2}{2\sqrt{-bc}ab}\right)}{(b^2-2\,bc+c^2)\sqrt{-bc}b}+\frac{(b+c)\log\left(|bx|\right)}{b^2-2\,bc+c^2}$$
$$-\frac{4\left(\sqrt{bc}\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2a(b+c)|b|-(b^3-2\,b^2c\cdots\right.}{\left(\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^4-2\,(b^2+bc)\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)\cdots\right.}$$
$$-\frac{(bx+a)b+ab+(bx+a)c-ac}{(b^2-2\,bc+c^2)bx}$$

input
```
integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

output
```
2*sqrt(b*c)*abs(b)*log((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b
*c - a*b*c))^2)/(b^3 - 2*b^2*c + b*c^2) + 2*sqrt(b*c)*(b + c)*abs(b)*arcta
n(-1/2*(a*b^2 + a*b*c - (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*
b*c - a*b*c))^2)/(sqrt(-b*c)*a*b))/((b^2 - 2*b*c + c^2)*sqrt(-b*c)*b) + (b
 + c)*log(abs(b*x))/(b^2 - 2*b*c + c^2) - 4*(sqrt(b*c)*(sqrt(b*c)*sqrt(b*x
 + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*(b + c)*abs(b) - (b^3 - 2
*b^2*c + b*c^2)*sqrt(b*c)*a^2*abs(b))/(((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*
b^2 + (b*x + a)*b*c - a*b*c))^4 - 2*(b^2 + b*c)*(sqrt(b*c)*sqrt(b*x + a) -
 sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a + (b^4 - 2*b^3*c + b^2*c^2)*a^2)
*(b^2 - 2*b*c + c^2)) - ((b*x + a)*b + a*b + (b*x + a)*c - a*c)/((b^2 - 2*
b*c + c^2)*b*x)
```

3.435.9 Mupad [B] (verification not implemented)

Time = 33.21 (sec) , antiderivative size = 4285, normalized size of antiderivative = 31.05

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((a + b*x)^(1/2) + (a + c*x)^(1/2))^2,x)
```

output
```
(atan((((b*c)^(1/2)*((4*(b*c)^(1/2)*((4*(b^4*c^12 + 16*b^5*c^11 - 42*b^6*c
^10 + 25*b^7*c^9 + 25*b^8*c^8 - 42*b^9*c^7 + 16*b^10*c^6 + b^11*c^5))/(b^4
 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(4*b^5*c^12 -
 36*b^7*c^10 + 64*b^8*c^9 - 36*b^9*c^8 + 4*b^11*c^6))/(b^4 - 4*b^3*c - 4*b
*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(4*b^5*c^13 - b^4*c^14 - 5*b^
6*c^12 + b^7*c^11 + b^8*c^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4*b^12*c^
6 - b^13*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)
^(1/2) - a^(1/2))*(4*b^3*c^15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6*c^12
+ 516*b^7*c^11 - 618*b^8*c^10 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11*c^7
- 31*b^12*c^6 + 4*b^13*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c -
 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*
(4*b^3*c^14 - 27*b^4*c^13 + 99*b^5*c^12 - 175*b^6*c^11 + 99*b^7*c^10 + 99*
b^8*c^9 - 175*b^9*c^8 + 99*b^10*c^7 - 27*b^11*c^6 + 4*b^12*c^5))/(((a + c*
x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)
^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(73*b^4*c^12 - 278*b^5*c^11 + 503*b^6*
c^10 - 596*b^7*c^9 + 503*b^8*c^8 - 278*b^9*c^7 + 73*b^10*c^6))/(((a + c*x)
^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2
 - (4*(4*b^5*c^10 + 24*b^6*c^9 + 40*b^7*c^8 + 24*b^8*c^7 + 4*b^9*c^6))/(b^
4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*
(65*b^4*c^11 - 167*b^5*c^10 + 198*b^6*c^9 + 198*b^7*c^8 - 167*b^8*c^7 +...
```

3.436 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

3.436.1 Optimal result

Integrand size = 25, antiderivative size = 123

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx=-\frac{a}{(b-c)^2x^2}-\frac{b+c}{(b-c)^2x}+\frac{\sqrt{a+bx}\sqrt{a+cx}}{2a(b-c)x}+\frac{\sqrt{a+bx}(a+cx)^{3/2}}{a(b-c)^2x^2}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{2a}$$

output
```
-a/(b-c)^2/x^2+(-b-c)/(b-c)^2/x-1/2*arctanh((b*x+a)^(1/2)/(c*x+a)^(1/2))/a
+(c*x+a)^(3/2)*(b*x+a)^(1/2)/a/(b-c)^2/x^2+1/2*(b*x+a)^(1/2)*(c*x+a)^(1/2)
/a/(b-c)/x
```

3.436.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx=\frac{-2a^2+(b+c)x\sqrt{a+bx}\sqrt{a+cx}+2a\left(-bx-cx+\sqrt{a+bx}\sqrt{a+cx}\right)-(b-c)^2x^2\operatorname{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a+bx}}\right)}{2a(b-c)^2x^2}$$

input
```
Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

output
```
(-2*a^2 + (b + c)*x*Sqrt[a + b*x]*Sqrt[a + c*x] + 2*a*(-(b*x) - c*x + Sqrt
[a + b*x]*Sqrt[a + c*x]) - (b - c)^2*x^2*ArcTanh[Sqrt[a + c*x]/Sqrt[a + b*
x]])/(2*a*(b - c)^2*x^2)
```

3.436.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7241, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}dx$$

$$\Big\downarrow \text{7241}$$

$$\frac{\int \left(\frac{2a}{x^3}-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{x^3}+\frac{b+c}{x^2}\right)dx}{(b-c)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{(b-c)^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{2a}+\frac{\sqrt{a+bx}(a+cx)^{3/2}}{ax^2}+\frac{(b-c)\sqrt{a+bx}\sqrt{a+cx}}{2ax}-\frac{a}{x^2}-\frac{b+c}{x}}{(b-c)^2}$$

input
```
Int[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

output
```
(-(a/x^2) - (b + c)/x + ((b - c)*Sqrt[a + b*x]*Sqrt[a + c*x])/(2*a*x) + (S
qrt[a + b*x]*(a + c*x)^(3/2))/(a*x^2) - ((b - c)^2*ArcTanh[Sqrt[a + b*x]/S
qrt[a + c*x]])/(2*a))/(b - c)^2
```

3.436.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7241
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(b*e^2 - d*f^2)^m   Int[ExpandIntegran
d[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

3.436.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.54

method	result
default	$-\frac{b}{x(b-c)^2}-\frac{c}{x(b-c)^2}-\frac{a}{(b-c)^2x^2}+\frac{\sqrt{bx+a}\,\sqrt{cx+a}\left(-\ln\left(\frac{a\left(2\operatorname{csgn}(a)\sqrt{bc\,x^2+abx+acx+a^2}+bx+cx+2a\right)}{x}\right)x^2b^2+2\ln\left(\frac{a\left(2\operatorname{csgn}(\right.}{}\right.\right.}{}$

input
```
int(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x/(b-c)^2*b-1/x/(b-c)^2*c-a/(b-c)^2/x^2+1/4/(b-c)^2*(b*x+a)^(1/2)*(c*x+
a)^(1/2)/a*(-ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)+b*x+c*x+2*a)/
x)*x^2*b^2+2*ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)+b*x+c*x+2*a)/
x)*x^2*b*c-ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)+b*x+c*x+2*a)/x)
*x^2*c^2+2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*x*b+2*csgn(a)*(b*c*x^2+
a*b*x+a*c*x+a^2)^(1/2)*x*c+4*csgn(a)*a*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2))*cs
gn(a)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/x^2
```

3.436.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$
$$=\frac{4\,(b^2-2\,bc+c^2)x^2\log\left(-\frac{(b+c)x-2\sqrt{bx+a}\sqrt{cx+a}+2\,a}{x}\right)+(b^2+2\,bc+c^2)x^2+8\left((b+c)x+2\,a\right)\sqrt{bx+a}\sqrt{cx+}}{16\,(ab^2-2\,abc+ac^2)x^2}$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/16*(4*(b^2 - 2*b*c + c^2)*x^2*log(-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x
 + a) + 2*a)/x) + (b^2 + 2*b*c + c^2)*x^2 + 8*((b + c)*x + 2*a)*sqrt(b*x +
 a)*sqrt(c*x + a) - 16*a^2 - 16*(a*b + a*c)*x)/((a*b^2 - 2*a*b*c + a*c^2)*
x^2)
```

3.436.6 Sympy [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx=\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

input
```
integrate(1/x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

output
```
Integral(1/(x*(sqrt(a + b*x) + sqrt(a + c*x))**2), x)
```

3.436.7 Maxima [F]

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx=\int \frac{1}{x\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(1/(x*(sqrt(b*x + a) + sqrt(c*x + a))^2), x)
```

3.436.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(107) = 214$.

Time = 2.84 (sec) , antiderivative size = 532, normalized size of antiderivative = 4.33

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = -\frac{\sqrt{bc}|b|\arctan\left(-\frac{ab^2+abc-\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2}{2\sqrt{-bc}ab}\right)}{2\sqrt{-bc}ab}$$
$$-\frac{(b^2+6\,bc+c^2)\sqrt{bc}\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^6|b|-(3\,b^4+5\,b^3c+5\,b^2c^2+3\,bc^3)\sqrt{bc}\left(\right.}{\left(\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+\right.\right.}$$
$$-\frac{(bx+a)b^2+(bx+a)bc-abc}{(b^2-2\,bc+c^2)b^2x^2}$$

input
```
integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

output
```
-1/2*sqrt(b*c)*abs(b)*arctan(-1/2*(a*b^2 + a*b*c - (sqrt(b*c)*sqrt(b*x + a
) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2)/(sqrt(-b*c)*a*b))/(sqrt(-b*c)*
a*b) - ((b^2 + 6*b*c + c^2)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^
2 + (b*x + a)*b*c - a*b*c))^6*abs(b) - (3*b^4 + 5*b^3*c + 5*b^2*c^2 + 3*b*
c^3)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b
*c))^4*a*abs(b) + (3*b^6 - 4*b^5*c + 2*b^4*c^2 - 4*b^3*c^3 + 3*b^2*c^4)*sq
rt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*
a^2*abs(b) - (b^8 - 3*b^7*c + 2*b^6*c^2 + 2*b^5*c^3 - 3*b^4*c^4 + b^3*c^5)
*sqrt(b*c)*a^3*abs(b))/(((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)
*b*c - a*b*c))^4 - 2*(b^2 + b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (
b*x + a)*b*c - a*b*c))^2*a + (b^4 - 2*b^3*c + b^2*c^2)*a^2)^2*(b^2 - 2*b*c
 + c^2)) - ((b*x + a)*b^2 + (b*x + a)*b*c - a*b*c)/((b^2 - 2*b*c + c^2)*b^
2*x^2)
```

3.436. $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

3.436.9 Mupad [B] (verification not implemented)

Time = 30.04 (sec) , antiderivative size = 787, normalized size of antiderivative = 6.40

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{\ln\left(\frac{\left(\sqrt{a+b\,x}-\sqrt{a+c\,x}\right)\left(b-\frac{c\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}\right)}{\sqrt{a+c\,x}-\sqrt{a}}\right)}{4\,a}$$

$$-\frac{\frac{b^4}{2}+\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4\left(-\frac{b^4}{2}+4\,b^3\,c+\frac{3\,b^2\,c^2}{2}+4\,b\,c^3-\frac{c^4}{2}\right)}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^4}-\frac{\left(2\,b^4+2\,c\,b^3\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}-\frac{\left(b^2\,c^2+b\,c^3\right)\left(\cdots\right.}{\left(\sqrt{a+c\cdots\right.}}}{\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4\left(8\,a\,b^4+16\,a\,b^3\,c-48\,a\,b^2\,c^2+16\,a\,b\,c^3+8\,a\,c^4\right)}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^4}-\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3\left(16\,a\,b^4-16\,a\,b^3\,c-16\,a\,b^2\,c^2+16\,a\,b\,c^3\right)}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^3}-\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)\cdots}{}}$$

$$-\frac{\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)}{4\,a}-\frac{a+x\,(b+c)}{x^2\,\left(b^2-2\,b\,c+c^2\right)}$$

$$-\frac{c^2\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{16\,a\,(b-c)^2\,\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}+\frac{c\,(b+c)\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{8\,a\,(b-c)^2\,\left(\sqrt{a+c\,x}-\sqrt{a}\right)}$$

input
```
int(1/(x*((a + b*x)^(1/2) + (a + c*x)^(1/2))^2),x)
```

output
```
log((((a + b*x)^(1/2) - (a + c*x)^(1/2))*(b - (c*((a + b*x)^(1/2) - a^(1/2
)))/((a + c*x)^(1/2) - a^(1/2))))/((a + c*x)^(1/2) - a^(1/2)))/(4*a) - (b^
4/2 + (((a + b*x)^(1/2) - a^(1/2))^4*(4*b*c^3 + 4*b^3*c - b^4/2 - c^4/2 +
(3*b^2*c^2)/2))/((a + c*x)^(1/2) - a^(1/2))^4 - ((2*b^3*c + 2*b^4)*((a + b
*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - ((b*c^3 + b^2*c^2)*((a
 + b*x)^(1/2) - a^(1/2))^5)/((a + c*x)^(1/2) - a^(1/2))^5 + (((a + b*x)^(1
/2) - a^(1/2))^2*(6*b^3*c + (5*b^4)/2 + (5*b^2*c^2)/2))/((a + c*x)^(1/2) -
 a^(1/2))^2 - (((a + b*x)^(1/2) - a^(1/2))^3*(b*c^3 + 6*b^3*c + b^4 + 6*b^
2*c^2))/((a + c*x)^(1/2) - a^(1/2))^3)/((((a + b*x)^(1/2) - a^(1/2))^4*(8*
a*b^4 + 8*a*c^4 - 48*a*b^2*c^2 + 16*a*b*c^3 + 16*a*b^3*c))/((a + c*x)^(1/2
) - a^(1/2))^4 - (((a + b*x)^(1/2) - a^(1/2))^3*(16*a*b^4 - 16*a*b^2*c^2 +
 16*a*b*c^3 - 16*a*b^3*c))/((a + c*x)^(1/2) - a^(1/2))^3 - (((a + b*x)^(1/
2) - a^(1/2))^5*(16*a*c^4 - 16*a*b^2*c^2 - 16*a*b*c^3 + 16*a*b^3*c))/((a +
 c*x)^(1/2) - a^(1/2))^5 + (((a + b*x)^(1/2) - a^(1/2))^2*(8*a*b^4 + 8*a*b
^2*c^2 - 16*a*b^3*c))/((a + c*x)^(1/2) - a^(1/2))^2 + (((a + b*x)^(1/2) -
a^(1/2))^6*(8*a*c^4 + 8*a*b^2*c^2 - 16*a*b*c^3))/((a + c*x)^(1/2) - a^(1/2
))^6) - log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))/(4*a)
 - (a + x*(b + c))/(x^2*(b^2 - 2*b*c + c^2)) - (c^2*((a + b*x)^(1/2) - a^(
1/2))^2)/(16*a*(b - c)^2*((a + c*x)^(1/2) - a^(1/2))^2) + (c*(b + c)*((a +
 b*x)^(1/2) - a^(1/2)))/(8*a*(b - c)^2*((a + c*x)^(1/2) - a^(1/2)))
```

3.437 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

3.437.1 Optimal result

Integrand size = 25, antiderivative size = 174

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = -\frac{2a}{3(b-c)^2x^3}-\frac{b+c}{2(b-c)^2x^2}-\frac{(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4a^2(b-c)x} - \frac{(b+c)\sqrt{a+bx}(a+cx)^{3/2}}{2a^2(b-c)^2x^2} + \frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2(b-c)^2x^3}+\frac{(b+c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{4a^2}$$

output
```
-2/3*a/(b-c)^2/x^3+1/2*(-b-c)/(b-c)^2/x^2+2/3*(b*x+a)^(3/2)*(c*x+a)^(3/2)/
a^2/(b-c)^2/x^3+1/4*(b+c)*arctanh((b*x+a)^(1/2)/(c*x+a)^(1/2))/a^2-1/2*(b+
c)*(c*x+a)^(3/2)*(b*x+a)^(1/2)/a^2/(b-c)^2/x^2-1/4*(b+c)*(b*x+a)^(1/2)*(c*
x+a)^(1/2)/a^2/(b-c)/x
```

3.437.2 Mathematica [A] (verified)

Time = 10.18 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

$$= \frac{-8a^3+2a(b+c)x\sqrt{a+bx}\sqrt{a+cx}+\left(-3b^2+2bc-3c^2\right)x^2\sqrt{a+bx}\sqrt{a+cx}+a^2\left(-6bx-6cx+8\sqrt{a+\right.}{12a^2(b-c)^2x^3}$$

input
```
Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

output
```
(-8*a^3 + 2*a*(b + c)*x*Sqrt[a + b*x]*Sqrt[a + c*x] + (-3*b^2 + 2*b*c - 3*
c^2)*x^2*Sqrt[a + b*x]*Sqrt[a + c*x] + a^2*(-6*b*x - 6*c*x + 8*Sqrt[a + b*
x]*Sqrt[a + c*x]) + 3*(b - c)^2*(b + c)*x^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a +
 c*x]])/(12*a^2*(b - c)^2*x^3)
```

3.437.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.92, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7241, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2} \, dx$$

$$\Big\downarrow \text{ 7241}$$

$$\frac{\int \left(\frac{2a}{x^4}-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{x^4}+\frac{b+c}{x^3}\right)dx}{(b-c)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{(b+c)(b-c)^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{4a^2}-\frac{(b^2-c^2)\sqrt{a+bx}\sqrt{a+cx}}{4a^2x}+\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2x^3}-\frac{(b+c)\sqrt{a+bx}(a+cx)^{3/2}}{2a^2x^2}-\frac{2a}{3x^3}-\frac{b+c}{2x^2}}{(b-c)^2}$$

input
```
Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

output
```
((-2*a)/(3*x^3) - (b + c)/(2*x^2) - ((b^2 - c^2)*Sqrt[a + b*x]*Sqrt[a + c*
x])/(4*a^2*x) - ((b + c)*Sqrt[a + b*x]*(a + c*x)^(3/2))/(2*a^2*x^2) + (2*(
a + b*x)^(3/2)*(a + c*x)^(3/2))/(3*a^2*x^3) + ((b - c)^2*(b + c)*ArcTanh[S
qrt[a + b*x]/Sqrt[a + c*x]])/(4*a^2))/(b - c)^2
```

3.437.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7241
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(b*e^2 - d*f^2)^m   Int[ExpandIntegran
d[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

3.437.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 457, normalized size of antiderivative = 2.63

method	result
default	$-\frac{b}{2x^2(b-c)^2}-\frac{c}{2x^2(b-c)^2}-\frac{2a}{3(b-c)^2x^3}-\frac{\sqrt{bx+a}\,\sqrt{cx+a}\left(-3\ln\left(\frac{a\left(2\,\operatorname{csgn}(a)\sqrt{bc\,x^2+abx+acx+a^2}+bx+cx+2a\right)}{x}\right)x^3b^3+3\ln\left(\frac{a(}{}\right.\right.}{}$

input
```
int(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2/(b-c)^2*b-1/2/x^2/(b-c)^2*c-2/3*a/(b-c)^2/x^3-1/24/(b-c)^2*(b*x+a
)^(1/2)*(c*x+a)^(1/2)/a^2*(-3*ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1
/2)+b*x+c*x+2*a)/x)*x^3*b^3+3*ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1
/2)+b*x+c*x+2*a)/x)*x^3*b^2*c+3*ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^
(1/2)+b*x+c*x+2*a)/x)*x^3*b*c^2-3*ln(a*(2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2
)^(1/2)+b*x+c*x+2*a)/x)*x^3*c^3+6*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*
x^2*b^2-4*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*x^2*b*c+6*csgn(a)*(b*c*x
^2+a*b*x+a*c*x+a^2)^(1/2)*x^2*c^2-4*csgn(a)*a*(b*c*x^2+a*b*x+a*c*x+a^2)^(1
/2)*x*b-4*csgn(a)*a*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*x*c-16*(b*c*x^2+a*b*x+
a*c*x+a^2)^(1/2)*a^2*csgn(a))*csgn(a)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/x^3
```

3.437.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx =$$
$$-\frac{12\,(b^3-b^2c-bc^2+c^3)x^3\log\left(-\frac{(b+c)x-2\,\sqrt{bx+a}\sqrt{cx+a}+2\,a}{x}\right)+(5\,b^3+3\,b^2c+3\,bc^2+5\,c^3)x^3+64\,a^3+8\,((}{96\,(a^2b^2-2\,a^2bc+a^2c^2)x}$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

output
```
-1/96*(12*(b^3 - b^2*c - b*c^2 + c^3)*x^3*log(-((b + c)*x - 2*sqrt(b*x + a
)*sqrt(c*x + a) + 2*a)/x) + (5*b^3 + 3*b^2*c + 3*b*c^2 + 5*c^3)*x^3 + 64*a
^3 + 8*((3*b^2 - 2*b*c + 3*c^2)*x^2 - 8*a^2 - 2*(a*b + a*c)*x)*sqrt(b*x +
a)*sqrt(c*x + a) + 48*(a^2*b + a^2*c)*x)/((a^2*b^2 - 2*a^2*b*c + a^2*c^2)*
x^3)
```

3.437.6 Sympy [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

input
```
integrate(1/x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

output
```
Integral(1/(x**2*(sqrt(a + b*x) + sqrt(a + c*x))**2), x)
```

3.437.7 Maxima [F]

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \int \frac{1}{x^2\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^2}\,dx$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate(1/(x^2*(sqrt(b*x + a) + sqrt(c*x + a))^2), x)
```

3.437.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 802 vs. $2(146) = 292$.

Time = 2.46 (sec) , antiderivative size = 802, normalized size of antiderivative = 4.61

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

$$= \frac{\sqrt{bc}(b+c)|b|\arctan\left(-\frac{ab^2+abc-\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2}{2\sqrt{-bc}ab}\right)}{4\sqrt{-bc}a^2b}$$

$$+\frac{3\left(b^3-b^2c-bc^2+c^3\right)\sqrt{bc}\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^{10}|b|-3\left(5\,b^5+22\,b^3c^2+5\,bc^4\right)\sqrt{}}{}$$

$$-\frac{3\,(bx+a)b^3+ab^3+3\,(bx+a)b^2c-3\,ab^2c}{6\,(b^2-2\,bc+c^2)b^3x^3}$$

input
```
integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

output
```
1/4*sqrt(b*c)*(b + c)*abs(b)*arctan(-1/2*(a*b^2 + a*b*c - (sqrt(b*c)*sqrt(
b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2)/(sqrt(-b*c)*a*b))/(sqrt
(-b*c)*a^2*b) + 1/6*(3*(b^3 - b^2*c - b*c^2 + c^3)*sqrt(b*c)*(sqrt(b*c)*sq
rt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^10*abs(b) - 3*(5*b^5 +
22*b^3*c^2 + 5*b*c^4)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b
*x + a)*b*c - a*b*c))^8*a*abs(b) + 2*(15*b^7 - b^6*c + 18*b^5*c^2 + 18*b^4
*c^3 - b^3*c^4 + 15*b^2*c^5)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b
^2 + (b*x + a)*b*c - a*b*c))^6*a^2*abs(b) - 6*(5*b^9 - 6*b^8*c - 5*b^7*c^2
 + 12*b^6*c^3 - 5*b^5*c^4 - 6*b^4*c^5 + 5*b^3*c^6)*sqrt(b*c)*(sqrt(b*c)*sq
rt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^4*a^3*abs(b) + 3*(5*b^1
1 - 17*b^10*c + 21*b^9*c^2 - 9*b^8*c^3 - 9*b^7*c^4 + 21*b^6*c^5 - 17*b^5*c
^6 + 5*b^4*c^7)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a
)*b*c - a*b*c))^2*a^4*abs(b) - (3*b^13 - 20*b^12*c + 60*b^11*c^2 - 108*b^1
0*c^3 + 130*b^9*c^4 - 108*b^8*c^5 + 60*b^7*c^6 - 20*b^6*c^7 + 3*b^5*c^8)*s
qrt(b*c)*a^5*abs(b))/(((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b
*c - a*b*c))^4 - 2*(b^2 + b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*
x + a)*b*c - a*b*c))^2*a + (b^4 - 2*b^3*c + b^2*c^2)*a^2)^3*(b^2 - 2*b*c +
 c^2)*a) - 1/6*(3*(b*x + a)*b^3 + a*b^3 + 3*(b*x + a)*b^2*c - 3*a*b^2*c)/(
(b^2 - 2*b*c + c^2)*b^3*x^3)
```

3.437.9 Mupad [B] (verification not implemented)

Time = 48.12 (sec) , antiderivative size = 1290, normalized size of antiderivative = 7.41

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x^2*((a + b*x)^(1/2) + (a + c*x)^(1/2))^2),x)
```

output
```
(log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*(b + c))/(8*
a^2) - ((((a + b*x)^(1/2) - a^(1/2))^7*(3*b^5*c - 15*b*c^5 + 3*c^6 + 3*b^2
*c^4 + 3*b^3*c^3 - 15*b^4*c^2))/((a + c*x)^(1/2) - a^(1/2))^7 - (((a + b*x
)^(1/2) - a^(1/2))^5*(26*b^5*c - b*c^5 - b^6 + 26*b^2*c^4 + 4*b^3*c^3 + 4*
b^4*c^2))/((a + c*x)^(1/2) - a^(1/2))^5 - b^6/3 + ((b^5*c + b^6)*((a + b*x
)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (((a + b*x)^(1/2) - a^(1
/2))^8*(c^6 - 6*b*c^5 + 7*b^2*c^4 - 6*b^3*c^3 + b^4*c^2))/((a + c*x)^(1/2)
 - a^(1/2))^8 + (((a + b*x)^(1/2) - a^(1/2))^6*(6*b*c^5 + 6*b^5*c - (5*b^6
)/3 - (5*c^6)/3 + 30*b^2*c^4 - 24*b^3*c^3 + 30*b^4*c^2))/((a + c*x)^(1/2)
- a^(1/2))^6 - (((17*b^6)/3 + (17*b^3*c^3)/3)*((a + b*x)^(1/2) - a^(1/2))^
3)/((a + c*x)^(1/2) - a^(1/2))^3 + (((a + b*x)^(1/2) - a^(1/2))^2*(b^6 - 4
*b^5*c + b^4*c^2))/((a + c*x)^(1/2) - a^(1/2))^2 + (((a + b*x)^(1/2) - a^(
1/2))^4*(18*b^5*c + 5*b^6 + 5*b^2*c^4 + 18*b^3*c^3 - 6*b^4*c^2))/((a + c*x
)^(1/2) - a^(1/2))^4)/((((a + b*x)^(1/2) - a^(1/2))^5*(96*a^2*b^5 + 96*a^2
*b*c^4 + 96*a^2*b^4*c + 96*a^2*b^2*c^3 - 384*a^2*b^3*c^2))/((a + c*x)^(1/2
) - a^(1/2))^5 - (((a + b*x)^(1/2) - a^(1/2))^8*(96*a^2*c^5 - 96*a^2*b*c^4
 - 96*a^2*b^2*c^3 + 96*a^2*b^3*c^2))/((a + c*x)^(1/2) - a^(1/2))^8 - (((a
+ b*x)^(1/2) - a^(1/2))^6*(32*a^2*b^5 + 32*a^2*c^5 + 224*a^2*b*c^4 + 224*a
^2*b^4*c - 256*a^2*b^2*c^3 - 256*a^2*b^3*c^2))/((a + c*x)^(1/2) - a^(1/2))
^6 - (((a + b*x)^(1/2) - a^(1/2))^4*(96*a^2*b^5 - 96*a^2*b^4*c + 96*a^2...
```

3.438 $\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$

3.438.1 Optimal result

Integrand size = 25, antiderivative size = 277

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = -\frac{8a^2(a+bx)^{3/2}}{3b^2(b-c)^3}+\frac{2a^2(b+3c)(a+bx)^{3/2}}{3b^3(b-c)^3}+\frac{8a(a+bx)^{5/2}}{5b^2(b-c)^3} - \frac{4a(b+3c)(a+bx)^{5/2}}{5b^3(b-c)^3}+\frac{2(b+3c)(a+bx)^{7/2}}{7b^3(b-c)^3} + \frac{8a^2(a+cx)^{3/2}}{3(b-c)^3c^2}-\frac{2a^2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c^3}-\frac{8a(a+cx)^{5/2}}{5(b-c)^3c^2} + \frac{4a(3b+c)(a+cx)^{5/2}}{5(b-c)^3c^3}-\frac{2(3b+c)(a+cx)^{7/2}}{7(b-c)^3c^3}$$

output
```
-8/3*a^2*(b*x+a)^(3/2)/b^2/(b-c)^3+2/3*a^2*(b+3*c)*(b*x+a)^(3/2)/b^3/(b-c)
^3+8/5*a*(b*x+a)^(5/2)/b^2/(b-c)^3-4/5*a*(b+3*c)*(b*x+a)^(5/2)/b^3/(b-c)^3
+2/7*(b+3*c)*(b*x+a)^(7/2)/b^3/(b-c)^3+8/3*a^2*(c*x+a)^(3/2)/(b-c)^3/c^2-2
/3*a^2*(3*b+c)*(c*x+a)^(3/2)/(b-c)^3/c^3-8/5*a*(c*x+a)^(5/2)/(b-c)^3/c^2+4
/5*a*(3*b+c)*(c*x+a)^(5/2)/(b-c)^3/c^3-2/7*(3*b+c)*(c*x+a)^(7/2)/(b-c)^3/c
^3
```

3.438.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2321 vs. $2(277) = 554$.

Time = 6.82 (sec) , antiderivative size = 2321, normalized size of antiderivative = 8.38

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\, dx = \text{Result too large to show}$$

input
```
Integrate[x^4/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

output
```
(-2*(a + c*x)*(5*a^3*b^3*Sqrt[a - (a*b)/c]*c^6*x^5*(b^2*x + 3*b*c*x - 3*b*
Sqrt[a + b*x]*Sqrt[a + c*x] - c*Sqrt[a + b*x]*Sqrt[a + c*x]) + a^9*(b^4*c*
(-139*Sqrt[a - (a*b)/c] + 413*Sqrt[a + b*x] - 216*Sqrt[a + c*x]) + b^5*(5*
Sqrt[a - (a*b)/c] - 35*Sqrt[a + b*x] + 8*Sqrt[a + c*x]) - 32*c^5*(35*Sqrt[
a - (a*b)/c] - 35*Sqrt[a + b*x] + 32*Sqrt[a + c*x]) + 7*b^3*c^2*(125*Sqrt[
a - (a*b)/c] - 251*Sqrt[a + b*x] + 176*Sqrt[a + c*x]) + 4*b*c^4*(665*Sqrt[
a - (a*b)/c] - 805*Sqrt[a + b*x] + 704*Sqrt[a + c*x]) - b^2*c^3*(2289*Sqrt
[a - (a*b)/c] - 3479*Sqrt[a + b*x] + 2816*Sqrt[a + c*x])) + a^4*b^2*c^5*x^
4*(-120*Sqrt[a - (a*b)/c]*c^2*Sqrt[a + b*x]*Sqrt[a + c*x] - 5*b^3*x*(22*Sq
rt[a - (a*b)/c] - 7*Sqrt[a + b*x] + 21*Sqrt[a + c*x]) + b*(-279*Sqrt[a - (
a*b)/c]*c*Sqrt[a + b*x]*Sqrt[a + c*x] + 7*c^2*x*(54*Sqrt[a - (a*b)/c] - 15
*Sqrt[a + b*x] + 5*Sqrt[a + c*x])) + b^2*(327*Sqrt[a - (a*b)/c]*Sqrt[a + b
*x]*Sqrt[a + c*x] + c*x*(-176*Sqrt[a - (a*b)/c] + 70*Sqrt[a + b*x] + 70*Sq
rt[a + c*x]))) + a^5*b*c^4*x^3*(-400*Sqrt[a - (a*b)/c]*c^3*Sqrt[a + b*x]*S
qrt[a + c*x] + b^4*x*(285*Sqrt[a - (a*b)/c] - 210*Sqrt[a + b*x] + 609*Sqrt
[a + c*x]) + b^3*(-785*Sqrt[a - (a*b)/c]*Sqrt[a + b*x]*Sqrt[a + c*x] + c*x
*(-573*Sqrt[a - (a*b)/c] + 63*Sqrt[a + b*x] - 1442*Sqrt[a + c*x])) + b*(-1
216*Sqrt[a - (a*b)/c]*c^2*Sqrt[a + b*x]*Sqrt[a + c*x] + 7*c^3*x*(233*Sqrt[
a - (a*b)/c] - 123*Sqrt[a + b*x] + 40*Sqrt[a + c*x])) + b^2*(2313*Sqrt[a -
 (a*b)/c]*c*Sqrt[a + b*x]*Sqrt[a + c*x] + c^2*x*(-1183*Sqrt[a - (a*b)/c...
```

3.438.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.78, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7241, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

$$\Big\downarrow \ 7241$$

$$\frac{\int \left((b+3c)\sqrt{a+bx}x^2-(3b+c)\sqrt{a+cx}x^2+4a\sqrt{a+bx}x-4a\sqrt{a+cx}x\right)dx}{(b-c)^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{2a^2(b+3c)(a+bx)^{3/2}}{3b^3}-\frac{8a^2(a+bx)^{3/2}}{3b^2}-\frac{2a^2(3b+c)(a+cx)^{3/2}}{3c^3}+\frac{8a^2(a+cx)^{3/2}}{3c^2}+\frac{2(b+3c)(a+bx)^{7/2}}{7b^3}-\frac{4a(b+3c)(a+bx)^{5/2}}{5b^3}+\frac{8a(a+bx)^{5/\cdots}}{5b^2}\cdots}{(b-c)^3}$$

input
```
Int[x^4/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

output
```
((-8*a^2*(a + b*x)^(3/2))/(3*b^2) + (2*a^2*(b + 3*c)*(a + b*x)^(3/2))/(3*b
^3) + (8*a*(a + b*x)^(5/2))/(5*b^2) - (4*a*(b + 3*c)*(a + b*x)^(5/2))/(5*b
^3) + (2*(b + 3*c)*(a + b*x)^(7/2))/(7*b^3) + (8*a^2*(a + c*x)^(3/2))/(3*c
^2) - (2*a^2*(3*b + c)*(a + c*x)^(3/2))/(3*c^3) - (8*a*(a + c*x)^(5/2))/(5
*c^2) + (4*a*(3*b + c)*(a + c*x)^(5/2))/(5*c^3) - (2*(3*b + c)*(a + c*x)^(
7/2))/(7*c^3))/(b - c)^3
```

3.438.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7241
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(b*e^2 - d*f^2)^m   Int[ExpandIntegran
d[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

3.438.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.89

method	result
default	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{4(bx+a)^{\frac{5}{2}}a}{5}+\frac{2(bx+a)^{\frac{3}{2}}a^2}{3}}{(b-c)^3b^2}+\frac{8a\left(\frac{(bx+a)^{\frac{5}{2}}}{5}-\frac{(bx+a)^{\frac{3}{2}}a}{3}\right)}{(b-c)^3b^2}-\frac{8a\left(\frac{(cx+a)^{\frac{5}{2}}}{5}-\frac{(cx+a)^{\frac{3}{2}}a}{3}\right)}{(b-c)^3c^2}+\frac{6c\left(\frac{(bx+a)^{\frac{7}{2}}}{7}-\frac{2(bx+a)^{\frac{5}{2}}a}{5}+\right.}{(b-c)^3b^3}$

input
```
int(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
2/(b-c)^3/b^2*(1/7*(b*x+a)^(7/2)-2/5*(b*x+a)^(5/2)*a+1/3*(b*x+a)^(3/2)*a^2
)+8/(b-c)^3*a/b^2*(1/5*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a)-8/(b-c)^3*a/c^2*
(1/5*(c*x+a)^(5/2)-1/3*(c*x+a)^(3/2)*a)+6/(b-c)^3*c/b^3*(1/7*(b*x+a)^(7/2)
-2/5*(b*x+a)^(5/2)*a+1/3*(b*x+a)^(3/2)*a^2)-6/(b-c)^3*b/c^3*(1/7*(c*x+a)^(
7/2)-2/5*(c*x+a)^(5/2)*a+1/3*(c*x+a)^(3/2)*a^2)-2/(b-c)^3/c^2*(1/7*(c*x+a)
^(7/2)-2/5*(c*x+a)^(5/2)*a+1/3*(c*x+a)^(3/2)*a^2)
```

3.438.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.81

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = -\frac{2\left((16\,a^3bc^3-8\,a^3c^4-5\,(b^4c^3+3\,b^3c^4)x^3-(29\,ab^3c^3+3\,ab^2c^4)x^2-4\,(2\,a^2b^2c^3-a^2bc^4)x\right)\sqrt{bx+a}+(\right.}{35\,(b^6c^3-3\,b^5c^4+}$$

input
```
integrate(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

output
```
-2/35*((16*a^3*b*c^3 - 8*a^3*c^4 - 5*(b^4*c^3 + 3*b^3*c^4)*x^3 - (29*a*b^3
*c^3 + 3*a*b^2*c^4)*x^2 - 4*(2*a^2*b^2*c^3 - a^2*b*c^4)*x)*sqrt(b*x + a) +
 (8*a^3*b^4 - 16*a^3*b^3*c + 5*(3*b^4*c^3 + b^3*c^4)*x^3 + (3*a*b^4*c^2 +
29*a*b^3*c^3)*x^2 - 4*(a^2*b^4*c - 2*a^2*b^3*c^2)*x)*sqrt(c*x + a))/(b^6*c
^3 - 3*b^5*c^4 + 3*b^4*c^5 - b^3*c^6)
```

3.438.6 Sympy [F]

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx=\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

input
```
integrate(x**4/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

output
```
Integral(x**4/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

3.438.7 Maxima [F]

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx=\int \frac{x^4}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

input
```
integrate(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(x^4/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

3.438.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 932 vs. 2(237) = 474.

Time = 1.06 (sec) , antiderivative size = 932, normalized size of antiderivative = 3.36

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

output
```
-2/35*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*(((b*x + a)*(5*(3*b^22*c^5*abs(b
) - 17*b^21*c^6*abs(b) + 39*b^20*c^7*abs(b) - 45*b^19*c^8*abs(b) + 25*b^18
*c^9*abs(b) - 3*b^17*c^10*abs(b) - 3*b^16*c^11*abs(b) + b^15*c^12*abs(b))*
(b*x + a)/(b^29*c^5 - 9*b^28*c^6 + 36*b^27*c^7 - 84*b^26*c^8 + 126*b^25*c^
9 - 126*b^24*c^10 + 84*b^23*c^11 - 36*b^22*c^12 + 9*b^21*c^13 - b^20*c^14)
 + (3*a*b^23*c^4*abs(b) - 34*a*b^22*c^5*abs(b) + 126*a*b^21*c^6*abs(b) - 2
10*a*b^20*c^7*abs(b) + 140*a*b^19*c^8*abs(b) + 42*a*b^18*c^9*abs(b) - 126*
a*b^17*c^10*abs(b) + 74*a*b^16*c^11*abs(b) - 15*a*b^15*c^12*abs(b))/(b^29*
c^5 - 9*b^28*c^6 + 36*b^27*c^7 - 84*b^26*c^8 + 126*b^25*c^9 - 126*b^24*c^1
0 + 84*b^23*c^11 - 36*b^22*c^12 + 9*b^21*c^13 - b^20*c^14)) - (4*a^2*b^24*
c^3*abs(b) - 26*a^2*b^23*c^4*abs(b) + 85*a^2*b^22*c^5*abs(b) - 203*a^2*b^2
1*c^6*abs(b) + 385*a^2*b^20*c^7*abs(b) - 539*a^2*b^19*c^8*abs(b) + 511*a^2
*b^18*c^9*abs(b) - 305*a^2*b^17*c^10*abs(b) + 103*a^2*b^16*c^11*abs(b) - 1
5*a^2*b^15*c^12*abs(b))/(b^29*c^5 - 9*b^28*c^6 + 36*b^27*c^7 - 84*b^26*c^8
 + 126*b^25*c^9 - 126*b^24*c^10 + 84*b^23*c^11 - 36*b^22*c^12 + 9*b^21*c^1
3 - b^20*c^14))*(b*x + a) + (8*a^3*b^25*c^2*abs(b) - 60*a^3*b^24*c^3*abs(b
) + 187*a^3*b^23*c^4*abs(b) - 296*a^3*b^22*c^5*abs(b) + 196*a^3*b^21*c^6*a
bs(b) + 112*a^3*b^20*c^7*abs(b) - 350*a^3*b^19*c^8*abs(b) + 328*a^3*b^18*c
^9*abs(b) - 164*a^3*b^17*c^10*abs(b) + 44*a^3*b^16*c^11*abs(b) - 5*a^3*b^1
5*c^12*abs(b))/(b^29*c^5 - 9*b^28*c^6 + 36*b^27*c^7 - 84*b^26*c^8 + 126...
```

3.438. $\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$

3.438.9 Mupad [B] (verification not implemented)

Time = 17.55 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.55

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{x^2\left(\frac{12\,a\,(3\,b+c)}{7\,(b-c)^3}-\frac{2\,a\,(3\,b+5\,c)}{(b-c)^3}\right)\sqrt{a+c\,x}}{5\,c} - \frac{2\,a\left(\frac{8\,a^2}{(b-c)^3}-\frac{4\,a\left(\frac{2\,a\,(5\,b+3\,c)}{(b-c)^3}-\frac{12\,a\,\left(b^2+3\,c\,b\right)}{7\,b\,(b-c)^3}\right)}{5\,b}\right)\sqrt{a+b\,x}}{3\,b^2} + \frac{x\left(\frac{8\,a^2}{(b-c)^3}-\frac{4\,a\left(\frac{2\,a\,(5\,b+3\,c)}{(b-c)^3}-\frac{12\,a\,\left(b^2+3\,c\,b\right)}{7\,b\,(b-c)^3}\right)}{5\,b}\right)\sqrt{a+b\,x}}{3\,b} + \frac{2\,a\left(\frac{8\,a^2}{(b-c)^3}+\frac{4\,a\left(\frac{12\,a\,(3\,b+c)}{7\,(b-c)^3}-\frac{2\,a\,(3\,b+5\,c)}{(b-c)^3}\right)}{5\,c}\right)\sqrt{a+c\,x}}{3\,c^2} + \frac{x^2\left(\frac{2\,a\,(5\,b+3\,c)}{(b-c)^3}-\frac{12\,a\,\left(b^2+3\,c\,b\right)}{7\,b\,(b-c)^3}\right)\sqrt{a+b\,x}}{5\,b} - \frac{x\left(\frac{8\,a^2}{(b-c)^3}+\frac{4\,a\left(\frac{12\,a\,(3\,b+c)}{7\,(b-c)^3}-\frac{2\,a\,(3\,b+5\,c)}{(b-c)^3}\right)}{5\,c}\right)\sqrt{a+c\,x}}{3\,c} - \frac{2\,x^3\,(3\,b+c)\,\sqrt{a+c\,x}}{7\,(b-c)^3} + \frac{2\,x^3\,\left(b^2+3\,c\,b\right)\sqrt{a+b\,x}}{7\,b\,(b-c)^3}$$

input
```
int(x^4/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

output
```
(x^2*((12*a*(3*b + c))/(7*(b - c)^3) - (2*a*(3*b + 5*c))/(b - c)^3)*(a + c
*x)^(1/2))/(5*c) - (2*a*((8*a^2)/(b - c)^3 - (4*a*((2*a*(5*b + 3*c))/(b -
c)^3 - (12*a*(3*b*c + b^2))/(7*b*(b - c)^3)))/(5*b))*(a + b*x)^(1/2))/(3*b
^2) + (x*((8*a^2)/(b - c)^3 - (4*a*((2*a*(5*b + 3*c))/(b - c)^3 - (12*a*(3
*b*c + b^2))/(7*b*(b - c)^3)))/(5*b))*(a + b*x)^(1/2))/(3*b) + (2*a*((8*a^
2)/(b - c)^3 + (4*a*((12*a*(3*b + c))/(7*(b - c)^3) - (2*a*(3*b + 5*c))/(b
 - c)^3))/(5*c))*(a + c*x)^(1/2))/(3*c^2) + (x^2*((2*a*(5*b + 3*c))/(b - c
)^3 - (12*a*(3*b*c + b^2))/(7*b*(b - c)^3))*(a + b*x)^(1/2))/(5*b) - (x*((
8*a^2)/(b - c)^3 + (4*a*((12*a*(3*b + c))/(7*(b - c)^3) - (2*a*(3*b + 5*c)
)/(b - c)^3))/(5*c))*(a + c*x)^(1/2))/(3*c) - (2*x^3*(3*b + c)*(a + c*x)^(
1/2))/(7*(b - c)^3) + (2*x^3*(3*b*c + b^2)*(a + b*x)^(1/2))/(7*b*(b - c)^3
)
```

3.439 $\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$

3.439.1 Optimal result

Integrand size = 25, antiderivative size = 163

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{8a(a+bx)^{3/2}}{3b(b-c)^3} - \frac{2a(b+3c)(a+bx)^{3/2}}{3b^2(b-c)^3} + \frac{2(b+3c)(a+bx)^{5/2}}{5b^2(b-c)^3} - \frac{8a(a+cx)^{3/2}}{3(b-c)^3c} + \frac{2a(3b+c)(a+cx)^{3/2}}{3(b-c)^3c^2} - \frac{2(3b+c)(a+cx)^{5/2}}{5(b-c)^3c^2}$$

output
```
8/3*a*(b*x+a)^(3/2)/b/(b-c)^3-2/3*a*(b+3*c)*(b*x+a)^(3/2)/b^2/(b-c)^3+2/5*
(b+3*c)*(b*x+a)^(5/2)/b^2/(b-c)^3-8/3*a*(c*x+a)^(3/2)/(b-c)^3/c+2/3*a*(3*b
+c)*(c*x+a)^(3/2)/(b-c)^3/c^2-2/5*(3*b+c)*(c*x+a)^(5/2)/(b-c)^3/c^2
```

3.439.2 Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.21

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{2\sqrt{a-\frac{ab}{c}+\frac{b(a+cx)}{c}}\left(a^2b^3-4a^2b^2c+5a^2bc^2-2a^2c^3-2ab^3(a+cx)+ab^2c(a+cx)+abc^2(a+cx)+b^3(a\right.}{5b^2(b-c)^3c^2} + \frac{2\left(5ab(a+cx)^{3/2}-5ac(a+cx)^{3/2}-3b(a+cx)^{5/2}-c(a+cx)^{5/2}\right)}{5(b-c)^3c^2}$$

input
```
Integrate[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

output
```
(2*Sqrt[a - (a*b)/c + (b*(a + c*x))/c]*(a^2*b^3 - 4*a^2*b^2*c + 5*a^2*b*c^
2 - 2*a^2*c^3 - 2*a*b^3*(a + c*x) + a*b^2*c*(a + c*x) + a*b*c^2*(a + c*x)
+ b^3*(a + c*x)^2 + 3*b^2*c*(a + c*x)^2))/(5*b^2*(b - c)^3*c^2) + (2*(5*a*
b*(a + c*x)^(3/2) - 5*a*c*(a + c*x)^(3/2) - 3*b*(a + c*x)^(5/2) - c*(a + c
*x)^(5/2)))/(5*(b - c)^3*c^2)
```

3.439.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.79, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7241, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}dx$$

$$\Big\downarrow \text{7241}$$

$$\frac{\int \left(4\sqrt{a+bx}a-4\sqrt{a+cx}a+(b+3c)x\sqrt{a+bx}-(3b+c)x\sqrt{a+cx}\right)dx}{(b-c)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2(b+3c)(a+bx)^{5/2}}{5b^2}-\frac{2a(b+3c)(a+bx)^{3/2}}{3b^2}-\frac{2(3b+c)(a+cx)^{5/2}}{5c^2}+\frac{2a(3b+c)(a+cx)^{3/2}}{3c^2}+\frac{8a(a+bx)^{3/2}}{3b}-\frac{8a(a+cx)^{3/2}}{3c}}{(b-c)^3}$$

input
```
Int[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

output
```
((8*a*(a + b*x)^(3/2))/(3*b) - (2*a*(b + 3*c)*(a + b*x)^(3/2))/(3*b^2) + (
2*(b + 3*c)*(a + b*x)^(5/2))/(5*b^2) - (8*a*(a + c*x)^(3/2))/(3*c) + (2*a*
(3*b + c)*(a + c*x)^(3/2))/(3*c^2) - (2*(3*b + c)*(a + c*x)^(5/2))/(5*c^2)
)/(b - c)^3
```

3.439.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7241
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(b*e^2 - d*f^2)^m   Int[ExpandIntegran
d[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

3.439.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.06

method	result
default	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{2(bx+a)^{\frac{3}{2}}a}{3}}{(b-c)^3 b}+\frac{8a(bx+a)^{\frac{3}{2}}}{3b(b-c)^3}-\frac{8a(cx+a)^{\frac{3}{2}}}{3(b-c)^3 c}+\frac{6c\left(\frac{(bx+a)^{\frac{5}{2}}}{5}-\frac{(bx+a)^{\frac{3}{2}}a}{3}\right)}{(b-c)^3 b^2}-\frac{6b\left(\frac{(cx+a)^{\frac{5}{2}}}{5}-\frac{(cx+a)^{\frac{3}{2}}a}{3}\right)}{(b-c)^3 c^2}-\frac{2\left(\frac{(cx+a)}{5}\right.}{(}$

input
```
int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
2/(b-c)^3/b*(1/5*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a)+8/3*a*(b*x+a)^(3/2)/b/
(b-c)^3-8/3*a*(c*x+a)^(3/2)/(b-c)^3/c+6/(b-c)^3*c/b^2*(1/5*(b*x+a)^(5/2)-1
/3*(b*x+a)^(3/2)*a)-6/(b-c)^3*b/c^2*(1/5*(c*x+a)^(5/2)-1/3*(c*x+a)^(3/2)*a
)-2/(b-c)^3/c*(1/5*(c*x+a)^(5/2)-1/3*(c*x+a)^(3/2)*a)
```

3.439.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$
$$=\frac{2\left((6\,a^2bc^2-2\,a^2c^3+(b^3c^2+3\,b^2c^3)x^2+(7\,ab^2c^2+abc^3)x)\sqrt{bx+a}+(2\,a^2b^3-6\,a^2b^2c-(3\,b^3c^2+b^2c^3)x\right.}{5\,(b^5c^2-3\,b^4c^3+3\,b^3c^4-b^2c^5)}$$

input
```
integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

output
```
2/5*((6*a^2*b*c^2 - 2*a^2*c^3 + (b^3*c^2 + 3*b^2*c^3)*x^2 + (7*a*b^2*c^2 +
 a*b*c^3)*x)*sqrt(b*x + a) + (2*a^2*b^3 - 6*a^2*b^2*c - (3*b^3*c^2 + b^2*c
^3)*x^2 - (a*b^3*c + 7*a*b^2*c^2)*x)*sqrt(c*x + a))/(b^5*c^2 - 3*b^4*c^3 +
 3*b^3*c^4 - b^2*c^5)
```

3.439.6 Sympy [F]

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx=\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

input
```
integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

output
```
Integral(x**3/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

3.439.7 Maxima [F]

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx=\int \frac{x^3}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

input
```
integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

3.439.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(139) = 278$.

Time = 1.02 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.94

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = -\frac{2}{5}\sqrt{ab^2+(bx+a)bc-abc}\left((bx+a)\left(\frac{(3\,b^{12}c^3|b|-8\,b^{11}c^4|b|+6\,b^{10}c^5|b|-b^8c^7|b|)(bx+a)}{b^{18}c^3-6\,b^{17}c^4+15\,b^{16}c^5-20\,b^{15}c^6+15\,b^{14}c^7-6\,b^{13}c^8+b^{12}c^9}\right.\right.$$
$$+\frac{2\left((bx+a)^{\frac{5}{2}}b+5\,(bx+a)^{\frac{3}{2}}ab+3\,(bx+a)^{\frac{5}{2}}c-5\,(bx+a)^{\frac{3}{2}}ac\right)}{5\,(b^5-3\,b^4c+3\,b^3c^2-b^2c^3)}$$

input
```
integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

output
```
-2/5*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*((b*x + a)*((3*b^12*c^3*abs(b) -
8*b^11*c^4*abs(b) + 6*b^10*c^5*abs(b) - b^8*c^7*abs(b))*(b*x + a)/(b^18*c^
3 - 6*b^17*c^4 + 15*b^16*c^5 - 20*b^15*c^6 + 15*b^14*c^7 - 6*b^13*c^8 + b^
12*c^9) + (a*b^13*c^2*abs(b) - 2*a*b^12*c^3*abs(b) - 2*a*b^11*c^4*abs(b) +
 8*a*b^10*c^5*abs(b) - 7*a*b^9*c^6*abs(b) + 2*a*b^8*c^7*abs(b))/(b^18*c^3
- 6*b^17*c^4 + 15*b^16*c^5 - 20*b^15*c^6 + 15*b^14*c^7 - 6*b^13*c^8 + b^12
*c^9)) - (2*a^2*b^14*c*abs(b) - 11*a^2*b^13*c^2*abs(b) + 25*a^2*b^12*c^3*a
bs(b) - 30*a^2*b^11*c^4*abs(b) + 20*a^2*b^10*c^5*abs(b) - 7*a^2*b^9*c^6*ab
s(b) + a^2*b^8*c^7*abs(b))/(b^18*c^3 - 6*b^17*c^4 + 15*b^16*c^5 - 20*b^15*
c^6 + 15*b^14*c^7 - 6*b^13*c^8 + b^12*c^9)) + 2/5*((b*x + a)^(5/2)*b + 5*(
b*x + a)^(3/2)*a*b + 3*(b*x + a)^(5/2)*c - 5*(b*x + a)^(3/2)*a*c)/(b^5 - 3
*b^4*c + 3*b^3*c^2 - b^2*c^3)
```

3.439.9 Mupad [B] (verification not implemented)

Time = 16.86 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.64

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{\left(\frac{8\,a^2}{(b-c)^3}+\frac{2\,a\left(\frac{8\,a\,(b+3\,c)}{5\,(b-c)^3}-\frac{2\,a\,(5\,b+3\,c)}{(b-c)^3}\right)}{3\,b}\right)\sqrt{a+b\,x}}{b} - \frac{\left(\frac{8\,a^2}{(b-c)^3}+\frac{2\,a\left(\frac{8\,a\,(3\,b+c)}{5\,(b-c)^3}-\frac{2\,a\,(3\,b+5\,c)}{(b-c)^3}\right)}{3\,c}\right)\sqrt{a+c\,x}}{c} - \frac{x\left(\frac{8\,a\,(b+3\,c)}{5\,(b-c)^3}-\frac{2\,a\,(5\,b+3\,c)}{(b-c)^3}\right)\sqrt{a+b\,x}}{3\,b} + \frac{x\left(\frac{8\,a\,(3\,b+c)}{5\,(b-c)^3}-\frac{2\,a\,(3\,b+5\,c)}{(b-c)^3}\right)\sqrt{a+c\,x}}{3\,c} + \frac{2\,x^2\,(b+3\,c)\,\sqrt{a+b\,x}}{5\,{(b-c)}^3} - \frac{2\,x^2\,(3\,b+c)\,\sqrt{a+c\,x}}{5\,{(b-c)}^3}$$

input
```
int(x^3/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

output
```
(((8*a^2)/(b - c)^3 + (2*a*((8*a*(b + 3*c))/(5*(b - c)^3) - (2*a*(5*b + 3*
c))/(b - c)^3))/(3*b))*(a + b*x)^(1/2))/b - (((8*a^2)/(b - c)^3 + (2*a*((8
*a*(3*b + c))/(5*(b - c)^3) - (2*a*(3*b + 5*c))/(b - c)^3))/(3*c))*(a + c*
x)^(1/2))/c - (x*((8*a*(b + 3*c))/(5*(b - c)^3) - (2*a*(5*b + 3*c))/(b - c
)^3)*(a + b*x)^(1/2))/(3*b) + (x*((8*a*(3*b + c))/(5*(b - c)^3) - (2*a*(3*
b + 5*c))/(b - c)^3)*(a + c*x)^(1/2))/(3*c) + (2*x^2*(b + 3*c)*(a + b*x)^(
1/2))/(5*(b - c)^3) - (2*x^2*(3*b + c)*(a + c*x)^(1/2))/(5*(b - c)^3)
```

3.439. $\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$

3.440 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$

3.440.1 Optimal result

Integrand size = 25, antiderivative size = 155

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{8a\sqrt{a+bx}}{(b-c)^3}+\frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a\sqrt{a+cx}}{(b-c)^3}-\frac{2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c}-\frac{8a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{8a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}$$

output
```
2/3*(b+3*c)*(b*x+a)^(3/2)/b/(b-c)^3-2/3*(3*b+c)*(c*x+a)^(3/2)/(b-c)^3/c-8*
a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))/(b-c)^3+8*a^(3/2)*arctanh((c*x+a)^(
1/2)/a^(1/2))/(b-c)^3+8*a*(b*x+a)^(1/2)/(b-c)^3-8*a*(c*x+a)^(1/2)/(b-c)^3
```

3.440.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1004 vs. 2(155) = 310.

Time = 5.57 (sec) , antiderivative size = 1004, normalized size of antiderivative = 6.48

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{-2a\sqrt{b-c}\left(b\sqrt{a-\frac{ab}{c}}c^3x^2\left(b^2x+3bcx-3b\sqrt{a+bx}\sqrt{a+cx}-c\sqrt{a+bx}\sqrt{a+cx}\right)+a^3\left(bc^2\left(12\sqrt{a-\frac{ab}{c}}\right.\right.\right.}{}$$

input
```
Integrate[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

output
```
(-2*a*Sqrt[b - c]*(b*Sqrt[a - (a*b)/c]*c^3*x^2*(b^2*x + 3*b*c*x - 3*b*Sqrt
[a + b*x]*Sqrt[a + c*x] - c*Sqrt[a + b*x]*Sqrt[a + c*x]) + a^3*(b*c^2*(12*
Sqrt[a - (a*b)/c] - 3*Sqrt[a + b*x] - 53*Sqrt[a + c*x]) + b^2*c*(-15*Sqrt[
a - (a*b)/c] + 24*Sqrt[a + b*x] - 2*Sqrt[a + c*x]) + b^3*(Sqrt[a - (a*b)/c
] - 3*Sqrt[a + b*x] + 3*Sqrt[a + c*x]) + 2*c^3*(9*Sqrt[a - (a*b)/c] - 9*Sq
rt[a + b*x] + 26*Sqrt[a + c*x])) + a*c^2*x*(-4*Sqrt[a - (a*b)/c]*c^2*Sqrt[
a + b*x]*Sqrt[a + c*x] + b^3*x*(-3*Sqrt[a - (a*b)/c] + 3*Sqrt[a + b*x] - 9
*Sqrt[a + c*x]) + b*(-22*Sqrt[a - (a*b)/c]*c*Sqrt[a + b*x]*Sqrt[a + c*x] +
 3*c^2*x*(6*Sqrt[a - (a*b)/c] - 3*Sqrt[a + b*x] + Sqrt[a + c*x])) + b^2*(6
*Sqrt[a - (a*b)/c]*Sqrt[a + b*x]*Sqrt[a + c*x] + c*x*(9*Sqrt[a - (a*b)/c]
+ 6*Sqrt[a + b*x] + 6*Sqrt[a + c*x]))) + a^2*(-3*b^3*c*x*(Sqrt[a - (a*b)/c
] + 2*Sqrt[a + c*x]) + b^2*(9*Sqrt[a - (a*b)/c]*c*Sqrt[a + b*x]*Sqrt[a + c
*x] + c^2*x*(-9*Sqrt[a - (a*b)/c] + 30*Sqrt[a + b*x] - 44*Sqrt[a + c*x]))
+ 2*c^3*(-26*Sqrt[a - (a*b)/c]*Sqrt[a + b*x]*Sqrt[a + c*x] + c*x*(9*Sqrt[a
 - (a*b)/c] - 9*Sqrt[a + b*x] + 2*Sqrt[a + c*x])) + b*(27*Sqrt[a - (a*b)/c
]*c^2*Sqrt[a + b*x]*Sqrt[a + c*x] + c^3*x*(30*Sqrt[a - (a*b)/c] - 12*Sqrt[
a + b*x] + 46*Sqrt[a + c*x])))) - 48*a^3*c^(3/2)*(-b + c)*(b*c*x*(3*Sqrt[a
 - (a*b)/c] - Sqrt[a + b*x]) + a*(-(b*Sqrt[a - (a*b)/c]) + 4*Sqrt[a - (a*b
)/c]*c + 3*b*Sqrt[a + b*x] - 4*c*Sqrt[a + b*x]))*ArcTan[(Sqrt[b - c]*Sqrt[
a + c*x])/(Sqrt[c]*(-Sqrt[a - (a*b)/c] + Sqrt[a + b*x] + Sqrt[a + c*x])...
```

3.440.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.78, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {7241, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}dx$$

$$\Big\downarrow \text{ 7241}$$

$$\frac{\int \left(\frac{4\sqrt{a+bx}a}{x}-\frac{4\sqrt{a+cx}a}{x}+(b+3c)\sqrt{a+bx}-(3b+c)\sqrt{a+cx}\right)dx}{(b-c)^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-8a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+8a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)+\frac{2(b+3c)(a+bx)^{3/2}}{3b}-\frac{2(3b+c)(a+cx)^{3/2}}{3c}+8a\sqrt{a+bx}-8a\sqrt{a+cx}}{(b-c)^3}$$

input
```
Int[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

output
```
(8*a*Sqrt[a + b*x] + (2*(b + 3*c)*(a + b*x)^(3/2))/(3*b) - 8*a*Sqrt[a + c*
x] - (2*(3*b + c)*(a + c*x)^(3/2))/(3*c) - 8*a^(3/2)*ArcTanh[Sqrt[a + b*x]
/Sqrt[a]] + 8*a^(3/2)*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(b - c)^3
```

3.440.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7241
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(b*e^2 - d*f^2)^m   Int[ExpandIntegran
d[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

3.440.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

method	result
default	$\frac{2(bx+a)^{\frac{3}{2}}}{3(b-c)^3}+\frac{2c(bx+a)^{\frac{3}{2}}}{(b-c)^3 b}-\frac{2b(cx+a)^{\frac{3}{2}}}{(b-c)^3 c}-\frac{2(cx+a)^{\frac{3}{2}}}{3(b-c)^3}+\frac{4a\left(2\sqrt{bx+a}-2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{(b-c)^3}-\frac{4a\left(2\sqrt{cx+a}-2\sqrt{a}\,\text{arctanh}\right.}{(b-c)^3}$

input
```
int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
2/3/(b-c)^3*(b*x+a)^(3/2)+2/(b-c)^3*c*(b*x+a)^(3/2)/b-2/(b-c)^3*b*(c*x+a)^
(3/2)/c-2/3/(b-c)^3*(c*x+a)^(3/2)+4*a/(b-c)^3*(2*(b*x+a)^(1/2)-2*a^(1/2)*a
rctanh((b*x+a)^(1/2)/a^(1/2)))-4*a/(b-c)^3*(2*(c*x+a)^(1/2)-2*a^(1/2)*arct
anh((c*x+a)^(1/2)/a^(1/2)))
```

3.440.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.07

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$
$$= \left[-\frac{2\left(6\,a^{\frac{3}{2}}bc\log\left(\frac{bx+2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+6\,a^{\frac{3}{2}}bc\log\left(\frac{cx-2\,\sqrt{cx+a}\sqrt{a}+2\,a}{x}\right)-(13\,abc+3\,ac^2+(b^2c+3\,bc^2)x)\sqrt{b}\right.}{3\,(b^4c-3\,b^3c^2+3\,b^2c^3-bc^4)}\right.$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

output
```
[-2/3*(6*a^(3/2)*b*c*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 6*a^(3
/2)*b*c*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) - (13*a*b*c + 3*a*c^2
 + (b^2*c + 3*b*c^2)*x)*sqrt(b*x + a) + (3*a*b^2 + 13*a*b*c + (3*b^2*c + b
*c^2)*x)*sqrt(c*x + a))/(b^4*c - 3*b^3*c^2 + 3*b^2*c^3 - b*c^4), 2/3*(12*s
qrt(-a)*a*b*c*arctan(sqrt(b*x + a)*sqrt(-a)/a) - 12*sqrt(-a)*a*b*c*arctan(
sqrt(c*x + a)*sqrt(-a)/a) + (13*a*b*c + 3*a*c^2 + (b^2*c + 3*b*c^2)*x)*sqr
t(b*x + a) - (3*a*b^2 + 13*a*b*c + (3*b^2*c + b*c^2)*x)*sqrt(c*x + a))/(b^
4*c - 3*b^3*c^2 + 3*b^2*c^3 - b*c^4)]
```

3.440.6 Sympy [F]

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

input
```
integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

output
```
Integral(x**2/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

3.440.7 Maxima [F]

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \int \frac{x^2}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

3.440.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2374 vs. $2(131) = 262$.

Time = 1.58 (sec) , antiderivative size = 2374, normalized size of antiderivative = 15.32

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

output
```
-2/3*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*((3*b^7*c*abs(b) - 8*b^6*c^2*abs(
b) + 6*b^5*c^3*abs(b) - b^3*c^5*abs(b))*(b*x + a)/(b^12*c - 6*b^11*c^2 + 1
5*b^10*c^3 - 20*b^9*c^4 + 15*b^8*c^5 - 6*b^7*c^6 + b^6*c^7) + (3*a*b^8*abs
(b) + a*b^7*c*abs(b) - 22*a*b^6*c^2*abs(b) + 30*a*b^5*c^3*abs(b) - 13*a*b^
4*c^4*abs(b) + a*b^3*c^5*abs(b))/(b^12*c - 6*b^11*c^2 + 15*b^10*c^3 - 20*b
^9*c^4 + 15*b^8*c^5 - 6*b^7*c^6 + b^6*c^7)) + 8*a^2*arctan(sqrt(b*x + a)/s
qrt(-a))/((b^3 - 3*b^2*c + 3*b*c^2 - c^3)*sqrt(-a)) + 2/3*((b*x + a)^(3/2)
*b^9 + 12*sqrt(b*x + a)*a*b^9 - 3*(b*x + a)^(3/2)*b^8*c - 72*sqrt(b*x + a)
*a*b^8*c - 3*(b*x + a)^(3/2)*b^7*c^2 + 180*sqrt(b*x + a)*a*b^7*c^2 + 25*(b
*x + a)^(3/2)*b^6*c^3 - 240*sqrt(b*x + a)*a*b^6*c^3 - 45*(b*x + a)^(3/2)*b
^5*c^4 + 180*sqrt(b*x + a)*a*b^5*c^4 + 39*(b*x + a)^(3/2)*b^4*c^5 - 72*sqr
t(b*x + a)*a*b^4*c^5 - 17*(b*x + a)^(3/2)*b^3*c^6 + 12*sqrt(b*x + a)*a*b^3
*c^6 + 3*(b*x + a)^(3/2)*b^2*c^7)/(b^12 - 9*b^11*c + 36*b^10*c^2 - 84*b^9*
c^3 + 126*b^8*c^4 - 126*b^7*c^5 + 84*b^6*c^6 - 36*b^5*c^7 + 9*b^4*c^8 - b^
3*c^9) - 8*(2*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(a*b^3*c - a*b
^2*c^2)*sqrt(-a)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + 2*(a*b^4 - 3*
a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(a*b^3 - a*b^2*c)*sqrt(-a*b*c)*abs(b) +
 (a^2*b^7 - 5*a^2*b^6*c + 10*a^2*b^5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^3*c^4
- a^2*b^2*c^5)*sqrt(-a*b*c)*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3
)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^2*b^8 - 5*a^2*b^7*c + ...
```

3.440.9 Mupad [B] (verification not implemented)

Time = 20.35 (sec) , antiderivative size = 762, normalized size of antiderivative = 4.92

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$
$$= \frac{4\,a^{3/2}\,b^4 - \frac{4\,a^{3/2}\,c^4\left(\frac{4\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^3}-\frac{15\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^4}+\frac{24\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)^5}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^5}+\frac{6\,\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)^6}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^6}\right)}{3} - \frac{4\,a^{3/2}\,b^2\,c^2\left(\frac{24\,(\sqrt{a+b}}{\sqrt{a+c\,x}}\right.}{}}{}$$

input
```
int(x^2/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

output
```
(4*a^(3/2)*b^4 - (4*a^(3/2)*c^4*((4*((a + b*x)^(1/2) - a^(1/2))^3)/((a + c
*x)^(1/2) - a^(1/2))^3 - (15*((a + b*x)^(1/2) - a^(1/2))^4)/((a + c*x)^(1/
2) - a^(1/2))^4 + (24*((a + b*x)^(1/2) - a^(1/2))^5)/((a + c*x)^(1/2) - a^
(1/2))^5 + (6*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))
*((a + b*x)^(1/2) - a^(1/2))^6)/((a + c*x)^(1/2) - a^(1/2))^6))/3 - (4*a^(
3/2)*b^2*c^2*((24*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2))
 + (12*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 + (12*
((a + b*x)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3 - (15*((a + b
*x)^(1/2) - a^(1/2))^4)/((a + c*x)^(1/2) - a^(1/2))^4 + (18*log(((a + b*x)
^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*((a + b*x)^(1/2) - a^(1/2))
^2)/((a + c*x)^(1/2) - a^(1/2))^2 - 3))/3 + (4*a^(3/2)*b*c^3*((6*((a + b*x
)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 - (12*((a + b*x)^(1/2)
 - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3 + (66*((a + b*x)^(1/2) - a^(1
/2))^4)/((a + c*x)^(1/2) - a^(1/2))^4 - (24*((a + b*x)^(1/2) - a^(1/2))^5)
/((a + c*x)^(1/2) - a^(1/2))^5 + (18*log(((a + b*x)^(1/2) - a^(1/2))/((a +
 c*x)^(1/2) - a^(1/2)))*((a + b*x)^(1/2) - a^(1/2))^4)/((a + c*x)^(1/2) -
a^(1/2))^4))/3 + (4*a^(3/2)*b^3*c*(6*log(((a + b*x)^(1/2) - a^(1/2))/((a +
 c*x)^(1/2) - a^(1/2))) - (24*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2
) - a^(1/2)) + (6*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2
))^2 - (4*((a + b*x)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3 ...
```

3.441 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3} dx$

3.441.1 Optimal result

Integrand size = 23, antiderivative size = 157

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3} dx = \frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3}-\frac{4a\sqrt{a+bx}}{(b-c)^3x}-\frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3}+\frac{4a\sqrt{a+cx}}{(b-c)^3x}-\frac{6\sqrt{a}(b+c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{6\sqrt{a}(b+c)\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}$$

output
```
-6*(b+c)*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)/(b-c)^3+6*(b+c)*arctanh((c
*x+a)^(1/2)/a^(1/2))*a^(1/2)/(b-c)^3+2*(b+3*c)*(b*x+a)^(1/2)/(b-c)^3-4*a*(
b*x+a)^(1/2)/(b-c)^3/x-2*(3*b+c)*(c*x+a)^(1/2)/(b-c)^3+4*a*(c*x+a)^(1/2)/(
b-c)^3/x
```

3.441.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 690 vs. $2(157) = 314$.

Time = 10.68 (sec) , antiderivative size = 690, normalized size of antiderivative = 4.39

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

$$= \frac{2\sqrt{b-c}\left(b\sqrt{a-\frac{ab}{c}}c^2x\left(bx+cx-2\sqrt{a+bx}\sqrt{a+cx}\right)+a\left(4b\sqrt{a-\frac{ab}{c}}c\sqrt{a+bx}\sqrt{a+cx}+6\sqrt{a-\frac{ab}{c}}c^2\sqrt{\cdots}\right.\right.}{\cdots}$$

input
```
Integrate[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

output
```
(2*Sqrt[b - c]*(b*Sqrt[a - (a*b)/c]*c^2*x*(b*x + c*x - 2*Sqrt[a + b*x]*Sqr
t[a + c*x]) + a*(4*b*Sqrt[a - (a*b)/c]*c*Sqrt[a + b*x]*Sqrt[a + c*x] + 6*S
qrt[a - (a*b)/c]*c^2*Sqrt[a + b*x]*Sqrt[a + c*x] + 2*c^3*x*(Sqrt[a - (a*b)
/c] - Sqrt[a + b*x]) + b^2*c*x*(Sqrt[a - (a*b)/c] + Sqrt[a + b*x] - 5*Sqrt
[a + c*x]) + b*c^2*x*(7*Sqrt[a - (a*b)/c] - 5*Sqrt[a + b*x] - Sqrt[a + c*x
])) + a^2*(2*c^2*(Sqrt[a - (a*b)/c] - Sqrt[a + b*x] - 3*Sqrt[a + c*x]) + b
*c*(6*Sqrt[a - (a*b)/c] - 5*Sqrt[a + b*x] - Sqrt[a + c*x]) + b^2*(Sqrt[a +
 b*x] + Sqrt[a + c*x]))) - 12*a*Sqrt[c]*(b + c)*(2*Sqrt[a - (a*b)/c]*c*Sqr
t[a + b*x]*Sqrt[a + c*x] + 2*a*c*(Sqrt[a - (a*b)/c] - Sqrt[a + b*x] - Sqrt
[a + c*x]) + b*c*x*(2*Sqrt[a - (a*b)/c] - Sqrt[a + b*x] - Sqrt[a + c*x]) +
 a*b*(Sqrt[a + b*x] + Sqrt[a + c*x]))*ArcTan[(Sqrt[b - c]*Sqrt[a + c*x])/(
Sqrt[c]*(-Sqrt[a - (a*b)/c] + Sqrt[a + b*x] + Sqrt[a + c*x]))])/((b - c)^(
5/2)*c*(a*(b - c) + Sqrt[a - (a*b)/c]*c*Sqrt[a + b*x])*(a + b*x - Sqrt[a -
 (a*b)/c]*Sqrt[a + b*x] - Sqrt[a - (a*b)/c]*Sqrt[a + c*x] + Sqrt[a + b*x]*
Sqrt[a + c*x]))
```

3.441.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {7241, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}dx$$

$$\Big\downarrow \text{ 7241}$$

$$\frac{\int \left(\frac{4\sqrt{a+bx}a}{x^2} - \frac{4\sqrt{a+cx}a}{x^2} + \frac{(b+3c)\sqrt{a+bx}}{x} - \frac{(3b+c)\sqrt{a+cx}}{x}\right) dx}{(b-c)^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{-2\sqrt{a}(b+3c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right) + 2\sqrt{a}(3b+c)\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right) - 4\sqrt{a}b\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right) + 4\sqrt{a}c\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}$$

input
```
Int[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

output
```
(2*(b + 3*c)*Sqrt[a + b*x] - (4*a*Sqrt[a + b*x])/x - 2*(3*b + c)*Sqrt[a +
c*x] + (4*a*Sqrt[a + c*x])/x - 4*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
- 2*Sqrt[a]*(b + 3*c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]] + 4*Sqrt[a]*c*ArcTanh
[Sqrt[a + c*x]/Sqrt[a]] + 2*Sqrt[a]*(3*b + c)*ArcTanh[Sqrt[a + c*x]/Sqrt[a
]])/(b - c)^3
```

3.441.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7241
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(b*e^2 - d*f^2)^m   Int[ExpandIntegran
d[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

3.441.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.51

method	result
default	$\frac{b\left(2\sqrt{bx+a}-2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{(b-c)^3} + \frac{8ab\left(-\frac{\sqrt{bx+a}}{2xb} - \frac{\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{(b-c)^3} - \frac{8ac\left(-\frac{\sqrt{cx+a}}{2cx} - \frac{\text{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{(b-c)^3} + \frac{3c\left(2\sqrt{bx\cdots\right.}{}$

input
```
int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
1/(b-c)^3*b*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))+8*a
/(b-c)^3*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1
/2))-8*a/(b-c)^3*c*(-1/2*(c*x+a)^(1/2)/c/x-1/2/a^(1/2)*arctanh((c*x+a)^(1/
2)/a^(1/2)))+3/(b-c)^3*c*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/
a^(1/2)))-3/(b-c)^3*b*(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(
1/2)))-1/(b-c)^3*c*(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2
)))
```

3.441.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.66

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$
$$= \left[-\frac{3\sqrt{a}(b+c)x\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+3\sqrt{a}(b+c)x\log\left(\frac{cx-2\sqrt{cx+a}\sqrt{a}+2a}{x}\right)-2\left((b+3c)x-2a\right)\sqrt{bx+}}{(b^3-3b^2c+3bc^2-c^3)x}\right.$$

input
```
integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

output
```
[-(3*sqrt(a)*(b + c)*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 3*sq
rt(a)*(b + c)*x*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) - 2*((b + 3*c
)*x - 2*a)*sqrt(b*x + a) + 2*((3*b + c)*x - 2*a)*sqrt(c*x + a))/((b^3 - 3*
b^2*c + 3*b*c^2 - c^3)*x), 2*(3*sqrt(-a)*(b + c)*x*arctan(sqrt(b*x + a)*sq
rt(-a)/a) - 3*sqrt(-a)*(b + c)*x*arctan(sqrt(c*x + a)*sqrt(-a)/a) + ((b +
3*c)*x - 2*a)*sqrt(b*x + a) - ((3*b + c)*x - 2*a)*sqrt(c*x + a))/((b^3 - 3
*b^2*c + 3*b*c^2 - c^3)*x)]
```

3.441.6 Sympy [F]

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

input
```
integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

output
```
Integral(x/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

3.441.7 Maxima [F]

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx=\int \frac{x}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

input
```
integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

3.441.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2318 vs. $2(137) = 274$.

Time = 9.20 (sec) , antiderivative size = 2318, normalized size of antiderivative = 14.76

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

output
```
-2*(sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*(3*b*abs(b) + c*abs(b))/(b^4 - 3*b
^3*c + 3*b^2*c^2 - b*c^3) + 2*sqrt(b*x + a)*a*b/((b^3 - 3*b^2*c + 3*b*c^2
- c^3)*x) - 3*(a*b^2 + a*b*c)*arctan(sqrt(b*x + a)/sqrt(-a))/((b^3 - 3*b^2
*c + 3*b*c^2 - c^3)*sqrt(-a)) - (sqrt(b*x + a)*b^2 + 3*sqrt(b*x + a)*b*c)/
(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + 4*((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2
 + (b*x + a)*b*c - a*b*c))*a^2*b^3*c*abs(b) - (sqrt(b*c)*sqrt(b*x + a) - s
qrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a^2*b^2*c^2*abs(b) + (sqrt(b*c)*sqrt(b
*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*a*b*c*abs(b))/((a^2*b^4 -
 2*a^2*b^3*c + a^2*b^2*c^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*
x + a)*b*c - a*b*c))^2*a*b^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (
b*x + a)*b*c - a*b*c))^2*a*b*c + (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (
b*x + a)*b*c - a*b*c))^4)*(b^3 - 3*b^2*c + 3*b*c^2 - c^3)) + 3*(2*(a*b^4*c
 - a*b^2*c^3)*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*sqrt(-a)*abs(b
)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + 2*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2
- a*b*c^3)^2*(a*b^4 - a*b^2*c^2)*sqrt(-a*b*c)*abs(b) + (a^2*b^8 - 4*a^2*b^
7*c + 5*a^2*b^6*c^2 - 5*a^2*b^4*c^4 + 4*a^2*b^3*c^5 - a^2*b^2*c^6)*sqrt(-a
*b*c)*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b)*sgn(b^3 - 3*b
^2*c + 3*b*c^2 - c^3) + (a^2*b^9 - 4*a^2*b^8*c + 5*a^2*b^7*c^2 - 5*a^2*b^5
*c^4 + 4*a^2*b^4*c^5 - a^2*b^3*c^6)*sqrt(-a)*abs(-a*b^4 + 3*a*b^3*c - 3*a*
b^2*c^2 + a*b*c^3)*abs(b) + (a^3*b^12*c - 5*a^3*b^11*c^2 + 8*a^3*b^10*c...
```

3.441.9 Mupad [B] (verification not implemented)

Time = 21.49 (sec) , antiderivative size = 559, normalized size of antiderivative = 3.56

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$
$$= \frac{2\sqrt{a}\,b^2\left(\sqrt{a+c\,x}-\sqrt{a}\right)\left(\frac{8\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}-\frac{2\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}+\frac{3\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}+1\right)-2\sqrt{a}\,c^2\left(\sqrt{a\right.}}{}$$

input
```
int(x/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

output
```
(2*a^(1/2)*b^2*((a + c*x)^(1/2) - a^(1/2))*((8*((a + b*x)^(1/2) - a^(1/2))
)/((a + c*x)^(1/2) - a^(1/2)) - (2*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*
x)^(1/2) - a^(1/2))^2 + (3*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2
) - a^(1/2)))*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) + 1
) - 2*a^(1/2)*c^2*((a + c*x)^(1/2) - a^(1/2))*((2*((a + b*x)^(1/2) - a^(1/
2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 - ((a + b*x)^(1/2) - a^(1/2))^4/((a +
 c*x)^(1/2) - a^(1/2))^4 + (3*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(
1/2) - a^(1/2)))*((a + b*x)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2)
)^3) + 2*a^(1/2)*b*c*((a + c*x)^(1/2) - a^(1/2))*((8*((a + b*x)^(1/2) - a^
(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (14*((a + b*x)^(1/2) - a^(1/2))^2)/(
(a + c*x)^(1/2) - a^(1/2))^2 + (3*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*
x)^(1/2) - a^(1/2)))*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/
2)) - (3*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*((a
+ b*x)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3))/((b - c)^3*((a
+ b*x)^(1/2) - a^(1/2))*(b - (c*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^
(1/2) - a^(1/2))^2))
```

3.442 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$

3.442.1 Optimal result

Integrand size = 21, antiderivative size = 164

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = -\frac{2a\sqrt{a+bx}}{(b-c)^3x^2}-\frac{(2b+3c)\sqrt{a+bx}}{(b-c)^3x}+\frac{2a\sqrt{a+cx}}{(b-c)^3x^2}+\frac{(3b+2c)\sqrt{a+cx}}{(b-c)^3x}-\frac{3bc\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3}+\frac{3bc\operatorname{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3}$$

output
```
-3*b*c*arctanh((b*x+a)^(1/2)/a^(1/2))/(b-c)^3/a^(1/2)+3*b*c*arctanh((c*x+a
)^(1/2)/a^(1/2))/(b-c)^3/a^(1/2)-2*a*(b*x+a)^(1/2)/(b-c)^3/x^2-(2*b+3*c)*(
b*x+a)^(1/2)/(b-c)^3/x+2*a*(c*x+a)^(1/2)/(b-c)^3/x^2+(3*b+2*c)*(c*x+a)^(1/
2)/(b-c)^3/x
```

3.442.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.21 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.11

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{-\frac{3(b+3c)\left(a+bx+bx\sqrt{1+\frac{bx}{a}}\operatorname{arctanh}\left(\sqrt{1+\frac{bx}{a}}\right)\right)}{x\sqrt{a+bx}}+\frac{3(3b+c)\left(a+cx+cx\sqrt{1+\frac{cx}{a}}\operatorname{arctanh}\left(\sqrt{1+\frac{cx}{a}}\right)\right)}{x\sqrt{a+cx}}-\frac{8b^2(a+bx)^{3/2}\,\text{Hypergeometric}}{a^2}}{3(b-c)^3}$$

input
```
Integrate[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-3),x]
```

output
```
((-3*(b + 3*c)*(a + b*x + b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]]
))/(x*Sqrt[a + b*x]) + (3*(3*b + c)*(a + c*x + c*x*Sqrt[1 + (c*x)/a]*ArcTa
nh[Sqrt[1 + (c*x)/a]]))/(x*Sqrt[a + c*x]) - (8*b^2*(a + b*x)^(3/2)*Hyperge
ometric2F1[3/2, 3, 5/2, 1 + (b*x)/a])/a^2 + (8*c^2*(a + c*x)^(3/2)*Hyperge
ometric2F1[3/2, 3, 5/2, 1 + (c*x)/a])/a^2)/(3*(b - c)^3)
```

3.442.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.30, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {7241, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

$$\Big\downarrow \text{7241}$$

$$\frac{\int \left(\frac{4\sqrt{a+bx}a}{x^3}-\frac{4\sqrt{a+cx}a}{x^3}+\frac{(b+3c)\sqrt{a+bx}}{x^2}-\frac{(3b+c)\sqrt{a+cx}}{x^2}\right)dx}{(b-c)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{b(b+3c)\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{c(3b+c)\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{c^2\text{arctanh}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{(b+3c)\sqrt{a+bx}}{x}+\frac{(3b+c)\sqrt{a\cdots}}{x}}{(b-c)^3}$$

input
```
Int[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-3),x]
```

output
```
((-2*a*Sqrt[a + b*x])/x^2 - (b*Sqrt[a + b*x])/x - ((b + 3*c)*Sqrt[a + b*x]
)/x + (2*a*Sqrt[a + c*x])/x^2 + (c*Sqrt[a + c*x])/x + ((3*b + c)*Sqrt[a +
c*x])/x + (b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a] - (b*(b + 3*c)*ArcT
anh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a] - (c^2*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])
/Sqrt[a] + (c*(3*b + c)*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/Sqrt[a])/(b - c)^3
```

3.442.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7241
```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Simp[(b*e^2 - d*f^2)^m   Int[ExpandIntegran
d[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; Free
Q[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

3.442.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(144) = 288$.

Time = 0.05 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.83

method	result
default	$\frac{2b^{2}\left(-\frac{\sqrt{bx+a}}{2xb}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{(b-c)^{3}}+\frac{8a\,b^{2}\left(\frac{-\frac{(bx+a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{bx+a}}{8}}{x^{2}b^{2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)}{(b-c)^{3}}-\frac{8a\,c^{2}\left(\frac{-\frac{(cx+a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{cx+a}}{8}}{c^{2}x^{2}}+\frac{\operatorname{arctanh}\left(\sqrt{}\right.}{8a^{\frac{3}{2}}}\right.}{(b-c)^{3}}$

input
```
int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
2/(b-c)^3*b^2*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a
^(1/2))+8/(b-c)^3*a*b^2*((-1/8/a*(b*x+a)^(3/2)-1/8*(b*x+a)^(1/2))/x^2/b^2+
1/8/a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-8/(b-c)^3*a*c^2*((-1/8/a*(c*x+
a)^(3/2)-1/8*(c*x+a)^(1/2))/c^2/x^2+1/8/a^(3/2)*arctanh((c*x+a)^(1/2)/a^(1
/2)))+6/(b-c)^3*c*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a^(1
/2))/a^(1/2))-6/(b-c)^3*b*c*(-1/2*(c*x+a)^(1/2)/c/x-1/2/a^(1/2)*arctanh((c
*x+a)^(1/2)/a^(1/2)))-2/(b-c)^3*c^2*(-1/2*(c*x+a)^(1/2)/c/x-1/2/a^(1/2)*ar
ctanh((c*x+a)^(1/2)/a^(1/2)))
```

3.442.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.81

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$
$$= \left[-\frac{3\sqrt{a}bcx^2\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+3\sqrt{a}bcx^2\log\left(\frac{cx-2\sqrt{cx+a}\sqrt{a}+2a}{x}\right)+2\left(2a^2+(2ab+3ac)x\right)\sqrt{bx+a}}{2\left(ab^3-3ab^2c+3abc^2-ac^3\right)x^2}\right.$$

input
```
integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

output
```
[-1/2*(3*sqrt(a)*b*c*x^2*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 3*
sqrt(a)*b*c*x^2*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) + 2*(2*a^2 +
(2*a*b + 3*a*c)*x)*sqrt(b*x + a) - 2*(2*a^2 + (3*a*b + 2*a*c)*x)*sqrt(c*x
+ a))/((a*b^3 - 3*a*b^2*c + 3*a*b*c^2 - a*c^3)*x^2), (3*sqrt(-a)*b*c*x^2*a
rctan(sqrt(b*x + a)*sqrt(-a)/a) - 3*sqrt(-a)*b*c*x^2*arctan(sqrt(c*x + a)*
sqrt(-a)/a) - (2*a^2 + (2*a*b + 3*a*c)*x)*sqrt(b*x + a) + (2*a^2 + (3*a*b
+ 2*a*c)*x)*sqrt(c*x + a))/((a*b^3 - 3*a*b^2*c + 3*a*b*c^2 - a*c^3)*x^2)]
```

3.442.6 Sympy [F]

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

input
```
integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

output
```
Integral((sqrt(a + b*x) + sqrt(a + c*x))**(-3), x)
```

3.442.7 Maxima [F]

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx=\int \frac{1}{\left(\sqrt{bx+a}+\sqrt{cx+a}\right)^3}\,dx$$

input
```
integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate((sqrt(b*x + a) + sqrt(c*x + a))^(-3), x)
```

3.442.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2766 vs. $2(144) = 288$.

Time = 17.84 (sec) , antiderivative size = 2766, normalized size of antiderivative = 16.87

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

output
```
3*b*c*arctan(sqrt(b*x + a)/sqrt(-a))/((b^3 - 3*b^2*c + 3*b*c^2 - c^3)*sqrt
(-a)) - 2*(3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c
))*a^3*b^7*c*abs(b) - 7*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*
b*c - a*b*c))*a^3*b^6*c^2*abs(b) + 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2
 + (b*x + a)*b*c - a*b*c))*a^3*b^5*c^3*abs(b) + 3*(sqrt(b*c)*sqrt(b*x + a)
 - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a^3*b^4*c^4*abs(b) - 2*(sqrt(b*c)*
sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a^3*b^3*c^5*abs(b) -
3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*a^2*b^
5*c*abs(b) - 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b
*c))^3*a^2*b^3*c^3*abs(b) + 6*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x
 + a)*b*c - a*b*c))^3*a^2*b^2*c^4*abs(b) - 3*(sqrt(b*c)*sqrt(b*x + a) - sq
rt(a*b^2 + (b*x + a)*b*c - a*b*c))^5*a*b^3*c*abs(b) - 3*(sqrt(b*c)*sqrt(b*
x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^5*a*b^2*c^2*abs(b) - 6*(sqrt
(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^5*a*b*c^3*abs(b
) + 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^7*b*
c*abs(b) + 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c
))^7*c^2*abs(b))/((a^2*b^4 - 2*a^2*b^3*c + a^2*b^2*c^2 - 2*(sqrt(b*c)*sqrt
(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*b^2 - 2*(sqrt(b*c)*sq
rt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*b*c + (sqrt(b*c)*sq
rt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^4)^2*(b^3 - 3*b^2*c ...
```

3.442.9 Mupad [B] (verification not implemented)

Time = 19.71 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.75

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$
$$= \frac{c^2\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{4\sqrt{a}\,(b-c)^3\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2} - \frac{\left(\frac{\sqrt{a}\,b^2}{4\,(a\,b^3-3\,a\,b^2\,c+3\,a\,b\,c^2-a\,c^3)}-\frac{\sqrt{a}\,(b^2+c\,b)\,(\sqrt{a+b\,x}-\sqrt{a})}{(\sqrt{a+c\,x}-\sqrt{a})\,(a\,b^3-3\,a\,b^2\,c+3\,a\,b\,c^2-a\,c^3)}\right)\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2} + \frac{3\,b\,c\,\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)}{\sqrt{a}\,\left(b^3-3\,b^2\,c+3\,b\,c^2-c^3\right)} - \frac{c\,(b+c)\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a}\,(b-c)^3\,\left(\sqrt{a+c\,x}-\sqrt{a}\right)}$$

input
```
int(1/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

output
```
(c^2*((a + b*x)^(1/2) - a^(1/2))^2)/(4*a^(1/2)*(b - c)^3*((a + c*x)^(1/2)
- a^(1/2))^2) - (((a^(1/2)*b^2)/(4*(a*b^3 - a*c^3 + 3*a*b*c^2 - 3*a*b^2*c)
) - (a^(1/2)*(b*c + b^2)*((a + b*x)^(1/2) - a^(1/2)))/(((a + c*x)^(1/2) -
a^(1/2))*(a*b^3 - a*c^3 + 3*a*b*c^2 - 3*a*b^2*c)))*((a + c*x)^(1/2) - a^(1
/2))^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (3*b*c*log(((a + b*x)^(1/2) - a^(1
/2))/((a + c*x)^(1/2) - a^(1/2))))/(a^(1/2)*(3*b*c^2 - 3*b^2*c + b^3 - c^3
)) - (c*(b + c)*((a + b*x)^(1/2) - a^(1/2)))/(a^(1/2)*(b - c)^3*((a + c*x)
^(1/2) - a^(1/2)))
```

3.443 $\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$

3.443.1 Optimal result

Integrand size = 27, antiderivative size = 31

$$\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx=x-\frac{x^2}{2}+\frac{1}{2}x\sqrt{1-x^2}+\frac{\arcsin(x)}{2}$$

output
```
x-1/2*x^2+1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

3.443.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.45

$$\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx=x-\frac{x^2}{2}+\frac{1}{2}x\sqrt{1-x^2}-\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[Sqrt[1 - x]*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

output
```
x - x^2/2 + (x*Sqrt[1 - x^2])/2 - ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

3.443.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {7239, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{x+1}\right) \, dx$$

$$\Big\downarrow \text{7239}$$

$$\int \left(\sqrt{1-x^2}-x+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arcsin(x)}{2}-\frac{x^2}{2}+\frac{1}{2}\sqrt{1-x^2}x+x$$

input
```
Int[Sqrt[1 - x]*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

output
```
x - x^2/2 + (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

3.443.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

3.443.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(23) = 46.

Time = 0.94 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.03

method	result	size
default	$x-\frac{x^{2}}{2}+\frac{\sqrt{1-x}\,(x+1)^{\frac{3}{2}}}{2}-\frac{\sqrt{1-x}\,\sqrt{x+1}}{2}+\frac{\sqrt{(1-x)(x+1)}\,\arcsin(x)}{2\sqrt{x+1}\,\sqrt{1-x}}$	63

input
```
int((1-x)^(1/2)*((1-x)^(1/2)+(x+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x-1/2*x^2+1/2*(1-x)^(1/2)*(x+1)^(3/2)-1/2*(1-x)^(1/2)*(x+1)^(1/2)+1/2*((1-
x)*(x+1))^(1/2)/(x+1)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

3.443.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = -\frac{1}{2}\,x^{2}+\frac{1}{2}\,\sqrt{x+1}x\sqrt{-x+1}+x - \arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
```

output
```
-1/2*x^2 + 1/2*sqrt(x + 1)*x*sqrt(-x + 1) + x - arctan((sqrt(x + 1)*sqrt(-
x + 1) - 1)/x)
```

3.443.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(22) = 44.

Time = 1.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.55

$$\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = -\frac{(1-x)^{2}}{2}-2\sqrt{x+1}\left(\frac{(1-x)^{\frac{3}{2}}}{4}-\frac{\sqrt{1-x}}{4}\right) - \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{1-x}}{2}\right)$$

input
```
integrate((1-x)**(1/2)*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

output
```
-(1 - x)**2/2 - 2*sqrt(x + 1)*((1 - x)**(3/2)/4 - sqrt(1 - x)/4) - asin(sq
rt(2)*sqrt(1 - x)/2)
```

3.443.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx=-\frac{1}{2}\,x^2+\frac{1}{2}\,\sqrt{-x^2+1}x+x+\frac{1}{2}\,\arcsin\left(x\right)$$

input
```
integrate((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
```

output
```
-1/2*x^2 + 1/2*sqrt(-x^2 + 1)*x + x + 1/2*arcsin(x)
```

3.443.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(23) = 46.

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx=-\frac{1}{2}\,(x-1)^2+\frac{1}{2}\,(x+2)\sqrt{x+1}\sqrt{-x+1}-\sqrt{x+1}\sqrt{-x+1}-\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{-x+1}\right)$$

input
```
integrate((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
```

output
```
-1/2*(x - 1)^2 + 1/2*(x + 2)*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x + 1)*sqrt(-
x + 1) - arcsin(1/2*sqrt(2)*sqrt(-x + 1))
```

3.443.9 Mupad [B] (verification not implemented)

Time = 21.80 (sec) , antiderivative size = 209, normalized size of antiderivative = 6.74

$$\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$

$$= x-2\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-\frac{\frac{2\,\left(\sqrt{1-x}-1\right)}{\sqrt{x+1}-1}-\frac{14\,{\left(\sqrt{1-x}-1\right)}^3}{{\left(\sqrt{x+1}-1\right)}^3}+\frac{14\,{\left(\sqrt{1-x}-1\right)}^5}{{\left(\sqrt{x+1}-1\right)}^5}-\frac{2\,{\left(\sqrt{1-x}-1\right)}^7}{{\left(\sqrt{x+1}-1\right)}^7}}{\frac{4\,{\left(\sqrt{1-x}-1\right)}^2}{{\left(\sqrt{x+1}-1\right)}^2}+\frac{6\,{\left(\sqrt{1-x}-1\right)}^4}{{\left(\sqrt{x+1}-1\right)}^4}+\frac{4\,{\left(\sqrt{1-x}-1\right)}^6}{{\left(\sqrt{x+1}-1\right)}^6}+\frac{{\left(\sqrt{1-x}-1\right)}^8}{{\left(\sqrt{x+1}-1\right)}^8}+1}-\frac{x^2}{2}$$

input
```
int(((x + 1)^(1/2) + (1 - x)^(1/2))*(1 - x)^(1/2),x)
```

output
```
x - 2*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - ((2*((1 - x)^(1/2) -
 1))/((x + 1)^(1/2) - 1) - (14*((1 - x)^(1/2) - 1)^3)/((x + 1)^(1/2) - 1)^
3 + (14*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (2*((1 - x)^(1/2) -
 1)^7)/((x + 1)^(1/2) - 1)^7)/((4*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) -
1)^2 + (6*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (4*((1 - x)^(1/2)
 - 1)^6)/((x + 1)^(1/2) - 1)^6 + ((1 - x)^(1/2) - 1)^8/((x + 1)^(1/2) - 1)
^8 + 1) - x^2/2
```

3.444 $\int x^3(-\sqrt{1-x}-\sqrt{1+x})\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$

3.444.1 Optimal result

Integrand size = 42, antiderivative size = 38

$$\int x^3\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx=-\frac{x^4}{2}+\frac{2}{3}\left(1-x^2\right)^{3/2}-\frac{2}{5}\left(1-x^2\right)^{5/2}$$

output
```
-1/2*x^4+2/3*(-x^2+1)^(3/2)-2/5*(-x^2+1)^(5/2)
```

3.444.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int x^3\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$=-\frac{1}{30}\left(-1+x^2\right)\left(15+8\sqrt{1-x^2}+3x^2\left(5+4\sqrt{1-x^2}\right)\right)$$

input
```
Integrate[x^3*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

output
```
-1/30*((-1 + x^2)*(15 + 8*Sqrt[1 - x^2] + 3*x^2*(5 + 4*Sqrt[1 - x^2])))
```

3.444.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {7239, 25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(-\sqrt{1-x}-\sqrt{x+1}\right)\left(\sqrt{1-x}+\sqrt{x+1}\right)dx$$

$$\Big\downarrow \text{ 7239}$$

$$\int -x^3\left(\sqrt{1-x}+\sqrt{x+1}\right)^2dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int x^3\left(\sqrt{1-x}+\sqrt{x+1}\right)^2dx$$

$$\Big\downarrow \text{ 7293}$$

$$-\int \left(2\sqrt{1-x^2}x^3+2x^3\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{x^4}{2}-\frac{2}{5}\left(1-x^2\right)^{5/2}+\frac{2}{3}\left(1-x^2\right)^{3/2}$$

input
```
Int[x^3*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

output
```
-1/2*x^4 + (2*(1 - x^2)^(3/2))/3 - (2*(1 - x^2)^(5/2))/5
```

3.444.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.444.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{x^{4}}{2}-\frac{2\sqrt{1-x}\,\sqrt{x+1}\,\left(x^{2}-1\right)\left(3x^{2}+2\right)}{15}$	33

input
```
int(x^3*(-(1-x)^(1/2)-(x+1)^(1/2))*((1-x)^(1/2)+(x+1)^(1/2)),x,method=_RET
URNVERBOSE)
```

output
```
-1/2*x^4-2/15*(1-x)^(1/2)*(x+1)^(1/2)*(x^2-1)*(3*x^2+2)
```

3.444.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int x^3\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = -\frac{1}{2}\,x^{4}-\frac{2}{15}\left(3\,x^{4}-x^{2}-2\right)\sqrt{x+1}\sqrt{-x+1}$$

input
```
integrate(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algo
rithm="fricas")
```

output
```
-1/2*x^4 - 2/15*(3*x^4 - x^2 - 2)*sqrt(x + 1)*sqrt(-x + 1)
```

3.444.6 Sympy [F]

$$\int x^3\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)dx=-\int 2x^3\,dx-\int 2x^3\sqrt{1-x}\sqrt{x+1}\,dx$$

input
```
integrate(x**3*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

output
```
-Integral(2*x**3, x) - Integral(2*x**3*sqrt(1 - x)*sqrt(x + 1), x)
```

3.444.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int x^3\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)dx$$
$$=-\frac{1}{2}x^4+\frac{2}{5}\left(-x^2+1\right)^{\frac{3}{2}}x^2+\frac{4}{15}\left(-x^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algo
rithm="maxima")
```

output
```
-1/2*x^4 + 2/5*(-x^2 + 1)^(3/2)*x^2 + 4/15*(-x^2 + 1)^(3/2)
```

3.444.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(28) = 56.

Time = 0.32 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.03

$$\int x^3\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)dx$$
$$=-\frac{1}{2}x^4-\frac{1}{60}\left((2(3(4x-17)(x+1)+133)(x+1)-295)(x+1)+195\right)\sqrt{x+1}\sqrt{-x+1}$$
$$-\frac{1}{12}\left((2(3x-10)(x+1)+43)(x+1)-39\right)\sqrt{x+1}\sqrt{-x+1}$$

input
```
integrate(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algo
rithm="giac")
```

output
```
-1/2*x^4 - 1/60*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) +
195)*sqrt(x + 1)*sqrt(-x + 1) - 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1)
- 39)*sqrt(x + 1)*sqrt(-x + 1)
```

3.444.9 Mupad [B] (verification not implemented)

Time = 17.65 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int x^3\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$=\sqrt{1-x}\left(\frac{4\sqrt{x+1}}{15}+\frac{2x^2\sqrt{x+1}}{15}-\frac{2x^4\sqrt{x+1}}{5}\right)-\frac{x^4}{2}$$

input
```
int(-x^3*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

output
```
(1 - x)^(1/2)*((4*(x + 1)^(1/2))/15 + (2*x^2*(x + 1)^(1/2))/15 - (2*x^4*(x
 + 1)^(1/2))/5) - x^4/2
```

3.445 $\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$

3.445.1 Optimal result

Integrand size = 42, antiderivative size = 48

$$\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = -\frac{2x^3}{3}+\frac{1}{4}x\sqrt{1-x^2}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{\arcsin(x)}{4}$$

output
```
-2/3*x^3-1/4*arcsin(x)+1/4*x*(-x^2+1)^(1/2)-1/2*x^3*(-x^2+1)^(1/2)
```

3.445.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.42

$$\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = \frac{1}{12}\left(-8+3x\sqrt{1-x^2}-x^3\left(8+6\sqrt{1-x^2}\right)-12\arctan\left(\frac{-\sqrt{2}+\sqrt{1+x}}{\sqrt{1-x}}\right)\right)$$

input
```
Integrate[x^2*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

output
```
(-8 + 3*x*Sqrt[1 - x^2] - x^3*(8 + 6*Sqrt[1 - x^2]) - 12*ArcTan[(-Sqrt[2]
+ Sqrt[1 + x])/Sqrt[1 - x]])/12
```

3.445.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {7239, 25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(-\sqrt{1-x}-\sqrt{x+1}\right)\left(\sqrt{1-x}+\sqrt{x+1}\right)dx$$

$$\Big\downarrow \text{7239}$$

$$\int -x^2\left(\sqrt{1-x}+\sqrt{x+1}\right)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int x^2\left(\sqrt{1-x}+\sqrt{x+1}\right)^2dx$$

$$\Big\downarrow \text{7293}$$

$$-\int \left(2\sqrt{1-x^2}x^2+2x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arcsin(x)}{4}-\frac{2x^3}{3}+\frac{1}{4}\sqrt{1-x^2}x-\frac{1}{2}\sqrt{1-x^2}x^3$$

input
```
Int[x^2*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

output
```
(-2*x^3)/3 + (x*Sqrt[1 - x^2])/4 - (x^3*Sqrt[1 - x^2])/2 - ArcSin[x]/4
```

3.445.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.445.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

method	result	size
default	$-\frac{2x^{3}}{3}-\frac{\sqrt{1-x}\,\sqrt{x+1}\,\left(2x^{3}\sqrt{-x^{2}+1}-x\sqrt{-x^{2}+1}+\arcsin\left(x\right)\right)}{4\sqrt{-x^{2}+1}}$	59

input
```
int(x^2*(-(1-x)^(1/2)-(x+1)^(1/2))*((1-x)^(1/2)+(x+1)^(1/2)),x,method=_RET
URNVERBOSE)
```

output
```
-2/3*x^3-1/4*(1-x)^(1/2)*(x+1)^(1/2)*(2*x^3*(-x^2+1)^(1/2)-x*(-x^2+1)^(1/2
)+arcsin(x))/(-x^2+1)^(1/2)
```

3.445.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$=-\frac{2}{3}\,x^{3}-\frac{1}{4}\,\left(2\,x^{3}-x\right)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{2}\,\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algo
rithm="fricas")
```

output
```
-2/3*x^3 - 1/4*(2*x^3 - x)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*arctan((sqrt(x +
 1)*sqrt(-x + 1) - 1)/x)
```

3.445.6 Sympy [F]

$$\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)dx = -\int 2x^2\,dx - \int 2x^2\sqrt{1-x}\sqrt{x+1}\,dx$$

input
```
integrate(x**2*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

output
```
-Integral(2*x**2, x) - Integral(2*x**2*sqrt(1 - x)*sqrt(x + 1), x)
```

3.445.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.71

$$\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)dx = -\frac{2}{3}x^3+\frac{1}{2}\left(-x^2+1\right)^{\frac{3}{2}}x-\frac{1}{4}\sqrt{-x^2+1}x-\frac{1}{4}\arcsin\left(x\right)$$

input
```
integrate(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algo
rithm="maxima")
```

output
```
-2/3*x^3 + 1/2*(-x^2 + 1)^(3/2)*x - 1/4*sqrt(-x^2 + 1)*x - 1/4*arcsin(x)
```

3.445.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(36) = 72.

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.58

$$\begin{aligned}\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)dx &= -\frac{2}{3}x^3-\frac{1}{12}\left(\left(2\left(3x-10\right)(x+1)+43\right)(x+1)-39\right)\sqrt{x+1}\sqrt{-x+1} \\ &\quad -\frac{1}{3}\left(\left(2x-5\right)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{2}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)\end{aligned}$$

input
```
integrate(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algo
rithm="giac")
```

output
```
-2/3*x^3 - 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqr
t(-x + 1) - 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*arc
sin(1/2*sqrt(2)*sqrt(x + 1))
```

3.445.9 Mupad [B] (verification not implemented)

Time = 26.49 (sec) , antiderivative size = 381, normalized size of antiderivative = 7.94

$$\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = \operatorname{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right) - \frac{\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}-\frac{35\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{273\left(\sqrt{1-x}-1\right)^5}{\left(\sqrt{x+1}-1\right)^5}-\frac{715\left(\sqrt{1-x}-1\right)^7}{\left(\sqrt{x+1}-1\right)^7}+\frac{715\left(\sqrt{1-x}-1\right)^9}{\left(\sqrt{x+1}-1\right)^9}-\frac{273\left(\sqrt{1-x}-1\right)^{11}}{\left(\sqrt{x+1}-1\right)^{11}}+\frac{35\left(\sqrt{1-x}-1\right)^{13}}{\left(\sqrt{x+1}-1\right)^{13}}-}{\frac{8\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{28\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{56\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{70\left(\sqrt{1-x}-1\right)^8}{\left(\sqrt{x+1}-1\right)^8}+\frac{56\left(\sqrt{1-x}-1\right)^{10}}{\left(\sqrt{x+1}-1\right)^{10}}+\frac{28\left(\sqrt{1-x}-1\right)^{12}}{\left(\sqrt{x+1}-1\right)^{12}}+\frac{8\left(\sqrt{1-x}-1\right)^{14}}{\left(\sqrt{x+1}-1\right)^{14}}+} - \frac{2x^3}{3}$$

input
```
int(-x^2*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

output
```
atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - (((1 - x)^(1/2) - 1)/((x +
 1)^(1/2) - 1) - (35*((1 - x)^(1/2) - 1)^3)/((x + 1)^(1/2) - 1)^3 + (273*(
(1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (715*((1 - x)^(1/2) - 1)^7)/
((x + 1)^(1/2) - 1)^7 + (715*((1 - x)^(1/2) - 1)^9)/((x + 1)^(1/2) - 1)^9
- (273*((1 - x)^(1/2) - 1)^11)/((x + 1)^(1/2) - 1)^11 + (35*((1 - x)^(1/2)
 - 1)^13)/((x + 1)^(1/2) - 1)^13 - ((1 - x)^(1/2) - 1)^15/((x + 1)^(1/2) -
 1)^15)/((8*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)^2 + (28*((1 - x)^(1
/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (56*((1 - x)^(1/2) - 1)^6)/((x + 1)^(1
/2) - 1)^6 + (70*((1 - x)^(1/2) - 1)^8)/((x + 1)^(1/2) - 1)^8 + (56*((1 -
x)^(1/2) - 1)^10)/((x + 1)^(1/2) - 1)^10 + (28*((1 - x)^(1/2) - 1)^12)/((x
 + 1)^(1/2) - 1)^12 + (8*((1 - x)^(1/2) - 1)^14)/((x + 1)^(1/2) - 1)^14 +
((1 - x)^(1/2) - 1)^16/((x + 1)^(1/2) - 1)^16 + 1) - (2*x^3)/3
```

3.446 $\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$

3.446.1 Optimal result

Integrand size = 40, antiderivative size = 21

$$\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx=-x^2+\frac{2}{3}(1-x^2)^{3/2}$$

output
```
-x^2+2/3*(-x^2+1)^(3/2)
```

3.446.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx=-\frac{1}{3}(-1+x)(1+x)\left(3+2\sqrt{1-x^2}\right)$$

input
```
Integrate[x*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

output
```
-1/3*((-1 + x)*(1 + x)*(3 + 2*Sqrt[1 - x^2]))
```

3.446.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {7239, 25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(-\sqrt{1-x}-\sqrt{x+1}\right)\left(\sqrt{1-x}+\sqrt{x+1}\right)dx$$

$$\Big\downarrow \text{7239}$$

$$\int -x\left(\sqrt{1-x}+\sqrt{x+1}\right)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int x\left(\sqrt{1-x}+\sqrt{x+1}\right)^2dx$$

$$\Big\downarrow \text{7293}$$

$$-\int \left(2\sqrt{1-x^2}x+2x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}\left(1-x^2\right)^{3/2}-x^2$$

input
```
Int[x*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

output
```
-x^2 + (2*(1 - x^2)^(3/2))/3
```

3.446.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.446.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

method	result	size
default	$-x^{2}-\frac{2 \sqrt{1-x}\, \sqrt{x+1}\, \left(x^{2}-1\right)}{3}$	26

input
```
int(x*(-(1-x)^(1/2)-(x+1)^(1/2))*((1-x)^(1/2)+(x+1)^(1/2)),x,method=_RETUR
NVERBOSE)
```

output
```
-x^2-2/3*(1-x)^(1/2)*(x+1)^(1/2)*(x^2-1)
```

3.446.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx=-x^2-\frac{2}{3}\left(x^2-1\right)\sqrt{x+1}\sqrt{-x+1}$$

input
```
integrate(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algori
thm="fricas")
```

output
```
-x^2 - 2/3*(x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)
```

3.446.6 Sympy [F]

$$\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx=-\int 2x\,dx-\int 2x\sqrt{1-x}\sqrt{x+1}\,dx$$

input
```
integrate(x*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

output
```
-Integral(2*x, x) - Integral(2*x*sqrt(1 - x)*sqrt(x + 1), x)
```

3.446.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx=-x^2+\frac{2}{3}\left(-x^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algori
thm="maxima")
```

output
```
-x^2 + 2/3*(-x^2 + 1)^(3/2)
```

3.446.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(17) = 34.

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.57

$$\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$=-(x+1)^2-\frac{1}{3}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1}-\sqrt{x+1}(x-2)\sqrt{-x+1}+2x+2$$

input
```
integrate(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algori
thm="giac")
```

output
```
-(x + 1)^2 - 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x
 + 1)*(x - 2)*sqrt(-x + 1) + 2*x + 2
```

3.446.9 Mupad [B] (verification not implemented)

Time = 17.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx=-x^2-\frac{2\left(x^2-1\right)\sqrt{1-x}\sqrt{x+1}}{3}$$

input
```
int(-x*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

output
```
- x^2 - (2*(x^2 - 1)*(1 - x)^(1/2)*(x + 1)^(1/2))/3
```

3.447 $\int \left(-\sqrt{1-x}-\sqrt{1+x}\right) \left(\sqrt{1-x}+\sqrt{1+x}\right) dx$

3.447.1 Optimal result

Integrand size = 39, antiderivative size = 22

$$\int \left(-\sqrt{1-x}-\sqrt{1+x}\right) \left(\sqrt{1-x}+\sqrt{1+x}\right) dx = -2x - x\sqrt{1-x^2} - \arcsin(x)$$

output
```
-2*x-arcsin(x)-x*(-x^2+1)^(1/2)
```

3.447.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 46 vs. 2(22) = 44.

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \left(-\sqrt{1-x}-\sqrt{1+x}\right) \left(\sqrt{1-x}+\sqrt{1+x}\right) dx$$
$$= -2 - x\left(2+\sqrt{1-x^2}\right) - 4\arctan\left(\frac{-\sqrt{2}+\sqrt{1+x}}{\sqrt{1-x}}\right)$$

input
```
Integrate[(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

output
```
-2 - x*(2 + Sqrt[1 - x^2]) - 4*ArcTan[(-Sqrt[2] + Sqrt[1 + x])/Sqrt[1 - x]
]
```

3.447.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {7239, 25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-\sqrt{1-x}-\sqrt{x+1}\right)\left(\sqrt{1-x}+\sqrt{x+1}\right)dx$$

$$\Big\downarrow \text{7239}$$

$$\int -\left(\sqrt{1-x}+\sqrt{x+1}\right)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int \left(\sqrt{1-x}+\sqrt{x+1}\right)^2dx$$

$$\Big\downarrow \text{7293}$$

$$-\int \left(2\sqrt{1-x^2}+2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\arcsin(x)-\sqrt{1-x^2}x-2x$$

input
```
Int[(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

output
```
-2*x - x*Sqrt[1 - x^2] - ArcSin[x]
```

3.447.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.447.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(20) = 40.

Time = 0.94 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.68

method	result	size
default	$-2x-\sqrt{1-x}\,(x+1)^{\frac{3}{2}}+\sqrt{1-x}\,\sqrt{x+1}-\frac{\sqrt{(1-x)(x+1)}\,\arcsin(x)}{\sqrt{x+1}\,\sqrt{1-x}}$	59

input
```
int((-(1-x)^(1/2)-(x+1)^(1/2))*((1-x)^(1/2)+(x+1)^(1/2)),x,method=_RETURNV
ERBOSE)
```

output
```
-2*x-(1-x)^(1/2)*(x+1)^(3/2)+(1-x)^(1/2)*(x+1)^(1/2)-((1-x)*(x+1))^(1/2)/(
x+1)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

3.447.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(20) = 40.

Time = 0.33 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86

$$\int \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = -\sqrt{x+1}x\sqrt{-x+1}-2\,x+2\,\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorith
m="fricas")
```

output
```
-sqrt(x + 1)*x*sqrt(-x + 1) - 2*x + 2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1
)/x)
```

3.447.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(17) = 34$.

Time = 1.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$= -2x - 4\sqrt{1-x}\left(\frac{(x+1)^{\frac{3}{2}}}{4} - \frac{\sqrt{x+1}}{4}\right) - 2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - 2$$

input
```
integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

output
```
-2*x - 4*sqrt(1 - x)*((x + 1)**(3/2)/4 - sqrt(x + 1)/4) - 2*asin(sqrt(2)*s
qrt(x + 1)/2) - 2
```

3.447.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx = -\sqrt{-x^2+1}x - 2\,x - \arcsin(x)$$

input
```
integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorith
m="maxima")
```

output
```
-sqrt(-x^2 + 1)*x - 2*x - arcsin(x)
```

3.447.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(20) = 40$.

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$= -\sqrt{x+1}(x-2)\sqrt{-x+1} - 2\,x - 2\,\sqrt{x+1}\sqrt{-x+1} - 2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\sqrt{x+1}\right) - 2$$

input
```
integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorith
m="giac")
```

output
```
-sqrt(x + 1)*(x - 2)*sqrt(-x + 1) - 2*x - 2*sqrt(x + 1)*sqrt(-x + 1) - 2*a
rcsin(1/2*sqrt(2)*sqrt(x + 1)) - 2
```

3.447.9 Mupad [B] (verification not implemented)

Time = 17.97 (sec) , antiderivative size = 205, normalized size of antiderivative = 9.32

$$\int \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx$$
$$=4\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-2\,x+\frac{\frac{4\left(\sqrt{1-x}-1\right)}{\sqrt{x+1}-1}-\frac{28\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{28\left(\sqrt{1-x}-1\right)^5}{\left(\sqrt{x+1}-1\right)^5}-\frac{4\left(\sqrt{1-x}-1\right)^7}{\left(\sqrt{x+1}-1\right)^7}}{\frac{4\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{6\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{4\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{\left(\sqrt{1-x}-1\right)^8}{\left(\sqrt{x+1}-1\right)^8}+1}$$

input
```
int(-((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

output
```
4*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - 2*x + ((4*((1 - x)^(1/2)
 - 1))/((x + 1)^(1/2) - 1) - (28*((1 - x)^(1/2) - 1)^3)/((x + 1)^(1/2) - 1
)^3 + (28*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (4*((1 - x)^(1/2)
 - 1)^7)/((x + 1)^(1/2) - 1)^7)/((4*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2)
- 1)^2 + (6*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (4*((1 - x)^(1/
2) - 1)^6)/((x + 1)^(1/2) - 1)^6 + ((1 - x)^(1/2) - 1)^8/((x + 1)^(1/2) -
1)^8 + 1)
```

3.448 $\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx$

3.448.1 Optimal result

Integrand size = 42, antiderivative size = 32

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx = -2\sqrt{1-x^2}+2\text{arctanh}\left(\sqrt{1-x^2}\right)-2\log(x)$$

output
```
2*arctanh((-x^2+1)^(1/2))-2*ln(x)-2*(-x^2+1)^(1/2)
```

3.448.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 72 vs. 2(32) = 64.

Time = 0.01 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.25

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx = -2\left(\sqrt{1-x^2}+2\log\left(\sqrt{2}-\sqrt{1+x}\right)+2\log\left(\sqrt{1-x}-\sqrt{1+x}\right)-2\log\left(-2+\sqrt{2}\sqrt{1+x}\right)\right)$$

input
```
Integrate[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x,x]
```

output
```
-2*(Sqrt[1 - x^2] + 2*Log[Sqrt[2] - Sqrt[1 + x]] + 2*Log[Sqrt[1 - x] - Sqr
t[1 + x]] - 2*Log[-2 + Sqrt[2]*Sqrt[1 + x]])
```

3.448.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {7239, 25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{x+1}\right)\left(\sqrt{1-x}+\sqrt{x+1}\right)}{x} \, dx$$

$$\Big\downarrow \ 7239$$

$$\int -\frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x}dx$$

$$\Big\downarrow \ 7293$$

$$-\int \left(\frac{2\sqrt{1-x^2}}{x}+\frac{2}{x}\right)dx$$

$$\Big\downarrow \ 2009$$

$$2\text{arctanh}\left(\sqrt{1-x^2}\right)-2\sqrt{1-x^2}-2\log(x)$$

input
```
Int[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x,x]
```

output
```
-2*Sqrt[1 - x^2] + 2*ArcTanh[Sqrt[1 - x^2]] - 2*Log[x]
```

3.448.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.448.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.59

method	result	size
default	$-2\ln\left(x\right)-\frac{2\sqrt{1-x}\,\sqrt{x+1}\,\left(\sqrt{-x^{2}+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)\right)}{\sqrt{-x^{2}+1}}$	51

input
```
int((-(1-x)^(1/2)-(x+1)^(1/2))*((1-x)^(1/2)+(x+1)^(1/2))/x,x,method=_RETUR
NVERBOSE)
```

output
```
-2*ln(x)-2*(1-x)^(1/2)*(x+1)^(1/2)/(-x^2+1)^(1/2)*((-x^2+1)^(1/2)-arctanh(
1/(-x^2+1)^(1/2)))
```

3.448.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx = -2\sqrt{x+1}\sqrt{-x+1}-2\log\left(x\right)-2\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

input
```
integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x, algori
thm="fricas")
```

output
```
-2*sqrt(x + 1)*sqrt(-x + 1) - 2*log(x) - 2*log((sqrt(x + 1)*sqrt(-x + 1) -
 1)/x)
```

3.448.6 Sympy [F]

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx = -\int \frac{2}{x}\,dx - \int \frac{2\sqrt{1-x}\sqrt{x+1}}{x}\,dx$$

input
```
integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x,x)
```

output
```
-Integral(2/x, x) - Integral(2*sqrt(1 - x)*sqrt(x + 1)/x, x)
```

3.448.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx$$
$$= -2\sqrt{-x^2+1} - 2\log(x) + 2\log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

input
```
integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x, algori
thm="maxima")
```

output
```
-2*sqrt(-x^2 + 1) - 2*log(x) + 2*log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

3.448.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(28) = 56$.

Time = 0.34 (sec) , antiderivative size = 130, normalized size of antiderivative = 4.06

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx$$
$$= -2\sqrt{x+1}\sqrt{-x+1} - 2\log\left(\sqrt{x+1}+1\right) - 2\log\left(\left|\sqrt{x+1}-1\right|\right)$$
$$+ 2\log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} + \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}} + 2\right|\right)$$
$$- 2\log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} + \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}} - 2\right|\right)$$

input
```
integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x, algori
thm="giac")
```

output
```
-2*sqrt(x + 1)*sqrt(-x + 1) - 2*log(sqrt(x + 1) + 1) - 2*log(abs(sqrt(x +
1) - 1)) + 2*log(abs(-(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + sqrt(x + 1)/(
sqrt(2) - sqrt(-x + 1)) + 2)) - 2*log(abs(-(sqrt(2) - sqrt(-x + 1))/sqrt(x
 + 1) + sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) - 2))
```

3.448.9 Mupad [B] (verification not implemented)

Time = 18.14 (sec) , antiderivative size = 122, normalized size of antiderivative = 3.81

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx = 2\,\ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-2\,\ln\left(\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-1\right) - 2\,\ln\left(x\right)-\frac{16\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2\left(\frac{2\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+1\right)}$$

input
```
int(-((x + 1)^(1/2) + (1 - x)^(1/2))^2/x,x)
```

output
```
2*log(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - 2*log(((1 - x)^(1/2) - 1)
^2/((x + 1)^(1/2) - 1)^2 - 1) - 2*log(x) - (16*((1 - x)^(1/2) - 1)^2)/(((x
 + 1)^(1/2) - 1)^2*((2*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)^2 + ((1
- x)^(1/2) - 1)^4/((x + 1)^(1/2) - 1)^4 + 1))
```

3.449 $\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx$

3.449.1 Optimal result

Integrand size = 42, antiderivative size = 26

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx=\frac{2}{x}+\frac{2\sqrt{1-x^2}}{x}+2\arcsin(x)$$

output
```
2/x+2*arcsin(x)+2*(-x^2+1)^(1/2)/x
```

3.449.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx=\frac{2\left(1+\sqrt{1-x^2}-4x\arctan\left(\frac{\sqrt{1+x}}{\sqrt{2}-\sqrt{1-x}}\right)\right)}{x}$$

input
```
Integrate[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^2,x
]
```

output
```
(2*(1 + Sqrt[1 - x^2] - 4*x*ArcTan[Sqrt[1 + x]/(Sqrt[2] - Sqrt[1 - x])]))/
x
```

3.449.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {7239, 25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{x+1}\right)\left(\sqrt{1-x}+\sqrt{x+1}\right)}{x^2}dx$$

$$\Big\downarrow \text{7239}$$

$$\int -\frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x^2}dx$$

$$\Big\downarrow \text{7293}$$

$$-\int \left(\frac{2\sqrt{1-x^2}}{x^2}+\frac{2}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2\arcsin(x)+\frac{2\sqrt{1-x^2}}{x}+\frac{2}{x}$$

input
```
Int[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^2,x]
```

output
```
2/x + (2*Sqrt[1 - x^2])/x + 2*ArcSin[x]
```

3.449.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.449.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(24) = 48$.

Time = 1.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

method	result	size
default	$\frac{2}{x}-\frac{2\left(-\arcsin\left(x\right)x-\sqrt{-x^{2}+1}\right)\sqrt{1-x}\,\sqrt{x+1}}{x\sqrt{-x^{2}+1}}$	50

input
```
int((-(1-x)^(1/2)-(x+1)^(1/2))*((1-x)^(1/2)+(x+1)^(1/2))/x^2,x,method=_RET
URNVERBOSE)
```

output
```
2/x-2*(-arcsin(x)*x-(-x^2+1)^(1/2))*(1-x)^(1/2)*(x+1)^(1/2)/x/(-x^2+1)^(1/
2)
```

3.449.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.69

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx$$
$$=-\frac{2\left(2\,x\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)-\sqrt{x+1}\sqrt{-x+1}-1\right)}{x}$$

input
```
integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^2,x, algo
rithm="fricas")
```

output
```
-2*(2*x*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - sqrt(x + 1)*sqrt(-x + 1
) - 1)/x
```

3.449.6 Sympy [F]

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx = -\int \frac{2}{x^2}\,dx - \int \frac{2\sqrt{1-x}\sqrt{x+1}}{x^2}\,dx$$

input
```
integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x**2,x)
```

output
```
-Integral(2/x**2, x) - Integral(2*sqrt(1 - x)*sqrt(x + 1)/x**2, x)
```

3.449.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx = \frac{2\sqrt{-x^2+1}}{x} + \frac{2}{x} + 2\arcsin(x)$$

input
```
integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^2,x, algo
rithm="maxima")
```

output
```
2*sqrt(-x^2 + 1)/x + 2/x + 2*arcsin(x)
```

3.449.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(24) = 48.

Time = 0.35 (sec) , antiderivative size = 149, normalized size of antiderivative = 5.73

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx$$
$$= 2\pi + \frac{8\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} - \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)}{\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} - \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^2 - 4} + \frac{2}{x} + 4\arctan\left(\frac{\sqrt{x+1}\left(\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1} - 1\right)}{2\left(\sqrt{2}-\sqrt{-x+1}\right)}\right)$$

input
```
integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^2,x, algo
rithm="giac")
```

output
```
2*pi + 8*((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - sq
rt(-x + 1)))/(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2)
 - sqrt(-x + 1)))^2 - 4) + 2/x + 4*arctan(1/2*sqrt(x + 1)*((sqrt(2) - sqrt
(-x + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1)))
```

3.449.9 Mupad [B] (verification not implemented)

Time = 17.87 (sec) , antiderivative size = 118, normalized size of antiderivative = 4.54

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx$$
$$=\frac{\frac{5\left(\sqrt{1-x}-1\right)^2}{2\left(\sqrt{x+1}-1\right)^2}-\frac{1}{2}}{\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}-\frac{\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}}-8\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)+\frac{\sqrt{1-x}-1}{2\left(\sqrt{x+1}-1\right)}+\frac{2}{x}$$

input
```
int(-((x + 1)^(1/2) + (1 - x)^(1/2))^2/x^2,x)
```

output
```
((5*((1 - x)^(1/2) - 1)^2)/(2*((x + 1)^(1/2) - 1)^2) - 1/2)/(((1 - x)^(1/2
) - 1)/((x + 1)^(1/2) - 1) - ((1 - x)^(1/2) - 1)^3/((x + 1)^(1/2) - 1)^3)
- 8*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) + ((1 - x)^(1/2) - 1)/(2
*((x + 1)^(1/2) - 1)) + 2/x
```

3.450 $\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3} \, dx$

3.450.1 Optimal result

Integrand size = 42, antiderivative size = 33

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3} \, dx=\frac{1}{x^2}+\frac{\sqrt{1-x^2}}{x^2}-\text{arctanh}\left(\sqrt{1-x^2}\right)$$

output
```
1/x^2-arctanh((-x^2+1)^(1/2))+(-x^2+1)^(1/2)/x^2
```

3.450.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 148 vs. 2(33) = 66.

Time = 0.03 (sec) , antiderivative size = 148, normalized size of antiderivative = 4.48

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3} \, dx = -2\text{arctanh}\left(\frac{2-\sqrt{2}+2\sqrt{1-x}+2\sqrt{1+x}-\sqrt{2}\sqrt{1+x}}{-2+\sqrt{2}+\sqrt{2}\sqrt{1+x}}\right)-\log\left(\sqrt{2}-\sqrt{1+x}\right)+\frac{1+\sqrt{1-x^2}+x^2\log\left(-2-\sqrt{2}+\sqrt{1-x}+\sqrt{1+x}+\sqrt{2}\sqrt{1+x}\right)}{x^2}$$

input
```
Integrate[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^3,x
]
```

output
```
-2*ArcTanh[(2 - Sqrt[2] + 2*Sqrt[1 - x] + 2*Sqrt[1 + x] - Sqrt[2]*Sqrt[1 +
 x])/(-2 + Sqrt[2] + Sqrt[2]*Sqrt[1 + x])] - Log[Sqrt[2] - Sqrt[1 + x]] +
(1 + Sqrt[1 - x^2] + x^2*Log[-2 - Sqrt[2] + Sqrt[1 - x] + Sqrt[1 + x] + Sq
rt[2]*Sqrt[1 + x]])/x^2
```

3.450.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {7239, 25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{x+1}\right)\left(\sqrt{1-x}+\sqrt{x+1}\right)}{x^3} \, dx$$

$$\Big\downarrow \text{7239}$$

$$\int -\frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x^3}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x^3}dx$$

$$\Big\downarrow \text{7293}$$

$$-\int \left(\frac{2\sqrt{1-x^2}}{x^3}+\frac{2}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\text{arctanh}\left(\sqrt{1-x^2}\right)+\frac{\sqrt{1-x^2}}{x^2}+\frac{1}{x^2}$$

input
```
Int[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^3,x]
```

output
```
x^(-2) + Sqrt[1 - x^2]/x^2 - ArcTanh[Sqrt[1 - x^2]]
```

3.450.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.450.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.73

method	result	size
default	$\frac{1}{x^{2}}-\frac{\sqrt{1-x}\,\sqrt{x+1}\,\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right) x^{2}-\sqrt{-x^{2}+1}\right)}{x^{2} \sqrt{-x^{2}+1}}$	57

input
```
int((-(1-x)^(1/2)-(x+1)^(1/2))*((1-x)^(1/2)+(x+1)^(1/2))/x^3,x,method=_RET
URNVERBOSE)
```

output
```
1/x^2-(1-x)^(1/2)*(x+1)^(1/2)*(arctanh(1/(-x^2+1)^(1/2))*x^2-(-x^2+1)^(1/2
))/x^2/(-x^2+1)^(1/2)
```

3.450.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx = \frac{x^{2}\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+\sqrt{x+1}\sqrt{-x+1}+1}{x^{2}}$$

input
```
integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x, algo
rithm="fricas")
```

output
```
(x^2*log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + sqrt(x + 1)*sqrt(-x + 1) + 1)
/x^2
```

3.450.6 Sympy [F]

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx = -\int \frac{2}{x^3}\,dx - \int \frac{2\sqrt{1-x}\sqrt{x+1}}{x^3}\,dx$$

input
```
integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x**3,x)
```

output
```
-Integral(2/x**3, x) - Integral(2*sqrt(1 - x)*sqrt(x + 1)/x**3, x)
```

3.450.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx = \sqrt{-x^2+1} + \frac{(-x^2+1)^{\frac{3}{2}}}{x^2} + \frac{1}{x^2} - \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

input
```
integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x, algo
rithm="maxima")
```

output
```
sqrt(-x^2 + 1) + (-x^2 + 1)^(3/2)/x^2 + 1/x^2 - log(2*sqrt(-x^2 + 1)/abs(x
) + 2/abs(x))
```

3.450.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(29) = 58.

Time = 0.42 (sec) , antiderivative size = 233, normalized size of antiderivative = 7.06

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx$$
$$= -\frac{4\left(\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^3+\frac{4\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}}-\frac{4\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)}{\left(\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^2-4\right)^2}$$
$$+\frac{1}{x^2}-\log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}+\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}+2\right|\right)$$
$$+\log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}+\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}-2\right|\right)$$

input
```
integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x, algo
rithm="giac")
```

output
```
-4*(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - sqrt(-x
 + 1)))^3 + 4*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 4*sqrt(x + 1)/(sqrt(2
) - sqrt(-x + 1)))/(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(s
qrt(2) - sqrt(-x + 1)))^2 - 4)^2 + 1/x^2 - log(abs(-(sqrt(2) - sqrt(-x + 1
))/sqrt(x + 1) + sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) + 2)) + log(abs(-(sq
rt(2) - sqrt(-x + 1))/sqrt(x + 1) + sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) -
 2))
```

3.450.9 Mupad [B] (verification not implemented)

Time = 19.13 (sec) , antiderivative size = 186, normalized size of antiderivative = 5.64

$$\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx$$
$$=\ln\left(\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)$$
$$-\frac{\left(\sqrt{1-x}-1\right)^2}{16\left(\sqrt{x+1}-1\right)^2}+\frac{\frac{\left(\sqrt{1-x}-1\right)^2}{8\left(\sqrt{x+1}-1\right)^2}+\frac{15\left(\sqrt{1-x}-1\right)^4}{16\left(\sqrt{x+1}-1\right)^4}-\frac{1}{16}}{\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-\frac{2\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}}+\frac{1}{x^2}$$

input
```
int(-((x + 1)^(1/2) + (1 - x)^(1/2))^2/x^3,x)
```

output
```
log(((1 - x)^(1/2) - 1)^2/((x + 1)^(1/2) - 1)^2 - 1) - log(((1 - x)^(1/2)
- 1)/((x + 1)^(1/2) - 1)) - ((1 - x)^(1/2) - 1)^2/(16*((x + 1)^(1/2) - 1)^
2) + (((1 - x)^(1/2) - 1)^2/(8*((x + 1)^(1/2) - 1)^2) + (15*((1 - x)^(1/2)
 - 1)^4)/(16*((x + 1)^(1/2) - 1)^4) - 1/16)/(((1 - x)^(1/2) - 1)^2/((x + 1
)^(1/2) - 1)^2 - (2*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + ((1 - x
)^(1/2) - 1)^6/((x + 1)^(1/2) - 1)^6) + 1/x^2
```

3.450. $\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx$

3.451 $\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx$

3.451.1 Optimal result

Integrand size = 39, antiderivative size = 28

$$\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx=\sqrt{1-x^2}-\operatorname{arctanh}\left(\sqrt{1-x^2}\right)+\log(x)$$

output
```
-arctanh((-x^2+1)^(1/2))+ln(x)+(-x^2+1)^(1/2)
```

3.451.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 68 vs. 2(28) = 56.

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.43

$$\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx=\sqrt{1-x^2}-2\log\left(-2+\sqrt{2-2x}\right)+2\log\left(\sqrt{2}-\sqrt{1-x}\right)+2\log\left(-\sqrt{1-x}+\sqrt{1+x}\right)$$

input
```
Integrate[(Sqrt[1 - x] + Sqrt[1 + x])/(-Sqrt[1 - x] + Sqrt[1 + x]),x]
```

output
```
Sqrt[1 - x^2] - 2*Log[-2 + Sqrt[2 - 2*x]] + 2*Log[Sqrt[2] - Sqrt[1 - x]] +
 2*Log[-Sqrt[1 - x] + Sqrt[1 + x]]
```

3.451.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 69 vs. $2(28) = 56$.

Time = 0.56 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.46, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {2528, 7239, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x}+\sqrt{x+1}}{\sqrt{x+1}-\sqrt{1-x}}dx$$

$$\Big\downarrow \text{2528}$$

$$\frac{1}{2}\int \frac{\sqrt{1-x}(\sqrt{1-x}+\sqrt{x+1})}{x}dx+\frac{1}{2}\int \frac{\sqrt{x+1}(\sqrt{1-x}+\sqrt{x+1})}{x}dx$$

$$\Big\downarrow \text{7239}$$

$$\frac{1}{2}\int \frac{-x+\sqrt{1-x^2}+1}{x}dx+\frac{1}{2}\int \frac{x+\sqrt{1-x^2}+1}{x}dx$$

$$\Big\downarrow \text{2010}$$

$$\frac{1}{2}\int \left(\frac{\sqrt{1-x^2}}{x}+\frac{1}{x}-1\right)dx+\frac{1}{2}\int \left(\frac{\sqrt{1-x^2}}{x}+\frac{1}{x}+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\text{arctanh}\left(\sqrt{1-x^2}\right)+\sqrt{1-x^2}-x+\log(x)\right)+\frac{1}{2}\left(-\text{arctanh}\left(\sqrt{1-x^2}\right)+\sqrt{1-x^2}+x+\log(x)\right)$$

input
```
Int[(Sqrt[1 - x] + Sqrt[1 + x])/(-Sqrt[1 - x] + Sqrt[1 + x]),x]
```

output
```
(-x + Sqrt[1 - x^2] - ArcTanh[Sqrt[1 - x^2]] + Log[x])/2 + (x + Sqrt[1 - x
^2] - ArcTanh[Sqrt[1 - x^2]] + Log[x])/2
```

3.451.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2528
```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Simp[c/(e*(b*c - a*d))   Int[(u*Sqrt[a + b*x])/x, x], x] - Si
mp[a/(f*(b*c - a*d))   Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

3.451.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

method	result	size
default	$\ln\left(x\right)+\frac{\sqrt{1-x}\,\sqrt{x+1}\,\left(\sqrt{-x^{2}+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)\right)}{\sqrt{-x^{2}+1}}$	48

input
```
int(((1-x)^(1/2)+(x+1)^(1/2))/(-(1-x)^(1/2)+(x+1)^(1/2)),x,method=_RETURNV
ERBOSE)
```

output
```
ln(x)+(1-x)^(1/2)*(x+1)^(1/2)/(-x^2+1)^(1/2)*((-x^2+1)^(1/2)-arctanh(1/(-x
^2+1)^(1/2)))
```

3.451.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx = \sqrt{x+1}\sqrt{-x+1}+\log{(x)}+\log{\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)}$$

input
```
integrate((((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x, algorith
m="fricas")
```

output
```
sqrt(x + 1)*sqrt(-x + 1) + log(x) + log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

3.451.6 Sympy [F]

$$\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx = -\int \frac{\sqrt{1-x}}{\sqrt{1-x}-\sqrt{x+1}}\,dx - \int \frac{\sqrt{x+1}}{\sqrt{1-x}-\sqrt{x+1}}\,dx$$

input
```
integrate((((1-x)**(1/2)+(1+x)**(1/2))/(-(1-x)**(1/2)+(1+x)**(1/2)),x)
```

output
```
-Integral(sqrt(1 - x)/(sqrt(1 - x) - sqrt(x + 1)), x) - Integral(sqrt(x +
1)/(sqrt(1 - x) - sqrt(x + 1)), x)
```

3.451.7 Maxima [F]

$$\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx = \int \frac{\sqrt{x+1}+\sqrt{-x+1}}{\sqrt{x+1}-\sqrt{-x+1}}\,dx$$

input
```
integrate((((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x, algorith
m="maxima")
```

output
```
integrate((sqrt(x + 1) + sqrt(-x + 1))/(sqrt(x + 1) - sqrt(-x + 1)), x)
```

3.451.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(24) = 48$.

Time = 0.37 (sec) , antiderivative size = 123, normalized size of antiderivative = 4.39

$$\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx = \sqrt{x+1}\sqrt{-x+1}+\log\left(\sqrt{x+1}+1\right)+\log\left(\left|\sqrt{x+1}-1\right|\right) - \log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}+\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}+2\right|\right) + \log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}+\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}-2\right|\right)$$

input
```
integrate(((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x, algorith
m="giac")
```

output
```
sqrt(x + 1)*sqrt(-x + 1) + log(sqrt(x + 1) + 1) + log(abs(sqrt(x + 1) - 1)
) - log(abs(-(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + sqrt(x + 1)/(sqrt(2) -
 sqrt(-x + 1)) + 2)) + log(abs(-(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + sqr
t(x + 1)/(sqrt(2) - sqrt(-x + 1)) - 2))
```

3.451.9 Mupad [B] (verification not implemented)

Time = 19.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.32

$$\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx = \ln\left(\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-1\right) - \ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right) + \ln(x) - \frac{8\left(x-2\sqrt{x+1}+2\right)\left(x+2\sqrt{1-x}-2\right)}{\left(2\sqrt{x+1}+2\sqrt{1-x}-4\right)^2}$$

input
```
int(((x + 1)^(1/2) + (1 - x)^(1/2))/((x + 1)^(1/2) - (1 - x)^(1/2)),x)
```

output
```
log(((1 - x)^(1/2) - 1)^2/((x + 1)^(1/2) - 1)^2 - 1) - log(((1 - x)^(1/2)
- 1)/((x + 1)^(1/2) - 1)) + log(x) - (8*(x - 2*(x + 1)^(1/2) + 2)*(x + 2*(
1 - x)^(1/2) - 2))/(2*(x + 1)^(1/2) + 2*(1 - x)^(1/2) - 4)^2
```

3.452 $\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}} \, dx$

3.452.1 Optimal result

Integrand size = 35, antiderivative size = 33

$$\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}} \, dx = \frac{x^2}{2} - \frac{1}{2}\sqrt{-1+x}x\sqrt{1+x} + \frac{\operatorname{arccosh}(x)}{2}$$

output
```
1/2*x^2+1/2*arccosh(x)-1/2*x*(-1+x)^(1/2)*(1+x)^(1/2)
```

3.452.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}} \, dx = \frac{1}{2}\left(-1+x^2-\sqrt{-1+x}x\sqrt{1+x}-2\log\left(\sqrt{-1+x}-\sqrt{1+x}\right)\right)$$

input
```
Integrate[(-Sqrt[-1 + x] + Sqrt[1 + x])/(Sqrt[-1 + x] + Sqrt[1 + x]),x]
```

output
```
(-1 + x^2 - Sqrt[-1 + x]*x*Sqrt[1 + x] - 2*Log[Sqrt[-1 + x] - Sqrt[1 + x]]
)/2
```

3.452.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 79 vs. $2(33) = 66$.

Time = 0.35 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.39, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2529, 25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x-1}+\sqrt{x+1}}\,dx$$

$$\Big\downarrow \ 2529$$

$$\frac{1}{2}\int -\sqrt{x+1}\left(\sqrt{x-1}-\sqrt{x+1}\right)dx-\frac{1}{2}\int -\sqrt{x-1}\left(\sqrt{x-1}-\sqrt{x+1}\right)dx$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\int \sqrt{x-1}\left(\sqrt{x-1}-\sqrt{x+1}\right)dx-\frac{1}{2}\int \sqrt{x+1}\left(\sqrt{x-1}-\sqrt{x+1}\right)dx$$

$$\Big\downarrow \ 7293$$

$$\frac{1}{2}\int \left(x-\sqrt{x-1}\sqrt{x+1}-1\right)dx-\frac{1}{2}\int \left(-x+\sqrt{x-1}\sqrt{x+1}-1\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{\operatorname{arccosh}(x)}{2}+\frac{x^2}{2}-\frac{1}{2}\sqrt{x-1}\sqrt{x+1}x-x\right)+\frac{1}{2}\left(\frac{\operatorname{arccosh}(x)}{2}+\frac{x^2}{2}-\frac{1}{2}\sqrt{x-1}\sqrt{x+1}x+x\right)$$

input
```
Int[(-Sqrt[-1 + x] + Sqrt[1 + x])/(Sqrt[-1 + x] + Sqrt[1 + x]),x]
```

output
```
(-x + x^2/2 - (Sqrt[-1 + x]*x*Sqrt[1 + x])/2 + ArcCosh[x]/2)/2 + (x + x^2/
2 - (Sqrt[-1 + x]*x*Sqrt[1 + x])/2 + ArcCosh[x]/2)/2
```

3.452.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2529
```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Simp[-d/(e*(b*c - a*d))   Int[u*Sqrt[a + b*x], x], x] + Simp[
b/(f*(b*c - a*d))   Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f}
, x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.452.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(23) = 46$.

Time = 0.92 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.88

method	result	size
default	$-\frac{\sqrt{x-1}\,(x+1)^{\frac{3}{2}}}{2}+\frac{\sqrt{x-1}\,\sqrt{x+1}}{2}+\frac{\sqrt{(x-1)(x+1)}\,\ln\left(x+\sqrt{x^{2}-1}\right)}{2\sqrt{x+1}\,\sqrt{x-1}}+\frac{x^{2}}{2}$	62

input
```
int((-(x-1)^(1/2)+(x+1)^(1/2))/((x-1)^(1/2)+(x+1)^(1/2)),x,method=_RETURNV
ERBOSE)
```

output
```
-1/2*(x-1)^(1/2)*(x+1)^(3/2)+1/2*(x-1)^(1/2)*(x+1)^(1/2)+1/2*((x-1)*(x+1))
^(1/2)/(x+1)^(1/2)/(x-1)^(1/2)*ln(x+(x^2-1)^(1/2))+1/2*x^2
```

3.452.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = -\frac{1}{2}\sqrt{x+1}\sqrt{x-1}x+\frac{1}{2}x^2-\frac{1}{2}\log\left(\sqrt{x+1}\sqrt{x-1}-x\right)$$

input
```
integrate((-(-1+x)^(1/2)+(1+x)^(1/2))/((-1+x)^(1/2)+(1+x)^(1/2)),x, algori
thm="fricas")
```

output
```
-1/2*sqrt(x + 1)*sqrt(x - 1)*x + 1/2*x^2 - 1/2*log(sqrt(x + 1)*sqrt(x - 1)
 - x)
```

3.452.6 Sympy [A] (verification not implemented)

Time = 11.61 (sec) , antiderivative size = 223, normalized size of antiderivative = 6.76

$$\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx = -\frac{(x-1)^{\frac{5}{2}}}{4\sqrt{x+1}}-\frac{3(x-1)^{\frac{3}{2}}}{4\sqrt{x+1}}-\frac{\sqrt{x-1}}{2\sqrt{x+1}}+\frac{(x-1)^2}{4} + \begin{cases} \frac{(x+1)^2}{4}+\frac{\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2}-\frac{(x+1)^{\frac{5}{2}}}{4\sqrt{x-1}}+\frac{3(x+1)^{\frac{3}{2}}}{4\sqrt{x-1}}-\frac{\sqrt{x+1}}{2\sqrt{x-1}} & \text{for } |x+1|>2 \\ \frac{(x+1)^2}{4}-\frac{i\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2}+\frac{i(x+1)^{\frac{5}{2}}}{4\sqrt{1-x}}-\frac{3i(x+1)^{\frac{3}{2}}}{4\sqrt{1-x}}+\frac{i\sqrt{x+1}}{2\sqrt{1-x}} & \text{otherwise} \end{cases} + \frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{x-1}}{2}\right)}{2}$$

input
```
integrate((-(-1+x)**(1/2)+(1+x)**(1/2))/((-1+x)**(1/2)+(1+x)**(1/2)),x)
```

output
```
-(x - 1)**(5/2)/(4*sqrt(x + 1)) - 3*(x - 1)**(3/2)/(4*sqrt(x + 1)) - sqrt(
x - 1)/(2*sqrt(x + 1)) + (x - 1)**2/4 + Piecewise(((x + 1)**2/4 + acosh(sq
rt(2)*sqrt(x + 1)/2)/2 - (x + 1)**(5/2)/(4*sqrt(x - 1)) + 3*(x + 1)**(3/2)
/(4*sqrt(x - 1)) - sqrt(x + 1)/(2*sqrt(x - 1)), Abs(x + 1) > 2), ((x + 1)*
*2/4 - I*asin(sqrt(2)*sqrt(x + 1)/2)/2 + I*(x + 1)**(5/2)/(4*sqrt(1 - x))
- 3*I*(x + 1)**(3/2)/(4*sqrt(1 - x)) + I*sqrt(x + 1)/(2*sqrt(1 - x)), True
)) + asinh(sqrt(2)*sqrt(x - 1)/2)/2
```

3.452.7 Maxima [F]

$$\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx=\int \frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}\,dx$$

input
```
integrate((-(-1+x)^(1/2)+(1+x)^(1/2))/((-1+x)^(1/2)+(1+x)^(1/2)),x, algori
thm="maxima")
```

output
```
integrate((sqrt(x + 1) - sqrt(x - 1))/(sqrt(x + 1) + sqrt(x - 1)), x)
```

3.452.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

$$\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx=\frac{1}{2}\,(x+1)^2-\frac{1}{2}\,\sqrt{x+1}\sqrt{x-1}x-x-\log\left(\sqrt{x+1}-\sqrt{x-1}\right)-1$$

input
```
integrate((-(-1+x)^(1/2)+(1+x)^(1/2))/((-1+x)^(1/2)+(1+x)^(1/2)),x, algori
thm="giac")
```

output
```
1/2*(x + 1)^2 - 1/2*sqrt(x + 1)*sqrt(x - 1)*x - x - log(sqrt(x + 1) - sqrt
(x - 1)) - 1
```

3.452.9 Mupad [B] (verification not implemented)

Time = 27.82 (sec) , antiderivative size = 200, normalized size of antiderivative = 6.06

$$\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx=\operatorname{acosh}(x)-2\operatorname{atanh}\left(\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}\right)+\frac{\frac{14\,(\sqrt{x-1}-\mathrm{i})^3}{(\sqrt{x+1}-1)^3}+\frac{14\,(\sqrt{x-1}-\mathrm{i})^5}{(\sqrt{x+1}-1)^5}+\frac{2\,(\sqrt{x-1}-\mathrm{i})^7}{(\sqrt{x+1}-1)^7}+\frac{2\,(\sqrt{x-1}-\mathrm{i})}{\sqrt{x+1}-1}}{1+\frac{6\,(\sqrt{x-1}-\mathrm{i})^4}{(\sqrt{x+1}-1)^4}-\frac{4\,(\sqrt{x-1}-\mathrm{i})^6}{(\sqrt{x+1}-1)^6}+\frac{(\sqrt{x-1}-\mathrm{i})^8}{(\sqrt{x+1}-1)^8}-\frac{4\,(\sqrt{x-1}-\mathrm{i})^2}{(\sqrt{x+1}-1)^2}}+\frac{x^2}{2}$$

input
```
int(-((x - 1)^(1/2) - (x + 1)^(1/2))/((x - 1)^(1/2) + (x + 1)^(1/2)),x)
```

output
```
acosh(x) - 2*atanh(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1)) + ((14*((x -
1)^(1/2) - 1i)^3)/((x + 1)^(1/2) - 1)^3 + (14*((x - 1)^(1/2) - 1i)^5)/((x
+ 1)^(1/2) - 1)^5 + (2*((x - 1)^(1/2) - 1i)^7)/((x + 1)^(1/2) - 1)^7 + (2*
((x - 1)^(1/2) - 1i))/((x + 1)^(1/2) - 1))/((6*((x - 1)^(1/2) - 1i)^4)/((x
 + 1)^(1/2) - 1)^4 - (4*((x - 1)^(1/2) - 1i)^2)/((x + 1)^(1/2) - 1)^2 - (4
*((x - 1)^(1/2) - 1i)^6)/((x + 1)^(1/2) - 1)^6 + ((x - 1)^(1/2) - 1i)^8/((
x + 1)^(1/2) - 1)^8 + 1) + x^2/2
```

3.452. $\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx$

3.453 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n dx$

3.453.1 Optimal result

Integrand size = 25, antiderivative size = 121

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)} + \frac{af^2\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{1+n}\text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}{d}\right)}{2d^2e(1+n)}$$

output
```
1/2*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1+n)/e/(1+n)+1/2*a*f^2*hypergeom([2,
1+n],[2+n],(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))/d)*(d+e*x+f*(a+e^2*x^2/f^2)^(1/
2))^(1+n)/d^2/e/(1+n)
```

3.453.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.71

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$=\frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{1+n}\left(d^2+af^2\,\text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}{d}\right)\right)}{2d^2e(1+n)}$$

input
```
Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^n,x]
```

output
```
((d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(1 + n)*(d^2 + a*f^2*Hypergeometric
2F1[2, 1 + n, 2 + n, (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])/d]))/(2*d^2*e*(
1 + n))
```

3.453.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2542, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^n dx$$
$$\Big\downarrow \text{2542}$$
$$\frac{\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^n\left(d^2-2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)d+af^2+\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2\right)}{\left(-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex\right)^2}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)}{2e}$$
$$\Big\downarrow \text{1195}$$
$$\frac{\int \left(\frac{af^2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^n}{\left(-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex\right)^2}+\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^n\right)d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)}{2e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}{d}\right)}{d^2(n+1)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{n+1}}{2e}$$

input
```
Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^n,x]
```

output
```
((d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(1 + n)/(1 + n) + (a*f^2*(d + e*x +
 f*Sqrt[a + (e^2*x^2)/f^2])^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, (d
+ e*x + f*Sqrt[a + (e^2*x^2)/f^2])/d])/(d^2*(1 + n)))/(2*e)
```

3.453.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2542
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Simp[1/(2*e)   Subst[Int[(g + h*x^n)^p*((d^2 + a*f
^2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fr
eeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.453.4 Maple [F]

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n dx$$

input
```
int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^n,x)
```

output
```
int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^n,x)
```

3.453.5 Fricas [F]

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n dx = \int \left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^n dx$$

input
```
integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^n,x, algorithm="fricas")
```

output
```
integral((e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)^n, x)
```

3.453.6 Sympy [F]

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n dx = \int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n dx$$

input
```
integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**n,x)
```

output
```
Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**n, x)
```

3.453.7 Maxima [F]

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n dx = \int \left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^n dx$$

input
```
integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^n,x, algorithm="maxima")
```

output
```
integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^n, x)
```

3.453.8 Giac [F]

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n dx = \int \left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^n dx$$

input
```
integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^n,x, algorithm="giac")
```

output
```
integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^n, x)
```

3.453.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n dx = \int \left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)^n dx$$

input
```
int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^n,x)
```

output
```
int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^n, x)
```

3.454 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3 dx$

3.454.1 Optimal result

Integrand size = 25, antiderivative size = 175

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3 dx = -\frac{ad^3f^2}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{adf^2\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{e}+\frac{af^2\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}{4e}+\frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^4}{8e}+\frac{3ad^2f^2\log\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

output
```
3/2*a*d^2*f^2*ln(e*x+f*(a+e^2*x^2/f^2)^(1/2))/e-1/2*a*d^3*f^2/e/(e*x+f*(a+
e^2*x^2/f^2)^(1/2))+a*d*f^2*(e*x+f*(a+e^2*x^2/f^2)^(1/2))/e+1/4*a*f^2*(d+e
*x+f*(a+e^2*x^2/f^2)^(1/2))^2/e+1/8*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^4/e
```

3.454.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.10

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3 dx$$
$$= \frac{ex(2d^3+6adf^2+3d^2ex+3aef^2x+4de^2x^2+2e^3x^3)+\sqrt{a+\frac{e^2x^2}{f^2}}(2af^3(2d+ex)+efx(3d^2+4dex+2e}{2e}$$

input
```
Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3,x]
```

output
```
(e*x*(2*d^3 + 6*a*d*f^2 + 3*d^2*e*x + 3*a*e*f^2*x + 4*d*e^2*x^2 + 2*e^3*x^
3) + Sqrt[a + (e^2*x^2)/f^2]*(2*a*f^3*(2*d + e*x) + e*f*x*(3*d^2 + 4*d*e*x
 + 2*e^2*x^2)) - 3*a*d^2*f^2*Log[e*(Sqrt[a]*f + e*x - f*Sqrt[a + (e^2*x^2)
/f^2])] + 3*a*d^2*f^2*Log[-(Sqrt[a]*f) + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])
/(2*e)
```

3.454.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2542, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^3 dx$$
$$\Big\downarrow \text{2542}$$
$$\frac{\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^3\left(d^2-2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)d+af^2+\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2\right)}{\left(-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex\right)^2}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)}{2e}$$
$$\Big\downarrow \text{1195}$$

$$\frac{\int \left(\frac{af^2d^3}{\left(-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex\right)^2}-\frac{3af^2d^2}{-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex}+2af^2d+\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^3+af^2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)\right)d\left(d\right.}{2e}$$

$\big\downarrow$ 2009

$$\frac{\frac{ad^3f^2}{f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex}+3ad^2f^2\log\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)+\frac{1}{4}\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^4+\frac{1}{2}af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+\right.}{2e}$$

input
```
Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3,x]
```

output
```
((a*d^3*f^2)/(-(e*x) - f*Sqrt[a + (e^2*x^2)/f^2]) + 2*a*d*f^2*(d + e*x + f
*Sqrt[a + (e^2*x^2)/f^2]) + (a*f^2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2
)/2 + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^4/4 + 3*a*d^2*f^2*Log[-(e*x) -
 f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e)
```

3.454.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2542
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Simp[1/(2*e)   Subst[Int[(g + h*x^n)^p*((d^2 + a*f
^2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fr
eeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.454. $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3 dx$

3.454.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(157) = 314$.

Time = 0.93 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.95

method	result
default	$f^3\left(\frac{x\left(a+\frac{e^2x^2}{f^2}\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{a+\frac{e^2x^2}{f^2}}}{2}+\frac{a\ln\left(\frac{e^2x}{f^2\sqrt{\frac{e^2}{f^2}}}+\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2\sqrt{\frac{e^2}{f^2}}}\right)}{4}\right)+3f^2\left(\frac{e^3x^4}{4f^2}+\frac{d\,e^2x^3}{3f^2}+\frac{ae\,x^2}{2}+adx\right)+3f$

input
```
int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
f^3*(1/4*x*(a+e^2*x^2/f^2)^(3/2)+3/4*a*(1/2*x*(a+e^2*x^2/f^2)^(1/2)+1/2*a*
ln(e^2*x/f^2/(e^2/f^2)^(1/2)+(a+e^2*x^2/f^2)^(1/2))/(e^2/f^2)^(1/2)))+3*f^
2*(1/4*e^3/f^2*x^4+1/3*d*e^2/f^2*x^3+1/2*a*e*x^2+a*d*x)+3*f*(d^2*(1/2*x*(a
+e^2*x^2/f^2)^(1/2)+1/2*a*ln(e^2*x/f^2/(e^2/f^2)^(1/2)+(a+e^2*x^2/f^2)^(1/
2))/(e^2/f^2)^(1/2))+e^2*(1/4*x*(a+e^2*x^2/f^2)^(3/2)/e^2*f^2-1/4*a/e^2*f^
2*(1/2*x*(a+e^2*x^2/f^2)^(1/2)+1/2*a*ln(e^2*x/f^2/(e^2/f^2)^(1/2)+(a+e^2*x
^2/f^2)^(1/2))/(e^2/f^2)^(1/2)))+2/3/e*d*f^2*((e^2*x^2+a*f^2)/f^2)^(3/2))+
1/4*(e*x+d)^4/e
```

3.454.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.92

$$\int\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3dx$$
$$=\frac{2\,e^4x^4+4\,de^3x^3-3\,ad^2f^2\log\left(-ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)+3\,(ae^2f^2+d^2e^2)x^2+2\,(3\,adef^2+d^3e)x+(2\,e^3fx}{2\,e}$$

input
```
integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="fricas")
```

output
```
1/2*(2*e^4*x^4 + 4*d*e^3*x^3 - 3*a*d^2*f^2*log(-e*x + f*sqrt((e^2*x^2 + a*
f^2)/f^2)) + 3*(a*e^2*f^2 + d^2*e^2)*x^2 + 2*(3*a*d*e*f^2 + d^3*e)*x + (2*
e^3*f*x^3 + 4*d*e^2*f*x^2 + 4*a*d*f^3 + (2*a*e*f^3 + 3*d^2*e*f)*x)*sqrt((e
^2*x^2 + a*f^2)/f^2))/e
```

3.454.6 Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 459, normalized size of antiderivative = 2.62

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3 dx = 3adf^2x+\frac{3aef^2x^2}{2} + af^3\left(\begin{cases} \frac{a\left(\begin{cases} \frac{\log\left(\frac{2e^2x}{f^2}+2\sqrt{\frac{e^2}{f^2}}\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{\sqrt{\frac{e^2}{f^2}}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{\frac{e^2x^2}{f^2}}} & \text{otherwise} \end{cases}\right)}{2}+\frac{x\sqrt{a+\frac{e^2x^2}{f^2}}}{2} & \text{for } \frac{e^2}{f^2}\neq 0 \\ \sqrt{a}x & \text{otherwise} \end{cases}\right)+d^3x + \frac{3d^2ex^2}{2}+3d^2f\left(\begin{cases} \frac{a\left(\begin{cases} \frac{\log\left(\frac{2e^2x}{f^2}+2\sqrt{\frac{e^2}{f^2}}\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{\sqrt{\frac{e^2}{f^2}}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{\frac{e^2x^2}{f^2}}} & \text{otherwise} \end{cases}\right)}{2}+\frac{x\sqrt{a+\frac{e^2x^2}{f^2}}}{2} & \text{for } \frac{e^2}{f^2}\neq 0 \\ \sqrt{a}x & \text{otherwise} \end{cases}\right) + 2de^2x^3+6def\left(\begin{cases} \sqrt{a+\frac{e^2x^2}{f^2}}\left(\frac{af^2}{3e^2}+\frac{x^2}{3}\right) & \text{for } \frac{e^2}{f^2}\neq 0 \\ \frac{\sqrt{a}x^2}{2} & \text{otherwise} \end{cases}\right)+e^3x^4 + 4e^2f\left(\begin{cases} -\frac{a^2f^2\left(\begin{cases} \frac{\log\left(\frac{2e^2x}{f^2}+2\sqrt{\frac{e^2}{f^2}}\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{\sqrt{\frac{e^2}{f^2}}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{\frac{e^2x^2}{f^2}}} & \text{otherwise} \end{cases}\right)}{8e^2}+\sqrt{a+\frac{e^2x^2}{f^2}}\left(\frac{af^2x}{8e^2}+\frac{x^3}{4}\right) & \text{for } \frac{e^2}{f^2}\neq 0 \\ \frac{\sqrt{a}x^3}{3} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**3,x)
```

output
```
3*a*d*f**2*x + 3*a*e*f**2*x**2/2 + a*f**3*Piecewise((a*Piecewise((log(2*e*
*2*x/f**2 + 2*sqrt(e**2/f**2)*sqrt(a + e**2*x**2/f**2))/sqrt(e**2/f**2), N
e(a, 0)), (x*log(x)/sqrt(e**2*x**2/f**2), True))/2 + x*sqrt(a + e**2*x**2/
f**2)/2, Ne(e**2/f**2, 0)), (sqrt(a)*x, True)) + d**3*x + 3*d**2*e*x**2/2
+ 3*d**2*f*Piecewise((a*Piecewise((log(2*e**2*x/f**2 + 2*sqrt(e**2/f**2)*s
qrt(a + e**2*x**2/f**2))/sqrt(e**2/f**2), Ne(a, 0)), (x*log(x)/sqrt(e**2*x
**2/f**2), True))/2 + x*sqrt(a + e**2*x**2/f**2)/2, Ne(e**2/f**2, 0)), (sq
rt(a)*x, True)) + 2*d*e**2*x**3 + 6*d*e*f*Piecewise((sqrt(a + e**2*x**2/f*
*2)*(a*f**2/(3*e**2) + x**2/3), Ne(e**2/f**2, 0)), (sqrt(a)*x**2/2, True))
 + e**3*x**4 + 4*e**2*f*Piecewise((-a**2*f**2*Piecewise((log(2*e**2*x/f**2
 + 2*sqrt(e**2/f**2)*sqrt(a + e**2*x**2/f**2))/sqrt(e**2/f**2), Ne(a, 0)),
 (x*log(x)/sqrt(e**2*x**2/f**2), True))/(8*e**2) + sqrt(a + e**2*x**2/f**2
)*(a*f**2*x/(8*e**2) + x**3/4), Ne(e**2/f**2, 0)), (sqrt(a)*x**3/3, True))
```

3.454.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.67

$$\begin{aligned}
&\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3 dx \\
&= \frac{1}{4}e^3x^4 + \frac{3\left(\frac{e^2x^2}{f^2}+a\right)^2 f^4}{4e} \\
&\quad - \frac{3}{8}\left(\frac{a^2f^3\operatorname{arsinh}\left(\frac{e^2x}{\sqrt{ae^2}f}\right)}{\sqrt{e^2}e^2} - \frac{2\left(\frac{e^2x^2}{f^2}+a\right)^{\frac{3}{2}}f^2x}{e^2} + \frac{\sqrt{\frac{e^2x^2}{f^2}+a}af^2x}{e^2}\right)e^2f \\
&\quad + \frac{1}{8}\left(\frac{3a^2f\operatorname{arsinh}\left(\frac{e^2x}{\sqrt{ae^2}f}\right)}{\sqrt{e^2}} + 2\left(\frac{e^2x^2}{f^2}+a\right)^{\frac{3}{2}}x + 3\sqrt{\frac{e^2x^2}{f^2}+a}ax\right)f^3 \\
&\quad + d^3x + \frac{3}{2}\left(ex^2 + \left(\frac{af\operatorname{arsinh}\left(\frac{e^2x}{\sqrt{ae^2}f}\right)}{\sqrt{e^2}} + \sqrt{\frac{e^2x^2}{f^2}+a}x\right)f\right)d^2 \\
&\quad + \left(e^2x^3 + \frac{2\left(\frac{e^2x^2}{f^2}+a\right)^{\frac{3}{2}}f^3}{e} + \left(\frac{e^2x^3}{f^2}+3ax\right)f^2\right)d
\end{aligned}$$

input
```
integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="maxima")
```

output
```
1/4*e^3*x^4 + 3/4*(e^2*x^2/f^2 + a)^2*f^4/e - 3/8*(a^2*f^3*arcsinh(e^2*x/(
sqrt(a*e^2)*f))/(sqrt(e^2)*e^2) - 2*(e^2*x^2/f^2 + a)^(3/2)*f^2*x/e^2 + sq
rt(e^2*x^2/f^2 + a)*a*f^2*x/e^2)*e^2*f + 1/8*(3*a^2*f*arcsinh(e^2*x/(sqrt(
a*e^2)*f))/sqrt(e^2) + 2*(e^2*x^2/f^2 + a)^(3/2)*x + 3*sqrt(e^2*x^2/f^2 +
a)*a*x)*f^3 + d^3*x + 3/2*(e*x^2 + (a*f*arcsinh(e^2*x/(sqrt(a*e^2)*f))/sqr
t(e^2) + sqrt(e^2*x^2/f^2 + a)*x)*f)*d^2 + (e^2*x^3 + 2*(e^2*x^2/f^2 + a)^
(3/2)*f^3/e + (e^2*x^3/f^2 + 3*a*x)*f^2)*d
```

3.454.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.98

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3\,dx$$
$$= e^3x^4 + \frac{3}{2}aef^2x^2 + 2\,de^2x^3 + 3\,adf^2x + \frac{3}{2}d^2ex^2 - \frac{3\,ad^2f|f|\log\left(\left|-x|e| + \sqrt{e^2x^2+af^2}\right|\right)}{2\,|e|} + d^3x + \frac{1}{2}\sqrt{e^2x^2+af^2}\left(\frac{4\,adf|f|}{e} + \left(2\left(\frac{e^2x|f|}{f} + \frac{2\,de|f|}{f}\right)x + \frac{2\,ae^4f^4|f| + 3\,d^2e^4f^2|f|}{e^4f^3}\right)x\right)$$

input
```
integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")
```

output
```
e^3*x^4 + 3/2*a*e*f^2*x^2 + 2*d*e^2*x^3 + 3*a*d*f^2*x + 3/2*d^2*e*x^2 - 3/
2*a*d^2*f*abs(f)*log(abs(-x*abs(e) + sqrt(e^2*x^2 + a*f^2)))/abs(e) + d^3*
x + 1/2*sqrt(e^2*x^2 + a*f^2)*(4*a*d*f*abs(f)/e + (2*(e^2*x*abs(f)/f + 2*d
*e*abs(f)/f)*x + (2*a*e^4*f^4*abs(f) + 3*d^2*e^4*f^2*abs(f))/(e^4*f^3))*x)
```

3.454.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3 dx = \int \left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)^3 dx$$

input
```
int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^3,x)
```

output
```
int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^3, x)
```

3.455 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2 dx$

3.455.1 Optimal result

Integrand size = 25, antiderivative size = 136

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2 dx = -\frac{ad^2f^2}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{af^2\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}+\frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}{6e}+\frac{adf^2\log\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{e}$$

output
```
a*d*f^2*ln(e*x+f*(a+e^2*x^2/f^2)^(1/2))/e-1/2*a*d^2*f^2/e/(e*x+f*(a+e^2*x^
2/f^2)^(1/2))+1/2*a*f^2*(e*x+f*(a+e^2*x^2/f^2)^(1/2))/e+1/6*(d+e*x+f*(a+e^
2*x^2/f^2)^(1/2))^3/e
```

3.455.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.83

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2 dx = d^2x+af^2x+dex^2+\frac{2e^2x^3}{3} + \frac{\sqrt{a+\frac{e^2x^2}{f^2}}\,(2af^3+efx(3d+2ex))}{3e} + \frac{2adf^2\operatorname{arctanh}\left(\frac{f\left(-\sqrt{a}+\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{ex}\right)}{e}$$

input
```
Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2,x]
```

output
```
d^2*x + a*f^2*x + d*e*x^2 + (2*e^2*x^3)/3 + (Sqrt[a + (e^2*x^2)/f^2]*(2*a*
f^3 + e*f*x*(3*d + 2*e*x)))/(3*e) + (2*a*d*f^2*ArcTanh[(f*(-Sqrt[a] + Sqrt
[a + (e^2*x^2)/f^2]))/(e*x)])/e
```

3.455.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2542, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^2 dx$$

$$\Big\downarrow \; 2542$$

$$\frac{\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2\left(d^2-2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)d+af^2+\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2\right)}{\left(-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex\right)^2}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)}{2e}$$

$$\Big\downarrow \; 1195$$

$$\frac{\int \left(af^2 - \frac{2adf^2}{-\sqrt{\frac{e^2x^2}{f^2}+af-ex}} + \frac{ad^2f^2}{\left(-\sqrt{\frac{e^2x^2}{f^2}+af-ex}\right)^2} + \left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2\right) d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)}{2e}$$

$\downarrow$ 2009

$$\frac{\frac{ad^2f^2}{f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex} + \frac{1}{3}\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^3 + af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right) + 2adf^2\log\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right.}{2e}$$

input
```
Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2,x]
```

output
```
((a*d^2*f^2)/(-(e*x) - f*Sqrt[a + (e^2*x^2)/f^2]) + a*f^2*(d + e*x + f*Sqr
t[a + (e^2*x^2)/f^2]) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3/3 + 2*a*d*
f^2*Log[-(e*x) - f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e)
```

3.455.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2542
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Simp[1/(2*e)   Subst[Int[(g + h*x^n)^p*((d^2 + a*f
^2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fr
eeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.455.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{e^{2} x^{3}}{3}+a\,f^{2} x+2 f \left(d \left(\frac{x \sqrt{a+\frac{e^{2} x^{2}}{f^{2}}}}{2}+\frac{a \ln \left(\frac{e^{2} x}{f^{2} \sqrt{\frac{e^{2}}{f^{2}}}}+\sqrt{a+\frac{e^{2} x^{2}}{f^{2}}}\right)}{2 \sqrt{\frac{e^{2}}{f^{2}}}}\right)+\frac{f^{2} \left(\frac{e^{2} x^{2}+a\,f^{2}}{f^{2}}\right)^{\frac{3}{2}}}{3 e}\right)+\frac{\left(e x +d \right)^{3}}{3 e}$	124

input
```
int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*e^2*x^3+a*f^2*x+2*f*(d*(1/2*x*(a+e^2*x^2/f^2)^(1/2)+1/2*a*ln(e^2*x/f^2
/(e^2/f^2)^(1/2)+(a+e^2*x^2/f^2)^(1/2))/(e^2/f^2)^(1/2))+1/3/e*f^2*((e^2*x
^2+a*f^2)/f^2)^(3/2))+1/3*(e*x+d)^3/e
```

3.455.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.84

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2\,dx$$
$$=\frac{2\,e^{3}x^{3}+3\,de^{2}x^{2}-3\,adf^{2}\log\left(-ex+f\sqrt{\frac{e^{2}x^{2}+af^{2}}{f^{2}}}\right)+3\,(aef^{2}+d^{2}e)x+(2\,e^{2}fx^{2}+2\,af^{3}+3\,defx)\sqrt{\frac{e^{2}x^{2}}{}}}{3\,e}$$

input
```
integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/3*(2*e^3*x^3 + 3*d*e^2*x^2 - 3*a*d*f^2*log(-e*x + f*sqrt((e^2*x^2 + a*f^
2)/f^2)) + 3*(a*e*f^2 + d^2*e)*x + (2*e^2*f*x^2 + 2*a*f^3 + 3*d*e*f*x)*sqr
t((e^2*x^2 + a*f^2)/f^2))/e
```

3.455.6 Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.36

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2 dx$$
$$= af^2x+d^2x+dex^2 + 2df\left(\begin{cases} \frac{a\left(\begin{cases} \frac{\log\left(\frac{2e^2x}{f^2}+2\sqrt{\frac{e^2}{f^2}}\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{\sqrt{\frac{e^2}{f^2}}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{\frac{e^2x^2}{f^2}}} & \text{otherwise}\end{cases}\right)}{2} + \frac{x\sqrt{a+\frac{e^2x^2}{f^2}}}{2} & \text{for } \frac{e^2}{f^2}\neq 0 \\ \sqrt{a}x & \text{otherwise}\end{cases}\right) + \frac{2e^2x^3}{3} + 2ef\left(\begin{cases}\sqrt{a+\frac{e^2x^2}{f^2}}\left(\frac{af^2}{3e^2}+\frac{x^2}{3}\right) & \text{for } \frac{e^2}{f^2}\neq 0 \\ \frac{\sqrt{a}x^2}{2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**2,x)
```

output
```
a*f**2*x + d**2*x + d*e*x**2 + 2*d*f*Piecewise((a*Piecewise((log(2*e**2*x/
f**2 + 2*sqrt(e**2/f**2)*sqrt(a + e**2*x**2/f**2))/sqrt(e**2/f**2), Ne(a,
0)), (x*log(x)/sqrt(e**2*x**2/f**2), True))/2 + x*sqrt(a + e**2*x**2/f**2)
/2, Ne(e**2/f**2, 0)), (sqrt(a)*x, True)) + 2*e**2*x**3/3 + 2*e*f*Piecewis
e((sqrt(a + e**2*x**2/f**2)*(a*f**2/(3*e**2) + x**2/3), Ne(e**2/f**2, 0)),
 (sqrt(a)*x**2/2, True))
```

3.455.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.79

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2 dx = \frac{1}{3}e^2x^3 + \frac{2\left(\frac{e^2x^2}{f^2}+a\right)^{\frac{3}{2}}f^3}{3e} + \frac{1}{3}\left(\frac{e^2x^3}{f^2}+3ax\right)f^2 + d^2x + \left(ex^2 + \left(\frac{af\,\mathrm{arsinh}\left(\frac{e^2x}{\sqrt{ae^2}f}\right)}{\sqrt{e^2}} + \sqrt{\frac{e^2x^2}{f^2}+ax}\right)f\right)d$$

input
```
integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="maxima")
```

output
```
1/3*e^2*x^3 + 2/3*(e^2*x^2/f^2 + a)^(3/2)*f^3/e + 1/3*(e^2*x^3/f^2 + 3*a*x
)*f^2 + d^2*x + (e*x^2 + (a*f*arcsinh(e^2*x/(sqrt(a*e^2)*f))/sqrt(e^2) + s
qrt(e^2*x^2/f^2 + a)*x)*f)*d
```

3.455.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.79

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2 dx = \frac{2}{3}e^2x^3 + af^2x + dex^2 - \frac{adf|f|\log\left(\left|-x|e|+\sqrt{e^2x^2+af^2}\right|\right)}{|e|} + d^2x + \frac{1}{3}\sqrt{e^2x^2+af^2}\left(\left(\frac{2ex|f|}{f}+\frac{3d|f|}{f}\right)x+\frac{2af|f|}{e}\right)$$

input
```
integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")
```

output
```
2/3*e^2*x^3 + a*f^2*x + d*e*x^2 - a*d*f*abs(f)*log(abs(-x*abs(e) + sqrt(e^
2*x^2 + a*f^2)))/abs(e) + d^2*x + 1/3*sqrt(e^2*x^2 + a*f^2)*((2*e*x*abs(f)
/f + 3*d*abs(f)/f)*x + 2*a*f*abs(f)/e)
```

3.455.9 Mupad [B] (verification not implemented)

Time = 18.79 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.54

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2 dx = \begin{cases} x\left(d+\sqrt{a}\,f\right)^2 \\ x\left(d^2+a\,f^2\right)+\frac{2\,e^2\,x^3}{3}+d\,e\,x^2+\frac{2\,a\,f^3\,\sqrt{a+\frac{e^2\,x^2}{f^2}}}{e}-\frac{2\,f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\,\left(2\,a\,f^2-e^2\,x^2\right)}{3\,e}+d\,f\,x\,\sqrt{a+\frac{e^2\,x^2}{f^2}}+\frac{2\,a\,d\,f\,\ln\left(x\,\sqrt{\right.}\right.}{} \end{cases}$$

input
```
int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^2,x)
```

output
```
piecewise(e == 0, x*(d + a^(1/2)*f)^2, e ~= 0, x*(a*f^2 + d^2) + (2*e^2*x^
3)/3 + d*e*x^2 + (2*a*f^3*(a + (e^2*x^2)/f^2)^(1/2))/e - (2*f*(a + (e^2*x^
2)/f^2)^(1/2)*(2*a*f^2 - e^2*x^2))/(3*e) + d*f*x*(a + (e^2*x^2)/f^2)^(1/2)
 + (2*a*d*f*log(x*(e^2/f^2)^(1/2) + (a + (e^2*x^2)/f^2)^(1/2)))/(e^2/f^2)^
(1/2) - (a*d*e^2*log(2*x*(e^2/f^2)^(1/2) + 2*(a + (e^2*x^2)/f^2)^(1/2)))/(
f*(e^2/f^2)^(3/2)))
```

3.455. $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2 dx$

3.456 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\,dx$

3.456.1 Optimal result

Integrand size = 23, antiderivative size = 68

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\,dx=dx+\frac{ex^2}{2}+\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{af^2\text{arctanh}\left(\frac{ex}{f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e}$$

output
```
d*x+1/2*e*x^2+1/2*a*f^2*arctanh(e*x/f/(a+e^2*x^2/f^2)^(1/2))/e+1/2*f*x*(a+
e^2*x^2/f^2)^(1/2)
```

3.456.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\,dx=dx+\frac{ex^2}{2}+\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{af^2\text{arctanh}\left(\frac{-\frac{\sqrt{a}f}{e}+\frac{f\sqrt{a+\frac{e^2x^2}{f^2}}}{e}}{x}\right)}{e}$$

input
```
Integrate[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2],x]
```

output
```
d*x + (e*x^2)/2 + (f*x*Sqrt[a + (e^2*x^2)/f^2])/2 + (a*f^2*ArcTanh[(-((Sqr
t[a]*f)/e) + (f*Sqrt[a + (e^2*x^2)/f^2])/e)/x])/e
```

3.456.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{af^2\text{arctanh}\left(\frac{ex}{f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e}+\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}+dx+\frac{ex^2}{2}$$

input
```
Int[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2],x]
```

output
```
d*x + (e*x^2)/2 + (f*x*Sqrt[a + (e^2*x^2)/f^2])/2 + (a*f^2*ArcTanh[(e*x)/(
f*Sqrt[a + (e^2*x^2)/f^2])])/(2*e)
```

3.456.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.456.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.10

method	result	size
default	$dx+\frac{e\,x^2}{2}+\frac{fx\sqrt{a+\frac{e^2x^2}{f^2}}}{2}+\frac{fa\ln\left(\frac{e^2x}{f^2\sqrt{\frac{e^2}{f^2}}}+\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2\sqrt{\frac{e^2}{f^2}}}$	75
parts	$dx+\frac{e\,x^2}{2}+\frac{fx\sqrt{a+\frac{e^2x^2}{f^2}}}{2}+\frac{fa\ln\left(\frac{e^2x}{f^2\sqrt{\frac{e^2}{f^2}}}+\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2\sqrt{\frac{e^2}{f^2}}}$	75

input
```
int(d+e*x+f*(a+e^2*x^2/f^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
d*x+1/2*e*x^2+1/2*f*x*(a+e^2*x^2/f^2)^(1/2)+1/2*f*a*ln(e^2*x/f^2/(e^2/f^2)
^(1/2)+(a+e^2*x^2/f^2)^(1/2))/(e^2/f^2)^(1/2)
```

3.456.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\,dx = \frac{e^2x^2-af^2\log\left(-ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)+efx\sqrt{\frac{e^2x^2+af^2}{f^2}}+2\,dex}{2\,e}$$

input
```
integrate(d+e*x+f*(a+e^2*x^2/f^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(e^2*x^2 - a*f^2*log(-e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2)) + e*f*x*sqr
t((e^2*x^2 + a*f^2)/f^2) + 2*d*e*x)/e
```

3.456.6 Sympy [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\,dx = dx+\frac{ex^2}{2}+f\left(\frac{\sqrt{a}x\sqrt{1+\frac{e^2x^2}{af^2}}}{2}+\frac{af\operatorname{asinh}\left(\frac{ex}{\sqrt{a}f}\right)}{2e}\right)$$

input
```
integrate(d+e*x+f*(a+e**2*x**2/f**2)**(1/2),x)
```

output
```
d*x + e*x**2/2 + f*(sqrt(a)*x*sqrt(1 + e**2*x**2/(a*f**2))/2 + a*f*asinh(e
*x/(sqrt(a)*f))/(2*e))
```

3.456.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\,dx=\frac{1}{2}\,ex^2+\frac{1}{2}\left(\frac{af\,\operatorname{arsinh}\left(\frac{e^2x}{\sqrt{ae^2}f}\right)}{\sqrt{e^2}}+\sqrt{\frac{e^2x^2}{f^2}+a}x\right)f+dx$$

input
```
integrate(d+e*x+f*(a+e^2*x^2/f^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*e*x^2 + 1/2*(a*f*arcsinh(e^2*x/(sqrt(a*e^2)*f))/sqrt(e^2) + sqrt(e^2*x
^2/f^2 + a)*x)*f + d*x
```

3.456.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\,dx=\frac{1}{2}\,ex^2+dx-\frac{\left(\frac{af^2\log\left(\left|-x|e|+\sqrt{e^2x^2+af^2}\right|\right)}{|e|}-\sqrt{e^2x^2+af^2}x\right)|f|}{2\,f}$$

input
```
integrate(d+e*x+f*(a+e^2*x^2/f^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*e*x^2 + d*x - 1/2*(a*f^2*log(abs(-x*abs(e) + sqrt(e^2*x^2 + a*f^2)))/a
bs(e) - sqrt(e^2*x^2 + a*f^2)*x)*abs(f)/f
```

3.456.9 Mupad [B] (verification not implemented)

Time = 17.64 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.00

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\,dx$$
$$=\begin{cases} x\left(d+\sqrt{a}\,f\right) & \text{if } e=0\\ d\,x+\frac{e\,x^2}{2}+\frac{f\,x\,\sqrt{a+\frac{e^2\,x^2}{f^2}}}{2}+\frac{a\,e^2\,\ln\left(x\,\sqrt{\frac{e^2}{f^2}}+\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)}{f\,{\left(\frac{e^2}{f^2}\right)}^{3/2}}-\frac{a\,e^2\,\ln\left(2\,x\,\sqrt{\frac{e^2}{f^2}}+2\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)}{2\,f\,{\left(\frac{e^2}{f^2}\right)}^{3/2}} & \text{if } e\neq 0 \end{cases}$$

input
```
int(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2),x)
```

output
```
piecewise(e == 0, x*(d + a^(1/2)*f), e ~= 0, d*x + (e*x^2)/2 + (f*x*(a + (
e^2*x^2)/f^2)^(1/2))/2 + (a*e^2*log(x*(e^2/f^2)^(1/2) + (a + (e^2*x^2)/f^2
)^(1/2)))/(f*(e^2/f^2)^(3/2)) - (a*e^2*log(2*x*(e^2/f^2)^(1/2) + 2*(a + (e
^2*x^2)/f^2)^(1/2)))/(2*f*(e^2/f^2)^(3/2)))
```

3.456. $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right) dx$

3.457 $\int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$

3.457.1 Optimal result

Integrand size = 25, antiderivative size = 117

$$\int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx = -\frac{af^2}{2de\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}-\frac{af^2\log\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2d^2e}+\frac{\left(1+\frac{af^2}{d^2}\right)\log\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

output
```
-1/2*a*f^2*ln(e*x+f*(a+e^2*x^2/f^2)^(1/2))/d^2/e+1/2*(1+a*f^2/d^2)*ln(d+e*
x+f*(a+e^2*x^2/f^2)^(1/2))/e-1/2*a*f^2/d/e/(e*x+f*(a+e^2*x^2/f^2)^(1/2))
```

3.457.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 337 vs. 2(117) = 234.

Time = 0.86 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.88

$$\int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx = \frac{2de^2x-2def\sqrt{a+\frac{e^2x^2}{f^2}}-\left(af^2\left(e-\sqrt{\frac{e^2}{f^2}}f\right)+d^2\left(e+\sqrt{\frac{e^2}{f^2}}f\right)\right)\log\left(-\sqrt{\frac{e^2}{f^2}}x+\sqrt{a+\frac{e^2x^2}{f^2}}\right)+\sqrt{\frac{e^2}{f^2}}f(d^2}{}$$

input
```
Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-1),x]
```

output
```
(2*d*e^2*x - 2*d*e*f*Sqrt[a + (e^2*x^2)/f^2] - (a*f^2*(e - Sqrt[e^2/f^2]*f
) + d^2*(e + Sqrt[e^2/f^2]*f))*Log[-(Sqrt[e^2/f^2]*x) + Sqrt[a + (e^2*x^2)
/f^2]] + Sqrt[e^2/f^2]*f*(d^2 + a*f^2)*Log[a*f + d*(-(Sqrt[e^2/f^2]*x) + S
qrt[a + (e^2*x^2)/f^2])] + e*(d^2 + a*f^2)*Log[d*e*(a*f + d*(-(Sqrt[e^2/f^
2]*x) + Sqrt[a + (e^2*x^2)/f^2]))] - Sqrt[e^2/f^2]*f*(d^2 + a*f^2)*Log[d +
 f*(-(Sqrt[e^2/f^2]*x) + Sqrt[a + (e^2*x^2)/f^2])] + e*(d^2 + a*f^2)*Log[d
^2*e*(d + f*(-(Sqrt[e^2/f^2]*x) + Sqrt[a + (e^2*x^2)/f^2]))])/(4*d^2*e^2)
```

3.457.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2542, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}dx$$

$$\Big\downarrow \text{2542}$$

$$\frac{\int \frac{d^2-2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)d+af^2+\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2}{\left(-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex\right)^2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)}{2e}$$

$$\Big\downarrow \text{1195}$$

$$\frac{\int \left(\frac{af^2}{d^2\left(-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex\right)}+\frac{af^2}{d\left(-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex\right)^2}+\frac{d^2+af^2}{d^2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)}\right)d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)}{2e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{af^2\log\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}{d^2}+\left(\frac{af^2}{d^2}+1\right)\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)+\frac{af^2}{d\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}}{2e}$$

input
```
Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-1),x]
```

output
```
((a*f^2)/(d*(-(e*x) - f*Sqrt[a + (e^2*x^2)/f^2])) - (a*f^2*Log[-(e*x) - f*
Sqrt[a + (e^2*x^2)/f^2]])/d^2 + (1 + (a*f^2)/d^2)*Log[d + e*x + f*Sqrt[a +
 (e^2*x^2)/f^2]])/(2*e)
```

3.457.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2542
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Simp[1/(2*e)   Subst[Int[(g + h*x^n)^p*((d^2 + a*f
^2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fr
eeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.457.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 650 vs. $2(105) = 210$.

Time = 0.06 (sec) , antiderivative size = 651, normalized size of antiderivative = 5.56

method	result
default	$-\frac{f\left(\frac{\sqrt{\frac{4e^2\left(x+\frac{-a f^2+d^2}{2ed}\right)^2}{f^2}+\frac{4e\left(a f^2-d^2\right)\left(x+\frac{-a f^2+d^2}{2ed}\right)}{d f^2}+\frac{a^2 f^4+2a d^2 f^2+d^4}{d^2 f^2}}}{2}+\frac{e\left(a f^2-d^2\right)\ln\left(\frac{\frac{e\left(a f^2-d^2\right)}{2d f^2}+\frac{e^2\left(x+\frac{-a f^2+d^2}{2ed}\right)}{f^2}}{\sqrt{\frac{e^2}{f^2}}}+\sqrt{\cdots}\right.}{\cdots}\right.}{\cdots}$

input
```
int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*f/e/d*(1/2*(4*e^2*(x+1/2*(-a*f^2+d^2)/e/d)^2/f^2+4*e*(a*f^2-d^2)/d/f^
2*(x+1/2*(-a*f^2+d^2)/e/d)+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)+1/2*e*
(a*f^2-d^2)/d/f^2*ln((1/2*e*(a*f^2-d^2)/d/f^2+e^2/f^2*(x+1/2*(-a*f^2+d^2)/
e/d))/(e^2/f^2)^(1/2)+(e^2*(x+1/2*(-a*f^2+d^2)/e/d)^2/f^2+e*(a*f^2-d^2)/d/
f^2*(x+1/2*(-a*f^2+d^2)/e/d)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))
/(e^2/f^2)^(1/2)-1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2/((a^2*f^4+2*a*d^2*f
^2+d^4)/d^2/f^2)^(1/2)*ln((1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2+e*(a*f^2-
d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/e/d)+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2
)^(1/2)*(4*e^2*(x+1/2*(-a*f^2+d^2)/e/d)^2/f^2+4*e*(a*f^2-d^2)/d/f^2*(x+1/2
*(-a*f^2+d^2)/e/d)+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(x+1/2*(-a*f^
2+d^2)/e/d))+1/2*ln(a*f^2-2*d*e*x-d^2)/e-e*(-1/2/e/d*x+1/4*(-a*f^2+d^2)/e
^2/d^2*ln(-a*f^2+2*d*e*x+d^2))
```

3.457.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.60

$$\int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx = \frac{2\,dex - 2\,df\sqrt{\frac{e^2x^2+af^2}{f^2}} + (af^2+d^2)\log\left(af^2-dex+df\sqrt{\frac{e^2x^2+af^2}{f^2}}\right) + (af^2+d^2)\log\left(-af^2+2\,dex+d^2\right.}{4\,d^2e}$$

input
```
integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)),x, algorithm="fricas")
```

output
```
1/4*(2*d*e*x - 2*d*f*sqrt((e^2*x^2 + a*f^2)/f^2) + (a*f^2 + d^2)*log(a*f^2
 - d*e*x + d*f*sqrt((e^2*x^2 + a*f^2)/f^2)) + (a*f^2 + d^2)*log(-a*f^2 + 2
*d*e*x + d^2) - (a*f^2 + d^2)*log(-e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) - d
) + (a*f^2 - d^2)*log(-e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2)))/(d^2*e)
```

3.457.6 Sympy [F]

$$\int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx = \int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2)),x)
```

output
```
Integral(1/(d + e*x + f*sqrt(a + e**2*x**2/f**2)), x)
```

3.457.7 Maxima [F]

$$\int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx=\int \frac{1}{ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

3.457.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(105) = 210$.

Time = 0.50 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.56

$$\int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$$
$$=\frac{x}{2d}+\frac{(af^2+d^2)\log(|-af^2+2dex+d^2|)}{4d^2e}-\frac{\sqrt{e^2x^2+af^2}|f|}{2def}$$
$$+\frac{(af^2|f|-d^2|f|)\log\left(\left|-x|e|+\sqrt{e^2x^2+af^2}\right|\right)}{4d^2f|e|}$$
$$-\frac{(a^2e^2f^4|f|+2ad^2e^2f^2|f|+d^4e^2|f|)\log\left(\frac{\left|aef^2-d^2e-2\left(x|e|-\sqrt{e^2x^2+af^2}\right)d|e|-|aef^2+d^2e|\right|}{\left|aef^2-d^2e-2\left(x|e|-\sqrt{e^2x^2+af^2}\right)d|e|+|aef^2+d^2e|\right|}\right)}{4d^2ef|aef^2+d^2e||e|}$$

input
```
integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)),x, algorithm="giac")
```

output
```
1/2*x/d + 1/4*(a*f^2 + d^2)*log(abs(-a*f^2 + 2*d*e*x + d^2))/(d^2*e) - 1/2
*sqrt(e^2*x^2 + a*f^2)*abs(f)/(d*e*f) + 1/4*(a*f^2*abs(f) - d^2*abs(f))*lo
g(abs(-x*abs(e) + sqrt(e^2*x^2 + a*f^2)))/(d^2*f*abs(e)) - 1/4*(a^2*e^2*f^
4*abs(f) + 2*a*d^2*e^2*f^2*abs(f) + d^4*e^2*abs(f))*log(abs(a*e*f^2 - d^2*
e - 2*(x*abs(e) - sqrt(e^2*x^2 + a*f^2))*d*abs(e) - abs(a*e*f^2 + d^2*e))/
abs(a*e*f^2 - d^2*e - 2*(x*abs(e) - sqrt(e^2*x^2 + a*f^2))*d*abs(e) + abs(
a*e*f^2 + d^2*e)))/(d^2*e*f*abs(a*e*f^2 + d^2*e)*abs(e))
```

3.457.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx = \int \frac{1}{d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}}\,dx$$

input
```
int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2)),x)
```

output
```
int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2)), x)
```

3.458 $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx$

3.458.1 Optimal result

Integrand size = 25, antiderivative size = 151

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx=-\frac{af^2}{2d^2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}-\frac{1+\frac{af^2}{d^2}}{2e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}-\frac{af^2\log\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{d^3e}+\frac{af^2\log\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{d^3e}$$

output
```
-a*f^2*ln(e*x+f*(a+e^2*x^2/f^2)^(1/2))/d^3/e+a*f^2*ln(d+e*x+f*(a+e^2*x^2/f
^2)^(1/2))/d^3/e-1/2*a*f^2/d^2/e/(e*x+f*(a+e^2*x^2/f^2)^(1/2))+1/2*(-1-a*f
^2/d^2)/e/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))
```

3.458.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.51

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx = \frac{-d\left(d^3+d^2ex+af^2\left(ex-f\sqrt{a+\frac{e^2x^2}{f^2}}\right)+dex\left(-ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)+af^2(-d^2+af^2-2dex)\log\left(-\right.}{d^3e}$$

input
```
Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-2),x]
```

output
```
(-(d*(d^3 + d^2*e*x + a*f^2*(e*x - f*Sqrt[a + (e^2*x^2)/f^2]) + d*e*x*(-(e
*x) + f*Sqrt[a + (e^2*x^2)/f^2]))) + a*f^2*(-d^2 + a*f^2 - 2*d*e*x)*Log[-(
Sqrt[a]*f) + e*x + f*Sqrt[a + (e^2*x^2)/f^2]] + a*f^2*(d^2 - a*f^2 + 2*d*e
*x)*Log[-(a*f^2) + d*(e*x - f*Sqrt[a + (e^2*x^2)/f^2]) + Sqrt[a]*f*(d + e*
x + f*Sqrt[a + (e^2*x^2)/f^2])])/(d^3*e*(d^2 - a*f^2 + 2*d*e*x))
```

3.458.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2542, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^2}dx$$

$$\Big\downarrow \text{2542}$$

$$\frac{\int \frac{d^2-2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)d+af^2+\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2}{\left(-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex\right)^2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)}{2e}$$

$$\Big\downarrow \text{1195}$$

$$\frac{\int \left(\frac{2af^2}{d^3\left(-\sqrt{\frac{e^2x^2}{f^2}+af}-ex\right)}+\frac{2af^2}{d^3\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)}+\frac{af^2}{d^2\left(-\sqrt{\frac{e^2x^2}{f^2}+af}-ex\right)^2}+\frac{d^2+af^2}{d^2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2}\right)d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}}\right.}{2e}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{2af^2\log\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}{d^3}+\frac{2af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{d^3}+\frac{af^2}{d^2\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}-\frac{\frac{af^2}{d^2}+1}{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e}$$

input
```
Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-2),x]
```

output
```
((a*f^2)/(d^2*(-(e*x) - f*Sqrt[a + (e^2*x^2)/f^2])) - (1 + (a*f^2)/d^2)/(d
 + e*x + f*Sqrt[a + (e^2*x^2)/f^2]) - (2*a*f^2*Log[-(e*x) - f*Sqrt[a + (e^
2*x^2)/f^2]])/d^3 + (2*a*f^2*Log[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/d^3
)/(2*e)
```

3.458.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2542
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Simp[1/(2*e)   Subst[Int[(g + h*x^n)^p*((d^2 + a*f
^2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fr
eeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.458. $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx$

3.458.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3181 vs. 2(140) = 280.

Time = 0.08 (sec) , antiderivative size = 3182, normalized size of antiderivative = 21.07

method	result	size
default	Expression too large to display	3182

input
```
int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*d/(a*f^2-2*d*e*x-d^2)/e+1/2*a*f^2/(a*f^2-2*d*e*x-d^2)/e/d+2*e^2*(1/4/e
^2/d^2*x+1/4/e^3/d^3*(a*f^2-d^2)*ln(-a*f^2+2*d*e*x+d^2)-1/8*(a^2*f^4-2*a*d
^2*f^2+d^4)/e^3/d^3/(-a*f^2+2*d*e*x+d^2))+2*e*d*(1/4/e^2/d^2*ln(-a*f^2+2*d
*e*x+d^2)-1/4*(a*f^2-d^2)/(-a*f^2+2*d*e*x+d^2)/e^2/d^2)-1/2/d*f/e^2*(-4/(a
^2*f^4+2*a*d^2*f^2+d^4)*d^2*f^2/(x+1/2*(-a*f^2+d^2)/e/d)*(e^2*(x+1/2*(-a*f
^2+d^2)/e/d)^2/f^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/e/d)+1/4*(a^2*f
^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(3/2)+2*e*(a*f^2-d^2)*d/(a^2*f^4+2*a*d^2*f^2+
d^4)*(1/2*(4*e^2*(x+1/2*(-a*f^2+d^2)/e/d)^2/f^2+4*e*(a*f^2-d^2)/d/f^2*(x+1
/2*(-a*f^2+d^2)/e/d)+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)+1/2*e*(a*f^2
-d^2)/d/f^2*ln((1/2*e*(a*f^2-d^2)/d/f^2+e^2/f^2*(x+1/2*(-a*f^2+d^2)/e/d))/
(e^2/f^2)^(1/2)+(e^2*(x+1/2*(-a*f^2+d^2)/e/d)^2/f^2+e*(a*f^2-d^2)/d/f^2*(x
+1/2*(-a*f^2+d^2)/e/d)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(e^2/
f^2)^(1/2)-1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2/((a^2*f^4+2*a*d^2*f^2+d^4
)/d^2/f^2)^(1/2)*ln((1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2+e*(a*f^2-d^2)/d
/f^2*(x+1/2*(-a*f^2+d^2)/e/d)+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2
)*(4*e^2*(x+1/2*(-a*f^2+d^2)/e/d)^2/f^2+4*e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f
^2+d^2)/e/d)+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(x+1/2*(-a*f^2+d^2)
/e/d)))+8*e^2/(a^2*f^4+2*a*d^2*f^2+d^4)*d^2*(1/4*(2*e^2/f^2*(x+1/2*(-a*f^2
+d^2)/e/d)+e*(a*f^2-d^2)/d/f^2)/e^2*f^2*(e^2*(x+1/2*(-a*f^2+d^2)/e/d)^2/f^
2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/e/d)+1/4*(a^2*f^4+2*a*d^2*f^2...
```

3.458.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(139) = 278.

Time = 0.38 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.88

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx$$

$$= \frac{a^2f^4 - 2\,d^2e^2x^2 + ad^2f^2 - 2\,d^3ex + (a^2f^4 - 2\,adef^2x - ad^2f^2)\log\left(-aef^2x + 2\,de^2x^2 + adf^2 + (af^3 - 2\right.}{}$$

input
```
integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/2*(a^2*f^4 - 2*d^2*e^2*x^2 + a*d^2*f^2 - 2*d^3*e*x + (a^2*f^4 - 2*a*d*e*
f^2*x - a*d^2*f^2)*log(-a*e*f^2*x + 2*d*e^2*x^2 + a*d*f^2 + (a*f^3 - 2*d*e
*f*x)*sqrt((e^2*x^2 + a*f^2)/f^2)) + (a^2*f^4 - 2*a*d*e*f^2*x - a*d^2*f^2)
*log(-a*f^2 + 2*d*e*x + d^2) - (a^2*f^4 - 2*a*d*e*f^2*x - a*d^2*f^2)*log(-
e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) - d) - 2*(a*d*f^3 - d^2*e*f*x)*sqrt((e
^2*x^2 + a*f^2)/f^2))/(a*d^3*e*f^2 - 2*d^4*e^2*x - d^5*e)
```

3.458.6 Sympy [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx=\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**2,x)
```

output
```
Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-2), x)
```

3.458.7 Maxima [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx=\int \frac{1}{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^2}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-2), x)
```

3.458.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx = \frac{af^2\log\left(\left|-af^2+2\,dex+d^2\right|\right)}{2\,d^3e} + \frac{af|f|\log\left(\left|-x|e|+\sqrt{e^2x^2+af^2}\right|\right)}{2\,d^3|e|} + \frac{x}{2\,d^2} - \frac{\sqrt{e^2x^2+af^2}|f|}{2\,d^2ef} + \frac{a^2f^4+2\,ad^2f^2+d^4}{4\,(af^2-2\,dex-d^2)d^3e}$$

input
```
integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")
```

output
```
1/2*a*f^2*log(abs(-a*f^2 + 2*d*e*x + d^2))/(d^3*e) + 1/2*a*f*abs(f)*log(ab
s(-x*abs(e) + sqrt(e^2*x^2 + a*f^2)))/(d^3*abs(e)) + 1/2*x/d^2 - 1/2*sqrt(
e^2*x^2 + a*f^2)*abs(f)/(d^2*e*f) + 1/4*(a^2*f^4 + 2*a*d^2*f^2 + d^4)/((a*
f^2 - 2*d*e*x - d^2)*d^3*e)
```

3.458.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx = \int \frac{1}{\left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)^2}\,dx$$

input
```
int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^2,x)
```

output
```
int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^2, x)
```

3.459 $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}\,dx$

3.459.1 Optimal result

Integrand size = 25, antiderivative size = 193

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}\,dx = -\frac{af^2}{2d^3e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{1+\frac{af^2}{d^2}}{4e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2} - \frac{af^2}{d^3e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{3af^2\log\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2d^4e} + \frac{3af^2\log\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2d^4e}$$

output
```
-3/2*a*f^2*ln(e*x+f*(a+e^2*x^2/f^2)^(1/2))/d^4/e+3/2*a*f^2*ln(d+e*x+f*(a+e
^2*x^2/f^2)^(1/2))/d^4/e-1/2*a*f^2/d^3/e/(e*x+f*(a+e^2*x^2/f^2)^(1/2))+1/4
*(-1-a*f^2/d^2)/e/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2-a*f^2/d^3/e/(d+e*x+f*(
a+e^2*x^2/f^2)^(1/2))
```

3.459.2 Mathematica [A] (verified)

Time = 1.91 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.44

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}\,dx = -\frac{\frac{d\sqrt{a+\frac{e^2x^2}{f^2}}\left(3a^2f^5+d^2efx(3d+4ex)-adf^3(5d+9ex)\right)}{(d^2-af^2+2dex)^2}+\frac{d\left(2d^5+6d^4ex-3a^2ef^4x+3d^3e^2x^2+9ade^2f^2x^2+d^2\left(3aef^2x-4e^3x^3\right)\right)}{(d^2-af^2+2dex)^2}+3af^2\text{l}\cdots}{}$$

input
```
Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3),x]
```

output
```
-1/2*((d*Sqrt[a + (e^2*x^2)/f^2]*(3*a^2*f^5 + d^2*e*f*x*(3*d + 4*e*x) - a*
d*f^3*(5*d + 9*e*x)))/(d^2 - a*f^2 + 2*d*e*x)^2 + (d*(2*d^5 + 6*d^4*e*x -
3*a^2*e*f^4*x + 3*d^3*e^2*x^2 + 9*a*d*e^2*f^2*x^2 + d^2*(3*a*e*f^2*x - 4*e
^3*x^3)))/(d^2 - a*f^2 + 2*d*e*x)^2 + 3*a*f^2*Log[-(Sqrt[a]*f) + e*x + f*S
qrt[a + (e^2*x^2)/f^2]] - 3*a*f^2*Log[-(a*f^2) + d*(e*x - f*Sqrt[a + (e^2*
x^2)/f^2]) + Sqrt[a]*f*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])])/(d^4*e)
```

3.459.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2542, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^3}dx$$

$$\Big\downarrow \text{2542}$$

$$\frac{\int \frac{d^2-2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)d+af^2+\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2}{\left(-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex\right)^2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^3}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)}{2e}$$

$$\Big\downarrow \text{1195}$$

$$\frac{\int \left(\frac{3af^2}{d^4\left(-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex\right)}+\frac{3af^2}{d^4\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)}+\frac{af^2}{d^3\left(-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex\right)^2}+\frac{2af^2}{d^3\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2}+\frac{d^2+af^2}{d^2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a\right.}}\right.}{2e}$$

$\downarrow$ 2009

$$\frac{-\frac{3af^2\log\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}{d^4}+\frac{3af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{d^4}+\frac{af^2}{d^3\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}-\frac{2af^2}{d^3\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}-\frac{}{2\left(f\sqrt{a+\right.}}}{2e}$$

input
```
Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3),x]
```

output
```
((a*f^2)/(d^3*(-(e*x) - f*Sqrt[a + (e^2*x^2)/f^2])) - (1 + (a*f^2)/d^2)/(2
*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2) - (2*a*f^2)/(d^3*(d + e*x + f*Sq
rt[a + (e^2*x^2)/f^2])) - (3*a*f^2*Log[-(e*x) - f*Sqrt[a + (e^2*x^2)/f^2]]
)/d^4 + (3*a*f^2*Log[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/d^4)/(2*e)
```

3.459.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2542
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Simp[1/(2*e)   Subst[Int[(g + h*x^n)^p*((d^2 + a*f
^2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fr
eeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.459. $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}\,dx$

3.459.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 11351 vs. 2(176) = 352.

Time = 0.13 (sec) , antiderivative size = 11352, normalized size of antiderivative = 58.82

method	result	size
default	Expression too large to display	11352

input
```
int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.459.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. 2(175) = 350.

Time = 0.55 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.78

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}\,dx = \frac{5\,a^3f^6+8\,d^3e^3x^3-6\,a^2d^2f^4-3\,ad^4f^2+2\,(ad^2e^2f^2+5\,d^4e^2)x^2-2\,(7\,a^2def^4+ad^3ef^2-2\,d^5e)x+3\,(a^3}{}$$

input
```
integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="fricas")
```

output
```
1/4*(5*a^3*f^6 + 8*d^3*e^3*x^3 - 6*a^2*d^2*f^4 - 3*a*d^4*f^2 + 2*(a*d^2*e^
2*f^2 + 5*d^4*e^2)*x^2 - 2*(7*a^2*d*e*f^4 + a*d^3*e*f^2 - 2*d^5*e)*x + 3*(
a^3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d^2*f^4 + a*d^4*f^2 - 4*(a^2*d*e*f^4
 - a*d^3*e*f^2)*x)*log(-a*e*f^2*x + 2*d*e^2*x^2 + a*d*f^2 + (a*f^3 - 2*d*e
*f*x)*sqrt((e^2*x^2 + a*f^2)/f^2)) + 3*(a^3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*
a^2*d^2*f^4 + a*d^4*f^2 - 4*(a^2*d*e*f^4 - a*d^3*e*f^2)*x)*log(-a*f^2 + 2*
d*e*x + d^2) - 3*(a^3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d^2*f^4 + a*d^4*f^
2 - 4*(a^2*d*e*f^4 - a*d^3*e*f^2)*x)*log(-e*x + f*sqrt((e^2*x^2 + a*f^2)/f
^2) - d) - 2*(3*a^2*d*f^5 + 4*d^3*e^2*f*x^2 - 5*a*d^3*f^3 - 3*(3*a*d^2*e*f
^3 - d^4*e*f)*x)*sqrt((e^2*x^2 + a*f^2)/f^2))/(a^2*d^4*e*f^4 + 4*d^6*e^3*x
^2 - 2*a*d^6*e*f^2 + d^8*e - 4*(a*d^5*e^2*f^2 - d^7*e^2)*x)
```

3.459.6 Sympy [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}\,dx=\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**3,x)
```

output
```
Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-3), x)
```

3.459.7 Maxima [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}\,dx=\int \frac{1}{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^3}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-3), x)
```

3.459.8 Giac [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.94

$$\begin{aligned}&\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}\,dx\\&=\frac{3\,af^2\log\left(\left|-af^2+2\,dex+d^2\right|\right)}{4\,d^4e}+\frac{3\,af|f|\log\left(\left|-x|e|+\sqrt{e^2x^2+af^2}\right|\right)}{4\,d^4|e|}+\frac{x}{2\,d^3}\\&\quad-\frac{\sqrt{e^2x^2+af^2}|f|}{2\,d^3ef}+\frac{5\,a^3f^6-3\,a^2d^2f^4-9\,ad^4f^2-d^6-12\,(a^2def^4+ad^3ef^2)x}{8\,(af^2-2\,dex-d^2)^2d^4e}\end{aligned}$$

input
```
integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")
```

output
```
3/4*a*f^2*log(abs(-a*f^2 + 2*d*e*x + d^2))/(d^4*e) + 3/4*a*f*abs(f)*log(ab
s(-x*abs(e) + sqrt(e^2*x^2 + a*f^2)))/(d^4*abs(e)) + 1/2*x/d^3 - 1/2*sqrt(
e^2*x^2 + a*f^2)*abs(f)/(d^3*e*f) + 1/8*(5*a^3*f^6 - 3*a^2*d^2*f^4 - 9*a*d
^4*f^2 - d^6 - 12*(a^2*d*e*f^4 + a*d^3*e*f^2)*x)/((a*f^2 - 2*d*e*x - d^2)^
2*d^4*e)
```

3.459.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}\,dx=\int \frac{1}{\left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)^3}\,dx$$

input
```
int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^3,x)
```

output
```
int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^3, x)
```

3.460 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2} dx$

3.460.1 Optimal result

Integrand size = 27, antiderivative size = 225

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = \frac{2adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{ad^2f^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{af^2\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e} + \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{7/2}}{7e} - \frac{5ad^{3/2}f^2\text{arctanh}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2e}$$

output
```
-5/2*a*d^(3/2)*f^2*arctanh((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/d^(1/2))/
e+1/3*a*f^2*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2)/e+1/7*(d+e*x+f*(a+e^2*x^
2/f^2)^(1/2))^(7/2)/e+2*a*d*f^2*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/e-1/
2*a*d^2*f^2*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/e/(e*x+f*(a+e^2*x^2/f^2)
^(1/2))
```

3.460.2 Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.94

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = \frac{\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\left(20a^2f^4+6(d+2ex)^3\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)+af^2\left(-3d^2+4ex\left(19ex+13f\sqrt{a+\frac{e^2x^2}{f^2}}\right)+4d\left(38\right.\right.\right.}{ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{42e}$$

input
```
Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2),x]
```

output
```
((Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]*(20*a^2*f^4 + 6*(d + 2*e*x)^3*
(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + a*f^2*(-3*d^2 + 4*e*x*(19*e*x + 13*f*S
qrt[a + (e^2*x^2)/f^2]) + 4*d*(38*e*x + 29*f*Sqrt[a + (e^2*x^2)/f^2]))))/(
e*x + f*Sqrt[a + (e^2*x^2)/f^2]) - 105*a*d^(3/2)*f^2*ArcTanh[Sqrt[d + e*x
+ f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(42*e)
```

3.460.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2542, 1192, 1580, 25, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2} dx$$

$$\Big\downarrow \text{2542}$$

$$\frac{\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^{5/2}\left(d^2-2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)d+af^2+\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2\right)}{\left(-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex\right)^2}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)}{2e}$$

$$\Big\downarrow \text{1192}$$

$$\frac{\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^3\left(d^2-2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)d+af^2+\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2\right)}{\left(-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex\right)^2}d\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}}{e}$$

$\Big\downarrow$ 1580

$$\frac{\frac{1}{2}\int -\frac{2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^4-2d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^3+2af^2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2+2adf^2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)+ad^2f^2}{-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex}d\sqrt{d+ex+}}{e}$$

$\Big\downarrow$ 25

$$\frac{\frac{ad^2f^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}-\frac{1}{2}\int \frac{2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^4-2d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^3+2af^2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2+2adf^2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}}\right.}{-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex}}{e}$$

$\Big\downarrow$ 2341

$$\frac{\frac{ad^2f^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}-\frac{1}{2}\int \left(-2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^3-2af^2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)-4adf^2+\frac{5ad^2f^2}{-\sqrt{\frac{e^2x^2}{f^2}+a}f}\right.}{e}$$

$\Big\downarrow$ 2009

$$\frac{\frac{1}{2}\left(-5ad^{3/2}f^2\text{arctanh}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)+\frac{2}{7}\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{7/2}+\frac{2}{3}af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}+4\right.}{e}$$

input
```
Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2),x]
```

output
```
((a*d^2*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*(-(e*x) - f*Sqrt
[a + (e^2*x^2)/f^2])) + (4*a*d*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2
]] + (2*a*f^2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2))/3 + (2*(d + e*x
 + f*Sqrt[a + (e^2*x^2)/f^2])^(7/2))/7 - 5*a*d^(3/2)*f^2*ArcTanh[Sqrt[d +
e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/2)/e
```

3.460. $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}dx$

3.460.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2542
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Simp[1/(2*e)   Subst[Int[(g + h*x^n)^p*((d^2 + a*f
^2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fr
eeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.460.4 Maple [F]

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}dx$$

input
```
int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x)
```

3.460. $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2} dx$

output
```
int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x)
```

3.460.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.85

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = \left[\frac{105\, ad^{\frac{3}{2}} f^2 \log\left(af^2-2\,dex+2\,df\sqrt{\frac{e^2x^2+af^2}{f^2}}+2\left(\sqrt{d}ex-\sqrt{d}f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex\right.\right.}{}\right.$$

input
```
integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="fricas")
```

output
```
[1/84*(105*a*d^(3/2)*f^2*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^2*x^2 + a*f^2
)/f^2) + 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x
+ f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)) + 2*(24*e^3*x^3 + 36*d*e^2*x^2 + 116
*a*d*f^2 + 6*d^3 + (32*a*e*f^2 + 39*d^2*e)*x + (24*e^2*f*x^2 + 20*a*f^3 +
36*d*e*f*x - 3*d^2*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*
x^2 + a*f^2)/f^2) + d))/e, 1/42*(105*a*sqrt(-d)*d*f^2*arctan(sqrt(e*x + f*
sqrt((e^2*x^2 + a*f^2)/f^2) + d)*sqrt(-d)/d) + (24*e^3*x^3 + 36*d*e^2*x^2
+ 116*a*d*f^2 + 6*d^3 + (32*a*e*f^2 + 39*d^2*e)*x + (24*e^2*f*x^2 + 20*a*f
^3 + 36*d*e*f*x - 3*d^2*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt(
(e^2*x^2 + a*f^2)/f^2) + d))/e]
```

3.460.6 Sympy [F]

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = \int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}} dx$$

input
```
integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(5/2),x)
```

output
```
Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(5/2), x)
```

3.460.7 Maxima [F]

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = \int \left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{5}{2}} dx$$

input
```
integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(5/2), x)
```

3.460.8 Giac [F]

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = \int \left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{5}{2}} dx$$

input
```
integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="giac")
```

output
```
integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(5/2), x)
```

3.460.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = \int \left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)^{5/2} dx$$

input
```
int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(5/2),x)
```

output
```
int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(5/2), x)
```

3.461 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2} dx$

3.461.1 Optimal result

Integrand size = 27, antiderivative size = 183

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}{5e} - \frac{3a\sqrt{d}f^2\text{arctanh}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2e}$$

output
```
-3/2*a*f^2*arctanh((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/d^(1/2))*d^(1/2)/
e+1/5*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2)/e+a*f^2*(d+e*x+f*(a+e^2*x^2/f^
2)^(1/2))^(1/2)/e-1/2*a*d*f^2*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/e/(e*x
+f*(a+e^2*x^2/f^2)^(1/2))
```

3.461.2 Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.93

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \frac{\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\left(2(d+2ex)^2\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)+af^2\left(-d+16ex+12f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)}{ex+f\sqrt{a+\frac{e^2x^2}{f^2}}} - 15a\sqrt{d}f^2\text{arct}}{10e}$$

input
```
Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2),x]
```

output
```
((Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]*(2*(d + 2*e*x)^2*(e*x + f*Sqrt
[a + (e^2*x^2)/f^2]) + a*f^2*(-d + 16*e*x + 12*f*Sqrt[a + (e^2*x^2)/f^2]))
)/(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) - 15*a*Sqrt[d]*f^2*ArcTanh[Sqrt[d + e*
x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(10*e)
```

3.461.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2542, 1192, 1580, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2} dx$$

$$\Big\downarrow \text{2542}$$

$$\frac{\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^{3/2}\left(d^2-2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)d+af^2+\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2\right)}{\left(-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex\right)^2}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)}{2e}$$

$$\Big\downarrow \text{1192}$$

$$\frac{\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2\left(d^2-2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)d+af^2+\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2\right)}{\left(-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex\right)^2}d\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}}{e}$$

$\downarrow$ 1580

$$\frac{\frac{adf^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}-\frac{1}{2}\int \frac{2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^3-2d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2+2af^2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)+adf^2}{-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex}d\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}}{e}$$

$\downarrow$ 2341

$$\frac{\frac{adf^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}-\frac{1}{2}\int \left(-2af^2+\frac{3adf^2}{-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex}-2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2\right)d\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}}{e}$$

$\downarrow$ 2009

$$\frac{\frac{1}{2}\left(-3a\sqrt{d}f^2\text{arctanh}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)+\frac{2}{5}\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}+2af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}\right)+\frac{adf^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}}{e}$$

input
```
Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2),x]
```

output
```
((a*d*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*(-(e*x) - f*Sqrt[a
 + (e^2*x^2)/f^2])) + (2*a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]] +
 (2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2))/5 - 3*a*Sqrt[d]*f^2*ArcTa
nh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/2)/e
```

3.461. $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2} dx$

3.461.3.1 Defintions of rubi rules used

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2542
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Simp[1/(2*e)   Subst[Int[(g + h*x^n)^p*((d^2 + a*f
^2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fr
eeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.461.4 Maple [F]

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}dx$$

input
```
int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x)
```

output
```
int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x)
```

3.461.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.84

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \left[\frac{15\,a\sqrt{d}f^2\log\left(af^2-2\,dex+2\,df\sqrt{\frac{e^2x^2+af^2}{f^2}}+2\left(\sqrt{d}ex-\sqrt{d}f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex\cdots}\right.}{\cdots}\right.$$

input
```
integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
[1/20*(15*a*sqrt(d)*f^2*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^2*x^2 + a*f^2)
/f^2) + 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x +
 f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)) + 2*(4*e^2*x^2 + 12*a*f^2 + 9*d*e*x +
 2*d^2 + (4*e*f*x - d*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e
^2*x^2 + a*f^2)/f^2) + d))/e, 1/10*(15*a*sqrt(-d)*f^2*arctan(sqrt(e*x + f*
sqrt((e^2*x^2 + a*f^2)/f^2) + d)*sqrt(-d)/d) + (4*e^2*x^2 + 12*a*f^2 + 9*d
*e*x + 2*d^2 + (4*e*f*x - d*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*s
qrt((e^2*x^2 + a*f^2)/f^2) + d))/e]
```

3.461.6 Sympy [F]

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}} dx$$

input
```
integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(3/2),x)
```

output
```
Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(3/2), x)
```

3.461.7 Maxima [F]

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \int \left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{3}{2}} dx$$

input
```
integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(3/2), x)
```

3.461.8 Giac [F]

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \int \left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{3}{2}} dx$$

input
```
integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(3/2), x)
```

3.461.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \int \left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)^{3/2} dx$$

input
```
int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(3/2),x)
```

output
```
int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(3/2), x)
```

3.462 $\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$

3.462.1 Optimal result

Integrand size = 27, antiderivative size = 147

$$\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx=-\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e}-\frac{af^2\text{arctanh}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2\sqrt{d}e}$$

output
```
-1/2*a*f^2*arctanh((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/d^(1/2))/e/d^(1/2
)+1/3*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2)/e-1/2*a*f^2*(d+e*x+f*(a+e^2*x^
2/f^2)^(1/2))^(1/2)/e/(e*x+f*(a+e^2*x^2/f^2)^(1/2))
```

3.462.2 Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.97

$$\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$$
$$=\frac{\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\left(-af^2+2(d+2ex)\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)}{ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}-\frac{3af^2\text{arctanh}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{\sqrt{d}}}{6e}$$

input
```
Integrate[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```

output
```
((Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]*(-(a*f^2) + 2*(d + 2*e*x)*(e*x
 + f*Sqrt[a + (e^2*x^2)/f^2])))/(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) - (3*a*f
^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/Sqrt[d])/(6
*e)
```

3.462.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2542, 1192, 1580, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}\,dx$$
$$\Big\downarrow \text{2542}$$
$$\frac{\int \frac{\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}\left(d^2-2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)d+af^2+\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2\right)}{\left(-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex\right)^2}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)}{2e}$$
$$\Big\downarrow \text{1192}$$

$$\frac{\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)\left(d^2-2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)d+af^2+\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2\right)}{\left(-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex\right)^2}d\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}}{e}$$

$$\Big\downarrow \text{1580}$$

$$\frac{\frac{1}{2}\int -\frac{af^2+2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2-2d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)}{-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex}d\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}+\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}}{e}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}-\frac{1}{2}\int \frac{af^2+2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2-2d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)}{-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex}d\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}}{e}$$

$$\Big\downarrow \text{1467}$$

$$\frac{\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}-\frac{1}{2}\int \left(\frac{af^2}{-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex}-2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)\right)d\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\left(\frac{2}{3}\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}-\frac{af^2\text{arctanh}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}\right)+\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}}{e}$$

input
```
Int[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```

output
```
((a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*(-(e*x) - f*Sqrt[a +
 (e^2*x^2)/f^2])) + ((2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2))/3 - (
a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/Sqrt[d])
/2)/e
```

3.462. $\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$

3.462.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2542
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Simp[1/(2*e)   Subst[Int[(g + h*x^n)^p*((d^2 + a*f
^2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fr
eeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.462. $\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$

3.462.4 Maple [F]

$$\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}dx$$

input
```
int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x)
```

output
```
int((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x)
```

3.462.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.05

$$\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$$
$$= \left[\frac{3\,a\sqrt{d}f^2\log\left(af^2-2\,dex+2\,df\sqrt{\frac{e^2x^2+af^2}{f^2}}+2\left(\sqrt{d}ex-\sqrt{d}f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\right)+2}{12\,de}\right.$$

input
```
integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*a*sqrt(d)*f^2*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^2*x^2 + a*f^2)/
f^2) + 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x +
f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)) + 2*(5*d*e*x - d*f*sqrt((e^2*x^2 + a*f
^2)/f^2) + 2*d^2)*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/(d*e), 1/
6*(3*a*sqrt(-d)*f^2*arctan(sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)*s
qrt(-d)/d) + (5*d*e*x - d*f*sqrt((e^2*x^2 + a*f^2)/f^2) + 2*d^2)*sqrt(e*x
+ f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/(d*e)]
```

3.462.6 Sympy [F]

$$\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx = \int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$$

input
```
integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(1/2),x)
```

output
```
Integral(sqrt(d + e*x + f*sqrt(a + e**2*x**2/f**2)), x)
```

3.462.7 Maxima [F]

$$\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx = \int \sqrt{ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d}\,dx$$

input
```
integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

3.462.8 Giac [F]

$$\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx = \int \sqrt{ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d}\,dx$$

input
```
integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

3.462.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx=\int \sqrt{d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}}\,dx$$

input
```
int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(1/2),x)
```

output
```
int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(1/2), x)
```

3.463 $\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx$

3.463.1 Optimal result

Integrand size = 27, antiderivative size = 147

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx = \frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2de\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{af^2\text{arctanh}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2d^{3/2}e}$$

output
```
1/2*a*f^2*arctanh((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/d^(1/2))/d^(3/2)/e
+(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/e-1/2*a*f^2*(d+e*x+f*(a+e^2*x^2/f^2
)^(1/2))^(1/2)/d/e/(e*x+f*(a+e^2*x^2/f^2)^(1/2))
```

3.463.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx$$
$$= \frac{\frac{\sqrt{d}\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\left(-af^2+2d\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)}{ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}+af^2\text{arctanh}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2d^{3/2}e}$$

input
```
Integrate[1/Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```

output
```
((Sqrt[d]*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]*(-(a*f^2) + 2*d*(e*x +
 f*Sqrt[a + (e^2*x^2)/f^2])))/(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + a*f^2*Ar
cTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*d^(3/2)*e)
```

3.463.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2542, 1192, 1471, 25, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}dx$$
$$\Big\downarrow \text{2542}$$
$$\frac{\int \frac{d^2-2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)d+af^2+\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2}{\left(-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex\right)^2\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)}{2e}$$
$$\Big\downarrow \text{1192}$$

$$\frac{\int \frac{d^2-2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)d+af^2+\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2}{\left(-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex\right)^2}d\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}}{e}$$

$$\Big\downarrow \; 1471$$

$$\frac{\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}-\frac{\int -\frac{2d^2-2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)d+af^2}{-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex}d\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}}{2d}}{e}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{2d^2-2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)d+af^2}{-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex}d\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}}{2d}+\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}}{e}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{af^2\int \frac{1}{-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex}d\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}+2d\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d}+\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}}{e}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{af^2\text{arctanh}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}+2d\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d}+\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}}{e}$$

input
```
Int[1/Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```

output
```
((a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d*(-(e*x) - f*Sqrt[a
 + (e^2*x^2)/f^2])) + (2*d*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]] + (a*
f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/Sqrt[d])/(
2*d))/e
```

3.463. $\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx$

3.463.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2542
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Simp[1/(2*e)   Subst[Int[(g + h*x^n)^p*((d^2 + a*f
^2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fr
eeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.463. $\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx$

3.463.4 Maple [F]

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}dx$$

input
```
int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x)
```

output
```
int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x)
```

3.463.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.03

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx = \left[\frac{a\sqrt{d}f^2\log\left(af^2-2\,dex+2\,df\sqrt{\frac{e^2x^2+af^2}{f^2}}-2\left(\sqrt{d}ex-\sqrt{d}f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\right)+2\,(}{4\,d^2e} - \frac{a\sqrt{-d}f^2\arctan\left(\frac{\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\sqrt{-d}}{d}\right)-\left(dex-df\sqrt{\frac{e^2x^2+af^2}{f^2}}+2\,d^2\right)\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}}{2\,d^2e}\right]$$

input
```
integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(a*sqrt(d)*f^2*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^2*x^2 + a*f^2)/f^2
) - 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*s
qrt((e^2*x^2 + a*f^2)/f^2) + d)) + 2*(d*e*x - d*f*sqrt((e^2*x^2 + a*f^2)/f
^2) + 2*d^2)*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/(d^2*e), -1/2*
(a*sqrt(-d)*f^2*arctan(sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)*sqrt(
-d)/d) - (d*e*x - d*f*sqrt((e^2*x^2 + a*f^2)/f^2) + 2*d^2)*sqrt(e*x + f*sq
rt((e^2*x^2 + a*f^2)/f^2) + d))/(d^2*e)]
```

3.463.6 Sympy [F]

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx = \int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(1/2),x)
```

output
```
Integral(1/sqrt(d + e*x + f*sqrt(a + e**2*x**2/f**2)), x)
```

3.463.7 Maxima [F]

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx = \int \frac{1}{\sqrt{ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d}}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

3.463.8 Giac [F]

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx=\int \frac{1}{\sqrt{ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d}}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

3.463.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx=\int \frac{1}{\sqrt{d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}}}\,dx$$

input
```
int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(1/2),x)
```

output
```
int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(1/2), x)
```

3.464 $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx$

3.464.1 Optimal result

Integrand size = 27, antiderivative size = 158

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx=-\frac{1+\frac{af^2}{d^2}}{e\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}-\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{3af^2\operatorname{arctanh}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2d^{5/2}e}$$

output
```
3/2*a*f^2*arctanh((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/d^(1/2))/d^(5/2)/e
+(-1-a*f^2/d^2)/e/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)-1/2*a*f^2*(d+e*x+f
*(a+e^2*x^2/f^2)^(1/2))^(1/2)/d^2/e/(e*x+f*(a+e^2*x^2/f^2)^(1/2))
```

3.464.2 Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = \frac{-\frac{\sqrt{d}\left(2d^2\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)+af^2\left(d+3ex+3f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)}{\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}+3af^2\text{arctanh}\left(\frac{\sqrt{d+ex+\ldots}}{\ldots}\right)}{2d^{5/2}e}$$

input
```
Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3/2),x]
```

output
```
(-((Sqrt[d]*(2*d^2*(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + a*f^2*(d + 3*e*x +
3*f*Sqrt[a + (e^2*x^2)/f^2])))/((e*x + f*Sqrt[a + (e^2*x^2)/f^2])*Sqrt[d +
 e*x + f*Sqrt[a + (e^2*x^2)/f^2]])) + 3*a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqr
t[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*d^(5/2)*e)
```

3.464.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2542, 1192, 1582, 25, 359, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}dx$$

$$\Big\downarrow \text{2542}$$

$$\frac{\int \frac{d^2-2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)d+af^2+\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2}{\left(-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex\right)^2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^{3/2}}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)}{2e}$$

$$\Big\downarrow \text{1192}$$

$$\frac{\int \frac{d^2-2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)d+af^2+\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2}{\left(-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex\right)^2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)}d\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}}{e}$$

$$\Big\downarrow \ 1582$$

$$\frac{\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^2\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}-\frac{\int -\frac{2d\left(d^2+af^2\right)-\left(2d^2-af^2\right)\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)}{\left(-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex\right)\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)}d\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}}{2d^2}}{e}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{2d\left(d^2+af^2\right)-\left(2d^2-af^2\right)\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)}{\left(-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex\right)\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)}d\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}}{2d^2}+\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^2\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}}{e}$$

$$\Big\downarrow \ 359$$

$$\frac{\frac{3af^2\int \frac{1}{-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex}d\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}-\frac{2\left(af^2+d^2\right)}{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}}{2d^2}+\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^2\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}}{e}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\frac{3af^2\text{arctanh}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{2\left(af^2+d^2\right)}{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}}{2d^2}+\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^2\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}}{e}$$

input
```
Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3/2),x]
```

output
```
((a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^2*(-(e*x) - f*Sqrt
[a + (e^2*x^2)/f^2])) + ((-2*(d^2 + a*f^2))/Sqrt[d + e*x + f*Sqrt[a + (e^2
*x^2)/f^2]] + (3*a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/S
qrt[d]])/Sqrt[d])/(2*d^2))/e
```

3.464. $\displaystyle\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx$

3.464.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 2542
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Simp[1/(2*e)   Subst[Int[(g + h*x^n)^p*((d^2 + a*f
^2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fr
eeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.464. $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx$

3.464.4 Maple [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}}\,dx$$

input
```
int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x)
```

output
```
int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x)
```

3.464.5 Fricas [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 487, normalized size of antiderivative = 3.08

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = \left[\frac{3\left(a^2f^4-2\,adef^2x-ad^2f^2\right)\sqrt{d}\log\left(af^2-2\,dex+2\,df\sqrt{\frac{e^2x^2+af^2}{f^2}}-\cdots\right.}{\cdots}\right.$$

$$-\frac{3\left(a^2f^4-2\,adef^2x-ad^2f^2\right)\sqrt{-d}\arctan\left(\frac{\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\sqrt{-d}}{d}\right)+\left(2\,d^2e^2x^2-2\,ad^2f^2-2\,d^4-(3\,ade\cdots\right.}{2\left(ad^3ef^2-2\,d^4e^2x-d^5e\right)}$$

input
```
integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(3*(a^2*f^4 - 2*a*d*e*f^2*x - a*d^2*f^2)*sqrt(d)*log(a*f^2 - 2*d*e*x
+ 2*d*f*sqrt((e^2*x^2 + a*f^2)/f^2) - 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2
*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)) - 2*(2*
d^2*e^2*x^2 - 2*a*d^2*f^2 - 2*d^4 - (3*a*d*e*f^2 + d^3*e)*x + (3*a*d*f^3 -
 2*d^2*e*f*x + d^3*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*
x^2 + a*f^2)/f^2) + d))/(a*d^3*e*f^2 - 2*d^4*e^2*x - d^5*e), -1/2*(3*(a^2*
f^4 - 2*a*d*e*f^2*x - a*d^2*f^2)*sqrt(-d)*arctan(sqrt(e*x + f*sqrt((e^2*x^
2 + a*f^2)/f^2) + d)*sqrt(-d)/d) + (2*d^2*e^2*x^2 - 2*a*d^2*f^2 - 2*d^4 -
(3*a*d*e*f^2 + d^3*e)*x + (3*a*d*f^3 - 2*d^2*e*f*x + d^3*f)*sqrt((e^2*x^2
+ a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/(a*d^3*e*f^2
 - 2*d^4*e^2*x - d^5*e)]
```

3.464.6 Sympy [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = \int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(3/2),x)
```

output
```
Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-3/2), x)
```

3.464.7 Maxima [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = \int \frac{1}{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-3/2), x)
```

3.464.8 Giac [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = \int \frac{1}{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-3/2), x)
```

3.464.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = \int \frac{1}{\left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)^{3/2}}\,dx$$

input
```
int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(3/2),x)
```

output
```
int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(3/2), x)
```

3.465 $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx$

3.465.1 Optimal result

Integrand size = 27, antiderivative size = 199

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx = -\frac{1+\frac{af^2}{d^2}}{3e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}} - \frac{2af^2}{d^3e\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}} - \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^3e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{5af^2\,\text{arctanh}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2d^{7/2}e}$$

output
```
5/2*a*f^2*arctanh((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/d^(1/2))/d^(7/2)/e
+1/3*(-1-a*f^2/d^2)/e/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2)-2*a*f^2/d^3/e/
(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)-1/2*a*f^2*(d+e*x+f*(a+e^2*x^2/f^2)^(
1/2))^(1/2)/d^3/e/(e*x+f*(a+e^2*x^2/f^2)^(1/2))
```

3.465.2 Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.05

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx = \frac{-\frac{\sqrt{d}\left(15a^2f^4+2d^3\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)+af^2\left(3d^2+20d\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)+30ex\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right.\right.\right.}{\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}}{6d^{7/2}e}$$

input
```
Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-5/2),x]
```

output
```
(-((Sqrt[d]*(15*a^2*f^4 + 2*d^3*(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + a*f^2*
(3*d^2 + 20*d*(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + 30*e*x*(e*x + f*Sqrt[a +
 (e^2*x^2)/f^2]))))/((e*x + f*Sqrt[a + (e^2*x^2)/f^2])*(d + e*x + f*Sqrt[a
 + (e^2*x^2)/f^2])^(3/2))) + 15*a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e
^2*x^2)/f^2]]/Sqrt[d]])/(6*d^(7/2)*e)
```

3.465.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2542, 1192, 1582, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 2542}$$

$$\frac{\int \frac{d^2-2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)d+af^2+\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2}{\left(-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex\right)^2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^{5/2}}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)}{2e}$$

$$\Big\downarrow \text{ 1192}$$

$$\frac{\int \frac{d^2-2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)d+af^2+\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2}{\left(-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex\right)^2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2}d\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}}{e}$$

$$\downarrow \text{ 1582}$$

$$\frac{\frac{\int \frac{2\left(d^2+af^2\right)d^2-2\left(d^2-af^2\right)\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)d+af^2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2}{\left(-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex\right)\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2}d\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}}{2d^3}+\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^3\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}}{e}$$

$$\downarrow \text{ 1584}$$

$$\frac{\frac{\int \left(\frac{5af^2}{-\sqrt{\frac{e^2x^2}{f^2}+a}f-ex}+\frac{4af^2}{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}+\frac{2\left(d^3+af^2d\right)}{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}\right)^2}\right)d\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}}{2d^3}+\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^3\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}}{e}$$

$$\downarrow \text{ 2009}$$

$$\frac{\frac{\frac{5af^2\text{arctanh}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{2d\left(af^2+d^2\right)}{3\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}-\frac{4af^2}{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}}{2d^3}+\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^3\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}}{e}$$

input
```
Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-5/2),x]
```

output
```
((a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^3*(-(e*x) - f*Sqrt
[a + (e^2*x^2)/f^2])) + ((-2*d*(d^2 + a*f^2))/(3*(d + e*x + f*Sqrt[a + (e^
2*x^2)/f^2])^(3/2)) - (4*a*f^2)/Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]
+ (5*a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/Sqr
t[d])/(2*d^3))/e
```

3.465. $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx$

3.465.3.1 Defintions of rubi rules used

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2542
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Simp[1/(2*e)   Subst[Int[(g + h*x^n)^p*((d^2 + a*f
^2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fr
eeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.465.4 Maple [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x)
```

output
```
int(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x)
```

3.465.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. 2(171) = 342.

Time = 0.66 (sec) , antiderivative size = 812, normalized size of antiderivative = 4.08

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx = \left[\frac{15\left(a^3f^6+4\,ad^2e^2f^2x^2-2\,a^2d^2f^4+ad^4f^2-4\left(a^2def^4-ad^3ef^2\right)x\right)\sqrt{}}{}\right.$$

$$-\frac{15\left(a^3f^6+4\,ad^2e^2f^2x^2-2\,a^2d^2f^4+ad^4f^2-4\left(a^2def^4-ad^3ef^2\right)x\right)\sqrt{-d}\arctan\left(\frac{\sqrt{ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}+d}\sqrt{-d}}{d}\right)}{}$$

input
```
integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(15*(a^3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d^2*f^4 + a*d^4*f^2 - 4*(
a^2*d*e*f^4 - a*d^3*e*f^2)*x)*sqrt(d)*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^
2*x^2 + a*f^2)/f^2) - 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^
2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)) + 2*(12*d^3*e^3*x^3 + 1
0*a^2*d^2*f^4 - 16*a*d^4*f^2 - 2*d^6 - 8*(5*a*d^2*e^2*f^2 - d^4*e^2)*x^2 +
 (15*a^2*d*e*f^4 - 46*a*d^3*e*f^2 - d^5*e)*x - (15*a^2*d*f^5 + 12*d^3*e^2*
f*x^2 - 22*a*d^3*f^3 - d^5*f - 8*(5*a*d^2*e*f^3 - d^4*e*f)*x)*sqrt((e^2*x^
2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/(a^2*d^4*e
*f^4 + 4*d^6*e^3*x^2 - 2*a*d^6*e*f^2 + d^8*e - 4*(a*d^5*e^2*f^2 - d^7*e^2)
*x), -1/6*(15*(a^3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d^2*f^4 + a*d^4*f^2 -
 4*(a^2*d*e*f^4 - a*d^3*e*f^2)*x)*sqrt(-d)*arctan(sqrt(e*x + f*sqrt((e^2*x
^2 + a*f^2)/f^2) + d)*sqrt(-d)/d) - (12*d^3*e^3*x^3 + 10*a^2*d^2*f^4 - 16*
a*d^4*f^2 - 2*d^6 - 8*(5*a*d^2*e^2*f^2 - d^4*e^2)*x^2 + (15*a^2*d*e*f^4 -
46*a*d^3*e*f^2 - d^5*e)*x - (15*a^2*d*f^5 + 12*d^3*e^2*f*x^2 - 22*a*d^3*f^
3 - d^5*f - 8*(5*a*d^2*e*f^3 - d^4*e*f)*x)*sqrt((e^2*x^2 + a*f^2)/f^2))*sq
rt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/(a^2*d^4*e*f^4 + 4*d^6*e^3*x^
2 - 2*a*d^6*e*f^2 + d^8*e - 4*(a*d^5*e^2*f^2 - d^7*e^2)*x)]
```

3.465.6 Sympy [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx = \int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(5/2),x)
```

output
```
Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-5/2), x)
```

3.465.7 Maxima [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx = \int \frac{1}{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-5/2), x)
```

3.465.8 Giac [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx = \int \frac{1}{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="giac")
```

output
```
integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-5/2), x)
```

3.465.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx = \int \frac{1}{\left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)^{5/2}}\,dx$$

input
```
int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(5/2),x)
```

output
```
int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(5/2), x)
```

3.466 $\int \sqrt{x-\sqrt{-4+x^2}}\,dx$

3.466.1 Optimal result

Integrand size = 17, antiderivative size = 41

$$\int \sqrt{x-\sqrt{-4+x^2}}\,dx=\frac{4}{\sqrt{x-\sqrt{-4+x^2}}}+\frac{1}{3}\left(x-\sqrt{-4+x^2}\right)^{3/2}$$

output
```
1/3*(x-(x^2-4)^(1/2))^(3/2)+4/(x-(x^2-4)^(1/2))^(1/2)
```

3.466.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \sqrt{x-\sqrt{-4+x^2}}\,dx=\frac{4}{\sqrt{x-\sqrt{-4+x^2}}}+\frac{1}{3}\left(x-\sqrt{-4+x^2}\right)^{3/2}$$

input
```
Integrate[Sqrt[x - Sqrt[-4 + x^2]],x]
```

output
```
4/Sqrt[x - Sqrt[-4 + x^2]] + (x - Sqrt[-4 + x^2])^(3/2)/3
```

3.466.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2542, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x-\sqrt{x^2-4}}dx$$

$$\Big\downarrow \text{2542}$$

$$\frac{1}{2}\int -\frac{4-\left(x-\sqrt{x^2-4}\right)^2}{\left(x-\sqrt{x^2-4}\right)^{3/2}}d\left(x-\sqrt{x^2-4}\right)$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{4-\left(x-\sqrt{x^2-4}\right)^2}{\left(x-\sqrt{x^2-4}\right)^{3/2}}d\left(x-\sqrt{x^2-4}\right)$$

$$\Big\downarrow \text{244}$$

$$-\frac{1}{2}\int \left(\frac{4}{\left(x-\sqrt{x^2-4}\right)^{3/2}}-\sqrt{x-\sqrt{x^2-4}}\right)d\left(x-\sqrt{x^2-4}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2}{3}\left(x-\sqrt{x^2-4}\right)^{3/2}+\frac{8}{\sqrt{x-\sqrt{x^2-4}}}\right)$$

input
```
Int[Sqrt[x - Sqrt[-4 + x^2]],x]
```

output
```
(8/Sqrt[x - Sqrt[-4 + x^2]] + (2*(x - Sqrt[-4 + x^2])^(3/2))/3)/2
```

3.466.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2542
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Simp[1/(2*e)   Subst[Int[(g + h*x^n)^p*((d^2 + a*f
^2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fr
eeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.466.4 Maple [F]

$$\int \sqrt{x-\sqrt{x^2-4}}dx$$

input
```
int((x-(x^2-4)^(1/2))^(1/2),x)
```

output
```
int((x-(x^2-4)^(1/2))^(1/2),x)
```

3.466.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.63

$$\int \sqrt{x-\sqrt{-4+x^2}}\,dx = \frac{2}{3}\left(2\,x+\sqrt{x^2-4}\right)\sqrt{x-\sqrt{x^2-4}}$$

input
```
integrate((x-(x^2-4)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
2/3*(2*x + sqrt(x^2 - 4))*sqrt(x - sqrt(x^2 - 4))
```

3.466.6 Sympy [F]

$$\int \sqrt{x-\sqrt{-4+x^2}}\,dx=\int \sqrt{x-\sqrt{x^2-4}}\,dx$$

input
```
integrate((x-(x**2-4)**(1/2))**(1/2),x)
```

output
```
Integral(sqrt(x - sqrt(x**2 - 4)), x)
```

3.466.7 Maxima [F]

$$\int \sqrt{x-\sqrt{-4+x^2}}\,dx=\int \sqrt{x-\sqrt{x^2-4}}\,dx$$

input
```
integrate((x-(x^2-4)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x - sqrt(x^2 - 4)), x)
```

3.466.8 Giac [F]

$$\int \sqrt{x-\sqrt{-4+x^2}}\,dx=\int \sqrt{x-\sqrt{x^2-4}}\,dx$$

input
```
integrate((x-(x^2-4)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x - sqrt(x^2 - 4)), x)
```

3.466.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{x - \sqrt{-4 + x^2}}\, dx = \int \sqrt{x - \sqrt{x^2 - 4}}\, dx$$

input
```
int((x - (x^2 - 4)^(1/2))^(1/2),x)
```

output
```
int((x - (x^2 - 4)^(1/2))^(1/2), x)
```

3.467 $\int \sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}\,dx$

3.467.1 Optimal result

Integrand size = 26, antiderivative size = 69

$$\int \sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}\,dx=-\frac{b^2c}{a\sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}}+\frac{\left(ax+b\sqrt{c+\frac{a^2x^2}{b^2}}\right)^{3/2}}{3a}$$

output
```
1/3*(a*x+b*(c+a^2*x^2/b^2)^(1/2))^(3/2)/a-b^2*c/a/(a*x+b*(c+a^2*x^2/b^2)^(
1/2))^(1/2)
```

3.467.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}\,dx=-\frac{b^2c}{a\sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}}+\frac{\left(ax+b\sqrt{c+\frac{a^2x^2}{b^2}}\right)^{3/2}}{3a}$$

input
```
Integrate[Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]],x]
```

output
```
-((b^2*c)/(a*Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]])) + (a*x + b*Sqrt[c + (
a^2*x^2)/b^2])^(3/2)/(3*a)
```

3.467.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2542, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\sqrt{\frac{a^2x^2}{b^2}+c}+ax}\,dx$$

$$\Big\downarrow \text{ 2542}$$

$$\frac{\int \frac{cb^2+\left(\sqrt{\frac{a^2x^2}{b^2}+cb}+ax\right)^2}{\left(\sqrt{\frac{a^2x^2}{b^2}+cb}+ax\right)^{3/2}}d\left(\sqrt{\frac{a^2x^2}{b^2}+cb}+ax\right)}{2a}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{\int \left(\frac{cb^2}{\left(\sqrt{\frac{a^2x^2}{b^2}+cb}+ax\right)^{3/2}}+\sqrt{\sqrt{\frac{a^2x^2}{b^2}+cb}+ax}\right)d\left(\sqrt{\frac{a^2x^2}{b^2}+cb}+ax\right)}{2a}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{2}{3}\left(b\sqrt{\frac{a^2x^2}{b^2}+c}+ax\right)^{3/2}-\frac{2b^2c}{\sqrt{b\sqrt{\frac{a^2x^2}{b^2}+c}+ax}}}{2a}$$

input
```
Int[Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]],x]
```

output
```
((-2*b^2*c)/Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]] + (2*(a*x + b*Sqrt[c + (
a^2*x^2)/b^2])^(3/2))/3)/(2*a)
```

3.467.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2542
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Simp[1/(2*e)   Subst[Int[(g + h*x^n)^p*((d^2 + a*f
^2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fr
eeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.467.4 Maple [F]

$$\int \sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}dx$$

input
```
int((a*x+b*(c+a^2/b^2*x^2)^(1/2))^(1/2),x)
```

output
```
int((a*x+b*(c+a^2/b^2*x^2)^(1/2))^(1/2),x)
```

3.467.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}\,dx = \frac{2\left(2\,ax-b\sqrt{\frac{a^2x^2+b^2c}{b^2}}\right)\sqrt{ax+b\sqrt{\frac{a^2x^2+b^2c}{b^2}}}}{3\,a}$$

input
```
integrate((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
2/3*(2*a*x - b*sqrt((a^2*x^2 + b^2*c)/b^2))*sqrt(a*x + b*sqrt((a^2*x^2 + b
^2*c)/b^2))/a
```

3.467.6 Sympy [F]

$$\int \sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}\,dx=\int \sqrt{ax+b\sqrt{\frac{a^2x^2}{b^2}+c}}\,dx$$

input
```
integrate((a*x+b*(c+a**2*x**2/b**2)**(1/2))**(1/2),x)
```

output
```
Integral(sqrt(a*x + b*sqrt(a**2*x**2/b**2 + c)), x)
```

3.467.7 Maxima [F]

$$\int \sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}\,dx=\int \sqrt{ax+\sqrt{\frac{a^2x^2}{b^2}+c}\,b}\,dx$$

input
```
integrate((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x + sqrt(a^2*x^2/b^2 + c)*b), x)
```

3.467.8 Giac [F]

$$\int \sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}\,dx=\int \sqrt{ax+\sqrt{\frac{a^2x^2}{b^2}+c}\,b}\,dx$$

input
```
integrate((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x + sqrt(a^2*x^2/b^2 + c)*b), x)
```

3.467.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}\,dx=\int \sqrt{a\,x+b\,\sqrt{c+\frac{a^2\,x^2}{b^2}}}\,dx$$

input
```
int((a*x + b*(c + (a^2*x^2)/b^2)^(1/2))^(1/2),x)
```

output
```
int((a*x + b*(c + (a^2*x^2)/b^2)^(1/2))^(1/2), x)
```

3.468 $\int \sqrt{1+\sqrt{1-x^2}}\,dx$

3.468.1 Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \sqrt{1+\sqrt{1-x^2}}\,dx=-\frac{2x^3}{3\left(1+\sqrt{1-x^2}\right)^{3/2}}+\frac{2x}{\sqrt{1+\sqrt{1-x^2}}}$$

output
```
-2/3*x^3/(1+(-x^2+1)^(1/2))^(3/2)+2*x/(1+(-x^2+1)^(1/2))^(1/2)
```

3.468.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \sqrt{1+\sqrt{1-x^2}}\,dx=\frac{2x\left(2+\sqrt{1-x^2}\right)}{3\sqrt{1+\sqrt{1-x^2}}}$$

input
```
Integrate[Sqrt[1 + Sqrt[1 - x^2]],x]
```

output
```
(2*x*(2 + Sqrt[1 - x^2]))/(3*Sqrt[1 + Sqrt[1 - x^2]])
```

3.468.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2554}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sqrt{1-x^2}+1}\,dx$$

$$\Big\downarrow \; 2554$$

$$\frac{2x}{\sqrt{\sqrt{1-x^2}+1}} - \frac{2x^3}{3\left(\sqrt{1-x^2}+1\right)^{3/2}}$$

input
```
Int[Sqrt[1 + Sqrt[1 - x^2]],x]
```

output
```
(-2*x^3)/(3*(1 + Sqrt[1 - x^2])^(3/2)) + (2*x)/Sqrt[1 + Sqrt[1 - x^2]]
```

3.468.3.1 Defintions of rubi rules used

rule 2554
```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[2*b^2*d
*(x^3/(3*(a + b*Sqrt[c + d*x^2])^(3/2))), x] + Simp[2*a*(x/Sqrt[a + b*Sqrt[
c + d*x^2]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

3.468.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

method	result	size
meijerg	$\frac{i\left(\frac{32i\sqrt{\pi}\,\sqrt{2}\,x^3\cos\left(\frac{3\arcsin(x)}{2}\right)}{3}-\frac{8i\sqrt{\pi}\,\sqrt{2}\,\left(-\frac{4}{3}x^4+\frac{2}{3}x^2+\frac{2}{3}\right)\sin\left(\frac{3\arcsin(x)}{2}\right)}{\sqrt{-x^2+1}}\right)}{8\sqrt{\pi}}$	60

input
```
int((1+(-x^2+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*I/Pi^(1/2)*(32/3*I*Pi^(1/2)*2^(1/2)*x^3*cos(3/2*arcsin(x))-8*I*Pi^(1/2
)*2^(1/2)*(-4/3*x^4+2/3*x^2+2/3)*sin(3/2*arcsin(x))/(-x^2+1)^(1/2))
```

3.468.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \sqrt{1+\sqrt{1-x^2}}\,dx = \frac{2\left(x^2-\sqrt{-x^2+1}+1\right)\sqrt{\sqrt{-x^2+1}+1}}{3\,x}$$

input
```
integrate((1+(-x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
2/3*(x^2 - sqrt(-x^2 + 1) + 1)*sqrt(sqrt(-x^2 + 1) + 1)/x
```

3.468.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 415, normalized size of antiderivative = 9.22

$$\int \sqrt{1+\sqrt{1-x^2}}\,dx$$
$$= \begin{cases} -\frac{\sqrt{2}ix^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2-1}\sqrt{i\sqrt{x^2-1}+1}-12i\pi\sqrt{i\sqrt{x^2-1}+1}} - \frac{3\sqrt{2}x\sqrt{x^2-1}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2-1}\sqrt{i\sqrt{x^2-1}+1}-12i\pi\sqrt{i\sqrt{x^2-1}+1}} + \frac{3\sqrt{2}ix\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2-1}\sqrt{i\sqrt{x^2-1}+1}-12i\pi\sqrt{i\sqrt{x^2-}}} \\ \frac{\sqrt{2}x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{1-x^2}\sqrt{\sqrt{1-x^2}+1}+12\pi\sqrt{\sqrt{1-x^2}+1}} - \frac{3\sqrt{2}x\sqrt{1-x^2}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{1-x^2}\sqrt{\sqrt{1-x^2}+1}+12\pi\sqrt{\sqrt{1-x^2}+1}} - \frac{3\sqrt{2}x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{1-x^2}\sqrt{\sqrt{1-x^2}+1}+12\pi\sqrt{\sqrt{1-x^2}+1}} \end{cases}$$

input
```
integrate((1+(-x**2+1)**(1/2))**(1/2),x)
```

output
```
Piecewise((-sqrt(2)*I*x**3*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 - 1)*sq
rt(I*sqrt(x**2 - 1) + 1) - 12*I*pi*sqrt(I*sqrt(x**2 - 1) + 1)) - 3*sqrt(2)
*x*sqrt(x**2 - 1)*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 - 1)*sqrt(I*sqrt
(x**2 - 1) + 1) - 12*I*pi*sqrt(I*sqrt(x**2 - 1) + 1)) + 3*sqrt(2)*I*x*gamm
a(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 - 1)*sqrt(I*sqrt(x**2 - 1) + 1) - 12*I
*pi*sqrt(I*sqrt(x**2 - 1) + 1)), Abs(x**2) > 1), (sqrt(2)*x**3*gamma(-1/4)
*gamma(1/4)/(12*pi*sqrt(1 - x**2)*sqrt(sqrt(1 - x**2) + 1) + 12*pi*sqrt(sq
rt(1 - x**2) + 1)) - 3*sqrt(2)*x*sqrt(1 - x**2)*gamma(-1/4)*gamma(1/4)/(12
*pi*sqrt(1 - x**2)*sqrt(sqrt(1 - x**2) + 1) + 12*pi*sqrt(sqrt(1 - x**2) +
1)) - 3*sqrt(2)*x*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(1 - x**2)*sqrt(sqrt(1
 - x**2) + 1) + 12*pi*sqrt(sqrt(1 - x**2) + 1)), True))
```

3.468.7 Maxima [F]

$$\int \sqrt{1+\sqrt{1-x^2}}\,dx = \int \sqrt{\sqrt{-x^2+1}+1}\,dx$$

input
```
integrate((1+(-x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(-x^2 + 1) + 1), x)
```

3.468.8 Giac [F]

$$\int \sqrt{1+\sqrt{1-x^2}}\,dx = \int \sqrt{\sqrt{-x^2+1}+1}\,dx$$

input
```
integrate((1+(-x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sqrt(-x^2 + 1) + 1), x)
```

3.468.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+\sqrt{1-x^2}}\,dx = \int \sqrt{\sqrt{1-x^2}+1}\,dx$$

input
```
int(((1 - x^2)^(1/2) + 1)^(1/2),x)
```

output
```
int(((1 - x^2)^(1/2) + 1)^(1/2), x)
```

3.469 $\int \sqrt{1+\sqrt{1+x^2}}\,dx$

3.469.1 Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \sqrt{1+\sqrt{1+x^2}}\,dx=\frac{2x^3}{3\left(1+\sqrt{1+x^2}\right)^{3/2}}+\frac{2x}{\sqrt{1+\sqrt{1+x^2}}}$$

output
```
2/3*x^3/(1+(x^2+1)^(1/2))^(3/2)+2*x/(1+(x^2+1)^(1/2))^(1/2)
```

3.469.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \sqrt{1+\sqrt{1+x^2}}\,dx=\frac{2x\left(2+\sqrt{1+x^2}\right)}{3\sqrt{1+\sqrt{1+x^2}}}$$

input
```
Integrate[Sqrt[1 + Sqrt[1 + x^2]],x]
```

output
```
(2*x*(2 + Sqrt[1 + x^2]))/(3*Sqrt[1 + Sqrt[1 + x^2]])
```

3.469.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2554}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sqrt{x^2+1}+1}\,dx$$

$$\Big\downarrow \text{2554}$$

$$\frac{2x}{\sqrt{\sqrt{x^2+1}+1}}+\frac{2x^3}{3\left(\sqrt{x^2+1}+1\right)^{3/2}}$$

input
```
Int[Sqrt[1 + Sqrt[1 + x^2]],x]
```

output
```
(2*x^3)/(3*(1 + Sqrt[1 + x^2])^(3/2)) + (2*x)/Sqrt[1 + Sqrt[1 + x^2]]
```

3.469.3.1 Defintions of rubi rules used

rule 2554
```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[2*b^2*d
*(x^3/(3*(a + b*Sqrt[c + d*x^2])^(3/2))), x] + Simp[2*a*(x/Sqrt[a + b*Sqrt[
c + d*x^2]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

3.469.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

method	result	size
meijerg	$-\frac{-\frac{32\sqrt{\pi}\,\sqrt{2}\,x^{3}\cosh\left(\frac{3\,\operatorname{arcsinh}(x)}{2}\right)}{3}-\frac{8\sqrt{\pi}\,\sqrt{2}\,\left(-\frac{4}{3}x^{4}-\frac{2}{3}x^{2}+\frac{2}{3}\right)\sinh\left(\frac{3\,\operatorname{arcsinh}(x)}{2}\right)}{\sqrt{x^{2}+1}}}{8\sqrt{\pi}}$	55

input
```
int((1+(x^2+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/Pi^(1/2)*(-32/3*Pi^(1/2)*2^(1/2)*x^3*cosh(3/2*arcsinh(x))-8*Pi^(1/2)*
2^(1/2)*(-4/3*x^4-2/3*x^2+2/3)*sinh(3/2*arcsinh(x))/(x^2+1)^(1/2))
```

3.469.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int \sqrt{1+\sqrt{1+x^2}}\,dx = \frac{2\left(x^2+\sqrt{x^2+1}-1\right)\sqrt{\sqrt{x^2+1}+1}}{3\,x}$$

input
```
integrate((1+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
2/3*(x^2 + sqrt(x^2 + 1) - 1)*sqrt(sqrt(x^2 + 1) + 1)/x
```

3.469.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(36) = 72.

Time = 0.66 (sec) , antiderivative size = 197, normalized size of antiderivative = 4.80

$$\int \sqrt{1+\sqrt{1+x^2}}\,dx = -\frac{\sqrt{2}x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+1}\sqrt{\sqrt{x^2+1}+1}+12\pi\sqrt{\sqrt{x^2+1}+1}} - \frac{3\sqrt{2}x\sqrt{x^2+1}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+1}\sqrt{\sqrt{x^2+1}+1}+12\pi\sqrt{\sqrt{x^2+1}+1}} - \frac{3\sqrt{2}x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+1}\sqrt{\sqrt{x^2+1}+1}+12\pi\sqrt{\sqrt{x^2+1}+1}}$$

input
```
integrate((1+(x**2+1)**(1/2))**(1/2),x)
```

output
```
-sqrt(2)*x**3*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 1)*sqrt(sqrt(x**2
+ 1) + 1) + 12*pi*sqrt(sqrt(x**2 + 1) + 1)) - 3*sqrt(2)*x*sqrt(x**2 + 1)*g
amma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 1)*sqrt(sqrt(x**2 + 1) + 1) + 12*
pi*sqrt(sqrt(x**2 + 1) + 1)) - 3*sqrt(2)*x*gamma(-1/4)*gamma(1/4)/(12*pi*s
qrt(x**2 + 1)*sqrt(sqrt(x**2 + 1) + 1) + 12*pi*sqrt(sqrt(x**2 + 1) + 1))
```

3.469.7 Maxima [F]

$$\int \sqrt{1+\sqrt{1+x^2}}\,dx = \int \sqrt{\sqrt{x^2+1}+1}\,dx$$

input
```
integrate((1+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(x^2 + 1) + 1), x)
```

3.469.8 Giac [F]

$$\int \sqrt{1+\sqrt{1+x^2}}\,dx = \int \sqrt{\sqrt{x^2+1}+1}\,dx$$

input
```
integrate((1+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sqrt(x^2 + 1) + 1), x)
```

3.469.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+\sqrt{1+x^2}}\,dx = \int \sqrt{\sqrt{x^2+1}+1}\,dx$$

input
```
int(((x^2 + 1)^(1/2) + 1)^(1/2),x)
```

output
```
int(((x^2 + 1)^(1/2) + 1)^(1/2), x)
```

3.470 $\int \sqrt{5+\sqrt{25+x^2}}\,dx$

3.470.1 Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \sqrt{5+\sqrt{25+x^2}}\,dx=\frac{2x^3}{3\left(5+\sqrt{25+x^2}\right)^{3/2}}+\frac{10x}{\sqrt{5+\sqrt{25+x^2}}}$$

output
```
2/3*x^3/(5+(x^2+25)^(1/2))^(3/2)+10*x/(5+(x^2+25)^(1/2))^(1/2)
```

3.470.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \sqrt{5+\sqrt{25+x^2}}\,dx=\frac{2x\left(10+\sqrt{25+x^2}\right)}{3\sqrt{5+\sqrt{25+x^2}}}$$

input
```
Integrate[Sqrt[5 + Sqrt[25 + x^2]],x]
```

output
```
(2*x*(10 + Sqrt[25 + x^2]))/(3*Sqrt[5 + Sqrt[25 + x^2]])
```

3.470.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2554}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sqrt{x^2+25}+5}\,dx$$

$$\Big\downarrow \ 2554$$

$$\frac{10x}{\sqrt{\sqrt{x^2+25}+5}}+\frac{2x^3}{3\left(\sqrt{x^2+25}+5\right)^{3/2}}$$

input
```
Int[Sqrt[5 + Sqrt[25 + x^2]],x]
```

output
```
(2*x^3)/(3*(5 + Sqrt[25 + x^2])^(3/2)) + (10*x)/Sqrt[5 + Sqrt[25 + x^2]]
```

3.470.3.1 Defintions of rubi rules used

rule 2554
```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[2*b^2*d
*(x^3/(3*(a + b*Sqrt[c + d*x^2])^(3/2))), x] + Simp[2*a*(x/Sqrt[a + b*Sqrt[
c + d*x^2]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

3.470.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.56

method	result	size
meijerg	$-\frac{5\sqrt{5}\left(-\frac{32\sqrt{\pi}\,\sqrt{2}\,x^{3}\cosh\left(\frac{3\,\text{arcsinh}\left(\frac{x}{5}\right)}{2}\right)}{375}-\frac{8\sqrt{\pi}\,\sqrt{2}\,\left(-\frac{4}{1875}x^{4}-\frac{2}{75}x^{2}+\frac{2}{3}\right)\sinh\left(\frac{3\,\text{arcsinh}\left(\frac{x}{5}\right)}{2}\right)}{\sqrt{\frac{x^{2}}{25}+1}}\right)}{8\sqrt{\pi}}$	64

input
```
int((5+(x^2+25)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-5/8*5^(1/2)/Pi^(1/2)*(-32/375*Pi^(1/2)*2^(1/2)*x^3*cosh(3/2*arcsinh(1/5*x
))-8*Pi^(1/2)*2^(1/2)*(-4/1875*x^4-2/75*x^2+2/3)*sinh(3/2*arcsinh(1/5*x))/
(1/25*x^2+1)^(1/2))
```

3.470.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \sqrt{5+\sqrt{25+x^2}}\,dx = \frac{2\left(x^2+5\sqrt{x^2+25}-25\right)\sqrt{\sqrt{x^2+25}+5}}{3\,x}$$

input
```
integrate((5+(x^2+25)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
2/3*(x^2 + 5*sqrt(x^2 + 25) - 25)*sqrt(sqrt(x^2 + 25) + 5)/x
```

3.470.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(36) = 72.

Time = 0.67 (sec) , antiderivative size = 197, normalized size of antiderivative = 4.80

$$\int \sqrt{5+\sqrt{25+x^2}}\,dx = -\frac{\sqrt{2}x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+25}\sqrt{\sqrt{x^2+25}+5}+60\pi\sqrt{\sqrt{x^2+25}+5}} - \frac{15\sqrt{2}x\sqrt{x^2+25}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+25}\sqrt{\sqrt{x^2+25}+5}+60\pi\sqrt{\sqrt{x^2+25}+5}} - \frac{75\sqrt{2}x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+25}\sqrt{\sqrt{x^2+25}+5}+60\pi\sqrt{\sqrt{x^2+25}+5}}$$

input
```
integrate((5+(x**2+25)**(1/2))**(1/2),x)
```

output
```
-sqrt(2)*x**3*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 25)*sqrt(sqrt(x**2
 + 25) + 5) + 60*pi*sqrt(sqrt(x**2 + 25) + 5)) - 15*sqrt(2)*x*sqrt(x**2 +
25)*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 25)*sqrt(sqrt(x**2 + 25) + 5
) + 60*pi*sqrt(sqrt(x**2 + 25) + 5)) - 75*sqrt(2)*x*gamma(-1/4)*gamma(1/4)
/(12*pi*sqrt(x**2 + 25)*sqrt(sqrt(x**2 + 25) + 5) + 60*pi*sqrt(sqrt(x**2 +
 25) + 5))
```

3.470.7 Maxima [F]

$$\int \sqrt{5+\sqrt{25+x^2}}\,dx = \int \sqrt{\sqrt{x^2+25}+5}\,dx$$

input
```
integrate((5+(x^2+25)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(x^2 + 25) + 5), x)
```

3.470.8 Giac [F]

$$\int \sqrt{5+\sqrt{25+x^2}}\,dx = \int \sqrt{\sqrt{x^2+25}+5}\,dx$$

input
```
integrate((5+(x^2+25)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sqrt(x^2 + 25) + 5), x)
```

3.470.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{5+\sqrt{25+x^2}}\,dx = \int \sqrt{\sqrt{x^2+25}+5}\,dx$$

input
```
int(((x^2 + 25)^(1/2) + 5)^(1/2),x)
```

output
```
int(((x^2 + 25)^(1/2) + 5)^(1/2), x)
```

3.471 $\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx$

3.471.1 Optimal result

Integrand size = 25, antiderivative size = 66

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx=\frac{2b^2cx^3}{3\left(a+b\sqrt{\frac{a^2}{b^2}+cx^2}\right)^{3/2}}+\frac{2ax}{\sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}}$$

output
```
2/3*b^2*c*x^3/(a+b*(a^2/b^2+c*x^2)^(1/2))^(3/2)+2*a*x/(a+b*(a^2/b^2+c*x^2)
^(1/2))^(1/2)
```

3.471.2 Mathematica [A] (verified)

Time = 4.00 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx=\frac{2\sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\left(-a^2+b^2cx^2+ab\sqrt{\frac{a^2}{b^2}+cx^2}\right)}{3b^2cx}$$

input
```
Integrate[Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]],x]
```

output
```
(2*Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]]*(-a^2 + b^2*c*x^2 + a*b*Sqrt[a^2/b^2
+ c*x^2]))/(3*b^2*c*x)
```

3.471.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2554}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{b\sqrt{\frac{a^2}{b^2}+cx^2}+a}\,dx$$

$$\Big\downarrow \text{2554}$$

$$\frac{2ax}{\sqrt{b\sqrt{\frac{a^2}{b^2}+cx^2}+a}}+\frac{2b^2cx^3}{3\left(b\sqrt{\frac{a^2}{b^2}+cx^2}+a\right)^{3/2}}$$

input
```
Int[Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]],x]
```

output
```
(2*b^2*c*x^3)/(3*(a + b*Sqrt[a^2/b^2 + c*x^2])^(3/2)) + (2*a*x)/Sqrt[a + b
*Sqrt[a^2/b^2 + c*x^2]]
```

3.471.3.1 Defintions of rubi rules used

rule 2554
```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[2*b^2*d
*(x^3/(3*(a + b*Sqrt[c + d*x^2])^(3/2))), x] + Simp[2*a*(x/Sqrt[a + b*Sqrt[
c + d*x^2]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

3.471.4 Maple [F]

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+c\,x^2}}dx$$

input
```
int((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x)
```

output
```
int((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x)
```

3.471.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx = \frac{2\left(b^2cx^2+ab\sqrt{\frac{b^2cx^2+a^2}{b^2}}-a^2\right)\sqrt{b\sqrt{\frac{b^2cx^2+a^2}{b^2}}+a}}{3\,b^2cx}$$

input
```
integrate((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
2/3*(b^2*c*x^2 + a*b*sqrt((b^2*c*x^2 + a^2)/b^2) - a^2)*sqrt(b*sqrt((b^2*c
*x^2 + a^2)/b^2) + a)/(b^2*c*x)
```

3.471.6 Sympy [F]

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx = \int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx$$

input
```
integrate((a+b*(a**2/b**2+c*x**2)**(1/2))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sqrt(a**2/b**2 + c*x**2)), x)
```

3.471.7 Maxima [F]

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx = \int \sqrt{\sqrt{cx^2+\frac{a^2}{b^2}}b+a}\,dx$$

input
```
integrate((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(c*x^2 + a^2/b^2)*b + a), x)
```

3.471.8 Giac [F]

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx = \int \sqrt{\sqrt{cx^2+\frac{a^2}{b^2}}b+a}\,dx$$

input
```
integrate((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sqrt(c*x^2 + a^2/b^2)*b + a), x)
```

3.471.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx = \int \sqrt{a+b\,\sqrt{c\,x^2+\frac{a^2}{b^2}}}\,dx$$

input
```
int((a + b*(c*x^2 + a^2/b^2)^(1/2))^(1/2),x)
```

output
```
int((a + b*(c*x^2 + a^2/b^2)^(1/2))^(1/2), x)
```

3.472 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n dx$

3.472.1 Optimal result

Integrand size = 28, antiderivative size = 166

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)} + \frac{f^2(4ae^2-b^2f^2)\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{1+n} \text{Hypergeometric2F1}\left(2,1+n,2+n,\frac{2e\left(d+ex+f\sqrt{a+bx}\right.}{2de-bf^2}\right.}{2e\left(2de-bf^2\right)^2(1+n)}$$

output
```
1/2*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1+n)/e/(1+n)+1/2*f^2*(-b^2*f^2+4*
a*e^2)*hypergeom([2, 1+n],[2+n],2*e*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))/(-
b*f^2+2*d*e))*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1+n)/e/(-b*f^2+2*d*e)^2
/(1+n)
```

3.472.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.81

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n dx$$

$$= \frac{\left(d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)^{1+n}\left(\left(-2de+bf^2\right)^2+\left(4ae^2f^2-b^2f^4\right)\text{Hypergeometric2F1}\left(2,1+n,2\right.\right.}{2e\left(-2de+bf^2\right)^2(1+n)}$$

input
```
Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^n,x]
```

output
```
((d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])^(1 + n)*((-2*d*e + b*f^2)^2 +
 (4*a*e^2*f^2 - b^2*f^4)*Hypergeometric2F1[2, 1 + n, 2 + n, (2*e*(d + e*x
+ f*Sqrt[a + x*(b + (e^2*x)/f^2)]))/(2*d*e - b*f^2)]))/(2*e*(-2*d*e + b*f^
2)^2*(1 + n))
```

3.472.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2541, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^n dx$$

$$\Big\downarrow \text{2541}$$

$$2\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^n\left(ed^2-bf^2d+aef^2+e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2-(2de-bf^2)\left(d+ex\right.\right.}{\left(-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\right)^2}$$

$$\Big\downarrow \text{1195}$$

$$2\int \left(\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^n}{4e}+\frac{(4ae^2f^2-b^2f^4)\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^n}{4e\left(-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\right)^2}\right)d\left(d+ex+f\sqrt{\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{f^2(4ae^2-b^2f^2)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right.}{2de-bf^2}\right.}{4e(n+1)(2de-bf^2)^2}\right.$$

input
```
Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^n,x]
```

output
```
2*((d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(1 + n)/(4*e*(1 + n)) + (f^
2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(1 + n)*
Hypergeometric2F1[2, 1 + n, 2 + n, (2*e*(d + e*x + f*Sqrt[a + b*x + (e^2*x
^2)/f^2]))/(2*d*e - b*f^2)])/(4*e*(2*d*e - b*f^2)^2*(1 + n)))
```

3.472.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2541
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Simp[2   Subst[Int[(g + h*x^n)^p*((d
^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x
)^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e
, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.472.4 Maple [F]

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n dx$$

input
```
int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^n,x)
```

output
```
int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^n,x)
```

3.472.5 Fricas [F(-1)]

Timed out.

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n dx = \text{Timed out}$$

input
```
integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^n,x, algorithm="fricas")
```

output
```
Timed out
```

3.472.6 Sympy [F(-2)]

Exception generated.

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**n,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.472.7 Maxima [F]

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n dx = \int \left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d\right)^n dx$$

input
```
integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^n,x, algorithm="maxima")
```

output
```
integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^n, x)
```

3.472.8 Giac [F]

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n dx = \int \left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d\right)^n dx$$

input
```
integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^n,x, algorithm="giac")
```

output
```
integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^n, x)
```

3.472.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n dx = \int \left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)^n dx$$

input
```
int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^n,x)
```

output
```
int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^n, x)
```

3.473 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3 dx$

3.473.1 Optimal result

Integrand size = 28, antiderivative size = 303

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3 dx = \frac{f^2(2de-bf^2)\left(4ae^2-b^2f^2\right)\left(ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{8e^4} + \frac{f^2(4ae^2-b^2f^2)\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}{16e^3} + \frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^4}{8e} - \frac{f^2(2de-bf^2)^3\left(4ae^2-b^2f^2\right)}{32e^5\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} + \frac{3f^2(2de-bf^2)^2\left(4ae^2-b^2f^2\right)\log\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}{32e^5}$$

output
```
3/32*f^2*(-b*f^2+2*d*e)^2*(-b^2*f^2+4*a*e^2)*ln(b*f^2+2*e*(e*x+f*(a+x*(b*f
^2+e^2*x)/f^2)^(1/2)))/e^5+1/8*f^2*(-b*f^2+2*d*e)*(-b^2*f^2+4*a*e^2)*(e*x+
f*(a+b*x+e^2*x^2/f^2)^(1/2))/e^4+1/16*f^2*(-b^2*f^2+4*a*e^2)*(d+e*x+f*(a+b
*x+e^2*x^2/f^2)^(1/2))^2/e^3+1/8*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^4/e-1
/32*f^2*(-b*f^2+2*d*e)^3*(-b^2*f^2+4*a*e^2)/e^5/(b*f^2+2*e*(e*x+f*(a+x*(b*
f^2+e^2*x)/f^2)^(1/2)))
```

3.473.2 Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.86

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3 dx$$

$$= \frac{1}{16}\left(8x\left(2d^3+3d^2ex+ex\left(3af^2+2x\left(bf^2+e^2x\right)\right)+d\left(6af^2+x\left(3bf^2+4e^2x\right)\right)\right)\right.$$

$$+\frac{\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}(3b^3f^7-2b^2ef^5(6d+ex)+4be^2f^3(3d^2-2af^2+2dex+2e^2x^2)+8e^3f(2af^2(2d+e}{e^4}$$

$$\left.+\frac{3(4ae^2-b^2f^2)\left(-2def+bf^3\right)^2\operatorname{arctanh}\left(\frac{ex}{f\left(-\sqrt{a}+\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)}\right)}{e^5}\right)$$

input
```
Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^3,x]
```

output
```
(8*x*(2*d^3 + 3*d^2*e*x + e*x*(3*a*f^2 + 2*x*(b*f^2 + e^2*x)) + d*(6*a*f^2
 + x*(3*b*f^2 + 4*e^2*x))) + (Sqrt[a + x*(b + (e^2*x)/f^2)]*(3*b^3*f^7 - 2
*b^2*e*f^5*(6*d + e*x) + 4*b*e^2*f^3*(3*d^2 - 2*a*f^2 + 2*d*e*x + 2*e^2*x^
2) + 8*e^3*f*(2*a*f^2*(2*d + e*x) + e*x*(3*d^2 + 4*d*e*x + 2*e^2*x^2))))/e
^4 + (3*(4*a*e^2 - b^2*f^2)*(-2*d*e*f + b*f^3)^2*ArcTanh[(e*x)/(f*(-Sqrt[a
] + Sqrt[a + x*(b + (e^2*x)/f^2)]))])/e^5)/16
```

3.473.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2541, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.473. $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3 dx$

$$\int \left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^3 dx$$

$\Big\downarrow$ 2541

$$2\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^3\left(ed^2-bf^2d+aef^2+e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2-(2de-bf^2)\left(d+ex\right.\right.}{\left(-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\right)^2}$$

$\Big\downarrow$ 1195

$$2\int \left(\frac{f^2(4ae^2-b^2f^2)(2de-bf^2)^3}{32e^4\left(-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\right)^2}+\frac{f^2(4ae^2-b^2f^2)(2de-bf^2)}{16e^4}+\frac{\left(d+ex+f\sqrt{\frac{e^2}{}}\right.}{4e}\right.$$

$\Big\downarrow$ 2009

$$2\left(\frac{f^2(4ae^2-b^2f^2)(2de-bf^2)^3}{64e^5\left(-2e\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)-bf^2+2de\right)}+\frac{3f^2(4ae^2-b^2f^2)(2de-bf^2)^2\log\left(-2e\left(f\sqrt{a+bx}\right.\right.}{64e^5}\right.$$

input
```
Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^3,x]
```

output
```
2*((f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + b*x + (e
^2*x^2)/f^2]))/(16*e^4) + (f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + b
*x + (e^2*x^2)/f^2])^2)/(32*e^3) + (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f
^2])^4/(16*e) + (f^2*(2*d*e - b*f^2)^3*(4*a*e^2 - b^2*f^2))/(64*e^5*(2*d*e
 - b*f^2 - 2*e*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]))) + (3*f^2*(2*d
*e - b*f^2)^2*(4*a*e^2 - b^2*f^2)*Log[2*d*e - b*f^2 - 2*e*(d + e*x + f*Sqr
t[a + b*x + (e^2*x^2)/f^2])])/(64*e^5))
```

3.473. $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3 dx$

3.473.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2541
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Simp[2   Subst[Int[(g + h*x^n)^p*((d
^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x
)^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e
, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.473.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(283) = 566$.

Time = 1.26 (sec) , antiderivative size = 809, normalized size of antiderivative = 2.67

method	result
default	$f^3 \left(\frac{\left(b+\frac{2e^2x}{f^2}\right)f^2\left(a+bx+\frac{e^2x^2}{f^2}\right)^{\frac{3}{2}}}{8e^2} + \frac{3\left(\frac{4e^2a}{f^2}-b^2\right)f^2\left(\frac{\left(b+\frac{2e^2x}{f^2}\right)f^2\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+\frac{\left(\frac{4e^2a}{f^2}-b^2\right)f^2\ln\left(\frac{\frac{b}{2}+\frac{e^2x}{f^2}}{\sqrt{\frac{e^2}{f^2}}}+\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{8e^2\sqrt{\frac{e^2}{f^2}}}\right)}{16e^2} \right.$

input
```
int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
f^3*(1/8*(b+2*e^2*x/f^2)/e^2*f^2*(a+b*x+e^2*x^2/f^2)^(3/2)+3/16*(4*e^2/f^2
*a-b^2)/e^2*f^2*(1/4*(b+2*e^2*x/f^2)/e^2*f^2*(a+b*x+e^2*x^2/f^2)^(1/2)+1/8
*(4*e^2/f^2*a-b^2)/e^2*f^2*ln((1/2*b+e^2*x/f^2)/(e^2/f^2)^(1/2)+(a+b*x+e^2
*x^2/f^2)^(1/2))/(e^2/f^2)^(1/2)))+3*f^2*(1/4*e^3/f^2*x^4+1/3*(d*e^2/f^2+b
*e)*x^3+1/2*(a*e+b*d)*x^2+a*d*x)+3*f*(d^2*(1/4*(b+2*e^2*x/f^2)/e^2*f^2*(a+
b*x+e^2*x^2/f^2)^(1/2)+1/8*(4*e^2/f^2*a-b^2)/e^2*f^2*ln((1/2*b+e^2*x/f^2)/
(e^2/f^2)^(1/2)+(a+b*x+e^2*x^2/f^2)^(1/2))/(e^2/f^2)^(1/2))+e^2*(1/4*x*(a+
b*x+e^2*x^2/f^2)^(3/2)/e^2*f^2-5/8*b/e^2*f^2*(1/3*(a+b*x+e^2*x^2/f^2)^(3/2
)/e^2*f^2-1/2*b/e^2*f^2*(1/4*(b+2*e^2*x/f^2)/e^2*f^2*(a+b*x+e^2*x^2/f^2)^(
1/2)+1/8*(4*e^2/f^2*a-b^2)/e^2*f^2*ln((1/2*b+e^2*x/f^2)/(e^2/f^2)^(1/2)+(a
+b*x+e^2*x^2/f^2)^(1/2))/(e^2/f^2)^(1/2)))-1/4*a/e^2*f^2*(1/4*(b+2*e^2*x/f
^2)/e^2*f^2*(a+b*x+e^2*x^2/f^2)^(1/2)+1/8*(4*e^2/f^2*a-b^2)/e^2*f^2*ln((1/
2*b+e^2*x/f^2)/(e^2/f^2)^(1/2)+(a+b*x+e^2*x^2/f^2)^(1/2))/(e^2/f^2)^(1/2))
)+2*e*d*(1/3*(a+b*x+e^2*x^2/f^2)^(3/2)/e^2*f^2-1/2*b/e^2*f^2*(1/4*(b+2*e^2
*x/f^2)/e^2*f^2*(a+b*x+e^2*x^2/f^2)^(1/2)+1/8*(4*e^2/f^2*a-b^2)/e^2*f^2*ln
((1/2*b+e^2*x/f^2)/(e^2/f^2)^(1/2)+(a+b*x+e^2*x^2/f^2)^(1/2))/(e^2/f^2)^(1
/2))))+1/4*(e*x+d)^4/e
```

3.473.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.14

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3 dx$$
$$= \frac{32\,e^8x^4+32\,(be^6f^2+2\,de^7)x^3+48\,(d^2e^6+(bde^5+ae^6)f^2)x^2+32\,(3\,ade^5f^2+d^3e^5)x+3\,(b^4f^8-16\,ad^2}{}$$

input
```
integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="fricas")
```

output
```
1/32*(32*e^8*x^4 + 32*(b*e^6*f^2 + 2*d*e^7)*x^3 + 48*(d^2*e^6 + (b*d*e^5 +
 a*e^6)*f^2)*x^2 + 32*(3*a*d*e^5*f^2 + d^3*e^5)*x + 3*(b^4*f^8 - 16*a*d^2*
e^4*f^2 - 4*(b^3*d*e + a*b^2*e^2)*f^6 + 4*(b^2*d^2*e^2 + 4*a*b*d*e^3)*f^4)
*log(-b*f^2 - 2*e^2*x + 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 2*(
3*b^3*e*f^7 + 16*e^7*f*x^3 - 4*(3*b^2*d*e^2 + 2*a*b*e^3)*f^5 + 4*(3*b*d^2*
e^3 + 8*a*d*e^4)*f^3 + 8*(b*e^5*f^3 + 4*d*e^6*f)*x^2 - 2*(b^2*e^3*f^5 - 12
*d^2*e^5*f - 4*(b*d*e^4 + 2*a*e^5)*f^3)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2
)/f^2))/e^5
```

3.473.6 Sympy [A] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 1363, normalized size of antiderivative = 4.50

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3 dx = \text{Too large to display}$$

input
```
integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**3,x)
```

output
```
3*a*d*f**2*x + 3*a*e*f**2*x**2/2 + a*f**3*Piecewise(((a/2 - b**2*f**2/(8*e
**2))*Piecewise((log(b + 2*e**2*x/f**2 + 2*sqrt(e**2/f**2)*sqrt(a + b*x +
e**2*x**2/f**2))/sqrt(e**2/f**2), Ne(a - b**2*f**2/(4*e**2), 0)), ((b*f**2
/(2*e**2) + x)*log(b*f**2/(2*e**2) + x)/sqrt(e**2*(b*f**2/(2*e**2) + x)**2
/f**2), True)) + (b*f**2/(4*e**2) + x/2)*sqrt(a + b*x + e**2*x**2/f**2), N
e(e**2/f**2, 0)), (2*(a + b*x)**(3/2)/(3*b), Ne(b, 0)), (sqrt(a)*x, True))
 + 3*b*d*f**2*x**2/2 + b*e*f**2*x**3 + b*f**3*Piecewise(((-a*b*f**2/(12*e*
*2) - b*f**2*(a/3 - b**2*f**2/(8*e**2))/(2*e**2))*Piecewise((log(b + 2*e**
2*x/f**2 + 2*sqrt(e**2/f**2)*sqrt(a + b*x + e**2*x**2/f**2))/sqrt(e**2/f**
2), Ne(a - b**2*f**2/(4*e**2), 0)), ((b*f**2/(2*e**2) + x)*log(b*f**2/(2*e
**2) + x)/sqrt(e**2*(b*f**2/(2*e**2) + x)**2/f**2), True)) + sqrt(a + b*x
+ e**2*x**2/f**2)*(b*f**2*x/(12*e**2) + x**2/3 + f**2*(a/3 - b**2*f**2/(8*
e**2))/e**2), Ne(e**2/f**2, 0)), (2*(-a*(a + b*x)**(3/2)/3 + (a + b*x)**(5
/2)/5)/b**2, Ne(b, 0)), (sqrt(a)*x**2/2, True)) + d**3*x + 3*d**2*e*x**2/2
 + 3*d**2*f*Piecewise(((a/2 - b**2*f**2/(8*e**2))*Piecewise((log(b + 2*e**
2*x/f**2 + 2*sqrt(e**2/f**2)*sqrt(a + b*x + e**2*x**2/f**2))/sqrt(e**2/f**
2), Ne(a - b**2*f**2/(4*e**2), 0)), ((b*f**2/(2*e**2) + x)*log(b*f**2/(2*e
**2) + x)/sqrt(e**2*(b*f**2/(2*e**2) + x)**2/f**2), True)) + (b*f**2/(4*e*
*2) + x/2)*sqrt(a + b*x + e**2*x**2/f**2), Ne(e**2/f**2, 0)), (2*(a + b*x)
**(3/2)/(3*b), Ne(b, 0)), (sqrt(a)*x, True)) + 2*d*e**2*x**3 + 6*d*e*f*...
```

3.473.7 Maxima [F(-2)]

Exception generated.

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3 dx = \text{Exception raised: ValueError}$$

input
```
integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b^2*f^2-4*a*e^2>0)', see `assume
?` for mor
```

3.473.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.31

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3 dx$$
$$= bef^2x^3+e^3x^4+\frac{3}{2}bdf^2x^2+\frac{3}{2}aef^2x^2+2de^2x^3+3adf^2x+\frac{3}{2}d^2ex^2+d^3x$$
$$+\frac{1}{16}\sqrt{bf^2x+e^2x^2+af^2}\left(2\left(4\left(\frac{2e^2x|f|}{f}+\frac{be^6f^4|f|+4de^7f^2|f|}{e^6f^3}\right)x-\frac{b^2e^4f^6|f|-4bde^5f^4|f|-8ae^6\cdots}{e^6f^3}\right.\right.$$
$$+\frac{3\left(b^4f^7|f|-4b^3def^5|f|-4ab^2e^2f^5|f|+4b^2d^2e^2f^3|f|+16abde^3f^3|f|-16ad^2e^4f|f|\right)\log\left(\left|-bf^2-2\cdots\right.\right.}{32e^4|e|}$$

input
```
integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")
```

output
```
b*e*f^2*x^3 + e^3*x^4 + 3/2*b*d*f^2*x^2 + 3/2*a*e*f^2*x^2 + 2*d*e^2*x^3 +
3*a*d*f^2*x + 3/2*d^2*e*x^2 + d^3*x + 1/16*sqrt(b*f^2*x + e^2*x^2 + a*f^2)
*(2*(4*(2*e^2*x*abs(f)/f + (b*e^6*f^4*abs(f) + 4*d*e^7*f^2*abs(f))/(e^6*f^
3))*x - (b^2*e^4*f^6*abs(f) - 4*b*d*e^5*f^4*abs(f) - 8*a*e^6*f^4*abs(f) -
12*d^2*e^6*f^2*abs(f))/(e^6*f^3))*x + (3*b^3*e^2*f^8*abs(f) - 12*b^2*d*e^3
*f^6*abs(f) - 8*a*b*e^4*f^6*abs(f) + 12*b*d^2*e^4*f^4*abs(f) + 32*a*d*e^5*
f^4*abs(f))/(e^6*f^3)) + 3/32*(b^4*f^7*abs(f) - 4*b^3*d*e*f^5*abs(f) - 4*a
*b^2*e^2*f^5*abs(f) + 4*b^2*d^2*e^2*f^3*abs(f) + 16*a*b*d*e^3*f^3*abs(f) -
 16*a*d^2*e^4*f*abs(f))*log(abs(-b*f^2 - 2*(x*abs(e) - sqrt(b*f^2*x + e^2*
x^2 + a*f^2))*abs(e)))/(e^4*abs(e))
```

3.473.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3\,dx=\int \left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)^3\,dx$$

input
```
int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^3,x)
```

output
```
int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^3, x)
```

3.474 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2 dx$

3.474.1 Optimal result

Integrand size = 28, antiderivative size = 237

$$\begin{aligned}\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2 dx &= \frac{f^2(4ae^2-b^2f^2)\left(ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{8e^3}+\frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}{6e}\\&\quad-\frac{f^2(2de-bf^2)^2(4ae^2-b^2f^2)}{16e^4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}\\&\quad+\frac{f^2(2de-bf^2)(4ae^2-b^2f^2)\log\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}{8e^4}\end{aligned}$$

output
```
1/8*f^2*(-b*f^2+2*d*e)*(-b^2*f^2+4*a*e^2)*ln(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+
e^2*x)/f^2)^(1/2)))/e^4+1/8*f^2*(-b^2*f^2+4*a*e^2)*(e*x+f*(a+b*x+e^2*x^2/f
^2)^(1/2))/e^3+1/6*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3/e-1/16*f^2*(-b*f^
2+2*d*e)^2*(-b^2*f^2+4*a*e^2)/e^4/(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2
)^(1/2)))
```

3.474.2 Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.74

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2 dx$$
$$=\frac{1}{12}\left(2x\left(6d^2+6af^2+6dex+x\left(3bf^2+4e^2x\right)\right)+\frac{\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}(-3b^2f^5+2bef^3(3d+ex)+4e^2f(2af^2+ex(3d+2ex)))}{e^3}+\frac{3f^2(-2de+bf^2)\left(-4ae^2+b^2f^2\right)\operatorname{arctanh}\left(\frac{ex}{f\left(-\sqrt{a}+\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)}\right)}{e^4}\right)$$

input
```
Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^2,x]
```

output
```
(2*x*(6*d^2 + 6*a*f^2 + 6*d*e*x + x*(3*b*f^2 + 4*e^2*x)) + (Sqrt[a + x*(b
+ (e^2*x)/f^2)]*(-3*b^2*f^5 + 2*b*e*f^3*(3*d + e*x) + 4*e^2*f*(2*a*f^2 + e
*x*(3*d + 2*e*x))))/e^3 + (3*f^2*(-2*d*e + b*f^2)*(-4*a*e^2 + b^2*f^2)*Arc
Tanh[(e*x)/(f*(-Sqrt[a] + Sqrt[a + x*(b + (e^2*x)/f^2)]))])/e^4)/12
```

3.474.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2541, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^2 dx$$

$\big\downarrow$ 2541

$$2\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2\left(ed^2-bf^2d+aef^2+e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2-(2de-bf^2)\left(d+ex\right.\right.}{\left(-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\right)^2}$$

$\big\downarrow$ 1195

$$2\int \left(-\frac{(2de-bf^2)(4ae^2-b^2f^2)f^2}{8e^3\left(-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\right)}+\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2}{4e}+\frac{4ae^2f^2-b^2f^4}{16e^3}\right.$$

$\big\downarrow$ 2009

$$2\left(\frac{f^2(4ae^2-b^2f^2)(2de-bf^2)^2}{32e^4\left(-2e\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)-bf^2+2de\right)}+\frac{f^2(4ae^2-b^2f^2)(2de-bf^2)\log\left(-2e\left(f\sqrt{a+bx+\right.\right.}{16e^4}\right.$$

input
```
Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^2,x]
```

output
```
2*((f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]))/(
16*e^3) + (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^3/(12*e) + (f^2*(2*d
*e - b*f^2)^2*(4*a*e^2 - b^2*f^2))/(32*e^4*(2*d*e - b*f^2 - 2*e*(d + e*x +
 f*Sqrt[a + b*x + (e^2*x^2)/f^2]))) + (f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*
f^2)*Log[2*d*e - b*f^2 - 2*e*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])])
/(16*e^4))
```

3.474.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.474. $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2 dx$

rule 2541
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Simp[2   Subst[Int[(g + h*x^n)^p*((d
^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x
)^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e
, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.474.4 Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.27

method	result
default	$\frac{b x^2 f^2}{2}+\frac{e^2 x^3}{3}+a f^2 x+2 f \left(d \left(\frac{\left(b+\frac{2 e^2 x}{f^2}\right) f^2 \sqrt{a+b x+\frac{e^2 x^2}{f^2}}}{4 e^2}+\frac{\left(\frac{4 e^2 a}{f^2}-b^2\right) f^2 \ln \left(\frac{\frac{b}{2}+\frac{e^2 x}{f^2}}{\sqrt{\frac{e^2}{f^2}}}+\sqrt{a+b x+\frac{e^2 x^2}{f^2}}\right)}{8 e^2 \sqrt{\frac{e^2}{f^2}}}\right)+e \left(\right.\right.$

input
```
int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*b*x^2*f^2+1/3*e^2*x^3+a*f^2*x+2*f*(d*(1/4*(b+2*e^2*x/f^2)/e^2*f^2*(a+b
*x+e^2*x^2/f^2)^(1/2)+1/8*(4*e^2/f^2*a-b^2)/e^2*f^2*ln((1/2*b+e^2*x/f^2)/(
e^2/f^2)^(1/2)+(a+b*x+e^2*x^2/f^2)^(1/2))/(e^2/f^2)^(1/2))+e*(1/3*(a+b*x+e
^2*x^2/f^2)^(3/2)/e^2*f^2-1/2*b/e^2*f^2*(1/4*(b+2*e^2*x/f^2)/e^2*f^2*(a+b*
x+e^2*x^2/f^2)^(1/2)+1/8*(4*e^2/f^2*a-b^2)/e^2*f^2*ln((1/2*b+e^2*x/f^2)/(e
^2/f^2)^(1/2)+(a+b*x+e^2*x^2/f^2)^(1/2))/(e^2/f^2)^(1/2)))+1/3*(e*x+d)^3/
e
```

3.474.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.92

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2 dx$$

$$=\frac{16 e^6x^3+12 (be^4f^2+2 de^5)x^2+24 (ae^4f^2+d^2e^4)x-3 (b^3f^6+8 ade^3f^2-2 (b^2de+2 abe^2)f^4) \log \left(-bf\right.}{}$$

input
```
integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/24*(16*e^6*x^3 + 12*(b*e^4*f^2 + 2*d*e^5)*x^2 + 24*(a*e^4*f^2 + d^2*e^4)
*x - 3*(b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4)*log(-b*f^2
- 2*e^2*x + 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) - 2*(3*b^2*e*f^5
- 8*e^5*f*x^2 - 2*(3*b*d*e^2 + 4*a*e^3)*f^3 - 2*(b*e^3*f^3 + 6*d*e^4*f)*x)
*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))/e^4
```

3.474.6 Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 490, normalized size of antiderivative = 2.07

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2 dx = af^2x + \frac{bf^2x^2}{2} + d^2x + dex^2$$
$$+2df\begin{cases}\left(\frac{a}{2}-\frac{b^2f^2}{8e^2}\right)\left(\begin{cases}\frac{\log\left(b+\frac{2e^2x}{f^2}+2\sqrt{\frac{e^2}{f^2}}\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{\sqrt{\frac{e^2}{f^2}}} & \text{for } a-\frac{b^2f^2}{4e^2}\neq 0\\ \frac{\left(\frac{bf^2}{2e^2}+x\right)\log\left(\frac{bf^2}{2e^2}+x\right)}{\sqrt{\frac{e^2\left(\frac{bf^2}{2e^2}+x\right)^2}{f^2}}} & \text{otherwise}\end{cases}\right)+\left(\frac{bf^2}{4e^2}+\frac{x}{2}\right)\sqrt{a+bx+\frac{e^2x^2}{f^2}} \\ \frac{2(a+bx)^{\frac{3}{2}}}{3b} \\ \sqrt{a}x\end{cases}$$
$$+\frac{2e^2x^3}{3}$$
$$+2ef\begin{cases}\left(-\frac{abf^2}{12e^2}-\frac{bf^2\left(\frac{a}{3}-\frac{b^2f^2}{8e^2}\right)}{2e^2}\right)\left(\begin{cases}\frac{\log\left(b+\frac{2e^2x}{f^2}+2\sqrt{\frac{e^2}{f^2}}\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{\sqrt{\frac{e^2}{f^2}}} & \text{for } a-\frac{b^2f^2}{4e^2}\neq 0\\ \frac{\left(\frac{bf^2}{2e^2}+x\right)\log\left(\frac{bf^2}{2e^2}+x\right)}{\sqrt{\frac{e^2\left(\frac{bf^2}{2e^2}+x\right)^2}{f^2}}} & \text{otherwise}\end{cases}\right)+\sqrt{a+bx+\frac{e^2x^2}{f^2}} \\ \frac{2\left(-\frac{a(a+bx)^{\frac{3}{2}}}{3}+\frac{(a+bx)^{\frac{5}{2}}}{5}\right)}{b^2} \\ \frac{\sqrt{a}x^2}{2}\end{cases}$$

input
```
integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**2,x)
```

output
```
a*f**2*x + b*f**2*x**2/2 + d**2*x + d*e*x**2 + 2*d*f*Piecewise(((a/2 - b**
2*f**2/(8*e**2))*Piecewise((log(b + 2*e**2*x/f**2 + 2*sqrt(e**2/f**2)*sqrt
(a + b*x + e**2*x**2/f**2))/sqrt(e**2/f**2), Ne(a - b**2*f**2/(4*e**2), 0)
), ((b*f**2/(2*e**2) + x)*log(b*f**2/(2*e**2) + x)/sqrt(e**2*(b*f**2/(2*e*
*2) + x)**2/f**2), True)) + (b*f**2/(4*e**2) + x/2)*sqrt(a + b*x + e**2*x*
*2/f**2), Ne(e**2/f**2, 0)), (2*(a + b*x)**(3/2)/(3*b), Ne(b, 0)), (sqrt(a
)*x, True)) + 2*e**2*x**3/3 + 2*e*f*Piecewise(((-a*b*f**2/(12*e**2) - b*f*
*2*(a/3 - b**2*f**2/(8*e**2))/(2*e**2))*Piecewise((log(b + 2*e**2*x/f**2 +
 2*sqrt(e**2/f**2)*sqrt(a + b*x + e**2*x**2/f**2))/sqrt(e**2/f**2), Ne(a -
 b**2*f**2/(4*e**2), 0)), ((b*f**2/(2*e**2) + x)*log(b*f**2/(2*e**2) + x)/
sqrt(e**2*(b*f**2/(2*e**2) + x)**2/f**2), True)) + sqrt(a + b*x + e**2*x**
2/f**2)*(b*f**2*x/(12*e**2) + x**2/3 + f**2*(a/3 - b**2*f**2/(8*e**2))/e**
2), Ne(e**2/f**2, 0)), (2*(-a*(a + b*x)**(3/2)/3 + (a + b*x)**(5/2)/5)/b**
2, Ne(b, 0)), (sqrt(a)*x**2/2, True))
```

3.474.7 Maxima [F(-2)]

Exception generated.

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2 dx = \text{Exception raised: ValueError}$$

input
```
integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b^2*f^2-4*a*e^2>0)', see `assume
?` for mor
```

3.474.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2 dx = \frac{1}{2}bf^2x^2 + \frac{2}{3}e^2x^3 + af^2x + dex^2 + d^2x + \frac{1}{12}\sqrt{bf^2x+e^2x^2+af^2}\left(2\left(\frac{4ex|f|}{f} + \frac{be^3f^3|f| + 6de^4f|f|}{e^4f^2}\right)x - \frac{3b^2ef^5|f| - 6bde^2f^3|f| - 8ae^3f^3|f|}{e^4f^2} \right. - \frac{\left(b^3f^5|f| - 2b^2def^3|f| - 4abe^2f^3|f| + 8ade^3f|f|\right)\log\left(\left|-bf^2 - 2\left(x|e| - \sqrt{bf^2x+e^2x^2+af^2}\right)|e|\right|\right)}{8e^3|e|}$$

input
```
integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")
```

output
```
1/2*b*f^2*x^2 + 2/3*e^2*x^3 + a*f^2*x + d*e*x^2 + d^2*x + 1/12*sqrt(b*f^2*
x + e^2*x^2 + a*f^2)*(2*(4*e*x*abs(f)/f + (b*e^3*f^3*abs(f) + 6*d*e^4*f*ab
s(f))/(e^4*f^2))*x - (3*b^2*e*f^5*abs(f) - 6*b*d*e^2*f^3*abs(f) - 8*a*e^3*
f^3*abs(f))/(e^4*f^2)) - 1/8*(b^3*f^5*abs(f) - 2*b^2*d*e*f^3*abs(f) - 4*a*
b*e^2*f^3*abs(f) + 8*a*d*e^3*f*abs(f))*log(abs(-b*f^2 - 2*(x*abs(e) - sqrt
(b*f^2*x + e^2*x^2 + a*f^2))*abs(e)))/(e^3*abs(e))
```

3.474.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2 dx = \int \left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)^2 dx$$

input
```
int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^2,x)
```

output
```
int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^2, x)
```

3.475 $\int \left(d+e x+f \sqrt{a+b x+\frac{e^2 x^2}{f^2}}\right) \, dx$

3.475.1 Optimal result

Integrand size = 26, antiderivative size = 118

$$\int \left(d+e x+f \sqrt{a+b x+\frac{e^2 x^2}{f^2}}\right) \, dx=d x+\frac{e x^2}{2}+\frac{f \left(b f^2+2 e^2 x\right) \sqrt{a+b x+\frac{e^2 x^2}{f^2}}}{4 e^2}+\frac{f^2 \left(4 a e^2-b^2 f^2\right) \operatorname{arctanh}\left(\frac{b f^2+2 e^2 x}{2 e f \sqrt{a+b x+\frac{e^2 x^2}{f^2}}}\right)}{8 e^3}$$

output
```
d*x+1/2*e*x^2+1/8*f^2*(-b^2*f^2+4*a*e^2)*arctanh(1/2*(b*f^2+2*e^2*x)/e/f/(
a+b*x+e^2*x^2/f^2)^(1/2))/e^3+1/4*f*(b*f^2+2*e^2*x)*(a+b*x+e^2*x^2/f^2)^(1
/2)/e^2
```

3.475.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.53

$$\int \left(d+e x+f \sqrt{a+b x+\frac{e^2 x^2}{f^2}}\right) \, dx$$
$$=\frac{8 d e^3 x+4 e^4 x^2+2 b e f^3 \sqrt{a+x \left(b+\frac{e^2 x}{f^2}\right)}+4 e^3 f x \sqrt{a+x \left(b+\frac{e^2 x}{f^2}\right)}+\left(-4 a e^2 f^2+b^2 f^4\right) \log \left(e^3 \left(\sqrt{a} f+\right.\right.}{8 e^3}$$

input
```
Integrate[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2],x]
```

output
```
(8*d*e^3*x + 4*e^4*x^2 + 2*b*e*f^3*Sqrt[a + x*(b + (e^2*x)/f^2)] + 4*e^3*f
*x*Sqrt[a + x*(b + (e^2*x)/f^2)] + (-4*a*e^2*f^2 + b^2*f^4)*Log[e^3*(Sqrt[
a]*f + e*x - f*Sqrt[a + x*(b + (e^2*x)/f^2)])] + (4*a*e^2*f^2 - b^2*f^4)*L
og[-(Sqrt[a]*f) + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/(8*e^3)
```

3.475.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^2\left(4ae^2-b^2f^2\right)\text{arctanh}\left(\frac{bf^2+2e^2x}{2ef\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\right)}{8e^3}+\frac{f\left(bf^2+2e^2x\right)\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+dx+\frac{ex^2}{2}$$

input
```
Int[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2],x]
```

output
```
d*x + (e*x^2)/2 + (f*(b*f^2 + 2*e^2*x)*Sqrt[a + b*x + (e^2*x^2)/f^2])/(4*e
^2) + (f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(b*f^2 + 2*e^2*x)/(2*e*f*Sqrt[a + b
*x + (e^2*x^2)/f^2])])/(8*e^3)
```

3.475.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.475.4 Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.04

method	result	size
default	$dx+\frac{e\,x^{2}}{2}+f\left(\frac{\left(b+\frac{2e^{2}x}{f^{2}}\right)f^{2}\sqrt{a+bx+\frac{e^{2}x^{2}}{f^{2}}}}{4e^{2}}+\frac{\left(\frac{4e^{2}a}{f^{2}}-b^{2}\right)f^{2}\ln\left(\frac{\frac{b}{2}+\frac{e^{2}x}{f^{2}}}{\sqrt{\frac{e^{2}}{f^{2}}}}+\sqrt{a+bx+\frac{e^{2}x^{2}}{f^{2}}}\right)}{8e^{2}\sqrt{\frac{e^{2}}{f^{2}}}}\right)$	123
parts	$dx+\frac{e\,x^{2}}{2}+f\left(\frac{\left(b+\frac{2e^{2}x}{f^{2}}\right)f^{2}\sqrt{a+bx+\frac{e^{2}x^{2}}{f^{2}}}}{4e^{2}}+\frac{\left(\frac{4e^{2}a}{f^{2}}-b^{2}\right)f^{2}\ln\left(\frac{\frac{b}{2}+\frac{e^{2}x}{f^{2}}}{\sqrt{\frac{e^{2}}{f^{2}}}}+\sqrt{a+bx+\frac{e^{2}x^{2}}{f^{2}}}\right)}{8e^{2}\sqrt{\frac{e^{2}}{f^{2}}}}\right)$	123

input
```
int(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
d*x+1/2*e*x^2+f*(1/4*(b+2*e^2*x/f^2)/e^2*f^2*(a+b*x+e^2*x^2/f^2)^(1/2)+1/8
*(4*e^2/f^2*a-b^2)/e^2*f^2*ln((1/2*b+e^2*x/f^2)/(e^2/f^2)^(1/2)+(a+b*x+e^2
*x^2/f^2)^(1/2))/(e^2/f^2)^(1/2))
```

3.475.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.04

$$\int\left(d+ex+f\sqrt{a+bx+\frac{e^{2}x^{2}}{f^{2}}}\right)dx$$
$$=\frac{4\,e^{4}x^{2}+8\,de^{3}x+\left(b^{2}f^{4}-4\,ae^{2}f^{2}\right)\log\left(-bf^{2}-2\,e^{2}x+2\,ef\sqrt{\frac{bf^{2}x+e^{2}x^{2}+af^{2}}{f^{2}}}\right)+2\left(bef^{3}+2\,e^{3}fx\right)\sqrt{\frac{bf^{2}x+e^{2}x^{2}+af^{2}}{f^{2}}}}{8\,e^{3}}$$

input
```
integrate(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(4*e^4*x^2 + 8*d*e^3*x + (b^2*f^4 - 4*a*e^2*f^2)*log(-b*f^2 - 2*e^2*x
+ 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 2*(b*e*f^3 + 2*e^3*f*x)*s
qrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))/e^3
```

3.475.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.67

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)\,dx = dx+\frac{ex^2}{2}$$
$$+f\left(\begin{cases}\left(\frac{a}{2}-\frac{b^2f^2}{8e^2}\right)\left(\begin{cases}\frac{\log\left(b+\frac{2e^2x}{f^2}+2\sqrt{\frac{e^2}{f^2}}\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{\sqrt{\frac{e^2}{f^2}}} & \text{for } a-\frac{b^2f^2}{4e^2}\neq 0\\ \frac{\left(\frac{bf^2}{2e^2}+x\right)\log\left(\frac{bf^2}{2e^2}+x\right)}{\sqrt{\frac{e^2\left(\frac{bf^2}{2e^2}+x\right)^2}{f^2}}} & \text{otherwise}\end{cases}\right)+\left(\frac{bf^2}{4e^2}+\frac{x}{2}\right)\sqrt{a+bx+\frac{e^2x^2}{f^2}} & \text{fo}\\ \frac{2(a+bx)^{\frac{3}{2}}}{3b} & \text{fo}\\ \sqrt{a}x & \text{ot}\end{cases}\right)$$

input
```
integrate(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2),x)
```

output
```
d*x + e*x**2/2 + f*Piecewise(((a/2 - b**2*f**2/(8*e**2))*Piecewise((log(b
+ 2*e**2*x/f**2 + 2*sqrt(e**2/f**2)*sqrt(a + b*x + e**2*x**2/f**2))/sqrt(e
**2/f**2), Ne(a - b**2*f**2/(4*e**2), 0)), ((b*f**2/(2*e**2) + x)*log(b*f*
*2/(2*e**2) + x)/sqrt(e**2*(b*f**2/(2*e**2) + x)**2/f**2), True)) + (b*f**
2/(4*e**2) + x/2)*sqrt(a + b*x + e**2*x**2/f**2), Ne(e**2/f**2, 0)), (2*(a
 + b*x)**(3/2)/(3*b), Ne(b, 0)), (sqrt(a)*x, True))
```

3.475.7 Maxima [F(-2)]

Exception generated.

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b^2*f^2-4*a*e^2>0)', see `assume
?` for mor
```

3.475.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.01

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)\,dx = \frac{1}{2}\,ex^2+dx + \frac{\left(2\,\sqrt{bf^2x+e^2x^2+af^2}\left(\frac{bf^2}{e^2}+2\,x\right)+\frac{(b^2f^4-4\,ae^2f^2)\log\left(\left|-bf^2-2\,\left(x|e|-\sqrt{bf^2x+e^2x^2+af^2}\right)|e|\right|\right)}{e^2|e|}\right)|f|}{8\,f}$$

input
```
integrate(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*e*x^2 + d*x + 1/8*(2*sqrt(b*f^2*x + e^2*x^2 + a*f^2)*(b*f^2/e^2 + 2*x)
 + (b^2*f^4 - 4*a*e^2*f^2)*log(abs(-b*f^2 - 2*(x*abs(e) - sqrt(b*f^2*x + e
^2*x^2 + a*f^2))*abs(e)))/(e^2*abs(e)))*abs(f)/f
```

3.475.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)\,dx=\int d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\,dx$$

input
```
int(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2),x)
```

output
```
int(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2), x)
```

3.476 $\int \frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$

3.476.1 Optimal result

Integrand size = 28, antiderivative size = 215

$$\int \frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx = -\frac{f^2(4ae^2-b^2f^2)}{2e(2de-bf^2)\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} + \frac{2(d^2e-bdf^2+aef^2)\log\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{(2de-bf^2)^2} - \frac{f^2(4ae^2-b^2f^2)\log\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}{2e(2de-bf^2)^2}$$

output
```
2*(a*e*f^2-b*d*f^2+d^2*e)*ln(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))/(-b*f^2+2*
d*e)^2-1/2*f^2*(-b^2*f^2+4*a*e^2)*ln(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f
^2)^(1/2)))/e/(-b*f^2+2*d*e)^2-1/2*f^2*(-b^2*f^2+4*a*e^2)/e/(-b*f^2+2*d*e)
/(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(1/2)))
```

3.476.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.24

$$\int \frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$$
$$= \frac{2e^2(2de-bf^2)\,x+2ef(-2de+bf^2)\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}-(-2de+bf^2)^2\log\left(e\left(\sqrt{a}f+ex-f\sqrt{a+x\left(\right.\right.\right.}}{}$$

input
```
Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-1),x]
```

output
```
(2*e^2*(2*d*e - b*f^2)*x + 2*e*f*(-2*d*e + b*f^2)*Sqrt[a + x*(b + (e^2*x)/
f^2)] - (-2*d*e + b*f^2)^2*Log[e*(Sqrt[a]*f + e*x - f*Sqrt[a + x*(b + (e^2
*x)/f^2)])] - (4*a*e^2*f^2 - b^2*f^4)*Log[-(Sqrt[a]*f) + e*x + f*Sqrt[a +
x*(b + (e^2*x)/f^2)]] + 4*e*(d^2*e - b*d*f^2 + a*e*f^2)*Log[-(a*f^2) + d*e
*x - b*f^2*x - d*f*Sqrt[a + x*(b + (e^2*x)/f^2)] + Sqrt[a]*f*(d + e*x + f*
Sqrt[a + x*(b + (e^2*x)/f^2)])])/(2*e*(-2*d*e + b*f^2)^2)
```

3.476.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2541, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}\,dx$$

$$\Big\downarrow \text{ 2541}$$

$$2\int \frac{ed^2-bf^2d+aef^2+e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2-(2de-bf^2)\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\left(-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\right)^2}d\left(d+ex+\right.$$

$$\Big\downarrow \text{ 1195}$$

$$2\int\left(\frac{ed^2-bf^2d+aef^2}{(2de-bf^2)^2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}+\frac{4ae^2f^2-b^2f^4}{2(2de-bf^2)^2\left(-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right.\right.}\right.$$

$\downarrow$ 2009

$$2\left(\frac{f^2(4ae^2-b^2f^2)}{4e(2de-bf^2)\left(-2e\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)-bf^2+2de\right)}-\frac{f^2(4ae^2-b^2f^2)\log\left(-2e\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right.\right.}{4e(2de-bf^2)^2}\right.$$

input
```
Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-1),x]
```

output
```
2*((f^2*(4*a*e^2 - b^2*f^2))/(4*e*(2*d*e - b*f^2)*(2*d*e - b*f^2 - 2*e*(d
+ e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]))) + ((d^2*e - b*d*f^2 + a*e*f^2)*
Log[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)^2 - (f^2*(
4*a*e^2 - b^2*f^2)*Log[2*d*e - b*f^2 - 2*e*(d + e*x + f*Sqrt[a + b*x + (e^
2*x^2)/f^2]])])/(4*e*(2*d*e - b*f^2)^2))
```

3.476.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2541
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Simp[2   Subst[Int[(g + h*x^n)^p*((d
^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x
)^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e
, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.476.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1262 vs. 2(205) = 410.

Time = 0.08 (sec) , antiderivative size = 1263, normalized size of antiderivative = 5.87

method	result	size
default	Expression too large to display	1263

input
```
int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
f/(b*f^2-2*d*e)*((e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^
2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e
))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^
4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)-1/2*(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*
d^2*e^2)/f^2/(b*f^2-2*d*e)*ln((-1/2*(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^
2*e^2)/f^2/(b*f^2-2*d*e)+e^2/f^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e)))/(e^2/f^2)^
(1/2)+(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d
*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(a^2*e^2
*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/
(b*f^2-2*d*e)^2)^(1/2))/(e^2/f^2)^(1/2)-(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2
*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2/((a^2*e^2*
f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(
b*f^2-2*d*e)^2)^(1/2)*ln((2*(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2
*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^2*f^2+
2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+2*(
(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e
^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-
b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2
)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*
b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(x+(a*f^2-d^2)/(b*f^2-...
```

3.476.5 Fricas [A] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.73

$$\int \frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx =$$

$$-\frac{2\,(be^2f^2-2\,de^3)x-2\,(d^2e^2-(bde-ae^2)f^2)\log\left((bd-2\,ae)f^2-(bef^2-2\,de^2)x+(bf^3-2\,def)\sqrt{\frac{b}{}}\right.}{}$$

input
```
integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2)),x, algorithm="fricas")
```

output
```
-1/2*(2*(b*e^2*f^2 - 2*d*e^3)*x - 2*(d^2*e^2 - (b*d*e - a*e^2)*f^2)*log((b
*d - 2*a*e)*f^2 - (b*e*f^2 - 2*d*e^2)*x + (b*f^3 - 2*d*e*f)*sqrt((b*f^2*x
+ e^2*x^2 + a*f^2)/f^2)) - 2*(d^2*e^2 - (b*d*e - a*e^2)*f^2)*log(a*f^2 - d
^2 + (b*f^2 - 2*d*e)*x) + (b^2*f^4 + 2*d^2*e^2 - 2*(b*d*e + a*e^2)*f^2)*lo
g(-b*f^2 - 2*e^2*x + 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 2*(d^2
*e^2 - (b*d*e - a*e^2)*f^2)*log(-e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/
f^2) - d) - 2*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))
/(b^2*e*f^4 - 4*b*d*e^2*f^2 + 4*d^2*e^3)
```

3.476.6 Sympy [F]

$$\int \frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx = \int \frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2)),x)
```

output
```
Integral(1/(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2)), x)
```

3.476.7 Maxima [F]

$$\int \frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx = \int \frac{1}{ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

3.476.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx=\text{Timed out}$$

input
```
integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2)),x, algorithm="giac")
```

output
```
Timed out
```

3.476.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx=\int \frac{1}{d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}}\,dx$$

input
```
int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2)),x)
```

output
```
int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2)), x)
```

3.477 $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx$

3.477.1 Optimal result

Integrand size = 28, antiderivative size = 266

$$\begin{aligned}
&\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx \\
&= -\frac{2(d^2e-bdf^2+aef^2)}{(2de-bf^2)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)} \\
&\quad -\frac{f^2(4ae^2-b^2f^2)}{(2de-bf^2)^2\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} \\
&\quad +\frac{2f^2(4ae^2-b^2f^2)\log\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{(2de-bf^2)^3} \\
&\quad -\frac{2f^2(4ae^2-b^2f^2)\log\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}{(2de-bf^2)^3}
\end{aligned}$$

output
```
2*f^2*(-b^2*f^2+4*a*e^2)*ln(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))/(-b*f^2+2*d
*e)^3-2*f^2*(-b^2*f^2+4*a*e^2)*ln(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)
^(1/2)))/(-b*f^2+2*d*e)^3-2*(a*e*f^2-b*d*f^2+d^2*e)/(-b*f^2+2*d*e)^2/(d+e*
x+f*(a+b*x+e^2*x^2/f^2)^(1/2))-f^2*(-b^2*f^2+4*a*e^2)/(-b*f^2+2*d*e)^2/(b*
f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(1/2)))
```

3.477.2 Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.61

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx$$

$$= \frac{2(bf^3(-d+ex)+2ef(af^2-dex))\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}{(-2de+bf^2)^2\,(d^2+2dex-f^2(a+bx))}$$

$$-\frac{2(4a^2e^3f^2x+bdx(2d^2e^2-2bdef^2+b^2f^4+2de^3x-be^2f^2x)+a(4d^3e^2+2be^3f^2x^2+d^2(-4bef^2+4e^3}{(bd-2ae)\,(-2de+bf^2)^2\,(-d^2-2dex+f^2(a+bx))}$$

$$-\frac{2(4ae^2f^2-b^2f^4)\log\left(-\sqrt{a}f+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)}{(2de-bf^2)^3}$$

$$+\frac{2(4ae^2f^2-b^2f^4)\log\left(-af^2+dex-bf^2x-df\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+\sqrt{a}f\left(d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right.\right.\right.}{(2de-bf^2)^3}$$

input
```
Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-2),x]
```

output
```
(2*(b*f^3*(-d + e*x) + 2*e*f*(a*f^2 - d*e*x))*Sqrt[a + x*(b + (e^2*x)/f^2)
])/((-2*d*e + b*f^2)^2*(d^2 + 2*d*e*x - f^2*(a + b*x))) - (2*(4*a^2*e^3*f^
2*x + b*d*x*(2*d^2*e^2 - 2*b*d*e*f^2 + b^2*f^4 + 2*d*e^3*x - b*e^2*f^2*x)
+ a*(4*d^3*e^2 + 2*b*e^3*f^2*x^2 + d^2*(-4*b*e*f^2 + 4*e^3*x) + d*(b^2*f^4
 - 6*b*e^2*f^2*x - 4*e^4*x^2))))/((b*d - 2*a*e)*(-2*d*e + b*f^2)^2*(-d^2 -
 2*d*e*x + f^2*(a + b*x))) - (2*(4*a*e^2*f^2 - b^2*f^4)*Log[-(Sqrt[a]*f) +
 e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/(2*d*e - b*f^2)^3 + (2*(4*a*e^2*f
^2 - b^2*f^4)*Log[-(a*f^2) + d*e*x - b*f^2*x - d*f*Sqrt[a + x*(b + (e^2*x)
/f^2)] + Sqrt[a]*f*(d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])])/(2*d*e -
b*f^2)^3
```

3.477.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2541, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^2}dx$$

$$\Big\downarrow \text{2541}$$

$$2\int \frac{ed^2-bf^2d+aef^2+e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2-(2de-bf^2)\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2\left(-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\right)^2}d\left(d+ex+\right.$$

$$\Big\downarrow \text{1195}$$

$$2\int \left(\frac{ed^2-bf^2d+aef^2}{(2de-bf^2)^2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2}+\frac{4ae^2f^2-b^2f^4}{(2de-bf^2)^3\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}+\frac{}{(2de-bf^2)^3\left(\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{f^2(4ae^2-b^2f^2)}{2(2de-bf^2)^2\left(-2e\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)-bf^2+2de\right)}+\frac{f^2(4ae^2-b^2f^2)\log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d\right.}{(2de-bf^2)^3}\right.$$

input
```
Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-2),x]
```

output
```
2*(-((d^2*e - b*d*f^2 + a*e*f^2)/((2*d*e - b*f^2)^2*(d + e*x + f*Sqrt[a +
b*x + (e^2*x^2)/f^2]))) + (f^2*(4*a*e^2 - b^2*f^2))/(2*(2*d*e - b*f^2)^2*(
2*d*e - b*f^2 - 2*e*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]))) + (f^2*(
4*a*e^2 - b^2*f^2)*Log[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e
- b*f^2)^3 - (f^2*(4*a*e^2 - b^2*f^2)*Log[2*d*e - b*f^2 - 2*e*(d + e*x + f
*Sqrt[a + b*x + (e^2*x^2)/f^2])])/(2*d*e - b*f^2)^3)
```

3.477.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2541
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Simp[2   Subst[Int[(g + h*x^n)^p*((d
^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x
)^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e
, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.477.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 6302 vs. 2(258) = 516.

Time = 0.11 (sec) , antiderivative size = 6303, normalized size of antiderivative = 23.70

method	result	size
default	Expression too large to display	6303

input
```
int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.477.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 826 vs. 2(251) = 502.

Time = 1.67 (sec) , antiderivative size = 826, normalized size of antiderivative = 3.11

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx =$$

$$-\frac{ab^2f^6+(3\,b^2d^2-14\,abde+8\,a^2e^2)f^4-2\,(bd^3e-4\,ad^2e^2)f^2-4\,(b^2e^2f^4-4\,bde^3f^2+4\,d^2e^4)x^2+(b^3f}{}$$

input
```
integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="fricas")
```

output
```
-1/2*(a*b^2*f^6 + (3*b^2*d^2 - 14*a*b*d*e + 8*a^2*e^2)*f^4 - 2*(b*d^3*e -
4*a*d^2*e^2)*f^2 - 4*(b^2*e^2*f^4 - 4*b*d*e^3*f^2 + 4*d^2*e^4)*x^2 + (b^3*
f^6 - 8*b^2*d*e*f^4 + 20*b*d^2*e^2*f^2 - 16*d^3*e^3)*x - 2*(a*b^2*f^6 + 4*
a*d^2*e^2*f^2 - (b^2*d^2 + 4*a^2*e^2)*f^4 + (b^3*f^6 + 8*a*d*e^3*f^2 - 2*(
b^2*d*e + 2*a*b*e^2)*f^4)*x)*log(-4*a*d*e^2*f^2 - (b^2*d - 4*a*b*e)*f^4 +
4*(b*e^3*f^2 - 2*d*e^4)*x^2 + (3*b^2*e*f^4 - 4*(2*b*d*e^2 - a*e^3)*f^2)*x
- (b^2*f^5 - 4*(b*d*e - a*e^2)*f^3 + 4*(b*e^2*f^3 - 2*d*e^3*f)*x)*sqrt((b*
f^2*x + e^2*x^2 + a*f^2)/f^2)) - 2*(a*b^2*f^6 + 4*a*d^2*e^2*f^2 - (b^2*d^2
 + 4*a^2*e^2)*f^4 + (b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4
)*x)*log(a*f^2 - d^2 + (b*f^2 - 2*d*e)*x) + 2*(a*b^2*f^6 + 4*a*d^2*e^2*f^2
 - (b^2*d^2 + 4*a^2*e^2)*f^4 + (b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a
*b*e^2)*f^4)*x)*log(-e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - d) -
4*((b^2*d - 2*a*b*e)*f^5 - 2*(b*d^2*e - 2*a*d*e^2)*f^3 - (b^2*e*f^5 - 4*b*
d*e^2*f^3 + 4*d^2*e^3*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))/(a*b^3*
f^8 + 8*d^5*e^3 - (b^3*d^2 + 6*a*b^2*d*e)*f^6 + 6*(b^2*d^3*e + 2*a*b*d^2*e
^2)*f^4 - 4*(3*b*d^4*e^2 + 2*a*d^3*e^3)*f^2 + (b^4*f^8 - 8*b^3*d*e*f^6 + 2
4*b^2*d^2*e^2*f^4 - 32*b*d^3*e^3*f^2 + 16*d^4*e^4)*x)
```

3.477.6 Sympy [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx=\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**2,x)
```

output
```
Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-2), x)
```

3.477.7 Maxima [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx=\int \frac{1}{\left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d\right)^2}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-2), x)
```

3.477.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1618 vs. 2(251) = 502.

Time = 6.66 (sec) , antiderivative size = 1618, normalized size of antiderivative = 6.08

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx=\text{Too large to display}$$

input
```
integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")
```

output
```
2*e^2*x/(b^2*f^4 - 4*b*d*e*f^2 + 4*d^2*e^2) + 1/5*(b^2*e*f^3*abs(f) - 4*a*
e^3*f*abs(f))*log(abs((x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))^4*b^3*f
^6 - 2*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))^2*b^3*d^2*f^6 + b^3*d^
4*f^6 + 4*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))^2*a*b^2*d*e*f^6 - 4
*a*b^2*d^3*e*f^6 + 4*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))^2*a^2*b*
e^2*f^6 + 4*a^2*b*d^2*e^2*f^6 + 4*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f
^2))^3*a*b^2*f^6*abs(e) - 4*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))*a
*b^2*d^2*f^6*abs(e) + 8*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))*a^2*b
*d*e*f^6*abs(e) - 4*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))^4*b^2*d*e
*f^4 + 4*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))^2*b^2*d^3*e*f^4 + 8*
(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))^4*a*b*e^2*f^4 - 24*(x*abs(e)
- sqrt(b*f^2*x + e^2*x^2 + a*f^2))^2*a*b*d^2*e^2*f^4 + 16*(x*abs(e) - sqrt
(b*f^2*x + e^2*x^2 + a*f^2))^2*a^2*d*e^3*f^4 + 2*(x*abs(e) - sqrt(b*f^2*x
+ e^2*x^2 + a*f^2))^5*b^2*f^4*abs(e) - 8*(x*abs(e) - sqrt(b*f^2*x + e^2*x^
2 + a*f^2))^3*b^2*d^2*f^4*abs(e) + 6*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 +
a*f^2))*b^2*d^4*f^4*abs(e) - 16*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2
))*a*b*d^3*e*f^4*abs(e) + 8*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))^3
*a^2*e^2*f^4*abs(e) + 8*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))*a^2*d
^2*e^2*f^4*abs(e) - 4*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))^4*b*d^2
*e^2*f^2 + 12*(x*abs(e) - sqrt(b*f^2*x + e^2*x^2 + a*f^2))^2*b*d^4*e^2*...
```

3.477.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx=\int \frac{1}{\left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)^2}\,dx$$

input
```
int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^2,x)
```

output
```
int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^2, x)
```

3.478 $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}\,dx$

3.478.1 Optimal result

Integrand size = 28, antiderivative size = 330

$$\begin{aligned}\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}\,dx &= -\frac{d^2e-bdf^2+aef^2}{(2de-bf^2)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2} \\ &\quad -\frac{2f^2(4ae^2-b^2f^2)}{(2de-bf^2)^3\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)} \\ &\quad -\frac{2ef^2(4ae^2-b^2f^2)}{(2de-bf^2)^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} \\ &\quad +\frac{6ef^2(4ae^2-b^2f^2)\log\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{(2de-bf^2)^4} \\ &\quad -\frac{6ef^2(4ae^2-b^2f^2)\log\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}{(2de-bf^2)^4}\end{aligned}$$

output
```
6*e*f^2*(-b^2*f^2+4*a*e^2)*ln(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))/(-b*f^2+2
*d*e)^4-6*e*f^2*(-b^2*f^2+4*a*e^2)*ln(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/
f^2)^(1/2)))/(-b*f^2+2*d*e)^4+(-a*e*f^2+b*d*f^2-d^2*e)/(-b*f^2+2*d*e)^2/(d
+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2-2*f^2*(-b^2*f^2+4*a*e^2)/(-b*f^2+2*d*e
)^3/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))-2*e*f^2*(-b^2*f^2+4*a*e^2)/(-b*f^2
+2*d*e)^3/(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(1/2)))
```

3.478.2 Mathematica [A] (verified)

Time = 10.51 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}\,dx =$$

$$-\frac{\frac{(-2de+bf^2)^2(d^2e-bdf^2+aef^2)}{\left(d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)^2}+\frac{2f^2(-2de+bf^2)(-4ae^2+b^2f^2)}{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}+\frac{2ef^2(2de-bf^2)(4ae^2-b^2f^2)}{bf^2+2e\left(ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)}-6ef^2(4ae^2-b^2f^2)\log}{(-2de+}$$

input
```
Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3),x]
```

output
```
-((((-2*d*e + b*f^2)^2*(d^2*e - b*d*f^2 + a*e*f^2))/(d + e*x + f*Sqrt[a +
x*(b + (e^2*x)/f^2)])^2 + (2*f^2*(-2*d*e + b*f^2)*(-4*a*e^2 + b^2*f^2))/(d
 + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + (2*e*f^2*(2*d*e - b*f^2)*(4*a*
e^2 - b^2*f^2))/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) - 6*
e*f^2*(4*a*e^2 - b^2*f^2)*Log[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]] +
 6*e*f^2*(4*a*e^2 - b^2*f^2)*Log[-(b*f^2) - 2*e*(e*x + f*Sqrt[a + x*(b + (
e^2*x)/f^2)])])/(-2*d*e + b*f^2)^4)
```

3.478.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2541, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^3}dx$$

$$\Big\downarrow \text{2541}$$

$$2\int \frac{ed^2-bf^2d+aef^2+e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2-(2de-bf^2)\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^3\left(-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\right)^2}d\left(d+ex+\right.$$

$$\Big\downarrow \text{1195}$$

$$2\int\left(\frac{ed^2-bf^2d+aef^2}{(2de-bf^2)^2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^3}+\frac{3(4ae^3f^2-b^2ef^4)}{(2de-bf^2)^4\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}+\frac{}{(2de-bf^2)^4\left(\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(-\frac{f^2(4ae^2-b^2f^2)}{(2de-bf^2)^3\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}+\frac{ef^2(4ae^2-b^2f^2)}{(2de-bf^2)^3\left(-2e\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)-bf^2+2de\right)}\right.$$

input
```
Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3),x]
```

output
```
2*(-1/2*(d^2*e - b*d*f^2 + a*e*f^2)/((2*d*e - b*f^2)^2*(d + e*x + f*Sqrt[a
 + b*x + (e^2*x^2)/f^2])^2) - (f^2*(4*a*e^2 - b^2*f^2))/((2*d*e - b*f^2)^3
*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])) + (e*f^2*(4*a*e^2 - b^2*f^2)
)/((2*d*e - b*f^2)^3*(2*d*e - b*f^2 - 2*e*(d + e*x + f*Sqrt[a + b*x + (e^2
*x^2)/f^2]))) + (3*e*f^2*(4*a*e^2 - b^2*f^2)*Log[d + e*x + f*Sqrt[a + b*x
+ (e^2*x^2)/f^2]])/(2*d*e - b*f^2)^4 - (3*e*f^2*(4*a*e^2 - b^2*f^2)*Log[2*
d*e - b*f^2 - 2*e*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])])/(2*d*e - b
*f^2)^4)
```

3.478. $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}dx$

3.478.3.1 Defintions of rubi rules used

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2541
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Simp[2   Subst[Int[(g + h*x^n)^p*((d
^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x
)^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e
, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.478.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 29132 vs. $2(320) = 640$.

Time = 0.29 (sec) , antiderivative size = 29133, normalized size of antiderivative = 88.28

method	result	size
default	Expression too large to display	29133

input
```
int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.478.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1954 vs. $2(311) = 622$.

Time = 12.00 (sec) , antiderivative size = 1954, normalized size of antiderivative = 5.92

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="fricas")
```

output
```
((3*a*b^3*d - 4*a^2*b^2*e)*f^8 - (b^3*d^3 + 4*a*b^2*d^2*e + 10*a^2*b*d*e^2
 - 20*a^3*e^3)*f^6 - 4*(b^2*d^4*e - 8*a*b*d^3*e^2 + 6*a^2*d^2*e^3)*f^4 - 4
*(b^3*e^3*f^6 - 6*b^2*d*e^4*f^4 + 12*b*d^2*e^5*f^2 - 8*d^3*e^6)*x^3 + 2*(b
*d^5*e^2 - 6*a*d^4*e^3)*f^2 - (b^4*e*f^8 - 2*a*b^2*e^3*f^6 - 40*d^4*e^5 -
2*(11*b^2*d^2*e^3 - 4*a*b*d*e^4)*f^4 + 8*(7*b*d^3*e^4 - a*d^2*e^5)*f^2)*x^
2 + (16*d^5*e^4 + (3*b^4*d - 5*a*b^3*e)*f^8 - (7*b^3*d^2*e + 10*a*b^2*d*e^
2 - 28*a^2*b*e^3)*f^6 + 2*(5*b^2*d^3*e^2 + 22*a*b*d^2*e^3 - 28*a^2*d*e^4)*
f^4 - 8*(3*b*d^4*e^3 + a*d^3*e^4)*f^2)*x - 3*(a^2*b^2*e*f^8 - 4*a*d^4*e^3*
f^2 - 2*(a*b^2*d^2*e + 2*a^3*e^3)*f^6 + (b^2*d^4*e + 8*a^2*d^2*e^3)*f^4 +
(b^4*e*f^8 - 16*a*d^2*e^5*f^2 - 4*(b^3*d*e^2 + a*b^2*e^3)*f^6 + 4*(b^2*d^2
*e^3 + 4*a*b*d*e^4)*f^4)*x^2 + 2*(a*b^3*e*f^8 - 8*a*d^3*e^4*f^2 - (b^3*d^2
*e + 2*a*b^2*d*e^2 + 4*a^2*b*e^3)*f^6 + 2*(b^2*d^3*e^2 + 2*a*b*d^2*e^3 + 4
*a^2*d*e^4)*f^4)*x)*log(-4*a*d*e^2*f^2 - (b^2*d - 4*a*b*e)*f^4 + 4*(b*e^3*
f^2 - 2*d*e^4)*x^2 + (3*b^2*e*f^4 - 4*(2*b*d*e^2 - a*e^3)*f^2)*x - (b^2*f^
5 - 4*(b*d*e - a*e^2)*f^3 + 4*(b*e^2*f^3 - 2*d*e^3*f)*x)*sqrt((b*f^2*x + e
^2*x^2 + a*f^2)/f^2)) - 3*(a^2*b^2*e*f^8 - 4*a*d^4*e^3*f^2 - 2*(a*b^2*d^2*
e + 2*a^3*e^3)*f^6 + (b^2*d^4*e + 8*a^2*d^2*e^3)*f^4 + (b^4*e*f^8 - 16*a*d
^2*e^5*f^2 - 4*(b^3*d*e^2 + a*b^2*e^3)*f^6 + 4*(b^2*d^2*e^3 + 4*a*b*d*e^4)
*f^4)*x^2 + 2*(a*b^3*e*f^8 - 8*a*d^3*e^4*f^2 - (b^3*d^2*e + 2*a*b^2*d*e^2
+ 4*a^2*b*e^3)*f^6 + 2*(b^2*d^3*e^2 + 2*a*b*d^2*e^3 + 4*a^2*d*e^4)*f^4)...
```

3.478.6 Sympy [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}\,dx=\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**3,x)
```

output
```
Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-3), x)
```

3.478.7 Maxima [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}\,dx = \int \frac{1}{\left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d\right)^3}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-3), x)
```

3.478.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")
```

output
```
Timed out
```

3.478.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}\,dx = \int \frac{1}{\left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)^3}\,dx$$

input
```
int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^3,x)
```

output
```
int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^3, x)
```

3.479 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2} dx$

3.479.1 Optimal result

Integrand size = 30, antiderivative size = 370

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = \frac{f^2(2de-bf^2)(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4e^4} + \frac{f^2(4ae^2-b^2f^2)\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}{12e^3} + \frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{7/2}}{7e} - \frac{f^2(2de-bf^2)^2(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{16e^4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} - \frac{5f^2(2de-bf^2)^{3/2}(4ae^2-b^2f^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\sqrt{2de-bf^2}}\right)}{16\sqrt{2}e^{9/2}}$$

output
```
-5/32*f^2*(-b*f^2+2*d*e)^(3/2)*(-b^2*f^2+4*a*e^2)*arctanh(2^(1/2)*e^(1/2)*
(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/(-b*f^2+2*d*e)^(1/2))/e^(9/2)*2^
(1/2)+1/12*f^2*(-b^2*f^2+4*a*e^2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2
)/e^3+1/7*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(7/2)/e+1/4*f^2*(-b*f^2+2*d*
e)*(-b^2*f^2+4*a*e^2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/e^4-1/16*f
^2*(-b*f^2+2*d*e)^2*(-b^2*f^2+4*a*e^2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))
^(1/2)/e^4/(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(1/2)))
```

3.479.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 985 vs. $2(370) = 740$.

3.479. $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2} dx$

Time = 4.55 (sec) , antiderivative size = 985, normalized size of antiderivative = 2.66

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = \frac{1}{32}\left(\frac{2\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}\left(105b^4f^8+28b^3ef^6\left(-10d+3ex+5f\right.\right.}{}\right.$$

$$-\frac{20b^2d^2f^4\arctan\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{e^{5/2}\sqrt{-de+\frac{bf^2}{2}}}$$

$$-\frac{80abdf^4\arctan\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{e^{3/2}\sqrt{-de+\frac{bf^2}{2}}}$$

$$+\frac{20b^3df^6\arctan\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{e^{7/2}\sqrt{-de+\frac{bf^2}{2}}}$$

$$+\frac{20ab^2f^6\arctan\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{e^{5/2}\sqrt{-de+\frac{bf^2}{2}}}-\frac{5b^4f^8\arctan\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{e^{9/2}\sqrt{-de+\frac{bf^2}{2}}}$$

$$\left.+\frac{80\sqrt{2}ad^2f^2\arctan\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{\sqrt{e}\sqrt{-2de+bf^2}}\right)$$

input
```
Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2),x]
```

3.479. $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2} dx$

output
```
((2*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]]*(105*b^4*f^8 + 28*b^3*
e*f^6*(-10*d + 3*e*x + 5*f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 4*b^2*e^2*f^4*
(21*d^2 - 119*a*f^2 + 16*e*x*(2*e*x - f*Sqrt[a + x*(b + (e^2*x)/f^2)]) - 2
*d*(31*e*x + 49*f*Sqrt[a + x*(b + (e^2*x)/f^2)])) + 16*b*e^3*f^2*(3*d^3 +
79*a*d*f^2 + 36*d*e*x*(2*e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 9*d^2*(3
*e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 4*(15*e^3*x^3 - 8*a*f^3*Sqrt[a +
 x*(b + (e^2*x)/f^2)] + 9*e^2*f*x^2*Sqrt[a + x*(b + (e^2*x)/f^2)])) + 16*e
^4*(20*a^2*f^4 + 6*(d + 2*e*x)^3*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) +
 a*f^2*(-3*d^2 + 4*e*x*(19*e*x + 13*f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 4*d
*(38*e*x + 29*f*Sqrt[a + x*(b + (e^2*x)/f^2)])))))/(21*e^4*(b*f^2 + 2*e*(e
*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]))) - (20*b^2*d^2*f^4*ArcTan[(Sqrt[2]*
Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f
^2]])/(e^(5/2)*Sqrt[-(d*e) + (b*f^2)/2]) - (80*a*b*d*f^4*ArcTan[(Sqrt[2]*S
qrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^
2]])/(e^(3/2)*Sqrt[-(d*e) + (b*f^2)/2]) + (20*b^3*d*f^6*ArcTan[(Sqrt[2]*Sq
rt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2
]])/(e^(7/2)*Sqrt[-(d*e) + (b*f^2)/2]) + (20*a*b^2*f^6*ArcTan[(Sqrt[2]*Sqr
t[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]
])/(e^(5/2)*Sqrt[-(d*e) + (b*f^2)/2]) - (5*b^4*f^8*ArcTan[(Sqrt[2]*Sqrt[e]
*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]]...
```

3.479.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2541, 1192, 1580, 25, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}dx$$

$$\Big\downarrow \text{2541}$$

$$2\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^{5/2}\left(ed^2-bf^2d+aef^2+e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2-\left(2de-bf^2\right)\left(d+e\right.\right.}{\left(-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\right)^2}$$

$$\Big\downarrow \text{1192}$$

$$4\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^3\left(ed^2-bf^2d+aef^2+e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2-(2de-bf^2)\left(d+ex\right.\right.}{\left(-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\right)^2}$$

$\Big\downarrow$ 1580

$$4\left(\frac{\int -\frac{32\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^4e^5-16(2de-bf^2)\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^3e^4+8f^2(4ae^2-b^2f^2)\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2e^3+4f^2(2de}{-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}}{64e^5}\right.$$

$\Big\downarrow$ 25

$$4\left(\frac{f^2(4ae^2-b^2f^2)(2de-bf^2)^2\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{64e^4\left(-2e\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)-bf^2+2de\right)}-\frac{\int\frac{32\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^4e^5-16(2de-bf^2)\left(d+ex+f\right.}{}}{}\right.$$

$\Big\downarrow$ 2341

$$4\left(\frac{f^2(4ae^2-b^2f^2)(2de-bf^2)^2\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{64e^4\left(-2e\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)-bf^2+2de\right)}-\frac{\int\left(-16\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^3e^4-4f^2\right.}{}\right.$$

$\Big\downarrow$ 2009

$$4\left(\frac{f^2(4ae^2-b^2f^2)(2de-bf^2)^2\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{64e^4\left(-2e\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)-bf^2+2de\right)}-\frac{\frac{5\sqrt{e}f^2(4ae^2-b^2f^2)(2de-bf^2)^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+b}}}{\sqrt{2de}}\right.}{\sqrt{2}}}{}\right.$$

3.479. $\int\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}dx$

input
```
Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2),x]
```

output
```
4*((f^2*(2*d*e - b*f^2)^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*
x + (e^2*x^2)/f^2]])/(64*e^4*(2*d*e - b*f^2 - 2*e*(d + e*x + f*Sqrt[a + b*
x + (e^2*x^2)/f^2]))) - (-4*e*f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Sqrt
[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]] - (4*e^2*f^2*(4*a*e^2 - b^2*f^
2)*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2))/3 - (16*e^4*(d + e*x
 + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(7/2))/7 + (5*Sqrt[e]*f^2*(2*d*e - b*f
^2)^(3/2)*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sq
rt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2]])/Sqrt[2])/(64*e^5))
```

3.479.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.479. $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2} dx$

rule 2541
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Simp[2   Subst[Int[(g + h*x^n)^p*((d
^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x
)^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e
, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.479.4 Maple [F]

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}dx$$

input
```
int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x)
```

output
```
int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x)
```

3.479.5 Fricas [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 923, normalized size of antiderivative = 2.49

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}dx = \left[\frac{105\sqrt{\frac{1}{2}}(b^3f^6+8\,ade^3f^2-2\,(b^2de+2\,abe^2)f^4)\sqrt{-\frac{bf^2-2\,de}{e}}\log\left(-b^2f^4+4\,(\right.}{}\right.$$

$$-\frac{105\sqrt{\frac{1}{2}}(b^3f^6+8\,ade^3f^2-2\,(b^2de+2\,abe^2)f^4)\sqrt{\frac{bf^2-2\,de}{e}}\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{ex+f\sqrt{\frac{bf^2x+e^2x^2+af^2}{f^2}}+de}\sqrt{\frac{bf^2-2\,de}{e}}}{bf^2-2\,de}\right)-}{}$$

input
```
integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="fricas")
```

output
```
[1/672*(105*sqrt(1/2)*(b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f
^4)*sqrt(-(b*f^2 - 2*d*e)/e)*log(-b^2*f^4 + 4*(b*d*e - a*e^2)*f^2 - 4*(b*e
^2*f^2 - 2*d*e^3)*x + 4*(2*sqrt(1/2)*e^2*f*sqrt(-(b*f^2 - 2*d*e)/e)*sqrt((
b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(1/2)*(b*e*f^2 + 2*e^3*x)*sqrt(-(b*f
^2 - 2*d*e)/e))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d) +
4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 2*(105*b^
3*f^6 + 192*e^6*x^3 + 48*d^3*e^3 - 56*(5*b^2*d*e + 6*a*b*e^2)*f^4 + 4*(21*
b*d^2*e^2 + 232*a*d*e^3)*f^2 + 144*(b*e^4*f^2 + 2*d*e^5)*x^2 + 2*(7*b^2*e^
2*f^4 + 156*d^2*e^4 - 4*(3*b*d*e^3 - 32*a*e^4)*f^2)*x - 2*(35*b^2*e*f^5 -
96*e^5*f*x^2 + 12*d^2*e^3*f - 4*(21*b*d*e^2 + 20*a*e^3)*f^3 - 24*(b*e^3*f^
3 + 6*d*e^4*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt
((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/e^4, -1/336*(105*sqrt(1/2)*(b^3*f^
6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4)*sqrt((b*f^2 - 2*d*e)/e)*a
rctan(2*sqrt(1/2)*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d)*
e*sqrt((b*f^2 - 2*d*e)/e)/(b*f^2 - 2*d*e)) - (105*b^3*f^6 + 192*e^6*x^3 +
48*d^3*e^3 - 56*(5*b^2*d*e + 6*a*b*e^2)*f^4 + 4*(21*b*d^2*e^2 + 232*a*d*e^
3)*f^2 + 144*(b*e^4*f^2 + 2*d*e^5)*x^2 + 2*(7*b^2*e^2*f^4 + 156*d^2*e^4 -
4*(3*b*d*e^3 - 32*a*e^4)*f^2)*x - 2*(35*b^2*e*f^5 - 96*e^5*f*x^2 + 12*d^2*
e^3*f - 4*(21*b*d*e^2 + 20*a*e^3)*f^3 - 24*(b*e^3*f^3 + 6*d*e^4*f)*x)*sqrt
((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2...
```

3.479.6 Sympy [F]

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}\,dx=\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(5/2),x)
```

output
```
Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(5/2), x)
```

3.479.7 Maxima [F]

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = \int \left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{5}{2}} dx$$

input
```
integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(5/2), x)
```

3.479.8 Giac [F]

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = \int \left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{5}{2}} dx$$

input
```
integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="giac")
```

output
```
integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(5/2), x)
```

3.479.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = \int \left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)^{5/2} dx$$

input
```
int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(5/2),x)
```

output
```
int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(5/2), x)
```

3.480 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2} dx$

3.480.1 Optimal result

Integrand size = 30, antiderivative size = 302

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \frac{f^2(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4e^3} + \frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}{5e} - \frac{f^2(2de-bf^2)(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{8e^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} - \frac{3f^2\sqrt{2de-bf^2}(4ae^2-b^2f^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\sqrt{2de-bf^2}}\right)}{8\sqrt{2}e^{7/2}}$$

output
```
-3/16*f^2*(-b^2*f^2+4*a*e^2)*arctanh(2^(1/2)*e^(1/2)*(d+e*x+f*(a+b*x+e^2*x
^2/f^2)^(1/2))^(1/2)/(-b*f^2+2*d*e)^(1/2))*(-b*f^2+2*d*e)^(1/2)/e^(7/2)*2^
(1/2)+1/5*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2)/e+1/4*f^2*(-b^2*f^2+4*
a*e^2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/e^3-1/8*f^2*(-b*f^2+2*d*e
)*(-b^2*f^2+4*a*e^2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/e^3/(b*f^2+
2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(1/2)))
```

3.480.2 Mathematica [A] (verified)

Time = 2.57 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.47

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \frac{\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}\left(-15b^3f^6-2b^2ef^4\left(-5d+6ex+10f\sqrt{a+\right.\right.}{} - \frac{3af^2\sqrt{-de+\frac{bf^2}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{2e^{3/2}} + \frac{3b^2f^4\sqrt{-de+\frac{bf^2}{2}}\arctan\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{8e^{7/2}}$$

input
```
Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2),x]
```

output
```
(Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]]*(-15*b^3*f^6 - 2*b^2*e*f^
4*(-5*d + 6*e*x + 10*f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 4*b*e^2*f^2*(2*d^2
 + 17*a*f^2 + 8*e*x*(2*e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 4*d*(3*e*x
 + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) + 8*e^3*(2*(d + 2*e*x)^2*(e*x + f*Sqr
t[a + x*(b + (e^2*x)/f^2)]) + a*f^2*(-d + 16*e*x + 12*f*Sqrt[a + x*(b + (e
^2*x)/f^2)]))))/(40*e^3*(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)
]))) - (3*a*f^2*Sqrt[-(d*e) + (b*f^2)/2]*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[d +
e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]]])/Sqrt[-2*d*e + b*f^2]])/(2*e^(3/2))
 + (3*b^2*f^4*Sqrt[-(d*e) + (b*f^2)/2]*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*
x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]]])/Sqrt[-2*d*e + b*f^2]])/(8*e^(7/2))
```

3.480.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2541, 1192, 1580, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}dx$$

$$\Big\downarrow \text{2541}$$

$$2\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^{3/2}\left(ed^2-bf^2d+aef^2+e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2-(2de-bf^2)\left(d+e\right.\right.}{\left(-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\right)^2}$$

$$\Big\downarrow \text{1192}$$

$$4\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^{2}\left(ed^2-bf^2d+aef^2+e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2-(2de-bf^2)\left(d+ex\right.\right.}{\left(-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\right)^2}$$

$$\Big\downarrow \text{1580}$$

$$4\left(\frac{f^2(4ae^2-b^2f^2)(2de-bf^2)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{32e^3\left(-2e\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)-bf^2+2de\right)}-\frac{\int\frac{16\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^3e^4-8(2de-bf^2)\left(d+ex+f\sqrt{\right.}}{-b}}{}\right.$$

$$\Big\downarrow \text{2341}$$

$$4\left(\frac{f^2(4ae^2-b^2f^2)(2de-bf^2)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{32e^3\left(-2e\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)-bf^2+2de\right)}-\frac{\int\left(-8\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2e^3-2f^2(4\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$4\left(\frac{f^2(4ae^2-b^2f^2)(2de-bf^2)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{32e^3\left(-2e\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)-bf^2+2de\right)}-\frac{3\sqrt{e}f^2(4ae^2-b^2f^2)\sqrt{2de-bf^2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2}{}}}}{\sqrt{2de-bf^2}}\right.}{\sqrt{2}}\right.$$

input
```
Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2),x]
```

output
```
4*((f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x
+ (e^2*x^2)/f^2]])/(32*e^3*(2*d*e - b*f^2 - 2*e*(d + e*x + f*Sqrt[a + b*x
+ (e^2*x^2)/f^2]))) - (-2*e*f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[
a + b*x + (e^2*x^2)/f^2]] - (8*e^3*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f
^2])^(5/2))/5 + (3*Sqrt[e]*f^2*Sqrt[2*d*e - b*f^2]*(4*a*e^2 - b^2*f^2)*Arc
Tanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqr
t[2*d*e - b*f^2]])/Sqrt[2])/(32*e^4))
```

3.480.3.1 Defintions of rubi rules used

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.480. $\int\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}dx$

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2541
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Simp[2   Subst[Int[(g + h*x^n)^p*((d
^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x
)^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e
, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.480.4 Maple [F]

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}dx$$

input
```
int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x)
```

output
```
int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x)
```

3.480.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 657, normalized size of antiderivative = 2.18

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}dx = \left[-\frac{15\sqrt{\frac{1}{2}}(b^2f^4-4ae^2f^2)\sqrt{-\frac{bf^2-2de}{e}}\log\left(-b^2f^4+4(bde-ae^2)f^2-4(be^2\right.}{}\right.$$

input
```
integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="fricas")
```

output
```
[-1/80*(15*sqrt(1/2)*(b^2*f^4 - 4*a*e^2*f^2)*sqrt(-(b*f^2 - 2*d*e)/e)*log(
-b^2*f^4 + 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x + 4*(2*sqrt(1
/2)*e^2*f*sqrt(-(b*f^2 - 2*d*e)/e)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) -
 sqrt(1/2)*(b*e*f^2 + 2*e^3*x)*sqrt(-(b*f^2 - 2*d*e)/e))*sqrt(e*x + f*sqrt
((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d) + 4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f
^2*x + e^2*x^2 + a*f^2)/f^2)) + 2*(15*b^2*f^4 - 16*e^4*x^2 - 8*d^2*e^2 - 2
*(5*b*d*e + 24*a*e^2)*f^2 + 2*(b*e^2*f^2 - 18*d*e^3)*x - 2*(5*b*e*f^3 + 8*
e^3*f*x - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*s
qrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/e^3, 1/40*(15*sqrt(1/2)*(b^2*f^
4 - 4*a*e^2*f^2)*sqrt((b*f^2 - 2*d*e)/e)*arctan(2*sqrt(1/2)*sqrt(e*x + f*s
qrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d)*e*sqrt((b*f^2 - 2*d*e)/e)/(b*f^2
 - 2*d*e)) - (15*b^2*f^4 - 16*e^4*x^2 - 8*d^2*e^2 - 2*(5*b*d*e + 24*a*e^2)
*f^2 + 2*(b*e^2*f^2 - 18*d*e^3)*x - 2*(5*b*e*f^3 + 8*e^3*f*x - 2*d*e^2*f)*
sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^
2 + a*f^2)/f^2) + d))/e^3]
```

3.480.6 Sympy [F]

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}} dx$$

input
```
integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(3/2),x)
```

output
```
Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(3/2), x)
```

3.480.7 Maxima [F]

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \int \left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{3}{2}} dx$$

input
```
integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(3/2), x)
```

3.480.8 Giac [F]

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \int \left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{3}{2}} dx$$

input
```
integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(3/2), x)
```

3.480.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \int \left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)^{3/2} dx$$

input
```
int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(3/2),x)
```

output
```
int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(3/2), x)
```

3.481 $\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$

3.481.1 Optimal result

Integrand size = 30, antiderivative size = 233

$$\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx=\frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e}-\frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}-\frac{f^2(4ae^2-b^2f^2)\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\sqrt{2de-bf^2}}\right)}{4\sqrt{2}e^{5/2}\sqrt{2de-bf^2}}$$

output
```
-1/8*f^2*(-b^2*f^2+4*a*e^2)*arctanh(2^(1/2)*e^(1/2)*(d+e*x+f*(a+b*x+e^2*x^
2/f^2)^(1/2))^(1/2)/(-b*f^2+2*d*e)^(1/2))/e^(5/2)*2^(1/2)/(-b*f^2+2*d*e)^(
1/2)+1/3*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2)/e-1/4*f^2*(4*a-b^2*f^2/
e^2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/(b*f^2+2*e*(e*x+f*(a+x*(b*f
^2+e^2*x)/f^2)^(1/2)))
```

3.481.2 Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.40

$$\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$$
$$=\frac{\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}\left(3b^2f^4+4bef^2\left(d+3ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)+4e^2\left(-af^2+2(d+2\cdots\right.\right.}{12e^2\left(bf^2+2e\left(ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)}$$
$$+\frac{af^2\arctan\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{\sqrt{2}\sqrt{e}\sqrt{-2de+bf^2}}-\frac{b^2f^4\arctan\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{4e^{5/2}\sqrt{-4de+2bf^2}}$$

input
```
Integrate[Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

output
```
(Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]]*(3*b^2*f^4 + 4*b*e*f^2*(d
 + 3*e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 4*e^2*(-(a*f^2) + 2*(d + 2*e
*x)*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]))))/(12*e^2*(b*f^2 + 2*e*(e*x +
 f*Sqrt[a + x*(b + (e^2*x)/f^2)]))) + (a*f^2*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[
d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]])/(Sqrt[2
]*Sqrt[e]*Sqrt[-2*d*e + b*f^2]) - (b^2*f^4*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[d
+ e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]])/(4*e^(5/2
)*Sqrt[-4*d*e + 2*b*f^2])
```

3.481.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2541, 1192, 1580, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}\,dx$$

$\Big\downarrow$ 2541

$$2\int \frac{\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}}\left(ed^2-bf^2d+aef^2+e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2-(2de-bf^2)\left(d+ex+\right.\right.}{\left(-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\right)^2}$$

$\Big\downarrow$ 1192

$$4\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\left(ed^2-bf^2d+aef^2+e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2-(2de-bf^2)\left(d+ex\right.\right.}{\left(-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\right)^2}$$

$\Big\downarrow$ 1580

$$4\left(\frac{\int -\frac{8\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2e^3-4(2de-bf^2)\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)e^2+f^2(4ae^2-b^2f^2)e}{-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}d\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}}}{16e^3}+\right.$$

$\Big\downarrow$ 25

$$4\left(\frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{16\left(-2e\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)-bf^2+2de\right)}-\frac{\int \frac{8\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2e^3-4(2de-bf^2)\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx\right.}}{-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}}{16e^3}\right.$$

$\Big\downarrow$ 1467

$$4\left(\frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{16\left(-2e\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)-bf^2+2de\right)}-\frac{\int\left(\frac{ef^2(4ae^2-b^2f^2)}{-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}-4e^2\left(d+ex+f\right.\right.}{16e^3}\right.$$

$\Big\downarrow$ 2009

3.481. $\displaystyle\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$

$$4\left(\frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{16\left(-2e\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)-bf^2+2de\right)}-\frac{\frac{\sqrt{e}f^2(4ae^2-b^2f^2)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{2}\sqrt{2de-bf^2}}-\frac{4}{3}e^2\right.$$

input
```
Int[Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

output
```
4*((f^2*(4*a - (b^2*f^2)/e^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^
2]])/(16*(2*d*e - b*f^2 - 2*e*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]))
) - ((-4*e^2*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2))/3 + (Sqrt[
e]*f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[
a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2]])/(Sqrt[2]*Sqrt[2*d*e - b*f
^2]))/(16*e^3))
```

3.481.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

3.481. $\int\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2541
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Simp[2   Subst[Int[(g + h*x^n)^p*((d
^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x
)^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e
, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.481.4 Maple [F]

$$\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}dx$$

input
```
int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x)
```

output
```
int((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x)
```

3.481.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 692, normalized size of antiderivative = 2.97

$$\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$$

$$= \left[-\frac{3\,(b^2f^4-4\,ae^2f^2)\sqrt{-2\,bef^2+4\,de^2}\log\left(-b^2f^4+4\,(bde-ae^2)f^2-4\,(be^2f^2-2\,de^3)x-2\left(2\sqrt{-2\,be\right.\right.}{}\right.$$

input
```
integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*(b^2*f^4 - 4*a*e^2*f^2)*sqrt(-2*b*e*f^2 + 4*d*e^2)*log(-b^2*f^4
+ 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x - 2*(2*sqrt(-2*b*e*f^2
 + 4*d*e^2)*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(-2*b*e*f^2 +
4*d*e^2)*(b*f^2 + 2*e^2*x))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/
f^2) + d) + 4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))
 - 4*(3*b^2*e*f^4 - 2*b*d*e^2*f^2 - 8*d^2*e^3 + 10*(b*e^3*f^2 - 2*d*e^4)*x
 - 2*(b*e^2*f^3 - 2*d*e^3*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e
*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(b*e^3*f^2 - 2*d*e^4),
1/24*(3*(b^2*f^4 - 4*a*e^2*f^2)*sqrt(2*b*e*f^2 - 4*d*e^2)*arctan(1/2*sqrt(
e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d)*(sqrt(2*b*e*f^2 - 4*d*e
^2)*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(2*b*e*f^2 - 4*d*e^2)*(e
*x + d))/(a*e*f^2 - d^2*e + (b*e*f^2 - 2*d*e^2)*x)) + 2*(3*b^2*e*f^4 - 2*b
*d*e^2*f^2 - 8*d^2*e^3 + 10*(b*e^3*f^2 - 2*d*e^4)*x - 2*(b*e^2*f^3 - 2*d*e
^3*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x +
e^2*x^2 + a*f^2)/f^2) + d))/(b*e^3*f^2 - 2*d*e^4)]
```

3.481.6 Sympy [F]

$$\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx = \int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$$

input
```
integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(1/2),x)
```

output
```
Integral(sqrt(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2)), x)
```

3.481.7 Maxima [F]

$$\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx = \int \sqrt{ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d}\,dx$$

input
```
integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

3.481.8 Giac [F]

$$\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx = \int \sqrt{ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d}\,dx$$

input
```
integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

3.481.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx=\int \sqrt{d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}}\,dx$$

input
```
int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(1/2),x)
```

output
```
int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(1/2), x)
```

3.482 $\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx$

3.482.1 Optimal result

Integrand size = 30, antiderivative size = 244

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx = \frac{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{e} - \frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{2\left(2de-bf^2\right)\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} + \frac{f^2(4ae^2-b^2f^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\sqrt{2de-bf^2}}\right)}{2\sqrt{2}e^{3/2}\left(2de-bf^2\right)^{3/2}}$$

output
```
1/4*f^2*(-b^2*f^2+4*a*e^2)*arctanh(2^(1/2)*e^(1/2)*(d+e*x+f*(a+b*x+e^2*x^2
/f^2)^(1/2))^(1/2)/(-b*f^2+2*d*e)^(1/2))/e^(3/2)/(-b*f^2+2*d*e)^(3/2)*2^(1
/2)+(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/e-1/2*f^2*(4*a*e-b^2*f^2/e)*
(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/(-b*f^2+2*d*e)/(b*f^2+2*e*(e*x+f
*(a+x*(b*f^2+e^2*x)/f^2)^(1/2)))
```

3.482.2 Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.40

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx$$

$$= \frac{\frac{2\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}\left(-b^2f^4-4bef^2\left(-d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)+4e^2\left(-af^2+2dex+2df\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)}{(2de-bf^2)\left(bf^2+2e\left(ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)}+\frac{4\sqrt{2}ae^2f^2\arctan}{}}{4e^{3/2}}$$

input
```
Integrate[1/Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

output
```
((2*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]]*(-(b^2*f^4) -
4*b*e*f^2*(-d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 4*e^2*(-(a*f^2) +
 2*d*e*x + 2*d*f*Sqrt[a + x*(b + (e^2*x)/f^2)])))/((2*d*e - b*f^2)*(b*f^2
+ 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]))) + (4*Sqrt[2]*a*e^2*f^2*Arc
Tan[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt
[-2*d*e + b*f^2]])/(-2*d*e + b*f^2)^(3/2) - (Sqrt[2]*b^2*f^4*ArcTan[(Sqrt[
2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e +
b*f^2]])/(-2*d*e + b*f^2)^(3/2))/(4*e^(3/2))
```

3.482.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2541, 1192, 1471, 27, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}\,dx$$

$$\Big\downarrow \; 2541$$

$$2\int \frac{ed^2-bf^2d+aef^2+e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2-(2de-bf^2)\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}{\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}}\left(-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\right)^2}d\left(d+ex+\right.$$

$$\Big\downarrow \text{1192}$$

$$4\int \frac{ed^2-bf^2d+aef^2+e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2-(2de-bf^2)\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}{\left(-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\right)^2}d\sqrt{d+ex+}$$

$$\Big\downarrow \text{1471}$$

$$4\left(\frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{8(2de-bf^2)\left(-2e\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)-bf^2+2de\right)}-\frac{\int -\frac{\frac{b^2f^4}{e}-8bdf^2+4aef^2+8d^2e-4(2de-bf^2)\left(d+ex+f\right.}{4\left(-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx-}\right.\right.}}{2(2de}\right.$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{\int \frac{\frac{b^2f^4}{e}-8bdf^2+4aef^2+8d^2e-4(2de-bf^2)\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}{-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}d\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}}}{8(2de-bf^2)}+\frac{f^2\left(4ae-\frac{b^2}{e}\right.}{8(2de-bf^2)\left(-2e\left(\right.\right.}\right.$$

$$\Big\downarrow \text{299}$$

$$4\left(\frac{\frac{f^2(4ae^2-b^2f^2)\int \frac{1}{-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}d\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}}}{e}+\frac{2(2de-bf^2)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{e}}{8(2de-bf^2)}+\frac{}{8(2de-b}\right.$$

$$\Big\downarrow \text{221}$$

3.482. $\displaystyle\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx$

$$4\left(\frac{\frac{f^2(4ae^2-b^2f^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{2}e^{3/2}\sqrt{2de-bf^2}}+\frac{2(2de-bf^2)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{e}}{8(2de-bf^2)}+\frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{\cdots}}{8(2de-bf^2)\left(-2e\left(f\sqrt{a+\cdots}\right.\right.}\right.$$

input
```
Int[1/Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

output
```
4*((f^2*(4*a*e - (b^2*f^2)/e)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^
2]])/(8*(2*d*e - b*f^2)*(2*d*e - b*f^2 - 2*e*(d + e*x + f*Sqrt[a + b*x + (
e^2*x^2)/f^2]))) + ((2*(2*d*e - b*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^
2*x^2)/f^2]])/e + (f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d
 + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2]])/(Sqrt[2]*
e^(3/2)*Sqrt[2*d*e - b*f^2]))/(8*(2*d*e - b*f^2)))
```

3.482.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

3.482. $\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx$

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2541
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Simp[2   Subst[Int[(g + h*x^n)^p*((d
^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x
)^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e
, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.482.4 Maple [F]

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}dx$$

input
```
int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x)
```

output
```
int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x)
```

3.482.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 716, normalized size of antiderivative = 2.93

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}dx$$

$$= \left[\frac{(b^2f^4 - 4\,ae^2f^2)\sqrt{-2\,bef^2 + 4\,de^2}\log\left(-b^2f^4 + 4\,(bde - ae^2)f^2 - 4\,(be^2f^2 - 2\,de^3)x + 2\left(2\sqrt{-2\,bef^2\right.\right.}{} \right.$$

input
```
integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="fricas
")
```

output
```
[1/8*((b^2*f^4 - 4*a*e^2*f^2)*sqrt(-2*b*e*f^2 + 4*d*e^2)*log(-b^2*f^4 + 4*
(b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x + 2*(2*sqrt(-2*b*e*f^2 + 4
*d*e^2)*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(-2*b*e*f^2 + 4*d*
e^2)*(b*f^2 + 2*e^2*x))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)
 + d) + 4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 4
*(b^2*e*f^4 - 6*b*d*e^2*f^2 + 8*d^2*e^3 - 2*(b*e^3*f^2 - 2*d*e^4)*x + 2*(b
*e^2*f^3 - 2*d*e^3*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*
sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(b^2*e^2*f^4 - 4*b*d*e^3*f^2 +
 4*d^2*e^4), 1/4*((b^2*f^4 - 4*a*e^2*f^2)*sqrt(2*b*e*f^2 - 4*d*e^2)*arctan
(1/2*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d)*(sqrt(2*b*e*f
^2 - 4*d*e^2)*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(2*b*e*f^2 - 4
*d*e^2)*(e*x + d))/(a*e*f^2 - d^2*e + (b*e*f^2 - 2*d*e^2)*x)) + 2*(b^2*e*f
^4 - 6*b*d*e^2*f^2 + 8*d^2*e^3 - 2*(b*e^3*f^2 - 2*d*e^4)*x + 2*(b*e^2*f^3
- 2*d*e^3*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f
^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(b^2*e^2*f^4 - 4*b*d*e^3*f^2 + 4*d^2*e^
4)]
```

3.482.6 Sympy [F]

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx = \int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(1/2),x)
```

output
```
Integral(1/sqrt(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2)), x)
```

3.482.7 Maxima [F]

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx = \int \frac{1}{\sqrt{ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\,f+d}}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

3.482.8 Giac [F]

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx = \int \frac{1}{\sqrt{ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\,f+d}}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)
```

3.482.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx = \int \frac{1}{\sqrt{d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}}}\,dx$$

input
```
int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(1/2),x)
```

output
```
int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(1/2), x)
```

3.483 $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx$

3.483.1 Optimal result

Integrand size = 30, antiderivative size = 269

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = -\frac{4(d^2e-bdf^2+aef^2)}{(2de-bf^2)^2\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}} - \frac{f^2(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{(2de-bf^2)^2\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} + \frac{3f^2(4ae^2-b^2f^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\sqrt{2de-bf^2}}\right)}{\sqrt{2}\sqrt{e}\,(2de-bf^2)^{5/2}}$$

output
```
3/2*f^2*(-b^2*f^2+4*a*e^2)*arctanh(2^(1/2)*e^(1/2)*(d+e*x+f*(a+b*x+e^2*x^2
/f^2)^(1/2))^(1/2)/(-b*f^2+2*d*e)^(1/2))/(-b*f^2+2*d*e)^(5/2)*2^(1/2)/e^(1
/2)-4*(a*e*f^2-b*d*f^2+d^2*e)/(-b*f^2+2*d*e)^2/(d+e*x+f*(a+b*x+e^2*x^2/f^2
)^(1/2))^(1/2)-f^2*(-b^2*f^2+4*a*e^2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^
(1/2)/(-b*f^2+2*d*e)^2/(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(1/2)))
```

3.483.2 Mathematica [A] (verified)

Time = 2.41 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.47

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = \frac{b^2f^4\left(5d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)-4bef^2\left(d^2+af^2-2d\left(ex\right.\right.}{(-2de+bf^2)^2\sqrt{d+e}}$$

$$-\frac{6\sqrt{2}ae^{3/2}f^2\arctan\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{(-2de+bf^2)^{5/2}}$$

$$+\frac{3b^2f^4\arctan\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{\sqrt{2}\sqrt{e}\,(-2de+bf^2)^{5/2}}$$

input
```
Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3/2),x]
```

output
```
(b^2*f^4*(5*d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) - 4*b*e*f^2*(d^2 +
a*f^2 - 2*d*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) - 4*e^2*(2*d^2*(e*x +
 f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + a*f^2*(d + 3*e*x + 3*f*Sqrt[a + x*(b +
 (e^2*x)/f^2)])))/((-2*d*e + b*f^2)^2*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^
2*x)/f^2)]]*(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]))) - (6*Sq
rt[2]*a*e^(3/2)*f^2*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b
 + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]])/(-2*d*e + b*f^2)^(5/2) + (3*b^2*
f^4*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]
])/Sqrt[-2*d*e + b*f^2]])/(Sqrt[2]*Sqrt[e]*(-2*d*e + b*f^2)^(5/2))
```

3.483.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2541, 1192, 1582, 27, 359, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}dx$$

$$\Big\downarrow \text{2541}$$

$$2\int \frac{ed^2-bf^2d+aef^2+e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2-(2de-bf^2)\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^{3/2}\left(-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\right)^2}d\left(d+ex+\right.$$

$$\Big\downarrow \text{1192}$$

$$4\int \frac{ed^2-bf^2d+aef^2+e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2-(2de-bf^2)\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\left(-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\right)^2}d\sqrt{d+ex}$$

$$\Big\downarrow \text{1582}$$

$$4\left(\frac{f^2(4ae^2-b^2f^2)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{4(2de-bf^2)^2\left(-2e\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)-bf^2+2de\right)}-\frac{\int -\frac{2e^2\left(4(2de-bf^2)(ed^2-bf^2d+aef^2)-(3b^2f^4-4ae\right.}{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\left(-bf^2+\right.}}{}\right.$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{\int \frac{4(2de-bf^2)(ed^2-bf^2d+aef^2)-(3b^2f^4-4ae^2f^2-8bdef^2+8d^2e^2)\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\left(-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\right)}d\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}}}{4(2de-bf^2)^2}+\frac{}{4}\right.$$

$$\Big\downarrow \text{359}$$

$$4\left(\frac{3f^2(4ae^2-b^2f^2)\int \frac{1}{-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}d\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}}-\frac{4(aef^2-bdf^2+d^2e)}{\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}}{4(2de-bf^2)^2}+\right.$$

$$\Big\downarrow \text{221}$$

3.483. $\displaystyle\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}dx$

$$4\left(\frac{\frac{3f^2(4ae^2-b^2f^2)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{2}\sqrt{e}\sqrt{2de-bf^2}}-\frac{4(aef^2-bdf^2+d^2e)}{\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}}{4\left(2de-bf^2\right)^2}+\frac{f^2\left(4ae^2-b^2f^2\right)\sqrt{f\sqrt{a+b}}}{4\left(2de-bf^2\right)^2\left(-2e\left(f\sqrt{a+bx+\frac{e^2}{}}\right.\right.}\right.$$

input
```
Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3/2),x]
```

output
```
4*((f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]
])/(4*(2*d*e - b*f^2)^2*(2*d*e - b*f^2 - 2*e*(d + e*x + f*Sqrt[a + b*x + (
e^2*x^2)/f^2]))) + ((-4*(d^2*e - b*d*f^2 + a*e*f^2))/Sqrt[d + e*x + f*Sqrt
[a + b*x + (e^2*x^2)/f^2]] + (3*f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*S
qrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2
]])/(Sqrt[2]*Sqrt[e]*Sqrt[2*d*e - b*f^2]))/(4*(2*d*e - b*f^2)^2))
```

3.483.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

3.483. $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx$

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 2541
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Simp[2   Subst[Int[(g + h*x^n)^p*((d
^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x
)^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e
, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.483.4 Maple [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x)
```

output
```
int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x)
```

3.483.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 696 vs. $2(239) = 478$.

Time = 0.79 (sec) , antiderivative size = 1456, normalized size of antiderivative = 5.41

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="fricas
")
```

output
```
[1/4*(3*(a*b^2*f^6 + 4*a*d^2*e^2*f^2 - (b^2*d^2 + 4*a^2*e^2)*f^4 + (b^3*f^
6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4)*x)*sqrt(-2*b*e*f^2 + 4*d*
e^2)*log(-b^2*f^4 + 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x - 2*
(2*sqrt(-2*b*e*f^2 + 4*d*e^2)*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) -
sqrt(-2*b*e*f^2 + 4*d*e^2)*(b*f^2 + 2*e^2*x))*sqrt(e*x + f*sqrt((b*f^2*x +
 e^2*x^2 + a*f^2)/f^2) + d) + 4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*
x^2 + a*f^2)/f^2)) + 4*(a*b^2*e*f^6 - 8*d^4*e^3 - (5*b^2*d^2*e - 2*a*b*d*e
^2)*f^4 + 2*(7*b*d^3*e^2 - 4*a*d^2*e^3)*f^2 + 2*(b^2*e^3*f^4 - 4*b*d*e^4*f
^2 + 4*d^2*e^5)*x^2 + (b^3*e*f^6 - 4*d^3*e^4 - 2*(4*b^2*d*e^2 - 3*a*b*e^3)
*f^4 + 2*(7*b*d^2*e^3 - 6*a*d*e^4)*f^2)*x + 2*(2*d^3*e^3*f + (2*b^2*d*e -
3*a*b*e^2)*f^5 - (5*b*d^2*e^2 - 6*a*d*e^3)*f^3 - (b^2*e^2*f^5 - 4*b*d*e^3*
f^3 + 4*d^2*e^4*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*
sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(a*b^3*e*f^8 + 8*d^5*e^4 - (b^
3*d^2*e + 6*a*b^2*d*e^2)*f^6 + 6*(b^2*d^3*e^2 + 2*a*b*d^2*e^3)*f^4 - 4*(3*
b*d^4*e^3 + 2*a*d^3*e^4)*f^2 + (b^4*e*f^8 - 8*b^3*d*e^2*f^6 + 24*b^2*d^2*e
^3*f^4 - 32*b*d^3*e^4*f^2 + 16*d^4*e^5)*x), -1/2*(3*(a*b^2*f^6 + 4*a*d^2*e
^2*f^2 - (b^2*d^2 + 4*a^2*e^2)*f^4 + (b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e
 + 2*a*b*e^2)*f^4)*x)*sqrt(2*b*e*f^2 - 4*d*e^2)*arctan(1/2*sqrt(e*x + f*sq
rt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d)*(sqrt(2*b*e*f^2 - 4*d*e^2)*f*sqrt
((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(2*b*e*f^2 - 4*d*e^2)*(e*x + d)...
```

3.483.6 Sympy [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = \int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(3/2),x)
```

output
```
Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-3/2), x)
```

3.483.7 Maxima [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = \int \frac{1}{\left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\,f+d\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="maxima
")
```

output
```
integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-3/2), x)
```

3.483.8 Giac [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = \int \frac{1}{\left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}\,f+d\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-3/2), x)
```

3.483.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = \int \frac{1}{\left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)^{3/2}}\,dx$$

input
```
int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(3/2),x)
```

output
```
int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(3/2), x)
```

3.484 $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx$

3.484.1 Optimal result

Integrand size = 30, antiderivative size = 335

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx = -\frac{4(d^2e-bdf^2+aef^2)}{3\left(2de-bf^2\right)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}} - \frac{4f^2(4ae^2-b^2f^2)}{\left(2de-bf^2\right)^3\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}} - \frac{2ef^2(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\left(2de-bf^2\right)^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} + \frac{5\sqrt{2}\sqrt{e}f^2(4ae^2-b^2f^2)\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{\sqrt{2de-bf^2}}\right)}{\left(2de-bf^2\right)^{7/2}}$$

output
```
5*f^2*(-b^2*f^2+4*a*e^2)*arctanh(2^(1/2)*e^(1/2)*(d+e*x+f*(a+b*x+e^2*x^2/f
^2)^(1/2))^(1/2)/(-b*f^2+2*d*e)^(1/2))*2^(1/2)*e^(1/2)/(-b*f^2+2*d*e)^(7/2
)-4/3*(a*e*f^2-b*d*f^2+d^2*e)/(-b*f^2+2*d*e)^2/(d+e*x+f*(a+b*x+e^2*x^2/f^2
)^(1/2))^(3/2)-4*f^2*(-b^2*f^2+4*a*e^2)/(-b*f^2+2*d*e)^3/(d+e*x+f*(a+b*x+e
^2*x^2/f^2)^(1/2))^(1/2)-2*e*f^2*(-b^2*f^2+4*a*e^2)*(d+e*x+f*(a+b*x+e^2*x^
2/f^2)^(1/2))^(1/2)/(-b*f^2+2*d*e)^3/(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/
f^2)^(1/2)))
```

3.484.2 Mathematica [A] (verified)

Time = 3.34 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.66

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx = \frac{2b^3f^6\left(4d+21ex+6f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)+2b^2ef^4\left(9d^2+17af^2\right.}{}$$

$$+\frac{20\sqrt{2}ae^{5/2}f^2\arctan\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{(-2de+bf^2)^{7/2}}$$

$$-\frac{5\sqrt{2}b^2\sqrt{e}f^4\arctan\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{(-2de+bf^2)^{7/2}}$$

input
```
Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-5/2),x]
```

output
```
(2*b^3*f^6*(4*d + 21*e*x + 6*f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 2*b^2*e*f^
4*(9*d^2 + 17*a*f^2 + 14*d*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 30*e*
x*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) - 8*b*e^2*f^2*(d^3 + 7*a*d*f^2
- 3*d^2*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 5*a*f^2*(4*e*x + f*Sqrt[
a + x*(b + (e^2*x)/f^2)])) - 8*e^3*(15*a^2*f^4 + 2*d^3*(e*x + f*Sqrt[a + x
*(b + (e^2*x)/f^2)]) + a*f^2*(3*d^2 + 20*d*(e*x + f*Sqrt[a + x*(b + (e^2*x
)/f^2)]) + 30*e*x*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]))))/(3*(2*d*e - b
*f^2)^3*(d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])^(3/2)*(b*f^2 + 2*e*(e*
x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]))) + (20*Sqrt[2]*a*e^(5/2)*f^2*ArcTan[
(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*
d*e + b*f^2]])/(-2*d*e + b*f^2)^(7/2) - (5*Sqrt[2]*b^2*Sqrt[e]*f^4*ArcTan[
(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*
d*e + b*f^2]])/(-2*d*e + b*f^2)^(7/2)
```

3.484.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2541, 1192, 1582, 27, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}}dx$$

$$\Big\downarrow \text{2541}$$

$$2\int \frac{ed^2-bf^2d+aef^2+e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2-\left(2de-bf^2\right)\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^{5/2}\left(-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\right)^2}d\left(d+ex+\right.$$

$$\Big\downarrow \text{1192}$$

$$4\int \frac{ed^2-bf^2d+aef^2+e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2-\left(2de-bf^2\right)\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^{2}\left(-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\right)^2}d\sqrt{d+ex}$$

$$\Big\downarrow \text{1582}$$

$$4\left(\frac{\int \frac{4\left(f^2(4ae^2-b^2f^2)\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2e^3+2(2de-bf^2)^2(ed^2-bf^2d+aef^2)e^2-2(2de-bf^2)(b^2f^4-2ae^2f^2-2bdef^2+2d^2e^2)\left(d+ex\right.}{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2\left(-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\right)}}{8e^2(2de-bf^2)^3}\right.$$

$$\Big\downarrow \; 27$$

$$4\left(\frac{\int \frac{f^2(4ae^2-b^2f^2)\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2e^3+2(2de-bf^2)^2(ed^2-bf^2d+aef^2)e^2-2(2de-bf^2)(b^2f^4-2ae^2f^2-2bdef^2+2d^2e^2)\left(d+ex+f\right.}{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2\left(-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)\right)}}{2e^2(2de-bf^2)^3}\right.$$

$$\Big\downarrow \; 1584$$

$$4\left(\frac{\int \left(\frac{2(2de-bf^2)(ed^2-bf^2d+aef^2)e^2}{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)^2}+\frac{2(4ae^4f^2-b^2e^2f^4)}{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}}+\frac{5(4ae^5f^2-b^2e^3f^4)}{-bf^2+2de-2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}\right)d\sqrt{d+ex+f\sqrt{\frac{e^2x^2}{f^2}}}}{2e^2(2de-bf^2)^3}\right.$$

$$\Big\downarrow \; 2009$$

$$4\left(\frac{\frac{5e^{5/2}f^2(4ae^2-b^2f^2)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{2}\sqrt{2de-bf^2}}-\frac{2e^2f^2(4ae^2-b^2f^2)}{\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}-\frac{2e^2(2de-bf^2)(aef^2-bdf^2+d^2e)}{3\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}}{2e^2(2de-bf^2)^3}+\frac{}{2(2d}\right.$$

input
```
Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-5/2),x]
```

3.484. $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx$

output
```
4*((e*f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^
2]])/(2*(2*d*e - b*f^2)^3*(2*d*e - b*f^2 - 2*e*(d + e*x + f*Sqrt[a + b*x +
 (e^2*x^2)/f^2]))) + ((-2*e^2*(2*d*e - b*f^2)*(d^2*e - b*d*f^2 + a*e*f^2))
/(3*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2)) - (2*e^2*f^2*(4*a*e
^2 - b^2*f^2))/Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]] + (5*e^(5/2
)*f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a
 + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2]])/(Sqrt[2]*Sqrt[2*d*e - b*f^
2]))/(2*e^2*(2*d*e - b*f^2)^3))
```

3.484.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.484. $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx$

rule 2541
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Simp[2   Subst[Int[(g + h*x^n)^p*((d
^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x
)^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e
, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.484.4 Maple [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x)
```

output
```
int(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x)
```

3.484.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1223 vs. $2(298) = 596$.

Time = 1.96 (sec) , antiderivative size = 2514, normalized size of antiderivative = 7.50

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="fricas
")
```

output
```
[-1/6*(15*sqrt(2)*(a^2*b^2*f^8 - 4*a*d^4*e^2*f^2 - 2*(a*b^2*d^2 + 2*a^3*e^
2)*f^6 + (b^2*d^4 + 8*a^2*d^2*e^2)*f^4 + (b^4*f^8 - 16*a*d^2*e^4*f^2 - 4*(
b^3*d*e + a*b^2*e^2)*f^6 + 4*(b^2*d^2*e^2 + 4*a*b*d*e^3)*f^4)*x^2 + 2*(a*b
^3*f^8 - 8*a*d^3*e^3*f^2 - (b^3*d^2 + 2*a*b^2*d*e + 4*a^2*b*e^2)*f^6 + 2*(
b^2*d^3*e + 2*a*b*d^2*e^2 + 4*a^2*d*e^3)*f^4)*x)*sqrt(-e/(b*f^2 - 2*d*e))*
log(-b^2*f^4 + 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x - 2*(2*sq
rt(2)*(b*e*f^3 - 2*d*e^2*f)*sqrt(-e/(b*f^2 - 2*d*e))*sqrt((b*f^2*x + e^2*x
^2 + a*f^2)/f^2) - sqrt(2)*(b^2*f^4 - 2*b*d*e*f^2 + 2*(b*e^2*f^2 - 2*d*e^3
)*x)*sqrt(-e/(b*f^2 - 2*d*e)))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^
2)/f^2) + d) + 4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^
2)) - 4*(4*d^5*e^2 + (8*a*b^2*d - 5*a^2*b*e)*f^6 - 2*(2*b^2*d^3 + a*b*d^2*
e + 10*a^2*d*e^2)*f^4 - 6*(b^2*e^3*f^4 - 4*b*d*e^4*f^2 + 4*d^2*e^5)*x^3 -
(9*b*d^4*e - 32*a*d^3*e^2)*f^2 + (3*b^3*e*f^6 - 16*d^3*e^4 + 4*(b^2*d*e^2
- 10*a*b*e^3)*f^4 - 4*(3*b*d^2*e^3 - 20*a*d*e^4)*f^2)*x^2 + 2*(d^4*e^3 + (
4*b^3*d - a*b^2*e)*f^6 - (7*b^2*d^2*e + 6*a*b*d*e^2 + 15*a^2*e^3)*f^4 - 2*
(5*b*d^3*e^2 - 23*a*d^2*e^3)*f^2)*x - 2*(3*a*b^2*f^7 + d^4*e^2*f - (b^2*d^
2 + 2*a*b*d*e + 15*a^2*e^2)*f^5 - 2*(3*b*d^3*e - 11*a*d^2*e^2)*f^3 - 3*(b^
2*e^2*f^5 - 4*b*d*e^3*f^3 + 4*d^2*e^4*f)*x^2 + (3*b^3*f^7 + 40*a*d*e^3*f^3
 - 8*d^3*e^3*f - 4*(b^2*d*e + 5*a*b*e^2)*f^5)*x)*sqrt((b*f^2*x + e^2*x^2 +
 a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/...
```

3.484.6 Sympy [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx=\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(5/2),x)
```

output
```
Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-5/2), x)
```

3.484.7 Maxima [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx=\int \frac{1}{\left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="maxima
")
```

output
```
integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-5/2), x)
```

3.484.8 Giac [F]

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx=\int \frac{1}{\left(ex+\sqrt{bx+\frac{e^2x^2}{f^2}+a}f+d\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="giac")
```

output
```
integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-5/2), x)
```

3.484.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx=\int \frac{1}{\left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)^{5/2}}\,dx$$

input
```
int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(5/2),x)
```

output
```
int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(5/2), x)
```

3.485 $\int (a+x^2)^2 \left(x+\sqrt{a+x^2}\right)^n dx$

3.485.1 Optimal result

Integrand size = 21, antiderivative size = 164

$$\int (a+x^2)^2 \left(x+\sqrt{a+x^2}\right)^n dx = -\frac{a^5\left(x+\sqrt{a+x^2}\right)^{-5+n}}{32(5-n)} - \frac{5a^4\left(x+\sqrt{a+x^2}\right)^{-3+n}}{32(3-n)} - \frac{5a^3\left(x+\sqrt{a+x^2}\right)^{-1+n}}{16(1-n)} + \frac{5a^2\left(x+\sqrt{a+x^2}\right)^{1+n}}{16(1+n)} + \frac{5a\left(x+\sqrt{a+x^2}\right)^{3+n}}{32(3+n)} + \frac{\left(x+\sqrt{a+x^2}\right)^{5+n}}{32(5+n)}$$

output
```
-1/32*a^5*(x+(x^2+a)^(1/2))^(-5+n)/(5-n)-5/32*a^4*(x+(x^2+a)^(1/2))^(-3+n)
/(3-n)-5/16*a^3*(x+(x^2+a)^(1/2))^(-1+n)/(1-n)+5/16*a^2*(x+(x^2+a)^(1/2))^
(1+n)/(1+n)+5/32*a*(x+(x^2+a)^(1/2))^(3+n)/(3+n)+1/32*(x+(x^2+a)^(1/2))^(5
+n)/(5+n)
```

3.485.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.84

$$\int (a+x^2)^2 \left(x+\sqrt{a+x^2}\right)^n dx = \frac{1}{32}\left(x+\sqrt{a+x^2}\right)^{-5+n}\left(\frac{a^5}{-5+n} + \frac{5a^4\left(x+\sqrt{a+x^2}\right)^2}{-3+n} + \frac{10a^3\left(x+\sqrt{a+x^2}\right)^4}{-1+n} + \frac{10a^2\left(x+\sqrt{a+x^2}\right)^6}{1+n} + \frac{5a\left(x+\sqrt{a+x^2}\right)^8}{3+n} + \frac{\left(x+\sqrt{a+x^2}\right)^{10}}{5+n}\right)$$

input
```
Integrate[(a + x^2)^2*(x + Sqrt[a + x^2])^n,x]
```

output
```
((x + Sqrt[a + x^2])^(-5 + n)*(a^5/(-5 + n) + (5*a^4*(x + Sqrt[a + x^2])^2
)/(-3 + n) + (10*a^3*(x + Sqrt[a + x^2])^4)/(-1 + n) + (10*a^2*(x + Sqrt[a
 + x^2])^6)/(1 + n) + (5*a*(x + Sqrt[a + x^2])^8)/(3 + n) + (x + Sqrt[a +
x^2])^10/(5 + n)))/32
```

3.485.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2547, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+x^2)^2 \left(\sqrt{a+x^2}+x\right)^n dx$$

$$\Big\downarrow \text{2547}$$

$$\frac{1}{32}\int \left(x+\sqrt{x^2+a}\right)^{n-6}\left(\left(x+\sqrt{x^2+a}\right)^2+a\right)^5 d\left(x+\sqrt{x^2+a}\right)$$

$$\Big\downarrow \text{244}$$

$$\frac{1}{32}\int \left(a^5\left(x+\sqrt{x^2+a}\right)^{n-6}+5a^4\left(x+\sqrt{x^2+a}\right)^{n-4}+10a^3\left(x+\sqrt{x^2+a}\right)^{n-2}+10a^2\left(x+\sqrt{x^2+a}\right)^{n}+5a(\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{32}\left(-\frac{a^5\left(\sqrt{a+x^2}+x\right)^{n-5}}{5-n}-\frac{5a^4\left(\sqrt{a+x^2}+x\right)^{n-3}}{3-n}-\frac{10a^3\left(\sqrt{a+x^2}+x\right)^{n-1}}{1-n}+\frac{10a^2\left(\sqrt{a+x^2}+x\right)^{n+1}}{n+1}+\frac{5a}{}\right.$$

input
```
Int[(a + x^2)^2*(x + Sqrt[a + x^2])^n,x]
```

output
```
(-((a^5*(x + Sqrt[a + x^2])^(-5 + n))/(5 - n)) - (5*a^4*(x + Sqrt[a + x^2]
)^(-3 + n))/(3 - n) - (10*a^3*(x + Sqrt[a + x^2])^(-1 + n))/(1 - n) + (10*
a^2*(x + Sqrt[a + x^2])^(1 + n))/(1 + n) + (5*a*(x + Sqrt[a + x^2])^(3 + n
))/(3 + n) + (x + Sqrt[a + x^2])^(5 + n)/(5 + n))/32
```

3.485.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2547
```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m
Subst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
 x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
 x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Intege
rQ[m] || GtQ[i/c, 0])
```

3.485.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.96 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.32

method	result
meijerg	$\frac{2^{n} x^{5+n} {}_{3}F_{2}\left(-\frac{n}{2},-\frac{5}{2}-\frac{n}{2},\frac{1}{2}-\frac{n}{2};1-n,-\frac{3}{2}-\frac{n}{2};-\frac{a}{x^{2}}\right)}{5+n}+\frac{2^{1+n} a\, x^{3+n} {}_{3}F_{2}\left(-\frac{n}{2},-\frac{3}{2}-\frac{n}{2},\frac{1}{2}-\frac{n}{2};1-n,-\frac{1}{2}-\frac{n}{2};-\frac{a}{x^{2}}\right)}{3+n}+\frac{a^{\frac{5}{2}+\frac{n}{2}} n \left(\frac{8\sqrt{\pi}\, x^{1}\cdots}{\cdots}\right.}{\cdots}$

input
```
int((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x,method=_RETURNVERBOSE)
```

output
```
2^n/(5+n)*x^(5+n)*hypergeom([-1/2*n,-5/2-1/2*n,1/2-1/2*n],[1-n,-3/2-1/2*n]
,-a/x^2)+2^(1+n)*a/(3+n)*x^(3+n)*hypergeom([-1/2*n,-3/2-1/2*n,1/2-1/2*n],[
1-n,-1/2-1/2*n],-a/x^2)+1/4*a^(5/2+1/2*n)/Pi^(1/2)*n*(8*Pi^(1/2)/(1+n)/n*x
^(1+n)*a^(-1/2-1/2*n)*(a/x^2*n+n-1)/(-2+2*n)*((1+a/x^2)^(1/2)+1)^(-1+n)+4*
Pi^(1/2)/(1+n)/n*x^(1+n)*a^(-1/2-1/2*n)*(1+a/x^2)^(1/2)*((1+a/x^2)^(1/2)+1
)^(-1+n))
```

3.485.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.96

$$\int (a+x^2)^2 \left(x+\sqrt{a+x^2}\right)^n dx = -\frac{(5(n^4-10n^2+9)x^5+10(an^4-16an^2+15a)x^3+5(a^2n^4-22a^2n^2+45a^2)x-(a^2n^5-30a^2n^3+}{n^6-35n^4+259n^2}$$

input
```
integrate((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

output
```
-(5*(n^4 - 10*n^2 + 9)*x^5 + 10*(a*n^4 - 16*a*n^2 + 15*a)*x^3 + 5*(a^2*n^4
 - 22*a^2*n^2 + 45*a^2)*x - (a^2*n^5 - 30*a^2*n^3 + (n^5 - 10*n^3 + 9*n)*x
^4 + 149*a^2*n + 2*(a*n^5 - 20*a*n^3 + 19*a*n)*x^2)*sqrt(x^2 + a))*(x + sq
rt(x^2 + a))^n/(n^6 - 35*n^4 + 259*n^2 - 225)
```

3.485.6 Sympy [F(-1)]

Timed out.

$$\int (a+x^2)^2 \left(x+\sqrt{a+x^2}\right)^n dx = \text{Timed out}$$

input
```
integrate((x**2+a)**2*(x+(x**2+a)**(1/2))**n,x)
```

output
```
Timed out
```

3.485.7 Maxima [F]

$$\int (a+x^2)^2 \left(x+\sqrt{a+x^2}\right)^n dx = \int (x^2+a)^2 \left(x+\sqrt{x^2+a}\right)^n dx$$

input
```
integrate((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

output
```
integrate((x^2 + a)^2*(x + sqrt(x^2 + a))^n, x)
```

3.485.8 Giac [F]

$$\int \left(a+x^2\right)^2 \left(x+\sqrt{a+x^2}\right)^n \, dx = \int \left(x^2+a\right)^2 \left(x+\sqrt{x^2+a}\right)^n \, dx$$

input
```
integrate((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
```

output
```
integrate((x^2 + a)^2*(x + sqrt(x^2 + a))^n, x)
```

3.485.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+x^2\right)^2 \left(x+\sqrt{a+x^2}\right)^n \, dx = \int \left(x^2+a\right)^2 \left(x+\sqrt{x^2+a}\right)^n \, dx$$

input
```
int((a + x^2)^2*(x + (a + x^2)^(1/2))^n,x)
```

output
```
int((a + x^2)^2*(x + (a + x^2)^(1/2))^n, x)
```

3.486 $\int (a+x^2)\left(x+\sqrt{a+x^2}\right)^n\,dx$

3.486.1 Optimal result

Integrand size = 19, antiderivative size = 108

$$\int (a+x^2)\left(x+\sqrt{a+x^2}\right)^n\,dx=-\frac{a^3\left(x+\sqrt{a+x^2}\right)^{-3+n}}{8(3-n)}-\frac{3a^2\left(x+\sqrt{a+x^2}\right)^{-1+n}}{8(1-n)}+\frac{3a\left(x+\sqrt{a+x^2}\right)^{1+n}}{8(1+n)}+\frac{\left(x+\sqrt{a+x^2}\right)^{3+n}}{8(3+n)}$$

output
```
-1/8*a^3*(x+(x^2+a)^(1/2))^(-3+n)/(3-n)-3/8*a^2*(x+(x^2+a)^(1/2))^(-1+n)/(
1-n)+3/8*a*(x+(x^2+a)^(1/2))^(1+n)/(1+n)+1/8*(x+(x^2+a)^(1/2))^(3+n)/(3+n)
```

3.486.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int (a+x^2)\left(x+\sqrt{a+x^2}\right)^n\,dx=\frac{1}{8}\left(x+\sqrt{a+x^2}\right)^{-3+n}\left(\frac{a^3}{-3+n}+\frac{3a^2\left(x+\sqrt{a+x^2}\right)^2}{-1+n}+\frac{3a\left(x+\sqrt{a+x^2}\right)^4}{1+n}+\frac{\left(x+\sqrt{a+x^2}\right)^6}{3+n}\right)$$

input
```
Integrate[(a + x^2)*(x + Sqrt[a + x^2])^n,x]
```

output
```
((x + Sqrt[a + x^2])^(-3 + n)*(a^3/(-3 + n) + (3*a^2*(x + Sqrt[a + x^2])^2
)/(-1 + n) + (3*a*(x + Sqrt[a + x^2])^4)/(1 + n) + (x + Sqrt[a + x^2])^6/(
3 + n)))/8
```

3.486.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2547, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+x^2)\left(\sqrt{a+x^2}+x\right)^n\,dx$$

$$\Big\downarrow \text{2547}$$

$$\frac{1}{8}\int \left(x+\sqrt{x^2+a}\right)^{n-4}\left(\left(x+\sqrt{x^2+a}\right)^2+a\right)^3d\left(x+\sqrt{x^2+a}\right)$$

$$\Big\downarrow \text{244}$$

$$\frac{1}{8}\int \left(a^3\left(x+\sqrt{x^2+a}\right)^{n-4}+3a^2\left(x+\sqrt{x^2+a}\right)^{n-2}+3a\left(x+\sqrt{x^2+a}\right)^{n}+\left(x+\sqrt{x^2+a}\right)^{n+2}\right)d\left(x+\sqrt{x^2\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}\left(-\frac{a^3\left(\sqrt{a+x^2}+x\right)^{n-3}}{3-n}-\frac{3a^2\left(\sqrt{a+x^2}+x\right)^{n-1}}{1-n}+\frac{3a\left(\sqrt{a+x^2}+x\right)^{n+1}}{n+1}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+3}}{n+3}\right)$$

input
```
Int[(a + x^2)*(x + Sqrt[a + x^2])^n,x]
```

output
```
(-((a^3*(x + Sqrt[a + x^2])^(-3 + n))/(3 - n)) - (3*a^2*(x + Sqrt[a + x^2]
)^(-1 + n))/(1 - n) + (3*a*(x + Sqrt[a + x^2])^(1 + n))/(1 + n) + (x + Sqr
t[a + x^2])^(3 + n)/(3 + n))/8
```

3.486.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2547
```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m
Subst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
 x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
 x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Intege
rQ[m] || GtQ[i/c, 0])
```

3.486.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.55

method	result
meijerg	$\frac{2^{n} x^{3+n} {}_{3}F_{2}\left(-\frac{n}{2},-\frac{3}{2}-\frac{n}{2},\frac{1}{2}-\frac{n}{2};1-n,-\frac{1}{2}-\frac{n}{2};-\frac{a}{x^{2}}\right)}{3+n}+\frac{a^{\frac{3}{2}+\frac{n}{2}} n \left(\frac{8\sqrt{\pi}\, x^{1+n} a^{-\frac{1}{2}-\frac{n}{2}} \left(\frac{a n}{x^{2}}+n-1\right)\left(\sqrt{1+\frac{a}{x^{2}}}+1\right)^{-1+n}}{(1+n) n (-2+2n)}+\frac{4\sqrt{\pi}\, x^{1+n} a^{-\frac{1}{2}-}}{}\right.}{4\sqrt{\pi}}$

input
```
int((x^2+a)*(x+(x^2+a)^(1/2))^n,x,method=_RETURNVERBOSE)
```

output
```
2^n/(3+n)*x^(3+n)*hypergeom([-1/2*n,-3/2-1/2*n,1/2-1/2*n],[1-n,-1/2-1/2*n]
,-a/x^2)+1/4*a^(3/2+1/2*n)/Pi^(1/2)*n*(8*Pi^(1/2)/(1+n)/n*x^(1+n)*a^(-1/2-
1/2*n)*(a/x^2*n+n-1)/(-2+2*n)*((1+a/x^2)^(1/2)+1)^(-1+n)+4*Pi^(1/2)/(1+n)/
n*x^(1+n)*a^(-1/2-1/2*n)*(1+a/x^2)^(1/2)*((1+a/x^2)^(1/2)+1)^(-1+n))
```

3.486.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.72

$$\int (a+x^2)\left(x+\sqrt{a+x^2}\right)^n dx = -\frac{\left(3\left(n^2-1\right)x^3+3\left(an^2-3a\right)x-\left(an^3+\left(n^3-n\right)x^2-7an\right)\sqrt{x^2+a}\right)\left(x+\sqrt{x^2+a}\right)^n}{n^4-10n^2+9}$$

input
```
integrate((x^2+a)*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

output
```
-(3*(n^2 - 1)*x^3 + 3*(a*n^2 - 3*a)*x - (a*n^3 + (n^3 - n)*x^2 - 7*a*n)*sq
rt(x^2 + a))*(x + sqrt(x^2 + a))^n/(n^4 - 10*n^2 + 9)
```

3.486.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2237 vs. 2(85) = 170.

Time = 14.58 (sec) , antiderivative size = 15302, normalized size of antiderivative = 141.69

$$\int (a+x^2)\left(x+\sqrt{a+x^2}\right)^n dx = \text{Too large to display}$$

input
```
integrate((x**2+a)*(x+(x**2+a)**(1/2))**n,x)
```

output
```
a*Piecewise((2*a**(9/2)*a**(n/2 + 1/2)*n*cosh(n*asinh(x/sqrt(a)) + asinh(x
/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gam
ma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(
1 - n/2)) - 2*a**(9/2)*a**(n/2 + 1/2)*n*gamma(1 - n/2)/(2*a**(9/2)*n**2*ga
mma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 -
n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) - 2*a**(7/2)*a**(n/2 + 1/2)*n*x**2*
sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2
)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)
*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) + 4*a**(7/2)*a
**(n/2 + 1/2)*n*x**2*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 -
 n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(
7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) - 2*a**(7/
2)*a**(n/2 + 1/2)*n*x**2*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) -
2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/
2)*x**2*gamma(1 - n/2)) - 2*a**(7/2)*a**(n/2 + 1/2)*x**2*sqrt(a/x**2 + 1)*
sinh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**
2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(
1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) + 2*a**(7/2)*a**(n/2 + 1/2)*x**
2*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n
**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*g...
```

3.486.7 Maxima [F]

$$\int (a+x^2) \left(x+\sqrt{a+x^2}\right)^n dx = \int (x^2+a) \left(x+\sqrt{x^2+a}\right)^n dx$$

input
```
integrate((x^2+a)*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

output
```
integrate((x^2 + a)*(x + sqrt(x^2 + a))^n, x)
```

3.486.8 Giac [F]

$$\int \left(a+x^2\right) \left(x+\sqrt{a+x^2}\right)^n \, dx = \int \left(x^2+a\right)\left(x+\sqrt{x^2+a}\right)^n \, dx$$

input
```
integrate((x^2+a)*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
```

output
```
integrate((x^2 + a)*(x + sqrt(x^2 + a))^n, x)
```

3.486.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+x^2\right) \left(x+\sqrt{a+x^2}\right)^n \, dx = \int \left(x^2+a\right) \left(x+\sqrt{x^2+a}\right)^n \, dx$$

input
```
int((a + x^2)*(x + (a + x^2)^(1/2))^n,x)
```

output
```
int((a + x^2)*(x + (a + x^2)^(1/2))^n, x)
```

3.487 $\int \left(x+\sqrt{a+x^2}\right)^n \, dx$

3.487.1 Optimal result

Integrand size = 13, antiderivative size = 52

$$\int \left(x+\sqrt{a+x^2}\right)^n \, dx = -\frac{a\left(x+\sqrt{a+x^2}\right)^{-1+n}}{2(1-n)}+\frac{\left(x+\sqrt{a+x^2}\right)^{1+n}}{2(1+n)}$$

output
```
-1/2*a*(x+(x^2+a)^(1/2))^(-1+n)/(1-n)+1/2*(x+(x^2+a)^(1/2))^(1+n)/(1+n)
```

3.487.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \left(x+\sqrt{a+x^2}\right)^n \, dx = \frac{\left(x+\sqrt{a+x^2}\right)^{-1+n}\left(an+(-1+n)x\left(x+\sqrt{a+x^2}\right)\right)}{-1+n^2}$$

input
```
Integrate[(x + Sqrt[a + x^2])^n,x]
```

output
```
((x + Sqrt[a + x^2])^(-1 + n)*(a*n + (-1 + n)*x*(x + Sqrt[a + x^2])))/(-1
+ n^2)
```

3.487.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2542, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sqrt{a+x^2}+x\right)^n dx$$

$$\Big\downarrow \text{2542}$$

$$\frac{1}{2}\int \left(x+\sqrt{x^2+a}\right)^{n-2}\left(\left(x+\sqrt{x^2+a}\right)^2+a\right)d\left(x+\sqrt{x^2+a}\right)$$

$$\Big\downarrow \text{244}$$

$$\frac{1}{2}\int \left(a\left(x+\sqrt{x^2+a}\right)^{n-2}+\left(x+\sqrt{x^2+a}\right)^n\right)d\left(x+\sqrt{x^2+a}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\left(\sqrt{a+x^2}+x\right)^{n+1}}{n+1}-\frac{a\left(\sqrt{a+x^2}+x\right)^{n-1}}{1-n}\right)$$

input
```
Int[(x + Sqrt[a + x^2])^n,x]
```

output
```
(-((a*(x + Sqrt[a + x^2])^(-1 + n))/(1 - n)) + (x + Sqrt[a + x^2])^(1 + n)
/(1 + n))/2
```

3.487.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2542
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Simp[1/(2*e)   Subst[Int[(g + h*x^n)^p*((d^2 + a*f
^2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fr
eeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.487.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(44) = 88$.

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.31

method	result	size
meijerg	$\frac{a^{\frac{1}{2}+\frac{n}{2}} n \left(\frac{8\sqrt{\pi}\, x^{1+n} a^{-\frac{1}{2}-\frac{n}{2}} \left(\frac{a n}{x^{2}}+n-1\right) \left(\sqrt{1+\frac{a}{x^{2}}}+1\right)^{-1+n}}{(1+n) n (-2+2n)}+\frac{4\sqrt{\pi}\, x^{1+n} a^{-\frac{1}{2}-\frac{n}{2}} \sqrt{1+\frac{a}{x^{2}}}\, \left(\sqrt{1+\frac{a}{x^{2}}}+1\right)^{-1+n}}{(1+n) n}\right)}{4\sqrt{\pi}}$	120

input
```
int((x+(x^2+a)^(1/2))^n,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^(1/2+1/2*n)/Pi^(1/2)*n*(8*Pi^(1/2)/(1+n)/n*x^(1+n)*a^(-1/2-1/2*n)*(a
/x^2*n+n-1)/(-2+2*n)*((1+a/x^2)^(1/2)+1)^(-1+n)+4*Pi^(1/2)/(1+n)/n*x^(1+n)
*a^(-1/2-1/2*n)*(1+a/x^2)^(1/2)*((1+a/x^2)^(1/2)+1)^(-1+n))
```

3.487.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.62

$$\int \left(x+\sqrt{a+x^2}\right)^n\,dx=\frac{\left(\sqrt{x^2+a}n-x\right)\left(x+\sqrt{x^2+a}\right)^n}{n^2-1}$$

input
```
integrate((x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

output
```
(sqrt(x^2 + a)*n - x)*(x + sqrt(x^2 + a))^n/(n^2 - 1)
```

3.487.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2236 vs. 2(37) = 74.

Time = 1.71 (sec) , antiderivative size = 2236, normalized size of antiderivative = 43.00

$$\int \left(x+\sqrt{a+x^2}\right)^n\,dx = \text{Too large to display}$$

input
```
integrate((x+(x**2+a)**(1/2))**n,x)
```

output
```
Piecewise((2*a**(9/2)*a**(n/2 + 1/2)*n*cosh(n*asinh(x/sqrt(a)) + asinh(x/s
qrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma
(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1
- n/2)) - 2*a**(9/2)*a**(n/2 + 1/2)*n*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamm
a(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/
2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) - 2*a**(7/2)*a**(n/2 + 1/2)*n*x**2*sq
rt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/
(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n
**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) + 4*a**(7/2)*a**
(n/2 + 1/2)*n*x**2*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n
/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/
2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) - 2*a**(7/2)
*a**(n/2 + 1/2)*n*x**2*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*
a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)
*x**2*gamma(1 - n/2)) - 2*a**(7/2)*a**(n/2 + 1/2)*x**2*sqrt(a/x**2 + 1)*si
nh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*
gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1
- n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) + 2*a**(7/2)*a**(n/2 + 1/2)*x**2*
cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**
2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gam...
```

3.487.7 Maxima [F]

$$\int \left(x+\sqrt{a+x^2}\right)^n\,dx = \int \left(x+\sqrt{x^2+a}\right)^n\,dx$$

input
```
integrate((x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

output
```
integrate((x + sqrt(x^2 + a))^n, x)
```

3.487.8 Giac [F]

$$\int \left(x+\sqrt{a+x^2}\right)^n dx = \int \left(x+\sqrt{x^2+a}\right)^n dx$$

input
```
integrate((x+(x^2+a)^(1/2))^n,x, algorithm="giac")
```

output
```
integrate((x + sqrt(x^2 + a))^n, x)
```

3.487.9 Mupad [F(-1)]

Timed out.

$$\int \left(x+\sqrt{a+x^2}\right)^n dx = \int \left(x+\sqrt{x^2+a}\right)^n dx$$

input
```
int((x + (a + x^2)^(1/2))^n,x)
```

output
```
int((x + (a + x^2)^(1/2))^n, x)
```

3.488 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2}\,dx$

3.488.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2}\,dx = \frac{2\left(x+\sqrt{a+x^2}\right)^{1+n}\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a(1+n)}$$

output
```
2*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-(x+(x^2+a)^(1/2))^2/a)*(x+(x^2+a)^
(1/2))^(1+n)/a/(1+n)
```

3.488.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2}\,dx = \frac{2\left(x+\sqrt{a+x^2}\right)^{1+n}\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},1+\frac{1+n}{2},-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a(1+n)}$$

input
```
Integrate[(x + Sqrt[a + x^2])^n/(a + x^2),x]
```

output
```
(2*(x + Sqrt[a + x^2])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, 1 + (1 + n)
/2, -((x + Sqrt[a + x^2])^2/a)])/(a*(1 + n))
```

3.488.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2547, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\sqrt{a+x^2}+x\right)^n}{a+x^2}\,dx$$

$$\Big\downarrow \text{ 2547}$$

$$2\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x+\sqrt{x^2+a}\right)^2+a}d\left(x+\sqrt{x^2+a}\right)$$

$$\Big\downarrow \text{ 278}$$

$$\frac{2\left(\sqrt{a+x^2}+x\right)^{n+1}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a(n+1)}$$

input
```
Int[(x + Sqrt[a + x^2])^n/(a + x^2),x]
```

output
```
(2*(x + Sqrt[a + x^2])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2,
-((x + Sqrt[a + x^2])^2/a)])/(a*(1 + n))
```

3.488.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2547
```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m
Subst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
 x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
 x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Intege
rQ[m] || GtQ[i/c, 0])
```

3.488.4 Maple [F]

$$\int \frac{\left(x+\sqrt{x^{2}+a}\right)^{n}}{x^{2}+a}dx$$

input
```
int((x+(x^2+a)^(1/2))^n/(x^2+a),x)
```

output
```
int((x+(x^2+a)^(1/2))^n/(x^2+a),x)
```

3.488.5 Fricas [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2}\,dx=\int \frac{\left(x+\sqrt{x^{2}+a}\right)^{n}}{x^{2}+a}\,dx$$

input
```
integrate((x+(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="fricas")
```

output
```
integral((x + sqrt(x^2 + a))^n/(x^2 + a), x)
```

3.488.6 Sympy [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2}\,dx = \int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2}\,dx$$

input
```
integrate((x+(x**2+a)**(1/2))**n/(x**2+a),x)
```

output
```
Integral((x + sqrt(a + x**2))**n/(a + x**2), x)
```

3.488.7 Maxima [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2}\,dx = \int \frac{\left(x+\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

input
```
integrate((x+(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="maxima")
```

output
```
integrate((x + sqrt(x^2 + a))^n/(x^2 + a), x)
```

3.488.8 Giac [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2}\,dx = \int \frac{\left(x+\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

input
```
integrate((x+(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="giac")
```

output
```
integrate((x + sqrt(x^2 + a))^n/(x^2 + a), x)
```

3.488.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2}\,dx=\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

input
```
int((x + (a + x^2)^(1/2))^n/(a + x^2),x)
```

output
```
int((x + (a + x^2)^(1/2))^n/(a + x^2), x)
```

3.489 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2} dx$

3.489.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2} dx = \frac{8\left(x+\sqrt{a+x^2}\right)^{3+n} \text{Hypergeometric2F1}\left(3,\frac{3+n}{2},\frac{5+n}{2},-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^3(3+n)}$$

output
```
8*hypergeom([3, 3/2+1/2*n],[5/2+1/2*n],-(x+(x^2+a)^(1/2))^2/a)*(x+(x^2+a)^
(1/2))^(3+n)/a^3/(3+n)
```

3.489.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2} dx = \frac{8\left(x+\sqrt{a+x^2}\right)^{3+n} \text{Hypergeometric2F1}\left(3,\frac{3+n}{2},1+\frac{3+n}{2},-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^3(3+n)}$$

input
```
Integrate[(x + Sqrt[a + x^2])^n/(a + x^2)^2,x]
```

output
```
(8*(x + Sqrt[a + x^2])^(3 + n)*Hypergeometric2F1[3, (3 + n)/2, 1 + (3 + n)
/2, -((x + Sqrt[a + x^2])^2/a)])/(a^3*(3 + n))
```

3.489.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2547, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\sqrt{a+x^2}+x\right)^n}{(a+x^2)^2}dx$$

$$\Big\downarrow \text{ 2547}$$

$$8\int \frac{\left(x+\sqrt{x^2+a}\right)^{n+2}}{\left(\left(x+\sqrt{x^2+a}\right)^2+a\right)^3}d\left(x+\sqrt{x^2+a}\right)$$

$$\Big\downarrow \text{ 278}$$

$$\frac{8\left(\sqrt{a+x^2}+x\right)^{n+3}\text{Hypergeometric2F1}\left(3,\frac{n+3}{2},\frac{n+5}{2},-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^3(n+3)}$$

input
```
Int[(x + Sqrt[a + x^2])^n/(a + x^2)^2,x]
```

output
```
(8*(x + Sqrt[a + x^2])^(3 + n)*Hypergeometric2F1[3, (3 + n)/2, (5 + n)/2,
-((x + Sqrt[a + x^2])^2/a)])/(a^3*(3 + n))
```

3.489.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2547
```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m
Subst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
 x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
 x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Intege
rQ[m] || GtQ[i/c, 0])
```

3.489.4 Maple [F]

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}dx$$

input
```
int((x+(x^2+a)^(1/2))^n/(x^2+a)^2,x)
```

output
```
int((x+(x^2+a)^(1/2))^n/(x^2+a)^2,x)
```

3.489.5 Fricas [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx=\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

input
```
integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="fricas")
```

output
```
integral((x + sqrt(x^2 + a))^n/(x^4 + 2*a*x^2 + a^2), x)
```

3.489.6 Sympy [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx=\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx$$

input
```
integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**2,x)
```

output
```
Integral((x + sqrt(a + x**2))**n/(a + x**2)**2, x)
```

3.489.7 Maxima [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx=\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

input
```
integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^2, x)
```

3.489.8 Giac [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx=\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

input
```
integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^2, x)
```

3.489.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx=\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

input
```
int((x + (a + x^2)^(1/2))^n/(a + x^2)^2,x)
```

output
```
int((x + (a + x^2)^(1/2))^n/(a + x^2)^2, x)
```

3.490 $\int (a+x^2)^2 \left(x-\sqrt{a+x^2}\right)^n\,dx$

3.490.1 Optimal result

Integrand size = 23, antiderivative size = 176

$$\int (a+x^2)^2 \left(x-\sqrt{a+x^2}\right)^n\,dx = -\frac{a^5\left(x-\sqrt{a+x^2}\right)^{-5+n}}{32(5-n)}-\frac{5a^4\left(x-\sqrt{a+x^2}\right)^{-3+n}}{32(3-n)}-\frac{5a^3\left(x-\sqrt{a+x^2}\right)^{-1+n}}{16(1-n)}+\frac{5a^2\left(x-\sqrt{a+x^2}\right)^{1+n}}{16(1+n)}+\frac{5a\left(x-\sqrt{a+x^2}\right)^{3+n}}{32(3+n)}+\frac{\left(x-\sqrt{a+x^2}\right)^{5+n}}{32(5+n)}$$

output
```
-1/32*a^5*(x-(x^2+a)^(1/2))^(-5+n)/(5-n)-5/32*a^4*(x-(x^2+a)^(1/2))^(-3+n)
/(3-n)-5/16*a^3*(x-(x^2+a)^(1/2))^(-1+n)/(1-n)+5/16*a^2*(x-(x^2+a)^(1/2))^
(1+n)/(1+n)+5/32*a*(x-(x^2+a)^(1/2))^(3+n)/(3+n)+1/32*(x-(x^2+a)^(1/2))^(5
+n)/(5+n)
```

3.490.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.85

$$\int (a+x^2)^2 \left(x-\sqrt{a+x^2}\right)^n\,dx = \frac{1}{32}\left(x-\sqrt{a+x^2}\right)^{-5+n}\left(\frac{a^5}{-5+n}+\frac{5a^4\left(x-\sqrt{a+x^2}\right)^2}{-3+n}+\frac{10a^3\left(x-\sqrt{a+x^2}\right)^4}{-1+n}+\frac{10a^2\left(x-\sqrt{a+x^2}\right)^6}{1+n}+\frac{5a\left(x-\sqrt{a+x^2}\right)^8}{3+n}+\frac{\left(x-\sqrt{a+x^2}\right)^{10}}{5+n}\right)$$

input
```
Integrate[(a + x^2)^2*(x - Sqrt[a + x^2])^n,x]
```

output
```
((x - Sqrt[a + x^2])^(-5 + n)*(a^5/(-5 + n) + (5*a^4*(x - Sqrt[a + x^2])^2
)/(-3 + n) + (10*a^3*(x - Sqrt[a + x^2])^4)/(-1 + n) + (10*a^2*(x - Sqrt[a
 + x^2])^6)/(1 + n) + (5*a*(x - Sqrt[a + x^2])^8)/(3 + n) + (x - Sqrt[a +
x^2])^10/(5 + n)))/32
```

3.490.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2547, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+x^2\right)^2 \left(x-\sqrt{a+x^2}\right)^n \, dx$$

$$\Big\downarrow \ 2547$$

$$\frac{1}{32}\int \left(x-\sqrt{x^2+a}\right)^{n-6}\left(\left(x-\sqrt{x^2+a}\right)^2+a\right)^5 d\left(x-\sqrt{x^2+a}\right)$$

$$\Big\downarrow \ 244$$

$$\frac{1}{32}\int \left(a^5\left(x-\sqrt{x^2+a}\right)^{n-6}+5a^4\left(x-\sqrt{x^2+a}\right)^{n-4}+10a^3\left(x-\sqrt{x^2+a}\right)^{n-2}+10a^2\left(x-\sqrt{x^2+a}\right)^{n}+5a\left(\right.\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{32}\left(-\frac{a^5\left(x-\sqrt{a+x^2}\right)^{n-5}}{5-n}-\frac{5a^4\left(x-\sqrt{a+x^2}\right)^{n-3}}{3-n}-\frac{10a^3\left(x-\sqrt{a+x^2}\right)^{n-1}}{1-n}+\frac{10a^2\left(x-\sqrt{a+x^2}\right)^{n+1}}{n+1}+\frac{5a}{}\right.$$

input
```
Int[(a + x^2)^2*(x - Sqrt[a + x^2])^n,x]
```

output
```
(-((a^5*(x - Sqrt[a + x^2])^(-5 + n))/(5 - n)) - (5*a^4*(x - Sqrt[a + x^2]
)^(-3 + n))/(3 - n) - (10*a^3*(x - Sqrt[a + x^2])^(-1 + n))/(1 - n) + (10*
a^2*(x - Sqrt[a + x^2])^(1 + n))/(1 + n) + (5*a*(x - Sqrt[a + x^2])^(3 + n
))/(3 + n) + (x - Sqrt[a + x^2])^(5 + n)/(5 + n))/32
```

3.490.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2547
```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m
Subst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
 x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
 x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Intege
rQ[m] || GtQ[i/c, 0])
```

3.490.4 Maple [F]

$$\int \left(x^2+a\right)^2 \left(x-\sqrt{x^2+a}\right)^n dx$$

input
```
int((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x)
```

output
```
int((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x)
```

3.490.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.90

$$\int \left(a+x^2\right)^2 \left(x-\sqrt{a+x^2}\right)^n dx = -\frac{(5\,(n^4-10\,n^2+9)x^5+10\,(an^4-16\,an^2+15\,a)x^3+5\,(a^2n^4-22\,a^2n^2+45\,a^2)x+(a^2n^5-30\,a^2n^3+}{n^6-35\,n^4+259\,n^2}$$

input
```
integrate((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

output
```
-(5*(n^4 - 10*n^2 + 9)*x^5 + 10*(a*n^4 - 16*a*n^2 + 15*a)*x^3 + 5*(a^2*n^4
 - 22*a^2*n^2 + 45*a^2)*x + (a^2*n^5 - 30*a^2*n^3 + (n^5 - 10*n^3 + 9*n)*x
^4 + 149*a^2*n + 2*(a*n^5 - 20*a*n^3 + 19*a*n)*x^2)*sqrt(x^2 + a))*(x - sq
rt(x^2 + a))^n/(n^6 - 35*n^4 + 259*n^2 - 225)
```

3.490.6 Sympy [F]

$$\int (a+x^2)^2 \left(x-\sqrt{a+x^2}\right)^n dx = \int (a+x^2)^2 \left(x-\sqrt{a+x^2}\right)^n dx$$

input
```
integrate((x**2+a)**2*(x-(x**2+a)**(1/2))**n,x)
```

output
```
Integral((a + x**2)**2*(x - sqrt(a + x**2))**n, x)
```

3.490.7 Maxima [F]

$$\int (a+x^2)^2 \left(x-\sqrt{a+x^2}\right)^n dx = \int (x^2+a)^2 \left(x-\sqrt{x^2+a}\right)^n dx$$

input
```
integrate((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

output
```
integrate((x^2 + a)^2*(x - sqrt(x^2 + a))^n, x)
```

3.490.8 Giac [F]

$$\int (a+x^2)^2 \left(x-\sqrt{a+x^2}\right)^n dx = \int (x^2+a)^2 \left(x-\sqrt{x^2+a}\right)^n dx$$

input
```
integrate((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
```

output
```
integrate((x^2 + a)^2*(x - sqrt(x^2 + a))^n, x)
```

3.490.9 Mupad [F(-1)]

Timed out.

$$\int (a+x^2)^2 \left(x-\sqrt{a+x^2}\right)^n dx = \int \left(x-\sqrt{x^2+a}\right)^n (x^2+a)^2 dx$$

input
```
int((x - (a + x^2)^(1/2))^n*(a + x^2)^2,x)
```

output
```
int((x - (a + x^2)^(1/2))^n*(a + x^2)^2, x)
```

3.491 $\int (a+x^2) \left(x-\sqrt{a+x^2}\right)^n \, dx$

3.491.1 Optimal result

Integrand size = 21, antiderivative size = 116

$$\int (a+x^2) \left(x-\sqrt{a+x^2}\right)^n \, dx = -\frac{a^3\left(x-\sqrt{a+x^2}\right)^{-3+n}}{8(3-n)} - \frac{3a^2\left(x-\sqrt{a+x^2}\right)^{-1+n}}{8(1-n)} + \frac{3a\left(x-\sqrt{a+x^2}\right)^{1+n}}{8(1+n)} + \frac{\left(x-\sqrt{a+x^2}\right)^{3+n}}{8(3+n)}$$

output
```
-1/8*a^3*(x-(x^2+a)^(1/2))^(-3+n)/(3-n)-3/8*a^2*(x-(x^2+a)^(1/2))^(-1+n)/(
1-n)+3/8*a*(x-(x^2+a)^(1/2))^(1+n)/(1+n)+1/8*(x-(x^2+a)^(1/2))^(3+n)/(3+n)
```

3.491.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.86

$$\int (a+x^2) \left(x-\sqrt{a+x^2}\right)^n \, dx = \frac{1}{8}\left(x-\sqrt{a+x^2}\right)^{-3+n} \left(\frac{a^3}{-3+n} + \frac{3a^2\left(x-\sqrt{a+x^2}\right)^2}{-1+n} + \frac{3a\left(x-\sqrt{a+x^2}\right)^4}{1+n} + \frac{\left(x-\sqrt{a+x^2}\right)^6}{3+n}\right)$$

input
```
Integrate[(a + x^2)*(x - Sqrt[a + x^2])^n,x]
```

output
```
((x - Sqrt[a + x^2])^(-3 + n)*(a^3/(-3 + n) + (3*a^2*(x - Sqrt[a + x^2])^2
)/(-1 + n) + (3*a*(x - Sqrt[a + x^2])^4)/(1 + n) + (x - Sqrt[a + x^2])^6/(
3 + n)))/8
```

3.491.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2547, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+x^2\right)\left(x-\sqrt{a+x^2}\right)^n dx$$

$$\Big\downarrow \text{2547}$$

$$\frac{1}{8}\int \left(x-\sqrt{x^2+a}\right)^{n-4}\left(\left(x-\sqrt{x^2+a}\right)^2+a\right)^3 d\left(x-\sqrt{x^2+a}\right)$$

$$\Big\downarrow \text{244}$$

$$\frac{1}{8}\int \left(a^3\left(x-\sqrt{x^2+a}\right)^{n-4}+3a^2\left(x-\sqrt{x^2+a}\right)^{n-2}+3a\left(x-\sqrt{x^2+a}\right)^{n}+\left(x-\sqrt{x^2+a}\right)^{n+2}\right)d\left(x-\sqrt{x^2+a}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}\left(-\frac{a^3\left(x-\sqrt{a+x^2}\right)^{n-3}}{3-n}-\frac{3a^2\left(x-\sqrt{a+x^2}\right)^{n-1}}{1-n}+\frac{3a\left(x-\sqrt{a+x^2}\right)^{n+1}}{n+1}+\frac{\left(x-\sqrt{a+x^2}\right)^{n+3}}{n+3}\right)$$

input
```
Int[(a + x^2)*(x - Sqrt[a + x^2])^n,x]
```

output
```
(-((a^3*(x - Sqrt[a + x^2])^(-3 + n))/(3 - n)) - (3*a^2*(x - Sqrt[a + x^2]
)^(-1 + n))/(1 - n) + (3*a*(x - Sqrt[a + x^2])^(1 + n))/(1 + n) + (x - Sqr
t[a + x^2])^(3 + n)/(3 + n))/8
```

3.491.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2547
```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m
Subst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
 x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
 x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Intege
rQ[m] || GtQ[i/c, 0])
```

3.491.4 Maple [F]

$$\int (x^2 + a) \left(x - \sqrt{x^2 + a}\right)^n dx$$

input
```
int((x^2+a)*(x-(x^2+a)^(1/2))^n,x)
```

output
```
int((x^2+a)*(x-(x^2+a)^(1/2))^n,x)
```

3.491.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

$$\int (a + x^2) \left(x - \sqrt{a + x^2}\right)^n dx = -\frac{\left(3 (n^2 - 1)x^3 + 3 (an^2 - 3 a)x + (an^3 + (n^3 - n)x^2 - 7 an)\sqrt{x^2 + a}\right) \left(x - \sqrt{x^2 + a}\right)^n}{n^4 - 10 n^2 + 9}$$

input
```
integrate((x^2+a)*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

output
```
-(3*(n^2 - 1)*x^3 + 3*(a*n^2 - 3*a)*x + (a*n^3 + (n^3 - n)*x^2 - 7*a*n)*sq
rt(x^2 + a))*(x - sqrt(x^2 + a))^n/(n^4 - 10*n^2 + 9)
```

3.491.6 Sympy [F]

$$\int (a+x^2)\left(x-\sqrt{a+x^2}\right)^n dx = \int (a+x^2)\left(x-\sqrt{a+x^2}\right)^n dx$$

input
```
integrate((x**2+a)*(x-(x**2+a)**(1/2))**n,x)
```

output
```
Integral((a + x**2)*(x - sqrt(a + x**2))**n, x)
```

3.491.7 Maxima [F]

$$\int (a+x^2)\left(x-\sqrt{a+x^2}\right)^n dx = \int (x^2+a)\left(x-\sqrt{x^2+a}\right)^n dx$$

input
```
integrate((x^2+a)*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

output
```
integrate((x^2 + a)*(x - sqrt(x^2 + a))^n, x)
```

3.491.8 Giac [F]

$$\int (a+x^2)\left(x-\sqrt{a+x^2}\right)^n dx = \int (x^2+a)\left(x-\sqrt{x^2+a}\right)^n dx$$

input
```
integrate((x^2+a)*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
```

output
```
integrate((x^2 + a)*(x - sqrt(x^2 + a))^n, x)
```

3.491.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+x^2\right)\left(x-\sqrt{a+x^2}\right)^n\,dx=\int \left(x-\sqrt{x^2+a}\right)^n\left(x^2+a\right)\,dx$$

input
```
int((x - (a + x^2)^(1/2))^n*(a + x^2),x)
```

output
```
int((x - (a + x^2)^(1/2))^n*(a + x^2), x)
```

3.492 $\int \left(x-\sqrt{a+x^2}\right)^n \, dx$

3.492.1 Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \left(x-\sqrt{a+x^2}\right)^n \, dx=-\frac{a\left(x-\sqrt{a+x^2}\right)^{-1+n}}{2(1-n)}+\frac{\left(x-\sqrt{a+x^2}\right)^{1+n}}{2(1+n)}$$

output
```
-1/2*a*(x-(x^2+a)^(1/2))^(-1+n)/(1-n)+1/2*(x-(x^2+a)^(1/2))^(1+n)/(1+n)
```

3.492.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \left(x-\sqrt{a+x^2}\right)^n \, dx=\frac{1}{2}\left(x-\sqrt{a+x^2}\right)^{-1+n}\left(\frac{a}{-1+n}+\frac{\left(x-\sqrt{a+x^2}\right)^2}{1+n}\right)$$

input
```
Integrate[(x - Sqrt[a + x^2])^n,x]
```

output
```
((x - Sqrt[a + x^2])^(-1 + n)*(a/(-1 + n) + (x - Sqrt[a + x^2])^2/(1 + n))
)/2
```

3.492.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2542, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x-\sqrt{a+x^2}\right)^n dx$$

$$\Big\downarrow \text{2542}$$

$$\frac{1}{2}\int \left(x-\sqrt{x^2+a}\right)^{n-2}\left(\left(x-\sqrt{x^2+a}\right)^2+a\right)d\left(x-\sqrt{x^2+a}\right)$$

$$\Big\downarrow \text{244}$$

$$\frac{1}{2}\int \left(a\left(x-\sqrt{x^2+a}\right)^{n-2}+\left(x-\sqrt{x^2+a}\right)^n\right)d\left(x-\sqrt{x^2+a}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\left(x-\sqrt{a+x^2}\right)^{n+1}}{n+1}-\frac{a\left(x-\sqrt{a+x^2}\right)^{n-1}}{1-n}\right)$$

input
```
Int[(x - Sqrt[a + x^2])^n,x]
```

output
```
(-((a*(x - Sqrt[a + x^2])^(-1 + n))/(1 - n)) + (x - Sqrt[a + x^2])^(1 + n)
/(1 + n))/2
```

3.492.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2542
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Simp[1/(2*e)   Subst[Int[(g + h*x^n)^p*((d^2 + a*f
^2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fr
eeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.492.4 Maple [F]

$$\int \left(x-\sqrt{x^2+a}\right)^n dx$$

input
```
int((x-(x^2+a)^(1/2))^n,x)
```

output
```
int((x-(x^2+a)^(1/2))^n,x)
```

3.492.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.59

$$\int \left(x-\sqrt{a+x^2}\right)^n dx = -\frac{\left(\sqrt{x^2+a}n+x\right)\left(x-\sqrt{x^2+a}\right)^n}{n^2-1}$$

input
```
integrate((x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

output
```
-(sqrt(x^2 + a)*n + x)*(x - sqrt(x^2 + a))^n/(n^2 - 1)
```

3.492.6 Sympy [F]

$$\int \left(x-\sqrt{a+x^2}\right)^n dx = \int \left(x-\sqrt{a+x^2}\right)^n dx$$

input
```
integrate((x-(x**2+a)**(1/2))**n,x)
```

output
```
Integral((x - sqrt(a + x**2))**n, x)
```

3.492.7 Maxima [F]

$$\int \left(x-\sqrt{a+x^2}\right)^n dx = \int \left(x-\sqrt{x^2+a}\right)^n dx$$

input
```
integrate((x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

output
```
integrate((x - sqrt(x^2 + a))^n, x)
```

3.492.8 Giac [F]

$$\int \left(x-\sqrt{a+x^2}\right)^n dx = \int \left(x-\sqrt{x^2+a}\right)^n dx$$

input
```
integrate((x-(x^2+a)^(1/2))^n,x, algorithm="giac")
```

output
```
integrate((x - sqrt(x^2 + a))^n, x)
```

3.492.9 Mupad [F(-1)]

Timed out.

$$\int \left(x-\sqrt{a+x^2}\right)^n dx = \int \left(x-\sqrt{x^2+a}\right)^n dx$$

input
```
int((x - (a + x^2)^(1/2))^n,x)
```

output
```
int((x - (a + x^2)^(1/2))^n, x)
```

3.493 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2} \, dx$

3.493.1 Optimal result

Integrand size = 23, antiderivative size = 63

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2} \, dx = \frac{2\left(x-\sqrt{a+x^2}\right)^{1+n} \text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a(1+n)}$$

output
```
2*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-(x-(x^2+a)^(1/2))^2/a)*(x-(x^2+a)^
(1/2))^(1+n)/a/(1+n)
```

3.493.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2} \, dx = \frac{2\left(x-\sqrt{a+x^2}\right)^{1+n} \text{Hypergeometric2F1}\left(1,\frac{1+n}{2},1+\frac{1+n}{2},-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a(1+n)}$$

input
```
Integrate[(x - Sqrt[a + x^2])^n/(a + x^2),x]
```

output
```
(2*(x - Sqrt[a + x^2])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, 1 + (1 + n)
/2, -((x - Sqrt[a + x^2])^2/a)])/(a*(1 + n))
```

3.493.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2547, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2} \, dx$$

$$\Big\downarrow \ 2547$$

$$2\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x-\sqrt{x^2+a}\right)^2+a}d\left(x-\sqrt{x^2+a}\right)$$

$$\Big\downarrow \ 278$$

$$\frac{2\left(x-\sqrt{a+x^2}\right)^{n+1}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a(n+1)}$$

input
```
Int[(x - Sqrt[a + x^2])^n/(a + x^2),x]
```

output
```
(2*(x - Sqrt[a + x^2])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2,
-((x - Sqrt[a + x^2])^2/a)])/(a*(1 + n))
```

3.493.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2547
```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m
Subst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
 x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
 x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Intege
rQ[m] || GtQ[i/c, 0])
```

3.493.4 Maple [F]

$$\int \frac{\left(x-\sqrt{x^{2}+a}\right)^{n}}{x^{2}+a}dx$$

input
```
int((x-(x^2+a)^(1/2))^n/(x^2+a),x)
```

output
```
int((x-(x^2+a)^(1/2))^n/(x^2+a),x)
```

3.493.5 Fricas [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2}\,dx=\int \frac{\left(x-\sqrt{x^2+a}\right)^{n}}{x^{2}+a}\,dx$$

input
```
integrate((x-(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="fricas")
```

output
```
integral((x - sqrt(x^2 + a))^n/(x^2 + a), x)
```

3.493.6 Sympy [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2}\,dx=\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2}\,dx$$

input
```
integrate((x-(x**2+a)**(1/2))**n/(x**2+a),x)
```

output
```
Integral((x - sqrt(a + x**2))**n/(a + x**2), x)
```

3.493.7 Maxima [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2}\,dx=\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

input
```
integrate((x-(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="maxima")
```

output
```
integrate((x - sqrt(x^2 + a))^n/(x^2 + a), x)
```

3.493.8 Giac [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2}\,dx=\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

input
```
integrate((x-(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="giac")
```

output
```
integrate((x - sqrt(x^2 + a))^n/(x^2 + a), x)
```

3.493.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2}\,dx=\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

input
```
int((x - (a + x^2)^(1/2))^n/(a + x^2),x)
```

output
```
int((x - (a + x^2)^(1/2))^n/(a + x^2), x)
```

3.494 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx$

3.494.1 Optimal result

Integrand size = 23, antiderivative size = 63

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx = \frac{8\left(x-\sqrt{a+x^2}\right)^{3+n}\text{Hypergeometric2F1}\left(3,\frac{3+n}{2},\frac{5+n}{2},-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^3(3+n)}$$

output
```
8*hypergeom([3, 3/2+1/2*n],[5/2+1/2*n],-(x-(x^2+a)^(1/2))^2/a)*(x-(x^2+a)^
(1/2))^(3+n)/a^3/(3+n)
```

3.494.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx = \frac{8\left(x-\sqrt{a+x^2}\right)^{3+n}\text{Hypergeometric2F1}\left(3,\frac{3+n}{2},1+\frac{3+n}{2},-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^3(3+n)}$$

input
```
Integrate[(x - Sqrt[a + x^2])^n/(a + x^2)^2,x]
```

output
```
(8*(x - Sqrt[a + x^2])^(3 + n)*Hypergeometric2F1[3, (3 + n)/2, 1 + (3 + n)
/2, -((x - Sqrt[a + x^2])^2/a)])/(a^3*(3 + n))
```

3.494.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2547, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^2}dx$$

$$\Big\downarrow \text{2547}$$

$$8\int \frac{\left(x-\sqrt{x^2+a}\right)^{n+2}}{\left(\left(x-\sqrt{x^2+a}\right)^2+a\right)^3}d\left(x-\sqrt{x^2+a}\right)$$

$$\Big\downarrow \text{278}$$

$$\frac{8\left(x-\sqrt{a+x^2}\right)^{n+3}\text{Hypergeometric2F1}\left(3,\frac{n+3}{2},\frac{n+5}{2},-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^3(n+3)}$$

input
```
Int[(x - Sqrt[a + x^2])^n/(a + x^2)^2,x]
```

output
```
(8*(x - Sqrt[a + x^2])^(3 + n)*Hypergeometric2F1[3, (3 + n)/2, (5 + n)/2,
-((x - Sqrt[a + x^2])^2/a)])/(a^3*(3 + n))
```

3.494.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2547
```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m
Subst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
 x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
 x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Intege
rQ[m] || GtQ[i/c, 0])
```

3.494.4 Maple [F]

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}dx$$

input
```
int((x-(x^2+a)^(1/2))^n/(x^2+a)^2,x)
```

output
```
int((x-(x^2+a)^(1/2))^n/(x^2+a)^2,x)
```

3.494.5 Fricas [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx=\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

input
```
integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="fricas")
```

output
```
integral((x - sqrt(x^2 + a))^n/(x^4 + 2*a*x^2 + a^2), x)
```

3.494.6 Sympy [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx=\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx$$

input
```
integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**2,x)
```

output
```
Integral((x - sqrt(a + x**2))**n/(a + x**2)**2, x)
```

3.494.7 Maxima [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx=\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

input
```
integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^2, x)
```

3.494.8 Giac [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx=\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

input
```
integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="giac")
```

output
```
integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^2, x)
```

3.494.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx=\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{{\left(x^2+a\right)}^2}\,dx$$

input
```
int((x - (a + x^2)^(1/2))^n/(a + x^2)^2,x)
```

output
```
int((x - (a + x^2)^(1/2))^n/(a + x^2)^2, x)
```

3.495 $\int (a+x^2)^{5/2} \left(x+\sqrt{a+x^2}\right)^n dx$

3.495.1 Optimal result

Integrand size = 23, antiderivative size = 187

$$\int (a+x^2)^{5/2} \left(x+\sqrt{a+x^2}\right)^n dx = -\frac{a^6\left(x+\sqrt{a+x^2}\right)^{-6+n}}{64(6-n)} - \frac{3a^5\left(x+\sqrt{a+x^2}\right)^{-4+n}}{32(4-n)} - \frac{15a^4\left(x+\sqrt{a+x^2}\right)^{-2+n}}{64(2-n)} + \frac{5a^3\left(x+\sqrt{a+x^2}\right)^{n}}{16n} + \frac{15a^2\left(x+\sqrt{a+x^2}\right)^{2+n}}{64(2+n)} + \frac{3a\left(x+\sqrt{a+x^2}\right)^{4+n}}{32(4+n)} + \frac{\left(x+\sqrt{a+x^2}\right)^{6+n}}{64(6+n)}$$

output
```
-1/64*a^6*(x+(x^2+a)^(1/2))^(-6+n)/(6-n)-3/32*a^5*(x+(x^2+a)^(1/2))^(-4+n)
/(4-n)-15/64*a^4*(x+(x^2+a)^(1/2))^(-2+n)/(2-n)+5/16*a^3*(x+(x^2+a)^(1/2))
^n/n+15/64*a^2*(x+(x^2+a)^(1/2))^(2+n)/(2+n)+3/32*a*(x+(x^2+a)^(1/2))^(4+n
)/(4+n)+1/64*(x+(x^2+a)^(1/2))^(6+n)/(6+n)
```

3.495.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.84

$$\int (a+x^2)^{5/2}\left(x+\sqrt{a+x^2}\right)^n\,dx=\frac{1}{64}\left(x+\sqrt{a+x^2}\right)^n\left(\frac{20a^3}{n}+\frac{a^6}{(-6+n)\left(x+\sqrt{a+x^2}\right)^6}+\frac{6a^5}{(-4+n)\left(x+\sqrt{a+x^2}\right)^4}+\frac{15a^4}{(-2+n)\left(x+\sqrt{a+x^2}\right)^2}+\frac{15a^2\left(x+\sqrt{a+x^2}\right)^2}{2+n}+\frac{6a\left(x+\sqrt{a+x^2}\right)^4}{4+n}+\frac{\left(x+\sqrt{a+x^2}\right)^6}{6+n}\right)$$

input
```
Integrate[(a + x^2)^(5/2)*(x + Sqrt[a + x^2])^n,x]
```

output
```
((x + Sqrt[a + x^2])^n*((20*a^3)/n + a^6/((-6 + n)*(x + Sqrt[a + x^2])^6)
+ (6*a^5)/((-4 + n)*(x + Sqrt[a + x^2])^4) + (15*a^4)/((-2 + n)*(x + Sqrt[
a + x^2])^2) + (15*a^2*(x + Sqrt[a + x^2])^2)/(2 + n) + (6*a*(x + Sqrt[a +
 x^2])^4)/(4 + n) + (x + Sqrt[a + x^2])^6/(6 + n)))/64
```

3.495.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2547, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+x^2)^{5/2}\left(\sqrt{a+x^2}+x\right)^n\,dx$$

$$\Big\downarrow \ 2547$$

$$\frac{1}{64}\int \left(x+\sqrt{x^2+a}\right)^{n-7}\left(\left(x+\sqrt{x^2+a}\right)^2+a\right)^6 d\left(x+\sqrt{x^2+a}\right)$$

$$\Big\downarrow \ 244$$

$$\frac{1}{64}\int \left(a^6\left(x+\sqrt{x^2+a}\right)^{n-7}+6a^5\left(x+\sqrt{x^2+a}\right)^{n-5}+15a^4\left(x+\sqrt{x^2+a}\right)^{n-3}+20a^3\left(x+\sqrt{x^2+a}\right)^{n-1}+15$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{64}\left(-\frac{a^6\left(\sqrt{a+x^2}+x\right)^{n-6}}{6-n}-\frac{6a^5\left(\sqrt{a+x^2}+x\right)^{n-4}}{4-n}-\frac{15a^4\left(\sqrt{a+x^2}+x\right)^{n-2}}{2-n}+\frac{20a^3\left(\sqrt{a+x^2}+x\right)^{n}}{n}+\frac{15a^2}{}\right.$$

input
```
Int[(a + x^2)^(5/2)*(x + Sqrt[a + x^2])^n,x]
```

output
```
(-((a^6*(x + Sqrt[a + x^2])^(-6 + n))/(6 - n)) - (6*a^5*(x + Sqrt[a + x^2]
)^(-4 + n))/(4 - n) - (15*a^4*(x + Sqrt[a + x^2])^(-2 + n))/(2 - n) + (20*
a^3*(x + Sqrt[a + x^2])^n)/n + (15*a^2*(x + Sqrt[a + x^2])^(2 + n))/(2 + n
) + (6*a*(x + Sqrt[a + x^2])^(4 + n))/(4 + n) + (x + Sqrt[a + x^2])^(6 + n
)/(6 + n))/64
```

3.495.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2547
```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m
Subst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
 x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
 x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Intege
rQ[m] || GtQ[i/c, 0])
```

3.495.4 Maple [F]

$$\int \left(x^2+a\right)^{\frac{5}{2}} \left(x+\sqrt{x^2+a}\right)^n dx$$

input
```
int((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x)
```

output
```
int((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x)
```

3.495.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.07

$$\int (a+x^2)^{5/2} \left(x+\sqrt{a+x^2}\right)^n dx = \frac{(a^3n^6-50\,a^3n^4+(n^6-20\,n^4+64\,n^2)x^6+544\,a^3n^2+3\,(an^6-30\,an^4+104\,an^2)x^4-7}{}$$

input
```
integrate((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

output
```
(a^3*n^6 - 50*a^3*n^4 + (n^6 - 20*n^4 + 64*n^2)*x^6 + 544*a^3*n^2 + 3*(a*n
^6 - 30*a*n^4 + 104*a*n^2)*x^4 - 720*a^3 + 3*(a^2*n^6 - 40*a^2*n^4 + 264*a
^2*n^2)*x^2 - 6*((n^5 - 20*n^3 + 64*n)*x^5 + 2*(a*n^5 - 30*a*n^3 + 104*a*n
)*x^3 + (a^2*n^5 - 40*a^2*n^3 + 264*a^2*n)*x)*sqrt(x^2 + a))*(x + sqrt(x^2
 + a))^n/(n^7 - 56*n^5 + 784*n^3 - 2304*n)
```

3.495.6 Sympy [F(-2)]

Exception generated.

$$\int (a+x^2)^{5/2} \left(x+\sqrt{a+x^2}\right)^n dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((x**2+a)**(5/2)*(x+(x**2+a)**(1/2))**n,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.495.7 Maxima [F]

$$\int (a+x^2)^{5/2} \left(x+\sqrt{a+x^2}\right)^n dx = \int (x^2+a)^{\frac{5}{2}} \left(x+\sqrt{x^2+a}\right)^n dx$$

input
```
integrate((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

output
```
integrate((x^2 + a)^(5/2)*(x + sqrt(x^2 + a))^n, x)
```

3.495.8 Giac [F]

$$\int \left(a+x^2\right)^{5/2} \left(x+\sqrt{a+x^2}\right)^n \, dx=\int \left(x^2+a\right)^{\frac{5}{2}} \left(x+\sqrt{x^2+a}\right)^n \, dx$$

input
```
integrate((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
```

output
```
integrate((x^2 + a)^(5/2)*(x + sqrt(x^2 + a))^n, x)
```

3.495.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+x^2\right)^{5/2} \left(x+\sqrt{a+x^2}\right)^n \, dx=\int \left(x^2+a\right)^{5/2} \left(x+\sqrt{x^2+a}\right)^n \, dx$$

input
```
int((a + x^2)^(5/2)*(x + (a + x^2)^(1/2))^n,x)
```

output
```
int((a + x^2)^(5/2)*(x + (a + x^2)^(1/2))^n, x)
```

3.496 $\int (a+x^2)^{3/2} \left(x+\sqrt{a+x^2}\right)^n \, dx$

3.496.1 Optimal result

Integrand size = 23, antiderivative size = 131

$$\int (a+x^2)^{3/2} \left(x+\sqrt{a+x^2}\right)^n \, dx = -\frac{a^4\left(x+\sqrt{a+x^2}\right)^{-4+n}}{16(4-n)} - \frac{a^3\left(x+\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)} + \frac{3a^2\left(x+\sqrt{a+x^2}\right)^n}{8n} + \frac{a\left(x+\sqrt{a+x^2}\right)^{2+n}}{4(2+n)} + \frac{\left(x+\sqrt{a+x^2}\right)^{4+n}}{16(4+n)}$$

output
```
-1/16*a^4*(x+(x^2+a)^(1/2))^(-4+n)/(4-n)-1/4*a^3*(x+(x^2+a)^(1/2))^(-2+n)/
(2-n)+3/8*a^2*(x+(x^2+a)^(1/2))^n/n+1/4*a*(x+(x^2+a)^(1/2))^(2+n)/(2+n)+1/
16*(x+(x^2+a)^(1/2))^(4+n)/(4+n)
```

3.496.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.85

$$\int (a+x^2)^{3/2} \left(x+\sqrt{a+x^2}\right)^n \, dx = \frac{1}{16}\left(x+\sqrt{a+x^2}\right)^n \left(\frac{6a^2}{n} + \frac{a^4}{(-4+n)\left(x+\sqrt{a+x^2}\right)^4} + \frac{4a^3}{(-2+n)\left(x+\sqrt{a+x^2}\right)^2} + \frac{4a\left(x+\sqrt{a+x^2}\right)^2}{2+n} + \frac{\left(x+\sqrt{a+x^2}\right)^4}{4+n}\right)$$

input
```
Integrate[(a + x^2)^(3/2)*(x + Sqrt[a + x^2])^n,x]
```

output
```
((x + Sqrt[a + x^2])^n*((6*a^2)/n + a^4/((-4 + n)*(x + Sqrt[a + x^2])^4) +
 (4*a^3)/((-2 + n)*(x + Sqrt[a + x^2])^2) + (4*a*(x + Sqrt[a + x^2])^2)/(2
 + n) + (x + Sqrt[a + x^2])^4/(4 + n)))/16
```

3.496.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2547, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+x^2\right)^{3/2} \left(\sqrt{a+x^2}+x\right)^n \, dx$$

$$\Big\downarrow \text{2547}$$

$$\frac{1}{16} \int \left(x+\sqrt{x^2+a}\right)^{n-5} \left(\left(x+\sqrt{x^2+a}\right)^2+a\right)^4 d\left(x+\sqrt{x^2+a}\right)$$

$$\Big\downarrow \text{244}$$

$$\frac{1}{16} \int \left(a^4\left(x+\sqrt{x^2+a}\right)^{n-5}+4a^3\left(x+\sqrt{x^2+a}\right)^{n-3}+6a^2\left(x+\sqrt{x^2+a}\right)^{n-1}+4a\left(x+\sqrt{x^2+a}\right)^{n+1}+\left(x+\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{16}\left(-\frac{a^4\left(\sqrt{a+x^2}+x\right)^{n-4}}{4-n}-\frac{4a^3\left(\sqrt{a+x^2}+x\right)^{n-2}}{2-n}+\frac{6a^2\left(\sqrt{a+x^2}+x\right)^{n}}{n}+\frac{4a\left(\sqrt{a+x^2}+x\right)^{n+2}}{n+2}+\frac{\left(\sqrt{a+\right.}}{n}\right.$$

input
```
Int[(a + x^2)^(3/2)*(x + Sqrt[a + x^2])^n,x]
```

output
```
(-((a^4*(x + Sqrt[a + x^2])^(-4 + n))/(4 - n)) - (4*a^3*(x + Sqrt[a + x^2]
)^(-2 + n))/(2 - n) + (6*a^2*(x + Sqrt[a + x^2])^n)/n + (4*a*(x + Sqrt[a +
 x^2])^(2 + n))/(2 + n) + (x + Sqrt[a + x^2])^(4 + n)/(4 + n))/16
```

3.496.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2547
```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m
Subst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
 x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
 x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Intege
rQ[m] || GtQ[i/c, 0])
```

3.496.4 Maple [F]

$$\int \left(x^{2}+a \right)^{\frac{3}{2}} \left(x +\sqrt{x^{2}+a}\right)^{n}d x$$

input
```
int((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x)
```

output
```
int((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x)
```

3.496.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.84

$$\int (a+x^2)^{3/2} \left(x+\sqrt{a+x^2}\right)^n dx = \frac{(a^2n^4 + (n^4-4\,n^2)x^4 - 16\,a^2n^2 + 2\,(an^4-10\,an^2)x^2 + 24\,a^2 - 4\,((n^3-4\,n)x^3 + (an^3 -}{n^5-20\,n^3+64\,n}$$

input
```
integrate((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

output
```
(a^2*n^4 + (n^4 - 4*n^2)*x^4 - 16*a^2*n^2 + 2*(a*n^4 - 10*a*n^2)*x^2 + 24*
a^2 - 4*((n^3 - 4*n)*x^3 + (a*n^3 - 10*a*n)*x)*sqrt(x^2 + a))*(x + sqrt(x^
2 + a))^n/(n^5 - 20*n^3 + 64*n)
```

3.496.6 Sympy [F]

$$\int \left(a+x^2\right)^{3/2} \left(x+\sqrt{a+x^2}\right)^n \, dx = \int \left(a+x^2\right)^{\frac{3}{2}} \left(x+\sqrt{a+x^2}\right)^n \, dx$$

input
```
integrate((x**2+a)**(3/2)*(x+(x**2+a)**(1/2))**n,x)
```

output
```
Integral((a + x**2)**(3/2)*(x + sqrt(a + x**2))**n, x)
```

3.496.7 Maxima [F]

$$\int \left(a+x^2\right)^{3/2} \left(x+\sqrt{a+x^2}\right)^n \, dx = \int \left(x^2+a\right)^{\frac{3}{2}} \left(x+\sqrt{x^2+a}\right)^n \, dx$$

input
```
integrate((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

output
```
integrate((x^2 + a)^(3/2)*(x + sqrt(x^2 + a))^n, x)
```

3.496.8 Giac [F]

$$\int \left(a+x^2\right)^{3/2} \left(x+\sqrt{a+x^2}\right)^n \, dx = \int \left(x^2+a\right)^{\frac{3}{2}} \left(x+\sqrt{x^2+a}\right)^n \, dx$$

input
```
integrate((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
```

output
```
integrate((x^2 + a)^(3/2)*(x + sqrt(x^2 + a))^n, x)
```

3.496.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+x^2\right)^{3/2} \left(x+\sqrt{a+x^2}\right)^n \, dx = \int \left(x^2+a\right)^{3/2} \left(x+\sqrt{x^2+a}\right)^n \, dx$$

input
```
int((a + x^2)^(3/2)*(x + (a + x^2)^(1/2))^n,x)
```

output
```
int((a + x^2)^(3/2)*(x + (a + x^2)^(1/2))^n, x)
```

3.497 $\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n\,dx$

3.497.1 Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n\,dx = -\frac{a^2\left(x+\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)} + \frac{a\left(x+\sqrt{a+x^2}\right)^n}{2n} + \frac{\left(x+\sqrt{a+x^2}\right)^{2+n}}{4(2+n)}$$

output
```
-1/4*a^2*(x+(x^2+a)^(1/2))^(-2+n)/(2-n)+1/2*a*(x+(x^2+a)^(1/2))^n/n+1/4*(x
+(x^2+a)^(1/2))^(2+n)/(2+n)
```

3.497.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.87

$$\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n\,dx = \frac{1}{4}\left(x+\sqrt{a+x^2}\right)^n\left(\frac{2a}{n}+\frac{a^2}{(-2+n)\left(x+\sqrt{a+x^2}\right)^2}+\frac{\left(x+\sqrt{a+x^2}\right)^2}{2+n}\right)$$

input
```
Integrate[Sqrt[a + x^2]*(x + Sqrt[a + x^2])^n,x]
```

output
```
((x + Sqrt[a + x^2])^n*((2*a)/n + a^2/((-2 + n)*(x + Sqrt[a + x^2])^2) + (
x + Sqrt[a + x^2])^2/(2 + n)))/4
```

3.497.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2547, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+x^2}\left(\sqrt{a+x^2}+x\right)^n \, dx$$

$$\Big\downarrow \text{2547}$$

$$\frac{1}{4}\int \left(x+\sqrt{x^2+a}\right)^{n-3}\left(\left(x+\sqrt{x^2+a}\right)^2+a\right)^2 d\left(x+\sqrt{x^2+a}\right)$$

$$\Big\downarrow \text{244}$$

$$\frac{1}{4}\int \left(a^2\left(x+\sqrt{x^2+a}\right)^{n-3}+2a\left(x+\sqrt{x^2+a}\right)^{n-1}+\left(x+\sqrt{x^2+a}\right)^{n+1}\right) d\left(x+\sqrt{x^2+a}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\frac{a^2\left(\sqrt{a+x^2}+x\right)^{n-2}}{2-n}+\frac{2a\left(\sqrt{a+x^2}+x\right)^n}{n}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+2}}{n+2}\right)$$

input
```
Int[Sqrt[a + x^2]*(x + Sqrt[a + x^2])^n,x]
```

output
```
(-((a^2*(x + Sqrt[a + x^2])^(-2 + n))/(2 - n)) + (2*a*(x + Sqrt[a + x^2])^
n)/n + (x + Sqrt[a + x^2])^(2 + n)/(2 + n))/4
```

3.497.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2547
```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m
Subst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
 x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
 x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Intege
rQ[m] || GtQ[i/c, 0])
```

3.497.4 Maple [F]

$$\int \sqrt{x^2+a}\left(x+\sqrt{x^2+a}\right)^n dx$$

input
```
int((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x)
```

output
```
int((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x)
```

3.497.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.64

$$\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n dx = \frac{\left(n^2x^2+an^2-2\sqrt{x^2+a}nx-2a\right)\left(x+\sqrt{x^2+a}\right)^n}{n^3-4n}$$

input
```
integrate((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

output
```
(n^2*x^2 + a*n^2 - 2*sqrt(x^2 + a)*n*x - 2*a)*(x + sqrt(x^2 + a))^n/(n^3 -
 4*n)
```

3.497.6 Sympy [F]

$$\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n dx = \int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n dx$$

input
```
integrate((x**2+a)**(1/2)*(x+(x**2+a)**(1/2))**n,x)
```

output
```
Integral(sqrt(a + x**2)*(x + sqrt(a + x**2))**n, x)
```

3.497.7 Maxima [F]

$$\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n dx = \int \sqrt{x^2+a}\left(x+\sqrt{x^2+a}\right)^n dx$$

input
```
integrate((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n, x)
```

3.497.8 Giac [F]

$$\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n dx = \int \sqrt{x^2+a}\left(x+\sqrt{x^2+a}\right)^n dx$$

input
```
integrate((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n, x)
```

3.497.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n dx = \int \sqrt{x^2+a}\,\left(x+\sqrt{x^2+a}\right)^n dx$$

input
```
int((a + x^2)^(1/2)*(x + (a + x^2)^(1/2))^n,x)
```

output
```
int((a + x^2)^(1/2)*(x + (a + x^2)^(1/2))^n, x)
```

3.498 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}} \, dx$

3.498.1 Optimal result

Integrand size = 23, antiderivative size = 17

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}} \, dx = \frac{\left(x+\sqrt{a+x^2}\right)^n}{n}$$

output
```
(x+(x^2+a)^(1/2))^n/n
```

3.498.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}} \, dx = \frac{\left(x+\sqrt{a+x^2}\right)^n}{n}$$

input
```
Integrate[(x + Sqrt[a + x^2])^n/Sqrt[a + x^2],x]
```

output
```
(x + Sqrt[a + x^2])^n/n
```

3.498.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2547, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\sqrt{a+x^2}+x\right)^n}{\sqrt{a+x^2}}dx$$

$$\Big\downarrow \text{ 2547}$$

$$\int \left(\sqrt{a+x^2}+x\right)^{n-1}d\left(\sqrt{a+x^2}+x\right)$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\left(\sqrt{a+x^2}+x\right)^n}{n}$$

input
```
Int[(x + Sqrt[a + x^2])^n/Sqrt[a + x^2],x]
```

output
```
(x + Sqrt[a + x^2])^n/n
```

3.498.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2547
```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m
Subst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
 x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
 x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Intege
rQ[m] || GtQ[i/c, 0])
```

3.498.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\left(x+\sqrt{x^{2}+a}\right)^{n}}{n}$	16
default	$\frac{\left(x+\sqrt{x^{2}+a}\right)^{n}}{n}$	16

input
```
int((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x+(x^2+a)^(1/2))^n/n
```

3.498.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx=\frac{\left(x+\sqrt{x^2+a}\right)^n}{n}$$

input
```
integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
(x + sqrt(x^2 + a))^n/n
```

3.498.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(12) = 24$.

Time = 1.51 (sec) , antiderivative size = 311, normalized size of antiderivative = 18.29

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx = \begin{cases} \frac{\sqrt{a}a^{\frac{n}{2}}\sinh\left(n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{nx\sqrt{\frac{a}{x^2}+1}}-\frac{2a^{\frac{n}{2}}\cosh\left(n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\Gamma\left(1-\frac{n}{2}\right)}{n^2\Gamma\left(-\frac{n}{2}\right)}+\frac{a^{\frac{n}{2}}x\cosh\left(n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a}n}+\frac{a^{\frac{n}{2}}x\sinh\left(\cdots\right.}{\cdots} \\ \frac{a^{\frac{n}{2}}\sinh\left(n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{n\sqrt{1+\frac{x^2}{a}}}-\frac{2a^{\frac{n}{2}}\cosh\left(n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\Gamma\left(1-\frac{n}{2}\right)}{n^2\Gamma\left(-\frac{n}{2}\right)}+\frac{a^{\frac{n}{2}}x^2\sinh\left(n\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)-\operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{an\sqrt{1+\frac{x^2}{a}}}+\frac{a^{\frac{n}{2}}x\cosh\left(n\cdots\right.}{\cdots} \end{cases}$$

input
```
integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a)*a**(n/2)*sinh(n*asinh(x/sqrt(a)) - asinh(x/sqrt(a)))/(n
*x*sqrt(a/x**2 + 1)) - 2*a**(n/2)*cosh(n*asinh(x/sqrt(a)))*gamma(1 - n/2)/
(n**2*gamma(-n/2)) + a**(n/2)*x*cosh(n*asinh(x/sqrt(a)) - asinh(x/sqrt(a))
)/(sqrt(a)*n) + a**(n/2)*x**sinh(n*asinh(x/sqrt(a)) - asinh(x/sqrt(a)))/(sq
rt(a)*n*sqrt(a/x**2 + 1)), Abs(x**2/a) > 1), (a**(n/2)*sinh(n*asinh(x/sqrt
(a)) - asinh(x/sqrt(a)))/(n*sqrt(1 + x**2/a)) - 2*a**(n/2)*cosh(n*asinh(x/
sqrt(a)))*gamma(1 - n/2)/(n**2*gamma(-n/2)) + a**(n/2)*x**2*sinh(n*asinh(x
/sqrt(a)) - asinh(x/sqrt(a)))/(a*n*sqrt(1 + x**2/a)) + a**(n/2)*x*cosh(n*a
sinh(x/sqrt(a)) - asinh(x/sqrt(a)))/(sqrt(a)*n), True))
```

3.498.7 Maxima [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx = \int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\sqrt{x^2+a}}\,dx$$

input
```
integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + sqrt(x^2 + a))^n/sqrt(x^2 + a), x)
```

3.498.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx = \frac{\left(x+\sqrt{x^2+a}\right)^n}{n}$$

input
```
integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="giac")
```

output
```
(x + sqrt(x^2 + a))^n/n
```

3.498.9 Mupad [B] (verification not implemented)

Time = 16.97 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx=\frac{\left(x+\sqrt{x^2+a}\right)^n}{n}$$

input
```
int((x + (a + x^2)^(1/2))^n/(a + x^2)^(1/2),x)
```

output
```
(x + (a + x^2)^(1/2))^n/n
```

3.499 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}}\,dx$

3.499.1 Optimal result

Integrand size = 23, antiderivative size = 59

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}}\,dx=\frac{4\left(x+\sqrt{a+x^2}\right)^{2+n}\text{Hypergeometric2F1}\left(2,\frac{2+n}{2},\frac{4+n}{2},-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^2(2+n)}$$

output
```
4*hypergeom([2, 1+1/2*n],[2+1/2*n],-(x+(x^2+a)^(1/2))^2/a)*(x+(x^2+a)^(1/2
))^(2+n)/a^2/(2+n)
```

3.499.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}}\,dx=\frac{4\left(x+\sqrt{a+x^2}\right)^{2+n}\text{Hypergeometric2F1}\left(2,\frac{2+n}{2},1+\frac{2+n}{2},-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^2(2+n)}$$

input
```
Integrate[(x + Sqrt[a + x^2])^n/(a + x^2)^(3/2),x]
```

output
```
(4*(x + Sqrt[a + x^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, 1 + (2 + n)
/2, -((x + Sqrt[a + x^2])^2/a)])/(a^2*(2 + n))
```

3.499.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2547, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\sqrt{a+x^2}+x\right)^n}{(a+x^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 2547$$

$$4\int \frac{\left(x+\sqrt{x^2+a}\right)^{n+1}}{\left(\left(x+\sqrt{x^2+a}\right)^2+a\right)^2}d\left(x+\sqrt{x^2+a}\right)$$

$$\Big\downarrow \ 278$$

$$\frac{4\left(\sqrt{a+x^2}+x\right)^{n+2}\text{Hypergeometric2F1}\left(2,\frac{n+2}{2},\frac{n+4}{2},-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^2(n+2)}$$

input
```
Int[(x + Sqrt[a + x^2])^n/(a + x^2)^(3/2),x]
```

output
```
(4*(x + Sqrt[a + x^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2,
-((x + Sqrt[a + x^2])^2/a)])/(a^2*(2 + n))
```

3.499.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2547
```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m
Subst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
 x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
 x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Intege
rQ[m] || GtQ[i/c, 0])
```

3.499.4 Maple [F]

$$\int \frac{\left(x+\sqrt{x^{2}+a}\right)^{n}}{\left(x^{2}+a\right)^{\frac{3}{2}}}dx$$

input
```
int((x+(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x)
```

output
```
int((x+(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x)
```

3.499.5 Fricas [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{(a+x^2)^{3/2}}\,dx = \int \frac{\left(x+\sqrt{x^2+a}\right)^n}{(x^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n/(x^4 + 2*a*x^2 + a^2), x)
```

3.499.6 Sympy [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{(a+x^2)^{3/2}}\,dx = \int \frac{\left(x+\sqrt{a+x^2}\right)^n}{(a+x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**(3/2),x)
```

output
```
Integral((x + sqrt(a + x**2))**n/(a + x**2)**(3/2), x)
```

3.499.7 Maxima [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{(a+x^2)^{3/2}}\,dx = \int \frac{\left(x+\sqrt{x^2+a}\right)^n}{(x^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(3/2), x)
```

3.499.8 Giac [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{(a+x^2)^{3/2}}\,dx = \int \frac{\left(x+\sqrt{x^2+a}\right)^n}{(x^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(3/2), x)
```

3.499.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{(a+x^2)^{3/2}}\,dx = \int \frac{\left(x+\sqrt{x^2+a}\right)^n}{(x^2+a)^{3/2}}\,dx$$

input
```
int((x + (a + x^2)^(1/2))^n/(a + x^2)^(3/2),x)
```

output
```
int((x + (a + x^2)^(1/2))^n/(a + x^2)^(3/2), x)
```

3.500 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}}\,dx$

3.500.1 Optimal result

Integrand size = 23, antiderivative size = 59

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}}\,dx=\frac{16\left(x+\sqrt{a+x^2}\right)^{4+n}\text{Hypergeometric2F1}\left(4,\frac{4+n}{2},\frac{6+n}{2},-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^4(4+n)}$$

output
```
16*hypergeom([4, 2+1/2*n],[3+1/2*n],-(x+(x^2+a)^(1/2))^2/a)*(x+(x^2+a)^(1/
2))^(4+n)/a^4/(4+n)
```

3.500.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}}\,dx=\frac{16\left(x+\sqrt{a+x^2}\right)^{4+n}\text{Hypergeometric2F1}\left(4,\frac{4+n}{2},1+\frac{4+n}{2},-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^4(4+n)}$$

input
```
Integrate[(x + Sqrt[a + x^2])^n/(a + x^2)^(5/2),x]
```

output
```
(16*(x + Sqrt[a + x^2])^(4 + n)*Hypergeometric2F1[4, (4 + n)/2, 1 + (4 + n
)/2, -((x + Sqrt[a + x^2])^2/a)])/(a^4*(4 + n))
```

3.500.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2547, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\sqrt{a+x^2}+x\right)^n}{(a+x^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{2547}$$

$$16\int \frac{\left(x+\sqrt{x^2+a}\right)^{n+3}}{\left(\left(x+\sqrt{x^2+a}\right)^2+a\right)^4}d\left(x+\sqrt{x^2+a}\right)$$

$$\Big\downarrow \text{278}$$

$$\frac{16\left(\sqrt{a+x^2}+x\right)^{n+4} \text{Hypergeometric2F1}\left(4,\frac{n+4}{2},\frac{n+6}{2},-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^4(n+4)}$$

input
```
Int[(x + Sqrt[a + x^2])^n/(a + x^2)^(5/2),x]
```

output
```
(16*(x + Sqrt[a + x^2])^(4 + n)*Hypergeometric2F1[4, (4 + n)/2, (6 + n)/2,
 -((x + Sqrt[a + x^2])^2/a)])/(a^4*(4 + n))
```

3.500.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2547
```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m
Subst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
 x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
 x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Intege
rQ[m] || GtQ[i/c, 0])
```

3.500.4 Maple [F]

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{5}{2}}}dx$$

input
```
int((x+(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x)
```

output
```
int((x+(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x)
```

3.500.5 Fricas [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}}\,dx=\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n/(x^6 + 3*a*x^4 + 3*a^2*x^2 +
a^3), x)
```

3.500.6 Sympy [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}}\,dx=\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**(5/2),x)
```

output
```
Integral((x + sqrt(a + x**2))**n/(a + x**2)**(5/2), x)
```

3.500.7 Maxima [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}}\,dx=\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(5/2), x)
```

3.500.8 Giac [F]

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}}\,dx=\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(5/2), x)
```

3.500.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}}\,dx=\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{5/2}}\,dx$$

input
```
int((x + (a + x^2)^(1/2))^n/(a + x^2)^(5/2),x)
```

output
```
int((x + (a + x^2)^(1/2))^n/(a + x^2)^(5/2), x)
```

3.501 $\int (a+x^2)^{5/2} \left(x-\sqrt{a+x^2}\right)^n \, dx$

3.501.1 Optimal result

Integrand size = 25, antiderivative size = 201

$$\begin{aligned}\int (a+x^2)^{5/2} \left(x-\sqrt{a+x^2}\right)^n \, dx &= \frac{a^6 \left(x-\sqrt{a+x^2}\right)^{-6+n}}{64(6-n)} \\ &+ \frac{3a^5 \left(x-\sqrt{a+x^2}\right)^{-4+n}}{32(4-n)} + \frac{15a^4 \left(x-\sqrt{a+x^2}\right)^{-2+n}}{64(2-n)} - \frac{5a^3 \left(x-\sqrt{a+x^2}\right)^{n}}{16n} \\ &- \frac{15a^2 \left(x-\sqrt{a+x^2}\right)^{2+n}}{64(2+n)} - \frac{3a \left(x-\sqrt{a+x^2}\right)^{4+n}}{32(4+n)} - \frac{\left(x-\sqrt{a+x^2}\right)^{6+n}}{64(6+n)}\end{aligned}$$

output
```
1/64*a^6*(x-(x^2+a)^(1/2))^(-6+n)/(6-n)+3/32*a^5*(x-(x^2+a)^(1/2))^(-4+n)/
(4-n)+15/64*a^4*(x-(x^2+a)^(1/2))^(-2+n)/(2-n)-5/16*a^3*(x-(x^2+a)^(1/2))^
n/n-15/64*a^2*(x-(x^2+a)^(1/2))^(2+n)/(2+n)-3/32*a*(x-(x^2+a)^(1/2))^(4+n)
/(4+n)-1/64*(x-(x^2+a)^(1/2))^(6+n)/(6+n)
```

3.501.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.86

$$\int \left(a+x^2\right)^{5/2} \left(x-\sqrt{a+x^2}\right)^n \, dx = \frac{1}{64}\left(x-\sqrt{a+x^2}\right)^n \left(-\frac{20a^3}{n} - \frac{a^6}{(-6+n)\left(x-\sqrt{a+x^2}\right)^6} - \frac{6a^5}{(-4+n)\left(x-\sqrt{a+x^2}\right)^4} - \frac{15a^4}{(-2+n)\left(x-\sqrt{a+x^2}\right)^2} - \frac{15a^2\left(x-\sqrt{a+x^2}\right)^2}{2+n} - \frac{6a\left(x-\sqrt{a+x^2}\right)^4}{4+n} - \frac{\left(x-\sqrt{a+x^2}\right)^6}{6+n}\right)$$

input
```
Integrate[(a + x^2)^(5/2)*(x - Sqrt[a + x^2])^n,x]
```

output
```
((x - Sqrt[a + x^2])^n*((-20*a^3)/n - a^6/((-6 + n)*(x - Sqrt[a + x^2])^6)
 - (6*a^5)/((-4 + n)*(x - Sqrt[a + x^2])^4) - (15*a^4)/((-2 + n)*(x - Sqrt
[a + x^2])^2) - (15*a^2*(x - Sqrt[a + x^2])^2)/(2 + n) - (6*a*(x - Sqrt[a
+ x^2])^4)/(4 + n) - (x - Sqrt[a + x^2])^6/(6 + n)))/64
```

3.501.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2547, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+x^2\right)^{5/2} \left(x-\sqrt{a+x^2}\right)^n \, dx$$

$$\Big\downarrow \text{2547}$$

$$-\frac{1}{64}\int \left(x-\sqrt{x^2+a}\right)^{n-7}\left(\left(x-\sqrt{x^2+a}\right)^2+a\right)^6 d\left(x-\sqrt{x^2+a}\right)$$

$$\Big\downarrow \text{244}$$

$$-\frac{1}{64}\int \left(a^6\left(x-\sqrt{x^2+a}\right)^{n-7}+6a^5\left(x-\sqrt{x^2+a}\right)^{n-5}+15a^4\left(x-\sqrt{x^2+a}\right)^{n-3}+20a^3\left(x-\sqrt{x^2+a}\right)^{n-1}+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{64}\left(\frac{a^6\left(x-\sqrt{a+x^2}\right)^{n-6}}{6-n}+\frac{6a^5\left(x-\sqrt{a+x^2}\right)^{n-4}}{4-n}+\frac{15a^4\left(x-\sqrt{a+x^2}\right)^{n-2}}{2-n}-\frac{20a^3\left(x-\sqrt{a+x^2}\right)^{n}}{n}-\frac{15a^2\left(\right.}{}\right.$$

input
```
Int[(a + x^2)^(5/2)*(x - Sqrt[a + x^2])^n,x]
```

output
```
((a^6*(x - Sqrt[a + x^2])^(-6 + n))/(6 - n) + (6*a^5*(x - Sqrt[a + x^2])^(
-4 + n))/(4 - n) + (15*a^4*(x - Sqrt[a + x^2])^(-2 + n))/(2 - n) - (20*a^3
*(x - Sqrt[a + x^2])^n)/n - (15*a^2*(x - Sqrt[a + x^2])^(2 + n))/(2 + n) -
 (6*a*(x - Sqrt[a + x^2])^(4 + n))/(4 + n) - (x - Sqrt[a + x^2])^(6 + n)/(
6 + n))/64
```

3.501.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2547
```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m
Subst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
 x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
 x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Intege
rQ[m] || GtQ[i/c, 0])
```

3.501.4 Maple [F]

$$\int \left(x^2+a\right)^{\frac{5}{2}}\left(x-\sqrt{x^2+a}\right)^n dx$$

input
```
int((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x)
```

output
```
int((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x)
```

3.501.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.01

$$\int (a+x^2)^{5/2}\left(x-\sqrt{a+x^2}\right)^n dx = -\frac{(a^3n^6 - 50\,a^3n^4 + (n^6 - 20\,n^4 + 64\,n^2)x^6 + 544\,a^3n^2 + 3\,(an^6 - 30\,an^4 + 104\,an^2)x^4 - 720\,a^3 + 3\,(a^2n^6 -}{}$$

input
```
integrate((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

output
```
-(a^3*n^6 - 50*a^3*n^4 + (n^6 - 20*n^4 + 64*n^2)*x^6 + 544*a^3*n^2 + 3*(a*
n^6 - 30*a*n^4 + 104*a*n^2)*x^4 - 720*a^3 + 3*(a^2*n^6 - 40*a^2*n^4 + 264*
a^2*n^2)*x^2 + 6*((n^5 - 20*n^3 + 64*n)*x^5 + 2*(a*n^5 - 30*a*n^3 + 104*a*
n)*x^3 + (a^2*n^5 - 40*a^2*n^3 + 264*a^2*n)*x)*sqrt(x^2 + a))*(x - sqrt(x^
2 + a))^n/(n^7 - 56*n^5 + 784*n^3 - 2304*n)
```

3.501.6 Sympy [F(-2)]

Exception generated.

$$\int (a+x^2)^{5/2}\left(x-\sqrt{a+x^2}\right)^n dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((x**2+a)**(5/2)*(x-(x**2+a)**(1/2))**n,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.501.7 Maxima [F]

$$\int (a+x^2)^{5/2}\left(x-\sqrt{a+x^2}\right)^n dx = \int (x^2+a)^{\frac{5}{2}}\left(x-\sqrt{x^2+a}\right)^n dx$$

input
```
integrate((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

output
```
integrate((x^2 + a)^(5/2)*(x - sqrt(x^2 + a))^n, x)
```

3.501.8 Giac [F]

$$\int (a+x^2)^{5/2}\left(x-\sqrt{a+x^2}\right)^n\,dx=\int (x^2+a)^{\frac{5}{2}}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

input
```
integrate((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
```

output
```
integrate((x^2 + a)^(5/2)*(x - sqrt(x^2 + a))^n, x)
```

3.501.9 Mupad [F(-1)]

Timed out.

$$\int (a+x^2)^{5/2}\left(x-\sqrt{a+x^2}\right)^n\,dx=\int \left(x-\sqrt{x^2+a}\right)^n(x^2+a)^{5/2}\,dx$$

input
```
int((x - (a + x^2)^(1/2))^n*(a + x^2)^(5/2),x)
```

output
```
int((x - (a + x^2)^(1/2))^n*(a + x^2)^(5/2), x)
```

3.502 $\int (a+x^2)^{3/2} \left(x-\sqrt{a+x^2}\right)^n \, dx$

3.502.1 Optimal result

Integrand size = 25, antiderivative size = 141

$$\int (a+x^2)^{3/2} \left(x-\sqrt{a+x^2}\right)^n \, dx = \frac{a^4\left(x-\sqrt{a+x^2}\right)^{-4+n}}{16(4-n)}+\frac{a^3\left(x-\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)} -\frac{3a^2\left(x-\sqrt{a+x^2}\right)^{n}}{8n}-\frac{a\left(x-\sqrt{a+x^2}\right)^{2+n}}{4(2+n)}-\frac{\left(x-\sqrt{a+x^2}\right)^{4+n}}{16(4+n)}$$

output
```
1/16*a^4*(x-(x^2+a)^(1/2))^(-4+n)/(4-n)+1/4*a^3*(x-(x^2+a)^(1/2))^(-2+n)/(
2-n)-3/8*a^2*(x-(x^2+a)^(1/2))^n/n-1/4*a*(x-(x^2+a)^(1/2))^(2+n)/(2+n)-1/1
6*(x-(x^2+a)^(1/2))^(4+n)/(4+n)
```

3.502.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.87

$$\int (a+x^2)^{3/2} \left(x-\sqrt{a+x^2}\right)^n \, dx = \frac{1}{16}\left(x-\sqrt{a+x^2}\right)^n \left(-\frac{6a^2}{n}-\frac{a^4}{(-4+n)\left(x-\sqrt{a+x^2}\right)^4} -\frac{4a^3}{(-2+n)\left(x-\sqrt{a+x^2}\right)^2}-\frac{4a\left(x-\sqrt{a+x^2}\right)^2}{2+n}-\frac{\left(x-\sqrt{a+x^2}\right)^4}{4+n}\right)$$

input
```
Integrate[(a + x^2)^(3/2)*(x - Sqrt[a + x^2])^n,x]
```

output
```
((x - Sqrt[a + x^2])^n*((-6*a^2)/n - a^4/((-4 + n)*(x - Sqrt[a + x^2])^4)
- (4*a^3)/((-2 + n)*(x - Sqrt[a + x^2])^2) - (4*a*(x - Sqrt[a + x^2])^2)/(
2 + n) - (x - Sqrt[a + x^2])^4/(4 + n)))/16
```

3.502.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2547, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+x^2)^{3/2} \left(x-\sqrt{a+x^2}\right)^n dx$$

$$\Big\downarrow \text{2547}$$

$$-\frac{1}{16}\int \left(x-\sqrt{x^2+a}\right)^{n-5}\left(\left(x-\sqrt{x^2+a}\right)^2+a\right)^4 d\left(x-\sqrt{x^2+a}\right)$$

$$\Big\downarrow \text{244}$$

$$-\frac{1}{16}\int \left(a^4\left(x-\sqrt{x^2+a}\right)^{n-5}+4a^3\left(x-\sqrt{x^2+a}\right)^{n-3}+6a^2\left(x-\sqrt{x^2+a}\right)^{n-1}+4a\left(x-\sqrt{x^2+a}\right)^{n+1}+\left(x\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{16}\left(\frac{a^4\left(x-\sqrt{a+x^2}\right)^{n-4}}{4-n}+\frac{4a^3\left(x-\sqrt{a+x^2}\right)^{n-2}}{2-n}-\frac{6a^2\left(x-\sqrt{a+x^2}\right)^{n}}{n}-\frac{4a\left(x-\sqrt{a+x^2}\right)^{n+2}}{n+2}-\frac{\left(x-\sqrt{a\right.}}{n-}\right.$$

input
```
Int[(a + x^2)^(3/2)*(x - Sqrt[a + x^2])^n,x]
```

output
```
((a^4*(x - Sqrt[a + x^2])^(-4 + n))/(4 - n) + (4*a^3*(x - Sqrt[a + x^2])^(
-2 + n))/(2 - n) - (6*a^2*(x - Sqrt[a + x^2])^n)/n - (4*a*(x - Sqrt[a + x^
2])^(2 + n))/(2 + n) - (x - Sqrt[a + x^2])^(4 + n)/(4 + n))/16
```

3.502.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2547
```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m
Subst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
 x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
 x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Intege
rQ[m] || GtQ[i/c, 0])
```

3.502.4 Maple [F]

$$\int \left(x^2+a\right)^{\frac{3}{2}} \left(x-\sqrt{x^2+a}\right)^n dx$$

input
```
int((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x)
```

output
```
int((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x)
```

3.502.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.80

$$\int \left(a+x^2\right)^{3/2} \left(x-\sqrt{a+x^2}\right)^n dx = -\frac{\left(a^2n^4+(n^4-4\,n^2)x^4-16\,a^2n^2+2\,(an^4-10\,an^2)x^2+24\,a^2+4\,((n^3-4\,n)x^3+(an^3-10\,an)x)\sqrt{x^2+\cdots}\right.}{n^5-20\,n^3+64\,n}$$

input
```
integrate((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

output
```
-(a^2*n^4 + (n^4 - 4*n^2)*x^4 - 16*a^2*n^2 + 2*(a*n^4 - 10*a*n^2)*x^2 + 24
*a^2 + 4*((n^3 - 4*n)*x^3 + (a*n^3 - 10*a*n)*x)*sqrt(x^2 + a))*(x - sqrt(x
^2 + a))^n/(n^5 - 20*n^3 + 64*n)
```

3.502.6 Sympy [F]

$$\int \left(a+x^2\right)^{3/2} \left(x-\sqrt{a+x^2}\right)^n \, dx=\int \left(a+x^{2}\right)^{\frac{3}{2}} \left(x-\sqrt{a+x^{2}}\right)^{n}\, dx$$

input
```
integrate((x**2+a)**(3/2)*(x-(x**2+a)**(1/2))**n,x)
```

output
```
Integral((a + x**2)**(3/2)*(x - sqrt(a + x**2))**n, x)
```

3.502.7 Maxima [F]

$$\int \left(a+x^2\right)^{3/2} \left(x-\sqrt{a+x^2}\right)^n \, dx=\int \left(x^{2}+a\right)^{\frac{3}{2}} \left(x-\sqrt{x^{2}+a}\right)^{n}\, dx$$

input
```
integrate((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

output
```
integrate((x^2 + a)^(3/2)*(x - sqrt(x^2 + a))^n, x)
```

3.502.8 Giac [F]

$$\int \left(a+x^2\right)^{3/2} \left(x-\sqrt{a+x^2}\right)^n \, dx=\int \left(x^{2}+a\right)^{\frac{3}{2}} \left(x-\sqrt{x^{2}+a}\right)^{n}\, dx$$

input
```
integrate((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
```

output
```
integrate((x^2 + a)^(3/2)*(x - sqrt(x^2 + a))^n, x)
```

3.502.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+x^2\right)^{3/2}\left(x-\sqrt{a+x^2}\right)^n\,dx=\int \left(x-\sqrt{x^2+a}\right)^n\left(x^2+a\right)^{3/2}\,dx$$

input
```
int((x - (a + x^2)^(1/2))^n*(a + x^2)^(3/2),x)
```

output
```
int((x - (a + x^2)^(1/2))^n*(a + x^2)^(3/2), x)
```

3.503 $\int \sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^n dx$

3.503.1 Optimal result

Integrand size = 25, antiderivative size = 81

$$\int \sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^n dx=\frac{a^2\left(x-\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)}-\frac{a\left(x-\sqrt{a+x^2}\right)^n}{2n}-\frac{\left(x-\sqrt{a+x^2}\right)^{2+n}}{4(2+n)}$$

output
```
1/4*a^2*(x-(x^2+a)^(1/2))^(-2+n)/(2-n)-1/2*a*(x-(x^2+a)^(1/2))^n/n-1/4*(x-
(x^2+a)^(1/2))^(2+n)/(2+n)
```

3.503.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^n dx=\frac{1}{4}\left(x-\sqrt{a+x^2}\right)^n\left(-\frac{2a}{n}-\frac{a^2}{(-2+n)\left(x-\sqrt{a+x^2}\right)^2}-\frac{\left(x-\sqrt{a+x^2}\right)^2}{2+n}\right)$$

input
```
Integrate[Sqrt[a + x^2]*(x - Sqrt[a + x^2])^n,x]
```

output
```
((x - Sqrt[a + x^2])^n*((-2*a)/n - a^2/((-2 + n)*(x - Sqrt[a + x^2])^2) -
(x - Sqrt[a + x^2])^2/(2 + n)))/4
```

3.503.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2547, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^n dx$$

$$\Big\downarrow \text{2547}$$

$$-\frac{1}{4}\int \left(x-\sqrt{x^2+a}\right)^{n-3}\left(\left(x-\sqrt{x^2+a}\right)^2+a\right)^2 d\left(x-\sqrt{x^2+a}\right)$$

$$\Big\downarrow \text{244}$$

$$-\frac{1}{4}\int \left(a^2\left(x-\sqrt{x^2+a}\right)^{n-3}+2a\left(x-\sqrt{x^2+a}\right)^{n-1}+\left(x-\sqrt{x^2+a}\right)^{n+1}\right) d\left(x-\sqrt{x^2+a}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(\frac{a^2\left(x-\sqrt{a+x^2}\right)^{n-2}}{2-n}-\frac{2a\left(x-\sqrt{a+x^2}\right)^n}{n}-\frac{\left(x-\sqrt{a+x^2}\right)^{n+2}}{n+2}\right)$$

input
```
Int[Sqrt[a + x^2]*(x - Sqrt[a + x^2])^n,x]
```

output
```
((a^2*(x - Sqrt[a + x^2])^(-2 + n))/(2 - n) - (2*a*(x - Sqrt[a + x^2])^n)/
n - (x - Sqrt[a + x^2])^(2 + n)/(2 + n))/4
```

3.503.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2547
```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m
Subst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
 x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
 x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Intege
rQ[m] || GtQ[i/c, 0])
```

3.503.4 Maple [F]

$$\int \sqrt{x^2+a}\left(x-\sqrt{x^2+a}\right)^n dx$$

input
```
int((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x)
```

output
```
int((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x)
```

3.503.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.63

$$\int \sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^n dx = -\frac{(n^2x^2+an^2+2\sqrt{x^2+a}nx-2a)\left(x-\sqrt{x^2+a}\right)^n}{n^3-4n}$$

input
```
integrate((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

output
```
-(n^2*x^2 + a*n^2 + 2*sqrt(x^2 + a)*n*x - 2*a)*(x - sqrt(x^2 + a))^n/(n^3
- 4*n)
```

3.503.6 Sympy [F]

$$\int \sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^n dx = \int \sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^n dx$$

input
```
integrate((x**2+a)**(1/2)*(x-(x**2+a)**(1/2))**n,x)
```

output
```
Integral(sqrt(a + x**2)*(x - sqrt(a + x**2))**n, x)
```

3.503.7 Maxima [F]

$$\int \sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^n\,dx=\int \sqrt{x^2+a}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

input
```
integrate((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n, x)
```

3.503.8 Giac [F]

$$\int \sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^n\,dx=\int \sqrt{x^2+a}\left(x-\sqrt{x^2+a}\right)^n\,dx$$

input
```
integrate((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n, x)
```

3.503.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^n\,dx=\int \left(x-\sqrt{x^2+a}\right)^n\sqrt{x^2+a}\,dx$$

input
```
int((x - (a + x^2)^(1/2))^n*(a + x^2)^(1/2),x)
```

output
```
int((x - (a + x^2)^(1/2))^n*(a + x^2)^(1/2), x)
```

3.504 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx$

3.504.1 Optimal result

Integrand size = 25, antiderivative size = 20

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx=-\frac{\left(x-\sqrt{a+x^2}\right)^n}{n}$$

output
```
-(x-(x^2+a)^(1/2))^n/n
```

3.504.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx=-\frac{\left(x-\sqrt{a+x^2}\right)^n}{n}$$

input
```
Integrate[(x - Sqrt[a + x^2])^n/Sqrt[a + x^2],x]
```

output
```
-((x - Sqrt[a + x^2])^n/n)
```

3.504.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2547, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}dx$$

$$\Big\downarrow \text{2547}$$

$$-\int \left(x-\sqrt{x^2+a}\right)^{n-1}d\left(x-\sqrt{x^2+a}\right)$$

$$\Big\downarrow \text{15}$$

$$-\frac{\left(x-\sqrt{a+x^2}\right)^n}{n}$$

input
```
Int[(x - Sqrt[a + x^2])^n/Sqrt[a + x^2],x]
```

output
```
-((x - Sqrt[a + x^2])^n/n)
```

3.504.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2547
```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m
Subst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
 x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
 x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Intege
rQ[m] || GtQ[i/c, 0])
```

3.504.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$-\frac{\left(x-\sqrt{x^{2}+a}\right)^{n}}{n}$	19
default	$-\frac{\left(x-\sqrt{x^{2}+a}\right)^{n}}{n}$	19

input
```
int((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(x-(x^2+a)^(1/2))^n/n
```

3.504.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx=-\frac{\left(x-\sqrt{x^2+a}\right)^n}{n}$$

input
```
integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-(x - sqrt(x^2 + a))^n/n
```

3.504.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(14) = 28.

Time = 0.95 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.05

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx=\begin{cases} -\frac{\left(x-\sqrt{a+x^2}\right)^n}{n} & \text{for } n\neq 0 \\ \begin{cases} \log\left(2x+2\sqrt{a+x^2}\right) & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{x^2}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**(1/2),x)
```

output
```
Piecewise((-(x - sqrt(a + x**2))**n/n, Ne(n, 0)), (Piecewise((log(2*x + 2*
sqrt(a + x**2)), Ne(a, 0)), (x*log(x)/sqrt(x**2), True)), True))
```

3.504.7 Maxima [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx = \int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\sqrt{x^2+a}}\,dx$$

input
```
integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x - sqrt(x^2 + a))^n/sqrt(x^2 + a), x)
```

3.504.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx = -\frac{\left(x-\sqrt{x^2+a}\right)^n}{n}$$

input
```
integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-(x - sqrt(x^2 + a))^n/n
```

3.504.9 Mupad [B] (verification not implemented)

Time = 17.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx = -\frac{\left(x-\sqrt{x^2+a}\right)^n}{n}$$

input
```
int((x - (a + x^2)^(1/2))^n/(a + x^2)^(1/2),x)
```

output
```
-(x - (a + x^2)^(1/2))^n/n
```

3.505 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}}\,dx$

3.505.1 Optimal result

Integrand size = 25, antiderivative size = 63

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}}\,dx=-\frac{4\left(x-\sqrt{a+x^2}\right)^{2+n}\text{Hypergeometric2F1}\left(2,\frac{2+n}{2},\frac{4+n}{2},-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^2(2+n)}$$

output
```
-4*hypergeom([2, 1+1/2*n],[2+1/2*n],-(x-(x^2+a)^(1/2))^2/a)*(x-(x^2+a)^(1/
2))^(2+n)/a^2/(2+n)
```

3.505.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}}\,dx = -\frac{4\left(x-\sqrt{a+x^2}\right)^{2+n}\text{Hypergeometric2F1}\left(2,\frac{2+n}{2},1+\frac{2+n}{2},-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^2(2+n)}$$

input
```
Integrate[(x - Sqrt[a + x^2])^n/(a + x^2)^(3/2),x]
```

output
```
(-4*(x - Sqrt[a + x^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, 1 + (2 + n
)/2, -((x - Sqrt[a + x^2])^2/a)])/(a^2*(2 + n))
```

3.505.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2547, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^{3/2}}dx$$
$$\Big\downarrow \text{ 2547}$$
$$-4\int \frac{\left(x-\sqrt{x^2+a}\right)^{n+1}}{\left(\left(x-\sqrt{x^2+a}\right)^2+a\right)^2}d\left(x-\sqrt{x^2+a}\right)$$
$$\Big\downarrow \text{ 278}$$
$$-\frac{4\left(x-\sqrt{a+x^2}\right)^{n+2}\text{Hypergeometric2F1}\left(2,\frac{n+2}{2},\frac{n+4}{2},-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^2(n+2)}$$

input
```
Int[(x - Sqrt[a + x^2])^n/(a + x^2)^(3/2),x]
```

output
```
(-4*(x - Sqrt[a + x^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2,
 -((x - Sqrt[a + x^2])^2/a)])/(a^2*(2 + n))
```

3.505.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2547
```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m
Subst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
 x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
 x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Intege
rQ[m] || GtQ[i/c, 0])
```

3.505.4 Maple [F]

$$\int \frac{\left(x-\sqrt{x^{2}+a}\right)^{n}}{\left(x^{2}+a\right)^{\frac{3}{2}}}dx$$

input
```
int((x-(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x)
```

output
```
int((x-(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x)
```

3.505.5 Fricas [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}}\,dx=\int \frac{\left(x-\sqrt{x^{2}+a}\right)^{n}}{\left(x^{2}+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n/(x^4 + 2*a*x^2 + a^2), x)
```

3.505.6 Sympy [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^{3/2}}\,dx = \int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**(3/2),x)
```

output
```
Integral((x - sqrt(a + x**2))**n/(a + x**2)**(3/2), x)
```

3.505.7 Maxima [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^{3/2}}\,dx = \int \frac{\left(x-\sqrt{x^2+a}\right)^n}{(x^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(3/2), x)
```

3.505.8 Giac [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^{3/2}}\,dx = \int \frac{\left(x-\sqrt{x^2+a}\right)^n}{(x^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(3/2), x)
```

3.505.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}}\,dx=\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{3/2}}\,dx$$

input
```
int((x - (a + x^2)^(1/2))^n/(a + x^2)^(3/2),x)
```

output
```
int((x - (a + x^2)^(1/2))^n/(a + x^2)^(3/2), x)
```

3.506 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}}\,dx$

3.506.1 Optimal result

Integrand size = 25, antiderivative size = 63

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}}\,dx = -\frac{16\left(x-\sqrt{a+x^2}\right)^{4+n}\text{Hypergeometric2F1}\left(4,\frac{4+n}{2},\frac{6+n}{2},-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^4(4+n)}$$

output
```
-16*hypergeom([4, 2+1/2*n],[3+1/2*n],-(x-(x^2+a)^(1/2))^2/a)*(x-(x^2+a)^(1
/2))^(4+n)/a^4/(4+n)
```

3.506.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}}\,dx = -\frac{16\left(x-\sqrt{a+x^2}\right)^{4+n}\text{Hypergeometric2F1}\left(4,\frac{4+n}{2},1+\frac{4+n}{2},-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^4(4+n)}$$

input
```
Integrate[(x - Sqrt[a + x^2])^n/(a + x^2)^(5/2),x]
```

output
```
(-16*(x - Sqrt[a + x^2])^(4 + n)*Hypergeometric2F1[4, (4 + n)/2, 1 + (4 +
n)/2, -((x - Sqrt[a + x^2])^2/a)])/(a^4*(4 + n))
```

3.506.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2547, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 2547}$$

$$-16\int \frac{\left(x-\sqrt{x^2+a}\right)^{n+3}}{\left(\left(x-\sqrt{x^2+a}\right)^2+a\right)^4}d\left(x-\sqrt{x^2+a}\right)$$

$$\Big\downarrow \text{ 278}$$

$$-\frac{16\left(x-\sqrt{a+x^2}\right)^{n+4}\text{Hypergeometric2F1}\left(4,\frac{n+4}{2},\frac{n+6}{2},-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^4(n+4)}$$

input
```
Int[(x - Sqrt[a + x^2])^n/(a + x^2)^(5/2),x]
```

output
```
(-16*(x - Sqrt[a + x^2])^(4 + n)*Hypergeometric2F1[4, (4 + n)/2, (6 + n)/2
, -((x - Sqrt[a + x^2])^2/a)])/(a^4*(4 + n))
```

3.506.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2547
```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m
Subst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
 x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
 x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Intege
rQ[m] || GtQ[i/c, 0])
```

3.506.4 Maple [F]

$$\int \frac{\left(x-\sqrt{x^{2}+a}\right)^{n}}{\left(x^{2}+a\right)^{\frac{5}{2}}}dx$$

input
```
int((x-(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x)
```

output
```
int((x-(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x)
```

3.506.5 Fricas [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^{5/2}}\,dx=\int \frac{\left(x-\sqrt{x^{2}+a}\right)^{n}}{\left(x^{2}+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n/(x^6 + 3*a*x^4 + 3*a^2*x^2 +
a^3), x)
```

3.506.6 Sympy [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^{5/2}}\,dx=\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**(5/2),x)
```

output
```
Integral((x - sqrt(a + x**2))**n/(a + x**2)**(5/2), x)
```

3.506.7 Maxima [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^{5/2}}\,dx=\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{(x^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(5/2), x)
```

3.506.8 Giac [F]

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^{5/2}}\,dx=\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{(x^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(5/2), x)
```

3.506.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}}\,dx=\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{5/2}}\,dx$$

input
```
int((x - (a + x^2)^(1/2))^n/(a + x^2)^(5/2),x)
```

output
```
int((x - (a + x^2)^(1/2))^n/(a + x^2)^(5/2), x)
```

3.507 $\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2 \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n$

3.507.1 Optimal result

Integrand size = 56, antiderivative size = 365

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2 \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$=\frac{(d^2-af^2)^5\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-5+n}}{32ef^4(5-n)}$$
$$-\frac{5(d^2-af^2)^4\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-3+n}}{32ef^4(3-n)}$$
$$+\frac{5(d^2-af^2)^3\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-1+n}}{16ef^4(1-n)}$$
$$+\frac{5(d^2-af^2)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{1+n}}{16ef^4(1+n)}$$
$$-\frac{5(d^2-af^2)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{3+n}}{32ef^4(3+n)}+\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{5+n}}{32ef^4(5+n)}$$

output
```
1/32*(-a*f^2+d^2)^5*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-5+n)/e/f
^4/(5-n)-5/32*(-a*f^2+d^2)^4*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(
-3+n)/e/f^4/(3-n)+5/16*(-a*f^2+d^2)^3*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)
^(1/2))^(-1+n)/e/f^4/(1-n)+5/16*(-a*f^2+d^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2
*x^2/f^2)^(1/2))^(1+n)/e/f^4/(1+n)-5/32*(-a*f^2+d^2)*(d+e*x+f*(a+2*d*e*x/f
^2+e^2*x^2/f^2)^(1/2))^(3+n)/e/f^4/(3+n)+1/32*(d+e*x+f*(a+2*d*e*x/f^2+e^2*
x^2/f^2)^(1/2))^(5+n)/e/f^4/(5+n)
```

3.507.2 Mathematica [A] (verified)

Time = 4.20 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.77

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

$$= \frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^{-5+n}\left(-\frac{(d^2-af^2)^5}{-5+n}+\frac{5(d^2-af^2)^4\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{-3+n}-\frac{10(d^2-af^2)^3\left(d+ex+f\sqrt{a+\frac{e}{}}\right.}{-1+n}\right.}{32e_{\cdot}}$$

input
```
Integrate[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^2*(d + e*x + f*Sqrt[a + (2*d
*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

output
```
((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(-5 + n)*(-((d^2 - a*f^2)^5
/(-5 + n)) + (5*(d^2 - a*f^2)^4*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^
2])^2)/(-3 + n) - (10*(d^2 - a*f^2)^3*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*
x))/f^2])^4)/(-1 + n) + (10*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (e*x*(2*
d + e*x))/f^2])^6)/(1 + n) - (5*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (e*x*(
2*d + e*x))/f^2])^8)/(3 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2
])^10/(5 + n)))/(32*e*f^4)
```

3.507.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2546, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2 \left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n dx$$

$\big\downarrow$ 2546

$$\frac{2\int -\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-6}\left(d^2-af^2-\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2\right)^5}{64e}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)}{f^4}$$

$\big\downarrow$ 27

$$-\frac{\int \left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-6}\left(d^2-af^2-\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2\right)^5 d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2d\dots}{f\dots}}\right.}{32ef^4}$$

$\big\downarrow$ 244

$$-\frac{\int \left(\left(d^2-af^2\right)^5\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-6}-5\left(d^2-af^2\right)^4\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-4}+10\left(d^2\dots\right.\right.}{}$$

$\big\downarrow$ 2009

$$-\frac{-\frac{\left(d^2-af^2\right)^5\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-5}}{5-n}+\frac{5\left(d^2-af^2\right)^4\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-3}}{3-n}-\frac{10\left(d^2-af^2\right)^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\dots\right.}{1-n}}{32e\dots}$$

input
```
Int[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/
f^2 + (e^2*x^2)/f^2])^n,x]
```

output
```
-1/32*(-(((d^2 - a*f^2)^5*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/
f^2])^(-5 + n))/(5 - n)) + (5*(d^2 - a*f^2)^4*(d + e*x + f*Sqrt[a + (2*d*e
*x)/f^2 + (e^2*x^2)/f^2])^(-3 + n))/(3 - n) - (10*(d^2 - a*f^2)^3*(d + e*x
 + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-1 + n))/(1 - n) - (10*(d^2
 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1 + n))
/(1 + n) + (5*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2
)/f^2])^(3 + n))/(3 + n) - (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)
/f^2])^(5 + n)/(5 + n))/(e*f^4)
```

3.507. $\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2 \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$

3.507.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2546
```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(2/f^(2*m)
)*(i/c)^m   Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e
*x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*S
qrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && Eq
Q[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m
] && (IntegerQ[m] || GtQ[i/c, 0])
```

3.507.4 Maple [F]

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

input
```
int((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/
2))^n,x)
```

output
```
int((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/
2))^n,x)
```

3.507.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 654, normalized size of antiderivative = 1.79

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx=$$

$$-\frac{\left(5\,a^2df^4n^4+225\,a^2df^4-300\,ad^3f^2+5\,(e^5n^4-10\,e^5n^2+9\,e^5)x^5+120\,d^5+25\,(de^4n^4-10\,de^4n^2+9\right.}{}$$

input
```
integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^
2)^(1/2))^n,x, algorithm="fricas")
```

output
```
-(5*a^2*d*f^4*n^4 + 225*a^2*d*f^4 - 300*a*d^3*f^2 + 5*(e^5*n^4 - 10*e^5*n^
2 + 9*e^5)*x^5 + 120*d^5 + 25*(d*e^4*n^4 - 10*d*e^4*n^2 + 9*d*e^4)*x^4 + 1
0*(15*a*e^3*f^2 + 30*d^2*e^3 + (a*e^3*f^2 + 4*d^2*e^3)*n^4 - 2*(8*a*e^3*f^
2 + 17*d^2*e^3)*n^2)*x^3 - 10*(11*a^2*d*f^4 - 6*a*d^3*f^2)*n^2 + 10*(45*a*
d*e^2*f^2 + (3*a*d*e^2*f^2 + 2*d^3*e^2)*n^4 - 2*(24*a*d*e^2*f^2 + d^3*e^2)
*n^2)*x^2 + 5*(45*a^2*e*f^4 + (a^2*e*f^4 + 4*a*d^2*e*f^2)*n^4 - 2*(11*a^2*
e*f^4 + 26*a*d^2*e*f^2 - 12*d^4*e)*n^2)*x - (a^2*f^5*n^5 + (e^4*f*n^5 - 10
*e^4*f*n^3 + 9*e^4*f*n)*x^4 - 10*(3*a^2*f^5 - 2*a*d^2*f^3)*n^3 + 4*(d*e^3*
f*n^5 - 10*d*e^3*f*n^3 + 9*d*e^3*f*n)*x^3 + 2*((a*e^2*f^3 + 2*d^2*e^2*f)*n
^5 - 10*(2*a*e^2*f^3 + d^2*e^2*f)*n^3 + (19*a*e^2*f^3 + 8*d^2*e^2*f)*n)*x^
2 + (149*a^2*f^5 - 260*a*d^2*f^3 + 120*d^4*f)*n + 4*(a*d*e*f^3*n^5 - 10*(2
*a*d*e*f^3 - d^3*e*f)*n^3 + (19*a*d*e*f^3 - 10*d^3*e*f)*n)*x)*sqrt((e^2*x^
2 + a*f^2 + 2*d*e*x)/f^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2)
+ d)^n/(e*f^4*n^6 - 35*e*f^4*n^4 + 259*e*f^4*n^2 - 225*e*f^4)
```

3.507.6 Sympy [F(-2)]

Exception generated.

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx$$
$$=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)**2*(d+e*x+f*(a+2*d*e*x/f**2+e**2
*x**2/f**2)**(1/2))**n,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.507.7 Maxima [F]

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$=\int \left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^2\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n dx$$

input
```
integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^
2)^(1/2))^n,x, algorithm="maxima")
```

output
```
integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^2*(e*x + sqrt(e^2*x^2/f^2 + a +
2*d*e*x/f^2)*f + d)^n, x)
```

3.507.8 Giac [F]

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$=\int \left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^2\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n dx$$

input
```
integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^
2)^(1/2))^n,x, algorithm="giac")
```

output
```
integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^2*(e*x + sqrt(e^2*x^2/f^2 + a +
2*d*e*x/f^2)*f + d)^n, x)
```

3.507.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$=\int \left(d+f\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n\left(a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}\right)^2 dx$$

input
```
int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^
2)/f^2 + (2*d*e*x)/f^2)^2,x)
```

output
```
int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^
2)/f^2 + (2*d*e*x)/f^2)^2, x)
```

3.508 $\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n$

3.508.1 Optimal result

Integrand size = 54, antiderivative size = 239

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$=\frac{(d^2-af^2)^3\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-3+n}}{8ef^2(3-n)}-\frac{3(d^2-af^2)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-1+n}}{8ef^2(1-n)}-\frac{3(d^2-af^2)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{1+n}}{8ef^2(1+n)}+\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{3+n}}{8ef^2(3+n)}$$

output
```
1/8*(-a*f^2+d^2)^3*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-3+n)/e/f^
2/(3-n)-3/8*(-a*f^2+d^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-1
+n)/e/f^2/(1-n)-3/8*(-a*f^2+d^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2
))^(1+n)/e/f^2/(1+n)+1/8*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(3+n)
/e/f^2/(3+n)
```

3.508.2 Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.78

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

$$=\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^{-3+n}\left(-\frac{(d^2-af^2)^3}{-3+n}+\frac{3(d^2-af^2)^2\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{-1+n}-\frac{3(d^2-af^2)\left(d+ex+f\sqrt{a+\frac{ex(}{}}\right.}{1+n}\right.}{8ef^2}$$

input
```
Integrate[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)*(d + e*x + f*Sqrt[a + (2*d*e
*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

output
```
((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(-3 + n)*(-((d^2 - a*f^2)^3
/(-3 + n)) + (3*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^
2])^2)/(-1 + n) - (3*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))
/f^2])^4)/(1 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^6/(3 + n
)))/(8*e*f^2)
```

3.508.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2546, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n dx$$

$$\Big\downarrow \text{2546}$$

$$\frac{2\int -\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-4}\left(d^2-af^2-\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2\right)^3}{16e}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)}{f^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-4}\left(d^2-af^2-\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2\right)^3 d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2de}{f}}\right.}{8ef^2}$$

$\big\downarrow$ 244

$$-\frac{\int \left(\left(d^2-af^2\right)^3\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-4}-3\left(d^2-af^2\right)^2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-2}+3(d^2-\right.}{8e}$$

$\big\downarrow$ 2009

$$-\frac{-\frac{\left(d^2-af^2\right)^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-3}}{3-n}+\frac{3\left(d^2-af^2\right)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{1-n}+\frac{3\left(d^2-af^2\right)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n}}{n+1}}{8ef^2}$$

input
```
Int[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^
2 + (e^2*x^2)/f^2])^n,x]
```

output
```
-1/8*(-(((d^2 - a*f^2)^3*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f
^2])^(-3 + n))/(3 - n)) + (3*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*
x)/f^2 + (e^2*x^2)/f^2])^(-1 + n))/(1 - n) + (3*(d^2 - a*f^2)*(d + e*x + f
*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1 + n))/(1 + n) - (d + e*x + f*
Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(3 + n)/(3 + n))/(e*f^2)
```

3.508.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.508. $\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$

rule 2546
```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(2/f^(2*m)
)*(i/c)^m   Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e
*x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*S
qrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && Eq
Q[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m
] && (IntegerQ[m] || GtQ[i/c, 0])
```

3.508.4 Maple [F]

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

input
```
int((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)
)^n,x)
```

output
```
int((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)
)^n,x)
```

3.508.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx =$$
$$-\frac{\left(3\,adf^2n^2-9\,adf^2+3\,(e^3n^2-e^3)x^3+6\,d^3+9\,(de^2n^2-de^2)x^2-3\,(3\,aef^2-(aef^2+2\,d^2e)n^2)x-(a\cdots\right.}{ef^2n^4\cdots}$$

input
```
integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)
^(1/2))^n,x, algorithm="fricas")
```

output
```
-(3*a*d*f^2*n^2 - 9*a*d*f^2 + 3*(e^3*n^2 - e^3)*x^3 + 6*d^3 + 9*(d*e^2*n^2
 - d*e^2)*x^2 - 3*(3*a*e*f^2 - (a*e*f^2 + 2*d^2*e)*n^2)*x - (a*f^3*n^3 + (
e^2*f*n^3 - e^2*f*n)*x^2 - (7*a*f^3 - 6*d^2*f)*n + 2*(d*e*f*n^3 - d*e*f*n)
*x)*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2
+ 2*d*e*x)/f^2) + d)^n/(e*f^2*n^4 - 10*e*f^2*n^2 + 9*e*f^2)
```

3.508.6 Sympy [F(-2)]

Exception generated.

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)*(d+e*x+f*(a+2*d*e*x/f**2+e**2*x*
*2/f**2)**(1/2))**n,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.508.7 Maxima [F]

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$= \int \left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n dx$$

input
```
integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)
^(1/2))^n,x, algorithm="maxima")
```

output
```
integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a + 2*
d*e*x/f^2)*f + d)^n, x)
```

3.508.8 Giac [F]

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$= \int \left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n dx$$

input
```
integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)
^(1/2))^n,x, algorithm="giac")
```

output
```
integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a + 2*
d*e*x/f^2)*f + d)^n, x)
```

3.508.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$=\int \left(d+f\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n\left(a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}\right)dx$$

input
```
int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^
2)/f^2 + (2*d*e*x)/f^2),x)
```

output
```
int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^
2)/f^2 + (2*d*e*x)/f^2), x)
```

3.509 $\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$

3.509.1 Optimal result

Integrand size = 33, antiderivative size = 107

$$\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx=\frac{(d^2-af^2)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-1+n}}{2e(1-n)}+\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)}$$

output
```
1/2*(-a*f^2+d^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-1+n)/e/(1-n
)+1/2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(1+n)/e/(1+n)
```

3.509.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.83

$$\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx=\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^{-1+n}\left(\frac{-d^2+af^2}{-1+n}+\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{1+n}\right)}{2e}$$

input
```
Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

output
```
((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(-1 + n)*((-d^2 + a*f^2)/(-
1 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2/(1 + n)))/(2*e)
```

3.509.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2541, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n dx$$

$$\Big\downarrow \text{2541}$$

$$2\int -\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-2}\left(d^2-af^2-\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2\right)}{4e}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\right.$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-2}\left(d^2-af^2-\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2\right)d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2de}{f^2}}\right.}{2e}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\left(d^2-af^2\right)\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-2}-\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^n\right)d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2}{}}\right.}{2e}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{\left(d^2-af^2\right)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{1-n}-\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{n+1}}{2e}$$

input
```
Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

output
```
-1/2*(-(((d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2
])^(-1 + n))/(1 - n)) - (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^
2])^(1 + n)/(1 + n))/e
```

3.509.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2541
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Simp[2   Subst[Int[(g + h*x^n)^p*((d
^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x
)^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e
, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.509.4 Maple [F]

$$\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

input
```
int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x)
```

output
```
int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x)
```

3.509.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.75

$$\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{\left(fn\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}-ex-d\right)\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n}{en^2-e}$$

input
```
integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="fric
as")
```

output
```
(f*n*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) - e*x - d)*(e*x + f*sqrt((e^2*x
^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n/(e*n^2 - e)
```

3.509.6 Sympy [F]

$$\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

input
```
integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n,x)
```

output
```
Integral((d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**n, x)
```

3.509.7 Maxima [F]

$$\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \int \left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n dx$$

input
```
integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="maxi
ma")
```

output
```
integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

3.509.8 Giac [F]

$$\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx=\int \left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n dx$$

input
```
integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="giac
")
```

output
```
integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

3.509.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx=\int \left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n dx$$

input
```
int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n,x)
```

output
```
int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n, x)
```

3.510 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx$

3.510.1 Optimal result

Integrand size = 56, antiderivative size = 122

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx = -\frac{2f^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{1+n}\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^2}{d^2-af^2}\right)}{e\left(d^2-af^2\right)(1+n)}$$

output
```
-2*f^2*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^
2/f^2)^(1/2))^2/(-a*f^2+d^2))*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^
(1+n)/e/(-a*f^2+d^2)/(1+n)
```

3.510.2 Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.92

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx = -\frac{2f^2\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^{1+n}\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{d^2-af^2}\right)}{e\,(d^2-af^2)\,(1+n)}$$

input
```
Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*
d*e*x)/f^2 + (e^2*x^2)/f^2),x]
```

output
```
(-2*f^2*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(1 + n)*Hypergeometr
ic2F1[1, (1 + n)/2, (3 + n)/2, (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2
])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)*(1 + n))
```

3.510.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2546, 25, 27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}dx$$

$$\Big\downarrow \; 2546$$

$$2f^2\int -\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^n}{e\left(d^2-af^2-\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2\right)}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)$$

$$\Big\downarrow \; 25$$

$$-2f^2\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^n}{e\left(d^2-af^2-\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2\right)}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)$$

$$\Big\downarrow \; 27$$

$$-\frac{2f^2\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^n}{d^2-af^2-\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)}{e}$$

$$\Big\downarrow \; 278$$

$$-\frac{2f^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+1)\left(d^2-af^2\right)}$$

input
```
Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d*e*x)
/f^2 + (e^2*x^2)/f^2),x]
```

output
```
(-2*f^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1 + n)*Hype
rgeometric2F1[1, (1 + n)/2, (3 + n)/2, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2
 + (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)*(1 + n))
```

3.510.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

3.510. $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx$

rule 2546
```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(2/f^(2*m)
)*(i/c)^m   Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e
*x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*S
qrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && Eq
Q[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m
] && (IntegerQ[m] || GtQ[i/c, 0])
```

3.510.4 Maple [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}dx$$

input
```
int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f
^2),x)
```

output
```
int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f
^2),x)
```

3.510.5 Fricas [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx = \int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,f+d\right)^n}{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,dx$$

input
```
integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2
*x^2/f^2),x, algorithm="fricas")
```

output
```
integral((e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^2/(e^2*x^
2 + a*f^2 + 2*d*e*x), x)
```

3.510.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/f
**2+e**2*x**2/f**2),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.510.7 Maxima [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx = \int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,dx$$

input
```
integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2
*x^2/f^2),x, algorithm="maxima")
```

output
```
integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2
 + a + 2*d*e*x/f^2), x)
```

3.510.8 Giac [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx = \int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,dx$$

input
```
integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2
*x^2/f^2),x, algorithm="giac")
```

output
```
integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2
 + a + 2*d*e*x/f^2), x)
```

3.510.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx=\int \frac{\left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n}{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}\,dx$$

input
```
int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^
2)/f^2 + (2*d*e*x)/f^2),x)
```

output
```
int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^
2)/f^2 + (2*d*e*x)/f^2), x)
```

3.511 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}\,dx$

3.511.1 Optimal result

Integrand size = 56, antiderivative size = 122

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}\,dx = -\frac{8f^4\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{3+n}\text{Hypergeometric2F1}\left(3,\frac{3+n}{2},\frac{5+n}{2},\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^2}{d^2-af^2}\right)}{e\left(d^2-af^2\right)^3(3+n)}$$

output
```
-8*f^4*hypergeom([3, 3/2+1/2*n],[5/2+1/2*n],(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^
2/f^2)^(1/2))^2/(-a*f^2+d^2))*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^
(3+n)/e/(-a*f^2+d^2)^3/(3+n)
```

3.511.2 Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.92

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}\,dx = -\frac{8f^4\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^{3+n}\text{Hypergeometric2F1}\left(3,\frac{3+n}{2},\frac{5+n}{2},\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{d^2-af^2}\right)}{e\left(d^2-af^2\right)^3(3+n)}$$

input
```
Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*
d*e*x)/f^2 + (e^2*x^2)/f^2)^2,x]
```

output
```
(-8*f^4*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(3 + n)*Hypergeometr
ic2F1[3, (3 + n)/2, (5 + n)/2, (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2
])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^3*(3 + n))
```

3.511.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2546, 27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}dx$$

$$\Big\downarrow \text{2546}$$

$$2f^4\int -\frac{4\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n+2}}{e\left(d^2-af^2-\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2\right)^3}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{8f^4\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n+2}}{\left(d^2-af^2-\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2\right)^3}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)}{e}$$

$$\Big\downarrow \text{ 278}$$

$$-\frac{8f^4\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+3}\text{Hypergeometric2F1}\left(3,\frac{n+3}{2},\frac{n+5}{2},\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+3)\left(d^2-af^2\right)^3}$$

input
```
Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d*e*x)
/f^2 + (e^2*x^2)/f^2)^2,x]
```

output
```
(-8*f^4*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(3 + n)*Hype
rgeometric2F1[3, (3 + n)/2, (5 + n)/2, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2
 + (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^3*(3 + n))
```

3.511.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2546
```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(2/f^(2*m)
)*(i/c)^m   Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e
*x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*S
qrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && Eq
Q[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m
] && (IntegerQ[m] || GtQ[i/c, 0])
```

3.511. $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}\,dx$

3.511.4 Maple [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}dx$$

input
```
int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f
^2)^2,x)
```

output
```
int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f
^2)^2,x)
```

3.511.5 Fricas [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}\,dx = \int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^2}\,dx$$

input
```
integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2
*x^2/f^2)^2,x, algorithm="fricas")
```

output
```
integral((e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^4/(e^4*x^
4 + 4*d*e^3*x^3 + a^2*f^4 + 4*a*d*e*f^2*x + 2*(a*e^2*f^2 + 2*d^2*e^2)*x^2)
, x)
```

3.511.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/f
**2+e**2*x**2/f**2)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.511.7 Maxima [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}\,dx=\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^2}\,dx$$

input
```
integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2
*x^2/f^2)^2,x, algorithm="maxima")
```

output
```
integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2
 + a + 2*d*e*x/f^2)^2, x)
```

3.511.8 Giac [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}\,dx=\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^2}\,dx$$

input
```
integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2
*x^2/f^2)^2,x, algorithm="giac")
```

output
```
integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2
 + a + 2*d*e*x/f^2)^2, x)
```

3.511.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}\,dx=\int \frac{\left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n}{\left(a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}\right)^2}\,dx$$

input
```
int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^
2)/f^2 + (2*d*e*x)/f^2)^2,x)
```

output
```
int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^
2)/f^2 + (2*d*e*x)/f^2)^2, x)
```

3.511. $$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}\,dx$$

3.512 $\int \left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n dx$

3.512.1 Optimal result

Integrand size = 33, antiderivative size = 107

$$\int \left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n dx=\frac{(d^2-af^2)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-1+n}}{2e(1-n)}+\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)}$$

output
```
1/2*(-a*f^2+d^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-1+n)/e/(1-n
)+1/2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(1+n)/e/(1+n)
```

3.512.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.83

$$\int \left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n dx=\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^{-1+n}\left(\frac{-d^2+af^2}{-1+n}+\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{1+n}\right)}{2e}$$

input
```
Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n,x]
```

output
```
((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(-1 + n)*((-d^2 + a*f^2)/(-
1 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2/(1 + n)))/(2*e)
```

3.512.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2543, 2541, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}+d+ex\right)^n dx$$

$$\Big\downarrow \text{2543}$$

$$\int \left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n dx$$

$$\Big\downarrow \text{2541}$$

$$2\int -\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-2}\left(d^2-af^2-\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2\right)}{4e}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\right.$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-2}\left(d^2-af^2-\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2\right)d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}\right.}{2e}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(\left(d^2-af^2\right)\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-2}-\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^n\right)d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+2\right.}{2e}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{1-n}-\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{n+1}}{2e}$$

input
```
Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n,x]
```

output
```
-1/2*(-(((d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2
])^(-1 + n))/(1 - n)) - (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^
2])^(1 + n)/(1 + n))/e
```

3.512.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2541
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Simp[2   Subst[Int[(g + h*x^n)^p*((d
^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x
)^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e
, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

rule 2543
```
Int[((g_.) + (h_.)*((u_) + (f_.)*Sqrt[v_])^(n_))^(p_.), x_Symbol] :> Int[(g
 + h*(ExpandToSum[u, x] + f*Sqrt[ExpandToSum[v, x]])^n)^p, x] /; FreeQ[{f,
g, h, n}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !(LinearMatchQ[u, x]
&& QuadraticMatchQ[v, x]) && EqQ[Coefficient[u, x, 1]^2 - Coefficient[v, x,
 2]*f^2, 0] && IntegerQ[p]
```

3.512. $\int \left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n dx$

3.512.4 Maple [F]

$$\int \left(d + ex + f\sqrt{\frac{a\,f^2 + ex\,(ex + 2d)}{f^2}}\right)^n dx$$

input
```
int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x)
```

output
```
int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x)
```

3.512.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.75

$$\int \left(d + ex + f\sqrt{\frac{af^2 + ex(2d + ex)}{f^2}}\right)^n dx = \frac{\left(fn\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}} - ex - d\right)\left(ex + f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}} + d\right)^n}{en^2 - e}$$

input
```
integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x, algorithm="fric
as")
```

output
```
(f*n*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) - e*x - d)*(e*x + f*sqrt((e^2*x
^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n/(e*n^2 - e)
```

3.512.6 Sympy [F(-1)]

Timed out.

$$\int \left(d + ex + f\sqrt{\frac{af^2 + ex(2d + ex)}{f^2}}\right)^n dx = \text{Timed out}$$

input
```
integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n,x)
```

output
```
Timed out
```

3.512.7 Maxima [F]

$$\int \left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n dx = \int \left(ex+f\left(\frac{\sqrt{af^2+(ex+2\,d)ex}}{f}\right)+d\right)^n dx$$

input
```
integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x, algorithm="maxi
ma")
```

output
```
integrate((e*x + d + sqrt(a*f^2 + (e*x + 2*d)*e*x))^n, x)
```

3.512.8 Giac [F]

$$\int \left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n dx = \int \left(ex+f\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}+d\right)^n dx$$

input
```
integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x, algorithm="giac
")
```

output
```
integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n, x)
```

3.512.9 Mupad [F(-1)]

Timed out.

$$\int \left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n dx = \int \left(d+e\,x+f\,\sqrt{\frac{a\,f^2+e\,x\,\left(2\,d+e\,x\right)}{f^2}}\right)^n dx$$

input
```
int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n,x)
```

output
```
int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n, x)
```

3.513 $\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx$

3.513.1 Optimal result

Integrand size = 56, antiderivative size = 122

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx = -\frac{2f^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{1+n}\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^2}{d^2-af^2}\right)}{e\left(d^2-af^2\right)(1+n)}$$

output
```
-2*f^2*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^
2/f^2)^(1/2))^2/(-a*f^2+d^2))*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^
(1+n)/e/(-a*f^2+d^2)/(1+n)
```

3.513.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.92

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx = -\frac{2f^2\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^{1+n}\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{d^2-af^2}\right)}{e\left(d^2-af^2\right)(1+n)}$$

input
```
Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/(a + (2*d*e*
x)/f^2 + (e^2*x^2)/f^2),x]
```

output
```
(-2*f^2*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(1 + n)*Hypergeometr
ic2F1[1, (1 + n)/2, (3 + n)/2, (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2
])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)*(1 + n))
```

3.513.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.089, Rules used = {2552, 2546, 25, 27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}+d+ex\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}dx$$

$$\Big\downarrow \ 2552$$

$$\int \frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}dx$$

$$\Big\downarrow \ 2546$$

$$2f^2\int -\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^n}{e\left(d^2-af^2-\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2\right)}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)$$

$$\Big\downarrow \text{ 25}$$

$$-2f^2\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^n}{e\left(d^2-af^2-\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2\right)}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{2f^2\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^n}{d^2-af^2-\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)}{e}$$

$$\Big\downarrow \text{ 278}$$

$$-\frac{2f^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+1)\left(d^2-af^2\right)}$$

input
```
Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/(a + (2*d*e*x)/f^2
 + (e^2*x^2)/f^2),x]
```

output
```
(-2*f^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1 + n)*Hype
rgeometric2F1[1, (1 + n)/2, (3 + n)/2, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2
 + (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)*(1 + n))
```

3.513.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.513. $\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}dx$

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2546
```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(2/f^(2*m)
)*(i/c)^m   Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e
*x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*S
qrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && Eq
Q[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m
] && (IntegerQ[m] || GtQ[i/c, 0])
```

rule 2552
```
Int[((u_) + (f_.)*((j_.) + (k_.)*Sqrt[v_]))^(n_.)*(w_)^(m_.), x_Symbol] :>
Int[ExpandToSum[w, x]^m*(ExpandToSum[u + f*j, x] + f*k*Sqrt[ExpandToSum[v,
x]])^n, x] /; FreeQ[{f, j, k, m, n}, x] && LinearQ[u, x] && QuadraticQ[{v,
w}, x] &&  !(LinearMatchQ[u, x] && QuadraticMatchQ[{v, w}, x] && (EqQ[j, 0]
 || EqQ[f, 1])) && EqQ[Coefficient[u, x, 1]^2 - Coefficient[v, x, 2]*f^2*k^
2, 0]
```

3.513.4 Maple [F]

$$\int \frac{\left(d+ex+f\sqrt{\frac{a\,f^2+ex(ex+2d)}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}dx$$

input
```
int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f
^2),x)
```

output
```
int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f
^2),x)
```

3.513.5 Fricas [F]

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx = \int \frac{\left(ex+f\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}+d\right)^n}{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,dx$$

input
```
integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2
*x^2/f^2),x, algorithm="fricas")
```

output
```
integral((e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^2/(e^2*x^
2 + a*f^2 + 2*d*e*x), x)
```

3.513.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx = \text{Timed out}$$

input
```
integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n/(a+2*d*e*x/f**
2+e**2*x**2/f**2),x)
```

output
```
Timed out
```

3.513.7 Maxima [F]

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx = \int \frac{\left(ex+d+\sqrt{af^2+(ex+2\,d)ex}\right)^n}{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,dx$$

input
```
integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2
*x^2/f^2),x, algorithm="maxima")
```

output
```
integrate((e*x + d + sqrt(a*f^2 + (e*x + 2*d)*e*x))^n/(e^2*x^2/f^2 + a + 2
*d*e*x/f^2), x)
```

3.513.8 Giac [F]

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx=\int \frac{\left(ex+f\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}+d\right)^n}{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,dx$$

input
```
integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2
*x^2/f^2),x, algorithm="giac")
```

output
```
integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/(e^2*x^2/f^2
 + a + 2*d*e*x/f^2), x)
```

3.513.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx=\int \frac{\left(d+e\,x+f\,\sqrt{\frac{a\,f^2+e\,x\,(2\,d+e\,x)}{f^2}}\right)^n}{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}\,dx$$

input
```
int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n/(a + (e^2*x^2)/f
^2 + (2*d*e*x)/f^2),x)
```

output
```
int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n/(a + (e^2*x^2)/f
^2 + (2*d*e*x)/f^2), x)
```

3.514 $\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$

3.514.1 Optimal result

Integrand size = 58, antiderivative size = 297

$$\begin{aligned}
&\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \\
&-\frac{(d^2-af^2)^4\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-4+n}}{16ef^3(4-n)} \\
&+\frac{(d^2-af^2)^3\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-2+n}}{4ef^3(2-n)} \\
&+\frac{3(d^2-af^2)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{n}}{8ef^3n} \\
&-\frac{(d^2-af^2)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{2+n}}{4ef^3(2+n)} \\
&+\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{4+n}}{16ef^3(4+n)}
\end{aligned}$$

output
```
-1/16*(-a*f^2+d^2)^4*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-4+n)/e/
f^3/(4-n)+1/4*(-a*f^2+d^2)^3*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(
-2+n)/e/f^3/(2-n)+3/8*(-a*f^2+d^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^
(1/2))^n/e/f^3/n-1/4*(-a*f^2+d^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/
2))^(2+n)/e/f^3/(2+n)+1/16*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(4+
n)/e/f^3/(4+n)
```

3.514.2 Mathematica [A] (verified)

Time = 6.54 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.77

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2} \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^n \left(\frac{6(d^2-af^2)^2}{n}+\frac{(d^2-af^2)^4}{(-4+n)\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^4}-\cdots\right.}{\cdots}$$

input
```
Integrate[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^(3/2)*(d + e*x + f*Sqrt[a +
(2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

output
```
((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n*((6*(d^2 - a*f^2)^2)/n +
(d^2 - a*f^2)^4/((-4 + n)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^4)
 - (4*(d^2 - a*f^2)^3)/((-2 + n)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f
^2])^2) - (4*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2
)/(2 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^4/(4 + n)))/(16*
e*f^3)
```

3.514.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2546, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n\,dx$$

$\downarrow$ 2546

$$\frac{2\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-5}\left(d^2-af^2-\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2\right)^4}{32e}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)}{f^3}$$

$\downarrow$ 27

$$\frac{\int \left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-5}\left(d^2-af^2-\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2\right)^4 d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}}\right.}{16ef^3}$$

$\downarrow$ 244

$$\int \left(\left(d^2-af^2\right)^4\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-5}-4\left(d^2-af^2\right)^3\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-3}+6\left(d^2-a\right.\right.$$

$\downarrow$ 2009

$$\frac{-\frac{\left(d^2-af^2\right)^4\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-4}}{4-n}+\frac{4\left(d^2-af^2\right)^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{2-n}+\frac{6\left(d^2-af^2\right)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n}}{n}}{16ef^3}$$

input
```
Int[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^(3/2)*(d + e*x + f*Sqrt[a + (2*d*e
*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

output
```
(-(((d^2 - a*f^2)^4*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^
(-4 + n))/(4 - n)) + (4*(d^2 - a*f^2)^3*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^
2 + (e^2*x^2)/f^2])^(-2 + n))/(2 - n) + (6*(d^2 - a*f^2)^2*(d + e*x + f*Sq
rt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/n - (4*(d^2 - a*f^2)*(d + e*x +
f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(2 + n))/(2 + n) + (d + e*x + f
*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(4 + n)/(4 + n))/(16*e*f^3)
```

3.514. $\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx$

3.514.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2546
```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(2/f^(2*m)
)*(i/c)^m   Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e
*x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*S
qrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && Eq
Q[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m
] && (IntegerQ[m] || GtQ[i/c, 0])
```

3.514.4 Maple [F]

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{\frac{3}{2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

input
```
int((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)
^(1/2))^n,x)
```

output
```
int((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)
^(1/2))^n,x)
```

3.514.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.27

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{\left(a^2f^4n^4+24\,a^2f^4-48\,ad^2f^2+(e^4n^4-4\,e^4n^2)x^4+24\,d^4+4\,(de^3n^4-4\,de\right.}{}$$

input
```
integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^
2/f^2)^(1/2))^n,x, algorithm="fricas")
```

output
```
(a^2*f^4*n^4 + 24*a^2*f^4 - 48*a*d^2*f^2 + (e^4*n^4 - 4*e^4*n^2)*x^4 + 24*
d^4 + 4*(d*e^3*n^4 - 4*d*e^3*n^2)*x^3 - 4*(4*a^2*f^4 - 3*a*d^2*f^2)*n^2 +
2*((a*e^2*f^2 + 2*d^2*e^2)*n^4 - 2*(5*a*e^2*f^2 + d^2*e^2)*n^2)*x^2 + 4*(a
*d*e*f^2*n^4 - 2*(5*a*d*e*f^2 - 3*d^3*e)*n^2)*x - 4*(a*d*f^3*n^3 + (e^3*f*
n^3 - 4*e^3*f*n)*x^3 + 3*(d*e^2*f*n^3 - 4*d*e^2*f*n)*x^2 - 2*(5*a*d*f^3 -
3*d^3*f)*n + ((a*e*f^3 + 2*d^2*e*f)*n^3 - 2*(5*a*e*f^3 + d^2*e*f)*n)*x)*sq
rt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*
e*x)/f^2) + d)^n/(e*f^3*n^5 - 20*e*f^3*n^3 + 64*e*f^3*n)
```

3.514.6 Sympy [F(-2)]

Exception generated.

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)**(3/2)*(d+e*x+f*(a+2*d*e*x/f**2+
e**2*x**2/f**2)**(1/2))**n,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.514.7 Maxima [F]

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx=\int \left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^{\frac{3}{2}}\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n dx$$

input
```
integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^
2/f^2)^(1/2))^n,x, algorithm="maxima")
```

output
```
integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^(3/2)*(e*x + sqrt(e^2*x^2/f^2 +
a + 2*d*e*x/f^2)*f + d)^n, x)
```

3.514.8 Giac [F]

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx=\int \left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^{\frac{3}{2}}\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n dx$$

input
```
integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^
2/f^2)^(1/2))^n,x, algorithm="giac")
```

output
```
integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^(3/2)*(e*x + sqrt(e^2*x^2/f^2 +
a + 2*d*e*x/f^2)*f + d)^n, x)
```

3.514.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx=\int \left(d+f\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n\left(a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}\right)^{3/2} dx$$

input
```
int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^
2)/f^2 + (2*d*e*x)/f^2)^(3/2),x)
```

output
```
int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^
2)/f^2 + (2*d*e*x)/f^2)^(3/2), x)
```

3.515 $\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n d$

3.515.1 Optimal result

Integrand size = 58, antiderivative size = 171

$$\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = -\frac{(d^2-af^2)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-2+n}}{4ef(2-n)} - \frac{(d^2-af^2)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{n}}{2efn} + \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{2+n}}{4ef(2+n)}$$

output
```
-1/4*(-a*f^2+d^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-2+n)/e/f
/(2-n)-1/2*(-a*f^2+d^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/e/f/
n+1/4*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(2+n)/e/f/(2+n)
```

3.515.2 Mathematica [A] (verified)

Time = 2.24 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.79

$$\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^n\left(\frac{2(-d^2+af^2)}{n}+\frac{(d^2-af^2)^2}{(-2+n)\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}+\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{2+n}\right)}{4ef}$$

input
```
Integrate[Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2
*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

output
```
((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n*((2*(-d^2 + a*f^2))/n + (
d^2 - a*f^2)^2/((-2 + n)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2)
+ (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2/(2 + n)))/(4*e*f)
```

3.515.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2546, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n dx$$

$$\Big\downarrow \text{2546}$$

$$\frac{2\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-3}\left(d^2-af^2-\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2\right)^2}{8e}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-3}\left(d^2-af^2-\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2\right)^2 d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}}\right.}{4ef}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(\left(d^2-af^2\right)^2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-3}-2\left(d^2-af^2\right)\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-1}+\left(d+ex\right.\right.}{4ef}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\left(d^2-af^2\right)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{2-n}-\frac{2\left(d^2-af^2\right)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{n}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}}{n+2}}{4ef}$$

input
```
Int[Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x
)/f^2 + (e^2*x^2)/f^2])^n,x]
```

output
```
(-(((d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^
(-2 + n))/(2 - n)) - (2*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2
+ (e^2*x^2)/f^2])^n)/n + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f
^2])^(2 + n)/(2 + n))/(4*e*f)
```

3.515.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2546
```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(2/f^(2*m)
)*(i/c)^m   Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e
*x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*S
qrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && Eq
Q[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m
] && (IntegerQ[m] || GtQ[i/c, 0])
```

3.515.4 Maple [F]

$$\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

input
```
int((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)
^(1/2))^n,x)
```

output
```
int((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)
^(1/2))^n,x)
```

3.515.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.71

$$\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$=\frac{\left(e^2n^2x^2+af^2n^2+2\,den^2x-2\,af^2+2\,d^2-2\,(efnx+dfn)\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}\right)\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right.}{efn^3-4\,efn}$$

input
```
integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^
2/f^2)^(1/2))^n,x, algorithm="fricas")
```

output
```
(e^2*n^2*x^2 + a*f^2*n^2 + 2*d*e*n^2*x - 2*a*f^2 + 2*d^2 - 2*(e*f*n*x + d*
f*n)*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2
 + 2*d*e*x)/f^2) + d)^n/(e*f*n^3 - 4*e*f*n)
```

3.515.6 Sympy [F(-2)]

Exception generated.

$$\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2)*(d+e*x+f*(a+2*d*e*x/f**2+
e**2*x**2/f**2)**(1/2))**n,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.515.7 Maxima [F]

$$\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$= \int \sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n dx$$

input
```
integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^
2/f^2)^(1/2))^n,x, algorithm="maxima")
```

output
```
integrate(sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a
+ 2*d*e*x/f^2)*f + d)^n, x)
```

3.515.8 Giac [F]

$$\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$= \int \sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n dx$$

input
```
integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^
2/f^2)^(1/2))^n,x, algorithm="giac")
```

output
```
integrate(sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a
+ 2*d*e*x/f^2)*f + d)^n, x)
```

3.515.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$=\int \left(d+f\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}\,dx$$

input
```
int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^
2)/f^2 + (2*d*e*x)/f^2)^(1/2),x)
```

output
```
int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^
2)/f^2 + (2*d*e*x)/f^2)^(1/2), x)
```

3.516 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx$

3.516.1 Optimal result

Integrand size = 58, antiderivative size = 41

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx=\frac{f\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en}$$

output
```
f*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/e/n
```

3.516.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx=\frac{f\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^n}{en}$$

input
```
Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a +
 (2*d*e*x)/f^2 + (e^2*x^2)/f^2],x]
```

output
```
(f*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n)/(e*n)
```

3.516.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2546, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}dx$$

$$\Big\downarrow \text{ 2546}$$

$$2f\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-1}}{2e}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)$$

$$\Big\downarrow \text{ 15}$$

$$\frac{f\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en}$$

input
```
Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a + (2*d*
e*x)/f^2 + (e^2*x^2)/f^2],x]
```

output
```
(f*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(e*n)
```

3.516.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2546
```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(2/f^(2*m)
)*(i/c)^m   Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e
*x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*S
qrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && Eq
Q[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m
] && (IntegerQ[m] || GtQ[i/c, 0])
```

3.516.4 Maple [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx$$

input
```
int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f
^2)^(1/2),x)
```

output
```
int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f
^2)^(1/2),x)
```

3.516.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx=\frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^nf}{en}$$

input
```
integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2
*x^2/f^2)^(1/2),x, algorithm="fricas")
```

output
```
(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f/(e*n)
```

3.516.6 Sympy [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx=\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx$$

input
```
integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/f
**2+e**2*x**2/f**2)**(1/2),x)
```

output
```
Integral((d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**n/sqrt(a +
 2*d*e*x/f**2 + e**2*x**2/f**2), x)
```

3.516.7 Maxima [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx = \int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}}\,dx$$

input
```
integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2
*x^2/f^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*x^2
/f^2 + a + 2*d*e*x/f^2), x)
```

3.516.8 Giac [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx = \int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}}\,dx$$

input
```
integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2
*x^2/f^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*x^2
/f^2 + a + 2*d*e*x/f^2), x)
```

3.516.9 Mupad [B] (verification not implemented)

Time = 17.34 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx = \frac{f\left(d+e\,x+f\,\sqrt{\frac{e^2\,x^2+2\,d\,e\,x+a\,f^2}{f^2}}\right)^n}{e\,n}$$

input
```
int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^
2)/f^2 + (2*d*e*x)/f^2)^(1/2),x)
```

output
```
(f*(d + e*x + f*((a*f^2 + e^2*x^2 + 2*d*e*x)/f^2)^(1/2))^n)/(e*n)
```

3.517 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx$

3.517.1 Optimal result

Integrand size = 58, antiderivative size = 122

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx=\frac{4f^3\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{2+n}\text{Hypergeometric2F1}\left(2,\frac{2+n}{2}\right.}{e\left(d^2-af^2\right)^2(2+n)}$$

output
```
4*f^3*hypergeom([2, 1+1/2*n],[2+1/2*n],(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2
)^(1/2))^2/(-a*f^2+d^2))*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(2+n)
/e/(-a*f^2+d^2)^2/(2+n)
```

3.517.2 Mathematica [A] (verified)

Time = 3.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.92

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx=\frac{4f^3\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^{2+n}\text{Hypergeometric2F1}\left(2,\frac{2+n}{2},\right.}{e\left(d^2-af^2\right)^2(2+n)}$$

input
```
Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*
d*e*x)/f^2 + (e^2*x^2)/f^2)^(3/2),x]
```

output
```
(4*f^3*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(2 + n)*Hypergeometri
c2F1[2, (2 + n)/2, (4 + n)/2, (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2]
)^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^2*(2 + n))
```

3.517.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052, Rules used = {2546, 27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2546}$$

$$2f^3\int \frac{2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n+1}}{e\left(d^2-af^2-\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2\right)^2}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{4f^3\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n+1}}{\left(d^2-af^2-\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2\right)^2}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)}{e}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{4f^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}\text{Hypergeometric2F1}\left(2,\frac{n+2}{2},\frac{n+4}{2},\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+2)\left(d^2-af^2\right)^2}$$

input
```
Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d*e*x)
/f^2 + (e^2*x^2)/f^2)^(3/2),x]
```

output
```
(4*f^3*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(2 + n)*Hyper
geometric2F1[2, (2 + n)/2, (4 + n)/2, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2
+ (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^2*(2 + n))
```

3.517.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2546
```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(2/f^(2*m)
)*(i/c)^m   Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e
*x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*S
qrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && Eq
Q[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m
] && (IntegerQ[m] || GtQ[i/c, 0])
```

3.517.4 Maple [F]

$$\int \frac{\left(d + ex + f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)^n}{\left(a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}\right)^{\frac{3}{2}}}dx$$

input
```
int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f
^2)^(3/2),x)
```

output
```
int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f
^2)^(3/2),x)
```

3.517. $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}} dx$

3.517.5 Fricas [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx = \int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2
*x^2/f^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^4*sqrt((e
^2*x^2 + a*f^2 + 2*d*e*x)/f^2)/(e^4*x^4 + 4*d*e^3*x^3 + a^2*f^4 + 4*a*d*e*
f^2*x + 2*(a*e^2*f^2 + 2*d^2*e^2)*x^2), x)
```

3.517.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/f
**2+e**2*x**2/f**2)**(3/2),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.517.7 Maxima [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx = \int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2
*x^2/f^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2
 + a + 2*d*e*x/f^2)^(3/2), x)
```

3.517.8 Giac [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx=\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\left(\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2
*x^2/f^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*x^2/f^2
 + a + 2*d*e*x/f^2)^(3/2), x)
```

3.517.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx=\int \frac{\left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n}{\left(a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}\right)^{3/2}}\,dx$$

input
```
int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^
2)/f^2 + (2*d*e*x)/f^2)^(3/2),x)
```

output
```
int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^
2)/f^2 + (2*d*e*x)/f^2)^(3/2), x)
```

3.518 $\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}}\,dx$

3.518.1 Optimal result

Integrand size = 58, antiderivative size = 41

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}}\,dx=\frac{f\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en}$$

output
```
f*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/e/n
```

3.518.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}}\,dx=\frac{f\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^n}{en}$$

input
```
Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2
+ e*x*(2*d + e*x))/f^2],x]
```

output
```
(f*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n)/(e*n)
```

3.518.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.052$, Rules used = {2552, 2546, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}+d+ex\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}}dx$$

$$\Big\downarrow \text{2552}$$

$$\int \frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}dx$$

$$\Big\downarrow \text{2546}$$

$$2f\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-1}}{2e}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)$$

$$\Big\downarrow \text{15}$$

$$\frac{f\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en}$$

input
```
Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2 + e*x*
(2*d + e*x))/f^2],x]
```

output
```
(f*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(e*n)
```

3.518.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2546
```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(2/f^(2*m)
)*(i/c)^m   Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e
*x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*S
qrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && Eq
Q[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m
] && (IntegerQ[m] || GtQ[i/c, 0])
```

rule 2552
```
Int[((u_) + (f_.)*((j_.) + (k_.)*Sqrt[v_]))^(n_.)*(w_)^(m_.), x_Symbol] :>
Int[ExpandToSum[w, x]^m*(ExpandToSum[u + f*j, x] + f*k*Sqrt[ExpandToSum[v,
x]])^n, x] /; FreeQ[{f, j, k, m, n}, x] && LinearQ[u, x] && QuadraticQ[{v,
w}, x] &&  !(LinearMatchQ[u, x] && QuadraticMatchQ[{v, w}, x] && (EqQ[j, 0]
 || EqQ[f, 1])) && EqQ[Coefficient[u, x, 1]^2 - Coefficient[v, x, 2]*f^2*k^
2, 0]
```

3.518.4 Maple [F]

$$\int \frac{\left(d+ex+f\sqrt{\frac{a\,f^2+ex(ex+2d)}{f^2}}\right)^n}{\sqrt{\frac{a\,f^2+ex(ex+2d)}{f^2}}}dx$$

input
```
int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2*d))/f
^2)^(1/2),x)
```

output
```
int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2*d))/f
^2)^(1/2),x)
```

3.518.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}}\,dx = \frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n f}{en}$$

input
```
integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2
*d))/f^2)^(1/2),x, algorithm="fricas")
```

output
```
(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f/(e*n)
```

3.518.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}}\,dx = \text{Timed out}$$

input
```
integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n/((a*f**2+e*x*(
e*x+2*d))/f**2)**(1/2),x)
```

output
```
Timed out
```

3.518.7 Maxima [F]

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}}\,dx = \int \frac{\left(ex+d+\sqrt{af^2+(ex+2\,d)ex}\right)^n}{\frac{\sqrt{af^2+(ex+2\,d)ex}}{f}}\,dx$$

input
```
integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2
*d))/f^2)^(1/2),x, algorithm="maxima")
```

output
```
f*integrate((e*x + d + sqrt(a*f^2 + (e*x + 2*d)*e*x))^n/sqrt(a*f^2 + (e*x
+ 2*d)*e*x), x)
```

3.518.8 Giac [F]

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}}\,dx = \int \frac{\left(ex+f\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}+d\right)^n}{\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}}\,dx$$

input
```
integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2
*d))/f^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/sqrt((a*f^2
+ (e*x + 2*d)*e*x)/f^2), x)
```

3.518.9 Mupad [B] (verification not implemented)

Time = 16.69 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}}\,dx = \frac{f\left(d+e\,x+f\,\sqrt{\frac{a\,f^2+e\,x\,(2\,d+e\,x)}{f^2}}\right)^n}{e\,n}$$

input
```
int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n/((a*f^2 + e*x*(2
*d + e*x))/f^2)^(1/2),x)
```

output
```
(f*(d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n)/(e*n)
```

3.519 $\int \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$

3.519.1 Optimal result

Integrand size = 62, antiderivative size = 327

$$\int \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$= -\frac{(d^2-af^2)^2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-2+n}}{4ef(2-n)\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$
$$-\frac{(d^2-af^2)\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{n}}{2efn\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$
$$+\frac{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{2+n}}{4ef(2+n)\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$

output
```
-1/4*(-a*f^2+d^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-2+n)*(a*
g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)/e/f/(2-n)/(a+2*d*e*x/f^2+e^2*x^2/f^2)
^(1/2)-1/2*(-a*f^2+d^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n*(a*g
+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)/e/f/n/(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2
)+1/4*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(2+n)*(a*g+2*d*e*g*x/f^2
+e^2*g*x^2/f^2)^(1/2)/e/f/(2+n)/(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)
```

3.519.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.54

$$\int \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx$$
$$=\frac{\sqrt{g\left(a+\frac{ex(2d+ex)}{f^2}\right)}\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^n\left(\frac{2(-d^2+af^2)}{n}+\frac{(d^2-af^2)^2}{(-2+n)\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}+\frac{\left(d+ex+f\sqrt{a\cdots}\right.}{2\cdots}\right.}{4ef\sqrt{a+\frac{ex(2d+ex)}{f^2}}}$$

input
```
Integrate[Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[
a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

output
```
(Sqrt[g*(a + (e*x*(2*d + e*x))/f^2)]*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x
))/f^2])^n*((2*(-d^2 + a*f^2))/n + (d^2 - a*f^2)^2/((-2 + n)*(d + e*x + f*
Sqrt[a + (e*x*(2*d + e*x))/f^2])^2) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*
x))/f^2])^2/(2 + n)))/(4*e*f*Sqrt[a + (e*x*(2*d + e*x))/f^2])
```

3.519.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.64, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {2548, 2546, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n\,dx$$
$$\Big\downarrow \text{2548}$$
$$\frac{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\int \sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^n\,dx}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$
$$\Big\downarrow \text{2546}$$

$$\frac{2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\int\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-3}\left(d^2-af^2-\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2\right)^2}{8e}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}\cdots}\right.}{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$

$\big\downarrow$ 27

$$\frac{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\int\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-3}\left(d^2-af^2-\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2\right)^2d\left(d+\cdots\right.}{4ef\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$

$\big\downarrow$ 244

$$\frac{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\int\left(\left(d^2-af^2\right)^2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-3}-2\left(d^2-af^2\right)\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}\cdots}\right.\right.}{4ef\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$

$\big\downarrow$ 2009

$$\frac{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(-\frac{\left(d^2-af^2\right)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{2-n}-\frac{2\left(d^2-af^2\right)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n}}{n}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\cdots}\right.}{n+2\cdots}\right.}{4ef\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$

input
```
Int[Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2
*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

output
```
(Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(-(((d^2 - a*f^2)^2*(d + e*
x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-2 + n))/(2 - n)) - (2*(d^
2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/n + (d
 + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(2 + n)/(2 + n)))/(4*e
*f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])
```

3.519. $\int\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$

3.519.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2546
```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(2/f^(2*m)
)*(i/c)^m   Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e
*x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*S
qrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && Eq
Q[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m
] && (IntegerQ[m] || GtQ[i/c, 0])
```

rule 2548
```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(i/c)^(m -
 1/2)*(Sqrt[g + h*x + i*x^2]/Sqrt[a + b*x + c*x^2])   Int[(a + b*x + c*x^2)
^m*(d + e*x + f*Sqrt[a + b*x + c*x^2])^n, x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h -
b*i, 0] && IGtQ[m + 1/2, 0] &&  !GtQ[i/c, 0]
```

3.519.4 Maple [F]

$$\int \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2g\,x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

input
```
int((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^
2/f^2)^(1/2))^n,x)
```

output
```
int((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^
2/f^2)^(1/2))^n,x)
```

3.519.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.71

$$\int \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx =$$
$$-\frac{\left(2\,e^3nx^3+6\,de^2nx^2+2\,adf^2n+2\,(aef^2+2\,d^2e)nx-(e^2fn^2x^2+af^3n^2+2\,defn^2x-2\,af^3+2\,d^2f)\right.}{aef^2n^3-4\,aef^2n+(e^3n^3-4\,e^3n)x^2+2\,(de^2n^3}$$

input
```
integrate((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+
e^2*x^2/f^2)^(1/2))^n,x, algorithm="fricas")
```

output
```
-(2*e^3*n*x^3 + 6*d*e^2*n*x^2 + 2*a*d*f^2*n + 2*(a*e*f^2 + 2*d^2*e)*n*x -
(e^2*f*n^2*x^2 + a*f^3*n^2 + 2*d*e*f*n^2*x - 2*a*f^3 + 2*d^2*f)*sqrt((e^2*
x^2 + a*f^2 + 2*d*e*x)/f^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2
) + d)^n*sqrt((e^2*g*x^2 + a*f^2*g + 2*d*e*g*x)/f^2)/(a*e*f^2*n^3 - 4*a*e*
f^2*n + (e^3*n^3 - 4*e^3*n)*x^2 + 2*(d*e^2*n^3 - 4*d*e^2*n)*x)
```

3.519.6 Sympy [F(-2)]

Exception generated.

$$\int \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((a*g+2*d*e*g*x/f**2+e**2*g*x**2/f**2)**(1/2)*(d+e*x+f*(a+2*d*e*x
/f**2+e**2*x**2/f**2)**(1/2))**n,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.519.7 Maxima [F]

$$\int \sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}} \left(d + ex + f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)^n dx$$
$$= \int \sqrt{\frac{e^2gx^2}{f^2} + ag + \frac{2\,degx}{f^2}} \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}f + d\right)^n dx$$

input
```
integrate((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+
e^2*x^2/f^2)^(1/2))^n,x, algorithm="maxima")
```

output
```
integrate(sqrt(e^2*g*x^2/f^2 + a*g + 2*d*e*g*x/f^2)*(e*x + sqrt(e^2*x^2/f^
2 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

3.519.8 Giac [F]

$$\int \sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}} \left(d + ex + f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)^n dx$$
$$= \int \sqrt{\frac{e^2gx^2}{f^2} + ag + \frac{2\,degx}{f^2}} \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2\,dex}{f^2}}f + d\right)^n dx$$

input
```
integrate((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+
e^2*x^2/f^2)^(1/2))^n,x, algorithm="giac")
```

output
```
integrate(sqrt(e^2*g*x^2/f^2 + a*g + 2*d*e*g*x/f^2)*(e*x + sqrt(e^2*x^2/f^
2 + a + 2*d*e*x/f^2)*f + d)^n, x)
```

3.519.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$
$$=\int \left(d+f\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n\sqrt{a\,g+\frac{e^2\,g\,x^2}{f^2}+\frac{2\,d\,e\,g\,x}{f^2}}\,dx$$

input
```
int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a*g + (e^2*
g*x^2)/f^2 + (2*d*e*g*x)/f^2)^(1/2),x)
```

output
```
int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a*g + (e^2*
g*x^2)/f^2 + (2*d*e*g*x)/f^2)^(1/2), x)
```

3.520 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx$

3.520.1 Optimal result

Integrand size = 62, antiderivative size = 93

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx=\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

output
```
f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(
1/2))^n/e/n/(a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)
```

3.520.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.82

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx=\frac{f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^n}{en\sqrt{g\left(a+\frac{ex(2d+ex)}{f^2}\right)}}$$

input
```
Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a*g
 + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2],x]
```

output
```
(f*Sqrt[a + (e*x*(2*d + e*x))/f^2]*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))
/f^2])^n)/(e*n*Sqrt[g*(a + (e*x*(2*d + e*x))/f^2)])
```

3.520.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2550, 2546, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}dx$$

$$\Big\downarrow \text{2550}$$

$$\frac{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^n}{\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}}dx}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$\Big\downarrow \text{2546}$$

$$\frac{2f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-1}}{2e}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$\Big\downarrow \text{15}$$

$$\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

input
```
Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a*g + (2*
d*e*g*x)/f^2 + (e^2*g*x^2)/f^2],x]
```

output
```
(f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)
/f^2 + (e^2*x^2)/f^2])^n)/(e*n*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^
2])
```

3.520. $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}dx$

3.520.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2546
```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(2/f^(2*m)
)*(i/c)^m   Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e
*x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*S
qrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && Eq
Q[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m
] && (IntegerQ[m] || GtQ[i/c, 0])
```

rule 2550
```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(i/c)^(m +
 1/2)*(Sqrt[a + b*x + c*x^2]/Sqrt[g + h*x + i*x^2])   Int[(a + b*x + c*x^2)
^m*(d + e*x + f*Sqrt[a + b*x + c*x^2])^n, x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h -
b*i, 0] && ILtQ[m - 1/2, 0] &&  !GtQ[i/c, 0]
```

3.520.4 Maple [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2g\,x^2}{f^2}}}dx$$

input
```
int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+e^2*g
*x^2/f^2)^(1/2),x)
```

output
```
int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+e^2*g
*x^2/f^2)^(1/2),x)
```

3.520.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.26

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx = \frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n f^3\sqrt{\frac{e^2gx^2+af^2g+2\,degx}{f^2}}\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}}{e^3gnx^2+aef^2gn+2\,de^2gnx}$$

input
```
integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2
+e^2*g*x^2/f^2)^(1/2),x, algorithm="fricas")
```

output
```
(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^3*sqrt((e^2*g*x^2
+ a*f^2*g + 2*d*e*g*x)/f^2)*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2)/(e^3*g*n
*x^2 + a*e*f^2*g*n + 2*d*e^2*g*n*x)
```

3.520.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a*g+2*d*e*g
*x/f**2+e**2*g*x**2/f**2)**(1/2),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.520.7 Maxima [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx=\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\sqrt{\frac{e^2gx^2}{f^2}+ag+\frac{2\,degx}{f^2}}}\,dx$$

input
```
integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2
+e^2*g*x^2/f^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*g*x
^2/f^2 + a*g + 2*d*e*g*x/f^2), x)
```

3.520.8 Giac [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx=\int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\sqrt{\frac{e^2gx^2}{f^2}+ag+\frac{2\,degx}{f^2}}}\,dx$$

input
```
integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2
+e^2*g*x^2/f^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*g*x
^2/f^2 + a*g + 2*d*e*g*x/f^2), x)
```

3.520.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx=\int \frac{\left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n}{\sqrt{a\,g+\frac{e^2\,g\,x^2}{f^2}+\frac{2\,d\,e\,g\,x}{f^2}}}\,dx$$

input
```
int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a*g + (e^2*
g*x^2)/f^2 + (2*d*e*g*x)/f^2)^(1/2),x)
```

output
```
int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a*g + (e^2*
g*x^2)/f^2 + (2*d*e*g*x)/f^2)^(1/2), x)
```

3.521 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{3/2}}\,dx$

3.521.1 Optimal result

Integrand size = 62, antiderivative size = 177

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{3/2}}\,dx = \frac{4f^3\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{2+n}\text{Hypergeom}}{e\left(d^2-af^2\right)^2 g(2+n)\sqrt{ag+}}$$

output
```
4*f^3*hypergeom([2, 1+1/2*n],[2+1/2*n],(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2
)^(1/2))^2/(-a*f^2+d^2))*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d
*e*x/f^2+e^2*x^2/f^2)^(1/2))^(2+n)/e/(-a*f^2+d^2)^2/g/(2+n)/(a*g+2*d*e*g*x
/f^2+e^2*g*x^2/f^2)^(1/2)
```

3.521.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.86

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{3/2}}\,dx = \frac{4f^3\left(a+\frac{ex(2d+ex)}{f^2}\right)^{3/2}\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^{2+n}\text{Hypergeom}}{e\left(d^2-af^2\right)^2(2+n)\left(g\left(a+\frac{ex}{}\right.\right.}$$

input
```
Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a*g + (
2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2)^(3/2),x]
```

output
```
(4*f^3*(a + (e*x*(2*d + e*x))/f^2)^(3/2)*(d + e*x + f*Sqrt[a + (e*x*(2*d +
 e*x))/f^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2, (d + e*x +
 f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^2*(
2 + n)*(g*(a + (e*x*(2*d + e*x))/f^2))^(3/2))
```

3.521.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2550, 2546, 27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2550}$$

$$\frac{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^n}{\left(\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a\right)^{3/2}}dx}{g\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$\Big\downarrow \text{2546}$$

$$\frac{2f^3\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\int \frac{2\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n+1}}{e\left(d^2-af^2-\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2\right)^2}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)}{g\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4f^3\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n+1}}{\left(d^2-af^2-\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2\right)^2}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)}{eg\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$\Big\downarrow 278$$

$$\frac{4f^3\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}\text{Hypergeometric2F1}\left(2,\frac{n+2}{2},\frac{n+4}{2},\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right.}{d^2-af^2}\right.}{eg(n+2)\left(d^2-af^2\right)^2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

input
```
Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a*g + (2*d*e*
g*x)/f^2 + (e^2*g*x^2)/f^2)^(3/2),x]
```

output
```
(4*f^3*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*
e*x)/f^2 + (e^2*x^2)/f^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, (4 + n)
/2, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^2/(d^2 - a*f^2)]
)/(e*(d^2 - a*f^2)^2*g*(2 + n)*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^
2])
```

3.521.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 2546
```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(2/f^(2*m)
)*(i/c)^m   Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e
*x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*S
qrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && Eq
Q[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m
] && (IntegerQ[m] || GtQ[i/c, 0])
```

3.521. $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{3/2}}\,dx$

rule 2550
```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(i/c)^(m +
 1/2)*(Sqrt[a + b*x + c*x^2]/Sqrt[g + h*x + i*x^2])   Int[(a + b*x + c*x^2)
^m*(d + e*x + f*Sqrt[a + b*x + c*x^2])^n, x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h -
b*i, 0] && ILtQ[m - 1/2, 0] &&  !GtQ[i/c, 0]
```

3.521.4 Maple [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2g\,x^2}{f^2}\right)^{\frac{3}{2}}}dx$$

input
```
int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+e^2*g
*x^2/f^2)^(3/2),x)
```

output
```
int((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+e^2*g
*x^2/f^2)^(3/2),x)
```

3.521.5 Fricas [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{3/2}}\,dx = \int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}f+d\right)^n}{\left(\frac{e^2gx^2}{f^2}+ag+\frac{2\,degx}{f^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2
+e^2*g*x^2/f^2)^(3/2),x, algorithm="fricas")
```

output
```
integral((e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^4*sqrt((e
^2*g*x^2 + a*f^2*g + 2*d*e*g*x)/f^2)/(e^4*g^2*x^4 + 4*d*e^3*g^2*x^3 + a^2*
f^4*g^2 + 4*a*d*e*f^2*g^2*x + 2*(a*e^2*f^2 + 2*d^2*e^2)*g^2*x^2), x)
```

3.521.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{3/2}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a*g+2*d*e*g
*x/f**2+e**2*g*x**2/f**2)**(3/2),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.521.7 Maxima [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{3/2}}\,dx = \int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,f+d\right)^n}{\left(\frac{e^2gx^2}{f^2}+ag+\frac{2\,degx}{f^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2
+e^2*g*x^2/f^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*g*x^2/f
^2 + a*g + 2*d*e*g*x/f^2)^(3/2), x)
```

3.521.8 Giac [F]

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{3/2}}\,dx = \int \frac{\left(ex+\sqrt{\frac{e^2x^2}{f^2}+a+\frac{2\,dex}{f^2}}\,f+d\right)^n}{\left(\frac{e^2gx^2}{f^2}+ag+\frac{2\,degx}{f^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2
+e^2*g*x^2/f^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/(e^2*g*x^2/f
^2 + a*g + 2*d*e*g*x/f^2)^(3/2), x)
```

3.521.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{3/2}}\,dx=\int \frac{\left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n}{\left(a\,g+\frac{e^2\,g\,x^2}{f^2}+\frac{2\,d\,e\,g\,x}{f^2}\right)^{3/2}}\,dx$$

input
```
int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a*g + (e^2*
g*x^2)/f^2 + (2*d*e*g*x)/f^2)^(3/2),x)
```

output
```
int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a*g + (e^2*
g*x^2)/f^2 + (2*d*e*g*x)/f^2)^(3/2), x)
```

3.522 $\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}\,dx$

3.522.1 Optimal result

Integrand size = 60, antiderivative size = 93

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}\,dx=\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

output
```
f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(
1/2))^n/e/n/(a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)
```

3.522.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.82

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}\,dx=\frac{f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^n}{en\sqrt{g\left(a+\frac{ex(2d+ex)}{f^2}\right)}}$$

input
```
Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2*
g + e*g*x*(2*d + e*x))/f^2],x]
```

output
```
(f*Sqrt[a + (e*x*(2*d + e*x))/f^2]*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))
/f^2])^n)/(e*n*Sqrt[g*(a + (e*x*(2*d + e*x))/f^2)])
```

3.522.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2552, 2550, 2546, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}+d+ex\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}dx$$

$$\Big\downarrow \text{ 2552}$$

$$\int \frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}dx$$

$$\Big\downarrow \text{ 2550}$$

$$\frac{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^n}{\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}}dx}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$\Big\downarrow \text{ 2546}$$

$$\frac{2f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\int \frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^{n-1}}{2e}d\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

input
```
Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2*g + e*
g*x*(2*d + e*x))/f^2],x]
```

3.522. $\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}dx$

output
```
(f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)
/f^2 + (e^2*x^2)/f^2])^n)/(e*n*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^
2])
```

3.522.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2546
```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(2/f^(2*m)
)*(i/c)^m   Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e
*x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*S
qrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && Eq
Q[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m
] && (IntegerQ[m] || GtQ[i/c, 0])
```

rule 2550
```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Simp[(i/c)^(m +
 1/2)*(Sqrt[a + b*x + c*x^2]/Sqrt[g + h*x + i*x^2])   Int[(a + b*x + c*x^2)
^m*(d + e*x + f*Sqrt[a + b*x + c*x^2])^n, x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h -
b*i, 0] && ILtQ[m - 1/2, 0] &&  !GtQ[i/c, 0]
```

rule 2552
```
Int[((u_) + (f_.)*((j_.) + (k_.)*Sqrt[v_]))^(n_.)*(w_)^(m_.), x_Symbol] :>
Int[ExpandToSum[w, x]^m*(ExpandToSum[u + f*j, x] + f*k*Sqrt[ExpandToSum[v,
x]])^n, x] /; FreeQ[{f, j, k, m, n}, x] && LinearQ[u, x] && QuadraticQ[{v,
w}, x] &&  !(LinearMatchQ[u, x] && QuadraticMatchQ[{v, w}, x] && (EqQ[j, 0]
 || EqQ[f, 1])) && EqQ[Coefficient[u, x, 1]^2 - Coefficient[v, x, 2]*f^2*k^
2, 0]
```

3.522. $\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}\,dx$

3.522.4 Maple [F]

$$\int \frac{\left(d+ex+f\sqrt{\frac{a\,f^2+ex(ex+2d)}{f^2}}\right)^n}{\sqrt{\frac{a\,f^2g+egx(ex+2d)}{f^2}}}dx$$

input
```
int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e*x+2*d
))/f^2)^(1/2),x)
```

output
```
int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e*x+2*d
))/f^2)^(1/2),x)
```

3.522.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.26

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}dx = \frac{\left(ex+f\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}+d\right)^n f^3\sqrt{\frac{e^2gx^2+af^2g+2\,degx}{f^2}}\sqrt{\frac{e^2x^2+af^2+2\,dex}{f^2}}}{e^3gnx^2+aef^2gn+2\,de^2gnx}$$

input
```
integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e
*x+2*d))/f^2)^(1/2),x, algorithm="fricas")
```

output
```
(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^3*sqrt((e^2*g*x^2
+ a*f^2*g + 2*d*e*g*x)/f^2)*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2)/(e^3*g*n
*x^2 + a*e*f^2*g*n + 2*d*e^2*g*n*x)
```

3.522.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}dx = \text{Timed out}$$

input
```
integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n/((a*f**2*g+e*g
*x*(e*x+2*d))/f**2)**(1/2),x)
```

output
```
Timed out
```

3.522.7 Maxima [F]

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}\,dx = \int \frac{\left(ex+d+\sqrt{af^2+(ex+2\,d)ex}\right)^n}{\frac{\sqrt{af^2g+(ex+2\,d)egx}}{f}}\,dx$$

input
```
integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e
*x+2*d))/f^2)^(1/2),x, algorithm="maxima")
```

output
```
f*integrate((e*x + d + sqrt(a*f^2 + (e*x + 2*d)*e*x))^n/sqrt(a*f^2*g + (e*
x + 2*d)*e*g*x), x)
```

3.522.8 Giac [F]

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}\,dx = \int \frac{\left(ex+f\sqrt{\frac{af^2+(ex+2\,d)ex}{f^2}}+d\right)^n}{\sqrt{\frac{af^2g+(ex+2\,d)egx}{f^2}}}\,dx$$

input
```
integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e
*x+2*d))/f^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/sqrt((a*f^2*
g + (e*x + 2*d)*e*g*x)/f^2), x)
```

3.522.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}\,dx=\int \frac{\left(d+e\,x+f\,\sqrt{\frac{a\,f^2+e\,x\,(2\,d+e\,x)}{f^2}}\right)^n}{\sqrt{\frac{a\,g\,f^2+e\,g\,x\,(2\,d+e\,x)}{f^2}}}\,dx$$

input
```
int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n/((a*f^2*g + e*g*
x*(2*d + e*x))/f^2)^(1/2),x)
```

output
```
int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n/((a*f^2*g + e*g*
x*(2*d + e*x))/f^2)^(1/2), x)
```

3.523 $\int \frac{1}{(a+bx)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

3.523.1 Optimal result

Integrand size = 30, antiderivative size = 191

$$\int \frac{1}{(a+bx)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = -\frac{b\operatorname{arctanh}\left(\frac{\sqrt{b^2e+a^2f}\sqrt{c+dx^2}}{\sqrt{b^2c+a^2d}\sqrt{e+fx^2}}\right)}{\sqrt{b^2c+a^2d}\sqrt{b^2e+a^2f}} + \frac{\sqrt{-c}\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\text{EllipticPi}\left(-\frac{b^2c}{a^2d},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right),\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

output
```
-b*arctanh((a^2*f+b^2*e)^(1/2)*(d*x^2+c)^(1/2)/(a^2*d+b^2*c)^(1/2)/(f*x^2+
e)^(1/2))/(a^2*d+b^2*c)^(1/2)/(a^2*f+b^2*e)^(1/2)+EllipticPi(x*d^(1/2)/(-c
)^(1/2),-b^2*c/a^2/d,(c*f/d/e)^(1/2))*(-c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^
2/e)^(1/2)/a/d^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

3.523.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.19 (sec) , antiderivative size = 772, normalized size of antiderivative = 4.04

$$\int \frac{1}{(a+bx)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{2\sqrt{d}\left(\sqrt{c}+i\sqrt{d}x\right)\sqrt{\frac{\left(\sqrt{d}\sqrt{e}-\sqrt{c}\sqrt{f}\right)\left(i\sqrt{c}+\sqrt{d}x\right)}{\left(\sqrt{d}\sqrt{e}+\sqrt{c}\sqrt{f}\right)\left(-i\sqrt{c}+\sqrt{d}x\right)}}\left(\sqrt{e}+i\sqrt{f}x\right)\sqrt{\frac{\sqrt{c}\sqrt{d}\left(i\sqrt{e}+\sqrt{f}x\right)}{\left(-\sqrt{d}\sqrt{e}+\sqrt{c}\sqrt{f}\right)\left(-i\sqrt{c}+\sqrt{d}x\right)}}\left(\left(b\sqrt{c}+ia\sqrt{d}\right)\mathrm{E}\right.}{\left(b\sqrt{c}-ia\sqrt{d}\right)\left(b\right.}$$

input
```
Integrate[1/((a + b*x)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

output
```
(2*Sqrt[d]*(Sqrt[c] + I*Sqrt[d]*x)*Sqrt[((Sqrt[d]*Sqrt[e] - Sqrt[c]*Sqrt[f
])*(I*Sqrt[c] + Sqrt[d]*x))/((Sqrt[d]*Sqrt[e] + Sqrt[c]*Sqrt[f])*((-I)*Sqr
t[c] + Sqrt[d]*x))]*(Sqrt[e] + I*Sqrt[f]*x)*Sqrt[(Sqrt[c]*Sqrt[d]*(I*Sqrt[
e] + Sqrt[f]*x))/((-(Sqrt[d]*Sqrt[e]) + Sqrt[c]*Sqrt[f])*((-I)*Sqrt[c] + S
qrt[d]*x))]*((b*Sqrt[c] + I*a*Sqrt[d])*EllipticF[ArcSin[Sqrt[((Sqrt[d]*Sqr
t[e] - Sqrt[c]*Sqrt[f])*(I*Sqrt[c] + Sqrt[d]*x))/((Sqrt[d]*Sqrt[e] + Sqrt[
c]*Sqrt[f])*((-I)*Sqrt[c] + Sqrt[d]*x))]], (Sqrt[d]*Sqrt[e] + Sqrt[c]*Sqrt
[f])^2/(Sqrt[d]*Sqrt[e] - Sqrt[c]*Sqrt[f])^2] - 2*b*Sqrt[c]*EllipticPi[((b
*Sqrt[c] - I*a*Sqrt[d])*(Sqrt[d]*Sqrt[e] + Sqrt[c]*Sqrt[f]))/((b*Sqrt[c] +
 I*a*Sqrt[d])*(-(Sqrt[d]*Sqrt[e]) + Sqrt[c]*Sqrt[f])), ArcSin[Sqrt[((Sqrt[
d]*Sqrt[e] - Sqrt[c]*Sqrt[f])*(I*Sqrt[c] + Sqrt[d]*x))/((Sqrt[d]*Sqrt[e] +
 Sqrt[c]*Sqrt[f])*((-I)*Sqrt[c] + Sqrt[d]*x))]], (Sqrt[d]*Sqrt[e] + Sqrt[c
]*Sqrt[f])^2/(Sqrt[d]*Sqrt[e] - Sqrt[c]*Sqrt[f])^2]))/((b*Sqrt[c] - I*a*Sq
rt[d])*(b*Sqrt[c] + I*a*Sqrt[d])*(-(Sqrt[d]*Sqrt[e]) + Sqrt[c]*Sqrt[f])*Sq
rt[(Sqrt[c]*Sqrt[d]*(Sqrt[e] + I*Sqrt[f]*x))/((Sqrt[d]*Sqrt[e] + Sqrt[c]*S
qrt[f])*(Sqrt[c] + I*Sqrt[d]*x))]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

3.523.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {2538, 413, 413, 412, 435, 104, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

$$\Big\downarrow \text{2538}$$

$$a\int \frac{1}{(a^2-b^2x^2)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx-b\int \frac{x}{(a^2-b^2x^2)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx$$

$$\Big\downarrow \text{413}$$

$$\frac{a\sqrt{\frac{dx^2}{c}+1}\int \frac{1}{(a^2-b^2x^2)\sqrt{\frac{dx^2}{c}+1}\sqrt{fx^2+e}}dx}{\sqrt{c+dx^2}}-b\int \frac{x}{(a^2-b^2x^2)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx$$

$$\Big\downarrow \text{413}$$

$$\frac{a\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\int \frac{1}{(a^2-b^2x^2)\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}}dx}{\sqrt{c+dx^2}\sqrt{e+fx^2}}-b\int \frac{x}{(a^2-b^2x^2)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx$$

$$\Big\downarrow \text{412}$$

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\,\text{EllipticPi}\left(-\frac{b^2c}{a^2d},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right),\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}-b\int \frac{x}{(a^2-b^2x^2)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx$$

$$\Big\downarrow \text{435}$$

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\,\text{EllipticPi}\left(-\frac{b^2c}{a^2d},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right),\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}-\frac{1}{2}b\int \frac{1}{(a^2-b^2x^2)\sqrt{dx^2+c}\sqrt{fx^2+e}}dx^2$$

$$\Big\downarrow \text{104}$$

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\,\text{EllipticPi}\left(-\frac{b^2c}{a^2d},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right),\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}-b\int \frac{1}{-((fa^2+b^2e)x^4)+b^2c+a^2d}d\frac{\sqrt{dx^2+c}}{\sqrt{fx^2+e}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\,\text{EllipticPi}\left(-\frac{b^2c}{a^2d},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right),\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}\sqrt{a^2f+b^2e}}{\sqrt{e+fx^2}\sqrt{a^2d+b^2c}}\right)}{\sqrt{a^2d+b^2c}\sqrt{a^2f+b^2e}}$$

input
```
Int[1/((a + b*x)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

3.523. $\int \frac{1}{(a+bx)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

output
```
-((b*ArcTanh[(Sqrt[b^2*e + a^2*f]*Sqrt[c + d*x^2])/(Sqrt[b^2*c + a^2*d]*Sq
rt[e + f*x^2])])/(Sqrt[b^2*c + a^2*d]*Sqrt[b^2*e + a^2*f])) + (Sqrt[-c]*Sq
rt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[-((b^2*c)/(a^2*d)), ArcSi
n[(Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(a*Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2])
```

3.523.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 2538
```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[a   Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] - Simp[b   Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

3.523.4 Maple [A] (verified)

Time = 4.54 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.42

method	result
elliptic	$\frac{\sqrt{(dx^2+c)(fx^2+e)}\left(-\frac{\operatorname{arctanh}\left(\frac{\frac{(cf+de)a^2}{b^2}+2ce+(cf+de)x^2+\frac{2dfx^2a^2}{b^2}}{2\sqrt{\frac{dfa^4}{b^4}+\frac{(cf+de)a^2}{b^2}+ce}\sqrt{dfx^4+cfx^2+edx^2+ce}}\right)}{2\sqrt{\frac{dfa^4}{b^4}+\frac{(cf+de)a^2}{b^2}+ce}}+\frac{b\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(x\sqrt{-\frac{d}{c}},-\frac{b^2c}{a^2d},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)}{\sqrt{-\frac{d}{c}}\,a\sqrt{dfx^4+cfx^2+edx^2+ce}}\right)}{\sqrt{dx^2+c}\sqrt{fx^2+e}\,b}$
default	$\frac{\left(2b\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{fx^2+e}{e}}\,\Pi\left(x\sqrt{-\frac{d}{c}},-\frac{b^2c}{a^2d},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)\sqrt{\frac{a^4df+a^2b^2cf+a^2b^2de+b^4ec}{b^4}}-\operatorname{arctanh}\left(\frac{2a^2dfx^2+b^2cfx^2+b^2dex^2+a^2cf+a^2de+}{2b^2\sqrt{\frac{a^4df+a^2b^2cf+a^2b^2de+b^4ec}{b^4}}\sqrt{(dx^2+c)}}\right.\right.}{2ba\sqrt{\frac{a^4df+a^2b^2cf+a^2b^2de+b^4ec}{b^4}}\sqrt{-\frac{d}{c}}\,(dfx^4+cfx^2+edx^2+ce)}$

input
```
int(1/(b*x+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)/b*(-1/2/(d*f*a
^4/b^4+(c*f+d*e)*a^2/b^2+c*e)^(1/2)*arctanh(1/2*((c*f+d*e)*a^2/b^2+2*c*e+(
c*f+d*e)*x^2+2*d*f*x^2*a^2/b^2)/(d*f*a^4/b^4+(c*f+d*e)*a^2/b^2+c*e)^(1/2)/
(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2))+1/(-d/c)^(1/2)/a*b*(1+1/c*d*x^2)^(1/2
)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticPi(x*(-d/c
)^(1/2),-b^2*c/a^2/d,(-f/e)^(1/2)/(-d/c)^(1/2)))
```

3.523.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.523.6 Sympy [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(a+bx)\,\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

output
```
Integral(1/((a + b*x)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

3.523.7 Maxima [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}(bx+a)}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*(b*x + a)), x)
```

3.523.8 Giac [F]

$$\int \frac{1}{(a+bx)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{dx^2+c}\sqrt{fx^2+e}(bx+a)}\,dx$$

input
```
integrate(1/(b*x+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*(b*x + a)), x)
```

3.523.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}\,\left(a+b\,x\right)}\,dx$$

input
```
int(1/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)*(a + b*x)),x)
```

output
```
int(1/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)*(a + b*x)), x)
```

3.524 $\int \frac{e-2fx^2}{e^2+4dfx^2+4efx^2+4f^2x^4}\,dx$

3.524.1 Optimal result

Integrand size = 37, antiderivative size = 81

$$\int \frac{e-2fx^2}{e^2+4dfx^2+4efx^2+4f^2x^4}\,dx=-\frac{\log\left(e-2\sqrt{-d}\sqrt{f}x+2fx^2\right)}{4\sqrt{-d}\sqrt{f}}+\frac{\log\left(e+2\sqrt{-d}\sqrt{f}x+2fx^2\right)}{4\sqrt{-d}\sqrt{f}}$$

output
```
-1/4*ln(e+2*f*x^2-2*x*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)+1/4*ln(e+2*f*
x^2+2*x*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)
```

3.524.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 191 vs. 2(81) = 162.

Time = 0.09 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.36

$$\int \frac{e-2fx^2}{e^2+4dfx^2+4efx^2+4f^2x^4}\,dx=\frac{-\frac{\left(-d-2e+\sqrt{d}\sqrt{d+2e}\right)\arctan\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{d+e-\sqrt{d}\sqrt{d+2e}}}\right)}{\sqrt{d+e-\sqrt{d}\sqrt{d+2e}}}-\frac{\left(d+2e+\sqrt{d}\sqrt{d+2e}\right)\arctan\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{d+e+\sqrt{d}\sqrt{d+2e}}}\right)}{\sqrt{d+e+\sqrt{d}\sqrt{d+2e}}}}{2\sqrt{2}\sqrt{d}\sqrt{d+2e}\sqrt{f}}$$

input
```
Integrate[(e - 2*f*x^2)/(e^2 + 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),x]
```

output
```
(-(((-d - 2*e + Sqrt[d]*Sqrt[d + 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[d +
 e - Sqrt[d]*Sqrt[d + 2*e]]])/Sqrt[d + e - Sqrt[d]*Sqrt[d + 2*e]]) - ((d +
 2*e + Sqrt[d]*Sqrt[d + 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[d + e + Sqrt
[d]*Sqrt[d + 2*e]]])/Sqrt[d + e + Sqrt[d]*Sqrt[d + 2*e]])/(2*Sqrt[2]*Sqrt[
d]*Sqrt[d + 2*e]*Sqrt[f])
```

3.524.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6, 1478, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e-2fx^2}{4dfx^2+e^2+4efx^2+4f^2x^4}dx$$

$$\Big\downarrow \text{ 6}$$

$$\int \frac{e-2fx^2}{4fx^2(d+e)+e^2+4f^2x^4}dx$$

$$\Big\downarrow \text{ 1478}$$

$$-\frac{\int -\frac{2(\sqrt{-d}-2\sqrt{f}x)}{\sqrt{f}\left(2x^2-\frac{2\sqrt{-d}x}{\sqrt{f}}+\frac{e}{f}\right)}dx}{4\sqrt{-d}\sqrt{f}}-\frac{\int -\frac{2(2\sqrt{f}x+\sqrt{-d})}{\sqrt{f}\left(2x^2+\frac{2\sqrt{-d}x}{\sqrt{f}}+\frac{e}{f}\right)}dx}{4\sqrt{-d}\sqrt{f}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sqrt{-d}-2\sqrt{f}x}{2x^2-\frac{2\sqrt{-d}x}{\sqrt{f}}+\frac{e}{f}}dx}{2\sqrt{-d}f}+\frac{\int \frac{2\sqrt{f}x+\sqrt{-d}}{2x^2+\frac{2\sqrt{-d}x}{\sqrt{f}}+\frac{e}{f}}dx}{2\sqrt{-d}f}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{\log\left(2\sqrt{-d}\sqrt{f}x+e+2fx^2\right)}{4\sqrt{-d}\sqrt{f}}-\frac{\log\left(-2\sqrt{-d}\sqrt{f}x+e+2fx^2\right)}{4\sqrt{-d}\sqrt{f}}$$

input
```
Int[(e - 2*f*x^2)/(e^2 + 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),x]
```

output
```
-1/4*Log[e - 2*Sqrt[-d]*Sqrt[f]*x + 2*f*x^2]/(Sqrt[-d]*Sqrt[f]) + Log[e +
2*Sqrt[-d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[-d]*Sqrt[f])
```

3.524.3.1 Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[Fx, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] && !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

3.524.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{\ln\left(-2\sqrt{-df}\, f\, x^{2}-2dfx-\sqrt{-df}\, e\right)}{4\sqrt{-df}}+\frac{\ln\left(-2\sqrt{-df}\, f\, x^{2}+2dfx-\sqrt{-df}\, e\right)}{4\sqrt{-df}}$
default	$f^{2}\left(-\frac{\left(df+2ef-\sqrt{d\, f^{2}(d+2e)}\right)\sqrt{2}\, \operatorname{arctanh}\left(\frac{fx\sqrt{2}}{\sqrt{-df-ef+\sqrt{d\, f^{2}(d+2e)}}}\right)}{4f^{2}\sqrt{d\, f^{2}(d+2e)}\, \sqrt{-df-ef+\sqrt{d\, f^{2}(d+2e)}}}+\frac{\left(-df-2ef-\sqrt{d\, f^{2}(d+2e)}\right)\sqrt{2}\, \arctan\left(\frac{fx\sqrt{2}}{\sqrt{df+ef+\sqrt{d\, f^{2}(d+2e)}}}\right)}{4f^{2}\sqrt{d\, f^{2}(d+2e)}\, \sqrt{df+ef+\sqrt{d\, f^{2}(d+2e)}}}\right.$

input
```
int((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2),x,method=_RETURNVERBO
SE)
```

output
```
-1/4/(-d*f)^(1/2)*ln(-2*(-d*f)^(1/2)*f*x^2-2*d*f*x-(-d*f)^(1/2)*e)+1/4/(-d
*f)^(1/2)*ln(-2*(-d*f)^(1/2)*f*x^2+2*d*f*x-(-d*f)^(1/2)*e)
```

3.524.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.74

$$\int \frac{e-2fx^2}{e^2+4dfx^2+4efx^2+4f^2x^4}\,dx = \left[-\frac{\sqrt{-df}\log\left(\frac{4\,f^2x^4-4\,(d-e)fx^2+e^2+4\,(2\,fx^3+ex)\sqrt{-df}}{4\,f^2x^4+4\,(d+e)fx^2+e^2}\right)}{4\,df}, -\frac{\sqrt{df}\arctan\left(\frac{\sqrt{df}x}{d}\right)-\sqrt{df}\arctan\left(\frac{(2\,fx^3+(2\,d+e)x)\sqrt{df}}{de}\right)}{2\,df}\right]$$

input
```
integrate((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="f
ricas")
```

output
```
[-1/4*sqrt(-d*f)*log((4*f^2*x^4 - 4*(d - e)*f*x^2 + e^2 + 4*(2*f*x^3 + e*x
)*sqrt(-d*f))/(4*f^2*x^4 + 4*(d + e)*f*x^2 + e^2))/(d*f), -1/2*(sqrt(d*f)*
arctan(sqrt(d*f)*x/d) - sqrt(d*f)*arctan((2*f*x^3 + (2*d + e)*x)*sqrt(d*f)
/(d*e)))/(d*f)]
```

3.524.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{e-2fx^2}{e^2+4dfx^2+4efx^2+4f^2x^4}\,dx = \frac{\sqrt{-\frac{1}{df}}\log\left(-dx\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^2\right)}{4} - \frac{\sqrt{-\frac{1}{df}}\log\left(dx\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^2\right)}{4}$$

input
```
integrate((-2*f*x**2+e)/(4*f**2*x**4+4*d*f*x**2+4*e*f*x**2+e**2),x)
```

output
```
sqrt(-1/(d*f))*log(-d*x*sqrt(-1/(d*f)) + e/(2*f) + x**2)/4 - sqrt(-1/(d*f)
)*log(d*x*sqrt(-1/(d*f)) + e/(2*f) + x**2)/4
```

3.524.7 Maxima [F]

$$\int \frac{e-2fx^2}{e^2+4dfx^2+4efx^2+4f^2x^4}\,dx = \int -\frac{2\,fx^2-e}{4\,f^2x^4+4\,dfx^2+4\,efx^2+e^2}\,dx$$

input
```
integrate((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="m
axima")
```

output
```
-integrate((2*f*x^2 - e)/(4*f^2*x^4 + 4*d*f*x^2 + 4*e*f*x^2 + e^2), x)
```

3.524.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(61) = 122$.

Time = 0.40 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.28

$$\int \frac{e-2fx^2}{e^2+4dfx^2+4efx^2+4f^2x^4}\,dx$$
$$= \frac{\sqrt{2}\left(def|f|-\sqrt{d^2+2\,de}(d+e)f^2+\sqrt{d^2+2\,de}df^2\right)\arctan\left(\frac{4\sqrt{\frac{1}{2}}x}{\sqrt{\frac{4\,df+4\,ef+\sqrt{-16\,e^2f^2+16\,(df+ef)^2}}{f^2}}}\right)}{4\left(d^2+de-\sqrt{d^2+2\,de}d\right)\sqrt{\left(d+e+\sqrt{d^2+2\,de}\right)f}f^2}$$
$$+\frac{\sqrt{2}\left(def|f|+\sqrt{d^2+2\,de}(d+e)f^2-\sqrt{d^2+2\,de}df^2\right)\arctan\left(\frac{4\sqrt{\frac{1}{2}}x}{\sqrt{\frac{4\,df+4\,ef-\sqrt{-16\,e^2f^2+16\,(df+ef)^2}}{f^2}}}\right)}{4\left(d^2+de+\sqrt{d^2+2\,de}d\right)\sqrt{\left(d+e-\sqrt{d^2+2\,de}\right)f}f^2}$$

input
```
integrate((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="g
iac")
```

output
```
1/4*sqrt(2)*(d*e*f*abs(f) - sqrt(d^2 + 2*d*e)*(d + e)*f^2 + sqrt(d^2 + 2*d
*e)*d*f^2)*arctan(4*sqrt(1/2)*x/sqrt((4*d*f + 4*e*f + sqrt(-16*e^2*f^2 + 1
6*(d*f + e*f)^2))/f^2))/((d^2 + d*e - sqrt(d^2 + 2*d*e)*d)*sqrt((d + e + s
qrt(d^2 + 2*d*e))*f)*f^2) + 1/4*sqrt(2)*(d*e*f*abs(f) + sqrt(d^2 + 2*d*e)*
(d + e)*f^2 - sqrt(d^2 + 2*d*e)*d*f^2)*arctan(4*sqrt(1/2)*x/sqrt((4*d*f +
4*e*f - sqrt(-16*e^2*f^2 + 16*(d*f + e*f)^2))/f^2))/((d^2 + d*e + sqrt(d^2
 + 2*d*e)*d)*sqrt((d + e - sqrt(d^2 + 2*d*e))*f)*f^2)
```

3.524.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int \frac{e-2fx^2}{e^2+4dfx^2+4efx^2+4f^2x^4}\,dx = \frac{\operatorname{atan}\left(\frac{2\,f^{3/2}\,x^3+2\,d\,\sqrt{f}\,x+e\,\sqrt{f}\,x}{\sqrt{d}\,e}\right)-\operatorname{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{d}}\right)}{2\,\sqrt{d}\,\sqrt{f}}$$

input
```
int((e - 2*f*x^2)/(e^2 + 4*f^2*x^4 + 4*d*f*x^2 + 4*e*f*x^2),x)
```

output
```
(atan((2*f^(3/2)*x^3 + 2*d*f^(1/2)*x + e*f^(1/2)*x)/(d^(1/2)*e)) - atan((f
^(1/2)*x)/d^(1/2)))/(2*d^(1/2)*f^(1/2))
```

3.525 $\int \frac{e-2fx^2}{e^2-4dfx^2+4efx^2+4f^2x^4}\,dx$

3.525.1 Optimal result

Integrand size = 37, antiderivative size = 73

$$\int \frac{e-2fx^2}{e^2-4dfx^2+4efx^2+4f^2x^4}\,dx = -\frac{\log\left(e-2\sqrt{d}\sqrt{f}x+2fx^2\right)}{4\sqrt{d}\sqrt{f}} + \frac{\log\left(e+2\sqrt{d}\sqrt{f}x+2fx^2\right)}{4\sqrt{d}\sqrt{f}}$$

output
```
-1/4*ln(e+2*f*x^2-2*x*d^(1/2)*f^(1/2))/d^(1/2)/f^(1/2)+1/4*ln(e+2*f*x^2+2*
x*d^(1/2)*f^(1/2))/d^(1/2)/f^(1/2)
```

3.525.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.19

$$\int \frac{e-2fx^2}{e^2-4dfx^2+4efx^2+4f^2x^4}\,dx = \frac{-\frac{\left(-id+2ie+\sqrt{d}\sqrt{-d+2e}\right)\arctan\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{-d+e-i\sqrt{d}\sqrt{-d+2e}}}\right)}{\sqrt{-d+e-i\sqrt{d}\sqrt{-d+2e}}}-\frac{\left(id-2ie+\sqrt{d}\sqrt{-d+2e}\right)\arctan\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{-d+e+i\sqrt{d}\sqrt{-d+2e}}}\right)}{\sqrt{-d+e+i\sqrt{d}\sqrt{-d+2e}}}}{2\sqrt{2}\sqrt{d}\sqrt{-d+2e}\sqrt{f}}$$

input
```
Integrate[(e - 2*f*x^2)/(e^2 - 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),x]
```

output
```
(-((((-I)*d + (2*I)*e + Sqrt[d]*Sqrt[-d + 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]*x)
/Sqrt[-d + e - I*Sqrt[d]*Sqrt[-d + 2*e]]])/Sqrt[-d + e - I*Sqrt[d]*Sqrt[-d
 + 2*e]]) - ((I*d - (2*I)*e + Sqrt[d]*Sqrt[-d + 2*e])*ArcTan[(Sqrt[2]*Sqrt
[f]*x)/Sqrt[-d + e + I*Sqrt[d]*Sqrt[-d + 2*e]]])/Sqrt[-d + e + I*Sqrt[d]*S
qrt[-d + 2*e]])/(2*Sqrt[2]*Sqrt[d]*Sqrt[-d + 2*e]*Sqrt[f])
```

3.525.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {6, 1478, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e-2fx^2}{-4dfx^2+e^2+4efx^2+4f^2x^4}dx$$
$$\Big\downarrow \text{ 6}$$
$$\int \frac{e-2fx^2}{fx^2(4e-4d)+e^2+4f^2x^4}dx$$
$$\Big\downarrow \text{ 1478}$$
$$-\frac{\int -\frac{2\left(\sqrt{d}-2\sqrt{f}x\right)}{\sqrt{f}\left(2x^2-\frac{2\sqrt{d}x}{\sqrt{f}}+\frac{e}{f}\right)}dx}{4\sqrt{d}\sqrt{f}}-\frac{\int -\frac{2\left(2\sqrt{f}x+\sqrt{d}\right)}{\sqrt{f}\left(2x^2+\frac{2\sqrt{d}x}{\sqrt{f}}+\frac{e}{f}\right)}dx}{4\sqrt{d}\sqrt{f}}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\int \frac{\sqrt{d}-2\sqrt{f}x}{2x^2-\frac{2\sqrt{d}x}{\sqrt{f}}+\frac{e}{f}}dx}{2\sqrt{d}f}+\frac{\int \frac{2\sqrt{f}x+\sqrt{d}}{2x^2+\frac{2\sqrt{d}x}{\sqrt{f}}+\frac{e}{f}}dx}{2\sqrt{d}f}$$
$$\Big\downarrow \text{ 1103}$$
$$\frac{\log\left(2\sqrt{d}\sqrt{f}x+e+2fx^2\right)}{4\sqrt{d}\sqrt{f}}-\frac{\log\left(-2\sqrt{d}\sqrt{f}x+e+2fx^2\right)}{4\sqrt{d}\sqrt{f}}$$

input
```
Int[(e - 2*f*x^2)/(e^2 - 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),x]
```

output
```
-1/4*Log[e - 2*Sqrt[d]*Sqrt[f]*x + 2*f*x^2]/(Sqrt[d]*Sqrt[f]) + Log[e + 2*
Sqrt[d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[d]*Sqrt[f])
```

3.525.3.1 Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

3.525.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

method	result
risch	$\frac{\ln\left(2\sqrt{df}\, f\, x^{2}+2 d f x+\sqrt{df}\, e\right)}{4 \sqrt{df}}-\frac{\ln\left(2\sqrt{df}\, f\, x^{2}-2 d f x+\sqrt{df}\, e\right)}{4 \sqrt{df}}$
default	$f^{2}\left(-\frac{\left(-df+2ef-\sqrt{d\,f^{2}(d-2e)}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{f x\sqrt{2}}{\sqrt{df-ef+\sqrt{d\,f^{2}(d-2e)}}}\right)}{4f^{2}\sqrt{d\,f^{2}(d-2e)}\,\sqrt{df-ef+\sqrt{d\,f^{2}(d-2e)}}}+\frac{\left(df-2ef-\sqrt{d\,f^{2}(d-2e)}\right)\sqrt{2}\,\arctan\left(\frac{f x\sqrt{2}}{\sqrt{-df+ef+\sqrt{d\,\ldots}}}\right.}{4f^{2}\sqrt{d\,f^{2}(d-2e)}\,\sqrt{-df+ef+\sqrt{d\,f^{2}(d-2e}}}\right.$

input
```
int((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2),x,method=_RETURNVERBO
SE)
```

output
```
1/4/(d*f)^(1/2)*ln(2*(d*f)^(1/2)*f*x^2+2*d*f*x+(d*f)^(1/2)*e)-1/4/(d*f)^(1
/2)*ln(2*(d*f)^(1/2)*f*x^2-2*d*f*x+(d*f)^(1/2)*e)
```

3.525.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.00

$$\int \frac{e-2fx^2}{e^2-4dfx^2+4efx^2+4f^2x^4}\,dx = \left[\frac{\sqrt{df}\log\left(\frac{4f^2x^4+4(d+e)fx^2+e^2+4(2fx^3+ex)\sqrt{df}}{4f^2x^4-4(d-e)fx^2+e^2}\right)}{4df}, -\frac{\sqrt{-df}\arctan\left(\frac{\sqrt{-df}x}{d}\right)-\sqrt{-df}\arctan\left(\frac{(2fx^3-(2d-e)x)\sqrt{-df}}{de}\right)}{2df}\right]$$

input
```
integrate((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="f
ricas")
```

output
```
[1/4*sqrt(d*f)*log((4*f^2*x^4 + 4*(d + e)*f*x^2 + e^2 + 4*(2*f*x^3 + e*x)*
sqrt(d*f))/(4*f^2*x^4 - 4*(d - e)*f*x^2 + e^2))/(d*f), -1/2*(sqrt(-d*f)*ar
ctan(sqrt(-d*f)*x/d) - sqrt(-d*f)*arctan((2*f*x^3 - (2*d - e)*x)*sqrt(-d*f
)/(d*e)))/(d*f)]
```

3.525.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{e-2fx^2}{e^2-4dfx^2+4efx^2+4f^2x^4}\,dx = -\frac{\sqrt{\frac{1}{df}}\log\left(-dx\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^2\right)}{4} + \frac{\sqrt{\frac{1}{df}}\log\left(dx\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^2\right)}{4}$$

input
```
integrate((-2*f*x**2+e)/(4*f**2*x**4-4*d*f*x**2+4*e*f*x**2+e**2),x)
```

output
```
-sqrt(1/(d*f))*log(-d*x*sqrt(1/(d*f)) + e/(2*f) + x**2)/4 + sqrt(1/(d*f))*
log(d*x*sqrt(1/(d*f)) + e/(2*f) + x**2)/4
```

3.525.7 Maxima [F]

$$\int \frac{e-2fx^2}{e^2-4dfx^2+4efx^2+4f^2x^4}\,dx=\int -\frac{2\,fx^2-e}{4\,f^2x^4-4\,dfx^2+4\,efx^2+e^2}\,dx$$

input
```
integrate((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="m
axima")
```

output
```
-integrate((2*f*x^2 - e)/(4*f^2*x^4 - 4*d*f*x^2 + 4*e*f*x^2 + e^2), x)
```

3.525.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(53) = 106$.

Time = 0.41 (sec) , antiderivative size = 282, normalized size of antiderivative = 3.86

$$\int \frac{e-2fx^2}{e^2-4dfx^2+4efx^2+4f^2x^4}\,dx=$$

$$-\frac{\sqrt{2}\left(def|f|+\sqrt{d^2-2\,de}(d-e)f^2-\sqrt{d^2-2\,de}df^2\right)\arctan\left(\frac{4\sqrt{\frac{1}{2}}x}{\sqrt{-\frac{4\,df-4\,ef+\sqrt{-16\,e^2f^2+16\,(df-ef)^2}}{f^2}}}\right)}{4\left(d^2-de-\sqrt{d^2-2\,de}d\right)\sqrt{-\left(d-e+\sqrt{d^2-2\,de}\right)f}f^2}$$
$$-\frac{\sqrt{2}\left(def|f|-\sqrt{d^2-2\,de}(d-e)f^2+\sqrt{d^2-2\,de}df^2\right)\arctan\left(\frac{4\sqrt{\frac{1}{2}}x}{\sqrt{-\frac{4\,df-4\,ef-\sqrt{-16\,e^2f^2+16\,(df-ef)^2}}{f^2}}}\right)}{4\left(d^2-de+\sqrt{d^2-2\,de}d\right)\sqrt{-\left(d-e-\sqrt{d^2-2\,de}\right)f}f^2}$$

input
```
integrate((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="g
iac")
```

output
```
-1/4*sqrt(2)*(d*e*f*abs(f) + sqrt(d^2 - 2*d*e)*(d - e)*f^2 - sqrt(d^2 - 2*
d*e)*d*f^2)*arctan(4*sqrt(1/2)*x/sqrt(-(4*d*f - 4*e*f + sqrt(-16*e^2*f^2 +
 16*(d*f - e*f)^2))/f^2))/((d^2 - d*e - sqrt(d^2 - 2*d*e)*d)*sqrt(-(d - e
+ sqrt(d^2 - 2*d*e))*f)*f^2) - 1/4*sqrt(2)*(d*e*f*abs(f) - sqrt(d^2 - 2*d*
e)*(d - e)*f^2 + sqrt(d^2 - 2*d*e)*d*f^2)*arctan(4*sqrt(1/2)*x/sqrt(-(4*d*
f - 4*e*f - sqrt(-16*e^2*f^2 + 16*(d*f - e*f)^2))/f^2))/((d^2 - d*e + sqrt
(d^2 - 2*d*e)*d)*sqrt(-(d - e - sqrt(d^2 - 2*d*e))*f)*f^2)
```

3.525.9 Mupad [B] (verification not implemented)

Time = 17.81 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.38

$$\int \frac{e-2fx^2}{e^2-4dfx^2+4efx^2+4f^2x^4}\,dx=\frac{\operatorname{atanh}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x}{2\,f\,x^2+e}\right)}{2\,\sqrt{d}\,\sqrt{f}}$$

input
```
int((e - 2*f*x^2)/(e^2 + 4*f^2*x^4 - 4*d*f*x^2 + 4*e*f*x^2),x)
```

output
```
atanh((2*d^(1/2)*f^(1/2)*x)/(e + 2*f*x^2))/(2*d^(1/2)*f^(1/2))
```

3.526 $\int \frac{e-4fx^3}{e^2+4dfx^2+4efx^3+4f^2x^6}\,dx$

3.526.1 Optimal result

Integrand size = 37, antiderivative size = 38

$$\int \frac{e-4fx^3}{e^2+4dfx^2+4efx^3+4f^2x^6}\,dx=\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

output
```
1/2*arctan(2*x*d^(1/2)*f^(1/2)/(2*f*x^3+e))/d^(1/2)/f^(1/2)
```

3.526.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.29

$$\int \frac{e-4fx^3}{e^2+4dfx^2+4efx^3+4f^2x^6}\,dx = -\frac{\text{RootSum}\left[e^2+4df\#1^2+4ef\#1^3+4f^2\#1^6\&,\frac{-e\log(x-\#1)+4f\log(x-\#1)\#1^3}{2d\#1+3e\#1^2+6f\#1^5}\&\right]}{4f}$$

input
```
Integrate[(e - 4*f*x^3)/(e^2 + 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),x]
```

output
```
-1/4*RootSum[e^2 + 4*d*f*#1^2 + 4*e*f*#1^3 + 4*f^2*#1^6 & , (-(e*Log[x - #
1]) + 4*f*Log[x - #1]*#1^3)/(2*d*#1 + 3*e*#1^2 + 6*f*#1^5) & ]/f
```

3.526.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2519, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e-4fx^3}{4dfx^2+e^2+4efx^3+4f^2x^6}dx$$

$$\Big\downarrow \text{2519}$$

$$2e^2\int \frac{1}{\frac{4dfx^2e^2}{(2fx^3+e)^2}+e^2}d\frac{x}{2(2fx^3+e)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

input
```
Int[(e - 4*f*x^3)/(e^2 + 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),x]
```

output
```
ArcTan[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

3.526.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2519
```
Int[((A_) + (B_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^(n_) + (d_.)
*(x_)^(n2_)), x_Symbol] :> Simp[A^2*(n - 1)   Subst[Int[1/(a + A^2*b*(n - 1
)^2*x^2), x], x, x/(A*(n - 1) - B*x^n)], x] /; FreeQ[{a, b, c, d, A, B, n},
 x] && EqQ[n2, 2*n] && NeQ[n, 2] && EqQ[a*B^2 - A^2*d*(n - 1)^2, 0] && EqQ[
B*c + 2*A*d*(n - 1), 0]
```

3.526.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.84

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(4f^2\_Z^6+4ef\_Z^3+4df\_Z^2+e^2\right)}\frac{\left(-4\_R^3f+e\right)\ln\left(x-\_R\right)}{6f\_R^5+3e\_R^2+2d\_R}}{4f}$
risch	$-\frac{\ln\left(\left(32f(-df)^{\frac{3}{2}}d+54e^2f^2d\right)x^3+\left(54e^2(-df)^{\frac{3}{2}}-32d^3f^2\right)x+16e(-df)^{\frac{3}{2}}d+27e^3fd\right)}{4\sqrt{-df}}+\frac{\ln\left(\left(32f(-df)^{\frac{3}{2}}d-54e^2f^2d\right)x^3+\left(54e^2(-\right.\right.}{4\sqrt{}}$

input
```
int((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x,method=_RETURNVERBO
SE)
```

output
```
1/4/f*sum((-4*_R^3*f+e)/(6*_R^5*f+3*_R^2*e+2*_R*d)*ln(x-_R),_R=RootOf(4*_Z
^6*f^2+4*_Z^3*e*f+4*_Z^2*d*f+e^2))
```

3.526.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(28) = 56.

Time = 0.33 (sec) , antiderivative size = 153, normalized size of antiderivative = 4.03

$$\int \frac{e-4fx^3}{e^2+4dfx^2+4efx^3+4f^2x^6}\,dx = \left[-\frac{\sqrt{-df}\log\left(\frac{4f^2x^6+4efx^3-4dfx^2+e^2+4\left(2fx^4+ex\right)\sqrt{-df}}{4f^2x^6+4efx^3+4dfx^2+e^2}\right)}{4df},\; -\frac{\sqrt{df}\arctan\left(\frac{\sqrt{df}x^2}{d}\right)-\sqrt{df}\arctan\left(\frac{\left(2fx^5+ex^2+2dx\right)\sqrt{df}}{de}\right)}{2df}\right]$$

input
```
integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x, algorithm="f
ricas")
```

output
```
[-1/4*sqrt(-d*f)*log((4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^2 + e^2 + 4*(2*f*x^4
 + e*x)*sqrt(-d*f))/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^2 + e^2))/(d*f), -1/2
*(sqrt(d*f)*arctan(sqrt(d*f)*x^2/d) - sqrt(d*f)*arctan((2*f*x^5 + e*x^2 +
2*d*x)*sqrt(d*f)/(d*e)))/(d*f)]
```

3.526.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(34) = 68$.

Time = 0.38 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.84

$$\int \frac{e-4fx^3}{e^2+4dfx^2+4efx^3+4f^2x^6}\,dx = \frac{\sqrt{-\frac{1}{df}}\log\left(-dx\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4} - \frac{\sqrt{-\frac{1}{df}}\log\left(dx\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}$$

input
```
integrate((-4*f*x**3+e)/(4*f**2*x**6+4*e*f*x**3+4*d*f*x**2+e**2),x)
```

output
```
sqrt(-1/(d*f))*log(-d*x*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4 - sqrt(-1/(d*f)
)*log(d*x*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4
```

3.526.7 Maxima [F]

$$\int \frac{e-4fx^3}{e^2+4dfx^2+4efx^3+4f^2x^6}\,dx = \int -\frac{4\,fx^3-e}{4\,f^2x^6+4\,efx^3+4\,dfx^2+e^2}\,dx$$

input
```
integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x, algorithm="m
axima")
```

output
```
-integrate((4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^2 + e^2), x)
```

3.526.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(28) = 56$.

Time = 0.33 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \frac{e-4fx^3}{e^2+4dfx^2+4efx^3+4f^2x^6}\,dx = -\frac{\sqrt{-df}\log\left(\left|2\,fx^3+2\,\sqrt{-df}x+e\right|\right)}{4\,df} + \frac{\sqrt{-df}\log\left(\left|2\,fx^3-2\,\sqrt{-df}x+e\right|\right)}{4\,df}$$

input
```
integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x, algorithm="g
iac")
```

output
```
-1/4*sqrt(-d*f)*log(abs(2*f*x^3 + 2*sqrt(-d*f)*x + e))/(d*f) + 1/4*sqrt(-d
*f)*log(abs(2*f*x^3 - 2*sqrt(-d*f)*x + e))/(d*f)
```

3.526.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int \frac{e-4fx^3}{e^2+4dfx^2+4efx^3+4f^2x^6}\,dx=\frac{\operatorname{atan}\left(\frac{2\,f^{3/2}\,x^5+2\,d\,\sqrt{f}\,x+e\,\sqrt{f}\,x^2}{\sqrt{d}\,e}\right)-\operatorname{atan}\left(\frac{\sqrt{f}\,x^2}{\sqrt{d}}\right)}{2\,\sqrt{d}\,\sqrt{f}}$$

input
```
int((e - 4*f*x^3)/(e^2 + 4*f^2*x^6 + 4*d*f*x^2 + 4*e*f*x^3),x)
```

output
```
(atan((2*f^(3/2)*x^5 + 2*d*f^(1/2)*x + e*f^(1/2)*x^2)/(d^(1/2)*e)) - atan(
(f^(1/2)*x^2)/d^(1/2)))/(2*d^(1/2)*f^(1/2))
```

3.527 $\int \frac{e-4fx^3}{e^2-4dfx^2+4efx^3+4f^2x^6}\,dx$

3.527.1 Optimal result

Integrand size = 37, antiderivative size = 38

$$\int \frac{e-4fx^3}{e^2-4dfx^2+4efx^3+4f^2x^6}\,dx=\frac{\operatorname{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

output
```
1/2*arctanh(2*x*d^(1/2)*f^(1/2)/(2*f*x^3+e))/d^(1/2)/f^(1/2)
```

3.527.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.29

$$\int \frac{e-4fx^3}{e^2-4dfx^2+4efx^3+4f^2x^6}\,dx$$
$$=-\frac{\text{RootSum}\left[e^2-4df\#1^2+4ef\#1^3+4f^2\#1^6\&,\frac{-e\log(x-\#1)+4f\log(x-\#1)\#1^3}{-2d\#1+3e\#1^2+6f\#1^5}\&\right]}{4f}$$

input
```
Integrate[(e - 4*f*x^3)/(e^2 - 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),x]
```

output
```
-1/4*RootSum[e^2 - 4*d*f*#1^2 + 4*e*f*#1^3 + 4*f^2*#1^6 & , (-(e*Log[x - #
1]) + 4*f*Log[x - #1]*#1^3)/(-2*d*#1 + 3*e*#1^2 + 6*f*#1^5) & ]/f
```

3.527.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {2519, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e-4fx^3}{-4dfx^2+e^2+4efx^3+4f^2x^6}\,dx$$

$$\Big\downarrow \text{2519}$$

$$2e^2\int \frac{1}{e^2-\frac{4de^2fx^2}{(2fx^3+e)^2}}d\frac{x}{2\left(2fx^3+e\right)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\operatorname{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

input
```
Int[(e - 4*f*x^3)/(e^2 - 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),x]
```

output
```
ArcTanh[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

3.527.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2519
```
Int[((A_) + (B_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^(n_) + (d_.)
*(x_)^(n2_)), x_Symbol] :> Simp[A^2*(n - 1)   Subst[Int[1/(a + A^2*b*(n - 1
)^2*x^2), x], x, x/(A*(n - 1) - B*x^n)], x] /; FreeQ[{a, b, c, d, A, B, n},
 x] && EqQ[n2, 2*n] && NeQ[n, 2] && EqQ[a*B^2 - A^2*d*(n - 1)^2, 0] && EqQ[
B*c + 2*A*d*(n - 1), 0]
```

3.527.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.89

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(4f^2\_Z^6+4ef\_Z^3-4df\_Z^2+e^2\right)}\frac{\left(4\_R^3f-e\right)\ln\left(x-\_R\right)}{-6f\_R^5-3e\_R^2+2d\_R}}{4f}$
risch	$\frac{\ln\left(\left(-32f(df)^{\frac{3}{2}}d+54e^2f^2d\right)x^3+\left(54e^2(df)^{\frac{3}{2}}-32d^3f^2\right)x-16e(df)^{\frac{3}{2}}d+27e^3fd\right)}{4\sqrt{df}}-\frac{\ln\left(\left(-32f(df)^{\frac{3}{2}}d-54e^2f^2d\right)x^3+\left(54e^2(df)^{\frac{3}{2}}+3\right.}{4\sqrt{df}}$

input
```
int((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x,method=_RETURNVERBO
SE)
```

output
```
1/4/f*sum((4*_R^3*f-e)/(-6*_R^5*f-3*_R^2*e+2*_R*d)*ln(x-_R),_R=RootOf(4*_Z
^6*f^2+4*_Z^3*e*f-4*_Z^2*d*f+e^2))
```

3.527.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(28) = 56$.

Time = 0.32 (sec) , antiderivative size = 155, normalized size of antiderivative = 4.08

$$\int \frac{e-4fx^3}{e^2-4dfx^2+4efx^3+4f^2x^6}\,dx = \left[\frac{\sqrt{df}\log\left(\frac{4f^2x^6+4efx^3+4dfx^2+e^2+4(2fx^4+ex)\sqrt{df}}{4f^2x^6+4efx^3-4dfx^2+e^2}\right)}{4df},\right.$$
$$\left.-\frac{\sqrt{-df}\arctan\left(\frac{\sqrt{-df}x^2}{d}\right)-\sqrt{-df}\arctan\left(\frac{(2fx^5+ex^2-2dx)\sqrt{-df}}{de}\right)}{2df}\right]$$

input
```
integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x, algorithm="f
ricas")
```

output
```
[1/4*sqrt(d*f)*log((4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^2 + e^2 + 4*(2*f*x^4 +
 e*x)*sqrt(d*f))/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^2 + e^2))/(d*f), -1/2*(s
qrt(-d*f)*arctan(sqrt(-d*f)*x^2/d) - sqrt(-d*f)*arctan((2*f*x^5 + e*x^2 -
2*d*x)*sqrt(-d*f)/(d*e)))/(d*f)]
```

3.527.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.66

$$\int \frac{e-4fx^3}{e^2-4dfx^2+4efx^3+4f^2x^6}\,dx = -\frac{\sqrt{\frac{1}{df}}\log\left(-dx\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4} + \frac{\sqrt{\frac{1}{df}}\log\left(dx\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}$$

input
```
integrate((-4*f*x**3+e)/(4*f**2*x**6+4*e*f*x**3-4*d*f*x**2+e**2),x)
```

output
```
-sqrt(1/(d*f))*log(-d*x*sqrt(1/(d*f)) + e/(2*f) + x**3)/4 + sqrt(1/(d*f))*
log(d*x*sqrt(1/(d*f)) + e/(2*f) + x**3)/4
```

3.527.7 Maxima [F]

$$\int \frac{e-4fx^3}{e^2-4dfx^2+4efx^3+4f^2x^6}\,dx = \int -\frac{4\,fx^3-e}{4\,f^2x^6+4\,efx^3-4\,dfx^2+e^2}\,dx$$

input
```
integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x, algorithm="m
axima")
```

output
```
-integrate((4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^2 + e^2), x)
```

3.527.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(28) = 56.

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.66

$$\int \frac{e-4fx^3}{e^2-4dfx^2+4efx^3+4f^2x^6}\,dx = \frac{\sqrt{df}\log\left(\left|2\,fx^3+2\,\sqrt{df}x+e\right|\right)}{4\,df} - \frac{\sqrt{df}\log\left(\left|2\,fx^3-2\,\sqrt{df}x+e\right|\right)}{4\,df}$$

input
```
integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x, algorithm="g
iac")
```

output
```
1/4*sqrt(d*f)*log(abs(2*f*x^3 + 2*sqrt(d*f)*x + e))/(d*f) - 1/4*sqrt(d*f)*
log(abs(2*f*x^3 - 2*sqrt(d*f)*x + e))/(d*f)
```

3.527.9 Mupad [B] (verification not implemented)

Time = 17.55 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \frac{e-4fx^3}{e^2-4dfx^2+4efx^3+4f^2x^6}\,dx = -\frac{\operatorname{atanh}\left(\frac{-32\,f\,d^2\,x+27\,e^3+54\,f\,e^2\,x^3}{16\,d^{3/2}\,e\,\sqrt{f}+32\,d^{3/2}\,f^{3/2}\,x^3-54\,\sqrt{d}\,e^2\,\sqrt{f}\,x}\right)}{2\,\sqrt{d}\,\sqrt{f}}$$

input
```
int((e - 4*f*x^3)/(e^2 + 4*f^2*x^6 - 4*d*f*x^2 + 4*e*f*x^3),x)
```

output
```
-atanh((27*e^3 - 32*d^2*f*x + 54*e^2*f*x^3)/(16*d^(3/2)*e*f^(1/2) + 32*d^(
3/2)*f^(3/2)*x^3 - 54*d^(1/2)*e^2*f^(1/2)*x))/(2*d^(1/2)*f^(1/2))
```

3.528 $\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx$

3.528.1 Optimal result

Integrand size = 42, antiderivative size = 38

$$\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx=\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

output
```
1/2*arctan(2*x*d^(1/2)*f^(1/2)/(e+2*f*x^n))/d^(1/2)/f^(1/2)
```

3.528.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx=\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

input
```
Integrate[(e - 2*f*(-1 + n)*x^n)/(e^2 + 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2
*n)),x]
```

output
```
ArcTan[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

3.528.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2519, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e-2f(n-1)x^n}{4dfx^2+e^2+4efx^n+4f^2x^{2n}}dx$$
$$\Big\downarrow \text{2519}$$
$$-e^2(1-n)\int \frac{1}{\frac{4df(1-n)^2x^2e^2}{(2f(1-n)x^n+e(1-n))^2}+e^2}d\left(-\frac{x}{2f(1-n)x^n+e(1-n)}\right)$$
$$\Big\downarrow \text{218}$$
$$\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}(1-n)x}{e(1-n)+2f(1-n)x^n}\right)}{2\sqrt{d}\sqrt{f}}$$

input
```
Int[(e - 2*f*(-1 + n)*x^n)/(e^2 + 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)),x
]
```

output
```
ArcTan[(2*Sqrt[d]*Sqrt[f]*(1 - n)*x)/(e*(1 - n) + 2*f*(1 - n)*x^n)]/(2*Sqr
t[d]*Sqrt[f])
```

3.528.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2519
```
Int[((A_) + (B_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^(n_) + (d_.)
*(x_)^(n2_)), x_Symbol] :> Simp[A^2*(n - 1)   Subst[Int[1/(a + A^2*b*(n - 1
)^2*x^2), x], x, x/(A*(n - 1) - B*x^n)], x] /; FreeQ[{a, b, c, d, A, B, n},
 x] && EqQ[n2, 2*n] && NeQ[n, 2] && EqQ[a*B^2 - A^2*d*(n - 1)^2, 0] && EqQ[
B*c + 2*A*d*(n - 1), 0]
```

3.528.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(28) = 56.

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.05

method	result	size
risch	$-\frac{\ln\left(x^{n}+\frac{2dfx+\sqrt{-df}\, e}{2\sqrt{-df}\, f}\right)}{4\sqrt{-df}}+\frac{\ln\left(x^{n}+\frac{-2dfx+\sqrt{-df}\, e}{2\sqrt{-df}\, f}\right)}{4\sqrt{-df}}$	78

input
```
int((e-2*f*(-1+n)*x^n)/(e^2+4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x,method=_R
ETURNVERBOSE)
```

output
```
-1/4/(-d*f)^(1/2)*ln(x^n+1/2*(2*d*f*x+(-d*f)^(1/2)*e)/(-d*f)^(1/2)/f)+1/4/
(-d*f)^(1/2)*ln(x^n+1/2*(-2*d*f*x+(-d*f)^(1/2)*e)/(-d*f)^(1/2)/f)
```

3.528.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.79

$$\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx = \left[-\frac{\sqrt{-df}\log\left(-\frac{4\,dfx^2-4\,f^2x^{2\,n}-4\,\sqrt{-df}ex-e^2-4\,\left(2\,\sqrt{-df}fx+ef\right)x^n}{4\,dfx^2+4\,f^2x^{2\,n}+4\,efx^n+e^2}\right)}{4\,df},\ -\frac{\sqrt{df}\arctan\left(\frac{2\,\sqrt{df}fx^n+\sqrt{df}e}{2\,dfx}\right)}{2\,df}\right]$$

input
```
integrate((e-2*f*(-1+n)*x^n)/(e^2+4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, al
gorithm="fricas")
```

output
```
[-1/4*sqrt(-d*f)*log(-(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*sqrt(-d*f)*e*x - e^2
- 4*(2*sqrt(-d*f)*f*x + e*f)*x^n)/(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*e*f*x^n +
 e^2))/(d*f), -1/2*sqrt(d*f)*arctan(1/2*(2*sqrt(d*f)*f*x^n + sqrt(d*f)*e)/
(d*f*x))/(d*f)]
```

3.528.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(34) = 68.

Time = 15.54 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.87

$$\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx = \begin{cases} \frac{x}{e} & \text{for } d=0\wedge f=0 \\ \frac{x}{e+2fx^n} & \text{for } d=0 \\ \frac{x}{e} & \text{for } f=0 \\ \frac{\log\left(-\frac{e\sqrt{-\frac{1}{df}}}{2}-fx^n\sqrt{-\frac{1}{df}}+x\right)}{4df\sqrt{-\frac{1}{df}}}-\frac{\log\left(\frac{e\sqrt{-\frac{1}{df}}}{2}+fx^n\sqrt{-\frac{1}{df}}+x\right)}{4df\sqrt{-\frac{1}{df}}} & \text{otherwise} \end{cases}$$

input
```
integrate((e-2*f*(-1+n)*x**n)/(e**2+4*d*f*x**2+4*e*f*x**n+4*f**2*x**(2*n))
,x)
```

output
```
Piecewise((x/e, Eq(d, 0) & Eq(f, 0)), (x/(e + 2*f*x**n), Eq(d, 0)), (x/e,
Eq(f, 0)), (log(-e*sqrt(-1/(d*f))/2 - f*x**n*sqrt(-1/(d*f)) + x)/(4*d*f*sq
rt(-1/(d*f))) - log(e*sqrt(-1/(d*f))/2 + f*x**n*sqrt(-1/(d*f)) + x)/(4*d*f
*sqrt(-1/(d*f))), True))
```

3.528.7 Maxima [F]

$$\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx = \int -\frac{2\,f(n-1)x^n-e}{4\,dfx^2+4\,f^2x^{2\,n}+4\,efx^n+e^2}\,dx$$

input
```
integrate((e-2*f*(-1+n)*x^n)/(e^2+4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, al
gorithm="maxima")
```

output
```
-integrate((2*f*(n - 1)*x^n - e)/(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*e*f*x^n +
e^2), x)
```

3.528.8 Giac [F]

$$\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx=\int -\frac{2\,f(n-1)x^n-e}{4\,dfx^2+4\,f^2x^{2\,n}+4\,efx^n+e^2}\,dx$$

input
```
integrate((e-2*f*(-1+n)*x^n)/(e^2+4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, al
gorithm="giac")
```

output
```
integrate(-(2*f*(n - 1)*x^n - e)/(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*e*f*x^n +
e^2), x)
```

3.528.9 Mupad [B] (verification not implemented)

Time = 17.51 (sec) , antiderivative size = 196, normalized size of antiderivative = 5.16

$$\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx=\frac{\ln\left(-\frac{e+2\,f\,x^n-2\,f\,n\,x^n}{4\,f^2}-\frac{e^2\,n-4\,d\,f\,x^2+4\,d\,f\,n\,x^2+2\,e\,f\,n\,x^n}{8\,\sqrt{-d}\,f^{5/2}\,x}\right)}{4\,\sqrt{-d}\,\sqrt{f}}-\frac{\mathrm{atan}\left(\frac{x\,\left(8\,d\,f\,n^2-16\,d\,f\,n+8\,d\,f\right)}{4\,\sqrt{d}\,\sqrt{f}\,\left(e\,n-e\,n^2\right)}\right)}{2\,\sqrt{d}\,\sqrt{f}}-\frac{\ln\left(\frac{e^2\,n-4\,d\,f\,x^2+4\,d\,f\,n\,x^2+2\,e\,f\,n\,x^n}{8\,\sqrt{-d}\,f^{5/2}\,x}-\frac{e+2\,f\,x^n-2\,f\,n\,x^n}{4\,f^2}\right)}{4\,\sqrt{-d}\,\sqrt{f}}$$

input
```
int((e - 2*f*x^n*(n - 1))/(e^2 + 4*f^2*x^(2*n) + 4*d*f*x^2 + 4*e*f*x^n),x)
```

output
```
log(- (e + 2*f*x^n - 2*f*n*x^n)/(4*f^2) - (e^2*n - 4*d*f*x^2 + 4*d*f*n*x^2
 + 2*e*f*n*x^n)/(8*(-d)^(1/2)*f^(5/2)*x))/(4*(-d)^(1/2)*f^(1/2)) - atan((x
*(8*d*f - 16*d*f*n + 8*d*f*n^2))/(4*d^(1/2)*f^(1/2)*(e*n - e*n^2)))/(2*d^(
1/2)*f^(1/2)) - log((e^2*n - 4*d*f*x^2 + 4*d*f*n*x^2 + 2*e*f*n*x^n)/(8*(-d
)^(1/2)*f^(5/2)*x) - (e + 2*f*x^n - 2*f*n*x^n)/(4*f^2))/(4*(-d)^(1/2)*f^(1
/2))
```

3.529 $\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx$

3.529.1 Optimal result

Integrand size = 42, antiderivative size = 38

$$\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx=\frac{\operatorname{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

output
```
1/2*arctanh(2*x*d^(1/2)*f^(1/2)/(e+2*f*x^n))/d^(1/2)/f^(1/2)
```

3.529.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx=\frac{\operatorname{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

input
```
Integrate[(e - 2*f*(-1 + n)*x^n)/(e^2 - 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2
*n)),x]
```

output
```
ArcTanh[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

3.529.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2519, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e-2f(n-1)x^n}{-4dfx^2+e^2+4efx^n+4f^2x^{2n}}dx$$

$$\Big\downarrow \text{2519}$$

$$-e^2(1-n)\int \frac{1}{e^2-\frac{4de^2f(1-n)^2x^2}{(2f(1-n)x^n+e(1-n))^2}}d\left(-\frac{x}{2f(1-n)x^n+e(1-n)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{\text{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}(1-n)x}{e(1-n)+2f(1-n)x^n}\right)}{2\sqrt{d}\sqrt{f}}$$

input
```
Int[(e - 2*f*(-1 + n)*x^n)/(e^2 - 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)),x
]
```

output
```
ArcTanh[(2*Sqrt[d]*Sqrt[f]*(1 - n)*x)/(e*(1 - n) + 2*f*(1 - n)*x^n)]/(2*Sq
rt[d]*Sqrt[f])
```

3.529.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2519
```
Int[((A_) + (B_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^(n_) + (d_.)
*(x_)^(n2_)), x_Symbol] :> Simp[A^2*(n - 1)   Subst[Int[1/(a + A^2*b*(n - 1
)^2*x^2), x], x, x/(A*(n - 1) - B*x^n)], x] /; FreeQ[{a, b, c, d, A, B, n},
 x] && EqQ[n2, 2*n] && NeQ[n, 2] && EqQ[a*B^2 - A^2*d*(n - 1)^2, 0] && EqQ[
B*c + 2*A*d*(n - 1), 0]
```

3.529.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(28) = 56.

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.89

method	result	size
risch	$\frac{\ln\left(x^{n}+\frac{2 d f x+\sqrt{d f}\, e}{2 \sqrt{d f}\, f}\right)}{4 \sqrt{d f}}-\frac{\ln\left(x^{n}+\frac{-2 d f x+\sqrt{d f}\, e}{2 \sqrt{d f}\, f}\right)}{4 \sqrt{d f}}$	72

input
```
int((e-2*f*(-1+n)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x,method=_R
ETURNVERBOSE)
```

output
```
1/4/(d*f)^(1/2)*ln(x^n+1/2*(2*d*f*x+(d*f)^(1/2)*e)/(d*f)^(1/2)/f)-1/4/(d*f
)^(1/2)*ln(x^n+1/2*(-2*d*f*x+(d*f)^(1/2)*e)/(d*f)^(1/2)/f)
```

3.529.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.79

$$\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx = \left[\frac{\sqrt{df}\log\left(-\frac{4\,dfx^2+4\,f^2x^{2\,n}+4\,\sqrt{df}ex+e^2+4\,(2\,\sqrt{df}fx+ef)x^n}{4\,dfx^2-4\,f^2x^{2\,n}-4\,efx^n-e^2}\right)}{4\,df}, -\frac{\sqrt{-df}\arctan\left(\frac{2\,\sqrt{-df}fx^n+\sqrt{-df}e}{2\,dfx}\right)}{2\,df}\right]$$

input
```
integrate((e-2*f*(-1+n)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, al
gorithm="fricas")
```

output
```
[1/4*sqrt(d*f)*log(-(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*sqrt(d*f)*e*x + e^2 + 4
*(2*sqrt(d*f)*f*x + e*f)*x^n)/(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*e*f*x^n - e^2
))/(d*f), -1/2*sqrt(-d*f)*arctan(1/2*(2*sqrt(-d*f)*f*x^n + sqrt(-d*f)*e)/(
d*f*x))/(d*f)]
```

3.529.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(34) = 68$.

Time = 15.59 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.61

$$\int \frac{e - 2f(-1+n)x^n}{e^2 - 4dfx^2 + 4efx^n + 4f^2x^{2n}}\,dx = \begin{cases} \frac{x}{e} & \text{for } d = 0 \wedge f = 0 \\ \frac{x}{e+2fx^n} & \text{for } d = 0 \\ \frac{x}{e} & \text{for } f = 0 \\ -\frac{\log\left(-\frac{e\sqrt{\frac{1}{df}}}{2} - fx^n\sqrt{\frac{1}{df}} + x\right)}{4df\sqrt{\frac{1}{df}}} + \frac{\log\left(\frac{e\sqrt{\frac{1}{df}}}{2} + fx^n\sqrt{\frac{1}{df}} + x\right)}{4df\sqrt{\frac{1}{df}}} & \text{otherwise} \end{cases}$$

input
```
integrate((e-2*f*(-1+n)*x**n)/(e**2-4*d*f*x**2+4*e*f*x**n+4*f**2*x**(2*n))
,x)
```

output
```
Piecewise((x/e, Eq(d, 0) & Eq(f, 0)), (x/(e + 2*f*x**n), Eq(d, 0)), (x/e,
Eq(f, 0)), (-log(-e*sqrt(1/(d*f))/2 - f*x**n*sqrt(1/(d*f)) + x)/(4*d*f*sqr
t(1/(d*f))) + log(e*sqrt(1/(d*f))/2 + f*x**n*sqrt(1/(d*f)) + x)/(4*d*f*sqr
t(1/(d*f))), True))
```

3.529.7 Maxima [F]

$$\int \frac{e - 2f(-1+n)x^n}{e^2 - 4dfx^2 + 4efx^n + 4f^2x^{2n}}\,dx = \int \frac{2\,f(n-1)x^n - e}{4\,dfx^2 - 4\,f^2x^{2\,n} - 4\,efx^n - e^2}\,dx$$

input
```
integrate((e-2*f*(-1+n)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, al
gorithm="maxima")
```

output
```
integrate((2*f*(n - 1)*x^n - e)/(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*e*f*x^n - e
^2), x)
```

3.529.8 Giac [F]

$$\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx=\int \frac{2\,f(n-1)x^n-e}{4\,dfx^2-4\,f^2x^{2\,n}-4\,efx^n-e^2}\,dx$$

input
```
integrate((e-2*f*(-1+n)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, al
gorithm="giac")
```

output
```
integrate((2*f*(n - 1)*x^n - e)/(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*e*f*x^n - e
^2), x)
```

3.529.9 Mupad [B] (verification not implemented)

Time = 20.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.66

$$\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx=\frac{\ln\left(\frac{\left(e+2\,f\,x^n+2\,\sqrt{d}\,\sqrt{f}\,x\right)\,\left(e\,n+2\,\sqrt{d}\,\sqrt{f}\,x-2\,\sqrt{d}\,\sqrt{f}\,n\,x\right)}{x}\right)}{4\,\sqrt{d}\,\sqrt{f}}-\frac{\ln\left(\frac{\left(e+2\,f\,x^n-2\,\sqrt{d}\,\sqrt{f}\,x\right)\,\left(e\,n-2\,\sqrt{d}\,\sqrt{f}\,x+2\,\sqrt{d}\,\sqrt{f}\,n\,x\right)}{x}\right)}{4\,\sqrt{d}\,\sqrt{f}}-\frac{\mathrm{atan}\left(\frac{2\,\sqrt{d}\,\sqrt{f}\,x\,\left(n\,1\mathrm{i}-\mathrm{i}\right)}{e\,n}\right)\,1\mathrm{i}}{2\,\sqrt{d}\,\sqrt{f}}$$

input
```
int((e - 2*f*x^n*(n - 1))/(e^2 + 4*f^2*x^(2*n) - 4*d*f*x^2 + 4*e*f*x^n),x)
```

output
```
log(((e + 2*f*x^n + 2*d^(1/2)*f^(1/2)*x)*(e*n + 2*d^(1/2)*f^(1/2)*x - 2*d^
(1/2)*f^(1/2)*n*x))/x)/(4*d^(1/2)*f^(1/2)) - log(((e + 2*f*x^n - 2*d^(1/2)
*f^(1/2)*x)*(e*n - 2*d^(1/2)*f^(1/2)*x + 2*d^(1/2)*f^(1/2)*n*x))/x)/(4*d^(
1/2)*f^(1/2)) - (atan((2*d^(1/2)*f^(1/2)*x*(n*1i - 1i))/(e*n))*1i)/(2*d^(1
/2)*f^(1/2))
```

3.530 $\int \frac{x}{e^2+4efx^2+4dfx^4+4f^2x^4}\,dx$

3.530.1 Optimal result

Integrand size = 30, antiderivative size = 42

$$\int \frac{x}{e^2+4efx^2+4dfx^4+4f^2x^4}\,dx=\frac{\arctan\left(\frac{\sqrt{f}(e+2(d+f)x^2)}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}$$

output
```
1/4*arctan((e+2*(d+f)*x^2)*f^(1/2)/e/d^(1/2))/e/d^(1/2)/f^(1/2)
```

3.530.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{x}{e^2+4efx^2+4dfx^4+4f^2x^4}\,dx=\frac{\arctan\left(\frac{\sqrt{f}(e+2(d+f)x^2)}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}$$

input
```
Integrate[x/(e^2 + 4*e*f*x^2 + 4*d*f*x^4 + 4*f^2*x^4),x]
```

output
```
ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^2))/(Sqrt[d]*e)]/(4*Sqrt[d]*e*Sqrt[f])
```

3.530.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6, 1432, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{4dfx^4+e^2+4efx^2+4f^2x^4}dx$$

$$\Big\downarrow \; 6$$

$$\int \frac{x}{4x^4(df+f^2)+e^2+4efx^2}dx$$

$$\Big\downarrow \; 1432$$

$$\frac{1}{2}\int \frac{1}{4f(d+f)x^4+4efx^2+e^2}dx^2$$

$$\Big\downarrow \; 1083$$

$$-\int \frac{1}{-x^4-16de^2f}d\left(8f(d+f)x^2+4ef\right)$$

$$\Big\downarrow \; 217$$

$$\frac{\arctan\left(\frac{8fx^2(d+f)+4ef}{4\sqrt{d}e\sqrt{f}}\right)}{4\sqrt{d}e\sqrt{f}}$$

input
```
Int[x/(e^2 + 4*e*f*x^2 + 4*d*f*x^4 + 4*f^2*x^4),x]
```

output
```
ArcTan[(4*e*f + 8*f*(d + f)*x^2)/(4*Sqrt[d]*e*Sqrt[f])]/(4*Sqrt[d]*e*Sqrt[
f])
```

3.530.3.1 Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[Fx, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

3.530.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\arctan\left(\frac{2\left(4df+4f^2\right)x^2+4ef}{4\sqrt{df}\,e}\right)}{4\sqrt{df}\,e}$	42
risch	$-\frac{\ln\left(\left(2\sqrt{-df}-2f\right)x^2-e\right)}{8\sqrt{-df}\,e}+\frac{\ln\left(\left(2\sqrt{-df}+2f\right)x^2+e\right)}{8\sqrt{-df}\,e}$	64

input
```
int(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x,method=_RETURNVERBOSE)
```

output
```
1/4/(d*f)^(1/2)/e*arctan(1/4*(2*(4*d*f+4*f^2)*x^2+4*e*f)/(d*f)^(1/2)/e)
```

3.530.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.69

$$\int \frac{x}{e^2+4efx^2+4dfx^4+4f^2x^4}\,dx = \left[-\frac{\sqrt{-df}\log\left(\frac{4\,(d^2f+2\,df^2+f^3)x^4-de^2+e^2f+4\,(def+ef^2)x^2-2\,(2\,(de+ef)x^2+e^2)\sqrt{-df}}{4\,(df+f^2)x^4+4\,efx^2+e^2}\right)}{8\,def}, \frac{\sqrt{df}\arctan\left(\frac{(2\,(d+f)x^2+e)\sqrt{df}}{de}\right)}{4\,def}\right]$$

input
```
integrate(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="fricas")
```

output
```
[-1/8*sqrt(-d*f)*log((4*(d^2*f + 2*d*f^2 + f^3)*x^4 - d*e^2 + e^2*f + 4*(d
*e*f + e*f^2)*x^2 - 2*(2*(d*e + e*f)*x^2 + e^2)*sqrt(-d*f))/(4*(d*f + f^2)
*x^4 + 4*e*f*x^2 + e^2))/(d*e*f), 1/4*sqrt(d*f)*arctan((2*(d + f)*x^2 + e)
*sqrt(d*f)/(d*e))/(d*e*f)]
```

3.530.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(37) = 74.

Time = 0.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.86

$$\int \frac{x}{e^2+4efx^2+4dfx^4+4f^2x^4}\,dx = \frac{-\frac{\sqrt{-\frac{1}{df}}\log\left(x^2+\frac{-de\sqrt{-\frac{1}{df}}+e}{2d+2f}\right)}{8}+\frac{\sqrt{-\frac{1}{df}}\log\left(x^2+\frac{de\sqrt{-\frac{1}{df}}+e}{2d+2f}\right)}{8}}{e}$$

input
```
integrate(x/(4*d*f*x**4+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

output
```
(-sqrt(-1/(d*f))*log(x**2 + (-d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f))/8 + sqr
t(-1/(d*f))*log(x**2 + (d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f))/8)/e
```

3.530.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{x}{e^2 + 4efx^2 + 4dfx^4 + 4f^2x^4}\,dx = \frac{\arctan\left(\frac{2\,(df+f^2)x^2+ef}{\sqrt{df}e}\right)}{4\sqrt{df}e}$$

input
```
integrate(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="maxima")
```

output
```
1/4*arctan((2*(d*f + f^2)*x^2 + e*f)/(sqrt(d*f)*e))/(sqrt(d*f)*e)
```

3.530.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \frac{x}{e^2 + 4efx^2 + 4dfx^4 + 4f^2x^4}\,dx = \frac{\arctan\left(\frac{2\,dfx^2+2\,f^2x^2+ef}{\sqrt{df}e}\right)}{4\sqrt{df}e}$$

input
```
integrate(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="giac")
```

output
```
1/4*arctan((2*d*f*x^2 + 2*f^2*x^2 + e*f)/(sqrt(d*f)*e))/(sqrt(d*f)*e)
```

3.530.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{x}{e^2 + 4efx^2 + 4dfx^4 + 4f^2x^4}\,dx = \frac{\mathrm{atan}\left(\frac{e\,\sqrt{f}+2\,f^{3/2}\,x^2+2\,d\,\sqrt{f}\,x^2}{\sqrt{d}\,e}\right)}{4\,\sqrt{d}\,e\,\sqrt{f}}$$

input
```
int(x/(e^2 + 4*f^2*x^4 + 4*d*f*x^4 + 4*e*f*x^2),x)
```

output
```
atan((e*f^(1/2) + 2*f^(3/2)*x^2 + 2*d*f^(1/2)*x^2)/(d^(1/2)*e))/(4*d^(1/2)
*e*f^(1/2))
```

3.531 $\int \frac{x}{e^2+4efx^2-4dfx^4+4f^2x^4} dx$

3.531.1 Optimal result

Integrand size = 30, antiderivative size = 44

$$\int \frac{x}{e^2+4efx^2-4dfx^4+4f^2x^4} dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}(e-2(d-f)x^2)}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}$$

output
```
-1/4*arctanh((e-2*(d-f)*x^2)*f^(1/2)/e/d^(1/2))/e/d^(1/2)/f^(1/2)
```

3.531.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{x}{e^2+4efx^2-4dfx^4+4f^2x^4} dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}(e-2dx^2+2fx^2)}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}$$

input
```
Integrate[x/(e^2 + 4*e*f*x^2 - 4*d*f*x^4 + 4*f^2*x^4),x]
```

output
```
-1/4*ArcTanh[(Sqrt[f]*(e - 2*d*x^2 + 2*f*x^2))/(Sqrt[d]*e)]/(Sqrt[d]*e*Sqr
t[f])
```

3.531.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6, 1432, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{-4dfx^4+e^2+4efx^2+4f^2x^4}dx$$

$$\Big\downarrow \ 6$$

$$\int \frac{x}{x^4(4f^2-4df)+e^2+4efx^2}dx$$

$$\Big\downarrow \ 1432$$

$$\frac{1}{2}\int \frac{1}{-4(d-f)fx^4+4efx^2+e^2}dx^2$$

$$\Big\downarrow \ 1083$$

$$-\int \frac{1}{16de^2f-x^4}d\left(4ef-8(d-f)fx^2\right)$$

$$\Big\downarrow \ 219$$

$$-\frac{\text{arctanh}\left(\frac{4ef-8fx^2(d-f)}{4\sqrt{d}e\sqrt{f}}\right)}{4\sqrt{d}e\sqrt{f}}$$

input
```
Int[x/(e^2 + 4*e*f*x^2 - 4*d*f*x^4 + 4*f^2*x^4),x]
```

output
```
-1/4*ArcTanh[(4*e*f - 8*(d - f)*f*x^2)/(4*Sqrt[d]*e*Sqrt[f])]/(Sqrt[d]*e*S
qrt[f])
```

3.531.3.1 Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[Fx, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

3.531.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{2\left(4df-4f^{2}\right)x^{2}-4ef}{4\sqrt{df}\,e}\right)}{4\sqrt{df}\,e}$	42
risch	$\frac{\ln\left(\left(-2\sqrt{df}-2f\right)x^{2}-e\right)}{8\sqrt{df}\,e}-\frac{\ln\left(\left(-2\sqrt{df}+2f\right)x^{2}+e\right)}{8\sqrt{df}\,e}$	60

input
```
int(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x,method=_RETURNVERBOSE)
```

output
```
1/4/(d*f)^(1/2)/e*arctanh(1/4*(2*(4*d*f-4*f^2)*x^2-4*e*f)/(d*f)^(1/2)/e)
```

3.531.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 168, normalized size of antiderivative = 3.82

$$\int \frac{x}{e^2 + 4efx^2 - 4dfx^4 + 4f^2x^4} \, dx$$
$$= \left[\frac{\sqrt{df} \log \left(-\frac{4 \left(d^2 f - 2 \, d f^2 + f^3\right) x^4 + d e^2 + e^2 f - 4 \left(d e f - e f^2\right) x^2 + 2 \left(2 \left(d e - e f\right) x^2 - e^2\right) \sqrt{df}}{4 \left(df - f^2\right) x^4 - 4 \, e f x^2 - e^2} \right)}{8 \, d e f}, \frac{\sqrt{-df} \arctan \left(-\frac{\left(2 \left(d - f\right) x^2 - e\right) \sqrt{-df}}{d e} \right)}{4 \, d e f} \right.$$

input
```
integrate(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="fricas")
```

output
```
[1/8*sqrt(d*f)*log(-(4*(d^2*f - 2*d*f^2 + f^3)*x^4 + d*e^2 + e^2*f - 4*(d*
e*f - e*f^2)*x^2 + 2*(2*(d*e - e*f)*x^2 - e^2)*sqrt(d*f))/(4*(d*f - f^2)*x
^4 - 4*e*f*x^2 - e^2))/(d*e*f), 1/4*sqrt(-d*f)*arctan(-(2*(d - f)*x^2 - e)
*sqrt(-d*f)/(d*e))/(d*e*f)]
```

3.531.6 Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.70

$$\int \frac{x}{e^2 + 4efx^2 - 4dfx^4 + 4f^2x^4} \, dx = -\frac{\frac{\sqrt{\frac{1}{df}} \log \left(x^2 + \frac{-de\sqrt{\frac{1}{df}} - e}{2d - 2f} \right)}{8} - \frac{\sqrt{\frac{1}{df}} \log \left(x^2 + \frac{de\sqrt{\frac{1}{df}} - e}{2d - 2f} \right)}{8}}{e}$$

input
```
integrate(x/(-4*d*f*x**4+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

output
```
-(sqrt(1/(d*f))*log(x**2 + (-d*e*sqrt(1/(d*f)) - e)/(2*d - 2*f))/8 - sqrt(
1/(d*f))*log(x**2 + (d*e*sqrt(1/(d*f)) - e)/(2*d - 2*f))/8)/e
```

3.531.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.52

$$\int \frac{x}{e^2+4efx^2-4dfx^4+4f^2x^4}\,dx=\frac{\log\left(\frac{2\,(df-f^2)x^2-ef+\sqrt{df}e}{2\,(df-f^2)x^2-ef-\sqrt{df}e}\right)}{8\,\sqrt{df}e}$$

input
```
integrate(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="maxima")
```

output
```
1/8*log((2*(d*f - f^2)*x^2 - e*f + sqrt(d*f)*e)/(2*(d*f - f^2)*x^2 - e*f -
 sqrt(d*f)*e))/(sqrt(d*f)*e)
```

3.531.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{x}{e^2+4efx^2-4dfx^4+4f^2x^4}\,dx=-\frac{\arctan\left(\frac{2\,dfx^2-2\,f^2x^2-ef}{\sqrt{-df}e}\right)}{4\,\sqrt{-df}e}$$

input
```
integrate(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="giac")
```

output
```
-1/4*arctan((2*d*f*x^2 - 2*f^2*x^2 - e*f)/(sqrt(-d*f)*e))/(sqrt(-d*f)*e)
```

3.531.9 Mupad [B] (verification not implemented)

Time = 17.48 (sec) , antiderivative size = 199, normalized size of antiderivative = 4.52

$$\int \frac{x}{e^2+4efx^2-4dfx^4+4f^2x^4}\,dx$$
$$=\frac{\mathrm{atanh}\left(\frac{16\,d^{3/2}\,f^{3/2}\,x^2}{\frac{8\,e\,f^3}{d}-32\,f^3\,x^2-16\,e\,f^2+16\,d\,f^2\,x^2+8\,d\,e\,f+\frac{16\,f^4\,x^2}{d}}-\frac{32\,\sqrt{d}\,f^{5/2}\,x^2}{\frac{8\,e\,f^3}{d}-32\,f^3\,x^2-16\,e\,f^2+16\,d\,f^2\,x^2+8\,d\,e\,f+\frac{16\,f^4\,x^2}{d}}+\frac{}{\sqrt{d}\left(\frac{8\,e\,f^3}{d}-32\,f^3\right.}\right.}{4\,\sqrt{d}\,e\,\sqrt{f}}$$

input
```
int(x/(e^2 + 4*f^2*x^4 - 4*d*f*x^4 + 4*e*f*x^2),x)
```

output
```
atanh((16*d^(3/2)*f^(3/2)*x^2)/((8*e*f^3)/d - 32*f^3*x^2 - 16*e*f^2 + 16*d
*f^2*x^2 + 8*d*e*f + (16*f^4*x^2)/d) - (32*d^(1/2)*f^(5/2)*x^2)/((8*e*f^3)
/d - 32*f^3*x^2 - 16*e*f^2 + 16*d*f^2*x^2 + 8*d*e*f + (16*f^4*x^2)/d) + (1
6*f^(7/2)*x^2)/(d^(1/2)*((8*e*f^3)/d - 32*f^3*x^2 - 16*e*f^2 + 16*d*f^2*x^
2 + 8*d*e*f + (16*f^4*x^2)/d)))/(4*d^(1/2)*e*f^(1/2))
```

3.531. $\int \frac{x}{e^2+4efx^2-4dfx^4+4f^2x^4}\,dx$

3.532 $\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4+4dfx^6}\,dx$

3.532.1 Optimal result

Integrand size = 42, antiderivative size = 40

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4+4dfx^6}\,dx=\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

output
```
1/2*arctan(2*x^3*d^(1/2)*f^(1/2)/(2*f*x^2+e))/d^(1/2)/f^(1/2)
```

3.532.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.12

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4+4dfx^6}\,dx =\frac{\text{RootSum}\left[e^2+4ef\#1^2+4f^2\#1^4+4df\#1^6\&,\frac{3e\log(x-\#1)\#1+2f\log(x-\#1)\#1^3}{e+2f\#1^2+3d\#1^4}\&\right]}{8f}$$

input
```
Integrate[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d*f*x^6),
x]
```

output
```
RootSum[e^2 + 4*e*f*#1^2 + 4*f^2*#1^4 + 4*d*f*#1^6 & , (3*e*Log[x - #1]*#1
 + 2*f*Log[x - #1]*#1^3)/(e + 2*f*#1^2 + 3*d*#1^4) & ]/(8*f)
```

3.532.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2520, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(3e+2fx^2)}{4dfx^6+e^2+4efx^2+4f^2x^4}dx$$

$$\Big\downarrow \text{ 2520}$$

$$3e^2\int \frac{1}{\frac{4de^2fx^6}{(2fx^2+e)^2}+e^2}d\frac{x^3}{3(2fx^2+e)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

input
```
Int[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d*f*x^6),x]
```

output
```
ArcTan[(2*Sqrt[d]*Sqrt[f]*x^3)/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

3.532.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2520
```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Simp[A^2*((m - n + 1)/(m + 1)
)   Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
 1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2,
2*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0]
&& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

3.532.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.85

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(4df\_Z^6+4f^2\_Z^4+4ef\_Z^2+e^2\right)}\frac{\left(2\_R^4f+3e\_R^2\right)\ln\left(x-\_R\right)}{3d\_R^5+2\_R^3f+e\_R}}{8f}$	74
risch	$-\frac{\ln\left(-2d^2f^2x^3+2(-df)^{\frac{3}{2}}fx^2+(-df)^{\frac{3}{2}}e\right)}{4\sqrt{-df}}+\frac{\ln\left(2d^2f^2x^3+2(-df)^{\frac{3}{2}}fx^2+(-df)^{\frac{3}{2}}e\right)}{4\sqrt{-df}}$	84

input
```
int(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x,method=_RETURN
VERBOSE)
```

output
```
1/8/f*sum((2*_R^4*f+3*_R^2*e)/(3*_R^5*d+2*_R^3*f+_R*e)*ln(x-_R),_R=RootOf(
4*_Z^6*d*f+4*_Z^4*f^2+4*_Z^2*e*f+e^2))
```

3.532.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(30) = 60.

Time = 0.31 (sec) , antiderivative size = 208, normalized size of antiderivative = 5.20

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4+4dfx^6}\,dx$$
$$=\left[-\frac{\sqrt{-df}\log\left(\frac{4\,dfx^6-4\,f^2x^4-4\,efx^2-e^2-4\,(2\,fx^5+ex^3)\sqrt{-df}}{4\,dfx^6+4\,f^2x^4+4\,efx^2+e^2}\right)}{4\,df}, \frac{\sqrt{df}\arctan\left(\frac{\sqrt{df}x}{f}\right)-\sqrt{df}\arctan\left(\frac{2\,(2\,dfx^5-(de-2\,f^2)\right.}{de^2}\right.}{2\,df}\right.$$

input
```
integrate(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorit
hm="fricas")
```

output
```
[-1/4*sqrt(-d*f)*log((4*d*f*x^6 - 4*f^2*x^4 - 4*e*f*x^2 - e^2 - 4*(2*f*x^5
 + e*x^3)*sqrt(-d*f))/(4*d*f*x^6 + 4*f^2*x^4 + 4*e*f*x^2 + e^2))/(d*f), 1/
2*(sqrt(d*f)*arctan(sqrt(d*f)*x/f) - sqrt(d*f)*arctan(2*(2*d*f*x^5 - (d*e
- 2*f^2)*x^3 + e*f*x)*sqrt(d*f)/(d*e^2)) + sqrt(d*f)*arctan((2*d*f*x^3 - (
d*e - 2*f^2)*x)*sqrt(d*f)/(d*e*f)))/(d*f)]
```

3.532.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(36) = 72$.

Time = 0.65 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.25

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4+4dfx^6}\,dx = -\frac{\sqrt{-\frac{1}{df}}\log\left(-\frac{e\sqrt{-\frac{1}{df}}}{2}-fx^2\sqrt{-\frac{1}{df}}+x^3\right)}{4} + \frac{\sqrt{-\frac{1}{df}}\log\left(\frac{e\sqrt{-\frac{1}{df}}}{2}+fx^2\sqrt{-\frac{1}{df}}+x^3\right)}{4}$$

input
```
integrate(x**2*(2*f*x**2+3*e)/(4*d*f*x**6+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

output
```
-sqrt(-1/(d*f))*log(-e*sqrt(-1/(d*f))/2 - f*x**2*sqrt(-1/(d*f)) + x**3)/4
+ sqrt(-1/(d*f))*log(e*sqrt(-1/(d*f))/2 + f*x**2*sqrt(-1/(d*f)) + x**3)/4
```

3.532.7 Maxima [F]

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4+4dfx^6}\,dx = \int \frac{(2\,fx^2+3\,e)x^2}{4\,dfx^6+4\,f^2x^4+4\,efx^2+e^2}\,dx$$

input
```
integrate(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorit
hm="maxima")
```

output
```
integrate((2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 + 4*f^2*x^4 + 4*e*f*x^2 + e^2), x
)
```

3.532.8 Giac [F]

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4+4dfx^6}\,dx = \int \frac{(2\,fx^2+3\,e)x^2}{4\,dfx^6+4\,f^2x^4+4\,efx^2+e^2}\,dx$$

input
```
integrate(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorit
hm="giac")
```

output
```
sage0*x
```

3.532.9 Mupad [B] (verification not implemented)

Time = 17.47 (sec) , antiderivative size = 278, normalized size of antiderivative = 6.95

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4+4dfx^6}\,dx = \frac{\operatorname{atan}\left(\frac{2\,f^2\,x+2\,d\,f\,x^3-d\,e\,x}{\sqrt{d}\,e\,\sqrt{f}}\right)-\operatorname{atan}\left(\frac{1984\,d^{3/2}\,f^{9/2}\,x^3}{432\,d^2\,e^2\,f^2-128\,d\,e\,f^4}+\frac{1728\,d^{5/2}\,f^{7/2}\,x^5}{432\,d^2\,e^2\,f^2-128\,d\,e\,f^4}+\frac{512\,\sqrt{d}\,f^{13/2}\,x^3}{128\,d\,e^2\,f^4-432\,d^2\,e^3\,f^2}+\frac{512\,d^{3/2}\,f^{1}}{128\,d\,e^2\,f^4-43}\right.}{2\,\sqrt{d}\,\sqrt{f}}$$

input

```
int((x^2*(3*e + 2*f*x^2))/(e^2 + 4*f^2*x^4 + 4*d*f*x^6 + 4*e*f*x^2),x)
```

output

```
(atan((2*f^2*x + 2*d*f*x^3 - d*e*x)/(d^(1/2)*e*f^(1/2))) - atan((1984*d^(3
/2)*f^(9/2)*x^3)/(432*d^2*e^2*f^2 - 128*d*e*f^4) + (1728*d^(5/2)*f^(7/2)*x
^5)/(432*d^2*e^2*f^2 - 128*d*e*f^4) + (512*d^(1/2)*f^(13/2)*x^3)/(128*d*e^
2*f^4 - 432*d^2*e^3*f^2) + (512*d^(3/2)*f^(11/2)*x^5)/(128*d*e^2*f^4 - 432
*d^2*e^3*f^2) - (256*d^(1/2)*f^(11/2)*x)/(432*d^2*e^2*f^2 - 128*d*e*f^4) +
 (864*d^(3/2)*e*f^(7/2)*x)/(432*d^2*e^2*f^2 - 128*d*e*f^4) - (864*d^(5/2)*
e*f^(5/2)*x^3)/(432*d^2*e^2*f^2 - 128*d*e*f^4)) + atan((d^(1/2)*x)/f^(1/2)
))/(2*d^(1/2)*f^(1/2))
```

3.533 $\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4-4dfx^6} dx$

3.533.1 Optimal result

Integrand size = 42, antiderivative size = 40

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4-4dfx^6} dx = \frac{\operatorname{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

output
```
1/2*arctanh(2*x^3*d^(1/2)*f^(1/2)/(2*f*x^2+e))/d^(1/2)/f^(1/2)
```

3.533.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.12

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4-4dfx^6} dx = \frac{\text{RootSum}\left[e^2+4ef\#1^2+4f^2\#1^4-4df\#1^6\&,\frac{3e\log(x-\#1)\#1+2f\log(x-\#1)\#1^3}{e+2f\#1^2-3d\#1^4}\&\right]}{8f}$$

input
```
Integrate[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d*f*x^6),
x]
```

output
```
RootSum[e^2 + 4*e*f*#1^2 + 4*f^2*#1^4 - 4*d*f*#1^6 & , (3*e*Log[x - #1]*#1
 + 2*f*Log[x - #1]*#1^3)/(e + 2*f*#1^2 - 3*d*#1^4) & ]/(8*f)
```

3.533.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2520, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(3e+2fx^2)}{-4dfx^6+e^2+4efx^2+4f^2x^4}dx$$

$$\Big\downarrow \text{2520}$$

$$3e^2\int \frac{1}{e^2-\frac{4de^2fx^6}{(2fx^2+e)^2}}d\frac{x^3}{3(2fx^2+e)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\text{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

input
```
Int[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d*f*x^6),x]
```

output
```
ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^3)/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

3.533.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2520
```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Simp[A^2*((m - n + 1)/(m + 1)
)   Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
 1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2,
2*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0]
&& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

3.533.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.92

method	result	size
default	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(4df\,\_Z^6-4f^2\,\_Z^4-4ef\,\_Z^2-e^2\right)}\frac{\left(2\_R^4f+3e\_R^2\right)\ln\left(x-\_R\right)}{3d\_R^5-2\_R^3f-e\_R}}{8f}$	77
risch	$\frac{\ln\left(-2d^2f^2x^3-2(df)^{\frac{3}{2}}fx^2-(df)^{\frac{3}{2}}e\right)}{4\sqrt{df}}-\frac{\ln\left(2d^2f^2x^3-2(df)^{\frac{3}{2}}fx^2-(df)^{\frac{3}{2}}e\right)}{4\sqrt{df}}$	80

input
```
int(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x,method=_RETUR
NVERBOSE)
```

output
```
-1/8/f*sum((2*_R^4*f+3*_R^2*e)/(3*_R^5*d-2*_R^3*f-_R*e)*ln(x-_R),_R=RootOf
(4*_Z^6*d*f-4*_Z^4*f^2-4*_Z^2*e*f-e^2))
```

3.533.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(30) = 60.

Time = 0.30 (sec) , antiderivative size = 213, normalized size of antiderivative = 5.32

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4-4dfx^6}\,dx = \left[\frac{\sqrt{df}\log\left(\frac{4\,dfx^6+4\,f^2x^4+4\,efx^2+e^2+4\,(2\,fx^5+ex^3)\sqrt{df}}{4\,dfx^6-4\,f^2x^4-4\,efx^2-e^2}\right)}{4\,df},\right.$$
$$-\frac{\sqrt{-df}\arctan\left(\frac{\sqrt{-df}x}{f}\right)-\sqrt{-df}\arctan\left(\frac{2\,(2\,dfx^5-(de+2\,f^2)x^3-efx)\sqrt{-df}}{de^2}\right)+\sqrt{-df}\arctan\left(\frac{(2\,dfx^3-(de+2\,f^2)x)}{def}\right.}{2\,df}$$

input
```
integrate(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algori
thm="fricas")
```

output
```
[1/4*sqrt(d*f)*log((4*d*f*x^6 + 4*f^2*x^4 + 4*e*f*x^2 + e^2 + 4*(2*f*x^5 +
 e*x^3)*sqrt(d*f))/(4*d*f*x^6 - 4*f^2*x^4 - 4*e*f*x^2 - e^2))/(d*f), -1/2*
(sqrt(-d*f)*arctan(sqrt(-d*f)*x/f) - sqrt(-d*f)*arctan(2*(2*d*f*x^5 - (d*e
 + 2*f^2)*x^3 - e*f*x)*sqrt(-d*f)/(d*e^2)) + sqrt(-d*f)*arctan((2*d*f*x^3
- (d*e + 2*f^2)*x)*sqrt(-d*f)/(d*e*f)))/(d*f)]
```

3.533.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(36) = 72$.

Time = 0.68 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.00

$$\int \frac{x^2(3e + 2fx^2)}{e^2 + 4efx^2 + 4f^2x^4 - 4dfx^6} \, dx = -\frac{\sqrt{\frac{1}{df}} \log \left(-\frac{e\sqrt{\frac{1}{df}}}{2} - fx^2\sqrt{\frac{1}{df}} + x^3\right)}{4} + \frac{\sqrt{\frac{1}{df}} \log \left(\frac{e\sqrt{\frac{1}{df}}}{2} + fx^2\sqrt{\frac{1}{df}} + x^3\right)}{4}$$

input
```
integrate(x**2*(2*f*x**2+3*e)/(-4*d*f*x**6+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

output
```
-sqrt(1/(d*f))*log(-e*sqrt(1/(d*f))/2 - f*x**2*sqrt(1/(d*f)) + x**3)/4 + s
qrt(1/(d*f))*log(e*sqrt(1/(d*f))/2 + f*x**2*sqrt(1/(d*f)) + x**3)/4
```

3.533.7 Maxima [F]

$$\int \frac{x^2(3e + 2fx^2)}{e^2 + 4efx^2 + 4f^2x^4 - 4dfx^6} \, dx = \int -\frac{(2\,fx^2 + 3\,e)x^2}{4\,dfx^6 - 4\,f^2x^4 - 4\,efx^2 - e^2} \, dx$$

input
```
integrate(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algori
thm="maxima")
```

output
```
-integrate((2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 - 4*f^2*x^4 - 4*e*f*x^2 - e^2),
x)
```

3.533.8 Giac [F]

$$\int \frac{x^2(3e + 2fx^2)}{e^2 + 4efx^2 + 4f^2x^4 - 4dfx^6} \, dx = \int -\frac{(2\,fx^2 + 3\,e)x^2}{4\,dfx^6 - 4\,f^2x^4 - 4\,efx^2 - e^2} \, dx$$

input
```
integrate(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algori
thm="giac")
```

output
```
sage0*x
```

3.533.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.75

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4-4dfx^6}\,dx = \frac{\operatorname{atanh}\left(\frac{2\sqrt{d}\sqrt{f}\,x^3}{2\,f\,x^2+e}\right)}{2\sqrt{d}\sqrt{f}}$$

input
```
int((x^2*(3*e + 2*f*x^2))/(e^2 + 4*f^2*x^4 - 4*d*f*x^6 + 4*e*f*x^2),x)
```

output
```
atanh((2*d^(1/2)*f^(1/2)*x^3)/(e + 2*f*x^2))/(2*d^(1/2)*f^(1/2))
```

3.534 $\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4+4dfx^{2+2m}}\,dx$

3.534.1 Optimal result

Integrand size = 51, antiderivative size = 42

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4+4dfx^{2+2m}}\,dx=\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

output
```
1/2*arctan(2*x^(1+m)*d^(1/2)*f^(1/2)/(2*f*x^2+e))/d^(1/2)/f^(1/2)
```

3.534.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4+4dfx^{2+2m}}\,dx=\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

input
```
Integrate[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^
4 + 4*d*f*x^(2 + 2*m)),x]
```

output
```
ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

3.534.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.86, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {2520, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(e(m+1)+2f(m-1)x^2)}{4dfx^{2m+2}+e^2+4efx^2+4f^2x^4}dx$$

$$\Big\downarrow \text{2520}$$

$$-e^2(1-m)(m+1)\int \frac{1}{\frac{4de^2fx^{2m+2}}{(2fx^2+e)^2}+e^2}d\left(-\frac{x^{m+1}}{(1-m)(m+1)(2fx^2+e)}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{(1-m)(m+1)\arctan\left(\frac{2\sqrt{d}\sqrt{f}(1-m^2)x^{m+1}}{(1-m)(m+1)(e+2fx^2)}\right)}{2\sqrt{d}\sqrt{f}(1-m^2)}$$

input
```
Int[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*
d*f*x^(2 + 2*m)),x]
```

output
```
((1 - m)*(1 + m)*ArcTan[(2*Sqrt[d]*Sqrt[f]*(1 - m^2)*x^(1 + m))/((1 - m)*(
1 + m)*(e + 2*f*x^2))])/(2*Sqrt[d]*Sqrt[f]*(1 - m^2))
```

3.534.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2520
```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Simp[A^2*((m - n + 1)/(m + 1)
)   Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
 1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2,
2*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0]
&& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

3.534.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(32) = 64.

Time = 1.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.86

method	result	size
risch	$-\frac{\ln\left(x^{m}+\frac{\left(2 f \,x^{2}+e \right) \sqrt{-d f}}{2 d f x}\right)}{4 \sqrt{-d f}}+\frac{\ln\left(x^{m}-\frac{\left(2 f \,x^{2}+e \right) \sqrt{-d f}}{2 d f x}\right)}{4 \sqrt{-d f}}$	78

input
```
int(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2+2*m))
,x,method=_RETURNVERBOSE)
```

output
```
-1/4/(-d*f)^(1/2)*ln(x^m+1/2*(2*f*x^2+e)*(-d*f)^(1/2)/d/f/x)+1/4/(-d*f)^(1
/2)*ln(x^m-1/2*(2*f*x^2+e)*(-d*f)^(1/2)/d/f/x)
```

3.534.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.48

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4+4dfx^{2+2m}}\,dx = \left[-\frac{\sqrt{-df}\log\left(-\frac{4\,f^2x^4-4\,dfx^2x^{2\,m}+4\,efx^2+4\,(2\,fx^3+ex)\sqrt{-df}x^m+e^2}{4\,f^2x^4+4\,dfx^2x^{2\,m}+4\,efx^2+e^2}\right)}{4\,df},\ -\frac{\sqrt{df}\arctan\left(\frac{(2\,fx^2+e)\sqrt{df}}{2\,dfxx^m}\right)}{2\,df}\right]$$

input
```
integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2
+2*m)),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-d*f)*log(-(4*f^2*x^4 - 4*d*f*x^2*x^(2*m) + 4*e*f*x^2 + 4*(2*f*
x^3 + e*x)*sqrt(-d*f)*x^m + e^2)/(4*f^2*x^4 + 4*d*f*x^2*x^(2*m) + 4*e*f*x^
2 + e^2))/(d*f), -1/2*sqrt(d*f)*arctan(1/2*(2*f*x^2 + e)*sqrt(d*f)/(d*f*x*
x^m))/(d*f)]
```

3.534.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(37) = 74.

Time = 98.47 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.90

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4+4dfx^{2+2m}}\,dx = \begin{cases} \frac{xx^m}{e} & \text{for } d=0 \wedge f=0 \\ \frac{xx^m}{e+2fx^2} & \text{for } d=0 \\ \frac{xx^m}{e} & \text{for } f=0 \\ \frac{\log\left(-\frac{e\sqrt{-\frac{1}{df}}}{2x}-fx\sqrt{-\frac{1}{df}}+x^m\right)}{4df\sqrt{-\frac{1}{df}}} - \frac{\log\left(\frac{e\sqrt{-\frac{1}{df}}}{2x}+fx\sqrt{-\frac{1}{df}}+x^m\right)}{4df\sqrt{-\frac{1}{df}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*(e*(1+m)+2*f*(-1+m)*x**2)/(e**2+4*e*f*x**2+4*f**2*x**4+4*d*
f*x**(2+2*m)),x)
```

output
```
Piecewise((x*x**m/e, Eq(d, 0) & Eq(f, 0)), (x*x**m/(e + 2*f*x**2), Eq(d, 0
)), (x*x**m/e, Eq(f, 0)), (log(-e*sqrt(-1/(d*f))/(2*x) - f*x*sqrt(-1/(d*f)
) + x**m)/(4*d*f*sqrt(-1/(d*f))) - log(e*sqrt(-1/(d*f))/(2*x) + f*x*sqrt(-
1/(d*f)) + x**m)/(4*d*f*sqrt(-1/(d*f))), True))
```

3.534.7 Maxima [F]

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4+4dfx^{2+2m}}\,dx = \int \frac{(2\,f(m-1)x^2+e(m+1))x^m}{4\,f^2x^4+4\,efx^2+4\,dfx^{2\,m+2}+e^2}\,dx$$

input
```
integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2
+2*m)),x, algorithm="maxima")
```

output
```
integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 + 4*d*f
*x^(2*m + 2) + e^2), x)
```

3.534.8 Giac [F]

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4+4dfx^{2+2m}}\,dx = \int \frac{(2\,f(m-1)x^2+e(m+1))x^m}{4\,f^2x^4+4\,efx^2+4\,dfx^{2\,m+2}+e^2}\,dx$$

input
```
integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2
+2*m)),x, algorithm="giac")
```

output
```
integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 + 4*d*f
*x^(2*m + 2) + e^2), x)
```

3.534.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4+4dfx^{2+2m}}\,dx = \int \frac{x^m\,(2\,f\,(m-1)\,x^2+e\,(m+1))}{e^2+4\,f^2\,x^4+4\,e\,f\,x^2+4\,d\,f\,x^{2\,m+2}}\,dx$$

input
```
int((x^m*(e*(m + 1) + 2*f*x^2*(m - 1)))/(e^2 + 4*f^2*x^4 + 4*e*f*x^2 + 4*d
*f*x^(2*m + 2)),x)
```

output
```
int((x^m*(e*(m + 1) + 2*f*x^2*(m - 1)))/(e^2 + 4*f^2*x^4 + 4*e*f*x^2 + 4*d
*f*x^(2*m + 2)), x)
```

3.535 $\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4-4dfx^{2+2m}}\,dx$

3.535.1 Optimal result

Integrand size = 51, antiderivative size = 42

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4-4dfx^{2+2m}}\,dx=\frac{\operatorname{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

output
```
1/2*arctanh(2*x^(1+m)*d^(1/2)*f^(1/2)/(2*f*x^2+e))/d^(1/2)/f^(1/2)
```

3.535.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4-4dfx^{2+2m}}\,dx=\frac{\operatorname{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

input
```
Integrate[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^
4 - 4*d*f*x^(2 + 2*m)),x]
```

output
```
ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

3.535.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.86, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2520, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(e(m+1)+2f(m-1)x^2)}{-4dfx^{2m+2}+e^2+4efx^2+4f^2x^4}\,dx$$

$$\Big\downarrow \text{2520}$$

$$-e^2(1-m)(m+1)\int \frac{1}{e^2-\frac{4de^2fx^{2m+2}}{(2fx^2+e)^2}}d\left(-\frac{x^{m+1}}{(1-m)(m+1)\left(2fx^2+e\right)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{(1-m)(m+1)\text{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}(1-m^2)x^{m+1}}{(1-m)(m+1)(e+2fx^2)}\right)}{2\sqrt{d}\sqrt{f}\left(1-m^2\right)}$$

input
```
Int[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*
d*f*x^(2 + 2*m)),x]
```

output
```
((1 - m)*(1 + m)*ArcTanh[(2*Sqrt[d]*Sqrt[f]*(1 - m^2)*x^(1 + m))/((1 - m)*
(1 + m)*(e + 2*f*x^2))])/(2*Sqrt[d]*Sqrt[f]*(1 - m^2))
```

3.535.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2520
```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Simp[A^2*((m - n + 1)/(m + 1)
)   Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
 1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2,
2*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0]
&& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

3.535.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(32) = 64$.

Time = 1.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.76

method	result	size
risch	$\frac{\ln\left(x^{m}+\frac{\left(2 f \,x^{2}+e \right) \sqrt{d f}}{2 d f x}\right)}{4 \sqrt{d f}}-\frac{\ln\left(x^{m}-\frac{\left(2 f \,x^{2}+e \right) \sqrt{d f}}{2 d f x}\right)}{4 \sqrt{d f}}$	74

input
```
int(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2+2*m))
,x,method=_RETURNVERBOSE)
```

output
```
1/4/(d*f)^(1/2)*ln(x^m+1/2*(2*f*x^2+e)*(d*f)^(1/2)/d/f/x)-1/4/(d*f)^(1/2)*
ln(x^m-1/2*(2*f*x^2+e)*(d*f)^(1/2)/d/f/x)
```

3.535.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.48

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4-4dfx^{2+2m}}\,dx$$
$$=\left[\frac{\sqrt{df}\log\left(-\frac{4\,f^2x^4+4\,dfx^2x^{2\,m}+4\,efx^2+4\,(2\,fx^3+ex)\sqrt{df}x^m+e^2}{4\,f^2x^4-4\,dfx^2x^{2\,m}+4\,efx^2+e^2}\right)}{4\,df},\ -\frac{\sqrt{-df}\arctan\left(\frac{(2\,fx^2+e)\sqrt{-df}}{2\,dfxx^m}\right)}{2\,df}\right]$$

input
```
integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2
+2*m)),x, algorithm="fricas")
```

output
```
[1/4*sqrt(d*f)*log(-(4*f^2*x^4 + 4*d*f*x^2*x^(2*m) + 4*e*f*x^2 + 4*(2*f*x^
3 + e*x)*sqrt(d*f)*x^m + e^2)/(4*f^2*x^4 - 4*d*f*x^2*x^(2*m) + 4*e*f*x^2 +
 e^2))/(d*f), -1/2*sqrt(-d*f)*arctan(1/2*(2*f*x^2 + e)*sqrt(-d*f)/(d*f*x*x
^m))/(d*f)]
```

3.535.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(37) = 74.

Time = 97.94 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.67

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4-4dfx^{2+2m}}\,dx = \begin{cases} \frac{xx^m}{e} & \text{for } d=0 \wedge f=0 \\ \frac{xx^m}{e+2fx^2} & \text{for } d=0 \\ \frac{xx^m}{e} & \text{for } f=0 \\ -\frac{\log\left(-\frac{e\sqrt{\frac{1}{df}}}{2x}-fx\sqrt{\frac{1}{df}}+x^m\right)}{4df\sqrt{\frac{1}{df}}}+\frac{\log\left(\frac{e\sqrt{\frac{1}{df}}}{2x}+fx\sqrt{\frac{1}{df}}+x^m\right)}{4df\sqrt{\frac{1}{df}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*(e*(1+m)+2*f*(-1+m)*x**2)/(e**2+4*e*f*x**2+4*f**2*x**4-4*d*
f*x**(2+2*m)),x)
```

output
```
Piecewise((x*x**m/e, Eq(d, 0) & Eq(f, 0)), (x*x**m/(e + 2*f*x**2), Eq(d, 0
)), (x*x**m/e, Eq(f, 0)), (-log(-e*sqrt(1/(d*f))/(2*x) - f*x*sqrt(1/(d*f))
 + x**m)/(4*d*f*sqrt(1/(d*f))) + log(e*sqrt(1/(d*f))/(2*x) + f*x*sqrt(1/(d
*f)) + x**m)/(4*d*f*sqrt(1/(d*f))), True))
```

3.535.7 Maxima [F]

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4-4dfx^{2+2m}}\,dx = \int \frac{(2\,f(m-1)x^2+e(m+1))x^m}{4\,f^2x^4+4\,efx^2-4\,dfx^{2\,m+2}+e^2}\,dx$$

input
```
integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2
+2*m)),x, algorithm="maxima")
```

output
```
integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 - 4*d*f
*x^(2*m + 2) + e^2), x)
```

3.535.8 Giac [F]

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4-4dfx^{2+2m}}\,dx = \int \frac{(2\,f(m-1)x^2+e(m+1))x^m}{4\,f^2x^4+4\,efx^2-4\,dfx^{2\,m+2}+e^2}\,dx$$

input
```
integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2
+2*m)),x, algorithm="giac")
```

output
```
integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 - 4*d*f
*x^(2*m + 2) + e^2), x)
```

3.535.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4-4dfx^{2+2m}}\,dx = \int \frac{x^m\,(2\,f\,(m-1)\,x^2+e\,(m+1))}{e^2+4\,f^2\,x^4+4\,e\,f\,x^2-4\,d\,f\,x^{2\,m+2}}\,dx$$

input
```
int((x^m*(e*(m + 1) + 2*f*x^2*(m - 1)))/(e^2 + 4*f^2*x^4 + 4*e*f*x^2 - 4*d
*f*x^(2*m + 2)),x)
```

output
```
int((x^m*(e*(m + 1) + 2*f*x^2*(m - 1)))/(e^2 + 4*f^2*x^4 + 4*e*f*x^2 - 4*d
*f*x^(2*m + 2)), x)
```

3.536 $\int \frac{x(2e-2fx^3)}{e^2+4efx^3+4dfx^4+4f^2x^6}\,dx$

3.536.1 Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3+4dfx^4+4f^2x^6}\,dx = \frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

output
```
1/2*arctan(2*x^2*d^(1/2)*f^(1/2)/(2*f*x^3+e))/d^(1/2)/f^(1/2)
```

3.536.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.15

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3+4dfx^4+4f^2x^6}\,dx = -\frac{\text{RootSum}\left[e^2+4ef\#1^3+4df\#1^4+4f^2\#1^6\&,\frac{-e\log(x-\#1)+f\log(x-\#1)\#1^3}{3e\#1+4d\#1^2+6f\#1^4}\&\right]}{2f}$$

input
```
Integrate[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 + 4*d*f*x^4 + 4*f^2*x^6),x]
```

output
```
-1/2*RootSum[e^2 + 4*e*f*#1^3 + 4*d*f*#1^4 + 4*f^2*#1^6 & , (-(e*Log[x - #
1]) + f*Log[x - #1]*#1^3)/(3*e*#1 + 4*d*#1^2 + 6*f*#1^4) & ]/f
```

3.536.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2520, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(2e-2fx^3)}{4dfx^4+e^2+4efx^3+4f^2x^6}dx$$

$$\Big\downarrow \text{2520}$$

$$-2e^2\int \frac{1}{\frac{4de^2fx^4}{(2fx^3+e)^2}+e^2}d\left(-\frac{x^2}{2(2fx^3+e)}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

input
```
Int[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 + 4*d*f*x^4 + 4*f^2*x^6),x]
```

output
```
ArcTan[(2*Sqrt[d]*Sqrt[f]*x^2)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

3.536.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2520
```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Simp[A^2*((m - n + 1)/(m + 1)
)   Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
 1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2,
2*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0]
&& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

3.536.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.85

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(4f^2\_Z^6+4df\_Z^4+4ef\_Z^3+e^2\right)} \frac{\left(-\_R^4 f+e\_R\right)\ln\left(x-\_R\right)}{6f\_R^5+4d\_R^3+3e\_R^2}}{2f}$
risch	$-\frac{\ln\left(\left(16f(-df)^{\frac{3}{2}}d+54df^3e\right)x^3+\left(54(-df)^{\frac{3}{2}}ef-16d^3f^2\right)x^2+8e(-df)^{\frac{3}{2}}d+27e^2f^2d\right)}{4\sqrt{-df}}+\frac{\ln\left(\left(16f(-df)^{\frac{3}{2}}d-54df^3e\right)x^3+\left(54(-d\right.\right.}{4\sqrt{}}$

input
```
int(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2),x,method=_RETURNV
ERBOSE)
```

output
```
1/2/f*sum((-_R^4*f+_R*e)/(6*_R^5*f+4*_R^3*d+3*_R^2*e)*ln(x-_R),_R=RootOf(4
*_Z^6*f^2+4*_Z^4*d*f+4*_Z^3*e*f+e^2))
```

3.536.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(30) = 60$.

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.82

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3+4dfx^4+4f^2x^6}\,dx = \left[-\frac{\sqrt{-df}\log\left(\frac{4f^2x^6-4dfx^4+4efx^3+e^2+4(2fx^5+ex^2)\sqrt{-df}}{4f^2x^6+4dfx^4+4efx^3+e^2}\right)}{4df}, -\frac{\sqrt{df}\arctan\left(\frac{\sqrt{df}x}{d}\right)-\sqrt{df}\arctan\left(\frac{(2fx^4+2dx^2+ex)\sqrt{df}}{de}\right)}{2df}\right]$$

input
```
integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2),x, algorith
m="fricas")
```

output
```
[-1/4*sqrt(-d*f)*log((4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3 + e^2 + 4*(2*f*x^5
 + e*x^2)*sqrt(-d*f))/(4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3 + e^2))/(d*f), -1
/2*(sqrt(d*f)*arctan(sqrt(d*f)*x/d) - sqrt(d*f)*arctan((2*f*x^4 + 2*d*x^2
+ e*x)*sqrt(d*f)/(d*e)))/(d*f)]
```

3.536.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(36) = 72$.

Time = 0.61 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.82

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3+4dfx^4+4f^2x^6}\,dx = \frac{\sqrt{-\frac{1}{df}}\log\left(-dx^2\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4} - \frac{\sqrt{-\frac{1}{df}}\log\left(dx^2\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}$$

input
```
integrate(x*(-2*f*x**3+2*e)/(4*f**2*x**6+4*d*f*x**4+4*e*f*x**3+e**2),x)
```

output
```
sqrt(-1/(d*f))*log(-d*x**2*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4 - sqrt(-1/(d
*f))*log(d*x**2*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4
```

3.536.7 Maxima [F]

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3+4dfx^4+4f^2x^6}\,dx = \int -\frac{2\,(fx^3-e)x}{4\,f^2x^6+4\,dfx^4+4\,efx^3+e^2}\,dx$$

input
```
integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2),x, algorith
m="maxima")
```

output
```
-2*integrate((f*x^3 - e)*x/(4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3 + e^2), x)
```

3.536.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(30) = 60$.

Time = 0.36 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.78

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3+4dfx^4+4f^2x^6}\,dx = -\frac{\sqrt{-df}\log\left(\left|2\,fx^3+2\,\sqrt{-df}x^2+e\right|\right)}{4\,df} + \frac{\sqrt{-df}\log\left(\left|2\,fx^3-2\,\sqrt{-df}x^2+e\right|\right)}{4\,df}$$

input
```
integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2),x, algorith
m="giac")
```

output
```
-1/4*sqrt(-d*f)*log(abs(2*f*x^3 + 2*sqrt(-d*f)*x^2 + e))/(d*f) + 1/4*sqrt(
-d*f)*log(abs(2*f*x^3 - 2*sqrt(-d*f)*x^2 + e))/(d*f)
```

3.536.9 Mupad [B] (verification not implemented)

Time = 16.92 (sec) , antiderivative size = 233, normalized size of antiderivative = 5.82

$$\int \frac{x(2e - 2fx^3)}{e^2 + 4efx^3 + 4dfx^4 + 4f^2x^6}\,dx = \frac{\operatorname{atan}\left(\frac{128\,d^{7/2}\,\sqrt{f}\,x^2}{64\,d^3\,e+729\,f\,e^3} - \frac{216\,d^{3/2}\,e^2\,\sqrt{f}}{64\,d^3\,e+729\,f\,e^3} + \frac{128\,d^{5/2}\,f^{3/2}\,x^4}{64\,d^3\,e+729\,f\,e^3} + \frac{216\,d^{3/2}\,e\,\sqrt{f}}{64\,d^3+729\,f\,e^2} + \frac{729\,d^{3/2}\,e^2\,f^{3/2}\,x}{64\,d^5+729\,f\,d^2\,e^2} + \frac{1458\,d^{3/2}\,e\,f^{5/2}\,x^4}{64\,d^5+729\,f\,d^2\,e^2} + \frac{64\,d^{5/2}}{64\,d^3\,e-}\right.}{2\,\sqrt{d}\,\sqrt{f}}$$

input
```
int((x*(2*e - 2*f*x^3))/(e^2 + 4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3),x)
```

output
```
(atan((128*d^(7/2)*f^(1/2)*x^2)/(64*d^3*e + 729*e^3*f) - (216*d^(3/2)*e^2*
f^(1/2))/(64*d^3*e + 729*e^3*f) + (128*d^(5/2)*f^(3/2)*x^4)/(64*d^3*e + 72
9*e^3*f) + (216*d^(3/2)*e*f^(1/2))/(729*e^2*f + 64*d^3) + (729*d^(3/2)*e^2
*f^(3/2)*x)/(64*d^5 + 729*d^2*e^2*f) + (1458*d^(3/2)*e*f^(5/2)*x^4)/(64*d^
5 + 729*d^2*e^2*f) + (64*d^(5/2)*e*f^(1/2)*x)/(64*d^3*e + 729*e^3*f) + (14
58*d^(3/2)*e*f^(3/2)*x^2)/(64*d^4 + 729*d*e^2*f)) - atan((f^(1/2)*x)/d^(1/
2)))/(2*d^(1/2)*f^(1/2))
```

3.537 $\int \frac{x(2e-2fx^3)}{e^2+4efx^3-4dfx^4+4f^2x^6} dx$

3.537.1 Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3-4dfx^4+4f^2x^6} dx = \frac{\operatorname{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

output
```
1/2*arctanh(2*x^2*d^(1/2)*f^(1/2)/(2*f*x^3+e))/d^(1/2)/f^(1/2)
```

3.537.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.15

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3-4dfx^4+4f^2x^6} dx = -\frac{\text{RootSum}\left[e^2+4ef\#1^3-4df\#1^4+4f^2\#1^6\&,\frac{-e\log(x-\#1)+f\log(x-\#1)\#1^3}{3e\#1-4d\#1^2+6f\#1^4}\&\right]}{2f}$$

input
```
Integrate[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 - 4*d*f*x^4 + 4*f^2*x^6),x]
```

output
```
-1/2*RootSum[e^2 + 4*e*f*#1^3 - 4*d*f*#1^4 + 4*f^2*#1^6 & , (-(e*Log[x - #
1]) + f*Log[x - #1]*#1^3)/(3*e*#1 - 4*d*#1^2 + 6*f*#1^4) & ]/f
```

3.537.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2520, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(2e-2fx^3)}{-4dfx^4+e^2+4efx^3+4f^2x^6}\,dx$$
$$\Big\downarrow \text{2520}$$
$$-2e^2\int \frac{1}{e^2-\frac{4de^2fx^4}{(2fx^3+e)^2}}d\left(-\frac{x^2}{2(2fx^3+e)}\right)$$
$$\Big\downarrow \text{221}$$
$$\frac{\text{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

input
```
Int[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 - 4*d*f*x^4 + 4*f^2*x^6),x]
```

output
```
ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^2)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

3.537.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2520
```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Simp[A^2*((m - n + 1)/(m + 1)
)   Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
 1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2,
2*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0]
&& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

3.537.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.85

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(4f^2\_Z^6-4df\_Z^4+4ef\_Z^3+e^2\right)}\frac{\left(\_R^4f-e\_R\right)\ln\left(x-\_R\right)}{-6f\_R^5+4d\_R^3-3e\_R^2}}{2f}$
risch	$\frac{\ln\left(\left(-16f(df)^{\frac{3}{2}}d+54df^3e\right)x^3+\left(54(df)^{\frac{3}{2}}ef-16d^3f^2\right)x^2-8e(df)^{\frac{3}{2}}d+27e^2f^2d\right)}{4\sqrt{df}}-\frac{\ln\left(\left(-16f(df)^{\frac{3}{2}}d-54df^3e\right)x^3+\left(54(df)^{\frac{3}{2}}ef+\right.\right.}{4\sqrt{df}}$

input
```
int(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2),x,method=_RETURNV
ERBOSE)
```

output
```
1/2/f*sum((_R^4*f-_R*e)/(-6*_R^5*f+4*_R^3*d-3*_R^2*e)*ln(x-_R),_R=RootOf(4
*_Z^6*f^2-4*_Z^4*d*f+4*_Z^3*e*f+e^2))
```

3.537.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(30) = 60.

Time = 0.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.88

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3-4dfx^4+4f^2x^6}\,dx = \left[\frac{\sqrt{df}\log\left(\frac{4f^2x^6+4dfx^4+4efx^3+e^2+4(2fx^5+ex^2)\sqrt{df}}{4f^2x^6-4dfx^4+4efx^3+e^2}\right)}{4df},\right. \\ \left.-\frac{\sqrt{-df}\arctan\left(\frac{\sqrt{-df}x}{d}\right)-\sqrt{-df}\arctan\left(\frac{(2fx^4-2dx^2+ex)\sqrt{-df}}{de}\right)}{2df}\right]$$

input
```
integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2),x, algorith
m="fricas")
```

output
```
[1/4*sqrt(d*f)*log((4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3 + e^2 + 4*(2*f*x^5 +
 e*x^2)*sqrt(d*f))/(4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3 + e^2))/(d*f), -1/2*
(sqrt(-d*f)*arctan(sqrt(-d*f)*x/d) - sqrt(-d*f)*arctan((2*f*x^4 - 2*d*x^2
+ e*x)*sqrt(-d*f)/(d*e)))/(d*f)]
```

3.537.6 Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.65

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3-4dfx^4+4f^2x^6}\,dx = -\frac{\sqrt{\frac{1}{df}}\log\left(-dx^2\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4} + \frac{\sqrt{\frac{1}{df}}\log\left(dx^2\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}$$

input
```
integrate(x*(-2*f*x**3+2*e)/(4*f**2*x**6-4*d*f*x**4+4*e*f*x**3+e**2),x)
```

output
```
-sqrt(1/(d*f))*log(-d*x**2*sqrt(1/(d*f)) + e/(2*f) + x**3)/4 + sqrt(1/(d*f
))*log(d*x**2*sqrt(1/(d*f)) + e/(2*f) + x**3)/4
```

3.537.7 Maxima [F]

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3-4dfx^4+4f^2x^6}\,dx = \int -\frac{2\,(fx^3-e)x}{4\,f^2x^6-4\,dfx^4+4\,efx^3+e^2}\,dx$$

input
```
integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2),x, algorith
m="maxima")
```

output
```
-2*integrate((f*x^3 - e)*x/(4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3 + e^2), x)
```

3.537.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(30) = 60.

Time = 0.34 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.68

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3-4dfx^4+4f^2x^6}\,dx = \frac{\sqrt{df}\log\left(\left|2\,fx^3+2\,\sqrt{df}x^2+e\right|\right)}{4\,df} - \frac{\sqrt{df}\log\left(\left|2\,fx^3-2\,\sqrt{df}x^2+e\right|\right)}{4\,df}$$

input
```
integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2),x, algorith
m="giac")
```

output
```
1/4*sqrt(d*f)*log(abs(2*f*x^3 + 2*sqrt(d*f)*x^2 + e))/(d*f) - 1/4*sqrt(d*f
)*log(abs(2*f*x^3 - 2*sqrt(d*f)*x^2 + e))/(d*f)
```

3.537.9 Mupad [B] (verification not implemented)

Time = 17.37 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.68

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3-4dfx^4+4f^2x^6}\,dx = -\frac{\operatorname{atanh}\left(\frac{27\,e^2\,\sqrt{f}+54\,e\,f^{3/2}\,x^3-16\,d^2\,\sqrt{f}\,x^2}{8\,d^{3/2}\,e+16\,d^{3/2}\,f\,x^3-54\,\sqrt{d}\,e\,f\,x^2}\right)}{2\,\sqrt{d}\,\sqrt{f}}$$

input
```
int((x*(2*e - 2*f*x^3))/(e^2 + 4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3),x)
```

output
```
-atanh((27*e^2*f^(1/2) + 54*e*f^(3/2)*x^3 - 16*d^2*f^(1/2)*x^2)/(8*d^(3/2)
*e + 16*d^(3/2)*f*x^3 - 54*d^(1/2)*e*f*x^2))/(2*d^(1/2)*f^(1/2))
```

3.538 $\int \frac{x^2}{e^2+4efx^3+4dfx^6+4f^2x^6}\,dx$

3.538.1 Optimal result

Integrand size = 32, antiderivative size = 42

$$\int \frac{x^2}{e^2+4efx^3+4dfx^6+4f^2x^6}\,dx=\frac{\arctan\left(\frac{\sqrt{f}(e+2(d+f)x^3)}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}$$

output
```
1/6*arctan((e+2*(d+f)*x^3)*f^(1/2)/e/d^(1/2))/e/d^(1/2)/f^(1/2)
```

3.538.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{e^2+4efx^3+4dfx^6+4f^2x^6}\,dx=\frac{\arctan\left(\frac{\sqrt{f}(e+2(d+f)x^3)}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}$$

input
```
Integrate[x^2/(e^2 + 4*e*f*x^3 + 4*d*f*x^6 + 4*f^2*x^6),x]
```

output
```
ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^3))/(Sqrt[d]*e)]/(6*Sqrt[d]*e*Sqrt[f])
```

3.538.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6, 1690, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{4dfx^6+e^2+4efx^3+4f^2x^6}dx$$

$$\Big\downarrow \; 6$$

$$\int \frac{x^2}{4x^6(df+f^2)+e^2+4efx^3}dx$$

$$\Big\downarrow \; 1690$$

$$\frac{1}{3}\int \frac{1}{4f(d+f)x^6+4efx^3+e^2}dx^3$$

$$\Big\downarrow \; 1083$$

$$-\frac{2}{3}\int \frac{1}{-x^6-16de^2f}d\left(8f(d+f)x^3+4ef\right)$$

$$\Big\downarrow \; 217$$

$$\frac{\arctan\left(\frac{8fx^3(d+f)+4ef}{4\sqrt{d}e\sqrt{f}}\right)}{6\sqrt{d}e\sqrt{f}}$$

input
```
Int[x^2/(e^2 + 4*e*f*x^3 + 4*d*f*x^6 + 4*f^2*x^6),x]
```

output
```
ArcTan[(4*e*f + 8*f*(d + f)*x^3)/(4*Sqrt[d]*e*Sqrt[f])]/(6*Sqrt[d]*e*Sqrt[
f])
```

3.538.3.1 Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.538.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\arctan\left(\frac{2\left(4df+4f^2\right)x^3+4ef}{4\sqrt{df}\,e}\right)}{6\sqrt{df}\,e}$	42
risch	$-\frac{\ln\left(\left(2\sqrt{-df}-2f\right)x^3-e\right)}{12\sqrt{-df}\,e}+\frac{\ln\left(\left(2\sqrt{-df}+2f\right)x^3+e\right)}{12\sqrt{-df}\,e}$	64

input
```
int(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x,method=_RETURNVERBOSE)
```

output
```
1/6/(d*f)^(1/2)/e*arctan(1/4*(2*(4*d*f+4*f^2)*x^3+4*e*f)/(d*f)^(1/2)/e)
```

3.538.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.69

$$\int \frac{x^2}{e^2 + 4efx^3 + 4dfx^6 + 4f^2x^6}\,dx$$
$$= \left[-\frac{\sqrt{-df}\log\left(\frac{4\,(d^2f+2\,df^2+f^3)x^6+4\,(def+ef^2)x^3-de^2+e^2f-2\,(2\,(de+ef)x^3+e^2)\sqrt{-df}}{4\,(df+f^2)x^6+4\,efx^3+e^2}\right)}{12\,def},\ \frac{\sqrt{df}\arctan\left(\frac{(2\,(d+f)x^3+e)\sqrt{df}}{de}\right)}{6\,def}\right]$$

input
```
integrate(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="fricas")
```

output
```
[-1/12*sqrt(-d*f)*log((4*(d^2*f + 2*d*f^2 + f^3)*x^6 + 4*(d*e*f + e*f^2)*x
^3 - d*e^2 + e^2*f - 2*(2*(d*e + e*f)*x^3 + e^2)*sqrt(-d*f))/(4*(d*f + f^2
)*x^6 + 4*e*f*x^3 + e^2))/(d*e*f), 1/6*sqrt(d*f)*arctan((2*(d + f)*x^3 + e
)*sqrt(d*f)/(d*e))/(d*e*f)]
```

3.538.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(37) = 74$.

Time = 0.45 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.86

$$\int \frac{x^2}{e^2 + 4efx^3 + 4dfx^6 + 4f^2x^6}\,dx = \frac{-\frac{\sqrt{-\frac{1}{df}}\log\left(x^3+\frac{-de\sqrt{-\frac{1}{df}}+e}{2d+2f}\right)}{12}+\frac{\sqrt{-\frac{1}{df}}\log\left(x^3+\frac{de\sqrt{-\frac{1}{df}}+e}{2d+2f}\right)}{12}}{e}$$

input
```
integrate(x**2/(4*d*f*x**6+4*f**2*x**6+4*e*f*x**3+e**2),x)
```

output
```
(-sqrt(-1/(d*f))*log(x**3 + (-d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f))/12 + sq
rt(-1/(d*f))*log(x**3 + (d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f))/12)/e
```

3.538.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{e^2 + 4efx^3 + 4dfx^6 + 4f^2x^6}\,dx = \frac{\arctan\left(\frac{2\,(df+f^2)x^3+ef}{\sqrt{df}e}\right)}{6\,\sqrt{df}e}$$

input
```
integrate(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="maxima")
```

output
```
1/6*arctan((2*(d*f + f^2)*x^3 + e*f)/(sqrt(d*f)*e))/(sqrt(d*f)*e)
```

3.538.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{e^2 + 4efx^3 + 4dfx^6 + 4f^2x^6}\,dx = \frac{\arctan\left(\frac{2\,dfx^3+2\,f^2x^3+ef}{\sqrt{df}e}\right)}{6\,\sqrt{df}e}$$

input
```
integrate(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="giac")
```

output
```
1/6*arctan((2*d*f*x^3 + 2*f^2*x^3 + e*f)/(sqrt(d*f)*e))/(sqrt(d*f)*e)
```

3.538.9 Mupad [B] (verification not implemented)

Time = 17.47 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{e^2 + 4efx^3 + 4dfx^6 + 4f^2x^6}\,dx = \frac{\operatorname{atan}\left(\frac{e\,\sqrt{f}+2\,f^{3/2}\,x^3+2\,d\,\sqrt{f}\,x^3}{\sqrt{d}\,e}\right)}{6\,\sqrt{d}\,e\,\sqrt{f}}$$

input
```
int(x^2/(e^2 + 4*f^2*x^6 + 4*d*f*x^6 + 4*e*f*x^3),x)
```

output
```
atan((e*f^(1/2) + 2*f^(3/2)*x^3 + 2*d*f^(1/2)*x^3)/(d^(1/2)*e))/(6*d^(1/2)
*e*f^(1/2))
```

3.539 $\int \frac{x^2}{e^2+4efx^3-4dfx^6+4f^2x^6}\,dx$

3.539.1 Optimal result

Integrand size = 32, antiderivative size = 44

$$\int \frac{x^2}{e^2+4efx^3-4dfx^6+4f^2x^6}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}(e-2(d-f)x^3)}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}$$

output
```
-1/6*arctanh((e-2*(d-f)*x^3)*f^(1/2)/e/d^(1/2))/e/d^(1/2)/f^(1/2)
```

3.539.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{x^2}{e^2+4efx^3-4dfx^6+4f^2x^6}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{f}(e-2dx^3+2fx^3)}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}$$

input
```
Integrate[x^2/(e^2 + 4*e*f*x^3 - 4*d*f*x^6 + 4*f^2*x^6),x]
```

output
```
-1/6*ArcTanh[(Sqrt[f]*(e - 2*d*x^3 + 2*f*x^3))/(Sqrt[d]*e)]/(Sqrt[d]*e*Sqr
t[f])
```

3.539.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6, 1690, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{-4dfx^6+e^2+4efx^3+4f^2x^6} \, dx$$

$$\Big\downarrow \; 6$$

$$\int \frac{x^2}{x^6(4f^2-4df)+e^2+4efx^3}dx$$

$$\Big\downarrow \; 1690$$

$$\frac{1}{3}\int \frac{1}{-4(d-f)fx^6+4efx^3+e^2}dx^3$$

$$\Big\downarrow \; 1083$$

$$-\frac{2}{3}\int \frac{1}{16de^2f-x^6}d\left(4ef-8(d-f)fx^3\right)$$

$$\Big\downarrow \; 219$$

$$-\frac{\operatorname{arctanh}\left(\frac{4ef-8fx^3(d-f)}{4\sqrt{d}e\sqrt{f}}\right)}{6\sqrt{d}e\sqrt{f}}$$

input
```
Int[x^2/(e^2 + 4*e*f*x^3 - 4*d*f*x^6 + 4*f^2*x^6),x]
```

output
```
-1/6*ArcTanh[(4*e*f - 8*(d - f)*f*x^3)/(4*Sqrt[d]*e*Sqrt[f])]/(Sqrt[d]*e*S
qrt[f])
```

3.539.3.1 Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.539.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{2\left(4df-4f^{2}\right)x^{3}-4ef}{4\sqrt{df}\,e}\right)}{6\sqrt{df}\,e}$	42
risch	$\frac{\ln\left(\left(-2\sqrt{df}-2f\right)x^{3}-e\right)}{12\sqrt{df}\,e}-\frac{\ln\left(\left(-2\sqrt{df}+2f\right)x^{3}+e\right)}{12\sqrt{df}\,e}$	60

input
```
int(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x,method=_RETURNVERBOSE)
```

output
```
1/6/(d*f)^(1/2)/e*arctanh(1/4*(2*(4*d*f-4*f^2)*x^3-4*e*f)/(d*f)^(1/2)/e)
```

3.539.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 168, normalized size of antiderivative = 3.82

$$\int \frac{x^2}{e^2+4efx^3-4dfx^6+4f^2x^6}\,dx = \left[\frac{\sqrt{df}\log\left(-\frac{4\left(d^2f-2\,df^2+f^3\right)x^6-4\left(def-ef^2\right)x^3+de^2+e^2f+2\left(2\left(de-ef\right)x^3-e^2\right)\sqrt{df}}{4\left(df-f^2\right)x^6-4\,efx^3-e^2}\right)}{12\,def}, \frac{\sqrt{-df}\arctan\left(-\frac{\left(2\left(d-f\right)x^3-e\right)\sqrt{-df}}{de}\right)}{6\,def}\right.$$

input
```
integrate(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="fricas")
```

output
```
[1/12*sqrt(d*f)*log(-(4*(d^2*f - 2*d*f^2 + f^3)*x^6 - 4*(d*e*f - e*f^2)*x^
3 + d*e^2 + e^2*f + 2*(2*(d*e - e*f)*x^3 - e^2)*sqrt(d*f))/(4*(d*f - f^2)*
x^6 - 4*e*f*x^3 - e^2))/(d*e*f), 1/6*sqrt(-d*f)*arctan(-(2*(d - f)*x^3 - e
)*sqrt(-d*f)/(d*e))/(d*e*f)]
```

3.539.6 Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.70

$$\int \frac{x^2}{e^2+4efx^3-4dfx^6+4f^2x^6}\,dx = -\frac{\frac{\sqrt{\frac{1}{df}}\log\left(x^3+\frac{-de\sqrt{\frac{1}{df}}-e}{2d-2f}\right)}{12}-\frac{\sqrt{\frac{1}{df}}\log\left(x^3+\frac{de\sqrt{\frac{1}{df}}-e}{2d-2f}\right)}{12}}{e}$$

input
```
integrate(x**2/(-4*d*f*x**6+4*f**2*x**6+4*e*f*x**3+e**2),x)
```

output
```
-(sqrt(1/(d*f))*log(x**3 + (-d*e*sqrt(1/(d*f)) - e)/(2*d - 2*f))/12 - sqrt
(1/(d*f))*log(x**3 + (d*e*sqrt(1/(d*f)) - e)/(2*d - 2*f))/12)/e
```

3.539.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.52

$$\int \frac{x^2}{e^2+4efx^3-4dfx^6+4f^2x^6}\,dx=\frac{\log\left(\frac{2\,(df-f^2)x^3-ef+\sqrt{df}e}{2\,(df-f^2)x^3-ef-\sqrt{df}e}\right)}{12\,\sqrt{df}e}$$

input
```
integrate(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="maxima")
```

output
```
1/12*log((2*(d*f - f^2)*x^3 - e*f + sqrt(d*f)*e)/(2*(d*f - f^2)*x^3 - e*f
- sqrt(d*f)*e))/(sqrt(d*f)*e)
```

3.539.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{e^2+4efx^3-4dfx^6+4f^2x^6}\,dx=-\frac{\arctan\left(\frac{2\,dfx^3-2\,f^2x^3-ef}{\sqrt{-df}e}\right)}{6\,\sqrt{-df}e}$$

input
```
integrate(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="giac")
```

output
```
-1/6*arctan((2*d*f*x^3 - 2*f^2*x^3 - e*f)/(sqrt(-d*f)*e))/(sqrt(-d*f)*e)
```

3.539.9 Mupad [B] (verification not implemented)

Time = 17.36 (sec) , antiderivative size = 923, normalized size of antiderivative = 20.98

$$\int \frac{x^2}{e^2+4efx^3-4dfx^6+4f^2x^6}\,dx$$

$$=\frac{\mathrm{atan}\left(\frac{\frac{\left(x^3\,\left(32\,d^3\,f^3-96\,d^2\,f^4+96\,d\,f^5-32\,f^6\right)+\frac{x^3\,\left(-64\,e\,d^3\,f^4+192\,e\,d^2\,f^5-192\,e\,d\,f^6+64\,e\,f^7\right)+16\,e^2\,f^6-48\,d\,e^2\,f^5+48\,d^2\,e^2\,f^4-16\,d^3\,e^2\,f^3-\frac{x^3\,\left(-384\cdots\right.}{\sqrt{d}\,e\,\sqrt{f}\cdots}\cdots}{\cdots}\right.}{\sqrt{d}\,e\,\sqrt{f}}}{\frac{x^3\,\left(32\,d^3\,f^3-96\,d^2\,f^4+96\,d\,f^5-32\,f^6\right)+\frac{x^3\,\left(-64\,e\,d^3\,f^4+192\,e\,d^2\,f^5-192\,e\,d\,f^6+64\,e\,f^7\right)+16\,e^2\,f^6-48\,d\,e^2\,f^5+48\,d^2\,e^2\,f^4-16\,d^3\,e^2\,f^3-\frac{x^3\,\left(\cdots\right.}{\sqrt{d}\,e\cdots}\cdots}{\cdots}}{\sqrt{d}\,e\,\sqrt{f}}}\cdots\right)}{\cdots}$$

input
```
int(x^2/(e^2 + 4*f^2*x^6 - 4*d*f*x^6 + 4*e*f*x^3),x)
```

output
```
(atan((((x^3*(96*d*f^5 - 32*f^6 - 96*d^2*f^4 + 32*d^3*f^3) + (x^3*(64*e*f^
7 + 192*d^2*e*f^5 - 64*d^3*e*f^4 - 192*d*e*f^6) + 16*e^2*f^6 - 48*d*e^2*f^
5 + 48*d^2*e^2*f^4 - 16*d^3*e^2*f^3 - ((x^3*(384*e^2*f^8 - 1152*d*e^2*f^7
+ 1152*d^2*e^2*f^6 - 384*d^3*e^2*f^5))/12 + 16*e^3*f^7 - 48*d*e^3*f^6 + 48
*d^2*e^3*f^5 - 16*d^3*e^3*f^4)/(d^(1/2)*e*f^(1/2)))/(d^(1/2)*e*f^(1/2)))*1
i)/(d^(1/2)*e*f^(1/2)) + ((x^3*(96*d*f^5 - 32*f^6 - 96*d^2*f^4 + 32*d^3*f^
3) - (x^3*(64*e*f^7 + 192*d^2*e*f^5 - 64*d^3*e*f^4 - 192*d*e*f^6) + 16*e^2
*f^6 - 48*d*e^2*f^5 + 48*d^2*e^2*f^4 - 16*d^3*e^2*f^3 + ((x^3*(384*e^2*f^8
 - 1152*d*e^2*f^7 + 1152*d^2*e^2*f^6 - 384*d^3*e^2*f^5))/12 + 16*e^3*f^7 -
 48*d*e^3*f^6 + 48*d^2*e^3*f^5 - 16*d^3*e^3*f^4)/(d^(1/2)*e*f^(1/2)))/(d^(
1/2)*e*f^(1/2)))*1i)/(d^(1/2)*e*f^(1/2)))/((x^3*(96*d*f^5 - 32*f^6 - 96*d^
2*f^4 + 32*d^3*f^3) + (x^3*(64*e*f^7 + 192*d^2*e*f^5 - 64*d^3*e*f^4 - 192*
d*e*f^6) + 16*e^2*f^6 - 48*d*e^2*f^5 + 48*d^2*e^2*f^4 - 16*d^3*e^2*f^3 - (
(x^3*(384*e^2*f^8 - 1152*d*e^2*f^7 + 1152*d^2*e^2*f^6 - 384*d^3*e^2*f^5))/
12 + 16*e^3*f^7 - 48*d*e^3*f^6 + 48*d^2*e^3*f^5 - 16*d^3*e^3*f^4)/(d^(1/2)
*e*f^(1/2)))/(d^(1/2)*e*f^(1/2)))/(d^(1/2)*e*f^(1/2)) - (x^3*(96*d*f^5 - 3
2*f^6 - 96*d^2*f^4 + 32*d^3*f^3) - (x^3*(64*e*f^7 + 192*d^2*e*f^5 - 64*d^3
*e*f^4 - 192*d*e*f^6) + 16*e^2*f^6 - 48*d*e^2*f^5 + 48*d^2*e^2*f^4 - 16*d^
3*e^2*f^3 + ((x^3*(384*e^2*f^8 - 1152*d*e^2*f^7 + 1152*d^2*e^2*f^6 - 384*d
^3*e^2*f^5))/12 + 16*e^3*f^7 - 48*d*e^3*f^6 + 48*d^2*e^3*f^5 - 16*d^3*e...
```

3.539. $\int \frac{x^2}{e^2+4efx^3-4dfx^6+4f^2x^6}\,dx$

3.540 $\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}}\,dx$

3.540.1 Optimal result

Integrand size = 51, antiderivative size = 42

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}}\,dx=\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

output
```
1/2*arctan(2*x^(1+m)*d^(1/2)*f^(1/2)/(2*f*x^3+e))/d^(1/2)/f^(1/2)
```

3.540.2 Mathematica [A] (verified)

Time = 2.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}}\,dx=\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

input
```
Integrate[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^
6 + 4*d*f*x^(2 + 2*m)),x]
```

output
```
ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

3.540.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2520, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (e(m+1)+2f(m-2)x^3)}{4dfx^{2m+2}+e^2+4efx^3+4f^2x^6}dx$$

$$\Big\downarrow \text{2520}$$

$$-e^2(2-m)(m+1)\int \frac{1}{\frac{4de^2fx^{2m+2}}{(2fx^3+e)^2}+e^2}d\left(-\frac{x^{m+1}}{(2-m)(m+1)\left(2fx^3+e\right)}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

input
```
Int[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6 + 4*
d*f*x^(2 + 2*m)),x]
```

output
```
ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

3.540.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2520
```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Simp[A^2*((m - n + 1)/(m + 1)
)   Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
 1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2,
2*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0]
&& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

3.540.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(32) = 64.

Time = 2.57 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.86

method	result	size
risch	$-\frac{\ln\left(x^{m}+\frac{\left(2 f\,x^{3}+e \right) \sqrt{-d f}}{2 d f x}\right)}{4 \sqrt{-d f}}+\frac{\ln\left(x^{m}-\frac{\left(2 f\,x^{3}+e \right) \sqrt{-d f}}{2 d f x}\right)}{4 \sqrt{-d f}}$	78

input
```
int(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2+2*m))
,x,method=_RETURNVERBOSE)
```

output
```
-1/4/(-d*f)^(1/2)*ln(x^m+1/2*(2*f*x^3+e)*(-d*f)^(1/2)/d/f/x)+1/4/(-d*f)^(1
/2)*ln(x^m-1/2*(2*f*x^3+e)*(-d*f)^(1/2)/d/f/x)
```

3.540.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.48

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}}\,dx = \left[-\frac{\sqrt{-df}\log\left(-\frac{4\,f^2x^6-4\,dfx^2x^{2\,m}+4\,efx^3+4\,\left(2\,fx^4+ex\right)\sqrt{-df}x^m+e^2}{4\,f^2x^6+4\,dfx^2x^{2\,m}+4\,efx^3+e^2}\right)}{4\,df}, -\frac{\sqrt{df}\arctan\left(\frac{\left(2\,fx^3+e\right)\sqrt{df}}{2\,dfxx^m}\right)}{2\,df}\right]$$

input
```
integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2
+2*m)),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-d*f)*log(-(4*f^2*x^6 - 4*d*f*x^2*x^(2*m) + 4*e*f*x^3 + 4*(2*f*
x^4 + e*x)*sqrt(-d*f)*x^m + e^2)/(4*f^2*x^6 + 4*d*f*x^2*x^(2*m) + 4*e*f*x^
3 + e^2))/(d*f), -1/2*sqrt(d*f)*arctan(1/2*(2*f*x^3 + e)*sqrt(d*f)/(d*f*x*
x^m))/(d*f)]
```

3.540.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(e*(1+m)+2*f*(-2+m)*x**3)/(e**2+4*e*f*x**3+4*f**2*x**6+4*d*
f*x**(2+2*m)),x)
```

output
```
Timed out
```

3.540.7 Maxima [F]

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}}\,dx = \int \frac{(2\,f(m-2)x^3+e(m+1))x^m}{4\,f^2x^6+4\,efx^3+4\,dfx^{2\,m+2}+e^2}\,dx$$

input
```
integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2
+2*m)),x, algorithm="maxima")
```

output
```
integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f
*x^(2*m + 2) + e^2), x)
```

3.540.8 Giac [F]

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}}\,dx = \int \frac{(2\,f(m-2)x^3+e(m+1))x^m}{4\,f^2x^6+4\,efx^3+4\,dfx^{2\,m+2}+e^2}\,dx$$

input
```
integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2
+2*m)),x, algorithm="giac")
```

output
```
integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f
*x^(2*m + 2) + e^2), x)
```

3.540.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}}\,dx=\int \frac{x^m\,\left(2\,f\,\left(m-2\right)\,x^3+e\,\left(m+1\right)\right)}{e^2+4\,f^2\,x^6+4\,e\,f\,x^3+4\,d\,f\,x^{2\,m+2}}\,dx$$

input
```
int((x^m*(e*(m + 1) + 2*f*x^3*(m - 2)))/(e^2 + 4*f^2*x^6 + 4*e*f*x^3 + 4*d
*f*x^(2*m + 2)),x)
```

output
```
int((x^m*(e*(m + 1) + 2*f*x^3*(m - 2)))/(e^2 + 4*f^2*x^6 + 4*e*f*x^3 + 4*d
*f*x^(2*m + 2)), x)
```

3.541 $\int \frac{x^m\left(e(1+m)+2f(-2+m)x^3\right)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}}\,dx$

3.541.1 Optimal result

Integrand size = 51, antiderivative size = 42

$$\int \frac{x^m\left(e(1+m)+2f(-2+m)x^3\right)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}}\,dx=\frac{\operatorname{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

output
```
1/2*arctanh(2*x^(1+m)*d^(1/2)*f^(1/2)/(2*f*x^3+e))/d^(1/2)/f^(1/2)
```

3.541.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{x^m\left(e(1+m)+2f(-2+m)x^3\right)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}}\,dx=\frac{\operatorname{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

input
```
Integrate[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^
6 - 4*d*f*x^(2 + 2*m)),x]
```

output
```
ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

3.541.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {2520, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(e(m+1)+2f(m-2)x^3)}{-4dfx^{2m+2}+e^2+4efx^3+4f^2x^6}dx$$
$$\Big\downarrow \text{2520}$$
$$-e^2(2-m)(m+1)\int \frac{1}{e^2-\frac{4de^2fx^{2m+2}}{(2fx^3+e)^2}}d\left(-\frac{x^{m+1}}{(2-m)(m+1)(2fx^3+e)}\right)$$
$$\Big\downarrow \text{221}$$
$$\frac{\text{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

input
```
Int[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6 - 4*
d*f*x^(2 + 2*m)),x]
```

output
```
ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

3.541.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2520
```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Simp[A^2*((m - n + 1)/(m + 1)
)   Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
 1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2,
2*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0]
&& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

3.541.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(32) = 64$.

Time = 2.59 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.76

method	result	size
risch	$\frac{\ln\left(x^{m}+\frac{\left(2 f \,x^{3}+e \right) \sqrt{d f}}{2 d f x}\right)}{4 \sqrt{d f}}-\frac{\ln\left(x^{m}-\frac{\left(2 f \,x^{3}+e \right) \sqrt{d f}}{2 d f x}\right)}{4 \sqrt{d f}}$	74

input
```
int(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2+2*m))
,x,method=_RETURNVERBOSE)
```

output
```
1/4/(d*f)^(1/2)*ln(x^m+1/2*(2*f*x^3+e)*(d*f)^(1/2)/d/f/x)-1/4/(d*f)^(1/2)*
ln(x^m-1/2*(2*f*x^3+e)*(d*f)^(1/2)/d/f/x)
```

3.541.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.48

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}}\,dx$$
$$= \left[\frac{\sqrt{df}\log\left(-\frac{4\,f^2x^6+4\,dfx^2x^{2\,m}+4\,efx^3+4\,(2\,fx^4+ex)\sqrt{df}x^m+e^2}{4\,f^2x^6-4\,dfx^2x^{2\,m}+4\,efx^3+e^2}\right)}{4\,df},\ -\frac{\sqrt{-df}\arctan\left(\frac{(2\,fx^3+e)\sqrt{-df}}{2\,dfxx^m}\right)}{2\,df}\right]$$

input
```
integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2
+2*m)),x, algorithm="fricas")
```

output
```
[1/4*sqrt(d*f)*log(-(4*f^2*x^6 + 4*d*f*x^2*x^(2*m) + 4*e*f*x^3 + 4*(2*f*x^
4 + e*x)*sqrt(d*f)*x^m + e^2)/(4*f^2*x^6 - 4*d*f*x^2*x^(2*m) + 4*e*f*x^3 +
 e^2))/(d*f), -1/2*sqrt(-d*f)*arctan(1/2*(2*f*x^3 + e)*sqrt(-d*f)/(d*f*x*x
^m))/(d*f)]
```

3.541.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(e*(1+m)+2*f*(-2+m)*x**3)/(e**2+4*e*f*x**3+4*f**2*x**6-4*d*
f*x**(2+2*m)),x)
```

output
```
Timed out
```

3.541.7 Maxima [F]

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}}\,dx = \int \frac{(2\,f(m-2)x^3+e(m+1))x^m}{4\,f^2x^6+4\,efx^3-4\,dfx^{2\,m+2}+e^2}\,dx$$

input
```
integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2
+2*m)),x, algorithm="maxima")
```

output
```
integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f
*x^(2*m + 2) + e^2), x)
```

3.541.8 Giac [F]

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}}\,dx = \int \frac{(2\,f(m-2)x^3+e(m+1))x^m}{4\,f^2x^6+4\,efx^3-4\,dfx^{2\,m+2}+e^2}\,dx$$

input
```
integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2
+2*m)),x, algorithm="giac")
```

output
```
integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f
*x^(2*m + 2) + e^2), x)
```

3.541.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}}\,dx = \int \frac{x^m\,(2\,f\,(m-2)\,x^3+e\,(m+1))}{e^2+4\,f^2\,x^6+4\,e\,f\,x^3-4\,d\,f\,x^{2\,m+2}}\,dx$$

input
```
int((x^m*(e*(m + 1) + 2*f*x^3*(m - 2)))/(e^2 + 4*f^2*x^6 + 4*e*f*x^3 - 4*d
*f*x^(2*m + 2)),x)
```

output
```
int((x^m*(e*(m + 1) + 2*f*x^3*(m - 2)))/(e^2 + 4*f^2*x^6 + 4*e*f*x^3 - 4*d
*f*x^(2*m + 2)), x)
```

3.542 $\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx$

3.542.1 Optimal result

Integrand size = 56, antiderivative size = 42

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx=\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

output
```
1/2*arctan(2*x^(1+m)*d^(1/2)*f^(1/2)/(e+2*f*x^n))/d^(1/2)/f^(1/2)
```

3.542.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx=\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

input
```
Integrate[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 + 4*d*f*x^(2 + 2*m)
 + 4*e*f*x^n + 4*f^2*x^(2*n)),x]
```

output
```
ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

3.542.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.67, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2520, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(e(m+1)+2f(m-n+1)x^n)}{4dfx^{2m+2}+e^2+4efx^n+4f^2x^{2n}}\,dx$$

$$\Big\downarrow \; 2520$$

$$e^2(m+1)(m-n+$$

$$1)\int \frac{1}{\frac{4de^2f(m+1)^2(m-n+1)^2x^{2m+2}}{(2f(m+1)(m-n+1)x^n+e(m+1)(m-n+1))^2}+e^2}d\frac{x^{m+1}}{2f(m+1)(m-n+1)x^n+e(m+1)(m-n+1)}$$

$$\Big\downarrow \; 218$$

$$\frac{\arctan\left(\frac{2\sqrt{d}\sqrt{f}(m+1)(m-n+1)x^{m+1}}{e(m+1)(m-n+1)+2f(m+1)(m-n+1)x^n}\right)}{2\sqrt{d}\sqrt{f}}$$

input
```
Int[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 + 4*d*f*x^(2 + 2*m) + 4*e
*f*x^n + 4*f^2*x^(2*n)),x]
```

output
```
ArcTan[(2*Sqrt[d]*Sqrt[f]*(1 + m)*(1 + m - n)*x^(1 + m))/(e*(1 + m)*(1 + m
 - n) + 2*f*(1 + m)*(1 + m - n)*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

3.542.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2520
```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Simp[A^2*((m - n + 1)/(m + 1)
)   Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
 1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2,
2*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0]
&& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

3.542.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(32) = 64$.

Time = 3.54 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.00

method	result	size
risch	$-\frac{\ln\left(x^{n}+\frac{2 x^{m} d f x+\sqrt{-d f}\, e}{2 \sqrt{-d f}\, f}\right)}{4 \sqrt{-d f}}+\frac{\ln\left(x^{n}+\frac{-2 x^{m} d f x+\sqrt{-d f}\, e}{2 \sqrt{-d f}\, f}\right)}{4 \sqrt{-d f}}$	84

input
```
int(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^2*x^(
2*n)),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(-d*f)^(1/2)*ln(x^n+1/2*(2*x^m*d*f*x+(-d*f)^(1/2)*e)/(-d*f)^(1/2)/f)+
1/4/(-d*f)^(1/2)*ln(x^n+1/2*(-2*x^m*d*f*x+(-d*f)^(1/2)*e)/(-d*f)^(1/2)/f)
```

3.542.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.93

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx = \left[-\frac{\sqrt{-df}\log\left(-\frac{4\,dfx^{2}x^{2\,m}-4\,\sqrt{-df}exx^{m}-4\,f^{2}x^{2\,n}-e^{2}-4\,\left(2\,\sqrt{-df}\,fxx^{m}+ef\right)x^{n}}{4\,dfx^{2}x^{2\,m}+4\,f^{2}x^{2\,n}+4\,efx^{n}+e^{2}}\right)}{4\,df}, -\frac{\sqrt{df}\arctan\left(\frac{2\,\sqrt{df}\,fx^{n}+\sqrt{df}e}{2\,dfxx^{m}}\right)}{2\,df}\right]$$

input
```
integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f
^2*x^(2*n)),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-d*f)*log(-(4*d*f*x^2*x^(2*m) - 4*sqrt(-d*f)*e*x*x^m - 4*f^2*x^
(2*n) - e^2 - 4*(2*sqrt(-d*f)*f*x*x^m + e*f)*x^n)/(4*d*f*x^2*x^(2*m) + 4*f
^2*x^(2*n) + 4*e*f*x^n + e^2))/(d*f), -1/2*sqrt(d*f)*arctan(1/2*(2*sqrt(d*
f)*f*x^n + sqrt(d*f)*e)/(d*f*x*x^m))/(d*f)]
```

3.542.6 Sympy [F]

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx=\int \frac{x^m(em+e+2fmx^n-2fnx^n+2fx^n)}{4dfx^{2m+2}+e^2+4efx^n+4f^2x^{2n}}\,dx$$

input
```
integrate(x**m*(e*(1+m)+2*f*(1+m-n)*x**n)/(e**2+4*d*f*x**(2+2*m)+4*e*f*x**
n+4*f**2*x**(2*n)),x)
```

output
```
Integral(x**m*(e*m + e + 2*f*m*x**n - 2*f*n*x**n + 2*f*x**n)/(4*d*f*x**(2*
m + 2) + e**2 + 4*e*f*x**n + 4*f**2*x**(2*n)), x)
```

3.542.7 Maxima [F]

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx=\int \frac{(2\,f(m-n+1)x^n+e(m+1))x^m}{4\,dfx^{2\,m+2}+4\,f^2x^{2\,n}+4\,efx^n+e^2}\,dx$$

input
```
integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f
^2*x^(2*n)),x, algorithm="maxima")
```

output
```
integrate((2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2) + 4*f^2
*x^(2*n) + 4*e*f*x^n + e^2), x)
```

3.542.8 Giac [F]

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx=\int \frac{(2\,f(m-n+1)x^n+e(m+1))x^m}{4\,dfx^{2\,m+2}+4\,f^2x^{2\,n}+4\,efx^n+e^2}\,dx$$

input
```
integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f
^2*x^(2*n)),x, algorithm="giac")
```

output
```
integrate((2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2) + 4*f^2
*x^(2*n) + 4*e*f*x^n + e^2), x)
```

3.542.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx = \int \frac{x^m\,(e\,(m+1)+2\,f\,x^n\,(m-n+1))}{e^2+4\,f^2\,x^{2\,n}+4\,d\,f\,x^{2\,m+2}+4\,e\,f\,x^n}\,dx$$

input
```
int((x^m*(e*(m + 1) + 2*f*x^n*(m - n + 1)))/(e^2 + 4*f^2*x^(2*n) + 4*d*f*x
^(2*m + 2) + 4*e*f*x^n),x)
```

output
```
int((x^m*(e*(m + 1) + 2*f*x^n*(m - n + 1)))/(e^2 + 4*f^2*x^(2*n) + 4*d*f*x
^(2*m + 2) + 4*e*f*x^n), x)
```

3.543 $\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx$

3.543.1 Optimal result

Integrand size = 56, antiderivative size = 42

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx=\frac{\operatorname{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

output
```
1/2*arctanh(2*x^(1+m)*d^(1/2)*f^(1/2)/(e+2*f*x^n))/d^(1/2)/f^(1/2)
```

3.543.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx=\frac{\operatorname{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

input
```
Integrate[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 - 4*d*f*x^(2 + 2*m)
 + 4*e*f*x^n + 4*f^2*x^(2*n)),x]
```

output
```
ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

3.543.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.67, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2520, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(e(m+1)+2f(m-n+1)x^n)}{-4dfx^{2m+2}+e^2+4efx^n+4f^2x^{2n}}dx$$

$$\Big\downarrow \text{2520}$$

$$e^2(m+1)(m-n+1)\int \frac{1}{e^2-\frac{4de^2f(m+1)^2(m-n+1)^2x^{2m+2}}{(2f(m+1)(m-n+1)x^n+e(m+1)(m-n+1))^2}}d\frac{x^{m+1}}{2f(m+1)(m-n+1)x^n+e(m+1)(m-n+1)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\text{arctanh}\left(\frac{2\sqrt{d}\sqrt{f}(m+1)(m-n+1)x^{m+1}}{e(m+1)(m-n+1)+2f(m+1)(m-n+1)x^n}\right)}{2\sqrt{d}\sqrt{f}}$$

input
```
Int[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 - 4*d*f*x^(2 + 2*m) + 4*e
*f*x^n + 4*f^2*x^(2*n)),x]
```

output
```
ArcTanh[(2*Sqrt[d]*Sqrt[f]*(1 + m)*(1 + m - n)*x^(1 + m))/(e*(1 + m)*(1 +
m - n) + 2*f*(1 + m)*(1 + m - n)*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

3.543.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2520
```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Simp[A^2*((m - n + 1)/(m + 1)
)   Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
 1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2,
2*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0]
&& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

3.543.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(32) = 64.

Time = 3.46 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.86

method	result	size
risch	$\frac{\ln\left(x^n+\frac{2x^m dfx+\sqrt{df}\,e}{2\sqrt{df}\,f}\right)}{4\sqrt{df}}-\frac{\ln\left(x^n+\frac{-2x^m dfx+\sqrt{df}\,e}{2\sqrt{df}\,f}\right)}{4\sqrt{df}}$	78

input
```
int(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^2*x^(
2*n)),x,method=_RETURNVERBOSE)
```

output
```
1/4/(d*f)^(1/2)*ln(x^n+1/2*(2*x^m*d*f*x+(d*f)^(1/2)*e)/(d*f)^(1/2)/f)-1/4/
(d*f)^(1/2)*ln(x^n+1/2*(-2*x^m*d*f*x+(d*f)^(1/2)*e)/(d*f)^(1/2)/f)
```

3.543.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.93

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx$$
$$=\left[\frac{\sqrt{df}\log\left(-\frac{4\,dfx^2x^{2\,m}+4\,\sqrt{df}exx^m+4\,f^2x^{2\,n}+e^2+4\,(2\,\sqrt{df}\,fxx^m+ef)x^n}{4\,dfx^2x^{2\,m}-4\,f^2x^{2\,n}-4\,efx^n-e^2}\right)}{4\,df},\right.$$
$$\left.-\frac{\sqrt{-df}\,\arctan\left(\frac{2\,\sqrt{-df}\,fx^n+\sqrt{-df}e}{2\,dfxx^m}\right)}{2\,df}\right]$$

input
```
integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f
^2*x^(2*n)),x, algorithm="fricas")
```

output
```
[1/4*sqrt(d*f)*log(-(4*d*f*x^2*x^(2*m) + 4*sqrt(d*f)*e*x*x^m + 4*f^2*x^(2*
n) + e^2 + 4*(2*sqrt(d*f)*f*x*x^m + e*f)*x^n)/(4*d*f*x^2*x^(2*m) - 4*f^2*x
^(2*n) - 4*e*f*x^n - e^2))/(d*f), -1/2*sqrt(-d*f)*arctan(1/2*(2*sqrt(-d*f)
*f*x^n + sqrt(-d*f)*e)/(d*f*x*x^m))/(d*f)]
```

3.543.6 Sympy [F]

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx = -\int \frac{ex^m}{4dfx^{2m+2}-e^2-4efx^n-4f^2x^{2n}}\,dx - \int \frac{emx^m}{4dfx^{2m+2}-e^2-4efx^n-4f^2x^{2n}}\,dx - \int \frac{2fx^mx^n}{4dfx^{2m+2}-e^2-4efx^n-4f^2x^{2n}}\,dx - \int \frac{2fmx^mx^n}{4dfx^{2m+2}-e^2-4efx^n-4f^2x^{2n}}\,dx - \int \left(-\frac{2fnx^mx^n}{4dfx^{2m+2}-e^2-4efx^n-4f^2x^{2n}}\right)dx$$

input
```
integrate(x**m*(e*(1+m)+2*f*(1+m-n)*x**n)/(e**2-4*d*f*x**(2+2*m)+4*e*f*x**
n+4*f**2*x**(2*n)),x)
```

output
```
-Integral(e*x**m/(4*d*f*x**(2*m + 2) - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)
), x) - Integral(e*m*x**m/(4*d*f*x**(2*m + 2) - e**2 - 4*e*f*x**n - 4*f**2
*x**(2*n)), x) - Integral(2*f*x**m*x**n/(4*d*f*x**(2*m + 2) - e**2 - 4*e*f
*x**n - 4*f**2*x**(2*n)), x) - Integral(2*f*m*x**m*x**n/(4*d*f*x**(2*m + 2
) - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)), x) - Integral(-2*f*n*x**m*x**n/(
4*d*f*x**(2*m + 2) - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)), x)
```

3.543.7 Maxima [F]

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx = \int -\frac{(2\,f(m-n+1)x^n+e(m+1))x^m}{4\,dfx^{2\,m+2}-4\,f^2x^{2\,n}-4\,efx^n-e^2}\,dx$$

input
```
integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f
^2*x^(2*n)),x, algorithm="maxima")
```

output
```
-integrate((2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2) - 4*f^
2*x^(2*n) - 4*e*f*x^n - e^2), x)
```

3.543.8 Giac [F]

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx=\int -\frac{(2\,f(m-n+1)x^n+e(m+1))x^m}{4\,dfx^{2\,m+2}-4\,f^2x^{2\,n}-4\,efx^n-e^2}\,dx$$

input
```
integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f
^2*x^(2*n)),x, algorithm="giac")
```

output
```
integrate(-(2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2) - 4*f^
2*x^(2*n) - 4*e*f*x^n - e^2), x)
```

3.543.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx=\int \frac{x^m\,(e\,(m+1)+2\,f\,x^n\,(m-n+1))}{e^2+4\,f^2\,x^{2\,n}-4\,d\,f\,x^{2\,m+2}+4\,e\,f\,x^n}\,dx$$

input
```
int((x^m*(e*(m + 1) + 2*f*x^n*(m - n + 1)))/(e^2 + 4*f^2*x^(2*n) - 4*d*f*x
^(2*m + 2) + 4*e*f*x^n),x)
```

output
```
int((x^m*(e*(m + 1) + 2*f*x^n*(m - n + 1)))/(e^2 + 4*f^2*x^(2*n) - 4*d*f*x
^(2*m + 2) + 4*e*f*x^n), x)
```

3.544 $\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

3.544.1 Optimal result

Integrand size = 29, antiderivative size = 134

$$\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = -\frac{(2ac^2-d^2)\,x^2}{2b^2c^3}+\frac{d(2ac^2-d^2)\,\sqrt{a+bx^2}}{b^3c^4}-\frac{d(a+bx^2)^{3/2}}{3b^3c^2}+\frac{(a+bx^2)^2}{4b^3c}+\frac{(ac^2-d^2)^2\log\left(d+c\sqrt{a+bx^2}\right)}{b^3c^5}$$

output
```
-1/2*(2*a*c^2-d^2)*x^2/b^2/c^3-1/3*d*(b*x^2+a)^(3/2)/b^3/c^2+1/4*(b*x^2+a)
^2/b^3/c+(a*c^2-d^2)^2*ln(d+c*(b*x^2+a)^(1/2))/b^3/c^5+d*(2*a*c^2-d^2)*(b*
x^2+a)^(1/2)/b^3/c^4
```

3.544.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \frac{3c^2(a+bx^2)\left(-3ac^2+2d^2+bc^2x^2\right)-4cd\sqrt{a+bx^2}(-5ac^2+3d^2+bc^2x^2)+12(-ac^2+d^2)^2\log\left(d+c\sqrt{a\right.}}{12b^3c^5}$$

input
```
Integrate[x^5/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

output
```
(3*c^2*(a + b*x^2)*(-3*a*c^2 + 2*d^2 + b*c^2*x^2) - 4*c*d*Sqrt[a + b*x^2]*
(-5*a*c^2 + 3*d^2 + b*c^2*x^2) + 12*(-(a*c^2) + d^2)^2*Log[d + c*Sqrt[a +
b*x^2]])/(12*b^3*c^5)
```

3.544.3 Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2586, 7267, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{d\sqrt{a+bx^2}+ac+bcx^2}dx$$

$$\Big\downarrow \text{2586}$$

$$\frac{1}{2}\int \frac{x^4}{bcx^2+ac+d\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \text{7267}$$

$$\frac{\int \frac{(a-x^4)^2}{\sqrt{bx^2+ac+d}}d\sqrt{bx^2+a}}{b^3}$$

$$\Big\downarrow \text{476}$$

$$\frac{\int \left(\frac{x^6}{c}-\frac{dx^4}{c^2}+\frac{2ac^2d-d^3}{c^4}-\frac{(2ac^2-d^2)\sqrt{bx^2+a}}{c^3}+\frac{(ac^2-d^2)^2}{c^4\left(\sqrt{bx^2+ac}+d\right)}\right)d\sqrt{bx^2+a}}{b^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(ac^2-d^2)^2\log\left(c\sqrt{a+bx^2}+d\right)}{c^5}+\frac{d\sqrt{a+bx^2}(2ac^2-d^2)}{c^4}-\frac{x^4(2ac^2-d^2)}{2c^3}-\frac{dx^6}{3c^2}+\frac{x^8}{4c}}{b^3}$$

input
```
Int[x^5/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

output
```
(-1/2*((2*a*c^2 - d^2)*x^4)/c^3 - (d*x^6)/(3*c^2) + x^8/(4*c) + (d*(2*a*c^
2 - d^2)*Sqrt[a + b*x^2])/c^4 + ((a*c^2 - d^2)^2*Log[d + c*Sqrt[a + b*x^2]
])/c^5)/b^3
```

3.544. $\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}}dx$

3.544.3.1 Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2586
```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Simp[1/n   Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a
+ b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*
d, 0] && IntegerQ[(m + 1)/n]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.544.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1811 vs. 2(122) = 244.

Time = 0.14 (sec) , antiderivative size = 1812, normalized size of antiderivative = 13.52

method	result	size
default	Expression too large to display	1812

input
```
int(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
d*(-1/3/b^3/c^2*(b*x^2+a)^(3/2)-1/2/b^2*c^2*a^2/((-a*b)^(1/2)*c^2+(-(a*c^2
-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((b*(x-1
/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2)+(-a*b)^(1/2)
*ln(((x-1/b*(-a*b)^(1/2))*b+(-a*b)^(1/2))/b^(1/2)+(b*(x-1/b*(-a*b)^(1/2))^
2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2))-1/2/b^2*c^2*a^2/((-
a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)
*b*c^2)^(1/2))*((b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/
2)))^(1/2)-(-a*b)^(1/2)*ln(((x+1/b*(-a*b)^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+(
b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2))/b^(1/
2))-1/2*(-a^2*c^4+2*a*c^2*d^2-d^4)/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c
^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/c^2*((b*(x+(-(a*c
^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2
-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2)-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*ln
((-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/
b^(1/2)+(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(
1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/b^(1/2)-d^2/
c^2/(d^2/c^2)^(1/2)*ln((2*d^2/c^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a
*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)
^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(
1/2)/b/c^2)+d^2/c^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)))-1/2*...
```

3.544.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.74

$$\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$$
$$= \frac{3\,b^2c^4x^4-6\,(abc^4-bc^2d^2)x^2+6\,(a^2c^4-2\,ac^2d^2+d^4)\log\left(bc^2x^2+ac^2-d^2\right)+3\,(a^2c^4-2\,ac^2d^2+d^4)\log}{}$$

input
```
integrate(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

output
```
1/12*(3*b^2*c^4*x^4 - 6*(a*b*c^4 - b*c^2*d^2)*x^2 + 6*(a^2*c^4 - 2*a*c^2*d
^2 + d^4)*log(b*c^2*x^2 + a*c^2 - d^2) + 3*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*l
og(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - 3*(a^2*c^4 -
2*a*c^2*d^2 + d^4)*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)/
x^2) - 4*(b*c^3*d*x^2 - 5*a*c^3*d + 3*c*d^3)*sqrt(b*x^2 + a))/(b^3*c^5)
```

3.544.6 Sympy [A] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.03

$$\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{2\left(\frac{(a+bx^2)^2}{8c}-\frac{d(a+bx^2)^{\frac{3}{2}}}{6c^2}+\frac{(a+bx^2)(-2ac^2+d^2)}{4c^3}+\frac{\sqrt{a+bx^2}\cdot(2ac^2d-d^3)}{2c^4}+\frac{(ac^2-d^2)^2\left(\begin{cases}\frac{\sqrt{a+bx^2}}{d} & \text{for } c=0\\ \frac{\log\left(c\sqrt{a+bx^2}+d\right)}{c} & \text{otherwise}\end{cases}\right)}{2c^4}\right)}{b^3} & \text{for } b \neq \\ \frac{x^6}{3\cdot(2\sqrt{a}d+2ac)} & \text{otherw} \end{cases}$$

input
```
integrate(x**5/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

output
```
Piecewise((2*((a + b*x**2)**2/(8*c) - d*(a + b*x**2)**(3/2)/(6*c**2) + (a
+ b*x**2)*(-2*a*c**2 + d**2)/(4*c**3) + sqrt(a + b*x**2)*(2*a*c**2*d - d**
3)/(2*c**4) + (a*c**2 - d**2)**2*Piecewise((sqrt(a + b*x**2)/d, Eq(c, 0)),
 (log(c*sqrt(a + b*x**2) + d)/c, True))/(2*c**4))/b**3, Ne(b, 0)), (x**6/(
3*(2*sqrt(a)*d + 2*a*c)), True))
```

3.544.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.93

$$\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \frac{\frac{3\,(bx^2+a)^2c^3-4\,(bx^2+a)^{\frac{3}{2}}c^2d-6\,(2\,ac^3-cd^2)\,(bx^2+a)+12\,(2\,ac^2d-d^3)\sqrt{bx^2+a}}{c^4}+\frac{12\,(a^2c^4-2\,ac^2d^2+d^4)\log\left(\sqrt{bx^2+a}c+d\right)}{c^5}}{12\,b^3}$$

input
```
integrate(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

output
```
1/12*((3*(b*x^2 + a)^2*c^3 - 4*(b*x^2 + a)^(3/2)*c^2*d - 6*(2*a*c^3 - c*d^
2)*(b*x^2 + a) + 12*(2*a*c^2*d - d^3)*sqrt(b*x^2 + a))/c^4 + 12*(a^2*c^4 -
 2*a*c^2*d^2 + d^4)*log(sqrt(b*x^2 + a)*c + d)/c^5)/b^3
```

3.544.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.16

$$\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \frac{(a^2c^4-2\,ac^2d^2+d^4)\log\left(\left|\sqrt{bx^2+a}c+d\right|\right)}{b^3c^5} + \frac{3\,(bx^2+a)^2b^9c^3-12\,(bx^2+a)ab^9c^3-4\,(bx^2+a)^{\frac{3}{2}}b^9c^2d+24\,\sqrt{bx^2+a}ab^9c^2d+6\,(bx^2+a)b^9cd^2-12\ldots}{12\,b^{12}c^4}$$

input
```
integrate(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

output
```
(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(abs(sqrt(b*x^2 + a)*c + d))/(b^3*c^5) +
1/12*(3*(b*x^2 + a)^2*b^9*c^3 - 12*(b*x^2 + a)*a*b^9*c^3 - 4*(b*x^2 + a)^(
3/2)*b^9*c^2*d + 24*sqrt(b*x^2 + a)*a*b^9*c^2*d + 6*(b*x^2 + a)*b^9*c*d^2
- 12*sqrt(b*x^2 + a)*b^9*d^3)/(b^12*c^4)
```

3.544.9 Mupad [B] (verification not implemented)

Time = 17.48 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.25

$$\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \frac{x^4}{4\,b\,c}-\sqrt{b\,x^2+a}\left(\frac{d^3}{b^3\,c^4}-\frac{2\,a\,d}{b^3\,c^2}\right)-\frac{d\,(b\,x^2+a)^{3/2}}{3\,b^3\,c^2} - \frac{x^2\,(a\,c^2-d^2)}{2\,b^2\,c^3}+\frac{\mathrm{atanh}\left(\frac{c\,\sqrt{b\,x^2+a}}{d}\right)\,(a\,c^2-d^2)^2}{b^3\,c^5} + \frac{\ln\left(b\,c^2\,x^2+a\,c^2-d^2\right)\,(a^2\,c^4-2\,a\,c^2\,d^2+d^4)}{2\,b^3\,c^5}$$

input
```
int(x^5/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2),x)
```

output
```
x^4/(4*b*c) - (a + b*x^2)^(1/2)*(d^3/(b^3*c^4) - (2*a*d)/(b^3*c^2)) - (d*(
a + b*x^2)^(3/2))/(3*b^3*c^2) - (x^2*(a*c^2 - d^2))/(2*b^2*c^3) + (atanh((
c*(a + b*x^2)^(1/2))/d)*(a*c^2 - d^2)^2)/(b^3*c^5) + (log(a*c^2 - d^2 + b*
c^2*x^2)*(d^4 + a^2*c^4 - 2*a*c^2*d^2))/(2*b^3*c^5)
```

3.545 $\int \frac{x^3}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

3.545.1 Optimal result

Integrand size = 29, antiderivative size = 69

$$\int \frac{x^3}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \frac{x^2}{2bc}-\frac{d\sqrt{a+bx^2}}{b^2c^2}-\frac{(ac^2-d^2)\log\left(d+c\sqrt{a+bx^2}\right)}{b^2c^3}$$

output
```
1/2*x^2/b/c-(a*c^2-d^2)*ln(d+c*(b*x^2+a)^(1/2))/b^2/c^3-d*(b*x^2+a)^(1/2)/
b^2/c^2
```

3.545.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{x^3}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \frac{c\left(ac+bcx^2-2d\sqrt{a+bx^2}\right)+(-2ac^2+2d^2)\log\left(d+c\sqrt{a+bx^2}\right)}{2b^2c^3}$$

input
```
Integrate[x^3/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

output
```
(c*(a*c + b*c*x^2 - 2*d*Sqrt[a + b*x^2]) + (-2*a*c^2 + 2*d^2)*Log[d + c*Sq
rt[a + b*x^2]])/(2*b^2*c^3)
```

3.545.3 Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2586, 7267, 25, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{d\sqrt{a+bx^2}+ac+bcx^2}\,dx$$

$$\Big\downarrow \; 2586$$

$$\frac{1}{2}\int \frac{x^2}{bcx^2+ac+d\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \; 7267$$

$$\frac{\int -\frac{a-x^4}{\sqrt{bx^2+ac}+d}d\sqrt{bx^2+a}}{b^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{a-x^4}{\sqrt{bx^2+ac}+d}d\sqrt{bx^2+a}}{b^2}$$

$$\Big\downarrow \; 476$$

$$-\frac{\int \left(\frac{d}{c^2}-\frac{\sqrt{bx^2+a}}{c}+\frac{ac^2-d^2}{c^2\left(\sqrt{bx^2+ac}+d\right)}\right)d\sqrt{bx^2+a}}{b^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{d\sqrt{a+bx^2}}{c^2}-\frac{(ac^2-d^2)\log\left(c\sqrt{a+bx^2}+d\right)}{c^3}+\frac{x^4}{2c}}{b^2}$$

input
```
Int[x^3/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

output
```
(x^4/(2*c) - (d*Sqrt[a + b*x^2])/c^2 - ((a*c^2 - d^2)*Log[d + c*Sqrt[a + b
*x^2]])/c^3)/b^2
```

3.545.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2586
```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Simp[1/n   Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a
+ b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*
d, 0] && IntegerQ[(m + 1)/n]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.545.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1698 vs. $2(63) = 126$.

Time = 0.08 (sec) , antiderivative size = 1699, normalized size of antiderivative = 24.62

method	result	size
default	Expression too large to display	1699

input
```
int(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
d*(-1/2*c^2*a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)
*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b*((b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/
2)*(x-1/b*(-a*b)^(1/2)))^(1/2)+(-a*b)^(1/2)*ln(((x-1/b*(-a*b)^(1/2))*b+(-a
*b)^(1/2))/b^(1/2)+(b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^
(1/2)))^(1/2))/b^(1/2))-1/2*c^2*a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(
1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b*((b*(x+1/b*(-a*b)^(
1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2)-(-a*b)^(1/2)*ln(((x+1/b
*(-a*b)^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+(b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^
(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2))+1/2*(a*c^2-d^2)/((-a*b)^(1/2)*
c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1
/2))/b*((b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(
1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2)+(-(a*c^2-d^2)
*b*c^2)^(1/2)/c^2*ln(((-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+(x-(-(a*c^2-d^2)*b*c^
2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(
a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^
(1/2))/b^(1/2)-d^2/c^2/(d^2/c^2)^(1/2)*ln((2*d^2/c^2+2*(-(a*c^2-d^2)*b*c^2
)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x-(
-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(
a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/
2)/b/c^2)))+1/2*(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2...
```

3.545.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(63) = 126$.

Time = 0.30 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.33

$$\int \frac{x^3}{ac + bcx^2 + d\sqrt{a + bx^2}}\,dx = \frac{2\,bc^2x^2 - 4\,\sqrt{bx^2 + a}cd - 2\,(ac^2 - d^2)\log\left(bc^2x^2 + ac^2 - d^2\right) - (ac^2 - d^2)\log\left(-\frac{bc^2x^2 + ac^2 + 2\,\sqrt{bx^2 + a}cd + d^2}{x^2}\right) + \cdots}{4\,b^2c^3}$$

input
```
integrate(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

output
```
1/4*(2*b*c^2*x^2 - 4*sqrt(b*x^2 + a)*c*d - 2*(a*c^2 - d^2)*log(b*c^2*x^2 +
 a*c^2 - d^2) - (a*c^2 - d^2)*log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*
c*d + d^2)/x^2) + (a*c^2 - d^2)*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a
)*c*d + d^2)/x^2))/(b^2*c^3)
```

3.545.6 Sympy [A] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.23

$$\int \frac{x^3}{ac + bcx^2 + d\sqrt{a + bx^2}}\,dx = \begin{cases} \frac{2\left(\frac{a+bx^2}{4c} - \frac{d\sqrt{a+bx^2}}{2c^2} - \frac{\left(ac^2-d^2\right)\left(\begin{cases} \frac{\sqrt{a+bx^2}}{d} & \text{for } c = 0 \\ \frac{\log\left(c\sqrt{a+bx^2}+d\right)}{c} & \text{otherwise} \end{cases}\right)}{2c^2}\right)}{b^2} & \text{for } b \neq 0 \\ \frac{x^4}{2\cdot\left(2\sqrt{a}d+2ac\right)} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

output
```
Piecewise((2*((a + b*x**2)/(4*c) - d*sqrt(a + b*x**2)/(2*c**2) - (a*c**2 -
 d**2)*Piecewise((sqrt(a + b*x**2)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**2) +
 d)/c, True))/(2*c**2))/b**2, Ne(b, 0)), (x**4/(2*(2*sqrt(a)*d + 2*a*c)),
True))
```

3.545.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{ac + bcx^2 + d\sqrt{a + bx^2}}\,dx = \frac{\frac{(bx^2+a)c-2\sqrt{bx^2+a}d}{c^2} - \frac{2\left(ac^2-d^2\right)\log\left(\sqrt{bx^2+a}c+d\right)}{c^3}}{2b^2}$$

input
```
integrate(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

output
```
1/2*(((b*x^2 + a)*c - 2*sqrt(b*x^2 + a)*d)/c^2 - 2*(a*c^2 - d^2)*log(sqrt(
b*x^2 + a)*c + d)/c^3)/b^2
```

3.545.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.04

$$\int \frac{x^3}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = -\frac{\frac{2\,(ac^2-d^2)\log\left(\left|\sqrt{bx^2+a}c+d\right|\right)}{bc^3}-\frac{(bx^2+a)bc-2\,\sqrt{bx^2+a}bd}{b^2c^2}}{2\,b}$$

input
```
integrate(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

output
```
-1/2*(2*(a*c^2 - d^2)*log(abs(sqrt(b*x^2 + a)*c + d))/(b*c^3) - ((b*x^2 +
a)*b*c - 2*sqrt(b*x^2 + a)*b*d)/(b^2*c^2))/b
```

3.545.9 Mupad [B] (verification not implemented)

Time = 17.31 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.78

$$\int \frac{x^3}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \frac{x^2}{2\,b\,c}-\frac{d\,\sqrt{b\,x^2+a}}{b^2\,c^2}+\frac{\mathrm{atanh}\left(\frac{c\,(a\,c^2-d^2)\,\sqrt{b\,x^2+a}}{d^3-a\,c^2\,d}\right)\,(a\,c^2-d^2)}{b^2\,c^3} - \frac{\ln\left(b\,c^2\,x^2+a\,c^2-d^2\right)\,(a\,c^2-d^2)}{2\,b^2\,c^3}$$

input
```
int(x^3/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2),x)
```

output
```
x^2/(2*b*c) - (d*(a + b*x^2)^(1/2))/(b^2*c^2) + (atanh((c*(a*c^2 - d^2)*(a
 + b*x^2)^(1/2))/(d^3 - a*c^2*d))*(a*c^2 - d^2))/(b^2*c^3) - (log(a*c^2 -
d^2 + b*c^2*x^2)*(a*c^2 - d^2))/(2*b^2*c^3)
```

3.546 $\int \frac{x}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

3.546.1 Optimal result

Integrand size = 27, antiderivative size = 23

$$\int \frac{x}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx=\frac{\log\left(d+c\sqrt{a+bx^2}\right)}{bc}$$

output
```
ln(d+c*(b*x^2+a)^(1/2))/b/c
```

3.546.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{x}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx=\frac{\log\left(bd+bc\sqrt{a+bx^2}\right)}{bc}$$

input
```
Integrate[x/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

output
```
Log[b*d + b*c*Sqrt[a + b*x^2]]/(b*c)
```

3.546.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2586, 7267, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{d\sqrt{a+bx^2}+ac+bcx^2}\,dx$$

$$\Big\downarrow \text{ 2586}$$

$$\frac{1}{2}\int \frac{1}{bcx^2+ac+d\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \text{ 7267}$$

$$\frac{\int \frac{1}{\sqrt{bx^2+ac+d}}d\sqrt{bx^2+a}}{b}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\log\left(c\sqrt{a+bx^2}+d\right)}{bc}$$

input
```
Int[x/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

output
```
Log[d + c*Sqrt[a + b*x^2]]/(b*c)
```

3.546.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2586
```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Simp[1/n   Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a
+ b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*
d, 0] && IntegerQ[(m + 1)/n]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.546.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1627 vs. $2(21) = 42$.

Time = 0.07 (sec) , antiderivative size = 1628, normalized size of antiderivative = 70.78

method	result	size
default	Expression too large to display	1628

input
```
int(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
d*(1/2*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^
2+(-(a*c^2-d^2)*b*c^2)^(1/2))*((b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x
-1/b*(-a*b)^(1/2)))^(1/2)+(-a*b)^(1/2)*ln(((x-1/b*(-a*b)^(1/2))*b+(-a*b)^(
1/2))/b^(1/2)+(b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)
))^(1/2))/b^(1/2))+1/2*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(
-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*((b*(x+1/b*(-a*b)^(1/2))^2-2
*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2)-(-a*b)^(1/2)*ln(((x+1/b*(-a*b)^(
1/2))*b-(-a*b)^(1/2))/b^(1/2)+(b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+
1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2))-1/2*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)
*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*((b*(x+(-(a*
c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^
2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2)-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*l
n((-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)
/b^(1/2)+(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^
(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/b^(1/2)-d^2
/c^2/(d^2/c^2)^(1/2)*ln((2*d^2/c^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(
a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2
)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^
(1/2)/b/c^2)+d^2/c^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)))-1/2*c^
2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a...
```

3.546.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(21) = 42.

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 4.57

$$\int \frac{x}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \frac{2\log\left(bc^2x^2+ac^2-d^2\right)+\log\left(-\frac{bc^2x^2+ac^2+2\sqrt{bx^2+a}cd+d^2}{x^2}\right)-\log\left(-\frac{bc^2x^2+ac^2-2\sqrt{bx^2+a}cd+d^2}{x^2}\right)}{4\,bc}$$

input
```
integrate(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

output
```
1/4*(2*log(b*c^2*x^2 + a*c^2 - d^2) + log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x
^2 + a)*c*d + d^2)/x^2) - log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d
+ d^2)/x^2))/(b*c)
```

3.546.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(17) = 34.

Time = 1.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.00

$$\int \frac{x}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{\begin{cases} \frac{\sqrt{a+bx^2}}{d} & \text{for } c=0 \\ \frac{\log\left(c\sqrt{a+bx^2}+d\right)}{c} & \text{otherwise} \end{cases}}{b} & \text{for } b\neq 0 \\ \frac{x^2}{2\sqrt{a}d+2ac} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

output
```
Piecewise((Piecewise((sqrt(a + b*x**2)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**
2) + d)/c, True))/b, Ne(b, 0)), (x**2/(2*sqrt(a)*d + 2*a*c), True))
```

3.546.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx=\frac{\log\left(\sqrt{bx^2+a}c+d\right)}{bc}$$

input
```
integrate(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

output
```
log(sqrt(b*x^2 + a)*c + d)/(b*c)
```

3.546.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx=\frac{\log\left(\left|\sqrt{bx^2+a}c+d\right|\right)}{bc}$$

input
```
integrate(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

output
```
log(abs(sqrt(b*x^2 + a)*c + d))/(b*c)
```

3.546.9 Mupad [B] (verification not implemented)

Time = 18.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.96

$$\int \frac{x}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx=\frac{\mathrm{atanh}\left(\frac{c\sqrt{b\,x^2+a}}{d}\right)+\frac{\ln\left(b\,c^2\,x^2+a\,c^2-d^2\right)}{2}}{b\,c}$$

input
```
int(x/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2),x)
```

output
```
(atanh((c*(a + b*x^2)^(1/2))/d) + log(a*c^2 - d^2 + b*c^2*x^2)/2)/(b*c)
```

3.547 $\int \frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$

3.547.1 Optimal result

Integrand size = 29, antiderivative size = 88

$$\int \frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx=\frac{d\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,(ac^2-d^2)}+\frac{c\log(x)}{ac^2-d^2}-\frac{c\log\left(d+c\sqrt{a+bx^2}\right)}{ac^2-d^2}$$

output
```
c*ln(x)/(a*c^2-d^2)-c*ln(d+c*(b*x^2+a)^(1/2))/(a*c^2-d^2)+d*arctanh((b*x^2
+a)^(1/2)/a^(1/2))/(a*c^2-d^2)/a^(1/2)
```

3.547.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78

$$\int \frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx=\frac{\frac{2d\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}+c\log\left(bx^2\right)-2c\log\left(d+c\sqrt{a+bx^2}\right)}{2ac^2-2d^2}$$

input
```
Integrate[1/(x*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

output
```
((2*d*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a] + c*Log[b*x^2] - 2*c*Log[d
 + c*Sqrt[a + b*x^2]])/(2*a*c^2 - 2*d^2)
```

3.547.3 Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2586, 7267, 25, 479, 452, 219, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(d\sqrt{a+bx^2}+ac+bcx^2\right)}dx$$

$$\Big\downarrow \text{ 2586}$$

$$\frac{1}{2}\int \frac{1}{x^2\left(bcx^2+ac+d\sqrt{bx^2+a}\right)}dx^2$$

$$\Big\downarrow \text{ 7267}$$

$$\int -\frac{1}{(a-x^4)\left(c\sqrt{a+bx^2}+d\right)}d\sqrt{a+bx^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{1}{(a-x^4)\left(\sqrt{bx^2+a}c+d\right)}d\sqrt{bx^2+a}$$

$$\Big\downarrow \text{ 479}$$

$$\frac{\int \frac{d-c\sqrt{bx^2+a}}{a-x^4}d\sqrt{bx^2+a}}{ac^2-d^2}-\frac{c\log\left(c\sqrt{a+bx^2}+d\right)}{ac^2-d^2}$$

$$\Big\downarrow \text{ 452}$$

$$\frac{d\int \frac{1}{a-x^4}d\sqrt{bx^2+a}-c\int \frac{\sqrt{bx^2+a}}{a-x^4}d\sqrt{bx^2+a}}{ac^2-d^2}-\frac{c\log\left(c\sqrt{a+bx^2}+d\right)}{ac^2-d^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{d\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-c\int \frac{\sqrt{bx^2+a}}{a-x^4}d\sqrt{bx^2+a}}{ac^2-d^2}-\frac{c\log\left(c\sqrt{a+bx^2}+d\right)}{ac^2-d^2}$$

$$\Big\downarrow \text{ 240}$$

$$\frac{\frac{d\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{1}{2}c\log\left(a-x^4\right)}{ac^2-d^2}-\frac{c\log\left(c\sqrt{a+bx^2}+d\right)}{ac^2-d^2}$$

input
```
Int[1/(x*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

output
```
((d*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a] + (c*Log[a - x^4])/2)/(a*c^2
 - d^2) - (c*Log[d + c*Sqrt[a + b*x^2]])/(a*c^2 - d^2)
```

3.547.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 479
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[d*(Log
[RemoveContent[c + d*x, x]]/(b*c^2 + a*d^2)), x] + Simp[b/(b*c^2 + a*d^2)
 Int[(c - d*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2586
```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Simp[1/n   Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a
+ b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*
d, 0] && IntegerQ[(m + 1)/n]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.547. $\int \frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$

3.547.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1696 vs. $2(80) = 160$.

Time = 0.08 (sec) , antiderivative size = 1697, normalized size of antiderivative = 19.28

method	result	size
default	Expression too large to display	1697

input
```
int(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*c^3/(a*c^2-d^2)/d^2*ln(b*c^2*x^2+a*c^2-d^2)+c*ln(x)/(a*c^2-d^2)+1/2
*c/d^2*ln(b*c^2*x^2+a*c^2-d^2)-d*(1/a/(a*c^2-d^2)*((b*x^2+a)^(1/2)-a^(1/2)
*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))-1/2*b*c^2/a/((-a*b)^(1/2)*c^2+(-(a
*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((b*
(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2)+(-a*b)^(
1/2)*ln(((x-1/b*(-a*b)^(1/2))*b+(-a*b)^(1/2))/b^(1/2)+(b*(x-1/b*(-a*b)^(1/
2))^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2))-1/2*b*c^2/a/((-
a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)
*b*c^2)^(1/2))*((b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/
2)))^(1/2)-(-a*b)^(1/2)*ln(((x+1/b*(-a*b)^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+(
b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2))/b^(1/
2))+1/2*b*c^4/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((
-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((b*(x-(-(a*c^2-d^2)*b*c^2)^(1
/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(1/2
)/b/c^2)+d^2/c^2)^(1/2)+(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*ln(((-(a*c^2-d^2)*b
*c^2)^(1/2)/c^2+(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x-(-(a
*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c
^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/b^(1/2)-d^2/c^2/(d^2/c^2)^(1/2
)*ln((2*d^2/c^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^(
1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2...
```

3.547.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.59

$$\int \frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$$
$$= \left[-\frac{2\,ac\log\left(bc^2x^2+ac^2-d^2\right)-4\,ac\log\left(x\right)+ac\log\left(-\frac{bc^2x^2+ac^2+2\sqrt{bx^2+a}cd+d^2}{x^2}\right)-ac\log\left(-\frac{bc^2x^2+ac^2-2\sqrt{bx}}{x^2}\right.}{4\left(a^2c^2-ad^2\right)}\right.$$
$$-\frac{2\,ac\log\left(bc^2x^2+ac^2-d^2\right)-4\,ac\log\left(x\right)+ac\log\left(-\frac{bc^2x^2+ac^2+2\sqrt{bx^2+a}cd+d^2}{x^2}\right)-ac\log\left(-\frac{bc^2x^2+ac^2-2\sqrt{bx^2}}{x^2}\right.}{4\left(a^2c^2-ad^2\right)}$$

input
```
integrate(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

output
```
[-1/4*(2*a*c*log(b*c^2*x^2 + a*c^2 - d^2) - 4*a*c*log(x) + a*c*log(-(b*c^2
*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - a*c*log(-(b*c^2*x^2 + a
*c^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) + 2*sqrt(a)*d*log(-(b*x^2 - 2*sqr
t(b*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a^2*c^2 - a*d^2), -1/4*(2*a*c*log(b*c^2
*x^2 + a*c^2 - d^2) - 4*a*c*log(x) + a*c*log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(
b*x^2 + a)*c*d + d^2)/x^2) - a*c*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 +
a)*c*d + d^2)/x^2) + 4*sqrt(-a)*d*arctan(sqrt(-a)/sqrt(b*x^2 + a)))/(a^2*c
^2 - a*d^2)]
```

3.547.6 Sympy [A] (verification not implemented)

Time = 2.93 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.58

$$\int \frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$$
$$= \begin{cases} \frac{2\left(-\frac{bc^2\left(\begin{cases}\frac{\sqrt{a+bx^2}}{d} & \text{for}\ c=0\\ \frac{\log\left(c\sqrt{a+bx^2}+d\right)}{c} & \text{otherwise}\end{cases}\right)}{2\left(ac^2-d^2\right)}-\frac{b\left(-\frac{c\log\left(-bx^2\right)}{2}+\frac{d\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{\sqrt{-a}}\right)}{2\left(ac^2-d^2\right)}\right)}{b} & \text{for}\ b\neq 0\\ \begin{cases}\frac{x^2\log\left(x^2\right)}{2\sqrt{a}dx^2+2acx^2} & \text{for}\ 2\sqrt{a}d+2ac\neq 0\\ \tilde{\infty}x^2 & \text{otherwise}\end{cases} & \text{otherwise}\end{cases}$$

input
```
integrate(1/x/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

output
```
Piecewise((2*(-b*c**2*Piecewise((sqrt(a + b*x**2)/d, Eq(c, 0)), (log(c*sqr
t(a + b*x**2) + d)/c, True))/(2*(a*c**2 - d**2)) - b*(-c*log(-b*x**2)/2 +
d*atan(sqrt(a + b*x**2)/sqrt(-a))/sqrt(-a))/(2*(a*c**2 - d**2)))/b, Ne(b,
0)), (Piecewise((x**2*log(x**2)/(2*sqrt(a)*d*x**2 + 2*a*c*x**2), Ne(2*sqrt
(a)*d + 2*a*c, 0)), (zoo*x**2, True)), True))
```

3.547.7 Maxima [F]

$$\int \frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx=\int \frac{1}{\left(bcx^2+ac+\sqrt{bx^2+a}\,d\right)x}\,dx$$

input
```
integrate(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x), x)
```

3.547.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx=-\frac{c^2\log\left(\left|\sqrt{bx^2+a}\,c+d\right|\right)}{ac^3-cd^2}+\frac{c\log\left(bx^2\right)}{2\left(ac^2-d^2\right)}-\frac{d\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\left(ac^2-d^2\right)\sqrt{-a}}$$

input
```
integrate(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

output
```
-c^2*log(abs(sqrt(b*x^2 + a)*c + d))/(a*c^3 - c*d^2) + 1/2*c*log(b*x^2)/(a
*c^2 - d^2) - d*arctan(sqrt(b*x^2 + a)/sqrt(-a))/((a*c^2 - d^2)*sqrt(-a))
```

3.547.9 Mupad [B] (verification not implemented)

Time = 18.19 (sec) , antiderivative size = 1270, normalized size of antiderivative = 14.43

$$\int \frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx = \frac{c\ln(x)}{a\,c^2-d^2}$$

$$-\frac{c\,\mathrm{atan}\left(\frac{\frac{c\left(4\,c^6\,d^2\,\sqrt{b\,x^2+a}+\frac{c\left(4\,c^4\,d^5-8\,a\,c^6\,d^3+4\,a^2\,c^8\,d-\frac{c\,\sqrt{b\,x^2+a}\left(8\,a^3\,c^{10}-8\,a^2\,c^8\,d^2-8\,a\,c^6\,d^4+8\,c^4\,d^6\right)}{2\left(a\,c^2-d^2\right)}\right)}{2\left(a\,c^2-d^2\right)}\right)1\mathrm{i}}{2\left(a\,c^2-d^2\right)}+\frac{c\left(4\,c^6\,d^2\,\sqrt{b\,x^2+a}-\frac{c\left(4\,c^4 \cdots\right.}{\cdots}\right.}{\cdots}}{\frac{c\left(4\,c^6\,d^2\,\sqrt{b\,x^2+a}+\frac{c\left(4\,c^4\,d^5-8\,a\,c^6\,d^3+4\,a^2\,c^8\,d-\frac{c\,\sqrt{b\,x^2+a}\left(8\,a^3\,c^{10}-8\,a^2\,c^8\,d^2-8\,a\,c^6\,d^4+8\,c^4\,d^6\right)}{2\left(a\,c^2-d^2\right)}\right)}{2\left(a\,c^2-d^2\right)}\right)}{2\left(a\,c^2-d^2\right)}-\frac{c\left(4\,c^6\,d^2\,\sqrt{b\,x^2+a}-\frac{c\left(4\,c^4 \cdots\right.}{\cdots}\right.}{\cdots}}\right.}{a\,c^2-d^2}$$

$$-\frac{c\ln\left(b\,c^2\,x^2+a\,c^2-d^2\right)}{2\,a\,c^2-2\,d^2}$$

$$-\frac{d\,\mathrm{atan}\left(\frac{\frac{d\left(4\,c^6\,d^2\,\sqrt{b\,x^2+a}+\frac{d\left(4\,c^4\,d^5-8\,a\,c^6\,d^3+4\,a^2\,c^8\,d-\frac{d\,\sqrt{b\,x^2+a}\left(8\,a^3\,c^{10}-8\,a^2\,c^8\,d^2-8\,a\,c^6\,d^4+8\,c^4\,d^6\right)}{\sqrt{a}\left(2\,a\,c^2-2\,d^2\right)}\right)}{\sqrt{a}\left(2\,a\,c^2-2\,d^2\right)}\right)1\mathrm{i}}{\sqrt{a}\left(2\,a\,c^2-2\,d^2\right)}+\frac{d\left(4\,c^6\,d^2\,\sqrt{b\,x^2+a}-\frac{d\left(4\,c^4 \cdots\right.}{\cdots}\right.}{\cdots}}{\frac{d\left(4\,c^6\,d^2\,\sqrt{b\,x^2+a}+\frac{d\left(4\,c^4\,d^5-8\,a\,c^6\,d^3+4\,a^2\,c^8\,d-\frac{d\,\sqrt{b\,x^2+a}\left(8\,a^3\,c^{10}-8\,a^2\,c^8\,d^2-8\,a\,c^6\,d^4+8\,c^4\,d^6\right)}{\sqrt{a}\left(2\,a\,c^2-2\,d^2\right)}\right)}{\sqrt{a}\left(2\,a\,c^2-2\,d^2\right)}\right)}{\sqrt{a}\left(2\,a\,c^2-2\,d^2\right)}-\frac{d\left(4\,c^6\,d^2\,\sqrt{b\,x^2+a}-\frac{d\left(4\,c^4 \cdots\right.}{\cdots}\right.}{\cdots}}\right.}{\sqrt{a}\left(a\,c^2-d^2\right)}$$

input
```
int(1/(x*(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2)),x)
```

output
```
(c*log(x))/(a*c^2 - d^2) - (c*atan(((c*(4*c^6*d^2*(a + b*x^2)^(1/2) + (c*(
4*c^4*d^5 - 8*a*c^6*d^3 + 4*a^2*c^8*d - (c*(a + b*x^2)^(1/2)*(8*a^3*c^10 +
 8*c^4*d^6 - 8*a*c^6*d^4 - 8*a^2*c^8*d^2))/(2*(a*c^2 - d^2))))/(2*(a*c^2 -
 d^2)))*1i)/(2*(a*c^2 - d^2)) + (c*(4*c^6*d^2*(a + b*x^2)^(1/2) - (c*(4*c^
4*d^5 - 8*a*c^6*d^3 + 4*a^2*c^8*d + (c*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8*c
^4*d^6 - 8*a*c^6*d^4 - 8*a^2*c^8*d^2))/(2*(a*c^2 - d^2))))/(2*(a*c^2 - d^2
)))*1i)/(2*(a*c^2 - d^2)))/((c*(4*c^6*d^2*(a + b*x^2)^(1/2) + (c*(4*c^4*d^
5 - 8*a*c^6*d^3 + 4*a^2*c^8*d - (c*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8*c^4*d
^6 - 8*a*c^6*d^4 - 8*a^2*c^8*d^2))/(2*(a*c^2 - d^2))))/(2*(a*c^2 - d^2))))
/(2*(a*c^2 - d^2)) - (c*(4*c^6*d^2*(a + b*x^2)^(1/2) - (c*(4*c^4*d^5 - 8*a
*c^6*d^3 + 4*a^2*c^8*d + (c*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8*c^4*d^6 - 8*
a*c^6*d^4 - 8*a^2*c^8*d^2))/(2*(a*c^2 - d^2))))/(2*(a*c^2 - d^2))))/(2*(a*
c^2 - d^2))))*1i)/(a*c^2 - d^2) - (c*log(a*c^2 - d^2 + b*c^2*x^2))/(2*a*c^
2 - 2*d^2) - (d*atan(((d*(4*c^6*d^2*(a + b*x^2)^(1/2) + (d*(4*c^4*d^5 - 8*
a*c^6*d^3 + 4*a^2*c^8*d - (d*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8*c^4*d^6 - 8
*a*c^6*d^4 - 8*a^2*c^8*d^2))/(a^(1/2)*(2*a*c^2 - 2*d^2))))/(a^(1/2)*(2*a*c
^2 - 2*d^2)))*1i)/(a^(1/2)*(2*a*c^2 - 2*d^2)) + (d*(4*c^6*d^2*(a + b*x^2)^
(1/2) - (d*(4*c^4*d^5 - 8*a*c^6*d^3 + 4*a^2*c^8*d + (d*(a + b*x^2)^(1/2)*(
8*a^3*c^10 + 8*c^4*d^6 - 8*a*c^6*d^4 - 8*a^2*c^8*d^2))/(a^(1/2)*(2*a*c^2 -
 2*d^2))))/(a^(1/2)*(2*a*c^2 - 2*d^2)))*1i)/(a^(1/2)*(2*a*c^2 - 2*d^2))...
```

3.547. $\int \frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$

3.548 $\int \frac{1}{x^3\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$

3.548.1 Optimal result

Integrand size = 29, antiderivative size = 151

$$\int \frac{1}{x^3\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx = -\frac{ac-d\sqrt{a+bx^2}}{2a\left(ac^2-d^2\right)x^2} - \frac{bd(3ac^2-d^2)\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}\left(ac^2-d^2\right)^2} - \frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2} + \frac{bc^3\log\left(d+c\sqrt{a+bx^2}\right)}{\left(ac^2-d^2\right)^2}$$

output
```
-1/2*b*d*(3*a*c^2-d^2)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/(a*c^2-d^2
)^2-b*c^3*ln(x)/(a*c^2-d^2)^2+b*c^3*ln(d+c*(b*x^2+a)^(1/2))/(a*c^2-d^2)^2+
1/2*(-a*c+d*(b*x^2+a)^(1/2))/a/(a*c^2-d^2)/x^2
```

3.548.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx = \frac{bd(-3ac^2+d^2)\,x^2\mathrm{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\sqrt{a}\left(-\left(\left(ac^2-d^2\right)\left(ac-d\sqrt{a+bx^2}\right)\right)-abc^3x^2\log\left(bx^2\right)+2abc^3x^2\right.}{2a^{3/2}\left(-ac^2+d^2\right)^2x^2}$$

input
```
Integrate[1/(x^3*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

output
```
(b*d*(-3*a*c^2 + d^2)*x^2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]] + Sqrt[a]*(-((a
*c^2 - d^2)*(a*c - d*Sqrt[a + b*x^2])) - a*b*c^3*x^2*Log[b*x^2] + 2*a*b*c^
3*x^2*Log[d + c*Sqrt[a + b*x^2]]))/(2*a^(3/2)*(-(a*c^2) + d^2)^2*x^2)
```

3.548.3 Rubi [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2586, 7267, 496, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(d\sqrt{a+bx^2}+ac+bcx^2\right)}dx$$

$$\Big\downarrow \text{ 2586}$$

$$\frac{1}{2}\int \frac{1}{x^4\left(bcx^2+ac+d\sqrt{bx^2+a}\right)}dx^2$$

$$\Big\downarrow \text{ 7267}$$

$$b\int \frac{1}{\left(a-x^4\right)^2\left(\sqrt{bx^2+a}c+d\right)}d\sqrt{bx^2+a}$$

$$\Big\downarrow \text{ 496}$$

$$b\left(\frac{ac-d\sqrt{a+bx^2}}{2a\left(a-x^4\right)\left(ac^2-d^2\right)}-\frac{\int -\frac{2ac^2-d\sqrt{bx^2+a}c-d^2}{\left(a-x^4\right)\left(\sqrt{bx^2+a}c+d\right)}d\sqrt{bx^2+a}}{2a\left(ac^2-d^2\right)}\right)$$

$$\Big\downarrow \text{ 25}$$

$$b\left(\frac{\int \frac{2ac^2-d\sqrt{bx^2+a}c-d^2}{\left(a-x^4\right)\left(\sqrt{bx^2+a}c+d\right)}d\sqrt{bx^2+a}}{2a\left(ac^2-d^2\right)}+\frac{ac-d\sqrt{a+bx^2}}{2a\left(a-x^4\right)\left(ac^2-d^2\right)}\right)$$

$$\Big\downarrow \text{ 657}$$

$$b\left(\frac{\int \left(\frac{2ac^4}{\left(ac^2-d^2\right)\left(\sqrt{bx^2+a}c+d\right)}+\frac{2a\sqrt{bx^2+a}c^3-3adc^2+d^3}{\left(ac^2-d^2\right)\left(a-x^4\right)}\right)d\sqrt{bx^2+a}}{2a\left(ac^2-d^2\right)}+\frac{ac-d\sqrt{a+bx^2}}{2a\left(a-x^4\right)\left(ac^2-d^2\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$b\left(\frac{-\frac{d(3ac^2-d^2)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}(ac^2-d^2)}+\frac{2ac^3\log\left(c\sqrt{a+bx^2}+d\right)}{ac^2-d^2}-\frac{ac^3\log(a-x^4)}{ac^2-d^2}}{2a\,(ac^2-d^2)}+\frac{ac-d\sqrt{a+bx^2}}{2a\,(a-x^4)\,(ac^2-d^2)}\right)$$

input
```
Int[1/(x^3*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

output
```
b*((a*c - d*Sqrt[a + b*x^2])/(2*a*(a*c^2 - d^2)*(a - x^4)) + (-((d*(3*a*c^
2 - d^2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(Sqrt[a]*(a*c^2 - d^2))) - (a*c
^3*Log[a - x^4])/(a*c^2 - d^2) + (2*a*c^3*Log[d + c*Sqrt[a + b*x^2]])/(a*c
^2 - d^2))/(2*a*(a*c^2 - d^2)))
```

3.548.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2586
```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Simp[1/n   Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a
+ b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*
d, 0] && IntegerQ[(m + 1)/n]
```

3.548. $\int \frac{1}{x^3\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.548.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1909 vs. $2(137) = 274$.

Time = 0.10 (sec) , antiderivative size = 1910, normalized size of antiderivative = 12.65

method	result	size
default	Expression too large to display	1910

input
```
int(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
a*c*(1/2*b*c^4/(a*c^2-d^2)^2/d^2*ln(b*c^2*x^2+a*c^2-d^2)-1/2/a/(a*c^2-d^2)
/x^2-b*(2*a*c^2-d^2)/a^2/(a*c^2-d^2)^2*ln(x)-1/2*b/a^2/d^2*ln(b*x^2+a))+b*
c*(-1/2*c^2/(a*c^2-d^2)/d^2*ln(b*c^2*x^2+a*c^2-d^2)+1/a/(a*c^2-d^2)*ln(x)+
1/2/a/d^2*ln(b*x^2+a))-d*(1/a/(a*c^2-d^2)*(-1/2/a/x^2*(b*x^2+a)^(3/2)+1/2*
b/a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))-b*(2*
a*c^2-d^2)/a^2/(a*c^2-d^2)^2*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b
*x^2+a)^(1/2))/x))+1/2*b^2*c^2/a^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^
(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((b*(x-1/b*(-a*b)^(1/
2))^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2)+(-a*b)^(1/2)*ln(((x-1/b*(
-a*b)^(1/2))*b+(-a*b)^(1/2))/b^(1/2)+(b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1
/2)*(x-1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2))+1/2*b^2*c^2/a^2/((-a*b)^(1/2)*c^
2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2)
)*((b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2)-(-
a*b)^(1/2)*ln(((x+1/b*(-a*b)^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+(b*(x+1/b*(-a*
b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2))-1/2*b^2*c
^6/(a*c^2-d^2)^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/
2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2
)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+
d^2/c^2)^(1/2)-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*ln((-(-(a*c^2-d^2)*b*c^2)^(1
/2)/c^2+(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x+(-(a*c^2-...
```

3.548.5 Fricas [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 530, normalized size of antiderivative = 3.51

$$\int \frac{1}{x^3\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$$
$$= \left[\frac{2\,a^2bc^3x^2\log\left(bc^2x^2+ac^2-d^2\right)-4\,a^2bc^3x^2\log\left(x\right)+a^2bc^3x^2\log\left(-\frac{bc^2x^2+ac^2+2\,\sqrt{bx^2+a}acd+d^2}{x^2}\right)-a^2bc^3x^2\,\mathrm{lo}}{4\,(a}\right.$$

input
```
integrate(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

output
```
[1/4*(2*a^2*b*c^3*x^2*log(b*c^2*x^2 + a*c^2 - d^2) - 4*a^2*b*c^3*x^2*log(x
) + a^2*b*c^3*x^2*log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d + d^2)/x
^2) - a^2*b*c^3*x^2*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)
/x^2) - 2*a^3*c^3 + 2*a^2*c*d^2 - (3*a*b*c^2*d - b*d^3)*sqrt(a)*x^2*log(-(
b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(a^2*c^2*d - a*d^3)*sqrt
(b*x^2 + a))/((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4)*x^2), 1/4*(2*a^2*b*c^3*x
^2*log(b*c^2*x^2 + a*c^2 - d^2) - 4*a^2*b*c^3*x^2*log(x) + a^2*b*c^3*x^2*l
og(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - a^2*b*c^3*x^2
*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - 2*a^3*c^3 +
 2*a^2*c*d^2 + 2*(3*a*b*c^2*d - b*d^3)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(b
*x^2 + a)) + 2*(a^2*c^2*d - a*d^3)*sqrt(b*x^2 + a))/((a^4*c^4 - 2*a^3*c^2*
d^2 + a^2*d^4)*x^2)]
```

3.548.6 Sympy [F]

$$\int \frac{1}{x^3\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx=\int \frac{1}{x^3\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$$

input
```
integrate(1/x**3/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

output
```
Integral(1/(x**3*(a*c + b*c*x**2 + d*sqrt(a + b*x**2))), x)
```

3.548.7 Maxima [F]

$$\int \frac{1}{x^3\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx=\int \frac{1}{\left(bcx^2+ac+\sqrt{bx^2+a}d\right)x^3}\,dx$$

input
```
integrate(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x^3), x)
```

3.548.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^3\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx=\frac{bc^4\log\left(\left|\sqrt{bx^2+a}c+d\right|\right)}{a^2c^5-2\,ac^3d^2+cd^4}-\frac{bc^3\log\left(-bx^2\right)}{2\left(a^2c^4-2\,ac^2d^2+d^4\right)}+\frac{\left(3\,abc^2d-bd^3\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\left(a^3c^4-2\,a^2c^2d^2+ad^4\right)\sqrt{-a}}-\frac{a^2bc^3-abcd^2-\left(abc^2d-bd^3\right)\sqrt{bx^2+a}}{2\left(ac^2-d^2\right)^2abx^2}$$

input
```
integrate(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

output
```
b*c^4*log(abs(sqrt(b*x^2 + a)*c + d))/(a^2*c^5 - 2*a*c^3*d^2 + c*d^4) - 1/
2*b*c^3*log(-b*x^2)/(a^2*c^4 - 2*a*c^2*d^2 + d^4) + 1/2*(3*a*b*c^2*d - b*d
^3)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/((a^3*c^4 - 2*a^2*c^2*d^2 + a*d^4)*sq
rt(-a)) - 1/2*(a^2*b*c^3 - a*b*c*d^2 - (a*b*c^2*d - b*d^3)*sqrt(b*x^2 + a)
)/((a*c^2 - d^2)^2*a*b*x^2)
```

3.548.9 Mupad [B] (verification not implemented)

Time = 19.54 (sec) , antiderivative size = 4602, normalized size of antiderivative = 30.48

$$\int \frac{1}{x^3 \left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx = \text{Too large to display}$$

input
```
int(1/(x^3*(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2)),x)
```

output
```
(atan((((((((12*a^2*b*c^6*d^9 - 28*a^3*b*c^8*d^7 + 32*a^4*b*c^10*d^5 - 18*a
^5*b*c^12*d^3 - 2*a*b*c^4*d^11 + 4*a^6*b*c^14*d)/(16*(a^5*c^6 - a^2*d^6 +
3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) - ((a + b*x^2)^(1/2)*(16*(b^2*d^6 - 6*a*b^
2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*
c^4*d^4 - 4*a^6*c^6*d^2))^(1/2)*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c^
6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12*d^2))/(512*(a^4*c^4
 + a^2*d^4 - 2*a^3*c^2*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4
*d^4 - 4*a^6*c^6*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2
)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/
2))/(16*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2
)) + ((a + b*x^2)^(1/2)*(b^2*c^6*d^6 - 6*a*b^2*c^8*d^4 + 13*a^2*b^2*c^10*d
^2))/(32*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*
d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^
4 - 4*a^6*c^6*d^2))^(1/2)*1i)/(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c
^4*d^4 - 4*a^6*c^6*d^2) - (((((12*a^2*b*c^6*d^9 - 28*a^3*b*c^8*d^7 + 32*a^
4*b*c^10*d^5 - 18*a^5*b*c^12*d^3 - 2*a*b*c^4*d^11 + 4*a^6*b*c^14*d)/(16*(a
^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) + ((a + b*x^2)^(1/2)*(1
6*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a
^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2)*(16*a^7*c^14 + 16*a^2*c
^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^...
```

3.549 $\int \frac{x^2}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

3.549.1 Optimal result

Integrand size = 29, antiderivative size = 147

$$\int \frac{x^2}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx=\frac{x}{bc}-\frac{\sqrt{ac^2-d^2}\arctan\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}+\frac{\sqrt{ac^2-d^2}\arctan\left(\frac{\sqrt{b}dx}{\sqrt{ac^2-d^2}\sqrt{a+bx^2}}\right)}{b^{3/2}c^2}-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}c^2}$$

output
```
x/b/c-d*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)/c^2-arctan(c*x*b^(1/2)/
(a*c^2-d^2)^(1/2))*(a*c^2-d^2)^(1/2)/b^(3/2)/c^2+arctan(d*x*b^(1/2)/(a*c^2
-d^2)^(1/2)/(b*x^2+a)^(1/2))*(a*c^2-d^2)^(1/2)/b^(3/2)/c^2
```

3.549.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx=\frac{\sqrt{b}cx+2\sqrt{ac^2-d^2}\arctan\left(\frac{d+c\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{ac^2-d^2}}\right)+d\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}c^2}$$

input
```
Integrate[x^2/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[b]*c*x + 2*Sqrt[a*c^2 - d^2]*ArcTan[(d + c*(-(Sqrt[b]*x) + Sqrt[a +
b*x^2]))/Sqrt[a*c^2 - d^2]] + d*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(b^(3
/2)*c^2)
```

3.549.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2587, 27, 262, 218, 385, 224, 219, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{d\sqrt{a+bx^2}+ac+bcx^2}dx$$

$$\Big\downarrow \text{ 2587}$$

$$ac\int \frac{x^2}{a\left(bx^2c^2+ac^2-d^2\right)}dx-ad\int \frac{x^2}{a\sqrt{bx^2+a}\left(bx^2c^2+ac^2-d^2\right)}dx$$

$$\Big\downarrow \text{ 27}$$

$$c\int \frac{x^2}{bx^2c^2+ac^2-d^2}dx-d\int \frac{x^2}{\sqrt{bx^2+a}\left(bx^2c^2+ac^2-d^2\right)}dx$$

$$\Big\downarrow \text{ 262}$$

$$c\left(\frac{x}{bc^2}-\frac{\left(ac^2-d^2\right)\int \frac{1}{bx^2c^2+ac^2-d^2}dx}{bc^2}\right)-d\int \frac{x^2}{\sqrt{bx^2+a}\left(bx^2c^2+ac^2-d^2\right)}dx$$

$$\Big\downarrow \text{ 218}$$

$$c\left(\frac{x}{bc^2}-\frac{\sqrt{ac^2-d^2}\arctan\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^3}\right)-d\int \frac{x^2}{\sqrt{bx^2+a}\left(bx^2c^2+ac^2-d^2\right)}dx$$

$$\Big\downarrow \text{ 385}$$

$$c\left(\frac{x}{bc^2}-\frac{\sqrt{ac^2-d^2}\arctan\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^3}\right)-d\left(\frac{\int \frac{1}{\sqrt{bx^2+a}}dx}{bc^2}-\frac{\left(ac^2-d^2\right)\int \frac{1}{\sqrt{bx^2+a}\left(bx^2c^2+ac^2-d^2\right)}dx}{bc^2}\right)$$

$$\Big\downarrow \text{ 224}$$

$$c\left(\frac{x}{bc^2}-\frac{\sqrt{ac^2-d^2}\arctan\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^3}\right)-d\left(\frac{\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{bc^2}-\frac{(ac^2-d^2)\int\frac{1}{\sqrt{bx^2+a}(bx^2c^2+ac^2-d^2)}dx}{bc^2}\right)$$

$$\Big\downarrow \text{219}$$

$$c\left(\frac{x}{bc^2}-\frac{\sqrt{ac^2-d^2}\arctan\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^3}\right)-d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}c^2}-\frac{(ac^2-d^2)\int\frac{1}{\sqrt{bx^2+a}(bx^2c^2+ac^2-d^2)}dx}{bc^2}\right)$$

$$\Big\downarrow \text{291}$$

$$c\left(\frac{x}{bc^2}-\frac{\sqrt{ac^2-d^2}\arctan\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^3}\right)-d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}c^2}-\frac{(ac^2-d^2)\int\frac{1}{ac^2-d^2-\frac{\left(b\left(ac^2-d^2\right)-abc^2\right)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{bc^2}\right)$$

$$\Big\downarrow \text{218}$$

$$c\left(\frac{x}{bc^2}-\frac{\sqrt{ac^2-d^2}\arctan\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^3}\right)-d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}c^2}-\frac{\sqrt{ac^2-d^2}\arctan\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2d}\right)$$

input
```
Int[x^2/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

output
```
c*(x/(b*c^2) - (Sqrt[a*c^2 - d^2]*ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]])
/(b^(3/2)*c^3)) - d*(-((Sqrt[a*c^2 - d^2]*ArcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2
 - d^2]*Sqrt[a + b*x^2])])/(b^(3/2)*c^2*d)) + ArcTanh[(Sqrt[b]*x)/Sqrt[a +
 b*x^2]]/(b^(3/2)*c^2))
```

3.549. $\int \frac{x^2}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

3.549.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 385
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2),
x_Symbol] :> Simp[e^2/b   Int[(e*x)^(m - 2)*(c + d*x^2)^q, x], x] - Simp[a*
(e^2/b)   Int[(e*x)^(m - 2)*((c + d*x^2)^q/(a + b*x^2)), x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3] && IntBinomial
Q[a, b, c, d, e, m, 2, -1, q, x]
```

rule 2587
```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Simp[c   Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Simp[a*e   Int[
u/((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e
, n}, x] && EqQ[b*c - a*d, 0]
```

3.549. $\int \frac{x^2}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

3.549.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1779 vs. $2(123) = 246$.

Time = 0.08 (sec) , antiderivative size = 1780, normalized size of antiderivative = 12.11

method	result	size
default	Expression too large to display	1780

input
```
int(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
d*(-1/2*c^2*a/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(
-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*((b*(x-1/b*(-a*b)^(1/2))^2+2
*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2)+(-a*b)^(1/2)*ln(((x-1/b*(-a*b)^(
1/2))*b+(-a*b)^(1/2))/b^(1/2)+(b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-
1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2))+1/2*c^2*a/(-a*b)^(1/2)/((-a*b)^(1/2)*c^
2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2
))*((b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2)-(
-a*b)^(1/2)*ln(((x+1/b*(-a*b)^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+(b*(x+1/b*(-a
*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2))-1/2*c^2*
(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c
^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*((b*(x+(-(a*c^2-
d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^
2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2)-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*ln((-
(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2+(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(
1/2)+(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2
)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/b^(1/2)-d^2/c^2
/(d^2/c^2)^(1/2)*ln((2*d^2/c^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^
2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1
/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2
)/b/c^2)+d^2/c^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))+1/2*c^2...
```

3.549.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 1168, normalized size of antiderivative = 7.95

$$\int \frac{x^2}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$$

$$= \left[\frac{4\,bcx + 2\,\sqrt{b}d\log\left(-2\,bx^2 + 2\,\sqrt{bx^2+a}\sqrt{b}x - a\right) + b\sqrt{-\frac{ac^2-d^2}{b}}\log\left(\frac{a^4c^4-2\,a^3c^2d^2+a^2d^4+\left(a^2b^2c^4-8\,ab^2c^2d^2+8\,b\cdots\right.}{\cdots}\right.}{\cdots}\right.$$

input
```
integrate(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

output
```
[1/4*(4*b*c*x + 2*sqrt(b)*d*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a
) + b*sqrt(-(a*c^2 - d^2)/b)*log((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4 + (a^2
*b^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*a^2*b*c^2*d
^2 + 4*a*b*d^4)*x^2 - 4*((a*b^2*c^2*d - 2*b^2*d^3)*x^3 + (a^2*b*c^2*d - a*
b*d^3)*x)*sqrt(b*x^2 + a)*sqrt(-(a*c^2 - d^2)/b))/(b^2*c^4*x^4 + a^2*c^4 -
 2*a*c^2*d^2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)*x^2)) + 2*b*sqrt(-(a*c^2 - d^
2)/b)*log((b*c^2*x^2 - 2*b*c*x*sqrt(-(a*c^2 - d^2)/b) - a*c^2 + d^2)/(b*c^
2*x^2 + a*c^2 - d^2)))/(b^2*c^2), 1/4*(4*b*c*x + 4*sqrt(-b)*d*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) + b*sqrt(-(a*c^2 - d^2)/b)*log((a^4*c^4 - 2*a^3*c^2
*d^2 + a^2*d^4 + (a^2*b^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*
b*c^4 - 5*a^2*b*c^2*d^2 + 4*a*b*d^4)*x^2 - 4*((a*b^2*c^2*d - 2*b^2*d^3)*x^
3 + (a^2*b*c^2*d - a*b*d^3)*x)*sqrt(b*x^2 + a)*sqrt(-(a*c^2 - d^2)/b))/(b^
2*c^4*x^4 + a^2*c^4 - 2*a*c^2*d^2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)*x^2)) +
2*b*sqrt(-(a*c^2 - d^2)/b)*log((b*c^2*x^2 - 2*b*c*x*sqrt(-(a*c^2 - d^2)/b)
 - a*c^2 + d^2)/(b*c^2*x^2 + a*c^2 - d^2)))/(b^2*c^2), 1/2*(2*b*c*x + 2*b*
sqrt((a*c^2 - d^2)/b)*arctan(-b*c*x*sqrt((a*c^2 - d^2)/b)/(a*c^2 - d^2)) -
 b*sqrt((a*c^2 - d^2)/b)*arctan(1/2*(a^2*c^2 - a*d^2 + (a*b*c^2 - 2*b*d^2)
*x^2)*sqrt(b*x^2 + a)*sqrt((a*c^2 - d^2)/b)/((a*b*c^2*d - b*d^3)*x^3 + (a^
2*c^2*d - a*d^3)*x)) + sqrt(b)*d*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*
x - a))/(b^2*c^2), 1/2*(2*b*c*x + 2*b*sqrt((a*c^2 - d^2)/b)*arctan(-b*c...
```

3.549.6 Sympy [F]

$$\int \frac{x^2}{ac + bcx^2 + d\sqrt{a + bx^2}}\, dx = \int \frac{x^2}{ac + bcx^2 + d\sqrt{a + bx^2}}\, dx$$

input
```
integrate(x**2/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

output
```
Integral(x**2/(a*c + b*c*x**2 + d*sqrt(a + b*x**2)), x)
```

3.549.7 Maxima [F]

$$\int \frac{x^2}{ac + bcx^2 + d\sqrt{a + bx^2}} \, dx = \int \frac{x^2}{bcx^2 + ac + \sqrt{bx^2 + a}d} \, dx$$

input
```
integrate(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d), x)
```

3.549.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{ac + bcx^2 + d\sqrt{a + bx^2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.549.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{ac + bcx^2 + d\sqrt{a + bx^2}} \, dx = \int \frac{x^2}{a\,c + d\,\sqrt{b\,x^2 + a} + b\,c\,x^2} \, dx$$

input
```
int(x^2/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2),x)
```

output
```
int(x^2/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2), x)
```

3.550 $\int \frac{1}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

3.550.1 Optimal result

Integrand size = 25, antiderivative size = 103

$$\int \frac{1}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx=\frac{\arctan\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}-\frac{\arctan\left(\frac{\sqrt{b}dx}{\sqrt{ac^2-d^2}\sqrt{a+bx^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}$$

output
```
arctan(c*x*b^(1/2)/(a*c^2-d^2)^(1/2))/b^(1/2)/(a*c^2-d^2)^(1/2)-arctan(d*x
*b^(1/2)/(a*c^2-d^2)^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)/(a*c^2-d^2)^(1/2)
```

3.550.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.61

$$\int \frac{1}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx=-\frac{2\arctan\left(\frac{d+c\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}$$

input
```
Integrate[(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])^(-1),x]
```

output
```
(-2*ArcTan[(d + c*(-(Sqrt[b]*x) + Sqrt[a + b*x^2]))/Sqrt[a*c^2 - d^2]])/(S
qrt[b]*Sqrt[a*c^2 - d^2])
```

3.550.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2587, 27, 218, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{d\sqrt{a+bx^2}+ac+bcx^2}dx$$

$$\Big\downarrow \text{2587}$$

$$ac\int \frac{1}{abc^2x^2+a(ac^2-d^2)}dx-ad\int \frac{1}{a\sqrt{bx^2+a}\,(bx^2c^2+ac^2-d^2)}dx$$

$$\Big\downarrow \text{27}$$

$$ac\int \frac{1}{abc^2x^2+a(ac^2-d^2)}dx-d\int \frac{1}{\sqrt{bx^2+a}\,(bx^2c^2+ac^2-d^2)}dx$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}-d\int \frac{1}{\sqrt{bx^2+a}\,(bx^2c^2+ac^2-d^2)}dx$$

$$\Big\downarrow \text{291}$$

$$\frac{\arctan\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}-d\int \frac{1}{ac^2-d^2-\frac{(b(ac^2-d^2)-abc^2)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}-\frac{\arctan\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}$$

input
```
Int[(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])^(-1),x]
```

output
```
ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]]/(Sqrt[b]*Sqrt[a*c^2 - d^2]) - ArcT
an[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[a + b*x^2])]/(Sqrt[b]*Sqrt[a*c^2
- d^2])
```

3.550.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 2587
```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Simp[c   Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Simp[a*e   Int[
u/((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e
, n}, x] && EqQ[b*c - a*d, 0]
```

3.550.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1717 vs. $2(85) = 170$.

Time = 0.08 (sec) , antiderivative size = 1718, normalized size of antiderivative = 16.68

method	result	size
default	Expression too large to display	1718

input
```
int(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
d*(1/2*b*c^2/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-
(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*((b*(x-1/b*(-a*b)^(1/2))^2+2*
(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2)+(-a*b)^(1/2)*ln(((x-1/b*(-a*b)^(1
/2))*b+(-a*b)^(1/2))/b^(1/2)+(b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1
/b*(-a*b)^(1/2)))^(1/2))/b^(1/2))-1/2*b*c^2/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2
+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2)
)*((b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2)-(-
a*b)^(1/2)*ln(((x+1/b*(-a*b)^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+(b*(x+1/b*(-a*
b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2))+1/2*b*c^4
/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2
-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*((b*(x+(-(a*c^2-d^2)*b*c^2)
^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(
1/2)/b/c^2)+d^2/c^2)^(1/2)-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*ln((-(-(a*c^2-d^
2)*b*c^2)^(1/2)/c^2+(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x+
(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-
(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/b^(1/2)-d^2/c^2/(d^2/c^2)^
(1/2)*ln((2*d^2/c^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^
2)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^
2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^
2/c^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))-1/2*b*c^4/((-a*b)^...
```

3.550.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(85) = 170$.

Time = 0.31 (sec) , antiderivative size = 510, normalized size of antiderivative = 4.95

$$\int \frac{1}{ac + bcx^2 + d\sqrt{a + bx^2}}\,dx$$
$$= \left[-\frac{\sqrt{-abc^2 + bd^2}\log\left(\frac{a^4c^4 - 2\,a^3c^2d^2 + a^2d^4 + (a^2b^2c^4 - 8\,ab^2c^2d^2 + 8\,b^2d^4)x^4 + 2\,(a^3bc^4 - 5\,a^2bc^2d^2 + 4\,abd^4)x^2 - 4\,\sqrt{-abc^2 + bd^2}\left((abc^2d- \cdots\right.}{b^2c^4x^4 + a^2c^4 - 2\,ac^2d^2 + d^4 + 2\,(abc^4 - bc^2d^2)x^2}\right.}{4\,(abc^2 - bd^2)}\right.$$
$$\left. - \frac{2\,\sqrt{abc^2 - bd^2}\arctan\left(-\frac{\sqrt{abc^2 - bd^2}cx}{ac^2 - d^2}\right) - \sqrt{abc^2 - bd^2}\arctan\left(\frac{(a^2c^2 - ad^2 + (abc^2 - 2\,bd^2)x^2)\sqrt{abc^2 - bd^2}\sqrt{bx^2 + a}}{2\,((ab^2c^2d - b^2d^3)x^3 + (a^2bc^2d - abd^3)x)}\right)}{2\,(abc^2 - bd^2)}\right]$$

input
```
integrate(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

output
```
[-1/4*(sqrt(-a*b*c^2 + b*d^2)*log((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4 + (a^
2*b^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*a^2*b*c^2*
d^2 + 4*a*b*d^4)*x^2 - 4*sqrt(-a*b*c^2 + b*d^2)*((a*b*c^2*d - 2*b*d^3)*x^3
 + (a^2*c^2*d - a*d^3)*x)*sqrt(b*x^2 + a))/(b^2*c^4*x^4 + a^2*c^4 - 2*a*c^
2*d^2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)*x^2)) + 2*sqrt(-a*b*c^2 + b*d^2)*log
((b*c^2*x^2 - a*c^2 - 2*sqrt(-a*b*c^2 + b*d^2)*c*x + d^2)/(b*c^2*x^2 + a*c
^2 - d^2)))/(a*b*c^2 - b*d^2), -1/2*(2*sqrt(a*b*c^2 - b*d^2)*arctan(-sqrt(
a*b*c^2 - b*d^2)*c*x/(a*c^2 - d^2)) - sqrt(a*b*c^2 - b*d^2)*arctan(1/2*(a^
2*c^2 - a*d^2 + (a*b*c^2 - 2*b*d^2)*x^2)*sqrt(a*b*c^2 - b*d^2)*sqrt(b*x^2
+ a)/((a*b^2*c^2*d - b^2*d^3)*x^3 + (a^2*b*c^2*d - a*b*d^3)*x)))/(a*b*c^2
- b*d^2)]
```

3.550.6 Sympy [F]

$$\int \frac{1}{ac + bcx^2 + d\sqrt{a + bx^2}}\, dx = \int \frac{1}{ac + bcx^2 + d\sqrt{a + bx^2}}\, dx$$

input
```
integrate(1/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

output
```
Integral(1/(a*c + b*c*x**2 + d*sqrt(a + b*x**2)), x)
```

3.550.7 Maxima [F]

$$\int \frac{1}{ac + bcx^2 + d\sqrt{a + bx^2}}\, dx = \int \frac{1}{bcx^2 + ac + \sqrt{bx^2 + a}d}\, dx$$

input
```
integrate(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d), x)
```

3.550.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04

$$\int \frac{1}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx=\frac{\arctan\left(\frac{bcx}{\sqrt{abc^2-bd^2}}\right)}{\sqrt{abc^2-bd^2}}+\frac{\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2c^2+ac^2-2\,d^2}{2\,\sqrt{ac^2-d^2}d}\right)}{\sqrt{ac^2-d^2}\sqrt{b}}$$

input
```
integrate(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

output
```
arctan(b*c*x/sqrt(a*b*c^2 - b*d^2))/sqrt(a*b*c^2 - b*d^2) + arctan(1/2*((s
qrt(b)*x - sqrt(b*x^2 + a))^2*c^2 + a*c^2 - 2*d^2)/(sqrt(a*c^2 - d^2)*d))/
(sqrt(a*c^2 - d^2)*sqrt(b))
```

3.550.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{\mathrm{atan}\left(\frac{b\,c\,x}{\sqrt{a\,b\,c^2-b\,d^2}}\right)}{\sqrt{a\,b\,c^2-b\,d^2}}-\frac{d\,x}{\sqrt{a}\,(a\,c^2-d^2)} & \text{if } b=0\vee d=0 \\ \frac{\mathrm{atan}\left(\frac{b\,c\,x}{\sqrt{a\,b\,c^2-b\,d^2}}\right)}{\sqrt{a\,b\,c^2-b\,d^2}}-\frac{d\,\mathrm{atan}\left(\frac{x\,\sqrt{a\,b\,c^2-b\,\left(a\,c^2-d^2\right)}}{\sqrt{a\,c^2-d^2}\,\sqrt{b\,x^2+a}}\right)}{\sqrt{-(a\,c^2-d^2)\,(b\,(a\,c^2-d^2)-a\,b\,c^2)}} & \text{if } 0<b\,d^2 \\ \frac{\mathrm{atan}\left(\frac{b\,c\,x}{\sqrt{a\,b\,c^2-b\,d^2}}\right)}{\sqrt{a\,b\,c^2-b\,d^2}}-\frac{d\,\ln\left(\frac{\sqrt{\left(a\,c^2-d^2\right)\,\left(b\,x^2+a\right)}+x\,\sqrt{b\,\left(a\,c^2-d^2\right)-a\,b\,c^2}}{\sqrt{\left(a\,c^2-d^2\right)\,\left(b\,x^2+a\right)}-x\,\sqrt{b\,\left(a\,c^2-d^2\right)-a\,b\,c^2}}\right)}{2\,\sqrt{(a\,c^2-d^2)\,(b\,(a\,c^2-d^2)-a\,b\,c^2)}} & \text{if } b\,d^2<0 \\ \int \frac{1}{a\,c+d\,\sqrt{b\,x^2+a}+b\,c\,x^2}\,dx & \text{if } b\,d^2\notin \mathbb{R} \end{cases}$$

input
```
int(1/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2),x)
```

output
```
piecewise(b == 0 | d == 0, atan((b*c*x)/(- b*d^2 + a*b*c^2)^(1/2))/(- b*d^
2 + a*b*c^2)^(1/2) - (d*x)/(a^(1/2)*(a*c^2 - d^2)), 0 < b*d^2, atan((b*c*x
)/(- b*d^2 + a*b*c^2)^(1/2))/(- b*d^2 + a*b*c^2)^(1/2) - (d*atan((x*(- b*(
a*c^2 - d^2) + a*b*c^2)^(1/2))/((a*c^2 - d^2)^(1/2)*(a + b*x^2)^(1/2))))/(
-(a*c^2 - d^2)*(b*(a*c^2 - d^2) - a*b*c^2))^(1/2), b*d^2 < 0, atan((b*c*x)
/(- b*d^2 + a*b*c^2)^(1/2))/(- b*d^2 + a*b*c^2)^(1/2) - (d*log((((a*c^2 -
d^2)*(a + b*x^2))^(1/2) + x*(b*(a*c^2 - d^2) - a*b*c^2)^(1/2))/(((a*c^2 -
d^2)*(a + b*x^2))^(1/2) - x*(b*(a*c^2 - d^2) - a*b*c^2)^(1/2))))/(2*((a*c^
2 - d^2)*(b*(a*c^2 - d^2) - a*b*c^2))^(1/2)), ~in(b*d^2, 'real'), int(1/(a
*c + d*(a + b*x^2)^(1/2) + b*c*x^2), x))
```

3.550. $\int \frac{1}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

3.551 $\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$

3.551.1 Optimal result

Integrand size = 29, antiderivative size = 160

$$\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx=-\frac{c}{(ac^2-d^2)\,x}+\frac{d\sqrt{a+bx^2}}{a\,(ac^2-d^2)\,x}-\frac{\sqrt{b}c^2\arctan\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{(ac^2-d^2)^{3/2}}+\frac{\sqrt{b}c^2\arctan\left(\frac{\sqrt{b}dx}{\sqrt{ac^2-d^2}\sqrt{a+bx^2}}\right)}{(ac^2-d^2)^{3/2}}$$

output
```
-c/(a*c^2-d^2)/x-c^2*arctan(c*x*b^(1/2)/(a*c^2-d^2)^(1/2))*b^(1/2)/(a*c^2-
d^2)^(3/2)+c^2*arctan(d*x*b^(1/2)/(a*c^2-d^2)^(1/2)/(b*x^2+a)^(1/2))*b^(1/
2)/(a*c^2-d^2)^(3/2)+d*(b*x^2+a)^(1/2)/a/(a*c^2-d^2)/x
```

3.551.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx=-\frac{ac-d\sqrt{a+bx^2}}{a^2c^2x-ad^2x}+\frac{2\sqrt{b}c^2\arctan\left(\frac{d+c\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{ac^2-d^2}}\right)}{(ac^2-d^2)^{3/2}}$$

input
```
Integrate[1/(x^2*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

output
```
-((a*c - d*Sqrt[a + b*x^2])/(a^2*c^2*x - a*d^2*x)) + (2*Sqrt[b]*c^2*ArcTan
[(d + c*(-(Sqrt[b]*x) + Sqrt[a + b*x^2]))/Sqrt[a*c^2 - d^2]])/(a*c^2 - d^2
)^(3/2)
```

3.551.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2587, 27, 264, 218, 382, 25, 27, 291, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(d\sqrt{a+bx^2}+ac+bcx^2\right)}dx$$

$$\Big\downarrow \text{2587}$$

$$ac\int \frac{1}{ax^2(bx^2c^2+ac^2-d^2)}dx-ad\int \frac{1}{ax^2\sqrt{bx^2+a}\,(bx^2c^2+ac^2-d^2)}dx$$

$$\Big\downarrow \text{27}$$

$$c\int \frac{1}{x^2(bx^2c^2+ac^2-d^2)}dx-d\int \frac{1}{x^2\sqrt{bx^2+a}\,(bx^2c^2+ac^2-d^2)}dx$$

$$\Big\downarrow \text{264}$$

$$c\left(-\frac{bc^2\int \frac{1}{bx^2c^2+ac^2-d^2}dx}{ac^2-d^2}-\frac{1}{x(ac^2-d^2)}\right)-d\int \frac{1}{x^2\sqrt{bx^2+a}\,(bx^2c^2+ac^2-d^2)}dx$$

$$\Big\downarrow \text{218}$$

$$c\left(-\frac{\sqrt{b}c\arctan\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{(ac^2-d^2)^{3/2}}-\frac{1}{x(ac^2-d^2)}\right)-d\int \frac{1}{x^2\sqrt{bx^2+a}\,(bx^2c^2+ac^2-d^2)}dx$$

$$\Big\downarrow \text{382}$$

$$c\left(-\frac{\sqrt{b}c\arctan\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{(ac^2-d^2)^{3/2}}-\frac{1}{x(ac^2-d^2)}\right)-d\left(\frac{\int -\frac{abc^2}{\sqrt{bx^2+a}(bx^2c^2+ac^2-d^2)}dx}{a(ac^2-d^2)}-\frac{\sqrt{a+bx^2}}{ax(ac^2-d^2)}\right)$$

$$\Big\downarrow \text{25}$$

$$c\left(-\frac{\sqrt{b}c\arctan\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{(ac^2-d^2)^{3/2}}-\frac{1}{x(ac^2-d^2)}\right)-d\left(-\frac{\int \frac{abc^2}{\sqrt{bx^2+a}(bx^2c^2+ac^2-d^2)}dx}{a(ac^2-d^2)}-\frac{\sqrt{a+bx^2}}{ax(ac^2-d^2)}\right)$$

$$\Big\downarrow \text{27}$$

$$c\left(-\frac{\sqrt{b}c\arctan\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{(ac^2-d^2)^{3/2}}-\frac{1}{x(ac^2-d^2)}\right)-d\left(-\frac{bc^2\int\frac{1}{\sqrt{bx^2+a}(bx^2c^2+ac^2-d^2)}dx}{ac^2-d^2}-\frac{\sqrt{a+bx^2}}{ax(ac^2-d^2)}\right)$$

$$\Big\downarrow \text{291}$$

$$c\left(-\frac{\sqrt{b}c\arctan\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{(ac^2-d^2)^{3/2}}-\frac{1}{x(ac^2-d^2)}\right)-d\left(-\frac{bc^2\int\frac{1}{ac^2-d^2-\frac{(b(ac^2-d^2)-abc^2)x^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{ac^2-d^2}-\frac{\sqrt{a+bx^2}}{ax(ac^2-d^2)}\right)$$

$$\Big\downarrow \text{218}$$

$$c\left(-\frac{\sqrt{b}c\arctan\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{(ac^2-d^2)^{3/2}}-\frac{1}{x(ac^2-d^2)}\right)-d\left(-\frac{\sqrt{b}c^2\arctan\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{d(ac^2-d^2)^{3/2}}-\frac{\sqrt{a+bx^2}}{ax(ac^2-d^2)}\right)$$

input
```
Int[1/(x^2*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

output
```
c*(-(1/((a*c^2 - d^2)*x)) - (Sqrt[b]*c*ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d
^2]])/(a*c^2 - d^2)^(3/2)) - d*(-(Sqrt[a + b*x^2]/(a*(a*c^2 - d^2)*x)) - (
Sqrt[b]*c^2*ArcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[a + b*x^2])])/(d*
(a*c^2 - d^2)^(3/2)))
```

3.551.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.551. $\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 2587
```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Simp[c   Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Simp[a*e   Int[
u/((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e
, n}, x] && EqQ[b*c - a*d, 0]
```

3.551.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1810 vs. $2(140) = 280$.

Time = 0.09 (sec) , antiderivative size = 1811, normalized size of antiderivative = 11.32

method	result	size
default	Expression too large to display	1811

input
```
int(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
b*c^2/d^2/(b*(a*c^2-d^2))^(1/2)*arctan(x*b*c/(b*(a*c^2-d^2))^(1/2))-c/(a*c
^2-d^2)/x-a*c^4/(a*c^2-d^2)*b/d^2/(b*(a*c^2-d^2))^(1/2)*arctan(x*b*c/(b*(a
*c^2-d^2))^(1/2))-d*(1/(a*c^2-d^2)/a*(-1/a/x*(b*x^2+a)^(3/2)+2*b/a*(1/2*x*
(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))-1/2*b^2*c^2/
a/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)
*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)
*(x-1/b*(-a*b)^(1/2)))^(1/2)+(-a*b)^(1/2)*ln(((x-1/b*(-a*b)^(1/2))*b+(-a*b
)^(1/2))/b^(1/2)+(b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1
/2)))^(1/2))/b^(1/2))+1/2*b^2*c^2/a/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-(a*c^
2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((b*(x+
1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2)-(-a*b)^(1/2
)*ln(((x+1/b*(-a*b)^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+(b*(x+1/b*(-a*b)^(1/2))
^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2))-1/2*b^2*c^6/(a*c^2
-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a
*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*((b*(x+(-(a*c^2-d^2)*b*
c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^
2)^(1/2)/b/c^2)+d^2/c^2)^(1/2)-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*ln((-(-(a*c^
2-d^2)*b*c^2)^(1/2)/c^2+(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b
*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(
x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/b^(1/2)-d^2/c^2/(d^...
```

3.551.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 581, normalized size of antiderivative = 3.63

$$\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$$
$$= \left[-\frac{ac^2x\sqrt{-\frac{b}{ac^2-d^2}}\log\left(\frac{a^4c^4-2\,a^3c^2d^2+a^2d^4+\left(a^2b^2c^4-8\,ab^2c^2d^2+8\,b^2d^4\right)x^4+2\,\left(a^3bc^4-5\,a^2bc^2d^2+4\,abd^4\right)x^2+4\,\left(\left(a^2bc^4d-3\,abc^2d^3+\cdots\right.\right.}{b^2c^4x^4+a^2c^4-2\,ac^2d^2+d^4+2\,\left(abc^4-bc^2d^2\right)x^2}\cdots\right.}{\cdots}\right.$$
$$-\frac{2\,ac^2x\sqrt{\frac{b}{ac^2-d^2}}\arctan\left(cx\sqrt{\frac{b}{ac^2-d^2}}\right)-ac^2x\sqrt{\frac{b}{ac^2-d^2}}\arctan\left(-\frac{\left(a^2c^2-ad^2+\left(abc^2-2\,bd^2\right)x^2\right)\sqrt{bx^2+a}\sqrt{\frac{b}{ac^2-d^2}}}{2\,\left(b^2dx^3+abdx\right)}\right)+\cdots}{2\,\left(a^2c^2-ad^2\right)x}$$

input
```
integrate(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

output
```
[-1/4*(a*c^2*x*sqrt(-b/(a*c^2 - d^2))*log((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d
^4 + (a^2*b^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*a^
2*b*c^2*d^2 + 4*a*b*d^4)*x^2 + 4*((a^2*b*c^4*d - 3*a*b*c^2*d^3 + 2*b*d^5)*
x^3 + (a^3*c^4*d - 2*a^2*c^2*d^3 + a*d^5)*x)*sqrt(b*x^2 + a)*sqrt(-b/(a*c^
2 - d^2)))/(b^2*c^4*x^4 + a^2*c^4 - 2*a*c^2*d^2 + d^4 + 2*(a*b*c^4 - b*c^2
*d^2)*x^2)) + 2*a*c^2*x*sqrt(-b/(a*c^2 - d^2))*log((b*c^2*x^2 - a*c^2 + 2*
(a*c^3 - c*d^2)*x*sqrt(-b/(a*c^2 - d^2)) + d^2)/(b*c^2*x^2 + a*c^2 - d^2))
 + 4*a*c - 4*sqrt(b*x^2 + a)*d)/((a^2*c^2 - a*d^2)*x), -1/2*(2*a*c^2*x*sqr
t(b/(a*c^2 - d^2))*arctan(c*x*sqrt(b/(a*c^2 - d^2))) - a*c^2*x*sqrt(b/(a*c
^2 - d^2))*arctan(-1/2*(a^2*c^2 - a*d^2 + (a*b*c^2 - 2*b*d^2)*x^2)*sqrt(b*
x^2 + a)*sqrt(b/(a*c^2 - d^2))/(b^2*d*x^3 + a*b*d*x)) + 2*a*c - 2*sqrt(b*x
^2 + a)*d)/((a^2*c^2 - a*d^2)*x)]
```

3.551.6 Sympy [F]

$$\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx=\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$$

input
```
integrate(1/x**2/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

output
```
Integral(1/(x**2*(a*c + b*c*x**2 + d*sqrt(a + b*x**2))), x)
```

3.551.7 Maxima [F]

$$\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx=\int \frac{1}{\left(bcx^2+ac+\sqrt{bx^2+a}d\right)x^2}\,dx$$

input
```
integrate(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x^2), x)
```

3.551.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^2 \left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx = -b^{\frac{3}{2}}d\left(\frac{c^2\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2c^2+ac^2-2\,d^2}{2\,\sqrt{ac^2-d^2}d}\right)}{(abc^2-bd^2)\sqrt{ac^2-d^2}d}+\frac{2}{(abc^2-bd^2)\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)}\right) - \frac{bc^2\arctan\left(\frac{bcx}{\sqrt{abc^2-bd^2}}\right)}{\sqrt{abc^2-bd^2}(ac^2-d^2)} - \frac{c}{(ac^2-d^2)x}$$

input
```
integrate(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

output
```
-b^(3/2)*d*(c^2*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*c^2 + a*c^2 -
2*d^2)/(sqrt(a*c^2 - d^2)*d))/((a*b*c^2 - b*d^2)*sqrt(a*c^2 - d^2)*d) + 2/
((a*b*c^2 - b*d^2)*((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a))) - b*c^2*arctan(
b*c*x/sqrt(a*b*c^2 - b*d^2))/(sqrt(a*b*c^2 - b*d^2)*(a*c^2 - d^2)) - c/((a
*c^2 - d^2)*x)
```

3.551.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx = \int \frac{1}{x^2\,\left(a\,c+d\,\sqrt{b\,x^2+a}+b\,c\,x^2\right)}\,dx$$

input
```
int(1/(x^2*(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2)),x)
```

output
```
int(1/(x^2*(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2)), x)
```

3.552 $\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

3.552.1 Optimal result

Integrand size = 29, antiderivative size = 140

$$\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = -\frac{(2ac^2-d^2)\,x^3}{3b^2c^3}+\frac{2d(2ac^2-d^2)\sqrt{a+bx^3}}{3b^3c^4}-\frac{2d(a+bx^3)^{3/2}}{9b^3c^2}+\frac{(a+bx^3)^2}{6b^3c}+\frac{2(ac^2-d^2)^2\log\left(d+c\sqrt{a+bx^3}\right)}{3b^3c^5}$$

output
```
-1/3*(2*a*c^2-d^2)*x^3/b^2/c^3-2/9*d*(b*x^3+a)^(3/2)/b^3/c^2+1/6*(b*x^3+a)
^2/b^3/c+2/3*(a*c^2-d^2)^2*ln(d+c*(b*x^3+a)^(1/2))/b^3/c^5+2/3*d*(2*a*c^2-
d^2)*(b*x^3+a)^(1/2)/b^3/c^4
```

3.552.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.77

$$\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \frac{3c^2(a+bx^3)\left(-3ac^2+2d^2+bc^2x^3\right)-4cd\sqrt{a+bx^3}(-5ac^2+3d^2+bc^2x^3)+12(-ac^2+d^2)^2\log\left(d+c\sqrt{a\right.}}{18b^3c^5}$$

input
```
Integrate[x^8/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

output
```
(3*c^2*(a + b*x^3)*(-3*a*c^2 + 2*d^2 + b*c^2*x^3) - 4*c*d*Sqrt[a + b*x^3]*
(-5*a*c^2 + 3*d^2 + b*c^2*x^3) + 12*(-(a*c^2) + d^2)^2*Log[d + c*Sqrt[a +
b*x^3]])/(18*b^3*c^5)
```

3.552.3 Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2586, 7267, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{d\sqrt{a+bx^3}+ac+bcx^3}dx$$

$$\Big\downarrow \; 2586$$

$$\frac{1}{3}\int \frac{x^6}{bcx^3+ac+d\sqrt{bx^3+a}}dx^3$$

$$\Big\downarrow \; 7267$$

$$\frac{2\int \frac{(a-x^6)^2}{\sqrt{bx^3+ac+d}}d\sqrt{bx^3+a}}{3b^3}$$

$$\Big\downarrow \; 476$$

$$\frac{2\int \left(\frac{x^9}{c}-\frac{dx^6}{c^2}+\frac{2ac^2d-d^3}{c^4}-\frac{(2ac^2-d^2)\sqrt{bx^3+a}}{c^3}+\frac{(ac^2-d^2)^2}{c^4\left(\sqrt{bx^3+ac+d}\right)}\right)d\sqrt{bx^3+a}}{3b^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{2\left(\frac{(ac^2-d^2)^2\log\left(c\sqrt{a+bx^3}+d\right)}{c^5}+\frac{d\sqrt{a+bx^3}(2ac^2-d^2)}{c^4}-\frac{x^6(2ac^2-d^2)}{2c^3}-\frac{dx^9}{3c^2}+\frac{x^{12}}{4c}\right)}{3b^3}$$

input
```
Int[x^8/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

output
```
(2*(-1/2*((2*a*c^2 - d^2)*x^6)/c^3 - (d*x^9)/(3*c^2) + x^12/(4*c) + (d*(2*
a*c^2 - d^2)*Sqrt[a + b*x^3])/c^4 + ((a*c^2 - d^2)^2*Log[d + c*Sqrt[a + b*
x^3]])/c^5))/(3*b^3)
```

3.552.3.1 Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2586
```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Simp[1/n   Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a
+ b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*
d, 0] && IntegerQ[(m + 1)/n]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.552.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(124) = 248.

Time = 0.39 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.37

method	result
default	$d\left(-\frac{2(b x^3+a)^{\frac{3}{2}}}{9b^3c^2}+\frac{2a^2\sqrt{b x^3+a}}{3b^3d^2}-\frac{(a^2c^4-2a c^2d^2+d^4)\left(d\ln\left(c\sqrt{b x^3+a}-d\right)-d\ln\left(d+c\sqrt{b x^3+a}\right)+2c\sqrt{b x^3+a}\right)}{3d^2c^5b^3}\right)-ac\Big(-$
elliptic	$\sqrt{b x^3+a}\left(d+c\sqrt{b x^3+a}\right)\left(c\left(\frac{\frac{1}{2}b c^2x^6-c^2x^3a+d^2x^3}{3b^2c^4}+\frac{\left(a^2c^4-2a c^2d^2+d^4\right)\ln\left(b c^2x^3+a c^2-d^2\right)}{3b^3c^6}\right)-\frac{2d x^3\sqrt{b x^3+a}}{9b^2c^2}+\frac{2\left(\frac{\left(a c^2-d^2\right)d}{b^2c^4}+\frac{2da}{3b^2c^2}\right.}{3b}\right.$

input
```
int(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
d*(-2/9/b^3/c^2*(b*x^3+a)^(3/2)+2/3*a^2/b^3/d^2*(b*x^3+a)^(1/2)-1/3*(a^2*c
^4-2*a*c^2*d^2+d^4)/d^2/c^5/b^3*(d*ln(c*(b*x^3+a)^(1/2)-d)-d*ln(d+c*(b*x^3
+a)^(1/2))+2*c*(b*x^3+a)^(1/2)))-a*c*(-1/3/b^2/c^2*x^3+1/3*(-a^2*c^4+2*a*c
^2*d^2-d^4)/d^2/c^4/b^3*ln(b*c^2*x^3+a*c^2-d^2)+1/3*a^2/b^3/d^2*ln(b*x^3+a
))-b*c*(-1/3/c^4/b^3*(1/2*b*c^2*x^6-2*c^2*x^3*a+d^2*x^3)+1/3/c^6/b^4*(a^3*
c^6-3*a^2*c^4*d^2+3*a*c^2*d^4-d^6)/d^2*ln(b*c^2*x^3+a*c^2-d^2)-1/3/b^4*a^3
/d^2*ln(b*x^3+a))
```

3.552. $\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

3.552.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.36

$$\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \frac{3\,b^2c^4x^6-6\,(abc^4-bc^2d^2)x^3+6\,(a^2c^4-2\,ac^2d^2+d^4)\log\left(bc^2x^3+ac^2-d^2\right)+6\,(a^2c^4-2\,ac^2d^2+d^4)\log}{18\,b^3}$$

input
```
integrate(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

output
```
1/18*(3*b^2*c^4*x^6 - 6*(a*b*c^4 - b*c^2*d^2)*x^3 + 6*(a^2*c^4 - 2*a*c^2*d
^2 + d^4)*log(b*c^2*x^3 + a*c^2 - d^2) + 6*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*l
og(sqrt(b*x^3 + a)*c + d) - 6*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(sqrt(b*x^3
 + a)*c - d) - 4*(b*c^3*d*x^3 - 5*a*c^3*d + 3*c*d^3)*sqrt(b*x^3 + a))/(b^3
*c^5)
```

3.552.6 Sympy [A] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.04

$$\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \begin{cases} \frac{2\left(\frac{(a+bx^3)^2}{12c}-\frac{d(a+bx^3)^{\frac{3}{2}}}{9c^2}+\frac{(a+bx^3)(-2ac^2+d^2)}{6c^3}+\frac{\sqrt{a+bx^3}\cdot(2ac^2d-d^3)}{3c^4}+\frac{(ac^2-d^2)^2\left(\begin{cases}\frac{\sqrt{a+bx^3}}{d} & \text{for } c=0\\ \frac{\log\left(c\sqrt{a+bx^3}+d\right)}{c} & \text{otherwise}\end{cases}\right)}{3c^4}\right)}{b^3} & \text{for } b\neq \\ \frac{x^9}{3\cdot(3\sqrt{a}d+3ac)} & \text{otherw} \end{cases}$$

input
```
integrate(x**8/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

output
```
Piecewise((2*((a + b*x**3)**2/(12*c) - d*(a + b*x**3)**(3/2)/(9*c**2) + (a
 + b*x**3)*(-2*a*c**2 + d**2)/(6*c**3) + sqrt(a + b*x**3)*(2*a*c**2*d - d*
*3)/(3*c**4) + (a*c**2 - d**2)**2*Piecewise((sqrt(a + b*x**3)/d, Eq(c, 0))
, (log(c*sqrt(a + b*x**3) + d)/c, True))/(3*c**4))/b**3, Ne(b, 0)), (x**9/
(3*(3*sqrt(a)*d + 3*a*c)), True))
```

3.552.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.89

$$\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$$
$$= \frac{\frac{3\,(bx^3+a)^2c^3-4\,(bx^3+a)^{\frac{3}{2}}c^2d-6\,(2\,ac^3-cd^2)\,(bx^3+a)+12\,(2\,ac^2d-d^3)\,\sqrt{bx^3+a}}{c^4}+\frac{12\,(a^2c^4-2\,ac^2d^2+d^4)\,\log\left(\sqrt{bx^3+a}c+d\right)}{c^5}}{18\,b^3}$$

input
```
integrate(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

output
```
1/18*((3*(b*x^3 + a)^2*c^3 - 4*(b*x^3 + a)^(3/2)*c^2*d - 6*(2*a*c^3 - c*d^
2)*(b*x^3 + a) + 12*(2*a*c^2*d - d^3)*sqrt(b*x^3 + a))/c^4 + 12*(a^2*c^4 -
 2*a*c^2*d^2 + d^4)*log(sqrt(b*x^3 + a)*c + d)/c^5)/b^3
```

3.552.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.11

$$\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \frac{2\,(a^2c^4-2\,ac^2d^2+d^4)\log\left(\left|\sqrt{bx^3+a}c+d\right|\right)}{3\,b^3c^5}$$
$$+\frac{3\,(bx^3+a)^2b^9c^3-12\,(bx^3+a)ab^9c^3-4\,(bx^3+a)^{\frac{3}{2}}b^9c^2d+24\,\sqrt{bx^3+a}ab^9c^2d+6\,(bx^3+a)b^9cd^2-12}{18\,b^{12}c^4}$$

input
```
integrate(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

output
```
2/3*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(abs(sqrt(b*x^3 + a)*c + d))/(b^3*c^5
) + 1/18*(3*(b*x^3 + a)^2*b^9*c^3 - 12*(b*x^3 + a)*a*b^9*c^3 - 4*(b*x^3 +
a)^(3/2)*b^9*c^2*d + 24*sqrt(b*x^3 + a)*a*b^9*c^2*d + 6*(b*x^3 + a)*b^9*c*
d^2 - 12*sqrt(b*x^3 + a)*b^9*d^3)/(b^12*c^4)
```

3.552.9 Mupad [B] (verification not implemented)

Time = 17.83 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.43

$$\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \frac{\left(\frac{2\,d\,\left(a\,c^2-d^2\right)}{b^2\,c^4}+\frac{4\,a\,d}{3\,b^2\,c^2}\right)\,\sqrt{b\,x^3+a}}{3\,b}+\frac{x^6}{6\,b\,c} - \frac{x^3\,\left(a\,c^2-d^2\right)}{3\,b^2\,c^3}+\frac{\ln\left(\frac{d+c\,\sqrt{b\,x^3+a}}{d-c\,\sqrt{b\,x^3+a}}\right)\,\left(a\,c^2-d^2\right)^2}{3\,b^3\,c^5} + \frac{\ln\left(b\,c^2\,x^3+a\,c^2-d^2\right)\,\left(a^2\,c^4-2\,a\,c^2\,d^2+d^4\right)}{3\,b^3\,c^5} - \frac{2\,d\,x^3\,\sqrt{b\,x^3+a}}{9\,b^2\,c^2}$$

input
```
int(x^8/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)
```

output
```
(((2*d*(a*c^2 - d^2))/(b^2*c^4) + (4*a*d)/(3*b^2*c^2))*(a + b*x^3)^(1/2))/
(3*b) + x^6/(6*b*c) - (x^3*(a*c^2 - d^2))/(3*b^2*c^3) + (log((d + c*(a + b
*x^3)^(1/2))/(d - c*(a + b*x^3)^(1/2)))*(a*c^2 - d^2)^2)/(3*b^3*c^5) + (lo
g(a*c^2 - d^2 + b*c^2*x^3)*(d^4 + a^2*c^4 - 2*a*c^2*d^2))/(3*b^3*c^5) - (2
*d*x^3*(a + b*x^3)^(1/2))/(9*b^2*c^2)
```

3.553 $\int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

3.553.1 Optimal result

Integrand size = 29, antiderivative size = 73

$$\int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx=\frac{x^3}{3bc}-\frac{2d\sqrt{a+bx^3}}{3b^2c^2}-\frac{2(ac^2-d^2)\log\left(d+c\sqrt{a+bx^3}\right)}{3b^2c^3}$$

output
```
1/3*x^3/b/c-2/3*(a*c^2-d^2)*ln(d+c*(b*x^3+a)^(1/2))/b^2/c^3-2/3*d*(b*x^3+a
)^(1/2)/b^2/c^2
```

3.553.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx =\frac{c\left(ac+bcx^3-2d\sqrt{a+bx^3}\right)+(-2ac^2+2d^2)\log\left(d+c\sqrt{a+bx^3}\right)}{3b^2c^3}$$

input
```
Integrate[x^5/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

output
```
(c*(a*c + b*c*x^3 - 2*d*Sqrt[a + b*x^3]) + (-2*a*c^2 + 2*d^2)*Log[d + c*Sq
rt[a + b*x^3]])/(3*b^2*c^3)
```

3.553.3 Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2586, 7267, 25, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{d\sqrt{a+bx^3}+ac+bcx^3}\,dx$$

$$\Big\downarrow \text{2586}$$

$$\frac{1}{3}\int \frac{x^3}{bcx^3+ac+d\sqrt{bx^3+a}}dx^3$$

$$\Big\downarrow \text{7267}$$

$$\frac{2\int -\frac{a-x^6}{\sqrt{bx^3+ac}+d}d\sqrt{bx^3+a}}{3b^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\int \frac{a-x^6}{\sqrt{bx^3+ac}+d}d\sqrt{bx^3+a}}{3b^2}$$

$$\Big\downarrow \text{476}$$

$$-\frac{2\int \left(\frac{d}{c^2}-\frac{\sqrt{bx^3+a}}{c}+\frac{ac^2-d^2}{c^2\left(\sqrt{bx^3+ac}+d\right)}\right)d\sqrt{bx^3+a}}{3b^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(-\frac{d\sqrt{a+bx^3}}{c^2}-\frac{(ac^2-d^2)\log\left(c\sqrt{a+bx^3}+d\right)}{c^3}+\frac{x^6}{2c}\right)}{3b^2}$$

input
```
Int[x^5/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

output
```
(2*(x^6/(2*c) - (d*Sqrt[a + b*x^3])/c^2 - ((a*c^2 - d^2)*Log[d + c*Sqrt[a
+ b*x^3]])/c^3))/(3*b^2)
```

3.553.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2586
```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Simp[1/n   Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a
+ b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*
d, 0] && IntegerQ[(m + 1)/n]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.553.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(63) = 126$.

Time = 0.16 (sec) , antiderivative size = 246, normalized size of antiderivative = 3.37

method	result
default	$d\left(-\frac{2a\sqrt{b\,x^3+a}}{3b^2d^2}+\frac{\left(a\,c^2-d^2\right)\left(d\ln\left(c\sqrt{b\,x^3+a}-d\right)-d\ln\left(d+c\sqrt{b\,x^3+a}\right)+2c\sqrt{b\,x^3+a}\right)}{3b^2d^2c^3}\right)-ac\left(\frac{\left(a\,c^2-d^2\right)\ln\left(b\,c^2x^3+a\,c^2-d^2\right)}{3b^2d^2c^2}\right.$
elliptic	$\sqrt{b\,x^3+a}\left(d+c\sqrt{b\,x^3+a}\right)\left(c\left(\frac{x^3}{3b\,c^2}+\frac{\left(-a\,c^2+d^2\right)\ln\left(b\,c^2x^3+a\,c^2-d^2\right)}{3b^2c^4}\right)-\frac{2d\sqrt{b\,x^3+a}}{3b^2c^2}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\,c^2\_Z^3+a\,c^2-d^2\right)}\frac{\left(a\,c^2-d^2\right)\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.$

input
```
int(x^5/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
d*(-2/3*a/b^2/d^2*(b*x^3+a)^(1/2)+1/3*(a*c^2-d^2)/b^2/d^2*(d*ln(c*(b*x^3+a
)^(1/2)-d)-d*ln(d+c*(b*x^3+a)^(1/2))+2*c*(b*x^3+a)^(1/2))/c^3)-a*c*(1/3*(a
*c^2-d^2)/b^2/d^2/c^2*ln(b*c^2*x^3+a*c^2-d^2)-1/3*a/b^2/d^2*ln(b*x^3+a))-b
*c*(-1/3/b^2/c^2*x^3+1/3*(-a^2*c^4+2*a*c^2*d^2-d^4)/d^2/c^4/b^3*ln(b*c^2*x
^3+a*c^2-d^2)+1/3*a^2/b^3/d^2*ln(b*x^3+a))
```

3.553. $\int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

3.553.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.62

$$\int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \frac{bc^2x^3-2\sqrt{bx^3+a}cd-(ac^2-d^2)\log(bc^2x^3+ac^2-d^2)-(ac^2-d^2)\log\left(\sqrt{bx^3+a}c+d\right)+(ac^2-d^2)\,\mathrm{l}}{3\,b^2c^3}$$

input
```
integrate(x^5/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

output
```
1/3*(b*c^2*x^3 - 2*sqrt(b*x^3 + a)*c*d - (a*c^2 - d^2)*log(b*c^2*x^3 + a*c
^2 - d^2) - (a*c^2 - d^2)*log(sqrt(b*x^3 + a)*c + d) + (a*c^2 - d^2)*log(s
qrt(b*x^3 + a)*c - d))/(b^2*c^3)
```

3.553.6 Sympy [A] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.26

$$\int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \begin{cases} \frac{2\left(\frac{a+bx^3}{6c}-\frac{d\sqrt{a+bx^3}}{3c^2}-\frac{(ac^2-d^2)\left(\begin{cases}\frac{\sqrt{a+bx^3}}{d} & \text{for } c=0\\ \frac{\log\left(c\sqrt{a+bx^3}+d\right)}{c} & \text{otherwise}\end{cases}\right)}{3c^2}\right)}{b^2} & \text{for } b\neq 0\\ \frac{x^6}{2\cdot(3\sqrt{a}d+3ac)} & \text{otherwise}\end{cases}$$

input
```
integrate(x**5/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

output
```
Piecewise((2*((a + b*x**3)/(6*c) - d*sqrt(a + b*x**3)/(3*c**2) - (a*c**2 -
 d**2)*Piecewise((sqrt(a + b*x**3)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**3) +
 d)/c, True))/(3*c**2))/b**2, Ne(b, 0)), (x**6/(2*(3*sqrt(a)*d + 3*a*c)),
True))
```

3.553.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

$$\int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \frac{\frac{(bx^3+a)c-2\sqrt{bx^3+a}d}{c^2} - \frac{2(ac^2-d^2)\log\left(\sqrt{bx^3+a}c+d\right)}{c^3}}{3\,b^2}$$

input
```
integrate(x^5/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

output
```
1/3*(((b*x^3 + a)*c - 2*sqrt(b*x^3 + a)*d)/c^2 - 2*(a*c^2 - d^2)*log(sqrt(
b*x^3 + a)*c + d)/c^3)/b^2
```

3.553.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99

$$\int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = -\frac{\frac{2(ac^2-d^2)\log\left(\left|\sqrt{bx^3+a}c+d\right|\right)}{bc^3} - \frac{(bx^3+a)bc-2\sqrt{bx^3+a}bd}{b^2c^2}}{3\,b}$$

input
```
integrate(x^5/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

output
```
-1/3*(2*(a*c^2 - d^2)*log(abs(sqrt(b*x^3 + a)*c + d))/(b*c^3) - ((b*x^3 +
a)*b*c - 2*sqrt(b*x^3 + a)*b*d)/(b^2*c^2))/b
```

3.553.9 Mupad [B] (verification not implemented)

Time = 18.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.63

$$\int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \frac{x^3}{3\,b\,c} - \frac{2\,d\,\sqrt{b\,x^3+a}}{3\,b^2\,c^2} + \frac{\ln\left(\frac{d-c\,\sqrt{b\,x^3+a}}{d+c\,\sqrt{b\,x^3+a}}\right)\,\left(a\,c^2-d^2\right)}{3\,b^2\,c^3} - \frac{\ln\left(b\,c^2\,x^3+a\,c^2-d^2\right)\,\left(a\,c^2-d^2\right)}{3\,b^2\,c^3}$$

input
```
int(x^5/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)
```

output
```
x^3/(3*b*c) - (2*d*(a + b*x^3)^(1/2))/(3*b^2*c^2) + (log((d - c*(a + b*x^3
)^(1/2))/(d + c*(a + b*x^3)^(1/2)))*(a*c^2 - d^2))/(3*b^2*c^3) - (log(a*c^
2 - d^2 + b*c^2*x^3)*(a*c^2 - d^2))/(3*b^2*c^3)
```

3.554 $\int \frac{x^2}{ac+bcx^3+d\sqrt{a+bx^3}} dx$

3.554.1 Optimal result

Integrand size = 29, antiderivative size = 26

$$\int \frac{x^2}{ac+bcx^3+d\sqrt{a+bx^3}} dx = \frac{2\log\left(d+c\sqrt{a+bx^3}\right)}{3bc}$$

output
```
2/3*ln(d+c*(b*x^3+a)^(1/2))/b/c
```

3.554.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{ac+bcx^3+d\sqrt{a+bx^3}} dx = \frac{2\log\left(bd+bc\sqrt{a+bx^3}\right)}{3bc}$$

input
```
Integrate[x^2/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

output
```
(2*Log[b*d + b*c*Sqrt[a + b*x^3]])/(3*b*c)
```

3.554.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2586, 7267, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{d\sqrt{a+bx^3}+ac+bcx^3}dx$$

$$\Big\downarrow \; 2586$$

$$\frac{1}{3}\int \frac{1}{bcx^3+ac+d\sqrt{bx^3+a}}dx^3$$

$$\Big\downarrow \; 7267$$

$$\frac{2\int \frac{1}{\sqrt{bx^3+ac+d}}d\sqrt{bx^3+a}}{3b}$$

$$\Big\downarrow \; 16$$

$$\frac{2\log\left(c\sqrt{a+bx^3}+d\right)}{3bc}$$

input
```
Int[x^2/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

output
```
(2*Log[d + c*Sqrt[a + b*x^3]])/(3*b*c)
```

3.554.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2586
```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Simp[1/n   Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a
 + b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*
d, 0] && IntegerQ[(m + 1)/n]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.554.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(22) = 44$.

Time = 0.15 (sec) , antiderivative size = 160, normalized size of antiderivative = 6.15

method	result
default	$\frac{\ln\left(d+c\sqrt{b\,x^3+a}\right)}{3bc}-\frac{\ln\left(c\sqrt{b\,x^3+a}-d\right)}{3cb}+\frac{ac\ln\left(b\,c^2x^3+a\,c^2-d^2\right)}{3d^2b}-\frac{ac\ln\left(b\,x^3+a\right)}{3d^2b}-bc\left(\frac{\left(a\,c^2-d^2\right)\ln\left(b\,c^2x^3+a\,c^2-d^2\right)}{3b^2d^2c^2}-\right.$
elliptic	$\sqrt{b\,x^3+a}\left(d+c\sqrt{b\,x^3+a}\right)\left(\frac{\ln\left(b\,c^2x^3+a\,c^2-d^2\right)}{3bc}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\,c^2\_Z^3+a\,c^2-d^2\right)}\frac{\left(-b^2a\right)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{\left(-b^2a\right)^{\frac{1}{3}}-i\sqrt{3}\left(-b^2a\right)^{\frac{1}{3}}}{b}\right)}{\left(-b^2a\right)^{\frac{1}{3}}}}}{\ }\right.}{\ }\right.$

3.554. $\int \frac{x^2}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

input
```
int(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(d+c*(b*x^3+a)^(1/2))/b/c-1/3/c/b*ln(c*(b*x^3+a)^(1/2)-d)+1/3*a*c/d^
2/b*ln(b*c^2*x^3+a*c^2-d^2)-1/3*a*c/d^2/b*ln(b*x^3+a)-b*c*(1/3*(a*c^2-d^2)
/b^2/d^2/c^2*ln(b*c^2*x^3+a*c^2-d^2)-1/3*a/b^2/d^2*ln(b*x^3+a))
```

3.554.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(22) = 44$.

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.35

$$\int \frac{x^2}{ac + bcx^3 + d\sqrt{a + bx^3}}\,dx = \frac{\log\left(bc^2x^3 + ac^2 - d^2\right) + \log\left(\sqrt{bx^3 + a}c + d\right) - \log\left(\sqrt{bx^3 + a}c - d\right)}{3\,bc}$$

input
```
integrate(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

output
```
1/3*(log(b*c^2*x^3 + a*c^2 - d^2) + log(sqrt(b*x^3 + a)*c + d) - log(sqrt(
b*x^3 + a)*c - d))/(b*c)
```

3.554.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(20) = 40$.

Time = 1.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \frac{x^2}{ac + bcx^3 + d\sqrt{a + bx^3}}\,dx = \begin{cases} \frac{2\left(\begin{cases} \frac{\sqrt{a+bx^3}}{d} & \text{for}\: c = 0 \\ \frac{\log\left(c\sqrt{a+bx^3}+d\right)}{c} & \text{otherwise} \end{cases}\right)}{3b} & \text{for}\: b \neq 0 \\ \frac{x^3}{3\sqrt{a}d+3ac} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

output
```
Piecewise((2*Piecewise((sqrt(a + b*x**3)/d, Eq(c, 0)), (log(c*sqrt(a + b*x
**3) + d)/c, True))/(3*b), Ne(b, 0)), (x**3/(3*sqrt(a)*d + 3*a*c), True))
```

3.554.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx=\frac{2\,\log\left(\sqrt{bx^3+a}c+d\right)}{3\,bc}$$

input
```
integrate(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

output
```
2/3*log(sqrt(b*x^3 + a)*c + d)/(b*c)
```

3.554.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx=\frac{2\,\log\left(\left|\sqrt{bx^3+a}c+d\right|\right)}{3\,bc}$$

input
```
integrate(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

output
```
2/3*log(abs(sqrt(b*x^3 + a)*c + d))/(b*c)
```

3.554.9 Mupad [B] (verification not implemented)

Time = 17.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.31

$$\int \frac{x^2}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx=\frac{\ln\left(\frac{d+c\,\sqrt{b\,x^3+a}}{d-c\,\sqrt{b\,x^3+a}}\right)+\ln\left(b\,c^2\,x^3+a\,c^2-d^2\right)}{3\,b\,c}$$

input
```
int(x^2/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)
```

output
```
(log((d + c*(a + b*x^3)^(1/2))/(d - c*(a + b*x^3)^(1/2))) + log(a*c^2 - d^
2 + b*c^2*x^3))/(3*b*c)
```

3.555 $\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)} dx$

3.555.1 Optimal result

Integrand size = 29, antiderivative size = 93

$$\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)} dx = \frac{2d\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}\,(ac^2-d^2)} + \frac{c\log(x)}{ac^2-d^2} - \frac{2c\log\left(d+c\sqrt{a+bx^3}\right)}{3\,(ac^2-d^2)}$$

output
```
c*ln(x)/(a*c^2-d^2)-2/3*c*ln(d+c*(b*x^3+a)^(1/2))/(a*c^2-d^2)+2/3*d*arctan
h((b*x^3+a)^(1/2)/a^(1/2))/(a*c^2-d^2)/a^(1/2)
```

3.555.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.74

$$\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)} dx = \frac{\frac{2d\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{\sqrt{a}} + c\log\left(bx^3\right) - 2c\log\left(d+c\sqrt{a+bx^3}\right)}{3ac^2-3d^2}$$

input
```
Integrate[1/(x*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

output
```
((2*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/Sqrt[a] + c*Log[b*x^3] - 2*c*Log[d
 + c*Sqrt[a + b*x^3]])/(3*a*c^2 - 3*d^2)
```

3.555.3 Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2586, 7267, 25, 479, 452, 219, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(d\sqrt{a+bx^3}+ac+bcx^3\right)}dx$$

$$\Big\downarrow \text{2586}$$

$$\frac{1}{3}\int \frac{1}{x^3\left(bcx^3+ac+d\sqrt{bx^3+a}\right)}dx^3$$

$$\Big\downarrow \text{7267}$$

$$\frac{2}{3}\int -\frac{1}{(a-x^6)\left(\sqrt{bx^3+a}c+d\right)}d\sqrt{bx^3+a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2}{3}\int \frac{1}{(a-x^6)\left(\sqrt{bx^3+a}c+d\right)}d\sqrt{bx^3+a}$$

$$\Big\downarrow \text{479}$$

$$\frac{2}{3}\left(\frac{\int \frac{d-c\sqrt{bx^3+a}}{a-x^6}d\sqrt{bx^3+a}}{ac^2-d^2}-\frac{c\log\left(c\sqrt{a+bx^3}+d\right)}{ac^2-d^2}\right)$$

$$\Big\downarrow \text{452}$$

$$\frac{2}{3}\left(\frac{d\int \frac{1}{a-x^6}d\sqrt{bx^3+a}-c\int \frac{\sqrt{bx^3+a}}{a-x^6}d\sqrt{bx^3+a}}{ac^2-d^2}-\frac{c\log\left(c\sqrt{a+bx^3}+d\right)}{ac^2-d^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{2}{3}\left(\frac{\frac{d\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{\sqrt{a}}-c\int \frac{\sqrt{bx^3+a}}{a-x^6}d\sqrt{bx^3+a}}{ac^2-d^2}-\frac{c\log\left(c\sqrt{a+bx^3}+d\right)}{ac^2-d^2}\right)$$

$$\Big\downarrow \text{240}$$

$$\frac{2}{3}\left(\frac{\frac{d\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{1}{2}c\log\left(a-x^6\right)}{ac^2-d^2}-\frac{c\log\left(c\sqrt{a+bx^3}+d\right)}{ac^2-d^2}\right)$$

input
```
Int[1/(x*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

output
```
(2*(((d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/Sqrt[a] + (c*Log[a - x^6])/2)/(a
*c^2 - d^2) - (c*Log[d + c*Sqrt[a + b*x^3]])/(a*c^2 - d^2)))/3
```

3.555.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 479
```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[d*(Log
[RemoveContent[c + d*x, x]]/(b*c^2 + a*d^2)), x] + Simp[b/(b*c^2 + a*d^2)
 Int[(c - d*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2586
```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Simp[1/n   Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a
+ b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*
d, 0] && IntegerQ[(m + 1)/n]
```

3.555. $\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.555.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(81) = 162$.

Time = 1.11 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.40

method	result
default	$\frac{c\ln(x)}{a c^2-d^2}-\frac{a c^3\ln(b c^2x^3+a c^2-d^2)}{3(a c^2-d^2)d^2}+\frac{c\ln(b c^2x^3+a c^2-d^2)}{3d^2}-d\left(\frac{\frac{2\sqrt{b x^3+a}}{3}-\frac{2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{b x^3+a}}{\sqrt{a}}\right)}{3}}{a(a c^2-d^2)}+\frac{2\sqrt{b x^3+a}}{3a d^2}-\frac{c\left(d\,l\right.}{}\right.$
elliptic	Expression too large to display

input
```
int(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
c*ln(x)/(a*c^2-d^2)-1/3*a*c^3/(a*c^2-d^2)/d^2*ln(b*c^2*x^3+a*c^2-d^2)+1/3*
c/d^2*ln(b*c^2*x^3+a*c^2-d^2)-d*(1/a/(a*c^2-d^2)*(2/3*(b*x^3+a)^(1/2)-2/3*
a^(1/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+2/3/a/d^2*(b*x^3+a)^(1/2)-1/3*c/
(a*c^2-d^2)/d^2*(d*ln(c*(b*x^3+a)^(1/2)-d)-d*ln(d+c*(b*x^3+a)^(1/2))+2*c*(
b*x^3+a)^(1/2)))
```

3.555.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.49

$$\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$$
$$= \left[-\frac{ac\log\left(bc^2x^3+ac^2-d^2\right)+ac\log\left(\sqrt{bx^3+a}c+d\right)-ac\log\left(\sqrt{bx^3+a}c-d\right)-3\,ac\log\left(x\right)-\sqrt{a}d\log}{3\left(a^2c^2-ad^2\right)}\right.$$
$$-\frac{ac\log\left(bc^2x^3+ac^2-d^2\right)+ac\log\left(\sqrt{bx^3+a}c+d\right)-ac\log\left(\sqrt{bx^3+a}c-d\right)-3\,ac\log\left(x\right)+2\sqrt{-a}d\,\mathrm{a}}{3\left(a^2c^2-ad^2\right)}$$

input
```
integrate(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

output
```
[-1/3*(a*c*log(b*c^2*x^3 + a*c^2 - d^2) + a*c*log(sqrt(b*x^3 + a)*c + d) -
 a*c*log(sqrt(b*x^3 + a)*c - d) - 3*a*c*log(x) - sqrt(a)*d*log((b*x^3 + 2*
sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3))/(a^2*c^2 - a*d^2), -1/3*(a*c*log(b*c^
2*x^3 + a*c^2 - d^2) + a*c*log(sqrt(b*x^3 + a)*c + d) - a*c*log(sqrt(b*x^3
 + a)*c - d) - 3*a*c*log(x) + 2*sqrt(-a)*d*arctan(sqrt(b*x^3 + a)*sqrt(-a)
/a))/(a^2*c^2 - a*d^2)]
```

3.555.6 Sympy [A] (verification not implemented)

Time = 2.97 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.61

$$\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$$
$$=\begin{cases} \frac{2\left(-\frac{bc^2\left(\begin{cases}\frac{\sqrt{a+bx^3}}{d} & \text{for } c=0\\ \frac{\log\left(c\sqrt{a+bx^3}+d\right)}{c} & \text{otherwise}\end{cases}\right)}{3(ac^2-d^2)}-\frac{b\left(-\frac{c\log\left(-bx^3\right)}{2}+\frac{d\operatorname{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{\sqrt{-a}}\right)}{3(ac^2-d^2)}\right)}{b} & \text{for } b\neq 0\\ \begin{cases}\frac{x^3\log\left(x^3\right)}{3\sqrt{a}dx^3+3acx^3} & \text{for } 3\sqrt{a}d+3ac\neq 0\\ \tilde{\infty}x^3 & \text{otherwise}\end{cases} & \text{otherwise}\end{cases}$$

input
```
integrate(1/x/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

output
```
Piecewise((2*(-b*c**2*Piecewise((sqrt(a + b*x**3)/d, Eq(c, 0)), (log(c*sqr
t(a + b*x**3) + d)/c, True))/(3*(a*c**2 - d**2)) - b*(-c*log(-b*x**3)/2 +
d*atan(sqrt(a + b*x**3)/sqrt(-a))/sqrt(-a))/(3*(a*c**2 - d**2)))/b, Ne(b,
0)), (Piecewise((x**3*log(x**3)/(3*sqrt(a)*d*x**3 + 3*a*c*x**3), Ne(3*sqrt
(a)*d + 3*a*c, 0)), (zoo*x**3, True)), True))
```

3.555.7 Maxima [F]

$$\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=\int \frac{1}{\left(bcx^3+ac+\sqrt{bx^3+a}d\right)x}\,dx$$

input
```
integrate(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x), x)
```

3.555.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01

$$\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=-\frac{2\,c^2\log\left(\left|\sqrt{bx^3+a}c+d\right|\right)}{3\,(ac^3-cd^2)}+\frac{c\log\left(bx^3\right)}{3\,(ac^2-d^2)}-\frac{2\,d\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,(ac^2-d^2)\sqrt{-a}}$$

input
```
integrate(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

output
```
-2/3*c^2*log(abs(sqrt(b*x^3 + a)*c + d))/(a*c^3 - c*d^2) + 1/3*c*log(b*x^3
)/(a*c^2 - d^2) - 2/3*d*arctan(sqrt(b*x^3 + a)/sqrt(-a))/((a*c^2 - d^2)*sq
rt(-a))
```

3.555.9 Mupad [B] (verification not implemented)

Time = 17.76 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.68

$$\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=\frac{c\,\ln(x)}{a\,c^2-d^2}+\frac{c\,\ln\left(\frac{d-c\,\sqrt{b\,x^3+a}}{d+c\,\sqrt{b\,x^3+a}}\right)}{3\,\left(a\,c^2-d^2\right)}-\frac{c\,\ln\left(b\,c^2\,x^3+a\,c^2-d^2\right)}{3\,a\,c^2-3\,d^2}+\frac{d\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\,{\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}^3}{x^6}\right)}{3\,\sqrt{a}\,\left(a\,c^2-d^2\right)}$$

input
```
int(1/(x*(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3)),x)
```

output
```
(c*log(x))/(a*c^2 - d^2) + (c*log((d - c*(a + b*x^3)^(1/2))/(d + c*(a + b*
x^3)^(1/2))))/(3*(a*c^2 - d^2)) - (c*log(a*c^2 - d^2 + b*c^2*x^3))/(3*a*c^
2 - 3*d^2) + (d*log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^
(1/2))^3)/x^6))/(3*a^(1/2)*(a*c^2 - d^2))
```

3.555. $\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

3.556 $\int \frac{1}{x^4\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

3.556.1 Optimal result

Integrand size = 29, antiderivative size = 154

$$\int \frac{1}{x^4\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=-\frac{ac-d\sqrt{a+bx^3}}{3a\left(ac^2-d^2\right)x^3}-\frac{bd(3ac^2-d^2)\,\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}\left(ac^2-d^2\right)^2}-\frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2}+\frac{2bc^3\log\left(d+c\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)^2}$$

output
```
-1/3*b*d*(3*a*c^2-d^2)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)/(a*c^2-d^2
)^2-b*c^3*ln(x)/(a*c^2-d^2)^2+2/3*b*c^3*ln(d+c*(b*x^3+a)^(1/2))/(a*c^2-d^2
)^2+1/3*(-a*c+d*(b*x^3+a)^(1/2))/a/(a*c^2-d^2)/x^3
```

3.556.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^4\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=\frac{bd(-3ac^2+d^2)\,x^3\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\sqrt{a}\left(-\left(\left(ac^2-d^2\right)\left(ac-d\sqrt{a+bx^3}\right)\right)-abc^3x^3\log\left(bx^3\right)+2abc^3x^3\right.}{3a^{3/2}\left(-ac^2+d^2\right)^2x^3}$$

input
```
Integrate[1/(x^4*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

output
```
(b*d*(-3*a*c^2 + d^2)*x^3*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] + Sqrt[a]*(-((a
*c^2 - d^2)*(a*c - d*Sqrt[a + b*x^3])) - a*b*c^3*x^3*Log[b*x^3] + 2*a*b*c^
3*x^3*Log[d + c*Sqrt[a + b*x^3]]))/(3*a^(3/2)*(-(a*c^2) + d^2)^2*x^3)
```

3.556.3 Rubi [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2586, 7267, 496, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\left(d\sqrt{a+bx^3}+ac+bcx^3\right)}dx$$

$$\Big\downarrow \text{2586}$$

$$\frac{1}{3}\int \frac{1}{x^6\left(bcx^3+ac+d\sqrt{bx^3+a}\right)}dx^3$$

$$\Big\downarrow \text{7267}$$

$$\frac{2}{3}b\int \frac{1}{\left(a-x^6\right)^2\left(\sqrt{bx^3+a}c+d\right)}d\sqrt{bx^3+a}$$

$$\Big\downarrow \text{496}$$

$$\frac{2}{3}b\left(\frac{ac-d\sqrt{a+bx^3}}{2a\left(a-x^6\right)\left(ac^2-d^2\right)}-\frac{\int -\frac{2ac^2-d\sqrt{bx^3+a}c-d^2}{\left(a-x^6\right)\left(\sqrt{bx^3+a}c+d\right)}d\sqrt{bx^3+a}}{2a\left(ac^2-d^2\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{3}b\left(\frac{\int \frac{2ac^2-d\sqrt{bx^3+a}c-d^2}{\left(a-x^6\right)\left(\sqrt{bx^3+a}c+d\right)}d\sqrt{bx^3+a}}{2a\left(ac^2-d^2\right)}+\frac{ac-d\sqrt{a+bx^3}}{2a\left(a-x^6\right)\left(ac^2-d^2\right)}\right)$$

$$\Big\downarrow \text{657}$$

$$\frac{2}{3}b\left(\frac{\int \left(\frac{2ac^4}{\left(ac^2-d^2\right)\left(\sqrt{bx^3+a}c+d\right)}+\frac{2a\sqrt{bx^3+a}c^3-3adc^2+d^3}{\left(ac^2-d^2\right)\left(a-x^6\right)}\right)d\sqrt{bx^3+a}}{2a\left(ac^2-d^2\right)}+\frac{ac-d\sqrt{a+bx^3}}{2a\left(a-x^6\right)\left(ac^2-d^2\right)}\right)$$

$$\downarrow 2009$$

$$\frac{2}{3}b\left(\frac{-\frac{d(3ac^2-d^2)\text{arctanh}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{\sqrt{a}(ac^2-d^2)}+\frac{2ac^3\log\left(c\sqrt{a+bx^3}+d\right)}{ac^2-d^2}-\frac{ac^3\log(a-x^6)}{ac^2-d^2}}{2a\left(ac^2-d^2\right)}+\frac{ac-d\sqrt{a+bx^3}}{2a\left(a-x^6\right)\left(ac^2-d^2\right)}\right)$$

input
```
Int[1/(x^4*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

output
```
(2*b*((a*c - d*Sqrt[a + b*x^3])/(2*a*(a*c^2 - d^2)*(a - x^6)) + (-((d*(3*a
*c^2 - d^2)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(Sqrt[a]*(a*c^2 - d^2))) - (
a*c^3*Log[a - x^6])/(a*c^2 - d^2) + (2*a*c^3*Log[d + c*Sqrt[a + b*x^3]])/(
a*c^2 - d^2))/(2*a*(a*c^2 - d^2))))/3
```

3.556.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2586
```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Simp[1/n   Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a
+ b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*
d, 0] && IntegerQ[(m + 1)/n]
```

3.556. $\int \frac{1}{x^4\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.556.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(138) = 276$.

Time = 0.97 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.63

method	result
default	$ac\left(-\frac{b\ln(b\,x^3+a)}{3a^2d^2}-\frac{1}{3a(a\,c^2-d^2)x^3}-\frac{b(2a\,c^2-d^2)\ln(x)}{a^2(a\,c^2-d^2)^2}+\frac{b\,c^4\ln(b\,c^2x^3+a\,c^2-d^2)}{3(a\,c^2-d^2)^2d^2}\right)+bc\left(\frac{\ln(b\,x^3+a)}{3a\,d^2}+\frac{\ln(x)}{a(a\,c^2-d^2)}-\right.$
elliptic	Expression too large to display

input
```
int(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
a*c*(-1/3*b/a^2/d^2*ln(b*x^3+a)-1/3/a/(a*c^2-d^2)/x^3-b*(2*a*c^2-d^2)/a^2/
(a*c^2-d^2)^2*ln(x)+1/3*b*c^4/(a*c^2-d^2)^2/d^2*ln(b*c^2*x^3+a*c^2-d^2))+b
*c*(1/3/a/d^2*ln(b*x^3+a)+1/a/(a*c^2-d^2)*ln(x)-1/3*c^2/(a*c^2-d^2)/d^2*ln
(b*c^2*x^3+a*c^2-d^2))-d*(1/a/(a*c^2-d^2)*(-1/3*(b*x^3+a)^(1/2)/x^3-1/3*b*
arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2))-2/3*b/a^2/d^2*(b*x^3+a)^(1/2)-b*
(2*a*c^2-d^2)/a^2/(a*c^2-d^2)^2*(2/3*(b*x^3+a)^(1/2)-2/3*a^(1/2)*arctanh((
b*x^3+a)^(1/2)/a^(1/2)))+1/3*b*c^3/(a*c^2-d^2)^2/d^2*(d*ln(c*(b*x^3+a)^(1/
2)-d)-d*ln(d+c*(b*x^3+a)^(1/2))+2*c*(b*x^3+a)^(1/2)))
```

3.556.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 445, normalized size of antiderivative = 2.89

$$\int \frac{1}{x^4\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx = \left[\frac{2\,a^2bc^3x^3\log\left(bc^2x^3+ac^2-d^2\right)+2\,a^2bc^3x^3\log\left(\sqrt{bx^3+a}c+d\right)-2\,a^2bc^3x^3\log\left(\sqrt{bx^3+a}c-d\right)-6\,a^2}{6\,(a^4c^4-2\,a^3}\right.$$

input
```
integrate(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

output
```
[1/6*(2*a^2*b*c^3*x^3*log(b*c^2*x^3 + a*c^2 - d^2) + 2*a^2*b*c^3*x^3*log(s
qrt(b*x^3 + a)*c + d) - 2*a^2*b*c^3*x^3*log(sqrt(b*x^3 + a)*c - d) - 6*a^2
*b*c^3*x^3*log(x) - 2*a^3*c^3 - (3*a*b*c^2*d - b*d^3)*sqrt(a)*x^3*log((b*x
^3 + 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*a^2*c*d^2 + 2*(a^2*c^2*d -
a*d^3)*sqrt(b*x^3 + a))/((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4)*x^3), 1/3*(a^
2*b*c^3*x^3*log(b*c^2*x^3 + a*c^2 - d^2) + a^2*b*c^3*x^3*log(sqrt(b*x^3 +
a)*c + d) - a^2*b*c^3*x^3*log(sqrt(b*x^3 + a)*c - d) - 3*a^2*b*c^3*x^3*log
(x) - a^3*c^3 + (3*a*b*c^2*d - b*d^3)*sqrt(-a)*x^3*arctan(sqrt(b*x^3 + a)*
sqrt(-a)/a) + a^2*c*d^2 + (a^2*c^2*d - a*d^3)*sqrt(b*x^3 + a))/((a^4*c^4 -
 2*a^3*c^2*d^2 + a^2*d^4)*x^3)]
```

3.556.6 Sympy [F]

$$\int \frac{1}{x^4 \left(ac + bcx^3 + d\sqrt{a + bx^3}\right)}\,dx = \int \frac{1}{x^4 \left(ac + bcx^3 + d\sqrt{a + bx^3}\right)}\,dx$$

input
```
integrate(1/x**4/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

output
```
Integral(1/(x**4*(a*c + b*c*x**3 + d*sqrt(a + b*x**3))), x)
```

3.556.7 Maxima [F]

$$\int \frac{1}{x^4 \left(ac + bcx^3 + d\sqrt{a + bx^3}\right)}\,dx = \int \frac{1}{\left(bcx^3 + ac + \sqrt{bx^3 + a}d\right)x^4}\,dx$$

input
```
integrate(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^4), x)
```

3.556.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.37

$$\int \frac{1}{x^4\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx = \frac{2\,bc^4\log\left(\left|\sqrt{bx^3+a}c+d\right|\right)}{3\left(a^2c^5-2\,ac^3d^2+cd^4\right)} - \frac{bc^3\log\left(-bx^3\right)}{3\left(a^2c^4-2\,ac^2d^2+d^4\right)} + \frac{\left(3\,abc^2d-bd^3\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\left(a^3c^4-2\,a^2c^2d^2+ad^4\right)\sqrt{-a}} - \frac{a^2bc^3-abcd^2-\left(abc^2d-bd^3\right)\sqrt{bx^3+a}}{3\left(ac^2-d^2\right)^2abx^3}$$

input
```
integrate(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

output
```
2/3*b*c^4*log(abs(sqrt(b*x^3 + a)*c + d))/(a^2*c^5 - 2*a*c^3*d^2 + c*d^4)
- 1/3*b*c^3*log(-b*x^3)/(a^2*c^4 - 2*a*c^2*d^2 + d^4) + 1/3*(3*a*b*c^2*d -
 b*d^3)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/((a^3*c^4 - 2*a^2*c^2*d^2 + a*d^4
)*sqrt(-a)) - 1/3*(a^2*b*c^3 - a*b*c*d^2 - (a*b*c^2*d - b*d^3)*sqrt(b*x^3
+ a))/((a*c^2 - d^2)^2*a*b*x^3)
```

3.556.9 Mupad [B] (verification not implemented)

Time = 18.90 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.61

$$\int \frac{1}{x^4\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx = \frac{b\,c^3\,\ln\left(b\,c^2\,x^3+a\,c^2-d^2\right)}{3\,a^2\,c^4-6\,a\,c^2\,d^2+3\,d^4} - \frac{b\,c^3\,\ln\left(x\right)}{a^2\,c^4-2\,a\,c^2\,d^2+d^4} - \frac{c}{3\,x^3\,\left(a\,c^2-d^2\right)} + \frac{b\,c^3\,\ln\left(\frac{d+c\,\sqrt{b\,x^3+a}}{d-c\,\sqrt{b\,x^3+a}}\right)}{3\,\left(a\,c^2-d^2\right)^2} + \frac{d\,\sqrt{b\,x^3+a}}{3\,a\,x^3\,\left(a\,c^2-d^2\right)} + \frac{b\,d\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)\,\left(3\,a\,c^2-d^2\right)}{6\,a^{3/2}\,\left(a\,c^2-d^2\right)^2}$$

input
```
int(1/(x^4*(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3)),x)
```

output
```
(b*c^3*log(a*c^2 - d^2 + b*c^2*x^3))/(3*d^4 + 3*a^2*c^4 - 6*a*c^2*d^2) - (
b*c^3*log(x))/(d^4 + a^2*c^4 - 2*a*c^2*d^2) - c/(3*x^3*(a*c^2 - d^2)) + (b
*c^3*log((d + c*(a + b*x^3)^(1/2))/(d - c*(a + b*x^3)^(1/2))))/(3*(a*c^2 -
 d^2)^2) + (d*(a + b*x^3)^(1/2))/(3*a*x^3*(a*c^2 - d^2)) + (b*d*log((((a +
 b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)*(3*a*c^2 -
d^2))/(6*a^(3/2)*(a*c^2 - d^2)^2)
```

3.556. $\int \frac{1}{x^4\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

3.557 $\int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

3.557.1 Optimal result

Integrand size = 29, antiderivative size = 311

$$\begin{aligned}\int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx &= \frac{x}{bc}-\frac{dx^4\sqrt{1+\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4(ac^2-d^2)\sqrt{a+bx^3}} \\ &+\frac{\sqrt[3]{ac^2-d^2}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}c^{5/3}}-\frac{\sqrt[3]{ac^2-d^2}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{4/3}c^{5/3}} \\ &+\frac{\sqrt[3]{ac^2-d^2}\log\left((ac^2-d^2)^{2/3}-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+b^{2/3}c^{4/3}x^2\right)}{6b^{4/3}c^{5/3}}\end{aligned}$$

output
```
x/b/c-1/3*(a*c^2-d^2)^(1/3)*ln((a*c^2-d^2)^(1/3)+b^(1/3)*c^(2/3)*x)/b^(4/3
)/c^(5/3)+1/6*(a*c^2-d^2)^(1/3)*ln((a*c^2-d^2)^(2/3)-b^(1/3)*c^(2/3)*(a*c^
2-d^2)^(1/3)*x+b^(2/3)*c^(4/3)*x^2)/b^(4/3)/c^(5/3)+1/3*(a*c^2-d^2)^(1/3)*
arctan(1/3*(1-2*b^(1/3)*c^(2/3)*x/(a*c^2-d^2)^(1/3))*3^(1/2))/b^(4/3)/c^(5
/3)*3^(1/2)-1/4*d*x^4*AppellF1(4/3,1/2,1,7/3,-b*x^3/a,-b*c^2*x^3/(a*c^2-d^
2))*(1+b*x^3/a)^(1/2)/(a*c^2-d^2)/(b*x^3+a)^(1/2)
```

3.557.2 Mathematica [A] (verified)

Time = 10.43 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.95

$$\int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = -\frac{dx^4\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{(4ac^2-4d^2)\sqrt{a+bx^3}} + \frac{6\sqrt[3]{b}c^{2/3}x-2\sqrt{3}\sqrt[3]{ac^2-d^2}\arctan\left(\frac{-1+\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)-2\sqrt[3]{ac^2-d^2}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)+\sqrt[3]{a}}{6b^{4/3}c^{5/3}}$$

input
```
Integrate[x^3/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

output
```
-((d*x^4*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), -((b
*c^2*x^3)/(a*c^2 - d^2))])/((4*a*c^2 - 4*d^2)*Sqrt[a + b*x^3])) + (6*b^(1/
3)*c^(2/3)*x - 2*Sqrt[3]*(a*c^2 - d^2)^(1/3)*ArcTan[(-1 + (2*b^(1/3)*c^(2/
3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]] - 2*(a*c^2 - d^2)^(1/3)*Log[(a*c^2 - d
^2)^(1/3) + b^(1/3)*c^(2/3)*x] + (a*c^2 - d^2)^(1/3)*Log[(a*c^2 - d^2)^(2/
3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2])/(6*b^(4
/3)*c^(5/3))
```

3.557.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$, Rules used = {2587, 27, 843, 750, 16, 1013, 1012, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{d\sqrt{a+bx^3}+ac+bcx^3}\,dx$$

$$\Big\downarrow \text{ 2587}$$

$$ac\int \frac{x^3}{a(bc^2x^3+ac^2-d^2)}dx-ad\int \frac{x^3}{a\sqrt{bx^3+a}\,(bc^2x^3+ac^2-d^2)}dx$$

$$\Big\downarrow \text{ 27}$$

$$c\int \frac{x^3}{bc^2x^3+ac^2-d^2}dx-d\int \frac{x^3}{\sqrt{bx^3+a}\,(bc^2x^3+ac^2-d^2)}dx$$

$$\Big\downarrow \; 843$$

$$c\left(\frac{x}{bc^2}-\frac{\left(ac^2-d^2\right)\int \frac{1}{bc^2x^3+ac^2-d^2}dx}{bc^2}\right)-d\int \frac{x^3}{\sqrt{bx^3+a}\left(bc^2x^3+ac^2-d^2\right)}dx$$

$$\Big\downarrow \; 750$$

$$c\left(\frac{x}{bc^2}-\frac{\left(ac^2-d^2\right)\left(\frac{\int \frac{2\sqrt[3]{ac^2-d^2}-\sqrt[3]{b}c^{2/3}x}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx}{3\left(ac^2-d^2\right)^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}c^{2/3}x+\sqrt[3]{ac^2-d^2}}dx}{3\left(ac^2-d^2\right)^{2/3}}\right)}{bc^2}\right)-d\int \frac{x^3}{\sqrt{bx^3+a}\left(bc^2x^3+ac^2-d^2\right)}dx$$

$$\Big\downarrow \; 16$$

$$c\left(\frac{x}{bc^2}-\frac{\left(ac^2-d^2\right)\left(\frac{\int \frac{2\sqrt[3]{ac^2-d^2}-\sqrt[3]{b}c^{2/3}x}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx}{3\left(ac^2-d^2\right)^{2/3}}+\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\sqrt[3]{b}c^{2/3}\left(ac^2-d^2\right)^{2/3}}\right)}{bc^2}\right)-d\int \frac{x^3}{\sqrt{bx^3+a}\left(bc^2x^3+ac^2-d^2\right)}dx$$

$$\Big\downarrow \; 1013$$

3.557. $\int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

$$c\left(\frac{x}{bc^2}-\frac{(ac^2-d^2)\left(\frac{\int \frac{2\sqrt[3]{ac^2-d^2}-\sqrt[3]{b}c^{2/3}x}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+(ac^2-d^2)^{2/3}}dx}{3(ac^2-d^2)^{2/3}}+\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\sqrt[3]{b}c^{2/3}(ac^2-d^2)^{2/3}}\right)}{bc^2}\right)-\frac{d\sqrt{\frac{bx^3}{a}+1}\int \frac{x^3}{\sqrt{\frac{bx^3}{a}+1}(bc^2x^3+ac^2-d^2)}dx}{\sqrt{a+bx^3}}$$

$\big\downarrow$ 1012

$$c\left(\frac{x}{bc^2}-\frac{(ac^2-d^2)\left(\frac{\int \frac{2\sqrt[3]{ac^2-d^2}-\sqrt[3]{b}c^{2/3}x}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+(ac^2-d^2)^{2/3}}dx}{3(ac^2-d^2)^{2/3}}+\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\sqrt[3]{b}c^{2/3}(ac^2-d^2)^{2/3}}\right)}{bc^2}\right)-\frac{dx^4\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\sqrt{a+bx^3}\,(ac^2-d^2)}$$

$\big\downarrow$ 1142

3.557. $\int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

$$c\left(\frac{x}{bc^2}-\frac{(ac^2-d^2)\left(\frac{\frac{3}{2}\sqrt[3]{ac^2-d^2}\int\frac{1}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+(ac^2-d^2)^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}c^{2/3}\left(\sqrt[3]{ac^2-d^2}-2\sqrt[3]{b}c^{2/3}\right.}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+(a}}{2\sqrt[3]{b}c^{2/3}}}{3(ac^2-d^2)^{2/3}}\right.}{bc^2}\right.$$

$$\frac{dx^4\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\sqrt{a+bx^3}\,(ac^2-d^2)}$$

$$\Big\downarrow \text{25}$$

$$c\left(\frac{x}{bc^2}-\frac{(ac^2-d^2)\left(\frac{\frac{3}{2}\sqrt[3]{ac^2-d^2}\int\frac{1}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+(ac^2-d^2)^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}c^{2/3}\left(\sqrt[3]{ac^2-d^2}-2\sqrt[3]{b}c^{2/3}\right.}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+(ac}}{2\sqrt[3]{b}c^{2/3}}}{3(ac^2-d^2)^{2/3}}\right.}{bc^2}\right.$$

$$\frac{dx^4\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\sqrt{a+bx^3}\,(ac^2-d^2)}$$

$$\Big\downarrow \text{27}$$

3.557. $\int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

$$c\left(\frac{x}{bc^2}-\frac{\left(ac^2-d^2\right)\left(\frac{\frac{3}{2}\sqrt[3]{ac^2-d^2}\int\frac{1}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{ac^2-d^2}-2\sqrt[3]{b}c^{2/3}x}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+(}}{3\left(ac^2-d^2\right)^{2/3}}\right.}{bc^2}\right.$$

$$\frac{dx^4\sqrt{\frac{bx^3}{a}+1}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\sqrt{a+bx^3}\left(ac^2-d^2\right)}$$

$\downarrow$ 1082

$$c\left(\frac{x}{bc^2}-\frac{\left(ac^2-d^2\right)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{ac^2-d^2}-2\sqrt[3]{b}c^{2/3}x}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}\right)}{\sqrt[3]{b}c^{2/3}}}{3\left(ac^2-d^2\right)^{2/3}}+\cdots\right.}{bc^2}\right.$$

$$\frac{dx^4\sqrt{\frac{bx^3}{a}+1}\operatorname{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\sqrt{a+bx^3}\left(ac^2-d^2\right)}$$

$\downarrow$ 217

3.557. $\int\frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

$$c\left(\frac{x}{bc^2}-\frac{\left(ac^2-d^2\right)\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{ac^2-d^2}-2\sqrt[3]{b}c^{2/3}x}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt[3]{b}c^{2/3}}}{3\left(ac^2-d^2\right)^{2/3}}+\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{\cdots}\right.}{3\sqrt[3]{b}c^{2/3}\left(ac^2-d\cdots\right.}\right.}{bc^2}\right.$$

$$\frac{dx^4\sqrt{\frac{bx^3}{a}+1}\,\mathrm{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\sqrt{a+bx^3}\left(ac^2-d^2\right)}$$

$$\Big\downarrow \; 1103$$

3.557. $\int\frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

$$c\left(\frac{x}{bc^2}-\frac{\left(ac^2-d^2\right)\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt[3]{b}c^{2/3}}-\frac{\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{2\sqrt[3]{b}c^{2/3}}}{3\left(ac^2-d^2\right)^{2/3}}+\frac{\log\left(\sqrt[3]{ac^2-d^2}\cdots\right.}{3\sqrt[3]{b}c^{2/3}\left(ac^2\cdots\right.}\right.}{bc^2}\right.$$

$$\frac{dx^4\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\sqrt{a+bx^3}\left(ac^2-d^2\right)}$$

input
```
Int[x^3/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

output
```
-1/4*(d*x^4*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), -
((b*c^2*x^3)/(a*c^2 - d^2))])/((a*c^2 - d^2)*Sqrt[a + b*x^3]) + c*(x/(b*c^
2) - ((a*c^2 - d^2)*(Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x]/(3*b^(1/
3)*c^(2/3)*(a*c^2 - d^2)^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*c^(2/
3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]])/(b^(1/3)*c^(2/3))) - Log[(a*c^2 - d^2
)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2]/(2*
b^(1/3)*c^(2/3)))/(3*(a*c^2 - d^2)^(2/3))))/(b*c^2))
```

3.557.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.557. $\int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2587
```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Simp[c   Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Simp[a*e   Int[
u/((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e
, n}, x] && EqQ[b*c - a*d, 0]
```

3.557.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 0.99 (sec) , antiderivative size = 994, normalized size of antiderivative = 3.20

method	result	size
elliptic	Expression too large to display	994
default	Expression too large to display	1672

input
```
int(x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
(b*x^3+a)^(1/2)*(d+c*(b*x^3+a)^(1/2))/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2))*(c*(
1/b/c^2*x+(1/3/b/c^2/((a*c^2-d^2)/b/c^2)^(2/3)*ln(x+((a*c^2-d^2)/b/c^2)^(1
/3))-1/6/b/c^2/((a*c^2-d^2)/b/c^2)^(2/3)*ln(x^2-((a*c^2-d^2)/b/c^2)^(1/3)*
x+((a*c^2-d^2)/b/c^2)^(2/3))+1/3/b/c^2/((a*c^2-d^2)/b/c^2)^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2/((a*c^2-d^2)/b/c^2)^(1/3)*x-1)))*(-a*c^2+d^2)/b/c^2)+
2/3*I*d/b^2/c^2*3^(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^
(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(
1/3))/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x
+1/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(
1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/
3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1
/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)
))^(1/2))+1/3*I/d/b^4/c^2*2^(1/2)*sum((a*c^2-d^2)/_alpha^2*(-b^2*a)^(1/3)*
(1/2*I*b*(2*x+1/b*((-b^2*a)^(1/3)-I*3^(1/2)*(-b^2*a)^(1/3)))/(-b^2*a)^(1/3
))^(1/2)*(b*(x-1/b*(-b^2*a)^(1/3))/(-3*(-b^2*a)^(1/3)+I*3^(1/2)*(-b^2*a)^(
1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-b^2*a)^(1/3)+I*3^(1/2)*(-b^2*a)^(1/3)))
/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-b^2*a)^(1/3)*3^(1/2)*_alpha*b-
I*(-b^2*a)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-b^2*a)^(1/3)*_alpha*b-(-b^2*a)^(
2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-
b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),-1/2/b*c^2*(2*I*(-b^2*a)^...
```

3.557.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

output
```
Timed out
```

3.557.6 Sympy [F]

$$\int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx=\int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$$

input
```
integrate(x**3/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

output
```
Integral(x**3/(a*c + b*c*x**3 + d*sqrt(a + b*x**3)), x)
```

3.557.7 Maxima [F]

$$\int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx=\int \frac{x^3}{bcx^3+ac+\sqrt{bx^3+a}\,d}\,dx$$

input
```
integrate(x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x^3/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

3.557.8 Giac [F]

$$\int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx=\int \frac{x^3}{bcx^3+ac+\sqrt{bx^3+a}\,d}\,dx$$

input
```
integrate(x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

output
```
integrate(x^3/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

3.557.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx=\int \frac{x^3}{a\,c+d\,\sqrt{b\,x^3+a}+b\,c\,x^3}\,dx$$

input
```
int(x^3/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)
```

output
```
int(x^3/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3), x)
```

3.558 $\int \frac{x}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

3.558.1 Optimal result

Integrand size = 27, antiderivative size = 304

$$\int \frac{x}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = -\frac{dx^2\sqrt{1+\frac{bx^3}{a}}\operatorname{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\left(ac^2-d^2\right)\sqrt{a+bx^3}} - \frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}} - \frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}} + \frac{\log\left(\left(ac^2-d^2\right)^{2/3}-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+b^{2/3}c^{4/3}x^2\right)}{6b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}$$

output
```
-1/3*ln((a*c^2-d^2)^(1/3)+b^(1/3)*c^(2/3)*x)/b^(2/3)/c^(1/3)/(a*c^2-d^2)^(
1/3)+1/6*ln((a*c^2-d^2)^(2/3)-b^(1/3)*c^(2/3)*(a*c^2-d^2)^(1/3)*x+b^(2/3)*
c^(4/3)*x^2)/b^(2/3)/c^(1/3)/(a*c^2-d^2)^(1/3)-1/3*arctan(1/3*(1-2*b^(1/3)
*c^(2/3)*x/(a*c^2-d^2)^(1/3))*3^(1/2))/b^(2/3)/c^(1/3)/(a*c^2-d^2)^(1/3)*3
^(1/2)-1/2*d*x^2*AppellF1(2/3,1/2,1,5/3,-b*x^3/a,-b*c^2*x^3/(a*c^2-d^2))*(
1+b*x^3/a)^(1/2)/(a*c^2-d^2)/(b*x^3+a)^(1/2)
```

3.558.2 Mathematica [A] (verified)

Time = 10.21 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.83

$$\int \frac{x}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = -\frac{dx^2\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{(2ac^2-2d^2)\sqrt{a+bx^3}} + \frac{2\sqrt{3}\arctan\left(\frac{-1+\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)+\log\left((ac^2-d^2)^{2/3}-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}\cdots\right.}{6b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}$$

input
```
Integrate[x/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

output
```
-((d*x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b
*c^2*x^3)/(a*c^2 - d^2))])/((2*a*c^2 - 2*d^2)*Sqrt[a + b*x^3])) + (2*Sqrt[
3]*ArcTan[(-1 + (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]] - 2*Lo
g[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x] + Log[(a*c^2 - d^2)^(2/3) - b^(
1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2]])/(6*b^(2/3)*c^(1
/3)*(a*c^2 - d^2)^(1/3))
```

3.558.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2587, 27, 821, 16, 1013, 1012, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{d\sqrt{a+bx^3}+ac+bcx^3}\,dx$$

$$\Big\downarrow \text{2587}$$

$$ac\int \frac{x}{a(bc^2x^3+ac^2-d^2)}dx - ad\int \frac{x}{a\sqrt{bx^3+a}\,(bc^2x^3+ac^2-d^2)}dx$$

$$\Big\downarrow \text{27}$$

$$c\int \frac{x}{bc^2x^3+ac^2-d^2}dx - d\int \frac{x}{\sqrt{bx^3+a}\,(bc^2x^3+ac^2-d^2)}dx$$

$$\Big\downarrow \text{821}$$

$$c\left(\frac{\int \frac{\sqrt[3]{b}c^{2/3}x+\sqrt[3]{ac^2-d^2}}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+(ac^2-d^2)^{2/3}}dx}{3\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}}-\frac{\int \frac{1}{\sqrt[3]{b}c^{2/3}x+\sqrt[3]{ac^2-d^2}}dx}{3\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}}\right)-d\int \frac{x}{\sqrt{bx^3+a}\,(bc^2x^3+ac^2-d^2)}dx$$

$$\Big\downarrow \text{16}$$

$$c\left(\frac{\int \frac{\sqrt[3]{b}c^{2/3}x+\sqrt[3]{ac^2-d^2}}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+(ac^2-d^2)^{2/3}}dx}{3\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}}-\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{2/3}c^{4/3}\sqrt[3]{ac^2-d^2}}\right)-d\int \frac{x}{\sqrt{bx^3+a}\,(bc^2x^3+ac^2-d^2)}dx$$

$$\Big\downarrow \text{1013}$$

$$c\left(\frac{\int \frac{\sqrt[3]{b}c^{2/3}x+\sqrt[3]{ac^2-d^2}}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+(ac^2-d^2)^{2/3}}dx}{3\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}}-\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{2/3}c^{4/3}\sqrt[3]{ac^2-d^2}}\right)-\frac{d\sqrt{\frac{bx^3}{a}+1}\int \frac{x}{\sqrt{\frac{bx^3}{a}+1}(bc^2x^3+ac^2-d^2)}dx}{\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{1012}$$

$$c\left(\frac{\int \frac{\sqrt[3]{b}c^{2/3}x+\sqrt[3]{ac^2-d^2}}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+(ac^2-d^2)^{2/3}}dx}{3\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}}-\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{2/3}c^{4/3}\sqrt[3]{ac^2-d^2}}\right)-\frac{dx^2\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\sqrt{a+bx^3}\,(ac^2-d^2)}$$

$$\Big\downarrow \text{1142}$$

3.558. $\int \frac{x}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

$$c\left(\frac{\frac{3}{2}\sqrt[3]{ac^2-d^2}\int \frac{1}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+(ac^2-d^2)^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}c^{2/3}\left(\sqrt[3]{ac^2-d^2}-2\sqrt[3]{b}c^{2/3}x\right)}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx}{2\sqrt[3]{b}c^{2/3}}}{3\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}}-\frac{\text{lo}}{}\right.$$

$$\frac{dx^2\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\sqrt{a+bx^3}\,(ac^2-d^2)}$$

$$\Big\downarrow \text{25}$$

$$c\left(\frac{\frac{3}{2}\sqrt[3]{ac^2-d^2}\int \frac{1}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+(ac^2-d^2)^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}c^{2/3}\left(\sqrt[3]{ac^2-d^2}-2\sqrt[3]{b}c^{2/3}x\right)}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx}{2\sqrt[3]{b}c^{2/3}}}{3\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}}-\frac{\log}{}\right.$$

$$\frac{dx^2\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\sqrt{a+bx^3}\,(ac^2-d^2)}$$

$$\Big\downarrow \text{27}$$

$$c\left(\frac{\frac{3}{2}\sqrt[3]{ac^2-d^2}\int \frac{1}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+(ac^2-d^2)^{2/3}}dx-\frac{1}{2}\int \frac{\sqrt[3]{ac^2-d^2}-2\sqrt[3]{b}c^{2/3}x}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+(ac^2-d^2)^{2/3}}dx}{3\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}}-\right.$$

$$\frac{dx^2\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\sqrt{a+bx^3}\,(ac^2-d^2)}$$

$$\Big\downarrow \text{1082}$$

3.558. $\int \frac{x}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

$$c\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}\right)}{\sqrt[3]{b}c^{2/3}}-\frac{1}{2}\int \frac{\sqrt[3]{ac^2-d^2}-2\sqrt[3]{b}c^{2/3}x}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+(ac^2-d^2)^{2/3}}dx}{3\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}}-\frac{\log\left(\sqrt[3]{ac^2-d^2}\right.}{3b^{2/3}c^{4/3}\sqrt[3]{}}\right.$$

$$\frac{dx^2\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\sqrt{a+bx^3}\,(ac^2-d^2)}$$

$$\Big\downarrow \text{ 217}$$

$$c\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{ac^2-d^2}-2\sqrt[3]{b}c^{2/3}x}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+(ac^2-d^2)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt[3]{b}c^{2/3}}}{3\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}}-\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{2/3}c^{4/3}\sqrt[3]{ac^2-d^2}}\right)$$

$$\frac{dx^2\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\sqrt{a+bx^3}\,(ac^2-d^2)}$$

$$\Big\downarrow \text{ 1103}$$

$$c\left(\frac{\frac{\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+(ac^2-d^2)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{2\sqrt[3]{b}c^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt[3]{b}c^{2/3}}}{3\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}}-\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{2/3}c^{4/3}\sqrt[3]{ac^2-d^2}}\right)-$$

$$\frac{dx^2\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\sqrt{a+bx^3}\,(ac^2-d^2)}$$

3.558. $\int \frac{x}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

input
```
Int[x/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

output
```
-1/2*(d*x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -
((b*c^2*x^3)/(a*c^2 - d^2))])/((a*c^2 - d^2)*Sqrt[a + b*x^3]) + c*(-1/3*Lo
g[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x]/(b^(2/3)*c^(4/3)*(a*c^2 - d^2)^
(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3)
)/Sqrt[3]])/(b^(1/3)*c^(2/3))) + Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)
*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2]/(2*b^(1/3)*c^(2/3)))/(3*b^(1
/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)))
```

3.558.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2587
```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Simp[c   Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Simp[a*e   Int[
u/((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e
, n}, x] && EqQ[b*c - a*d, 0]
```

3.558.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 0.88 (sec) , antiderivative size = 665, normalized size of antiderivative = 2.19

method	result
elliptic	$\frac{\sqrt{b\,x^{3}+a}\left(d+c\sqrt{b\,x^{3}+a}\right)\left(-\frac{\ln\left(x+\left(\frac{a\,c^{2}-d^{2}}{b\,c^{2}}\right)^{\frac{1}{3}}\right)}{3bc\left(\frac{a\,c^{2}-d^{2}}{b\,c^{2}}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^{2}-\left(\frac{a\,c^{2}-d^{2}}{b\,c^{2}}\right)^{\frac{1}{3}}x+\left(\frac{a\,c^{2}-d^{2}}{b\,c^{2}}\right)^{\frac{2}{3}}\right)}{6bc\left(\frac{a\,c^{2}-d^{2}}{b\,c^{2}}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a\,c^{2}-d^{2}}{b\,c^{2}}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3bc\left(\frac{a\,c^{2}-d^{2}}{b\,c^{2}}\right)^{\frac{1}{3}}}-\ldots i\sqrt{\ldots}\right.}{\ldots}$
default	Expression too large to display

input
```
int(x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)
```

3.558. $\int \frac{x}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

output
```
(b*x^3+a)^(1/2)*(d+c*(b*x^3+a)^(1/2))/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2))*(-1/
3/b/c/((a*c^2-d^2)/b/c^2)^(1/3)*ln(x+((a*c^2-d^2)/b/c^2)^(1/3))+1/6/b/c/((
a*c^2-d^2)/b/c^2)^(1/3)*ln(x^2-((a*c^2-d^2)/b/c^2)^(1/3)*x+((a*c^2-d^2)/b/
c^2)^(2/3))+1/3/b/c*3^(1/2)/((a*c^2-d^2)/b/c^2)^(1/3)*arctan(1/3*3^(1/2)*(
2/((a*c^2-d^2)/b/c^2)^(1/3)*x-1))-1/3*I/d/b^3*2^(1/2)*sum(1/_alpha*(-b^2*a
)^(1/3)*(1/2*I*b*(2*x+1/b*((-b^2*a)^(1/3)-I*3^(1/2)*(-b^2*a)^(1/3)))/(-b^2
*a)^(1/3))^(1/2)*(b*(x-1/b*(-b^2*a)^(1/3))/(-3*(-b^2*a)^(1/3)+I*3^(1/2)*(-
b^2*a)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-b^2*a)^(1/3)+I*3^(1/2)*(-b^2*a)
^(1/3)))/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-b^2*a)^(1/3)*3^(1/2)*_
alpha*b-I*(-b^2*a)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-b^2*a)^(1/3)*_alpha*b-(-
b^2*a)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1
/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),-1/2/b*c^2*(2*I*(-b^
2*a)^(1/3)*3^(1/2)*_alpha^2*b-I*(-b^2*a)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*a*
b-3*(-b^2*a)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(
-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b
*c^2+a*c^2-d^2)))
```

3.558.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx=\text{Timed out}$$

input
```
integrate(x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

output
```
Timed out
```

3.558.6 Sympy [F]

$$\int \frac{x}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx=\int \frac{x}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$$

input
```
integrate(x/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

output
```
Integral(x/(a*c + b*c*x**3 + d*sqrt(a + b*x**3)), x)
```

3.558.7 Maxima [F]

$$\int \frac{x}{ac + bcx^3 + d\sqrt{a + bx^3}}\,dx = \int \frac{x}{bcx^3 + ac + \sqrt{bx^3 + a}d}\,dx$$

input
```
integrate(x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

3.558.8 Giac [F]

$$\int \frac{x}{ac + bcx^3 + d\sqrt{a + bx^3}}\,dx = \int \frac{x}{bcx^3 + ac + \sqrt{bx^3 + a}d}\,dx$$

input
```
integrate(x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

output
```
integrate(x/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

3.558.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{ac + bcx^3 + d\sqrt{a + bx^3}}\,dx = \int \frac{x}{a\,c + d\,\sqrt{b\,x^3 + a} + b\,c\,x^3}\,dx$$

input
```
int(x/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)
```

output
```
int(x/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3), x)
```

3.559 $\int \frac{1}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

3.559.1 Optimal result

Integrand size = 25, antiderivative size = 300

$$\int \frac{1}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = -\frac{dx\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{(ac^2-d^2)\sqrt{a+bx^3}} - \frac{\sqrt[3]{c}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}\,(ac^2-d^2)^{2/3}} + \frac{\sqrt[3]{c}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\sqrt[3]{b}\,(ac^2-d^2)^{2/3}} - \frac{\sqrt[3]{c}\log\left((ac^2-d^2)^{2/3}-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+b^{2/3}c^{4/3}x^2\right)}{6\sqrt[3]{b}\,(ac^2-d^2)^{2/3}}$$

output
```
1/3*c^(1/3)*ln((a*c^2-d^2)^(1/3)+b^(1/3)*c^(2/3)*x)/b^(1/3)/(a*c^2-d^2)^(2
/3)-1/6*c^(1/3)*ln((a*c^2-d^2)^(2/3)-b^(1/3)*c^(2/3)*(a*c^2-d^2)^(1/3)*x+b
^(2/3)*c^(4/3)*x^2)/b^(1/3)/(a*c^2-d^2)^(2/3)-1/3*c^(1/3)*arctan(1/3*(1-2*
b^(1/3)*c^(2/3)*x/(a*c^2-d^2)^(1/3))*3^(1/2))/b^(1/3)/(a*c^2-d^2)^(2/3)*3^
(1/2)-d*x*AppellF1(1/3,1/2,1,4/3,-b*x^3/a,-b*c^2*x^3/(a*c^2-d^2))*(1+b*x^3
/a)^(1/2)/(a*c^2-d^2)/(b*x^3+a)^(1/2)
```

3.559.2 Mathematica [A] (warning: unable to verify)

Time = 10.46 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.42

$$\int \frac{1}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx =$$

$$-\frac{8ad(ac^2-d^2)\,x\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\right.}{\sqrt{a+bx^3}\,(ac^2-d^2+bc^2x^3)\left(8a\,(ac^2-d^2)\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)-3bx^3\left(2ac^2\,\text{AppellF1}\right.\right.}$$

$$+\frac{\sqrt[3]{c}\left(2\sqrt{3}\arctan\left(\frac{-1+\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)+2\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)-\log\left((ac^2-d^2)^{2/3}-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac\right.\right.}{6\sqrt[3]{b}\,(ac^2-d^2)^{2/3}}$$

input
```
Integrate[(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])^(-1),x]
```

output
```
(-8*a*d*(a*c^2 - d^2)*x*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*c^2*
x^3)/(a*c^2 - d^2))])/(Sqrt[a + b*x^3]*(a*c^2 - d^2 + b*c^2*x^3)*(8*a*(a*c
^2 - d^2)*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 -
d^2))] - 3*b*x^3*(2*a*c^2*AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*c^
2*x^3)/(a*c^2 - d^2))] + (a*c^2 - d^2)*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3
)/a), -((b*c^2*x^3)/(a*c^2 - d^2))]))) + (c^(1/3)*(2*Sqrt[3]*ArcTan[(-1 +
(2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]] + 2*Log[(a*c^2 - d^2)^
(1/3) + b^(1/3)*c^(2/3)*x] - Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*
c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2]))/(6*b^(1/3)*(a*c^2 - d^2)^(2/3)
)
```

3.559.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {2587, 27, 750, 16, 27, 937, 936, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{d\sqrt{a+bx^3}+ac+bcx^3}\,dx$$

$$\Big\downarrow \; 2587$$

$$ac\int \frac{1}{abc^2x^3+a\left(ac^2-d^2\right)}dx-ad\int \frac{1}{a\sqrt{bx^3+a}\left(bc^2x^3+ac^2-d^2\right)}dx$$

$$\Big\downarrow \text{27}$$

$$ac\int \frac{1}{abc^2x^3+a\left(ac^2-d^2\right)}dx-d\int \frac{1}{\sqrt{bx^3+a}\left(bc^2x^3+ac^2-d^2\right)}dx$$

$$\Big\downarrow \text{750}$$

$$ac\left(\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{ac^2-d^2}-\sqrt[3]{b}c^{2/3}x\right)}{a^{2/3}b^{2/3}c^{4/3}x^2-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+a^{2/3}\left(ac^2-d^2\right)^{2/3}}dx}{3a^{2/3}\left(ac^2-d^2\right)^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}c^{2/3}x+\sqrt[3]{a}\sqrt[3]{ac^2-d^2}}dx}{3a^{2/3}\left(ac^2-d^2\right)^{2/3}}\right)-d\int \frac{1}{\sqrt{bx^3+a}\left(bc^2x^3+ac^2-d^2\right)}dx$$

$$\Big\downarrow \text{16}$$

$$ac\left(\frac{\int \frac{\sqrt[3]{a}\left(2\sqrt[3]{ac^2-d^2}-\sqrt[3]{b}c^{2/3}x\right)}{a^{2/3}b^{2/3}c^{4/3}x^2-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+a^{2/3}\left(ac^2-d^2\right)^{2/3}}dx}{3a^{2/3}\left(ac^2-d^2\right)^{2/3}}+\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3a\sqrt[3]{b}c^{2/3}\left(ac^2-d^2\right)^{2/3}}\right)-d\int \frac{1}{\sqrt{bx^3+a}\left(bc^2x^3+ac^2-d^2\right)}dx$$

$$\Big\downarrow \text{27}$$

$$ac\left(\frac{\int \frac{2\sqrt[3]{ac^2-d^2}-\sqrt[3]{b}c^{2/3}x}{a^{2/3}b^{2/3}c^{4/3}x^2-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+a^{2/3}\left(ac^2-d^2\right)^{2/3}}dx}{3\sqrt[3]{a}\left(ac^2-d^2\right)^{2/3}}+\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3a\sqrt[3]{b}c^{2/3}\left(ac^2-d^2\right)^{2/3}}\right)-d\int \frac{1}{\sqrt{bx^3+a}\left(bc^2x^3+ac^2-d^2\right)}dx$$

$$\Big\downarrow \text{937}$$

$$ac\left(\frac{\int \frac{2\sqrt[3]{ac^2-d^2}-\sqrt[3]{b}c^{2/3}x}{a^{2/3}b^{2/3}c^{4/3}x^2-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+a^{2/3}\left(ac^2-d^2\right)^{2/3}}dx}{3\sqrt[3]{a}\left(ac^2-d^2\right)^{2/3}}+\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3a\sqrt[3]{b}c^{2/3}\left(ac^2-d^2\right)^{2/3}}\right)-\frac{d\sqrt{\frac{bx^3}{a}+1}\int \frac{1}{\sqrt{\frac{bx^3}{a}+1}\left(bc^2x^3+ac^2-d^2\right)}dx}{\sqrt{a+bx^3}}$$

$$\Big\downarrow \text{936}$$

3.559. $\int \frac{1}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

$$ac\left(\frac{\int \frac{2\sqrt[3]{ac^2-d^2}-\sqrt[3]{b}c^{2/3}x}{a^{2/3}b^{2/3}c^{4/3}x^2-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+a^{2/3}(ac^2-d^2)^{2/3}}dx}{3\sqrt[3]{a}\left(ac^2-d^2\right)^{2/3}}+\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3a\sqrt[3]{b}c^{2/3}\left(ac^2-d^2\right)^{2/3}}\right)-$$

$$\frac{dx\sqrt{\frac{bx^3}{a}+1}\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\sqrt{a+bx^3}\left(ac^2-d^2\right)}$$

$\downarrow$ 1142

$$ac\left(\frac{\frac{3}{2}\sqrt[3]{ac^2-d^2}\int \frac{1}{a^{2/3}b^{2/3}c^{4/3}x^2-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+a^{2/3}(ac^2-d^2)^{2/3}}dx-\frac{\int -\frac{a^{2/3}\sqrt[3]{b}c^{2/3}\left(\sqrt[3]{ac^2-d^2}-2\sqrt[3]{\right.}}{a^{2/3}b^{2/3}c^{4/3}x^2-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x}}{2a^{2/3}\sqrt[3]{b}c^{2/3}}}{3\sqrt[3]{a}\left(ac^2-d^2\right)^{2/3}}\right.$$

$$\frac{dx\sqrt{\frac{bx^3}{a}+1}\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\sqrt{a+bx^3}\left(ac^2-d^2\right)}$$

$\downarrow$ 25

$$ac\left(\frac{\frac{\int \frac{\sqrt[3]{b}c^{2/3}\left(\sqrt[3]{ac^2-d^2}-2\sqrt[3]{b}c^{2/3}x\right)}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx}{2a^{2/3}\sqrt[3]{b}c^{2/3}}+\frac{3}{2}\sqrt[3]{ac^2-d^2}\int \frac{1}{a^{2/3}b^{2/3}c^{4/3}x^2-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+a^{2/3}(ac^2-d^2)}}{3\sqrt[3]{a}\left(ac^2-d^2\right)^{2/3}}\right.$$

$$\frac{dx\sqrt{\frac{bx^3}{a}+1}\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\sqrt{a+bx^3}\left(ac^2-d^2\right)}$$

$\downarrow$ 27

3.559. $\int \frac{1}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

$$ac\left(\frac{\frac{\int \frac{\sqrt[3]{ac^2-d^2}-2\sqrt[3]{b}c^{2/3}x}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx}{2a^{2/3}}+\frac{3}{2}\sqrt[3]{ac^2-d^2}\int \frac{1}{a^{2/3}b^{2/3}c^{4/3}x^2-a^{2/3}\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+a^{2/3}(ac^2-d^2)}}{3\sqrt[3]{a}\left(ac^2-d^2\right)^{2/3}}\right.$$

$$\frac{dx\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\sqrt{a+bx^3}\left(ac^2-d^2\right)}$$

$$\Big\downarrow \text{1082}$$

$$ac\left(\frac{\frac{\int \frac{\sqrt[3]{ac^2-d^2}-2\sqrt[3]{b}c^{2/3}x}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx}{2a^{2/3}}+\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}\right)}{a^{2/3}\sqrt[3]{b}c^{2/3}}}{3\sqrt[3]{a}\left(ac^2-d^2\right)^{2/3}}+\frac{\log\left(\sqrt[3]{ac^2-d^2}\right.}{3a\sqrt[3]{b}c^{2/3}\left(ac^2\right.}\right.$$

$$\frac{dx\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\sqrt{a+bx^3}\left(ac^2-d^2\right)}$$

$$\Big\downarrow \text{217}$$

$$ac\left(\frac{\frac{\int \frac{\sqrt[3]{ac^2-d^2}-2\sqrt[3]{b}c^{2/3}x}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx}{2a^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{a^{2/3}\sqrt[3]{b}c^{2/3}}}{3\sqrt[3]{a}\left(ac^2-d^2\right)^{2/3}}+\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3a\sqrt[3]{b}c^{2/3}\left(ac^2-d^2\right)^{2/3}}\right)-$$

$$\frac{dx\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\sqrt{a+bx^3}\left(ac^2-d^2\right)}$$

$$\Big\downarrow \text{1103}$$

3.559. $\int \frac{1}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

$$ac\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{a^{2/3}\sqrt[3]{b}c^{2/3}}-\frac{\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+(ac^2-d^2)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{2a^{2/3}\sqrt[3]{b}c^{2/3}}}{3\sqrt[3]{a}\,(ac^2-d^2)^{2/3}}+\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3a\sqrt[3]{b}c^{2/3}\,(ac^2-d^2)^{2/3}}\right.$$

$$\frac{dx\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\sqrt{a+bx^3}\,(ac^2-d^2)}$$

input
```
Int[(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])^(-1),x]
```

output
```
-((d*x*Sqrt[1 + (b*x^3)/a]*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*c
^2*x^3)/(a*c^2 - d^2))])/((a*c^2 - d^2)*Sqrt[a + b*x^3])) + a*c*(Log[(a*c^
2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x]/(3*a*b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(2/
3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/S
qrt[3]])/(a^(2/3)*b^(1/3)*c^(2/3))) - Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^
(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2]/(2*a^(2/3)*b^(1/3)*c^(2
/3)))/(3*a^(1/3)*(a*c^2 - d^2)^(2/3)))
```

3.559.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2587
```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Simp[c   Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Simp[a*e   Int[
u/((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e
, n}, x] && EqQ[b*c - a*d, 0]
```

3.559. $\int \frac{1}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

3.559.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 0.86 (sec) , antiderivative size = 665, normalized size of antiderivative = 2.22

method	result
elliptic	$\sqrt{b x^3+a}\left(d+c\sqrt{b x^3+a}\right)\left(\frac{\ln\left(x+\left(\frac{a c^2-d^2}{b c^2}\right)^{\frac{1}{3}}\right)}{3bc\left(\frac{a c^2-d^2}{b c^2}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{a c^2-d^2}{b c^2}\right)^{\frac{1}{3}}x+\left(\frac{a c^2-d^2}{b c^2}\right)^{\frac{2}{3}}\right)}{6bc\left(\frac{a c^2-d^2}{b c^2}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a c^2-d^2}{b c^2}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3bc\left(\frac{a c^2-d^2}{b c^2}\right)^{\frac{2}{3}}}-\right.$ ... $i\sqrt{2}$
default	Expression too large to display

input
```
int(1/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
(b*x^3+a)^(1/2)*(d+c*(b*x^3+a)^(1/2))/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2))*(1/3
/b/c/((a*c^2-d^2)/b/c^2)^(2/3)*ln(x+((a*c^2-d^2)/b/c^2)^(1/3))-1/6/b/c/((a
*c^2-d^2)/b/c^2)^(2/3)*ln(x^2-((a*c^2-d^2)/b/c^2)^(1/3)*x+((a*c^2-d^2)/b/c
^2)^(2/3))+1/3/b/c/((a*c^2-d^2)/b/c^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
/((a*c^2-d^2)/b/c^2)^(1/3)*x-1))-1/3*I/d/b^3*2^(1/2)*sum(1/_alpha^2*(-b^2*
a)^(1/3)*(1/2*I*b*(2*x+1/b*((-b^2*a)^(1/3)-I*3^(1/2)*(-b^2*a)^(1/3)))/(-b^
2*a)^(1/3))^(1/2)*(b*(x-1/b*(-b^2*a)^(1/3))/(-3*(-b^2*a)^(1/3)+I*3^(1/2)*(
-b^2*a)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-b^2*a)^(1/3)+I*3^(1/2)*(-b^2*a
)^(1/3)))/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-b^2*a)^(1/3)*3^(1/2)*
_alpha*b-I*(-b^2*a)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-b^2*a)^(1/3)*_alpha*b-(
-b^2*a)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(
1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),-1/2/b*c^2*(2*I*(-b
^2*a)^(1/3)*3^(1/2)*_alpha^2*b-I*(-b^2*a)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*a
*b-3*(-b^2*a)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*
(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*
b*c^2+a*c^2-d^2)))
```

3.559.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   Not
integrable (provided residues have no relations)
```

3.559.6 Sympy [F]

$$\int \frac{1}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \int \frac{1}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$$

input
```
integrate(1/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

output
```
Integral(1/(a*c + b*c*x**3 + d*sqrt(a + b*x**3)), x)
```

3.559.7 Maxima [F]

$$\int \frac{1}{ac + bcx^3 + d\sqrt{a + bx^3}} \, dx = \int \frac{1}{bcx^3 + ac + \sqrt{bx^3 + a}d} \, dx$$

input
```
integrate(1/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

3.559.8 Giac [F]

$$\int \frac{1}{ac + bcx^3 + d\sqrt{a + bx^3}} \, dx = \int \frac{1}{bcx^3 + ac + \sqrt{bx^3 + a}d} \, dx$$

input
```
integrate(1/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

output
```
integrate(1/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

3.559.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{ac + bcx^3 + d\sqrt{a + bx^3}} \, dx = \int \frac{1}{a\,c + d\,\sqrt{b\,x^3 + a} + b\,c\,x^3} \, dx$$

input
```
int(1/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)
```

output
```
int(1/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3), x)
```

3.560 $\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

3.560.1 Optimal result

Integrand size = 29, antiderivative size = 319

$$\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx = -\frac{c}{(ac^2-d^2)\,x}+\frac{d\sqrt{1+\frac{bx^3}{a}}\operatorname{AppellF1}\left(-\frac{1}{3},\frac{1}{2},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{(ac^2-d^2)\,x\sqrt{a+bx^3}}+\frac{\sqrt[3]{b}c^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\,(ac^2-d^2)^{4/3}}+\frac{\sqrt[3]{b}c^{5/3}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\,(ac^2-d^2)^{4/3}}-\frac{\sqrt[3]{b}c^{5/3}\log\left((ac^2-d^2)^{2/3}-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}\,x+b^{2/3}c^{4/3}x^2\right)}{6\,(ac^2-d^2)^{4/3}}$$

output
```
-c/(a*c^2-d^2)/x+1/3*b^(1/3)*c^(5/3)*ln((a*c^2-d^2)^(1/3)+b^(1/3)*c^(2/3)*
x)/(a*c^2-d^2)^(4/3)-1/6*b^(1/3)*c^(5/3)*ln((a*c^2-d^2)^(2/3)-b^(1/3)*c^(2
/3)*(a*c^2-d^2)^(1/3)*x+b^(2/3)*c^(4/3)*x^2)/(a*c^2-d^2)^(4/3)+1/3*b^(1/3)
*c^(5/3)*arctan(1/3*(1-2*b^(1/3)*c^(2/3)*x/(a*c^2-d^2)^(1/3))*3^(1/2))/(a*
c^2-d^2)^(4/3)*3^(1/2)+d*AppellF1(-1/3,1/2,1,2/3,-b*x^3/a,-b*c^2*x^3/(a*c^
2-d^2))*(1+b*x^3/a)^(1/2)/(a*c^2-d^2)/x/(b*x^3+a)^(1/2)
```

3.560.2 Mathematica [A] (verified)

Time = 10.52 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.55

$$\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$$

$$= \frac{15bd\sqrt[3]{ac^2-d^2}(ac^2+d^2)\,x^3\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{2},1,\frac{5}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)-6b^2c^2d\sqrt[3]{ac^2-d^2}x^6\sqrt{1+\frac{bx^3}{a}}}{}$$

input
```
Integrate[1/(x^2*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

output
```
(15*b*d*(a*c^2 - d^2)^(1/3)*(a*c^2 + d^2)*x^3*Sqrt[1 + (b*x^3)/a]*AppellF1
[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))] - 6*b^2*c^2
*d*(a*c^2 - d^2)^(1/3)*x^6*Sqrt[1 + (b*x^3)/a]*AppellF1[5/3, 1/2, 1, 8/3,
-((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))] - 10*(a*c^2 - d^2)*(-6*a*d*(a*
c^2 - d^2)^(1/3) - 6*b*d*(a*c^2 - d^2)^(1/3)*x^3 + 6*a*c*(a*c^2 - d^2)^(1/
3)*Sqrt[a + b*x^3] + 2*Sqrt[3]*a*b^(1/3)*c^(5/3)*x*Sqrt[a + b*x^3]*ArcTan[
(-1 + (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]] - 2*a*b^(1/3)*c^
(5/3)*x*Sqrt[a + b*x^3]*Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x] + a*b
^(1/3)*c^(5/3)*x*Sqrt[a + b*x^3]*Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)
*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2]))/(60*a*(a*c^2 - d^2)^(7/3)*
x*Sqrt[a + b*x^3])
```

3.560.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {2587, 27, 847, 821, 16, 1013, 1012, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(d\sqrt{a+bx^3}+ac+bcx^3\right)}\,dx$$

$$\Big\downarrow \text{2587}$$

$$ac\int \frac{1}{ax^2\left(bc^2x^3+ac^2-d^2\right)}dx-ad\int \frac{1}{ax^2\sqrt{bx^3+a}\left(bc^2x^3+ac^2-d^2\right)}dx$$

$$\Big\downarrow \text{27}$$

$$c\int \frac{1}{x^2\left(bc^2x^3+ac^2-d^2\right)}dx-d\int \frac{1}{x^2\sqrt{bx^3+a}\left(bc^2x^3+ac^2-d^2\right)}dx$$

$$\Big\downarrow \text{847}$$

$$c\left(-\frac{bc^2\int \frac{x}{bc^2x^3+ac^2-d^2}dx}{ac^2-d^2}-\frac{1}{x\left(ac^2-d^2\right)}\right)-d\int \frac{1}{x^2\sqrt{bx^3+a}\left(bc^2x^3+ac^2-d^2\right)}dx$$

$$\Big\downarrow \text{821}$$

$$c\left(-\frac{bc^2\left(\frac{\int \frac{\sqrt[3]{b}c^{2/3}x+\sqrt[3]{ac^2-d^2}}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx}{3\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}}-\frac{\int \frac{1}{\sqrt[3]{b}c^{2/3}x+\sqrt[3]{ac^2-d^2}}dx}{3\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}}\right)}{ac^2-d^2}-\frac{1}{x\left(ac^2-d^2\right)}\right)-d\int \frac{1}{x^2\sqrt{bx^3+a}\left(bc^2x^3+ac^2-d^2\right)}dx$$

$$\Big\downarrow \text{16}$$

$$c\left(-\frac{bc^2\left(\frac{\int \frac{\sqrt[3]{b}c^{2/3}x+\sqrt[3]{ac^2-d^2}}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx}{3\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}}-\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{2/3}c^{4/3}\sqrt[3]{ac^2-d^2}}\right)}{ac^2-d^2}-\frac{1}{x\left(ac^2-d^2\right)}\right)-d\int \frac{1}{x^2\sqrt{bx^3+a}\left(bc^2x^3+ac^2-d^2\right)}dx$$

$$\Big\downarrow \text{1013}$$

3.560. $\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

$$c\left(-\frac{bc^2\left(\frac{\int \frac{\sqrt[3]{b}c^{2/3}x+\sqrt[3]{ac^2-d^2}}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx}{3\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}}-\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{2/3}c^{4/3}\sqrt[3]{ac^2-d^2}}\right)}{ac^2-d^2}-\frac{1}{x\left(ac^2-d^2\right)}\right)-$$

$$\frac{d\sqrt{\frac{bx^3}{a}+1}\int \frac{1}{x^2\sqrt{\frac{bx^3}{a}+1}(bc^2x^3+ac^2-d^2)}dx}{\sqrt{a+bx^3}}$$

$\downarrow$ 1012

$$c\left(-\frac{bc^2\left(\frac{\int \frac{\sqrt[3]{b}c^{2/3}x+\sqrt[3]{ac^2-d^2}}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx}{3\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}}-\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{2/3}c^{4/3}\sqrt[3]{ac^2-d^2}}\right)}{ac^2-d^2}-\frac{1}{x\left(ac^2-d^2\right)}\right)+$$

$$\frac{d\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(-\frac{1}{3},\frac{1}{2},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{x\sqrt{a+bx^3}\left(ac^2-d^2\right)}$$

$\downarrow$ 1142

3.560. $\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

$$c\left(-\frac{bc^2\left(\frac{\frac{3}{2}\sqrt[3]{ac^2-d^2}\int \frac{1}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx+\frac{\int -\frac{\sqrt[3]{b}c^{2/3}\left(\sqrt[3]{ac^2-d^2}-2\sqrt[3]{b}c^{2/3}x\right)}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx}{2\sqrt[3]{b}c^{2/3}}}{3\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}}-\right.}{ac^2-d^2}\right.$$

$$\frac{d\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(-\frac{1}{3},\frac{1}{2},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{x\sqrt{a+bx^3}\left(ac^2-d^2\right)}$$

$\Big\downarrow$ 25

$$c\left(-\frac{bc^2\left(\frac{\frac{3}{2}\sqrt[3]{ac^2-d^2}\int \frac{1}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx-\frac{\int \frac{\sqrt[3]{b}c^{2/3}\left(\sqrt[3]{ac^2-d^2}-2\sqrt[3]{b}c^{2/3}x\right)}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx}{2\sqrt[3]{b}c^{2/3}}}{3\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}}-\right.}{ac^2-d^2}\right.$$

$$\frac{d\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(-\frac{1}{3},\frac{1}{2},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{x\sqrt{a+bx^3}\left(ac^2-d^2\right)}$$

$\Big\downarrow$ 27

3.560. $\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

$$c\left(-\frac{bc^2\left(\frac{\frac{3}{2}\sqrt[3]{ac^2-d^2}\int\frac{1}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx-\frac{1}{2}\int\frac{\sqrt[3]{ac^2-d^2}-2\sqrt[3]{b}c^{2/3}x}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx}{3\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}}-\cdots\right.}{ac^2-d^2}\cdots\right.$$

$$\frac{d\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(-\frac{1}{3},\frac{1}{2},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{x\sqrt{a+bx^3}\left(ac^2-d^2\right)}$$

$$\Big\downarrow \; 1082$$

$$c\left(-\frac{bc^2\left(\frac{\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}\right)}{\sqrt[3]{b}c^{2/3}}-\frac{1}{2}\int\frac{\sqrt[3]{ac^2-d^2}-2\sqrt[3]{b}c^{2/3}x}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx}{3\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}}-\frac{\log\left(\sqrt[3]{ac^2-\cdots}\right.}{3b^{2/3}c^{4/3}\sqrt[3]{\cdots}}\cdots\right.}{ac^2-d^2}\cdots\right.$$

$$\frac{d\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(-\frac{1}{3},\frac{1}{2},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{x\sqrt{a+bx^3}\left(ac^2-d^2\right)}$$

$$\Big\downarrow \; 217$$

3.560. $\int\frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}dx$

$$c\left(-\frac{bc^2\left(\frac{-\frac{1}{2}\int\frac{\sqrt[3]{ac^2-d^2}-2\sqrt[3]{b}c^{2/3}x}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt[3]{b}c^{2/3}}}{3\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}}-\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{2/3}c^{4/3}\sqrt[3]{ac^2-d^2}}\right)}{ac^2-d^2}\right.$$

$$\frac{d\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(-\frac{1}{3},\frac{1}{2},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{x\sqrt{a+bx^3}\,(ac^2-d^2)}$$

$$\Big\downarrow \ 1103$$

3.560. $\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

$$\frac{d\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(-\frac{1}{3},\frac{1}{2},1,\frac{2}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{x\sqrt{a+bx^3}\,(ac^2-d^2)}+$$

$$c\left(-\frac{bc^2\left(\frac{\frac{\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{2\sqrt[3]{b}c^{2/3}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt[3]{b}c^{2/3}}}{3\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}}-\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3b^{2/3}c^{4/3}\sqrt[3]{ac^2-d^2}}\right)}{ac^2-d^2}-\right.$$

input
```
Int[1/(x^2*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

output
```
(d*Sqrt[1 + (b*x^3)/a]*AppellF1[-1/3, 1/2, 1, 2/3, -((b*x^3)/a), -((b*c^2*
x^3)/(a*c^2 - d^2))])/((a*c^2 - d^2)*x*Sqrt[a + b*x^3]) + c*(-(1/((a*c^2 -
 d^2)*x)) - (b*c^2*(-1/3*Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x]/(b^(
2/3)*c^(4/3)*(a*c^2 - d^2)^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*c^(
2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]])/(b^(1/3)*c^(2/3))) + Log[(a*c^2 - d
^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2]/(
2*b^(1/3)*c^(2/3)))/(3*b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3))))/(a*c^2 - d^2
))
```

3.560.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.560. $\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2587
```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Simp[c   Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Simp[a*e   Int[
u/((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e
, n}, x] && EqQ[b*c - a*d, 0]
```

3.560.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 0.83 (sec) , antiderivative size = 1200, normalized size of antiderivative = 3.76

method	result	size
elliptic	Expression too large to display	1200
default	Expression too large to display	2404

input
```
int(1/x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
(b*x^3+a)^(1/2)*(d+c*(b*x^3+a)^(1/2))/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2))*(c*(
-1/(a*c^2-d^2)/x-(-1/3/b/c^2/((a*c^2-d^2)/b/c^2)^(1/3)*ln(x+((a*c^2-d^2)/b
/c^2)^(1/3))+1/6/b/c^2/((a*c^2-d^2)/b/c^2)^(1/3)*ln(x^2-((a*c^2-d^2)/b/c^2
)^(1/3)*x+((a*c^2-d^2)/b/c^2)^(2/3))+1/3*3^(1/2)/b/c^2/((a*c^2-d^2)/b/c^2)
^(1/3)*arctan(1/3*3^(1/2)*(2/((a*c^2-d^2)/b/c^2)^(1/3)*x-1)))*b*c^2/(a*c^2
-d^2))+d/a/(a*c^2-d^2)*(b*x^3+a)^(1/2)/x+1/3*I*d/a/(a*c^2-d^2)*3^(1/2)*(-b
^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(
1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^2*a)^(1/3
)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1/3)+1/2*I
*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3+a)^(1/2)
*((-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*EllipticE(1/3*3^(
1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/
(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+1
/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/b*(-b^2*a)^(1/3)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b
/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/3)/(-3/2/b*(-b^2*a)^(1/3)+
1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)))+1/3*I/d/b^2*c^2*2^(1/2)*sum(1/(a*
c^2-d^2)/_alpha*(-b^2*a)^(1/3)*(1/2*I*b*(2*x+1/b*((-b^2*a)^(1/3)-I*3^(1/2)
*(-b^2*a)^(1/3)))/(-b^2*a)^(1/3))^(1/2)*(b*(x-1/b*(-b^2*a)^(1/3))/(-3*(-b^
2*a)^(1/3)+I*3^(1/2)*(-b^2*a)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-b^2*a...
```

3.560.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(ac + bcx^3 + d\sqrt{a + bx^3}\right)} \, dx = \text{Timed out}$$

input
```
integrate(1/x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

output
```
Timed out
```

3.560.6 Sympy [F]

$$\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$$

input
```
integrate(1/x**2/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

output
```
Integral(1/(x**2*(a*c + b*c*x**3 + d*sqrt(a + b*x**3))), x)
```

3.560.7 Maxima [F]

$$\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=\int \frac{1}{\left(bcx^3+ac+\sqrt{bx^3+a}d\right)x^2}\,dx$$

input
```
integrate(1/x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^2), x)
```

3.560.8 Giac [F]

$$\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=\int \frac{1}{\left(bcx^3+ac+\sqrt{bx^3+a}d\right)x^2}\,dx$$

input
```
integrate(1/x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

output
```
integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^2), x)
```

3.560.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(ac + bcx^3 + d\sqrt{a + bx^3}\right)} \, dx = \int \frac{1}{x^2 \left(a\,c + d\,\sqrt{b\,x^3 + a} + b\,c\,x^3\right)} \, dx$$

input
```
int(1/(x^2*(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3)),x)
```

output
```
int(1/(x^2*(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3)), x)
```

3.561 $\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

3.561.1 Optimal result

Integrand size = 29, antiderivative size = 324

$$\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx = -\frac{c}{2\left(ac^2-d^2\right)x^2}+\frac{d\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(-\frac{2}{3},\frac{1}{2},1,\frac{1}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\left(ac^2-d^2\right)x^2\sqrt{a+bx^3}}+\frac{b^{2/3}c^{7/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\left(ac^2-d^2\right)^{5/3}}-\frac{b^{2/3}c^{7/3}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\left(ac^2-d^2\right)^{5/3}}+\frac{b^{2/3}c^{7/3}\log\left(\left(ac^2-d^2\right)^{2/3}-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+b^{2/3}c^{4/3}x^2\right)}{6\left(ac^2-d^2\right)^{5/3}}$$

output
```
-1/2*c/(a*c^2-d^2)/x^2-1/3*b^(2/3)*c^(7/3)*ln((a*c^2-d^2)^(1/3)+b^(1/3)*c^
(2/3)*x)/(a*c^2-d^2)^(5/3)+1/6*b^(2/3)*c^(7/3)*ln((a*c^2-d^2)^(2/3)-b^(1/3
)*c^(2/3)*(a*c^2-d^2)^(1/3)*x+b^(2/3)*c^(4/3)*x^2)/(a*c^2-d^2)^(5/3)+1/3*b
^(2/3)*c^(7/3)*arctan(1/3*(1-2*b^(1/3)*c^(2/3)*x/(a*c^2-d^2)^(1/3))*3^(1/2
))/(a*c^2-d^2)^(5/3)*3^(1/2)+1/2*d*AppellF1(-2/3,1/2,1,1/3,-b*x^3/a,-b*c^2
*x^3/(a*c^2-d^2))*(1+b*x^3/a)^(1/2)/(a*c^2-d^2)/x^2/(b*x^3+a)^(1/2)
```

3.561.2 Mathematica [A] (warning: unable to verify)

Time = 13.07 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.86

$$\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=\frac{b^2c^2dx^4\sqrt{1+\frac{bx^3}{a}}\,\text{AppellF1}\left(\frac{4}{3},\frac{1}{2},1,\frac{7}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{16a\left(-ac^2+d^2\right)^2\sqrt{a+bx^3}}$$
$$+\frac{2bd(-5ac^2+d^2)\,x\,\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},\right.}{\sqrt{a+bx^3}\left(ac^2-d^2+bc^2x^3\right)\left(8a\left(-ac^2+d^2\right)\text{AppellF1}\left(\frac{1}{3},\frac{1}{2},1,\frac{4}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)+3bx^3\left(2ac^2\,\text{AppellF}\right.\right.}$$
$$+\frac{-3ac(ac^2-d^2)^{2/3}+3d(ac^2-d^2)^{2/3}\sqrt{a+bx^3}-2\sqrt{3}ab^{2/3}c^{7/3}x^2\arctan\left(\frac{-1+\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)-2ab^{2/3}c^{7/}}{6a\left(ac^2-d\right.}$$

input
```
Integrate[1/(x^3*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

output
```
(b^2*c^2*d*x^4*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a)
, -((b*c^2*x^3)/(a*c^2 - d^2))])/(16*a*(-(a*c^2) + d^2)^2*Sqrt[a + b*x^3])
 + (2*b*d*(-5*a*c^2 + d^2)*x*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b
*c^2*x^3)/(a*c^2 - d^2))])/(Sqrt[a + b*x^3]*(a*c^2 - d^2 + b*c^2*x^3)*(8*a
*(-(a*c^2) + d^2)*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*c^2*x^3)/(
a*c^2 - d^2))] + 3*b*x^3*(2*a*c^2*AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a),
 -((b*c^2*x^3)/(a*c^2 - d^2))] + (a*c^2 - d^2)*AppellF1[4/3, 3/2, 1, 7/3,
-((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))]))) + (-3*a*c*(a*c^2 - d^2)^(2/
3) + 3*d*(a*c^2 - d^2)^(2/3)*Sqrt[a + b*x^3] - 2*Sqrt[3]*a*b^(2/3)*c^(7/3)
*x^2*ArcTan[(-1 + (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]] - 2*
a*b^(2/3)*c^(7/3)*x^2*Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x] + a*b^(
2/3)*c^(7/3)*x^2*Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(
1/3)*x + b^(2/3)*c^(4/3)*x^2])/(6*a*(a*c^2 - d^2)^(5/3)*x^2)
```

3.561.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {2587, 27, 847, 750, 16, 1013, 1012, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(d\sqrt{a+bx^3}+ac+bcx^3\right)}dx$$

$$\Big\downarrow \text{ 2587}$$

$$ac\int \frac{1}{ax^3\left(bc^2x^3+ac^2-d^2\right)}dx-ad\int \frac{1}{ax^3\sqrt{bx^3+a}\left(bc^2x^3+ac^2-d^2\right)}dx$$

$$\Big\downarrow \text{ 27}$$

$$c\int \frac{1}{x^3\left(bc^2x^3+ac^2-d^2\right)}dx-d\int \frac{1}{x^3\sqrt{bx^3+a}\left(bc^2x^3+ac^2-d^2\right)}dx$$

$$\Big\downarrow \text{ 847}$$

$$c\left(-\frac{bc^2\int \frac{1}{bc^2x^3+ac^2-d^2}dx}{ac^2-d^2}-\frac{1}{2x^2\left(ac^2-d^2\right)}\right)-d\int \frac{1}{x^3\sqrt{bx^3+a}\left(bc^2x^3+ac^2-d^2\right)}dx$$

$$\Big\downarrow \text{ 750}$$

$$c\left(-\frac{bc^2\left(\frac{\int \frac{2\sqrt[3]{ac^2-d^2}-\sqrt[3]{b}c^{2/3}x}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx}{3\left(ac^2-d^2\right)^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{b}c^{2/3}x+\sqrt[3]{ac^2-d^2}}dx}{3\left(ac^2-d^2\right)^{2/3}}\right)}{ac^2-d^2}-\frac{1}{2x^2\left(ac^2-d^2\right)}\right)-$$

$$d\int \frac{1}{x^3\sqrt{bx^3+a}\left(bc^2x^3+ac^2-d^2\right)}dx$$

$$\Big\downarrow \text{ 16}$$

$$c\left(-\frac{bc^2\left(\frac{\int \frac{2\sqrt[3]{ac^2-d^2}-\sqrt[3]{b}c^{2/3}x}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx}{3\left(ac^2-d^2\right)^{2/3}}+\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\sqrt[3]{b}c^{2/3}\left(ac^2-d^2\right)^{2/3}}\right)}{ac^2-d^2}-\frac{1}{2x^2\left(ac^2-d^2\right)}\right)-$$

$$d\int \frac{1}{x^3\sqrt{bx^3+a}\left(bc^2x^3+ac^2-d^2\right)}dx$$

$$\Big\downarrow \text{ 1013}$$

3.561. $\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}dx$

$$c\left(-\frac{bc^2\left(\frac{\int \frac{2\sqrt[3]{ac^2-d^2}-\sqrt[3]{b}c^{2/3}x}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx}{3(ac^2-d^2)^{2/3}}+\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\sqrt[3]{b}c^{2/3}(ac^2-d^2)^{2/3}}\right)}{ac^2-d^2}-\frac{1}{2x^2\left(ac^2-d^2\right)}\right)-$$

$$\frac{d\sqrt{\frac{bx^3}{a}+1}\int \frac{1}{x^3\sqrt{\frac{bx^3}{a}+1}(bc^2x^3+ac^2-d^2)}dx}{\sqrt{a+bx^3}}$$

$\downarrow$ 1012

$$c\left(-\frac{bc^2\left(\frac{\int \frac{2\sqrt[3]{ac^2-d^2}-\sqrt[3]{b}c^{2/3}x}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx}{3(ac^2-d^2)^{2/3}}+\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\sqrt[3]{b}c^{2/3}(ac^2-d^2)^{2/3}}\right)}{ac^2-d^2}-\frac{1}{2x^2\left(ac^2-d^2\right)}\right)+$$

$$\frac{d\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(-\frac{2}{3},\frac{1}{2},1,\frac{1}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2x^2\sqrt{a+bx^3}\left(ac^2-d^2\right)}$$

$\downarrow$ 1142

3.561. $\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

$$c\left(-\frac{bc^2\left(\frac{\frac{3}{2}\sqrt[3]{ac^2-d^2}\int\frac{1}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx-\frac{\int-\frac{\sqrt[3]{b}c^{2/3}\left(\sqrt[3]{ac^2-d^2}-2\sqrt[3]{b}c^{2/3}x\right)}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx}{2\sqrt[3]{b}c^{2/3}}}{3\left(ac^2-d^2\right)^{2/3}}+\right.}{ac^2-d^2}\right.$$

$$\frac{d\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(-\frac{2}{3},\frac{1}{2},1,\frac{1}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2x^2\sqrt{a+bx^3}\,(ac^2-d^2)}$$

$$\Big\downarrow \; 25$$

$$c\left(-\frac{bc^2\left(\frac{\frac{3}{2}\sqrt[3]{ac^2-d^2}\int\frac{1}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx+\frac{\int\frac{\sqrt[3]{b}c^{2/3}\left(\sqrt[3]{ac^2-d^2}-2\sqrt[3]{b}c^{2/3}x\right)}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx}{2\sqrt[3]{b}c^{2/3}}}{3\left(ac^2-d^2\right)^{2/3}}+\right.}{ac^2-d^2}\right.$$

$$\frac{d\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(-\frac{2}{3},\frac{1}{2},1,\frac{1}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2x^2\sqrt{a+bx^3}\,(ac^2-d^2)}$$

$$\Big\downarrow \; 27$$

3.561. $\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

$$c\left(-\frac{bc^2\left(\frac{\frac{3}{2}\sqrt[3]{ac^2-d^2}\int\frac{1}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx+\frac{1}{2}\int\frac{\sqrt[3]{ac^2-d^2}-2\sqrt[3]{b}c^{2/3}x}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx}{3\left(ac^2-d^2\right)^{2/3}} -$$

$$\frac{d\sqrt{\frac{bx^3}{a}+1}\operatorname{AppellF1}\left(-\frac{2}{3},\frac{1}{2},1,\frac{1}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2x^2\sqrt{a+bx^3}\left(ac^2-d^2\right)}$$

$\downarrow$ 1082

$$c\left(-\frac{bc^2\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{ac^2-d^2}-2\sqrt[3]{b}c^{2/3}x}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx+\frac{3\int\frac{1}{-\left(1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}\right)}{\sqrt[3]{b}c^{2/3}}}{3\left(ac^2-d^2\right)^{2/3}}+\frac{\log\left(\sqrt[3]{ac^2-d^2}\right.}{3\sqrt[3]{b}c^{2/3}(a}\right.}{ac^2-d^2}\right.$$

$$\frac{d\sqrt{\frac{bx^3}{a}+1}\operatorname{AppellF1}\left(-\frac{2}{3},\frac{1}{2},1,\frac{1}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2x^2\sqrt{a+bx^3}\left(ac^2-d^2\right)}$$

$\downarrow$ 217

3.561. $\int\frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

$$c\left(-\frac{bc^2\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{ac^2-d^2}-2\sqrt[3]{b}c^{2/3}x}{b^{2/3}c^{4/3}x^2-\sqrt[3]{b}c^{2/3}\sqrt[3]{ac^2-d^2}x+\left(ac^2-d^2\right)^{2/3}}dx-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt[3]{b}c^{2/3}}}{3\left(ac^2-d^2\right)^{2/3}}+\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\sqrt[3]{b}c^{2/3}\left(ac^2-d^2\right)^{2/3}}\right)}{ac^2-d^2}-\right.$$

$$\frac{d\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(-\frac{2}{3},\frac{1}{2},1,\frac{1}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2x^2\sqrt{a+bx^3}\left(ac^2-d^2\right)}$$

$$\Big\downarrow \; 1103$$

3.561. $\int\frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

$$\frac{d\sqrt{\frac{bx^3}{a}+1}\,\text{AppellF1}\left(-\frac{2}{3},\frac{1}{2},1,\frac{1}{3},-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2x^2\sqrt{a+bx^3}\,(ac^2-d^2)}+c\left(-\frac{bc^2\left(\frac{-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt[3]{b}c^{2/3}}-\frac{\log\left(-\sqrt[3]{b}c^{2/3}x\sqrt[3]{ac^2-d^2}+(ac^2-d^2)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{2\sqrt[3]{b}c^{2/3}}}{3(ac^2-d^2)^{2/3}}+\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}c^{2/3}x\right)}{3\sqrt[3]{b}c^{2/3}(ac^2-d^2)^{2/3}}\right)}{ac^2-d^2}\right.$$

input
```
Int[1/(x^3*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

output
```
(d*Sqrt[1 + (b*x^3)/a]*AppellF1[-2/3, 1/2, 1, 1/3, -((b*x^3)/a), -((b*c^2*
x^3)/(a*c^2 - d^2))])/(2*(a*c^2 - d^2)*x^2*Sqrt[a + b*x^3]) + c*(-1/2*1/((
a*c^2 - d^2)*x^2) - (b*c^2*(Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x]/(
3*b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3
)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]])/(b^(1/3)*c^(2/3))) - Log[(a*c^
2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x
^2]/(2*b^(1/3)*c^(2/3)))/(3*(a*c^2 - d^2)^(2/3))))/(a*c^2 - d^2))
```

3.561.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

3.561. $\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.561. $\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2587
```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Simp[c   Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Simp[a*e   Int[
u/((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e
, n}, x] && EqQ[b*c - a*d, 0]
```

3.561.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 6.

Time = 0.91 (sec) , antiderivative size = 1049, normalized size of antiderivative = 3.24

method	result	size
elliptic	Expression too large to display	1049
default	Expression too large to display	1948

input
```
int(1/x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
(b*x^3+a)^(1/2)*(d+c*(b*x^3+a)^(1/2))/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2))*(c*(
-1/2/(a*c^2-d^2)/x^2-(1/3/b/c^2/((a*c^2-d^2)/b/c^2)^(2/3)*ln(x+((a*c^2-d^2
)/b/c^2)^(1/3))-1/6/b/c^2/((a*c^2-d^2)/b/c^2)^(2/3)*ln(x^2-((a*c^2-d^2)/b/
c^2)^(1/3)*x+((a*c^2-d^2)/b/c^2)^(2/3))+1/3/b/c^2/((a*c^2-d^2)/b/c^2)^(2/3
)*3^(1/2)*arctan(1/3*3^(1/2)*(2/((a*c^2-d^2)/b/c^2)^(1/3)*x-1)))*b*c^2/(a*
c^2-d^2))+1/2*d/a/(a*c^2-d^2)*(b*x^3+a)^(1/2)/x^2-1/6*I*d/a/(a*c^2-d^2)*3^
(1/2)*(-b^2*a)^(1/3)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(
1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)*((x-1/b*(-b^2*a)^(1/3))/(-3/2/b*(-b^
2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-b^2*a)^(1
/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2)/(b*x^3
+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b
*(-b^2*a)^(1/3))*3^(1/2)*b/(-b^2*a)^(1/3))^(1/2),(I*3^(1/2)/b*(-b^2*a)^(1/
3)/(-3/2/b*(-b^2*a)^(1/3)+1/2*I*3^(1/2)/b*(-b^2*a)^(1/3)))^(1/2))+1/3*I/d/
b^2*c^2*2^(1/2)*sum(1/_alpha^2/(a*c^2-d^2)*(-b^2*a)^(1/3)*(1/2*I*b*(2*x+1/
b*((-b^2*a)^(1/3)-I*3^(1/2)*(-b^2*a)^(1/3)))/(-b^2*a)^(1/3))^(1/2)*(b*(x-1
/b*(-b^2*a)^(1/3))/(-3*(-b^2*a)^(1/3)+I*3^(1/2)*(-b^2*a)^(1/3)))^(1/2)*(-1
/2*I*b*(2*x+1/b*((-b^2*a)^(1/3)+I*3^(1/2)*(-b^2*a)^(1/3)))/(-b^2*a)^(1/3))
^(1/2)/(b*x^3+a)^(1/2)*(I*(-b^2*a)^(1/3)*3^(1/2)*_alpha*b-I*(-b^2*a)^(2/3)
*3^(1/2)+2*_alpha^2*b^2-(-b^2*a)^(1/3)*_alpha*b-(-b^2*a)^(2/3))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2/b*(-b^2*a)^(1/3)-1/2*I*3^(1/2)/b*(-b^2*a)^(1/3))...
```

3.561.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

output
```
Timed out
```

3.561.6 Sympy [F]

$$\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$$

input
```
integrate(1/x**3/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

output
```
Integral(1/(x**3*(a*c + b*c*x**3 + d*sqrt(a + b*x**3))), x)
```

3.561.7 Maxima [F]

$$\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=\int \frac{1}{\left(bcx^3+ac+\sqrt{bx^3+a}d\right)x^3}\,dx$$

input
```
integrate(1/x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^3), x)
```

3.561.8 Giac [F]

$$\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=\int \frac{1}{\left(bcx^3+ac+\sqrt{bx^3+a}d\right)x^3}\,dx$$

input
```
integrate(1/x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

output
```
integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^3), x)
```

3.561.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=\int \frac{1}{x^3\,\left(a\,c+d\,\sqrt{b\,x^3+a}+b\,c\,x^3\right)}\,dx$$

input
```
int(1/(x^3*(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3)),x)
```

output
```
int(1/(x^3*(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3)), x)
```

3.562 $\int \frac{1}{ac+bcx^n+d\sqrt{a+bx^n}} dx$

3.562.1 Optimal result

Integrand size = 25, antiderivative size = 135

$$\int \frac{1}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx=-\frac{dx\sqrt{1+\frac{bx^n}{a}}\operatorname{AppellF1}\left(\frac{1}{n},\frac{1}{2},1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{(ac^2-d^2)\sqrt{a+bx^n}}+\frac{cx\operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bc^2x^n}{ac^2-d^2}\right)}{ac^2-d^2}$$

output
```
c*x*hypergeom([1, 1/n],[1+1/n],-b*c^2*x^n/(a*c^2-d^2))/(a*c^2-d^2)-d*x*App
ellF1(1/n,1/2,1,1+1/n,-b*x^n/a,-b*c^2*x^n/(a*c^2-d^2))*(1+b*x^n/a)^(1/2)/(
a*c^2-d^2)/(a+b*x^n)^(1/2)
```

3.562.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 320 vs. 2(135) = 270.

Time = 0.55 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.37

$$\int \frac{1}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx=-\frac{2ad(ac^2-d^2)(1+n)x\operatorname{AppellF}}{\sqrt{a+bx^n}\,(ac^2-d^2+bc^2x^n)\left(-2abc^2nx^n\operatorname{AppellF1}\left(1+\frac{1}{n},\frac{1}{2},2,2+\frac{1}{n},-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)+(ac^2-d^2)\left(-\right.\right.}+\frac{cx\operatorname{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bc^2x^n}{ac^2-d^2}\right)}{ac^2-d^2}$$

input
```
Integrate[(a*c + b*c*x^n + d*Sqrt[a + b*x^n])^(-1),x]
```

output
```
(-2*a*d*(a*c^2 - d^2)*(1 + n)*x*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*
x^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))])/(Sqrt[a + b*x^n]*(a*c^2 - d^2 + b*
c^2*x^n)*(-2*a*b*c^2*n*x^n*AppellF1[1 + n^(-1), 1/2, 2, 2 + n^(-1), -((b*x
^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))] + (a*c^2 - d^2)*(-(b*n*x^n*AppellF1[
1 + n^(-1), 3/2, 1, 2 + n^(-1), -((b*x^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))
]) + 2*a*(1 + n)*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x^n)/a), -((b*c
^2*x^n)/(a*c^2 - d^2))]))) + (c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1),
 -((b*c^2*x^n)/(a*c^2 - d^2))])/(a*c^2 - d^2)
```

3.562.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2587, 778, 937, 936}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{d\sqrt{a+bx^n}+ac+bcx^n}dx$$

$$\Big\downarrow \text{2587}$$

$$ac\int \frac{1}{abc^2x^n+a\left(ac^2-d^2\right)}dx-ad\int \frac{1}{\sqrt{bx^n+a}\left(abc^2x^n+a\left(ac^2-d^2\right)\right)}dx$$

$$\Big\downarrow \text{778}$$

$$\frac{cx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bc^2x^n}{ac^2-d^2}\right)}{ac^2-d^2}-ad\int \frac{1}{\sqrt{bx^n+a}\left(abc^2x^n+a\left(ac^2-d^2\right)\right)}dx$$

$$\Big\downarrow \text{937}$$

$$\frac{cx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bc^2x^n}{ac^2-d^2}\right)}{ac^2-d^2}-\frac{ad\sqrt{\frac{bx^n}{a}+1}\int \frac{1}{\sqrt{\frac{bx^n}{a}+1}\left(abc^2x^n+a\left(ac^2-d^2\right)\right)}dx}{\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{936}$$

$$\frac{cx\,\text{Hypergeometric2F1}\left(1,\frac{1}{n},1+\frac{1}{n},-\frac{bc^2x^n}{ac^2-d^2}\right)}{ac^2-d^2}-\frac{dx\sqrt{\frac{bx^n}{a}+1}\,\text{AppellF1}\left(\frac{1}{n},\frac{1}{2},1,1+\frac{1}{n},-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{(ac^2-d^2)\sqrt{a+bx^n}}$$

input
```
Int[(a*c + b*c*x^n + d*Sqrt[a + b*x^n])^(-1),x]
```

output
```
-((d*x*Sqrt[1 + (b*x^n)/a]*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x^n)/
a), -((b*c^2*x^n)/(a*c^2 - d^2))])/((a*c^2 - d^2)*Sqrt[a + b*x^n])) + (c*x
*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*c^2*x^n)/(a*c^2 - d^2))])/(
a*c^2 - d^2)
```

3.562.3.1 Defintions of rubi rules used

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 936
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 937
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])
  Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q
}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2587
```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Simp[c   Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Simp[a*e   Int[
u/((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e
, n}, x] && EqQ[b*c - a*d, 0]
```

3.562.4 Maple [F]

$$\int \frac{1}{ac+bc\,x^{n}+d\sqrt{a+b\,x^{n}}}dx$$

input
```
int(1/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
```

output
```
int(1/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
```

3.562.5 Fricas [F]

$$\int \frac{1}{ac+bcx^{n}+d\sqrt{a+bx^{n}}}\,dx=\int \frac{1}{bcx^{n}+ac+\sqrt{bx^{n}+a}d}\,dx$$

input
```
integrate(1/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="fricas")
```

output
```
integral((b*c*x^n + a*c - sqrt(b*x^n + a)*d)/(b^2*c^2*x^(2*n) + a^2*c^2 -
a*d^2 + (2*a*b*c^2 - b*d^2)*x^n), x)
```

3.562.6 Sympy [F]

$$\int \frac{1}{ac+bcx^{n}+d\sqrt{a+bx^{n}}}\,dx=\int \frac{1}{ac+bcx^{n}+d\sqrt{a+bx^{n}}}\,dx$$

input
```
integrate(1/(a*c+b*c*x**n+d*(a+b*x**n)**(1/2)),x)
```

output
```
Integral(1/(a*c + b*c*x**n + d*sqrt(a + b*x**n)), x)
```

3.562.7 Maxima [F]

$$\int \frac{1}{ac + bcx^n + d\sqrt{a + bx^n}}\, dx = \int \frac{1}{bcx^n + ac + \sqrt{bx^n + a}d}\, dx$$

input
```
integrate(1/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(b*c*x^n + a*c + sqrt(b*x^n + a)*d), x)
```

3.562.8 Giac [F]

$$\int \frac{1}{ac + bcx^n + d\sqrt{a + bx^n}}\, dx = \int \frac{1}{bcx^n + ac + \sqrt{bx^n + a}d}\, dx$$

input
```
integrate(1/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="giac")
```

output
```
integrate(1/(b*c*x^n + a*c + sqrt(b*x^n + a)*d), x)
```

3.562.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{ac + bcx^n + d\sqrt{a + bx^n}}\, dx = \int \frac{1}{a\,c + d\,\sqrt{a + b\,x^n} + b\,c\,x^n}\, dx$$

input
```
int(1/(a*c + d*(a + b*x^n)^(1/2) + b*c*x^n),x)
```

output
```
int(1/(a*c + d*(a + b*x^n)^(1/2) + b*c*x^n), x)
```

3.563 $\int \frac{x^m}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx$

3.563.1 Optimal result

Integrand size = 29, antiderivative size = 167

$$\begin{aligned}\int \frac{x^m}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx &= -\frac{dx^{1+m}\sqrt{1+\frac{bx^n}{a}}\,\text{AppellF1}\left(\frac{1+m}{n},\frac{1}{2},1,\frac{1+m+n}{n},-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{(ac^2-d^2)\,(1+m)\sqrt{a+bx^n}}\\&\quad+\frac{cx^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{n},\frac{1+m+n}{n},-\frac{bc^2x^n}{ac^2-d^2}\right)}{(ac^2-d^2)\,(1+m)}\end{aligned}$$

output
```
c*x^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*c^2*x^n/(a*c^2-d^2))/(a*c^
2-d^2)/(1+m)-d*x^(1+m)*AppellF1((1+m)/n,1/2,1,(1+m+n)/n,-b*x^n/a,-b*c^2*x^
n/(a*c^2-d^2))*(1+b*x^n/a)^(1/2)/(a*c^2-d^2)/(1+m)/(a+b*x^n)^(1/2)
```

3.563.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.93

$$\begin{aligned}&\int \frac{x^m}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx\\&=\frac{x^{1+m}\left(-d\sqrt{1+\frac{bx^n}{a}}\,\text{AppellF1}\left(\frac{1+m}{n},\frac{1}{2},1,\frac{1+m+n}{n},-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)+c\sqrt{a+bx^n}\,\text{Hypergeometric2F1}\left(1,\frac{1+}{n}\right.\right.}{(ac^2-d^2)\,(1+m)\sqrt{a+bx^n}}\end{aligned}$$

input
```
Integrate[x^m/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]),x]
```

output
```
(x^(1 + m)*(-(d*Sqrt[1 + (b*x^n)/a]*AppellF1[(1 + m)/n, 1/2, 1, (1 + m + n
)/n, -((b*x^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))]) + c*Sqrt[a + b*x^n]*Hype
rgeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*c^2*x^n)/(a*c^2 - d^2))]))
/((a*c^2 - d^2)*(1 + m)*Sqrt[a + b*x^n])
```

3.563.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2587, 888, 1013, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{d\sqrt{a+bx^n}+ac+bcx^n}dx$$

$$\Big\downarrow \text{2587}$$

$$ac\int \frac{x^m}{abc^2x^n+a\left(ac^2-d^2\right)}dx-ad\int \frac{x^m}{\sqrt{bx^n+a}\left(abc^2x^n+a\left(ac^2-d^2\right)\right)}dx$$

$$\Big\downarrow \text{888}$$

$$\frac{cx^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bc^2x^n}{ac^2-d^2}\right)}{(m+1)\left(ac^2-d^2\right)}-ad\int \frac{x^m}{\sqrt{bx^n+a}\left(abc^2x^n+a\left(ac^2-d^2\right)\right)}dx$$

$$\Big\downarrow \text{1013}$$

$$\frac{cx^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bc^2x^n}{ac^2-d^2}\right)}{(m+1)\left(ac^2-d^2\right)}-\frac{ad\sqrt{\frac{bx^n}{a}+1}\int \frac{x^m}{\sqrt{\frac{bx^n}{a}+1}\left(abc^2x^n+a\left(ac^2-d^2\right)\right)}dx}{\sqrt{a+bx^n}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{cx^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{n},\frac{m+n+1}{n},-\frac{bc^2x^n}{ac^2-d^2}\right)}{(m+1)\left(ac^2-d^2\right)}-\frac{dx^{m+1}\sqrt{\frac{bx^n}{a}+1}\,\text{AppellF1}\left(\frac{m+1}{n},\frac{1}{2},1,\frac{m+n+1}{n},-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{(m+1)\left(ac^2-d^2\right)\sqrt{a+bx^n}}$$

input
```
Int[x^m/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]),x]
```

output
```
-((d*x^(1 + m)*Sqrt[1 + (b*x^n)/a]*AppellF1[(1 + m)/n, 1/2, 1, (1 + m + n)
/n, -((b*x^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))])/((a*c^2 - d^2)*(1 + m)*Sq
rt[a + b*x^n])) + (c*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)
/n, -((b*c^2*x^n)/(a*c^2 - d^2))])/((a*c^2 - d^2)*(1 + m))
```

3.563.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2587
```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Simp[c   Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Simp[a*e   Int[
u/((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e
, n}, x] && EqQ[b*c - a*d, 0]
```

3.563. $\int \frac{x^m}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx$

3.563.4 Maple [F]

$$\int \frac{x^m}{ac+bc\,x^n+d\sqrt{a+b\,x^n}}dx$$

input
```
int(x^m/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
```

output
```
int(x^m/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
```

3.563.5 Fricas [F]

$$\int \frac{x^m}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx = \int \frac{x^m}{bcx^n+ac+\sqrt{bx^n+a}d}\,dx$$

input
```
integrate(x^m/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="fricas")
```

output
```
integral((b*c*x^m*x^n + a*c*x^m - sqrt(b*x^n + a)*d*x^m)/(b^2*c^2*x^(2*n)
+ a^2*c^2 - a*d^2 + (2*a*b*c^2 - b*d^2)*x^n), x)
```

3.563.6 Sympy [F]

$$\int \frac{x^m}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx = \int \frac{x^m}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx$$

input
```
integrate(x**m/(a*c+b*c*x**n+d*(a+b*x**n)**(1/2)),x)
```

output
```
Integral(x**m/(a*c + b*c*x**n + d*sqrt(a + b*x**n)), x)
```

3.563.7 Maxima [F]

$$\int \frac{x^m}{ac + bcx^n + d\sqrt{a + bx^n}}\,dx = \int \frac{x^m}{bcx^n + ac + \sqrt{bx^n + a}d}\,dx$$

input
```
integrate(x^m/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x^m/(b*c*x^n + a*c + sqrt(b*x^n + a)*d), x)
```

3.563.8 Giac [F]

$$\int \frac{x^m}{ac + bcx^n + d\sqrt{a + bx^n}}\,dx = \int \frac{x^m}{bcx^n + ac + \sqrt{bx^n + a}d}\,dx$$

input
```
integrate(x^m/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="giac")
```

output
```
integrate(x^m/(b*c*x^n + a*c + sqrt(b*x^n + a)*d), x)
```

3.563.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{ac + bcx^n + d\sqrt{a + bx^n}}\,dx = \int \frac{x^m}{a\,c + d\,\sqrt{a + b\,x^n} + b\,c\,x^n}\,dx$$

input
```
int(x^m/(a*c + d*(a + b*x^n)^(1/2) + b*c*x^n),x)
```

output
```
int(x^m/(a*c + d*(a + b*x^n)^(1/2) + b*c*x^n), x)
```

3.564 $\int \frac{x^{-1+n}}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx$

3.564.1 Optimal result

Integrand size = 31, antiderivative size = 27

$$\int \frac{x^{-1+n}}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx=\frac{2\log\left(d+c\sqrt{a+bx^n}\right)}{bcn}$$

output
```
2*ln(d+c*(a+b*x^n)^(1/2))/b/c/n
```

3.564.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{x^{-1+n}}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx=\frac{2\log\left(bdn+bcn\sqrt{a+bx^n}\right)}{bcn}$$

input
```
Integrate[x^(-1 + n)/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]),x]
```

output
```
(2*Log[b*d*n + b*c*n*Sqrt[a + b*x^n]])/(b*c*n)
```

3.564.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {2586, 7267, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{n-1}}{d\sqrt{a+bx^n}+ac+bcx^n}dx$$
$$\Big\downarrow \text{2586}$$
$$\frac{\int \frac{1}{bcx^n+ac+d\sqrt{bx^n+a}}dx^n}{n}$$
$$\Big\downarrow \text{7267}$$
$$\frac{2\int \frac{1}{\sqrt{bx^n+ac+d}}d\sqrt{bx^n+a}}{bn}$$
$$\Big\downarrow \text{16}$$
$$\frac{2\log\left(c\sqrt{a+bx^n}+d\right)}{bcn}$$

input
```
Int[x^(-1 + n)/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]),x]
```

output
```
(2*Log[d + c*Sqrt[a + b*x^n]])/(b*c*n)
```

3.564.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2586
```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Simp[1/n   Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a
+ b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*
d, 0] && IntegerQ[(m + 1)/n]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.564.4 Maple [F]

$$\int \frac{x^{-1+n}}{ac+bc\,x^n+d\sqrt{a+b\,x^n}}dx$$

input
```
int(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
```

output
```
int(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
```

3.564.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^{-1+n}}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx=\frac{2\,\log\left(\sqrt{bx^n+a}c+d\right)}{bcn}$$

input
```
integrate(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="fricas")
```

output
```
2*log(sqrt(b*x^n + a)*c + d)/(b*c*n)
```

3.564.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(20) = 40.

Time = 6.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int \frac{x^{-1+n}}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx=\begin{cases}\frac{2\left(\begin{cases}\frac{\sqrt{a+bx^n}}{d} & \text{for}\; c=0\\ \frac{\log\left(c\sqrt{a+bx^n}+d\right)}{c} & \text{otherwise}\end{cases}\right)}{bn} & \text{for}\; b\neq 0\\ \frac{x^n}{\sqrt{adn+acn}} & \text{otherwise}\end{cases}$$

input
```
integrate(x**(-1+n)/(a*c+b*c*x**n+d*(a+b*x**n)**(1/2)),x)
```

output
```
Piecewise((2*Piecewise((sqrt(a + b*x**n)/d, Eq(c, 0)), (log(c*sqrt(a + b*x
**n) + d)/c, True))/(b*n), Ne(b, 0)), (x**n/(sqrt(a)*d*n + a*c*n), True))
```

3.564.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(25) = 50$.

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.26

$$\int \frac{x^{-1+n}}{ac + bcx^n + d\sqrt{a + bx^n}}\,dx = -\frac{\log\left(\frac{bx^n+a}{b}\right)}{bcn} + \frac{2\,\log\left(\frac{bcx^n+ac+\sqrt{bx^n+a}d}{d}\right)}{bcn}$$

input
```
integrate(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="maxima")
```

output
```
-log((b*x^n + a)/b)/(b*c*n) + 2*log((b*c*x^n + a*c + sqrt(b*x^n + a)*d)/d)
/(b*c*n)
```

3.564.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{x^{-1+n}}{ac + bcx^n + d\sqrt{a + bx^n}}\,dx = \frac{2\,\log\left(\left|\sqrt{bx^n + a}c + d\right|\right)}{bcn}$$

input
```
integrate(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="giac")
```

output
```
2*log(abs(sqrt(b*x^n + a)*c + d))/(b*c*n)
```

3.564.9 Mupad [B] (verification not implemented)

Time = 16.92 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{x^{-1+n}}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx=\frac{2\,\ln\left(d+c\sqrt{a+b\,x^n}\right)}{b\,c\,n}$$

input
```
int(x^(n - 1)/(a*c + d*(a + b*x^n)^(1/2) + b*c*x^n),x)
```

output
```
(2*log(d + c*(a + b*x^n)^(1/2)))/(b*c*n)
```

3.565 $\int \frac{1}{\sqrt{x}+4x^{3/2}}\,dx$

3.565.1 Optimal result

Integrand size = 15, antiderivative size = 8

$$\int \frac{1}{\sqrt{x}+4x^{3/2}}\,dx = \arctan\left(2\sqrt{x}\right)$$

output
```
arctan(2*x^(1/2))
```

3.565.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}+4x^{3/2}}\,dx = \arctan\left(2\sqrt{x}\right)$$

input
```
Integrate[(Sqrt[x] + 4*x^(3/2))^(-1),x]
```

output
```
ArcTan[2*Sqrt[x]]
```

3.565.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2027, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{4x^{3/2}+\sqrt{x}}dx$$
$$\Big\downarrow \text{2027}$$
$$\int \frac{1}{\sqrt{x}(4x+1)}dx$$
$$\Big\downarrow \text{73}$$
$$2\int \frac{1}{4x+1}d\sqrt{x}$$
$$\Big\downarrow \text{216}$$
$$\arctan\left(2\sqrt{x}\right)$$

input
```
Int[(Sqrt[x] + 4*x^(3/2))^(-1),x]
```

output
```
ArcTan[2*Sqrt[x]]
```

3.565.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.565.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\arctan\left(2\sqrt{x}\right)$	7
default	$\arctan\left(2\sqrt{x}\right)$	7
meijerg	$\arctan\left(2\sqrt{x}\right)$	7
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-4\operatorname{RootOf}\left(\_Z^2+1\right)x+4\sqrt{x}+\operatorname{RootOf}\left(\_Z^2+1\right)}{1+4x}\right)}{2}$	39

input
```
int(1/(4*x^(3/2)+x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
arctan(2*x^(1/2))
```

3.565.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x}+4x^{3/2}}\,dx = \arctan\left(2\sqrt{x}\right)$$

input
```
integrate(1/(4*x^(3/2)+x^(1/2)),x, algorithm="fricas")
```

output
```
arctan(2*sqrt(x))
```

3.565.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{x}+4x^{3/2}}\,dx = \operatorname{atan}\left(2\sqrt{x}\right)$$

input
```
integrate(1/(4*x**(3/2)+x**(1/2)),x)
```

output
```
atan(2*sqrt(x))
```

3.565.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x}+4x^{3/2}}\,dx = \arctan\left(2\sqrt{x}\right)$$

input
```
integrate(1/(4*x^(3/2)+x^(1/2)),x, algorithm="maxima")
```

output
```
arctan(2*sqrt(x))
```

3.565.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x}+4x^{3/2}}\,dx = \arctan\left(2\sqrt{x}\right)$$

input
```
integrate(1/(4*x^(3/2)+x^(1/2)),x, algorithm="giac")
```

output
```
arctan(2*sqrt(x))
```

3.565.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x}+4x^{3/2}}\,dx = \operatorname{atan}\left(2\sqrt{x}\right)$$

input
```
int(1/(x^(1/2) + 4*x^(3/2)),x)
```

output
```
atan(2*x^(1/2))
```

3.566 $\int \frac{1}{\sqrt{x}-x^{5/2}}\,dx$

3.566.1 Optimal result

Integrand size = 15, antiderivative size = 13

$$\int \frac{1}{\sqrt{x}-x^{5/2}}\,dx = \arctan\left(\sqrt{x}\right)+\text{arctanh}\left(\sqrt{x}\right)$$

output
```
arctan(x^(1/2))+arctanh(x^(1/2))
```

3.566.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}-x^{5/2}}\,dx = \arctan\left(\sqrt{x}\right)+\text{arctanh}\left(\sqrt{x}\right)$$

input
```
Integrate[(Sqrt[x] - x^(5/2))^(-1),x]
```

output
```
ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

3.566.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2027, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}-x^{5/2}}dx$$
$$\Big\downarrow \text{2027}$$
$$\int \frac{1}{\sqrt{x}\,(1-x^2)}dx$$
$$\Big\downarrow \text{266}$$
$$2\int \frac{1}{1-x^2}d\sqrt{x}$$
$$\Big\downarrow \text{756}$$
$$2\left(\frac{1}{2}\int \frac{1}{1-x}d\sqrt{x}+\frac{1}{2}\int \frac{1}{x+1}d\sqrt{x}\right)$$
$$\Big\downarrow \text{216}$$
$$2\left(\frac{1}{2}\int \frac{1}{1-x}d\sqrt{x}+\frac{\arctan\left(\sqrt{x}\right)}{2}\right)$$
$$\Big\downarrow \text{219}$$
$$2\left(\frac{\arctan\left(\sqrt{x}\right)}{2}+\frac{\text{arctanh}\left(\sqrt{x}\right)}{2}\right)$$

input
```
Int[(Sqrt[x] - x^(5/2))^(-1),x]
```

output
```
2*(ArcTan[Sqrt[x]]/2 + ArcTanh[Sqrt[x]]/2)
```

3.566.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.566.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\arctan\left(\sqrt{x}\right)+\operatorname{arctanh}\left(\sqrt{x}\right)$	10
default	$\arctan\left(\sqrt{x}\right)+\operatorname{arctanh}\left(\sqrt{x}\right)$	10
meijerg	$-\frac{\sqrt{x}\left(\ln\left(1-\left(x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(x^{2}\right)^{\frac{1}{4}}\right)-2\arctan\left(\left(x^{2}\right)^{\frac{1}{4}}\right)\right)}{2\left(x^{2}\right)^{\frac{1}{4}}}$	40
trager	$-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)x+2\sqrt{x}-\operatorname{RootOf}\left(\_Z^{2}+1\right)}{x+1}\right)}{2}+\frac{\ln\left(\frac{2\sqrt{x}+1+x}{x-1}\right)}{2}$	56

3.566. $\int \frac{1}{\sqrt{x}-x^{5/2}}\,dx$

input
```
int(1/(-x^(5/2)+x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
arctan(x^(1/2))+arctanh(x^(1/2))
```

3.566.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(9) = 18$.

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{x} - x^{5/2}}\, dx = \arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x} + 1\right) - \frac{1}{2}\log\left(\sqrt{x} - 1\right)$$

input
```
integrate(1/(-x^(5/2)+x^(1/2)),x, algorithm="fricas")
```

output
```
arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

3.566.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{x} - x^{5/2}}\, dx = -\frac{\log\left(\sqrt{x} - 1\right)}{2} + \frac{\log\left(\sqrt{x} + 1\right)}{2} + \operatorname{atan}\left(\sqrt{x}\right)$$

input
```
integrate(1/(-x**(5/2)+x**(1/2)),x)
```

output
```
-log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 + atan(sqrt(x))
```

3.566.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(9) = 18.

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{x}-x^{5/2}}\,dx = \arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\sqrt{x}-1\right)$$

input
```
integrate(1/(-x^(5/2)+x^(1/2)),x, algorithm="maxima")
```

output
```
arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

3.566.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(9) = 18.

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{1}{\sqrt{x}-x^{5/2}}\,dx = \arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\left|\sqrt{x}-1\right|\right)$$

input
```
integrate(1/(-x^(5/2)+x^(1/2)),x, algorithm="giac")
```

output
```
arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(abs(sqrt(x) - 1))
```

3.566.9 Mupad [B] (verification not implemented)

Time = 16.49 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{x}-x^{5/2}}\,dx = \operatorname{atan}\left(\sqrt{x}\right) + \operatorname{atanh}\left(\sqrt{x}\right)$$

input
```
int(1/(x^(1/2) - x^(5/2)),x)
```

output
```
atan(x^(1/2)) + atanh(x^(1/2))
```

3.567 $\int \frac{1}{-\sqrt[4]{x}+\sqrt{x}}\,dx$

3.567.1 Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{1}{-\sqrt[4]{x}+\sqrt{x}}\,dx = 4\sqrt[4]{x}+2\sqrt{x}+4\log\left(1-\sqrt[4]{x}\right)$$

output
```
4*x^(1/4)+4*ln(1-x^(1/4))+2*x^(1/2)
```

3.567.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{-\sqrt[4]{x}+\sqrt{x}}\,dx = 2\left(2+\sqrt[4]{x}\right)\sqrt[4]{x}+4\log\left(-1+\sqrt[4]{x}\right)$$

input
```
Integrate[(-x^(1/4) + Sqrt[x])^(-1),x]
```

output
```
2*(2 + x^(1/4))*x^(1/4) + 4*Log[-1 + x^(1/4)]
```

3.567.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2027, 798, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}-\sqrt[4]{x}}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{1}{\left(\sqrt[4]{x}-1\right)\sqrt[4]{x}}dx$$

$$\Big\downarrow \text{798}$$

$$4\int -\frac{\sqrt{x}}{1-\sqrt[4]{x}}d\sqrt[4]{x}$$

$$\Big\downarrow \text{25}$$

$$-4\int \frac{\sqrt{x}}{1-\sqrt[4]{x}}d\sqrt[4]{x}$$

$$\Big\downarrow \text{49}$$

$$-4\int \left(-\sqrt[4]{x}+\frac{1}{1-\sqrt[4]{x}}-1\right)d\sqrt[4]{x}$$

$$\Big\downarrow \text{2009}$$

$$4\left(\frac{\sqrt{x}}{2}+\sqrt[4]{x}+\log\left(1-\sqrt[4]{x}\right)\right)$$

input
```
Int[(-x^(1/4) + Sqrt[x])^(-1),x]
```

output
```
4*(x^(1/4) + Sqrt[x]/2 + Log[1 - x^(1/4)])
```

3.567.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.567.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$2\sqrt{x}+4x^{\frac{1}{4}}+4\ln\left(x^{\frac{1}{4}}-1\right)$	20
default	$2\sqrt{x}+4x^{\frac{1}{4}}+4\ln\left(x^{\frac{1}{4}}-1\right)$	20
meijerg	$\frac{2x^{\frac{1}{4}}\left(6+3x^{\frac{1}{4}}\right)}{3}+4\ln\left(1-x^{\frac{1}{4}}\right)$	24

input
```
int(1/(-x^(1/4)+x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)+4*x^(1/4)+4*ln(x^(1/4)-1)
```

3.567.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{1}{-\sqrt[4]{x}+\sqrt{x}}\,dx = 2\sqrt{x} + 4\,x^{\frac{1}{4}} + 4\,\log\left(x^{\frac{1}{4}} - 1\right)$$

input
```
integrate(1/(-x^(1/4)+x^(1/2)),x, algorithm="fricas")
```

output
```
2*sqrt(x) + 4*x^(1/4) + 4*log(x^(1/4) - 1)
```

3.567.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1}{-\sqrt[4]{x}+\sqrt{x}}\,dx = 4\sqrt[4]{x} + 2\sqrt{x} + 4\log\left(\sqrt[4]{x} - 1\right)$$

input
```
integrate(1/(-x**(1/4)+x**(1/2)),x)
```

output
```
4*x**(1/4) + 2*sqrt(x) + 4*log(x**(1/4) - 1)
```

3.567.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{1}{-\sqrt[4]{x}+\sqrt{x}}\,dx = 2\sqrt{x} + 4\,x^{\frac{1}{4}} + 4\,\log\left(x^{\frac{1}{4}} - 1\right)$$

input
```
integrate(1/(-x^(1/4)+x^(1/2)),x, algorithm="maxima")
```

output
```
2*sqrt(x) + 4*x^(1/4) + 4*log(x^(1/4) - 1)
```

3.567.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{1}{-\sqrt[4]{x}+\sqrt{x}}\,dx = 2\sqrt{x}+4\,x^{\frac{1}{4}}+4\,\log\left(\left|x^{\frac{1}{4}}-1\right|\right)$$

input
```
integrate(1/(-x^(1/4)+x^(1/2)),x, algorithm="giac")
```

output
```
2*sqrt(x) + 4*x^(1/4) + 4*log(abs(x^(1/4) - 1))
```

3.567.9 Mupad [B] (verification not implemented)

Time = 16.55 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{1}{-\sqrt[4]{x}+\sqrt{x}}\,dx = 4\,\ln\left(x^{1/4}-1\right)+2\sqrt{x}+4\,x^{1/4}$$

input
```
int(1/(x^(1/2) - x^(1/4)),x)
```

output
```
4*log(x^(1/4) - 1) + 2*x^(1/2) + 4*x^(1/4)
```

3.568 $\int \frac{1}{\sqrt[3]{x}+\sqrt{x}} \, dx$

3.568.1 Optimal result

Integrand size = 13, antiderivative size = 32

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}} \, dx = 6\sqrt[6]{x}-3\sqrt[3]{x}+2\sqrt{x}-6\log\left(1+\sqrt[6]{x}\right)$$

output
```
6*x^(1/6)-3*x^(1/3)-6*ln(1+x^(1/6))+2*x^(1/2)
```

3.568.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}} \, dx = \left(6-3\sqrt[6]{x}+2\sqrt[3]{x}\right)\sqrt[6]{x}-6\log\left(1+\sqrt[6]{x}\right)$$

input
```
Integrate[(x^(1/3) + Sqrt[x])^(-1),x]
```

output
```
(6 - 3*x^(1/6) + 2*x^(1/3))*x^(1/6) - 6*Log[1 + x^(1/6)]
```

3.568.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2027, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}+\sqrt[3]{x}}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{1}{\left(\sqrt[6]{x}+1\right)\sqrt[3]{x}}dx$$

$$\Big\downarrow \text{798}$$

$$6\int \frac{\sqrt{x}}{\sqrt[6]{x}+1}d\sqrt[6]{x}$$

$$\Big\downarrow \text{49}$$

$$6\int \left(\sqrt[3]{x}-\sqrt[6]{x}+\frac{1}{-\sqrt[6]{x}-1}+1\right)d\sqrt[6]{x}$$

$$\Big\downarrow \text{2009}$$

$$6\left(\frac{\sqrt{x}}{3}-\frac{\sqrt[3]{x}}{2}+\sqrt[6]{x}-\log\left(\sqrt[6]{x}+1\right)\right)$$

input
```
Int[(x^(1/3) + Sqrt[x])^(-1),x]
```

output
```
6*(x^(1/6) - x^(1/3)/2 + Sqrt[x]/3 - Log[1 + x^(1/6)])
```

3.568.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.568.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

method	result
derivativedivides	$6x^{\frac{1}{6}}-3x^{\frac{1}{3}}-6\ln\left(1+x^{\frac{1}{6}}\right)+2\sqrt{x}$
meijerg	$\frac{x^{\frac{1}{6}}\left(4x^{\frac{1}{3}}-6x^{\frac{1}{6}}+12\right)}{2}-6\ln\left(1+x^{\frac{1}{6}}\right)$
default	$2\ln\left(x^{\frac{1}{6}}-1\right)-\ln\left(x^{\frac{1}{3}}+x^{\frac{1}{6}}+1\right)+\ln\left(1-x^{\frac{1}{6}}+x^{\frac{1}{3}}\right)-2\ln\left(1+x^{\frac{1}{6}}\right)+2\sqrt{x}+\ln\left(-\right.$

input
```
int(1/(x^(1/3)+x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
6*x^(1/6)-3*x^(1/3)-6*ln(1+x^(1/6))+2*x^(1/2)
```

3.568.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = 2\,\sqrt{x}-3\,x^{\frac{1}{3}}+6\,x^{\frac{1}{6}}-6\,\log\left(x^{\frac{1}{6}}+1\right)$$

input
```
integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="fricas")
```

output
```
2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

3.568.6 Sympy [F]

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = \int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx$$

input
```
integrate(1/(x**(1/3)+x**(1/2)),x)
```

output
```
Integral(1/(x**(1/3) + sqrt(x)), x)
```

3.568.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = 2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}}+1\right)$$

input
```
integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="maxima")
```

output
```
2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

3.568.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = 2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}}+1\right)$$

input
```
integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="giac")
```

output
```
2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

3.568.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = 2\sqrt{x} - 6\ln\left(x^{1/6}+1\right) - 3x^{1/3} + 6x^{1/6}$$

input
```
int(1/(x^(1/2) + x^(1/3)),x)
```

output
```
2*x^(1/2) - 6*log(x^(1/6) + 1) - 3*x^(1/3) + 6*x^(1/6)
```

3.569 $\int \frac{1}{\sqrt[4]{x}+\sqrt{x}}\,dx$

3.569.1 Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \frac{1}{\sqrt[4]{x}+\sqrt{x}}\,dx=-4\sqrt[4]{x}+2\sqrt{x}+4\log\left(1+\sqrt[4]{x}\right)$$

output
```
-4*x^(1/4)+4*ln(1+x^(1/4))+2*x^(1/2)
```

3.569.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[4]{x}+\sqrt{x}}\,dx=2\left(-2+\sqrt[4]{x}\right)\sqrt[4]{x}+4\log\left(1+\sqrt[4]{x}\right)$$

input
```
Integrate[(x^(1/4) + Sqrt[x])^(-1),x]
```

output
```
2*(-2 + x^(1/4))*x^(1/4) + 4*Log[1 + x^(1/4)]
```

3.569.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2027, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}+\sqrt[4]{x}}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{1}{\left(\sqrt[4]{x}+1\right)\sqrt[4]{x}}dx$$

$$\Big\downarrow \text{798}$$

$$4\int \frac{\sqrt{x}}{\sqrt[4]{x}+1}d\sqrt[4]{x}$$

$$\Big\downarrow \text{49}$$

$$4\int \left(\sqrt[4]{x}+\frac{1}{\sqrt[4]{x}+1}-1\right)d\sqrt[4]{x}$$

$$\Big\downarrow \text{2009}$$

$$4\left(\frac{\sqrt{x}}{2}-\sqrt[4]{x}+\log\left(\sqrt[4]{x}+1\right)\right)$$

input
```
Int[(x^(1/4) + Sqrt[x])^(-1),x]
```

output
```
4*(-x^(1/4) + Sqrt[x]/2 + Log[1 + x^(1/4)])
```

3.569.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.569.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-4x^{\frac{1}{4}}+4\ln\left(1+x^{\frac{1}{4}}\right)+2\sqrt{x}$	20
default	$-4x^{\frac{1}{4}}+4\ln\left(1+x^{\frac{1}{4}}\right)+2\sqrt{x}$	20
meijerg	$-\frac{2x^{\frac{1}{4}}\left(-3x^{\frac{1}{4}}+6\right)}{3}+4\ln\left(1+x^{\frac{1}{4}}\right)$	22

input
```
int(1/(x^(1/4)+x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-4*x^(1/4)+4*ln(1+x^(1/4))+2*x^(1/2)
```

3.569.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt[4]{x}+\sqrt{x}}\,dx = 2\sqrt{x}-4\,x^{\frac{1}{4}}+4\,\log\left(x^{\frac{1}{4}}+1\right)$$

input
```
integrate(1/(x^(1/4)+x^(1/2)),x, algorithm="fricas")
```

output
```
2*sqrt(x) - 4*x^(1/4) + 4*log(x^(1/4) + 1)
```

3.569.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt[4]{x}+\sqrt{x}}\,dx = -4\sqrt[4]{x} + 2\sqrt{x} + 4\log\left(\sqrt[4]{x}+1\right)$$

input
```
integrate(1/(x**(1/4)+x**(1/2)),x)
```

output
```
-4*x**(1/4) + 2*sqrt(x) + 4*log(x**(1/4) + 1)
```

3.569.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt[4]{x}+\sqrt{x}}\,dx = 2\sqrt{x} - 4x^{\frac{1}{4}} + 4\log\left(x^{\frac{1}{4}}+1\right)$$

input
```
integrate(1/(x^(1/4)+x^(1/2)),x, algorithm="maxima")
```

output
```
2*sqrt(x) - 4*x^(1/4) + 4*log(x^(1/4) + 1)
```

3.569.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt[4]{x}+\sqrt{x}}\,dx = 2\sqrt{x} - 4x^{\frac{1}{4}} + 4\log\left(x^{\frac{1}{4}}+1\right)$$

input
```
integrate(1/(x^(1/4)+x^(1/2)),x, algorithm="giac")
```

output
```
2*sqrt(x) - 4*x^(1/4) + 4*log(x^(1/4) + 1)
```

3.569.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt[4]{x}+\sqrt{x}}\,dx = 4\,\ln\left(x^{1/4}+1\right)+2\,\sqrt{x}-4\,x^{1/4}$$

input
```
int(1/(x^(1/2) + x^(1/4)),x)
```

output
```
4*log(x^(1/4) + 1) + 2*x^(1/2) - 4*x^(1/4)
```

3.570 $\int \frac{1}{-\sqrt[3]{x}+x^{2/3}} \, dx$

3.570.1 Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{1}{-\sqrt[3]{x}+x^{2/3}} \, dx = 3\sqrt[3]{x}+3\log \left(1-\sqrt[3]{x}\right)$$

output
```
3*x^(1/3)+3*ln(1-x^(1/3))
```

3.570.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{-\sqrt[3]{x}+x^{2/3}} \, dx = 3\sqrt[3]{x}+3\log \left(-1+\sqrt[3]{x}\right)$$

input
```
Integrate[(-x^(1/3) + x^(2/3))^(-1),x]
```

output
```
3*x^(1/3) + 3*Log[-1 + x^(1/3)]
```

3.570.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2027, 798, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{2/3}-\sqrt[3]{x}}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{1}{\left(\sqrt[3]{x}-1\right)\sqrt[3]{x}}dx$$

$$\Big\downarrow \text{798}$$

$$3\int -\frac{\sqrt[3]{x}}{1-\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{25}$$

$$-3\int \frac{\sqrt[3]{x}}{1-\sqrt[3]{x}}d\sqrt[3]{x}$$

$$\Big\downarrow \text{49}$$

$$-3\int \left(\frac{1}{1-\sqrt[3]{x}}-1\right)d\sqrt[3]{x}$$

$$\Big\downarrow \text{2009}$$

$$3\left(\sqrt[3]{x}+\log\left(1-\sqrt[3]{x}\right)\right)$$

input
```
Int[(-x^(1/3) + x^(2/3))^(-1),x]
```

output
```
3*(x^(1/3) + Log[1 - x^(1/3)])
```

3.570.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.570.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$3x^{\frac{1}{3}}+3\ln\left(x^{\frac{1}{3}}-1\right)$	15
default	$3x^{\frac{1}{3}}+3\ln\left(x^{\frac{1}{3}}-1\right)$	15
meijerg	$3x^{\frac{1}{3}}+3\ln\left(1-x^{\frac{1}{3}}\right)$	17
trager	$3x^{\frac{1}{3}}+\ln\left(-3x^{\frac{2}{3}}+3x^{\frac{1}{3}}+x-1\right)$	21

input
```
int(1/(-x^(1/3)+x^(2/3)),x,method=_RETURNVERBOSE)
```

output
```
3*x^(1/3)+3*ln(x^(1/3)-1)
```

3.570.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{1}{-\sqrt[3]{x}+x^{2/3}}\,dx = 3\,x^{\frac{1}{3}} + 3\,\log\left(x^{\frac{1}{3}} - 1\right)$$

input
```
integrate(1/(-x^(1/3)+x^(2/3)),x, algorithm="fricas")
```

output
```
3*x^(1/3) + 3*log(x^(1/3) - 1)
```

3.570.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{1}{-\sqrt[3]{x}+x^{2/3}}\,dx = 3\sqrt[3]{x} + 3\log\left(\sqrt[3]{x} - 1\right)$$

input
```
integrate(1/(-x**(1/3)+x**(2/3)),x)
```

output
```
3*x**(1/3) + 3*log(x**(1/3) - 1)
```

3.570.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{1}{-\sqrt[3]{x}+x^{2/3}}\,dx = 3\,x^{\frac{1}{3}} + 3\,\log\left(x^{\frac{1}{3}} - 1\right)$$

input
```
integrate(1/(-x^(1/3)+x^(2/3)),x, algorithm="maxima")
```

output
```
3*x^(1/3) + 3*log(x^(1/3) - 1)
```

3.570.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{1}{-\sqrt[3]{x}+x^{2/3}}\,dx = 3\,x^{\frac{1}{3}} + 3\,\log\left(\left|x^{\frac{1}{3}} - 1\right|\right)$$

input
```
integrate(1/(-x^(1/3)+x^(2/3)),x, algorithm="giac")
```

output
```
3*x^(1/3) + 3*log(abs(x^(1/3) - 1))
```

3.570.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{1}{-\sqrt[3]{x}+x^{2/3}}\,dx = 3\,\ln\left(x^{1/3} - 1\right) + 3\,x^{1/3}$$

input
```
int(-1/(x^(1/3) - x^(2/3)),x)
```

output
```
3*log(x^(1/3) - 1) + 3*x^(1/3)
```

3.571 $\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx$

3.571.1 Optimal result

Integrand size = 13, antiderivative size = 62

$$\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx=2\sqrt{x}+\frac{4\arctan\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{4}{3}\log\left(1+\sqrt[4]{x}\right)-\frac{2}{3}\log\left(1-\sqrt[4]{x}+\sqrt{x}\right)$$

output
```
4/3*ln(1+x^(1/4))-2/3*ln(1-x^(1/4)+x^(1/2))+4/3*arctan(1/3*(1-2*x^(1/4))*3
^(1/2))*3^(1/2)+2*x^(1/2)
```

3.571.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx=\frac{2}{3}\left(3\sqrt{x}+2\sqrt{3}\arctan\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)+2\log\left(1+\sqrt[4]{x}\right)-\log\left(1-\sqrt[4]{x}+\sqrt{x}\right)\right)$$

input
```
Integrate[(x^(-1/4) + Sqrt[x])^(-1),x]
```

output
```
(2*(3*Sqrt[x] + 2*Sqrt[3]*ArcTan[(1 - 2*x^(1/4))/Sqrt[3]] + 2*Log[1 + x^(1
/4)] - Log[1 - x^(1/4) + Sqrt[x]]))/3
```

3.571.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {2027, 864, 843, 821, 16, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}+\frac{1}{\sqrt[4]{x}}}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{\sqrt[4]{x}}{x^{3/4}+1}dx$$

$$\Big\downarrow \text{864}$$

$$4\int \frac{x}{x^{3/4}+1}d\sqrt[4]{x}$$

$$\Big\downarrow \text{843}$$

$$4\left(\frac{\sqrt{x}}{2}-\int \frac{\sqrt[4]{x}}{x^{3/4}+1}d\sqrt[4]{x}\right)$$

$$\Big\downarrow \text{821}$$

$$4\left(\frac{1}{3}\int \frac{1}{\sqrt[4]{x}+1}d\sqrt[4]{x}-\frac{1}{3}\int \frac{\sqrt[4]{x}+1}{\sqrt{x}-\sqrt[4]{x}+1}d\sqrt[4]{x}+\frac{\sqrt{x}}{2}\right)$$

$$\Big\downarrow \text{16}$$

$$4\left(-\frac{1}{3}\int \frac{\sqrt[4]{x}+1}{\sqrt{x}-\sqrt[4]{x}+1}d\sqrt[4]{x}+\frac{\sqrt{x}}{2}+\frac{1}{3}\log\left(\sqrt[4]{x}+1\right)\right)$$

$$\Big\downarrow \text{1142}$$

$$4\left(\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{\sqrt{x}-\sqrt[4]{x}+1}d\sqrt[4]{x}-\frac{1}{2}\int -\frac{1-2\sqrt[4]{x}}{\sqrt{x}-\sqrt[4]{x}+1}d\sqrt[4]{x}\right)+\frac{\sqrt{x}}{2}+\frac{1}{3}\log\left(\sqrt[4]{x}+1\right)\right)$$

$$\Big\downarrow \text{25}$$

$$4\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2\sqrt[4]{x}}{\sqrt{x}-\sqrt[4]{x}+1}d\sqrt[4]{x}-\frac{3}{2}\int \frac{1}{\sqrt{x}-\sqrt[4]{x}+1}d\sqrt[4]{x}\right)+\frac{\sqrt{x}}{2}+\frac{1}{3}\log\left(\sqrt[4]{x}+1\right)\right)$$

$$\Big\downarrow \text{1083}$$

$$4\left(\frac{1}{3}\left(3\int \frac{1}{-\sqrt{x}-3}d(2\sqrt[4]{x}-1)+\frac{1}{2}\int \frac{1-2\sqrt[4]{x}}{\sqrt{x}-\sqrt[4]{x}+1}d\sqrt[4]{x}\right)+\frac{\sqrt{x}}{2}+\frac{1}{3}\log\left(\sqrt[4]{x}+1\right)\right)$$

$$\Big\downarrow 217$$

$$4\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{1-2\sqrt[4]{x}}{\sqrt{x}-\sqrt[4]{x}+1}d\sqrt[4]{x}-\sqrt{3}\arctan\left(\frac{2\sqrt[4]{x}-1}{\sqrt{3}}\right)\right)+\frac{\sqrt{x}}{2}+\frac{1}{3}\log\left(\sqrt[4]{x}+1\right)\right)$$

$$\Big\downarrow 1103$$

$$4\left(\frac{1}{3}\left(-\sqrt{3}\arctan\left(\frac{2\sqrt[4]{x}-1}{\sqrt{3}}\right)-\frac{1}{2}\log\left(\sqrt{x}-\sqrt[4]{x}+1\right)\right)+\frac{\sqrt{x}}{2}+\frac{1}{3}\log\left(\sqrt[4]{x}+1\right)\right)$$

input
```
Int[(x^(-1/4) + Sqrt[x])^(-1),x]
```

output
```
4*(Sqrt[x]/2 + Log[1 + x^(1/4)]/3 + (-(Sqrt[3]*ArcTan[(-1 + 2*x^(1/4))/Sqr
t[3]]) - Log[1 - x^(1/4) + Sqrt[x]]/2)/3)
```

3.571.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

3.571. $\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx$

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.571.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$2\sqrt{x}+\frac{4\ln\left(1+x^{\frac{1}{4}}\right)}{3}-\frac{2\ln\left(1-x^{\frac{1}{4}}+\sqrt{x}\right)}{3}-\frac{4\sqrt{3}\,\arctan\left(\frac{\left(2x^{\frac{1}{4}}-1\right)\sqrt{3}}{3}\right)}{3}$	46
default	$2\sqrt{x}+\frac{4\ln\left(1+x^{\frac{1}{4}}\right)}{3}-\frac{2\ln\left(1-x^{\frac{1}{4}}+\sqrt{x}\right)}{3}-\frac{4\sqrt{3}\,\arctan\left(\frac{\left(2x^{\frac{1}{4}}-1\right)\sqrt{3}}{3}\right)}{3}$	46
meijerg	$2\sqrt{x}-\frac{4\sqrt{x}\left(-\frac{\ln\left(1+x^{\frac{1}{4}}\right)}{\sqrt{x}}+\frac{\ln\left(1-x^{\frac{1}{4}}+\sqrt{x}\right)}{2\sqrt{x}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x^{\frac{1}{4}}}{2-x^{\frac{1}{4}}}\right)}{\sqrt{x}}\right)}{3}$	65

3.571. $\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx$

input
```
int(1/(1/x^(1/4)+x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)+4/3*ln(1+x^(1/4))-2/3*ln(1-x^(1/4)+x^(1/2))-4/3*3^(1/2)*arctan(1
/3*(2*x^(1/4)-1)*3^(1/2))
```

3.571.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

$$\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx = -\frac{4}{3}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}x^{\frac{1}{4}}-\frac{1}{3}\,\sqrt{3}\right)+2\,\sqrt{x} - \frac{2}{3}\,\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\,\log\left(x^{\frac{1}{4}}+1\right)$$

input
```
integrate(1/(1/x^(1/4)+x^(1/2)),x, algorithm="fricas")
```

output
```
-4/3*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/4) - 1/3*sqrt(3)) + 2*sqrt(x) - 2/3*l
og(sqrt(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

3.571.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10

$$\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx = 2\sqrt{x}+\frac{4\log\left(\sqrt[4]{x}+1\right)}{3}-\frac{2\log\left(-4\sqrt[4]{x}+4\sqrt{x}+4\right)}{3} - \frac{4\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[4]{x}}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate(1/(1/x**(1/4)+x**(1/2)),x)
```

output
```
2*sqrt(x) + 4*log(x**(1/4) + 1)/3 - 2*log(-4*x**(1/4) + 4*sqrt(x) + 4)/3 -
 4*sqrt(3)*atan(2*sqrt(3)*x**(1/4)/3 - sqrt(3)/3)/3
```

3.571.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx = -\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{4}}-1\right)\right)+2\sqrt{x} - \frac{2}{3}\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\log\left(x^{\frac{1}{4}}+1\right)$$

input
```
integrate(1/(1/x^(1/4)+x^(1/2)),x, algorithm="maxima")
```

output
```
-4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/4) - 1)) + 2*sqrt(x) - 2/3*log(sqr
t(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

3.571.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx = -\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{4}}-1\right)\right)+2\sqrt{x} - \frac{2}{3}\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\log\left(x^{\frac{1}{4}}+1\right)$$

input
```
integrate(1/(1/x^(1/4)+x^(1/2)),x, algorithm="giac")
```

output
```
-4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/4) - 1)) + 2*sqrt(x) - 2/3*log(sqr
t(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

3.571.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18

$$\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx = \frac{4\,\ln\left(16\,x^{1/4}+16\right)}{3}+\ln\left(9\left(-\frac{2}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right)^2+16\,x^{1/4}\right)\left(-\frac{2}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right) - \ln\left(9\left(\frac{2}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right)^2+16\,x^{1/4}\right)\left(\frac{2}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right)+2\sqrt{x}$$

input
```
int(1/(x^(1/2) + 1/x^(1/4)),x)
```

output
```
(4*log(16*x^(1/4) + 16))/3 + log(9*((3^(1/2)*2i)/3 - 2/3)^2 + 16*x^(1/4))*
((3^(1/2)*2i)/3 - 2/3) - log(9*((3^(1/2)*2i)/3 + 2/3)^2 + 16*x^(1/4))*((3^
(1/2)*2i)/3 + 2/3) + 2*x^(1/2)
```

3.571. $\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx$

3.572 $\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}} dx$

3.572.1 Optimal result

Integrand size = 13, antiderivative size = 73

$$\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}} dx = -12\sqrt[12]{x}+6\sqrt[6]{x}-4\sqrt[4]{x}+3\sqrt[3]{x}-\frac{12x^{5/12}}{5} +2\sqrt{x}-\frac{12x^{7/12}}{7}+\frac{3x^{2/3}}{2}+12\log\left(1+\sqrt[12]{x}\right)$$

output
```
-12*x^(1/12)+6*x^(1/6)-4*x^(1/4)+3*x^(1/3)-12/5*x^(5/12)-12/7*x^(7/12)+3/2
*x^(2/3)+12*ln(1+x^(1/12))+2*x^(1/2)
```

3.572.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}} dx = \frac{1}{70}\left(-840\sqrt[12]{x}+420\sqrt[6]{x}-280\sqrt[4]{x}+210\sqrt[3]{x}-168x^{5/12}+140\sqrt{x} -120x^{7/12}+105x^{2/3}\right)+12\log\left(1+\sqrt[12]{x}\right)$$

input
```
Integrate[(x^(1/4) + x^(1/3))^(-1),x]
```

output
```
(-840*x^(1/12) + 420*x^(1/6) - 280*x^(1/4) + 210*x^(1/3) - 168*x^(5/12) +
140*Sqrt[x] - 120*x^(7/12) + 105*x^(2/3))/70 + 12*Log[1 + x^(1/12)]
```

3.572.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2027, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{x}+\sqrt[4]{x}}dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int \frac{1}{\left(\sqrt[12]{x}+1\right)\sqrt[4]{x}}dx$$

$$\Big\downarrow \text{ 798}$$

$$12\int \frac{x^{2/3}}{\sqrt[12]{x}+1}d\sqrt[12]{x}$$

$$\Big\downarrow \text{ 49}$$

$$12\int \left(x^{7/12}-\sqrt{x}+x^{5/12}-\sqrt[3]{x}+\sqrt[4]{x}-\sqrt[6]{x}+\sqrt[12]{x}+\frac{1}{\sqrt[12]{x}+1}-1\right)d\sqrt[12]{x}$$

$$\Big\downarrow \text{ 2009}$$

$$12\left(\frac{x^{2/3}}{8}-\frac{x^{7/12}}{7}-\frac{x^{5/12}}{5}+\frac{\sqrt{x}}{6}+\frac{\sqrt[3]{x}}{4}-\frac{\sqrt[4]{x}}{3}+\frac{\sqrt[6]{x}}{2}-\sqrt[12]{x}+\log\left(\sqrt[12]{x}+1\right)\right)$$

input
```
Int[(x^(1/4) + x^(1/3))^(-1),x]
```

output
```
12*(-x^(1/12) + x^(1/6)/2 - x^(1/4)/3 + x^(1/3)/4 - x^(5/12)/5 + Sqrt[x]/6
 - x^(7/12)/7 + x^(2/3)/8 + Log[1 + x^(1/12)])
```

3.572.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.572.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

method	result
derivativedivides	$-12x^{\frac{1}{12}}+6x^{\frac{1}{6}}-4x^{\frac{1}{4}}+3x^{\frac{1}{3}}-\frac{12x^{\frac{5}{12}}}{5}-\frac{12x^{\frac{7}{12}}}{7}+\frac{3x^{\frac{2}{3}}}{2}+12\ln\left(1+x^{\frac{1}{12}}\right)+2\sqrt{x}$
meijerg	$-\frac{x^{\frac{1}{12}}\left(-315x^{\frac{7}{12}}+360\sqrt{x}-420x^{\frac{5}{12}}+504x^{\frac{1}{3}}-630x^{\frac{1}{4}}+840x^{\frac{1}{6}}-1260x^{\frac{1}{12}}+2520\right)}{210}+12\ln\left(1+x^{\frac{1}{12}}\right)$
default	$2\sqrt{x}-4x^{\frac{1}{4}}+3x^{\frac{1}{3}}+6x^{\frac{1}{6}}+\frac{3x^{\frac{2}{3}}}{2}+\ln\left(x-1\right)+2\ln\left(x^{\frac{1}{6}}-1\right)-\ln\left(x^{\frac{1}{3}}+x^{\frac{1}{6}}+1\right)-2\ln$

input
```
int(1/(x^(1/4)+x^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
-12*x^(1/12)+6*x^(1/6)-4*x^(1/4)+3*x^(1/3)-12/5*x^(5/12)-12/7*x^(7/12)+3/2
*x^(2/3)+12*ln(1+x^(1/12))+2*x^(1/2)
```

3.572.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx = \frac{3}{2}\,x^{\frac{2}{3}} - \frac{12}{7}\,x^{\frac{7}{12}} + 2\,\sqrt{x} - \frac{12}{5}\,x^{\frac{5}{12}} + 3\,x^{\frac{1}{3}} - 4\,x^{\frac{1}{4}} + 6\,x^{\frac{1}{6}} - 12\,x^{\frac{1}{12}} + 12\,\log\left(x^{\frac{1}{12}} + 1\right)$$

input
```
integrate(1/(x^(1/4)+x^(1/3)),x, algorithm="fricas")
```

output
```
3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3) - 4*x^
(1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

3.572.6 Sympy [F]

$$\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx = \int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx$$

input
```
integrate(1/(x**(1/4)+x**(1/3)),x)
```

output
```
Integral(1/(x**(1/4) + x**(1/3)), x)
```

3.572.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx = \frac{3}{2}\,x^{\frac{2}{3}} - \frac{12}{7}\,x^{\frac{7}{12}} + 2\,\sqrt{x} - \frac{12}{5}\,x^{\frac{5}{12}} + 3\,x^{\frac{1}{3}} - 4\,x^{\frac{1}{4}} + 6\,x^{\frac{1}{6}} - 12\,x^{\frac{1}{12}} + 12\,\log\left(x^{\frac{1}{12}} + 1\right)$$

input
```
integrate(1/(x^(1/4)+x^(1/3)),x, algorithm="maxima")
```

output
```
3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3) - 4*x^
(1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

3.572.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx = \frac{3}{2}\,x^{\frac{2}{3}} - \frac{12}{7}\,x^{\frac{7}{12}} + 2\,\sqrt{x} - \frac{12}{5}\,x^{\frac{5}{12}} + 3\,x^{\frac{1}{3}} - 4\,x^{\frac{1}{4}} + 6\,x^{\frac{1}{6}} - 12\,x^{\frac{1}{12}} + 12\,\log\left(x^{\frac{1}{12}} + 1\right)$$

input
```
integrate(1/(x^(1/4)+x^(1/3)),x, algorithm="giac")
```

output
```
3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3) - 4*x^
(1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

3.572.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx = 12\,\ln\left(x^{1/12}+1\right) + 2\,\sqrt{x} + 3\,x^{1/3} - 4\,x^{1/4} + \frac{3\,x^{2/3}}{2} + 6\,x^{1/6} - 12\,x^{1/12} - \frac{12\,x^{5/12}}{5} - \frac{12\,x^{7/12}}{7}$$

input
```
int(1/(x^(1/3) + x^(1/4)),x)
```

output
```
12*log(x^(1/12) + 1) + 2*x^(1/2) + 3*x^(1/3) - 4*x^(1/4) + (3*x^(2/3))/2 +
 6*x^(1/6) - 12*x^(1/12) - (12*x^(5/12))/5 - (12*x^(7/12))/7
```

3.573 $\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}} dx$

3.573.1 Optimal result

Integrand size = 13, antiderivative size = 130

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}} dx = 12 \sqrt[12]{x} - 6\sqrt[6]{x} + 4\sqrt[4]{x} - 3\sqrt[3]{x} + \frac{12x^{5/12}}{5} - 2\sqrt{x} + \frac{12x^{7/12}}{7} - \frac{3x^{2/3}}{2} + \frac{4x^{3/4}}{3} - \frac{6x^{5/6}}{5} + \frac{12x^{11/12}}{11} - x + \frac{12x^{13/12}}{13} - \frac{6x^{7/6}}{7} + \frac{4x^{5/4}}{5} - 12\log\left(1+\sqrt[12]{x}\right)$$

output
```
12*x^(1/12)-6*x^(1/6)+4*x^(1/4)-3*x^(1/3)+12/5*x^(5/12)+12/7*x^(7/12)-3/2*
x^(2/3)+4/3*x^(3/4)-6/5*x^(5/6)+12/11*x^(11/12)-x+12/13*x^(13/12)-6/7*x^(7
/6)+4/5*x^(5/4)-12*ln(1+x^(1/12))-2*x^(1/2)
```

3.573.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.90

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}} dx = \frac{360360 \sqrt[12]{x} - 180180\sqrt[6]{x} + 120120\sqrt[4]{x} - 90090\sqrt[3]{x} + 72072x^{5/12} - 60060\sqrt{x} + 51480x^{7/12} - 45045x^{2/3} +}{30030} - 12\log\left(1+\sqrt[12]{x}\right)$$

input
```
Integrate[(x^(-1/3) + x^(-1/4))^(-1),x]
```

output
```
(360360*x^(1/12) - 180180*x^(1/6) + 120120*x^(1/4) - 90090*x^(1/3) + 72072
*x^(5/12) - 60060*Sqrt[x] + 51480*x^(7/12) - 45045*x^(2/3) + 40040*x^(3/4)
 - 36036*x^(5/6) + 32760*x^(11/12) - 30030*x + 27720*x^(13/12) - 25740*x^(
7/6) + 24024*x^(5/4))/30030 - 12*Log[1 + x^(1/12)]
```

3.573.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2027, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\frac{1}{\sqrt[3]{x}}}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{\sqrt[3]{x}}{\sqrt[12]{x}+1}dx$$

$$\Big\downarrow \text{798}$$

$$12\int \frac{x^{5/4}}{\sqrt[12]{x}+1}d\sqrt[12]{x}$$

$$\Big\downarrow \text{49}$$

$$12\int \left(x^{7/6}-x^{13/12}+x-x^{11/12}+x^{5/6}-x^{3/4}+x^{2/3}-x^{7/12}+\sqrt{x}-x^{5/12}+\sqrt[3]{x}-\sqrt[4]{x}+\sqrt[6]{x}-\sqrt[12]{x}+\frac{1}{-\sqrt[12]{x}-$$

$$\Big\downarrow \text{2009}$$

$$12\left(\frac{x^{5/4}}{15}-\frac{x^{7/6}}{14}+\frac{x^{13/12}}{13}+\frac{x^{11/12}}{11}-\frac{x^{5/6}}{10}+\frac{x^{3/4}}{9}-\frac{x^{2/3}}{8}+\frac{x^{7/12}}{7}+\frac{x^{5/12}}{5}-\frac{x}{12}-\frac{\sqrt{x}}{6}-\frac{\sqrt[3]{x}}{4}+\frac{\sqrt[4]{x}}{3}-\frac{\sqrt[6]{x}}{2}+\sqrt[12]{}\right.$$

input
```
Int[(x^(-1/3) + x^(-1/4))^(-1),x]
```

output
```
12*(x^(1/12) - x^(1/6)/2 + x^(1/4)/3 - x^(1/3)/4 + x^(5/12)/5 - Sqrt[x]/6
+ x^(7/12)/7 - x^(2/3)/8 + x^(3/4)/9 - x^(5/6)/10 + x^(11/12)/11 - x/12 +
x^(13/12)/13 - x^(7/6)/14 + x^(5/4)/15 - Log[1 + x^(1/12)])
```

3.573.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.573.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.64

method	result
derivativedivides	$12x^{\frac{1}{12}}-6x^{\frac{1}{6}}+4x^{\frac{1}{4}}-3x^{\frac{1}{3}}+\frac{12x^{\frac{5}{12}}}{5}+\frac{12x^{\frac{7}{12}}}{7}-\frac{3x^{\frac{2}{3}}}{2}+\frac{4x^{\frac{3}{4}}}{3}-\frac{6x^{\frac{5}{6}}}{5}+\frac{12x^{\frac{11}{12}}}{11}-x+\frac{12x^{\frac{13}{12}}}{13}-6$
default	$12x^{\frac{1}{12}}-6x^{\frac{1}{6}}+4x^{\frac{1}{4}}-3x^{\frac{1}{3}}+\frac{12x^{\frac{5}{12}}}{5}+\frac{12x^{\frac{7}{12}}}{7}-\frac{3x^{\frac{2}{3}}}{2}+\frac{4x^{\frac{3}{4}}}{3}-\frac{6x^{\frac{5}{6}}}{5}+\frac{12x^{\frac{11}{12}}}{11}-x+\frac{12x^{\frac{13}{12}}}{13}-6$
meijerg	$\frac{x^{\frac{1}{12}}\left(48048x^{\frac{7}{6}}-51480x^{\frac{13}{12}}+55440x-60060x^{\frac{11}{12}}+65520x^{\frac{5}{6}}-72072x^{\frac{3}{4}}+80080x^{\frac{2}{3}}-90090x^{\frac{7}{12}}+102960\sqrt{x}-120120x^{\frac{5}{12}}+\right.}{60060}$

input
```
int(1/(1/x^(1/3)+1/x^(1/4)),x,method=_RETURNVERBOSE)
```

output
```
12*x^(1/12)-6*x^(1/6)+4*x^(1/4)-3*x^(1/3)+12/5*x^(5/12)+12/7*x^(7/12)-3/2*
x^(2/3)+4/3*x^(3/4)-6/5*x^(5/6)+12/11*x^(11/12)-x+12/13*x^(13/12)-6/7*x^(7
/6)+4/5*x^(5/4)-12*ln(1+x^(1/12))-2*x^(1/2)
```

3.573.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.58

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}}\,dx=\frac{4}{5}\,(x+5)x^{\frac{1}{4}}-\frac{6}{7}\,(x+7)x^{\frac{1}{6}}+\frac{12}{13}\,(x+13)x^{\frac{1}{12}}-x+\frac{12}{11}\,x^{\frac{11}{12}}-\frac{6}{5}\,x^{\frac{5}{6}}+\frac{4}{3}\,x^{\frac{3}{4}}-\frac{3}{2}\,x^{\frac{2}{3}}+\frac{12}{7}\,x^{\frac{7}{12}}-2\,\sqrt{x}+\frac{12}{5}\,x^{\frac{5}{12}}-3\,x^{\frac{1}{3}}-12\,\log\left(x^{\frac{1}{12}}+1\right)$$

input
```
integrate(1/(1/x^(1/3)+1/x^(1/4)),x, algorithm="fricas")
```

output
```
4/5*(x + 5)*x^(1/4) - 6/7*(x + 7)*x^(1/6) + 12/13*(x + 13)*x^(1/12) - x +
12/11*x^(11/12) - 6/5*x^(5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12)
- 2*sqrt(x) + 12/5*x^(5/12) - 3*x^(1/3) - 12*log(x^(1/12) + 1)
```

3.573.6 Sympy [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.93

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}}\,dx=\frac{12x^{\frac{13}{12}}}{13}+\frac{12x^{\frac{11}{12}}}{11}+\frac{12x^{\frac{7}{12}}}{7}+\frac{12x^{\frac{5}{12}}}{5}+12\sqrt[12]{x}-\frac{6x^{\frac{7}{6}}}{7}-\frac{6x^{\frac{5}{6}}}{5}-6\sqrt[6]{x}+\frac{4x^{\frac{5}{4}}}{5}+\frac{4x^{\frac{3}{4}}}{3}+4\sqrt[4]{x}-\frac{3x^{\frac{2}{3}}}{2}-3\sqrt[3]{x}-2\sqrt{x}-x-12\log\left(\sqrt[12]{x}+1\right)$$

input
```
integrate(1/(1/x**(1/3)+1/x**(1/4)),x)
```

output
```
12*x**(13/12)/13 + 12*x**(11/12)/11 + 12*x**(7/12)/7 + 12*x**(5/12)/5 + 12
*x**(1/12) - 6*x**(7/6)/7 - 6*x**(5/6)/5 - 6*x**(1/6) + 4*x**(5/4)/5 + 4*x
**(3/4)/3 + 4*x**(1/4) - 3*x**(2/3)/2 - 3*x**(1/3) - 2*sqrt(x) - x - 12*lo
g(x**(1/12) + 1)
```

3.573.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.63

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}}\,dx = \frac{4}{5}\,x^{\frac{5}{4}} - \frac{6}{7}\,x^{\frac{7}{6}} + \frac{12}{13}\,x^{\frac{13}{12}} - x + \frac{12}{11}\,x^{\frac{11}{12}} - \frac{6}{5}\,x^{\frac{5}{6}} + \frac{4}{3}\,x^{\frac{3}{4}} - \frac{3}{2}\,x^{\frac{2}{3}} + \frac{12}{7}\,x^{\frac{7}{12}} - 2\,\sqrt{x} + \frac{12}{5}\,x^{\frac{5}{12}} - 3\,x^{\frac{1}{3}} + 4\,x^{\frac{1}{4}} - 6\,x^{\frac{1}{6}} + 12\,x^{\frac{1}{12}} - 12\,\log\left(x^{\frac{1}{12}} + 1\right)$$

input
```
integrate(1/(1/x^(1/3)+1/x^(1/4)),x, algorithm="maxima")
```

output
```
4/5*x^(5/4) - 6/7*x^(7/6) + 12/13*x^(13/12) - x + 12/11*x^(11/12) - 6/5*x^
(5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) - 2*sqrt(x) + 12/5*x^(5/
12) - 3*x^(1/3) + 4*x^(1/4) - 6*x^(1/6) + 12*x^(1/12) - 12*log(x^(1/12) +
1)
```

3.573.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.63

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}}\,dx = \frac{4}{5}\,x^{\frac{5}{4}} - \frac{6}{7}\,x^{\frac{7}{6}} + \frac{12}{13}\,x^{\frac{13}{12}} - x + \frac{12}{11}\,x^{\frac{11}{12}} - \frac{6}{5}\,x^{\frac{5}{6}} + \frac{4}{3}\,x^{\frac{3}{4}} - \frac{3}{2}\,x^{\frac{2}{3}} + \frac{12}{7}\,x^{\frac{7}{12}} - 2\,\sqrt{x} + \frac{12}{5}\,x^{\frac{5}{12}} - 3\,x^{\frac{1}{3}} + 4\,x^{\frac{1}{4}} - 6\,x^{\frac{1}{6}} + 12\,x^{\frac{1}{12}} - 12\,\log\left(x^{\frac{1}{12}} + 1\right)$$

input
```
integrate(1/(1/x^(1/3)+1/x^(1/4)),x, algorithm="giac")
```

output
```
4/5*x^(5/4) - 6/7*x^(7/6) + 12/13*x^(13/12) - x + 12/11*x^(11/12) - 6/5*x^
(5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) - 2*sqrt(x) + 12/5*x^(5/
12) - 3*x^(1/3) + 4*x^(1/4) - 6*x^(1/6) + 12*x^(1/12) - 12*log(x^(1/12) +
1)
```

3.573.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.63

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}}\,dx = 4\,x^{1/4} - 12\,\ln\left(x^{1/12}+1\right) - 2\,\sqrt{x} - 3\,x^{1/3} - x - \frac{3\,x^{2/3}}{2} - 6\,x^{1/6} + \frac{4\,x^{3/4}}{3} + \frac{4\,x^{5/4}}{5} - \frac{6\,x^{5/6}}{5} + 12\,x^{1/12} - \frac{6\,x^{7/6}}{7} + \frac{12\,x^{5/12}}{5} + \frac{12\,x^{7/12}}{7} + \frac{12\,x^{11/12}}{11} + \frac{12\,x^{13/12}}{13}$$

input
```
int(1/(1/x^(1/3) + 1/x^(1/4)),x)
```

output
```
4*x^(1/4) - 12*log(x^(1/12) + 1) - 2*x^(1/2) - 3*x^(1/3) - x - (3*x^(2/3))
/2 - 6*x^(1/6) + (4*x^(3/4))/3 + (4*x^(5/4))/5 - (6*x^(5/6))/5 + 12*x^(1/1
2) - (6*x^(7/6))/7 + (12*x^(5/12))/5 + (12*x^(7/12))/7 + (12*x^(11/12))/11
 + (12*x^(13/12))/13
```

3.574 $\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}} \, dx$

3.574.1 Optimal result

Integrand size = 15, antiderivative size = 200

$$\begin{aligned}\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}} \, dx &= 2\sqrt{x}+\frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\arctan\left(\frac{1-\sqrt{5}+4\sqrt[6]{x}}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) \\ &\quad -\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{5}+4\sqrt[6]{x}\right)\right) \\ &\quad +\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2+\sqrt[6]{x}-\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right) \\ &\quad -\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2+\sqrt[6]{x}+\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)\end{aligned}$$

output
```
6/5*ln(1-x^(1/6))-3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))*(-5^(1/2)+1
)-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*5^(1/2))*(5^(1/2)+1)+2*x^(1/2)+3/5*a
rctan((1+4*x^(1/6)-5^(1/2))/(10+2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)-3/5
*arctan(1/20*(1+4*x^(1/6)+5^(1/2))*(50+10*5^(1/2))^(1/2))*(10+2*5^(1/2))^(
1/2)
```

3.574.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.63

$$\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = 2\sqrt{x}+\frac{6}{5}\log\left(-1+\sqrt[6]{x}\right)-\frac{6}{5}\text{RootSum}\left[1+\#1+\#1^2+\#1^3+\#1^4\&,\frac{-\log\left(\sqrt[6]{x}-\#1\right)-2\log\left(\sqrt[6]{x}-\#1\right)\#1+2\log\left(\sqrt[6]{x}-\#1\right)\#1^2+\log\left(\sqrt[6]{x}-\#1\right)\#1^3}{1+2\#1+3\#1^2+4\#1^3}\&\right]$$

input
```
Integrate[(-x^(-1/3) + Sqrt[x])^(-1),x]
```

output
```
2*Sqrt[x] + (6*Log[-1 + x^(1/6)])/5 - (6*RootSum[1 + #1 + #1^2 + #1^3 + #1
^4 & , (-Log[x^(1/6) - #1] - 2*Log[x^(1/6) - #1]*#1 + 2*Log[x^(1/6) - #1]*
#1^2 + Log[x^(1/6) - #1]*#1^3)/(1 + 2*#1 + 3*#1^2 + 4*#1^3) & ])/5
```

3.574.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {2027, 864, 25, 843, 823, 16, 27, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}-\frac{1}{\sqrt[3]{x}}}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{\sqrt[3]{x}}{x^{5/6}-1}dx$$

$$\Big\downarrow \text{864}$$

$$6\int -\frac{x^{7/6}}{1-x^{5/6}}d\sqrt[6]{x}$$

$$\Big\downarrow \text{25}$$

$$-6\int \frac{x^{7/6}}{1-x^{5/6}}d\sqrt[6]{x}$$

$$\Big\downarrow \text{843}$$

$$6\left(\frac{\sqrt{x}}{3}-\int \frac{\sqrt[3]{x}}{1-x^{5/6}}d\sqrt[6]{x}\right)$$

$$\Big\downarrow \text{823}$$

$$6\left(-\frac{1}{5}\int \frac{1}{1-\sqrt[6]{x}}d\sqrt[6]{x}-\frac{2}{5}\int -\frac{-\left(\left(1+\sqrt{5}\right)\sqrt[6]{x}\right)+\sqrt{5}+1}{2\left(2\sqrt[3]{x}+\left(1-\sqrt{5}\right)\sqrt[6]{x}+2\right)}d\sqrt[6]{x}-\frac{2}{5}\int -\frac{-\left(\left(1-\sqrt{5}\right)\sqrt[6]{x}\right)-\sqrt{5}+1}{2\left(2\sqrt[3]{x}+\left(1+\sqrt{5}\right)\sqrt[6]{x}+2\right)}d\sqrt[6]{x}+\frac{\sqrt{x}}{3}\right)$$

$$\Big\downarrow \text{16}$$

$$6\left(-\frac{2}{5}\int -\frac{-\left(\left(1+\sqrt{5}\right)\sqrt[6]{x}\right)+\sqrt{5}+1}{2\left(2\sqrt[3]{x}+\left(1-\sqrt{5}\right)\sqrt[6]{x}+2\right)}d\sqrt[6]{x}-\frac{2}{5}\int -\frac{-\left(\left(1-\sqrt{5}\right)\sqrt[6]{x}\right)-\sqrt{5}+1}{2\left(2\sqrt[3]{x}+\left(1+\sqrt{5}\right)\sqrt[6]{x}+2\right)}d\sqrt[6]{x}+\frac{\sqrt{x}}{3}+\frac{1}{5}\log\left(1-\sqrt[6]{x}\right)\right)$$

$$\Big\downarrow \text{27}$$

$$6\left(\frac{1}{5}\int \frac{-\left(\left(1+\sqrt{5}\right)\sqrt[6]{x}\right)+\sqrt{5}+1}{2\sqrt[3]{x}+\left(1-\sqrt{5}\right)\sqrt[6]{x}+2}d\sqrt[6]{x}+\frac{1}{5}\int \frac{-\left(\left(1-\sqrt{5}\right)\sqrt[6]{x}\right)-\sqrt{5}+1}{2\sqrt[3]{x}+\left(1+\sqrt{5}\right)\sqrt[6]{x}+2}d\sqrt[6]{x}+\frac{\sqrt{x}}{3}+\frac{1}{5}\log\left(1-\sqrt[6]{x}\right)\right)$$

$$\Big\downarrow \text{1142}$$

$$6\left(\frac{1}{5}\left(\sqrt{5}\int \frac{1}{2\sqrt[3]{x}+\left(1-\sqrt{5}\right)\sqrt[6]{x}+2}d\sqrt[6]{x}-\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{4\sqrt[6]{x}-\sqrt{5}+1}{2\sqrt[3]{x}+\left(1-\sqrt{5}\right)\sqrt[6]{x}+2}d\sqrt[6]{x}\right)+\frac{1}{5}\left(-\sqrt{5}\int \frac{}{2\sqrt[3]{x}+}\right.\right.$$

$$\Big\downarrow \text{1083}$$

$$6\left(\frac{1}{5}\left(-2\sqrt{5}\int \frac{1}{-\sqrt[3]{x}-2\left(5+\sqrt{5}\right)}d\left(4\sqrt[6]{x}-\sqrt{5}+1\right)-\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{4\sqrt[6]{x}-\sqrt{5}+1}{2\sqrt[3]{x}+\left(1-\sqrt{5}\right)\sqrt[6]{x}+2}d\sqrt[6]{x}\right)+\frac{1}{5}\left(2\sqrt{5}\right.\right.$$

$$\Big\downarrow \text{217}$$

$$6\left(\frac{1}{5}\left(\sqrt{\frac{10}{5+\sqrt{5}}}\arctan\left(\frac{4\sqrt[6]{x}-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{1}{4}\left(1+\sqrt{5}\right)\int \frac{4\sqrt[6]{x}-\sqrt{5}+1}{2\sqrt[3]{x}+\left(1-\sqrt{5}\right)\sqrt[6]{x}+2}d\sqrt[6]{x}\right)+\frac{1}{5}\left(-\frac{1}{4}\left(1-\sqrt{5}\right)\right.\right.$$

$$\Big\downarrow \text{1103}$$

$$6\left(\frac{1}{5}\left(\sqrt{\frac{10}{5+\sqrt{5}}}\arctan\left(\frac{4\sqrt[6]{x}-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{1}{4}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\left(1-\sqrt{5}\right)\sqrt[6]{x}+2\right)\right)+\frac{1}{5}\left(-\sqrt{\frac{10}{5-\sqrt{5}}}\text{ar}\right.\right.$$

3.574. $\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx$

input
```
Int[(-x^(-1/3) + Sqrt[x])^(-1),x]
```

output
```
6*(Sqrt[x]/3 + Log[1 - x^(1/6)]/5 + (Sqrt[10/(5 + Sqrt[5])]*ArcTan[(1 - Sq
rt[5] + 4*x^(1/6))/Sqrt[2*(5 + Sqrt[5])]] - ((1 + Sqrt[5])*Log[2 + (1 - Sq
rt[5])*x^(1/6) + 2*x^(1/3)])/4)/5 + (-(Sqrt[10/(5 - Sqrt[5])]*ArcTan[(1 +
Sqrt[5] + 4*x^(1/6))/Sqrt[2*(5 - Sqrt[5])]]) - ((1 - Sqrt[5])*Log[2 + (1 +
 Sqrt[5])*x^(1/6) + 2*x^(1/3)])/4)/5)
```

3.574.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 823
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*
k - 1)*m*(Pi/n)] + s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k
 - 1)*(Pi/n)]*x + s^2*x^2), x]; r^(m + 1)/(a*n*s^m)   Int[1/(r - s*x), x] -
 2*((-r)^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 1)/2}], x]] /; FreeQ[{a, b
}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && NegQ[a/b]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

3.574. $\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx$

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.574.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.62

method	result
meijerg	$-\frac{6(-1)^{\frac{2}{5}}\left(\frac{5\sqrt{x}\,(-1)^{\frac{3}{5}}}{3}+(-1)^{\frac{3}{5}}\left(\ln\left(1-x^{\frac{1}{6}}\right)-\cos\left(\frac{\pi}{5}\right)\ln\left(1-2\cos\left(\frac{2\pi}{5}\right)x^{\frac{1}{6}}+x^{\frac{1}{3}}\right)+2\sin\left(\frac{\pi}{5}\right)\arctan\left(\frac{\sin\left(\frac{2\pi}{5}\right)x^{\frac{1}{6}}}{1-\cos\left(\frac{2\pi}{5}\right)x^{\frac{1}{6}}}\right)+\text{c}\right.\right.}{5}$
derivativedivides	$2\sqrt{x}+\frac{3\left(\sqrt{5}-1\right)\ln\left(2+x^{\frac{1}{6}}+2x^{\frac{1}{3}}+x^{\frac{1}{6}}\sqrt{5}\right)}{10}+\frac{12\left(-\sqrt{5}+1-\frac{\left(\sqrt{5}-1\right)\left(\sqrt{5}+1\right)}{4}\right)\arctan\left(\frac{1+4x^{\frac{1}{6}}+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}-\frac{3\ln\left(2+x\right.}{}$
default	$2\sqrt{x}+\frac{3\left(\sqrt{5}-1\right)\ln\left(2+x^{\frac{1}{6}}+2x^{\frac{1}{3}}+x^{\frac{1}{6}}\sqrt{5}\right)}{10}+\frac{12\left(-\sqrt{5}+1-\frac{\left(\sqrt{5}-1\right)\left(\sqrt{5}+1\right)}{4}\right)\arctan\left(\frac{1+4x^{\frac{1}{6}}+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}-\frac{3\ln\left(2+x\right.}{}$

input
```
int(1/(-1/x^(1/3)+x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-6/5*(-1)^(2/5)*(5/3*x^(1/2)*(-1)^(3/5)+(-1)^(3/5)*(ln(1-x^(1/6))-cos(1/5*
Pi)*ln(1-2*cos(2/5*Pi)*x^(1/6)+x^(1/3))+2*sin(1/5*Pi)*arctan(sin(2/5*Pi)*x
^(1/6)/(1-cos(2/5*Pi)*x^(1/6)))+cos(2/5*Pi)*ln(1+2*cos(1/5*Pi)*x^(1/6)+x^(
1/3))-2*sin(2/5*Pi)*arctan(sin(1/5*Pi)*x^(1/6)/(1+cos(1/5*Pi)*x^(1/6)))))
```

3.574.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(133) = 266$.

Time = 0.98 (sec) , antiderivative size = 638, normalized size of antiderivative = 3.19

$$\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx$$

$$= \frac{1}{10}\left(3\sqrt{5} - \sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2 + \frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right) + \frac{9}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)^2} + 3\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2 + \frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right) - \frac{27}{4}\left(\right.} + 72x^{\frac{1}{6}} + 36\right)$$

$$+ \frac{1}{10}\left(3\sqrt{5} + \sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2 + \frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-\right.} + \frac{9}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)^2 - 3\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2 + \frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right) - \frac{27}{4}\left(\right.} + 72x^{\frac{1}{6}} + 36\right)$$

$$- \frac{3}{10}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)\log\left(-\frac{9}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2 + 36x^{\frac{1}{6}}\right)$$

$$+ \frac{3}{10}\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)\log\left(-\frac{9}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)^2 + 36x^{\frac{1}{6}}\right)$$

$$+ 2\sqrt{x} + \frac{6}{5}\log\left(x^{\frac{1}{6}} - 1\right)$$

input
```
integrate(1/(-1/x^(1/3)+x^(1/2)),x, algorithm="fricas")
```

output
```
1/10*(3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 +
 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5)
- sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqr
t(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) - 3)*log(9/4*(sqrt(2)*sqrt(sqrt(
5) - 5) + sqrt(5) + 1)^2 + 9/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2
 + 3*sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)
*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1
) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqr
t(5) - 5) + 18*sqrt(5) - 90)*(sqrt(5) - 1) + 72*x^(1/6) + 36) + 1/10*(3*sq
rt(5) + sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt
(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5)
- 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(
sqrt(5) - 5) + 18*sqrt(5) - 90) - 3)*log(9/4*(sqrt(2)*sqrt(sqrt(5) - 5) +
sqrt(5) + 1)^2 + 9/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 - 3*sqrt(
-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt
(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(
sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5)
+ 18*sqrt(5) - 90)*(sqrt(5) - 1) + 72*x^(1/6) + 36) - 3/10*(sqrt(2)*sqrt(s
qrt(5) - 5) + sqrt(5) + 1)*log(-9/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) +
 1)^2 + 36*x^(1/6)) + 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)*lo...
```

3.574.6 Sympy [F]

$$\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx=\int \frac{\sqrt[3]{x}}{\left(\sqrt[6]{x}-1\right)\left(\sqrt[6]{x}+x^{\frac{2}{3}}+\sqrt[3]{x}+\sqrt{x}+1\right)}\,dx$$

input
```
integrate(1/(-1/x**(1/3)+x**(1/2)),x)
```

output
```
Integral(x**(1/3)/((x**(1/6) - 1)*(x**(1/6) + x**(2/3) + x**(1/3) + sqrt(x
) + 1)), x)
```

3.574.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(133) = 266$.

Time = 0.27 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.36

$$\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = -\frac{6}{5}(-1)^{\frac{3}{5}}\log\left((-1)^{\frac{1}{5}}+x^{\frac{1}{6}}\right) - \frac{6\sqrt{5}(-1)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}\right)}{5\sqrt{2\sqrt{5}-10}} + \frac{6\sqrt{5}(-1)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}\right)}{5\sqrt{-2\sqrt{5}-10}} + 2\sqrt{x} + \frac{6\log\left(-x^{\frac{1}{6}}\left(\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\right)+2(-1)^{\frac{2}{5}}+2x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}(-1)^{\frac{2}{5}}+(-1)^{\frac{2}{5}}\right)} - \frac{6\log\left(x^{\frac{1}{6}}\left(\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\right)+2(-1)^{\frac{2}{5}}+2x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}(-1)^{\frac{2}{5}}-(-1)^{\frac{2}{5}}\right)}$$

input
```
integrate(1/(-1/x^(1/3)+x^(1/2)),x, algorithm="maxima")
```

output
```
-6/5*(-1)^(3/5)*log((-1)^(1/5) + x^(1/6)) - 6/5*sqrt(5)*(-1)^(3/5)*log((sq
rt(5)*(-1)^(1/5) + (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6
))/(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*
x^(1/6)))/sqrt(2*sqrt(5) - 10) + 6/5*sqrt(5)*(-1)^(3/5)*log((sqrt(5)*(-1)^
(1/5) - (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6))/(sqrt(5
)*(-1)^(1/5) + (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6)))
/sqrt(-2*sqrt(5) - 10) + 2*sqrt(x) + 6/5*log(-x^(1/6)*(sqrt(5)*(-1)^(1/5)
+ (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(5)*(-1)^(2/5) + (-1)^(2/5)
) - 6/5*log(x^(1/6)*(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x
^(1/3))/(sqrt(5)*(-1)^(2/5) - (-1)^(2/5))
```

3.574.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.70

$$\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = \frac{3}{5}\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4\,x^{\frac{1}{6}}-1}{\sqrt{2\sqrt{5}+10}}\right) - \frac{3}{5}\sqrt{2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4\,x^{\frac{1}{6}}+1}{\sqrt{-2\sqrt{5}+10}}\right) + \frac{3}{10}\sqrt{5}\log\left(\frac{1}{2}\,x^{\frac{1}{6}}\left(\sqrt{5}+1\right)+x^{\frac{1}{3}}+1\right) - \frac{3}{10}\sqrt{5}\log\left(-\frac{1}{2}\,x^{\frac{1}{6}}\left(\sqrt{5}-1\right)+x^{\frac{1}{3}}+1\right) + 2\sqrt{x} - \frac{3}{10}\log\left(x^{\frac{2}{3}}+\sqrt{x}+x^{\frac{1}{3}}+x^{\frac{1}{6}}+1\right) + \frac{6}{5}\log\left(\left|x^{\frac{1}{6}}-1\right|\right)$$

input
```
integrate(1/(-1/x^(1/3)+x^(1/2)),x, algorithm="giac")
```

output
```
3/5*sqrt(-2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*x^(1/6) - 1)/sqrt(2*sqrt(5)
 + 10)) - 3/5*sqrt(2*sqrt(5) + 10)*arctan((sqrt(5) + 4*x^(1/6) + 1)/sqrt(-
2*sqrt(5) + 10)) + 3/10*sqrt(5)*log(1/2*x^(1/6)*(sqrt(5) + 1) + x^(1/3) +
1) - 3/10*sqrt(5)*log(-1/2*x^(1/6)*(sqrt(5) - 1) + x^(1/3) + 1) + 2*sqrt(x
) - 3/10*log(x^(2/3) + sqrt(x) + x^(1/3) + x^(1/6) + 1) + 6/5*log(abs(x^(1
/6) - 1))
```

3.574.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.12

$$\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = \frac{6\ln\left(1296\,x^{1/6}-1296\right)}{5} - \ln\left(\left(-750\,x^{1/6}\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-5}}{10}-\frac{3\sqrt{5}}{10}+\frac{3}{10}\right)^3 - 1296\right)\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-5}}{10}-\frac{3\sqrt{5}}{10}+\frac{3}{10}\right)\right) + \ln\left(\left(750\,x^{1/6}\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-5}}{10}+\frac{3\sqrt{5}}{10}-\frac{3}{10}\right)^3 - 1296\right)\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-5}}{10}+\frac{3\sqrt{5}}{10}-\frac{3}{10}\right)\right) - \ln\left(-750\right.$$

input
```
int(1/(x^(1/2) - 1/x^(1/3)),x)
```

output
```
(6*log(1296*x^(1/6) - 1296))/5 - log(- 750*x^(1/6)*((3*2^(1/2)*(- 5^(1/2)
- 5)^(1/2))/10 - (3*5^(1/2))/10 + 3/10)^3 - 1296)*((3*2^(1/2)*(- 5^(1/2) -
 5)^(1/2))/10 - (3*5^(1/2))/10 + 3/10) + log(750*x^(1/6)*((3*2^(1/2)*(- 5^
(1/2) - 5)^(1/2))/10 + (3*5^(1/2))/10 - 3/10)^3 - 1296)*((3*2^(1/2)*(- 5^(
1/2) - 5)^(1/2))/10 + (3*5^(1/2))/10 - 3/10) - log(- 750*x^(1/6)*((3*5^(1/
2))/10 - (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10)^3 - 1296)*((3*5^(1/2))
/10 - (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10) - log(- 750*x^(1/6)*((3*5
^(1/2))/10 + (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10)^3 - 1296)*((3*5^(1
/2))/10 + (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10) + 2*x^(1/2)
```

3.574. $\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx$

3.575 $\int \frac{\sqrt{x}}{x+x^2} dx$

3.575.1 Optimal result

Integrand size = 13, antiderivative size = 8

$$\int \frac{\sqrt{x}}{x+x^2} dx = 2\arctan\left(\sqrt{x}\right)$$

output
```
2*arctan(x^(1/2))
```

3.575.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}}{x+x^2} dx = 2\arctan\left(\sqrt{x}\right)$$

input
```
Integrate[Sqrt[x]/(x + x^2),x]
```

output
```
2*ArcTan[Sqrt[x]]
```

3.575.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {9, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{x^2+x}\,dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{1}{\sqrt{x}(x+1)}dx$$
$$\Big\downarrow \; 73$$
$$2\int \frac{1}{x+1}d\sqrt{x}$$
$$\Big\downarrow \; 216$$
$$2\arctan\left(\sqrt{x}\right)$$

input
```
Int[Sqrt[x]/(x + x^2),x]
```

output
```
2*ArcTan[Sqrt[x]]
```

3.575.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.575.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$2 \arctan\left(\sqrt{x}\right)$	7
default	$2 \arctan\left(\sqrt{x}\right)$	7
meijerg	$2 \arctan\left(\sqrt{x}\right)$	7
trager	$\operatorname{RootOf}\left(\_Z^2+1\right) \ln\left(\frac{2 \operatorname{RootOf}\left(\_Z^2+1\right) \sqrt{x}+x-1}{x+1}\right)$	29

input
```
int(x^(1/2)/(x^2+x),x,method=_RETURNVERBOSE)
```

output
```
2*arctan(x^(1/2))
```

3.575.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{x}}{x+x^2}\,dx = 2 \arctan\left(\sqrt{x}\right)$$

input
```
integrate(x^(1/2)/(x^2+x),x, algorithm="fricas")
```

output
```
2*arctan(sqrt(x))
```

3.575.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{x}}{x+x^2}\,dx = 2\,\mathrm{atan}\left(\sqrt{x}\right)$$

input
```
integrate(x**(1/2)/(x**2+x),x)
```

output
```
2*atan(sqrt(x))
```

3.575.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{x}}{x+x^2}\,dx = 2\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate(x^(1/2)/(x^2+x),x, algorithm="maxima")
```

output
```
2*arctan(sqrt(x))
```

3.575.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{x}}{x+x^2}\,dx = 2\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate(x^(1/2)/(x^2+x),x, algorithm="giac")
```

output
```
2*arctan(sqrt(x))
```

3.575.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{x}}{x+x^2}\,dx = 2\,\mathrm{atan}\left(\sqrt{x}\right)$$

input

```
int(x^(1/2)/(x + x^2),x)
```

output

```
2*atan(x^(1/2))
```

3.576 $\int \frac{x}{4\sqrt{x}+x}\,dx$

3.576.1 Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{x}{4\sqrt{x}+x}\,dx = -8\sqrt{x}+x+32\log\left(4+\sqrt{x}\right)$$

output
```
x+32*ln(4+x^(1/2))-8*x^(1/2)
```

3.576.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x}{4\sqrt{x}+x}\,dx = -8\sqrt{x}+x+32\log\left(4+\sqrt{x}\right)$$

input
```
Integrate[x/(4*Sqrt[x] + x),x]
```

output
```
-8*Sqrt[x] + x + 32*Log[4 + Sqrt[x]]
```

3.576.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {10, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x+4\sqrt{x}}dx$$

$$\Big\downarrow \text{ 10}$$

$$\int \frac{\sqrt{x}}{\sqrt{x}+4}dx$$

$$\Big\downarrow \text{ 798}$$

$$2\int \frac{x}{\sqrt{x}+4}d\sqrt{x}$$

$$\Big\downarrow \text{ 49}$$

$$2\int \left(\sqrt{x}+\frac{16}{\sqrt{x}+4}-4\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{x}{2}-4\sqrt{x}+16\log\left(\sqrt{x}+4\right)\right)$$

input
```
Int[x/(4*Sqrt[x] + x),x]
```

output
```
2*(-4*Sqrt[x] + x/2 + 16*Log[4 + Sqrt[x]])
```

3.576.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.576.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$x+32\ln\left(4+\sqrt{x}\right)-8\sqrt{x}$	16
default	$x+32\ln\left(4+\sqrt{x}\right)-8\sqrt{x}$	16
trager	$x-1-8\sqrt{x}+16\ln\left(8\sqrt{x}+16+x\right)$	20
meijerg	$-\frac{4\sqrt{x}\left(-\frac{3\sqrt{x}}{4}+6\right)}{3}+32\ln\left(1+\frac{\sqrt{x}}{4}\right)$	24

input
```
int(x/(x+4*x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x+32*ln(4+x^(1/2))-8*x^(1/2)
```

3.576.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x}{4\sqrt{x}+x}\,dx = x-8\sqrt{x}+32\log\left(\sqrt{x}+4\right)$$

input
```
integrate(x/(x+4*x^(1/2)),x, algorithm="fricas")
```

output
```
x - 8*sqrt(x) + 32*log(sqrt(x) + 4)
```

3.576.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x}{4\sqrt{x}+x}\,dx = -8\sqrt{x}+x+32\log\left(\sqrt{x}+4\right)$$

input
```
integrate(x/(x+4*x**(1/2)),x)
```

output
```
-8*sqrt(x) + x + 32*log(sqrt(x) + 4)
```

3.576.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x}{4\sqrt{x}+x}\,dx = x-8\sqrt{x}+32\log\left(\sqrt{x}+4\right)$$

input
```
integrate(x/(x+4*x^(1/2)),x, algorithm="maxima")
```

output
```
x - 8*sqrt(x) + 32*log(sqrt(x) + 4)
```

3.576.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x}{4\sqrt{x}+x}\,dx = x-8\sqrt{x}+32\log\left(\sqrt{x}+4\right)$$

input
```
integrate(x/(x+4*x^(1/2)),x, algorithm="giac")
```

output
```
x - 8*sqrt(x) + 32*log(sqrt(x) + 4)
```

3.576.9 Mupad [B] (verification not implemented)

Time = 17.76 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x}{4\sqrt{x}+x}\,dx = x + 32\,\ln\left(\sqrt{x}+4\right) - 8\sqrt{x}$$

input
```
int(x/(x + 4*x^(1/2)),x)
```

output
```
x + 32*log(x^(1/2) + 4) - 8*x^(1/2)
```

3.577 $\int \frac{\sqrt{x}}{\sqrt[3]{x}+x} \, dx$

3.577.1 Optimal result

Integrand size = 15, antiderivative size = 108

$$\int \frac{\sqrt{x}}{\sqrt[3]{x}+x} \, dx = 2\sqrt{x}+\frac{3\arctan\left(1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}}-\frac{3\arctan\left(1+\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}} -\frac{3\log\left(1-\sqrt{2}\sqrt[6]{x}+\sqrt[3]{x}\right)}{2\sqrt{2}}+\frac{3\log\left(1+\sqrt{2}\sqrt[6]{x}+\sqrt[3]{x}\right)}{2\sqrt{2}}$$

output
```
-3/2*arctan(-1+x^(1/6)*2^(1/2))*2^(1/2)-3/2*arctan(1+x^(1/6)*2^(1/2))*2^(1
/2)-3/4*ln(1+x^(1/3)-x^(1/6)*2^(1/2))*2^(1/2)+3/4*ln(1+x^(1/3)+x^(1/6)*2^(
1/2))*2^(1/2)+2*x^(1/2)
```

3.577.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{x}}{\sqrt[3]{x}+x} \, dx = 2\sqrt{x}-\frac{3\arctan\left(\frac{-1+\sqrt[3]{x}}{\sqrt{2}\sqrt[6]{x}}\right)}{\sqrt{2}}+\frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt[6]{x}}{1+\sqrt[3]{x}}\right)}{\sqrt{2}}$$

input
```
Integrate[Sqrt[x]/(x^(1/3) + x),x]
```

output
```
2*Sqrt[x] - (3*ArcTan[(-1 + x^(1/3))/(Sqrt[2]*x^(1/6))])/Sqrt[2] + (3*ArcT
anh[(Sqrt[2]*x^(1/6))/(1 + x^(1/3))])/Sqrt[2]
```

3.577.3 Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.15, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {10, 864, 262, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{x+\sqrt[3]{x}}dx$$

$$\Big\downarrow \text{ 10}$$

$$\int \frac{\sqrt[6]{x}}{x^{2/3}+1}dx$$

$$\Big\downarrow \text{ 864}$$

$$3\int \frac{x^{5/6}}{x^{2/3}+1}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 262}$$

$$3\left(\frac{2\sqrt{x}}{3}-\int \frac{\sqrt[6]{x}}{x^{2/3}+1}d\sqrt[3]{x}\right)$$

$$\Big\downarrow \text{ 266}$$

$$3\left(\frac{2\sqrt{x}}{3}-2\int \frac{x^{2/3}}{x^{4/3}+1}d\sqrt[6]{x}\right)$$

$$\Big\downarrow \text{ 826}$$

$$3\left(\frac{2\sqrt{x}}{3}-2\left(\frac{1}{2}\int \frac{x^{2/3}+1}{x^{4/3}+1}d\sqrt[6]{x}-\frac{1}{2}\int \frac{1-x^{2/3}}{x^{4/3}+1}d\sqrt[6]{x}\right)\right)$$

$$\Big\downarrow \text{ 1476}$$

$$3\left(\frac{2\sqrt{x}}{3}-2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^{2/3}-\sqrt{2}\sqrt[6]{x}+1}d\sqrt[6]{x}+\frac{1}{2}\int \frac{1}{x^{2/3}+\sqrt{2}\sqrt[6]{x}+1}d\sqrt[6]{x}\right)-\frac{1}{2}\int \frac{1-x^{2/3}}{x^{4/3}+1}d\sqrt[6]{x}\right)\right)$$

$$\Big\downarrow \text{ 1082}$$

$$3\left(\frac{2\sqrt{x}}{3}-2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-x^{2/3}-1}d\left(1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-x^{2/3}-1}d\left(\sqrt{2}\sqrt[6]{x}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x^{2/3}}{x^{4/3}+1}d\sqrt[6]{x}\right)\right)$$

$$\Big\downarrow \text{ 217}$$

$$3\left(\frac{2\sqrt{x}}{3}-2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt[6]{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-x^{2/3}}{x^{4/3}+1}d\sqrt[6]{x}\right)\right)$$

$\Big\downarrow$ 1479

$$3\left(\frac{2\sqrt{x}}{3}-2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2\sqrt[6]{x}}{x^{2/3}-\sqrt{2}\sqrt[6]{x}+1}d\sqrt[6]{x}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[6]{x}+1\right)}{x^{2/3}+\sqrt{2}\sqrt[6]{x}+1}d\sqrt[6]{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt[6]{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\right.}{\sqrt{2}}\right.\right.\right.$$

$\Big\downarrow$ 25

$$3\left(\frac{2\sqrt{x}}{3}-2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt[6]{x}}{x^{2/3}-\sqrt{2}\sqrt[6]{x}+1}d\sqrt[6]{x}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[6]{x}+1\right)}{x^{2/3}+\sqrt{2}\sqrt[6]{x}+1}d\sqrt[6]{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt[6]{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt[6]{\right.}}{\sqrt{2}}\right.\right.\right.$$

$\Big\downarrow$ 27

$$3\left(\frac{2\sqrt{x}}{3}-2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt[6]{x}}{x^{2/3}-\sqrt{2}\sqrt[6]{x}+1}d\sqrt[6]{x}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\sqrt{2}\sqrt[6]{x}+1}{x^{2/3}+\sqrt{2}\sqrt[6]{x}+1}d\sqrt[6]{x}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt[6]{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\right.}{\sqrt{}}\right.\right.\right.$$

$\Big\downarrow$ 1103

$$3\left(\frac{2\sqrt{x}}{3}-2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt[6]{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x^{2/3}-\sqrt{2}\sqrt[6]{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x^{2/3}+\sqrt{2}\sqrt[6]{x}\right.}{2\sqrt{2}}\right.\right.\right.$$

input
```
Int[Sqrt[x]/(x^(1/3) + x),x]
```

output
```
3*((2*Sqrt[x])/3 - 2*((-(ArcTan[1 - Sqrt[2]*x^(1/6)]/Sqrt[2]) + ArcTan[1 +
 Sqrt[2]*x^(1/6)]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*x^(1/6) + x^(2/3)]/(2*Sqrt
[2]) - Log[1 + Sqrt[2]*x^(1/6) + x^(2/3)]/(2*Sqrt[2]))/2))
```

3.577. $\int \frac{\sqrt{x}}{\sqrt[3]{x}+x}\,dx$

3.577.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

3.577. $\int \frac{\sqrt{x}}{\sqrt[3]{x}+x}\,dx$

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.577.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.61

method	result	siz
derivativedivides	$2\sqrt{x}-\frac{3\sqrt{2}\left(\ln\left(\frac{1+x^{\frac{1}{3}}-x^{\frac{1}{6}}\sqrt{2}}{1+x^{\frac{1}{3}}+x^{\frac{1}{6}}\sqrt{2}}\right)+2\arctan\left(1+x^{\frac{1}{6}}\sqrt{2}\right)+2\arctan\left(-1+x^{\frac{1}{6}}\sqrt{2}\right)\right)}{4}$	66
default	$2\sqrt{x}-\frac{3\sqrt{2}\left(\ln\left(\frac{1+x^{\frac{1}{3}}-x^{\frac{1}{6}}\sqrt{2}}{1+x^{\frac{1}{3}}+x^{\frac{1}{6}}\sqrt{2}}\right)+2\arctan\left(1+x^{\frac{1}{6}}\sqrt{2}\right)+2\arctan\left(-1+x^{\frac{1}{6}}\sqrt{2}\right)\right)}{4}$	66
meijerg	$2\sqrt{x}-\frac{3\sqrt{x}\left(\frac{\sqrt{2}\,\ln\left(1+x^{\frac{1}{3}}-x^{\frac{1}{6}}\sqrt{2}\right)}{2\sqrt{x}}+\frac{\sqrt{2}\,\arctan\left(\frac{x^{\frac{1}{6}}\sqrt{2}}{2-x^{\frac{1}{6}}\sqrt{2}}\right)}{\sqrt{x}}-\frac{\sqrt{2}\,\ln\left(1+x^{\frac{1}{3}}+x^{\frac{1}{6}}\sqrt{2}\right)}{2\sqrt{x}}+\frac{\sqrt{2}\,\arctan\left(\frac{x^{\frac{1}{6}}\sqrt{2}}{2+x^{\frac{1}{6}}\sqrt{2}}\right)}{\sqrt{x}}\right)}{2}$	11

input
```
int(x^(1/2)/(x^(1/3)+x),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)-3/4*2^(1/2)*(ln((1+x^(1/3)-x^(1/6)*2^(1/2))/(1+x^(1/3)+x^(1/6)*2
^(1/2)))+2*arctan(1+x^(1/6)*2^(1/2))+2*arctan(-1+x^(1/6)*2^(1/2)))
```

3.577.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{x}}{\sqrt[3]{x}+x}\,dx = -\left(\frac{3}{4}i-\frac{3}{4}\right)\sqrt{2}\log\left((i+1)\sqrt{2}+2x^{\frac{1}{6}}\right) + \left(\frac{3}{4}i+\frac{3}{4}\right)\sqrt{2}\log\left(-(i-1)\sqrt{2}+2x^{\frac{1}{6}}\right) - \left(\frac{3}{4}i+\frac{3}{4}\right)\sqrt{2}\log\left((i-1)\sqrt{2}+2x^{\frac{1}{6}}\right) + \left(\frac{3}{4}i-\frac{3}{4}\right)\sqrt{2}\log\left(-(i+1)\sqrt{2}+2x^{\frac{1}{6}}\right) + 2\sqrt{x}$$

input
```
integrate(x^(1/2)/(x^(1/3)+x),x, algorithm="fricas")
```

output
```
-(3/4*I - 3/4)*sqrt(2)*log((I + 1)*sqrt(2) + 2*x^(1/6)) + (3/4*I + 3/4)*sq
rt(2)*log(-(I - 1)*sqrt(2) + 2*x^(1/6)) - (3/4*I + 3/4)*sqrt(2)*log((I - 1
)*sqrt(2) + 2*x^(1/6)) + (3/4*I - 3/4)*sqrt(2)*log(-(I + 1)*sqrt(2) + 2*x^
(1/6)) + 2*sqrt(x)
```

3.577.6 Sympy [F]

$$\int \frac{\sqrt{x}}{\sqrt[3]{x}+x}\,dx = \int \frac{\sqrt{x}}{\sqrt[3]{x}+x}\,dx$$

input
```
integrate(x**(1/2)/(x**(1/3)+x),x)
```

output
```
Integral(sqrt(x)/(x**(1/3) + x), x)
```

3.577.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{x}}{\sqrt[3]{x}+x}\,dx = -\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,x^{\frac{1}{6}}\right)\right) - \frac{3}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,x^{\frac{1}{6}}\right)\right) + \frac{3}{4}\sqrt{2}\log\left(\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right) - \frac{3}{4}\sqrt{2}\log\left(-\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right) + 2\sqrt{x}$$

input
```
integrate(x^(1/2)/(x^(1/3)+x),x, algorithm="maxima")
```

output
```
-3/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*x^(1/6))) - 3/2*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*x^(1/6))) + 3/4*sqrt(2)*log(sqrt(2)*x^(1/6) +
x^(1/3) + 1) - 3/4*sqrt(2)*log(-sqrt(2)*x^(1/6) + x^(1/3) + 1) + 2*sqrt(x)
```

3.577.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{x}}{\sqrt[3]{x}+x}\,dx = -\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,x^{\frac{1}{6}}\right)\right) - \frac{3}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,x^{\frac{1}{6}}\right)\right) + \frac{3}{4}\sqrt{2}\log\left(\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right) - \frac{3}{4}\sqrt{2}\log\left(-\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right) + 2\sqrt{x}$$

input
```
integrate(x^(1/2)/(x^(1/3)+x),x, algorithm="giac")
```

output
```
-3/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*x^(1/6))) - 3/2*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*x^(1/6))) + 3/4*sqrt(2)*log(sqrt(2)*x^(1/6) +
x^(1/3) + 1) - 3/4*sqrt(2)*log(-sqrt(2)*x^(1/6) + x^(1/3) + 1) + 2*sqrt(x)
```

3.577.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{x}}{\sqrt[3]{x}+x}\,dx = 2\sqrt{x} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x^{1/6}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{3}{2}+\frac{3}{2}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x^{1/6}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{3}{2}-\frac{3}{2}\mathrm{i}\right)$$

input
```
int(x^(1/2)/(x + x^(1/3)),x)
```

output
```
2*x^(1/2) - 2^(1/2)*atan(2^(1/2)*x^(1/6)*(1/2 + 1i/2))*(3/2 + 3i/2) - 2^(1
/2)*atan(2^(1/2)*x^(1/6)*(1/2 - 1i/2))*(3/2 - 3i/2)
```

3.577. $\int \frac{\sqrt{x}}{\sqrt[3]{x}+x} \, dx$

3.578 $\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx$

3.578.1 Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx = -12\sqrt[12]{x}+3\sqrt[3]{x}-\frac{12x^{7/12}}{7}+\frac{6x^{5/6}}{5} - 4\sqrt{3}\arctan\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right)+6\log\left(1+\sqrt[12]{x}\right)-2\log\left(1+\sqrt[4]{x}\right)$$

output
```
-12*x^(1/12)+3*x^(1/3)-12/7*x^(7/12)+6/5*x^(5/6)+6*ln(1+x^(1/12))-2*ln(1+x
^(1/4))-4*arctan(1/3*(1-2*x^(1/12))*3^(1/2))*3^(1/2)
```

3.578.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx = -12\sqrt[12]{x}+3\sqrt[3]{x}-\frac{12x^{7/12}}{7}+\frac{6x^{5/6}}{5} - 4\sqrt{3}\arctan\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right)+4\log\left(1+\sqrt[12]{x}\right)-2\log\left(1-\sqrt[12]{x}+\sqrt[6]{x}\right)$$

input
```
Integrate[x^(1/3)/(x^(1/4) + Sqrt[x]),x]
```

output
```
-12*x^(1/12) + 3*x^(1/3) - (12*x^(7/12))/7 + (6*x^(5/6))/5 - 4*Sqrt[3]*Arc
Tan[(1 - 2*x^(1/12))/Sqrt[3]] + 4*Log[1 + x^(1/12)] - 2*Log[1 - x^(1/12) +
 x^(1/6)]
```

3.578.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {10, 864, 60, 60, 60, 60, 70, 16, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{x}}{\sqrt{x}+\sqrt[4]{x}}dx$$

$$\Big\downarrow \text{10}$$

$$\int \frac{\sqrt[12]{x}}{\sqrt[4]{x}+1}dx$$

$$\Big\downarrow \text{864}$$

$$4\int \frac{x^{5/6}}{\sqrt[4]{x}+1}d\sqrt[4]{x}$$

$$\Big\downarrow \text{60}$$

$$4\left(\frac{3x^{5/6}}{10}-\int \frac{x^{7/12}}{\sqrt[4]{x}+1}d\sqrt[4]{x}\right)$$

$$\Big\downarrow \text{60}$$

$$4\left(\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+1}d\sqrt[4]{x}+\frac{3x^{5/6}}{10}-\frac{3x^{7/12}}{7}\right)$$

$$\Big\downarrow \text{60}$$

$$4\left(-\int \frac{\sqrt[12]{x}}{\sqrt[4]{x}+1}d\sqrt[4]{x}+\frac{3x^{5/6}}{10}-\frac{3x^{7/12}}{7}+\frac{3\sqrt[3]{x}}{4}\right)$$

$$\Big\downarrow \text{60}$$

$$4\left(\int \frac{1}{\left(\sqrt[4]{x}+1\right)\sqrt[6]{x}}d\sqrt[4]{x}+\frac{3x^{5/6}}{10}-\frac{3x^{7/12}}{7}+\frac{3\sqrt[3]{x}}{4}-3\sqrt[12]{x}\right)$$

$$\Big\downarrow \text{70}$$

$$4\left(\frac{3}{2}\int \frac{1}{\sqrt[12]{x}+1}d\sqrt[12]{x}+\frac{3}{2}\int \frac{1}{\sqrt{x}-\sqrt[12]{x}+1}d\sqrt[12]{x}+\frac{3x^{5/6}}{10}-\frac{3x^{7/12}}{7}+\frac{3\sqrt[3]{x}}{4}-3\sqrt[12]{x}-\frac{1}{2}\log\left(\sqrt[4]{x}+1\right)\right)$$

$$\Big\downarrow \; 16$$

$$4\left(\frac{3}{2}\int \frac{1}{\sqrt{x}-\sqrt[12]{x}+1}d\sqrt[12]{x}+\frac{3x^{5/6}}{10}-\frac{3x^{7/12}}{7}+\frac{3\sqrt[3]{x}}{4}-3\sqrt[12]{x}+\frac{3}{2}\log\left(\sqrt[12]{x}+1\right)-\frac{1}{2}\log\left(\sqrt[4]{x}+1\right)\right)$$

$$\Big\downarrow \; 1083$$

$$4\left(-3\int \frac{1}{-\sqrt{x}-3}d\left(2\sqrt[12]{x}-1\right)+\frac{3x^{5/6}}{10}-\frac{3x^{7/12}}{7}+\frac{3\sqrt[3]{x}}{4}-3\sqrt[12]{x}+\frac{3}{2}\log\left(\sqrt[12]{x}+1\right)-\frac{1}{2}\log\left(\sqrt[4]{x}+1\right)\right)$$

$$\Big\downarrow \; 217$$

$$4\left(\sqrt{3}\arctan\left(\frac{2\sqrt[12]{x}-1}{\sqrt{3}}\right)+\frac{3x^{5/6}}{10}-\frac{3x^{7/12}}{7}+\frac{3\sqrt[3]{x}}{4}-3\sqrt[12]{x}+\frac{3}{2}\log\left(\sqrt[12]{x}+1\right)-\frac{1}{2}\log\left(\sqrt[4]{x}+1\right)\right)$$

input
```
Int[x^(1/3)/(x^(1/4) + Sqrt[x]),x]
```

output
```
4*(-3*x^(1/12) + (3*x^(1/3))/4 - (3*x^(7/12))/7 + (3*x^(5/6))/10 + Sqrt[3]
*ArcTan[(-1 + 2*x^(1/12))/Sqrt[3]] + (3*Log[1 + x^(1/12)])/2 - Log[1 + x^(
1/4)]/2)
```

3.578.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

3.578. $\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx$

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 70
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Simp[3/(2*b*q)   Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1
/3)], x] + Simp[3/(2*b*q^2)   Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.578.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{6x^{\frac{5}{6}}}{5}-\frac{12x^{\frac{7}{12}}}{7}+3x^{\frac{1}{3}}-12x^{\frac{1}{12}}+4\ln\left(1+x^{\frac{1}{12}}\right)-2\ln\left(1-x^{\frac{1}{12}}+x^{\frac{1}{6}}\right)+4\sqrt{3}\,\arctan\left(\frac{\left(2x\right.}{}\right.$
default	$\frac{6x^{\frac{5}{6}}}{5}-\frac{12x^{\frac{7}{12}}}{7}+3x^{\frac{1}{3}}-12x^{\frac{1}{12}}+4\ln\left(1+x^{\frac{1}{12}}\right)-2\ln\left(1-x^{\frac{1}{12}}+x^{\frac{1}{6}}\right)+4\sqrt{3}\,\arctan\left(\frac{\left(2x\right.}{}\right.$
meijerg	$-\frac{3x^{\frac{1}{12}}\left(-182x^{\frac{3}{4}}+260\sqrt{x}-455x^{\frac{1}{4}}+1820\right)}{455}+4x^{\frac{1}{12}}\left(\frac{\ln\left(1+x^{\frac{1}{12}}\right)}{x^{\frac{1}{12}}}-\frac{\ln\left(1-x^{\frac{1}{12}}+x^{\frac{1}{6}}\right)}{2x^{\frac{1}{12}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x^{\frac{1}{12}}}{2-x^{\frac{1}{12}}}\right)}{x^{\frac{1}{12}}}\right.$

input
```
int(x^(1/3)/(x^(1/4)+x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
6/5*x^(5/6)-12/7*x^(7/12)+3*x^(1/3)-12*x^(1/12)+4*ln(1+x^(1/12))-2*ln(1-x^
(1/12)+x^(1/6))+4*3^(1/2)*arctan(1/3*(2*x^(1/12)-1)*3^(1/2))
```

3.578.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx = 4\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{12}}-\frac{1}{3}\sqrt{3}\right)+\frac{6}{5}x^{\frac{5}{6}}-\frac{12}{7}x^{\frac{7}{12}}+3x^{\frac{1}{3}} -12x^{\frac{1}{12}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)+4\log\left(x^{\frac{1}{12}}+1\right)$$

input
```
integrate(x^(1/3)/(x^(1/4)+x^(1/2)),x, algorithm="fricas")
```

output
```
4*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/12) - 1/3*sqrt(3)) + 6/5*x^(5/6) - 12/7*
x^(7/12) + 3*x^(1/3) - 12*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) + 4*log
(x^(1/12) + 1)
```

3.578.6 Sympy [F]

$$\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx = \int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx$$

input
```
integrate(x**(1/3)/(x**(1/4)+x**(1/2)),x)
```

output
```
Integral(x**(1/3)/(x**(1/4) + sqrt(x)), x)
```

3.578.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx = 4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}}-1\right)\right)+\frac{6}{5}x^{\frac{5}{6}}-\frac{12}{7}x^{\frac{7}{12}}+3x^{\frac{1}{3}} - 12x^{\frac{1}{12}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)+4\log\left(x^{\frac{1}{12}}+1\right)$$

input
```
integrate(x^(1/3)/(x^(1/4)+x^(1/2)),x, algorithm="maxima")
```

output
```
4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 6/5*x^(5/6) - 12/7*x^(7/1
2) + 3*x^(1/3) - 12*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) + 4*log(x^(1/
12) + 1)
```

3.578.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx = 4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}}-1\right)\right)+\frac{6}{5}x^{\frac{5}{6}}-\frac{12}{7}x^{\frac{7}{12}}+3x^{\frac{1}{3}} - 12x^{\frac{1}{12}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)+4\log\left(x^{\frac{1}{12}}+1\right)$$

input
```
integrate(x^(1/3)/(x^(1/4)+x^(1/2)),x, algorithm="giac")
```

output
```
4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 6/5*x^(5/6) - 12/7*x^(7/1
2) + 3*x^(1/3) - 12*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) + 4*log(x^(1/
12) + 1)
```

3.578.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx$$
$$= 4\ln\left(144\,x^{1/12}+144\right)-\ln\left(18-36\,x^{1/12}+\sqrt{3}\,18\mathrm{i}\right)\left(2+\sqrt{3}\,2\mathrm{i}\right)$$
$$+\ln\left(36\,x^{1/12}-18+\sqrt{3}\,18\mathrm{i}\right)\left(-2+\sqrt{3}\,2\mathrm{i}\right)+3\,x^{1/3}+\frac{6\,x^{5/6}}{5}-12\,x^{1/12}-\frac{12\,x^{7/12}}{7}$$

input
```
int(x^(1/3)/(x^(1/2) + x^(1/4)),x)
```

output
```
4*log(144*x^(1/12) + 144) - log(3^(1/2)*18i - 36*x^(1/12) + 18)*(3^(1/2)*2
i + 2) + log(3^(1/2)*18i + 36*x^(1/12) - 18)*(3^(1/2)*2i - 2) + 3*x^(1/3)
+ (6*x^(5/6))/5 - 12*x^(1/12) - (12*x^(7/12))/7
```

3.579 $\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}} \, dx$

3.579.1 Optimal result

Integrand size = 19, antiderivative size = 119

$$\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}} \, dx = -12\sqrt[12]{x}+6\sqrt[6]{x}-4\sqrt[4]{x}+3\sqrt[3]{x}-\frac{12x^{5/12}}{5}+2\sqrt{x}-\frac{12x^{7/12}}{7}+\frac{3x^{2/3}}{2} -\frac{4x^{3/4}}{3}+\frac{6x^{5/6}}{5}-\frac{12x^{11/12}}{11}+x-\frac{12x^{13/12}}{13}+\frac{6x^{7/6}}{7}+12\log\left(1+\sqrt[12]{x}\right)$$

output
```
-12*x^(1/12)+6*x^(1/6)-4*x^(1/4)+3*x^(1/3)-12/5*x^(5/12)-12/7*x^(7/12)+3/2
*x^(2/3)-4/3*x^(3/4)+6/5*x^(5/6)-12/11*x^(11/12)+x-12/13*x^(13/12)+6/7*x^(
7/6)+12*ln(1+x^(1/12))+2*x^(1/2)
```

3.579.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}} \, dx = \frac{-360360\sqrt[12]{x}+180180\sqrt[6]{x}-120120\sqrt[4]{x}+90090\sqrt[3]{x}-72072x^{5/12}+60060\sqrt{x}-51480x^{7/12}+45045x^{2/3}}{30030} + 12\log\left(1+\sqrt[12]{x}\right)$$

input
```
Integrate[Sqrt[x]/(x^(1/4) + x^(1/3)),x]
```

output
```
(-360360*x^(1/12) + 180180*x^(1/6) - 120120*x^(1/4) + 90090*x^(1/3) - 7207
2*x^(5/12) + 60060*Sqrt[x] - 51480*x^(7/12) + 45045*x^(2/3) - 40040*x^(3/4
) + 36036*x^(5/6) - 32760*x^(11/12) + 30030*x - 27720*x^(13/12) + 25740*x^
(7/6))/30030 + 12*Log[1 + x^(1/12)]
```

3.579.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {10, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt[3]{x}+\sqrt[4]{x}}dx$$

$$\Big\downarrow \text{10}$$

$$\int \frac{\sqrt[4]{x}}{\sqrt[12]{x}+1}dx$$

$$\Big\downarrow \text{798}$$

$$12\int \frac{x^{7/6}}{\sqrt[12]{x}+1}d\sqrt[12]{x}$$

$$\Big\downarrow \text{49}$$

$$12\int \left(x^{13/12}-x+x^{11/12}-x^{5/6}+x^{3/4}-x^{2/3}+x^{7/12}-\sqrt{x}+x^{5/12}-\sqrt[3]{x}+\sqrt[4]{x}-\sqrt[6]{x}+\sqrt[12]{x}+\frac{1}{\sqrt[12]{x}+1}-1\right)$$

$$\Big\downarrow \text{2009}$$

$$12\left(\frac{x^{7/6}}{14}-\frac{x^{13/12}}{13}-\frac{x^{11/12}}{11}+\frac{x^{5/6}}{10}-\frac{x^{3/4}}{9}+\frac{x^{2/3}}{8}-\frac{x^{7/12}}{7}-\frac{x^{5/12}}{5}+\frac{x}{12}+\frac{\sqrt{x}}{6}+\frac{\sqrt[3]{x}}{4}-\frac{\sqrt[4]{x}}{3}+\frac{\sqrt[6]{x}}{2}-\sqrt[12]{x}+\log\right.$$

input
```
Int[Sqrt[x]/(x^(1/4) + x^(1/3)),x]
```

output
```
12*(-x^(1/12) + x^(1/6)/2 - x^(1/4)/3 + x^(1/3)/4 - x^(5/12)/5 + Sqrt[x]/6
 - x^(7/12)/7 + x^(2/3)/8 - x^(3/4)/9 + x^(5/6)/10 - x^(11/12)/11 + x/12 -
 x^(13/12)/13 + x^(7/6)/14 + Log[1 + x^(1/12)])
```

3.579. $\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}}dx$

3.579.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.579.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.64

method	result
derivativedivides	$-12x^{\frac{1}{12}}+6x^{\frac{1}{6}}-4x^{\frac{1}{4}}+3x^{\frac{1}{3}}-\frac{12x^{\frac{5}{12}}}{5}-\frac{12x^{\frac{7}{12}}}{7}+\frac{3x^{\frac{2}{3}}}{2}-\frac{4x^{\frac{3}{4}}}{3}+\frac{6x^{\frac{5}{6}}}{5}-\frac{12x^{\frac{11}{12}}}{11}+x-\frac{12x^{\frac{13}{12}}}{13}+$
default	$-12x^{\frac{1}{12}}+6x^{\frac{1}{6}}-4x^{\frac{1}{4}}+3x^{\frac{1}{3}}-\frac{12x^{\frac{5}{12}}}{5}-\frac{12x^{\frac{7}{12}}}{7}+\frac{3x^{\frac{2}{3}}}{2}-\frac{4x^{\frac{3}{4}}}{3}+\frac{6x^{\frac{5}{6}}}{5}-\frac{12x^{\frac{11}{12}}}{11}+x-\frac{12x^{\frac{13}{12}}}{13}+$
meijerg	$-\frac{x^{\frac{1}{12}}\left(-25740x^{\frac{13}{12}}+27720x-30030x^{\frac{11}{12}}+32760x^{\frac{5}{6}}-36036x^{\frac{3}{4}}+40040x^{\frac{2}{3}}-45045x^{\frac{7}{12}}+51480\sqrt{x}-60060x^{\frac{5}{12}}+72072x^{\frac{1}{3}}\right.}{30030}$

input
```
int(x^(1/2)/(x^(1/4)+x^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
-12*x^(1/12)+6*x^(1/6)-4*x^(1/4)+3*x^(1/3)-12/5*x^(5/12)-12/7*x^(7/12)+3/2
*x^(2/3)-4/3*x^(3/4)+6/5*x^(5/6)-12/11*x^(11/12)+x-12/13*x^(13/12)+6/7*x^(
7/6)+12*ln(1+x^(1/12))+2*x^(1/2)
```

3.579.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx = \frac{6}{7}\,(x+7)x^{\frac{1}{6}} - \frac{12}{13}\,(x+13)x^{\frac{1}{12}} + x - \frac{12}{11}\,x^{\frac{11}{12}} + \frac{6}{5}\,x^{\frac{5}{6}} - \frac{4}{3}\,x^{\frac{3}{4}} + \frac{3}{2}\,x^{\frac{2}{3}} - \frac{12}{7}\,x^{\frac{7}{12}} + 2\sqrt{x} - \frac{12}{5}\,x^{\frac{5}{12}} + 3\,x^{\frac{1}{3}} - 4\,x^{\frac{1}{4}} + 12\,\log\left(x^{\frac{1}{12}}+1\right)$$

input
```
integrate(x^(1/2)/(x^(1/4)+x^(1/3)),x, algorithm="fricas")
```

output
```
6/7*(x + 7)*x^(1/6) - 12/13*(x + 13)*x^(1/12) + x - 12/11*x^(11/12) + 6/5*
x^(5/6) - 4/3*x^(3/4) + 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(
5/12) + 3*x^(1/3) - 4*x^(1/4) + 12*log(x^(1/12) + 1)
```

3.579.6 Sympy [F]

$$\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx = \int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx$$

input
```
integrate(x**(1/2)/(x**(1/4)+x**(1/3)),x)
```

output
```
Integral(sqrt(x)/(x**(1/4) + x**(1/3)), x)
```

3.579.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx = \frac{6}{7}\,x^{\frac{7}{6}} - \frac{12}{13}\,x^{\frac{13}{12}} + x - \frac{12}{11}\,x^{\frac{11}{12}} + \frac{6}{5}\,x^{\frac{5}{6}} - \frac{4}{3}\,x^{\frac{3}{4}} + \frac{3}{2}\,x^{\frac{2}{3}} - \frac{12}{7}\,x^{\frac{7}{12}} + 2\sqrt{x} - \frac{12}{5}\,x^{\frac{5}{12}} + 3\,x^{\frac{1}{3}} - 4\,x^{\frac{1}{4}} + 6\,x^{\frac{1}{6}} - 12\,x^{\frac{1}{12}} + 12\,\log\left(x^{\frac{1}{12}}+1\right)$$

input
```
integrate(x^(1/2)/(x^(1/4)+x^(1/3)),x, algorithm="maxima")
```

output
```
6/7*x^(7/6) - 12/13*x^(13/12) + x - 12/11*x^(11/12) + 6/5*x^(5/6) - 4/3*x^
(3/4) + 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3
) - 4*x^(1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

3.579.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx = \frac{6}{7}\,x^{\frac{7}{6}} - \frac{12}{13}\,x^{\frac{13}{12}} + x - \frac{12}{11}\,x^{\frac{11}{12}} + \frac{6}{5}\,x^{\frac{5}{6}} - \frac{4}{3}\,x^{\frac{3}{4}} + \frac{3}{2}\,x^{\frac{2}{3}} - \frac{12}{7}\,x^{\frac{7}{12}} + 2\sqrt{x} - \frac{12}{5}\,x^{\frac{5}{12}} + 3\,x^{\frac{1}{3}} - 4\,x^{\frac{1}{4}} + 6\,x^{\frac{1}{6}} - 12\,x^{\frac{1}{12}} + 12\,\log\left(x^{\frac{1}{12}} + 1\right)$$

input
```
integrate(x^(1/2)/(x^(1/4)+x^(1/3)),x, algorithm="giac")
```

output
```
6/7*x^(7/6) - 12/13*x^(13/12) + x - 12/11*x^(11/12) + 6/5*x^(5/6) - 4/3*x^
(3/4) + 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3
) - 4*x^(1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

3.579.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx = x + 12\,\ln\left(x^{1/12} + 1\right) + 2\sqrt{x} + 3\,x^{1/3} - 4\,x^{1/4} + \frac{3\,x^{2/3}}{2} + 6\,x^{1/6} - \frac{4\,x^{3/4}}{3} + \frac{6\,x^{5/6}}{5} - 12\,x^{1/12} + \frac{6\,x^{7/6}}{7} - \frac{12\,x^{5/12}}{5} - \frac{12\,x^{7/12}}{7} - \frac{12\,x^{11/12}}{11} - \frac{12\,x^{13/12}}{13}$$

input
```
int(x^(1/2)/(x^(1/3) + x^(1/4)),x)
```

output
```
x + 12*log(x^(1/12) + 1) + 2*x^(1/2) + 3*x^(1/3) - 4*x^(1/4) + (3*x^(2/3))
/2 + 6*x^(1/6) - (4*x^(3/4))/3 + (6*x^(5/6))/5 - 12*x^(1/12) + (6*x^(7/6))
/7 - (12*x^(5/12))/5 - (12*x^(7/12))/7 - (12*x^(11/12))/11 - (12*x^(13/12)
)/13
```

3.580 $\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx$

3.580.1 Optimal result

Integrand size = 21, antiderivative size = 201

$$\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = 6\sqrt[6]{x}+x-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\arctan\left(\frac{1-\sqrt{5}+4\sqrt[6]{x}}{\sqrt{2\left(5+\sqrt{5}\right)}}\right) - \frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\arctan\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{5}+4\sqrt[6]{x}\right)\right) + \frac{6}{5}\log\left(1-\sqrt[6]{x}\right) - \frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2+\sqrt[6]{x}-\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right) - \frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2+\sqrt[6]{x}+\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)$$

output
```
6*x^(1/6)+x+6/5*ln(1-x^(1/6))-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*5^(1/2))
*(-5^(1/2)+1)-3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))*(5^(1/2)+1)-3/5
*arctan(1/20*(1+4*x^(1/6)+5^(1/2))*(50+10*5^(1/2))^(1/2))*(10-2*5^(1/2))^(
1/2)-3/5*arctan((1+4*x^(1/6)-5^(1/2))/(10+2*5^(1/2))^(1/2))*(10+2*5^(1/2))
^(1/2)
```

3.580.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = 6\sqrt[6]{x}+x+\frac{6}{5}\log\left(-1+\sqrt[6]{x}\right)-\frac{6}{5}\text{RootSum}\left[1+\#1+\#1^2+\#1^3+\#1^4\&,\frac{4\log\left(\sqrt[6]{x}-\#1\right)+3\log\left(\sqrt[6]{x}-\#1\right)\#1+2\log\left(\sqrt[6]{x}-\#1\right)\#1^2+\log\left(\sqrt[6]{x}-\#1\right)\#1^3}{1+2\#1+3\#1^2+4\#1^3}\&\right]$$

input
```
Integrate[Sqrt[x]/(-x^(-1/3) + Sqrt[x]),x]
```

output
```
6*x^(1/6) + x + (6*Log[-1 + x^(1/6)])/5 - (6*RootSum[1 + #1 + #1^2 + #1^3
+ #1^4 & , (4*Log[x^(1/6) - #1] + 3*Log[x^(1/6) - #1]*#1 + 2*Log[x^(1/6) -
 #1]*#1^2 + Log[x^(1/6) - #1]*#1^3)/(1 + 2*#1 + 3*#1^2 + 4*#1^3) & ])/5
```

3.580.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {10, 25, 864, 831, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{x}-\frac{1}{\sqrt[3]{x}}}dx$$

$$\Big\downarrow \text{10}$$

$$\int -\frac{x^{5/6}}{1-x^{5/6}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{x^{5/6}}{1-x^{5/6}}dx$$

$$\Big\downarrow \text{864}$$

$$-6\int \frac{x^{5/3}}{1-x^{5/6}}d\sqrt[6]{x}$$

$$\Big\downarrow \text{831}$$

$$-6\int\left(-x^{5/6}+\frac{1}{1-x^{5/6}}-1\right)d\sqrt[6]{x}$$

$$\Big\downarrow \text{2009}$$

$$-6\left(\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\arctan\left(\frac{4\sqrt[6]{x}-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)+\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\arctan\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(4\sqrt[6]{x}+\sqrt{5}+1\right)\right)\right.$$

input
```
Int[Sqrt[x]/(-x^(-1/3) + Sqrt[x]),x]
```

output
```
-6*(-x^(1/6) - x/6 + (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(1 - Sqrt[5] + 4*x^(1/6
))/Sqrt[2*(5 + Sqrt[5])]])/5 + (Sqrt[(5 - Sqrt[5])/2]*ArcTan[(Sqrt[(5 + Sq
rt[5])/10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 - Log[1 - x^(1/6)]/5 + ((1 - S
qrt[5])*Log[1 + ((1 - Sqrt[5])*x^(1/6))/2 + x^(1/3)])/20 + ((1 + Sqrt[5])*
Log[1 + ((1 + Sqrt[5])*x^(1/6))/2 + x^(1/3)])/20)
```

3.580.3.1 Defintions of rubi rules used

rule 10
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x
_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a + b*x^(s - r))^p, x],
 x] /; FreeQ[{a, b, e, m, r, s}, x] && IntegerQ[p] && (IntegerQ[p*r] || GtQ
[e, 0]) && PosQ[s - r]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 831
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.580. $\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx$

3.580.4 Maple [A] (warning: unable to verify)

Time = 1.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.66

method	result
meijerg	$\frac{6(-1)^{\frac{4}{5}}\left(-\frac{5x^{\frac{1}{6}}(-1)^{\frac{1}{5}}\left(11x^{\frac{5}{6}}+66\right)}{66}-(-1)^{\frac{1}{5}}\left(\ln\left(1-x^{\frac{1}{6}}\right)+\cos\left(\frac{2\pi}{5}\right)\ln\left(1-2\cos\left(\frac{2\pi}{5}\right)x^{\frac{1}{6}}+x^{\frac{1}{3}}\right)-2\sin\left(\frac{2\pi}{5}\right)\arctan\left(\frac{\sin\left(\frac{2}{}\right.}{1-\cos\left(\right.}\right.\right.\right.}{5}$
derivativedivides	$x+6x^{\frac{1}{6}}+\frac{6\ln\left(x^{\frac{1}{6}}-1\right)}{5}-\frac{3\ln\left(2+x^{\frac{1}{6}}+2x^{\frac{1}{3}}-x^{\frac{1}{6}}\sqrt{5}\right)\left(-\sqrt{5}+1\right)}{10}-\frac{12\left(4-\frac{\left(-\sqrt{5}+1\right)^{2}}{4}\right)\arctan\left(\frac{1+4x^{\frac{1}{6}}-\sqrt{5}}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}-$
default	$x+6x^{\frac{1}{6}}+\frac{6\ln\left(x^{\frac{1}{6}}-1\right)}{5}-\frac{3\ln\left(2+x^{\frac{1}{6}}+2x^{\frac{1}{3}}-x^{\frac{1}{6}}\sqrt{5}\right)\left(-\sqrt{5}+1\right)}{10}-\frac{12\left(4-\frac{\left(-\sqrt{5}+1\right)^{2}}{4}\right)\arctan\left(\frac{1+4x^{\frac{1}{6}}-\sqrt{5}}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}-$

input
```
int(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
6/5*(-1)^(4/5)*(-5/66*x^(1/6)*(-1)^(1/5)*(11*x^(5/6)+66)-(-1)^(1/5)*(ln(1-
x^(1/6))+cos(2/5*Pi)*ln(1-2*cos(2/5*Pi)*x^(1/6)+x^(1/3))-2*sin(2/5*Pi)*arc
tan(sin(2/5*Pi)*x^(1/6)/(1-cos(2/5*Pi)*x^(1/6)))-cos(1/5*Pi)*ln(1+2*cos(1/
5*Pi)*x^(1/6)+x^(1/3))-2*sin(1/5*Pi)*arctan(sin(1/5*Pi)*x^(1/6)/(1+cos(1/5
*Pi)*x^(1/6)))))
```

3.580.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. 2(134) = 268.

Time = 0.94 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.72

$$\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = -\frac{3}{10}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)\log\left(\frac{3}{2}\sqrt{2}\sqrt{\sqrt{5}-5}+\frac{3}{2}\sqrt{5}+6x^{\frac{1}{6}}+\frac{3}{2}\right)$$
$$+\frac{3}{10}\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)\log\left(-\frac{3}{2}\sqrt{2}\sqrt{\sqrt{5}-5}+\frac{3}{2}\sqrt{5}+6x^{\frac{1}{6}}+\frac{3}{2}\right)$$
$$+\frac{1}{10}\left(3\sqrt{5}-\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^{2}+\frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-\right.}\right.$$
$$+\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^{2}+\frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)-\frac{27}{4}\left(\sqrt{\right.}}$$
$$\left.+12x^{\frac{1}{6}}+3\right)$$
$$+\frac{1}{10}\left(3\sqrt{5}+\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^{2}+\frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-\right.}\right.$$
$$-\sqrt{-\frac{27}{4}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^{2}+\frac{9}{2}\left(\sqrt{2}\sqrt{\sqrt{5}-5}+\sqrt{5}-3\right)\left(\sqrt{2}\sqrt{\sqrt{5}-5}-\sqrt{5}-1\right)-\frac{27}{4}\left(\sqrt{\right.}}$$
$$\left.+12x^{\frac{1}{6}}+3\right)+x+6x^{\frac{1}{6}}+\frac{6}{5}\log\left(x^{\frac{1}{6}}-1\right)$$

input
```
integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x, algorithm="fricas")
```

3.580. $\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx$

output
```
-3/10*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)*log(3/2*sqrt(2)*sqrt(sqrt(
5) - 5) + 3/2*sqrt(5) + 6*x^(1/6) + 3/2) + 3/10*(sqrt(2)*sqrt(sqrt(5) - 5)
 - sqrt(5) - 1)*log(-3/2*sqrt(2)*sqrt(sqrt(5) - 5) + 3/2*sqrt(5) + 6*x^(1/
6) + 3/2) + 1/10*(3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt
(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(s
qrt(5) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1
)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) - 3)*log(-3*sqrt(5)
+ sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sq
rt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) -
 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5
) - 5) + 18*sqrt(5) - 90) + 12*x^(1/6) + 3) + 1/10*(3*sqrt(5) + sqrt(-27/4
*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) -
 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sqrt(
2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*
sqrt(5) - 90) - 3)*log(-3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5)
+ sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*
sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(
5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) + 12*x^(1/6) +
 3) + x + 6*x^(1/6) + 6/5*log(x^(1/6) - 1)
```

3.580.6 Sympy [F]

$$\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}} + \sqrt{x}}\, dx = \int \frac{x^{\frac{5}{6}}}{\left(\sqrt[6]{x} - 1\right)\left(\sqrt[6]{x} + x^{\frac{2}{3}} + \sqrt[3]{x} + \sqrt{x} + 1\right)}\, dx$$

input
```
integrate(x**(1/2)/(-1/x**(1/3)+x**(1/2)),x)
```

output
```
Integral(x**(5/6)/((x**(1/6) - 1)*(x**(1/6) + x**(2/3) + x**(1/3) + sqrt(x
) + 1)), x)
```

3.580.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(134) = 268.

Time = 0.28 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = -\frac{3\sqrt{5}(-1)^{\frac{1}{5}}\left(\sqrt{5}-1\right)\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}\right)}{5\sqrt{2\sqrt{5}-10}} - \frac{3\sqrt{5}(-1)^{\frac{1}{5}}\left(\sqrt{5}+1\right)\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}\right)}{5\sqrt{-2\sqrt{5}-10}} - \frac{6}{5}(-1)^{\frac{1}{5}}\log\left((-1)^{\frac{1}{5}}+x^{\frac{1}{6}}\right)+x - \frac{3\left(\sqrt{5}+3\right)\log\left(-x^{\frac{1}{6}}\left(\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\right)+2(-1)^{\frac{2}{5}}+2x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}(-1)^{\frac{4}{5}}+(-1)^{\frac{4}{5}}\right)} - \frac{3\left(\sqrt{5}-3\right)\log\left(x^{\frac{1}{6}}\left(\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\right)+2(-1)^{\frac{2}{5}}+2x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}(-1)^{\frac{4}{5}}-(-1)^{\frac{4}{5}}\right)} + 6x^{\frac{1}{6}}$$

input
```
integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x, algorithm="maxima")
```

output
```
-3/5*sqrt(5)*(-1)^(1/5)*(sqrt(5) - 1)*log((sqrt(5)*(-1)^(1/5) + (-1)^(1/5)
*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6))/(sqrt(5)*(-1)^(1/5) - (-1)
^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6)))/sqrt(2*sqrt(5) - 10
) - 3/5*sqrt(5)*(-1)^(1/5)*(sqrt(5) + 1)*log((sqrt(5)*(-1)^(1/5) - (-1)^(1
/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6))/(sqrt(5)*(-1)^(1/5) +
(-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6)))/sqrt(-2*sqrt(5
) - 10) - 6/5*(-1)^(1/5)*log((-1)^(1/5) + x^(1/6)) + x - 3/5*(sqrt(5) + 3)
*log(-x^(1/6)*(sqrt(5)*(-1)^(1/5) + (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3)
)/(sqrt(5)*(-1)^(4/5) + (-1)^(4/5)) - 3/5*(sqrt(5) - 3)*log(x^(1/6)*(sqrt(
5)*(-1)^(1/5) - (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(5)*(-1)^(4/5
) - (-1)^(4/5)) + 6*x^(1/6)
```

3.580.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = -\frac{3}{5}\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4x^{\frac{1}{6}}-1}{\sqrt{2\sqrt{5}+10}}\right) - \frac{3}{5}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4x^{\frac{1}{6}}+1}{\sqrt{-2\sqrt{5}+10}}\right) - \frac{3}{10}\sqrt{5}\log\left(\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}+1\right)+x^{\frac{1}{3}}+1\right) + \frac{3}{10}\sqrt{5}\log\left(-\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}-1\right)+x^{\frac{1}{3}}+1\right)+x+6x^{\frac{1}{6}} - \frac{3}{10}\log\left(x^{\frac{2}{3}}+\sqrt{x}+x^{\frac{1}{3}}+x^{\frac{1}{6}}+1\right)+\frac{6}{5}\log\left(\left|x^{\frac{1}{6}}-1\right|\right)$$

input
```
integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x, algorithm="giac")
```

output
```
-3/5*sqrt(2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*x^(1/6) - 1)/sqrt(2*sqrt(5)
 + 10)) - 3/5*sqrt(-2*sqrt(5) + 10)*arctan((sqrt(5) + 4*x^(1/6) + 1)/sqrt(
-2*sqrt(5) + 10)) - 3/10*sqrt(5)*log(1/2*x^(1/6)*(sqrt(5) + 1) + x^(1/3) +
 1) + 3/10*sqrt(5)*log(-1/2*x^(1/6)*(sqrt(5) - 1) + x^(1/3) + 1) + x + 6*x
^(1/6) - 3/10*log(x^(2/3) + sqrt(x) + x^(1/3) + x^(1/6) + 1) + 6/5*log(abs
(x^(1/6) - 1))
```

3.580.9 Mupad [B] (verification not implemented)

Time = 17.04 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = x+\frac{6\ln\left(1296x^{1/6}-1296\right)}{5} - \ln\left(270\sqrt{2}\sqrt{-\sqrt{5}-5}-270\sqrt{5}+1080x^{1/6}+270\right)\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-5}}{10}-\frac{3\sqrt{5}}{10}+\frac{3}{10}\right) + \ln\left(270\sqrt{2}\sqrt{-\sqrt{5}-5}+270\sqrt{5}-1080x^{1/6}-270\right)\left(\frac{3\sqrt{2}\sqrt{-\sqrt{5}-5}}{10}+\frac{3\sqrt{5}}{10}-\frac{3}{10}\right)+6x^{1/6}-\ln\Big(270$$

input
```
int(x^(1/2)/(x^(1/2) - 1/x^(1/3)),x)
```

output
```
x + (6*log(1296*x^(1/6) - 1296))/5 - log(270*2^(1/2)*(- 5^(1/2) - 5)^(1/2)
 - 270*5^(1/2) + 1080*x^(1/6) + 270)*((3*2^(1/2)*(- 5^(1/2) - 5)^(1/2))/10
 - (3*5^(1/2))/10 + 3/10) + log(270*2^(1/2)*(- 5^(1/2) - 5)^(1/2) + 270*5^
(1/2) - 1080*x^(1/6) - 270)*((3*2^(1/2)*(- 5^(1/2) - 5)^(1/2))/10 + (3*5^(
1/2))/10 - 3/10) + 6*x^(1/6) - log(270*5^(1/2) + 1080*x^(1/6) - 270*2^(1/2
)*(5^(1/2) - 5)^(1/2) + 270)*((3*5^(1/2))/10 - (3*2^(1/2)*(5^(1/2) - 5)^(1
/2))/10 + 3/10) - log(270*5^(1/2) + 1080*x^(1/6) + 270*2^(1/2)*(5^(1/2) -
5)^(1/2) + 270)*((3*5^(1/2))/10 + (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/1
0)
```

3.580. $\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx$

3.581 $\int \frac{\sqrt{b-\frac{a}{x}}x^m}{\sqrt{a-bx}}\,dx$

3.581.1 Optimal result

Integrand size = 26, antiderivative size = 36

$$\int \frac{\sqrt{b-\frac{a}{x}}x^m}{\sqrt{a-bx}}\,dx=\frac{2\sqrt{b-\frac{a}{x}}x^{1+m}}{(1+2m)\sqrt{a-bx}}$$

output
```
2*x^(1+m)*(b-a/x)^(1/2)/(1+2*m)/(-b*x+a)^(1/2)
```

3.581.2 Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{b-\frac{a}{x}}x^m}{\sqrt{a-bx}}\,dx=\frac{\sqrt{b-\frac{a}{x}}x^{1+m}}{\left(\frac{1}{2}+m\right)\sqrt{a-bx}}$$

input
```
Integrate[(Sqrt[b - a/x]*x^m)/Sqrt[a - b*x],x]
```

output
```
(Sqrt[b - a/x]*x^(1 + m))/((1/2 + m)*Sqrt[a - b*x])
```

3.581.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1017, 37, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}dx$$

$$\Big\downarrow \text{1017}$$

$$\frac{\sqrt{x}\sqrt{b-\frac{a}{x}}\int \frac{x^{m-\frac{1}{2}}\sqrt{bx-a}}{\sqrt{a-bx}}dx}{\sqrt{bx-a}}$$

$$\Big\downarrow \text{37}$$

$$\frac{\sqrt{x}\sqrt{b-\frac{a}{x}}\int x^{m-\frac{1}{2}}dx}{\sqrt{a-bx}}$$

$$\Big\downarrow \text{15}$$

$$\frac{2x^{m+1}\sqrt{b-\frac{a}{x}}}{(2m+1)\sqrt{a-bx}}$$

input
```
Int[(Sqrt[b - a/x]*x^m)/Sqrt[a - b*x],x]
```

output
```
(2*Sqrt[b - a/x]*x^(1 + m))/((1 + 2*m)*Sqrt[a - b*x])
```

3.581.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 37
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> S
imp[(a + b*x)^m/(c + d*x)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a + b*x, c + d*x]
```

rule 1017
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_), x_Symbol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n
)^FracPart[q])   Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

3.581.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{2x^{1+m}\sqrt{-\frac{-bx+a}{x}}}{(1+2m)\sqrt{-bx+a}}$	36

input
```
int(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1+m)/(1+2*m)/(-b*x+a)^(1/2)*(-(-b*x+a)/x)^(1/2)
```

3.581.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{b-\frac{a}{x}}x^m}{\sqrt{a-bx}}\,dx = \frac{2\sqrt{-bx+a}xx^m\sqrt{\frac{bx-a}{x}}}{2\,am-(2\,bm+b)x+a}$$

input
```
integrate(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(-b*x + a)*x*x^m*sqrt((b*x - a)/x)/(2*a*m - (2*b*m + b)*x + a)
```

3.581.6 Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x}}\,x^m}{\sqrt{a-bx}}\,dx = \int \frac{x^m\sqrt{-\frac{a}{x}+b}}{\sqrt{a-bx}}\,dx$$

input
```
integrate(x**m*(b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
```

output
```
Integral(x**m*sqrt(-a/x + b)/sqrt(a - b*x), x)
```

3.581.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{b-\frac{a}{x}}\,x^m}{\sqrt{a-bx}}\,dx = \frac{2\sqrt{x}x^m}{2i\,m+i}$$

input
```
integrate(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(x)*x^m/(2*I*m + I)
```

3.581.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{b-\frac{a}{x}}\,x^m}{\sqrt{a-bx}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.581.9 Mupad [B] (verification not implemented)

Time = 17.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{b-\frac{a}{x}}x^m}{\sqrt{a-bx}}\,dx = \frac{2\,x^{m+1}\,\sqrt{b-\frac{a}{x}}}{(2\,m+1)\,\sqrt{a-b\,x}}$$

input
```
int((x^m*(b - a/x)^(1/2))/(a - b*x)^(1/2),x)
```

output
```
(2*x^(m + 1)*(b - a/x)^(1/2))/((2*m + 1)*(a - b*x)^(1/2))
```

3.582 $\int \frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}} dx$

3.582.1 Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}} dx = \frac{2\sqrt{b-\frac{a}{x}}x^3}{5\sqrt{a-bx}}$$

output
```
2/5*x^3*(b-a/x)^(1/2)/(-b*x+a)^(1/2)
```

3.582.2 Mathematica [A] (verified)

Time = 8.77 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}} dx = -\frac{2x^2\sqrt{a-bx}}{5\sqrt{b-\frac{a}{x}}}$$

input
```
Integrate[(Sqrt[b - a/x]*x^2)/Sqrt[a - b*x],x]
```

output
```
(-2*x^2*Sqrt[a - b*x])/(5*Sqrt[b - a/x])
```

3.582.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1017, 37, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx$$

$$\Big\downarrow \text{1017}$$

$$\frac{\sqrt{x}\sqrt{b-\frac{a}{x}}\int \frac{x^{3/2}\sqrt{bx-a}}{\sqrt{a-bx}}dx}{\sqrt{bx-a}}$$

$$\Big\downarrow \text{37}$$

$$\frac{\sqrt{x}\sqrt{b-\frac{a}{x}}\int x^{3/2}dx}{\sqrt{a-bx}}$$

$$\Big\downarrow \text{15}$$

$$\frac{2x^3\sqrt{b-\frac{a}{x}}}{5\sqrt{a-bx}}$$

input
```
Int[(Sqrt[b - a/x]*x^2)/Sqrt[a - b*x],x]
```

output
```
(2*Sqrt[b - a/x]*x^3)/(5*Sqrt[a - b*x])
```

3.582.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 37
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> S
imp[(a + b*x)^m/(c + d*x)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !SimplerQ[a + b*x, c + d*x]
```

rule 1017
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_), x_Symbol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n
)^FracPart[q])   Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

3.582.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{2x^{3}\sqrt{-\frac{-bx+a}{x}}}{5\sqrt{-bx+a}}$	27
default	$\frac{2x^{3}\sqrt{-\frac{-bx+a}{x}}}{5\sqrt{-bx+a}}$	27
risch	$-\frac{2\sqrt{-\frac{-bx+a}{x}}\,\sqrt{-\left(-bx+a\right)x}\,\sqrt{\frac{x\left(-bx+a\right)}{bx-a}}\,\left(bx-a\right)x^{3}}{5\left(-bx+a\right)^{\frac{3}{2}}\sqrt{\left(bx-a\right)x}\,\sqrt{-x}}$	80

input
```
int(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*x^3*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

3.582.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}}\,dx=-\frac{2\,\sqrt{-bx+a}x^3\sqrt{\frac{bx-a}{x}}}{5\,(bx-a)}$$

input
```
integrate(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/5*sqrt(-b*x + a)*x^3*sqrt((b*x - a)/x)/(b*x - a)
```

3.582.6 Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}}\,dx = \int \frac{x^2\sqrt{-\frac{a}{x}+b}}{\sqrt{a-bx}}\,dx$$

input
```
integrate(x**2*(b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
```

output
```
Integral(x**2*sqrt(-a/x + b)/sqrt(a - b*x), x)
```

3.582.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}}\,dx = -\frac{2}{5}i\,x^{\frac{5}{2}}$$

input
```
integrate(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-2/5*I*x^(5/2)
```

3.582.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(23) = 46.

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.62

$$\int \frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}}\,dx = \frac{2\sqrt{-ab}a^2|b|\text{sgn}(x)}{5\,b^4} - \frac{2\left(\sqrt{-ab}a^2 - \frac{((bx-a)b+ab)^2\sqrt{-(bx-a)b-ab}}{b^2}\right)|b|\text{sgn}(x)}{5\,b^4}$$

input
```
integrate(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/5*sqrt(-a*b)*a^2*abs(b)*sgn(x)/b^4 - 2/5*(sqrt(-a*b)*a^2 - ((b*x - a)*b
+ a*b)^2*sqrt(-(b*x - a)*b - a*b)/b^2)*abs(b)*sgn(x)/b^4
```

3.582.9 Mupad [B] (verification not implemented)

Time = 16.95 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{b-\frac{a}{x}}x^2}{\sqrt{a-bx}}\,dx=\frac{2\,x^3\,\sqrt{b-\frac{a}{x}}}{5\,\sqrt{a-b\,x}}$$

input
```
int((x^2*(b - a/x)^(1/2))/(a - b*x)^(1/2),x)
```

output
```
(2*x^3*(b - a/x)^(1/2))/(5*(a - b*x)^(1/2))
```

3.583 $\int \frac{\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}\,dx$

3.583.1 Optimal result

Integrand size = 24, antiderivative size = 29

$$\int \frac{\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}\,dx = \frac{2\sqrt{b-\frac{a}{x}}x^2}{3\sqrt{a-bx}}$$

output
```
2/3*x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2)
```

3.583.2 Mathematica [A] (verified)

Time = 5.58 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}\,dx = -\frac{2x\sqrt{a-bx}}{3\sqrt{b-\frac{a}{x}}}$$

input
```
Integrate[(Sqrt[b - a/x]*x)/Sqrt[a - b*x],x]
```

output
```
(-2*x*Sqrt[a - b*x])/(3*Sqrt[b - a/x])
```

3.583.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1017, 37, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}dx$$

$$\Big\downarrow \; 1017$$

$$\frac{\sqrt{x}\sqrt{b-\frac{a}{x}}\int \frac{\sqrt{x}\sqrt{bx-a}}{\sqrt{a-bx}}dx}{\sqrt{bx-a}}$$

$$\Big\downarrow \; 37$$

$$\frac{\sqrt{x}\sqrt{b-\frac{a}{x}}\int \sqrt{x}dx}{\sqrt{a-bx}}$$

$$\Big\downarrow \; 15$$

$$\frac{2x^2\sqrt{b-\frac{a}{x}}}{3\sqrt{a-bx}}$$

input
```
Int[(Sqrt[b - a/x]*x)/Sqrt[a - b*x],x]
```

output
```
(2*Sqrt[b - a/x]*x^2)/(3*Sqrt[a - b*x])
```

3.583.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 37
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> S
imp[(a + b*x)^m/(c + d*x)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a + b*x, c + d*x]
```

rule 1017
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_), x_Symbol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n
)^FracPart[q])   Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

3.583.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{2x^{2}\sqrt{-\frac{-bx+a}{x}}}{3\sqrt{-bx+a}}$	27
default	$\frac{2x^{2}\sqrt{-\frac{-bx+a}{x}}}{3\sqrt{-bx+a}}$	27
risch	$-\frac{2\sqrt{-\frac{-bx+a}{x}}\,\sqrt{-(-bx+a)x}\,\sqrt{\frac{x(-bx+a)}{bx-a}}\,(bx-a)x^{2}}{3(-bx+a)^{\frac{3}{2}}\sqrt{(bx-a)x}\,\sqrt{-x}}$	80

input
```
int(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^2*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

3.583.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}\,dx = -\frac{2\sqrt{-bx+a}x^{2}\sqrt{\frac{bx-a}{x}}}{3\,(bx-a)}$$

input
```
integrate(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(-b*x + a)*x^2*sqrt((b*x - a)/x)/(b*x - a)
```

3.583.6 Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}\,dx = \int \frac{x\sqrt{-\frac{a}{x}+b}}{\sqrt{a-bx}}\,dx$$

input
```
integrate(x*(b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
```

output
```
Integral(x*sqrt(-a/x + b)/sqrt(a - b*x), x)
```

3.583.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}\,dx = -\frac{2}{3}i\,x^{\frac{3}{2}}$$

input
```
integrate(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-2/3*I*x^(3/2)
```

3.583.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(23) = 46.

Time = 0.35 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.93

$$\int \frac{\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}\,dx = \frac{2\sqrt{-ab}a|b|\mathrm{sgn}(x)}{3\,b^3} - \frac{2\left(\sqrt{-ab}a + \frac{(-(bx-a)b-ab)^{\frac{3}{2}}}{b}\right)|b|\mathrm{sgn}(x)}{3\,b^3}$$

input
```
integrate(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/3*sqrt(-a*b)*a*abs(b)*sgn(x)/b^3 - 2/3*(sqrt(-a*b)*a + (-(b*x - a)*b - a
*b)^(3/2)/b)*abs(b)*sgn(x)/b^3
```

3.583.9 Mupad [B] (verification not implemented)

Time = 17.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{b-\frac{a}{x}}\,x}{\sqrt{a-bx}}\,dx = \frac{2\,x^2\,\sqrt{b-\frac{a}{x}}}{3\,\sqrt{a-b\,x}}$$

input
```
int((x*(b - a/x)^(1/2))/(a - b*x)^(1/2),x)
```

output
```
(2*x^2*(b - a/x)^(1/2))/(3*(a - b*x)^(1/2))
```

3.584 $\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx$

3.584.1 Optimal result

Integrand size = 23, antiderivative size = 25

$$\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx=\frac{2\sqrt{b-\frac{a}{x}}x}{\sqrt{a-bx}}$$

output
```
2*x*(b-a/x)^(1/2)/(-b*x+a)^(1/2)
```

3.584.2 Mathematica [A] (verified)

Time = 4.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx=-\frac{2\sqrt{a-bx}}{\sqrt{b-\frac{a}{x}}}$$

input
```
Integrate[Sqrt[b - a/x]/Sqrt[a - b*x],x]
```

output
```
(-2*Sqrt[a - b*x])/Sqrt[b - a/x]
```

3.584.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {942, 37, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx$$

$$\Big\downarrow \text{ 942}$$

$$\frac{\sqrt{x}\sqrt{b-\frac{a}{x}}\int \frac{\sqrt{bx-a}}{\sqrt{x}\sqrt{a-bx}}dx}{\sqrt{bx-a}}$$

$$\Big\downarrow \text{ 37}$$

$$\frac{\sqrt{x}\sqrt{b-\frac{a}{x}}\int \frac{1}{\sqrt{x}}dx}{\sqrt{a-bx}}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{2x\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

input
```
Int[Sqrt[b - a/x]/Sqrt[a - b*x],x]
```

output
```
(2*Sqrt[b - a/x]*x)/Sqrt[a - b*x]
```

3.584.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 37
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> S
imp[(a + b*x)^m/(c + d*x)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a + b*x, c + d*x]
```

rule 942
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symb
ol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n)^FracPart
[q])   Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x], x] /; FreeQ[{a, b, c,
 d, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

3.584.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{2x\sqrt{-\frac{-bx+a}{x}}}{\sqrt{-bx+a}}$	25
default	$\frac{2x\sqrt{-\frac{-bx+a}{x}}}{\sqrt{-bx+a}}$	25
risch	$-\frac{2\sqrt{-\frac{-bx+a}{x}}\,\sqrt{-(-bx+a)x}\,\sqrt{\frac{x(-bx+a)}{bx-a}}\,(bx-a)x}{(-bx+a)^{\frac{3}{2}}\sqrt{(bx-a)x}\,\sqrt{-x}}$	78

input
```
int((b-a/x)^(1/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

3.584.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx = -\frac{2\sqrt{-bx+a}x\sqrt{\frac{bx-a}{x}}}{bx-a}$$

input
```
integrate((b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(-b*x + a)*x*sqrt((b*x - a)/x)/(b*x - a)
```

3.584.6 Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx = \int \frac{\sqrt{-\frac{a}{x}+b}}{\sqrt{a-bx}}\,dx$$

input
```
integrate((b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(-a/x + b)/sqrt(a - b*x), x)
```

3.584.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx = -2i\sqrt{x}$$

input
```
integrate((b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
-2*I*sqrt(x)
```

3.584.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(21) = 42.

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx = \frac{2\left(\sqrt{-(bx-a)b-ab}-\sqrt{-ab}\right)|b|\mathrm{sgn}(x)}{b^2} + \frac{2\sqrt{-ab}|b|\mathrm{sgn}(x)}{b^2}$$

input
```
integrate((b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2*(sqrt(-(b*x - a)*b - a*b) - sqrt(-a*b))*abs(b)*sgn(x)/b^2 + 2*sqrt(-a*b)
*abs(b)*sgn(x)/b^2
```

3.584.9 Mupad [B] (verification not implemented)

Time = 17.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx=\frac{2\,x\,\sqrt{b-\frac{a}{x}}}{\sqrt{a-b\,x}}$$

input
```
int((b - a/x)^(1/2)/(a - b*x)^(1/2),x)
```

output
```
(2*x*(b - a/x)^(1/2))/(a - b*x)^(1/2)
```

3.585 $\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}} dx$

3.585.1 Optimal result

Integrand size = 26, antiderivative size = 24

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}} dx = -\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

output
```
-2*(b-a/x)^(1/2)/(-b*x+a)^(1/2)
```

3.585.2 Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}} dx = -\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

input
```
Integrate[Sqrt[b - a/x]/(x*Sqrt[a - b*x]),x]
```

output
```
(-2*Sqrt[b - a/x])/Sqrt[a - b*x]
```

3.585.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1017, 37, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx$$

$$\Big\downarrow \ 1017$$

$$\frac{\sqrt{x}\sqrt{b-\frac{a}{x}}\int \frac{\sqrt{bx-a}}{x^{3/2}\sqrt{a-bx}}dx}{\sqrt{bx-a}}$$

$$\Big\downarrow \ 37$$

$$\frac{\sqrt{x}\sqrt{b-\frac{a}{x}}\int \frac{1}{x^{3/2}}dx}{\sqrt{a-bx}}$$

$$\Big\downarrow \ 15$$

$$-\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

input
```
Int[Sqrt[b - a/x]/(x*Sqrt[a - b*x]),x]
```

output
```
(-2*Sqrt[b - a/x])/Sqrt[a - b*x]
```

3.585.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 37
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> S
imp[(a + b*x)^m/(c + d*x)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a + b*x, c + d*x]
```

rule 1017
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_), x_Symbol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n
)^FracPart[q])   Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

3.585.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{2\sqrt{-\frac{-bx+a}{x}}}{\sqrt{-bx+a}}$	24
default	$-\frac{2\sqrt{-\frac{-bx+a}{x}}}{\sqrt{-bx+a}}$	24
risch	$\frac{2\sqrt{-\frac{-bx+a}{x}}\,\sqrt{-(-bx+a)x}\,\sqrt{\frac{x(-bx+a)}{bx-a}}\,(bx-a)}{(-bx+a)^{\frac{3}{2}}\sqrt{(bx-a)x}\,\sqrt{-x}}$	77

input
```
int((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

3.585.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx = \frac{2\sqrt{-bx+a}\sqrt{\frac{bx-a}{x}}}{bx-a}$$

input
```
integrate((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(-b*x + a)*sqrt((b*x - a)/x)/(b*x - a)
```

3.585.6 Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx = \int \frac{\sqrt{-\frac{a}{x}+b}}{x\sqrt{a-bx}}\,dx$$

input
```
integrate((b-a/x)**(1/2)/x/(-b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(-a/x + b)/(x*sqrt(a - b*x)), x)
```

3.585.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx = \frac{2i}{\sqrt{x}}$$

input
```
integrate((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2*I/sqrt(x)
```

3.585.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(20) = 40.

Time = 0.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx = \frac{2\left(\frac{b^3}{\sqrt{-(bx-a)b-ab}}-\frac{b^3}{\sqrt{-ab}}\right)|b|\mathrm{sgn}(x)}{b^3}$$

input
```
integrate((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2*(b^3/sqrt(-(b*x - a)*b - a*b) - b^3/sqrt(-a*b))*abs(b)*sgn(x)/b^3
```

3.585.9 Mupad [B] (verification not implemented)

Time = 16.88 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx = -\frac{2\,\sqrt{b-\frac{a}{x}}}{\sqrt{a-b\,x}}$$

input
```
int((b - a/x)^(1/2)/(x*(a - b*x)^(1/2)),x)
```

output
```
-(2*(b - a/x)^(1/2))/(a - b*x)^(1/2)
```

3.586 $\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx$

3.586.1 Optimal result

Integrand size = 26, antiderivative size = 29

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx=-\frac{2\sqrt{b-\frac{a}{x}}}{3x\sqrt{a-bx}}$$

output
```
-2/3*(b-a/x)^(1/2)/x/(-b*x+a)^(1/2)
```

3.586.2 Mathematica [A] (verified)

Time = 3.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx=\frac{2\left(b-\frac{a}{x}\right)^{3/2}}{3(a-bx)^{3/2}}$$

input
```
Integrate[Sqrt[b - a/x]/(x^2*Sqrt[a - b*x]),x]
```

output
```
(2*(b - a/x)^(3/2))/(3*(a - b*x)^(3/2))
```

3.586.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1017, 37, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}dx$$

$$\Big\downarrow \ 1017$$

$$\frac{\sqrt{x}\sqrt{b-\frac{a}{x}}\int \frac{\sqrt{bx-a}}{x^{5/2}\sqrt{a-bx}}dx}{\sqrt{bx-a}}$$

$$\Big\downarrow \ 37$$

$$\frac{\sqrt{x}\sqrt{b-\frac{a}{x}}\int \frac{1}{x^{5/2}}dx}{\sqrt{a-bx}}$$

$$\Big\downarrow \ 15$$

$$-\frac{2\sqrt{b-\frac{a}{x}}}{3x\sqrt{a-bx}}$$

input
```
Int[Sqrt[b - a/x]/(x^2*Sqrt[a - b*x]),x]
```

output
```
(-2*Sqrt[b - a/x])/(3*x*Sqrt[a - b*x])
```

3.586.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 37
```
Int[(u_.)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> S
imp[(a + b*x)^m/(c + d*x)^m   Int[u*(c + d*x)^(m + n), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a + b*x, c + d*x]
```

rule 1017
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_), x_Symbol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n
)^FracPart[q])   Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

3.586.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{2\sqrt{-\frac{-bx+a}{x}}}{3x\sqrt{-bx+a}}$	27
default	$-\frac{2\sqrt{-\frac{-bx+a}{x}}}{3x\sqrt{-bx+a}}$	27
risch	$\frac{2\sqrt{-\frac{-bx+a}{x}}\,\sqrt{-(-bx+a)x}\,\sqrt{\frac{x(-bx+a)}{bx-a}}\,(bx-a)}{3(-bx+a)^{\frac{3}{2}}\sqrt{(bx-a)x}\,\sqrt{-x}\,x}$	80

input
```
int((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-(-b*x+a)/x)^(1/2)/x/(-b*x+a)^(1/2)
```

3.586.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx = \frac{2\sqrt{-bx+a}\sqrt{\frac{bx-a}{x}}}{3\,(bx^2-ax)}$$

input
```
integrate((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(-b*x + a)*sqrt((b*x - a)/x)/(b*x^2 - a*x)
```

3.586.6 Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx = \int \frac{\sqrt{-\frac{a}{x}+b}}{x^2\sqrt{a-bx}}\,dx$$

input
```
integrate((b-a/x)**(1/2)/x**2/(-b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(-a/x + b)/(x**2*sqrt(a - b*x)), x)
```

3.586.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx = \frac{2i}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
2/3*I/x^(3/2)
```

3.586.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(23) = 46.

Time = 0.38 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.07

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx = \frac{2\left(\frac{b^5}{((bx-a)b+ab)\sqrt{-(bx-a)b-ab}}-\frac{b^4}{\sqrt{-ab}a}\right)|b|\mathrm{sgn}(x)}{3\,b^3}$$

input
```
integrate((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2/3*(b^5/(((b*x - a)*b + a*b)*sqrt(-(b*x - a)*b - a*b)) - b^4/(sqrt(-a*b)*
a))*abs(b)*sgn(x)/b^3
```

3.586.9 Mupad [B] (verification not implemented)

Time = 16.74 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx = -\frac{2\,\sqrt{b-\frac{a}{x}}}{3\,x\,\sqrt{a-b\,x}}$$

input
```
int((b - a/x)^(1/2)/(x^2*(a - b*x)^(1/2)),x)
```

output
```
-(2*(b - a/x)^(1/2))/(3*x*(a - b*x)^(1/2))
```

3.587 $\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx$

3.587.1 Optimal result

Integrand size = 17, antiderivative size = 80

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx = \frac{\left(a+\frac{b}{x}\right)^m x\left(1+\frac{ax}{b}\right)^{-m}(c+dx)^n\left(1+\frac{dx}{c}\right)^{-n}\operatorname{AppellF1}\left(1-m,-m,-n,2-m,-\frac{ax}{b},-\frac{dx}{c}\right)}{1-m}$$

output
```
(a+b/x)^m*x*(d*x+c)^n*AppellF1(1-m,-m,-n,2-m,-a*x/b,-d*x/c)/(1-m)/((1+a*x/
b)^m)/((1+d*x/c)^n)
```

3.587.2 Mathematica [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx = \int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx$$

input
```
Integrate[(a + b/x)^m*(c + d*x)^n,x]
```

output
```
Integrate[(a + b/x)^m*(c + d*x)^n, x]
```

3.587.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {942, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n \, dx$$
$$\Big\downarrow \text{942}$$
$$x^m \left(a+\frac{b}{x}\right)^m (ax+b)^{-m} \int x^{-m} (b+ax)^m (c+dx)^n dx$$
$$\Big\downarrow \text{152}$$
$$x^m \left(a+\frac{b}{x}\right)^m \left(\frac{ax}{b}+1\right)^{-m} \int x^{-m} \left(\frac{ax}{b}+1\right)^m (c+dx)^n dx$$
$$\Big\downarrow \text{152}$$
$$x^m \left(a+\frac{b}{x}\right)^m \left(\frac{ax}{b}+1\right)^{-m} (c+dx)^n \left(\frac{dx}{c}+1\right)^{-n} \int x^{-m} \left(\frac{ax}{b}+1\right)^m \left(\frac{dx}{c}+1\right)^n dx$$
$$\Big\downarrow \text{150}$$
$$\frac{x \left(a+\frac{b}{x}\right)^m \left(\frac{ax}{b}+1\right)^{-m} (c+dx)^n \left(\frac{dx}{c}+1\right)^{-n} \text{AppellF1}\left(1-m,-m,-n,2-m,-\frac{ax}{b},-\frac{dx}{c}\right)}{1-m}$$

input
```
Int[(a + b/x)^m*(c + d*x)^n,x]
```

output
```
((a + b/x)^m*x*(c + d*x)^n*AppellF1[1 - m, -m, -n, 2 - m, -((a*x)/b), -((d
*x)/c)])/((1 - m)*(1 + (a*x)/b)^m*(1 + (d*x)/c)^n)
```

3.587.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 942
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symb
ol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n)^FracPart
[q])   Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x], x] /; FreeQ[{a, b, c,
 d, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

3.587.4 Maple [F]

$$\int \left(a+\frac{b}{x}\right)^{m} (dx+c)^{n} \, dx$$

input
```
int((a+b/x)^m*(d*x+c)^n,x)
```

output
```
int((a+b/x)^m*(d*x+c)^n,x)
```

3.587.5 Fricas [F]

$$\int \left(a+\frac{b}{x}\right)^{m} (c+dx)^{n} \, dx = \int (dx+c)^{n} \left(a+\frac{b}{x}\right)^{m} \, dx$$

input
```
integrate((a+b/x)^m*(d*x+c)^n,x, algorithm="fricas")
```

output
```
integral((d*x + c)^n*((a*x + b)/x)^m, x)
```

3.587.6 Sympy [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx=\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx$$

input
```
integrate((a+b/x)**m*(d*x+c)**n,x)
```

output
```
Integral((a + b/x)**m*(c + d*x)**n, x)
```

3.587.7 Maxima [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx=\int (dx+c)^n\left(a+\frac{b}{x}\right)^m\,dx$$

input
```
integrate((a+b/x)^m*(d*x+c)^n,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^n*(a + b/x)^m, x)
```

3.587.8 Giac [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx=\int (dx+c)^n\left(a+\frac{b}{x}\right)^m\,dx$$

input
```
integrate((a+b/x)^m*(d*x+c)^n,x, algorithm="giac")
```

output
```
integrate((d*x + c)^n*(a + b/x)^m, x)
```

3.587.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx=\int \left(a+\frac{b}{x}\right)^m (c+d\,x)^n\,dx$$

input
```
int((a + b/x)^m*(c + d*x)^n,x)
```

output
```
int((a + b/x)^m*(c + d*x)^n, x)
```

3.588 $\int \left(a+\frac{b}{x}\right)^m (c+dx)^2\,dx$

3.588.1 Optimal result

Integrand size = 17, antiderivative size = 138

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^2\,dx=\frac{d(6ac-bd(2-m))\left(a+\frac{b}{x}\right)^{1+m}x^2}{6a^2}+\frac{d^2\left(a+\frac{b}{x}\right)^{1+m}x^3}{3a} -\frac{b(6a^2c^2-6abcd(1-m)+b^2d^2(2-3m+m^2))\left(a+\frac{b}{x}\right)^{1+m}\text{Hypergeometric2F1}\left(2,1+m,2+m,1+\frac{b}{ax}\right)}{6a^4(1+m)}$$

output
```
1/6*d*(6*a*c-b*d*(2-m))*(a+b/x)^(1+m)*x^2/a^2+1/3*d^2*(a+b/x)^(1+m)*x^3/a-
1/6*b*(6*a^2*c^2-6*a*b*c*d*(1-m)+b^2*d^2*(m^2-3*m+2))*(a+b/x)^(1+m)*hyperg
eom([2, 1+m],[2+m],1+b/a/x)/a^4/(1+m)
```

3.588.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.81

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^2\,dx =\frac{\left(a+\frac{b}{x}\right)^m(b+ax)\left(a^2d(1+m)x^2(bd(-2+m)+2a(3c+dx))-b(6a^2c^2+6abcd(-1+m)+b^2d^2(2-3m+m^2))\ldots\right)}{6a^4(1+m)x}$$

input
```
Integrate[(a + b/x)^m*(c + d*x)^2,x]
```

output
```
((a + b/x)^m*(b + a*x)*(a^2*d*(1 + m)*x^2*(b*d*(-2 + m) + 2*a*(3*c + d*x))
 - b*(6*a^2*c^2 + 6*a*b*c*d*(-1 + m) + b^2*d^2*(2 - 3*m + m^2))*Hypergeome
tric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)]))/(6*a^4*(1 + m)*x)
```

3.588.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {941, 948, 100, 87, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \left(a+\frac{b}{x}\right)^m dx$$

$$\Big\downarrow \text{ 941}$$

$$\int x^2 \left(\frac{c}{x}+d\right)^2 \left(a+\frac{b}{x}\right)^m dx$$

$$\Big\downarrow \text{ 948}$$

$$-\int \left(a+\frac{b}{x}\right)^m \left(\frac{c}{x}+d\right)^2 x^4 d\frac{1}{x}$$

$$\Big\downarrow \text{ 100}$$

$$\frac{d^2x^3\left(a+\frac{b}{x}\right)^{m+1}}{3a}-\frac{\int \left(a+\frac{b}{x}\right)^m \left(\frac{3ac^2}{x}+d(6ac-bd(2-m))\right) x^3 d\frac{1}{x}}{3a}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{d^2x^3\left(a+\frac{b}{x}\right)^{m+1}}{3a}-\frac{\frac{(6a^2c^2-bd(1-m)(6ac-bd(2-m)))\int \left(a+\frac{b}{x}\right)^m x^2 d\frac{1}{x}}{2a}-\frac{dx^2\left(a+\frac{b}{x}\right)^{m+1}(6ac-bd(2-m))}{2a}}{3a}$$

$$\Big\downarrow \text{ 75}$$

$$\frac{d^2x^3\left(a+\frac{b}{x}\right)^{m+1}}{3a}-\frac{\frac{b\left(a+\frac{b}{x}\right)^{m+1}(6a^2c^2-bd(1-m)(6ac-bd(2-m)))\,\text{Hypergeometric2F1}\left(2,m+1,m+2,\frac{b}{ax}+1\right)}{2a^3(m+1)}-\frac{dx^2\left(a+\frac{b}{x}\right)^{m+1}(6ac-bd(2-m))}{2a}}{3a}$$

input
```
Int[(a + b/x)^m*(c + d*x)^2,x]
```

output
```
(d^2*(a + b/x)^(1 + m)*x^3)/(3*a) - (-1/2*(d*(6*a*c - b*d*(2 - m))*(a + b/
x)^(1 + m)*x^2)/a + (b*(6*a^2*c^2 - b*d*(6*a*c - b*d*(2 - m))*(1 - m))*(a
+ b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)])/(2*a^3*(1
+ m)))/(3*a)
```

3.588.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.588. $\int \left(a+\frac{b}{x}\right)^m (c+dx)^2 \, dx$

3.588.4 Maple [F]

$$\int \left(a+\frac{b}{x}\right)^m (dx+c)^2\, dx$$

input
```
int((a+b/x)^m*(d*x+c)^2,x)
```

output
```
int((a+b/x)^m*(d*x+c)^2,x)
```

3.588.5 Fricas [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^2\, dx = \int (dx+c)^2 \left(a+\frac{b}{x}\right)^m dx$$

input
```
integrate((a+b/x)^m*(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^2 + 2*c*d*x + c^2)*((a*x + b)/x)^m, x)
```

3.588.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.74 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^2\, dx = \frac{b^m c^2 x^{1-m}\Gamma(1-m)\, {}_2F_1\left(\begin{matrix} -m, 1-m \\ 2-m \end{matrix}\middle| \frac{axe^{i\pi}}{b}\right)}{\Gamma(2-m)} + \frac{2b^m c d x^{2-m}\Gamma(2-m)\, {}_2F_1\left(\begin{matrix} -m, 2-m \\ 3-m \end{matrix}\middle| \frac{axe^{i\pi}}{b}\right)}{\Gamma(3-m)} + \frac{b^m d^2 x^{3-m}\Gamma(3-m)\, {}_2F_1\left(\begin{matrix} -m, 3-m \\ 4-m \end{matrix}\middle| \frac{axe^{i\pi}}{b}\right)}{\Gamma(4-m)}$$

input
```
integrate((a+b/x)**m*(d*x+c)**2,x)
```

output
```
b**m*c**2*x**(1 - m)*gamma(1 - m)*hyper((-m, 1 - m), (2 - m,), a*x*exp_pol
ar(I*pi)/b)/gamma(2 - m) + 2*b**m*c*d*x**(2 - m)*gamma(2 - m)*hyper((-m, 2
 - m), (3 - m,), a*x*exp_polar(I*pi)/b)/gamma(3 - m) + b**m*d**2*x**(3 - m
)*gamma(3 - m)*hyper((-m, 3 - m), (4 - m,), a*x*exp_polar(I*pi)/b)/gamma(4
 - m)
```

3.588.7 Maxima [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^2\,dx=\int (dx+c)^2\left(a+\frac{b}{x}\right)^m\,dx$$

input
```
integrate((a+b/x)^m*(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2*(a + b/x)^m, x)
```

3.588.8 Giac [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^2\,dx=\int (dx+c)^2\left(a+\frac{b}{x}\right)^m\,dx$$

input
```
integrate((a+b/x)^m*(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*(a + b/x)^m, x)
```

3.588.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^2\,dx=\int \left(a+\frac{b}{x}\right)^m (c+d\,x)^2\,dx$$

input
```
int((a + b/x)^m*(c + d*x)^2,x)
```

output
```
int((a + b/x)^m*(c + d*x)^2, x)
```

3.589 $\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx$

3.589.1 Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx = \frac{d\left(a+\frac{b}{x}\right)^{1+m}x^2}{2a} - \frac{b(2ac-bd(1-m))\left(a+\frac{b}{x}\right)^{1+m}\text{Hypergeometric2F1}\left(2,1+m,2+m,1+\frac{b}{ax}\right)}{2a^3(1+m)}$$

output
```
1/2*d*(a+b/x)^(1+m)*x^2/a-1/2*b*(2*a*c-b*d*(1-m))*(a+b/x)^(1+m)*hypergeom(
[2, 1+m],[2+m],1+b/a/x)/a^3/(1+m)
```

3.589.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx = \frac{\left(a+\frac{b}{x}\right)^m(b+ax)\left(a^2d(1+m)x^2+b(-2ac-bd(-1+m))\,\text{Hypergeometric2F1}\left(2,1+m,2+m,1+\frac{b}{ax}\right)\right)}{2a^3(1+m)x}$$

input
```
Integrate[(a + b/x)^m*(c + d*x),x]
```

output
```
((a + b/x)^m*(b + a*x)*(a^2*d*(1 + m)*x^2 + b*(-2*a*c - b*d*(-1 + m))*Hype
rgeometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)]))/(2*a^3*(1 + m)*x)
```

3.589.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {941, 948, 87, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\left(a+\frac{b}{x}\right)^m dx$$

$$\Big\downarrow \text{941}$$

$$\int x\left(\frac{c}{x}+d\right)\left(a+\frac{b}{x}\right)^m dx$$

$$\Big\downarrow \text{948}$$

$$-\int \left(a+\frac{b}{x}\right)^m\left(\frac{c}{x}+d\right)x^3d\frac{1}{x}$$

$$\Big\downarrow \text{87}$$

$$\frac{dx^2\left(a+\frac{b}{x}\right)^{m+1}}{2a}-\frac{(2ac-bd(1-m))\int \left(a+\frac{b}{x}\right)^m x^2d\frac{1}{x}}{2a}$$

$$\Big\downarrow \text{75}$$

$$\frac{dx^2\left(a+\frac{b}{x}\right)^{m+1}}{2a}-\frac{b\left(a+\frac{b}{x}\right)^{m+1}(2ac-bd(1-m))\,\text{Hypergeometric2F1}\left(2,m+1,m+2,\frac{b}{ax}+1\right)}{2a^3(m+1)}$$

input
```
Int[(a + b/x)^m*(c + d*x),x]
```

output
```
(d*(a + b/x)^(1 + m)*x^2)/(2*a) - (b*(2*a*c - b*d*(1 - m))*(a + b/x)^(1 +
m)*Hypergeometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)])/(2*a^3*(1 + m))
```

3.589.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.589.4 Maple [F]

$$\int \left(a+\frac{b}{x}\right)^{m} (dx+c)\, dx$$

input
```
int((a+b/x)^m*(d*x+c),x)
```

output
```
int((a+b/x)^m*(d*x+c),x)
```

3.589.5 Fricas [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx=\int (dx+c)\left(a+\frac{b}{x}\right)^m\,dx$$

input
```
integrate((a+b/x)^m*(d*x+c),x, algorithm="fricas")
```

output
```
integral((d*x + c)*((a*x + b)/x)^m, x)
```

3.589.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.67 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx=\frac{b^m c x^{1-m}\Gamma(1-m)\,{}_2F_1\left(\begin{matrix}-m,1-m\\2-m\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{\Gamma(2-m)}+\frac{b^m d x^{2-m}\Gamma(2-m)\,{}_2F_1\left(\begin{matrix}-m,2-m\\3-m\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{\Gamma(3-m)}$$

input
```
integrate((a+b/x)**m*(d*x+c),x)
```

output
```
b**m*c*x**(1 - m)*gamma(1 - m)*hyper((-m, 1 - m), (2 - m,), a*x*exp_polar(
I*pi)/b)/gamma(2 - m) + b**m*d*x**(2 - m)*gamma(2 - m)*hyper((-m, 2 - m),
(3 - m,), a*x*exp_polar(I*pi)/b)/gamma(3 - m)
```

3.589.7 Maxima [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx=\int (dx+c)\left(a+\frac{b}{x}\right)^m\,dx$$

input
```
integrate((a+b/x)^m*(d*x+c),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(a + b/x)^m, x)
```

3.589.8 Giac [F]

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx=\int (dx+c)\left(a+\frac{b}{x}\right)^m dx$$

input
```
integrate((a+b/x)^m*(d*x+c),x, algorithm="giac")
```

output
```
integrate((d*x + c)*(a + b/x)^m, x)
```

3.589.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx=\int \left(a+\frac{b}{x}\right)^m (c+d\,x)\ dx$$

input
```
int((a + b/x)^m*(c + d*x),x)
```

output
```
int((a + b/x)^m*(c + d*x), x)
```

3.590 $\int \left(a+\frac{b}{x}\right)^m \, dx$

3.590.1 Optimal result

Integrand size = 9, antiderivative size = 40

$$\int \left(a+\frac{b}{x}\right)^m \, dx=-\frac{b\left(a+\frac{b}{x}\right)^{1+m} \text{Hypergeometric2F1}\left(2,1+m,2+m,1+\frac{b}{ax}\right)}{a^2(1+m)}$$

output
```
-b*(a+b/x)^(1+m)*hypergeom([2, 1+m],[2+m],1+b/a/x)/a^2/(1+m)
```

3.590.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int \left(a+\frac{b}{x}\right)^m \, dx=-\frac{\left(a+\frac{b}{x}\right)^m x\left(1+\frac{ax}{b}\right)^{-m} \text{Hypergeometric2F1}\left(1-m,-m,2-m,-\frac{ax}{b}\right)}{-1+m}$$

input
```
Integrate[(a + b/x)^m,x]
```

output
```
-(((a + b/x)^m*x*Hypergeometric2F1[1 - m, -m, 2 - m, -((a*x)/b)])/((-1 + m
)*(1 + (a*x)/b)^m))
```

3.590.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {773, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+\frac{b}{x}\right)^m dx$$

$$\Big\downarrow \ 773$$

$$-\int \left(a+\frac{b}{x}\right)^m x^2 d\frac{1}{x}$$

$$\Big\downarrow \ 75$$

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1} \text{Hypergeometric2F1}\left(2,m+1,m+2,\frac{b}{ax}+1\right)}{a^2(m+1)}$$

input
```
Int[(a + b/x)^m,x]
```

output
```
-((b*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)])/(a
^2*(1 + m)))
```

3.590.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

3.590.4 Maple [F]

$$\int \left(a+\frac{b}{x}\right)^m dx$$

input
```
int((a+b/x)^m,x)
```

output
```
int((a+b/x)^m,x)
```

3.590.5 Fricas [F]

$$\int \left(a+\frac{b}{x}\right)^m dx = \int \left(a+\frac{b}{x}\right)^m dx$$

input
```
integrate((a+b/x)^m,x, algorithm="fricas")
```

output
```
integral(((a*x + b)/x)^m, x)
```

3.590.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \left(a+\frac{b}{x}\right)^m dx = \frac{b^m x^{1-m}\Gamma(1-m)\,{}_2F_1\left(\begin{matrix} -m, 1-m \\ 2-m \end{matrix}\middle| \frac{axe^{i\pi}}{b}\right)}{\Gamma(2-m)}$$

input
```
integrate((a+b/x)**m,x)
```

output
```
b**m*x**(1 - m)*gamma(1 - m)*hyper((-m, 1 - m), (2 - m,), a*x*exp_polar(I*
pi)/b)/gamma(2 - m)
```

3.590.7 Maxima [F]

$$\int \left(a+\frac{b}{x}\right)^m dx=\int \left(a+\frac{b}{x}\right)^m dx$$

input
```
integrate((a+b/x)^m,x, algorithm="maxima")
```

output
```
integrate((a + b/x)^m, x)
```

3.590.8 Giac [F]

$$\int \left(a+\frac{b}{x}\right)^m dx=\int \left(a+\frac{b}{x}\right)^m dx$$

input
```
integrate((a+b/x)^m,x, algorithm="giac")
```

output
```
integrate((a + b/x)^m, x)
```

3.590.9 Mupad [B] (verification not implemented)

Time = 16.89 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int \left(a+\frac{b}{x}\right)^m dx=-\frac{x\left(a+\frac{b}{x}\right)^m {}_2F_1\left(1-m,-m;\ 2-m;\ -\frac{a\,x}{b}\right)}{\left(\frac{a\,x}{b}+1\right)^m (m-1)}$$

input
```
int((a + b/x)^m,x)
```

output
```
-(x*(a + b/x)^m*hypergeom([1 - m, -m], 2 - m, -(a*x)/b))/(((a*x)/b + 1)^m*
(m - 1))
```

3.591 $\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx$

3.591.1 Optimal result

Integrand size = 17, antiderivative size = 101

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx = -\frac{c\left(a+\frac{b}{x}\right)^{1+m}\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(ac-bd)(1+m)} + \frac{\left(a+\frac{b}{x}\right)^{1+m}\text{Hypergeometric2F1}\left(1,1+m,2+m,1+\frac{b}{ax}\right)}{ad(1+m)}$$

output
```
-c*(a+b/x)^(1+m)*hypergeom([1, 1+m],[2+m],c*(a+b/x)/(a*c-b*d))/d/(a*c-b*d)
/(1+m)+(a+b/x)^(1+m)*hypergeom([1, 1+m],[2+m],1+b/a/x)/a/d/(1+m)
```

3.591.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx$$
$$= \frac{\left(a+\frac{b}{x}\right)^m(b+ax)\left(ac\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)+(-ac+bd)\,\text{Hypergeometric2F1}\right.}{ad(-ac+bd)(1+m)x}$$

input
```
Integrate[(a + b/x)^m/(c + d*x),x]
```

output
```
((a + b/x)^m*(b + a*x)*(a*c*Hypergeometric2F1[1, 1 + m, 2 + m, (c*(a + b/x
))/(a*c - b*d)] + (-(a*c) + b*d)*Hypergeometric2F1[1, 1 + m, 2 + m, 1 + b/
(a*x)]))/(a*d*(-(a*c) + b*d)*(1 + m)*x)
```

3.591.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {941, 948, 97, 75, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}dx$$

$$\Big\downarrow \text{941}$$

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{x\left(\frac{c}{x}+d\right)}dx$$

$$\Big\downarrow \text{948}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^m x}{\frac{c}{x}+d}d\frac{1}{x}$$

$$\Big\downarrow \text{97}$$

$$\frac{c\int \frac{\left(a+\frac{b}{x}\right)^m}{\frac{c}{x}+d}d\frac{1}{x}}{d}-\frac{\int \left(a+\frac{b}{x}\right)^m xd\frac{1}{x}}{d}$$

$$\Big\downarrow \text{75}$$

$$\frac{c\int \frac{\left(a+\frac{b}{x}\right)^m}{\frac{c}{x}+d}d\frac{1}{x}}{d}+\frac{\left(a+\frac{b}{x}\right)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{b}{ax}+1\right)}{ad(m+1)}$$

$$\Big\downarrow \text{78}$$

$$\frac{\left(a+\frac{b}{x}\right)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{b}{ax}+1\right)}{ad(m+1)}-\frac{c\left(a+\frac{b}{x}\right)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(m+1)(ac-bd)}$$

input
```
Int[(a + b/x)^m/(c + d*x),x]
```

output
```
-((c*(a + b/x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (c*(a + b/x))/(a
*c - b*d)])/(d*(a*c - b*d)*(1 + m))) + ((a + b/x)^(1 + m)*Hypergeometric2F
1[1, 1 + m, 2 + m, 1 + b/(a*x)])/(a*d*(1 + m))
```

3.591.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.591. $\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx} \, dx$

3.591.4 Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{dx+c}dx$$

input
```
int((a+b/x)^m/(d*x+c),x)
```

output
```
int((a+b/x)^m/(d*x+c),x)
```

3.591.5 Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{dx+c}\,dx$$

input
```
integrate((a+b/x)^m/(d*x+c),x, algorithm="fricas")
```

output
```
integral(((a*x + b)/x)^m/(d*x + c), x)
```

3.591.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx$$

input
```
integrate((a+b/x)**m/(d*x+c),x)
```

output
```
Integral((a + b/x)**m/(c + d*x), x)
```

3.591.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{dx+c}\,dx$$

input
```
integrate((a+b/x)^m/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((a + b/x)^m/(d*x + c), x)
```

3.591.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{dx+c}\,dx$$

input
```
integrate((a+b/x)^m/(d*x+c),x, algorithm="giac")
```

output
```
integrate((a + b/x)^m/(d*x + c), x)
```

3.591.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{c+d\,x}\,dx$$

input
```
int((a + b/x)^m/(c + d*x),x)
```

output
```
int((a + b/x)^m/(c + d*x), x)
```

3.592 $\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx$

3.592.1 Optimal result

Integrand size = 17, antiderivative size = 56

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx=-\frac{b\left(a+\frac{b}{x}\right)^{1+m}\text{Hypergeometric2F1}\left(2,1+m,2+m,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)^2(1+m)}$$

output
```
-b*(a+b/x)^(1+m)*hypergeom([2, 1+m],[2+m],c*(a+b/x)/(a*c-b*d))/(a*c-b*d)^2
/(1+m)
```

3.592.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx=-\frac{b\left(a+\frac{b}{x}\right)^{1+m}\text{Hypergeometric2F1}\left(2,1+m,2+m,-\frac{c\left(a+\frac{b}{x}\right)}{-ac+bd}\right)}{(-ac+bd)^2(1+m)}$$

input
```
Integrate[(a + b/x)^m/(c + d*x)^2,x]
```

output
```
-((b*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((c*(a + b/x))/
(-(a*c) + b*d))])/((-(a*c) + b*d)^2*(1 + m)))
```

3.592.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {941, 946, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}dx$$

$$\Big\downarrow \text{941}$$

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{x^2\left(\frac{c}{x}+d\right)^2}dx$$

$$\Big\downarrow \text{946}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^m}{\left(\frac{c}{x}+d\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{78}$$

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}\text{Hypergeometric2F1}\left(2,m+1,m+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(m+1)(ac-bd)^2}$$

input
```
Int[(a + b/x)^m/(c + d*x)^2,x]
```

output
```
-((b*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, (c*(a + b/x))/(a
*c - b*d)])/((a*c - b*d)^2*(1 + m)))
```

3.592.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

3.592.4 Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{m}}{\left(dx+c\right)^{2}}dx$$

input
```
int((a+b/x)^m/(d*x+c)^2,x)
```

output
```
int((a+b/x)^m/(d*x+c)^2,x)
```

3.592.5 Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{m}}{(c+dx)^{2}}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^{m}}{\left(dx+c\right)^{2}}\,dx$$

input
```
integrate((a+b/x)^m/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(((a*x + b)/x)^m/(d^2*x^2 + 2*c*d*x + c^2), x)
```

3.592.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{\left(c+dx\right)^2}\,dx$$

input
```
integrate((a+b/x)**m/(d*x+c)**2,x)
```

output
```
Integral((a + b/x)**m/(c + d*x)**2, x)
```

3.592.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{\left(dx+c\right)^2}\,dx$$

input
```
integrate((a+b/x)^m/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((a + b/x)^m/(d*x + c)^2, x)
```

3.592.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx = \int \frac{\left(a+\frac{b}{x}\right)^m}{\left(dx+c\right)^2}\,dx$$

input
```
integrate((a+b/x)^m/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((a + b/x)^m/(d*x + c)^2, x)
```

3.592.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{\left(c+d\,x\right)^2}\,dx$$

input
```
int((a + b/x)^m/(c + d*x)^2,x)
```

output
```
int((a + b/x)^m/(c + d*x)^2, x)
```

3.593 $\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx$

3.593.1 Optimal result

Integrand size = 17, antiderivative size = 112

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx = -\frac{d\left(a+\frac{b}{x}\right)^{1+m}}{2c(ac-bd)\left(d+\frac{c}{x}\right)^2} - \frac{b(2ac-bd(1+m))\left(a+\frac{b}{x}\right)^{1+m}\text{Hypergeometric2F1}\left(2,1+m,2+m,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{2c(ac-bd)^3(1+m)}$$

output
```
-1/2*d*(a+b/x)^(1+m)/c/(a*c-b*d)/(d+c/x)^2-1/2*b*(2*a*c-b*d*(1+m))*(a+b/x)
^(1+m)*hypergeom([2, 1+m],[2+m],c*(a+b/x)/(a*c-b*d))/c/(a*c-b*d)^3/(1+m)
```

3.593.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx = \frac{\left(a+\frac{b}{x}\right)^{1+m}\left(-\frac{dx^2}{(c+dx)^2}+\frac{b(-2ac+bd(1+m))\,\text{Hypergeometric2F1}\left(2,1+m,2+m,\frac{bc+acx}{acx-bdx}\right)}{(ac-bd)^2(1+m)}\right)}{2c(ac-bd)}$$

input
```
Integrate[(a + b/x)^m/(c + d*x)^3,x]
```

output
```
((a + b/x)^(1 + m)*(-((d*x^2)/(c + d*x)^2) + (b*(-2*a*c + b*d*(1 + m))*Hyp
ergeometric2F1[2, 1 + m, 2 + m, (b*c + a*c*x)/(a*c*x - b*d*x)])/((a*c - b*
d)^2*(1 + m))))/(2*c*(a*c - b*d))
```

3.593.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {941, 948, 87, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}dx$$

$$\Big\downarrow \text{941}$$

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{x^3\left(\frac{c}{x}+d\right)^3}dx$$

$$\Big\downarrow \text{948}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^m}{\left(\frac{c}{x}+d\right)^3 x}d\frac{1}{x}$$

$$\Big\downarrow \text{87}$$

$$-\frac{(2ac-bd(m+1))\int \frac{\left(a+\frac{b}{x}\right)^m}{\left(\frac{c}{x}+d\right)^2}d\frac{1}{x}}{2c(ac-bd)}-\frac{d\left(a+\frac{b}{x}\right)^{m+1}}{2c\left(\frac{c}{x}+d\right)^2(ac-bd)}$$

$$\Big\downarrow \text{78}$$

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}(2ac-bd(m+1))\text{Hypergeometric2F1}\left(2,m+1,m+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{2c(m+1)(ac-bd)^3}-\frac{d\left(a+\frac{b}{x}\right)^{m+1}}{2c\left(\frac{c}{x}+d\right)^2(ac-bd)}$$

input
```
Int[(a + b/x)^m/(c + d*x)^3,x]
```

output
```
-1/2*(d*(a + b/x)^(1 + m))/(c*(a*c - b*d)*(d + c/x)^2) - (b*(2*a*c - b*d*(
1 + m))*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, (c*(a + b/x))
/(a*c - b*d)])/(2*c*(a*c - b*d)^3*(1 + m))
```

3.593.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && !IntegerQ[m] && IntegerQ[n]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.593.4 Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^3}dx$$

input
```
int((a+b/x)^m/(d*x+c)^3,x)
```

output
```
int((a+b/x)^m/(d*x+c)^3,x)
```

3.593. $\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx$

3.593.5 Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^3}\,dx$$

input
```
integrate((a+b/x)^m/(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral(((a*x + b)/x)^m/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

3.593.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx$$

input
```
integrate((a+b/x)**m/(d*x+c)**3,x)
```

output
```
Integral((a + b/x)**m/(c + d*x)**3, x)
```

3.593.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^3}\,dx$$

input
```
integrate((a+b/x)^m/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((a + b/x)^m/(d*x + c)^3, x)
```

3.593.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^3}\,dx$$

input
```
integrate((a+b/x)^m/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((a + b/x)^m/(d*x + c)^3, x)
```

3.593.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+d\,x)^3}\,dx$$

input
```
int((a + b/x)^m/(c + d*x)^3,x)
```

output
```
int((a + b/x)^m/(c + d*x)^3, x)
```

3.594 $\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4} \, dx$

3.594.1 Optimal result

Integrand size = 17, antiderivative size = 185

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4} \, dx = \frac{d^2\left(a+\frac{b}{x}\right)^{1+m}}{3c^2(ac-bd)\left(d+\frac{c}{x}\right)^3} - \frac{d(6ac-bd(4+m))\left(a+\frac{b}{x}\right)^{1+m}}{6c^2(ac-bd)^2\left(d+\frac{c}{x}\right)^2} - \frac{b(6a^2c^2-6abcd(1+m)+b^2d^2(2+3m+m^2))\left(a+\frac{b}{x}\right)^{1+m} \text{Hypergeometric2F1}\left(2,1+m,2+m,\frac{c(a+}{ac-}\right.}{6c^2(ac-bd)^4(1+m)}$$

output
```
1/3*d^2*(a+b/x)^(1+m)/c^2/(a*c-b*d)/(d+c/x)^3-1/6*d*(6*a*c-b*d*(4+m))*(a+b
/x)^(1+m)/c^2/(a*c-b*d)^2/(d+c/x)^2-1/6*b*(6*a^2*c^2-6*a*b*c*d*(1+m)+b^2*d
^2*(m^2+3*m+2))*(a+b/x)^(1+m)*hypergeom([2, 1+m],[2+m],c*(a+b/x)/(a*c-b*d)
)/c^2/(a*c-b*d)^4/(1+m)
```

3.594.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.84

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4} \, dx = \frac{\left(a+\frac{b}{x}\right)^{1+m}\left(\frac{2d^2(ac-bd)x^3}{(c+dx)^3}+\frac{d(-6ac+bd(4+m))x^2}{(c+dx)^2}-\frac{b(6a^2c^2-6abcd(1+m)+b^2d^2(2+3m+m^2))\text{Hypergeometric2F1}\left(2,1+m,2+m,\frac{b}{a}\right.}{(ac-bd)^2(1+m)}\right.}{6c^2(ac-bd)^2}$$

input
```
Integrate[(a + b/x)^m/(c + d*x)^4,x]
```

output
```
((a + b/x)^(1 + m)*((2*d^2*(a*c - b*d)*x^3)/(c + d*x)^3 + (d*(-6*a*c + b*d
*(4 + m))*x^2)/(c + d*x)^2 - (b*(6*a^2*c^2 - 6*a*b*c*d*(1 + m) + b^2*d^2*(
2 + 3*m + m^2))*Hypergeometric2F1[2, 1 + m, 2 + m, (b*c + a*c*x)/(a*c*x -
b*d*x)])/((a*c - b*d)^2*(1 + m))))/(6*c^2*(a*c - b*d)^2)
```

3.594.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {941, 948, 100, 25, 87, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}dx$$

$$\Big\downarrow \text{ 941}$$

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{x^4\left(\frac{c}{x}+d\right)^4}dx$$

$$\Big\downarrow \text{ 948}$$

$$-\int \frac{\left(a+\frac{b}{x}\right)^m}{\left(\frac{c}{x}+d\right)^4x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{ 100}$$

$$\frac{d^2\left(a+\frac{b}{x}\right)^{m+1}}{3c^2\left(\frac{c}{x}+d\right)^3(ac-bd)}-\frac{\int -\frac{\left(a+\frac{b}{x}\right)^m\left(d(3ac-bd(m+1))-\frac{3c(ac-bd)}{x}\right)}{\left(\frac{c}{x}+d\right)^3}d\frac{1}{x}}{3c^2(ac-bd)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\left(a+\frac{b}{x}\right)^m\left(d(3ac-bd(m+1))-\frac{3c(ac-bd)}{x}\right)}{\left(\frac{c}{x}+d\right)^3}d\frac{1}{x}}{3c^2(ac-bd)}+\frac{d^2\left(a+\frac{b}{x}\right)^{m+1}}{3c^2\left(\frac{c}{x}+d\right)^3(ac-bd)}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{-\frac{(6a^2c^2-6abcd(m+1)+b^2d^2(m^2+3m+2))\int \frac{\left(a+\frac{b}{x}\right)^m}{\left(\frac{c}{x}+d\right)^2}d\frac{1}{x}}{2(ac-bd)}-\frac{d\left(a+\frac{b}{x}\right)^{m+1}(6ac-bd(m+4))}{2\left(\frac{c}{x}+d\right)^2(ac-bd)}}{3c^2(ac-bd)}+\frac{d^2\left(a+\frac{b}{x}\right)^{m+1}}{3c^2\left(\frac{c}{x}+d\right)^3(ac-bd)}$$

$$\Big\downarrow \text{78}$$

$$\frac{-\frac{b\left(a+\frac{b}{x}\right)^{m+1}(6a^2c^2-6abcd(m+1)+b^2d^2(m^2+3m+2))\text{Hypergeometric2F1}\left(2,m+1,m+2,\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{2(m+1)(ac-bd)^3}-\frac{d\left(a+\frac{b}{x}\right)^{m+1}(6ac-bd(m+4))}{2\left(\frac{c}{x}+d\right)^2(ac-bd)}}{3c^2(ac-bd)}+\frac{d^2\left(a+\frac{b}{x}\right)^{m+1}}{3c^2\left(\frac{c}{x}+d\right)^3(ac-bd)}$$

input
```
Int[(a + b/x)^m/(c + d*x)^4,x]
```

output
```
(d^2*(a + b/x)^(1 + m))/(3*c^2*(a*c - b*d)*(d + c/x)^3) + (-1/2*(d*(6*a*c
- b*d*(4 + m))*(a + b/x)^(1 + m))/((a*c - b*d)*(d + c/x)^2) - (b*(6*a^2*c^
2 - 6*a*b*c*d*(1 + m) + b^2*d^2*(2 + 3*m + m^2))*(a + b/x)^(1 + m)*Hyperge
ometric2F1[2, 1 + m, 2 + m, (c*(a + b/x))/(a*c - b*d)])/(2*(a*c - b*d)^3*(
1 + m)))/(3*c^2*(a*c - b*d))
```

3.594.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

3.594. $\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx$

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.594.4 Maple [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{m}}{\left(dx+c\right)^{4}}dx$$

input
```
int((a+b/x)^m/(d*x+c)^4,x)
```

output
```
int((a+b/x)^m/(d*x+c)^4,x)
```

3.594.5 Fricas [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^{m}}{(c+dx)^{4}}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^{m}}{\left(dx+c\right)^{4}}\,dx$$

input
```
integrate((a+b/x)^m/(d*x+c)^4,x, algorithm="fricas")
```

output
```
integral(((a*x + b)/x)^m/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*
x + c^4), x)
```

3.594.6 Sympy [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx$$

input
```
integrate((a+b/x)**m/(d*x+c)**4,x)
```

output
```
Integral((a + b/x)**m/(c + d*x)**4, x)
```

3.594.7 Maxima [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^4}\,dx$$

input
```
integrate((a+b/x)^m/(d*x+c)^4,x, algorithm="maxima")
```

output
```
integrate((a + b/x)^m/(d*x + c)^4, x)
```

3.594.8 Giac [F]

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^4}\,dx$$

input
```
integrate((a+b/x)^m/(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate((a + b/x)^m/(d*x + c)^4, x)
```

3.594.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{{\left(c+d\,x\right)}^4}\,dx$$

input
```
int((a + b/x)^m/(c + d*x)^4,x)
```

output
```
int((a + b/x)^m/(c + d*x)^4, x)
```

3.595 $\int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{a-bx^2}}\,dx$

3.595.1 Optimal result

Integrand size = 28, antiderivative size = 33

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{a-bx^2}}\,dx=\frac{\sqrt{b-\frac{a}{x^2}}x^{1+m}}{m\sqrt{a-bx^2}}$$

output
```
x^(1+m)*(b-a/x^2)^(1/2)/m/(-b*x^2+a)^(1/2)
```

3.595.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{a-bx^2}}\,dx=\frac{\sqrt{b-\frac{a}{x^2}}x^{1+m}}{m\sqrt{a-bx^2}}$$

input
```
Integrate[(Sqrt[b - a/x^2]*x^m)/Sqrt[a - b*x^2],x]
```

output
```
(Sqrt[b - a/x^2]*x^(1 + m))/(m*Sqrt[a - b*x^2])
```

3.595.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1017, 283, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx$$

$$\Big\downarrow \text{1017}$$

$$\frac{x\sqrt{b-\frac{a}{x^2}}\int \frac{x^{m-1}\sqrt{bx^2-a}}{\sqrt{a-bx^2}}dx}{\sqrt{bx^2-a}}$$

$$\Big\downarrow \text{283}$$

$$\frac{x\sqrt{b-\frac{a}{x^2}}\int x^{m-1}dx}{\sqrt{a-bx^2}}$$

$$\Big\downarrow \text{15}$$

$$\frac{x^{m+1}\sqrt{b-\frac{a}{x^2}}}{m\sqrt{a-bx^2}}$$

input
```
Int[(Sqrt[b - a/x^2]*x^m)/Sqrt[a - b*x^2],x]
```

output
```
(Sqrt[b - a/x^2]*x^(1 + m))/(m*Sqrt[a - b*x^2])
```

3.595.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 283
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Sy
mbol] :> Simp[(a + b*x^n)^p/(c + d*x^n)^p   Int[u*(c + d*x^n)^(p + q), x],
x] /; FreeQ[{a, b, c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a
+ b*x^n, c + d*x^n]
```

rule 1017
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_), x_Symbol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n
)^FracPart[q])   Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

3.595.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{x^{1+m} \sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}}{m \sqrt{-b\,x^{2}+a}}$	35
risch	$\frac{i \sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}\, \left(b\,x^{2}-a \right) x \sqrt{\frac{-b\,x^{2}+a}{b\,x^{2}-a}}\, x^{m}}{\left(-b\,x^{2}+a \right)^{\frac{3}{2}} m}$	67

input
```
int(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x^(1+m)/m/(-b*x^2+a)^(1/2)*(-(-b*x^2+a)/x^2)^(1/2)
```

3.595.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{-bx^2+a}xx^m\sqrt{\frac{bx^2-a}{x^2}}}{bmx^2-am}$$

input
```
integrate(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-b*x^2 + a)*x*x^m*sqrt((b*x^2 - a)/x^2)/(b*m*x^2 - a*m)
```

3.595.6 Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{a-bx^2}}\,dx = \int \frac{x^m\sqrt{-\frac{a}{x^2}+b}}{\sqrt{a-bx^2}}\,dx$$

input
```
integrate(x**m*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(x**m*sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

3.595.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{a-bx^2}}\,dx = -\frac{i\,x^m}{m}$$

input
```
integrate(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-I*x^m/m
```

3.595.8 Giac [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b - a/x^2)*x^m/sqrt(-b*x^2 + a), x)
```

3.595.9 Mupad [B] (verification not implemented)

Time = 16.81 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^m}{\sqrt{a-bx^2}}\,dx=\frac{x^{m+1}\,\sqrt{b-\frac{a}{x^2}}}{m\,\sqrt{a-b\,x^2}}$$

input
```
int((x^m*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2),x)
```

output
```
(x^(m + 1)*(b - a/x^2)^(1/2))/(m*(a - b*x^2)^(1/2))
```

3.596 $\int \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}} \, dx$

3.596.1 Optimal result

Integrand size = 28, antiderivative size = 31

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}} \, dx = \frac{\sqrt{b-\frac{a}{x^2}}x^3}{2\sqrt{a-bx^2}}$$

output
```
1/2*x^3*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

3.596.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}} \, dx = -\frac{\sqrt{b-\frac{a}{x^2}}x\sqrt{a-bx^2}}{2b}$$

input
```
Integrate[(Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2],x]
```

output
```
-1/2*(Sqrt[b - a/x^2]*x*Sqrt[a - b*x^2])/b
```

3.596.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1017, 283, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{1017}$$

$$\frac{x\sqrt{b-\frac{a}{x^2}}\int \frac{x\sqrt{bx^2-a}}{\sqrt{a-bx^2}}dx}{\sqrt{bx^2-a}}$$

$$\Big\downarrow \text{283}$$

$$\frac{x\sqrt{b-\frac{a}{x^2}}\int xdx}{\sqrt{a-bx^2}}$$

$$\Big\downarrow \text{15}$$

$$\frac{x^3\sqrt{b-\frac{a}{x^2}}}{2\sqrt{a-bx^2}}$$

input
```
Int[(Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2],x]
```

output
```
(Sqrt[b - a/x^2]*x^3)/(2*Sqrt[a - b*x^2])
```

3.596.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 283
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Sy
mbol] :> Simp[(a + b*x^n)^p/(c + d*x^n)^p   Int[u*(c + d*x^n)^(p + q), x],
x] /; FreeQ[{a, b, c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a
+ b*x^n, c + d*x^n]
```

rule 1017
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_), x_Symbol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n
)^FracPart[q])   Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

3.596.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{x^{3} \sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}}{2 \sqrt{-b\,x^{2}+a}}$	31
default	$\frac{x^{3} \sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}}{2 \sqrt{-b\,x^{2}+a}}$	31
risch	$\frac{i x^{3} \sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}\, \left(b\,x^{2}-a \right) \sqrt{\frac{-b\,x^{2}+a}{b\,x^{2}-a}}}{2 \left(-b\,x^{2}+a \right)^{\frac{3}{2}}}$	63

input
```
int(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^3*(-(-b*x^2+a)/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

3.596.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{-bx^2+a}x^3\sqrt{\frac{bx^2-a}{x^2}}}{2\,(bx^2-a)}$$

input
```
integrate(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-b*x^2 + a)*x^3*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a)
```

3.596.6 Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}\,dx=\int \frac{x^2\sqrt{-\frac{a}{x^2}+b}}{\sqrt{a-bx^2}}\,dx$$

input
```
integrate(x**2*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(x**2*sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

3.596.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.16

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}\,dx=-\frac{1}{2}i\,x^2$$

input
```
integrate(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*I*x^2
```

3.596.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}\,dx=-\frac{i\,bx^2-i\,a}{2\,b}$$

input
```
integrate(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(I*b*x^2 - I*a)/b
```

3.596.9 Mupad [B] (verification not implemented)

Time = 16.67 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}\,dx=\frac{x^3\sqrt{b-\frac{a}{x^2}}}{2\sqrt{a-b\,x^2}}$$

input
```
int((x^2*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2),x)
```

output
```
(x^3*(b - a/x^2)^(1/2))/(2*(a - b*x^2)^(1/2))
```

3.597 $\int \frac{\sqrt{b-\frac{a}{x^2}}x}{\sqrt{a-bx^2}}\,dx$

3.597.1 Optimal result

Integrand size = 26, antiderivative size = 28

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x}{\sqrt{a-bx^2}}\,dx = \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}$$

output
```
x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

3.597.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x}{\sqrt{a-bx^2}}\,dx = \frac{\sqrt{b-\frac{a}{x^2}}x^2}{\sqrt{a-bx^2}}$$

input
```
Integrate[(Sqrt[b - a/x^2]*x)/Sqrt[a - b*x^2],x]
```

output
```
(Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2]
```

3.597.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1017, 283, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{1017}$$

$$\frac{x\sqrt{b-\frac{a}{x^2}}\int \frac{\sqrt{bx^2-a}}{\sqrt{a-bx^2}}dx}{\sqrt{bx^2-a}}$$

$$\Big\downarrow \text{283}$$

$$\frac{x\sqrt{b-\frac{a}{x^2}}\int 1dx}{\sqrt{a-bx^2}}$$

$$\Big\downarrow \text{24}$$

$$\frac{x^2\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

input
```
Int[(Sqrt[b - a/x^2]*x)/Sqrt[a - b*x^2],x]
```

output
```
(Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2]
```

3.597.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 283
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Sy
mbol] :> Simp[(a + b*x^n)^p/(c + d*x^n)^p   Int[u*(c + d*x^n)^(p + q), x],
x] /; FreeQ[{a, b, c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a
+ b*x^n, c + d*x^n]
```

rule 1017
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_), x_Symbol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n
)^FracPart[q])   Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

3.597.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{x^{2} \sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}}{\sqrt{-b\,x^{2}+a}}$	30
risch	$\frac{i x^{2} \sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}\, \left(b\,x^{2}-a \right) \sqrt{\frac{-b\,x^{2}+a}{b\,x^{2}-a}}}{\left(-b\,x^{2}+a \right)^{\frac{3}{2}}}$	63

input
```
int(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x^2*(-(-b*x^2+a)/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

3.597.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{b-\frac{a}{x^2}}x}{\sqrt{a-bx^2}}\,dx=-\frac{\sqrt{-bx^2+a}x^2\sqrt{\frac{bx^2-a}{x^2}}}{bx^2-a}$$

input
```
integrate(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-b*x^2 + a)*x^2*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a)
```

3.597.6 Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\,x}{\sqrt{a-bx^2}}\,dx = \int \frac{x\sqrt{-\frac{a}{x^2}+b}}{\sqrt{a-bx^2}}\,dx$$

input
```
integrate(x*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(x*sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

3.597.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.25

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\,x}{\sqrt{a-bx^2}}\,dx = -i\sqrt{x^2}$$

input
```
integrate(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-I*sqrt(x^2)
```

3.597.8 Giac [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\,x}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{b-\frac{a}{x^2}}\,x}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b - a/x^2)*x/sqrt(-b*x^2 + a), x)
```

3.597.9 Mupad [B] (verification not implemented)

Time = 17.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\,x}{\sqrt{a-bx^2}}\,dx = \frac{\sqrt{b\,x^2-a}\,\sqrt{x^2}}{\sqrt{a-b\,x^2}}$$

input
```
int((x*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2),x)
```

output
```
((b*x^2 - a)^(1/2)*(x^2)^(1/2))/(a - b*x^2)^(1/2)
```

3.598 $\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx$

3.598.1 Optimal result

Integrand size = 25, antiderivative size = 28

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx=\frac{\sqrt{b-\frac{a}{x^2}}\,x\log(x)}{\sqrt{a-bx^2}}$$

output
```
x*ln(x)*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

3.598.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx=\frac{\sqrt{b-\frac{a}{x^2}}\,x\log(x)}{\sqrt{a-bx^2}}$$

input
```
Integrate[Sqrt[b - a/x^2]/Sqrt[a - b*x^2],x]
```

output
```
(Sqrt[b - a/x^2]*x*Log[x])/Sqrt[a - b*x^2]
```

3.598.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {942, 283, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{942}$$

$$\frac{x\sqrt{b-\frac{a}{x^2}}\int \frac{\sqrt{bx^2-a}}{x\sqrt{a-bx^2}}dx}{\sqrt{bx^2-a}}$$

$$\Big\downarrow \text{283}$$

$$\frac{x\sqrt{b-\frac{a}{x^2}}\int \frac{1}{x}dx}{\sqrt{a-bx^2}}$$

$$\Big\downarrow \text{14}$$

$$\frac{x\log(x)\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

input
```
Int[Sqrt[b - a/x^2]/Sqrt[a - b*x^2],x]
```

output
```
(Sqrt[b - a/x^2]*x*Log[x])/Sqrt[a - b*x^2]
```

3.598.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 283
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Sy
mbol] :> Simp[(a + b*x^n)^p/(c + d*x^n)^p   Int[u*(c + d*x^n)^(p + q), x],
x] /; FreeQ[{a, b, c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a
+ b*x^n, c + d*x^n]
```

rule 942
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symb
ol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n)^FracPart
[q])   Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x], x] /; FreeQ[{a, b, c,
 d, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

3.598.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{\sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}\,x\ln\left(x\right)}{\sqrt{-b\,x^{2}+a}}$	30
risch	$\frac{i\sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}\,\left(b\,x^{2}-a\right)x\sqrt{\frac{-b\,x^{2}+a}{b\,x^{2}-a}}\,\ln\left(x\right)}{\left(-b\,x^{2}+a\right)^{\frac{3}{2}}}$	63

input
```
int((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b*x^2+a)/x^2)^(1/2)*x/(-b*x^2+a)^(1/2)*ln(x)
```

3.598.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(24) = 48.

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.82

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx=-\arctan\left(\frac{\sqrt{-bx^2+a}(x^3+x)\sqrt{\frac{bx^2-a}{x^2}}}{bx^4-(a+b)x^2+a}\right)$$

input
```
integrate((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-arctan(sqrt(-b*x^2 + a)*(x^3 + x)*sqrt((b*x^2 - a)/x^2)/(b*x^4 - (a + b)*
x^2 + a))
```

3.598.6 Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx=\int \frac{\sqrt{-\frac{a}{x^2}+b}}{\sqrt{a-bx^2}}\,dx$$

input
```
integrate((b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

3.598.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.14

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx=-i\log(x)$$

input
```
integrate((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-I*log(x)
```

3.598.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

3.598.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-b\,x^2}}\,dx$$

input
```
int((b - a/x^2)^(1/2)/(a - b*x^2)^(1/2),x)
```

output
```
int((b - a/x^2)^(1/2)/(a - b*x^2)^(1/2), x)
```

3.599 $\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx$

3.599.1 Optimal result

Integrand size = 28, antiderivative size = 26

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx=-\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

output
```
-(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

3.599.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx=-\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

input
```
Integrate[Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2]),x]
```

output
```
-(Sqrt[b - a/x^2]/Sqrt[a - b*x^2])
```

3.599.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1017, 283, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{ 1017}$$

$$\frac{x\sqrt{b-\frac{a}{x^2}}\int \frac{\sqrt{bx^2-a}}{x^2\sqrt{a-bx^2}}dx}{\sqrt{bx^2-a}}$$

$$\Big\downarrow \text{ 283}$$

$$\frac{x\sqrt{b-\frac{a}{x^2}}\int \frac{1}{x^2}dx}{\sqrt{a-bx^2}}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

input
```
Int[Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2]),x]
```

output
```
-(Sqrt[b - a/x^2]/Sqrt[a - b*x^2])
```

3.599.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 283
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Sy
mbol] :> Simp[(a + b*x^n)^p/(c + d*x^n)^p   Int[u*(c + d*x^n)^(p + q), x],
x] /; FreeQ[{a, b, c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a
+ b*x^n, c + d*x^n]
```

rule 1017
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_), x_Symbol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n
)^FracPart[q])   Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

3.599.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

method	result	size
gosper	$-\frac{\sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}}{\sqrt{-b\,x^{2}+a}}$	28
default	$-\frac{\sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}}{\sqrt{-b\,x^{2}+a}}$	28
risch	$-\frac{i\sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}\,\left(b\,x^{2}-a\right)\sqrt{\frac{-b\,x^{2}+a}{b\,x^{2}-a}}}{\left(-b\,x^{2}+a\right)^{\frac{3}{2}}}$	60

input
```
int((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-b*x^2+a)/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

3.599.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{-bx^2+a}(x-1)\sqrt{\frac{bx^2-a}{x^2}}}{bx^2-a}$$

input
```
integrate((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-b*x^2 + a)*(x - 1)*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a)
```

3.599.6 Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{-\frac{a}{x^2}+b}}{x\sqrt{a-bx^2}}\,dx$$

input
```
integrate((b-a/x**2)**(1/2)/x/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(-a/x**2 + b)/(x*sqrt(a - b*x**2)), x)
```

3.599.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.27

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx = \frac{i}{\sqrt{x^2}}$$

input
```
integrate((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
I/sqrt(x^2)
```

3.599.8 Giac [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{-bx^2+a}x}\,dx$$

input
```
integrate((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b - a/x^2)/(sqrt(-b*x^2 + a)*x), x)
```

3.599.9 Mupad [B] (verification not implemented)

Time = 16.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-b\,x^2}}$$

input
```
int((b - a/x^2)^(1/2)/(x*(a - b*x^2)^(1/2)),x)
```

output
```
-(b - a/x^2)^(1/2)/(a - b*x^2)^(1/2)
```

3.600 $\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx$

3.600.1 Optimal result

Integrand size = 28, antiderivative size = 31

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{b-\frac{a}{x^2}}}{2x\sqrt{a-bx^2}}$$

output
```
-1/2*(b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2)
```

3.600.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{b-\frac{a}{x^2}}}{2x\sqrt{a-bx^2}}$$

input
```
Integrate[Sqrt[b - a/x^2]/(x^2*Sqrt[a - b*x^2]),x]
```

output
```
-1/2*Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2])
```

3.600.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1017, 283, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \ 1017$$

$$\frac{x\sqrt{b-\frac{a}{x^2}}\int \frac{\sqrt{bx^2-a}}{x^3\sqrt{a-bx^2}}dx}{\sqrt{bx^2-a}}$$

$$\Big\downarrow \ 283$$

$$\frac{x\sqrt{b-\frac{a}{x^2}}\int \frac{1}{x^3}dx}{\sqrt{a-bx^2}}$$

$$\Big\downarrow \ 15$$

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{2x\sqrt{a-bx^2}}$$

input
```
Int[Sqrt[b - a/x^2]/(x^2*Sqrt[a - b*x^2]),x]
```

output
```
-1/2*Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2])
```

3.600.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 283
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Sy
mbol] :> Simp[(a + b*x^n)^p/(c + d*x^n)^p   Int[u*(c + d*x^n)^(p + q), x],
x] /; FreeQ[{a, b, c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a
+ b*x^n, c + d*x^n]
```

rule 1017
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_), x_Symbol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n
)^FracPart[q])   Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

3.600.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{\sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}}{2x\sqrt{-b\,x^{2}+a}}$	31
default	$-\frac{\sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}}{2x\sqrt{-b\,x^{2}+a}}$	31
risch	$-\frac{i\sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}\,\left(b\,x^{2}-a\right)\sqrt{\frac{-b\,x^{2}+a}{b\,x^{2}-a}}}{2x\left(-b\,x^{2}+a\right)^{\frac{3}{2}}}$	63

input
```
int((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-(-b*x^2+a)/x^2)^(1/2)/x/(-b*x^2+a)^(1/2)
```

3.600.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{-bx^2+a}(x^2-1)\sqrt{\frac{bx^2-a}{x^2}}}{2\,(bx^3-ax)}$$

input
```
integrate((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-b*x^2 + a)*(x^2 - 1)*sqrt((b*x^2 - a)/x^2)/(b*x^3 - a*x)
```

3.600.6 Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx=\int \frac{\sqrt{-\frac{a}{x^2}+b}}{x^2\sqrt{a-bx^2}}\,dx$$

input
```
integrate((b-a/x**2)**(1/2)/x**2/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(-a/x**2 + b)/(x**2*sqrt(a - b*x**2)), x)
```

3.600.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.16

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx=\frac{i}{2\,x^2}$$

input
```
integrate((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*I/x^2
```

3.600.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.16

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx=\frac{i}{2\,x^2}$$

input
```
integrate((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*I/x^2
```

3.600.9 Mupad [B] (verification not implemented)

Time = 17.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{b-\frac{a}{x^2}}}{2\,x\,\sqrt{a-b\,x^2}}$$

input
```
int((b - a/x^2)^(1/2)/(x^2*(a - b*x^2)^(1/2)),x)
```

output
```
-(b - a/x^2)^(1/2)/(2*x*(a - b*x^2)^(1/2))
```

3.601 $\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx$

3.601.1 Optimal result

Integrand size = 21, antiderivative size = 406

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx=\frac{2c\sqrt{c+dx}(b+ax^2)}{5a\sqrt{a+\frac{b}{x^2}}x}+\frac{2(c+dx)^{3/2}(b+ax^2)}{5a\sqrt{a+\frac{b}{x^2}}x}+\frac{2\sqrt{b}(ac^2-3bd^2)\sqrt{c+dx}\sqrt{1+\frac{ax^2}{b}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{-a}x}{\sqrt{b}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{-a}\sqrt{b}d}{ac-\sqrt{-a}\sqrt{b}d}\right)}{5(-a)^{3/2}d\sqrt{a+\frac{b}{x^2}}x\sqrt{\frac{a(c+dx)}{ac-\sqrt{-a}\sqrt{b}d}}}-\frac{2\sqrt{b}c(ac^2+bd^2)\sqrt{\frac{a(c+dx)}{ac-\sqrt{-a}\sqrt{b}d}}\sqrt{1+\frac{ax^2}{b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{-a}x}{\sqrt{b}}}}{\sqrt{2}}\right),-\frac{2\sqrt{-a}\sqrt{b}d}{ac-\sqrt{-a}\sqrt{b}d}\right)}{5(-a)^{3/2}d\sqrt{a+\frac{b}{x^2}}x\sqrt{c+dx}}$$

output
```
2/5*(d*x+c)^(3/2)*(a*x^2+b)/a/x/(a+b/x^2)^(1/2)+2/5*c*(a*x^2+b)*(d*x+c)^(1
/2)/a/x/(a+b/x^2)^(1/2)+2/5*(a*c^2-3*b*d^2)*EllipticE(1/2*(1-x*(-a)^(1/2)/
b^(1/2))^(1/2)*2^(1/2),(-2*d*(-a)^(1/2)*b^(1/2)/(a*c-d*(-a)^(1/2)*b^(1/2))
)^(1/2))*b^(1/2)*(d*x+c)^(1/2)*(1+a*x^2/b)^(1/2)/(-a)^(3/2)/d/x/(a+b/x^2)^
(1/2)/(a*(d*x+c)/(a*c-d*(-a)^(1/2)*b^(1/2)))^(1/2)-2/5*c*(a*c^2+b*d^2)*Ell
ipticF(1/2*(1-x*(-a)^(1/2)/b^(1/2))^(1/2)*2^(1/2),(-2*d*(-a)^(1/2)*b^(1/2)
/(a*c-d*(-a)^(1/2)*b^(1/2)))^(1/2))*b^(1/2)*(1+a*x^2/b)^(1/2)*(a*(d*x+c)/(
a*c-d*(-a)^(1/2)*b^(1/2)))^(1/2)/(-a)^(3/2)/d/x/(a+b/x^2)^(1/2)/(d*x+c)^(1
/2)
```

3.601. $\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx$

3.601.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.58 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.29

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx = \frac{\sqrt{c+dx}\left(\frac{2(2c+dx)(b+ax^2)}{a}+\frac{2\left(d^2\sqrt{-c-\frac{i\sqrt{b}d}{\sqrt{a}}}(ac^2-3bd^2)(b+ax^2)+\sqrt{a}\left(-ia^{3/2}c^3+a\sqrt{b}c^2d+3i\sqrt{a}bcd^2-3b^{3/2}\right.\right.}{}\right.}{}$$

input
```
Integrate[(c + d*x)^(3/2)/Sqrt[a + b/x^2],x]
```

output
```
(Sqrt[c + d*x]*((2*(2*c + d*x)*(b + a*x^2))/a + (2*(d^2*Sqrt[-c - (I*Sqrt[
b]*d)/Sqrt[a]]*(a*c^2 - 3*b*d^2)*(b + a*x^2) + Sqrt[a]*((-I)*a^(3/2)*c^3 +
 a*Sqrt[b]*c^2*d + (3*I)*Sqrt[a]*b*c*d^2 - 3*b^(3/2)*d^3)*Sqrt[(d*((I*Sqrt
[b])/Sqrt[a] + x))/(c + d*x)]*Sqrt[-(((I*Sqrt[b]*d)/Sqrt[a] - d*x)/(c + d*
x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c - (I*Sqrt[b]*d)/Sqrt[a]]/
Sqrt[c + d*x]], (Sqrt[a]*c - I*Sqrt[b]*d)/(Sqrt[a]*c + I*Sqrt[b]*d)] - Sqr
t[a]*Sqrt[b]*d*(a*c^2 + (4*I)*Sqrt[a]*Sqrt[b]*c*d - 3*b*d^2)*Sqrt[(d*((I*S
qrt[b])/Sqrt[a] + x))/(c + d*x)]*Sqrt[-(((I*Sqrt[b]*d)/Sqrt[a] - d*x)/(c +
 d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c - (I*Sqrt[b]*d)/Sqrt[a
]]/Sqrt[c + d*x]], (Sqrt[a]*c - I*Sqrt[b]*d)/(Sqrt[a]*c + I*Sqrt[b]*d)]))/
(a^2*d^2*Sqrt[-c - (I*Sqrt[b]*d)/Sqrt[a]]*(c + d*x))))/(5*Sqrt[a + b/x^2]*
x)
```

3.601.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 719, normalized size of antiderivative = 1.77, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1780, 596, 687, 27, 599, 25, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx$$

$$\Big\downarrow \text{1780}$$

$$\frac{\sqrt{ax^2+b}\int \frac{x(c+dx)^{3/2}}{\sqrt{ax^2+b}}dx}{x\sqrt{a+\frac{b}{x^2}}}$$

$$\Big\downarrow \text{596}$$

$$\frac{\sqrt{ax^2+b}\left(\frac{2\sqrt{ax^2+b}(c+dx)^{3/2}}{5a}-\frac{3\int \frac{(bd-acx)\sqrt{c+dx}}{\sqrt{ax^2+b}}dx}{5a}\right)}{x\sqrt{a+\frac{b}{x^2}}}$$

$$\Big\downarrow \text{687}$$

$$\frac{\sqrt{ax^2+b}\left(\frac{2\sqrt{ax^2+b}(c+dx)^{3/2}}{5a}-\frac{3\left(\frac{2\int \frac{a\left(4bcd-\left(ac^2-3bd^2\right)x\right)}{2\sqrt{c+dx}\sqrt{ax^2+b}}dx}{3a}-\frac{2}{3}c\sqrt{ax^2+b}\sqrt{c+dx}\right)}{5a}\right)}{x\sqrt{a+\frac{b}{x^2}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{ax^2+b}\left(\frac{2\sqrt{ax^2+b}(c+dx)^{3/2}}{5a}-\frac{3\left(\frac{1}{3}\int \frac{4bcd-\left(ac^2-3bd^2\right)x}{\sqrt{c+dx}\sqrt{ax^2+b}}dx-\frac{2}{3}c\sqrt{ax^2+b}\sqrt{c+dx}\right)}{5a}\right)}{x\sqrt{a+\frac{b}{x^2}}}$$

$$\Big\downarrow \text{599}$$

$$\frac{\sqrt{ax^2+b}\left(\frac{2\sqrt{ax^2+b}(c+dx)^{3/2}}{5a}-\frac{3\left(-\frac{2\int -\frac{c\left(ac^2+bd^2\right)-\left(ac^2-3bd^2\right)(c+dx)}{\sqrt{\frac{ac^2}{d^2}-\frac{2a(c+dx)c}{d^2}+\frac{a(c+dx)^2}{d^2}+b}}d\sqrt{c+dx}}{3d^2}-\frac{2}{3}c\sqrt{ax^2+b}\sqrt{c+dx}\right)}{5a}\right)}{x\sqrt{a+\frac{b}{x^2}}}$$

$$\Big\downarrow \text{25}$$

3.601. $\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx$

$$\frac{\sqrt{ax^2+b}\left(\frac{2\sqrt{ax^2+b}(c+dx)^{3/2}}{5a}-\frac{3\left(\frac{2\int \frac{c\left(ac^2+bd^2\right)-\left(ac^2-3bd^2\right)(c+dx)}{\sqrt{\frac{ac^2}{d^2}-\frac{2a(c+dx)c}{d^2}+\frac{a(c+dx)^2}{d^2}+b}}d\sqrt{c+dx}}{3d^2}-\frac{2}{3}c\sqrt{ax^2+b}\sqrt{c+dx}\right)}{5a}\right)}{x\sqrt{a+\frac{b}{x^2}}}$$

$\downarrow$ 1511

$$\frac{\sqrt{ax^2+b}\left(\frac{2\sqrt{ax^2+b}(c+dx)^{3/2}}{5a}-\frac{3\left(-\frac{2\left(\frac{\sqrt{ac^2+bd^2}\left(-\sqrt{a}c\sqrt{ac^2+bd^2}+ac^2-3bd^2\right)\int \frac{1}{\sqrt{\frac{ac^2}{d^2}-\frac{2a(c+dx)c}{d^2}+\frac{a(c+dx)^2}{d^2}+b}}d\sqrt{c+dx}}{\sqrt{a}}-\frac{\left(ac^2-3bd^2\right)\sqrt{ac^2+\cdots}}{\cdots}\cdots\right)}{3d^2}\cdots\right)}{5a}\cdots\right)}{x\sqrt{a+\frac{b}{x^2}}}$$

$\downarrow$ 1416

3.601. $\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx$

$$\frac{\sqrt{ax^2+b}\left(\frac{2\sqrt{ax^2+b}(c+dx)^{3/2}}{5a}-\frac{3\left(-\frac{2\left(\frac{\left(ac^2+bd^2\right)^{3/4}\left(-\sqrt{a}c\sqrt{ac^2+bd^2}+ac^2-3bd^2\right)\left(\frac{\sqrt{a}(c+dx)}{\sqrt{ac^2+bd^2}}+1\right)\sqrt{\frac{\frac{ac^2}{d^2}-\frac{2ac(c+dx)}{d^2}+\frac{a(c+dx)^2}{d^2}+b}{\left(\frac{ac^2}{d^2}+b\right)\left(\frac{\sqrt{a}(c+dx)}{\sqrt{ac^2+bd^2}}+1\right)^2}}\text{EllipticF}\left(\cdots\right.}{2a^{3/4}\sqrt{\frac{ac^2}{d^2}-\frac{2ac(c+dx)}{d^2}+\frac{a(c+dx)^2}{d^2}+b}}\cdots}{\cdots}\cdots\right)}{\cdots}\right)}{x\sqrt{a+\cdots}}$$

$\downarrow$ 1509

3.601. $\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx$

$$\sqrt{ax^2+b}\left(\frac{2\sqrt{ax^2+b}(c+dx)^{3/2}}{5a}-\frac{3\left(-\frac{2\left(\frac{\left(ac^2+bd^2\right)^{3/4}\left(-\sqrt{a}c\sqrt{ac^2+bd^2}+ac^2-3bd^2\right)\left(\frac{\sqrt{a}(c+dx)}{\sqrt{ac^2+bd^2}}+1\right)\sqrt{\frac{\frac{ac^2}{d^2}-\frac{2ac(c+dx)}{d^2}+\frac{a(c+dx)^2}{d^2}+b}{\left(\frac{ac^2}{d^2}+b\right)\left(\frac{\sqrt{a}(c+dx)}{\sqrt{ac^2+bd^2}}+1\right)^2}}\text{EllipticF}\left(\right.}{2a^{3/4}\sqrt{\frac{ac^2}{d^2}-\frac{2ac(c+dx)}{d^2}+\frac{a(c+dx)^2}{d^2}+b}}\right)}{}\right)}{}\right)$$

input
```
Int[(c + d*x)^(3/2)/Sqrt[a + b/x^2],x]
```

3.601. $\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx$

output
```
(Sqrt[b + a*x^2]*((2*(c + d*x)^(3/2)*Sqrt[b + a*x^2])/(5*a) - (3*((-2*c*Sq
rt[c + d*x]*Sqrt[b + a*x^2])/3 - (2*(-(((a*c^2 - 3*b*d^2)*Sqrt[a*c^2 + b*d
^2]*(-((Sqrt[c + d*x]*Sqrt[b + (a*c^2)/d^2 - (2*a*c*(c + d*x))/d^2 + (a*(c
 + d*x)^2)/d^2])/((b + (a*c^2)/d^2)*(1 + (Sqrt[a]*(c + d*x))/Sqrt[a*c^2 +
b*d^2]))) + ((a*c^2 + b*d^2)^(1/4)*(1 + (Sqrt[a]*(c + d*x))/Sqrt[a*c^2 + b
*d^2])*Sqrt[(b + (a*c^2)/d^2 - (2*a*c*(c + d*x))/d^2 + (a*(c + d*x)^2)/d^2
)/((b + (a*c^2)/d^2)*(1 + (Sqrt[a]*(c + d*x))/Sqrt[a*c^2 + b*d^2])^2)]*Ell
ipticE[2*ArcTan[(a^(1/4)*Sqrt[c + d*x])/(a*c^2 + b*d^2)^(1/4)], (1 + (Sqrt
[a]*c)/Sqrt[a*c^2 + b*d^2])/2])/(a^(1/4)*Sqrt[b + (a*c^2)/d^2 - (2*a*c*(c
+ d*x))/d^2 + (a*(c + d*x)^2)/d^2]))))/Sqrt[a]) + ((a*c^2 + b*d^2)^(3/4)*(a
*c^2 - 3*b*d^2 - Sqrt[a]*c*Sqrt[a*c^2 + b*d^2])*(1 + (Sqrt[a]*(c + d*x))/S
qrt[a*c^2 + b*d^2])*Sqrt[(b + (a*c^2)/d^2 - (2*a*c*(c + d*x))/d^2 + (a*(c
+ d*x)^2)/d^2)/((b + (a*c^2)/d^2)*(1 + (Sqrt[a]*(c + d*x))/Sqrt[a*c^2 + b*
d^2])^2)]*EllipticF[2*ArcTan[(a^(1/4)*Sqrt[c + d*x])/(a*c^2 + b*d^2)^(1/4)
], (1 + (Sqrt[a]*c)/Sqrt[a*c^2 + b*d^2])/2])/(2*a^(3/4)*Sqrt[b + (a*c^2)/d
^2 - (2*a*c*(c + d*x))/d^2 + (a*(c + d*x)^2)/d^2])))/(3*d^2)))/(5*a)))/(Sq
rt[a + b/x^2]*x)
```

3.601.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 596
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 2))), x] - Simp[n/(b*(n
 + 2*p + 2))   Int[(c + d*x)^(n - 1)*(a + b*x^2)^p*(a*d - b*c*x), x], x] /;
 FreeQ[{a, b, c, d, p}, x] && GtQ[n, 0] && NeQ[n + 2*p + 2, 0]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

3.601. $\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx$

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1780
```
Int[((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_Sy
mbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(c + a*x^(2*
n))^FracPart[p])   Int[((d + e*x^n)^q*(c + a*x^(2*n))^p)/x^(2*n*p), x], x]
/; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[mn2, -2*n] &&  !IntegerQ[p] &&  !
IntegerQ[q] && PosQ[n]
```

3.601. $\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx$

3.601.4 Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 623, normalized size of antiderivative = 1.53

method	result
risch	$\frac{2(dx+2c)(a\,x^2+b)\sqrt{dx+c}}{5a\sqrt{\frac{a\,x^2+b}{x^2}}\,x}+\frac{\left(-\frac{8bcd\left(\frac{c}{d}-\frac{\sqrt{-ab}}{a}\right)\sqrt{\frac{\frac{c}{d}+x}{\frac{c}{d}-\frac{\sqrt{-ab}}{a}}}\sqrt{\frac{x-\frac{\sqrt{-ab}}{a}}{-\frac{c}{d}-\frac{\sqrt{-ab}}{a}}}\sqrt{\frac{x+\frac{\sqrt{-ab}}{a}}{-\frac{c}{d}+\frac{\sqrt{-ab}}{a}}}\,F\left(\sqrt{\frac{\frac{c}{d}+x}{\frac{c}{d}-\frac{\sqrt{-ab}}{a}}},\sqrt{\frac{-\frac{c}{d}+\frac{\sqrt{-ab}}{a}}{-\frac{c}{d}-\frac{\sqrt{-ab}}{a}}}\right)}{\sqrt{a d\,x^3+a c\,x^2+b d x+b c}}+\frac{2\left(a\,c^2-\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((d*x+c)^(3/2)/(a+b/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(d*x+2*c)*(a*x^2+b)*(d*x+c)^(1/2)/a/((a*x^2+b)/x^2)^(1/2)/x+1/5/a*(-8*
b*c*d*(c/d-1/a*(-a*b)^(1/2))*((c/d+x)/(c/d-1/a*(-a*b)^(1/2)))^(1/2)*((x-1/
a*(-a*b)^(1/2))/(-c/d-1/a*(-a*b)^(1/2)))^(1/2)*((x+1/a*(-a*b)^(1/2))/(-c/d
+1/a*(-a*b)^(1/2)))^(1/2)/(a*d*x^3+a*c*x^2+b*d*x+b*c)^(1/2)*EllipticF(((c/
d+x)/(c/d-1/a*(-a*b)^(1/2)))^(1/2),((-c/d+1/a*(-a*b)^(1/2))/(-c/d-1/a*(-a*
b)^(1/2)))^(1/2))+2*(a*c^2-3*b*d^2)*(c/d-1/a*(-a*b)^(1/2))*((c/d+x)/(c/d-1
/a*(-a*b)^(1/2)))^(1/2)*((x-1/a*(-a*b)^(1/2))/(-c/d-1/a*(-a*b)^(1/2)))^(1/
2)*((x+1/a*(-a*b)^(1/2))/(-c/d+1/a*(-a*b)^(1/2)))^(1/2)/(a*d*x^3+a*c*x^2+b
*d*x+b*c)^(1/2)*((-c/d-1/a*(-a*b)^(1/2))*EllipticE(((c/d+x)/(c/d-1/a*(-a*b
)^(1/2)))^(1/2),((-c/d+1/a*(-a*b)^(1/2))/(-c/d-1/a*(-a*b)^(1/2)))^(1/2))+1
/a*(-a*b)^(1/2)*EllipticF(((c/d+x)/(c/d-1/a*(-a*b)^(1/2)))^(1/2),((-c/d+1/
a*(-a*b)^(1/2))/(-c/d-1/a*(-a*b)^(1/2)))^(1/2))))/((a*x^2+b)/x^2)^(1/2)/x*
((a*x^2+b)*(d*x+c))^(1/2)/(d*x+c)^(1/2)
```

3.601.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.58

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx =$$

$$-\frac{2\left((ac^3+9\,bcd^2)\sqrt{ad}\text{weierstrassPInverse}\left(\frac{4\,(ac^2-3\,bd^2)}{3\,ad^2},-\frac{8\,(ac^3+9\,bcd^2)}{27\,ad^3},\frac{3\,dx+c}{3\,d}\right)+3\,(ac^2d-3\,bd^3)\sqrt{ad}\text{weierst}\right.}{}$$

input
```
integrate((d*x+c)^(3/2)/(a+b/x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/15*((a*c^3 + 9*b*c*d^2)*sqrt(a*d)*weierstrassPInverse(4/3*(a*c^2 - 3*b*
d^2)/(a*d^2), -8/27*(a*c^3 + 9*b*c*d^2)/(a*d^3), 1/3*(3*d*x + c)/d) + 3*(a
*c^2*d - 3*b*d^3)*sqrt(a*d)*weierstrassZeta(4/3*(a*c^2 - 3*b*d^2)/(a*d^2),
 -8/27*(a*c^3 + 9*b*c*d^2)/(a*d^3), weierstrassPInverse(4/3*(a*c^2 - 3*b*d
^2)/(a*d^2), -8/27*(a*c^3 + 9*b*c*d^2)/(a*d^3), 1/3*(3*d*x + c)/d)) - 3*(a
*d^3*x^2 + 2*a*c*d^2*x)*sqrt(d*x + c)*sqrt((a*x^2 + b)/x^2))/(a^2*d^2)
```

3.601.6 Sympy [F]

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx=\int \frac{(c+dx)^{\frac{3}{2}}}{\sqrt{a+\frac{b}{x^2}}}\,dx$$

input
```
integrate((d*x+c)**(3/2)/(a+b/x**2)**(1/2),x)
```

output
```
Integral((c + d*x)**(3/2)/sqrt(a + b/x**2), x)
```

3.601.7 Maxima [F]

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx=\int \frac{(dx+c)^{\frac{3}{2}}}{\sqrt{a+\frac{b}{x^2}}}\,dx$$

input
```
integrate((d*x+c)^(3/2)/(a+b/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(3/2)/sqrt(a + b/x^2), x)
```

3.601.8 Giac [F]

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx = \int \frac{(dx+c)^{\frac{3}{2}}}{\sqrt{a+\frac{b}{x^2}}}\,dx$$

input
```
integrate((d*x+c)^(3/2)/(a+b/x^2)^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(3/2)/sqrt(a + b/x^2), x)
```

3.601.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx = \int \frac{(c+d\,x)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx$$

input
```
int((c + d*x)^(3/2)/(a + b/x^2)^(1/2),x)
```

output
```
int((c + d*x)^(3/2)/(a + b/x^2)^(1/2), x)
```

3.602 $\int \frac{-1+x^3}{(-4x+x^4)^{2/3}}\,dx$

3.602.1 Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \frac{-1+x^3}{(-4x+x^4)^{2/3}}\,dx = \frac{3}{4}\sqrt[3]{-4x+x^4}$$

output
```
3/4*(x^4-4*x)^(1/3)
```

3.602.2 Mathematica [A] (verified)

Time = 10.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^3}{(-4x+x^4)^{2/3}}\,dx = \frac{3}{4}\sqrt[3]{x(-4+x^3)}$$

input
```
Integrate[(-1 + x^3)/(-4*x + x^4)^(2/3),x]
```

output
```
(3*(x*(-4 + x^3))^(1/3))/4
```

3.602.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-1}{\left(x^4-4x\right)^{2/3}}\,dx$$

$$\Big\downarrow \ 2021$$

$$\frac{3}{4}\sqrt[3]{x^4-4x}$$

input
```
Int[(-1 + x^3)/(-4*x + x^4)^(2/3),x]
```

output
```
(3*(-4*x + x^4)^(1/3))/4
```

3.602.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.602.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{3(x^4-4x)^{\frac{1}{3}}}{4}$	12
trager	$\frac{3(x^4-4x)^{\frac{1}{3}}}{4}$	12
pseudoelliptic	$\frac{3(x(x^3-4))^{\frac{1}{3}}}{4}$	12
gosper	$\frac{3x(x^3-4)}{4(x^4-4x)^{\frac{2}{3}}}$	18
risch	$\frac{3x(x^3-4)}{4(x(x^3-4))^{\frac{2}{3}}}$	18
meijerg	$-\frac{3\,2^{\frac{2}{3}}\left(-\operatorname{signum}\left(-1+\frac{x^3}{4}\right)\right)^{\frac{2}{3}} x^{\frac{1}{3}}\,{}_2F_1\left(\frac{1}{9},\frac{2}{3};\frac{10}{9};\frac{x^3}{4}\right)}{4\operatorname{signum}\left(-1+\frac{x^3}{4}\right)^{\frac{2}{3}}}+\frac{3\,2^{\frac{2}{3}}\left(-\operatorname{signum}\left(-1+\frac{x^3}{4}\right)\right)^{\frac{2}{3}} x^{\frac{10}{3}}\,{}_2F_1\left(\frac{2}{3},\frac{10}{9};\frac{19}{9};\frac{x^3}{4}\right)}{40\operatorname{signum}\left(-1+\frac{x^3}{4}\right)^{\frac{2}{3}}}$	84

input
```
int((x^3-1)/(x^4-4*x)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
3/4*(x^4-4*x)^(1/3)
```

3.602.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{-1+x^3}{(-4x+x^4)^{2/3}}\,dx = \frac{3}{4}\left(x^4-4\,x\right)^{\frac{1}{3}}$$

input
```
integrate((x^3-1)/(x^4-4*x)^(2/3),x, algorithm="fricas")
```

output
```
3/4*(x^4 - 4*x)^(1/3)
```

3.602.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{-1+x^3}{(-4x+x^4)^{2/3}}\,dx = \frac{3\sqrt[3]{x^4-4x}}{4}$$

input
```
integrate((x**3-1)/(x**4-4*x)**(2/3),x)
```

output
```
3*(x**4 - 4*x)**(1/3)/4
```

3.602.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{-1+x^3}{(-4x+x^4)^{2/3}}\,dx = \frac{3}{4}\left(x^4-4\,x\right)^{\frac{1}{3}}$$

input
```
integrate((x^3-1)/(x^4-4*x)^(2/3),x, algorithm="maxima")
```

output
```
3/4*(x^4 - 4*x)^(1/3)
```

3.602.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{-1+x^3}{(-4x+x^4)^{2/3}}\,dx = \frac{3}{4}\left(x^4-4\,x\right)^{\frac{1}{3}}$$

input
```
integrate((x^3-1)/(x^4-4*x)^(2/3),x, algorithm="giac")
```

output
```
3/4*(x^4 - 4*x)^(1/3)
```

3.602.9 Mupad [B] (verification not implemented)

Time = 17.83 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{-1+x^3}{(-4x+x^4)^{2/3}}\,dx = \frac{3\,(x^4-4\,x)^{1/3}}{4}$$

input
```
int((x^3 - 1)/(x^4 - 4*x)^(2/3),x)
```

output
```
(3*(x^4 - 4*x)^(1/3))/4
```

3.603 $\int (2-x^2) \sqrt[4]{6x-x^3}\, dx$

3.603.1 Optimal result

Integrand size = 21, antiderivative size = 17

$$\int (2-x^2) \sqrt[4]{6x-x^3}\, dx = \frac{4}{15}(6x-x^3)^{5/4}$$

output
```
4/15*(-x^3+6*x)^(5/4)
```

3.603.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2 in optimal.

Time = 10.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 4.24

$$\int (2-x^2) \sqrt[4]{6x-x^3}\, dx = -\frac{4\sqrt[4]{-x(-6+x^2)}\left(-26x\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{8},\frac{13}{8},\frac{x^2}{6}\right)+5x^3\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{13}{8},\frac{21}{8},\frac{x^2}{6}\right)\right)}{65\sqrt[4]{1-\frac{x^2}{6}}}$$

input
```
Integrate[(2 - x^2)*(6*x - x^3)^(1/4),x]
```

output
```
(-4*(-(x*(-6 + x^2)))^(1/4)*(-26*x*Hypergeometric2F1[-1/4, 5/8, 13/8, x^2/
6] + 5*x^3*Hypergeometric2F1[-1/4, 13/8, 21/8, x^2/6]))/(65*(1 - x^2/6)^(1
/4))
```

3.603.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2-x^2\right) \sqrt[4]{6x-x^3} \, dx$$
$$\Big\downarrow \; 2021$$
$$\frac{4}{15} \left(6x-x^3\right)^{5/4}$$

input
```
Int[(2 - x^2)*(6*x - x^3)^(1/4),x]
```

output
```
(4*(6*x - x^3)^(5/4))/15
```

3.603.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.603.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{4(-x^3+6x)^{\frac{5}{4}}}{15}$	14
risch	$-\frac{4(-x(x^2-6))^{\frac{1}{4}}x(x^2-6)}{15}$	19
pseudoelliptic	$-\frac{4(-x(x^2-6))^{\frac{1}{4}}x(x^2-6)}{15}$	19
gosper	$-\frac{4(-x^3+6x)^{\frac{1}{4}}x(x^2-6)}{15}$	20
trager	$-\frac{4(-x^3+6x)^{\frac{1}{4}}x(x^2-6)}{15}$	20
meijerg	$\frac{8\,6^{\frac{1}{4}}x^{\frac{5}{4}}\,{}_2F_1\left(-\frac{1}{4},\frac{5}{8};\frac{13}{8};\frac{x^2}{6}\right)}{5}-\frac{4\,6^{\frac{1}{4}}x^{\frac{13}{4}}\,{}_2F_1\left(-\frac{1}{4},\frac{13}{8};\frac{21}{8};\frac{x^2}{6}\right)}{13}$	40

input
```
int((-x^2+2)*(-x^3+6*x)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
4/15*(-x^3+6*x)^(5/4)
```

3.603.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int (2-x^2)\sqrt[4]{6x-x^3}\,dx = -\frac{4}{15}\left(x^3-6\,x\right)\left(-x^3+6\,x\right)^{\frac{1}{4}}$$

input
```
integrate((-x^2+2)*(-x^3+6*x)^(1/4),x, algorithm="fricas")
```

output
```
-4/15*(x^3 - 6*x)*(-x^3 + 6*x)^(1/4)
```

3.603.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82

$$\int (2-x^2)\sqrt[4]{6x-x^3}\,dx = -\frac{4x^3\sqrt[4]{-x^3+6x}}{15}+\frac{8x\sqrt[4]{-x^3+6x}}{5}$$

input
```
integrate((-x**2+2)*(-x**3+6*x)**(1/4),x)
```

output
```
-4*x**3*(-x**3 + 6*x)**(1/4)/15 + 8*x*(-x**3 + 6*x)**(1/4)/5
```

3.603.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (2-x^2)\sqrt[4]{6x-x^3}\,dx = \frac{4}{15}\left(-x^3+6x\right)^{\frac{5}{4}}$$

input
```
integrate((-x^2+2)*(-x^3+6*x)^(1/4),x, algorithm="maxima")
```

output
```
4/15*(-x^3 + 6*x)^(5/4)
```

3.603.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (2-x^2)\sqrt[4]{6x-x^3}\,dx = \frac{4}{15}\left(-x^3+6x\right)^{\frac{5}{4}}$$

input
```
integrate((-x^2+2)*(-x^3+6*x)^(1/4),x, algorithm="giac")
```

output
```
4/15*(-x^3 + 6*x)^(5/4)
```

3.603.9 Mupad [B] (verification not implemented)

Time = 17.77 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \left(2-x^2\right) \sqrt[4]{6x-x^3}\,dx=-\frac{4\,x\,\left(x^2-6\right)\,{\left(6\,x-x^3\right)}^{1/4}}{15}$$

input
```
int(-(x^2 - 2)*(6*x - x^3)^(1/4),x)
```

output
```
-(4*x*(x^2 - 6)*(6*x - x^3)^(1/4))/15
```

3.604 $\int (1+x^4) \sqrt{5x+x^5}\,dx$

3.604.1 Optimal result

Integrand size = 17, antiderivative size = 15

$$\int (1+x^4) \sqrt{5x+x^5}\,dx = \frac{2}{15}\left(5x+x^5\right)^{3/2}$$

output
```
2/15*(x^5+5*x)^(3/2)
```

3.604.2 Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (1+x^4) \sqrt{5x+x^5}\,dx = \frac{2}{15}\left(x\left(5+x^4\right)\right)^{3/2}$$

input
```
Integrate[(1 + x^4)*Sqrt[5*x + x^5],x]
```

output
```
(2*(x*(5 + x^4))^(3/2))/15
```

3.604.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x^4+1)\sqrt{x^5+5x}\,dx$$
$$\Big\downarrow \; 2021$$
$$\frac{2}{15}(x^5+5x)^{3/2}$$

input
```
Int[(1 + x^4)*Sqrt[5*x + x^5],x]
```

output
```
(2*(5*x + x^5)^(3/2))/15
```

3.604.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.604.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{2(x^5+5x)^{\frac{3}{2}}}{15}$	12
gosper	$\frac{2x(x^4+5)\sqrt{x^5+5x}}{15}$	18
trager	$\frac{2x(x^4+5)\sqrt{x^5+5x}}{15}$	18
pseudoelliptic	$\frac{2x(x^4+5)\sqrt{x(x^4+5)}}{15}$	18
risch	$\frac{2x^2(x^4+5)^2}{15\sqrt{x(x^4+5)}}$	22
meijerg	$\frac{2\sqrt{5}\,x^{\frac{3}{2}}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{8};\frac{11}{8};-\frac{x^4}{5}\right)}{3}+\frac{2\sqrt{5}\,x^{\frac{11}{2}}\,{}_2F_1\left(-\frac{1}{2},\frac{11}{8};\frac{19}{8};-\frac{x^4}{5}\right)}{11}$	40

input
```
int((x^4+1)*(x^5+5*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(x^5+5*x)^(3/2)
```

3.604.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int (1+x^4)\sqrt{5x+x^5}\,dx=\frac{2}{15}\left(x^5+5\,x\right)^{\frac{3}{2}}$$

input
```
integrate((x^4+1)*(x^5+5*x)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(x^5 + 5*x)^(3/2)
```

3.604.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int (1+x^4)\sqrt{5x+x^5}\,dx = \frac{2x^5\sqrt{x^5+5x}}{15} + \frac{2x\sqrt{x^5+5x}}{3}$$

input
```
integrate((x**4+1)*(x**5+5*x)**(1/2),x)
```

output
```
2*x**5*sqrt(x**5 + 5*x)/15 + 2*x*sqrt(x**5 + 5*x)/3
```

3.604.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int (1+x^4)\sqrt{5x+x^5}\,dx = \frac{2}{15}\left(x^5+5\,x\right)^{\frac{3}{2}}$$

input
```
integrate((x^4+1)*(x^5+5*x)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(x^5 + 5*x)^(3/2)
```

3.604.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int (1+x^4)\sqrt{5x+x^5}\,dx = \frac{2}{15}\left(x^5+5\,x\right)^{\frac{3}{2}}$$

input
```
integrate((x^4+1)*(x^5+5*x)^(1/2),x, algorithm="giac")
```

output
```
2/15*(x^5 + 5*x)^(3/2)
```

3.604.9 Mupad [B] (verification not implemented)

Time = 17.50 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int (1+x^4)\sqrt{5x+x^5}\,dx=\frac{2\,{\left(x^5+5\,x\right)}^{3/2}}{15}$$

input
```
int((5*x + x^5)^(1/2)*(x^4 + 1),x)
```

output
```
(2*(5*x + x^5)^(3/2))/15
```

3.605 $\int (2+5x^4) \sqrt{2x+x^5} \, dx$

3.605.1 Optimal result

Integrand size = 19, antiderivative size = 15

$$\int (2+5x^4) \sqrt{2x+x^5} \, dx = \frac{2}{3}(2x+x^5)^{3/2}$$

output
```
2/3*(x^5+2*x)^(3/2)
```

3.605.2 Mathematica [A] (verified)

Time = 10.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (2+5x^4) \sqrt{2x+x^5} \, dx = \frac{2}{3}(x(2+x^4))^{3/2}$$

input
```
Integrate[(2 + 5*x^4)*Sqrt[2*x + x^5],x]
```

output
```
(2*(x*(2 + x^4))^(3/2))/3
```

3.605.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5x^4+2)\sqrt{x^5+2x}\,dx$$

$$\Big\downarrow \text{ 2021}$$

$$\frac{2}{3}\left(x^5+2x\right)^{3/2}$$

input
```
Int[(2 + 5*x^4)*Sqrt[2*x + x^5],x]
```

output
```
(2*(2*x + x^5)^(3/2))/3
```

3.605.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.605.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{2(x^5+2x)^{\frac{3}{2}}}{3}$	12
default	$\frac{2(x^5+2x)^{\frac{3}{2}}}{3}$	12
gosper	$\frac{2x(x^4+2)\sqrt{x^5+2x}}{3}$	18
trager	$\frac{2x(x^4+2)\sqrt{x^5+2x}}{3}$	18
pseudoelliptic	$\frac{2x(x^4+2)\sqrt{x(x^4+2)}}{3}$	18
risch	$\frac{2x^2(x^4+2)^2}{3\sqrt{x(x^4+2)}}$	22
meijerg	$\frac{4\sqrt{2}\,x^{\frac{3}{2}}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{8};\frac{11}{8};-\frac{x^4}{2}\right)}{3}+\frac{10\sqrt{2}\,x^{\frac{11}{2}}\,{}_2F_1\left(-\frac{1}{2},\frac{11}{8};\frac{19}{8};-\frac{x^4}{2}\right)}{11}$	40

input
```
int((5*x^4+2)*(x^5+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(x^5+2*x)^(3/2)
```

3.605.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int (2+5x^4)\sqrt{2x+x^5}\,dx=\frac{2}{3}\left(x^5+2\,x\right)^{\frac{3}{2}}$$

input
```
integrate((5*x^4+2)*(x^5+2*x)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(x^5 + 2*x)^(3/2)
```

3.605.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \left(2+5x^4\right)\sqrt{2x+x^5}\,dx=\frac{2x^5\sqrt{x^5+2x}}{3}+\frac{4x\sqrt{x^5+2x}}{3}$$

input
```
integrate((5*x**4+2)*(x**5+2*x)**(1/2),x)
```

output
```
2*x**5*sqrt(x**5 + 2*x)/3 + 4*x*sqrt(x**5 + 2*x)/3
```

3.605.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \left(2+5x^4\right)\sqrt{2x+x^5}\,dx=\frac{2}{3}\left(x^5+2\,x\right)^{\frac{3}{2}}$$

input
```
integrate((5*x^4+2)*(x^5+2*x)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(x^5 + 2*x)^(3/2)
```

3.605.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \left(2+5x^4\right)\sqrt{2x+x^5}\,dx=\frac{2}{3}\left(x^5+2\,x\right)^{\frac{3}{2}}$$

input
```
integrate((5*x^4+2)*(x^5+2*x)^(1/2),x, algorithm="giac")
```

output
```
2/3*(x^5 + 2*x)^(3/2)
```

3.605.9 Mupad [B] (verification not implemented)

Time = 17.40 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \left(2+5x^4\right)\sqrt{2x+x^5}\,dx=\frac{2\,{\left(x^5+2\,x\right)}^{3/2}}{3}$$

input
```
int((2*x + x^5)^(1/2)*(5*x^4 + 2),x)
```

output
```
(2*(2*x + x^5)^(3/2))/3
```

3.606 $\int \frac{x+3x^2}{\sqrt{x^2+2x^3}}\,dx$

3.606.1 Optimal result

Integrand size = 21, antiderivative size = 13

$$\int \frac{x+3x^2}{\sqrt{x^2+2x^3}}\,dx = \sqrt{x^2+2x^3}$$

output
```
(2*x^3+x^2)^(1/2)
```

3.606.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x+3x^2}{\sqrt{x^2+2x^3}}\,dx = \sqrt{x^2(1+2x)}$$

input
```
Integrate[(x + 3*x^2)/Sqrt[x^2 + 2*x^3],x]
```

output
```
Sqrt[x^2*(1 + 2*x)]
```

3.606.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{3x^2+x}{\sqrt{2x^3+x^2}}\,dx$$

$$\Big\downarrow \text{ 2021}$$

$$\sqrt{2x^3+x^2}$$

input
```
Int[(x + 3*x^2)/Sqrt[x^2 + 2*x^3],x]
```

output
```
Sqrt[x^2 + 2*x^3]
```

3.606.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.606.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
trager	$\sqrt{2x^3+x^2}$	12
pseudoelliptic	$\sqrt{x^2(1+2x)}$	12
gosper	$\frac{x^2(1+2x)}{\sqrt{2x^3+x^2}}$	21
default	$\frac{x^2(1+2x)}{\sqrt{2x^3+x^2}}$	21
risch	$\frac{x^2(1+2x)}{\sqrt{x^2(1+2x)}}$	21
meijerg	$\frac{\sqrt{\pi}-\frac{\sqrt{\pi}(-8x+8)\sqrt{1+2x}}{8}}{\sqrt{\pi}}+\frac{-2\sqrt{\pi}+2\sqrt{\pi}\sqrt{1+2x}}{2\sqrt{\pi}}$	53

input
```
int((3*x^2+x)/(2*x^3+x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(2*x^3+x^2)^(1/2)
```

3.606.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x+3x^2}{\sqrt{x^2+2x^3}}\,dx = \sqrt{2\,x^3+x^2}$$

input
```
integrate((3*x^2+x)/(2*x^3+x^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(2*x^3 + x^2)
```

3.606.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{x+3x^2}{\sqrt{x^2+2x^3}}\,dx = \sqrt{2x^3+x^2}$$

input
```
integrate((3*x**2+x)/(2*x**3+x**2)**(1/2),x)
```

output
```
sqrt(2*x**3 + x**2)
```

3.606.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x+3x^2}{\sqrt{x^2+2x^3}}\,dx = \sqrt{2\,x^3+x^2}$$

input
```
integrate((3*x^2+x)/(2*x^3+x^2)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(2*x^3 + x^2)
```

3.606.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{x+3x^2}{\sqrt{x^2+2x^3}}\,dx = \frac{(2\,x+1)^{\frac{3}{2}}-\sqrt{2\,x+1}}{2\,\mathrm{sgn}\left(x\right)}$$

input
```
integrate((3*x^2+x)/(2*x^3+x^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*((2*x + 1)^(3/2) - sqrt(2*x + 1))/sgn(x)
```

3.606.9 Mupad [B] (verification not implemented)

Time = 17.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{x+3x^2}{\sqrt{x^2+2x^3}}\,dx = |x|\,\sqrt{2\,x+1}$$

input
```
int((x + 3*x^2)/(x^2 + 2*x^3)^(1/2),x)
```

output
```
abs(x)*(2*x + 1)^(1/2)
```

3.607 $\int \frac{2+\sqrt[3]{1-5x}}{3+\sqrt[3]{1-5x}}\,dx$

3.607.1 Optimal result

Integrand size = 25, antiderivative size = 44

$$\int \frac{2+\sqrt[3]{1-5x}}{3+\sqrt[3]{1-5x}}\,dx=-\frac{9}{5}\sqrt[3]{1-5x}+\frac{3}{10}(1-5x)^{2/3}+x+\frac{27}{5}\log\left(3+\sqrt[3]{1-5x}\right)$$

output
```
-9/5*(1-5*x)^(1/3)+3/10*(1-5*x)^(2/3)+x+27/5*ln(3+(1-5*x)^(1/3))
```

3.607.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \frac{2+\sqrt[3]{1-5x}}{3+\sqrt[3]{1-5x}}\,dx=\frac{1}{10}\left(-2-18\sqrt[3]{1-5x}+3(1-5x)^{2/3}+10x+54\log\left(3+\sqrt[3]{1-5x}\right)\right)$$

input
```
Integrate[(2 + (1 - 5*x)^(1/3))/(3 + (1 - 5*x)^(1/3)),x]
```

output
```
(-2 - 18*(1 - 5*x)^(1/3) + 3*(1 - 5*x)^(2/3) + 10*x + 54*Log[3 + (1 - 5*x)
^(1/3)])/10
```

3.607.3 Rubi [A] (warning: unable to verify)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {938, 900, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{1-5x}+2}{\sqrt[3]{1-5x}+3}dx$$

$$\Big\downarrow \text{938}$$

$$-\frac{1}{5}\int \frac{\sqrt[3]{1-5x}+2}{\sqrt[3]{1-5x}+3}d(1-5x)$$

$$\Big\downarrow \text{900}$$

$$-\frac{3}{5}\int \frac{(1-5x)^{2/3}(3-5x)}{4-5x}d\sqrt[3]{1-5x}$$

$$\Big\downarrow \text{86}$$

$$-\frac{3}{5}\int \left(5x+(1-5x)^{2/3}-\frac{9}{4-5x}+2\right)d\sqrt[3]{1-5x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{5}\left(\frac{1}{3}(1-5x)-\frac{1}{2}(1-5x)^{2/3}+3\sqrt[3]{1-5x}-9\log(4-5x)\right)$$

input
```
Int[(2 + (1 - 5*x)^(1/3))/(3 + (1 - 5*x)^(1/3)),x]
```

output
```
(-3*(3*(1 - 5*x)^(1/3) - (1 - 5*x)^(2/3)/2 + (1 - 5*x)/3 - 9*Log[4 - 5*x])
)/5
```

3.607.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 900
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(a + b*x^(g*n)
)^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
 && NeQ[b*c - a*d, 0] && FractionQ[n]
```

rule 938
```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Simp[1/Coefficient[u, x, 1]   Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
 x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u
, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.607.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-\frac{1}{5}+x+\frac{3(1-5x)^{\frac{2}{3}}}{10}-\frac{9(1-5x)^{\frac{1}{3}}}{5}+\frac{27\ln\left(3+(1-5x)^{\frac{1}{3}}\right)}{5}$	34
default	$-\frac{1}{5}+x+\frac{3(1-5x)^{\frac{2}{3}}}{10}-\frac{9(1-5x)^{\frac{1}{3}}}{5}+\frac{27\ln\left(3+(1-5x)^{\frac{1}{3}}\right)}{5}$	34
trager	$x-\frac{9(1-5x)^{\frac{1}{3}}}{5}+\frac{3(1-5x)^{\frac{2}{3}}}{10}+\frac{9\ln\left(9(1-5x)^{\frac{2}{3}}+27(1-5x)^{\frac{1}{3}}-5x+28\right)}{5}$	47

input
```
int((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
-1/5+x+3/10*(1-5*x)^(2/3)-9/5*(1-5*x)^(1/3)+27/5*ln(3+(1-5*x)^(1/3))
```

3.607.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \frac{2+\sqrt[3]{1-5x}}{3+\sqrt[3]{1-5x}}\,dx = x+\frac{3}{10}\,(-5\,x+1)^{\frac{2}{3}}-\frac{9}{5}\,(-5\,x+1)^{\frac{1}{3}}+\frac{27}{5}\,\log\left((-5\,x+1)^{\frac{1}{3}}+3\right)$$

input
```
integrate((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x, algorithm="fricas")
```

output
```
x + 3/10*(-5*x + 1)^(2/3) - 9/5*(-5*x + 1)^(1/3) + 27/5*log((-5*x + 1)^(1/
3) + 3)
```

3.607.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{2+\sqrt[3]{1-5x}}{3+\sqrt[3]{1-5x}}\,dx = x+\frac{3(1-5x)^{\frac{2}{3}}}{10}-\frac{9\sqrt[3]{1-5x}}{5}+\frac{27\log\left(\sqrt[3]{1-5x}+3\right)}{5}$$

input
```
integrate((2+(1-5*x)**(1/3))/(3+(1-5*x)**(1/3)),x)
```

output
```
x + 3*(1 - 5*x)**(2/3)/10 - 9*(1 - 5*x)**(1/3)/5 + 27*log((1 - 5*x)**(1/3)
 + 3)/5
```

3.607.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

$$\int \frac{2+\sqrt[3]{1-5x}}{3+\sqrt[3]{1-5x}}\,dx = x+\frac{3}{10}\,(-5\,x+1)^{\frac{2}{3}}-\frac{9}{5}\,(-5\,x+1)^{\frac{1}{3}}+\frac{27}{5}\,\log\left((-5\,x+1)^{\frac{1}{3}}+3\right)-\frac{1}{5}$$

input
```
integrate((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x, algorithm="maxima")
```

output
```
x + 3/10*(-5*x + 1)^(2/3) - 9/5*(-5*x + 1)^(1/3) + 27/5*log((-5*x + 1)^(1/
3) + 3) - 1/5
```

3.607.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

$$\int \frac{2+\sqrt[3]{1-5x}}{3+\sqrt[3]{1-5x}}\,dx = x+\frac{3}{10}\,(-5\,x+1)^{\frac{2}{3}}-\frac{9}{5}\,(-5\,x+1)^{\frac{1}{3}}+\frac{27}{5}\,\log\left((-5\,x+1)^{\frac{1}{3}}+3\right)-\frac{1}{5}$$

input
```
integrate((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x, algorithm="giac")
```

output
```
x + 3/10*(-5*x + 1)^(2/3) - 9/5*(-5*x + 1)^(1/3) + 27/5*log((-5*x + 1)^(1/
3) + 3) - 1/5
```

3.607.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \frac{2+\sqrt[3]{1-5x}}{3+\sqrt[3]{1-5x}}\,dx = x+\frac{27\,\ln\left((1-5\,x)^{1/3}+3\right)}{5}-\frac{9\,(1-5\,x)^{1/3}}{5}+\frac{3\,(1-5\,x)^{2/3}}{10}$$

input
```
int(((1 - 5*x)^(1/3) + 2)/((1 - 5*x)^(1/3) + 3),x)
```

output
```
x + (27*log((1 - 5*x)^(1/3) + 3))/5 - (9*(1 - 5*x)^(1/3))/5 + (3*(1 - 5*x)
^(2/3))/10
```

3.608 $\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\,dx$

3.608.1 Optimal result

Integrand size = 17, antiderivative size = 21

$$\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\,dx = 4\sqrt{x}+x+4\log\left(1-\sqrt{x}\right)$$

output
```
x+4*ln(1-x^(1/2))+4*x^(1/2)
```

3.608.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\,dx = 4\sqrt{x}+x+4\log\left(-1+\sqrt{x}\right)$$

input
```
Integrate[(1 + Sqrt[x])/(-1 + Sqrt[x]),x]
```

output
```
4*Sqrt[x] + x + 4*Log[-1 + Sqrt[x]]
```

3.608.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {900, 25, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}+1}{\sqrt{x}-1}dx$$

$$\Big\downarrow \text{900}$$

$$2\int -\frac{\left(\sqrt{x}+1\right)\sqrt{x}}{1-\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{25}$$

$$-2\int \frac{\left(\sqrt{x}+1\right)\sqrt{x}}{1-\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{86}$$

$$-2\int \left(-\sqrt{x}-\frac{2}{\sqrt{x}-1}-2\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{x}{2}+2\sqrt{x}+2\log\left(1-\sqrt{x}\right)\right)$$

input
```
Int[(1 + Sqrt[x])/(-1 + Sqrt[x]),x]
```

output
```
2*(2*Sqrt[x] + x/2 + 2*Log[1 - Sqrt[x]])
```

3.608.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 900
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(a + b*x^(g*n)
)^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
 && NeQ[b*c - a*d, 0] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.608.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$x+4\sqrt{x}+4\ln\left(-1+\sqrt{x}\right)$	16
default	$x+4\sqrt{x}+4\ln\left(-1+\sqrt{x}\right)$	16
trager	$x-2+4\sqrt{x}+2\ln\left(2\sqrt{x}-1-x\right)$	22
meijerg	$2\sqrt{x}+4\ln\left(1-\sqrt{x}\right)+\frac{\sqrt{x}\,(6+3\sqrt{x})}{3}$	29

input
```
int((1+x^(1/2))/(-1+x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x+4*x^(1/2)+4*ln(-1+x^(1/2))
```

3.608.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\,dx = x+4\,\sqrt{x}+4\,\log\left(\sqrt{x}-1\right)$$

input
```
integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="fricas")
```

output
```
x + 4*sqrt(x) + 4*log(sqrt(x) - 1)
```

3.608.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\,dx = 4\sqrt{x}+x+4\log\left(\sqrt{x}-1\right)$$

input
```
integrate((1+x**(1/2))/(-1+x**(1/2)),x)
```

output
```
4*sqrt(x) + x + 4*log(sqrt(x) - 1)
```

3.608.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\,dx = x+4\,\sqrt{x}+4\,\log\left(\sqrt{x}-1\right)$$

input
```
integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="maxima")
```

output
```
x + 4*sqrt(x) + 4*log(sqrt(x) - 1)
```

3.608.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\,dx = x+4\sqrt{x}+4\log\left(\left|\sqrt{x}-1\right|\right)$$

input
```
integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="giac")
```

output
```
x + 4*sqrt(x) + 4*log(abs(sqrt(x) - 1))
```

3.608.9 Mupad [B] (verification not implemented)

Time = 17.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\,dx = x+4\ln\left(\sqrt{x}-1\right)+4\sqrt{x}$$

input
```
int((x^(1/2) + 1)/(x^(1/2) - 1),x)
```

output
```
x + 4*log(x^(1/2) - 1) + 4*x^(1/2)
```

3.609 $\int \frac{1-\sqrt{2+3x}}{1+\sqrt{2+3x}} \, dx$

3.609.1 Optimal result

Integrand size = 27, antiderivative size = 33

$$\int \frac{1-\sqrt{2+3x}}{1+\sqrt{2+3x}} \, dx = -x+\frac{4}{3}\sqrt{2+3x}-\frac{4}{3}\log \left(1+\sqrt{2+3x}\right)$$

output
```
-x-4/3*ln(1+(2+3*x)^(1/2))+4/3*(2+3*x)^(1/2)
```

3.609.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{1-\sqrt{2+3x}}{1+\sqrt{2+3x}} \, dx = \frac{1}{3}\left(-2-3x+4\sqrt{2+3x}-4\log \left(1+\sqrt{2+3x}\right)\right)$$

input
```
Integrate[(1 - Sqrt[2 + 3*x])/(1 + Sqrt[2 + 3*x]),x]
```

output
```
(-2 - 3*x + 4*Sqrt[2 + 3*x] - 4*Log[1 + Sqrt[2 + 3*x]])/3
```

3.609.3 Rubi [A] (warning: unable to verify)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {938, 900, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-\sqrt{3x+2}}{\sqrt{3x+2}+1}dx$$

$$\Big\downarrow \text{938}$$

$$\frac{1}{3}\int \frac{1-\sqrt{3x+2}}{\sqrt{3x+2}+1}d(3x+2)$$

$$\Big\downarrow \text{900}$$

$$\frac{2}{3}\int \frac{(-3x-1)\sqrt{3x+2}}{3x+3}d\sqrt{3x+2}$$

$$\Big\downarrow \text{86}$$

$$\frac{2}{3}\int \left(-3x-\frac{2}{3x+3}\right)d\sqrt{3x+2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}\left(\frac{1}{2}(-3x-2)+2\sqrt{3x+2}-2\log(3x+3)\right)$$

input
```
Int[(1 - Sqrt[2 + 3*x])/(1 + Sqrt[2 + 3*x]),x]
```

output
```
(2*((-2 - 3*x)/2 + 2*Sqrt[2 + 3*x] - 2*Log[3 + 3*x]))/3
```

3.609.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))))
```

rule 900
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(a + b*x^(g*n)
)^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
 && NeQ[b*c - a*d, 0] && FractionQ[n]
```

rule 938
```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Simp[1/Coefficient[u, x, 1]   Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
 x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u
, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.609.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-x-\frac{2}{3}+\frac{4\sqrt{3x+2}}{3}-\frac{4\ln\left(1+\sqrt{3x+2}\right)}{3}$	27
default	$-x-\frac{2}{3}+\frac{4\sqrt{3x+2}}{3}-\frac{4\ln\left(1+\sqrt{3x+2}\right)}{3}$	27
trager	$-x+\frac{4\sqrt{3x+2}}{3}-\frac{2\ln\left(2\sqrt{3x+2}+3+3x\right)}{3}$	31

input
```
int((1-(3*x+2)^(1/2))/(1+(3*x+2)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-x-2/3+4/3*(3*x+2)^(1/2)-4/3*ln(1+(3*x+2)^(1/2))
```

3.609.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1-\sqrt{2+3x}}{1+\sqrt{2+3x}}\,dx=-x+\frac{4}{3}\sqrt{3\,x+2}-\frac{4}{3}\log\left(\sqrt{3\,x+2}+1\right)$$

input
```
integrate((1-(2+3*x)^(1/2))/(1+(2+3*x)^(1/2)),x, algorithm="fricas")
```

output
```
-x + 4/3*sqrt(3*x + 2) - 4/3*log(sqrt(3*x + 2) + 1)
```

3.609.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{1-\sqrt{2+3x}}{1+\sqrt{2+3x}}\,dx = -x + \frac{4\sqrt{3x+2}}{3} - \frac{4\log\left(\sqrt{3x+2}+1\right)}{3}$$

input
```
integrate((1-(2+3*x)**(1/2))/(1+(2+3*x)**(1/2)),x)
```

output
```
-x + 4*sqrt(3*x + 2)/3 - 4*log(sqrt(3*x + 2) + 1)/3
```

3.609.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{1-\sqrt{2+3x}}{1+\sqrt{2+3x}}\,dx = -x + \frac{4}{3}\sqrt{3\,x+2} - \frac{4}{3}\log\left(\sqrt{3\,x+2}+1\right) - \frac{2}{3}$$

input
```
integrate((1-(2+3*x)^(1/2))/(1+(2+3*x)^(1/2)),x, algorithm="maxima")
```

output
```
-x + 4/3*sqrt(3*x + 2) - 4/3*log(sqrt(3*x + 2) + 1) - 2/3
```

3.609.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{1-\sqrt{2+3x}}{1+\sqrt{2+3x}}\,dx = -x + \frac{4}{3}\sqrt{3\,x+2} - \frac{4}{3}\log\left(\sqrt{3\,x+2}+1\right) - \frac{2}{3}$$

input
```
integrate((1-(2+3*x)^(1/2))/(1+(2+3*x)^(1/2)),x, algorithm="giac")
```

output
```
-x + 4/3*sqrt(3*x + 2) - 4/3*log(sqrt(3*x + 2) + 1) - 2/3
```

3.609.9 Mupad [B] (verification not implemented)

Time = 17.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1-\sqrt{2+3x}}{1+\sqrt{2+3x}}\,dx = \frac{4\sqrt{3\,x+2}}{3} - \frac{4\,\ln\left(\sqrt{3\,x+2}+1\right)}{3} - x$$

input
```
int(-((3*x + 2)^(1/2) - 1)/((3*x + 2)^(1/2) + 1),x)
```

output
```
(4*(3*x + 2)^(1/2))/3 - (4*log((3*x + 2)^(1/2) + 1))/3 - x
```

3.610 $\int \frac{-1+\sqrt{a+bx}}{1+\sqrt{a+bx}}\,dx$

3.610.1 Optimal result

Integrand size = 25, antiderivative size = 33

$$\int \frac{-1+\sqrt{a+bx}}{1+\sqrt{a+bx}}\,dx = x-\frac{4\sqrt{a+bx}}{b}+\frac{4\log\left(1+\sqrt{a+bx}\right)}{b}$$

output
```
x+4*ln(1+(b*x+a)^(1/2))/b-4*(b*x+a)^(1/2)/b
```

3.610.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{-1+\sqrt{a+bx}}{1+\sqrt{a+bx}}\,dx = \frac{a+bx-4\sqrt{a+bx}+4\log\left(b\left(1+\sqrt{a+bx}\right)\right)}{b}$$

input
```
Integrate[(-1 + Sqrt[a + b*x])/(1 + Sqrt[a + b*x]),x]
```

output
```
(a + b*x - 4*Sqrt[a + b*x] + 4*Log[b*(1 + Sqrt[a + b*x])])/b
```

3.610.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {938, 25, 900, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx}-1}{\sqrt{a+bx}+1}dx$$

$$\Big\downarrow \text{938}$$

$$\frac{\int -\frac{1-\sqrt{a+bx}}{\sqrt{a+bx}+1}d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{1-\sqrt{a+bx}}{\sqrt{a+bx}+1}d(a+bx)}{b}$$

$$\Big\downarrow \text{900}$$

$$-\frac{2\int \frac{(-a-bx+1)\sqrt{a+bx}}{\sqrt{a+bx}+1}d\sqrt{a+bx}}{b}$$

$$\Big\downarrow \text{86}$$

$$-\frac{2\int \left(-a-bx-\frac{2}{\sqrt{a+bx}+1}+2\right)d\sqrt{a+bx}}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(\frac{1}{2}(-a-bx)+2\sqrt{a+bx}-2\log\left(\sqrt{a+bx}+1\right)\right)}{b}$$

input
```
Int[(-1 + Sqrt[a + b*x])/(1 + Sqrt[a + b*x]),x]
```

output
```
(-2*((-a - b*x)/2 + 2*Sqrt[a + b*x] - 2*Log[1 + Sqrt[a + b*x]]))/b
```

3.610.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 900
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(a + b*x^(g*n)
)^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
 && NeQ[b*c - a*d, 0] && FractionQ[n]
```

rule 938
```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Simp[1/Coefficient[u, x, 1]   Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
 x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u
, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.610.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{bx+a-4\sqrt{bx+a}+4\ln\left(1+\sqrt{bx+a}\right)}{b}$	35
default	$\frac{bx+a-4\sqrt{bx+a}+4\ln\left(1+\sqrt{bx+a}\right)}{b}$	35

input
```
int((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/b*(1/2*b*x+1/2*a-2*(b*x+a)^(1/2)+2*ln(1+(b*x+a)^(1/2)))
```

3.610.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{-1+\sqrt{a+bx}}{1+\sqrt{a+bx}}\,dx = \frac{bx-4\sqrt{bx+a}+4\log\left(\sqrt{bx+a}+1\right)}{b}$$

input
```
integrate((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)),x, algorithm="fricas")
```

output
```
(b*x - 4*sqrt(b*x + a) + 4*log(sqrt(b*x + a) + 1))/b
```

3.610.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \frac{-1+\sqrt{a+bx}}{1+\sqrt{a+bx}}\,dx = \begin{cases} x-\frac{4\sqrt{a+bx}}{b}+\frac{4\log\left(\sqrt{a+bx}+1\right)}{b} & \text{for } b\neq 0 \\ \frac{x(\sqrt{a}-1)}{\sqrt{a}+1} & \text{otherwise} \end{cases}$$

input
```
integrate((-1+(b*x+a)**(1/2))/(1+(b*x+a)**(1/2)),x)
```

output
```
Piecewise((x - 4*sqrt(a + b*x)/b + 4*log(sqrt(a + b*x) + 1)/b, Ne(b, 0)),
(x*(sqrt(a) - 1)/(sqrt(a) + 1), True))
```

3.610.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{-1+\sqrt{a+bx}}{1+\sqrt{a+bx}}\,dx = \frac{bx+a-4\sqrt{bx+a}+4\log\left(\sqrt{bx+a}+1\right)}{b}$$

input
```
integrate((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)),x, algorithm="maxima")
```

output
```
(b*x + a - 4*sqrt(b*x + a) + 4*log(sqrt(b*x + a) + 1))/b
```

3.610.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int \frac{-1+\sqrt{a+bx}}{1+\sqrt{a+bx}}\,dx = \frac{4\log\left(\sqrt{bx+a}+1\right)}{b} + \frac{(bx+a)b-4\sqrt{bx+a}b}{b^2}$$

input
```
integrate((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)),x, algorithm="giac")
```

output
```
4*log(sqrt(b*x + a) + 1)/b + ((b*x + a)*b - 4*sqrt(b*x + a)*b)/b^2
```

3.610.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{-1+\sqrt{a+bx}}{1+\sqrt{a+bx}}\,dx = x + \frac{4\ln\left(\sqrt{a+b\,x}+1\right)}{b} - \frac{4\sqrt{a+b\,x}}{b}$$

input
```
int(((a + b*x)^(1/2) - 1)/((a + b*x)^(1/2) + 1),x)
```

output
```
x + (4*log((a + b*x)^(1/2) + 1))/b - (4*(a + b*x)^(1/2))/b
```

3.611 $\int \frac{a+bnx^{-1+n}}{ax+bx^n} dx$

3.611.1 Optimal result

Integrand size = 22, antiderivative size = 10

$$\int \frac{a+bnx^{-1+n}}{ax+bx^n} dx = \log(ax+bx^n)$$

output
```
ln(a*x+b*x^n)
```

3.611.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{a+bnx^{-1+n}}{ax+bx^n} dx = \log(ax+bx^n)$$

input
```
Integrate[(a + b*n*x^(-1 + n))/(a*x + b*x^n),x]
```

output
```
Log[a*x + b*x^n]
```

3.611.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(10) = 20.

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.70, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2027, 1016, 948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bnx^{n-1}}{ax+bx^n}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x^{-n}\left(a+bnx^{n-1}\right)}{ax^{1-n}+b}dx$$

$$\Big\downarrow \text{1016}$$

$$\int \frac{ax^{1-n}+bn}{x\left(ax^{1-n}+b\right)}dx$$

$$\Big\downarrow \text{948}$$

$$\frac{\int \frac{x^{n-1}\left(ax^{1-n}+bn\right)}{ax^{1-n}+b}dx^{1-n}}{1-n}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(nx^{n-1}+\frac{a-an}{ax^{1-n}+b}\right)dx^{1-n}}{1-n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(1-n)\log\left(ax^{1-n}+b\right)+n\log\left(x^{1-n}\right)}{1-n}$$

input
```
Int[(a + b*n*x^(-1 + n))/(a*x + b*x^n),x]
```

output
```
(n*Log[x^(1 - n)] + (1 - n)*Log[b + a*x^(1 - n)])/(1 - n)
```

3.611.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.611.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

method	result	size
risch	$\ln\left(x^{n}+\frac{a x}{b}\right)$	12
norman	$\ln\left(a x +b\,{\mathrm e}^{n \ln\left(x \right)}\right)$	13

input
```
int((a+b*n*x^(-1+n))/(a*x+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
ln(x^n+a*x/b)
```

3.611.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{a+bnx^{-1+n}}{ax+bx^n}\,dx=\log\left(ax+bx^n\right)$$

input
```
integrate((a+b*n*x^(-1+n))/(a*x+b*x^n),x, algorithm="fricas")
```

output
```
log(a*x + b*x^n)
```

3.611.6 Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{a+bnx^{-1+n}}{ax+bx^n}\,dx=\begin{cases}\log\left(x+\frac{bx^n}{a}\right) & \text{for } a\neq 0\\ n\log\left(x\right) & \text{otherwise}\end{cases}$$

input
```
integrate((a+b*n*x**(-1+n))/(a*x+b*x**n),x)
```

output
```
Piecewise((log(x + b*x**n/a), Ne(a, 0)), (n*log(x), True))
```

3.611.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{a+bnx^{-1+n}}{ax+bx^n}\,dx=\log\left(ax+bx^n\right)$$

input
```
integrate((a+b*n*x^(-1+n))/(a*x+b*x^n),x, algorithm="maxima")
```

output
```
log(a*x + b*x^n)
```

3.611.8 Giac [F]

$$\int \frac{a + bnx^{-1+n}}{ax + bx^n} \, dx = \int \frac{bnx^{n-1} + a}{ax + bx^n} \, dx$$

input
```
integrate((a+b*n*x^(-1+n))/(a*x+b*x^n),x, algorithm="giac")
```

output
```
integrate((b*n*x^(n - 1) + a)/(a*x + b*x^n), x)
```

3.611.9 Mupad [B] (verification not implemented)

Time = 17.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{a + bnx^{-1+n}}{ax + bx^n} \, dx = \ln \left(b\, x^n + a\, x\right)$$

input
```
int((a + b*n*x^(n - 1))/(b*x^n + a*x),x)
```

output
```
log(b*x^n + a*x)
```

3.612 $\int \frac{x^{-n}\left(a+bnx^{-1+n}\right)}{b+ax^{1-n}}\,dx$

3.612.1 Optimal result

Integrand size = 29, antiderivative size = 17

$$\int \frac{x^{-n}(a+bnx^{-1+n})}{b+ax^{1-n}}\,dx = n\log(x)+\log\left(b+ax^{1-n}\right)$$

output
```
n*ln(x)+ln(b+a*x^(1-n))
```

3.612.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \frac{x^{-n}(a+bnx^{-1+n})}{b+ax^{1-n}}\,dx = \log\left(ax+bx^n\right)$$

input
```
Integrate[(a + b*n*x^(-1 + n))/(x^n*(b + a*x^(1 - n))),x]
```

output
```
Log[a*x + b*x^n]
```

3.612.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1016, 948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{-n}\left(a+bnx^{n-1}\right)}{ax^{1-n}+b}dx$$
$$\Big\downarrow \text{ 1016}$$
$$\int \frac{ax^{1-n}+bn}{x\left(ax^{1-n}+b\right)}dx$$
$$\Big\downarrow \text{ 948}$$
$$\frac{\int \frac{x^{n-1}\left(ax^{1-n}+bn\right)}{ax^{1-n}+b}dx^{1-n}}{1-n}$$
$$\Big\downarrow \text{ 86}$$
$$\frac{\int \left(nx^{n-1}+\frac{a-an}{ax^{1-n}+b}\right)dx^{1-n}}{1-n}$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{(1-n)\log\left(ax^{1-n}+b\right)+n\log\left(x^{1-n}\right)}{1-n}$$

input
```
Int[(a + b*n*x^(-1 + n))/(x^n*(b + a*x^(1 - n))),x]
```

output
```
(n*Log[x^(1 - n)] + (1 - n)*Log[b + a*x^(1 - n)])/(1 - n)
```

3.612.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.612.4 Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

method	result	size
risch	$\ln\left(x^n+\frac{ax}{b}\right)$	12
norman	$\ln\left(ax+b\,\mathrm{e}^{n\ln(x)}\right)$	13

input
```
int((a+b*n*x^(-1+n))/(x^n)/(b+a*x^(1-n)),x,method=_RETURNVERBOSE)
```

output
```
ln(x^n+a*x/b)
```

3.612.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \frac{x^{-n}(a+bnx^{-1+n})}{b+ax^{1-n}}\,dx=\log\left(ax+bx^{n}\right)$$

input
```
integrate((a+b*n*x^(-1+n))/(x^n)/(b+a*x^(1-n)),x, algorithm="fricas")
```

output
```
log(a*x + b*x^n)
```

3.612.6 Sympy [A] (verification not implemented)

Time = 12.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{x^{-n}(a+bnx^{-1+n})}{b+ax^{1-n}}\,dx=\begin{cases} n\log\left(x\right)+\log\left(x\right)+\log\left(x^{-n}+\frac{b}{ax}\right) & \text{for } a\neq 0\\ n\log\left(x\right) & \text{otherwise}\end{cases}$$

input
```
integrate((a+b*n*x**(-1+n))/(x**n)/(b+a*x**(1-n)),x)
```

output
```
Piecewise((n*log(x) + log(x) + log(x**(-n) + b/(a*x)), Ne(a, 0)), (n*log(x
), True))
```

3.612.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(17) = 34.

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 5.06

$$\int \frac{x^{-n}(a+bnx^{-1+n})}{b+ax^{1-n}}\,dx=bn\left(\frac{\log\left(x\right)}{b}-\frac{n\log\left(x\right)}{b(n-1)}+\frac{\log\left(\frac{ax+bx^{n}}{b}\right)}{b(n-1)}\right)+a\left(\frac{n\log\left(x\right)}{a(n-1)}-\frac{\log\left(\frac{ax+bx^{n}}{b}\right)}{a(n-1)}\right)$$

input
```
integrate((a+b*n*x^(-1+n))/(x^n)/(b+a*x^(1-n)),x, algorithm="maxima")
```

output
```
b*n*(log(x)/b - n*log(x)/(b*(n - 1)) + log((a*x + b*x^n)/b)/(b*(n - 1))) +
 a*(n*log(x)/(a*(n - 1)) - log((a*x + b*x^n)/b)/(a*(n - 1)))
```

3.612.8 Giac [F]

$$\int \frac{x^{-n}(a+bnx^{-1+n})}{b+ax^{1-n}}\,dx = \int \frac{bnx^{n-1}+a}{(ax^{-n+1}+b)x^n}\,dx$$

input
```
integrate((a+b*n*x^(-1+n))/(x^n)/(b+a*x^(1-n)),x, algorithm="giac")
```

output
```
integrate((b*n*x^(n - 1) + a)/((a*x^(-n + 1) + b)*x^n), x)
```

3.612.9 Mupad [B] (verification not implemented)

Time = 17.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.29

$$\int \frac{x^{-n}(a+bnx^{-1+n})}{b+ax^{1-n}}\,dx = -\frac{\ln\left(b+a\,x^{1-n}\right)-2\,n\,\mathrm{atanh}\left(\frac{2\,a\,x^{1-n}}{b}+1\right)}{n-1}$$

input
```
int((a + b*n*x^(n - 1))/(x^n*(b + a*x^(1 - n))),x)
```

output
```
-(log(b + a*x^(1 - n)) - 2*n*atanh((2*a*x^(1 - n))/b + 1))/(n - 1)
```

3.613 $\int x(a+bx+cx^2)^m\left(d+ex+fx^2+gx^3\right)^n(2ad+(3$

3.613.1 Optimal result

Integrand size = 176, antiderivative size = 37

$$\int x\left(a+bx+cx^2\right)^m\left(d+ex+fx^2+gx^3\right)^n\left(2ad+(3bd+3ae+bdm+aen)x+(4cd+4be+4af+2cdm+bem+ben+2afn)x^2+(5ce+5bf+5ag+2cem+bfm+cen+2bfn+3agn)x^3+(6cf+6bg+2cfm+bgm+2cfn+3bgn)x^4+cg(7+2m+3n)x^5\right)\,dx=x^2\left(a+bx+cx^2\right)^{1+m}\left(d+ex+fx^2+gx^3\right)^{1+n}$$

output
```
x^2*(c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)
```

3.613.2 Mathematica [A] (verified)

Time = 3.89 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int x\left(a+bx+cx^2\right)^m\left(d+ex+fx^2+gx^3\right)^n\left(2ad+(3bd+3ae+bdm+aen)x+(4cd+4be+4af+2cdm+bem+ben+2afn)x^2+(5ce+5bf+5ag+2cem+bfm+cen+2bfn+3agn)x^3+(6cf+6bg+2cfm+bgm+2cfn+3bgn)x^4+cg(7+2m+3n)x^5\right)\,dx=x^2(a+x(b+cx))^{1+m}(d+x(e+x(f+gx)))^{1+n}$$

input
```
Integrate[x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(2*a*d + (3*b*
d + 3*a*e + b*d*m + a*e*n)*x + (4*c*d + 4*b*e + 4*a*f + 2*c*d*m + b*e*m +
b*e*n + 2*a*f*n)*x^2 + (5*c*e + 5*b*f + 5*a*g + 2*c*e*m + b*f*m + c*e*n +
2*b*f*n + 3*a*g*n)*x^3 + (6*c*f + 6*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*
g*n)*x^4 + c*g*(7 + 2*m + 3*n)*x^5),x]
```

output
```
x^2*(a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n)
```

3.613.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.006$, Rules used = {2023}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx+cx^2)^m\left(d+ex+fx^2+gx^3\right)^n\left(x^2(2afn+4af+bem+ben+4be+2cdm+4cd)+x^3(3agn+5ag\right.$$

$$\Big\downarrow \ 2023$$

$$x^2\left(a+bx+cx^2\right)^{m+1}\left(d+ex+fx^2+gx^3\right)^{n+1}$$

input
```
Int[x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(2*a*d + (3*b*d + 3*
a*e + b*d*m + a*e*n)*x + (4*c*d + 4*b*e + 4*a*f + 2*c*d*m + b*e*m + b*e*n
+ 2*a*f*n)*x^2 + (5*c*e + 5*b*f + 5*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*
n + 3*a*g*n)*x^3 + (6*c*f + 6*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x
^4 + c*g*(7 + 2*m + 3*n)*x^5),x]
```

output
```
x^2*(a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

3.613.3.1 Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

3.613.4 Maple [A] (verified)

Time = 56.81 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

method	result
gosper	$x^2(c\,x^2+bx+a)^{1+m}\,(g\,x^3+f\,x^2+ex+d)^{1+n}$
risch	$x^2(cg\,x^5+bg\,x^4+cf\,x^4+ag\,x^3+bf\,x^3+c\,x^3e+af\,x^2+e\,x^2b+x^2cd+aex+bdx+ad)$
parallelrisch	$\frac{x^7(c\,x^2+bx+a)^m(g\,x^3+f\,x^2+ex+d)^n c^2g^2+x^6(c\,x^2+bx+a)^m(g\,x^3+f\,x^2+ex+d)^n bc\,g^2+x^6(c\,x^2+bx+a)^m(g\,x^3+f\,x^2+ex+}{}$

input
```
int(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*e+3*b*
d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x^2+(3*a*g*n+b*f*m+2*
b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+
6*b*g+6*c*f)*x^4+c*g*(7+2*m+3*n)*x^5),x,method=_RETURNVERBOSE)
```

output
```
x^2*(c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)
```

3.613.5 Fricas [F(-1)]

Timed out.

$$\int x(a+bx+cx^2)^m\,(d+ex+fx^2+gx^3)^n\,(2ad+(3bd+3ae+bdm+aen)x + (4cd+4be+4af+2cdm+bem+ben+2afn)x^2 + (5ce+5bf+5ag+2cem+bfm+cen+2bfn+3agn)x^3 + (6cf+6bg+2cfm+bgm+2cfn+3bgn)x^4+cg(7+2m+3n)x^5)\,dx = \text{Timed out}$$

input
```
integrate(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*
e+3*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x^2+(3*a*g*n+b*
f*m+2*b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*
c*f*n+6*b*g+6*c*f)*x^4+c*g*(7+2*m+3*n)*x^5),x, algorithm="fricas")
```

output
```
Timed out
```

3.613.6 Sympy [F(-1)]

Timed out.

$$\int x(a+bx+cx^2)^m\left(d+ex+fx^2+gx^3\right)^n\left(2ad+(3bd+3ae+bdm+aen)x+(4cd+4be+4af+2cdm+bem+ben+2afn)x^2+(5ce+5bf+5ag+2cem+bfm+cen+2bfn+3agn)x^3+(6cf+6bg+2cfm+bgm+2cfn+3bgn)x^4+cg(7+2m+3n)x^5\right)\,dx=\text{Timed out}$$

input
```
integrate(x*(c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(2*a*d+(a*e*n+b*d*m
+3*a*e+3*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x**2+(3*a*
g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x**3+(b*g*m+3*b*g*n+2*c
*f*m+2*c*f*n+6*b*g+6*c*f)*x**4+c*g*(7+2*m+3*n)*x**5),x)
```

output
```
Timed out
```

3.613.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(37) = 74.

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.62

$$\int x(a+bx+cx^2)^m\left(d+ex+fx^2+gx^3\right)^n\left(2ad+(3bd+3ae+bdm+aen)x+(4cd+4be+4af+2cdm+bem+ben+2afn)x^2+(5ce+5bf+5ag+2cem+bfm+cen+2bfn+3agn)x^3+(6cf+6bg+2cfm+bgm+2cfn+3bgn)x^4+cg(7+2m+3n)x^5\right)\,dx=\left(cgx^7+(cf+bg)x^6+(ce+bf+ag)x^5+(cd+be+af)x^4+adx^2+(bd+ae)x^3\right)e^{(n\log(gx^3+fx^2+ex+d)+m}$$

input
```
integrate(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*
e+3*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x^2+(3*a*g*n+b*
f*m+2*b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*
c*f*n+6*b*g+6*c*f)*x^4+c*g*(7+2*m+3*n)*x^5),x, algorithm="maxima")
```

output
```
(c*g*x^7 + (c*f + b*g)*x^6 + (c*e + b*f + a*g)*x^5 + (c*d + b*e + a*f)*x^4
 + a*d*x^2 + (b*d + a*e)*x^3)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*
x^2 + b*x + a))
```

3.613.8 Giac [F(-1)]

Timed out.

$$\int x(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(2ad+(3bd+3ae+bdm+aen)x + (4cd+4be+4af+2cdm+bem+ben+2afn)x^2 + (5ce+5bf+5ag+2cem+bfm+cen+2bfn+3agn)x^3 + (6cf+6bg+2cfm+bgm+2cfn+3bgn)x^4+cg(7+2m+3n)x^5)\,dx = \text{Timed out}$$

input
```
integrate(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*
e+3*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x^2+(3*a*g*n+b*
f*m+2*b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*
c*f*n+6*b*g+6*c*f)*x^4+c*g*(7+2*m+3*n)*x^5),x, algorithm="giac")
```

output
```
Timed out
```

3.613.9 Mupad [F(-1)]

Timed out.

$$\int x(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(2ad+(3bd+3ae+bdm+aen)x + (4cd+4be+4af+2cdm+bem+ben+2afn)x^2 + (5ce+5bf+5ag+2cem+bfm+cen+2bfn+3agn)x^3 + (6cf+6bg+2cfm+bgm+2cfn+3bgn)x^4+cg(7+2m+3n)x^5)\,dx = \text{Hanged}$$

3.613. $\int x(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(2ad+(3bd+3ae+bdm+aen)x+(4cd+4be+4af+2cdm+$

input
```
int(x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(2*a*d + x^4*(6*b*g
+ 6*c*f + b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) + x^2*(4*a*f + 4*b*e + 4*c*
d + b*e*m + 2*c*d*m + 2*a*f*n + b*e*n) + x*(3*a*e + 3*b*d + b*d*m + a*e*n)
 + x^3*(5*a*g + 5*b*f + 5*c*e + b*f*m + 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*
n) + c*g*x^5*(2*m + 3*n + 7)),x)
```

output
```
\text{Hanged}
```

3.613.

$\int x(a+bx+cx^2)^m\left(d+ex+fx^2+gx^3\right)^n\left(2ad+(3bd+3ae+bdm+aen)x+(4cd+4be+4af+2cdm+\right.$

3.614 $\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (ad+(2bd$

3.614.1 Optimal result

Integrand size = 174, antiderivative size = 35

$$\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (ad+(2bd+2ae+bdm+aen)x + (3cd+3be+3af+2cdm+bem+ben+2afn)x^2 + (4ce+4bf+4ag+2cem+bfm+cen+2bfn+3agn)x^3 + (5cf+5bg+2cfm+bgm+2cfn+3bgn)x^4+cg(6+2m+3n)x^5)\,dx = x(a+bx+cx^2)^{1+m}(d+ex+fx^2+gx^3)^{1+n}$$

output
```
x*(c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)
```

3.614.2 Mathematica [A] (verified)

Time = 10.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (ad+(2bd+2ae+bdm+aen)x + (3cd+3be+3af+2cdm+bem+ben+2afn)x^2 + (4ce+4bf+4ag+2cem+bfm+cen+2bfn+3agn)x^3 + (5cf+5bg+2cfm+bgm+2cfn+3bgn)x^4+cg(6+2m+3n)x^5)\,dx = x(a+x(b+cx))^{1+m}(d+x(e+x(f+gx)))^{1+n}$$

input
```
Integrate[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*d + (2*b*d +
2*a*e + b*d*m + a*e*n)*x + (3*c*d + 3*b*e + 3*a*f + 2*c*d*m + b*e*m + b*e*
n + 2*a*f*n)*x^2 + (4*c*e + 4*b*f + 4*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*
f*n + 3*a*g*n)*x^3 + (5*c*f + 5*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)
*x^4 + c*g*(6 + 2*m + 3*n)*x^5),x]
```

output
```
x*(a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n)
```

3.614.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.006$, Rules used = {2023}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (x^2(2afn+3af+bem+ben+3be+2cdm+3cd)+x^3(3agn+4ag$$

$$\Big\downarrow \text{2023}$$

$$x(a+bx+cx^2)^{m+1}(d+ex+fx^2+gx^3)^{n+1}$$

input
```
Int[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*d + (2*b*d + 2*a*e
+ b*d*m + a*e*n)*x + (3*c*d + 3*b*e + 3*a*f + 2*c*d*m + b*e*m + b*e*n + 2*
a*f*n)*x^2 + (4*c*e + 4*b*f + 4*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n +
3*a*g*n)*x^3 + (5*c*f + 5*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 +
 c*g*(6 + 2*m + 3*n)*x^5),x]
```

output
```
x*(a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

3.614.3.1 Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

3.614.4 Maple [A] (verified)

Time = 43.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

method	result
gosper	$x(c\,x^2+bx+a)^{1+m}\,(g\,x^3+f\,x^2+ex+d)^{1+n}$
risch	$x(cg\,x^5+bg\,x^4+cf\,x^4+ag\,x^3+bf\,x^3+c\,x^3e+af\,x^2+e\,x^2b+x^2cd+aex+bdx+ad)\,($
parallelrisch	$\frac{x^6(c\,x^2+bx+a)^m(g\,x^3+f\,x^2+ex+d)^n c^2g^2+x^5(c\,x^2+bx+a)^m(g\,x^3+f\,x^2+ex+d)^n bc\,g^2+x^5(c\,x^2+bx+a)^m(g\,x^3+f\,x^2+ex+}{}$

input
```
int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*b*d)*x
+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*
n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+5*b*
g+5*c*f)*x^4+c*g*(6+2*m+3*n)*x^5),x,method=_RETURNVERBOSE)
```

output
```
x*(c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)
```

3.614.5 Fricas [F(-1)]

Timed out.

$$\int \left(a+bx+cx^2\right)^m \left(d+ex+fx^2+gx^3\right)^n \left(ad+(2bd+2ae+bdm+aen)x + (3cd+3be+3af+2cdm+bem+ben+2afn)x^2 + (4ce+4bf+4ag+2cem+bfm+cen+2bfn+3agn)x^3 + (5cf+5bg+2cfm+bgm+2cfn+3bgn)x^4+cg(6+2m+3n)x^5\right)\,dx = \text{Timed out}$$

3.614.

$\int (a+bx+cx^2)^m \left(d+ex+fx^2+gx^3\right)^n \left(ad+(2bd+2ae+bdm+aen)x+(3cd+3be+3af+2cdm+b\right.$

input
```
integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*
b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x^2+(3*a*g*n+b*f*m+
2*b*f*n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*
n+5*b*g+5*c*f)*x^4+c*g*(6+2*m+3*n)*x^5),x, algorithm="fricas")
```

output
```
Timed out
```

3.614.6 Sympy [F(-1)]

Timed out.

$$\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (ad+(2bd+2ae+bdm+aen)x + (3cd+3be+3af+2cdm+bem+ben+2afn)x^2 + (4ce+4bf+4ag+2cem+bfm+cen+2bfn+3agn)x^3 + (5cf+5bg+2cfm+bgm+2cfn+3bgn)x^4+cg(6+2m+3n)x^5)\, dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(a*d+(a*e*n+b*d*m+2*a
*e+2*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x**2+(3*a*g*n+
b*f*m+2*b*f*n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x**3+(b*g*m+3*b*g*n+2*c*f*m
+2*c*f*n+5*b*g+5*c*f)*x**4+c*g*(6+2*m+3*n)*x**5),x)
```

output
```
Timed out
```

3.614.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(35) = 70$.

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.71

$$\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (ad+(2bd+2ae+bdm+aen)x + (3cd+3be+3af+2cdm+bem+ben+2afn)x^2 + (4ce+4bf+4ag+2cem+bfm+cen+2bfn+3agn)x^3 + (5cf+5bg+2cfm+bgm+2cfn+3bgn)x^4+cg(6+2m+3n)x^5)\, dx = (cgx^6+(cf+bg)x^5+(ce+bf+ag)x^4+(cd+be+af)x^3+adx+(bd+ae)x^2)e^{(n\log(gx^3+fx^2+ex+d)+m\log}$$

input
```
integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*
b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x^2+(3*a*g*n+b*f*m+
2*b*f*n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*
n+5*b*g+5*c*f)*x^4+c*g*(6+2*m+3*n)*x^5),x, algorithm="maxima")
```

output
```
(c*g*x^6 + (c*f + b*g)*x^5 + (c*e + b*f + a*g)*x^4 + (c*d + b*e + a*f)*x^3
 + a*d*x + (b*d + a*e)*x^2)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*x^
2 + b*x + a))
```

3.614.8 Giac [F(-1)]

Timed out.

$$\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (ad+(2bd+2ae+bdm+aen)x + (3cd+3be+3af+2cdm+bem+ben+2afn)x^2 + (4ce+4bf+4ag+2cem+bfm+cen+2bfn+3agn)x^3 + (5cf+5bg+2cfm+bgm+2cfn+3bgn)x^4+cg(6+2m+3n)x^5)\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*
b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x^2+(3*a*g*n+b*f*m+
2*b*f*n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*
n+5*b*g+5*c*f)*x^4+c*g*(6+2*m+3*n)*x^5),x, algorithm="giac")
```

output
```
Timed out
```

3.614.9 Mupad [F(-1)]

Timed out.

$$\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (ad+(2bd+2ae+bdm+aen)x + (3cd+3be+3af+2cdm+bem+ben+2afn)x^2 + (4ce+4bf+4ag+2cem+bfm+cen+2bfn+3agn)x^3 + (5cf+5bg+2cfm+bgm+2cfn+3bgn)x^4+cg(6+2m+3n)x^5)\,dx = \text{Hanged}$$

3.614.

$\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (ad+(2bd+2ae+bdm+aen)x+(3cd+3be+3af+2cdm+b$

input
```
int((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*d + x^4*(5*b*g + 5*
c*f + b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) + x^2*(3*a*f + 3*b*e + 3*c*d +
b*e*m + 2*c*d*m + 2*a*f*n + b*e*n) + x*(2*a*e + 2*b*d + b*d*m + a*e*n) + x
^3*(4*a*g + 4*b*f + 4*c*e + b*f*m + 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) +
 c*g*x^5*(2*m + 3*n + 6)),x)
```

output
```
\text{Hanged}
```

3.614.

$\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (ad+(2bd+2ae+bdm+aen)x+(3cd+3be+3af+2cdm+b$

3.615 $\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (bd+ae+$

3.615.1 Optimal result

Integrand size = 164, antiderivative size = 34

$$\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (bd+ae+bdm+aen+(2cd+2be+2af+2cdm+bem+ben+2afn)x+(3ce+3bf+3ag+2cem+bfm+cen+2bfn+3agn)x^2+(4cf+4bg+2cfm+bgm+2cfn+3bgn)x^3+cg(5+2m+3n)x^4)\,dx=(a+bx+cx^2)^{1+m}(d+ex+fx^2+gx^3)^{1+n}$$

output
```
(c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)
```

3.615.2 Mathematica [A] (verified)

Time = 6.96 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (bd+ae+bdm+aen+(2cd+2be+2af+2cdm+bem+ben+2afn)x+(3ce+3bf+3ag+2cem+bfm+cen+2bfn+3agn)x^2+(4cf+4bg+2cfm+bgm+2cfn+3bgn)x^3+cg(5+2m+3n)x^4)\,dx=(a+x(b+cx))^{1+m}(d+x(e+x(f+gx)))^{1+n}$$

input
```
Integrate[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(b*d + a*e + b*d
*m + a*e*n + (2*c*d + 2*b*e + 2*a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x
 + (3*c*e + 3*b*f + 3*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x
^2 + (4*c*f + 4*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^3 + c*g*(5 +
2*m + 3*n)*x^4),x]
```

output
```
(a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n)
```

3.615.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.006$, Rules used = {2023}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (x(2afn+2af+bem+ben+2be+2cdm+2cd)+x^2(3agn+3ag+$$

$$\Big\downarrow \text{2023}$$

$$(a+bx+cx^2)^{m+1} (d+ex+fx^2+gx^3)^{n+1}$$

input
```
Int[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(b*d + a*e + b*d*m + a
*e*n + (2*c*d + 2*b*e + 2*a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x + (3*
c*e + 3*b*f + 3*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^2 + (
4*c*f + 4*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^3 + c*g*(5 + 2*m +
3*n)*x^4),x]
```

output
```
(a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

3.615.

$\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (bd+ae+bdm+aen+(2cd+2be+2af+2cdm+bem+ben+$

3.615.3.1 Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

3.615.4 Maple [A] (verified)

Time = 32.99 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

method	result
gosper	$\left(c\,x^{2}+bx+a\right)^{1+m} \left(g\,x^{3}+f\,x^{2}+ex+d\right)^{1+n}$
risch	$\left(cg\,x^{5}+bg\,x^{4}+cf\,x^{4}+ag\,x^{3}+bf\,x^{3}+c\,x^{3}e+af\,x^{2}+e\,x^{2}b+x^{2}cd+aex+bdx+ad\right)(c$
parallelrisch	$\frac{x^{5}\left(c\,x^{2}+bx+a\right)^{m}\left(g\,x^{3}+f\,x^{2}+ex+d\right)^{n}c^{2}g^{2}+x^{4}\left(c\,x^{2}+bx+a\right)^{m}\left(g\,x^{3}+f\,x^{2}+ex+d\right)^{n}bc\,g^{2}+x^{4}\left(c\,x^{2}+bx+a\right)^{m}\left(g\,x^{3}+f\,x^{2}+ex+\right.}{}$

input
```
int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*f*n+b*
e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*
n+3*a*g+3*b*f+3*c*e)*x^2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)*x^3+c
*g*(5+2*m+3*n)*x^4),x,method=_RETURNVERBOSE)
```

output
```
(c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)
```

3.615.5 Fricas [F(-1)]

Timed out.

$$\int \left(a+bx+cx^2\right)^m \left(d+ex+fx^2+gx^3\right)^n \left(bd+ae+bdm+aen+(2cd+2be+2af+2cdm+bem+ben+2afn)x+(3ce+3bf+3ag+2cem+bfm+cen+2bfn+3agn)x^2+(4cf+4bg+2cfm+bgm+2cfn+3bgn)x^3+cg(5+2m+3n)x^4\right)\,dx=\text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*
f*n+b*e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*
m+c*e*n+3*a*g+3*b*f+3*c*e)*x^2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)
*x^3+c*g*(5+2*m+3*n)*x^4),x, algorithm="fricas")
```

output
```
Timed out
```

3.615.6 Sympy [F(-1)]

Timed out.

$$\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (bd+ae+bdm+aen+(2cd+2be+2af+2cdm+bem+ben+2afn)x+(3ce+3bf+3ag+2cem+bfm+cen+2bfn+3agn)x^2+(4cf+4bg+2cfm+bgm+2cfn+3bgn)x^3+cg(5+2m+3n)x^4)\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(b*d+a*e+b*d*m+a*e*n+
(2*a*f*n+b*e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2
*c*e*m+c*e*n+3*a*g+3*b*f+3*c*e)*x**2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+
4*c*f)*x**3+c*g*(5+2*m+3*n)*x**4),x)
```

output
```
Timed out
```

3.615.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(34) = 68.

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.71

$$\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (bd+ae+bdm+aen+(2cd+2be+2af+2cdm+bem+ben+2afn)x+(3ce+3bf+3ag+2cem+bfm+cen+2bfn+3agn)x^2+(4cf+4bg+2cfm+bgm+2cfn+3bgn)x^3+cg(5+2m+3n)x^4)\,dx = (cgx^5+(cf+bg)x^4+(ce+bf+ag)x^3+(cd+be+af)x^2+ad+(bd+ae)x)e^{(n\log(gx^3+fx^2+ex+d)+m\log(}$$

input
```
integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*
f*n+b*e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*
m+c*e*n+3*a*g+3*b*f+3*c*e)*x^2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)
*x^3+c*g*(5+2*m+3*n)*x^4),x, algorithm="maxima")
```

output
```
(c*g*x^5 + (c*f + b*g)*x^4 + (c*e + b*f + a*g)*x^3 + (c*d + b*e + a*f)*x^2
 + a*d + (b*d + a*e)*x)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*x^2 +
b*x + a))
```

3.615.8 Giac [F(-1)]

Timed out.

$$\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (bd+ae+bdm+aen+(2cd+2be+2af+2cdm+bem+ben+2afn)x+(3ce+3bf+3ag+2cem+bfm+cen+2bfn+3agn)x^2+(4cf+4bg+2cfm+bgm+2cfn+3bgn)x^3+cg(5+2m+3n)x^4)\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*
f*n+b*e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*
m+c*e*n+3*a*g+3*b*f+3*c*e)*x^2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)
*x^3+c*g*(5+2*m+3*n)*x^4),x, algorithm="giac")
```

output
```
Timed out
```

3.615.9 Mupad [B] (verification not implemented)

Time = 27.14 (sec) , antiderivative size = 148, normalized size of antiderivative = 4.35

$$\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (bd+ae+bdm+aen+(2cd+2be+2af+2cdm+bem+ben+2afn)x+(3ce+3bf+3ag+2cem+bfm+cen+2bfn+3agn)x^2+(4cf+4bg+2cfm+bgm+2cfn+3bgn)x^3+cg(5+2m+3n)x^4)\,dx = (g\,x^3+f\,x^2+e\,x+d)^n\,\left(x^4\,(b\,g+c\,f)\,(c\,x^2+b\,x+a)^m+x^2\,(c\,x^2+b\,x+a)^m\,(a\,f+b\,e+c\,d)+x^3\,(c\,x^2+b\,x+a)^m\,(a\,g+b\,f+c\,e)+a\,d\,(c\,x^2+b\,x+a)^m+x\,(a\,e+b\,d)\,(c\,x^2+b\,x+a)^m+c\,g\,x^5\,(c\,x^2+b\,x+a)^m\right)$$

input
```
int((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*e + b*d + x^3*(4*b*
g + 4*c*f + b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) + x^2*(3*a*g + 3*b*f + 3*
c*e + b*f*m + 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) + x*(2*a*f + 2*b*e + 2*
c*d + b*e*m + 2*c*d*m + 2*a*f*n + b*e*n) + b*d*m + a*e*n + c*g*x^4*(2*m +
3*n + 5)),x)
```

output
```
(d + e*x + f*x^2 + g*x^3)^n*(x^4*(b*g + c*f)*(a + b*x + c*x^2)^m + x^2*(a
+ b*x + c*x^2)^m*(a*f + b*e + c*d) + x^3*(a + b*x + c*x^2)^m*(a*g + b*f +
c*e) + a*d*(a + b*x + c*x^2)^m + x*(a*e + b*d)*(a + b*x + c*x^2)^m + c*g*x
^5*(a + b*x + c*x^2)^m)
```

3.615.

$\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (bd+ae+bdm+aen+(2cd+2be+2af+2cdm+bem+ben+$

3.616 $\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-ad+(bdm+aen)x+(cd+be+af+$

3.616.1 Optimal result

Integrand size = 167, antiderivative size = 37

$$\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2$$

$$= \frac{(a+bx+cx^2)^{1+m}(d+ex+fx^2+gx^3)^{1+n}}{x}$$

output
```
(c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)/x
```

3.616.2 Mathematica [A] (verified)

Time = 8.77 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2$$

$$= \frac{(a+x(b+cx))^{1+m}(d+x(e+x(f+gx)))^{1+n}}{x}$$

input
```
Integrate[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-(a*d) + (b*d*
m + a*e*n)*x + (c*d + b*e + a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 +
 (2*c*e + 2*b*f + 2*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3
 + (3*c*f + 3*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(4 + 2*
m + 3*n)*x^5))/x^2,x]
```

output
```
((a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n))/x
```

3.616.

$\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2afn)x^2+(2ce+2bf+2ag+2cem+bfm+cen+2bfn+3a}{x^2}$

3.616.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (x^2(2afn+af+bem+ben+be+2cdm+cd)+x^3(3agn+2ag+bf$$

$$\Big\downarrow \text{7293}$$

$$\int \Big(cgx^3(2m+3n+4)(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n + x^2(a+bx+cx^2)^m (bg(m+3n+3)+cf(2m$$

$$\Big\downarrow \text{2009}$$

$$(af(2n+1)+be(m+n+1)+c(2dm+d))\int (cx^2+bx+a)^m (gx^3+fx^2+ex+d)^n dx - ad\int \frac{(cx^2+bx+a)^m (gx^3+fx^2+ex+d)^n}{x^2}dx + (aen+bdm)\int \frac{(cx^2+bx+a)^m (gx^3+fx^2+ex+d)^n}{x}dx + (ag(3n+2)+bf(m+2n+2)+ce(2m+n+2))\int x(cx^2+bx+a)^m (gx^3+fx^2+ex+d)^n dx + (bg(m+3n+3)+cf(2m+2n+3))\int x^2(cx^2+bx+a)^m (gx^3+fx^2+ex+d)^n dx + cg(2m+3n+4)\int x^3(cx^2+bx+a)^m (gx^3+fx^2+ex+d)^n dx$$

input
```
Int[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-(a*d) + (b*d*m + a*
e*n)*x + (c*d + b*e + a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (2*c*
e + 2*b*f + 2*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 + (3*
c*f + 3*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(4 + 2*m + 3*
n)*x^5))/x^2,x]
```

output
```
$Aborted
```

3.616.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.616.4 Maple [A] (verified)

Time = 84.55 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

method	result
gosper	$\frac{(c x^2+bx+a)^{1+m} (g x^3+f x^2+ex+d)^{1+n}}{x}$
risch	$\frac{(cg x^5+bg x^4+cf x^4+ag x^3+bf x^3+c x^3 e+af x^2+e x^2 b+x^2 cd+aex+bdx+ad) (c x^2+bx+a)^m (g x^3+f x^2+ex+d)^n}{x}$
parallelrisch	$x^5 (c x^2+bx+a)^m (g x^3+f x^2+ex+d)^n c^2 g^2+x^4 (c x^2+bx+a)^m (g x^3+f x^2+ex+d)^n bc g^2+x^4 (c x^2+bx+a)^m (g x^3+f x^2+ex+$

input
```
int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a*f*n+b
*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+2
*a*g+2*b*f+2*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x^4+c*g*
(4+2*m+3*n)*x^5)/x^2,x,method=_RETURNVERBOSE)
```

output
```
(c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)/x
```

3.616.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2$$

= Timed out

input
```
integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a
*f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c
*e*n+2*a*g+2*b*f+2*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x^
4+c*g*(4+2*m+3*n)*x^5)/x^2,x, algorithm="fricas")
```

output
```
Timed out
```

3.616.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2$$

= Timed out

input
```
integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(-a*d+(a*e*n+b*d*m)*x
+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x**2+(3*a*g*n+b*f*m+2*b*f*n+2*c
*e*m+c*e*n+2*a*g+2*b*f+2*c*e)*x**3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*
c*f)*x**4+c*g*(4+2*m+3*n)*x**5)/x**2,x)
```

output
```
Timed out
```

3.616.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(37) = 74.

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.57

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2$$

$$= \frac{(cgx^5+(cf+bg)x^4+(ce+bf+ag)x^3+(cd+be+af)x^2+ad+(bd+ae)x)e^{(n\log(gx^3+fx^2+ex+d)+m\log(}}{x}$$

input
```
integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a
*f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c
*e*n+2*a*g+2*b*f+2*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x^
4+c*g*(4+2*m+3*n)*x^5)/x^2,x, algorithm="maxima")
```

output
```
(c*g*x^5 + (c*f + b*g)*x^4 + (c*e + b*f + a*g)*x^3 + (c*d + b*e + a*f)*x^2
 + a*d + (b*d + a*e)*x)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*x^2 +
b*x + a))/x
```

3.616.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2$$

$= \text{Timed out}$

input
```
integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a
*f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c
*e*n+2*a*g+2*b*f+2*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x^
4+c*g*(4+2*m+3*n)*x^5)/x^2,x, algorithm="giac")
```

output
```
Timed out
```

3.616.9 Mupad [B] (verification not implemented)

Time = 25.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2$$

$$= \frac{(c\,x^2+b\,x+a)^{m+1}\,(g\,x^3+f\,x^2+e\,x+d)^{n+1}}{x}$$

input
```
int(((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(x^4*(3*b*g + 3*c*f +
 b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) - a*d + x^2*(a*f + b*e + c*d + b*e*m
 + 2*c*d*m + 2*a*f*n + b*e*n) + x*(b*d*m + a*e*n) + x^3*(2*a*g + 2*b*f + 2
*c*e + b*f*m + 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) + c*g*x^5*(2*m + 3*n +
 4)))/x^2,x)
```

output
```
((a + b*x + c*x^2)^(m + 1)*(d + e*x + f*x^2 + g*x^3)^(n + 1))/x
```

3.616.

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2afn)x^2+(2ce+2bf+2ag+2cem+bfm+cen+2bfn+3ag}{x^2}$$

3.617 $\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-2ad+(-bd-ae+bdm+aen)x+(2$

3.617.1 Optimal result

Integrand size = 163, antiderivative size = 37

$$\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2af$$

$$=\frac{(a+bx+cx^2)^{1+m}(d+ex+fx^2+gx^3)^{1+n}}{x^2}$$

output
```
(c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)/x^2
```

3.617.2 Mathematica [A] (verified)

Time = 9.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2af$$

$$=\frac{(a+x(b+cx))^{1+m}(d+x(e+x(f+gx)))^{1+n}}{x^2}$$

input
```
Integrate[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-2*a*d + (-(b*
d) - a*e + b*d*m + a*e*n)*x + (2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (c
*e + b*f + a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 + (2*c*f
 + 2*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(3 + 2*m + 3*n)*
x^5))/x^3,x]
```

output
```
((a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n))/x^2
```

3.617. $\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2afn)x^2+(ce+bf+ag+2cem+bfm+cen+2bfn+3agn)x^3}{x^3}$

3.617.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (x^2(2afn+bem+ben+2cdm)+x^3(3agn+ag+bfm+2bfn+bf$$

$$\Big\downarrow \ 7293$$

$$\int \Big(cgx^2(2m+3n+3)(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n + ce(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n \Big(\frac{a(}{}$$

$$\Big\downarrow \ 2009$$

$$(ag(3n+1)+bf(m+2n+1)+ce(2m+n+1))\int (cx^2+bx+a)^m (gx^3+fx^2+ex+d)^n dx - 2ad\int \frac{(cx^2+bx+a)^m (gx^3+fx^2+ex+d)^n}{x^3}dx - (ae(1-n)+bd(1-m))\int \frac{(cx^2+bx+a)^m (gx^3+fx^2+ex+d)^n}{x^2}dx + (2afn+be(m+n)+2cdm)\int \frac{(cx^2+bx+a)^m (gx^3+fx^2+ex+d)^n}{x}dx + (bg(m+3n+2)+2cf(m+n+1))\int x(cx^2+bx+a)^m (gx^3+fx^2+ex+d)^n dx + cg(2m+3n+3)\int x^2(cx^2+bx+a)^m (gx^3+fx^2+ex+d)^n dx$$

input
```
Int[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-2*a*d + (-(b*d) - a
*e + b*d*m + a*e*n)*x + (2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (c*e + b
*f + a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 + (2*c*f + 2*b
*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(3 + 2*m + 3*n)*x^5))/
x^3,x]
```

output
```
$Aborted
```

3.617.
$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2afn)x^2+(ce+bf+ag+2cem+bfm+cen+2bfn+3agn)x}{x^3}$$

3.617.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.617.4 Maple [A] (verified)

Time = 82.80 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

method	result
gosper	$\frac{(c\,x^2+bx+a)^{1+m}(g\,x^3+f\,x^2+ex+d)^{1+n}}{x^2}$
risch	$\frac{(cg\,x^5+bg\,x^4+cf\,x^4+ag\,x^3+bf\,x^3+c\,x^3e+af\,x^2+e\,x^2b+x^2cd+aex+bdx+ad)(c\,x^2+bx+a)^m(g\,x^3+f\,x^2+ex+d)^n}{x^2}$
parallelrisch	$x^5(c\,x^2+bx+a)^m(g\,x^3+f\,x^2+ex+d)^n c^2g^2+x^4(c\,x^2+bx+a)^m(g\,x^3+f\,x^2+ex+d)^n bc\,g^2+x^4(c\,x^2+bx+a)^m(g\,x^3+f\,x^2+ex+$

input
```
int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-b*d)*x+
(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+a*g
+b*f+c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x^4+c*g*(3+2*m+3
*n)*x^5)/x^3,x,method=_RETURNVERBOSE)
```

output
```
(c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)/x^2
```

3.617.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2af$$
$$= \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-b
*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e
*n+a*g+b*f+c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x^4+c*g*(3
+2*m+3*n)*x^5)/x^3,x, algorithm="fricas")
```

output
```
Timed out
```

3.617.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2af$$

= Timed out

input
```
integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(-2*a*d+(a*e*n+b*d*m-
a*e-b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x**2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e
*m+c*e*n+a*g+b*f+c*e)*x**3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x**
4+c*g*(3+2*m+3*n)*x**5)/x**3,x)
```

output
```
Timed out
```

3.617.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(37) = 74$.

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.57

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2af$$

$$= \frac{(cgx^5+(cf+bg)x^4+(ce+bf+ag)x^3+(cd+be+af)x^2+ad+(bd+ae)x)e^{(n\log(gx^3+fx^2+ex+d)+m\log(}}{x^2}$$

input
```
integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-b
*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e
*n+a*g+b*f+c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x^4+c*g*(3
+2*m+3*n)*x^5)/x^3,x, algorithm="maxima")
```

output
```
(c*g*x^5 + (c*f + b*g)*x^4 + (c*e + b*f + a*g)*x^3 + (c*d + b*e + a*f)*x^2
 + a*d + (b*d + a*e)*x)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*x^2 +
b*x + a))/x^2
```

3.617. $\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2afn)x^2+(ce+bf+ag+2cem+bfm+cen+2bfn+3agn)x}{x^3}$

3.617.8 Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2af$$

$= \text{Timed out}$

input
```
integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-b
*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e
*n+a*g+b*f+c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x^4+c*g*(3
+2*m+3*n)*x^5)/x^3,x, algorithm="giac")
```

output
```
Timed out
```

3.617.9 Mupad [B] (verification not implemented)

Time = 26.02 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.95

$$\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2af$$

$$= (cx^2+bx+a)^m(gx^3+fx^2+ex+d)^n(af+be+cd+cgx^3+agx+bfx+cex+bgx^2+cfx^2)+\frac{(ae+bd)(cx^2+bx+a)^m(gx^3+fx^2+ex+d)^n}{x}+\frac{ad(cx^2+bx+a)^m(gx^3+fx^2+ex+d)^n}{x^2}$$

input
```
int(((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(x^4*(2*b*g + 2*c*f +
 b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) - 2*a*d - x*(a*e + b*d - b*d*m - a*e
*n) + x^2*(b*e*m + 2*c*d*m + 2*a*f*n + b*e*n) + x^3*(a*g + b*f + c*e + b*f
*m + 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) + c*g*x^5*(2*m + 3*n + 3)))/x^3,
x)
```

output
```
(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*f + b*e + c*d + c*g*x^3
 + a*g*x + b*f*x + c*e*x + b*g*x^2 + c*f*x^2) + ((a*e + b*d)*(a + b*x + c*
x^2)^m*(d + e*x + f*x^2 + g*x^3)^n)/x + (a*d*(a + b*x + c*x^2)^m*(d + e*x
+ f*x^2 + g*x^3)^n)/x^2
```

3.618 $\int x^3 \left(a+b\sqrt{c+dx}\right)^2 \, dx$

3.618.1 Optimal result

Integrand size = 19, antiderivative size = 185

$$\int x^3 \left(a+b\sqrt{c+dx}\right)^2 \, dx = -\frac{a^2c^3x}{d^3} - \frac{4abc^3(c+dx)^{3/2}}{3d^4} + \frac{c^2(3a^2-b^2c)\,(c+dx)^2}{2d^4} + \frac{12abc^2(c+dx)^{5/2}}{5d^4} - \frac{c(a^2-b^2c)\,(c+dx)^3}{d^4} - \frac{12abc(c+dx)^{7/2}}{7d^4} + \frac{(a^2-3b^2c)\,(c+dx)^4}{4d^4} + \frac{4ab(c+dx)^{9/2}}{9d^4} + \frac{b^2(c+dx)^5}{5d^4}$$

output
```
-a^2*c^3*x/d^3-4/3*a*b*c^3*(d*x+c)^(3/2)/d^4+1/2*c^2*(-b^2*c+3*a^2)*(d*x+c
)^2/d^4+12/5*a*b*c^2*(d*x+c)^(5/2)/d^4-c*(-b^2*c+a^2)*(d*x+c)^3/d^4-12/7*a
*b*c*(d*x+c)^(7/2)/d^4+1/4*(-3*b^2*c+a^2)*(d*x+c)^4/d^4+4/9*a*b*(d*x+c)^(9
/2)/d^4+1/5*b^2*(d*x+c)^5/d^4
```

3.618.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.57

$$\int x^3 \left(a+b\sqrt{c+dx}\right)^2 \, dx = -\frac{16ab\sqrt{c+dx}(16c^4-8c^3dx+6c^2d^2x^2-5cd^3x^3-35d^4x^4)+315a^2(c^4-d^4x^4)+63b^2(c^5-5cd^4x^4-4d}{1260d^4}$$

input
```
Integrate[x^3*(a + b*Sqrt[c + d*x])^2,x]
```

output
```
-1/1260*(16*a*b*Sqrt[c + d*x]*(16*c^4 - 8*c^3*d*x + 6*c^2*d^2*x^2 - 5*c*d^
3*x^3 - 35*d^4*x^4) + 315*a^2*(c^4 - d^4*x^4) + 63*b^2*(c^5 - 5*c*d^4*x^4
- 4*d^5*x^5))/d^4
```

3.618.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\sqrt{c+dx}\right)^2 dx$$

$$\Big\downarrow \ 896$$

$$\frac{\int d^3x^3\left(a+b\sqrt{c+dx}\right)^2 d(c+dx)}{d^4}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int -d^3x^3\left(a+b\sqrt{c+dx}\right)^2 d(c+dx)}{d^4}$$

$$\Big\downarrow \ 1732$$

$$-\frac{2\int -d^3x^3\sqrt{c+dx}\left(a+b\sqrt{c+dx}\right)^2 d\sqrt{c+dx}}{d^4}$$

$$\Big\downarrow \ 522$$

$$-\frac{2\int \left(-b^2(c+dx)^{9/2}-2ab(c+dx)^4-\left(a^2-3b^2c\right)(c+dx)^{7/2}+6abc(c+dx)^3-3c\left(b^2c-a^2\right)(c+dx)^{5/2}-6abc$$
$$d^4$$

$$\Big\downarrow \ 2009$$

$$-\frac{2\left(-\frac{1}{4}c^2\left(3a^2-b^2c\right)(c+dx)^2-\frac{1}{8}\left(a^2-3b^2c\right)(c+dx)^4+\frac{1}{2}c\left(a^2-b^2c\right)(c+dx)^3+\frac{1}{2}a^2c^3(c+dx)+\frac{2}{3}abc^3(c+d$$
$$d^4$$

input
```
Int[x^3*(a + b*Sqrt[c + d*x])^2,x]
```

output
```
(-2*((a^2*c^3*(c + d*x))/2 + (2*a*b*c^3*(c + d*x)^(3/2))/3 - (c^2*(3*a^2 -
 b^2*c)*(c + d*x)^2)/4 - (6*a*b*c^2*(c + d*x)^(5/2))/5 + (c*(a^2 - b^2*c)*
(c + d*x)^3)/2 + (6*a*b*c*(c + d*x)^(7/2))/7 - ((a^2 - 3*b^2*c)*(c + d*x)^
4)/8 - (2*a*b*(c + d*x)^(9/2))/9 - (b^2*(c + d*x)^5)/10))/d^4
```

3.618.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.618.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.42

method	result
default	$b^{2}\left(\frac{1}{5}x^{5}d+\frac{1}{4}c\,x^{4}\right)+\frac{4ab\left(\frac{(dx+c)^{\frac{9}{2}}}{9}-\frac{3c(dx+c)^{\frac{7}{2}}}{7}+\frac{3c^{2}(dx+c)^{\frac{5}{2}}}{5}-\frac{c^{3}(dx+c)^{\frac{3}{2}}}{3}\right)}{d^{4}}+\frac{a^{2}x^{4}}{4}$
trager	$\frac{\left(4b^{2}dx+5b^{2}c+5a^{2}\right)x^{4}}{20}-\frac{4ab\left(-35d^{4}x^{4}-5d^{3}c\,x^{3}+6c^{2}x^{2}d^{2}-8c^{3}dx+16c^{4}\right)\sqrt{dx+c}}{315d^{4}}$
derivativedivides	$\frac{\frac{b^{2}(dx+c)^{5}}{5}+\frac{4ab(dx+c)^{\frac{9}{2}}}{9}+\frac{\left(-3b^{2}c+a^{2}\right)(dx+c)^{4}}{4}-\frac{12cab(dx+c)^{\frac{7}{2}}}{7}+\frac{\left(3b^{2}c^{2}-3c\,a^{2}\right)(dx+c)^{3}}{3}+\frac{12c^{2}ab(dx+c)^{\frac{5}{2}}}{5}+\frac{\left(-b^{2}c^{3}+3a^{2}c^{2}\right)}{2}}{d^{4}}$

input
```
int(x^3*(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
b^2*(1/5*x^5*d+1/4*c*x^4)+4*a*b/d^4*(1/9*(d*x+c)^(9/2)-3/7*c*(d*x+c)^(7/2)
+3/5*c^2*(d*x+c)^(5/2)-1/3*c^3*(d*x+c)^(3/2))+1/4*a^2*x^4
```

3.618.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.51

$$\int x^3 \left(a+b\sqrt{c+dx}\right)^2 \, dx$$
$$= \frac{252\, b^2 d^5 x^5 + 315\, (b^2 c + a^2) d^4 x^4 + 16\, (35\, abd^4 x^4 + 5\, abcd^3 x^3 - 6\, abc^2 d^2 x^2 + 8\, abc^3 dx - 16\, abc^4)\sqrt{dx+c}}{1260\, d^4}$$

input
```
integrate(x^3*(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/1260*(252*b^2*d^5*x^5 + 315*(b^2*c + a^2)*d^4*x^4 + 16*(35*a*b*d^4*x^4 +
 5*a*b*c*d^3*x^3 - 6*a*b*c^2*d^2*x^2 + 8*a*b*c^3*d*x - 16*a*b*c^4)*sqrt(d*
x + c))/d^4
```

3.618.6 Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.96

$$\int x^3 \left(a+b\sqrt{c+dx}\right)^2 \, dx$$
$$= \begin{cases} \frac{2\left(-\frac{a^2c^3(c+dx)}{2} - \frac{2abc^3(c+dx)^{\frac{3}{2}}}{3} + \frac{6abc^2(c+dx)^{\frac{5}{2}}}{5} - \frac{6abc(c+dx)^{\frac{7}{2}}}{7} + \frac{2ab(c+dx)^{\frac{9}{2}}}{9} + \frac{b^2(c+dx)^5}{10} + \frac{(a^2-3b^2c)(c+dx)^4}{8} + \frac{(c+dx)^3(-3a^2c+3b^2c^2)}{6} + \frac{(c+dx)^2 \cdots}{\cdots}\right.}{d^4} \\ \frac{x^4(a+b\sqrt{c})^2}{4} \end{cases}$$

input
```
integrate(x**3*(a+b*(d*x+c)**(1/2))**2,x)
```

output
```
Piecewise((2*(-a**2*c**3*(c + d*x)/2 - 2*a*b*c**3*(c + d*x)**(3/2)/3 + 6*a
*b*c**2*(c + d*x)**(5/2)/5 - 6*a*b*c*(c + d*x)**(7/2)/7 + 2*a*b*(c + d*x)*
*(9/2)/9 + b**2*(c + d*x)**5/10 + (a**2 - 3*b**2*c)*(c + d*x)**4/8 + (c +
d*x)**3*(-3*a**2*c + 3*b**2*c**2)/6 + (c + d*x)**2*(3*a**2*c**2 - b**2*c**
3)/4)/d**4, Ne(d, 0)), (x**4*(a + b*sqrt(c))**2/4, True))
```

3.618.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.82

$$\int x^3\left(a+b\sqrt{c+dx}\right)^2\,dx$$
$$=\frac{252\,(dx+c)^5b^2+560\,(dx+c)^{\frac{9}{2}}ab-2160\,(dx+c)^{\frac{7}{2}}abc+3024\,(dx+c)^{\frac{5}{2}}abc^2-1680\,(dx+c)^{\frac{3}{2}}abc^3-126}{1260}$$

input
```
integrate(x^3*(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
1/1260*(252*(d*x + c)^5*b^2 + 560*(d*x + c)^(9/2)*a*b - 2160*(d*x + c)^(7/
2)*a*b*c + 3024*(d*x + c)^(5/2)*a*b*c^2 - 1680*(d*x + c)^(3/2)*a*b*c^3 - 1
260*(d*x + c)*a^2*c^3 - 315*(3*b^2*c - a^2)*(d*x + c)^4 + 1260*(b^2*c^2 -
a^2*c)*(d*x + c)^3 - 630*(b^2*c^3 - 3*a^2*c^2)*(d*x + c)^2)/d^4
```

3.618.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.82

$$\int x^3\left(a+b\sqrt{c+dx}\right)^2\,dx$$
$$=\frac{252\,b^2d^2x^5+315\,b^2cdx^4+315\,a^2dx^4+\frac{144\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}c^3\right)abc}{d^3}+\frac{16\left(35\,(dx+c)^{\frac{9}{2}}-180\right.}{}}{1260\,d}$$

input
```
integrate(x^3*(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
1/1260*(252*b^2*d^2*x^5 + 315*b^2*c*d*x^4 + 315*a^2*d*x^4 + 144*(5*(d*x +
c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c
)*c^3)*a*b*c/d^3 + 16*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d
*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b/d
^3)/d
```

3.618.9 Mupad [B] (verification not implemented)

Time = 17.65 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.90

$$\int x^3\left(a+b\sqrt{c+dx}\right)^2\,dx=\frac{b^2\,(c+d\,x)^5}{5\,d^4}-\frac{(6\,b^2\,c-2\,a^2)\,\,(c+d\,x)^4}{8\,d^4}+\frac{(6\,a^2\,c^2-2\,b^2\,c^3)\,\,(c+d\,x)^2}{4\,d^4}-\frac{a^2\,c^3\,x}{d^3}+\frac{4\,a\,b\,(c+d\,x)^{9/2}}{9\,d^4}+\frac{c\,(b^2\,c-a^2)\,\,(c+d\,x)^3}{d^4}-\frac{4\,a\,b\,c^3\,(c+d\,x)^{3/2}}{3\,d^4}+\frac{12\,a\,b\,c^2\,(c+d\,x)^{5/2}}{5\,d^4}-\frac{12\,a\,b\,c\,(c+d\,x)^{7/2}}{7\,d^4}$$

input
```
int(x^3*(a + b*(c + d*x)^(1/2))^2,x)
```

output
```
(b^2*(c + d*x)^5)/(5*d^4) - ((6*b^2*c - 2*a^2)*(c + d*x)^4)/(8*d^4) + ((6*
a^2*c^2 - 2*b^2*c^3)*(c + d*x)^2)/(4*d^4) - (a^2*c^3*x)/d^3 + (4*a*b*(c +
d*x)^(9/2))/(9*d^4) + (c*(b^2*c - a^2)*(c + d*x)^3)/d^4 - (4*a*b*c^3*(c +
d*x)^(3/2))/(3*d^4) + (12*a*b*c^2*(c + d*x)^(5/2))/(5*d^4) - (12*a*b*c*(c
+ d*x)^(7/2))/(7*d^4)
```

3.619 $\int x^2\left(a+b\sqrt{c+dx}\right)^2\,dx$

3.619.1 Optimal result

Integrand size = 19, antiderivative size = 138

$$\int x^2\left(a+b\sqrt{c+dx}\right)^2\,dx=\frac{a^2c^2x}{d^2}+\frac{4abc^2(c+dx)^{3/2}}{3d^3}-\frac{c(2a^2-b^2c)\,(c+dx)^2}{2d^3}-\frac{8abc(c+dx)^{5/2}}{5d^3}+\frac{(a^2-2b^2c)\,(c+dx)^3}{3d^3}+\frac{4ab(c+dx)^{7/2}}{7d^3}+\frac{b^2(c+dx)^4}{4d^3}$$

output
```
a^2*c^2*x/d^2+4/3*a*b*c^2*(d*x+c)^(3/2)/d^3-1/2*c*(-b^2*c+2*a^2)*(d*x+c)^2
/d^3-8/5*a*b*c*(d*x+c)^(5/2)/d^3+1/3*(-2*b^2*c+a^2)*(d*x+c)^3/d^3+4/7*a*b*
(d*x+c)^(7/2)/d^3+1/4*b^2*(d*x+c)^4/d^3
```

3.619.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.67

$$\int x^2\left(a+b\sqrt{c+dx}\right)^2\,dx=\frac{140a^2(c^3+d^3x^3)+16ab\sqrt{c+dx}(8c^3-4c^2dx+3cd^2x^2+15d^3x^3)+35b^2(c^4+4cd^3x^3+3d^4x^4)}{420d^3}$$

input
```
Integrate[x^2*(a + b*Sqrt[c + d*x])^2,x]
```

output
```
(140*a^2*(c^3 + d^3*x^3) + 16*a*b*Sqrt[c + d*x]*(8*c^3 - 4*c^2*d*x + 3*c*d
^2*x^2 + 15*d^3*x^3) + 35*b^2*(c^4 + 4*c*d^3*x^3 + 3*d^4*x^4))/(420*d^3)
```

3.619.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {896, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\sqrt{c+dx}\right)^2 dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int d^2x^2\left(a+b\sqrt{c+dx}\right)^2 d(c+dx)}{d^3}$$

$$\Big\downarrow \text{1732}$$

$$\frac{2\int d^2x^2\sqrt{c+dx}\left(a+b\sqrt{c+dx}\right)^2 d\sqrt{c+dx}}{d^3}$$

$$\Big\downarrow \text{522}$$

$$\frac{2\int \left(b^2(c+dx)^{7/2}+2ab(c+dx)^3+\left(a^2-2b^2c\right)(c+dx)^{5/2}-4abc(c+dx)^2+c\left(b^2c-2a^2\right)(c+dx)^{3/2}+2abc^2(c}{d^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{1}{6}\left(a^2-2b^2c\right)(c+dx)^3-\frac{1}{4}c\left(2a^2-b^2c\right)(c+dx)^2+\frac{1}{2}a^2c^2(c+dx)+\frac{2}{3}abc^2(c+dx)^{3/2}+\frac{2}{7}ab(c+dx)^{7/2}-\frac{4}{5}abc\right.}{d^3}$$

input
```
Int[x^2*(a + b*Sqrt[c + d*x])^2,x]
```

output
```
(2*((a^2*c^2*(c + d*x))/2 + (2*a*b*c^2*(c + d*x)^(3/2))/3 - (c*(2*a^2 - b^
2*c)*(c + d*x)^2)/4 - (4*a*b*c*(c + d*x)^(5/2))/5 + ((a^2 - 2*b^2*c)*(c +
d*x)^3)/6 + (2*a*b*(c + d*x)^(7/2))/7 + (b^2*(c + d*x)^4)/8))/d^3
```

3.619.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.619.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.48

method	result	siz
default	$b^{2} \left(\frac{1}{4} d\,x^{4}+\frac{1}{3} c\,x^{3}\right)+\frac{4 a b \left(\frac{\left(d x +c \right)^{\frac{7}{2}}}{7}-\frac{2 c \left(d x +c \right)^{\frac{5}{2}}}{5}+\frac{c^{2} \left(d x +c \right)^{\frac{3}{2}}}{3}\right)}{d^{3}}+\frac{a^{2} x^{3}}{3}$	66
trager	$\frac{\left(3 b^{2} d x +4 b^{2} c +4 a^{2}\right) x^{3}}{12}+\frac{4 a b \left(15 d^{3} x^{3}+3 c\,d^{2} x^{2}-4 c^{2} d x +8 c^{3}\right) \sqrt{d x +c}}{105 d^{3}}$	70
derivativedivides	$\frac{\frac{b^{2} \left(d x +c \right)^{4}}{4}+\frac{4 a b \left(d x +c \right)^{\frac{7}{2}}}{7}+\frac{\left(-2 b^{2} c +a^{2}\right) \left(d x +c \right)^{3}}{3}-\frac{8 c a b \left(d x +c \right)^{\frac{5}{2}}}{5}+\frac{\left(b^{2} c^{2}-2 c\,a^{2}\right) \left(d x +c \right)^{2}}{2}+\frac{4 c^{2} a b \left(d x +c \right)^{\frac{3}{2}}}{3}+c^{2} a^{2} \left(d x +c \right)}{d^{3}}$	11

input
```
int(x^2*(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
b^2*(1/4*d*x^4+1/3*c*x^3)+4*a*b/d^3*(1/7*(d*x+c)^(7/2)-2/5*c*(d*x+c)^(5/2)
+1/3*c^2*(d*x+c)^(3/2))+1/3*a^2*x^3
```

3.619.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.59

$$\int x^2\left(a+b\sqrt{c+dx}\right)^2\,dx = \frac{105\,b^2d^4x^4+140\,(b^2c+a^2)d^3x^3+16\,(15\,abd^3x^3+3\,abcd^2x^2-4\,abc^2dx+8\,abc^3)\sqrt{dx+c}}{420\,d^3}$$

input
```
integrate(x^2*(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/420*(105*b^2*d^4*x^4 + 140*(b^2*c + a^2)*d^3*x^3 + 16*(15*a*b*d^3*x^3 +
3*a*b*c*d^2*x^2 - 4*a*b*c^2*d*x + 8*a*b*c^3)*sqrt(d*x + c))/d^3
```

3.619.6 Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

$$\int x^2\left(a+b\sqrt{c+dx}\right)^2\,dx = \begin{cases} \frac{2\left(\frac{a^2c^2(c+dx)}{2}+\frac{2abc^2(c+dx)^{\frac{3}{2}}}{3}-\frac{4abc(c+dx)^{\frac{5}{2}}}{5}+\frac{2ab(c+dx)^{\frac{7}{2}}}{7}+\frac{b^2(c+dx)^4}{8}+\frac{(a^2-2b^2c)(c+dx)^3}{6}+\frac{(c+dx)^2(-2a^2c+b^2c^2)}{4}\right)}{d^3} & \text{for } d\neq 0 \\ \frac{x^3(a+b\sqrt{c})^2}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(a+b*(d*x+c)**(1/2))**2,x)
```

output
```
Piecewise((2*(a**2*c**2*(c + d*x)/2 + 2*a*b*c**2*(c + d*x)**(3/2)/3 - 4*a*
b*c*(c + d*x)**(5/2)/5 + 2*a*b*(c + d*x)**(7/2)/7 + b**2*(c + d*x)**4/8 +
(a**2 - 2*b**2*c)*(c + d*x)**3/6 + (c + d*x)**2*(-2*a**2*c + b**2*c**2)/4)
/d**3, Ne(d, 0)), (x**3*(a + b*sqrt(c))**2/3, True))
```

3.619.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.81

$$\int x^2\left(a+b\sqrt{c+dx}\right)^2\,dx$$
$$=\frac{105\,(dx+c)^4b^2+240\,(dx+c)^{\frac{7}{2}}ab-672\,(dx+c)^{\frac{5}{2}}abc+560\,(dx+c)^{\frac{3}{2}}abc^2+420\,(dx+c)a^2c^2-140\,(2\,b^2}{420\,d^3}$$

input
```
integrate(x^2*(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
1/420*(105*(d*x + c)^4*b^2 + 240*(d*x + c)^(7/2)*a*b - 672*(d*x + c)^(5/2)
*a*b*c + 560*(d*x + c)^(3/2)*a*b*c^2 + 420*(d*x + c)*a^2*c^2 - 140*(2*b^2*
c - a^2)*(d*x + c)^3 + 210*(b^2*c^2 - 2*a^2*c)*(d*x + c)^2)/d^3
```

3.619.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.92

$$\int x^2\left(a+b\sqrt{c+dx}\right)^2\,dx$$
$$=\frac{105\,b^2d^2x^4+140\,b^2cdx^3+140\,a^2dx^3+\frac{112\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}c^2\right)abc}{d^2}+\frac{48\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(d}{d^2}}{420\,d}$$

input
```
integrate(x^2*(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
1/420*(105*b^2*d^2*x^4 + 140*b^2*c*d*x^3 + 140*a^2*d*x^3 + 112*(3*(d*x + c
)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*b*c/d^2 + 48*(5*(
d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d
*x + c)*c^3)*a*b/d^2)/d
```

3.619.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.90

$$\int x^2\left(a+b\sqrt{c+dx}\right)^2\,dx=\frac{b^2\,(c+d\,x)^4}{4\,d^3}-\frac{(4\,a^2\,c-2\,b^2\,c^2)\,(c+d\,x)^2}{4\,d^3}-\frac{(4\,b^2\,c-2\,a^2)\,(c+d\,x)^3}{6\,d^3}+\frac{a^2\,c^2\,x}{d^2}+\frac{4\,a\,b\,(c+d\,x)^{7/2}}{7\,d^3}+\frac{4\,a\,b\,c^2\,(c+d\,x)^{3/2}}{3\,d^3}-\frac{8\,a\,b\,c\,(c+d\,x)^{5/2}}{5\,d^3}$$

input
```
int(x^2*(a + b*(c + d*x)^(1/2))^2,x)
```

output
```
(b^2*(c + d*x)^4)/(4*d^3) - ((4*a^2*c - 2*b^2*c^2)*(c + d*x)^2)/(4*d^3) -
((4*b^2*c - 2*a^2)*(c + d*x)^3)/(6*d^3) + (a^2*c^2*x)/d^2 + (4*a*b*(c + d*
x)^(7/2))/(7*d^3) + (4*a*b*c^2*(c + d*x)^(3/2))/(3*d^3) - (8*a*b*c*(c + d*
x)^(5/2))/(5*d^3)
```

3.620 $\int x\left(a+b\sqrt{c+dx}\right)^2\,dx$

3.620.1 Optimal result

Integrand size = 17, antiderivative size = 89

$$\int x\left(a+b\sqrt{c+dx}\right)^2\,dx=-\frac{a^2cx}{d}-\frac{4abc(c+dx)^{3/2}}{3d^2}+\frac{(a^2-b^2c)\,(c+dx)^2}{2d^2}+\frac{4ab(c+dx)^{5/2}}{5d^2}+\frac{b^2(c+dx)^3}{3d^2}$$

output
```
-a^2*c*x/d-4/3*a*b*c*(d*x+c)^(3/2)/d^2+1/2*(-b^2*c+a^2)*(d*x+c)^2/d^2+4/5*
a*b*(d*x+c)^(5/2)/d^2+1/3*b^2*(d*x+c)^3/d^2
```

3.620.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int x\left(a+b\sqrt{c+dx}\right)^2\,dx=\frac{(c+dx)\left(-15a^2(c-dx)-8ab(2c-3dx)\sqrt{c+dx}+5b^2(-c^2+cdx+2d^2x^2)\right)}{30d^2}$$

input
```
Integrate[x*(a + b*Sqrt[c + d*x])^2,x]
```

output
```
((c + d*x)*(-15*a^2*(c - d*x) - 8*a*b*(2*c - 3*d*x)*Sqrt[c + d*x] + 5*b^2*
(-c^2 + c*d*x + 2*d^2*x^2)))/(30*d^2)
```

3.620.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sqrt{c+dx}\right)^2 dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int dx\left(a+b\sqrt{c+dx}\right)^2 d(c+dx)}{d^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -dx\left(a+b\sqrt{c+dx}\right)^2 d(c+dx)}{d^2}$$

$$\Big\downarrow \text{1732}$$

$$-\frac{2\int -dx\sqrt{c+dx}\left(a+b\sqrt{c+dx}\right)^2 d\sqrt{c+dx}}{d^2}$$

$$\Big\downarrow \text{522}$$

$$-\frac{2\int \left(-b^2(c+dx)^{5/2}-2ab(c+dx)^2-\left(a^2-b^2c\right)(c+dx)^{3/2}+2abc(c+dx)+a^2c\sqrt{c+dx}\right) d\sqrt{c+dx}}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(-\frac{1}{4}\left(a^2-b^2c\right)(c+dx)^2+\frac{1}{2}a^2c(c+dx)-\frac{2}{5}ab(c+dx)^{5/2}+\frac{2}{3}abc(c+dx)^{3/2}-\frac{1}{6}b^2(c+dx)^3\right)}{d^2}$$

input
```
Int[x*(a + b*Sqrt[c + d*x])^2,x]
```

output
```
(-2*((a^2*c*(c + d*x))/2 + (2*a*b*c*(c + d*x)^(3/2))/3 - ((a^2 - b^2*c)*(c
 + d*x)^2)/4 - (2*a*b*(c + d*x)^(5/2))/5 - (b^2*(c + d*x)^3)/6))/d^2
```

3.620.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.620.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

method	result	size
default	$b^{2}\left(\frac{1}{3}x^{3}d+\frac{1}{2}c\,x^{2}\right)+\frac{4ab\left(\frac{(dx+c)^{\frac{5}{2}}}{5}-\frac{c(dx+c)^{\frac{3}{2}}}{3}\right)}{d^{2}}+\frac{a^{2}x^{2}}{2}$	54
trager	$\frac{(2b^{2}dx+3b^{2}c+3a^{2})x^{2}}{6}-\frac{4ab(-3d^{2}x^{2}-cdx+2c^{2})\sqrt{dx+c}}{15d^{2}}$	59
derivativedivides	$\frac{\frac{b^{2}(dx+c)^{3}}{3}+\frac{4ab(dx+c)^{\frac{5}{2}}}{5}+\frac{\left(-b^{2}c+a^{2}\right)(dx+c)^{2}}{2}-\frac{4cab(dx+c)^{\frac{3}{2}}}{3}-c\,a^{2}(dx+c)}{d^{2}}$	72

input
```
int(x*(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
b^2*(1/3*x^3*d+1/2*c*x^2)+4*a*b/d^2*(1/5*(d*x+c)^(5/2)-1/3*c*(d*x+c)^(3/2)
)+1/2*a^2*x^2
```

3.620.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.75

$$\int x\left(a+b\sqrt{c+dx}\right)^2\,dx = \frac{10\,b^2d^3x^3+15\,(b^2c+a^2)d^2x^2+8\,(3\,abd^2x^2+abcdx-2\,abc^2)\sqrt{dx+c}}{30\,d^2}$$

input
```
integrate(x*(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/30*(10*b^2*d^3*x^3 + 15*(b^2*c + a^2)*d^2*x^2 + 8*(3*a*b*d^2*x^2 + a*b*c
*d*x - 2*a*b*c^2)*sqrt(d*x + c))/d^2
```

3.620.6 Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.99

$$\int x\left(a+b\sqrt{c+dx}\right)^2\,dx = \begin{cases} \frac{2\left(-\frac{a^2c(c+dx)}{2}-\frac{2abc(c+dx)^{\frac{3}{2}}}{3}+\frac{2ab(c+dx)^{\frac{5}{2}}}{5}+\frac{b^2(c+dx)^3}{6}+\frac{(a^2-b^2c)(c+dx)^2}{4}\right)}{d^2} & \text{for } d\neq 0 \\ \frac{x^2(a+b\sqrt{c})^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(a+b*(d*x+c)**(1/2))**2,x)
```

output
```
Piecewise((2*(-a**2*c*(c + d*x)/2 - 2*a*b*c*(c + d*x)**(3/2)/3 + 2*a*b*(c
+ d*x)**(5/2)/5 + b**2*(c + d*x)**3/6 + (a**2 - b**2*c)*(c + d*x)**2/4)/d*
*2, Ne(d, 0)), (x**2*(a + b*sqrt(c))**2/2, True))
```

3.620.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.81

$$\int x\left(a+b\sqrt{c+dx}\right)^2\,dx$$
$$=\frac{10\,(dx+c)^3b^2+24\,(dx+c)^{\frac{5}{2}}ab-40\,(dx+c)^{\frac{3}{2}}abc-30\,(dx+c)a^2c-15\,(b^2c-a^2)(dx+c)^2}{30\,d^2}$$

input
```
integrate(x*(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
1/30*(10*(d*x + c)^3*b^2 + 24*(d*x + c)^(5/2)*a*b - 40*(d*x + c)^(3/2)*a*b
*c - 30*(d*x + c)*a^2*c - 15*(b^2*c - a^2)*(d*x + c)^2)/d^2
```

3.620.8 Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.47

$$\int x\left(a+b\sqrt{c+dx}\right)^2\,dx$$
$$=\frac{10\,b^2d^2x^3+\frac{40\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}c\right)abc}{d}+\frac{15\left((dx+c)^2-2\,(dx+c)c\right)b^2c}{d}+\frac{15\left((dx+c)^2-2\,(dx+c)c\right)a^2}{d}+\frac{8\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}\right.}{d}}{30\,d}$$

input
```
integrate(x*(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
1/30*(10*b^2*d^2*x^3 + 40*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b*c/d +
15*((d*x + c)^2 - 2*(d*x + c)*c)*b^2*c/d + 15*((d*x + c)^2 - 2*(d*x + c)*c
)*a^2/d + 8*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c
^2)*a*b/d)/d
```

3.620.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

$$\int x\left(a+b\sqrt{c+dx}\right)^2\,dx=\frac{b^2\,{\left(c+d\,x\right)}^3}{3\,d^2}-\frac{\left(2\,b^2\,c-2\,a^2\right)\,{\left(c+d\,x\right)}^2}{4\,d^2}+\frac{4\,a\,b\,{\left(c+d\,x\right)}^{5/2}}{5\,d^2}-\frac{a^2\,c\,x}{d}-\frac{4\,a\,b\,c\,{\left(c+d\,x\right)}^{3/2}}{3\,d^2}$$

input
```
int(x*(a + b*(c + d*x)^(1/2))^2,x)
```

output
```
(b^2*(c + d*x)^3)/(3*d^2) - ((2*b^2*c - 2*a^2)*(c + d*x)^2)/(4*d^2) + (4*a
*b*(c + d*x)^(5/2))/(5*d^2) - (a^2*c*x)/d - (4*a*b*c*(c + d*x)^(3/2))/(3*d
^2)
```

3.621 $\int \left(a+b\sqrt{c+dx}\right)^2 dx$

3.621.1 Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \left(a+b\sqrt{c+dx}\right)^2 dx=a^2x+\frac{4ab(c+dx)^{3/2}}{3d}+\frac{b^2(c+dx)^2}{2d}$$

output
```
a^2*x+4/3*a*b*(d*x+c)^(3/2)/d+1/2*b^2*(d*x+c)^2/d
```

3.621.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \left(a+b\sqrt{c+dx}\right)^2 dx=\frac{(c+dx)\left(6a^2+8ab\sqrt{c+dx}+3b^2(c+dx)\right)}{6d}$$

input
```
Integrate[(a + b*Sqrt[c + d*x])^2,x]
```

output
```
((c + d*x)*(6*a^2 + 8*a*b*Sqrt[c + d*x] + 3*b^2*(c + d*x)))/(6*d)
```

3.621.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {239, 774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt{c+dx}\right)^2 dx$$

$$\Big\downarrow \ 239$$

$$\frac{\int \left(a+b\sqrt{c+dx}\right)^2 d(c+dx)}{d}$$

$$\Big\downarrow \ 774$$

$$\frac{2\int \sqrt{c+dx}\left(a+b\sqrt{c+dx}\right)^2 d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \ 49$$

$$\frac{2\int \left(\sqrt{c+dx}a^2+2b(c+dx)a+b^2(c+dx)^{3/2}\right) d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{2\left(\frac{1}{2}a^2(c+dx)+\frac{2}{3}ab(c+dx)^{3/2}+\frac{1}{4}b^2(c+dx)^2\right)}{d}$$

input
```
Int[(a + b*Sqrt[c + d*x])^2,x]
```

output
```
(2*((a^2*(c + d*x))/2 + (2*a*b*(c + d*x)^(3/2))/3 + (b^2*(c + d*x)^2)/4))/
d
```

3.621.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.621.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

method	result	size
default	$b^{2}\left(cx+\frac{1}{2}d\,x^{2}\right)+\frac{4ab\left(dx+c\right)^{\frac{3}{2}}}{3d}+a^{2}x$	35
trager	$\frac{\left(b^{2}dx+2b^{2}c+2a^{2}\right)x}{2}+\frac{4ab\left(dx+c\right)^{\frac{3}{2}}}{3d}$	37
derivativedivides	$\frac{\frac{b^{2}\left(dx+c\right)^{2}}{2}+\frac{4ab\left(dx+c\right)^{\frac{3}{2}}}{3}+a^{2}\left(dx+c\right)}{d}$	40

input
```
int((a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
b^2*(c*x+1/2*d*x^2)+4/3*a*b*(d*x+c)^(3/2)/d+a^2*x
```

3.621.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int\left(a+b\sqrt{c+dx}\right)^{2}\,dx=\frac{3\,b^{2}d^{2}x^{2}+6\,(b^{2}c+a^{2})dx+8\,(abdx+abc)\sqrt{dx+c}}{6\,d}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/6*(3*b^2*d^2*x^2 + 6*(b^2*c + a^2)*d*x + 8*(a*b*d*x + a*b*c)*sqrt(d*x +
c))/d
```

3.621.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.66

$$\int \left(a+b\sqrt{c+dx}\right)^2\,dx=\begin{cases} a^2x+\frac{4abc\sqrt{c+dx}}{3d}+\frac{4abx\sqrt{c+dx}}{3}+b^2cx+\frac{b^2dx^2}{2} & \text{for } d\neq 0\\ x\left(a+b\sqrt{c}\right)^2 & \text{otherwise}\end{cases}$$

input
```
integrate((a+b*(d*x+c)**(1/2))**2,x)
```

output
```
Piecewise((a**2*x + 4*a*b*c*sqrt(c + d*x)/(3*d) + 4*a*b*x*sqrt(c + d*x)/3
+ b**2*c*x + b**2*d*x**2/2, Ne(d, 0)), (x*(a + b*sqrt(c))**2, True))
```

3.621.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \left(a+b\sqrt{c+dx}\right)^2\,dx=\frac{1}{2}\left(dx^2+2\,cx\right)b^2+a^2x+\frac{4\left(dx+c\right)^{\frac{3}{2}}ab}{3\,d}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
1/2*(d*x^2 + 2*c*x)*b^2 + a^2*x + 4/3*(d*x + c)^(3/2)*a*b/d
```

3.621.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(35) = 70.

Time = 0.39 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.00

$$\int \left(a+b\sqrt{c+dx}\right)^2\,dx$$
$$=\frac{6\left(dx+c\right)b^2c+24\sqrt{dx+c}abc+6\left(dx+c\right)a^2+8\left(\left(dx+c\right)^{\frac{3}{2}}-3\sqrt{dx+c}c\right)ab+3\left(\left(dx+c\right)^2-2\left(dx+c\right.\right.}{6\,d}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
1/6*(6*(d*x + c)*b^2*c + 24*sqrt(d*x + c)*a*b*c + 6*(d*x + c)*a^2 + 8*((d*
x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b + 3*((d*x + c)^2 - 2*(d*x + c)*c)*b^
2)/d
```

3.621.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \left(a+b\sqrt{c+dx}\right)^2\,dx=\frac{3\,b^2\,{\left(c+d\,x\right)}^2+8\,a\,b\,{\left(c+d\,x\right)}^{3/2}+6\,a^2\,d\,x}{6\,d}$$

input
```
int((a + b*(c + d*x)^(1/2))^2,x)
```

output
```
(3*b^2*(c + d*x)^2 + 8*a*b*(c + d*x)^(3/2) + 6*a^2*d*x)/(6*d)
```

3.622 $\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx$

3.622.1 Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx=b^2dx+4ab\sqrt{c+dx}-4ab\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+\left(a^2+b^2c\right)\log(x)$$

output
```
b^2*d*x+(b^2*c+a^2)*ln(x)-4*a*b*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)+4*a
*b*(d*x+c)^(1/2)
```

3.622.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx=b\left(bc+bdx+4a\sqrt{c+dx}\right)-4ab\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+\left(a^2+b^2c\right)\log(-dx)$$

input
```
Integrate[(a + b*Sqrt[c + d*x])^2/x,x]
```

output
```
b*(b*c + b*d*x + 4*a*Sqrt[c + d*x]) - 4*a*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/
Sqrt[c]] + (a^2 + b^2*c)*Log[-(d*x)]
```

3.622.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.25, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {896, 25, 1732, 525, 25, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x} \, dx$$

$$\Big\downarrow \text{896}$$

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{dx} d(c+dx)$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{\left(a+b\sqrt{c+dx}\right)^2}{dx} d(c+dx)$$

$$\Big\downarrow \text{1732}$$

$$-2\int -\frac{\sqrt{c+dx}\left(a+b\sqrt{c+dx}\right)^2}{dx} d\sqrt{c+dx}$$

$$\Big\downarrow \text{525}$$

$$-2\left(-\int \frac{\sqrt{c+dx}\left(a^2+2b\sqrt{c+dx}a+b^2c\right)}{dx} d\sqrt{c+dx} - \frac{1}{2}b^2(c+dx)\right)$$

$$\Big\downarrow \text{25}$$

$$-2\left(\int -\frac{\sqrt{c+dx}\left(a^2+2b\sqrt{c+dx}a+b^2c\right)}{dx} d\sqrt{c+dx} - \frac{1}{2}b^2(c+dx)\right)$$

$$\Big\downarrow \text{523}$$

$$-2\left(\int \left(-2ab - \frac{2abc+\left(a^2+b^2c\right)\sqrt{c+dx}}{dx}\right) d\sqrt{c+dx} - \frac{1}{2}b^2(c+dx)\right)$$

$$\Big\downarrow \text{2009}$$

$$-2\left(-\frac{1}{2}\left(a^2+b^2c\right)\log(-dx) + 2ab\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right) - 2ab\sqrt{c+dx} - \frac{1}{2}b^2(c+dx)\right)$$

input
```
Int[(a + b*Sqrt[c + d*x])^2/x,x]
```

3.622. $\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x} \, dx$

output
```
-2*(-2*a*b*Sqrt[c + d*x] - (b^2*(c + d*x))/2 + 2*a*b*Sqrt[c]*ArcTanh[Sqrt[
c + d*x]/Sqrt[c]] - ((a^2 + b^2*c)*Log[-(d*x)])/2)
```

3.622.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 525
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_))^(n_))/((a_) + (b_.)*(x_)^2), x_Symbol]
:> Simp[d^n*(x^(m + n - 1)/(b*(m + n - 1))), x] + Simp[1/b   Int[x^m*(Expan
dToSum[b*(c + d*x)^n - b*d^n*x^n - a*d^n*x^(n - 2), x]/(a + b*x^2)), x], x]
 /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 1] && IGtQ[m, -2] && NeQ[m + n - 1, 0
]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.622. $\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx$

3.622.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

method	result	size
default	$b^2(dx+c\ln(x))+2ab\left(2\sqrt{dx+c}-2\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)\right)+a^2\ln(x)$	51
derivativedivides	$(dx+c)\,b^2+4ab\sqrt{dx+c}-(-b^2c-a^2)\ln(-dx)-4ab\,\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)\sqrt{c}$	60

input
```
int((a+b*(d*x+c)^(1/2))^2/x,x,method=_RETURNVERBOSE)
```

output
```
b^2*(d*x+c*ln(x))+2*a*b*(2*(d*x+c)^(1/2)-2*c^(1/2)*arctanh((d*x+c)^(1/2)/c
^(1/2)))+a^2*ln(x)
```

3.622.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.07

$$\int\frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx=\left[b^2dx+2\,ab\sqrt{c}\log\left(\frac{dx-2\,\sqrt{dx+c}\sqrt{c}+2\,c}{x}\right)+4\,\sqrt{dx+c}ab+\left(b^2c+a^2\right)\log\left(x\right),b^2dx+4\,ab\sqrt{-c}\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right)+4\,\sqrt{dx+c}ab+\left(b^2c+a^2\right)\log\left(x\right)\right]$$

input
```
integrate((a+b*(d*x+c)^(1/2))^2/x,x, algorithm="fricas")
```

output
```
[b^2*d*x + 2*a*b*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 4*
sqrt(d*x + c)*a*b + (b^2*c + a^2)*log(x), b^2*d*x + 4*a*b*sqrt(-c)*arctan(
sqrt(d*x + c)*sqrt(-c)/c) + 4*sqrt(d*x + c)*a*b + (b^2*c + a^2)*log(x)]
```

3.622.6 Sympy [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.53

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx = \begin{cases} \frac{4abc\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}} + 4ab\sqrt{c+dx} + b^2(c+dx) - 2\left(-\frac{a^2}{2} - \frac{b^2c}{2}\right)\log(-dx) & \text{for } d \neq 0 \\ \left(a+b\sqrt{c}\right)^2\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*(d*x+c)**(1/2))**2/x,x)
```

output
```
Piecewise((4*a*b*c*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + 4*a*b*sqrt(c +
d*x) + b**2*(c + d*x) - 2*(-a**2/2 - b**2*c/2)*log(-d*x), Ne(d, 0)), ((a +
 b*sqrt(c))**2*log(x), True))
```

3.622.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx = 2\,ab\sqrt{c}\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right) + (dx+c)b^2 + 4\sqrt{dx+c}ab + \left(b^2c+a^2\right)\log(dx)$$

input
```
integrate((a+b*(d*x+c)^(1/2))^2/x,x, algorithm="maxima")
```

output
```
2*a*b*sqrt(c)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))) + (
d*x + c)*b^2 + 4*sqrt(d*x + c)*a*b + (b^2*c + a^2)*log(d*x)
```

3.622.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx = \frac{4\,abc\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + (dx+c)b^2 + 4\,\sqrt{dx+c}ab + \left(b^2c+a^2\right)\log\left(dx\right)$$

input
```
integrate((a+b*(d*x+c)^(1/2))^2/x,x, algorithm="giac")
```

output
```
4*a*b*c*arctan(sqrt(d*x + c)/sqrt(-c))/sqrt(-c) + (d*x + c)*b^2 + 4*sqrt(d
*x + c)*a*b + (b^2*c + a^2)*log(d*x)
```

3.622.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.28

$$\begin{aligned}\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx &= \ln\left(\left(2\,a^2+2\,c\,b^2\right)\,\sqrt{c+d\,x}-2\,\left(a+b\,\sqrt{c}\right)^2\sqrt{c+d\,x}\right. \\ &\quad \left.+4\,a\,b\,c\right)\,\left(a+b\,\sqrt{c}\right)^2 \\ &\quad + \ln\left(\left(2\,a^2+2\,c\,b^2\right)\,\sqrt{c+d\,x}-2\,\left(a-b\,\sqrt{c}\right)^2\sqrt{c+d\,x}\right. \\ &\quad \left.+4\,a\,b\,c\right)\,\left(a-b\,\sqrt{c}\right)^2+4\,a\,b\,\sqrt{c+d\,x}+b^2\,d\,x\end{aligned}$$

input
```
int((a + b*(c + d*x)^(1/2))^2/x,x)
```

output
```
log((2*b^2*c + 2*a^2)*(c + d*x)^(1/2) - 2*(a + b*c^(1/2))^2*(c + d*x)^(1/2
) + 4*a*b*c)*(a + b*c^(1/2))^2 + log((2*b^2*c + 2*a^2)*(c + d*x)^(1/2) - 2
*(a - b*c^(1/2))^2*(c + d*x)^(1/2) + 4*a*b*c)*(a - b*c^(1/2))^2 + 4*a*b*(c
 + d*x)^(1/2) + b^2*d*x
```

3.623 $\int \frac{(a+b\sqrt{c+dx})^2}{x^2} \, dx$

3.623.1 Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \frac{(a+b\sqrt{c+dx})^2}{x^2} \, dx = -\frac{(a+b\sqrt{c+dx})^2}{x} - \frac{2abd\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}} + b^2 d\log(x)$$

output
```
b^2*d*ln(x)-2*a*b*d*arctanh((d*x+c)^(1/2)/c^(1/2))/c^(1/2)-(a+b*(d*x+c)^(1
/2))^2/x
```

3.623.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19

$$\int \frac{(a+b\sqrt{c+dx})^2}{x^2} \, dx = -\frac{a^2+b^2c+2ab\sqrt{c+dx}}{x} - \frac{2abd\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}} + b^2 d\log(-dx)$$

input
```
Integrate[(a + b*Sqrt[c + d*x])^2/x^2,x]
```

output
```
-((a^2 + b^2*c + 2*a*b*Sqrt[c + d*x])/x) - (2*a*b*d*ArcTanh[Sqrt[c + d*x]/
Sqrt[c]])/Sqrt[c] + b^2*d*Log[-(d*x)]
```

3.623.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {896, 1732, 531, 27, 452, 219, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}dx$$

$$\Big\downarrow \ 896$$

$$d\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{d^2x^2}d(c+dx)$$

$$\Big\downarrow \ 1732$$

$$2d\int \frac{\sqrt{c+dx}\left(a+b\sqrt{c+dx}\right)^2}{d^2x^2}d\sqrt{c+dx}$$

$$\Big\downarrow \ 531$$

$$2d\left(\frac{\int \frac{2bc\left(a+b\sqrt{c+dx}\right)}{dx}d\sqrt{c+dx}}{2c}-\frac{\left(a+b\sqrt{c+dx}\right)^2}{2dx}\right)$$

$$\Big\downarrow \ 27$$

$$2d\left(-b\int -\frac{a+b\sqrt{c+dx}}{dx}d\sqrt{c+dx}-\frac{\left(a+b\sqrt{c+dx}\right)^2}{2dx}\right)$$

$$\Big\downarrow \ 452$$

$$2d\left(-b\left(a\int -\frac{1}{dx}d\sqrt{c+dx}+b\int -\frac{\sqrt{c+dx}}{dx}d\sqrt{c+dx}\right)-\frac{\left(a+b\sqrt{c+dx}\right)^2}{2dx}\right)$$

$$\Big\downarrow \ 219$$

$$2d\left(-b\left(b\int -\frac{\sqrt{c+dx}}{dx}d\sqrt{c+dx}+\frac{a\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\left(a+b\sqrt{c+dx}\right)^2}{2dx}\right)$$

$$\Big\downarrow \ 240$$

$$2d\left(-b\left(\frac{a\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{1}{2}b\log(-dx)\right)-\frac{\left(a+b\sqrt{c+dx}\right)^2}{2dx}\right)$$

input
```
Int[(a + b*Sqrt[c + d*x])^2/x^2,x]
```

output
```
2*d*(-1/2*(a + b*Sqrt[c + d*x])^2/(d*x) - b*((a*ArcTanh[Sqrt[c + d*x]/Sqrt
[c]])/Sqrt[c] - (b*Log[-(d*x)])/2))
```

3.623.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 531
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m, a + b*x^2, x], e = Coeff[Polynomi
alRemainder[x^m, a + b*x^2, x], x, 0], f = Coeff[PolynomialRemainder[x^m, a
 + b*x^2, x], x, 1]}, Simp[(c + d*x)^n*(a*f - b*e*x)*((a + b*x^2)^(p + 1)/(
2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b
*x^2)^(p + 1)*ExpandToSum[2*a*b*(p + 1)*(c + d*x)*Qx - a*d*f*n + b*c*e*(2*p
 + 3) + b*d*e*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGt
Q[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && GtQ[n, 1] && IntegerQ[2*p]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

3.623. $\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}\,dx$

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

3.623.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

method	result	size
default	$b^2\left(d\ln\left(x\right)-\frac{c}{x}\right)+4abd\left(-\frac{\sqrt{dx+c}}{2dx}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)-\frac{a^2}{x}$	63
derivativedivides	$2d\left(-\frac{ab\sqrt{dx+c}+\frac{b^2c}{2}+\frac{a^2}{2}}{dx}+b\left(\frac{b\ln\left(-dx\right)}{2}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)\right)$	64

input
```
int((a+b*(d*x+c)^(1/2))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
b^2*(d*ln(x)-1/x*c)+4*a*b*d*(-1/2*(d*x+c)^(1/2)/d/x-1/2/c^(1/2)*arctanh((d
*x+c)^(1/2)/c^(1/2)))-a^2/x
```

3.623.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.72

$$\int\frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}\,dx$$
$$=\left[\frac{b^2cdx\log\left(x\right)+ab\sqrt{c}dx\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)-b^2c^2-2\sqrt{dx+c}abc-a^2c}{cx},\frac{b^2cdx\log\left(x\right)+2ab\sqrt{-c}dx\right.$$

input
```
integrate((a+b*(d*x+c)^(1/2))^2/x^2,x, algorithm="fricas")
```

output
```
[(b^2*c*d*x*log(x) + a*b*sqrt(c)*d*x*log((d*x - 2*sqrt(d*x + c)*sqrt(c) +
2*c)/x) - b^2*c^2 - 2*sqrt(d*x + c)*a*b*c - a^2*c)/(c*x), (b^2*c*d*x*log(x
) + 2*a*b*sqrt(-c)*d*x*arctan(sqrt(d*x + c)*sqrt(-c)/c) - b^2*c^2 - 2*sqrt
(d*x + c)*a*b*c - a^2*c)/(c*x)]
```

3.623.6 Sympy [A] (verification not implemented)

Time = 22.67 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.35

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}\,dx = -\frac{a^2}{x} - \frac{2ab\sqrt{d}\sqrt{\frac{c}{dx}+1}}{\sqrt{x}} - \frac{2abd\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\sqrt{x}}\right)}{\sqrt{c}} - \frac{b^2c}{x} + b^2d\log(x)$$

input
```
integrate((a+b*(d*x+c)**(1/2))**2/x**2,x)
```

output
```
-a**2/x - 2*a*b*sqrt(d)*sqrt(c/(d*x) + 1)/sqrt(x) - 2*a*b*d*asinh(sqrt(c)/
(sqrt(d)*sqrt(x)))/sqrt(c) - b**2*c/x + b**2*d*log(x)
```

3.623.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.35

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}\,dx = \left(b^2\log(dx) + \frac{ab\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{\sqrt{c}} - \frac{b^2c+2\sqrt{dx+c}ab+a^2}{dx}\right)d$$

input
```
integrate((a+b*(d*x+c)^(1/2))^2/x^2,x, algorithm="maxima")
```

output
```
(b^2*log(d*x) + a*b*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)
))/sqrt(c) - (b^2*c + 2*sqrt(d*x + c)*a*b + a^2)/(d*x))*d
```

3.623.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.48

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}\,dx = \frac{b^2d^2\log(dx) + \frac{2abd^2\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{b^2cd^2+2\sqrt{dx+c}abd^2+a^2d^2}{dx}}{d}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^2/x^2,x, algorithm="giac")
```

output
```
(b^2*d^2*log(d*x) + 2*a*b*d^2*arctan(sqrt(d*x + c)/sqrt(-c))/sqrt(-c) - (b
^2*c*d^2 + 2*sqrt(d*x + c)*a*b*d^2 + a^2*d^2)/(d*x))/d
```

3.623.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.43

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}\,dx = b\,d\,\ln\left(2\,b\,d\,\left(b+\frac{a}{\sqrt{c}}\right)\,\sqrt{c+d\,x}-2\,b^2\,d\,\sqrt{c+d\,x}-2\,a\,b\,d\right)\,\left(b+\frac{a}{\sqrt{c}}\right)-\frac{a^2\,d+b^2\,c\,d+2\,a\,b\,d\,\sqrt{c+d\,x}}{d\,x}+b\,d\,\ln\left(2\,b\,d\,\left(b-\frac{a}{\sqrt{c}}\right)\,\sqrt{c+d\,x}-2\,b^2\,d\,\sqrt{c+d\,x}-2\,a\,b\,d\right)\,\left(b-\frac{a}{\sqrt{c}}\right)$$

input
```
int((a + b*(c + d*x)^(1/2))^2/x^2,x)
```

output
```
b*d*log(2*b*d*(b + a/c^(1/2))*(c + d*x)^(1/2) - 2*b^2*d*(c + d*x)^(1/2) -
2*a*b*d)*(b + a/c^(1/2)) - (a^2*d + b^2*c*d + 2*a*b*d*(c + d*x)^(1/2))/(d*
x) + b*d*log(2*b*d*(b - a/c^(1/2))*(c + d*x)^(1/2) - 2*b^2*d*(c + d*x)^(1/
2) - 2*a*b*d)*(b - a/c^(1/2))
```

3.624 $\int \frac{(a+b\sqrt{c+dx})^2}{x^3}\,dx$

3.624.1 Optimal result

Integrand size = 19, antiderivative size = 80

$$\int \frac{(a+b\sqrt{c+dx})^2}{x^3}\,dx = -\frac{bd(bc+a\sqrt{c+dx})}{2cx} - \frac{(a+b\sqrt{c+dx})^2}{2x^2} + \frac{abd^2\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}}$$

output
```
1/2*a*b*d^2*arctanh((d*x+c)^(1/2)/c^(1/2))/c^(3/2)-1/2*b*d*(b*c+a*(d*x+c)^
(1/2))/c/x-1/2*(a+b*(d*x+c)^(1/2))^2/x^2
```

3.624.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\sqrt{c+dx})^2}{x^3}\,dx = -\frac{a^2c+ab\sqrt{c+dx}(2c+dx)+b^2c(c+2dx)}{2cx^2} + \frac{abd^2\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}}$$

input
```
Integrate[(a + b*Sqrt[c + d*x])^2/x^3,x]
```

output
```
-1/2*(a^2*c + a*b*Sqrt[c + d*x]*(2*c + d*x) + b^2*c*(c + 2*d*x))/(c*x^2) +
 (a*b*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(2*c^(3/2))
```

3.624.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {896, 25, 1732, 531, 27, 454, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}dx$$
$$\Big\downarrow \text{896}$$
$$d^2\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{d^3x^3}d(c+dx)$$
$$\Big\downarrow \text{25}$$
$$-d^2\int -\frac{\left(a+b\sqrt{c+dx}\right)^2}{d^3x^3}d(c+dx)$$
$$\Big\downarrow \text{1732}$$
$$-2d^2\int -\frac{\sqrt{c+dx}\left(a+b\sqrt{c+dx}\right)^2}{d^3x^3}d\sqrt{c+dx}$$
$$\Big\downarrow \text{531}$$
$$-2d^2\left(\frac{\int -\frac{2bc\left(a+b\sqrt{c+dx}\right)}{d^2x^2}d\sqrt{c+dx}}{4c}+\frac{\left(a+b\sqrt{c+dx}\right)^2}{4d^2x^2}\right)$$
$$\Big\downarrow \text{27}$$
$$-2d^2\left(\frac{\left(a+b\sqrt{c+dx}\right)^2}{4d^2x^2}-\frac{1}{2}b\int \frac{a+b\sqrt{c+dx}}{d^2x^2}d\sqrt{c+dx}\right)$$
$$\Big\downarrow \text{454}$$
$$-2d^2\left(\frac{\left(a+b\sqrt{c+dx}\right)^2}{4d^2x^2}-\frac{1}{2}b\left(\frac{a\int -\frac{1}{dx}d\sqrt{c+dx}}{2c}-\frac{a\sqrt{c+dx}+bc}{2cdx}\right)\right)$$
$$\Big\downarrow \text{219}$$
$$-2d^2\left(\frac{\left(a+b\sqrt{c+dx}\right)^2}{4d^2x^2}-\frac{1}{2}b\left(\frac{a\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}}-\frac{a\sqrt{c+dx}+bc}{2cdx}\right)\right)$$

input
```
Int[(a + b*Sqrt[c + d*x])^2/x^3,x]
```

output
```
-2*d^2*((a + b*Sqrt[c + d*x])^2/(4*d^2*x^2) - (b*(-1/2*(b*c + a*Sqrt[c + d
*x])/(c*d*x) + (a*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(2*c^(3/2))))/2)
```

3.624.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 531
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m, a + b*x^2, x], e = Coeff[Polynomi
alRemainder[x^m, a + b*x^2, x], x, 0], f = Coeff[PolynomialRemainder[x^m, a
 + b*x^2, x], x, 1]}, Simp[(c + d*x)^n*(a*f - b*e*x)*((a + b*x^2)^(p + 1)/(
2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b
*x^2)^(p + 1)*ExpandToSum[2*a*b*(p + 1)*(c + d*x)*Qx - a*d*f*n + b*c*e*(2*p
 + 3) + b*d*e*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGt
Q[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && GtQ[n, 1] && IntegerQ[2*p]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

3.624. $\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx$

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

3.624.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.01

method	result	size
derivativedivides	$2d^2\left(-\frac{-\frac{ab(dx+c)^{\frac{3}{2}}}{4c}+\frac{(dx+c)b^2}{2}+\frac{ab\sqrt{dx+c}}{4}-\frac{b^2c}{4}+\frac{a^2}{4}}{d^2x^2}+\frac{ab\,\mathrm{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{4c^{\frac{3}{2}}}\right)$	81
default	$b^2\left(-\frac{c}{2x^2}-\frac{d}{x}\right)+4ab\,d^2\left(-\frac{-\frac{(dx+c)^{\frac{3}{2}}}{8c}+\frac{\sqrt{dx+c}}{8}}{d^2x^2}+\frac{\mathrm{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{8c^{\frac{3}{2}}}\right)-\frac{a^2}{2x^2}$	82

input
```
int((a+b*(d*x+c)^(1/2))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
2*d^2*(-(1/4*a*b/c*(d*x+c)^(3/2)+1/2*(d*x+c)*b^2+1/4*a*b*(d*x+c)^(1/2)-1/4
*b^2*c+1/4*a^2)/d^2/x^2+1/4*a*b/c^(3/2)*arctanh((d*x+c)^(1/2)/c^(1/2)))
```

3.624.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.26

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx$$
$$=\left[\frac{ab\sqrt{c}d^2x^2\log\left(\frac{dx+2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)-4b^2c^2dx-2b^2c^3-2a^2c^2-2\left(abcdx+2abc^2\right)\sqrt{dx+c}}{4c^2x^2},\right.$$
$$\left.-\frac{ab\sqrt{-c}d^2x^2\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right)+2b^2c^2dx+b^2c^3+a^2c^2+\left(abcdx+2abc^2\right)\sqrt{dx+c}}{2c^2x^2}\right]$$

input
```
integrate((a+b*(d*x+c)^(1/2))^2/x^3,x, algorithm="fricas")
```

output
```
[1/4*(a*b*sqrt(c)*d^2*x^2*log((d*x + 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 4
*b^2*c^2*d*x - 2*b^2*c^3 - 2*a^2*c^2 - 2*(a*b*c*d*x + 2*a*b*c^2)*sqrt(d*x
+ c))/(c^2*x^2), -1/2*(a*b*sqrt(-c)*d^2*x^2*arctan(sqrt(d*x + c)*sqrt(-c)/
c) + 2*b^2*c^2*d*x + b^2*c^3 + a^2*c^2 + (a*b*c*d*x + 2*a*b*c^2)*sqrt(d*x
+ c))/(c^2*x^2)]
```

3.624.6 Sympy [A] (verification not implemented)

Time = 90.95 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.68

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx = -\frac{a^2}{2x^2} - \frac{abc}{\sqrt{d}x^{\frac{5}{2}}\sqrt{\frac{c}{dx}+1}} - \frac{3ab\sqrt{d}}{2x^{\frac{3}{2}}\sqrt{\frac{c}{dx}+1}} - \frac{abd^{\frac{3}{2}}}{2c\sqrt{x}\sqrt{\frac{c}{dx}+1}} + \frac{abd^2\operatorname{asinh}\left(\frac{\sqrt{c}}{\sqrt{d}\sqrt{x}}\right)}{2c^{\frac{3}{2}}} - \frac{b^2c}{2x^2} - \frac{b^2d}{x}$$

input
```
integrate((a+b*(d*x+c)**(1/2))**2/x**3,x)
```

output
```
-a**2/(2*x**2) - a*b*c/(sqrt(d)*x**(5/2)*sqrt(c/(d*x) + 1)) - 3*a*b*sqrt(d
)/(2*x**(3/2)*sqrt(c/(d*x) + 1)) - a*b*d**(3/2)/(2*c*sqrt(x)*sqrt(c/(d*x)
+ 1)) + a*b*d**2*asinh(sqrt(c)/(sqrt(d)*sqrt(x)))/(2*c**(3/2)) - b**2*c/(2
*x**2) - b**2*d/x
```

3.624.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.41

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx = -\frac{1}{4}\left(\frac{ab\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{c^{\frac{3}{2}}} + \frac{2\left(2\,(dx+c)b^2c - b^2c^2 + (dx+c)^{\frac{3}{2}}ab + \sqrt{dx+c}abc + a^2c\right)}{(dx+c)^2c - 2\,(dx+c)c^2 + c^3}\right)d^2$$

input
```
integrate((a+b*(d*x+c)^(1/2))^2/x^3,x, algorithm="maxima")
```

output
```
-1/4*(a*b*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/c^(3/2)
 + 2*(2*(d*x + c)*b^2*c - b^2*c^2 + (d*x + c)^(3/2)*a*b + sqrt(d*x + c)*a*
b*c + a^2*c)/((d*x + c)^2*c - 2*(d*x + c)*c^2 + c^3))*d^2
```

3.624.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.31

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx = -\frac{\frac{abd^3\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}c}+\frac{2\,(dx+c)b^2cd^3-b^2c^2d^3+(dx+c)^{\frac{3}{2}}abd^3+\sqrt{dx+c}abcd^3+a^2cd^3}{cd^2x^2}}{2\,d}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^2/x^3,x, algorithm="giac")
```

output
```
-1/2*(a*b*d^3*arctan(sqrt(d*x + c)/sqrt(-c))/(sqrt(-c)*c) + (2*(d*x + c)*b
^2*c*d^3 - b^2*c^2*d^3 + (d*x + c)^(3/2)*a*b*d^3 + sqrt(d*x + c)*a*b*c*d^3
 + a^2*c*d^3)/(c*d^2*x^2))/d
```

3.624.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx = \frac{a\,b\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}}{\sqrt{c}}\right)}{2\,c^{3/2}}-\frac{b^2\,c}{2\,x^2}-\frac{b^2\,d}{x}-\frac{a\,b\,\sqrt{c+d\,x}}{2\,x^2}-\frac{a\,b\,{\left(c+d\,x\right)}^{3/2}}{2\,c\,x^2}-\frac{a^2}{2\,x^2}$$

input
```
int((a + b*(c + d*x)^(1/2))^2/x^3,x)
```

output
```
(a*b*d^2*atanh((c + d*x)^(1/2)/c^(1/2)))/(2*c^(3/2)) - (b^2*c)/(2*x^2) - (
b^2*d)/x - (a*b*(c + d*x)^(1/2))/(2*x^2) - (a*b*(c + d*x)^(3/2))/(2*c*x^2)
 - a^2/(2*x^2)
```

3.625 $\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx$

3.625.1 Optimal result

Integrand size = 21, antiderivative size = 326

$$\begin{aligned}\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx = &-\frac{4a(a^2-b^2c)^3\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^8d^4}\\ &+\frac{4(a^2-b^2c)^2(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^8d^4}\\ &-\frac{12a(7a^2-3b^2c)(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^8d^4}\\ &+\frac{4(35a^4-30a^2b^2c+3b^4c^2)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^8d^4}\\ &-\frac{20a(7a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8d^4}\\ &+\frac{12(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^8d^4}\\ &-\frac{28a\left(a+b\sqrt{c+dx}\right)^{15/2}}{15b^8d^4}+\frac{4\left(a+b\sqrt{c+dx}\right)^{17/2}}{17b^8d^4}\end{aligned}$$

output
```
-4/3*a*(-b^2*c+a^2)^3*(a+b*(d*x+c)^(1/2))^(3/2)/b^8/d^4+4/5*(-b^2*c+a^2)^2
*(-b^2*c+7*a^2)*(a+b*(d*x+c)^(1/2))^(5/2)/b^8/d^4-12/7*a*(-3*b^2*c+7*a^2)*
(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(7/2)/b^8/d^4+4/9*(3*b^4*c^2-30*a^2*b^2*c
+35*a^4)*(a+b*(d*x+c)^(1/2))^(9/2)/b^8/d^4-20/11*a*(-3*b^2*c+7*a^2)*(a+b*(
d*x+c)^(1/2))^(11/2)/b^8/d^4+12/13*(-b^2*c+7*a^2)*(a+b*(d*x+c)^(1/2))^(13/
2)/b^8/d^4-28/15*a*(a+b*(d*x+c)^(1/2))^(15/2)/b^8/d^4+4/17*(a+b*(d*x+c)^(1
/2))^(17/2)/b^8/d^4
```

3.625.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.71

$$\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$=\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(-14336a^7+3840a^5b^2(10c-7dx)+21504a^6b\sqrt{c+dx}-640a^4b^3(104c-49dx)\sqrt{c+\right.}{}$$

input
```
Integrate[x^3*Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(4*(a + b*Sqrt[c + d*x])^(3/2)*(-14336*a^7 + 3840*a^5*b^2*(10*c - 7*d*x) +
 21504*a^6*b*Sqrt[c + d*x] - 640*a^4*b^3*(104*c - 49*d*x)*Sqrt[c + d*x] -
48*a^3*b^4*(616*c^2 - 1080*c*d*x + 735*d^2*x^2) + 24*a^2*b^5*Sqrt[c + d*x]
*(2960*c^2 - 2716*c*d*x + 1617*d^2*x^2) + 6*a*b^6*(320*c^3 - 3936*c^2*d*x
+ 5754*c*d^2*x^2 - 7007*d^3*x^3) - 231*b^7*Sqrt[c + d*x]*(128*c^3 - 160*c^
2*d*x + 180*c*d^2*x^2 - 195*d^3*x^3)))/(765765*b^8*d^4)
```

3.625.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$\Big\downarrow \text{896}$$
$$\frac{\int d^3x^3\sqrt{a+b\sqrt{c+dx}}d(c+dx)}{d^4}$$
$$\Big\downarrow \text{25}$$
$$-\frac{\int -d^3x^3\sqrt{a+b\sqrt{c+dx}}d(c+dx)}{d^4}$$
$$\Big\downarrow \text{1732}$$
$$-\frac{2\int -d^3x^3\sqrt{c+dx}\sqrt{a+b\sqrt{c+dx}}d\sqrt{c+dx}}{d^4}$$

$$\Big\downarrow \text{522}$$

$$-\frac{2\int\left(-\frac{\left(a+b\sqrt{c+dx}\right)^{15/2}}{b^7}+\frac{7a\left(a+b\sqrt{c+dx}\right)^{13/2}}{b^7}+\frac{3\left(b^2c-7a^2\right)\left(a+b\sqrt{c+dx}\right)^{11/2}}{b^7}-\frac{5\left(3ab^2c-7a^3\right)\left(a+b\sqrt{c+dx}\right)^{9/2}}{b^7}+\frac{\left(-35a^4+30b^2ca^2\cdots\right.}{}}{}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(-\frac{6\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^8}+\frac{10a\left(7a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8}+\frac{6a\left(7a^2-3b^2c\right)\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^8}-\frac{2\left(a^2-b^2c\right)^2\left(7a^2-b^2\cdots\right.}{5b\cdots}}{}$$

input
```
Int[x^3*Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(-2*((2*a*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^8) - (2*(a^2 -
 b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^8) + (6*a*(7*a
^2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^8) - (2*(35*
a^4 - 30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^8) + (10
*a*(7*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(11/2))/(11*b^8) - (6*(7*a^2 -
b^2*c)*(a + b*Sqrt[c + d*x])^(13/2))/(13*b^8) + (14*a*(a + b*Sqrt[c + d*x]
)^(15/2))/(15*b^8) - (2*(a + b*Sqrt[c + d*x])^(17/2))/(17*b^8)))/d^4
```

3.625.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

3.625. $\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx$

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.625.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.18

method	result
derivativedivides	$-\frac{4\left(-\frac{\left(a+b\sqrt{dx+c}\right)^{\frac{17}{2}}}{17}+\frac{7a\left(a+b\sqrt{dx+c}\right)^{\frac{15}{2}}}{15}+\frac{\left(3b^2c-21a^2\right)\left(a+b\sqrt{dx+c}\right)^{\frac{13}{2}}}{13}+\frac{\left(8\left(-b^2c+a^2\right)a+2a\left(-2b^2c+6a^2\right)+\left(-3b^2c+15a^2\right)\right.}{11}\right.}{}$
default	$-\frac{4\left(-\frac{\left(a+b\sqrt{dx+c}\right)^{\frac{17}{2}}}{17}+\frac{7a\left(a+b\sqrt{dx+c}\right)^{\frac{15}{2}}}{15}+\frac{\left(3b^2c-21a^2\right)\left(a+b\sqrt{dx+c}\right)^{\frac{13}{2}}}{13}+\frac{\left(8\left(-b^2c+a^2\right)a+2a\left(-2b^2c+6a^2\right)+\left(-3b^2c+15a^2\right)\right.}{11}\right.}{}$

input
```
int(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/d^4/b^8*(-1/17*(a+b*(d*x+c)^(1/2))^(17/2)+7/15*a*(a+b*(d*x+c)^(1/2))^(1
5/2)+1/13*(3*b^2*c-21*a^2)*(a+b*(d*x+c)^(1/2))^(13/2)+1/11*(8*(-b^2*c+a^2)
*a+2*a*(-2*b^2*c+6*a^2)+(-3*b^2*c+15*a^2)*a)*(a+b*(d*x+c)^(1/2))^(11/2)+1/
9*(-(-b^2*c+a^2)*(-2*b^2*c+6*a^2)-8*a^2*(-b^2*c+a^2)-(-b^2*c+a^2)^2+(-8*(-
b^2*c+a^2)*a-2*a*(-2*b^2*c+6*a^2))*a)*(a+b*(d*x+c)^(1/2))^(9/2)+1/7*(6*(-b
^2*c+a^2)^2*a+((-b^2*c+a^2)*(-2*b^2*c+6*a^2)+8*a^2*(-b^2*c+a^2)+(-b^2*c+a^
2)^2)*a)*(a+b*(d*x+c)^(1/2))^(7/2)+1/5*(-(-b^2*c+a^2)^3-6*(-b^2*c+a^2)^2*a
^2)*(a+b*(d*x+c)^(1/2))^(5/2)+1/3*(-b^2*c+a^2)^3*a*(a+b*(d*x+c)^(1/2))^(3/
2))
```

3.625.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.88

$$\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$=\frac{4\left(45045\,b^8d^4x^4-29568\,b^8c^4+72960\,a^2b^6c^3-96128\,a^4b^4c^2+59904\,a^6b^2c-14336\,a^8+231\,(15\,b^8c-14\,a\right.}{}$$

input
```
integrate(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
4/765765*(45045*b^8*d^4*x^4 - 29568*b^8*c^4 + 72960*a^2*b^6*c^3 - 96128*a^
4*b^4*c^2 + 59904*a^6*b^2*c - 14336*a^8 + 231*(15*b^8*c - 14*a^2*b^6)*d^3*
x^3 - 28*(165*b^8*c^2 - 291*a^2*b^6*c + 140*a^4*b^4)*d^2*x^2 + 32*(231*b^8
*c^3 - 555*a^2*b^6*c^2 + 520*a^4*b^4*c - 168*a^6*b^2)*d*x + (3003*a*b^7*d^
3*x^3 - 27648*a*b^7*c^3 + 41472*a^3*b^5*c^2 - 28160*a^5*b^3*c + 7168*a^7*b
 - 3528*(2*a*b^7*c - a^3*b^5)*d^2*x^2 + 32*(417*a*b^7*c^2 - 417*a^3*b^5*c
+ 140*a^5*b^3)*d*x)*sqrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^8*d^4)
```

3.625.6 Sympy [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.10

$$\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$=\begin{cases}\frac{2\left(\begin{cases}\frac{2\left(-\frac{7a\left(a+b\sqrt{c+dx}\right)^{\frac{15}{2}}}{15b^6}+\frac{\left(a+b\sqrt{c+dx}\right)^{\frac{17}{2}}}{17b^6}+\frac{\left(a+b\sqrt{c+dx}\right)^{\frac{13}{2}}\cdot\left(21a^2-3b^2c\right)}{13b^6}+\frac{\left(a+b\sqrt{c+dx}\right)^{\frac{11}{2}}\left(-35a^3+15ab^2c\right)}{11b^6}+\frac{\left(a+b\sqrt{c+dx}\right)^{\frac{9}{2}}\cdot\left(35a^4-30a^2b^2c+3b\right.}{9b^6}}{} \\ \frac{\sqrt{a}d^4x^4}{8}\end{cases}\right.}{} \\ \frac{x^4\sqrt{a+b\sqrt{c}}}{4}\end{cases}$$

input
```
integrate(x**3*(a+b*(d*x+c)**(1/2))**(1/2),x)
```

output
```
Piecewise((2*Piecewise((2*(-7*a*(a + b*sqrt(c + d*x))**(15/2)/(15*b**6) +
(a + b*sqrt(c + d*x))**(17/2)/(17*b**6) + (a + b*sqrt(c + d*x))**(13/2)*(2
1*a**2 - 3*b**2*c)/(13*b**6) + (a + b*sqrt(c + d*x))**(11/2)*(-35*a**3 + 1
5*a*b**2*c)/(11*b**6) + (a + b*sqrt(c + d*x))**(9/2)*(35*a**4 - 30*a**2*b*
*2*c + 3*b**4*c**2)/(9*b**6) + (a + b*sqrt(c + d*x))**(7/2)*(-21*a**5 + 30
*a**3*b**2*c - 9*a*b**4*c**2)/(7*b**6) + (a + b*sqrt(c + d*x))**(5/2)*(7*a
**6 - 15*a**4*b**2*c + 9*a**2*b**4*c**2 - b**6*c**3)/(5*b**6) + (a + b*sqr
t(c + d*x))**(3/2)*(-a**7 + 3*a**5*b**2*c - 3*a**3*b**4*c**2 + a*b**6*c**3
)/(3*b**6))/b**2, Ne(b, 0)), (sqrt(a)*d**4*x**4/8, True))/d**4, Ne(d, 0)),
 (x**4*sqrt(a + b*sqrt(c))/4, True))
```

3.625.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.82

$$\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$= \frac{4\left(45045\left(\sqrt{dx+c}b+a\right)^{\frac{17}{2}}-357357\left(\sqrt{dx+c}b+a\right)^{\frac{15}{2}}a-176715\left(b^2c-7a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{13}{2}}+34807\right.}{}$$

input
```
integrate(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
4/765765*(45045*(sqrt(d*x + c)*b + a)^(17/2) - 357357*(sqrt(d*x + c)*b + a
)^(15/2)*a - 176715*(b^2*c - 7*a^2)*(sqrt(d*x + c)*b + a)^(13/2) + 348075*
(3*a*b^2*c - 7*a^3)*(sqrt(d*x + c)*b + a)^(11/2) + 85085*(3*b^4*c^2 - 30*a
^2*b^2*c + 35*a^4)*(sqrt(d*x + c)*b + a)^(9/2) - 328185*(3*a*b^4*c^2 - 10*
a^3*b^2*c + 7*a^5)*(sqrt(d*x + c)*b + a)^(7/2) - 153153*(b^6*c^3 - 9*a^2*b
^4*c^2 + 15*a^4*b^2*c - 7*a^6)*(sqrt(d*x + c)*b + a)^(5/2) + 255255*(a*b^6
*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*(sqrt(d*x + c)*b + a)^(3/2))/(b^
8*d^4)
```

3.625.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 915 vs. $2(279) = 558$.

Time = 0.42 (sec) , antiderivative size = 915, normalized size of antiderivative = 2.81

$$\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
-4/765765*(17*(15015*(sqrt(d*x + c)*b + a)^(3/2)*b^6*c^3 - 45045*sqrt(sqrt
(d*x + c)*b + a)*a*b^6*c^3 - 19305*(sqrt(d*x + c)*b + a)^(7/2)*b^4*c^2 + 8
1081*(sqrt(d*x + c)*b + a)^(5/2)*a*b^4*c^2 - 135135*(sqrt(d*x + c)*b + a)^
(3/2)*a^2*b^4*c^2 + 135135*sqrt(sqrt(d*x + c)*b + a)*a^3*b^4*c^2 + 12285*(
sqrt(d*x + c)*b + a)^(11/2)*b^2*c - 75075*(sqrt(d*x + c)*b + a)^(9/2)*a*b^
2*c + 193050*(sqrt(d*x + c)*b + a)^(7/2)*a^2*b^2*c - 270270*(sqrt(d*x + c)
*b + a)^(5/2)*a^3*b^2*c + 225225*(sqrt(d*x + c)*b + a)^(3/2)*a^4*b^2*c - 1
35135*sqrt(sqrt(d*x + c)*b + a)*a^5*b^2*c - 3003*(sqrt(d*x + c)*b + a)^(15
/2) + 24255*(sqrt(d*x + c)*b + a)^(13/2)*a - 85995*(sqrt(d*x + c)*b + a)^(
11/2)*a^2 + 175175*(sqrt(d*x + c)*b + a)^(9/2)*a^3 - 225225*(sqrt(d*x + c)
*b + a)^(7/2)*a^4 + 189189*(sqrt(d*x + c)*b + a)^(5/2)*a^5 - 105105*(sqrt(
d*x + c)*b + a)^(3/2)*a^6 + 45045*sqrt(sqrt(d*x + c)*b + a)*a^7)*a/(b^7*d^
3) + (153153*(sqrt(d*x + c)*b + a)^(5/2)*b^6*c^3 - 510510*(sqrt(d*x + c)*b
 + a)^(3/2)*a*b^6*c^3 + 765765*sqrt(sqrt(d*x + c)*b + a)*a^2*b^6*c^3 - 255
255*(sqrt(d*x + c)*b + a)^(9/2)*b^4*c^2 + 1312740*(sqrt(d*x + c)*b + a)^(7
/2)*a*b^4*c^2 - 2756754*(sqrt(d*x + c)*b + a)^(5/2)*a^2*b^4*c^2 + 3063060*
(sqrt(d*x + c)*b + a)^(3/2)*a^3*b^4*c^2 - 2297295*sqrt(sqrt(d*x + c)*b + a
)*a^4*b^4*c^2 + 176715*(sqrt(d*x + c)*b + a)^(13/2)*b^2*c - 1253070*(sqrt(
d*x + c)*b + a)^(11/2)*a*b^2*c + 3828825*(sqrt(d*x + c)*b + a)^(9/2)*a^2*b
^2*c - 6563700*(sqrt(d*x + c)*b + a)^(7/2)*a^3*b^2*c + 6891885*(sqrt(d*...
```

3.625.9 Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx = \int x^3\,\sqrt{a+b\,\sqrt{c+d\,x}}\,dx$$

input
```
int(x^3*(a + b*(c + d*x)^(1/2))^(1/2),x)
```

output
```
int(x^3*(a + b*(c + d*x)^(1/2))^(1/2), x)
```

3.626 $\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx$

3.626.1 Optimal result

Integrand size = 21, antiderivative size = 224

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx=-\frac{4a(a^2-b^2c)^2\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3}+\frac{4(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3}-\frac{8a(5a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6d^3}+\frac{8(5a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^6d^3}-\frac{20a\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^6d^3}+\frac{4\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^6d^3}$$

output
```
-4/3*a*(-b^2*c+a^2)^2*(a+b*(d*x+c)^(1/2))^(3/2)/b^6/d^3+4/5*(b^4*c^2-6*a^2
*b^2*c+5*a^4)*(a+b*(d*x+c)^(1/2))^(5/2)/b^6/d^3-8/7*a*(-3*b^2*c+5*a^2)*(a+
b*(d*x+c)^(1/2))^(7/2)/b^6/d^3+8/9*(-b^2*c+5*a^2)*(a+b*(d*x+c)^(1/2))^(9/2
)/b^6/d^3-20/11*a*(a+b*(d*x+c)^(1/2))^(11/2)/b^6/d^3+4/13*(a+b*(d*x+c)^(1/
2))^(13/2)/b^6/d^3
```

3.626.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.66

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$=\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(-1280a^5+32a^3b^2(68c-75dx)+1920a^4b\sqrt{c+dx}+16a^2b^3\sqrt{c+dx}(-254c+175d\right.}{45045b^6d^3}$$

input
```
Integrate[x^2*Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(4*(a + b*Sqrt[c + d*x])^(3/2)*(-1280*a^5 + 32*a^3*b^2*(68*c - 75*d*x) + 1
920*a^4*b*Sqrt[c + d*x] + 16*a^2*b^3*Sqrt[c + d*x]*(-254*c + 175*d*x) + 77
*b^5*Sqrt[c + d*x]*(32*c^2 - 40*c*d*x + 45*d^2*x^2) - 6*a*b^4*(96*c^2 - 38
0*c*d*x + 525*d^2*x^2)))/(45045*b^6*d^3)
```

3.626.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {896, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$\Big\downarrow \text{896}$$
$$\frac{\int d^2x^2\sqrt{a+b\sqrt{c+dx}}d(c+dx)}{d^3}$$
$$\Big\downarrow \text{1732}$$
$$\frac{2\int d^2x^2\sqrt{c+dx}\sqrt{a+b\sqrt{c+dx}}d\sqrt{c+dx}}{d^3}$$
$$\Big\downarrow \text{522}$$
$$\frac{2\int\left(\frac{\left(a+b\sqrt{c+dx}\right)^{11/2}}{b^5}-\frac{5a\left(a+b\sqrt{c+dx}\right)^{9/2}}{b^5}-\frac{2\left(b^2c-5a^2\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{b^5}-\frac{2\left(5a^3-3ab^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{b^5}+\frac{\left(5a^4-6b^2ca^2+b^4c^2\right)(a+}{b^5}\right.}{d^3}$$

$\downarrow$ 2009

$$\frac{2\left(\frac{4(5a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^6}-\frac{4a(5a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6}-\frac{2a(a^2-b^2c)^2\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6}+\frac{2(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6}\right.}{d^3}$$

input
```
Int[x^2*Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(2*((-2*a*(a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^6) + (2*(5*a^4
 - 6*a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^6) - (4*a*(5*a
^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^6) + (4*(5*a^2 - b^2*c)*(a
 + b*Sqrt[c + d*x])^(9/2))/(9*b^6) - (10*a*(a + b*Sqrt[c + d*x])^(11/2))/(
11*b^6) + (2*(a + b*Sqrt[c + d*x])^(13/2))/(13*b^6)))/d^3
```

3.626.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.626. $\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx$

3.626.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{\frac{4(a+b\sqrt{dx+c})^{\frac{13}{2}}}{13}-\frac{20a(a+b\sqrt{dx+c})^{\frac{11}{2}}}{11}-\frac{4(2b^2c-10a^2)(a+b\sqrt{dx+c})^{\frac{9}{2}}}{9}-\frac{4(4(-b^2c+a^2)a+a(-2b^2c+6a^2))(a+b\sqrt{dx+c})^{\frac{7}{2}}}{7}-\frac{4(}{}}{d^3b^6}$
default	$\frac{\frac{4(a+b\sqrt{dx+c})^{\frac{13}{2}}}{13}-\frac{20a(a+b\sqrt{dx+c})^{\frac{11}{2}}}{11}-\frac{4(2b^2c-10a^2)(a+b\sqrt{dx+c})^{\frac{9}{2}}}{9}-\frac{4(4(-b^2c+a^2)a+a(-2b^2c+6a^2))(a+b\sqrt{dx+c})^{\frac{7}{2}}}{7}-\frac{4(}{}}{d^3b^6}$

input
```
int(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/d^3/b^6*(1/13*(a+b*(d*x+c)^(1/2))^(13/2)-5/11*a*(a+b*(d*x+c)^(1/2))^(11/
2)-1/9*(2*b^2*c-10*a^2)*(a+b*(d*x+c)^(1/2))^(9/2)-1/7*(4*(-b^2*c+a^2)*a+a*
(-2*b^2*c+6*a^2))*(a+b*(d*x+c)^(1/2))^(7/2)-1/5*(-(-b^2*c+a^2)^2-4*a^2*(-b
^2*c+a^2))*(a+b*(d*x+c)^(1/2))^(5/2)-1/3*(-b^2*c+a^2)^2*a*(a+b*(d*x+c)^(1/
2))^(3/2))
```

3.626.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.82

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$= \frac{4\left(3465\,b^6d^3x^3+2464\,b^6c^3-4640\,a^2b^4c^2+4096\,a^4b^2c-1280\,a^6+35\,(11\,b^6c-10\,a^2b^4)d^2x^2-8\,(77\,b^6c^2\right.}{}$$

input
```
integrate(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
4/45045*(3465*b^6*d^3*x^3 + 2464*b^6*c^3 - 4640*a^2*b^4*c^2 + 4096*a^4*b^2
*c - 1280*a^6 + 35*(11*b^6*c - 10*a^2*b^4)*d^2*x^2 - 8*(77*b^6*c^2 - 127*a
^2*b^4*c + 60*a^4*b^2)*d*x + (315*a*b^5*d^2*x^2 + 1888*a*b^5*c^2 - 1888*a^
3*b^3*c + 640*a^5*b - 400*(2*a*b^5*c - a^3*b^3)*d*x)*sqrt(d*x + c))*sqrt(s
qrt(d*x + c)*b + a)/(b^6*d^3)
```

3.626.6 Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.08

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx$$

$$= \begin{cases} \frac{2\left(\begin{cases} \frac{2\left(-\frac{5a(a+b\sqrt{c+dx})^{\frac{11}{2}}}{11b^4}+\frac{(a+b\sqrt{c+dx})^{\frac{13}{2}}}{13b^4}+\frac{(a+b\sqrt{c+dx})^{\frac{9}{2}}\cdot(10a^2-2b^2c)}{9b^4}+\frac{(a+b\sqrt{c+dx})^{\frac{7}{2}}(-10a^3+6ab^2c)}{7b^4}+\frac{(a+b\sqrt{c+dx})^{\frac{5}{2}}\cdot(5a^4-6a^2b^2c+b^4c^2)}{5b^4} \right.}{b^2} \\ \frac{\sqrt{a}d^3x^3}{6} \end{cases}\right.}{d^3} \\ \frac{x^3\sqrt{a+b\sqrt{c}}}{3} \end{cases}$$

input
```
integrate(x**2*(a+b*(d*x+c)**(1/2))**(1/2),x)
```

output
```
Piecewise((2*Piecewise((2*(-5*a*(a + b*sqrt(c + d*x))**(11/2)/(11*b**4) +
(a + b*sqrt(c + d*x))**(13/2)/(13*b**4) + (a + b*sqrt(c + d*x))**(9/2)*(10
*a**2 - 2*b**2*c)/(9*b**4) + (a + b*sqrt(c + d*x))**(7/2)*(-10*a**3 + 6*a*
b**2*c)/(7*b**4) + (a + b*sqrt(c + d*x))**(5/2)*(5*a**4 - 6*a**2*b**2*c +
b**4*c**2)/(5*b**4) + (a + b*sqrt(c + d*x))**(3/2)*(-a**5 + 2*a**3*b**2*c
- a*b**4*c**2)/(3*b**4))/b**2, Ne(b, 0)), (sqrt(a)*d**3*x**3/6, True))/d**
3, Ne(d, 0)), (x**3*sqrt(a + b*sqrt(c))/3, True))
```

3.626.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.75

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx$$

$$= \frac{4\left(3465\left(\sqrt{dx+c}b+a\right)^{\frac{13}{2}}-20475\left(\sqrt{dx+c}b+a\right)^{\frac{11}{2}}a-10010\left(b^2c-5a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{9}{2}}+12870\left(3a\right.\right.}{}$$

input
```
integrate(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
4/45045*(3465*(sqrt(d*x + c)*b + a)^(13/2) - 20475*(sqrt(d*x + c)*b + a)^(
11/2)*a - 10010*(b^2*c - 5*a^2)*(sqrt(d*x + c)*b + a)^(9/2) + 12870*(3*a*b
^2*c - 5*a^3)*(sqrt(d*x + c)*b + a)^(7/2) + 9009*(b^4*c^2 - 6*a^2*b^2*c +
5*a^4)*(sqrt(d*x + c)*b + a)^(5/2) - 15015*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)
*(sqrt(d*x + c)*b + a)^(3/2))/(b^6*d^3)
```

3.626.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(188) = 376$.

Time = 0.39 (sec) , antiderivative size = 549, normalized size of antiderivative = 2.45

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx$$

$$= \frac{4\left(\frac{13\left(1155\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}b^4c^2-3465\sqrt{\sqrt{dx+c}b+a}ab^4c^2-990\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}}b^2c+4158\left(\sqrt{dx+c}b+a\right)^{\frac{5}{2}}ab^2c-6930\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}a^2b^2c+693}{}\right.\right.}{}$$

input
```
integrate(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
4/45045*(13*(1155*(sqrt(d*x + c)*b + a)^(3/2)*b^4*c^2 - 3465*sqrt(sqrt(d*x
 + c)*b + a)*a*b^4*c^2 - 990*(sqrt(d*x + c)*b + a)^(7/2)*b^2*c + 4158*(sqr
t(d*x + c)*b + a)^(5/2)*a*b^2*c - 6930*(sqrt(d*x + c)*b + a)^(3/2)*a^2*b^2
*c + 6930*sqrt(sqrt(d*x + c)*b + a)*a^3*b^2*c + 315*(sqrt(d*x + c)*b + a)^
(11/2) - 1925*(sqrt(d*x + c)*b + a)^(9/2)*a + 4950*(sqrt(d*x + c)*b + a)^(
7/2)*a^2 - 6930*(sqrt(d*x + c)*b + a)^(5/2)*a^3 + 5775*(sqrt(d*x + c)*b +
a)^(3/2)*a^4 - 3465*sqrt(sqrt(d*x + c)*b + a)*a^5)*a/(b^5*d^2) + (9009*(sq
rt(d*x + c)*b + a)^(5/2)*b^4*c^2 - 30030*(sqrt(d*x + c)*b + a)^(3/2)*a*b^4
*c^2 + 45045*sqrt(sqrt(d*x + c)*b + a)*a^2*b^4*c^2 - 10010*(sqrt(d*x + c)*
b + a)^(9/2)*b^2*c + 51480*(sqrt(d*x + c)*b + a)^(7/2)*a*b^2*c - 108108*(s
qrt(d*x + c)*b + a)^(5/2)*a^2*b^2*c + 120120*(sqrt(d*x + c)*b + a)^(3/2)*a
^3*b^2*c - 90090*sqrt(sqrt(d*x + c)*b + a)*a^4*b^2*c + 3465*(sqrt(d*x + c)
*b + a)^(13/2) - 24570*(sqrt(d*x + c)*b + a)^(11/2)*a + 75075*(sqrt(d*x +
c)*b + a)^(9/2)*a^2 - 128700*(sqrt(d*x + c)*b + a)^(7/2)*a^3 + 135135*(sqr
t(d*x + c)*b + a)^(5/2)*a^4 - 90090*(sqrt(d*x + c)*b + a)^(3/2)*a^5 + 4504
5*sqrt(sqrt(d*x + c)*b + a)*a^6)/(b^5*d^2))/(b*d)
```

3.626.9 Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx = \int x^2\,\sqrt{a+b\,\sqrt{c+d\,x}}\,dx$$

input
```
int(x^2*(a + b*(c + d*x)^(1/2))^(1/2),x)
```

output
```
int(x^2*(a + b*(c + d*x)^(1/2))^(1/2), x)
```

3.627 $\int x\sqrt{a+b\sqrt{c+dx}}\,dx$

3.627.1 Optimal result

Integrand size = 19, antiderivative size = 133

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx = -\frac{4a(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2} + \frac{4(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2} - \frac{12a\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2} + \frac{4\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^4d^2}$$

output
```
-4/3*a*(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(3/2)/b^4/d^2+4/5*(-b^2*c+3*a^2)*(
a+b*(d*x+c)^(1/2))^(5/2)/b^4/d^2-12/7*a*(a+b*(d*x+c)^(1/2))^(7/2)/b^4/d^2+
4/9*(a+b*(d*x+c)^(1/2))^(9/2)/b^4/d^2
```

3.627.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.63

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx = \frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(-16a^3+6ab^2(2c-5dx)+24a^2b\sqrt{c+dx}+7b^3\sqrt{c+dx}(-4c+5dx)\right)}{315b^4d^2}$$

input
```
Integrate[x*Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(4*(a + b*Sqrt[c + d*x])^(3/2)*(-16*a^3 + 6*a*b^2*(2*c - 5*d*x) + 24*a^2*b
*Sqrt[c + d*x] + 7*b^3*Sqrt[c + d*x]*(-4*c + 5*d*x)))/(315*b^4*d^2)
```

3.627.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int dx\sqrt{a+b\sqrt{c+dx}}d(c+dx)}{d^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -dx\sqrt{a+b\sqrt{c+dx}}d(c+dx)}{d^2}$$

$$\Big\downarrow \text{1732}$$

$$-\frac{2\int -dx\sqrt{c+dx}\sqrt{a+b\sqrt{c+dx}}d\sqrt{c+dx}}{d^2}$$

$$\Big\downarrow \text{522}$$

$$-\frac{2\int \left(-\frac{\left(a+b\sqrt{c+dx}\right)^{7/2}}{b^3}+\frac{3a\left(a+b\sqrt{c+dx}\right)^{5/2}}{b^3}+\frac{(b^2c-3a^2)\left(a+b\sqrt{c+dx}\right)^{3/2}}{b^3}+\frac{(a^3-ab^2c)\sqrt{a+b\sqrt{c+dx}}}{b^3}\right)d\sqrt{c+dx}}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(-\frac{2(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4}+\frac{2a(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4}-\frac{2\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^4}+\frac{6a\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4}\right)}{d^2}$$

input
```
Int[x*Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(-2*((2*a*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^4) - (2*(3*a^2 -
 b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^4) + (6*a*(a + b*Sqrt[c + d*x])^
(7/2))/(7*b^4) - (2*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^4)))/d^2
```

3.627.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.627.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$-\frac{4\left(-\frac{\left(a+b\sqrt{dx+c}\right)^{\frac{9}{2}}}{9}+\frac{3a\left(a+b\sqrt{dx+c}\right)^{\frac{7}{2}}}{7}+\frac{\left(b^{2}c-3a^{2}\right)\left(a+b\sqrt{dx+c}\right)^{\frac{5}{2}}}{5}+\frac{\left(-b^{2}c+a^{2}\right)a\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}\right)}{d^{2}b^{4}}$	93
default	$-\frac{4\left(-\frac{\left(a+b\sqrt{dx+c}\right)^{\frac{9}{2}}}{9}+\frac{3a\left(a+b\sqrt{dx+c}\right)^{\frac{7}{2}}}{7}+\frac{\left(b^{2}c-3a^{2}\right)\left(a+b\sqrt{dx+c}\right)^{\frac{5}{2}}}{5}+\frac{\left(-b^{2}c+a^{2}\right)a\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}\right)}{d^{2}b^{4}}$	93

input
```
int(x*(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/d^2/b^4*(-1/9*(a+b*(d*x+c)^(1/2))^(9/2)+3/7*a*(a+b*(d*x+c)^(1/2))^(7/2)
+1/5*(b^2*c-3*a^2)*(a+b*(d*x+c)^(1/2))^(5/2)+1/3*(-b^2*c+a^2)*a*(a+b*(d*x+
c)^(1/2))^(3/2))
```

3.627.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.77

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$=\frac{4\left(35\,b^4d^2x^2-28\,b^4c^2+36\,a^2b^2c-16\,a^4+(7\,b^4c-6\,a^2b^2)dx+(5\,ab^3dx-16\,ab^3c+8\,a^3b)\sqrt{dx+c}\right)\sqrt{\sqrt{c}}}{315\,b^4d^2}$$

input
```
integrate(x*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
4/315*(35*b^4*d^2*x^2 - 28*b^4*c^2 + 36*a^2*b^2*c - 16*a^4 + (7*b^4*c - 6*
a^2*b^2)*d*x + (5*a*b^3*d*x - 16*a*b^3*c + 8*a^3*b)*sqrt(d*x + c))*sqrt(sq
rt(d*x + c)*b + a)/(b^4*d^2)
```

3.627.6 Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.17

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$=\begin{cases}\frac{2\left(\begin{cases}\frac{2\left(-\frac{3a\left(a+b\sqrt{c+dx}\right)^{\frac{7}{2}}}{7b^2}+\frac{\left(a+b\sqrt{c+dx}\right)^{\frac{9}{2}}}{9b^2}+\frac{\left(a+b\sqrt{c+dx}\right)^{\frac{5}{2}}\cdot\left(3a^2-b^2c\right)}{5b^2}+\frac{\left(a+b\sqrt{c+dx}\right)^{\frac{3}{2}}\left(-a^3+ab^2c\right)}{3b^2}\right)}{b^2} & \text{for}\ b\neq 0\\ \sqrt{a}\left(-\frac{c(c+dx)}{2}+\frac{(c+dx)^2}{4}\right) & \text{otherwise}\end{cases}\right)}{d^2} & \text{for}\ d\neq 0\\ \frac{x^2\sqrt{a+b\sqrt{c}}}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*(a+b*(d*x+c)**(1/2))**(1/2),x)
```

output
```
Piecewise((2*Piecewise((2*(-3*a*(a + b*sqrt(c + d*x))**(7/2)/(7*b**2) + (a
 + b*sqrt(c + d*x))**(9/2)/(9*b**2) + (a + b*sqrt(c + d*x))**(5/2)*(3*a**2
 - b**2*c)/(5*b**2) + (a + b*sqrt(c + d*x))**(3/2)*(-a**3 + a*b**2*c)/(3*b
**2))/b**2, Ne(b, 0)), (sqrt(a)*(-c*(c + d*x)/2 + (c + d*x)**2/4), True))/
d**2, Ne(d, 0)), (x**2*sqrt(a + b*sqrt(c))/2, True))
```

3.627.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.70

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx$$
$$= \frac{4\left(35\left(\sqrt{dx+c}b+a\right)^{\frac{9}{2}}-135\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}}a-63\left(b^2c-3a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{5}{2}}+105\left(ab^2c-a^3\right)\left(\sqrt{d\right.}\right.}{315\,b^4d^2}$$

input
```
integrate(x*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
4/315*(35*(sqrt(d*x + c)*b + a)^(9/2) - 135*(sqrt(d*x + c)*b + a)^(7/2)*a
- 63*(b^2*c - 3*a^2)*(sqrt(d*x + c)*b + a)^(5/2) + 105*(a*b^2*c - a^3)*(sq
rt(d*x + c)*b + a)^(3/2))/(b^4*d^2)
```

3.627.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(109) = 218.

Time = 0.39 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.10

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx =$$
$$-\frac{4\left(\frac{3\left(35\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}b^2c-105\sqrt{\sqrt{dx+c}b+a}ab^2c-15\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}}+63\left(\sqrt{dx+c}b+a\right)^{\frac{5}{2}}a-105\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}a^2+105\sqrt{\sqrt{dx+c}b+a}a^3\right)}{b^3d}\right.}{}$$

input
```
integrate(x*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
-4/315*(3*(35*(sqrt(d*x + c)*b + a)^(3/2)*b^2*c - 105*sqrt(sqrt(d*x + c)*b
 + a)*a*b^2*c - 15*(sqrt(d*x + c)*b + a)^(7/2) + 63*(sqrt(d*x + c)*b + a)^
(5/2)*a - 105*(sqrt(d*x + c)*b + a)^(3/2)*a^2 + 105*sqrt(sqrt(d*x + c)*b +
 a)*a^3)*a/(b^3*d) + (63*(sqrt(d*x + c)*b + a)^(5/2)*b^2*c - 210*(sqrt(d*x
 + c)*b + a)^(3/2)*a*b^2*c + 315*sqrt(sqrt(d*x + c)*b + a)*a^2*b^2*c - 35*
(sqrt(d*x + c)*b + a)^(9/2) + 180*(sqrt(d*x + c)*b + a)^(7/2)*a - 378*(sqr
t(d*x + c)*b + a)^(5/2)*a^2 + 420*(sqrt(d*x + c)*b + a)^(3/2)*a^3 - 315*sq
rt(sqrt(d*x + c)*b + a)*a^4)/(b^3*d))/(b*d)
```

3.627.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx=\int x\,\sqrt{a+b\,\sqrt{c+d\,x}}\,dx$$

input
```
int(x*(a + b*(c + d*x)^(1/2))^(1/2),x)
```

output
```
int(x*(a + b*(c + d*x)^(1/2))^(1/2), x)
```

3.628 $\int \sqrt{a+b\sqrt{c+dx}}\,dx$

3.628.1 Optimal result

Integrand size = 17, antiderivative size = 56

$$\int \sqrt{a+b\sqrt{c+dx}}\,dx=-\frac{4a\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}+\frac{4\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^2d}$$

output
```
-4/3*a*(a+b*(d*x+c)^(1/2))^(3/2)/b^2/d+4/5*(a+b*(d*x+c)^(1/2))^(5/2)/b^2/d
```

3.628.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \sqrt{a+b\sqrt{c+dx}}\,dx=\frac{4\sqrt{a+b\sqrt{c+dx}}\left(-2a^2+ab\sqrt{c+dx}+3b^2(c+dx)\right)}{15b^2d}$$

input
```
Integrate[Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(4*Sqrt[a + b*Sqrt[c + d*x]]*(-2*a^2 + a*b*Sqrt[c + d*x] + 3*b^2*(c + d*x)
))/(15*b^2*d)
```

3.628.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {239, 774, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{239}$$

$$\frac{\int \sqrt{a+b\sqrt{c+dx}}d(c+dx)}{d}$$

$$\Big\downarrow \text{774}$$

$$\frac{2\int \sqrt{c+dx}\sqrt{a+b\sqrt{c+dx}}d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{53}$$

$$\frac{2\int \left(\frac{\left(a+b\sqrt{c+dx}\right)^{3/2}}{b}-\frac{a\sqrt{a+b\sqrt{c+dx}}}{b}\right)d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{2\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^2}-\frac{2a\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2}\right)}{d}$$

input

```
Int[Sqrt[a + b*Sqrt[c + d*x]],x]
```

output

```
(2*((-2*a*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^2) + (2*(a + b*Sqrt[c + d*x])^
(5/2))/(5*b^2)))/d
```

3.628.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.628.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{\frac{4\left(a+b\sqrt{dx+c}\right)^{\frac{5}{2}}}{5}-\frac{4a\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}}{b^{2}d}$	41
default	$\frac{\frac{4\left(a+b\sqrt{dx+c}\right)^{\frac{5}{2}}}{5}-\frac{4a\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}}{b^{2}d}$	41

input
```
int((a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/d/b^2*(1/5*(a+b*(d*x+c)^(1/2))^(5/2)-1/3*a*(a+b*(d*x+c)^(1/2))^(3/2))
```

3.628.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \sqrt{a+b\sqrt{c+dx}}\,dx = \frac{4\left(3\,b^2dx+3\,b^2c+\sqrt{dx+c}ab-2\,a^2\right)\sqrt{\sqrt{dx+c}b+a}}{15\,b^2d}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
4/15*(3*b^2*d*x + 3*b^2*c + sqrt(d*x + c)*a*b - 2*a^2)*sqrt(sqrt(d*x + c)*
b + a)/(b^2*d)
```

3.628.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.25

$$\int \sqrt{a+b\sqrt{c+dx}}\,dx = \begin{cases} \frac{2\left(\begin{cases} \frac{2\left(-\frac{a\left(a+b\sqrt{c+dx}\right)^{\frac{3}{2}}}{3}+\frac{\left(a+b\sqrt{c+dx}\right)^{\frac{5}{2}}}{5}\right)}{b^2} & \text{for } b\neq 0 \\ \frac{\sqrt{a}(c+dx)}{2} & \text{otherwise} \end{cases}\right)}{d} & \text{for } d\neq 0 \\ x\sqrt{a+b\sqrt{c}} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*(d*x+c)**(1/2))**(1/2),x)
```

output
```
Piecewise((2*Piecewise((2*(-a*(a + b*sqrt(c + d*x))**(3/2)/3 + (a + b*sqrt
(c + d*x))**(5/2)/5)/b**2, Ne(b, 0)), (sqrt(a)*(c + d*x)/2, True))/d, Ne(d
, 0)), (x*sqrt(a + b*sqrt(c)), True))
```

3.628.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \sqrt{a+b\sqrt{c+dx}}\,dx = \frac{4\left(\frac{3\left(\sqrt{dx+c}b+a\right)^{\frac{5}{2}}}{b^2}-\frac{5\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}a}{b^2}\right)}{15\,d}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
4/15*(3*(sqrt(d*x + c)*b + a)^(5/2)/b^2 - 5*(sqrt(d*x + c)*b + a)^(3/2)*a/
b^2)/d
```

3.628.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(44) = 88$.

Time = 0.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.77

$$\int \sqrt{a+b\sqrt{c+dx}}\,dx = \frac{4\left(\frac{5\left(\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}-3\sqrt{\sqrt{dx+c}b+a}a\right)a}{b}+\frac{3\left(\sqrt{dx+c}b+a\right)^{\frac{5}{2}}-10\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}a+15\sqrt{\sqrt{dx+c}b+a}a^{2}}{b}\right)}{15\,bd}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
4/15*(5*((sqrt(d*x + c)*b + a)^(3/2) - 3*sqrt(sqrt(d*x + c)*b + a)*a)*a/b
+ (3*(sqrt(d*x + c)*b + a)^(5/2) - 10*(sqrt(d*x + c)*b + a)^(3/2)*a + 15*s
qrt(sqrt(d*x + c)*b + a)*a^2)/b)/(b*d)
```

3.628.9 Mupad [B] (verification not implemented)

Time = 17.90 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int \sqrt{a+b\sqrt{c+dx}}\,dx = \frac{4\left(a+b\sqrt{c+d\,x}\right)^{5/2}}{5\,b^2\,d}-\frac{4\,a\left(a+b\sqrt{c+d\,x}\right)^{3/2}}{3\,b^2\,d}$$

input
```
int((a + b*(c + d*x)^(1/2))^(1/2),x)
```

output
```
(4*(a + b*(c + d*x)^(1/2))^(5/2))/(5*b^2*d) - (4*a*(a + b*(c + d*x)^(1/2))
^(3/2))/(3*b^2*d)
```

3.629 $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx$

3.629.1 Optimal result

Integrand size = 21, antiderivative size = 116

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx = 4\sqrt{a+b\sqrt{c+dx}} - 2\sqrt{a-b\sqrt{c}}\,\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right) - 2\sqrt{a+b\sqrt{c}}\,\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)$$

output
```
-2*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a-b*c^(1/2))^(1/2))*(a-b*c^(1/2))^(1
/2)-2*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a+b*c^(1/2))^(1/2))*(a+b*c^(1/2))
^(1/2)+4*(a+b*(d*x+c)^(1/2))^(1/2)
```

3.629.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx = 4\sqrt{a+b\sqrt{c+dx}} - 2\sqrt{-a-b\sqrt{c}}\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a-b\sqrt{c}}}\right) - 2\sqrt{-a+b\sqrt{c}}\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a+b\sqrt{c}}}\right)$$

input
```
Integrate[Sqrt[a + b*Sqrt[c + d*x]]/x,x]
```

output
```
4*Sqrt[a + b*Sqrt[c + d*x]] - 2*Sqrt[-a - b*Sqrt[c]]*ArcTan[Sqrt[a + b*Sqr
t[c + d*x]]/Sqrt[-a - b*Sqrt[c]]] - 2*Sqrt[-a + b*Sqrt[c]]*ArcTan[Sqrt[a +
 b*Sqrt[c + d*x]]/Sqrt[-a + b*Sqrt[c]]]
```

3.629.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {896, 25, 1732, 561, 27, 1602, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}dx$$

$$\Big\downarrow \text{896}$$

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{dx}d(c+dx)$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{\sqrt{a+b\sqrt{c+dx}}}{dx}d(c+dx)$$

$$\Big\downarrow \text{1732}$$

$$-2\int -\frac{\sqrt{c+dx}\sqrt{a+b\sqrt{c+dx}}}{dx}d\sqrt{c+dx}$$

$$\Big\downarrow \text{561}$$

$$-\frac{4\int \frac{(a-c-dx)(c+dx)}{b\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}d\sqrt{a+b\sqrt{c+dx}}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4\int \frac{(a-c-dx)(c+dx)}{\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c}d\sqrt{a+b\sqrt{c+dx}}}{b^2}$$

$$\Big\downarrow \text{1602}$$

$$-\frac{4\left(b^2\left(-\int -\frac{b^2\left(\frac{a^2}{b^2}-c\right)-a(c+dx)}{b^2\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}d\sqrt{a+b\sqrt{c+dx}}\right)-b^2\sqrt{a+b\sqrt{c+dx}}\right)}{b^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{4\left(b^2\int \frac{a^2-(c+dx)a-b^2c}{b^2\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}d\sqrt{a+b\sqrt{c+dx}}-b^2\sqrt{a+b\sqrt{c+dx}}\right)}{b^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{4\left(\int \frac{a^2-(c+dx)a-b^2c}{\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c}d\sqrt{a+b\sqrt{c+dx}}-b^2\sqrt{a+b\sqrt{c+dx}}\right)}{b^2}$$

$$\Big\downarrow \; 1480$$

$$-\frac{4\left(-\frac{1}{2}\left(a-b\sqrt{c}\right)\int \frac{1}{\frac{c+dx}{b^2}-\frac{a-b\sqrt{c}}{b^2}}d\sqrt{a+b\sqrt{c+dx}}-\frac{1}{2}\left(a+b\sqrt{c}\right)\int \frac{1}{\frac{c+dx}{b^2}-\frac{a+b\sqrt{c}}{b^2}}d\sqrt{a+b\sqrt{c+dx}}+b^2\left(-\sqrt{a+b\sqrt{c}}\right.\right.}{b^2}$$

$$\Big\downarrow \; 221$$

$$-\frac{4\left(\frac{1}{2}b^2\sqrt{a-b\sqrt{c}}\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)+\frac{1}{2}b^2\sqrt{a+b\sqrt{c}}\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)-b^2\sqrt{a+b\sqrt{c+dx}}\right)}{b^2}$$

input
```
Int[Sqrt[a + b*Sqrt[c + d*x]]/x,x]
```

output
```
(-4*(-(b^2*Sqrt[a + b*Sqrt[c + d*x]]) + (b^2*Sqrt[a - b*Sqrt[c]]*ArcTanh[S
qrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/2 + (b^2*Sqrt[a + b*Sqrt[c]
]*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/2))/b^2
```

3.629.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.629. $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx$

rule 561
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(n + 1) - 1)*(-c
/d + x^k/d)^m*Simp[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^k/d^2) + b*(x^(2*k)/d^2),
 x]^p, x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, m, p}, x] && Frac
tionQ[n] && IntegerQ[p] && IntegerQ[m]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

3.629. $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx$

3.629.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.33

method	result	siz
derivativedivides	$4\sqrt{a+b\sqrt{dx+c}}-\frac{2\left(-b^2c-a\sqrt{b^2c}\right)\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^2c}-a}}\right)}{\sqrt{b^2c}\,\sqrt{-\sqrt{b^2c}-a}}-\frac{2\left(b^2c-a\sqrt{b^2c}\right)\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)}{\sqrt{b^2c}\,\sqrt{\sqrt{b^2c}-a}}$	15
default	$4\sqrt{a+b\sqrt{dx+c}}-\frac{2\left(-b^2c-a\sqrt{b^2c}\right)\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^2c}-a}}\right)}{\sqrt{b^2c}\,\sqrt{-\sqrt{b^2c}-a}}-\frac{2\left(b^2c-a\sqrt{b^2c}\right)\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)}{\sqrt{b^2c}\,\sqrt{\sqrt{b^2c}-a}}$	15

input
```
int((a+b*(d*x+c)^(1/2))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
4*(a+b*(d*x+c)^(1/2))^(1/2)-2*(-b^2*c-a*(b^2*c)^(1/2))/(b^2*c)^(1/2)/(-(b^
2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)-a)^(1
/2))-2*(b^2*c-a*(b^2*c)^(1/2))/(b^2*c)^(1/2)/((b^2*c)^(1/2)-a)^(1/2)*arcta
n((a+b*(d*x+c)^(1/2))^(1/2)/((b^2*c)^(1/2)-a)^(1/2))
```

3.629.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(88) = 176$.

Time = 0.35 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.67

$$\begin{aligned}\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx = & -\sqrt{a+\sqrt{b^2c}}\log\left(2\sqrt{\sqrt{dx+c}b+a}+2\sqrt{a+\sqrt{b^2c}}\right)\\ & +\sqrt{a+\sqrt{b^2c}}\log\left(2\sqrt{\sqrt{dx+c}b+a}-2\sqrt{a+\sqrt{b^2c}}\right)\\ & -\sqrt{a-\sqrt{b^2c}}\log\left(2\sqrt{\sqrt{dx+c}b+a}+2\sqrt{a-\sqrt{b^2c}}\right)\\ & +\sqrt{a-\sqrt{b^2c}}\log\left(2\sqrt{\sqrt{dx+c}b+a}-2\sqrt{a-\sqrt{b^2c}}\right)\\ & +4\sqrt{\sqrt{dx+c}b+a}\end{aligned}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

output
```
-sqrt(a + sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b + a) + 2*sqrt(a + sqrt(b
^2*c))) + sqrt(a + sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b + a) - 2*sqrt(a
 + sqrt(b^2*c))) - sqrt(a - sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b + a) +
 2*sqrt(a - sqrt(b^2*c))) + sqrt(a - sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)
*b + a) - 2*sqrt(a - sqrt(b^2*c))) + 4*sqrt(sqrt(d*x + c)*b + a)
```

3.629.6 Sympy [F]

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx=\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx$$

input
```
integrate((a+b*(d*x+c)**(1/2))**(1/2)/x,x)
```

output
```
Integral(sqrt(a + b*sqrt(c + d*x))/x, x)
```

3.629.7 Maxima [F]

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx=\int \frac{\sqrt{\sqrt{dx+c}b+a}}{x}\,dx$$

input
```
integrate((a+b*(d*x+c)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(d*x + c)*b + a)/x, x)
```

3.629.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx = \frac{2\left(2\sqrt{\sqrt{dx+c}b+a}b-\frac{(b^3c-a^2b)\sqrt{b\sqrt{c}-a}\arctan\left(\frac{\sqrt{\sqrt{dx+c}b+a}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{b^2c-a^2}-\frac{(b^3c-a^2b)\sqrt{-b\sqrt{c}-a}\arctan\left(\frac{\sqrt{\sqrt{dx+c}b+a}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{b^2c-a^2}\right)}{b}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^(1/2)/x,x, algorithm="giac")
```

output
```
2*(2*sqrt(sqrt(d*x + c)*b + a)*b - (b^3*c - a^2*b)*sqrt(b*sqrt(c) - a)*arc
tan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a + sqrt(b^2*c)))/(b^2*c - a^2) - (b^3
*c - a^2*b)*sqrt(-b*sqrt(c) - a)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a
- sqrt(b^2*c)))/(b^2*c - a^2))/b
```

3.629.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx = \int \frac{\sqrt{a+b\,\sqrt{c+d\,x}}}{x}\,dx$$

input
```
int((a + b*(c + d*x)^(1/2))^(1/2)/x,x)
```

output
```
int((a + b*(c + d*x)^(1/2))^(1/2)/x, x)
```

3.630 $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx$

3.630.1 Optimal result

Integrand size = 21, antiderivative size = 137

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx=-\frac{\sqrt{a+b\sqrt{c+dx}}}{x}+\frac{bd\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{a-b\sqrt{c}}\sqrt{c}}-\frac{bd\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{a+b\sqrt{c}}\sqrt{c}}$$

output
```
1/2*b*d*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a-b*c^(1/2))^(1/2))/c^(1/2)/(a-
b*c^(1/2))^(1/2)-1/2*b*d*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a+b*c^(1/2))^(
1/2))/c^(1/2)/(a+b*c^(1/2))^(1/2)-(a+b*(d*x+c)^(1/2))^(1/2)/x
```

3.630.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx=\frac{1}{2}\left(-\frac{2\sqrt{a+b\sqrt{c+dx}}}{x}+\frac{bd\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a-b\sqrt{c}}}\right)}{\sqrt{-a-b\sqrt{c}}\sqrt{c}}-\frac{bd\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a+b\sqrt{c}}}\right)}{\sqrt{-a+b\sqrt{c}}\sqrt{c}}\right)$$

input
```
Integrate[Sqrt[a + b*Sqrt[c + d*x]]/x^2,x]
```

output
```
((-2*Sqrt[a + b*Sqrt[c + d*x]])/x + (b*d*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/
Sqrt[-a - b*Sqrt[c]]])/(Sqrt[-a - b*Sqrt[c]]*Sqrt[c]) - (b*d*ArcTan[Sqrt[a
 + b*Sqrt[c + d*x]]/Sqrt[-a + b*Sqrt[c]]])/(Sqrt[-a + b*Sqrt[c]]*Sqrt[c]))
/2
```

3.630.3 Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.36, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {896, 1732, 561, 25, 27, 1598, 27, 1406, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}dx$$

$$\Big\downarrow \text{896}$$

$$d\int \frac{\sqrt{a+b\sqrt{c+dx}}}{d^2x^2}d(c+dx)$$

$$\Big\downarrow \text{1732}$$

$$2d\int \frac{\sqrt{c+dx}\sqrt{a+b\sqrt{c+dx}}}{d^2x^2}d\sqrt{c+dx}$$

$$\Big\downarrow \text{561}$$

$$\frac{4d\int -\frac{(a-c-dx)(c+dx)}{b\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}d\sqrt{a+b\sqrt{c+dx}}}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{4d\int \frac{(a-c-dx)(c+dx)}{b\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}d\sqrt{a+b\sqrt{c+dx}}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4d\int \frac{(a-c-dx)(c+dx)}{\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}d\sqrt{a+b\sqrt{c+dx}}}{b^2}$$

$$\Big\downarrow \textbf{1598}$$

$$-\frac{4d\left(\frac{b^2\int -\frac{2c}{\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c}d\sqrt{a+b\sqrt{c+dx}}}{8c}+\frac{b^2\sqrt{a+b\sqrt{c+dx}}}{4\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}\right)}{b^2}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{4d\left(\frac{b^2\sqrt{a+b\sqrt{c+dx}}}{4\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}-\frac{1}{4}b^2\int \frac{1}{\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c}d\sqrt{a+b\sqrt{c+dx}}\right)}{b^2}$$

$$\Big\downarrow \textbf{1406}$$

$$-\frac{4d\left(\frac{b^2\sqrt{a+b\sqrt{c+dx}}}{4\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}-\frac{1}{4}b^2\left(\frac{\int \frac{1}{\frac{c+dx}{b^2}-\frac{a+b\sqrt{c}}{b^2}}d\sqrt{a+b\sqrt{c+dx}}}{2b\sqrt{c}}-\frac{\int \frac{1}{\frac{c+dx}{b^2}-\frac{a-b\sqrt{c}}{b^2}}d\sqrt{a+b\sqrt{c+dx}}}{2b\sqrt{c}}\right)\right)}{b^2}$$

$$\Big\downarrow \textbf{221}$$

$$-\frac{4d\left(\frac{b^2\sqrt{a+b\sqrt{c+dx}}}{4\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}-\frac{1}{4}b^2\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a-b\sqrt{c}}}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a+b\sqrt{c}}}\right)\right)}{b^2}$$

input
```
Int[Sqrt[a + b*Sqrt[c + d*x]]/x^2,x]
```

output
```
(-4*d*((b^2*Sqrt[a + b*Sqrt[c + d*x]])/(4*(a^2/b^2 - c - (2*a*(c + d*x))/b
^2 + (c + d*x)^2/b^2)) - (b^2*((b*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a
 - b*Sqrt[c]]])/(2*Sqrt[a - b*Sqrt[c]]*Sqrt[c]) - (b*ArcTanh[Sqrt[a + b*Sq
rt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/(2*Sqrt[a + b*Sqrt[c]]*Sqrt[c])))/4))/b
^2
```

3.630.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.630. $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx$

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 561
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(n + 1) - 1)*(-c
/d + x^k/d)^m*Simp[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^k/d^2) + b*(x^(2*k)/d^2),
 x]^p, x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, m, p}, x] && Frac
tionQ[n] && IntegerQ[p] && IntegerQ[m]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

3.630. $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx$

3.630.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.21

method	result
derivativedivides	$4d\,b^{2}\left(-\frac{\sqrt{a+b\sqrt{dx+c}}}{4\left(\left(a+b\sqrt{dx+c}\right)^{2}-2a\left(a+b\sqrt{dx+c}\right)-b^{2}c+a^{2}\right)}-\frac{\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^{2}c}-a}}\right)}{8\sqrt{b^{2}c}\,\sqrt{\sqrt{b^{2}c}-a}}+\frac{\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^{2}c}-a}}\right)}{8\sqrt{b^{2}c}\,\sqrt{-\sqrt{b^{2}c}-a}}\right)$
default	$4d\,b^{2}\left(-\frac{\sqrt{a+b\sqrt{dx+c}}}{4\left(\left(a+b\sqrt{dx+c}\right)^{2}-2a\left(a+b\sqrt{dx+c}\right)-b^{2}c+a^{2}\right)}-\frac{\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^{2}c}-a}}\right)}{8\sqrt{b^{2}c}\,\sqrt{\sqrt{b^{2}c}-a}}+\frac{\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^{2}c}-a}}\right)}{8\sqrt{b^{2}c}\,\sqrt{-\sqrt{b^{2}c}-a}}\right)$

input
```
int((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
4*d*b^2*(-1/4*(a+b*(d*x+c)^(1/2))^(1/2)/((a+b*(d*x+c)^(1/2))^2-2*a*(a+b*(d
*x+c)^(1/2))-b^2*c+a^2)-1/8/(b^2*c)^(1/2)/((b^2*c)^(1/2)-a)^(1/2)*arctan((
a+b*(d*x+c)^(1/2))^(1/2)/((b^2*c)^(1/2)-a)^(1/2))+1/8/(b^2*c)^(1/2)/(-(b^2
*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)-a)^(1/
2)))
```

3.630.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1003 vs. 2(101) = 202.

Time = 0.35 (sec) , antiderivative size = 1003, normalized size of antiderivative = 7.32

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx =$$

$$-\frac{x\sqrt{-\frac{ab^2d^2+\sqrt{\frac{b^6d^4}{b^4c^3-2\,a^2b^2c^2+a^4c}}\left(b^2c^2-a^2c\right)}{b^2c^2-a^2c}}\log\left(\sqrt{\sqrt{dx+c}b+a}b^4d^3+\left(b^4cd^2-\sqrt{\frac{b^6d^4}{b^4c^3-2\,a^2b^2c^2+a^4c}}\left(ab^2c^2-a^3\right.\right.\right.}{}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
-1/4*(x*sqrt(-(a*b^2*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))
*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*log(sqrt(sqrt(d*x + c)*b + a)*b^4*d
^3 + (b^4*c*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c
^2 - a^3*c))*sqrt(-(a*b^2*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^
4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))) - x*sqrt(-(a*b^2*d^2 + sqrt(b
^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^
2*c))*log(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3 - (b^4*c*d^2 - sqrt(b^6*d^4/(b
^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c^2 - a^3*c))*sqrt(-(a*b^2*d^2 + s
qrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2
 - a^2*c))) + x*sqrt(-(a*b^2*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 +
 a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*log(sqrt(sqrt(d*x + c)*b +
a)*b^4*d^3 + (b^4*c*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*
(a*b^2*c^2 - a^3*c))*sqrt(-(a*b^2*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*
c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))) - x*sqrt(-(a*b^2*d^2
- sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*
c^2 - a^2*c))*log(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3 - (b^4*c*d^2 + sqrt(b^
6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c^2 - a^3*c))*sqrt(-(a*b^2
*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/
(b^2*c^2 - a^2*c))) + 4*sqrt(sqrt(d*x + c)*b + a))/x
```

3.630.6 Sympy [F]

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx = \int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx$$

input
```
integrate((a+b*(d*x+c)**(1/2))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(a + b*sqrt(c + d*x))/x**2, x)
```

3.630.7 Maxima [F]

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx = \int \frac{\sqrt{\sqrt{dx+c}b+a}}{x^2}\,dx$$

input
```
integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(d*x + c)*b + a)/x^2, x)
```

3.630.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(101) = 202$.

Time = 0.41 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.69

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx = \frac{\frac{2\sqrt{\sqrt{dx+c}b+a}\,b^3d^2}{b^2c-\left(\sqrt{dx+c}b+a\right)^2+2\left(\sqrt{dx+c}b+a\right)a-a^2} - \frac{\left(b^3cd^2|b|+ab^3\sqrt{c}d^2\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}b+a}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{\left(bc^{\frac{3}{2}}+ac\right)\sqrt{b\sqrt{c}-a}|b|} + \frac{\left(b^3cd^2|b|-ab^3\sqrt{c}d^2\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}b+a}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{\left(bc^{\frac{3}{2}}-ac\right)\sqrt{-b\sqrt{c}-a}|b|}}{2\,bd}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

output
```
1/2*(2*sqrt(sqrt(d*x + c)*b + a)*b^3*d^2/(b^2*c - (sqrt(d*x + c)*b + a)^2
+ 2*(sqrt(d*x + c)*b + a)*a - a^2) - (b^3*c*d^2*abs(b) + a*b^3*sqrt(c)*d^2
)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a + sqrt(b^2*c)))/((b*c^(3/2) + a
*c)*sqrt(b*sqrt(c) - a)*abs(b)) + (b^3*c*d^2*abs(b) - a*b^3*sqrt(c)*d^2)*a
rctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a - sqrt(b^2*c)))/((b*c^(3/2) - a*c)
*sqrt(-b*sqrt(c) - a)*abs(b)))/(b*d)
```

3.630.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx = \int \frac{\sqrt{a+b\sqrt{c+d\,x}}}{x^2}\,dx$$

input
```
int((a + b*(c + d*x)^(1/2))^(1/2)/x^2,x)
```

output
```
int((a + b*(c + d*x)^(1/2))^(1/2)/x^2, x)
```

3.631 $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx$

3.631.1 Optimal result

Integrand size = 21, antiderivative size = 224

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx = -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2}+\frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8c\,(a^2-b^2c)\,x} - \frac{b\left(2a-3b\sqrt{c}\right)d^2\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16\left(a-b\sqrt{c}\right)^{3/2}c^{3/2}} + \frac{b\left(2a+3b\sqrt{c}\right)d^2\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{16\left(a+b\sqrt{c}\right)^{3/2}c^{3/2}}$$

output
```
-1/16*b*d^2*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a-b*c^(1/2))^(1/2))*(2*a-3*
b*c^(1/2))/c^(3/2)/(a-b*c^(1/2))^(3/2)+1/16*b*d^2*arctanh((a+b*(d*x+c)^(1/
2))^(1/2)/(a+b*c^(1/2))^(1/2))*(2*a+3*b*c^(1/2))/c^(3/2)/(a+b*c^(1/2))^(3/
2)-1/2*(a+b*(d*x+c)^(1/2))^(1/2)/x^2+1/8*b*d*(b*c-a*(d*x+c)^(1/2))*(a+b*(d
*x+c)^(1/2))^(1/2)/c/(-b^2*c+a^2)/x
```

3.631.2 Mathematica [A] (verified)

Time = 1.75 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx$$
$$= \frac{-\frac{2\sqrt{c}\sqrt{a+b\sqrt{c+dx}}\left(4a^2c+abdx\sqrt{c+dx}-b^2c(4c+dx)\right)}{(a^2-b^2c)x^2}+\frac{b(2a+3b\sqrt{c})d^2\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a-b\sqrt{c}}}\right)}{\left(-a-b\sqrt{c}\right)^{3/2}}+\frac{b(-2a+3b\sqrt{c})d^2\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a+b\sqrt{c}}}\right)}{\left(-a+b\sqrt{c}\right)^{3/2}}}{16c^{3/2}}$$

input
```
Integrate[Sqrt[a + b*Sqrt[c + d*x]]/x^3,x]
```

output
```
((-2*Sqrt[c]*Sqrt[a + b*Sqrt[c + d*x]]*(4*a^2*c + a*b*d*x*Sqrt[c + d*x] -
b^2*c*(4*c + d*x)))/((a^2 - b^2*c)*x^2) + (b*(2*a + 3*b*Sqrt[c])*d^2*ArcTa
n[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a - b*Sqrt[c]]])/(-a - b*Sqrt[c])^(3/2)
+ (b*(-2*a + 3*b*Sqrt[c])*d^2*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a + b
*Sqrt[c]]])/(-a + b*Sqrt[c])^(3/2))/(16*c^(3/2))
```

3.631.3 Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.50, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {896, 25, 1732, 561, 27, 1598, 27, 1405, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx$$

$$\Big\downarrow \text{ 896}$$

$$d^2\int \frac{\sqrt{a+b\sqrt{c+dx}}}{d^3x^3}d(c+dx)$$

$$\Big\downarrow \text{ 25}$$

$$-d^2\int -\frac{\sqrt{a+b\sqrt{c+dx}}}{d^3x^3}d(c+dx)$$

$$\Big\downarrow \text{ 1732}$$

$$-2d^2\int -\frac{\sqrt{c+dx}\sqrt{a+b\sqrt{c+dx}}}{d^3x^3}d\sqrt{c+dx}$$

$\Big\downarrow$ 561

$$-\frac{4d^2\int \frac{(a-c-dx)(c+dx)}{b\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^3}d\sqrt{a+b\sqrt{c+dx}}}{b}$$

$\Big\downarrow$ 27

$$-\frac{4d^2\int \frac{(a-c-dx)(c+dx)}{\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^3}d\sqrt{a+b\sqrt{c+dx}}}{b^2}$$

$\Big\downarrow$ 1598

$$-\frac{4d^2\left(\frac{b^2\int -\frac{2c}{\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}d\sqrt{a+b\sqrt{c+dx}}}{16c}+\frac{b^2\sqrt{a+b\sqrt{c+dx}}}{8\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}\right)}{b^2}$$

$\Big\downarrow$ 27

$$-\frac{4d^2\left(\frac{b^2\sqrt{a+b\sqrt{c+dx}}}{8\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}-\frac{1}{8}b^2\int \frac{1}{\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}d\sqrt{a+b\sqrt{c+dx}}\right)}{b^2}$$

$\Big\downarrow$ 1405

$$-\frac{4d^2\left(\frac{b^2\sqrt{a+b\sqrt{c+dx}}}{8\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}-\frac{1}{8}b^2\left(\frac{\sqrt{a+b\sqrt{c+dx}}\,(a^2-a(c+dx)+b^2c)}{4c(a^2-b^2c)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}-\frac{b^4\int \frac{2\left(a^2+(c+dx)a-3b^2c\right)}{b^4\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}d\sqrt{a+b\sqrt{c+}}}{8c(a^2-b^2c)}\right.\right.}{b^2}$$

$\Big\downarrow$ 27

$$-\frac{4d^2\left(\frac{b^2\sqrt{a+b\sqrt{c+dx}}}{8\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}-\frac{1}{8}b^2\left(\frac{\sqrt{a+b\sqrt{c+dx}}\,(a^2-a(c+dx)+b^2c)}{4c(a^2-b^2c)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}-\frac{\int \frac{a^2+(c+dx)a-3b^2c}{\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c}d\sqrt{a+b\sqrt{c+dx}}}{4c(a^2-b^2c)}\right)\right.}{b^2}$$

$\Big\downarrow$ 1480

$$-\frac{4d^2\left(\frac{b^2\sqrt{a+b\sqrt{c+dx}}}{8\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}-\frac{1}{8}b^2\left(\frac{\sqrt{a+b\sqrt{c+dx}}\,(a^2-a(c+dx)+b^2c)}{4c(a^2-b^2c)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}-\frac{\frac{1}{2}\left(-\frac{2a^2}{b\sqrt{c}}+a+3b\sqrt{c}\right)\int \frac{1}{\frac{c+dx}{b^2}-\frac{a-b\sqrt{c}}{b^2}}d\sqrt{a+b\sqrt{c}}}{4}\right.\right.}{b^2}$$

3.631. $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx$

$$\Big\downarrow \text{221}$$

$$-\frac{4d^2\left(\frac{b^2\sqrt{a+b\sqrt{c+dx}}}{8\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}-\frac{1}{8}b^2\left(\frac{\sqrt{a+b\sqrt{c+dx}}\left(a^2-a(c+dx)+b^2c\right)}{4c(a^2-b^2c)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}-\frac{-\frac{b^2\left(-\frac{2a^2}{b\sqrt{c}}+a+3b\sqrt{c}\right)\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{a-b\sqrt{c}}}\cdots}{4c(a^2-\cdots}\right.\cdots}{b^2}$$

input
```
Int[Sqrt[a + b*Sqrt[c + d*x]]/x^3,x]
```

output
```
(-4*d^2*((b^2*Sqrt[a + b*Sqrt[c + d*x]])/(8*(a^2/b^2 - c - (2*a*(c + d*x))
/b^2 + (c + d*x)^2/b^2)^2) - (b^2*((Sqrt[a + b*Sqrt[c + d*x]]*(a^2 + b^2*c
 - a*(c + d*x)))/(4*c*(a^2 - b^2*c)*(a^2/b^2 - c - (2*a*(c + d*x))/b^2 + (
c + d*x)^2/b^2)) - (-1/2*(b^2*(a - (2*a^2)/(b*Sqrt[c]) + 3*b*Sqrt[c])*ArcT
anh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/Sqrt[a - b*Sqrt[c]] -
(b^2*(a + (2*a^2)/(b*Sqrt[c]) - 3*b*Sqrt[c])*ArcTanh[Sqrt[a + b*Sqrt[c + d
*x]]/Sqrt[a + b*Sqrt[c]]])/(2*Sqrt[a + b*Sqrt[c]]))/(4*c*(a^2 - b^2*c))))/
8))/b^2
```

3.631.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 561
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(n + 1) - 1)*(-c
/d + x^k/d)^m*Simp[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^k/d^2) + b*(x^(2*k)/d^2),
 x]^p, x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, m, p}, x] && Frac
tionQ[n] && IntegerQ[p] && IntegerQ[m]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1598
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Simp[f
^2/(2*(p + 1)*(b^2 - 4*a*c))   Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
 GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

3.631. $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx$

3.631.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(174) = 348$.

Time = 0.30 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.66

method	result
derivativedivides	$-4d^2b^4\left(\frac{\frac{a(a+b\sqrt{dx+c})^{\frac{7}{2}}}{32b^2c(-b^2c+a^2)}-\frac{(b^2c+3a^2)(a+b\sqrt{dx+c})^{\frac{5}{2}}}{32b^2c(-b^2c+a^2)}+\frac{a(b^2c+3a^2)(a+b\sqrt{dx+c})^{\frac{3}{2}}}{32b^2c(-b^2c+a^2)}-\frac{(-3b^2c+a^2)\sqrt{a+b\sqrt{dx+c}}}{32b^2c}}{\left((a+b\sqrt{dx+c})^2-2a(a+b\sqrt{dx+c})-b^2c+a^2\right)^2}+\frac{(3b^2c}{}\right.$
default	$-4d^2b^4\left(\frac{\frac{a(a+b\sqrt{dx+c})^{\frac{7}{2}}}{32b^2c(-b^2c+a^2)}-\frac{(b^2c+3a^2)(a+b\sqrt{dx+c})^{\frac{5}{2}}}{32b^2c(-b^2c+a^2)}+\frac{a(b^2c+3a^2)(a+b\sqrt{dx+c})^{\frac{3}{2}}}{32b^2c(-b^2c+a^2)}-\frac{(-3b^2c+a^2)\sqrt{a+b\sqrt{dx+c}}}{32b^2c}}{\left((a+b\sqrt{dx+c})^2-2a(a+b\sqrt{dx+c})-b^2c+a^2\right)^2}+\frac{(3b^2c}{}\right.$

input
```
int((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-4*d^2*b^4*((1/32*a/b^2/c/(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(7/2)-1/32*(b^2
*c+3*a^2)/b^2/c/(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(5/2)+1/32*a*(b^2*c+3*a^2
)/b^2/c/(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(3/2)-1/32*(-3*b^2*c+a^2)/b^2/c*(
a+b*(d*x+c)^(1/2))^(1/2))/((a+b*(d*x+c)^(1/2))^2-2*a*(a+b*(d*x+c)^(1/2))-b
^2*c+a^2)^2+1/32/b^2/c/(-b^2*c+a^2)*(1/2*(3*b^2*c+a*(b^2*c)^(1/2)-2*a^2)/(
b^2*c)^(1/2)/((b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/((b^
2*c)^(1/2)-a)^(1/2))+1/2*(-3*b^2*c+a*(b^2*c)^(1/2)+2*a^2)/(b^2*c)^(1/2)/(-
(b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)-a)
^(1/2))))
```

3.631.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2856 vs. $2(175) = 350$.

Time = 0.56 (sec) , antiderivative size = 2856, normalized size of antiderivative = 12.75

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
1/32*((b^2*c^2 - a^2*c)*x^2*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^
2)*d^4 + (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14
*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^
4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))
/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))*log((81*b^10*c^2 - 8
1*a^2*b^8*c + 20*a^4*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^6 + ((27*b^10*c^4 -
24*a^2*b^8*c^3 + 5*a^4*b^6*c^2)*d^4 - 2*(2*a*b^8*c^7 - 7*a^3*b^6*c^6 + 9*a
^5*b^4*c^5 - 5*a^7*b^2*c^4 + a^9*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c +
25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*
c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))*sqrt(-((15*a*b^6*c^2 -
 15*a^3*b^4*c + 4*a^5*b^2)*d^4 + (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4
- a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9
- 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^
10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^
3))) - (b^2*c^2 - a^2*c)*x^2*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b
^2)*d^4 + (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^1
4*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a
^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3))
)/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))*log((81*b^10*c^2 -
81*a^2*b^8*c + 20*a^4*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^6 - ((27*b^10*c^...
```

3.631.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + b\sqrt{c + dx}}}{x^3}\, dx = \text{Timed out}$$

input
```
integrate((a+b*(d*x+c)**(1/2))**(1/2)/x**3,x)
```

output
```
Timed out
```

3.631.7 Maxima [F]

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx = \int \frac{\sqrt{\sqrt{dx+c}b+a}}{x^3}\,dx$$

input
```
integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(sqrt(d*x + c)*b + a)/x^3, x)
```

3.631.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 895 vs. $2(175) = 350$.

Time = 0.61 (sec) , antiderivative size = 895, normalized size of antiderivative = 4.00

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx$$
$$= \frac{\frac{\left(\left(b^3c^2-a^2bc\right)^2ab^3\sqrt{c}d^3-\left(3\,b^7c^3-4\,a^2b^5c^2+a^4b^3c\right)d^3\left|b^3c^2-a^2bc\right|+\left(3\,ab^9c^{\frac{9}{2}}-8\,a^3b^7c^{\frac{7}{2}}+7\,a^5b^5c^{\frac{5}{2}}-2\,a^7b^3c^{\frac{3}{2}}\right)d^3\right)\arctan\left(\frac{}{\sqrt{-\frac{ab^2c^2-a^3c+\sqrt{(ab}}{}}}\right.}{\left(b^5c^{\frac{9}{2}}-ab^4c^4-2\,a^2b^3c^{\frac{7}{2}}+2\,a^3b^2c^3+a^4bc^{\frac{5}{2}}-a^5c^2\right)\sqrt{-b\sqrt{c}-a}\left|b^3c^2-a^2bc\right|}}{}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x, algorithm="giac")
```

output
```
1/16*(((b^3*c^2 - a^2*b*c)^2*a*b^3*sqrt(c)*d^3 - (3*b^7*c^3 - 4*a^2*b^5*c^
2 + a^4*b^3*c)*d^3*abs(b^3*c^2 - a^2*b*c) + (3*a*b^9*c^(9/2) - 8*a^3*b^7*c
^(7/2) + 7*a^5*b^5*c^(5/2) - 2*a^7*b^3*c^(3/2))*d^3)*arctan(sqrt(sqrt(d*x
+ c)*b + a)/sqrt(-(a*b^2*c^2 - a^3*c + sqrt((a*b^2*c^2 - a^3*c)^2 + (b^4*c
^3 - 2*a^2*b^2*c^2 + a^4*c)*(b^2*c^2 - a^2*c)))/(b^2*c^2 - a^2*c)))/((b^5*
c^(9/2) - a*b^4*c^4 - 2*a^2*b^3*c^(7/2) + 2*a^3*b^2*c^3 + a^4*b*c^(5/2) -
a^5*c^2)*sqrt(-b*sqrt(c) - a)*abs(b^3*c^2 - a^2*b*c)) + ((b^3*c^2 - a^2*b*
c)^2*a*b^3*d^3 + (3*b^7*c^(5/2) - 4*a^2*b^5*c^(3/2) + a^4*b^3*sqrt(c))*d^3
*abs(b^3*c^2 - a^2*b*c) + (3*a*b^9*c^4 - 8*a^3*b^7*c^3 + 7*a^5*b^5*c^2 - 2
*a^7*b^3*c)*d^3)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^2*c^2 - a^3*c
 - sqrt((a*b^2*c^2 - a^3*c)^2 + (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*(b^2*c^2
 - a^2*c)))/(b^2*c^2 - a^2*c)))/((b^5*c^4 + a*b^4*c^(7/2) - 2*a^2*b^3*c^3
- 2*a^3*b^2*c^(5/2) + a^4*b*c^2 + a^5*c^(3/2))*sqrt(b*sqrt(c) - a)*abs(b^3
*c^2 - a^2*b*c)) - 2*(3*sqrt(sqrt(d*x + c)*b + a)*b^7*c^2*d^3 + (sqrt(d*x
+ c)*b + a)^(5/2)*b^5*c*d^3 - (sqrt(d*x + c)*b + a)^(3/2)*a*b^5*c*d^3 - 4*
sqrt(sqrt(d*x + c)*b + a)*a^2*b^5*c*d^3 - (sqrt(d*x + c)*b + a)^(7/2)*a*b^
3*d^3 + 3*(sqrt(d*x + c)*b + a)^(5/2)*a^2*b^3*d^3 - 3*(sqrt(d*x + c)*b + a
)^(3/2)*a^3*b^3*d^3 + sqrt(sqrt(d*x + c)*b + a)*a^4*b^3*d^3)/((b^2*c^2 - a
^2*c)*(b^2*c - (sqrt(d*x + c)*b + a)^2 + 2*(sqrt(d*x + c)*b + a)*a - a^2)^
2))/(b*d)
```

3.631.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx = \int \frac{\sqrt{a+b\,\sqrt{c+d\,x}}}{x^3}\,dx$$

input
```
int((a + b*(c + d*x)^(1/2))^(1/2)/x^3,x)
```

output
```
int((a + b*(c + d*x)^(1/2))^(1/2)/x^3, x)
```

3.632 $\int \frac{x^3}{a+b\sqrt{c+dx}} dx$

3.632.1 Optimal result

Integrand size = 19, antiderivative size = 230

$$\begin{aligned}\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx = & -\frac{a(a^4-3a^2b^2c+3b^4c^2)\,x}{b^6d^3}+\frac{2(a^2-b^2c)^3\sqrt{c+dx}}{b^7d^4} \\ & +\frac{2(a^4-3a^2b^2c+3b^4c^2)\,(c+dx)^{3/2}}{3b^5d^4}-\frac{a(a^2-3b^2c)\,(c+dx)^2}{2b^4d^4} \\ & +\frac{2(a^2-3b^2c)\,(c+dx)^{5/2}}{5b^3d^4}-\frac{a(c+dx)^3}{3b^2d^4} \\ & +\frac{2(c+dx)^{7/2}}{7bd^4}-\frac{2a(a^2-b^2c)^3\log\left(a+b\sqrt{c+dx}\right)}{b^8d^4}\end{aligned}$$

output
```
-a*(3*b^4*c^2-3*a^2*b^2*c+a^4)*x/b^6/d^3+2/3*(3*b^4*c^2-3*a^2*b^2*c+a^4)*(
d*x+c)^(3/2)/b^5/d^4-1/2*a*(-3*b^2*c+a^2)*(d*x+c)^2/b^4/d^4+2/5*(-3*b^2*c+
a^2)*(d*x+c)^(5/2)/b^3/d^4-1/3*a*(d*x+c)^3/b^2/d^4+2/7*(d*x+c)^(7/2)/b/d^4
-2*a*(-b^2*c+a^2)^3*ln(a+b*(d*x+c)^(1/2))/b^8/d^4+2*(-b^2*c+a^2)^3*(d*x+c)
^(1/2)/b^7/d^4
```

3.632.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.02

$$\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx = \frac{b\left(420a^6\sqrt{c+dx}-140a^4b^2(8c-dx)\sqrt{c+dx}-210a^5b(c+dx)+105a^3b^3\left(5c^2+4cdx-d^2x^2\right)+84a^2b^4\sqrt{\right.}{}$$

input
```
Integrate[x^3/(a + b*Sqrt[c + d*x]),x]
```

output
```
(b*(420*a^6*Sqrt[c + d*x] - 140*a^4*b^2*(8*c - d*x)*Sqrt[c + d*x] - 210*a^
5*b*(c + d*x) + 105*a^3*b^3*(5*c^2 + 4*c*d*x - d^2*x^2) + 84*a^2*b^4*Sqrt[
c + d*x]*(11*c^2 - 3*c*d*x + d^2*x^2) - 35*a*b^5*(11*c^3 + 6*c^2*d*x - 3*c
*d^2*x^2 + 2*d^3*x^3) + 12*b^6*Sqrt[c + d*x]*(-16*c^3 + 8*c^2*d*x - 6*c*d^
2*x^2 + 5*d^3*x^3)) - 420*a*(a^2 - b^2*c)^3*Log[a + b*Sqrt[c + d*x]])/(210
*b^8*d^4)
```

3.632.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{ 896}$$

$$\frac{\int \frac{d^3x^3}{a+b\sqrt{c+dx}}d(c+dx)}{d^4}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int -\frac{d^3x^3}{a+b\sqrt{c+dx}}d(c+dx)}{d^4}$$

$$\Big\downarrow \text{ 1732}$$

$$-\frac{2\int -\frac{d^3x^3\sqrt{c+dx}}{a+b\sqrt{c+dx}}d\sqrt{c+dx}}{d^4}$$

$$\Big\downarrow \; 522$$

$$-\frac{2\int \left(\frac{a(a^2-b^2c)^3}{b^7(a+b\sqrt{c+dx})}+\frac{(b^2c-a^2)^3}{b^7}-\frac{(c+dx)^3}{b}+\frac{a(c+dx)^{5/2}}{b^2}+\frac{(3b^2c-a^2)(c+dx)^2}{b^3}+\frac{a(a^2-3b^2c)(c+dx)^{3/2}}{b^4}-\frac{(a^4-3b^2ca^2+3b^4c^2)(c}{b^5}\right.}{d^4}$$

$$\Big\downarrow \; 2009$$

$$-\frac{2\left(\frac{a(a^2-b^2c)^3\log(a+b\sqrt{c+dx})}{b^8}-\frac{(a^2-b^2c)^3\sqrt{c+dx}}{b^7}+\frac{a(a^2-3b^2c)(c+dx)^2}{4b^4}-\frac{(a^2-3b^2c)(c+dx)^{5/2}}{5b^3}+\frac{a(a^4-3a^2b^2c+3b^4c^2)(c+dx)}{2b^6}-\right.}{d^4}$$

input
```
Int[x^3/(a + b*Sqrt[c + d*x]),x]
```

output
```
(-2*(-(((a^2 - b^2*c)^3*Sqrt[c + d*x])/b^7) + (a*(a^4 - 3*a^2*b^2*c + 3*b^
4*c^2)*(c + d*x))/(2*b^6) - ((a^4 - 3*a^2*b^2*c + 3*b^4*c^2)*(c + d*x)^(3/
2))/(3*b^5) + (a*(a^2 - 3*b^2*c)*(c + d*x)^2)/(4*b^4) - ((a^2 - 3*b^2*c)*(
c + d*x)^(5/2))/(5*b^3) + (a*(c + d*x)^3)/(6*b^2) - (c + d*x)^(7/2)/(7*b)
+ (a*(a^2 - b^2*c)^3*Log[a + b*Sqrt[c + d*x]])/b^8))/d^4
```

3.632.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.632.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{2\left(\frac{(dx+c)^{\frac{7}{2}} b^6}{7}-\frac{a(dx+c)^3 b^5}{6}-\frac{3b^6 c(dx+c)^{\frac{5}{2}}}{5}+\frac{a^2 b^4 (dx+c)^{\frac{5}{2}}}{5}+\frac{3a\,b^5 c(dx+c)^2}{4}+b^6 c^2 (dx+c)^{\frac{3}{2}}-\frac{a^3 b^3 (dx+c)^2}{4}-a^2 b^4 c(dx+c)^{\frac{3}{2}}-\frac{3a\,b}{}\right.}{b^7}$
default	$\frac{2\left(\frac{(dx+c)^{\frac{7}{2}} b^6}{7}-\frac{a(dx+c)^3 b^5}{6}-\frac{3b^6 c(dx+c)^{\frac{5}{2}}}{5}+\frac{a^2 b^4 (dx+c)^{\frac{5}{2}}}{5}+\frac{3a\,b^5 c(dx+c)^2}{4}+b^6 c^2 (dx+c)^{\frac{3}{2}}-\frac{a^3 b^3 (dx+c)^2}{4}-a^2 b^4 c(dx+c)^{\frac{3}{2}}-\frac{3a\,b}{}\right.}{b^7}$

input
```
int(x^3/(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/d^4*(1/b^7*(1/7*(d*x+c)^(7/2)*b^6-1/6*a*(d*x+c)^3*b^5-3/5*b^6*c*(d*x+c)^
(5/2)+1/5*a^2*b^4*(d*x+c)^(5/2)+3/4*a*b^5*c*(d*x+c)^2+b^6*c^2*(d*x+c)^(3/2
)-1/4*a^3*b^3*(d*x+c)^2-a^2*b^4*c*(d*x+c)^(3/2)-3/2*a*b^5*c^2*(d*x+c)-b^6*
c^3*(d*x+c)^(1/2)+1/3*a^4*b^2*(d*x+c)^(3/2)+3/2*a^3*b^3*c*(d*x+c)+3*a^2*b^
4*c^2*(d*x+c)^(1/2)-1/2*a^5*b*(d*x+c)-3*a^4*b^2*c*(d*x+c)^(1/2)+a^6*(d*x+c
)^(1/2))-a*(-b^6*c^3+3*a^2*b^4*c^2-3*a^4*b^2*c+a^6)/b^8*ln(a+b*(d*x+c)^(1/
2)))
```

3.632.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.99

$$\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx = -\frac{70\,ab^6d^3x^3-105\,(ab^6c-a^3b^4)d^2x^2+210\,(ab^6c^2-2\,a^3b^4c+a^5b^2)dx-420\,(ab^6c^3-3\,a^3b^4c^2+3\,a^5b^2c\cdots}{}$$

input
```
integrate(x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
-1/210*(70*a*b^6*d^3*x^3 - 105*(a*b^6*c - a^3*b^4)*d^2*x^2 + 210*(a*b^6*c^
2 - 2*a^3*b^4*c + a^5*b^2)*d*x - 420*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^
2*c - a^7)*log(sqrt(d*x + c)*b + a) - 4*(15*b^7*d^3*x^3 - 48*b^7*c^3 + 231
*a^2*b^5*c^2 - 280*a^4*b^3*c + 105*a^6*b - 3*(6*b^7*c - 7*a^2*b^5)*d^2*x^2
 + (24*b^7*c^2 - 63*a^2*b^5*c + 35*a^4*b^3)*d*x)*sqrt(d*x + c))/(b^8*d^4)
```

3.632.6 Sympy [A] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.10

$$\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx = \begin{cases} \frac{2\left(-\frac{a(c+dx)^3}{6b^2} - \frac{a\left(a^2-b^2c\right)^3\left(\begin{cases}\frac{\sqrt{c+dx}}{a} & \text{for } b=0 \\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^7} + \frac{(c+dx)^{\frac{7}{2}}}{7b} + \frac{\left(a^2-3b^2c\right)(c+dx)^{\frac{5}{2}}}{5b^3} + \frac{\left(-a^3+3ab^2c\right)(c+dx)^2}{4b^4} + \frac{(c+dx)^{\frac{3}{2}}\left(a^4-3a^2b\right.}{3b^5}\right.}{d^4} \\ \frac{x^4}{4\left(a+b\sqrt{c}\right)} \end{cases}$$

input
```
integrate(x**3/(a+b*(d*x+c)**(1/2)),x)
```

output
```
Piecewise((2*(-a*(c + d*x)**3/(6*b**2) - a*(a**2 - b**2*c)**3*Piecewise((s
qrt(c + d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b, True))/b**7 + (c +
 d*x)**(7/2)/(7*b) + (a**2 - 3*b**2*c)*(c + d*x)**(5/2)/(5*b**3) + (-a**3
+ 3*a*b**2*c)*(c + d*x)**2/(4*b**4) + (c + d*x)**(3/2)*(a**4 - 3*a**2*b**2
*c + 3*b**4*c**2)/(3*b**5) + (c + d*x)*(-a**5 + 3*a**3*b**2*c - 3*a*b**4*c
**2)/(2*b**6) + sqrt(c + d*x)*(a**6 - 3*a**4*b**2*c + 3*a**2*b**4*c**2 - b
**6*c**3)/b**7)/d**4, Ne(d, 0)), (x**4/(4*(a + b*sqrt(c))), True))
```

3.632.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.06

$$\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx = \frac{\frac{60\,(dx+c)^{\frac{7}{2}}b^6-70\,(dx+c)^3ab^5-84\,(3\,b^6c-a^2b^4)(dx+c)^{\frac{5}{2}}+105\,(3\,ab^5c-a^3b^3)(dx+c)^2+140\,(3\,b^6c^2-3\,a^2b^4c+a^4b^2)(dx+c)^{\frac{3}{2}}-210\,(3\,ab^5c^2-3\,a^3b}{b^7}}{210\,d^4}$$

input
```
integrate(x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
1/210*((60*(d*x + c)^(7/2)*b^6 - 70*(d*x + c)^3*a*b^5 - 84*(3*b^6*c - a^2*
b^4)*(d*x + c)^(5/2) + 105*(3*a*b^5*c - a^3*b^3)*(d*x + c)^2 + 140*(3*b^6*
c^2 - 3*a^2*b^4*c + a^4*b^2)*(d*x + c)^(3/2) - 210*(3*a*b^5*c^2 - 3*a^3*b^
3*c + a^5*b)*(d*x + c) - 420*(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6)
*sqrt(d*x + c))/b^7 + 420*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*
log(sqrt(d*x + c)*b + a)/b^8)/d^4
```

3.632.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.48

$$\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx = \frac{2\,(ab^6c^3-3\,a^3b^4c^2+3\,a^5b^2c-a^7)\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^8d^4} + \frac{60\,(dx+c)^{\frac{7}{2}}b^6d^{24}-252\,(dx+c)^{\frac{5}{2}}b^6cd^{24}+420\,(dx+c)^{\frac{3}{2}}b^6c^2d^{24}-420\,\sqrt{dx+c}b^6c^3d^{24}-70\,(dx+c)^3ab}{}$$

input
```
integrate(x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

output
```
2*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*log(abs(sqrt(d*x + c)*b
+ a))/(b^8*d^4) + 1/210*(60*(d*x + c)^(7/2)*b^6*d^24 - 252*(d*x + c)^(5/2)
*b^6*c*d^24 + 420*(d*x + c)^(3/2)*b^6*c^2*d^24 - 420*sqrt(d*x + c)*b^6*c^3
*d^24 - 70*(d*x + c)^3*a*b^5*d^24 + 315*(d*x + c)^2*a*b^5*c*d^24 - 630*(d*
x + c)*a*b^5*c^2*d^24 + 84*(d*x + c)^(5/2)*a^2*b^4*d^24 - 420*(d*x + c)^(3
/2)*a^2*b^4*c*d^24 + 1260*sqrt(d*x + c)*a^2*b^4*c^2*d^24 - 105*(d*x + c)^2
*a^3*b^3*d^24 + 630*(d*x + c)*a^3*b^3*c*d^24 + 140*(d*x + c)^(3/2)*a^4*b^2
*d^24 - 1260*sqrt(d*x + c)*a^4*b^2*c*d^24 - 210*(d*x + c)*a^5*b*d^24 + 420
*sqrt(d*x + c)*a^6*d^24)/(b^7*d^28)
```

3.632.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.38

$$\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx = \frac{2\,{\left(c+d\,x\right)}^{7/2}}{7\,b\,d^4}-\left(\frac{a^2\,\left(\frac{a^2\,\left(\frac{6\,c}{b\,d^4}-\frac{2\,a^2}{b^3\,d^4}\right)}{b^2}-\frac{6\,c^2}{b\,d^4}\right)}{b^2}+\frac{2\,c^3}{b\,d^4}\right)\,\sqrt{c+d\,x}-\left(\frac{a^2\,\left(\frac{6\,c}{b\,d^4}-\frac{2\,a^2}{b^3\,d^4}\right)}{3\,b^2}-\frac{2\,c^2}{b\,d^4}\right)\,{\left(c+d\,x\right)}^{3/2}-\left(\frac{6\,c}{5\,b\,d^4}-\frac{2\,a^2}{5\,b^3\,d^4}\right)\,{\left(c+d\,x\right)}^{5/2}+\frac{a\,\left(\frac{6\,c}{b\,d^4}-\frac{2\,a^2}{b^3\,d^4}\right)\,{\left(c+d\,x\right)}^2}{4\,b}-\frac{a\,{\left(c+d\,x\right)}^3}{3\,b^2\,d^4}-\frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(2\,a^7-6\,a^5\,b^2\,c+6\,a^3\,b^4\,c^2-2\,a\,b^6\,c^3\right)}{b^8\,d^4}+\frac{a\,d\,x\,\left(\frac{a^2\,\left(\frac{6\,c}{b\,d^4}-\frac{2\,a^2}{b^3\,d^4}\right)}{b^2}-\frac{6\,c^2}{b\,d^4}\right)}{2\,b}$$

input

```
int(x^3/(a + b*(c + d*x)^(1/2)),x)
```

output

```
(2*(c + d*x)^(7/2))/(7*b*d^4) - ((a^2*((a^2*((6*c)/(b*d^4) - (2*a^2)/(b^3*
d^4)))/b^2 - (6*c^2)/(b*d^4)))/b^2 + (2*c^3)/(b*d^4))*(c + d*x)^(1/2) - ((
a^2*((6*c)/(b*d^4) - (2*a^2)/(b^3*d^4)))/(3*b^2) - (2*c^2)/(b*d^4))*(c + d
*x)^(3/2) - ((6*c)/(5*b*d^4) - (2*a^2)/(5*b^3*d^4))*(c + d*x)^(5/2) + (a*(
(6*c)/(b*d^4) - (2*a^2)/(b^3*d^4))*(c + d*x)^2)/(4*b) - (a*(c + d*x)^3)/(3
*b^2*d^4) - (log(a + b*(c + d*x)^(1/2))*(2*a^7 - 6*a^5*b^2*c - 2*a*b^6*c^3
 + 6*a^3*b^4*c^2))/(b^8*d^4) + (a*d*x*((a^2*((6*c)/(b*d^4) - (2*a^2)/(b^3*
d^4)))/b^2 - (6*c^2)/(b*d^4)))/(2*b)
```

3.633 $\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx$

3.633.1 Optimal result

Integrand size = 19, antiderivative size = 151

$$\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx = -\frac{a(a^2-2b^2c)\,x}{b^4d^2}+\frac{2(a^2-b^2c)^2\sqrt{c+dx}}{b^5d^3}+\frac{2(a^2-2b^2c)\,(c+dx)^{3/2}}{3b^3d^3} - \frac{a(c+dx)^2}{2b^2d^3}+\frac{2(c+dx)^{5/2}}{5bd^3}-\frac{2a(a^2-b^2c)^2\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}$$

output
```
-a*(-2*b^2*c+a^2)*x/b^4/d^2+2/3*(-2*b^2*c+a^2)*(d*x+c)^(3/2)/b^3/d^3-1/2*a
*(d*x+c)^2/b^2/d^3+2/5*(d*x+c)^(5/2)/b/d^3-2*a*(-b^2*c+a^2)^2*ln(a+b*(d*x+
c)^(1/2))/b^6/d^3+2*(-b^2*c+a^2)^2*(d*x+c)^(1/2)/b^5/d^3
```

3.633.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx = \frac{b\left(60a^4\sqrt{c+dx}-20a^2b^2(5c-dx)\sqrt{c+dx}-30a^3b(c+dx)+15ab^3(3c^2+2cdx-d^2x^2)+4b^4\sqrt{c+dx}(8c\right.}{30b^6d^3}$$

input
```
Integrate[x^2/(a + b*Sqrt[c + d*x]),x]
```

output
```
(b*(60*a^4*Sqrt[c + d*x] - 20*a^2*b^2*(5*c - d*x)*Sqrt[c + d*x] - 30*a^3*b
*(c + d*x) + 15*a*b^3*(3*c^2 + 2*c*d*x - d^2*x^2) + 4*b^4*Sqrt[c + d*x]*(8
*c^2 - 4*c*d*x + 3*d^2*x^2)) - 60*a*(a^2 - b^2*c)^2*Log[a + b*Sqrt[c + d*x
]])/(30*b^6*d^3)
```

3.633.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {896, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+b\sqrt{c+dx}}dx$$

$$\Big\downarrow \; 896$$

$$\frac{\int \frac{d^2x^2}{a+b\sqrt{c+dx}}d(c+dx)}{d^3}$$

$$\Big\downarrow \; 1732$$

$$\frac{2\int \frac{d^2x^2\sqrt{c+dx}}{a+b\sqrt{c+dx}}d\sqrt{c+dx}}{d^3}$$

$$\Big\downarrow \; 522$$

$$\frac{2\int \left(-\frac{a\left(a^2-b^2c\right)^2}{b^5\left(a+b\sqrt{c+dx}\right)}+\frac{\left(b^2c-a^2\right)^2}{b^5}+\frac{(c+dx)^2}{b}-\frac{a(c+dx)^{3/2}}{b^2}-\frac{\left(2b^2c-a^2\right)(c+dx)}{b^3}-\frac{a\left(a^2-2b^2c\right)\sqrt{c+dx}}{b^4}\right)d\sqrt{c+dx}}{d^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{2\left(-\frac{a\left(a^2-b^2c\right)^2\log\left(a+b\sqrt{c+dx}\right)}{b^6}+\frac{\left(a^2-b^2c\right)^2\sqrt{c+dx}}{b^5}-\frac{a\left(a^2-2b^2c\right)(c+dx)}{2b^4}+\frac{\left(a^2-2b^2c\right)(c+dx)^{3/2}}{3b^3}-\frac{a(c+dx)^2}{4b^2}+\frac{(c+dx)^{5/2}}{5b}\right)}{d^3}$$

input
```
Int[x^2/(a + b*Sqrt[c + d*x]),x]
```

output
```
(2*(((a^2 - b^2*c)^2*Sqrt[c + d*x])/b^5 - (a*(a^2 - 2*b^2*c)*(c + d*x))/(2
*b^4) + ((a^2 - 2*b^2*c)*(c + d*x)^(3/2))/(3*b^3) - (a*(c + d*x)^2)/(4*b^2
) + (c + d*x)^(5/2)/(5*b) - (a*(a^2 - b^2*c)^2*Log[a + b*Sqrt[c + d*x]])/b
^6))/d^3
```

3.633.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.633.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{\frac{2\left(\frac{(dx+c)^{\frac{5}{2}} b^4}{5}-\frac{a(dx+c)^2 b^3}{4}-\frac{2b^4 c(dx+c)^{\frac{3}{2}}}{3}+\frac{a^2 b^2 (dx+c)^{\frac{3}{2}}}{3}+a\,b^3 c(dx+c)+b^4 c^2 \sqrt{dx+c}-\frac{a^3 b(dx+c)}{2}-2a^2 b^2 c\sqrt{dx+c}+a^4 \sqrt{dx+c}\right)}{b^5}}{d^3}$
default	$\frac{\frac{2\left(\frac{(dx+c)^{\frac{5}{2}} b^4}{5}-\frac{a(dx+c)^2 b^3}{4}-\frac{2b^4 c(dx+c)^{\frac{3}{2}}}{3}+\frac{a^2 b^2 (dx+c)^{\frac{3}{2}}}{3}+a\,b^3 c(dx+c)+b^4 c^2 \sqrt{dx+c}-\frac{a^3 b(dx+c)}{2}-2a^2 b^2 c\sqrt{dx+c}+a^4 \sqrt{dx+c}\right)}{b^5}}{d^3}$

input
```
int(x^2/(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/d^3*(1/b^5*(1/5*(d*x+c)^(5/2)*b^4-1/4*a*(d*x+c)^2*b^3-2/3*b^4*c*(d*x+c)^
(3/2)+1/3*a^2*b^2*(d*x+c)^(3/2)+a*b^3*c*(d*x+c)+b^4*c^2*(d*x+c)^(1/2)-1/2*
a^3*b*(d*x+c)-2*a^2*b^2*c*(d*x+c)^(1/2)+a^4*(d*x+c)^(1/2))-a*(b^4*c^2-2*a^
2*b^2*c+a^4)/b^6*ln(a+b*(d*x+c)^(1/2)))
```

3.633.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx = -\frac{15\,ab^4d^2x^2 - 30\,(ab^4c - a^3b^2)dx + 60\,(ab^4c^2 - 2\,a^3b^2c + a^5)\log\left(\sqrt{dx+c}b + a\right) - 4\,(3\,b^5d^2x^2 + 8\,b^5c^2 - \cdots}{30\,b^6d^3}$$

input
```
integrate(x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
-1/30*(15*a*b^4*d^2*x^2 - 30*(a*b^4*c - a^3*b^2)*d*x + 60*(a*b^4*c^2 - 2*a
^3*b^2*c + a^5)*log(sqrt(d*x + c)*b + a) - 4*(3*b^5*d^2*x^2 + 8*b^5*c^2 -
25*a^2*b^3*c + 15*a^4*b - (4*b^5*c - 5*a^2*b^3)*d*x)*sqrt(d*x + c))/(b^6*d
^3)
```

3.633.6 Sympy [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx = \begin{cases} \frac{2\left(-\frac{a(c+dx)^2}{4b^2} - \frac{a\left(a^2-b^2c\right)^2\left(\begin{cases}\frac{\sqrt{c+dx}}{a} & \text{for } b=0\\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^5} + \frac{(c+dx)^{\frac{5}{2}}}{5b} + \frac{\left(a^2-2b^2c\right)(c+dx)^{\frac{3}{2}}}{3b^3} + \frac{\left(-a^3+2ab^2c\right)(c+dx)}{2b^4} + \frac{\sqrt{c+dx}\left(a^4-2a^2b^2c+\cdots\right)}{b^5} \cdots\right)}{d^3} & \cdots \\ \frac{x^3}{3\left(a+b\sqrt{c}\right)} & \cdots \end{cases}$$

input
```
integrate(x**2/(a+b*(d*x+c)**(1/2)),x)
```

output
```
Piecewise((2*(-a*(c + d*x)**2/(4*b**2) - a*(a**2 - b**2*c)**2*Piecewise((s
qrt(c + d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b, True))/b**5 + (c +
 d*x)**(5/2)/(5*b) + (a**2 - 2*b**2*c)*(c + d*x)**(3/2)/(3*b**3) + (-a**3
+ 2*a*b**2*c)*(c + d*x)/(2*b**4) + sqrt(c + d*x)*(a**4 - 2*a**2*b**2*c + b
**4*c**2)/b**5)/d**3, Ne(d, 0)), (x**3/(3*(a + b*sqrt(c))), True))
```

3.633.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.98

$$\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx = \frac{\frac{12\,(dx+c)^{\frac{5}{2}}b^4-15\,(dx+c)^2ab^3-20\,(2\,b^4c-a^2b^2)\,(dx+c)^{\frac{3}{2}}+30\,(2\,ab^3c-a^3b)\,(dx+c)+60\,(b^4c^2-2\,a^2b^2c+a^4)\,\sqrt{dx+c}}{b^5}-\frac{60\,(ab^4c^2-2\,a^3b^2c+a^5)\,\mathrm{lo}}{b^6}}{30\,d^3}$$

input
```
integrate(x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
1/30*((12*(d*x + c)^(5/2)*b^4 - 15*(d*x + c)^2*a*b^3 - 20*(2*b^4*c - a^2*b
^2)*(d*x + c)^(3/2) + 30*(2*a*b^3*c - a^3*b)*(d*x + c) + 60*(b^4*c^2 - 2*a
^2*b^2*c + a^4)*sqrt(d*x + c))/b^5 - 60*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*lo
g(sqrt(d*x + c)*b + a)/b^6)/d^3
```

3.633.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.31

$$\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx = -\frac{2\,(ab^4c^2-2\,a^3b^2c+a^5)\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^6d^3} + \frac{12\,(dx+c)^{\frac{5}{2}}b^4d^{12}-40\,(dx+c)^{\frac{3}{2}}b^4cd^{12}+60\,\sqrt{dx+c}b^4c^2d^{12}-15\,(dx+c)^2ab^3d^{12}+60\,(dx+c)ab^3cd^{12}}{30\,b^5d^{15}}$$

input
```
integrate(x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

output
```
-2*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*log(abs(sqrt(d*x + c)*b + a))/(b^6*d^3)
 + 1/30*(12*(d*x + c)^(5/2)*b^4*d^12 - 40*(d*x + c)^(3/2)*b^4*c*d^12 + 60*
sqrt(d*x + c)*b^4*c^2*d^12 - 15*(d*x + c)^2*a*b^3*d^12 + 60*(d*x + c)*a*b^
3*c*d^12 + 20*(d*x + c)^(3/2)*a^2*b^2*d^12 - 120*sqrt(d*x + c)*a^2*b^2*c*d
^12 - 30*(d*x + c)*a^3*b*d^12 + 60*sqrt(d*x + c)*a^4*d^12)/(b^5*d^15)
```

3.633.9 Mupad [B] (verification not implemented)

Time = 17.68 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx = \frac{2\,{\left(c+d\,x\right)}^{5/2}}{5\,b\,d^3}-\left(\frac{a^2\,\left(\frac{4\,c}{b\,d^3}-\frac{2\,a^2}{b^3\,d^3}\right)}{b^2}-\frac{2\,c^2}{b\,d^3}\right)\,\sqrt{c+d\,x} - \left(\frac{4\,c}{3\,b\,d^3}-\frac{2\,a^2}{3\,b^3\,d^3}\right)\,{\left(c+d\,x\right)}^{3/2} - \frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(2\,a^5-4\,a^3\,b^2\,c+2\,a\,b^4\,c^2\right)}{b^6\,d^3} - \frac{a\,{\left(c+d\,x\right)}^2}{2\,b^2\,d^3}+\frac{a\,d\,x\,\left(\frac{4\,c}{b\,d^3}-\frac{2\,a^2}{b^3\,d^3}\right)}{2\,b}$$

input
```
int(x^2/(a + b*(c + d*x)^(1/2)),x)
```

output
```
(2*(c + d*x)^(5/2))/(5*b*d^3) - ((a^2*((4*c)/(b*d^3) - (2*a^2)/(b^3*d^3)))
/b^2 - (2*c^2)/(b*d^3))*(c + d*x)^(1/2) - ((4*c)/(3*b*d^3) - (2*a^2)/(3*b^
3*d^3))*(c + d*x)^(3/2) - (log(a + b*(c + d*x)^(1/2))*(2*a^5 - 4*a^3*b^2*c
 + 2*a*b^4*c^2))/(b^6*d^3) - (a*(c + d*x)^2)/(2*b^2*d^3) + (a*d*x*((4*c)/(
b*d^3) - (2*a^2)/(b^3*d^3)))/(2*b)
```

3.634 $\int \frac{x}{a+b\sqrt{c+dx}}\,dx$

3.634.1 Optimal result

Integrand size = 17, antiderivative size = 90

$$\int \frac{x}{a+b\sqrt{c+dx}}\,dx = -\frac{ax}{b^2d}+\frac{2(a^2-b^2c)\sqrt{c+dx}}{b^3d^2}+\frac{2(c+dx)^{3/2}}{3bd^2} - \frac{2a(a^2-b^2c)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}$$

output
```
-a*x/b^2/d+2/3*(d*x+c)^(3/2)/b/d^2-2*a*(-b^2*c+a^2)*ln(a+b*(d*x+c)^(1/2))/
b^4/d^2+2*(-b^2*c+a^2)*(d*x+c)^(1/2)/b^3/d^2
```

3.634.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

$$\int \frac{x}{a+b\sqrt{c+dx}}\,dx = \frac{b\left(6a^2\sqrt{c+dx}+2b^2(-2c+dx)\sqrt{c+dx}-3ab(c+dx)\right)-6(a^3-ab^2c)\log\left(a+b\sqrt{c+dx}\right)}{3b^4d^2}$$

input
```
Integrate[x/(a + b*Sqrt[c + d*x]),x]
```

output
```
(b*(6*a^2*Sqrt[c + d*x] + 2*b^2*(-2*c + d*x)*Sqrt[c + d*x] - 3*a*b*(c + d*
x)) - 6*(a^3 - a*b^2*c)*Log[a + b*Sqrt[c + d*x]])/(3*b^4*d^2)
```

3.634.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+b\sqrt{c+dx}}dx$$
$$\Big\downarrow \text{ 896}$$
$$\frac{\int \frac{dx}{a+b\sqrt{c+dx}}d(c+dx)}{d^2}$$
$$\Big\downarrow \text{ 25}$$
$$-\frac{\int -\frac{dx}{a+b\sqrt{c+dx}}d(c+dx)}{d^2}$$
$$\Big\downarrow \text{ 1732}$$
$$-\frac{2\int -\frac{dx\sqrt{c+dx}}{a+b\sqrt{c+dx}}d\sqrt{c+dx}}{d^2}$$
$$\Big\downarrow \text{ 522}$$
$$-\frac{2\int \left(\frac{\sqrt{c+dx}a}{b^2}+\frac{b^2c-a^2}{b^3}-\frac{c+dx}{b}+\frac{a^3-ab^2c}{b^3\left(a+b\sqrt{c+dx}\right)}\right)d\sqrt{c+dx}}{d^2}$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{2\left(\frac{a\left(a^2-b^2c\right)\log\left(a+b\sqrt{c+dx}\right)}{b^4}-\frac{\left(a^2-b^2c\right)\sqrt{c+dx}}{b^3}+\frac{a(c+dx)}{2b^2}-\frac{(c+dx)^{3/2}}{3b}\right)}{d^2}$$

input
```
Int[x/(a + b*Sqrt[c + d*x]),x]
```

output
```
(-2*(-(((a^2 - b^2*c)*Sqrt[c + d*x])/b^3) + (a*(c + d*x))/(2*b^2) - (c + d
*x)^(3/2)/(3*b) + (a*(a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])/b^4))/d^2
```

3.634.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.634.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\frac{2\left(\frac{(dx+c)^{\frac{3}{2}}b^2}{3}-\frac{a(dx+c)b}{2}-b^2c\sqrt{dx+c}+a^2\sqrt{dx+c}\right)}{b^3}-\frac{2a\left(-b^2c+a^2\right)\ln\left(a+b\sqrt{dx+c}\right)}{b^4}}{d^2}$	85
default	$\frac{\frac{2\left(\frac{(dx+c)^{\frac{3}{2}}b^2}{3}-\frac{a(dx+c)b}{2}-b^2c\sqrt{dx+c}+a^2\sqrt{dx+c}\right)}{b^3}-\frac{2a\left(-b^2c+a^2\right)\ln\left(a+b\sqrt{dx+c}\right)}{b^4}}{d^2}$	85

input
```
int(x/(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/d^2*(1/b^3*(1/3*(d*x+c)^(3/2)*b^2-1/2*a*(d*x+c)*b-b^2*c*(d*x+c)^(1/2)+a^
2*(d*x+c)^(1/2))-a*(-b^2*c+a^2)/b^4*ln(a+b*(d*x+c)^(1/2)))
```

3.634.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.79

$$\int \frac{x}{a+b\sqrt{c+dx}}\,dx = -\frac{3\,ab^2dx-6\,(ab^2c-a^3)\log\left(\sqrt{dx+c}b+a\right)-2\,(b^3dx-2\,b^3c+3\,a^2b)\sqrt{dx+c}}{3\,b^4d^2}$$

input
```
integrate(x/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
-1/3*(3*a*b^2*d*x - 6*(a*b^2*c - a^3)*log(sqrt(d*x + c)*b + a) - 2*(b^3*d*
x - 2*b^3*c + 3*a^2*b)*sqrt(d*x + c))/(b^4*d^2)
```

3.634.6 Sympy [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.16

$$\int \frac{x}{a+b\sqrt{c+dx}}\,dx = \begin{cases} \frac{2\left(-\frac{a(c+dx)}{2b^2}-\frac{a\left(a^2-b^2c\right)\left(\begin{cases}\frac{\sqrt{c+dx}}{a} & \text{for } b=0\\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^3}+\frac{(c+dx)^{\frac{3}{2}}}{3b}+\frac{\left(a^2-b^2c\right)\sqrt{c+dx}}{b^3}\right)}{d^2} & \text{for } d\neq 0\\ \frac{x^2}{2\left(a+b\sqrt{c}\right)} & \text{otherwise}\end{cases}$$

input
```
integrate(x/(a+b*(d*x+c)**(1/2)),x)
```

output
```
Piecewise((2*(-a*(c + d*x)/(2*b**2) - a*(a**2 - b**2*c)*Piecewise((sqrt(c
+ d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b, True))/b**3 + (c + d*x)*
*(3/2)/(3*b) + (a**2 - b**2*c)*sqrt(c + d*x)/b**3)/d**2, Ne(d, 0)), (x**2/
(2*(a + b*sqrt(c))), True))
```

3.634.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.90

$$\int \frac{x}{a+b\sqrt{c+dx}}\,dx = \frac{\frac{2\,(dx+c)^{\frac{3}{2}}b^2-3\,(dx+c)ab-6\,(b^2c-a^2)\sqrt{dx+c}}{b^3}+\frac{6\,(ab^2c-a^3)\log\left(\sqrt{dx+c}b+a\right)}{b^4}}{3\,d^2}$$

input
```
integrate(x/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
1/3*((2*(d*x + c)^(3/2)*b^2 - 3*(d*x + c)*a*b - 6*(b^2*c - a^2)*sqrt(d*x +
 c))/b^3 + 6*(a*b^2*c - a^3)*log(sqrt(d*x + c)*b + a)/b^4)/d^2
```

3.634.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.17

$$\int \frac{x}{a+b\sqrt{c+dx}}\,dx = \frac{\frac{6\,(ab^2c-a^3)\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^4d}+\frac{2\,(dx+c)^{\frac{3}{2}}b^2d^2-6\,\sqrt{dx+c}b^2cd^2-3\,(dx+c)abd^2+6\,\sqrt{dx+c}a^2d^2}{b^3d^3}}{3\,d}$$

input
```
integrate(x/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

output
```
1/3*(6*(a*b^2*c - a^3)*log(abs(sqrt(d*x + c)*b + a))/(b^4*d) + (2*(d*x + c
)^(3/2)*b^2*d^2 - 6*sqrt(d*x + c)*b^2*c*d^2 - 3*(d*x + c)*a*b*d^2 + 6*sqrt
(d*x + c)*a^2*d^2)/(b^3*d^3))/d
```

3.634.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99

$$\int \frac{x}{a+b\sqrt{c+dx}}\,dx = \frac{2\,(c+d\,x)^{3/2}}{3\,b\,d^2}-\left(\frac{2\,c}{b\,d^2}-\frac{2\,a^2}{b^3\,d^2}\right)\sqrt{c+d\,x} - \frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(2\,a^3-2\,a\,b^2\,c\right)}{b^4\,d^2}-\frac{a\,x}{b^2\,d}$$

input
```
int(x/(a + b*(c + d*x)^(1/2)),x)
```

output
```
(2*(c + d*x)^(3/2))/(3*b*d^2) - ((2*c)/(b*d^2) - (2*a^2)/(b^3*d^2))*(c + d
*x)^(1/2) - (log(a + b*(c + d*x)^(1/2))*(2*a^3 - 2*a*b^2*c))/(b^4*d^2) - (
a*x)/(b^2*d)
```

3.634. $\int \frac{x}{a+b\sqrt{c+dx}}\,dx$

3.635 $\int \frac{1}{a+b\sqrt{c+dx}}\,dx$

3.635.1 Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \frac{1}{a+b\sqrt{c+dx}}\,dx = \frac{2\sqrt{c+dx}}{bd} - \frac{2a\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

output
```
-2*a*ln(a+b*(d*x+c)^(1/2))/b^2/d+2*(d*x+c)^(1/2)/b/d
```

3.635.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{1}{a+b\sqrt{c+dx}}\,dx = \frac{2b\sqrt{c+dx} - 2a\log\left(bd\left(a+b\sqrt{c+dx}\right)\right)}{b^2d}$$

input
```
Integrate[(a + b*Sqrt[c + d*x])^(-1),x]
```

output
```
(2*b*Sqrt[c + d*x] - 2*a*Log[b*d*(a + b*Sqrt[c + d*x])])/(b^2*d)
```

3.635.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {239, 774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{239}$$

$$\frac{\int \frac{1}{a+b\sqrt{c+dx}}d(c+dx)}{d}$$

$$\Big\downarrow \text{774}$$

$$\frac{2\int \frac{\sqrt{c+dx}}{a+b\sqrt{c+dx}}d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{49}$$

$$\frac{2\int \left(\frac{1}{b}-\frac{a}{b\left(a+b\sqrt{c+dx}\right)}\right)d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{\sqrt{c+dx}}{b}-\frac{a\log\left(a+b\sqrt{c+dx}\right)}{b^2}\right)}{d}$$

input
```
Int[(a + b*Sqrt[c + d*x])^(-1),x]
```

output
```
(2*(Sqrt[c + d*x]/b - (a*Log[a + b*Sqrt[c + d*x]])/b^2))/d
```

3.635.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.635.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\frac{2\sqrt{dx+c}}{b}-\frac{2a\ln\left(a+b\sqrt{dx+c}\right)}{b^2}}{d}$	36
default	$\frac{2\sqrt{dx+c}}{bd}+\frac{a\ln\left(-a+b\sqrt{dx+c}\right)}{b^2d}-\frac{a\ln\left(a+b\sqrt{dx+c}\right)}{b^2d}-\frac{a\ln\left(b^2dx+b^2c-a^2\right)}{b^2d}$	87

input
```
int(1/(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/b*(d*x+c)^(1/2)-a/b^2*ln(a+b*(d*x+c)^(1/2)))
```

3.635.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{1}{a+b\sqrt{c+dx}}\,dx=-\frac{2\left(a\log\left(\sqrt{dx+c}b+a\right)-\sqrt{dx+c}b\right)}{b^2d}$$

input
```
integrate(1/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
-2*(a*log(sqrt(d*x + c)*b + a) - sqrt(d*x + c)*b)/(b^2*d)
```

3.635.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\sqrt{c+dx}}\,dx = \begin{cases} \frac{x}{a} & \text{for } b=0 \wedge (b=0 \vee d=0) \\ \frac{x}{a+b\sqrt{c}} & \text{for } d=0 \\ -\frac{2a\log\left(\frac{a}{b}+\sqrt{c+dx}\right)}{b^2d} + \frac{2\sqrt{c+dx}}{bd} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*(d*x+c)**(1/2)),x)
```

output
```
Piecewise((x/a, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x/(a + b*sqrt(c)), Eq(
d, 0)), (-2*a*log(a/b + sqrt(c + d*x))/(b**2*d) + 2*sqrt(c + d*x)/(b*d), T
rue))
```

3.635.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{1}{a+b\sqrt{c+dx}}\,dx = -\frac{2\left(\frac{a\log\left(\sqrt{dx+c}b+a\right)}{b^2} - \frac{\sqrt{dx+c}}{b}\right)}{d}$$

input
```
integrate(1/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
-2*(a*log(sqrt(d*x + c)*b + a)/b^2 - sqrt(d*x + c)/b)/d
```

3.635.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93

$$\int \frac{1}{a+b\sqrt{c+dx}}\,dx = -\frac{2\,a\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^2d} + \frac{2\,\sqrt{dx+c}}{bd}$$

input
```
integrate(1/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

output
```
-2*a*log(abs(sqrt(d*x + c)*b + a))/(b^2*d) + 2*sqrt(d*x + c)/(b*d)
```

3.635.9 Mupad [B] (verification not implemented)

Time = 17.45 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{1}{a+b\sqrt{c+dx}}\,dx=-\frac{2\left(a\,\ln\left(a+b\,\sqrt{c+d\,x}\right)-b\,\sqrt{c+d\,x}\right)}{b^2\,d}$$

input
```
int(1/(a + b*(c + d*x)^(1/2)),x)
```

output
```
-(2*(a*log(a + b*(c + d*x)^(1/2)) - b*(c + d*x)^(1/2)))/(b^2*d)
```

3.636 $\int \frac{1}{x(a+b\sqrt{c+dx})} dx$

3.636.1 Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)} dx = \frac{2b\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c} + \frac{a\log(x)}{a^2-b^2c} - \frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}$$

output
```
a*ln(x)/(-b^2*c+a^2)-2*a*ln(a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)+2*b*arctanh((d
*x+c)^(1/2)/c^(1/2))*c^(1/2)/(-b^2*c+a^2)
```

3.636.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)} dx = \frac{2b\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right) + a\log(-dx) - 2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}$$

input
```
Integrate[1/(x*(a + b*Sqrt[c + d*x])),x]
```

output
```
(2*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]] + a*Log[-(d*x)] - 2*a*Log[a +
b*Sqrt[c + d*x]])/(a^2 - b^2*c)
```

3.636.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {896, 25, 1732, 587, 16, 452, 219, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}dx$$

$$\Big\downarrow \text{896}$$

$$\int \frac{1}{dx\left(a+b\sqrt{c+dx}\right)}d(c+dx)$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{1}{dx\left(a+b\sqrt{c+dx}\right)}d(c+dx)$$

$$\Big\downarrow \text{1732}$$

$$-2\int -\frac{\sqrt{c+dx}}{dx\left(a+b\sqrt{c+dx}\right)}d\sqrt{c+dx}$$

$$\Big\downarrow \text{587}$$

$$-2\left(\frac{ab\int \frac{1}{a+b\sqrt{c+dx}}d\sqrt{c+dx}}{a^2-b^2c}-\frac{\int -\frac{bc-a\sqrt{c+dx}}{dx}d\sqrt{c+dx}}{a^2-b^2c}\right)$$

$$\Big\downarrow \text{16}$$

$$-2\left(\frac{a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}-\frac{\int -\frac{bc-a\sqrt{c+dx}}{dx}d\sqrt{c+dx}}{a^2-b^2c}\right)$$

$$\Big\downarrow \text{452}$$

$$-2\left(\frac{a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}-\frac{bc\int -\frac{1}{dx}d\sqrt{c+dx}-a\int -\frac{\sqrt{c+dx}}{dx}d\sqrt{c+dx}}{a^2-b^2c}\right)$$

$$\Big\downarrow \text{219}$$

$$-2\left(\frac{a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}-\frac{b\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)-a\int -\frac{\sqrt{c+dx}}{dx}d\sqrt{c+dx}}{a^2-b^2c}\right)$$

$$\Big\downarrow \text{240}$$

$$-2\left(\frac{a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}-\frac{\frac{1}{2}a\log(-dx)+b\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c}\right)$$

input
```
Int[1/(x*(a + b*Sqrt[c + d*x])),x]
```

output
```
-2*(-((b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]] + (a*Log[-(d*x)])/2)/(a^2
- b^2*c)) + (a*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c))
```

3.636.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 587
```
Int[(x_.)/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[(-
c)*(d/(b*c^2 + a*d^2))   Int[1/(c + d*x), x], x] + Simp[1/(b*c^2 + a*d^2)
 Int[(a*d + b*c*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
^2 + a*d^2, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

3.636.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{a\ln(-dx)+2\sqrt{c}\,b\ \operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{-b^2c+a^2}-\frac{2a\ln\left(a+b\sqrt{dx+c}\right)}{-b^2c+a^2}$	69
default	$\frac{a\ln(-dx)+2\sqrt{c}\,b\ \operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{-b^2c+a^2}-\frac{2a\ln\left(a+b\sqrt{dx+c}\right)}{-b^2c+a^2}$	69

input
```
int(1/x/(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/(-b^2*c+a^2)*(1/2*a*ln(-d*x)+c^(1/2)*b*arctanh((d*x+c)^(1/2)/c^(1/2)))-2
*a*ln(a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)
```

3.636.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.52

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx$$
$$=\left[\frac{b\sqrt{c}\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)+2a\log\left(\sqrt{dx+c}b+a\right)-a\log\left(x\right)}{b^2c-a^2},\frac{2b\sqrt{-c}\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right)+2a\log\left(\sqrt{\right.}\right.}{b^2c-a^2}\right.$$

input
```
integrate(1/x/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
[(b*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*a*log(sqrt(d*
x + c)*b + a) - a*log(x))/(b^2*c - a^2), (2*b*sqrt(-c)*arctan(sqrt(d*x + c
)*sqrt(-c)/c) + 2*a*log(sqrt(d*x + c)*b + a) - a*log(x))/(b^2*c - a^2)]
```

3.636.6 Sympy [A] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx = \begin{cases} -\frac{2ab\left(\begin{cases} \frac{\sqrt{c+dx}}{a} & \text{for}\ b=0 \\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise} \end{cases}\right)}{a^2-b^2c} - \frac{2\left(-\frac{a\log(-dx)}{2}+\frac{bc\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}\right)}{a^2-b^2c} & \text{for}\ d\neq 0 \\ \frac{\log(x)}{a+b\sqrt{c}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(a+b*(d*x+c)**(1/2)),x)
```

output
```
Piecewise((-2*a*b*Piecewise((sqrt(c + d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c
 + d*x))/b, True))/(a**2 - b**2*c) - 2*(-a*log(-d*x)/2 + b*c*atan(sqrt(c +
 d*x)/sqrt(-c))/sqrt(-c))/(a**2 - b**2*c), Ne(d, 0)), (log(x)/(a + b*sqrt(
c)), True))
```

3.636.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.16

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx = \frac{b\sqrt{c}\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{b^2c-a^2} - \frac{a\log(dx)}{b^2c-a^2} + \frac{2\,a\log\left(\sqrt{dx+c}b+a\right)}{b^2c-a^2}$$

input
```
integrate(1/x/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
b*sqrt(c)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/(b^2*c
- a^2) - a*log(d*x)/(b^2*c - a^2) + 2*a*log(sqrt(d*x + c)*b + a)/(b^2*c -
a^2)
```

3.636.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx = \frac{2\,ab\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^3c-a^2b} + \frac{2\,bc\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{(b^2c-a^2)\sqrt{-c}} - \frac{a\log\left(dx\right)}{b^2c-a^2}$$

input
```
integrate(1/x/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

output
```
2*a*b*log(abs(sqrt(d*x + c)*b + a))/(b^3*c - a^2*b) + 2*b*c*arctan(sqrt(d*
x + c)/sqrt(-c))/((b^2*c - a^2)*sqrt(-c)) - a*log(d*x)/(b^2*c - a^2)
```

3.636.9 Mupad [B] (verification not implemented)

Time = 17.83 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.21

$$\begin{aligned}\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx &= \frac{\ln\left(2\,b^3\,c^{3/2}-2\,b^3\,c\,\sqrt{c+d\,x}-6\,a\,b^2\,c+6\,a\,b^2\,\sqrt{c}\,\sqrt{c+d\,x}\right)}{a+b\,\sqrt{c}} \\ &\quad + \frac{\ln\left(-2\,b^3\,c^{3/2}-2\,b^3\,c\,\sqrt{c+d\,x}-6\,a\,b^2\,c-6\,a\,b^2\,\sqrt{c}\,\sqrt{c+d\,x}\right)}{a-b\,\sqrt{c}} \\ &\quad + \frac{2\,a\,\ln\left(4\,b^5\,c^2\,\sqrt{c+d\,x}-36\,a^3\,b^2\,c+4\,a\,b^4\,c^2-36\,a^2\,b^3\,c\,\sqrt{c+d\,x}\right)}{b^2\,c-a^2}\end{aligned}$$

input
```
int(1/(x*(a + b*(c + d*x)^(1/2))),x)
```

output
```
log(2*b^3*c^(3/2) - 2*b^3*c*(c + d*x)^(1/2) - 6*a*b^2*c + 6*a*b^2*c^(1/2)*
(c + d*x)^(1/2))/(a + b*c^(1/2)) + log(- 2*b^3*c^(3/2) - 2*b^3*c*(c + d*x)
^(1/2) - 6*a*b^2*c - 6*a*b^2*c^(1/2)*(c + d*x)^(1/2))/(a - b*c^(1/2)) + (2
*a*log(4*b^5*c^2*(c + d*x)^(1/2) - 36*a^3*b^2*c + 4*a*b^4*c^2 - 36*a^2*b^3
*c*(c + d*x)^(1/2)))/(b^2*c - a^2)
```

3.637 $\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx$

3.637.1 Optimal result

Integrand size = 19, antiderivative size = 130

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx = -\frac{a-b\sqrt{c+dx}}{(a^2-b^2c)\,x}+\frac{b(a^2+b^2c)\,d\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\,(a^2-b^2c)^2} + \frac{ab^2d\log(x)}{(a^2-b^2c)^2}-\frac{2ab^2d\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^2}$$

output
```
a*b^2*d*ln(x)/(-b^2*c+a^2)^2-2*a*b^2*d*ln(a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)^
2+b*(b^2*c+a^2)*d*arctanh((d*x+c)^(1/2)/c^(1/2))/(-b^2*c+a^2)^2/c^(1/2)+(-
a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)/x
```

3.637.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx = \frac{b(a^2+b^2c)\,dx\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+\sqrt{c}\left(-\left((a^2-b^2c)\left(a-b\sqrt{c+dx}\right)\right)+ab^2dx\log(-dx)-2ab^2dx\log\left(a+\right.\right.}{\sqrt{c}\,(a^2-b^2c)^2\,x}$$

input
```
Integrate[1/(x^2*(a + b*Sqrt[c + d*x])),x]
```

output
```
(b*(a^2 + b^2*c)*d*x*ArcTanh[Sqrt[c + d*x]/Sqrt[c]] + Sqrt[c]*(-((a^2 - b^
2*c)*(a - b*Sqrt[c + d*x])) + a*b^2*d*x*Log[-(d*x)] - 2*a*b^2*d*x*Log[a +
b*Sqrt[c + d*x]]))/(Sqrt[c]*(a^2 - b^2*c)^2*x)
```

3.637.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {896, 1732, 593, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+b\sqrt{c+dx}\right)} \, dx$$

$$\Big\downarrow \text{ 896}$$

$$d\int \frac{1}{d^2x^2 \left(a+b\sqrt{c+dx}\right)}d(c+dx)$$

$$\Big\downarrow \text{ 1732}$$

$$2d\int \frac{\sqrt{c+dx}}{d^2x^2 \left(a+b\sqrt{c+dx}\right)}d\sqrt{c+dx}$$

$$\Big\downarrow \text{ 593}$$

$$2d\left(-\frac{b\int \frac{a-b\sqrt{c+dx}}{dx\left(a+b\sqrt{c+dx}\right)}d\sqrt{c+dx}}{2\left(a^2-b^2c\right)}-\frac{a-b\sqrt{c+dx}}{2dx\left(a^2-b^2c\right)}\right)$$

$$\Big\downarrow \text{ 25}$$

$$2d\left(\frac{b\int -\frac{a-b\sqrt{c+dx}}{dx\left(a+b\sqrt{c+dx}\right)}d\sqrt{c+dx}}{2\left(a^2-b^2c\right)}-\frac{a-b\sqrt{c+dx}}{2dx\left(a^2-b^2c\right)}\right)$$

$$\Big\downarrow \text{ 657}$$

$$2d\left(\frac{b\int \left(-\frac{2ab^2}{\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}-\frac{a^2-2b\sqrt{c+dx}a+b^2c}{\left(a^2-b^2c\right)dx}\right)d\sqrt{c+dx}}{2\left(a^2-b^2c\right)}-\frac{a-b\sqrt{c+dx}}{2dx\left(a^2-b^2c\right)}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$2d\left(\frac{b\left(\frac{(a^2+b^2c)\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}(a^2-b^2c)}+\frac{ab\log(-dx)}{a^2-b^2c}-\frac{2ab\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}\right)}{2\left(a^2-b^2c\right)}-\frac{a-b\sqrt{c+dx}}{2dx\left(a^2-b^2c\right)}\right)$$

input
```
Int[1/(x^2*(a + b*Sqrt[c + d*x])),x]
```

output
```
2*d*(-1/2*(a - b*Sqrt[c + d*x])/((a^2 - b^2*c)*d*x) + (b*(((a^2 + b^2*c)*A
rcTanh[Sqrt[c + d*x]/Sqrt[c]])/(Sqrt[c]*(a^2 - b^2*c)) + (a*b*Log[-(d*x)])
/(a^2 - b^2*c) - (2*a*b*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)))/(2*(a^2
- b^2*c)))
```

3.637.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 593
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[(c + d*x)^(n + 1)*(c - d*x)*((a + b*x^2)^(p + 1)/(2*(p + 1)*(b*c^2 +
a*d^2))), x] - Simp[d/(2*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*
x^2)^(p + 1)*(c*n - d*(n + 2*p + 4)*x), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.637.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

method	result	s
derivativedivides	$2d\left(-\frac{a\,b^2\ln\left(a+b\sqrt{dx+c}\right)}{\left(-b^2c+a^2\right)^2}+\frac{-\frac{\left(\frac{1}{2}b^3c-\frac{1}{2}a^2b\right)\sqrt{dx+c}-\frac{a\,b^2c}{2}+\frac{a^3}{2}}{dx}+\frac{b\left(ab\ln\left(-dx\right)+\frac{\left(b^2c+a^2\right)\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{2}}{\left(-b^2c+a^2\right)^2}\right)$	1
default	$2d\left(-\frac{a\,b^2\ln\left(a+b\sqrt{dx+c}\right)}{\left(-b^2c+a^2\right)^2}+\frac{-\frac{\left(\frac{1}{2}b^3c-\frac{1}{2}a^2b\right)\sqrt{dx+c}-\frac{a\,b^2c}{2}+\frac{a^3}{2}}{dx}+\frac{b\left(ab\ln\left(-dx\right)+\frac{\left(b^2c+a^2\right)\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{2}}{\left(-b^2c+a^2\right)^2}\right)$	1

input
```
int(1/x^2/(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2*d*(-a*b^2/(-b^2*c+a^2)^2*ln(a+b*(d*x+c)^(1/2))+1/(-b^2*c+a^2)^2*(-((1/2*
b^3*c-1/2*a^2*b)*(d*x+c)^(1/2)-1/2*a*b^2*c+1/2*a^3)/d/x+1/2*b*(a*b*ln(-d*x
)+(b^2*c+a^2)/c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2)))))
```

3.637.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.18

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx$$
$$=\left[-\frac{4\,ab^2cdx\log\left(\sqrt{dx+c}b+a\right)-2\,ab^2cdx\log\left(x\right)-2\,ab^2c^2-(b^3c+a^2b)\sqrt{c}dx\log\left(\frac{dx+2\,\sqrt{dx+c}\sqrt{c}+2\,c}{x}\right)+2}{2\,(b^4c^3-2\,a^2b^2c^2+a^4c)x}\right.$$
$$-\frac{2\,ab^2cdx\log\left(\sqrt{dx+c}b+a\right)-ab^2cdx\log\left(x\right)-ab^2c^2+(b^3c+a^2b)\sqrt{-c}dx\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right)+a^3c+}{(b^4c^3-2\,a^2b^2c^2+a^4c)x}$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
[-1/2*(4*a*b^2*c*d*x*log(sqrt(d*x + c)*b + a) - 2*a*b^2*c*d*x*log(x) - 2*a
*b^2*c^2 - (b^3*c + a^2*b)*sqrt(c)*d*x*log((d*x + 2*sqrt(d*x + c)*sqrt(c)
+ 2*c)/x) + 2*a^3*c + 2*(b^3*c^2 - a^2*b*c)*sqrt(d*x + c))/((b^4*c^3 - 2*a
^2*b^2*c^2 + a^4*c)*x), -(2*a*b^2*c*d*x*log(sqrt(d*x + c)*b + a) - a*b^2*c
*d*x*log(x) - a*b^2*c^2 + (b^3*c + a^2*b)*sqrt(-c)*d*x*arctan(sqrt(d*x + c
)*sqrt(-c)/c) + a^3*c + (b^3*c^2 - a^2*b*c)*sqrt(d*x + c))/((b^4*c^3 - 2*a
^2*b^2*c^2 + a^4*c)*x)]
```

3.637.6 Sympy [F]

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx=\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx$$

input
```
integrate(1/x**2/(a+b*(d*x+c)**(1/2)),x)
```

output
```
Integral(1/(x**2*(a + b*sqrt(c + d*x))), x)
```

3.637.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.47

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx$$
$$=\frac{1}{2}\left(\frac{2\,ab^2\log\left(dx\right)}{b^4c^2-2\,a^2b^2c+a^4}-\frac{4\,ab^2\log\left(\sqrt{dx+c}b+a\right)}{b^4c^2-2\,a^2b^2c+a^4}-\frac{\left(b^3c+a^2b\right)\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{\left(b^4c^2-2\,a^2b^2c+a^4\right)\sqrt{c}}+\frac{2\left(\sqrt{dx+c}b-\right.}{b^2c^2-a^2c-\left(b^2c-\right.}\right.$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
1/2*(2*a*b^2*log(d*x)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - 4*a*b^2*log(sqrt(d*x
 + c)*b + a)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - (b^3*c + a^2*b)*log((sqrt(d*x
 + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/((b^4*c^2 - 2*a^2*b^2*c + a^4)
*sqrt(c)) + 2*(sqrt(d*x + c)*b - a)/(b^2*c^2 - a^2*c - (b^2*c - a^2)*(d*x
+ c)))*d
```

3.637.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.47

$$\int \frac{1}{x^2 \left(a+b\sqrt{c+dx}\right)}\,dx = -\frac{2\,ab^3d\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^5c^2-2\,a^2b^3c+a^4b}+\frac{ab^2d\log\left(-dx\right)}{b^4c^2-2\,a^2b^2c+a^4} - \frac{(b^3cd+a^2bd)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{(b^4c^2-2\,a^2b^2c+a^4)\sqrt{-c}} + \frac{ab^2cd-a^3d-(b^3cd-a^2bd)\sqrt{dx+c}}{(b^2c-a^2)^2dx}$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

output
```
-2*a*b^3*d*log(abs(sqrt(d*x + c)*b + a))/(b^5*c^2 - 2*a^2*b^3*c + a^4*b) +
 a*b^2*d*log(-d*x)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - (b^3*c*d + a^2*b*d)*arc
tan(sqrt(d*x + c)/sqrt(-c))/((b^4*c^2 - 2*a^2*b^2*c + a^4)*sqrt(-c)) + (a*
b^2*c*d - a^3*d - (b^3*c*d - a^2*b*d)*sqrt(d*x + c))/((b^2*c - a^2)^2*d*x)
```

3.637.9 Mupad [B] (verification not implemented)

Time = 18.52 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.69

$$\int \frac{1}{x^2 \left(a+b\sqrt{c+dx}\right)}\,dx = \frac{\ln\left(\sqrt{c+d\,x}-\sqrt{c}\right)\,\left(4\,a\,b^2\,c\,d-b\,\sqrt{c}\,d\,\left(2\,a^2+2\,c\,b^2\right)\right)}{4\,a^4\,c-8\,a^2\,b^2\,c^2+4\,b^4\,c^3} + \frac{\ln\left(\sqrt{c+d\,x}+\sqrt{c}\right)\,\left(4\,a\,b^2\,c\,d+b\,\sqrt{c}\,d\,\left(2\,a^2+2\,c\,b^2\right)\right)}{4\,a^4\,c-8\,a^2\,b^2\,c^2+4\,b^4\,c^3} + \frac{\frac{a\,d}{b^2\,c-a^2}-\frac{b\,d\,\sqrt{c+d\,x}}{b^2\,c-a^2}}{d\,x} - \frac{2\,a\,b^2\,d\,\ln\left(a+b\,\sqrt{c+d\,x}\right)}{\left(b^2\,c-a^2\right)^2}$$

input
```
int(1/(x^2*(a + b*(c + d*x)^(1/2))),x)
```

output
```
(log((c + d*x)^(1/2) - c^(1/2))*(4*a*b^2*c*d - b*c^(1/2)*d*(2*b^2*c + 2*a^
2)))/(4*a^4*c + 4*b^4*c^3 - 8*a^2*b^2*c^2) + (log((c + d*x)^(1/2) + c^(1/2
))*(4*a*b^2*c*d + b*c^(1/2)*d*(2*b^2*c + 2*a^2)))/(4*a^4*c + 4*b^4*c^3 - 8
*a^2*b^2*c^2) + ((a*d)/(b^2*c - a^2) - (b*d*(c + d*x)^(1/2))/(b^2*c - a^2)
)/(d*x) - (2*a*b^2*d*log(a + b*(c + d*x)^(1/2)))/(b^2*c - a^2)^2
```

3.638 $\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx$

3.638.1 Optimal result

Integrand size = 19, antiderivative size = 204

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx = -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2}-\frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4c\left(a^2-b^2c\right)^2x} - \frac{b\left(a^4-6a^2b^2c-3b^4c^2\right)d^2\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{4c^{3/2}\left(a^2-b^2c\right)^3} + \frac{ab^4d^2\log(x)}{\left(a^2-b^2c\right)^3}-\frac{2ab^4d^2\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^3}$$

output
```
-1/4*b*(-3*b^4*c^2-6*a^2*b^2*c+a^4)*d^2*arctanh((d*x+c)^(1/2)/c^(1/2))/c^(
3/2)/(-b^2*c+a^2)^3+a*b^4*d^2*ln(x)/(-b^2*c+a^2)^3-2*a*b^4*d^2*ln(a+b*(d*x
+c)^(1/2))/(-b^2*c+a^2)^3+1/2*(-a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)/x^2-1/4*b*
d*(4*a*b*c-(3*b^2*c+a^2)*(d*x+c)^(1/2))/c/(-b^2*c+a^2)^2/x
```

3.638.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx = \frac{b\left(a^4-6a^2b^2c-3b^4c^2\right)d^2x^2\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+\sqrt{c}\left(\left(a^2-b^2c\right)\left(2a^3c-2ab^2c(c-2dx)+b^3c(2c-3dx)\sqrt{c\cdots}\right.\right.}{4c^{3/2}\left(-a^2+b^2c\right)^3x\cdots}$$

input
```
Integrate[1/(x^3*(a + b*Sqrt[c + d*x])),x]
```

output
```
(b*(a^4 - 6*a^2*b^2*c - 3*b^4*c^2)*d^2*x^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]]
+ Sqrt[c]*((a^2 - b^2*c)*(2*a^3*c - 2*a*b^2*c*(c - 2*d*x) + b^3*c*(2*c - 3
*d*x)*Sqrt[c + d*x] - a^2*b*Sqrt[c + d*x]*(2*c + d*x)) - 4*a*b^4*c*d^2*x^2
*Log[-(d*x)] + 8*a*b^4*c*d^2*x^2*Log[a + b*Sqrt[c + d*x]]))/(4*c^(3/2)*(-a
^2 + b^2*c)^3*x^2)
```

3.638.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {896, 25, 1732, 593, 25, 686, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}dx$$

$$\Big\downarrow \text{ 896}$$

$$d^2\int \frac{1}{d^3x^3\left(a+b\sqrt{c+dx}\right)}d(c+dx)$$

$$\Big\downarrow \text{ 25}$$

$$-d^2\int -\frac{1}{d^3x^3\left(a+b\sqrt{c+dx}\right)}d(c+dx)$$

$$\Big\downarrow \text{ 1732}$$

$$-2d^2\int -\frac{\sqrt{c+dx}}{d^3x^3\left(a+b\sqrt{c+dx}\right)}d\sqrt{c+dx}$$

$$\Big\downarrow \text{ 593}$$

$$-2d^2\left(\frac{a-b\sqrt{c+dx}}{4d^2x^2\left(a^2-b^2c\right)}-\frac{b\int -\frac{a-3b\sqrt{c+dx}}{d^2x^2\left(a+b\sqrt{c+dx}\right)}d\sqrt{c+dx}}{4\left(a^2-b^2c\right)}\right)$$

$$\Big\downarrow \text{ 25}$$

$$-2d^2\left(\frac{b\int \frac{a-3b\sqrt{c+dx}}{d^2x^2\left(a+b\sqrt{c+dx}\right)}d\sqrt{c+dx}}{4\left(a^2-b^2c\right)}+\frac{a-b\sqrt{c+dx}}{4d^2x^2\left(a^2-b^2c\right)}\right)$$

$$\Big\downarrow \; 686$$

$$-2d^2\left(\frac{b\left(\frac{4abc-(a^2+3b^2c)\sqrt{c+dx}}{2cdx(a^2-b^2c)}-\frac{\int\frac{a(a^2-5b^2c)+b(a^2+3b^2c)\sqrt{c+dx}}{dx(a+b\sqrt{c+dx})}d\sqrt{c+dx}}{2c(a^2-b^2c)}\right)}{4(a^2-b^2c)}+\frac{a-b\sqrt{c+dx}}{4d^2x^2(a^2-b^2c)}\right)$$

$$\Big\downarrow \; 25$$

$$-2d^2\left(\frac{b\left(\frac{\int-\frac{a(a^2-5b^2c)+b(a^2+3b^2c)\sqrt{c+dx}}{dx(a+b\sqrt{c+dx})}d\sqrt{c+dx}}{2c(a^2-b^2c)}+\frac{4abc-(a^2+3b^2c)\sqrt{c+dx}}{2cdx(a^2-b^2c)}\right)}{4(a^2-b^2c)}+\frac{a-b\sqrt{c+dx}}{4d^2x^2(a^2-b^2c)}\right)$$

$$\Big\downarrow \; 657$$

$$-2d^2\left(\frac{b\left(\frac{\int\left(\frac{8ab^4c}{(a^2-b^2c)(a+b\sqrt{c+dx})}-\frac{a^4-6b^2ca^2+8b^3c\sqrt{c+dx}a-3b^4c^2}{(a^2-b^2c)dx}\right)d\sqrt{c+dx}}{2c(a^2-b^2c)}+\frac{4abc-(a^2+3b^2c)\sqrt{c+dx}}{2cdx(a^2-b^2c)}\right)}{4(a^2-b^2c)}+\frac{a-b\sqrt{c+dx}}{4d^2x^2(a^2-b^2c)}\right)$$

$$\Big\downarrow \; 2009$$

$$-2d^2\left(\frac{a-b\sqrt{c+dx}}{4d^2x^2(a^2-b^2c)}+\frac{b\left(\frac{4abc-(a^2+3b^2c)\sqrt{c+dx}}{2cdx(a^2-b^2c)}+\frac{-\frac{4ab^3c\log(-dx)}{a^2-b^2c}+\frac{8ab^3c\log(a+b\sqrt{c+dx})}{a^2-b^2c}+\frac{(a^4-6a^2b^2c-3b^4c^2)\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}(a^2-b^2c)}}{2c(a^2-b^2c)}\right.}{4(a^2-b^2c)}\right.$$

input
```
Int[1/(x^3*(a + b*Sqrt[c + d*x])),x]
```

3.638. $\int \frac{1}{x^3(a+b\sqrt{c+dx})}\,dx$

output
```
-2*d^2*((a - b*Sqrt[c + d*x])/(4*(a^2 - b^2*c)*d^2*x^2) + (b*((4*a*b*c - (
a^2 + 3*b^2*c)*Sqrt[c + d*x])/(2*c*(a^2 - b^2*c)*d*x) + (((a^4 - 6*a^2*b^2
*c - 3*b^4*c^2)*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(Sqrt[c]*(a^2 - b^2*c)) -
(4*a*b^3*c*Log[-(d*x)])/(a^2 - b^2*c) + (8*a*b^3*c*Log[a + b*Sqrt[c + d*x]
])/(a^2 - b^2*c))/(2*c*(a^2 - b^2*c))))/(4*(a^2 - b^2*c)))
```

3.638.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 593
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[(c + d*x)^(n + 1)*(c - d*x)*((a + b*x^2)^(p + 1)/(2*(p + 1)*(b*c^2 +
a*d^2))), x] - Simp[d/(2*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*
x^2)^(p + 1)*(c*n - d*(n + 2*p + 4)*x), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.638.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.11

method	result
derivativedivides	$2d^2\left(-\frac{a\,b^4\ln\left(a+b\sqrt{dx+c}\right)}{\left(-b^2c+a^2\right)^3}-\frac{\frac{-\frac{b\left(-3b^4c^2+2a^2b^2c+a^4\right)(dx+c)^{\frac{3}{2}}}{8c}+\left(-\frac{1}{2}a\,b^4c+\frac{1}{2}a^3b^2\right)(dx+c)+\left(\frac{3}{4}a^2b^3c-\frac{1}{8}b\,a^4-\frac{5}{8}c^2b^5\right)\sqrt{}}{d^2x^2}}{\left(-b^2\right.}\right.$
default	$2d^2\left(-\frac{a\,b^4\ln\left(a+b\sqrt{dx+c}\right)}{\left(-b^2c+a^2\right)^3}-\frac{\frac{-\frac{b\left(-3b^4c^2+2a^2b^2c+a^4\right)(dx+c)^{\frac{3}{2}}}{8c}+\left(-\frac{1}{2}a\,b^4c+\frac{1}{2}a^3b^2\right)(dx+c)+\left(\frac{3}{4}a^2b^3c-\frac{1}{8}b\,a^4-\frac{5}{8}c^2b^5\right)\sqrt{}}{d^2x^2}}{\left(-b^2\right.}\right.$

input
```
int(1/x^3/(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2*d^2*(-a*b^4/(-b^2*c+a^2)^3*ln(a+b*(d*x+c)^(1/2))-1/(-b^2*c+a^2)^3*((-1/8
*b*(-3*b^4*c^2+2*a^2*b^2*c+a^4)/c*(d*x+c)^(3/2)+(-1/2*a*b^4*c+1/2*a^3*b^2)
*(d*x+c)+(3/4*a^2*b^3*c-1/8*b*a^4-5/8*c^2*b^5)*(d*x+c)^(1/2)+3/4*a*b^4*c^2
-a^3*b^2*c+1/4*a^5)/d^2/x^2+1/8*b/c*(-4*a*b^3*c*ln(-d*x)+(-3*b^4*c^2-6*a^2
*b^2*c+a^4)/c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2)))))
```

3.638.5 Fricas [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 534, normalized size of antiderivative = 2.62

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx$$
$$= \left[\frac{16\,ab^4c^2d^2x^2\log\left(\sqrt{dx+c}b+a\right)-8\,ab^4c^2d^2x^2\log(x)+4\,ab^4c^4-8\,a^3b^2c^3+4\,a^5c^2+\left(3\,b^5c^2+6\,a^2b^3c-\right.}{8\left(b^6c^5\right.}\right.$$

input
```
integrate(1/x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
[1/8*(16*a*b^4*c^2*d^2*x^2*log(sqrt(d*x + c)*b + a) - 8*a*b^4*c^2*d^2*x^2*
log(x) + 4*a*b^4*c^4 - 8*a^3*b^2*c^3 + 4*a^5*c^2 + (3*b^5*c^2 + 6*a^2*b^3*
c - a^4*b)*sqrt(c)*d^2*x^2*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) -
8*(a*b^4*c^3 - a^3*b^2*c^2)*d*x - 2*(2*b^5*c^4 - 4*a^2*b^3*c^3 + 2*a^4*b*c
^2 - (3*b^5*c^3 - 2*a^2*b^3*c^2 - a^4*b*c)*d*x)*sqrt(d*x + c))/((b^6*c^5 -
 3*a^2*b^4*c^4 + 3*a^4*b^2*c^3 - a^6*c^2)*x^2), 1/4*(8*a*b^4*c^2*d^2*x^2*l
og(sqrt(d*x + c)*b + a) - 4*a*b^4*c^2*d^2*x^2*log(x) + 2*a*b^4*c^4 - 4*a^3
*b^2*c^3 + 2*a^5*c^2 + (3*b^5*c^2 + 6*a^2*b^3*c - a^4*b)*sqrt(-c)*d^2*x^2*
arctan(sqrt(d*x + c)*sqrt(-c)/c) - 4*(a*b^4*c^3 - a^3*b^2*c^2)*d*x - (2*b^
5*c^4 - 4*a^2*b^3*c^3 + 2*a^4*b*c^2 - (3*b^5*c^3 - 2*a^2*b^3*c^2 - a^4*b*c
)*d*x)*sqrt(d*x + c))/((b^6*c^5 - 3*a^2*b^4*c^4 + 3*a^4*b^2*c^3 - a^6*c^2)
*x^2)]
```

3.638.6 Sympy [F]

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx=\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx$$

input
```
integrate(1/x**3/(a+b*(d*x+c)**(1/2)),x)
```

output
```
Integral(1/(x**3*(a + b*sqrt(c + d*x))), x)
```

3.638.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.80

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx = -\frac{1}{8}\left(\frac{8\,ab^4\log\left(dx\right)}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6}-\frac{16\,ab^4\log\left(\sqrt{dx+c}b+a\right)}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6}-\frac{\left(3\,b^5c^2+6\,a^2b^3c-a^4b\right)\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{\left(b^6c^4-3\,a^2b^4c^3+3\,a^4b^2c^2-a^6c\right)\sqrt{c}}\right.$$

input
```
integrate(1/x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
-1/8*(8*a*b^4*log(d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6) - 16*
a*b^4*log(sqrt(d*x + c)*b + a)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^
6) - (3*b^5*c^2 + 6*a^2*b^3*c - a^4*b)*log((sqrt(d*x + c) - sqrt(c))/(sqrt
(d*x + c) + sqrt(c)))/((b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*s
qrt(c)) + 2*(4*(d*x + c)*a*b^2*c - 6*a*b^2*c^2 + 2*a^3*c - (3*b^3*c + a^2*
b)*(d*x + c)^(3/2) + (5*b^3*c^2 - a^2*b*c)*sqrt(d*x + c))/(b^4*c^5 - 2*a^2
*b^2*c^4 + a^4*c^3 + (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*(d*x + c)^2 - 2*(b^
4*c^4 - 2*a^2*b^2*c^3 + a^4*c^2)*(d*x + c)))*d^2
```

3.638.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.84

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx = \frac{2\,ab^5d^2\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^7c^3-3\,a^2b^5c^2+3\,a^4b^3c-a^6b}$$
$$-\frac{ab^4d^2\log\left(dx\right)}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6}+\frac{\left(3\,b^5c^2d^2+6\,a^2b^3cd^2-a^4bd^2\right)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{4\left(b^6c^4-3\,a^2b^4c^3+3\,a^4b^2c^2-a^6c\right)\sqrt{-c}}$$
$$+\frac{6\,ab^4c^3d^2-8\,a^3b^2c^2d^2+2\,a^5cd^2+\left(3\,b^5c^2d^2-2\,a^2b^3cd^2-a^4bd^2\right)(dx+c)^{\frac{3}{2}}-4\left(ab^4c^2d^2-a^3b^2cd^2\right)(dx+\cdots}{4\left(b^2c-a^2\right)^3cd^2x^2}$$

input
```
integrate(1/x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

output
```
2*a*b^5*d^2*log(abs(sqrt(d*x + c)*b + a))/(b^7*c^3 - 3*a^2*b^5*c^2 + 3*a^4
*b^3*c - a^6*b) - a*b^4*d^2*log(d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*
c - a^6) + 1/4*(3*b^5*c^2*d^2 + 6*a^2*b^3*c*d^2 - a^4*b*d^2)*arctan(sqrt(d
*x + c)/sqrt(-c))/((b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt(
-c)) + 1/4*(6*a*b^4*c^3*d^2 - 8*a^3*b^2*c^2*d^2 + 2*a^5*c*d^2 + (3*b^5*c^2
*d^2 - 2*a^2*b^3*c*d^2 - a^4*b*d^2)*(d*x + c)^(3/2) - 4*(a*b^4*c^2*d^2 - a
^3*b^2*c*d^2)*(d*x + c) - (5*b^5*c^3*d^2 - 6*a^2*b^3*c^2*d^2 + a^4*b*c*d^2
)*sqrt(d*x + c))/((b^2*c - a^2)^3*c*d^2*x^2)
```

3.638.9 Mupad [B] (verification not implemented)

Time = 19.57 (sec) , antiderivative size = 1094, normalized size of antiderivative = 5.36

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx$$

$$= \frac{\ln\left(\frac{b^5 d^4 (a^2+3 c b^2)^2 \sqrt{c+d x}}{16 c^2 (b^2 c-a^2)^4} - \frac{a b^4 d^4 (-a^4+2 a^2 b^2 c+15 b^4 c^2)}{16 c^2 (b^2 c-a^2)^4} - \frac{b d^2 \sqrt{c^3}\left(\frac{b^2 d^2 \left(3 b^2 c-a^2\right)}{4 c \left(b^2 c-a^2\right)} + \frac{b^2 d^2 \sqrt{c^3}\left(a^2 \sqrt{c+d x}+4 a b c+3 b^2 c \sqrt{c+d x}\right)}{4 c^3 \left(b^2 c-a^2 \right.}\cdots\right.}{\cdots}\right.}{8\,(a^6 \cdots}$$

$$- \frac{\frac{a^3 d^2-3 a b^2 c d^2}{2 (a^4-2 a^2 b^2 c+b^4 c^2)} - \frac{(a^2 b d^2+3 c b^3 d^2) (c+d x)^{3/2}}{4 c (a^4-2 a^2 b^2 c+b^4 c^2)} + \frac{b d^2 (5 b^2 c-a^2) \sqrt{c+d x}}{4 (a^4-2 a^2 b^2 c+b^4 c^2)} + \frac{a b^2 d^2 (c+d x)}{a^4-2 a^2 b^2 c+b^4 c^2}}{(c+d\,x)^2 - 2\,c\,(c+d\,x) + c^2}$$

$$+ \frac{\ln\left(\frac{b^5 d^4 (a^2+3 c b^2)^2 \sqrt{c+d x}}{16 c^2 (b^2 c-a^2)^4} - \frac{a b^4 d^4 (-a^4+2 a^2 b^2 c+15 b^4 c^2)}{16 c^2 (b^2 c-a^2)^4} - \frac{b d^2 \sqrt{c^3}\left(\frac{b^2 d^2 \left(3 b^2 c-a^2\right)}{4 c \left(b^2 c-a^2\right)} + \frac{b^2 d^2 \sqrt{c^3}\left(a^2 \sqrt{c+d x}+4 a b c+3 b^2 c \sqrt{c+d \cdots}\right.}{4 c^3 \left(b^2 c \cdots\right.}\cdots\right.}{\cdots}\right.}{8\,(\cdots}$$

$$+ \frac{2\,a\,b^4\,d^2\,\ln\left(a+b\,\sqrt{c+d\,x}\right)}{\left(b^2\,c-a^2\right)^3}$$

input
```
int(1/(x^3*(a + b*(c + d*x)^(1/2))),x)
```

output
```
(log((b^5*d^4*(3*b^2*c + a^2)^2*(c + d*x)^(1/2))/(16*c^2*(b^2*c - a^2)^4)
- (a*b^4*d^4*(15*b^4*c^2 - a^4 + 2*a^2*b^2*c))/(16*c^2*(b^2*c - a^2)^4) -
(b*d^2*(c^3)^(1/2)*((b^2*d^2*(3*b^2*c - a^2))/(4*c*(b^2*c - a^2)) + (b^2*d
^2*(c^3)^(1/2)*(a^2*(c + d*x)^(1/2) + 4*a*b*c + 3*b^2*c*(c + d*x)^(1/2))*(
3*b^4*c^2 - a^4 + 6*a^2*b^2*c + 8*a*b^3*(c^3)^(1/2)))/(4*c^3*(b^2*c - a^2)
^3) - (a*b^3*d^2*(9*b^2*c - a^2)*(c + d*x)^(1/2))/(2*c*(b^2*c - a^2)^2))*(
3*b^4*c^2 - a^4 + 6*a^2*b^2*c + 8*a*b^3*(c^3)^(1/2)))/(8*c^3*(b^2*c - a^2)
^3))*(8*a*b^4*c^3*d^2 - a^4*b*d^2*(c^3)^(1/2) + 3*b^5*c^2*d^2*(c^3)^(1/2)
+ 6*a^2*b^3*c*d^2*(c^3)^(1/2)))/(8*(a^6*c^3 - b^6*c^6 - 3*a^4*b^2*c^4 + 3*
a^2*b^4*c^5)) - ((a^3*d^2 - 3*a*b^2*c*d^2)/(2*(a^4 + b^4*c^2 - 2*a^2*b^2*c
)) - ((a^2*b*d^2 + 3*b^3*c*d^2)*(c + d*x)^(3/2))/(4*c*(a^4 + b^4*c^2 - 2*a
^2*b^2*c)) + (b*d^2*(5*b^2*c - a^2)*(c + d*x)^(1/2))/(4*(a^4 + b^4*c^2 - 2
*a^2*b^2*c)) + (a*b^2*d^2*(c + d*x))/(a^4 + b^4*c^2 - 2*a^2*b^2*c))/((c +
d*x)^2 - 2*c*(c + d*x) + c^2) + (log((b^5*d^4*(3*b^2*c + a^2)^2*(c + d*x)^
(1/2))/(16*c^2*(b^2*c - a^2)^4) - (a*b^4*d^4*(15*b^4*c^2 - a^4 + 2*a^2*b^2
*c))/(16*c^2*(b^2*c - a^2)^4) - (b*d^2*(c^3)^(1/2)*((b^2*d^2*(3*b^2*c - a^
2))/(4*c*(b^2*c - a^2)) + (b^2*d^2*(c^3)^(1/2)*(a^2*(c + d*x)^(1/2) + 4*a*
b*c + 3*b^2*c*(c + d*x)^(1/2))*(a^4 - 3*b^4*c^2 - 6*a^2*b^2*c + 8*a*b^3*(c
^3)^(1/2)))/(4*c^3*(b^2*c - a^2)^3) - (a*b^3*d^2*(9*b^2*c - a^2)*(c + d*x)
^(1/2))/(2*c*(b^2*c - a^2)^2))*(a^4 - 3*b^4*c^2 - 6*a^2*b^2*c + 8*a*b^3...
```

3.638. $\int \frac{1}{x^3(a+b\sqrt{c+dx})}\,dx$

3.639 $\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$

3.639.1 Optimal result

Integrand size = 19, antiderivative size = 240

$$\begin{aligned}\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx &= \frac{(5a^4-9a^2b^2c+3b^4c^2)\,x}{b^6d^3}-\frac{12a(a^2-b^2c)^2\sqrt{c+dx}}{b^7d^4}\\ &\quad-\frac{4a(2a^2-3b^2c)\,(c+dx)^{3/2}}{3b^5d^4}+\frac{3(a^2-b^2c)\,(c+dx)^2}{2b^4d^4}\\ &\quad-\frac{4a(c+dx)^{5/2}}{5b^3d^4}+\frac{(c+dx)^3}{3b^2d^4}+\frac{2a(a^2-b^2c)^3}{b^8d^4\left(a+b\sqrt{c+dx}\right)}\\ &\quad+\frac{2(a^2-b^2c)^2\,(7a^2-b^2c)\log\left(a+b\sqrt{c+dx}\right)}{b^8d^4}\end{aligned}$$

output
```
(3*b^4*c^2-9*a^2*b^2*c+5*a^4)*x/b^6/d^3-4/3*a*(-3*b^2*c+2*a^2)*(d*x+c)^(3/
2)/b^5/d^4+3/2*(-b^2*c+a^2)*(d*x+c)^2/b^4/d^4-4/5*a*(d*x+c)^(5/2)/b^3/d^4+
1/3*(d*x+c)^3/b^2/d^4+2*(-b^2*c+a^2)^2*(-b^2*c+7*a^2)*ln(a+b*(d*x+c)^(1/2)
)/b^8/d^4-12*a*(-b^2*c+a^2)^2*(d*x+c)^(1/2)/b^7/d^4+2*a*(-b^2*c+a^2)^3/b^8
/d^4/(a+b*(d*x+c)^(1/2))
```

3.639.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.18

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$= \frac{60a^7-360a^6b\sqrt{c+dx}-30a^5b^2(13c+7dx)+10a^4b^3\sqrt{c+dx}(79c+7dx)-3a^2b^5\sqrt{c+dx}(163c^2+36cdx\ldots}{\ldots}$$

input
```
Integrate[x^3/(a + b*Sqrt[c + d*x])^2,x]
```

output
```
(60*a^7 - 360*a^6*b*Sqrt[c + d*x] - 30*a^5*b^2*(13*c + 7*d*x) + 10*a^4*b^3
*Sqrt[c + d*x]*(79*c + 7*d*x) - 3*a^2*b^5*Sqrt[c + d*x]*(163*c^2 + 36*c*d*
x - 7*d^2*x^2) + 5*a^3*b^4*(119*c^2 + 76*c*d*x - 7*d^2*x^2) + 5*b^7*Sqrt[c
 + d*x]*(11*c^3 + 6*c^2*d*x - 3*c*d^2*x^2 + 2*d^3*x^3) - a*b^6*(269*c^3 +
162*c^2*d*x - 33*c*d^2*x^2 + 14*d^3*x^3) + 60*(a^2 - b^2*c)^2*(7*a^2 - b^2
*c)*(a + b*Sqrt[c + d*x])*Log[a + b*Sqrt[c + d*x]])/(30*b^8*d^4*(a + b*Sqr
t[c + d*x]))
```

3.639.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$\Big\downarrow \; 896$$
$$\frac{\int \frac{d^3x^3}{\left(a+b\sqrt{c+dx}\right)^2}d(c+dx)}{d^4}$$
$$\Big\downarrow \; 25$$
$$-\frac{\int -\frac{d^3x^3}{\left(a+b\sqrt{c+dx}\right)^2}d(c+dx)}{d^4}$$
$$\Big\downarrow \; 1732$$

$$-\frac{2\int -\frac{d^3x^3\sqrt{c+dx}}{\left(a+b\sqrt{c+dx}\right)^2}d\sqrt{c+dx}}{d^4}$$

$$\Big\downarrow 522$$

$$-\frac{2\int \left(\frac{a\left(a^2-b^2c\right)^3}{b^7\left(a+b\sqrt{c+dx}\right)^2}+\frac{6a\left(a^2-b^2c\right)^2}{b^7}-\frac{(c+dx)^{5/2}}{b^2}+\frac{2a(c+dx)^2}{b^3}+\frac{3\left(b^2c-a^2\right)(c+dx)^{3/2}}{b^4}+\frac{2a\left(2a^2-3b^2c\right)(c+dx)}{b^5}-\frac{\left(5a^4-9b^2ca^2+3\right.}{b^6}\right.}{d^4}$$

$$\Big\downarrow 2009$$

$$-\frac{2\left(-\frac{a\left(a^2-b^2c\right)^3}{b^8\left(a+b\sqrt{c+dx}\right)}-\frac{\left(7a^2-b^2c\right)\left(a^2-b^2c\right)^2\log\left(a+b\sqrt{c+dx}\right)}{b^8}+\frac{6a\left(a^2-b^2c\right)^2\sqrt{c+dx}}{b^7}+\frac{2a\left(2a^2-3b^2c\right)(c+dx)^{3/2}}{3b^5}-\frac{3\left(a^2-b^2c\right)(c+dx)^2}{4b^4}\right.}{d^4}$$

input
```
Int[x^3/(a + b*Sqrt[c + d*x])^2,x]
```

output
```
(-2*((6*a*(a^2 - b^2*c)^2*Sqrt[c + d*x])/b^7 - ((5*a^4 - 9*a^2*b^2*c + 3*b
^4*c^2)*(c + d*x))/(2*b^6) + (2*a*(2*a^2 - 3*b^2*c)*(c + d*x)^(3/2))/(3*b^
5) - (3*(a^2 - b^2*c)*(c + d*x)^2)/(4*b^4) + (2*a*(c + d*x)^(5/2))/(5*b^3)
 - (c + d*x)^3/(6*b^2) - (a*(a^2 - b^2*c)^3)/(b^8*(a + b*Sqrt[c + d*x])) -
 ((a^2 - b^2*c)^2*(7*a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])/b^8))/d^4
```

3.639.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.639.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.15

method	result
derivativedivides	$-\frac{2\left(-\frac{(dx+c)^3 b^5}{6}+\frac{2a(dx+c)^{\frac{5}{2}} b^4}{5}+\frac{3b^5 c(dx+c)^2}{4}-\frac{3a^2b^3(dx+c)^2}{4}-2a\,b^4c(dx+c)^{\frac{3}{2}}-\frac{3b^5c^2(dx+c)}{2}+\frac{4a^3b^2(dx+c)^{\frac{3}{2}}}{3}+\frac{9a^2b^3c(dx+c)}{2}\right.}{b^7}$
default	$-\frac{2\left(-\frac{(dx+c)^3 b^5}{6}+\frac{2a(dx+c)^{\frac{5}{2}} b^4}{5}+\frac{3b^5 c(dx+c)^2}{4}-\frac{3a^2b^3(dx+c)^2}{4}-2a\,b^4c(dx+c)^{\frac{3}{2}}-\frac{3b^5c^2(dx+c)}{2}+\frac{4a^3b^2(dx+c)^{\frac{3}{2}}}{3}+\frac{9a^2b^3c(dx+c)}{2}\right.}{b^7}$

input
```
int(x^3/(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
2/d^4*(-1/b^7*(-1/6*(d*x+c)^3*b^5+2/5*a*(d*x+c)^(5/2)*b^4+3/4*b^5*c*(d*x+c
)^2-3/4*a^2*b^3*(d*x+c)^2-2*a*b^4*c*(d*x+c)^(3/2)-3/2*b^5*c^2*(d*x+c)+4/3*
a^3*b^2*(d*x+c)^(3/2)+9/2*a^2*b^3*c*(d*x+c)+6*a*c^2*b^4*(d*x+c)^(1/2)-5/2*
a^4*b*(d*x+c)-12*a^3*c*b^2*(d*x+c)^(1/2)+6*a^5*(d*x+c)^(1/2))+1/b^8*(-b^6*
c^3+9*a^2*b^4*c^2-15*a^4*b^2*c+7*a^6)*ln(a+b*(d*x+c)^(1/2))+a*(-b^6*c^3+3*
a^2*b^4*c^2-3*a^4*b^2*c+a^6)/b^8/(a+b*(d*x+c)^(1/2)))
```

3.639.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.63

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$=\frac{10\,b^8d^4x^4+55\,b^8c^4-220\,a^2b^6c^3+195\,a^4b^4c^2+30\,a^6b^2c-60\,a^8-5\,(b^8c-7\,a^2b^6)d^3x^3+15\,(b^8c^2-8\,a^2b^6}{}$$

input
```
integrate(x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/30*(10*b^8*d^4*x^4 + 55*b^8*c^4 - 220*a^2*b^6*c^3 + 195*a^4*b^4*c^2 + 30
*a^6*b^2*c - 60*a^8 - 5*(b^8*c - 7*a^2*b^6)*d^3*x^3 + 15*(b^8*c^2 - 8*a^2*
b^6*c + 7*a^4*b^4)*d^2*x^2 + 5*(17*b^8*c^3 - 87*a^2*b^6*c^2 + 96*a^4*b^4*c
 - 30*a^6*b^2)*d*x - 60*(b^8*c^4 - 10*a^2*b^6*c^3 + 24*a^4*b^4*c^2 - 22*a^
6*b^2*c + 7*a^8 + (b^8*c^3 - 9*a^2*b^6*c^2 + 15*a^4*b^4*c - 7*a^6*b^2)*d*x
)*log(sqrt(d*x + c)*b + a) - 4*(6*a*b^7*d^3*x^3 + 81*a*b^7*c^3 - 271*a^3*b
^5*c^2 + 295*a^5*b^3*c - 105*a^7*b - 2*(6*a*b^7*c - 7*a^3*b^5)*d^2*x^2 + 2
*(24*a*b^7*c^2 - 61*a^3*b^5*c + 35*a^5*b^3)*d*x)*sqrt(d*x + c))/(b^10*d^5*
x + (b^10*c - a^2*b^8)*d^4)
```

3.639.6 Sympy [A] (verification not implemented)

Time = 5.69 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.11

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$$

$$= \begin{cases} \frac{2\left(-\frac{2a(c+dx)^{\frac{5}{2}}}{5b^3} - \frac{a\left(a^2-b^2c\right)^3\left(\begin{cases}\frac{\sqrt{c+dx}}{a^2} & \text{for } b=0 \\ -\frac{1}{b(a+b\sqrt{c+dx})} & \text{otherwise}\end{cases}\right)}{b^7} + \frac{(c+dx)^3}{6b^2} + \frac{\left(3a^2-3b^2c\right)(c+dx)^2}{4b^4} + \frac{\left(-4a^3+6ab^2c\right)(c+dx)^{\frac{3}{2}}}{3b^5} + \frac{(c+dx)\left(5a^4-9a\right.}{2b^6}\cdots\right)}{d^4} \\ \frac{x^4}{4\left(a+b\sqrt{c}\right)^2} \end{cases}$$

input
```
integrate(x**3/(a+b*(d*x+c)**(1/2))**2,x)
```

output
```
Piecewise((2*(-2*a*(c + d*x)**(5/2)/(5*b**3) - a*(a**2 - b**2*c)**3*Piecew
ise((sqrt(c + d*x)/a**2, Eq(b, 0)), (-1/(b*(a + b*sqrt(c + d*x))), True))/
b**7 + (c + d*x)**3/(6*b**2) + (3*a**2 - 3*b**2*c)*(c + d*x)**2/(4*b**4) +
 (-4*a**3 + 6*a*b**2*c)*(c + d*x)**(3/2)/(3*b**5) + (c + d*x)*(5*a**4 - 9*
a**2*b**2*c + 3*b**4*c**2)/(2*b**6) + (a**2 - b**2*c)**2*(7*a**2 - b**2*c)
*Piecewise((sqrt(c + d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b, True)
)/b**7 + sqrt(c + d*x)*(-6*a**5 + 12*a**3*b**2*c - 6*a*b**4*c**2)/b**7)/d*
*4, Ne(d, 0)), (x**4/(4*(a + b*sqrt(c))**2), True))
```

3.639.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.05

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx =$$

$$-\frac{\frac{60\,(ab^6c^3-3\,a^3b^4c^2+3\,a^5b^2c-a^7)}{\sqrt{dx+c}b^9+ab^8} - \frac{10\,(dx+c)^3b^5-24\,(dx+c)^{\frac{5}{2}}ab^4-45\,(b^5c-a^2b^3)(dx+c)^2+40\,(3\,ab^4c-2\,a^3b^2)(dx+c)^{\frac{3}{2}}+30\,(3\,b^5c^2-9\,a^2}{b^7}}{30\,d^4}$$

input
```
integrate(x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
-1/30*(60*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)/(sqrt(d*x + c)*b
^9 + a*b^8) - (10*(d*x + c)^3*b^5 - 24*(d*x + c)^(5/2)*a*b^4 - 45*(b^5*c -
 a^2*b^3)*(d*x + c)^2 + 40*(3*a*b^4*c - 2*a^3*b^2)*(d*x + c)^(3/2) + 30*(3
*b^5*c^2 - 9*a^2*b^3*c + 5*a^4*b)*(d*x + c) - 360*(a*b^4*c^2 - 2*a^3*b^2*c
 + a^5)*sqrt(d*x + c))/b^7 + 60*(b^6*c^3 - 9*a^2*b^4*c^2 + 15*a^4*b^2*c -
7*a^6)*log(sqrt(d*x + c)*b + a)/b^8)/d^4
```

3.639.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.35

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = -\frac{2\,(b^6c^3-9\,a^2b^4c^2+15\,a^4b^2c-7\,a^6)\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^8d^4}$$
$$-\frac{2\,(ab^6c^3-3\,a^3b^4c^2+3\,a^5b^2c-a^7)}{\left(\sqrt{dx+c}b+a\right)b^8d^4}$$
$$+\frac{10\,(dx+c)^3b^{10}d^{20}-45\,(dx+c)^2b^{10}cd^{20}+90\,(dx+c)b^{10}c^2d^{20}-24\,(dx+c)^{\frac{5}{2}}ab^9d^{20}+120\,(dx+c)^{\frac{3}{2}}ab^9}{}$$

input
```
integrate(x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
-2*(b^6*c^3 - 9*a^2*b^4*c^2 + 15*a^4*b^2*c - 7*a^6)*log(abs(sqrt(d*x + c)*
b + a))/(b^8*d^4) - 2*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)/((sq
rt(d*x + c)*b + a)*b^8*d^4) + 1/30*(10*(d*x + c)^3*b^10*d^20 - 45*(d*x + c
)^2*b^10*c*d^20 + 90*(d*x + c)*b^10*c^2*d^20 - 24*(d*x + c)^(5/2)*a*b^9*d^
20 + 120*(d*x + c)^(3/2)*a*b^9*c*d^20 - 360*sqrt(d*x + c)*a*b^9*c^2*d^20 +
 45*(d*x + c)^2*a^2*b^8*d^20 - 270*(d*x + c)*a^2*b^8*c*d^20 - 80*(d*x + c)
^(3/2)*a^3*b^7*d^20 + 720*sqrt(d*x + c)*a^3*b^7*c*d^20 + 150*(d*x + c)*a^4
*b^6*d^20 - 360*sqrt(d*x + c)*a^5*b^5*d^20)/(b^12*d^24)
```

3.639.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.92

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \left(\frac{4\,a^3}{3\,b^5\,d^4}+\frac{2\,a\,\left(\frac{6\,c}{b^2\,d^4}-\frac{6\,a^2}{b^4\,d^4}\right)}{3\,b}\right)\,{\left(c+d\,x\right)}^{3/2} - \left(\frac{3\,c}{2\,b^2\,d^4}-\frac{3\,a^2}{2\,b^4\,d^4}\right)\,{\left(c+d\,x\right)}^2 - \left(\frac{2\,a\,\left(\frac{a^2\,\left(\frac{6\,c}{b^2\,d^4}-\frac{6\,a^2}{b^4\,d^4}\right)}{b^2}-\frac{2\,a\,\left(\frac{4\,a^3}{b^5\,d^4}+\frac{2\,a\,\left(\frac{6\,c}{b^2\,d^4}-\frac{6\,a^2}{b^4\,d^4}\right)}{b}\right)}{b}+\frac{6\,c^2}{b^2\,d^4}\right)}{b} + \frac{a^2\,\left(\frac{4\,a^3}{b^5\,d^4}+\frac{2\,a\,\left(\frac{6\,c}{b^2\,d^4}-\frac{6\,a^2}{b^4\,d^4}\right)}{b}\right)}{b^2}\right)\,\sqrt{c+d\,x} + \frac{{\left(c+d\,x\right)}^3}{3\,b^2\,d^4}+\frac{2\,\left(a^7-3\,a^5\,b^2\,c+3\,a^3\,b^4\,c^2-a\,b^6\,c^3\right)}{b\,\left(b^8\,d^4\,\sqrt{c+d\,x}+a\,b^7\,d^4\right)} + d\,x\,\left(\frac{a^2\,\left(\frac{6\,c}{b^2\,d^4}-\frac{6\,a^2}{b^4\,d^4}\right)}{2\,b^2}-\frac{a\,\left(\frac{4\,a^3}{b^5\,d^4}+\frac{2\,a\,\left(\frac{6\,c}{b^2\,d^4}-\frac{6\,a^2}{b^4\,d^4}\right)}{b}\right)}{b}+\frac{3\,c^2}{b^2\,d^4}\right) + \frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(14\,a^6-30\,a^4\,b^2\,c+18\,a^2\,b^4\,c^2-2\,b^6\,c^3\right)}{b^8\,d^4} - \frac{4\,a\,{\left(c+d\,x\right)}^{5/2}}{5\,b^3\,d^4}$$

input
```
int(x^3/(a + b*(c + d*x)^(1/2))^2,x)
```

output
```
((4*a^3)/(3*b^5*d^4) + (2*a*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/(3*b))*
(c + d*x)^(3/2) - ((3*c)/(2*b^2*d^4) - (3*a^2)/(2*b^4*d^4))*(c + d*x)^2 -
((2*a*((a^2*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/b^2 - (2*a*((4*a^3)/(b^
5*d^4) + (2*a*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/b))/b + (6*c^2)/(b^2*
d^4)))/b + (a^2*((4*a^3)/(b^5*d^4) + (2*a*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*
d^4)))/b))/b^2)*(c + d*x)^(1/2) + (c + d*x)^3/(3*b^2*d^4) + (2*(a^7 - 3*a^
5*b^2*c - a*b^6*c^3 + 3*a^3*b^4*c^2))/(b*(b^8*d^4*(c + d*x)^(1/2) + a*b^7*
d^4)) + d*x*((a^2*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/(2*b^2) - (a*((4*
a^3)/(b^5*d^4) + (2*a*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/b))/b + (3*c^
2)/(b^2*d^4)) + (log(a + b*(c + d*x)^(1/2))*(14*a^6 - 2*b^6*c^3 - 30*a^4*b
^2*c + 18*a^2*b^4*c^2))/(b^8*d^4) - (4*a*(c + d*x)^(5/2))/(5*b^3*d^4)
```

3.639. $\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$

3.640 $\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2} dx$

3.640.1 Optimal result

Integrand size = 19, antiderivative size = 166

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2} dx = \frac{(3a^2-2b^2c)\,x}{b^4d^2} - \frac{8a(a^2-b^2c)\sqrt{c+dx}}{b^5d^3} - \frac{4a(c+dx)^{3/2}}{3b^3d^3} + \frac{(c+dx)^2}{2b^2d^3} + \frac{2a(a^2-b^2c)^2}{b^6d^3\left(a+b\sqrt{c+dx}\right)} + \frac{2(5a^4-6a^2b^2c+b^4c^2)\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}$$

output
```
(-2*b^2*c+3*a^2)*x/b^4/d^2-4/3*a*(d*x+c)^(3/2)/b^3/d^3+1/2*(d*x+c)^2/b^2/d
^3+2*(b^4*c^2-6*a^2*b^2*c+5*a^4)*ln(a+b*(d*x+c)^(1/2))/b^6/d^3-8*a*(-b^2*c
+a^2)*(d*x+c)^(1/2)/b^5/d^3+2*a*(-b^2*c+a^2)^2/b^6/d^3/(a+b*(d*x+c)^(1/2))
```

3.640.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.17

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2} dx = \frac{12a^5-48a^4b\sqrt{c+dx}-6a^3b^2(9c+5dx)+2a^2b^3\sqrt{c+dx}(29c+5dx)+ab^4(43c^2+26cdx-5d^2x^2)+3b^5}{6b^6d^3\left(a+b\sqrt{c+\right.}}$$

input
```
Integrate[x^2/(a + b*Sqrt[c + d*x])^2,x]
```

output
```
(12*a^5 - 48*a^4*b*Sqrt[c + d*x] - 6*a^3*b^2*(9*c + 5*d*x) + 2*a^2*b^3*Sqr
t[c + d*x]*(29*c + 5*d*x) + a*b^4*(43*c^2 + 26*c*d*x - 5*d^2*x^2) + 3*b^5*
Sqrt[c + d*x]*(-3*c^2 - 2*c*d*x + d^2*x^2) + 12*(5*a^4 - 6*a^2*b^2*c + b^4
*c^2)*(a + b*Sqrt[c + d*x])*Log[a + b*Sqrt[c + d*x]])/(6*b^6*d^3*(a + b*Sq
rt[c + d*x]))
```

3.640.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {896, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}dx$$

$$\Big\downarrow \text{ 896}$$

$$\frac{\int \frac{d^2x^2}{\left(a+b\sqrt{c+dx}\right)^2}d(c+dx)}{d^3}$$

$$\Big\downarrow \text{ 1732}$$

$$\frac{2\int \frac{d^2x^2\sqrt{c+dx}}{\left(a+b\sqrt{c+dx}\right)^2}d\sqrt{c+dx}}{d^3}$$

$$\Big\downarrow \text{ 522}$$

$$\frac{2\int \left(-\frac{a\left(a^2-b^2c\right)^2}{b^5\left(a+b\sqrt{c+dx}\right)^2}-\frac{4a\left(a^2-b^2c\right)}{b^5}+\frac{(c+dx)^{3/2}}{b^2}-\frac{2a(c+dx)}{b^3}-\frac{\left(2b^2c-3a^2\right)\sqrt{c+dx}}{b^4}+\frac{5a^4-6b^2ca^2+b^4c^2}{b^5\left(a+b\sqrt{c+dx}\right)}\right)d\sqrt{c+dx}}{d^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2\left(\frac{a\left(a^2-b^2c\right)^2}{b^6\left(a+b\sqrt{c+dx}\right)}-\frac{4a\left(a^2-b^2c\right)\sqrt{c+dx}}{b^5}+\frac{\left(3a^2-2b^2c\right)(c+dx)}{2b^4}+\frac{\left(5a^4-6a^2b^2c+b^4c^2\right)\log\left(a+b\sqrt{c+dx}\right)}{b^6}-\frac{2a(c+dx)^{3/2}}{3b^3}+\frac{(c+dx)^2}{4b^2}\right)}{d^3}$$

3.640. $\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}dx$

input
```
Int[x^2/(a + b*Sqrt[c + d*x])^2,x]
```

output
```
(2*((-4*a*(a^2 - b^2*c)*Sqrt[c + d*x])/b^5 + ((3*a^2 - 2*b^2*c)*(c + d*x))
/(2*b^4) - (2*a*(c + d*x)^(3/2))/(3*b^3) + (c + d*x)^2/(4*b^2) + (a*(a^2 -
 b^2*c)^2)/(b^6*(a + b*Sqrt[c + d*x])) + ((5*a^4 - 6*a^2*b^2*c + b^4*c^2)*
Log[a + b*Sqrt[c + d*x]])/b^6))/d^3
```

3.640.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.640.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{(dx+c)^2 b^3}{4}+\frac{2a(dx+c)^{\frac{3}{2}} b^2}{3}+b^3 c(dx+c)-\frac{3a^2 b(dx+c)}{2}-4ac\,b^2\sqrt{dx+c}+4a^3\sqrt{dx+c}\right)}{b^5}+\frac{2a\left(b^4c^2-2a^2b^2c+a^4\right)}{b^6\left(a+b\sqrt{dx+c}\right)}+\frac{2\left(b^4c^2-6a^2b^{\cdots}\right.}{\cdots}}{d^3}$
default	$\frac{-\frac{2\left(-\frac{(dx+c)^2 b^3}{4}+\frac{2a(dx+c)^{\frac{3}{2}} b^2}{3}+b^3 c(dx+c)-\frac{3a^2 b(dx+c)}{2}-4ac\,b^2\sqrt{dx+c}+4a^3\sqrt{dx+c}\right)}{b^5}+\frac{2a\left(b^4c^2-2a^2b^2c+a^4\right)}{b^6\left(a+b\sqrt{dx+c}\right)}+\frac{2\left(b^4c^2-6a^2b^{\cdots}\right.}{\cdots}}{d^3}$

input
```
int(x^2/(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
2/d^3*(-1/b^5*(-1/4*(d*x+c)^2*b^3+2/3*a*(d*x+c)^(3/2)*b^2+b^3*c*(d*x+c)-3/
2*a^2*b*(d*x+c)-4*a*c*b^2*(d*x+c)^(1/2)+4*a^3*(d*x+c)^(1/2))+a*(b^4*c^2-2*
a^2*b^2*c+a^4)/b^6/(a+b*(d*x+c)^(1/2))+1/b^6*(b^4*c^2-6*a^2*b^2*c+5*a^4)*l
n(a+b*(d*x+c)^(1/2)))
```

3.640.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.62

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$= \frac{3\,b^6d^3x^3 - 9\,b^6c^3 + 15\,a^2b^4c^2 + 6\,a^4b^2c - 12\,a^6 - 3\,(b^6c - 5\,a^2b^4)d^2x^2 - 3\,(5\,b^6c^2 - 14\,a^2b^4c + 6\,a^4b^2)dx + }{}$$

input
```
integrate(x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/6*(3*b^6*d^3*x^3 - 9*b^6*c^3 + 15*a^2*b^4*c^2 + 6*a^4*b^2*c - 12*a^6 - 3
*(b^6*c - 5*a^2*b^4)*d^2*x^2 - 3*(5*b^6*c^2 - 14*a^2*b^4*c + 6*a^4*b^2)*d*
x + 12*(b^6*c^3 - 7*a^2*b^4*c^2 + 11*a^4*b^2*c - 5*a^6 + (b^6*c^2 - 6*a^2*
b^4*c + 5*a^4*b^2)*d*x)*log(sqrt(d*x + c)*b + a) - 4*(2*a*b^5*d^2*x^2 - 13
*a*b^5*c^2 + 28*a^3*b^3*c - 15*a^5*b - 2*(4*a*b^5*c - 5*a^3*b^3)*d*x)*sqrt
(d*x + c))/(b^8*d^4*x + (b^8*c - a^2*b^6)*d^3)
```

3.640.6 Sympy [A] (verification not implemented)

Time = 4.54 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.14

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$= \begin{cases} \frac{2\left(-\frac{2a(c+dx)^{\frac{3}{2}}}{3b^3}-\frac{a\left(a^2-b^2c\right)^2\left(\begin{cases}\frac{\sqrt{c+dx}}{a^2} & \text{for } b=0\\ -\frac{1}{b\left(a+b\sqrt{c+dx}\right)} & \text{otherwise}\end{cases}\right)}{b^5}+\frac{(c+dx)^2}{4b^2}+\frac{\left(3a^2-2b^2c\right)(c+dx)}{2b^4}+\frac{\left(a^2-b^2c\right)\left(5a^2-b^2c\right)\left(\begin{cases}\frac{\sqrt{c+dx}}{a} \\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b}\end{cases}\right.}{b^5}\right.}{d^3} \\ \frac{x^3}{3\left(a+b\sqrt{c}\right)^2} \end{cases}$$

input
```
integrate(x**2/(a+b*(d*x+c)**(1/2))**2,x)
```

output
```
Piecewise((2*(-2*a*(c + d*x)**(3/2)/(3*b**3) - a*(a**2 - b**2*c)**2*Piecew
ise((sqrt(c + d*x)/a**2, Eq(b, 0)), (-1/(b*(a + b*sqrt(c + d*x))), True))/
b**5 + (c + d*x)**2/(4*b**2) + (3*a**2 - 2*b**2*c)*(c + d*x)/(2*b**4) + (a
**2 - b**2*c)*(5*a**2 - b**2*c)*Piecewise((sqrt(c + d*x)/a, Eq(b, 0)), (lo
g(a + b*sqrt(c + d*x))/b, True))/b**5 + (-4*a**3 + 4*a*b**2*c)*sqrt(c + d*
x)/b**5)/d**3, Ne(d, 0)), (x**3/(3*(a + b*sqrt(c))**2), True))
```

3.640.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$= \frac{\frac{12\left(ab^4c^2-2a^3b^2c+a^5\right)}{\sqrt{dx+c}\,b^7+ab^6}+\frac{3(dx+c)^2b^3-8(dx+c)^{\frac{3}{2}}ab^2-6\left(2b^3c-3a^2b\right)(dx+c)+48\left(ab^2c-a^3\right)\sqrt{dx+c}}{b^5}+\frac{12\left(b^4c^2-6a^2b^2c+5a^4\right)\log\left(\sqrt{dx+c}\right.}{b^6}}{6\,d^3}$$

input
```
integrate(x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
1/6*(12*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)/(sqrt(d*x + c)*b^7 + a*b^6) + (3*(
d*x + c)^2*b^3 - 8*(d*x + c)^(3/2)*a*b^2 - 6*(2*b^3*c - 3*a^2*b)*(d*x + c)
 + 48*(a*b^2*c - a^3)*sqrt(d*x + c))/b^5 + 12*(b^4*c^2 - 6*a^2*b^2*c + 5*a
^4)*log(sqrt(d*x + c)*b + a)/b^6)/d^3
```

3.640.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.15

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$= \frac{2\left(b^4c^2-6\,a^2b^2c+5\,a^4\right)\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^6d^3}+\frac{2\left(ab^4c^2-2\,a^3b^2c+a^5\right)}{\left(\sqrt{dx+c}b+a\right)b^6d^3}$$
$$+\frac{3\,(dx+c)^2b^6d^9-12\,(dx+c)b^6cd^9-8\,(dx+c)^{\frac{3}{2}}ab^5d^9+48\,\sqrt{dx+c}ab^5cd^9+18\,(dx+c)a^2b^4d^9-48\,\sqrt{}}{6\,b^8d^{12}}$$

input
```
integrate(x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
2*(b^4*c^2 - 6*a^2*b^2*c + 5*a^4)*log(abs(sqrt(d*x + c)*b + a))/(b^6*d^3)
+ 2*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)/((sqrt(d*x + c)*b + a)*b^6*d^3) + 1/6*
(3*(d*x + c)^2*b^6*d^9 - 12*(d*x + c)*b^6*c*d^9 - 8*(d*x + c)^(3/2)*a*b^5*
d^9 + 48*sqrt(d*x + c)*a*b^5*c*d^9 + 18*(d*x + c)*a^2*b^4*d^9 - 48*sqrt(d*
x + c)*a^3*b^3*d^9)/(b^8*d^12)
```

3.640.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.19

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \left(\frac{4\,a^3}{b^5\,d^3}+\frac{2\,a\left(\frac{4\,c}{b^2\,d^3}-\frac{6\,a^2}{b^4\,d^3}\right)}{b}\right)\sqrt{c+d\,x}$$
$$+\frac{2\left(a^5-2\,a^3\,b^2\,c+a\,b^4\,c^2\right)}{b\left(b^6\,d^3\,\sqrt{c+d\,x}+a\,b^5\,d^3\right)}+\frac{\left(c+d\,x\right)^2}{2\,b^2\,d^3}$$
$$-d\,x\left(\frac{2\,c}{b^2\,d^3}-\frac{3\,a^2}{b^4\,d^3}\right)-\frac{4\,a\,{\left(c+d\,x\right)}^{3/2}}{3\,b^3\,d^3}$$
$$+\frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(10\,a^4-12\,a^2\,b^2\,c+2\,b^4\,c^2\right)}{b^6\,d^3}$$

input
```
int(x^2/(a + b*(c + d*x)^(1/2))^2,x)
```

output
```
((4*a^3)/(b^5*d^3) + (2*a*((4*c)/(b^2*d^3) - (6*a^2)/(b^4*d^3)))/b)*(c + d
*x)^(1/2) + (2*(a^5 - 2*a^3*b^2*c + a*b^4*c^2))/(b*(b^6*d^3*(c + d*x)^(1/2
) + a*b^5*d^3)) + (c + d*x)^2/(2*b^2*d^3) - d*x*((2*c)/(b^2*d^3) - (3*a^2)
/(b^4*d^3)) - (4*a*(c + d*x)^(3/2))/(3*b^3*d^3) + (log(a + b*(c + d*x)^(1/
2))*(10*a^4 + 2*b^4*c^2 - 12*a^2*b^2*c))/(b^6*d^3)
```

3.640. $\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$

3.641 $\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$

3.641.1 Optimal result

Integrand size = 17, antiderivative size = 95

$$\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx=\frac{x}{b^2d}-\frac{4a\sqrt{c+dx}}{b^3d^2}+\frac{2a(a^2-b^2c)}{b^4d^2\left(a+b\sqrt{c+dx}\right)}+\frac{2(3a^2-b^2c)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}$$

output
```
x/b^2/d+2*(-b^2*c+3*a^2)*ln(a+b*(d*x+c)^(1/2))/b^4/d^2-4*a*(d*x+c)^(1/2)/b
^3/d^2+2*a*(-b^2*c+a^2)/b^4/d^2/(a+b*(d*x+c)^(1/2))
```

3.641.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.14

$$\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx=\frac{2a^3-2ab^2c-4a^2b\sqrt{c+dx}-3ab^2(c+dx)+b^3(c+dx)^{3/2}}{b^4d^2\left(a+b\sqrt{c+dx}\right)}-\frac{2(-3a^2+b^2c)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}$$

input
```
Integrate[x/(a + b*Sqrt[c + d*x])^2,x]
```

output
```
(2*a^3 - 2*a*b^2*c - 4*a^2*b*Sqrt[c + d*x] - 3*a*b^2*(c + d*x) + b^3*(c +
d*x)^(3/2))/(b^4*d^2*(a + b*Sqrt[c + d*x])) - (2*(-3*a^2 + b^2*c)*Log[a +
b*Sqrt[c + d*x]])/(b^4*d^2)
```

3.641.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int \frac{dx}{\left(a+b\sqrt{c+dx}\right)^2}d(c+dx)}{d^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{dx}{\left(a+b\sqrt{c+dx}\right)^2}d(c+dx)}{d^2}$$

$$\Big\downarrow \text{1732}$$

$$-\frac{2\int -\frac{dx\sqrt{c+dx}}{\left(a+b\sqrt{c+dx}\right)^2}d\sqrt{c+dx}}{d^2}$$

$$\Big\downarrow \text{522}$$

$$-\frac{2\int \left(\frac{2a}{b^3}-\frac{\sqrt{c+dx}}{b^2}+\frac{b^2c-3a^2}{b^3\left(a+b\sqrt{c+dx}\right)}+\frac{a^3-ab^2c}{b^3\left(a+b\sqrt{c+dx}\right)^2}\right)d\sqrt{c+dx}}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(-\frac{a(a^2-b^2c)}{b^4\left(a+b\sqrt{c+dx}\right)}-\frac{(3a^2-b^2c)\log\left(a+b\sqrt{c+dx}\right)}{b^4}+\frac{2a\sqrt{c+dx}}{b^3}-\frac{c+dx}{2b^2}\right)}{d^2}$$

input
```
Int[x/(a + b*Sqrt[c + d*x])^2,x]
```

output
```
(-2*((2*a*Sqrt[c + d*x])/b^3 - (c + d*x)/(2*b^2) - (a*(a^2 - b^2*c))/(b^4*
(a + b*Sqrt[c + d*x])) - ((3*a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])/b^4))/
d^2
```

3.641.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.641.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{-\frac{2\left(-\frac{(dx+c)b}{2}+2a\sqrt{dx+c}\right)}{b^{3}}+\frac{2\left(-b^{2}c+3a^{2}\right)\ln\left(a+b\sqrt{dx+c}\right)}{b^{4}}+\frac{2a\left(-b^{2}c+a^{2}\right)}{b^{4}\left(a+b\sqrt{dx+c}\right)}}{d^{2}}$	87
default	$\frac{-\frac{2\left(-\frac{(dx+c)b}{2}+2a\sqrt{dx+c}\right)}{b^{3}}+\frac{2\left(-b^{2}c+3a^{2}\right)\ln\left(a+b\sqrt{dx+c}\right)}{b^{4}}+\frac{2a\left(-b^{2}c+a^{2}\right)}{b^{4}\left(a+b\sqrt{dx+c}\right)}}{d^{2}}$	87

input
```
int(x/(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
2/d^2*(-1/b^3*(-1/2*(d*x+c)*b+2*a*(d*x+c)^(1/2))+1/b^4*(-b^2*c+3*a^2)*ln(a
+b*(d*x+c)^(1/2))+a*(-b^2*c+a^2)/b^4/(a+b*(d*x+c)^(1/2)))
```

3.641.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.72

$$\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{b^4d^2x^2+b^4c^2+a^2b^2c-2\,a^4+(2\,b^4c-a^2b^2)dx-2\,(b^4c^2-4\,a^2b^2c+3\,a^4+(b^4c-3\,a^2b^2)dx)\log\left(\sqrt{dx+\cdots}\right.}{b^6d^3x+(b^6c-a^2b^4)d^2}$$

input
```
integrate(x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
(b^4*d^2*x^2 + b^4*c^2 + a^2*b^2*c - 2*a^4 + (2*b^4*c - a^2*b^2)*d*x - 2*(
b^4*c^2 - 4*a^2*b^2*c + 3*a^4 + (b^4*c - 3*a^2*b^2)*d*x)*log(sqrt(d*x + c)
*b + a) - 2*(2*a*b^3*d*x + 3*a*b^3*c - 3*a^3*b)*sqrt(d*x + c))/(b^6*d^3*x
+ (b^6*c - a^2*b^4)*d^2)
```

3.641.6 Sympy [A] (verification not implemented)

Time = 3.50 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.33

$$\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \begin{cases} \frac{2\left(-\frac{a\left(a^2-b^2c\right)\left(\begin{cases}\frac{\sqrt{c+dx}}{a^2} & \text{for } b=0\\ -\frac{1}{b\left(a+b\sqrt{c+dx}\right)} & \text{otherwise}\end{cases}\right)}{b^3}-\frac{2a\sqrt{c+dx}}{b^3}+\frac{c+dx}{2b^2}+\frac{\left(3a^2-b^2c\right)\left(\begin{cases}\frac{\sqrt{c+dx}}{a} & \text{for } b=0\\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^3}\right)}{d^2} & \text{for } d\neq \cdots \\ \frac{x^2}{2\left(a+b\sqrt{c}\right)^2} & \text{otherw}\cdots \end{cases}$$

input
```
integrate(x/(a+b*(d*x+c)**(1/2))**2,x)
```

output
```
Piecewise((2*(-a*(a**2 - b**2*c)*Piecewise((sqrt(c + d*x)/a**2, Eq(b, 0)),
 (-1/(b*(a + b*sqrt(c + d*x))), True))/b**3 - 2*a*sqrt(c + d*x)/b**3 + (c
+ d*x)/(2*b**2) + (3*a**2 - b**2*c)*Piecewise((sqrt(c + d*x)/a, Eq(b, 0)),
 (log(a + b*sqrt(c + d*x))/b, True))/b**3)/d**2, Ne(d, 0)), (x**2/(2*(a +
b*sqrt(c))**2), True))
```

3.641.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95

$$\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = -\frac{\frac{2\,(ab^2c-a^3)}{\sqrt{dx+c}b^5+ab^4} - \frac{(dx+c)b-4\,\sqrt{dx+c}a}{b^3} + \frac{2\,(b^2c-3\,a^2)\log\left(\sqrt{dx+c}b+a\right)}{b^4}}{d^2}$$

input
```
integrate(x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
-(2*(a*b^2*c - a^3)/(sqrt(d*x + c)*b^5 + a*b^4) - ((d*x + c)*b - 4*sqrt(d*
x + c)*a)/b^3 + 2*(b^2*c - 3*a^2)*log(sqrt(d*x + c)*b + a)/b^4)/d^2
```

3.641.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.07

$$\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = -\frac{\frac{2\,(b^2c-3\,a^2)\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^4d} - \frac{(dx+c)b^2d-4\,\sqrt{dx+c}abd}{b^4d^2} + \frac{2\,(ab^2c-a^3)}{\left(\sqrt{dx+c}b+a\right)b^4d}}{d}$$

input
```
integrate(x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
-(2*(b^2*c - 3*a^2)*log(abs(sqrt(d*x + c)*b + a))/(b^4*d) - ((d*x + c)*b^2
*d - 4*sqrt(d*x + c)*a*b*d)/(b^4*d^2) + 2*(a*b^2*c - a^3)/((sqrt(d*x + c)*
b + a)*b^4*d))/d
```

3.641.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.03

$$\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{x}{b^2\,d} + \frac{2\,(a^3-a\,b^2\,c)}{b\,\left(b^4\,d^2\,\sqrt{c+d\,x}+a\,b^3\,d^2\right)} - \frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(2\,b^2\,c-6\,a^2\right)}{b^4\,d^2} - \frac{4\,a\,\sqrt{c+d\,x}}{b^3\,d^2}$$

input
```
int(x/(a + b*(c + d*x)^(1/2))^2,x)
```

output
```
x/(b^2*d) + (2*(a^3 - a*b^2*c))/(b*(b^4*d^2*(c + d*x)^(1/2) + a*b^3*d^2))
- (log(a + b*(c + d*x)^(1/2))*(2*b^2*c - 6*a^2))/(b^4*d^2) - (4*a*(c + d*x
)^(1/2))/(b^3*d^2)
```

3.641. $\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2} \, dx$

3.642 $\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2} \, dx$

3.642.1 Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2} \, dx = \frac{2a}{b^2d\left(a+b\sqrt{c+dx}\right)}+\frac{2\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

output
```
2*ln(a+b*(d*x+c)^(1/2))/b^2/d+2*a/b^2/d/(a+b*(d*x+c)^(1/2))
```

3.642.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2} \, dx = \frac{2\left(\frac{a}{a+b\sqrt{c+dx}}+\log\left(bd\left(a+b\sqrt{c+dx}\right)\right)\right)}{b^2d}$$

input
```
Integrate[(a + b*Sqrt[c + d*x])^(-2),x]
```

output
```
(2*(a/(a + b*Sqrt[c + d*x]) + Log[b*d*(a + b*Sqrt[c + d*x])]))/(b^2*d)
```

3.642.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {239, 774, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}dx$$

$$\Big\downarrow \ 239$$

$$\frac{\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}d(c+dx)}{d}$$

$$\Big\downarrow \ 774$$

$$\frac{2\int \frac{\sqrt{c+dx}}{\left(a+b\sqrt{c+dx}\right)^2}d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \ 49$$

$$\frac{2\int \left(\frac{1}{b\left(a+b\sqrt{c+dx}\right)}-\frac{a}{b\left(a+b\sqrt{c+dx}\right)^2}\right)d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{2\left(\frac{a}{b^2\left(a+b\sqrt{c+dx}\right)}+\frac{\log\left(a+b\sqrt{c+dx}\right)}{b^2}\right)}{d}$$

input
```
Int[(a + b*Sqrt[c + d*x])^(-2),x]
```

output
```
(2*(a/(b^2*(a + b*Sqrt[c + d*x])) + Log[a + b*Sqrt[c + d*x]]/b^2))/d
```

3.642.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.642.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{\frac{2\ln(a+b\sqrt{dx+c})}{b^2}+\frac{2a}{b^2(a+b\sqrt{dx+c})}}{d}$
default	$\frac{a^2}{(-b^2dx-b^2c+a^2)b^2d}+\frac{c}{(-b^2dx-b^2c+a^2)d}+b^2d\left(-\frac{-b^2c+a^2}{b^4d^2(b^2dx+b^2c-a^2)}+\frac{\ln(b^2dx+b^2c-a^2)}{b^4d^2}\right)+\frac{a}{b^2d(-a+b\sqrt{}}$

input
```
int(1/(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/b^2*ln(a+b*(d*x+c)^(1/2))+a/b^2/(a+b*(d*x+c)^(1/2)))
```

3.642.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

$$\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{2\left(\sqrt{dx+c}ab-a^2+\left(b^2dx+b^2c-a^2\right)\log\left(\sqrt{dx+c}b+a\right)\right)}{b^4d^2x+\left(b^4c-a^2b^2\right)d}$$

input
```
integrate(1/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
2*(sqrt(d*x + c)*a*b - a^2 + (b^2*d*x + b^2*c - a^2)*log(sqrt(d*x + c)*b +
 a))/(b^4*d^2*x + (b^4*c - a^2*b^2)*d)
```

3.642.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(39) = 78.

Time = 0.56 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.64

$$\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \begin{cases} \frac{x}{a^2} & \text{for } b=0\wedge(b=0\vee d=0) \\ \frac{x}{\left(a+b\sqrt{c}\right)^2} & \text{for } d=0 \\ \frac{2a\log\left(\frac{a}{b}+\sqrt{c+dx}\right)}{ab^2d+b^3d\sqrt{c+dx}}+\frac{2a}{ab^2d+b^3d\sqrt{c+dx}}+\frac{2b\sqrt{c+dx}\log\left(\frac{a}{b}+\sqrt{c+dx}\right)}{ab^2d+b^3d\sqrt{c+dx}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*(d*x+c)**(1/2))**2,x)
```

output
```
Piecewise((x/a**2, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x/(a + b*sqrt(c))**
2, Eq(d, 0)), (2*a*log(a/b + sqrt(c + d*x))/(a*b**2*d + b**3*d*sqrt(c + d*
x)) + 2*a/(a*b**2*d + b**3*d*sqrt(c + d*x)) + 2*b*sqrt(c + d*x)*log(a/b +
sqrt(c + d*x))/(a*b**2*d + b**3*d*sqrt(c + d*x)), True))
```

3.642.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{2\left(\frac{a}{\sqrt{dx+c}b^3+ab^2}+\frac{\log\left(\sqrt{dx+c}b+a\right)}{b^2}\right)}{d}$$

input
```
integrate(1/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
2*(a/(sqrt(d*x + c)*b^3 + a*b^2) + log(sqrt(d*x + c)*b + a)/b^2)/d
```

3.642.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{2\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^2d}+\frac{2a}{\left(\sqrt{dx+c}b+a\right)b^2d}$$

input
```
integrate(1/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
2*log(abs(sqrt(d*x + c)*b + a))/(b^2*d) + 2*a/((sqrt(d*x + c)*b + a)*b^2*d
)
```

3.642.9 Mupad [B] (verification not implemented)

Time = 17.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{2\ln\left(a+b\sqrt{c+d\,x}\right)}{b^2\,d}+\frac{2\,a}{b^2\,\left(a\,d+b\,d\,\sqrt{c+d\,x}\right)}$$

input
```
int(1/(a + b*(c + d*x)^(1/2))^2,x)
```

output
```
(2*log(a + b*(c + d*x)^(1/2)))/(b^2*d) + (2*a)/(b^2*(a*d + b*d*(c + d*x)^(
1/2)))
```

3.643 $\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx$

3.643.1 Optimal result

Integrand size = 19, antiderivative size = 129

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{2a}{(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)}+\frac{4ab\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{(a^2-b^2c)^2}+\frac{(a^2+b^2c)\log(x)}{(a^2-b^2c)^2}-\frac{2(a^2+b^2c)\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^2}$$

output
```
(b^2*c+a^2)*ln(x)/(-b^2*c+a^2)^2-2*(b^2*c+a^2)*ln(a+b*(d*x+c)^(1/2))/(-b^2
*c+a^2)^2+4*a*b*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)/(-b^2*c+a^2)^2+2*a/
(-b^2*c+a^2)/(a+b*(d*x+c)^(1/2))
```

3.643.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{\frac{2a(a^2-b^2c)}{a+b\sqrt{c+dx}}+4ab\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+(a^2+b^2c)\log(-dx)-2(a^2+b^2c)\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^2}$$

input
```
Integrate[1/(x*(a + b*Sqrt[c + d*x])^2),x]
```

output
```
((2*a*(a^2 - b^2*c))/(a + b*Sqrt[c + d*x]) + 4*a*b*Sqrt[c]*ArcTanh[Sqrt[c
+ d*x]/Sqrt[c]] + (a^2 + b^2*c)*Log[-(d*x)] - 2*(a^2 + b^2*c)*Log[a + b*Sq
rt[c + d*x]])/(a^2 - b^2*c)^2
```

3.643.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {896, 25, 1732, 594, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}dx$$

$$\Big\downarrow \text{896}$$

$$\int \frac{1}{dx\left(a+b\sqrt{c+dx}\right)^2}d(c+dx)$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{1}{dx\left(a+b\sqrt{c+dx}\right)^2}d(c+dx)$$

$$\Big\downarrow \text{1732}$$

$$-2\int -\frac{\sqrt{c+dx}}{dx\left(a+b\sqrt{c+dx}\right)^2}d\sqrt{c+dx}$$

$$\Big\downarrow \text{594}$$

$$-2\left(\frac{\int \frac{bc-a\sqrt{c+dx}}{dx\left(a+b\sqrt{c+dx}\right)}d\sqrt{c+dx}}{a^2-b^2c}-\frac{a}{(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)}\right)$$

$$\Big\downarrow \text{25}$$

$$-2\left(-\frac{\int -\frac{bc-a\sqrt{c+dx}}{dx\left(a+b\sqrt{c+dx}\right)}d\sqrt{c+dx}}{a^2-b^2c}-\frac{a}{(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)}\right)$$

$$\Big\downarrow \text{657}$$

$$-2\left(-\frac{\int\left(-\frac{b(a^2+b^2c)}{(a^2-b^2c)(a+b\sqrt{c+dx})}-\frac{2abc-(a^2+b^2c)\sqrt{c+dx}}{(a^2-b^2c)dx}\right)d\sqrt{c+dx}}{a^2-b^2c}-\frac{a}{(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)}\right)$$

$$\Big\downarrow \text{2009}$$

$$-2\left(-\frac{\frac{2ab\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c}+\frac{(a^2+b^2c)\log(-dx)}{2(a^2-b^2c)}-\frac{(a^2+b^2c)\log(a+b\sqrt{c+dx})}{a^2-b^2c}}{a^2-b^2c}-\frac{a}{(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)}\right)$$

input
```
Int[1/(x*(a + b*Sqrt[c + d*x])^2),x]
```

output
```
-2*(-(a/((a^2 - b^2*c)*(a + b*Sqrt[c + d*x]))) - ((2*a*b*Sqrt[c]*ArcTanh[S
qrt[c + d*x]/Sqrt[c]])/(a^2 - b^2*c) + ((a^2 + b^2*c)*Log[-(d*x)])/(2*(a^2
 - b^2*c)) - ((a^2 + b^2*c)*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c))/(a^2
- b^2*c))
```

3.643.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 594
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2)))
, x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)
^p*(a*d*(n + 1) + b*c*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, p}, x]
 && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.643.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{2a}{(-b^2c+a^2)(a+b\sqrt{dx+c})}-\frac{2(b^2c+a^2)\ln(a+b\sqrt{dx+c})}{(-b^2c+a^2)^2}+\frac{-(-b^2c-a^2)\ln(-dx)+4ab\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)\sqrt{c}}{(-b^2c+a^2)^2}$	118
default	$\frac{2a}{(-b^2c+a^2)(a+b\sqrt{dx+c})}-\frac{2(b^2c+a^2)\ln(a+b\sqrt{dx+c})}{(-b^2c+a^2)^2}+\frac{-(-b^2c-a^2)\ln(-dx)+4ab\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)\sqrt{c}}{(-b^2c+a^2)^2}$	118

input
```
int(1/x/(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
2*a/(-b^2*c+a^2)/(a+b*(d*x+c)^(1/2))-2*(b^2*c+a^2)*ln(a+b*(d*x+c)^(1/2))/(
-b^2*c+a^2)^2+2/(-b^2*c+a^2)^2*(-1/2*(-b^2*c-a^2)*ln(-d*x)+2*a*b*arctanh((
d*x+c)^(1/2)/c^(1/2))*c^(1/2))
```

3.643.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.44

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$=\left[\frac{2\,a^2b^2c-2\,a^4+2\,(ab^3dx+ab^3c-a^3b)\sqrt{c}\log\left(\frac{dx+2\,\sqrt{dx+c}\sqrt{c}+2\,c}{x}\right)-2\,(b^4c^2-a^4+(b^4c+a^2b^2)dx)\log\left(\sqrt{\right.}}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6+(b^6c^2-2\,c}\right.$$

input
```
integrate(1/x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
[(2*a^2*b^2*c - 2*a^4 + 2*(a*b^3*d*x + a*b^3*c - a^3*b)*sqrt(c)*log((d*x +
 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 2*(b^4*c^2 - a^4 + (b^4*c + a^2*b^2)*
d*x)*log(sqrt(d*x + c)*b + a) + (b^4*c^2 - a^4 + (b^4*c + a^2*b^2)*d*x)*lo
g(x) - 2*(a*b^3*c - a^3*b)*sqrt(d*x + c))/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4
*b^2*c - a^6 + (b^6*c^2 - 2*a^2*b^4*c + a^4*b^2)*d*x), (2*a^2*b^2*c - 2*a^
4 - 4*(a*b^3*d*x + a*b^3*c - a^3*b)*sqrt(-c)*arctan(sqrt(d*x + c)*sqrt(-c)
/c) - 2*(b^4*c^2 - a^4 + (b^4*c + a^2*b^2)*d*x)*log(sqrt(d*x + c)*b + a) +
 (b^4*c^2 - a^4 + (b^4*c + a^2*b^2)*d*x)*log(x) - 2*(a*b^3*c - a^3*b)*sqrt
(d*x + c))/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6 + (b^6*c^2 - 2*a^2
*b^4*c + a^4*b^2)*d*x)]
```

3.643.6 Sympy [A] (verification not implemented)

Time = 4.64 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.29

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$= \begin{cases} -\frac{2ab\left(\begin{cases}\frac{\sqrt{c+dx}}{a^2} & \text{for } b=0\\ -\frac{1}{b(a+b\sqrt{c+dx})} & \text{otherwise}\end{cases}\right)}{a^2-b^2c} - \frac{2b(a^2+b^2c)\left(\begin{cases}\frac{\sqrt{c+dx}}{a} & \text{for } b=0\\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise}\end{cases}\right)}{(a^2-b^2c)^2} - \frac{2\cdot\left(\frac{2abc\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\left(-\frac{a^2}{2}-\right.\right.}{(a^2-b^2c)^2} \\ \frac{\log(x)}{\left(a+b\sqrt{c}\right)^2} \end{cases}$$

input
```
integrate(1/x/(a+b*(d*x+c)**(1/2))**2,x)
```

output
```
Piecewise((-2*a*b*Piecewise((sqrt(c + d*x)/a**2, Eq(b, 0)), (-1/(b*(a + b*
sqrt(c + d*x))), True))/(a**2 - b**2*c) - 2*b*(a**2 + b**2*c)*Piecewise((s
qrt(c + d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b, True))/(a**2 - b**
2*c)**2 - 2*(2*a*b*c*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + (-a**2/2 - b*
*2*c/2)*log(-d*x))/(a**2 - b**2*c)**2, Ne(d, 0)), (log(x)/(a + b*sqrt(c))*
*2, True))
```

3.643.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.36

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx = -\frac{2\,ab\sqrt{c}\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{b^4c^2-2\,a^2b^2c+a^4} + \frac{\left(b^2c+a^2\right)\log\left(dx\right)}{b^4c^2-2\,a^2b^2c+a^4} - \frac{2\left(b^2c+a^2\right)\log\left(\sqrt{dx+c}b+a\right)}{b^4c^2-2\,a^2b^2c+a^4} - \frac{2\,a}{ab^2c-a^3+\left(b^3c-a^2b\right)\sqrt{dx+c}}$$

input
```
integrate(1/x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
-2*a*b*sqrt(c)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/(b
^4*c^2 - 2*a^2*b^2*c + a^4) + (b^2*c + a^2)*log(d*x)/(b^4*c^2 - 2*a^2*b^2*
c + a^4) - 2*(b^2*c + a^2)*log(sqrt(d*x + c)*b + a)/(b^4*c^2 - 2*a^2*b^2*c
 + a^4) - 2*a/(a*b^2*c - a^3 + (b^3*c - a^2*b)*sqrt(d*x + c))
```

3.643.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.35

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx = -\frac{4\,abc\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\left(b^4c^2-2\,a^2b^2c+a^4\right)\sqrt{-c}} + \frac{\left(b^2c+a^2\right)\log\left(-dx\right)}{b^4c^2-2\,a^2b^2c+a^4} - \frac{2\left(b^3c+a^2b\right)\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^5c^2-2\,a^2b^3c+a^4b} - \frac{2\left(ab^2c-a^3\right)}{\left(b^2c-a^2\right)^2\left(\sqrt{dx+c}b+a\right)}$$

input
```
integrate(1/x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
-4*a*b*c*arctan(sqrt(d*x + c)/sqrt(-c))/((b^4*c^2 - 2*a^2*b^2*c + a^4)*sqr
t(-c)) + (b^2*c + a^2)*log(-d*x)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - 2*(b^3*c
+ a^2*b)*log(abs(sqrt(d*x + c)*b + a))/(b^5*c^2 - 2*a^2*b^3*c + a^4*b) - 2
*(a*b^2*c - a^3)/((b^2*c - a^2)^2*(sqrt(d*x + c)*b + a))
```

3.643.9 Mupad [B] (verification not implemented)

Time = 17.95 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.97

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{\ln\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\left(a+b\,\sqrt{c}\right)^2} + \ln\left(a+b\,\sqrt{c+d\,x}\right)\left(\frac{2}{b^2\,c-a^2}-\frac{4\,b^2\,c}{\left(b^2\,c-a^2\right)^2}\right) + \frac{\ln\left(\sqrt{c+d\,x}+\sqrt{c}\right)}{\left(a-b\,\sqrt{c}\right)^2} - \frac{2\,a}{\left(b^2\,c-a^2\right)\,\left(a+b\,\sqrt{c+d\,x}\right)}$$

input
```
int(1/(x*(a + b*(c + d*x)^(1/2))^2),x)
```

output
```
log((c + d*x)^(1/2) - c^(1/2))/(a + b*c^(1/2))^2 + log(a + b*(c + d*x)^(1/
2))*(2/(b^2*c - a^2) - (4*b^2*c)/(b^2*c - a^2)^2) + log((c + d*x)^(1/2) +
c^(1/2))/(a - b*c^(1/2))^2 - (2*a)/((b^2*c - a^2)*(a + b*(c + d*x)^(1/2)))
```

3.644 $\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx$

3.644.1 Optimal result

Integrand size = 19, antiderivative size = 202

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{4ab^2d}{(a^2-b^2c)^2\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{(a^2-b^2c)\,x\left(a+b\sqrt{c+dx}\right)} + \frac{2ab(a^2+3b^2c)\,d\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\,(a^2-b^2c)^3} + \frac{b^2(3a^2+b^2c)\,d\log(x)}{(a^2-b^2c)^3} - \frac{2b^2(3a^2+b^2c)\,d\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^3}$$

output
```
b^2*(b^2*c+3*a^2)*d*ln(x)/(-b^2*c+a^2)^3-2*b^2*(b^2*c+3*a^2)*d*ln(a+b*(d*x
+c)^(1/2))/(-b^2*c+a^2)^3+2*a*b*(3*b^2*c+a^2)*d*arctanh((d*x+c)^(1/2)/c^(1
/2))/(-b^2*c+a^2)^3/c^(1/2)+4*a*b^2*d/(-b^2*c+a^2)^2/(a+b*(d*x+c)^(1/2))+(
-a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)/x/(a+b*(d*x+c)^(1/2))
```

3.644.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{\frac{(a^2-b^2c)\left(-a^3+a^2b\sqrt{c+dx}-b^3c\sqrt{c+dx}+ab^2(c+4dx)\right)}{x\left(a+b\sqrt{c+dx}\right)} + \frac{2ab(a^2+3b^2c)d\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}} + b^2(3a^2+b^2c)\,d\log(-dx) - 2b^2(3}{(a^2-b^2c)^3}$$

input
```
Integrate[1/(x^2*(a + b*Sqrt[c + d*x])^2),x]
```

output
```
(((a^2 - b^2*c)*(-a^3 + a^2*b*Sqrt[c + d*x] - b^3*c*Sqrt[c + d*x] + a*b^2*
(c + 4*d*x)))/(x*(a + b*Sqrt[c + d*x])) + (2*a*b*(a^2 + 3*b^2*c)*d*ArcTanh
[Sqrt[c + d*x]/Sqrt[c]])/Sqrt[c] + b^2*(3*a^2 + b^2*c)*d*Log[-(d*x)] - 2*b
^2*(3*a^2 + b^2*c)*d*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^3
```

3.644.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {896, 1732, 593, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}dx$$

$$\Big\downarrow \text{896}$$

$$d\int \frac{1}{d^2x^2\left(a+b\sqrt{c+dx}\right)^2}d(c+dx)$$

$$\Big\downarrow \text{1732}$$

$$2d\int \frac{\sqrt{c+dx}}{d^2x^2\left(a+b\sqrt{c+dx}\right)^2}d\sqrt{c+dx}$$

$$\Big\downarrow \text{593}$$

$$2d\left(-\frac{b\int \frac{2\left(a-b\sqrt{c+dx}\right)}{dx\left(a+b\sqrt{c+dx}\right)^2}d\sqrt{c+dx}}{2\left(a^2-b^2c\right)}-\frac{a-b\sqrt{c+dx}}{2dx\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$2d\left(\frac{b\int -\frac{a-b\sqrt{c+dx}}{dx\left(a+b\sqrt{c+dx}\right)^2}d\sqrt{c+dx}}{a^2-b^2c}-\frac{a-b\sqrt{c+dx}}{2dx\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)}\right)$$

$$\Big\downarrow \text{657}$$

$$2d\left(\frac{b\int\left(-\frac{(3a^2+b^2c)b^2}{(a^2-b^2c)^2(a+b\sqrt{c+dx})}-\frac{2ab^2}{(a^2-b^2c)(a+b\sqrt{c+dx})^2}-\frac{a(a^2+3b^2c)-b(3a^2+b^2c)\sqrt{c+dx}}{(a^2-b^2c)^2dx}\right)d\sqrt{c+dx}}{a^2-b^2c}-\frac{a-b\sqrt{c}}{2dx\,(a^2-b^2c)\,(a}\right.$$

$\downarrow$ 2009

$$2d\left(\frac{b\left(\frac{a(a^2+3b^2c)\text{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}(a^2-b^2c)^2}+\frac{2ab}{(a^2-b^2c)(a+b\sqrt{c+dx})}+\frac{b(3a^2+b^2c)\log(-dx)}{2(a^2-b^2c)^2}-\frac{b(3a^2+b^2c)\log(a+b\sqrt{c+dx})}{(a^2-b^2c)^2}\right)}{a^2-b^2c}-\frac{}{2dx\,(a^2-}\right.$$

input
```
Int[1/(x^2*(a + b*Sqrt[c + d*x])^2),x]
```

output
```
2*d*(-1/2*(a - b*Sqrt[c + d*x])/((a^2 - b^2*c)*d*x*(a + b*Sqrt[c + d*x]))
+ (b*((2*a*b)/((a^2 - b^2*c)*(a + b*Sqrt[c + d*x])) + (a*(a^2 + 3*b^2*c)*A
rcTanh[Sqrt[c + d*x]/Sqrt[c]])/(Sqrt[c]*(a^2 - b^2*c)^2) + (b*(3*a^2 + b^2
*c)*Log[-(d*x)])/(2*(a^2 - b^2*c)^2) - (b*(3*a^2 + b^2*c)*Log[a + b*Sqrt[c
 + d*x]])/(a^2 - b^2*c)^2))/(a^2 - b^2*c))
```

3.644.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 593
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[(c + d*x)^(n + 1)*(c - d*x)*((a + b*x^2)^(p + 1)/(2*(p + 1)*(b*c^2 +
a*d^2))), x] - Simp[d/(2*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*
x^2)^(p + 1)*(c*n - d*(n + 2*p + 4)*x), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.644.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.90

method	result
derivativedivides	$2d\Bigg(\frac{a\,b^{2}}{\left(-b^{2}c+a^{2}\right)^{2}\left(a+b\sqrt{dx+c}\right)}-\frac{b^{2}\left(b^{2}c+3a^{2}\right)\ln\left(a+b\sqrt{dx+c}\right)}{\left(-b^{2}c+a^{2}\right)^{3}}+\frac{-\frac{\left(a\,b^{3}c-a^{3}b\right)\sqrt{dx+c}-\frac{b^{4}c^{2}}{2}+\frac{a^{4}}{2}}{dx}+b\Bigg(-\frac{\left(-b^{3}c-3a^{2}\right.}{2}}{(-b^{2}c+a}$
default	$2d\Bigg(\frac{a\,b^{2}}{\left(-b^{2}c+a^{2}\right)^{2}\left(a+b\sqrt{dx+c}\right)}-\frac{b^{2}\left(b^{2}c+3a^{2}\right)\ln\left(a+b\sqrt{dx+c}\right)}{\left(-b^{2}c+a^{2}\right)^{3}}+\frac{-\frac{\left(a\,b^{3}c-a^{3}b\right)\sqrt{dx+c}-\frac{b^{4}c^{2}}{2}+\frac{a^{4}}{2}}{dx}+b\Bigg(-\frac{\left(-b^{3}c-3a^{2}\right.}{2}}{(-b^{2}c+a}$

input
```
int(1/x^2/(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
2*d*(a*b^2/(-b^2*c+a^2)^2/(a+b*(d*x+c)^(1/2))-b^2*(b^2*c+3*a^2)/(-b^2*c+a^
2)^3*ln(a+b*(d*x+c)^(1/2))+1/(-b^2*c+a^2)^3*(-((a*b^3*c-a^3*b)*(d*x+c)^(1/
2)-1/2*b^4*c^2+1/2*a^4)/d/x+b*(-1/2*(-b^3*c-3*a^2*b)*ln(-d*x)+(3*a*b^2*c+a
^3)/c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2)))))
```

3.644.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(195) = 390$.

Time = 0.53 (sec) , antiderivative size = 854, normalized size of antiderivative = 4.23

$$\int \frac{1}{x^2 \left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$= \left[-\frac{b^6c^4 - a^2b^4c^3 - a^4b^2c^2 + a^6c + (b^6c^3 + 2\,a^2b^4c^2 - 3\,a^4b^2c)dx - ((3\,ab^5c + a^3b^3)d^2x^2 + (3\,ab^5c^2 - 2\,a^3b^3}{}\right.$$
$$-\frac{b^6c^4 - a^2b^4c^3 - a^4b^2c^2 + a^6c + (b^6c^3 + 2\,a^2b^4c^2 - 3\,a^4b^2c)dx - 2\,((3\,ab^5c + a^3b^3)d^2x^2 + (3\,ab^5c^2 - 2\,a^3b}{}$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
[-(b^6*c^4 - a^2*b^4*c^3 - a^4*b^2*c^2 + a^6*c + (b^6*c^3 + 2*a^2*b^4*c^2
- 3*a^4*b^2*c)*d*x - ((3*a*b^5*c + a^3*b^3)*d^2*x^2 + (3*a*b^5*c^2 - 2*a^3
*b^3*c - a^5*b)*d*x)*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x)
- 2*((b^6*c^2 + 3*a^2*b^4*c)*d^2*x^2 + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^
2*c)*d*x)*log(sqrt(d*x + c)*b + a) + ((b^6*c^2 + 3*a^2*b^4*c)*d^2*x^2 + (b
^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x)*log(x) - 2*(a*b^5*c^3 - 2*a^3*b
^3*c^2 + a^5*b*c + 2*(a*b^5*c^2 - a^3*b^3*c)*d*x)*sqrt(d*x + c))/((b^8*c^4
 - 3*a^2*b^6*c^3 + 3*a^4*b^4*c^2 - a^6*b^2*c)*d*x^2 + (b^8*c^5 - 4*a^2*b^6
*c^4 + 6*a^4*b^4*c^3 - 4*a^6*b^2*c^2 + a^8*c)*x), -(b^6*c^4 - a^2*b^4*c^3
- a^4*b^2*c^2 + a^6*c + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x - 2*((
3*a*b^5*c + a^3*b^3)*d^2*x^2 + (3*a*b^5*c^2 - 2*a^3*b^3*c - a^5*b)*d*x)*sq
rt(-c)*arctan(sqrt(d*x + c)*sqrt(-c)/c) - 2*((b^6*c^2 + 3*a^2*b^4*c)*d^2*x
^2 + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x)*log(sqrt(d*x + c)*b + a)
 + ((b^6*c^2 + 3*a^2*b^4*c)*d^2*x^2 + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2
*c)*d*x)*log(x) - 2*(a*b^5*c^3 - 2*a^3*b^3*c^2 + a^5*b*c + 2*(a*b^5*c^2 -
a^3*b^3*c)*d*x)*sqrt(d*x + c))/((b^8*c^4 - 3*a^2*b^6*c^3 + 3*a^4*b^4*c^2 -
 a^6*b^2*c)*d*x^2 + (b^8*c^5 - 4*a^2*b^6*c^4 + 6*a^4*b^4*c^3 - 4*a^6*b^2*c
^2 + a^8*c)*x)]
```

3.644.6 Sympy [F]

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx$$

input
```
integrate(1/x**2/(a+b*(d*x+c)**(1/2))**2,x)
```

output
```
Integral(1/(x**2*(a + b*sqrt(c + d*x))**2), x)
```

3.644.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx=$$
$$-d\left(\frac{(b^4c+3\,a^2b^2)\log{(dx)}}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6}-\frac{2\,(b^4c+3\,a^2b^2)\log{\left(\sqrt{dx+c}b+a\right)}}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6}-\frac{(3\,ab^3c+a^3b)\log{\left(\frac{\sqrt{dx+c}-}{\sqrt{dx+c}+}\right.}}{(b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-}\right.$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
-d*((b^4*c + 3*a^2*b^2)*log(d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c -
a^6) - 2*(b^4*c + 3*a^2*b^2)*log(sqrt(d*x + c)*b + a)/(b^6*c^3 - 3*a^2*b^4
*c^2 + 3*a^4*b^2*c - a^6) - (3*a*b^3*c + a^3*b)*log((sqrt(d*x + c) - sqrt(
c))/(sqrt(d*x + c) + sqrt(c)))/((b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a
^6)*sqrt(c)) + (4*(d*x + c)*a*b^2 - 3*a*b^2*c - a^3 - (b^3*c - a^2*b)*sqrt
(d*x + c))/(a*b^4*c^3 - 2*a^3*b^2*c^2 + a^5*c - (b^5*c^2 - 2*a^2*b^3*c + a
^4*b)*(d*x + c)^(3/2) - (a*b^4*c^2 - 2*a^3*b^2*c + a^5)*(d*x + c) + (b^5*c
^3 - 2*a^2*b^3*c^2 + a^4*b*c)*sqrt(d*x + c)))
```

3.644.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx = -\frac{(b^4cd+3\,a^2b^2d)\log(-dx)}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6}+\frac{2\,(b^5cd+3\,a^2b^3d)\log\left(\left|-\sqrt{dx+c}b-a\right|\right)}{b^7c^3-3\,a^2b^5c^2+3\,a^4b^3c-a^6b}+\frac{2\,(3\,ab^3cd+a^3bd)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{(b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6)\sqrt{-c}}-\frac{\sqrt{dx+c}b^3cd-4\,(dx+c)ab^2d+3\,ab^2cd-\sqrt{dx+c}a^2bd+a^3d}{(b^4c^2-2\,a^2b^2c+a^4)\left((dx+c)^{\frac{3}{2}}b-\sqrt{dx+c}bc+(dx+c)a-ac\right)}$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
-(b^4*c*d + 3*a^2*b^2*d)*log(-d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c
- a^6) + 2*(b^5*c*d + 3*a^2*b^3*d)*log(abs(-sqrt(d*x + c)*b - a))/(b^7*c^3
 - 3*a^2*b^5*c^2 + 3*a^4*b^3*c - a^6*b) + 2*(3*a*b^3*c*d + a^3*b*d)*arctan
(sqrt(d*x + c)/sqrt(-c))/((b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6)*sq
rt(-c)) - (sqrt(d*x + c)*b^3*c*d - 4*(d*x + c)*a*b^2*d + 3*a*b^2*c*d - sqr
t(d*x + c)*a^2*b*d + a^3*d)/((b^4*c^2 - 2*a^2*b^2*c + a^4)*((d*x + c)^(3/2
)*b - sqrt(d*x + c)*b*c + (d*x + c)*a - a*c))
```

3.644.9 Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.36

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{b\,d\,\ln\left(\sqrt{c+d\,x}+\sqrt{c}\right)}{a^3\,\sqrt{c}-b^3\,c^2+3\,a\,b^2\,c^{3/2}-3\,a^2\,b\,c}-\frac{\frac{a\,d\,\left(a^2+3\,c\,b^2\right)}{{\left(b^2\,c-a^2\right)}^2}+\frac{b\,d\,\sqrt{c+d\,x}}{b^2\,c-a^2}-\frac{4\,a\,b^2\,d\,\left(c+d\,x\right)}{a^4-2\,a^2\,b^2\,c+b^4\,c^2}}{b\,{\left(c+d\,x\right)}^{3/2}-a\,c+a\,\left(c+d\,x\right)-b\,c\,\sqrt{c+d\,x}}-\frac{b\,d\,\ln\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{a^3\,\sqrt{c}+b^3\,c^2+3\,a\,b^2\,c^{3/2}+3\,a^2\,b\,c}-\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(\frac{6\,b^2\,d}{{\left(b^2\,c-a^2\right)}^2}-\frac{8\,b^4\,c\,d}{{\left(b^2\,c-a^2\right)}^3}\right)$$

input
```
int(1/(x^2*(a + b*(c + d*x)^(1/2))^2),x)
```

output
```
(b*d*log((c + d*x)^(1/2) + c^(1/2)))/(a^3*c^(1/2) - b^3*c^2 + 3*a*b^2*c^(3
/2) - 3*a^2*b*c) - ((a*d*(3*b^2*c + a^2))/(b^2*c - a^2)^2 + (b*d*(c + d*x)
^(1/2))/(b^2*c - a^2) - (4*a*b^2*d*(c + d*x))/(a^4 + b^4*c^2 - 2*a^2*b^2*c
))/(b*(c + d*x)^(3/2) - a*c + a*(c + d*x) - b*c*(c + d*x)^(1/2)) - (b*d*lo
g((c + d*x)^(1/2) - c^(1/2)))/(a^3*c^(1/2) + b^3*c^2 + 3*a*b^2*c^(3/2) + 3
*a^2*b*c) - log(a + b*(c + d*x)^(1/2))*((6*b^2*d)/(b^2*c - a^2)^2 - (8*b^4
*c*d)/(b^2*c - a^2)^3)
```

3.644. $\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx$

3.645 $\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx$

3.645.1 Optimal result

Integrand size = 19, antiderivative size = 306

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{ab^2(a^2+11b^2c)\,d^2}{2c\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} - \frac{bd\left(3abc-\left(a^2+2b^2c\right)\sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)^2x\left(a+b\sqrt{c+dx}\right)} - \frac{ab(a^4-10a^2b^2c-15b^4c^2)\,d^2\operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}\left(a^2-b^2c\right)^4} + \frac{b^4(5a^2+b^2c)\,d^2\log(x)}{\left(a^2-b^2c\right)^4} - \frac{2b^4(5a^2+b^2c)\,d^2\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^4}$$

output
```
-1/2*a*b*(-15*b^4*c^2-10*a^2*b^2*c+a^4)*d^2*arctanh((d*x+c)^(1/2)/c^(1/2))
/c^(3/2)/(-b^2*c+a^2)^4+b^4*(b^2*c+5*a^2)*d^2*ln(x)/(-b^2*c+a^2)^4-2*b^4*(
b^2*c+5*a^2)*d^2*ln(a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)^4+1/2*a*b^2*(11*b^2*c+
a^2)*d^2/c/(-b^2*c+a^2)^3/(a+b*(d*x+c)^(1/2))+1/2*(-a+b*(d*x+c)^(1/2))/(-b
^2*c+a^2)/x^2/(a+b*(d*x+c)^(1/2))-1/2*b*d*(3*a*b*c-(2*b^2*c+a^2)*(d*x+c)^(
1/2))/c/(-b^2*c+a^2)^2/x/(a+b*(d*x+c)^(1/2))
```

3.645.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^3 \left(a+b\sqrt{c+dx}\right)^2} \, dx$$
$$= \frac{1}{2} \left(\frac{a^5c - b^5c^2(c-2dx)\sqrt{c+dx} + a^2b^3c(2c-dx)\sqrt{c+dx} - a^4b(c+dx)^{3/2} + ab^4c(c^2-3cdx-11d^2x^2) -}{c\left(-a^2+b^2c\right)^3 x^2 \left(a+b\sqrt{c+dx}\right)} \right.$$
$$+ \frac{\left(-a^5b+10a^3b^3c+15ab^5c^2\right) d^2 \operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{c^{3/2}\left(a^2-b^2c\right)^4} + \frac{2b^4(5a^2+b^2c)\, d^2 \log(-dx)}{\left(a^2-b^2c\right)^4}$$
$$\left. - \frac{4b^4(5a^2+b^2c)\, d^2 \log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^4} \right)$$

input
```
Integrate[1/(x^3*(a + b*Sqrt[c + d*x])^2),x]
```

output
```
((a^5*c - b^5*c^2*(c - 2*d*x)*Sqrt[c + d*x] + a^2*b^3*c*(2*c - d*x)*Sqrt[c
 + d*x] - a^4*b*(c + d*x)^(3/2) + a*b^4*c*(c^2 - 3*c*d*x - 11*d^2*x^2) - a
^3*b^2*(2*c^2 - 3*c*d*x + d^2*x^2))/(c*(-a^2 + b^2*c)^3*x^2*(a + b*Sqrt[c
+ d*x])) + ((-(a^5*b) + 10*a^3*b^3*c + 15*a*b^5*c^2)*d^2*ArcTanh[Sqrt[c +
d*x]/Sqrt[c]])/(c^(3/2)*(a^2 - b^2*c)^4) + (2*b^4*(5*a^2 + b^2*c)*d^2*Log[
-(d*x)])/(a^2 - b^2*c)^4 - (4*b^4*(5*a^2 + b^2*c)*d^2*Log[a + b*Sqrt[c + d
*x]])/(a^2 - b^2*c)^4)/2
```

3.645.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {896, 25, 1732, 593, 27, 686, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+b\sqrt{c+dx}\right)^2} \, dx$$
$$\Big\downarrow \; 896$$

$$d^2 \int \frac{1}{d^3x^3 \left(a+b\sqrt{c+dx}\right)^2}d(c+dx)$$

$\Big\downarrow$ 25

$$-d^2 \int -\frac{1}{d^3x^3 \left(a+b\sqrt{c+dx}\right)^2}d(c+dx)$$

$\Big\downarrow$ 1732

$$-2d^2 \int -\frac{\sqrt{c+dx}}{d^3x^3 \left(a+b\sqrt{c+dx}\right)^2}d\sqrt{c+dx}$$

$\Big\downarrow$ 593

$$-2d^2 \left(\frac{a-b\sqrt{c+dx}}{4d^2x^2 (a^2-b^2c) \left(a+b\sqrt{c+dx}\right)}-\frac{b\int -\frac{2\left(a-2b\sqrt{c+dx}\right)}{d^2x^2 \left(a+b\sqrt{c+dx}\right)^2}d\sqrt{c+dx}}{4 (a^2-b^2c)}\right)$$

$\Big\downarrow$ 27

$$-2d^2 \left(\frac{b\int \frac{a-2b\sqrt{c+dx}}{d^2x^2 \left(a+b\sqrt{c+dx}\right)^2}d\sqrt{c+dx}}{2 (a^2-b^2c)}+\frac{a-b\sqrt{c+dx}}{4d^2x^2 (a^2-b^2c) \left(a+b\sqrt{c+dx}\right)}\right)$$

$\Big\downarrow$ 686

$$-2d^2 \left(\frac{b \left(\frac{3abc-(a^2+2b^2c)\sqrt{c+dx}}{2cdx(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)}-\frac{\int \frac{a\left(a^2-7b^2c\right)+2b\left(a^2+2b^2c\right)\sqrt{c+dx}}{dx\left(a+b\sqrt{c+dx}\right)^2}d\sqrt{c+dx}}{2c(a^2-b^2c)}\right)}{2 (a^2-b^2c)}+\frac{a-b\sqrt{c+dx}}{4d^2x^2 (a^2-b^2c) \left(a+b\sqrt{c+dx}\right)}\right)$$

$\Big\downarrow$ 25

$$-2d^2 \left(\frac{b \left(\frac{\int -\frac{a\left(a^2-7b^2c\right)+2b\left(a^2+2b^2c\right)\sqrt{c+dx}}{dx\left(a+b\sqrt{c+dx}\right)^2}d\sqrt{c+dx}}{2c(a^2-b^2c)}+\frac{3abc-(a^2+2b^2c)\sqrt{c+dx}}{2cdx(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)}\right)}{2 (a^2-b^2c)}+\frac{a-b\sqrt{c+dx}}{4d^2x^2 (a^2-b^2c) \left(a+b\sqrt{c+dx}\right)}\right)$$

$\Big\downarrow$ 657

3.645. $\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx$

$$-2d^2\left(\frac{b\left(\frac{\int\left(\frac{a(a^2+11b^2c)b^2}{(a^2-b^2c)(a+b\sqrt{c+dx})^2}-\frac{4c(5a^2+b^2c)\sqrt{c+dx}b^3+a(a^4-10b^2ca^2-15b^4c^2)}{(a^2-b^2c)^2dx}+\frac{4c(cb^6+5a^2b^4)}{(a^2-b^2c)^2(a+b\sqrt{c+dx})}\right)d\sqrt{c+dx}}{2c(a^2-b^2c)}+\frac{3abc-(a^2+2b^2c)\sqrt{\ldots}}{2cdx(a^2-b^2c)(a+b\ldots}\right)}{2(a^2-b^2c)}\ldots\right.$$

$\downarrow$ 2009

$$-2d^2\left(\frac{a-b\sqrt{c+dx}}{4d^2x^2(a^2-b^2c)(a+b\sqrt{c+dx})}+\frac{b\left(\frac{3abc-(a^2+2b^2c)\sqrt{c+dx}}{2cdx(a^2-b^2c)(a+b\sqrt{c+dx})}+\frac{-\frac{ab(a^2+11b^2c)}{(a^2-b^2c)(a+b\sqrt{c+dx})}-\frac{2b^3c(5a^2+b^2c)\log(-dx)}{(a^2-b^2c)^2}+\frac{4b^3c\ldots}{\ldots}}{\ldots}\right)}{2(a^2-b^2c)}\ldots\right.$$

input
```
Int[1/(x^3*(a + b*Sqrt[c + d*x])^2),x]
```

output
```
-2*d^2*((a - b*Sqrt[c + d*x])/(4*(a^2 - b^2*c)*d^2*x^2*(a + b*Sqrt[c + d*x
])) + (b*((3*a*b*c - (a^2 + 2*b^2*c)*Sqrt[c + d*x])/(2*c*(a^2 - b^2*c)*d*x
*(a + b*Sqrt[c + d*x])) + (-((a*b*(a^2 + 11*b^2*c))/((a^2 - b^2*c)*(a + b*
Sqrt[c + d*x]))) + (a*(a^4 - 10*a^2*b^2*c - 15*b^4*c^2)*ArcTanh[Sqrt[c + d
*x]/Sqrt[c]])/(Sqrt[c]*(a^2 - b^2*c)^2) - (2*b^3*c*(5*a^2 + b^2*c)*Log[-(d
*x)])/(a^2 - b^2*c)^2 + (4*b^3*c*(5*a^2 + b^2*c)*Log[a + b*Sqrt[c + d*x]])
/(a^2 - b^2*c)^2)/(2*c*(a^2 - b^2*c))))/(2*(a^2 - b^2*c)))
```

3.645.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 593
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[(c + d*x)^(n + 1)*(c - d*x)*((a + b*x^2)^(p + 1)/(2*(p + 1)*(b*c^2 +
a*d^2))), x] - Simp[d/(2*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*
x^2)^(p + 1)*(c*n - d*(n + 2*p + 4)*x), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.645. $\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx$

3.645.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.99

method	result
derivativedivides	$2d^2\left(-\frac{-\frac{ab\left(-7b^4c^2+6a^2b^2c+a^4\right)(dx+c)^{\frac{3}{2}}}{4c}+\left(-\frac{1}{2}c^2b^6-a^2b^4c+\frac{3}{2}b^2a^4\right)(dx+c)+\left(-\frac{9}{4}a\,b^5c^2+\frac{5}{2}a^3b^3c-\frac{1}{4}a^5b\right)\sqrt{dx+c}+\frac{3b^6c^3}{4}+}{d^2x^2}}{(-b^2c+a^2)^4}\right.$
default	$2d^2\left(-\frac{-\frac{ab\left(-7b^4c^2+6a^2b^2c+a^4\right)(dx+c)^{\frac{3}{2}}}{4c}+\left(-\frac{1}{2}c^2b^6-a^2b^4c+\frac{3}{2}b^2a^4\right)(dx+c)+\left(-\frac{9}{4}a\,b^5c^2+\frac{5}{2}a^3b^3c-\frac{1}{4}a^5b\right)\sqrt{dx+c}+\frac{3b^6c^3}{4}+}{d^2x^2}}{(-b^2c+a^2)^4}\right.$

input
```
int(1/x^3/(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
2*d^2*(-1/(-b^2*c+a^2)^4*((-1/4*a*b*(-7*b^4*c^2+6*a^2*b^2*c+a^4)/c*(d*x+c)
^(3/2)+(-1/2*c^2*b^6-a^2*b^4*c+3/2*b^2*a^4)*(d*x+c)+(-9/4*a*b^5*c^2+5/2*a^
3*b^3*c-1/4*a^5*b)*(d*x+c)^(1/2)+3/4*b^6*c^3+3/4*a^2*b^4*c^2-7/4*a^4*b^2*c
+1/4*a^6)/d^2/x^2+1/4*b/c*(-1/2*(4*b^5*c^2+20*a^2*b^3*c)*ln(-d*x)+(-15*a*b
^4*c^2-10*a^3*b^2*c+a^5)/c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2))))+b^4/(-b^
2*c+a^2)^3*a/(a+b*(d*x+c)^(1/2))-b^4*(b^2*c+5*a^2)/(-b^2*c+a^2)^4*ln(a+b*(
d*x+c)^(1/2)))
```

3.645.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. 2(289) = 578.

Time = 1.25 (sec) , antiderivative size = 1252, normalized size of antiderivative = 4.09

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*b^8*c^6 - 4*a^2*b^6*c^5 + 4*a^6*b^2*c^3 - 2*a^8*c^2 - 4*(b^8*c^4
+ 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2 - 2*(b^8*c^5 + 3*a^2*b^6*c^4 - 9*
a^4*b^4*c^3 + 5*a^6*b^2*c^2)*d*x - ((15*a*b^7*c^2 + 10*a^3*b^5*c - a^5*b^3
)*d^3*x^3 + (15*a*b^7*c^3 - 5*a^3*b^5*c^2 - 11*a^5*b^3*c + a^7*b)*d^2*x^2)
*sqrt(c)*log((d*x + 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 8*((b^8*c^3 + 5*a^
2*b^6*c^2)*d^3*x^3 + (b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2)*lo
g(sqrt(d*x + c)*b + a) - 4*((b^8*c^3 + 5*a^2*b^6*c^2)*d^3*x^3 + (b^8*c^4 +
 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2)*log(x) - 2*(2*a*b^7*c^5 - 6*a^3*b
^5*c^4 + 6*a^5*b^3*c^3 - 2*a^7*b*c^2 - (11*a*b^7*c^3 - 10*a^3*b^5*c^2 - a^
5*b^3*c)*d^2*x^2 - (5*a*b^7*c^4 - 9*a^3*b^5*c^3 + 3*a^5*b^3*c^2 + a^7*b*c)
*d*x)*sqrt(d*x + c))/((b^10*c^6 - 4*a^2*b^8*c^5 + 6*a^4*b^6*c^4 - 4*a^6*b^
4*c^3 + a^8*b^2*c^2)*d*x^3 + (b^10*c^7 - 5*a^2*b^8*c^6 + 10*a^4*b^6*c^5 -
10*a^6*b^4*c^4 + 5*a^8*b^2*c^3 - a^10*c^2)*x^2), -1/2*(b^8*c^6 - 2*a^2*b^6
*c^5 + 2*a^6*b^2*c^3 - a^8*c^2 - 2*(b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^
2)*d^2*x^2 - (b^8*c^5 + 3*a^2*b^6*c^4 - 9*a^4*b^4*c^3 + 5*a^6*b^2*c^2)*d*x
 + ((15*a*b^7*c^2 + 10*a^3*b^5*c - a^5*b^3)*d^3*x^3 + (15*a*b^7*c^3 - 5*a^
3*b^5*c^2 - 11*a^5*b^3*c + a^7*b)*d^2*x^2)*sqrt(-c)*arctan(sqrt(d*x + c)*s
qrt(-c)/c) + 4*((b^8*c^3 + 5*a^2*b^6*c^2)*d^3*x^3 + (b^8*c^4 + 4*a^2*b^6*c
^3 - 5*a^4*b^4*c^2)*d^2*x^2)*log(sqrt(d*x + c)*b + a) - 2*((b^8*c^3 + 5*a^
2*b^6*c^2)*d^3*x^3 + (b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2)...
```

3.645.6 Sympy [F]

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx=\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx$$

input
```
integrate(1/x**3/(a+b*(d*x+c)**(1/2))**2,x)
```

output
```
Integral(1/(x**3*(a + b*sqrt(c + d*x))**2), x)
```

3.645.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 659 vs. $2(289) = 578$.

Time = 0.29 (sec) , antiderivative size = 659, normalized size of antiderivative = 2.15

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$= \frac{1}{4}d^2\left(\frac{4\left(b^6c+5\,a^2b^4\right)\log\left(dx\right)}{b^8c^4-4\,a^2b^6c^3+6\,a^4b^4c^2-4\,a^6b^2c+a^8}-\frac{8\left(b^6c+5\,a^2b^4\right)\log\left(\sqrt{dx+c}b+a\right)}{b^8c^4-4\,a^2b^6c^3+6\,a^4b^4c^2-4\,a^6b^2c+a^8}-\frac{\left(15\,ab^5c^2\right.}{\left(b^8c^5-4\,a^2\right.}\right.$$

input
```
integrate(1/x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

output
```
1/4*d^2*(4*(b^6*c + 5*a^2*b^4)*log(d*x)/(b^8*c^4 - 4*a^2*b^6*c^3 + 6*a^4*b
^4*c^2 - 4*a^6*b^2*c + a^8) - 8*(b^6*c + 5*a^2*b^4)*log(sqrt(d*x + c)*b +
a)/(b^8*c^4 - 4*a^2*b^6*c^3 + 6*a^4*b^4*c^2 - 4*a^6*b^2*c + a^8) - (15*a*b
^5*c^2 + 10*a^3*b^3*c - a^5*b)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c
) + sqrt(c)))/((b^8*c^5 - 4*a^2*b^6*c^4 + 6*a^4*b^4*c^3 - 4*a^6*b^2*c^2 +
a^8*c)*sqrt(c)) - 2*(7*a*b^4*c^3 + 6*a^3*b^2*c^2 - a^5*c + (11*a*b^4*c + a
^3*b^2)*(d*x + c)^2 - (2*b^5*c^2 - a^2*b^3*c - a^4*b)*(d*x + c)^(3/2) - (1
9*a*b^4*c^2 + 5*a^3*b^2*c)*(d*x + c) + 3*(b^5*c^3 - a^2*b^3*c^2)*sqrt(d*x
+ c))/(a*b^6*c^6 - 3*a^3*b^4*c^5 + 3*a^5*b^2*c^4 - a^7*c^3 + (b^7*c^4 - 3*
a^2*b^5*c^3 + 3*a^4*b^3*c^2 - a^6*b*c)*(d*x + c)^(5/2) + (a*b^6*c^4 - 3*a^
3*b^4*c^3 + 3*a^5*b^2*c^2 - a^7*c)*(d*x + c)^2 - 2*(b^7*c^5 - 3*a^2*b^5*c^
4 + 3*a^4*b^3*c^3 - a^6*b*c^2)*(d*x + c)^(3/2) - 2*(a*b^6*c^5 - 3*a^3*b^4*
c^4 + 3*a^5*b^2*c^3 - a^7*c^2)*(d*x + c) + (b^7*c^6 - 3*a^2*b^5*c^5 + 3*a^
4*b^3*c^4 - a^6*b*c^3)*sqrt(d*x + c)))
```

3.645.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.70

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx$$
$$= \frac{\left(b^6cd^2+5\,a^2b^4d^2\right)\log\left(-dx\right)}{b^8c^4-4\,a^2b^6c^3+6\,a^4b^4c^2-4\,a^6b^2c+a^8}-\frac{2\left(b^7cd^2+5\,a^2b^5d^2\right)\log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^9c^4-4\,a^2b^7c^3+6\,a^4b^5c^2-4\,a^6b^3c+a^8b}$$
$$-\frac{\left(15\,ab^5c^2d^2+10\,a^3b^3cd^2-a^5bd^2\right)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{2\left(b^8c^5-4\,a^2b^6c^4+6\,a^4b^4c^3-4\,a^6b^2c^2+a^8c\right)\sqrt{-c}}$$
$$-\frac{7\,ab^6c^4d^2-a^3b^4c^3d^2-7\,a^5b^2c^2d^2+a^7cd^2+\left(11\,ab^6c^2d^2-10\,a^3b^4cd^2-a^5b^2d^2\right)\left(dx+c\right)^2-\left(2\,b^7c^3d^2-\right.}{2\left(b^2c-\right.}$$

input
```
integrate(1/x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

output
```
(b^6*c*d^2 + 5*a^2*b^4*d^2)*log(-d*x)/(b^8*c^4 - 4*a^2*b^6*c^3 + 6*a^4*b^4
*c^2 - 4*a^6*b^2*c + a^8) - 2*(b^7*c*d^2 + 5*a^2*b^5*d^2)*log(abs(sqrt(d*x
 + c)*b + a))/(b^9*c^4 - 4*a^2*b^7*c^3 + 6*a^4*b^5*c^2 - 4*a^6*b^3*c + a^8
*b) - 1/2*(15*a*b^5*c^2*d^2 + 10*a^3*b^3*c*d^2 - a^5*b*d^2)*arctan(sqrt(d*
x + c)/sqrt(-c))/((b^8*c^5 - 4*a^2*b^6*c^4 + 6*a^4*b^4*c^3 - 4*a^6*b^2*c^2
 + a^8*c)*sqrt(-c)) - 1/2*(7*a*b^6*c^4*d^2 - a^3*b^4*c^3*d^2 - 7*a^5*b^2*c
^2*d^2 + a^7*c*d^2 + (11*a*b^6*c^2*d^2 - 10*a^3*b^4*c*d^2 - a^5*b^2*d^2)*(
d*x + c)^2 - (2*b^7*c^3*d^2 - 3*a^2*b^5*c^2*d^2 + a^6*b*d^2)*(d*x + c)^(3/
2) - (19*a*b^6*c^3*d^2 - 14*a^3*b^4*c^2*d^2 - 5*a^5*b^2*c*d^2)*(d*x + c) +
 3*(b^7*c^4*d^2 - 2*a^2*b^5*c^3*d^2 + a^4*b^3*c^2*d^2)*sqrt(d*x + c))/((b^
2*c - a^2)^4*(sqrt(d*x + c)*b + a)*c*d^2*x^2)
```

3.645.9 Mupad [B] (verification not implemented)

Time = 21.42 (sec) , antiderivative size = 1441, normalized size of antiderivative = 4.71

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(x^3*(a + b*(c + d*x)^(1/2))^2),x)
```

output
```
(((5*a^3*b^2*d^2 + 19*a*b^4*c*d^2)*(c + d*x))/(2*(b^2*c - a^2)*(a^4 + b^4*
c^2 - 2*a^2*b^2*c)) + ((a^3*b^2*d^2 + 11*a*b^4*c*d^2)*(c + d*x)^2)/(2*c*(a
^6 - b^6*c^3 - 3*a^4*b^2*c + 3*a^2*b^4*c^2)) - (a*(7*b^4*c^2*d^2 - a^4*d^2
 + 6*a^2*b^2*c*d^2))/(2*(b^2*c - a^2)*(a^4 + b^4*c^2 - 2*a^2*b^2*c)) + (b*
(a^2*d^2 + 2*b^2*c*d^2)*(c + d*x)^(3/2))/(2*c*(a^4 + b^4*c^2 - 2*a^2*b^2*c
)) - (3*b^3*c*d^2*(c + d*x)^(1/2))/(2*(a^4 + b^4*c^2 - 2*a^2*b^2*c)))/(a*(
c + d*x)^2 + b*(c + d*x)^(5/2) + a*c^2 - 2*a*c*(c + d*x) - 2*b*c*(c + d*x)
^(3/2) + b*c^2*(c + d*x)^(1/2)) + log(a + b*(c + d*x)^(1/2))*((10*b^4*d^2)
/(b^2*c - a^2)^3 - (12*b^6*c*d^2)/(b^2*c - a^2)^4) + (log((a*b^4*d^4*(a^6
- 44*b^6*c^3 + 2*a^4*b^2*c - 103*a^2*b^4*c^2))/(4*c^2*(b^2*c - a^2)^6) - (
b*d^2*((b^2*d^2*(a^2*(c + d*x)^(1/2) + 4*a*b*c + 3*b^2*c*(c + d*x)^(1/2))*
(a^5*(c^3)^(1/2) + 4*b^5*c^4 + 20*a^2*b^3*c^3 - 10*a^3*b^2*c*(c^3)^(1/2) -
 15*a*b^4*c^2*(c^3)^(1/2)))/(2*c^3*(b^2*c - a^2)^4) - (b^3*d^2*(c + d*x)^(
1/2)*(6*b^4*c^2 - a^4 + 19*a^2*b^2*c))/(c*(b^2*c - a^2)^3) + (a*b^2*d^2*(7
*b^2*c - a^2))/(2*c*(b^2*c - a^2)^2))*(a^5*(c^3)^(1/2) + 4*b^5*c^4 + 20*a^
2*b^3*c^3 - 10*a^3*b^2*c*(c^3)^(1/2) - 15*a*b^4*c^2*(c^3)^(1/2)))/(4*c^3*(
b^2*c - a^2)^4) + (a^2*b^5*d^4*(11*b^2*c + a^2)^2*(c + d*x)^(1/2))/(4*c^2*
(b^2*c - a^2)^6))*(4*b^6*c^4*d^2 + 20*a^2*b^4*c^3*d^2 + a^5*b*d^2*(c^3)^(1
/2) - 10*a^3*b^3*c*d^2*(c^3)^(1/2) - 15*a*b^5*c^2*d^2*(c^3)^(1/2)))/(4*(a^
8*c^3 + b^8*c^7 - 4*a^6*b^2*c^4 + 6*a^4*b^4*c^5 - 4*a^2*b^6*c^6)) + (lo...
```

3.645. $\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx$

3.646 $\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

3.646.1 Optimal result

Integrand size = 21, antiderivative size = 324

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx = &-\frac{4a(a^2-b^2c)^3\sqrt{a+b\sqrt{c+dx}}}{b^8d^4} \\ &+\frac{4(a^2-b^2c)^2(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^8d^4} \\ &-\frac{12a(7a^2-3b^2c)(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^8d^4} \\ &+\frac{4(35a^4-30a^2b^2c+3b^4c^2)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^8d^4} \\ &-\frac{20a(7a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^8d^4} \\ &+\frac{12(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8d^4} \\ &-\frac{28a\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^8d^4}+\frac{4\left(a+b\sqrt{c+dx}\right)^{15/2}}{15b^8d^4}\end{aligned}$$

output
```
4/3*(-b^2*c+a^2)^2*(-b^2*c+7*a^2)*(a+b*(d*x+c)^(1/2))^(3/2)/b^8/d^4-12/5*a
*(-3*b^2*c+7*a^2)*(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(5/2)/b^8/d^4+4/7*(3*b^
4*c^2-30*a^2*b^2*c+35*a^4)*(a+b*(d*x+c)^(1/2))^(7/2)/b^8/d^4-20/9*a*(-3*b^
2*c+7*a^2)*(a+b*(d*x+c)^(1/2))^(9/2)/b^8/d^4+12/11*(-b^2*c+7*a^2)*(a+b*(d*
x+c)^(1/2))^(11/2)/b^8/d^4-28/13*a*(a+b*(d*x+c)^(1/2))^(13/2)/b^8/d^4+4/15
*(a+b*(d*x+c)^(1/2))^(15/2)/b^8/d^4-4*a*(-b^2*c+a^2)^3*(a+b*(d*x+c)^(1/2))
^(1/2)/b^8/d^4
```

3.646.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.72

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$
$$= \frac{4\sqrt{a+b\sqrt{c+dx}}\left(-14336a^7+768a^5b^2(58c-7dx)+7168a^6b\sqrt{c+dx}-640a^4b^3(32c-7dx)\sqrt{c+dx}+2\right.}{}$$

input
```
Integrate[x^3/Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(4*Sqrt[a + b*Sqrt[c + d*x]]*(-14336*a^7 + 768*a^5*b^2*(58*c - 7*d*x) + 71
68*a^6*b*Sqrt[c + d*x] - 640*a^4*b^3*(32*c - 7*d*x)*Sqrt[c + d*x] + 24*a^2
*b^5*Sqrt[c + d*x]*(784*c^2 - 356*c*d*x + 147*d^2*x^2) - 16*a^3*b^4*(2936*
c^2 - 680*c*d*x + 245*d^2*x^2) + 6*a*b^6*(2880*c^3 - 928*c^2*d*x + 658*c*d
^2*x^2 - 539*d^3*x^3) - 39*b^7*Sqrt[c + d*x]*(128*c^3 - 96*c^2*d*x + 84*c*
d^2*x^2 - 77*d^3*x^3)))/(45045*b^8*d^4)
```

3.646.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$
$$\Big\downarrow \text{ 896}$$
$$\frac{\int \frac{d^3x^3}{\sqrt{a+b\sqrt{c+dx}}}d(c+dx)}{d^4}$$
$$\Big\downarrow \text{ 25}$$
$$-\frac{\int -\frac{d^3x^3}{\sqrt{a+b\sqrt{c+dx}}}d(c+dx)}{d^4}$$
$$\Big\downarrow \text{ 1732}$$

$$-\frac{2\int -\frac{d^3x^3\sqrt{c+dx}}{\sqrt{a+b\sqrt{c+dx}}}d\sqrt{c+dx}}{d^4}$$

$\Big\downarrow$ 522

$$-\frac{2\int \left(-\frac{\left(a+b\sqrt{c+dx}\right)^{13/2}}{b^7}+\frac{7a\left(a+b\sqrt{c+dx}\right)^{11/2}}{b^7}+\frac{3\left(b^2c-7a^2\right)\left(a+b\sqrt{c+dx}\right)^{9/2}}{b^7}-\frac{5\left(3ab^2c-7a^3\right)\left(a+b\sqrt{c+dx}\right)^{7/2}}{b^7}+\frac{\left(-35a^4+30b^2ca^2-\right.}{}$$

$\Big\downarrow$ 2009

$$-\frac{2\left(-\frac{6\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8}+\frac{10a\left(7a^2-3b^2c\right)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^8}+\frac{6a\left(7a^2-3b^2c\right)\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^8}-\frac{2\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right.}{3b^8}\right.}{}$$

input
```
Int[x^3/Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(-2*((2*a*(a^2 - b^2*c)^3*Sqrt[a + b*Sqrt[c + d*x]])/b^8 - (2*(a^2 - b^2*c
)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^8) + (6*a*(7*a^2 - 3
*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^8) - (2*(35*a^4 -
30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^8) + (10*a*(7*
a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^8) - (6*(7*a^2 - b^2*c)*(
a + b*Sqrt[c + d*x])^(11/2))/(11*b^8) + (14*a*(a + b*Sqrt[c + d*x])^(13/2)
)/(13*b^8) - (2*(a + b*Sqrt[c + d*x])^(15/2))/(15*b^8)))/d^4
```

3.646.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

3.646. $\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.646.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.19

method	result
derivativedivides	$-\frac{4\left(-\frac{(a+b\sqrt{dx+c})^{\frac{15}{2}}}{15}+\frac{7a(a+b\sqrt{dx+c})^{\frac{13}{2}}}{13}+\frac{(3b^2c-21a^2)(a+b\sqrt{dx+c})^{\frac{11}{2}}}{11}+\frac{\left(8(-b^2c+a^2)a+2a(-2b^2c+6a^2)+(-3b^2c+15a^2)\right.}{9}\right.}{}$
default	$-\frac{4\left(-\frac{(a+b\sqrt{dx+c})^{\frac{15}{2}}}{15}+\frac{7a(a+b\sqrt{dx+c})^{\frac{13}{2}}}{13}+\frac{(3b^2c-21a^2)(a+b\sqrt{dx+c})^{\frac{11}{2}}}{11}+\frac{\left(8(-b^2c+a^2)a+2a(-2b^2c+6a^2)+(-3b^2c+15a^2)\right.}{9}\right.}{}$

input
```
int(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/d^4/b^8*(-1/15*(a+b*(d*x+c)^(1/2))^(15/2)+7/13*a*(a+b*(d*x+c)^(1/2))^(1
3/2)+1/11*(3*b^2*c-21*a^2)*(a+b*(d*x+c)^(1/2))^(11/2)+1/9*(8*(-b^2*c+a^2)*
a+2*a*(-2*b^2*c+6*a^2)+(-3*b^2*c+15*a^2)*a)*(a+b*(d*x+c)^(1/2))^(9/2)+1/7*
(-(-b^2*c+a^2)*(-2*b^2*c+6*a^2)-8*a^2*(-b^2*c+a^2)-(-b^2*c+a^2)^2+(-8*(-b^
2*c+a^2)*a-2*a*(-2*b^2*c+6*a^2))*a)*(a+b*(d*x+c)^(1/2))^(7/2)+1/5*(6*(-b^2
*c+a^2)^2*a+((-b^2*c+a^2)*(-2*b^2*c+6*a^2)+8*a^2*(-b^2*c+a^2)+(-b^2*c+a^2)
^2)*a)*(a+b*(d*x+c)^(1/2))^(5/2)+1/3*(-(-b^2*c+a^2)^3-6*(-b^2*c+a^2)^2*a^2
)*(a+b*(d*x+c)^(1/2))^(3/2)+(-b^2*c+a^2)^3*a*(a+b*(d*x+c)^(1/2))^(1/2))
```

3.646.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx = -\frac{4\left(3234\,ab^6d^3x^3-17280\,ab^6c^3+46976\,a^3b^4c^2-44544\,a^5b^2c+14336\,a^7-28\,(141\,ab^6c-140\,a^3b^4)d^2x^2\right.}{}$$

input
```
integrate(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-4/45045*(3234*a*b^6*d^3*x^3 - 17280*a*b^6*c^3 + 46976*a^3*b^4*c^2 - 44544
*a^5*b^2*c + 14336*a^7 - 28*(141*a*b^6*c - 140*a^3*b^4)*d^2*x^2 + 64*(87*a
*b^6*c^2 - 170*a^3*b^4*c + 84*a^5*b^2)*d*x - (3003*b^7*d^3*x^3 - 4992*b^7*
c^3 + 18816*a^2*b^5*c^2 - 20480*a^4*b^3*c + 7168*a^6*b - 252*(13*b^7*c - 1
4*a^2*b^5)*d^2*x^2 + 32*(117*b^7*c^2 - 267*a^2*b^5*c + 140*a^4*b^3)*d*x)*s
qrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^8*d^4)
```

3.646.6 Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.10

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$

$$= \begin{cases} \frac{2\left(\begin{cases} \frac{2\left(-\frac{7a\left(a+b\sqrt{c+dx}\right)^{\frac{13}{2}}}{13b^6}+\frac{\left(a+b\sqrt{c+dx}\right)^{\frac{15}{2}}}{15b^6}+\frac{\left(a+b\sqrt{c+dx}\right)^{\frac{11}{2}}\cdot\left(21a^2-3b^2c\right)}{11b^6}+\frac{\left(a+b\sqrt{c+dx}\right)^{\frac{9}{2}}\left(-35a^3+15ab^2c\right)}{9b^6}+\frac{\left(a+b\sqrt{c+dx}\right)^{\frac{7}{2}}\cdot\left(35a^4-30a^2b^2c+3b^4\right.}{7b^6}\right.}{} \\ \frac{d^4x^4}{8\sqrt{a}} \end{cases}\right.}{} \\ \frac{x^4}{4\sqrt{a+b\sqrt{c}}} \end{cases}$$

input
```
integrate(x**3/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

output
```
Piecewise((2*Piecewise((2*(-7*a*(a + b*sqrt(c + d*x))**(13/2)/(13*b**6) +
(a + b*sqrt(c + d*x))**(15/2)/(15*b**6) + (a + b*sqrt(c + d*x))**(11/2)*(2
1*a**2 - 3*b**2*c)/(11*b**6) + (a + b*sqrt(c + d*x))**(9/2)*(-35*a**3 + 15
*a*b**2*c)/(9*b**6) + (a + b*sqrt(c + d*x))**(7/2)*(35*a**4 - 30*a**2*b**2
*c + 3*b**4*c**2)/(7*b**6) + (a + b*sqrt(c + d*x))**(5/2)*(-21*a**5 + 30*a
**3*b**2*c - 9*a*b**4*c**2)/(5*b**6) + (a + b*sqrt(c + d*x))**(3/2)*(7*a**
6 - 15*a**4*b**2*c + 9*a**2*b**4*c**2 - b**6*c**3)/(3*b**6) + sqrt(a + b*s
qrt(c + d*x))*(-a**7 + 3*a**5*b**2*c - 3*a**3*b**4*c**2 + a*b**6*c**3)/b**
6)/b**2, Ne(b, 0)), (d**4*x**4/(8*sqrt(a)), True))/d**4, Ne(d, 0)), (x**4/
(4*sqrt(a + b*sqrt(c))), True))
```

3.646.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4\left(3003\left(\sqrt{dx+c}b+a\right)^{\frac{15}{2}} - 24255\left(\sqrt{dx+c}b+a\right)^{\frac{13}{2}}a - 12285\left(b^2c-7a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{11}{2}} + 25025\left(3\right.\right.}{}$$

input
```
integrate(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
4/45045*(3003*(sqrt(d*x + c)*b + a)^(15/2) - 24255*(sqrt(d*x + c)*b + a)^(
13/2)*a - 12285*(b^2*c - 7*a^2)*(sqrt(d*x + c)*b + a)^(11/2) + 25025*(3*a*
b^2*c - 7*a^3)*(sqrt(d*x + c)*b + a)^(9/2) + 6435*(3*b^4*c^2 - 30*a^2*b^2*
c + 35*a^4)*(sqrt(d*x + c)*b + a)^(7/2) - 27027*(3*a*b^4*c^2 - 10*a^3*b^2*
c + 7*a^5)*(sqrt(d*x + c)*b + a)^(5/2) - 15015*(b^6*c^3 - 9*a^2*b^4*c^2 +
15*a^4*b^2*c - 7*a^6)*(sqrt(d*x + c)*b + a)^(3/2) + 45045*(a*b^6*c^3 - 3*a
^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*sqrt(sqrt(d*x + c)*b + a))/(b^8*d^4)
```

3.646.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.26

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx = -\frac{4\left(15015\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}b^6c^3 - 45045\sqrt{\sqrt{dx+c}b+a}\,ab^6c^3 - 19305\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}}b^4c^2 + 81081\left(\right.\right.}{}$$

input
```
integrate(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
-4/45045*(15015*(sqrt(d*x + c)*b + a)^(3/2)*b^6*c^3 - 45045*sqrt(sqrt(d*x
+ c)*b + a)*a*b^6*c^3 - 19305*(sqrt(d*x + c)*b + a)^(7/2)*b^4*c^2 + 81081*
(sqrt(d*x + c)*b + a)^(5/2)*a*b^4*c^2 - 135135*(sqrt(d*x + c)*b + a)^(3/2)
*a^2*b^4*c^2 + 135135*sqrt(sqrt(d*x + c)*b + a)*a^3*b^4*c^2 + 12285*(sqrt(
d*x + c)*b + a)^(11/2)*b^2*c - 75075*(sqrt(d*x + c)*b + a)^(9/2)*a*b^2*c +
 193050*(sqrt(d*x + c)*b + a)^(7/2)*a^2*b^2*c - 270270*(sqrt(d*x + c)*b +
a)^(5/2)*a^3*b^2*c + 225225*(sqrt(d*x + c)*b + a)^(3/2)*a^4*b^2*c - 135135
*sqrt(sqrt(d*x + c)*b + a)*a^5*b^2*c - 3003*(sqrt(d*x + c)*b + a)^(15/2) +
 24255*(sqrt(d*x + c)*b + a)^(13/2)*a - 85995*(sqrt(d*x + c)*b + a)^(11/2)
*a^2 + 175175*(sqrt(d*x + c)*b + a)^(9/2)*a^3 - 225225*(sqrt(d*x + c)*b +
a)^(7/2)*a^4 + 189189*(sqrt(d*x + c)*b + a)^(5/2)*a^5 - 105105*(sqrt(d*x +
 c)*b + a)^(3/2)*a^6 + 45045*sqrt(sqrt(d*x + c)*b + a)*a^7)/(b^8*d^4)
```

3.646.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{x^3}{\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

input
```
int(x^3/(a + b*(c + d*x)^(1/2))^(1/2),x)
```

output
```
int(x^3/(a + b*(c + d*x)^(1/2))^(1/2), x)
```

3.647 $\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

3.647.1 Optimal result

Integrand size = 21, antiderivative size = 222

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx = -\frac{4a(a^2-b^2c)^2\sqrt{a+b\sqrt{c+dx}}}{b^6d^3} + \frac{4(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3} - \frac{8a(5a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3} + \frac{8(5a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6d^3} - \frac{20a\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^6d^3} + \frac{4\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^6d^3}$$

output
```
4/3*(b^4*c^2-6*a^2*b^2*c+5*a^4)*(a+b*(d*x+c)^(1/2))^(3/2)/b^6/d^3-8/5*a*(-
3*b^2*c+5*a^2)*(a+b*(d*x+c)^(1/2))^(5/2)/b^6/d^3+8/7*(-b^2*c+5*a^2)*(a+b*(
d*x+c)^(1/2))^(7/2)/b^6/d^3-20/9*a*(a+b*(d*x+c)^(1/2))^(9/2)/b^6/d^3+4/11*
(a+b*(d*x+c)^(1/2))^(11/2)/b^6/d^3-4*a*(-b^2*c+a^2)^2*(a+b*(d*x+c)^(1/2))^
(1/2)/b^6/d^3
```

3.647.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.66

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4\sqrt{a+b\sqrt{c+dx}}\left(-1280a^5+96a^3b^2(28c-5dx)+640a^4b\sqrt{c+dx}-16a^2b^3(74c-25dx)\sqrt{c+dx}+15b^5\right.}{3465b^6d^3}$$

input
```
Integrate[x^2/Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(4*Sqrt[a + b*Sqrt[c + d*x]]*(-1280*a^5 + 96*a^3*b^2*(28*c - 5*d*x) + 640*
a^4*b*Sqrt[c + d*x] - 16*a^2*b^3*(74*c - 25*d*x)*Sqrt[c + d*x] + 15*b^5*Sq
rt[c + d*x]*(32*c^2 - 24*c*d*x + 21*d^2*x^2) - 2*a*b^4*(736*c^2 - 244*c*d*
x + 175*d^2*x^2)))/(3465*b^6*d^3)
```

3.647.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {896, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int \frac{d^2x^2}{\sqrt{a+b\sqrt{c+dx}}}d(c+dx)}{d^3}$$

$$\Big\downarrow \text{1732}$$

$$\frac{2\int \frac{d^2x^2\sqrt{c+dx}}{\sqrt{a+b\sqrt{c+dx}}}d\sqrt{c+dx}}{d^3}$$

$$\Big\downarrow \text{522}$$

$$\frac{2\int \left(\frac{\left(a+b\sqrt{c+dx}\right)^{9/2}}{b^5}-\frac{5a\left(a+b\sqrt{c+dx}\right)^{7/2}}{b^5}-\frac{2\left(b^2c-5a^2\right)\left(a+b\sqrt{c+dx}\right)^{5/2}}{b^5}-\frac{2\left(5a^3-3ab^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{b^5}+\frac{\left(5a^4-6b^2ca^2+b^4c^2\right)\sqrt{a+\right.}{b^5}}{d^3}$$

$$\downarrow \text{2009}$$

$$2\left(\frac{4(5a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6}-\frac{4a(5a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6}-\frac{2a(a^2-b^2c)^2\sqrt{a+b\sqrt{c+dx}}}{b^6}+\frac{2(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6}+\right.$$
$$\overline{\qquad\qquad d^3 \qquad\qquad}$$

input
```
Int[x^2/Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(2*((-2*a*(a^2 - b^2*c)^2*Sqrt[a + b*Sqrt[c + d*x]])/b^6 + (2*(5*a^4 - 6*a
^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^6) - (4*a*(5*a^2 - 3
*b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^6) + (4*(5*a^2 - b^2*c)*(a + b*S
qrt[c + d*x])^(7/2))/(7*b^6) - (10*a*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^6)
+ (2*(a + b*Sqrt[c + d*x])^(11/2))/(11*b^6)))/d^3
```

3.647.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.647.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\frac{4(a+b\sqrt{dx+c})^{\frac{11}{2}}}{11}-\frac{20a(a+b\sqrt{dx+c})^{\frac{9}{2}}}{9}-\frac{4(2b^2c-10a^2)(a+b\sqrt{dx+c})^{\frac{7}{2}}}{7}-\frac{4(4(-b^2c+a^2)a+a(-2b^2c+6a^2))(a+b\sqrt{dx+c})^{\frac{5}{2}}}{5}-\frac{4(-}{}}{d^3b^6}$
default	$\frac{\frac{4(a+b\sqrt{dx+c})^{\frac{11}{2}}}{11}-\frac{20a(a+b\sqrt{dx+c})^{\frac{9}{2}}}{9}-\frac{4(2b^2c-10a^2)(a+b\sqrt{dx+c})^{\frac{7}{2}}}{7}-\frac{4(4(-b^2c+a^2)a+a(-2b^2c+6a^2))(a+b\sqrt{dx+c})^{\frac{5}{2}}}{5}-\frac{4(-}{}}{d^3b^6}$

input
```
int(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/d^3/b^6*(1/11*(a+b*(d*x+c)^(1/2))^(11/2)-5/9*a*(a+b*(d*x+c)^(1/2))^(9/2)
-1/7*(2*b^2*c-10*a^2)*(a+b*(d*x+c)^(1/2))^(7/2)-1/5*(4*(-b^2*c+a^2)*a+a*(-
2*b^2*c+6*a^2))*(a+b*(d*x+c)^(1/2))^(5/2)-1/3*(-(-b^2*c+a^2)^2-4*a^2*(-b^2
*c+a^2))*(a+b*(d*x+c)^(1/2))^(3/2)-(-b^2*c+a^2)^2*a*(a+b*(d*x+c)^(1/2))^(1
/2))
```

3.647.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx =$$
$$-\frac{4\left(350\,ab^4d^2x^2+1472\,ab^4c^2-2688\,a^3b^2c+1280\,a^5-8\,(61\,ab^4c-60\,a^3b^2)dx-(315\,b^5d^2x^2+480\,b^5c^2\right.}{3465\,b^6d^3}$$

input
```
integrate(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-4/3465*(350*a*b^4*d^2*x^2 + 1472*a*b^4*c^2 - 2688*a^3*b^2*c + 1280*a^5 -
8*(61*a*b^4*c - 60*a^3*b^2)*d*x - (315*b^5*d^2*x^2 + 480*b^5*c^2 - 1184*a^
2*b^3*c + 640*a^4*b - 40*(9*b^5*c - 10*a^2*b^3)*d*x)*sqrt(d*x + c))*sqrt(s
qrt(d*x + c)*b + a)/(b^6*d^3)
```

3.647.6 Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$

$$= \begin{cases} \frac{2\left(\begin{cases} \frac{2\left(-\frac{5a(a+b\sqrt{c+dx})^{\frac{9}{2}}}{9b^4}+\frac{(a+b\sqrt{c+dx})^{\frac{11}{2}}}{11b^4}+\frac{(a+b\sqrt{c+dx})^{\frac{7}{2}}\cdot(10a^2-2b^2c)}{7b^4}+\frac{(a+b\sqrt{c+dx})^{\frac{5}{2}}(-10a^3+6ab^2c)}{5b^4}+\frac{(a+b\sqrt{c+dx})^{\frac{3}{2}}\cdot(5a^4-6a^2b^2c+b^4c^2)}{3b^4}+ \right.}{b^2} \\ \frac{d^3x^3}{6\sqrt{a}} \end{cases}\right)}{d^3} \\ \frac{x^3}{3\sqrt{a+b\sqrt{c}}} \end{cases}$$

input
```
integrate(x**2/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

output
```
Piecewise((2*Piecewise((2*(-5*a*(a + b*sqrt(c + d*x))**(9/2)/(9*b**4) + (a
 + b*sqrt(c + d*x))**(11/2)/(11*b**4) + (a + b*sqrt(c + d*x))**(7/2)*(10*a
**2 - 2*b**2*c)/(7*b**4) + (a + b*sqrt(c + d*x))**(5/2)*(-10*a**3 + 6*a*b*
*2*c)/(5*b**4) + (a + b*sqrt(c + d*x))**(3/2)*(5*a**4 - 6*a**2*b**2*c + b*
*4*c**2)/(3*b**4) + sqrt(a + b*sqrt(c + d*x))*(-a**5 + 2*a**3*b**2*c - a*b
**4*c**2)/b**4)/b**2, Ne(b, 0)), (d**3*x**3/(6*sqrt(a)), True))/d**3, Ne(d
, 0)), (x**3/(3*sqrt(a + b*sqrt(c))), True))
```

3.647.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$

$$= \frac{4\left(315\left(\sqrt{dx+c}b+a\right)^{\frac{11}{2}}-1925\left(\sqrt{dx+c}b+a\right)^{\frac{9}{2}}a-990\left(b^2c-5a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}}+1386\left(3ab^2c-\right.\right.}{}$$

input
```
integrate(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
4/3465*(315*(sqrt(d*x + c)*b + a)^(11/2) - 1925*(sqrt(d*x + c)*b + a)^(9/2
)*a - 990*(b^2*c - 5*a^2)*(sqrt(d*x + c)*b + a)^(7/2) + 1386*(3*a*b^2*c -
5*a^3)*(sqrt(d*x + c)*b + a)^(5/2) + 1155*(b^4*c^2 - 6*a^2*b^2*c + 5*a^4)*
(sqrt(d*x + c)*b + a)^(3/2) - 3465*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*sqrt(sq
rt(d*x + c)*b + a))/(b^6*d^3)
```

3.647.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$
$$= \frac{4\left(1155\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}b^4c^2-3465\sqrt{\sqrt{dx+c}b+a}\,ab^4c^2-990\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}}b^2c+4158\left(\sqrt{dx+c}b\right.\right.}{}$$

input
```
integrate(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
4/3465*(1155*(sqrt(d*x + c)*b + a)^(3/2)*b^4*c^2 - 3465*sqrt(sqrt(d*x + c)
*b + a)*a*b^4*c^2 - 990*(sqrt(d*x + c)*b + a)^(7/2)*b^2*c + 4158*(sqrt(d*x
 + c)*b + a)^(5/2)*a*b^2*c - 6930*(sqrt(d*x + c)*b + a)^(3/2)*a^2*b^2*c +
6930*sqrt(sqrt(d*x + c)*b + a)*a^3*b^2*c + 315*(sqrt(d*x + c)*b + a)^(11/2
) - 1925*(sqrt(d*x + c)*b + a)^(9/2)*a + 4950*(sqrt(d*x + c)*b + a)^(7/2)*
a^2 - 6930*(sqrt(d*x + c)*b + a)^(5/2)*a^3 + 5775*(sqrt(d*x + c)*b + a)^(3
/2)*a^4 - 3465*sqrt(sqrt(d*x + c)*b + a)*a^5)/(b^6*d^3)
```

3.647.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{x^2}{\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

input
```
int(x^2/(a + b*(c + d*x)^(1/2))^(1/2),x)
```

output
```
int(x^2/(a + b*(c + d*x)^(1/2))^(1/2), x)
```

3.648 $\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}} dx$

3.648.1 Optimal result

Integrand size = 19, antiderivative size = 131

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}} dx = -\frac{4a(a^2-b^2c)\sqrt{a+b\sqrt{c+dx}}}{b^4d^2} + \frac{4(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2} - \frac{12a\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2} + \frac{4\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2}$$

output
```
4/3*(-b^2*c+3*a^2)*(a+b*(d*x+c)^(1/2))^(3/2)/b^4/d^2-12/5*a*(a+b*(d*x+c)^(
1/2))^(5/2)/b^4/d^2+4/7*(a+b*(d*x+c)^(1/2))^(7/2)/b^4/d^2-4*a*(-b^2*c+a^2)
*(a+b*(d*x+c)^(1/2))^(1/2)/b^4/d^2
```

3.648.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.64

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}} dx = \frac{4\sqrt{a+b\sqrt{c+dx}}\left(-48a^3+2ab^2(26c-9dx)+24a^2b\sqrt{c+dx}+5b^3\sqrt{c+dx}(-4c+3dx)\right)}{105b^4d^2}$$

input
```
Integrate[x/Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(4*Sqrt[a + b*Sqrt[c + d*x]]*(-48*a^3 + 2*a*b^2*(26*c - 9*d*x) + 24*a^2*b*
Sqrt[c + d*x] + 5*b^3*Sqrt[c + d*x]*(-4*c + 3*d*x)))/(105*b^4*d^2)
```

3.648.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int \frac{dx}{\sqrt{a+b\sqrt{c+dx}}}d(c+dx)}{d^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{dx}{\sqrt{a+b\sqrt{c+dx}}}d(c+dx)}{d^2}$$

$$\Big\downarrow \text{1732}$$

$$-\frac{2\int -\frac{dx\sqrt{c+dx}}{\sqrt{a+b\sqrt{c+dx}}}d\sqrt{c+dx}}{d^2}$$

$$\Big\downarrow \text{522}$$

$$-\frac{2\int \left(-\frac{\left(a+b\sqrt{c+dx}\right)^{5/2}}{b^3}+\frac{3a\left(a+b\sqrt{c+dx}\right)^{3/2}}{b^3}+\frac{\left(b^2c-3a^2\right)\sqrt{a+b\sqrt{c+dx}}}{b^3}+\frac{a^3-ab^2c}{b^3\sqrt{a+b\sqrt{c+dx}}}\right)d\sqrt{c+dx}}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(-\frac{2\left(3a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4}+\frac{2a\left(a^2-b^2c\right)\sqrt{a+b\sqrt{c+dx}}}{b^4}-\frac{2\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4}+\frac{6a\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4}\right)}{d^2}$$

input
```
Int[x/Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(-2*((2*a*(a^2 - b^2*c)*Sqrt[a + b*Sqrt[c + d*x]])/b^4 - (2*(3*a^2 - b^2*c
)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^4) + (6*a*(a + b*Sqrt[c + d*x])^(5/2))
/(5*b^4) - (2*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^4)))/d^2
```

3.648.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.648.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$-\frac{4\left(-\frac{\left(a+b\sqrt{dx+c}\right)^{\frac{7}{2}}}{7}+\frac{3a\left(a+b\sqrt{dx+c}\right)^{\frac{5}{2}}}{5}+\frac{\left(b^{2}c-3a^{2}\right)\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}+\left(-b^{2}c+a^{2}\right)a\sqrt{a+b\sqrt{dx+c}}\right)}{d^{2}b^{4}}$	92
default	$-\frac{4\left(-\frac{\left(a+b\sqrt{dx+c}\right)^{\frac{7}{2}}}{7}+\frac{3a\left(a+b\sqrt{dx+c}\right)^{\frac{5}{2}}}{5}+\frac{\left(b^{2}c-3a^{2}\right)\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}+\left(-b^{2}c+a^{2}\right)a\sqrt{a+b\sqrt{dx+c}}\right)}{d^{2}b^{4}}$	92

input
```
int(x/(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/d^2/b^4*(-1/7*(a+b*(d*x+c)^(1/2))^(7/2)+3/5*a*(a+b*(d*x+c)^(1/2))^(5/2)
+1/3*(b^2*c-3*a^2)*(a+b*(d*x+c)^(1/2))^(3/2)+(-b^2*c+a^2)*a*(a+b*(d*x+c)^(
1/2))^(1/2))
```

3.648.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.54

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$
$$= -\frac{4\left(18\,ab^2dx-52\,ab^2c+48\,a^3-\left(15\,b^3dx-20\,b^3c+24\,a^2b\right)\sqrt{dx+c}\right)\sqrt{\sqrt{dx+c}b+a}}{105\,b^4d^2}$$

input
```
integrate(x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-4/105*(18*a*b^2*d*x - 52*a*b^2*c + 48*a^3 - (15*b^3*d*x - 20*b^3*c + 24*a
^2*b)*sqrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^4*d^2)
```

3.648.6 Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.18

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$
$$= \begin{cases} \frac{2\left(\begin{cases} \frac{2\left(-\frac{3a\left(a+b\sqrt{c+dx}\right)^{\frac{5}{2}}}{5b^2}+\frac{\left(a+b\sqrt{c+dx}\right)^{\frac{7}{2}}}{7b^2}+\frac{\left(a+b\sqrt{c+dx}\right)^{\frac{3}{2}}\cdot\left(3a^2-b^2c\right)}{3b^2}+\frac{\sqrt{a+b\sqrt{c+dx}}\left(-a^3+ab^2c\right)}{b^2}\right)}{b^2} & \text{for}\ b\neq 0 \\ \frac{-\frac{c(c+dx)}{2}+\frac{(c+dx)^2}{4}}{\sqrt{a}} & \text{otherwise} \end{cases}\right)}{d^2} & \text{for}\ d\neq 0 \\ \frac{x^2}{2\sqrt{a+b\sqrt{c}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

output
```
Piecewise((2*Piecewise((2*(-3*a*(a + b*sqrt(c + d*x))**(5/2)/(5*b**2) + (a
 + b*sqrt(c + d*x))**(7/2)/(7*b**2) + (a + b*sqrt(c + d*x))**(3/2)*(3*a**2
 - b**2*c)/(3*b**2) + sqrt(a + b*sqrt(c + d*x))*(-a**3 + a*b**2*c)/b**2)/b
**2, Ne(b, 0)), ((-c*(c + d*x)/2 + (c + d*x)**2/4)/sqrt(a), True))/d**2, N
e(d, 0)), (x**2/(2*sqrt(a + b*sqrt(c))), True))
```

3.648.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.71

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4\left(15\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}}-63\left(\sqrt{dx+c}b+a\right)^{\frac{5}{2}}a-35\left(b^2c-3a^2\right)\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}+105\left(ab^2c-a^3\right)\sqrt{\sqrt{d}}\right.}{105\,b^4d^2}$$

input
```
integrate(x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
4/105*(15*(sqrt(d*x + c)*b + a)^(7/2) - 63*(sqrt(d*x + c)*b + a)^(5/2)*a -
 35*(b^2*c - 3*a^2)*(sqrt(d*x + c)*b + a)^(3/2) + 105*(a*b^2*c - a^3)*sqrt
(sqrt(d*x + c)*b + a))/(b^4*d^2)
```

3.648.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.88

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx = -\frac{4\left(35\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}b^2c-105\sqrt{\sqrt{dx+c}b+a}\,ab^2c-15\left(\sqrt{dx+c}b+a\right)^{\frac{7}{2}}+63\left(\sqrt{dx+c}b+a\right)^{\frac{5}{2}}a-\right.}{105\,b^4d^2}$$

input
```
integrate(x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
-4/105*(35*(sqrt(d*x + c)*b + a)^(3/2)*b^2*c - 105*sqrt(sqrt(d*x + c)*b +
a)*a*b^2*c - 15*(sqrt(d*x + c)*b + a)^(7/2) + 63*(sqrt(d*x + c)*b + a)^(5/
2)*a - 105*(sqrt(d*x + c)*b + a)^(3/2)*a^2 + 105*sqrt(sqrt(d*x + c)*b + a)
*a^3)/(b^4*d^2)
```

3.648.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{x}{\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

input
```
int(x/(a + b*(c + d*x)^(1/2))^(1/2),x)
```

output
```
int(x/(a + b*(c + d*x)^(1/2))^(1/2), x)
```

3.649 $\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}} \, dx$

3.649.1 Optimal result

Integrand size = 17, antiderivative size = 54

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}} \, dx = -\frac{4a\sqrt{a+b\sqrt{c+dx}}}{b^2d} + \frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}$$

output
```
4/3*(a+b*(d*x+c)^(1/2))^(3/2)/b^2/d-4*a*(a+b*(d*x+c)^(1/2))^(1/2)/b^2/d
```

3.649.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}} \, dx = \frac{4\left(-2a+b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{3b^2d}$$

input
```
Integrate[1/Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(4*(-2*a + b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/(3*b^2*d)
```

3.649.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {239, 774, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}dx$$

$$\Big\downarrow \text{239}$$

$$\frac{\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}d(c+dx)}{d}$$

$$\Big\downarrow \text{774}$$

$$\frac{2\int \frac{\sqrt{c+dx}}{\sqrt{a+b\sqrt{c+dx}}}d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{53}$$

$$\frac{2\int \left(\frac{\sqrt{a+b\sqrt{c+dx}}}{b}-\frac{a}{b\sqrt{a+b\sqrt{c+dx}}}\right)d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{2\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2}-\frac{2a\sqrt{a+b\sqrt{c+dx}}}{b^2}\right)}{d}$$

input
```
Int[1/Sqrt[a + b*Sqrt[c + d*x]],x]
```

output
```
(2*((-2*a*Sqrt[a + b*Sqrt[c + d*x]])/b^2 + (2*(a + b*Sqrt[c + d*x])^(3/2))
/(3*b^2)))/d
```

3.649.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.649.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{\frac{4\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}-4a\sqrt{a+b\sqrt{dx+c}}}{b^{2}d}$	41
default	$\frac{\frac{4\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}-4a\sqrt{a+b\sqrt{dx+c}}}{b^{2}d}$	41

input
```
int(1/(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/d/b^2*(1/3*(a+b*(d*x+c)^(1/2))^(3/2)-a*(a+b*(d*x+c)^(1/2))^(1/2))
```

3.649.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4\sqrt{\sqrt{dx+c}b+a}\left(\sqrt{dx+c}b-2a\right)}{3b^2d}$$

input
```
integrate(1/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
4/3*sqrt(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b - 2*a)/(b^2*d)
```

3.649.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \begin{cases} \frac{2\left(\begin{cases} \frac{2\left(-a\sqrt{a+b\sqrt{c+dx}}+\frac{(a+b\sqrt{c+dx})^{\frac{3}{2}}}{3}\right)}{b^2} & \text{for } b\neq 0 \\ \frac{c+dx}{2\sqrt{a}} & \text{otherwise} \end{cases}\right)}{d} & \text{for } d\neq 0 \\ \frac{x}{\sqrt{a+b\sqrt{c}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

output
```
Piecewise((2*Piecewise((2*(-a*sqrt(a + b*sqrt(c + d*x)) + (a + b*sqrt(c +
d*x))**(3/2)/3)/b**2, Ne(b, 0)), ((c + d*x)/(2*sqrt(a)), True))/d, Ne(d, 0
)), (x/sqrt(a + b*sqrt(c)), True))
```

3.649.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4\left(\frac{(\sqrt{dx+c}b+a)^{\frac{3}{2}}}{b^2}-\frac{3\sqrt{\sqrt{dx+c}b+a}a}{b^2}\right)}{3d}$$

input
```
integrate(1/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
4/3*((sqrt(d*x + c)*b + a)^(3/2)/b^2 - 3*sqrt(sqrt(d*x + c)*b + a)*a/b^2)/
d
```

3.649.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4\left(\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}-3\sqrt{\sqrt{dx+c}b+a}a\right)}{3\,b^2d}$$

input
```
integrate(1/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
4/3*((sqrt(d*x + c)*b + a)^(3/2) - 3*sqrt(sqrt(d*x + c)*b + a)*a)/(b^2*d)
```

3.649.9 Mupad [B] (verification not implemented)

Time = 17.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{4\left(a+b\sqrt{c+d\,x}\right)^{3/2}}{3\,b^2\,d}-\frac{4\,a\,\sqrt{a+b\sqrt{c+d\,x}}}{b^2\,d}$$

input
```
int(1/(a + b*(c + d*x)^(1/2))^(1/2),x)
```

output
```
(4*(a + b*(c + d*x)^(1/2))^(3/2))/(3*b^2*d) - (4*a*(a + b*(c + d*x)^(1/2))
^(1/2))/(b^2*d)
```

3.650 $\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx$

3.650.1 Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{\sqrt{a-b\sqrt{c}}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{\sqrt{a+b\sqrt{c}}}$$

output
```
-2*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a-b*c^(1/2))^(1/2))/(a-b*c^(1/2))^(1
/2)-2*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a+b*c^(1/2))^(1/2))/(a+b*c^(1/2))
^(1/2)
```

3.650.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a-b\sqrt{c}}}\right)}{\sqrt{-a-b\sqrt{c}}}+\frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a+b\sqrt{c}}}\right)}{\sqrt{-a+b\sqrt{c}}}$$

input
```
Integrate[1/(x*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

output
```
(2*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a - b*Sqrt[c]]])/Sqrt[-a - b*Sqr
t[c]] + (2*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a + b*Sqrt[c]]])/Sqrt[-a
 + b*Sqrt[c]]
```

3.650.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {896, 25, 1732, 561, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}dx$$

$$\Big\downarrow \text{ 896}$$

$$\int \frac{1}{dx\sqrt{a+b\sqrt{c+dx}}}d(c+dx)$$

$$\Big\downarrow \text{ 25}$$

$$-\int -\frac{1}{dx\sqrt{a+b\sqrt{c+dx}}}d(c+dx)$$

$$\Big\downarrow \text{ 1732}$$

$$-2\int -\frac{\sqrt{c+dx}}{dx\sqrt{a+b\sqrt{c+dx}}}d\sqrt{c+dx}$$

$$\Big\downarrow \text{ 561}$$

$$-\frac{4\int \frac{a-c-dx}{b\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}d\sqrt{a+b\sqrt{c+dx}}}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{4\int \frac{a-c-dx}{\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c}d\sqrt{a+b\sqrt{c+dx}}}{b^2}$$

$$\Big\downarrow \text{ 1480}$$

$$-\frac{4\left(-\frac{1}{2}\int \frac{1}{\frac{c+dx}{b^2}-\frac{a-b\sqrt{c}}{b^2}}d\sqrt{a+b\sqrt{c+dx}}-\frac{1}{2}\int \frac{1}{\frac{c+dx}{b^2}-\frac{a+b\sqrt{c}}{b^2}}d\sqrt{a+b\sqrt{c+dx}}\right)}{b^2}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{4\left(\frac{b^2\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{a-b\sqrt{c}}}+\frac{b^2\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{a+b\sqrt{c}}}\right)}{b^2}$$

input
```
Int[1/(x*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

output
```
(-4*((b^2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/(2*Sqrt[
a - b*Sqrt[c]]) + (b^2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c
]]])/(2*Sqrt[a + b*Sqrt[c]])))/b^2
```

3.650.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 561
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(n + 1) - 1)*(-c
/d + x^k/d)^m*Simp[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^k/d^2) + b*(x^(2*k)/d^2),
 x]^p, x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, m, p}, x] && Frac
tionQ[n] && IntegerQ[p] && IntegerQ[m]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

3.650. $\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx$

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

3.650.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^2c}-a}}\right)}{\sqrt{-\sqrt{b^2c}-a}}+\frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)}{\sqrt{\sqrt{b^2c}-a}}$	92
default	$\frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^2c}-a}}\right)}{\sqrt{-\sqrt{b^2c}-a}}+\frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)}{\sqrt{\sqrt{b^2c}-a}}$	92

input
```
int(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(-(b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2
)-a)^(1/2))+2/((b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/((b
^2*c)^(1/2)-a)^(1/2))
```

3.650.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 743 vs. $2(73) = 146$.

Time = 0.33 (sec) , antiderivative size = 743, normalized size of antiderivative = 7.66

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx$$

$$= \sqrt{-\frac{(b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}+a}{b^2c-a^2}}\log\left(4\left((b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}-a\right)\sqrt{-\frac{(b^2c-a^2)\sqrt{\frac{b}{b^4c^2-2}}}{b^2c-a^2}} + 4\sqrt{\sqrt{dx+c}b+a}\right)$$

$$-\sqrt{-\frac{(b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}+a}{b^2c-a^2}}\log\left(-4\left((b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}-a\right)\sqrt{-\frac{(b^2c-a^2)\sqrt{\frac{}{b^4}}}{b^2c}} + 4\sqrt{\sqrt{dx+c}b+a}\right)$$

$$-\sqrt{\frac{(b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}-a}{b^2c-a^2}}\log\left(4\left((b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}+a\right)\sqrt{\frac{(b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2}}}{b^2c-a^2}} + 4\sqrt{\sqrt{dx+c}b+a}\right)$$

$$+\sqrt{\frac{(b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}-a}{b^2c-a^2}}\log\left(-4\left((b^2c-a^2)\sqrt{\frac{b^2c}{b^4c^2-2\,a^2b^2c+a^4}}+a\right)\sqrt{\frac{(b^2c-a^2)\sqrt{\frac{b}{b^4c^2-2}}}{b^2c-a}} + 4\sqrt{\sqrt{dx+c}b+a}\right)$$

input
```
integrate(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

3.650. $\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx$

output
```
sqrt(-((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c
 - a^2))*log(4*((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) -
a)*sqrt(-((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^
2*c - a^2)) + 4*sqrt(sqrt(d*x + c)*b + a)) - sqrt(-((b^2*c - a^2)*sqrt(b^2
*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c - a^2))*log(-4*((b^2*c - a^2
)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)*sqrt(-((b^2*c - a^2)*sqrt
(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c - a^2)) + 4*sqrt(sqrt(d*
x + c)*b + a)) - sqrt(((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a
^4)) - a)/(b^2*c - a^2))*log(4*((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*
b^2*c + a^4)) + a)*sqrt(((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c +
 a^4)) - a)/(b^2*c - a^2)) + 4*sqrt(sqrt(d*x + c)*b + a)) + sqrt(((b^2*c -
 a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)/(b^2*c - a^2))*log(-4
*((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)*sqrt(((b^2*
c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)/(b^2*c - a^2)) + 4
*sqrt(sqrt(d*x + c)*b + a))
```

3.650.6 Sympy [F]

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx$$

input
```
integrate(1/x/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a + b*sqrt(c + d*x))), x)
```

3.650.7 Maxima [F]

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{1}{\sqrt{\sqrt{dx+c}b+ax}}\,dx$$

input
```
integrate(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x), x)
```

3.650.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.44

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{2\left(\frac{\left(b^2\sqrt{c}|b|+ab^2\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}b+a}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{\left(b\sqrt{c}+a\right)\sqrt{b\sqrt{c}-a}}+\frac{\left(b^2\sqrt{c}|b|-ab^2\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}b+a}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{\left(b\sqrt{c}-a\right)\sqrt{-b\sqrt{c}-a}}\right)}{b^2}$$

input
```
integrate(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
2*((b^2*sqrt(c)*abs(b) + a*b^2)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a +
 sqrt(b^2*c)))/((b*sqrt(c) + a)*sqrt(b*sqrt(c) - a)) + (b^2*sqrt(c)*abs(b)
 - a*b^2)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a - sqrt(b^2*c)))/((b*sqr
t(c) - a)*sqrt(-b*sqrt(c) - a)))/b^2
```

3.650.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{1}{x\,\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

input
```
int(1/(x*(a + b*(c + d*x)^(1/2))^(1/2)),x)
```

output
```
int(1/(x*(a + b*(c + d*x)^(1/2))^(1/2)), x)
```

3.651 $\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx$

3.651.1 Optimal result

Integrand size = 21, antiderivative size = 163

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx = -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{(a^2-b^2c)\,x} - \frac{bd\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\left(a-b\sqrt{c}\right)^{3/2}\sqrt{c}} + \frac{bd\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\left(a+b\sqrt{c}\right)^{3/2}\sqrt{c}}$$

output
```
-1/2*b*d*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a-b*c^(1/2))^(1/2))/c^(1/2)/(a
-b*c^(1/2))^(3/2)+1/2*b*d*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a+b*c^(1/2))^
(1/2))/c^(1/2)/(a+b*c^(1/2))^(3/2)-(a-b*(d*x+c)^(1/2))*(a+b*(d*x+c)^(1/2))
^(1/2)/(-b^2*c+a^2)/x
```

3.651.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{1}{2}\left(-\frac{2\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{(a^2-b^2c)\,x} + \frac{bd\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a-b\sqrt{c}}}\right)}{\left(-a-b\sqrt{c}\right)^{3/2}\sqrt{c}} - \frac{bd\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a+b\sqrt{c}}}\right)}{\left(-a+b\sqrt{c}\right)^{3/2}\sqrt{c}}\right)$$

input
```
Integrate[1/(x^2*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

output
```
((-2*(a - b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/((a^2 - b^2*c)*x) +
(b*d*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a - b*Sqrt[c]]])/((-a - b*Sqrt
[c])^(3/2)*Sqrt[c]) - (b*d*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a + b*Sq
rt[c]]])/((-a + b*Sqrt[c])^(3/2)*Sqrt[c]))/2
```

3.651.3 Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.44, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {896, 1732, 561, 25, 27, 1492, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}dx$$

$$\Big\downarrow \text{896}$$

$$d\int \frac{1}{d^2x^2\sqrt{a+b\sqrt{c+dx}}}d(c+dx)$$

$$\Big\downarrow \text{1732}$$

$$2d\int \frac{\sqrt{c+dx}}{d^2x^2\sqrt{a+b\sqrt{c+dx}}}d\sqrt{c+dx}$$

$$\Big\downarrow \text{561}$$

$$\frac{4d\int -\frac{a-c-dx}{b\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}d\sqrt{a+b\sqrt{c+dx}}}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{4d\int \frac{a-c-dx}{b\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}d\sqrt{a+b\sqrt{c+dx}}}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4d\int \frac{a-c-dx}{\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}d\sqrt{a+b\sqrt{c+dx}}}{b^2}$$

$$\Big\downarrow \; 1492$$

$$-\frac{4d\left(\frac{b^2(2a-c-dx)\sqrt{a+b\sqrt{c+dx}}}{4(a^2-b^2c)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}-\frac{b^4\int -\frac{2c(2a-c-dx)}{b^2\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}d\sqrt{a+b\sqrt{c+dx}}}{8c(a^2-b^2c)}\right)}{b^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{4d\left(\frac{b^2\int \frac{2a-c-dx}{\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c}d\sqrt{a+b\sqrt{c+dx}}}{4(a^2-b^2c)}+\frac{b^2(2a-c-dx)\sqrt{a+b\sqrt{c+dx}}}{4(a^2-b^2c)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}\right)}{b^2}$$

$$\Big\downarrow \; 1480$$

$$-\frac{4d\left(\frac{b^2\left(-\frac{\left(\frac{a}{\sqrt{c}}+b\right)\int \frac{1}{\frac{c+dx}{b^2}-\frac{a-b\sqrt{c}}{b^2}}d\sqrt{a+b\sqrt{c+dx}}}{2b}-\frac{1}{2}\left(1-\frac{a}{b\sqrt{c}}\right)\int \frac{1}{\frac{c+dx}{b^2}-\frac{a+b\sqrt{c}}{b^2}}d\sqrt{a+b\sqrt{c+dx}}\right)}{4(a^2-b^2c)}+\frac{b^2(2a-c-dx)\sqrt{a+b\sqrt{c+dx}}}{4(a^2-b^2c)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}\right)}{b^2}$$

$$\Big\downarrow \; 221$$

$$-\frac{4d\left(\frac{b^2\left(\frac{b^2\left(1-\frac{a}{b\sqrt{c}}\right)\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{a+b\sqrt{c}}}+\frac{b\left(\frac{a}{\sqrt{c}}+b\right)\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{a-b\sqrt{c}}}\right)}{4(a^2-b^2c)}+\frac{b^2(2a-c-dx)\sqrt{a+b\sqrt{c+dx}}}{4(a^2-b^2c)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}\right)}{b^2}$$

input
```
Int[1/(x^2*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

output
```
(-4*d*((b^2*(2*a - c - d*x)*Sqrt[a + b*Sqrt[c + d*x]])/(4*(a^2 - b^2*c)*(a
^2/b^2 - c - (2*a*(c + d*x))/b^2 + (c + d*x)^2/b^2)) + (b^2*((b*(b + a/Sqr
t[c])*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/(2*Sqrt[a -
b*Sqrt[c]]) + (b^2*(1 - a/(b*Sqrt[c]))*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/S
qrt[a + b*Sqrt[c]]])/(2*Sqrt[a + b*Sqrt[c]])))/(4*(a^2 - b^2*c))))/b^2
```

3.651. $\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx$

3.651.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 561
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(n + 1) - 1)*(-c
/d + x^k/d)^m*Simp[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^k/d^2) + b*(x^(2*k)/d^2),
 x]^p, x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, m, p}, x] && Frac
tionQ[n] && IntegerQ[p] && IntegerQ[m]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

3.651. $\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx$

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

3.651.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(125) = 250$.

Time = 0.29 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.59

method	result
derivativedivides	$4d\,b^2\left(-\frac{\sqrt{b^2c}\left(\frac{2\sqrt{a+b\sqrt{dx+c}}}{\left(4\sqrt{b^2c}-4a\right)\left(b\sqrt{dx+c}+\sqrt{b^2c}\right)}+\frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)}{\left(4\sqrt{b^2c}-4a\right)\sqrt{\sqrt{b^2c}-a}}\right)}{4b^2c}+\frac{\sqrt{b^2c}\left(-\frac{2\sqrt{a+b\sqrt{dx+c}}}{\left(-4\sqrt{b^2c}-4a\right)\left(-b\sqrt{dx+c}+\sqrt{b^2\right.}}\right.}{4b^2c}$
default	$4d\,b^2\left(-\frac{\sqrt{b^2c}\left(\frac{2\sqrt{a+b\sqrt{dx+c}}}{\left(4\sqrt{b^2c}-4a\right)\left(b\sqrt{dx+c}+\sqrt{b^2c}\right)}+\frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)}{\left(4\sqrt{b^2c}-4a\right)\sqrt{\sqrt{b^2c}-a}}\right)}{4b^2c}+\frac{\sqrt{b^2c}\left(-\frac{2\sqrt{a+b\sqrt{dx+c}}}{\left(-4\sqrt{b^2c}-4a\right)\left(-b\sqrt{dx+c}+\sqrt{b^2\right.}}\right.}{4b^2c}$

input
```
int(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4*d*b^2*(-1/4*(b^2*c)^(1/2)/b^2/c*(2*(a+b*(d*x+c)^(1/2))^(1/2)/(4*(b^2*c)^
(1/2)-4*a)/(b*(d*x+c)^(1/2)+(b^2*c)^(1/2))+2/(4*(b^2*c)^(1/2)-4*a)/((b^2*c
)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/((b^2*c)^(1/2)-a)^(1/2))
)+1/4*(b^2*c)^(1/2)/b^2/c*(-2*(a+b*(d*x+c)^(1/2))^(1/2)/(-4*(b^2*c)^(1/2)-
4*a)/(-b*(d*x+c)^(1/2)+(b^2*c)^(1/2))+2/(-4*(b^2*c)^(1/2)-4*a)/(-(b^2*c)^(
1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2)))
```

3.651.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2493 vs. $2(127) = 254$.

Time = 0.41 (sec) , antiderivative size = 2493, normalized size of antiderivative = 15.29

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/4*((b^2*c - a^2)*x*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b
^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*
d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*
b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*
c^2 - a^6*c))*log((b^6*c + 3*a^2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 + (2*(
a*b^6*c^2 + 3*a^3*b^4*c)*d^2 - (b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2 -
a^8*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^1
0*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2
+ a^12*c)))*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b^4*c^3 +
3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12
*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 -
 6*a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6
*c))) - (b^2*c - a^2)*x*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^
2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^
6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a
^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b
^2*c^2 - a^6*c))*log((b^6*c + 3*a^2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 - (
2*(a*b^6*c^2 + 3*a^3*b^4*c)*d^2 - (b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2
 - a^8*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*
b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^...
```

3.651.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx$$

input
```
integrate(1/x**2/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a + b*sqrt(c + d*x))), x)
```

3.651.7 Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{1}{\sqrt{\sqrt{dx+c}b+a}\,x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x^2), x)
```

3.651.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(127) = 254$.

Time = 0.62 (sec) , antiderivative size = 644, normalized size of antiderivative = 3.95

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx$$

$$= \frac{\frac{\left((b^3c-a^2b)^2b^4cd^2-2\left(ab^6c^{\frac{3}{2}}-a^3b^4\sqrt{c}\right)d^2|-b^3c+a^2b|+(a^2b^8c^2-2a^4b^6c+a^6b^4)d^2\right)\arctan\left(\frac{\sqrt{\sqrt{dx+cb+a}}}{\sqrt{-\frac{ab^2c-a^3+\sqrt{\left(ab^2c-a^3\right)^2+\left(b^4c^2-2a^2b^2c+a^4\right)\left(b^2c\right.}}{b^2c-a^2}}}\right.}{\left(b^5c^3+ab^4c^{\frac{5}{2}}-2a^2b^3c^2-2a^3b^2c^{\frac{3}{2}}+a^4bc+a^5\sqrt{c}\right)\sqrt{b\sqrt{c}-a}|-b^3c+a^2b|}}{}$$

input
```
integrate(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
1/2*(((b^3*c - a^2*b)^2*b^4*c*d^2 - 2*(a*b^6*c^(3/2) - a^3*b^4*sqrt(c))*d^
2*abs(-b^3*c + a^2*b) + (a^2*b^8*c^2 - 2*a^4*b^6*c + a^6*b^4)*d^2)*arctan(
sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^2*c - a^3 + sqrt((a*b^2*c - a^3)^2 +
(b^4*c^2 - 2*a^2*b^2*c + a^4)*(b^2*c - a^2)))/(b^2*c - a^2)))/((b^5*c^3 +
a*b^4*c^(5/2) - 2*a^2*b^3*c^2 - 2*a^3*b^2*c^(3/2) + a^4*b*c + a^5*sqrt(c))
*sqrt(b*sqrt(c) - a)*abs(-b^3*c + a^2*b)) + ((b^3*c - a^2*b)^2*b^4*c*d^2 +
 2*(a*b^6*c^(3/2) - a^3*b^4*sqrt(c))*d^2*abs(-b^3*c + a^2*b) + (a^2*b^8*c^
2 - 2*a^4*b^6*c + a^6*b^4)*d^2)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*
b^2*c - a^3 - sqrt((a*b^2*c - a^3)^2 + (b^4*c^2 - 2*a^2*b^2*c + a^4)*(b^2*
c - a^2)))/(b^2*c - a^2)))/((b^5*c^3 - a*b^4*c^(5/2) - 2*a^2*b^3*c^2 + 2*a
^3*b^2*c^(3/2) + a^4*b*c - a^5*sqrt(c))*sqrt(-b*sqrt(c) - a)*abs(-b^3*c +
a^2*b)) + 2*((sqrt(d*x + c)*b + a)^(3/2)*b^4*d^2 - 2*sqrt(sqrt(d*x + c)*b
+ a)*a*b^4*d^2)/((b^2*c - (sqrt(d*x + c)*b + a)^2 + 2*(sqrt(d*x + c)*b + a
)*a - a^2)*(b^2*c - a^2)))/(b^2*d)
```

3.651.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{1}{x^2\,\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

input
```
int(1/(x^2*(a + b*(c + d*x)^(1/2))^(1/2)),x)
```

output
```
int(1/(x^2*(a + b*(c + d*x)^(1/2))^(1/2)), x)
```

3.652 $\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx$

3.652.1 Optimal result

Integrand size = 21, antiderivative size = 261

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx = -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\sqrt{a+b\sqrt{c+dx}}\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+dx}\right)}{8c\left(a^2-b^2c\right)^2x} + \frac{b\left(2a-5b\sqrt{c}\right)d^2\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16\left(a-b\sqrt{c}\right)^{5/2}c^{3/2}} - \frac{b\left(2a+5b\sqrt{c}\right)d^2\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{16\left(a+b\sqrt{c}\right)^{5/2}c^{3/2}}$$

output
```
1/16*b*d^2*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a-b*c^(1/2))^(1/2))*(2*a-5*b
*c^(1/2))/c^(3/2)/(a-b*c^(1/2))^(5/2)-1/16*b*d^2*arctanh((a+b*(d*x+c)^(1/2
))^(1/2)/(a+b*c^(1/2))^(1/2))*(2*a+5*b*c^(1/2))/c^(3/2)/(a+b*c^(1/2))^(5/2
)-1/2*(a-b*(d*x+c)^(1/2))*(a+b*(d*x+c)^(1/2))^(1/2)/(-b^2*c+a^2)/x^2-1/8*b
*d*(6*a*b*c-(5*b^2*c+a^2)*(d*x+c)^(1/2))*(a+b*(d*x+c)^(1/2))^(1/2)/c/(-b^2
*c+a^2)^2/x
```

3.652.2 Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{-\frac{2\sqrt{c}\sqrt{a+b\sqrt{c+dx}}\left(4a^3c+b^3c(4c-5dx)\sqrt{c+dx}-a^2b\sqrt{c+dx}(4c+dx)+2ab^2c(-2c+3dx)\right)}{(a^2-b^2c)^2x^2}+\frac{b(2a+5b\sqrt{c})d^2\arctan\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a-b\sqrt{c}}}\right)}{(-a-b\sqrt{c})^{5/2}}+\frac{b(-2a+}{}}{16c^{3/2}}$$

input
```
Integrate[1/(x^3*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

output
```
((-2*Sqrt[c]*Sqrt[a + b*Sqrt[c + d*x]]*(4*a^3*c + b^3*c*(4*c - 5*d*x)*Sqrt
[c + d*x] - a^2*b*Sqrt[c + d*x]*(4*c + d*x) + 2*a*b^2*c*(-2*c + 3*d*x)))/(
(a^2 - b^2*c)^2*x^2) + (b*(2*a + 5*b*Sqrt[c])*d^2*ArcTan[Sqrt[a + b*Sqrt[c
 + d*x]]/Sqrt[-a - b*Sqrt[c]]])/(-a - b*Sqrt[c])^(5/2) + (b*(-2*a + 5*b*Sq
rt[c])*d^2*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a + b*Sqrt[c]]])/(-a + b
*Sqrt[c])^(5/2))/(16*c^(3/2))
```

3.652.3 Rubi [A] (warning: unable to verify)

Time = 0.61 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.50, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {896, 25, 1732, 561, 27, 1492, 27, 1492, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx$$

$$\Big\downarrow \text{ 896}$$

$$d^2\int \frac{1}{d^3x^3\sqrt{a+b\sqrt{c+dx}}}d(c+dx)$$

$$\Big\downarrow \text{ 25}$$

$$-d^2\int -\frac{1}{d^3x^3\sqrt{a+b\sqrt{c+dx}}}d(c+dx)$$

$$\Big\downarrow \text{ 1732}$$

$$-2d^2\int -\frac{\sqrt{c+dx}}{d^3x^3\sqrt{a+b\sqrt{c+dx}}}d\sqrt{c+dx}$$

$\Big\downarrow$ 561

$$-\frac{4d^2\int \frac{a-c-dx}{b\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^3}d\sqrt{a+b\sqrt{c+dx}}}{b}$$

$\Big\downarrow$ 27

$$-\frac{4d^2\int \frac{a-c-dx}{\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^3}d\sqrt{a+b\sqrt{c+dx}}}{b^2}$$

$\Big\downarrow$ 1492

$$-\frac{4d^2\left(\frac{b^2(2a-c-dx)\sqrt{a+b\sqrt{c+dx}}}{8(a^2-b^2c)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}-\frac{b^4\int -\frac{2c(6a-5(c+dx))}{b^2\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}d\sqrt{a+b\sqrt{c+dx}}}{16c(a^2-b^2c)}\right)}{b^2}$$

$\Big\downarrow$ 27

$$-\frac{4d^2\left(\frac{b^2\int \frac{6a-5(c+dx)}{\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}d\sqrt{a+b\sqrt{c+dx}}}{8(a^2-b^2c)}+\frac{b^2(2a-c-dx)\sqrt{a+b\sqrt{c+dx}}}{8(a^2-b^2c)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)^2}\right)}{b^2}$$

$\Big\downarrow$ 1492

$$-\frac{4d^2\left(\frac{b^2\left(\frac{\sqrt{a+b\sqrt{c+dx}}\left(a\left(a^2+11b^2c\right)-\left(a^2+5b^2c\right)(c+dx)\right)}{4c\left(a^2-b^2c\right)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}-\frac{b^4\int \frac{2\left(a\left(a^2-13b^2c\right)+\left(a^2+5b^2c\right)(c+dx)\right)}{b^4\left(\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}d\sqrt{a+b\sqrt{c+dx}}}{8c\left(a^2-b^2c\right)}\right)}{8(a^2-b^2c)}+\frac{b^2(2a-c-dx)\sqrt{a+b}}{8(a^2-b^2c)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\right.}\right.}{b^2}$$

$\Big\downarrow$ 27

3.652. $\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx$

$$-\frac{4d^2\left(\frac{b^2\left(\frac{\sqrt{a+b\sqrt{c+dx}}\left(a\left(a^2+11b^2c\right)-\left(a^2+5b^2c\right)(c+dx)\right)}{4c\left(a^2-b^2c\right)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}-\frac{\int \frac{a\left(a^2-13b^2c\right)+\left(a^2+5b^2c\right)(c+dx)}{\frac{a^2}{b^2}-\frac{2(c+dx)a}{b^2}+\frac{(c+dx)^2}{b^2}-c}d\sqrt{a+b\sqrt{c+dx}}}{4c\left(a^2-b^2c\right)}\right)}{8(a^2-b^2c)}+\frac{b^2(2a-c-dx)\sqrt{a+b\sqrt{c+a}}}{8(a^2-b^2c)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+d}{b^2}\right.}\right.}{b^2}$$

$\Big\downarrow$ 1480

$$-\frac{4d^2\left(\frac{b^2\left(\frac{\sqrt{a+b\sqrt{c+dx}}\left(a\left(a^2+11b^2c\right)-\left(a^2+5b^2c\right)(c+dx)\right)}{4c\left(a^2-b^2c\right)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}-\frac{\frac{(a-b\sqrt{c})^2(2a+5b\sqrt{c})\int \frac{1}{\frac{c+dx}{b^2}-\frac{a+b\sqrt{c}}{b^2}}d\sqrt{a+b\sqrt{c+dx}}}{2b\sqrt{c}}-\frac{(2a-5b\sqrt{c})(a+b\sqrt{c})^2\int \frac{1}{\frac{c+dx}{b^2}-\frac{a-b\sqrt{}}{b^2}}}{2b\sqrt{c}}}{4c\left(a^2-b^2c\right)}\right.}{8(a^2-b^2c)}\right.}{b^2}$$

$\Big\downarrow$ 221

$$-\frac{4d^2\left(\frac{b^2\left(\frac{\sqrt{a+b\sqrt{c+dx}}\left(a\left(a^2+11b^2c\right)-\left(a^2+5b^2c\right)(c+dx)\right)}{4c\left(a^2-b^2c\right)\left(\frac{a^2}{b^2}-\frac{2a(c+dx)}{b^2}+\frac{(c+dx)^2}{b^2}-c\right)}-\frac{\frac{b(2a-5b\sqrt{c})(a+b\sqrt{c})^2\text{arctanh}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a-b\sqrt{c}}}-\frac{b(a-b\sqrt{c})^2(2a+5b\sqrt{c})\text{arctanh}\left(\frac{\sqrt{a+}}{\sqrt{a}}\right.}{2\sqrt{c}\sqrt{a+b\sqrt{c}}}}{4c\left(a^2-b^2c\right)}\right.}{8(a^2-b^2c)}\right.}{b^2}$$

input
```
Int[1/(x^3*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

output
```
(-4*d^2*((b^2*(2*a - c - d*x)*Sqrt[a + b*Sqrt[c + d*x]])/(8*(a^2 - b^2*c)*
(a^2/b^2 - c - (2*a*(c + d*x))/b^2 + (c + d*x)^2/b^2)^2) + (b^2*((Sqrt[a +
 b*Sqrt[c + d*x]]*(a*(a^2 + 11*b^2*c) - (a^2 + 5*b^2*c)*(c + d*x)))/(4*c*(
a^2 - b^2*c)*(a^2/b^2 - c - (2*a*(c + d*x))/b^2 + (c + d*x)^2/b^2)) - ((b*
(2*a - 5*b*Sqrt[c])*(a + b*Sqrt[c])^2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sq
rt[a - b*Sqrt[c]]])/(2*Sqrt[a - b*Sqrt[c]]*Sqrt[c]) - (b*(a - b*Sqrt[c])^2
*(2*a + 5*b*Sqrt[c])*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]
])/(2*Sqrt[a + b*Sqrt[c]]*Sqrt[c]))/(4*c*(a^2 - b^2*c))))/(8*(a^2 - b^2*c)
)))/b^2
```

3.652. $\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx$

3.652.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 561
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(n + 1) - 1)*(-c
/d + x^k/d)^m*Simp[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^k/d^2) + b*(x^(2*k)/d^2),
 x]^p, x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, m, p}, x] && Frac
tionQ[n] && IntegerQ[p] && IntegerQ[m]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

3.652. $\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx$

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

3.652.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(209) = 418$.

Time = 0.42 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.64

method	result
derivativedivides	$-4d^2b^4\left(\frac{\frac{-\frac{\left(5\sqrt{b^2c}+2a\right)\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{4\left(b^2c+2a\sqrt{b^2c}+a^2\right)}+\frac{\left(7\sqrt{b^2c}+2a\right)\sqrt{a+b\sqrt{dx+c}}}{4\sqrt{b^2c}+4a}}{\left(-b\sqrt{dx+c}+\sqrt{b^2c}\right)^2}-\frac{\left(5\sqrt{b^2c}+2a\right)\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^2c}-a}}\right)}{4\left(b^2c+2a\sqrt{b^2c}+a^2\right)\sqrt{-\sqrt{b^2c}-a}}}{16b^2c\sqrt{b^2c}}-\frac{-\frac{\left(-5\sqrt{b^2c\right.}}{4\left(b^2\right.}}{}\right.$
default	$-4d^2b^4\left(\frac{\frac{-\frac{\left(5\sqrt{b^2c}+2a\right)\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{4\left(b^2c+2a\sqrt{b^2c}+a^2\right)}+\frac{\left(7\sqrt{b^2c}+2a\right)\sqrt{a+b\sqrt{dx+c}}}{4\sqrt{b^2c}+4a}}{\left(-b\sqrt{dx+c}+\sqrt{b^2c}\right)^2}-\frac{\left(5\sqrt{b^2c}+2a\right)\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^2c}-a}}\right)}{4\left(b^2c+2a\sqrt{b^2c}+a^2\right)\sqrt{-\sqrt{b^2c}-a}}}{16b^2c\sqrt{b^2c}}-\frac{-\frac{\left(-5\sqrt{b^2c\right.}}{4\left(b^2\right.}}{}\right.$

input
```
int(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4*d^2*b^4*(1/16/b^2/c/(b^2*c)^(1/2)*((-1/4*(5*(b^2*c)^(1/2)+2*a)/(b^2*c+2
*a*(b^2*c)^(1/2)+a^2)*(a+b*(d*x+c)^(1/2))^(3/2)+1/4*(7*(b^2*c)^(1/2)+2*a)/
((b^2*c)^(1/2)+a)*(a+b*(d*x+c)^(1/2))^(1/2))/(-b*(d*x+c)^(1/2)+(b^2*c)^(1/
2))^2-1/4*(5*(b^2*c)^(1/2)+2*a)/(b^2*c+2*a*(b^2*c)^(1/2)+a^2)/(-(b^2*c)^(1
/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2)))-1
/16/b^2/c/(b^2*c)^(1/2)*((-1/4*(-5*(b^2*c)^(1/2)+2*a)/(b^2*c-2*a*(b^2*c)^(
1/2)+a^2)*(a+b*(d*x+c)^(1/2))^(3/2)+1/4*(-7*(b^2*c)^(1/2)+2*a)/(-(b^2*c)^(
1/2)+a)*(a+b*(d*x+c)^(1/2))^(1/2))/(-b*(d*x+c)^(1/2)-(b^2*c)^(1/2))^2-1/4*
(5*(b^2*c)^(1/2)-2*a)/(-b^2*c+2*a*(b^2*c)^(1/2)-a^2)/((b^2*c)^(1/2)-a)^(1/
2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/((b^2*c)^(1/2)-a)^(1/2)))
```

3.652.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4390 vs. $2(212) = 424$.

Time = 1.22 (sec) , antiderivative size = 4390, normalized size of antiderivative = 16.82

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-1/32*((b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*x^2*sqrt(-((105*a*b^8*c^3 + 70*a^
3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 + (b^10*c^8 - 5*a^2*b^8*c^7 + 10
*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c
^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*
b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*
c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*
b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^
2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))*l
og((625*b^12*c^3 + 3750*a^2*b^10*c^2 - 1491*a^4*b^8*c + 140*a^6*b^6)*sqrt(
sqrt(d*x + c)*b + a)*d^6 + ((325*a*b^12*c^5 + 1977*a^3*b^10*c^4 - 609*a^5*
b^8*c^3 + 35*a^7*b^6*c^2)*d^4 - (5*b^14*c^10 - 16*a^2*b^12*c^9 + 3*a^4*b^1
0*c^8 + 50*a^6*b^8*c^7 - 85*a^8*b^6*c^6 + 60*a^10*b^4*c^5 - 19*a^12*b^2*c^
4 + 2*a^14*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^
2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 +
45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^
8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^
4 + a^20*c^3)))*sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*
a^7*b^2)*d^4 + (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5
 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 2196
6*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*...
```

3.652.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**3/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

output
```
Timed out
```

3.652.7 Maxima [F]

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{1}{\sqrt{\sqrt{dx+c}b+ax^3}}\,dx$$

input
```
integrate(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x^3), x)
```

3.652.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1303 vs. $2(212) = 424$.

Time = 0.44 (sec) , antiderivative size = 1303, normalized size of antiderivative = 4.99

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
1/16*(((b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c)^2*(5*b^6*c + a^2*b^4)*d^3 - (13
*a*b^10*c^(7/2) - 27*a^3*b^8*c^(5/2) + 15*a^5*b^6*c^(3/2) - a^7*b^4*sqrt(c
))*d^3*abs(b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c) + 2*(4*a^2*b^14*c^6 - 17*a^4
*b^12*c^5 + 28*a^6*b^10*c^4 - 22*a^8*b^8*c^3 + 8*a^10*b^6*c^2 - a^12*b^4*c
)*d^3)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^4*c^3 - 2*a^3*b^2*c^2 +
 a^5*c + sqrt((a*b^4*c^3 - 2*a^3*b^2*c^2 + a^5*c)^2 + (b^6*c^4 - 3*a^2*b^4
*c^3 + 3*a^4*b^2*c^2 - a^6*c)*(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))/(b^4*c^3
 - 2*a^2*b^2*c^2 + a^4*c)))/((b^9*c^6 - a*b^8*c^(11/2) - 4*a^2*b^7*c^5 + 4
*a^3*b^6*c^(9/2) + 6*a^4*b^5*c^4 - 6*a^5*b^4*c^(7/2) - 4*a^6*b^3*c^3 + 4*a
^7*b^2*c^(5/2) + a^8*b*c^2 - a^9*c^(3/2))*sqrt(-b*sqrt(c) - a)*abs(b^5*c^3
 - 2*a^2*b^3*c^2 + a^4*b*c)) + ((b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c)^2*(5*b
^6*c + a^2*b^4)*d^3 + (13*a*b^10*c^(7/2) - 27*a^3*b^8*c^(5/2) + 15*a^5*b^6
*c^(3/2) - a^7*b^4*sqrt(c))*d^3*abs(b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c) + 2
*(4*a^2*b^14*c^6 - 17*a^4*b^12*c^5 + 28*a^6*b^10*c^4 - 22*a^8*b^8*c^3 + 8*
a^10*b^6*c^2 - a^12*b^4*c)*d^3)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*
b^4*c^3 - 2*a^3*b^2*c^2 + a^5*c - sqrt((a*b^4*c^3 - 2*a^3*b^2*c^2 + a^5*c)
^2 + (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*(b^4*c^3 - 2*a^2*b^
2*c^2 + a^4*c)))/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))/((b^9*c^6 + a*b^8*c^(
11/2) - 4*a^2*b^7*c^5 - 4*a^3*b^6*c^(9/2) + 6*a^4*b^5*c^4 + 6*a^5*b^4*c^(7
/2) - 4*a^6*b^3*c^3 - 4*a^7*b^2*c^(5/2) + a^8*b*c^2 + a^9*c^(3/2))*sqrt...
```

3.652.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx = \int \frac{1}{x^3\,\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

input
```
int(1/(x^3*(a + b*(c + d*x)^(1/2))^(1/2)),x)
```

output
```
int(1/(x^3*(a + b*(c + d*x)^(1/2))^(1/2)), x)
```

3.653 $\int x^3 \left(a+b\sqrt{c+dx}\right)^p \, dx$

3.653.1 Optimal result

Integrand size = 19, antiderivative size = 350

$$\begin{aligned}\int x^3 \left(a+b\sqrt{c+dx}\right)^p \, dx = &-\frac{2a(a^2-b^2c)^3 \left(a+b\sqrt{c+dx}\right)^{1+p}}{b^8d^4(1+p)} \\ &+ \frac{2(a^2-b^2c)^2 (7a^2-b^2c) \left(a+b\sqrt{c+dx}\right)^{2+p}}{b^8d^4(2+p)} \\ &- \frac{6a(7a^2-3b^2c) (a^2-b^2c) \left(a+b\sqrt{c+dx}\right)^{3+p}}{b^8d^4(3+p)} \\ &+ \frac{2(35a^4-30a^2b^2c+3b^4c^2) \left(a+b\sqrt{c+dx}\right)^{4+p}}{b^8d^4(4+p)} \\ &- \frac{10a(7a^2-3b^2c) \left(a+b\sqrt{c+dx}\right)^{5+p}}{b^8d^4(5+p)} \\ &+ \frac{6(7a^2-b^2c) \left(a+b\sqrt{c+dx}\right)^{6+p}}{b^8d^4(6+p)} \\ &- \frac{14a\left(a+b\sqrt{c+dx}\right)^{7+p}}{b^8d^4(7+p)} + \frac{2\left(a+b\sqrt{c+dx}\right)^{8+p}}{b^8d^4(8+p)}\end{aligned}$$

output
```
-2*a*(-b^2*c+a^2)^3*(a+b*(d*x+c)^(1/2))^(p+1)/b^8/d^4/(p+1)+2*(-b^2*c+a^2)
^2*(-b^2*c+7*a^2)*(a+b*(d*x+c)^(1/2))^(2+p)/b^8/d^4/(2+p)-6*a*(-3*b^2*c+7*
a^2)*(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(3+p)/b^8/d^4/(3+p)+2*(3*b^4*c^2-30*
a^2*b^2*c+35*a^4)*(a+b*(d*x+c)^(1/2))^(4+p)/b^8/d^4/(4+p)-10*a*(-3*b^2*c+7
*a^2)*(a+b*(d*x+c)^(1/2))^(5+p)/b^8/d^4/(5+p)+6*(-b^2*c+7*a^2)*(a+b*(d*x+c
)^(1/2))^(6+p)/b^8/d^4/(6+p)-14*a*(a+b*(d*x+c)^(1/2))^(7+p)/b^8/d^4/(7+p)+
2*(a+b*(d*x+c)^(1/2))^(8+p)/b^8/d^4/(8+p)
```

3.653.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.59

$$\int x^3 \left(a+b\sqrt{c+dx}\right)^p dx =$$
$$-\frac{2\left(a+b\sqrt{c+dx}\right)^{1+p}\left(5040a^7-5040a^6b(1+p)\sqrt{c+dx}+360a^5b^2(6c(-7+p+p^2)+7d(2+3p+p^2)x\right.}{}$$

input
```
Integrate[x^3*(a + b*Sqrt[c + d*x])^p,x]
```

output
```
(-2*(a + b*Sqrt[c + d*x])^(1 + p)*(5040*a^7 - 5040*a^6*b*(1 + p)*Sqrt[c +
d*x] + 360*a^5*b^2*(6*c*(-7 + p + p^2) + 7*d*(2 + 3*p + p^2)*x) - 120*a^4*
b^3*(1 + p)*Sqrt[c + d*x]*(2*c*(-63 - 5*p + 2*p^2) + 7*d*(6 + 5*p + p^2)*x
) + 6*a^3*b^4*(8*c^2*(315 - 124*p - 139*p^2 - 14*p^3 + p^4) + 40*c*d*(-42
- 61*p - 16*p^2 + 4*p^3 + p^4)*x + 35*d^2*(24 + 50*p + 35*p^2 + 10*p^3 + p
^4)*x^2) - 6*a^2*b^5*(1 + p)*Sqrt[c + d*x]*(-24*c^2*(-105 - 24*p + 5*p^2 +
 p^3) + 4*c*d*(-420 - 386*p - 94*p^2 - p^3 + p^4)*x + 7*d^2*(120 + 154*p +
 71*p^2 + 14*p^3 + p^4)*x^2) - b^7*(105 + 176*p + 86*p^2 + 16*p^3 + p^4)*S
qrt[c + d*x]*(-48*c^3 + 24*c^2*d*(2 + p)*x - 6*c*d^2*(8 + 6*p + p^2)*x^2 +
 d^3*(48 + 44*p + 12*p^2 + p^3)*x^3) + a*b^6*(48*c^3*(-105 + 103*p + 138*p
^2 + 38*p^3 + 3*p^4) - 24*c^2*d*(-210 - 283*p - 21*p^2 + 74*p^3 + 24*p^4 +
 2*p^5)*x + 6*c*d^2*(-840 - 1726*p - 1151*p^2 - 265*p^3 + 10*p^4 + 11*p^5
+ p^6)*x^2 + 7*d^3*(720 + 1764*p + 1624*p^2 + 735*p^3 + 175*p^4 + 21*p^5 +
 p^6)*x^3)))/(b^8*d^4*(1 + p)*(2 + p)*(3 + p)*(4 + p)*(5 + p)*(6 + p)*(7 +
 p)*(8 + p))
```

3.653.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b\sqrt{c+dx}\right)^p dx$$
$$\Big\downarrow \ 896$$

$$\frac{\int d^3x^3\left(a+b\sqrt{c+dx}\right)^p d(c+dx)}{d^4}$$

$\Big\downarrow$ 25

$$-\frac{\int -d^3x^3\left(a+b\sqrt{c+dx}\right)^p d(c+dx)}{d^4}$$

$\Big\downarrow$ 1732

$$-\frac{2\int -d^3x^3\sqrt{c+dx}\left(a+b\sqrt{c+dx}\right)^p d\sqrt{c+dx}}{d^4}$$

$\Big\downarrow$ 522

$$-\frac{2\int\left(\frac{a\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)^p}{b^7}+\frac{\left(b^2c-7a^2\right)\left(b^2c-a^2\right)^2\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^7}+\frac{3\left(7a^5-10b^2ca^3+3b^4c^2a\right)\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^7}+\frac{\left(-35a^4+30b^2c\right.}{}\right.}{}$$

$\Big\downarrow$ 2009

$$-\frac{2\left(\frac{a\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^8(p+1)}-\frac{\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^8(p+2)}+\frac{3a\left(7a^2-3b^2c\right)\left(a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^8(p+3)}+\frac{5a\left(7a^2-3b^2c\right)\left(a\right.}{b^8(p+}\right.}{d^4}$$

input
```
Int[x^3*(a + b*Sqrt[c + d*x])^p,x]
```

output
```
(-2*((a*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])^(1 + p))/(b^8*(1 + p)) - ((a
^2 - b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(2 + p))/(b^8*(2 + p))
 + (3*a*(7*a^2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3 + p))/(b^
8*(3 + p)) - ((35*a^4 - 30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c + d*x])^(4
 + p))/(b^8*(4 + p)) + (5*a*(7*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(5 + p
))/(b^8*(5 + p)) - (3*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(6 + p))/(b^8*
(6 + p)) + (7*a*(a + b*Sqrt[c + d*x])^(7 + p))/(b^8*(7 + p)) - (a + b*Sqrt
[c + d*x])^(8 + p)/(b^8*(8 + p))))/d^4
```

3.653. $\int x^3\left(a+b\sqrt{c+dx}\right)^p\,dx$

3.653.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.653.4 Maple [F]

$$\int x^{3}\left(a+b\sqrt{dx+c}\right)^{p}dx$$

input
```
int(x^3*(a+b*(d*x+c)^(1/2))^p,x)
```

output
```
int(x^3*(a+b*(d*x+c)^(1/2))^p,x)
```

3.653.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1416 vs. $2(335) = 670$.

Time = 0.41 (sec) , antiderivative size = 1416, normalized size of antiderivative = 4.05

$$\int x^3 \left(a + b\sqrt{c + dx}\right)^p \, dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")
```

output
```
-2*(5040*b^8*c^4 - 20160*a^2*b^6*c^3 + 30240*a^4*b^4*c^2 - 20160*a^6*b^2*c
 + 5040*a^8 + 48*(b^8*c^4 + 6*a^2*b^6*c^3 + a^4*b^4*c^2)*p^4 - (b^8*d^4*p^
7 + 28*b^8*d^4*p^6 + 322*b^8*d^4*p^5 + 1960*b^8*d^4*p^4 + 6769*b^8*d^4*p^3
 + 13132*b^8*d^4*p^2 + 13068*b^8*d^4*p + 5040*b^8*d^4)*x^4 + 384*(2*b^8*c^
4 + 7*a^2*b^6*c^3 - 3*a^4*b^4*c^2)*p^3 - (b^8*c*d^3*p^7 + (22*b^8*c - 7*a^
2*b^6)*d^3*p^6 + 5*(38*b^8*c - 21*a^2*b^6)*d^3*p^5 + 5*(164*b^8*c - 119*a^
2*b^6)*d^3*p^4 + (1849*b^8*c - 1575*a^2*b^6)*d^3*p^3 + 2*(1019*b^8*c - 959
*a^2*b^6)*d^3*p^2 + 840*(b^8*c - a^2*b^6)*d^3*p)*x^3 + 48*(86*b^8*c^4 + 81
*a^2*b^6*c^3 - 124*a^4*b^4*c^2 + 45*a^6*b^2*c)*p^2 + 6*(18*b^8*c^2*d^2*p^5
 + (b^8*c^2 + a^2*b^6*c)*d^2*p^6 + (118*b^8*c^2 - 95*a^2*b^6*c + 35*a^4*b^
4)*d^2*p^4 + 6*(58*b^8*c^2 - 80*a^2*b^6*c + 35*a^4*b^4)*d^2*p^3 + (457*b^8
*c^2 - 806*a^2*b^6*c + 385*a^4*b^4)*d^2*p^2 + 210*(b^8*c^2 - 2*a^2*b^6*c +
 a^4*b^4)*d^2*p)*x^2 + 192*(44*b^8*c^4 - 71*a^2*b^6*c^3 + 54*a^4*b^4*c^2 -
 15*a^6*b^2*c)*p - 24*((b^8*c^3 + 3*a^2*b^6*c^2)*d*p^5 + 2*(8*b^8*c^3 + 9*
a^2*b^6*c^2 - 5*a^4*b^4*c)*d*p^4 + (86*b^8*c^3 - 57*a^2*b^6*c^2 + 15*a^4*b
^4*c)*d*p^3 + (176*b^8*c^3 - 387*a^2*b^6*c^2 + 340*a^4*b^4*c - 105*a^6*b^2
)*d*p^2 + 105*(b^8*c^3 - 3*a^2*b^6*c^2 + 3*a^4*b^4*c - a^6*b^2)*d*p)*x + (
192*(a*b^7*c^3 + a^3*b^5*c^2)*p^4 + 96*(27*a*b^7*c^3 + 2*a^3*b^5*c^2 - 5*a
^5*b^3*c)*p^3 - (a*b^7*d^3*p^7 + 21*a*b^7*d^3*p^6 + 175*a*b^7*d^3*p^5 + 73
5*a*b^7*d^3*p^4 + 1624*a*b^7*d^3*p^3 + 1764*a*b^7*d^3*p^2 + 720*a*b^7*d...
```

3.653.6 Sympy [F]

$$\int x^3 \left(a + b\sqrt{c + dx}\right)^p \, dx = \int x^3 \left(a + b\sqrt{c + dx}\right)^p \, dx$$

input
```
integrate(x**3*(a+b*(d*x+c)**(1/2))**p,x)
```

output
```
Integral(x**3*(a + b*sqrt(c + d*x))**p, x)
```

3.653.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(335) = 670$.

Time = 0.22 (sec) , antiderivative size = 728, normalized size of antiderivative = 2.08

$$\int x^3\left(a+b\sqrt{c+dx}\right)^p\,dx =$$

$$-\frac{2\left(\frac{\left((dx+c)b^2(p+1)+\sqrt{dx+c}abp-a^2\right)\left(\sqrt{dx+c}b+a\right)^p c^3}{(p^2+3p+2)b^2}-\frac{3\left((p^3+6p^2+11p+6)(dx+c)^2b^4+(p^3+3p^2+2p)(dx+c)^{\frac{3}{2}}ab^3-3\,(p^2+p)(dx+c)a\right.}{(p^4+10p^3+35p^2+50p+24)b^4}\right.}{}$$

input
```
integrate(x^3*(a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")
```

output
```
-2*(((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b +
 a)^p*c^3/((p^2 + 3*p + 2)*b^2) - 3*((p^3 + 6*p^2 + 11*p + 6)*(d*x + c)^2*
b^4 + (p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a*b^3 - 3*(p^2 + p)*(d*x + c)*a^
2*b^2 + 6*sqrt(d*x + c)*a^3*b*p - 6*a^4)*(sqrt(d*x + c)*b + a)^p*c^2/((p^4
 + 10*p^3 + 35*p^2 + 50*p + 24)*b^4) + 3*((p^5 + 15*p^4 + 85*p^3 + 225*p^2
 + 274*p + 120)*(d*x + c)^3*b^6 + (p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p)*
(d*x + c)^(5/2)*a*b^5 - 5*(p^4 + 6*p^3 + 11*p^2 + 6*p)*(d*x + c)^2*a^2*b^4
 + 20*(p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a^3*b^3 - 60*(p^2 + p)*(d*x + c)
*a^4*b^2 + 120*sqrt(d*x + c)*a^5*b*p - 120*a^6)*(sqrt(d*x + c)*b + a)^p*c/
((p^6 + 21*p^5 + 175*p^4 + 735*p^3 + 1624*p^2 + 1764*p + 720)*b^6) - ((p^7
 + 28*p^6 + 322*p^5 + 1960*p^4 + 6769*p^3 + 13132*p^2 + 13068*p + 5040)*(d
*x + c)^4*b^8 + (p^7 + 21*p^6 + 175*p^5 + 735*p^4 + 1624*p^3 + 1764*p^2 +
720*p)*(d*x + c)^(7/2)*a*b^7 - 7*(p^6 + 15*p^5 + 85*p^4 + 225*p^3 + 274*p^
2 + 120*p)*(d*x + c)^3*a^2*b^6 + 42*(p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p
)*(d*x + c)^(5/2)*a^3*b^5 - 210*(p^4 + 6*p^3 + 11*p^2 + 6*p)*(d*x + c)^2*a
^4*b^4 + 840*(p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a^5*b^3 - 2520*(p^2 + p)*
(d*x + c)*a^6*b^2 + 5040*sqrt(d*x + c)*a^7*b*p - 5040*a^8)*(sqrt(d*x + c)*
b + a)^p/((p^8 + 36*p^7 + 546*p^6 + 4536*p^5 + 22449*p^4 + 67284*p^3 + 118
124*p^2 + 109584*p + 40320)*b^8))/d^4
```

3.653.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5699 vs. 2(335) = 670.

Time = 0.51 (sec) , antiderivative size = 5699, normalized size of antiderivative = 16.28

$$\int x^3\left(a+b\sqrt{c+dx}\right)^p\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")
```

output
```
-2*((sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^7 - (sqrt(d*
x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^6*c^3*p^7 + 34*(sqrt(d*x + c)*b
+ a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^6 - 35*(sqrt(d*x + c)*b + a)*(sqr
t(d*x + c)*b + a)^p*a*b^6*c^3*p^6 - 3*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x +
c)*b + a)^p*b^4*c^2*p^7 + 9*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^
p*a*b^4*c^2*p^7 - 9*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^
4*c^2*p^7 + 3*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^4*c^2*p^
7 + 478*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^5 - 511*
(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^6*c^3*p^5 - 96*(sqrt(d*x
 + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^6 + 297*(sqrt(d*x + c)*b
+ a)^3*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p^6 - 306*(sqrt(d*x + c)*b + a)^2
*(sqrt(d*x + c)*b + a)^p*a^2*b^4*c^2*p^6 + 105*(sqrt(d*x + c)*b + a)*(sqrt
(d*x + c)*b + a)^p*a^3*b^4*c^2*p^6 + 3*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x +
 c)*b + a)^p*b^2*c*p^7 - 15*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^
p*a*b^2*c*p^7 + 30*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*b^2
*c*p^7 - 30*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^7
+ 15*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4*b^2*c*p^7 - 3*(sq
rt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5*b^2*c*p^7 + 3580*(sqrt(d*x
+ c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^4 - 4025*(sqrt(d*x + c)*b
+ a)*(sqrt(d*x + c)*b + a)^p*a*b^6*c^3*p^4 - 1254*(sqrt(d*x + c)*b + a)...
```

3.653.9 Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\sqrt{c+dx}\right)^p\,dx = \int x^3\,{\left(a+b\,\sqrt{c+d\,x}\right)}^p \,d x$$

input
```
int(x^3*(a + b*(c + d*x)^(1/2))^p,x)
```

output
```
int(x^3*(a + b*(c + d*x)^(1/2))^p, x)
```

3.654 $\int x^2 \left(a+b\sqrt{c+dx}\right)^p \, dx$

3.654.1 Optimal result

Integrand size = 19, antiderivative size = 242

$$\int x^2 \left(a+b\sqrt{c+dx}\right)^p \, dx = -\frac{2a(a^2-b^2c)^2 \left(a+b\sqrt{c+dx}\right)^{1+p}}{b^6d^3(1+p)} + \frac{2(5a^4-6a^2b^2c+b^4c^2) \left(a+b\sqrt{c+dx}\right)^{2+p}}{b^6d^3(2+p)} - \frac{4a(5a^2-3b^2c) \left(a+b\sqrt{c+dx}\right)^{3+p}}{b^6d^3(3+p)} + \frac{4(5a^2-b^2c) \left(a+b\sqrt{c+dx}\right)^{4+p}}{b^6d^3(4+p)} - \frac{10a\left(a+b\sqrt{c+dx}\right)^{5+p}}{b^6d^3(5+p)} + \frac{2\left(a+b\sqrt{c+dx}\right)^{6+p}}{b^6d^3(6+p)}$$

output
```
-2*a*(-b^2*c+a^2)^2*(a+b*(d*x+c)^(1/2))^(p+1)/b^6/d^3/(p+1)+2*(b^4*c^2-6*a
^2*b^2*c+5*a^4)*(a+b*(d*x+c)^(1/2))^(2+p)/b^6/d^3/(2+p)-4*a*(-3*b^2*c+5*a^
2)*(a+b*(d*x+c)^(1/2))^(3+p)/b^6/d^3/(3+p)+4*(-b^2*c+5*a^2)*(a+b*(d*x+c)^(
1/2))^(4+p)/b^6/d^3/(4+p)-10*a*(a+b*(d*x+c)^(1/2))^(5+p)/b^6/d^3/(5+p)+2*(
a+b*(d*x+c)^(1/2))^(6+p)/b^6/d^3/(6+p)
```

3.654.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.17

$$\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx =$$
$$-\frac{2\left(a+b\sqrt{c+dx}\right)^{1+p}\left(120a^5-120a^4b(1+p)\sqrt{c+dx}+12a^3b^2\left(4c(-5+p+p^2)+5d(2+3p+p^2)\,x\right)-\right.}{}$$

input
```
Integrate[x^2*(a + b*Sqrt[c + d*x])^p,x]
```

output
```
(-2*(a + b*Sqrt[c + d*x])^(1 + p)*(120*a^5 - 120*a^4*b*(1 + p)*Sqrt[c + d*
x] + 12*a^3*b^2*(4*c*(-5 + p + p^2) + 5*d*(2 + 3*p + p^2)*x) - 4*a^2*b^3*(
1 + p)*Sqrt[c + d*x]*(2*c*(-30 - 4*p + p^2) + 5*d*(6 + 5*p + p^2)*x) - b^5
*(15 + 23*p + 9*p^2 + p^3)*Sqrt[c + d*x]*(8*c^2 - 4*c*d*(2 + p)*x + d^2*(8
 + 6*p + p^2)*x^2) + a*b^4*(-8*c^2*(-15 + 10*p + 12*p^2 + 2*p^3) + 4*c*d*(
-30 - 43*p - 10*p^2 + 4*p^3 + p^4)*x + 5*d^2*(24 + 50*p + 35*p^2 + 10*p^3
+ p^4)*x^2)))/(b^6*d^3*(1 + p)*(2 + p)*(3 + p)*(4 + p)*(5 + p)*(6 + p))
```

3.654.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {896, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx$$
$$\Big\downarrow \; 896$$
$$\frac{\int d^2x^2\left(a+b\sqrt{c+dx}\right)^p d(c+dx)}{d^3}$$
$$\Big\downarrow \; 1732$$
$$\frac{2\int d^2x^2\sqrt{c+dx}\left(a+b\sqrt{c+dx}\right)^p d\sqrt{c+dx}}{d^3}$$
$$\Big\downarrow \; 522$$

$$\frac{2\int\left(-\frac{a(a^2-b^2c)^2(a+b\sqrt{c+dx})^p}{b^5}+\frac{(5a^4-6b^2ca^2+b^4c^2)(a+b\sqrt{c+dx})^{p+1}}{b^5}-\frac{2(5a^3-3ab^2c)(a+b\sqrt{c+dx})^{p+2}}{b^5}-\frac{2(b^2c-5a^2)(a+b\sqrt{c+dx})^{p-}}{b^5}\right.}{d^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(-\frac{a(a^2-b^2c)^2(a+b\sqrt{c+dx})^{p+1}}{b^6(p+1)}-\frac{2a(5a^2-3b^2c)(a+b\sqrt{c+dx})^{p+3}}{b^6(p+3)}+\frac{2(5a^2-b^2c)(a+b\sqrt{c+dx})^{p+4}}{b^6(p+4)}+\frac{(5a^4-6a^2b^2c+b^4c^2)(a+b\sqrt{c+dx})^{p+}}{b^6(p+2)}\right.}{d^3}$$

input
```
Int[x^2*(a + b*Sqrt[c + d*x])^p,x]
```

output
```
(2*(-((a*(a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])^(1 + p))/(b^6*(1 + p))) + (
(5*a^4 - 6*a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(2 + p))/(b^6*(2 + p
)) - (2*a*(5*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(3 + p))/(b^6*(3 + p)) +
 (2*(5*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(4 + p))/(b^6*(4 + p)) - (5*a*(a
 + b*Sqrt[c + d*x])^(5 + p))/(b^6*(5 + p)) + (a + b*Sqrt[c + d*x])^(6 + p)
/(b^6*(6 + p))))/d^3
```

3.654.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.654.4 Maple [F]

$$\int x^2 \left(a+b\sqrt{dx+c}\right)^p dx$$

input
```
int(x^2*(a+b*(d*x+c)^(1/2))^p,x)
```

output
```
int(x^2*(a+b*(d*x+c)^(1/2))^p,x)
```

3.654.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 712 vs. 2(230) = 460.

Time = 0.33 (sec) , antiderivative size = 712, normalized size of antiderivative = 2.94

$$\int x^2 \left(a+b\sqrt{c+dx}\right)^p dx = \frac{2\left(120\,b^6c^3-360\,a^2b^4c^2+360\,a^4b^2c-120\,a^6+8\,(b^6c^3+3\,a^2b^4c^2)p^3+(b^6d^3p^5+15\,b^6d^3p^4+85\,b^6d^3p^3+\cdots\right.}{}$$

input
```
integrate(x^2*(a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")
```

output
```
2*(120*b^6*c^3 - 360*a^2*b^4*c^2 + 360*a^4*b^2*c - 120*a^6 + 8*(b^6*c^3 +
3*a^2*b^4*c^2)*p^3 + (b^6*d^3*p^5 + 15*b^6*d^3*p^4 + 85*b^6*d^3*p^3 + 225*
b^6*d^3*p^2 + 274*b^6*d^3*p + 120*b^6*d^3)*x^3 + 24*(3*b^6*c^3 + 3*a^2*b^4
*c^2 - 2*a^4*b^2*c)*p^2 + (b^6*c*d^2*p^5 + (11*b^6*c - 5*a^2*b^4)*d^2*p^4
+ (41*b^6*c - 30*a^2*b^4)*d^2*p^3 + (61*b^6*c - 55*a^2*b^4)*d^2*p^2 + 30*(
b^6*c - a^2*b^4)*d^2*p)*x^2 + 8*(23*b^6*c^3 - 24*a^2*b^4*c^2 + 9*a^4*b^2*c
)*p - 4*((b^6*c^2 + a^2*b^4*c)*d*p^4 + 3*(3*b^6*c^2 - a^2*b^4*c)*d*p^3 + (
23*b^6*c^2 - 34*a^2*b^4*c + 15*a^4*b^2)*d*p^2 + 15*(b^6*c^2 - 2*a^2*b^4*c
+ a^4*b^2)*d*p)*x + (8*(3*a*b^5*c^2 + a^3*b^3*c)*p^3 + 24*(7*a*b^5*c^2 - 3
*a^3*b^3*c)*p^2 + (a*b^5*d^2*p^5 + 10*a*b^5*d^2*p^4 + 35*a*b^5*d^2*p^3 + 5
0*a*b^5*d^2*p^2 + 24*a*b^5*d^2*p)*x^2 + 8*(33*a*b^5*c^2 - 40*a^3*b^3*c + 1
5*a^5*b)*p - 4*(2*a*b^5*c*d*p^4 + 5*(3*a*b^5*c - a^3*b^3)*d*p^3 + (31*a*b^
5*c - 15*a^3*b^3)*d*p^2 + 2*(9*a*b^5*c - 5*a^3*b^3)*d*p)*x)*sqrt(d*x + c))
*(sqrt(d*x + c)*b + a)^p/(b^6*d^3*p^6 + 21*b^6*d^3*p^5 + 175*b^6*d^3*p^4 +
 735*b^6*d^3*p^3 + 1624*b^6*d^3*p^2 + 1764*b^6*d^3*p + 720*b^6*d^3)
```

3.654.6 Sympy [F]

$$\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx=\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx$$

input
```
integrate(x**2*(a+b*(d*x+c)**(1/2))**p,x)
```

output
```
Integral(x**2*(a + b*sqrt(c + d*x))**p, x)
```

3.654.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.66

$$\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx$$
$$=\frac{2\left(\frac{\left((dx+c)b^2(p+1)+\sqrt{dx+c}\,abp-a^2\right)\left(\sqrt{dx+c}\,b+a\right)^p c^2}{(p^2+3\,p+2)b^2}-\frac{2\left((p^3+6\,p^2+11\,p+6)(dx+c)^2b^4+(p^3+3\,p^2+2\,p)(dx+c)^{\frac{3}{2}}ab^3-3\,(p^2+p)(dx+c)a^2b^2\cdots\right.}{(p^4+10\,p^3+35\,p^2+50\,p+24)b^4}\cdots\right.}{\cdots}$$

input
```
integrate(x^2*(a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")
```

output
```
2*(((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b +
a)^p*c^2/((p^2 + 3*p + 2)*b^2) - 2*((p^3 + 6*p^2 + 11*p + 6)*(d*x + c)^2*b
^4 + (p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a*b^3 - 3*(p^2 + p)*(d*x + c)*a^2
*b^2 + 6*sqrt(d*x + c)*a^3*b*p - 6*a^4)*(sqrt(d*x + c)*b + a)^p*c/((p^4 +
10*p^3 + 35*p^2 + 50*p + 24)*b^4) + ((p^5 + 15*p^4 + 85*p^3 + 225*p^2 + 27
4*p + 120)*(d*x + c)^3*b^6 + (p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p)*(d*x
+ c)^(5/2)*a*b^5 - 5*(p^4 + 6*p^3 + 11*p^2 + 6*p)*(d*x + c)^2*a^2*b^4 + 20
*(p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a^3*b^3 - 60*(p^2 + p)*(d*x + c)*a^4*
b^2 + 120*sqrt(d*x + c)*a^5*b*p - 120*a^6)*(sqrt(d*x + c)*b + a)^p/((p^6 +
 21*p^5 + 175*p^4 + 735*p^3 + 1624*p^2 + 1764*p + 720)*b^6))/d^3
```

3.654.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2511 vs. $2(230) = 460$.

Time = 0.36 (sec) , antiderivative size = 2511, normalized size of antiderivative = 10.38

$$\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")
```

output
```
2*((sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^5 - (sqrt(d*x
 + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p^5 + 19*(sqrt(d*x + c)*b +
 a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^4 - 20*(sqrt(d*x + c)*b + a)*(sqrt
(d*x + c)*b + a)^p*a*b^4*c^2*p^4 - 2*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c
)*b + a)^p*b^2*c*p^5 + 6*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a
*b^2*c*p^5 - 6*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p
^5 + 2*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^5 + 137*(
sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^3 - 155*(sqrt(d*x
 + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p^3 - 34*(sqrt(d*x + c)*b +
 a)^4*(sqrt(d*x + c)*b + a)^p*b^2*c*p^4 + 108*(sqrt(d*x + c)*b + a)^3*(sqr
t(d*x + c)*b + a)^p*a*b^2*c*p^4 - 114*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x +
c)*b + a)^p*a^2*b^2*c*p^4 + 40*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)
^p*a^3*b^2*c*p^4 + (sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*p^5 - 5
*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*p^5 + 10*(sqrt(d*x + c)
*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*p^5 - 10*(sqrt(d*x + c)*b + a)^3*(sq
rt(d*x + c)*b + a)^p*a^3*p^5 + 5*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b
+ a)^p*a^4*p^5 - (sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5*p^5 + 4
61*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^2 - 580*(sqrt
(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p^2 - 214*(sqrt(d*x + c
)*b + a)^4*(sqrt(d*x + c)*b + a)^p*b^2*c*p^3 + 726*(sqrt(d*x + c)*b + a...
```

3.654.9 Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx = \int x^2\,\left(a+b\,\sqrt{c+d\,x}\right)^p\,dx$$

input
```
int(x^2*(a + b*(c + d*x)^(1/2))^p,x)
```

output
```
int(x^2*(a + b*(c + d*x)^(1/2))^p, x)
```

3.655 $\int x\left(a+b\sqrt{c+dx}\right)^p\,dx$

3.655.1 Optimal result

Integrand size = 17, antiderivative size = 145

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx=-\frac{2a(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^4d^2(1+p)}+\frac{2(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^4d^2(2+p)}-\frac{6a\left(a+b\sqrt{c+dx}\right)^{3+p}}{b^4d^2(3+p)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{4+p}}{b^4d^2(4+p)}$$

output
```
-2*a*(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(p+1)/b^4/d^2/(p+1)+2*(-b^2*c+3*a^2)
*(a+b*(d*x+c)^(1/2))^(2+p)/b^4/d^2/(2+p)-6*a*(a+b*(d*x+c)^(1/2))^(3+p)/b^4
/d^2/(3+p)+2*(a+b*(d*x+c)^(1/2))^(4+p)/b^4/d^2/(4+p)
```

3.655.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.88

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx=$$

$$-\frac{2\left(a+b\sqrt{c+dx}\right)^{1+p}\left(6a^3-6a^2b(1+p)\sqrt{c+dx}-b^3(3+4p+p^2)\sqrt{c+dx}(-2c+d(2+p)x)+ab^2(2c\right.}{b^4d^2(1+p)(2+p)(3+p)(4+p)}$$

input
```
Integrate[x*(a + b*Sqrt[c + d*x])^p,x]
```

output
```
(-2*(a + b*Sqrt[c + d*x])^(1 + p)*(6*a^3 - 6*a^2*b*(1 + p)*Sqrt[c + d*x] -
 b^3*(3 + 4*p + p^2)*Sqrt[c + d*x]*(-2*c + d*(2 + p)*x) + a*b^2*(2*c*(-3 +
 p + p^2) + 3*d*(2 + 3*p + p^2)*x)))/(b^4*d^2*(1 + p)*(2 + p)*(3 + p)*(4 +
 p))
```

3.655.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx$$

$$\Big\downarrow \text{896}$$

$$\frac{\int dx\left(a+b\sqrt{c+dx}\right)^p d(c+dx)}{d^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -dx\left(a+b\sqrt{c+dx}\right)^p d(c+dx)}{d^2}$$

$$\Big\downarrow \text{1732}$$

$$-\frac{2\int -dx\sqrt{c+dx}\left(a+b\sqrt{c+dx}\right)^p d\sqrt{c+dx}}{d^2}$$

$$\Big\downarrow \text{522}$$

$$-\frac{2\int \left(\frac{(a^3-ab^2c)\left(a+b\sqrt{c+dx}\right)^p}{b^3}+\frac{(b^2c-3a^2)\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^3}+\frac{3a\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^3}-\frac{\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^3}\right)d\sqrt{c+dx}}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\left(\frac{a(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^4(p+1)}-\frac{(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^4(p+2)}+\frac{3a\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^4(p+3)}-\frac{\left(a+b\sqrt{c+dx}\right)^{p+4}}{b^4(p+4)}\right)}{d^2}$$

input
```
Int[x*(a + b*Sqrt[c + d*x])^p,x]
```

output
```
(-2*((a*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(1 + p))/(b^4*(1 + p)) - ((3*a
^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(2 + p))/(b^4*(2 + p)) + (3*a*(a + b*Sqr
t[c + d*x])^(3 + p))/(b^4*(3 + p)) - (a + b*Sqrt[c + d*x])^(4 + p)/(b^4*(4
 + p))))/d^2
```

3.655.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.655.4 Maple [F]

$$\int x\left(a+b\sqrt{dx+c}\right)^p\,dx$$

input
```
int(x*(a+b*(d*x+c)^(1/2))^p,x)
```

output
```
int(x*(a+b*(d*x+c)^(1/2))^p,x)
```

3.655.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(137) = 274.

Time = 0.31 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.03

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx = -\frac{2\left(6\,b^4c^2-12\,a^2b^2c+6\,a^4+2\,(b^4c^2+a^2b^2c)p^2-(b^4d^2p^3+6\,b^4d^2p^2+11\,b^4d^2p+6\,b^4d^2)x^2+4\,(2\,b^4c^2-\right.}{}$$

input
```
integrate(x*(a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")
```

output
```
-2*(6*b^4*c^2 - 12*a^2*b^2*c + 6*a^4 + 2*(b^4*c^2 + a^2*b^2*c)*p^2 - (b^4*
d^2*p^3 + 6*b^4*d^2*p^2 + 11*b^4*d^2*p + 6*b^4*d^2)*x^2 + 4*(2*b^4*c^2 - a
^2*b^2*c)*p - (b^4*c*d*p^3 + (4*b^4*c - 3*a^2*b^2)*d*p^2 + 3*(b^4*c - a^2*
b^2)*d*p)*x + (4*a*b^3*c*p^2 + 2*(5*a*b^3*c - 3*a^3*b)*p - (a*b^3*d*p^3 +
3*a*b^3*d*p^2 + 2*a*b^3*d*p)*x)*sqrt(d*x + c))*(sqrt(d*x + c)*b + a)^p/(b^
4*d^2*p^4 + 10*b^4*d^2*p^3 + 35*b^4*d^2*p^2 + 50*b^4*d^2*p + 24*b^4*d^2)
```

3.655.6 Sympy [F]

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx=\int x\left(a+b\sqrt{c+dx}\right)^p\,dx$$

input
```
integrate(x*(a+b*(d*x+c)**(1/2))**p,x)
```

output
```
Integral(x*(a + b*sqrt(c + d*x))**p, x)
```

3.655.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.29

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx = -\frac{2\left(\frac{((dx+c)b^2(p+1)+\sqrt{dx+c}abp-a^2)(\sqrt{dx+c}b+a)^pc}{(p^2+3p+2)b^2}-\frac{\left((p^3+6p^2+11p+6)(dx+c)^2b^4+(p^3+3p^2+2p)(dx+c)^{\frac{3}{2}}ab^3-3(p^2+p)(dx+c)a^2b\right.}{(p^4+10p^3+35p^2+50p+24)b^4}\right.}{d^2}$$

input
```
integrate(x*(a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")
```

output
```
-2*((((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b +
 a)^p*c/((p^2 + 3*p + 2)*b^2) - ((p^3 + 6*p^2 + 11*p + 6)*(d*x + c)^2*b^4
+ (p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a*b^3 - 3*(p^2 + p)*(d*x + c)*a^2*b^
2 + 6*sqrt(d*x + c)*a^3*b*p - 6*a^4)*(sqrt(d*x + c)*b + a)^p/((p^4 + 10*p^
3 + 35*p^2 + 50*p + 24)*b^4))/d^2
```

3.655.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 806 vs. 2(137) = 274.

Time = 0.35 (sec) , antiderivative size = 806, normalized size of antiderivative = 5.56

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx = -\frac{2\left(\left(\sqrt{dx+c}b+a\right)^2\left(\sqrt{dx+c}b+a\right)^pb^2cp^3-\left(\sqrt{dx+c}b+a\right)\left(\sqrt{dx+c}b+a\right)^pab^2cp^3+8\left(\sqrt{dx+c}b+a\right.\right.}{}$$

input
```
integrate(x*(a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")
```

output
```
-2*((sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^2*c*p^3 - (sqrt(d*x
+ c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p^3 + 8*(sqrt(d*x + c)*b + a)^
2*(sqrt(d*x + c)*b + a)^p*b^2*c*p^2 - 9*(sqrt(d*x + c)*b + a)*(sqrt(d*x +
c)*b + a)^p*a*b^2*c*p^2 - (sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*
p^3 + 3*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*p^3 - 3*(sqrt(d*
x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*p^3 + (sqrt(d*x + c)*b + a)*(s
qrt(d*x + c)*b + a)^p*a^3*p^3 + 19*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*
b + a)^p*b^2*c*p - 26*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^2*
c*p - 6*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*p^2 + 21*(sqrt(d*x
 + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*p^2 - 24*(sqrt(d*x + c)*b + a)^2*
(sqrt(d*x + c)*b + a)^p*a^2*p^2 + 9*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b
 + a)^p*a^3*p^2 + 12*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^2*c
 - 24*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^2*c - 11*(sqrt(d*x
 + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*p + 42*(sqrt(d*x + c)*b + a)^3*(sqr
t(d*x + c)*b + a)^p*a*p - 57*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)
^p*a^2*p + 26*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*p - 6*(sqr
t(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p + 24*(sqrt(d*x + c)*b + a)^3*(
sqrt(d*x + c)*b + a)^p*a - 36*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a
)^p*a^2 + 24*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3)/((b^2*p^4
+ 10*b^2*p^3 + 35*b^2*p^2 + 50*b^2*p + 24*b^2)*b^2*d^2)
```

3.655.9 Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx=\int x\,\left(a+b\,\sqrt{c+d\,x}\right)^p\,dx$$

input
```
int(x*(a + b*(c + d*x)^(1/2))^p,x)
```

output
```
int(x*(a + b*(c + d*x)^(1/2))^p, x)
```

3.656 $\int \left(a+b\sqrt{c+dx}\right)^p\,dx$

3.656.1 Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \left(a+b\sqrt{c+dx}\right)^p\,dx=-\frac{2a\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^2d(1+p)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^2d(2+p)}$$

output
```
-2*a*(a+b*(d*x+c)^(1/2))^(p+1)/b^2/d/(p+1)+2*(a+b*(d*x+c)^(1/2))^(2+p)/b^2
/d/(2+p)
```

3.656.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

$$\int \left(a+b\sqrt{c+dx}\right)^p\,dx=\frac{2\left(a+b\sqrt{c+dx}\right)^{1+p}\left(-a+b(1+p)\sqrt{c+dx}\right)}{b^2d(1+p)(2+p)}$$

input
```
Integrate[(a + b*Sqrt[c + d*x])^p,x]
```

output
```
(2*(a + b*Sqrt[c + d*x])^(1 + p)*(-a + b*(1 + p)*Sqrt[c + d*x]))/(b^2*d*(1
 + p)*(2 + p))
```

3.656.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {239, 774, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\sqrt{c+dx}\right)^p dx$$

$$\Big\downarrow \text{239}$$

$$\frac{\int \left(a+b\sqrt{c+dx}\right)^p d(c+dx)}{d}$$

$$\Big\downarrow \text{774}$$

$$\frac{2\int \sqrt{c+dx}\left(a+b\sqrt{c+dx}\right)^p d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{53}$$

$$\frac{2\int \left(\frac{\left(a+b\sqrt{c+dx}\right)^{p+1}}{b}-\frac{a\left(a+b\sqrt{c+dx}\right)^p}{b}\right)d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^2(p+2)}-\frac{a\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^2(p+1)}\right)}{d}$$

input
```
Int[(a + b*Sqrt[c + d*x])^p,x]
```

output
```
(2*(-((a*(a + b*Sqrt[c + d*x])^(1 + p))/(b^2*(1 + p))) + (a + b*Sqrt[c + d
*x])^(2 + p)/(b^2*(2 + p))))/d
```

3.656.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.656.4 Maple [F]

$$\int \left(a+b\sqrt{dx+c}\right)^{p} dx$$

input
```
int((a+b*(d*x+c)^(1/2))^p,x)
```

output
```
int((a+b*(d*x+c)^(1/2))^p,x)
```

3.656.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.31

$$\int \left(a+b\sqrt{c+dx}\right)^p dx = \frac{2\left(b^2cp+\sqrt{dx+c}abp+b^2c-a^2+(b^2dp+b^2d)x\right)\left(\sqrt{dx+c}b+a\right)^p}{b^2dp^2+3\,b^2dp+2\,b^2d}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")
```

output
```
2*(b^2*c*p + sqrt(d*x + c)*a*b*p + b^2*c - a^2 + (b^2*d*p + b^2*d)*x)*(sqr
t(d*x + c)*b + a)^p/(b^2*d*p^2 + 3*b^2*d*p + 2*b^2*d)
```

3.656.6 Sympy [F]

$$\int \left(a+b\sqrt{c+dx}\right)^p\,dx=\int \left(a+b\sqrt{c+dx}\right)^p\,dx$$

input
```
integrate((a+b*(d*x+c)**(1/2))**p,x)
```

output
```
Integral((a + b*sqrt(c + d*x))**p, x)
```

3.656.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \left(a+b\sqrt{c+dx}\right)^p\,dx=\frac{2\left((dx+c)b^2(p+1)+\sqrt{dx+c}abp-a^2\right)\left(\sqrt{dx+c}b+a\right)^p}{(p^2+3\,p+2)b^2d}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")
```

output
```
2*((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b + a
)^p/((p^2 + 3*p + 2)*b^2*d)
```

3.656.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(58) = 116.

Time = 0.57 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.08

$$\int \left(a+b\sqrt{c+dx}\right)^p\,dx$$
$$=\frac{2\left(\left(\sqrt{dx+c}b+a\right)^2\left(\sqrt{dx+c}b+a\right)^pp-\left(\sqrt{dx+c}b+a\right)\left(\sqrt{dx+c}b+a\right)^pap+\left(\sqrt{dx+c}b+a\right)^2\left(\sqrt{dx+c}\right.}{(p^2+3\,p+2)b^2d}$$

input
```
integrate((a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")
```

output
```
2*((sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*p - (sqrt(d*x + c)*b +
a)*(sqrt(d*x + c)*b + a)^p*a*p + (sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b
+ a)^p - 2*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a)/((p^2 + 3*p +
2)*b^2*d)
```

3.656.9 Mupad [B] (verification not implemented)

Time = 17.52 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.35

$$\int \left(a+b\sqrt{c+dx}\right)^p\,dx=\begin{cases} -\frac{2\,a\,\ln\left(a+b\,\sqrt{c+d\,x}\right)-2\,b\,\sqrt{c+d\,x}}{b^2\,d} & \text{if } p=-1\\ \frac{2\,\left(\ln\left(a+b\,\sqrt{c+d\,x}\right)+\frac{a}{a+b\,\sqrt{c+d\,x}}\right)}{b^2\,d} & \text{if } p=-2\\ \frac{4\,\left(a+b\,\sqrt{c+d\,x}\right)^{p+2}}{b^2\,d\,(2\,p+4)}-\frac{4\,a\,\left(a+b\,\sqrt{c+d\,x}\right)^{p+1}}{b^2\,d\,(2\,p+2)} & \text{if } p\neq -1\wedge p\neq -2 \end{cases}$$

input
```
int((a + b*(c + d*x)^(1/2))^p,x)
```

output
```
piecewise(p == -1, -(2*a*log(a + b*(c + d*x)^(1/2)) - 2*b*(c + d*x)^(1/2))
/(b^2*d), p == -2, (2*(log(a + b*(c + d*x)^(1/2)) + a/(a + b*(c + d*x)^(1/
2))))/(b^2*d), p ~= -1 & p ~= -2, (4*(a + b*(c + d*x)^(1/2))^(p + 2))/(b^2
*d*(2*p + 4)) - (4*a*(a + b*(c + d*x)^(1/2))^(p + 1))/(b^2*d*(2*p + 2)))
```

3.657 $\int \frac{(a+b\sqrt{c+dx})^p}{x}\,dx$

3.657.1 Optimal result

Integrand size = 19, antiderivative size = 139

$$\int \frac{(a+b\sqrt{c+dx})^p}{x}\,dx = -\frac{(a+b\sqrt{c+dx})^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sqrt{c+dx}}{a-b\sqrt{c}}\right)}{(a-b\sqrt{c})(1+p)} - \frac{(a+b\sqrt{c+dx})^{1+p}\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sqrt{c+dx}}{a+b\sqrt{c}}\right)}{(a+b\sqrt{c})(1+p)}$$

output
```
-hypergeom([1, p+1],[2+p],(a+b*(d*x+c)^(1/2))/(a-b*c^(1/2)))*(a+b*(d*x+c)^
(1/2))^(p+1)/(p+1)/(a-b*c^(1/2))-hypergeom([1, p+1],[2+p],(a+b*(d*x+c)^(1/
2))/(a+b*c^(1/2)))*(a+b*(d*x+c)^(1/2))^(p+1)/(p+1)/(a+b*c^(1/2))
```

3.657.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.98

$$\int \frac{(a+b\sqrt{c+dx})^p}{x}\,dx = -\frac{(a+b\sqrt{c+dx})^{1+p}\left((a+b\sqrt{c})\,\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{a+b\sqrt{c+dx}}{a-b\sqrt{c}}\right)+(a-b\sqrt{c})\,\text{Hypergeo}\right.}{(a-b\sqrt{c})(a+b\sqrt{c})(1+p)}$$

input
```
Integrate[(a + b*Sqrt[c + d*x])^p/x,x]
```

output
```
-(((a + b*Sqrt[c + d*x])^(1 + p)*((a + b*Sqrt[c])*Hypergeometric2F1[1, 1 +
 p, 2 + p, (a + b*Sqrt[c + d*x])/(a - b*Sqrt[c])] + (a - b*Sqrt[c])*Hyperg
eometric2F1[1, 1 + p, 2 + p, (a + b*Sqrt[c + d*x])/(a + b*Sqrt[c])]))/((a
- b*Sqrt[c])*(a + b*Sqrt[c])*(1 + p)))
```

3.657.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {896, 25, 1732, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}dx$$

$$\Big\downarrow \text{896}$$

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{dx}d(c+dx)$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{\left(a+b\sqrt{c+dx}\right)^p}{dx}d(c+dx)$$

$$\Big\downarrow \text{1732}$$

$$-2\int -\frac{\sqrt{c+dx}\left(a+b\sqrt{c+dx}\right)^p}{dx}d\sqrt{c+dx}$$

$$\Big\downarrow \text{615}$$

$$-2\int \left(\frac{\left(a+b\sqrt{c+dx}\right)^p}{2\left(-c+\sqrt{c}-dx\right)}-\frac{\left(a+b\sqrt{c+dx}\right)^p}{2\left(\sqrt{c}+\sqrt{c+dx}\right)}\right)d\sqrt{c+dx}$$

$$\Big\downarrow \text{2009}$$

$$-2\left(\frac{\left(a+b\sqrt{c+dx}\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{a+b\sqrt{c+dx}}{a-b\sqrt{c}}\right)}{2(p+1)\left(a-b\sqrt{c}\right)}+\frac{\left(a+b\sqrt{c+dx}\right)^{p+1}\text{Hypergeometric2F}}{2(p+1)\left(a+\right.}\right.$$

input
```
Int[(a + b*Sqrt[c + d*x])^p/x,x]
```

output
```
-2*(((a + b*Sqrt[c + d*x])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a +
 b*Sqrt[c + d*x])/(a - b*Sqrt[c])])/(2*(a - b*Sqrt[c])*(1 + p)) + ((a + b*
Sqrt[c + d*x])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sqrt[c +
d*x])/(a + b*Sqrt[c])])/(2*(a + b*Sqrt[c])*(1 + p)))
```

3.657.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.657. $\int \frac{(a+b\sqrt{c+dx})^p}{x}\,dx$

3.657.4 Maple [F]

$$\int \frac{\left(a+b\sqrt{dx+c}\right)^{p}}{x}dx$$

input
```
int((a+b*(d*x+c)^(1/2))^p/x,x)
```

output
```
int((a+b*(d*x+c)^(1/2))^p/x,x)
```

3.657.5 Fricas [F]

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^{p}}{x}\,dx=\int \frac{\left(\sqrt{dx+c}b+a\right)^{p}}{x}\,dx$$

input
```
integrate((a+b*(d*x+c)^(1/2))^p/x,x, algorithm="fricas")
```

output
```
integral((sqrt(d*x + c)*b + a)^p/x, x)
```

3.657.6 Sympy [F]

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^{p}}{x}\,dx=\int \frac{\left(a+b\sqrt{c+dx}\right)^{p}}{x}\,dx$$

input
```
integrate((a+b*(d*x+c)**(1/2))**p/x,x)
```

output
```
Integral((a + b*sqrt(c + d*x))**p/x, x)
```

3.657.7 Maxima [F]

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx=\int \frac{\left(\sqrt{dx+c}b+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*(d*x+c)^(1/2))^p/x,x, algorithm="maxima")
```

output
```
integrate((sqrt(d*x + c)*b + a)^p/x, x)
```

3.657.8 Giac [F]

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx=\int \frac{\left(\sqrt{dx+c}b+a\right)^p}{x}\,dx$$

input
```
integrate((a+b*(d*x+c)^(1/2))^p/x,x, algorithm="giac")
```

output
```
integrate((sqrt(d*x + c)*b + a)^p/x, x)
```

3.657.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx=\int \frac{\left(a+b\,\sqrt{c+d\,x}\right)^p}{x}\,dx$$

input
```
int((a + b*(c + d*x)^(1/2))^p/x,x)
```

output
```
int((a + b*(c + d*x)^(1/2))^p/x, x)
```

3.658 $\int \frac{(a+b(cx)^n)^{5/2}}{x} dx$

3.658.1 Optimal result

Integrand size = 17, antiderivative size = 93

$$\int \frac{(a+b(cx)^n)^{5/2}}{x} dx = \frac{2a^2\sqrt{a+b(cx)^n}}{n} + \frac{2a(a+b(cx)^n)^{3/2}}{3n} + \frac{2(a+b(cx)^n)^{5/2}}{5n} - \frac{2a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

output
```
2/3*a*(a+b*(c*x)^n)^(3/2)/n+2/5*(a+b*(c*x)^n)^(5/2)/n-2*a^(5/2)*arctanh((a
+b*(c*x)^n)^(1/2)/a^(1/2))/n+2*a^2*(a+b*(c*x)^n)^(1/2)/n
```

3.658.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.83

$$\int \frac{(a+b(cx)^n)^{5/2}}{x} dx = \frac{2\sqrt{a+b(cx)^n}(23a^2+11ab(cx)^n+3b^2(cx)^{2n}) - 30a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{15n}$$

input
```
Integrate[(a + b*(c*x)^n)^(5/2)/x,x]
```

output
```
(2*Sqrt[a + b*(c*x)^n]*(23*a^2 + 11*a*b*(c*x)^n + 3*b^2*(c*x)^(2*n)) - 30*
a^(5/2)*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(15*n)
```

3.658.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {891, 27, 798, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b(cx)^n)^{5/2}}{x} \, dx$$

$$\Big\downarrow \ 891$$

$$\frac{\int \frac{(b(cx)^n+a)^{5/2}}{x}d(cx)}{c}$$

$$\Big\downarrow \ 27$$

$$\int \frac{(a+b(cx)^n)^{5/2}}{cx}d(cx)$$

$$\Big\downarrow \ 798$$

$$\frac{\int \frac{(b(cx)^n+a)^{5/2}}{cx}d(cx)^n}{n}$$

$$\Big\downarrow \ 60$$

$$\frac{a\int \frac{(b(cx)^n+a)^{3/2}}{cx}d(cx)^n+\frac{2}{5}(a+b(cx)^n)^{5/2}}{n}$$

$$\Big\downarrow \ 60$$

$$\frac{a\left(a\int \frac{\sqrt{b(cx)^n+a}}{cx}d(cx)^n+\frac{2}{3}(a+b(cx)^n)^{3/2}\right)+\frac{2}{5}(a+b(cx)^n)^{5/2}}{n}$$

$$\Big\downarrow \ 60$$

$$\frac{a\left(a\left(a\int \frac{1}{cx\sqrt{b(cx)^n+a}}d(cx)^n+2\sqrt{a+b(cx)^n}\right)+\frac{2}{3}(a+b(cx)^n)^{3/2}\right)+\frac{2}{5}(a+b(cx)^n)^{5/2}}{n}$$

$$\Big\downarrow \ 73$$

$$\frac{a\left(a\left(\frac{2a\int \frac{1}{\frac{c^2x^2}{b}-\frac{a}{b}}d\sqrt{b(cx)^n+a}}{b}+2\sqrt{a+b(cx)^n}\right)+\frac{2}{3}(a+b(cx)^n)^{3/2}\right)+\frac{2}{5}(a+b(cx)^n)^{5/2}}{n}$$

$$\Big\downarrow \ 221$$

$$\frac{a\left(a\left(2\sqrt{a+b(cx)^n}-2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a+b(cx)^n)^{3/2}\right)+\frac{2}{5}(a+b(cx)^n)^{5/2}}{n}$$

input
```
Int[(a + b*(c*x)^n)^(5/2)/x,x]
```

output
```
((2*(a + b*(c*x)^n)^(5/2))/5 + a*((2*(a + b*(c*x)^n)^(3/2))/3 + a*(2*Sqrt[
a + b*(c*x)^n] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])))/n
```

3.658.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.658. $\int \frac{(a+b(cx)^n)^{5/2}}{x}\,dx$

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

3.658.4 Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\frac{2(a+b(cx)^n)^{\frac{5}{2}}}{5}+\frac{2a(a+b(cx)^n)^{\frac{3}{2}}}{3}+2a^2\sqrt{a+b(cx)^n}-2a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	70
default	$\frac{\frac{2(a+b(cx)^n)^{\frac{5}{2}}}{5}+\frac{2a(a+b(cx)^n)^{\frac{3}{2}}}{3}+2a^2\sqrt{a+b(cx)^n}-2a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	70
risch	$\frac{2(3b^2e^{2n\ln(cx)}+11a\,e^{n\ln(cx)}b+23a^2)\sqrt{a+b\,e^{n\ln(cx)}}}{15n}-\frac{2a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b\,e^{n\ln(cx)}}}{\sqrt{a}}\right)}{n}$	77

input
```
int((a+b*(c*x)^n)^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(2/5*(a+b*(c*x)^n)^(5/2)+2/3*a*(a+b*(c*x)^n)^(3/2)+2*a^2*(a+b*(c*x)^n)
^(1/2)-2*a^(5/2)*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2)))
```

3.658.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.76

$$\int \frac{(a+b(cx)^n)^{5/2}}{x}\,dx = \left[\frac{15\,a^{\frac{5}{2}}\log\left(\frac{(cx)^nb-2\sqrt{(cx)^nb+a}\sqrt{a}+2\,a}{(cx)^n}\right)+2\left(11\,(cx)^n\,ab+3\,(cx)^{2\,n}\,b^2+23\,a^2\right)\sqrt{(cx)^n\,b}}{15\,n}\right.$$

input
```
integrate((a+b*(c*x)^n)^(5/2)/x,x, algorithm="fricas")
```

output
```
[1/15*(15*a^(5/2)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c
*x)^n) + 2*(11*(c*x)^n*a*b + 3*(c*x)^(2*n)*b^2 + 23*a^2)*sqrt((c*x)^n*b +
a))/n, 2/15*(15*sqrt(-a)*a^2*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a) + (11*
(c*x)^n*a*b + 3*(c*x)^(2*n)*b^2 + 23*a^2)*sqrt((c*x)^n*b + a))/n]
```

3.658.6 Sympy [A] (verification not implemented)

Time = 18.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.44

$$\int \frac{(a+b(cx)^n)^{5/2}}{x}\,dx = \begin{cases} \frac{\begin{cases} \frac{2a^3\operatorname{atan}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2a^2\sqrt{a+b\,(cx)^n} + \frac{2a(a+b(cx)^n)^{\frac{3}{2}}}{3} + \frac{2(a+b(cx)^n)^{\frac{5}{2}}}{5} & \text{for } b\neq 0 \\ a^{\frac{5}{2}}\log\left((cx)^n\right) & \text{otherwise} \end{cases}}{n} \\ -\left(-a^2\sqrt{a+b} - 2ab\sqrt{a+b} - b^2\sqrt{a+b}\right)\log(cx) \end{cases}$$

input
```
integrate((a+b*(c*x)**n)**(5/2)/x,x)
```

output
```
Piecewise((Piecewise((2*a**3*atan(sqrt(a + b*(c*x)**n)/sqrt(-a))/sqrt(-a)
+ 2*a**2*sqrt(a + b*(c*x)**n) + 2*a*(a + b*(c*x)**n)**(3/2)/3 + 2*(a + b*(
c*x)**n)**(5/2)/5, Ne(b, 0)), (a**(5/2)*log((c*x)**n), True))/n, Ne(n, 0))
, (-(-a**2*sqrt(a + b) - 2*a*b*sqrt(a + b) - b**2*sqrt(a + b))*log(c*x), T
rue))
```

3.658.7 Maxima [F]

$$\int \frac{(a+b(cx)^n)^{5/2}}{x}\,dx = \int \frac{\left((cx)^n b + a\right)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate((a+b*(c*x)^n)^(5/2)/x,x, algorithm="maxima")
```

output
```
integrate(((c*x)^n*b + a)^(5/2)/x, x)
```

3.658.8 Giac [F]

$$\int \frac{(a+b(cx)^n)^{5/2}}{x}\,dx = \int \frac{\left((cx)^n b + a\right)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate((a+b*(c*x)^n)^(5/2)/x,x, algorithm="giac")
```

output
```
integrate(((c*x)^n*b + a)^(5/2)/x, x)
```

3.658.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b(cx)^n)^{5/2}}{x}\,dx = \int \frac{\left(a+b\,\left(c\,x\right)^n\right)^{5/2}}{x}\,dx$$

input
```
int((a + b*(c*x)^n)^(5/2)/x,x)
```

output
```
int((a + b*(c*x)^n)^(5/2)/x, x)
```

3.659 $\int \frac{(a+b(cx)^n)^{3/2}}{x} dx$

3.659.1 Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \frac{(a+b(cx)^n)^{3/2}}{x} dx = \frac{2a\sqrt{a+b(cx)^n}}{n} + \frac{2(a+b(cx)^n)^{3/2}}{3n} - \frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

output
```
2/3*(a+b*(c*x)^n)^(3/2)/n-2*a^(3/2)*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))/n
+2*a*(a+b*(c*x)^n)^(1/2)/n
```

3.659.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int \frac{(a+b(cx)^n)^{3/2}}{x} dx = \frac{2\sqrt{a+b(cx)^n}(4a+b(cx)^n) - 6a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{3n}$$

input
```
Integrate[(a + b*(c*x)^n)^(3/2)/x,x]
```

output
```
(2*Sqrt[a + b*(c*x)^n]*(4*a + b*(c*x)^n) - 6*a^(3/2)*ArcTanh[Sqrt[a + b*(c
*x)^n]/Sqrt[a]])/(3*n)
```

3.659.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {891, 27, 798, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b(cx)^n)^{3/2}}{x} \, dx$$
$$\Big\downarrow \text{891}$$
$$\frac{\int \frac{(b(cx)^n+a)^{3/2}}{x}d(cx)}{c}$$
$$\Big\downarrow \text{27}$$
$$\int \frac{(a+b(cx)^n)^{3/2}}{cx}d(cx)$$
$$\Big\downarrow \text{798}$$
$$\frac{\int \frac{(b(cx)^n+a)^{3/2}}{cx}d(cx)^n}{n}$$
$$\Big\downarrow \text{60}$$
$$\frac{a\int \frac{\sqrt{b(cx)^n+a}}{cx}d(cx)^n+\frac{2}{3}(a+b(cx)^n)^{3/2}}{n}$$
$$\Big\downarrow \text{60}$$
$$\frac{a\left(a\int \frac{1}{cx\sqrt{b(cx)^n+a}}d(cx)^n+2\sqrt{a+b(cx)^n}\right)+\frac{2}{3}(a+b(cx)^n)^{3/2}}{n}$$
$$\Big\downarrow \text{73}$$
$$\frac{a\left(\frac{2a\int \frac{1}{\frac{c^2x^2}{b}-\frac{a}{b}}d\sqrt{b(cx)^n+a}}{b}+2\sqrt{a+b(cx)^n}\right)+\frac{2}{3}(a+b(cx)^n)^{3/2}}{n}$$
$$\Big\downarrow \text{221}$$
$$\frac{a\left(2\sqrt{a+b(cx)^n}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a+b(cx)^n)^{3/2}}{n}$$

input
```
Int[(a + b*(c*x)^n)^(3/2)/x,x]
```

output
```
((2*(a + b*(c*x)^n)^(3/2))/3 + a*(2*Sqrt[a + b*(c*x)^n] - 2*Sqrt[a]*ArcTan
h[Sqrt[a + b*(c*x)^n]/Sqrt[a]]))/n
```

3.659.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

3.659. $\int \frac{(a+b(cx)^n)^{3/2}}{x} dx$

3.659.4 Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{\frac{2(a+b(cx)^n)^{\frac{3}{2}}}{3}+2a\sqrt{a+b(cx)^n}-2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	54
default	$\frac{\frac{2(a+b(cx)^n)^{\frac{3}{2}}}{3}+2a\sqrt{a+b(cx)^n}-2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	54
risch	$\frac{2(b\,e^{n\ln(cx)}+4a)\sqrt{a+b\,e^{n\ln(cx)}}}{3n}-\frac{2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b\,e^{n\ln(cx)}}}{\sqrt{a}}\right)}{n}$	59

input
```
int((a+b*(c*x)^n)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(2/3*(a+b*(c*x)^n)^(3/2)+2*a*(a+b*(c*x)^n)^(1/2)-2*a^(3/2)*arctanh((a+
b*(c*x)^n)^(1/2)/a^(1/2)))
```

3.659.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.86

$$\int \frac{(a+b(cx)^n)^{3/2}}{x}\,dx = \left[\frac{3\,a^{\frac{3}{2}}\log\left(\frac{(cx)^n b-2\,\sqrt{(cx)^n b+a}\sqrt{a}+2\,a}{(cx)^n}\right)+2\left((cx)^n b+4\,a\right)\sqrt{(cx)^n b+a}}{3\,n}, \frac{2\left(3\,\sqrt{-a}a\,\mathrm{arct}\right.}{}\right.$$

input
```
integrate((a+b*(c*x)^n)^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/3*(3*a^(3/2)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x
)^n) + 2*((c*x)^n*b + 4*a)*sqrt((c*x)^n*b + a))/n, 2/3*(3*sqrt(-a)*a*arcta
n(sqrt((c*x)^n*b + a)*sqrt(-a)/a) + ((c*x)^n*b + 4*a)*sqrt((c*x)^n*b + a))
/n]
```

3.659.6 Sympy [A] (verification not implemented)

Time = 12.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.36

$$\int \frac{(a+b(cx)^n)^{3/2}}{x}\,dx = \begin{cases} \frac{\begin{cases} \frac{2a^2 \operatorname{atan}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2a\sqrt{a+b\,(cx)^n} + \frac{2(a+b(cx)^n)^{\frac{3}{2}}}{3} & \text{for}\ b \neq 0 \\ a^{\frac{3}{2}} \log\left((cx)^n\right) & \text{otherwise} \end{cases}}{n} & \text{for}\ n \neq 0 \\ \left(a\sqrt{a+b} + b\sqrt{a+b}\right)\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*(c*x)**n)**(3/2)/x,x)
```

output
```
Piecewise((Piecewise((2*a**2*atan(sqrt(a + b*(c*x)**n)/sqrt(-a))/sqrt(-a)
+ 2*a*sqrt(a + b*(c*x)**n) + 2*(a + b*(c*x)**n)**(3/2)/3, Ne(b, 0)), (a**(
3/2)*log((c*x)**n), True))/n, Ne(n, 0)), ((a*sqrt(a + b) + b*sqrt(a + b))*
log(x), True))
```

3.659.7 Maxima [F]

$$\int \frac{(a+b(cx)^n)^{3/2}}{x}\,dx = \int \frac{((cx)^n b + a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+b*(c*x)^n)^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate(((c*x)^n*b + a)^(3/2)/x, x)
```

3.659.8 Giac [F]

$$\int \frac{(a+b(cx)^n)^{3/2}}{x}\,dx = \int \frac{((cx)^n b + a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+b*(c*x)^n)^(3/2)/x,x, algorithm="giac")
```

output
```
integrate(((c*x)^n*b + a)^(3/2)/x, x)
```

3.659.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b(cx)^n)^{3/2}}{x}\,dx = \int \frac{\left(a+b\,{\left(c\,x\right)}^n\right)^{3/2}}{x}\,dx$$

input
```
int((a + b*(c*x)^n)^(3/2)/x,x)
```

output
```
int((a + b*(c*x)^n)^(3/2)/x, x)
```

3.660 $\int \frac{\sqrt{a+b(cx)^n}}{x} dx$

3.660.1 Optimal result

Integrand size = 17, antiderivative size = 49

$$\int \frac{\sqrt{a+b(cx)^n}}{x} dx = \frac{2\sqrt{a+b(cx)^n}}{n} - \frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

output
```
-2*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))*a^(1/2)/n+2*(a+b*(c*x)^n)^(1/2)/n
```

3.660.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+b(cx)^n}}{x} dx = \frac{2\left(\sqrt{a+b(cx)^n} - \sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)\right)}{n}$$

input
```
Integrate[Sqrt[a + b*(c*x)^n]/x,x]
```

output
```
(2*(Sqrt[a + b*(c*x)^n] - Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]]))/n
```

3.660.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {891, 27, 798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b(cx)^n}}{x} \, dx$$

$$\Big\downarrow \text{891}$$

$$\frac{\int \frac{\sqrt{b(cx)^n+a}}{x}d(cx)}{c}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\sqrt{a+b(cx)^n}}{cx}d(cx)$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{\sqrt{b(cx)^n+a}}{cx}d(cx)^n}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{a\int \frac{1}{cx\sqrt{b(cx)^n+a}}d(cx)^n+2\sqrt{a+b(cx)^n}}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2a\int \frac{1}{\frac{c^2x^2}{b}-\frac{a}{b}}d\sqrt{b(cx)^n+a}}{b}+2\sqrt{a+b(cx)^n}}{n}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{a+b(cx)^n}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

input
```
Int[Sqrt[a + b*(c*x)^n]/x,x]
```

output
```
(2*Sqrt[a + b*(c*x)^n] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/n
```

3.660.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

3.660. $\int \frac{\sqrt{a+b(cx)^n}}{x}\,dx$

3.660.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{2\sqrt{a+b(cx)^n}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	40
default	$\frac{2\sqrt{a+b(cx)^n}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	40
risch	$\frac{2\sqrt{a+b\,{\mathrm e}^{n\ln(cx)}}}{n}-\frac{2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,{\mathrm e}^{n\ln(cx)}}}{\sqrt{a}}\right)}{n}$	46

input
```
int((a+b*(c*x)^n)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(2*(a+b*(c*x)^n)^(1/2)-2*a^(1/2)*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2)))
```

3.660.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.10

$$\int \frac{\sqrt{a+b(cx)^n}}{x}\,dx = \left[\frac{\sqrt{a}\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b+a}\sqrt{a}+2a}{(cx)^n}\right)+2\sqrt{(cx)^n b+a}}{n}, \frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{(cx)^n b+a}\sqrt{-a}}{a}\right)+\sqrt{(cx)^n b+a}\right)}{n}\right]$$

input
```
integrate((a+b*(c*x)^n)^(1/2)/x,x, algorithm="fricas")
```

output
```
[(sqrt(a)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)^n) +
 2*sqrt((c*x)^n*b + a))/n, 2*(sqrt(-a)*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)
/a) + sqrt((c*x)^n*b + a))/n]
```

3.660.6 Sympy [F]

$$\int \frac{\sqrt{a+b(cx)^n}}{x}\,dx = \int \frac{\sqrt{a+b\,(cx)^n}}{x}\,dx$$

input
```
integrate((a+b*(c*x)**n)**(1/2)/x,x)
```

output
```
Integral(sqrt(a + b*(c*x)**n)/x, x)
```

3.660.7 Maxima [F]

$$\int \frac{\sqrt{a+b(cx)^n}}{x}\,dx = \int \frac{\sqrt{(cx)^n\,b+a}}{x}\,dx$$

input
```
integrate((a+b*(c*x)^n)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt((c*x)^n*b + a)/x, x)
```

3.660.8 Giac [F]

$$\int \frac{\sqrt{a+b(cx)^n}}{x}\,dx = \int \frac{\sqrt{(cx)^n\,b+a}}{x}\,dx$$

input
```
integrate((a+b*(c*x)^n)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt((c*x)^n*b + a)/x, x)
```

3.660.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b(cx)^n}}{x}\,dx=\int \frac{\sqrt{a+b\,{\left(c\,x\right)}^n}}{x} \,d x$$

input
```
int((a + b*(c*x)^n)^(1/2)/x,x)
```

output
```
int((a + b*(c*x)^n)^(1/2)/x, x)
```

3.661 $\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx$

3.661.1 Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

output
```
-2*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

3.661.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

input
```
Integrate[1/(x*Sqrt[a + b*(c*x)^n]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

3.661.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {891, 27, 798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+b(cx)^n}}dx$$

$$\Big\downarrow \text{891}$$

$$\frac{\int \frac{1}{x\sqrt{b(cx)^n+a}}d(cx)}{c}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{cx\sqrt{a+b(cx)^n}}d(cx)$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{1}{cx\sqrt{b(cx)^n+a}}d(cx)^n}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\int \frac{1}{\frac{c^2x^2}{b}-\frac{a}{b}}d\sqrt{b(cx)^n+a}}{bn}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

input
```
Int[1/(x*Sqrt[a + b*(c*x)^n]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

3.661.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

3.661.4 Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n\sqrt{a}}$	25
default	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n\sqrt{a}}$	25

input
```
int(1/x/(a+b*(c*x)^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

3.661.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.60

$$\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx = \left[\frac{\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b+a}\sqrt{a}+2a}{(cx)^n}\right)}{\sqrt{a}n}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{(cx)^n b+a}\sqrt{-a}}{a}\right)}{an}\right]$$

input
```
integrate(1/x/(a+b*(c*x)^n)^(1/2),x, algorithm="fricas")
```

output
```
[log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)^n)/(sqrt(a)*n
), 2*sqrt(-a)*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a)/(a*n)]
```

3.661.6 Sympy [F]

$$\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx = \int \frac{1}{x\sqrt{a+b\,(cx)^n}}\,dx$$

input
```
integrate(1/x/(a+b*(c*x)**n)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a + b*(c*x)**n)), x)
```

3.661.7 Maxima [F]

$$\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx = \int \frac{1}{\sqrt{(cx)^n\,b+a}\,x}\,dx$$

input
```
integrate(1/x/(a+b*(c*x)^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt((c*x)^n*b + a)*x), x)
```

3.661.8 Giac [F]

$$\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx = \int \frac{1}{\sqrt{(cx)^n b + ax}}\,dx$$

input
```
integrate(1/x/(a+b*(c*x)^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt((c*x)^n*b + a)*x), x)
```

3.661.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx = \int \frac{1}{x\,\sqrt{a+b\,(c\,x)^n}}\,dx$$

input
```
int(1/(x*(a + b*(c*x)^n)^(1/2)),x)
```

output
```
int(1/(x*(a + b*(c*x)^n)^(1/2)), x)
```

3.662 $\int \frac{1}{x(a+b(cx)^n)^{3/2}} dx$

3.662.1 Optimal result

Integrand size = 17, antiderivative size = 52

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{3/2}} dx = \frac{2}{an\sqrt{a+b(cx)^n}} - \frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

output
```
-2*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))/a^(3/2)/n+2/a/n/(a+b*(c*x)^n)^(1/2
)
```

3.662.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{3/2}} dx = \frac{2}{an\sqrt{a+b(cx)^n}} - \frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

input
```
Integrate[1/(x*(a + b*(c*x)^n)^(3/2)),x]
```

output
```
2/(a*n*Sqrt[a + b*(c*x)^n]) - (2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(a^
(3/2)*n)
```

3.662.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {891, 27, 798, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+b(cx)^n)^{3/2}}dx$$
$$\Big\downarrow \text{891}$$
$$\frac{\int \frac{1}{x(b(cx)^n+a)^{3/2}}d(cx)}{c}$$
$$\Big\downarrow \text{27}$$
$$\int \frac{1}{cx(a+b(cx)^n)^{3/2}}d(cx)$$
$$\Big\downarrow \text{798}$$
$$\frac{\int \frac{1}{cx(b(cx)^n+a)^{3/2}}d(cx)^n}{n}$$
$$\Big\downarrow \text{61}$$
$$\frac{\frac{\int \frac{1}{cx\sqrt{b(cx)^n+a}}d(cx)^n}{a}+\frac{2}{a\sqrt{a+b(cx)^n}}}{n}$$
$$\Big\downarrow \text{73}$$
$$\frac{\frac{2\int \frac{1}{\frac{c^2x^2}{b}-\frac{a}{b}}d\sqrt{b(cx)^n+a}}{ab}+\frac{2}{a\sqrt{a+b(cx)^n}}}{n}$$
$$\Big\downarrow \text{221}$$
$$\frac{\frac{2}{a\sqrt{a+b(cx)^n}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}}}{n}$$

input
```
Int[1/(x*(a + b*(c*x)^n)^(3/2)),x]
```

output
```
(2/(a*Sqrt[a + b*(c*x)^n]) - (2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/a^(3
/2))/n
```

3.662.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

3.662.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\frac{2}{a\sqrt{a+b(cx)^n}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{n}$	43
default	$\frac{\frac{2}{a\sqrt{a+b(cx)^n}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{n}$	43

input
```
int(1/x/(a+b*(c*x)^n)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/n*(2/a/(a+b*(c*x)^n)^(1/2)-2/a^(3/2)*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2)
))
```

3.662.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 164, normalized size of antiderivative = 3.15

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{3/2}}\,dx = \left[\frac{\left((cx)^n\sqrt{a}b+a^{\frac{3}{2}}\right)\log\left(\frac{(cx)^nb-2\sqrt{(cx)^nb+a}\sqrt{a}+2a}{(cx)^n}\right)+2\sqrt{(cx)^nb+a}a}{(cx)^na^2bn+a^3n}, \frac{2\left(\left((cx)^n\sqrt{\right.\right.}{}$$

input
```
integrate(1/x/(a+b*(c*x)^n)^(3/2),x, algorithm="fricas")
```

output
```
[(((c*x)^n*sqrt(a)*b + a^(3/2))*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqr
t(a) + 2*a)/(c*x)^n) + 2*sqrt((c*x)^n*b + a)*a)/((c*x)^n*a^2*b*n + a^3*n),
 2*(((c*x)^n*sqrt(-a)*b + sqrt(-a)*a)*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/
a) + sqrt((c*x)^n*b + a)*a)/((c*x)^n*a^2*b*n + a^3*n)]
```

3.662.6 Sympy [A] (verification not implemented)

Time = 3.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.27

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{b}{an\sqrt{a+b(cx)^n}}+\frac{b\,\mathrm{atan}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{-a}}\right)}{an\sqrt{-a}}\right)}{b} & \text{for } b\neq 0 \\ \frac{\log\left((cx)^n\right)}{a^{\frac{3}{2}}n} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(a+b*(c*x)**n)**(3/2),x)
```

output
```
Piecewise((2*(b/(a*n*sqrt(a + b*(c*x)**n)) + b*atan(sqrt(a + b*(c*x)**n)/s
qrt(-a))/(a*n*sqrt(-a)))/b, Ne(b, 0)), (log((c*x)**n)/(a**(3/2)*n), True))
```

3.662.7 Maxima [F]

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{3/2}}\,dx = \int \frac{1}{\left((cx)^n b+a\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+b*(c*x)^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(((c*x)^n*b + a)^(3/2)*x), x)
```

3.662.8 Giac [F]

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{3/2}}\,dx = \int \frac{1}{\left((cx)^n b+a\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+b*(c*x)^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/(((c*x)^n*b + a)^(3/2)*x), x)
```

3.662.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{3/2}}\,dx=\int \frac{1}{x\,{\left(a+b\,{\left(c\,x\right)}^n\right)}^{3/2}}\,dx$$

input
```
int(1/(x*(a + b*(c*x)^n)^(3/2)),x)
```

output
```
int(1/(x*(a + b*(c*x)^n)^(3/2)), x)
```

3.663 $\int \frac{1}{x(a+b(cx)^n)^{5/2}} dx$

3.663.1 Optimal result

Integrand size = 17, antiderivative size = 75

$$\int \frac{1}{x(a+b(cx)^n)^{5/2}} dx = \frac{2}{3an(a+b(cx)^n)^{3/2}} + \frac{2}{a^2n\sqrt{a+b(cx)^n}} - \frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}$$

output
```
2/3/a/n/(a+b*(c*x)^n)^(3/2)-2*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))/a^(5/2)
/n+2/a^2/n/(a+b*(c*x)^n)^(1/2)
```

3.663.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

$$\int \frac{1}{x(a+b(cx)^n)^{5/2}} dx = \frac{2(a+3(a+b(cx)^n))}{3a^2n(a+b(cx)^n)^{3/2}} - \frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}$$

input
```
Integrate[1/(x*(a + b*(c*x)^n)^(5/2)),x]
```

output
```
(2*(a + 3*(a + b*(c*x)^n)))/(3*a^2*n*(a + b*(c*x)^n)^(3/2)) - (2*ArcTanh[S
qrt[a + b*(c*x)^n]/Sqrt[a]])/(a^(5/2)*n)
```

3.663.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {891, 27, 798, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{5/2}}dx$$
$$\Big\downarrow \text{891}$$
$$\frac{\int \frac{1}{x(b(cx)^n+a)^{5/2}}d(cx)}{c}$$
$$\Big\downarrow \text{27}$$
$$\int \frac{1}{cx\left(a+b(cx)^n\right)^{5/2}}d(cx)$$
$$\Big\downarrow \text{798}$$
$$\frac{\int \frac{1}{cx(b(cx)^n+a)^{5/2}}d(cx)^n}{n}$$
$$\Big\downarrow \text{61}$$
$$\frac{\frac{\int \frac{1}{cx(b(cx)^n+a)^{3/2}}d(cx)^n}{a}+\frac{2}{3a(a+b(cx)^n)^{3/2}}}{n}$$
$$\Big\downarrow \text{61}$$
$$\frac{\frac{\frac{\int \frac{1}{cx\sqrt{b(cx)^n+a}}d(cx)^n}{a}+\frac{2}{a\sqrt{a+b(cx)^n}}}{a}+\frac{2}{3a(a+b(cx)^n)^{3/2}}}{n}$$
$$\Big\downarrow \text{73}$$
$$\frac{\frac{\frac{2\int \frac{1}{\frac{c^2x^2}{b}-\frac{a}{b}}d\sqrt{b(cx)^n+a}}{ab}+\frac{2}{a\sqrt{a+b(cx)^n}}}{a}+\frac{2}{3a(a+b(cx)^n)^{3/2}}}{n}$$
$$\Big\downarrow \text{221}$$
$$\frac{\frac{\frac{2}{a\sqrt{a+b(cx)^n}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a(a+b(cx)^n)^{3/2}}}{n}$$

3.663. $\int \frac{1}{x(a+b(cx)^n)^{5/2}}dx$

input
```
Int[1/(x*(a + b*(c*x)^n)^(5/2)),x]
```

output
```
(2/(3*a*(a + b*(c*x)^n)^(3/2)) + (2/(a*Sqrt[a + b*(c*x)^n]) - (2*ArcTanh[S
qrt[a + b*(c*x)^n]/Sqrt[a]])/a^(3/2))/a)/n
```

3.663.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

3.663. $\int \frac{1}{x(a+b(cx)^n)^{5/2}}\,dx$

3.663.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\frac{2}{a^2\sqrt{a+b(cx)^n}}+\frac{2}{3a(a+b(cx)^n)^{\frac{3}{2}}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}}{n}$	59
default	$\frac{\frac{2}{a^2\sqrt{a+b(cx)^n}}+\frac{2}{3a(a+b(cx)^n)^{\frac{3}{2}}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}}{n}$	59

input
```
int(1/x/(a+b*(c*x)^n)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/n*(2/a^2/(a+b*(c*x)^n)^(1/2)+2/3/a/(a+b*(c*x)^n)^(3/2)-2/a^(5/2)*arctanh
((a+b*(c*x)^n)^(1/2)/a^(1/2)))
```

3.663.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(63) = 126.

Time = 0.31 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.49

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{5/2}}\,dx = \left[\frac{3\left(2\,(cx)^n\,a^{\frac{3}{2}}b+(cx)^{2\,n}\,\sqrt{a}b^2+a^{\frac{5}{2}}\right)\log\left(\frac{(cx)^n b-2\,\sqrt{(cx)^n b+a}\sqrt{a}+2\,a}{(cx)^n}\right)+2\left(3\,(cx)^n\,ab\right.}{3\left(2\,(cx)^n\,a^4bn+(cx)^{2\,n}\,a^3b^2n+a^5n\right)}\right.$$

input
```
integrate(1/x/(a+b*(c*x)^n)^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(2*(c*x)^n*a^(3/2)*b + (c*x)^(2*n)*sqrt(a)*b^2 + a^(5/2))*log(((c*
x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)^n) + 2*(3*(c*x)^n*a*b
+ 4*a^2)*sqrt((c*x)^n*b + a))/(2*(c*x)^n*a^4*b*n + (c*x)^(2*n)*a^3*b^2*n +
 a^5*n), 2/3*(3*(2*(c*x)^n*sqrt(-a)*a*b + (c*x)^(2*n)*sqrt(-a)*b^2 + sqrt(
-a)*a^2)*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a) + (3*(c*x)^n*a*b + 4*a^2)*
sqrt((c*x)^n*b + a))/(2*(c*x)^n*a^4*b*n + (c*x)^(2*n)*a^3*b^2*n + a^5*n)]
```

3.663.6 Sympy [A] (verification not implemented)

Time = 4.67 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{5/2}}\,dx = \begin{cases} \frac{2\left(\frac{b}{3an(a+b(cx)^n)^{\frac{3}{2}}}+\frac{b}{a^2n\sqrt{a+b(cx)^n}}+\frac{b\,\mathrm{atan}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{-a}}\right)}{a^2n\sqrt{-a}}\right)}{b} & \text{for } b\neq 0 \\ \frac{\log\left((cx)^n\right)}{a^{\frac{5}{2}}n} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(a+b*(c*x)**n)**(5/2),x)
```

output
```
Piecewise((2*(b/(3*a*n*(a + b*(c*x)**n)**(3/2)) + b/(a**2*n*sqrt(a + b*(c*
x)**n)) + b*atan(sqrt(a + b*(c*x)**n)/sqrt(-a))/(a**2*n*sqrt(-a)))/b, Ne(b
, 0)), (log((c*x)**n)/(a**(5/2)*n), True))
```

3.663.7 Maxima [F]

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{5/2}}\,dx = \int \frac{1}{\left((cx)^n b+a\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(1/x/(a+b*(c*x)^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/(((c*x)^n*b + a)^(5/2)*x), x)
```

3.663.8 Giac [F]

$$\int \frac{1}{x\left(a+b(cx)^n\right)^{5/2}}\,dx = \int \frac{1}{\left((cx)^n b+a\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(1/x/(a+b*(c*x)^n)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/(((c*x)^n*b + a)^(5/2)*x), x)
```

3.663.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x(a+b(cx)^n)^{5/2}}\,dx = \int \frac{1}{x\,(a+b\,(c\,x)^n)^{5/2}}\,dx$$

input
```
int(1/(x*(a + b*(c*x)^n)^(5/2)),x)
```

output
```
int(1/(x*(a + b*(c*x)^n)^(5/2)), x)
```

3.664 $\int \frac{(-a+b(cx)^n)^{5/2}}{x} dx$

3.664.1 Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \frac{(-a+b(cx)^n)^{5/2}}{x} dx = \frac{2a^2\sqrt{-a+b(cx)^n}}{n} - \frac{2a(-a+b(cx)^n)^{3/2}}{3n} + \frac{2(-a+b(cx)^n)^{5/2}}{5n} - \frac{2a^{5/2}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

output
```
-2/3*a*(-a+b*(c*x)^n)^(3/2)/n+2/5*(-a+b*(c*x)^n)^(5/2)/n-2*a^(5/2)*arctan(
(-a+b*(c*x)^n)^(1/2)/a^(1/2))/n+2*a^2*(-a+b*(c*x)^n)^(1/2)/n
```

3.664.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

$$\int \frac{(-a+b(cx)^n)^{5/2}}{x} dx = \frac{2\sqrt{-a+b(cx)^n}(23a^2 - 11ab(cx)^n + 3b^2(cx)^{2n}) - 30a^{5/2}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{15n}$$

input
```
Integrate[(-a + b*(c*x)^n)^(5/2)/x,x]
```

output
```
(2*Sqrt[-a + b*(c*x)^n]*(23*a^2 - 11*a*b*(c*x)^n + 3*b^2*(c*x)^(2*n)) - 30
*a^(5/2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(15*n)
```

3.664.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {891, 27, 798, 60, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b(cx)^n-a)^{5/2}}{x}dx$$

$$\Big\downarrow \ 891$$

$$\frac{\int \frac{(b(cx)^n-a)^{5/2}}{x}d(cx)}{c}$$

$$\Big\downarrow \ 27$$

$$\int \frac{(b(cx)^n-a)^{5/2}}{cx}d(cx)$$

$$\Big\downarrow \ 798$$

$$\frac{\int \frac{(b(cx)^n-a)^{5/2}}{cx}d(cx)^n}{n}$$

$$\Big\downarrow \ 60$$

$$\frac{\frac{2}{5}(b(cx)^n-a)^{5/2}-a\int \frac{(b(cx)^n-a)^{3/2}}{cx}d(cx)^n}{n}$$

$$\Big\downarrow \ 60$$

$$\frac{\frac{2}{5}(b(cx)^n-a)^{5/2}-a\left(\frac{2}{3}(b(cx)^n-a)^{3/2}-a\int \frac{\sqrt{b(cx)^n-a}}{cx}d(cx)^n\right)}{n}$$

$$\Big\downarrow \ 60$$

$$\frac{\frac{2}{5}(b(cx)^n-a)^{5/2}-a\left(\frac{2}{3}(b(cx)^n-a)^{3/2}-a\left(2\sqrt{b(cx)^n-a}-a\int \frac{1}{cx\sqrt{b(cx)^n-a}}d(cx)^n\right)\right)}{n}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{2}{5}(b(cx)^n-a)^{5/2}-a\left(\frac{2}{3}(b(cx)^n-a)^{3/2}-a\left(2\sqrt{b(cx)^n-a}-\frac{2a\int \frac{1}{\frac{c^2x^2}{b}+\frac{a}{b}}d\sqrt{b(cx)^n-a}}{b}\right)\right)}{n}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{2}{5}(b(cx)^n-a)^{5/2}-a\left(\frac{2}{3}(b(cx)^n-a)^{3/2}-a\left(2\sqrt{b(cx)^n-a}-2\sqrt{a}\arctan\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)\right)\right)}{n}$$

input
```
Int[(-a + b*(c*x)^n)^(5/2)/x,x]
```

output
```
((2*(-a + b*(c*x)^n)^(5/2))/5 - a*((2*(-a + b*(c*x)^n)^(3/2))/3 - a*(2*Sqr
t[-a + b*(c*x)^n] - 2*Sqrt[a]*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])))/n
```

3.664.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

3.664.4 Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{\frac{2(-a+b(cx)^n)^{\frac{5}{2}}}{5}-\frac{2a(-a+b(cx)^n)^{\frac{3}{2}}}{3}+2a^2\sqrt{-a+b(cx)^n}-2a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	78
default	$\frac{\frac{2(-a+b(cx)^n)^{\frac{5}{2}}}{5}-\frac{2a(-a+b(cx)^n)^{\frac{3}{2}}}{3}+2a^2\sqrt{-a+b(cx)^n}-2a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$	78
risch	$-\frac{2\left(3b^2e^{2n\ln(cx)}-11a\,e^{n\ln(cx)}b+23a^2\right)\left(a-b\,e^{n\ln(cx)}\right)}{15n\sqrt{-a+b\,e^{n\ln(cx)}}}-\frac{2a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-a+b\,e^{n\ln(cx)}}}{\sqrt{a}}\right)}{n}$	93

input
```
int((-a+b*(c*x)^n)^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(2/5*(-a+b*(c*x)^n)^(5/2)-2/3*a*(-a+b*(c*x)^n)^(3/2)+2*a^2*(-a+b*(c*x)
^n)^(1/2)-2*a^(5/2)*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2)))
```

3.664.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.67

$$\int \frac{(-a+b(cx)^n)^{5/2}}{x}\,dx = \left[\frac{15\sqrt{-a}a^2\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b-a}\sqrt{-a}-2a}{(cx)^n}\right)-2\left(11\,(cx)^n\,ab-3\,(cx)^{2n}\,b^2-23\,a^2\right)\cdots}{15\,n}\right.$$

$$\left.-\frac{2\left(15\,a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{(cx)^n b-a}}{\sqrt{a}}\right)+\left(11\,(cx)^n\,ab-3\,(cx)^{2n}\,b^2-23\,a^2\right)\sqrt{(cx)^n\,b-a}\right)}{15\,n}\right]$$

input
```
integrate((-a+b*(c*x)^n)^(5/2)/x,x, algorithm="fricas")
```

output
```
[1/15*(15*sqrt(-a)*a^2*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2
*a)/(c*x)^n) - 2*(11*(c*x)^n*a*b - 3*(c*x)^(2*n)*b^2 - 23*a^2)*sqrt((c*x)^
n*b - a))/n, -2/15*(15*a^(5/2)*arctan(sqrt((c*x)^n*b - a)/sqrt(a)) + (11*(
c*x)^n*a*b - 3*(c*x)^(2*n)*b^2 - 23*a^2)*sqrt((c*x)^n*b - a))/n]
```

3.664.6 Sympy [A] (verification not implemented)

Time = 17.82 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.30

$$\int \frac{(-a+b(cx)^n)^{5/2}}{x}\,dx = \begin{cases} \frac{\begin{cases} -2a^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)+2a^2\sqrt{-a+b\,(cx)^n}-\frac{2a(-a+b(cx)^n)^{\frac{3}{2}}}{3}+\frac{2(-a+b(cx)^n)^{\frac{5}{2}}}{5} \\ a^2\sqrt{-a}\log\left((cx)^n\right) \end{cases}}{n} \\ -\left(-a^2\sqrt{-a+b}+2ab\sqrt{-a+b}-b^2\sqrt{-a+b}\right)\log\left(cx\right) \end{cases}$$

input
```
integrate((-a+b*(c*x)**n)**(5/2)/x,x)
```

output
```
Piecewise((Piecewise((-2*a**(5/2)*atan(sqrt(-a + b*(c*x)**n)/sqrt(a)) + 2*
a**2*sqrt(-a + b*(c*x)**n) - 2*a*(-a + b*(c*x)**n)**(3/2)/3 + 2*(-a + b*(c
*x)**n)**(5/2)/5, Ne(b, 0)), (a**2*sqrt(-a)*log((c*x)**n), True))/n, Ne(n,
 0)), (-(-a**2*sqrt(-a + b) + 2*a*b*sqrt(-a + b) - b**2*sqrt(-a + b))*log(
c*x), True))
```

3.664.7 Maxima [F]

$$\int \frac{(-a+b(cx)^n)^{5/2}}{x}\,dx = \int \frac{\left((cx)^n\,b-a\right)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate((-a+b*(c*x)^n)^(5/2)/x,x, algorithm="maxima")
```

output
```
integrate(((c*x)^n*b - a)^(5/2)/x, x)
```

3.664.8 Giac [F]

$$\int \frac{(-a+b(cx)^n)^{5/2}}{x}\,dx=\int \frac{((cx)^n b-a)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate((-a+b*(c*x)^n)^(5/2)/x,x, algorithm="giac")
```

output
```
integrate(((c*x)^n*b - a)^(5/2)/x, x)
```

3.664.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(-a+b(cx)^n)^{5/2}}{x}\,dx=\int \frac{(b\,(c\,x)^n-a)^{5/2}}{x}\,dx$$

input
```
int((b*(c*x)^n - a)^(5/2)/x,x)
```

output
```
int((b*(c*x)^n - a)^(5/2)/x, x)
```

3.665 $\int \frac{(-a+b(cx)^n)^{3/2}}{x} dx$

3.665.1 Optimal result

Integrand size = 19, antiderivative size = 76

$$\int \frac{(-a+b(cx)^n)^{3/2}}{x} dx = -\frac{2a\sqrt{-a+b(cx)^n}}{n} + \frac{2(-a+b(cx)^n)^{3/2}}{3n} + \frac{2a^{3/2}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

output
```
2/3*(-a+b*(c*x)^n)^(3/2)/n+2*a^(3/2)*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/
n-2*a*(-a+b*(c*x)^n)^(1/2)/n
```

3.665.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \frac{(-a+b(cx)^n)^{3/2}}{x} dx = \frac{-2(4a-b(cx)^n)\sqrt{-a+b(cx)^n} + 6a^{3/2}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{3n}$$

input
```
Integrate[(-a + b*(c*x)^n)^(3/2)/x,x]
```

output
```
(-2*(4*a - b*(c*x)^n)*Sqrt[-a + b*(c*x)^n] + 6*a^(3/2)*ArcTan[Sqrt[-a + b*
(c*x)^n]/Sqrt[a]])/(3*n)
```

3.665.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {891, 27, 798, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b(cx)^n-a)^{3/2}}{x}dx$$

$$\Big\downarrow \text{ 891}$$

$$\frac{\int \frac{(b(cx)^n-a)^{3/2}}{x}d(cx)}{c}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(b(cx)^n-a)^{3/2}}{cx}d(cx)$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\int \frac{(b(cx)^n-a)^{3/2}}{cx}d(cx)^n}{n}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{\frac{2}{3}(b(cx)^n-a)^{3/2}-a\int \frac{\sqrt{b(cx)^n-a}}{cx}d(cx)^n}{n}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{\frac{2}{3}(b(cx)^n-a)^{3/2}-a\left(2\sqrt{b(cx)^n-a}-a\int \frac{1}{cx\sqrt{b(cx)^n-a}}d(cx)^n\right)}{n}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\frac{2}{3}(b(cx)^n-a)^{3/2}-a\left(2\sqrt{b(cx)^n-a}-\frac{2a\int \frac{1}{\frac{c^2x^2}{b}+\frac{a}{b}}d\sqrt{b(cx)^n-a}}{b}\right)}{n}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{2}{3}(b(cx)^n-a)^{3/2}-a\left(2\sqrt{b(cx)^n-a}-2\sqrt{a}\arctan\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)\right)}{n}$$

3.665. $\int \frac{(-a+b(cx)^n)^{3/2}}{x}\,dx$

input
```
Int[(-a + b*(c*x)^n)^(3/2)/x,x]
```

output
```
((2*(-a + b*(c*x)^n)^(3/2))/3 - a*(2*Sqrt[-a + b*(c*x)^n] - 2*Sqrt[a]*ArcT
an[Sqrt[-a + b*(c*x)^n]/Sqrt[a]]))/n
```

3.665.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

3.665. $\int \frac{(-a+b(cx)^n)^{3/2}}{x} \, dx$

3.665.4 Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\frac{2\left(-a+b(cx)^{n}\right)^{\frac{3}{2}}}{3}-2a\sqrt{-a+b(cx)^{n}}+2a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-a+b(cx)^{n}}}{\sqrt{a}}\right)}{n}$	60
default	$\frac{\frac{2\left(-a+b(cx)^{n}\right)^{\frac{3}{2}}}{3}-2a\sqrt{-a+b(cx)^{n}}+2a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-a+b(cx)^{n}}}{\sqrt{a}}\right)}{n}$	60
risch	$\frac{2\left(-b\,e^{n\ln(cx)}+4a\right)\left(a-b\,e^{n\ln(cx)}\right)}{3n\sqrt{-a+b\,e^{n\ln(cx)}}}+\frac{2a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-a+b\,e^{n\ln(cx)}}}{\sqrt{a}}\right)}{n}$	76

input
```
int((-a+b*(c*x)^n)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(2/3*(-a+b*(c*x)^n)^(3/2)-2*a*(-a+b*(c*x)^n)^(1/2)+2*a^(3/2)*arctan((-
a+b*(c*x)^n)^(1/2)/a^(1/2)))
```

3.665.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.78

$$\int \frac{(-a+b(cx)^n)^{3/2}}{x}\,dx = \left[\frac{3\sqrt{-a}a\log\left(\frac{(cx)^n b+2\sqrt{(cx)^n b-a}\sqrt{-a}-2a}{(cx)^n}\right)+2\sqrt{(cx)^n b-a}\left((cx)^n b-4a\right)}{3n}, \frac{2\left(3a^{\frac{3}{2}}a\right.}{}\right.$$

input
```
integrate((-a+b*(c*x)^n)^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/3*(3*sqrt(-a)*a*log(((c*x)^n*b + 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/
(c*x)^n) + 2*sqrt((c*x)^n*b - a)*((c*x)^n*b - 4*a))/n, 2/3*(3*a^(3/2)*arct
an(sqrt((c*x)^n*b - a)/sqrt(a)) + sqrt((c*x)^n*b - a)*((c*x)^n*b - 4*a))/n
]
```

3.665.6 Sympy [A] (verification not implemented)

Time = 11.99 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.24

$$\int \frac{(-a+b(cx)^n)^{3/2}}{x}\,dx = \begin{cases} \frac{\begin{cases} 2a^{\frac{3}{2}}\,\mathrm{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right) - 2a\sqrt{-a+b\,(cx)^n} + \frac{2(-a+b(cx)^n)^{\frac{3}{2}}}{3} & \text{for } b \neq 0 \\ -a\sqrt{-a}\log\left((cx)^n\right) & \text{otherwise} \end{cases}}{n} & \text{for } n \neq 0 \\ \left(-a\sqrt{-a+b}+b\sqrt{-a+b}\right)\log(x) & \text{otherwise} \end{cases}$$

input
```
integrate((-a+b*(c*x)**n)**(3/2)/x,x)
```

output
```
Piecewise((Piecewise((2*a**(3/2)*atan(sqrt(-a + b*(c*x)**n)/sqrt(a)) - 2*a
*sqrt(-a + b*(c*x)**n) + 2*(-a + b*(c*x)**n)**(3/2)/3, Ne(b, 0)), (-a*sqrt
(-a)*log((c*x)**n), True))/n, Ne(n, 0)), ((-a*sqrt(-a + b) + b*sqrt(-a + b
))*log(x), True))
```

3.665.7 Maxima [F]

$$\int \frac{(-a+b(cx)^n)^{3/2}}{x}\,dx = \int \frac{((cx)^n\,b-a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((-a+b*(c*x)^n)^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate(((c*x)^n*b - a)^(3/2)/x, x)
```

3.665.8 Giac [F]

$$\int \frac{(-a+b(cx)^n)^{3/2}}{x}\,dx = \int \frac{((cx)^n\,b-a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((-a+b*(c*x)^n)^(3/2)/x,x, algorithm="giac")
```

output
```
integrate(((c*x)^n*b - a)^(3/2)/x, x)
```

3.665.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(-a+b(cx)^n)^{3/2}}{x}\,dx = \int \frac{\left(b\,(c\,x)^n-a\right)^{3/2}}{x}\,dx$$

input
```
int((b*(c*x)^n - a)^(3/2)/x,x)
```

output
```
int((b*(c*x)^n - a)^(3/2)/x, x)
```

3.666 $\int \frac{\sqrt{-a+b(cx)^n}}{x} \, dx$

3.666.1 Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \frac{\sqrt{-a+b(cx)^n}}{x} \, dx = \frac{2\sqrt{-a+b(cx)^n}}{n} - \frac{2\sqrt{a}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

output
```
-2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))*a^(1/2)/n+2*(-a+b*(c*x)^n)^(1/2)/n
```

3.666.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{-a+b(cx)^n}}{x} \, dx = \frac{2\left(\sqrt{-a+b(cx)^n} - \sqrt{a}\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)\right)}{n}$$

input
```
Integrate[Sqrt[-a + b*(c*x)^n]/x,x]
```

output
```
(2*(Sqrt[-a + b*(c*x)^n] - Sqrt[a]*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]]))/
n
```

3.666.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {891, 27, 798, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b(cx)^n-a}}{x}\,dx$$

$$\Big\downarrow \text{891}$$

$$\frac{\int \frac{\sqrt{b(cx)^n-a}}{x}d(cx)}{c}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\sqrt{b(cx)^n-a}}{cx}d(cx)$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{\sqrt{b(cx)^n-a}}{cx}d(cx)^n}{n}$$

$$\Big\downarrow \text{60}$$

$$\frac{2\sqrt{b(cx)^n-a}-a\int \frac{1}{cx\sqrt{b(cx)^n-a}}d(cx)^n}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\sqrt{b(cx)^n-a}-\frac{2a\int \frac{1}{\frac{c^2x^2}{b}+\frac{a}{b}}d\sqrt{b(cx)^n-a}}{b}}{n}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\sqrt{b(cx)^n-a}-2\sqrt{a}\arctan\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}$$

input
```
Int[Sqrt[-a + b*(c*x)^n]/x,x]
```

output
```
(2*Sqrt[-a + b*(c*x)^n] - 2*Sqrt[a]*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/
n
```

3.666.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

3.666. $\int \frac{\sqrt{-a+b(cx)^n}}{x} \, dx$

3.666.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{2\sqrt{-a+b(cx)^{n}}-2\sqrt{a}\,\arctan\left(\frac{\sqrt{-a+b(cx)^{n}}}{\sqrt{a}}\right)}{n}$	44
default	$\frac{2\sqrt{-a+b(cx)^{n}}-2\sqrt{a}\,\arctan\left(\frac{\sqrt{-a+b(cx)^{n}}}{\sqrt{a}}\right)}{n}$	44
risch	$-\frac{2\left(a-b\,{\mathrm e}^{n\ln(cx)}\right)}{n\sqrt{-a+b\,{\mathrm e}^{n\ln(cx)}}}-\frac{2\sqrt{a}\,\arctan\left(\frac{\sqrt{-a+b\,{\mathrm e}^{n\ln(cx)}}}{\sqrt{a}}\right)}{n}$	62

input
```
int((-a+b*(c*x)^n)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(2*(-a+b*(c*x)^n)^(1/2)-2*a^(1/2)*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))
)
```

3.666.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.08

$$\int \frac{\sqrt{-a+b(cx)^n}}{x}\,dx = \left[\frac{\sqrt{-a}\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b-a}\sqrt{-a}-2a}{(cx)^n}\right)+2\sqrt{(cx)^n b-a}}{n}, -\frac{2\left(\sqrt{a}\arctan\left(\frac{\sqrt{(cx)^n b-a}}{\sqrt{a}}\right)-\sqrt{(cx)^n b-a}\right)}{n}\right]$$

input
```
integrate((-a+b*(c*x)^n)^(1/2)/x,x, algorithm="fricas")
```

output
```
[(sqrt(-a)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(c*x)^n)
 + 2*sqrt((c*x)^n*b - a))/n, -2*(sqrt(a)*arctan(sqrt((c*x)^n*b - a)/sqrt(a
)) - sqrt((c*x)^n*b - a))/n]
```

3.666.6 Sympy [F]

$$\int \frac{\sqrt{-a+b(cx)^n}}{x}\,dx=\int \frac{\sqrt{-a+b\,(cx)^n}}{x}\,dx$$

input
```
integrate((-a+b*(c*x)**n)**(1/2)/x,x)
```

output
```
Integral(sqrt(-a + b*(c*x)**n)/x, x)
```

3.666.7 Maxima [F]

$$\int \frac{\sqrt{-a+b(cx)^n}}{x}\,dx=\int \frac{\sqrt{(cx)^n\,b-a}}{x}\,dx$$

input
```
integrate((-a+b*(c*x)^n)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt((c*x)^n*b - a)/x, x)
```

3.666.8 Giac [F]

$$\int \frac{\sqrt{-a+b(cx)^n}}{x}\,dx=\int \frac{\sqrt{(cx)^n\,b-a}}{x}\,dx$$

input
```
integrate((-a+b*(c*x)^n)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt((c*x)^n*b - a)/x, x)
```

3.666.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-a+b(cx)^n}}{x}\,dx = \int \frac{\sqrt{b\,(c\,x)^n-a}}{x}\,dx$$

input
```
int((b*(c*x)^n - a)^(1/2)/x,x)
```

output
```
int((b*(c*x)^n - a)^(1/2)/x, x)
```

3.667 $\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx$

3.667.1 Optimal result

Integrand size = 19, antiderivative size = 32

$$\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx=\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

output
```
2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

3.667.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx=\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

input
```
Integrate[1/(x*Sqrt[-a + b*(c*x)^n]),x]
```

output
```
(2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

3.667.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {891, 27, 798, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{b(cx)^n-a}}dx$$

$$\Big\downarrow \text{891}$$

$$\frac{\int \frac{1}{x\sqrt{b(cx)^n-a}}d(cx)}{c}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{cx\sqrt{b(cx)^n-a}}d(cx)$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{1}{cx\sqrt{b(cx)^n-a}}d(cx)^n}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\int \frac{1}{\frac{c^2x^2}{b}+\frac{a}{b}}d\sqrt{b(cx)^n-a}}{bn}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\arctan\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

input
```
Int[1/(x*Sqrt[-a + b*(c*x)^n]),x]
```

output
```
(2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

3.667.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

3.667.4 Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n\sqrt{a}}$	27
default	$\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n\sqrt{a}}$	27

input
```
int(1/x/(-a+b*(c*x)^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

3.667.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.50

$$\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx = \left[-\frac{\sqrt{-a}\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b-a}\sqrt{-a}-2a}{(cx)^n}\right)}{an}, \frac{2\arctan\left(\frac{\sqrt{(cx)^n b-a}}{\sqrt{a}}\right)}{\sqrt{a}n}\right]$$

input
```
integrate(1/x/(-a+b*(c*x)^n)^(1/2),x, algorithm="fricas")
```

output
```
[-sqrt(-a)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(c*x)^n)
/(a*n), 2*arctan(sqrt((c*x)^n*b - a)/sqrt(a))/(sqrt(a)*n)]
```

3.667.6 Sympy [F]

$$\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx = \int \frac{1}{x\sqrt{-a+b\,(cx)^n}}\,dx$$

input
```
integrate(1/x/(-a+b*(c*x)**n)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(-a + b*(c*x)**n)), x)
```

3.667.7 Maxima [F]

$$\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx = \int \frac{1}{\sqrt{(cx)^n\,b-a}\,x}\,dx$$

input
```
integrate(1/x/(-a+b*(c*x)^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt((c*x)^n*b - a)*x), x)
```

3.667.8 Giac [F]

$$\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx = \int \frac{1}{\sqrt{(cx)^n b - a}x}\,dx$$

input
```
integrate(1/x/(-a+b*(c*x)^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt((c*x)^n*b - a)*x), x)
```

3.667.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx = \int \frac{1}{x\,\sqrt{b\,(c\,x)^n - a}}\,dx$$

input
```
int(1/(x*(b*(c*x)^n - a)^(1/2)),x)
```

output
```
int(1/(x*(b*(c*x)^n - a)^(1/2)), x)
```

3.668 $\int \frac{1}{x(-a+b(cx)^n)^{3/2}}\,dx$

3.668.1 Optimal result

Integrand size = 19, antiderivative size = 56

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{3/2}}\,dx=-\frac{2}{an\sqrt{-a+b(cx)^n}}-\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

output
```
-2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/a^(3/2)/n-2/a/n/(-a+b*(c*x)^n)^(1/
2)
```

3.668.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{3/2}}\,dx=-\frac{2}{an\sqrt{-a+b(cx)^n}}-\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

input
```
Integrate[1/(x*(-a + b*(c*x)^n)^(3/2)),x]
```

output
```
-2/(a*n*Sqrt[-a + b*(c*x)^n]) - (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(
a^(3/2)*n)
```

3.668.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {891, 27, 798, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(b(cx)^n-a\right)^{3/2}}dx$$

$$\Big\downarrow \text{891}$$

$$\frac{\int \frac{1}{x(b(cx)^n-a)^{3/2}}d(cx)}{c}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{cx\left(b(cx)^n-a\right)^{3/2}}d(cx)$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{1}{cx(b(cx)^n-a)^{3/2}}d(cx)^n}{n}$$

$$\Big\downarrow \text{61}$$

$$\frac{-\frac{\int \frac{1}{cx\sqrt{b(cx)^n-a}}d(cx)^n}{a}-\frac{2}{a\sqrt{b(cx)^n-a}}}{n}$$

$$\Big\downarrow \text{73}$$

$$\frac{-\frac{2\int \frac{1}{\frac{c^2x^2}{b}+\frac{a}{b}}d\sqrt{b(cx)^n-a}}{ab}-\frac{2}{a\sqrt{b(cx)^n-a}}}{n}$$

$$\Big\downarrow \text{218}$$

$$\frac{-\frac{2\arctan\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{b(cx)^n-a}}}{n}$$

input
```
Int[1/(x*(-a + b*(c*x)^n)^(3/2)),x]
```

output
```
(-2/(a*Sqrt[-a + b*(c*x)^n]) - (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/a^
(3/2))/n
```

3.668.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

3.668. $\int \frac{1}{x(-a+b(cx)^n)^{3/2}}\,dx$

3.668.4 Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{-\frac{2}{a\sqrt{-a+b(cx)^n}}-\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{n}$	47
default	$\frac{-\frac{2}{a\sqrt{-a+b(cx)^n}}-\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{n}$	47

input
```
int(1/x/(-a+b*(c*x)^n)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/n*(-2/a/(-a+b*(c*x)^n)^(1/2)-2/a^(3/2)*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/
2)))
```

3.668.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 3.12

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{3/2}}\,dx = \left[-\frac{\left((cx)^n\sqrt{-a}b-\sqrt{-a}a\right)\log\left(\frac{(cx)^nb+2\sqrt{(cx)^nb-a}\sqrt{-a}-2a}{(cx)^n}\right)+2\sqrt{(cx)^nb-a}a}{(cx)^na^2bn-a^3n},\right.$$
$$\left.-\frac{2\left(\left((cx)^n\sqrt{a}b-a^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{(cx)^nb-a}}{\sqrt{a}}\right)+\sqrt{(cx)^nb-a}a\right)}{(cx)^na^2bn-a^3n}\right]$$

input
```
integrate(1/x/(-a+b*(c*x)^n)^(3/2),x, algorithm="fricas")
```

output
```
[-(((c*x)^n*sqrt(-a)*b - sqrt(-a)*a)*log(((c*x)^n*b + 2*sqrt((c*x)^n*b - a
)*sqrt(-a) - 2*a)/(c*x)^n) + 2*sqrt((c*x)^n*b - a)*a)/((c*x)^n*a^2*b*n - a
^3*n), -2*(((c*x)^n*sqrt(a)*b - a^(3/2))*arctan(sqrt((c*x)^n*b - a)/sqrt(a
)) + sqrt((c*x)^n*b - a)*a)/((c*x)^n*a^2*b*n - a^3*n)]
```

3.668.6 Sympy [A] (verification not implemented)

Time = 4.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.18

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{3/2}}\,dx = -\begin{cases} \frac{2\left(\frac{b}{an\sqrt{-a+b(cx)^n}}+\frac{b\operatorname{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}n}\right)}{b} & \text{for } b\neq 0 \\ \frac{\log\left((cx)^n\right)}{an\sqrt{-a}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(-a+b*(c*x)**n)**(3/2),x)
```

output
```
-Piecewise((2*(b/(a*n*sqrt(-a + b*(c*x)**n)) + b*atan(sqrt(-a + b*(c*x)**n
)/sqrt(a))/(a**(3/2)*n))/b, Ne(b, 0)), (log((c*x)**n)/(a*n*sqrt(-a)), True
))
```

3.668.7 Maxima [F]

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{3/2}}\,dx = \int \frac{1}{\left((cx)^n b-a\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(-a+b*(c*x)^n)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(((c*x)^n*b - a)^(3/2)*x), x)
```

3.668.8 Giac [F]

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{3/2}}\,dx = \int \frac{1}{\left((cx)^n b-a\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(-a+b*(c*x)^n)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/(((c*x)^n*b - a)^(3/2)*x), x)
```

3.668.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{3/2}}\,dx = \int \frac{1}{x\,\left(b\,{\left(c\,x\right)}^n-a\right)^{3/2}}\,dx$$

input
```
int(1/(x*(b*(c*x)^n - a)^(3/2)),x)
```

output
```
int(1/(x*(b*(c*x)^n - a)^(3/2)), x)
```

3.669 $\int \frac{1}{x(-a+b(cx)^n)^{5/2}}\,dx$

3.669.1 Optimal result

Integrand size = 19, antiderivative size = 81

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{5/2}}\,dx=-\frac{2}{3an\left(-a+b(cx)^n\right)^{3/2}}+\frac{2}{a^2n\sqrt{-a+b(cx)^n}}+\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}$$

output
```
-2/3/a/n/(-a+b*(c*x)^n)^(3/2)+2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/a^(5/
2)/n+2/a^2/n/(-a+b*(c*x)^n)^(1/2)
```

3.669.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{5/2}}\,dx=\frac{2\left(\frac{\sqrt{a}(-4a+3b(cx)^n)}{(-a+b(cx)^n)^{3/2}}+3\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)\right)}{3a^{5/2}n}$$

input
```
Integrate[1/(x*(-a + b*(c*x)^n)^(5/2)),x]
```

output
```
(2*((Sqrt[a]*(-4*a + 3*b*(c*x)^n))/(-a + b*(c*x)^n)^(3/2) + 3*ArcTan[Sqrt[
-a + b*(c*x)^n]/Sqrt[a]]))/(3*a^(5/2)*n)
```

3.669.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {891, 27, 798, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(b(cx)^n-a\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 891}$$

$$\frac{\int \frac{1}{x(b(cx)^n-a)^{5/2}}d(cx)}{c}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1}{cx\left(b(cx)^n-a\right)^{5/2}}d(cx)$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\int \frac{1}{cx(b(cx)^n-a)^{5/2}}d(cx)^n}{n}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{-\frac{\int \frac{1}{cx(b(cx)^n-a)^{3/2}}d(cx)^n}{a}-\frac{2}{3a(b(cx)^n-a)^{3/2}}}{n}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{-\frac{-\frac{\int \frac{1}{cx\sqrt{b(cx)^n-a}}d(cx)^n}{a}-\frac{2}{a\sqrt{b(cx)^n-a}}}{a}-\frac{2}{3a(b(cx)^n-a)^{3/2}}}{n}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{-\frac{-\frac{2\int \frac{1}{\frac{c^2x^2}{b}+\frac{a}{b}}d\sqrt{b(cx)^n-a}}{ab}-\frac{2}{a\sqrt{b(cx)^n-a}}}{a}-\frac{2}{3a(b(cx)^n-a)^{3/2}}}{n}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{-\frac{-\frac{2\arctan\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{b(cx)^n-a}}}{a}-\frac{2}{3a(b(cx)^n-a)^{3/2}}}{n}$$

input
```
Int[1/(x*(-a + b*(c*x)^n)^(5/2)),x]
```

output
```
(-2/(3*a*(-a + b*(c*x)^n)^(3/2)) - (-2/(a*Sqrt[-a + b*(c*x)^n]) - (2*ArcTa
n[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/a^(3/2))/a)/n
```

3.669.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

3.669. $\int \frac{1}{x(-a+b(cx)^n)^{5/2}}\,dx$

3.669.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{-\frac{2}{3a(-a+b(cx)^n)^{\frac{3}{2}}}+\frac{2}{a^2\sqrt{-a+b(cx)^n}}+\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}}{n}$	65
default	$\frac{-\frac{2}{3a(-a+b(cx)^n)^{\frac{3}{2}}}+\frac{2}{a^2\sqrt{-a+b(cx)^n}}+\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}}{n}$	65

input
```
int(1/x/(-a+b*(c*x)^n)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/n*(-2/3/a/(-a+b*(c*x)^n)^(3/2)+2/a^2/(-a+b*(c*x)^n)^(1/2)+2/a^(5/2)*arct
an((-a+b*(c*x)^n)^(1/2)/a^(1/2)))
```

3.669.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 277, normalized size of antiderivative = 3.42

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{5/2}}\,dx=\left[-\frac{3\left(2\,(cx)^n\sqrt{-a}ab-(cx)^{2\,n}\sqrt{-a}b^2-\sqrt{-a}a^2\right)\log\left(\frac{(cx)^nb-2\sqrt{(cx)^nb-a}\sqrt{-a}-2\,a}{(cx)^n}\right)}{3\left(2\,(cx)^n\,a^4bn-(cx)^{2\,n}\,a^3b^2n-a^5n\right)}\right.$$

input
```
integrate(1/x/(-a+b*(c*x)^n)^(5/2),x, algorithm="fricas")
```

output
```
[-1/3*(3*(2*(c*x)^n*sqrt(-a)*a*b - (c*x)^(2*n)*sqrt(-a)*b^2 - sqrt(-a)*a^2
)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(c*x)^n) + 2*(3*(
c*x)^n*a*b - 4*a^2)*sqrt((c*x)^n*b - a))/(2*(c*x)^n*a^4*b*n - (c*x)^(2*n)*
a^3*b^2*n - a^5*n), 2/3*(3*(2*(c*x)^n*a^(3/2)*b - (c*x)^(2*n)*sqrt(a)*b^2
- a^(5/2))*arctan(sqrt((c*x)^n*b - a)/sqrt(a)) - (3*(c*x)^n*a*b - 4*a^2)*s
qrt((c*x)^n*b - a))/(2*(c*x)^n*a^4*b*n - (c*x)^(2*n)*a^3*b^2*n - a^5*n)]
```

3.669.6 Sympy [A] (verification not implemented)

Time = 4.69 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{5/2}}\,dx = \begin{cases} \frac{2\left(-\frac{b}{3an(-a+b(cx)^n)^{\frac{3}{2}}}+\frac{b}{a^2n\sqrt{-a+b(cx)^n}}+\frac{b\operatorname{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}n}\right)}{b} & \text{for } b\neq 0 \\ \frac{\log\left((cx)^n\right)}{a^2n\sqrt{-a}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(-a+b*(c*x)**n)**(5/2),x)
```

output
```
Piecewise((2*(-b/(3*a*n*(-a + b*(c*x)**n)**(3/2)) + b/(a**2*n*sqrt(-a + b*
(c*x)**n)) + b*atan(sqrt(-a + b*(c*x)**n)/sqrt(a))/(a**(5/2)*n))/b, Ne(b,
0)), (log((c*x)**n)/(a**2*n*sqrt(-a)), True))
```

3.669.7 Maxima [F]

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{5/2}}\,dx = \int \frac{1}{\left((cx)^n b-a\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(1/x/(-a+b*(c*x)^n)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/(((c*x)^n*b - a)^(5/2)*x), x)
```

3.669.8 Giac [F]

$$\int \frac{1}{x\left(-a+b(cx)^n\right)^{5/2}}\,dx = \int \frac{1}{\left((cx)^n b-a\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(1/x/(-a+b*(c*x)^n)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/(((c*x)^n*b - a)^(5/2)*x), x)
```

3.669.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\,(-a+b(cx)^n)^{5/2}}\,dx=\int \frac{1}{x\,{\left(b\,{\left(c\,x\right)}^n-a\right)}^{5/2}} \,d x$$

input
```
int(1/(x*(b*(c*x)^n - a)^(5/2)),x)
```

output
```
int(1/(x*(b*(c*x)^n - a)^(5/2)), x)
```

3.670 $\int \frac{1}{x\sqrt{a+bx}}\,dx$

3.670.1 Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
-2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

3.670.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[1/(x*Sqrt[a + b*x]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

3.670.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+bx}}\,dx$$

$$\Big\downarrow \text{73}$$

$$\frac{2\int \frac{1}{\frac{a+bx}{b}-\frac{a}{b}}d\sqrt{a+bx}}{b}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Int[1/(x*Sqrt[a + b*x]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

3.670.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.670.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18
default	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18
pseudoelliptic	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18

input
```
int(1/x/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

3.670.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.43

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = \left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)}{a}\right]$$

input
```
integrate(1/x/(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(s
qrt(b*x + a)*sqrt(-a)/a)/a]
```

3.670.6 Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = -\frac{2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

input
```
integrate(1/x/(b*x+a)**(1/2),x)
```

output
```
-2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)
```

3.670.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = \frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
integrate(1/x/(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a)
```

3.670.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = \frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

input
```
integrate(1/x/(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

3.670.9 Mupad [B] (verification not implemented)

Time = 16.70 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
int(1/(x*(a + b*x)^(1/2)),x)
```

output
```
-(2*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

3.671 $\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx$

3.671.1 Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}m}$$

output
```
-2*arctanh((a+b*(c*x)^m)^(1/2)/a^(1/2))/m/a^(1/2)
```

3.671.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}m}$$

input
```
Integrate[1/(x*Sqrt[a + b*(c*x)^m]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*(c*x)^m]/Sqrt[a]])/(Sqrt[a]*m)
```

3.671.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {891, 27, 798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+b(cx)^m}}dx$$
$$\Big\downarrow \ 891$$
$$\frac{\int \frac{1}{x\sqrt{b(cx)^m+a}}d(cx)}{c}$$
$$\Big\downarrow \ 27$$
$$\int \frac{1}{cx\sqrt{a+b(cx)^m}}d(cx)$$
$$\Big\downarrow \ 798$$
$$\frac{\int \frac{1}{cx\sqrt{b(cx)^m+a}}d(cx)^m}{m}$$
$$\Big\downarrow \ 73$$
$$\frac{2\int \frac{1}{\frac{c^2x^2}{b}-\frac{a}{b}}d\sqrt{b(cx)^m+a}}{bm}$$
$$\Big\downarrow \ 221$$
$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}m}$$

input
```
Int[1/(x*Sqrt[a + b*(c*x)^m]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*(c*x)^m]/Sqrt[a]])/(Sqrt[a]*m)
```

3.671.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

3.671.4 Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{m\sqrt{a}}$	25
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{m\sqrt{a}}$	25

input
```
int(1/x/(a+b*(c*x)^m)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((a+b*(c*x)^m)^(1/2)/a^(1/2))/m/a^(1/2)
```

3.671.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.60

$$\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx = \left[\frac{\log\left(\frac{(cx)^m b-2\sqrt{(cx)^m b+a}\sqrt{a}+2a}{(cx)^m}\right)}{\sqrt{a}m}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{(cx)^m b+a}\sqrt{-a}}{a}\right)}{am}\right]$$

input
```
integrate(1/x/(a+b*(c*x)^m)^(1/2),x, algorithm="fricas")
```

output
```
[log(((c*x)^m*b - 2*sqrt((c*x)^m*b + a)*sqrt(a) + 2*a)/(c*x)^m)/(sqrt(a)*m
), 2*sqrt(-a)*arctan(sqrt((c*x)^m*b + a)*sqrt(-a)/a)/(a*m)]
```

3.671.6 Sympy [F]

$$\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx = \int \frac{1}{x\sqrt{a+b\,(cx)^m}}\,dx$$

input
```
integrate(1/x/(a+b*(c*x)**m)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a + b*(c*x)**m)), x)
```

3.671.7 Maxima [F]

$$\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx = \int \frac{1}{\sqrt{(cx)^m\,b+a}x}\,dx$$

input
```
integrate(1/x/(a+b*(c*x)^m)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt((c*x)^m*b + a)*x), x)
```

3.671.8 Giac [F]

$$\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx = \int \frac{1}{\sqrt{(cx)^m\,b+ax}}\,dx$$

input
```
integrate(1/x/(a+b*(c*x)^m)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt((c*x)^m*b + a)*x), x)
```

3.671.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx = \int \frac{1}{x\,\sqrt{a+b\,(c\,x)^m}}\,dx$$

input
```
int(1/(x*(a + b*(c*x)^m)^(1/2)),x)
```

output
```
int(1/(x*(a + b*(c*x)^m)^(1/2)), x)
```

3.672 $\int \frac{1}{x\sqrt{a+b(c(dx)^m)^n}}\,dx$

3.672.1 Optimal result

Integrand size = 21, antiderivative size = 37

$$\int \frac{1}{x\sqrt{a+b\left(c(dx)^m\right)^n}}\,dx=-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a}mn}$$

output
```
-2*arctanh((a+b*(c*(d*x)^m)^n)^(1/2)/a^(1/2))/m/n/a^(1/2)
```

3.672.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+b\left(c(dx)^m\right)^n}}\,dx=-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a}mn}$$

input
```
Integrate[1/(x*Sqrt[a + b*(c*(d*x)^m)^n]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*(c*(d*x)^m)^n]/Sqrt[a]])/(Sqrt[a]*m*n)
```

3.672.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {7282, 891, 27, 798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+b\left(c(dx)^m\right)^n}}dx$$

$$\Big\downarrow \text{7282}$$

$$\frac{\int \frac{(dx)^{-m}}{\sqrt{b(c(dx)^m)^n+a}}d(dx)^m}{m}$$

$$\Big\downarrow \text{891}$$

$$\frac{\int \frac{c(dx)^{-m}}{\sqrt{b(c(dx)^m)^n+a}}d(c(dx)^m)}{cm}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(dx)^{-m}}{\sqrt{b(c(dx)^m)^n+a}}d(c(dx)^m)}{m}$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{(dx)^{-m}}{\sqrt{b(c(dx)^m)^n+a}}d(c(dx)^m)^n}{mn}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\int \frac{1}{\frac{(dx)^{2m}}{b}-\frac{a}{b}}d\sqrt{b\left(c(dx)^m\right)^n+a}}{bmn}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a}mn}$$

input
```
Int[1/(x*Sqrt[a + b*(c*(d*x)^m)^n]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*(c*(d*x)^m)^n]/Sqrt[a]])/(Sqrt[a]*m*n)
```

3.672. $\int \frac{1}{x\sqrt{a+b(c(dx)^m)^n}}\,dx$

3.672.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

3.672.4 Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b\left(c(dx)^{m}\right)^{n}}}{\sqrt{a}}\right)}{m n \sqrt{a}}$	32
default	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b\left(c(dx)^{m}\right)^{n}}}{\sqrt{a}}\right)}{m n \sqrt{a}}$	32

input
```
int(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((a+b*(c*(d*x)^m)^n)^(1/2)/a^(1/2))/m/n/a^(1/2)
```

3.672.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.14

$$\int \frac{1}{x\sqrt{a+b\left(c(dx)^m\right)^n}}\,dx$$
$$= \left[\frac{\log\left(\left(be^{(mn\log(dx)+n\log(c))}-2\sqrt{be^{(mn\log(dx)+n\log(c))}+a}\sqrt{a}+2\,a\right)e^{(-mn\log(dx)-n\log(c))}\right)}{\sqrt{a}mn}, \frac{2\sqrt{-a}\arctan\left(\right.}{}\right.$$

input
```
integrate(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x, algorithm="fricas")
```

output
```
[log((b*e^(m*n*log(d*x) + n*log(c)) - 2*sqrt(b*e^(m*n*log(d*x) + n*log(c))
 + a)*sqrt(a) + 2*a)*e^(-m*n*log(d*x) - n*log(c)))/(sqrt(a)*m*n), 2*sqrt(-
a)*arctan(sqrt(b*e^(m*n*log(d*x) + n*log(c)) + a)*sqrt(-a)/a)/(a*m*n)]
```

3.672.6 Sympy [F]

$$\int \frac{1}{x\sqrt{a+b\left(c(dx)^m\right)^n}}\,dx = \int \frac{1}{x\sqrt{a+b\left(c\left(dx\right)^m\right)^n}}\,dx$$

input
```
integrate(1/x/(a+b*(c*(d*x)**m)**n)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a + b*(c*(d*x)**m)**n)), x)
```

3.672.7 Maxima [F]

$$\int \frac{1}{x\sqrt{a+b\,(c(dx)^m)^n}}\,dx = \int \frac{1}{\sqrt{((dx)^m\,c)^n\,b+a}x}\,dx$$

input
```
integrate(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(((d*x)^m*c)^n*b + a)*x), x)
```

3.672.8 Giac [F]

$$\int \frac{1}{x\sqrt{a+b\,(c(dx)^m)^n}}\,dx = \int \frac{1}{\sqrt{((dx)^m\,c)^n\,b+a}x}\,dx$$

input
```
integrate(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(((d*x)^m*c)^n*b + a)*x), x)
```

3.672.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+b\,(c(dx)^m)^n}}\,dx = \int \frac{1}{x\,\sqrt{a+b\,(c\,(d\,x)^m)^n}}\,dx$$

input
```
int(1/(x*(a + b*(c*(d*x)^m)^n)^(1/2)),x)
```

output
```
int(1/(x*(a + b*(c*(d*x)^m)^n)^(1/2)), x)
```

3.673 $\int \frac{1}{x\sqrt{a+b(c(d(ex)^m)^n)^p}}\,dx$

3.673.1 Optimal result

Integrand size = 25, antiderivative size = 44

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx=-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(c(d(ex)^m)^n)^p}}{\sqrt{a}}\right)}{\sqrt{a}mnp}$$

output
```
-2*arctanh((a+b*(c*(d*(e*x)^m)^n)^p)^(1/2)/a^(1/2))/m/n/p/a^(1/2)
```

3.673.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx=-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(c(d(ex)^m)^n)^p}}{\sqrt{a}}\right)}{\sqrt{a}mnp}$$

input
```
Integrate[1/(x*Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*(c*(d*(e*x)^m)^n)^p/Sqrt[a]])/(Sqrt[a]*m*n*p)
```

3.673.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {7282, 7282, 891, 27, 798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}dx$$

$$\Big\downarrow \text{7282}$$

$$\frac{\int \frac{(ex)^{-m}}{\sqrt{b(c(d(ex)^m)^n)^p+a}}d(ex)^m}{m}$$

$$\Big\downarrow \text{7282}$$

$$\frac{\int \frac{(ex)^{-m}}{\sqrt{b(c(d(ex)^m)^n)^p+a}}d(d(ex)^m)^n}{mn}$$

$$\Big\downarrow \text{891}$$

$$\frac{\int \frac{c(ex)^{-m}}{\sqrt{b(c(d(ex)^m)^n)^p+a}}d(c(d(ex)^m)^n)}{cmn}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(ex)^{-m}}{\sqrt{b(c(d(ex)^m)^n)^p+a}}d(c(d(ex)^m)^n)}{mn}$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{(ex)^{-m}}{\sqrt{b(c(d(ex)^m)^n)^p+a}}d(c(d(ex)^m)^n)^p}{mnp}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\int \frac{1}{\frac{(ex)^{2m}}{b}-\frac{a}{b}}d\sqrt{b\left(c\left(d(ex)^m\right)^n\right)^p+a}}{bmnp}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(c(d(ex)^m)^n)^p}}{\sqrt{a}}\right)}{\sqrt{a}mnp}$$

input
```
Int[1/(x*Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]/Sqrt[a]])/(Sqrt[a]*m*n*p)
```

3.673.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

3.673. $\int \frac{1}{x\sqrt{a+b(c(d(ex)^m)^n)^p}}\,dx$

3.673.4 Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{a+b(c(d(ex)^m)^n)^p}}{\sqrt{a}}\right)}{mnp\sqrt{a}}$	39
default	$-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{a+b(c(d(ex)^m)^n)^p}}{\sqrt{a}}\right)}{mnp\sqrt{a}}$	39

input
```
int(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((a+b*(c*(d*(e*x)^m)^n)^p)^(1/2)/a^(1/2))/m/n/p/a^(1/2)
```

3.673.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 147, normalized size of antiderivative = 3.34

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx$$
$$= \left[\frac{\log\left(\left(be^{(mnp\log(ex)+np\log(d)+p\log(c))} - 2\sqrt{be^{(mnp\log(ex)+np\log(d)+p\log(c))}+a}\sqrt{a}+2a\right)e^{(-mnp\log(ex)-np\log(d)-p\right.}}{\sqrt{a}mnp}\right.$$

input
```
integrate(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x, algorithm="fricas")
```

output
```
[log((b*e^(m*n*p*log(e*x) + n*p*log(d) + p*log(c)) - 2*sqrt(b*e^(m*n*p*log
(e*x) + n*p*log(d) + p*log(c)) + a)*sqrt(a) + 2*a)*e^(-m*n*p*log(e*x) - n*
p*log(d) - p*log(c)))/(sqrt(a)*m*n*p), 2*sqrt(-a)*arctan(sqrt(b*e^(m*n*p*l
og(e*x) + n*p*log(d) + p*log(c)) + a)*sqrt(-a)/a)/(a*m*n*p)]
```

3.673.6 Sympy [F]

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx = \int \frac{1}{x\sqrt{a+b\left(c\left(d\left(ex\right)^m\right)^n\right)^p}}\,dx$$

input
```
integrate(1/x/(a+b*(c*(d*(e*x)**m)**n)**p)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a + b*(c*(d*(e*x)**m)**n)**p)), x)
```

3.673.7 Maxima [F]

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx = \int \frac{1}{\sqrt{\left(\left((ex)^m d\right)^n c\right)^p b+a}\,x}\,dx$$

input
```
integrate(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt((((e*x)^m*d)^n*c)^p*b + a)*x), x)
```

3.673.8 Giac [F]

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx = \int \frac{1}{\sqrt{\left(\left((ex)^m d\right)^n c\right)^p b+a}\,x}\,dx$$

input
```
integrate(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt((((e*x)^m*d)^n*c)^p*b + a)*x), x)
```

3.673.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx=\int \frac{1}{x\,\sqrt{a+b\,{\left(c\,{\left(d\,{\left(e\,x\right)}^m\right)}^n\right)}^p}} \,d x$$

input
```
int(1/(x*(a + b*(c*(d*(e*x)^m)^n)^p)^(1/2)),x)
```

output
```
int(1/(x*(a + b*(c*(d*(e*x)^m)^n)^p)^(1/2)), x)
```

3.674 $\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}\,dx$

3.674.1 Optimal result

Integrand size = 29, antiderivative size = 51

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}{\sqrt{a}}\right)}{\sqrt{a}mnpq}$$

output
```
-2*arctanh((a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2)/a^(1/2))/m/n/p/q/a^(1/2)
```

3.674.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}{\sqrt{a}}\right)}{\sqrt{a}mnpq}$$

input
```
Integrate[1/(x*Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]/Sqrt[a]])/(Sqrt[a]*m*n*p
*q)
```

3.674.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {7282, 7282, 7282, 891, 27, 798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}dx$$

$$\Big\downarrow \text{7282}$$

$$\frac{\int \frac{(fx)^{-m}}{\sqrt{b(c(d(e(fx)^m)^n)^p)^q+a}}d(fx)^m}{m}$$

$$\Big\downarrow \text{7282}$$

$$\frac{\int \frac{(fx)^{-m}}{\sqrt{b(c(d(e(fx)^m)^n)^p)^q+a}}d(e(fx)^m)^n}{mn}$$

$$\Big\downarrow \text{7282}$$

$$\frac{\int \frac{(fx)^{-m}}{\sqrt{b(c(d(e(fx)^m)^n)^p)^q+a}}d(d(e(fx)^m)^n)^p}{mnp}$$

$$\Big\downarrow \text{891}$$

$$\frac{\int \frac{c(fx)^{-m}}{\sqrt{b(c(d(e(fx)^m)^n)^p)^q+a}}d(c(d(e(fx)^m)^n)^p)}{cmnp}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(fx)^{-m}}{\sqrt{b(c(d(e(fx)^m)^n)^p)^q+a}}d(c(d(e(fx)^m)^n)^p)}{mnp}$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{(fx)^{-m}}{\sqrt{b(c(d(e(fx)^m)^n)^p)^q+a}}d(c(d(e(fx)^m)^n)^p)^q}{mnpq}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\int \frac{1}{\frac{(fx)^{2m}}{b}-\frac{a}{b}}d\sqrt{b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q+a}}{bmnpq}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b(c(d(e(fx)^m)^n)^p)^q}}{\sqrt{a}}\right)}{\sqrt{a}mnpq}$$

input
```
Int[1/(x*Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]),x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]/Sqrt[a]])/(Sqrt[a]*m*n*p
*q)
```

3.674.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 891
```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Simp[1/c   Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a
, b, c, d, m, n, p}, x]
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

3.674.4 Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(c\left(d\left(e(fx)^{m}\right)^{n}\right)^{p}\right)^{q}}}{\sqrt{a}}\right)}{mnpq\sqrt{a}}$	46
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(c\left(d\left(e(fx)^{m}\right)^{n}\right)^{p}\right)^{q}}}{\sqrt{a}}\right)}{mnpq\sqrt{a}}$	46

input
```
int(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2)/a^(1/2))/m/n/p/q/a^(1/2)
```

3.674.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.57

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e(fx)^{m}\right)^{n}\right)^{p}\right)^{q}}}\,dx$$
$$= \left[\frac{\log\left(\left(be^{(mnpq\log(fx)+npq\log(e)+pq\log(d)+q\log(c))} - 2\sqrt{be^{(mnpq\log(fx)+npq\log(e)+pq\log(d)+q\log(c))}+a}\sqrt{a}+2\,a\right)e^{(-}\right.}{\sqrt{a}mnpq}\right.$$

input
```
integrate(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x, algorithm="fricas")
```

output
```
[log((b*e^(m*n*p*q*log(f*x) + n*p*q*log(e) + p*q*log(d) + q*log(c)) - 2*sq
rt(b*e^(m*n*p*q*log(f*x) + n*p*q*log(e) + p*q*log(d) + q*log(c)) + a)*sqrt
(a) + 2*a)*e^(-m*n*p*q*log(f*x) - n*p*q*log(e) - p*q*log(d) - q*log(c)))/(
sqrt(a)*m*n*p*q), 2*sqrt(-a)*arctan(sqrt(b*e^(m*n*p*q*log(f*x) + n*p*q*log
(e) + p*q*log(d) + q*log(c)) + a)*sqrt(-a)/a)/(a*m*n*p*q)]
```

3.674.6 Sympy [F]

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}\,dx = \int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e\left(fx\right)^m\right)^n\right)^p\right)^q}}\,dx$$

input
```
integrate(1/x/(a+b*(c*(d*(e*(f*x)**m)**n)**p)**q)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a + b*(c*(d*(e*(f*x)**m)**n)**p)**q)), x)
```

3.674.7 Maxima [F]

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}\,dx = \int \frac{1}{\sqrt{\left(\left(\left((fx)^m e\right)^n d\right)^p c\right)^q b+a}x}\,dx$$

input
```
integrate(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(((((f*x)^m*e)^n*d)^p*c)^q*b + a)*x), x)
```

3.674.8 Giac [F]

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}\,dx = \int \frac{1}{\sqrt{\left(\left(\left((fx)^m e\right)^n d\right)^p c\right)^q b+a}x}\,dx$$

input
```
integrate(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(((((f*x)^m*e)^n*d)^p*c)^q*b + a)*x), x)
```

3.674.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}\,dx=\int \frac{1}{x\,\sqrt{a+b\,{\left(c\,{\left(d\,{\left(e\,{\left(f\,x\right)}^m\right)}^n\right)}^p\right)}^q}}\,dx$$

input
```
int(1/(x*(a + b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2)),x)
```

output
```
int(1/(x*(a + b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2)), x)
```

3.675 $\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^3}{x} \, dx$

3.675.1 Optimal result

Integrand size = 20, antiderivative size = 76

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^3}{x} \, dx = \frac{35}{16}\sqrt{-1+\frac{1}{x^2}} - \frac{35}{48}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2 - \frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 - \frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2} x^6 - \frac{35}{16}\arctan\left(\sqrt{-1+\frac{1}{x^2}}\right)$$

output
```
-35/48*(-1+1/x^2)^(3/2)*x^2-7/24*(-1+1/x^2)^(5/2)*x^4-1/6*(-1+1/x^2)^(7/2)
*x^6-35/16*arctan((-1+1/x^2)^(1/2))+35/16*(-1+1/x^2)^(1/2)
```

3.675.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^3}{x} \, dx = \frac{1}{48}\sqrt{-1+\frac{1}{x^2}}(48+87x^2-38x^4+8x^6) - \frac{35\sqrt{-1+\frac{1}{x^2}}\, x \operatorname{arctanh}\left(\frac{\sqrt{-1+x^2}}{-1+x}\right)}{8\sqrt{-1+x^2}}$$

input
```
Integrate[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^3)/x,x]
```

output
```
(Sqrt[-1 + x^(-2)]*(48 + 87*x^2 - 38*x^4 + 8*x^6))/48 - (35*Sqrt[-1 + x^(-
2)]*x*ArcTanh[Sqrt[-1 + x^2]/(-1 + x)])/(8*Sqrt[-1 + x^2])
```

3.675.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {1016, 281, 798, 51, 51, 51, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{1}{x^2}-1}\left(x^2-1\right)^3}{x}dx$$
$$\Big\downarrow \text{1016}$$
$$\int \left(1-\frac{1}{x^2}\right)^3\sqrt{\frac{1}{x^2}-1}x^5dx$$
$$\Big\downarrow \text{281}$$
$$-\int \left(\frac{1}{x^2}-1\right)^{7/2}x^5dx$$
$$\Big\downarrow \text{798}$$
$$\frac{1}{2}\int \left(\frac{1}{x^2}-1\right)^{7/2}x^8d\frac{1}{x^2}$$
$$\Big\downarrow \text{51}$$
$$\frac{1}{2}\left(\frac{7}{6}\int \left(\frac{1}{x^2}-1\right)^{5/2}x^6d\frac{1}{x^2}-\frac{1}{3}\left(\frac{1}{x^2}-1\right)^{7/2}x^6\right)$$
$$\Big\downarrow \text{51}$$
$$\frac{1}{2}\left(\frac{7}{6}\left(\frac{5}{4}\int \left(\frac{1}{x^2}-1\right)^{3/2}x^4d\frac{1}{x^2}-\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{5/2}x^4\right)-\frac{1}{3}\left(\frac{1}{x^2}-1\right)^{7/2}x^6\right)$$
$$\Big\downarrow \text{51}$$
$$\frac{1}{2}\left(\frac{7}{6}\left(\frac{5}{4}\left(\frac{3}{2}\int \sqrt{\frac{1}{x^2}-1}x^2d\frac{1}{x^2}-\left(\frac{1}{x^2}-1\right)^{3/2}x^2\right)-\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{5/2}x^4\right)-\frac{1}{3}\left(\frac{1}{x^2}-1\right)^{7/2}x^6\right)$$
$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{7}{6}\left(\frac{5}{4}\left(\frac{3}{2}\left(2\sqrt{\frac{1}{x^2}-1}-\int\frac{x^2}{\sqrt{\frac{1}{x^2}-1}}d\frac{1}{x^2}\right)-\left(\frac{1}{x^2}-1\right)^{3/2}x^2\right)-\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{5/2}x^4\right)-\frac{1}{3}\left(\frac{1}{x^2}-1\right)^{7/2}x^6\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{7}{6}\left(\frac{5}{4}\left(\frac{3}{2}\left(2\sqrt{\frac{1}{x^2}-1}-2\int\frac{1}{1+\frac{1}{x^4}}d\sqrt{\frac{1}{x^2}-1}\right)-\left(\frac{1}{x^2}-1\right)^{3/2}x^2\right)-\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{5/2}x^4\right)-\frac{1}{3}\left(\frac{1}{x^2}-1\right)^{7/2}x$$

$$\Big\downarrow \; 216$$

$$\frac{1}{2}\left(\frac{7}{6}\left(\frac{5}{4}\left(\frac{3}{2}\left(2\sqrt{\frac{1}{x^2}-1}-2\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)\right)-\left(\frac{1}{x^2}-1\right)^{3/2}x^2\right)-\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{5/2}x^4\right)-\frac{1}{3}\left(\frac{1}{x^2}-1\right)^{7/2}x$$

input
```
Int[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^3)/x,x]
```

output
```
(-1/3*((-1 + x^(-2))^(7/2)*x^6) + (7*(-1/2*((-1 + x^(-2))^(5/2)*x^4) + (5*
(-((-1 + x^(-2))^(3/2)*x^2) + (3*(2*Sqrt[-1 + x^(-2)] - 2*ArcTan[Sqrt[-1 +
 x^(-2)]]))/2))/4))/6)/2
```

3.675.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

3.675. $\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)^3}{x}\,dx$

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

3.675.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.83

method	result	size
trager	$2\left(\frac{1}{12}x^6-\frac{19}{48}x^4+\frac{29}{32}x^2+\frac{1}{2}\right)\sqrt{-\frac{x^2-1}{x^2}}-\frac{35\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\left(\sqrt{-\frac{x^2-1}{x^2}}+\mathrm{RootOf}\left(\_Z^2+1\right)\right)x\right)}{16}$	63
risch	$\frac{\left(8x^8-46x^6+125x^4-39x^2-48\right)\sqrt{-\frac{x^2-1}{x^2}}}{48x^2-48}-\frac{35\arcsin\left(x\right)\sqrt{-\frac{x^2-1}{x^2}}\,x\sqrt{-x^2+1}}{16\left(x^2-1\right)}$	78
default	$\frac{\sqrt{-\frac{x^2-1}{x^2}}\left(-8x^4\left(-x^2+1\right)^{\frac{3}{2}}+30x^2\left(-x^2+1\right)^{\frac{3}{2}}+48\left(-x^2+1\right)^{\frac{3}{2}}+105x^2\sqrt{-x^2+1}+105\arcsin\left(x\right)x\right)}{48\sqrt{-x^2+1}}$	83

3.675. $\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)^3}{x}\,dx$

input
```
int((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*(1/12*x^6-19/48*x^4+29/32*x^2+1/2)*(-(x^2-1)/x^2)^(1/2)-35/16*RootOf(_Z^
2+1)*ln(((-(x^2-1)/x^2)^(1/2)+RootOf(_Z^2+1))*x)
```

3.675.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^3}{x}\,dx = \frac{1}{48}\left(8\,x^6-38\,x^4+87\,x^2+48\right)\sqrt{-\frac{x^2-1}{x^2}} - \frac{35}{8}\arctan\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}-1}{x}\right)$$

input
```
integrate((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
1/48*(8*x^6 - 38*x^4 + 87*x^2 + 48)*sqrt(-(x^2 - 1)/x^2) - 35/8*arctan((x*
sqrt(-(x^2 - 1)/x^2) - 1)/x)
```

3.675.6 Sympy [A] (verification not implemented)

Time = 51.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^3}{x}\,dx = -\frac{x^6\left(-1+\frac{1}{x^2}\right)^{\frac{3}{2}}}{6} - \frac{5x^4\sqrt{-1+\frac{1}{x^2}}\cdot\left(2-\frac{1}{x^2}\right)}{16} + \frac{3x^2\sqrt{-1+\frac{1}{x^2}}}{2} + \sqrt{-1+\frac{1}{x^2}} - \frac{35\,\mathrm{atan}\left(\sqrt{-1+\frac{1}{x^2}}\right)}{16}$$

input
```
integrate((x**2-1)**3*(-1+1/x**2)**(1/2)/x,x)
```

output
```
-x**6*(-1 + x**(-2))**(3/2)/6 - 5*x**4*sqrt(-1 + x**(-2))*(2 - 1/x**2)/16
+ 3*x**2*sqrt(-1 + x**(-2))/2 + sqrt(-1 + x**(-2)) - 35*atan(sqrt(-1 + x**
(-2)))/16
```

3.675.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(56) = 112.

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^3}{x}\,dx = \frac{3}{2}\,x^2\sqrt{\frac{1}{x^2}-1}+\sqrt{\frac{1}{x^2}-1} - \frac{3\left(\frac{1}{x^2}-1\right)^{\frac{5}{2}}+8\left(\frac{1}{x^2}-1\right)^{\frac{3}{2}}-3\sqrt{\frac{1}{x^2}-1}}{48\left(\left(\frac{1}{x^2}-1\right)^3+3\left(\frac{1}{x^2}-1\right)^2+\frac{3}{x^2}-2\right)} + \frac{3\left(\left(\frac{1}{x^2}-1\right)^{\frac{3}{2}}-\sqrt{\frac{1}{x^2}-1}\right)}{8\left(\left(\frac{1}{x^2}-1\right)^2+\frac{2}{x^2}-1\right)} - \frac{35}{16}\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)$$

input
```
integrate((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
3/2*x^2*sqrt(1/x^2 - 1) + sqrt(1/x^2 - 1) - 1/48*(3*(1/x^2 - 1)^(5/2) + 8*
(1/x^2 - 1)^(3/2) - 3*sqrt(1/x^2 - 1))/((1/x^2 - 1)^3 + 3*(1/x^2 - 1)^2 +
3/x^2 - 2) + 3/8*((1/x^2 - 1)^(3/2) - sqrt(1/x^2 - 1))/((1/x^2 - 1)^2 + 2/
x^2 - 1) - 35/16*arctan(sqrt(1/x^2 - 1))
```

3.675.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^3}{x}\,dx = \frac{1}{48}\left(2\left(4\,x^2\mathrm{sgn}(x)-19\,\mathrm{sgn}(x)\right)x^2+87\,\mathrm{sgn}(x)\right)\sqrt{-x^2+1}x + \frac{35}{16}\arcsin\left(x\right)\mathrm{sgn}(x) - \frac{x\mathrm{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)} + \frac{\left(\sqrt{-x^2+1}-1\right)\mathrm{sgn}(x)}{2\,x}$$

input
```
integrate((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x, algorithm="giac")
```

output
```
1/48*(2*(4*x^2*sgn(x) - 19*sgn(x))*x^2 + 87*sgn(x))*sqrt(-x^2 + 1)*x + 35/
16*arcsin(x)*sgn(x) - 1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 +
 1) - 1)*sgn(x)/x
```

3.675. $\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^3}{x}\,dx$

3.675.9 Mupad [B] (verification not implemented)

Time = 17.76 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^3}{x}\,dx = \sqrt{\frac{1}{x^2}-1} - \frac{35\,\mathrm{atan}\left(\sqrt{\frac{1}{x^2}-1}\right)}{16} + \frac{19\,x^6\sqrt{\frac{1}{x^2}-1}}{16} + \frac{17\,x^6\left(\frac{1}{x^2}-1\right)^{3/2}}{6} + \frac{29\,x^6\left(\frac{1}{x^2}-1\right)^{5/2}}{16}$$

input
```
int(((1/x^2 - 1)^(1/2)*(x^2 - 1)^3)/x,x)
```

output
```
(1/x^2 - 1)^(1/2) - (35*atan((1/x^2 - 1)^(1/2)))/16 + (19*x^6*(1/x^2 - 1)^
(1/2))/16 + (17*x^6*(1/x^2 - 1)^(3/2))/6 + (29*x^6*(1/x^2 - 1)^(5/2))/16
```

3.676 $\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)^2}{x} \, dx$

3.676.1 Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)^2}{x} \, dx = -\frac{15}{8}\sqrt{-1+\frac{1}{x^2}}+\frac{5}{8}\left(-1+\frac{1}{x^2}\right)^{3/2}x^2 +\frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2}x^4+\frac{15}{8}\arctan\left(\sqrt{-1+\frac{1}{x^2}}\right)$$

output
```
5/8*(-1+1/x^2)^(3/2)*x^2+1/4*(-1+1/x^2)^(5/2)*x^4+15/8*arctan((-1+1/x^2)^(
1/2))-15/8*(-1+1/x^2)^(1/2)
```

3.676.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)^2}{x} \, dx = \frac{1}{8}\sqrt{-1+\frac{1}{x^2}}\left(-8-9x^2+2x^4\right) +\frac{15\sqrt{-1+\frac{1}{x^2}}x\text{arctanh}\left(\frac{\sqrt{-1+x^2}}{-1+x}\right)}{4\sqrt{-1+x^2}}$$

input
```
Integrate[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^2)/x,x]
```

output
```
(Sqrt[-1 + x^(-2)]*(-8 - 9*x^2 + 2*x^4))/8 + (15*Sqrt[-1 + x^(-2)]*x*ArcTa
nh[Sqrt[-1 + x^2]/(-1 + x)])/(4*Sqrt[-1 + x^2])
```

3.676.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1016, 281, 798, 51, 51, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{1}{x^2}-1}\left(x^2-1\right)^2}{x}dx$$

$$\Big\downarrow \text{1016}$$

$$\int \left(1-\frac{1}{x^2}\right)^2\sqrt{\frac{1}{x^2}-1}x^3dx$$

$$\Big\downarrow \text{281}$$

$$\int \left(\frac{1}{x^2}-1\right)^{5/2}x^3dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{2}\int \left(\frac{1}{x^2}-1\right)^{5/2}x^6d\frac{1}{x^2}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{5/2}x^4-\frac{5}{4}\int \left(\frac{1}{x^2}-1\right)^{3/2}x^4d\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{5/2}x^4-\frac{5}{4}\left(\frac{3}{2}\int \sqrt{\frac{1}{x^2}-1}x^2d\frac{1}{x^2}-\left(\frac{1}{x^2}-1\right)^{3/2}x^2\right)\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{5/2}x^4-\frac{5}{4}\left(\frac{3}{2}\left(2\sqrt{\frac{1}{x^2}-1}-\int \frac{x^2}{\sqrt{\frac{1}{x^2}-1}}d\frac{1}{x^2}\right)-\left(\frac{1}{x^2}-1\right)^{3/2}x^2\right)\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{5/2}x^4-\frac{5}{4}\left(\frac{3}{2}\left(2\sqrt{\frac{1}{x^2}-1}-2\int\frac{1}{1+\frac{1}{x^4}}d\sqrt{\frac{1}{x^2}-1}\right)-\left(\frac{1}{x^2}-1\right)^{3/2}x^2\right)\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{5/2}x^4-\frac{5}{4}\left(\frac{3}{2}\left(2\sqrt{\frac{1}{x^2}-1}-2\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)\right)-\left(\frac{1}{x^2}-1\right)^{3/2}x^2\right)\right)$$

input
```
Int[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^2)/x,x]
```

output
```
(((-1 + x^(-2))^(5/2)*x^4)/2 - (5*(-((-1 + x^(-2))^(3/2)*x^2) + (3*(2*Sqrt
[-1 + x^(-2)] - 2*ArcTan[Sqrt[-1 + x^(-2)]]))/2))/4)/2
```

3.676.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.676. $\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^2}{x}\,dx$

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

3.676.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.99 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

method	result	size
trager	$2\left(\frac{1}{8}x^{4}-\frac{9}{16}x^{2}-\frac{1}{2}\right)\sqrt{-\frac{x^{2}-1}{x^{2}}}+\frac{15\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(\left(\sqrt{-\frac{x^{2}-1}{x^{2}}}+\mathrm{RootOf}\left(\_Z^{2}+1\right)\right)x\right)}{8}$	58
default	$-\frac{\sqrt{-\frac{x^{2}-1}{x^{2}}}\left(2x^{2}\left(-x^{2}+1\right)^{\frac{3}{2}}+8\left(-x^{2}+1\right)^{\frac{3}{2}}+15x^{2}\sqrt{-x^{2}+1}+15\arcsin\left(x\right)x\right)}{8\sqrt{-x^{2}+1}}$	69
risch	$\frac{\left(2x^{6}-11x^{4}+x^{2}+8\right)\sqrt{-\frac{x^{2}-1}{x^{2}}}}{8x^{2}-8}+\frac{15\arcsin\left(x\right)\sqrt{-\frac{x^{2}-1}{x^{2}}}\,x\sqrt{-x^{2}+1}}{8\left(x^{2}-1\right)}$	71

input
```
int((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*(1/8*x^4-9/16*x^2-1/2)*(-(x^2-1)/x^2)^(1/2)+15/8*RootOf(_Z^2+1)*ln(((-(x
^2-1)/x^2)^(1/2)+RootOf(_Z^2+1))*x)
```

3.676.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^2}{x}\,dx = \frac{1}{8}\left(2x^4-9x^2-8\right)\sqrt{-\frac{x^2-1}{x^2}}+\frac{15}{4}\arctan\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}-1}{x}\right)$$

input
```
integrate((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
1/8*(2*x^4 - 9*x^2 - 8)*sqrt(-(x^2 - 1)/x^2) + 15/4*arctan((x*sqrt(-(x^2 -
 1)/x^2) - 1)/x)
```

3.676.6 Sympy [A] (verification not implemented)

Time = 23.56 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^2}{x}\,dx = \frac{x^4\sqrt{-1+\frac{1}{x^2}}\cdot\left(2-\frac{1}{x^2}\right)}{8}-x^2\sqrt{-1+\frac{1}{x^2}} - \sqrt{-1+\frac{1}{x^2}}+\frac{15\,\mathrm{atan}\left(\sqrt{-1+\frac{1}{x^2}}\right)}{8}$$

input
```
integrate((x**2-1)**2*(-1+1/x**2)**(1/2)/x,x)
```

output
```
x**4*sqrt(-1 + x**(-2))*(2 - 1/x**2)/8 - x**2*sqrt(-1 + x**(-2)) - sqrt(-1
 + x**(-2)) + 15*atan(sqrt(-1 + x**(-2)))/8
```

3.676.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^2}{x}\,dx = -x^2\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}-1}-\frac{\left(\frac{1}{x^2}-1\right)^{\frac{3}{2}}-\sqrt{\frac{1}{x^2}-1}}{8\left(\left(\frac{1}{x^2}-1\right)^2+\frac{2}{x^2}-1\right)} + \frac{15}{8}\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)$$

input
```
integrate((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
-x^2*sqrt(1/x^2 - 1) - sqrt(1/x^2 - 1) - 1/8*((1/x^2 - 1)^(3/2) - sqrt(1/x
^2 - 1))/((1/x^2 - 1)^2 + 2/x^2 - 1) + 15/8*arctan(sqrt(1/x^2 - 1))
```

3.676.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)^2}{x}\,dx = \frac{1}{8}\left(2\,x^2\mathrm{sgn}(x)-9\,\mathrm{sgn}(x)\right)\sqrt{-x^2+1}x-\frac{15}{8}\arcsin\left(x\right)\mathrm{sgn}(x)+\frac{x\mathrm{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)}-\frac{\left(\sqrt{-x^2+1}-1\right)\mathrm{sgn}(x)}{2\,x}$$

input
```
integrate((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x, algorithm="giac")
```

output
```
1/8*(2*x^2*sgn(x) - 9*sgn(x))*sqrt(-x^2 + 1)*x - 15/8*arcsin(x)*sgn(x) + 1
/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) - 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x
```

3.676.9 Mupad [B] (verification not implemented)

Time = 17.65 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}\left(-1+x^2\right)^2}{x}\,dx = \frac{15\,\mathrm{atan}\left(\sqrt{\frac{1}{x^2}-1}\right)}{8}-\sqrt{\frac{1}{x^2}-1}-\frac{7\,x^4\,\sqrt{\frac{1}{x^2}-1}}{8}-\frac{9\,x^4\,{\left(\frac{1}{x^2}-1\right)}^{3/2}}{8}$$

input
```
int(((1/x^2 - 1)^(1/2)*(x^2 - 1)^2)/x,x)
```

output
```
(15*atan((1/x^2 - 1)^(1/2)))/8 - (1/x^2 - 1)^(1/2) - (7*x^4*(1/x^2 - 1)^(1
/2))/8 - (9*x^4*(1/x^2 - 1)^(3/2))/8
```

3.677 $\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)}{x}\,dx$

3.677.1 Optimal result

Integrand size = 18, antiderivative size = 44

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)}{x}\,dx = \frac{3}{2}\sqrt{-1+\frac{1}{x^2}}-\frac{1}{2}\left(-1+\frac{1}{x^2}\right)^{3/2}x^2-\frac{3}{2}\arctan\left(\sqrt{-1+\frac{1}{x^2}}\right)$$

output
```
-1/2*(-1+1/x^2)^(3/2)*x^2-3/2*arctan((-1+1/x^2)^(1/2))+3/2*(-1+1/x^2)^(1/2
)
```

3.677.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)}{x}\,dx = \frac{1}{2}\sqrt{-1+\frac{1}{x^2}}\left(2+x^2-\frac{6x\text{arctanh}\left(\frac{\sqrt{-1+x^2}}{-1+x}\right)}{\sqrt{-1+x^2}}\right)$$

input
```
Integrate[(Sqrt[-1 + x^(-2)]*(-1 + x^2))/x,x]
```

output
```
(Sqrt[-1 + x^(-2)]*(2 + x^2 - (6*x*ArcTanh[Sqrt[-1 + x^2]/(-1 + x)])/Sqrt[
-1 + x^2]))/2
```

3.677.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1016, 281, 798, 51, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{1}{x^2}-1}(x^2-1)}{x}\,dx$$

$$\Big\downarrow \text{ 1016}$$

$$\int \left(1-\frac{1}{x^2}\right)\sqrt{\frac{1}{x^2}-1}xdx$$

$$\Big\downarrow \text{ 281}$$

$$-\int \left(\frac{1}{x^2}-1\right)^{3/2}xdx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{2}\int \left(\frac{1}{x^2}-1\right)^{3/2}x^4d\frac{1}{x^2}$$

$$\Big\downarrow \text{ 51}$$

$$\frac{1}{2}\left(\frac{3}{2}\int \sqrt{\frac{1}{x^2}-1}x^2d\frac{1}{x^2}-\left(\frac{1}{x^2}-1\right)^{3/2}x^2\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2}\left(\frac{3}{2}\left(2\sqrt{\frac{1}{x^2}-1}-\int \frac{x^2}{\sqrt{\frac{1}{x^2}-1}}d\frac{1}{x^2}\right)-\left(\frac{1}{x^2}-1\right)^{3/2}x^2\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(\frac{3}{2}\left(2\sqrt{\frac{1}{x^2}-1}-2\int \frac{1}{1+\frac{1}{x^4}}d\sqrt{\frac{1}{x^2}-1}\right)-\left(\frac{1}{x^2}-1\right)^{3/2}x^2\right)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{2}\left(\frac{3}{2}\left(2\sqrt{\frac{1}{x^2}-1}-2\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)\right)-\left(\frac{1}{x^2}-1\right)^{3/2}x^2\right)$$

3.677. $\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)}{x}\,dx$

input
```
Int[(Sqrt[-1 + x^(-2)]*(-1 + x^2))/x,x]
```

output
```
(-((-1 + x^(-2))^(3/2)*x^2) + (3*(2*Sqrt[-1 + x^(-2)] - 2*ArcTan[Sqrt[-1 +
 x^(-2)]]))/2)/2
```

3.677.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

3.677. $\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)}{x}\,dx$

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

3.677.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

method	result	size
trager	$2\left(\frac{x^2}{4}+\frac{1}{2}\right)\sqrt{-\frac{x^2-1}{x^2}}-\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\left(\sqrt{-\frac{x^2-1}{x^2}}+\mathrm{RootOf}\left(\_Z^2+1\right)\right)x\right)}{2}$	53
default	$\frac{\sqrt{-\frac{x^2-1}{x^2}}\left(2\left(-x^2+1\right)^{\frac{3}{2}}+3x^2\sqrt{-x^2+1}+3\arcsin(x)x\right)}{2\sqrt{-x^2+1}}$	55
risch	$\frac{(x^4+x^2-2)\sqrt{-\frac{x^2-1}{x^2}}}{2x^2-2}-\frac{3\arcsin(x)\sqrt{-\frac{x^2-1}{x^2}}\,x\sqrt{-x^2+1}}{2(x^2-1)}$	64

input
```
int((x^2-1)*(-1+1/x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2*(1/4*x^2+1/2)*(-(x^2-1)/x^2)^(1/2)-3/2*RootOf(_Z^2+1)*ln(((-(x^2-1)/x^2)
^(1/2)+RootOf(_Z^2+1))*x)
```

3.677.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)}{x}\,dx=\frac{1}{2}\left(x^2+2\right)\sqrt{-\frac{x^2-1}{x^2}}-3\arctan\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}-1}{x}\right)$$

input
```
integrate((x^2-1)*(-1+1/x^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
1/2*(x^2 + 2)*sqrt(-(x^2 - 1)/x^2) - 3*arctan((x*sqrt(-(x^2 - 1)/x^2) - 1)
/x)
```

3.677.6 Sympy [A] (verification not implemented)

Time = 10.70 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)}{x}\,dx = \frac{x^2\sqrt{-1+\frac{1}{x^2}}}{2}+\sqrt{-1+\frac{1}{x^2}}-\frac{3\,\mathrm{atan}\left(\sqrt{-1+\frac{1}{x^2}}\right)}{2}$$

input
```
integrate((x**2-1)*(-1+1/x**2)**(1/2)/x,x)
```

output
```
x**2*sqrt(-1 + x**(-2))/2 + sqrt(-1 + x**(-2)) - 3*atan(sqrt(-1 + x**(-2))
)/2
```

3.677.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)}{x}\,dx = \frac{1}{2}\,x^2\sqrt{\frac{1}{x^2}-1}+\sqrt{\frac{1}{x^2}-1}-\frac{3}{2}\,\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)$$

input
```
integrate((x^2-1)*(-1+1/x^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
1/2*x^2*sqrt(1/x^2 - 1) + sqrt(1/x^2 - 1) - 3/2*arctan(sqrt(1/x^2 - 1))
```

3.677.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)}{x}\,dx = \frac{1}{2}\,\sqrt{-x^2+1}x\mathrm{sgn}(x)+\frac{3}{2}\,\arcsin\left(x\right)\mathrm{sgn}(x) - \frac{x\mathrm{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)}+\frac{\left(\sqrt{-x^2+1}-1\right)\mathrm{sgn}(x)}{2\,x}$$

input
```
integrate((x^2-1)*(-1+1/x^2)^(1/2)/x,x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 + 1)*x*sgn(x) + 3/2*arcsin(x)*sgn(x) - 1/2*x*sgn(x)/(sqrt(-x
^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x
```

3.677.9 Mupad [B] (verification not implemented)

Time = 18.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)}{x}\,dx = \sqrt{\frac{1}{x^2}-1} - \frac{3\,\mathrm{atan}\left(\sqrt{\frac{1}{x^2}-1}\right)}{2} + \frac{x^2\sqrt{\frac{1}{x^2}-1}}{2}$$

input
```
int(((1/x^2 - 1)^(1/2)*(x^2 - 1))/x,x)
```

output
```
(1/x^2 - 1)^(1/2) - (3*atan((1/x^2 - 1)^(1/2)))/2 + (x^2*(1/x^2 - 1)^(1/2)
)/2
```

3.678 $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)}\,dx$

3.678.1 Optimal result

Integrand size = 20, antiderivative size = 9

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\,(-1+x^2)}\,dx=\sqrt{-1+\frac{1}{x^2}}$$

output
```
(-1+1/x^2)^(1/2)
```

3.678.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\,(-1+x^2)}\,dx=\sqrt{-1+\frac{1}{x^2}}$$

input
```
Integrate[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)),x]
```

output
```
Sqrt[-1 + x^(-2)]
```

3.678.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1016, 281, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{1}{x^2}-1}}{x(x^2-1)}dx$$

$$\Big\downarrow \ 1016$$

$$\int \frac{\sqrt{\frac{1}{x^2}-1}}{\left(1-\frac{1}{x^2}\right)x^3}dx$$

$$\Big\downarrow \ 281$$

$$-\int \frac{1}{\sqrt{\frac{1}{x^2}-1x^3}}dx$$

$$\Big\downarrow \ 793$$

$$\sqrt{\frac{1}{x^2}-1}$$

input
```
Int[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)),x]
```

output
```
Sqrt[-1 + x^(-2)]
```

3.678.3.1 Defintions of rubi rules used

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

3.678.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.44

method	result	size
gosper	$\sqrt{-\frac{x^2-1}{x^2}}$	13
default	$\sqrt{-\frac{x^2-1}{x^2}}$	13
trager	$\sqrt{-\frac{x^2-1}{x^2}}$	13
risch	$\sqrt{-\frac{x^2-1}{x^2}}$	13

input
```
int((-1+1/x^2)^(1/2)/x/(x^2-1),x,method=_RETURNVERBOSE)
```

output
```
(-(x^2-1)/x^2)^(1/2)
```

3.678.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)}\,dx = \sqrt{-\frac{x^2-1}{x^2}}$$

input
```
integrate((-1+1/x^2)^(1/2)/x/(x^2-1),x, algorithm="fricas")
```

output
```
sqrt(-(x^2 - 1)/x^2)
```

3.678.6 Sympy [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)}\,dx = \sqrt{-1+\frac{1}{x^2}}$$

input
```
integrate((-1+1/x**2)**(1/2)/x/(x**2-1),x)
```

output
```
sqrt(-1 + x**(-2))
```

3.678.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(7) = 14.

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)}\,dx = \frac{\sqrt{x+1}\sqrt{-x+1}}{x}$$

input
```
integrate((-1+1/x^2)^(1/2)/x/(x^2-1),x, algorithm="maxima")
```

output
```
sqrt(x + 1)*sqrt(-x + 1)/x
```

3.678.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(7) = 14.

Time = 0.33 (sec) , antiderivative size = 37, normalized size of antiderivative = 4.11

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)}\,dx = -\frac{x\mathrm{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)} + \frac{\left(\sqrt{-x^2+1}-1\right)\mathrm{sgn}(x)}{2\,x}$$

input
```
integrate((-1+1/x^2)^(1/2)/x/(x^2-1),x, algorithm="giac")
```

output
```
-1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x
```

3.678. $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)}\,dx$

3.678.9 Mupad [B] (verification not implemented)

Time = 17.76 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)}\,dx = \frac{\sqrt{1-x^2}}{|x|}$$

input
```
int((1/x^2 - 1)^(1/2)/(x*(x^2 - 1)),x)
```

output
```
(1 - x^2)^(1/2)/abs(x)
```

3.679 $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)^2} \, dx$

3.679.1 Optimal result

Integrand size = 20, antiderivative size = 21

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)^2} \, dx = \frac{1}{\sqrt{-1+\frac{1}{x^2}}} - \sqrt{-1+\frac{1}{x^2}}$$

output
```
1/(-1+1/x^2)^(1/2)-(-1+1/x^2)^(1/2)
```

3.679.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)^2} \, dx = \frac{\sqrt{-1+\frac{1}{x^2}}(1-2x^2)}{-1+x^2}$$

input
```
Integrate[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^2),x]
```

output
```
(Sqrt[-1 + x^(-2)]*(1 - 2*x^2))/(-1 + x^2)
```

3.679.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1016, 281, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{1}{x^2}-1}}{x\left(x^2-1\right)^2}dx$$

$$\Big\downarrow \text{1016}$$

$$\int \frac{\sqrt{\frac{1}{x^2}-1}}{\left(1-\frac{1}{x^2}\right)^2x^5}dx$$

$$\Big\downarrow \text{281}$$

$$\int \frac{1}{\left(\frac{1}{x^2}-1\right)^{3/2}x^5}dx$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{2}\int \frac{1}{\left(\frac{1}{x^2}-1\right)^{3/2}x^2}d\frac{1}{x^2}$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{2}\int \left(\frac{1}{\sqrt{\frac{1}{x^2}-1}}+\frac{1}{\left(\frac{1}{x^2}-1\right)^{3/2}}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2}{\sqrt{\frac{1}{x^2}-1}}-2\sqrt{\frac{1}{x^2}-1}\right)$$

input
```
Int[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^2),x]
```

output
```
(2/Sqrt[-1 + x^(-2)] - 2*Sqrt[-1 + x^(-2)])/2
```

3.679. $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)^2}\,dx$

3.679.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.679.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

method	result	size
gosper	$-\frac{(2x^2-1)\sqrt{-\frac{x^2-1}{x^2}}}{x^2-1}$	29
trager	$-\frac{(2x^2-1)\sqrt{-\frac{x^2-1}{x^2}}}{x^2-1}$	29
risch	$-\frac{(2x^2-1)\sqrt{-\frac{x^2-1}{x^2}}}{x^2-1}$	29
default	$-\frac{(2x^2-1)\sqrt{-\frac{x^2-1}{x^2}}}{(x-1)(x+1)}$	32

3.679. $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)^2}\,dx$

input
```
int((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x,method=_RETURNVERBOSE)
```

output
```
-(2*x^2-1)*(-(x^2-1)/x^2)^(1/2)/(x^2-1)
```

3.679.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^2}\,dx = -\frac{\left(2\,x^2-1\right)\sqrt{-\frac{x^2-1}{x^2}}}{x^2-1}$$

input
```
integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x, algorithm="fricas")
```

output
```
-(2*x^2 - 1)*sqrt(-(x^2 - 1)/x^2)/(x^2 - 1)
```

3.679.6 Sympy [A] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^2}\,dx = -\sqrt{-1+\frac{1}{x^2}}+\frac{1}{\sqrt{-1+\frac{1}{x^2}}}$$

input
```
integrate((-1+1/x**2)**(1/2)/x/(x**2-1)**2,x)
```

output
```
-sqrt(-1 + x**(-2)) + 1/sqrt(-1 + x**(-2))
```

3.679.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^2}\,dx = -\frac{\left(2\,x^2-1\right)\sqrt{x+1}\sqrt{-x+1}}{x^3-x}$$

input
```
integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x, algorithm="maxima")
```

output
```
-(2*x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)/(x^3 - x)
```

3.679.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(17) = 34$.

Time = 0.35 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.76

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^2}\,dx = -\frac{\sqrt{-x^2+1}x\mathrm{sgn}(x)}{x^2-1}+\frac{x\mathrm{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)}-\frac{\left(\sqrt{-x^2+1}-1\right)\mathrm{sgn}(x)}{2\,x}$$

input
```
integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x, algorithm="giac")
```

output
```
-sqrt(-x^2 + 1)*x*sgn(x)/(x^2 - 1) + 1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) - 1
/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x
```

3.679.9 Mupad [B] (verification not implemented)

Time = 18.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^2}\,dx = \frac{x\,\sqrt{\frac{1}{x^2}-1}\,\left(2\,x^2-1\right)}{x-x^3}$$

input
```
int((1/x^2 - 1)^(1/2)/(x*(x^2 - 1)^2),x)
```

output
```
(x*(1/x^2 - 1)^(1/2)*(2*x^2 - 1))/(x - x^3)
```

3.680 $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)^3} dx$

3.680.1 Optimal result

Integrand size = 20, antiderivative size = 34

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)^3} dx = -\frac{1}{3\left(-1+\frac{1}{x^2}\right)^{3/2}} - \frac{2}{\sqrt{-1+\frac{1}{x^2}}} + \sqrt{-1+\frac{1}{x^2}}$$

output
```
-1/3/(-1+1/x^2)^(3/2)-2/(-1+1/x^2)^(1/2)+(-1+1/x^2)^(1/2)
```

3.680.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)^3} dx = \frac{\sqrt{-1+\frac{1}{x^2}}(3-12x^2+8x^4)}{3(-1+x^2)^2}$$

input
```
Integrate[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^3),x]
```

output
```
(Sqrt[-1 + x^(-2)]*(3 - 12*x^2 + 8*x^4))/(3*(-1 + x^2)^2)
```

3.680.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1016, 281, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{1}{x^2}-1}}{x\left(x^2-1\right)^3}dx$$

$$\Big\downarrow \text{1016}$$

$$\int \frac{\sqrt{\frac{1}{x^2}-1}}{\left(1-\frac{1}{x^2}\right)^3x^7}dx$$

$$\Big\downarrow \text{281}$$

$$-\int \frac{1}{\left(\frac{1}{x^2}-1\right)^{5/2}x^7}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{2}\int \frac{1}{\left(\frac{1}{x^2}-1\right)^{5/2}x^4}d\frac{1}{x^2}$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{1}{\sqrt{\frac{1}{x^2}-1}}+\frac{2}{\left(\frac{1}{x^2}-1\right)^{3/2}}+\frac{1}{\left(\frac{1}{x^2}-1\right)^{5/2}}\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(2\sqrt{\frac{1}{x^2}-1}-\frac{4}{\sqrt{\frac{1}{x^2}-1}}-\frac{2}{3\left(\frac{1}{x^2}-1\right)^{3/2}}\right)$$

input
```
Int[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^3),x]
```

output
```
(-2/(3*(-1 + x^(-2))^(3/2)) - 4/Sqrt[-1 + x^(-2)] + 2*Sqrt[-1 + x^(-2)])/2
```

3.680. $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)^3}\,dx$

3.680.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.680.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{\left(8x^{4}-12x^{2}+3\right)\sqrt{-\frac{x^{2}-1}{x^{2}}}}{3\left(x^{2}-1\right)^{2}}$	34
trager	$\frac{\left(8x^{4}-12x^{2}+3\right)\sqrt{-\frac{x^{2}-1}{x^{2}}}}{3\left(x^{2}-1\right)^{2}}$	34
risch	$\frac{\left(8x^{4}-12x^{2}+3\right)\sqrt{-\frac{x^{2}-1}{x^{2}}}}{3\left(x^{2}-1\right)^{2}}$	34
default	$\frac{\left(8x^{4}-12x^{2}+3\right)\sqrt{-\frac{x^{2}-1}{x^{2}}}}{3\left(x-1\right)^{2}\left(x+1\right)^{2}}$	37

3.680. $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^3}\,dx$

input
```
int((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*(8*x^4-12*x^2+3)*(-(x^2-1)/x^2)^(1/2)/(x^2-1)^2
```

3.680.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^3}\,dx = \frac{(8\,x^4-12\,x^2+3)\sqrt{-\frac{x^2-1}{x^2}}}{3\,(x^4-2\,x^2+1)}$$

input
```
integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x, algorithm="fricas")
```

output
```
1/3*(8*x^4 - 12*x^2 + 3)*sqrt(-(x^2 - 1)/x^2)/(x^4 - 2*x^2 + 1)
```

3.680.6 Sympy [A] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^3}\,dx = \sqrt{-1+\frac{1}{x^2}} - \frac{2}{\sqrt{-1+\frac{1}{x^2}}} - \frac{1}{3\left(-1+\frac{1}{x^2}\right)^{\frac{3}{2}}}$$

input
```
integrate((-1+1/x**2)**(1/2)/x/(x**2-1)**3,x)
```

output
```
sqrt(-1 + x**(-2)) - 2/sqrt(-1 + x**(-2)) - 1/(3*(-1 + x**(-2))**(3/2))
```

3.680.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^3}\,dx = \frac{(8\,x^4-12\,x^2+3)\sqrt{x+1}\sqrt{-x+1}}{3\,(x^5-2\,x^3+x)}$$

input
```
integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x, algorithm="maxima")
```

output
```
1/3*(8*x^4 - 12*x^2 + 3)*sqrt(x + 1)*sqrt(-x + 1)/(x^5 - 2*x^3 + x)
```

3.680.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(26) = 52$.

Time = 0.49 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)^3}\,dx = -\frac{x\mathrm{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)} + \frac{\left(\sqrt{-x^2+1}-1\right)\mathrm{sgn}(x)}{2\,x} - \frac{\left(5\,x^2\mathrm{sgn}(x)-6\,\mathrm{sgn}(x)\right)x}{3\,(x^2-1)\sqrt{-x^2+1}}$$

input
```
integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x, algorithm="giac")
```

output
```
-1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x - 1
/3*(5*x^2*sgn(x) - 6*sgn(x))*x/((x^2 - 1)*sqrt(-x^2 + 1))
```

3.680.9 Mupad [B] (verification not implemented)

Time = 18.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)^3}\,dx = \frac{\sqrt{\frac{1}{x^2}-1}\,(8\,x^4-12\,x^2+3)}{3\,(x^2-1)^2}$$

input
```
int((1/x^2 - 1)^(1/2)/(x*(x^2 - 1)^3),x)
```

output
```
((1/x^2 - 1)^(1/2)*(8*x^4 - 12*x^2 + 3))/(3*(x^2 - 1)^2)
```

3.681 $\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2}\,dx$

3.681.1 Optimal result

Integrand size = 18, antiderivative size = 9

$$\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2}\,dx=\frac{1}{\sqrt{1+\frac{1}{x^2}}}$$

output
```
1/(1+1/x^2)^(1/2)
```

3.681.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 20 vs. 2(9) = 18.

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.22

$$\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2}\,dx=\frac{\sqrt{1+\frac{1}{x^2}}x^2}{1+x^2}$$

input
```
Integrate[(Sqrt[1 + x^(-2)]*x)/(1 + x^2)^2,x]
```

output
```
(Sqrt[1 + x^(-2)]*x^2)/(1 + x^2)
```

3.681.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1016, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{1}{x^2}+1}x}{(x^2+1)^2}dx$$

$$\Big\downarrow \text{1016}$$

$$\int \frac{1}{\left(\frac{1}{x^2}+1\right)^{3/2}x^3}dx$$

$$\Big\downarrow \text{793}$$

$$\frac{1}{\sqrt{\frac{1}{x^2}+1}}$$

input
```
Int[(Sqrt[1 + x^(-2)]*x)/(1 + x^2)^2,x]
```

output
```
1/Sqrt[1 + x^(-2)]
```

3.681.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

3.681.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(7) = 14$.

Time = 0.98 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.56

method	result	size
gosper	$\frac{x^{2}\sqrt{\frac{x^{2}+1}{x^{2}}}}{x^{2}+1}$	23
default	$\frac{x^{2}\sqrt{\frac{x^{2}+1}{x^{2}}}}{x^{2}+1}$	23
risch	$\frac{x^{2}\sqrt{\frac{x^{2}+1}{x^{2}}}}{x^{2}+1}$	23
trager	$\frac{x^{2}\sqrt{-\frac{-x^{2}-1}{x^{2}}}}{x^{2}+1}$	26

input
```
int(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(x^2+1)*x^2*((x^2+1)/x^2)^(1/2)
```

3.681.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(7) = 14$.

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 3.11

$$\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2}\,dx = \frac{x^2\sqrt{\frac{x^2+1}{x^2}}+x^2+1}{x^2+1}$$

input
```
integrate(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
(x^2*sqrt((x^2 + 1)/x^2) + x^2 + 1)/(x^2 + 1)
```

3.681.6 Sympy [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2}\,dx = \frac{x}{\sqrt{x^2+1}}$$

input
```
integrate(x*(1+1/x**2)**(1/2)/(x**2+1)**2,x)
```

output
```
x/sqrt(x**2 + 1)
```

3.681.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2}\,dx = \frac{1}{\sqrt{\frac{x^2+1}{x^2}}}$$

input
```
integrate(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
1/sqrt((x^2 + 1)/x^2)
```

3.681.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2}\,dx = \frac{x\mathrm{sgn}(x)}{\sqrt{x^2+1}}$$

input
```
integrate(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x, algorithm="giac")
```

output
```
x*sgn(x)/sqrt(x^2 + 1)
```

3.681.9 Mupad [B] (verification not implemented)

Time = 18.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{1+\frac{1}{x^2}}\,x}{\left(1+x^2\right)^2}\,dx=\frac{x^2\,\sqrt{\frac{1}{x^2}+1}}{x^2+1}$$

input
```
int((x*(1/x^2 + 1)^(1/2))/(x^2 + 1)^2,x)
```

output
```
(x^2*(1/x^2 + 1)^(1/2))/(x^2 + 1)
```

3.682 $\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x(1+x^2)}\,dx$

3.682.1 Optimal result

Integrand size = 20, antiderivative size = 9

$$\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x(1+x^2)}\,dx = \frac{1}{\sqrt{1+\frac{1}{x^2}}}$$

output
```
1/(1+1/x^2)^(1/2)
```

3.682.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 20 vs. 2(9) = 18.

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.22

$$\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x(1+x^2)}\,dx = \frac{\sqrt{1+\frac{1}{x^2}}x^2}{1+x^2}$$

input
```
Integrate[1/(Sqrt[1 + x^(-2)]*x*(1 + x^2)),x]
```

output
```
(Sqrt[1 + x^(-2)]*x^2)/(1 + x^2)
```

3.682.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1016, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{1}{x^2}+1}x\left(x^2+1\right)}dx$$

$$\Big\downarrow \text{1016}$$

$$\int \frac{1}{\left(\frac{1}{x^2}+1\right)^{3/2}x^3}dx$$

$$\Big\downarrow \text{793}$$

$$\frac{1}{\sqrt{\frac{1}{x^2}+1}}$$

input
```
Int[1/(Sqrt[1 + x^(-2)]*x*(1 + x^2)),x]
```

output
```
1/Sqrt[1 + x^(-2)]
```

3.682.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

3.682.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

method	result	size
gosper	$\frac{1}{\sqrt{\frac{x^2+1}{x^2}}}$	12
default	$\frac{1}{\sqrt{\frac{x^2+1}{x^2}}}$	12
risch	$\frac{1}{\sqrt{\frac{x^2+1}{x^2}}}$	12
trager	$\frac{x^2\sqrt{-\frac{-x^2-1}{x^2}}}{x^2+1}$	26

input
```
int(1/x/(x^2+1)/(1+1/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/((x^2+1)/x^2)^(1/2)
```

3.682.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(7) = 14$.

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 3.11

$$\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x\,(1+x^2)}\,dx = \frac{x^2\sqrt{\frac{x^2+1}{x^2}}+x^2+1}{x^2+1}$$

input
```
integrate(1/x/(x^2+1)/(1+1/x^2)^(1/2),x, algorithm="fricas")
```

output
```
(x^2*sqrt((x^2 + 1)/x^2) + x^2 + 1)/(x^2 + 1)
```

3.682.6 Sympy [A] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x(1+x^2)}\,dx = \frac{1}{\sqrt{1+\frac{1}{x^2}}}$$

input
```
integrate(1/x/(x**2+1)/(1+1/x**2)**(1/2),x)
```

output
```
1/sqrt(1 + x**(-2))
```

3.682.7 Maxima [F]

$$\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x(1+x^2)}\,dx = \int \frac{1}{(x^2+1)x\sqrt{\frac{1}{x^2}+1}}\,dx$$

input
```
integrate(1/x/(x^2+1)/(1+1/x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 + 1)*x*sqrt(1/x^2 + 1)), x)
```

3.682.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x(1+x^2)}\,dx = \frac{x}{\sqrt{x^2+1}\mathrm{sgn}(x)}$$

input
```
integrate(1/x/(x^2+1)/(1+1/x^2)^(1/2),x, algorithm="giac")
```

output
```
x/(sqrt(x^2 + 1)*sgn(x))
```

3.682.9 Mupad [B] (verification not implemented)

Time = 18.37 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x\,(1+x^2)}\,dx = \frac{x^2\,\sqrt{\frac{1}{x^2}+1}}{x^2+1}$$

input
```
int(1/(x*(1/x^2 + 1)^(1/2)*(x^2 + 1)),x)
```

output
```
(x^2*(1/x^2 + 1)^(1/2))/(x^2 + 1)
```

3.683 $\int \frac{x}{a+bx^2+\sqrt{a+bx^2}} dx$

3.683.1 Optimal result

Integrand size = 22, antiderivative size = 18

$$\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx=\frac{\log\left(1+\sqrt{a+bx^2}\right)}{b}$$

output
```
ln(1+(b*x^2+a)^(1/2))/b
```

3.683.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx=\frac{\log\left(b+b\sqrt{a+bx^2}\right)}{b}$$

input
```
Integrate[x/(a + b*x^2 + Sqrt[a + b*x^2]),x]
```

output
```
Log[b + b*Sqrt[a + b*x^2]]/b
```

3.683.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2586, 7267, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bx^2}+a+bx^2}dx$$
$$\Big\downarrow \text{2586}$$
$$\frac{1}{2}\int \frac{1}{bx^2+a+\sqrt{bx^2+a}}dx^2$$
$$\Big\downarrow \text{7267}$$
$$\frac{\int \frac{1}{\sqrt{bx^2+a}+1}d\sqrt{bx^2+a}}{b}$$
$$\Big\downarrow \text{16}$$
$$\frac{\log \left(\sqrt{a+bx^2}+1\right)}{b}$$

input
```
Int[x/(a + b*x^2 + Sqrt[a + b*x^2]),x]
```

output
```
Log[1 + Sqrt[a + b*x^2]]/b
```

3.683.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2586
```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Simp[1/n   Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a
+ b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*
d, 0] && IntegerQ[(m + 1)/n]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.683.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 894 vs. $2(16) = 32$.

Time = 0.09 (sec) , antiderivative size = 895, normalized size of antiderivative = 49.72

method	result
default	$-\frac{\sqrt{b\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}+2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}+\frac{\sqrt{-ab}\,\ln\left(\frac{\left(x-\frac{\sqrt{-ab}}{b}\right)b+\sqrt{-ab}}{\sqrt{b}}+\sqrt{b\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}+2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}\right)}{\sqrt{b}}}{2\left(\sqrt{-(a-1)b}+\sqrt{-ab}\right)\left(-\sqrt{-(a-1)b}+\sqrt{-ab}\right)}-\frac{\sqrt{b\left(x+\frac{\sqrt{-ab}}{b}\right)^{2}}}{}$

input
```
int(x/(a+b*x^2+(b*x^2+a)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/((-(a-1)*b)^(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*((b*
(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2)+(-a*b)^(
1/2)*ln(((x-1/b*(-a*b)^(1/2))*b+(-a*b)^(1/2))/b^(1/2)+(b*(x-1/b*(-a*b)^(1/
2))^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2))-1/2/((-(a-1)*b)
^(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*((b*(x+1/b*(-a*b)^(1
/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2)-(-a*b)^(1/2)*ln(((x+1/b*
(-a*b)^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+(b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(
1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2))+1/2/((-(a-1)*b)^(1/2)+(-a*b)^(1
/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*((b*(x+(-(a-1)*b)^(1/2)/b)^2-2*(-(a-
1)*b)^(1/2)*(x+(-(a-1)*b)^(1/2)/b)+1)^(1/2)-(-(a-1)*b)^(1/2)*ln(((x+(-(a-1
)*b)^(1/2)/b)*b-(-(a-1)*b)^(1/2))/b^(1/2)+(b*(x+(-(a-1)*b)^(1/2)/b)^2-2*(-
(a-1)*b)^(1/2)*(x+(-(a-1)*b)^(1/2)/b)+1)^(1/2))/b^(1/2)-arctanh(1/2*(2-2*(
-(a-1)*b)^(1/2)*(x+(-(a-1)*b)^(1/2)/b))/(b*(x+(-(a-1)*b)^(1/2)/b)^2-2*(-(a
-1)*b)^(1/2)*(x+(-(a-1)*b)^(1/2)/b)+1)^(1/2)))+1/2/((-(a-1)*b)^(1/2)+(-a*b
)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*((b*(x-(-(a-1)*b)^(1/2)/b)^2+2*(
-(a-1)*b)^(1/2)*(x-(-(a-1)*b)^(1/2)/b)+1)^(1/2)+(-(a-1)*b)^(1/2)*ln(((x-(-
(a-1)*b)^(1/2)/b)*b+(-(a-1)*b)^(1/2))/b^(1/2)+(b*(x-(-(a-1)*b)^(1/2)/b)^2+
2*(-(a-1)*b)^(1/2)*(x-(-(a-1)*b)^(1/2)/b)+1)^(1/2))/b^(1/2)-arctanh(1/2*(2
+2*(-(a-1)*b)^(1/2)*(x-(-(a-1)*b)^(1/2)/b))/(b*(x-(-(a-1)*b)^(1/2)/b)^2+2*
(-(a-1)*b)^(1/2)*(x-(-(a-1)*b)^(1/2)/b)+1)^(1/2)))+1/2*a/b*ln(b*x^2+a-1...
```

3.683.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(16) = 32$.

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.72

$$\int \frac{x}{a + bx^2 + \sqrt{a + bx^2}}\,dx = \frac{2\,\log\left(bx^2 + a - 1\right) + \log\left(\frac{bx^2 + a + 2\sqrt{bx^2 + a} + 1}{x^2}\right) - \log\left(\frac{bx^2 + a - 2\sqrt{bx^2 + a} + 1}{x^2}\right)}{4\,b}$$

input
```
integrate(x/(a+b*x^2+(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

output
```
1/4*(2*log(b*x^2 + a - 1) + log((b*x^2 + a + 2*sqrt(b*x^2 + a) + 1)/x^2) -
 log((b*x^2 + a - 2*sqrt(b*x^2 + a) + 1)/x^2))/b
```

3.683.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(14) = 28$.

Time = 0.74 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.94

$$\int \frac{x}{a + bx^2 + \sqrt{a + bx^2}}\,dx = \begin{cases} \frac{2\left(-\frac{\log\left(2\sqrt{a+bx^2}\right)}{4} + \frac{\log\left(2\sqrt{a+bx^2}+2\right)}{4} + \frac{\log\left(2a+2bx^2+2\sqrt{a+bx^2}\right)}{4}\right)}{b} & \text{for } b \neq 0 \\ \frac{x^2}{2\sqrt{a}+2a} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(a+b*x**2+(b*x**2+a)**(1/2)),x)
```

output
```
Piecewise((2*(-log(2*sqrt(a + b*x**2))/4 + log(2*sqrt(a + b*x**2) + 2)/4 +
 log(2*a + 2*b*x**2 + 2*sqrt(a + b*x**2))/4)/b, Ne(b, 0)), (x**2/(2*sqrt(a
) + 2*a), True))
```

3.683.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx=\frac{\log\left(\sqrt{bx^2+a}+1\right)}{b}$$

input
```
integrate(x/(a+b*x^2+(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

output
```
log(sqrt(b*x^2 + a) + 1)/b
```

3.683.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx=\frac{\log\left(\sqrt{bx^2+a}+1\right)}{b}$$

input
```
integrate(x/(a+b*x^2+(b*x^2+a)^(1/2)),x, algorithm="giac")
```

output
```
log(sqrt(b*x^2 + a) + 1)/b
```

3.683.9 Mupad [B] (verification not implemented)

Time = 19.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx=\frac{\mathrm{atanh}\left(\sqrt{b\,x^2+a}\right)+\frac{\ln\left(b\,x^2+a-1\right)}{2}}{b}$$

input
```
int(x/(a + b*x^2 + (a + b*x^2)^(1/2)),x)
```

output
```
(atanh((a + b*x^2)^(1/2)) + log(a + b*x^2 - 1)/2)/b
```

3.684 $\int \frac{x}{x^2-\sqrt[3]{x^2}}\,dx$

3.684.1 Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \frac{x}{x^2-\sqrt[3]{x^2}}\,dx = \frac{3}{4}\log\left(1-\left(x^2\right)^{2/3}\right)$$

output
```
3/4*ln(1-(x^2)^(2/3))
```

3.684.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{x}{x^2-\sqrt[3]{x^2}}\,dx = \frac{3}{4}\log\left(-1+\sqrt[3]{x^2}\right)+\frac{3}{4}\log\left(1+\sqrt[3]{x^2}\right)$$

input
```
Integrate[x/(x^2 - (x^2)^(1/3)),x]
```

output
```
(3*Log[-1 + (x^2)^(1/3)])/4 + (3*Log[1 + (x^2)^(1/3)])/4
```

3.684.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {7266, 2027, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^2-\sqrt[3]{x^2}}dx$$

$$\Big\downarrow \text{7266}$$

$$\frac{1}{2}\int \frac{1}{x^2-\sqrt[3]{x^2}}dx^2$$

$$\Big\downarrow \text{2027}$$

$$\frac{1}{2}\int \frac{1}{\sqrt[3]{x^2}\left(\left(x^2\right)^{2/3}-1\right)}dx^2$$

$$\Big\downarrow \text{792}$$

$$\frac{3}{4}\log\left(1-\left(x^2\right)^{2/3}\right)$$

input
```
Int[x/(x^2 - (x^2)^(1/3)),x]
```

output
```
(3*Log[1 - (x^2)^(2/3)])/4
```

3.684.3.1 Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.684.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

method	result
meijerg	$\frac{3\ln\left(1-\frac{x^{2}}{\left(x^{2}\right)^{\frac{1}{3}}}\right)}{4}$
derivativedivides	$\frac{\ln\left(x^{2}-1\right)}{4}+\frac{\ln\left(x^{2}+1\right)}{4}-\frac{\ln\left(\left(x^{2}\right)^{\frac{2}{3}}+\left(x^{2}\right)^{\frac{1}{3}}+1\right)}{4}+\frac{\ln\left(\left(x^{2}\right)^{\frac{1}{3}}-1\right)}{2}-\frac{\ln\left(\left(x^{2}\right)^{\frac{2}{3}}-\left(x^{2}\right)^{\frac{1}{3}}+1\right)}{4}+\frac{\ln\left(\left(x^{2}\right)^{\frac{1}{3}}+1\right)}{2}$
default	$\frac{\ln\left(x^{2}-1\right)}{4}+\frac{\ln\left(x^{2}+1\right)}{4}-\frac{\ln\left(\left(x^{2}\right)^{\frac{2}{3}}+\left(x^{2}\right)^{\frac{1}{3}}+1\right)}{4}+\frac{\ln\left(\left(x^{2}\right)^{\frac{1}{3}}-1\right)}{2}-\frac{\ln\left(\left(x^{2}\right)^{\frac{2}{3}}-\left(x^{2}\right)^{\frac{1}{3}}+1\right)}{4}+\frac{\ln\left(\left(x^{2}\right)^{\frac{1}{3}}+1\right)}{2}$
trager	$-\frac{\ln\left(-\frac{x^{8}+3\left(x^{2}\right)^{\frac{1}{3}}x^{6}+6\left(x^{2}\right)^{\frac{2}{3}}x^{4}+7x^{4}+6x^{2}\left(x^{2}\right)^{\frac{1}{3}}+3\left(x^{2}\right)^{\frac{2}{3}}+1}{\left(x-1\right)^{3}\left(x+1\right)^{3}\left(x^{2}+1\right)^{3}}\right)}{4}$

input
```
int(x/(x^2-(x^2)^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
3/4*ln(1-x^2/(x^2)^(1/3))
```

3.684.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \frac{x}{x^2-\sqrt[3]{x^2}}\,dx = -3\,\log\left(\frac{\left(x^{2}\right)^{\frac{1}{3}}}{x}\right)+\frac{3}{4}\,\log\left(-\frac{x^{2}-\left(x^{2}\right)^{\frac{1}{3}}}{x^{2}}\right)$$

input
```
integrate(x/(x^2-(x^2)^(1/3)),x, algorithm="fricas")
```

output
```
-3*log((x^2)^(1/3)/x) + 3/4*log(-(x^2 - (x^2)^(1/3))/x^2)
```

3.684.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{x}{x^2 - \sqrt[3]{x^2}}\, dx = -\frac{\log(x)}{2} + \frac{3\log\left(-x^2 + \sqrt[3]{x^2}\right)}{4}$$

input
```
integrate(x/(x**2-(x**2)**(1/3)),x)
```

output
```
-log(x)/2 + 3*log(-x**2 + (x**2)**(1/3))/4
```

3.684.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int \frac{x}{x^2 - \sqrt[3]{x^2}}\, dx = \frac{3}{4}\log\left(\left(x^2\right)^{\frac{1}{3}} + 1\right) + \frac{3}{4}\log\left(\left(x^2\right)^{\frac{1}{3}} - 1\right)$$

input
```
integrate(x/(x^2-(x^2)^(1/3)),x, algorithm="maxima")
```

output
```
3/4*log((x^2)^(1/3) + 1) + 3/4*log((x^2)^(1/3) - 1)
```

3.684.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{x^2 - \sqrt[3]{x^2}}\, dx = \frac{3}{4}\log\left(\left|(x\mathrm{sgn}(x))^{\frac{1}{3}}\, x\mathrm{sgn}(x) - 1\right|\right)$$

input
```
integrate(x/(x^2-(x^2)^(1/3)),x, algorithm="giac")
```

output
```
3/4*log(abs((x*sgn(x))^(1/3)*x*sgn(x) - 1))
```

3.684.9 Mupad [B] (verification not implemented)

Time = 18.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{x}{x^2 - \sqrt[3]{x^2}}\,dx = \frac{3\ln\left((x^2)^{2/3} - 1\right)}{4}$$

input
```
int(-x/((x^2)^(1/3) - x^2),x)
```

output
```
(3*log((x^2)^(2/3) - 1))/4
```

3.685 $\int x(1+x^2)^3 \sqrt{2+2x^2+x^4}\,dx$

3.685.1 Optimal result

Integrand size = 23, antiderivative size = 44

$$\int x(1+x^2)^3 \sqrt{2+2x^2+x^4}\,dx = -\frac{1}{15}(2+2x^2+x^4)^{3/2} + \frac{1}{10}(1+x^2)^2 (2+2x^2+x^4)^{3/2}$$

output
```
-1/15*(x^4+2*x^2+2)^(3/2)+1/10*(x^2+1)^2*(x^4+2*x^2+2)^(3/2)
```

3.685.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int x(1+x^2)^3 \sqrt{2+2x^2+x^4}\,dx = \frac{1}{30}\sqrt{2+2x^2+x^4}(2+14x^2+19x^4+12x^6+3x^8)$$

input
```
Integrate[x*(1 + x^2)^3*Sqrt[2 + 2*x^2 + x^4],x]
```

output
```
(Sqrt[2 + 2*x^2 + x^4]*(2 + 14*x^2 + 19*x^4 + 12*x^6 + 3*x^8))/30
```

3.685.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1576, 1116, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(x^2+1)^3 \sqrt{x^4+2x^2+2}\,dx$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int (x^2+1)^3 \sqrt{x^4+2x^2+2}dx^2$$

$$\Big\downarrow \text{1116}$$

$$\frac{1}{2}\left(\frac{1}{5}(x^2+1)^2 (x^4+2x^2+2)^{3/2}-\frac{2}{5}\int (x^2+1) \sqrt{x^4+2x^2+2}dx^2\right)$$

$$\Big\downarrow \text{1104}$$

$$\frac{1}{2}\left(\frac{1}{5}(x^2+1)^2 (x^4+2x^2+2)^{3/2}-\frac{2}{15}(x^4+2x^2+2)^{3/2}\right)$$

input
```
Int[x*(1 + x^2)^3*Sqrt[2 + 2*x^2 + x^4],x]
```

output
```
((-2*(2 + 2*x^2 + x^4)^(3/2))/15 + ((1 + x^2)^2*(2 + 2*x^2 + x^4)^(3/2))/5
)/2
```

3.685.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1116
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[2*d*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(b*(m + 2*p
 + 1))), x] + Simp[d^2*(m - 1)*((b^2 - 4*a*c)/(b^2*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[m, 1] && NeQ[m + 2*p
+ 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) || OddQ[m])
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

3.685.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{(x^4+2x^2+2)^{\frac{3}{2}}(3x^4+6x^2+1)}{30}$	27
elliptic	$\frac{(x^4+2x^2+2)^{\frac{3}{2}}(3x^4+6x^2+1)}{30}$	27
pseudoelliptic	$\frac{(x^4+2x^2+2)^{\frac{3}{2}}(3x^4+6x^2+1)}{30}$	27
trager	$\left(\frac{1}{10}x^8+\frac{2}{5}x^6+\frac{19}{30}x^4+\frac{7}{15}x^2+\frac{1}{15}\right)\sqrt{x^4+2x^2+2}$	36
risch	$\frac{(3x^8+12x^6+19x^4+14x^2+2)\sqrt{x^4+2x^2+2}}{30}$	37
default	$\frac{x^4(x^4+2x^2+2)^{\frac{3}{2}}}{10}+\frac{x^2(x^4+2x^2+2)^{\frac{3}{2}}}{5}+\frac{(x^4+2x^2+2)^{\frac{3}{2}}}{30}$	50

input
```
int(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/30*(x^4+2*x^2+2)^(3/2)*(3*x^4+6*x^2+1)
```

3.685.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int x(1+x^2)^3\sqrt{2+2x^2+x^4}\,dx = \frac{1}{30}\left(3\,x^8+12\,x^6+19\,x^4+14\,x^2+2\right)\sqrt{x^4+2\,x^2+2}$$

input
```
integrate(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2),x, algorithm="fricas")
```

output
```
1/30*(3*x^8 + 12*x^6 + 19*x^4 + 14*x^2 + 2)*sqrt(x^4 + 2*x^2 + 2)
```

3.685.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(36) = 72.

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.14

$$\int x(1+x^2)^3\sqrt{2+2x^2+x^4}\,dx = \frac{x^8\sqrt{x^4+2x^2+2}}{10}+\frac{2x^6\sqrt{x^4+2x^2+2}}{5}+\frac{19x^4\sqrt{x^4+2x^2+2}}{30}+\frac{7x^2\sqrt{x^4+2x^2+2}}{15}+\frac{\sqrt{x^4+2x^2+2}}{15}$$

input
```
integrate(x*(x**2+1)**3*(x**4+2*x**2+2)**(1/2),x)
```

output
```
x**8*sqrt(x**4 + 2*x**2 + 2)/10 + 2*x**6*sqrt(x**4 + 2*x**2 + 2)/5 + 19*x*
*4*sqrt(x**4 + 2*x**2 + 2)/30 + 7*x**2*sqrt(x**4 + 2*x**2 + 2)/15 + sqrt(x
**4 + 2*x**2 + 2)/15
```

3.685.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int x(1+x^2)^3\sqrt{2+2x^2+x^4}\,dx = \frac{1}{10}\left(x^4+2\,x^2+2\right)^{\frac{3}{2}}x^4+\frac{1}{5}\left(x^4+2\,x^2+2\right)^{\frac{3}{2}}x^2+\frac{1}{30}\left(x^4+2\,x^2+2\right)^{\frac{3}{2}}$$

input
```
integrate(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2),x, algorithm="maxima")
```

output
```
1/10*(x^4 + 2*x^2 + 2)^(3/2)*x^4 + 1/5*(x^4 + 2*x^2 + 2)^(3/2)*x^2 + 1/30*
(x^4 + 2*x^2 + 2)^(3/2)
```

3.685.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

$$\int x(1+x^2)^3\sqrt{2+2x^2+x^4}\,dx = \frac{1}{10}\left(x^4+2\,x^2+2\right)^{\frac{5}{2}} - \frac{1}{6}\left(x^4+2\,x^2+2\right)^{\frac{3}{2}}$$

input
```
integrate(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2),x, algorithm="giac")
```

output
```
1/10*(x^4 + 2*x^2 + 2)^(5/2) - 1/6*(x^4 + 2*x^2 + 2)^(3/2)
```

3.685.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

$$\int x(1+x^2)^3\sqrt{2+2x^2+x^4}\,dx = \frac{\left(x^4+2\,x^2+2\right)^{3/2}\left(3\,x^4+6\,x^2+1\right)}{30}$$

input
```
int(x*(x^2 + 1)^3*(2*x^2 + x^4 + 2)^(1/2),x)
```

output
```
((2*x^2 + x^4 + 2)^(3/2)*(6*x^2 + 3*x^4 + 1))/30
```

3.686 $\int x^5\sqrt{1-x^3}\left(1+x^9\right)^2\,dx$

3.686.1 Optimal result

Integrand size = 22, antiderivative size = 121

$$\begin{aligned}\int x^5\sqrt{1-x^3}\left(1+x^9\right)^2\,dx &= -\frac{8}{9}\left(1-x^3\right)^{3/2}+\frac{32}{15}\left(1-x^3\right)^{5/2}-\frac{22}{7}\left(1-x^3\right)^{7/2}\\&\quad+\frac{86}{27}\left(1-x^3\right)^{9/2}-\frac{74}{33}\left(1-x^3\right)^{11/2}+\frac{14}{13}\left(1-x^3\right)^{13/2}-\frac{14}{45}\left(1-x^3\right)^{15/2}+\frac{2}{51}\left(1-x^3\right)^{17/2}\end{aligned}$$

output
```
-8/9*(-x^3+1)^(3/2)+32/15*(-x^3+1)^(5/2)-22/7*(-x^3+1)^(7/2)+86/27*(-x^3+1
)^(9/2)-74/33*(-x^3+1)^(11/2)+14/13*(-x^3+1)^(13/2)-14/45*(-x^3+1)^(15/2)+
2/51*(-x^3+1)^(17/2)
```

3.686.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.47

$$\int x^5\sqrt{1-x^3}\left(1+x^9\right)^2\,dx = \frac{2\sqrt{1-x^3}(-173014-86507x^3+126561x^6-22160x^9-19390x^{12}+135702x^{15}-3234x^{18}-3003x^{21}+4}{2297295}$$

input
```
Integrate[x^5*Sqrt[1 - x^3]*(1 + x^9)^2,x]
```

output
```
(2*Sqrt[1 - x^3]*(-173014 - 86507*x^3 + 126561*x^6 - 22160*x^9 - 19390*x^1
2 + 135702*x^15 - 3234*x^18 - 3003*x^21 + 45045*x^24))/2297295
```

3.686.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2361, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{1-x^3}\left(x^9+1\right)^2\,dx$$

$$\Big\downarrow \text{2361}$$

$$\frac{1}{3}\int x^3\sqrt{1-x^3}\left(x^9+1\right)^2\,dx^3$$

$$\Big\downarrow \text{2123}$$

$$\frac{1}{3}\int \left(-\left(1-x^3\right)^{15/2}+7\left(1-x^3\right)^{13/2}-21\left(1-x^3\right)^{11/2}+37\left(1-x^3\right)^{9/2}-43\left(1-x^3\right)^{7/2}+33\left(1-x^3\right)^{5/2}-16\left(1-\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{2}{17}\left(1-x^3\right)^{17/2}-\frac{14}{15}\left(1-x^3\right)^{15/2}+\frac{42}{13}\left(1-x^3\right)^{13/2}-\frac{74}{11}\left(1-x^3\right)^{11/2}+\frac{86}{9}\left(1-x^3\right)^{9/2}-\frac{66}{7}\left(1-x^3\right)^{7/2}+\frac{32}{5}\right.$$

input
```
Int[x^5*Sqrt[1 - x^3]*(1 + x^9)^2,x]
```

output
```
((-8*(1 - x^3)^(3/2))/3 + (32*(1 - x^3)^(5/2))/5 - (66*(1 - x^3)^(7/2))/7
+ (86*(1 - x^3)^(9/2))/9 - (74*(1 - x^3)^(11/2))/11 + (42*(1 - x^3)^(13/2)
)/13 - (14*(1 - x^3)^(15/2))/15 + (2*(1 - x^3)^(17/2))/17)/3
```

3.686.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2361
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Simp[1/n
  Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[S
implify[(m + 1)/n]]
```

3.686.4 Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.40

method	result
pseudoelliptic	$-\frac{2\left(-x^{3}+1\right)^{\frac{3}{2}}\left(45045x^{21}+42042x^{18}+38808x^{15}+174510x^{12}+155120x^{9}+132960x^{6}+259521x^{3}+173014\right)}{2297295}$
trager	$\left(\frac{2}{51}x^{24}-\frac{2}{765}x^{21}-\frac{28}{9945}x^{18}+\frac{1436}{12155}x^{15}-\frac{1108}{65637}x^{12}-\frac{8864}{459459}x^{9}+\frac{84374}{765765}x^{6}-\frac{173014}{2297295}x^{3}-\frac{346028}{2297295}\right.$
gosper	$\frac{2\sqrt{-x^{3}+1}\left(45045x^{21}+42042x^{18}+38808x^{15}+174510x^{12}+155120x^{9}+132960x^{6}+259521x^{3}+173014\right)(x-1)\left(x^{2}+x+1\right)}{2297295}$
risch	$-\frac{2\left(45045x^{24}-3003x^{21}-3234x^{18}+135702x^{15}-19390x^{12}-22160x^{9}+126561x^{6}-86507x^{3}-173014\right)\left(x^{3}-1\right)}{2297295\sqrt{-x^{3}+1}}$
default	$\frac{84374x^{6}\sqrt{-x^{3}+1}}{765765}-\frac{173014x^{3}\sqrt{-x^{3}+1}}{2297295}-\frac{346028\sqrt{-x^{3}+1}}{2297295}+\frac{2x^{24}\sqrt{-x^{3}+1}}{51}-\frac{2x^{21}\sqrt{-x^{3}+1}}{765}-\frac{28x^{18}\sqrt{-x^{3}+1}}{9945}+$
elliptic	$\frac{84374x^{6}\sqrt{-x^{3}+1}}{765765}-\frac{173014x^{3}\sqrt{-x^{3}+1}}{2297295}-\frac{346028\sqrt{-x^{3}+1}}{2297295}+\frac{2x^{24}\sqrt{-x^{3}+1}}{51}-\frac{2x^{21}\sqrt{-x^{3}+1}}{765}-\frac{28x^{18}\sqrt{-x^{3}+1}}{9945}+$
meijerg	$-\frac{-\frac{8192\sqrt{\pi}}{109395}+\frac{4\sqrt{\pi}\left(-x^{3}+1\right)^{\frac{3}{2}}\left(6435x^{21}+6006x^{18}+5544x^{15}+5040x^{12}+4480x^{9}+3840x^{6}+3072x^{3}+2048\right)}{109395}}{6\sqrt{\pi}}+\frac{\frac{512\sqrt{\pi}}{3465}-\frac{4\sqrt{\pi}\left(-x^{3}+1\right)^{\frac{3}{2}}}{}}{}$

input
```
int(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/2297295*(-x^3+1)^(3/2)*(45045*x^21+42042*x^18+38808*x^15+174510*x^12+15
5120*x^9+132960*x^6+259521*x^3+173014)
```

3.686.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.44

$$\int x^5\sqrt{1-x^3}\left(1+x^9\right)^2\,dx$$
$$=\frac{2}{2297295}\left(45045\,x^{24}-3003\,x^{21}-3234\,x^{18}+135702\,x^{15}-19390\,x^{12}-22160\,x^{9}+126561\,x^{6}-86507\,x^{3}\right.$$

input
```
integrate(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x, algorithm="fricas")
```

output
```
2/2297295*(45045*x^24 - 3003*x^21 - 3234*x^18 + 135702*x^15 - 19390*x^12 -
 22160*x^9 + 126561*x^6 - 86507*x^3 - 173014)*sqrt(-x^3 + 1)
```

3.686.6 Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.10

$$\int x^5\sqrt{1-x^3}\left(1+x^9\right)^2\,dx = \frac{2x^{24}\sqrt{1-x^3}}{51} - \frac{2x^{21}\sqrt{1-x^3}}{765} - \frac{28x^{18}\sqrt{1-x^3}}{9945} + \frac{1436x^{15}\sqrt{1-x^3}}{12155} - \frac{1108x^{12}\sqrt{1-x^3}}{65637} - \frac{8864x^9\sqrt{1-x^3}}{459459} + \frac{84374x^6\sqrt{1-x^3}}{765765} - \frac{173014x^3\sqrt{1-x^3}}{2297295} - \frac{346028\sqrt{1-x^3}}{2297295}$$

input
```
integrate(x**5*(x**9+1)**2*(-x**3+1)**(1/2),x)
```

output
```
2*x**24*sqrt(1 - x**3)/51 - 2*x**21*sqrt(1 - x**3)/765 - 28*x**18*sqrt(1 -
 x**3)/9945 + 1436*x**15*sqrt(1 - x**3)/12155 - 1108*x**12*sqrt(1 - x**3)/
65637 - 8864*x**9*sqrt(1 - x**3)/459459 + 84374*x**6*sqrt(1 - x**3)/765765
 - 173014*x**3*sqrt(1 - x**3)/2297295 - 346028*sqrt(1 - x**3)/2297295
```

3.686.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int x^5\sqrt{1-x^3}\left(1+x^9\right)^2\,dx = \frac{2}{51}\left(-x^3+1\right)^{\frac{17}{2}} - \frac{14}{45}\left(-x^3+1\right)^{\frac{15}{2}} + \frac{14}{13}\left(-x^3+1\right)^{\frac{13}{2}} - \frac{74}{33}\left(-x^3+1\right)^{\frac{11}{2}} + \frac{86}{27}\left(-x^3+1\right)^{\frac{9}{2}} - \frac{22}{7}\left(-x^3+1\right)^{\frac{7}{2}} + \frac{32}{15}\left(-x^3+1\right)^{\frac{5}{2}} - \frac{8}{9}\left(-x^3+1\right)^{\frac{3}{2}}$$

input
```
integrate(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x, algorithm="maxima")
```

output
```
2/51*(-x^3 + 1)^(17/2) - 14/45*(-x^3 + 1)^(15/2) + 14/13*(-x^3 + 1)^(13/2)
 - 74/33*(-x^3 + 1)^(11/2) + 86/27*(-x^3 + 1)^(9/2) - 22/7*(-x^3 + 1)^(7/2
) + 32/15*(-x^3 + 1)^(5/2) - 8/9*(-x^3 + 1)^(3/2)
```

3.686.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.14

$$\int x^5\sqrt{1-x^3}(1+x^9)^2\,dx = \frac{2}{51}(x^3-1)^8\sqrt{-x^3+1}+\frac{14}{45}(x^3-1)^7\sqrt{-x^3+1} + \frac{14}{13}(x^3-1)^6\sqrt{-x^3+1}+\frac{74}{33}(x^3-1)^5\sqrt{-x^3+1} + \frac{86}{27}(x^3-1)^4\sqrt{-x^3+1}+\frac{22}{7}(x^3-1)^3\sqrt{-x^3+1} + \frac{32}{15}(x^3-1)^2\sqrt{-x^3+1}-\frac{8}{9}(-x^3+1)^{\frac{3}{2}}$$

input
```
integrate(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x, algorithm="giac")
```

output
```
2/51*(x^3 - 1)^8*sqrt(-x^3 + 1) + 14/45*(x^3 - 1)^7*sqrt(-x^3 + 1) + 14/13
*(x^3 - 1)^6*sqrt(-x^3 + 1) + 74/33*(x^3 - 1)^5*sqrt(-x^3 + 1) + 86/27*(x^
3 - 1)^4*sqrt(-x^3 + 1) + 22/7*(x^3 - 1)^3*sqrt(-x^3 + 1) + 32/15*(x^3 - 1
)^2*sqrt(-x^3 + 1) - 8/9*(-x^3 + 1)^(3/2)
```

3.686.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int x^5\sqrt{1-x^3}(1+x^9)^2\,dx = \frac{84374\,x^6\sqrt{1-x^3}}{765765}-\frac{173014\,x^3\sqrt{1-x^3}}{2297295}-\frac{8864\,x^9\sqrt{1-x^3}}{459459} - \frac{1108\,x^{12}\sqrt{1-x^3}}{65637}+\frac{1436\,x^{15}\sqrt{1-x^3}}{12155}-\frac{28\,x^{18}\sqrt{1-x^3}}{9945} - \frac{2\,x^{21}\sqrt{1-x^3}}{765}+\frac{2\,x^{24}\sqrt{1-x^3}}{51}-\frac{346028\sqrt{1-x^3}}{2297295}$$

input
```
int(x^5*(1 - x^3)^(1/2)*(x^9 + 1)^2,x)
```

output
```
(84374*x^6*(1 - x^3)^(1/2))/765765 - (173014*x^3*(1 - x^3)^(1/2))/2297295
- (8864*x^9*(1 - x^3)^(1/2))/459459 - (1108*x^12*(1 - x^3)^(1/2))/65637 +
(1436*x^15*(1 - x^3)^(1/2))/12155 - (28*x^18*(1 - x^3)^(1/2))/9945 - (2*x^
21*(1 - x^3)^(1/2))/765 + (2*x^24*(1 - x^3)^(1/2))/51 - (346028*(1 - x^3)^
(1/2))/2297295
```

3.687 $\int \left(\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right) dx$

3.687.1 Optimal result

Integrand size = 34, antiderivative size = 50

$$\int \left(\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right) dx = -\frac{1}{b\sqrt{a+bx^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

output
```
-arctanh((b*x^2+a)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)-1/b/(b*x^2+a)^(1/2)
```

3.687.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \left(\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right) dx = -\frac{1}{b\sqrt{a+bx^2}}+\frac{\arctan\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

input
```
Integrate[x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2]),x]
```

output
```
-(1/(b*Sqrt[a + b*x^2])) + ArcTan[Sqrt[a + b*x^2]/Sqrt[-a + b]]/Sqrt[-a +
b]
```

3.687.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{(x^2+1)\sqrt{a+bx^2}}+\frac{x}{(a+bx^2)^{3/2}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{1}{b\sqrt{a+bx^2}}$$

input
```
Int[x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2]),x]
```

output
```
-(1/(b*Sqrt[a + b*x^2])) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]/Sqrt[a - b
]
```

3.687.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.687.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{1}{b\sqrt{b\,x^2+a}}+\frac{\arctan\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$	42

input
```
int(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/b/(b*x^2+a)^(1/2)+1/(-a+b)^(1/2)*arctan((b*x^2+a)^(1/2)/(-a+b)^(1/2))
```

3.687.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(42) = 84$.

Time = 0.32 (sec) , antiderivative size = 268, normalized size of antiderivative = 5.36

$$\int \left(\frac{x}{(a+bx^2)^{3/2}} + \frac{x}{(1+x^2)\sqrt{a+bx^2}}\right) dx = \left[\frac{(b^2x^2+ab)\sqrt{a-b}\log\left(\frac{b^2x^4+2\,(4\,ab-3\,b^2)x^2-4\,(bx^2+2\,a-b)\sqrt{bx^2+a}\sqrt{a-b}+8\,a^2-8\,ab+b^2}{x^4+2\,x^2+1}\right) - \right. \right.$$
$$\left. -\frac{(b^2x^2+ab)\sqrt{-a+b}\arctan\left(-\frac{(bx^2+2\,a-b)\sqrt{bx^2+a}\sqrt{-a+b}}{2\,((ab-b^2)x^2+a^2-ab)}\right)+2\sqrt{bx^2+a}(a-b)}{2\,(a^2b-ab^2+(ab^2-b^3)x^2)}\right]$$

input
```
integrate(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
[1/4*((b^2*x^2 + a*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a*b - 3*b^2)*x^2 - 4
*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(x^4
 + 2*x^2 + 1)) - 4*sqrt(b*x^2 + a)*(a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)
*x^2), -1/2*((b^2*x^2 + a*b)*sqrt(-a + b)*arctan(-1/2*(b*x^2 + 2*a - b)*sq
rt(b*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 + a^2 - a*b)) + 2*sqrt(b*x^2 +
 a)*(a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)*x^2)]
```

3.687.6 Sympy [A] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.66

$$\int \left(\frac{x}{(a+bx^2)^{3/2}} + \frac{x}{(1+x^2)\sqrt{a+bx^2}}\right) dx = \begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases} + \begin{cases} \frac{\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} & \text{for } b \neq 0 \\ \begin{cases} \tilde{\infty}x^2 & \text{for } \sqrt{a} = 0 \\ \frac{\log\left(2\sqrt{a}x^2+2\sqrt{a}\right)}{2\sqrt{a}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(b*x**2+a)**(3/2)+x/(x**2+1)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True))
+ Piecewise((atan(sqrt(a + b*x**2)/sqrt(-a + b))/sqrt(-a + b), Ne(b, 0)),
(Piecewise((zoo*x**2, Eq(sqrt(a), 0)), (log(2*sqrt(a)*x**2 + 2*sqrt(a))/(2
*sqrt(a)), True)), True))
```

3.687.7 Maxima [F(-2)]

Exception generated.

$$\int \left(\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for m
ore detail
```

3.687.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \left(\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)\,dx = \frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{1}{\sqrt{bx^2+a}b}$$

input
```
integrate(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/(sqrt(b*x^2 + a)*b)
```

3.687.9 Mupad [B] (verification not implemented)

Time = 18.79 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \left(\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)\,dx=-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{1}{b\,\sqrt{b\,x^2+a}}$$

input
```
int(x/(a + b*x^2)^(3/2) + x/((x^2 + 1)*(a + b*x^2)^(1/2)),x)
```

output
```
- atanh((a + b*x^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2) - 1/(b*(a + b*x^2)^
(1/2))
```

3.688 $\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx$

3.688.1 Optimal result

Integrand size = 31, antiderivative size = 50

$$\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx = -\frac{1}{b\sqrt{a+bx^2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

output
```
-arctanh((b*x^2+a)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)-1/b/(b*x^2+a)^(1/2)
```

3.688.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx = -\frac{1}{b\sqrt{a+bx^2}} + \frac{\arctan\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

input
```
Integrate[(x*(1 + a + x^2 + b*x^2))/((1 + x^2)*(a + b*x^2)^(3/2)),x]
```

output
```
-(1/(b*Sqrt[a + b*x^2])) + ArcTan[Sqrt[a + b*x^2]/Sqrt[-a + b]]/Sqrt[-a +
b]
```

3.688.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {6, 435, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx^2+x^2+1)}{(x^2+1)(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 6}$$

$$\int \frac{x(a+(b+1)x^2+1)}{(x^2+1)(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 435}$$

$$\frac{1}{2}\int \frac{(b+1)x^2+a+1}{(x^2+1)(bx^2+a)^{3/2}}dx^2$$

$$\Big\downarrow \text{ 87}$$

$$\frac{1}{2}\left(\int \frac{1}{(x^2+1)\sqrt{bx^2+a}}dx^2-\frac{2}{b\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(\frac{2\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}+1}d\sqrt{bx^2+a}}{b}-\frac{2}{b\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{2}\left(-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{2}{b\sqrt{a+bx^2}}\right)$$

input
```
Int[(x*(1 + a + x^2 + b*x^2))/((1 + x^2)*(a + b*x^2)^(3/2)),x]
```

output
```
(-2/(b*Sqrt[a + b*x^2]) - (2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]])/Sqrt[a
 - b])/2
```

3.688.3.1 Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[Fx, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

3.688.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$-\frac{1}{b\sqrt{b\,x^2+a}}+\frac{\arctan\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$	42
default	$-\frac{b+1}{b\sqrt{b\,x^2+a}}+(a-b)\left(\frac{1}{(a-b)\sqrt{b\,x^2+a}}+\frac{\arctan\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{-a+b}}\right)}{(a-b)\sqrt{-a+b}}\right)$	76

input
```
int(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/b/(b*x^2+a)^(1/2)+1/(-a+b)^(1/2)*arctan((b*x^2+a)^(1/2)/(-a+b)^(1/2))
```

3.688.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(42) = 84$.

Time = 0.31 (sec) , antiderivative size = 268, normalized size of antiderivative = 5.36

$$\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)\left(a+bx^2\right)^{3/2}}\,dx = \left[\frac{(b^2x^2+ab)\sqrt{a-b}\log\left(\frac{b^2x^4+2\,(4\,ab-3\,b^2)x^2-4\,(bx^2+2\,a-b)\sqrt{bx^2+a}\sqrt{a-b}+8\,a^2-8\,ab+b^2}{x^4+2\,x^2+1}\right)}{4\,(a^2b-ab^2+(ab^2-b^3)x^2)}\right.$$
$$\left. -\frac{(b^2x^2+ab)\sqrt{-a+b}\arctan\left(-\frac{(bx^2+2\,a-b)\sqrt{bx^2+a}\sqrt{-a+b}}{2\,((ab-b^2)x^2+a^2-ab)}\right)+2\,\sqrt{bx^2+a}(a-b)}{2\,(a^2b-ab^2+(ab^2-b^3)x^2)}\right]$$

input
```
integrate(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((b^2*x^2 + a*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a*b - 3*b^2)*x^2 - 4
*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(x^4
 + 2*x^2 + 1)) - 4*sqrt(b*x^2 + a)*(a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)
*x^2), -1/2*((b^2*x^2 + a*b)*sqrt(-a + b)*arctan(-1/2*(b*x^2 + 2*a - b)*sq
rt(b*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 + a^2 - a*b)) + 2*sqrt(b*x^2 +
 a)*(a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)*x^2)]
```

3.688.6 Sympy [F]

$$\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)\left(a+bx^2\right)^{3/2}}\,dx = \int \frac{x(a+bx^2+x^2+1)}{(a+bx^2)^{\frac{3}{2}}\,(x^2+1)}\,dx$$

input
```
integrate(x*(b*x**2+x**2+a+1)/(x**2+1)/(b*x**2+a)**(3/2),x)
```

output
```
Integral(x*(a + b*x**2 + x**2 + 1)/((a + b*x**2)**(3/2)*(x**2 + 1)), x)
```

3.688.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for m
ore detail
```

3.688.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} - \frac{1}{\sqrt{bx^2+a}b}$$

input
```
integrate(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/(sqrt(b*x^2 + a)*b)
```

3.688.9 Mupad [B] (verification not implemented)

Time = 19.35 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.92

$$\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx = \frac{1}{\sqrt{b\,x^2+a}\,(a-b)} - \frac{a}{\sqrt{b\,x^2+a}\,(a\,b-b^2)} - \frac{a\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}} + \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}$$

input
```
int((x*(a + b*x^2 + x^2 + 1))/((x^2 + 1)*(a + b*x^2)^(3/2)),x)
```

output
```
1/((a + b*x^2)^(1/2)*(a - b)) - a/((a + b*x^2)^(1/2)*(a*b - b^2)) - (a*ata
nh((a + b*x^2)^(1/2)/(a - b)^(1/2)))/(a - b)^(3/2) + (b*atanh((a + b*x^2)^
(1/2)/(a - b)^(1/2)))/(a - b)^(3/2)
```

3.688. $\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx$

3.689 $\int \left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right) dx$

3.689.1 Optimal result

Integrand size = 47, antiderivative size = 68

$$\int \left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right) dx = -\frac{1}{3b(a+bx^2)^{3/2}}-\frac{1}{b\sqrt{a+bx^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

output
```
-1/3/b/(b*x^2+a)^(3/2)-arctanh((b*x^2+a)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)-1/
b/(b*x^2+a)^(1/2)
```

3.689.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right) dx = \frac{-1-3a-3bx^2}{3b(a+bx^2)^{3/2}}+\frac{\arctan\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

input
```
Integrate[x/(a + b*x^2)^(5/2) + x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a
+ b*x^2]),x]
```

output
```
(-1 - 3*a - 3*b*x^2)/(3*b*(a + b*x^2)^(3/2)) + ArcTan[Sqrt[a + b*x^2]/Sqrt
[-a + b]]/Sqrt[-a + b]
```

3.689.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{(x^2+1)\sqrt{a+bx^2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(a+bx^2)^{5/2}}\right)dx$$
$$\Big\downarrow \ 2009$$
$$-\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{1}{b\sqrt{a+bx^2}}-\frac{1}{3b(a+bx^2)^{3/2}}$$

input
```
Int[x/(a + b*x^2)^(5/2) + x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^
2]),x]
```

output
```
-1/3*1/(b*(a + b*x^2)^(3/2)) - 1/(b*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*
x^2]/Sqrt[a - b]]/Sqrt[a - b]
```

3.689.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.689.4 Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{1}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{1}{b\sqrt{b\,x^{2}+a}}+\frac{\arctan\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$	56

input
```
int(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x,method
=_RETURNVERBOSE)
```

output
```
-1/3/b/(b*x^2+a)^(3/2)-1/b/(b*x^2+a)^(1/2)+1/(-a+b)^(1/2)*arctan((b*x^2+a)
^(1/2)/(-a+b)^(1/2))
```

3.689.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(56) = 112.

Time = 0.33 (sec) , antiderivative size = 382, normalized size of antiderivative = 5.62

$$\int\left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx=\left[\frac{3\,(b^3x^4+2\,ab^2x^2+a^2b)\sqrt{a-b}\log\left(\frac{b^2x^4+2\,(4\,ab-3\,b^2)x^2-4\,(bx^2+2\,a-b)\sqrt{bx^2+a}\sqrt{a-b}+8}{x^4+2\,x^2+1}\right.}{12\,((ab^3-b^4)x^4+a^3b-a^2b^2+}\right.$$

input
```
integrate(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x,
 algorithm="fricas")
```

output
```
[1/12*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a
*b - 3*b^2)*x^2 - 4*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2
- 8*a*b + b^2)/(x^4 + 2*x^2 + 1)) - 4*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a +
1)*b + a)*sqrt(b*x^2 + a))/((a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b
^2 - a*b^3)*x^2), -1/6*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(-a + b)*arc
tan(-1/2*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 +
 a^2 - a*b)) + 2*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a + 1)*b + a)*sqrt(b*x^2
+ a))/((a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b^2 - a*b^3)*x^2)]
```

3.689.6 Sympy [A] (verification not implemented)

Time = 2.92 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.93

$$\int \left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx = \begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases} + \begin{cases} \frac{\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} & \text{for } b\neq 0 \\ \begin{cases} \tilde{\infty}x^2 & \text{for } \sqrt{a}=0 \\ \frac{\log(2\sqrt{a}x^2+2\sqrt{a})}{2\sqrt{a}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases} + \begin{cases} -\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(b*x**2+a)**(5/2)+x/(b*x**2+a)**(3/2)+x/(x**2+1)/(b*x**2+a)**(
1/2),x)
```

output
```
Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True))
+ Piecewise((atan(sqrt(a + b*x**2)/sqrt(-a + b))/sqrt(-a + b), Ne(b, 0)),
(Piecewise((zoo*x**2, Eq(sqrt(a), 0)), (log(2*sqrt(a)*x**2 + 2*sqrt(a))/(2
*sqrt(a)), True)), True)) + Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2
*x**2*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(5/2)), True))
```

3.689.7 Maxima [F(-2)]

Exception generated.

$$\int \left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x,
 algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for m
ore detail
```

3.689.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

$$\int \left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx=\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{1}{\sqrt{bx^2+ab}}-\frac{1}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

input
```
integrate(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x,
 algorithm="giac")
```

output
```
arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/(sqrt(b*x^2 + a)*b)
- 1/3/((b*x^2 + a)^(3/2)*b)
```

3.689.9 Mupad [B] (verification not implemented)

Time = 18.38 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx=-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{1}{b\,\sqrt{b\,x^2+a}}-\frac{1}{3\,b\,(b\,x^2+a)^{3/2}}$$

input
```
int(x/(a + b*x^2)^(3/2) + x/(a + b*x^2)^(5/2) + x/((x^2 + 1)*(a + b*x^2)^(
1/2)),x)
```

output
```
- atanh((a + b*x^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2) - 1/(b*(a + b*x^2)^
(1/2)) - 1/(3*b*(a + b*x^2)^(3/2))
```

3.690 $\int \frac{x\left(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4\right)}{\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}\,dx$

3.690.1 Optimal result

Integrand size = 58, antiderivative size = 68

$$\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx = -\frac{1}{3b\left(a+bx^2\right)^{3/2}}-\frac{1}{b\sqrt{a+bx^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

output
```
-1/3/b/(b*x^2+a)^(3/2)-arctanh((b*x^2+a)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)-1/
b/(b*x^2+a)^(1/2)
```

3.690.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx = \frac{-1-3a-3bx^2}{3b\left(a+bx^2\right)^{3/2}}+\frac{\arctan\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

input
```
Integrate[(x*(1 + a + a^2 + x^2 + a*x^2 + b*x^2 + 2*a*b*x^2 + b*x^4 + b^2*
x^4))/((1 + x^2)*(a + b*x^2)^(5/2)),x]
```

output
```
(-1 - 3*a - 3*b*x^2)/(3*b*(a + b*x^2)^(3/2)) + ArcTan[Sqrt[a + b*x^2]/Sqrt
[-a + b]]/Sqrt[-a + b]
```

3.690.3 Rubi [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {6, 6, 6, 6, 7266, 1192, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a^2+2abx^2+ax^2+a+b^2x^4+bx^4+bx^2+x^2+1\right)}{(x^2+1)\,(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \; 6$$

$$\int \frac{x\left(a^2+2abx^2+(a+1)x^2+a+b^2x^4+bx^4+bx^2+1\right)}{(x^2+1)\,(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \; 6$$

$$\int \frac{x\left(a^2+2abx^2+x^2(a+b+1)+a+b^2x^4+bx^4+1\right)}{(x^2+1)\,(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \; 6$$

$$\int \frac{x\left(a^2+x^2(2ab+a+b+1)+a+b^2x^4+bx^4+1\right)}{(x^2+1)\,(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \; 6$$

$$\int \frac{x\left(a^2+x^2(2ab+a+b+1)+a+\left(b^2+b\right)x^4+1\right)}{(x^2+1)\,(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \; 7266$$

$$\frac{1}{2}\int \frac{b(b+1)x^4+(2ba+a+b+1)x^2+a^2+a+1}{(x^2+1)\,(bx^2+a)^{5/2}}dx^2$$

$$\Big\downarrow \; 1192$$

$$\frac{\int \frac{-b(b+1)x^8-b(-a+b+1)x^4+(a-b)b}{x^8(-x^4+a-b)}d\sqrt{bx^2+a}}{b^2}$$

$$\Big\downarrow \; 1584$$

$$\frac{\int \left(\frac{b^2}{x^4-a+b}+\frac{b}{x^4}+\frac{b}{x^8}\right)d\sqrt{bx^2+a}}{b^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{b^2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{b}{3x^6}-\frac{b}{x^2}}{b^2}$$

input
```
Int[(x*(1 + a + a^2 + x^2 + a*x^2 + b*x^2 + 2*a*b*x^2 + b*x^4 + b^2*x^4))/
((1 + x^2)*(a + b*x^2)^(5/2)),x]
```

output
```
(-1/3*b/x^6 - b/x^2 - (b^2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]])/Sqrt[a -
b])/b^2
```

3.690.3.1 Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[Fx, x]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.690. $\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)(a+bx^2)^{5/2}}\,dx$

3.690.4 Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\frac{\arctan\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{-a+b}}\right) b\left(b\,x^{2}+a\right)^{\frac{3}{2}}-\sqrt{-a+b}\,\left(b\,x^{2}+a+\frac{1}{3}\right)}{\sqrt{-a+b}\,\left(b\,x^{2}+a\right)^{\frac{3}{2}} b}$
default	$\left(b^{2}+b\right)\left(-\frac{x^{2}}{b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{2a}{3b^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}\right)-\frac{2ab-b^{2}+a+1}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\left(a^{2}-2ab+b^{2}\right)\left(\frac{1}{\left(a-b\right)^{2}\sqrt{b\,x^{2}+a}}+\frac{\arctan}{(a-}\right.$

input
```
int(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^
(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-a+b)^(1/2)*(arctan((b*x^2+a)^(1/2)/(-a+b)^(1/2))*b*(b*x^2+a)^(3/2)-(-a
+b)^(1/2)*(b*x^2+a+1/3))/(b*x^2+a)^(3/2)/b
```

3.690.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(56) = 112.

Time = 0.32 (sec) , antiderivative size = 382, normalized size of antiderivative = 5.62

$$\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)\,(a+bx^2)^{5/2}}\,dx = \left[\frac{3\,(b^3x^4+2\,ab^2x^2+a^2b)\sqrt{a-b}\log\left(\frac{b^2x^4+2\,(\cdots}{\cdots}\right.}{\cdots}\right.$$

$$-\frac{3\,(b^3x^4+2\,ab^2x^2+a^2b)\sqrt{-a+b}\arctan\left(-\frac{(bx^2+2\,a-b)\sqrt{bx^2+a}\sqrt{-a+b}}{2\,((ab-b^2)x^2+a^2-ab)}\right)+2\,(3\,(ab-b^2)x^2+3\,a^2-(3\,a+1)b\cdots}{6\,((ab^3-b^4)x^4+a^3b-a^2b^2+2\,(a^2b^2-ab^3)x^2)}$$

input
```
integrate(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x
^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a
*b - 3*b^2)*x^2 - 4*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2
- 8*a*b + b^2)/(x^4 + 2*x^2 + 1)) - 4*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a +
1)*b + a)*sqrt(b*x^2 + a))/((a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b
^2 - a*b^3)*x^2), -1/6*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(-a + b)*arc
tan(-1/2*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 +
 a^2 - a*b)) + 2*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a + 1)*b + a)*sqrt(b*x^2
+ a))/((a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b^2 - a*b^3)*x^2)]
```

3.690.6 Sympy [F]

$$\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx = \int \frac{x(a^2+2abx^2+ax^2+a+b^2x^4+bx^4+bx}{\left(a+bx^2\right)^{\frac{5}{2}}\left(x^2+1\right)}$$

input
```
integrate(x*(b**2*x**4+b*x**4+2*a*b*x**2+a*x**2+b*x**2+a**2+x**2+a+1)/(x**
2+1)/(b*x**2+a)**(5/2),x)
```

output
```
Integral(x*(a**2 + 2*a*b*x**2 + a*x**2 + a + b**2*x**4 + b*x**4 + b*x**2 +
 x**2 + 1)/((a + b*x**2)**(5/2)*(x**2 + 1)), x)
```

3.690.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x
^2+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for m
ore detail
```

3.690.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)\,(a+bx^2)^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} - \frac{3\,bx^2+3\,a+1}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

input
```
integrate(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x
^2+a)^(5/2),x, algorithm="giac")
```

output
```
arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/3*(3*b*x^2 + 3*a + 1
)/((b*x^2 + a)^(3/2)*b)
```

3.690.9 Mupad [B] (verification not implemented)

Time = 18.90 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)\,(a+bx^2)^{5/2}}\,dx = -\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} - \frac{b\,x^2+a+\frac{1}{3}}{b\,(b\,x^2+a)^{3/2}}$$

input
```
int((x*(a + a*x^2 + b*x^2 + b*x^4 + a^2 + x^2 + b^2*x^4 + 2*a*b*x^2 + 1))/
((x^2 + 1)*(a + b*x^2)^(5/2)),x)
```

output
```
- atanh((a + b*x^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2) - (a + b*x^2 + 1/3)
/(b*(a + b*x^2)^(3/2))
```

3.691 $\int \frac{1}{\sqrt{\sqrt{x}+x}}\,dx$

3.691.1 Optimal result

Integrand size = 11, antiderivative size = 34

$$\int \frac{1}{\sqrt{\sqrt{x}+x}}\,dx = 2\sqrt{\sqrt{x}+x} - 2\text{arctanh}\left(\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)$$

output
```
-2*arctanh(x^(1/2)/(x+x^(1/2))^(1/2))+2*(x+x^(1/2))^(1/2)
```

3.691.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{\sqrt{x}+x}}\,dx = 2\sqrt{\sqrt{x}+x} + \log\left(-1 - 2\sqrt{x} + 2\sqrt{\sqrt{x}+x}\right)$$

input
```
Integrate[1/Sqrt[Sqrt[x] + x],x]
```

output
```
2*Sqrt[Sqrt[x] + x] + Log[-1 - 2*Sqrt[x] + 2*Sqrt[Sqrt[x] + x]]
```

3.691.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1916, 1919, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x+\sqrt{x}}}dx$$
$$\Big\downarrow \text{1916}$$
$$2\sqrt{x+\sqrt{x}}-\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{x+\sqrt{x}}}dx$$
$$\Big\downarrow \text{1919}$$
$$2\sqrt{x+\sqrt{x}}-\int \frac{1}{\sqrt{x+\sqrt{x}}}d\sqrt{x}$$
$$\Big\downarrow \text{1091}$$
$$2\sqrt{x+\sqrt{x}}-2\int \frac{1}{1-x}d\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}$$
$$\Big\downarrow \text{219}$$
$$2\sqrt{x+\sqrt{x}}-2\text{arctanh}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

input
```
Int[1/Sqrt[Sqrt[x] + x],x]
```

output
```
2*Sqrt[Sqrt[x] + x] - 2*ArcTanh[Sqrt[x]/Sqrt[Sqrt[x] + x]]
```

3.691.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1916
```
Int[1/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[-2*(Sqrt
[a*x^j + b*x^n]/(b*(n - 2)*x^(n - 1))), x] - Simp[a*((2*n - j - 2)/(b*(n -
2)))   Int[1/(x^(n - j)*Sqrt[a*x^j + b*x^n]), x], x] /; FreeQ[{a, b}, x] &&
 LtQ[2*(n - 1), j, n]
```

rule 1919
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
 && EqQ[Simplify[m - n + 1], 0]
```

3.691.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$2\sqrt{x+\sqrt{x}}-\ln\left(\frac{1}{2}+\sqrt{x}+\sqrt{x+\sqrt{x}}\right)$	26
meijerg	$\frac{2\sqrt{\pi}\,x^{\frac{1}{4}}\sqrt{1+\sqrt{x}}-2\sqrt{\pi}\,\operatorname{arcsinh}\left(x^{\frac{1}{4}}\right)}{\sqrt{\pi}}$	30
default	$\frac{\sqrt{x+\sqrt{x}}\left(2\sqrt{x+\sqrt{x}}-\ln\left(\frac{1}{2}+\sqrt{x}+\sqrt{x+\sqrt{x}}\right)\right)}{\sqrt{\sqrt{x}\,(1+\sqrt{x})}}$	45

input
```
int(1/(x+x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(x+x^(1/2))^(1/2)-ln(1/2+x^(1/2)+(x+x^(1/2))^(1/2))
```

3.691.5 Fricas [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int \frac{1}{\sqrt{\sqrt{x}+x}}\,dx = 2\sqrt{x+\sqrt{x}} + \frac{1}{2}\log\left(4\sqrt{x+\sqrt{x}}(2\sqrt{x}+1) - 8x - 8\sqrt{x} - 1\right)$$

input
```
integrate(1/(x+x^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(x + sqrt(x)) + 1/2*log(4*sqrt(x + sqrt(x))*(2*sqrt(x) + 1) - 8*x -
8*sqrt(x) - 1)
```

3.691.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{\sqrt{x}+x}}\,dx = 2\sqrt{\sqrt{x}+x} - \log\left(2\sqrt{x} + 2\sqrt{\sqrt{x}+x} + 1\right)$$

input
```
integrate(1/(x+x**(1/2))**(1/2),x)
```

output
```
2*sqrt(sqrt(x) + x) - log(2*sqrt(x) + 2*sqrt(sqrt(x) + x) + 1)
```

3.691.7 Maxima [F]

$$\int \frac{1}{\sqrt{\sqrt{x}+x}}\,dx = \int \frac{1}{\sqrt{x+\sqrt{x}}}\,dx$$

input
```
integrate(1/(x+x^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(x + sqrt(x)), x)
```

3.691.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{\sqrt{x}+x}}\,dx = 2\sqrt{x+\sqrt{x}} + \log\left(-2\sqrt{x+\sqrt{x}} + 2\sqrt{x} + 1\right)$$

input
```
integrate(1/(x+x^(1/2))^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(x + sqrt(x)) + log(-2*sqrt(x + sqrt(x)) + 2*sqrt(x) + 1)
```

3.691.9 Mupad [B] (verification not implemented)

Time = 18.75 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int \frac{1}{\sqrt{\sqrt{x}+x}}\,dx = \frac{2\sqrt{x}\left(\sqrt{x}+1\right) + x^{1/4}\,\mathrm{asin}\left(x^{1/4}\,1\mathrm{i}\right)\,\sqrt{\sqrt{x}+1}\,2\mathrm{i}}{\sqrt{x+\sqrt{x}}}$$

input
```
int(1/(x + x^(1/2))^(1/2),x)
```

output
```
(2*x^(1/2)*(x^(1/2) + 1) + x^(1/4)*asin(x^(1/4)*1i)*(x^(1/2) + 1)^(1/2)*2i
)/(x + x^(1/2))^(1/2)
```

3.692 $\int \sqrt{\sqrt{x}+x}\,dx$

3.692.1 Optimal result

Integrand size = 11, antiderivative size = 74

$$\int \sqrt{\sqrt{x}+x}\,dx=-\frac{1}{4}\sqrt{\sqrt{x}+x}+\frac{1}{6}\sqrt{x}\sqrt{\sqrt{x}+x}+\frac{2}{3}x\sqrt{\sqrt{x}+x}+\frac{1}{4}\text{arctanh}\left(\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)$$

output
```
1/4*arctanh(x^(1/2)/(x+x^(1/2))^(1/2))-1/4*(x+x^(1/2))^(1/2)+2/3*x*(x+x^(1
/2))^(1/2)+1/6*x^(1/2)*(x+x^(1/2))^(1/2)
```

3.692.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

$$\int \sqrt{\sqrt{x}+x}\,dx=\frac{1}{12}\sqrt{\sqrt{x}+x}(-3+2\sqrt{x}+8x)+\frac{1}{4}\text{arctanh}\left(\frac{\sqrt{\sqrt{x}+x}}{\sqrt{x}}\right)$$

input
```
Integrate[Sqrt[Sqrt[x] + x],x]
```

output
```
(Sqrt[Sqrt[x] + x]*(-3 + 2*Sqrt[x] + 8*x))/12 + ArcTanh[Sqrt[Sqrt[x] + x]/
Sqrt[x]]/4
```

3.692.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {1910, 1924, 1134, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x+\sqrt{x}}\,dx$$

$$\Big\downarrow \text{1910}$$

$$\frac{1}{6}\int \frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}dx+\frac{2}{3}\sqrt{x+\sqrt{x}}x$$

$$\Big\downarrow \text{1924}$$

$$\frac{1}{3}\int \frac{x}{\sqrt{x+\sqrt{x}}}d\sqrt{x}+\frac{2}{3}\sqrt{x+\sqrt{x}}x$$

$$\Big\downarrow \text{1134}$$

$$\frac{1}{3}\left(\frac{1}{2}\sqrt{x}\sqrt{x+\sqrt{x}}-\frac{3}{4}\int \frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}d\sqrt{x}\right)+\frac{2}{3}\sqrt{x+\sqrt{x}}x$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{3}\left(\frac{1}{2}\sqrt{x}\sqrt{x+\sqrt{x}}-\frac{3}{4}\left(\sqrt{x+\sqrt{x}}-\frac{1}{2}\int \frac{1}{\sqrt{x+\sqrt{x}}}d\sqrt{x}\right)\right)+\frac{2}{3}\sqrt{x+\sqrt{x}}x$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{3}\left(\frac{1}{2}\sqrt{x}\sqrt{x+\sqrt{x}}-\frac{3}{4}\left(\sqrt{x+\sqrt{x}}-\int \frac{1}{1-x}d\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)\right)+\frac{2}{3}\sqrt{x+\sqrt{x}}x$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{1}{2}\sqrt{x}\sqrt{x+\sqrt{x}}-\frac{3}{4}\left(\sqrt{x+\sqrt{x}}-\text{arctanh}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)\right)\right)+\frac{2}{3}\sqrt{x+\sqrt{x}}x$$

input
```
Int[Sqrt[Sqrt[x] + x],x]
```

output
```
(2*x*Sqrt[Sqrt[x] + x])/3 + ((Sqrt[x]*Sqrt[Sqrt[x] + x])/2 - (3*(Sqrt[Sqrt
[x] + x] - ArcTanh[Sqrt[x]/Sqrt[Sqrt[x] + x]]))/4)/3
```

3.692.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

3.692. $\int \sqrt{\sqrt{x}+x}\,dx$

3.692.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.55

method	result	size
meijerg	$-\frac{\frac{\sqrt{\pi}\, x^{\frac{1}{4}} \left(-40 x-10 \sqrt{x}+15\right) \sqrt{1+\sqrt{x}}}{60}-\frac{\sqrt{\pi}\, \operatorname{arcsinh}\left(x^{\frac{1}{4}}\right)}{4}}{\sqrt{\pi}}$	41
derivativedivides	$\frac{2 \left(x+\sqrt{x}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2 \sqrt{x}\right) \sqrt{x+\sqrt{x}}}{4}+\frac{\ln\left(\frac{1}{2}+\sqrt{x}+\sqrt{x+\sqrt{x}}\right)}{8}$	42
default	$\frac{2 \left(x+\sqrt{x}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2 \sqrt{x}\right) \sqrt{x+\sqrt{x}}}{4}+\frac{\ln\left(\frac{1}{2}+\sqrt{x}+\sqrt{x+\sqrt{x}}\right)}{8}$	42

input
```
int((x+x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/Pi^(1/2)*(1/60*Pi^(1/2)*x^(1/4)*(-40*x-10*x^(1/2)+15)*(1+x^(1/2))^(1/2)
-1/4*Pi^(1/2)*arcsinh(x^(1/4)))
```

3.692.5 Fricas [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.66

$$\int \sqrt{\sqrt{x}+x}\, dx = \frac{1}{12} \left(8\, x + 2\, \sqrt{x} - 3\right) \sqrt{x + \sqrt{x}} + \frac{1}{16} \log\left(4\, \sqrt{x + \sqrt{x}} (2\, \sqrt{x} + 1) + 8\, x + 8\, \sqrt{x} + 1\right)$$

input
```
integrate((x+x^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/12*(8*x + 2*sqrt(x) - 3)*sqrt(x + sqrt(x)) + 1/16*log(4*sqrt(x + sqrt(x)
)*(2*sqrt(x) + 1) + 8*x + 8*sqrt(x) + 1)
```

3.692.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.65

$$\int \sqrt{\sqrt{x}+x}\,dx = 2\sqrt{\sqrt{x}+x}\left(\frac{\sqrt{x}}{12}+\frac{x}{3}-\frac{1}{8}\right)+\frac{\log\left(2\sqrt{x}+2\sqrt{\sqrt{x}+x}+1\right)}{8}$$

input
```
integrate((x+x**(1/2))**(1/2),x)
```

output
```
2*sqrt(sqrt(x) + x)*(sqrt(x)/12 + x/3 - 1/8) + log(2*sqrt(x) + 2*sqrt(sqrt
(x) + x) + 1)/8
```

3.692.7 Maxima [F]

$$\int \sqrt{\sqrt{x}+x}\,dx = \int \sqrt{x+\sqrt{x}}\,dx$$

input
```
integrate((x+x^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x + sqrt(x)), x)
```

3.692.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.58

$$\int \sqrt{\sqrt{x}+x}\,dx = \frac{1}{12}\left(2\sqrt{x}(4\sqrt{x}+1)-3\right)\sqrt{x+\sqrt{x}}-\frac{1}{8}\log\left(-2\sqrt{x+\sqrt{x}}+2\sqrt{x}+1\right)$$

input
```
integrate((x+x^(1/2))^(1/2),x, algorithm="giac")
```

output
```
1/12*(2*sqrt(x)*(4*sqrt(x) + 1) - 3)*sqrt(x + sqrt(x)) - 1/8*log(-2*sqrt(x
 + sqrt(x)) + 2*sqrt(x) + 1)
```

3.692.9 Mupad [B] (verification not implemented)

Time = 18.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.36

$$\int \sqrt{\sqrt{x}+x}\,dx = \frac{4\,x\,\sqrt{x+\sqrt{x}}\,{}_2F_1\left(-\frac{1}{2},\frac{5}{2};\ \frac{7}{2};\ -\sqrt{x}\right)}{5\,\sqrt{\sqrt{x}+1}}$$

input
```
int((x + x^(1/2))^(1/2),x)
```

output
```
(4*x*(x + x^(1/2))^(1/2)*hypergeom([-1/2, 5/2], 7/2, -x^(1/2)))/(5*(x^(1/2
) + 1)^(1/2))
```

3.693 $\int \sqrt{-x}\left(\sqrt{-x}+x\right)\,dx$

3.693.1 Optimal result

Integrand size = 17, antiderivative size = 19

$$\int \sqrt{-x}\left(\sqrt{-x}+x\right)\,dx=\frac{2}{5}(-x)^{5/2}-\frac{x^2}{2}$$

output
```
2/5*(-x)^(5/2)-1/2*x^2
```

3.693.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \sqrt{-x}\left(\sqrt{-x}+x\right)\,dx=\frac{1}{10}\left(-5+4\sqrt{-x}\right)x^2$$

input
```
Integrate[Sqrt[-x]*(Sqrt[-x] + x),x]
```

output
```
((-5 + 4*Sqrt[-x])*x^2)/10
```

3.693.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-x}(x+\sqrt{-x}) \, dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(-(-x)^{3/2}-x\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}(-x)^{5/2}-\frac{x^2}{2}$$

input
```
Int[Sqrt[-x]*(Sqrt[-x] + x),x]
```

output
```
(2*(-x)^(5/2))/5 - x^2/2
```

3.693.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

3.693.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$\frac{2(-x)^{\frac{5}{2}}}{5}-\frac{x^{2}}{2}$	14
default	$\frac{2(-x)^{\frac{5}{2}}}{5}-\frac{x^{2}}{2}$	14
trager	$-\frac{(x-1)(x+1)}{2}+\frac{2x^{2}\sqrt{-x}}{5}$	20

input
```
int((-x)^(1/2)*(x+(-x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/5*(-x)^(5/2)-1/2*x^2
```

3.693.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \sqrt{-x}\left(\sqrt{-x}+x\right)\,dx=\frac{2}{5}\,\sqrt{-x}x^{2}-\frac{1}{2}\,x^{2}$$

input
```
integrate((-x)^(1/2)*(x+(-x)^(1/2)),x, algorithm="fricas")
```

output
```
2/5*sqrt(-x)*x^2 - 1/2*x^2
```

3.693.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \sqrt{-x}\left(\sqrt{-x}+x\right)\,dx=\frac{2x^{2}\sqrt{-x}}{5}-\frac{x^{2}}{2}$$

input
```
integrate((-x)**(1/2)*(x+(-x)**(1/2)),x)
```

output
```
2*x**2*sqrt(-x)/5 - x**2/2
```

3.693.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \sqrt{-x}\left(\sqrt{-x}+x\right)\, dx = \frac{2}{5}\,(-x)^{\frac{5}{2}} - \frac{1}{2}\,x^2$$

input
```
integrate((-x)^(1/2)*(x+(-x)^(1/2)),x, algorithm="maxima")
```

output
```
2/5*(-x)^(5/2) - 1/2*x^2
```

3.693.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \sqrt{-x}\left(\sqrt{-x}+x\right)\, dx = \frac{2}{5}\,\sqrt{-x}x^2 - \frac{1}{2}\,x^2$$

input
```
integrate((-x)^(1/2)*(x+(-x)^(1/2)),x, algorithm="giac")
```

output
```
2/5*sqrt(-x)*x^2 - 1/2*x^2
```

3.693.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \sqrt{-x}\left(\sqrt{-x}+x\right)\, dx = \frac{2\,(-x)^{5/2}}{5} - \frac{x^2}{2}$$

input
```
int((-x)^(1/2)*(x + (-x)^(1/2)),x)
```

output
```
(2*(-x)^(5/2))/5 - x^2/2
```

3.694 $\int \frac{5+\sqrt[4]{x}}{-6+x}\,dx$

3.694.1 Optimal result

Integrand size = 13, antiderivative size = 54

$$\int \frac{5+\sqrt[4]{x}}{-6+x}\,dx = 4\sqrt[4]{x}-2\sqrt[4]{6}\arctan\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)-2\sqrt[4]{6}\text{arctanh}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)+5\log(6-x)$$

output
```
4*x^(1/4)-2*6^(1/4)*arctan(1/6*x^(1/4)*6^(3/4))-2*6^(1/4)*arctanh(1/6*x^(1
/4)*6^(3/4))+5*ln(6-x)
```

3.694.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{5+\sqrt[4]{x}}{-6+x}\,dx = 4\sqrt[4]{x}-2\sqrt[4]{6}\arctan\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)-2\sqrt[4]{6}\text{arctanh}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)+5\log(-6+x)$$

input
```
Integrate[(5 + x^(1/4))/(-6 + x),x]
```

output
```
4*x^(1/4) - 2*6^(1/4)*ArcTan[x^(1/4)/6^(1/4)] - 2*6^(1/4)*ArcTanh[x^(1/4)/
6^(1/4)] + 5*Log[-6 + x]
```

3.694.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {7267, 25, 2370, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{x}+5}{x-6} \, dx$$

$$\Big\downarrow \text{7267}$$

$$4\int -\frac{\left(\sqrt[4]{x}+5\right)x^{3/4}}{6-x}d\sqrt[4]{x}$$

$$\Big\downarrow \text{25}$$

$$-4\int \frac{\left(\sqrt[4]{x}+5\right)x^{3/4}}{6-x}d\sqrt[4]{x}$$

$$\Big\downarrow \text{2370}$$

$$-4\int \left(\frac{x}{6-x}+\frac{5x^{3/4}}{6-x}\right)d\sqrt[4]{x}$$

$$\Big\downarrow \text{2009}$$

$$4\left(-\frac{\sqrt[4]{3}\arctan\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)}{2^{3/4}}-\frac{\sqrt[4]{3}\text{arctanh}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)}{2^{3/4}}+\sqrt[4]{x}+\frac{5}{4}\log(6-x)\right)$$

input
```
Int[(5 + x^(1/4))/(-6 + x),x]
```

output
```
4*(x^(1/4) - (3^(1/4)*ArcTan[x^(1/4)/6^(1/4)])/2^(3/4) - (3^(1/4)*ArcTanh[
x^(1/4)/6^(1/4)])/2^(3/4) + (5*Log[6 - x])/4)
```

3.694.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2370
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.694.4 Maple [A] (verified)

Time = 9.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

method	result
derivativedivides	$4x^{\frac{1}{4}}-6^{\frac{1}{4}}\left(\ln\left(\frac{x^{\frac{1}{4}}+6^{\frac{1}{4}}}{x^{\frac{1}{4}}-6^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x^{\frac{1}{4}}6^{\frac{3}{4}}}{6}\right)\right)+5\ln\left(-6+x\right)$
default	$4x^{\frac{1}{4}}-6^{\frac{1}{4}}\left(\ln\left(\frac{x^{\frac{1}{4}}+6^{\frac{1}{4}}}{x^{\frac{1}{4}}-6^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x^{\frac{1}{4}}6^{\frac{3}{4}}}{6}\right)\right)+5\ln\left(-6+x\right)$
meijerg	$5\ln\left(1-\frac{x}{6}\right)-6^{\frac{1}{4}}(-1)^{\frac{3}{4}}\left(\frac{2\,8^{\frac{1}{4}}3^{\frac{3}{4}}x^{\frac{1}{4}}(-1)^{\frac{1}{4}}}{3}+(-1)^{\frac{1}{4}}\left(\ln\left(1-\frac{x^{\frac{1}{4}}6^{\frac{3}{4}}}{6}\right)-\ln\left(1+\frac{x^{\frac{1}{4}}6^{\frac{3}{4}}}{6}\right)-2\,\mathrm{a}\right.\right.$
trager	Expression too large to display

input
```
int((5+x^(1/4))/(-6+x),x,method=_RETURNVERBOSE)
```

output
```
4*x^(1/4)-6^(1/4)*(ln((x^(1/4)+6^(1/4))/(x^(1/4)-6^(1/4)))+2*arctan(1/6*x^
(1/4)*6^(3/4)))+5*ln(-6+x)
```

3.694.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(42) = 84.

Time = 0.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.78

$$\int \frac{5+\sqrt[4]{x}}{-6+x}\,dx = -\left(\sqrt{-\sqrt{6}}-5\right)\log\left(2\sqrt{-\sqrt{6}}+2\,x^{\frac{1}{4}}\right) + \left(\sqrt{-\sqrt{6}}+5\right)\log\left(-2\sqrt{-\sqrt{6}}+2\,x^{\frac{1}{4}}\right) - \left(6^{\frac{1}{4}}-5\right)\log\left(2\cdot 6^{\frac{1}{4}}+2\,x^{\frac{1}{4}}\right) + \left(6^{\frac{1}{4}}+5\right)\log\left(-2\cdot 6^{\frac{1}{4}}+2\,x^{\frac{1}{4}}\right) + 4\,x^{\frac{1}{4}}$$

input
```
integrate((5+x^(1/4))/(-6+x),x, algorithm="fricas")
```

output
```
-(sqrt(-sqrt(6)) - 5)*log(2*sqrt(-sqrt(6)) + 2*x^(1/4)) + (sqrt(-sqrt(6))
+ 5)*log(-2*sqrt(-sqrt(6)) + 2*x^(1/4)) - (6^(1/4) - 5)*log(2*6^(1/4) + 2*
x^(1/4)) + (6^(1/4) + 5)*log(-2*6^(1/4) + 2*x^(1/4)) + 4*x^(1/4)
```

3.694.6 Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.85

$$\int \frac{5+\sqrt[4]{x}}{-6+x}\,dx = 4\sqrt[4]{x}+\sqrt[4]{6}\log\left(\sqrt[4]{x}-\sqrt[4]{6}\right)+5\log\left(\sqrt[4]{x}-\sqrt[4]{6}\right)-\sqrt[4]{6}\log\left(\sqrt[4]{x}+\sqrt[4]{6}\right) + 5\log\left(\sqrt[4]{x}+\sqrt[4]{6}\right)+5\log\left(\sqrt{x}+\sqrt{6}\right)-2\cdot\sqrt[4]{6}\,\mathrm{atan}\left(\frac{6^{\frac{3}{4}}\sqrt[4]{x}}{6}\right)$$

input
```
integrate((5+x**(1/4))/(-6+x),x)
```

output
```
4*x**(1/4) + 6**(1/4)*log(x**(1/4) - 6**(1/4)) + 5*log(x**(1/4) - 6**(1/4)
) - 6**(1/4)*log(x**(1/4) + 6**(1/4)) + 5*log(x**(1/4) + 6**(1/4)) + 5*log
(sqrt(x) + sqrt(6)) - 2*6**(1/4)*atan(6**(3/4)*x**(1/4)/6)
```

3.694.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.24

$$\int \frac{5+\sqrt[4]{x}}{-6+x}\,dx = -2\cdot 6^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 6^{\frac{3}{4}}x^{\frac{1}{4}}\right)+6^{\frac{1}{4}}\log\left(-\frac{6^{\frac{1}{4}}-x^{\frac{1}{4}}}{6^{\frac{1}{4}}+x^{\frac{1}{4}}}\right) + 4\,x^{\frac{1}{4}} + 5\,\log\left(\sqrt{6}+\sqrt{x}\right) + 5\,\log\left(-\sqrt{6}+\sqrt{x}\right)$$

input
```
integrate((5+x^(1/4))/(-6+x),x, algorithm="maxima")
```

output
```
-2*6^(1/4)*arctan(1/6*6^(3/4)*x^(1/4)) + 6^(1/4)*log(-(6^(1/4) - x^(1/4))/
(6^(1/4) + x^(1/4))) + 4*x^(1/4) + 5*log(sqrt(6) + sqrt(x)) + 5*log(-sqrt(
6) + sqrt(x))
```

3.694.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{5+\sqrt[4]{x}}{-6+x}\,dx = -2\cdot 6^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 6^{\frac{3}{4}}x^{\frac{1}{4}}\right)-6^{\frac{1}{4}}\log\left(6^{\frac{1}{4}}+x^{\frac{1}{4}}\right) + 6^{\frac{1}{4}}\log\left(\left|-6^{\frac{1}{4}}+x^{\frac{1}{4}}\right|\right)+4\,x^{\frac{1}{4}}+5\,\log\left(|x-6|\right)$$

input
```
integrate((5+x^(1/4))/(-6+x),x, algorithm="giac")
```

output
```
-2*6^(1/4)*arctan(1/6*6^(3/4)*x^(1/4)) - 6^(1/4)*log(6^(1/4) + x^(1/4)) +
6^(1/4)*log(abs(-6^(1/4) + x^(1/4))) + 4*x^(1/4) + 5*log(abs(x - 6))
```

3.694.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 162, normalized size of antiderivative = 3.00

$$\int \frac{5+\sqrt[4]{x}}{-6+x}\,dx = \ln\left(11520\,x^{1/4} - \left(6^{1/4}+5\right)\,\left(2304\,x^{1/4}-2304\,6^{1/4}+11520\right)+57600\right)\,\left(6^{1/4}+5\right)-\ln\left(\left(6^{1/4}-5\right)\,\left(2304\,6^{1/4}+2304\,x^{1/4}+1152\right.\right.$$

input
```
int((x^(1/4) + 5)/(x - 6),x)
```

output
```
log(11520*x^(1/4) - (6^(1/4) + 5)*(2304*x^(1/4) - 2304*6^(1/4) + 11520) +
57600)*(6^(1/4) + 5) - log((6^(1/4) - 5)*(2304*6^(1/4) + 2304*x^(1/4) + 11
520) + 11520*x^(1/4) + 57600)*(6^(1/4) - 5) - log(11520*x^(1/4) + ((-6^(1/
2))^(1/2) - 5)*(2304*(-6^(1/2))^(1/2) + 2304*x^(1/4) + 11520) + 57600)*((-
6^(1/2))^(1/2) - 5) + log(11520*x^(1/4) - ((-6^(1/2))^(1/2) + 5)*(2304*x^(
1/4) - 2304*(-6^(1/2))^(1/2) + 11520) + 57600)*((-6^(1/2))^(1/2) + 5) + 4*
x^(1/4)
```

3.694. $\int \frac{5+\sqrt[4]{x}}{-6+x}\,dx$

3.695 $\int \frac{1}{4+\sqrt{4-x}-x} \, dx$

3.695.1 Optimal result

Integrand size = 16, antiderivative size = 14

$$\int \frac{1}{4+\sqrt{4-x}-x} \, dx = -2\log \left(1+\sqrt{4-x}\right)$$

output
```
-2*ln(1+(4-x)^(1/2))
```

3.695.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{4+\sqrt{4-x}-x} \, dx = -2\log \left(1+\sqrt{4-x}\right)$$

input
```
Integrate[(4 + Sqrt[4 - x] - x)^(-1),x]
```

output
```
-2*Log[1 + Sqrt[4 - x]]
```

3.695.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {7267, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{-x+\sqrt{4-x}+4} \, dx$$
$$\Big\downarrow \; 7267$$
$$-2\int \frac{1}{\sqrt{4-x}+1} d\sqrt{4-x}$$
$$\Big\downarrow \; 16$$
$$-2\log\left(\sqrt{4-x}+1\right)$$

input
```
Int[(4 + Sqrt[4 - x] - x)^(-1),x]
```

output
```
-2*Log[1 + Sqrt[4 - x]]
```

3.695.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.695.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-2\ln\left(1+\sqrt{4-x}\right)$	13
default	$-\ln\left(-3+x\right)-2\,\operatorname{arctanh}\left(\sqrt{4-x}\right)$	18
trager	$-\ln\left(2\sqrt{4-x}+5-x\right)$	18

input
```
int(1/(4-x+(4-x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-2*ln(1+(4-x)^(1/2))
```

3.695.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{4+\sqrt{4-x}-x}\,dx = -2\,\log\left(\sqrt{-x+4}+1\right)$$

input
```
integrate(1/(4-x+(4-x)^(1/2)),x, algorithm="fricas")
```

output
```
-2*log(sqrt(-x + 4) + 1)
```

3.695.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(12) = 24$.

Time = 0.65 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.29

$$\int \frac{1}{4+\sqrt{4-x}-x}\,dx = \log\left(2\sqrt{4-x}\right)-\log\left(2\sqrt{4-x}+2\right)-\log\left(-x+\sqrt{4-x}+4\right)$$

input
```
integrate(1/(4-x+(4-x)**(1/2)),x)
```

output
```
log(2*sqrt(4 - x)) - log(2*sqrt(4 - x) + 2) - log(-x + sqrt(4 - x) + 4)
```

3.695.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{4+\sqrt{4-x}-x}\,dx = -2\log\left(\sqrt{-x+4}+1\right)$$

input
```
integrate(1/(4-x+(4-x)^(1/2)),x, algorithm="maxima")
```

output
```
-2*log(sqrt(-x + 4) + 1)
```

3.695.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{4+\sqrt{4-x}-x}\,dx = -2\log\left(\sqrt{-x+4}+1\right)$$

input
```
integrate(1/(4-x+(4-x)^(1/2)),x, algorithm="giac")
```

output
```
-2*log(sqrt(-x + 4) + 1)
```

3.695.9 Mupad [B] (verification not implemented)

Time = 17.96 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{4+\sqrt{4-x}-x}\,dx = -2\ln\left(\sqrt{4-x}+1\right)$$

input
```
int(1/((4 - x)^(1/2) - x + 4),x)
```

output
```
-2*log((4 - x)^(1/2) + 1)
```

3.696 $\int \frac{1}{1+x-\sqrt{2+x}}\,dx$

3.696.1 Optimal result

Integrand size = 14, antiderivative size = 61

$$\int \frac{1}{1+x-\sqrt{2+x}}\,dx = \frac{1}{5}\left(5-\sqrt{5}\right)\log\left(1-\sqrt{5}-2\sqrt{2+x}\right) + \frac{1}{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2\sqrt{2+x}\right)$$

output
```
1/5*ln(1-5^(1/2)-2*(2+x)^(1/2))*(5-5^(1/2))+1/5*ln(1+5^(1/2)-2*(2+x)^(1/2)
)*(5+5^(1/2))
```

3.696.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{1}{1+x-\sqrt{2+x}}\,dx = \frac{1}{5}\left(\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2\sqrt{2+x}\right) - \left(-5+\sqrt{5}\right)\log\left(-1+\sqrt{5}+2\sqrt{2+x}\right)\right)$$

input
```
Integrate[(1 + x - Sqrt[2 + x])^(-1),x]
```

output
```
((5 + Sqrt[5])*Log[1 + Sqrt[5] - 2*Sqrt[2 + x]] - (-5 + Sqrt[5])*Log[-1 +
Sqrt[5] + 2*Sqrt[2 + x]])/5
```

3.696.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {7267, 25, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x-\sqrt{x+2}+1}dx$$

$$\Big\downarrow \text{ 7267}$$

$$2\int -\frac{\sqrt{x+2}}{-x+\sqrt{x+2}-1}d\sqrt{x+2}$$

$$\Big\downarrow \text{ 25}$$

$$-2\int \frac{\sqrt{x+2}}{-x+\sqrt{x+2}-1}d\sqrt{x+2}$$

$$\Big\downarrow \text{ 1141}$$

$$2\int \left(-\frac{5+\sqrt{5}}{5\left(-2\sqrt{x+2}+\sqrt{5}+1\right)}-\frac{5-\sqrt{5}}{5\left(-2\sqrt{x+2}-\sqrt{5}+1\right)}\right)d\sqrt{x+2}$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{1}{10}\left(5-\sqrt{5}\right)\log\left(-2\sqrt{x+2}-\sqrt{5}+1\right)+\frac{1}{10}\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+2}+\sqrt{5}+1\right)\right)$$

input
```
Int[(1 + x - Sqrt[2 + x])^(-1),x]
```

output
```
2*(((5 - Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[2 + x]])/10 + ((5 + Sqrt[5])*Lo
g[1 + Sqrt[5] - 2*Sqrt[2 + x]])/10)
```

3.696.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.696.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.54

method	result
derivativedivides	$\ln\left(1+x-\sqrt{x+2}\right)-\frac{2\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2\sqrt{x+2}-1\right)\sqrt{5}}{5}\right)}{5}$
default	$-\frac{\operatorname{arctanh}\left(\frac{(1+2x)\sqrt{5}}{5}\right)\sqrt{5}}{5}+\frac{\ln\left(x^{2}+x-1\right)}{2}+\frac{\ln\left(1+x-\sqrt{x+2}\right)}{2}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2\sqrt{x+2}-1\right)\sqrt{5}}{5}\right)}{5}-\frac{\ln\left(x+1+\sqrt{x+2}$
trager	$-\ln\left(-1-x+\sqrt{x+2}\right)\operatorname{RootOf}\left(5\_Z^{2}-10\_Z+4\right)+\ln\Big(150\operatorname{RootOf}\left(5\_Z^{2}-10\_$

input
```
int(1/(1+x-(x+2)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
ln(1+x-(x+2)^(1/2))-2/5*5^(1/2)*arctanh(1/5*(2*(x+2)^(1/2)-1)*5^(1/2))
```

3.696.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

$$\int \frac{1}{1+x-\sqrt{2+x}}\,dx = \frac{1}{5}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}(x+3)-\left(\sqrt{5}(2x+1)-5\right)\sqrt{x+2}+7x+3}{x^2+x-1}\right) + \log\left(x-\sqrt{x+2}+1\right)$$

input
```
integrate(1/(1+x-(2+x)^(1/2)),x, algorithm="fricas")
```

output
```
1/5*sqrt(5)*log((2*x^2 - sqrt(5)*(x + 3) - (sqrt(5)*(2*x + 1) - 5)*sqrt(x
+ 2) + 7*x + 3)/(x^2 + x - 1)) + log(x - sqrt(x + 2) + 1)
```

3.696.6 Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \frac{1}{1+x-\sqrt{2+x}}\,dx = \frac{\sqrt{5}\left(-\log\left(\sqrt{x+2}-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)+\log\left(\sqrt{x+2}-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\right)}{5} + \log\left(x-\sqrt{x+2}+1\right)$$

input
```
integrate(1/(1+x-(2+x)**(1/2)),x)
```

output
```
sqrt(5)*(-log(sqrt(x + 2) - 1/2 + sqrt(5)/2) + log(sqrt(x + 2) - sqrt(5)/2
 - 1/2))/5 + log(x - sqrt(x + 2) + 1)
```

3.696.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \frac{1}{1+x-\sqrt{2+x}}\,dx = \frac{1}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{x+2}+1}{\sqrt{5}+2\sqrt{x+2}-1}\right)+\log\left(x-\sqrt{x+2}+1\right)$$

input
```
integrate(1/(1+x-(2+x)^(1/2)),x, algorithm="maxima")
```

output
```
1/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(x + 2) + 1)/(sqrt(5) + 2*sqrt(x + 2) -
1)) + log(x - sqrt(x + 2) + 1)
```

3.696.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{1}{1+x-\sqrt{2+x}}\,dx = \frac{1}{5}\sqrt{5}\log\left(\frac{\left|-\sqrt{5}+2\sqrt{x+2}-1\right|}{\left|\sqrt{5}+2\sqrt{x+2}-1\right|}\right)+\log\left(\left|x-\sqrt{x+2}+1\right|\right)$$

input
```
integrate(1/(1+x-(2+x)^(1/2)),x, algorithm="giac")
```

output
```
1/5*sqrt(5)*log(abs(-sqrt(5) + 2*sqrt(x + 2) - 1)/abs(sqrt(5) + 2*sqrt(x +
 2) - 1)) + log(abs(x - sqrt(x + 2) + 1))
```

3.696.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.16

$$\int \frac{1}{1+x-\sqrt{2+x}}\,dx = \ln\left(2\sqrt{x+2}-\left(\frac{\sqrt{5}}{5}+1\right)\left(2\sqrt{x+2}-1\right)\right)\left(\frac{\sqrt{5}}{5}+1\right) - \ln\left(2\sqrt{x+2}+\left(\frac{\sqrt{5}}{5}-1\right)\left(2\sqrt{x+2}-1\right)\right)\left(\frac{\sqrt{5}}{5}-1\right)$$

input
```
int(1/(x - (x + 2)^(1/2) + 1),x)
```

output
```
log(2*(x + 2)^(1/2) - (5^(1/2)/5 + 1)*(2*(x + 2)^(1/2) - 1))*(5^(1/2)/5 +
1) - log(2*(x + 2)^(1/2) + (5^(1/2)/5 - 1)*(2*(x + 2)^(1/2) - 1))*(5^(1/2)
/5 - 1)
```

3.697 $\int \frac{1}{4+x+\sqrt{1+x}}\,dx$

3.697.1 Optimal result

Integrand size = 12, antiderivative size = 37

$$\int \frac{1}{4+x+\sqrt{1+x}}\,dx = -\frac{2\arctan\left(\frac{1+2\sqrt{1+x}}{\sqrt{11}}\right)}{\sqrt{11}} + \log\left(4+x+\sqrt{1+x}\right)$$

output
```
ln(4+x+(1+x)^(1/2))-2/11*arctan(1/11*(1+2*(1+x)^(1/2))*11^(1/2))*11^(1/2)
```

3.697.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{1}{4+x+\sqrt{1+x}}\,dx = -\frac{2\arctan\left(\frac{1+2\sqrt{1+x}}{\sqrt{11}}\right)}{\sqrt{11}} + \log\left(4+x+\sqrt{1+x}\right)$$

input
```
Integrate[(4 + x + Sqrt[1 + x])^(-1),x]
```

output
```
(-2*ArcTan[(1 + 2*Sqrt[1 + x])/Sqrt[11]])/Sqrt[11] + Log[4 + x + Sqrt[1 +
x]]
```

3.697.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {7267, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x+\sqrt{x+1}+4}dx$$
$$\Big\downarrow \text{7267}$$
$$2\int \frac{\sqrt{x+1}}{x+\sqrt{x+1}+4}d\sqrt{x+1}$$
$$\Big\downarrow \text{1142}$$
$$2\left(\frac{1}{2}\int \frac{2\sqrt{x+1}+1}{x+\sqrt{x+1}+4}d\sqrt{x+1}-\frac{1}{2}\int \frac{1}{x+\sqrt{x+1}+4}d\sqrt{x+1}\right)$$
$$\Big\downarrow \text{1083}$$
$$2\left(\int \frac{1}{-x-12}d\left(2\sqrt{x+1}+1\right)+\frac{1}{2}\int \frac{2\sqrt{x+1}+1}{x+\sqrt{x+1}+4}d\sqrt{x+1}\right)$$
$$\Big\downarrow \text{217}$$
$$2\left(\frac{1}{2}\int \frac{2\sqrt{x+1}+1}{x+\sqrt{x+1}+4}d\sqrt{x+1}-\frac{\arctan\left(\frac{2\sqrt{x+1}+1}{\sqrt{11}}\right)}{\sqrt{11}}\right)$$
$$\Big\downarrow \text{1103}$$
$$2\left(\frac{1}{2}\log\left(x+\sqrt{x+1}+4\right)-\frac{\arctan\left(\frac{2\sqrt{x+1}+1}{\sqrt{11}}\right)}{\sqrt{11}}\right)$$

input
```
Int[(4 + x + Sqrt[1 + x])^(-1),x]
```

output
```
2*(-(ArcTan[(1 + 2*Sqrt[1 + x])/Sqrt[11]]/Sqrt[11]) + Log[4 + x + Sqrt[1 +
 x]]/2)
```

3.697.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.697.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\ln\left(4+x+\sqrt{x+1}\right)-\frac{2\arctan\left(\frac{(1+2\sqrt{x+1})\sqrt{11}}{11}\right)\sqrt{11}}{11}$
default	$\frac{\ln(4+x+\sqrt{x+1})}{2}-\frac{\arctan\left(\frac{(1+2\sqrt{x+1})\sqrt{11}}{11}\right)\sqrt{11}}{11}-\frac{\ln(x+4-\sqrt{x+1})}{2}-\frac{\sqrt{11}\arctan\left(\frac{(2\sqrt{x+1}-1)\sqrt{11}}{11}\right)}{11}+\frac{\sqrt{11}\text{ ar}}{}$
trager	$-\ln\left(4+x+\sqrt{x+1}\right)\text{RootOf}\left(11\_Z^2-22\_Z+12\right)+\ln\Big(-847\,\text{RootOf}\left(11\_Z^2-2\right.$

input
```
int(1/(4+x+(x+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
ln(4+x+(x+1)^(1/2))-2/11*arctan(1/11*(1+2*(x+1)^(1/2))*11^(1/2))*11^(1/2)
```

3.697.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{1}{4+x+\sqrt{1+x}}\,dx = -\frac{2}{11}\sqrt{11}\arctan\left(\frac{2}{11}\sqrt{11}\sqrt{x+1}+\frac{1}{11}\sqrt{11}\right)+\log\left(x+\sqrt{x+1}+4\right)$$

input
```
integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="fricas")
```

output
```
-2/11*sqrt(11)*arctan(2/11*sqrt(11)*sqrt(x + 1) + 1/11*sqrt(11)) + log(x +
 sqrt(x + 1) + 4)
```

3.697.6 Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int \frac{1}{4+x+\sqrt{1+x}}\,dx = \log\left(x+\sqrt{x+1}+4\right)-\frac{2\sqrt{11}\,\operatorname{atan}\left(\frac{2\sqrt{11}\left(\sqrt{x+1}+\frac{1}{2}\right)}{11}\right)}{11}$$

input
```
integrate(1/(4+x+(1+x)**(1/2)),x)
```

output
```
log(x + sqrt(x + 1) + 4) - 2*sqrt(11)*atan(2*sqrt(11)*(sqrt(x + 1) + 1/2)/
11)/11
```

3.697.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{1}{4+x+\sqrt{1+x}}\,dx = -\frac{2}{11}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\left(2\sqrt{x+1}+1\right)\right)+\log\left(x+\sqrt{x+1}+4\right)$$

input
```
integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="maxima")
```

output
```
-2/11*sqrt(11)*arctan(1/11*sqrt(11)*(2*sqrt(x + 1) + 1)) + log(x + sqrt(x
+ 1) + 4)
```

3.697.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{1}{4+x+\sqrt{1+x}}\,dx = -\frac{2}{11}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\left(2\sqrt{x+1}+1\right)\right)+\log\left(x+\sqrt{x+1}+4\right)$$

input
```
integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="giac")
```

output
```
-2/11*sqrt(11)*arctan(1/11*sqrt(11)*(2*sqrt(x + 1) + 1)) + log(x + sqrt(x
+ 1) + 4)
```

3.697.9 Mupad [B] (verification not implemented)

Time = 18.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{1}{4+x+\sqrt{1+x}}\,dx = \ln\left(x+\sqrt{x+1}+4\right)-\frac{2\sqrt{11}\,\mathrm{atan}\left(\frac{\sqrt{11}}{11}+\frac{2\sqrt{11}\sqrt{x+1}}{11}\right)}{11}$$

input
```
int(1/(x + (x + 1)^(1/2) + 4),x)
```

output
```
log(x + (x + 1)^(1/2) + 4) - (2*11^(1/2)*atan(11^(1/2)/11 + (2*11^(1/2)*(x
 + 1)^(1/2))/11))/11
```

3.698 $\int \frac{1}{x-\sqrt{1+x}}\,dx$

3.698.1 Optimal result

Integrand size = 13, antiderivative size = 61

$$\int \frac{1}{x-\sqrt{1+x}}\,dx=\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(1-\sqrt{5}-2\sqrt{1+x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2\sqrt{1+x}\right)$$

output
```
1/5*ln(1-5^(1/2)-2*(1+x)^(1/2))*(5-5^(1/2))+1/5*ln(1+5^(1/2)-2*(1+x)^(1/2)
)*(5+5^(1/2))
```

3.698.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{1}{x-\sqrt{1+x}}\,dx=\frac{1}{5}\left(\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2\sqrt{1+x}\right)-\left(-5+\sqrt{5}\right)\log\left(-1+\sqrt{5}+2\sqrt{1+x}\right)\right)$$

input
```
Integrate[(x - Sqrt[1 + x])^(-1),x]
```

output
```
((5 + Sqrt[5])*Log[1 + Sqrt[5] - 2*Sqrt[1 + x]] - (-5 + Sqrt[5])*Log[-1 +
Sqrt[5] + 2*Sqrt[1 + x]])/5
```

3.698.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {7267, 25, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x-\sqrt{x+1}}dx$$

$$\Big\downarrow \text{7267}$$

$$2\int -\frac{\sqrt{x+1}}{\sqrt{x+1}-x}d\sqrt{x+1}$$

$$\Big\downarrow \text{25}$$

$$-2\int \frac{\sqrt{x+1}}{\sqrt{x+1}-x}d\sqrt{x+1}$$

$$\Big\downarrow \text{1141}$$

$$2\int \left(-\frac{5+\sqrt{5}}{5\left(-2\sqrt{x+1}+\sqrt{5}+1\right)}-\frac{5-\sqrt{5}}{5\left(-2\sqrt{x+1}-\sqrt{5}+1\right)}\right)d\sqrt{x+1}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{1}{10}\left(5-\sqrt{5}\right)\log\left(-2\sqrt{x+1}-\sqrt{5}+1\right)+\frac{1}{10}\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+1}+\sqrt{5}+1\right)\right)$$

input
```
Int[(x - Sqrt[1 + x])^(-1),x]
```

output
```
2*(((5 - Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[1 + x]])/10 + ((5 + Sqrt[5])*Lo
g[1 + Sqrt[5] - 2*Sqrt[1 + x]])/10)
```

3.698.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.698.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.52

method	result
derivativedivides	$\ln\left(x-\sqrt{x+1}\right)-\frac{2\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2\sqrt{x+1}-1)\sqrt{5}}{5}\right)}{5}$
default	$\frac{\ln(x^2-x-1)}{2}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x-1)\sqrt{5}}{5}\right)}{5}+\frac{\ln(x-\sqrt{x+1})}{2}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2\sqrt{x+1}-1)\sqrt{5}}{5}\right)}{5}-\frac{\ln(x+\sqrt{x+1})}{2}-$
trager	$\operatorname{RootOf}\left(5\_Z^2-10\_Z+4\right)\ln\left(x-\sqrt{x+1}\right)-\ln\left(5\operatorname{RootOf}\left(5\_Z^2-10\_Z+4\right)^2x-\right.$

input
```
int(1/(x-(x+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
ln(x-(x+1)^(1/2))-2/5*5^(1/2)*arctanh(1/5*(2*(x+1)^(1/2)-1)*5^(1/2))
```

3.698.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05

$$\int \frac{1}{x-\sqrt{1+x}}\,dx = \frac{1}{5}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}(x+2)-\left(\sqrt{5}(2x-1)-5\right)\sqrt{x+1}+3x-2}{x^2-x-1}\right) + \log\left(x-\sqrt{x+1}\right)$$

input
```
integrate(1/(x-(1+x)^(1/2)),x, algorithm="fricas")
```

output
```
1/5*sqrt(5)*log((2*x^2 - sqrt(5)*(x + 2) - (sqrt(5)*(2*x - 1) - 5)*sqrt(x
+ 1) + 3*x - 2)/(x^2 - x - 1)) + log(x - sqrt(x + 1))
```

3.698.6 Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{1}{x-\sqrt{1+x}}\,dx = \frac{\sqrt{5}\left(-\log\left(\sqrt{x+1}-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)+\log\left(\sqrt{x+1}-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)\right)}{5} + \log\left(x-\sqrt{x+1}\right)$$

input
```
integrate(1/(x-(1+x)**(1/2)),x)
```

output
```
sqrt(5)*(-log(sqrt(x + 1) - 1/2 + sqrt(5)/2) + log(sqrt(x + 1) - sqrt(5)/2
 - 1/2))/5 + log(x - sqrt(x + 1))
```

3.698.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \frac{1}{x-\sqrt{1+x}}\,dx = \frac{1}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{x+1}+1}{\sqrt{5}+2\sqrt{x+1}-1}\right) + \log\left(x-\sqrt{x+1}\right)$$

input
```
integrate(1/(x-(1+x)^(1/2)),x, algorithm="maxima")
```

output
```
1/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(x + 1) + 1)/(sqrt(5) + 2*sqrt(x + 1) -
1)) + log(x - sqrt(x + 1))
```

3.698.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \frac{1}{x-\sqrt{1+x}}\,dx = \frac{1}{5}\sqrt{5}\log\left(\frac{\left|-\sqrt{5}+2\sqrt{x+1}-1\right|}{\left|\sqrt{5}+2\sqrt{x+1}-1\right|}\right)+\log\left(\left|x-\sqrt{x+1}\right|\right)$$

input
```
integrate(1/(x-(1+x)^(1/2)),x, algorithm="giac")
```

output
```
1/5*sqrt(5)*log(abs(-sqrt(5) + 2*sqrt(x + 1) - 1)/abs(sqrt(5) + 2*sqrt(x +
 1) - 1)) + log(abs(x - sqrt(x + 1)))
```

3.698.9 Mupad [B] (verification not implemented)

Time = 17.77 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.16

$$\int \frac{1}{x-\sqrt{1+x}}\,dx = \ln\left(2\sqrt{x+1}-\left(\frac{\sqrt{5}}{5}+1\right)\left(2\sqrt{x+1}-1\right)\right)\left(\frac{\sqrt{5}}{5}+1\right) - \ln\left(2\sqrt{x+1}+\left(\frac{\sqrt{5}}{5}-1\right)\left(2\sqrt{x+1}-1\right)\right)\left(\frac{\sqrt{5}}{5}-1\right)$$

input
```
int(1/(x - (x + 1)^(1/2)),x)
```

output
```
log(2*(x + 1)^(1/2) - (5^(1/2)/5 + 1)*(2*(x + 1)^(1/2) - 1))*(5^(1/2)/5 +
1) - log(2*(x + 1)^(1/2) + (5^(1/2)/5 - 1)*(2*(x + 1)^(1/2) - 1))*(5^(1/2)
/5 - 1)
```

3.699 $\int \frac{1}{x-\sqrt{2+x}}\,dx$

3.699.1 Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{1}{x-\sqrt{2+x}}\,dx = \frac{4}{3}\log\left(2-\sqrt{2+x}\right)+\frac{2}{3}\log\left(1+\sqrt{2+x}\right)$$

output
```
4/3*ln(2-(2+x)^(1/2))+2/3*ln(1+(2+x)^(1/2))
```

3.699.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{1}{x-\sqrt{2+x}}\,dx = \frac{4}{3}\log\left(-2+\sqrt{2+x}\right)+\frac{2}{3}\log\left(1+\sqrt{2+x}\right)$$

input
```
Integrate[(x - Sqrt[2 + x])^(-1),x]
```

output
```
(4*Log[-2 + Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3
```

3.699.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {7267, 25, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x-\sqrt{x+2}}dx$$
$$\Big\downarrow \ 7267$$
$$2\int -\frac{\sqrt{x+2}}{\sqrt{x+2}-x}d\sqrt{x+2}$$
$$\Big\downarrow \ 25$$
$$-2\int \frac{\sqrt{x+2}}{\sqrt{x+2}-x}d\sqrt{x+2}$$
$$\Big\downarrow \ 1141$$
$$2\int \left(\frac{1}{3\left(\sqrt{x+2}+1\right)}-\frac{2}{3\left(2-\sqrt{x+2}\right)}\right)d\sqrt{x+2}$$
$$\Big\downarrow \ 2009$$
$$2\left(\frac{2}{3}\log\left(2-\sqrt{x+2}\right)+\frac{1}{3}\log\left(\sqrt{x+2}+1\right)\right)$$

input
```
Int[(x - Sqrt[2 + x])^(-1),x]
```

output
```
2*((2*Log[2 - Sqrt[2 + x]])/3 + Log[1 + Sqrt[2 + x]]/3)
```

3.699.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.699.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{2\ln\left(1+\sqrt{x+2}\right)}{3}+\frac{4\ln\left(\sqrt{x+2}-2\right)}{3}$	22
trager	$\frac{\ln\left(6\sqrt{x+2}\,x^{2}-x^{3}+16\sqrt{x+2}\,x-15x^{2}+8\sqrt{x+2}-24x-12\right)}{3}$	44
default	$\frac{\ln(x+1)}{3}+\frac{2\ln(x-2)}{3}+\frac{\ln\left(1+\sqrt{x+2}\right)}{3}-\frac{2\ln\left(\sqrt{x+2}+2\right)}{3}-\frac{\ln\left(\sqrt{x+2}-1\right)}{3}+\frac{2\ln\left(\sqrt{x+2}-2\right)}{3}$	54

input
```
int(1/(x-(x+2)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/3*ln(1+(x+2)^(1/2))+4/3*ln((x+2)^(1/2)-2)
```

3.699.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{1}{x-\sqrt{2+x}}\,dx=\frac{2}{3}\,\log\left(\sqrt{x+2}+1\right)+\frac{4}{3}\,\log\left(\sqrt{x+2}-2\right)$$

input
```
integrate(1/(x-(2+x)^(1/2)),x, algorithm="fricas")
```

output
```
2/3*log(sqrt(x + 2) + 1) + 4/3*log(sqrt(x + 2) - 2)
```

3.699.6 Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{1}{x-\sqrt{2+x}}\,dx = \log\left(x-\sqrt{x+2}\right)+\frac{\log\left(2\sqrt{x+2}-4\right)}{3}-\frac{\log\left(2\sqrt{x+2}+2\right)}{3}$$

input
```
integrate(1/(x-(2+x)**(1/2)),x)
```

output
```
log(x - sqrt(x + 2)) + log(2*sqrt(x + 2) - 4)/3 - log(2*sqrt(x + 2) + 2)/3
```

3.699.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{1}{x-\sqrt{2+x}}\,dx = \frac{2}{3}\log\left(\sqrt{x+2}+1\right)+\frac{4}{3}\log\left(\sqrt{x+2}-2\right)$$

input
```
integrate(1/(x-(2+x)^(1/2)),x, algorithm="maxima")
```

output
```
2/3*log(sqrt(x + 2) + 1) + 4/3*log(sqrt(x + 2) - 2)
```

3.699.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{1}{x-\sqrt{2+x}}\,dx = \frac{2}{3}\log\left(\sqrt{x+2}+1\right)+\frac{4}{3}\log\left(\left|\sqrt{x+2}-2\right|\right)$$

input
```
integrate(1/(x-(2+x)^(1/2)),x, algorithm="giac")
```

output
```
2/3*log(sqrt(x + 2) + 1) + 4/3*log(abs(sqrt(x + 2) - 2))
```

3.699.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{x-\sqrt{2+x}}\,dx = \frac{2\ln\left(\frac{2\sqrt{x+2}}{3}+\frac{2}{3}\right)}{3}+\frac{4\ln\left(\frac{4}{3}-\frac{2\sqrt{x+2}}{3}\right)}{3}$$

input
```
int(1/(x - (x + 2)^(1/2)),x)
```

output
```
(2*log((2*(x + 2)^(1/2))/3 + 2/3))/3 + (4*log(4/3 - (2*(x + 2)^(1/2))/3))/
3
```

3.700 $\int \frac{1}{-\sqrt{1-x}+x}\,dx$

3.700.1 Optimal result

Integrand size = 15, antiderivative size = 65

$$\int \frac{1}{-\sqrt{1-x}+x}\,dx = \frac{1}{5}\left(5-\sqrt{5}\right)\log\left(1-\sqrt{5}+2\sqrt{1-x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}+2\sqrt{1-x}\right)$$

output
```
1/5*ln(1-5^(1/2)+2*(1-x)^(1/2))*(5-5^(1/2))+1/5*ln(1+5^(1/2)+2*(1-x)^(1/2)
)*(5+5^(1/2))
```

3.700.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{1}{-\sqrt{1-x}+x}\,dx = \frac{1}{5}\left(-\left(\left(-5+\sqrt{5}\right)\log\left(-1+\sqrt{5}-2\sqrt{1-x}\right)\right)+\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}+2\sqrt{1-x}\right)\right)$$

input
```
Integrate[(-Sqrt[1 - x] + x)^(-1),x]
```

output
```
(-((-5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*Sqrt[1 - x]]) + (5 + Sqrt[5])*Log[1
 + Sqrt[5] + 2*Sqrt[1 - x]])/5
```

3.700.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {7267, 25, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x-\sqrt{1-x}}dx$$

$$\Big\downarrow \text{ 7267}$$

$$2\int -\frac{\sqrt{1-x}}{x-\sqrt{1-x}}d\sqrt{1-x}$$

$$\Big\downarrow \text{ 25}$$

$$-2\int \frac{\sqrt{1-x}}{x-\sqrt{1-x}}d\sqrt{1-x}$$

$$\Big\downarrow \text{ 1141}$$

$$2\int \left(\frac{1+\sqrt{5}}{2\sqrt{5}\sqrt{1-x}+\sqrt{5}+5}+\frac{5-\sqrt{5}}{5\left(2\sqrt{1-x}-\sqrt{5}+1\right)}\right)d\sqrt{1-x}$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{1}{10}\left(5-\sqrt{5}\right)\log\left(2\sqrt{1-x}-\sqrt{5}+1\right)+\frac{1}{10}\left(5+\sqrt{5}\right)\log\left(2\sqrt{1-x}+\sqrt{5}+1\right)\right)$$

input
```
Int[(-Sqrt[1 - x] + x)^(-1),x]
```

output
```
2*(((5 - Sqrt[5])*Log[1 - Sqrt[5] + 2*Sqrt[1 - x]])/10 + ((5 + Sqrt[5])*Lo
g[1 + Sqrt[5] + 2*Sqrt[1 - x]])/10)
```

3.700.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.700.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.55

method	result
derivativedivides	$\ln\left(-x+\sqrt{1-x}\right)+\frac{2\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2\sqrt{1-x}+1\right)\sqrt{5}}{5}\right)}{5}$
default	$\frac{\ln\left(x^{2}+x-1\right)}{2}+\frac{\operatorname{arctanh}\left(\frac{(1+2x)\sqrt{5}}{5}\right)\sqrt{5}}{5}-\frac{\ln\left(-x-\sqrt{1-x}\right)}{2}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2\sqrt{1-x}-1\right)\sqrt{5}}{5}\right)}{5}+\frac{\ln\left(-x+\sqrt{1-x}\right)}{2}+$
trager	$-\ln\left(x-\sqrt{1-x}\right)\operatorname{RootOf}\left(5\_Z^{2}-10\_Z+4\right)+\ln\left(5\operatorname{RootOf}\left(5\_Z^{2}-10\_Z+4\right)^{2}x\right.$

input
```
int(1/(x-(1-x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
ln(-x+(1-x)^(1/2))+2/5*5^(1/2)*arctanh(1/5*(2*(1-x)^(1/2)+1)*5^(1/2))
```

3.700.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{1}{-\sqrt{1-x}+x}\,dx = \frac{1}{5}\sqrt{5}\log\left(\frac{2x^2+\sqrt{5}(x-2)-\left(\sqrt{5}(2x+1)+5\right)\sqrt{-x+1}-3x-2}{x^2+x-1}\right) + \log\left(-x+\sqrt{-x+1}\right)$$

input
```
integrate(1/(x-(1-x)^(1/2)),x, algorithm="fricas")
```

output
```
1/5*sqrt(5)*log((2*x^2 + sqrt(5)*(x - 2) - (sqrt(5)*(2*x + 1) + 5)*sqrt(-x
 + 1) - 3*x - 2)/(x^2 + x - 1)) + log(-x + sqrt(-x + 1))
```

3.700.6 Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int \frac{1}{-\sqrt{1-x}+x}\,dx = -\frac{\sqrt{5}\left(-\log\left(\sqrt{1-x}+\frac{1}{2}+\frac{\sqrt{5}}{2}\right)+\log\left(\sqrt{1-x}-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)\right)}{5} + \log\left(x-\sqrt{1-x}\right)$$

input
```
integrate(1/(x-(1-x)**(1/2)),x)
```

output
```
-sqrt(5)*(-log(sqrt(1 - x) + 1/2 + sqrt(5)/2) + log(sqrt(1 - x) - sqrt(5)/
2 + 1/2))/5 + log(x - sqrt(1 - x))
```

3.700.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \frac{1}{-\sqrt{1-x}+x}\,dx = -\frac{1}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{-x+1}-1}{\sqrt{5}+2\sqrt{-x+1}+1}\right) + \log\left(-x+\sqrt{-x+1}\right)$$

input
```
integrate(1/(x-(1-x)^(1/2)),x, algorithm="maxima")
```

output
```
-1/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(-x + 1) - 1)/(sqrt(5) + 2*sqrt(-x + 1)
 + 1)) + log(-x + sqrt(-x + 1))
```

3.700.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int \frac{1}{-\sqrt{1-x}+x}\,dx = -\frac{1}{5}\sqrt{5}\log\left(\frac{\left|-\sqrt{5}+2\sqrt{-x+1}+1\right|}{\sqrt{5}+2\sqrt{-x+1}+1}\right) + \log\left(\left|-x+\sqrt{-x+1}\right|\right)$$

input
```
integrate(1/(x-(1-x)^(1/2)),x, algorithm="giac")
```

output
```
-1/5*sqrt(5)*log(abs(-sqrt(5) + 2*sqrt(-x + 1) + 1)/(sqrt(5) + 2*sqrt(-x +
 1) + 1)) + log(abs(-x + sqrt(-x + 1)))
```

3.700.9 Mupad [B] (verification not implemented)

Time = 18.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.22

$$\int \frac{1}{-\sqrt{1-x}+x}\,dx = \ln\left(2\sqrt{1-x}-\left(\frac{\sqrt{5}}{5}+1\right)\left(2\sqrt{1-x}+1\right)\right)\left(\frac{\sqrt{5}}{5}+1\right) - \ln\left(2\sqrt{1-x}+\left(\frac{\sqrt{5}}{5}-1\right)\left(2\sqrt{1-x}+1\right)\right)\left(\frac{\sqrt{5}}{5}-1\right)$$

input
```
int(1/(x - (1 - x)^(1/2)),x)
```

output
```
log(2*(1 - x)^(1/2) - (5^(1/2)/5 + 1)*(2*(1 - x)^(1/2) + 1))*(5^(1/2)/5 +
1) - log(2*(1 - x)^(1/2) + (5^(1/2)/5 - 1)*(2*(1 - x)^(1/2) + 1))*(5^(1/2)
/5 - 1)
```

3.701 $\int \sqrt{1+\sqrt{x}+x}\,dx$

3.701.1 Optimal result

Integrand size = 12, antiderivative size = 62

$$\int \sqrt{1+\sqrt{x}+x}\,dx=-\frac{1}{4}(1+2\sqrt{x})\sqrt{1+\sqrt{x}+x}+\frac{2}{3}(1+\sqrt{x}+x)^{3/2}-\frac{3}{8}\text{arcsinh}\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)$$

output
```
-3/8*arcsinh(1/3*(1+2*x^(1/2))*3^(1/2))+2/3*(1+x+x^(1/2))^(3/2)-1/4*(1+2*x
^(1/2))*(1+x+x^(1/2))^(1/2)
```

3.701.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \sqrt{1+\sqrt{x}+x}\,dx=\frac{1}{12}\sqrt{1+\sqrt{x}+x}(5+2\sqrt{x}+8x)+\frac{3}{8}\log\left(-1-2\sqrt{x}+2\sqrt{1+\sqrt{x}+x}\right)$$

input
```
Integrate[Sqrt[1 + Sqrt[x] + x],x]
```

output
```
(Sqrt[1 + Sqrt[x] + x]*(5 + 2*Sqrt[x] + 8*x))/12 + (3*Log[-1 - 2*Sqrt[x] +
 2*Sqrt[1 + Sqrt[x] + x]])/8
```

3.701.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1680, 1160, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x+\sqrt{x}+1}\,dx$$

$$\Big\downarrow \text{1680}$$

$$2\int \sqrt{x}\sqrt{x+\sqrt{x}+1}d\sqrt{x}$$

$$\Big\downarrow \text{1160}$$

$$2\left(\frac{1}{3}\left(x+\sqrt{x}+1\right)^{3/2}-\frac{1}{2}\int \sqrt{x+\sqrt{x}+1}d\sqrt{x}\right)$$

$$\Big\downarrow \text{1087}$$

$$2\left(\frac{1}{2}\left(-\frac{3}{8}\int \frac{1}{\sqrt{x+\sqrt{x}+1}}d\sqrt{x}-\frac{1}{4}\sqrt{x+\sqrt{x}+1}\left(2\sqrt{x}+1\right)\right)+\frac{1}{3}\left(x+\sqrt{x}+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{1090}$$

$$2\left(\frac{1}{2}\left(-\frac{1}{8}\sqrt{3}\int \frac{1}{\sqrt{\frac{x}{3}+1}}d\left(2\sqrt{x}+1\right)-\frac{1}{4}\sqrt{x+\sqrt{x}+1}\left(2\sqrt{x}+1\right)\right)+\frac{1}{3}\left(x+\sqrt{x}+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{222}$$

$$2\left(\frac{1}{2}\left(-\frac{3}{8}\text{arcsinh}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)-\frac{1}{4}\sqrt{x+\sqrt{x}+1}\left(2\sqrt{x}+1\right)\right)+\frac{1}{3}\left(x+\sqrt{x}+1\right)^{3/2}\right)$$

input
```
Int[Sqrt[1 + Sqrt[x] + x],x]
```

output
```
2*((1 + Sqrt[x] + x)^(3/2)/3 + (-1/4*((1 + 2*Sqrt[x])*Sqrt[1 + Sqrt[x] + x
]) - (3*ArcSinh[(1 + 2*Sqrt[x])/Sqrt[3]])/8)/2)
```

3.701.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

3.701.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$\frac{2\left(1+x+\sqrt{x}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{x}\right)\sqrt{1+x+\sqrt{x}}}{4}-\frac{3\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,\left(\sqrt{x}+\frac{1}{2}\right)}{3}\right)}{8}$	42
default	$\frac{2\left(1+x+\sqrt{x}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{x}\right)\sqrt{1+x+\sqrt{x}}}{4}-\frac{3\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,\left(\sqrt{x}+\frac{1}{2}\right)}{3}\right)}{8}$	42

input
```
int((1+x+x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(1+x+x^(1/2))^(3/2)-1/4*(1+2*x^(1/2))*(1+x+x^(1/2))^(1/2)-3/8*arcsinh(
2/3*3^(1/2)*(x^(1/2)+1/2))
```

3.701.5 Fricas [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int \sqrt{1+\sqrt{x}+x}\,dx = \frac{1}{12}\left(8\,x+2\,\sqrt{x}+5\right)\sqrt{x+\sqrt{x}+1} + \frac{3}{16}\,\log\left(4\,\sqrt{x+\sqrt{x}+1}\left(2\,\sqrt{x}+1\right)-8\,x-8\,\sqrt{x}-5\right)$$

input
```
integrate((1+x+x^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/12*(8*x + 2*sqrt(x) + 5)*sqrt(x + sqrt(x) + 1) + 3/16*log(4*sqrt(x + sqr
t(x) + 1)*(2*sqrt(x) + 1) - 8*x - 8*sqrt(x) - 5)
```

3.701.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.77

$$\int \sqrt{1+\sqrt{x}+x}\,dx = 2\left(\frac{\sqrt{x}}{12}+\frac{x}{3}+\frac{5}{24}\right)\sqrt{\sqrt{x}+x+1} - \frac{3\,\operatorname{asinh}\left(\frac{2\sqrt{3}\left(\sqrt{x}+\frac{1}{2}\right)}{3}\right)}{8}$$

input
```
integrate((1+x+x**(1/2))**(1/2),x)
```

output
```
2*(sqrt(x)/12 + x/3 + 5/24)*sqrt(sqrt(x) + x + 1) - 3*asinh(2*sqrt(3)*(sqr
t(x) + 1/2)/3)/8
```

3.701.7 Maxima [F]

$$\int \sqrt{1+\sqrt{x}+x}\,dx = \int \sqrt{x+\sqrt{x}+1}\,dx$$

input
```
integrate((1+x+x^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x + sqrt(x) + 1), x)
```

3.701.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \sqrt{1+\sqrt{x}+x}\,dx = \frac{1}{12}\left(2\sqrt{x}(4\sqrt{x}+1)+5\right)\sqrt{x+\sqrt{x}+1} + \frac{3}{8}\log\left(2\sqrt{x+\sqrt{x}+1}-2\sqrt{x}-1\right)$$

input
```
integrate((1+x+x^(1/2))^(1/2),x, algorithm="giac")
```

output
```
1/12*(2*sqrt(x)*(4*sqrt(x) + 1) + 5)*sqrt(x + sqrt(x) + 1) + 3/8*log(2*sqr
t(x + sqrt(x) + 1) - 2*sqrt(x) - 1)
```

3.701.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+\sqrt{x}+x}\,dx = \int \sqrt{x+\sqrt{x}+1}\,dx$$

input
```
int((x + x^(1/2) + 1)^(1/2),x)
```

output
```
int((x + x^(1/2) + 1)^(1/2), x)
```

3.702 $\int \sqrt{1+x+\sqrt{1+x}}\,dx$

3.702.1 Optimal result

Integrand size = 14, antiderivative size = 75

$$\int \sqrt{1+x+\sqrt{1+x}}\,dx = \frac{2}{3}\left(1+x+\sqrt{1+x}\right)^{3/2}-\frac{1}{4}\sqrt{1+x+\sqrt{1+x}}\left(1+2\sqrt{1+x}\right)+\frac{1}{4}\text{arctanh}\left(\frac{\sqrt{1+x}}{\sqrt{1+x+\sqrt{1+x}}}\right)$$

output
```
1/4*arctanh((1+x)^(1/2)/(1+x+(1+x)^(1/2))^(1/2))+2/3*(1+x+(1+x)^(1/2))^(3/
2)-1/4*(1+2*(1+x)^(1/2))*(1+x+(1+x)^(1/2))^(1/2)
```

3.702.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

$$\int \sqrt{1+x+\sqrt{1+x}}\,dx = \frac{1}{12}\left(\sqrt{1+x+\sqrt{1+x}}\left(5+8x+2\sqrt{1+x}\right)+3\text{arctanh}\left(\frac{\sqrt{1+x+\sqrt{1+x}}}{\sqrt{1+x}}\right)\right)$$

input
```
Integrate[Sqrt[1 + x + Sqrt[1 + x]],x]
```

output
```
(Sqrt[1 + x + Sqrt[1 + x]]*(5 + 8*x + 2*Sqrt[1 + x]) + 3*ArcTanh[Sqrt[1 +
x + Sqrt[1 + x]]/Sqrt[1 + x]])/12
```

3.702.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {7267, 2048, 1160, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x+\sqrt{x+1}+1}\,dx$$

$$\Big\downarrow \text{7267}$$

$$2\int \sqrt{x+1}\sqrt{\sqrt{x+1}\left(\sqrt{x+1}+1\right)}d\sqrt{x+1}$$

$$\Big\downarrow \text{2048}$$

$$2\int \sqrt{x+1}\sqrt{x+\sqrt{x+1}+1}d\sqrt{x+1}$$

$$\Big\downarrow \text{1160}$$

$$2\left(\frac{1}{3}\left(x+\sqrt{x+1}+1\right)^{3/2}-\frac{1}{2}\int \sqrt{x+\sqrt{x+1}+1}d\sqrt{x+1}\right)$$

$$\Big\downarrow \text{1087}$$

$$2\left(\frac{1}{2}\left(\frac{1}{8}\int \frac{1}{\sqrt{x+\sqrt{x+1}+1}}d\sqrt{x+1}-\frac{1}{4}\sqrt{x+\sqrt{x+1}+1}\left(2\sqrt{x+1}+1\right)\right)+\frac{1}{3}\left(x+\sqrt{x+1}+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{1091}$$

$$2\left(\frac{1}{2}\left(\frac{1}{4}\int -\frac{1}{x}d\frac{\sqrt{x+1}}{\sqrt{x+\sqrt{x+1}+1}}-\frac{1}{4}\sqrt{x+\sqrt{x+1}+1}\left(2\sqrt{x+1}+1\right)\right)+\frac{1}{3}\left(x+\sqrt{x+1}+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{1}{2}\left(\frac{1}{4}\text{arctanh}\left(\frac{\sqrt{x+1}}{\sqrt{x+\sqrt{x+1}+1}}\right)-\frac{1}{4}\sqrt{x+\sqrt{x+1}+1}\left(2\sqrt{x+1}+1\right)\right)+\frac{1}{3}\left(x+\sqrt{x+1}+1\right)^{3/2}\right)$$

input
```
Int[Sqrt[1 + x + Sqrt[1 + x]],x]
```

output
```
2*((1 + x + Sqrt[1 + x])^(3/2)/3 + (-1/4*(Sqrt[1 + x + Sqrt[1 + x]]*(1 + 2
*Sqrt[1 + x])) + ArcTanh[Sqrt[1 + x]/Sqrt[1 + x + Sqrt[1 + x]]]/4)/2)
```

3.702.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.702. $\int \sqrt{1+x+\sqrt{1+x}}\,dx$

3.702.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{2\left(1+x+\sqrt{x+1}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{x+1}\right)\sqrt{1+x+\sqrt{x+1}}}{4}+\frac{\ln\left(\frac{1}{2}+\sqrt{x+1}+\sqrt{1+x+\sqrt{x+1}}\right)}{8}$	55
default	$\frac{2\left(1+x+\sqrt{x+1}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{x+1}\right)\sqrt{1+x+\sqrt{x+1}}}{4}+\frac{\ln\left(\frac{1}{2}+\sqrt{x+1}+\sqrt{1+x+\sqrt{x+1}}\right)}{8}$	55

input
```
int((1+x+(x+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(1+x+(x+1)^(1/2))^(3/2)-1/4*(1+2*(x+1)^(1/2))*(1+x+(x+1)^(1/2))^(1/2)+
1/8*ln(1/2+(x+1)^(1/2)+(1+x+(x+1)^(1/2))^(1/2))
```

3.702.5 Fricas [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int\sqrt{1+x+\sqrt{1+x}}\,dx=\frac{1}{12}\left(8x+2\sqrt{x+1}+5\right)\sqrt{x+\sqrt{x+1}+1}+\frac{1}{16}\log\left(-4\sqrt{x+\sqrt{x+1}+1}\left(2\sqrt{x+1}+1\right)-8x-8\sqrt{x+1}-9\right)$$

input
```
integrate((1+x+(1+x)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/12*(8*x + 2*sqrt(x + 1) + 5)*sqrt(x + sqrt(x + 1) + 1) + 1/16*log(-4*sqr
t(x + sqrt(x + 1) + 1)*(2*sqrt(x + 1) + 1) - 8*x - 8*sqrt(x + 1) - 9)
```

3.702.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.77

$$\int \sqrt{1+x+\sqrt{1+x}}\,dx = 2\left(\frac{x}{3}+\frac{\sqrt{x+1}}{12}+\frac{5}{24}\right)\sqrt{x+\sqrt{x+1}+1} + \frac{\log\left(2\sqrt{x+1}+2\sqrt{x+\sqrt{x+1}+1}+1\right)}{8}$$

input
```
integrate((1+x+(1+x)**(1/2))**(1/2),x)
```

output
```
2*(x/3 + sqrt(x + 1)/12 + 5/24)*sqrt(x + sqrt(x + 1) + 1) + log(2*sqrt(x +
 1) + 2*sqrt(x + sqrt(x + 1) + 1) + 1)/8
```

3.702.7 Maxima [F]

$$\int \sqrt{1+x+\sqrt{1+x}}\,dx = \int \sqrt{x+\sqrt{x+1}+1}\,dx$$

input
```
integrate((1+x+(1+x)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x + sqrt(x + 1) + 1), x)
```

3.702.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.73

$$\int \sqrt{1+x+\sqrt{1+x}}\,dx = \frac{1}{12}\left(2\sqrt{x+1}\left(4\sqrt{x+1}+1\right)-3\right)\sqrt{x+\sqrt{x+1}+1} - \frac{1}{8}\log\left(-2\sqrt{x+\sqrt{x+1}+1}+2\sqrt{x+1}+1\right)$$

input
```
integrate((1+x+(1+x)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
1/12*(2*sqrt(x + 1)*(4*sqrt(x + 1) + 1) - 3)*sqrt(x + sqrt(x + 1) + 1) - 1
/8*log(-2*sqrt(x + sqrt(x + 1) + 1) + 2*sqrt(x + 1) + 1)
```

3.702.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+x+\sqrt{1+x}}\,dx = \int \sqrt{x+\sqrt{x+1}+1}\,dx$$

input
```
int((x + (x + 1)^(1/2) + 1)^(1/2),x)
```

output
```
int((x + (x + 1)^(1/2) + 1)^(1/2), x)
```

3.703 $\int \sqrt{\sqrt{-1+x}+x}\,dx$

3.703.1 Optimal result

Integrand size = 13, antiderivative size = 68

$$\int \sqrt{\sqrt{-1+x}+x}\,dx=-\frac{1}{4}\left(1+2\sqrt{-1+x}\right)\sqrt{\sqrt{-1+x}+x}+\frac{2}{3}\left(\sqrt{-1+x}+x\right)^{3/2}-\frac{3}{8}\operatorname{arcsinh}\left(\frac{1+2\sqrt{-1+x}}{\sqrt{3}}\right)$$

output
```
-3/8*arcsinh(1/3*(1+2*(-1+x)^(1/2))*3^(1/2))+2/3*(x+(-1+x)^(1/2))^(3/2)-1/
4*(1+2*(-1+x)^(1/2))*(x+(-1+x)^(1/2))^(1/2)
```

3.703.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int \sqrt{\sqrt{-1+x}+x}\,dx=\frac{1}{12}\left(5+2\sqrt{-1+x}+8(-1+x)\right)\sqrt{\sqrt{-1+x}+x}+\frac{3}{8}\log\left(-1-2\sqrt{-1+x}+2\sqrt{\sqrt{-1+x}+x}\right)$$

input
```
Integrate[Sqrt[Sqrt[-1 + x] + x],x]
```

output
```
((5 + 2*Sqrt[-1 + x] + 8*(-1 + x))*Sqrt[Sqrt[-1 + x] + x])/12 + (3*Log[-1
- 2*Sqrt[-1 + x] + 2*Sqrt[Sqrt[-1 + x] + x]])/8
```

3.703.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {7267, 1160, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x+\sqrt{x-1}}\,dx$$

$$\Big\downarrow \text{7267}$$

$$2\int \sqrt{x-1}\sqrt{x+\sqrt{x-1}}d\sqrt{x-1}$$

$$\Big\downarrow \text{1160}$$

$$2\left(\frac{1}{3}\left(x+\sqrt{x-1}\right)^{3/2}-\frac{1}{2}\int \sqrt{x+\sqrt{x-1}}d\sqrt{x-1}\right)$$

$$\Big\downarrow \text{1087}$$

$$2\left(\frac{1}{2}\left(-\frac{3}{8}\int \frac{1}{\sqrt{x+\sqrt{x-1}}}d\sqrt{x-1}-\frac{1}{4}\sqrt{x+\sqrt{x-1}}\left(2\sqrt{x-1}+1\right)\right)+\frac{1}{3}\left(x+\sqrt{x-1}\right)^{3/2}\right)$$

$$\Big\downarrow \text{1090}$$

$$2\left(\frac{1}{2}\left(-\frac{1}{8}\sqrt{3}\int \frac{1}{\sqrt{\frac{x-1}{3}+1}}d\left(2\sqrt{x-1}+1\right)-\frac{1}{4}\sqrt{x+\sqrt{x-1}}\left(2\sqrt{x-1}+1\right)\right)+\frac{1}{3}\left(x+\sqrt{x-1}\right)^{3/2}\right)$$

$$\Big\downarrow \text{222}$$

$$2\left(\frac{1}{2}\left(-\frac{3}{8}\text{arcsinh}\left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right)-\frac{1}{4}\sqrt{x+\sqrt{x-1}}\left(2\sqrt{x-1}+1\right)\right)+\frac{1}{3}\left(x+\sqrt{x-1}\right)^{3/2}\right)$$

input
```
Int[Sqrt[Sqrt[-1 + x] + x],x]
```

output
```
2*((Sqrt[-1 + x] + x)^(3/2)/3 + (-1/4*((1 + 2*Sqrt[-1 + x])*Sqrt[Sqrt[-1 +
 x] + x]) - (3*ArcSinh[(1 + 2*Sqrt[-1 + x])/Sqrt[3]])/8)/2)
```

3.703.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.703.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{2\left(x+\sqrt{x-1}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{x-1}\right)\sqrt{x+\sqrt{x-1}}}{4}-\frac{3\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,\left(\sqrt{x-1}+\frac{1}{2}\right)}{3}\right)}{8}$	48
default	$\frac{2\left(x+\sqrt{x-1}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{x-1}\right)\sqrt{x+\sqrt{x-1}}}{4}-\frac{3\,\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,\left(\sqrt{x-1}+\frac{1}{2}\right)}{3}\right)}{8}$	48

input
```
int((x+(x-1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(x+(x-1)^(1/2))^(3/2)-1/4*(1+2*(x-1)^(1/2))*(x+(x-1)^(1/2))^(1/2)-3/8*
arcsinh(2/3*3^(1/2)*((x-1)^(1/2)+1/2))
```

3.703.5 Fricas [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \sqrt{\sqrt{-1+x}+x}\,dx = \frac{1}{12}\left(8\,x+2\,\sqrt{x-1}-3\right)\sqrt{x+\sqrt{x-1}} + \frac{3}{16}\log\left(-4\,\sqrt{x+\sqrt{x-1}}\left(2\,\sqrt{x-1}+1\right)+8\,x+8\,\sqrt{x-1}-3\right)$$

input
```
integrate((x+(-1+x)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/12*(8*x + 2*sqrt(x - 1) - 3)*sqrt(x + sqrt(x - 1)) + 3/16*log(-4*sqrt(x
+ sqrt(x - 1))*(2*sqrt(x - 1) + 1) + 8*x + 8*sqrt(x - 1) - 3)
```

3.703.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.75

$$\int \sqrt{\sqrt{-1+x}+x}\,dx = 2\sqrt{x+\sqrt{x-1}}\left(\frac{x}{3}+\frac{\sqrt{x-1}}{12}-\frac{1}{8}\right)-\frac{3\,\operatorname{asinh}\left(\frac{2\sqrt{3}\left(\sqrt{x-1}+\frac{1}{2}\right)}{3}\right)}{8}$$

input
```
integrate((x+(-1+x)**(1/2))**(1/2),x)
```

output
```
2*sqrt(x + sqrt(x - 1))*(x/3 + sqrt(x - 1)/12 - 1/8) - 3*asinh(2*sqrt(3)*(
sqrt(x - 1) + 1/2)/3)/8
```

3.703.7 Maxima [F]

$$\int \sqrt{\sqrt{-1+x}+x}\,dx = \int \sqrt{x+\sqrt{x-1}}\,dx$$

input
```
integrate((x+(-1+x)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x + sqrt(x - 1)), x)
```

3.703.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

$$\int \sqrt{\sqrt{-1+x}+x}\,dx = \frac{1}{12}\left(2\sqrt{x-1}(4\sqrt{x-1}+1)+5\right)\sqrt{x+\sqrt{x-1}} + \frac{3}{8}\log\left(2\sqrt{x+\sqrt{x-1}}-2\sqrt{x-1}-1\right)$$

input
```
integrate((x+(-1+x)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
1/12*(2*sqrt(x - 1)*(4*sqrt(x - 1) + 1) + 5)*sqrt(x + sqrt(x - 1)) + 3/8*l
og(2*sqrt(x + sqrt(x - 1)) - 2*sqrt(x - 1) - 1)
```

3.703.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sqrt{-1+x}+x}\,dx = \int \sqrt{x+\sqrt{x-1}}\,dx$$

input
```
int((x + (x - 1)^(1/2))^(1/2),x)
```

output
```
int((x + (x - 1)^(1/2))^(1/2), x)
```

3.704 $\int \sqrt{2x+\sqrt{-1+2x}}\,dx$

3.704.1 Optimal result

Integrand size = 17, antiderivative size = 80

$$\int \sqrt{2x+\sqrt{-1+2x}}\,dx=\frac{1}{3}\left(2x+\sqrt{-1+2x}\right)^{3/2}-\frac{1}{8}\sqrt{2x+\sqrt{-1+2x}}\left(1+2\sqrt{-1+2x}\right)-\frac{3}{16}\text{arcsinh}\left(\frac{1+2\sqrt{-1+2x}}{\sqrt{3}}\right)$$

output
```
-3/16*arcsinh(1/3*(1+2*(-1+2*x)^(1/2))*3^(1/2))+1/3*(2*x+(-1+2*x)^(1/2))^(
3/2)-1/8*(1+2*(-1+2*x)^(1/2))*(2*x+(-1+2*x)^(1/2))^(1/2)
```

3.704.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \sqrt{2x+\sqrt{-1+2x}}\,dx=\frac{1}{48}\left(2\sqrt{2x+\sqrt{-1+2x}}\left(-3+16x+2\sqrt{-1+2x}\right)+9\log\left(-1-2\sqrt{-1+2x}+2\sqrt{2x+\sqrt{-1+2x}}\right)\right)$$

input
```
Integrate[Sqrt[2*x + Sqrt[-1 + 2*x]],x]
```

output
```
(2*Sqrt[2*x + Sqrt[-1 + 2*x]]*(-3 + 16*x + 2*Sqrt[-1 + 2*x]) + 9*Log[-1 -
2*Sqrt[-1 + 2*x] + 2*Sqrt[2*x + Sqrt[-1 + 2*x]]])/48
```

3.704.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {7267, 1160, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{2x+\sqrt{2x-1}}dx$$

$$\Big\downarrow \text{7267}$$

$$\int \sqrt{2x-1}\sqrt{2x+\sqrt{2x-1}}d\sqrt{2x-1}$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{3}\left(2x+\sqrt{2x-1}\right)^{3/2}-\frac{1}{2}\int \sqrt{2x+\sqrt{2x-1}}d\sqrt{2x-1}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(-\frac{3}{8}\int \frac{1}{\sqrt{2x+\sqrt{2x-1}}}d\sqrt{2x-1}-\frac{1}{4}\sqrt{2x+\sqrt{2x-1}}\left(2\sqrt{2x-1}+1\right)\right)+\frac{1}{3}\left(2x+\sqrt{2x-1}\right)^{3/2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{2}\left(-\frac{1}{8}\sqrt{3}\int \frac{1}{\sqrt{\frac{1}{3}(2x-1)+1}}d\left(2\sqrt{2x-1}+1\right)-\frac{1}{4}\sqrt{2x+\sqrt{2x-1}}\left(2\sqrt{2x-1}+1\right)\right)+\frac{1}{3}\left(2x+\sqrt{2x-1}\right)^{3/2}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\left(-\frac{3}{8}\text{arcsinh}\left(\frac{2\sqrt{2x-1}+1}{\sqrt{3}}\right)-\frac{1}{4}\sqrt{2x+\sqrt{2x-1}}\left(2\sqrt{2x-1}+1\right)\right)+\frac{1}{3}\left(2x+\sqrt{2x-1}\right)^{3/2}$$

input
```
Int[Sqrt[2*x + Sqrt[-1 + 2*x]],x]
```

output
```
(2*x + Sqrt[-1 + 2*x])^(3/2)/3 + (-1/4*(Sqrt[2*x + Sqrt[-1 + 2*x]]*(1 + 2*
Sqrt[-1 + 2*x])) - (3*ArcSinh[(1 + 2*Sqrt[-1 + 2*x])/Sqrt[3]])/8)/2
```

3.704.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.704.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\left(2x+\sqrt{2x-1}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{2x-1}\right)\sqrt{2x+\sqrt{2x-1}}}{8}-\frac{3\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\,\left(\sqrt{2x-1}+\frac{1}{2}\right)}{3}\right)}{16}$	60
default	$\frac{\left(2x+\sqrt{2x-1}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{2x-1}\right)\sqrt{2x+\sqrt{2x-1}}}{8}-\frac{3\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\,\left(\sqrt{2x-1}+\frac{1}{2}\right)}{3}\right)}{16}$	60

input
```
int((2*x+(2*x-1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*x+(2*x-1)^(1/2))^(3/2)-1/8*(1+2*(2*x-1)^(1/2))*(2*x+(2*x-1)^(1/2))^
(1/2)-3/16*arcsinh(2/3*3^(1/2)*((2*x-1)^(1/2)+1/2))
```

3.704.5 Fricas [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

$$\int \sqrt{2x+\sqrt{-1+2x}}\,dx = \frac{1}{24}\left(16\,x+2\,\sqrt{2\,x-1}-3\right)\sqrt{2\,x+\sqrt{2\,x-1}} + \frac{3}{32}\,\log\left(-4\,\sqrt{2\,x+\sqrt{2\,x-1}}\left(2\,\sqrt{2\,x-1}+1\right)+16\,x + 8\,\sqrt{2\,x-1}-3\right)$$

input
```
integrate((2*x+(-1+2*x)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/24*(16*x + 2*sqrt(2*x - 1) - 3)*sqrt(2*x + sqrt(2*x - 1)) + 3/32*log(-4*
sqrt(2*x + sqrt(2*x - 1))*(2*sqrt(2*x - 1) + 1) + 16*x + 8*sqrt(2*x - 1) -
 3)
```

3.704.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

$$\int \sqrt{2x+\sqrt{-1+2x}}\,dx = \sqrt{2x+\sqrt{2x-1}}\cdot\left(\frac{2x}{3}+\frac{\sqrt{2x-1}}{12}-\frac{1}{8}\right)-\frac{3\,\operatorname{asinh}\left(\frac{2\sqrt{3}\left(\sqrt{2x-1}+\frac{1}{2}\right)}{3}\right)}{16}$$

input
```
integrate((2*x+(-1+2*x)**(1/2))**(1/2),x)
```

output
```
sqrt(2*x + sqrt(2*x - 1))*(2*x/3 + sqrt(2*x - 1)/12 - 1/8) - 3*asinh(2*sqr
t(3)*(sqrt(2*x - 1) + 1/2)/3)/16
```

3.704.7 Maxima [F]

$$\int \sqrt{2x+\sqrt{-1+2x}}\,dx = \int \sqrt{2\,x+\sqrt{2\,x-1}}\,dx$$

input
```
integrate((2*x+(-1+2*x)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(2*x + sqrt(2*x - 1)), x)
```

3.704.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

$$\int \sqrt{2x+\sqrt{-1+2x}}\,dx = \frac{1}{24}\left(2\sqrt{2\,x-1}(4\sqrt{2\,x-1}+1)+5\right)\sqrt{2\,x+\sqrt{2\,x-1}} + \frac{3}{16}\log\left(2\sqrt{2\,x+\sqrt{2\,x-1}}-2\sqrt{2\,x-1}-1\right)$$

input
```
integrate((2*x+(-1+2*x)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
1/24*(2*sqrt(2*x - 1)*(4*sqrt(2*x - 1) + 1) + 5)*sqrt(2*x + sqrt(2*x - 1))
 + 3/16*log(2*sqrt(2*x + sqrt(2*x - 1)) - 2*sqrt(2*x - 1) - 1)
```

3.704.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{2x+\sqrt{-1+2x}}\,dx = \int \sqrt{2\,x+\sqrt{2\,x-1}}\,dx$$

input
```
int((2*x + (2*x - 1)^(1/2))^(1/2),x)
```

output
```
int((2*x + (2*x - 1)^(1/2))^(1/2), x)
```

3.705 $\int \sqrt{3x+\sqrt{-7+8x}}\,dx$

3.705.1 Optimal result

Integrand size = 17, antiderivative size = 109

$$\int \sqrt{3x+\sqrt{-7+8x}}\,dx = -\frac{\left(4+3\sqrt{-7+8x}\right)\sqrt{21-3(7-8x)+8\sqrt{-7+8x}}}{36\sqrt{2}} + \frac{\left(21-3(7-8x)+8\sqrt{-7+8x}\right)^{3/2}}{72\sqrt{2}} - \frac{47\operatorname{arcsinh}\left(\frac{4+3\sqrt{-7+8x}}{\sqrt{47}}\right)}{36\sqrt{6}}$$

output
```
-47/216*arcsinh(1/47*(4+3*(-7+8*x)^(1/2))*47^(1/2))*6^(1/2)+1/144*(24*x+8*
(-7+8*x)^(1/2))^(3/2)*2^(1/2)-1/36*(4+3*(-7+8*x)^(1/2))*(6*x+2*(-7+8*x)^(1
/2))^(1/2)*2^(1/2)
```

3.705.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.76

$$\int \sqrt{3x+\sqrt{-7+8x}}\,dx = \frac{1}{18}\sqrt{3x+\sqrt{-7+8x}}\left(-4+12x+\sqrt{-7+8x}\right) + \frac{47\log\left(-4-3\sqrt{-7+8x}+2\sqrt{6}\sqrt{3x+\sqrt{-7+8x}}\right)}{36\sqrt{6}}$$

input
```
Integrate[Sqrt[3*x + Sqrt[-7 + 8*x]],x]
```

output
```
(Sqrt[3*x + Sqrt[-7 + 8*x]]*(-4 + 12*x + Sqrt[-7 + 8*x]))/18 + (47*Log[-4
- 3*Sqrt[-7 + 8*x] + 2*Sqrt[6]*Sqrt[3*x + Sqrt[-7 + 8*x]]])/(36*Sqrt[6])
```

3.705.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {7267, 27, 1160, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3x+\sqrt{8x-7}}\,dx$$

$$\Big\downarrow \text{7267}$$

$$\frac{1}{4}\int \frac{\sqrt{8x-7}\sqrt{3(8x-7)+8\sqrt{8x-7}+21}}{2\sqrt{2}}d\sqrt{8x-7}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt{8x-7}\sqrt{3(8x-7)+8\sqrt{8x-7}+21}d\sqrt{8x-7}}{8\sqrt{2}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{\frac{1}{9}\left(3(8x-7)+8\sqrt{8x-7}+21\right)^{3/2}-\frac{4}{3}\int \sqrt{3(8x-7)+8\sqrt{8x-7}+21}d\sqrt{8x-7}}{8\sqrt{2}}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\frac{1}{9}\left(3(8x-7)+8\sqrt{8x-7}+21\right)^{3/2}-\frac{4}{3}\left(\frac{47}{6}\int \frac{1}{\sqrt{3(8x-7)+8\sqrt{8x-7}+21}}d\sqrt{8x-7}+\frac{1}{6}\sqrt{3(8x-7)+8\sqrt{8x-7}+21}(3\sqrt{8}\right.}{8\sqrt{2}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{\frac{1}{9}\left(3(8x-7)+8\sqrt{8x-7}+21\right)^{3/2}-\frac{4}{3}\left(\frac{1}{12}\sqrt{\frac{47}{3}}\int \frac{1}{\sqrt{\frac{1}{188}(8x-7)+1}}d\left(6\sqrt{8x-7}+8\right)+\frac{1}{6}\sqrt{3(8x-7)+8\sqrt{8x-7}+21}\right.}{8\sqrt{2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{\frac{1}{9}(3(8x-7)+8\sqrt{8x-7}+21)^{3/2}-\frac{4}{3}\left(\frac{47\text{arcsinh}\left(\frac{6\sqrt{8x-7}+8}{2\sqrt{47}}\right)}{6\sqrt{3}}+\frac{1}{6}\sqrt{3(8x-7)+8\sqrt{8x-7}+21}(3\sqrt{8x-7}+4)\right)}{8\sqrt{2}}$$

input
```
Int[Sqrt[3*x + Sqrt[-7 + 8*x]],x]
```

output
```
((21 + 8*Sqrt[-7 + 8*x] + 3*(-7 + 8*x))^(3/2)/9 - (4*(((4 + 3*Sqrt[-7 + 8*
x])*Sqrt[21 + 8*Sqrt[-7 + 8*x] + 3*(-7 + 8*x)])/6 + (47*ArcSinh[(8 + 6*Sqr
t[-7 + 8*x])/(2*Sqrt[47])])/(6*Sqrt[3])))/3)/(8*Sqrt[2])
```

3.705.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.705.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.61

method	result	size
derivativedivides	$\frac{\left(48x+16\sqrt{-7+8x}\right)^{\frac{3}{2}}}{288}-\frac{\left(12\sqrt{-7+8x}+16\right)\sqrt{48x+16\sqrt{-7+8x}}}{288}-\frac{47\sqrt{6}\,\operatorname{arcsinh}\left(\frac{3\sqrt{47}\left(\sqrt{-7+8x}+\frac{4}{3}\right)}{47}\right)}{216}$	67
default	$\frac{\left(48x+16\sqrt{-7+8x}\right)^{\frac{3}{2}}}{288}-\frac{\left(12\sqrt{-7+8x}+16\right)\sqrt{48x+16\sqrt{-7+8x}}}{288}-\frac{47\sqrt{6}\,\operatorname{arcsinh}\left(\frac{3\sqrt{47}\left(\sqrt{-7+8x}+\frac{4}{3}\right)}{47}\right)}{216}$	67

input
```
int((3*x+(-7+8*x)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/288*(48*x+16*(-7+8*x)^(1/2))^(3/2)-1/288*(12*(-7+8*x)^(1/2)+16)*(48*x+16
*(-7+8*x)^(1/2))^(1/2)-47/216*6^(1/2)*arcsinh(3/47*47^(1/2)*((-7+8*x)^(1/2
)+4/3))
```

3.705.5 Fricas [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\begin{aligned}&\int\sqrt{3x+\sqrt{-7+8x}}\,dx\\&=\frac{1}{18}\left(12\,x+\sqrt{8\,x-7}-4\right)\sqrt{3\,x+\sqrt{8\,x-7}}\\&\quad+\frac{47}{864}\sqrt{6}\log\Bigg(-41472\,x^2-192\,(144\,x-47)\sqrt{8\,x-7}\\&\qquad+8\left(3\sqrt{6}(144\,x+17)\sqrt{8\,x-7}+4\sqrt{6}(432\,x-299)\right)\sqrt{3\,x+\sqrt{8\,x-7}}-9792\,x\\&\qquad+30047\Bigg)\end{aligned}$$

input
```
integrate((3*x+(-7+8*x)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/18*(12*x + sqrt(8*x - 7) - 4)*sqrt(3*x + sqrt(8*x - 7)) + 47/864*sqrt(6)
*log(-41472*x^2 - 192*(144*x - 47)*sqrt(8*x - 7) + 8*(3*sqrt(6)*(144*x + 1
7)*sqrt(8*x - 7) + 4*sqrt(6)*(432*x - 299))*sqrt(3*x + sqrt(8*x - 7)) - 97
92*x + 30047)
```

3.705.6 Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.61

$$\int \sqrt{3x+\sqrt{-7+8x}}\,dx = \frac{\sqrt{3x+\sqrt{8x-7}}\cdot\left(\frac{8x}{3}+\frac{2\sqrt{8x-7}}{9}-\frac{8}{9}\right)}{4} - \frac{47\sqrt{6}\operatorname{asinh}\left(\frac{3\sqrt{47}\left(\sqrt{8x-7}+\frac{4}{3}\right)}{47}\right)}{216}$$

input
```
integrate((3*x+(-7+8*x)**(1/2))**(1/2),x)
```

output
```
sqrt(3*x + sqrt(8*x - 7))*(8*x/3 + 2*sqrt(8*x - 7)/9 - 8/9)/4 - 47*sqrt(6)
*asinh(3*sqrt(47)*(sqrt(8*x - 7) + 4/3)/47)/216
```

3.705.7 Maxima [F]

$$\int \sqrt{3x+\sqrt{-7+8x}}\,dx = \int \sqrt{3\,x+\sqrt{8\,x-7}}\,dx$$

input
```
integrate((3*x+(-7+8*x)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(3*x + sqrt(8*x - 7)), x)
```

3.705.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.81

$$\int \sqrt{3x+\sqrt{-7+8x}}\,dx$$
$$=\frac{1}{216}\sqrt{2}\left(3\sqrt{2}\left(\sqrt{8x-7}\left(3\sqrt{8x-7}+2\right)+13\right)\sqrt{3x+\sqrt{8x-7}}+47\sqrt{3}\log\left(-\sqrt{3}\left(\sqrt{3}\sqrt{8x-7}-2\sqrt{\right.\right.\right.$$

input
```
integrate((3*x+(-7+8*x)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
1/216*sqrt(2)*(3*sqrt(2)*(sqrt(8*x - 7)*(3*sqrt(8*x - 7) + 2) + 13)*sqrt(3
*x + sqrt(8*x - 7)) + 47*sqrt(3)*log(-sqrt(3)*(sqrt(3)*sqrt(8*x - 7) - 2*s
qrt(2)*sqrt(3*x + sqrt(8*x - 7))) - 4))
```

3.705.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{3x+\sqrt{-7+8x}}\,dx=\int \sqrt{3\,x+\sqrt{8\,x-7}}\,dx$$

input
```
int((3*x + (8*x - 7)^(1/2))^(1/2),x)
```

output
```
int((3*x + (8*x - 7)^(1/2))^(1/2), x)
```

3.706 $\int \frac{1}{\sqrt{x+\sqrt{1+x}}}\,dx$

3.706.1 Optimal result

Integrand size = 13, antiderivative size = 47

$$\int \frac{1}{\sqrt{x+\sqrt{1+x}}}\,dx = 2\sqrt{x+\sqrt{1+x}} - \operatorname{arctanh}\left(\frac{1+2\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)$$

output
```
-arctanh(1/2*(1+2*(1+x)^(1/2))/(x+(1+x)^(1/2))^(1/2))+2*(x+(1+x)^(1/2))^(1
/2)
```

3.706.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{x+\sqrt{1+x}}}\,dx = 2\sqrt{x+\sqrt{1+x}} + \log\left(-1-2\sqrt{1+x}+2\sqrt{x+\sqrt{1+x}}\right)$$

input
```
Integrate[1/Sqrt[x + Sqrt[1 + x]],x]
```

output
```
2*Sqrt[x + Sqrt[1 + x]] + Log[-1 - 2*Sqrt[1 + x] + 2*Sqrt[x + Sqrt[1 + x]]
]
```

3.706.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {7267, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x+\sqrt{x+1}}}dx$$

$$\Big\downarrow \text{7267}$$

$$2\int \frac{\sqrt{x+1}}{\sqrt{x+\sqrt{x+1}}}d\sqrt{x+1}$$

$$\Big\downarrow \text{1160}$$

$$2\left(\sqrt{x+\sqrt{x+1}}-\frac{1}{2}\int \frac{1}{\sqrt{x+\sqrt{x+1}}}d\sqrt{x+1}\right)$$

$$\Big\downarrow \text{1092}$$

$$2\left(\sqrt{x+\sqrt{x+1}}-\int \frac{1}{3-x}d\frac{2\sqrt{x+1}+1}{\sqrt{x+\sqrt{x+1}}}\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\sqrt{x+\sqrt{x+1}}-\frac{1}{2}\text{arctanh}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)\right)$$

input
```
Int[1/Sqrt[x + Sqrt[1 + x]],x]
```

output
```
2*(Sqrt[x + Sqrt[1 + x]] - ArcTanh[(1 + 2*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1
+ x]])]/2)
```

3.706.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.706.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$2\sqrt{x+\sqrt{x+1}}-\ln\left(\frac{1}{2}+\sqrt{x+1}+\sqrt{x+\sqrt{x+1}}\right)$	32
default	$2\sqrt{x+\sqrt{x+1}}-\ln\left(\frac{1}{2}+\sqrt{x+1}+\sqrt{x+\sqrt{x+1}}\right)$	32

input
```
int(1/(x+(x+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(x+(x+1)^(1/2))^(1/2)-ln(1/2+(x+1)^(1/2)+(x+(x+1)^(1/2))^(1/2))
```

3.706.5 Fricas [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x+\sqrt{1+x}}}\,dx = 2\sqrt{x+\sqrt{x+1}} + \frac{1}{2}\log\left(4\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}+1\right)-8\,x-8\sqrt{x+1}-5\right)$$

input
```
integrate(1/(x+(1+x)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(x + sqrt(x + 1)) + 1/2*log(4*sqrt(x + sqrt(x + 1))*(2*sqrt(x + 1) +
 1) - 8*x - 8*sqrt(x + 1) - 5)
```

3.706.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{x+\sqrt{1+x}}}\,dx = 2\sqrt{x+\sqrt{x+1}} - \log\left(2\sqrt{x+1}+2\sqrt{x+\sqrt{x+1}}+1\right)$$

input
```
integrate(1/(x+(1+x)**(1/2))**(1/2),x)
```

output
```
2*sqrt(x + sqrt(x + 1)) - log(2*sqrt(x + 1) + 2*sqrt(x + sqrt(x + 1)) + 1)
```

3.706.7 Maxima [F]

$$\int \frac{1}{\sqrt{x+\sqrt{1+x}}}\,dx = \int \frac{1}{\sqrt{x+\sqrt{x+1}}}\,dx$$

input
```
integrate(1/(x+(1+x)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(x + sqrt(x + 1)), x)
```

3.706.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{x+\sqrt{1+x}}}\,dx = 2\sqrt{x+\sqrt{x+1}}+\log\left(-2\sqrt{x+\sqrt{x+1}}+2\sqrt{x+1}+1\right)$$

input
```
integrate(1/(x+(1+x)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(x + sqrt(x + 1)) + log(-2*sqrt(x + sqrt(x + 1)) + 2*sqrt(x + 1) + 1
)
```

3.706.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x+\sqrt{1+x}}}\,dx = \int \frac{1}{\sqrt{x+\sqrt{x+1}}}\,dx$$

input
```
int(1/(x + (x + 1)^(1/2))^(1/2),x)
```

output
```
int(1/(x + (x + 1)^(1/2))^(1/2), x)
```

3.707 $\int \frac{1+x}{4+x+\sqrt{-9+6x}}\,dx$

3.707.1 Optimal result

Integrand size = 18, antiderivative size = 67

$$\int \frac{1+x}{4+x+\sqrt{-9+6x}}\,dx = x-2\sqrt{3}\sqrt{-3+2x}+4\sqrt{6}\arctan\left(\frac{3+\sqrt{-9+6x}}{2\sqrt{6}}\right)+3\log\left(4+x+\sqrt{3}\sqrt{-3+2x}\right)$$

output
```
x+3*ln(4+x+(-3+2*x)^(1/2)*3^(1/2))+4*arctan(1/12*(3+(-9+6*x)^(1/2))*6^(1/2
))*6^(1/2)-2*(-3+2*x)^(1/2)*3^(1/2)
```

3.707.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \frac{1+x}{4+x+\sqrt{-9+6x}}\,dx = -\frac{3}{2}+x-2\sqrt{-9+6x}+4\sqrt{6}\arctan\left(\frac{\sqrt{3}+\sqrt{-3+2x}}{2\sqrt{2}}\right)+3\log\left(4+x+\sqrt{-9+6x}\right)$$

input
```
Integrate[(1 + x)/(4 + x + Sqrt[-9 + 6*x]),x]
```

output
```
-3/2 + x - 2*Sqrt[-9 + 6*x] + 4*Sqrt[6]*ArcTan[(Sqrt[3] + Sqrt[-3 + 2*x])/
(2*Sqrt[2])] + 3*Log[4 + x + Sqrt[-9 + 6*x]]
```

3.707.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {7267, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{x+\sqrt{6x-9}+4}dx$$

$$\Big\downarrow \text{7267}$$

$$\frac{1}{3}\int \frac{\sqrt{6x-9}(6x+6)}{6x+6\sqrt{6x-9}+24}d\sqrt{6x-9}$$

$$\Big\downarrow \text{2159}$$

$$\frac{1}{3}\int \left(\frac{18\left(\sqrt{6x-9}+11\right)}{6x+6\sqrt{6x-9}+24}+\sqrt{6x-9}-6\right)d\sqrt{6x-9}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(12\sqrt{6}\arctan\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)+\frac{1}{2}(6x-9)-6\sqrt{6x-9}+9\log\left(6x+6\sqrt{6x-9}+24\right)\right)$$

input
```
Int[(1 + x)/(4 + x + Sqrt[-9 + 6*x]),x]
```

output
```
(-6*Sqrt[-9 + 6*x] + (-9 + 6*x)/2 + 12*Sqrt[6]*ArcTan[(3 + Sqrt[-9 + 6*x])
/(2*Sqrt[6])] + 9*Log[24 + 6*x + 6*Sqrt[-9 + 6*x]])/3
```

3.707.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.707.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

method	result
derivativedivides	$-\frac{3}{2}+x-2\sqrt{-9+6x}+3\ln\left(24+6x+6\sqrt{-9+6x}\right)+4\sqrt{6}\,\arctan\left(\frac{(2\sqrt{-9+6x}+6)\sqrt{6}}{24}\right)$
default	$-\frac{3}{2}+x-2\sqrt{-9+6x}+3\ln\left(24+6x+6\sqrt{-9+6x}\right)+4\sqrt{6}\,\arctan\left(\frac{(2\sqrt{-9+6x}+6)\sqrt{6}}{24}\right)$
trager	$x-2\sqrt{-9+6x}+\mathrm{RootOf}\left(\_Z^2-6\_Z+33\right)\ln\left(4+x+\sqrt{-9+6x}\right)-\ln\left(161\,\mathrm{RootO}\right.$

input
```
int((x+1)/(4+x+(-9+6*x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-3/2+x-2*(-9+6*x)^(1/2)+3*ln(24+6*x+6*(-9+6*x)^(1/2))+4*6^(1/2)*arctan(1/2
4*(2*(-9+6*x)^(1/2)+6)*6^(1/2))
```

3.707.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \frac{1+x}{4+x+\sqrt{-9+6x}}\,dx = 4\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}\sqrt{6x-9}+\frac{1}{4}\sqrt{6}\right)+x - 2\sqrt{6x-9}+3\log\left(x+\sqrt{6x-9}+4\right)$$

input
```
integrate((1+x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="fricas")
```

output
```
4*sqrt(6)*arctan(1/12*sqrt(6)*sqrt(6*x - 9) + 1/4*sqrt(6)) + x - 2*sqrt(6*
x - 9) + 3*log(x + sqrt(6*x - 9) + 4)
```

3.707.6 Sympy [A] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

$$\int \frac{1+x}{4+x+\sqrt{-9+6x}}\,dx = x-2\sqrt{6x-9}+3\log\left(6x+6\sqrt{6x-9}+24\right) + 4\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\left(\sqrt{6x-9}+3\right)}{12}\right)-\frac{3}{2}$$

input
```
integrate((1+x)/(4+x+(-9+6*x)**(1/2)),x)
```

output
```
x - 2*sqrt(6*x - 9) + 3*log(6*x + 6*sqrt(6*x - 9) + 24) + 4*sqrt(6)*atan(s
qrt(6)*(sqrt(6*x - 9) + 3)/12) - 3/2
```

3.707.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

$$\int \frac{1+x}{4+x+\sqrt{-9+6x}}\,dx = 4\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}\left(\sqrt{6x-9}+3\right)\right)+x - 2\sqrt{6x-9}+3\log\left(6x+6\sqrt{6x-9}+24\right)-\frac{3}{2}$$

input
```
integrate((1+x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="maxima")
```

output
```
4*sqrt(6)*arctan(1/12*sqrt(6)*(sqrt(6*x - 9) + 3)) + x - 2*sqrt(6*x - 9) +
 3*log(6*x + 6*sqrt(6*x - 9) + 24) - 3/2
```

3.707.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.90

$$\int \frac{1+x}{4+x+\sqrt{-9+6x}}\,dx = 4\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(\sqrt{3}+\sqrt{2x-3}\right)\right)-2\sqrt{3}\sqrt{2x-3} + x+3\log\left(2\sqrt{3}\sqrt{2x-3}+2x+8\right)-\frac{3}{2}$$

input
```
integrate((1+x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="giac")
```

output
```
4*sqrt(3)*sqrt(2)*arctan(1/4*sqrt(2)*(sqrt(3) + sqrt(2*x - 3))) - 2*sqrt(3
)*sqrt(2*x - 3) + x + 3*log(2*sqrt(3)*sqrt(2*x - 3) + 2*x + 8) - 3/2
```

3.707.9 Mupad [B] (verification not implemented)

Time = 17.32 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.52

$$\int \frac{1+x}{4+x+\sqrt{-9+6x}}\,dx = x+3\ \ln\left(\left(6\sqrt{6\,x-9}+\left(-3+\sqrt{6}\,2\mathrm{i}\right)\ \left(2\sqrt{6\,x-9}+6\right)+66\right)\ \left(6\sqrt{6\,x-9}-\left(3+\sqrt{6}\,2\mathrm{i}\right)\ \left(2\sqrt{6\,x-9}+6\right)+66\right)\right)+4\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\sqrt{6\,x-9}}{12}+\frac{\sqrt{6}}{4}\right)-2\sqrt{6\,x-9}$$

input
```
int((x + 1)/(x + (6*x - 9)^(1/2) + 4),x)
```

output
```
x + 3*log((6*(6*x - 9)^(1/2) + (6^(1/2)*2i - 3)*(2*(6*x - 9)^(1/2) + 6) +
66)*(6*(6*x - 9)^(1/2) - (6^(1/2)*2i + 3)*(2*(6*x - 9)^(1/2) + 6) + 66)) +
 4*6^(1/2)*atan((6^(1/2)*(6*x - 9)^(1/2))/12 + 6^(1/2)/4) - 2*(6*x - 9)^(1
/2)
```

3.708 $\int \frac{12-x}{4+x+\sqrt{-9+6x}}\,dx$

3.708.1 Optimal result

Integrand size = 20, antiderivative size = 71

$$\int \frac{12-x}{4+x+\sqrt{-9+6x}}\,dx = -x+2\sqrt{3}\sqrt{-3+2x}-21\sqrt{\frac{3}{2}}\arctan\left(\frac{3+\sqrt{-9+6x}}{2\sqrt{6}}\right)+10\log\left(4+x+\sqrt{3}\sqrt{-3+2x}\right)$$

output
```
-x+10*ln(4+x+(-3+2*x)^(1/2)*3^(1/2))-21/2*arctan(1/12*(3+(-9+6*x)^(1/2))*6
^(1/2))*6^(1/2)+2*(-3+2*x)^(1/2)*3^(1/2)
```

3.708.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$\int \frac{12-x}{4+x+\sqrt{-9+6x}}\,dx = \frac{1}{2}\left(3-2x+4\sqrt{-9+6x}-21\sqrt{6}\arctan\left(\frac{\sqrt{3}+\sqrt{-3+2x}}{2\sqrt{2}}\right)+20\log\left(4+x+\sqrt{-9+6x}\right)\right)$$

input
```
Integrate[(12 - x)/(4 + x + Sqrt[-9 + 6*x]),x]
```

output
```
(3 - 2*x + 4*Sqrt[-9 + 6*x] - 21*Sqrt[6]*ArcTan[(Sqrt[3] + Sqrt[-3 + 2*x])
/(2*Sqrt[2])] + 20*Log[4 + x + Sqrt[-9 + 6*x]])/2
```

3.708.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7267, 25, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{12-x}{x+\sqrt{6x-9}+4}dx$$

$$\Big\downarrow \text{7267}$$

$$-\frac{1}{3}\int -\frac{(72-6x)\sqrt{6x-9}}{6x+6\sqrt{6x-9}+24}d\sqrt{6x-9}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int \frac{(72-6x)\sqrt{6x-9}}{6x+6\sqrt{6x-9}+24}d\sqrt{6x-9}$$

$$\Big\downarrow \text{2159}$$

$$\frac{1}{3}\int \left(-\frac{6\left(33-10\sqrt{6x-9}\right)}{6x+6\sqrt{6x-9}+24}-\sqrt{6x-9}+6\right)d\sqrt{6x-9}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(-63\sqrt{\frac{3}{2}}\arctan\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)+\frac{1}{2}(9-6x)+6\sqrt{6x-9}+30\log\left(6x+6\sqrt{6x-9}+24\right)\right)$$

input
```
Int[(12 - x)/(4 + x + Sqrt[-9 + 6*x]),x]
```

output
```
((9 - 6*x)/2 + 6*Sqrt[-9 + 6*x] - 63*Sqrt[3/2]*ArcTan[(3 + Sqrt[-9 + 6*x])
/(2*Sqrt[6])] + 30*Log[24 + 6*x + 6*Sqrt[-9 + 6*x]])/3
```

3.708.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.708.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{3}{2}-x+2\sqrt{-9+6x}+10\ln\left(24+6x+6\sqrt{-9+6x}\right)-\frac{21\sqrt{6}\,\arctan\left(\frac{(2\sqrt{-9+6x}+6)\sqrt{6}}{24}\right)}{2}$
default	$\frac{3}{2}-x+2\sqrt{-9+6x}+10\ln\left(24+6x+6\sqrt{-9+6x}\right)-\frac{21\sqrt{6}\,\arctan\left(\frac{(2\sqrt{-9+6x}+6)\sqrt{6}}{24}\right)}{2}$
trager	$-x+2\sqrt{-9+6x}-\ln\left(4+x+\sqrt{-9+6x}\right)\operatorname{RootOf}\left(8\_Z^2-160\_Z+2123\right)+\ln\left(18\right.$

input
```
int((12-x)/(4+x+(-9+6*x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
3/2-x+2*(-9+6*x)^(1/2)+10*ln(24+6*x+6*(-9+6*x)^(1/2))-21/2*6^(1/2)*arctan(
1/24*(2*(-9+6*x)^(1/2)+6)*6^(1/2))
```

3.708.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int \frac{12-x}{4+x+\sqrt{-9+6x}}\,dx = -\frac{21}{2}\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{12}\sqrt{3}\sqrt{2}\sqrt{6x-9}+\frac{1}{4}\sqrt{3}\sqrt{2}\right) - x + 2\sqrt{6x-9} + 10\log\left(x+\sqrt{6x-9}+4\right)$$

input
```
integrate((12-x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="fricas")
```

output
```
-21/2*sqrt(3)*sqrt(2)*arctan(1/12*sqrt(3)*sqrt(2)*sqrt(6*x - 9) + 1/4*sqrt
(3)*sqrt(2)) - x + 2*sqrt(6*x - 9) + 10*log(x + sqrt(6*x - 9) + 4)
```

3.708.6 Sympy [A] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.85

$$\int \frac{12-x}{4+x+\sqrt{-9+6x}}\,dx = -x + 2\sqrt{6x-9} + 10\log\left(6x+6\sqrt{6x-9}+24\right) - \frac{21\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}(\sqrt{6x-9}+3)}{12}\right)}{2} + \frac{3}{2}$$

input
```
integrate((12-x)/(4+x+(-9+6*x)**(1/2)),x)
```

output
```
-x + 2*sqrt(6*x - 9) + 10*log(6*x + 6*sqrt(6*x - 9) + 24) - 21*sqrt(6)*ata
n(sqrt(6)*(sqrt(6*x - 9) + 3)/12)/2 + 3/2
```

3.708.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.72

$$\int \frac{12-x}{4+x+\sqrt{-9+6x}}\,dx = -\frac{21}{2}\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}\left(\sqrt{6x-9}+3\right)\right) - x + 2\sqrt{6x-9} + 10\log\left(6x+6\sqrt{6x-9}+24\right) + \frac{3}{2}$$

input
```
integrate((12-x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="maxima")
```

output
```
-21/2*sqrt(6)*arctan(1/12*sqrt(6)*(sqrt(6*x - 9) + 3)) - x + 2*sqrt(6*x -
9) + 10*log(6*x + 6*sqrt(6*x - 9) + 24) + 3/2
```

3.708.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.87

$$\int \frac{12-x}{4+x+\sqrt{-9+6x}}\,dx = -\frac{21}{2}\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(\sqrt{3}+\sqrt{2x-3}\right)\right) + 2\sqrt{3}\sqrt{2x-3} - x + 10\log\left(2\sqrt{3}\sqrt{2x-3}+2x+8\right) + \frac{3}{2}$$

input
```
integrate((12-x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="giac")
```

output
```
-21/2*sqrt(3)*sqrt(2)*arctan(1/4*sqrt(2)*(sqrt(3) + sqrt(2*x - 3))) + 2*sq
rt(3)*sqrt(2*x - 3) - x + 10*log(2*sqrt(3)*sqrt(2*x - 3) + 2*x + 8) + 3/2
```

3.708.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.66

$$\int \frac{12-x}{4+x+\sqrt{-9+6x}}\,dx = 2\sqrt{6x-9} + 10\ln\left(\left(\left(2\sqrt{6x-9}+6\right)\left(-10+\frac{\sqrt{2}\sqrt{3}\,21\mathrm{i}}{4}\right) + 20\sqrt{6x-9} - 66\right)\left(\left(2\sqrt{6x-9}+6\right)\left(10+\frac{\sqrt{2}\sqrt{3}\,21\mathrm{i}}{4}\right) - 20\sqrt{6x-9} + 66\right)\right) - x - \frac{21\sqrt{2}\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{3}}{4}+\frac{\sqrt{2}\sqrt{3}\sqrt{6x-9}}{12}\right)}{2}$$

input
```
int(-(x - 12)/(x + (6*x - 9)^(1/2) + 4),x)
```

output
```
10*log(((2*(6*x - 9)^(1/2) + 6)*((2^(1/2)*3^(1/2)*21i)/4 - 10) + 20*(6*x -
 9)^(1/2) - 66)*((2*(6*x - 9)^(1/2) + 6)*((2^(1/2)*3^(1/2)*21i)/4 + 10) -
20*(6*x - 9)^(1/2) + 66)) - x + 2*(6*x - 9)^(1/2) - (21*2^(1/2)*3^(1/2)*at
an((2^(1/2)*3^(1/2))/4 + (2^(1/2)*3^(1/2)*(6*x - 9)^(1/2))/12))/2
```

3.708. $\int \frac{12-x}{4+x+\sqrt{-9+6x}}\,dx$

3.709 $\int \frac{-1+x^3}{\sqrt{x}(1+x^2)} dx$

3.709.1 Optimal result

Integrand size = 18, antiderivative size = 52

$$\int \frac{-1+x^3}{\sqrt{x}\,(1+x^2)}\,dx = \frac{2x^{3/2}}{3}+\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{x}\right)-\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{x}\right)$$

output
```
2/3*x^(3/2)-arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-arctan(1+2^(1/2)*x^(1/2))*2
^(1/2)
```

3.709.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.62

$$\int \frac{-1+x^3}{\sqrt{x}\,(1+x^2)}\,dx = \frac{2x^{3/2}}{3}-\sqrt{2}\arctan\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right)$$

input
```
Integrate[(-1 + x^3)/(Sqrt[x]*(1 + x^2)),x]
```

output
```
(2*x^(3/2))/3 - Sqrt[2]*ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])]
```

3.709.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2035, 25, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-1}{\sqrt{x}\left(x^2+1\right)}dx$$

$$\Big\downarrow \text{2035}$$

$$2\int -\frac{1-x^3}{x^2+1}d\sqrt{x}$$

$$\Big\downarrow \text{25}$$

$$-2\int \frac{1-x^3}{x^2+1}d\sqrt{x}$$

$$\Big\downarrow \text{2426}$$

$$-2\int \left(\frac{x+1}{x^2+1}-x\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}+\frac{x^{3/2}}{3}\right)$$

input
```
Int[(-1 + x^3)/(Sqrt[x]*(1 + x^2)),x]
```

output
```
2*(x^(3/2)/3 + ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sq
rt[x]]/Sqrt[2])
```

3.709.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.709.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.17

method	result
trager	$\frac{2x^{\frac{3}{2}}}{3}-\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+2\right)x^2-4\operatorname{RootOf}\left(\_Z^2+2\right)x-4x^{\frac{3}{2}}+\operatorname{RootOf}\left(\_Z^2+2\right)+4\sqrt{x}}{x^2+1}\right)}{2}$
risch	$\frac{2x^{\frac{3}{2}}}{3}-\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}-\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}-\frac{\sqrt{2}\,\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)}{4}-\frac{\sqrt{2}\,\ln\left(\right.}{}$
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3}-\frac{\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{4}-\frac{\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\,\sqrt{x}+1}{x+\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\right.\right.}{4}$
default	$\frac{2x^{\frac{3}{2}}}{3}-\frac{\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{4}-\frac{\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\,\sqrt{x}+1}{x+\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\right.\right.}{4}$
meijerg	$\frac{2x^{\frac{3}{2}}}{3}-\frac{x^{\frac{3}{2}}\left(\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\left(x^2\right)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2\left(x^2\right)^{\frac{3}{4}}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^2\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^2\right)^{\frac{1}{4}}}\right)}{\left(x^2\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\,\ln\left(1+\sqrt{2}\left(x^2\right)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2\left(x^2\right)^{\frac{3}{4}}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^2\right.}{2+\sqrt{2}\left(x^2\right.}\right)}{\left(x^2\right)^{\frac{3}{4}}}\right.}{2}$

input
```
int((x^3-1)/(x^2+1)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^(3/2)-1/2*RootOf(_Z^2+2)*ln(-(RootOf(_Z^2+2)*x^2-4*RootOf(_Z^2+2)*x-
4*x^(3/2)+RootOf(_Z^2+2)+4*x^(1/2))/(x^2+1))
```

3.709.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.44

$$\int \frac{-1+x^3}{\sqrt{x}\,(1+x^2)}\,dx = \frac{2}{3}\,x^{\frac{3}{2}} - \sqrt{2}\arctan\left(\frac{\sqrt{2}(x-1)}{2\,\sqrt{x}}\right)$$

input
```
integrate((x^3-1)/(x^2+1)/x^(1/2),x, algorithm="fricas")
```

output
```
2/3*x^(3/2) - sqrt(2)*arctan(1/2*sqrt(2)*(x - 1)/sqrt(x))
```

3.709.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{-1+x^3}{\sqrt{x}\,(1+x^2)}\,dx = \frac{2x^{\frac{3}{2}}}{3} - \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right) - \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)$$

input
```
integrate((x**3-1)/(x**2+1)/x**(1/2),x)
```

output
```
2*x**(3/2)/3 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1) - sqrt(2)*atan(sqrt(2)*sq
rt(x) + 1)
```

3.709.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \frac{-1+x^3}{\sqrt{x}\,(1+x^2)}\,dx = {} & \frac{2}{3}\,x^{\frac{3}{2}} - \sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{x}\right)\right) \\ & - \sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\sqrt{x}\right)\right)\end{aligned}$$

input
```
integrate((x^3-1)/(x^2+1)/x^(1/2),x, algorithm="maxima")
```

output
```
2/3*x^(3/2) - sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x)))
```

3.709.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{-1+x^3}{\sqrt{x}\,(1+x^2)}\,dx = \frac{2}{3}\,x^{\frac{3}{2}} - \sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{x}\right)\right) - \sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\sqrt{x}\right)\right)$$

input
```
integrate((x^3-1)/(x^2+1)/x^(1/2),x, algorithm="giac")
```

output
```
2/3*x^(3/2) - sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x)))
```

3.709.9 Mupad [B] (verification not implemented)

Time = 18.59 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \frac{-1+x^3}{\sqrt{x}\,(1+x^2)}\,dx = \frac{2\,x^{3/2}}{3} - \frac{\sqrt{2}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{x}}{2}+\frac{\sqrt{2}\,x^{3/2}}{2}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{x}}{2}\right)\right)}{2}$$

input
```
int((x^3 - 1)/(x^(1/2)*(x^2 + 1)),x)
```

output
```
(2*x^(3/2))/3 - (2^(1/2)*(2*atan((2^(1/2)*x^(1/2))/2 + (2^(1/2)*x^(3/2))/2
) + 2*atan((2^(1/2)*x^(1/2))/2)))/2
```

3.710 $\int \frac{1}{2\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx$

3.710.1 Optimal result

Integrand size = 26, antiderivative size = 20

$$\int \frac{1}{2\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = -\text{arcsinh}\left(\frac{1-2\sqrt{-1+x}}{\sqrt{3}}\right)$$

output
```
-arcsinh(1/3*(1-2*(-1+x)^(1/2))*3^(1/2))
```

3.710.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{1}{2\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = -\log\left(1-2\sqrt{-1+x}+2\sqrt{-\sqrt{-1+x}+x}\right)$$

input
```
Integrate[1/(2*Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]
```

output
```
-Log[1 - 2*Sqrt[-1 + x] + 2*Sqrt[-Sqrt[-1 + x] + x]]
```

3.710.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {27, 7267, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2\sqrt{x-1}\sqrt{x-\sqrt{x-1}}}dx$$
$$\Big\downarrow \ 27$$
$$\frac{1}{2}\int \frac{1}{\sqrt{x-1}\sqrt{x-\sqrt{x-1}}}dx$$
$$\Big\downarrow \ 7267$$
$$\int \frac{1}{\sqrt{x-\sqrt{x-1}}}d\sqrt{x-1}$$
$$\Big\downarrow \ 1090$$
$$\frac{\int \frac{1}{\sqrt{\frac{x-1}{3}+1}}d\left(2\sqrt{x-1}-1\right)}{\sqrt{3}}$$
$$\Big\downarrow \ 222$$
$$\text{arcsinh}\left(\frac{2\sqrt{x-1}-1}{\sqrt{3}}\right)$$

input
```
Int[1/(2*Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]
```

output
```
ArcSinh[(-1 + 2*Sqrt[-1 + x])/Sqrt[3]]
```

3.710.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.710.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,\left(\sqrt{x-1}-\frac{1}{2}\right)}{3}\right)$	14
default	$\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,\left(\sqrt{x-1}-\frac{1}{2}\right)}{3}\right)$	14

input
```
int(1/2/(x-1)^(1/2)/(x-(x-1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsinh(2/3*3^(1/2)*((x-1)^(1/2)-1/2))
```

3.710.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(17) = 34.

Time = 0.45 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \frac{1}{2\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = \frac{1}{2}\,\log\left(4\,\sqrt{x-\sqrt{x-1}}(2\,\sqrt{x-1}-1)+8\,x-8\,\sqrt{x-1}-3\right)$$

input
```
integrate(1/2/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/2*log(4*sqrt(x - sqrt(x - 1))*(2*sqrt(x - 1) - 1) + 8*x - 8*sqrt(x - 1)
- 3)
```

3.710.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{2\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = \operatorname{asinh}\left(\frac{2\sqrt{3}\left(\sqrt{x-1}-\frac{1}{2}\right)}{3}\right)$$

input
```
integrate(1/2/(-1+x)**(1/2)/(x-(-1+x)**(1/2))**(1/2),x)
```

output
```
asinh(2*sqrt(3)*(sqrt(x - 1) - 1/2)/3)
```

3.710.7 Maxima [F]

$$\int \frac{1}{2\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = \int \frac{1}{2\sqrt{x-\sqrt{x-1}}\sqrt{x-1}}\,dx$$

input
```
integrate(1/2/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
1/2*integrate(1/(sqrt(x - sqrt(x - 1))*sqrt(x - 1)), x)
```

3.710.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{1}{2\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = -\log\left(2\sqrt{x-\sqrt{x-1}}-2\sqrt{x-1}+1\right)$$

input
```
integrate(1/2/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
-log(2*sqrt(x - sqrt(x - 1)) - 2*sqrt(x - 1) + 1)
```

3.710.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{2\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = \int \frac{1}{2\,\sqrt{x-\sqrt{x-1}}\,\sqrt{x-1}}\,dx$$

input
```
int(1/(2*(x - (x - 1)^(1/2))^(1/2)*(x - 1)^(1/2)),x)
```

output
```
int(1/(2*(x - (x - 1)^(1/2))^(1/2)*(x - 1)^(1/2)), x)
```

3.711 $\int \frac{1+x^{7/2}}{1-x^2} \, dx$

3.711.1 Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{1+x^{7/2}}{1-x^2} \, dx = -2\sqrt{x} - \frac{2x^{5/2}}{5} + \arctan\left(\sqrt{x}\right) - \log\left(1-\sqrt{x}\right) + \frac{1}{2}\log(1+x)$$

output
```
-2/5*x^(5/2)+arctan(x^(1/2))+1/2*ln(1+x)-ln(1-x^(1/2))-2*x^(1/2)
```

3.711.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{1+x^{7/2}}{1-x^2} \, dx = -\frac{2}{5}\sqrt{x}(5+x^2) + \arctan\left(\sqrt{x}\right) - \log\left(-1+\sqrt{x}\right) + \frac{1}{2}\log(1+x)$$

input
```
Integrate[(1 + x^(7/2))/(1 - x^2),x]
```

output
```
(-2*Sqrt[x]*(5 + x^2))/5 + ArcTan[Sqrt[x]] - Log[-1 + Sqrt[x]] + Log[1 + x
]/2
```

3.711.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {7267, 2372, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}+1}{1-x^2} \, dx$$

$$\Big\downarrow \text{7267}$$

$$2 \int \frac{\sqrt{x}\left(x^{7/2}+1\right)}{1-x^2}d\sqrt{x}$$

$$\Big\downarrow \text{2372}$$

$$2 \int \left(\frac{x^4}{1-x^2}+\frac{\sqrt{x}}{1-x^2}\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2 \left(\frac{\arctan\left(\sqrt{x}\right)}{2}+\frac{\text{arctanh}\left(\sqrt{x}\right)}{2}+\frac{\text{arctanh}(x)}{2}-\frac{x^{5/2}}{5}-\sqrt{x}\right)$$

input
```
Int[(1 + x^(7/2))/(1 - x^2),x]
```

output
```
2*(-Sqrt[x] - x^(5/2)/5 + ArcTan[Sqrt[x]]/2 + ArcTanh[Sqrt[x]]/2 + ArcTanh
[x]/2)
```

3.711.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2372
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.711.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{2x^{\frac{5}{2}}}{5}-2\sqrt{x}+\frac{\ln(x+1)}{2}+\arctan\left(\sqrt{x}\right)-\ln\left(-1+\sqrt{x}\right)$
default	$-\frac{2x^{\frac{5}{2}}}{5}-2\sqrt{x}-\frac{\ln(-1+\sqrt{x})}{2}+\frac{\ln(1+\sqrt{x})}{2}+\arctan\left(\sqrt{x}\right)+\operatorname{arctanh}\left(x\right)$
meijerg	$\operatorname{arctanh}\left(x\right)-\frac{(-1)^{\frac{3}{4}}\left(-\frac{4\sqrt{x}\,(-1)^{\frac{1}{4}}\left(9x^{2}+45\right)}{45}-\frac{\sqrt{x}\,(-1)^{\frac{1}{4}}\left(\ln\left(1-\left(x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(x^{2}\right)^{\frac{1}{4}}\right)-2\arctan\left(\left(x^{2}\right)^{\frac{1}{4}}\right)\right)}{\left(x^{2}\right)^{\frac{1}{4}}}\right)}{2}$
trager	$\left(-\frac{2x^{2}}{5}-2\right)\sqrt{x}-\ln\left(-\frac{-24\operatorname{RootOf}\left(8\_Z^{2}-4\_Z+1\right)^{2}x+16\operatorname{RootOf}\left(8\_Z^{2}-4\_Z+1\right)\sqrt{x}+48\operatorname{RootOf}\left(8\right.}{}\right.$

input
```
int((1+x^(7/2))/(-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-2/5*x^(5/2)-2*x^(1/2)+1/2*ln(x+1)+arctan(x^(1/2))-ln(-1+x^(1/2))
```

3.711.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{1+x^{7/2}}{1-x^2}\,dx=-\frac{2}{5}\left(x^{2}+5\right)\sqrt{x}+\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(x+1\right)-\log\left(\sqrt{x}-1\right)$$

input
```
integrate((1+x^(7/2))/(-x^2+1),x, algorithm="fricas")
```

output
```
-2/5*(x^2 + 5)*sqrt(x) + arctan(sqrt(x)) + 1/2*log(x + 1) - log(sqrt(x) -
1)
```

3.711.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{1+x^{7/2}}{1-x^2}\,dx = -\frac{2x^{\frac{5}{2}}}{5} - 2\sqrt{x} - \log\left(\sqrt{x}-1\right) + \frac{\log\left(x+1\right)}{2} + \operatorname{atan}\left(\sqrt{x}\right)$$

input
```
integrate((1+x**(7/2))/(-x**2+1),x)
```

output
```
-2*x**(5/2)/5 - 2*sqrt(x) - log(sqrt(x) - 1) + log(x + 1)/2 + atan(sqrt(x)
)
```

3.711.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{1+x^{7/2}}{1-x^2}\,dx = -\frac{2}{5}\,x^{\frac{5}{2}} - 2\,\sqrt{x} + \arctan\left(\sqrt{x}\right) + \frac{1}{2}\,\log\left(x+1\right) - \log\left(\sqrt{x}-1\right)$$

input
```
integrate((1+x^(7/2))/(-x^2+1),x, algorithm="maxima")
```

output
```
-2/5*x^(5/2) - 2*sqrt(x) + arctan(sqrt(x)) + 1/2*log(x + 1) - log(sqrt(x)
- 1)
```

3.711.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70

$$\int \frac{1+x^{7/2}}{1-x^2}\,dx = -\frac{2}{5}\,x^{\frac{5}{2}} - 2\,\sqrt{x} + \arctan\left(\sqrt{x}\right) + \frac{1}{2}\,\log\left(x+1\right) - \log\left(\left|\sqrt{x}-1\right|\right)$$

input
```
integrate((1+x^(7/2))/(-x^2+1),x, algorithm="giac")
```

output
```
-2/5*x^(5/2) - 2*sqrt(x) + arctan(sqrt(x)) + 1/2*log(x + 1) - log(abs(sqrt
(x) - 1))
```

3.711.9 Mupad [B] (verification not implemented)

Time = 18.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

$$\int \frac{1+x^{7/2}}{1-x^2}\,dx = -\ln\left(10\sqrt{x}-10\right)-2\sqrt{x}-\frac{2\,x^{5/2}}{5}$$
$$+\ln\left(1+\sqrt{x}\,(-3-\mathrm{i})-3\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)+\ln\left(1+\sqrt{x}\,(-3+1\mathrm{i})+3\mathrm{i}\right)\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

input
```
int(-(x^(7/2) + 1)/(x^2 - 1),x)
```

output
```
log((1 - 3i) - x^(1/2)*(3 + 1i))*(1/2 + 1i/2) - log(10*x^(1/2) - 10) + log
((1 + 3i) - x^(1/2)*(3 - 1i))*(1/2 - 1i/2) - 2*x^(1/2) - (2*x^(5/2))/5
```

3.712 $\int \frac{4+2x}{\sqrt[3]{-1+2x}+\sqrt{-1+2x}}\,dx$

3.712.1 Optimal result

Integrand size = 27, antiderivative size = 116

$$\int \frac{4+2x}{\sqrt[3]{-1+2x}+\sqrt{-1+2x}}\,dx = -x+18\sqrt[6]{-1+2x}-9\sqrt[3]{-1+2x}+6\sqrt{-1+2x} -\frac{3}{4}(-1+2x)^{2/3}+\frac{3}{5}(-1+2x)^{5/6}+\frac{3}{7}(-1+2x)^{7/6} -\frac{3}{8}(-1+2x)^{4/3}+\frac{1}{3}(-1+2x)^{3/2}-18\log\left(1+\sqrt[6]{-1+2x}\right)$$

output
```
-x+18*(-1+2*x)^(1/6)-9*(-1+2*x)^(1/3)-3/4*(-1+2*x)^(2/3)+3/5*(-1+2*x)^(5/6
)+3/7*(-1+2*x)^(7/6)-3/8*(-1+2*x)^(4/3)+1/3*(-1+2*x)^(3/2)-18*ln(1+(-1+2*x
)^(1/6))+6*(-1+2*x)^(1/2)
```

3.712.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.12

$$\int \frac{4+2x}{\sqrt[3]{-1+2x}+\sqrt{-1+2x}}\,dx = 2\left(\frac{1}{4}+\frac{123}{14}\sqrt[6]{-1+2x}-\frac{69}{16}\sqrt[3]{-1+2x}+\frac{17}{6}\sqrt{-1+2x}-\frac{3}{8}(-1+2x)^{2/3}+\frac{3}{10}(-1+2x)^{5/6} +x\left(-\frac{1}{2}+\frac{3}{7}\sqrt[6]{-1+2x}-\frac{3}{8}\sqrt[3]{-1+2x}+\frac{1}{3}\sqrt{-1+2x}\right)-9\log\left(1+\sqrt[6]{-1+2x}\right)\right)$$

input
```
Integrate[(4 + 2*x)/((-1 + 2*x)^(1/3) + Sqrt[-1 + 2*x]),x]
```

output
```
2*(1/4 + (123*(-1 + 2*x)^(1/6))/14 - (69*(-1 + 2*x)^(1/3))/16 + (17*Sqrt[-
1 + 2*x])/6 - (3*(-1 + 2*x)^(2/3))/8 + (3*(-1 + 2*x)^(5/6))/10 + x*(-1/2 +
 (3*(-1 + 2*x)^(1/6))/7 - (3*(-1 + 2*x)^(1/3))/8 + Sqrt[-1 + 2*x]/3) - 9*L
og[1 + (-1 + 2*x)^(1/6)])
```

3.712.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7267, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+4}{\sqrt{2x-1}+\sqrt[3]{2x-1}}dx$$

$$\Big\downarrow \text{7267}$$

$$3\int \frac{\sqrt{2x-1}(2x+4)}{\sqrt[6]{2x-1}+1}d\sqrt[6]{2x-1}$$

$$\Big\downarrow \text{2123}$$

$$3\int \left((2x-1)^{4/3}-(2x-1)^{7/6}-(2x-1)^{5/6}+(2x-1)^{2/3}-\sqrt{2x-1}+6\sqrt[3]{2x-1}-6\sqrt[6]{2x-1}+2x-\frac{6}{\sqrt[6]{2x-1}\right.$$

$$\Big\downarrow \text{2009}$$

$$3\left(\frac{1}{9}(2x-1)^{3/2}-\frac{1}{8}(2x-1)^{4/3}+\frac{1}{7}(2x-1)^{7/6}+\frac{1}{5}(2x-1)^{5/6}-\frac{1}{4}(2x-1)^{2/3}+2\sqrt{2x-1}-3\sqrt[3]{2x-1}+6\sqrt[6]{2x\right.$$

input
```
Int[(4 + 2*x)/((-1 + 2*x)^(1/3) + Sqrt[-1 + 2*x]),x]
```

output
```
3*((1 - 2*x)/6 + 6*(-1 + 2*x)^(1/6) - 3*(-1 + 2*x)^(1/3) + 2*Sqrt[-1 + 2*x
] - (-1 + 2*x)^(2/3)/4 + (-1 + 2*x)^(5/6)/5 + (-1 + 2*x)^(7/6)/7 - (-1 + 2
*x)^(4/3)/8 + (-1 + 2*x)^(3/2)/9 - 6*Log[1 + (-1 + 2*x)^(1/6)])
```

3.712.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.712.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{(2x-1)^{\frac{3}{2}}}{3}-\frac{3(2x-1)^{\frac{4}{3}}}{8}+\frac{3(2x-1)^{\frac{7}{6}}}{7}-x+\frac{1}{2}+\frac{3(2x-1)^{\frac{5}{6}}}{5}-\frac{3(2x-1)^{\frac{2}{3}}}{4}+6\sqrt{2x-1}-9(2x-1)^{\frac{1}{3}}$
default	$\frac{(2x-1)^{\frac{3}{2}}}{3}-\frac{3(2x-1)^{\frac{4}{3}}}{8}+\frac{3(2x-1)^{\frac{7}{6}}}{7}-x+\frac{1}{2}+\frac{3(2x-1)^{\frac{5}{6}}}{5}-\frac{3(2x-1)^{\frac{2}{3}}}{4}+6\sqrt{2x-1}-9(2x-1)^{\frac{1}{3}}$

input
```
int((4+2*x)/((2*x-1)^(1/3)+(2*x-1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*x-1)^(3/2)-3/8*(2*x-1)^(4/3)+3/7*(2*x-1)^(7/6)-x+1/2+3/5*(2*x-1)^(5
/6)-3/4*(2*x-1)^(2/3)+6*(2*x-1)^(1/2)-9*(2*x-1)^(1/3)+18*(2*x-1)^(1/6)-18*
ln(1+(2*x-1)^(1/6))
```

3.712.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.66

$$\begin{aligned}\int \frac{4+2x}{\sqrt[3]{-1+2x}+\sqrt{-1+2x}}\,dx &= \frac{1}{3}\,(2\,x+17)\sqrt{2\,x-1}-\frac{3}{8}\,(2\,x+23)(2\,x-1)^{\frac{1}{3}} \\ &\quad+\frac{3}{7}\,(2\,x+41)(2\,x-1)^{\frac{1}{6}}-x+\frac{3}{5}\,(2\,x-1)^{\frac{5}{6}} \\ &\quad-\frac{3}{4}\,(2\,x-1)^{\frac{2}{3}}-18\,\log\left((2\,x-1)^{\frac{1}{6}}+1\right)\end{aligned}$$

input
```
integrate((4+2*x)/((-1+2*x)^(1/3)+(-1+2*x)^(1/2)),x, algorithm="fricas")
```

output
```
1/3*(2*x + 17)*sqrt(2*x - 1) - 3/8*(2*x + 23)*(2*x - 1)^(1/3) + 3/7*(2*x +
 41)*(2*x - 1)^(1/6) - x + 3/5*(2*x - 1)^(5/6) - 3/4*(2*x - 1)^(2/3) - 18*
log((2*x - 1)^(1/6) + 1)
```

3.712.6 Sympy [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

$$\int \frac{4+2x}{\sqrt[3]{-1+2x}+\sqrt{-1+2x}}\,dx = -x+\frac{3(2x-1)^{\frac{7}{6}}}{7}+\frac{3(2x-1)^{\frac{5}{6}}}{5}+18\sqrt[6]{2x-1} - \frac{3(2x-1)^{\frac{4}{3}}}{8}-\frac{3(2x-1)^{\frac{2}{3}}}{4}-9\sqrt[3]{2x-1} + \frac{(2x-1)^{\frac{3}{2}}}{3}+6\sqrt{2x-1}-18\log\left(\sqrt[6]{2x-1}+1\right)+\frac{1}{2}$$

input
```
integrate((4+2*x)/((-1+2*x)**(1/3)+(-1+2*x)**(1/2)),x)
```

output
```
-x + 3*(2*x - 1)**(7/6)/7 + 3*(2*x - 1)**(5/6)/5 + 18*(2*x - 1)**(1/6) - 3
*(2*x - 1)**(4/3)/8 - 3*(2*x - 1)**(2/3)/4 - 9*(2*x - 1)**(1/3) + (2*x - 1
)**(3/2)/3 + 6*sqrt(2*x - 1) - 18*log((2*x - 1)**(1/6) + 1) + 1/2
```

3.712.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

$$\int \frac{4+2x}{\sqrt[3]{-1+2x}+\sqrt{-1+2x}}\,dx = \frac{1}{3}\,(2\,x-1)^{\frac{3}{2}}-\frac{3}{8}\,(2\,x-1)^{\frac{4}{3}}+\frac{3}{7}\,(2\,x-1)^{\frac{7}{6}}-x + \frac{3}{5}\,(2\,x-1)^{\frac{5}{6}}-\frac{3}{4}\,(2\,x-1)^{\frac{2}{3}}+6\,\sqrt{2\,x-1}-9\,(2\,x-1)^{\frac{1}{3}} + 18\,(2\,x-1)^{\frac{1}{6}}-18\,\log\left((2\,x-1)^{\frac{1}{6}}+1\right)+\frac{1}{2}$$

input
```
integrate((4+2*x)/((-1+2*x)^(1/3)+(-1+2*x)^(1/2)),x, algorithm="maxima")
```

output
```
1/3*(2*x - 1)^(3/2) - 3/8*(2*x - 1)^(4/3) + 3/7*(2*x - 1)^(7/6) - x + 3/5*
(2*x - 1)^(5/6) - 3/4*(2*x - 1)^(2/3) + 6*sqrt(2*x - 1) - 9*(2*x - 1)^(1/3
) + 18*(2*x - 1)^(1/6) - 18*log((2*x - 1)^(1/6) + 1) + 1/2
```

3.712.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

$$\int \frac{4+2x}{\sqrt[3]{-1+2x}+\sqrt{-1+2x}}\,dx = \frac{1}{3}\,(2\,x-1)^{\frac{3}{2}} - \frac{3}{8}\,(2\,x-1)^{\frac{4}{3}} + \frac{3}{7}\,(2\,x-1)^{\frac{7}{6}} - x + \frac{3}{5}\,(2\,x-1)^{\frac{5}{6}} - \frac{3}{4}\,(2\,x-1)^{\frac{2}{3}} + 6\,\sqrt{2\,x-1} - 9\,(2\,x-1)^{\frac{1}{3}} + 18\,(2\,x-1)^{\frac{1}{6}} - 18\,\log\left((2\,x-1)^{\frac{1}{6}}+1\right) + \frac{1}{2}$$

input
```
integrate((4+2*x)/((-1+2*x)^(1/3)+(-1+2*x)^(1/2)),x, algorithm="giac")
```

output
```
1/3*(2*x - 1)^(3/2) - 3/8*(2*x - 1)^(4/3) + 3/7*(2*x - 1)^(7/6) - x + 3/5*
(2*x - 1)^(5/6) - 3/4*(2*x - 1)^(2/3) + 6*sqrt(2*x - 1) - 9*(2*x - 1)^(1/3
) + 18*(2*x - 1)^(1/6) - 18*log((2*x - 1)^(1/6) + 1) + 1/2
```

3.712.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.76

$$\int \frac{4+2x}{\sqrt[3]{-1+2x}+\sqrt{-1+2x}}\,dx = 6\,\sqrt{2\,x-1} - 18\,\ln\left((2\,x-1)^{1/6}+1\right) - x - 9\,(2\,x-1)^{1/3} - \frac{3\,(2\,x-1)^{2/3}}{4} + \frac{(2\,x-1)^{3/2}}{3} + 18\,(2\,x-1)^{1/6} - \frac{3\,(2\,x-1)^{4/3}}{8} + \frac{3\,(2\,x-1)^{5/6}}{5} + \frac{3\,(2\,x-1)^{7/6}}{7}$$

input
```
int((2*x + 4)/((2*x - 1)^(1/2) + (2*x - 1)^(1/3)),x)
```

output
```
6*(2*x - 1)^(1/2) - 18*log((2*x - 1)^(1/6) + 1) - x - 9*(2*x - 1)^(1/3) -
(3*(2*x - 1)^(2/3))/4 + (2*x - 1)^(3/2)/3 + 18*(2*x - 1)^(1/6) - (3*(2*x -
 1)^(4/3))/8 + (3*(2*x - 1)^(5/6))/5 + (3*(2*x - 1)^(7/6))/7
```

3.713 $\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}} \, dx$

3.713.1 Optimal result

Integrand size = 17, antiderivative size = 83

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}} \, dx = -48\sqrt{2+\sqrt{1+\sqrt{x}}}+\frac{88}{3}\left(2+\sqrt{1+\sqrt{x}}\right)^{3/2} -\frac{48}{5}\left(2+\sqrt{1+\sqrt{x}}\right)^{5/2}+\frac{8}{7}\left(2+\sqrt{1+\sqrt{x}}\right)^{7/2}$$

output
```
88/3*(2+(1+x^(1/2))^(1/2))^(3/2)-48/5*(2+(1+x^(1/2))^(1/2))^(5/2)+8/7*(2+(
1+x^(1/2))^(1/2))^(7/2)-48*(2+(1+x^(1/2))^(1/2))^(1/2)
```

3.713.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}} \, dx = \frac{8}{105}\sqrt{2+\sqrt{1+\sqrt{x}}}\left(-280+76\sqrt{1+\sqrt{x}} +3\left(-12+5\sqrt{1+\sqrt{x}}\right)\sqrt{x}\right)$$

input
```
Integrate[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

output
```
(8*Sqrt[2 + Sqrt[1 + Sqrt[x]]]*(-280 + 76*Sqrt[1 + Sqrt[x]] + 3*(-12 + 5*S
qrt[1 + Sqrt[x]])*Sqrt[x]))/105
```

3.713.3 Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {7267, 896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sqrt{\sqrt{x}+1}+2}}dx$$

$$\Big\downarrow \text{7267}$$

$$2\int \frac{\sqrt{x}}{\sqrt{\sqrt{\sqrt{x}+1}+2}}d\sqrt{x}$$

$$\Big\downarrow \text{896}$$

$$2\int \frac{\sqrt{x}}{\sqrt{\sqrt[4]{x}+2}}d\left(\sqrt{x}+1\right)$$

$$\Big\downarrow \text{25}$$

$$-2\int -\frac{\sqrt{x}}{\sqrt{\sqrt[4]{x}+2}}d\left(\sqrt{x}+1\right)$$

$$\Big\downarrow \text{1732}$$

$$-4\int \frac{(1-x)\sqrt[4]{x}}{\sqrt{\sqrt{x}+3}}d\sqrt[4]{x}$$

$$\Big\downarrow \text{522}$$

$$-4\int \left(-\left(\sqrt{x}+3\right)^{5/2}+6\left(\sqrt{x}+3\right)^{3/2}-11\sqrt{\sqrt{x}+3}+\frac{6}{\sqrt{\sqrt{x}+3}}\right)d\sqrt[4]{x}$$

$$\Big\downarrow \text{2009}$$

$$-4\left(-\frac{2}{7}\left(\sqrt{x}+3\right)^{7/2}+\frac{12}{5}\left(\sqrt{x}+3\right)^{5/2}-\frac{22}{3}\left(\sqrt{x}+3\right)^{3/2}+12\sqrt{\sqrt{x}+3}\right)$$

input
```
Int[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

output
```
-4*(12*Sqrt[3 + Sqrt[x]] - (22*(3 + Sqrt[x])^(3/2))/3 + (12*(3 + Sqrt[x])^
(5/2))/5 - (2*(3 + Sqrt[x])^(7/2))/7)
```

3.713.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.713.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.65

method	result	size
derivativedivides	$\frac{88\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{3}{2}}}{3}-\frac{48\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{5}{2}}}{5}+\frac{8\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{7}{2}}}{7}-48\sqrt{2+\sqrt{1+\sqrt{x}}}$	54
default	$\frac{88\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{3}{2}}}{3}-\frac{48\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{5}{2}}}{5}+\frac{8\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{7}{2}}}{7}-48\sqrt{2+\sqrt{1+\sqrt{x}}}$	54

input
```
int(1/(2+(1+x^(1/2))^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
88/3*(2+(1+x^(1/2))^(1/2))^(3/2)-48/5*(2+(1+x^(1/2))^(1/2))^(5/2)+8/7*(2+(
1+x^(1/2))^(1/2))^(7/2)-48*(2+(1+x^(1/2))^(1/2))^(1/2)
```

3.713.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx=\frac{8}{105}\left(\left(15\sqrt{x}+76\right)\sqrt{\sqrt{x}+1}-36\sqrt{x}-280\right)\sqrt{\sqrt{\sqrt{x}+1}+2}$$

input
```
integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
8/105*((15*sqrt(x) + 76)*sqrt(sqrt(x) + 1) - 36*sqrt(x) - 280)*sqrt(sqrt(s
qrt(x) + 1) + 2)
```

3.713.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx=\frac{8\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}}}{7}-\frac{48\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}}}{5}+\frac{88\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}}}{3}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

input
```
integrate(1/(2+(1+x**(1/2))**(1/2))**(1/2),x)
```

output
```
8*(sqrt(sqrt(x) + 1) + 2)**(7/2)/7 - 48*(sqrt(sqrt(x) + 1) + 2)**(5/2)/5 +
 88*(sqrt(sqrt(x) + 1) + 2)**(3/2)/3 - 48*sqrt(sqrt(sqrt(x) + 1) + 2)
```

3.713.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = \frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}} - \frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}} + \frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}} - 48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

input
```
integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
8/7*(sqrt(sqrt(x) + 1) + 2)^(7/2) - 48/5*(sqrt(sqrt(x) + 1) + 2)^(5/2) + 8
8/3*(sqrt(sqrt(x) + 1) + 2)^(3/2) - 48*sqrt(sqrt(sqrt(x) + 1) + 2)
```

3.713.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = \frac{8\left(15\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}} - 126\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}} + 385\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}} - 630\sqrt{\sqrt{\sqrt{x}+1}+2}\right)}{105\,\mathrm{sgn}\left(4\left(\sqrt{x}+1\right)^{2} - 8\sqrt{x} - 7\right)\mathrm{sgn}\left(4x-3\right)}$$

input
```
integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

output
```
8/105*(15*(sqrt(sqrt(x) + 1) + 2)^(7/2) - 126*(sqrt(sqrt(x) + 1) + 2)^(5/2
) + 385*(sqrt(sqrt(x) + 1) + 2)^(3/2) - 630*sqrt(sqrt(sqrt(x) + 1) + 2))/(
sgn(4*(sqrt(x) + 1)^2 - 8*sqrt(x) - 7)*sgn(4*x - 3))
```

3.713.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = \int \frac{1}{\sqrt{\sqrt{\sqrt{x}+1}+2}}\,dx$$

input
```
int(1/((x^(1/2) + 1)^(1/2) + 2)^(1/2),x)
```

output
```
int(1/((x^(1/2) + 1)^(1/2) + 2)^(1/2), x)
```

3.714 $\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx$

3.714.1 Optimal result

Integrand size = 17, antiderivative size = 64

$$\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx=\frac{64}{5}\left(2+\sqrt{4+\sqrt{x}}\right)^{5/2}-\frac{48}{7}\left(2+\sqrt{4+\sqrt{x}}\right)^{7/2}+\frac{8}{9}\left(2+\sqrt{4+\sqrt{x}}\right)^{9/2}$$

output
```
64/5*(2+(4+x^(1/2))^(1/2))^(5/2)-48/7*(2+(4+x^(1/2))^(1/2))^(7/2)+8/9*(2+(
4+x^(1/2))^(1/2))^(9/2)
```

3.714.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx=\frac{8}{315}\sqrt{2+\sqrt{4+\sqrt{x}}}\left(-64\left(2+\sqrt{4+\sqrt{x}}\right)+2\left(2+5\sqrt{4+\sqrt{x}}\right)\sqrt{x}+35x\right)$$

input
```
Integrate[Sqrt[2 + Sqrt[4 + Sqrt[x]]],x]
```

output
```
(8*Sqrt[2 + Sqrt[4 + Sqrt[x]]]*(-64*(2 + Sqrt[4 + Sqrt[x]]) + 2*(2 + 5*Sqr
t[4 + Sqrt[x]])*Sqrt[x] + 35*x))/315
```

3.714.3 Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {7267, 896, 25, 1388, 900, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sqrt{\sqrt{x}+4}+2}\,dx$$

$$\Big\downarrow \text{7267}$$

$$2\int \sqrt{\sqrt{\sqrt{x}+4}+2}\sqrt{x}d\sqrt{x}$$

$$\Big\downarrow \text{896}$$

$$2\int \sqrt{\sqrt[4]{x}+2}\sqrt{x}d(\sqrt{x}+4)$$

$$\Big\downarrow \text{25}$$

$$-2\int -\sqrt{\sqrt[4]{x}+2}\sqrt{x}d(\sqrt{x}+4)$$

$$\Big\downarrow \text{1388}$$

$$-2\int \left(2-\sqrt[4]{x}\right)\left(\sqrt[4]{x}+2\right)^{3/2}d(\sqrt{x}+4)$$

$$\Big\downarrow \text{900}$$

$$-4\int \left(-\sqrt{x}-2\right)\left(\sqrt{x}+6\right)^{3/2}\sqrt[4]{x}d\sqrt[4]{x}$$

$$\Big\downarrow \text{86}$$

$$-4\int \left(-\left(\sqrt{x}+6\right)^{7/2}+6\left(\sqrt{x}+6\right)^{5/2}-8\left(\sqrt{x}+6\right)^{3/2}\right)d\sqrt[4]{x}$$

$$\Big\downarrow \text{2009}$$

$$-4\left(-\frac{2}{9}\left(\sqrt{x}+6\right)^{9/2}+\frac{12}{7}\left(\sqrt{x}+6\right)^{7/2}-\frac{16}{5}\left(\sqrt{x}+6\right)^{5/2}\right)$$

input
```
Int[Sqrt[2 + Sqrt[4 + Sqrt[x]]],x]
```

3.714. $\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx$

output
```
-4*((-16*(6 + Sqrt[x])^(5/2))/5 + (12*(6 + Sqrt[x])^(7/2))/7 - (2*(6 + Sqr
t[x])^(9/2))/9)
```

3.714.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 900
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(a + b*x^(g*n)
)^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
 && NeQ[b*c - a*d, 0] && FractionQ[n]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]
```

3.714. $\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx$

3.714.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2.

Time = 0.42 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.27

method	result	size
meijerg	$2x\,{}_3F_2\left(-\frac{1}{4},\frac{1}{4},2;\frac{1}{2},3;-\frac{\sqrt{x}}{4}\right)$	17
derivativedivides	$\frac{64\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{5}{2}}}{5}-\frac{48\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{7}{2}}}{7}+\frac{8\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{9}{2}}}{9}$	41
default	$\frac{64\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{5}{2}}}{5}-\frac{48\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{7}{2}}}{7}+\frac{8\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{9}{2}}}{9}$	41

input
```
int((2+(4+x^(1/2))^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x*hypergeom([-1/4,1/4,2],[1/2,3],-1/4*x^(1/2))
```

3.714.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.61

$$\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx = \frac{8}{315}\left(2\left(5\sqrt{x}-32\right)\sqrt{\sqrt{x}+4}+35\,x+4\sqrt{x}-128\right)\sqrt{\sqrt{\sqrt{x}+4}+2}$$

input
```
integrate((2+(4+x^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
8/315*(2*(5*sqrt(x) - 32)*sqrt(sqrt(x) + 4) + 35*x + 4*sqrt(x) - 128)*sqrt
(sqrt(sqrt(x) + 4) + 2)
```

3.714.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(54) = 108.

Time = 1.40 (sec) , antiderivative size = 216, normalized size of antiderivative = 3.38

$$\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx = -\frac{2\sqrt{2}\sqrt{x}\sqrt{\sqrt{x}+4}\sqrt{\sqrt{\sqrt{x}+4}+2}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{63\pi} - \frac{4\sqrt{2}\sqrt{x}\sqrt{\sqrt{\sqrt{x}+4}+2}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{315\pi} - \frac{\sqrt{2}x\sqrt{\sqrt{\sqrt{x}+4}+2}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{9\pi} + \frac{64\sqrt{2}\sqrt{\sqrt{x}+4}\sqrt{\sqrt{\sqrt{x}+4}+2}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{315\pi} + \frac{128\sqrt{2}\sqrt{\sqrt{\sqrt{x}+4}+2}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{315\pi}$$

input
```
integrate((2+(4+x**(1/2))**(1/2))**(1/2),x)
```

output
```
-2*sqrt(2)*sqrt(x)*sqrt(sqrt(x) + 4)*sqrt(sqrt(sqrt(x) + 4) + 2)*gamma(-1/
4)*gamma(1/4)/(63*pi) - 4*sqrt(2)*sqrt(x)*sqrt(sqrt(sqrt(x) + 4) + 2)*gamm
a(-1/4)*gamma(1/4)/(315*pi) - sqrt(2)*x*sqrt(sqrt(sqrt(x) + 4) + 2)*gamma(
-1/4)*gamma(1/4)/(9*pi) + 64*sqrt(2)*sqrt(sqrt(x) + 4)*sqrt(sqrt(sqrt(x) +
 4) + 2)*gamma(-1/4)*gamma(1/4)/(315*pi) + 128*sqrt(2)*sqrt(sqrt(sqrt(x) +
 4) + 2)*gamma(-1/4)*gamma(1/4)/(315*pi)
```

3.714.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

$$\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx = \frac{8}{9}\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{9}{2}} - \frac{48}{7}\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{7}{2}} + \frac{64}{5}\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{5}{2}}$$

input
```
integrate((2+(4+x^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
8/9*(sqrt(sqrt(x) + 4) + 2)^(9/2) - 48/7*(sqrt(sqrt(x) + 4) + 2)^(7/2) + 6
4/5*(sqrt(sqrt(x) + 4) + 2)^(5/2)
```

3.714.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(40) = 80$.

Time = 0.39 (sec) , antiderivative size = 268, normalized size of antiderivative = 4.19

$$\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx$$
$$= \frac{8}{315}\left(\left(35\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{9}{2}} - 360\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{7}{2}} + 1512\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{5}{2}} - 3360\left(\sqrt{\sqrt{x}+4}+2\right.\right.\right.$$
$$- 15)$$

input
```
integrate((2+(4+x^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

output
```
8/315*((35*(sqrt(sqrt(x) + 4) + 2)^(9/2) - 360*(sqrt(sqrt(x) + 4) + 2)^(7/
2) + 1512*(sqrt(sqrt(x) + 4) + 2)^(5/2) - 3360*(sqrt(sqrt(x) + 4) + 2)^(3/
2) + 5040*sqrt(sqrt(sqrt(x) + 4) + 2))*sgn(4*(sqrt(x) + 4)^2 - 32*sqrt(x)
- 79) + 18*(5*(sqrt(sqrt(x) + 4) + 2)^(7/2) - 42*(sqrt(sqrt(x) + 4) + 2)^(
5/2) + 140*(sqrt(sqrt(x) + 4) + 2)^(3/2) - 280*sqrt(sqrt(sqrt(x) + 4) + 2)
)*sgn(4*(sqrt(x) + 4)^2 - 32*sqrt(x) - 79) - 84*(3*(sqrt(sqrt(x) + 4) + 2)
^(5/2) - 20*(sqrt(sqrt(x) + 4) + 2)^(3/2) + 60*sqrt(sqrt(sqrt(x) + 4) + 2)
)*sgn(4*(sqrt(x) + 4)^2 - 32*sqrt(x) - 79) - 840*((sqrt(sqrt(x) + 4) + 2)^
(3/2) - 6*sqrt(sqrt(sqrt(x) + 4) + 2))*sgn(4*(sqrt(x) + 4)^2 - 32*sqrt(x)
- 79))*sgn(4*x - 15)
```

3.714.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx = \int \sqrt{\sqrt{\sqrt{x}+4}+2}\,dx$$

input
```
int(((x^(1/2) + 4)^(1/2) + 2)^(1/2),x)
```

output
```
int(((x^(1/2) + 4)^(1/2) + 2)^(1/2), x)
```

3.715 $\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx$

3.715.1 Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx=\frac{64}{25}\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{5/2}-\frac{48}{35}\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{7/2}+\frac{8}{45}\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{9/2}$$

output
```
64/25*(2-(4+(-9+5*x)^(1/2))^(1/2))^(5/2)-48/35*(2-(4+(-9+5*x)^(1/2))^(1/2)
)^(7/2)+8/45*(2-(4+(-9+5*x)^(1/2))^(1/2))^(9/2)
```

3.715.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx = -\frac{8\sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\left(443-175x-4\sqrt{-9+5x}-64\sqrt{4+\sqrt{-9+5x}}+10\sqrt{-9+5x}\sqrt{4+\sqrt{-9\cdots}}\right.}{1575}$$

input
```
Integrate[Sqrt[2 - Sqrt[4 + Sqrt[-9 + 5*x]]],x]
```

output
```
(-8*Sqrt[2 - Sqrt[4 + Sqrt[-9 + 5*x]]]*(443 - 175*x - 4*Sqrt[-9 + 5*x] - 6
4*Sqrt[4 + Sqrt[-9 + 5*x]] + 10*Sqrt[-9 + 5*x]*Sqrt[4 + Sqrt[-9 + 5*x]]))/
1575
```

3.715.3 Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {7267, 896, 25, 1388, 900, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{2-\sqrt{\sqrt{5x-9}+4}}dx$$

$$\Big\downarrow \text{ 7267}$$

$$\frac{2}{5}\int \sqrt{5x-9}\sqrt{2-\sqrt{\sqrt{5x-9}+4}}d\sqrt{5x-9}$$

$$\Big\downarrow \text{ 896}$$

$$\frac{2}{5}\int \sqrt{5x-9}\sqrt{2-\sqrt[4]{5x-9}}d\left(\sqrt{5x-9}+4\right)$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{2}{5}\int -\sqrt{5x-9}\sqrt{2-\sqrt[4]{5x-9}}d\left(\sqrt{5x-9}+4\right)$$

$$\Big\downarrow \text{ 1388}$$

$$-\frac{2}{5}\int \left(2-\sqrt[4]{5x-9}\right)^{3/2}\left(\sqrt[4]{5x-9}+2\right)d\left(\sqrt{5x-9}+4\right)$$

$$\Big\downarrow \text{ 900}$$

$$-\frac{4}{5}\int \sqrt[4]{5x-9}\left(-\sqrt{5x-9}-2\right)^{3/2}\left(\sqrt{5x-9}+6\right)d\sqrt[4]{5x-9}$$

$$\Big\downarrow \text{ 86}$$

$$-\frac{4}{5}\int \left(\left(-\sqrt{5x-9}-2\right)^{7/2}-6\left(-\sqrt{5x-9}-2\right)^{5/2}+8\left(-\sqrt{5x-9}-2\right)^{3/2}\right)d\sqrt[4]{5x-9}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{4}{5}\left(-\frac{2}{9}\left(-\sqrt{5x-9}-2\right)^{9/2}+\frac{12}{7}\left(-\sqrt{5x-9}-2\right)^{7/2}-\frac{16}{5}\left(-\sqrt{5x-9}-2\right)^{5/2}\right)$$

---

input
```
Int[Sqrt[2 - Sqrt[4 + Sqrt[-9 + 5*x]]],x]
```

output
```
(-4*((-16*(-2 - Sqrt[-9 + 5*x])^(5/2))/5 + (12*(-2 - Sqrt[-9 + 5*x])^(7/2)
)/7 - (2*(-2 - Sqrt[-9 + 5*x])^(9/2))/9))/5
```

3.715.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 900
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(a + b*x^(g*n)
)^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
 && NeQ[b*c - a*d, 0] && FractionQ[n]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.715.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{64\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{\frac{5}{2}}}{25}-\frac{48\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{\frac{7}{2}}}{35}+\frac{8\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{\frac{9}{2}}}{45}$	59
default	$\frac{64\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{\frac{5}{2}}}{25}-\frac{48\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{\frac{7}{2}}}{35}+\frac{8\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{\frac{9}{2}}}{45}$	59

input
```
int((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
64/25*(2-(4+(-9+5*x)^(1/2))^(1/2))^(5/2)-48/35*(2-(4+(-9+5*x)^(1/2))^(1/2)
)^(7/2)+8/45*(2-(4+(-9+5*x)^(1/2))^(1/2))^(9/2)
```

3.715.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.70

$$\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx =$$
$$-\frac{8}{1575}\left(2\left(5\sqrt{5x-9}-32\right)\sqrt{\sqrt{5x-9}+4}-175x-4\sqrt{5x-9}+443\right)\sqrt{-\sqrt{\sqrt{5x-9}+4}+2}$$

input
```
integrate((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-8/1575*(2*(5*sqrt(5*x - 9) - 32)*sqrt(sqrt(5*x - 9) + 4) - 175*x - 4*sqrt
(5*x - 9) + 443)*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2)
```

3.715.6 Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx = \frac{8\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{\frac{9}{2}}}{45} - \frac{48\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{\frac{7}{2}}}{35} + \frac{64\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{\frac{5}{2}}}{25}$$

input
```
integrate((2-(4+(-9+5*x)**(1/2))**(1/2))**(1/2),x)
```

output
```
8*(2 - sqrt(sqrt(5*x - 9) + 4))**(9/2)/45 - 48*(2 - sqrt(sqrt(5*x - 9) + 4
))**(7/2)/35 + 64*(2 - sqrt(sqrt(5*x - 9) + 4))**(5/2)/25
```

3.715.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.71

$$\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx = \frac{8}{45}\left(-\sqrt{\sqrt{5\,x-9}+4}+2\right)^{\frac{9}{2}} - \frac{48}{35}\left(-\sqrt{\sqrt{5\,x-9}+4}+2\right)^{\frac{7}{2}} + \frac{64}{25}\left(-\sqrt{\sqrt{5\,x-9}+4}+2\right)^{\frac{5}{2}}$$

input
```
integrate((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
8/45*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(9/2) - 48/35*(-sqrt(sqrt(5*x - 9) + 4
) + 2)^(7/2) + 64/25*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(5/2)
```

3.715.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(58) = 116$.

Time = 0.41 (sec) , antiderivative size = 474, normalized size of antiderivative = 5.78

$$\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx =$$
$$-\frac{8}{1575}\left(\left(35\left(\sqrt{\sqrt{5\,x-9}+4}-2\right)^4\sqrt{-\sqrt{\sqrt{5\,x-9}+4}+2}+360\left(\sqrt{\sqrt{5\,x-9}+4}-2\right)^3\sqrt{-\sqrt{\sqrt{5\,x}}}\right.\right.$$
$$-51)$$

input
```
integrate((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

output
```
-8/1575*((35*(sqrt(sqrt(5*x - 9) + 4) - 2)^4*sqrt(-sqrt(sqrt(5*x - 9) + 4)
 + 2) + 360*(sqrt(sqrt(5*x - 9) + 4) - 2)^3*sqrt(-sqrt(sqrt(5*x - 9) + 4)
+ 2) + 1512*(sqrt(sqrt(5*x - 9) + 4) - 2)^2*sqrt(-sqrt(sqrt(5*x - 9) + 4)
+ 2) - 3360*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(3/2) + 5040*sqrt(-sqrt(sqrt(5*
x - 9) + 4) + 2))*sgn(-4*(sqrt(5*x - 9) + 4)^2 + 32*sqrt(5*x - 9) + 79) -
18*(5*(sqrt(sqrt(5*x - 9) + 4) - 2)^3*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2) +
 42*(sqrt(sqrt(5*x - 9) + 4) - 2)^2*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2) - 1
40*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(3/2) + 280*sqrt(-sqrt(sqrt(5*x - 9) + 4
) + 2))*sgn(-4*(sqrt(5*x - 9) + 4)^2 + 32*sqrt(5*x - 9) + 79) - 84*(3*(sqr
t(sqrt(5*x - 9) + 4) - 2)^2*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2) - 20*(-sqrt
(sqrt(5*x - 9) + 4) + 2)^(3/2) + 60*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2))*sg
n(-4*(sqrt(5*x - 9) + 4)^2 + 32*sqrt(5*x - 9) + 79) - 840*((-sqrt(sqrt(5*x
 - 9) + 4) + 2)^(3/2) - 6*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2))*sgn(-4*(sqrt
(5*x - 9) + 4)^2 + 32*sqrt(5*x - 9) + 79))*sgn(20*x - 51)
```

3.715.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx = \int \sqrt{2-\sqrt{\sqrt{5\,x-9}+4}}\,dx$$

input
```
int((2 - ((5*x - 9)^(1/2) + 4)^(1/2))^(1/2),x)
```

output
```
int((2 - ((5*x - 9)^(1/2) + 4)^(1/2))^(1/2), x)
```

3.716 $\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx$

3.716.1 Optimal result

Integrand size = 17, antiderivative size = 83

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = -48\sqrt{2+\sqrt{1+\sqrt{x}}}+\frac{88}{3}\left(2+\sqrt{1+\sqrt{x}}\right)^{3/2} - \frac{48}{5}\left(2+\sqrt{1+\sqrt{x}}\right)^{5/2}+\frac{8}{7}\left(2+\sqrt{1+\sqrt{x}}\right)^{7/2}$$

output
```
88/3*(2+(1+x^(1/2))^(1/2))^(3/2)-48/5*(2+(1+x^(1/2))^(1/2))^(5/2)+8/7*(2+(
1+x^(1/2))^(1/2))^(7/2)-48*(2+(1+x^(1/2))^(1/2))^(1/2)
```

3.716.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = \frac{8}{105}\sqrt{2+\sqrt{1+\sqrt{x}}}\left(-280+76\sqrt{1+\sqrt{x}} + 3\left(-12+5\sqrt{1+\sqrt{x}}\right)\sqrt{x}\right)$$

input
```
Integrate[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

output
```
(8*Sqrt[2 + Sqrt[1 + Sqrt[x]]]*(-280 + 76*Sqrt[1 + Sqrt[x]] + 3*(-12 + 5*S
qrt[1 + Sqrt[x]])*Sqrt[x]))/105
```

3.716.3 Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {7267, 896, 25, 1732, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sqrt{\sqrt{x}+1}+2}}dx$$

$$\Big\downarrow \text{7267}$$

$$2\int \frac{\sqrt{x}}{\sqrt{\sqrt{\sqrt{x}+1}+2}}d\sqrt{x}$$

$$\Big\downarrow \text{896}$$

$$2\int \frac{\sqrt{x}}{\sqrt{\sqrt[4]{x}+2}}d\left(\sqrt{x}+1\right)$$

$$\Big\downarrow \text{25}$$

$$-2\int -\frac{\sqrt{x}}{\sqrt{\sqrt[4]{x}+2}}d\left(\sqrt{x}+1\right)$$

$$\Big\downarrow \text{1732}$$

$$-4\int \frac{(1-x)\sqrt[4]{x}}{\sqrt{\sqrt{x}+3}}d\sqrt[4]{x}$$

$$\Big\downarrow \text{522}$$

$$-4\int \left(-\left(\sqrt{x}+3\right)^{5/2}+6\left(\sqrt{x}+3\right)^{3/2}-11\sqrt{\sqrt{x}+3}+\frac{6}{\sqrt{\sqrt{x}+3}}\right)d\sqrt[4]{x}$$

$$\Big\downarrow \text{2009}$$

$$-4\left(-\frac{2}{7}\left(\sqrt{x}+3\right)^{7/2}+\frac{12}{5}\left(\sqrt{x}+3\right)^{5/2}-\frac{22}{3}\left(\sqrt{x}+3\right)^{3/2}+12\sqrt{\sqrt{x}+3}\right)$$

input
```
Int[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

output
```
-4*(12*Sqrt[3 + Sqrt[x]] - (22*(3 + Sqrt[x])^(3/2))/3 + (12*(3 + Sqrt[x])^
(5/2))/5 - (2*(3 + Sqrt[x])^(7/2))/7)
```

3.716.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 1732
```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(d + e*x^(g*
n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}
, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.716.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.65

method	result	size
derivativedivides	$\frac{88\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{3}{2}}}{3}-\frac{48\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{5}{2}}}{5}+\frac{8\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{7}{2}}}{7}-48\sqrt{2+\sqrt{1+\sqrt{x}}}$	54
default	$\frac{88\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{3}{2}}}{3}-\frac{48\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{5}{2}}}{5}+\frac{8\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{7}{2}}}{7}-48\sqrt{2+\sqrt{1+\sqrt{x}}}$	54

input
```
int(1/(2+(1+x^(1/2))^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
88/3*(2+(1+x^(1/2))^(1/2))^(3/2)-48/5*(2+(1+x^(1/2))^(1/2))^(5/2)+8/7*(2+(
1+x^(1/2))^(1/2))^(7/2)-48*(2+(1+x^(1/2))^(1/2))^(1/2)
```

3.716.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = \frac{8}{105}\left((15\sqrt{x}+76)\sqrt{\sqrt{x}+1}-36\sqrt{x}-280\right)\sqrt{\sqrt{\sqrt{x}+1}+2}$$

input
```
integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
8/105*((15*sqrt(x) + 76)*sqrt(sqrt(x) + 1) - 36*sqrt(x) - 280)*sqrt(sqrt(s
qrt(x) + 1) + 2)
```

3.716.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = \frac{8\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}}}{7}-\frac{48\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}}}{5}+\frac{88\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}}}{3}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

input
```
integrate(1/(2+(1+x**(1/2))**(1/2))**(1/2),x)
```

output
```
8*(sqrt(sqrt(x) + 1) + 2)**(7/2)/7 - 48*(sqrt(sqrt(x) + 1) + 2)**(5/2)/5 +
 88*(sqrt(sqrt(x) + 1) + 2)**(3/2)/3 - 48*sqrt(sqrt(sqrt(x) + 1) + 2)
```

3.716.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = \frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}} - \frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}} + \frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}} - 48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

input
```
integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
8/7*(sqrt(sqrt(x) + 1) + 2)^(7/2) - 48/5*(sqrt(sqrt(x) + 1) + 2)^(5/2) + 8
8/3*(sqrt(sqrt(x) + 1) + 2)^(3/2) - 48*sqrt(sqrt(sqrt(x) + 1) + 2)
```

3.716.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = \frac{8\left(15\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}} - 126\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}} + 385\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}} - 630\sqrt{\sqrt{\sqrt{x}+1}+2}\right)}{105\,\mathrm{sgn}\left(4\left(\sqrt{x}+1\right)^{2} - 8\sqrt{x} - 7\right)\mathrm{sgn}\left(4\,x-3\right)}$$

input
```
integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

output
```
8/105*(15*(sqrt(sqrt(x) + 1) + 2)^(7/2) - 126*(sqrt(sqrt(x) + 1) + 2)^(5/2
) + 385*(sqrt(sqrt(x) + 1) + 2)^(3/2) - 630*sqrt(sqrt(sqrt(x) + 1) + 2))/(
sgn(4*(sqrt(x) + 1)^2 - 8*sqrt(x) - 7)*sgn(4*x - 3))
```

3.716.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = \int \frac{1}{\sqrt{\sqrt{\sqrt{x}+1}+2}}\,dx$$

input
```
int(1/((x^(1/2) + 1)^(1/2) + 2)^(1/2),x)
```

output
```
int(1/((x^(1/2) + 1)^(1/2) + 2)^(1/2), x)
```

3.717 $\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx$

3.717.1 Optimal result

Integrand size = 23, antiderivative size = 190

$$\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx = -\frac{32}{5}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{5/2} + \frac{48}{7}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{7/2} + \frac{112}{9}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{9/2} - \frac{320}{11}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{11/2} + \frac{288}{13}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{13/2} - \frac{112}{15}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{15/2} + \frac{16}{17}\left(\right.$$

output
```
-32/5*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(5/2)+48/7*(1+(1+(1+x^(1/2))^(1/2))^
(1/2))^(7/2)+112/9*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(9/2)-320/11*(1+(1+(1+x
^(1/2))^(1/2))^(1/2))^(11/2)+288/13*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(13/2)
-112/15*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(15/2)+16/17*(1+(1+(1+x^(1/2))^(1/
2))^(1/2))^(17/2)
```

3.717.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.88

$$\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx$$
$$=\frac{16\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\left(-8\left(3519-1094\sqrt{1+\sqrt{1+\sqrt{x}}}+163\sqrt{1+\sqrt{x}}+584\sqrt{1+\sqrt{1+\sqrt{x}}}\sqrt{1+\right.\right.}{7657}$$

input
```
Integrate[Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]],x]
```

output
```
(16*Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]]*(-8*(3519 - 1094*Sqrt[1 + Sqrt[1
 + Sqrt[x]]] + 163*Sqrt[1 + Sqrt[x]] + 584*Sqrt[1 + Sqrt[1 + Sqrt[x]]]*Sqr
t[1 + Sqrt[x]]) + 7*(659 - 504*Sqrt[1 + Sqrt[1 + Sqrt[x]]] + 33*Sqrt[1 + S
qrt[x]] + 429*Sqrt[1 + Sqrt[1 + Sqrt[x]]]*Sqrt[1 + Sqrt[x]])*Sqrt[x] + 450
45*x))/765765
```

3.717.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {7267, 7267, 25, 7267, 2003, 2115, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sqrt{\sqrt{\sqrt{x}+1}+1}+1}\,dx$$
$$\Big\downarrow \; 7267$$
$$2\int \sqrt{\sqrt{\sqrt{\sqrt{x}+1}+1}+1}\sqrt{x}d\sqrt{x}$$
$$\Big\downarrow \; 7267$$
$$4\int -\sqrt{\sqrt{\sqrt{\sqrt{x}+1}+1}+1}\sqrt{\sqrt{x}+1}(1-x)d\sqrt{\sqrt{x}+1}$$
$$\Big\downarrow \; 25$$

$$-4\int \sqrt{\sqrt{\sqrt{\sqrt{x}+1}+1}+1}\sqrt{\sqrt{x}+1}(1-x)d\sqrt{\sqrt{x}+1}$$

$$\Big\downarrow \text{7267}$$

$$8\int \sqrt{\sqrt{\sqrt{\sqrt{x}+1}+1}+1}(1-x)(2-x)x^{3/2}d\sqrt{\sqrt{\sqrt{x}+1}+1}$$

$$\Big\downarrow \text{2003}$$

$$8\int \left(1-\sqrt{\sqrt{\sqrt{x}+1}+1}\right)\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{3/2}(2-x)x^{3/2}d\sqrt{\sqrt{\sqrt{x}+1}+1}$$

$$\Big\downarrow \text{2115}$$

$$8\int \left(\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{15/2}-7\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{13/2}+18\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{11/2}-20\left(\sqrt{\sqrt{x}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$8\left(\frac{2}{17}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{17/2}-\frac{14}{15}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{15/2}+\frac{36}{13}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{13/2}-\frac{40}{11}\left(\sqrt{\sqrt{}}\right.\right.$$

input
```
Int[Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]],x]
```

output
```
8*((-4*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(5/2))/5 + (6*(1 + Sqrt[1 + Sqrt[
1 + Sqrt[x]]])^(7/2))/7 + (14*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(9/2))/9 -
 (40*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(11/2))/11 + (36*(1 + Sqrt[1 + Sqrt
[1 + Sqrt[x]]])^(13/2))/13 - (14*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(15/2))
/15 + (2*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(17/2))/17)
```

3.717. $\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx$

3.717.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2003
```
Int[(u_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :
> Int[u*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}
, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&
  !IntegerQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2115
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.717.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.64

method	result
derivativedivides	$-\frac{32\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{5}{2}}}{5}+\frac{48\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{7}{2}}}{7}+\frac{112\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{9}{2}}}{9}-\frac{320\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{11}{2}}}{11}$
default	$-\frac{32\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{5}{2}}}{5}+\frac{48\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{7}{2}}}{7}+\frac{112\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{9}{2}}}{9}-\frac{320\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{11}{2}}}{11}$

input
```
int((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-32/5*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(5/2)+48/7*(1+(1+(1+x^(1/2))^(1/2))^
(1/2))^(7/2)+112/9*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(9/2)-320/11*(1+(1+(1+x
^(1/2))^(1/2))^(1/2))^(11/2)+288/13*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(13/2)
-112/15*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(15/2)+16/17*(1+(1+(1+x^(1/2))^(1/
2))^(1/2))^(17/2)
```

3.717.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.40

$$\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx$$
$$= \frac{16}{765765}\left(\left(231\sqrt{x}-1304\right)\sqrt{\sqrt{x}+1}+\left(\left(3003\sqrt{x}-4672\right)\sqrt{\sqrt{x}+1}-3528\sqrt{x}+8752\right)\sqrt{\sqrt{\sqrt{x}+1}+\right.$$

input
```
integrate((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
16/765765*((231*sqrt(x) - 1304)*sqrt(sqrt(x) + 1) + ((3003*sqrt(x) - 4672)
*sqrt(sqrt(x) + 1) - 3528*sqrt(x) + 8752)*sqrt(sqrt(sqrt(x) + 1) + 1) + 45
045*x + 4613*sqrt(x) - 28152)*sqrt(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)
```

3.717.6 Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.87

$$\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx = \frac{16\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{17}{2}}}{17} - \frac{112\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{15}{2}}}{15} + \frac{288\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{13}{2}}}{13} - \frac{320\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{11}{2}}}{11} + \frac{112\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{9}{2}}}{9} + \frac{48\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{7}{2}}}{7} - \frac{32\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{5}{2}}}{5}$$

input
```
integrate((1+(1+(1+x**(1/2))**(1/2))**(1/2))**(1/2),x)
```

output
```
16*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)**(17/2)/17 - 112*(sqrt(sqrt(sqrt(x) +
 1) + 1) + 1)**(15/2)/15 + 288*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)**(13/2)/1
3 - 320*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)**(11/2)/11 + 112*(sqrt(sqrt(sqrt
(x) + 1) + 1) + 1)**(9/2)/9 + 48*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)**(7/2)/
7 - 32*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)**(5/2)/5
```

3.717.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.63

$$\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx = \frac{16}{17}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{17}{2}} - \frac{112}{15}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{15}{2}} + \frac{288}{13}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{13}{2}} - \frac{320}{11}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{11}{2}} + \frac{112}{9}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{9}{2}} + \frac{48}{7}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{7}{2}} - \frac{32}{5}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{5}{2}}$$

input
```
integrate((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
16/17*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(17/2) - 112/15*(sqrt(sqrt(sqrt(x)
 + 1) + 1) + 1)^(15/2) + 288/13*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(13/2) -
 320/11*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(11/2) + 112/9*(sqrt(sqrt(sqrt(x
) + 1) + 1) + 1)^(9/2) + 48/7*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(7/2) - 32
/5*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(5/2)
```

3.717.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7916 vs. 2(120) = 240.

Time = 43.80 (sec) , antiderivative size = 7916, normalized size of antiderivative = 41.66

$$\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx = \text{Too large to display}$$

input
```
integrate((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

output
```
16/765765*(7*(6435*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(17/2) - 58344*(sqrt(
sqrt(sqrt(x) + 1) + 1) + 1)^(15/2) + 235620*(sqrt(sqrt(sqrt(x) + 1) + 1) +
 1)^(13/2) - 556920*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(11/2) + 850850*(sqr
t(sqrt(sqrt(x) + 1) + 1) + 1)^(9/2) - 875160*(sqrt(sqrt(sqrt(x) + 1) + 1)
+ 1)^(7/2) + 612612*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(5/2) - 291720*(sqrt
(sqrt(sqrt(x) + 1) + 1) + 1)^(3/2) + 109395*sqrt(sqrt(sqrt(sqrt(x) + 1) +
1) + 1))*sgn(70368744177664*(sqrt(sqrt(x) + 1) + 1)^92 - 6473924464345088*
(sqrt(sqrt(x) + 1) + 1)^91 + 291326600895528960*(sqrt(sqrt(x) + 1) + 1)^90
 - 8545580292935516160*(sqrt(sqrt(x) + 1) + 1)^89 + 183728762437276532736*
(sqrt(sqrt(x) + 1) + 1)^88 - 3086556782054646743040*(sqrt(sqrt(x) + 1) + 1
)^87 + 42179809308639429132288*(sqrt(sqrt(x) + 1) + 1)^86 - 48197884682284
1400164352*(sqrt(sqrt(x) + 1) + 1)^85 + 4697911198078384159588352*(sqrt(sq
rt(x) + 1) + 1)^84 - 39651330432185076620984320*(sqrt(sqrt(x) + 1) + 1)^83
 + 293183639716003233721745408*(sqrt(sqrt(x) + 1) + 1)^82 - 19166563364402
69370174734336*(sqrt(sqrt(x) + 1) + 1)^81 + 11160164453620451334571425792*
(sqrt(sqrt(x) + 1) + 1)^80 - 58223902019906429347317153792*(sqrt(sqrt(x) +
 1) + 1)^79 + 273479024956137655533112918016*(sqrt(sqrt(x) + 1) + 1)^78 -
1160956607882993155309408616448*(sqrt(sqrt(x) + 1) + 1)^77 + 4467886822469
532994953426239488*(sqrt(sqrt(x) + 1) + 1)^76 - 15624039803063454614788052
615168*(sqrt(sqrt(x) + 1) + 1)^75 + 49728771914087708805425247813632*(s...
```

3.717.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx = \int \sqrt{\sqrt{\sqrt{\sqrt{x}+1}+1}+1}\,dx$$

input
```
int((((x^(1/2) + 1)^(1/2) + 1)^(1/2) + 1)^(1/2),x)
```

output
```
int((((x^(1/2) + 1)^(1/2) + 1)^(1/2) + 1)^(1/2), x)
```

3.717. $\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx$

3.718 $\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx$

3.718.1 Optimal result

Integrand size = 25, antiderivative size = 233

$$\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx = -\frac{16}{3}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{3/2} + \frac{136}{5}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{5/2} - \frac{480}{7}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{7/2} + \frac{304}{3}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{9/2} - \frac{760}{11}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{11/2} + \frac{300}{13}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{13/2}$$

output
```
-16/3*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(3/2)+136/5*(2+(3+(-1+2*x^(1/2))^
(1/2))^(1/2))^(5/2)-480/7*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(7/2)+304/3*(
2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(9/2)-760/11*(2+(3+(-1+2*x^(1/2))^(1/2))
^(1/2))^(11/2)+300/13*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(13/2)-56/15*(2+(
3+(-1+2*x^(1/2))^(1/2))^(1/2))^(15/2)+4/17*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/
2))^(17/2)
```

3.718.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.80

$$\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx$$
$$= \frac{8\sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\left(8\left(-15510-7428\sqrt{3+\sqrt{-1+2\sqrt{x}}}+211\sqrt{-1+2\sqrt{x}}+1700\sqrt{3+\sqrt{-1}\right.\right.}{}$$

input
```
Integrate[Sqrt[2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]],x]
```

output
```
(8*Sqrt[2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]]*(8*(-15510 - 7428*Sqrt[3 + Sqr
t[-1 + 2*Sqrt[x]]] + 211*Sqrt[-1 + 2*Sqrt[x]] + 1700*Sqrt[3 + Sqrt[-1 + 2*
Sqrt[x]]]*Sqrt[-1 + 2*Sqrt[x]]) + 7*(-549 - 672*Sqrt[3 + Sqrt[-1 + 2*Sqrt[
x]]] - 121*Sqrt[-1 + 2*Sqrt[x]] + 286*Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]*Sqrt[
-1 + 2*Sqrt[x]])*Sqrt[x] + 30030*x))/255255
```

3.718.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {7267, 7267, 7267, 25, 2091, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}\,dx$$
$$\Big\downarrow \text{7267}$$
$$2\int \sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}\sqrt{x}d\sqrt{x}$$
$$\Big\downarrow \text{7267}$$
$$\int \sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}\sqrt{2\sqrt{x}-1}(x+1)d\sqrt{2\sqrt{x}-1}$$
$$\Big\downarrow \text{7267}$$

$$2\int -\sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}\sqrt{\sqrt{2\sqrt{x}-1}+3}\left((x-3)^2+1\right)(3-x)d\sqrt{\sqrt{2\sqrt{x}-1}+3}$$

$$\Big\downarrow \text{25}$$

$$-2\int \sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}\sqrt{\sqrt{2\sqrt{x}-1}+3}\left((x-3)^2+1\right)(3-x)d\sqrt{\sqrt{2\sqrt{x}-1}+3}$$

$$\Big\downarrow \text{2091}$$

$$-2\int \sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}\sqrt{\sqrt{2\sqrt{x}-1}+3}(3-x)\left(x^2-6x+10\right)d\sqrt{\sqrt{2\sqrt{x}-1}+3}$$

$$\Big\downarrow \text{2123}$$

$$-2\int \left(-\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{15/2}+14\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{13/2}-75\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{11/2}+190\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{2}{17}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{17/2}-\frac{28}{15}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{15/2}+\frac{150}{13}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{13/2}-\frac{380}{11}\left(\right.\right.$$

input
```
Int[Sqrt[2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]],x]
```

output
```
2*((-8*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(3/2))/3 + (68*(2 + Sqrt[3 + S
qrt[-1 + 2*Sqrt[x]]])^(5/2))/5 - (240*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])
^(7/2))/7 + (152*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(9/2))/3 - (380*(2 +
 Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(11/2))/11 + (150*(2 + Sqrt[3 + Sqrt[-1 +
 2*Sqrt[x]]])^(13/2))/13 - (28*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(15/2)
)/15 + (2*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(17/2))/17)
```

3.718. $\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx$

3.718.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2091
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && PolyQ[Px, x] && BinomialQ[z, x
] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ[u, x])
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.718.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.66

method	result
derivativedivides	$-\frac{16\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{3}{2}}}{3}+\frac{136\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{5}{2}}}{5}-\frac{480\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{7}{2}}}{7}+\frac{304\left(2+\sqrt{3+\sqrt{-}\right.}{3}$
default	$-\frac{16\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{3}{2}}}{3}+\frac{136\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{5}{2}}}{5}-\frac{480\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{7}{2}}}{7}+\frac{304\left(2+\sqrt{3+\sqrt{-}\right.}{3}$

input
```
int((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-16/3*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(3/2)+136/5*(2+(3+(-1+2*x^(1/2))^
(1/2))^(1/2))^(5/2)-480/7*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(7/2)+304/3*(
2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(9/2)-760/11*(2+(3+(-1+2*x^(1/2))^(1/2))
^(1/2))^(11/2)+300/13*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(13/2)-56/15*(2+(
3+(-1+2*x^(1/2))^(1/2))^(1/2))^(15/2)+4/17*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/
2))^(17/2)
```

3.718.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.36

$$\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx =$$

$$-\frac{8}{255255}\left(\left(847\sqrt{x}-1688\right)\sqrt{2\sqrt{x}-1}-2\left(\left(1001\sqrt{x}+6800\right)\sqrt{2\sqrt{x}-1}-2352\sqrt{x}-29712\right)\sqrt{\sqrt{2}}\right.$$

input
```
integrate((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-8/255255*((847*sqrt(x) - 1688)*sqrt(2*sqrt(x) - 1) - 2*((1001*sqrt(x) + 6
800)*sqrt(2*sqrt(x) - 1) - 2352*sqrt(x) - 29712)*sqrt(sqrt(2*sqrt(x) - 1)
+ 3) - 30030*x + 3843*sqrt(x) + 124080)*sqrt(sqrt(sqrt(2*sqrt(x) - 1) + 3)
 + 2)
```

3.718.6 Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.87

$$\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx = \frac{4\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{17}{2}}}{17} - \frac{56\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{15}{2}}}{15} + \frac{300\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{13}{2}}}{13} - \frac{760\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{11}{2}}}{11} + \frac{304\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{9}{2}}}{3} - \frac{480\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{7}{2}}}{7} + \frac{136\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{5}{2}}}{5} - \frac{16\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{3}{2}}}{3}$$

input
```
integrate((2+(3+(-1+2*x**(1/2))**(1/2))**(1/2))**(1/2),x)
```

output
```
4*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)**(17/2)/17 - 56*(sqrt(sqrt(2*sqrt(x)
 - 1) + 3) + 2)**(15/2)/15 + 300*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)**(13/
2)/13 - 760*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)**(11/2)/11 + 304*(sqrt(sqr
t(2*sqrt(x) - 1) + 3) + 2)**(9/2)/3 - 480*(sqrt(sqrt(2*sqrt(x) - 1) + 3) +
 2)**(7/2)/7 + 136*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)**(5/2)/5 - 16*(sqrt
(sqrt(2*sqrt(x) - 1) + 3) + 2)**(3/2)/3
```

3.718.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.66

$$\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx = \frac{4}{17}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{17}{2}} - \frac{56}{15}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{15}{2}} + \frac{300}{13}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{13}{2}} - \frac{760}{11}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{11}{2}} + \frac{304}{3}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{9}{2}} - \frac{480}{7}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{7}{2}} + \frac{136}{5}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{5}{2}} - \frac{16}{3}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{3}{2}}$$

input
```
integrate((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
4/17*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(17/2) - 56/15*(sqrt(sqrt(2*sqrt(
x) - 1) + 3) + 2)^(15/2) + 300/13*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(13/
2) - 760/11*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(11/2) + 304/3*(sqrt(sqrt(
2*sqrt(x) - 1) + 3) + 2)^(9/2) - 480/7*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)
^(7/2) + 136/5*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(5/2) - 16/3*(sqrt(sqrt
(2*sqrt(x) - 1) + 3) + 2)^(3/2)
```

3.718.8 Giac [A] (verification not implemented)

Time = 4.88 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.16

$$\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx$$
$$=\frac{4}{255255}\left(15015\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{17}{2}}-238238\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{15}{2}}+1472625\left(\sqrt{\sqrt{2\sqrt{x}\right.\right.$$
$$+376832\,x^{22}+8224768\,x^{21}+113971200\,x^{20}+1130782720\,x^{19}+8582063104\,x^{18}$$
$$+51933387264\,x^{17}+257575619584\,x^{16}+1066188686592\,x^{15}+3723204389632\,x^{14}$$
$$+11019822890016\,x^{13}+27631512444352\,x^{12}+58424530490176\,x^{11}$$
$$+103336828749760\,x^{10}+151203890043312\,x^{9}+180411181747936\,x^{8}$$
$$+172287199292960\,x^{7}+128457231939048\,x^{6}+72257964298210\,x^{5}+29175203228012\,x^{4}$$
$$\left.+7830371130072\,x^{3}+1228114804752\,x^{2}+87490886400\,x+933120000\right)$$

input
```
integrate((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

output
```
4/255255*(15015*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(17/2) - 238238*(sqrt(
sqrt(2*sqrt(x) - 1) + 3) + 2)^(15/2) + 1472625*(sqrt(sqrt(2*sqrt(x) - 1) +
 3) + 2)^(13/2) - 4408950*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(11/2) + 646
6460*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(9/2) - 4375800*(sqrt(sqrt(2*sqrt
(x) - 1) + 3) + 2)^(7/2) + 1735734*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(5/
2) - 340340*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(3/2))*sgn(8192*x^23 + 376
832*x^22 + 8224768*x^21 + 113971200*x^20 + 1130782720*x^19 + 8582063104*x^
18 + 51933387264*x^17 + 257575619584*x^16 + 1066188686592*x^15 + 372320438
9632*x^14 + 11019822890016*x^13 + 27631512444352*x^12 + 58424530490176*x^1
1 + 103336828749760*x^10 + 151203890043312*x^9 + 180411181747936*x^8 + 172
287199292960*x^7 + 128457231939048*x^6 + 72257964298210*x^5 + 291752032280
12*x^4 + 7830371130072*x^3 + 1228114804752*x^2 + 87490886400*x + 933120000
)
```

3.718.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx = \int \sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}\,dx$$

input
```
int((((2*x^(1/2) - 1)^(1/2) + 3)^(1/2) + 2)^(1/2),x)
```

output
```
int((((2*x^(1/2) - 1)^(1/2) + 3)^(1/2) + 2)^(1/2), x)
```

3.719 $\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}\,dx$

3.719.1 Optimal result

Integrand size = 21, antiderivative size = 160

$$\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}\,dx = \frac{16}{5}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{5/2}-\frac{24}{7}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{7/2}+8\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{9/2} -\frac{160}{11}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{11/2}+\frac{144}{13}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{13/2}-\frac{56}{15}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{15/2}+\frac{8}{17}\left(1+\right.$$

output
```
16/5*(1+(1+(-1+x)^(1/2))^(1/2))^(5/2)-24/7*(1+(1+(-1+x)^(1/2))^(1/2))^(7/2
)+8*(1+(1+(-1+x)^(1/2))^(1/2))^(9/2)-160/11*(1+(1+(-1+x)^(1/2))^(1/2))^(11
/2)+144/13*(1+(1+(-1+x)^(1/2))^(1/2))^(13/2)-56/15*(1+(1+(-1+x)^(1/2))^(1/
2))^(15/2)+8/17*(1+(1+(-1+x)^(1/2))^(1/2))^(17/2)
```

3.719.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.59

$$\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}\,dx = \frac{8\sqrt{1+\sqrt{1+\sqrt{-1+x}}}\left(-8872+1109\sqrt{-1+x}+28231(-1+x)+77(-1+x)^{3/2}+15015(-1+x)^2+\right.}{255255}$$

input
```
Integrate[Sqrt[1 + Sqrt[1 + Sqrt[-1 + x]]]*x,x]
```

output
```
(8*Sqrt[1 + Sqrt[1 + Sqrt[-1 + x]]]*(-8872 + 1109*Sqrt[-1 + x] + 28231*(-1
 + x) + 77*(-1 + x)^(3/2) + 15015*(-1 + x)^2 + Sqrt[1 + Sqrt[-1 + x]]*(-76
96 + 4544*Sqrt[-1 + x] + 7*(-168 + 143*Sqrt[-1 + x])*x)))/255255
```

3.719.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {7267, 7267, 25, 2003, 2091, 2115, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sqrt{\sqrt{x-1}+1}+1}x\,dx$$

$$\Big\downarrow \text{7267}$$

$$2\int \sqrt{\sqrt{\sqrt{x-1}+1}+1}\sqrt{x-1}xd\sqrt{x-1}$$

$$\Big\downarrow \text{7267}$$

$$4\int -\sqrt{\sqrt{\sqrt{x-1}+1}+1}\left((x-2)^2+1\right)\sqrt{\sqrt{x-1}+1}(2-x)d\sqrt{\sqrt{x-1}+1}$$

$$\Big\downarrow \text{25}$$

$$-4\int \sqrt{\sqrt{\sqrt{x-1}+1}+1}\left((x-2)^2+1\right)\sqrt{\sqrt{x-1}+1}(2-x)d\sqrt{\sqrt{x-1}+1}$$

$$\Big\downarrow \text{2003}$$

$$-4\int \left(1-\sqrt{\sqrt{x-1}+1}\right)\left(\sqrt{\sqrt{x-1}+1}+1\right)^{3/2}\left((x-2)^2+1\right)\sqrt{\sqrt{x-1}+1}d\sqrt{\sqrt{x-1}+1}$$

$$\Big\downarrow \text{2091}$$

$$-4\int \left(1-\sqrt{\sqrt{x-1}+1}\right)\left(\sqrt{\sqrt{x-1}+1}+1\right)^{3/2}\sqrt{\sqrt{x-1}+1}\left((x-1)^2-2(x-1)+2\right)d\sqrt{\sqrt{x-1}+1}$$

$$\Big\downarrow \text{2115}$$

$$-4\int\left(-\left(\sqrt{\sqrt{x-1}+1}+1\right)^{15/2}+7\left(\sqrt{\sqrt{x-1}+1}+1\right)^{13/2}-18\left(\sqrt{\sqrt{x-1}+1}+1\right)^{11/2}+20\left(\sqrt{\sqrt{x-1}+\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$4\left(\frac{2}{17}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{17/2}-\frac{14}{15}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{15/2}+\frac{36}{13}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{13/2}-\frac{40}{11}\left(\sqrt{\sqrt{x-1}+1\right.\right.$$

input
```
Int[Sqrt[1 + Sqrt[1 + Sqrt[-1 + x]]]*x,x]
```

output
```
4*((4*(1 + Sqrt[1 + Sqrt[-1 + x]])^(5/2))/5 - (6*(1 + Sqrt[1 + Sqrt[-1 + x
]])^(7/2))/7 + 2*(1 + Sqrt[1 + Sqrt[-1 + x]])^(9/2) - (40*(1 + Sqrt[1 + Sq
rt[-1 + x]])^(11/2))/11 + (36*(1 + Sqrt[1 + Sqrt[-1 + x]])^(13/2))/13 - (1
4*(1 + Sqrt[1 + Sqrt[-1 + x]])^(15/2))/15 + (2*(1 + Sqrt[1 + Sqrt[-1 + x]]
)^(17/2))/17)
```

3.719.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2003
```
Int[(u_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :
> Int[u*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}
, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&
  !IntegerQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2091
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && PolyQ[Px, x] && BinomialQ[z, x
] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ[u, x])
```

rule 2115
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

3.719. $\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}x}\,dx$

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.719.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{16\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{5}{2}}}{5}-\frac{24\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{7}{2}}}{7}+8\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{9}{2}}-\frac{160\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{11}{2}}}{11}+$
default	$\frac{16\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{5}{2}}}{5}-\frac{24\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{7}{2}}}{7}+8\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{9}{2}}-\frac{160\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{11}{2}}}{11}+$

input
```
int(x*(1+(1+(x-1)^(1/2))^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
16/5*(1+(1+(x-1)^(1/2))^(1/2))^(5/2)-24/7*(1+(1+(x-1)^(1/2))^(1/2))^(7/2)+
8*(1+(1+(x-1)^(1/2))^(1/2))^(9/2)-160/11*(1+(1+(x-1)^(1/2))^(1/2))^(11/2)+
144/13*(1+(1+(x-1)^(1/2))^(1/2))^(13/2)-56/15*(1+(1+(x-1)^(1/2))^(1/2))^(1
5/2)+8/17*(1+(1+(x-1)^(1/2))^(1/2))^(17/2)
```

3.719.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.39

$$\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}x\,dx$$
$$=\frac{8}{255255}\left(15015\,x^2+(77\,x+1032)\sqrt{x-1}+\left((1001\,x+4544)\sqrt{x-1}-1176\,x-7696\right)\sqrt{\sqrt{x-1}+1}-\right.$$

input
```
integrate(x*(1+(1+(-1+x)^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
8/255255*(15015*x^2 + (77*x + 1032)*sqrt(x - 1) + ((1001*x + 4544)*sqrt(x
- 1) - 1176*x - 7696)*sqrt(sqrt(x - 1) + 1) - 1799*x - 22088)*sqrt(sqrt(sq
rt(x - 1) + 1) + 1)
```

3.719.6 Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.87

$$\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}x\,dx = \frac{8\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{17}{2}}}{17} - \frac{56\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{15}{2}}}{15} + \frac{144\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{13}{2}}}{13} - \frac{160\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{11}{2}}}{11} + 8\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{9}{2}} - \frac{24\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{7}{2}}}{7} + \frac{16\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{5}{2}}}{5}$$

input
```
integrate(x*(1+(1+(-1+x)**(1/2))**(1/2))**(1/2),x)
```

output
```
8*(sqrt(sqrt(x - 1) + 1) + 1)**(17/2)/17 - 56*(sqrt(sqrt(x - 1) + 1) + 1)*
*(15/2)/15 + 144*(sqrt(sqrt(x - 1) + 1) + 1)**(13/2)/13 - 160*(sqrt(sqrt(x
 - 1) + 1) + 1)**(11/2)/11 + 8*(sqrt(sqrt(x - 1) + 1) + 1)**(9/2) - 24*(sq
rt(sqrt(x - 1) + 1) + 1)**(7/2)/7 + 16*(sqrt(sqrt(x - 1) + 1) + 1)**(5/2)/
5
```

3.719.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.66

$$\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}x\,dx = \frac{8}{17}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{17}{2}} - \frac{56}{15}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{15}{2}} + \frac{144}{13}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{13}{2}} - \frac{160}{11}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{11}{2}} + 8\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{9}{2}} - \frac{24}{7}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{7}{2}} + \frac{16}{5}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{5}{2}}$$

input
```
integrate(x*(1+(1+(-1+x)^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
8/17*(sqrt(sqrt(x - 1) + 1) + 1)^(17/2) - 56/15*(sqrt(sqrt(x - 1) + 1) + 1
)^(15/2) + 144/13*(sqrt(sqrt(x - 1) + 1) + 1)^(13/2) - 160/11*(sqrt(sqrt(x
 - 1) + 1) + 1)^(11/2) + 8*(sqrt(sqrt(x - 1) + 1) + 1)^(9/2) - 24/7*(sqrt(
sqrt(x - 1) + 1) + 1)^(7/2) + 16/5*(sqrt(sqrt(x - 1) + 1) + 1)^(5/2)
```

3.719.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 859 vs. $2(106) = 212$.

Time = 0.43 (sec) , antiderivative size = 859, normalized size of antiderivative = 5.37

$$\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}x\,dx = \text{Too large to display}$$

input
```
integrate(x*(1+(1+(-1+x)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

output
```
8/765765*(7*(6435*(sqrt(sqrt(x - 1) + 1) + 1)^(17/2) - 58344*(sqrt(sqrt(x
- 1) + 1) + 1)^(15/2) + 235620*(sqrt(sqrt(x - 1) + 1) + 1)^(13/2) - 556920
*(sqrt(sqrt(x - 1) + 1) + 1)^(11/2) + 850850*(sqrt(sqrt(x - 1) + 1) + 1)^(
9/2) - 875160*(sqrt(sqrt(x - 1) + 1) + 1)^(7/2) + 612612*(sqrt(sqrt(x - 1)
 + 1) + 1)^(5/2) - 291720*(sqrt(sqrt(x - 1) + 1) + 1)^(3/2) + 109395*sqrt(
sqrt(sqrt(x - 1) + 1) + 1))*sgn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7)
 + 119*(429*(sqrt(sqrt(x - 1) + 1) + 1)^(15/2) - 3465*(sqrt(sqrt(x - 1) +
1) + 1)^(13/2) + 12285*(sqrt(sqrt(x - 1) + 1) + 1)^(11/2) - 25025*(sqrt(sq
rt(x - 1) + 1) + 1)^(9/2) + 32175*(sqrt(sqrt(x - 1) + 1) + 1)^(7/2) - 2702
7*(sqrt(sqrt(x - 1) + 1) + 1)^(5/2) + 15015*(sqrt(sqrt(x - 1) + 1) + 1)^(3
/2) - 6435*sqrt(sqrt(sqrt(x - 1) + 1) + 1))*sgn(4*(sqrt(x - 1) + 1)^2 - 8*
sqrt(x - 1) - 7) - 765*(231*(sqrt(sqrt(x - 1) + 1) + 1)^(13/2) - 1638*(sqr
t(sqrt(x - 1) + 1) + 1)^(11/2) + 5005*(sqrt(sqrt(x - 1) + 1) + 1)^(9/2) -
8580*(sqrt(sqrt(x - 1) + 1) + 1)^(7/2) + 9009*(sqrt(sqrt(x - 1) + 1) + 1)^
(5/2) - 6006*(sqrt(sqrt(x - 1) + 1) + 1)^(3/2) + 3003*sqrt(sqrt(sqrt(x - 1
) + 1) + 1))*sgn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7) - 3315*(63*(sq
rt(sqrt(x - 1) + 1) + 1)^(11/2) - 385*(sqrt(sqrt(x - 1) + 1) + 1)^(9/2) +
990*(sqrt(sqrt(x - 1) + 1) + 1)^(7/2) - 1386*(sqrt(sqrt(x - 1) + 1) + 1)^(
5/2) + 1155*(sqrt(sqrt(x - 1) + 1) + 1)^(3/2) - 693*sqrt(sqrt(sqrt(x - 1)
+ 1) + 1))*sgn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7) + 9724*(35*(s...
```

3.719.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}x\,dx = \int x\sqrt{\sqrt{\sqrt{x-1}+1}+1}\,dx$$

input
```
int(x*(((x - 1)^(1/2) + 1)^(1/2) + 1)^(1/2),x)
```

output
```
int(x*(((x - 1)^(1/2) + 1)^(1/2) + 1)^(1/2), x)
```

3.720 $\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}} \, dx$

3.720.1 Optimal result

Integrand size = 23, antiderivative size = 20

$$\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}} \, dx = -2\text{arcsinh}\left(\frac{1-2\sqrt{-1+x}}{\sqrt{3}}\right)$$

output
```
-2*arcsinh(1/3*(1-2*(-1+x)^(1/2))*3^(1/2))
```

3.720.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}} \, dx = -2\log\left(1-2\sqrt{-1+x}+2\sqrt{-\sqrt{-1+x}+x}\right)$$

input
```
Integrate[1/(Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]
```

output
```
-2*Log[1 - 2*Sqrt[-1 + x] + 2*Sqrt[-Sqrt[-1 + x] + x]]
```

3.720.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {7267, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x-1}\sqrt{x-\sqrt{x-1}}}dx$$
$$\Big\downarrow \text{7267}$$
$$2\int \frac{1}{\sqrt{x-\sqrt{x-1}}}d\sqrt{x-1}$$
$$\Big\downarrow \text{1090}$$
$$\frac{2\int \frac{1}{\sqrt{\frac{x-1}{3}+1}}d(2\sqrt{x-1}-1)}{\sqrt{3}}$$
$$\Big\downarrow \text{222}$$
$$2\text{arcsinh}\left(\frac{2\sqrt{x-1}-1}{\sqrt{3}}\right)$$

input
```
Int[1/(Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]
```

output
```
2*ArcSinh[(-1 + 2*Sqrt[-1 + x])/Sqrt[3]]
```

3.720.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.720.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$2 \operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(\sqrt{x-1}-\frac{1}{2}\right)}{3}\right)$	16
default	$2 \operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(\sqrt{x-1}-\frac{1}{2}\right)}{3}\right)$	16

input
```
int(1/(x-1)^(1/2)/(x-(x-1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arcsinh(2/3*3^(1/2)*((x-1)^(1/2)-1/2))
```

3.720.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(17) = 34.

Time = 0.45 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = \log\left(4\sqrt{x-\sqrt{x-1}}\left(2\sqrt{x-1}-1\right)+8x-8\sqrt{x-1}-3\right)$$

input
```
integrate(1/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
log(4*sqrt(x - sqrt(x - 1))*(2*sqrt(x - 1) - 1) + 8*x - 8*sqrt(x - 1) - 3)
```

3.720.6 Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = 2\,\mathrm{asinh}\left(\frac{2\sqrt{3}\left(\sqrt{x-1}-\frac{1}{2}\right)}{3}\right)$$

input
```
integrate(1/(-1+x)**(1/2)/(x-(-1+x)**(1/2))**(1/2),x)
```

output
```
2*asinh(2*sqrt(3)*(sqrt(x - 1) - 1/2)/3)
```

3.720.7 Maxima [F]

$$\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = \int \frac{1}{\sqrt{x-\sqrt{x-1}}\sqrt{x-1}}\,dx$$

input
```
integrate(1/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x - sqrt(x - 1))*sqrt(x - 1)), x)
```

3.720.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = -2\,\log\left(2\sqrt{x-\sqrt{x-1}}-2\sqrt{x-1}+1\right)$$

input
```
integrate(1/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
-2*log(2*sqrt(x - sqrt(x - 1)) - 2*sqrt(x - 1) + 1)
```

3.720.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = \int \frac{1}{\sqrt{x-\sqrt{x-1}}\,\sqrt{x-1}}\,dx$$

input
```
int(1/((x - (x - 1)^(1/2))^(1/2)*(x - 1)^(1/2)),x)
```

output
```
int(1/((x - (x - 1)^(1/2))^(1/2)*(x - 1)^(1/2)), x)
```

3.721 $\int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}}\,dx$

3.721.1 Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}}\,dx = 2\sqrt{1+x+\sqrt{-1+2x}}-\sqrt{2}\text{arcsinh}\left(\frac{1+\sqrt{-1+2x}}{\sqrt{2}}\right)$$

output
```
-arcsinh(1/2*(1+(-1+2*x)^(1/2))*2^(1/2))*2^(1/2)+2*(1+x+(-1+2*x)^(1/2))^(1
/2)
```

3.721.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.34

$$\int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}}\,dx = 2\sqrt{1+x+\sqrt{-1+2x}} + \sqrt{2}\log\left(-1-\sqrt{-1+2x}+\sqrt{2+2x+2\sqrt{-1+2x}}\right)$$

input
```
Integrate[1/Sqrt[1 + x + Sqrt[-1 + 2*x]],x]
```

output
```
2*Sqrt[1 + x + Sqrt[-1 + 2*x]] + Sqrt[2]*Log[-1 - Sqrt[-1 + 2*x] + Sqrt[2
+ 2*x + 2*Sqrt[-1 + 2*x]]]
```

3.721.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {7267, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x+\sqrt{2x-1}+1}}dx$$

$$\Big\downarrow \text{7267}$$

$$\int \frac{\sqrt{2}\sqrt{2x-1}}{\sqrt{2x+2\sqrt{2x-1}+2}}d\sqrt{2x-1}$$

$$\Big\downarrow \text{27}$$

$$\sqrt{2}\int \frac{\sqrt{2x-1}}{\sqrt{2x+2\sqrt{2x-1}+2}}d\sqrt{2x-1}$$

$$\Big\downarrow \text{1160}$$

$$\sqrt{2}\left(\sqrt{2x+2\sqrt{2x-1}+2}-\int \frac{1}{\sqrt{2x+2\sqrt{2x-1}+2}}d\sqrt{2x-1}\right)$$

$$\Big\downarrow \text{1090}$$

$$\sqrt{2}\left(\sqrt{2x+2\sqrt{2x-1}+2}-\frac{\int \frac{1}{\sqrt{\frac{1}{8}(2x-1)+1}}d\left(2\sqrt{2x-1}+2\right)}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{222}$$

$$\sqrt{2}\left(\sqrt{2x+2\sqrt{2x-1}+2}-\text{arcsinh}\left(\frac{2\sqrt{2x-1}+2}{2\sqrt{2}}\right)\right)$$

input
```
Int[1/Sqrt[1 + x + Sqrt[-1 + 2*x]],x]
```

output
```
Sqrt[2]*(Sqrt[2 + 2*x + 2*Sqrt[-1 + 2*x]] - ArcSinh[(2 + 2*Sqrt[-1 + 2*x])
/(2*Sqrt[2])])
```

3.721.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.721.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\sqrt{4x+4+4\sqrt{2x-1}}-\operatorname{arcsinh}\left(\frac{(1+\sqrt{2x-1})\sqrt{2}}{2}\right)\sqrt{2}$	38
default	$\sqrt{4x+4+4\sqrt{2x-1}}-\operatorname{arcsinh}\left(\frac{(1+\sqrt{2x-1})\sqrt{2}}{2}\right)\sqrt{2}$	38

input
```
int(1/(1+x+(2*x-1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(4*x+4+4*(2*x-1)^(1/2))^(1/2)-arcsinh(1/2*(1+(2*x-1)^(1/2))*2^(1/2))*2^(1/
2)
```

3.721.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(35) = 70.

Time = 0.65 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.93

$$\int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}}\,dx = \frac{1}{4}\sqrt{2}\log\left(-8\,x^2 - 8\,(2\,x+1)\sqrt{2\,x-1} + 2\left(\sqrt{2}(2\,x+3)\sqrt{2\,x-1} + \sqrt{2}(6\,x-1)\right)\sqrt{x+\sqrt{2\,x-1}+1} - 24\,x + 7\right) + 2\sqrt{x+\sqrt{2\,x-1}+1}$$

input
```
integrate(1/(1+x+(-1+2*x)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(-8*x^2 - 8*(2*x + 1)*sqrt(2*x - 1) + 2*(sqrt(2)*(2*x + 3)*
sqrt(2*x - 1) + sqrt(2)*(6*x - 1))*sqrt(x + sqrt(2*x - 1) + 1) - 24*x + 7)
 + 2*sqrt(x + sqrt(2*x - 1) + 1)
```

3.721.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}}\,dx = 2\sqrt{x+\sqrt{2x-1}+1} - \sqrt{2}\,\mathrm{asinh}\left(\frac{\sqrt{2}\left(\sqrt{2x-1}+1\right)}{2}\right)$$

input
```
integrate(1/(1+x+(-1+2*x)**(1/2))**(1/2),x)
```

output
```
2*sqrt(x + sqrt(2*x - 1) + 1) - sqrt(2)*asinh(sqrt(2)*(sqrt(2*x - 1) + 1)/
2)
```

3.721.7 Maxima [F]

$$\int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}}\,dx = \int \frac{1}{\sqrt{x+\sqrt{2\,x-1}+1}}\,dx$$

input
```
integrate(1/(1+x+(-1+2*x)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(x + sqrt(2*x - 1) + 1), x)
```

3.721.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}}\,dx = \sqrt{2}\left(\sqrt{2\,x+2\,\sqrt{2\,x-1}+2}+\log\left(\sqrt{2\,x+2\,\sqrt{2\,x-1}+2}-\sqrt{2\,x-1}-1\right)\right)$$

input
```
integrate(1/(1+x+(-1+2*x)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*(sqrt(2*x + 2*sqrt(2*x - 1) + 2) + log(sqrt(2*x + 2*sqrt(2*x - 1)
+ 2) - sqrt(2*x - 1) - 1))
```

3.721.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}}\,dx = \int \frac{1}{\sqrt{x+\sqrt{2\,x-1}+1}}\,dx$$

input
```
int(1/(x + (2*x - 1)^(1/2) + 1)^(1/2),x)
```

output
```
int(1/(x + (2*x - 1)^(1/2) + 1)^(1/2), x)
```

3.722 $\int \frac{q+px}{\sqrt{b+ax}(f+\sqrt{b+ax})} dx$

3.722.1 Optimal result

Integrand size = 28, antiderivative size = 54

$$\int \frac{q+px}{\sqrt{b+ax}\,(f+\sqrt{b+ax})}\,dx = \frac{px}{a} - \frac{2fp\sqrt{b+ax}}{a^2} - \frac{2(bp-f^2p-aq)\log\left(f+\sqrt{b+ax}\right)}{a^2}$$

output
```
p*x/a-2*(-f^2*p-a*q+b*p)*ln(f+(a*x+b)^(1/2))/a^2-2*f*p*(a*x+b)^(1/2)/a^2
```

3.722.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{q+px}{\sqrt{b+ax}\,(f+\sqrt{b+ax})}\,dx = \frac{p\left(b+ax-2f\sqrt{b+ax}\right)+2(-bp+f^2p+aq)\log\left(f+\sqrt{b+ax}\right)}{a^2}$$

input
```
Integrate[(q + p*x)/(Sqrt[b + a*x]*(f + Sqrt[b + a*x])),x]
```

output
```
(p*(b + a*x - 2*f*Sqrt[b + a*x]) + 2*(-(b*p) + f^2*p + a*q)*Log[f + Sqrt[b
 + a*x]])/a^2
```

3.722.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {7267, 25, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{px+q}{\sqrt{ax+b}\left(\sqrt{ax+b}+f\right)}dx$$

$$\Big\downarrow \text{7267}$$

$$\frac{2\int -\frac{bp-(b+ax)p-aq}{f+\sqrt{b+ax}}d\sqrt{b+ax}}{a^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\int \frac{bp-(b+ax)p-aq}{f+\sqrt{b+ax}}d\sqrt{b+ax}}{a^2}$$

$$\Big\downarrow \text{476}$$

$$-\frac{2\int \left(fp-\sqrt{b+ax}p+\frac{-pf^2+bp-aq}{f+\sqrt{b+ax}}\right)d\sqrt{b+ax}}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(-\left(-aq+bp+f^2(-p)\right)\log\left(\sqrt{ax+b}+f\right)-fp\sqrt{ax+b}+\frac{1}{2}p(ax+b)\right)}{a^2}$$

input
```
Int[(q + p*x)/(Sqrt[b + a*x]*(f + Sqrt[b + a*x])),x]
```

output
```
(2*(-(f*p*Sqrt[b + a*x]) + (p*(b + a*x))/2 - (b*p - f^2*p - a*q)*Log[f + S
qrt[b + a*x]]))/a^2
```

3.722.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.722.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{-2fp\sqrt{ax+b}+p(ax+b)+2(f^2p+aq-bp)\ln(f+\sqrt{ax+b})}{a^2}$	50
default	$\frac{-2fp\sqrt{ax+b}+p(ax+b)+2(f^2p+aq-bp)\ln(f+\sqrt{ax+b})}{a^2}$	50

input
```
int((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/a^2*(-f*p*(a*x+b)^(1/2)+1/2*p*(a*x+b)+(f^2*p+a*q-b*p)*ln(f+(a*x+b)^(1/2)
))
```

3.722.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{q+px}{\sqrt{b+ax}\left(f+\sqrt{b+ax}\right)}\,dx = \frac{apx-2\sqrt{ax+b}fp+2\left((f^2-b)p+aq\right)\log\left(f+\sqrt{ax+b}\right)}{a^2}$$

input
```
integrate((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x, algorithm="fricas")
```

output
```
(a*p*x - 2*sqrt(a*x + b)*f*p + 2*((f^2 - b)*p + a*q)*log(f + sqrt(a*x + b)
))/a^2
```

3.722.6 Sympy [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.35

$$\int \frac{q+px}{\sqrt{b+ax}\left(f+\sqrt{b+ax}\right)}\,dx = \begin{cases} \frac{2\left(-\frac{fp\sqrt{ax+b}}{a}+\frac{p(ax+b)}{2a}-\frac{\left(-aq+bp-f^2p\right)\log\left(f+\sqrt{ax+b}\right)}{a}\right)}{a} & \text{for } a\neq 0 \\ \frac{\frac{px^2}{2}+qx}{\sqrt{b}\left(\sqrt{b}+f\right)} & \text{otherwise} \end{cases}$$

input
```
integrate((p*x+q)/(a*x+b)**(1/2)/(f+(a*x+b)**(1/2)),x)
```

output
```
Piecewise((2*(-f*p*sqrt(a*x + b)/a + p*(a*x + b)/(2*a) - (-a*q + b*p - f**
2*p)*log(f + sqrt(a*x + b))/a)/a, Ne(a, 0)), ((p*x**2/2 + q*x)/(sqrt(b)*(s
qrt(b) + f)), True))
```

3.722.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int \frac{q+px}{\sqrt{b+ax}\left(f+\sqrt{b+ax}\right)}\,dx = \frac{\frac{2\left((f^2-b)p+aq\right)\log\left(f+\sqrt{ax+b}\right)}{a}-\frac{2\sqrt{ax+b}fp-(ax+b)p}{a}}{a}$$

input
```
integrate((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x, algorithm="maxima")
```

output
```
(2*((f^2 - b)*p + a*q)*log(f + sqrt(a*x + b))/a - (2*sqrt(a*x + b)*f*p - (
a*x + b)*p)/a)/a
```

3.722.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13

$$\int \frac{q+px}{\sqrt{b+ax}\left(f+\sqrt{b+ax}\right)}\,dx = \frac{2\left(f^2p-bp+aq\right)\log\left(\left|f+\sqrt{ax+b}\right|\right)}{a^2} - \frac{2\sqrt{ax+b}a^2fp-(ax+b)a^2p}{a^4}$$

input
```
integrate((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x, algorithm="giac")
```

output
```
2*(f^2*p - b*p + a*q)*log(abs(f + sqrt(a*x + b)))/a^2 - (2*sqrt(a*x + b)*a
^2*f*p - (a*x + b)*a^2*p)/a^4
```

3.722.9 Mupad [B] (verification not implemented)

Time = 18.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{q+px}{\sqrt{b+ax}\left(f+\sqrt{b+ax}\right)}\,dx = \frac{\ln\left(f+\sqrt{b+a\,x}\right)\,\left(2\,p\,f^2+2\,a\,q-2\,b\,p\right)}{a^2} + \frac{p\,x}{a} - \frac{2\,f\,p\,\sqrt{b+a\,x}}{a^2}$$

input
```
int((q + p*x)/((f + (b + a*x)^(1/2))*(b + a*x)^(1/2)),x)
```

output
```
(log(f + (b + a*x)^(1/2))*(2*a*q - 2*b*p + 2*f^2*p))/a^2 + (p*x)/a - (2*f*
p*(b + a*x)^(1/2))/a^2
```

3.723 $\int \sqrt{1-\sqrt{x}-x}\,dx$

3.723.1 Optimal result

Integrand size = 16, antiderivative size = 70

$$\int \sqrt{1-\sqrt{x}-x}\,dx=-\frac{1}{4}(1+2\sqrt{x})\sqrt{1-\sqrt{x}-x}-\frac{2}{3}(1-\sqrt{x}-x)^{3/2}-\frac{5}{8}\arcsin\left(\frac{1+2\sqrt{x}}{\sqrt{5}}\right)$$

output
```
-5/8*arcsin(1/5*(1+2*x^(1/2))*5^(1/2))-2/3*(1-x-x^(1/2))^(3/2)-1/4*(1+2*x^
(1/2))*(1-x-x^(1/2))^(1/2)
```

3.723.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.91

$$\int \sqrt{1-\sqrt{x}-x}\,dx=\frac{1}{12}\sqrt{1-\sqrt{x}-x}(-11+2\sqrt{x}+8x)-\frac{5}{4}\arctan\left(\frac{\sqrt{x}}{-1+\sqrt{1-\sqrt{x}-x}}\right)$$

input
```
Integrate[Sqrt[1 - Sqrt[x] - x],x]
```

output
```
(Sqrt[1 - Sqrt[x] - x]*(-11 + 2*Sqrt[x] + 8*x))/12 - (5*ArcTan[Sqrt[x]/(-1
 + Sqrt[1 - Sqrt[x] - x])])/4
```

3.723.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1680, 1160, 1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-x-\sqrt{x}+1}\,dx$$

$$\Big\downarrow \text{1680}$$

$$2\int \sqrt{-x-\sqrt{x}+1}\sqrt{x}d\sqrt{x}$$

$$\Big\downarrow \text{1160}$$

$$2\left(-\frac{1}{2}\int \sqrt{-x-\sqrt{x}+1}d\sqrt{x}-\frac{1}{3}\left(-x-\sqrt{x}+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{1087}$$

$$2\left(\frac{1}{2}\left(-\frac{5}{8}\int \frac{1}{\sqrt{-x-\sqrt{x}+1}}d\sqrt{x}-\frac{1}{4}\sqrt{-x-\sqrt{x}+1}(2\sqrt{x}+1)\right)-\frac{1}{3}\left(-x-\sqrt{x}+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{1090}$$

$$2\left(\frac{1}{2}\left(\frac{1}{8}\sqrt{5}\int \frac{1}{\sqrt{1-\frac{x}{5}}}d(-2\sqrt{x}-1)-\frac{1}{4}(2\sqrt{x}+1)\sqrt{-x-\sqrt{x}+1}\right)-\frac{1}{3}\left(-x-\sqrt{x}+1\right)^{3/2}\right)$$

$$\Big\downarrow \text{223}$$

$$2\left(\frac{1}{2}\left(\frac{5}{8}\arcsin\left(\frac{-2\sqrt{x}-1}{\sqrt{5}}\right)-\frac{1}{4}(2\sqrt{x}+1)\sqrt{-x-\sqrt{x}+1}\right)-\frac{1}{3}\left(-x-\sqrt{x}+1\right)^{3/2}\right)$$

input
```
Int[Sqrt[1 - Sqrt[x] - x],x]
```

output
```
2*(-1/3*(1 - Sqrt[x] - x)^(3/2) + (-1/4*((1 + 2*Sqrt[x])*Sqrt[1 - Sqrt[x]
- x]) + (5*ArcSin[(-1 - 2*Sqrt[x])/Sqrt[5]])/8)/2)
```

3.723.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1680
```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*
n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fr
actionQ[n]
```

3.723.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$-\frac{2\left(1-x-\sqrt{x}\right)^{\frac{3}{2}}}{3}+\frac{\left(-2\sqrt{x}-1\right)\sqrt{1-x-\sqrt{x}}}{4}-\frac{5\arcsin\left(\frac{2\sqrt{5}\left(\sqrt{x}+\frac{1}{2}\right)}{5}\right)}{8}$	50
default	$-\frac{2\left(1-x-\sqrt{x}\right)^{\frac{3}{2}}}{3}+\frac{\left(-2\sqrt{x}-1\right)\sqrt{1-x-\sqrt{x}}}{4}-\frac{5\arcsin\left(\frac{2\sqrt{5}\left(\sqrt{x}+\frac{1}{2}\right)}{5}\right)}{8}$	50

input
```
int((1-x-x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(1-x-x^(1/2))^(3/2)+1/4*(-2*x^(1/2)-1)*(1-x-x^(1/2))^(1/2)-5/8*arcsin
(2/5*5^(1/2)*(x^(1/2)+1/2))
```

3.723.5 Fricas [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.20

$$\int \sqrt{1-\sqrt{x}-x}\,dx = \frac{1}{12}\left(8\,x+2\,\sqrt{x}-11\right)\sqrt{-x-\sqrt{x}+1} + \frac{5}{16}\arctan\left(-\frac{\left(8\,x^2-\left(16\,x^2-38\,x+11\right)\sqrt{x}-9\,x+3\right)\sqrt{-x-\sqrt{x}+1}}{4\left(4\,x^3-13\,x^2+7\,x-1\right)}\right)$$

input
```
integrate((1-x-x^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/12*(8*x + 2*sqrt(x) - 11)*sqrt(-x - sqrt(x) + 1) + 5/16*arctan(-1/4*(8*x
^2 - (16*x^2 - 38*x + 11)*sqrt(x) - 9*x + 3)*sqrt(-x - sqrt(x) + 1)/(4*x^3
 - 13*x^2 + 7*x - 1))
```

3.723.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.69

$$\int \sqrt{1-\sqrt{x}-x}\,dx = 2\sqrt{-\sqrt{x}-x+1}\left(\frac{\sqrt{x}}{12}+\frac{x}{3}-\frac{11}{24}\right)-\frac{5\,\mathrm{asin}\left(\frac{2\sqrt{5}\left(\sqrt{x}+\frac{1}{2}\right)}{5}\right)}{8}$$

input
```
integrate((1-x-x**(1/2))**(1/2),x)
```

output
```
2*sqrt(-sqrt(x) - x + 1)*(sqrt(x)/12 + x/3 - 11/24) - 5*asin(2*sqrt(5)*(sq
rt(x) + 1/2)/5)/8
```

3.723.7 Maxima [F]

$$\int \sqrt{1-\sqrt{x}-x}\,dx = \int \sqrt{-x-\sqrt{x}+1}\,dx$$

input
```
integrate((1-x-x^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x - sqrt(x) + 1), x)
```

3.723.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.63

$$\int \sqrt{1-\sqrt{x}-x}\,dx = \frac{1}{12}\left(2\sqrt{x}\left(4\sqrt{x}+1\right)-11\right)\sqrt{-x-\sqrt{x}+1} - \frac{5}{8}\arcsin\left(\frac{1}{5}\sqrt{5}\left(2\sqrt{x}+1\right)\right)$$

input
```
integrate((1-x-x^(1/2))^(1/2),x, algorithm="giac")
```

output
```
1/12*(2*sqrt(x)*(4*sqrt(x) + 1) - 11)*sqrt(-x - sqrt(x) + 1) - 5/8*arcsin(
1/5*sqrt(5)*(2*sqrt(x) + 1))
```

3.723.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-\sqrt{x}-x}\,dx = \int \sqrt{1-\sqrt{x}-x}\,dx$$

input
```
int((1 - x^(1/2) - x)^(1/2),x)
```

output
```
int((1 - x^(1/2) - x)^(1/2), x)
```

3.724 $\int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x}\,dx$

3.724.1 Optimal result

Integrand size = 22, antiderivative size = 19

$$\int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x}\,dx = 4\sqrt{x}+x+2\log\left(4+\sqrt{x}\right)$$

output
```
x+2*ln(4+x^(1/2))+4*x^(1/2)
```

3.724.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x}\,dx = 4\sqrt{x}+x+2\log\left(4+\sqrt{x}\right)$$

input
```
Integrate[(9 + 6*Sqrt[x] + x)/(4*Sqrt[x] + x),x]
```

output
```
4*Sqrt[x] + x + 2*Log[4 + Sqrt[x]]
```

3.724.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1380, 1731, 9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+6\sqrt{x}+9}{x+4\sqrt{x}}dx$$

$$\Big\downarrow \text{1380}$$

$$\int \frac{\left(\sqrt{x}+3\right)^2}{x+4\sqrt{x}}dx$$

$$\Big\downarrow \text{1731}$$

$$2\int \frac{\left(\sqrt{x}+3\right)^2\sqrt{x}}{x+4\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{9}$$

$$2\int \frac{\left(\sqrt{x}+3\right)^2}{\sqrt{x}+4}d\sqrt{x}$$

$$\Big\downarrow \text{49}$$

$$2\int \left(\sqrt{x}+\frac{1}{\sqrt{x}+4}+2\right)d\sqrt{x}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{x}{2}+2\sqrt{x}+\log\left(\sqrt{x}+4\right)\right)$$

input
```
Int[(9 + 6*Sqrt[x] + x)/(4*Sqrt[x] + x),x]
```

output
```
2*(2*Sqrt[x] + x/2 + Log[4 + Sqrt[x]])
```

3.724. $\int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x}\,dx$

3.724.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 1731
```
Int[((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(
g - 1)*(d + e*x^(g*n))^q*(a + b*x^(g*n) + c*x^(2*g*n))^p, x], x, x^(1/g)],
x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.724.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$x+2\ln\left(4+\sqrt{x}\right)+4\sqrt{x}$	16
default	$x+2\ln\left(4+\sqrt{x}\right)+4\sqrt{x}$	16
trager	$x-1+4\sqrt{x}+\ln\left(8\sqrt{x}+16+x\right)$	18
meijerg	$2\ln\left(1+\frac{\sqrt{x}}{4}\right)-\frac{4\sqrt{x}\left(-\frac{3\sqrt{x}}{4}+6\right)}{3}+12\sqrt{x}$	29

input
```
int((9+x+6*x^(1/2))/(x+4*x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x+2*ln(4+x^(1/2))+4*x^(1/2)
```

3.724.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x}\,dx = x+4\,\sqrt{x}+2\,\log\left(\sqrt{x}+4\right)$$

input
```
integrate((9+x+6*x^(1/2))/(x+4*x^(1/2)),x, algorithm="fricas")
```

output
```
x + 4*sqrt(x) + 2*log(sqrt(x) + 4)
```

3.724.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x}\,dx = 4\sqrt{x}+x+2\log\left(\sqrt{x}+4\right)$$

input
```
integrate((9+x+6*x**(1/2))/(x+4*x**(1/2)),x)
```

output
```
4*sqrt(x) + x + 2*log(sqrt(x) + 4)
```

3.724.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x}\,dx = x+4\,\sqrt{x}+2\,\log\left(\sqrt{x}+4\right)$$

input
```
integrate((9+x+6*x^(1/2))/(x+4*x^(1/2)),x, algorithm="maxima")
```

output
```
x + 4*sqrt(x) + 2*log(sqrt(x) + 4)
```

3.724.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x}\,dx = x+4\sqrt{x}+2\log\left(\sqrt{x}+4\right)$$

input
```
integrate((9+x+6*x^(1/2))/(x+4*x^(1/2)),x, algorithm="giac")
```

output
```
x + 4*sqrt(x) + 2*log(sqrt(x) + 4)
```

3.724.9 Mupad [B] (verification not implemented)

Time = 18.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x}\,dx = x+2\ln\left(\sqrt{x}+4\right)+4\sqrt{x}$$

input
```
int((x + 6*x^(1/2) + 9)/(x + 4*x^(1/2)),x)
```

output
```
x + 2*log(x^(1/2) + 4) + 4*x^(1/2)
```

3.725 $\int \frac{6-8x^{7/2}}{5-9\sqrt{x}}\,dx$

3.725.1 Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \frac{6-8x^{7/2}}{5-9\sqrt{x}}\,dx = -\frac{56145628\sqrt{x}}{43046721}+\frac{125000x}{4782969}+\frac{50000x^{3/2}}{1594323}+\frac{2500x^2}{59049} +\frac{400x^{5/2}}{6561}+\frac{200x^3}{2187}+\frac{80x^{7/2}}{567}+\frac{2x^4}{9}-\frac{280728140\log\left(5-9\sqrt{x}\right)}{387420489}$$

output
```
125000/4782969*x+50000/1594323*x^(3/2)+2500/59049*x^2+400/6561*x^(5/2)+200
/2187*x^3+80/567*x^(7/2)+2/9*x^4-280728140/387420489*ln(5-9*x^(1/2))-56145
628/43046721*x^(1/2)
```

3.725.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int \frac{6-8x^{7/2}}{5-9\sqrt{x}}\,dx = \frac{2\sqrt{x}\left(-196509698+3937500\sqrt{x}+4725000x+6378750x^{3/2}+9185400x^2+13778100x^{5/}\right.}{301327047} -\frac{280728140\log\left(-5+9\sqrt{x}\right)}{387420489}$$

input
```
Integrate[(6 - 8*x^(7/2))/(5 - 9*Sqrt[x]),x]
```

output
```
(2*Sqrt[x]*(-196509698 + 3937500*Sqrt[x] + 4725000*x + 6378750*x^(3/2) + 9
185400*x^2 + 13778100*x^(5/2) + 21257640*x^3 + 33480783*x^(7/2)))/30132704
7 - (280728140*Log[-5 + 9*Sqrt[x]])/387420489
```

3.725.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2432, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{6-8x^{7/2}}{5-9\sqrt{x}}\,dx$$

$$\Big\downarrow \text{2432}$$

$$\int \left(\frac{8x^{7/2}}{9\sqrt{x}-5}-\frac{6}{9\sqrt{x}-5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{80x^{7/2}}{567}+\frac{400x^{5/2}}{6561}+\frac{50000x^{3/2}}{1594323}+\frac{2x^4}{9}+\frac{200x^3}{2187}+\frac{2500x^2}{59049}+\frac{125000x}{4782969}-\frac{56145628\sqrt{x}}{43046721}-\frac{280728140\log\left(5-9\sqrt{x}\right)}{387420489}$$

input
```
Int[(6 - 8*x^(7/2))/(5 - 9*Sqrt[x]),x]
```

output
```
(-56145628*Sqrt[x])/43046721 + (125000*x)/4782969 + (50000*x^(3/2))/159432
3 + (2500*x^2)/59049 + (400*x^(5/2))/6561 + (200*x^3)/2187 + (80*x^(7/2))/
567 + (2*x^4)/9 - (280728140*Log[5 - 9*Sqrt[x]])/387420489
```

3.725.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2432
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

3.725.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.65

method	result
derivativedivides	$\frac{2x^4}{9}+\frac{80x^{\frac{7}{2}}}{567}+\frac{200x^3}{2187}+\frac{400x^{\frac{5}{2}}}{6561}+\frac{2500x^2}{59049}+\frac{50000x^{\frac{3}{2}}}{1594323}+\frac{125000x}{4782969}-\frac{56145628\sqrt{x}}{43046721}-\frac{280728140\ln(-5+9\sqrt{\cdots}}{387420489}$
default	$\frac{2x^4}{9}+\frac{80x^{\frac{7}{2}}}{567}+\frac{200x^3}{2187}+\frac{400x^{\frac{5}{2}}}{6561}+\frac{2500x^2}{59049}+\frac{50000x^{\frac{3}{2}}}{1594323}+\frac{125000x}{4782969}-\frac{56145628\sqrt{x}}{43046721}-\frac{280728140\ln(-5+9\sqrt{\cdots}}{387420489}$
trager	$\frac{2(531441x^3+750141x^2+851391x+913891)(x-1)}{4782969}+2\left(\frac{40}{567}x^3+\frac{200}{6561}x^2+\frac{25000}{1594323}x-\frac{28072814}{43046721}\right)\sqrt{x}-14\cdots$
meijerg	$-\frac{4\sqrt{x}}{3}-\frac{280728140\ln\left(1-\frac{9\sqrt{x}}{5}\right)}{387420489}+\frac{31250\sqrt{x}\left(\frac{301327047x^{\frac{7}{2}}}{15625}+\frac{38263752x^3}{3125}+\frac{4960116x^{\frac{5}{2}}}{625}+\frac{3306744x^2}{625}+\frac{91854x^{\frac{3}{2}}}{25}+\frac{13608\cdots}{5}\cdots\right.}{2711943423}$

input
```
int((6-8*x^(7/2))/(5-9*x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/9*x^4+80/567*x^(7/2)+200/2187*x^3+400/6561*x^(5/2)+2500/59049*x^2+50000/
1594323*x^(3/2)+125000/4782969*x-56145628/43046721*x^(1/2)-280728140/38742
0489*ln(-5+9*x^(1/2))
```

3.725.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64

$$\int \frac{6-8x^{7/2}}{5-9\sqrt{x}}\,dx = \frac{2}{9}\,x^4+\frac{200}{2187}\,x^3+\frac{2500}{59049}\,x^2 + \frac{4}{301327047}\left(10628820\,x^3+4592700\,x^2+2362500\,x-98254849\right)\sqrt{x} + \frac{125000}{4782969}\,x-\frac{280728140}{387420489}\,\log\left(9\,\sqrt{x}-5\right)$$

input
```
integrate((6-8*x^(7/2))/(5-9*x^(1/2)),x, algorithm="fricas")
```

output
```
2/9*x^4 + 200/2187*x^3 + 2500/59049*x^2 + 4/301327047*(10628820*x^3 + 4592
700*x^2 + 2362500*x - 98254849)*sqrt(x) + 125000/4782969*x - 280728140/387
420489*log(9*sqrt(x) - 5)
```

3.725.6 Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int \frac{6-8x^{7/2}}{5-9\sqrt{x}}\,dx = \frac{80x^{\frac{7}{2}}}{567} + \frac{400x^{\frac{5}{2}}}{6561} + \frac{50000x^{\frac{3}{2}}}{1594323} - \frac{56145628\sqrt{x}}{43046721} + \frac{2x^4}{9} + \frac{200x^3}{2187} + \frac{2500x^2}{59049} + \frac{125000x}{4782969} - \frac{280728140\log\left(9\sqrt{x}-5\right)}{387420489}$$

input
```
integrate((6-8*x**(7/2))/(5-9*x**(1/2)),x)
```

output
```
80*x**(7/2)/567 + 400*x**(5/2)/6561 + 50000*x**(3/2)/1594323 - 56145628*sq
rt(x)/43046721 + 2*x**4/9 + 200*x**3/2187 + 2500*x**2/59049 + 125000*x/478
2969 - 280728140*log(9*sqrt(x) - 5)/387420489
```

3.725.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64

$$\int \frac{6-8x^{7/2}}{5-9\sqrt{x}}\,dx = \frac{2}{9}\,x^4 + \frac{80}{567}\,x^{\frac{7}{2}} + \frac{200}{2187}\,x^3 + \frac{400}{6561}\,x^{\frac{5}{2}} + \frac{2500}{59049}\,x^2 + \frac{50000}{1594323}\,x^{\frac{3}{2}} + \frac{125000}{4782969}\,x - \frac{56145628}{43046721}\,\sqrt{x} - \frac{280728140}{387420489}\,\log\left(9\,\sqrt{x}-5\right)$$

input
```
integrate((6-8*x^(7/2))/(5-9*x^(1/2)),x, algorithm="maxima")
```

output
```
2/9*x^4 + 80/567*x^(7/2) + 200/2187*x^3 + 400/6561*x^(5/2) + 2500/59049*x^
2 + 50000/1594323*x^(3/2) + 125000/4782969*x - 56145628/43046721*sqrt(x) -
 280728140/387420489*log(9*sqrt(x) - 5)
```

3.725.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.65

$$\int \frac{6-8x^{7/2}}{5-9\sqrt{x}}\,dx = \frac{2}{9}\,x^4 + \frac{80}{567}\,x^{\frac{7}{2}} + \frac{200}{2187}\,x^3 + \frac{400}{6561}\,x^{\frac{5}{2}} + \frac{2500}{59049}\,x^2 + \frac{50000}{1594323}\,x^{\frac{3}{2}} + \frac{125000}{4782969}\,x - \frac{56145628}{43046721}\,\sqrt{x} - \frac{280728140}{387420489}\,\log\left(\left|9\,\sqrt{x}-5\right|\right)$$

input
```
integrate((6-8*x^(7/2))/(5-9*x^(1/2)),x, algorithm="giac")
```

output
```
2/9*x^4 + 80/567*x^(7/2) + 200/2187*x^3 + 400/6561*x^(5/2) + 2500/59049*x^
2 + 50000/1594323*x^(3/2) + 125000/4782969*x - 56145628/43046721*sqrt(x) -
 280728140/387420489*log(abs(9*sqrt(x) - 5))
```

3.725.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.61

$$\int \frac{6-8x^{7/2}}{5-9\sqrt{x}}\,dx = \frac{125000\,x}{4782969} - \frac{280728140\,\ln\left(\sqrt{x}-\frac{5}{9}\right)}{387420489} + \frac{2500\,x^2}{59049} - \frac{56145628\,\sqrt{x}}{43046721} + \frac{200\,x^3}{2187} + \frac{2\,x^4}{9} + \frac{50000\,x^{3/2}}{1594323} + \frac{400\,x^{5/2}}{6561} + \frac{80\,x^{7/2}}{567}$$

input
```
int((8*x^(7/2) - 6)/(9*x^(1/2) - 5),x)
```

output
```
(125000*x)/4782969 - (280728140*log(x^(1/2) - 5/9))/387420489 + (2500*x^2)
/59049 - (56145628*x^(1/2))/43046721 + (200*x^3)/2187 + (2*x^4)/9 + (50000
*x^(3/2))/1594323 + (400*x^(5/2))/6561 + (80*x^(7/2))/567
```

3.726 $\int \frac{\sqrt{1+x}(1+x^3)}{1+x^2}\,dx$

3.726.1 Optimal result

Integrand size = 20, antiderivative size = 80

$$\int \frac{\sqrt{1+x}(1+x^3)}{1+x^2}\,dx = -2\sqrt{1+x}-\frac{2}{3}(1+x)^{3/2}+\frac{2}{5}(1+x)^{5/2} + (1-i)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{1+x}}{\sqrt{1-i}}\right)+(1+i)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{1+x}}{\sqrt{1+i}}\right)$$

output
```
-2/3*(1+x)^(3/2)+2/5*(1+x)^(5/2)+(1-I)^(3/2)*arctanh((1+x)^(1/2)/(1-I)^(1/
2))+(1+I)^(3/2)*arctanh((1+x)^(1/2)/(1+I)^(1/2))-2*(1+x)^(1/2)
```

3.726.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{1+x}(1+x^3)}{1+x^2}\,dx = \frac{2}{15}\sqrt{1+x}(-17+x+3x^2)+\sqrt{2+2i}\arctan\left(\sqrt{-\frac{1}{2}-\frac{i}{2}}\sqrt{1+x}\right) + \sqrt{2-2i}\arctan\left(\sqrt{-\frac{1}{2}+\frac{i}{2}}\sqrt{1+x}\right)$$

input
```
Integrate[(Sqrt[1 + x]*(1 + x^3))/(1 + x^2),x]
```

output
```
(2*Sqrt[1 + x]*(-17 + x + 3*x^2))/15 + Sqrt[2 + 2*I]*ArcTan[Sqrt[-1/2 - I/
2]*Sqrt[1 + x]] + Sqrt[2 - 2*I]*ArcTan[Sqrt[-1/2 + I/2]*Sqrt[1 + x]]
```

3.726.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 224 vs. 2(80) = 160.

Time = 0.54 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.80, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2156, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x+1}(x^3+1)}{x^2+1}\,dx$$

$$\Big\downarrow \; 2156$$

$$\int \frac{(x+1)^{3/2}(x^2-x+1)}{x^2+1}dx$$

$$\Big\downarrow \; 2160$$

$$\int \left((x+1)^{3/2}-\frac{x(x+1)^{3/2}}{x^2+1}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\sqrt{1+\sqrt{2}}\arctan\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{x+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\sqrt{1+\sqrt{2}}\arctan\left(\frac{2\sqrt{x+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}-2\sqrt{x+1}-\frac{\log\left(x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}}+\frac{\log\left(x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}}$$

input
```
Int[(Sqrt[1 + x]*(1 + x^3))/(1 + x^2),x]
```

output
```
-2*Sqrt[1 + x] - (2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5 - Sqrt[1 + Sqrt
[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + x])/Sqrt[2*(-1 + Sqrt[2])]
] + Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*Sqrt[1 + x])/Sqrt[
2*(-1 + Sqrt[2])]] - Log[1 + Sqrt[2] + x - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 +
x]]/(2*Sqrt[1 + Sqrt[2]]) + Log[1 + Sqrt[2] + x + Sqrt[2*(1 + Sqrt[2])]*Sq
rt[1 + x]]/(2*Sqrt[1 + Sqrt[2]])
```

3.726. $\int \frac{\sqrt{1+x}(1+x^3)}{1+x^2}\,dx$

3.726.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2156
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[(d + e*x)^(m + 1)*PolynomialQuotient[Pq, d + e*x, x]*(a + b*x^2)^p,
x] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[PolynomialRemaind
er[Pq, d + e*x, x], 0]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.726.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(58) = 116$.

Time = 1.37 (sec) , antiderivative size = 289, normalized size of antiderivative = 3.61

method	result
derivativedivides	$\frac{2(x+1)^{\frac{5}{2}}}{5}-\frac{2(x+1)^{\frac{3}{2}}}{3}-2\sqrt{x+1}-\frac{\left(-\sqrt{2+2\sqrt{2}}\,\sqrt{2}+2\sqrt{2+2\sqrt{2}}\right)\ln\left(x+1-\sqrt{x+1}\,\sqrt{2+2\sqrt{2}}+\sqrt{2}\right)}{4}-\frac{\left(-2\sqrt{2}\right.}{}$
default	$\frac{2(x+1)^{\frac{5}{2}}}{5}-\frac{2(x+1)^{\frac{3}{2}}}{3}-2\sqrt{x+1}-\frac{\left(-\sqrt{2+2\sqrt{2}}\,\sqrt{2}+2\sqrt{2+2\sqrt{2}}\right)\ln\left(x+1-\sqrt{x+1}\,\sqrt{2+2\sqrt{2}}+\sqrt{2}\right)}{4}-\frac{\left(-2\sqrt{2}\right.}{}$
trager	$\left(\frac{2}{5}x^2+\frac{2}{15}x-\frac{34}{15}\right)\sqrt{x+1}-\operatorname{RootOf}\left(\_Z^2+16\operatorname{RootOf}\left(512\_Z^4+32\_Z^2+1\right)^2+1\right)$
risch	$\frac{2(3x^2+x-17)\sqrt{x+1}}{15}-\frac{\ln\left(x+1+\sqrt{x+1}\,\sqrt{2+2\sqrt{2}}+\sqrt{2}\right)\sqrt{2+2\sqrt{2}}\,\sqrt{2}}{4}+\frac{\ln\left(x+1+\sqrt{x+1}\,\sqrt{2+2\sqrt{2}}+\sqrt{2}\right)\sqrt{2+2\sqrt{2}}}{2}$

input
```
int((x^3+1)*(x+1)^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
2/5*(x+1)^(5/2)-2/3*(x+1)^(3/2)-2*(x+1)^(1/2)-1/4*(-(2+2*2^(1/2))^(1/2)*2^
(1/2)+2*(2+2*2^(1/2))^(1/2))*ln(x+1-(x+1)^(1/2)*(2+2*2^(1/2))^(1/2)+2^(1/2
))-(-2*2^(1/2)+1/2*(-(2+2*2^(1/2))^(1/2)*2^(1/2)+2*(2+2*2^(1/2))^(1/2))*(2
+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(x+1)^(1/2)-(2+2*2^(1/2)
)^(1/2))/(-2+2*2^(1/2))^(1/2))+1/4*(-(2+2*2^(1/2))^(1/2)*2^(1/2)+2*(2+2*2^
(1/2))^(1/2))*ln(x+1+(x+1)^(1/2)*(2+2*2^(1/2))^(1/2)+2^(1/2))+(2*2^(1/2)-1
/2*(-(2+2*2^(1/2))^(1/2)*2^(1/2)+2*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2
))/(-2+2*2^(1/2))^(1/2)*arctan((2*(x+1)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2
^(1/2))^(1/2))
```

3.726.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{1+x}(1+x^3)}{1+x^2}\,dx = \frac{2}{15}\left(3\,x^2+x-17\right)\sqrt{x+1} + \frac{1}{2}\,\sqrt{2i-2}\log\left(-(i-1)\,\sqrt{2i-2}+2\,\sqrt{x+1}\right) - \frac{1}{2}\,\sqrt{2i-2}\log\left((i-1)\,\sqrt{2i-2}+2\,\sqrt{x+1}\right) + \frac{1}{2}\,\sqrt{-2i-2}\log\left((i+1)\,\sqrt{-2i-2}+2\,\sqrt{x+1}\right) - \frac{1}{2}\,\sqrt{-2i-2}\log\left(-(i+1)\,\sqrt{-2i-2}+2\,\sqrt{x+1}\right)$$

input
```
integrate((x^3+1)*(1+x)^(1/2)/(x^2+1),x, algorithm="fricas")
```

output
```
2/15*(3*x^2 + x - 17)*sqrt(x + 1) + 1/2*sqrt(2*I - 2)*log(-(I - 1)*sqrt(2*
I - 2) + 2*sqrt(x + 1)) - 1/2*sqrt(2*I - 2)*log((I - 1)*sqrt(2*I - 2) + 2*
sqrt(x + 1)) + 1/2*sqrt(-2*I - 2)*log((I + 1)*sqrt(-2*I - 2) + 2*sqrt(x +
1)) - 1/2*sqrt(-2*I - 2)*log(-(I + 1)*sqrt(-2*I - 2) + 2*sqrt(x + 1))
```

3.726.6 Sympy [F]

$$\int \frac{\sqrt{1+x}(1+x^3)}{1+x^2}\,dx = \int \frac{(x+1)^{\frac{3}{2}}\,(x^2-x+1)}{x^2+1}\,dx$$

input
```
integrate((x**3+1)*(1+x)**(1/2)/(x**2+1),x)
```

output
```
Integral((x + 1)**(3/2)*(x**2 - x + 1)/(x**2 + 1), x)
```

3.726.7 Maxima [F]

$$\int \frac{\sqrt{1+x}(1+x^3)}{1+x^2}\,dx = \int \frac{(x^3+1)\sqrt{x+1}}{x^2+1}\,dx$$

input
```
integrate((x^3+1)*(1+x)^(1/2)/(x^2+1),x, algorithm="maxima")
```

output
```
integrate((x^3 + 1)*sqrt(x + 1)/(x^2 + 1), x)
```

3.726.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(50) = 100$.

Time = 0.83 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.14

$$\begin{aligned}\int \frac{\sqrt{1+x}(1+x^3)}{1+x^2}\,dx &= \frac{2}{5}\,(x+1)^{\frac{5}{2}} - \frac{2}{3}\,(x+1)^{\frac{3}{2}} \\ &\quad + \sqrt{\sqrt{2}+1}\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{x+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right) \\ &\quad + \sqrt{\sqrt{2}+1}\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{x+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right) \\ &\quad + \frac{1}{2}\sqrt{\sqrt{2}-1}\log\left(2^{\frac{1}{4}}\sqrt{x+1}\sqrt{\sqrt{2}+2}+x+\sqrt{2}+1\right) \\ &\quad - \frac{1}{2}\sqrt{\sqrt{2}-1}\log\left(-2^{\frac{1}{4}}\sqrt{x+1}\sqrt{\sqrt{2}+2}+x+\sqrt{2}+1\right) - 2\sqrt{x+1}\end{aligned}$$

input
```
integrate((x^3+1)*(1+x)^(1/2)/(x^2+1),x, algorithm="giac")
```

output
```
2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2) + sqrt(sqrt(2) + 1)*arctan(1/2*2^(3/
4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*sqrt(x + 1))/sqrt(-sqrt(2) + 2)) + sqrt(
sqrt(2) + 1)*arctan(-1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(x + 1
))/sqrt(-sqrt(2) + 2)) + 1/2*sqrt(sqrt(2) - 1)*log(2^(1/4)*sqrt(x + 1)*sqr
t(sqrt(2) + 2) + x + sqrt(2) + 1) - 1/2*sqrt(sqrt(2) - 1)*log(-2^(1/4)*sqr
t(x + 1)*sqrt(sqrt(2) + 2) + x + sqrt(2) + 1) - 2*sqrt(x + 1)
```

3.726.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.19

$$\int \frac{\sqrt{1+x}(1+x^3)}{1+x^2}\,dx = \frac{2\,(x+1)^{5/2}}{5} - \frac{2\,(x+1)^{3/2}}{3} - 2\,\sqrt{x+1}$$
$$-\operatorname{atan}\left(\frac{\sqrt{2}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{x+1}\,64\mathrm{i}}{256\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}-64} - \frac{\sqrt{2}\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{x+1}\,64\mathrm{i}}{256\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}-64}\right)\left(\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\,2\mathrm{i}+\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\right.$$
$$+\operatorname{atan}\left(\frac{\sqrt{2}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{x+1}\,64\mathrm{i}}{256\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}+64} + \frac{\sqrt{2}\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{x+1}\,64\mathrm{i}}{256\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}+64}\right)\left(\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\,2\mathrm{i}-\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\right.$$

input
```
int(((x^3 + 1)*(x + 1)^(1/2))/(x^2 + 1),x)
```

output
```
(2*(x + 1)^(5/2))/5 - (2*(x + 1)^(3/2))/3 - 2*(x + 1)^(1/2) - atan((2^(1/2
)*(- 2^(1/2)/4 - 1/4)^(1/2)*(x + 1)^(1/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/2
)*(- 2^(1/2)/4 - 1/4)^(1/2) - 64) - (2^(1/2)*(2^(1/2)/4 - 1/4)^(1/2)*(x +
1)^(1/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2) - 64)
)*((- 2^(1/2)/4 - 1/4)^(1/2)*2i + (2^(1/2)/4 - 1/4)^(1/2)*2i) + atan((2^(1
/2)*(- 2^(1/2)/4 - 1/4)^(1/2)*(x + 1)^(1/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1
/2)*(- 2^(1/2)/4 - 1/4)^(1/2) + 64) + (2^(1/2)*(2^(1/2)/4 - 1/4)^(1/2)*(x
+ 1)^(1/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2) + 6
4))*((- 2^(1/2)/4 - 1/4)^(1/2)*2i - (2^(1/2)/4 - 1/4)^(1/2)*2i)
```

3.727 $\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}}\,dx$

3.727.1 Optimal result

Integrand size = 25, antiderivative size = 89

$$\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}}\,dx=\arctan\left(\frac{3-\sqrt{x}}{2\sqrt{-1-\sqrt{x}+x}}\right)-2\text{arctanh}\left(\frac{1-2\sqrt{x}}{2\sqrt{-1-\sqrt{x}+x}}\right)-\text{arctanh}\left(\frac{1+3\sqrt{x}}{2\sqrt{-1-\sqrt{x}+x}}\right)$$

output
```
arctan(1/2*(3-x^(1/2))/(-1+x-x^(1/2))^(1/2))-2*arctanh(1/2*(1-2*x^(1/2))/(
-1+x-x^(1/2))^(1/2))-arctanh(1/2*(1+3*x^(1/2))/(-1+x-x^(1/2))^(1/2))
```

3.727.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}}\,dx=-2\arctan\left(1-\sqrt{x}+\sqrt{-1-\sqrt{x}+x}\right)-2\text{arctanh}\left(1+\sqrt{x}-\sqrt{-1-\sqrt{x}+x}\right)-2\log\left(1-2\sqrt{x}+2\sqrt{-1-\sqrt{x}+x}\right)$$

input
```
Integrate[Sqrt[-1 - Sqrt[x] + x]/((-1 + x)*Sqrt[x]),x]
```

output
```
-2*ArcTan[1 - Sqrt[x] + Sqrt[-1 - Sqrt[x] + x]] - 2*ArcTanh[1 + Sqrt[x] -
Sqrt[-1 - Sqrt[x] + x]] - 2*Log[1 - 2*Sqrt[x] + 2*Sqrt[-1 - Sqrt[x] + x]]
```

3.727.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2035, 25, 1321, 25, 1092, 219, 1366, 25, 1154, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x-\sqrt{x}-1}}{(x-1)\sqrt{x}}dx$$

$$\Big\downarrow \text{2035}$$

$$2\int -\frac{\sqrt{x-\sqrt{x}-1}}{1-x}d\sqrt{x}$$

$$\Big\downarrow \text{25}$$

$$-2\int \frac{\sqrt{x-\sqrt{x}-1}}{1-x}d\sqrt{x}$$

$$\Big\downarrow \text{1321}$$

$$2\left(\int \frac{1}{\sqrt{x-\sqrt{x}-1}}d\sqrt{x}-\int -\frac{\sqrt{x}}{(1-x)\sqrt{x-\sqrt{x}-1}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\int \frac{1}{\sqrt{x-\sqrt{x}-1}}d\sqrt{x}+\int \frac{\sqrt{x}}{(1-x)\sqrt{x-\sqrt{x}-1}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{1092}$$

$$2\left(2\int \frac{1}{4-x}d\left(-\frac{1-2\sqrt{x}}{\sqrt{x-\sqrt{x}-1}}\right)+\int \frac{\sqrt{x}}{(1-x)\sqrt{x-\sqrt{x}-1}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\int \frac{\sqrt{x}}{(1-x)\sqrt{x-\sqrt{x}-1}}d\sqrt{x}-\text{arctanh}\left(\frac{1-2\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right)\right)$$

$$\Big\downarrow \text{1366}$$

$$2\left(\frac{1}{2}\int \frac{1}{(1-\sqrt{x})\sqrt{x-\sqrt{x}-1}}d\sqrt{x}+\frac{1}{2}\int -\frac{1}{(\sqrt{x}+1)\sqrt{x-\sqrt{x}-1}}d\sqrt{x}-\text{arctanh}\left(\frac{1-2\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{1}{2}\int \frac{1}{(1-\sqrt{x})\sqrt{x-\sqrt{x}-1}}d\sqrt{x}-\frac{1}{2}\int \frac{1}{(\sqrt{x}+1)\sqrt{x-\sqrt{x}-1}}d\sqrt{x}-\text{arctanh}\left(\frac{1-2\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right)\right)$$

$$\Big\downarrow \text{1154}$$

$$2\left(-\int \frac{1}{-x-4}d\frac{3-\sqrt{x}}{\sqrt{x-\sqrt{x}-1}}+\int \frac{1}{4-x}d\left(-\frac{3\sqrt{x}+1}{\sqrt{x-\sqrt{x}-1}}\right)-\text{arctanh}\left(\frac{1-2\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right)\right)$$

$$\Big\downarrow \text{217}$$

$$2\left(\int \frac{1}{4-x}d\left(-\frac{3\sqrt{x}+1}{\sqrt{x-\sqrt{x}-1}}\right)+\frac{1}{2}\arctan\left(\frac{3-\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right)-\text{arctanh}\left(\frac{1-2\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{1}{2}\arctan\left(\frac{3-\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right)-\text{arctanh}\left(\frac{1-2\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right)-\frac{1}{2}\text{arctanh}\left(\frac{3\sqrt{x}+1}{2\sqrt{x-\sqrt{x}-1}}\right)\right)$$

input
```
Int[Sqrt[-1 - Sqrt[x] + x]/((-1 + x)*Sqrt[x]),x]
```

output
```
2*(ArcTan[(3 - Sqrt[x])/(2*Sqrt[-1 - Sqrt[x] + x])]/2 - ArcTanh[(1 - 2*Sqr
t[x])/(2*Sqrt[-1 - Sqrt[x] + x])] - ArcTanh[(1 + 3*Sqrt[x])/(2*Sqrt[-1 - S
qrt[x] + x])]/2)
```

3.727.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

3.727. $\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}}\,dx$

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1321
```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol]
 :> Simp[c/f   Int[1/Sqrt[a + b*x + c*x^2], x], x] - Simp[1/f   Int[(c*d -
a*f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c,
 d, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

rule 1366
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[(h/2 + c*(g/(2*q
)))   Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[(h/2 - c*(g/(
2*q)))   Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d
, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

3.727. $\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}}\,dx$

3.727.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.46

method	result
derivativedivides	$-\sqrt{\left(1+\sqrt{x}\right)^{2}-3\sqrt{x}-2}+\frac{3\ln\left(-\frac{1}{2}+\sqrt{x}+\sqrt{\left(1+\sqrt{x}\right)^{2}-3\sqrt{x}-2}\right)}{2}+\operatorname{arctanh}\left(\frac{-1-3\sqrt{x}}{2\sqrt{\left(1+\sqrt{x}\right)^{2}-3\sqrt{x}-}}\right.$
default	$-\sqrt{\left(1+\sqrt{x}\right)^{2}-3\sqrt{x}-2}+\frac{3\ln\left(-\frac{1}{2}+\sqrt{x}+\sqrt{\left(1+\sqrt{x}\right)^{2}-3\sqrt{x}-2}\right)}{2}+\operatorname{arctanh}\left(\frac{-1-3\sqrt{x}}{2\sqrt{\left(1+\sqrt{x}\right)^{2}-3\sqrt{x}-}}\right.$

input
```
int((-1+x-x^(1/2))^(1/2)/(x-1)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((1+x^(1/2))^2-3*x^(1/2)-2)^(1/2)+3/2*ln(-1/2+x^(1/2)+((1+x^(1/2))^2-3*x^
(1/2)-2)^(1/2))+arctanh(1/2*(-1-3*x^(1/2))/((1+x^(1/2))^2-3*x^(1/2)-2)^(1/
2))+((-1+x^(1/2))^2+x^(1/2)-2)^(1/2)+1/2*ln(-1/2+x^(1/2)+((-1+x^(1/2))^2+x
^(1/2)-2)^(1/2))-arctan(1/2*(-3+x^(1/2))/((-1+x^(1/2))^2+x^(1/2)-2)^(1/2))
```

3.727.5 Fricas [A] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}}\,dx = -\arctan\left(\frac{\left((x-4)\sqrt{x}-2\,x+3\right)\sqrt{x-\sqrt{x}-1}}{2\,(x^{2}-3\,x+1)}\right) + \log\left(-\frac{8\,x^{2}+2\,\left((4\,x-5)\sqrt{x}+2\,x-1\right)\sqrt{x-\sqrt{x}-1}-17\,x-2\,\sqrt{x}+11}{x-1}\right)$$

input
```
integrate((-1+x-x^(1/2))^(1/2)/(-1+x)/x^(1/2),x, algorithm="fricas")
```

output
```
-arctan(1/2*((x - 4)*sqrt(x) - 2*x + 3)*sqrt(x - sqrt(x) - 1)/(x^2 - 3*x +
 1)) + log(-(8*x^2 + 2*((4*x - 5)*sqrt(x) + 2*x - 1)*sqrt(x - sqrt(x) - 1)
 - 17*x - 2*sqrt(x) + 11)/(x - 1))
```

3.727.6 Sympy [F]

$$\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}}\,dx = \int \frac{\sqrt{-\sqrt{x}+x-1}}{\sqrt{x}\,(x-1)}\,dx$$

input
```
integrate((-1+x-x**(1/2))**(1/2)/(-1+x)/x**(1/2),x)
```

output
```
Integral(sqrt(-sqrt(x) + x - 1)/(sqrt(x)*(x - 1)), x)
```

3.727.7 Maxima [F]

$$\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}}\,dx = \int \frac{\sqrt{x-\sqrt{x}-1}}{(x-1)\sqrt{x}}\,dx$$

input
```
integrate((-1+x-x^(1/2))^(1/2)/(-1+x)/x^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x - sqrt(x) - 1)/((x - 1)*sqrt(x)), x)
```

3.727.8 Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\begin{aligned}\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}}\,dx &= -2\arctan\left(\sqrt{x-\sqrt{x}-1}-\sqrt{x}+1\right)\\ &\quad -\log\left(-\sqrt{x-\sqrt{x}-1}+\sqrt{x}+2\right)+\log\left(-\sqrt{x-\sqrt{x}-1}+\sqrt{x}\right)\\ &\quad -2\log\left(\left|2\sqrt{x-\sqrt{x}-1}-2\sqrt{x}+1\right|\right)\end{aligned}$$

input
```
integrate((-1+x-x^(1/2))^(1/2)/(-1+x)/x^(1/2),x, algorithm="giac")
```

output
```
-2*arctan(sqrt(x - sqrt(x) - 1) - sqrt(x) + 1) - log(-sqrt(x - sqrt(x) - 1
) + sqrt(x) + 2) + log(-sqrt(x - sqrt(x) - 1) + sqrt(x)) - 2*log(abs(2*sqr
t(x - sqrt(x) - 1) - 2*sqrt(x) + 1))
```

3.727.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}}\,dx=\int \frac{\sqrt{x-\sqrt{x}-1}}{\sqrt{x}\,(x-1)}\,dx$$

input
```
int((x - x^(1/2) - 1)^(1/2)/(x^(1/2)*(x - 1)),x)
```

output
```
int((x - x^(1/2) - 1)^(1/2)/(x^(1/2)*(x - 1)), x)
```

3.728 $\int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}}\,dx$

3.728.1 Optimal result

Integrand size = 35, antiderivative size = 61

$$\int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}}\,dx = -\arctan\left(\frac{3+\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)+3\text{arctanh}\left(\frac{1-3\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)$$

output
```
-arctan(1/2*(3+(1+x)^(1/2))/(x+(1+x)^(1/2))^(1/2))+3*arctanh(1/2*(1-3*(1+x
)^(1/2))/(x+(1+x)^(1/2))^(1/2))
```

3.728.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

$$\int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}}\,dx = -2\arctan\left(1+\sqrt{1+x}-\sqrt{x+\sqrt{1+x}}\right) - 6\text{arctanh}\left(1-\sqrt{1+x}+\sqrt{x+\sqrt{1+x}}\right)$$

input
```
Integrate[(1 + 2*Sqrt[1 + x])/(x*Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]]),x]
```

output
```
-2*ArcTan[1 + Sqrt[1 + x] - Sqrt[x + Sqrt[1 + x]]] - 6*ArcTanh[1 - Sqrt[1
+ x] + Sqrt[x + Sqrt[1 + x]]]
```

3.728.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7267, 25, 1366, 25, 1154, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2\sqrt{x+1}+1}{x\sqrt{x+1}\sqrt{x+\sqrt{x+1}}}dx$$

$$\Big\downarrow \text{7267}$$

$$2\int \frac{2\sqrt{x+1}+1}{x\sqrt{x+\sqrt{x+1}}}d\sqrt{x+1}$$

$$\Big\downarrow \text{25}$$

$$-2\int -\frac{2\sqrt{x+1}+1}{x\sqrt{x+\sqrt{x+1}}}d\sqrt{x+1}$$

$$\Big\downarrow \text{1366}$$

$$2\left(-\frac{3}{2}\int \frac{1}{\left(1-\sqrt{x+1}\right)\sqrt{x+\sqrt{x+1}}}d\sqrt{x+1}-\frac{1}{2}\int -\frac{1}{\left(\sqrt{x+1}+1\right)\sqrt{x+\sqrt{x+1}}}d\sqrt{x+1}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{1}{2}\int \frac{1}{\left(\sqrt{x+1}+1\right)\sqrt{x+\sqrt{x+1}}}d\sqrt{x+1}-\frac{3}{2}\int \frac{1}{\left(1-\sqrt{x+1}\right)\sqrt{x+\sqrt{x+1}}}d\sqrt{x+1}\right)$$

$$\Big\downarrow \text{1154}$$

$$2\left(3\int \frac{1}{3-x}d\frac{1-3\sqrt{x+1}}{\sqrt{x+\sqrt{x+1}}}-\int \frac{1}{-x-5}d\left(-\frac{\sqrt{x+1}+3}{\sqrt{x+\sqrt{x+1}}}\right)\right)$$

$$\Big\downarrow \text{217}$$

$$2\left(3\int \frac{1}{3-x}d\frac{1-3\sqrt{x+1}}{\sqrt{x+\sqrt{x+1}}}-\frac{1}{2}\arctan\left(\frac{\sqrt{x+1}+3}{2\sqrt{x+\sqrt{x+1}}}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{3}{2}\text{arctanh}\left(\frac{1-3\sqrt{x+1}}{2\sqrt{x+\sqrt{x+1}}}\right)-\frac{1}{2}\arctan\left(\frac{\sqrt{x+1}+3}{2\sqrt{x+\sqrt{x+1}}}\right)\right)$$

input
```
Int[(1 + 2*Sqrt[1 + x])/(x*Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]]),x]
```

output
```
2*(-1/2*ArcTan[(3 + Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])] + (3*ArcTanh[(
1 - 3*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])])/2)
```

3.728.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1366
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[(h/2 + c*(g/(2*q
)))   Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[(h/2 - c*(g/(
2*q)))   Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d
, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.728.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\arctan\left(\frac{-3-\sqrt{x+1}}{2\sqrt{\left(1+\sqrt{x+1}\right)^{2}-\sqrt{x+1}-2}}\right)-3\,\operatorname{arctanh}\left(\frac{-1+3\sqrt{x+1}}{2\sqrt{\left(\sqrt{x+1}-1\right)^{2}+3\sqrt{x+1}-2}}\right)$	68
default	$\arctan\left(\frac{-3-\sqrt{x+1}}{2\sqrt{\left(1+\sqrt{x+1}\right)^{2}-\sqrt{x+1}-2}}\right)-3\,\operatorname{arctanh}\left(\frac{-1+3\sqrt{x+1}}{2\sqrt{\left(\sqrt{x+1}-1\right)^{2}+3\sqrt{x+1}-2}}\right)$	68

input
```
int((1+2*(x+1)^(1/2))/x/(x+1)^(1/2)/(x+(x+1)^(1/2))^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
arctan(1/2*(-3-(x+1)^(1/2))/((1+(x+1)^(1/2))^2-(x+1)^(1/2)-2)^(1/2))-3*arc
tanh(1/2*(-1+3*(x+1)^(1/2))/(((x+1)^(1/2)-1)^2+3*(x+1)^(1/2)-2)^(1/2))
```

3.728.5 Fricas [A] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02

$$\int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}}\,dx = \arctan\left(\frac{2\sqrt{x+\sqrt{x+1}}\left(\sqrt{x+1}-3\right)}{x-8}\right) + 3\log\left(\frac{2\sqrt{x+\sqrt{x+1}}\left(\sqrt{x+1}+1\right)-3x-2\sqrt{x+1}-2}{x}\right)$$

input
```
integrate((1+2*(1+x)^(1/2))/x/(1+x)^(1/2)/(x+(1+x)^(1/2))^(1/2),x, algorit
hm="fricas")
```

output
```
arctan(2*sqrt(x + sqrt(x + 1))*(sqrt(x + 1) - 3)/(x - 8)) + 3*log((2*sqrt(
x + sqrt(x + 1))*(sqrt(x + 1) + 1) - 3*x - 2*sqrt(x + 1) - 2)/x)
```

3.728.6 Sympy [F]

$$\int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}}\,dx = \int \frac{2\sqrt{x+1}+1}{x\sqrt{x+1}\sqrt{x+\sqrt{x+1}}}\,dx$$

input
```
integrate((1+2*(1+x)**(1/2))/x/(1+x)**(1/2)/(x+(1+x)**(1/2))**(1/2),x)
```

output
```
Integral((2*sqrt(x + 1) + 1)/(x*sqrt(x + 1)*sqrt(x + sqrt(x + 1))), x)
```

3.728.7 Maxima [F]

$$\int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}}\,dx = \int \frac{2\sqrt{x+1}+1}{\sqrt{x+\sqrt{x+1}}\sqrt{x+1}x}\,dx$$

input
```
integrate((1+2*(1+x)^(1/2))/x/(1+x)^(1/2)/(x+(1+x)^(1/2))^(1/2),x, algorit
hm="maxima")
```

output
```
integrate((2*sqrt(x + 1) + 1)/(sqrt(x + sqrt(x + 1))*sqrt(x + 1)*x), x)
```

3.728.8 Giac [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07

$$\int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}}\,dx = 2\arctan\left(\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}-1\right) - 3\log\left(\left|\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}+2\right|\right) + 3\log\left(\left|\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}\right|\right)$$

input
```
integrate((1+2*(1+x)^(1/2))/x/(1+x)^(1/2)/(x+(1+x)^(1/2))^(1/2),x, algorit
hm="giac")
```

output
```
2*arctan(sqrt(x + sqrt(x + 1)) - sqrt(x + 1) - 1) - 3*log(abs(sqrt(x + sqr
t(x + 1)) - sqrt(x + 1) + 2)) + 3*log(abs(sqrt(x + sqrt(x + 1)) - sqrt(x +
 1)))
```

3.728.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}}\,dx = \int \frac{2\,\sqrt{x+1}+1}{x\,\sqrt{x+\sqrt{x+1}}\,\sqrt{x+1}}\,dx$$

input
```
int((2*(x + 1)^(1/2) + 1)/(x*(x + (x + 1)^(1/2))^(1/2)*(x + 1)^(1/2)),x)
```

output
```
int((2*(x + 1)^(1/2) + 1)/(x*(x + (x + 1)^(1/2))^(1/2)*(x + 1)^(1/2)), x)
```

3.729 $\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx$

3.729.1 Optimal result

Integrand size = 13, antiderivative size = 8

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx = 2\text{arcsinh}\left(\sqrt{x}\right)$$

output
```
2*arcsinh(x^(1/2))
```

3.729.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 18 vs. 2(8) = 16.

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx = -2\log\left(-\sqrt{x}+\sqrt{1+x}\right)$$

input
```
Integrate[1/(Sqrt[x]*Sqrt[1 + x]),x]
```

output
```
-2*Log[-Sqrt[x] + Sqrt[1 + x]]
```

3.729.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt{x+1}}dx$$

$$\Big\downarrow \text{ 63}$$

$$2\int \frac{1}{\sqrt{x+1}}d\sqrt{x}$$

$$\Big\downarrow \text{ 222}$$

$$2\text{arcsinh}\left(\sqrt{x}\right)$$

input
```
Int[1/(Sqrt[x]*Sqrt[1 + x]),x]
```

output
```
2*ArcSinh[Sqrt[x]]
```

3.729.3.1 Defintions of rubi rules used

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

3.729.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
meijerg	$2 \operatorname{arcsinh}\left(\sqrt{x}\right)$	7
default	$\frac{\sqrt{(x+1)x}\,\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{\sqrt{x}\,\sqrt{x+1}}$	28

input
```
int(1/x^(1/2)/(x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arcsinh(x^(1/2))
```

3.729.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(6) = 12.

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx = -\log\left(2\sqrt{x+1}\sqrt{x} - 2x - 1\right)$$

input
```
integrate(1/x^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
-log(2*sqrt(x + 1)*sqrt(x) - 2*x - 1)
```

3.729.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 3.25

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx = \begin{cases} 2\operatorname{acosh}\left(\sqrt{x+1}\right) & \text{for } |x+1| > 1 \\ -2i\operatorname{asin}\left(\sqrt{x+1}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**(1/2)/(1+x)**(1/2),x)
```

output
```
Piecewise((2*acosh(sqrt(x + 1)), Abs(x + 1) > 1), (-2*I*asin(sqrt(x + 1)),
 True))
```

3.729.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(6) = 12$.

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 3.38

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx = \log\left(\frac{\sqrt{x+1}}{\sqrt{x}}+1\right) - \log\left(\frac{\sqrt{x+1}}{\sqrt{x}}-1\right)$$

input
```
integrate(1/x^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
log(sqrt(x + 1)/sqrt(x) + 1) - log(sqrt(x + 1)/sqrt(x) - 1)
```

3.729.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(6) = 12$.

Time = 0.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx = -2\log\left(\sqrt{x+1}-\sqrt{x}\right)$$

input
```
integrate(1/x^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
-2*log(sqrt(x + 1) - sqrt(x))
```

3.729.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx = 4\,\mathrm{atanh}\left(\frac{\sqrt{x+1}-1}{\sqrt{x}}\right)$$

input
```
int(1/(x^(1/2)*(x + 1)^(1/2)),x)
```

output
```
4*atanh(((x + 1)^(1/2) - 1)/x^(1/2))
```

3.730 $\int \frac{\sqrt{\frac{x}{1+x}}}{x} \, dx$

3.730.1 Optimal result

Integrand size = 15, antiderivative size = 8

$$\int \frac{\sqrt{\frac{x}{1+x}}}{x} \, dx = 2\text{arcsinh}\left(\sqrt{x}\right)$$

output
```
2*arcsinh(x^(1/2))
```

3.730.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 18 vs. 2(8) = 16.

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{\sqrt{\frac{x}{1+x}}}{x} \, dx = -2\log\left(-\sqrt{x}+\sqrt{1+x}\right)$$

input
```
Integrate[Sqrt[x/(1 + x)]/x,x]
```

output
```
-2*Log[-Sqrt[x] + Sqrt[1 + x]]
```

3.730.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2050, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{x}{x+1}}}{x} \, dx$$
$$\Big\downarrow \; 2050$$
$$\int \frac{1}{\sqrt{x}\sqrt{x+1}}dx$$
$$\Big\downarrow \; 63$$
$$2\int \frac{1}{\sqrt{x+1}}d\sqrt{x}$$
$$\Big\downarrow \; 222$$
$$2\text{arcsinh}\left(\sqrt{x}\right)$$

input
```
Int[Sqrt[x/(1 + x)]/x,x]
```

output
```
2*ArcSinh[Sqrt[x]]
```

3.730.3.1 Defintions of rubi rules used

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2050
```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

3.730. $\int \frac{\sqrt{\frac{x}{1+x}}}{x} \, dx$

3.730.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(6) = 12$.

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 4.00

method	result	size
default	$\frac{\sqrt{\frac{x}{x+1}}\,(x+1)\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{\sqrt{(x+1)x}}$	32
trager	$-\ln\left(2\sqrt{\frac{x}{x+1}}\,x+2\sqrt{\frac{x}{x+1}}-2x-1\right)$	32

input
```
int((x/(x+1))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(x/(x+1))^(1/2)*(x+1)/((x+1)*x)^(1/2)*ln(x+1/2+(x^2+x)^(1/2))
```

3.730.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(6) = 12$.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 3.38

$$\int \frac{\sqrt{\frac{x}{1+x}}}{x}\,dx = \log\left(\sqrt{\frac{x}{x+1}}+1\right)-\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

input
```
integrate((x/(1+x))^(1/2)/x,x, algorithm="fricas")
```

output
```
log(sqrt(x/(x + 1)) + 1) - log(sqrt(x/(x + 1)) - 1)
```

3.730.6 Sympy [F]

$$\int \frac{\sqrt{\frac{x}{1+x}}}{x}\,dx = \int \frac{\sqrt{\frac{x}{x+1}}}{x}\,dx$$

input
```
integrate((x/(1+x))**(1/2)/x,x)
```

output
```
Integral(sqrt(x/(x + 1))/x, x)
```

3.730.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(6) = 12.

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 3.38

$$\int \frac{\sqrt{\frac{x}{1+x}}}{x}\,dx = \log\left(\sqrt{\frac{x}{x+1}}+1\right) - \log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

input
```
integrate((x/(1+x))^(1/2)/x,x, algorithm="maxima")
```

output
```
log(sqrt(x/(x + 1)) + 1) - log(sqrt(x/(x + 1)) - 1)
```

3.730.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(6) = 12.

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.75

$$\int \frac{\sqrt{\frac{x}{1+x}}}{x}\,dx = -\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)\operatorname{sgn}(x+1)$$

input
```
integrate((x/(1+x))^(1/2)/x,x, algorithm="giac")
```

output
```
-log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x + 1)
```

3.730.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{\frac{x}{1+x}}}{x}\,dx = 2\,\mathrm{atanh}\left(\sqrt{\frac{x}{x+1}}\right)$$

input
```
int((x/(x + 1))^(1/2)/x,x)
```

output
```
2*atanh((x/(x + 1))^(1/2))
```

3.731 $\int \frac{\sqrt{x}}{\sqrt{1+x}} \, dx$

3.731.1 Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{\sqrt{x}}{\sqrt{1+x}} \, dx = \sqrt{x}\sqrt{1+x} - \operatorname{arcsinh}(\sqrt{x})$$

output
```
-arcsinh(x^(1/2))+x^(1/2)*(1+x)^(1/2)
```

3.731.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(22) = 44.

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{\sqrt{x}}{\sqrt{1+x}} \, dx = \frac{\sqrt{\frac{x}{1+x}}\left(\sqrt{x}(1+x) + \sqrt{1+x}\log\left(-\sqrt{x} + \sqrt{1+x}\right)\right)}{\sqrt{x}}$$

input
```
Integrate[Sqrt[x]/Sqrt[1 + x],x]
```

output
```
(Sqrt[x/(1 + x)]*(Sqrt[x]*(1 + x) + Sqrt[1 + x]*Log[-Sqrt[x] + Sqrt[1 + x]
]))/Sqrt[x]
```

3.731.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{x+1}}dx$$
$$\Big\downarrow \text{ 60}$$
$$\sqrt{x}\sqrt{x+1}-\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{x+1}}dx$$
$$\Big\downarrow \text{ 63}$$
$$\sqrt{x}\sqrt{x+1}-\int \frac{1}{\sqrt{x+1}}d\sqrt{x}$$
$$\Big\downarrow \text{ 222}$$
$$\sqrt{x}\sqrt{x+1}-\text{arcsinh}\left(\sqrt{x}\right)$$

input
```
Int[Sqrt[x]/Sqrt[1 + x],x]
```

output
```
Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

3.731.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

3.731.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

method	result	size
meijerg	$\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{x+1}-\sqrt{\pi}\,\operatorname{arcsinh}\left(\sqrt{x}\right)}{\sqrt{\pi}}$	27
default	$\sqrt{x}\,\sqrt{x+1}-\frac{\sqrt{(x+1)x}\,\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{2\sqrt{x}\,\sqrt{x+1}}$	39
risch	$\sqrt{x}\,\sqrt{x+1}-\frac{\sqrt{(x+1)x}\,\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{2\sqrt{x}\,\sqrt{x+1}}$	39

input
```
int(x^(1/2)/(x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/Pi^(1/2)*(Pi^(1/2)*x^(1/2)*(x+1)^(1/2)-Pi^(1/2)*arcsinh(x^(1/2)))
```

3.731.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx = \sqrt{x+1}\sqrt{x}+\frac{1}{2}\,\log\left(2\,\sqrt{x+1}\sqrt{x}-2\,x-1\right)$$

input
```
integrate(x^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(x + 1)*sqrt(x) + 1/2*log(2*sqrt(x + 1)*sqrt(x) - 2*x - 1)
```

3.731.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.86

$$\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx = \begin{cases} \sqrt{x}\sqrt{x+1} - \operatorname{acosh}\left(\sqrt{x+1}\right) & \text{for } |x+1| > 1 \\ i\operatorname{asin}\left(\sqrt{x+1}\right) - \frac{i(x+1)^{\frac{3}{2}}}{\sqrt{-x}} + \frac{i\sqrt{x+1}}{\sqrt{-x}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(1/2)/(1+x)**(1/2),x)
```

output
```
Piecewise((sqrt(x)*sqrt(x + 1) - acosh(sqrt(x + 1)), Abs(x + 1) > 1), (I*a
sin(sqrt(x + 1)) - I*(x + 1)**(3/2)/sqrt(-x) + I*sqrt(x + 1)/sqrt(-x), Tru
e))
```

3.731.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(16) = 32.

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx = \frac{\sqrt{x+1}}{\sqrt{x}\left(\frac{x+1}{x}-1\right)} - \frac{1}{2}\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}+1\right) + \frac{1}{2}\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}-1\right)$$

input
```
integrate(x^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x + 1)/(sqrt(x)*((x + 1)/x - 1)) - 1/2*log(sqrt(x + 1)/sqrt(x) + 1) +
 1/2*log(sqrt(x + 1)/sqrt(x) - 1)
```

3.731.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx = \sqrt{x+1}\sqrt{x} + \log\left(\sqrt{x+1}-\sqrt{x}\right)$$

input
```
integrate(x^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
sqrt(x + 1)*sqrt(x) + log(sqrt(x + 1) - sqrt(x))
```

3.731.9 Mupad [B] (verification not implemented)

Time = 19.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx = \sqrt{x}\,\sqrt{x+1} - 2\,\mathrm{atanh}\left(\frac{\sqrt{x}}{\sqrt{x+1}-1}\right)$$

input
```
int(x^(1/2)/(x + 1)^(1/2),x)
```

output
```
x^(1/2)*(x + 1)^(1/2) - 2*atanh(x^(1/2)/((x + 1)^(1/2) - 1))
```

3.732 $\int \sqrt{\frac{x}{1+x}}\,dx$

3.732.1 Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \sqrt{\frac{x}{1+x}}\,dx = \sqrt{x}\sqrt{1+x} - \operatorname{arcsinh}\left(\sqrt{x}\right)$$

output
```
-arcsinh(x^(1/2))+x^(1/2)*(1+x)^(1/2)
```

3.732.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(22) = 44.

Time = 0.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \sqrt{\frac{x}{1+x}}\,dx = \frac{\sqrt{\frac{x}{1+x}}\left(\sqrt{x}(1+x)+\sqrt{1+x}\log\left(-\sqrt{x}+\sqrt{1+x}\right)\right)}{\sqrt{x}}$$

input
```
Integrate[Sqrt[x/(1 + x)],x]
```

output
```
(Sqrt[x/(1 + x)]*(Sqrt[x]*(1 + x) + Sqrt[1 + x]*Log[-Sqrt[x] + Sqrt[1 + x]
]))/Sqrt[x]
```

3.732.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2050, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{x}{x+1}}dx$$

$$\Big\downarrow \text{ 2050}$$

$$\int \frac{\sqrt{x}}{\sqrt{x+1}}dx$$

$$\Big\downarrow \text{ 60}$$

$$\sqrt{x}\sqrt{x+1}-\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{x+1}}dx$$

$$\Big\downarrow \text{ 63}$$

$$\sqrt{x}\sqrt{x+1}-\int \frac{1}{\sqrt{x+1}}d\sqrt{x}$$

$$\Big\downarrow \text{ 222}$$

$$\sqrt{x}\sqrt{x+1}-\text{arcsinh}\left(\sqrt{x}\right)$$

input
```
Int[Sqrt[x/(1 + x)],x]
```

output
```
Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

3.732.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2050
```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

3.732.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(16) = 32$.

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.05

method	result	size
default	$\frac{\sqrt{\frac{x}{x+1}}\,(x+1)\left(2\sqrt{x^2+x}-\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)\right)}{2\sqrt{(x+1)x}}$	45
risch	$(x+1)\sqrt{\frac{x}{x+1}}-\frac{\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)\sqrt{\frac{x}{x+1}}\,\sqrt{(x+1)x}}{2x}$	47
trager	$2\left(\frac{x}{2}+\frac{1}{2}\right)\sqrt{\frac{x}{x+1}}-\frac{\ln\left(2\sqrt{\frac{x}{x+1}}\,x+2\sqrt{\frac{x}{x+1}}+2x+1\right)}{2}$	49

input
```
int((x/(x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x/(x+1))^(1/2)*(x+1)*(2*(x^2+x)^(1/2)-ln(x+1/2+(x^2+x)^(1/2)))/((x+1)
*x)^(1/2)
```

3.732.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(16) = 32$.

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \sqrt{\frac{x}{1+x}}\,dx = (x+1)\sqrt{\frac{x}{x+1}} - \frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}+1\right) + \frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

input
```
integrate((x/(1+x))^(1/2),x, algorithm="fricas")
```

output
```
(x + 1)*sqrt(x/(x + 1)) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(sqrt(x/(x
 + 1)) - 1)
```

3.732.6 Sympy [F]

$$\int \sqrt{\frac{x}{1+x}}\,dx = \int \sqrt{\frac{x}{x+1}}\,dx$$

input
```
integrate((x/(1+x))**(1/2),x)
```

output
```
Integral(sqrt(x/(x + 1)), x)
```

3.732.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(16) = 32$.

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \sqrt{\frac{x}{1+x}}\,dx = -\frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1} - \frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}+1\right) + \frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

input
```
integrate((x/(1+x))^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(x/(x + 1))/(x/(x + 1) - 1) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(
sqrt(x/(x + 1)) - 1)
```

3.732.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(16) = 32$.

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \sqrt{\frac{x}{1+x}}\,dx = \frac{1}{2}\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)\mathrm{sgn}(x+1)+\sqrt{x^2+x}\,\mathrm{sgn}(x+1)$$

input
```
integrate((x/(1+x))^(1/2),x, algorithm="giac")
```

output
```
1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x + 1) + sqrt(x^2 + x)*sgn(x
+ 1)
```

3.732.9 Mupad [B] (verification not implemented)

Time = 18.84 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \sqrt{\frac{x}{1+x}}\,dx = -\mathrm{atanh}\left(\sqrt{\frac{x}{x+1}}\right)-\frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1}$$

input
```
int((x/(x + 1))^(1/2),x)
```

output
```
- atanh((x/(x + 1))^(1/2)) - (x/(x + 1))^(1/2)/(x/(x + 1) - 1)
```

3.733 $\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx$

3.733.1 Optimal result

Integrand size = 18, antiderivative size = 36

$$\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx = -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + \arctan\left(\sqrt{-1+x}\sqrt{1+x}\right)$$

output
```
arctan((-1+x)^(1/2)*(1+x)^(1/2))-(-1+x)^(1/2)*(1+x)^(1/2)/x
```

3.733.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx = -\frac{\sqrt{\frac{-1+x}{1+x}}\left(\sqrt{-1+x}(1+x) + 2x\sqrt{1+x}\arctan\left(x-\sqrt{-1+x}\sqrt{1+x}\right)\right)}{\sqrt{-1+x}x}$$

input
```
Integrate[Sqrt[-1 + x]/(x^2*Sqrt[1 + x]),x]
```

output
```
-((Sqrt[(-1 + x)/(1 + x)]*(Sqrt[-1 + x]*(1 + x) + 2*x*Sqrt[1 + x]*ArcTan[x
 - Sqrt[-1 + x]*Sqrt[1 + x]]))/(Sqrt[-1 + x]*x))
```

3.733.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {105, 103, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x-1}}{x^2\sqrt{x+1}}dx$$

$$\Big\downarrow \text{105}$$

$$\int \frac{1}{\sqrt{x-1}x\sqrt{x+1}}dx-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

$$\Big\downarrow \text{103}$$

$$\int \frac{1}{(x-1)(x+1)+1}d\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

$$\Big\downarrow \text{216}$$

$$\arctan\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

input
```
Int[Sqrt[-1 + x]/(x^2*Sqrt[1 + x]),x]
```

output
```
-((Sqrt[-1 + x]*Sqrt[1 + x])/x) + ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

3.733.3.1 Defintions of rubi rules used

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.733.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

method	result	size
default	$\frac{\left(-\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)x-\sqrt{x^2-1}\right)\sqrt{x-1}\,\sqrt{x+1}}{x\sqrt{x^2-1}}$	43
risch	$-\frac{\sqrt{x-1}\,\sqrt{x+1}}{x}-\frac{\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)\sqrt{(x-1)(x+1)}}{\sqrt{x-1}\,\sqrt{x+1}}$	46

input
```
int((x-1)^(1/2)/x^2/(x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-arctan(1/(x^2-1)^(1/2))*x-(x^2-1)^(1/2))*(x-1)^(1/2)*(x+1)^(1/2)/x/(x^2-
1)^(1/2)
```

3.733.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx = \frac{2\,x\arctan\left(\sqrt{x+1}\sqrt{x-1}-x\right)-\sqrt{x+1}\sqrt{x-1}-x}{x}$$

input
```
integrate((-1+x)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
(2*x*arctan(sqrt(x + 1)*sqrt(x - 1) - x) - sqrt(x + 1)*sqrt(x - 1) - x)/x
```

3.733.6 Sympy [F]

$$\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx = \int \frac{\sqrt{x-1}}{x^2\sqrt{x+1}}\,dx$$

input
```
integrate((-1+x)**(1/2)/x**2/(1+x)**(1/2),x)
```

output
```
Integral(sqrt(x - 1)/(x**2*sqrt(x + 1)), x)
```

3.733.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx = -\frac{\sqrt{x^2-1}}{x}-\arcsin\left(\frac{1}{|x|}\right)$$

input
```
integrate((-1+x)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(x^2 - 1)/x - arcsin(1/abs(x))
```

3.733.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx = -\frac{8}{\left(\sqrt{x+1}-\sqrt{x-1}\right)^4+4} - 2\arctan\left(\frac{1}{2}\left(\sqrt{x+1}-\sqrt{x-1}\right)^2\right)$$

input
```
integrate((-1+x)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="giac")
```

output
```
-8/((sqrt(x + 1) - sqrt(x - 1))^4 + 4) - 2*arctan(1/2*(sqrt(x + 1) - sqrt(
x - 1))^2)
```

3.733.9 Mupad [B] (verification not implemented)

Time = 19.79 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.83

$$\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx = -\ln\left(\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^2}{\left(\sqrt{x+1}-1\right)^2}+1\right)1\mathrm{i} + \ln\left(\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}\right)1\mathrm{i} - \frac{\sqrt{x-1}-\mathrm{i}}{4\left(\sqrt{x+1}-1\right)} - \frac{\frac{5\left(\sqrt{x-1}-\mathrm{i}\right)^2}{\left(\sqrt{x+1}-1\right)^2}+1}{\frac{4\left(\sqrt{x-1}-\mathrm{i}\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{4\left(\sqrt{x-1}-\mathrm{i}\right)}{\sqrt{x+1}-1}}$$

input
```
int((x - 1)^(1/2)/(x^2*(x + 1)^(1/2)),x)
```

output
```
log(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1))*1i - log(((x - 1)^(1/2) - 1i
)^2/((x + 1)^(1/2) - 1)^2 + 1)*1i - ((x - 1)^(1/2) - 1i)/(4*((x + 1)^(1/2)
 - 1)) - ((5*((x - 1)^(1/2) - 1i)^2)/((x + 1)^(1/2) - 1)^2 + 1)/((4*((x -
1)^(1/2) - 1i)^3)/((x + 1)^(1/2) - 1)^3 + (4*((x - 1)^(1/2) - 1i))/((x + 1
)^(1/2) - 1))
```

3.734 $\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2} dx$

3.734.1 Optimal result

Integrand size = 17, antiderivative size = 36

$$\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2} dx = -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + \arctan\left(\sqrt{-1+x}\sqrt{1+x}\right)$$

output
```
arctan((-1+x)^(1/2)*(1+x)^(1/2))-(-1+x)^(1/2)*(1+x)^(1/2)/x
```

3.734.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2} dx = -\frac{\sqrt{\frac{-1+x}{1+x}}\left(\sqrt{-1+x}(1+x) + 2x\sqrt{1+x}\arctan\left(x - \sqrt{-1+x}\sqrt{1+x}\right)\right)}{\sqrt{-1+x}x}$$

input
```
Integrate[Sqrt[(-1 + x)/(1 + x)]/x^2,x]
```

output
```
-((Sqrt[(-1 + x)/(1 + x)]*(Sqrt[-1 + x]*(1 + x) + 2*x*Sqrt[1 + x]*ArcTan[x
 - Sqrt[-1 + x]*Sqrt[1 + x]]))/(Sqrt[-1 + x]*x))
```

3.734.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2050, 105, 103, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{x-1}{x+1}}}{x^2} \, dx$$
$$\Big\downarrow \ 2050$$
$$\int \frac{\sqrt{x-1}}{x^2\sqrt{x+1}}dx$$
$$\Big\downarrow \ 105$$
$$\int \frac{1}{\sqrt{x-1}x\sqrt{x+1}}dx-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$
$$\Big\downarrow \ 103$$
$$\int \frac{1}{(x-1)(x+1)+1}d\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$
$$\Big\downarrow \ 216$$
$$\arctan\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

input
```
Int[Sqrt[(-1 + x)/(1 + x)]/x^2,x]
```

output
```
-((Sqrt[-1 + x]*Sqrt[1 + x])/x) + ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

3.734.3.1 Defintions of rubi rules used

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2050
```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

3.734.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.56

method	result	size
risch	$-\frac{(x+1)\sqrt{\frac{x-1}{x+1}}}{x}-\frac{\arctan\left(\frac{1}{\sqrt{x^{2}-1}}\right)\sqrt{\frac{x-1}{x+1}}\,\sqrt{(x-1)(x+1)}}{x-1}$	56
default	$\frac{\sqrt{\frac{x-1}{x+1}}\,(x+1)\left(\left(x^{2}-1\right)^{\frac{3}{2}}-x^{2}\sqrt{x^{2}-1}-\arctan\left(\frac{1}{\sqrt{x^{2}-1}}\right)x\right)}{\sqrt{(x-1)(x+1)}\,x}$	59
trager	$-\frac{(x+1)\sqrt{-\frac{1-x}{x+1}}}{x}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-\frac{1-x}{x+1}}\,x+\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-\frac{1-x}{x+1}}-1}{x}\right)$	82

input
```
int(((x-1)/(x+1))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(x+1)/x*((x-1)/(x+1))^(1/2)-arctan(1/(x^2-1)^(1/2))*((x-1)/(x+1))^(1/2)*(
(x-1)*(x+1))^(1/2)/(x-1)
```

3.734.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2}\,dx = \frac{2\,x\arctan\left(\sqrt{\frac{x-1}{x+1}}\right) - (x+1)\sqrt{\frac{x-1}{x+1}}}{x}$$

input
```
integrate(((-1+x)/(1+x))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
(2*x*arctan(sqrt((x - 1)/(x + 1))) - (x + 1)*sqrt((x - 1)/(x + 1)))/x
```

3.734.6 Sympy [F]

$$\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2}\,dx = \int \frac{\sqrt{\frac{x-1}{x+1}}}{x^2}\,dx$$

input
```
integrate(((-1+x)/(1+x))**(1/2)/x**2,x)
```

output
```
Integral(sqrt((x - 1)/(x + 1))/x**2, x)
```

3.734.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2}\,dx = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}+1} + 2\,\arctan\left(\sqrt{\frac{x-1}{x+1}}\right)$$

input
```
integrate(((-1+x)/(1+x))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-2*sqrt((x - 1)/(x + 1))/((x - 1)/(x + 1) + 1) + 2*arctan(sqrt((x - 1)/(x
+ 1)))
```

3.734.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2}\,dx = -\frac{1}{2}\,(\pi-2)\text{sgn}(x+1) + 2\,\arctan\left(-x+\sqrt{x^2-1}\right)\text{sgn}(x+1) - \frac{2\,\text{sgn}(x+1)}{\left(x-\sqrt{x^2-1}\right)^2+1}$$

input
```
integrate(((-1+x)/(1+x))^(1/2)/x^2,x, algorithm="giac")
```

output
```
-1/2*(pi - 2)*sgn(x + 1) + 2*arctan(-x + sqrt(x^2 - 1))*sgn(x + 1) - 2*sgn
(x + 1)/((x - sqrt(x^2 - 1))^2 + 1)
```

3.734.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2}\,dx = 2\,\text{atan}\left(\sqrt{\frac{x-1}{x+1}}\right) - \frac{2\,\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}+1}$$

input
```
int(((x - 1)/(x + 1))^(1/2)/x^2,x)
```

output
```
2*atan(((x - 1)/(x + 1))^(1/2)) - (2*((x - 1)/(x + 1))^(1/2))/((x - 1)/(x
+ 1) + 1)
```

3.735 $\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}} \, dx$

3.735.1 Optimal result

Integrand size = 18, antiderivative size = 69

$$\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}} \, dx = -\frac{3}{8}\sqrt{-1+x}\sqrt{1+x}+\frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x} + \frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}+\frac{3\text{arccosh}(x)}{8}$$

output
```
3/8*arccosh(x)+1/24*(7-2*x)*(-1+x)^(3/2)*(1+x)^(1/2)+1/4*(-1+x)^(3/2)*x^2*
(1+x)^(1/2)-3/8*(-1+x)^(1/2)*(1+x)^(1/2)
```

3.735.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}} \, dx = \frac{\sqrt{\frac{-1+x}{1+x}}\left(\sqrt{-1+x}(-16-7x+x^2-2x^3+6x^4)-18\sqrt{1+x}\log\left(\sqrt{-1+x}-\sqrt{1+x}\right)\right)}{24\sqrt{-1+x}}$$

input
```
Integrate[(Sqrt[-1 + x]*x^3)/Sqrt[1 + x],x]
```

output
```
(Sqrt[(-1 + x)/(1 + x)]*(Sqrt[-1 + x]*(-16 - 7*x + x^2 - 2*x^3 + 6*x^4) -
18*Sqrt[1 + x]*Log[Sqrt[-1 + x] - Sqrt[1 + x]]))/(24*Sqrt[-1 + x])
```

3.735.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {111, 164, 60, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x-1}x^3}{\sqrt{x+1}}dx$$

$$\Big\downarrow \text{111}$$

$$\frac{1}{4}\int \frac{(2-x)\sqrt{x-1}x}{\sqrt{x+1}}dx+\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2$$

$$\Big\downarrow \text{164}$$

$$\frac{1}{4}\left(\frac{1}{6}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{2}\int \frac{\sqrt{x-1}}{\sqrt{x+1}}dx\right)+\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(\frac{1}{6}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{2}\left(\sqrt{x-1}\sqrt{x+1}-\int \frac{1}{\sqrt{x-1}\sqrt{x+1}}dx\right)\right)+\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2$$

$$\Big\downarrow \text{43}$$

$$\frac{1}{4}\left(\frac{1}{6}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{2}\left(\sqrt{x-1}\sqrt{x+1}-\text{arccosh}(x)\right)\right)+\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2$$

input
```
Int[(Sqrt[-1 + x]*x^3)/Sqrt[1 + x],x]
```

output
```
((-1 + x)^(3/2)*x^2*Sqrt[1 + x])/4 + (((7 - 2*x)*(-1 + x)^(3/2)*Sqrt[1 + x
])/6 - (3*(Sqrt[-1 + x]*Sqrt[1 + x] - ArcCosh[x]))/2)/4
```

3.735.3.1 Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

3.735. $\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx$

3.735.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

method	result	size
risch	$\frac{(6x^3-8x^2+9x-16)\sqrt{x-1}\,\sqrt{x+1}}{24}+\frac{3\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(x-1)(x+1)}}{8\sqrt{x-1}\,\sqrt{x+1}}$	60
default	$\frac{\sqrt{x-1}\,\sqrt{x+1}\left(6x^3\sqrt{x^2-1}-8x^2\sqrt{x^2-1}+9x\sqrt{x^2-1}+9\ln\left(x+\sqrt{x^2-1}\right)-16\sqrt{x^2-1}\right)}{24\sqrt{x^2-1}}$	76

input
```
int(x^3*(x-1)^(1/2)/(x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*x^3-8*x^2+9*x-16)*(x-1)^(1/2)*(x+1)^(1/2)+3/8*ln(x+(x^2-1)^(1/2))*
((x-1)*(x+1))^(1/2)/(x-1)^(1/2)/(x+1)^(1/2)
```

3.735.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx = \frac{1}{24}\left(6\,x^3-8\,x^2+9\,x-16\right)\sqrt{x+1}\sqrt{x-1}-\frac{3}{8}\,\log\left(\sqrt{x+1}\sqrt{x-1}-x\right)$$

input
```
integrate(x^3*(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
1/24*(6*x^3 - 8*x^2 + 9*x - 16)*sqrt(x + 1)*sqrt(x - 1) - 3/8*log(sqrt(x +
 1)*sqrt(x - 1) - x)
```

3.735.6 Sympy [F]

$$\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx = \int \frac{x^3\sqrt{x-1}}{\sqrt{x+1}}\,dx$$

input
```
integrate(x**3*(-1+x)**(1/2)/(1+x)**(1/2),x)
```

output
```
Integral(x**3*sqrt(x - 1)/sqrt(x + 1), x)
```

3.735.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx = \frac{1}{4}\left(x^2-1\right)^{\frac{3}{2}}x - \frac{1}{3}\left(x^2-1\right)^{\frac{3}{2}} + \frac{5}{8}\sqrt{x^2-1}x - \sqrt{x^2-1} + \frac{3}{8}\log\left(2\,x+2\sqrt{x^2-1}\right)$$

input
```
integrate(x^3*(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(x^2 - 1)^(3/2)*x - 1/3*(x^2 - 1)^(3/2) + 5/8*sqrt(x^2 - 1)*x - sqrt(x
^2 - 1) + 3/8*log(2*x + 2*sqrt(x^2 - 1))
```

3.735.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx = \frac{1}{24}\left((2\,(3\,x-10)(x+1)+43)(x+1)-39\right)\sqrt{x+1}\sqrt{x-1} - \frac{3}{4}\log\left(\sqrt{x+1}-\sqrt{x-1}\right)$$

input
```
integrate(x^3*(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
1/24*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(x - 1) -
3/4*log(sqrt(x + 1) - sqrt(x - 1))
```

3.735.9 Mupad [B] (verification not implemented)

Time = 32.74 (sec) , antiderivative size = 473, normalized size of antiderivative = 6.86

$$\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx = \frac{3\,\mathrm{atanh}\left(\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}\right)}{2} + \frac{\frac{23\left(\sqrt{x-1}-\mathrm{i}\right)^3}{2\left(\sqrt{x+1}-1\right)^3} - \frac{\left(\sqrt{x-1}-\mathrm{i}\right)^4 64\mathrm{i}}{\left(\sqrt{x+1}-1\right)^4} + \frac{333\left(\sqrt{x-1}-\mathrm{i}\right)^5}{2\left(\sqrt{x+1}-1\right)^5} + \frac{\left(\sqrt{x-1}-\mathrm{i}\right)^6 256\mathrm{i}}{3\left(\sqrt{x+1}-1\right)^6} + \frac{671\left(\sqrt{x-1}-\mathrm{i}\right)^7}{2\left(\sqrt{x+1}-1\right)^7} - \frac{\left(\sqrt{x-1}-\mathrm{i}\right)^8 128\mathrm{i}}{3\left(\sqrt{x+1}-1\right)^8} + \frac{671\left(\sqrt{x-1}-\mathrm{i}\right)^9}{2\left(\sqrt{x+1}-1\right)^9}}{1 + \frac{28\left(\sqrt{x-1}-\mathrm{i}\right)^4}{\left(\sqrt{x+1}-1\right)^4} - \frac{56\left(\sqrt{x-1}-\mathrm{i}\right)^6}{\left(\sqrt{x+1}-1\right)^6} + \frac{70\left(\sqrt{x-1}-\mathrm{i}\right)^8}{\left(\sqrt{x+1}-1\right)^8} - \frac{56\left(\sqrt{x-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{x+1}-1\right)^{10}} + \frac{28\,(}{(}}$$

input
```
int((x^3*(x - 1)^(1/2))/(x + 1)^(1/2),x)
```

output
```
(3*atanh(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1)))/2 + ((23*((x - 1)^(1/2
) - 1i)^3)/(2*((x + 1)^(1/2) - 1)^3) - (((x - 1)^(1/2) - 1i)^4*64i)/((x +
1)^(1/2) - 1)^4 + (333*((x - 1)^(1/2) - 1i)^5)/(2*((x + 1)^(1/2) - 1)^5) +
 (((x - 1)^(1/2) - 1i)^6*256i)/(3*((x + 1)^(1/2) - 1)^6) + (671*((x - 1)^(
1/2) - 1i)^7)/(2*((x + 1)^(1/2) - 1)^7) - (((x - 1)^(1/2) - 1i)^8*128i)/(3
*((x + 1)^(1/2) - 1)^8) + (671*((x - 1)^(1/2) - 1i)^9)/(2*((x + 1)^(1/2) -
 1)^9) + (((x - 1)^(1/2) - 1i)^10*256i)/(3*((x + 1)^(1/2) - 1)^10) + (333*
((x - 1)^(1/2) - 1i)^11)/(2*((x + 1)^(1/2) - 1)^11) - (((x - 1)^(1/2) - 1i
)^12*64i)/((x + 1)^(1/2) - 1)^12 + (23*((x - 1)^(1/2) - 1i)^13)/(2*((x + 1
)^(1/2) - 1)^13) - (3*((x - 1)^(1/2) - 1i)^15)/(2*((x + 1)^(1/2) - 1)^15)
- (3*((x - 1)^(1/2) - 1i))/(2*((x + 1)^(1/2) - 1)))/((28*((x - 1)^(1/2) -
1i)^4)/((x + 1)^(1/2) - 1)^4 - (8*((x - 1)^(1/2) - 1i)^2)/((x + 1)^(1/2) -
 1)^2 - (56*((x - 1)^(1/2) - 1i)^6)/((x + 1)^(1/2) - 1)^6 + (70*((x - 1)^(
1/2) - 1i)^8)/((x + 1)^(1/2) - 1)^8 - (56*((x - 1)^(1/2) - 1i)^10)/((x + 1
)^(1/2) - 1)^10 + (28*((x - 1)^(1/2) - 1i)^12)/((x + 1)^(1/2) - 1)^12 - (8
*((x - 1)^(1/2) - 1i)^14)/((x + 1)^(1/2) - 1)^14 + ((x - 1)^(1/2) - 1i)^16
/((x + 1)^(1/2) - 1)^16 + 1)
```

3.735. $\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx$

3.736 $\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx$

3.736.1 Optimal result

Integrand size = 17, antiderivative size = 69

$$\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx=-\frac{3}{8}\sqrt{-1+x}\sqrt{1+x}+\frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x}+\frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}+\frac{3\text{arccosh}(x)}{8}$$

output
```
3/8*arccosh(x)+1/24*(7-2*x)*(-1+x)^(3/2)*(1+x)^(1/2)+1/4*(-1+x)^(3/2)*x^2*
(1+x)^(1/2)-3/8*(-1+x)^(1/2)*(1+x)^(1/2)
```

3.736.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx=\frac{\sqrt{\frac{-1+x}{1+x}}\left(\sqrt{-1+x}(-16-7x+x^2-2x^3+6x^4)-18\sqrt{1+x}\log\left(\sqrt{-1+x}-\sqrt{1+x}\right)\right)}{24\sqrt{-1+x}}$$

input
```
Integrate[x^3*Sqrt[(-1 + x)/(1 + x)],x]
```

output
```
(Sqrt[(-1 + x)/(1 + x)]*(Sqrt[-1 + x]*(-16 - 7*x + x^2 - 2*x^3 + 6*x^4) -
18*Sqrt[1 + x]*Log[Sqrt[-1 + x] - Sqrt[1 + x]]))/(24*Sqrt[-1 + x])
```

3.736.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2050, 111, 164, 60, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{\frac{x-1}{x+1}}dx$$

$$\Big\downarrow \text{2050}$$

$$\int \frac{\sqrt{x-1}x^3}{\sqrt{x+1}}dx$$

$$\Big\downarrow \text{111}$$

$$\frac{1}{4}\int \frac{(2-x)\sqrt{x-1}x}{\sqrt{x+1}}dx+\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2$$

$$\Big\downarrow \text{164}$$

$$\frac{1}{4}\left(\frac{1}{6}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{2}\int \frac{\sqrt{x-1}}{\sqrt{x+1}}dx\right)+\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(\frac{1}{6}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{2}\left(\sqrt{x-1}\sqrt{x+1}-\int \frac{1}{\sqrt{x-1}\sqrt{x+1}}dx\right)\right)+\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2$$

$$\Big\downarrow \text{43}$$

$$\frac{1}{4}\left(\frac{1}{6}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{2}\left(\sqrt{x-1}\sqrt{x+1}-\text{arccosh}(x)\right)\right)+\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2$$

input
```
Int[x^3*Sqrt[(-1 + x)/(1 + x)],x]
```

output
```
((-1 + x)^(3/2)*x^2*Sqrt[1 + x])/4 + (((7 - 2*x)*(-1 + x)^(3/2)*Sqrt[1 + x
])/6 - (3*(Sqrt[-1 + x]*Sqrt[1 + x] - ArcCosh[x]))/2)/4
```

3.736.3.1 Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 2050
```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

3.736. $\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx$

3.736.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

method	result	size
risch	$\frac{(6x^3-8x^2+9x-16)(x+1)\sqrt{\frac{x-1}{x+1}}}{24}+\frac{3\ln\left(x+\sqrt{x^2-1}\right)\sqrt{\frac{x-1}{x+1}}\,\sqrt{(x-1)(x+1)}}{8(x-1)}$	70
trager	$\frac{(x+1)(6x^3-8x^2+9x-16)\sqrt{-\frac{1-x}{x+1}}}{24}-\frac{3\ln\left(-\sqrt{-\frac{1-x}{x+1}}\,x-\sqrt{-\frac{1-x}{x+1}}+x\right)}{8}$	74
default	$\frac{\sqrt{\frac{x-1}{x+1}}\,(x+1)\left(6x(x^2-1)^{\frac{3}{2}}-8((x-1)(x+1))^{\frac{3}{2}}+15x\sqrt{x^2-1}-24\sqrt{x^2-1}+9\ln\left(x+\sqrt{x^2-1}\right)\right)}{24\sqrt{(x-1)(x+1)}}$	79

input
```
int(x^3*((x-1)/(x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*x^3-8*x^2+9*x-16)*(x+1)*((x-1)/(x+1))^(1/2)+3/8*ln(x+(x^2-1)^(1/2)
)*((x-1)/(x+1))^(1/2)*((x-1)*(x+1))^(1/2)/(x-1)
```

3.736.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx=\frac{1}{24}\left(6\,x^4-2\,x^3+x^2-7\,x-16\right)\sqrt{\frac{x-1}{x+1}}+\frac{3}{8}\,\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{3}{8}\,\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(x^3*((-1+x)/(1+x))^(1/2),x, algorithm="fricas")
```

output
```
1/24*(6*x^4 - 2*x^3 + x^2 - 7*x - 16)*sqrt((x - 1)/(x + 1)) + 3/8*log(sqrt
((x - 1)/(x + 1)) + 1) - 3/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

3.736.6 Sympy [F]

$$\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx=\int x^3\sqrt{\frac{x-1}{x+1}}\,dx$$

input
```
integrate(x**3*((-1+x)/(1+x))**(1/2),x)
```

output
```
Integral(x**3*sqrt((x - 1)/(x + 1)), x)
```

3.736.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(49) = 98.

Time = 0.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.00

$$\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx=-\frac{39\left(\frac{x-1}{x+1}\right)^{\frac{7}{2}}-31\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}}+49\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}}-9\sqrt{\frac{x-1}{x+1}}}{12\left(\frac{4(x-1)}{x+1}-\frac{6(x-1)^2}{(x+1)^2}+\frac{4(x-1)^3}{(x+1)^3}-\frac{(x-1)^4}{(x+1)^4}-1\right)}+\frac{3}{8}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{3}{8}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(x^3*((-1+x)/(1+x))^(1/2),x, algorithm="maxima")
```

output
```
-1/12*(39*((x - 1)/(x + 1))^(7/2) - 31*((x - 1)/(x + 1))^(5/2) + 49*((x -
1)/(x + 1))^(3/2) - 9*sqrt((x - 1)/(x + 1)))/(4*(x - 1)/(x + 1) - 6*(x - 1
)^2/(x + 1)^2 + 4*(x - 1)^3/(x + 1)^3 - (x - 1)^4/(x + 1)^4 - 1) + 3/8*log
(sqrt((x - 1)/(x + 1)) + 1) - 3/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

3.736.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

$$\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx=-\frac{3}{8}\log\left(\left|-x+\sqrt{x^2-1}\right|\right)\operatorname{sgn}(x+1)+\frac{1}{24}\left((2\left(3\,x\operatorname{sgn}(x+1)-4\operatorname{sgn}(x+1)\right)x+9\operatorname{sgn}(x+1))x-16\operatorname{sgn}(x+1)\right)\sqrt{x^2-1}$$

input
```
integrate(x^3*((-1+x)/(1+x))^(1/2),x, algorithm="giac")
```

output
```
-3/8*log(abs(-x + sqrt(x^2 - 1)))*sgn(x + 1) + 1/24*((2*(3*x*sgn(x + 1) -
4*sgn(x + 1))*x + 9*sgn(x + 1))*x - 16*sgn(x + 1))*sqrt(x^2 - 1)
```

3.736.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.72

$$\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx = \frac{3\,\mathrm{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)}{4} - \frac{\frac{3\sqrt{\frac{x-1}{x+1}}}{4} - \frac{49\left(\frac{x-1}{x+1}\right)^{3/2}}{12} + \frac{31\left(\frac{x-1}{x+1}\right)^{5/2}}{12} - \frac{13\left(\frac{x-1}{x+1}\right)^{7/2}}{4}}{\frac{6\,(x-1)^2}{(x+1)^2} - \frac{4\,(x-1)}{x+1} - \frac{4\,(x-1)^3}{(x+1)^3} + \frac{(x-1)^4}{(x+1)^4} + 1}$$

input
```
int(x^3*((x - 1)/(x + 1))^(1/2),x)
```

output
```
(3*atanh(((x - 1)/(x + 1))^(1/2)))/4 - ((3*((x - 1)/(x + 1))^(1/2))/4 - (4
9*((x - 1)/(x + 1))^(3/2))/12 + (31*((x - 1)/(x + 1))^(5/2))/12 - (13*((x
- 1)/(x + 1))^(7/2))/4)/((6*(x - 1)^2)/(x + 1)^2 - (4*(x - 1))/(x + 1) - (
4*(x - 1)^3)/(x + 1)^3 + (x - 1)^4/(x + 1)^4 + 1)
```

3.737 $\int \frac{\sqrt{-\frac{x}{1+x}}}{x}\,dx$

3.737.1 Optimal result

Integrand size = 16, antiderivative size = 15

$$\int \frac{\sqrt{-\frac{x}{1+x}}}{x}\,dx = 2\arctan\left(\sqrt{-\frac{x}{1+x}}\right)$$

output
```
2*arctan((-x/(1+x))^(1/2))
```

3.737.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(15) = 30.

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.80

$$\int \frac{\sqrt{-\frac{x}{1+x}}}{x}\,dx = -\frac{2\sqrt{-\frac{x}{1+x}}\sqrt{1+x}\log\left(-\sqrt{x}+\sqrt{1+x}\right)}{\sqrt{x}}$$

input
```
Integrate[Sqrt[-(x/(1 + x))]/x,x]
```

output
```
(-2*Sqrt[-(x/(1 + x))]*Sqrt[1 + x]*Log[-Sqrt[x] + Sqrt[1 + x]])/Sqrt[x]
```

3.737.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2052, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-\frac{x}{x+1}}}{x} \, dx$$

$$\Big\downarrow \text{ 2052}$$

$$-2 \int \frac{1}{\frac{x}{x+1}-1} d\sqrt{-\frac{x}{x+1}}$$

$$\Big\downarrow \text{ 217}$$

$$2 \arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

input
```
Int[Sqrt[-(x/(1 + x))]/x,x]
```

output
```
2*ArcTan[Sqrt[-(x/(1 + x))]]
```

3.737.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

3.737.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(13) = 26.

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.20

method	result
default	$\frac{\sqrt{-\frac{x}{x+1}}\,(x+1)\ln\left(x+\frac{1}{2}+\sqrt{x^{2}+x}\right)}{\sqrt{(x+1)x}}$
trager	$-\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(2\sqrt{-\frac{x}{x+1}}\,x-2\operatorname{RootOf}\left(\_Z^{2}+1\right)x+2\sqrt{-\frac{x}{x+1}}-\operatorname{RootOf}\left(\_Z^{2}+1\right)\right)$

input
```
int((-x/(x+1))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(-x/(x+1))^(1/2)*(x+1)/((x+1)*x)^(1/2)*ln(x+1/2+(x^2+x)^(1/2))
```

3.737.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{-\frac{x}{1+x}}}{x}\,dx = 2\arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

input
```
integrate((-x/(1+x))^(1/2)/x,x, algorithm="fricas")
```

output
```
2*arctan(sqrt(-x/(x + 1)))
```

3.737.6 Sympy [F]

$$\int \frac{\sqrt{-\frac{x}{1+x}}}{x}\,dx = \int \frac{\sqrt{-\frac{x}{x+1}}}{x}\,dx$$

input
```
integrate((-x/(1+x))**(1/2)/x,x)
```

output
```
Integral(sqrt(-x/(x + 1))/x, x)
```

3.737.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{-\frac{x}{1+x}}}{x}\,dx = 2\arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

input
```
integrate((-x/(1+x))^(1/2)/x,x, algorithm="maxima")
```

output
```
2*arctan(sqrt(-x/(x + 1)))
```

3.737.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{-\frac{x}{1+x}}}{x}\,dx = -\frac{1}{2}\pi\mathrm{sgn}(x+1) - \arcsin\left(2\,x+1\right)\mathrm{sgn}(x+1)$$

input
```
integrate((-x/(1+x))^(1/2)/x,x, algorithm="giac")
```

output
```
-1/2*pi*sgn(x + 1) - arcsin(2*x + 1)*sgn(x + 1)
```

3.737.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{-\frac{x}{1+x}}}{x}\,dx = 2\,\mathrm{atan}\left(\sqrt{-\frac{x}{x+1}}\right)$$

input
```
int((-x/(x + 1))^(1/2)/x,x)
```

output
```
2*atan((-x/(x + 1))^(1/2))
```

3.738 $\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x} dx$

3.738.1 Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x} dx = 2\arctan\left(\sqrt{\frac{1-x}{1+x}}\right)$$

output
```
2*arctan(((1-x)/(1+x))^(1/2))
```

3.738.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(18) = 36.

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.83

$$\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x} dx = -\frac{2\sqrt{\frac{1-x}{1+x}}\sqrt{1-x^2}\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)}{-1+x}$$

input
```
Integrate[Sqrt[(1 - x)/(1 + x)]/(-1 + x),x]
```

output
```
(-2*Sqrt[(1 - x)/(1 + x)]*Sqrt[1 - x^2]*ArcTan[Sqrt[1 - x^2]/(-1 + x)])/(-
1 + x)
```

3.738.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2055, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{1-x}{x+1}}}{x-1} \, dx$$

$$\Big\downarrow \text{2055}$$

$$-4 \int \frac{1}{-\frac{2(1-x)}{x+1}-2} d\sqrt{\frac{1-x}{x+1}}$$

$$\Big\downarrow \text{217}$$

$$2 \arctan\left(\sqrt{\frac{1-x}{x+1}}\right)$$

input
```
Int[Sqrt[(1 - x)/(1 + x)]/(-1 + x),x]
```

output
```
2*ArcTan[Sqrt[(1 - x)/(1 + x)]]
```

3.738.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 2055
```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)
 Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/
(b*e - d*x^q)^(1/n + 1))*(u /. x -> ((-a)*e + c*x^q)^(1/n)/(b*e - d*x^q)^(1
/n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Int
egerQ[r]
```

3.738.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

method	result	size
default	$-\frac{\sqrt{-\frac{x-1}{x+1}}\,(x+1)\arcsin(x)}{\sqrt{-(x-1)(x+1)}}$	30
trager	$\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{x-1}{x+1}}\,x-\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{x-1}{x+1}}+x\right)$	52

input
```
int(((1-x)/(x+1))^(1/2)/(x-1),x,method=_RETURNVERBOSE)
```

output
```
-(-(x-1)/(x+1))^(1/2)*(x+1)/(-(x-1)*(x+1))^(1/2)*arcsin(x)
```

3.738.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx = 2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

input
```
integrate(((1-x)/(1+x))^(1/2)/(-1+x),x, algorithm="fricas")
```

output
```
2*arctan(sqrt(-(x - 1)/(x + 1)))
```

3.738.6 Sympy [F]

$$\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx = \int \frac{\sqrt{-\frac{x-1}{x+1}}}{x-1}\,dx$$

input
```
integrate(((1-x)/(1+x))**(1/2)/(-1+x),x)
```

output
```
Integral(sqrt(-(x - 1)/(x + 1))/(x - 1), x)
```

3.738.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx = 2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

input
```
integrate(((1-x)/(1+x))^(1/2)/(-1+x),x, algorithm="maxima")
```

output
```
2*arctan(sqrt(-(x - 1)/(x + 1)))
```

3.738.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx = -\frac{1}{2}\,\pi\mathrm{sgn}(x+1) - \arcsin\left(x\right)\mathrm{sgn}(x+1)$$

input
```
integrate(((1-x)/(1+x))^(1/2)/(-1+x),x, algorithm="giac")
```

output
```
-1/2*pi*sgn(x + 1) - arcsin(x)*sgn(x + 1)
```

3.738.9 Mupad [B] (verification not implemented)

Time = 17.87 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx = 2\,\mathrm{atan}\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

input
```
int((-(x - 1)/(x + 1))^(1/2)/(x - 1),x)
```

output
```
2*atan((-(x - 1)/(x + 1))^(1/2))
```

3.739 $\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx$

3.739.1 Optimal result

Integrand size = 26, antiderivative size = 24

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx = \frac{2\arctan\left(\sqrt{\frac{a+bx}{c-bx}}\right)}{b}$$

output
```
2*arctan(((b*x+a)/(-b*x+c))^(1/2))/b
```

3.739.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 63 vs. 2(24) = 48.

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx = \frac{2\sqrt{c-bx}\sqrt{\frac{a+bx}{c-bx}}\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{c-bx}}\right)}{b\sqrt{a+bx}}$$

input
```
Integrate[Sqrt[(a + b*x)/(c - b*x)]/(a + b*x),x]
```

output
```
(2*Sqrt[c - b*x]*Sqrt[(a + b*x)/(c - b*x)]*ArcTan[Sqrt[a + b*x]/Sqrt[c - b
*x]])/(b*Sqrt[a + b*x])
```

3.739.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2055, 27, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx} \, dx$$

$$\Big\downarrow \; 2055$$

$$2b(a+c) \int \frac{1}{b^2(a+c)\left(\frac{a+bx}{c-bx}+1\right)}d\sqrt{\frac{a+bx}{c-bx}}$$

$$\Big\downarrow \; 27$$

$$\frac{2\int \frac{1}{\frac{a+bx}{c-bx}+1}d\sqrt{\frac{a+bx}{c-bx}}}{b}$$

$$\Big\downarrow \; 216$$

$$\frac{2\arctan\left(\sqrt{\frac{a+bx}{c-bx}}\right)}{b}$$

input
```
Int[Sqrt[(a + b*x)/(c - b*x)]/(a + b*x),x]
```

output
```
(2*ArcTan[Sqrt[(a + b*x)/(c - b*x)]])/b
```

3.739.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2055
```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)
 Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/
(b*e - d*x^q)^(1/n + 1))*(u /. x -> ((-a)*e + c*x^q)^(1/n)/(b*e - d*x^q)^(1
/n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Int
egerQ[r]
```

3.739.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(22) = 44$.

Time = 1.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.54

method	result	size
default	$-\frac{\arctan\left(\frac{\sqrt{b^2}\,(2bx+a-c)}{2b\sqrt{-(bx+a)(bx-c)}}\right)(bx-c)\sqrt{-\frac{bx+a}{bx-c}}}{\sqrt{b^2}\,\sqrt{-(bx+a)(bx-c)}}$	85

input
```
int(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-arctan(1/2*(b^2)^(1/2)/b*(2*b*x+a-c)/(-(b*x+a)*(b*x-c))^(1/2))*(b*x-c)*(-
(b*x+a)/(b*x-c))^(1/2)/(b^2)^(1/2)/(-(b*x+a)*(b*x-c))^(1/2)
```

3.739.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx = \frac{2\arctan\left(\sqrt{-\frac{bx+a}{bx-c}}\right)}{b}$$

input
```
integrate(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x, algorithm="fricas")
```

output
```
2*arctan(sqrt(-(b*x + a)/(b*x - c)))/b
```

3.739.6 Sympy [F]

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx = \int \frac{\sqrt{\frac{a+bx}{-bx+c}}}{a+bx}\,dx$$

input
```
integrate(((b*x+a)/(-b*x+c))**(1/2)/(b*x+a),x)
```

output
```
Integral(sqrt((a + b*x)/(-b*x + c))/(a + b*x), x)
```

3.739.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx = \frac{2\,\arctan\left(\sqrt{-\frac{bx+a}{bx-c}}\right)}{b}$$

input
```
integrate(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x, algorithm="maxima")
```

output
```
2*arctan(sqrt(-(b*x + a)/(b*x - c)))/b
```

3.739.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx = -\frac{\arcsin\left(-\frac{2\,bx+a-c}{a+c}\right)\operatorname{sgn}(-ab-bc)\operatorname{sgn}(bx-c)}{|b|}$$

input
```
integrate(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x, algorithm="giac")
```

output
```
-arcsin(-(2*b*x + a - c)/(a + c))*sgn(-a*b - b*c)*sgn(b*x - c)/abs(b)
```

3.739.9 Mupad [B] (verification not implemented)

Time = 18.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx = -\frac{2\sqrt{-b}\,\mathrm{atanh}\left(\frac{\sqrt{-b}\,\sqrt{\frac{a+b\,x}{c-b\,x}}}{\sqrt{b}}\right)}{b^{3/2}}$$

input
```
int(((a + b*x)/(c - b*x))^(1/2)/(a + b*x),x)
```

output
```
-(2*(-b)^(1/2)*atanh(((-b)^(1/2)*((a + b*x)/(c - b*x))^(1/2))/b^(1/2)))/b^
(3/2)
```

3.740 $\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx} dx$

3.740.1 Optimal result

Integrand size = 25, antiderivative size = 41

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx} dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

output
```
2*arctanh(d^(1/2)*((b*x+a)/(d*x+c))^(1/2)/b^(1/2))/b^(1/2)/d^(1/2)
```

3.740.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.88

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx} dx = \frac{2\sqrt{\frac{a+bx}{c+dx}}\sqrt{c+dx}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{\sqrt{b}\sqrt{d}\sqrt{a+bx}}$$

input
```
Integrate[Sqrt[(a + b*x)/(c + d*x)]/(a + b*x),x]
```

output
```
(2*Sqrt[(a + b*x)/(c + d*x)]*Sqrt[c + d*x]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[a + b*x])])/(Sqrt[b]*Sqrt[d]*Sqrt[a + b*x])
```

3.740.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2055, 27, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx} \, dx$$

$$\Big\downarrow \; 2055$$

$$2(bc-ad)\int \frac{1}{(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\sqrt{\frac{a+bx}{c+dx}}$$

$$\Big\downarrow \; 27$$

$$2\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\sqrt{\frac{a+bx}{c+dx}}$$

$$\Big\downarrow \; 221$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

input
```
Int[Sqrt[(a + b*x)/(c + d*x)]/(a + b*x),x]
```

output
```
(2*ArcTanh[(Sqrt[d]*Sqrt[(a + b*x)/(c + d*x)])/Sqrt[b]])/(Sqrt[b]*Sqrt[d])
```

3.740.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2055
```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)
 Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/
(b*e - d*x^q)^(1/n + 1))*(u /. x -> ((-a)*e + c*x^q)^(1/n)/(b*e - d*x^q)^(1
/n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Int
egerQ[r]
```

3.740.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(31) = 62$.

Time = 1.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.95

method	result	size
default	$\frac{\ln\left(\frac{2bdx+2\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)(dx+c)\sqrt{\frac{bx+a}{dx+c}}}{\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}}$	80

input
```
int(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2)
)*(d*x+c)*((b*x+a)/(d*x+c))^(1/2)/((b*x+a)*(d*x+c))^(1/2)/(b*d)^(1/2)
```

3.740.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.56

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx = \left[\frac{\sqrt{bd}\log\left(2\,bdx+bc+ad+2\,\sqrt{bd}(dx+c)\sqrt{\frac{bx+a}{dx+c}}\right)}{bd},\ -\frac{2\,\sqrt{-bd}\arctan\left(\frac{\sqrt{-bd}(dx+c)\sqrt{\frac{bx+a}{dx+c}}}{bdx+ad}\right)}{bd}\right]$$

input
```
integrate(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x, algorithm="fricas")
```

output
```
[sqrt(b*d)*log(2*b*d*x + b*c + a*d + 2*sqrt(b*d)*(d*x + c)*sqrt((b*x + a)/
(d*x + c)))/(b*d), -2*sqrt(-b*d)*arctan(sqrt(-b*d)*(d*x + c)*sqrt((b*x + a
)/(d*x + c))/(b*d*x + a*d))/(b*d)]
```

3.740.6 Sympy [F]

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx = \int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx$$

input
```
integrate(((b*x+a)/(d*x+c))**(1/2)/(b*x+a),x)
```

output
```
Integral(sqrt((a + b*x)/(c + d*x))/(a + b*x), x)
```

3.740.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx = -\frac{\log\left(\frac{d\sqrt{\frac{bx+a}{dx+c}}-\sqrt{bd}}{d\sqrt{\frac{bx+a}{dx+c}}+\sqrt{bd}}\right)}{\sqrt{bd}}$$

input
```
integrate(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x, algorithm="maxima")
```

output
```
-log((d*sqrt((b*x + a)/(d*x + c)) - sqrt(b*d))/(d*sqrt((b*x + a)/(d*x + c)
) + sqrt(b*d)))/sqrt(b*d)
```

3.740.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx = -\frac{\log\left(\left|-bc-ad-2\sqrt{bd}\left(\sqrt{bd}x-\sqrt{bdx^2+bcx+adx+ac}\right)\right|\right)\operatorname{sgn}(dx+c)}{\sqrt{bd}}$$

input
```
integrate(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x, algorithm="giac")
```

output
```
-log(abs(-b*c - a*d - 2*sqrt(b*d)*(sqrt(b*d)*x - sqrt(b*d*x^2 + b*c*x + a*
d*x + a*c))))*sgn(d*x + c)/sqrt(b*d)
```

3.740.9 Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\sqrt{d}\,\sqrt{\frac{a+b\,x}{c+d\,x}}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{d}}$$

input
```
int(((a + b*x)/(c + d*x))^(1/2)/(a + b*x),x)
```

output
```
(2*atanh((d^(1/2)*((a + b*x)/(c + d*x))^(1/2))/b^(1/2)))/(b^(1/2)*d^(1/2))
```

3.741 $\int \sqrt{-\frac{x}{1+x}}\,dx$

3.741.1 Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \sqrt{-\frac{x}{1+x}}\,dx = \sqrt{-\frac{x}{1+x}}(1+x) - \arctan\left(\sqrt{-\frac{x}{1+x}}\right)$$

output
```
-arctan((-x/(1+x))^(1/2))+(1+x)*(-x/(1+x))^(1/2)
```

3.741.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

$$\int \sqrt{-\frac{x}{1+x}}\,dx = \frac{\sqrt{-\frac{x}{1+x}}\left(\sqrt{x}(1+x)+\sqrt{1+x}\log\left(-\sqrt{x}+\sqrt{1+x}\right)\right)}{\sqrt{x}}$$

input
```
Integrate[Sqrt[-(x/(1 + x))],x]
```

output
```
(Sqrt[-(x/(1 + x))]*(Sqrt[x]*(1 + x) + Sqrt[1 + x]*Log[-Sqrt[x] + Sqrt[1 +
 x]]))/Sqrt[x]
```

3.741.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.50, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2051, 252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-\frac{x}{x+1}}dx$$

$$\Big\downarrow \ 2051$$

$$-2\int -\frac{x}{(x+1)\left(1-\frac{x}{x+1}\right)^2}d\sqrt{-\frac{x}{x+1}}$$

$$\Big\downarrow \ 252$$

$$-2\left(\frac{1}{2}\int \frac{1}{1-\frac{x}{x+1}}d\sqrt{-\frac{x}{x+1}}-\frac{\sqrt{-\frac{x}{x+1}}}{2\left(1-\frac{x}{x+1}\right)}\right)$$

$$\Big\downarrow \ 216$$

$$-2\left(\frac{1}{2}\arctan\left(\sqrt{-\frac{x}{x+1}}\right)-\frac{\sqrt{-\frac{x}{x+1}}}{2\left(1-\frac{x}{x+1}\right)}\right)$$

input
```
Int[Sqrt[-(x/(1 + x))],x]
```

output
```
-2*(-1/2*Sqrt[-(x/(1 + x))]/(1 - x/(1 + x)) + ArcTan[Sqrt[-(x/(1 + x))]]/2
)
```

3.741.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2051
```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)   Subst[Int[
x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
 x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x]
&& FractionQ[p] && IntegerQ[1/n]
```

3.741.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.41

method	result	size
risch	$(x+1)\sqrt{-\frac{x}{x+1}}-\frac{\arcsin(1+2x)\sqrt{-\frac{x}{x+1}}\,\sqrt{-(x+1)x}}{2x}$	45
default	$\frac{\sqrt{-\frac{x}{x+1}}\,(x+1)\left(2\sqrt{x^2+x}-\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)\right)}{2\sqrt{(x+1)x}}$	46
trager	$2\left(\frac{x}{2}+\frac{1}{2}\right)\sqrt{-\frac{x}{x+1}}+\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(2\sqrt{-\frac{x}{x+1}}\,x-2\,\mathrm{RootOf}\left(\_Z^2+1\right)x+2\sqrt{-\frac{x}{x+1}}-\mathrm{RootOf}\left(\_Z^2+1\right)\right)}{2}$	71

input
```
int((-x/(x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x+1)*(-x/(x+1))^(1/2)-1/2*arcsin(1+2*x)*(-x/(x+1))^(1/2)/x*(-(x+1)*x)^(1/
2)
```

3.741.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int\sqrt{-\frac{x}{1+x}}\,dx=(x+1)\sqrt{-\frac{x}{x+1}}-\arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

input
```
integrate((-x/(1+x))^(1/2),x, algorithm="fricas")
```

output
```
(x + 1)*sqrt(-x/(x + 1)) - arctan(sqrt(-x/(x + 1)))
```

3.741.6 Sympy [F]

$$\int \sqrt{-\frac{x}{1+x}}\,dx = \int \sqrt{-\frac{x}{x+1}}\,dx$$

input
```
integrate((-x/(1+x))**(1/2),x)
```

output
```
Integral(sqrt(-x/(x + 1)), x)
```

3.741.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \sqrt{-\frac{x}{1+x}}\,dx = -\frac{\sqrt{-\frac{x}{x+1}}}{\frac{x}{x+1}-1} - \arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

input
```
integrate((-x/(1+x))^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(-x/(x + 1))/(x/(x + 1) - 1) - arctan(sqrt(-x/(x + 1)))
```

3.741.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \sqrt{-\frac{x}{1+x}}\,dx = \frac{1}{4}\,\pi\mathrm{sgn}(x+1) + \frac{1}{2}\,\arcsin\left(2\,x+1\right)\mathrm{sgn}(x+1) + \sqrt{-x^2-x}\mathrm{sgn}(x+1)$$

input
```
integrate((-x/(1+x))^(1/2),x, algorithm="giac")
```

output
```
1/4*pi*sgn(x + 1) + 1/2*arcsin(2*x + 1)*sgn(x + 1) + sqrt(-x^2 - x)*sgn(x
+ 1)
```

3.741.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \sqrt{-\frac{x}{1+x}}\,dx = -\mathrm{atan}\left(\sqrt{-\frac{x}{x+1}}\right) - \frac{\sqrt{-\frac{x}{x+1}}}{\frac{x}{x+1}-1}$$

input
```
int((-x/(x + 1))^(1/2),x)
```

output
```
- atan((-x/(x + 1))^(1/2)) - (-x/(x + 1))^(1/2)/(x/(x + 1) - 1)
```

3.742 $\int \sqrt{\frac{1-x}{1+x}}\,dx$

3.742.1 Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \sqrt{\frac{1-x}{1+x}}\,dx=\sqrt{\frac{1-x}{1+x}}(1+x)-2\arctan\left(\sqrt{\frac{1-x}{1+x}}\right)$$

output
```
-2*arctan(((1-x)/(1+x))^(1/2))+(1+x)*((1-x)/(1+x))^(1/2)
```

3.742.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.68

$$\int \sqrt{\frac{1-x}{1+x}}\,dx=\frac{\sqrt{\frac{1-x}{1+x}}\sqrt{1+x}\left(\sqrt{1-x^2}-2\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)\right)}{\sqrt{1-x}}$$

input
```
Integrate[Sqrt[(1 - x)/(1 + x)],x]
```

output
```
(Sqrt[(1 - x)/(1 + x)]*Sqrt[1 + x]*(Sqrt[1 - x^2] - 2*ArcTan[Sqrt[1 - x^2]
/(-1 + x)]))/Sqrt[1 - x]
```

3.742.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.50, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2051, 252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{1-x}{x+1}}dx$$
$$\Big\downarrow \text{ 2051}$$
$$-4\int \frac{1-x}{(x+1)\left(\frac{1-x}{x+1}+1\right)^2}d\sqrt{\frac{1-x}{x+1}}$$
$$\Big\downarrow \text{ 252}$$
$$-4\left(\frac{1}{2}\int \frac{1}{\frac{1-x}{x+1}+1}d\sqrt{\frac{1-x}{x+1}}-\frac{\sqrt{\frac{1-x}{x+1}}}{2\left(\frac{1-x}{x+1}+1\right)}\right)$$
$$\Big\downarrow \text{ 216}$$
$$-4\left(\frac{1}{2}\arctan\left(\sqrt{\frac{1-x}{x+1}}\right)-\frac{\sqrt{\frac{1-x}{x+1}}}{2\left(\frac{1-x}{x+1}+1\right)}\right)$$

input
```
Int[Sqrt[(1 - x)/(1 + x)],x]
```

output
```
-4*(-1/2*Sqrt[(1 - x)/(1 + x)]/(1 + (1 - x)/(1 + x)) + ArcTan[Sqrt[(1 - x)
/(1 + x)]]/2)
```

3.742.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2051
```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)   Subst[Int[
x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
 x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x]
&& FractionQ[p] && IntegerQ[1/n]
```

3.742.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

method	result
default	$\frac{\sqrt{-\frac{x-1}{x+1}}\,(x+1)\left(\sqrt{-x^{2}+1}+\arcsin(x)\right)}{\sqrt{-(x-1)(x+1)}}$
risch	$\sqrt{-\frac{x-1}{x+1}}\,(x+1)-\frac{\arcsin(x)\sqrt{-\frac{x-1}{x+1}}\,\sqrt{-(x-1)(x+1)}}{x-1}$
trager	$\sqrt{-\frac{x-1}{x+1}}\,(x+1)+\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-\frac{x-1}{x+1}}\,x+\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-\right.$

input
```
int(((1-x)/(x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(x-1)/(x+1))^(1/2)*(x+1)/(-(x-1)*(x+1))^(1/2)*((-x^2+1)^(1/2)+arcsin(x))
```

3.742.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \sqrt{\frac{1-x}{1+x}}\,dx=(x+1)\sqrt{-\frac{x-1}{x+1}}-2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

input
```
integrate(((1-x)/(1+x))^(1/2),x, algorithm="fricas")
```

output
```
(x + 1)*sqrt(-(x - 1)/(x + 1)) - 2*arctan(sqrt(-(x - 1)/(x + 1)))
```

3.742.6 Sympy [F]

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = \int \sqrt{\frac{1-x}{x+1}}\,dx$$

input
```
integrate(((1-x)/(1+x))**(1/2),x)
```

output
```
Integral(sqrt((1 - x)/(x + 1)), x)
```

3.742.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = -\frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1} - 2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

input
```
integrate(((1-x)/(1+x))^(1/2),x, algorithm="maxima")
```

output
```
-2*sqrt(-(x - 1)/(x + 1))/((x - 1)/(x + 1) - 1) - 2*arctan(sqrt(-(x - 1)/(
x + 1)))
```

3.742.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = \frac{1}{2}\pi\mathrm{sgn}(x+1) + \arcsin(x)\,\mathrm{sgn}(x+1) + \sqrt{-x^2+1}\mathrm{sgn}(x+1)$$

input
```
integrate(((1-x)/(1+x))^(1/2),x, algorithm="giac")
```

output
```
1/2*pi*sgn(x + 1) + arcsin(x)*sgn(x + 1) + sqrt(-x^2 + 1)*sgn(x + 1)
```

3.742.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = -2\,\mathrm{atan}\left(\sqrt{-\frac{x-1}{x+1}}\right) - \frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

input
```
int((-(x - 1)/(x + 1))^(1/2),x)
```

output
```
- 2*atan((-(x - 1)/(x + 1))^(1/2)) - (2*(-(x - 1)/(x + 1))^(1/2))/((x - 1)
/(x + 1) - 1)
```

3.743 $\int \sqrt{\frac{a+x}{a-x}}\,dx$

3.743.1 Optimal result

Integrand size = 15, antiderivative size = 42

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = -\left((a-x)\sqrt{\frac{a+x}{a-x}}\right) + 2a\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)$$

output
```
2*a*arctan(((a+x)/(a-x))^(1/2))-(a-x)*((a+x)/(a-x))^(1/2)
```

3.743.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.60

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = \frac{\sqrt{\frac{a+x}{a-x}}\left((-a+x)\sqrt{a+x} + 2a\sqrt{a-x}\arctan\left(\frac{\sqrt{a+x}}{\sqrt{a-x}}\right)\right)}{\sqrt{a+x}}$$

input
```
Integrate[Sqrt[(a + x)/(a - x)],x]
```

output
```
(Sqrt[(a + x)/(a - x)]*((-a + x)*Sqrt[a + x] + 2*a*Sqrt[a - x]*ArcTan[Sqrt
[a + x]/Sqrt[a - x]]))/Sqrt[a + x]
```

3.743.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2051, 252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{a+x}{a-x}}\,dx$$

$$\Big\downarrow \text{2051}$$

$$4a\int \frac{a+x}{(a-x)\left(\frac{a+x}{a-x}+1\right)^2}d\sqrt{\frac{a+x}{a-x}}$$

$$\Big\downarrow \text{252}$$

$$4a\left(\frac{1}{2}\int \frac{1}{\frac{a+x}{a-x}+1}d\sqrt{\frac{a+x}{a-x}}-\frac{\sqrt{\frac{a+x}{a-x}}}{2\left(\frac{a+x}{a-x}+1\right)}\right)$$

$$\Big\downarrow \text{216}$$

$$4a\left(\frac{1}{2}\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)-\frac{\sqrt{\frac{a+x}{a-x}}}{2\left(\frac{a+x}{a-x}+1\right)}\right)$$

input
```
Int[Sqrt[(a + x)/(a - x)],x]
```

output
```
4*a*(-1/2*Sqrt[(a + x)/(a - x)]/(1 + (a + x)/(a - x)) + ArcTan[Sqrt[(a + x
)/(a - x)]]/2)
```

3.743.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2051
```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)   Subst[Int[
x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
 x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x]
&& FractionQ[p] && IntegerQ[1/n]
```

3.743.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.45

method	result	size
default	$\frac{\sqrt{\frac{a+x}{a-x}}\,(a-x)\left(a\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)-\sqrt{a^2-x^2}\right)}{\sqrt{(a-x)(a+x)}}$	61
risch	$-\frac{(a-x)\sqrt{\frac{a+x}{a-x}}\,\sqrt{(a-x)(a+x)}}{\sqrt{-(-a+x)(a+x)}}+\frac{a\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)\sqrt{\frac{a+x}{a-x}}\,\sqrt{(a-x)(a+x)}}{a+x}$	90

input
```
int(((a+x)/(a-x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((a+x)/(a-x))^(1/2)*(a-x)*(a*arctan(x/(a^2-x^2)^(1/2))-(a^2-x^2)^(1/2))/((
a-x)*(a+x))^(1/2)
```

3.743.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int\sqrt{\frac{a+x}{a-x}}\,dx=2\,a\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

input
```
integrate(((a+x)/(a-x))^(1/2),x, algorithm="fricas")
```

output
```
2*a*arctan(sqrt((a + x)/(a - x))) - (a - x)*sqrt((a + x)/(a - x))
```

3.743.6 Sympy [F]

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = \int \sqrt{\frac{a+x}{a-x}}\,dx$$

input
```
integrate(((a+x)/(a-x))**(1/2),x)
```

output
```
Integral(sqrt((a + x)/(a - x)), x)
```

3.743.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = -2\,a\left(\frac{\sqrt{\frac{a+x}{a-x}}}{\frac{a+x}{a-x}+1} - \arctan\left(\sqrt{\frac{a+x}{a-x}}\right)\right)$$

input
```
integrate(((a+x)/(a-x))^(1/2),x, algorithm="maxima")
```

output
```
-2*a*(sqrt((a + x)/(a - x))/((a + x)/(a - x) + 1) - arctan(sqrt((a + x)/(a
 - x))))
```

3.743.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = a\arcsin\left(\frac{x}{a}\right)\operatorname{sgn}(a-x)\operatorname{sgn}(a) - \sqrt{a^2-x^2}\operatorname{sgn}(a-x)$$

input
```
integrate(((a+x)/(a-x))^(1/2),x, algorithm="giac")
```

output
```
a*arcsin(x/a)*sgn(a - x)*sgn(a) - sqrt(a^2 - x^2)*sgn(a - x)
```

3.743.9 Mupad [B] (verification not implemented)

Time = 17.92 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = 2\,a\,\mathrm{atan}\left(\sqrt{\frac{a+x}{a-x}}\right) - \frac{2\,a\,\sqrt{\frac{a+x}{a-x}}}{\frac{a+x}{a-x}+1}$$

input
```
int(((a + x)/(a - x))^(1/2),x)
```

output
```
2*a*atan(((a + x)/(a - x))^(1/2)) - (2*a*((a + x)/(a - x))^(1/2))/((a + x)
/(a - x) + 1)
```

3.744 $\int \sqrt{\frac{-a+x}{a+x}}\,dx$

3.744.1 Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \sqrt{\frac{-a+x}{a+x}}\,dx=\sqrt{-\frac{a-x}{a+x}}(a+x)-2a\text{arctanh}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

output
```
-2*a*arctanh(((-a+x)/(a+x))^(1/2))+(a+x)*((-a+x)/(a+x))^(1/2)
```

3.744.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.63

$$\int \sqrt{\frac{-a+x}{a+x}}\,dx=\frac{\sqrt{\frac{-a+x}{a+x}}\left(\sqrt{-a+x}(a+x)-2a\sqrt{a+x}\text{arctanh}\left(\frac{\sqrt{a+x}}{\sqrt{-a+x}}\right)\right)}{\sqrt{-a+x}}$$

input
```
Integrate[Sqrt[(-a + x)/(a + x)],x]
```

output
```
(Sqrt[(-a + x)/(a + x)]*(Sqrt[-a + x]*(a + x) - 2*a*Sqrt[a + x]*ArcTanh[Sq
rt[a + x]/Sqrt[-a + x]]))/Sqrt[-a + x]
```

3.744.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.46, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2051, 252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{x-a}{a+x}}dx$$

$$\Big\downarrow \text{2051}$$

$$4a\int -\frac{a-x}{(a+x)\left(\frac{a-x}{a+x}+1\right)^2}d\sqrt{-\frac{a-x}{a+x}}$$

$$\Big\downarrow \text{252}$$

$$4a\left(\frac{\sqrt{-\frac{a-x}{a+x}}}{2\left(\frac{a-x}{a+x}+1\right)}-\frac{1}{2}\int \frac{1}{\frac{a-x}{a+x}+1}d\sqrt{-\frac{a-x}{a+x}}\right)$$

$$\Big\downarrow \text{219}$$

$$4a\left(\frac{\sqrt{-\frac{a-x}{a+x}}}{2\left(\frac{a-x}{a+x}+1\right)}-\frac{1}{2}\text{arctanh}\left(\sqrt{-\frac{a-x}{a+x}}\right)\right)$$

input
```
Int[Sqrt[(-a + x)/(a + x)],x]
```

output
```
4*a*(Sqrt[-((a - x)/(a + x))]/(2*(1 + (a - x)/(a + x))) - ArcTanh[Sqrt[-((
a - x)/(a + x))]]/2)
```

3.744.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2051
```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)   Subst[Int[
x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
 x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x]
&& FractionQ[p] && IntegerQ[1/n]
```

3.744.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.51

method	result	size
default	$-\frac{\sqrt{-\frac{a-x}{a+x}}\,(a+x)\left(a\ln\left(x+\sqrt{-a^2+x^2}\right)-\sqrt{-a^2+x^2}\right)}{\sqrt{-(a-x)(a+x)}}$	62
risch	$\frac{(a+x)\sqrt{-\frac{a-x}{a+x}}\,\sqrt{-(a-x)(a+x)}}{\sqrt{(-a+x)(a+x)}}+\frac{a\ln\left(x+\sqrt{-a^2+x^2}\right)\sqrt{-\frac{a-x}{a+x}}\,\sqrt{-(a-x)(a+x)}}{a-x}$	92

input
```
int(((-a+x)/(a+x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(a-x)/(a+x))^(1/2)*(a+x)*(a*ln(x+(-a^2+x^2)^(1/2))-(-a^2+x^2)^(1/2))/(-
(a-x)*(a+x))^(1/2)
```

3.744.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

$$\int\sqrt{\frac{-a+x}{a+x}}\,dx=-a\log\left(\sqrt{-\frac{a-x}{a+x}}+1\right)+a\log\left(\sqrt{-\frac{a-x}{a+x}}-1\right)+(a+x)\sqrt{-\frac{a-x}{a+x}}$$

input
```
integrate(((-a+x)/(a+x))^(1/2),x, algorithm="fricas")
```

output
```
-a*log(sqrt(-(a - x)/(a + x)) + 1) + a*log(sqrt(-(a - x)/(a + x)) - 1) + (
a + x)*sqrt(-(a - x)/(a + x))
```

3.744.6 Sympy [F]

$$\int \sqrt{\frac{-a+x}{a+x}}\,dx = \int \sqrt{\frac{-a+x}{a+x}}\,dx$$

input
```
integrate(((-a+x)/(a+x))**(1/2),x)
```

output
```
Integral(sqrt((-a + x)/(a + x)), x)
```

3.744.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.71

$$\int \sqrt{\frac{-a+x}{a+x}}\,dx = a\left(\frac{2\sqrt{-\frac{a-x}{a+x}}}{\frac{a-x}{a+x}+1} - \log\left(\sqrt{-\frac{a-x}{a+x}}+1\right) + \log\left(\sqrt{-\frac{a-x}{a+x}}-1\right)\right)$$

input
```
integrate(((-a+x)/(a+x))^(1/2),x, algorithm="maxima")
```

output
```
a*(2*sqrt(-(a - x)/(a + x))/((a - x)/(a + x) + 1) - log(sqrt(-(a - x)/(a +
 x)) + 1) + log(sqrt(-(a - x)/(a + x)) - 1))
```

3.744.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \sqrt{\frac{-a+x}{a+x}}\,dx = a\log\left(\left|-x+\sqrt{-a^2+x^2}\right|\right)\operatorname{sgn}(a+x) + \sqrt{-a^2+x^2}\operatorname{sgn}(a+x)$$

input
```
integrate(((-a+x)/(a+x))^(1/2),x, algorithm="giac")
```

output
```
a*log(abs(-x + sqrt(-a^2 + x^2)))*sgn(a + x) + sqrt(-a^2 + x^2)*sgn(a + x)
```

3.744.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \sqrt{\frac{-a+x}{a+x}}\,dx = \frac{2\,a\,\sqrt{-\frac{a-x}{a+x}}}{\frac{a-x}{a+x}+1} - 2\,a\,\mathrm{atanh}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

input
```
int((-(a - x)/(a + x))^(1/2),x)
```

output
```
(2*a*(-(a - x)/(a + x))^(1/2))/((a - x)/(a + x) + 1) - 2*a*atanh((-(a - x)
/(a + x))^(1/2))
```

3.745 $\int \sqrt{\frac{a+bx}{c+dx}}\,dx$

3.745.1 Optimal result

Integrand size = 17, antiderivative size = 76

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx=\frac{\sqrt{\frac{a+bx}{c+dx}}(c+dx)}{d}-\frac{(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}d^{3/2}}$$

output
```
-(-a*d+b*c)*arctanh(d^(1/2)*((b*x+a)/(d*x+c))^(1/2)/b^(1/2))/d^(3/2)/b^(1/
2)+(d*x+c)*((b*x+a)/(d*x+c))^(1/2)/d
```

3.745.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.28

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx=\frac{\sqrt{\frac{a+bx}{c+dx}}\left(\sqrt{d}(c+dx)+\frac{(-bc+ad)\sqrt{c+dx}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{\sqrt{b}\sqrt{a+bx}}\right)}{d^{3/2}}$$

input
```
Integrate[Sqrt[(a + b*x)/(c + d*x)],x]
```

output
```
(Sqrt[(a + b*x)/(c + d*x)]*(Sqrt[d]*(c + d*x) + ((-(b*c) + a*d)*Sqrt[c + d
*x]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(Sqrt[b]*Sqr
t[a + b*x])))/d^(3/2)
```

3.745.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.28, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2051, 252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx$$

$$\Big\downarrow \text{ 2051}$$

$$2(bc-ad)\int \frac{a+bx}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\sqrt{\frac{a+bx}{c+dx}}$$

$$\Big\downarrow \text{ 252}$$

$$2(bc-ad)\left(\frac{\sqrt{\frac{a+bx}{c+dx}}}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\sqrt{\frac{a+bx}{c+dx}}}{2d}\right)$$

$$\Big\downarrow \text{ 221}$$

$$2(bc-ad)\left(\frac{\sqrt{\frac{a+bx}{c+dx}}}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{2\sqrt{b}d^{3/2}}\right)$$

input
```
Int[Sqrt[(a + b*x)/(c + d*x)],x]
```

output
```
2*(b*c - a*d)*(Sqrt[(a + b*x)/(c + d*x)]/(2*d*(b - (d*(a + b*x))/(c + d*x)
)) - ArcTanh[(Sqrt[d]*Sqrt[(a + b*x)/(c + d*x)])/Sqrt[b]]/(2*Sqrt[b]*d^(3/
2)))
```

3.745.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2051
```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)   Subst[Int[
x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
 x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x]
&& FractionQ[p] && IntegerQ[1/n]
```

3.745.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(64) = 128$.

Time = 0.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.00

method	result	s
default	$\frac{\sqrt{\frac{bx+a}{dx+c}}\,(dx+c)\left(\ln\left(\frac{2bdx+2\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)ad-\ln\left(\frac{2bdx+2\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)bc+2\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}\right)}{2\sqrt{(bx+a)(dx+c)}\,d\sqrt{bd}}$	1

input
```
int(((b*x+a)/(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((b*x+a)/(d*x+c))^(1/2)*(d*x+c)*(ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(
1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*d-ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x
+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*b*c+2*((b*x+a)*(d*x+c))^(1/2)
*(b*d)^(1/2))/((b*x+a)*(d*x+c))^(1/2)/d/(b*d)^(1/2)
```

3.745.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.37

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx = \left[-\frac{(bc-ad)\sqrt{bd}\log\left(2\,bdx+bc+ad+2\,\sqrt{bd}(dx+c)\sqrt{\frac{bx+a}{dx+c}}\right)-2\,(bd^2x+bcd)\sqrt{\frac{bx+a}{dx+c}}}{2\,bd^2}, \frac{(bc-ad)\sqrt{-bd}\,a}{}\right.$$

input
```
integrate(((b*x+a)/(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*((b*c - a*d)*sqrt(b*d)*log(2*b*d*x + b*c + a*d + 2*sqrt(b*d)*(d*x +
c)*sqrt((b*x + a)/(d*x + c))) - 2*(b*d^2*x + b*c*d)*sqrt((b*x + a)/(d*x +
c)))/(b*d^2), ((b*c - a*d)*sqrt(-b*d)*arctan(sqrt(-b*d)*(d*x + c)*sqrt((b*
x + a)/(d*x + c))/(b*d*x + a*d)) + (b*d^2*x + b*c*d)*sqrt((b*x + a)/(d*x +
 c)))/(b*d^2)]
```

3.745.6 Sympy [F]

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx = \int \sqrt{\frac{a+bx}{c+dx}}\,dx$$

input
```
integrate(((b*x+a)/(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt((a + b*x)/(c + d*x)), x)
```

3.745.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.55

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx = \frac{(bc-ad)\sqrt{\frac{bx+a}{dx+c}}}{bd-\frac{(bx+a)d^2}{dx+c}}+\frac{(bc-ad)\log\left(\frac{d\sqrt{\frac{bx+a}{dx+c}}-\sqrt{bd}}{d\sqrt{\frac{bx+a}{dx+c}}+\sqrt{bd}}\right)}{2\,\sqrt{bd}d}$$

input
```
integrate(((b*x+a)/(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
(b*c - a*d)*sqrt((b*x + a)/(d*x + c))/(b*d - (b*x + a)*d^2/(d*x + c)) + 1/
2*(b*c - a*d)*log((d*sqrt((b*x + a)/(d*x + c)) - sqrt(b*d))/(d*sqrt((b*x +
 a)/(d*x + c)) + sqrt(b*d)))/(sqrt(b*d)*d)
```

3.745.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.43

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx = \frac{(bc\text{sgn}(dx+c)-ad\text{sgn}(dx+c))\log\left(\left|-bc-ad-2\sqrt{bd}\left(\sqrt{bd}x-\sqrt{bdx^2+bcx+adx+ac}\right)\right|\right)}{2\sqrt{bd}d} + \frac{\sqrt{bdx^2+bcx+adx+ac}\,\text{sgn}(dx+c)}{d}$$

input
```
integrate(((b*x+a)/(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/2*(b*c*sgn(d*x + c) - a*d*sgn(d*x + c))*log(abs(-b*c - a*d - 2*sqrt(b*d)
*(sqrt(b*d)*x - sqrt(b*d*x^2 + b*c*x + a*d*x + a*c))))/(sqrt(b*d)*d) + sqr
t(b*d*x^2 + b*c*x + a*d*x + a*c)*sgn(d*x + c)/d
```

3.745.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.18

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx = \frac{\text{atanh}\left(\frac{\sqrt{d}\sqrt{\frac{a+b\,x}{c+d\,x}}}{\sqrt{b}}\right)(a\,d-b\,c)}{\sqrt{b}\,d^{3/2}} + \frac{(a\,d-b\,c)\sqrt{\frac{a+b\,x}{c+d\,x}}}{b\,d\left(\frac{d\,(a+b\,x)}{b\,(c+d\,x)}-1\right)}$$

input
```
int(((a + b*x)/(c + d*x))^(1/2),x)
```

output
```
(atanh((d^(1/2)*((a + b*x)/(c + d*x))^(1/2))/b^(1/2))*(a*d - b*c))/(b^(1/2
)*d^(3/2)) + ((a*d - b*c)*((a + b*x)/(c + d*x))^(1/2))/(b*d*((d*(a + b*x))
/(b*(c + d*x)) - 1))
```

3.746 $\int \sqrt{\frac{-1+x}{5+3x}}\,dx$

3.746.1 Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \sqrt{\frac{-1+x}{5+3x}}\,dx=\frac{1}{3}\sqrt{-1+x}\sqrt{5+3x}-\frac{8\text{arcsinh}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\sqrt{-1+x}\right)}{3\sqrt{3}}$$

output
```
-8/9*arcsinh(1/4*6^(1/2)*(-1+x)^(1/2))*3^(1/2)+1/3*(-1+x)^(1/2)*(5+3*x)^(1
/2)
```

3.746.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.53

$$\int \sqrt{\frac{-1+x}{5+3x}}\,dx=\frac{\sqrt{\frac{-1+x}{5+3x}}\left(3\sqrt{-1+x}(5+3x)-8\sqrt{15+9x}\text{arctanh}\left(\frac{\sqrt{5+3x}}{\sqrt{3}\sqrt{-1+x}}\right)\right)}{9\sqrt{-1+x}}$$

input
```
Integrate[Sqrt[(-1 + x)/(5 + 3*x)],x]
```

output
```
(Sqrt[(-1 + x)/(5 + 3*x)]*(3*Sqrt[-1 + x]*(5 + 3*x) - 8*Sqrt[15 + 9*x]*Arc
Tanh[Sqrt[5 + 3*x]/(Sqrt[3]*Sqrt[-1 + x])]))/(9*Sqrt[-1 + x])
```

3.746.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2050, 60, 64, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{x-1}{3x+5}}dx$$
$$\Big\downarrow \text{2050}$$
$$\int \frac{\sqrt{x-1}}{\sqrt{3x+5}}dx$$
$$\Big\downarrow \text{60}$$
$$\frac{1}{3}\sqrt{x-1}\sqrt{3x+5}-\frac{4}{3}\int \frac{1}{\sqrt{x-1}\sqrt{3x+5}}dx$$
$$\Big\downarrow \text{64}$$
$$\frac{1}{3}\sqrt{x-1}\sqrt{3x+5}-\frac{8}{3}\int \frac{1}{\sqrt{3(x-1)+8}}d\sqrt{x-1}$$
$$\Big\downarrow \text{222}$$
$$\frac{1}{3}\sqrt{x-1}\sqrt{3x+5}-\frac{8\text{arcsinh}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\sqrt{x-1}\right)}{3\sqrt{3}}$$

input
```
Int[Sqrt[(-1 + x)/(5 + 3*x)],x]
```

output
```
(Sqrt[-1 + x]*Sqrt[5 + 3*x])/3 - (8*ArcSinh[(Sqrt[3/2]*Sqrt[-1 + x])/2])/(
3*Sqrt[3])
```

3.746.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 64
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[2/b   Subst[Int[1/Sqrt[c - a*(d/b) + d*(x^2/b)], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[c - a*(d/b), 0] && ( !GtQ[a - c*(b/d), 0]
 || PosQ[b])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2050
```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

3.746.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(31) = 62$.

Time $= 0.23$ (sec) , antiderivative size $= 76$, normalized size of antiderivative $= 1.55$

method	result	si
default	$-\frac{\sqrt{\frac{x-1}{5+3x}}\,(5+3x)\left(4\ln\left(x\sqrt{3}+\frac{\sqrt{3}}{3}+\sqrt{3x^{2}+2x-5}\right)\sqrt{3}-3\sqrt{3x^{2}+2x-5}\right)}{9\sqrt{(5+3x)(x-1)}}$	7
risch	$\frac{(5+3x)\sqrt{\frac{x-1}{5+3x}}}{3}-\frac{4\ln\left(\frac{(1+3x)\sqrt{3}}{3}+\sqrt{3x^{2}+2x-5}\right)\sqrt{3}\,\sqrt{\frac{x-1}{5+3x}}\,\sqrt{(5+3x)(x-1)}}{9(x-1)}$	8
trager	$5\left(\frac{1}{3}+\frac{x}{5}\right)\sqrt{-\frac{1-x}{5+3x}}+\frac{4\,\mathrm{RootOf}\left(\_Z^{2}-3\right)\ln\left(9\sqrt{-\frac{1-x}{5+3x}}\,x-3\,\mathrm{RootOf}\left(\_Z^{2}-3\right)x+15\sqrt{-\frac{1-x}{5+3x}}-\mathrm{RootOf}\left(\_Z^{2}-3\right)\right)}{9}$	8

input
```
int(((x-1)/(5+3*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*((x-1)/(5+3*x))^(1/2)*(5+3*x)*(4*ln(x*3^(1/2)+1/3*3^(1/2)+(3*x^2+2*x-
5)^(1/2))*3^(1/2)-3*(3*x^2+2*x-5)^(1/2))/((5+3*x)*(x-1))^(1/2)
```

3.746.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10

$$\int \sqrt{\frac{-1+x}{5+3x}}\,dx = \frac{1}{3}\,(3\,x+5)\sqrt{\frac{x-1}{3\,x+5}} + \frac{4}{9}\,\sqrt{3}\log\left(\sqrt{3}(3\,x+5)\sqrt{\frac{x-1}{3\,x+5}} - 3\,x - 1\right)$$

input
```
integrate(((-1+x)/(5+3*x))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(3*x + 5)*sqrt((x - 1)/(3*x + 5)) + 4/9*sqrt(3)*log(sqrt(3)*(3*x + 5)*
sqrt((x - 1)/(3*x + 5)) - 3*x - 1)
```

3.746.6 Sympy [F]

$$\int \sqrt{\frac{-1+x}{5+3x}}\,dx = \int \sqrt{\frac{x-1}{3x+5}}\,dx$$

input
```
integrate(((-1+x)/(5+3*x))**(1/2),x)
```

output
```
Integral(sqrt((x - 1)/(3*x + 5)), x)
```

3.746.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(31) = 62.

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.63

$$\int \sqrt{\frac{-1+x}{5+3x}}\,dx = \frac{4}{9}\,\sqrt{3}\log\left(-\frac{\sqrt{3} - 3\,\sqrt{\frac{x-1}{3\,x+5}}}{\sqrt{3} + 3\,\sqrt{\frac{x-1}{3\,x+5}}}\right) - \frac{8\,\sqrt{\frac{x-1}{3\,x+5}}}{3\,\left(\frac{3\,(x-1)}{3\,x+5} - 1\right)}$$

input
```
integrate(((-1+x)/(5+3*x))^(1/2),x, algorithm="maxima")
```

output
```
4/9*sqrt(3)*log(-(sqrt(3) - 3*sqrt((x - 1)/(3*x + 5)))/(sqrt(3) + 3*sqrt((
x - 1)/(3*x + 5)))) - 8/3*sqrt((x - 1)/(3*x + 5))/(3*(x - 1)/(3*x + 5) - 1
)
```

3.746.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(31) = 62.

Time = 0.39 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.51

$$\begin{aligned}\int \sqrt{\frac{-1+x}{5+3x}}\,dx =& -\frac{8}{9}\sqrt{3}\log(2)\,\mathrm{sgn}(3\,x+5)\\ &+\frac{4}{9}\sqrt{3}\log\left(\left|-\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+2\,x-5}\right)-1\right|\right)\mathrm{sgn}(3\,x+5)\\ &+\frac{1}{3}\sqrt{3\,x^2+2\,x-5}\,\mathrm{sgn}(3\,x+5)\end{aligned}$$

input
```
integrate(((-1+x)/(5+3*x))^(1/2),x, algorithm="giac")
```

output
```
-8/9*sqrt(3)*log(2)*sgn(3*x + 5) + 4/9*sqrt(3)*log(abs(-sqrt(3)*(sqrt(3)*x
 - sqrt(3*x^2 + 2*x - 5)) - 1))*sgn(3*x + 5) + 1/3*sqrt(3*x^2 + 2*x - 5)*s
gn(3*x + 5)
```

3.746.9 Mupad [B] (verification not implemented)

Time = 18.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

$$\int \sqrt{\frac{-1+x}{5+3x}}\,dx = -\frac{8\sqrt{3}\,\mathrm{atanh}\left(\sqrt{3}\sqrt{\frac{x-1}{3\,x+5}}\right)}{9}-\frac{8\sqrt{\frac{x-1}{3\,x+5}}}{3\left(\frac{3\,x-3}{3\,x+5}-1\right)}$$

input
```
int(((x - 1)/(3*x + 5))^(1/2),x)
```

output
```
- (8*3^(1/2)*atanh(3^(1/2)*((x - 1)/(3*x + 5))^(1/2)))/9 - (8*((x - 1)/(3*
x + 5))^(1/2))/(3*((3*x - 3)/(3*x + 5) - 1))
```

3.747 $\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx$

3.747.1 Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx = -\frac{\sqrt{-1+5x}\sqrt{1+7x}}{x} - 12\arctan\left(\frac{\sqrt{1+7x}}{\sqrt{-1+5x}}\right)$$

output
```
-12*arctan((1+7*x)^(1/2)/(-1+5*x)^(1/2))-(-1+5*x)^(1/2)*(1+7*x)^(1/2)/x
```

3.747.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.91

$$\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx$$
$$= -\frac{\sqrt{\frac{-1+5x}{1+7x}}\left(\sqrt{-1+5x}(1+7x) + 12x\sqrt{1+7x}\arctan\left(\sqrt{35}x - \sqrt{-1+5x}\sqrt{1+7x}\right)\right)}{x\sqrt{-1+5x}}$$

input
```
Integrate[Sqrt[(-1 + 5*x)/(1 + 7*x)]/x^2,x]
```

output
```
-((Sqrt[(-1 + 5*x)/(1 + 7*x)]*(Sqrt[-1 + 5*x]*(1 + 7*x) + 12*x*Sqrt[1 + 7*
x]*ArcTan[Sqrt[35]*x - Sqrt[-1 + 5*x]*Sqrt[1 + 7*x]]))/(x*Sqrt[-1 + 5*x]))
```

3.747.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2050, 105, 104, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{5x-1}{7x+1}}}{x^2} \, dx$$

$$\Big\downarrow \text{ 2050}$$

$$\int \frac{\sqrt{5x-1}}{x^2\sqrt{7x+1}}dx$$

$$\Big\downarrow \text{ 105}$$

$$6\int \frac{1}{x\sqrt{5x-1}\sqrt{7x+1}}dx-\frac{\sqrt{5x-1}\sqrt{7x+1}}{x}$$

$$\Big\downarrow \text{ 104}$$

$$12\int \frac{1}{-\frac{7x+1}{5x-1}-1}d\frac{\sqrt{7x+1}}{\sqrt{5x-1}}-\frac{\sqrt{5x-1}\sqrt{7x+1}}{x}$$

$$\Big\downarrow \text{ 217}$$

$$-12\arctan\left(\frac{\sqrt{7x+1}}{\sqrt{5x-1}}\right)-\frac{\sqrt{5x-1}\sqrt{7x+1}}{x}$$

input
```
Int[Sqrt[(-1 + 5*x)/(1 + 7*x)]/x^2,x]
```

output
```
-((Sqrt[-1 + 5*x]*Sqrt[1 + 7*x])/x) - 12*ArcTan[Sqrt[1 + 7*x]/Sqrt[-1 + 5*
x]]
```

3.747. $\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2} \, dx$

3.747.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 2050
```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

3.747.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(38) = 76$.

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.83

method	result
risch	$-\frac{(1+7x)\sqrt{\frac{-1+5x}{1+7x}}}{x}+\frac{6\arctan\left(\frac{-2x-2}{2\sqrt{35x^2-2x-1}}\right)\sqrt{\frac{-1+5x}{1+7x}}\,\sqrt{(-1+5x)(1+7x)}}{-1+5x}$
trager	$-\frac{(1+7x)\sqrt{-\frac{1-5x}{1+7x}}}{x}+6\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{7\sqrt{-\frac{1-5x}{1+7x}}\,x+\mathrm{RootOf}\left(\_Z^2+1\right)x+\sqrt{-\frac{1-5x}{1+7x}}+\mathrm{RootOf}\left(\_Z^2+1\right)}{x}\right)$
default	$-\frac{\sqrt{\frac{-1+5x}{1+7x}}\,(1+7x)\left(-\left(35x^2-2x-1\right)^{\frac{3}{2}}+35\sqrt{35x^2-2x-1}\,x^2+6\arctan\left(\frac{x+1}{\sqrt{35x^2-2x-1}}\right)x-2\sqrt{35x^2-2x-1}\,x\right)}{\sqrt{(-1+5x)(1+7x)}\,x}$

3.747. $\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx$

input
```
int(((-1+5*x)/(1+7*x))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(1+7*x)/x*((-1+5*x)/(1+7*x))^(1/2)+6*arctan(1/2*(-2*x-2)/(35*x^2-2*x-1)^(
1/2))*((-1+5*x)/(1+7*x))^(1/2)*((-1+5*x)*(1+7*x))^(1/2)/(-1+5*x)
```

3.747.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx = \frac{12\,x\arctan\left(\sqrt{\frac{5\,x-1}{7\,x+1}}\right) - (7\,x+1)\sqrt{\frac{5\,x-1}{7\,x+1}}}{x}$$

input
```
integrate(((-1+5*x)/(1+7*x))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
(12*x*arctan(sqrt((5*x - 1)/(7*x + 1))) - (7*x + 1)*sqrt((5*x - 1)/(7*x +
1)))/x
```

3.747.6 Sympy [F]

$$\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx = \int \frac{\sqrt{\frac{5x-1}{7x+1}}}{x^2}\,dx$$

input
```
integrate(((-1+5*x)/(1+7*x))**(1/2)/x**2,x)
```

output
```
Integral(sqrt((5*x - 1)/(7*x + 1))/x**2, x)
```

3.747.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx = -\frac{12\sqrt{\frac{5x-1}{7x+1}}}{\frac{5x-1}{7x+1}+1} + 12\arctan\left(\sqrt{\frac{5x-1}{7x+1}}\right)$$

input
```
integrate(((-1+5*x)/(1+7*x))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-12*sqrt((5*x - 1)/(7*x + 1))/((5*x - 1)/(7*x + 1) + 1) + 12*arctan(sqrt((
5*x - 1)/(7*x + 1)))
```

3.747.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(38) = 76.

Time = 0.34 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.48

$$\begin{aligned}\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx &= \left(\sqrt{35} - 12\arctan\left(\frac{1}{7}\sqrt{35}\right)\right)\mathrm{sgn}(7x+1) \\ &\quad + 12\arctan\left(-\sqrt{35}x + \sqrt{35x^2-2x-1}\right)\mathrm{sgn}(7x+1) \\ &\quad - \frac{2\left(\left(\sqrt{35}x - \sqrt{35x^2-2x-1}\right)\mathrm{sgn}(7x+1) + \sqrt{35}\,\mathrm{sgn}(7x+1)\right)}{\left(\sqrt{35}x - \sqrt{35x^2-2x-1}\right)^2 + 1}\end{aligned}$$

input
```
integrate(((-1+5*x)/(1+7*x))^(1/2)/x^2,x, algorithm="giac")
```

output
```
(sqrt(35) - 12*arctan(1/7*sqrt(35)))*sgn(7*x + 1) + 12*arctan(-sqrt(35)*x
+ sqrt(35*x^2 - 2*x - 1))*sgn(7*x + 1) - 2*((sqrt(35)*x - sqrt(35*x^2 - 2*
x - 1))*sgn(7*x + 1) + sqrt(35)*sgn(7*x + 1))/((sqrt(35)*x - sqrt(35*x^2 -
 2*x - 1))^2 + 1)
```

3.747.9 Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx = 12\,\mathrm{atan}\left(\frac{\sqrt{5}\,\sqrt{7}\,\sqrt{35}\,\sqrt{\frac{5\,x-1}{7\,x+1}}}{35}\right) - \frac{12\,\sqrt{5}\,\sqrt{7}\,\sqrt{35}\,\sqrt{\frac{5\,x-1}{7\,x+1}}}{25\,\left(\frac{7\,x-\frac{7}{5}}{7\,x+1}+\frac{7}{5}\right)}$$

input
```
int(((5*x - 1)/(7*x + 1))^(1/2)/x^2,x)
```

output
```
12*atan((5^(1/2)*7^(1/2)*35^(1/2)*((5*x - 1)/(7*x + 1))^(1/2))/35) - (12*5
^(1/2)*7^(1/2)*35^(1/2)*((5*x - 1)/(7*x + 1))^(1/2))/(25*((7*x - 7/5)/(7*x
 + 1) + 7/5))
```

3.748 $\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)}\,dx$

3.748.1 Optimal result

Integrand size = 22, antiderivative size = 20

$$\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)}\,dx = -\sqrt{\frac{1-x}{1+x}}(1+x)$$

output
```
-(1+x)*((1-x)/(1+x))^(1/2)
```

3.748.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)}\,dx = \frac{-1+x}{\sqrt{\frac{1-x}{1+x}}}$$

input
```
Integrate[x/(Sqrt[(1 - x)/(1 + x)]*(1 + x)),x]
```

output
```
(-1 + x)/Sqrt[(1 - x)/(1 + x)]
```

3.748.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2056, 27, 297}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{\frac{1-x}{x+1}}(x+1)}dx$$

$$\Big\downarrow \text{2056}$$

$$-4\int \frac{1-\frac{1-x}{x+1}}{2\left(\frac{1-x}{x+1}+1\right)^2}d\sqrt{\frac{1-x}{x+1}}$$

$$\Big\downarrow \text{27}$$

$$-2\int \frac{1-\frac{1-x}{x+1}}{\left(\frac{1-x}{x+1}+1\right)^2}d\sqrt{\frac{1-x}{x+1}}$$

$$\Big\downarrow \text{297}$$

$$-\frac{2\sqrt{\frac{1-x}{x+1}}}{\frac{1-x}{x+1}+1}$$

input
```
Int[x/(Sqrt[(1 - x)/(1 + x)]*(1 + x)),x]
```

output
```
(-2*Sqrt[(1 - x)/(1 + x)])/(1 + (1 - x)/(1 + x))
```

3.748.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 297
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[c*
x*((a + b*x^2)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d
, 0] && EqQ[a*d - b*c*(2*p + 3), 0]
```

rule 2056
```
Int[(u_)^(r_.)*(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)
*(x_)^(n_.)))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c
- a*d)/n)   Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)
^((m + 1)/n - 1)/(b*e - d*x^q)^((m + 1)/n + 1))*(u /. x -> ((-a)*e + c*x^q)
^(1/n)/(b*e - d*x^q)^(1/n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/
q)], x]] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p]
&& IntegerQ[1/n] && IntegersQ[m, r]
```

3.748.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
gosper	$\frac{x-1}{\sqrt{-\frac{x-1}{x+1}}}$	17
risch	$\frac{x-1}{\sqrt{-\frac{x-1}{x+1}}}$	17
trager	$(-x-1)\sqrt{-\frac{x-1}{x+1}}$	19
default	$\frac{(x-1)\sqrt{-x^2+1}}{\sqrt{-\frac{x-1}{x+1}}\,\sqrt{-(x-1)(x+1)}}$	36

input
```
int(x/(x+1)/((1-x)/(x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x-1)/(-(x-1)/(x+1))^(1/2)
```

3.748.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)}\,dx = -(x+1)\sqrt{-\frac{x-1}{x+1}}$$

input
```
integrate(x/(1+x)/((1-x)/(1+x))^(1/2),x, algorithm="fricas")
```

output
```
-(x + 1)*sqrt(-(x - 1)/(x + 1))
```

3.748.6 Sympy [F]

$$\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)}\,dx = \int \frac{x}{\sqrt{-\frac{x-1}{x+1}}\,(x+1)}\,dx$$

input
```
integrate(x/(1+x)/((1-x)/(1+x))**(1/2),x)
```

output
```
Integral(x/(sqrt(-(x - 1)/(x + 1))*(x + 1)), x)
```

3.748.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)}\,dx = \frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

input
```
integrate(x/(1+x)/((1-x)/(1+x))^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(-(x - 1)/(x + 1))/((x - 1)/(x + 1) - 1)
```

3.748.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)}\,dx = -\frac{\sqrt{-x^2+1}}{\operatorname{sgn}(x+1)}$$

input
```
integrate(x/(1+x)/((1-x)/(1+x))^(1/2),x, algorithm="giac")
```

output
```
-sqrt(-x^2 + 1)/sgn(x + 1)
```

3.748.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)}\,dx = -\sqrt{-\frac{x-1}{x+1}}\,(x+1)$$

input
```
int(x/((-(x - 1)/(x + 1))^(1/2)*(x + 1)),x)
```

output
```
-(-(x - 1)/(x + 1))^(1/2)*(x + 1)
```

3.749 $\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx$

3.749.1 Optimal result

Integrand size = 20, antiderivative size = 18

$$\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx = -\left((1+x)\sqrt{-1+\frac{2}{1+x}}\right)$$

output
```
-(1+x)*(-1+2/(1+x))^(1/2)
```

3.749.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx = \frac{-1+x}{\sqrt{\frac{1-x}{1+x}}}$$

input
```
Integrate[x/((1 + x)*Sqrt[-1 + 2/(1 + x)]),x]
```

output
```
(-1 + x)/Sqrt[(1 - x)/(1 + x)]
```

3.749.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1015, 25, 1016, 899, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+1)\sqrt{\frac{2}{x+1}-1}}dx$$

$$\Big\downarrow \text{1015}$$

$$\int \frac{x}{(x+1)\sqrt{\frac{2}{x+1}-1}}d(x+1)$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{x}{(x+1)\sqrt{\frac{2}{x+1}-1}}d(x+1)$$

$$\Big\downarrow \text{1016}$$

$$-\int \frac{\frac{1}{x+1}-1}{\sqrt{\frac{2}{x+1}-1}}d(x+1)$$

$$\Big\downarrow \text{899}$$

$$\int \frac{x(x+1)^2}{\sqrt{\frac{2}{x+1}-1}}d\frac{1}{x+1}$$

$$\Big\downarrow \text{83}$$

$$-\left((x+1)\sqrt{\frac{2}{x+1}-1}\right)$$

input
```
Int[x/((1 + x)*Sqrt[-1 + 2/(1 + x)]),x]
```

output
```
-((1 + x)*Sqrt[-1 + 2/(1 + x)])
```

3.749.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1015
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_.)*((c_.) + (d_.)*(v_)^(n_))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/Coefficient[v, x, 1]^(m + 1)   Subst[Int[SimplifyI
ntegrand[(x - Coefficient[v, x, 0])^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x],
x, v], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[v, x] && IntegerQ[m
] && NeQ[v, x]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

3.749.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{x-1}{\sqrt{-\frac{x-1}{x+1}}}$	17
risch	$\frac{x-1}{\sqrt{-\frac{x-1}{x+1}}}$	17
trager	$\left(-x-1\right)\sqrt{-\frac{x-1}{x+1}}$	19
default	$-\frac{\sqrt{-\frac{x-1}{x+1}}\,(x+1)\sqrt{-x^{2}+1}}{\sqrt{-(x-1)(x+1)}}$	37

3.749. $\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx$

input
```
int(x/(x+1)/(-1+2/(x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x-1)/(-(x-1)/(x+1))^(1/2)
```

3.749.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx = -(x+1)\sqrt{-\frac{x-1}{x+1}}$$

input
```
integrate(x/(1+x)/(-1+2/(1+x))^(1/2),x, algorithm="fricas")
```

output
```
-(x + 1)*sqrt(-(x - 1)/(x + 1))
```

3.749.6 Sympy [F]

$$\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx = \int \frac{x}{\sqrt{-\frac{x-1}{x+1}}\,(x+1)}\,dx$$

input
```
integrate(x/(1+x)/(-1+2/(1+x))**(1/2),x)
```

output
```
Integral(x/(sqrt(-(x - 1)/(x + 1))*(x + 1)), x)
```

3.749.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx = \frac{\sqrt{x+1}(x-1)}{\sqrt{-x+1}}$$

input
```
integrate(x/(1+x)/(-1+2/(1+x))^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x + 1)*(x - 1)/sqrt(-x + 1)
```

3.749.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx = -\frac{\sqrt{-x^2+1}}{\operatorname{sgn}(x+1)}$$

input
```
integrate(x/(1+x)/(-1+2/(1+x))^(1/2),x, algorithm="giac")
```

output
```
-sqrt(-x^2 + 1)/sgn(x + 1)
```

3.749.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx = \int \frac{x}{(x+1)\,\sqrt{\frac{2}{x+1}-1}}\,dx$$

input
```
int(x/((x + 1)*(2/(x + 1) - 1)^(1/2)),x)
```

output
```
int(x/((x + 1)*(2/(x + 1) - 1)^(1/2)), x)
```

3.750 $\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx$

3.750.1 Optimal result

Integrand size = 20, antiderivative size = 54

$$\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx=\sqrt{2+x}\sqrt{3+x}-\operatorname{arcsinh}\left(\sqrt{2+x}\right)+2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{2+x}}{\sqrt{3+x}}\right)$$

output
```
-arcsinh((2+x)^(1/2))+2*arctanh(2^(1/2)*(2+x)^(1/2)/(3+x)^(1/2))*2^(1/2)+(
2+x)^(1/2)*(3+x)^(1/2)
```

3.750.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

$$\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx=\sqrt{2+x}\sqrt{3+x}+2\sqrt{2}\operatorname{arctanh}\left(\frac{-1-x+\sqrt{2+x}\sqrt{3+x}}{\sqrt{2}}\right)+\log\left(\sqrt{2+x}-\sqrt{3+x}\right)$$

input
```
Integrate[x/((1 + x)*Sqrt[(2 + x)/(3 + x)]),x]
```

output
```
Sqrt[2 + x]*Sqrt[3 + x] + 2*Sqrt[2]*ArcTanh[(-1 - x + Sqrt[2 + x]*Sqrt[3 +
 x])/Sqrt[2]] + Log[Sqrt[2 + x] - Sqrt[3 + x]]
```

3.750.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2050, 171, 27, 175, 64, 104, 220, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+1)\sqrt{\frac{x+2}{x+3}}}dx$$

$$\Big\downarrow \text{2050}$$

$$\int \frac{x\sqrt{x+3}}{(x+1)\sqrt{x+2}}dx$$

$$\Big\downarrow \text{171}$$

$$\int -\frac{x+5}{2(x+1)\sqrt{x+2}\sqrt{x+3}}dx+\sqrt{x+2}\sqrt{x+3}$$

$$\Big\downarrow \text{27}$$

$$\sqrt{x+2}\sqrt{x+3}-\frac{1}{2}\int \frac{x+5}{(x+1)\sqrt{x+2}\sqrt{x+3}}dx$$

$$\Big\downarrow \text{175}$$

$$\frac{1}{2}\left(-\int \frac{1}{\sqrt{x+2}\sqrt{x+3}}dx-4\int \frac{1}{(x+1)\sqrt{x+2}\sqrt{x+3}}dx\right)+\sqrt{x+2}\sqrt{x+3}$$

$$\Big\downarrow \text{64}$$

$$\frac{1}{2}\left(-2\int \frac{1}{\sqrt{x+3}}d\sqrt{x+2}-4\int \frac{1}{(x+1)\sqrt{x+2}\sqrt{x+3}}dx\right)+\sqrt{x+2}\sqrt{x+3}$$

$$\Big\downarrow \text{104}$$

$$\frac{1}{2}\left(-2\int \frac{1}{\sqrt{x+3}}d\sqrt{x+2}-8\int \frac{1}{\frac{2(x+2)}{x+3}-1}d\frac{\sqrt{x+2}}{\sqrt{x+3}}\right)+\sqrt{x+2}\sqrt{x+3}$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\left(4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x+2}}{\sqrt{x+3}}\right)-2\int \frac{1}{\sqrt{x+3}}d\sqrt{x+2}\right)+\sqrt{x+2}\sqrt{x+3}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\left(4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x+2}}{\sqrt{x+3}}\right)-2\text{arcsinh}\left(\sqrt{x+2}\right)\right)+\sqrt{x+2}\sqrt{x+3}$$

input
```
Int[x/((1 + x)*Sqrt[(2 + x)/(3 + x)]),x]
```

output
```
Sqrt[2 + x]*Sqrt[3 + x] + (-2*ArcSinh[Sqrt[2 + x]] + 4*Sqrt[2]*ArcTanh[(Sq
rt[2]*Sqrt[2 + x])/Sqrt[3 + x]])/2
```

3.750.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 64
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[2/b   Subst[Int[1/Sqrt[c - a*(d/b) + d*(x^2/b)], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[c - a*(d/b), 0] && ( !GtQ[a - c*(b/d), 0]
 || PosQ[b])
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2050
```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

3.750.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.46

method	result
default	$-\frac{(x+2)\left(-2\sqrt{2}\,\operatorname{arctanh}\left(\frac{(7+3x)\sqrt{2}}{4\sqrt{x^2+5x+6}}\right)+\ln\left(\frac{5}{2}+x+\sqrt{x^2+5x+6}\right)-2\sqrt{x^2+5x+6}\right)}{2\sqrt{\frac{x+2}{3+x}}\,\sqrt{(3+x)(x+2)}}$
risch	$\frac{x+2}{\sqrt{\frac{x+2}{3+x}}}+\frac{\left(-\frac{\ln\left(\frac{5}{2}+x+\sqrt{x^2+5x+6}\right)}{2}+\sqrt{2}\,\operatorname{arctanh}\left(\frac{(7+3x)\sqrt{2}}{4\sqrt{(x+1)^2+3x+5}}\right)\right)\sqrt{(3+x)(x+2)}}{\sqrt{\frac{x+2}{3+x}}\,(3+x)}$
trager	$3\left(1+\frac{x}{3}\right)\sqrt{-\frac{-x-2}{3+x}}-\frac{\ln\left(2\sqrt{-\frac{-x-2}{3+x}}\,x+6\sqrt{-\frac{-x-2}{3+x}}+2x+5\right)}{2}+\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{4\sqrt{-\frac{-x-2}{3+x}}\,x+3\operatorname{RootOf}\left(\_\right.}{}\right.$

input
```
int(x/(x+1)/((x+2)/(3+x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(x+2)*(-2*2^(1/2)*arctanh(1/4*(7+3*x)*2^(1/2)/(x^2+5*x+6)^(1/2))+ln(5
/2+x+(x^2+5*x+6)^(1/2))-2*(x^2+5*x+6)^(1/2))/((x+2)/(3+x))^(1/2)/((3+x)*(x
+2))^(1/2)
```

3.750.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(40) = 80$.

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.54

$$\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx = (x+3)\sqrt{\frac{x+2}{x+3}} + \sqrt{2}\log\left(\frac{2\sqrt{2}(x+3)\sqrt{\frac{x+2}{x+3}} + 3x + 7}{x+1}\right) - \frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}} + 1\right) + \frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}} - 1\right)$$

input
```
integrate(x/(1+x)/((2+x)/(3+x))^(1/2),x, algorithm="fricas")
```

output
```
(x + 3)*sqrt((x + 2)/(x + 3)) + sqrt(2)*log((2*sqrt(2)*(x + 3)*sqrt((x + 2
)/(x + 3)) + 3*x + 7)/(x + 1)) - 1/2*log(sqrt((x + 2)/(x + 3)) + 1) + 1/2*
log(sqrt((x + 2)/(x + 3)) - 1)
```

3.750.6 Sympy [F]

$$\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx = \int \frac{x}{\sqrt{\frac{x+2}{x+3}}\,(x+1)}\,dx$$

input
```
integrate(x/(1+x)/((2+x)/(3+x))**(1/2),x)
```

output
```
Integral(x/(sqrt((x + 2)/(x + 3))*(x + 1)), x)
```

3.750.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(40) = 80$.

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.91

$$\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx = -\sqrt{2}\log\left(-\frac{\sqrt{2} - 2\sqrt{\frac{x+2}{x+3}}}{\sqrt{2} + 2\sqrt{\frac{x+2}{x+3}}}\right) - \frac{\sqrt{\frac{x+2}{x+3}}}{\frac{x+2}{x+3} - 1} - \frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}} + 1\right) + \frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}} - 1\right)$$

input
```
integrate(x/(1+x)/((2+x)/(3+x))^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(2)*log(-(sqrt(2) - 2*sqrt((x + 2)/(x + 3)))/(sqrt(2) + 2*sqrt((x + 2
)/(x + 3)))) - sqrt((x + 2)/(x + 3))/((x + 2)/(x + 3) - 1) - 1/2*log(sqrt(
(x + 2)/(x + 3)) + 1) + 1/2*log(sqrt((x + 2)/(x + 3)) - 1)
```

3.750.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(40) = 80$.

Time = 0.38 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.39

$$\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx = \sqrt{2}\log\left(-\frac{\sqrt{2}-2}{\sqrt{2}+2}\right)\operatorname{sgn}(x+3) - \frac{\sqrt{2}\log\left(\frac{\left|-2\,x-2\,\sqrt{2}+2\,\sqrt{x^2+5\,x+6}-2\right|}{\left|-2\,x+2\,\sqrt{2}+2\,\sqrt{x^2+5\,x+6}-2\right|}\right)}{\operatorname{sgn}\left(x+3\right)} + \frac{\log\left(\left|-2\,x+2\,\sqrt{x^2+5\,x+6}-5\right|\right)}{2\operatorname{sgn}\left(x+3\right)} + \frac{\sqrt{x^2+5\,x+6}}{\operatorname{sgn}\left(x+3\right)}$$

input
```
integrate(x/(1+x)/((2+x)/(3+x))^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*log(-(sqrt(2) - 2)/(sqrt(2) + 2))*sgn(x + 3) - sqrt(2)*log(abs(-2*
x - 2*sqrt(2) + 2*sqrt(x^2 + 5*x + 6) - 2)/abs(-2*x + 2*sqrt(2) + 2*sqrt(x
^2 + 5*x + 6) - 2))/sgn(x + 3) + 1/2*log(abs(-2*x + 2*sqrt(x^2 + 5*x + 6)
- 5))/sgn(x + 3) + sqrt(x^2 + 5*x + 6)/sgn(x + 3)
```

3.750.9 Mupad [B] (verification not implemented)

Time = 18.00 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.15

$$\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx = 2\sqrt{2}\operatorname{atanh}\left(\sqrt{2}\sqrt{\frac{x+2}{x+3}}\right) - \frac{\sqrt{\frac{x+2}{x+3}}}{\frac{x+2}{x+3}-1} - \operatorname{atanh}\left(\sqrt{\frac{x+2}{x+3}}\right)$$

input
```
int(x/(((x + 2)/(x + 3))^(1/2)*(x + 1)),x)
```

output
```
2*2^(1/2)*atanh(2^(1/2)*((x + 2)/(x + 3))^(1/2)) - ((x + 2)/(x + 3))^(1/2)
/((x + 2)/(x + 3) - 1) - atanh(((x + 2)/(x + 3))^(1/2))
```

3.751 $\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2} dx$

3.751.1 Optimal result

Integrand size = 15, antiderivative size = 11

$$\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2} dx = \frac{2}{\sqrt{1+\frac{1}{x}}}$$

output
```
2/(1+1/x)^(1/2)
```

3.751.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2} dx = \frac{2x\sqrt{\frac{1+x}{x}}}{1+x}$$

input
```
Integrate[Sqrt[1 + x^(-1)]/(1 + x)^2,x]
```

output
```
(2*x*Sqrt[(1 + x)/x])/(1 + x)
```

3.751.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {941, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{1}{x}+1}}{(x+1)^2}dx$$

$$\Big\downarrow \text{941}$$

$$\int \frac{1}{\left(\frac{1}{x}+1\right)^{3/2}x^2}dx$$

$$\Big\downarrow \text{793}$$

$$\frac{2}{\sqrt{\frac{1}{x}+1}}$$

input
```
Int[Sqrt[1 + x^(-1)]/(1 + x)^2,x]
```

output
```
2/Sqrt[1 + x^(-1)]
```

3.751.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

3.751.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

method	result	size
gosper	$\frac{2x\sqrt{\frac{x+1}{x}}}{x+1}$	18
risch	$\frac{2x\sqrt{\frac{x+1}{x}}}{x+1}$	18
trager	$\frac{2x\sqrt{-\frac{-x-1}{x}}}{x+1}$	21
default	$\frac{2\sqrt{x^2+x}\,x\sqrt{\frac{x+1}{x}}}{(x+1)\sqrt{(x+1)x}}$	32

input
```
int((1+1/x)^(1/2)/(x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
2/(x+1)*x*((x+1)/x)^(1/2)
```

3.751.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2}\,dx = \frac{2\,x\sqrt{\frac{x+1}{x}}}{x+1}$$

input
```
integrate((1+1/x)^(1/2)/(1+x)^2,x, algorithm="fricas")
```

output
```
2*x*sqrt((x + 1)/x)/(x + 1)
```

3.751.6 Sympy [F]

$$\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2}\,dx = \int \frac{\sqrt{1+\frac{1}{x}}}{(x+1)^2}\,dx$$

input
```
integrate((1+1/x)**(1/2)/(1+x)**2,x)
```

output
```
Integral(sqrt(1 + 1/x)/(x + 1)**2, x)
```

3.751.7 Maxima [F]

$$\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2}\,dx = \int \frac{\sqrt{\frac{1}{x}+1}}{(x+1)^2}\,dx$$

input
```
integrate((1+1/x)^(1/2)/(1+x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(1/x + 1)/(x + 1)^2, x)
```

3.751.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(9) = 18$.

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2}\,dx = \frac{2\,\text{sgn}(x)}{x-\sqrt{x^2+x}+1} - 2\,\text{sgn}(x)$$

input
```
integrate((1+1/x)^(1/2)/(1+x)^2,x, algorithm="giac")
```

output
```
2*sgn(x)/(x - sqrt(x^2 + x) + 1) - 2*sgn(x)
```

3.751.9 Mupad [B] (verification not implemented)

Time = 18.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2}\,dx = \frac{2\,x\,\sqrt{\frac{1}{x}+1}}{x+1}$$

input
```
int((1/x + 1)^(1/2)/(x + 1)^2,x)
```

output
```
(2*x*(1/x + 1)^(1/2))/(x + 1)
```

3.752 $\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}} \, dx$

3.752.1 Optimal result

Integrand size = 21, antiderivative size = 29

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}} \, dx = -\frac{\sqrt{1+\frac{1}{x}}\sqrt{x}\arcsin(1-2x)}{\sqrt{1+x}}$$

output
```
arcsin(-1+2*x)*(1+1/x)^(1/2)*x^(1/2)/(1+x)^(1/2)
```

3.752.2 Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}} \, dx = -\arctan\left(\frac{\sqrt{\frac{1+x}{x}}(-1+2x)\sqrt{1-x^2}}{2(-1+x^2)}\right)$$

input
```
Integrate[Sqrt[1 + x^(-1)]/Sqrt[1 - x^2],x]
```

output
```
-ArcTan[(Sqrt[(1 + x)/x]*(-1 + 2*x)*Sqrt[1 - x^2])/(2*(-1 + x^2))]
```

3.752.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1778, 516, 62, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{1}{x}+1}}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{1778}$$

$$\frac{\sqrt{\frac{1}{x}+1}\sqrt{x}\int \frac{\sqrt{x+1}}{\sqrt{x}\sqrt{1-x^2}}dx}{\sqrt{x+1}}$$

$$\Big\downarrow \text{516}$$

$$\frac{\sqrt{\frac{1}{x}+1}\sqrt{x}\int \frac{1}{\sqrt{1-x}\sqrt{x}}dx}{\sqrt{x+1}}$$

$$\Big\downarrow \text{62}$$

$$\frac{\sqrt{\frac{1}{x}+1}\sqrt{x}\int \frac{1}{\sqrt{x-x^2}}dx}{\sqrt{x+1}}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{\sqrt{\frac{1}{x}+1}\sqrt{x}\int \frac{1}{\sqrt{1-(1-2x)^2}}d(1-2x)}{\sqrt{x+1}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{\sqrt{\frac{1}{x}+1}\sqrt{x}\arcsin(1-2x)}{\sqrt{x+1}}$$

input
```
Int[Sqrt[1 + x^(-1)]/Sqrt[1 - x^2],x]
```

output
```
-((Sqrt[1 + x^(-1)]*Sqrt[x]*ArcSin[1 - 2*x])/Sqrt[1 + x])
```

3.752. $\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\,dx$

3.752.3.1 Defintions of rubi rules used

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 516
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[(e*x)^m*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[n]))
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1778
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sym
bol] :> Simp[(e^IntPart[q]*((d + e*x^mn)^FracPart[q]/(1 + d*(1/(x^mn*e)))^F
racPart[q]))/x^(mn*FracPart[q])   Int[x^(mn*q)*(1 + d*(1/(x^mn*e)))^q*(a +
c*x^n2)^p, x], x] /; FreeQ[{a, c, d, e, mn, p, q}, x] && EqQ[n2, -2*mn] &&
 !IntegerQ[p] &&  !IntegerQ[q] && PosQ[n2]
```

3.752.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

method	result	size
default	$\frac{\sqrt{\frac{x+1}{x}}\, x\sqrt{-x^{2}+1}\, \arcsin\left(2x-1\right)}{\left(x+1\right)\sqrt{-\left(x-1\right)x}}$	40

input
```
int((1+1/x)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((x+1)/x)^(1/2)*x*(-x^2+1)^(1/2)/(x+1)/(-(x-1)*x)^(1/2)*arcsin(2*x-1)
```

3.752. $\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\, dx$

3.752.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\,dx = -\arctan\left(\frac{2\sqrt{-x^2+1}x\sqrt{\frac{x+1}{x}}}{2\,x^2+x-1}\right)$$

input
```
integrate((1+1/x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-arctan(2*sqrt(-x^2 + 1)*x*sqrt((x + 1)/x)/(2*x^2 + x - 1))
```

3.752.6 Sympy [F]

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{-(x-1)(x+1)}}\,dx$$

input
```
integrate((1+1/x)**(1/2)/(-x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(1 + 1/x)/sqrt(-(x - 1)*(x + 1)), x)
```

3.752.7 Maxima [F]

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{\frac{1}{x}+1}}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate((1+1/x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(1/x + 1)/sqrt(-x^2 + 1), x)
```

3.752.8 Giac [F]

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{\frac{1}{x}+1}}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate((1+1/x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(1/x + 1)/sqrt(-x^2 + 1), x)
```

3.752.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{\frac{1}{x}+1}}{\sqrt{1-x^2}}\,dx$$

input
```
int((1/x + 1)^(1/2)/(1 - x^2)^(1/2),x)
```

output
```
int((1/x + 1)^(1/2)/(1 - x^2)^(1/2), x)
```

3.753 $\int \frac{1}{x+\sqrt{3-2x-x^2}}\,dx$

3.753.1 Optimal result

Integrand size = 18, antiderivative size = 180

$$\begin{aligned}\int \frac{1}{x+\sqrt{3-2x-x^2}}\,dx &= \arctan\left(\frac{\sqrt{3}-\sqrt{3-2x-x^2}}{x}\right) \\ &\quad -\frac{1}{2}\log\left(-\frac{3-x-\sqrt{3}\sqrt{3-2x-x^2}}{x^2}\right) \\ &\quad +\frac{1}{14}\left(7+\sqrt{7}\right)\log\left(1+\sqrt{3}-\sqrt{7}-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right) \\ &\quad +\frac{1}{14}\left(7-\sqrt{7}\right)\log\left(1+\sqrt{3}+\sqrt{7}-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)\end{aligned}$$

output
```
arctan((3^(1/2)-(-x^2-2*x+3)^(1/2))/x)-1/2*ln((-3+x+3^(1/2)*(-x^2-2*x+3)^(
1/2))/x^2)+1/14*ln(1+3^(1/2)+7^(1/2)-3^(1/2)*(3^(1/2)-(-x^2-2*x+3)^(1/2))/
x)*(7-7^(1/2))+1/14*ln(1+3^(1/2)-7^(1/2)-3^(1/2)*(3^(1/2)-(-x^2-2*x+3)^(1/
2))/x)*(7+7^(1/2))
```

3.753.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.62

$$\int \frac{1}{x+\sqrt{3-2x-x^2}}\,dx = \frac{1}{14}\left(-14\arctan\left(\frac{\sqrt{3-2x-x^2}}{3+x}\right)-7\log(-1+x)-\left(-7+\sqrt{7}\right)\log\left(-2+\sqrt{7}(-1+x)+2x-\sqrt{3-2x-x^2}\right)+\left(7+\sqrt{7}\right)\log\left(2+\sqrt{7}(-1+x)-2x+\sqrt{3-2x-x^2}\right)\right)$$

input
```
Integrate[(x + Sqrt[3 - 2*x - x^2])^(-1),x]
```

output
```
(-14*ArcTan[Sqrt[3 - 2*x - x^2]/(3 + x)] - 7*Log[-1 + x] - (-7 + Sqrt[7])*
Log[-2 + Sqrt[7]*(-1 + x) + 2*x - Sqrt[3 - 2*x - x^2]] + (7 + Sqrt[7])*Log
[2 + Sqrt[7]*(-1 + x) - 2*x + Sqrt[3 - 2*x - x^2]])/14
```

3.753.3 Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {7285, 2142, 27, 452, 216, 240, 1142, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-x^2-2x+3}+x}\,dx$$

$$\Big\downarrow \text{7285}$$

$$2\int \frac{-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}+\frac{2\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}+\sqrt{3}}{\left(\frac{\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}+1\right)\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)}d\left(-\frac{\sqrt{3}-\sqrt{-x^2-2}}{x}\right.$$

$$\Big\downarrow \text{2142}$$

$$2\left(\frac{1}{32}\int -\frac{16\left(1-\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{x}\right)}{\frac{\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}+1}d\left(-\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{x}\right)+\frac{1}{32}\int \frac{16\left(-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}+\sqrt{}\right.}{\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x}\right.}{x}}\right.$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{1}{2}\int \frac{-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}+\sqrt{3}+2}{\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2}d\left(-\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{x}\right)-\frac{1}{2}\int \frac{1-\frac{\sqrt{3}-}{}}{\frac{\left(\sqrt{3}-\sqrt{-x^2}\right.}{x^2}}\right.$$

$$\Big\downarrow \; 452$$

$$2\left(\frac{1}{2}\left(-\int \frac{1}{\frac{\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}+1}d\left(-\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{x}\right)-\int -\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{x\left(\frac{\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}+1\right)}d\left(-\frac{\sqrt{3}-\sqrt{-}}{}\right.\right.\right.$$

$$\Big\downarrow \; 216$$

$$2\left(\frac{1}{2}\left(\arctan\left(\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{x}\right)-\int -\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{x\left(\frac{\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}+1\right)}d\left(-\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{x}\right)\right)+\frac{1}{2}\int\right.$$

$$\Big\downarrow \; 240$$

$$2\left(\frac{1}{2}\int \frac{-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}+\sqrt{3}+2}{\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2}d\left(-\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{x}\right)+\frac{1}{2}\left(\arctan\left(\frac{\sqrt{}}{}\right.\right.\right.$$

$$\Big\downarrow \; 1142$$

$$2\left(\frac{1}{2}\left(\int \frac{1}{\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2}d\left(-\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{x}\right)+\frac{1}{2}\int \frac{}{\sqrt{3}\left(\sqrt{3}-\sqrt{}\right.}\right.\right.$$

$$\Big\downarrow \; 27$$

3.753. $\int \frac{1}{x+\sqrt{3-2x-x^2}}\,dx$

$$2\left(\frac{1}{2}\left(\left(\int \frac{1}{\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2}d\left(-\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{x}\right)+\int \frac{}{\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-\right.}}{x}}\right.\right.\right.$$

$\Big\downarrow$ 1083

$$2\left(\frac{1}{2}\left(\left(\int \frac{-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}+\sqrt{3}+1}{\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2}d\left(-\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{x}\right)-2\int \frac{}{28-\frac{\left(\sqrt{3}\right.}{}}\right.\right.\right.$$

$\Big\downarrow$ 219

$$2\left(\frac{1}{2}\left(\left(\int \frac{-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}+\sqrt{3}+1}{\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2}d\left(-\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{x}\right)+\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}-}{}\right.}{\sqrt{}}\right.\right.\right.$$

$\Big\downarrow$ 1103

$$2\left(\frac{1}{2}\left(\arctan\left(\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{x}\right)-\frac{1}{2}\log\left(\frac{\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}+1\right)\right)+\frac{1}{2}\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}-\sqrt{-x^2-2x+}}{2\sqrt{7}x}\right.}{\sqrt{7}}\right.\right.$$

input
```
Int[(x + Sqrt[3 - 2*x - x^2])^(-1),x]
```

output
```
2*((ArcTan[(Sqrt[3] - Sqrt[3 - 2*x - x^2])/x] - Log[1 + (Sqrt[3] - Sqrt[3
- 2*x - x^2])^2/x^2]/2)/2 + (ArcTanh[(Sqrt[3] - Sqrt[3 - 2*x - x^2])/(2*Sq
rt[7]*x)]/Sqrt[7] + Log[2 - Sqrt[3] - (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3 -
 2*x - x^2]))/x + (Sqrt[3]*(Sqrt[3] - Sqrt[3 - 2*x - x^2])^2)/x^2]/2)/2)
```

3.753.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.753. $\int \frac{1}{x+\sqrt{3-2x-x^2}}\,dx$

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2142
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*(x_)^2)), x_Sym
bol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2]
, q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}, Simp[1/q   Int[(A*c^2*d - a
*c*C*d + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d + A*b*f -
 a*B*f)*x)/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[(c*C*d^2 + b*B*d*f -
A*c*d*f - a*C*d*f + a*A*f^2 - f*(B*c*d - b*C*d + A*b*f - a*B*f)*x)/(d + f*x
^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, f}, x] && PolyQ[Px, x, 2]
```

rule 7285
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfSquareRootOfQuadratic[u, x]}, Si
mp[2   Subst[Int[lst[[1]], x], x, lst[[2]]], x] /;  !FalseQ[lst] && EqQ[lst
[[3]], 1]] /; EulerIntegrandQ[u, x]
```

3.753. $\int \frac{1}{x+\sqrt{3-2x-x^2}}\,dx$

3.753.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(138) = 276$.

Time = 1.26 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.99

method	result
default	$-\frac{\sqrt{7}\left(\frac{\sqrt{-4\left(x+\frac{1}{2}-\frac{\sqrt{7}}{2}\right)^2+4\left(-1-\sqrt{7}\right)\left(x+\frac{1}{2}-\frac{\sqrt{7}}{2}\right)+8-2\sqrt{7}}}{4}+\frac{\left(-1-\sqrt{7}\right)\arcsin\left(\frac{x+1}{\sqrt{2-\frac{\sqrt{7}}{2}+\frac{\left(-1-\sqrt{7}\right)^2}{4}}}\right)}{4}-\frac{\left(2-\frac{\sqrt{7}}{2}\right)\operatorname{arctanh}\left(\frac{}{\left(-\frac{1}{2}+\frac{\sqrt{7}}{2}\right)}\right)}{}\right.}{7}$
trager	Expression too large to display

input
```
int(1/(x+(-x^2-2*x+3)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/7*7^(1/2)*(1/4*(-4*(x+1/2-1/2*7^(1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1
/2))+8-2*7^(1/2))^(1/2)+1/4*(-1-7^(1/2))*arcsin(1/(2-1/2*7^(1/2)+1/4*(-1-7
^(1/2))^2)^(1/2)*(x+1))-1/2*(2-1/2*7^(1/2))/(-1/2+1/2*7^(1/2))*arctanh((4-
7^(1/2)+(-1-7^(1/2))*(x+1/2-1/2*7^(1/2)))/(-1/2+1/2*7^(1/2))/(-4*(x+1/2-1/
2*7^(1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^(1/2))^(1/2)))+1/7*7
^(1/2)*(1/4*(-4*(x+1/2+1/2*7^(1/2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+8
+2*7^(1/2))^(1/2)+1/4*(-1+7^(1/2))*arcsin(1/(2+1/2*7^(1/2)+1/4*(-1+7^(1/2)
)^2)^(1/2)*(x+1))-1/2*(2+1/2*7^(1/2))/(1/2+1/2*7^(1/2))*arctanh((4+7^(1/2)
+(-1+7^(1/2))*(x+1/2+1/2*7^(1/2)))/(1/2+1/2*7^(1/2))/(-4*(x+1/2+1/2*7^(1/2
))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+8+2*7^(1/2))^(1/2)))+1/4*ln(2*x^2+
2*x-3)+1/14*7^(1/2)*arctanh(1/14*(4*x+2)*7^(1/2))
```

3.753.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(136) = 272$.

Time = 0.28 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.07

$$\int \frac{1}{x+\sqrt{3-2x-x^2}}\,dx$$
$$= \frac{1}{56}\sqrt{7}\log\left(\frac{24\,x^4+62\,x^3-153\,x^2+2\sqrt{7}(3\,x^4+x^3-45\,x^2+45\,x)-(14\,x^3-84\,x^2+\sqrt{7}(8\,x^3-30\,x^2\cdots}{4\,x^4+8\,x^3-8\,x^2-12\,x+9}\right.$$
$$+\frac{1}{56}\sqrt{7}\log\left(\frac{24\,x^4+62\,x^3-153\,x^2-2\sqrt{7}(3\,x^4+x^3-45\,x^2+45\,x)+(14\,x^3-84\,x^2-\sqrt{7}(8\,x^3-30\,x\cdots}{4\,x^4+8\,x^3-8\,x^2-12\,x+9}\right.$$
$$+\frac{1}{28}\sqrt{7}\log\left(\frac{2\,x^2+\sqrt{7}(2\,x+1)+2\,x+4}{2\,x^2+2\,x-3}\right)-\frac{1}{2}\arctan\left(\frac{\sqrt{-x^2-2\,x+3}(x+1)}{x^2+2\,x-3}\right)$$
$$+\frac{1}{4}\log\left(2\,x^2+2\,x-3\right)-\frac{1}{8}\log\left(\frac{2\sqrt{-x^2-2\,x+3}x+2\,x-3}{x^2}\right)$$
$$+\frac{1}{8}\log\left(-\frac{2\sqrt{-x^2-2\,x+3}x-2\,x+3}{x^2}\right)$$

input
```
integrate(1/(x+(-x^2-2*x+3)^(1/2)),x, algorithm="fricas")
```

output
```
1/56*sqrt(7)*log((24*x^4 + 62*x^3 - 153*x^2 + 2*sqrt(7)*(3*x^4 + x^3 - 45*
x^2 + 45*x) - (14*x^3 - 84*x^2 + sqrt(7)*(8*x^3 - 30*x^2 + 27*x - 27) + 12
6*x)*sqrt(-x^2 - 2*x + 3) + 180*x - 135)/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9
)) + 1/56*sqrt(7)*log((24*x^4 + 62*x^3 - 153*x^2 - 2*sqrt(7)*(3*x^4 + x^3
- 45*x^2 + 45*x) + (14*x^3 - 84*x^2 - sqrt(7)*(8*x^3 - 30*x^2 + 27*x - 27)
 + 126*x)*sqrt(-x^2 - 2*x + 3) + 180*x - 135)/(4*x^4 + 8*x^3 - 8*x^2 - 12*
x + 9)) + 1/28*sqrt(7)*log((2*x^2 + sqrt(7)*(2*x + 1) + 2*x + 4)/(2*x^2 +
2*x - 3)) - 1/2*arctan(sqrt(-x^2 - 2*x + 3)*(x + 1)/(x^2 + 2*x - 3)) + 1/4
*log(2*x^2 + 2*x - 3) - 1/8*log((2*sqrt(-x^2 - 2*x + 3)*x + 2*x - 3)/x^2)
+ 1/8*log(-(2*sqrt(-x^2 - 2*x + 3)*x - 2*x + 3)/x^2)
```

3.753.6 Sympy [F]

$$\int \frac{1}{x+\sqrt{3-2x-x^2}}\,dx=\int \frac{1}{x+\sqrt{-x^2-2x+3}}\,dx$$

input
```
integrate(1/(x+(-x**2-2*x+3)**(1/2)),x)
```

output
```
Integral(1/(x + sqrt(-x**2 - 2*x + 3)), x)
```

3.753.7 Maxima [F]

$$\int \frac{1}{x+\sqrt{3-2x-x^2}}\,dx=\int \frac{1}{x+\sqrt{-x^2-2\,x+3}}\,dx$$

input
```
integrate(1/(x+(-x^2-2*x+3)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(x + sqrt(-x^2 - 2*x + 3)), x)
```

3.753.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(136) = 272.

Time = 0.39 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.59

$$\begin{aligned}
&\int \frac{1}{x+\sqrt{3-2x-x^2}}\,dx \\
&= -\frac{1}{28}\sqrt{7}\log\left(\frac{\left|4\,x-2\,\sqrt{7}+2\right|}{\left|4\,x+2\,\sqrt{7}+2\right|}\right)+\frac{1}{28}\sqrt{7}\log\left(\frac{\left|-2\,\sqrt{7}+\frac{6\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1}+4\right|}{\left|2\,\sqrt{7}+\frac{6\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1}+4\right|}\right) \\
&\quad -\frac{1}{28}\sqrt{7}\log\left(\frac{\left|-2\,\sqrt{7}+\frac{2\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1}-4\right|}{\left|2\,\sqrt{7}+\frac{2\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1}-4\right|}\right) \\
&\quad +\frac{1}{2}\arcsin\left(\frac{1}{2}\,x+\frac{1}{2}\right)+\frac{1}{4}\log\left(\left|2\,x^2+2\,x-3\right|\right) \\
&\quad +\frac{1}{4}\log\left(\left|\frac{4\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1}+\frac{3\left(\sqrt{-x^2-2\,x+3}-2\right)^2}{(x+1)^2}-1\right|\right) \\
&\quad -\frac{1}{4}\log\left(\left|-\frac{4\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1}+\frac{\left(\sqrt{-x^2-2\,x+3}-2\right)^2}{(x+1)^2}-3\right|\right)
\end{aligned}$$

input
```
integrate(1/(x+(-x^2-2*x+3)^(1/2)),x, algorithm="giac")
```

output
```
-1/28*sqrt(7)*log(abs(4*x - 2*sqrt(7) + 2)/abs(4*x + 2*sqrt(7) + 2)) + 1/2
8*sqrt(7)*log(abs(-2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)/a
bs(2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)) - 1/28*sqrt(7)*l
og(abs(-2*sqrt(7) + 2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)/abs(2*sqrt(7
) + 2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)) + 1/2*arcsin(1/2*x + 1/2) +
 1/4*log(abs(2*x^2 + 2*x - 3)) + 1/4*log(abs(4*(sqrt(-x^2 - 2*x + 3) - 2)/
(x + 1) + 3*(sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 - 1)) - 1/4*log(abs(-4*
(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + (sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^
2 - 3))
```

3.753.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x+\sqrt{3-2x-x^2}}\,dx=\int \frac{1}{x+\sqrt{-x^2-2\,x+3}}\,dx$$

input
```
int(1/(x + (3 - x^2 - 2*x)^(1/2)),x)
```

output
```
int(1/(x + (3 - x^2 - 2*x)^(1/2)), x)
```

3.754 $\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^2}\,dx$

3.754.1 Optimal result

Integrand size = 18, antiderivative size = 172

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^2}\,dx = \frac{2\left(4-\sqrt{3}+\frac{3\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{7\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)} + \frac{8\operatorname{arctanh}\left(\frac{3-x-\sqrt{3}x-\sqrt{3}\sqrt{3-2x-x^2}}{\sqrt{7}x}\right)}{7\sqrt{7}}$$

output
```
8/49*arctanh(1/7*(3-x-x*3^(1/2)-3^(1/2)*(-x^2-2*x+3)^(1/2))/x*7^(1/2))*7^(
1/2)+2/7*(4-3^(1/2)+3*(3^(1/2)-(-x^2-2*x+3)^(1/2))/x)/(2-3^(1/2)-2*(1+3^(1
/2))*(3^(1/2)-(-x^2-2*x+3)^(1/2))/x+3^(1/2)*(3^(1/2)-(-x^2-2*x+3)^(1/2))^2
/x^2)
```

3.754.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.55

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^2}\,dx = \frac{3+6\sqrt{3-2x-x^2}-2x\left(4+\sqrt{3-2x-x^2}\right)}{14\left(-3+2x+2x^2\right)} + \frac{8\operatorname{arctanh}\left(\frac{2-2x+\sqrt{3-2x-x^2}}{\sqrt{7}(-1+x)}\right)}{7\sqrt{7}}$$

input
```
Integrate[(x + Sqrt[3 - 2*x - x^2])^(-2),x]
```

output
```
(3 + 6*Sqrt[3 - 2*x - x^2] - 2*x*(4 + Sqrt[3 - 2*x - x^2]))/(14*(-3 + 2*x
+ 2*x^2)) + (8*ArcTanh[(2 - 2*x + Sqrt[3 - 2*x - x^2])/(Sqrt[7]*(-1 + x))]
)/(7*Sqrt[7])
```

3.754.3 Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7285, 25, 2191, 27, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{-x^2-2x+3}+x\right)^2}dx$$

$$\Big\downarrow \text{7285}$$

$$2\int -\frac{-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}+\frac{2\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}+\sqrt{3}}{\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)^2}d\left(-\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{x}\right)$$

$$\Big\downarrow \text{25}$$

$$-2\int \frac{-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}+\frac{2\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}+\sqrt{3}}{\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)^2}d\left(-\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{x}\right)$$

$$\Big\downarrow \text{2191}$$

$$2\left(\frac{1}{28}\int \frac{16}{\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2}d\left(-\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{x}\right)+\frac{\cdots}{7\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-\cdots}\right.}{x\cdots}\cdots\right.}\cdots\right.$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{4}{7}\int \frac{1}{\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2}d\left(-\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{x}\right)+\frac{\cdots}{7\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2\cdots}\right.}{x^2}\right.}\right.$$

$\downarrow$ 1083

$$2\left(\frac{\frac{3\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+4}{7\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)}-\frac{8}{7}\int \frac{1}{28-\frac{\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}}d\left(2\left(1+\sqrt{3}\right)\cdots\right.\right.$$

$\downarrow$ 219

$$2\left(\frac{4\text{arctanh}\left(\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{2\sqrt{7}x}\right)}{7\sqrt{7}}+\frac{\frac{3\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+4}{7\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)}\right)$$

input
```
Int[(x + Sqrt[3 - 2*x - x^2])^(-2),x]
```

output
```
2*((4 - Sqrt[3] + (3*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x)/(7*(2 - Sqrt[3] -
 (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x + (Sqrt[3]*(Sqrt[3] -
 Sqrt[3 - 2*x - x^2])^2)/x^2)) + (4*ArcTanh[(Sqrt[3] - Sqrt[3 - 2*x - x^2]
)/(2*Sqrt[7]*x)])/(7*Sqrt[7]))
```

3.754.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.754. $\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^2}\,dx$

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 7285
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfSquareRootOfQuadratic[u, x]}, Si
mp[2   Subst[Int[lst[[1]], x], x, lst[[2]]], x] /;  !FalseQ[lst] && EqQ[lst
[[3]], 1]] /; EulerIntegrandQ[u, x]
```

3.754.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.60

method	result
trager	$\frac{(-3+x)x}{14x^2+14x-21}-\frac{(-3+x)\sqrt{-x^2-2x+3}}{7(2x^2+2x-3)}+\frac{4\,\mathrm{RootOf}\left(\_Z^2-7\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2-7\right)x-3\,\mathrm{RootOf}\left(\_Z^2-7\right)+7\sqrt{-x^2-2x+3}}{\mathrm{RootOf}\left(\_Z^2-7\right)x+x-3}\right)}{49}$
default	Expression too large to display

input
```
int(1/(x+(-x^2-2*x+3)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*(-3+x)*x/(2*x^2+2*x-3)-1/7*(-3+x)/(2*x^2+2*x-3)*(-x^2-2*x+3)^(1/2)+4/4
9*RootOf(_Z^2-7)*ln(-(RootOf(_Z^2-7)*x-3*RootOf(_Z^2-7)+7*(-x^2-2*x+3)^(1/
2))/(RootOf(_Z^2-7)*x+x-3))
```

3.754.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^2}\,dx = \frac{2\sqrt{7}(2x^2+2x-3)\log\left(\frac{x^4+44x^3-\sqrt{7}(3x^3+x^2-45x+45)\sqrt{-x^2-2x+3}+26x^2-276x+207}{4x^4+8x^3-8x^2-12x+9}\right)+4\sqrt{7}(2x^2+2x-3)\log}{98\,(2x^2+2x-3)}$$

input
```
integrate(1/(x+(-x^2-2*x+3)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/98*(2*sqrt(7)*(2*x^2 + 2*x - 3)*log((x^4 + 44*x^3 - sqrt(7)*(3*x^3 + x^2
 - 45*x + 45)*sqrt(-x^2 - 2*x + 3) + 26*x^2 - 276*x + 207)/(4*x^4 + 8*x^3
- 8*x^2 - 12*x + 9)) + 4*sqrt(7)*(2*x^2 + 2*x - 3)*log((2*x^2 + sqrt(7)*(2
*x + 1) + 2*x + 4)/(2*x^2 + 2*x - 3)) - 14*sqrt(-x^2 - 2*x + 3)*(x - 3) -
56*x + 21)/(2*x^2 + 2*x - 3)
```

3.754.6 Sympy [F]

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^2}\,dx = \int \frac{1}{\left(x+\sqrt{-x^2-2x+3}\right)^2}\,dx$$

input
```
integrate(1/(x+(-x**2-2*x+3)**(1/2))**2,x)
```

output
```
Integral((x + sqrt(-x**2 - 2*x + 3))**(-2), x)
```

3.754.7 Maxima [F]

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^2}\,dx = \int \frac{1}{\left(x+\sqrt{-x^2-2\,x+3}\right)^2}\,dx$$

input
```
integrate(1/(x+(-x^2-2*x+3)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate((x + sqrt(-x^2 - 2*x + 3))^(-2), x)
```

3.754.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(132) = 264$.

Time = 0.37 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.03

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^2}\,dx$$

$$= -\frac{2}{49}\sqrt{7}\log\left(\frac{|4x-2\sqrt{7}+2|}{|4x+2\sqrt{7}+2|}\right)+\frac{2}{49}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+4\right|}{\left|2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+4\right|}\right)$$
$$-\frac{2}{49}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}-4\right|}{\left|2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}-4\right|}\right)-\frac{8x-3}{14\,(2x^2+2x-3)}$$
$$-\frac{8\left(\frac{5\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+\frac{26\left(\sqrt{-x^2-2x+3}-2\right)^2}{(x+1)^2}+\frac{11\left(\sqrt{-x^2-2x+3}-2\right)^3}{(x+1)^3}-6\right)}{21\left(\frac{8\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+\frac{26\left(\sqrt{-x^2-2x+3}-2\right)^2}{(x+1)^2}+\frac{8\left(\sqrt{-x^2-2x+3}-2\right)^3}{(x+1)^3}-\frac{3\left(\sqrt{-x^2-2x+3}-2\right)^4}{(x+1)^4}-3\right)}$$

input
```
integrate(1/(x+(-x^2-2*x+3)^(1/2))^2,x, algorithm="giac")
```

output
```
-2/49*sqrt(7)*log(abs(4*x - 2*sqrt(7) + 2)/abs(4*x + 2*sqrt(7) + 2)) + 2/4
9*sqrt(7)*log(abs(-2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)/a
bs(2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)) - 2/49*sqrt(7)*l
og(abs(-2*sqrt(7) + 2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)/abs(2*sqrt(7
) + 2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)) - 1/14*(8*x - 3)/(2*x^2 + 2
*x - 3) - 8/21*(5*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 26*(sqrt(-x^2 - 2*x
 + 3) - 2)^2/(x + 1)^2 + 11*(sqrt(-x^2 - 2*x + 3) - 2)^3/(x + 1)^3 - 6)/(8
*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 26*(sqrt(-x^2 - 2*x + 3) - 2)^2/(x +
 1)^2 + 8*(sqrt(-x^2 - 2*x + 3) - 2)^3/(x + 1)^3 - 3*(sqrt(-x^2 - 2*x + 3)
 - 2)^4/(x + 1)^4 - 3)
```

3.754.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^2}\,dx=\int \frac{1}{\left(x+\sqrt{-x^2-2\,x+3}\right)^2}\,dx$$

input
```
int(1/(x + (3 - x^2 - 2*x)^(1/2))^2,x)
```

output
```
int(1/(x + (3 - x^2 - 2*x)^(1/2))^2, x)
```

3.755 $\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^3}\,dx$

3.755.1 Optimal result

Integrand size = 18, antiderivative size = 307

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^3}\,dx = -\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)^2} + \frac{2\left(18-43\sqrt{3}-\frac{\left(18+49\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{147\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)} + \frac{12\operatorname{arctanh}\left(\frac{3-x-\sqrt{3}x-\sqrt{3}\sqrt{3-2x-x^2}}{\sqrt{7}x}\right)}{49\sqrt{7}}$$

output
```
12/343*arctanh(1/7*(3-x-x*3^(1/2)-3^(1/2)*(-x^2-2*x+3)^(1/2))/x*7^(1/2))*7
^(1/2)-4/21*(9-5*3^(1/2)+(21+5*3^(1/2))*(3^(1/2)-(-x^2-2*x+3)^(1/2))/x)/(2
-3^(1/2)-2*(1+3^(1/2))*(3^(1/2)-(-x^2-2*x+3)^(1/2))/x+3^(1/2)*(3^(1/2)-(-x
^2-2*x+3)^(1/2))^2/x^2)^2+2/147*(18-43*3^(1/2)-(18+49*3^(1/2))*(3^(1/2)-(-
x^2-2*x+3)^(1/2))/x)/(2-3^(1/2)-2*(1+3^(1/2))*(3^(1/2)-(-x^2-2*x+3)^(1/2))
/x+3^(1/2)*(3^(1/2)-(-x^2-2*x+3)^(1/2))^2/x^2)
```

3.755.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.37

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^3}\,dx$$
$$=\frac{\frac{7(-279+300x+26x^2-48x^3)}{(-3+2x+2x^2)^2}+\frac{14\sqrt{3-2x-x^2}(15+83x-58x^2-34x^3)}{(-3+2x+2x^2)^2}+48\sqrt{7}\text{arctanh}\left(\frac{2-2x+\sqrt{3-2x-x^2}}{\sqrt{7}(-1+x)}\right)}{1372}$$

input
```
Integrate[(x + Sqrt[3 - 2*x - x^2])^(-3),x]
```

output
```
((7*(-279 + 300*x + 26*x^2 - 48*x^3))/(-3 + 2*x + 2*x^2)^2 + (14*Sqrt[3 -
2*x - x^2]*(15 + 83*x - 58*x^2 - 34*x^3))/(-3 + 2*x + 2*x^2)^2 + 48*Sqrt[7
]*ArcTanh[(2 - 2*x + Sqrt[3 - 2*x - x^2])/(Sqrt[7]*(-1 + x))])/1372
```

3.755.3 Rubi [A] (warning: unable to verify)

Time = 0.54 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {7285, 2191, 27, 2191, 27, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{-x^2-2x+3}+x\right)^3}dx$$

$$\Big\downarrow \; 7285$$

$$2\int \frac{-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^4}{x^4}+\frac{2\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^3}{x^3}+\frac{2\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}+\sqrt{3}}{\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)^3}d\left(-\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{x}\right)$$

$$\Big\downarrow \; 2191$$

$$2\left(-\frac{1}{56}\int -\frac{8\left(-\frac{21\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{42\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}+16\sqrt{3}+21\right)}{3\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)^2}d\left(-\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{x}\right)-\frac{}{21\left(\right.}\right.$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{1}{21}\int \frac{-\frac{21\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{42\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}+16\sqrt{3}+21}{\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)^2}d\left(-\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{x}\right)-\frac{}{21\left(\frac{\sqrt{3}\left(\right.}{}\right.}\right.$$

$$\Big\downarrow \; 2191$$

$$2\left(\frac{1}{21}\left(\frac{-\frac{\left(18+49\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-43\sqrt{3}+18}{7\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)}-\frac{1}{28}\int -\frac{72}{\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right.}{}}\right.\right.$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{1}{21}\left(\frac{18}{7}\int \frac{1}{\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2}d\left(-\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{x}\right)+\frac{}{7\left(\frac{\sqrt{3}\left(\right.}{}\right.}\right.\right.$$

$$\Big\downarrow \; 1083$$

$$2\left(\frac{1}{21}\left(\frac{-\frac{\left(18+49\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-43\sqrt{3}+18}{7\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)}-\frac{36}{7}\int \frac{1}{28-\frac{\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}}d\left(2\left(1+\right.\right.\right.\right.$$

$$\Big\downarrow \; 219$$

3.755. $\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^3}\,dx$

$$2\left(\frac{1}{21}\left(\frac{18\text{arctanh}\left(\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{2\sqrt{7}x}\right)}{7\sqrt{7}}+\frac{-\frac{\left(18+49\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-43\sqrt{3}+18}{7\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)}\right)-\frac{}{21\left(\right.}\right.$$

input
```
Int[(x + Sqrt[3 - 2*x - x^2])^(-3),x]
```

output
```
2*((-2*(9 - 5*Sqrt[3] + ((21 + 5*Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))
/x))/(21*(2 - Sqrt[3] - (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/
x + (Sqrt[3]*(Sqrt[3] - Sqrt[3 - 2*x - x^2])^2)/x^2)^2) + ((18 - 43*Sqrt[3
] - ((18 + 49*Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x)/(7*(2 - Sqrt[3]
 - (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x + (Sqrt[3]*(Sqrt[3]
 - Sqrt[3 - 2*x - x^2])^2)/x^2)) + (18*ArcTanh[(Sqrt[3] - Sqrt[3 - 2*x - x
^2])/(2*Sqrt[7]*x)])/(7*Sqrt[7]))/21)
```

3.755.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.755. $\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^3}\,dx$

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 7285
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfSquareRootOfQuadratic[u, x]}, Si
mp[2   Subst[Int[lst[[1]], x], x, lst[[2]]], x] /;  !FalseQ[lst] && EqQ[lst
[[3]], 1]] /; EulerIntegrandQ[u, x]
```

3.755.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.33 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.42

method	result
trager	$\frac{(62x^3+100x^2-111x-36)x}{98(2x^2+2x-3)^2} - \frac{(34x^3+58x^2-83x-15)\sqrt{-x^2-2x+3}}{98(2x^2+2x-3)^2} - \frac{6\,\text{RootOf}\left(\_Z^2-7\right)\ln\left(\frac{\text{RootOf}\left(\_Z^2-7\right)x-3\,\text{RootOf}\left(\_\right.}{\text{RootOf}\left(\_Z^2-7\right)}\right.}{343}$
default	Expression too large to display

input
```
int(1/(x+(-x^2-2*x+3)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
1/98*(62*x^3+100*x^2-111*x-36)*x/(2*x^2+2*x-3)^2-1/98*(34*x^3+58*x^2-83*x-
15)/(2*x^2+2*x-3)^2*(-x^2-2*x+3)^(1/2)-6/343*RootOf(_Z^2-7)*ln((RootOf(_Z^
2-7)*x-3*RootOf(_Z^2-7)-7*(-x^2-2*x+3)^(1/2))/(RootOf(_Z^2-7)*x-x+3))
```

3.755.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.73

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^3}\,dx =$$
$$-\frac{336\,x^3-6\,\sqrt{7}(4\,x^4+8\,x^3-8\,x^2-12\,x+9)\log\left(\frac{x^4+44\,x^3-\sqrt{7}(3\,x^3+x^2-45\,x+45)\sqrt{-x^2-2\,x+3}+26\,x^2-276\,x+207}{4\,x^4+8\,x^3-8\,x^2-12\,x+9}\right)}{}$$

input
```
integrate(1/(x+(-x^2-2*x+3)^(1/2))^3,x, algorithm="fricas")
```

output
```
-1/1372*(336*x^3 - 6*sqrt(7)*(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)*log((x^4 +
 44*x^3 - sqrt(7)*(3*x^3 + x^2 - 45*x + 45)*sqrt(-x^2 - 2*x + 3) + 26*x^2
- 276*x + 207)/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)) - 12*sqrt(7)*(4*x^4 + 8
*x^3 - 8*x^2 - 12*x + 9)*log((2*x^2 + sqrt(7)*(2*x + 1) + 2*x + 4)/(2*x^2
+ 2*x - 3)) - 182*x^2 + 14*(34*x^3 + 58*x^2 - 83*x - 15)*sqrt(-x^2 - 2*x +
 3) - 2100*x + 1953)/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)
```

3.755.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(x+(-x**2-2*x+3)**(1/2))**3,x)
```

output
```
Timed out
```

3.755.7 Maxima [F]

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^3}\,dx = \int \frac{1}{\left(x+\sqrt{-x^2-2\,x+3}\right)^3}\,dx$$

input
```
integrate(1/(x+(-x^2-2*x+3)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate((x + sqrt(-x^2 - 2*x + 3))^(-3), x)
```

3.755.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.47

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^3}\,dx$$

$$= -\frac{3}{343}\sqrt{7}\log\left(\frac{|4x-2\sqrt{7}+2|}{|4x+2\sqrt{7}+2|}\right)+\frac{3}{343}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+4\right|}{\left|2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+4\right|}\right)$$
$$-\frac{3}{343}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}-4\right|}{\left|2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}-4\right|}\right)-\frac{48x^3-26x^2-300x+279}{196\left(2x^2+2x-3\right)^2}$$
$$+\frac{4\left(\frac{231\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+\frac{3286\left(\sqrt{-x^2-2x+3}-2\right)^2}{(x+1)^2}-\frac{4441\left(\sqrt{-x^2-2x+3}-2\right)^3}{(x+1)^3}-\frac{18906\left(\sqrt{-x^2-2x+3}-2\right)^4}{(x+1)^4}-\frac{12487\left(\sqrt{-x^2-\right.}{(x+\right.}}{441\left(\frac{8\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+\frac{26\left(\sqrt{-x^2-2x+3}-2\right)^2}{(x+1)^2}+\frac{8\left(\sqrt{-x^2-2x+3}-2\right)^3}{(x+1)^3}-\right.}$$

input
```
integrate(1/(x+(-x^2-2*x+3)^(1/2))^3,x, algorithm="giac")
```

output
```
-3/343*sqrt(7)*log(abs(4*x - 2*sqrt(7) + 2)/abs(4*x + 2*sqrt(7) + 2)) + 3/
343*sqrt(7)*log(abs(-2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)
/abs(2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)) - 3/343*sqrt(7
)*log(abs(-2*sqrt(7) + 2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)/abs(2*sqr
t(7) + 2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)) - 1/196*(48*x^3 - 26*x^2
 - 300*x + 279)/(2*x^2 + 2*x - 3)^2 + 4/441*(231*(sqrt(-x^2 - 2*x + 3) - 2
)/(x + 1) + 3286*(sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 - 4441*(sqrt(-x^2
- 2*x + 3) - 2)^3/(x + 1)^3 - 18906*(sqrt(-x^2 - 2*x + 3) - 2)^4/(x + 1)^4
 - 12487*(sqrt(-x^2 - 2*x + 3) - 2)^5/(x + 1)^5 + 946*(sqrt(-x^2 - 2*x + 3
) - 2)^6/(x + 1)^6 + 1977*(sqrt(-x^2 - 2*x + 3) - 2)^7/(x + 1)^7 - 414)/(8
*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 26*(sqrt(-x^2 - 2*x + 3) - 2)^2/(x +
 1)^2 + 8*(sqrt(-x^2 - 2*x + 3) - 2)^3/(x + 1)^3 - 3*(sqrt(-x^2 - 2*x + 3)
 - 2)^4/(x + 1)^4 - 3)^2
```

3.755.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^3}\,dx=\int \frac{1}{\left(x+\sqrt{-x^2-2\,x+3}\right)^3}\,dx$$

input
```
int(1/(x + (3 - x^2 - 2*x)^(1/2))^3,x)
```

output
```
int(1/(x + (3 - x^2 - 2*x)^(1/2))^3, x)
```

3.756 $\int \frac{1}{x+\sqrt{-3-2x+x^2}} \, dx$

3.756.1 Optimal result

Integrand size = 16, antiderivative size = 65

$$\int \frac{1}{x+\sqrt{-3-2x+x^2}} \, dx = -\frac{2}{1-x-\sqrt{-3-2x+x^2}} + 2\log\left(1-x-\sqrt{-3-2x+x^2}\right) - \frac{3}{2}\log\left(x+\sqrt{-3-2x+x^2}\right)$$

output
```
2*ln(1-x-(x^2-2*x-3)^(1/2))-3/2*ln(x+(x^2-2*x-3)^(1/2))-2/(1-x-(x^2-2*x-3)
^(1/2))
```

3.756.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.20

$$\int \frac{1}{x+\sqrt{-3-2x+x^2}} \, dx = \frac{1}{2}\left(x-\sqrt{-3-2x+x^2} - \log\left(-1-x+\sqrt{-3-2x+x^2}\right) + 4\log\left(1+x+\sqrt{-3-2x+x^2}\right) - 3\log\left(3+3x+\sqrt{-3-2x+x^2}\right)\right)$$

input
```
Integrate[(x + Sqrt[-3 - 2*x + x^2])^(-1),x]
```

output
```
(x - Sqrt[-3 - 2*x + x^2] - Log[-1 - x + Sqrt[-3 - 2*x + x^2]] + 4*Log[1 +
 x + Sqrt[-3 - 2*x + x^2]] - 3*Log[3 + 3*x + Sqrt[-3 - 2*x + x^2]])/2
```

3.756.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2541, 27, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2-2x-3}+x}dx$$

$$\Big\downarrow \text{2541}$$

$$2\int -\frac{-\left(x+\sqrt{x^2-2x-3}\right)^2+2\left(x+\sqrt{x^2-2x-3}\right)+3}{4\left(-x-\sqrt{x^2-2x-3}+1\right)^2\left(x+\sqrt{x^2-2x-3}\right)}d\left(x+\sqrt{x^2-2x-3}\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}\int \frac{-\left(x+\sqrt{x^2-2x-3}\right)^2+2\left(x+\sqrt{x^2-2x-3}\right)+3}{\left(-x-\sqrt{x^2-2x-3}+1\right)^2\left(x+\sqrt{x^2-2x-3}\right)}d\left(x+\sqrt{x^2-2x-3}\right)$$

$$\Big\downarrow \text{1195}$$

$$-\frac{1}{2}\int \left(\frac{3}{x+\sqrt{x^2-2x-3}}-\frac{4}{x+\sqrt{x^2-2x-3}-1}+\frac{4}{\left(x+\sqrt{x^2-2x-3}-1\right)^2}\right)d\left(x+\sqrt{x^2-2x-3}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{4}{-\sqrt{x^2-2x-3}-x+1}+4\log\left(-\sqrt{x^2-2x-3}-x+1\right)-3\log\left(\sqrt{x^2-2x-3}+x\right)\right)$$

input
```
Int[(x + Sqrt[-3 - 2*x + x^2])^(-1),x]
```

output
```
(-4/(1 - x - Sqrt[-3 - 2*x + x^2]) + 4*Log[1 - x - Sqrt[-3 - 2*x + x^2]] -
 3*Log[x + Sqrt[-3 - 2*x + x^2]])/2
```

3.756.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2541
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Simp[2   Subst[Int[(g + h*x^n)^p*((d
^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x
)^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e
, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.756.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09

method	result	size
default	$-\frac{\sqrt{4\left(x+\frac{3}{2}\right)^{2}-20x-21}}{4}+\frac{5\ln\left(-1+x+\sqrt{\left(x+\frac{3}{2}\right)^{2}-5x-\frac{21}{4}}\right)}{4}+\frac{3\,\operatorname{arctanh}\left(\frac{-2-\frac{10x}{3}}{\sqrt{4\left(x+\frac{3}{2}\right)^{2}-20x-21}}\right)}{4}+\frac{x}{2}-\frac{3\ln\left(2x+3\right)}{4}$	71
trager	$\frac{x}{2}-\frac{\sqrt{x^{2}-2x-3}}{2}-\frac{\ln\left(\sqrt{x^{2}-2x-3}\,x^{3}-x^{4}+3\sqrt{x^{2}-2x-3}\,x^{2}-2x^{3}+\sqrt{x^{2}-2x-3}\,x+4x^{2}-3\sqrt{x^{2}-2x-3}+12x+9\right)}{2}$	93

input
```
int(1/(x+(x^2-2*x-3)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(4*(x+3/2)^2-20*x-21)^(1/2)+5/4*ln(-1+x+((x+3/2)^2-5*x-21/4)^(1/2))+3
/4*arctanh(2/3*(-3-5*x)/(4*(x+3/2)^2-20*x-21)^(1/2))+1/2*x-3/4*ln(2*x+3)
```

3.756.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.18

$$\int \frac{1}{x+\sqrt{-3-2x+x^2}}\,dx = \frac{1}{2}\,x - \frac{1}{2}\,\sqrt{x^2-2\,x-3} - \frac{3}{4}\,\log\left(2\,x+3\right) - \frac{5}{4}\,\log\left(-x+\sqrt{x^2-2\,x-3}+1\right) + \frac{3}{4}\,\log\left(-x+\sqrt{x^2-2\,x-3}\right) - \frac{3}{4}\,\log\left(-x+\sqrt{x^2-2\,x-3}-3\right)$$

input
```
integrate(1/(x+(x^2-2*x-3)^(1/2)),x, algorithm="fricas")
```

output
```
1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 3/4*log(2*x + 3) - 5/4*log(-x + sqrt(x^2
 - 2*x - 3) + 1) + 3/4*log(-x + sqrt(x^2 - 2*x - 3)) - 3/4*log(-x + sqrt(x
^2 - 2*x - 3) - 3)
```

3.756.6 Sympy [F]

$$\int \frac{1}{x+\sqrt{-3-2x+x^2}}\,dx = \int \frac{1}{x+\sqrt{x^2-2x-3}}\,dx$$

input
```
integrate(1/(x+(x**2-2*x-3)**(1/2)),x)
```

output
```
Integral(1/(x + sqrt(x**2 - 2*x - 3)), x)
```

3.756.7 Maxima [F]

$$\int \frac{1}{x+\sqrt{-3-2x+x^2}}\,dx = \int \frac{1}{x+\sqrt{x^2-2\,x-3}}\,dx$$

input
```
integrate(1/(x+(x^2-2*x-3)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(x + sqrt(x^2 - 2*x - 3)), x)
```

3.756.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

$$\int \frac{1}{x+\sqrt{-3-2x+x^2}}\,dx = \frac{1}{2}\,x - \frac{1}{2}\,\sqrt{x^2-2\,x-3} - \frac{3}{4}\,\log\left(|2\,x+3|\right) - \frac{5}{4}\,\log\left(\left|-x+\sqrt{x^2-2\,x-3}+1\right|\right) + \frac{3}{4}\,\log\left(\left|-x+\sqrt{x^2-2\,x-3}\right|\right) - \frac{3}{4}\,\log\left(\left|-x+\sqrt{x^2-2\,x-3}-3\right|\right)$$

input
```
integrate(1/(x+(x^2-2*x-3)^(1/2)),x, algorithm="giac")
```

output
```
1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 3/4*log(abs(2*x + 3)) - 5/4*log(abs(-x +
 sqrt(x^2 - 2*x - 3) + 1)) + 3/4*log(abs(-x + sqrt(x^2 - 2*x - 3))) - 3/4*
log(abs(-x + sqrt(x^2 - 2*x - 3) - 3))
```

3.756.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x+\sqrt{-3-2x+x^2}}\,dx = \frac{x}{2} - \frac{3\,\ln\left(x+\frac{3}{2}\right)}{4} - \int \frac{\sqrt{x^2-2\,x-3}}{2\,x+3}\,dx$$

input
```
int(1/(x + (x^2 - 2*x - 3)^(1/2)),x)
```

output
```
x/2 - (3*log(x + 3/2))/4 - int((x^2 - 2*x - 3)^(1/2)/(2*x + 3), x)
```

3.757 $\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^2} dx$

3.757.1 Optimal result

Integrand size = 16, antiderivative size = 83

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^2} dx = -\frac{2}{1-x-\sqrt{-3-2x+x^2}}+\frac{3}{2\left(x+\sqrt{-3-2x+x^2}\right)} +4\log\left(1-x-\sqrt{-3-2x+x^2}\right) -4\log\left(x+\sqrt{-3-2x+x^2}\right)$$

output
```
4*ln(1-x-(x^2-2*x-3)^(1/2))-4*ln(x+(x^2-2*x-3)^(1/2))-2/(1-x-(x^2-2*x-3)^(
1/2))+3/2/(x+(x^2-2*x-3)^(1/2))
```

3.757.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^2} dx = \frac{-9+6x+4x^2-4(3+x)\sqrt{-3-2x+x^2}-32(3+2x)\text{arctanh}\left(\frac{1+x}{2+2x+\sqrt{-3-2x+x^2}}\right)}{4(3+2x)}$$

input
```
Integrate[(x + Sqrt[-3 - 2*x + x^2])^(-2),x]
```

output
```
(-9 + 6*x + 4*x^2 - 4*(3 + x)*Sqrt[-3 - 2*x + x^2] - 32*(3 + 2*x)*ArcTanh[
(1 + x)/(2 + 2*x + Sqrt[-3 - 2*x + x^2])])/(4*(3 + 2*x))
```

3.757.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2541, 27, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{x^2-2x-3}+x\right)^2}dx$$

$$\Big\downarrow \text{2541}$$

$$2\int -\frac{-\left(x+\sqrt{x^2-2x-3}\right)^2+2\left(x+\sqrt{x^2-2x-3}\right)+3}{4\left(-x-\sqrt{x^2-2x-3}+1\right)^2\left(x+\sqrt{x^2-2x-3}\right)^2}d\left(x+\sqrt{x^2-2x-3}\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}\int \frac{-\left(x+\sqrt{x^2-2x-3}\right)^2+2\left(x+\sqrt{x^2-2x-3}\right)+3}{\left(-x-\sqrt{x^2-2x-3}+1\right)^2\left(x+\sqrt{x^2-2x-3}\right)^2}d\left(x+\sqrt{x^2-2x-3}\right)$$

$$\Big\downarrow \text{1195}$$

$$-\frac{1}{2}\int \left(\frac{8}{x+\sqrt{x^2-2x-3}}+\frac{3}{\left(x+\sqrt{x^2-2x-3}\right)^2}-\frac{8}{x+\sqrt{x^2-2x-3}-1}+\frac{4}{\left(x+\sqrt{x^2-2x-3}-1\right)^2}\right)d\left(x\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{4}{-\sqrt{x^2-2x-3}-x+1}+\frac{3}{\sqrt{x^2-2x-3}+x}+8\log\left(-\sqrt{x^2-2x-3}-x+1\right)-8\log\left(\sqrt{x^2-2x-3}+x\right.\right.$$

input
```
Int[(x + Sqrt[-3 - 2*x + x^2])^(-2),x]
```

output
```
(-4/(1 - x - Sqrt[-3 - 2*x + x^2]) + 3/(x + Sqrt[-3 - 2*x + x^2]) + 8*Log[
1 - x - Sqrt[-3 - 2*x + x^2]] - 8*Log[x + Sqrt[-3 - 2*x + x^2]])/2
```

3.757.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2541
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Simp[2   Subst[Int[(g + h*x^n)^p*((d
^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x
)^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e
, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.757.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

method	result
trager	$\frac{(3+x)x}{2x+3}-\frac{(3+x)\sqrt{x^2-2x-3}}{2x+3}+4\ln\left(-\frac{\sqrt{x^2-2x-3}+3+x}{2x+3}\right)$
default	$-2\ln\left(2x+3\right)+\frac{x}{2}-\frac{9}{4(2x+3)}-\frac{2\sqrt{4\left(x+\frac{3}{2}\right)^2-20x-21}}{3}+2\ln\left(-1+x+\sqrt{\left(x+\frac{3}{2}\right)^2-5x-\frac{21}{4}}\right)+2$

input
```
int(1/(x+(x^2-2*x-3)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
(3+x)*x/(2*x+3)-(3+x)/(2*x+3)*(x^2-2*x-3)^(1/2)+4*ln(-((x^2-2*x-3)^(1/2)+3
+x)/(2*x+3))
```

3.757.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.17

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^2} dx$$
$$= \frac{4\,x^2 - 8\,(2\,x+3)\log\left(x^2-\sqrt{x^2-2\,x-3}(x+1)-3\right) - 8\,(2\,x+3)\log\left(2\,x+3\right) + 8\,(2\,x+3)\log\left(-x+\right.}{4\,(2\,x+3)}$$

input
```
integrate(1/(x+(x^2-2*x-3)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/4*(4*x^2 - 8*(2*x + 3)*log(x^2 - sqrt(x^2 - 2*x - 3)*(x + 1) - 3) - 8*(2
*x + 3)*log(2*x + 3) + 8*(2*x + 3)*log(-x + sqrt(x^2 - 2*x - 3)) - 4*sqrt(
x^2 - 2*x - 3)*(x + 3) + 2*x - 15)/(2*x + 3)
```

3.757.6 Sympy [F]

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^2} dx = \int \frac{1}{\left(x+\sqrt{x^2-2x-3}\right)^2} dx$$

input
```
integrate(1/(x+(x**2-2*x-3)**(1/2))**2,x)
```

output
```
Integral((x + sqrt(x**2 - 2*x - 3))**(-2), x)
```

3.757.7 Maxima [F]

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^2} dx = \int \frac{1}{\left(x+\sqrt{x^2-2\,x-3}\right)^2} dx$$

input
```
integrate(1/(x+(x^2-2*x-3)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate((x + sqrt(x^2 - 2*x - 3))^(-2), x)
```

3.757.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(69) = 138$.

Time = 0.34 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.72

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^2}\,dx = \frac{1}{2}\,x - \frac{1}{2}\,\sqrt{x^2-2\,x-3} - \frac{3\,\left(5\,x-5\,\sqrt{x^2-2\,x-3}+3\right)}{4\,\left(\left(x-\sqrt{x^2-2\,x-3}\right)^2+3\,x-3\,\sqrt{x^2-2\,x-3}\right)} - \frac{9}{4\,(2\,x+3)} - 2\,\log\left(|2\,x+3|\right) - 2\,\log\left(\left|-x+\sqrt{x^2-2\,x-3}+1\right|\right) + 2\,\log\left(\left|-x+\sqrt{x^2-2\,x-3}\right|\right) - 2\,\log\left(\left|-x+\sqrt{x^2-2\,x-3}-3\right|\right)$$

input
```
integrate(1/(x+(x^2-2*x-3)^(1/2))^2,x, algorithm="giac")
```

output
```
1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 3/4*(5*x - 5*sqrt(x^2 - 2*x - 3) + 3)/((
x - sqrt(x^2 - 2*x - 3))^2 + 3*x - 3*sqrt(x^2 - 2*x - 3)) - 9/4/(2*x + 3)
- 2*log(abs(2*x + 3)) - 2*log(abs(-x + sqrt(x^2 - 2*x - 3) + 1)) + 2*log(a
bs(-x + sqrt(x^2 - 2*x - 3))) - 2*log(abs(-x + sqrt(x^2 - 2*x - 3) - 3))
```

3.757.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^2}\,dx = \int \frac{1}{\left(x+\sqrt{x^2-2\,x-3}\right)^2}\,dx$$

input
```
int(1/(x + (x^2 - 2*x - 3)^(1/2))^2,x)
```

output
```
int(1/(x + (x^2 - 2*x - 3)^(1/2))^2, x)
```

3.758 $\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^3}\,dx$

3.758.1 Optimal result

Integrand size = 16, antiderivative size = 101

$$\begin{aligned}\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^3}\,dx = {} & -\frac{2}{1-x-\sqrt{-3-2x+x^2}}+\frac{3}{4\left(x+\sqrt{-3-2x+x^2}\right)^2} \\ & +\frac{4}{x+\sqrt{-3-2x+x^2}}+6\log\left(1-x-\sqrt{-3-2x+x^2}\right) \\ & -6\log\left(x+\sqrt{-3-2x+x^2}\right)\end{aligned}$$

output
```
6*ln(1-x-(x^2-2*x-3)^(1/2))-6*ln(x+(x^2-2*x-3)^(1/2))-2/(1-x-(x^2-2*x-3)^(
1/2))+3/4/(x+(x^2-2*x-3)^(1/2))^2+4/(x+(x^2-2*x-3)^(1/2))
```

3.758.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^3}\,dx = -\frac{189+108x-48x^2-16x^3+4\sqrt{-3-2x+x^2}(33+31x+4x^2)+96(3+2x)^2\text{arctanh}\left(\frac{1+x}{2+2x+\sqrt{-3-2x+x^2}}\right.}{8(3+2x)^2}$$

input
```
Integrate[(x + Sqrt[-3 - 2*x + x^2])^(-3),x]
```

output
```
-1/8*(189 + 108*x - 48*x^2 - 16*x^3 + 4*Sqrt[-3 - 2*x + x^2]*(33 + 31*x +
4*x^2) + 96*(3 + 2*x)^2*ArcTanh[(1 + x)/(2 + 2*x + Sqrt[-3 - 2*x + x^2])])
/(3 + 2*x)^2
```

3.758.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2541, 27, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{x^2-2x-3}+x\right)^3}dx$$

$$\Big\downarrow \text{2541}$$

$$2\int -\frac{-\left(x+\sqrt{x^2-2x-3}\right)^2+2\left(x+\sqrt{x^2-2x-3}\right)+3}{4\left(-x-\sqrt{x^2-2x-3}+1\right)^2\left(x+\sqrt{x^2-2x-3}\right)^3}d\left(x+\sqrt{x^2-2x-3}\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}\int \frac{-\left(x+\sqrt{x^2-2x-3}\right)^2+2\left(x+\sqrt{x^2-2x-3}\right)+3}{\left(-x-\sqrt{x^2-2x-3}+1\right)^2\left(x+\sqrt{x^2-2x-3}\right)^3}d\left(x+\sqrt{x^2-2x-3}\right)$$

$$\Big\downarrow \text{1195}$$

$$-\frac{1}{2}\int \left(\frac{12}{x+\sqrt{x^2-2x-3}}+\frac{8}{\left(x+\sqrt{x^2-2x-3}\right)^2}+\frac{3}{\left(x+\sqrt{x^2-2x-3}\right)^3}-\frac{12}{x+\sqrt{x^2-2x-3}-1}+\frac{}{\left(x+\sqrt{}\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{4}{-\sqrt{x^2-2x-3}-x+1}+\frac{8}{\sqrt{x^2-2x-3}+x}+\frac{3}{2\left(\sqrt{x^2-2x-3}+x\right)^2}+12\log\left(-\sqrt{x^2-2x-3}-x+1\right)\right.$$

input
```
Int[(x + Sqrt[-3 - 2*x + x^2])^(-3),x]
```

3.758. $\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^3}dx$

output
```
(-4/(1 - x - Sqrt[-3 - 2*x + x^2]) + 3/(2*(x + Sqrt[-3 - 2*x + x^2])^2) +
8/(x + Sqrt[-3 - 2*x + x^2]) + 12*Log[1 - x - Sqrt[-3 - 2*x + x^2]] - 12*L
og[x + Sqrt[-3 - 2*x + x^2]])/2
```

3.758.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2541
```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Simp[2   Subst[Int[(g + h*x^n)^p*((d
^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x
)^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e
, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

3.758.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.68

method	result
trager	$\frac{(4x^2+33x+36)x}{2(2x+3)^2} - \frac{(4x^2+31x+33)\sqrt{x^2-2x-3}}{2(2x+3)^2} - 6\ln\left(x+3-\sqrt{x^2-2x-3}\right)$
default	$-\frac{9}{2x+3} - 3\ln(2x+3) + \frac{x}{2} + \frac{27}{8(2x+3)^2} - \frac{\left(\left(x+\frac{3}{2}\right)^2-5x-\frac{21}{4}\right)^{\frac{3}{2}}}{2\left(x+\frac{3}{2}\right)} - \sqrt{4\left(x+\frac{3}{2}\right)^2-20x-21} + 3\ \text{arctanh}$

input
```
int(1/(x+(x^2-2*x-3)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(4*x^2+33*x+36)*x/(2*x+3)^2-1/2*(4*x^2+31*x+33)/(2*x+3)^2*(x^2-2*x-3)^
(1/2)-6*ln(x+3-(x^2-2*x-3)^(1/2))
```

3.758.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.28

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^3}\,dx$$
$$= \frac{8\,x^3-10\,x^2-12\,(4\,x^2+12\,x+9)\log\left(x^2-\sqrt{x^2-2\,x-3}(x+1)-3\right)-12\,(4\,x^2+12\,x+9)\log(2\,x+3}{4\,(4\,x^2+1}$$

input
```
integrate(1/(x+(x^2-2*x-3)^(1/2))^3,x, algorithm="fricas")
```

output
```
1/4*(8*x^3 - 10*x^2 - 12*(4*x^2 + 12*x + 9)*log(x^2 - sqrt(x^2 - 2*x - 3)*
(x + 1) - 3) - 12*(4*x^2 + 12*x + 9)*log(2*x + 3) + 12*(4*x^2 + 12*x + 9)*
log(-x + sqrt(x^2 - 2*x - 3)) - 2*(4*x^2 + 31*x + 33)*sqrt(x^2 - 2*x - 3)
- 156*x - 171)/(4*x^2 + 12*x + 9)
```

3.758.6 Sympy [F]

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^3}\,dx = \int \frac{1}{\left(x+\sqrt{x^2-2x-3}\right)^3}\,dx$$

input
```
integrate(1/(x+(x**2-2*x-3)**(1/2))**3,x)
```

output
```
Integral((x + sqrt(x**2 - 2*x - 3))**(-3), x)
```

3.758.7 Maxima [F]

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^3}\,dx = \int \frac{1}{\left(x+\sqrt{x^2-2x-3}\right)^3}\,dx$$

input
```
integrate(1/(x+(x^2-2*x-3)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate((x + sqrt(x^2 - 2*x - 3))^(-3), x)
```

3.758.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(85) = 170$.

Time = 0.34 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.82

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^3}\,dx = \frac{1}{2}\,x - \frac{1}{2}\,\sqrt{x^2-2\,x-3} - \frac{104\,\left(x-\sqrt{x^2-2\,x-3}\right)^3 + 315\,\left(x-\sqrt{x^2-2\,x-3}\right)^2 + 162\,x - 162\,\sqrt{x^2-2\,x-3} + 27}{8\,\left(\left(x-\sqrt{x^2-2\,x-3}\right)^2 + 3\,x - 3\,\sqrt{x^2-2\,x-3}\right)^2} - \frac{9\,(16\,x+21)}{8\,(2\,x+3)^2} - 3\,\log\left(|2\,x+3|\right) - 3\,\log\left(\left|-x+\sqrt{x^2-2\,x-3}+1\right|\right) + 3\,\log\left(\left|-x+\sqrt{x^2-2\,x-3}\right|\right) - 3\,\log\left(\left|-x+\sqrt{x^2-2\,x-3}-3\right|\right)$$

input
```
integrate(1/(x+(x^2-2*x-3)^(1/2))^3,x, algorithm="giac")
```

output
```
1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 1/8*(104*(x - sqrt(x^2 - 2*x - 3))^3 + 3
15*(x - sqrt(x^2 - 2*x - 3))^2 + 162*x - 162*sqrt(x^2 - 2*x - 3) + 27)/((x
 - sqrt(x^2 - 2*x - 3))^2 + 3*x - 3*sqrt(x^2 - 2*x - 3))^2 - 9/8*(16*x + 2
1)/(2*x + 3)^2 - 3*log(abs(2*x + 3)) - 3*log(abs(-x + sqrt(x^2 - 2*x - 3)
+ 1)) + 3*log(abs(-x + sqrt(x^2 - 2*x - 3))) - 3*log(abs(-x + sqrt(x^2 - 2
*x - 3) - 3))
```

3.758.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^3}\,dx=\int \frac{1}{\left(x+\sqrt{x^2-2\,x-3}\right)^3}\,dx$$

input
```
int(1/(x + (x^2 - 2*x - 3)^(1/2))^3,x)
```

output
```
int(1/(x + (x^2 - 2*x - 3)^(1/2))^3, x)
```

3.759 $\int \frac{1}{x+\sqrt{-3-4x-x^2}} \, dx$

3.759.1 Optimal result

Integrand size = 18, antiderivative size = 108

$$\int \frac{1}{x+\sqrt{-3-4x-x^2}} \, dx = -\arctan\left(\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right) - \sqrt{2}\arctan\left(\frac{1-\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right) + \frac{1}{2}\log(3+x) + \frac{1}{2}\log\left(\frac{3\sqrt{-1-x}+\sqrt{-1-x}x+x\sqrt{3+x}}{(3+x)^{3/2}}\right)$$

output
```
-arctan((-1-x)^(1/2)/(3+x)^(1/2))+1/2*ln(3+x)+1/2*ln((3*(-1-x)^(1/2)+x*(-1
-x)^(1/2)+x*(3+x)^(1/2))/(3+x)^(3/2))-arctan(1/2*(1-3*(-1-x)^(1/2)/(3+x)^(
1/2))*2^(1/2))*2^(1/2)
```

3.759.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.75

$$\int \frac{1}{x+\sqrt{-3-4x-x^2}} \, dx = \frac{1}{2}\left(-2\arctan\left(\frac{\sqrt{-3-4x-x^2}}{3+x}\right) - 2\sqrt{2}\arctan\left(\frac{\sqrt{2}(1+x)}{1+x+\sqrt{-3-4x-x^2}}\right) + \log\left(x+\sqrt{-3-4x-x^2}\right)\right)$$

input
```
Integrate[(x + Sqrt[-3 - 4*x - x^2])^(-1),x]
```

output
```
(-2*ArcTan[Sqrt[-3 - 4*x - x^2]/(3 + x)] - 2*Sqrt[2]*ArcTan[(Sqrt[2]*(1 +
x))/(1 + x + Sqrt[-3 - 4*x - x^2])] + Log[x + Sqrt[-3 - 4*x - x^2]])/2
```

3.759.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.15, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {7287, 27, 1356, 27, 452, 216, 240, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-x^2-4x-3}+x}dx$$

$$\Big\downarrow \text{7287}$$

$$2\int \frac{2\sqrt{-x-1}}{\sqrt{x+3}\left(\frac{-x-1}{x+3}+1\right)\left(\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)}d\frac{\sqrt{-x-1}}{\sqrt{x+3}}$$

$$\Big\downarrow \text{27}$$

$$4\int \frac{\sqrt{-x-1}}{\sqrt{x+3}\left(\frac{-x-1}{x+3}+1\right)\left(\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)}d\frac{\sqrt{-x-1}}{\sqrt{x+3}}$$

$$\Big\downarrow \text{1356}$$

$$4\left(\frac{1}{8}\int -\frac{2\left(\frac{\sqrt{-x-1}}{\sqrt{x+3}}+1\right)}{\frac{-x-1}{x+3}+1}d\frac{\sqrt{-x-1}}{\sqrt{x+3}}+\frac{1}{8}\int \frac{2\left(\frac{3\sqrt{-x-1}}{\sqrt{x+3}}+1\right)}{\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1}d\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{1}{4}\int \frac{\frac{3\sqrt{-x-1}}{\sqrt{x+3}}+1}{\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1}d\frac{\sqrt{-x-1}}{\sqrt{x+3}}-\frac{1}{4}\int \frac{\frac{\sqrt{-x-1}}{\sqrt{x+3}}+1}{\frac{-x-1}{x+3}+1}d\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)$$

$$\Big\downarrow \text{452}$$

$$4\left(\frac{1}{4}\left(-\int \frac{1}{\frac{-x-1}{x+3}+1}d\frac{\sqrt{-x-1}}{\sqrt{x+3}}-\int \frac{\sqrt{-x-1}}{\sqrt{x+3}\left(\frac{-x-1}{x+3}+1\right)}d\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)+\frac{1}{4}\int \frac{\frac{3\sqrt{-x-1}}{\sqrt{x+3}}+1}{\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1}d\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)$$

$$\Big\downarrow \text{216}$$

$$4\left(\frac{1}{4}\left(-\int \frac{\sqrt{-x-1}}{\sqrt{x+3}\left(\frac{-x-1}{x+3}+1\right)}d\frac{\sqrt{-x-1}}{\sqrt{x+3}}-\arctan\left(\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)\right)+\frac{1}{4}\int \frac{\frac{3\sqrt{-x-1}}{\sqrt{x+3}}+1}{\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1}d\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)$$

$$\Big\downarrow \text{240}$$

$$4\left(\frac{1}{4}\int \frac{\frac{3\sqrt{-x-1}}{\sqrt{x+3}}+1}{\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1}d\frac{\sqrt{-x-1}}{\sqrt{x+3}}+\frac{1}{4}\left(-\arctan\left(\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)-\frac{1}{2}\log\left(\frac{-x-1}{x+3}+1\right)\right)\right)$$

$$\Big\downarrow \text{1142}$$

$$4\left(\frac{1}{4}\left(2\int \frac{1}{\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1}d\frac{\sqrt{-x-1}}{\sqrt{x+3}}+\frac{1}{2}\int -\frac{2\left(1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}\right)}{\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1}d\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)+\frac{1}{4}\left(-\arctan\left(\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right.\right.\right.$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{1}{4}\left(2\int \frac{1}{\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1}d\frac{\sqrt{-x-1}}{\sqrt{x+3}}-\int \frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1}d\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)+\frac{1}{4}\left(-\arctan\left(\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)\right.\right.$$

$$\Big\downarrow \text{1083}$$

$$4\left(\frac{1}{4}\left(-4\int \frac{1}{-\frac{-x-1}{x+3}-8}d\left(\frac{6\sqrt{-x-1}}{\sqrt{x+3}}-2\right)-\int \frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1}d\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)+\frac{1}{4}\left(-\arctan\left(\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)\right.\right.$$

$$\Big\downarrow \text{217}$$

$$4\left(\frac{1}{4}\left(\sqrt{2}\arctan\left(\frac{\frac{6\sqrt{-x-1}}{\sqrt{x+3}}-2}{2\sqrt{2}}\right)-\int \frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1}d\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)+\frac{1}{4}\left(-\arctan\left(\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)-\frac{1}{2}\log\left(\right.\right.\right.$$

$$\Big\downarrow \text{1103}$$

$$4\left(\frac{1}{4}\left(-\arctan\left(\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)-\frac{1}{2}\log\left(\frac{-x-1}{x+3}+1\right)\right)+\frac{1}{4}\left(\sqrt{2}\arctan\left(\frac{\frac{6\sqrt{-x-1}}{\sqrt{x+3}}-2}{2\sqrt{2}}\right)+\frac{1}{2}\log\left(\frac{3(-x-1)}{x+3}-2\right.\right.\right.$$

input
```
Int[(x + Sqrt[-3 - 4*x - x^2])^(-1),x]
```

3.759. $\int \frac{1}{x+\sqrt{-3-4x-x^2}}\,dx$

output
```
4*((-ArcTan[Sqrt[-1 - x]/Sqrt[3 + x]] - Log[1 + (-1 - x)/(3 + x)]/2)/4 + (
Sqrt[2]*ArcTan[(-2 + (6*Sqrt[-1 - x])/Sqrt[3 + x])/(2*Sqrt[2])] + Log[1 +
(3*(-1 - x))/(3 + x) - (2*Sqrt[-1 - x])/Sqrt[3 + x]]/2)/4)
```

3.759.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.759. $\int \frac{1}{x+\sqrt{-3-4x-x^2}}\,dx$

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1356
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*
(x_)^2)), x_Symbol] :> With[{q = Simplify[c^2*d^2 + b^2*d*f - 2*a*c*d*f + a
^2*f^2]}, Simp[1/q   Int[Simp[g*c^2*d + g*b^2*f - a*b*h*f - a*g*c*f + c*(h*
c*d + g*b*f - a*h*f)*x, x]/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[Simp[
b*h*d*f - g*c*d*f + a*g*f^2 - f*(h*c*d + g*b*f - a*h*f)*x, x]/(d + f*x^2),
x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c,
 0]
```

rule 7287
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfSquareRootOfQuadratic[u, x]}, Si
mp[2   Subst[Int[lst[[1]], x], x, lst[[2]]], x] /;  !FalseQ[lst] && EqQ[lst
[[3]], 3]] /; EulerIntegrandQ[u, x]
```

3.759.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(85) = 170$.

Time = 1.04 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.43

method	result
default	$\frac{\arcsin(x+2)}{2}-\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\sqrt{2}}{6}\right)-\operatorname{arctanh}\left(\frac{3x}{\left(-\frac{3}{2}-x\right)\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}}\right)\right)}{12\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(\frac{x}{-\frac{3}{2}-x}+1\right)^2}}\left(\frac{x}{-\frac{3}{2}-x}+1\right)}+\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{}}{}$
trager	$\operatorname{RootOf}\left(4\_Z^2-4\_Z+3\right)\ln\left(4\operatorname{RootOf}\left(2\_Z^2+2\_Z+1\right)^2\operatorname{RootOf}\left(4\_Z^2-4\_Z+3\right)^2x+4\right.$

input
```
int(1/(x+(-x^2-4*x-3)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsin(x+2)-1/12*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*
arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-arctanh(3*x/(-3/2-x)/(3*x^
2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(x/(-3/2-x)+1)^2)^(1/2)/(x/(-
3/2-x)+1)+1/3*3^(1/2)*4^(1/2)/((x^2/(-3/2-x)^2-4)/(x/(-3/2-x)+1)^2)^(1/2)/
(x/(-3/2-x)+1)*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2)*arctan(1/6*(3*x^2/(-3/2
-x)^2-12)^(1/2)*2^(1/2))-1/6*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(
2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))+arctanh(3*x/(-3/2-
x)/(3*x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(x/(-3/2-x)+1)^2)^(1/
2)/(x/(-3/2-x)+1)+1/4*ln(2*x^2+4*x+3)-1/2*2^(1/2)*arctan(1/4*(4+4*x)*2^(1/
2))
```

3.759.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(85) = 170$.

Time = 0.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.73

$$\begin{aligned}\int \frac{1}{x+\sqrt{-3-4x-x^2}}\,dx &= -\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}(x+1)\right)\\ &\quad+\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)\\ &\quad+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)\\ &\quad-\frac{1}{2}\arctan\left(\frac{\sqrt{-x^2-4x-3}(x+2)}{x^2+4x+3}\right)+\frac{1}{4}\log\left(2x^2+4x+3\right)\\ &\quad-\frac{1}{8}\log\left(-\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right)\\ &\quad+\frac{1}{8}\log\left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2}\right)\end{aligned}$$

input
```
integrate(1/(x+(-x^2-4*x-3)^(1/2)),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2)*arctan(sqrt(2)*(x + 1)) + 1/4*sqrt(2)*arctan(1/2*(sqrt(2)*x +
 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) + 1/4*sqrt(2)*arctan(-1/2*(sqr
t(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) - 1/2*arctan(sqrt(-x^2
 - 4*x - 3)*(x + 2)/(x^2 + 4*x + 3)) + 1/4*log(2*x^2 + 4*x + 3) - 1/8*log(
-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 1/8*log((2*sqrt(-x^2 - 4*x -
3)*x - 4*x - 3)/x^2)
```

3.759.6 Sympy [F]

$$\int \frac{1}{x+\sqrt{-3-4x-x^2}}\,dx = \int \frac{1}{x+\sqrt{-x^2-4x-3}}\,dx$$

input
```
integrate(1/(x+(-x**2-4*x-3)**(1/2)),x)
```

output
```
Integral(1/(x + sqrt(-x**2 - 4*x - 3)), x)
```

3.759.7 Maxima [F]

$$\int \frac{1}{x+\sqrt{-3-4x-x^2}}\,dx = \int \frac{1}{x+\sqrt{-x^2-4\,x-3}}\,dx$$

input
```
integrate(1/(x+(-x^2-4*x-3)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(x + sqrt(-x^2 - 4*x - 3)), x)
```

3.759.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(85) = 170.

Time = 0.33 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.82

$$\int \frac{1}{x+\sqrt{-3-4x-x^2}}\,dx = -\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}(x+1)\right) + \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right) + \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right) + \frac{1}{2}\arcsin\left(x+2\right) + \frac{1}{4}\log\left(2x^2+4x+3\right) + \frac{1}{4}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2} + \frac{3\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2} + 1\right) - \frac{1}{4}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2} + \frac{\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2} + 3\right)$$

input
```
integrate(1/(x+(-x^2-4*x-3)^(1/2)),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*arctan(sqrt(2)*(x + 1)) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(
sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(
(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*arcsin(x + 2) + 1/4*log(2*x
^2 + 4*x + 3) + 1/4*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^
2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) - 1/4*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/
(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

3.759.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x+\sqrt{-3-4x-x^2}}\,dx=\int \frac{1}{x+\sqrt{-x^2-4\,x-3}}\,dx$$

input
```
int(1/(x + (- 4*x - x^2 - 3)^(1/2)),x)
```

output
```
int(1/(x + (- 4*x - x^2 - 3)^(1/2)), x)
```

3.760 $\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^2}\,dx$

3.760.1 Optimal result

Integrand size = 18, antiderivative size = 87

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^2}\,dx=\frac{1-\frac{\sqrt{-1-x}}{\sqrt{3+x}}}{1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}}+\frac{\arctan\left(\frac{1-\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right)}{\sqrt{2}}$$

output
```
1/2*arctan(1/2*(1-3*(-1-x)^(1/2)/(3+x)^(1/2))*2^(1/2))*2^(1/2)+(1-(-1-x)^(
1/2)/(3+x)^(1/2))/(1-3*(1+x)/(3+x)-2*(-1-x)^(1/2)/(3+x)^(1/2))
```

3.760.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^2}\,dx$$
$$=\frac{3+x+(3+2x)\sqrt{-3-4x-x^2}+\sqrt{2}(3+4x+2x^2)\arctan\left(\frac{\sqrt{2}(1+x)}{1+x+\sqrt{-3-4x-x^2}}\right)}{2\left(3+4x+2x^2\right)}$$

input
```
Integrate[(x + Sqrt[-3 - 4*x - x^2])^(-2),x]
```

output
```
(3 + x + (3 + 2*x)*Sqrt[-3 - 4*x - x^2] + Sqrt[2]*(3 + 4*x + 2*x^2)*ArcTan
[(Sqrt[2]*(1 + x))/(1 + x + Sqrt[-3 - 4*x - x^2])])/(2*(3 + 4*x + 2*x^2))
```

3.760.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {7287, 27, 1159, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{-x^2-4x-3}+x\right)^2}dx$$

$$\Big\downarrow \text{ 7287}$$

$$2\int -\frac{2\sqrt{-x-1}}{\sqrt{x+3}\left(\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)^2}d\frac{\sqrt{-x-1}}{\sqrt{x+3}}$$

$$\Big\downarrow \text{ 27}$$

$$-4\int \frac{\sqrt{-x-1}}{\sqrt{x+3}\left(\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)^2}d\frac{\sqrt{-x-1}}{\sqrt{x+3}}$$

$$\Big\downarrow \text{ 1159}$$

$$-4\left(\frac{1}{4}\int \frac{1}{\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1}d\frac{\sqrt{-x-1}}{\sqrt{x+3}}-\frac{1-\frac{\sqrt{-x-1}}{\sqrt{x+3}}}{4\left(\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)}\right)$$

$$\Big\downarrow \text{ 1083}$$

$$-4\left(-\frac{1}{2}\int \frac{1}{-\frac{-x-1}{x+3}-8}d\left(\frac{6\sqrt{-x-1}}{\sqrt{x+3}}-2\right)-\frac{1-\frac{\sqrt{-x-1}}{\sqrt{x+3}}}{4\left(\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)}\right)$$

$$\Big\downarrow \text{ 217}$$

$$-4\left(\frac{\arctan\left(\frac{\frac{6\sqrt{-x-1}}{\sqrt{x+3}}-2}{2\sqrt{2}}\right)}{4\sqrt{2}}-\frac{1-\frac{\sqrt{-x-1}}{\sqrt{x+3}}}{4\left(\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)}\right)$$

input
```
Int[(x + Sqrt[-3 - 4*x - x^2])^(-2),x]
```

output
```
-4*(-1/4*(1 - Sqrt[-1 - x]/Sqrt[3 + x])/(1 + (3*(-1 - x))/(3 + x) - (2*Sqr
t[-1 - x])/Sqrt[3 + x]) + ArcTan[(-2 + (6*Sqrt[-1 - x])/Sqrt[3 + x])/(2*Sq
rt[2])]/(4*Sqrt[2]))
```

3.760.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 7287
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfSquareRootOfQuadratic[u, x]}, Si
mp[2   Subst[Int[lst[[1]], x], x, lst[[2]]], x] /;  !FalseQ[lst] && EqQ[lst
[[3]], 3]] /; EulerIntegrandQ[u, x]
```

3.760.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.34 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.26

method	result
trager	$-\frac{(2x+3)x}{2(2x^2+4x+3)}+\frac{(2x+3)\sqrt{-x^2-4x-3}}{4x^2+8x+6}+\frac{\mathrm{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{-2\,\mathrm{RootOf}\left(\_Z^2+2\right)x+2\sqrt{-x^2-4x-3}-3\,\mathrm{RootOf}\left(\_Z^2+2\right)}{\mathrm{RootOf}\left(\_Z^2+2\right)x+2x+3}\right)}{4}$
default	Expression too large to display

input
```
int(1/(x+(-x^2-4*x-3)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*x+3)*x/(2*x^2+4*x+3)+1/2*(2*x+3)/(2*x^2+4*x+3)*(-x^2-4*x-3)^(1/2)+
1/4*RootOf(_Z^2+2)*ln((-2*RootOf(_Z^2+2)*x+2*(-x^2-4*x-3)^(1/2)-3*RootOf(_
Z^2+2))/(RootOf(_Z^2+2)*x+2*x+3))
```

3.760.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.39

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^2}\,dx = \frac{2\sqrt{2}(2x^2+4x+3)\arctan\left(\sqrt{2}(x+1)\right)-\sqrt{2}(2x^2+4x+3)\arctan\left(\frac{\sqrt{2}(6x^2+20x+15)\sqrt{-x^2-4x-3}}{4(2x^3+11x^2+18x+9)}\right)+4\sqrt{-}}{8(2x^2+4x+3)}$$

input
```
integrate(1/(x+(-x^2-4*x-3)^(1/2))^2,x, algorithm="fricas")
```

output
```
1/8*(2*sqrt(2)*(2*x^2 + 4*x + 3)*arctan(sqrt(2)*(x + 1)) - sqrt(2)*(2*x^2
+ 4*x + 3)*arctan(1/4*sqrt(2)*(6*x^2 + 20*x + 15)*sqrt(-x^2 - 4*x - 3)/(2*
x^3 + 11*x^2 + 18*x + 9)) + 4*sqrt(-x^2 - 4*x - 3)*(2*x + 3) + 4*x + 12)/(
2*x^2 + 4*x + 3)
```

3.760.6 Sympy [F]

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^2}\,dx = \int \frac{1}{\left(x+\sqrt{-x^2-4x-3}\right)^2}\,dx$$

input
```
integrate(1/(x+(-x**2-4*x-3)**(1/2))**2,x)
```

output
```
Integral((x + sqrt(-x**2 - 4*x - 3))**(-2), x)
```

3.760.7 Maxima [F]

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^2}\,dx = \int \frac{1}{\left(x+\sqrt{-x^2-4\,x-3}\right)^2}\,dx$$

input
```
integrate(1/(x+(-x^2-4*x-3)^(1/2))^2,x, algorithm="maxima")
```

output
```
integrate((x + sqrt(-x^2 - 4*x - 3))^(-2), x)
```

3.760.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(72) = 144.

Time = 0.34 (sec) , antiderivative size = 263, normalized size of antiderivative = 3.02

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^2}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\sqrt{2}(x+1)\right) - \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+1\right)\right) - \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4\,x-3}-1}{x+2}+1\right)\right) + \frac{x+3}{2\left(2\,x^2+4\,x+3\right)} - \frac{\frac{10\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+\frac{7\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}-\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)^3}{(x+2)^3}+3}{3\left(\frac{8\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+\frac{14\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+\frac{8\left(\sqrt{-x^2-4\,x-3}-1\right)^3}{(x+2)^3}+\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)^4}{(x+2)^4}+3\right)}$$

input
```
integrate(1/(x+(-x^2-4*x-3)^(1/2))^2,x, algorithm="giac")
```

output
```
1/4*sqrt(2)*arctan(sqrt(2)*(x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*(s
qrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*((
sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*(x + 3)/(2*x^2 + 4*x + 3) -
1/3*(10*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 7*(sqrt(-x^2 - 4*x - 3) - 1)^
2/(x + 2)^2 - 2*(sqrt(-x^2 - 4*x - 3) - 1)^3/(x + 2)^3 + 3)/(8*(sqrt(-x^2
- 4*x - 3) - 1)/(x + 2) + 14*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 8*(s
qrt(-x^2 - 4*x - 3) - 1)^3/(x + 2)^3 + 3*(sqrt(-x^2 - 4*x - 3) - 1)^4/(x +
 2)^4 + 3)
```

3.760.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^2}\,dx=\int \frac{1}{\left(x+\sqrt{-x^2-4\,x-3}\right)^2}\,dx$$

input
```
int(1/(x + (- 4*x - x^2 - 3)^(1/2))^2,x)
```

output
```
int(1/(x + (- 4*x - x^2 - 3)^(1/2))^2, x)
```

3.761 $\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^3} dx$

3.761.1 Optimal result

Integrand size = 18, antiderivative size = 149

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^3} dx = -\frac{13-\frac{27\sqrt{-1-x}}{\sqrt{3+x}}}{18\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)} - \frac{2\left(2-\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)}{9\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)^2} - \frac{3\arctan\left(\frac{1-\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

output
```
-3/4*arctan(1/2*(1-3*(-1-x)^(1/2)/(3+x)^(1/2))*2^(1/2))*2^(1/2)+1/18*(-13+
27*(-1-x)^(1/2)/(3+x)^(1/2))/(1-3*(1+x)/(3+x)-2*(-1-x)^(1/2)/(3+x)^(1/2))-
2/9*(2-(-1-x)^(1/2)/(3+x)^(1/2))/(1-3*(1+x)/(3+x)-2*(-1-x)^(1/2)/(3+x)^(1/
2))^2
```

3.761.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.73

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^3} dx = -\frac{9+15x+16x^2+6x^3+\sqrt{-3-4x-x^2}(15+26x+22x^2+8x^3)+3\sqrt{2}(3+4x+2x^2)^2\arctan\left(\frac{\sqrt{\ }}{1+x+\sqrt{\ }}\right)}{4\left(3+4x+2x^2\right)^2}$$

input
```
Integrate[(x + Sqrt[-3 - 4*x - x^2])^(-3),x]
```

output
```
-1/4*(9 + 15*x + 16*x^2 + 6*x^3 + Sqrt[-3 - 4*x - x^2]*(15 + 26*x + 22*x^2
 + 8*x^3) + 3*Sqrt[2]*(3 + 4*x + 2*x^2)^2*ArcTan[(Sqrt[2]*(1 + x))/(1 + x
+ Sqrt[-3 - 4*x - x^2])])/(3 + 4*x + 2*x^2)^2
```

3.761.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {7287, 27, 2191, 27, 1159, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{-x^2-4x-3}+x\right)^3}dx$$

$$\Big\downarrow \text{7287}$$

$$2\int \frac{2\sqrt{-x-1}\left(\frac{-x-1}{x+3}+1\right)}{\sqrt{x+3}\left(\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)^3}d\frac{\sqrt{-x-1}}{\sqrt{x+3}}$$

$$\Big\downarrow \text{27}$$

$$4\int \frac{\sqrt{-x-1}\left(\frac{-x-1}{x+3}+1\right)}{\sqrt{x+3}\left(\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)^3}d\frac{\sqrt{-x-1}}{\sqrt{x+3}}$$

$$\Big\downarrow \text{2191}$$

$$4\left(\frac{1}{16}\int \frac{8\left(\frac{6\sqrt{-x-1}}{\sqrt{x+3}}+7\right)}{9\left(\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)^2}d\frac{\sqrt{-x-1}}{\sqrt{x+3}}-\frac{2-\frac{\sqrt{-x-1}}{\sqrt{x+3}}}{18\left(\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)^2}\right)$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{1}{18}\int \frac{\frac{6\sqrt{-x-1}}{\sqrt{x+3}}+7}{\left(\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)^2}d\frac{\sqrt{-x-1}}{\sqrt{x+3}}-\frac{2-\frac{\sqrt{-x-1}}{\sqrt{x+3}}}{18\left(\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)^2}\right)$$

$$\Big\downarrow \text{1159}$$

$$4\left(\frac{1}{18}\left(\frac{27}{4}\int \frac{1}{\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1}d\frac{\sqrt{-x-1}}{\sqrt{x+3}}-\frac{13-\frac{27\sqrt{-x-1}}{\sqrt{x+3}}}{4\left(\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)}\right)-\frac{2-\frac{\sqrt{-x-1}}{\sqrt{x+3}}}{18\left(\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)^2}\right)$$

$$\Big\downarrow \text{1083}$$

$$4\left(\frac{1}{18}\left(-\frac{27}{2}\int \frac{1}{-\frac{-x-1}{x+3}-8}d\left(\frac{6\sqrt{-x-1}}{\sqrt{x+3}}-2\right)-\frac{13-\frac{27\sqrt{-x-1}}{\sqrt{x+3}}}{4\left(\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)}\right)-\frac{2-\frac{\sqrt{-x-1}}{\sqrt{x+3}}}{18\left(\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)^2}\right)$$

$$\Big\downarrow \text{217}$$

$$4\left(\frac{1}{18}\left(\frac{27\arctan\left(\frac{\frac{6\sqrt{-x-1}}{\sqrt{x+3}}-2}{2\sqrt{2}}\right)}{4\sqrt{2}}-\frac{13-\frac{27\sqrt{-x-1}}{\sqrt{x+3}}}{4\left(\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)}\right)-\frac{2-\frac{\sqrt{-x-1}}{\sqrt{x+3}}}{18\left(\frac{3(-x-1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)^2}\right)$$

input
```
Int[(x + Sqrt[-3 - 4*x - x^2])^(-3),x]
```

output
```
4*(-1/18*(2 - Sqrt[-1 - x]/Sqrt[3 + x])/(1 + (3*(-1 - x))/(3 + x) - (2*Sqr
t[-1 - x])/Sqrt[3 + x])^2 + (-1/4*(13 - (27*Sqrt[-1 - x])/Sqrt[3 + x])/(1
+ (3*(-1 - x))/(3 + x) - (2*Sqrt[-1 - x])/Sqrt[3 + x]) + (27*ArcTan[(-2 +
(6*Sqrt[-1 - x])/Sqrt[3 + x])/(2*Sqrt[2])])/(4*Sqrt[2]))/18)
```

3.761.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.761. $\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^3}\,dx$

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 7287
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfSquareRootOfQuadratic[u, x]}, Si
mp[2   Subst[Int[lst[[1]], x], x, lst[[2]]], x] /;  !FalseQ[lst] && EqQ[lst
[[3]], 3]] /; EulerIntegrandQ[u, x]
```

3.761.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.57 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.87

method	result
trager	$\frac{(4x^3+10x^2+12x+9)x}{4(2x^2+4x+3)^2} - \frac{(8x^3+22x^2+26x+15)\sqrt{-x^2-4x-3}}{4(2x^2+4x+3)^2} - \frac{3\,\mathrm{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{-2\,\mathrm{RootOf}\left(\_Z^2+2\right)x+2\sqrt{-x^2-4x-3}-3}{\mathrm{RootOf}\left(\_Z^2+2\right)x+2x}\right.}{8}$
default	Expression too large to display

input
```
int(1/(x+(-x^2-4*x-3)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(4*x^3+10*x^2+12*x+9)*x/(2*x^2+4*x+3)^2-1/4*(8*x^3+22*x^2+26*x+15)/(2*
x^2+4*x+3)^2*(-x^2-4*x-3)^(1/2)-3/8*RootOf(_Z^2+2)*ln((-2*RootOf(_Z^2+2)*x
+2*(-x^2-4*x-3)^(1/2)-3*RootOf(_Z^2+2))/(RootOf(_Z^2+2)*x+2*x+3))
```

3.761. $\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^3}\,dx$

3.761.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^3}\,dx = -\frac{24\,x^3+6\,\sqrt{2}(4\,x^4+16\,x^3+28\,x^2+24\,x+9)\arctan\left(\sqrt{2}(x+1)\right)-3\,\sqrt{2}(4\,x^4+16\,x^3+28\,x^2+24\,x+}{16\,(4\,x^4+16\,x^3}$$

input
```
integrate(1/(x+(-x^2-4*x-3)^(1/2))^3,x, algorithm="fricas")
```

output
```
-1/16*(24*x^3 + 6*sqrt(2)*(4*x^4 + 16*x^3 + 28*x^2 + 24*x + 9)*arctan(sqrt
(2)*(x + 1)) - 3*sqrt(2)*(4*x^4 + 16*x^3 + 28*x^2 + 24*x + 9)*arctan(1/4*s
qrt(2)*(6*x^2 + 20*x + 15)*sqrt(-x^2 - 4*x - 3)/(2*x^3 + 11*x^2 + 18*x + 9
)) + 64*x^2 + 4*(8*x^3 + 22*x^2 + 26*x + 15)*sqrt(-x^2 - 4*x - 3) + 60*x +
 36)/(4*x^4 + 16*x^3 + 28*x^2 + 24*x + 9)
```

3.761.6 Sympy [F]

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^3}\,dx = \int \frac{1}{\left(x+\sqrt{-x^2-4x-3}\right)^3}\,dx$$

input
```
integrate(1/(x+(-x**2-4*x-3)**(1/2))**3,x)
```

output
```
Integral((x + sqrt(-x**2 - 4*x - 3))**(-3), x)
```

3.761.7 Maxima [F]

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^3}\,dx = \int \frac{1}{\left(x+\sqrt{-x^2-4\,x-3}\right)^3}\,dx$$

input
```
integrate(1/(x+(-x^2-4*x-3)^(1/2))^3,x, algorithm="maxima")
```

output
```
integrate((x + sqrt(-x^2 - 4*x - 3))^(-3), x)
```

3.761.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(119) = 238$.

Time = 0.33 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.46

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^3}\,dx$$
$$= -\frac{3}{8}\sqrt{2}\arctan\left(\sqrt{2}(x+1)\right)+\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)$$
$$+\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)-\frac{6x^3+16x^2+15x+9}{4(2x^2+4x+3)^2}$$
$$+\frac{\frac{618\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{1547\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+\frac{2362\left(\sqrt{-x^2-4x-3}-1\right)^3}{(x+2)^3}+\frac{2223\left(\sqrt{-x^2-4x-3}-1\right)^4}{(x+2)^4}+\frac{1174\left(\sqrt{-x^2-4x-3}\right.}{(x+2)^5}}{18\left(\frac{8\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{14\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+\frac{8\left(\sqrt{-x^2-4x-3}-1\right)^3}{(x+2)^3}+\frac{3\left(\sqrt{-}\right.}{}\right.}$$

input
```
integrate(1/(x+(-x^2-4*x-3)^(1/2))^3,x, algorithm="giac")
```

output
```
-3/8*sqrt(2)*arctan(sqrt(2)*(x + 1)) + 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(3*(
sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(
(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) - 1/4*(6*x^3 + 16*x^2 + 15*x + 9)
/(2*x^2 + 4*x + 3)^2 + 1/18*(618*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1547
*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 2362*(sqrt(-x^2 - 4*x - 3) - 1)^
3/(x + 2)^3 + 2223*(sqrt(-x^2 - 4*x - 3) - 1)^4/(x + 2)^4 + 1174*(sqrt(-x^
2 - 4*x - 3) - 1)^5/(x + 2)^5 + 377*(sqrt(-x^2 - 4*x - 3) - 1)^6/(x + 2)^6
 + 6*(sqrt(-x^2 - 4*x - 3) - 1)^7/(x + 2)^7 + 117)/(8*(sqrt(-x^2 - 4*x - 3
) - 1)/(x + 2) + 14*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 8*(sqrt(-x^2
- 4*x - 3) - 1)^3/(x + 2)^3 + 3*(sqrt(-x^2 - 4*x - 3) - 1)^4/(x + 2)^4 + 3
)^2
```

3.761.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^3}\,dx = \int \frac{1}{\left(x+\sqrt{-x^2-4\,x-3}\right)^3}\,dx$$

input
```
int(1/(x + (- 4*x - x^2 - 3)^(1/2))^3,x)
```

output
```
int(1/(x + (- 4*x - x^2 - 3)^(1/2))^3, x)
```

3.762 $\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx$

3.762.1 Optimal result

Integrand size = 35, antiderivative size = 42

$$\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx=-\frac{1}{15}\left(1-x^2-2x^3-x^4\right)^{3/2}\left(2+3x^2+6x^3+3x^4\right)$$

output
```
-1/15*(-x^4-2*x^3-x^2+1)^(3/2)*(3*x^4+6*x^3+3*x^2+2)
```

3.762.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx=\frac{1}{15}\left(-2-3x^2-6x^3-3x^4\right)\left(1-x^2-2x^3-x^4\right)^{3/2}$$

input
```
Integrate[x^3*(1 + x)^3*(1 + 2*x)*Sqrt[1 - x^2 - 2*x^3 - x^4],x]
```

output
```
((-2 - 3*x^2 - 6*x^3 - 3*x^4)*(1 - x^2 - 2*x^3 - x^4)^(3/2))/15
```

3.762.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.76, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2459, 2029, 2069, 1576, 27, 1116, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(x+1)^3(2x+1)\sqrt{-x^4-2x^3-x^2+1}\,dx$$

$$\Big\downarrow \text{2459}$$

$$\int \sqrt{-\left(x+\frac{1}{2}\right)^4+\frac{1}{2}\left(x+\frac{1}{2}\right)^2+\frac{15}{16}}\left(2\left(x+\frac{1}{2}\right)^7-\frac{3}{2}\left(x+\frac{1}{2}\right)^5+\frac{3}{8}\left(x+\frac{1}{2}\right)^3+\frac{1}{32}\left(-x-\frac{1}{2}\right)\right)d\left(x+\frac{1}{2}\right)$$

$$\Big\downarrow \text{2029}$$

$$\int \left(x+\frac{1}{2}\right)\sqrt{-\left(x+\frac{1}{2}\right)^4+\frac{1}{2}\left(x+\frac{1}{2}\right)^2+\frac{15}{16}}\left(2\left(x+\frac{1}{2}\right)^6-\frac{3}{2}\left(x+\frac{1}{2}\right)^4+\frac{3}{8}\left(x+\frac{1}{2}\right)^2-\frac{1}{32}\right)d\left(x+\frac{1}{2}\right)$$

$$\Big\downarrow \text{2069}$$

$$\int \left(x+\frac{1}{2}\right)\left(\sqrt[3]{2}\left(x+\frac{1}{2}\right)^2-\frac{1}{2\ 2^{2/3}}\right)^3\sqrt{-\left(x+\frac{1}{2}\right)^4+\frac{1}{2}\left(x+\frac{1}{2}\right)^2+\frac{15}{16}}d\left(x+\frac{1}{2}\right)$$

$$\Big\downarrow \text{1576}$$

$$\frac{1}{2}\int -\frac{1}{128}\left(1-4\left(x+\frac{1}{2}\right)^2\right)^3\sqrt{-16\left(x+\frac{1}{2}\right)^4+8\left(x+\frac{1}{2}\right)^2+15}d\left(x+\frac{1}{2}\right)^2$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{256}\int \left(1-4\left(x+\frac{1}{2}\right)^2\right)^3\sqrt{-16\left(x+\frac{1}{2}\right)^4+8\left(x+\frac{1}{2}\right)^2+15}d\left(x+\frac{1}{2}\right)^2$$

$$\Big\downarrow \text{1116}$$

$$\frac{1}{256}\left(-\frac{32}{5}\int \left(1-4\left(x+\frac{1}{2}\right)^2\right)\sqrt{-16\left(x+\frac{1}{2}\right)^4+8\left(x+\frac{1}{2}\right)^2+15}d\left(x+\frac{1}{2}\right)^2-\frac{1}{20}\left(-16\left(x+\frac{1}{2}\right)^4+8\left(x+\right.\right.\right.$$

$$\Big\downarrow \text{1104}$$

$$\frac{1}{256}\left(-\frac{1}{20}\left(-16\left(x+\frac{1}{2}\right)^4+8\left(x+\frac{1}{2}\right)^2+15\right)^{3/2}\left(1-4\left(x+\frac{1}{2}\right)^2\right)^2-\frac{8}{15}\left(-16\left(x+\frac{1}{2}\right)^4+8\left(x+\frac{1}{2}\right)^2+15\right.\right.$$

input
```
Int[x^3*(1 + x)^3*(1 + 2*x)*Sqrt[1 - x^2 - 2*x^3 - x^4],x]
```

output
```
((-8*(15 + 8*(1/2 + x)^2 - 16*(1/2 + x)^4)^(3/2))/15 - ((1 - 4*(1/2 + x)^2
)^2*(15 + 8*(1/2 + x)^2 - 16*(1/2 + x)^4)^(3/2))/20)/256
```

3.762.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1116
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[2*d*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(b*(m + 2*p
 + 1))), x] + Simp[d^2*(m - 1)*((b^2 - 4*a*c)/(b^2*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[m, 1] && NeQ[m + 2*p
+ 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) || OddQ[m])
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2029
```
Int[(Fx_.)*((d_.)*(x_)^(q_.) + (a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*
(x_)^(t_.))^(p_.), x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r)
+ d*x^(q - r))^p*Fx, x] /; FreeQ[{a, b, c, d, r, s, t, q}, x] && IntegerQ[p
] && PosQ[s - r] && PosQ[t - r] && PosQ[q - r] &&  !(EqQ[p, 1] && EqQ[u, 1]
)
```

3.762. $\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx$

rule 2069
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x^2, 0], Expon[Px, x^2
]], b = Rt[Coeff[Px, x^2, Expon[Px, x^2]], Expon[Px, x^2]]}, Int[u*(a + b*x
^2)^Expon[Px, x^2], x] /; EqQ[Px, (a + b*x^2)^Expon[Px, x^2]]] /; PolyQ[Px,
 x^2] && GtQ[Expon[Px, x^2], 1] && NeQ[Coeff[Px, x^2, 0], 0] &&  !MatchQ[Px
, (a_.)*(v_)^Expon[Px, x^2] /; FreeQ[a, x] && BinomialQ[v, x, 2]]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

3.762.4 Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$-\frac{(-x^4-2x^3-x^2+1)^{\frac{3}{2}}(x^4+2x^3+x^2+\frac{2}{3})}{5}$
gosper	$\frac{(x^2+x+1)(x^2+x-1)(3x^4+6x^3+3x^2+2)\sqrt{-x^4-2x^3-x^2+1}}{15}$
trager	$\left(\frac{1}{5}x^8+\frac{4}{5}x^7+\frac{6}{5}x^6+\frac{4}{5}x^5+\frac{2}{15}x^4-\frac{2}{15}x^3-\frac{1}{15}x^2-\frac{2}{15}\right)\sqrt{-x^4-2x^3-x^2+1}$
risch	$-\frac{(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2)(x^4+2x^3+x^2-1)}{15\sqrt{-x^4-2x^3-x^2+1}}$
default	$\frac{2x^4\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{2x^3\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{x^2\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{2\sqrt{-x^4-2x^3-x^2+1}}{15}+\frac{x^8\sqrt{-x^4-2x^3-x}}{5}$
elliptic	$\frac{2x^4\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{2x^3\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{x^2\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{2\sqrt{-x^4-2x^3-x^2+1}}{15}+\frac{x^8\sqrt{-x^4-2x^3-x}}{5}$

input
```
int(x^3*(x+1)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(-x^4-2*x^3-x^2+1)^(3/2)*(x^4+2*x^3+x^2+2/3)
```

3.762.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx$$
$$=\frac{1}{15}\left(3\,x^8+12\,x^7+18\,x^6+12\,x^5+2\,x^4-2\,x^3-x^2-2\right)\sqrt{-x^4-2\,x^3-x^2+1}$$

input
```
integrate(x^3*(1+x)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2),x, algorithm="frica
s")
```

output
```
1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(-x^
4 - 2*x^3 - x^2 + 1)
```

3.762.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(36) = 72.

Time = 0.16 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.33

$$\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx$$
$$=\frac{x^8\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{4x^7\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{6x^6\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{4x^5\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{2x^4\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{2x^3\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{x^2\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{2\sqrt{-x^4-2x^3-x^2+1}}{15}$$

input
```
integrate(x**3*(1+x)**3*(1+2*x)*(-x**4-2*x**3-x**2+1)**(1/2),x)
```

output
```
x**8*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**7*sqrt(-x**4 - 2*x**3 - x**2
 + 1)/5 + 6*x**6*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**5*sqrt(-x**4 - 2
*x**3 - x**2 + 1)/5 + 2*x**4*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 - 2*x**3*s
qrt(-x**4 - 2*x**3 - x**2 + 1)/15 - x**2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/1
5 - 2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15
```

3.762.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.40

$$\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx$$
$$=\frac{1}{15}\left(3\,x^8+12\,x^7+18\,x^6+12\,x^5+2\,x^4-2\,x^3-x^2-2\right)\sqrt{x^2+x+1}\sqrt{-x^2-x+1}$$

input
```
integrate(x^3*(1+x)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2),x, algorithm="maxim
a")
```

output
```
1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(x^2
 + x + 1)*sqrt(-x^2 - x + 1)
```

3.762.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx$$
$$=\frac{1}{5}\left(x^4+2\,x^3+x^2-1\right)^2\sqrt{-x^4-2\,x^3-x^2+1}-\frac{1}{3}\left(-x^4-2\,x^3-x^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(x^3*(1+x)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2),x, algorithm="giac"
)
```

output
```
1/5*(x^4 + 2*x^3 + x^2 - 1)^2*sqrt(-x^4 - 2*x^3 - x^2 + 1) - 1/3*(-x^4 - 2
*x^3 - x^2 + 1)^(3/2)
```

3.762.9 Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx$$
$$= -\frac{(3\,x^4+6\,x^3+3\,x^2+2)\,\left(-x^4-2\,x^3-x^2+1\right)^{3/2}}{15}$$

input
```
int(x^3*(2*x + 1)*(x + 1)^3*(1 - 2*x^3 - x^4 - x^2)^(1/2),x)
```

output
```
-((3*x^2 + 6*x^3 + 3*x^4 + 2)*(1 - 2*x^3 - x^4 - x^2)^(3/2))/15
```

3.763 $\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx$

3.763.1 Optimal result

Integrand size = 28, antiderivative size = 42

$$\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx=-\frac{1}{15}\left(1-x^2-2x^3-x^4\right)^{3/2}\left(2+3x^2+6x^3+3x^4\right)$$

output
```
-1/15*(-x^4-2*x^3-x^2+1)^(3/2)*(3*x^4+6*x^3+3*x^2+2)
```

3.763.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx=\frac{1}{15}\left(-2-3x^2-6x^3-3x^4\right)\left(1-x^2-2x^3-x^4\right)^{3/2}$$

input
```
Integrate[(1 + 2*x)*(x + x^2)^3*Sqrt[1 - (x + x^2)^2],x]
```

output
```
((-2 - 3*x^2 - 6*x^3 - 3*x^4)*(1 - x^2 - 2*x^3 - x^4)^(3/2))/15
```

3.763.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.76, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2027, 2459, 2029, 2069, 1576, 27, 1116, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x+1)\left(x^2+x\right)^3\sqrt{1-\left(x^2+x\right)^2}dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int x^3(x+1)^3(2x+1)\sqrt{1-\left(x^2+x\right)^2}dx$$

$$\Big\downarrow \text{ 2459}$$

$$\int \sqrt{-\left(x+\frac{1}{2}\right)^4+\frac{1}{2}\left(x+\frac{1}{2}\right)^2+\frac{15}{16}}\left(2\left(x+\frac{1}{2}\right)^7-\frac{3}{2}\left(x+\frac{1}{2}\right)^5+\frac{3}{8}\left(x+\frac{1}{2}\right)^3+\frac{1}{32}\left(-x-\frac{1}{2}\right)\right)d\left(x+\frac{1}{2}\right)$$

$$\Big\downarrow \text{ 2029}$$

$$\int \left(x+\frac{1}{2}\right)\sqrt{-\left(x+\frac{1}{2}\right)^4+\frac{1}{2}\left(x+\frac{1}{2}\right)^2+\frac{15}{16}}\left(2\left(x+\frac{1}{2}\right)^6-\frac{3}{2}\left(x+\frac{1}{2}\right)^4+\frac{3}{8}\left(x+\frac{1}{2}\right)^2-\frac{1}{32}\right)d\left(x+\frac{1}{2}\right)$$

$$\Big\downarrow \text{ 2069}$$

$$\int \left(x+\frac{1}{2}\right)\left(\sqrt[3]{2}\left(x+\frac{1}{2}\right)^2-\frac{1}{2\ 2^{2/3}}\right)^3\sqrt{-\left(x+\frac{1}{2}\right)^4+\frac{1}{2}\left(x+\frac{1}{2}\right)^2+\frac{15}{16}}d\left(x+\frac{1}{2}\right)$$

$$\Big\downarrow \text{ 1576}$$

$$\frac{1}{2}\int -\frac{1}{128}\left(1-4\left(x+\frac{1}{2}\right)^2\right)^3\sqrt{-16\left(x+\frac{1}{2}\right)^4+8\left(x+\frac{1}{2}\right)^2+15}d\left(x+\frac{1}{2}\right)^2$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{256}\int \left(1-4\left(x+\frac{1}{2}\right)^2\right)^3\sqrt{-16\left(x+\frac{1}{2}\right)^4+8\left(x+\frac{1}{2}\right)^2+15}d\left(x+\frac{1}{2}\right)^2$$

$$\Big\downarrow \text{ 1116}$$

3.763. $\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx$

$$\frac{1}{256}\left(-\frac{32}{5}\int\left(1-4\left(x+\frac{1}{2}\right)^2\right)\sqrt{-16\left(x+\frac{1}{2}\right)^4+8\left(x+\frac{1}{2}\right)^2+15}d\left(x+\frac{1}{2}\right)^2-\frac{1}{20}\left(-16\left(x+\frac{1}{2}\right)^4+8\left(x+\right.\right.\right.$$

$$\Big\downarrow \text{1104}$$

$$\frac{1}{256}\left(-\frac{1}{20}\left(-16\left(x+\frac{1}{2}\right)^4+8\left(x+\frac{1}{2}\right)^2+15\right)^{3/2}\left(1-4\left(x+\frac{1}{2}\right)^2\right)^2-\frac{8}{15}\left(-16\left(x+\frac{1}{2}\right)^4+8\left(x+\frac{1}{2}\right)^2+15\right.\right.$$

input
```
Int[(1 + 2*x)*(x + x^2)^3*Sqrt[1 - (x + x^2)^2],x]
```

output
```
((-8*(15 + 8*(1/2 + x)^2 - 16*(1/2 + x)^4)^(3/2))/15 - ((1 - 4*(1/2 + x)^2
)^2*(15 + 8*(1/2 + x)^2 - 16*(1/2 + x)^4)^(3/2))/20)/256
```

3.763.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1116
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[2*d*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(b*(m + 2*p
 + 1))), x] + Simp[d^2*(m - 1)*((b^2 - 4*a*c)/(b^2*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[m, 1] && NeQ[m + 2*p
+ 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) || OddQ[m])
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

3.763. $\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx$

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2029
```
Int[(Fx_.)*((d_.)*(x_)^(q_.) + (a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*
(x_)^(t_.))^(p_.), x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r)
+ d*x^(q - r))^p*Fx, x] /; FreeQ[{a, b, c, d, r, s, t, q}, x] && IntegerQ[p
] && PosQ[s - r] && PosQ[t - r] && PosQ[q - r] &&  !(EqQ[p, 1] && EqQ[u, 1]
)
```

rule 2069
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x^2, 0], Expon[Px, x^2
]], b = Rt[Coeff[Px, x^2, Expon[Px, x^2]], Expon[Px, x^2]]}, Int[u*(a + b*x
^2)^Expon[Px, x^2], x] /; EqQ[Px, (a + b*x^2)^Expon[Px, x^2]]] /; PolyQ[Px,
 x^2] && GtQ[Expon[Px, x^2], 1] && NeQ[Coeff[Px, x^2, 0], 0] &&  !MatchQ[Px
, (a_.)*(v_)^Expon[Px, x^2] /; FreeQ[a, x] && BinomialQ[v, x, 2]]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

3.763.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$-\frac{\left(-x^{4}-2x^{3}-x^{2}+1\right)^{\frac{3}{2}}\left(x^{4}+2x^{3}+x^{2}+\frac{2}{3}\right)}{5}$
gosper	$\frac{\left(x^{2}+x+1\right)\left(x^{2}+x-1\right)\left(3x^{4}+6x^{3}+3x^{2}+2\right)\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{15}$
trager	$\left(\frac{1}{5}x^{8}+\frac{4}{5}x^{7}+\frac{6}{5}x^{6}+\frac{4}{5}x^{5}+\frac{2}{15}x^{4}-\frac{2}{15}x^{3}-\frac{1}{15}x^{2}-\frac{2}{15}\right)\sqrt{-x^{4}-2x^{3}-x^{2}+1}$
risch	$-\frac{\left(3x^{8}+12x^{7}+18x^{6}+12x^{5}+2x^{4}-2x^{3}-x^{2}-2\right)\left(x^{4}+2x^{3}+x^{2}-1\right)}{15\sqrt{-x^{4}-2x^{3}-x^{2}+1}}$
default	$\frac{2x^{4}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{15}-\frac{2x^{3}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{15}-\frac{x^{2}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{15}-\frac{2\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{15}+\frac{x^{8}\sqrt{-x^{4}-2x^{3}-x}}{5}$
elliptic	$\frac{2x^{4}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{15}-\frac{2x^{3}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{15}-\frac{x^{2}\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{15}-\frac{2\sqrt{-x^{4}-2x^{3}-x^{2}+1}}{15}+\frac{x^{8}\sqrt{-x^{4}-2x^{3}-x}}{5}$

3.763. $\int(1+2x)\left(x+x^{2}\right)^{3}\sqrt{1-\left(x+x^{2}\right)^{2}}\,dx$

input
```
int((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(-x^4-2*x^3-x^2+1)^(3/2)*(x^4+2*x^3+x^2+2/3)
```

3.763.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx = \frac{1}{15}\left(3\,x^8+12\,x^7+18\,x^6+12\,x^5+2\,x^4-2\,x^3-x^2-2\right)\sqrt{-x^4-2\,x^3-x^2+1}$$

input
```
integrate((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(-x^
4 - 2*x^3 - x^2 + 1)
```

3.763.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(36) = 72.

Time = 0.77 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.33

$$\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx = \frac{x^8\sqrt{-x^4-2x^3-x^2+1}}{5} + \frac{4x^7\sqrt{-x^4-2x^3-x^2+1}}{5} + \frac{6x^6\sqrt{-x^4-2x^3-x^2+1}}{5} + \frac{4x^5\sqrt{-x^4-2x^3-x^2+1}}{5} + \frac{2x^4\sqrt{-x^4-2x^3-x^2+1}}{15} - \frac{2x^3\sqrt{-x^4-2x^3-x^2+1}}{15} - \frac{x^2\sqrt{-x^4-2x^3-x^2+1}}{15} - \frac{2\sqrt{-x^4-2x^3-x^2+1}}{15}$$

input
```
integrate((1+2*x)*(x**2+x)**3*(1-(x**2+x)**2)**(1/2),x)
```

output
```
x**8*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**7*sqrt(-x**4 - 2*x**3 - x**2
 + 1)/5 + 6*x**6*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**5*sqrt(-x**4 - 2
*x**3 - x**2 + 1)/5 + 2*x**4*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 - 2*x**3*s
qrt(-x**4 - 2*x**3 - x**2 + 1)/15 - x**2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/1
5 - 2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15
```

3.763.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.40

$$\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx = \frac{1}{15}\left(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2\right)\sqrt{x^2+x+1}\sqrt{-x^2-x+1}$$

input
```
integrate((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(x^2
 + x + 1)*sqrt(-x^2 - x + 1)
```

3.763.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx = \frac{1}{5}\left(x^4+2x^3+x^2-1\right)^2\sqrt{-x^4-2x^3-x^2+1} - \frac{1}{3}\left(-x^4-2x^3-x^2+1\right)^{\frac{3}{2}}$$

input
```
integrate((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/5*(x^4 + 2*x^3 + x^2 - 1)^2*sqrt(-x^4 - 2*x^3 - x^2 + 1) - 1/3*(-x^4 - 2
*x^3 - x^2 + 1)^(3/2)
```

3.763.9 Mupad [B] (verification not implemented)

Time = 18.47 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

$$\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx = \sqrt{1-\left(x^2+x\right)^2}\left(\frac{x^8}{5}+\frac{4\,x^7}{5}+\frac{6\,x^6}{5}+\frac{4\,x^5}{5}+\frac{2\,x^4}{15}-\frac{2\,x^3}{15}-\frac{x^2}{15}-\frac{2}{15}\right)$$

input

```
int((2*x + 1)*(1 - (x + x^2)^2)^(1/2)*(x + x^2)^3,x)
```

output

```
(1 - (x + x^2)^2)^(1/2)*((2*x^4)/15 - (2*x^3)/15 - x^2/15 + (4*x^5)/5 + (6
*x^6)/5 + (4*x^7)/5 + x^8/5 - 2/15)
```

3.764 $\int (8x-8x^2+4x^3-x^4)^{3/2}\,dx$

3.764.1 Optimal result

Integrand size = 23, antiderivative size = 102

$$\int (8x-8x^2+4x^3-x^4)^{3/2}\,dx=\frac{2}{35}\left(13-3(-1+x)^2\right)\sqrt{3-2(-1+x)^2-(-1+x)^4}(-1+x)+\frac{1}{7}\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)+\frac{16}{5}\sqrt{3}E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)-\frac{176}{35}\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)$$

output
```
1/7*(3-2*(-1+x)^2-(-1+x)^4)^(3/2)*(-1+x)-16/5*EllipticE(-1+x,1/3*I*3^(1/2)
)*3^(1/2)+176/35*EllipticF(-1+x,1/3*I*3^(1/2))*3^(1/2)+2/35*(13-3*(-1+x)^2
)*(-1+x)*(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

3.764.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 22.48 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.73

$$\int (8x-8x^2+4x^3-x^4)^{3/2}\,dx=\frac{896-1056x+352x^2+848x^3-1420x^4+1152x^5-602x^6+206x^7-45x^8+5x^9+\frac{112i\sqrt{2}(-2\cdots}{\cdots}}{35\sqrt{\cdots}}$$

input
```
Integrate[(8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

output
```
(896 - 1056*x + 352*x^2 + 848*x^3 - 1420*x^4 + 1152*x^5 - 602*x^6 + 206*x^
7 - 45*x^8 + 5*x^9 + ((112*I)*Sqrt[2]*(-2 + x)*x*Sqrt[(4 - 2*x + x^2)/x^2]
*EllipticE[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[
3])/(-I + Sqrt[3])])/Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)] - (304*I)*Sq
rt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*x^2*Sqrt[(4 - 2*x + x^2)/x^
2]*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqr
t[3])/(-I + Sqrt[3])])/(35*Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))])
```

3.764.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {2458, 1404, 27, 1490, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-x^4+4x^3-8x^2+8x\right)^{3/2} dx$$

$$\Big\downarrow \text{2458}$$

$$\int \left(-(x-1)^4-2(x-1)^2+3\right)^{3/2} d(x-1)$$

$$\Big\downarrow \text{1404}$$

$$\frac{3}{7}\int 2\left(3-(x-1)^2\right)\sqrt{-(x-1)^4-2(x-1)^2+3}d(x-1)+\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{7}\int \left(3-(x-1)^2\right)\sqrt{-(x-1)^4-2(x-1)^2+3}d(x-1)+\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{1490}$$

$$\frac{6}{7}\left(\frac{1}{15}\left(13-3(x-1)^2\right)\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)-\frac{1}{15}\int -\frac{8\left(12-7(x-1)^2\right)}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)\right)+\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{7}\left(\frac{8}{15}\int \frac{12-7(x-1)^2}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)+\frac{1}{15}\left(13-3(x-1)^2\right)\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)\right)+$$
$$\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{1494}$$

$$\frac{6}{7}\left(\frac{16}{15}\int \frac{12-7(x-1)^2}{2\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{1}{15}\left(13-3(x-1)^2\right)\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)\right)+$$
$$\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{7}\left(\frac{8}{15}\int \frac{12-7(x-1)^2}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{1}{15}\left(13-3(x-1)^2\right)\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)\right)+$$
$$\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{399}$$

$$\frac{6}{7}\left(\frac{8}{15}\left(33\int \frac{1}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)-7\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)\right)+\frac{1}{15}\left(13-3(x-1)^2\right)\sqrt{-(x$$
$$\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{321}$$

$$\frac{6}{7}\left(\frac{8}{15}\left(-7\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)-11\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{1}{15}\left(13-3(x-1)^2\right)\sqrt{-(x-1)$$
$$\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{327}$$

$$\frac{6}{7}\left(\frac{8}{15}\left(7\sqrt{3}E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)-11\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{1}{15}\left(13-3(x-1)^2\right)\sqrt{-(x-1)^4$$
$$\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

input
```
Int[(8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

3.764. $\int \left(8x-8x^2+4x^3-x^4\right)^{3/2}\,dx$

output
```
((3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/7 + (6*(((13 - 3*(-1 + x)
^2)*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/15 + (8*(7*Sqrt[3]*Ellip
ticE[ArcSin[1 - x], -1/3] - 11*Sqrt[3]*EllipticF[ArcSin[1 - x], -1/3]))/15
))/7
```

3.764.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

3.764. $\int (8x - 8x^2 + 4x^3 - x^4)^{3/2} \, dx$

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.764.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 957 vs. $2(86) = 172$.

Time = 2.64 (sec) , antiderivative size = 958, normalized size of antiderivative = 9.39

method	result	size
risch	Expression too large to display	958
default	Expression too large to display	1050
elliptic	Expression too large to display	1050

input
```
int((-x^4+4*x^3-8*x^2+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*(5*x^5-25*x^4+66*x^3-98*x^2+32*x+20)*x*(x^3-4*x^2+8*x-8)/(-x*(x^3-4*x
^2+8*x-8))^(1/2)+32/7*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2)
)^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2
))/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-
1-I*3^(1/2)))^(1/2)*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)
,((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+64/5*(
-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I
*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))
^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*(2*
EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-
I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(((-1+I*3^(1/2
))*x/(1+I*3^(1/2))/(x-2))^(1/2),(1+I*3^(1/2))/(-1+I*3^(1/2)),((1+I*3^(1/2)
)*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2)))-16/5*(x*(x-1+I*3^(1
/2))*(x-1-I*3^(1/2))+2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2
))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/
2))/(1+I*3^(1/2))/(x-2))^(1/2)*(1/2*(6+2*I*3^(1/2))/(-1+I*3^(1/2))*Ellipti
cF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/
2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+1/2*(-1+I*3^(1/2))*EllipticE(((-1
+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1
+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-4/(-1+I*3^(1/2))*EllipticPi(((-1+I*3^...
```

3.764.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.88

$$\int \left(8x-8x^2+4x^3-x^4\right)^{3/2}\,dx =$$
$$-\frac{112\,(-i\,x+i)E\left(\arcsin\left(\frac{1}{x-1}\right)\,|-3\right)+80\,(-i\,x+i)F\left(\arcsin\left(\frac{1}{x-1}\right)\,|-3\right)+\left(5\,x^6-30\,x^5+91\,x^4-164\,x^3\right.}{35\,(x-1)}$$

input
```
integrate((-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="fricas")
```

output
```
-1/35*(112*(-I*x + I)*elliptic_e(arcsin(1/(x - 1)), -3) + 80*(-I*x + I)*el
liptic_f(arcsin(1/(x - 1)), -3) + (5*x^6 - 30*x^5 + 91*x^4 - 164*x^3 + 130
*x^2 - 12*x - 132)*sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x))/(x - 1)
```

3.764.6 Sympy [F]

$$\int \left(8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int \left(-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-x**4+4*x**3-8*x**2+8*x)**(3/2),x)
```

output
```
Integral((-x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

3.764.7 Maxima [F]

$$\int \left(8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int \left(-x^4+4\,x^3-8\,x^2+8\,x\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(3/2), x)
```

3.764.8 Giac [F]

$$\int \left(8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int \left(-x^4+4\,x^3-8\,x^2+8\,x\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(3/2), x)
```

3.764.9 Mupad [F(-1)]

Timed out.

$$\int \left(8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int \left(-x^4+4\,x^3-8\,x^2+8\,x\right)^{3/2}\,dx$$

input
```
int((8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
```

output
```
int((8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

3.765 $\int \sqrt{8x-8x^2+4x^3-x^4}\,dx$

3.765.1 Optimal result

Integrand size = 23, antiderivative size = 62

$$\int \sqrt{8x-8x^2+4x^3-x^4}\,dx = \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}(-1+x) + \frac{2E\left(\arcsin(1-x)\,\middle|-\frac{1}{3}\right)}{\sqrt{3}} - \frac{4\,\mathrm{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{\sqrt{3}}$$

output
```
-2/3*EllipticE(-1+x,1/3*I*3^(1/2))*3^(1/2)+4/3*EllipticF(-1+x,1/3*I*3^(1/2
))*3^(1/2)+1/3*(-1+x)*(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

3.765.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 22.42 (sec) , antiderivative size = 256, normalized size of antiderivative = 4.13

$$\int \sqrt{8x-8x^2+4x^3-x^4}\,dx = -\frac{-16+24x-24x^2+14x^3-5x^4+x^5-\frac{2i\sqrt{2}(-2+x)x\sqrt{\frac{4-2x+x^2}{x^2}}E\left(\arcsin\left(\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)}{\sqrt{-\frac{i(-2+x)}{(-i+\sqrt{3})x}}}+8i\sqrt{2}\sqrt{-\frac{i(-2\cdots}{(-i+\cdots}}\cdots}{3\sqrt{-x\left(-8+8x-4x^2+x^3\right)}}$$

input
```
Integrate[Sqrt[8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output
```
-1/3*(-16 + 24*x - 24*x^2 + 14*x^3 - 5*x^4 + x^5 - ((2*I)*Sqrt[2]*(-2 + x)
*x*Sqrt[(4 - 2*x + x^2)/x^2]*EllipticE[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/
(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/Sqrt[((-I)*(-2 + x))/((-I
 + Sqrt[3])*x)] + (8*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*x
^2*Sqrt[(4 - 2*x + x^2)/x^2]*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/
(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/Sqrt[-(x*(-8 + 8*x - 4*x^
2 + x^3))]
```

3.765.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {2458, 1404, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-x^4+4x^3-8x^2+8x}\,dx$$

$$\Big\downarrow \text{2458}$$

$$\int \sqrt{-(x-1)^4-2(x-1)^2+3}d(x-1)$$

$$\Big\downarrow \text{1404}$$

$$\frac{1}{3}\int \frac{2\left(3-(x-1)^2\right)}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)+\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\int \frac{3-(x-1)^2}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)+\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

$$\Big\downarrow \text{1494}$$

$$\frac{4}{3}\int \frac{3-(x-1)^2}{2\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\int \frac{3-(x-1)^2}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

$$\Big\downarrow \text{399}$$

$$\frac{2}{3}\left(6\int \frac{1}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)-\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)\right)+\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

$$\Big\downarrow \text{321}$$

$$\frac{2}{3}\left(-\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)-2\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

$$\Big\downarrow \text{327}$$

$$\frac{2}{3}\left(\sqrt{3}E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)-2\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

input
```
Int[Sqrt[8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output
```
(Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/3 + (2*(Sqrt[3]*EllipticE[A
rcSin[1 - x], -1/3] - 2*Sqrt[3]*EllipticF[ArcSin[1 - x], -1/3]))/3
```

3.765.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.765. $\int \sqrt{8x-8x^2+4x^3-x^4}\,dx$

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.765.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 935 vs. 2(54) = 108.

Time = 2.00 (sec) , antiderivative size = 936, normalized size of antiderivative = 15.10

method	result
risch	$-\frac{(x-1)x(x^3-4x^2+8x-8)}{3\sqrt{-x(x^3-4x^2+8x-8)}}+\frac{8\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\,(x-2)^2\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\,\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(x-2)}}\,F\left(\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}},\sqrt{\frac{(1+i}{(-1+}}\right.}{3\left(-1+i\sqrt{3}\right)\sqrt{-x(x-2)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{3}\right)}}$
default	$\frac{x\sqrt{-x^4+4x^3-8x^2+8x}}{3}-\frac{\sqrt{-x^4+4x^3-8x^2+8x}}{3}+\frac{8\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\,(x-2)^2\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\,\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(x-2)}}\,F\left(\sqrt{\right.}{3\left(-1+i\sqrt{3}\right)\sqrt{-x(x-2)\left(x-1+i\sqrt{3}\right)\left(x-1-\right.}}$
elliptic	$\frac{x\sqrt{-x^4+4x^3-8x^2+8x}}{3}-\frac{\sqrt{-x^4+4x^3-8x^2+8x}}{3}+\frac{8\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\,(x-2)^2\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\,\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(x-2)}}\,F\left(\sqrt{\right.}{3\left(-1+i\sqrt{3}\right)\sqrt{-x(x-2)\left(x-1+i\sqrt{3}\right)\left(x-1-\right.}}$

input
```
int((-x^4+4*x^3-8*x^2+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(x-1)*x*(x^3-4*x^2+8*x-8)/(-x*(x^3-4*x^2+8*x-8))^(1/2)+8/3*(-1-I*3^(1
/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))
/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(-
1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*EllipticF(((
-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(
-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-2/3*(x*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))
+2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x
-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x
-2))^(1/2)*(1/2*(6+2*I*3^(1/2))/(-1+I*3^(1/2))*EllipticF(((-1+I*3^(1/2))*x
/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(
1-I*3^(1/2)))^(1/2))+1/2*(-1+I*3^(1/2))*EllipticE(((-1+I*3^(1/2))*x/(1+I*3
^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(
1/2)))^(1/2))-4/(-1+I*3^(1/2))*EllipticPi(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/
(x-2))^(1/2),(-1-I*3^(1/2))/(1-I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-
1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1
/2)))^(1/2)+8/3*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2
)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+
I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^
(1/2)))^(1/2)*(2*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((
1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-2*Elli...
```

3.765. $\int \sqrt{8x-8x^2+4x^3-x^4}\,dx$

3.765.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int \sqrt{8x-8x^2+4x^3-x^4}\,dx = -\frac{2(-ix+i)E\left(\arcsin\left(\frac{1}{x-1}\right)|-3\right)+4(-ix+i)F\left(\arcsin\left(\frac{1}{x-1}\right)|-3\right)-\sqrt{-x^4+4x^3-8x^2+8x}(x^2-}{3(x-1)}$$

input
```
integrate((-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(2*(-I*x + I)*elliptic_e(arcsin(1/(x - 1)), -3) + 4*(-I*x + I)*ellipt
ic_f(arcsin(1/(x - 1)), -3) - sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x)*(x^2 - 2*x
+ 3))/(x - 1)
```

3.765.6 Sympy [F]

$$\int \sqrt{8x-8x^2+4x^3-x^4}\,dx = \int \sqrt{-x^4+4x^3-8x^2+8x}\,dx$$

input
```
integrate((-x**4+4*x**3-8*x**2+8*x)**(1/2),x)
```

output
```
Integral(sqrt(-x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

3.765.7 Maxima [F]

$$\int \sqrt{8x-8x^2+4x^3-x^4}\,dx = \int \sqrt{-x^4+4x^3-8x^2+8x}\,dx$$

input
```
integrate((-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

3.765.8 Giac [F]

$$\int \sqrt{8x-8x^2+4x^3-x^4}\,dx=\int \sqrt{-x^4+4\,x^3-8\,x^2+8\,x}\,dx$$

input
```
integrate((-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

3.765.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{8x-8x^2+4x^3-x^4}\,dx=\int \sqrt{-x^4+4\,x^3-8\,x^2+8\,x}\,dx$$

input
```
int((8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
```

output
```
int((8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

3.766 $\int \frac{1}{\sqrt{8x-8x^2+4x^3-x^4}}\,dx$

3.766.1 Optimal result

Integrand size = 23, antiderivative size = 17

$$\int \frac{1}{\sqrt{8x-8x^2+4x^3-x^4}}\,dx = -\frac{\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{\sqrt{3}}$$

output
```
1/3*EllipticF(-1+x,1/3*I*3^(1/2))*3^(1/2)
```

3.766.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 32.15 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.18

$$\int \frac{1}{\sqrt{8x-8x^2+4x^3-x^4}}\,dx = \frac{\sqrt{-i+\sqrt{3}+\frac{4i}{x}}\sqrt{-\frac{i(-2+x)}{\left(-i+\sqrt{3}\right)x}}\,x\left(-4+x-i\sqrt{3}x\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right),\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)}{\sqrt{2}\sqrt{i+\sqrt{3}-\frac{4i}{x}}\sqrt{-x\left(-8+8x-4x^2+x^3\right)}}$$

input
```
Integrate[1/Sqrt[8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output
```
(Sqrt[-I + Sqrt[3] + (4*I)/x]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*x*(
-4 + x - I*Sqrt[3]*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2
]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/(Sqrt[2]*Sqrt[I + Sqrt[3] - (4*I
)/x]*Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))])
```

3.766.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2458, 1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-x^4+4x^3-8x^2+8x}}dx$$

$$\Big\downarrow \text{ 2458}$$

$$\int \frac{1}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)$$

$$\Big\downarrow \text{ 1408}$$

$$2\int \frac{1}{2\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)$$

$$\Big\downarrow \text{ 321}$$

$$-\frac{\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output
```
-(EllipticF[ArcSin[1 - x], -1/3]/Sqrt[3])
```

3.766.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.766.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(15) = 30$.

Time = 1.36 (sec) , antiderivative size = 200, normalized size of antiderivative = 11.76

method	result	size
default	$\frac{2\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\,(x-2)^{2}\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\,\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(x-2)}}\,F\left(\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}},\sqrt{\frac{\left(1+i\sqrt{3}\right)\left(-1-i\sqrt{3}\right)}{\left(-1+i\sqrt{3}\right)\left(1-i\sqrt{3}\right)}}\right)}{\left(-1+i\sqrt{3}\right)\sqrt{-x(x-2)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{3}\right)}}$	200
elliptic	$\frac{2\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\,(x-2)^{2}\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\,\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(x-2)}}\,F\left(\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}},\sqrt{\frac{\left(1+i\sqrt{3}\right)\left(-1-i\sqrt{3}\right)}{\left(-1+i\sqrt{3}\right)\left(1-i\sqrt{3}\right)}}\right)}{\left(-1+i\sqrt{3}\right)\sqrt{-x(x-2)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{3}\right)}}$	200

input
```
int(1/(-x^4+4*x^3-8*x^2+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-
1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-
2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*
EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-
I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))
```

3.766.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{8x-8x^2+4x^3-x^4}}\,dx = -\frac{1}{2}\sqrt{2}\text{weierstrassPInverse}\left(-\frac{2}{3},\frac{7}{54},-\frac{x-3}{3\,x}\right)$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2)*weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x)
```

3.766.6 Sympy [F]

$$\int \frac{1}{\sqrt{8x-8x^2+4x^3-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+4x^3-8x^2+8x}}\,dx$$

input
```
integrate(1/(-x**4+4*x**3-8*x**2+8*x)**(1/2),x)
```

output
```
Integral(1/sqrt(-x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

3.766.7 Maxima [F]

$$\int \frac{1}{\sqrt{8x-8x^2+4x^3-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+4\,x^3-8\,x^2+8\,x}}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

3.766.8 Giac [F]

$$\int \frac{1}{\sqrt{8x-8x^2+4x^3-x^4}}\,dx=\int \frac{1}{\sqrt{-x^4+4\,x^3-8\,x^2+8\,x}}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

3.766.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{8x-8x^2+4x^3-x^4}}\,dx=\int \frac{1}{\sqrt{-x^4+4\,x^3-8\,x^2+8\,x}}\,dx$$

input
```
int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
```

output
```
int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

3.767 $\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx$

3.767.1 Optimal result

Integrand size = 23, antiderivative size = 73

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \frac{(5+(-1+x)^2)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} + \frac{E\left(\arcsin(1-x)\middle|-\frac{1}{3}\right)}{8\sqrt{3}} - \frac{\operatorname{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{4\sqrt{3}}$$

output
```
-1/24*EllipticE(-1+x,1/3*I*3^(1/2))*3^(1/2)+1/12*EllipticF(-1+x,1/3*I*3^(1
/2))*3^(1/2)+1/24*(5+(-1+x)^2)*(-1+x)/(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

3.767.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 22.67 (sec) , antiderivative size = 261, normalized size of antiderivative = 3.58

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \frac{\sqrt{-x\left(-8+8x-4x^2+x^3\right)}\left(\frac{\sqrt{2}\left(-i+\sqrt{3}\right)\sqrt{-\frac{i(-2+x)}{\left(-i+\sqrt{3}\right)x}}E\left(\arcsin\left(\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{-i+\sqrt{}}\right.}{\sqrt{\frac{4-2x+x^2}{x^2}}}\right.}{24(-2+}$$

input
```
Integrate[(8*x - 8*x^2 + 4*x^3 - x^4)^(-3/2),x]
```

output
```
(Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))]*((Sqrt[2]*(-I + Sqrt[3])*Sqrt[((-I)*(
-2 + x))/((-I + Sqrt[3])*x)]*EllipticE[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/
(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/Sqrt[(4 - 2*x + x^2)/x^2]
 - (2 + x^2 - (4*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*x^2*S
qrt[(4 - 2*x + x^2)/x^2]*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqr
t[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/(4 - 2*x + x^2)))/(24*(-2 + x
)*x)
```

3.767.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2458, 1405, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-x^4+4x^3-8x^2+8x)^{3/2}}dx$$

$$\Big\downarrow \text{2458}$$

$$\int \frac{1}{(-(x-1)^4-2(x-1)^2+3)^{3/2}}d(x-1)$$

$$\Big\downarrow \text{1405}$$

$$\frac{((x-1)^2+5)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}-\frac{1}{48}\int -\frac{2(3-(x-1)^2)}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{24}\int \frac{3-(x-1)^2}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)+\frac{((x-1)^2+5)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{1494}$$

$$\frac{1}{12}\int \frac{3-(x-1)^2}{2\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{((x-1)^2+5)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{24}\int \frac{3-(x-1)^2}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{((x-1)^2+5)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{24}\left(6\int \frac{1}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)-\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)\right)+\frac{((x-1)^2+5)\,(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \; 321$$

$$\frac{1}{24}\left(-\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)-2\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{((x-1)^2+5)\,(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \; 327$$

$$\frac{1}{24}\left(\sqrt{3}E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)-2\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{((x-1)^2+5)\,(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}$$

input
```
Int[(8*x - 8*x^2 + 4*x^3 - x^4)^(-3/2),x]
```

output
```
((5 + (-1 + x)^2)*(-1 + x))/(24*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sq
rt[3]*EllipticE[ArcSin[1 - x], -1/3] - 2*Sqrt[3]*EllipticF[ArcSin[1 - x],
-1/3])/24
```

3.767.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.767. $\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx$

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.767.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 931 vs. $2(61) = 122$.

Time = 2.03 (sec) , antiderivative size = 932, normalized size of antiderivative = 12.77

method	result
risch	$\frac{x^3-3x^2+8x-6}{24\sqrt{-x(x^3-4x^2+8x-8)}}+\frac{\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\,(x-2)^2\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\,\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(x-2)}}\,F\left(\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}},\sqrt{\frac{\left(1+i\sqrt{3}\right)}{\left(-1+i\sqrt{3}\right)}}\right.}{6\left(-1+i\sqrt{3}\right)\sqrt{-x(x-2)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{3}\right)}}$
default	Expression too large to display
elliptic	Expression too large to display

3.767. $\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx$

input
```
int(1/(-x^4+4*x^3-8*x^2+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(x^3-3*x^2+8*x-6)/(-x*(x^3-4*x^2+8*x-8))^(1/2)+1/6*(-1-I*3^(1/2))*((-
1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^
(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1
/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*EllipticF(((-1+I*3^(
1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(
1/2))/(1-I*3^(1/2)))^(1/2))-1/24*(x*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))+2*(-1-
I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^
(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1
/2)*(1/2*(6+2*I*3^(1/2))/(-1+I*3^(1/2))*EllipticF(((-1+I*3^(1/2))*x/(1+I*3
^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(
1/2)))^(1/2))+1/2*(-1+I*3^(1/2))*EllipticE(((-1+I*3^(1/2))*x/(1+I*3^(1/2))
/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^
(1/2))-4/(-1+I*3^(1/2))*EllipticPi(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^
(1/2),(-1-I*3^(1/2))/(1-I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(
1/2))/(1-I*3^(1/2)))^(1/2))))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(
1/2)+1/6*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)
^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/
2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))
^(1/2)*(2*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(
1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(...
```

3.767.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.63

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx =$$
$$-\frac{5\sqrt{2}(x^4-4\,x^3+8\,x^2-8\,x)\text{weierstrassPInverse}\left(-\frac{2}{3},\frac{7}{54},-\frac{x-3}{3\,x}\right)-6\sqrt{2}(x^4-4\,x^3+8\,x^2-8\,x)\text{weierstrass}}{72\,(x^4-4\,x^3+8\,x^2-}$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="fricas")
```

output
```
-1/72*(5*sqrt(2)*(x^4 - 4*x^3 + 8*x^2 - 8*x)*weierstrassPInverse(-2/3, 7/5
4, -1/3*(x - 3)/x) - 6*sqrt(2)*(x^4 - 4*x^3 + 8*x^2 - 8*x)*weierstrassZeta
(-2/3, 7/54, weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x)) + 3*sqrt(-x^
4 + 4*x^3 - 8*x^2 + 8*x)*(x^2 + 2))/(x^4 - 4*x^3 + 8*x^2 - 8*x)
```

3.767.6 Sympy [F]

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+4x^3-8x^2+8x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x**4+4*x**3-8*x**2+8*x)**(3/2),x)
```

output
```
Integral((-x**4 + 4*x**3 - 8*x**2 + 8*x)**(-3/2), x)
```

3.767.7 Maxima [F]

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+4\,x^3-8\,x^2+8\,x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-3/2), x)
```

3.767.8 Giac [F]

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+4\,x^3-8\,x^2+8\,x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-3/2), x)
```

3.767.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx=\int \frac{1}{(-x^4+4\,x^3-8\,x^2+8\,x)^{3/2}}\,dx$$

input
```
int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
```

output
```
int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

3.768 $\int \frac{1}{(8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

3.768.1 Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \frac{(5+(-1+x)^2)(-1+x)}{72(3-2(-1+x)^2-(-1+x)^4)^{3/2}} + \frac{(26+7(-1+x)^2)(-1+x)}{432\sqrt{3-2(-1+x)^2-(-1+x)^4}} + \frac{7E\left(\arcsin(1-x)\,\middle|\,-\frac{1}{3}\right)}{144\sqrt{3}} - \frac{11\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{144\sqrt{3}}$$

output
```
1/72*(5+(-1+x)^2)*(-1+x)/(3-2*(-1+x)^2-(-1+x)^4)^(3/2)-7/432*EllipticE(-1+
x,1/3*I*3^(1/2))*3^(1/2)+11/432*EllipticF(-1+x,1/3*I*3^(1/2))*3^(1/2)+1/43
2*(26+7*(-1+x)^2)*(-1+x)/(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

3.768.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 22.85 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.73

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \frac{\frac{7i\sqrt{2}(-2+x)x^2\sqrt{\frac{4-2x+x^2}{x^2}}E\left(\arcsin\left(\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)}{\sqrt{-\frac{i(-2+x)}{(-i+\sqrt{3})x}}} + \frac{36-232x+274x^2-226x^3+115x^4-}{}}{432x\sqrt{-x\,(-}}$$

input
```
Integrate[(8*x - 8*x^2 + 4*x^3 - x^4)^(-5/2),x]
```

output
```
(((7*I)*Sqrt[2]*(-2 + x)*x^2*Sqrt[(4 - 2*x + x^2)/x^2]*EllipticE[ArcSin[Sq
rt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])
/Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)] + (36 - 232*x + 274*x^2 - 226*x^
3 + 115*x^4 - 37*x^5 + 7*x^6 - (19*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I +
Sqrt[3])*x)]*x^3*Sqrt[(4 - 2*x + x^2)/x^2]*(-8 + 8*x - 4*x^2 + x^3)*Ellipt
icF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I
 + Sqrt[3])])/(-8 + 8*x - 4*x^2 + x^3))/(432*x*Sqrt[-(x*(-8 + 8*x - 4*x^2
+ x^3))])
```

3.768.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {2458, 1405, 27, 1492, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-x^4+4x^3-8x^2+8x)^{5/2}}dx$$

$$\Big\downarrow \text{2458}$$

$$\int \frac{1}{(-(x-1)^4-2(x-1)^2+3)^{5/2}}d(x-1)$$

$$\Big\downarrow \text{1405}$$

$$\frac{((x-1)^2+5)(x-1)}{72(-(x-1)^4-2(x-1)^2+3)^{3/2}}-\frac{1}{144}\int -\frac{2(3(x-1)^2+19)}{(-(x-1)^4-2(x-1)^2+3)^{3/2}}d(x-1)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{72}\int \frac{3(x-1)^2+19}{(-(x-1)^4-2(x-1)^2+3)^{3/2}}d(x-1)+\frac{((x-1)^2+5)(x-1)}{72(-(x-1)^4-2(x-1)^2+3)^{3/2}}$$

$$\Big\downarrow \text{1492}$$

$$\frac{1}{72}\left(\frac{(7(x-1)^2+26)(x-1)}{6\sqrt{-(x-1)^4-2(x-1)^2+3}}-\frac{1}{48}\int -\frac{8(12-7(x-1)^2)}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)\right)+\frac{((x-1)^2+5)(x-1)}{72(-(x-1)^4-2(x-1)^2+3)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{72}\left(\frac{1}{6}\int \frac{12-7(x-1)^2}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)+\frac{\left(7(x-1)^2+26\right)(x-1)}{6\sqrt{-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \ 1494$$

$$\frac{1}{72}\left(\frac{1}{3}\int \frac{12-7(x-1)^2}{2\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{\left(7(x-1)^2+26\right)(x-1)}{6\sqrt{-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{72}\left(\frac{1}{6}\int \frac{12-7(x-1)^2}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{\left(7(x-1)^2+26\right)(x-1)}{6\sqrt{-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \ 399$$

$$\frac{1}{72}\left(\frac{1}{6}\left(33\int \frac{1}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)-7\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)\right)+\frac{\left(7(x-1)^2+26\right)(x-1)}{6\sqrt{-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \ 321$$

$$\frac{1}{72}\left(\frac{1}{6}\left(-7\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)-11\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{\left(7(x-1)^2+26\right)(x-1)}{6\sqrt{-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \ 327$$

$$\frac{1}{72}\left(\frac{1}{6}\left(7\sqrt{3}E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)-11\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{\left(7(x-1)^2+26\right)(x-1)}{6\sqrt{-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

3.768. $\int \frac{1}{(8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

input
```
Int[(8*x - 8*x^2 + 4*x^3 - x^4)^(-5/2),x]
```

output
```
((5 + (-1 + x)^2)*(-1 + x))/(72*(3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)) + (
((26 + 7*(-1 + x)^2)*(-1 + x))/(6*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]) + (
7*Sqrt[3]*EllipticE[ArcSin[1 - x], -1/3] - 11*Sqrt[3]*EllipticF[ArcSin[1 -
 x], -1/3])/6)/72
```

3.768.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

3.768. $\int \frac{1}{(8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.768.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 971 vs. 2(93) = 186.

Time = 2.07 (sec) , antiderivative size = 972, normalized size of antiderivative = 8.92

method	result	size
risch	Expression too large to display	972
default	Expression too large to display	1039
elliptic	Expression too large to display	1039

input
```
int(1/(-x^4+4*x^3-8*x^2+8*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/432*(7*x^7-49*x^6+187*x^5-445*x^4+670*x^3-622*x^2+216*x+36)/(-x*(x^3-4*x
^2+8*x-8))^(1/2)/x/(x^3-4*x^2+8*x-8)+5/216*(-1-I*3^(1/2))*((-1+I*3^(1/2))*
x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))
^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2
)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*EllipticF(((-1+I*3^(1/2))*x/(1+I*
3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^
(1/2)))^(1/2))+7/108*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))
^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2)
)/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1
-I*3^(1/2)))^(1/2)*(2*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/
2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-2*El
lipticPi(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),(1+I*3^(1/2))/(-1+I*
3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2)
))-7/432*(x*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))+2*(-1-I*3^(1/2))*((-1+I*3^(1/2
))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-
2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)*(1/2*(6+2*I*3^(1/2))
/(-1+I*3^(1/2))*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1
+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+1/2*(-1+I*
3^(1/2))*EllipticE(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1
/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-4/(-1+I*3^(1/2...
```

3.768.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.79

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{5/2}}\,dx =$$
$$-\frac{43\sqrt{2}(x^8-8\,x^7+32\,x^6-80\,x^5+128\,x^4-128\,x^3+64\,x^2)\text{weierstrassPInverse}\left(-\frac{2}{3},\frac{7}{54},-\frac{x-3}{3\,x}\right)-84\,\sqrt{2}(x$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(5/2),x, algorithm="fricas")
```

output
```
-1/2592*(43*sqrt(2)*(x^8 - 8*x^7 + 32*x^6 - 80*x^5 + 128*x^4 - 128*x^3 + 6
4*x^2)*weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x) - 84*sqrt(2)*(x^8 -
 8*x^7 + 32*x^6 - 80*x^5 + 128*x^4 - 128*x^3 + 64*x^2)*weierstrassZeta(-2/
3, 7/54, weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x)) + 6*(7*x^6 - 37*
x^5 + 115*x^4 - 226*x^3 + 274*x^2 - 232*x + 36)*sqrt(-x^4 + 4*x^3 - 8*x^2
+ 8*x))/(x^8 - 8*x^7 + 32*x^6 - 80*x^5 + 128*x^4 - 128*x^3 + 64*x^2)
```

3.768.6 Sympy [F]

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{1}{(-x^4+4x^3-8x^2+8x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-x**4+4*x**3-8*x**2+8*x)**(5/2),x)
```

output
```
Integral((-x**4 + 4*x**3 - 8*x**2 + 8*x)**(-5/2), x)
```

3.768.7 Maxima [F]

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{1}{(-x^4+4\,x^3-8\,x^2+8\,x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-5/2), x)
```

3.768.8 Giac [F]

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{1}{(-x^4+4\,x^3-8\,x^2+8\,x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-5/2), x)
```

3.768.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{1}{(-x^4+4\,x^3-8\,x^2+8\,x)^{5/2}}\,dx$$

input
```
int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x)
```

output
```
int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(5/2), x)
```

3.769 $\int ((2-x)x(4-2x+x^2))^{3/2}\,dx$

3.769.1 Optimal result

Integrand size = 19, antiderivative size = 102

$$\int ((2-x)x(4-2x+x^2))^{3/2}\,dx = \frac{2}{35}(13 - 3(-1+x)^2)\sqrt{3-2(-1+x)^2-(-1+x)^4}(-1+x) + \frac{1}{7}\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x) + \frac{16}{5}\sqrt{3}E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right) - \frac{176}{35}\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)$$

output

```
1/7*(3-2*(-1+x)^2-(-1+x)^4)^(3/2)*(-1+x)-16/5*EllipticE(-1+x,1/3*I*3^(1/2)
)*3^(1/2)+176/35*EllipticF(-1+x,1/3*I*3^(1/2))*3^(1/2)+2/35*(13-3*(-1+x)^2
)*(-1+x)*(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

3.769.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 24.42 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.73

$$\int ((2-x)x(4-2x+x^2))^{3/2}\,dx = \frac{\sqrt{-x(-8+8x-4x^2+x^3)}\left(\sqrt{\frac{4-2x+x^2}{x^2}}(-224+152x+44x^2-228x^3+230x^4-116x^5+35\right.}{}$$

input
```
Integrate[((2 - x)*x*(4 - 2*x + x^2))^(3/2),x]
```

output
```
(Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))]*(Sqrt[(4 - 2*x + x^2)/x^2]*(-224 + 15
2*x + 44*x^2 - 228*x^3 + 230*x^4 - 116*x^5 + 35*x^6 - 5*x^7) + 112*Sqrt[2]
*(-I + Sqrt[3])*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*EllipticE[ArcSin[
Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])
] + (304*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*EllipticF[Arc
Sin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt
[3])]))/(35*(-2 + x)*x*Sqrt[(4 - 2*x + x^2)/x^2])
```

3.769.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {2458, 1404, 27, 1490, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left((2-x)x\left(x^2-2x+4\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{2458}$$

$$\int \left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}d(x-1)$$

$$\Big\downarrow \text{1404}$$

$$\frac{3}{7}\int 2\left(3-(x-1)^2\right)\sqrt{-(x-1)^4-2(x-1)^2+3}d(x-1)+\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{7}\int \left(3-(x-1)^2\right)\sqrt{-(x-1)^4-2(x-1)^2+3}d(x-1)+\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{1490}$$

$$\frac{6}{7}\left(\frac{1}{15}\left(13-3(x-1)^2\right)\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)-\frac{1}{15}\int -\frac{8\left(12-7(x-1)^2\right)}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)\right)+\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{7}\left(\frac{8}{15}\int \frac{12-7(x-1)^2}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)+\frac{1}{15}\left(13-3(x-1)^2\right)\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)\right)+\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{1494}$$

$$\frac{6}{7}\left(\frac{16}{15}\int \frac{12-7(x-1)^2}{2\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{1}{15}\left(13-3(x-1)^2\right)\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)\right)+\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{7}\left(\frac{8}{15}\int \frac{12-7(x-1)^2}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{1}{15}\left(13-3(x-1)^2\right)\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)\right)+\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{399}$$

$$\frac{6}{7}\left(\frac{8}{15}\left(33\int \frac{1}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)-7\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)\right)+\frac{1}{15}\left(13-3(x-1)^2\right)\sqrt{-(x\right.$$
$$\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{321}$$

$$\frac{6}{7}\left(\frac{8}{15}\left(-7\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)-11\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{1}{15}\left(13-3(x-1)^2\right)\sqrt{-(x-1)\right.$$
$$\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{327}$$

$$\frac{6}{7}\left(\frac{8}{15}\left(7\sqrt{3}E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)-11\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{1}{15}\left(13-3(x-1)^2\right)\sqrt{-(x-1)^4\right.$$
$$\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

input
```
Int[((2 - x)*x*(4 - 2*x + x^2))^(3/2),x]
```

3.769. $\int \left((2-x)x(4-2x+x^2)\right)^{3/2}\,dx$

output
```
((3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/7 + (6*(((13 - 3*(-1 + x)
^2)*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/15 + (8*(7*Sqrt[3]*Ellip
ticE[ArcSin[1 - x], -1/3] - 11*Sqrt[3]*EllipticF[ArcSin[1 - x], -1/3]))/15
))/7
```

3.769.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.769.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 953 vs. $2(86) = 172$.

Time = 2.22 (sec) , antiderivative size = 954, normalized size of antiderivative = 9.35

method	result	size
risch	Expression too large to display	954
default	Expression too large to display	1050
elliptic	Expression too large to display	1050

input
```
int(((2-x)*x*(x^2-2*x+4))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*(5*x^5-25*x^4+66*x^3-98*x^2+32*x+20)*x*(x-2)*(x^2-2*x+4)/(-x*(x-2)*(x
^2-2*x+4))^(1/2)+32/7*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2)
)^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2
))/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-
1-I*3^(1/2)))^(1/2)*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)
,((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+64/5*(
-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I
*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))
^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*(2*
EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-
I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(((-1+I*3^(1/2
))*x/(1+I*3^(1/2))/(x-2))^(1/2),(1+I*3^(1/2))/(-1+I*3^(1/2)),((1+I*3^(1/2)
)*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2)))-16/5*(x*(x-1+I*3^(1
/2))*(x-1-I*3^(1/2))+2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2
))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/
2))/(1+I*3^(1/2))/(x-2))^(1/2)*(1/2*(6+2*I*3^(1/2))/(-1+I*3^(1/2))*Ellipti
cF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/
2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+1/2*(-1+I*3^(1/2))*EllipticE(((-1
+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1
+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-4/(-1+I*3^(1/2))*EllipticPi(((-1+I*3^...
```

3.769.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.88

$$\int \left((2-x)x\left(4-2x+x^2\right)\right)^{3/2} dx =$$
$$-\frac{112\,(-i\,x+i)E\left(\arcsin\left(\frac{1}{x-1}\right)\mid-3\right)+80\,(-i\,x+i)F\left(\arcsin\left(\frac{1}{x-1}\right)\mid-3\right)+\left(5\,x^6-30\,x^5+91\,x^4-164\,x^3\right.}{35\,(x-1)}$$

input
```
integrate(((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="fricas")
```

output
```
-1/35*(112*(-I*x + I)*elliptic_e(arcsin(1/(x - 1)), -3) + 80*(-I*x + I)*el
liptic_f(arcsin(1/(x - 1)), -3) + (5*x^6 - 30*x^5 + 91*x^4 - 164*x^3 + 130
*x^2 - 12*x - 132)*sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x))/(x - 1)
```

3.769.6 Sympy [F]

$$\int \left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}\,dx=\int \left(x(2-x)\left(x^2-2x+4\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(((2-x)*x*(x**2-2*x+4))**(3/2),x)
```

output
```
Integral((x*(2 - x)*(x**2 - 2*x + 4))**(3/2), x)
```

3.769.7 Maxima [F]

$$\int \left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}\,dx=\int \left(-\left(x^2-2\,x+4\right)(x-2)x\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(3/2), x)
```

3.769.8 Giac [F]

$$\int \left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}\,dx=\int \left(-\left(x^2-2\,x+4\right)(x-2)x\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="giac")
```

output
```
integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(3/2), x)
```

3.769.9 Mupad [F(-1)]

Timed out.

$$\int \left((2-x)x(4-2x+x^2)\right)^{3/2}\,dx=\int \left(-x\,(x-2)\,\left(x^2-2\,x+4\right)\right)^{3/2}\,dx$$

input
```
int((-x*(x - 2)*(x^2 - 2*x + 4))^(3/2),x)
```

output
```
int((-x*(x - 2)*(x^2 - 2*x + 4))^(3/2), x)
```

3.770 $\int \sqrt{(2-x)x(4-2x+x^2)}\,dx$

3.770.1 Optimal result

Integrand size = 19, antiderivative size = 62

$$\int \sqrt{(2-x)x(4-2x+x^2)}\,dx = \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}(-1+x) + \frac{2E\left(\arcsin(1-x)\,\middle|-\frac{1}{3}\right)}{\sqrt{3}} - \frac{4\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{\sqrt{3}}$$

output
```
-2/3*EllipticE(-1+x,1/3*I*3^(1/2))*3^(1/2)+4/3*EllipticF(-1+x,1/3*I*3^(1/2
))*3^(1/2)+1/3*(-1+x)*(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

3.770.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 18.91 (sec) , antiderivative size = 256, normalized size of antiderivative = 4.13

$$\int \sqrt{(2-x)x(4-2x+x^2)}\,dx = \frac{\sqrt{-x(-8+8x-4x^2+x^3)}\left(\sqrt{\frac{4-2x+x^2}{x^2}}(-4+4x-3x^2+x^3)+2\sqrt{2}(-i+\sqrt{3})\sqrt{-\frac{i(-2+x)}{(-i+\sqrt{3})x}}E\left(\arcsin\left(\right.\right.\right.}{3(-2+x)x\sqrt{\frac{4-2x+x^2}{x^2}}}$$

input
```
Integrate[Sqrt[(2 - x)*x*(4 - 2*x + x^2)],x]
```

output
```
(Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))]*(Sqrt[(4 - 2*x + x^2)/x^2]*(-4 + 4*x
- 3*x^2 + x^3) + 2*Sqrt[2]*(-I + Sqrt[3])*Sqrt[((-I)*(-2 + x))/((-I + Sqrt
[3])*x)]*EllipticE[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))],
(2*Sqrt[3])/(-I + Sqrt[3])] + (8*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sq
rt[3])*x)]*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))]
, (2*Sqrt[3])/(-I + Sqrt[3])]))/(3*(-2 + x)*x*Sqrt[(4 - 2*x + x^2)/x^2])
```

3.770.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2458, 1404, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{(2-x)x\left(x^2-2x+4\right)}dx$$

$$\Big\downarrow \text{2458}$$

$$\int \sqrt{-(x-1)^4-2(x-1)^2+3}d(x-1)$$

$$\Big\downarrow \text{1404}$$

$$\frac{1}{3}\int \frac{2\left(3-(x-1)^2\right)}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)+\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\int \frac{3-(x-1)^2}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)+\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

$$\Big\downarrow \text{1494}$$

$$\frac{4}{3}\int \frac{3-(x-1)^2}{2\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\int \frac{3-(x-1)^2}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

$$\Big\downarrow \text{399}$$

$$\frac{2}{3}\left(6\int \frac{1}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)-\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)\right)+ \frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

$$\Big\downarrow \text{321}$$

$$\frac{2}{3}\left(-\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)-2\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+ \frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

$$\Big\downarrow \text{327}$$

$$\frac{2}{3}\left(\sqrt{3}E\left(\arcsin(1-x)\middle|-\frac{1}{3}\right)-2\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+ \frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}(x-1)$$

input
```
Int[Sqrt[(2 - x)*x*(4 - 2*x + x^2)],x]
```

output
```
(Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/3 + (2*(Sqrt[3]*EllipticE[A
rcSin[1 - x], -1/3] - 2*Sqrt[3]*EllipticF[ArcSin[1 - x], -1/3]))/3
```

3.770.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

3.770. $\int \sqrt{(2-x)x\left(4-2x+x^2\right)}\,dx$

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.770.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 931 vs. 2(54) = 108.

Time = 1.95 (sec) , antiderivative size = 932, normalized size of antiderivative = 15.03

method	result
risch	$-\frac{(x-1)x(x-2)(x^2-2x+4)}{3\sqrt{-x(x-2)(x^2-2x+4)}}+\frac{8\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\,(x-2)^2\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(x-2)}}\,F\left(\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}},\sqrt{\frac{(1+}{(-1}}\right.}{3\left(-1+i\sqrt{3}\right)\sqrt{-x(x-2)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{3}\right)}}$
default	$\frac{x\sqrt{-x^4+4x^3-8x^2+8x}}{3}-\frac{\sqrt{-x^4+4x^3-8x^2+8x}}{3}+\frac{8\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\,(x-2)^2\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(x-2)}}\,F\left(\sqrt{\right.}{3\left(-1+i\sqrt{3}\right)\sqrt{-x(x-2)\left(x-1+i\sqrt{3}\right)\left(x-1-\right.}}$
elliptic	$\frac{x\sqrt{-x^4+4x^3-8x^2+8x}}{3}-\frac{\sqrt{-x^4+4x^3-8x^2+8x}}{3}+\frac{8\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\,(x-2)^2\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(x-2)}}\,F\left(\sqrt{\right.}{3\left(-1+i\sqrt{3}\right)\sqrt{-x(x-2)\left(x-1+i\sqrt{3}\right)\left(x-1-\right.}}$

input

```
int(((2-x)*x*(x^2-2*x+4))^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-1/3*(x-1)*x*(x-2)*(x^2-2*x+4)/(-x*(x-2)*(x^2-2*x+4))^(1/2)+8/3*(-1-I*3^(1
/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))
/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(-
1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*EllipticF(((
-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(
-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-2/3*(x*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))
+2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x
-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x
-2))^(1/2)*(1/2*(6+2*I*3^(1/2))/(-1+I*3^(1/2))*EllipticF(((-1+I*3^(1/2))*x
/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(
1-I*3^(1/2)))^(1/2))+1/2*(-1+I*3^(1/2))*EllipticE(((-1+I*3^(1/2))*x/(1+I*3
^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(
1/2)))^(1/2))-4/(-1+I*3^(1/2))*EllipticPi(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/
(x-2))^(1/2),(-1-I*3^(1/2))/(1-I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-
1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1
/2)))^(1/2)+8/3*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2
)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+
I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^
(1/2)))^(1/2)*(2*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((
1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-2*Elli...
```

3.770. $\int\sqrt{(2-x)x\left(4-2x+x^2\right)}\,dx$

3.770.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int \sqrt{(2-x)x\left(4-2x+x^2\right)}\,dx = -\frac{2\,(-i\,x+i)E(\arcsin\left(\frac{1}{x-1}\right)\,|-3)+4\,(-i\,x+i)F(\arcsin\left(\frac{1}{x-1}\right)\,|-3)-\sqrt{-x^4+4\,x^3-8\,x^2+8\,x}(x^2-}{3\,(x-1)}$$

input
```
integrate(((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="fricas")
```

output
```
-1/3*(2*(-I*x + I)*elliptic_e(arcsin(1/(x - 1)), -3) + 4*(-I*x + I)*ellipt
ic_f(arcsin(1/(x - 1)), -3) - sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x)*(x^2 - 2*x
+ 3))/(x - 1)
```

3.770.6 Sympy [F]

$$\int \sqrt{(2-x)x\left(4-2x+x^2\right)}\,dx = \int \sqrt{x\left(2-x\right)\left(x^2-2x+4\right)}\,dx$$

input
```
integrate(((2-x)*x*(x**2-2*x+4))**(1/2),x)
```

output
```
Integral(sqrt(x*(2 - x)*(x**2 - 2*x + 4)), x)
```

3.770.7 Maxima [F]

$$\int \sqrt{(2-x)x\left(4-2x+x^2\right)}\,dx = \int \sqrt{-(x^2-2\,x+4)(x-2)x}\,dx$$

input
```
integrate(((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-(x^2 - 2*x + 4)*(x - 2)*x), x)
```

3.770.8 Giac [F]

$$\int \sqrt{(2-x)x\,(4-2x+x^2)}\,dx = \int \sqrt{-(x^2-2\,x+4)(x-2)x}\,dx$$

input
```
integrate(((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-(x^2 - 2*x + 4)*(x - 2)*x), x)
```

3.770.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{(2-x)x\,(4-2x+x^2)}\,dx = \int \sqrt{-x\,\left(x-2\right)\,\left(x^2-2\,x+4\right)}\,dx$$

input
```
int((-x*(x - 2)*(x^2 - 2*x + 4))^(1/2),x)
```

output
```
int((-x*(x - 2)*(x^2 - 2*x + 4))^(1/2), x)
```

3.771 $\int \frac{1}{\sqrt{(2-x)x(4-2x+x^2)}}\,dx$

3.771.1 Optimal result

Integrand size = 19, antiderivative size = 17

$$\int \frac{1}{\sqrt{(2-x)x(4-2x+x^2)}}\,dx = -\frac{\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{\sqrt{3}}$$

output
```
1/3*EllipticF(-1+x,1/3*I*3^(1/2))*3^(1/2)
```

3.771.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.86 (sec) , antiderivative size = 100, normalized size of antiderivative = 5.88

$$\int \frac{1}{\sqrt{(2-x)x(4-2x+x^2)}}\,dx = -\frac{\sqrt[3]{-1}(-2+x)^2\sqrt{\frac{x\left(-1+i\sqrt{3}+x\right)}{(-2+x)^2}}\sqrt{\frac{-2+x-\sqrt[3]{-1}x}{-2+x}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{-\frac{(-1)^{2/3}x}{-2+x}}\right),(-1)^{2/3}\right)}{\sqrt{-x\left(-8+8x-4x^2+x^3\right)}}$$

input
```
Integrate[1/Sqrt[(2 - x)*x*(4 - 2*x + x^2)],x]
```

output
```
-(((-1)^(1/3)*(-2 + x)^2*Sqrt[(x*(-1 + I*Sqrt[3] + x))/(-2 + x)^2]*Sqrt[(-
2 + x - (-1)^(1/3)*x)/(-2 + x)]*EllipticF[ArcSin[Sqrt[-(((-1)^(2/3)*x)/(-2
 + x))]], (-1)^(2/3)])/Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))])
```

3.771.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2458, 1408, 27, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{(2-x)x(x^2-2x+4)}}dx$$
$$\Big\downarrow \text{ 2458}$$
$$\int \frac{1}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)$$
$$\Big\downarrow \text{ 1408}$$
$$2\int \frac{1}{2\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)$$
$$\Big\downarrow \text{ 27}$$
$$\int \frac{1}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)$$
$$\Big\downarrow \text{ 321}$$
$$-\frac{\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[(2 - x)*x*(4 - 2*x + x^2)],x]
```

output
```
-(EllipticF[ArcSin[1 - x], -1/3]/Sqrt[3])
```

3.771.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 1408
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
 - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[
c, 0]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.771.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(15) = 30$.

Time = 1.42 (sec) , antiderivative size = 200, normalized size of antiderivative = 11.76

method	result	size
default	$\frac{2\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\,(x-2)^{2}\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\,\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(x-2)}}\,F\left(\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}},\sqrt{\frac{\left(1+i\sqrt{3}\right)\left(-1-i\sqrt{3}\right)}{\left(-1+i\sqrt{3}\right)\left(1-i\sqrt{3}\right)}}\right)}{\left(-1+i\sqrt{3}\right)\sqrt{-x(x-2)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{3}\right)}}$	200
elliptic	$\frac{2\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\,(x-2)^{2}\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\,\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(x-2)}}\,F\left(\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}},\sqrt{\frac{\left(1+i\sqrt{3}\right)\left(-1-i\sqrt{3}\right)}{\left(-1+i\sqrt{3}\right)\left(1-i\sqrt{3}\right)}}\right)}{\left(-1+i\sqrt{3}\right)\sqrt{-x(x-2)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{3}\right)}}$	200

input
```
int(1/((2-x)*x*(x^2-2*x+4))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-
1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-
2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*
EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-
I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))
```

3.771.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{(2-x)x(4-2x+x^2)}}\,dx = -\frac{1}{2}\sqrt{2}\text{weierstrassPInverse}\left(-\frac{2}{3},\frac{7}{54},-\frac{x-3}{3\,x}\right)$$

input
```
integrate(1/((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2)*weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x)
```

3.771.6 Sympy [F]

$$\int \frac{1}{\sqrt{(2-x)x(4-2x+x^2)}}\,dx = \int \frac{1}{\sqrt{x\,(2-x)\,(x^2-2x+4)}}\,dx$$

input
```
integrate(1/((2-x)*x*(x**2-2*x+4))**(1/2),x)
```

output
```
Integral(1/sqrt(x*(2 - x)*(x**2 - 2*x + 4)), x)
```

3.771.7 Maxima [F]

$$\int \frac{1}{\sqrt{(2-x)x\,(4-2x+x^2)}}\,dx = \int \frac{1}{\sqrt{-(x^2-2\,x+4)(x-2)x}}\,dx$$

input
```
integrate(1/((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-(x^2 - 2*x + 4)*(x - 2)*x), x)
```

3.771.8 Giac [F]

$$\int \frac{1}{\sqrt{(2-x)x\,(4-2x+x^2)}}\,dx = \int \frac{1}{\sqrt{-(x^2-2\,x+4)(x-2)x}}\,dx$$

input
```
integrate(1/((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-(x^2 - 2*x + 4)*(x - 2)*x), x)
```

3.771.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{(2-x)x\,(4-2x+x^2)}}\,dx = \int \frac{1}{\sqrt{-x\,(x-2)\,(x^2-2\,x+4)}}\,dx$$

input
```
int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(1/2),x)
```

output
```
int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(1/2), x)
```

3.772 $\int \frac{1}{((2-x)x(4-2x+x^2))^{3/2}} dx$

3.772.1 Optimal result

Integrand size = 19, antiderivative size = 73

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{3/2}} dx = \frac{(5+(-1+x)^2)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} + \frac{E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)}{8\sqrt{3}} - \frac{\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{4\sqrt{3}}$$

output
```
-1/24*EllipticE(-1+x,1/3*I*3^(1/2))*3^(1/2)+1/12*EllipticF(-1+x,1/3*I*3^(1
/2))*3^(1/2)+1/24*(5+(-1+x)^2)*(-1+x)/(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

3.772.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 19.72 (sec) , antiderivative size = 298, normalized size of antiderivative = 4.08

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{3/2}} dx = \frac{(-2+x)^2x(4-2x+x^2)\left(2(-1+x)x-3(4-2x+x^2)-\frac{3x(4-2x+x^2)}{-2+x}\right.}{}$$

input
```
Integrate[((2 - x)*x*(4 - 2*x + x^2))^(-3/2),x]
```

output
```
((-2 + x)^2*x*(4 - 2*x + x^2)*(2*(-1 + x)*x - 3*(4 - 2*x + x^2) - (3*x*(4
- 2*x + x^2))/(-2 + x) - 4*(2 - x)*Sqrt[(4 - 2*x + x^2)/(-2 + x)^2]*(x*Sqr
t[(4 - 2*x + x^2)/(-2 + x)^2] - Sqrt[2]*(I + Sqrt[3])*Sqrt[(I*x)/((I + Sqr
t[3])*(-2 + x))]*EllipticE[ArcSin[Sqrt[-I + Sqrt[3] - (4*I)/(-2 + x)]/(Sqr
t[2]*3^(1/4))], (2*Sqrt[3])/(I + Sqrt[3])] + (4*I)*Sqrt[2]*Sqrt[(I*x)/((I
+ Sqrt[3])*(-2 + x))]*EllipticF[ArcSin[Sqrt[-I + Sqrt[3] - (4*I)/(-2 + x)]
/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(I + Sqrt[3])])))/(96*(-(x*(-8 + 8*x - 4*
x^2 + x^3)))^(3/2))
```

3.772.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2458, 1405, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{((2-x)x(x^2-2x+4))^{3/2}}\,dx$$

$$\Big\downarrow \ 2458$$

$$\int \frac{1}{(-(x-1)^4-2(x-1)^2+3)^{3/2}}d(x-1)$$

$$\Big\downarrow \ 1405$$

$$\frac{((x-1)^2+5)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}-\frac{1}{48}\int -\frac{2(3-(x-1)^2)}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{24}\int \frac{3-(x-1)^2}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)+\frac{((x-1)^2+5)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \ 1494$$

$$\frac{1}{12}\int \frac{3-(x-1)^2}{2\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{((x-1)^2+5)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{24}\int \frac{3-(x-1)^2}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{((x-1)^2+5)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{399}$$

$$\frac{1}{24}\left(6\int \frac{1}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)-\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)\right)+\frac{((x-1)^2+5)\,(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{321}$$

$$\frac{1}{24}\left(-\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)-2\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{((x-1)^2+5)\,(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{24}\left(\sqrt{3}E\left(\arcsin(1-x)\middle|-\frac{1}{3}\right)-2\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{((x-1)^2+5)\,(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}$$

input
```
Int[((2 - x)*x*(4 - 2*x + x^2))^(-3/2),x]
```

output
```
((5 + (-1 + x)^2)*(-1 + x))/(24*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sq
rt[3]*EllipticE[ArcSin[1 - x], -1/3] - 2*Sqrt[3]*EllipticF[ArcSin[1 - x],
-1/3])/24
```

3.772.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.772.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 962 vs. $2(61) = 122$.

Time = 1.45 (sec) , antiderivative size = 963, normalized size of antiderivative = 13.19

method	result	size
default	Expression too large to display	963
elliptic	Expression too large to display	963

3.772. $\int \frac{1}{((2-x)x(4-2x+x^2))^{3/2}}\,dx$

input
```
int(1/((2-x)*x*(x^2-2*x+4))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32*(-x^3+4*x^2-8*x+8)/(x*(-x^3+4*x^2-8*x+8))^(1/2)+2*x*(1/24+1/192*x^2)
/(-x*(x^3-4*x^2+8*x-8))^(1/2)+1/6*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^
(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((
x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*
3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/
(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(
1/2))+1/6*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2
)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1
/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))
)^(1/2)*(2*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^
(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(((
-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),(1+I*3^(1/2))/(-1+I*3^(1/2)),((
1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2)))-1/24*(x*
(x-1+I*3^(1/2))*(x-1-I*3^(1/2))+2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^
(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((
x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)*(1/2*(6+2*I*3^(1/2))/(-1+I*3^(1/
2))*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*
(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+1/2*(-1+I*3^(1/2))*Ell
ipticE(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3
^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-4/(-1+I*3^(1/2))*EllipticP...
```

3.772.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.63

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{3/2}}\,dx = -\frac{5\sqrt{2}(x^4-4x^3+8x^2-8x)\text{weierstrassPInverse}\left(-\frac{2}{3},\frac{7}{54},-\frac{x-3}{3x}\right)-6\sqrt{2}(x^4-4x^3+8x^2-8x)\text{weierstrass}}{72(x^4-4x^3+8x^2-}$$

input
```
integrate(1/((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="fricas")
```

output
```
-1/72*(5*sqrt(2)*(x^4 - 4*x^3 + 8*x^2 - 8*x)*weierstrassPInverse(-2/3, 7/5
4, -1/3*(x - 3)/x) - 6*sqrt(2)*(x^4 - 4*x^3 + 8*x^2 - 8*x)*weierstrassZeta
(-2/3, 7/54, weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x)) + 3*sqrt(-x^
4 + 4*x^3 - 8*x^2 + 8*x)*(x^2 + 2))/(x^4 - 4*x^3 + 8*x^2 - 8*x)
```

3.772.6 Sympy [F]

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{3/2}}\,dx = \int \frac{1}{(x(2-x)(x^2-2x+4))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((2-x)*x*(x**2-2*x+4))**(3/2),x)
```

output
```
Integral((x*(2 - x)*(x**2 - 2*x + 4))**(-3/2), x)
```

3.772.7 Maxima [F]

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{3/2}}\,dx = \int \frac{1}{(-(x^2-2x+4)(x-2)x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-3/2), x)
```

3.772.8 Giac [F]

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{3/2}}\,dx = \int \frac{1}{(-(x^2-2x+4)(x-2)x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="giac")
```

output
```
integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-3/2), x)
```

3.772.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{3/2}}\,dx = \int \frac{1}{\left(-x\,\left(x-2\right)\,\left(x^2-2\,x+4\right)\right)^{3/2}}\,dx$$

input
```
int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(3/2),x)
```

output
```
int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(3/2), x)
```

3.773 $\int \frac{1}{((2-x)x(4-2x+x^2))^{5/2}}\,dx$

3.773.1 Optimal result

Integrand size = 19, antiderivative size = 109

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{5/2}}\,dx = \frac{(5+(-1+x)^2)(-1+x)}{72(3-2(-1+x)^2-(-1+x)^4)^{3/2}} + \frac{(26+7(-1+x)^2)(-1+x)}{432\sqrt{3-2(-1+x)^2-(-1+x)^4}} + \frac{7E\left(\arcsin(1-x)\middle|-\frac{1}{3}\right)}{144\sqrt{3}} - \frac{11\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)}{144\sqrt{3}}$$

output
```
1/72*(5+(-1+x)^2)*(-1+x)/(3-2*(-1+x)^2-(-1+x)^4)^(3/2)-7/432*EllipticE(-1+
x,1/3*I*3^(1/2))*3^(1/2)+11/432*EllipticF(-1+x,1/3*I*3^(1/2))*3^(1/2)+1/43
2*(26+7*(-1+x)^2)*(-1+x)/(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

3.773.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 22.03 (sec) , antiderivative size = 327, normalized size of antiderivative = 3.00

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{5/2}}\,dx = \frac{(-2+x)^3x^2(4-2x+x^2)^2\left(-\frac{7x(4-2x+x^2)}{-2+x} + \frac{36+216x-622x^2+670x^3-445x^4+1\cdots}{(-2+x)^2x(4-2x+\cdots}\right.}{\cdots}$$

input
```
Integrate[((2 - x)*x*(4 - 2*x + x^2))^(-5/2),x]
```

output
```
((-2 + x)^3*x^2*(4 - 2*x + x^2)^2*((-7*x*(4 - 2*x + x^2))/(-2 + x) + (36 +
 216*x - 622*x^2 + 670*x^3 - 445*x^4 + 187*x^5 - 49*x^6 + 7*x^7)/((-2 + x)
^2*x*(4 - 2*x + x^2)) + ((7*I)*Sqrt[2]*x*Sqrt[(4 - 2*x + x^2)/(-2 + x)^2]*
EllipticE[ArcSin[Sqrt[-I + Sqrt[3] - (4*I)/(-2 + x)]/(Sqrt[2]*3^(1/4))], (
2*Sqrt[3])/(I + Sqrt[3])])/Sqrt[(I*x)/((I + Sqrt[3])*(-2 + x))] - (19*I)*S
qrt[2]*(-2 + x)*Sqrt[(I*x)/((I + Sqrt[3])*(-2 + x))]*Sqrt[(4 - 2*x + x^2)/
(-2 + x)^2]*EllipticF[ArcSin[Sqrt[-I + Sqrt[3] - (4*I)/(-2 + x)]/(Sqrt[2]*
3^(1/4))], (2*Sqrt[3])/(I + Sqrt[3])]))/(432*(-(x*(-8 + 8*x - 4*x^2 + x^3)
))^(5/2))
```

3.773.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {2458, 1405, 27, 1492, 27, 1494, 27, 399, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left((2-x)x\left(x^2-2x+4\right)\right)^{5/2}}dx$$

$$\Big\downarrow \; 2458$$

$$\int \frac{1}{\left(-(x-1)^4-2(x-1)^2+3\right)^{5/2}}d(x-1)$$

$$\Big\downarrow \; 1405$$

$$\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}-\frac{1}{144}\int -\frac{2\left(3(x-1)^2+19\right)}{\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}d(x-1)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{72}\int \frac{3(x-1)^2+19}{\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}d(x-1)+\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \; 1492$$

$$\frac{1}{72}\left(\frac{\left(7(x-1)^2+26\right)(x-1)}{6\sqrt{-(x-1)^4-2(x-1)^2+3}}-\frac{1}{48}\int -\frac{8\left(12-7(x-1)^2\right)}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)\right)+\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{72}\left(\frac{1}{6}\int \frac{12-7(x-1)^2}{\sqrt{-(x-1)^4-2(x-1)^2+3}}d(x-1)+\frac{\left(7(x-1)^2+26\right)(x-1)}{6\sqrt{-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \; 1494$$

$$\frac{1}{72}\left(\frac{1}{3}\int \frac{12-7(x-1)^2}{2\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{\left(7(x-1)^2+26\right)(x-1)}{6\sqrt{-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{72}\left(\frac{1}{6}\int \frac{12-7(x-1)^2}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)+\frac{\left(7(x-1)^2+26\right)(x-1)}{6\sqrt{-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \; 399$$

$$\frac{1}{72}\left(\frac{1}{6}\left(33\int \frac{1}{\sqrt{1-(x-1)^2}\sqrt{(x-1)^2+3}}d(x-1)-7\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)\right)+\frac{\left(7(x-1)^2+26\right)(x-1)}{6\sqrt{-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \; 321$$

$$\frac{1}{72}\left(\frac{1}{6}\left(-7\int \frac{\sqrt{(x-1)^2+3}}{\sqrt{1-(x-1)^2}}d(x-1)-11\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{\left(7(x-1)^2+26\right)(x-1)}{6\sqrt{-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \; 327$$

3.773. $\int \frac{1}{\left((2-x)x(4-2x+x^2)\right)^{5/2}}\,dx$

$$\frac{1}{72}\left(\frac{1}{6}\left(7\sqrt{3}E\left(\arcsin(1-x)\left|-\frac{1}{3}\right.\right)-11\sqrt{3}\,\text{EllipticF}\left(\arcsin(1-x),-\frac{1}{3}\right)\right)+\frac{\left(7(x-1)^2+26\right)(x-1)}{6\sqrt{-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\left((x-1)^2+5\right)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

input
```
Int[((2 - x)*x*(4 - 2*x + x^2))^(-5/2),x]
```

output
```
((5 + (-1 + x)^2)*(-1 + x))/(72*(3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)) + (
((26 + 7*(-1 + x)^2)*(-1 + x))/(6*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]) + (
7*Sqrt[3]*EllipticE[ArcSin[1 - x], -1/3] - 11*Sqrt[3]*EllipticF[ArcSin[1 -
 x], -1/3])/6)/72
```

3.773.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 399
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)
^2]), x_Symbol] :> Simp[f/b   Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2], x], x] +
 Simp[(b*e - a*f)/b   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; Fr
eeQ[{a, b, c, d, e, f}, x] &&  !((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] &&
(PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || SimplerSqrtQ[-b/a, -d/c])))))
```

3.773. $\int \frac{1}{((2-x)x(4-2x+x^2))^{5/2}}\,dx$

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1494
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*Sqrt[-c]   Int[(d + e*x^2)/(Sqr
t[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e
}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.773.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1038 vs. $2(93) = 186$.

Time = 1.49 (sec) , antiderivative size = 1039, normalized size of antiderivative = 9.53

method	result	size
default	Expression too large to display	1039
elliptic	Expression too large to display	1039

input
```
int(1/((2-x)*x*(x^2-2*x+4))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/768*(-x^4+4*x^3-8*x^2+8*x)^(1/2)/x^2-1/96*(-x^3+4*x^2-8*x+8)/(x*(-x^3+4
*x^2-8*x+8))^(1/2)+(1/36+1/288*x^2-1/96*x)*(-x^4+4*x^3-8*x^2+8*x)^(1/2)/(x
^3-4*x^2+8*x-8)^2+2*x*(53/3456+5/1728*x^2-19/4608*x)/(-x*(x^3-4*x^2+8*x-8)
)^(1/2)+5/216*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*
(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*
3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1
/2)))^(1/2)*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3
^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+7/108*(-1-I*3^
(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2
))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/
(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*(2*Ellipti
cF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/
2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(((-1+I*3^(1/2))*x/(1
+I*3^(1/2))/(x-2))^(1/2),(1+I*3^(1/2))/(-1+I*3^(1/2)),((1+I*3^(1/2))*(-1-I
*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2)))-7/432*(x*(x-1+I*3^(1/2))*(
x-1-I*3^(1/2))+2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/
2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1
+I*3^(1/2))/(x-2))^(1/2)*(1/2*(6+2*I*3^(1/2))/(-1+I*3^(1/2))*EllipticF(((-
1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-
1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+1/2*(-1+I*3^(1/2))*EllipticE(((-1+I*...
```

3.773.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.79

$$\int \frac{1}{((2-x)x(4-2x+x^2))^{5/2}}\,dx =$$
$$-\frac{43\sqrt{2}(x^8-8x^7+32x^6-80x^5+128x^4-128x^3+64x^2)\text{weierstrassPInverse}\left(-\frac{2}{3},\frac{7}{54},-\frac{x-3}{3x}\right)-84\sqrt{2}(x}{}$$

input
```
integrate(1/((2-x)*x*(x^2-2*x+4))^(5/2),x, algorithm="fricas")
```

output
```
-1/2592*(43*sqrt(2)*(x^8 - 8*x^7 + 32*x^6 - 80*x^5 + 128*x^4 - 128*x^3 + 6
4*x^2)*weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x) - 84*sqrt(2)*(x^8 -
 8*x^7 + 32*x^6 - 80*x^5 + 128*x^4 - 128*x^3 + 64*x^2)*weierstrassZeta(-2/
3, 7/54, weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x)) + 6*(7*x^6 - 37*
x^5 + 115*x^4 - 226*x^3 + 274*x^2 - 232*x + 36)*sqrt(-x^4 + 4*x^3 - 8*x^2
+ 8*x))/(x^8 - 8*x^7 + 32*x^6 - 80*x^5 + 128*x^4 - 128*x^3 + 64*x^2)
```

3.773.6 Sympy [F]

$$\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{5/2}}\,dx = \int \frac{1}{\left(x\left(2-x\right)\left(x^2-2x+4\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/((2-x)*x*(x**2-2*x+4))**(5/2),x)
```

output
```
Integral((x*(2 - x)*(x**2 - 2*x + 4))**(-5/2), x)
```

3.773.7 Maxima [F]

$$\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{5/2}}\,dx = \int \frac{1}{\left(-(x^2-2\,x+4)(x-2)x\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/((2-x)*x*(x^2-2*x+4))^(5/2),x, algorithm="maxima")
```

output
```
integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-5/2), x)
```

3.773.8 Giac [F]

$$\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{5/2}}\,dx = \int \frac{1}{\left(-(x^2-2\,x+4)(x-2)x\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/((2-x)*x*(x^2-2*x+4))^(5/2),x, algorithm="giac")
```

output
```
integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-5/2), x)
```

3.773.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{((2-x)x\,(4-2x+x^2))^{5/2}}\,dx = \int \frac{1}{(-x\,(x-2)\,(x^2-2\,x+4))^{5/2}}\,dx$$

input
```
int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(5/2),x)
```

output
```
int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(5/2), x)
```

3.774 $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}\,dx$

3.774.1 Optimal result

Integrand size = 31, antiderivative size = 730

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}\,dx=\frac{1}{7}\left(\frac{c}{d}+x\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}$$
$$+\frac{2c\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\left(7c^3+20ad^2-3cd^2\left(\frac{c}{d}+x\right)^2\right)}{35d^2}$$
$$-\frac{16c^3(c^3+8ad^2)\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{35d^2\sqrt{c^3+4ad^2}\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)}$$
$$+\frac{16c^{13/4}(c^3+4ad^2)^{3/4}(c^3+8ad^2)\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(c^3+4ad^2)\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)E\left(2\arctan\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right.\right.}{35d^5\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$
$$+\frac{8c^{7/4}(c^3+4ad^2)^{3/4}\left(\sqrt{c^3+4ad^2}(c^3+5ad^2)-c^{3/2}(c^3+8ad^2)\right)\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(c^3+4ad^2)\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)}{35d^5\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

output
```
1/7*(c/d+x)*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2)+2/35*c*(c/d+x)*(7*c^
3+20*a*d^2-3*c*d^2*(c/d+x)^2)*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)/d^
2-16/35*c^3*(8*a*d^2+c^3)*(c/d+x)*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2
)/d^2/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))/(4*a*d^2+c^3)^(1/2)+16/3
5*c^(13/4)*(4*a*d^2+c^3)^(3/4)*(8*a*d^2+c^3)*(cos(2*arctan((d*x+c)/c^(1/4)
/(4*a*d^2+c^3)^(1/4)))^2)^(1/2)/cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)
^(1/4)))*EllipticE(sin(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4))),1/2*
(2+2*c^(3/2)/(4*a*d^2+c^3)^(1/2))^(1/2))*(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c
^3)^(1/2))*(d^2*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/(c^(1/2)
+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))^2)^(1/2)/d^5/(d^2*x^4+4*c*d*x^3+4*c^2*
x^2+4*a*c)^(1/2)+8/35*c^(7/4)*(4*a*d^2+c^3)^(3/4)*(cos(2*arctan((d*x+c)/c^
(1/4)/(4*a*d^2+c^3)^(1/4)))^2)^(1/2)/cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2
+c^3)^(1/4)))*EllipticF(sin(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))
,1/2*(2+2*c^(3/2)/(4*a*d^2+c^3)^(1/2))^(1/2))*(c^(1/2)+d^2*(c/d+x)^2/(4*a*
d^2+c^3)^(1/2))*(-c^(3/2)*(8*a*d^2+c^3)+(5*a*d^2+c^3)*(4*a*d^2+c^3)^(1/2))
*(d^2*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/(c^(1/2)+d^2*(c/d+
x)^2/(4*a*d^2+c^3)^(1/2))^2)^(1/2)/d^5/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)
^(1/2)
```

3.774.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 16.14 (sec) , antiderivative size = 10468, normalized size of antiderivative = 14.34

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}\,dx=\text{Result too large to show}$$

input
```
Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(3/2),x]
```

output
```
Result too large to show
```

3.774.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 843, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {2458, 1404, 27, 1490, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}dx$$

$$\Big\downarrow \text{ 2458}$$

$$\int \left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)^{3/2}d\left(\frac{c}{d}+x\right)$$

$$\Big\downarrow \text{ 1404}$$

$$\frac{3}{7}\int 2c\left(\frac{c^3}{d^2}-\left(\frac{c}{d}+x\right)^2c+4a\right)\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}d\left(\frac{c}{d}+x\right)+\frac{1}{7}\left(\frac{c}{d}+x\right)\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)^{3/2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{6}{7}c\int \left(\frac{c^3}{d^2}-\left(\frac{c}{d}+x\right)^2c+4a\right)\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}d\left(\frac{c}{d}+x\right)+\frac{1}{7}\left(\frac{c}{d}+x\right)\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)^{3/2}$$

$$\Big\downarrow \text{ 1490}$$

$$\frac{6}{7}c\left(\frac{\int \frac{8c\left(\left(c^3+4ad^2\right)\left(c^3+5ad^2\right)-cd^2\left(c^3+8ad^2\right)\left(\frac{c}{d}+x\right)^2\right)}{d^2\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)}{15d^2}+\frac{\left(\frac{c}{d}+x\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}\,(\ldots}{15d^2}\right.$$

$$\frac{1}{7}\left(\frac{c}{d}+x\right)\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)^{3/2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{6}{7}c\left(\frac{8c\int \frac{\left(c^3+4ad^2\right)\left(c^3+5ad^2\right)-cd^2\left(c^3+8ad^2\right)\left(\frac{c}{d}+x\right)^2}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)}{15d^4}+\frac{\left(\frac{c}{d}+x\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}\left(20\right.}{15d^2}\right.$$

$$\frac{1}{7}\left(\frac{c}{d}+x\right)\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)^{3/2}$$

$\downarrow$ 1511

$$\frac{6}{7}c\left(\frac{8c\left(c^{3/2}\sqrt{4ad^2+c^3}\left(8ad^2+c^3\right)\int \frac{\sqrt{c}-\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}}{\sqrt{c}\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)-\sqrt{4ad^2+c^3}\left(8ac^{3/2}d^2-\sqrt{4ad\right.\right.}{15d^4}\right.$$

$$\frac{1}{7}\left(\frac{c}{d}+x\right)\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)^{3/2}$$

$\downarrow$ 27

$$\frac{6}{7}c\left(\frac{8c\left(c\sqrt{4ad^2+c^3}\left(8ad^2+c^3\right)\int \frac{\sqrt{c}-\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)-\sqrt{4ad^2+c^3}\left(8ac^{3/2}d^2-\sqrt{4ad^2+c\right.\right.}{15d^4}\right.$$

$$\frac{1}{7}\left(\frac{c}{d}+x\right)\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)^{3/2}$$

$\downarrow$ 1416

3.774. $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}\,dx$

$$\frac{6}{7}c\left(\frac{8c\left(c\sqrt{4ad^2+c^3}\left(8ad^2+c^3\right)\int\frac{\sqrt{c}-\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)-\frac{\left(4ad^2+c^3\right)^{3/4}\left(8ac^{3/2}d^2-\sqrt{4ad^2+c^3}\left(5ad^2+\right.\right.}{}\right.}{15}\right.$$

$$\frac{1}{7}\left(\frac{c}{d}+x\right)\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)^{3/2}$$

$$\Big\downarrow \; 1509$$

$$\frac{1}{7}\left(\frac{c}{d}+x\right)\left(d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)\right)^{3/2}+$$

$$\frac{6}{7}c\left(\frac{\left(\frac{c}{d}+x\right)\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}\left(7c^3-3d^2\left(\frac{c}{d}+x\right)^2c+20ad^2\right)}{15d^2}+\frac{8c\left(c\sqrt{c^3+4ad^2}\left(c^3+\right.\right.}{}\right.$$

input
```
Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(3/2),x]
```

3.774. $\int\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}\,dx$

output
```
((c/d + x)*(c*(4*a + c^3/d^2) - 2*c^2*(c/d + x)^2 + d^2*(c/d + x)^4)^(3/2)
)/7 + (6*c*(((c/d + x)*(7*c^3 + 20*a*d^2 - 3*c*d^2*(c/d + x)^2)*Sqrt[c*(4*
a + c^3/d^2) - 2*c^2*(c/d + x)^2 + d^2*(c/d + x)^4])/(15*d^2) + (8*c*(c*Sq
rt[c^3 + 4*a*d^2]*(c^3 + 8*a*d^2)*(-(((c/d + x)*Sqrt[c*(4*a + c^3/d^2) - 2
*c^2*(c/d + x)^2 + d^2*(c/d + x)^4])/((4*a + c^3/d^2)*(Sqrt[c] + (d^2*(c/d
 + x)^2)/Sqrt[c^3 + 4*a*d^2]))) + (c^(1/4)*(c^3 + 4*a*d^2)^(1/4)*(Sqrt[c]
+ (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*Sqrt[(d^2*(c*(4*a + c^3/d^2) - 2*
c^2*(c/d + x)^2 + d^2*(c/d + x)^4))/((c^3 + 4*a*d^2)*(Sqrt[c] + (d^2*(c/d
+ x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*EllipticE[2*ArcTan[(d*(c/d + x))/(c^(1/4)
*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(d*Sqrt[c*
(4*a + c^3/d^2) - 2*c^2*(c/d + x)^2 + d^2*(c/d + x)^4])) - ((c^3 + 4*a*d^2
)^(3/4)*(c^(9/2) + 8*a*c^(3/2)*d^2 - Sqrt[c^3 + 4*a*d^2]*(c^3 + 5*a*d^2))*
(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*Sqrt[(d^2*(c*(4*a + c^3/
d^2) - 2*c^2*(c/d + x)^2 + d^2*(c/d + x)^4))/((c^3 + 4*a*d^2)*(Sqrt[c] + (
d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*EllipticF[2*ArcTan[(d*(c/d + x))
/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(
2*c^(1/4)*d*Sqrt[c*(4*a + c^3/d^2) - 2*c^2*(c/d + x)^2 + d^2*(c/d + x)^4])
))/(15*d^4)))/7
```

3.774.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

3.774. $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2} dx$

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.774.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 5228 vs. $2(772) = 1544$.

Time = 6.08 (sec) , antiderivative size = 5229, normalized size of antiderivative = 7.16

method	result	size
default	Expression too large to display	5229
elliptic	Expression too large to display	5229
risch	Expression too large to display	6018

input
```
int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.774.5 Fricas [F]

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2} dx = \int \left(d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac\right)^{\frac{3}{2}} dx$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="fricas")
```

output
```
integral((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(3/2), x)
```

3.774.6 Sympy [F]

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2} dx = \int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{\frac{3}{2}} dx$$

input
```
integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(3/2),x)
```

output
```
Integral((4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4)**(3/2), x)
```

3.774.7 Maxima [F]

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2} dx = \int \left(d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac\right)^{\frac{3}{2}} dx$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(3/2), x)
```

3.774.8 Giac [F]

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}\,dx=\int \left(d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="giac")
```

output
```
integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(3/2), x)
```

3.774.9 Mupad [F(-1)]

Timed out.

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}\,dx=\int \left(4\,c^2\,x^2+4\,c\,d\,x^3+4\,a\,c+d^2\,x^4\right)^{3/2}\,dx$$

input
```
int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(3/2),x)
```

output
```
int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(3/2), x)
```

3.775 $\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$

3.775.1 Optimal result

Integrand size = 31, antiderivative size = 622

$$\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$$
$$=\frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}-\frac{2c^2\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{3\sqrt{c^3+4ad^2}\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)}$$
$$+\frac{2c^{9/4}(c^3+4ad^2)^{3/4}\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(c^3+4ad^2)\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)E\left(2\arctan\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(1+\right.\right.}{3d^3\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$
$$+\frac{c^{3/4}\sqrt[4]{c^3+4ad^2}\left(c^3+4ad^2-c^{3/2}\sqrt{c^3+4ad^2}\right)\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(c^3+4ad^2)\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)\text{EllipticF}\left(2\right.}{3d^3\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

output
```
1/3*(c/d+x)*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)-2/3*c^2*(c/d+x)*(d^2
*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)
^(1/2))/(4*a*d^2+c^3)^(1/2)+2/3*c^(9/4)*(4*a*d^2+c^3)^(3/4)*(cos(2*arctan(
(d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))^2)^(1/2)/cos(2*arctan((d*x+c)/c^(1/4
)/(4*a*d^2+c^3)^(1/4)))*EllipticE(sin(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^
3)^(1/4))),1/2*(2+2*c^(3/2)/(4*a*d^2+c^3)^(1/2))^(1/2))*(c^(1/2)+d^2*(c/d+
x)^2/(4*a*d^2+c^3)^(1/2))*(d^2*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^
2+c^3)/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))^2)^(1/2)/d^3/(d^2*x^4+4
*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)+1/3*c^(3/4)*(4*a*d^2+c^3)^(1/4)*(cos(2*arc
tan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))^2)^(1/2)/cos(2*arctan((d*x+c)/c^
(1/4)/(4*a*d^2+c^3)^(1/4)))*EllipticF(sin(2*arctan((d*x+c)/c^(1/4)/(4*a*d^
2+c^3)^(1/4))),1/2*(2+2*c^(3/2)/(4*a*d^2+c^3)^(1/2))^(1/2))*(c^(1/2)+d^2*(
c/d+x)^2/(4*a*d^2+c^3)^(1/2))*(c^3+4*a*d^2-c^(3/2)*(4*a*d^2+c^3)^(1/2))*(d
^2*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/(c^(1/2)+d^2*(c/d+x)^
2/(4*a*d^2+c^3)^(1/2))^2)^(1/2)/d^3/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1
/2)
```

3.775.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 16.07 (sec) , antiderivative size = 5218, normalized size of antiderivative = 8.39

$$\int \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}\, dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4],x]
```

output
```
Result too large to show
```

3.775.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 729, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {2458, 1404, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$$

$$\Big\downarrow \; 2458$$

$$\int \sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}d\left(\frac{c}{d}+x\right)$$

$$\Big\downarrow \; 1404$$

$$\frac{1}{3}\int \frac{2c\left(\frac{c^3}{d^2}-\left(\frac{c}{d}+x\right)^2c+4a\right)}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)+\frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}$$

$$\Big\downarrow \; 27$$

$$\frac{2}{3}c\int \frac{\frac{c^3}{d^2}-\left(\frac{c}{d}+x\right)^2c+4a}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)+\frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}$$

$$\Big\downarrow \; 1511$$

$$\frac{2}{3}c\left(\frac{c^{3/2}\sqrt{4ad^2+c^3}\int \frac{\sqrt{c}-\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}}{\sqrt{c}\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)}{d^2}+\frac{\left(-c^{3/2}\sqrt{4ad^2+c^3}+4ad^2+c^3\right)\int \frac{}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4}}}{d^2}\right.$$

$$\frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}$$

$$\Big\downarrow \; 27$$

$$\frac{2}{3}c\left(\frac{c\sqrt{4ad^2+c^3}\int \frac{\sqrt{c}-\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)}{d^2}+\frac{\left(-c^{3/2}\sqrt{4ad^2+c^3}+4ad^2+c^3\right)\int \frac{1}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2(}}}{d^2}\right.$$

$$\frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}$$

3.775. $\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$

$$\downarrow \text{1416}$$

$$\frac{2}{3}c\left(\frac{c\sqrt{4ad^2+c^3}\int \frac{\sqrt{c}-\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)}{d^2}+\frac{\sqrt[4]{4ad^2+c^3}\left(-c^{3/2}\sqrt{4ad^2+c^3}+4ad^2+c^3\right)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4a}}\right.}{}\right.$$

$$\frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}$$

$$\downarrow \text{1509}$$

$$\frac{2}{3}c\left(\frac{c\sqrt{4ad^2+c^3}\left(\frac{\sqrt[4]{c}\sqrt[4]{4ad^2+c^3}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)\sqrt{\frac{d^2\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)}{\left(4ad^2+c^3\right)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}E\left(2\arctan\left(\frac{d\left(\frac{c}{d}+x\right)}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\right.}{d\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}}\right.}{d^2}\right.$$

$$\frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}$$

input
```
Int[Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4],x]
```

3.775. $\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$

output
```
((c/d + x)*Sqrt[c*(4*a + c^3/d^2) - 2*c^2*(c/d + x)^2 + d^2*(c/d + x)^4])/
3 + (2*c*((c*Sqrt[c^3 + 4*a*d^2]*(-(((c/d + x)*Sqrt[c*(4*a + c^3/d^2) - 2*
c^2*(c/d + x)^2 + d^2*(c/d + x)^4])/((4*a + c^3/d^2)*(Sqrt[c] + (d^2*(c/d
+ x)^2)/Sqrt[c^3 + 4*a*d^2]))) + (c^(1/4)*(c^3 + 4*a*d^2)^(1/4)*(Sqrt[c] +
 (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*Sqrt[(d^2*(c*(4*a + c^3/d^2) - 2*c
^2*(c/d + x)^2 + d^2*(c/d + x)^4))/((c^3 + 4*a*d^2)*(Sqrt[c] + (d^2*(c/d +
 x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*EllipticE[2*ArcTan[(d*(c/d + x))/(c^(1/4)*
(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(d*Sqrt[c*(
4*a + c^3/d^2) - 2*c^2*(c/d + x)^2 + d^2*(c/d + x)^4])))/d^2 + ((c^3 + 4*a
*d^2)^(1/4)*(c^3 + 4*a*d^2 - c^(3/2)*Sqrt[c^3 + 4*a*d^2])*(Sqrt[c] + (d^2*
(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*Sqrt[(d^2*(c*(4*a + c^3/d^2) - 2*c^2*(c/
d + x)^2 + d^2*(c/d + x)^4))/((c^3 + 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)
/Sqrt[c^3 + 4*a*d^2])^2)]*EllipticF[2*ArcTan[(d*(c/d + x))/(c^(1/4)*(c^3 +
 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(2*c^(1/4)*d^3*Sq
rt[c*(4*a + c^3/d^2) - 2*c^2*(c/d + x)^2 + d^2*(c/d + x)^4])))/3
```

3.775.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

3.775. $\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.775.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 4864 vs. $2(668) = 1336$.

Time = 5.16 (sec) , antiderivative size = 4865, normalized size of antiderivative = 7.82

method	result	size
risch	Expression too large to display	4865
default	Expression too large to display	4890
elliptic	Expression too large to display	4890

input
```
int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(d*x+c)*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)/d+2/3*c/d*(8*a*d*((c
+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((-
(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x
-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/
d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))
/d))^(1/2)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)^2*((-(c+(2*d*(-a*c)^(1/2
)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(-2*d*(-a*c)^(
1/2)+c^2)^(1/2))/d)/((-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^
(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*((-(c+(
2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+
(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-
c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))
^(1/2)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/
2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/
2))/d)/(d^2*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(2*d*(-a*c)^(1/2
)+c^2)^(1/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*d*(-a*
c)^(1/2)+c^2)^(1/2))/d))^(1/2)*EllipticF(((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/
2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1
/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(
1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2),((-(c+(2*d*(-a*c...
```

3.775.5 Fricas [F]

$$\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx=\int \sqrt{d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac}\,dx$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

3.775.6 Sympy [F]

$$\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx=\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$$

input
```
integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(1/2),x)
```

output
```
Integral(sqrt(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4), x)
```

3.775.7 Maxima [F]

$$\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx=\int \sqrt{d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac}\,dx$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

3.775.8 Giac [F]

$$\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx=\int \sqrt{d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac}\,dx$$

input
```
integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

3.775.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx=\int \sqrt{4\,c^2\,x^2+4\,c\,d\,x^3+4\,a\,c+d^2\,x^4}\,dx$$

input
```
int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(1/2),x)
```

output
```
int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(1/2), x)
```

3.776 $\int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}\,dx$

3.776.1 Optimal result

Integrand size = 31, antiderivative size = 227

$$\int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}\,dx = \frac{\sqrt[4]{c^3+4ad^2}\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(c^3+4ad^2)\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)\text{EllipticF}\left(2\arctan\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right),\frac{1}{2}\left(1+\frac{}{\sqrt{c}}\right.\right.}{2\sqrt[4]{c}d\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

output
```
1/2*(4*a*d^2+c^3)^(1/4)*(cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4))
)^2)^(1/2)/cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))*EllipticF(si
n(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4))),1/2*(2+2*c^(3/2)/(4*a*d^2
+c^3)^(1/2))^(1/2))*(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))*(d^2*(d^2*
x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d
^2+c^3)^(1/2))^2)^(1/2)/c^(1/4)/d/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2
)
```

3.776.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.49 (sec) , antiderivative size = 822, normalized size of antiderivative = 3.62

$$\int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}\,dx$$
$$= \frac{2\left(-c+\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}-dx\right)\left(c+\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}+dx\right)\sqrt{-\frac{\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}\left(c-\sqrt{c^2+2i\sqrt{a}\sqrt{c}d}+dx\right)}{\left(\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}+\sqrt{c^2+2i\sqrt{a}\sqrt{c}d}\right)\left(-c+\sqrt{c^2-2i\sqrt{c}\right.}}}{d\sqrt{c^2-2i\sqrt{a}\sqrt{c}}}$$

input
```
Integrate[1/Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4],x]
```

output
```
(2*(-c + Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] - d*x)*(c + Sqrt[c^2 - (2*I)*
Sqrt[a]*Sqrt[c]*d] + d*x)*Sqrt[-((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d]*(c -
 Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d] + d*x))/((Sqrt[c^2 - (2*I)*Sqrt[a]*Sq
rt[c]*d] + Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(-c + Sqrt[c^2 - (2*I)*Sqr
t[a]*Sqrt[c]*d] - d*x)))]*Sqrt[-((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d]*(c +
 Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d] + d*x))/((Sqrt[c^2 - (2*I)*Sqrt[a]*Sq
rt[c]*d] - Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(-c + Sqrt[c^2 - (2*I)*Sqr
t[a]*Sqrt[c]*d] - d*x)))]*EllipticF[ArcSin[Sqrt[((Sqrt[c^2 - (2*I)*Sqrt[a]
*Sqrt[c]*d] - Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(c + Sqrt[c^2 - (2*I)*S
qrt[a]*Sqrt[c]*d] + d*x))/((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] + Sqrt[c^2
 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(-c + Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] - d
*x))]], (Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] + Sqrt[c^2 + (2*I)*Sqrt[a]*Sq
rt[c]*d])^2/(Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] - Sqrt[c^2 + (2*I)*Sqrt[a
]*Sqrt[c]*d])^2])/(d*Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d]*Sqrt[((Sqrt[c^2 -
 (2*I)*Sqrt[a]*Sqrt[c]*d] - Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(c + Sqrt
[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] + d*x))/((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]
*d] + Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(-c + Sqrt[c^2 - (2*I)*Sqrt[a]*
Sqrt[c]*d] - d*x))]*Sqrt[4*a*c + x^2*(2*c + d*x)^2])
```

3.776.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.14, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2458, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}dx$$

$$\Big\downarrow \text{2458}$$

$$\int \frac{1}{\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}}d\left(\frac{c}{d}+x\right)$$

$$\Big\downarrow \text{1416}$$

$$\frac{\sqrt[4]{4ad^2+c^3}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)\sqrt{\frac{d^2\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)}{(4ad^2+c^3)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{d\left(\frac{c}{d}+x\right)}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right),\frac{1}{2}\left(\frac{c}{\sqrt{c^3}}\right.\right.}{2\sqrt[4]{c}d\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}}$$

input
```
Int[1/Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4],x]
```

output
```
((c^3 + 4*a*d^2)^(1/4)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*S
qrt[(d^2*(c*(4*a + c^3/d^2) - 2*c^2*(c/d + x)^2 + d^2*(c/d + x)^4))/((c^3
+ 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*EllipticF
[2*ArcTan[(d*(c/d + x))/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqr
t[c^3 + 4*a*d^2])/2])/(2*c^(1/4)*d*Sqrt[c*(4*a + c^3/d^2) - 2*c^2*(c/d + x
)^2 + d^2*(c/d + x)^4])
```

3.776.3.1 Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.776.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1055 vs. $2(259) = 518$.

Time = 1.04 (sec) , antiderivative size = 1056, normalized size of antiderivative = 4.65

method	result	size
default	Expression too large to display	1056
elliptic	Expression too large to display	1056

input
```
int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*((c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d
)*((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/
d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1
/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(
1/2))/d))^(1/2)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)^2*((-(c+(2*d*(-a*c)
^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(-2*d*(-a
*c)^(1/2)+c^2)^(1/2))/d)/((-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-
a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*((
-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(
x+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))
/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d))^(1/2)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2
)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2
)^(1/2))/d)/(d^2*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(2*d*(-a*c)
^(1/2)+c^2)^(1/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*d
*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*EllipticF(((-(c+(-2*d*(-a*c)^(1/2)+c^2
)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^
2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c
^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2),((-(c+(2*d*(-a
*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((c+(-2*d...
```

3.776.5 Fricas [F]

$$\int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}\,dx=\int \frac{1}{\sqrt{d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac}}\,dx$$

input
```
integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="fricas
")
```

output
```
integral(1/sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

3.776.6 Sympy [F]

$$\int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}\,dx = \int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}\,dx$$

input
```
integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(1/2),x)
```

output
```
Integral(1/sqrt(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4), x)
```

3.776.7 Maxima [F]

$$\int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}\,dx = \int \frac{1}{\sqrt{d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac}}\,dx$$

input
```
integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

3.776.8 Giac [F]

$$\int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}\,dx = \int \frac{1}{\sqrt{d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac}}\,dx$$

input
```
integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

3.776.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}}\,dx = \int \frac{1}{\sqrt{4\,c^2\,x^2 + 4\,c\,d\,x^3 + 4\,a\,c + d^2\,x^4}}\,dx$$

input
```
int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(1/2),x)
```

output
```
int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(1/2), x)
```

3.777 $\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^{3/2}}\,dx$

3.777.1 Optimal result

Integrand size = 31, antiderivative size = 674

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^{3/2}}\,dx = -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\,(c^3+4ad^2)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}-\frac{d^2\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{8a\,(c^3+4ad^2)^{3/2}\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)}$$

$$+\frac{\sqrt[4]{c}\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(c^3+4ad^2)\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)E\left(2\arctan\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(1+\frac{c^{3/2}}{\sqrt{c^3+4ad^2}}\right)\right)}{8ad\sqrt[4]{c^3+4ad^2}\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

$$+\frac{\left(c^3+4ad^2-c^{3/2}\sqrt{c^3+4ad^2}\right)\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(c^3+4ad^2)\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)\text{EllipticF}\left(2\arctan\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+}}\right.\right.}{16ac^{5/4}d\,(c^3+4ad^2)^{3/4}\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

output
```
-1/8*(c/d+x)*(c^3-4*a*d^2-c*d^2*(c/d+x)^2)/a/c/(4*a*d^2+c^3)/(d^2*x^4+4*c*
d*x^3+4*c^2*x^2+4*a*c)^(1/2)-1/8*d^2*(c/d+x)*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+
4*a*c)^(1/2)/a/(4*a*d^2+c^3)^(3/2)/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1
/2))+1/8*c^(1/4)*(cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))^2)^(1
/2)/cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))*EllipticE(sin(2*arc
tan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4))),1/2*(2+2*c^(3/2)/(4*a*d^2+c^3)^(
1/2))^(1/2))*(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))*(d^2*(d^2*x^4+4*c
*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)
^(1/2))^2)^(1/2)/a/d/(4*a*d^2+c^3)^(1/4)/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*
c)^(1/2)+1/16*(cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))^2)^(1/2)
/cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))*EllipticF(sin(2*arctan
((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4))),1/2*(2+2*c^(3/2)/(4*a*d^2+c^3)^(1/2
))^(1/2))*(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))*(c^3+4*a*d^2-c^(3/2)
*(4*a*d^2+c^3)^(1/2))*(d^2*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+c^
3)/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))^2)^(1/2)/a/c^(5/4)/d/(4*a*d
^2+c^3)^(3/4)/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)
```

3.777.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 16.12 (sec) , antiderivative size = 5276, normalized size of antiderivative = 7.83

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-3/2),x]
```

output
```
Result too large to show
```

3.777.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 777, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {2458, 1405, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}}dx$$

$$\Big\downarrow \; 2458$$

$$\int \frac{1}{\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)^{3/2}}d\left(\frac{c}{d}+x\right)$$

$$\Big\downarrow \; 1405$$

$$\frac{\int \frac{2c\left(c^3-d^2\left(\frac{c}{d}+x\right)^2c+4ad^2\right)}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)}{16ac^2\left(4ad^2+c^3\right)}-\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(4ad^2+c^3\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{c^3-d^2\left(\frac{c}{d}+x\right)^2c+4ad^2}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)}{8ac\left(4ad^2+c^3\right)}-\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(4ad^2+c^3\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}}$$

$$\Big\downarrow \; 1511$$

$$\frac{c^{3/2}\sqrt{4ad^2+c^3}\int \frac{\sqrt{c}-\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}}{\sqrt{c}\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)-\sqrt{4ad^2+c^3}\left(c^{3/2}-\sqrt{4ad^2+c^3}\right)\int \frac{1}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\cdots\right.}}}{8ac\left(4ad^2+c^3\right)}-\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(4ad^2+c^3\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}}$$

$$\Big\downarrow \; 27$$

3.777. $\int \frac{1}{\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}}dx$

$$\frac{c\sqrt{4ad^2+c^3}\int \frac{\sqrt{c}-\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)-\sqrt{4ad^2+c^3}\left(c^{3/2}-\sqrt{4ad^2+c^3}\right)\int \frac{1}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2}}}{8ac\left(4ad^2+c^3\right)}$$

$$\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(4ad^2+c^3\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}}$$

$\downarrow$ 1416

$$\frac{c\sqrt{4ad^2+c^3}\int \frac{\sqrt{c}-\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}}{\sqrt{d^2\left(\frac{c}{d}+x\right)^4-2c^2\left(\frac{c}{d}+x\right)^2+c\left(\frac{c^3}{d^2}+4a\right)}}d\left(\frac{c}{d}+x\right)-\frac{\left(4ad^2+c^3\right)^{3/4}\left(c^{3/2}-\sqrt{4ad^2+c^3}\right)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)\sqrt{\frac{d^2\left(c\left(4a+\frac{c^3}{d^2}\right)-\right.}{\left(4ad^2+c^3\right)}}}{2\sqrt[4]{c}d\sqrt{c\left(4a\right.}}}{8ac\left(4ad^2+c^3\right)}$$

$$\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(4ad^2+c^3\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}}$$

$\downarrow$ 1509

$$\frac{c\sqrt{4ad^2+c^3}\left(\frac{\sqrt[4]{c}\sqrt[4]{4ad^2+c^3}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)\sqrt{\frac{d^2\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4\right)}{\left(4ad^2+c^3\right)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}E\left(2\arctan\left(\frac{d\left(\frac{c}{d}+x\right)}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(\frac{c}{\sqrt{c^3}}\right.\right.}{d\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}}\right.}{}$$

$$\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(4ad^2+c^3\right)\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2\left(\frac{c}{d}+x\right)^2+d^2\left(\frac{c}{d}+x\right)^4}}$$

input
```
Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-3/2),x]
```

3.777. $\int \frac{1}{\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}}\,dx$

output
```
-1/8*((c/d + x)*(c^3 - 4*a*d^2 - c*d^2*(c/d + x)^2))/(a*c*(c^3 + 4*a*d^2)*
Sqrt[c*(4*a + c^3/d^2) - 2*c^2*(c/d + x)^2 + d^2*(c/d + x)^4]) + (c*Sqrt[c
^3 + 4*a*d^2]*(-(((c/d + x)*Sqrt[c*(4*a + c^3/d^2) - 2*c^2*(c/d + x)^2 + d
^2*(c/d + x)^4])/((4*a + c^3/d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 +
4*a*d^2]))) + (c^(1/4)*(c^3 + 4*a*d^2)^(1/4)*(Sqrt[c] + (d^2*(c/d + x)^2)/
Sqrt[c^3 + 4*a*d^2])*Sqrt[(d^2*(c*(4*a + c^3/d^2) - 2*c^2*(c/d + x)^2 + d^
2*(c/d + x)^4))/((c^3 + 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4
*a*d^2])^2)]*EllipticE[2*ArcTan[(d*(c/d + x))/(c^(1/4)*(c^3 + 4*a*d^2)^(1/
4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(d*Sqrt[c*(4*a + c^3/d^2) - 2*
c^2*(c/d + x)^2 + d^2*(c/d + x)^4])) - ((c^3 + 4*a*d^2)^(3/4)*(c^(3/2) - S
qrt[c^3 + 4*a*d^2])*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*Sqrt
[(d^2*(c*(4*a + c^3/d^2) - 2*c^2*(c/d + x)^2 + d^2*(c/d + x)^4))/((c^3 + 4
*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*EllipticF[2*
ArcTan[(d*(c/d + x))/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c
^3 + 4*a*d^2])/2])/(2*c^(1/4)*d*Sqrt[c*(4*a + c^3/d^2) - 2*c^2*(c/d + x)^2
 + d^2*(c/d + x)^4]))/(8*a*c*(c^3 + 4*a*d^2))
```

3.777.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.777.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 5023 vs. $2(720) = 1440$.

Time = 1.03 (sec) , antiderivative size = 5024, normalized size of antiderivative = 7.45

method	result	size
default	Expression too large to display	5024
elliptic	Expression too large to display	5024

input
```
int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.777.5 Fricas [F]

$$\int \frac{1}{(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^{3/2}}\, dx = \int \frac{1}{(d^2x^4 + 4\, cdx^3 + 4\, c^2x^2 + 4\, ac)^{\frac{3}{2}}}\, dx$$

input
```
integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)/(d^4*x^8 + 8*c*d^3*
x^7 + 24*c^2*d^2*x^6 + 32*c^3*d*x^5 + 32*a*c^2*d*x^3 + 32*a*c^3*x^2 + 8*(2
*c^4 + a*c*d^2)*x^4 + 16*a^2*c^2), x)
```

3.777.6 Sympy [F]

$$\int \frac{1}{(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^{3/2}}\, dx = \int \frac{1}{(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^{\frac{3}{2}}}\, dx$$

input
```
integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(3/2),x)
```

output
```
Integral((4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4)**(-3/2), x)
```

3.777.7 Maxima [F]

$$\int \frac{1}{(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4)^{3/2}}\, dx = \int \frac{1}{(d^2x^4 + 4\, cdx^3 + 4\, c^2x^2 + 4\, ac)^{\frac{3}{2}}}\, dx$$

input
```
integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(-3/2), x)
```

3.777.8 Giac [F]

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^{3/2}}\,dx = \int \frac{1}{(d^2x^4+4\,cdx^3+4\,c^2x^2+4\,ac)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="giac")
```

output
```
integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(-3/2), x)
```

3.777.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^{3/2}}\,dx = \int \frac{1}{(4\,c^2\,x^2+4\,c\,d\,x^3+4\,a\,c+d^2\,x^4)^{3/2}}\,dx$$

input
```
int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(3/2),x)
```

output
```
int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(3/2), x)
```

3.778 $\int \sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\,dx$

3.778.1 Optimal result

Integrand size = 34, antiderivative size = 663

$$\int \sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\,dx$$
$$= \frac{1}{3}\left(\frac{d}{4e}+x\right)\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4} - \frac{2d^2\left(\frac{d}{4e}+x\right)\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}}{\sqrt{5d^4+256ae^3}\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)}$$
$$+ \frac{d^2(5d^4+256ae^3)^{3/4}\sqrt{\frac{e(8ae^2-d^3x+8de^2x^3+8e^3x^4)}{(5d^4+256ae^3)\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)^2}}\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)E\left(2\arctan\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right)|\frac{1}{2}\right.}{8\sqrt{2}e^2\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$
$$+ \frac{\sqrt[4]{5d^4+256ae^3}\left(5d^4+256ae^3-3d^2\sqrt{5d^4+256ae^3}\right)\sqrt{\frac{e(8ae^2-d^3x+8de^2x^3+8e^3x^4)}{(5d^4+256ae^3)\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)^2}}\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)\text{El}}{48\sqrt{2}e^2\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

output
```
1/3*(1/4*d/e+x)*(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)-2*d^2*(1/4*d/e
+x)*(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)/(1+16*e^2*(1/4*d/e+x)^2/(2
56*a*e^3+5*d^4)^(1/2))/(256*a*e^3+5*d^4)^(1/2)+1/16*d^2*(256*a*e^3+5*d^4)^
(3/4)*(cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))^2)^(1/2)/cos(2*arc
tan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))*EllipticE(sin(2*arctan((4*e*x+d)/(
256*a*e^3+5*d^4)^(1/4))),1/2*(2+6*d^2/(256*a*e^3+5*d^4)^(1/2))^(1/2))*(1+1
6*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))*(e*(8*e^3*x^4+8*d*e^2*x^3-d^3
*x+8*a*e^2)/(256*a*e^3+5*d^4)/(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1
/2))^2)^(1/2)/e^2*2^(1/2)/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)+1/96
*(256*a*e^3+5*d^4)^(1/4)*(cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))
^2)^(1/2)/cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))*EllipticF(sin(2
*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4))),1/2*(2+6*d^2/(256*a*e^3+5*d^4)
^(1/2))^(1/2))*(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))*(5*d^4+256
*a*e^3-3*d^2*(256*a*e^3+5*d^4)^(1/2))*(e*(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*
e^2)/(256*a*e^3+5*d^4)/(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))^2)
^(1/2)/e^2*2^(1/2)/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)
```

3.778.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7543 vs. 2(663) = 1326.

Time = 13.92 (sec) , antiderivative size = 7543, normalized size of antiderivative = 11.38

$$\int \sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\, dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4],x]
```

output
```
Result too large to show
```

3.778.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 814, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2458, 1404, 27, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx$$

$$\Big\downarrow \text{2458}$$

$$\int \sqrt{\frac{1}{32}\left(256ae^2+\frac{5d^4}{e}\right)-3d^2e\left(\frac{d}{4e}+x\right)^2+8e^3\left(\frac{d}{4e}+x\right)^4}d\left(\frac{d}{4e}+x\right)$$

$$\Big\downarrow \text{1404}$$

$$\frac{1}{3}\int \frac{\frac{5d^4}{e}-48e\left(\frac{d}{4e}+x\right)^2d^2+256ae^2}{2\sqrt{2}\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}}d\left(\frac{d}{4e}+x\right)+\frac{\left(\frac{d}{4e}+x\right)\sqrt{256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4}}{12\sqrt{2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\frac{5d^4}{e}-48e\left(\frac{d}{4e}+x\right)^2d^2+256ae^2}{\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}}d\left(\frac{d}{4e}+x\right)}{6\sqrt{2}}+\frac{\left(\frac{d}{4e}+x\right)\sqrt{256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4}}{12\sqrt{2}}$$

$$\Big\downarrow \text{1511}$$

$$\frac{\frac{3d^2\sqrt{256ae^3+5d^4}\int \frac{1-\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}}{\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}}d\left(\frac{d}{4e}+x\right)}{e}+\frac{\left(-3d^2\sqrt{256ae^3+5d^4}+256ae^3+5d^4\right)\int \frac{1}{\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e}}}{e}}{6\sqrt{2}}+\frac{\left(\frac{d}{4e}+x\right)\sqrt{256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4}}{12\sqrt{2}}$$

$$\Big\downarrow \text{1416}$$

$$\frac{\frac{3d^2\sqrt{256ae^3+5d^4}\int \frac{1-\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}}{\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}}d\left(\frac{d}{4e}+x\right)}{e}+\frac{\sqrt[4]{256ae^3+5d^4}\left(-3d^2\sqrt{256ae^3+5d^4}+256ae^3+5d^4\right)\left(\frac{16e^2\left(\frac{d}{4}\right.}{\sqrt{256ae}}\right.}{}}{6\sqrt{2}}+\frac{\left(\frac{d}{4e}+x\right)\sqrt{256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4}}{12\sqrt{2}}$$

$$\Big\downarrow \text{1509}$$

3.778. $\int \sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx$

$$\frac{\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}\left(\frac{d}{4e}+x\right)}{12\sqrt{2}}+$$

$$\frac{3\sqrt{5d^4+256ae^3}\left(\frac{\sqrt[4]{5d^4+256ae^3}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}+1\right)\sqrt{\frac{5d^4-96e^2\left(\frac{d}{4e}+x\right)^2d^2+256e^4\left(\frac{d}{4e}+x\right)^4+256ae^3}{\left(5d^4+256ae^3\right)\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}+1\right)^2}}E\left(2\arctan\left(\frac{4e\left(\frac{d}{4e}+x\right)}{\sqrt[4]{5d^4+256ae^3}}\right)|\frac{1}{2}\left(\frac{\ldots}{\sqrt{5\ldots}}\right.\right.}{4e\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}}\ldots}{e}\ldots$$

input
```
Int[Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4],x]
```

output
```
((d/(4*e) + x)*Sqrt[(5*d^4)/e + 256*a*e^2 - 96*d^2*e*(d/(4*e) + x)^2 + 256
*e^3*(d/(4*e) + x)^4])/(12*Sqrt[2]) + ((3*d^2*Sqrt[5*d^4 + 256*a*e^3]*(-((
e*(d/(4*e) + x)*Sqrt[(5*d^4)/e + 256*a*e^2 - 96*d^2*e*(d/(4*e) + x)^2 + 25
6*e^3*(d/(4*e) + x)^4])/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)^2)
/Sqrt[5*d^4 + 256*a*e^3]))) + ((5*d^4 + 256*a*e^3)^(1/4)*(1 + (16*e^2*(d/(
4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])*Sqrt[(5*d^4 + 256*a*e^3 - 96*d^2*e^2
*(d/(4*e) + x)^2 + 256*e^4*(d/(4*e) + x)^4)/((5*d^4 + 256*a*e^3)*(1 + (16*
e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])^2)]*EllipticE[2*ArcTan[(4*e*
(d/(4*e) + x))/(5*d^4 + 256*a*e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 + 256*a
*e^3])/2])/(4*e*Sqrt[(5*d^4)/e + 256*a*e^2 - 96*d^2*e*(d/(4*e) + x)^2 + 25
6*e^3*(d/(4*e) + x)^4])))/e + ((5*d^4 + 256*a*e^3)^(1/4)*(5*d^4 + 256*a*e^
3 - 3*d^2*Sqrt[5*d^4 + 256*a*e^3])*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^
4 + 256*a*e^3])*Sqrt[(5*d^4 + 256*a*e^3 - 96*d^2*e^2*(d/(4*e) + x)^2 + 256
*e^4*(d/(4*e) + x)^4)/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)^2)/S
qrt[5*d^4 + 256*a*e^3])^2)]*EllipticF[2*ArcTan[(4*e*(d/(4*e) + x))/(5*d^4
+ 256*a*e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/(8*e^2*Sqrt
[(5*d^4)/e + 256*a*e^2 - 96*d^2*e*(d/(4*e) + x)^2 + 256*e^3*(d/(4*e) + x)^
4]))/(6*Sqrt[2])
```

3.778. $\int \sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx$

3.778.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.778. $\int \sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\, dx$

3.778.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 7886 vs. 2(715) = 1430.

Time = 7.22 (sec) , antiderivative size = 7887, normalized size of antiderivative = 11.90

method	result	size
default	Expression too large to display	7887
elliptic	Expression too large to display	7887
risch	Expression too large to display	9561

input
```
int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.778.5 Fricas [F]

$$\int \sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx=\int \sqrt{8\,e^3x^4+8\,de^2x^3-d^3x+8\,ae^2}\,dx$$

input
```
integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="fricas
")
```

output
```
integral(sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

3.778.6 Sympy [F]

$$\int \sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx=\int \sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx$$

input
```
integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**(1/2),x)
```

output
```
Integral(sqrt(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4), x)
```

3.778.7 Maxima [F]

$$\int \sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx=\int \sqrt{8\,e^3x^4+8\,de^2x^3-d^3x+8\,ae^2}\,dx$$

input
```
integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="maxima
")
```

output
```
integrate(sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

3.778.8 Giac [F]

$$\int \sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx=\int \sqrt{8\,e^3x^4+8\,de^2x^3-d^3x+8\,ae^2}\,dx$$

input
```
integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

3.778.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx=\int \sqrt{-d^3\,x+8\,d\,e^2\,x^3+8\,e^3\,x^4+8\,a\,e^2}\,dx$$

input
```
int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(1/2),x)
```

output
```
int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(1/2), x)
```

3.779 $\int \frac{1}{\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}\,dx$

3.779.1 Optimal result

Integrand size = 34, antiderivative size = 235

$$\int \frac{1}{\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}\,dx = \frac{\sqrt[4]{5d^4+256ae^3}\sqrt{\frac{e(8ae^2-d^3x+8de^2x^3+8e^3x^4)}{(5d^4+256ae^3)\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)^2}}\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)\text{EllipticF}\left(2\arctan\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right),\frac{1}{2}\right.}{\sqrt{2}e\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

output
```
1/2*(256*a*e^3+5*d^4)^(1/4)*(cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4
)))^2)^(1/2)/cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))*EllipticF(si
n(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4))),1/2*(2+6*d^2/(256*a*e^3+5*d
^4)^(1/2))^(1/2))*(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))*(e*(8*e
^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)/(256*a*e^3+5*d^4)/(1+16*e^2*(1/4*d/e+x)^
2/(256*a*e^3+5*d^4)^(1/2))^2)^(1/2)/e*2^(1/2)/(8*e^3*x^4+8*d*e^2*x^3-d^3*x
+8*a*e^2)^(1/2)
```

3.779.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1065 vs. $2(235) = 470$.

Time = 11.41 (sec) , antiderivative size = 1065, normalized size of antiderivative = 4.53

$$\int \frac{1}{\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}}\,dx =$$

$$-\frac{\left(-d+\sqrt{3d^2-2\sqrt{d^4-64ae^3}}-4ex\right)\left(d-\sqrt{3d^2+2\sqrt{d^4-64ae^3}}+4ex\right)\sqrt{-\frac{\sqrt{3d^2-2\sqrt{d^4-64a}}}{\left(\sqrt{3d^2-2\sqrt{d^4-64ae^3}}-\sqrt{3d^2+}\right.}}}{}$$

input
```
Integrate[1/Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4],x]
```

output
```
-1/2*((-d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - 4*e*x)*(d - Sqrt[3*d^2
+ 2*Sqrt[d^4 - 64*a*e^3]] + 4*e*x)*Sqrt[-((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*
e^3]]*(d + Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]] + 4*e*x))/((Sqrt[3*d^2 - 2
*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])*(-d + Sqrt[
3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - 4*e*x)))]*Sqrt[(3*d^2 - 2*Sqrt[d^4 - 64*
a*e^3] - Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]*Sqrt[3*d^2 + 2*Sqrt[d^4 - 64
*a*e^3]] + d*(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt[d
^4 - 64*a*e^3]]) + 4*e*(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2
+ 2*Sqrt[d^4 - 64*a*e^3]])*x)/((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] + Sqr
t[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])*(-d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^
3]] - 4*e*x))]*EllipticF[ArcSin[Sqrt[((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]
] - Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])*(d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 6
4*a*e^3]] + 4*e*x))/((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] + Sqrt[3*d^2 +
2*Sqrt[d^4 - 64*a*e^3]])*(-d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - 4*e*
x))]], (Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] + Sqrt[3*d^2 + 2*Sqrt[d^4 - 6
4*a*e^3]])^2/(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt[d
^4 - 64*a*e^3]])^2])/(e*(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2
 + 2*Sqrt[d^4 - 64*a*e^3]])*Sqrt[(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]*(-d
 + Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]] - 4*e*x))/((Sqrt[3*d^2 - 2*Sqrt[d^
4 - 64*a*e^3]] + Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])*(-d + Sqrt[3*d^2...
```

3.779.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.19, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2458, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}\,dx$$

$$\Big\downarrow \ 2458$$

$$\int \frac{1}{\sqrt{\frac{1}{32}\left(256ae^2+\frac{5d^4}{e}\right)-3d^2e\left(\frac{d}{4e}+x\right)^2+8e^3\left(\frac{d}{4e}+x\right)^4}}d\left(\frac{d}{4e}+x\right)$$

$$\Big\downarrow \ 1416$$

$$\frac{\sqrt[4]{256ae^3+5d^4}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)\sqrt{\frac{256ae^3+5d^4-96d^2e^2\left(\frac{d}{4e}+x\right)^2+256e^4\left(\frac{d}{4e}+x\right)^4}{(256ae^3+5d^4)\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{4e\left(\frac{d}{4e}+x\right)}{\sqrt[4]{5d^4+256ae^{\cdots}}}\right.\right.}{\sqrt{2}e\sqrt{256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4}}$$

input
```
Int[1/Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4],x]
```

output
```
((5*d^4 + 256*a*e^3)^(1/4)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*
a*e^3])*Sqrt[(5*d^4 + 256*a*e^3 - 96*d^2*e^2*(d/(4*e) + x)^2 + 256*e^4*(d/
(4*e) + x)^4)/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^
4 + 256*a*e^3])^2)]*EllipticF[2*ArcTan[(4*e*(d/(4*e) + x))/(5*d^4 + 256*a*
e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/(Sqrt[2]*e*Sqrt[(5*
d^4)/e + 256*a*e^2 - 96*d^2*e*(d/(4*e) + x)^2 + 256*e^3*(d/(4*e) + x)^4])
```

3.779.3.1 Defintions of rubi rules used

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.779.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1703 vs. $2(273) = 546$.

Time = 1.23 (sec) , antiderivative size = 1704, normalized size of antiderivative = 7.25

method	result	size
default	Expression too large to display	1704
elliptic	Expression too large to display	1704

input
```
int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(1/4*(-d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(d*e
+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*((-1/4*(d*e+(3*d^2*e^
2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(d*e+(3*d^2*e^2+2*(-64*a*e^3
+d^4)^(1/2)*e^2)^(1/2))/e^2)*(x-1/4*(-d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/
2)*e^2)^(1/2))/e^2)/(-1/4*(d*e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/
2))/e^2-1/4*(-d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(x+1
/4*(d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2)*(x+1/4*
(d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)^2*((-1/4*(d*e+(3*
d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*d^2*e^2+2*(-6
4*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*(x-1/4*(-d*e+(3*d^2*e^2-2*(-64*a*e^3+d
^4)^(1/2)*e^2)^(1/2))/e^2)/(1/4*(-d*e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e
^2)^(1/2))/e^2-1/4*(-d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^
2)/(x+1/4*(d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2)*
((-1/4*(d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(
3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*(x+1/4*(d*e+(3*d^2*e^2-
2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(-1/4*(d*e+(3*d^2*e^2-2*(-64*a*e^
3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*
e^2)^(1/2))/e^2)/(x+1/4*(d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2)
)/e^2))^(1/2)/(-1/4*(d*e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^
2+1/4*(d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(-1/4*(d...
```

3.779.5 Fricas [F]

$$\int \frac{1}{\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}\,dx=\int \frac{1}{\sqrt{8\,e^3x^4+8\,de^2x^3-d^3x+8\,ae^2}}\,dx$$

input
```
integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="fric
as")
```

output
```
integral(1/sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

3.779.6 Sympy [F]

$$\int \frac{1}{\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}}\,dx = \int \frac{1}{\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}}\,dx$$

input
```
integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**(1/2),x)
```

output
```
Integral(1/sqrt(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4), x)
```

3.779.7 Maxima [F]

$$\int \frac{1}{\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}}\,dx = \int \frac{1}{\sqrt{8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2}}\,dx$$

input
```
integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate(1/sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

3.779.8 Giac [F]

$$\int \frac{1}{\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}}\,dx = \int \frac{1}{\sqrt{8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2}}\,dx$$

input
```
integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="giac
")
```

output
```
integrate(1/sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2), x)
```

3.779.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}\,dx=\int \frac{1}{\sqrt{-d^3\,x+8\,d\,e^2\,x^3+8\,e^3\,x^4+8\,a\,e^2}}\,dx$$

input
```
int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(1/2),x)
```

output
```
int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(1/2), x)
```

3.780 $\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^{3/2}}\,dx$

3.780.1 Optimal result

Integrand size = 34, antiderivative size = 748

$$\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^{3/2}}\,dx=\frac{4e\left(\frac{d}{4e}+x\right)\left(13d^4-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{(5d^8-64ad^4e^3-16384a^2e^6)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$
$$+\frac{384d^2e^2\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}{(d^4-64ae^3)\,(5d^4+256ae^3)^{3/2}\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)}$$
$$-\frac{12\sqrt{2}d^2\sqrt{\frac{e(8ae^2-d^3x+8de^2x^3+8e^3x^4)}{(5d^4+256ae^3)\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)^2}}\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)E\left(2\arctan\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right)|\frac{1}{2}\left(1+\frac{3d^2}{\sqrt{5d^4+256ae^3}}\cdots\right.\right.}{(d^4-64ae^3)\sqrt[4]{5d^4+256ae^3}\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$
$$-\frac{2\sqrt{2}\left(5d^4+256ae^3-3d^2\sqrt{5d^4+256ae^3}\right)\sqrt{\frac{e(8ae^2-d^3x+8de^2x^3+8e^3x^4)}{(5d^4+256ae^3)\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)^2}}\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)\text{EllipticF}\left(2\,\text{arc}\cdots\right.}{(d^4-64ae^3)\,(5d^4+256ae^3)^{3/4}\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x\cdots}}$$

output
```
4*e*(1/4*d/e+x)*(13*d^4-256*a*e^3-48*d^2*e^2*(1/4*d/e+x)^2)/(-16384*a^2*e^
6-64*a*d^4*e^3+5*d^8)/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)+384*d^2*
e^2*(1/4*d/e+x)*(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)/(-64*a*e^3+d^4
)/(256*a*e^3+5*d^4)^(3/2)/(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))
-12*d^2*(cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))^2)^(1/2)/cos(2*a
rctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))*EllipticE(sin(2*arctan((4*e*x+d)
/(256*a*e^3+5*d^4)^(1/4))),1/2*(2+6*d^2/(256*a*e^3+5*d^4)^(1/2))^(1/2))*2^
(1/2)*(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))*(e*(8*e^3*x^4+8*d*e
^2*x^3-d^3*x+8*a*e^2)/(256*a*e^3+5*d^4)/(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3
+5*d^4)^(1/2))^2)^(1/2)/(-64*a*e^3+d^4)/(256*a*e^3+5*d^4)^(1/4)/(8*e^3*x^4
+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)-2*(cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d
^4)^(1/4)))^2)^(1/2)/cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))*Elli
pticF(sin(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4))),1/2*(2+6*d^2/(256*a
*e^3+5*d^4)^(1/2))^(1/2))*2^(1/2)*(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4
)^(1/2))*(5*d^4+256*a*e^3-3*d^2*(256*a*e^3+5*d^4)^(1/2))*(e*(8*e^3*x^4+8*d
*e^2*x^3-d^3*x+8*a*e^2)/(256*a*e^3+5*d^4)/(1+16*e^2*(1/4*d/e+x)^2/(256*a*e
^3+5*d^4)^(1/2))^2)^(1/2)/(-64*a*e^3+d^4)/(256*a*e^3+5*d^4)^(3/4)/(8*e^3*x
^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)
```

3.780.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 7629 vs. 2(748) = 1496.

Time = 16.14 (sec) , antiderivative size = 7629, normalized size of antiderivative = 10.20

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^{3/2}}\, dx = \text{Result too large to show}$$

input
```
Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-3/2),x]
```

output
```
Result too large to show
```

3.780.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 882, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2458, 1405, 27, 1511, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^{3/2}}dx$$

$$\Big\downarrow \text{ 2458}$$

$$\int \frac{1}{\left(\frac{1}{32}\left(256ae^2+\frac{5d^4}{e}\right)-3d^2e\left(\frac{d}{4e}+x\right)^2+8e^3\left(\frac{d}{4e}+x\right)^4\right)^{3/2}}d\left(\frac{d}{4e}+x\right)$$

$$\Big\downarrow \text{ 1405}$$

$$\frac{16\sqrt{2}e\left(\frac{d}{4e}+x\right)\left(-256ae^3+13d^4-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{(-16384a^2e^6-64ad^4e^3+5d^8)\sqrt{256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4}}-\frac{8\int \frac{2\sqrt{2}e^2\left(5d^4-48e^2\left(\frac{d}{4e}+x\right)^2d^2+256ae^3\right)}{\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}}d\left(\frac{d}{4e}+x\right)}{e\left(-16384a^2e^6-64ad^4e^3+5d^8\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{16\sqrt{2}e\left(\frac{d}{4e}+x\right)\left(-256ae^3+13d^4-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{(-16384a^2e^6-64ad^4e^3+5d^8)\sqrt{256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4}}-\frac{16\sqrt{2}e\int \frac{5d^4-48e^2\left(\frac{d}{4e}+x\right)^2d^2+256ae^3}{\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}}d\left(\frac{d}{4e}+x\right)}{-16384a^2e^6-64ad^4e^3+5d^8}$$

$$\Big\downarrow \text{ 1511}$$

$$\frac{16\sqrt{2}e\left(\frac{d}{4e}+x\right)\left(-256ae^3+13d^4-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{(-16384a^2e^6-64ad^4e^3+5d^8)\sqrt{256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4}}-\frac{16\sqrt{2}e\left(3d^2\sqrt{256ae^3+5d^4}\int \frac{1-\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}}{\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}}d\left(\frac{d}{4e}+x\right)-\sqrt{256ae^3+5d^4}\left(3d^2-\sqrt{256ae^3}\right.\right.}{-16384a^2e^6-64ad^4e^3+5d^8}$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{16\sqrt{2}e\left(\frac{d}{4e}+x\right)\left(-256ae^3+13d^4-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{(-16384a^2e^6-64ad^4e^3+5d^8)\sqrt{256ae^2+\frac{5d^4}{e}-96d^2e\left(\frac{d}{4e}+x\right)^2+256e^3\left(\frac{d}{4e}+x\right)^4}}-$$

$$\frac{16\sqrt{2}e\left(3d^2\sqrt{256ae^3+5d^4}\int\frac{1-\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}}{\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}}d\left(\frac{d}{4e}+x\right)-\frac{(256ae^3+5d^4)^{3/4}\left(3d^2-\sqrt{256ae^3+5d^4}\right)}{\cdots}\right.}{-16384a^2e^6-64\cdots}$$

$\downarrow$ 1509

$$\frac{16\sqrt{2}e\left(\frac{d}{4e}+x\right)\left(13d^4-48e^2\left(\frac{d}{4e}+x\right)^2d^2-256ae^3\right)}{(5d^8-64ae^3d^4-16384a^2e^6)\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}}-$$

$$16\sqrt{2}e\left(3d^2\sqrt{5d^4+256ae^3}\left(\frac{\sqrt[4]{5d^4+256ae^3}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}+1\right)\sqrt{\frac{5d^4-96e^2\left(\frac{d}{4e}+x\right)^2d^2+256e^4\left(\frac{d}{4e}+x\right)^4+256ae^3}{(5d^4+256ae^3)\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}+1\right)^2}}E\left(2\arctan\left(\frac{4\cdots}{\sqrt[4]{5d\cdots}}\right.\right.}{4e\sqrt{\frac{5d^4}{e}-96e\left(\frac{d}{4e}+x\right)^2d^2+256e^3\left(\frac{d}{4e}+x\right)^4+256ae^2}}\right.\right.$$

input
```
Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-3/2),x]
```

3.780. $\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^{3/2}}\,dx$

output
```
(16*Sqrt[2]*e*(d/(4*e) + x)*(13*d^4 - 256*a*e^3 - 48*d^2*e^2*(d/(4*e) + x)
^2))/((5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)*Sqrt[(5*d^4)/e + 256*a*e^2 -
96*d^2*e*(d/(4*e) + x)^2 + 256*e^3*(d/(4*e) + x)^4]) - (16*Sqrt[2]*e*(3*d^
2*Sqrt[5*d^4 + 256*a*e^3]*(-((e*(d/(4*e) + x)*Sqrt[(5*d^4)/e + 256*a*e^2 -
 96*d^2*e*(d/(4*e) + x)^2 + 256*e^3*(d/(4*e) + x)^4])/((5*d^4 + 256*a*e^3)
*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3]))) + ((5*d^4 + 256*
a*e^3)^(1/4)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])*Sqrt[(
5*d^4 + 256*a*e^3 - 96*d^2*e^2*(d/(4*e) + x)^2 + 256*e^4*(d/(4*e) + x)^4)/
((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3]
)^2)]*EllipticE[2*ArcTan[(4*e*(d/(4*e) + x))/(5*d^4 + 256*a*e^3)^(1/4)], (
1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/(4*e*Sqrt[(5*d^4)/e + 256*a*e^2 -
 96*d^2*e*(d/(4*e) + x)^2 + 256*e^3*(d/(4*e) + x)^4])) - ((5*d^4 + 256*a*e
^3)^(3/4)*(3*d^2 - Sqrt[5*d^4 + 256*a*e^3])*(1 + (16*e^2*(d/(4*e) + x)^2)/
Sqrt[5*d^4 + 256*a*e^3])*Sqrt[(5*d^4 + 256*a*e^3 - 96*d^2*e^2*(d/(4*e) + x
)^2 + 256*e^4*(d/(4*e) + x)^4)/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e)
+ x)^2)/Sqrt[5*d^4 + 256*a*e^3])^2)]*EllipticF[2*ArcTan[(4*e*(d/(4*e) + x)
)/(5*d^4 + 256*a*e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/(8
*e*Sqrt[(5*d^4)/e + 256*a*e^2 - 96*d^2*e*(d/(4*e) + x)^2 + 256*e^3*(d/(4*e
) + x)^4])))/(5*d^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)
```

3.780.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

3.780. $\int \frac{1}{(8ae^2-d^3x+8de^2x^3+8e^3x^4)^{3/2}}\,dx$

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.780.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 8102 vs. 2(804) = 1608.

Time = 1.37 (sec) , antiderivative size = 8103, normalized size of antiderivative = 10.83

method	result	size
default	Expression too large to display	8103
elliptic	Expression too large to display	8103

input
```
int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.780.5 Fricas [F]

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^{3/2}} \, dx = \int \frac{1}{(8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x, algorithm="fric
as")
```

output
```
integral(sqrt(8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)/(64*e^6*x^8 + 128
*d*e^5*x^7 + 64*d^2*e^4*x^6 - 16*d^3*e^3*x^5 + 128*a*d*e^4*x^3 + d^6*x^2 -
 16*a*d^3*e^2*x + 64*a^2*e^4 - 16*(d^4*e^2 - 8*a*e^5)*x^4), x)
```

3.780.6 Sympy [F]

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^{3/2}} \, dx = \int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**(3/2),x)
```

output
```
Integral((8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)**(-3/2), x)
```

3.780.7 Maxima [F]

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^{3/2}} \, dx = \int \frac{1}{(8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x, algorithm="maxi
ma")
```

output
```
integrate((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^(-3/2), x)
```

3.780.8 Giac [F]

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^{3/2}}\,dx = \int \frac{1}{(8\,e^3x^4 + 8\,de^2x^3 - d^3x + 8\,ae^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x, algorithm="giac
")
```

output
```
integrate((8*e^3*x^4 + 8*d*e^2*x^3 - d^3*x + 8*a*e^2)^(-3/2), x)
```

3.780.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^{3/2}}\,dx = \int \frac{1}{(-d^3\,x + 8\,d\,e^2\,x^3 + 8\,e^3\,x^4 + 8\,a\,e^2)^{3/2}}\,dx$$

input
```
int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(3/2),x)
```

output
```
int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(3/2), x)
```

3.781 $\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx$

3.781.1 Optimal result

Integrand size = 24, antiderivative size = 452

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx = -\frac{16(7+2a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{35\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$
$$+\frac{2}{35}\left(13+5a-3(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x)$$
$$+\frac{1}{7}\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)$$
$$+\frac{16(7+2a)\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)E\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{35\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$
$$+\frac{4(3+a)(16+5a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\operatorname{EllipticF}\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right),-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{35\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

output
```
1/7*(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)*(-1+x)-16/35*(7+2*a)*(-1+x)*(1+(-1+x)^
2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+2/35*(1
3+5*a-3*(-1+x)^2)*(-1+x)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+4/35*(3+a)*(16+5*
a)*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/
2)*EllipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/
2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1
+(4+a)^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^
(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)+16/35*(7+2*a)*(1/(1+(-1+x)^2/(
1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((-1+x)
/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/
(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(1+(4
+a)^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/
2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

3.781.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 6287 vs. $2(452) = 904$.

Time = 16.10 (sec) , antiderivative size = 6287, normalized size of antiderivative = 13.91

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

output
```
Result too large to show
```

3.781.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2458, 1404, 27, 1490, 27, 1514, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-x^4+4x^3-8x^2+8x\right)^{3/2}\,dx$$

$$\Big\downarrow \; 2458$$

$$\int \left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}d(x-1)$$

$\Big\downarrow$ 1404

$$\frac{3}{7}\int 2\left(-(x-1)^2+a+3\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$\Big\downarrow$ 27

$$\frac{6}{7}\int \left(-(x-1)^2+a+3\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$\Big\downarrow$ 1490

$$\frac{6}{7}\left(\frac{1}{15}(x-1)\left(5a-3(x-1)^2+13\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}-\frac{1}{15}\int -\frac{2\left((a+3)(5a+16)-4(2a+7)(x-\right.}{\sqrt{-(x-1)^4-2(x-1)^2+a+}}\right.$$
$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$\Big\downarrow$ 27

$$\frac{6}{7}\left(\frac{2}{15}\int \frac{(a+3)(5a+16)-4(2a+7)(x-1)^2}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\frac{1}{15}(x-1)\left(5a-3(x-1)^2+13\right)\sqrt{a-(x-1)^4-2(x-\right.$$
$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$\Big\downarrow$ 1514

$$\frac{6}{7}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int \frac{(a+3)(5a+16)-4(2a+7)(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{1}{15}(x-1)\left(5a-3(x-1)^2+13\right)\sqrt{a-(x\right.$$
$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$\Big\downarrow$ 406

3.781. $\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx$

$$\frac{6}{7}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((a+3)(5a+16)\int\frac{1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)-4(2a+7)\int\frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\cdots}}\right.}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right.$$

$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$\downarrow$ 320

$$\frac{6}{7}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-4(2a+7)\cdots\right.}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right.$$

$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$\downarrow$ 388

$$\frac{6}{7}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-4(2a+7)\cdots\right.}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right.$$

$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$\downarrow$ 313

3.781. $\int\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx$

$$\frac{6}{7}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-4(2a+7)\right.}{15\sqrt{a-(x-1)^4-2(x-}}\right.$$

$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

input
```
Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

output
```
((3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/7 + (6*(((13 + 5*a -
3*(-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/15 + (2*Sq
rt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])
]*(-4*(7 + 2*a)*(((1 - Sqrt[4 + a])*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]
*(-1 + x))/Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])] - ((1 - Sqrt[4 + a])*Sqr
t[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*EllipticE[ArcTan
[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sq
rt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*
Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])])) + ((3 + a)*(16 + 5*a)*Sqrt[1 + Sq
rt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*EllipticF[ArcTan[(-1 + x
)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 +
(-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[1 +
 (-1 + x)^2/(1 + Sqrt[4 + a])])))/(15*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)
^4]))/7
```

3.781.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

3.781. $\int (a + 8x - 8x^2 + 4x^3 - x^4)^{3/2} \, dx$

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.781.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2654 vs. 2(506) = 1012.

Time = 5.24 (sec) , antiderivative size = 2655, normalized size of antiderivative = 5.87

method	result	size
default	Expression too large to display	2655
elliptic	Expression too large to display	2655
risch	Expression too large to display	3593

input
```
int((-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7*x^5*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+5/7*x^4*(-x^4+4*x^3-8*x^2+a+8*x)^(
1/2)-66/35*x^3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+14/5*x^2*(-x^4+4*x^3-8*x^2+a
+8*x)^(1/2)+(3/7*a-32/35)*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(-3/7*a-4/7)*(-
x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(a^2-(3/7*a-32/35)*a+12/7*a+16/7)*((-1-(a+4)^
(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(
1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+
4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))
^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+
4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2
)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2
))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1
-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-
(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))
^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^
(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/
2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(
-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+
(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-
(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-(64/35*a+32/5)*((-1-(a+
4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(...
```

3.781.5 Fricas [F]

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int \left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

3.781.6 Sympy [F]

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int \left(a-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

output
```
Integral((a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

3.781.7 Maxima [F]

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int \left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

3.781.8 Giac [F]

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int \left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

3.781.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int \left(-x^4+4\,x^3-8\,x^2+8\,x+a\right)^{3/2}\,dx$$

input
```
int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
```

output
```
int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

3.782 $\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

3.782.1 Optimal result

Integrand size = 24, antiderivative size = 397

$$\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx = -\frac{2\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{3\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}+\frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x)$$
$$+\frac{2\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)E\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{3\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$
$$+\frac{2(3+a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\text{EllipticF}\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right),-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{3\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

output
```
-2/3*(-1+x)*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a-2*(-1+x)^2-(
-1+x)^4)^(1/2)+1/3*(-1+x)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+2/3*(3+a)*(1/(1+
(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*Ellipt
icF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4
+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1
/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(
1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)+2/3*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1
/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((-1+x)/(1+(4+a)^(1/2))^(1
/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1
/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2))^(1/2)/(3
+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1
+(4+a)^(1/2))))^(1/2)
```

3.782.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3470 vs. $2(397) = 794$.

Time = 16.06 (sec) , antiderivative size = 3470, normalized size of antiderivative = 8.74

$$\int \sqrt{a + 8x - 8x^2 + 4x^3 - x^4}\, dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output
```
(-1/3 + x/3)*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4] + (2*((4*(-Sqrt[-1 - Sqrt
[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqr
t[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt
[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sq
rt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + S
qrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 -
 Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1
+ Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-
1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[
4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqr
t[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]]
 + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - S
qrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1
+ Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqr
t[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqr
t[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (2*a*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-
1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sq
rt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((
Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 +
a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + ...
```

3.782.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2458, 1404, 27, 1514, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-x^4+4x^3-8x^2+8x}\,dx$$

$$\Big\downarrow \text{2458}$$

$$\int \sqrt{a-(x-1)^4-2(x-1)^2+3}d(x-1)$$

$$\Big\downarrow \text{1404}$$

$$\frac{1}{3}\int \frac{2\left(-(x-1)^2+a+3\right)}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\int \frac{-(x-1)^2+a+3}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \; 1514$$

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int \frac{-(x-1)^2+a+3}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \; 406$$

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((a+3)\int \frac{1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)-\int \frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)\right)}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \; 320$$

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-\int \frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\cdots\right.}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}\cdots$$

$$\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \; 388$$

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((1-\sqrt{a+4})\int \frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}}{\left(\frac{(x-1)^2}{\sqrt{a+4}+1}+1\right)^{3/2}}d(x-1)+\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\cdots\right.\right.}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\cdots}\right.}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}\cdots$$

$$\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \; 313$$

3.782. $\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}+\frac{(1-\sqrt{a+4})\sqrt{\sqrt{a+4}+1}\sqrt{}}{\sqrt{}}\right.}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

input
```
Int[Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output
```
(Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/3 + (2*Sqrt[1 + (-1 + x
)^2/(1 - Sqrt[4 + a])]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])]*(-(((1 - Sqr
t[4 + a])*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*(-1 + x))/Sqrt[1 + (-1 +
x)^2/(1 + Sqrt[4 + a])]) + ((1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*Sqrt[1
 + (-1 + x)^2/(1 - Sqrt[4 + a])]*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4
 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - S
qrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 +
Sqrt[4 + a])]) + ((3 + a)*Sqrt[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - S
qrt[4 + a])]*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4
+ a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-
1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])])))/(3*
Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

3.782.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.782.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2518 vs. $2(455) = 910$.

Time = 4.20 (sec) , antiderivative size = 2519, normalized size of antiderivative = 6.35

method	result	size
default	Expression too large to display	2519
elliptic	Expression too large to display	2519
risch	Expression too large to display	3022

input
```
int((-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-1/3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(2
/3*a+4/3)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/
2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)
^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*
(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(
1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4
)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1
/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2)
)^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)
^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*
(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF
(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(
1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2
))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a
+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4
)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-4/
3*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2
)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^
(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1
+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))...
```

3.782.5 Fricas [F]

$$\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx = \int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}\,dx$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

3.782.6 Sympy [F]

$$\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx = \int \sqrt{a-x^4+4x^3-8x^2+8x}\,dx$$

input
```
integrate((-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

output
```
Integral(sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

3.782.7 Maxima [F]

$$\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx = \int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}\,dx$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

3.782.8 Giac [F]

$$\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx = \int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}\,dx$$

input
```
integrate((-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

3.782.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int \sqrt{-x^4+4\,x^3-8\,x^2+8\,x+a}\,dx$$

input
```
int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
```

output
```
int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

3.783 $\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$

3.783.1 Optimal result

Integrand size = 24, antiderivative size = 144

$$\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\text{EllipticF}\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right),-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

output
```
(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*
EllipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),
(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4
+a)^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/
2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

3.783.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 540 vs. 2(144) = 288.

Time = 11.03 (sec) , antiderivative size = 540, normalized size of antiderivative = 3.75

$$\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$$
$$= \frac{2\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)\sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(1+\sqrt{-1+\sqrt{4+a}}-x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}}\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\sqrt{\frac{\cdots}{(-\cdots}}\cdots}{\sqrt{-1-\sqrt{4+a}}\sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}}-\cdots\right.}{\left(\sqrt{-1-\sqrt{4+a}}+\cdots\right.}}\cdots}$$

input
```
Integrate[1/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output
```
(2*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(1 + Sqrt
[-1 + Sqrt[4 + a]] - x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]
)*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*Sqr
t[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((-Sqrt[-1 -
Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*
EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(
-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqr
t[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] +
Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]
)^2])/(Sqrt[-1 - Sqrt[4 + a]]*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sq
rt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] +
Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[a - x*(-8
+ 8*x - 4*x^2 + x^3)])
```

3.783.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2458, 1417, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-x^4+4x^3-8x^2+8x}}\,dx$$
$$\Big\downarrow \; 2458$$
$$\int \frac{1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}d(x-1)$$

$$\Big\downarrow \ 1417$$

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int \frac{1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \ 320$$

$$\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)\operatorname{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

input
```
Int[1/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output
```
(Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan
[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sq
rt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*
Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

3.783.3.1 Defintions of rubi rules used

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 1417
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(b + q
))]/Sqrt[a + b*x^2 + c*x^4])   Int[1/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 +
2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
&& NegQ[c/a]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.783. $\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$

3.783.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. 2(182) = 364.

Time = 0.88 (sec) , antiderivative size = 530, normalized size of antiderivative = 3.68

method	result
default	$-\frac{\left(\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\sqrt{\frac{\left(-\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\left(x-1-\sqrt{-1+\sqrt{a+4}}\right)}{\left(-\sqrt{-1-\sqrt{a+4}}-\sqrt{-1+\sqrt{a+4}}\right)\left(x-1+\sqrt{-1+\sqrt{a+4}}\right)}}\left(x-1+\sqrt{-1+\sqrt{a+4}}\right)^{2}\sqrt{-\frac{2\sqrt{-1+\sqrt{a+4}}}{\left(\sqrt{-1-\sqrt{a+4}}-\sqrt{-1}\right.}}}{\left(-\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\sqrt{-}}$
elliptic	$-\frac{\left(\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\sqrt{\frac{\left(-\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\left(x-1-\sqrt{-1+\sqrt{a+4}}\right)}{\left(-\sqrt{-1-\sqrt{a+4}}-\sqrt{-1+\sqrt{a+4}}\right)\left(x-1+\sqrt{-1+\sqrt{a+4}}\right)}}\left(x-1+\sqrt{-1+\sqrt{a+4}}\right)^{2}\sqrt{-\frac{2\sqrt{-1+\sqrt{a+4}}}{\left(\sqrt{-1-\sqrt{a+4}}-\sqrt{-1}\right.}}}{\left(-\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\sqrt{-}}$

input
```
int(1/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)
+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(
1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+
(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/
2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^
(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-
1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))
^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(
1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-
(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(
a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(
-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2))
)^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2)
)^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^
(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))
```

3.783.5 Fricas [F]

$$\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{1}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)/(x^4 - 4*x^3 + 8*x^2 - a -
8*x), x)
```

3.783.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx=\int \frac{1}{\sqrt{a-x^4+4x^3-8x^2+8x}}\,dx$$

input
```
integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

output
```
Integral(1/sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

3.783.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx=\int \frac{1}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

3.783.8 Giac [F]

$$\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx=\int \frac{1}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

3.783.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx=\int \frac{1}{\sqrt{-x^4+4\,x^3-8\,x^2+8\,x+a}}\,dx$$

input
```
int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
```

output
```
int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

3.784 $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx$

3.784.1 Optimal result

Integrand size = 24, antiderivative size = 437

$$\begin{aligned}\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx &= \frac{(5+a+(-1+x)^2)\,(-1+x)}{2\,(12+7a+a^2)\,\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\ &- \frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{2(3+a)(4+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\ &+ \frac{\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)E\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{2(3+a)(4+a)\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\ &+ \frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\text{EllipticF}\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right),-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{2(4+a)\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\end{aligned}$$

output
```
1/2*(5+a+(-1+x)^2)*(-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)-1/2
*(-1+x)*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(a^2+7*a+12)/(3+a-2*(
-1+x)^2-(-1+x)^4)^(1/2)+1/2*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+
x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+
x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1
+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^
4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
+1/2*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(
1/2)*EllipticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(
1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*
(1-(4+a)^(1/2))*(1+(4+a)^(1/2))^(1/2)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^
4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

3.784.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3526 vs. $2(437) = 874$.

Time = 16.07 (sec) , antiderivative size = 3526, normalized size of antiderivative = 8.07

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3/2),x]
```

output
```
((6 + a - 8*x - a*x + 3*x^2 - x^3)*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4])/(2
*(3 + a)*(4 + a)*(-a - 8*x + 8*x^2 - 4*x^3 + x^4)) + ((4*(-Sqrt[-1 - Sqrt[
4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt
[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[
4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqr
t[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sq
rt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 -
Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 +
 Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1
 - Sqrt[-1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4
 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt
[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]]
+ x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sq
rt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
 Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt
[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt
[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (2*a*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1
 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqr
t[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((S
qrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 ...
```

3.784.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2458, 1405, 27, 1514, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-x^4+4x^3-8x^2+8x)^{3/2}}\,dx$$

$$\Big\downarrow \text{2458}$$

$$\int \frac{1}{(a-(x-1)^4-2(x-1)^2+3)^{3/2}}d(x-1)$$

$$\Big\downarrow \text{1405}$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\int -\frac{2(-(x-1)^2+a+3)}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)}{4\left(a^2+7a+12\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{-(x-1)^2+a+3}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)}{2\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \ 1514$$

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int \frac{-(x-1)^2+a+3}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \ 406$$

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((a+3)\int \frac{1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)-\int \frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \ 320$$

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-\int \frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\right.}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \ 388$$

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\left(1-\sqrt{a+4}\right)\int \frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}}{\left(\frac{(x-1)^2}{\sqrt{a+4}+1}+1\right)^{3/2}}d(x-1)+\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\right.\right.}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\right.}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

3.784. $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx$

$$\Big\downarrow 313$$

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}+\frac{(1-\sqrt{a+4})\sqrt{\sqrt{a+4}+1}\sqrt{\ldots}}{\sqrt{\ldots}}\ldots\right.}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}\ldots}$$
$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

input
```
Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3/2),x]
```

output
```
((5 + a + (-1 + x)^2)*(-1 + x))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 + x
)^2 - (-1 + x)^4]) + (Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*Sqrt[1 + (-1
+ x)^2/(1 + Sqrt[4 + a])]*(-(((1 - Sqrt[4 + a])*Sqrt[1 + (-1 + x)^2/(1 - S
qrt[4 + a])]*(-1 + x))/Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])]) + ((1 - Sqr
t[4 + a])*Sqrt[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*Ell
ipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[
4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sq
rt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])]) + ((3 + a)*Sqrt[1 + S
qrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*EllipticF[ArcTan[(-1 +
x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 +
 (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[1
+ (-1 + x)^2/(1 + Sqrt[4 + a])])))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1
+ x)^2 - (-1 + x)^4])
```

3.784.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

3.784. $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx$

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.784. $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}} dx$

3.784.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2600 vs. $2(495) = 990$.

Time = 0.93 (sec) , antiderivative size = 2601, normalized size of antiderivative = 5.95

method	result	size
default	Expression too large to display	2601
elliptic	Expression too large to display	2601

input
```
int(1/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(1/4/(a^2+7*a+12)*x^3-3/4/(a^2+7*a+12)*x^2+1/4*(a+8)/(a^2+7*a+12)*x-1/4*
(6+a)/(a^2+7*a+12))/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-((a+5)/(a^2+7*a+12)-1/2
*(a+8)/(a^2+7*a+12))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-
1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/
(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/
2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-
(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x
-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4
)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+
(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)
)/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/
2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2
)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+
4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1
+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/
2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/
2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))
)^(1/2))-1/(a^2+7*a+12)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((
-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2
))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2...
```

3.784.5 Fricas [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)/(x^8 - 8*x^7 + 32*x^6 - 2*(a
 - 64)*x^4 - 80*x^5 + 8*(a - 16)*x^3 - 16*(a - 4)*x^2 + a^2 + 16*a*x), x)
```

3.784.6 Sympy [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{1}{(a-x^4+4x^3-8x^2+8x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

output
```
Integral((a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(-3/2), x)
```

3.784.7 Maxima [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-3/2), x)
```

3.784.8 Giac [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-3/2), x)
```

3.784.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{1}{(-x^4+4\,x^3-8\,x^2+8\,x+a)^{3/2}}\,dx$$

input
```
int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
```

output
```
int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

3.785 $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}} dx$

3.785.1 Optimal result

Integrand size = 24, antiderivative size = 517

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}} dx = \frac{(5+a+(-1+x)^2)(-1+x)}{6(12+7a+a^2)(3+a-2(-1+x)^2-(-1+x)^4)^{3/2}} + \frac{(104+47a+5a^2+4(7+2a)(-1+x)^2)(-1+x)}{12(3+a)^2(4+a)^2\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} - \frac{(7+2a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{3(3+a)^2(4+a)^2\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{(7+2a)\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)E\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)\middle|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{3(3+a)^2(4+a)^2\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{(16+5a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\text{EllipticF}\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right),-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{12(3+a)(4+a)^2\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

output
```
1/6*(5+a+(-1+x)^2)*(-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)+1/1
2*(104+47*a+5*a^2+4*(7+2*a)*(-1+x)^2)*(-1+x)/(a^2+7*a+12)^2/(3+a-2*(-1+x)^
2-(-1+x)^4)^(1/2)-1/3*(7+2*a)*(-1+x)*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)
^(1/2))/(a^2+7*a+12)^2/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/12*(16+5*a)*(1/(1
+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*Ellip
ticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(
4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)^(
1/2))^(1/2)/(3+a)/(4+a)^2/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-
(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)+1/3*(7+2*a)*(1/(1+(-1+x)
^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((-
1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1
/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(
1+(4+a)^(1/2))^(1/2)/(a^2+7*a+12)^2/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-
1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

3.785.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 6386 vs. $2(517) = 1034$.

Time = 16.14 (sec) , antiderivative size = 6386, normalized size of antiderivative = 12.35

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}} \, dx = \text{Result too large to show}$$

input
```
Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-5/2),x]
```

output
```
Result too large to show
```

3.785.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 597, normalized size of antiderivative = 1.15, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2458, 1405, 27, 1492, 27, 1514, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-x^4+4x^3-8x^2+8x)^{5/2}} \, dx$$

$$\Big\downarrow \ 2458$$

$$\int \frac{1}{\left(a-(x-1)^4-2(x-1)^2+3\right)^{5/2}}d(x-1)$$

$$\Big\downarrow \ 1405$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}-\frac{\int -\frac{2\left(3(x-1)^2+5a+19\right)}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}}d(x-1)}{12\left(a^2+7a+12\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{3(x-1)^2+5a+19}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}}d(x-1)}{6\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \ 1492$$

$$\frac{\frac{(x-1)\left(5a^2+4(2a+7)(x-1)^2+47a+104\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\int -\frac{2\left((a+3)(5a+16)-4(2a+7)(x-1)^2\right)}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)}{4\left(a^2+7a+12\right)}}{6\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{(a+3)(5a+16)-4(2a+7)(x-1)^2}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)}{2\left(a^2+7a+12\right)}+\frac{(x-1)\left(5a^2+4(2a+7)(x-1)^2+47a+104\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}}{6\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \ 1514$$

$$\frac{\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int \frac{(a+3)(5a+16)-4(2a+7)(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(5a^2+4(2a+7)(x-1)^2+47a+104\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}}{6\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \ 406$$

3.785. $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

$$\frac{\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((a+3)(5a+16)\int\frac{1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)-4(2a+7)\int\frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)\right)}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)(5a^2-}{2(a^2+7a+1}}{6\left(a^2+7a+12\right)}$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$\downarrow$ 320

$$\frac{\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-4(2a+7)\int\frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\right.}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}}{6\left(a^2+7a+12\right)}$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$\downarrow$ 388

$$\frac{\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-4(2a+7)\left(\frac{(1-\sqrt{a+4})(x-1)\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}}}{\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\right.\right.}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}}{6\left(a^2+7a+12\right)}$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$\downarrow$ 313

$$\frac{\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-4(2a+7)\left(\frac{(1-\sqrt{a+4})(x-1)\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}}}{\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\right.\right.}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}}{6\left(a^2+7a+12\right)}$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

3.785. $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

input
```
Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-5/2),x]
```

output
```
((5 + a + (-1 + x)^2)*(-1 + x))/(6*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2
- (-1 + x)^4)^(3/2)) + (((104 + 47*a + 5*a^2 + 4*(7 + 2*a)*(-1 + x)^2)*(-1
 + x))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sqr
t[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])]
*(-4*(7 + 2*a)*(((1 - Sqrt[4 + a])*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*
(-1 + x))/Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])] - ((1 - Sqrt[4 + a])*Sqrt
[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*EllipticE[ArcTan[
(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqr
t[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*S
qrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])])) + ((3 + a)*(16 + 5*a)*Sqrt[1 + Sqr
t[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*EllipticF[ArcTan[(-1 + x)
/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (
-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[1 +
(-1 + x)^2/(1 + Sqrt[4 + a])])))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 +
x)^2 - (-1 + x)^4]))/(6*(12 + 7*a + a^2))
```

3.785.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.785. $\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

3.785.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2756 vs. $2(571) = 1142$.

Time = 0.96 (sec) , antiderivative size = 2757, normalized size of antiderivative = 5.33

method	result	size
default	Expression too large to display	2757
elliptic	Expression too large to display	2757

input
```
int(1/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(1/6/(a^2+7*a+12)*x^3-1/2/(a^2+7*a+12)*x^2+1/6*(a+8)/(a^2+7*a+12)*x-1/6*(6
+a)/(a^2+7*a+12))*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)/(x^4-4*x^3+8*x^2-a-8*x)^2
+2*(1/6*(7+2*a)/(a^2+7*a+12)^2*x^3-1/2*(7+2*a)/(a^2+7*a+12)^2*x^2+1/24*(5*
a^2+71*a+188)/(a^2+7*a+12)^2*x-1/24*(5*a^2+55*a+132)/(a^2+7*a+12)^2)/(-x^4
+4*x^3-8*x^2+a+8*x)^(1/2)-(1/6*(5*a^2+47*a+104)/(a^2+7*a+12)^2-1/12*(5*a^2
+71*a+188)/(a^2+7*a+12)^2)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))
*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(
1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2
))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)
*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/
2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-
1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-
1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))
^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+
4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2))
)^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(
-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x
-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2
))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2
))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2...
```

3.785.5 Fricas [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx=\int \frac{1}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)/(x^12 - 12*x^11 + 72*x^10 -
 3*(a - 256)*x^8 - 280*x^9 + 24*(a - 64)*x^7 - 32*(3*a - 70)*x^6 + 48*(5*a
 - 48)*x^5 + 3*(a^2 - 128*a + 512)*x^4 - 4*(3*a^2 - 96*a + 128)*x^3 - a^3
- 24*a^2*x + 24*(a^2 - 8*a)*x^2), x)
```

3.785.6 Sympy [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx=\int \frac{1}{(a-x^4+4x^3-8x^2+8x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**(5/2),x)
```

output
```
Integral((a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(-5/2), x)
```

3.785.7 Maxima [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx=\int \frac{1}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-5/2), x)
```

3.785.8 Giac [F]

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{1}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-5/2), x)
```

3.785.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{1}{(-x^4+4\,x^3-8\,x^2+8\,x+a)^{5/2}}\,dx$$

input
```
int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x)
```

output
```
int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2), x)
```

3.786 $\int x(a+8x-8x^2+4x^3-x^4)^{3/2}\,dx$

3.786.1 Optimal result

Integrand size = 26, antiderivative size = 558

$$\int x(a+8x-8x^2+4x^3-x^4)^{3/2}\,dx = \frac{3}{16}(4+a)\left(1+(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4} + \frac{1}{8}\left(1+(-1+x)^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2} - \frac{16(7+2a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{35\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{2}{35}\left(13+5a-3(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x) + \frac{1}{7}\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x) + \frac{3}{16}(4+a)^2\arctan\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\right) + \frac{1}{}$$

output
```
1/8*(1+(-1+x)^2)*(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)+1/7*(3+a-2*(-1+x)^2-(-1+x
)^4)^(3/2)*(-1+x)+3/16*(4+a)^2*arctan((1+(-1+x)^2)/(3+a-2*(-1+x)^2-(-1+x)^
4)^(1/2))-16/35*(7+2*a)*(-1+x)*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2)
)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+3/16*(4+a)*(1+(-1+x)^2)*(3+a-2*(-1+x)^2-
(-1+x)^4)^(1/2)+2/35*(13+5*a-3*(-1+x)^2)*(-1+x)*(3+a-2*(-1+x)^2-(-1+x)^4)^
(1/2)+4/35*(3+a)*(16+5*a)*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)
^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)
^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x
)^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)
/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)+16/35*(
7+2*a)*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))
^(1/2)*EllipticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))
^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2))
)*(1-(4+a)^(1/2))*(1+(4+a)^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((
1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

3.786.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 7235 vs. $2(558) = 1116$.

Time = 17.10 (sec) , antiderivative size = 7235, normalized size of antiderivative = 12.97

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx = \text{Result too large to show}$$

input
```
Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

output
```
Result too large to show
```

3.786.3 Rubi [A] (warning: unable to verify)

Time = 0.84 (sec) , antiderivative size = 635, normalized size of antiderivative = 1.14, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {2459, 2202, 1404, 27, 1432, 1087, 1087, 1092, 217, 1490, 27, 1514, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a-x^4+4x^3-8x^2+8x\right)^{3/2}dx$$

$$\Big\downarrow \text{ 2459}$$

$$\int x\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}d(x-1)$$

$$\Big\downarrow \text{ 2202}$$

$$\int \left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}d(x-1)+\int \left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}(x-1)d(x-1)$$

$$\Big\downarrow \text{ 1404}$$

$$\frac{3}{7}\int 2\left(-(x-1)^2+a+3\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\int \left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}(x-1)d(x-1)+\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{6}{7}\int \left(-(x-1)^2+a+3\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\int \left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}(x-1)d(x-1)+\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{ 1432}$$

$$\frac{6}{7}\int \left(-(x-1)^2+a+3\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\frac{1}{2}\int \left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}d(x-1)^2+\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{1}{2}\left(\frac{3}{4}(a+4)\int \sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)^2+\frac{1}{4}x\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}\right)+\frac{6}{7}\int \left(-(x-1)^2+a+3\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{1}{2}\left(\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\int \frac{1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)^2+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{4}x\right.$$
$$\frac{6}{7}\int \left(-(x-1)^2+a+3\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

3.786. $\int x\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}dx$

$$\Big\downarrow 1092$$

$$\frac{1}{2}\left(\frac{3}{4}(a+4)\left((a+4)\int \frac{1}{-(x-1)^4-4}d\left(-\frac{2x}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2\cdot\right.\right.$$
$$\frac{6}{7}\int \left(-(x-1)^2+a+3\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\frac{1}{7}(x-$$
$$1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow 217$$

$$\frac{6}{7}\int \left(-(x-1)^2+a+3\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+$$
$$\frac{1}{2}\left(\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{4}x(a\right.$$
$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow 1490$$

$$\frac{6}{7}\left(\frac{1}{15}(x-1)\left(5a-3(x-1)^2+13\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}-\frac{1}{15}\int -\frac{2\big((a+3)(5a+16)-4(2a+7)(x-\right.}{\sqrt{-(x-1)^4-2(x-1)^2+a+}}$$
$$\frac{1}{2}\left(\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{4}x(a\right.$$
$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow 27$$

$$\frac{6}{7}\left(\frac{2}{15}\int \frac{(a+3)(5a+16)-4(2a+7)(x-1)^2}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\frac{1}{15}(x-1)\left(5a-3(x-1)^2+13\right)\sqrt{a-(x-1)^4-2(x\cdot\right.}$$
$$\frac{1}{2}\left(\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{4}x(a\right.$$
$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow 1514$$

3.786. $\int x\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx$

$$\frac{6}{7}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int\frac{(a+3)(5a+16)-4(2a+7)(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{1}{15}(x-1)\left(5a-3(x-1)^2+13\right)\sqrt{a-(x\right.$$

$$\frac{1}{2}\left(\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{4}x(a\right.$$

$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{406}$$

$$\frac{6}{7}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((a+3)(5a+16)\int\frac{1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)-4(2a+7)\int\frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\right.\right.}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right.$$

$$\frac{1}{2}\left(\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{4}x(a\right.$$

$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{320}$$

$$\frac{6}{7}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-4(2a+7)\right.}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right.$$

$$\frac{1}{2}\left(\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{4}x(a\right.$$

$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{388}$$

3.786. $\int x\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx$

$$\frac{6}{7}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-4(2a+7)\right.}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right.$$

$$\frac{1}{2}\left(\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{4}x(a\right.$$

$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow 313$$

$$\frac{1}{2}\left(\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{4}x(a\right.$$

$$\frac{6}{7}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-4(2a+7)\right.}{15\sqrt{a-(x-1)^4-2(x-}}\right.$$

$$\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

input
```
Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

3.786. $\int x\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx$

output
```
((3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/7 + (((3 + a - 2*(-1
+ x)^2 - (-1 + x)^4)^(3/2)*x)/4 + (3*(4 + a)*((Sqrt[3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4]*x)/2 + ((4 + a)*ArcTan[x/Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)
^4]])/2))/4)/2 + (6*(((13 + 5*a - 3*(-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2
- (-1 + x)^4]*(-1 + x))/15 + (2*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*Sqr
t[1 + (-1 + x)^2/(1 + Sqrt[4 + a])]*(-4*(7 + 2*a)*(((1 - Sqrt[4 + a])*Sqrt
[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*(-1 + x))/Sqrt[1 + (-1 + x)^2/(1 + Sqrt
[4 + a])] - ((1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(
1 - Sqrt[4 + a])]*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sq
rt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1
 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])]))
 + ((3 + a)*(16 + 5*a)*Sqrt[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt
[4 + a])]*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a
])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 +
 x)^2/(1 + Sqrt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])])))/(15*Sq
rt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]))/7
```

3.786.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

3.786. $\int x(a+8x-8x^2+4x^3-x^4)^{3/2}\,dx$

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

3.786. $\int x(a+8x-8x^2+4x^3-x^4)^{3/2}\,dx$

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

3.786.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2693 vs. $2(600) = 1200$.

Time = 4.90 (sec) , antiderivative size = 2694, normalized size of antiderivative = 4.83

method	result	size
default	Expression too large to display	2694
elliptic	Expression too large to display	2694
risch	Expression too large to display	3609

input
```
int(x*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*x^6*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+17/28*x^5*(-x^4+4*x^3-8*x^2+a+8*x)
^(1/2)-43/28*x^4*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+74/35*x^3*(-x^4+4*x^3-8*x^
2+a+8*x)^(1/2)+(5/16*a-9/20)*x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(-11/56*a-
29/70)*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(11/56*a+13/14)*(-x^4+4*x^3-8*x^2+
a+8*x)^(1/2)-(-(-11/56*a-29/70)*a-11/14*a-26/7)*((-1-(a+4)^(1/2))^(1/2)+(-
1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x
-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)
)/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(
-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)
-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)
^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a
+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(
1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2)
)^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-
1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)
^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(
a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))
^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1
/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1
/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-(a^2-2*(5/16*a-9/20)*a+55/14*a+62/5...
```

3.786.5 Fricas [F]

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int \left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(-(x^5 - 4*x^4 + 8*x^3 - a*x - 8*x^2)*sqrt(-x^4 + 4*x^3 - 8*x^2 +
a + 8*x), x)
```

3.786.6 Sympy [F]

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int x\left(a-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

output
```
Integral(x*(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

3.786.7 Maxima [F]

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int\left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x, x)
```

3.786.8 Giac [F]

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int\left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x, x)
```

3.786.9 Mupad [F(-1)]

Timed out.

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\int x\,\left(-x^4+4\,x^3-8\,x^2+8\,x+a\right)^{3/2}\,dx$$

input
```
int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
```

output
```
int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

3.787 $\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

3.787.1 Optimal result

Integrand size = 26, antiderivative size = 466

$$\begin{aligned}
&\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx\\
&=\frac{1}{4}\left(1+(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}\\
&\quad-\frac{2\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{3\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}+\frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x)\\
&\quad+\frac{1}{4}(4+a)\arctan\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\right)\\
&\quad+\frac{2\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)E\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{3\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\\
&\quad+\frac{2(3+a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\operatorname{EllipticF}\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right),-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{3\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}
\end{aligned}$$

output
```
1/4*(4+a)*arctan((1+(-1+x)^2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2))-2/3*(-1+x)*
(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/
2)+1/4*(1+(-1+x)^2)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/3*(-1+x)*(3+a-2*(-1+
x)^2-(-1+x)^4)^(1/2)+2/3*(3+a)*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(
-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(
-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+
(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^
(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)+2/
3*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2
)*EllipticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2
),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-
(4+a)^(1/2))*(1+(4+a)^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1
+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

3.787.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 4389 vs. $2(466) = 932$.

Time = 14.89 (sec) , antiderivative size = 4389, normalized size of antiderivative = 9.42

$$\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx = \text{Result too large to show}$$

input
```
Integrate[x*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output
```
(1/6 - x/6 + x^2/4)*Sqrt[a - x*(-8 + 8*x - 4*x^2 + x^3)] + (Sqrt[a - x*(-8
 + 8*x - 4*x^2 + x^3)]*((-8*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 +
a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] +
Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqr
t[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sq
rt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 -
Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]
*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1
 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x
))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a
]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
+ Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sqrt[-1 - Sqrt[4
+ a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-Sqrt[-1 - Sq
rt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1
- Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^
4]) + (2*a*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-
1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
+ a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[
-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - ...
```

3.787.3 Rubi [A] (warning: unable to verify)

Time = 0.75 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.17, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2459, 2202, 1404, 27, 1432, 1087, 1092, 217, 1514, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a-x^4+4x^3-8x^2+8x}\,dx$$

$$\Big\downarrow \text{2459}$$

$$\int x\sqrt{a-(x-1)^4-2(x-1)^2+3}d(x-1)$$

$$\Big\downarrow \text{2202}$$

$$\int \sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\int \sqrt{-(x-1)^4-2(x-1)^2+a+3}(x-1)d(x-1)$$

$$\Big\downarrow \text{1404}$$

$$\frac{1}{3}\int \frac{2(-(x-1)^2+a+3)}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\int \sqrt{-(x-1)^4-2(x-1)^2+a+3}(x-1)d(x-1)+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{3}\int \frac{-(x-1)^2+a+3}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\int \sqrt{-(x-1)^4-2(x-1)^2+a+3}(x-1)d(x-1)+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{1432}$$

$$\frac{2}{3}\int \frac{-(x-1)^2+a+3}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\frac{1}{2}\int \sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)^2+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}\left(\frac{1}{2}(a+4)\int \frac{1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)^2+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{2}{3}\int \frac{-(x-1)^2+a+3}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{1092}$$

$$\frac{1}{2}\left((a+4)\int \frac{1}{-(x-1)^4-4}d\left(-\frac{2x}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{2}{3}\int \frac{-(x-1)^2+a+3}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{217}$$

$$\frac{2}{3}\int \frac{-(x-1)^2+a+3}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\frac{1}{2}\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{1514}$$

3.787. $\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int \frac{-(x-1)^2+a+3}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{1}{2}\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$\Big\downarrow$ 406

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((a+3)\int \frac{1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)-\int \frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)\right)}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{1}{2}\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$\Big\downarrow$ 320

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-\int \frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1}{\sqrt{a+4}\cdots}}}\cdots\right.}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$
$$\frac{1}{2}\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$\Big\downarrow$ 388

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((1-\sqrt{a+4})\int \frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}}{\left(\frac{(x-1)^2}{\sqrt{a+4}+1}+1\right)^{3/2}}d(x-1)+\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\cdots\right.\right.}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\cdots}\right.}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$
$$\frac{1}{2}\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

3.787. $\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

$$\Big\downarrow \text{313}$$

$$\frac{1}{2}\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+$$

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}+\frac{(1-\sqrt{a+4})\sqrt{\sqrt{a+4}+1}\sqrt{}}{\sqrt{}}\right.}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

input

```
Int[x*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output

```
(Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/3 + ((Sqrt[3 + a - 2*(-
1 + x)^2 - (-1 + x)^4]*x)/2 + ((4 + a)*ArcTan[x/Sqrt[3 + a - 2*(-1 + x)^2
- (-1 + x)^4]])/2)/2 + (2*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*Sqrt[1 +
(-1 + x)^2/(1 + Sqrt[4 + a])]*(-(((1 - Sqrt[4 + a])*Sqrt[1 + (-1 + x)^2/(1
 - Sqrt[4 + a])]*(-1 + x))/Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])]) + ((1 -
 Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]
*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - S
qrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1
+ Sqrt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])]) + ((3 + a)*Sqrt[1
 + Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*EllipticF[ArcTan[(-
1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[
(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqr
t[1 + (-1 + x)^2/(1 + Sqrt[4 + a])])))/(3*Sqrt[3 + a - 2*(-1 + x)^2 - (-1
+ x)^4])
```

3.787.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1404
```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b
*x^2 + c*x^4)^p/(4*p + 1)), x] + Simp[2*(p/(4*p + 1))   Int[(2*a + b*x^2)*(
a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && GtQ[p, 0] && IntegerQ[2*p]
```

3.787. $\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

3.787.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2550 vs. $2(516) = 1032$.

Time = 4.76 (sec) , antiderivative size = 2551, normalized size of antiderivative = 5.47

method	result	size
default	Expression too large to display	2551
elliptic	Expression too large to display	2551
risch	Expression too large to display	3034

input
```
int(x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-1/6*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2
)+1/6*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(1/6*a-2/3)*((-1-(a+4)^(1/2))^(1/2)+(
-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(
x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2
))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*
(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2
)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4
)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(
a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^
(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2
))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(
-1-(a+4)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4
)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+
(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2)
)^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(
1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(
1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-(1/2*a+10/3)*((-1-(a+4)^(1/2))^(1/2)+
(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*
(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/
2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*...
```

3.787.5 Fricas [F]

$$\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}x\,dx$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x, x)
```

3.787.6 Sympy [F]

$$\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int x\sqrt{a-x^4+4x^3-8x^2+8x}\,dx$$

input
```
integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

output
```
Integral(x*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

3.787.7 Maxima [F]

$$\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}x\,dx$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x, x)
```

3.787.8 Giac [F]

$$\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}x\,dx$$

input
```
integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x, x)
```

3.787.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int x\,\sqrt{-x^4+4\,x^3-8\,x^2+8\,x+a}\,dx$$

input
```
int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
```

output
```
int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

3.788 $\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$

3.788.1 Optimal result

Integrand size = 26, antiderivative size = 179

$$\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \frac{1}{2}\arctan\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\right) + \frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\operatorname{EllipticF}\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right),-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

output
```
1/2*arctan((1+(-1+x)^2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2))+(1/(1+(-1+x)^2/(1
+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((-1+x)/
(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(
1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)/
(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/
(1+(4+a)^(1/2))))^(1/2)
```

3.788.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 813 vs. $2(179) = 358$.

Time = 12.96 (sec) , antiderivative size = 813, normalized size of antiderivative = 4.54

$$\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \frac{2\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)\sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(1+\sqrt{-1+\sqrt{4+a}}-x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}}\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\sqrt{\frac{}{(-}}}{}$$

input
```
Integrate[x/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output
```
(2*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(1 + Sqrt
[-1 + Sqrt[4 + a]] - x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]
)*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*Sqr
t[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((-Sqrt[-1 -
Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*
((1 + Sqrt[-1 - Sqrt[4 + a]])*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a
]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1
- Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))
]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4
+ a]] - Sqrt[-1 + Sqrt[4 + a]])^2] - 2*Sqrt[-1 - Sqrt[4 + a]]*EllipticPi[(
Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]]
+ Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1
+ Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]
] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1
- Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[
-1 + Sqrt[4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*Sqrt[((Sqrt[-1 - Sqrt[4 +
a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1
 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)
)]*Sqrt[a - x*(-8 + 8*x - 4*x^2 + x^3)])
```

3.788.3 Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2459, 2202, 1417, 320, 1432, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a-x^4+4x^3-8x^2+8x}}dx$$

$$\Big\downarrow \text{ 2459}$$

$$\int \frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}d(x-1)$$

$$\Big\downarrow \text{ 2202}$$

$$\int \frac{1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\int \frac{x-1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)$$

$$\Big\downarrow \text{ 1417}$$

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int \frac{1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\int \frac{x-1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)$$

$$\Big\downarrow \text{ 320}$$

$$\int \frac{x-1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{ 1432}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)^2+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{ 1092}$$

$$\int \frac{1}{-(x-1)^4-4}d\left(-\frac{2x}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}\right)+$$
$$\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+$$
$$\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

input
```
Int[x/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output
```
ArcTan[x/Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]]/2 + (Sqrt[1 + Sqrt[4 + a
]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + S
qrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(
1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 +
 x)^2 - (-1 + x)^4])
```

3.788.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1417
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(b + q
))]/Sqrt[a + b*x^2 + c*x^4])   Int[1/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 +
2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
&& NegQ[c/a]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

3.788.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 787 vs. $2(213) = 426$.

Time = 1.04 (sec) , antiderivative size = 788, normalized size of antiderivative = 4.40

method	result
default	$-\frac{\left(\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\sqrt{\frac{\left(-\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\left(x-1-\sqrt{-1+\sqrt{a+4}}\right)}{\left(-\sqrt{-1-\sqrt{a+4}}-\sqrt{-1+\sqrt{a+4}}\right)\left(x-1+\sqrt{-1+\sqrt{a+4}}\right)}}\left(x-1+\sqrt{-1+\sqrt{a+4}}\right)^2\sqrt{-\frac{2\sqrt{-1+\sqrt{a+4}}}{\left(\sqrt{-1-\sqrt{a+4}}-\sqrt{-1}\right.}}}{}$
elliptic	$-\frac{\left(\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\sqrt{\frac{\left(-\sqrt{-1-\sqrt{a+4}}+\sqrt{-1+\sqrt{a+4}}\right)\left(x-1-\sqrt{-1+\sqrt{a+4}}\right)}{\left(-\sqrt{-1-\sqrt{a+4}}-\sqrt{-1+\sqrt{a+4}}\right)\left(x-1+\sqrt{-1+\sqrt{a+4}}\right)}}\left(x-1+\sqrt{-1+\sqrt{a+4}}\right)^2\sqrt{-\frac{2\sqrt{-1+\sqrt{a+4}}}{\left(\sqrt{-1-\sqrt{a+4}}-\sqrt{-1}\right.}}}{}$

input
```
int(x/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)
+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(
1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+
(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/
2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^
(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-
1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))
^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(
1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-
(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*((1-(-1+(a+4)^(1/2
))^(1/2))*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1
-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/
(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1
/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1
/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2)
)^(1/2)))^(1/2))+2*(-1+(a+4)^(1/2))^(1/2)*EllipticPi(((-(-1-(a+4)^(1/2))^(
1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2
))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),(-(-1
-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(
a+4)^(1/2))^(1/2)),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((...
```

3.788.5 Fricas [F]

$$\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{x}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x/(x^4 - 4*x^3 + 8*x^2 - a
- 8*x), x)
```

3.788.6 Sympy [F]

$$\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx=\int \frac{x}{\sqrt{a-x^4+4x^3-8x^2+8x}}\,dx$$

input
```
integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

output
```
Integral(x/sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

3.788.7 Maxima [F]

$$\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx=\int \frac{x}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

3.788.8 Giac [F]

$$\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx=\int \frac{x}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

3.788.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{x}{\sqrt{-x^4+4\,x^3-8\,x^2+8\,x+a}}\,dx$$

input
```
int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
```

output
```
int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

3.789 $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx$

3.789.1 Optimal result

Integrand size = 26, antiderivative size = 474

$$\begin{aligned}\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx &= \frac{1+(-1+x)^2}{2(4+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\ &+ \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\ &- \frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{2(3+a)(4+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\ &+ \frac{\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)E\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)\middle|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{2(3+a)(4+a)\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\ &+ \frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\operatorname{EllipticF}\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right),-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{2(4+a)\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\end{aligned}$$

output
```
1/2*(1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/2*(5+a+(-1+x)^2)*
(-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)-1/2*(-1+x)*(1+(-1+x)^2
/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(
1/2)+1/2*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)
))^(1/2)*EllipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)
))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2
)))*(1+(4+a)^(1/2))^(1/2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)
^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)+1/2*(1/(1+(-1+x)^2
/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((-1+
x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2
)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(1+
(4+a)^(1/2))^(1/2)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)
^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

3.789.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3593 vs. $2(474) = 948$.

Time = 14.36 (sec) , antiderivative size = 3593, normalized size of antiderivative = 7.58

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

output
```
((-a - 2*x + a*x - a*x^2 - x^3)*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2)/(2*(3 +
 a)*(4 + a)*(-a - 8*x + 8*x^2 - 4*x^3 + x^4)*(a - x*(-8 + 8*x - 4*x^2 + x^
3))^(3/2)) + ((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2)*((4*(-Sqrt[-1 - Sqrt[4
 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[
((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4
 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt
[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqr
t[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - S
qrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 +
Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1
- Sqrt[-1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4
+ a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[
-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] +
 x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqr
t[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt[
-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt[
a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (2*a*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1
+ Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt
[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/(...
```

3.789.3 Rubi [A] (warning: unable to verify)

Time = 0.69 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {2459, 2202, 1405, 27, 1432, 1088, 1514, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a-x^4+4x^3-8x^2+8x)^{3/2}}\,dx$$

$$\Big\downarrow \; 2459$$

$$\int \frac{x}{(a-(x-1)^4-2(x-1)^2+3)^{3/2}}d(x-1)$$

$$\Big\downarrow \; 2202$$

$$\int \frac{1}{(-(x-1)^4-2(x-1)^2+a+3)^{3/2}}d(x-1)+\int \frac{x-1}{(-(x-1)^4-2(x-1)^2+a+3)^{3/2}}d(x-1)$$

$$\Big\downarrow \; 1405$$

$$-\frac{\int -\frac{2(-(x-1)^2+a+3)}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)}{4\left(a^2+7a+12\right)}+\int \frac{x-1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}}d(x-1)+\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-(x-1)^2+a+3}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)}{2\left(a^2+7a+12\right)}+\int \frac{x-1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}}d(x-1)+\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{1432}$$

$$\frac{\int \frac{-(x-1)^2+a+3}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)}{2\left(a^2+7a+12\right)}+\frac{1}{2}\int \frac{1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}}d(x-1)^2+\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{\int \frac{-(x-1)^2+a+3}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)}{2\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{2(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{1514}$$

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int \frac{-(x-1)^2+a+3}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{2(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{406}$$

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((a+3)\int \frac{1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)-\int \frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{2(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{320}$$

3.789. $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx$

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-\int\frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4+}}}}\right.}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{2(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$\downarrow$ 388

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\left(1-\sqrt{a+4}\right)\int\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}}{\left(\frac{(x-1)^2}{\sqrt{a+4}+1}+1\right)^{3/2}}d(x-1)+\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\text{EllipticF}\left(\arctan\left(\frac{}{\sqrt{}}\right.\right.}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\right.}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{2(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$\downarrow$ 313

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}+\frac{\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\sqrt{}}{\sqrt{}}\right.}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{2(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

input
```
Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

3.789. $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx$

output
```
((5 + a + (-1 + x)^2)*(-1 + x))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 + x
)^2 - (-1 + x)^4]) + x/(2*(4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
 + (Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4
 + a])]*(-(((1 - Sqrt[4 + a])*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*(-1 +
 x))/Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])]) + ((1 - Sqrt[4 + a])*Sqrt[1 +
 Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*EllipticE[ArcTan[(-1
+ x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1
 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[
1 + (-1 + x)^2/(1 + Sqrt[4 + a])]) + ((3 + a)*Sqrt[1 + Sqrt[4 + a]]*Sqrt[1
 + (-1 + x)^2/(1 - Sqrt[4 + a])]*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4
 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - S
qrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 +
Sqrt[4 + a])])))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^
4])
```

3.789.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

3.789. $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx$

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

3.789.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2615 vs. $2(528) = 1056$.

Time = 1.12 (sec) , antiderivative size = 2616, normalized size of antiderivative = 5.52

method	result	size
default	Expression too large to display	2616
elliptic	Expression too large to display	2616

input
```
int(x/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(1/4/(a^2+7*a+12)*x^3+1/4/(a^2+7*a+12)*a*x^2-1/4*(a-2)/(a^2+7*a+12)*x+1/
4/(a^2+7*a+12)*a)/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(2/(a^2+7*a+12)+1/2*(a-2)
/(a^2+7*a+12))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4
)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-
(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(
1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a
+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1
+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2
))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^
(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+
(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1
/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*Elli
pticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/
2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)
^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((
-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1
+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2
))-((1+a)/(a^2+7*a+12)-1/(a^2+7*a+12)*a)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)
^(1/2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+
(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x...
```

3.789.5 Fricas [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{x}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x/(x^8 - 8*x^7 + 32*x^6 - 2*
(a - 64)*x^4 - 80*x^5 + 8*(a - 16)*x^3 - 16*(a - 4)*x^2 + a^2 + 16*a*x), x
)
```

3.789.6 Sympy [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{x}{(a-x^4+4x^3-8x^2+8x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

output
```
Integral(x/(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

3.789.7 Maxima [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{x}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

3.789.8 Giac [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{x}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

3.789.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{x}{(-x^4+4\,x^3-8\,x^2+8\,x+a)^{3/2}}\,dx$$

input
```
int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
```

output
```
int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

3.790 $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

3.790.1 Optimal result

Integrand size = 26, antiderivative size = 591

$$\begin{aligned}
\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx &= \frac{1+(-1+x)^2}{6(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}} \\
&+ \frac{1+(-1+x)^2}{3(4+a)^2\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\
&+ \frac{\left(5+a+(-1+x)^2\right)(-1+x)}{6\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}} \\
&+ \frac{\left(104+47a+5a^2+4(7+2a)(-1+x)^2\right)(-1+x)}{12(3+a)^2(4+a)^2\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\
&- \frac{(7+2a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{3(3+a)^2(4+a)^2\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\
&+ \frac{(7+2a)\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)E\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)\middle|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{3(3+a)^2(4+a)^2\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\
&+ \frac{(16+5a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\operatorname{EllipticF}\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right),-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{12(3+a)(4+a)^2\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}
\end{aligned}$$

output
```
1/6*(1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)+1/6*(5+a+(-1+x)^2)*
(-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)+1/3*(1+(-1+x)^2)/(4+a)
^2/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/12*(104+47*a+5*a^2+4*(7+2*a)*(-1+x)^2
)*(-1+x)/(a^2+7*a+12)^2/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)-1/3*(7+2*a)*(-1+x)
*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(a^2+7*a+12)^2/(3+a-2*(-1+x)
^2-(-1+x)^4)^(1/2)+1/12*(16+5*a)*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1
+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1
+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(
1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)/(3+a)/(4+a)^2/(3+a-2*(-1
+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(
1/2))))^(1/2)+1/3*(7+2*a)*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)
^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)
^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x
)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2))^(1/2)/(a^2+7*a+12)^2/
(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/
(1+(4+a)^(1/2))))^(1/2)
```

3.790.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 6452 vs. 2(591) = 1182.

Time = 17.03 (sec) , antiderivative size = 6452, normalized size of antiderivative = 10.92

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x]
```

output
```
Result too large to show
```

3.790.3 Rubi [A] (warning: unable to verify)

Time = 0.83 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.12, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2459, 2202, 1405, 27, 1432, 1089, 1088, 1492, 27, 1514, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a-x^4+4x^3-8x^2+8x)^{5/2}}\,dx$$

$$\Big\downarrow \text{2459}$$

$$\int \frac{x}{(a-(x-1)^4-2(x-1)^2+3)^{5/2}}d(x-1)$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{1}{(-(x-1)^4-2(x-1)^2+a+3)^{5/2}}d(x-1)+\int \frac{x-1}{(-(x-1)^4-2(x-1)^2+a+3)^{5/2}}d(x-1)$$

$$\Big\downarrow \text{1405}$$

$$-\frac{\int -\frac{2\left(3(x-1)^2+5a+19\right)}{(-(x-1)^4-2(x-1)^2+a+3)^{3/2}}d(x-1)}{12\left(a^2+7a+12\right)}+\int \frac{x-1}{(-(x-1)^4-2(x-1)^2+a+3)^{5/2}}d(x-1)+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)(a-(x-1)^4-2(x-1)^2+3)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3(x-1)^2+5a+19}{(-(x-1)^4-2(x-1)^2+a+3)^{3/2}}d(x-1)}{6\left(a^2+7a+12\right)}+\int \frac{x-1}{(-(x-1)^4-2(x-1)^2+a+3)^{5/2}}d(x-1)+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)(a-(x-1)^4-2(x-1)^2+3)^{3/2}}$$

$$\Big\downarrow \text{1432}$$

$$\frac{\int \frac{3(x-1)^2+5a+19}{(-(x-1)^4-2(x-1)^2+a+3)^{3/2}}d(x-1)}{6\left(a^2+7a+12\right)}+\frac{1}{2}\int \frac{1}{(-(x-1)^4-2(x-1)^2+a+3)^{5/2}}d(x-1)^2+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)(a-(x-1)^4-2(x-1)^2+3)^{3/2}}$$

$$\Big\downarrow \text{1089}$$

$$\frac{\int \frac{3(x-1)^2+5a+19}{(-(x-1)^4-2(x-1)^2+a+3)^{3/2}}d(x-1)}{6\left(a^2+7a+12\right)}+\frac{1}{2}\left(\frac{2\int \frac{1}{(-(x-1)^4-2(x-1)^2+a+3)^{3/2}}d(x-1)^2}{3(a+4)}+\frac{x}{3(a+4)(a-(x-1)^4-2(x-1)^2+3)^{3/2}}\right)+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)(a-(x-1)^4-2(x-1)^2+3)^{3/2}}$$

$$\Big\downarrow \text{1088}$$

3.790. $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

$$\frac{\int \frac{3(x-1)^2+5a+19}{(-(x-1)^4-2(x-1)^2+a+3)^{3/2}}d(x-1)}{6\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{1}{2}\left(\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}\right)$$

$\downarrow$ 1492

$$\frac{\frac{(x-1)\left(5a^2+4(2a+7)(x-1)^2+47a+104\right)}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\int -\frac{2\left((a+3)(5a+16)-4(2a+7)(x-1)^2\right)}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)}{4(a^2+7a+12)}}{6\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{1}{2}\left(\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}\right)$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{(a+3)(5a+16)-4(2a+7)(x-1)^2}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)}{2(a^2+7a+12)}+\frac{(x-1)\left(5a^2+4(2a+7)(x-1)^2+47a+104\right)}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}}{6\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{1}{2}\left(\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}\right)$$

$\downarrow$ 1514

$$\frac{\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int \frac{(a+3)(5a+16)-4(2a+7)(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(5a^2+4(2a+7)(x-1)^2+47a+104\right)}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}}{6\left(a^2+7a+12\right)}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{1}{2}\left(\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}\right)$$

$\downarrow$ 406

3.790. $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

$$\frac{\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((a+3)(5a+16)\int\frac{1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)-4(2a+7)\int\frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)\right)}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)(5a^2-}{2(a^2+7a+1}}{6\left(a^2+7a+12\right)}$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+$$

$$\frac{1}{2}\left(\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}\right)$$

$$\Big\downarrow \text{320}$$

$$\frac{\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-4(2a+7)\int\frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\right.}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}}{6\left(a^2+7a+12\right)}$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+$$

$$\frac{1}{2}\left(\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}\right)$$

$$\Big\downarrow \text{388}$$

$$\frac{\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-4(2a+7)\left(\frac{(1-\sqrt{a+4})(x-1)\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+}}{\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\right.\right.}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}}{6\left(a^2+7a+12\right)}$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+$$

$$\frac{1}{2}\left(\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}\right)$$

$$\Big\downarrow \text{313}$$

3.790. $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

$$\frac{\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-4(2a+7)\left(\frac{(1-\sqrt{a+4})(x-1)\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}\cdots}}{\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\cdots\right.\right.}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}}{6\,(a^2+7a+12)}$$

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\,(a^2+7a+12)\,(a-(x-1)^4-2(x-1)^2+3)^{3/2}}+$$
$$\frac{1}{2}\left(\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\,(a-(x-1)^4-2(x-1)^2+3)^{3/2}}\right)$$

input
```
Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x]
```

output
```
((5 + a + (-1 + x)^2)*(-1 + x))/(6*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2
- (-1 + x)^4)^(3/2)) + (x/(3*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(
3/2)) + (2*x)/(3*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]))/2 + (
((104 + 47*a + 5*a^2 + 4*(7 + 2*a)*(-1 + x)^2)*(-1 + x))/(2*(12 + 7*a + a^
2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sqrt[1 + (-1 + x)^2/(1 - Sq
rt[4 + a])]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])]*(-4*(7 + 2*a)*(((1 - Sq
rt[4 + a])*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*(-1 + x))/Sqrt[1 + (-1 +
 x)^2/(1 + Sqrt[4 + a])] - ((1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*Sqrt[1
 + (-1 + x)^2/(1 - Sqrt[4 + a])]*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4
 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - S
qrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 +
Sqrt[4 + a])])) + ((3 + a)*(16 + 5*a)*Sqrt[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 +
 x)^2/(1 - Sqrt[4 + a])]*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]],
 (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 +
a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 +
 a])])))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]))/(6*
(12 + 7*a + a^2))
```

3.790. $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

3.790.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.790. $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}} \, dx$

rule 1405
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(b^2 - 2*a*c + 2*(p + 1)*(
b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fr
eeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

3.790. $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

3.790.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2776 vs. $2(637) = 1274$.

Time = 1.01 (sec) , antiderivative size = 2777, normalized size of antiderivative = 4.70

method	result	size
default	Expression too large to display	2777
elliptic	Expression too large to display	2777

input
```
int(x/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(1/6/(a^2+7*a+12)*x^3+1/6/(a^2+7*a+12)*a*x^2-1/6*(a-2)/(a^2+7*a+12)*x+1/6/
(a^2+7*a+12)*a)*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)/(x^4-4*x^3+8*x^2-a-8*x)^2+2
*(1/6*(7+2*a)/(a^2+7*a+12)^2*x^3+1/6*(a^2-12)/(a^2+7*a+12)^2*x^2-1/24*(3*a
^2-23*a-116)/(a^2+7*a+12)^2*x+1/24*(3*a^2-7*a-60)/(a^2+7*a+12)^2)/(-x^4+4*
x^3-8*x^2+a+8*x)^(1/2)-(1/6*(a^2+23*a+68)/(a^2+7*a+12)^2+1/12*(3*a^2-23*a-
116)/(a^2+7*a+12)^2)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-
1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/
(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/
2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-
(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x
-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4
)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+
(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2)
)/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/
2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2
)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+
4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1
+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/
2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/
2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/...
```

3.790.5 Fricas [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{x}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x/(x^12 - 12*x^11 + 72*x^10
 - 3*(a - 256)*x^8 - 280*x^9 + 24*(a - 64)*x^7 - 32*(3*a - 70)*x^6 + 48*(5
*a - 48)*x^5 + 3*(a^2 - 128*a + 512)*x^4 - 4*(3*a^2 - 96*a + 128)*x^3 - a^
3 - 24*a^2*x + 24*(a^2 - 8*a)*x^2), x)
```

3.790.6 Sympy [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{x}{(a-x^4+4x^3-8x^2+8x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**(5/2),x)
```

output
```
Integral(x/(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(5/2), x)
```

3.790.7 Maxima [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{x}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2), x)
```

3.790.8 Giac [F]

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx=\int \frac{x}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2), x)
```

3.790.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx=\int \frac{x}{(-x^4+4\,x^3-8\,x^2+8\,x+a)^{5/2}}\,dx$$

input
```
int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x)
```

output
```
int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2), x)
```

3.791 $\int x^2(a+8x-8x^2+4x^3-x^4)^{3/2}\,dx$

3.791.1 Optimal result

Integrand size = 28, antiderivative size = 585

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}dx=\frac{3}{8}(4+a)\left(1+(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}+\frac{1}{4}\left(1+(-1+x)^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}+\frac{4(140+111a+21a^2)\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{315\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}+\frac{2}{315}\left(2(80+27a)+3(20+7a)(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x)+\frac{1}{63}\left(15+7(-1+x)^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)+\frac{3}{8}(4+a)^2\arctan\left(\frac{1+(-1+x)}{\sqrt{3+a-2(-1+x)^2-}}\right.$$

output
```
1/4*(1+(-1+x)^2)*(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)+1/63*(15+7*(-1+x)^2)*(3+a
-2*(-1+x)^2-(-1+x)^4)^(3/2)*(-1+x)+3/8*(4+a)^2*arctan((1+(-1+x)^2)/(3+a-2*
(-1+x)^2-(-1+x)^4)^(1/2))+4/315*(21*a^2+111*a+140)*(-1+x)*(1+(-1+x)^2/(1-(
4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+3/8*(4+a)*(1+
(-1+x)^2)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+2/315*(160+54*a+3*(20+7*a)*(-1+x
)^2)*(-1+x)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+4/315*(3+a)*(100+33*a)*(1/(1+(
-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*Ellipti
cF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+
a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/
2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1
+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)-4/315*(21*a^2+111*a+140)*(1/(1+(-1+x)^2/
(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((-1+x
)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)
/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(1+(
4+a)^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1
/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

3.791.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 8500 vs. $2(585) = 1170$.

Time = 17.05 (sec) , antiderivative size = 8500, normalized size of antiderivative = 14.53

$$\int x^2 \left(a + 8x - 8x^2 + 4x^3 - x^4\right)^{3/2} dx = \text{Result too large to show}$$

input
```
Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

output
```
Result too large to show
```

3.791.3 Rubi [A] (warning: unable to verify)

Time = 0.92 (sec) , antiderivative size = 651, normalized size of antiderivative = 1.11, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {2459, 2006, 2202, 27, 1432, 1087, 1087, 1092, 217, 1490, 27, 1490, 27, 1514, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a - x^4 + 4x^3 - 8x^2 + 8x\right)^{3/2} dx$$

$$\Big\downarrow \text{ 2459}$$

$$\int \left((x-1)^2 + 2(x-1) + 1\right) \left(a - (x-1)^4 - 2(x-1)^2 + 3\right)^{3/2} d(x-1)$$

$$\Big\downarrow \text{ 2006}$$

$$\int x^2 \left(a - (x-1)^4 - 2(x-1)^2 + 3\right)^{3/2} d(x-1)$$

$$\Big\downarrow \text{ 2202}$$

$$\int \left((x-1)^2 + 1\right) \left(-(x-1)^4 - 2(x-1)^2 + a + 3\right)^{3/2} d(x-1) + \int 2\left(-(x-1)^4 - 2(x-1)^2 + a + 3\right)^{3/2} (x-1)d(x-1)$$

$$\Big\downarrow \text{ 27}$$

$$\int \left((x-1)^2 + 1\right) \left(-(x-1)^4 - 2(x-1)^2 + a + 3\right)^{3/2} d(x-1) + 2\int \left(-(x-1)^4 - 2(x-1)^2 + a + 3\right)^{3/2} (x-1)d(x-1)$$

$$\Big\downarrow \text{ 1432}$$

$$\int \left(-(x-1)^4 - 2(x-1)^2 + a + 3\right)^{3/2} d(x-1)^2 + \int \left((x-1)^2 + 1\right) \left(-(x-1)^4 - 2(x-1)^2 + a + 3\right)^{3/2} d(x-1)$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{3}{4}(a+4)\int \sqrt{-(x-1)^4 - 2(x-1)^2 + a + 3}d(x-1)^2 + \int \left((x-1)^2 + 1\right) \left(-(x-1)^4 - 2(x-1)^2 + a + 3\right)^{3/2} d(x-1) + \frac{1}{4}x\left(a - (x-1)^4 - 2(x-1)^2 + 3\right)^{3/2}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\int \frac{1}{\sqrt{-(x-1)^4 - 2(x-1)^2 + a + 3}}d(x-1)^2 + \frac{1}{2}x\sqrt{a - (x-1)^4 - 2(x-1)^2 + 3}\right) + \int \left((x-1)^2 + 1\right) \left(-(x-1)^4 - 2(x-1)^2 + a + 3\right)^{3/2} d(x-1) + \frac{1}{4}x\left(a - (x-1)^4 - 2(x-1)^2 + 3\right)^{3/2}$$

3.791. $\int x^2(a + 8x - 8x^2 + 4x^3 - x^4)^{3/2} \, dx$

$$\Big\downarrow \text{1092}$$

$$\frac{3}{4}(a+4)\left((a+4)\int \frac{1}{-(x-1)^4-4}d\left(-\frac{2x}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\int \left((x-1)^2+1\right)\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}d(x-1)+\frac{1}{4}x\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{217}$$

$$\int \left((x-1)^2+1\right)\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}d(x-1)+\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{4}x\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{1490}$$

$$-\frac{1}{21}\int -2\left((7a+20)(x-1)^2+8(a+3)\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{63}\left(7(x-1)^2+15\right)(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{1}{4}x\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{21}\int \left((7a+20)(x-1)^2+8(a+3)\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{63}\left(7(x-1)^2+15\right)(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{1}{4}x\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{1490}$$

$$\frac{2}{21}\left(\frac{1}{15}(x-1)\left(3(7a+20)(x-1)^2+2(27a+80)\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}-\frac{1}{15}\int -\frac{2\left(\left(21a^2+111a+14\right.\right.}{\sqrt{-(x-}}\right.$$

$$\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{63}\left(7(x-1)^2+15\right)(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{1}{4}x\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

3.791. $\int x^2(a+8x-8x^2+4x^3-x^4)^{3/2}\,dx$

$$\Big\downarrow \text{27}$$

$$\frac{2}{21}\left(\frac{2}{15}\int \frac{\left(21a^2+111a+140\right)(x-1)^2+(a+3)(33a+100)}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\frac{1}{15}(x-1)\left(3(7a+20)(x-1)^2+2(27a\right.\right.$$

$$\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{63}\left(7(x-1)^2+15\right)(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{1}{4}x\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{1514}$$

$$\frac{2}{21}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int \frac{(21a^2+111a+140)(x-1)^2+(a+3)(33a+100)}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{1}{15}(x-1)\left(3(7a+20)(x-1)^2\right.\right.$$

$$\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{63}\left(7(x-1)^2+15\right)(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{1}{4}x\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{406}$$

$$\frac{2}{21}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\left(21a^2+111a+140\right)\int \frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)+(a+3)(33a+100)\int\right.}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right.$$

$$\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+\frac{1}{63}\left(7(x-1)^2+15\right)(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{1}{4}x\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \text{320}$$

3.791. $\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx$

$$\frac{2}{21}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\left(21a^2+111a+140\right)\int\frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)+\frac{(a+3)(33a+100)\sqrt{\sqrt{a+4}\cdots}}{\cdots}\right.\right.}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right.$$

$$\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+$$
$$\frac{1}{63}\left(7(x-1)^2+15\right)(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{1}{4}x\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \; 388$$

$$\frac{2}{21}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\left(21a^2+111a+140\right)\left(\frac{\left(1-\sqrt{a+4}\right)(x-1)\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}}{\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-\left(1-\sqrt{a+4}\right)\int\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}\cdots}}{\left(\frac{(x-1)^2}{\sqrt{a+4}+1}+1\cdots\right.}\right.\right.\right.}{15\sqrt{a-(x-1)^4-2(x-1\cdots}}\right.$$

$$\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+$$
$$\frac{1}{63}\left(7(x-1)^2+15\right)(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{1}{4}x\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

$$\Big\downarrow \; 313$$

3.791. $\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx$

$$\frac{2}{21}\left(\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((21a^2+111a+140)\left(\frac{(1-\sqrt{a+4})(x-1)\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}}{\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-\frac{(1-\sqrt{a+4})\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}}}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}}{\frac{(x-1)^2}{\sqrt{a+4}+}}}}\right.\right.}{15\sqrt{a-(x-1)^4}}\right.$$

$$\frac{3}{4}(a+4)\left(\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}\right)+$$
$$\frac{1}{63}\left(7(x-1)^2+15\right)(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{1}{4}x\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}$$

input
```
Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

output
```
((15 + 7*(-1 + x)^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/6
3 + ((3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*x)/4 + (3*(4 + a)*((Sqrt[3
+ a - 2*(-1 + x)^2 - (-1 + x)^4]*x)/2 + ((4 + a)*ArcTan[x/Sqrt[3 + a - 2*(
-1 + x)^2 - (-1 + x)^4]])/2))/4 + (2*(((2*(80 + 27*a) + 3*(20 + 7*a)*(-1 +
 x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/15 + (2*Sqrt[1 +
(-1 + x)^2/(1 - Sqrt[4 + a])]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])]*((140
 + 111*a + 21*a^2)*(((1 - Sqrt[4 + a])*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a
])]*(-1 + x))/Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])] - ((1 - Sqrt[4 + a])*
Sqrt[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*EllipticE[Arc
Tan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/
(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a])
)]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])])) + ((3 + a)*(100 + 33*a)*Sqrt[1
 + Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*EllipticF[ArcTan[(-
1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[
(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqr
t[1 + (-1 + x)^2/(1 + Sqrt[4 + a])])))/(15*Sqrt[3 + a - 2*(-1 + x)^2 - (-1
 + x)^4])))/21
```

3.791. $\int x^2(a+8x-8x^2+4x^3-x^4)^{3/2}\,dx$

3.791.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

3.791. $\int x^2(a+8x-8x^2+4x^3-x^4)^{3/2}\,dx$

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

3.791. $\int x^2(a+8x-8x^2+4x^3-x^4)^{3/2}\,dx$

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

3.791.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2732 vs. 2(624) = 1248.

Time = 4.15 (sec) , antiderivative size = 2733, normalized size of antiderivative = 4.67

method	result	size
default	Expression too large to display	2733
elliptic	Expression too large to display	2733
risch	Expression too large to display	3625

input
```
int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*x^7*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+19/36*x^6*(-x^4+4*x^3-8*x^2+a+8*x)
^(1/2)-163/126*x^5*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+71/42*x^4*(-x^4+4*x^3-8*
x^2+a+8*x)^(1/2)+(11/45*a-16/63)*x^3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(-13/1
20*a-5/18)*x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(9/140*a+23/63)*x*(-x^4+4*x^
3-8*x^2+a+8*x)^(1/2)+(107/252*a+101/63)*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(-(
9/140*a+23/63)*a-107/63*a-404/63)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))
^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(
1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+
4)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2)
)^(1/2)*(x-1-(-1-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/
2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*
(x-1+(-1-(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/
2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)
^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+
(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))
^(1/2)))^(1/2)*EllipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))
*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1
/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+
4)^(1/2))^(1/2))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+
4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+...
```

3.791.5 Fricas [F]

$$\int x^2(a+8x-8x^2+4x^3-x^4)^{3/2}\,dx=\int(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(-(x^6 - 4*x^5 + 8*x^4 - a*x^2 - 8*x^3)*sqrt(-x^4 + 4*x^3 - 8*x^2
+ a + 8*x), x)
```

3.791.6 Sympy [F]

$$\int x^2 \left(a+8x-8x^2+4x^3-x^4\right)^{3/2} \, dx = \int x^2 \left(a-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}} \, dx$$

input
```
integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

output
```
Integral(x**2*(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

3.791.7 Maxima [F]

$$\int x^2 \left(a+8x-8x^2+4x^3-x^4\right)^{3/2} \, dx = \int \left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}} x^2 \, dx$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x^2, x)
```

3.791.8 Giac [F]

$$\int x^2 \left(a+8x-8x^2+4x^3-x^4\right)^{3/2} \, dx = \int \left(-x^4+4\,x^3-8\,x^2+a+8\,x\right)^{\frac{3}{2}} x^2 \, dx$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x^2, x)
```

3.791.9 Mupad [F(-1)]

Timed out.

$$\int x^2(a+8x-8x^2+4x^3-x^4)^{3/2}\,dx=\int x^2\left(-x^4+4\,x^3-8\,x^2+8\,x+a\right)^{3/2}dx$$

input
```
int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
```

output
```
int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

3.792 $\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

3.792.1 Optimal result

Integrand size = 28, antiderivative size = 485

$$\begin{aligned}
&\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx \\
&= \frac{1}{2}\left(1+(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4} \\
&\quad+\frac{2(8+3a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{15\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\
&\quad+\frac{1}{15}\left(7+3(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}(-1+x) \\
&\quad+\frac{1}{2}(4+a)\arctan\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\right) \\
&\quad-\frac{2(8+3a)\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)E\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{15\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\
&\quad+\frac{8(3+a)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\operatorname{EllipticF}\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right),-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{15\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}
\end{aligned}$$

output
```
1/2*(4+a)*arctan((1+(-1+x)^2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2))+2/15*(8+3*a
)*(-1+x)*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a-2*(-1+x)^2-(-1+
x)^4)^(1/2)+1/2*(1+(-1+x)^2)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/15*(7+3*(-1
+x)^2)*(-1+x)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+8/15*(3+a)*(1/(1+(-1+x)^2/(1
+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((-1+x)/
(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(
1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)/
(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/
(1+(4+a)^(1/2))))^(1/2)-2/15*(8+3*a)*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2
)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((-1+x)/(1+(4+a)^(1/2))^(1/2
)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2
))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2))^(1/2)/(3+a
-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(
4+a)^(1/2))))^(1/2)
```

3.792.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5647 vs. 2(485) = 970.

Time = 13.84 (sec) , antiderivative size = 5647, normalized size of antiderivative = 11.64

$$\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx = \text{Result too large to show}$$

input
```
Integrate[x^2*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output
```
Result too large to show
```

3.792.3 Rubi [A] (warning: unable to verify)

Time = 0.80 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.14, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.536$, Rules used = {2459, 2006, 2202, 27, 1432, 1087, 1092, 217, 1490, 27, 1514, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a-x^4+4x^3-8x^2+8x}\,dx$$

$$\Big\downarrow \text{2459}$$

$$\int \left((x-1)^2+2(x-1)+1\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}d(x-1)$$

$$\Big\downarrow \text{2006}$$

$$\int x^2\sqrt{a-(x-1)^4-2(x-1)^2+3}d(x-1)$$

$$\Big\downarrow \text{2202}$$

$$\int \left((x-1)^2+1\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\int 2\sqrt{-(x-1)^4-2(x-1)^2+a+3}(x-1)d(x-1)$$

$$\Big\downarrow \text{27}$$

$$\int \left((x-1)^2+1\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+2\int \sqrt{-(x-1)^4-2(x-1)^2+a+3}(x-1)d(x-1)$$

$$\Big\downarrow \text{1432}$$

$$\int \sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)^2+\int \left((x-1)^2+1\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{2}(a+4)\int \frac{1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)^2+\int \left((x-1)^2+1\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{1092}$$

$$(a+4)\int \frac{1}{-(x-1)^4-4}d\left(-\frac{2x}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}\right)+\int \left((x-1)^2+1\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{217}$$

$$\int \left((x-1)^2+1\right)\sqrt{-(x-1)^4-2(x-1)^2+a+3}d(x-1)+\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{1490}$$

3.792. $\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

$$-\frac{1}{15}\int -\frac{2\left((3a+8)(x-1)^2+4(a+3)\right)}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{15}\left(3(x-1)^2+7\right)(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \; 27$$

$$\frac{2}{15}\int \frac{(3a+8)(x-1)^2+4(a+3)}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{15}\left(3(x-1)^2+7\right)(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \; 1514$$

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int \frac{(3a+8)(x-1)^2+4(a+3)}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{15}\left(3(x-1)^2+7\right)(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \; 406$$

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(4(a+3)\int \frac{1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)+(3a+8)\int \frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-\right.}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{15}\left(3(x-1)^2+7\right)(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \; 320$$

3.792. $\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((3a+8)\int \frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)+\frac{4(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\operatorname{EllipticF}\left(\operatorname{arcta}\right.}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4+}}}}\right.}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{15}\left(3(x-1)^2+7\right)(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{388}$$

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((3a+8)\left(\frac{(1-\sqrt{a+4})(x-1)\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}}{\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-(1-\sqrt{a+4})\int \frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}}{\left(\frac{(x-1)^2}{\sqrt{a+4}+1}+1\right)^{3/2}}d(x-1)\right)+\right.}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{1}{15}\left(3(x-1)^2+7\right)(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

$$\Big\downarrow \text{313}$$

$$\frac{1}{2}(a+4)\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+$$

$$\frac{2\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{4(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\operatorname{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}+(3a+8)\left(\frac{(1-\sqrt{a+4}}{\sqrt{}}\right.\right.}{15\sqrt{a-(x-1)^4-2(x-1)^2+}}$$

$$\frac{1}{15}\left(3(x-1)^2+7\right)(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{2}x\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

input
```
Int[x^2*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

3.792. $\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

output
```
((7 + 3*(-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/15 +
 (Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*x)/2 + ((4 + a)*ArcTan[x/Sqrt[3
+ a - 2*(-1 + x)^2 - (-1 + x)^4]])/2 + (2*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4
+ a])]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])]*((8 + 3*a)*(((1 - Sqrt[4 + a
])*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*(-1 + x))/Sqrt[1 + (-1 + x)^2/(1
 + Sqrt[4 + a])] - ((1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 +
 x)^2/(1 - Sqrt[4 + a])]*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]],
 (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 +
a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 +
 a])])) + (4*(3 + a)*Sqrt[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4
 + a])]*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])
/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x
)^2/(1 + Sqrt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])])))/(15*Sqrt
[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

3.792.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

3.792. $\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1490
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Simp[2*(p/(c*(4*p + 1)*(4*p + 3)))
 Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3)
- b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

3.792. $\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

3.792.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2581 vs. 2(535) = 1070.

Time = 4.11 (sec) , antiderivative size = 2582, normalized size of antiderivative = 5.32

method	result	size
default	Expression too large to display	2582
elliptic	Expression too large to display	2582
risch	Expression too large to display	3044

input
```
int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-1/10*x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(
1/2)+1/15*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+1/3*(-x^4+4*x^3-8*x^2+a+8*x)^(1
/2)-(-1/15*a-4/3)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(
a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(
-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2))
)^(1/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1
-(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+
(-1+(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(
1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+
4)^(1/2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(
-1+(a+4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))
^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*E
llipticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^
(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a
+4)^(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))
*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+
(-1+(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(
1/2))-(1/5*a+28/15)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1
-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(
-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(...
```

3.792.5 Fricas [F]

$$\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}x^2\,dx$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2, x)
```

3.792.6 Sympy [F]

$$\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int x^2\sqrt{a-x^4+4x^3-8x^2+8x}\,dx$$

input
```
integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

output
```
Integral(x**2*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

3.792.7 Maxima [F]

$$\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}x^2\,dx$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2, x)
```

3.792.8 Giac [F]

$$\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int \sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}x^2\,dx$$

input
```
integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2, x)
```

3.792.9 Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx=\int x^2\,\sqrt{-x^4+4\,x^3-8\,x^2+8\,x+a}\,dx$$

input
```
int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
```

output
```
int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

3.793 $\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$

3.793.1 Optimal result

Integrand size = 28, antiderivative size = 388

$$\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}+\arctan\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\right)$$
$$-\frac{\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)E\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$
$$+\frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\text{EllipticF}\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right),-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

output
```
arctan((1+(-1+x)^2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2))+(-1+x)*(1+(-1+x)^2/(1
-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+(1/(1+(-1+x
)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((
-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(
1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^
(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1
+x)^2/(1+(4+a)^(1/2))))^(1/2)-(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-
1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-
1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(
-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2))^(1/2)/(3+a-2*(-1+
x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1
/2))))^(1/2)
```

3.793.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1145 vs. $2(388) = 776$.

Time = 15.19 (sec) , antiderivative size = 1145, normalized size of antiderivative = 2.95

$$\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$$

$$= \frac{\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\left(-1-\sqrt{-1+\sqrt{4+a}}+x\right)\left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)+\frac{2\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{}\right.}{}}{}$$

input
```
Integrate[x^2/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

output
```
((-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x)*(-1 +
 Sqrt[-1 + Sqrt[4 + a]] + x) + (2*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt
[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)^2*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*
(1 + Sqrt[-1 + Sqrt[4 + a]] - x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqr
t[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[((Sqrt[-1 - Sqrt[4 + a]
] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 -
 Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]
*Sqrt[-((Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[
-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] -
x)))]*((1 + Sqrt[-1 - Sqrt[4 + a]]*Sqrt[-1 + Sqrt[4 + a]])*EllipticE[ArcSi
n[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 -
Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 +
 Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[
4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2] - (1 + 2*S
qrt[-1 - Sqrt[4 + a]] + Sqrt[-1 - Sqrt[4 + a]]*Sqrt[-1 + Sqrt[4 + a]])*Ell
ipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1
+ Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqr
t[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2
] + 4*Sqrt[-1 - Sqrt[4 + a]]*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[...
```

3.793.3 Rubi [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.21, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2459, 2006, 2202, 27, 1432, 1092, 217, 1514, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a-x^4+4x^3-8x^2+8x}}dx$$

$$\Big\downarrow \text{2459}$$

$$\int \frac{(x-1)^2+2(x-1)+1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}d(x-1)$$

$$\Big\downarrow \text{2006}$$

$$\int \frac{x^2}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}d(x-1)$$

$$\Big\downarrow \text{2202}$$

$$\int \frac{(x-1)^2+1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\int \frac{2(x-1)}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)$$

$$\Big\downarrow \text{27}$$

$$\int \frac{(x-1)^2+1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+2\int \frac{x-1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)$$

$$\Big\downarrow \text{1432}$$

$$\int \frac{1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)^2+\int \frac{(x-1)^2+1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)$$

$$\Big\downarrow \text{1092}$$

$$2\int \frac{1}{-(x-1)^4-4}d\left(-\frac{2x}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}\right)+\int \frac{(x-1)^2+1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)$$

$$\Big\downarrow \text{217}$$

$$\int \frac{(x-1)^2+1}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)+\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)$$

$$\Big\downarrow \text{1514}$$

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int \frac{(x-1)^2+1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)$$

$$\Big\downarrow \text{406}$$

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\int \frac{1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)+\int \frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)\right)}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)$$

$$\Big\downarrow \text{320}$$

3.793. $\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\int \frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)+\frac{\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\right.}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\right.}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)$$

$$\Big\downarrow \; 388$$

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(-\left(1-\sqrt{a+4}\right)\int \frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}}{\left(\frac{(x-1)^2}{\sqrt{a+4}+1}+1\right)^{3/2}}d(x-1)+\frac{\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-}{\sqrt{\sqrt{a+}}}\right.\right.}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\right.}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)$$

$$\Big\downarrow \; 313$$

$$\arctan\left(\frac{x}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+$$

$$\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-\frac{\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+}}}}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+}}}{\frac{(x-1)^2}{\sqrt{a+4+}}}}}\right.}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

input
```
Int[x^2/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

3.793. $\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$

output
```
ArcTan[x/Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]] + (Sqrt[1 + (-1 + x)^2/(
1 - Sqrt[4 + a])]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])]*(((1 - Sqrt[4 + a
])*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*(-1 + x))/Sqrt[1 + (-1 + x)^2/(1
 + Sqrt[4 + a])] - ((1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 +
 x)^2/(1 - Sqrt[4 + a])]*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]],
 (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 +
a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 +
 a])]) + (Sqrt[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*Ell
ipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[
4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sq
rt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])])))/Sqrt[3 + a - 2*(-1
+ x)^2 - (-1 + x)^4]
```

3.793.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

3.793. $\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

3.793.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1146 vs. $2(454) = 908$.

Time = 1.44 (sec) , antiderivative size = 1147, normalized size of antiderivative = 2.96

method	result	size
default	Expression too large to display	1147
elliptic	Expression too large to display	1147

input
```
int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^
(1/2))^(1/2))+((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)
^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(
a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1
/2)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+
4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+
(a+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2)
)^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(
1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^
(1/2))-(1-(-1-(a+4)^(1/2))^(1/2))*(1+(-1+(a+4)^(1/2))^(1/2))+(1-(-1-(a+4)^
(1/2))^(1/2))*(1-(-1+(a+4)^(1/2))^(1/2))+(1-(-1+(a+4)^(1/2))^(1/2))^2)/(-(
-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+4)^(1/2))^(1/2)*Ellip
ticF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2
))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^
(1/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-
1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+
(a+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2)
)-1/2*(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*EllipticE(((-(-1-(a
+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-
1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2...
```

3.793.5 Fricas [F]

$$\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{x^2}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2/(x^4 - 4*x^3 + 8*x^2 -
a - 8*x), x)
```

3.793.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{x^2}{\sqrt{a-x^4+4x^3-8x^2+8x}}\,dx$$

input
```
integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

output
```
Integral(x**2/sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

3.793.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{x^2}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

3.793.8 Giac [F]

$$\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{x^2}{\sqrt{-x^4+4\,x^3-8\,x^2+a+8\,x}}\,dx$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

3.793.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \int \frac{x^2}{\sqrt{-x^4+4\,x^3-8\,x^2+8\,x+a}}\,dx$$

input
```
int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
```

output
```
int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

3.794 $\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx$

3.794.1 Optimal result

Integrand size = 28, antiderivative size = 311

$$\begin{aligned}
\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx &= \frac{1+(-1+x)^2}{(4+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\
&+ \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\
&- \frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{2(3+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} \\
&+ \frac{\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)E\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)\middle|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{2(3+a)\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}
\end{aligned}$$

output

```
(1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/2*(4+a)*(2+(-1+x)^2)*
(-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)-1/2*(-1+x)*(1+(-1+x)^2
/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/
2*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2
)*EllipticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2
),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-
(4+a)^(1/2))*(1+(4+a)^(1/2))^(1/2)/(3+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/(
(1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

3.794.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2941 vs. $2(311) = 622$.

Time = 14.51 (sec) , antiderivative size = 2941, normalized size of antiderivative = 9.46

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{3/2}} \, dx = \text{Result too large to show}$$

input
```
Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

output
```
((-a - 8*x - a*x + 6*x^2 + a*x^2 - 4*x^3 - a*x^3)*(a + 8*x - 8*x^2 + 4*x^3
 - x^4)^2)/(2*(3 + a)*(4 + a)*(-a - 8*x + 8*x^2 - 4*x^3 + x^4)*(a - x*(-8
+ 8*x - 4*x^2 + x^3))^(3/2)) - ((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2)*((2*
(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4
+ a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1
+ Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(
-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqr
t[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]
]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((
-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 +
 a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-
1 - Sqrt[4 + a]] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a
]])*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4
+ a]] - Sqrt[-1 + Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[
4 + a]]))])/(Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + S
qrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) - (4*(-Sqrt[-1 - Sqrt[4
+ a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[(
(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[...
```

3.794.3 Rubi [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.25, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {2459, 2006, 2202, 27, 1432, 1088, 1492, 27, 1460, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a-x^4+4x^3-8x^2+8x\right)^{3/2}}dx$$

$\Big\downarrow$ 2459

$$\int \frac{(x-1)^2+2(x-1)+1}{\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}d(x-1)$$

$\Big\downarrow$ 2006

$$\int \frac{x^2}{\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}d(x-1)$$

$\Big\downarrow$ 2202

$$\int \frac{(x-1)^2+1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}}d(x-1)+\int \frac{2(x-1)}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}}d(x-1)$$

$\Big\downarrow$ 27

$$\int \frac{(x-1)^2+1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}}d(x-1)+2\int \frac{x-1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}}d(x-1)$$

$\Big\downarrow$ 1432

$$\int \frac{1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}}d(x-1)^2+\int \frac{(x-1)^2+1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}}d(x-1)$$

$\Big\downarrow$ 1088

$$\int \frac{(x-1)^2+1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}}d(x-1)+\frac{x}{(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$\Big\downarrow$ 1492

$$-\frac{\int \frac{2(a+4)(x-1)^2}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)}{4\left(a^2+7a+12\right)}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$\Big\downarrow$ 27

$$-\frac{(a+4)\int \frac{(x-1)^2}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)}{2\left(a^2+7a+12\right)}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$\Big\downarrow$ 1460

3.794. $\int \frac{x^2}{\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}}dx$

$$-\frac{(a+4)\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int \frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{388}$$

$$-\frac{(a+4)\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{\left(1-\sqrt{a+4}\right)(x-1)\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}}{\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-\left(1-\sqrt{a+4}\right)\int \frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}}{\left(\frac{(x-1)^2}{\sqrt{a+4}+1}+1\right)^{3/2}}d(x-1)\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

$$\Big\downarrow \text{313}$$

$$-\frac{(a+4)\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{\left(1-\sqrt{a+4}\right)(x-1)\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}}{\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-\frac{\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}E\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right.\right.}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\right.}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$
$$\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

input
```
Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

output
```
((4 + a)*(2 + (-1 + x)^2)*(-1 + x))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1
 + x)^2 - (-1 + x)^4]) + x/((4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4
]) - ((4 + a)*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*Sqrt[1 + (-1 + x)^2/(
1 + Sqrt[4 + a])]*(((1 - Sqrt[4 + a])*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a]
)]*(-1 + x))/Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])] - ((1 - Sqrt[4 + a])*S
qrt[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*EllipticE[ArcT
an[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(
Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))
]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])])))/(2*(12 + 7*a + a^2)*Sqrt[3 + a
 - 2*(-1 + x)^2 - (-1 + x)^4])
```

3.794. $\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx$

3.794.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1460
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(
b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[x^2/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sq
rt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a
*c, 0] && NegQ[c/a]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

3.794. $\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{3/2}} dx$

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] && !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 && !PolyQ[Pn, x^2]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] && !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

3.794.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2606 vs. $2(331) = 662$.

Time = 1.28 (sec) , antiderivative size = 2607, normalized size of antiderivative = 8.38

method	result	size
default	Expression too large to display	2607
elliptic	Expression too large to display	2607

input
```
int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(1/4/(3+a)*x^3-1/4*(6+a)/(a^2+7*a+12)*x^2+1/4*(a+8)/(a^2+7*a+12)*x+1/4/(
a^2+7*a+12)*a)/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(2/(a^2+7*a+12)-1/2*(a+8)/(a
^2+7*a+12))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(
1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+
4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2
)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)
^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a
+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^
(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/
2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+
4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2)
)*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*Ellipti
cF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))
^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1
/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-
(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a
+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2))-
(-2/(a^2+7*a+12)+(6+a)/(a^2+7*a+12))*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/
2))^(1/2))*((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4
)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(...
```

3.794.5 Fricas [F]

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{x^2}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2/(x^8 - 8*x^7 + 32*x^6 -
2*(a - 64)*x^4 - 80*x^5 + 8*(a - 16)*x^3 - 16*(a - 4)*x^2 + a^2 + 16*a*x),
 x)
```

3.794.6 Sympy [F]

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{x^2}{(a-x^4+4x^3-8x^2+8x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

output
```
Integral(x**2/(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

3.794.7 Maxima [F]

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{x^2}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

3.794.8 Giac [F]

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{x^2}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

3.794.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \int \frac{x^2}{(-x^4+4\,x^3-8\,x^2+8\,x+a)^{3/2}}\,dx$$

input
```
int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
```

output
```
int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

3.795 $\int \frac{x^2}{\left(a+8x-8x^2+4x^3-x^4\right)^{5/2}}\,dx$

3.795.1 Optimal result

Integrand size = 28, antiderivative size = 582

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \frac{1+(-1+x)^2}{3(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}}$$
$$+\frac{2(1+(-1+x)^2)}{3(4+a)^2\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$
$$+\frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{6\left(12+7a+a^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}}$$
$$+\frac{\left(29+7a+(13+3a)(-1+x)^2\right)(-1+x)}{12(3+a)^2(4+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$
$$-\frac{(13+3a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{12(3+a)^2(4+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$
$$+\frac{(13+3a)\left(1-\sqrt{4+a}\right)\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)E\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{12(3+a)^2(4+a)\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$
$$+\frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)\operatorname{EllipticF}\left(\arctan\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right),-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{12\left(12+7a+a^2\right)\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

output
```
1/3*(1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)+1/6*(4+a)*(2+(-1+x)
^2)*(-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)+2/3*(1+(-1+x)^2)/(
4+a)^2/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/12*(29+7*a+(13+3*a)*(-1+x)^2)*(-1
+x)/(3+a)^2/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)-1/12*(13+3*a)*(-1+x)*(1+
(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a)^2/(4+a)/(3+a-2*(-1+x)^2-(-
1+x)^4)^(1/2)+1/12*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(
4+a)^(1/2)))^(1/2)*EllipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(
4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-
(4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)
^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)+1
/12*(13+3*a)*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(
1/2)))^(1/2)*EllipticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(
1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^
(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2))^(1/2)/(3+a)^2/(4+a)/(3+a-2*(-1+x)^
2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2)
)))^(1/2)
```

3.795.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5812 vs. $2(582) = 1164$.

Time = 17.14 (sec) , antiderivative size = 5812, normalized size of antiderivative = 9.99

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x]
```

output
```
Result too large to show
```

3.795.3 Rubi [A] (warning: unable to verify)

Time = 0.89 (sec) , antiderivative size = 653, normalized size of antiderivative = 1.12, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {2459, 2006, 2202, 27, 1432, 1089, 1088, 1492, 27, 1492, 27, 1514, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a-x^4+4x^3-8x^2+8x\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 2459}$$

$$\int \frac{(x-1)^2+2(x-1)+1}{\left(a-(x-1)^4-2(x-1)^2+3\right)^{5/2}}d(x-1)$$

$$\Big\downarrow \text{ 2006}$$

$$\int \frac{x^2}{\left(a-(x-1)^4-2(x-1)^2+3\right)^{5/2}}d(x-1)$$

$$\Big\downarrow \text{ 2202}$$

$$\int \frac{(x-1)^2+1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{5/2}}d(x-1)+\int \frac{2(x-1)}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{5/2}}d(x-1)$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{(x-1)^2+1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{5/2}}d(x-1)+2\int \frac{x-1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{5/2}}d(x-1)$$

$$\Big\downarrow \text{ 1432}$$

$$\int \frac{1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{5/2}}d(x-1)^2+\int \frac{(x-1)^2+1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{5/2}}d(x-1)$$

$$\Big\downarrow \text{ 1089}$$

$$\int \frac{(x-1)^2+1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{5/2}}d(x-1)+\frac{2\int \frac{1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}}d(x-1)^2}{3(a+4)}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \text{ 1088}$$

$$\int \frac{(x-1)^2+1}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{5/2}}d(x-1)+\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \text{ 1492}$$

$$-\frac{\int -\frac{2(a+4)\left(3(x-1)^2+4\right)}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}}d(x-1)}{12\left(a^2+7a+12\right)}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

3.795. $\int \frac{x^2}{\left(a+8x-8x^2+4x^3-x^4\right)^{5/2}}dx$

$$\frac{(a+4)\int \frac{3(x-1)^2+4}{\left(-(x-1)^4-2(x-1)^2+a+3\right)^{3/2}}d(x-1)}{6\left(a^2+7a+12\right)}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \text{1492}$$

$$\frac{(a+4)\left(\frac{(x-1)\left((3a+13)(x-1)^2+7a+29\right)}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\int -\frac{2\left(-\left((3a+13)(x-1)^2\right)+a+3\right)}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)}{4(a^2+7a+12)}\right)}{6\left(a^2+7a+12\right)}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a+4)\left(\frac{\int \frac{-\left((3a+13)(x-1)^2\right)+a+3}{\sqrt{-(x-1)^4-2(x-1)^2+a+3}}d(x-1)}{2(a^2+7a+12)}+\frac{(x-1)\left((3a+13)(x-1)^2+7a+29\right)}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)}{6\left(a^2+7a+12\right)}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \text{1514}$$

$$\frac{(a+4)\left(\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\int \frac{-\left((3a+13)(x-1)^2\right)+a+3}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left((3a+13)(x-1)^2+7a+29\right)}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)}{6\left(a^2+7a+12\right)}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \text{406}$$

3.795. $\int \frac{x^2}{\left(a+8x-8x^2+4x^3-x^4\right)^{5/2}}\,dx$

$$\frac{(a+4)\left(\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left((a+3)\int \frac{1}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)-(3a+13)\int \frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}d(x-1)\right)}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-}{2(a^2+7}\right.}{6\left(a^2+7a+12\right)}$$

$$\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \ 320$$

$$\frac{(a+4)\left(\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-(3a+13)\int \frac{(x-1)^2}{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1}{\sqrt{a+4}}}}\right.}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right.}{6\left(a^2+7a+12\right)}$$

$$\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \ 388$$

3.795. $\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

$$(a+4)\left(\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-(3a+13)\left(\frac{(1-\sqrt{a+4})(x-1)\sqrt{\frac{(x-}{1-\sqrt{}}}}{\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\right.\right.}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right.$$

$$\overline{\qquad 6\left(a^2+7a+12\right)}$$

$$\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

$$\Big\downarrow \textbf{313}$$

$$(a+4)\left(\frac{\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}\left(\frac{(a+3)\sqrt{\sqrt{a+4}+1}\sqrt{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right),-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}-(3a+13)\left(\frac{(1-\sqrt{a+4})(x-1)\sqrt{\frac{(x-}{1-\sqrt{}}}}{\sqrt{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\right.\right.}{2(a^2+7a+12)\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right.$$

$$\overline{\qquad 6\left(a^2+7a+12\right.}$$

$$\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{2x}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{x}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

input
```
Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x]
```

3.795. $\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

output
```
((4 + a)*(2 + (-1 + x)^2)*(-1 + x))/(6*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x
)^2 - (-1 + x)^4)^(3/2)) + x/(3*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4
)^(3/2)) + (2*x)/(3*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (
(4 + a)*(((29 + 7*a + (13 + 3*a)*(-1 + x)^2)*(-1 + x))/(2*(12 + 7*a + a^2)
*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sqrt[1 + (-1 + x)^2/(1 - Sqrt
[4 + a])]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])]*(-((13 + 3*a)*(((1 - Sqrt
[4 + a])*Sqrt[1 + (-1 + x)^2/(1 - Sqrt[4 + a])]*(-1 + x))/Sqrt[1 + (-1 + x
)^2/(1 + Sqrt[4 + a])] - ((1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*Sqrt[1 +
 (-1 + x)^2/(1 - Sqrt[4 + a])]*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 +
 a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqr
t[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 + Sq
rt[4 + a])]))) + ((3 + a)*Sqrt[1 + Sqrt[4 + a]]*Sqrt[1 + (-1 + x)^2/(1 - S
qrt[4 + a])]*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4
+ a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-
1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[1 + (-1 + x)^2/(1 + Sqrt[4 + a])])))/(2*
(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])))/(6*(12 + 7*a +
 a^2))
```

3.795.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

3.795. $\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 1492
```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2
 - 4*a*c))   Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p +
7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 LtQ[p, -1] && IntegerQ[2*p]
```

rule 1514
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4])   Int[(d + e*x^2)/(Sqrt[1
+ 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

rule 2006
```
Int[(u_.)*(Px_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] &&  !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

3.795. $\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

3.795.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2779 vs. $2(628) = 1256$.

Time = 1.51 (sec) , antiderivative size = 2780, normalized size of antiderivative = 4.78

method	result	size
default	Expression too large to display	2780
elliptic	Expression too large to display	2780

input
```
int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(1/6/(3+a)*x^3-1/6*(6+a)/(a^2+7*a+12)*x^2+1/6*(a+8)/(a^2+7*a+12)*x+1/6/(a^
2+7*a+12)*a)*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)/(x^4-4*x^3+8*x^2-a-8*x)^2+2*(1
/24*(13+3*a)/(3+a)/(a^2+7*a+12)*x^3-1/24*(a^2+27*a+84)/(a^2+7*a+12)^2*x^2+
1/6*(9*a+32)/(a^2+7*a+12)^2*x+1/12*(3*a^2+7*a-12)/(a^2+7*a+12)^2)/(-x^4+4*
x^3-8*x^2+a+8*x)^(1/2)-(-1/6*(a^2-9*a-44)/(a^2+7*a+12)^2-1/3*(9*a+32)/(a^2
+7*a+12)^2)*((-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*((-(-1-(a+4)^(
1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+
4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/2))^(1/2)))^(1/2
)*(x-1+(-1+(a+4)^(1/2))^(1/2))^2*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1-(-1-(a+4)
^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a
+4)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(a+4)^(1/2))^(1/2)*(x-1+(-1-(a+4)^(1/2))^
(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1/
2))^(1/2)))^(1/2)/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-1+(a+
4)^(1/2))^(1/2)/(-(x-1-(-1+(a+4)^(1/2))^(1/2))*(x-1+(-1+(a+4)^(1/2))^(1/2)
)*(x-1-(-1-(a+4)^(1/2))^(1/2))*(x-1+(-1-(a+4)^(1/2))^(1/2)))^(1/2)*Ellipti
cF(((-(-1-(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))*(x-1-(-1+(a+4)^(1/2))
^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))/(x-1+(-1+(a+4)^(1
/2))^(1/2)))^(1/2),((-(-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2))*((-1-
(a+4)^(1/2))^(1/2)+(-1+(a+4)^(1/2))^(1/2))/(-(-1-(a+4)^(1/2))^(1/2)+(-1+(a
+4)^(1/2))^(1/2))/((-1-(a+4)^(1/2))^(1/2)-(-1+(a+4)^(1/2))^(1/2)))^(1/2...
```

3.795.5 Fricas [F]

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{x^2}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2/(x^12 - 12*x^11 + 72*x^
10 - 3*(a - 256)*x^8 - 280*x^9 + 24*(a - 64)*x^7 - 32*(3*a - 70)*x^6 + 48*
(5*a - 48)*x^5 + 3*(a^2 - 128*a + 512)*x^4 - 4*(3*a^2 - 96*a + 128)*x^3 -
a^3 - 24*a^2*x + 24*(a^2 - 8*a)*x^2), x)
```

3.795.6 Sympy [F]

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{x^2}{(a-x^4+4x^3-8x^2+8x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**(5/2),x)
```

output
```
Integral(x**2/(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(5/2), x)
```

3.795.7 Maxima [F]

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{x^2}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2), x)
```

3.795.8 Giac [F]

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{x^2}{(-x^4+4\,x^3-8\,x^2+a+8\,x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2), x)
```

3.795.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \int \frac{x^2}{(-x^4+4\,x^3-8\,x^2+8\,x+a)^{5/2}}\,dx$$

input
```
int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x)
```

output
```
int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2), x)
```

3.796 $\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx$

3.796.1 Optimal result

Integrand size = 19, antiderivative size = 129

$$\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx = -\frac{x^2\sqrt{\frac{261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4}{\left(87+\frac{\sqrt{29}(4+x)^2}{x^2}\right)^2}}\left(87+\frac{\sqrt{29}(4+x)^2}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{4+x}{\sqrt{3}\sqrt[4]{29}x}\right),\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{8\sqrt{3}\sqrt[4]{29}\sqrt{8+8x-x^3+8x^4}}$$

output
```
-1/696*x^2*(cos(2*arctan(1/87*(4+x)*29^(3/4)/x*3^(1/2)))^2)^(1/2)/cos(2*ar
ctan(1/87*(4+x)*29^(3/4)/x*3^(1/2)))*EllipticF(sin(2*arctan(1/87*(4+x)*29^
(3/4)/x*3^(1/2))),1/58*(1682+58*29^(1/2))^(1/2))*(87+(4+x)^2*29^(1/2)/x^2)
*((261-6*(1+4/x)^2+(1+4/x)^4)/(87+(4+x)^2*29^(1/2)/x^2)^2)^(1/2)*29^(3/4)*
3^(1/2)/(8*x^4-x^3+8*x+8)^(1/2)
```

3.796.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 10.70 (sec) , antiderivative size = 927, normalized size of antiderivative = 7.19

$$\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/Sqrt[8 + 8*x - x^3 + 8*x^4],x]
```

output
```
(-2*EllipticF[ArcSin[Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(
Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 &
, 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 -
#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]], ((R
oot[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 +
8*#1^4 & , 4, 0]))/((Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*
#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - R
oot[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]*(x - Root[8 + 8*#1 - #1^3 + 8*#1
^4 & , 2, 0])^2*Sqrt[((Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 +
8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3,
0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^3
+ 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]))]*(Root[8 +
8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])*
Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(Root[8 + 8*#1 - #1^3
+ 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(x - Root[8
+ 8*#1 - #1^3 + 8*#1^4 & , 4, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0]
 - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 +
8*#1^4 & , 2, 0])^2*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*
#1 - #1^3 + 8*#1^4 & , 4, 0])^2)])/(Sqrt[8 + 8*x - x^3 + 8*x^4]*(-Root[...
```

3.796.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.30, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2504, 27, 7270, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{8x^4-x^3+8x+8}}dx$$

$$\Big\downarrow \text{2504}$$

$$-1024\int \frac{1}{128\sqrt{2}\left(1-4\left(\frac{1}{4}+\frac{1}{x}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}{\left(1-4\left(\frac{1}{4}+\frac{1}{x}\right)\right)^4}}}d\left(\frac{1}{4}+\frac{1}{x}\right)$$

$$\Big\downarrow \text{27}$$

$$-4\sqrt{2}\int \frac{1}{\left(1-4\left(\frac{1}{4}+\frac{1}{x}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}{\left(1-4\left(\frac{1}{4}+\frac{1}{x}\right)\right)^4}}}d\left(\frac{1}{4}+\frac{1}{x}\right)$$

$$\Big\downarrow \text{7270}$$

$$-\frac{4\sqrt{2}\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}\int \frac{1}{\sqrt{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}}d\left(\frac{1}{4}+\frac{1}{x}\right)}{\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}{\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^4}}}$$

$$\Big\downarrow \text{1416}$$

$$-\frac{\left(16\sqrt{29}\left(\frac{1}{x}+\frac{1}{4}\right)^2+87\right)\sqrt{\frac{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}{\left(16\sqrt{29}\left(\frac{1}{x}+\frac{1}{4}\right)^2+87\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{4\left(\frac{1}{4}+\frac{1}{x}\right)}{\sqrt{3}\sqrt[4]{29}}\right),\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{\sqrt{6}\sqrt[4]{29}\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}{\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^4}}}$$

input
```
Int[1/Sqrt[8 + 8*x - x^3 + 8*x^4],x]
```

output
```
-(((87 + 16*Sqrt[29]*(1/4 + x^(-1))^2)*Sqrt[(261 - 96*(1/4 + x^(-1))^2 + 2
56*(1/4 + x^(-1))^4)/(87 + 16*Sqrt[29]*(1/4 + x^(-1))^2)^2]*EllipticF[2*Ar
cTan[(4*(1/4 + x^(-1)))/(Sqrt[3]*29^(1/4))], (29 + Sqrt[29])/58])/(Sqrt[6]
*29^(1/4)*(1 - 4*(1/4 + x^(-1)))^2*Sqrt[(261 - 96*(1/4 + x^(-1))^2 + 256*(
1/4 + x^(-1))^4)/(1 - 4*(1/4 + x^(-1)))^4]))
```

3.796.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

3.796. $\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx$

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

3.796.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.13 (sec) , antiderivative size = 965, normalized size of antiderivative = 7.48

method	result	size
default	Expression too large to display	965
elliptic	Expression too large to display	965

input
```
int(1/(8*x^4-x^3+8*x+8)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)
)*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))
*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4
)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2
)))^(1/2)*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))^2*((RootOf(8*_Z^4-_Z^3+8*
_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*
_Z+8,index=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)-RootOf(8*_Z^4-_Z^3+8*_Z
+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*((RootOf(8*_Z^4
-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4
-_Z^3+8*_Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_
Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)/(RootOf
(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))/(RootOf(8
*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*2^(1/2)/((x
-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)
)*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,inde
x=4)))^(1/2)*EllipticF(((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-
_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^
4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^
4-_Z^3+8*_Z+8,index=2)))^(1/2),((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf
(8*_Z^4-_Z^3+8*_Z+8,index=3))*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootO...
```

3.796.5 Fricas [F]

$$\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx = \int \frac{1}{\sqrt{8\,x^4-x^3+8\,x+8}}\,dx$$

input
```
integrate(1/(8*x^4-x^3+8*x+8)^(1/2),x, algorithm="fricas")
```

output
```
integral(1/sqrt(8*x^4 - x^3 + 8*x + 8), x)
```

3.796.6 Sympy [F]

$$\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx = \int \frac{1}{\sqrt{8x^4-x^3+8x+8}}\,dx$$

input
```
integrate(1/(8*x**4-x**3+8*x+8)**(1/2),x)
```

output
```
Integral(1/sqrt(8*x**4 - x**3 + 8*x + 8), x)
```

3.796.7 Maxima [F]

$$\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx = \int \frac{1}{\sqrt{8\,x^4-x^3+8\,x+8}}\,dx$$

input
```
integrate(1/(8*x^4-x^3+8*x+8)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(8*x^4 - x^3 + 8*x + 8), x)
```

3.796.8 Giac [F]

$$\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx = \int \frac{1}{\sqrt{8\,x^4-x^3+8\,x+8}}\,dx$$

input
```
integrate(1/(8*x^4-x^3+8*x+8)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(8*x^4 - x^3 + 8*x + 8), x)
```

3.796.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx = \int \frac{1}{\sqrt{8\,x^4-x^3+8\,x+8}}\,dx$$

input
```
int(1/(8*x - x^3 + 8*x^4 + 8)^(1/2),x)
```

output
```
int(1/(8*x - x^3 + 8*x^4 + 8)^(1/2), x)
```

3.797 $\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx$

3.797.1 Optimal result

Integrand size = 19, antiderivative size = 431

$$\begin{aligned}\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx = & -\frac{\left(66-\left(1+\frac{4}{x}\right)^2\right)x^2}{1008\sqrt{8+8x-x^3+8x^4}} \\ & +\frac{\left(216-7\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)x^2}{12528\sqrt{8+8x-x^3+8x^4}}+\frac{7\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)\left(1+\frac{4}{x}\right)x^2}{432\sqrt{29}\sqrt{8+8x-x^3+8x^4}\left(87+\frac{\sqrt{29}(4+x)^2}{x^2}\right)} \\ & -\frac{7x^2\sqrt{\frac{261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4}{\left(87+\frac{\sqrt{29}(4+x)^2}{x^2}\right)^2}}\left(87+\frac{\sqrt{29}(4+x)^2}{x^2}\right)E\left(2\arctan\left(\frac{4+x}{\sqrt{3}\sqrt[4]{29}x}\right)|\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{144\sqrt{3}29^{3/4}\sqrt{8+8x-x^3+8x^4}} \\ & +\frac{\left(14-5\sqrt{29}\right)x^2\sqrt{\frac{261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4}{\left(87+\frac{\sqrt{29}(4+x)^2}{x^2}\right)^2}}\left(87+\frac{\sqrt{29}(4+x)^2}{x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{4+x}{\sqrt{3}\sqrt[4]{29}x}\right),\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{576\sqrt{3}29^{3/4}\sqrt{8+8x-x^3+8x^4}}\end{aligned}$$

output
```
-1/1008*(66-(1+4/x)^2)*x^2/(8*x^4-x^3+8*x+8)^(1/2)+1/12528*(216-7*(1+4/x)^
2)*(1+4/x)*x^2/(8*x^4-x^3+8*x+8)^(1/2)+7/12528*(261-6*(1+4/x)^2+(1+4/x)^4)
*(1+4/x)*x^2*29^(1/2)/(87+(4+x)^2*29^(1/2)/x^2)/(8*x^4-x^3+8*x+8)^(1/2)-7/
12528*x^2*(cos(2*arctan(1/87*(4+x)*29^(3/4)/x*3^(1/2)))^2)^(1/2)/cos(2*arc
tan(1/87*(4+x)*29^(3/4)/x*3^(1/2)))*EllipticE(sin(2*arctan(1/87*(4+x)*29^(
3/4)/x*3^(1/2))),1/58*(1682+58*29^(1/2))^(1/2))*(87+(4+x)^2*29^(1/2)/x^2)*
((261-6*(1+4/x)^2+(1+4/x)^4)/(87+(4+x)^2*29^(1/2)/x^2)^2)^(1/2)*29^(1/4)*3
^(1/2)/(8*x^4-x^3+8*x+8)^(1/2)+1/50112*x^2*(cos(2*arctan(1/87*(4+x)*29^(3/
4)/x*3^(1/2)))^2)^(1/2)/cos(2*arctan(1/87*(4+x)*29^(3/4)/x*3^(1/2)))*Ellip
ticF(sin(2*arctan(1/87*(4+x)*29^(3/4)/x*3^(1/2))),1/58*(1682+58*29^(1/2))^
(1/2))*(14-5*29^(1/2))*(87+(4+x)^2*29^(1/2)/x^2)*((261-6*(1+4/x)^2+(1+4/x)
^4)/(87+(4+x)^2*29^(1/2)/x^2)^2)^(1/2)*29^(1/4)*3^(1/2)/(8*x^4-x^3+8*x+8)^
(1/2)
```

3.797.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 16.15 (sec) , antiderivative size = 4865, normalized size of antiderivative = 11.29

$$\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(8 + 8*x - x^3 + 8*x^4)^(-3/2),x]
```

output
```
(544 + 1539*x - 1146*x^2 + 784*x^3)/(21924*Sqrt[8 + 8*x - x^3 + 8*x^4]) +
((28*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])^2*(-(EllipticF[ArcSin[S
qrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(Root[8 + 8*#1 - #1^3 +
 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8
 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0
] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]], -(((Root[8 + 8*#1 - #1^3
+ 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*
#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/(
(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4
& , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3
 + 8*#1^4 & , 4, 0])))]*Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0]) + Ellipti
cPi[(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#
1^4 & , 4, 0])/(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] + Root[8 + 8*#1 -
 #1^3 + 8*#1^4 & , 4, 0]), ArcSin[Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4
 & , 1, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^
3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Roo
t[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4
, 0]))]], -(((Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1
^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 +
 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , ...
```

3.797.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.28, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {2504, 27, 7270, 2202, 1576, 27, 1158, 2206, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(8x^4-x^3+8x+8)^{3/2}}dx$$

$$\Big\downarrow \text{2504}$$

$$-1024\int \frac{1}{1024\sqrt{2}\left(1-4\left(\frac{1}{4}+\frac{1}{x}\right)\right)^2\left(\frac{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}{\left(1-4\left(\frac{1}{4}+\frac{1}{x}\right)\right)^4}\right)^{3/2}}d\left(\frac{1}{4}+\frac{1}{x}\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{1}{\left(1-4\left(\frac{1}{4}+\frac{1}{x}\right)\right)^2\left(\frac{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}{\left(1-4\left(\frac{1}{4}+\frac{1}{x}\right)\right)^4}\right)^{3/2}}d\left(\frac{1}{4}+\frac{1}{x}\right)}{\sqrt{2}}$$

$$\Big\downarrow \; 7270$$

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}\int \frac{\left(1-4\left(\frac{1}{4}+\frac{1}{x}\right)\right)^4}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)^{3/2}}d\left(\frac{1}{4}+\frac{1}{x}\right)}{\sqrt{2}\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}{\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^4}}}$$

$$\Big\downarrow \; 2202$$

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}\left(\int \frac{256\left(\frac{1}{4}+\frac{1}{x}\right)^4+96\left(\frac{1}{4}+\frac{1}{x}\right)^2+1}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)^{3/2}}d\left(\frac{1}{4}+\frac{1}{x}\right)+\int \frac{\left(-256\left(\frac{1}{4}+\frac{1}{x}\right)^2-16\right)\left(\frac{1}{4}+\frac{1}{x}\right)}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)^{3/2}}\cdots\right.}{\sqrt{2}\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}{\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^4}}}$$

$$\Big\downarrow \; 1576$$

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}\left(\frac{1}{2}\int -\frac{16\left(16\left(\frac{1}{4}+\frac{1}{x}\right)^2+1\right)}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)^{3/2}}d\left(\frac{1}{4}+\frac{1}{x}\right)^2+\int \frac{256\left(\frac{1}{4}+\frac{1}{x}\right)^4+96\left(\frac{1}{4}+\frac{1}{x}\right)^2+\cdots}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+26\cdots\right.}\right.}{\sqrt{2}\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}{\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^4}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}\left(\int \frac{256\left(\frac{1}{4}+\frac{1}{x}\right)^4+96\left(\frac{1}{4}+\frac{1}{x}\right)^2+1}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)^{3/2}}d\left(\frac{1}{4}+\frac{1}{x}\right)-8\int \frac{16\left(\frac{1}{4}+\frac{1}{x}\right)^2+1}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)^{3\cdots}}\right.}{\sqrt{2}\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}{\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^4}}}$$

$$\Big\downarrow \; 1158$$

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}\left(\int \frac{256\left(\frac{1}{4}+\frac{1}{x}\right)^4+96\left(\frac{1}{4}+\frac{1}{x}\right)^2+1}{\left(256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261\right)^{3/2}}d\left(\frac{1}{4}+\frac{1}{x}\right)+\frac{33-8\left(\frac{1}{x}+\frac{1}{4}\right)^2}{63\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}}\right)}{\sqrt{2}\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}{\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^4}}}$$

$$\Big\downarrow \; 2206$$

3.797. $\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx$

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}\left(\frac{\int \frac{86016\left(435-224\left(\frac{1}{4}+\frac{1}{x}\right)^2\right)}{\sqrt{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}}d\left(\frac{1}{4}+\frac{1}{x}\right)}{67350528}+\frac{33-8\left(\frac{1}{x}+\frac{1}{4}\right)^2}{63\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}}-\frac{16(2}{783\sqrt{256}}\right.}{\sqrt{2}\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}{\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^4}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}\left(\frac{1}{783}\int \frac{435-224\left(\frac{1}{4}+\frac{1}{x}\right)^2}{\sqrt{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}}d\left(\frac{1}{4}+\frac{1}{x}\right)+\frac{33-8\left(\frac{1}{x}+\frac{1}{4}\right)^2}{63\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}}-\right.}{\sqrt{2}\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}{\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^4}}}$$

$$\Big\downarrow \; 1511$$

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}\left(\frac{1}{783}\left(3\left(145-14\sqrt{29}\right)\int \frac{1}{\sqrt{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}}d\left(\frac{1}{4}+\frac{1}{x}\right)+42\sqrt{29}\int \frac{}{87}\right.\right.}{\sqrt{2}\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\right.}{}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}\left(\frac{1}{783}\left(3\left(145-14\sqrt{29}\right)\int \frac{1}{\sqrt{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}}d\left(\frac{1}{4}+\frac{1}{x}\right)+\frac{14\int \frac{87-}{\sqrt{256\left(\frac{1}{4}+\right.}}}{}\right.\right.}{\sqrt{2}\left(1-4\left(\frac{1}{x}+\frac{1}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-}{\left(1-4(\right.}}}$$

$$\Big\downarrow \; 1416$$

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}\left(\frac{1}{783}\left(\frac{14\int \frac{87-16\sqrt{29}\left(\frac{1}{4}+\frac{1}{x}\right)^2}{\sqrt{256\left(\frac{1}{4}+\frac{1}{x}\right)^4-96\left(\frac{1}{4}+\frac{1}{x}\right)^2+261}}d\left(\frac{1}{4}+\frac{1}{x}\right)}{\sqrt{29}}+\frac{\sqrt{3}\left(145-14\sqrt{29}\right)\left(16\sqrt{29}\left(\frac{1}{x}+\frac{1}{4}\right)^2+87\right.}{}\right.\right.}{\sqrt{2}\left(1-4\left(\frac{1}{x}-\right.\right.}$$

$$\Big\downarrow \; 1509$$

3.797. $\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx$

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}\left(\frac{1}{783}\left(\frac{\sqrt{3}\left(145-14\sqrt{29}\right)\left(16\sqrt{29}\left(\frac{1}{x}+\frac{1}{4}\right)^2+87\right)\sqrt{\frac{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}{\left(16\sqrt{29}\left(\frac{1}{x}+\frac{1}{4}\right)^2+87\right)^2}}\operatorname{EllipticF}\left(2\operatorname{arc}\right.}{8\sqrt[4]{29}\sqrt{256\left(\frac{1}{x}+\frac{1}{4}\right)^4-96\left(\frac{1}{x}+\frac{1}{4}\right)^2+261}}\right.\right.}{}$$

input
```
Int[(8 + 8*x - x^3 + 8*x^4)^(-3/2),x]
```

output
```
-((Sqrt[261 - 96*(1/4 + x^(-1))^2 + 256*(1/4 + x^(-1))^4]*((33 - 8*(1/4 +
x^(-1))^2)/(63*Sqrt[261 - 96*(1/4 + x^(-1))^2 + 256*(1/4 + x^(-1))^4]) - (
16*(27 - 14*(1/4 + x^(-1))^2)*(1/4 + x^(-1)))/(783*Sqrt[261 - 96*(1/4 + x^
(-1))^2 + 256*(1/4 + x^(-1))^4]) + ((14*((-29*Sqrt[261 - 96*(1/4 + x^(-1))
^2 + 256*(1/4 + x^(-1))^4]*(1/4 + x^(-1)))/(87 + 16*Sqrt[29]*(1/4 + x^(-1)
)^2) + (Sqrt[3]*29^(3/4)*(87 + 16*Sqrt[29]*(1/4 + x^(-1))^2)*Sqrt[(261 - 9
6*(1/4 + x^(-1))^2 + 256*(1/4 + x^(-1))^4)/(87 + 16*Sqrt[29]*(1/4 + x^(-1)
)^2)^2]*EllipticE[2*ArcTan[(4*(1/4 + x^(-1)))/(Sqrt[3]*29^(1/4))], (29 + S
qrt[29])/58])/(4*Sqrt[261 - 96*(1/4 + x^(-1))^2 + 256*(1/4 + x^(-1))^4])))
/Sqrt[29] + (Sqrt[3]*(145 - 14*Sqrt[29])*(87 + 16*Sqrt[29]*(1/4 + x^(-1))^
2)*Sqrt[(261 - 96*(1/4 + x^(-1))^2 + 256*(1/4 + x^(-1))^4)/(87 + 16*Sqrt[2
9]*(1/4 + x^(-1))^2)^2]*EllipticF[2*ArcTan[(4*(1/4 + x^(-1)))/(Sqrt[3]*29^
(1/4))], (29 + Sqrt[29])/58])/(8*29^(1/4)*Sqrt[261 - 96*(1/4 + x^(-1))^2 +
 256*(1/4 + x^(-1))^4]))/783))/(Sqrt[2]*(1 - 4*(1/4 + x^(-1)))^2*Sqrt[(261
 - 96*(1/4 + x^(-1))^2 + 256*(1/4 + x^(-1))^4)/(1 - 4*(1/4 + x^(-1)))^4]))
```

3.797.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

3.797. $\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx$

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

3.797. $\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx$

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

3.797.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.70 (sec) , antiderivative size = 4426, normalized size of antiderivative = 10.27

method	result	size
default	Expression too large to display	4426
risch	Expression too large to display	4426
elliptic	Expression too large to display	4426

input
```
int(1/(8*x^4-x^3+8*x+8)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-16*(-17/10962-57/12992*x+191/58464*x^2-7/3132*x^3)/(8*x^4-x^3+8*x+8)^(1/2
)+421/12528*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,
index=4))*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,i
ndex=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8
,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8
,index=2)))^(1/2)*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))^2*((RootOf(8*_Z^4
-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4
-_Z^3+8*_Z+8,index=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)-RootOf(8*_Z^4-_
Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*((RootO
f(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootO
f(8*_Z^4-_Z^3+8*_Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(
8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)
/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))/(
RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*2^(
1/2)/((x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,
index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(x-RootOf(8*_Z^4-_Z^3+8*_
Z+8,index=4)))^(1/2)*EllipticF(((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf
(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(Root
Of(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-Root
Of(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2),((RootOf(8*_Z^4-_Z^3+8*_Z+8,inde...
```

3.797.5 Fricas [F]

$$\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx = \int \frac{1}{(8\,x^4-x^3+8\,x+8)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(8*x^4-x^3+8*x+8)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(8*x^4 - x^3 + 8*x + 8)/(64*x^8 - 16*x^7 + x^6 + 128*x^5 + 11
2*x^4 - 16*x^3 + 64*x^2 + 128*x + 64), x)
```

3.797.6 Sympy [F]

$$\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx = \int \frac{1}{(8x^4-x^3+8x+8)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(8*x**4-x**3+8*x+8)**(3/2),x)
```

output
```
Integral((8*x**4 - x**3 + 8*x + 8)**(-3/2), x)
```

3.797.7 Maxima [F]

$$\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx = \int \frac{1}{(8\,x^4-x^3+8\,x+8)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(8*x^4-x^3+8*x+8)^(3/2),x, algorithm="maxima")
```

output
```
integrate((8*x^4 - x^3 + 8*x + 8)^(-3/2), x)
```

3.797.8 Giac [F]

$$\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx = \int \frac{1}{(8\,x^4-x^3+8\,x+8)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(8*x^4-x^3+8*x+8)^(3/2),x, algorithm="giac")
```

output
```
integrate((8*x^4 - x^3 + 8*x + 8)^(-3/2), x)
```

3.797.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx = \int \frac{1}{\left(8\,x^4-x^3+8\,x+8\right)^{3/2}}\,dx$$

input
```
int(1/(8*x - x^3 + 8*x^4 + 8)^(3/2),x)
```

output
```
int(1/(8*x - x^3 + 8*x^4 + 8)^(3/2), x)
```

3.798 $\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx$

3.798.1 Optimal result

Integrand size = 19, antiderivative size = 108

$$\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx = -\frac{\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)\sqrt{\frac{5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4}{\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)^2}}x^2\,\text{EllipticF}\left(2\arctan\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right),\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{2\sqrt[4]{5}\sqrt{1+4x+4x^2+4x^4}}$$

output
```
-1/10*x^2*(cos(2*arctan(1/5*(1+1/x)*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*(1
+1/x)*5^(3/4)))*EllipticF(sin(2*arctan(1/5*(1+1/x)*5^(3/4))),1/10*(50+10*5
^(1/2))^(1/2))*((1+1/x)^2+5^(1/2))*((5-2*(1+1/x)^2+(1+1/x)^4)/((1+1/x)^2+5
^(1/2))^2)^(1/2)*5^(3/4)/(4*x^4+4*x^2+4*x+1)^(1/2)
```

3.798.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 10.38 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.31

$$\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx = \frac{(2-i)\sqrt{-\frac{1}{10}+\frac{i}{5}}\sqrt{\frac{\left(2i+\sqrt{-1-2i}-\sqrt{-1+2i}\right)\left(-i+\sqrt{-1-2i}-2x\right)}{\left(-2i+\sqrt{-1-2i}+\sqrt{-1+2i}\right)\left(i+\sqrt{-1-2i}+2x\right)}}\left(1+2x+2ix^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(2i+\sqrt{-1-2i}+\sqrt{-1+}}{\sqrt{-1+2i}(i+}}}{\sqrt{}}\right.\right.}{\sqrt{\frac{(1+2i)\left((-1+i)+\sqrt{-1-2i}\right)\left(1+2x+2ix^2\right)}{\left(i+\sqrt{-1-2i}+2x\right)^2}}\sqrt{1+4x+4x^2+4x^4}}$$

input
```
Integrate[1/Sqrt[1 + 4*x + 4*x^2 + 4*x^4],x]
```

output
```
((2 - I)*Sqrt[-1/10 + I/5]*Sqrt[((2*I + Sqrt[-1 - 2*I] - Sqrt[-1 + 2*I])*(
-I + Sqrt[-1 - 2*I] - 2*x))/((-2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(I +
 Sqrt[-1 - 2*I] + 2*x))]*(1 + 2*x + (2*I)*x^2)*EllipticF[ArcSin[Sqrt[((2*I
 + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(-I + Sqrt[-1 + 2*I] + 2*x))/(Sqrt[-1
+ 2*I]*(I + Sqrt[-1 - 2*I] + 2*x))]/Sqrt[2]], (5 - Sqrt[5])/2])/(Sqrt[((1
+ 2*I)*((-1 + I) + Sqrt[-1 - 2*I])*(1 + 2*x + (2*I)*x^2))/(I + Sqrt[-1 - 2
*I] + 2*x)^2]*Sqrt[1 + 4*x + 4*x^2 + 4*x^4])
```

3.798.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2504, 27, 7270, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{4x^4+4x^2+4x+1}}dx$$

$$\Big\downarrow \text{2504}$$

$$-16\int \frac{x^2}{16\sqrt{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)x^4}}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow \text{27}$$

$$-\int \frac{x^2}{\sqrt{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)x^4}}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow \text{7270}$$

$$-\frac{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}\,x^2\int \frac{1}{\sqrt{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}}d\left(1+\frac{1}{x}\right)}{\sqrt{\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)x^4}}$$

$$\Big\downarrow \text{1416}$$

$$-\frac{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{\frac{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)^2}}x^2\,\text{EllipticF}\left(2\arctan\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right),\frac{1}{10}(5+\sqrt{5})\right)}{2\sqrt[4]{5}\sqrt{\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)x^4}}$$

input
```
Int[1/Sqrt[1 + 4*x + 4*x^2 + 4*x^4],x]
```

output
```
-1/2*((Sqrt[5] + (1 + x^(-1))^2)*Sqrt[(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))
^4)/(Sqrt[5] + (1 + x^(-1))^2)^2]*x^2*EllipticF[2*ArcTan[(1 + x^(-1))/5^(1
/4)], (5 + Sqrt[5])/10])/(5^(1/4)*Sqrt[(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1)
)^4)*x^4])
```

3.798.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

3.798. $\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx$

3.798.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.83 (sec) , antiderivative size = 961, normalized size of antiderivative = 8.90

method	result	size
default	Expression too large to display	961
elliptic	Expression too large to display	961

input
```
int(1/(4*x^4+4*x^2+4*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)
)*((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index
=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1
,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*
_Z+1,index=2)))^(1/2)*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))^2*((RootOf(
4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-Roo
tOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3)-R
ootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=
2)))^(1/2)*((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_
Z+1,index=1))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))/(RootOf(4*_Z^4+4*_Z
^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+
4*_Z^2+4*_Z+1,index=2)))^(1/2)/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootO
f(4*_Z^4+4*_Z^2+4*_Z+1,index=2))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-Roo
tOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/((x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1
))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1
,index=3))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2)*EllipticF(((Roo
tOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x
-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=
4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,in
dex=2)))^(1/2),((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_...
```

3.798.5 Fricas [F]

$$\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx = \int \frac{1}{\sqrt{4\,x^4+4\,x^2+4\,x+1}}\,dx$$

input
```
integrate(1/(4*x^4+4*x^2+4*x+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(1/sqrt(4*x^4 + 4*x^2 + 4*x + 1), x)
```

3.798.6 Sympy [F]

$$\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx = \int \frac{1}{\sqrt{4x^4+4x^2+4x+1}}\,dx$$

input
```
integrate(1/(4*x**4+4*x**2+4*x+1)**(1/2),x)
```

output
```
Integral(1/sqrt(4*x**4 + 4*x**2 + 4*x + 1), x)
```

3.798.7 Maxima [F]

$$\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx = \int \frac{1}{\sqrt{4\,x^4+4\,x^2+4\,x+1}}\,dx$$

input
```
integrate(1/(4*x^4+4*x^2+4*x+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(4*x^4 + 4*x^2 + 4*x + 1), x)
```

3.798.8 Giac [F]

$$\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx=\int \frac{1}{\sqrt{4\,x^4+4\,x^2+4\,x+1}}\,dx$$

input
```
integrate(1/(4*x^4+4*x^2+4*x+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(4*x^4 + 4*x^2 + 4*x + 1), x)
```

3.798.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx=\int \frac{1}{\sqrt{4\,x^4+4\,x^2+4\,x+1}}\,dx$$

input
```
int(1/(4*x + 4*x^2 + 4*x^4 + 1)^(1/2),x)
```

output
```
int(1/(4*x + 4*x^2 + 4*x^4 + 1)^(1/2), x)
```

3.799 $\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}}\,dx$

3.799.1 Optimal result

Integrand size = 19, antiderivative size = 367

$$\begin{aligned}\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}}\,dx &= -\frac{\left(3-\left(1+\frac{1}{x}\right)^2\right)x^2}{\sqrt{1+4x+4x^2+4x^4}}\\
&+\frac{\left(13-9\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)x^2}{10\sqrt{1+4x+4x^2+4x^4}}+\frac{9\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)\left(1+\frac{1}{x}\right)x^2}{10\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)\sqrt{1+4x+4x^2+4x^4}}\\
&-\frac{9\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)\sqrt{\frac{5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4}{\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)^2}}x^2E\left(2\arctan\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right)|\frac{1}{10}(5+\sqrt{5})\right)}{2\,5^{3/4}\sqrt{1+4x+4x^2+4x^4}}\\
&+\frac{3(3-\sqrt{5})\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)\sqrt{\frac{5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4}{\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)^2}}x^2\,\mathrm{EllipticF}\left(2\arctan\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right),\frac{1}{10}(5+\sqrt{5})\right)}{4\,5^{3/4}\sqrt{1+4x+4x^2+4x^4}}\end{aligned}$$

output
```
-(3-(1+1/x)^2)*x^2/(4*x^4+4*x^2+4*x+1)^(1/2)+1/10*(13-9*(1+1/x)^2)*(1+1/x)
*x^2/(4*x^4+4*x^2+4*x+1)^(1/2)+9/10*(5-2*(1+1/x)^2+(1+1/x)^4)*(1+1/x)*x^2/
((1+1/x)^2+5^(1/2))/(4*x^4+4*x^2+4*x+1)^(1/2)-9/10*x^2*(cos(2*arctan(1/5*(
1+1/x)*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*(1+1/x)*5^(3/4)))*EllipticE(sin
(2*arctan(1/5*(1+1/x)*5^(3/4))),1/10*(50+10*5^(1/2))^(1/2))*((1+1/x)^2+5^(
1/2))*((5-2*(1+1/x)^2+(1+1/x)^4)/((1+1/x)^2+5^(1/2))^2)^(1/2)*5^(1/4)/(4*x
^4+4*x^2+4*x+1)^(1/2)+3/20*x^2*(cos(2*arctan(1/5*(1+1/x)*5^(3/4)))^2)^(1/2
)/cos(2*arctan(1/5*(1+1/x)*5^(3/4)))*EllipticF(sin(2*arctan(1/5*(1+1/x)*5^
(3/4))),1/10*(50+10*5^(1/2))^(1/2))*(3-5^(1/2))*((1+1/x)^2+5^(1/2))*((5-2*
(1+1/x)^2+(1+1/x)^4)/((1+1/x)^2+5^(1/2))^2)^(1/2)*5^(1/4)/(4*x^4+4*x^2+4*x
+1)^(1/2)
```

3.799.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 12.70 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.64

$$\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}}\,dx = \frac{19+42x-16x^2+36x^3+\frac{9}{2}\left(-i+\sqrt{-1-2i}-2x\right)\left(-i-\sqrt{-1+2i}+2x\right)}{}$$

input
```
Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^(-3/2),x]
```

output
```
(19 + 42*x - 16*x^2 + 36*x^3 + (9*(-I + Sqrt[-1 - 2*I] - 2*x)*(-I - Sqrt[-
1 + 2*I] + 2*x)*(-I + Sqrt[-1 + 2*I] + 2*x))/2 - ((9*I)*Sqrt[-2/5 + (4*I)/
5]*(-2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(2*I + Sqrt[-1 - 2*I] + Sqrt[-
1 + 2*I])*((I + Sqrt[-1 - 2*I])/2 + x)^2*Sqrt[((2*I + Sqrt[-1 - 2*I] - Sqr
t[-1 + 2*I])*(-I + Sqrt[-1 - 2*I] - 2*x))/((-2*I + Sqrt[-1 - 2*I] + Sqrt[-
1 + 2*I])*(I + Sqrt[-1 - 2*I] + 2*x))]*Sqrt[((1 + 2*I)*((-1 + I) + Sqrt[-1
 - 2*I])*(1 + 2*x + (2*I)*x^2))/(I + Sqrt[-1 - 2*I] + 2*x)^2]*EllipticE[Ar
cSin[Sqrt[((2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(-I + Sqrt[-1 + 2*I] +
2*x))/(Sqrt[-1 + 2*I]*(I + Sqrt[-1 - 2*I] + 2*x))]/Sqrt[2]], (5 - Sqrt[5])
/2])/((-1 + I) + Sqrt[-1 - 2*I]) + ((6 - 3*I)*Sqrt[-2/5 + (4*I)/5]*Sqrt[((
2*I + Sqrt[-1 - 2*I] - Sqrt[-1 + 2*I])*(-I + Sqrt[-1 - 2*I] - 2*x))/((-2*I
 + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(I + Sqrt[-1 - 2*I] + 2*x))]*(1 + 2*x
+ (2*I)*x^2)*EllipticF[ArcSin[Sqrt[((2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I]
)*(-I + Sqrt[-1 + 2*I] + 2*x))/(Sqrt[-1 + 2*I]*(I + Sqrt[-1 - 2*I] + 2*x))
]/Sqrt[2]], (5 - Sqrt[5])/2])/Sqrt[((1 + 2*I)*((-1 + I) + Sqrt[-1 - 2*I])*
(1 + 2*x + (2*I)*x^2))/(I + Sqrt[-1 - 2*I] + 2*x)^2])/(10*Sqrt[1 + 4*x + 4
*x^2 + 4*x^4])
```

3.799.3 Rubi [A] (warning: unable to verify)

Time = 0.70 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.09, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {2504, 27, 7270, 2202, 1576, 27, 1158, 2206, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(4x^4+4x^2+4x+1)^{3/2}}dx$$

$$\Big\downarrow \ 2504$$

$$-16\int \frac{x^2}{16\left(\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)x^4\right)^{3/2}}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow \ 27$$

$$-\int \frac{x^2}{\left(\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)x^4\right)^{3/2}}d\left(1+\frac{1}{x}\right)$$

$$\Big\downarrow \ 7270$$

$$-\frac{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}x^2\int \frac{1}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^{3/2}x^4}d\left(1+\frac{1}{x}\right)}{\sqrt{\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)x^4}}$$

$$\Big\downarrow \; 2202$$

$$-\frac{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}x^2\left(\int \frac{\left(1+\frac{1}{x}\right)^4+6\left(1+\frac{1}{x}\right)^2+1}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^{3/2}}d\left(1+\frac{1}{x}\right)+\int \frac{\left(-4\left(1+\frac{1}{x}\right)^2-4\right)\left(1+\frac{1}{x}\right)}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^{3/2}}d\left(1+\frac{1}{x}\right)\right)}{\sqrt{\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)x^4}}$$

$$\Big\downarrow \; 1576$$

$$-\frac{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}x^2\left(\int \frac{\left(1+\frac{1}{x}\right)^4+6\left(1+\frac{1}{x}\right)^2+1}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^{3/2}}d\left(1+\frac{1}{x}\right)+\frac{1}{2}\int -\frac{4\left(2+\frac{1}{x}\right)}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^{3/2}}d\left(1+\frac{1}{x}\right)^2\right)}{\sqrt{\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)x^4}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}x^2\left(\int \frac{\left(1+\frac{1}{x}\right)^4+6\left(1+\frac{1}{x}\right)^2+1}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^{3/2}}d\left(1+\frac{1}{x}\right)-2\int \frac{2+\frac{1}{x}}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^{3/2}}d\left(1+\frac{1}{x}\right)^2\right)}{\sqrt{\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)x^4}}$$

$$\Big\downarrow \; 1158$$

$$-\frac{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}x^2\left(\int \frac{\left(1+\frac{1}{x}\right)^4+6\left(1+\frac{1}{x}\right)^2+1}{\left(\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5\right)^{3/2}}d\left(1+\frac{1}{x}\right)+\frac{2-\frac{1}{x}}{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}}\right)}{\sqrt{\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)x^4}}$$

$$\Big\downarrow \; 2206$$

$$-\frac{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}x^2\left(\frac{1}{80}\int \frac{24\left(5-3\left(1+\frac{1}{x}\right)^2\right)}{\sqrt{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}}d\left(1+\frac{1}{x}\right)+\frac{2-\frac{1}{x}}{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}}-\frac{\left(13-9\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)}{10\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+}}\right)}{\sqrt{\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)x^4}}$$

$$\Big\downarrow \; 27$$

3.799. $\int \frac{1}{\left(1+4x+4x^2+4x^4\right)^{3/2}}\,dx$

$$-\frac{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}\,x^2\left(\frac{3}{10}\int \frac{5-3\left(1+\frac{1}{x}\right)^2}{\sqrt{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}}d\left(1+\frac{1}{x}\right)+\frac{2-\frac{1}{x}}{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}}-\frac{\left(13-9\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)}{10\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+}}\right.}{\sqrt{\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)x^4}}$$

$\downarrow$ 1511

$$-\frac{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}\,x^2\left(\frac{3}{10}\left(\left(5-3\sqrt{5}\right)\int \frac{1}{\sqrt{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}}d\left(1+\frac{1}{x}\right)+3\sqrt{5}\int \frac{\sqrt{5}-\left(1+\frac{1}{x}\right)^2}{\sqrt{5}\sqrt{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}}d\right.\right.}{\sqrt{\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)x^4}}$$

$\downarrow$ 27

$$-\frac{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}\,x^2\left(\frac{3}{10}\left(\left(5-3\sqrt{5}\right)\int \frac{1}{\sqrt{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}}d\left(1+\frac{1}{x}\right)+3\int \frac{\sqrt{5}-\left(1+\frac{1}{x}\right)^2}{\sqrt{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}}d\left(1+\frac{1}{x}\right.\right.\right.}{\sqrt{\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)x^4}}$$

$\downarrow$ 1416

$$-\frac{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}\,x^2\left(\frac{3}{10}\left(3\int \frac{\sqrt{5}-\left(1+\frac{1}{x}\right)^2}{\sqrt{\left(1+\frac{1}{x}\right)^4-2\left(1+\frac{1}{x}\right)^2+5}}d\left(1+\frac{1}{x}\right)+\frac{\left(5-3\sqrt{5}\right)\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{\frac{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)^2}}}{2\sqrt[4]{5}\sqrt{\left(\frac{1}{x}+1\right)^4-2}}\right.\right.}{\sqrt{\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)}}$$

$\downarrow$ 1509

$$-\frac{\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}\,x^2\left(\frac{3}{10}\left(\frac{\left(5-3\sqrt{5}\right)\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{\frac{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right),\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{2\sqrt[4]{5}\sqrt{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}}+\right.\right.}{}$$

input
```
Int[(1 + 4*x + 4*x^2 + 4*x^4)^(-3/2),x]
```

3.799. $\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}}\,dx$

output
```
-((Sqrt[5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4]*x^2*((2 - x^(-1))/Sqrt[5 -
2*(1 + x^(-1))^2 + (1 + x^(-1))^4] - ((13 - 9*(1 + x^(-1))^2)*(1 + x^(-1))
)/(10*Sqrt[5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4]) + (3*(3*(-((Sqrt[5 - 2*
(1 + x^(-1))^2 + (1 + x^(-1))^4]*(1 + x^(-1)))/(Sqrt[5] + (1 + x^(-1))^2))
 + (5^(1/4)*(Sqrt[5] + (1 + x^(-1))^2)*Sqrt[(5 - 2*(1 + x^(-1))^2 + (1 + x
^(-1))^4)/(Sqrt[5] + (1 + x^(-1))^2)^2]*EllipticE[2*ArcTan[(1 + x^(-1))/5^
(1/4)], (5 + Sqrt[5])/10])/Sqrt[5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4]) +
((5 - 3*Sqrt[5])*(Sqrt[5] + (1 + x^(-1))^2)*Sqrt[(5 - 2*(1 + x^(-1))^2 + (
1 + x^(-1))^4)/(Sqrt[5] + (1 + x^(-1))^2)^2]*EllipticF[2*ArcTan[(1 + x^(-1
))/5^(1/4)], (5 + Sqrt[5])/10])/(2*5^(1/4)*Sqrt[5 - 2*(1 + x^(-1))^2 + (1
+ x^(-1))^4])))/10))/Sqrt[(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)*x^4])
```

3.799.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

3.799. $\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}}\,dx$

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

3.799. $\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}} \, dx$

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

3.799.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.81 (sec) , antiderivative size = 2564, normalized size of antiderivative = 6.99

method	result	size
default	Expression too large to display	2564
risch	Expression too large to display	2564
elliptic	Expression too large to display	2564

input
```
int(1/(4*x^4+4*x^2+4*x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-8*(-9/20*x^3+1/5*x^2-21/40*x-19/80)/(4*x^4+4*x^2+4*x+1)^(1/2)+3/5*(RootOf
(4*_Z^4+4*_Z^2+4*_Z+1,index=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))*((Roo
tOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x
-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=
4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,in
dex=2)))^(1/2)*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))^2*((RootOf(4*_Z^4+
4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_
Z^4+4*_Z^2+4*_Z+1,index=3))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3)-RootOf(4
*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1
/2)*((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,ind
ex=1))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))/(RootOf(4*_Z^4+4*_Z^2+4*_Z
+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+
4*_Z+1,index=2)))^(1/2)/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^
4+4*_Z^2+4*_Z+1,index=2))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_
Z^4+4*_Z^2+4*_Z+1,index=1))/((x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-R
ootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=
3))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2)*EllipticF(((RootOf(4*_
Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf
(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-Root
Of(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=...
```

3.799.5 Fricas [F]

$$\int \frac{1}{\left(1+4x+4x^2+4x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(4\,x^4+4\,x^2+4\,x+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(4*x^4+4*x^2+4*x+1)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(4*x^4 + 4*x^2 + 4*x + 1)/(16*x^8 + 32*x^6 + 32*x^5 + 24*x^4
+ 32*x^3 + 24*x^2 + 8*x + 1), x)
```

3.799.6 Sympy [F]

$$\int \frac{1}{\left(1+4x+4x^2+4x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(4x^4+4x^2+4x+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(4*x**4+4*x**2+4*x+1)**(3/2),x)
```

output
```
Integral((4*x**4 + 4*x**2 + 4*x + 1)**(-3/2), x)
```

3.799.7 Maxima [F]

$$\int \frac{1}{\left(1+4x+4x^2+4x^4\right)^{3/2}}\,dx=\int \frac{1}{\left(4\,x^4+4\,x^2+4\,x+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(4*x^4+4*x^2+4*x+1)^(3/2),x, algorithm="maxima")
```

output
```
integrate((4*x^4 + 4*x^2 + 4*x + 1)^(-3/2), x)
```

3.799.8 Giac [F]

$$\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}}\,dx = \int \frac{1}{(4\,x^4+4\,x^2+4\,x+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(4*x^4+4*x^2+4*x+1)^(3/2),x, algorithm="giac")
```

output
```
integrate((4*x^4 + 4*x^2 + 4*x + 1)^(-3/2), x)
```

3.799.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}}\,dx = \int \frac{1}{(4\,x^4+4\,x^2+4\,x+1)^{3/2}}\,dx$$

input
```
int(1/(4*x + 4*x^2 + 4*x^4 + 1)^(3/2),x)
```

output
```
int(1/(4*x + 4*x^2 + 4*x^4 + 1)^(3/2), x)
```

3.800 $\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx$

3.800.1 Optimal result

Integrand size = 24, antiderivative size = 126

$$\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx = -\frac{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{\frac{517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4}{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)^2}}\,x^2\,\text{EllipticF}\left(2\arctan\left(\frac{4+3x}{\sqrt[4]{517}x}\right),\frac{517+19\sqrt{517}}{1034}\right)}{8\sqrt[4]{517}\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

output
```
-1/4136*x^2*(cos(2*arctan(1/517*(4+3*x)*517^(3/4)/x))^2)^(1/2)/cos(2*arcta
n(1/517*(4+3*x)*517^(3/4)/x))*EllipticF(sin(2*arctan(1/517*(4+3*x)*517^(3/
4)/x)),1/1034*(534578+19646*517^(1/2))^(1/2))*((3+4/x)^2+517^(1/2))*((517-
38*(3+4/x)^2+(3+4/x)^4)/((3+4/x)^2+517^(1/2))^2)^(1/2)*517^(3/4)/(8*x^4-15
*x^3+8*x^2+24*x+8)^(1/2)
```

3.800.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 10.62 (sec) , antiderivative size = 1148, normalized size of antiderivative = 9.11

$$\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx = \text{Too large to display}$$

input
```
Integrate[1/Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4],x]
```

output
```
(-2*EllipticF[ArcSin[Sqrt[((x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4
 & , 1, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0] - Root[8
 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 24*#1 + 8*
#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*
#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0]))]],
 ((Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0] - Root[8 + 24*#1 +
 8*#1^2 - 15*#1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 +
 8*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0]))
/((Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1 +
 8*#1^2 - 15*#1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 +
 8*#1^4 & , 2, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0]))
]*(x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0])^2*Sqrt[((Root
[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2
 - 15*#1^3 + 8*#1^4 & , 2, 0])*(x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*
#1^4 & , 3, 0]))/((x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0
])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1
+ 8*#1^2 - 15*#1^3 + 8*#1^4 & , 3, 0]))]*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^
3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0
])*Sqrt[((x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0])*(Root[
8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1...
```

3.800.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2504, 27, 7270, 1416}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{8x^4-15x^3+8x^2+24x+8}}dx$$

$$\Big\downarrow \text{2504}$$

$$-1024\int \frac{1}{128\sqrt{2}\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^2\sqrt{\frac{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^4}}}d\left(\frac{3}{4}+\frac{1}{x}\right)$$

$$\Big\downarrow \text{27}$$

$$-4\sqrt{2}\int \frac{1}{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^2\sqrt{\frac{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^4}}}d\left(\frac{3}{4}+\frac{1}{x}\right)$$

$$\Big\downarrow \text{7270}$$

$$-\frac{4\sqrt{2}\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\int \frac{1}{\sqrt{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^4}}}$$

$$\Big\downarrow \text{1416}$$

$$-\frac{\left(16\left(\frac{1}{x}+\frac{3}{4}\right)^2+\sqrt{517}\right)\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(16\left(\frac{1}{x}+\frac{3}{4}\right)^2+\sqrt{517}\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{4\left(\frac{3}{4}+\frac{1}{x}\right)}{\sqrt[4]{517}}\right),\frac{517+19\sqrt{517}}{1034}\right)}{\sqrt{2}\sqrt[4]{517}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^4}}}$$

input
```
Int[1/Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4],x]
```

output
```
-(((Sqrt[517] + 16*(3/4 + x^(-1))^2)*Sqrt[(517 - 608*(3/4 + x^(-1))^2 + 25
6*(3/4 + x^(-1))^4)/(Sqrt[517] + 16*(3/4 + x^(-1))^2)^2]*EllipticF[2*ArcTa
n[(4*(3/4 + x^(-1)))/517^(1/4)], (517 + 19*Sqrt[517])/1034])/(Sqrt[2]*517^
(1/4)*(3 - 4*(3/4 + x^(-1)))^2*Sqrt[(517 - 608*(3/4 + x^(-1))^2 + 256*(3/4
 + x^(-1))^4)/(3 - 4*(3/4 + x^(-1)))^4]))
```

3.800.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

3.800.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.16 (sec) , antiderivative size = 1180, normalized size of antiderivative = 9.37

method	result	size
default	Expression too large to display	1180
elliptic	Expression too large to display	1180

input
```
int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=1)-RootOf(8*_Z^4-15*_Z^3+8
*_Z^2+24*_Z+8,index=4))*((RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-Ro
otOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z
^2+24*_Z+8,index=1))/(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-RootOf
(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=1))/(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+2
4*_Z+8,index=2)))^(1/2)*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2))^
2*((RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-RootOf(8*_Z^4-15*_Z^3+8*
_Z^2+24*_Z+8,index=1))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=3))/(
RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=3)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2
+24*_Z+8,index=1))/(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)))^(1/2
)*((RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-RootOf(8*_Z^4-15*_Z^3+8*
_Z^2+24*_Z+8,index=1))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4))/(
RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2
+24*_Z+8,index=1))/(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)))^(1/2
)/(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-RootOf(8*_Z^4-15*_Z^3+8*_
Z^2+24*_Z+8,index=2))/(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-RootO
f(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=1))*2^(1/2)/((x-RootOf(8*_Z^4-15*_Z^
3+8*_Z^2+24*_Z+8,index=1))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2
))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=3))*(x-RootOf(8*_Z^4-15*_
Z^3+8*_Z^2+24*_Z+8,index=4)))^(1/2)*EllipticF(((RootOf(8*_Z^4-15*_Z^3+8...
```

3.800.5 Fricas [F]

$$\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx=\int \frac{1}{\sqrt{8\,x^4-15\,x^3+8\,x^2+24\,x+8}}\,dx$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2),x, algorithm="fricas")
```

output
```
integral(1/sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

3.800.6 Sympy [F]

$$\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx=\int \frac{1}{\sqrt{8x^4-15x^3+8x^2+24x+8}}\,dx$$

input
```
integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**(1/2),x)
```

output
```
Integral(1/sqrt(8*x**4 - 15*x**3 + 8*x**2 + 24*x + 8), x)
```

3.800.7 Maxima [F]

$$\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx=\int \frac{1}{\sqrt{8\,x^4-15\,x^3+8\,x^2+24\,x+8}}\,dx$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

3.800.8 Giac [F]

$$\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx = \int \frac{1}{\sqrt{8\,x^4-15\,x^3+8\,x^2+24\,x+8}}\,dx$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

3.800.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx = \int \frac{1}{\sqrt{8\,x^4-15\,x^3+8\,x^2+24\,x+8}}\,dx$$

input
```
int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(1/2),x)
```

output
```
int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(1/2), x)
```

3.801 $\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx$

3.801.1 Optimal result

Integrand size = 24, antiderivative size = 434

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx = -\frac{\left(172-7\left(3+\frac{4}{x}\right)^2\right)x^2}{208\sqrt{8+24x+8x^2-15x^3+8x^4}}+\frac{\left(50896-2455\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)x^2}{322608\sqrt{8+24x+8x^2-15x^3+8x^4}}$$
$$+\frac{2455\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)\left(3+\frac{4}{x}\right)x^2}{322608\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{8+24x+8x^2-15x^3+8x^4}}$$
$$-\frac{2455\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{\frac{517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4}{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)^2}}x^2E\left(2\arctan\left(\frac{4+3x}{\sqrt[4]{517}x}\right)\middle|\frac{517+19\sqrt{517}}{1034}\right)}{624\ 517^{3/4}\sqrt{8+24x+8x^2-15x^3+8x^4}}$$
$$+\frac{\left(4910-203\sqrt{517}\right)\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{\frac{517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4}{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)^2}}x^2\,\text{EllipticF}\left(2\arctan\left(\frac{4+3x}{\sqrt[4]{517}x}\right),\frac{517+19\sqrt{517}}{1034}\right)}{2496\ 517^{3/4}\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

output
```
-1/208*(172-7*(3+4/x)^2)*x^2/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)+1/322608*(5
0896-2455*(3+4/x)^2)*(3+4/x)*x^2/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)+2455/32
2608*(517-38*(3+4/x)^2+(3+4/x)^4)*(3+4/x)*x^2/((3+4/x)^2+517^(1/2))/(8*x^4
-15*x^3+8*x^2+24*x+8)^(1/2)-2455/322608*x^2*(cos(2*arctan(1/517*(4+3*x)*51
7^(3/4)/x))^2)^(1/2)/cos(2*arctan(1/517*(4+3*x)*517^(3/4)/x))*EllipticE(si
n(2*arctan(1/517*(4+3*x)*517^(3/4)/x)),1/1034*(534578+19646*517^(1/2))^(1/
2))*((3+4/x)^2+517^(1/2))*((517-38*(3+4/x)^2+(3+4/x)^4)/((3+4/x)^2+517^(1/
2))^2)^(1/2)*517^(1/4)/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)+1/1290432*x^2*(co
s(2*arctan(1/517*(4+3*x)*517^(3/4)/x))^2)^(1/2)/cos(2*arctan(1/517*(4+3*x)
*517^(3/4)/x))*EllipticF(sin(2*arctan(1/517*(4+3*x)*517^(3/4)/x)),1/1034*(
534578+19646*517^(1/2))^(1/2))*(4910-203*517^(1/2))*((3+4/x)^2+517^(1/2))*
((517-38*(3+4/x)^2+(3+4/x)^4)/((3+4/x)^2+517^(1/2))^2)^(1/2)*517^(1/4)/(8*
x^4-15*x^3+8*x^2+24*x+8)^(1/2)
```

3.801.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 16.10 (sec) , antiderivative size = 6019, normalized size of antiderivative = 13.87

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-3/2),x]
```

output
```
Result too large to show
```

3.801.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.22, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {2504, 27, 7270, 2202, 1576, 27, 1158, 2206, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(8x^4-15x^3+8x^2+24x+8)^{3/2}}\,dx$$

$$\Big\downarrow \; 2504$$

$$-1024\int \frac{1}{1024\sqrt{2}\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^2\left(\frac{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^4}\right)^{3/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)$$

$\Big\downarrow$ 27

$$-\frac{\int \frac{1}{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^2\left(\frac{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^4}\right)^{3/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{\sqrt{2}}$$

$\Big\downarrow$ 7270

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\int \frac{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^4}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^{3/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^4}}}$$

$\Big\downarrow$ 2202

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\int \frac{256\left(\frac{3}{4}+\frac{1}{x}\right)^4+864\left(\frac{3}{4}+\frac{1}{x}\right)^2+81}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^{3/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)+\int \frac{\left(-768\left(\frac{3}{4}+\frac{1}{x}\right)^2-432\right)\left(\frac{3}{4}+\frac{1}{x}\right)\cdots}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)\cdots}\cdots\right.}{\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^4}}}$$

$\Big\downarrow$ 1576

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{1}{2}\int -\frac{48\left(16\left(\frac{3}{4}+\frac{1}{x}\right)^2+9\right)}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^{3/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)^2+\int \frac{256\left(\frac{3}{4}+\frac{1}{x}\right)^4+864\left(\frac{3}{4}+\frac{1}{x}\right)\cdots}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2\cdots}\cdots\right.}{\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^4}}}$$

$\Big\downarrow$ 27

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\int \frac{256\left(\frac{3}{4}+\frac{1}{x}\right)^4+864\left(\frac{3}{4}+\frac{1}{x}\right)^2+81}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^{3/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)-24\int \frac{16\left(\frac{3}{4}+\frac{1}{x}\right)^2+9}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+5\cdots}\cdots\right.}{\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^4}}}$$

$\Big\downarrow$ 1158

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\int \frac{256\left(\frac{3}{4}+\frac{1}{x}\right)^4+864\left(\frac{3}{4}+\frac{1}{x}\right)^2+81}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^{3/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)+\frac{2\left(43-28\left(\frac{1}{x}+\frac{3}{4}\right)^2\right)\cdots}{13\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\cdots}\right.}{\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^4}}}$$

3.801. $\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx$

$$\Big\downarrow \text{2206}$$

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{\int \frac{4096\left(104951-78560\left(\frac{3}{4}+\frac{1}{x}\right)^2\right)}{\sqrt{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{82587648}+\frac{2\left(43-28\left(\frac{1}{x}+\frac{3}{4}\right)^2\right)}{13\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}}-\frac{3}{2016\ldots}\right.}{\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^4}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{\int \frac{104951-78560\left(\frac{3}{4}+\frac{1}{x}\right)^2}{\sqrt{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{20163}+\frac{2\left(43-28\left(\frac{1}{x}+\frac{3}{4}\right)^2\right)}{13\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}}-\frac{3}{2016\ldots}\right.}{\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^4}}}$$

$$\Big\downarrow \text{1511}$$

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{\left(104951-4910\sqrt{517}\right)\int \frac{1}{\sqrt{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}}d\left(\frac{3}{4}+\frac{1}{x}\right)+4910\sqrt{517}\int \frac{\sqrt{51\ldots}}{\sqrt{517}\sqrt{256\left(\frac{3}{4}\ldots\right.}}}{20163}\right.}{\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4\ldots}{\left(3-\ldots\right.}}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{\left(104951-4910\sqrt{517}\right)\int \frac{1}{\sqrt{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}}d\left(\frac{3}{4}+\frac{1}{x}\right)+4910\int \frac{\sqrt{517}-16\left(\frac{3}{4}\ldots\right.}{\sqrt{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\ldots}}}{20163}\right.}{\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\ldots}{\left(3-4\left(\frac{1}{x}+\ldots\right.\right.}}}$$

$$\Big\downarrow \text{1416}$$

3.801. $\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx$

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{4910\int\frac{\sqrt{517}-16\left(\frac{3}{4}+\frac{1}{x}\right)^2}{\sqrt{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}}d\left(\frac{3}{4}+\frac{1}{x}\right)+\frac{\left(104951-4910\sqrt{517}\right)\left(16\left(\frac{1}{x}+\frac{3}{4}\right)^2+\sqrt{517}\right)\sqrt{\frac{25\cdots}{\cdots}}}{8\sqrt[4]{5\cdots}}}{20163}\cdots\right.}{\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\cdots\right.}$$

$\downarrow$ 1509

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{\left(104951-4910\sqrt{517}\right)\left(16\left(\frac{1}{x}+\frac{3}{4}\right)^2+\sqrt{517}\right)\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(16\left(\frac{1}{x}+\frac{3}{4}\right)^2+\sqrt{517}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{4\left(\cdots\right)}{\sqrt[4]{\cdots}}\right)\cdots\right)}{8\sqrt[4]{517}\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}}\cdots\right.}{\cdots}$$

input
```
Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-3/2),x]
```

output
```
-((Sqrt[517 - 608*(3/4 + x^(-1))^2 + 256*(3/4 + x^(-1))^4]*((2*(43 - 28*(3
/4 + x^(-1))^2))/(13*Sqrt[517 - 608*(3/4 + x^(-1))^2 + 256*(3/4 + x^(-1))^
4]) - (32*(3181 - 2455*(3/4 + x^(-1))^2)*(3/4 + x^(-1)))/(20163*Sqrt[517 -
 608*(3/4 + x^(-1))^2 + 256*(3/4 + x^(-1))^4]) + (4910*((-517*Sqrt[517 - 6
08*(3/4 + x^(-1))^2 + 256*(3/4 + x^(-1))^4]*(3/4 + x^(-1)))/(517*Sqrt[517]
 + 8272*(3/4 + x^(-1))^2) + (517^(1/4)*(Sqrt[517] + 16*(3/4 + x^(-1))^2)*S
qrt[(517 - 608*(3/4 + x^(-1))^2 + 256*(3/4 + x^(-1))^4)/(Sqrt[517] + 16*(3
/4 + x^(-1))^2)^2]*EllipticE[2*ArcTan[(4*(3/4 + x^(-1)))/517^(1/4)], (517
+ 19*Sqrt[517])/1034])/(4*Sqrt[517 - 608*(3/4 + x^(-1))^2 + 256*(3/4 + x^(
-1))^4])) + ((104951 - 4910*Sqrt[517])*(Sqrt[517] + 16*(3/4 + x^(-1))^2)*S
qrt[(517 - 608*(3/4 + x^(-1))^2 + 256*(3/4 + x^(-1))^4)/(Sqrt[517] + 16*(3
/4 + x^(-1))^2)^2]*EllipticF[2*ArcTan[(4*(3/4 + x^(-1)))/517^(1/4)], (517
+ 19*Sqrt[517])/1034])/(8*517^(1/4)*Sqrt[517 - 608*(3/4 + x^(-1))^2 + 256*
(3/4 + x^(-1))^4]))/20163))/(Sqrt[2]*(3 - 4*(3/4 + x^(-1)))^2*Sqrt[(517 -
608*(3/4 + x^(-1))^2 + 256*(3/4 + x^(-1))^4)/(3 - 4*(3/4 + x^(-1)))^4]))
```

3.801. $\int\frac{1}{\left(8+24x+8x^2-15x^3+8x^4\right)^{3/2}}\,dx$

3.801.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

3.801. $\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx$

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

3.801.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.99 (sec) , antiderivative size = 5421, normalized size of antiderivative = 12.49

method	result	size
default	Expression too large to display	5421
risch	Expression too large to display	5421
elliptic	Expression too large to display	5421

input
```
int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.801. $\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx$

3.801.5 Fricas [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx=\int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)/(64*x^8 - 240*x^7 + 353*x
^6 + 144*x^5 - 528*x^4 + 144*x^3 + 704*x^2 + 384*x + 64), x)
```

3.801.6 Sympy [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx=\int \frac{1}{(8x^4-15x^3+8x^2+24x+8)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**(3/2),x)
```

output
```
Integral((8*x**4 - 15*x**3 + 8*x**2 + 24*x + 8)**(-3/2), x)
```

3.801.7 Maxima [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx=\int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(3/2),x, algorithm="maxima")
```

output
```
integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-3/2), x)
```

3.801.8 Giac [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx = \int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(3/2),x, algorithm="giac")
```

output
```
integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-3/2), x)
```

3.801.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx = \int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^{3/2}}\,dx$$

input
```
int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(3/2),x)
```

output
```
int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(3/2), x)
```

3.802 $\int \frac{1}{\left(8+24x+8x^2-15x^3+8x^4\right)^{5/2}}\,dx$

3.802.1 Optimal result

Integrand size = 24, antiderivative size = 577

$$\int \frac{1}{\left(8+24x+8x^2-15x^3+8x^4\right)^{5/2}}\,dx = -\frac{\left(124415-6308\left(3+\frac{4}{x}\right)^2\right)x^2}{97344\sqrt{8+24x+8x^2-15x^3+8x^4}}$$
$$-\frac{\left(64489-1399\left(3+\frac{4}{x}\right)^2\right)x^2}{624\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)\sqrt{8+24x+8x^2-15x^3+8x^4}}$$
$$+\frac{\left(18932921731-1086525994\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)x^2}{78056941248\sqrt{8+24x+8x^2-15x^3+8x^4}}$$
$$+\frac{\left(11921698-359497\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)x^2}{483912\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)\sqrt{8+24x+8x^2-15x^3+8x^4}}$$
$$+\frac{543262997\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)\left(3+\frac{4}{x}\right)x^2}{39028470624\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{8+24x+8x^2-15x^3+8x^4}}$$
$$-\frac{543262997\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{\frac{517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4}{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)^2}}\,x^2 E\left(2\arctan\left(\frac{4+3x}{\sqrt[4]{517}x}\right)|\frac{517+19\sqrt{517}}{1034}\right)}{75490272\;517^{3/4}\sqrt{8+24x+8x^2-15x^3+8x^4}}$$
$$+\frac{\left(4346103976-175318963\sqrt{517}\right)\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{\frac{517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4}{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)^2}}\,x^2\,\text{EllipticF}\left(2\arctan\left(\frac{4+3x}{\sqrt[4]{517}\right.\right.}{1207844352\;517^{3/4}\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

output
```
-1/97344*(124415-6308*(3+4/x)^2)*x^2/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)-1/6
24*(64489-1399*(3+4/x)^2)*x^2/(517-38*(3+4/x)^2+(3+4/x)^4)/(8*x^4-15*x^3+8
*x^2+24*x+8)^(1/2)+1/78056941248*(18932921731-1086525994*(3+4/x)^2)*(3+4/x
)*x^2/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)+1/483912*(11921698-359497*(3+4/x)^
2)*(3+4/x)*x^2/(517-38*(3+4/x)^2+(3+4/x)^4)/(8*x^4-15*x^3+8*x^2+24*x+8)^(1
/2)+543262997/39028470624*(517-38*(3+4/x)^2+(3+4/x)^4)*(3+4/x)*x^2/((3+4/x
)^2+517^(1/2))/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)-543262997/39028470624*x^2
*(cos(2*arctan(1/517*(4+3*x)*517^(3/4)/x))^2)^(1/2)/cos(2*arctan(1/517*(4+
3*x)*517^(3/4)/x))*EllipticE(sin(2*arctan(1/517*(4+3*x)*517^(3/4)/x)),1/10
34*(534578+19646*517^(1/2))^(1/2))*((3+4/x)^2+517^(1/2))*((517-38*(3+4/x)^
2+(3+4/x)^4)/((3+4/x)^2+517^(1/2))^2)^(1/2)*517^(1/4)/(8*x^4-15*x^3+8*x^2+
24*x+8)^(1/2)+1/624455529984*x^2*(cos(2*arctan(1/517*(4+3*x)*517^(3/4)/x))
^2)^(1/2)/cos(2*arctan(1/517*(4+3*x)*517^(3/4)/x))*EllipticF(sin(2*arctan(
1/517*(4+3*x)*517^(3/4)/x)),1/1034*(534578+19646*517^(1/2))^(1/2))*(434610
3976-175318963*517^(1/2))*((3+4/x)^2+517^(1/2))*((517-38*(3+4/x)^2+(3+4/x)
^4)/((3+4/x)^2+517^(1/2))^2)^(1/2)*517^(1/4)/(8*x^4-15*x^3+8*x^2+24*x+8)^(
1/2)
```

3.802.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 16.10 (sec) , antiderivative size = 6084, normalized size of antiderivative = 10.54

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-5/2),x]
```

output
```
Result too large to show
```

3.802.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 636, normalized size of antiderivative = 1.10, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.708$, Rules used = {2504, 27, 7270, 2202, 2194, 27, 2191, 27, 1158, 2206, 27, 2206, 27, 1511, 27, 1416, 1509}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.802. $\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx$

$$\int \frac{1}{(8x^4-15x^3+8x^2+24x+8)^{5/2}}dx$$

$$\Big\downarrow \; 2504$$

$$-1024\int \frac{1}{8192\sqrt{2}\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^2\left(\frac{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^4}\right)^{5/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{1}{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^2\left(\frac{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^4}\right)^{5/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{8\sqrt{2}}$$

$$\Big\downarrow \; 7270$$

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\int \frac{\left(3-4\left(\frac{3}{4}+\frac{1}{x}\right)\right)^8}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^{5/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{8\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^4}}}$$

$$\Big\downarrow \; 2202$$

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\int \frac{65536\left(\frac{3}{4}+\frac{1}{x}\right)^8+1032192\left(\frac{3}{4}+\frac{1}{x}\right)^6+1451520\left(\frac{3}{4}+\frac{1}{x}\right)^4+326592\left(\frac{3}{4}+\frac{1}{x}\right)^2+6561}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^{5/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)+\right.}{8\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^4}}}$$

$$\Big\downarrow \; 2194$$

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{1}{2}\int -\frac{96\left(4096\left(\frac{3}{4}+\frac{1}{x}\right)^6+16128\left(\frac{3}{4}+\frac{1}{x}\right)^4+9072\left(\frac{3}{4}+\frac{1}{x}\right)^2+729\right)}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^{5/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)^2+\int \frac{65536\left(\frac{3}{4}+\right.}{}\right.}{8\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^4}}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\int \frac{65536\left(\frac{3}{4}+\frac{1}{x}\right)^8+1032192\left(\frac{3}{4}+\frac{1}{x}\right)^6+1451520\left(\frac{3}{4}+\frac{1}{x}\right)^4+326592\left(\frac{3}{4}+\frac{1}{x}\right)^2+6561}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^{5/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)-\right.}{8\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^4}}}$$

$$\Big\downarrow \; 2191$$

3.802. $\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}dx$

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\int \frac{65536\left(\frac{3}{4}+\frac{1}{x}\right)^8+1032192\left(\frac{3}{4}+\frac{1}{x}\right)^6+1451520\left(\frac{3}{4}+\frac{1}{x}\right)^4+326592\left(\frac{3}{4}+\frac{1}{x}\right)^2+6561}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^{5/2}}d\left(\frac{3}{4}+\frac{1}{x}\right) \cdots}{8\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\cdots\right.}{\left(3-4\left(\frac{1}{x}+\cdots\right.\right.}}}$$

$\downarrow$ 27

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\int \frac{65536\left(\frac{3}{4}+\frac{1}{x}\right)^8+1032192\left(\frac{3}{4}+\frac{1}{x}\right)^6+1451520\left(\frac{3}{4}+\frac{1}{x}\right)^4+326592\left(\frac{3}{4}+\frac{1}{x}\right)^2+6561}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^{5/2}}d\left(\frac{3}{4}+\frac{1}{x}\right) \cdots}{8\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-\cdots}{\left(3-4\cdots\right.}}}$$

$\downarrow$ 1158

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\int \frac{65536\left(\frac{3}{4}+\frac{1}{x}\right)^8+1032192\left(\frac{3}{4}+\frac{1}{x}\right)^6+1451520\left(\frac{3}{4}+\frac{1}{x}\right)^4+326592\left(\frac{3}{4}+\frac{1}{x}\right)^2+6561}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^{5/2}}d\left(\frac{3}{4}+\frac{1}{x}\right) \cdots}{8\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\cdots\right.}{\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)\cdots}}}$$

$\downarrow$ 2206

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{\int \frac{4096\left(15485184\left(\frac{3}{4}+\frac{1}{x}\right)^4+832856352\left(\frac{3}{4}+\frac{1}{x}\right)^2+382261973\right)}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^{3/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{247762944}-48\left(-\frac{124415-\cdots}{73008\sqrt{256\left(\frac{1}{x}\cdots\right.}}\cdots\right.\right.}{8\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)\cdots}{\left(3\cdots\right.}}}$$

$\downarrow$ 27

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{\int \frac{15485184\left(\frac{3}{4}+\frac{1}{x}\right)^4+832856352\left(\frac{3}{4}+\frac{1}{x}\right)^2+382261973}{\left(256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517\right)^{3/2}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{60489}-48\left(-\frac{124415-10092\cdots}{73008\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-\cdots}}\cdots\right.\right.}{8\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-\cdots}{\left(3-4\cdots\right.}}}$$

$\downarrow$ 2206

3.802. $\int \frac{1}{\left(8+24x+8x^2-15x^3+8x^4\right)^{5/2}}\,dx$

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{\frac{\int \frac{4096\left(90639903871-69537663616\left(\frac{3}{4}+\frac{1}{x}\right)^2\right)}{\sqrt{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{82587648}-\frac{4\left(18932921731-17384415904\left(\frac{1}{x}+\frac{3}{4}\right)^2\right)\left(\frac{1}{x}\right.}{20163\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}}}{60489}\right.}{8\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)}$$

$\downarrow$ 27

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{\frac{\int \frac{90639903871-69537663616\left(\frac{3}{4}+\frac{1}{x}\right)^2}{\sqrt{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}}d\left(\frac{3}{4}+\frac{1}{x}\right)}{20163}-\frac{4\left(18932921731-17384415904\left(\frac{1}{x}+\frac{3}{4}\right)^2\right)\left(\frac{1}{x}+\frac{3}{4}\right)}{20163\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}}}{60489}-\right.}{8\sqrt{2}\left(3-4\left(\frac{1}{x}+\frac{3}{4}\right)\right)^2\sqrt{}}$$

$\downarrow$ 1511

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{\frac{\left(90639903871-4346103976\sqrt{517}\right)\int \frac{1}{\sqrt{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}}d\left(\frac{3}{4}+\frac{1}{x}\right)+4346103976\sqrt{517}\int -}{20163}}{6048}\right.}{}$$

$\downarrow$ 27

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{\frac{\left(90639903871-4346103976\sqrt{517}\right)\int \frac{1}{\sqrt{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}}d\left(\frac{3}{4}+\frac{1}{x}\right)+4346103976\int \frac{}{\sqrt{256(}}}{20163}}{60489}\right.}{}$$

$\downarrow$ 1416

3.802. $\int \frac{1}{\left(8+24x+8x^2-15x^3+8x^4\right)^{5/2}}\,dx$

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{4346103976\int\frac{\sqrt{517}-16\left(\frac{3}{4}+\frac{1}{x}\right)^2}{\sqrt{256\left(\frac{3}{4}+\frac{1}{x}\right)^4-608\left(\frac{3}{4}+\frac{1}{x}\right)^2+517}}d\left(\frac{3}{4}+\frac{1}{x}\right)+\frac{\left(90639903871-4346103976\sqrt{517}\right)\left(16\left(\frac{1}{x}+\right.\right.}{}}{20163}\right.}{}$$

$\downarrow$ 1509

$$-\frac{\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}\left(\frac{\left(90639903871-4346103976\sqrt{517}\right)\left(16\left(\frac{1}{x}+\frac{3}{4}\right)^2+\sqrt{517}\right)\sqrt{\frac{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}{\left(16\left(\frac{1}{x}+\frac{3}{4}\right)^2+\sqrt{517}\right)^2}}\text{EllipticF}\left(\right.}{8\sqrt[4]{517}\sqrt{256\left(\frac{1}{x}+\frac{3}{4}\right)^4-608\left(\frac{1}{x}+\frac{3}{4}\right)^2+517}}\right.}{}$$

input
```
Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-5/2),x]
```

3.802. $\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx$

output
```
-1/8*(Sqrt[517 - 608*(3/4 + x^(-1))^2 + 256*(3/4 + x^(-1))^4]*(-48*(-1/468
*(64489 - 22384*(3/4 + x^(-1))^2)/(517 - 608*(3/4 + x^(-1))^2 + 256*(3/4 +
 x^(-1))^4)^(3/2) - (124415 - 100928*(3/4 + x^(-1))^2)/(73008*Sqrt[517 - 6
08*(3/4 + x^(-1))^2 + 256*(3/4 + x^(-1))^4])) - (64*(5960849 - 2875976*(3/
4 + x^(-1))^2)*(3/4 + x^(-1)))/(60489*(517 - 608*(3/4 + x^(-1))^2 + 256*(3
/4 + x^(-1))^4)^(3/2)) + ((-4*(18932921731 - 17384415904*(3/4 + x^(-1))^2)
*(3/4 + x^(-1)))/(20163*Sqrt[517 - 608*(3/4 + x^(-1))^2 + 256*(3/4 + x^(-1
))^4]) + (4346103976*((-517*Sqrt[517 - 608*(3/4 + x^(-1))^2 + 256*(3/4 + x
^(-1))^4]*(3/4 + x^(-1)))/(517*Sqrt[517] + 8272*(3/4 + x^(-1))^2) + (517^(
1/4)*(Sqrt[517] + 16*(3/4 + x^(-1))^2)*Sqrt[(517 - 608*(3/4 + x^(-1))^2 +
256*(3/4 + x^(-1))^4)/(Sqrt[517] + 16*(3/4 + x^(-1))^2)^2]*EllipticE[2*Arc
Tan[(4*(3/4 + x^(-1)))/517^(1/4)], (517 + 19*Sqrt[517])/1034])/(4*Sqrt[517
 - 608*(3/4 + x^(-1))^2 + 256*(3/4 + x^(-1))^4])) + ((90639903871 - 434610
3976*Sqrt[517])*(Sqrt[517] + 16*(3/4 + x^(-1))^2)*Sqrt[(517 - 608*(3/4 + x
^(-1))^2 + 256*(3/4 + x^(-1))^4)/(Sqrt[517] + 16*(3/4 + x^(-1))^2)^2]*Elli
pticF[2*ArcTan[(4*(3/4 + x^(-1)))/517^(1/4)], (517 + 19*Sqrt[517])/1034])/
(8*517^(1/4)*Sqrt[517 - 608*(3/4 + x^(-1))^2 + 256*(3/4 + x^(-1))^4]))/201
63)/60489))/(Sqrt[2]*(3 - 4*(3/4 + x^(-1)))^2*Sqrt[(517 - 608*(3/4 + x^(-1
))^2 + 256*(3/4 + x^(-1))^4)/(3 - 4*(3/4 + x^(-1)))^4])
```

3.802.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

3.802. $\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx$

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2194
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Simp[1/2   Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)
^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ
[(m - 1)/2]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

3.802. $\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx$

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

3.802.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.84 (sec) , antiderivative size = 5441, normalized size of antiderivative = 9.43

method	result	size
risch	Expression too large to display	5441
default	Expression too large to display	5477
elliptic	Expression too large to display	5477

input
```
int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.802.5 Fricas [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx = \int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)/(512*x^12 - 2880*x^11 + 6
936*x^10 - 4527*x^9 - 8808*x^8 + 16776*x^7 + 5528*x^6 - 17856*x^5 - 384*x^
4 + 20160*x^3 + 15360*x^2 + 4608*x + 512), x)
```

3.802.6 Sympy [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx = \int \frac{1}{(8x^4-15x^3+8x^2+24x+8)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**(5/2),x)
```

output
```
Integral((8*x**4 - 15*x**3 + 8*x**2 + 24*x + 8)**(-5/2), x)
```

3.802.7 Maxima [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx = \int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(5/2),x, algorithm="maxima")
```

output
```
integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-5/2), x)
```

3.802.8 Giac [F]

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx = \int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(5/2),x, algorithm="giac")
```

output
```
integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-5/2), x)
```

3.802.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx = \int \frac{1}{(8\,x^4-15\,x^3+8\,x^2+24\,x+8)^{5/2}}\,dx$$

input
```
int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(5/2),x)
```

output
```
int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(5/2), x)
```

3.803 $\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx$

3.803.1 Optimal result

Integrand size = 24, antiderivative size = 130

$$\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx = -\frac{\sqrt{\frac{613-182\left(1-\frac{6}{x}\right)^2+\left(-1+\frac{6}{x}\right)^4}{\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)^2}}\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)x^2\,\text{EllipticF}\left(2\arctan\left(\frac{6-x}{\sqrt[4]{613}x}\right),\frac{613+91\sqrt{613}}{1226}\right)}{12\sqrt[4]{613}\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

output
```
-1/7356*x^2*(cos(2*arctan(1/613*(6-x)*613^(3/4)/x))^2)^(1/2)/cos(2*arctan(
1/613*(6-x)*613^(3/4)/x))*EllipticF(sin(2*arctan(1/613*(6-x)*613^(3/4)/x))
,1/1226*(751538+111566*613^(1/2))^(1/2))*((6-x)^2/x^2+613^(1/2))*((613-182
*(1-6/x)^2+(-1+6/x)^4)/((6-x)^2/x^2+613^(1/2))^2)^(1/2)*613^(3/4)/(3*x^4+1
5*x^3-44*x^2-6*x+9)^(1/2)
```

3.803.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 826, normalized size of antiderivative = 6.35

$$\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx = -\frac{2\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{(x-\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,1])(\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,2]-}{(x-\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,2])(\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,1]-}}\right.\right.}{}$$

input
```
Integrate[1/Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4],x]
```

output
```
(-2*EllipticF[ArcSin[Sqrt[((x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4
 & , 1, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9
 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))/((x - Root[9 - 6*#1 - 44*
#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*
#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))]],
 ((Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 -
44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 +
 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))
/((Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 -
44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 +
 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))
]*Sqrt[(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0])/(x - Roo
t[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])]*(x - Root[9 - 6*#1 - 4
4*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])^2*Sqrt[(x - Root[9 - 6*#1 - 44*#1^2 +
 15*#1^3 + 3*#1^4 & , 3, 0])/(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1
^4 & , 2, 0])]*Sqrt[(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4,
 0])/(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])])/Sqrt[(9
- 6*x - 44*x^2 + 15*x^3 + 3*x^4)*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1
^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Root
[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#...
```

3.803.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2504, 27, 7270, 1409}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^4+15x^3-44x^2-6x+9}}dx$$

$$\Big\downarrow \text{2504}$$

$$-1296\int \frac{1}{108\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\sqrt{\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^4}}}d\left(\frac{1}{x}-\frac{1}{6}\right)$$

$$\Big\downarrow \text{27}$$

$$-12\int \frac{1}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\sqrt{\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^4}}}d\left(\frac{1}{x}-\frac{1}{6}\right)$$

$$\Big\downarrow \text{7270}$$

$$-\frac{12\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}\int \frac{1}{\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}}d\left(\frac{1}{x}-\frac{1}{6}\right)}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\sqrt{\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^4}}}$$

$$\Big\downarrow \text{1409}$$

$$-\frac{\left(36\left(\frac{1}{x}-\frac{1}{6}\right)^2+\sqrt{613}\right)\sqrt{\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(36\left(\frac{1}{x}-\frac{1}{6}\right)^2+\sqrt{613}\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{6\left(\frac{1}{x}-\frac{1}{6}\right)}{\sqrt[4]{613}}\right),\frac{613+91\sqrt{613}}{1226}\right)}{\sqrt[4]{613}\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\sqrt{\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^4}}}$$

input
```
Int[1/Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4],x]
```

output
```
-(((Sqrt[613] + 36*(-1/6 + x^(-1))^2)*Sqrt[(613 - 6552*(-1/6 + x^(-1))^2 +
 1296*(-1/6 + x^(-1))^4)/(Sqrt[613] + 36*(-1/6 + x^(-1))^2)^2]*EllipticF[2
*ArcTan[(6*(-1/6 + x^(-1)))/613^(1/4)], (613 + 91*Sqrt[613])/1226])/(613^(
1/4)*(1 + 6*(-1/6 + x^(-1)))^2*Sqrt[(613 - 6552*(-1/6 + x^(-1))^2 + 1296*(
-1/6 + x^(-1))^4)/(1 + 6*(-1/6 + x^(-1)))^4]))
```

3.803.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1409
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

3.803.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.99 (sec) , antiderivative size = 1180, normalized size of antiderivative = 9.08

method	result	size
default	Expression too large to display	1180
elliptic	Expression too large to display	1180

input
```
int(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1)-RootOf(3*_Z^4+15*_Z^3-4
4*_Z^2-6*_Z+9,index=4))*((RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)-Ro
otOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2))*(x-RootOf(3*_Z^4+15*_Z^3-44*_
Z^2-6*_Z+9,index=1))/(RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)-RootOf
(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1))/(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-
6*_Z+9,index=2)))^(1/2)*(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2))^
2*((RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2)-RootOf(3*_Z^4+15*_Z^3-44
*_Z^2-6*_Z+9,index=1))*(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=3))/(
RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=3)-RootOf(3*_Z^4+15*_Z^3-44*_Z^
2-6*_Z+9,index=1))/(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2)))^(1/2
)*((RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2)-RootOf(3*_Z^4+15*_Z^3-44
*_Z^2-6*_Z+9,index=1))*(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4))/(
RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)-RootOf(3*_Z^4+15*_Z^3-44*_Z^
2-6*_Z+9,index=1))/(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2)))^(1/2
)/(RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)-RootOf(3*_Z^4+15*_Z^3-44*
_Z^2-6*_Z+9,index=2))/(RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2)-RootO
f(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1))*3^(1/2)/((x-RootOf(3*_Z^4+15*_Z^
3-44*_Z^2-6*_Z+9,index=1))*(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2
))*(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=3))*(x-RootOf(3*_Z^4+15*_
Z^3-44*_Z^2-6*_Z+9,index=4)))^(1/2)*EllipticF(((RootOf(3*_Z^4+15*_Z^3-4...
```

3.803.5 Fricas [F]

$$\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+15\,x^3-44\,x^2-6\,x+9}}\,dx$$

input
```
integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2),x, algorithm="fricas")
```

output
```
integral(1/sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9), x)
```

3.803.6 Sympy [F]

$$\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx = \int \frac{1}{\sqrt{3x^4+15x^3-44x^2-6x+9}}\,dx$$

input
```
integrate(1/(3*x**4+15*x**3-44*x**2-6*x+9)**(1/2),x)
```

output
```
Integral(1/sqrt(3*x**4 + 15*x**3 - 44*x**2 - 6*x + 9), x)
```

3.803.7 Maxima [F]

$$\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+15\,x^3-44\,x^2-6\,x+9}}\,dx$$

input
```
integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9), x)
```

3.803.8 Giac [F]

$$\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx = \int \frac{1}{\sqrt{3\,x^4+15\,x^3-44\,x^2-6\,x+9}}\,dx$$

input
```
integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9), x)
```

3.803.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx=\int \frac{1}{\sqrt{3\,x^4+15\,x^3-44\,x^2-6\,x+9}}\,dx$$

input
```
int(1/(15*x^3 - 44*x^2 - 6*x + 3*x^4 + 9)^(1/2),x)
```

output
```
int(1/(15*x^3 - 44*x^2 - 6*x + 3*x^4 + 9)^(1/2), x)
```

3.804 $\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx$

3.804.1 Optimal result

Integrand size = 24, antiderivative size = 444

$$\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx = -\frac{\left(176-23\left(1-\frac{6}{x}\right)^2\right)x^2}{51759\sqrt{9-6x-44x^2+15x^3+3x^4}}$$
$$+\frac{\left(45401-3722\left(1-\frac{6}{x}\right)^2\right)\left(1-\frac{6}{x}\right)x^2}{31728267\sqrt{9-6x-44x^2+15x^3+3x^4}}$$
$$+\frac{3722\left(613-182\left(1-\frac{6}{x}\right)^2+\left(-1+\frac{6}{x}\right)^4\right)\left(1-\frac{6}{x}\right)x^2}{31728267\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)\sqrt{9-6x-44x^2+15x^3+3x^4}}$$
$$+\frac{3722\sqrt{\frac{613-182\left(1-\frac{6}{x}\right)^2+\left(-1+\frac{6}{x}\right)^4}{\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)^2}}\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)x^2E\left(2\arctan\left(\frac{6-x}{\sqrt[4]{613}x}\right)\middle|\frac{613+91\sqrt{613}}{1226}\right)}{51759\ 613^{3/4}\sqrt{9-6x-44x^2+15x^3+3x^4}}$$
$$-\frac{\left(7444-145\sqrt{613}\right)\sqrt{\frac{613-182\left(1-\frac{6}{x}\right)^2+\left(-1+\frac{6}{x}\right)^4}{\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)^2}}\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)x^2\,\text{EllipticF}\left(2\arctan\left(\frac{6-x}{\sqrt[4]{613}x}\right),\frac{613+91\sqrt{613}}{1226}\right.}{207036\ 613^{3/4}\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

output
```
-1/51759*(176-23*(1-6/x)^2)*x^2/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2)+1/317282
67*(45401-3722*(1-6/x)^2)*(1-6/x)*x^2/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2)+37
22/31728267*(613-182*(1-6/x)^2+(-1+6/x)^4)*(1-6/x)*x^2/((6-x)^2/x^2+613^(1
/2))/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2)+3722/31728267*x^2*(cos(2*arctan(1/6
13*(6-x)*613^(3/4)/x))^2)^(1/2)/cos(2*arctan(1/613*(6-x)*613^(3/4)/x))*Ell
ipticE(sin(2*arctan(1/613*(6-x)*613^(3/4)/x)),1/1226*(751538+111566*613^(1
/2))^(1/2))*((6-x)^2/x^2+613^(1/2))*((613-182*(1-6/x)^2+(-1+6/x)^4)/((6-x)
^2/x^2+613^(1/2))^2)^(1/2)*613^(1/4)/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2)-1/1
26913068*x^2*(cos(2*arctan(1/613*(6-x)*613^(3/4)/x))^2)^(1/2)/cos(2*arctan
(1/613*(6-x)*613^(3/4)/x))*EllipticF(sin(2*arctan(1/613*(6-x)*613^(3/4)/x)
),1/1226*(751538+111566*613^(1/2))^(1/2))*(7444-145*613^(1/2))*((6-x)^2/x^
2+613^(1/2))*((613-182*(1-6/x)^2+(-1+6/x)^4)/((6-x)^2/x^2+613^(1/2))^2)^(1
/2)*613^(1/4)/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2)
```

3.804.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 15.99 (sec) , antiderivative size = 4974, normalized size of antiderivative = 11.20

$$\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4)^(-3/2),x]
```

output
```
(2*(106926 + 592639*x - 232005*x^2 - 44664*x^3 + 81441*EllipticF[ArcSin[Sq
rt[((x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0])*(Root[9 - 6
*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*
#1^3 + 3*#1^4 & , 4, 0]))/((x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4
 & , 2, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9
 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))]], ((Root[9 - 6*#1 - 44*#
1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#
1^4 & , 3, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Roo
t[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))/((Root[9 - 6*#1 - 44*#
1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#
1^4 & , 3, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Roo
t[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))]*Sqrt[(x - Root[9 - 6*
#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0])/(x - Root[9 - 6*#1 - 44*#1^2 +
15*#1^3 + 3*#1^4 & , 2, 0])]*(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1
^4 & , 2, 0])^2*Sqrt[(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3
, 0])/(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])]*Sqrt[(x
- Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0])/((x - Root[9 - 6*#
1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1
^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3,
0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*...
```

3.804.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.18, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {2504, 27, 7270, 2202, 1576, 27, 1158, 2206, 27, 1497, 27, 1409, 1496}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^4+15x^3-44x^2-6x+9)^{3/2}}dx$$

$$\Big\downarrow \text{2504}$$

$$-1296\int \frac{1}{972\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\left(\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^4}\right)^{3/2}}d\left(\frac{1}{x}-\frac{1}{6}\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{4}{3}\int \frac{1}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\left(\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^4}\right)^{3/2}}d\left(\frac{1}{x}-\frac{1}{6}\right)$$

$$\Big\downarrow \text{ 7270}$$

$$-\frac{4\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}\int \frac{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^4}{\left(1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613\right)^{3/2}}d\left(\frac{1}{x}-\frac{1}{6}\right)}{3\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\sqrt{\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^4}}}$$

$$\Big\downarrow \text{ 2202}$$

$$-\frac{4\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}\left(\int \frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4+216\left(\frac{1}{x}-\frac{1}{6}\right)^2+1}{\left(1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613\right)^{3/2}}d\left(\frac{1}{x}-\frac{1}{6}\right)+\int \frac{\left(864\left(\frac{1}{x}-\frac{1}{6}\right)^2+24\right)(\cdots}{\left(1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\cdots\right.\right.}\right.}{3\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\sqrt{\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^4}}}$$

$$\Big\downarrow \text{ 1576}$$

$$-\frac{4\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}\left(\frac{1}{2}\int \frac{24\left(36\left(\frac{1}{x}-\frac{1}{6}\right)^2+1\right)}{\left(1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613\right)^{3/2}}d\left(\frac{1}{x}-\frac{1}{6}\right)^2+\int \frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4+216\cdots}{\left(1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\cdots\right.}\right.}{3\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\sqrt{\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^4}}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{4\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}\left(12\int \frac{36\left(\frac{1}{x}-\frac{1}{6}\right)^2+1}{\left(1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613\right)^{3/2}}d\left(\frac{1}{x}-\frac{1}{6}\right)^2+\int \frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4+21\cdots}{\left(1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\cdots\right.}\right.}{3\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\sqrt{\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^4}}}$$

$$\Big\downarrow \text{ 1158}$$

$$-\frac{4\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}\left(\int \frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4+216\left(\frac{1}{x}-\frac{1}{6}\right)^2+1}{\left(1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613\right)^{3/2}}d\left(\frac{1}{x}-\frac{1}{6}\right)+\frac{4\left(44-207\left(\frac{1}{x}-\frac{1}{6}\right)^2\cdots\right.}{5751\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}\cdots\right.}}\right.}{3\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\sqrt{\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^4}}}$$

$$\Big\downarrow \text{ 2206}$$

3.804. $\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx$

$$-\frac{4\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}\left(-\frac{\int \frac{20736\left(88885-267984\left(\frac{1}{x}-\frac{1}{6}\right)^2\right)}{\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}}d\left(\frac{1}{x}-\frac{1}{6}\right)}{24367309056}+\frac{4\left(44-207\left(\frac{1}{x}-\frac{1}{6}\right)^2\right)}{5751\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+\cdots}}\cdots\right.}{3\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\sqrt{\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^4}}\cdots}$$

$\downarrow$ 27

$$-\frac{4\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}\left(-\frac{\int \frac{88885-267984\left(\frac{1}{x}-\frac{1}{6}\right)^2}{\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}}d\left(\frac{1}{x}-\frac{1}{6}\right)}{1175121}+\frac{4\left(44-207\left(\frac{1}{x}-\frac{1}{6}\right)^2\right)}{5751\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+\cdots}}\cdots\right.}{3\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\sqrt{\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^4}}\cdots}$$

$\downarrow$ 1497

$$-\frac{4\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}\left(\frac{-\left(\left(88885-7444\sqrt{613}\right)\int \frac{1}{\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}}d\left(\frac{1}{x}-\frac{1}{6}\right)\right)-7444\sqrt{613}\int \cdots}{1175121}\cdots\right.}{3\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\sqrt{\frac{1296\cdots}{\cdots}}\cdots}$$

$\downarrow$ 27

$$-\frac{4\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}\left(\frac{-\left(\left(88885-7444\sqrt{613}\right)\int \frac{1}{\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}}d\left(\frac{1}{x}-\frac{1}{6}\right)\right)-7444\int \frac{\cdots}{\sqrt{1296\left(\cdots\right.}}}{1175121}\cdots\right.}{3\left(6\left(\frac{1}{x}-\frac{1}{6}\right)+1\right)^2\sqrt{\frac{1296\left(\frac{1}{x}-\cdots\right.}{\cdots}}\cdots}$$

$\downarrow$ 1409

3.804. $\int \frac{1}{\left(9-6x-44x^2+15x^3+3x^4\right)^{3/2}}\,dx$

$$-\frac{4\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}\left(\frac{-7444\int\frac{\sqrt{613}-36\left(\frac{1}{x}-\frac{1}{6}\right)^2}{\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}}d\left(\frac{1}{x}-\frac{1}{6}\right)-\frac{\left(88885-7444\sqrt{613}\right)\left(36\left(\frac{1}{x}-\frac{1}{6}\right)^2+\sqrt{6}\right.}{}}{117512}\right.}{3\left(6\left(\frac{1}{x}-\frac{1}{6}\right.\right.}$$

$\downarrow$ 1496

$$-\frac{4\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}\left(-\frac{\left(88885-7444\sqrt{613}\right)\left(36\left(\frac{1}{x}-\frac{1}{6}\right)^2+\sqrt{613}\right)\sqrt{\frac{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}{\left(36\left(\frac{1}{x}-\frac{1}{6}\right)^2+\sqrt{613}\right)^2}}\text{EllipticF}\left(2\text{arct}\right.}{12\sqrt[4]{613}\sqrt{1296\left(\frac{1}{x}-\frac{1}{6}\right)^4-6552\left(\frac{1}{x}-\frac{1}{6}\right)^2+613}}\right.}{}$$

input
```
Int[(9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4)^(-3/2),x]
```

3.804. $\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx$

output
```
(-4*Sqrt[613 - 6552*(-1/6 + x^(-1))^2 + 1296*(-1/6 + x^(-1))^4]*((4*(44 -
207*(-1/6 + x^(-1))^2))/(5751*Sqrt[613 - 6552*(-1/6 + x^(-1))^2 + 1296*(-1
/6 + x^(-1))^4]) + (2*(45401 - 133992*(-1/6 + x^(-1))^2)*(-1/6 + x^(-1)))/
(1175121*Sqrt[613 - 6552*(-1/6 + x^(-1))^2 + 1296*(-1/6 + x^(-1))^4]) + (-
7444*((-613*Sqrt[613 - 6552*(-1/6 + x^(-1))^2 + 1296*(-1/6 + x^(-1))^4]*(-
1/6 + x^(-1)))/(613*Sqrt[613] + 22068*(-1/6 + x^(-1))^2) + (613^(1/4)*(Sqr
t[613] + 36*(-1/6 + x^(-1))^2)*Sqrt[(613 - 6552*(-1/6 + x^(-1))^2 + 1296*(
-1/6 + x^(-1))^4)/(Sqrt[613] + 36*(-1/6 + x^(-1))^2)^2]*EllipticE[2*ArcTan
[(6*(-1/6 + x^(-1)))/613^(1/4)], (613 + 91*Sqrt[613])/1226])/(6*Sqrt[613 -
 6552*(-1/6 + x^(-1))^2 + 1296*(-1/6 + x^(-1))^4])) - ((88885 - 7444*Sqrt[
613])*(Sqrt[613] + 36*(-1/6 + x^(-1))^2)*Sqrt[(613 - 6552*(-1/6 + x^(-1))^
2 + 1296*(-1/6 + x^(-1))^4)/(Sqrt[613] + 36*(-1/6 + x^(-1))^2)^2]*Elliptic
F[2*ArcTan[(6*(-1/6 + x^(-1)))/613^(1/4)], (613 + 91*Sqrt[613])/1226])/(12
*613^(1/4)*Sqrt[613 - 6552*(-1/6 + x^(-1))^2 + 1296*(-1/6 + x^(-1))^4]))/1
175121))/(3*(1 + 6*(-1/6 + x^(-1)))^2*Sqrt[(613 - 6552*(-1/6 + x^(-1))^2 +
 1296*(-1/6 + x^(-1))^4)/(1 + 6*(-1/6 + x^(-1)))^4])
```

3.804.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1409
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

rule 1496
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2
- 4*a*c, 0] && GtQ[c/a, 0] && LtQ[b/a, 0]
```

rule 1497
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ
[c/a, 0] && LtQ[b/a, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2202
```
Int[(Pn_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{n
 = Expon[Pn, x], k}, Int[Sum[Coeff[Pn, x, 2*k]*x^(2*k), {k, 0, n/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pn, x, 2*k + 1]*x^(2*k), {k, 0, (n -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pn, x]
 &&  !PolyQ[Pn, x^2]
```

rule 2206
```
Int[(Px_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Px, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Simp[1/(2*a*(p + 1)*(b^2 - 4*a*c))   Int[(a + b*x^2 + c
*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Px,
a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*
p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x
^2] && Expon[Px, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2504
```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Simp[-16*
a^2   Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 2
56*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
 x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a
^2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0
]
```

3.804. $\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx$

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

3.804.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.34 (sec) , antiderivative size = 5421, normalized size of antiderivative = 12.21

method	result	size
default	Expression too large to display	5421
risch	Expression too large to display	5421
elliptic	Expression too large to display	5421

input
```
int(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.804.5 Fricas [F]

$$\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+15\,x^3-44\,x^2-6\,x+9)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9)/(9*x^8 + 90*x^7 - 39*x^6
- 1356*x^5 + 1810*x^4 + 798*x^3 - 756*x^2 - 108*x + 81), x)
```

3.804.6 Sympy [F]

$$\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx=\int \frac{1}{(3x^4+15x^3-44x^2-6x+9)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x**4+15*x**3-44*x**2-6*x+9)**(3/2),x)
```

output
```
Integral((3*x**4 + 15*x**3 - 44*x**2 - 6*x + 9)**(-3/2), x)
```

3.804.7 Maxima [F]

$$\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx=\int \frac{1}{(3\,x^4+15\,x^3-44\,x^2-6\,x+9)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9)^(-3/2), x)
```

3.804.8 Giac [F]

$$\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx=\int \frac{1}{(3\,x^4+15\,x^3-44\,x^2-6\,x+9)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(3/2),x, algorithm="giac")
```

output
```
integrate((3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9)^(-3/2), x)
```

3.804.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx = \int \frac{1}{(3\,x^4+15\,x^3-44\,x^2-6\,x+9)^{3/2}}\,dx$$

input
```
int(1/(15*x^3 - 44*x^2 - 6*x + 3*x^4 + 9)^(3/2),x)
```

output
```
int(1/(15*x^3 - 44*x^2 - 6*x + 3*x^4 + 9)^(3/2), x)
```

3.805 $\int \frac{\left(2\sqrt{3-x}+\frac{3}{\sqrt{1+x}}\right)^2}{x} dx$

3.805.1 Optimal result

Integrand size = 27, antiderivative size = 56

$$\int \frac{\left(2\sqrt{3-x}+\frac{3}{\sqrt{1+x}}\right)^2}{x} dx = -4x+12\arcsin\left(\frac{1-x}{2}\right)-24\sqrt{3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt{1+x}}{\sqrt{3-x}}\right) + 21\log(x)-9\log(1+x)$$

output
```
-4*x-12*arcsin(-1/2+1/2*x)+21*ln(x)-9*ln(1+x)-24*arctanh(3^(1/2)*(1+x)^(1/
2)/(3-x)^(1/2))*3^(1/2)
```

3.805.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.67 (sec) , antiderivative size = 404, normalized size of antiderivative = 7.21

$$\int \frac{\left(2\sqrt{3-x}+\frac{3}{\sqrt{1+x}}\right)^2}{x}\,dx = 12-24i\pi-4x-48\arctan\left(\frac{\sqrt{1+x}}{2+\sqrt{3-x}}\right) - 42\log\left(-2+\sqrt{3-x}\right)-9\log(-1-x) + 21\log\left(4+\sqrt{3}(-2+\sqrt{3-x})-2\sqrt{3-x}-\sqrt{1+x}\right) - 12\sqrt{3}\log\left(4+\sqrt{3}(-2+\sqrt{3-x})-2\sqrt{3-x}-\sqrt{1+x}\right) + 21\log\left(-4+\sqrt{3}(-2+\sqrt{3-x})+2\sqrt{3-x}-\sqrt{1+x}\right) - 12\sqrt{3}\log\left(-4+\sqrt{3}(-2+\sqrt{3-x})+2\sqrt{3-x}-\sqrt{1+x}\right) + 21\log\left(4+\sqrt{3}(-2+\sqrt{3-x})-2\sqrt{3-x}+\sqrt{1+x}\right) + 12\sqrt{3}\log\left(4+\sqrt{3}(-2+\sqrt{3-x})-2\sqrt{3-x}+\sqrt{1+x}\right) + 21\log\left(-4+\sqrt{3}(-2+\sqrt{3-x})+2\sqrt{3-x}+\sqrt{1+x}\right) + 12\sqrt{3}\log\left(-4+\sqrt{3}(-2+\sqrt{3-x})+2\sqrt{3-x}+\sqrt{1+x}\right)$$

input
```
Integrate[(2*Sqrt[3 - x] + 3/Sqrt[1 + x])^2/x,x]
```

output
```
12 - (24*I)*Pi - 4*x - 48*ArcTan[Sqrt[1 + x]/(2 + Sqrt[3 - x])] - 42*Log[-
2 + Sqrt[3 - x]] - 9*Log[-1 - x] + 21*Log[4 + Sqrt[3]*(-2 + Sqrt[3 - x]) -
 2*Sqrt[3 - x] - Sqrt[1 + x]] - 12*Sqrt[3]*Log[4 + Sqrt[3]*(-2 + Sqrt[3 -
x]) - 2*Sqrt[3 - x] - Sqrt[1 + x]] + 21*Log[-4 + Sqrt[3]*(-2 + Sqrt[3 - x]
) + 2*Sqrt[3 - x] - Sqrt[1 + x]] - 12*Sqrt[3]*Log[-4 + Sqrt[3]*(-2 + Sqrt[
3 - x]) + 2*Sqrt[3 - x] - Sqrt[1 + x]] + 21*Log[4 + Sqrt[3]*(-2 + Sqrt[3 -
 x]) - 2*Sqrt[3 - x] + Sqrt[1 + x]] + 12*Sqrt[3]*Log[4 + Sqrt[3]*(-2 + Sqr
t[3 - x]) - 2*Sqrt[3 - x] + Sqrt[1 + x]] + 21*Log[-4 + Sqrt[3]*(-2 + Sqrt[
3 - x]) + 2*Sqrt[3 - x] + Sqrt[1 + x]] + 12*Sqrt[3]*Log[-4 + Sqrt[3]*(-2 +
 Sqrt[3 - x]) + 2*Sqrt[3 - x] + Sqrt[1 + x]]
```

3.805.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(2\sqrt{3-x}+\frac{3}{\sqrt{x+1}}\right)^2}{x} \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{12\sqrt{3-x}}{x\sqrt{x+1}}+\frac{12}{x}+\frac{9}{x(x+1)}-4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$12\arcsin\left(\frac{1-x}{2}\right)-24\sqrt{3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt{x+1}}{\sqrt{3-x}}\right)-4x+21\log(x)-9\log(x+1)$$

input
```
Int[(2*Sqrt[3 - x] + 3/Sqrt[1 + x])^2/x,x]
```

output
```
-4*x + 12*ArcSin[(1 - x)/2] - 24*Sqrt[3]*ArcTanh[(Sqrt[3]*Sqrt[1 + x])/Sqr
t[3 - x]] + 21*Log[x] - 9*Log[1 + x]
```

3.805.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.805.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.36

method	result	size
default	$-4x+21\ln\left(x\right)+\frac{12\sqrt{x+1}\,\sqrt{3-x}\,\left(-\arcsin\left(-\frac{1}{2}+\frac{x}{2}\right)-\sqrt{3}\,\operatorname{arctanh}\left(\frac{(3+x)\sqrt{3}}{3\sqrt{-x^{2}+2x+3}}\right)\right)}{\sqrt{-x^{2}+2x+3}}-9\ln\left(x+1\right)$	76

input
```
int((2*(3-x)^(1/2)+3/(x+1)^(1/2))^2/x,x,method=_RETURNVERBOSE)
```

output
```
-4*x+21*ln(x)+12*(x+1)^(1/2)*(3-x)^(1/2)/(-x^2+2*x+3)^(1/2)*(-arcsin(-1/2+
1/2*x)-3^(1/2)*arctanh(1/3*(3+x)*3^(1/2)/(-x^2+2*x+3)^(1/2)))-9*ln(x+1)
```

3.805.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.45

$$\int \frac{\left(2\sqrt{3-x}+\frac{3}{\sqrt{1+x}}\right)^2}{x}\,dx = 6\,\sqrt{3}\log\left(-\frac{\sqrt{3}(x+3)\sqrt{x+1}\sqrt{-x+3}+x^2-6\,x-9}{x^2}\right) - 4\,x + 12\,\arctan\left(\frac{\sqrt{x+1}(x-1)\sqrt{-x+3}}{x^2-2\,x-3}\right) - 9\,\log\left(x+1\right) + 21\,\log\left(x\right)$$

input
```
integrate((2*(3-x)^(1/2)+3/(1+x)^(1/2))^2/x,x, algorithm="fricas")
```

output
```
6*sqrt(3)*log(-(sqrt(3)*(x + 3)*sqrt(x + 1)*sqrt(-x + 3) + x^2 - 6*x - 9)/
x^2) - 4*x + 12*arctan(sqrt(x + 1)*(x - 1)*sqrt(-x + 3)/(x^2 - 2*x - 3)) -
 9*log(x + 1) + 21*log(x)
```

3.805.6 Sympy [F]

$$\int \frac{\left(2\sqrt{3-x}+\frac{3}{\sqrt{1+x}}\right)^2}{x}\,dx = \int \frac{\left(2\sqrt{3-x}\sqrt{x+1}+3\right)^2}{x\,(x+1)}\,dx$$

input
```
integrate((2*(3-x)**(1/2)+3/(1+x)**(1/2))**2/x,x)
```

output
```
Integral((2*sqrt(3 - x)*sqrt(x + 1) + 3)**2/(x*(x + 1)), x)
```

3.805.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{\left(2\sqrt{3-x}+\frac{3}{\sqrt{1+x}}\right)^2}{x}\,dx = -12\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{-x^2+2x+3}}{|x|}+\frac{6}{|x|}+2\right) - 4x + 12\arcsin\left(-\frac{1}{2}x+\frac{1}{2}\right) - 9\log(x+1) + 21\log(x)$$

input
```
integrate((2*(3-x)^(1/2)+3/(1+x)^(1/2))^2/x,x, algorithm="maxima")
```

output
```
-12*sqrt(3)*log(2*sqrt(3)*sqrt(-x^2 + 2*x + 3)/abs(x) + 6/abs(x) + 2) - 4*
x + 12*arcsin(-1/2*x + 1/2) - 9*log(x + 1) + 21*log(x)
```

3.805.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(44) = 88.

Time = 0.49 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.70

$$\int \frac{\left(2\sqrt{3-x}+\frac{3}{\sqrt{1+x}}\right)^2}{x}\,dx = 12\pi + 12\sqrt{3}\log\left(\frac{\left|-4\sqrt{3}+\frac{6(\sqrt{x+1}-2)}{\sqrt{-x+3}}-\frac{6\sqrt{-x+3}}{\sqrt{x+1}-2}\right|}{\left|4\sqrt{3}+\frac{6(\sqrt{x+1}-2)}{\sqrt{-x+3}}-\frac{6\sqrt{-x+3}}{\sqrt{x+1}-2}\right|}\right) - 4x + 24\arctan\left(\frac{\sqrt{-x+3}\left(\frac{(\sqrt{x+1}-2)^2}{x-3}+1\right)}{2\left(\sqrt{x+1}-2\right)}\right) + 21\log(|x|) - 9\log(|-x-1|) + 12$$

input
```
integrate((2*(3-x)^(1/2)+3/(1+x)^(1/2))^2/x,x, algorithm="giac")
```

output
```
12*pi + 12*sqrt(3)*log(abs(-4*sqrt(3) + 6*(sqrt(x + 1) - 2)/sqrt(-x + 3) -
 6*sqrt(-x + 3)/(sqrt(x + 1) - 2))/abs(4*sqrt(3) + 6*(sqrt(x + 1) - 2)/sqr
t(-x + 3) - 6*sqrt(-x + 3)/(sqrt(x + 1) - 2))) - 4*x + 24*arctan(1/2*sqrt(
-x + 3)*((sqrt(x + 1) - 2)^2/(x - 3) + 1)/(sqrt(x + 1) - 2)) + 21*log(abs(
x)) - 9*log(abs(-x - 1)) + 12
```

3.805.9 Mupad [B] (verification not implemented)

Time = 25.15 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.82

$$\int \frac{\left(2\sqrt{3-x}+\frac{3}{\sqrt{1+x}}\right)^2}{x}\,dx$$
$$= 48\,\mathrm{atan}\left(\frac{\sqrt{3-x}-4\sqrt{3}+3\sqrt{3}\sqrt{x+1}}{\sqrt{x+1}-3\sqrt{3}\sqrt{3-x}+8}\right)-9\ln(x+1)-4x+21\ln(x)$$
$$+12\sqrt{3}\ln\left(\frac{6x-12\sqrt{x+1}+4\sqrt{3}\sqrt{3-x}+2\sqrt{3}\sqrt{x+1}\sqrt{3-x}-6}{3x+6\sqrt{3}\sqrt{3-x}-18}\right)$$
$$-12\sqrt{3}\ln\left(\frac{\sqrt{x+1}-1}{\sqrt{3}-\sqrt{3-x}}\right)$$

input
```
int((3/(x + 1)^(1/2) + 2*(3 - x)^(1/2))^2/x,x)
```

output
```
48*atan(((3 - x)^(1/2) - 4*3^(1/2) + 3*3^(1/2)*(x + 1)^(1/2))/((x + 1)^(1/
2) - 3*3^(1/2)*(3 - x)^(1/2) + 8)) - 9*log(x + 1) - 4*x + 21*log(x) + 12*3
^(1/2)*log((6*x - 12*(x + 1)^(1/2) + 4*3^(1/2)*(3 - x)^(1/2) + 2*3^(1/2)*(
x + 1)^(1/2)*(3 - x)^(1/2) - 6)/(3*x + 6*3^(1/2)*(3 - x)^(1/2) - 18)) - 12
*3^(1/2)*log(((x + 1)^(1/2) - 1)/(3^(1/2) - (3 - x)^(1/2)))
```

3.806 $\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}} \, dx$

3.806.1 Optimal result

Integrand size = 20, antiderivative size = 65

$$\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}} \, dx = -\frac{1}{x} - x + \sqrt{1+x^2} + \frac{\sqrt{1+x^2}}{x} + \frac{1}{2}x\sqrt{1+x^2} - \frac{\operatorname{arcsinh}(x)}{2} - \log\left(1+\sqrt{1+x^2}\right)$$

output
```
-1/x-x-1/2*arcsinh(x)-ln(1+(x^2+1)^(1/2))+(x^2+1)^(1/2)+(x^2+1)^(1/2)/x+1/
2*x*(x^2+1)^(1/2)
```

3.806.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}} \, dx = \frac{-2(1+x^2) + \sqrt{1+x^2}(2+2x+x^2) + 3x\log\left(-x+\sqrt{1+x^2}\right) - 4x\log\left(1-x+\sqrt{1+x^2}\right)}{2x}$$

input
```
Integrate[(-1 + x + x^2)/(1 + Sqrt[1 + x^2]),x]
```

output
```
(-2*(1 + x^2) + Sqrt[1 + x^2]*(2 + 2*x + x^2) + 3*x*Log[-x + Sqrt[1 + x^2]
] - 4*x*Log[1 - x + Sqrt[1 + x^2]])/(2*x)
```

3.806.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x-1}{\sqrt{x^2+1}+1}dx$$
$$\Big\downarrow \text{7293}$$
$$\int \left(\frac{x^2}{\sqrt{x^2+1}+1}+\frac{x}{\sqrt{x^2+1}+1}-\frac{1}{\sqrt{x^2+1}+1}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\text{arcsinh}(x)}{2}+\frac{1}{2}\sqrt{x^2+1}x+\sqrt{x^2+1}+\frac{\sqrt{x^2+1}}{x}-\log\left(\sqrt{x^2+1}+1\right)-x-\frac{1}{x}$$

input
```
Int[(-1 + x + x^2)/(1 + Sqrt[1 + x^2]),x]
```

output
```
-x^(-1) - x + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x + (x*Sqrt[1 + x^2])/2 - ArcS
inh[x]/2 - Log[1 + Sqrt[1 + x^2]]
```

3.806.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.806.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

method	result	size
default	$-x-\frac{1}{x}-\frac{\operatorname{arcsinh}(x)}{2}-\frac{x\sqrt{x^{2}+1}}{2}+\sqrt{x^{2}+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{x^{2}+1}}\right)-\ln\left(x\right)+\frac{\left(x^{2}+1\right)^{\frac{3}{2}}}{x}$	56
meijerg	$-\frac{x\,{}_{3}F_{2}\left(\frac{1}{2},\frac{1}{2},1;\frac{3}{2},2;-x^{2}\right)}{2}+\frac{x^{3}\,{}_{3}F_{2}\left(\frac{1}{2},1,\frac{3}{2};2,\frac{5}{2};-x^{2}\right)}{6}+\frac{-4\sqrt{\pi}+4\sqrt{\pi}\,\sqrt{x^{2}+1}-4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^{2}+1}}{2}\right)}{4\sqrt{\pi}}$	76
trager	$-\frac{\left(x-1\right)^{2}}{x}+\frac{\left(x^{2}+2x+2\right)\sqrt{x^{2}+1}}{2x}+\frac{\ln\left(\frac{\sqrt{x^{2}+1}\,x^{2}-x^{3}+2x\sqrt{x^{2}+1}-2x^{2}+2\sqrt{x^{2}+1}-2x-2}{x^{4}}\right)}{2}$	84

input
```
int((x^2+x-1)/(1+(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-x-1/x-1/2*arcsinh(x)-1/2*x*(x^2+1)^(1/2)+(x^2+1)^(1/2)-arctanh(1/(x^2+1)^
(1/2))-ln(x)+1/x*(x^2+1)^(3/2)
```

3.806.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.29

$$\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}}\,dx = -\frac{2\,x^{2}+2\,x\log\left(x\right)+2\,x\log\left(-x+\sqrt{x^{2}+1}+1\right)-x\log\left(-x+\sqrt{x^{2}+1}\right)-2\,x\log\left(-x+\sqrt{x^{2}+1}-1\right)}{2\,x}$$

input
```
integrate((x^2+x-1)/(1+(x^2+1)^(1/2)),x, algorithm="fricas")
```

output
```
-1/2*(2*x^2 + 2*x*log(x) + 2*x*log(-x + sqrt(x^2 + 1) + 1) - x*log(-x + sq
rt(x^2 + 1)) - 2*x*log(-x + sqrt(x^2 + 1) - 1) - (x^2 + 2*x + 2)*sqrt(x^2
+ 1) - 2*x + 2)/x
```

3.806.6 Sympy [A] (verification not implemented)

Time = 2.63 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}}\,dx = \frac{x\sqrt{x^2+1}}{2} - x + \frac{x}{\sqrt{x^2+1}} + \sqrt{x^2+1} - \log\left(\sqrt{x^2+1}+1\right) - \frac{\operatorname{asinh}(x)}{2} - \frac{1}{x} + \frac{1}{x\sqrt{x^2+1}}$$

input
```
integrate((x**2+x-1)/(1+(x**2+1)**(1/2)),x)
```

output
```
x*sqrt(x**2 + 1)/2 - x + x/sqrt(x**2 + 1) + sqrt(x**2 + 1) - log(sqrt(x**2
 + 1) + 1) - asinh(x)/2 - 1/x + 1/(x*sqrt(x**2 + 1))
```

3.806.7 Maxima [F]

$$\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}}\,dx = \int \frac{x^2+x-1}{\sqrt{x^2+1}+1}\,dx$$

input
```
integrate((x^2+x-1)/(1+(x^2+1)^(1/2)),x, algorithm="maxima")
```

output
```
2*x - 5*arctan(1/2*x) + integrate((x^6 + x^5 - x^4)/(3*x^4 + 16*x^2 + (x^4
 + 8*x^2 + 16)*sqrt(x^2 + 1) + 16), x) + log(x^2 + 4)
```

3.806.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.37

$$\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}}\,dx = \frac{1}{2}\sqrt{x^2+1}(x+2) - x - \frac{2}{\left(x-\sqrt{x^2+1}\right)^2-1} - \frac{1}{x} + \frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right) - \log(|x|) - \log\left(\left|-x+\sqrt{x^2+1}+1\right|\right) + \log\left(\left|-x+\sqrt{x^2+1}-1\right|\right)$$

input
```
integrate((x^2+x-1)/(1+(x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
1/2*sqrt(x^2 + 1)*(x + 2) - x - 2/((x - sqrt(x^2 + 1))^2 - 1) - 1/x + 1/2*
log(-x + sqrt(x^2 + 1)) - log(abs(x)) - log(abs(-x + sqrt(x^2 + 1) + 1)) +
 log(abs(-x + sqrt(x^2 + 1) - 1))
```

3.806.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}}\,dx = \left(\frac{x}{2}+1\right)\sqrt{x^2+1} - \frac{\operatorname{asinh}(x)}{2} - \ln(x) - x + \frac{\sqrt{x^2+1}}{x} - \frac{1}{x} + \operatorname{atan}\left(\sqrt{x^2+1}\,1\mathrm{i}\right)\,1\mathrm{i}$$

input
```
int((x + x^2 - 1)/((x^2 + 1)^(1/2) + 1),x)
```

output
```
atan((x^2 + 1)^(1/2)*1i)*1i - x - asinh(x)/2 - log(x) + (x/2 + 1)*(x^2 + 1
)^(1/2) + (x^2 + 1)^(1/2)/x - 1/x
```

3.807 $\int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}}\,dx$

3.807.1 Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}}\,dx=\frac{1}{12}\left(6x^2+2x^3+\left(4-3x-2x^2\right)\sqrt{1+x^2}-3\text{arcsinh}(x)-6\log\left(1+\sqrt{1+x^2}\right)\right)$$

output
```
1/2*x^2+1/6*x^3-1/4*arcsinh(x)-1/2*ln(1+(x^2+1)^(1/2))+1/12*(-2*x^2-3*x+4)
*(x^2+1)^(1/2)
```

3.807.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

$$\int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}}\,dx=\frac{1}{12}\left(2x^2(3+x)+\left(4-3x-2x^2\right)\sqrt{1+x^2}+9\log\left(-x+\sqrt{1+x^2}\right)-12\log\left(1-x+\sqrt{1+x^2}\right)\right)$$

input
```
Integrate[(-1 + x + x^2)/(1 + x + Sqrt[1 + x^2]),x]
```

output
```
(2*x^2*(3 + x) + (4 - 3*x - 2*x^2)*Sqrt[1 + x^2] + 9*Log[-x + Sqrt[1 + x^2
]] - 12*Log[1 - x + Sqrt[1 + x^2]])/12
```

3.807.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.91, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x-1}{\sqrt{x^2+1}+x+1} \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{x^2}{\sqrt{x^2+1}+x+1}+\frac{x}{\sqrt{x^2+1}+x+1}-\frac{1}{\sqrt{x^2+1}+x+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{arcsinh}(x)}{4}+\frac{x^3}{6}+\frac{x^2}{2}-\frac{1}{4}\sqrt{x^2+1}x-\frac{1}{6}\left(x^2+1\right)^{3/2}+\frac{1}{2\left(\sqrt{x^2+1}+x\right)}+\frac{1}{2}\log\left(\sqrt{x^2+1}+x\right)-\log\left(\sqrt{x^2+1}+x+1\right)+\frac{x}{2}$$

input
```
Int[(-1 + x + x^2)/(1 + x + Sqrt[1 + x^2]),x]
```

output
```
x/2 + x^2/2 + x^3/6 - (x*Sqrt[1 + x^2])/4 - (1 + x^2)^(3/2)/6 + 1/(2*(x +
Sqrt[1 + x^2])) - ArcSinh[x]/4 + Log[x + Sqrt[1 + x^2]]/2 - Log[1 + x + Sq
rt[1 + x^2]]
```

3.807.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.807.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{x^2}{2}-\frac{\ln(x)}{2}+\frac{x^3}{6}-\frac{x\sqrt{x^2+1}}{4}-\frac{\operatorname{arcsinh}(x)}{4}-\frac{(x^2+1)^{\frac{3}{2}}}{6}+\frac{\sqrt{x^2+1}}{2}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)}{2}$	58
trager	$\frac{(x^2+4x+4)(x-1)}{6}+\frac{\left(-\frac{1}{3}x^2-\frac{1}{2}x+\frac{2}{3}\right)\sqrt{x^2+1}}{2}+\frac{\ln\left(\frac{\sqrt{x^2+1}\,x^2-x^3+2x\sqrt{x^2+1}-2x^2+2\sqrt{x^2+1}-2x-2}{x^4}\right)}{4}$	86

input
```
int((x^2+x-1)/(1+x+(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2-1/2*ln(x)+1/6*x^3-1/4*x*(x^2+1)^(1/2)-1/4*arcsinh(x)-1/6*(x^2+1)^(
3/2)+1/2*(x^2+1)^(1/2)-1/2*arctanh(1/(x^2+1)^(1/2))
```

3.807.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.47

$$\int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}}\,dx = \frac{1}{6}\,x^3+\frac{1}{2}\,x^2-\frac{1}{12}\left(2\,x^2+3\,x-4\right)\sqrt{x^2+1} - \frac{1}{2}\,\log\left(x\right)-\frac{1}{2}\,\log\left(-x+\sqrt{x^2+1}+1\right) + \frac{1}{4}\,\log\left(-x+\sqrt{x^2+1}\right)+\frac{1}{2}\,\log\left(-x+\sqrt{x^2+1}-1\right)$$

input
```
integrate((x^2+x-1)/(1+x+(x^2+1)^(1/2)),x, algorithm="fricas")
```

output
```
1/6*x^3 + 1/2*x^2 - 1/12*(2*x^2 + 3*x - 4)*sqrt(x^2 + 1) - 1/2*log(x) - 1/
2*log(-x + sqrt(x^2 + 1) + 1) + 1/4*log(-x + sqrt(x^2 + 1)) + 1/2*log(-x +
 sqrt(x^2 + 1) - 1)
```

3.807.6 Sympy [F]

$$\int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}}\,dx = \int \frac{x^2+x-1}{x+\sqrt{x^2+1}+1}\,dx$$

input
```
integrate((x**2+x-1)/(1+x+(x**2+1)**(1/2)),x)
```

output
```
Integral((x**2 + x - 1)/(x + sqrt(x**2 + 1) + 1), x)
```

3.807.7 Maxima [F]

$$\int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}}\,dx = \int \frac{x^2+x-1}{x+\sqrt{x^2+1}+1}\,dx$$

input
```
integrate((x^2+x-1)/(1+x+(x^2+1)^(1/2)),x, algorithm="maxima")
```

output
```
1/4*x^2 - 3/56*sqrt(7)*arctan(1/7*sqrt(7)*(4*x + 3)) + 1/4*x + integrate((
x^4 + x^3 - x^2)/(4*x^5 + 12*x^4 + 19*x^3 + 19*x^2 + (4*x^4 + 12*x^3 + 17*
x^2 + 12*x + 4)*sqrt(x^2 + 1) + 12*x + 4), x) - 7/16*log(2*x^2 + 3*x + 2)
```

3.807.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.51

$$\begin{aligned}\int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}}\,dx &= \frac{1}{6}\,x^3+\frac{1}{2}\,x^2-\frac{1}{12}\,((2\,x+3)x-4)\sqrt{x^2+1} \\ &\quad+\frac{1}{4}\,\log\left(-x+\sqrt{x^2+1}\right)-\frac{1}{2}\,\log\left(|x|\right) \\ &\quad-\frac{1}{2}\,\log\left(\left|-x+\sqrt{x^2+1}+1\right|\right)+\frac{1}{2}\,\log\left(\left|-x+\sqrt{x^2+1}-1\right|\right)\end{aligned}$$

input
```
integrate((x^2+x-1)/(1+x+(x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
1/6*x^3 + 1/2*x^2 - 1/12*((2*x + 3)*x - 4)*sqrt(x^2 + 1) + 1/4*log(-x + sq
rt(x^2 + 1)) - 1/2*log(abs(x)) - 1/2*log(abs(-x + sqrt(x^2 + 1) + 1)) + 1/
2*log(abs(-x + sqrt(x^2 + 1) - 1))
```

3.807.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}}\,dx = \frac{x^2}{2} - \frac{\ln(x)}{2} - \sqrt{x^2+1}\left(\frac{x^2}{6}+\frac{x}{4}-\frac{1}{3}\right) - \frac{\operatorname{asinh}(x)}{4} + \frac{x^3}{6} + \frac{\operatorname{atan}\left(\sqrt{x^2+1}\,1\mathrm{i}\right)\,1\mathrm{i}}{2}$$

input
```
int((x + x^2 - 1)/(x + (x^2 + 1)^(1/2) + 1),x)
```

output
```
(atan((x^2 + 1)^(1/2)*1i)*1i)/2 - asinh(x)/4 - log(x)/2 - (x^2 + 1)^(1/2)*
(x/4 + x^2/6 - 1/3) + x^2/2 + x^3/6
```

3.808 $\int \frac{2\sqrt{-1+x}+x}{\sqrt{-1+x}x} \, dx$

3.808.1 Optimal result

Integrand size = 22, antiderivative size = 14

$$\int \frac{2\sqrt{-1+x}+x}{\sqrt{-1+x}x} \, dx = 2\sqrt{-1+x}+2\log(x)$$

output
```
2*ln(x)+2*(-1+x)^(1/2)
```

3.808.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{2\sqrt{-1+x}+x}{\sqrt{-1+x}x} \, dx = 2\sqrt{-1+x}+2\log(x)$$

input
```
Integrate[(2*Sqrt[-1 + x] + x)/(Sqrt[-1 + x]*x),x]
```

output
```
2*Sqrt[-1 + x] + 2*Log[x]
```

3.808.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {7239, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+2\sqrt{x-1}}{\sqrt{x-1}x}\,dx$$
$$\Big\downarrow \text{7239}$$
$$\int \left(\frac{2}{x}+\frac{1}{\sqrt{x-1}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$2\sqrt{x-1}+2\log(x)$$

input
```
Int[(2*Sqrt[-1 + x] + x)/(Sqrt[-1 + x]*x),x]
```

output
```
2*Sqrt[-1 + x] + 2*Log[x]
```

3.808.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

3.808.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$2\ln(x)+2\sqrt{x-1}$	13
default	$2\ln(x)+2\sqrt{x-1}$	13
trager	$2\sqrt{x-1}-2\ln\left(\frac{1}{x}\right)$	15

input
```
int((x+2*(x-1)^(1/2))/x/(x-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*ln(x)+2*(x-1)^(1/2)
```

3.808.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{2\sqrt{-1+x}+x}{\sqrt{-1+x}x}\,dx = 2\,\sqrt{x-1}+2\,\log(x)$$

input
```
integrate((x+2*(-1+x)^(1/2))/x/(-1+x)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(x - 1) + 2*log(x)
```

3.808.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{2\sqrt{-1+x}+x}{\sqrt{-1+x}x}\,dx = 2\sqrt{x-1}+2\log(x)$$

input
```
integrate((x+2*(-1+x)**(1/2))/x/(-1+x)**(1/2),x)
```

output
```
2*sqrt(x - 1) + 2*log(x)
```

3.808.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{2\sqrt{-1+x}+x}{\sqrt{-1+x}x}\,dx = 2\sqrt{x-1}+2\log(x)$$

input
```
integrate((x+2*(-1+x)^(1/2))/x/(-1+x)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(x - 1) + 2*log(x)
```

3.808.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{2\sqrt{-1+x}+x}{\sqrt{-1+x}x}\,dx = 2\sqrt{x-1}+2\log(x)$$

input
```
integrate((x+2*(-1+x)^(1/2))/x/(-1+x)^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(x - 1) + 2*log(x)
```

3.808.9 Mupad [B] (verification not implemented)

Time = 20.38 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{2\sqrt{-1+x}+x}{\sqrt{-1+x}x}\,dx = 2\ln(x)+2\sqrt{x-1}$$

input
```
int((x + 2*(x - 1)^(1/2))/(x*(x - 1)^(1/2)),x)
```

output
```
2*log(x) + 2*(x - 1)^(1/2)
```

3.809 $\int \left(a+c\sqrt{x}+bx^{2/3}\right)^2\,dx$

3.809.1 Optimal result

Integrand size = 18, antiderivative size = 61

$$\int \left(a+c\sqrt{x}+bx^{2/3}\right)^2\,dx=a^2x+\frac{4}{3}acx^{3/2}+\frac{6}{5}abx^{5/3}+\frac{c^2x^2}{2}+\frac{12}{13}bcx^{13/6}+\frac{3}{7}b^2x^{7/3}$$

output
```
a^2*x+4/3*a*c*x^(3/2)+6/5*a*b*x^(5/3)+1/2*c^2*x^2+12/13*b*c*x^(13/6)+3/7*b
^2*x^(7/3)
```

3.809.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \left(a+c\sqrt{x}+bx^{2/3}\right)^2\,dx=\frac{2730a^2x+3640acx^{3/2}+3276abx^{5/3}+1365c^2x^2+2520bcx^{13/6}+1170b^2x^{7/3}}{2730}$$

input
```
Integrate[(a + c*Sqrt[x] + b*x^(2/3))^2,x]
```

output
```
(2730*a^2*x + 3640*a*c*x^(3/2) + 3276*a*b*x^(5/3) + 1365*c^2*x^2 + 2520*b*
c*x^(13/6) + 1170*b^2*x^(7/3))/2730
```

3.809.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7267, 2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^{2/3}+c\sqrt{x}\right)^2 dx$$

$$\Big\downarrow \text{ 7267}$$

$$6\int \left(a+\left(\sqrt[6]{x}b+c\right)\sqrt{x}\right)^2 x^{5/6}d\sqrt[6]{x}$$

$$\Big\downarrow \text{ 2465}$$

$$6\int \left(b^2x^{13/6}+2bcx^2+c^2x^{11/6}+2abx^{3/2}+2acx^{4/3}+a^2x^{5/6}\right)d\sqrt[6]{x}$$

$$\Big\downarrow \text{ 2009}$$

$$6\left(\frac{a^2x}{6}+\frac{1}{5}abx^{5/3}+\frac{2}{9}acx^{3/2}+\frac{1}{14}b^2x^{7/3}+\frac{2}{13}bcx^{13/6}+\frac{c^2x^2}{12}\right)$$

input
```
Int[(a + c*Sqrt[x] + b*x^(2/3))^2,x]
```

output
```
6*((a^2*x)/6 + (2*a*c*x^(3/2))/9 + (a*b*x^(5/3))/5 + (c^2*x^2)/12 + (2*b*c
*x^(13/6))/13 + (b^2*x^(7/3))/14)
```

3.809.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.809.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$a^2x+\frac{4acx^{\frac{3}{2}}}{3}+\frac{6abx^{\frac{5}{3}}}{5}+\frac{c^2x^2}{2}+\frac{12bcx^{\frac{13}{6}}}{13}+\frac{3b^2x^{\frac{7}{3}}}{7}$	44
default	$\frac{c^2x^2}{2}+2c\left(\frac{6bx^{\frac{13}{6}}}{13}+\frac{2ax^{\frac{3}{2}}}{3}\right)+a^2x+\frac{3b^2x^{\frac{7}{3}}}{7}+\frac{6abx^{\frac{5}{3}}}{5}$	46

input
```
int((a+b*x^(2/3)+c*x^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+4/3*a*c*x^(3/2)+6/5*a*b*x^(5/3)+1/2*c^2*x^2+12/13*b*c*x^(13/6)+3/7*b
^2*x^(7/3)
```

3.809.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

$$\int\left(a+c\sqrt{x}+bx^{2/3}\right)^2\,dx=\frac{3}{7}\,b^2x^{\frac{7}{3}}+\frac{12}{13}\,bcx^{\frac{13}{6}}+\frac{1}{2}\,c^2x^2+\frac{6}{5}\,abx^{\frac{5}{3}}+\frac{4}{3}\,acx^{\frac{3}{2}}+a^2x$$

input
```
integrate((a+b*x^(2/3)+c*x^(1/2))^2,x, algorithm="fricas")
```

output
```
3/7*b^2*x^(7/3) + 12/13*b*c*x^(13/6) + 1/2*c^2*x^2 + 6/5*a*b*x^(5/3) + 4/3
*a*c*x^(3/2) + a^2*x
```

3.809.6 Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int\left(a+c\sqrt{x}+bx^{2/3}\right)^2\,dx=a^2x+\frac{6abx^{\frac{5}{3}}}{5}+\frac{4acx^{\frac{3}{2}}}{3}+\frac{3b^2x^{\frac{7}{3}}}{7}+\frac{12bcx^{\frac{13}{6}}}{13}+\frac{c^2x^2}{2}$$

input
```
integrate((a+b*x**(2/3)+c*x**(1/2))**2,x)
```

output
```
a**2*x + 6*a*b*x**(5/3)/5 + 4*a*c*x**(3/2)/3 + 3*b**2*x**(7/3)/7 + 12*b*c*
x**(13/6)/13 + c**2*x**2/2
```

3.809.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \left(a+c\sqrt{x}+bx^{2/3}\right)^2\,dx=\frac{3}{7}\,b^2x^{\frac{7}{3}}+\frac{12}{13}\,bcx^{\frac{13}{6}}+\frac{1}{2}\,c^2x^2+a^2x+\frac{2}{15}\left(9\,bx^{\frac{5}{3}}+10\,cx^{\frac{3}{2}}\right)a$$

input
```
integrate((a+b*x^(2/3)+c*x^(1/2))^2,x, algorithm="maxima")
```

output
```
3/7*b^2*x^(7/3) + 12/13*b*c*x^(13/6) + 1/2*c^2*x^2 + a^2*x + 2/15*(9*b*x^(
5/3) + 10*c*x^(3/2))*a
```

3.809.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

$$\int \left(a+c\sqrt{x}+bx^{2/3}\right)^2\,dx=\frac{3}{7}\,b^2x^{\frac{7}{3}}+\frac{12}{13}\,bcx^{\frac{13}{6}}+\frac{1}{2}\,c^2x^2+\frac{6}{5}\,abx^{\frac{5}{3}}+\frac{4}{3}\,acx^{\frac{3}{2}}+a^2x$$

input
```
integrate((a+b*x^(2/3)+c*x^(1/2))^2,x, algorithm="giac")
```

output
```
3/7*b^2*x^(7/3) + 12/13*b*c*x^(13/6) + 1/2*c^2*x^2 + 6/5*a*b*x^(5/3) + 4/3
*a*c*x^(3/2) + a^2*x
```

3.809.9 Mupad [B] (verification not implemented)

Time = 21.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

$$\int \left(a+c\sqrt{x}+bx^{2/3}\right)^2\,dx=a^2\,x+\frac{3\,b^2\,x^{7/3}}{7}+\frac{c^2\,x^2}{2}+\frac{6\,a\,b\,x^{5/3}}{5}+\frac{4\,a\,c\,x^{3/2}}{3}+\frac{12\,b\,c\,x^{13/6}}{13}$$

input
```
int((a + b*x^(2/3) + c*x^(1/2))^2,x)
```

output
```
a^2*x + (3*b^2*x^(7/3))/7 + (c^2*x^2)/2 + (6*a*b*x^(5/3))/5 + (4*a*c*x^(3/
2))/3 + (12*b*c*x^(13/6))/13
```

3.810 $\int \left(a+c\sqrt{x}+bx^{2/3}\right)^3\,dx$

3.810.1 Optimal result

Integrand size = 18, antiderivative size = 114

$$\int \left(a+c\sqrt{x}+bx^{2/3}\right)^3\,dx=a^3x+2a^2cx^{3/2}+\frac{9}{5}a^2bx^{5/3}+\frac{3}{2}ac^2x^2+\frac{36}{13}abcx^{13/6}+\frac{9}{7}ab^2x^{7/3}+\frac{2}{5}c^3x^{5/2}+\frac{9}{8}bc^2x^{8/3}+\frac{18}{17}b^2cx^{17/6}+\frac{b^3x^3}{3}$$

output
```
a^3*x+2*a^2*c*x^(3/2)+9/5*a^2*b*x^(5/3)+3/2*a*c^2*x^2+36/13*a*b*c*x^(13/6)
+9/7*a*b^2*x^(7/3)+2/5*c^3*x^(5/2)+9/8*b*c^2*x^(8/3)+18/17*b^2*c*x^(17/6)+
1/3*b^3*x^3
```

3.810.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

$$\int \left(a+c\sqrt{x}+bx^{2/3}\right)^3\,dx=\frac{185640a^3x+371280a^2cx^{3/2}+334152a^2bx^{5/3}+278460ac^2x^2+514080abcx^{13/6}+238680ab^2x}{185640}$$

input
```
Integrate[(a + c*Sqrt[x] + b*x^(2/3))^3,x]
```

output
```
(185640*a^3*x + 371280*a^2*c*x^(3/2) + 334152*a^2*b*x^(5/3) + 278460*a*c^2
*x^2 + 514080*a*b*c*x^(13/6) + 238680*a*b^2*x^(7/3) + 74256*c^3*x^(5/2) +
208845*b*c^2*x^(8/3) + 196560*b^2*c*x^(17/6) + 61880*b^3*x^3)/185640
```

3.810.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {7267, 2465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^{2/3}+c\sqrt{x}\right)^3 dx$$

$$\Big\downarrow \text{7267}$$

$$6\int \left(a+\left(\sqrt[6]{x}b+c\right)\sqrt{x}\right)^3 x^{5/6}d\sqrt[6]{x}$$

$$\Big\downarrow \text{2465}$$

$$6\int \left(x^{5/6}a^3+3bx^{3/2}a^2+3cx^{4/3}a^2+3b^2x^{13/6}a+6bcx^2a+3c^2x^{11/6}a+b^3x^{17/6}+3b^2cx^{8/3}+3bc^2x^{5/2}+c^3x^{7/3}\right)d$$

$$\Big\downarrow \text{2009}$$

$$6\left(\frac{a^3x}{6}+\frac{3}{10}a^2bx^{5/3}+\frac{1}{3}a^2cx^{3/2}+\frac{3}{14}ab^2x^{7/3}+\frac{6}{13}abcx^{13/6}+\frac{1}{4}ac^2x^2+\frac{b^3x^3}{18}+\frac{3}{17}b^2cx^{17/6}+\frac{3}{16}bc^2x^{8/3}+\frac{1}{15}c^3x\right.$$

input
```
Int[(a + c*Sqrt[x] + b*x^(2/3))^3,x]
```

output
```
6*((a^3*x)/6 + (a^2*c*x^(3/2))/3 + (3*a^2*b*x^(5/3))/10 + (a*c^2*x^2)/4 +
(6*a*b*c*x^(13/6))/13 + (3*a*b^2*x^(7/3))/14 + (c^3*x^(5/2))/15 + (3*b*c^2
*x^(8/3))/16 + (3*b^2*c*x^(17/6))/17 + (b^3*x^3)/18)
```

3.810.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2465
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[ExpandToSum[u, Px^p, x], x] /; PolyQ
[Px, x] && GtQ[Expon[Px, x], 2] && !BinomialQ[Px, x] && !TrinomialQ[Px, x
] && IGtQ[p, 0]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.810.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.75

method	result
derivativedivides	$a^3x + 2a^2c\,x^{\frac{3}{2}} + \frac{9a^2b\,x^{\frac{5}{3}}}{5} + \frac{3a\,c^2x^2}{2} + \frac{36abc\,x^{\frac{13}{6}}}{13} + \frac{9a\,b^2x^{\frac{7}{3}}}{7} + \frac{2c^3x^{\frac{5}{2}}}{5} + \frac{9b\,c^2x^{\frac{8}{3}}}{8} + \frac{18b^2c\,x^{\frac{17}{6}}}{17} + \frac{b^3x^3}{3}$
default	$\frac{2c^3x^{\frac{5}{2}}}{5} + 3c^2\left(\frac{3b\,x^{\frac{8}{3}}}{8} + \frac{a\,x^2}{2}\right) + 3c\left(\frac{6b^2x^{\frac{17}{6}}}{17} + \frac{12ab\,x^{\frac{13}{6}}}{13} + \frac{2a^2x^{\frac{3}{2}}}{3}\right) + a^3x + \frac{b^3x^3}{3} + \frac{9a^2b\,x^{\frac{5}{3}}}{5} + \frac{9a\,b^2}{7}$

input
```
int((a+b*x^(2/3)+c*x^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*x+2*a^2*c*x^(3/2)+9/5*a^2*b*x^(5/3)+3/2*a*c^2*x^2+36/13*a*b*c*x^(13/6)
+9/7*a*b^2*x^(7/3)+2/5*c^3*x^(5/2)+9/8*b*c^2*x^(8/3)+18/17*b^2*c*x^(17/6)+
1/3*b^3*x^3
```

3.810.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.80

$$\int \left(a+c\sqrt{x}+bx^{2/3}\right)^3\,dx = \frac{1}{3}\,b^3x^3 + \frac{18}{17}\,b^2cx^{\frac{17}{6}} + \frac{9}{7}\,ab^2x^{\frac{7}{3}} + \frac{36}{13}\,abcx^{\frac{13}{6}} + \frac{3}{2}\,ac^2x^2 + a^3x + \frac{9}{40}\left(5\,bc^2x^2 + 8\,a^2bx\right)x^{\frac{2}{3}} + \frac{2}{5}\left(c^3x^2 + 5\,a^2cx\right)\sqrt{x}$$

input
```
integrate((a+b*x^(2/3)+c*x^(1/2))^3,x, algorithm="fricas")
```

output
```
1/3*b^3*x^3 + 18/17*b^2*c*x^(17/6) + 9/7*a*b^2*x^(7/3) + 36/13*a*b*c*x^(13
/6) + 3/2*a*c^2*x^2 + a^3*x + 9/40*(5*b*c^2*x^2 + 8*a^2*b*x)*x^(2/3) + 2/5
*(c^3*x^2 + 5*a^2*c*x)*sqrt(x)
```

3.810.6 Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02

$$\int \left(a+c\sqrt{x}+bx^{2/3}\right)^3\,dx = a^3x+\frac{9a^2bx^{\frac{5}{3}}}{5}+2a^2cx^{\frac{3}{2}}+\frac{9ab^2x^{\frac{7}{3}}}{7}+\frac{36abcx^{\frac{13}{6}}}{13}+\frac{3ac^2x^2}{2}+\frac{b^3x^3}{3}+\frac{18b^2cx^{\frac{17}{6}}}{17}+\frac{9bc^2x^{\frac{8}{3}}}{8}+\frac{2c^3x^{\frac{5}{2}}}{5}$$

input
```
integrate((a+b*x**(2/3)+c*x**(1/2))**3,x)
```

output
```
a**3*x + 9*a**2*b*x**(5/3)/5 + 2*a**2*c*x**(3/2) + 9*a*b**2*x**(7/3)/7 + 3
6*a*b*c*x**(13/6)/13 + 3*a*c**2*x**2/2 + b**3*x**3/3 + 18*b**2*c*x**(17/6)
/17 + 9*b*c**2*x**(8/3)/8 + 2*c**3*x**(5/2)/5
```

3.810.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.75

$$\int \left(a+c\sqrt{x}+bx^{2/3}\right)^3\,dx = \frac{1}{3}\,b^3x^3+\frac{18}{17}\,b^2cx^{\frac{17}{6}}+\frac{9}{8}\,bc^2x^{\frac{8}{3}}+\frac{2}{5}\,c^3x^{\frac{5}{2}}+a^3x+\frac{1}{5}\left(9\,bx^{\frac{5}{3}}+10\,cx^{\frac{3}{2}}\right)a^2+\frac{3}{182}\left(78\,b^2x^{\frac{7}{3}}+168\,bcx^{\frac{13}{6}}+91\,c^2x^2\right)a$$

input
```
integrate((a+b*x^(2/3)+c*x^(1/2))^3,x, algorithm="maxima")
```

output
```
1/3*b^3*x^3 + 18/17*b^2*c*x^(17/6) + 9/8*b*c^2*x^(8/3) + 2/5*c^3*x^(5/2) +
 a^3*x + 1/5*(9*b*x^(5/3) + 10*c*x^(3/2))*a^2 + 3/182*(78*b^2*x^(7/3) + 16
8*b*c*x^(13/6) + 91*c^2*x^2)*a
```

3.810.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.74

$$\int \left(a+c\sqrt{x}+bx^{2/3}\right)^3\,dx = \frac{1}{3}\,b^3x^3+\frac{18}{17}\,b^2cx^{\frac{17}{6}}+\frac{9}{8}\,bc^2x^{\frac{8}{3}}+\frac{2}{5}\,c^3x^{\frac{5}{2}}+\frac{9}{7}\,ab^2x^{\frac{7}{3}}+\frac{36}{13}\,abcx^{\frac{13}{6}}+\frac{3}{2}\,ac^2x^2+\frac{9}{5}\,a^2bx^{\frac{5}{3}}+2\,a^2cx^{\frac{3}{2}}+a^3x$$

input
```
integrate((a+b*x^(2/3)+c*x^(1/2))^3,x, algorithm="giac")
```

output
```
1/3*b^3*x^3 + 18/17*b^2*c*x^(17/6) + 9/8*b*c^2*x^(8/3) + 2/5*c^3*x^(5/2) +
 9/7*a*b^2*x^(7/3) + 36/13*a*b*c*x^(13/6) + 3/2*a*c^2*x^2 + 9/5*a^2*b*x^(5
/3) + 2*a^2*c*x^(3/2) + a^3*x
```

3.810.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.74

$$\int \left(a+c\sqrt{x}+bx^{2/3}\right)^3\,dx = a^3\,x+\frac{b^3\,x^3}{3}+\frac{2\,c^3\,x^{5/2}}{5}+\frac{9\,a^2\,b\,x^{5/3}}{5}+\frac{9\,a\,b^2\,x^{7/3}}{7}+\frac{3\,a\,c^2\,x^2}{2}+2\,a^2\,c\,x^{3/2}+\frac{9\,b\,c^2\,x^{8/3}}{8}+\frac{18\,b^2\,c\,x^{17/6}}{17}+\frac{36\,a\,b\,c\,x^{13/6}}{13}$$

input
```
int((a + b*x^(2/3) + c*x^(1/2))^3,x)
```

output
```
a^3*x + (b^3*x^3)/3 + (2*c^3*x^(5/2))/5 + (9*a^2*b*x^(5/3))/5 + (9*a*b^2*x
^(7/3))/7 + (3*a*c^2*x^2)/2 + 2*a^2*c*x^(3/2) + (9*b*c^2*x^(8/3))/8 + (18*
b^2*c*x^(17/6))/17 + (36*a*b*c*x^(13/6))/13
```

3.811 $\int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}x^3}\,dx$

3.811.1 Optimal result

Integrand size = 23, antiderivative size = 58

$$\int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}x^3}\,dx=\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

output
```
arctanh((a-b*(1-1/x^2))^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)+(a-b*(1-1/x^2))^(1/
2)/b
```

3.811.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.91

$$\int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}x^3}\,dx =\frac{\sqrt{-a+b}(b+ax^2-bx^2)-2bx\sqrt{b+ax^2-bx^2}\arctan\left(\frac{\sqrt{-a+b}x}{\sqrt{b}-\sqrt{b+(a-b)x^2}}\right)}{b\sqrt{-a+b}\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}x^2}$$

input
```
Integrate[(-1 + x^2)/(Sqrt[a - b + b/x^2]*x^3),x]
```

output
```
(Sqrt[-a + b]*(b + a*x^2 - b*x^2) - 2*b*x*Sqrt[b + a*x^2 - b*x^2]*ArcTan[(
Sqrt[-a + b]*x)/(Sqrt[b] - Sqrt[b + (a - b)*x^2])])/(b*Sqrt[-a + b]*Sqrt[a
 + b*(-1 + x^(-2))]*x^2)
```

3.811.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1016, 948, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-1}{x^3\sqrt{a+\frac{b}{x^2}-b}}dx$$

$$\Big\downarrow \text{1016}$$

$$\int \frac{1-\frac{1}{x^2}}{x\sqrt{a+\frac{b}{x^2}-b}}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \frac{\left(1-\frac{1}{x^2}\right)x^2}{\sqrt{a-b+\frac{b}{x^2}}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(\frac{2\sqrt{a+\frac{b}{x^2}-b}}{b}-\int \frac{x^2}{\sqrt{a-b+\frac{b}{x^2}}}d\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{2\sqrt{a+\frac{b}{x^2}-b}}{b}-\frac{2\int \frac{1}{-\frac{a}{b}+\frac{1}{bx^4}+1}d\sqrt{a-b+\frac{b}{x^2}}}{b}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}-b}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}+\frac{2\sqrt{a+\frac{b}{x^2}-b}}{b}\right)$$

input
```
Int[(-1 + x^2)/(Sqrt[a - b + b/x^2]*x^3),x]
```

output
```
((2*Sqrt[a - b + b/x^2])/b + (2*ArcTanh[Sqrt[a - b + b/x^2]/Sqrt[a - b]])/
Sqrt[a - b])/2
```

3.811.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

3.811.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(50) = 100$.

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.76

method	result	size
default	$\frac{\sqrt{a\,x^{2}-b\,x^{2}+b}\,\left(\ln\left(x\sqrt{a-b}+\sqrt{a\,x^{2}-b\,x^{2}+b}\right) b x+\sqrt{a\,x^{2}-b\,x^{2}+b}\,\sqrt{a-b}\right)}{\sqrt{\frac{a\,x^{2}-b\,x^{2}+b}{x^{2}}}\,x^{2}\sqrt{a-b}\,b}$	102
risch	$\frac{a\,x^{2}-b\,x^{2}+b}{b\,x^{2}\sqrt{\frac{a\,x^{2}-b\,x^{2}+b}{x^{2}}}}+\frac{\ln\left(x\sqrt{a-b}+\sqrt{x^{2}(a-b)+b}\right)\sqrt{a\,x^{2}-b\,x^{2}+b}}{\sqrt{a-b}\,\sqrt{\frac{a\,x^{2}-b\,x^{2}+b}{x^{2}}}\,x}$	110

input
```
int((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x^2-b*x^2+b)^(1/2)*(ln(x*(a-b)^(1/2)+(a*x^2-b*x^2+b)^(1/2))*b*x+(a*x^2-
b*x^2+b)^(1/2)*(a-b)^(1/2))/((a*x^2-b*x^2+b)/x^2)^(1/2)/x^2/(a-b)^(1/2)/b
```

3.811.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.10

$$\int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}x^3}\,dx$$
$$= \left[\frac{\sqrt{a-b}b\log\left(-2\,(a-b)x^2-2\sqrt{a-b}x^2\sqrt{\frac{(a-b)x^2+b}{x^2}}-b\right)+2\,(a-b)\sqrt{\frac{(a-b)x^2+b}{x^2}}}{2\,(ab-b^2)}, \frac{\sqrt{-a+b}b\arctan\left(-\right.}{}\right.$$

input
```
integrate((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a - b)*b*log(-2*(a - b)*x^2 - 2*sqrt(a - b)*x^2*sqrt(((a - b)*x
^2 + b)/x^2) - b) + 2*(a - b)*sqrt(((a - b)*x^2 + b)/x^2))/(a*b - b^2), (s
qrt(-a + b)*b*arctan(-sqrt(-a + b)*x^2*sqrt(((a - b)*x^2 + b)/x^2)/((a - b
)*x^2 + b)) + (a - b)*sqrt(((a - b)*x^2 + b)/x^2))/(a*b - b^2)]
```

3.811.6 Sympy [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.26

$$\int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}x^3}\,dx = -\frac{\begin{cases} -\frac{1}{\sqrt{a}x^2} & \text{for } b=0 \\ -\frac{2\sqrt{a-b+\frac{b}{x^2}}}{b} & \text{otherwise} \end{cases}}{2} - \frac{\begin{cases} \frac{2\operatorname{atan}\left(\frac{\sqrt{a-b+\frac{b}{x^2}}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} & \text{for } b\neq 0 \\ -\frac{\log(x^2)}{\sqrt{a}} & \text{otherwise} \end{cases}}{2}$$

input
```
integrate((x**2-1)/x**3/(a-b+b/x**2)**(1/2),x)
```

output
```
-Piecewise((-1/(sqrt(a)*x**2), Eq(b, 0)), (-2*sqrt(a - b + b/x**2)/b, True
))/2 - Piecewise((2*atan(sqrt(a - b + b/x**2)/sqrt(-a + b))/sqrt(-a + b),
Ne(b, 0)), (-log(x**2)/sqrt(a), True))/2
```

3.811.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for m
ore detail
```

3.811.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(44) = 88$.

Time = 0.42 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.59

$$\int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}x^3}\,dx = -\frac{\log\left(\left(\sqrt{a-b}x-\sqrt{ax^2-bx^2+b}\right)^2\right)}{2\sqrt{a-b}\operatorname{sgn}(x)} - \frac{2\sqrt{a-b}}{\left(\left(\sqrt{a-b}x-\sqrt{ax^2-bx^2+b}\right)^2-b\right)\operatorname{sgn}(x)}$$

input
```
integrate((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log((sqrt(a - b)*x - sqrt(a*x^2 - b*x^2 + b))^2)/(sqrt(a - b)*sgn(x))
 - 2*sqrt(a - b)/(((sqrt(a - b)*x - sqrt(a*x^2 - b*x^2 + b))^2 - b)*sgn(x)
)
```

3.811.9 Mupad [B] (verification not implemented)

Time = 21.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}x^3}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{a-b+\frac{b}{x^2}}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} + \frac{\sqrt{a-b+\frac{b}{x^2}}}{b}$$

input
```
int((x^2 - 1)/(x^3*(a - b + b/x^2)^(1/2)),x)
```

output
```
atanh((a - b + b/x^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2) + (a - b + b/x^2)
^(1/2)/b
```

3.812 $\int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}x^3}\,dx$

3.812.1 Optimal result

Integrand size = 22, antiderivative size = 58

$$\int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}x^3}\,dx=\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

output
```
arctanh((a-b*(1-1/x^2))^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)+(a-b*(1-1/x^2))^(1/
2)/b
```

3.812.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.91

$$\int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}x^3}\,dx$$
$$=\frac{\sqrt{-a+b}(b+ax^2-bx^2)-2bx\sqrt{b+ax^2-bx^2}\arctan\left(\frac{\sqrt{-a+b}x}{\sqrt{b}-\sqrt{b+(a-b)x^2}}\right)}{b\sqrt{-a+b}\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}x^2}$$

input
```
Integrate[(-1 + x^2)/(Sqrt[a + b*(-1 + x^(-2))]*x^3),x]
```

output
```
(Sqrt[-a + b]*(b + a*x^2 - b*x^2) - 2*b*x*Sqrt[b + a*x^2 - b*x^2]*ArcTan[(
Sqrt[-a + b]*x)/(Sqrt[b] - Sqrt[b + (a - b)*x^2])])/(b*Sqrt[-a + b]*Sqrt[a
 + b*(-1 + x^(-2))]*x^2)
```

3.812.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2090, 1016, 948, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-1}{x^3\sqrt{a+b\left(\frac{1}{x^2}-1\right)}}dx$$

$$\Big\downarrow \text{2090}$$

$$\int \frac{x^2-1}{x^3\sqrt{a+\frac{b}{x^2}-b}}dx$$

$$\Big\downarrow \text{1016}$$

$$\int \frac{1-\frac{1}{x^2}}{x\sqrt{a+\frac{b}{x^2}-b}}dx$$

$$\Big\downarrow \text{948}$$

$$-\frac{1}{2}\int \frac{\left(1-\frac{1}{x^2}\right)x^2}{\sqrt{a-b+\frac{b}{x^2}}}d\frac{1}{x^2}$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(\frac{2\sqrt{a+\frac{b}{x^2}-b}}{b}-\int \frac{x^2}{\sqrt{a-b+\frac{b}{x^2}}}d\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{2\sqrt{a+\frac{b}{x^2}-b}}{b}-\frac{2\int \frac{1}{-\frac{a}{b}+\frac{1}{bx^4}+1}d\sqrt{a-b+\frac{b}{x^2}}}{b}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{x^2}-b}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}+\frac{2\sqrt{a+\frac{b}{x^2}-b}}{b}\right)$$

input
```
Int[(-1 + x^2)/(Sqrt[a + b*(-1 + x^(-2))]*x^3),x]
```

output
```
((2*Sqrt[a - b + b/x^2])/b + (2*ArcTanh[Sqrt[a - b + b/x^2]/Sqrt[a - b]])/
Sqrt[a - b])/2
```

3.812.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

rule 2090
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && BinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && BinomialMatchQ[u
, x])
```

3.812.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(50) = 100$.

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.76

method	result	size
default	$\frac{\sqrt{a\,x^{2}-b\,x^{2}+b}\,\left(\ln\left(x\sqrt{a-b}+\sqrt{a\,x^{2}-b\,x^{2}+b}\right) b x+\sqrt{a\,x^{2}-b\,x^{2}+b}\,\sqrt{a-b}\right)}{\sqrt{\frac{a\,x^{2}-b\,x^{2}+b}{x^{2}}}\,x^{2} \sqrt{a-b}\, b}$	102
risch	$\frac{a\,x^{2}-b\,x^{2}+b}{b\,x^{2} \sqrt{\frac{a\,x^{2}-b\,x^{2}+b}{x^{2}}}}+\frac{\ln\left(x\sqrt{a-b}+\sqrt{x^{2} \left(a-b\right)+b}\right) \sqrt{a\,x^{2}-b\,x^{2}+b}}{\sqrt{a-b}\,\sqrt{\frac{a\,x^{2}-b\,x^{2}+b}{x^{2}}}\, x}$	110

input
```
int((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a*x^2-b*x^2+b)^(1/2)*(ln(x*(a-b)^(1/2)+(a*x^2-b*x^2+b)^(1/2))*b*x+(a*x^2-
b*x^2+b)^(1/2)*(a-b)^(1/2))/((a*x^2-b*x^2+b)/x^2)^(1/2)/x^2/(a-b)^(1/2)/b
```

3.812.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.10

$$\int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}x^3}\,dx = \left[\frac{\sqrt{a-b}b\log\left(-2\,(a-b)x^2-2\,\sqrt{a-b}x^2\sqrt{\frac{(a-b)x^2+b}{x^2}}-b\right)+2\,(a-b)\sqrt{\frac{(a-b)x^2+b}{x^2}}}{2\,(ab-b^2)}, \frac{\sqrt{-a+b}b\arctan\left(-\right.}{}\right.$$

input
```
integrate((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a - b)*b*log(-2*(a - b)*x^2 - 2*sqrt(a - b)*x^2*sqrt(((a - b)*x
^2 + b)/x^2) - b) + 2*(a - b)*sqrt(((a - b)*x^2 + b)/x^2))/(a*b - b^2), (s
qrt(-a + b)*b*arctan(-sqrt(-a + b)*x^2*sqrt(((a - b)*x^2 + b)/x^2)/((a - b
)*x^2 + b)) + (a - b)*sqrt(((a - b)*x^2 + b)/x^2))/(a*b - b^2)]
```

3.812.6 Sympy [A] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.26

$$\int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}x^3}\,dx = -\frac{\begin{cases} -\frac{1}{\sqrt{a}x^2} & \text{for}\: b = 0 \\ -\frac{2\sqrt{a-b+\frac{b}{x^2}}}{b} & \text{otherwise} \end{cases}}{2} - \frac{\begin{cases} \frac{2\operatorname{atan}\left(\frac{\sqrt{a-b+\frac{b}{x^2}}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} & \text{for}\: b \neq 0 \\ -\frac{\log\left(x^2\right)}{\sqrt{a}} & \text{otherwise} \end{cases}}{2}$$

input
```
integrate((x**2-1)/x**3/(a+b*(-1+1/x**2))**(1/2),x)
```

output
```
-Piecewise((-1/(sqrt(a)*x**2), Eq(b, 0)), (-2*sqrt(a - b + b/x**2)/b, True
))/2 - Piecewise((2*atan(sqrt(a - b + b/x**2)/sqrt(-a + b))/sqrt(-a + b),
Ne(b, 0)), (-log(x**2)/sqrt(a), True))/2
```

3.812.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for m
ore detail
```

3.812.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(44) = 88$.

Time = 0.43 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.59

$$\int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}x^3}\,dx = -\frac{\log\left(\left(\sqrt{a-b}x-\sqrt{ax^2-bx^2+b}\right)^2\right)}{2\sqrt{a-b}\operatorname{sgn}(x)} - \frac{2\sqrt{a-b}}{\left(\left(\sqrt{a-b}x-\sqrt{ax^2-bx^2+b}\right)^2-b\right)\operatorname{sgn}(x)}$$

input
```
integrate((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x, algorithm="giac")
```

output
```
-1/2*log((sqrt(a - b)*x - sqrt(a*x^2 - b*x^2 + b))^2)/(sqrt(a - b)*sgn(x))
 - 2*sqrt(a - b)/(((sqrt(a - b)*x - sqrt(a*x^2 - b*x^2 + b))^2 - b)*sgn(x)
)
```

3.812.9 Mupad [B] (verification not implemented)

Time = 21.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.07

$$\int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}x^3}\,dx = \frac{\sqrt{a+b\,\left(\frac{1}{x^2}-1\right)}}{b} + \frac{\ln\left(x^2\left(2\,a-2\,b+2\,\sqrt{a-b}\,\sqrt{a+b\,\left(\frac{1}{x^2}-1\right)}+\frac{b}{x^2}\right)\right)}{2\,\sqrt{a-b}}$$

input
```
int((x^2 - 1)/(x^3*(a + b*(1/x^2 - 1))^(1/2)),x)
```

output
```
(a + b*(1/x^2 - 1))^(1/2)/b + log(x^2*(2*a - 2*b + 2*(a - b)^(1/2)*(a + b*
(1/x^2 - 1))^(1/2) + b/x^2))/(2*(a - b)^(1/2))
```

3.813 $\int \frac{1+x}{(4+x^2)\sqrt{9+x^2}}\,dx$

3.813.1 Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{1+x}{(4+x^2)\sqrt{9+x^2}}\,dx=\frac{\arctan\left(\frac{\sqrt{5}x}{2\sqrt{9+x^2}}\right)}{2\sqrt{5}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{9+x^2}}{\sqrt{5}}\right)}{\sqrt{5}}$$

output
```
1/10*arctan(1/2*x*5^(1/2)/(x^2+9)^(1/2))*5^(1/2)-1/5*arctanh(1/5*(x^2+9)^(
1/2)*5^(1/2))*5^(1/2)
```

3.813.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04

$$\int \frac{1+x}{(4+x^2)\sqrt{9+x^2}}\,dx=-\frac{\arctan\left(\frac{4+x^2-x\sqrt{9+x^2}}{2\sqrt{5}}\right)+2\operatorname{arctanh}\left(\frac{\sqrt{9+x^2}}{\sqrt{5}}\right)}{2\sqrt{5}}$$

input
```
Integrate[(1 + x)/((4 + x^2)*Sqrt[9 + x^2]),x]
```

output
```
-1/2*(ArcTan[(4 + x^2 - x*Sqrt[9 + x^2])/(2*Sqrt[5])] + 2*ArcTanh[Sqrt[9 +
 x^2]/Sqrt[5]])/Sqrt[5]
```

3.813.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1343, 291, 216, 353, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{(x^2+4)\sqrt{x^2+9}}dx$$

$$\Big\downarrow \text{1343}$$

$$\int \frac{1}{(x^2+4)\sqrt{x^2+9}}dx+\int \frac{x}{(x^2+4)\sqrt{x^2+9}}dx$$

$$\Big\downarrow \text{291}$$

$$\int \frac{x}{(x^2+4)\sqrt{x^2+9}}dx+\int \frac{1}{\frac{5x^2}{x^2+9}+4}d\frac{x}{\sqrt{x^2+9}}$$

$$\Big\downarrow \text{216}$$

$$\int \frac{x}{(x^2+4)\sqrt{x^2+9}}dx+\frac{\arctan\left(\frac{\sqrt{5}x}{2\sqrt{x^2+9}}\right)}{2\sqrt{5}}$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{1}{(x^2+4)\sqrt{x^2+9}}dx^2+\frac{\arctan\left(\frac{\sqrt{5}x}{2\sqrt{x^2+9}}\right)}{2\sqrt{5}}$$

$$\Big\downarrow \text{73}$$

$$\int \frac{1}{x^4-5}d\sqrt{x^2+9}+\frac{\arctan\left(\frac{\sqrt{5}x}{2\sqrt{x^2+9}}\right)}{2\sqrt{5}}$$

$$\Big\downarrow \text{220}$$

$$\frac{\arctan\left(\frac{\sqrt{5}x}{2\sqrt{x^2+9}}\right)}{2\sqrt{5}}-\frac{\text{arctanh}\left(\frac{\sqrt{x^2+9}}{\sqrt{5}}\right)}{\sqrt{5}}$$

input
```
Int[(1 + x)/((4 + x^2)*Sqrt[9 + x^2]),x]
```

output
```
ArcTan[(Sqrt[5]*x)/(2*Sqrt[9 + x^2])]/(2*Sqrt[5]) - ArcTanh[Sqrt[9 + x^2]/
Sqrt[5]]/Sqrt[5]
```

3.813.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1343
```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Simp[g   Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Simp[h
  Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p,
q}, x]
```

3.813. $\int \frac{1+x}{(4+x^2)\sqrt{9+x^2}}\,dx$

3.813.4 Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

method	result
default	$\frac{\arctan\left(\frac{x\sqrt{5}}{2\sqrt{x^2+9}}\right)\sqrt{5}}{10}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{x^2+9}\,\sqrt{5}}{5}\right)\sqrt{5}}{5}$
trager	$\operatorname{RootOf}\left(1280\_Z^4-96\_Z^2+5\right)\ln\left(-\frac{-6400\operatorname{RootOf}\left(1280\_Z^4-96\_Z^2+5\right)^5 x+1120\operatorname{RootOf}\left(1280\_Z^4-96\_Z\right.}{}\right.$

input
```
int((x+1)/(x^2+4)/(x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/10*arctan(1/2*x*5^(1/2)/(x^2+9)^(1/2))*5^(1/2)-1/5*arctanh(1/5*(x^2+9)^(
1/2)*5^(1/2))*5^(1/2)
```

3.813.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.36

$$\int \frac{1+x}{(4+x^2)\sqrt{9+x^2}}\,dx = -\frac{1}{20}\sqrt{5}\sqrt{4i+3}\log\left(-(i-2)\sqrt{5}\sqrt{4i+3}-5x+5\sqrt{x^2+9}-10i\right) + \frac{1}{20}\sqrt{5}\sqrt{4i+3}\log\left((i-2)\sqrt{5}\sqrt{4i+3}-5x+5\sqrt{x^2+9}-10i\right) - \frac{1}{20}\sqrt{5}\sqrt{-4i+3}\log\left((i+2)\sqrt{5}\sqrt{-4i+3}-5x+5\sqrt{x^2+9}+10i\right) + \frac{1}{20}\sqrt{5}\sqrt{-4i+3}\log\left(-(i+2)\sqrt{5}\sqrt{-4i+3}-5x+5\sqrt{x^2+9}+10i\right)$$

input
```
integrate((1+x)/(x^2+4)/(x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
-1/20*sqrt(5)*sqrt(4*I + 3)*log(-(I - 2)*sqrt(5)*sqrt(4*I + 3) - 5*x + 5*s
qrt(x^2 + 9) - 10*I) + 1/20*sqrt(5)*sqrt(4*I + 3)*log((I - 2)*sqrt(5)*sqrt
(4*I + 3) - 5*x + 5*sqrt(x^2 + 9) - 10*I) - 1/20*sqrt(5)*sqrt(-4*I + 3)*lo
g((I + 2)*sqrt(5)*sqrt(-4*I + 3) - 5*x + 5*sqrt(x^2 + 9) + 10*I) + 1/20*sq
rt(5)*sqrt(-4*I + 3)*log(-(I + 2)*sqrt(5)*sqrt(-4*I + 3) - 5*x + 5*sqrt(x^
2 + 9) + 10*I)
```

3.813.6 Sympy [F]

$$\int \frac{1+x}{(4+x^2)\sqrt{9+x^2}}\,dx = \int \frac{x+1}{(x^2+4)\sqrt{x^2+9}}\,dx$$

input
```
integrate((1+x)/(x**2+4)/(x**2+9)**(1/2),x)
```

output
```
Integral((x + 1)/((x**2 + 4)*sqrt(x**2 + 9)), x)
```

3.813.7 Maxima [F]

$$\int \frac{1+x}{(4+x^2)\sqrt{9+x^2}}\,dx = \int \frac{x+1}{\sqrt{x^2+9}(x^2+4)}\,dx$$

input
```
integrate((1+x)/(x^2+4)/(x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + 1)/(sqrt(x^2 + 9)*(x^2 + 4)), x)
```

3.813.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(38) = 76$.

Time = 0.34 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.32

$$\int \frac{1+x}{(4+x^2)\sqrt{9+x^2}}\,dx = -\frac{1}{10}\sqrt{5}\arctan\left(\frac{1}{2}x-\frac{1}{2}\sqrt{5}-\frac{1}{2}\sqrt{x^2+9}\right) - \frac{1}{10}\sqrt{5}\arctan\left(-\frac{1}{2}x-\frac{1}{2}\sqrt{5}+\frac{1}{2}\sqrt{x^2+9}\right) + \frac{1}{10}\sqrt{5}\log\left(\left(x-\sqrt{x^2+9}\right)^2+2\sqrt{5}\left(x-\sqrt{x^2+9}\right)+9\right) - \frac{1}{10}\sqrt{5}\log\left(\left(x-\sqrt{x^2+9}\right)^2-2\sqrt{5}\left(x-\sqrt{x^2+9}\right)+9\right)$$

input
```
integrate((1+x)/(x^2+4)/(x^2+9)^(1/2),x, algorithm="giac")
```

output
```
-1/10*sqrt(5)*arctan(1/2*x - 1/2*sqrt(5) - 1/2*sqrt(x^2 + 9)) - 1/10*sqrt(
5)*arctan(-1/2*x - 1/2*sqrt(5) + 1/2*sqrt(x^2 + 9)) + 1/10*sqrt(5)*log((x
- sqrt(x^2 + 9))^2 + 2*sqrt(5)*(x - sqrt(x^2 + 9)) + 9) - 1/10*sqrt(5)*log
((x - sqrt(x^2 + 9))^2 - 2*sqrt(5)*(x - sqrt(x^2 + 9)) + 9)
```

3.813.9 Mupad [B] (verification not implemented)

Time = 20.64 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.26

$$\int \frac{1+x}{(4+x^2)\sqrt{9+x^2}}\,dx = \sqrt{5}\left(\ln(x-2i)-\ln\left(\sqrt{5}\sqrt{x^2+9}+9+x\,2i\right)\right)\left(\frac{1}{10}-\frac{1}{20}i\right) + \sqrt{5}\left(\ln(x+2i)-\ln\left(\sqrt{5}\sqrt{x^2+9}+9-x\,2i\right)\right)\left(\frac{1}{10}+\frac{1}{20}i\right)$$

input
```
int((x + 1)/((x^2 + 4)*(x^2 + 9)^(1/2)),x)
```

output
```
5^(1/2)*(log(x - 2i) - log(x*2i + 5^(1/2)*(x^2 + 9)^(1/2) + 9))*(1/10 - 1i
/20) + 5^(1/2)*(log(x + 2i) - log(5^(1/2)*(x^2 + 9)^(1/2) - x*2i + 9))*(1/
10 + 1i/20)
```

3.814 $\int x\left(1+\sqrt{1-x^2}\right)\,dx$

3.814.1 Optimal result

Integrand size = 15, antiderivative size = 23

$$\int x\left(1+\sqrt{1-x^2}\right)\,dx=\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

output
```
1/2*x^2-1/3*(-x^2+1)^(3/2)
```

3.814.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x\left(1+\sqrt{1-x^2}\right)\,dx=-\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}\left(-1+x^2\right)$$

input
```
Integrate[x*(1 + Sqrt[1 - x^2]),x]
```

output
```
-1/3*(1 - x^2)^(3/2) + (-1 + x^2)/2
```

3.814.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(\sqrt{1-x^2}+1\right)\,dx$$

$$\Big\downarrow \text{ 2010}$$

$$\int \left(\sqrt{1-x^2}x+x\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

input
```
Int[x*(1 + Sqrt[1 - x^2]),x]
```

output
```
x^2/2 - (1 - x^2)^(3/2)/3
```

3.814.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

3.814.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{x^2}{2}-\frac{(-x^2+1)^{\frac{3}{2}}}{3}$	18
derivativedivides	$\frac{x^2}{2}-\frac{1}{2}-\frac{(-x^2+1)^{\frac{3}{2}}}{3}$	19
trager	$\frac{x^2}{2}+\left(\frac{x^2}{3}-\frac{1}{3}\right)\sqrt{-x^2+1}$	24

input
```
int(x*(1+(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2-1/3*(-x^2+1)^(3/2)
```

3.814.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x\left(1+\sqrt{1-x^2}\right)\,dx=\frac{1}{2}x^2+\frac{1}{3}\left(x^2-1\right)\sqrt{-x^2+1}$$

input
```
integrate(x*(1+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

output
```
1/2*x^2 + 1/3*(x^2 - 1)*sqrt(-x^2 + 1)
```

3.814.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x\left(1+\sqrt{1-x^2}\right)\,dx=\frac{x^2\sqrt{1-x^2}}{3}+\frac{x^2}{2}-\frac{\sqrt{1-x^2}}{3}$$

input
```
integrate(x*(1+(-x**2+1)**(1/2)),x)
```

output
```
x**2*sqrt(1 - x**2)/3 + x**2/2 - sqrt(1 - x**2)/3
```

3.814.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int x\left(1+\sqrt{1-x^2}\right)\,dx=\frac{1}{2}\,x^2-\frac{1}{3}\,(-x^2+1)^{\frac{3}{2}}$$

input
```
integrate(x*(1+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

output
```
1/2*x^2 - 1/3*(-x^2 + 1)^(3/2)
```

3.814.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int x\left(1+\sqrt{1-x^2}\right)\,dx=\frac{1}{2}\,x^2-\frac{1}{3}\,(-x^2+1)^{\frac{3}{2}}-\frac{1}{2}$$

input
```
integrate(x*(1+(-x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
1/2*x^2 - 1/3*(-x^2 + 1)^(3/2) - 1/2
```

3.814.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x\left(1+\sqrt{1-x^2}\right)\,dx=\frac{x^2}{2}+\sqrt{1-x^2}\,\left(\frac{x^2}{3}-\frac{1}{3}\right)$$

input
```
int(x*((1 - x^2)^(1/2) + 1),x)
```

output
```
x^2/2 + (1 - x^2)^(1/2)*(x^2/3 - 1/3)
```

3.815 $\int x\left(1+\sqrt{1-x}\sqrt{1+x}\right)\,dx$

3.815.1 Optimal result

Integrand size = 21, antiderivative size = 23

$$\int x\left(1+\sqrt{1-x}\sqrt{1+x}\right)\,dx=\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

output
```
1/2*x^2-1/3*(-x^2+1)^(3/2)
```

3.815.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x\left(1+\sqrt{1-x}\sqrt{1+x}\right)\,dx=-\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}\left(-1+x^2\right)$$

input
```
Integrate[x*(1 + Sqrt[1 - x]*Sqrt[1 + x]),x]
```

output
```
-1/3*(1 - x^2)^(3/2) + (-1 + x^2)/2
```

3.815.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(\sqrt{1-x}\sqrt{x+1}+1\right)\,dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(\sqrt{1-x^2}x+x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

input
```
Int[x*(1 + Sqrt[1 - x]*Sqrt[1 + x]),x]
```

output
```
x^2/2 - (1 - x^2)^(3/2)/3
```

3.815.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

3.815.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

method	result	size
default	$\frac{\sqrt{1-x}\,\sqrt{x+1}\,(x^2-1)}{3}+\frac{x^2}{2}$	26

input
```
int(x*(1+(1-x)^(1/2)*(x+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/3*(1-x)^(1/2)*(x+1)^(1/2)*(x^2-1)+1/2*x^2
```

3.815.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x\left(1+\sqrt{1-x}\sqrt{1+x}\right)\,dx=\frac{1}{2}\,x^2+\frac{1}{3}\,(x^2-1)\sqrt{x+1}\sqrt{-x+1}$$

input
```
integrate(x*(1+(1-x)^(1/2)*(1+x)^(1/2)),x, algorithm="fricas")
```

output
```
1/2*x^2 + 1/3*(x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)
```

3.815.6 Sympy [F]

$$\int x\left(1+\sqrt{1-x}\sqrt{1+x}\right)\,dx=\int x\left(\sqrt{1-x}\sqrt{x+1}+1\right)\,dx$$

input
```
integrate(x*(1+(1-x)**(1/2)*(1+x)**(1/2)),x)
```

output
```
Integral(x*(sqrt(1 - x)*sqrt(x + 1) + 1), x)
```

3.815.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int x\left(1+\sqrt{1-x}\sqrt{1+x}\right)\,dx=\frac{1}{2}\,x^2-\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(x*(1+(1-x)^(1/2)*(1+x)^(1/2)),x, algorithm="maxima")
```

output
```
1/2*x^2 - 1/3*(-x^2 + 1)^(3/2)
```

3.815.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(17) = 34.

Time = 0.47 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.35

$$\int x\left(1+\sqrt{1-x}\sqrt{1+x}\right)\,dx=\frac{1}{2}\,(x+1)^2+\frac{1}{6}\,((2\,x-5)(x+1)+9)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{2}\,\sqrt{x+1}(x-2)\sqrt{-x+1}-x-1$$

input
```
integrate(x*(1+(1-x)^(1/2)*(1+x)^(1/2)),x, algorithm="giac")
```

output
```
1/2*(x + 1)^2 + 1/6*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 1/2
*sqrt(x + 1)*(x - 2)*sqrt(-x + 1) - x - 1
```

3.815.9 Mupad [B] (verification not implemented)

Time = 21.38 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

$$\int x\left(1+\sqrt{1-x}\sqrt{1+x}\right)\,dx=\frac{x^2}{2}-\frac{\sqrt{1-x}\left(-\frac{x^3}{3}-\frac{x^2}{3}+\frac{x}{3}+\frac{1}{3}\right)}{\sqrt{x+1}}$$

input
```
int(x*((1 - x)^(1/2)*(x + 1)^(1/2) + 1),x)
```

output
```
x^2/2 - ((1 - x)^(1/2)*(x/3 - x^2/3 - x^3/3 + 1/3))/(x + 1)^(1/2)
```

3.816 $\int x\left(1+\frac{1}{\sqrt{2+x}\sqrt{3+x}}\right)\,dx$

3.816.1 Optimal result

Integrand size = 19, antiderivative size = 33

$$\int x\left(1+\frac{1}{\sqrt{2+x}\sqrt{3+x}}\right)\,dx=\frac{x^2}{2}+\sqrt{2+x}\sqrt{3+x}-5\text{arcsinh}\left(\sqrt{2+x}\right)$$

output
```
1/2*x^2-5*arcsinh((2+x)^(1/2))+(2+x)^(1/2)*(3+x)^(1/2)
```

3.816.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.64

$$\int x\left(1+\frac{1}{\sqrt{2+x}\sqrt{3+x}}\right)\,dx=\frac{x^2}{2}+\frac{\sqrt{3+x}}{\sqrt{2+x}\left(-1+\frac{3+x}{2+x}\right)}-5\text{arctanh}\left(\frac{\sqrt{3+x}}{\sqrt{2+x}}\right)$$

input
```
Integrate[x*(1 + 1/(Sqrt[2 + x]*Sqrt[3 + x])),x]
```

output
```
x^2/2 + Sqrt[3 + x]/(Sqrt[2 + x]*(-1 + (3 + x)/(2 + x))) - 5*ArcTanh[Sqrt[
3 + x]/Sqrt[2 + x]]
```

3.816.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(\frac{1}{\sqrt{x+2}\sqrt{x+3}}+1\right)dx$$
$$\Big\downarrow \text{2010}$$
$$\int \left(\frac{x}{\sqrt{x+2}\sqrt{x+3}}+x\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-5\text{arcsinh}\left(\sqrt{x+2}\right)+\frac{x^2}{2}+\sqrt{x+2}\sqrt{x+3}$$

input
```
Int[x*(1 + 1/(Sqrt[2 + x]*Sqrt[3 + x])),x]
```

output
```
x^2/2 + Sqrt[2 + x]*Sqrt[3 + x] - 5*ArcSinh[Sqrt[2 + x]]
```

3.816.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

3.816.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(25) = 50$.

Time = 1.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.76

method	result	size
default	$-\frac{\sqrt{x+2}\,\sqrt{3+x}\,\left(-2\sqrt{x^{2}+5x+6}+5\ln\left(\frac{5}{2}+x+\sqrt{x^{2}+5x+6}\right)\right)}{2\sqrt{x^{2}+5x+6}}+\frac{x^{2}}{2}$	58

input
```
int(x*(1+1/(x+2)^(1/2)/(3+x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(x+2)^(1/2)*(3+x)^(1/2)*(-2*(x^2+5*x+6)^(1/2)+5*ln(5/2+x+(x^2+5*x+6)^
(1/2)))/(x^2+5*x+6)^(1/2)+1/2*x^2
```

3.816.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int x\left(1+\frac{1}{\sqrt{2+x}\sqrt{3+x}}\right)\,dx=\frac{1}{2}\,x^{2}+\sqrt{x+3}\sqrt{x+2}+\frac{5}{2}\,\log\left(2\,\sqrt{x+3}\sqrt{x+2}-2\,x-5\right)$$

input
```
integrate(x*(1+1/(2+x)^(1/2)/(3+x)^(1/2)),x, algorithm="fricas")
```

output
```
1/2*x^2 + sqrt(x + 3)*sqrt(x + 2) + 5/2*log(2*sqrt(x + 3)*sqrt(x + 2) - 2*
x - 5)
```

3.816.6 Sympy [F]

$$\int x\left(1+\frac{1}{\sqrt{2+x}\sqrt{3+x}}\right)\,dx=\int\frac{x\left(\sqrt{x+2}\sqrt{x+3}+1\right)}{\sqrt{x+2}\sqrt{x+3}}\,dx$$

input
```
integrate(x*(1+1/(2+x)**(1/2)/(3+x)**(1/2)),x)
```

output
```
Integral(x*(sqrt(x + 2)*sqrt(x + 3) + 1)/(sqrt(x + 2)*sqrt(x + 3)), x)
```

3.816.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int x\left(1+\frac{1}{\sqrt{2+x}\sqrt{3+x}}\right)\,dx=\frac{1}{2}\,x^2+\sqrt{x^2+5\,x+6}-\frac{5}{2}\,\log\left(2\,x+2\,\sqrt{x^2+5\,x+6}+5\right)$$

input
```
integrate(x*(1+1/(2+x)^(1/2)/(3+x)^(1/2)),x, algorithm="maxima")
```

output
```
1/2*x^2 + sqrt(x^2 + 5*x + 6) - 5/2*log(2*x + 2*sqrt(x^2 + 5*x + 6) + 5)
```

3.816.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int x\left(1+\frac{1}{\sqrt{2+x}\sqrt{3+x}}\right)\,dx=\frac{1}{2}\,(x+3)^2+\sqrt{x+3}\sqrt{x+2}-3\,x+5\,\log\left(\sqrt{x+3}-\sqrt{x+2}\right)-9$$

input
```
integrate(x*(1+1/(2+x)^(1/2)/(3+x)^(1/2)),x, algorithm="giac")
```

output
```
1/2*(x + 3)^2 + sqrt(x + 3)*sqrt(x + 2) - 3*x + 5*log(sqrt(x + 3) - sqrt(x
 + 2)) - 9
```

3.816.9 Mupad [B] (verification not implemented)

Time = 25.92 (sec) , antiderivative size = 180, normalized size of antiderivative = 5.45

$$\int x\left(1+\frac{1}{\sqrt{2+x}\sqrt{3+x}}\right)\,dx=\frac{\frac{10\left(\sqrt{x+2}-\sqrt{2}\right)}{\sqrt{x+3}-\sqrt{3}}+\frac{10\left(\sqrt{x+2}-\sqrt{2}\right)^3}{\left(\sqrt{x+3}-\sqrt{3}\right)^3}-\frac{8\sqrt{6}\left(\sqrt{x+2}-\sqrt{2}\right)^2}{\left(\sqrt{x+3}-\sqrt{3}\right)^2}}{\frac{\left(\sqrt{x+2}-\sqrt{2}\right)^4}{\left(\sqrt{x+3}-\sqrt{3}\right)^4}-\frac{2\left(\sqrt{x+2}-\sqrt{2}\right)^2}{\left(\sqrt{x+3}-\sqrt{3}\right)^2}+1}-10\,\mathrm{atanh}\left(\frac{\sqrt{x+2}-\sqrt{2}}{\sqrt{x+3}-\sqrt{3}}\right)+\frac{x^2}{2}$$

input
```
int(x*(1/((x + 2)^(1/2)*(x + 3)^(1/2)) + 1),x)
```

output
```
((10*((x + 2)^(1/2) - 2^(1/2)))/((x + 3)^(1/2) - 3^(1/2)) + (10*((x + 2)^(
1/2) - 2^(1/2))^3)/((x + 3)^(1/2) - 3^(1/2))^3 - (8*6^(1/2)*((x + 2)^(1/2)
 - 2^(1/2))^2)/((x + 3)^(1/2) - 3^(1/2))^2)/((((x + 2)^(1/2) - 2^(1/2))^4/(
(x + 3)^(1/2) - 3^(1/2))^4 - (2*((x + 2)^(1/2) - 2^(1/2))^2)/((x + 3)^(1/2
) - 3^(1/2))^2 + 1) - 10*atanh(((x + 2)^(1/2) - 2^(1/2))/((x + 3)^(1/2) -
3^(1/2))) + x^2/2
```

3.816. $\int x\left(1+\frac{1}{\sqrt{2+x}\sqrt{3+x}}\right)\,dx$

3.817 $\int \frac{x-\sqrt{x^6}}{x(1-x^4)}\,dx$

3.817.1 Optimal result

Integrand size = 24, antiderivative size = 45

$$\int \frac{x-\sqrt{x^6}}{x(1-x^4)}\,dx=\frac{\arctan(x)}{2}+\frac{\sqrt{x^6}\arctan(x)}{2x^3}+\frac{\text{arctanh}(x)}{2}-\frac{\sqrt{x^6}\text{arctanh}(x)}{2x^3}$$

output
```
1/2*arctan(x)+1/2*arctanh(x)+1/2*arctan(x)*(x^6)^(1/2)/x^3-1/2*arctanh(x)*
(x^6)^(1/2)/x^3
```

3.817.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$\int \frac{x-\sqrt{x^6}}{x(1-x^4)}\,dx=\frac{\arctan(x)}{2}-\frac{1}{2}\arctan\left(\frac{\sqrt{x^6}}{x^4}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt{x^6}}{x^4}\right)-\frac{1}{4}\log(1-x)+\frac{1}{4}\log(1+x)$$

input
```
Integrate[(x - Sqrt[x^6])/(x*(1 - x^4)),x]
```

output
```
ArcTan[x]/2 - ArcTan[Sqrt[x^6]/x^4]/2 - ArcTanh[Sqrt[x^6]/x^4]/2 - Log[1 -
 x]/4 + Log[1 + x]/4
```

3.817.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-\sqrt{x^6}}{x(1-x^4)} \, dx$$
$$\Big\downarrow \text{7276}$$
$$\int \left(\frac{1}{1-x^4}+\frac{\sqrt{x^6}}{x(x^4-1)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt{x^6}\arctan(x)}{2x^3}+\frac{\arctan(x)}{2}-\frac{\sqrt{x^6}\text{arctanh}(x)}{2x^3}+\frac{\text{arctanh}(x)}{2}$$

input
```
Int[(x - Sqrt[x^6])/(x*(1 - x^4)),x]
```

output
```
ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*Ar
cTanh[x])/(2*x^3)
```

3.817.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.817.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{\sqrt{x^6}\,(\ln(x-1)-\ln(x+1)+2\arctan(x))}{4x^3}+\frac{\arctan(x)}{2}+\frac{\operatorname{arctanh}(x)}{2}$	35
meijerg	$-\frac{x\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)-2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{1}{4}}}+\frac{\sqrt{x^6}\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)+2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{3}{4}}}$	80

input
```
int((x-(x^6)^(1/2))/x/(-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*(x^6)^(1/2)*(ln(x-1)-ln(x+1)+2*arctan(x))/x^3+1/2*arctan(x)+1/2*arctan
h(x)
```

3.817.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.04

$$\int \frac{x-\sqrt{x^6}}{x(1-x^4)}\,dx = \arctan(x)$$

input
```
integrate((x-(x^6)^(1/2))/x/(-x^4+1),x, algorithm="fricas")
```

output
```
arctan(x)
```

3.817.6 Sympy [F]

$$\int \frac{x-\sqrt{x^6}}{x(1-x^4)}\,dx = -\int \frac{x}{x^5-x}\,dx - \int \left(-\frac{\sqrt{x^6}}{x^5-x}\right)dx$$

input
```
integrate((x-(x**6)**(1/2))/x/(-x**4+1),x)
```

output
```
-Integral(x/(x**5 - x), x) - Integral(-sqrt(x**6)/(x**5 - x), x)
```

3.817.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.04

$$\int \frac{x-\sqrt{x^6}}{x\left(1-x^4\right)}\,dx=\arctan\left(x\right)$$

input
```
integrate((x-(x^6)^(1/2))/x/(-x^4+1),x, algorithm="maxima")
```

output
```
arctan(x)
```

3.817.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{x-\sqrt{x^6}}{x\left(1-x^4\right)}\,dx=\frac{1}{2}\left(\operatorname{sgn}(x)+1\right)\arctan\left(x\right)-\frac{1}{4}\left(\operatorname{sgn}(x)-1\right)\log\left(|x+1|\right)+\frac{1}{4}\left(\operatorname{sgn}(x)-1\right)\log\left(|x-1|\right)$$

input
```
integrate((x-(x^6)^(1/2))/x/(-x^4+1),x, algorithm="giac")
```

output
```
1/2*(sgn(x) + 1)*arctan(x) - 1/4*(sgn(x) - 1)*log(abs(x + 1)) + 1/4*(sgn(x
) - 1)*log(abs(x - 1))
```

3.817.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x-\sqrt{x^6}}{x\left(1-x^4\right)}\,dx=\int -\frac{x-\sqrt{x^6}}{x\,\left(x^4-1\right)}\,dx$$

input
```
int(-(x - (x^6)^(1/2))/(x*(x^4 - 1)),x)
```

output
```
int(-(x - (x^6)^(1/2))/(x*(x^4 - 1)), x)
```

3.818 $\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4}\,dx$

3.818.1 Optimal result

Integrand size = 24, antiderivative size = 45

$$\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4}\,dx=\frac{\arctan(x)}{2}+\frac{\sqrt{x^6}\arctan(x)}{2x^3}+\frac{\text{arctanh}(x)}{2}-\frac{\sqrt{x^6}\text{arctanh}(x)}{2x^3}$$

output
```
1/2*arctan(x)+1/2*arctanh(x)+1/2*arctan(x)*(x^6)^(1/2)/x^3-1/2*arctanh(x)*
(x^6)^(1/2)/x^3
```

3.818.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4}\,dx=\frac{\arctan(x)}{2}-\frac{1}{2}\arctan\left(\frac{\sqrt{x^6}}{x^4}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt{x^6}}{x^4}\right)-\frac{1}{4}\log(1-x)+\frac{1}{4}\log(1+x)$$

input
```
Integrate[(1 - Sqrt[x^6]/x)/(1 - x^4),x]
```

output
```
ArcTan[x]/2 - ArcTan[Sqrt[x^6]/x^4]/2 - ArcTanh[Sqrt[x^6]/x^4]/2 - Log[1 -
 x]/4 + Log[1 + x]/4
```

3.818.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4} \, dx$$

$$\Big\downarrow \; 7276$$

$$\int \left(\frac{1}{1-x^4}+\frac{\sqrt{x^6}}{x\left(x^4-1\right)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{x^6}\arctan(x)}{2x^3}+\frac{\arctan(x)}{2}-\frac{\sqrt{x^6}\text{arctanh}(x)}{2x^3}+\frac{\text{arctanh}(x)}{2}$$

input
```
Int[(1 - Sqrt[x^6]/x)/(1 - x^4),x]
```

output
```
ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*Ar
cTanh[x])/(2*x^3)
```

3.818.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.818.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{\sqrt{x^{6}}\,(\ln(x-1)-\ln(x+1)+2\arctan(x))}{4x^{3}}+\frac{\arctan(x)}{2}+\frac{\operatorname{arctanh}(x)}{2}$	35
meijerg	$-\frac{x\left(\ln\left(1-(x^{4})^{\frac{1}{4}}\right)-\ln\left(1+(x^{4})^{\frac{1}{4}}\right)-2\arctan\left((x^{4})^{\frac{1}{4}}\right)\right)}{4(x^{4})^{\frac{1}{4}}}+\frac{\sqrt{x^{6}}\left(\ln\left(1-(x^{4})^{\frac{1}{4}}\right)-\ln\left(1+(x^{4})^{\frac{1}{4}}\right)+2\arctan\left((x^{4})^{\frac{1}{4}}\right)\right)}{4(x^{4})^{\frac{3}{4}}}$	80

input
```
int((1-(x^6)^(1/2)/x)/(-x^4+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*(x^6)^(1/2)*(ln(x-1)-ln(x+1)+2*arctan(x))/x^3+1/2*arctan(x)+1/2*arctan
h(x)
```

3.818.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.04

$$\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4}\,dx = \arctan(x)$$

input
```
integrate((1-(x^6)^(1/2)/x)/(-x^4+1),x, algorithm="fricas")
```

output
```
arctan(x)
```

3.818.6 Sympy [F]

$$\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4}\,dx = -\int \frac{x}{x^5-x}\,dx - \int \left(-\frac{\sqrt{x^6}}{x^5-x}\right)dx$$

input
```
integrate((1-(x**6)**(1/2)/x)/(-x**4+1),x)
```

output
```
-Integral(x/(x**5 - x), x) - Integral(-sqrt(x**6)/(x**5 - x), x)
```

3.818.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.04

$$\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4}\,dx = \arctan(x)$$

input
```
integrate((1-(x^6)^(1/2)/x)/(-x^4+1),x, algorithm="maxima")
```

output
```
arctan(x)
```

3.818.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4}\,dx = \frac{1}{2}(\operatorname{sgn}(x)+1)\arctan(x) - \frac{1}{4}(\operatorname{sgn}(x)-1)\log(|x+1|) + \frac{1}{4}(\operatorname{sgn}(x)-1)\log(|x-1|)$$

input
```
integrate((1-(x^6)^(1/2)/x)/(-x^4+1),x, algorithm="giac")
```

output
```
1/2*(sgn(x) + 1)*arctan(x) - 1/4*(sgn(x) - 1)*log(abs(x + 1)) + 1/4*(sgn(x
) - 1)*log(abs(x - 1))
```

3.818.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4}\,dx = \int \frac{\frac{\sqrt{x^6}}{x}-1}{x^4-1}\,dx$$

input
```
int(((x^6)^(1/2)/x - 1)/(x^4 - 1),x)
```

output
```
int(((x^6)^(1/2)/x - 1)/(x^4 - 1), x)
```

3.819 $\int \frac{x-\sqrt{x^6}}{x-x^5} \, dx$

3.819.1 Optimal result

Integrand size = 21, antiderivative size = 45

$$\int \frac{x-\sqrt{x^6}}{x-x^5} \, dx = \frac{\arctan(x)}{2} + \frac{\sqrt{x^6}\arctan(x)}{2x^3} + \frac{\operatorname{arctanh}(x)}{2} - \frac{\sqrt{x^6}\operatorname{arctanh}(x)}{2x^3}$$

output
```
1/2*arctan(x)+1/2*arctanh(x)+1/2*arctan(x)*(x^6)^(1/2)/x^3-1/2*arctanh(x)*
(x^6)^(1/2)/x^3
```

3.819.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$\int \frac{x-\sqrt{x^6}}{x-x^5} \, dx = \frac{\arctan(x)}{2} - \frac{1}{2}\arctan\left(\frac{\sqrt{x^6}}{x^4}\right) - \frac{1}{2}\operatorname{arctanh}\left(\frac{\sqrt{x^6}}{x^4}\right) - \frac{1}{4}\log(1-x) + \frac{1}{4}\log(1+x)$$

input
```
Integrate[(x - Sqrt[x^6])/(x - x^5),x]
```

output
```
ArcTan[x]/2 - ArcTan[Sqrt[x^6]/x^4]/2 - ArcTanh[Sqrt[x^6]/x^4]/2 - Log[1 -
 x]/4 + Log[1 + x]/4
```

3.819.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2026, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-\sqrt{x^6}}{x-x^5} \, dx$$

$$\Big\downarrow \text{ 2026}$$

$$\int \frac{x-\sqrt{x^6}}{x (1-x^4)}dx$$

$$\Big\downarrow \text{ 7276}$$

$$\int \left(\frac{1}{1-x^4}+\frac{\sqrt{x^6}}{x (x^4-1)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{x^6} \arctan (x)}{2 x^3}+\frac{\arctan (x)}{2}-\frac{\sqrt{x^6}\text{arctanh}(x)}{2 x^3}+\frac{\text{arctanh}(x)}{2}$$

input
```
Int[(x - Sqrt[x^6])/(x - x^5),x]
```

output
```
ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*Ar
cTanh[x])/(2*x^3)
```

3.819.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.819.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{\sqrt{x^6}\,(-\ln(x-1)+\ln(x+1)-2\arctan(x))}{4x^3}+\frac{\arctan(x)}{2}+\frac{\operatorname{arctanh}(x)}{2}$	35
meijerg	$-\frac{x\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)-2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{1}{4}}}+\frac{\sqrt{x^6}\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)+2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{3}{4}}}$	80

input
```
int((x-(x^6)^(1/2))/(-x^5+x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(x^6)^(1/2)*(-ln(x-1)+ln(x+1)-2*arctan(x))/x^3+1/2*arctan(x)+1/2*arct
anh(x)
```

3.819.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.04

$$\int \frac{x-\sqrt{x^6}}{x-x^5}\,dx=\arctan(x)$$

input
```
integrate((x-(x^6)^(1/2))/(-x^5+x),x, algorithm="fricas")
```

output
```
arctan(x)
```

3.819.6 Sympy [F]

$$\int \frac{x-\sqrt{x^6}}{x-x^5}\,dx = -\int \frac{x}{x^5-x}\,dx - \int \left(-\frac{\sqrt{x^6}}{x^5-x}\right)dx$$

input
```
integrate((x-(x**6)**(1/2))/(-x**5+x),x)
```

output
```
-Integral(x/(x**5 - x), x) - Integral(-sqrt(x**6)/(x**5 - x), x)
```

3.819.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.04

$$\int \frac{x-\sqrt{x^6}}{x-x^5}\,dx = \arctan(x)$$

input
```
integrate((x-(x^6)^(1/2))/(-x^5+x),x, algorithm="maxima")
```

output
```
arctan(x)
```

3.819.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{x-\sqrt{x^6}}{x-x^5}\,dx = \frac{1}{2}(\mathrm{sgn}(x)+1)\arctan(x) - \frac{1}{4}(\mathrm{sgn}(x)-1)\log(|x+1|) + \frac{1}{4}(\mathrm{sgn}(x)-1)\log(|x-1|)$$

input
```
integrate((x-(x^6)^(1/2))/(-x^5+x),x, algorithm="giac")
```

output
```
1/2*(sgn(x) + 1)*arctan(x) - 1/4*(sgn(x) - 1)*log(abs(x + 1)) + 1/4*(sgn(x
) - 1)*log(abs(x - 1))
```

3.819.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x-\sqrt{x^6}}{x-x^5}\,dx=\int \frac{x-\sqrt{x^6}}{x-x^5}\,dx$$

input
```
int((x - (x^6)^(1/2))/(x - x^5),x)
```

output
```
int((x - (x^6)^(1/2))/(x - x^5), x)
```

3.820 $\int \frac{x}{x+\sqrt{x^6}}\,dx$

3.820.1 Optimal result

Integrand size = 13, antiderivative size = 45

$$\int \frac{x}{x+\sqrt{x^6}}\,dx=\frac{\arctan(x)}{2}+\frac{\sqrt{x^6}\arctan(x)}{2x^3}+\frac{\text{arctanh}(x)}{2}-\frac{\sqrt{x^6}\text{arctanh}(x)}{2x^3}$$

output
```
1/2*arctan(x)+1/2*arctanh(x)+1/2*arctan(x)*(x^6)^(1/2)/x^3-1/2*arctanh(x)*
(x^6)^(1/2)/x^3
```

3.820.2 Mathematica [A] (verified)

Time = 3.79 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$\int \frac{x}{x+\sqrt{x^6}}\,dx=\frac{\arctan(x)}{2}+\frac{1}{2}\arctan\left(\frac{\sqrt{x^6}}{x^2}\right)-\frac{1}{2}\text{arctanh}\left(\frac{\sqrt{x^6}}{x^2}\right)-\frac{1}{4}\log(1-x)+\frac{1}{4}\log(1+x)$$

input
```
Integrate[x/(x + Sqrt[x^6]),x]
```

output
```
ArcTan[x]/2 + ArcTan[Sqrt[x^6]/x^2]/2 - ArcTanh[Sqrt[x^6]/x^2]/2 - Log[1 -
 x]/4 + Log[1 + x]/4
```

3.820.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {7280, 9, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x^6}+x}dx$$

$$\Big\downarrow \text{7280}$$

$$\int \frac{x\left(x-\sqrt{x^6}\right)}{x^2-x^6}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{x-\sqrt{x^6}}{x\left(1-x^4\right)}dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{1}{1-x^4}+\frac{\sqrt{x^6}}{x\left(x^4-1\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{x^6}\arctan(x)}{2x^3}+\frac{\arctan(x)}{2}-\frac{\sqrt{x^6}\text{arctanh}(x)}{2x^3}+\frac{\text{arctanh}(x)}{2}$$

input
```
Int[x/(x + Sqrt[x^6]),x]
```

output
```
ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*Ar
cTanh[x])/(2*x^3)
```

3.820.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

rule 7280
```
Int[(u_.)/((a_.)*(x_)^(m_.) + (b_.)*Sqrt[(c_.)*(x_)^(n_)]), x_Symbol] :> In
t[u*((a*x^m - b*Sqrt[c*x^n])/(a^2*x^(2*m) - b^2*c*x^n)), x] /; FreeQ[{a, b,
 c, m, n}, x]
```

3.820.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.44

method	result	size
meijerg	$\frac{x^{\frac{3}{2}} \arctan\left(\frac{\left(x^{6}\right)^{\frac{1}{4}}}{\sqrt{x}}\right)}{\left(x^{6}\right)^{\frac{1}{4}}}$	20
default	$\frac{\arctan\left(\sqrt{\frac{\sqrt{x^{6}}}{x^{3}}}\, x\right)}{\sqrt{\frac{\sqrt{x^{6}}}{x^{3}}}}$	27

input
```
int(x/(x+(x^6)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x^(3/2)/(x^6)^(1/4)*arctan((x^6)^(1/4)/x^(1/2))
```

3.820.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.04

$$\int \frac{x}{x+\sqrt{x^6}}\,dx = \arctan(x)$$

input
```
integrate(x/(x+(x^6)^(1/2)),x, algorithm="fricas")
```

output
```
arctan(x)
```

3.820.6 Sympy [F]

$$\int \frac{x}{x+\sqrt{x^6}}\,dx = \int \frac{x}{x+\sqrt{x^6}}\,dx$$

input
```
integrate(x/(x+(x**6)**(1/2)),x)
```

output
```
Integral(x/(x + sqrt(x**6)), x)
```

3.820.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.04

$$\int \frac{x}{x+\sqrt{x^6}}\,dx = \arctan(x)$$

input
```
integrate(x/(x+(x^6)^(1/2)),x, algorithm="maxima")
```

output
```
arctan(x)
```

3.820.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.27

$$\int \frac{x}{x+\sqrt{x^6}}\,dx = \frac{\arctan\left(x\sqrt{\operatorname{sgn}(x)}\right)}{\sqrt{\operatorname{sgn}(x)}}$$

input
```
integrate(x/(x+(x^6)^(1/2)),x, algorithm="giac")
```

output
```
arctan(x*sqrt(sgn(x)))/sqrt(sgn(x))
```

3.820.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{x+\sqrt{x^6}}\,dx = \int \frac{x}{x+\sqrt{x^6}}\,dx$$

input
```
int(x/(x + (x^6)^(1/2)),x)
```

output
```
int(x/(x + (x^6)^(1/2)), x)
```

3.821 $\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3} dx$

3.821.1 Optimal result

Integrand size = 25, antiderivative size = 52

$$\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3} \, dx = \arctan\left(\sqrt{x}\right)+\frac{\sqrt{x^3}\arctan\left(\sqrt{x}\right)}{x^{3/2}}+\operatorname{arctanh}\left(\sqrt{x}\right)-\frac{\sqrt{x^3}\operatorname{arctanh}\left(\sqrt{x}\right)}{x^{3/2}}$$

output
```
arctan(x^(1/2))+arctanh(x^(1/2))+arctan(x^(1/2))*(x^3)^(1/2)/x^(3/2)-arcta
nh(x^(1/2))*(x^3)^(1/2)/x^(3/2)
```

3.821.2 Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3} \, dx = \arctan\left(\sqrt{x}\right)+\arctan\left(\frac{\sqrt{x^3}}{x}\right)+\operatorname{arctanh}\left(\sqrt{x}\right)-\operatorname{arctanh}\left(\frac{\sqrt{x^3}}{x}\right)$$

input
```
Integrate[(Sqrt[x] - Sqrt[x^3])/(x - x^3),x]
```

output
```
ArcTan[Sqrt[x]] + ArcTan[Sqrt[x^3]/x] + ArcTanh[Sqrt[x]] - ArcTanh[Sqrt[x^
3]/x]
```

3.821.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2026, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3} \, dx$$

$$\Big\downarrow \text{ 2026}$$

$$\int \frac{\sqrt{x}-\sqrt{x^3}}{x (1-x^2)}dx$$

$$\Big\downarrow \text{ 7276}$$

$$\int \left(\frac{\sqrt{x^3}}{x (x^2-1)}-\frac{1}{\sqrt{x} (x^2-1)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{x^3} \arctan \left(\sqrt{x}\right)}{x^{3/2}}+\arctan \left(\sqrt{x}\right)-\frac{\sqrt{x^3}\text{arctanh}\left(\sqrt{x}\right)}{x^{3/2}}+\text{arctanh}\left(\sqrt{x}\right)$$

input
```
Int[(Sqrt[x] - Sqrt[x^3])/(x - x^3),x]
```

output
```
ArcTan[Sqrt[x]] + (Sqrt[x^3]*ArcTan[Sqrt[x]])/x^(3/2) + ArcTanh[Sqrt[x]] -
 (Sqrt[x^3]*ArcTanh[Sqrt[x]])/x^(3/2)
```

3.821.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.821.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

method	result	si
default	$\arctan\left(\sqrt{x}\right)+\operatorname{arctanh}\left(\sqrt{x}\right)+\frac{\sqrt{x^{3}}\,\left(\ln\left(-1+\sqrt{x}\right)-\ln\left(1+\sqrt{x}\right)+2\arctan\left(\sqrt{x}\right)\right)}{2x^{\frac{3}{2}}}$	41
meijerg	$-\frac{\sqrt{x}\,\left(\ln\left(1-\left(x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(x^{2}\right)^{\frac{1}{4}}\right)-2\arctan\left(\left(x^{2}\right)^{\frac{1}{4}}\right)\right)}{2\left(x^{2}\right)^{\frac{1}{4}}}+\frac{\sqrt{x^{3}}\,\left(\ln\left(1-\left(x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(x^{2}\right)^{\frac{1}{4}}\right)+2\arctan\left(\left(x^{2}\right)^{\frac{1}{4}}\right)\right)}{2\left(x^{2}\right)^{\frac{3}{4}}}$	82

input
```
int((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x,method=_RETURNVERBOSE)
```

output
```
arctan(x^(1/2))+arctanh(x^(1/2))+1/2*(x^3)^(1/2)*(ln(-1+x^(1/2))-ln(1+x^(1
/2))+2*arctan(x^(1/2)))/x^(3/2)
```

3.821.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt{x}-\sqrt{x^{3}}}{x-x^{3}}\,dx = 2\arctan\left(\sqrt{x}\right)$$

input
```
integrate((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x, algorithm="fricas")
```

output
```
2*arctan(sqrt(x))
```

3.821.6 Sympy [F]

$$\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3}\,dx = -\int \frac{\sqrt{x}}{x^3-x}\,dx - \int \left(-\frac{\sqrt{x^3}}{x^3-x}\right)dx$$

input
```
integrate((x**(1/2)-(x**3)**(1/2))/(-x**3+x),x)
```

output
```
-Integral(sqrt(x)/(x**3 - x), x) - Integral(-sqrt(x**3)/(x**3 - x), x)
```

3.821.7 Maxima [F]

$$\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3}\,dx = \int \frac{\sqrt{x^3}-\sqrt{x}}{x^3-x}\,dx$$

input
```
integrate((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x, algorithm="maxima")
```

output
```
arctan(sqrt(x)) - integrate(1/2*sqrt(x)/(x + 1), x) + integrate(1/4/(sqrt(
x) + 1), x) + integrate(1/4/(sqrt(x) - 1), x) + 1/2*log(sqrt(x) + 1) - 1/2
*log(sqrt(x) - 1)
```

3.821.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3}\,dx = 2\arctan\left(\sqrt{x}\right)$$

input
```
integrate((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x, algorithm="giac")
```

output
```
2*arctan(sqrt(x))
```

3.821.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3}\,dx=-\int \frac{\sqrt{x^3}-\sqrt{x}}{x-x^3}\,dx$$

input
```
int(-((x^3)^(1/2) - x^(1/2))/(x - x^3),x)
```

output
```
-int(((x^3)^(1/2) - x^(1/2))/(x - x^3), x)
```

3.822 $\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx$

3.822.1 Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx = \arctan\left(\sqrt{x}\right) + \frac{\sqrt{x^3}\arctan\left(\sqrt{x}\right)}{x^{3/2}} + \operatorname{arctanh}\left(\sqrt{x}\right) - \frac{\sqrt{x^3}\operatorname{arctanh}\left(\sqrt{x}\right)}{x^{3/2}}$$

output
```
arctan(x^(1/2))+arctanh(x^(1/2))+arctan(x^(1/2))*(x^3)^(1/2)/x^(3/2)-arcta
nh(x^(1/2))*(x^3)^(1/2)/x^(3/2)
```

3.822.2 Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx = \arctan\left(\sqrt{x}\right) + \arctan\left(\frac{\sqrt{x^3}}{x}\right) + \operatorname{arctanh}\left(\sqrt{x}\right) - \operatorname{arctanh}\left(\frac{\sqrt{x^3}}{x}\right)$$

input
```
Integrate[(Sqrt[x] + Sqrt[x^3])^(-1),x]
```

output
```
ArcTan[Sqrt[x]] + ArcTan[Sqrt[x^3]/x] + ArcTanh[Sqrt[x]] - ArcTanh[Sqrt[x^
3]/x]
```

3.822.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {7280, 2026, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^3}+\sqrt{x}}dx$$

$$\Big\downarrow \text{7280}$$

$$\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{\sqrt{x}-\sqrt{x^3}}{x(1-x^2)}dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{\sqrt{x^3}}{x(x^2-1)}-\frac{1}{\sqrt{x}(x^2-1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{x^3}\arctan\left(\sqrt{x}\right)}{x^{3/2}}+\arctan\left(\sqrt{x}\right)-\frac{\sqrt{x^3}\text{arctanh}\left(\sqrt{x}\right)}{x^{3/2}}+\text{arctanh}\left(\sqrt{x}\right)$$

input
```
Int[(Sqrt[x] + Sqrt[x^3])^(-1),x]
```

output
```
ArcTan[Sqrt[x]] + (Sqrt[x^3]*ArcTan[Sqrt[x]])/x^(3/2) + ArcTanh[Sqrt[x]] -
 (Sqrt[x^3]*ArcTanh[Sqrt[x]])/x^(3/2)
```

3.822.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

rule 7280
```
Int[(u_.)/((a_.)*(x_)^(m_.) + (b_.)*Sqrt[(c_.)*(x_)^(n_)]), x_Symbol] :> In
t[u*((a*x^m - b*Sqrt[c*x^n])/(a^2*x^(2*m) - b^2*c*x^n)), x] /; FreeQ[{a, b,
 c, m, n}, x]
```

3.822.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.40

method	result	size
meijerg	$\frac{2x^{\frac{3}{4}} \arctan\left(\frac{\left(x^{3}\right)^{\frac{1}{4}}}{x^{\frac{1}{4}}}\right)}{\left(x^{3}\right)^{\frac{1}{4}}}$	21
default	$\frac{2 \arctan\left(\sqrt{\frac{\sqrt{x^{3}}}{x^{\frac{3}{2}}}}\, \sqrt{x}\right)}{\sqrt{\frac{\sqrt{x^{3}}}{x^{\frac{3}{2}}}}}$	30

input
```
int(1/(x^(1/2)+(x^3)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/(x^3)^(1/4)*x^(3/4)*arctan(1/x^(1/4)*(x^3)^(1/4))
```

3.822.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.12

$$\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx = 2\arctan\left(\sqrt{x}\right)$$

input
```
integrate(1/(x^(1/2)+(x^3)^(1/2)),x, algorithm="fricas")
```

output
```
2*arctan(sqrt(x))
```

3.822.6 Sympy [F]

$$\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx = \int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx$$

input
```
integrate(1/(x**(1/2)+(x**3)**(1/2)),x)
```

output
```
Integral(1/(sqrt(x) + sqrt(x**3)), x)
```

3.822.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.12

$$\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx = 2\arctan\left(\sqrt{x}\right)$$

input
```
integrate(1/(x^(1/2)+(x^3)^(1/2)),x, algorithm="maxima")
```

output
```
2*arctan(sqrt(x))
```

3.822.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.12

$$\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx = 2\arctan\left(\sqrt{x}\right)$$

input
```
integrate(1/(x^(1/2)+(x^3)^(1/2)),x, algorithm="giac")
```

output
```
2*arctan(sqrt(x))
```

3.822.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx = \int \frac{1}{\sqrt{x^3}+\sqrt{x}}\,dx$$

input
```
int(1/((x^3)^(1/2) + x^(1/2)),x)
```

output
```
int(1/((x^3)^(1/2) + x^(1/2)), x)
```

3.823 $\int \frac{1}{\sqrt{-1+x}+\sqrt{(-1+x)^3}}\,dx$

3.823.1 Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{(-1+x)^3}}\,dx = \arctan\left(\sqrt{-1+x}\right)+\frac{\sqrt{(-1+x)^3}\arctan\left(\sqrt{-1+x}\right)}{(-1+x)^{3/2}} + \operatorname{arctanh}\left(\sqrt{-1+x}\right)-\frac{\sqrt{(-1+x)^3}\operatorname{arctanh}\left(\sqrt{-1+x}\right)}{(-1+x)^{3/2}}$$

output
```
arctan((-1+x)^(1/2))+arctanh((-1+x)^(1/2))+arctan((-1+x)^(1/2))*((-1+x)^3)
^(1/2)/(-1+x)^(3/2)-arctanh((-1+x)^(1/2))*((-1+x)^3)^(1/2)/(-1+x)^(3/2)
```

3.823.2 Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{(-1+x)^3}}\,dx = \arctan\left(\sqrt{-1+x}\right)+\arctan\left(\frac{\sqrt{-1+3x-3x^2+x^3}}{-1+x}\right) + \operatorname{arctanh}\left(\sqrt{-1+x}\right)-\operatorname{arctanh}\left(\frac{\sqrt{-1+3x-3x^2+x^3}}{-1+x}\right)$$

input
```
Integrate[(Sqrt[-1 + x] + Sqrt[(-1 + x)^3])^(-1),x]
```

output
```
ArcTan[Sqrt[-1 + x]] + ArcTan[Sqrt[-1 + 3*x - 3*x^2 + x^3]/(-1 + x)] + Arc
Tanh[Sqrt[-1 + x]] - ArcTanh[Sqrt[-1 + 3*x - 3*x^2 + x^3]/(-1 + x)]
```

3.823.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {7281, 7280, 2026, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x-1}+\sqrt{(x-1)^3}}dx$$

$$\Big\downarrow \text{7281}$$

$$\int \frac{1}{\sqrt{x-1}+\sqrt{(x-1)^3}}d(x-1)$$

$$\Big\downarrow \text{7280}$$

$$\int \frac{\sqrt{x-1}-\sqrt{(x-1)^3}}{-(x-1)^3+x-1}d(x-1)$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{\sqrt{x-1}-\sqrt{(x-1)^3}}{\left(1-(x-1)^2\right)(x-1)}d(x-1)$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{\sqrt{(x-1)^3}}{\left((x-1)^2-1\right)(x-1)}-\frac{1}{\left((x-1)^2-1\right)\sqrt{x-1}}\right)d(x-1)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{(x-1)^3}\arctan\left(\sqrt{x-1}\right)}{(x-1)^{3/2}}+\arctan\left(\sqrt{x-1}\right)-\frac{\sqrt{(x-1)^3}\text{arctanh}\left(\sqrt{x-1}\right)}{(x-1)^{3/2}}+\text{arctanh}\left(\sqrt{x-1}\right)$$

input
```
Int[(Sqrt[-1 + x] + Sqrt[(-1 + x)^3])^(-1),x]
```

output
```
ArcTan[Sqrt[-1 + x]] + (Sqrt[(-1 + x)^3]*ArcTan[Sqrt[-1 + x]])/(-1 + x)^(3
/2) + ArcTanh[Sqrt[-1 + x]] - (Sqrt[(-1 + x)^3]*ArcTanh[Sqrt[-1 + x]])/(-1
 + x)^(3/2)
```

3.823.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

rule 7280
```
Int[(u_.)/((a_.)*(x_)^(m_.) + (b_.)*Sqrt[(c_.)*(x_)^(n_)]), x_Symbol] :> In
t[u*((a*x^m - b*Sqrt[c*x^n])/(a^2*x^(2*m) - b^2*c*x^n)), x] /; FreeQ[{a, b,
 c, m, n}, x]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.823.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{2\arctan\left(\sqrt{\frac{\sqrt{(x-1)^{3}}}{(x-1)^{\frac{3}{2}}}}\,\sqrt{x-1}\right)}{\sqrt{\frac{\sqrt{(x-1)^{3}}}{(x-1)^{\frac{3}{2}}}}}$	40

input
```
int(1/((x-1)^(1/2)+((x-1)^3)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/(((x-1)^3)^(1/2)/(x-1)^(3/2))^(1/2)*arctan((((x-1)^3)^(1/2)/(x-1)^(3/2))
^(1/2)*(x-1)^(1/2))
```

3.823.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.12

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{(-1+x)^3}}\,dx = 2\arctan\left(\sqrt{x-1}\right)$$

input
```
integrate(1/((-1+x)^(1/2)+((-1+x)^3)^(1/2)),x, algorithm="fricas")
```

output
```
2*arctan(sqrt(x - 1))
```

3.823.6 Sympy [F]

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{(-1+x)^3}}\,dx = \int \frac{1}{\sqrt{x-1}+\sqrt{(x-1)^3}}\,dx$$

input
```
integrate(1/((-1+x)**(1/2)+((-1+x)**3)**(1/2)),x)
```

output
```
Integral(1/(sqrt(x - 1) + sqrt((x - 1)**3)), x)
```

3.823.7 Maxima [F]

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{(-1+x)^3}}\,dx = \int \frac{1}{\sqrt{(x-1)^3}+\sqrt{x-1}}\,dx$$

input
```
integrate(1/((-1+x)^(1/2)+((-1+x)^3)^(1/2)),x, algorithm="maxima")
```

output
```
2*sqrt(x - 1) - integrate(sqrt(x - 1)/x, x)
```

3.823.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.12

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{(-1+x)^3}}\,dx = 2\arctan\left(\sqrt{x-1}\right)$$

input
```
integrate(1/((-1+x)^(1/2)+((-1+x)^3)^(1/2)),x, algorithm="giac")
```

output
```
2*arctan(sqrt(x - 1))
```

3.823.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{(-1+x)^3}}\,dx = \int -\frac{\sqrt{x-1}-\sqrt{(x-1)^3}}{(x-1)^3-x+1}\,dx$$

input
```
int(1/((x - 1)^(1/2) + ((x - 1)^3)^(1/2)),x)
```

output
```
int(-((x - 1)^(1/2) - ((x - 1)^3)^(1/2))/((x - 1)^3 - x + 1), x)
```

3.824 $\int \left(-\frac{3}{(4+5x)^2}-\frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\right) dx$

3.824.1 Optimal result

Integrand size = 35, antiderivative size = 31

$$\int \left(-\frac{3}{(4+5x)^2}-\frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\right) dx=\frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}$$

output
```
3/5/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)
```

3.824.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int \left(-\frac{3}{(4+5x)^2}-\frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\right) dx=\frac{3+\frac{5}{\sqrt{1-x^2}}-\frac{5x^2}{\sqrt{1-x^2}}}{20+25x}$$

input
```
Integrate[-3/(4 + 5*x)^2 - (5 + 4*x)/((4 + 5*x)^2*Sqrt[1 - x^2]),x]
```

output
```
(3 + 5/Sqrt[1 - x^2] - (5*x^2)/Sqrt[1 - x^2])/(20 + 25*x)
```

3.824.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-\frac{4x+5}{(5x+4)^2\sqrt{1-x^2}}-\frac{3}{(5x+4)^2}\right) dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

input
```
Int[-3/(4 + 5*x)^2 - (5 + 4*x)/((4 + 5*x)^2*Sqrt[1 - x^2]),x]
```

output
```
3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

3.824.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.824.4 Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
trager	$-\frac{3x}{4(4+5x)}+\frac{\sqrt{-x^2+1}}{4+5x}$	29
default	$\frac{\sqrt{-\left(\frac{4}{5}+x\right)^2+\frac{8x}{5}+\frac{41}{25}}}{4+5x}+\frac{3}{5(4+5x)}$	32
risch	$\frac{3}{25\left(\frac{4}{5}+x\right)}-\frac{x^2-1}{(4+5x)\sqrt{-x^2+1}}$	32

input
```
int(-3/(4+5*x)^2+(-4*x-5)/(4+5*x)^2/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-3/4*x/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)
```

3.824.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \left(-\frac{3}{(4+5x)^2}-\frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\right)\,dx=\frac{25\,x+20\,\sqrt{-x^2+1}+32}{20\,(5\,x+4)}$$

input
```
integrate(-3/(4+5*x)^2+(-5-4*x)/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="fri
cas")
```

output
```
1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

3.824.6 Sympy [F]

$$\int \left(-\frac{3}{(4+5x)^2}-\frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\right)\,dx$$
$$=-\int \frac{4x}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx$$
$$-\int \frac{3\sqrt{1-x^2}}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx$$
$$-\int \frac{5}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx$$

input
```
integrate(-3/(4+5*x)**2+(-5-4*x)/(4+5*x)**2/(-x**2+1)**(1/2),x)
```

output
```
-Integral(4*x/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1 - x**2) + 16*sqrt(1 -
x**2)), x) - Integral(3*sqrt(1 - x**2)/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt
(1 - x**2) + 16*sqrt(1 - x**2)), x) - Integral(5/(25*x**2*sqrt(1 - x**2) +
 40*x*sqrt(1 - x**2) + 16*sqrt(1 - x**2)), x)
```

3.824.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \left(-\frac{3}{(4+5x)^2}-\frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\right)\,dx=\frac{\sqrt{-x^2+1}}{5\,x+4}+\frac{3}{5\,(5\,x+4)}$$

input
```
integrate(-3/(4+5*x)^2+(-5-4*x)/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="max
ima")
```

output
```
sqrt(-x^2 + 1)/(5*x + 4) + 3/5/(5*x + 4)
```

3.824.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \left(-\frac{3}{(4+5x)^2}-\frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\right)\,dx=\frac{\sqrt{\frac{8}{5\,x+4}+\frac{9}{(5\,x+4)^2}-1}}{5\,\mathrm{sgn}\left(\frac{1}{5\,x+4}\right)}+\frac{3}{5\,(5\,x+4)}-\frac{1}{5}i\,\mathrm{sgn}\left(\frac{1}{5\,x+4}\right)$$

input
```
integrate(-3/(4+5*x)^2+(-5-4*x)/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="gia
c")
```

output
```
1/5*sqrt(8/(5*x + 4) + 9/(5*x + 4)^2 - 1)/sgn(1/(5*x + 4)) + 3/5/(5*x + 4)
 - 1/5*I*sgn(1/(5*x + 4))
```

3.824.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \left(-\frac{3}{(4+5x)^2}-\frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\right)\,dx=\frac{\sqrt{1-x^2}+\frac{3}{5}}{5\,x+4}$$

input
```
int(- 3/(5*x + 4)^2 - (4*x + 5)/((5*x + 4)^2*(1 - x^2)^(1/2)),x)
```

output
```
((1 - x^2)^(1/2) + 3/5)/(5*x + 4)
```

3.825 $\int \frac{-5-4x-3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}}\,dx$

3.825.1 Optimal result

Integrand size = 37, antiderivative size = 31

$$\int \frac{-5-4x-3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}}\,dx = \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}$$

output
```
3/5/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)
```

3.825.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{-5-4x-3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}}\,dx = \frac{3+5\sqrt{1-x^2}}{20+25x}$$

input
```
Integrate[(-5 - 4*x - 3*Sqrt[1 - x^2])/((4 + 5*x)^2*Sqrt[1 - x^2]),x]
```

output
```
(3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

3.825.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-3\sqrt{1-x^2}-4x-5}{(5x+4)^2\sqrt{1-x^2}}dx$$
$$\Big\downarrow \textbf{7293}$$
$$\int \left(-\frac{4x}{(5x+4)^2\sqrt{1-x^2}}-\frac{5}{(5x+4)^2\sqrt{1-x^2}}-\frac{3}{(5x+4)^2}\right)dx$$
$$\Big\downarrow \textbf{2009}$$
$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

input
```
Int[(-5 - 4*x - 3*Sqrt[1 - x^2])/((4 + 5*x)^2*Sqrt[1 - x^2]),x]
```

output
```
3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

3.825.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.825.4 Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
trager	$-\frac{3x}{4(4+5x)}+\frac{\sqrt{-x^2+1}}{4+5x}$	29
default	$\frac{\sqrt{-\left(\frac{4}{5}+x\right)^2+\frac{8x}{5}+\frac{41}{25}}}{4+5x}+\frac{3}{5(4+5x)}$	32

input
```
int((-5-4*x-3*(-x^2+1)^(1/2))/(4+5*x)^2/(-x^2+1)^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
-3/4*x/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)
```

3.825.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{-5-4x-3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}}\,dx = \frac{25\,x+20\,\sqrt{-x^2+1}+32}{20\,(5\,x+4)}$$

input
```
integrate((-5-4*x-3*(-x^2+1)^(1/2))/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm=
"fricas")
```

output
```
1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

3.825.6 Sympy [F]

$$\int \frac{-5-4x-3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}}\,dx = -\int \frac{4x}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx - \int \frac{3\sqrt{1-x^2}}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx - \int \frac{5}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx$$

input
```
integrate((-5-4*x-3*(-x**2+1)**(1/2))/(4+5*x)**2/(-x**2+1)**(1/2),x)
```

output
```
-Integral(4*x/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1 - x**2) + 16*sqrt(1 -
x**2)), x) - Integral(3*sqrt(1 - x**2)/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt
(1 - x**2) + 16*sqrt(1 - x**2)), x) - Integral(5/(25*x**2*sqrt(1 - x**2) +
 40*x*sqrt(1 - x**2) + 16*sqrt(1 - x**2)), x)
```

3.825.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{-5-4x-3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}}\,dx = \frac{5\,\sqrt{x+1}\sqrt{-x+1}+3}{5\,(5\,x+4)}$$

input
```
integrate((-5-4*x-3*(-x^2+1)^(1/2))/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm=
"maxima")
```

output
```
1/5*(5*sqrt(x + 1)*sqrt(-x + 1) + 3)/(5*x + 4)
```

3.825.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \frac{-5-4x-3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}}\,dx = \frac{\sqrt{\frac{8}{5\,x+4}+\frac{9}{(5\,x+4)^2}-1}}{5\,\mathrm{sgn}\left(\frac{1}{5\,x+4}\right)} + \frac{3}{5\,(5\,x+4)} - \frac{1}{5}i\,\mathrm{sgn}\left(\frac{1}{5\,x+4}\right)$$

input
```
integrate((-5-4*x-3*(-x^2+1)^(1/2))/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm=
"giac")
```

output
```
1/5*sqrt(8/(5*x + 4) + 9/(5*x + 4)^2 - 1)/sgn(1/(5*x + 4)) + 3/5/(5*x + 4)
 - 1/5*I*sgn(1/(5*x + 4))
```

3.825.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \frac{-5-4x-3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}}\,dx = \frac{\sqrt{1-x^2}+\frac{3}{5}}{5\,x+4}$$

input
```
int(-(4*x + 3*(1 - x^2)^(1/2) + 5)/((5*x + 4)^2*(1 - x^2)^(1/2)),x)
```

output
```
((1 - x^2)^(1/2) + 3/5)/(5*x + 4)
```

3.826 $\int \frac{1}{(-5-4x)\sqrt{1-x^2}+3(1-x^2)}\,dx$

3.826.1 Optimal result

Integrand size = 29, antiderivative size = 31

$$\int \frac{1}{(-5-4x)\sqrt{1-x^2}+3(1-x^2)}\,dx = \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}$$

output
```
3/5/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)
```

3.826.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{1}{(-5-4x)\sqrt{1-x^2}+3(1-x^2)}\,dx = \frac{3+5\sqrt{1-x^2}}{20+25x}$$

input
```
Integrate[((-5 - 4*x)*Sqrt[1 - x^2] + 3*(1 - x^2))^(-1),x]
```

output
```
(3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

3.826.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-x^2}(-4x-5)+3\left(1-x^2\right)}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{\sqrt{1-x^2}}{18(x-1)}-\frac{\sqrt{1-x^2}}{2(x+1)}+\frac{20\sqrt{1-x^2}}{9(5x+4)}-\frac{5\sqrt{1-x^2}}{(5x+4)^2}-\frac{3}{(5x+4)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

input
```
Int[((-5 - 4*x)*Sqrt[1 - x^2] + 3*(1 - x^2))^(-1),x]
```

output
```
3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

3.826.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.826.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
trager	$-\frac{3x}{4(4+5x)}+\frac{\sqrt{-x^2+1}}{4+5x}$	29
default	$\frac{3}{5(4+5x)}-\frac{\sqrt{-(x+1)^2+2x+2}}{2}+\frac{5\left(-\left(\frac{4}{5}+x\right)^2+\frac{8x}{5}+\frac{41}{25}\right)^{\frac{3}{2}}}{9\left(\frac{4}{5}+x\right)}+\frac{5x\sqrt{-\left(\frac{4}{5}+x\right)^2+\frac{8x}{5}+\frac{41}{25}}}{9}+\frac{\sqrt{-(x-1)^2-2x+2}}{18}$	81

input
```
int(1/(-3*x^2+3+(-4*x-5)*(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-3/4*x/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)
```

3.826.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{(-5-4x)\sqrt{1-x^2}+3(1-x^2)}\,dx = \frac{25\,x+20\,\sqrt{-x^2+1}+32}{20\,(5\,x+4)}$$

input
```
integrate(1/(-3*x^2+3+(-5-4*x)*(-x^2+1)^(1/2)),x, algorithm="fricas")
```

output
```
1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

3.826.6 Sympy [F]

$$\int \frac{1}{(-5-4x)\sqrt{1-x^2}+3(1-x^2)}\,dx = -\int \frac{1}{3x^2+4x\sqrt{1-x^2}+5\sqrt{1-x^2}-3}\,dx$$

input
```
integrate(1/(-3*x**2+3+(-5-4*x)*(-x**2+1)**(1/2)),x)
```

output
```
-Integral(1/(3*x**2 + 4*x*sqrt(1 - x**2) + 5*sqrt(1 - x**2) - 3), x)
```

3.826.7 Maxima [F]

$$\int \frac{1}{(-5-4x)\sqrt{1-x^2}+3(1-x^2)}\,dx = \int -\frac{1}{3\,x^2+\sqrt{-x^2+1}(4\,x+5)-3}\,dx$$

input
```
integrate(1/(-3*x^2+3+(-5-4*x)*(-x^2+1)^(1/2)),x, algorithm="maxima")
```

output
```
-integrate(1/(3*x^2 + sqrt(-x^2 + 1)*(4*x + 5) - 3), x)
```

3.826.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(27) = 54$.

Time = 0.37 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.19

$$\int \frac{1}{(-5-4x)\sqrt{1-x^2}+3(1-x^2)}\,dx = \frac{\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-4}{4\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-2\right)}+\frac{3}{5\,(5\,x+4)}$$

input
```
integrate(1/(-3*x^2+3+(-5-4*x)*(-x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
1/4*(5*(sqrt(-x^2 + 1) - 1)/x - 4)/(5*(sqrt(-x^2 + 1) - 1)/x - 2*(sqrt(-x^
2 + 1) - 1)^2/x^2 - 2) + 3/5/(5*x + 4)
```

3.826.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \frac{1}{(-5-4x)\sqrt{1-x^2}+3(1-x^2)}\,dx = \frac{\sqrt{1-x^2}+\frac{3}{5}}{5\,x+4}$$

input
```
int(-1/((4*x + 5)*(1 - x^2)^(1/2) + 3*x^2 - 3),x)
```

output
```
((1 - x^2)^(1/2) + 3/5)/(5*x + 4)
```

3.827 $\int \frac{1}{3-3x^2-5\sqrt{1-x^2}-4x\sqrt{1-x^2}}\,dx$

3.827.1 Optimal result

Integrand size = 36, antiderivative size = 31

$$\int \frac{1}{3-3x^2-5\sqrt{1-x^2}-4x\sqrt{1-x^2}}\,dx=\frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}$$

output
```
3/5/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)
```

3.827.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{1}{3-3x^2-5\sqrt{1-x^2}-4x\sqrt{1-x^2}}\,dx=\frac{3+5\sqrt{1-x^2}}{20+25x}$$

input
```
Integrate[(3 - 3*x^2 - 5*Sqrt[1 - x^2] - 4*x*Sqrt[1 - x^2])^(-1),x]
```

output
```
(3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

3.827.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{-3x^2-4\sqrt{1-x^2}x-5\sqrt{1-x^2}+3}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{\sqrt{1-x^2}}{18(x-1)}-\frac{\sqrt{1-x^2}}{2(x+1)}+\frac{20\sqrt{1-x^2}}{9(5x+4)}-\frac{5\sqrt{1-x^2}}{(5x+4)^2}-\frac{3}{(5x+4)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

input
```
Int[(3 - 3*x^2 - 5*Sqrt[1 - x^2] - 4*x*Sqrt[1 - x^2])^(-1),x]
```

output
```
3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

3.827.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.827.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
trager	$-\frac{3x}{4(4+5x)}+\frac{\sqrt{-x^2+1}}{4+5x}$	29
default	$\frac{3}{5(4+5x)}-\frac{\sqrt{-(x+1)^2+2x+2}}{2}+\frac{5\left(-\left(\frac{4}{5}+x\right)^2+\frac{8x}{5}+\frac{41}{25}\right)^{\frac{3}{2}}}{9\left(\frac{4}{5}+x\right)}+\frac{5x\sqrt{-\left(\frac{4}{5}+x\right)^2+\frac{8x}{5}+\frac{41}{25}}}{9}+\frac{\sqrt{-(x-1)^2-2x+2}}{18}$	81

input
```
int(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*x*(-x^2+1)^(1/2)),x,method=_RETURNVERBOS
E)
```

output
```
-3/4*x/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)
```

3.827.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{3-3x^2-5\sqrt{1-x^2}-4x\sqrt{1-x^2}}\,dx=\frac{25\,x+20\,\sqrt{-x^2+1}+32}{20\,(5\,x+4)}$$

input
```
integrate(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*x*(-x^2+1)^(1/2)),x, algorithm="fr
icas")
```

output
```
1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

3.827.6 Sympy [F]

$$\int \frac{1}{3-3x^2-5\sqrt{1-x^2}-4x\sqrt{1-x^2}}\,dx=-\int \frac{1}{3x^2+4x\sqrt{1-x^2}+5\sqrt{1-x^2}-3}\,dx$$

input
```
integrate(1/(3-3*x**2-5*(-x**2+1)**(1/2)-4*x*(-x**2+1)**(1/2)),x)
```

output
```
-Integral(1/(3*x**2 + 4*x*sqrt(1 - x**2) + 5*sqrt(1 - x**2) - 3), x)
```

3.827.7 Maxima [F]

$$\int \frac{1}{3-3x^2-5\sqrt{1-x^2}-4x\sqrt{1-x^2}}\,dx = \int -\frac{1}{3\,x^2+4\,\sqrt{-x^2+1}x+5\,\sqrt{-x^2+1}-3}\,dx$$

input
```
integrate(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*x*(-x^2+1)^(1/2)),x, algorithm="ma
xima")
```

output
```
-integrate(1/(3*x^2 + 4*sqrt(-x^2 + 1)*x + 5*sqrt(-x^2 + 1) - 3), x)
```

3.827.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(27) = 54$.

Time = 0.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.19

$$\int \frac{1}{3-3x^2-5\sqrt{1-x^2}-4x\sqrt{1-x^2}}\,dx = \frac{\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-4}{4\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-2\right)}+\frac{3}{5\,(5\,x+4)}$$

input
```
integrate(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*x*(-x^2+1)^(1/2)),x, algorithm="gi
ac")
```

output
```
1/4*(5*(sqrt(-x^2 + 1) - 1)/x - 4)/(5*(sqrt(-x^2 + 1) - 1)/x - 2*(sqrt(-x^
2 + 1) - 1)^2/x^2 - 2) + 3/5/(5*x + 4)
```

3.827.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \frac{1}{3-3x^2-5\sqrt{1-x^2}-4x\sqrt{1-x^2}}\,dx = \frac{\sqrt{1-x^2}+\frac{3}{5}}{5\,x+4}$$

input
```
int(-1/(4*x*(1 - x^2)^(1/2) + 3*x^2 + 5*(1 - x^2)^(1/2) - 3),x)
```

output
```
((1 - x^2)^(1/2) + 3/5)/(5*x + 4)
```

3.828 $\int \frac{-1+\sqrt{1-x^2}}{\sqrt{1-x^2}\left(2+x-2\sqrt{1-x^2}\right)^2}\,dx$

3.828.1 Optimal result

Integrand size = 43, antiderivative size = 31

$$\int \frac{-1+\sqrt{1-x^2}}{\sqrt{1-x^2}\left(2+x-2\sqrt{1-x^2}\right)^2}\,dx=\frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}$$

output
```
3/5/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)
```

3.828.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{-1+\sqrt{1-x^2}}{\sqrt{1-x^2}\left(2+x-2\sqrt{1-x^2}\right)^2}\,dx=\frac{3+5\sqrt{1-x^2}}{20+25x}$$

input
```
Integrate[(-1 + Sqrt[1 - x^2])/(Sqrt[1 - x^2]*(2 + x - 2*Sqrt[1 - x^2])^2)
,x]
```

output
```
(3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

3.828.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}\left(-2\sqrt{1-x^2}+x+2\right)^2}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{1}{\left(2\sqrt{1-x^2}-x-2\right)^2}-\frac{1}{\sqrt{1-x^2}\left(2\sqrt{1-x^2}-x-2\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

input
```
Int[(-1 + Sqrt[1 - x^2])/(Sqrt[1 - x^2]*(2 + x - 2*Sqrt[1 - x^2])^2),x]
```

output
```
3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

3.828.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.828.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
trager	$-\frac{3x}{4(4+5x)}+\frac{\sqrt{-x^2+1}}{4+5x}$	29
default	$\frac{\sqrt{-\left(\frac{4}{5}+x\right)^2+\frac{8x}{5}+\frac{41}{25}}}{4+5x}+\frac{3}{5(4+5x)}$	32

input
```
int(((-x^2+1)^(1/2)-1)/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
-3/4*x/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)
```

3.828.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{-1+\sqrt{1-x^2}}{\sqrt{1-x^2}\left(2+x-2\sqrt{1-x^2}\right)^2}\,dx=\frac{25\,x+20\,\sqrt{-x^2+1}+32}{20\,(5\,x+4)}$$

input
```
integrate((-1+(-x^2+1)^(1/2))/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2),x, a
lgorithm="fricas")
```

output
```
1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

3.828.6 Sympy [F]

$$\int \frac{-1+\sqrt{1-x^2}}{\sqrt{1-x^2}\left(2+x-2\sqrt{1-x^2}\right)^2}\,dx=\int \frac{\sqrt{1-x^2}-1}{\sqrt{-(x-1)(x+1)}\left(x-2\sqrt{1-x^2}+2\right)^2}\,dx$$

input
```
integrate((-1+(-x**2+1)**(1/2))/(2+x-2*(-x**2+1)**(1/2))**2/(-x**2+1)**(1/
2),x)
```

output
```
Integral((sqrt(1 - x**2) - 1)/(sqrt(-(x - 1)*(x + 1))*(x - 2*sqrt(1 - x**2
) + 2)**2), x)
```

3.828.7 Maxima [F]

$$\int \frac{-1+\sqrt{1-x^2}}{\sqrt{1-x^2}\left(2+x-2\sqrt{1-x^2}\right)^2}\,dx = \int \frac{\sqrt{-x^2+1}-1}{\sqrt{-x^2+1}\left(x-2\sqrt{-x^2+1}+2\right)^2}\,dx$$

input
```
integrate((-1+(-x^2+1)^(1/2))/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2),x, a
lgorithm="maxima")
```

output
```
-1/56*sqrt(7)*log((3*x - 2*sqrt(7) - 2)/(3*x + 2*sqrt(7) - 2)) - integrate
(-1/8*(100*x^7 + 285*x^6 + 264*x^5 + 80*x^4)/(21*x^9 + 278*x^8 + 283*x^7 -
 2022*x^6 - 3632*x^5 + 2256*x^4 + 7424*x^3 + 1536*x^2 - 8*(9*x^8 + 12*x^7
- 101*x^6 - 172*x^5 + 284*x^4 + 672*x^3 + 64*x^2 - 512*x - 256)*sqrt(x + 1
)*sqrt(-x + 1) - 4096*x - 2048), x) - 1/24*log(x + 2) + 1/16*log(x + 1) -
1/48*log(x - 1)
```

3.828.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(27) = 54$.

Time = 0.38 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.19

$$\int \frac{-1+\sqrt{1-x^2}}{\sqrt{1-x^2}\left(2+x-2\sqrt{1-x^2}\right)^2}\,dx = \frac{\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-4}{4\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-2\right)}+\frac{3}{5\,(5\,x+4)}$$

input
```
integrate((-1+(-x^2+1)^(1/2))/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2),x, a
lgorithm="giac")
```

output
```
1/4*(5*(sqrt(-x^2 + 1) - 1)/x - 4)/(5*(sqrt(-x^2 + 1) - 1)/x - 2*(sqrt(-x^
2 + 1) - 1)^2/x^2 - 2) + 3/5/(5*x + 4)
```

3.828.9 Mupad [B] (verification not implemented)

Time = 21.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \frac{-1+\sqrt{1-x^2}}{\sqrt{1-x^2}\left(2+x-2\sqrt{1-x^2}\right)^2}\,dx=\frac{\sqrt{1-x^2}+\frac{3}{5}}{5\,x+4}$$

input
```
int(((1 - x^2)^(1/2) - 1)/((1 - x^2)^(1/2)*(x - 2*(1 - x^2)^(1/2) + 2)^2),
x)
```

output
```
((1 - x^2)^(1/2) + 3/5)/(5*x + 4)
```

3.829 $\int \frac{a+bx^{-1+n}}{cx+dx^n}\,dx$

3.829.1 Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{a+bx^{-1+n}}{cx+dx^n}\,dx=\frac{b\log(x)}{d}-\frac{(bc-ad)\log\left(d+cx^{1-n}\right)}{cd(1-n)}$$

output
```
b*ln(x)/d-(-a*d+b*c)*ln(d+c*x^(1-n))/c/d/(1-n)
```

3.829.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^{-1+n}}{cx+dx^n}\,dx=\frac{b\log(x)+\frac{(bc-ad)\log\left(d+cx^{1-n}\right)}{c(-1+n)}}{d}$$

input
```
Integrate[(a + b*x^(-1 + n))/(c*x + d*x^n),x]
```

output
```
(b*Log[x] + ((b*c - a*d)*Log[d + c*x^(1 - n)])/(c*(-1 + n)))/d
```

3.829.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2027, 1016, 948, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^{n-1}}{cx+dx^n}dx$$
$$\Big\downarrow \text{2027}$$
$$\int \frac{x^{-n}\left(a+bx^{n-1}\right)}{cx^{1-n}+d}dx$$
$$\Big\downarrow \text{1016}$$
$$\int \frac{ax^{1-n}+b}{x\left(cx^{1-n}+d\right)}dx$$
$$\Big\downarrow \text{948}$$
$$\frac{\int \frac{x^{n-1}\left(ax^{1-n}+b\right)}{cx^{1-n}+d}dx^{1-n}}{1-n}$$
$$\Big\downarrow \text{86}$$
$$\frac{\int \left(\frac{bx^{n-1}}{d}+\frac{ad-bc}{d\left(cx^{1-n}+d\right)}\right)dx^{1-n}}{1-n}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\frac{b\log\left(x^{1-n}\right)}{d}-\frac{(bc-ad)\log\left(cx^{1-n}+d\right)}{cd}}{1-n}$$

input
```
Int[(a + b*x^(-1 + n))/(c*x + d*x^n),x]
```

output
```
((b*Log[x^(1 - n)])/d - ((b*c - a*d)*Log[d + c*x^(1 - n)])/(c*d))/(1 - n)
```

3.829.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.829.4 Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.35

method	result	size
norman	$\frac{(adn-bc)\ln(x)}{cd(-1+n)}-\frac{(ad-bc)\ln\left(cx+d\,\mathrm{e}^{n\ln(x)}\right)}{cd(-1+n)}$	58
risch	$\frac{b\ln(x)}{d}+\frac{n\ln(x)a}{c(-1+n)}-\frac{n\ln(x)b}{d(-1+n)}-\frac{\ln\left(x^n+\frac{xc}{d}\right)a}{c(-1+n)}+\frac{\ln\left(x^n+\frac{xc}{d}\right)b}{d(-1+n)}$	79

input
```
int((a+b*x^(-1+n))/(c*x+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
(a*d*n-b*c)/c/d/(-1+n)*ln(x)-(a*d-b*c)/c/d/(-1+n)*ln(c*x+d*exp(n*ln(x)))
```

3.829.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{a+bx^{-1+n}}{cx+dx^n}\,dx = \frac{(bc-ad)\log(cx+dx^n)+(adn-bc)\log(x)}{cdn-cd}$$

input
```
integrate((a+b*x^(-1+n))/(c*x+d*x^n),x, algorithm="fricas")
```

output
```
((b*c - a*d)*log(c*x + d*x^n) + (a*d*n - b*c)*log(x))/(c*d*n - c*d)
```

3.829.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(29) = 58$.

Time = 1.22 (sec) , antiderivative size = 172, normalized size of antiderivative = 4.00

$$\int \frac{a+bx^{-1+n}}{cx+dx^n}\,dx = \begin{cases} \tilde{\infty}(a+b)\log(x) & \text{for } c=0 \wedge d=0 \wedge n=1 \\ \frac{-\frac{ax}{nx^n-x^n}+\frac{bnx^n\log(x)}{nx^n-x^n}-\frac{bx^n\log(x)}{nx^n-x^n}}{d} & \text{for } c=0 \\ \frac{\frac{an\log(x)}{n-1}-\frac{a\log(x)}{n-1}+\frac{bx^{n-1}}{n-1}}{c} & \text{for } d=0 \\ \frac{(a+b)\log(x)}{c+d} & \text{for } n=1 \\ \frac{adn\log(x)}{cdn-cd}-\frac{ad\log\left(x+\frac{dx^n}{c}\right)}{cdn-cd}-\frac{bc\log(x)}{cdn-cd}+\frac{bc\log\left(x+\frac{dx^n}{c}\right)}{cdn-cd} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*x**(-1+n))/(c*x+d*x**n),x)
```

output
```
Piecewise((zoo*(a + b)*log(x), Eq(c, 0) & Eq(d, 0) & Eq(n, 1)), ((-a*x/(n*
x**n - x**n) + b*n*x**n*log(x)/(n*x**n - x**n) - b*x**n*log(x)/(n*x**n - x
**n))/d, Eq(c, 0)), ((a*n*log(x)/(n - 1) - a*log(x)/(n - 1) + b*x**(n - 1)
/(n - 1))/c, Eq(d, 0)), ((a + b)*log(x)/(c + d), Eq(n, 1)), (a*d*n*log(x)/
(c*d*n - c*d) - a*d*log(x + d*x**n/c)/(c*d*n - c*d) - b*c*log(x)/(c*d*n -
c*d) + b*c*log(x + d*x**n/c)/(c*d*n - c*d), True))
```

3.829.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(40) = 80$.

Time = 0.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.98

$$\int \frac{a + bx^{-1+n}}{cx + dx^n}\,dx = b\left(\frac{\log(x)}{d} - \frac{n\log(x)}{d(n-1)} + \frac{\log\left(\frac{cx+dx^n}{d}\right)}{d(n-1)}\right) + a\left(\frac{n\log(x)}{c(n-1)} - \frac{\log\left(\frac{cx+dx^n}{d}\right)}{c(n-1)}\right)$$

input
```
integrate((a+b*x^(-1+n))/(c*x+d*x^n),x, algorithm="maxima")
```

output
```
b*(log(x)/d - n*log(x)/(d*(n - 1)) + log((c*x + d*x^n)/d)/(d*(n - 1))) + a
*(n*log(x)/(c*(n - 1)) - log((c*x + d*x^n)/d)/(c*(n - 1)))
```

3.829.8 Giac [F]

$$\int \frac{a + bx^{-1+n}}{cx + dx^n}\,dx = \int \frac{bx^{n-1} + a}{cx + dx^n}\,dx$$

input
```
integrate((a+b*x^(-1+n))/(c*x+d*x^n),x, algorithm="giac")
```

output
```
integrate((b*x^(n - 1) + a)/(c*x + d*x^n), x)
```

3.829.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + bx^{-1+n}}{cx + dx^n}\,dx = \int \frac{a + b\,x^{n-1}}{d\,x^n + c\,x}\,dx$$

input
```
int((a + b*x^(n - 1))/(d*x^n + c*x),x)
```

output
```
int((a + b*x^(n - 1))/(d*x^n + c*x), x)
```

3.830 $\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx$

3.830.1 Optimal result

Integrand size = 27, antiderivative size = 42

$$\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx=-\frac{1}{2x}+x+\frac{\sqrt{1+2x^2}}{2x}-\frac{\operatorname{arcsinh}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

output
```
-1/2/x+x-1/2*arcsinh(x*2^(1/2))*2^(1/2)+1/2*(2*x^2+1)^(1/2)/x
```

3.830.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx=\frac{-1+2x^2+\sqrt{1+2x^2}+\sqrt{2}x\log\left(-\sqrt{2}x+\sqrt{1+2x^2}\right)}{2x}$$

input
```
Integrate[Sqrt[1 + 2*x^2]/(1 + Sqrt[1 + 2*x^2]),x]
```

output
```
(-1 + 2*x^2 + Sqrt[1 + 2*x^2] + Sqrt[2]*x*Log[-(Sqrt[2]*x) + Sqrt[1 + 2*x^
2]])/(2*x)
```

3.830.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {7291, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2+1}}{\sqrt{2x^2+1}+1}dx$$

$$\Big\downarrow \text{7291}$$

$$\int \left(\frac{1}{-\sqrt{2x^2+1}-1}+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{arcsinh}\left(\sqrt{2}x\right)}{\sqrt{2}}+\frac{\sqrt{2x^2+1}}{2x}+x-\frac{1}{2x}$$

input
```
Int[Sqrt[1 + 2*x^2]/(1 + Sqrt[1 + 2*x^2]),x]
```

output
```
-1/2*1/x + x + Sqrt[1 + 2*x^2]/(2*x) - ArcSinh[Sqrt[2]*x]/Sqrt[2]
```

3.830.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7291
```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

3.830.4 Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

method	result	size
default	$x-\frac{1}{2x}+\frac{(2x^2+1)^{\frac{3}{2}}}{2x}-x\sqrt{2x^2+1}-\frac{\operatorname{arcsinh}\left(x\sqrt{2}\right)\sqrt{2}}{2}$	45
trager	$\frac{(x-1)(1+2x)}{2x}+\frac{\sqrt{2x^2+1}}{2x}-\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(\operatorname{RootOf}\left(\_Z^2-2\right)x+\sqrt{2x^2+1}\right)}{2}$	56

input
```
int((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x-1/2/x+1/2/x*(2*x^2+1)^(3/2)-x*(2*x^2+1)^(1/2)-1/2*arcsinh(x*2^(1/2))*2^(
1/2)
```

3.830.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx=\frac{\sqrt{2}x\log\left(\sqrt{2}x-\sqrt{2\,x^2+1}\right)+2\,x^2+\sqrt{2\,x^2+1}-1}{2\,x}$$

input
```
integrate((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*x*log(sqrt(2)*x - sqrt(2*x^2 + 1)) + 2*x^2 + sqrt(2*x^2 + 1)
- 1)/x
```

3.830.6 Sympy [F]

$$\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx=\int \frac{\sqrt{2x^2+1}}{\sqrt{2x^2+1}+1}\,dx$$

input
```
integrate((2*x**2+1)**(1/2)/(1+(2*x**2+1)**(1/2)),x)
```

output
```
Integral(sqrt(2*x**2 + 1)/(sqrt(2*x**2 + 1) + 1), x)
```

3.830.7 Maxima [F]

$$\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx = \int \frac{\sqrt{2\,x^2+1}}{\sqrt{2\,x^2+1}+1}\,dx$$

input
```
integrate((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x, algorithm="maxima")
```

output
```
x - integrate(1/(sqrt(2*x^2 + 1) + 1), x)
```

3.830.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx = \frac{1}{2}\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{2\,x^2+1}\right)+x-\frac{\sqrt{2}}{\left(\sqrt{2}x-\sqrt{2\,x^2+1}\right)^2-1}-\frac{1}{2\,x}$$

input
```
integrate((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*log(-sqrt(2)*x + sqrt(2*x^2 + 1)) + x - sqrt(2)/((sqrt(2)*x -
sqrt(2*x^2 + 1))^2 - 1) - 1/2/x
```

3.830.9 Mupad [B] (verification not implemented)

Time = 20.71 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx = x-\frac{\sqrt{2}\,\mathrm{asinh}\left(\sqrt{2}\,x\right)}{2}+\frac{\frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{2}-\frac{1}{2}}{x}$$

input
```
int((2*x^2 + 1)^(1/2)/((2*x^2 + 1)^(1/2) + 1),x)
```

output
```
x - (2^(1/2)*asinh(2^(1/2)*x))/2 + ((2^(1/2)*(x^2 + 1/2)^(1/2))/2 - 1/2)/x
```

3.831 $\int \frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}}\,dx$

3.831.1 Optimal result

Integrand size = 27, antiderivative size = 65

$$\int \frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}}\,dx=\frac{4x}{3}-\frac{1}{3}\sqrt{-1+4x^2}-\frac{\operatorname{arctanh}\left(\sqrt{3}x\right)}{3\sqrt{3}}+\frac{\operatorname{arctanh}\left(\sqrt{3}\sqrt{-1+4x^2}\right)}{3\sqrt{3}}$$

output
```
4/3*x-1/9*arctanh(x*3^(1/2))*3^(1/2)+1/9*arctanh(3^(1/2)*(4*x^2-1)^(1/2))*
3^(1/2)-1/3*(4*x^2-1)^(1/2)
```

3.831.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}}\,dx=\frac{1}{9}\left(12x-3\sqrt{-1+4x^2}+2\sqrt{3}\operatorname{arctanh}\left(\frac{-2x+\sqrt{-1+4x^2}}{\sqrt{3}}\right)\right)$$

input
```
Integrate[Sqrt[-1 + 4*x^2]/(x + Sqrt[-1 + 4*x^2]),x]
```

output
```
(12*x - 3*Sqrt[-1 + 4*x^2] + 2*Sqrt[3]*ArcTanh[(-2*x + Sqrt[-1 + 4*x^2])/S
qrt[3]])/9
```

3.831.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{4x^2-1}}{\sqrt{4x^2-1}+x}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{4x^2-1}{3x^2-1}-\frac{x\sqrt{4x^2-1}}{3x^2-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\operatorname{arctanh}\left(\sqrt{3}\sqrt{4x^2-1}\right)}{3\sqrt{3}}-\frac{\operatorname{arctanh}\left(\sqrt{3}x\right)}{3\sqrt{3}}-\frac{1}{3}\sqrt{4x^2-1}+\frac{4x}{3}$$

input
```
Int[Sqrt[-1 + 4*x^2]/(x + Sqrt[-1 + 4*x^2]),x]
```

output
```
(4*x)/3 - Sqrt[-1 + 4*x^2]/3 - ArcTanh[Sqrt[3]*x]/(3*Sqrt[3]) + ArcTanh[Sq
rt[3]*Sqrt[-1 + 4*x^2]]/(3*Sqrt[3])
```

3.831.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.831.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

method	result
trager	$\frac{4x}{3}-\frac{\sqrt{4x^2-1}}{3}+\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2-3\right)+3\sqrt{4x^2-1}}{\operatorname{RootOf}\left(\_Z^2-3\right)x+1}\right)}{9}$
default	$\frac{4x}{3}-\frac{\operatorname{arctanh}\left(x\sqrt{3}\right)\sqrt{3}}{9}-\frac{\sqrt{36\left(x+\frac{\sqrt{3}}{3}\right)^2-24\sqrt{3}\left(x+\frac{\sqrt{3}}{3}\right)+3}}{18}+\frac{\sqrt{3}\ln\left(\sqrt{4}x+\sqrt{4\left(x+\frac{\sqrt{3}}{3}\right)^2-\frac{8\sqrt{3}\left(x+\frac{\sqrt{3}}{3}\right)}{3}+\frac{1}{3}}\right)\sqrt{4}}{18}+\frac{\sqrt{3}\ a}{}$

input
```
int((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
4/3*x-1/3*(4*x^2-1)^(1/2)+1/9*RootOf(_Z^2-3)*ln(-(RootOf(_Z^2-3)+3*(4*x^2-
1)^(1/2))/(RootOf(_Z^2-3)*x+1))
```

3.831.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.23

$$\int\frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}}\,dx=\frac{1}{18}\sqrt{3}\log\left(\frac{6\,x^2+\sqrt{3}\sqrt{4\,x^2-1}-1}{3\,x^2-1}\right)+\frac{1}{18}\sqrt{3}\log\left(\frac{3\,x^2-2\,\sqrt{3}x+1}{3\,x^2-1}\right)+\frac{4}{3}\,x-\frac{1}{3}\sqrt{4\,x^2-1}$$

input
```
integrate((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x, algorithm="fricas")
```

output
```
1/18*sqrt(3)*log((6*x^2 + sqrt(3)*sqrt(4*x^2 - 1) - 1)/(3*x^2 - 1)) + 1/18
*sqrt(3)*log((3*x^2 - 2*sqrt(3)*x + 1)/(3*x^2 - 1)) + 4/3*x - 1/3*sqrt(4*x
^2 - 1)
```

3.831.6 Sympy [F]

$$\int \frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}}\,dx = \int \frac{\sqrt{(2x-1)(2x+1)}}{x+\sqrt{4x^2-1}}\,dx$$

input
```
integrate((4*x**2-1)**(1/2)/(x+(4*x**2-1)**(1/2)),x)
```

output
```
Integral(sqrt((2*x - 1)*(2*x + 1))/(x + sqrt(4*x**2 - 1)), x)
```

3.831.7 Maxima [F]

$$\int \frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}}\,dx = \int \frac{\sqrt{4\,x^2-1}}{x+\sqrt{4\,x^2-1}}\,dx$$

input
```
integrate((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x, algorithm="maxima")
```

output
```
x - integrate(x/(sqrt(2*x + 1)*sqrt(2*x - 1) + x), x)
```

3.831.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(45) = 90$.

Time = 0.33 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.05

$$\int \frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}}\,dx = \frac{1}{18}\,\sqrt{3}\log\left(\frac{\left|6\,x-2\,\sqrt{3}\right|}{\left|6\,x+2\,\sqrt{3}\right|}\right) - \frac{1}{18}\,\sqrt{3}\log\left(-\frac{\left|-12\,x-4\,\sqrt{3}+6\,\sqrt{4\,x^2-1}+\frac{6}{2\,x-\sqrt{4\,x^2-1}}\right|}{2\left(6\,x-2\,\sqrt{3}-3\,\sqrt{4\,x^2-1}-\frac{3}{2\,x-\sqrt{4\,x^2-1}}\right)}\right) + \frac{4}{3}\,x - \frac{1}{3}\,\sqrt{4\,x^2-1}$$

input
```
integrate((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x, algorithm="giac")
```

output
```
1/18*sqrt(3)*log(abs(6*x - 2*sqrt(3))/abs(6*x + 2*sqrt(3))) - 1/18*sqrt(3)
*log(-1/2*abs(-12*x - 4*sqrt(3) + 6*sqrt(4*x^2 - 1) + 6/(2*x - sqrt(4*x^2
- 1)))/(6*x - 2*sqrt(3) - 3*sqrt(4*x^2 - 1) - 3/(2*x - sqrt(4*x^2 - 1))))
+ 4/3*x - 1/3*sqrt(4*x^2 - 1)
```

3.831.9 Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}}\,dx = \frac{4\,x}{3}+\frac{\sqrt{3}\,\ln\left(x-\frac{\sqrt{3}}{3}\right)}{18}-\frac{\sqrt{3}\,\ln\left(x+\frac{\sqrt{3}}{3}\right)}{18}+\frac{\sqrt{3}\,\mathrm{atanh}\left(\sqrt{3}\,\sqrt{4\,x^2-1}\right)}{9}-\frac{\sqrt{4\,x^2-1}}{3}$$

input
```
int((4*x^2 - 1)^(1/2)/(x + (4*x^2 - 1)^(1/2)),x)
```

output
```
(4*x)/3 + (3^(1/2)*log(x - 3^(1/2)/3))/18 - (3^(1/2)*log(x + 3^(1/2)/3))/1
8 + (3^(1/2)*atanh(3^(1/2)*(4*x^2 - 1)^(1/2)))/9 - (4*x^2 - 1)^(1/2)/3
```

3.832 $\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx$

3.832.1 Optimal result

Integrand size = 27, antiderivative size = 195

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx = -\frac{(cd^2-bde+ae^2)\sqrt{-1+x^2}}{2e(d^2-e^2)(d+ex)^2} + \frac{(c(d^3-4de^2)-e(3ade-b(d^2+2e^2)))\sqrt{-1+x^2}}{2e(d^2-e^2)^2(d+ex)} - \frac{(3bde-a(2d^2+e^2)-c(d^2+2e^2))\,\text{arctanh}\left(\frac{e+dx}{\sqrt{d^2-e^2}\sqrt{-1+x^2}}\right)}{2(d^2-e^2)^{5/2}}$$

output
```
-1/2*(3*b*d*e-a*(2*d^2+e^2)-c*(d^2+2*e^2))*arctanh((d*x+e)/(d^2-e^2)^(1/2)
/(x^2-1)^(1/2))/(d^2-e^2)^(5/2)-1/2*(a*e^2-b*d*e+c*d^2)*(x^2-1)^(1/2)/e/(d
^2-e^2)/(e*x+d)^2+1/2*(c*(d^3-4*d*e^2)-e*(3*a*d*e-b*(d^2+2*e^2)))*(x^2-1)^
(1/2)/e/(d^2-e^2)^2/(e*x+d)
```

3.832.2 Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.92

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx = \frac{\frac{(d-e)(d+e)\sqrt{-1+x^2}(ae(-4d^2+e^2-3dex)+cd(-3de+d^2x-4e^2x)+b(2d^3+de^2+d^2ex+2e^3x))}{(d+ex)^2} + 2\sqrt{-d^2+e^2}(-3bde+a(2d^2+e^2}{2(d-e)^3(d+e)^3}$$

input
```
Integrate[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[-1 + x^2]),x]
```

output
```
(((d - e)*(d + e)*Sqrt[-1 + x^2]*(a*e*(-4*d^2 + e^2 - 3*d*e*x) + c*d*(-3*d
*e + d^2*x - 4*e^2*x) + b*(2*d^3 + d*e^2 + d^2*e*x + 2*e^3*x)))/(d + e*x)^
2 + 2*Sqrt[-d^2 + e^2]*(-3*b*d*e + a*(2*d^2 + e^2) + c*(d^2 + 2*e^2))*ArcT
an[(d + e*(x - Sqrt[-1 + x^2]))/Sqrt[-d^2 + e^2]])/(2*(d - e)^3*(d + e)^3)
```

3.832.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2182, 25, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx+cx^2}{\sqrt{x^2-1}(d+ex)^3}dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{2(ad+cd-be)+\left(\frac{cd^2}{e}+bd-ae-2ce\right)x}{(d+ex)^2\sqrt{x^2-1}}dx}{2(d^2-e^2)}-\frac{\sqrt{x^2-1}(ae^2-bde+cd^2)}{2e(d^2-e^2)(d+ex)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2(ad+cd-be)+\left(\frac{cd^2}{e}+bd-ae-2ce\right)x}{(d+ex)^2\sqrt{x^2-1}}dx}{2(d^2-e^2)}-\frac{\sqrt{x^2-1}(ae^2-bde+cd^2)}{2e(d^2-e^2)(d+ex)^2}$$

$$\Big\downarrow \text{679}$$

$$\frac{\frac{\sqrt{x^2-1}(-3ade^2+bd^2e+2be^3+cd^3-4cde^2)}{e(d^2-e^2)(d+ex)}-\frac{(-a(2d^2+e^2)+3bde-c(d^2+2e^2))\int \frac{1}{(d+ex)\sqrt{x^2-1}}dx}{d^2-e^2}}{2(d^2-e^2)}-\frac{\sqrt{x^2-1}(ae^2-bde+cd^2)}{2e(d^2-e^2)(d+ex)^2}$$

$$\Big\downarrow \text{488}$$

$$\frac{\frac{(-a(2d^2+e^2)+3bde-c(d^2+2e^2))\int \frac{1}{d^2-e^2-\frac{(-e-dx)^2}{x^2-1}}d\frac{-e-dx}{\sqrt{x^2-1}}}{d^2-e^2}+\frac{\sqrt{x^2-1}(-3ade^2+bd^2e+2be^3+cd^3-4cde^2)}{e(d^2-e^2)(d+ex)}}{2(d^2-e^2)}-\frac{\sqrt{x^2-1}(ae^2-bde+cd^2)}{2e(d^2-e^2)(d+ex)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\text{arctanh}\left(\frac{-dx-e}{\sqrt{x^2-1}\sqrt{d^2-e^2}}\right)(-a(2d^2+e^2)+3bde-c(d^2+2e^2))}{(d^2-e^2)^{3/2}}+\frac{\sqrt{x^2-1}(-3ade^2+bd^2e+2be^3+cd^3-4cde^2)}{e(d^2-e^2)(d+ex)}}{2(d^2-e^2)}-\frac{\sqrt{x^2-1}(ae^2-bde+cd^2)}{2e(d^2-e^2)(d+ex)^2}$$

input
```
Int[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[-1 + x^2]),x]
```

output
```
-1/2*((c*d^2 - b*d*e + a*e^2)*Sqrt[-1 + x^2])/(e*(d^2 - e^2)*(d + e*x)^2)
+ (((c*d^3 + b*d^2*e - 3*a*d*e^2 - 4*c*d*e^2 + 2*b*e^3)*Sqrt[-1 + x^2])/(e
*(d^2 - e^2)*(d + e*x)) + ((3*b*d*e - a*(2*d^2 + e^2) - c*(d^2 + 2*e^2))*A
rcTanh[(-e - d*x)/(Sqrt[d^2 - e^2]*Sqrt[-1 + x^2])])/(d^2 - e^2)^(3/2))/(2
*(d^2 - e^2))
```

3.832.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

3.832.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(179) = 358$.

Time = 1.39 (sec) , antiderivative size = 729, normalized size of antiderivative = 3.74

method	result
default	$-\frac{c \ln\left(\frac{\frac{2d^2-2e^2}{e^2}-\frac{2d\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{d^2-e^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2-\frac{2d\left(x+\frac{d}{e}\right)}{e}+\frac{d^2-e^2}{e^2}}}{x+\frac{d}{e}}\right)}{e^3\sqrt{\frac{d^2-e^2}{e^2}}}+\frac{(be-2cd)\left(-\frac{e^2\sqrt{\left(x+\frac{d}{e}\right)^2-\frac{2d\left(x+\frac{d}{e}\right)}{e}+\frac{d^2-e^2}{e^2}}}{\left(d^2-e^2\right)\left(x+\frac{d}{e}\right)}-\frac{ed\ln\left(\frac{2d^2}{\cdots}\right.}{\cdots}\right.}{\cdots}$

input
```
int((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-c/e^3/((d^2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2/e*d*(x+1/e*d)+2*((d^2-e
^2)/e^2)^(1/2)*((x+1/e*d)^2-2/e*d*(x+1/e*d)+(d^2-e^2)/e^2)^(1/2))/(x+1/e*d
))+(b*e-2*c*d)/e^4*(-1/(d^2-e^2)*e^2/(x+1/e*d)*((x+1/e*d)^2-2/e*d*(x+1/e*d
)+(d^2-e^2)/e^2)^(1/2)-e*d/(d^2-e^2)/((d^2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)
/e^2-2/e*d*(x+1/e*d)+2*((d^2-e^2)/e^2)^(1/2)*((x+1/e*d)^2-2/e*d*(x+1/e*d)+
(d^2-e^2)/e^2)^(1/2))/(x+1/e*d)))+(a*e^2-b*d*e+c*d^2)/e^5*(-1/2/(d^2-e^2)*
e^2/(x+1/e*d)^2*((x+1/e*d)^2-2/e*d*(x+1/e*d)+(d^2-e^2)/e^2)^(1/2)+3/2*e*d/
(d^2-e^2)*(-1/(d^2-e^2)*e^2/(x+1/e*d)*((x+1/e*d)^2-2/e*d*(x+1/e*d)+(d^2-e^
2)/e^2)^(1/2)-e*d/(d^2-e^2)/((d^2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2/e*
d*(x+1/e*d)+2*((d^2-e^2)/e^2)^(1/2)*((x+1/e*d)^2-2/e*d*(x+1/e*d)+(d^2-e^2)
/e^2)^(1/2))/(x+1/e*d)))+1/2/(d^2-e^2)*e^2/((d^2-e^2)/e^2)^(1/2)*ln((2*(d^
2-e^2)/e^2-2/e*d*(x+1/e*d)+2*((d^2-e^2)/e^2)^(1/2)*((x+1/e*d)^2-2/e*d*(x+1
/e*d)+(d^2-e^2)/e^2)^(1/2))/(x+1/e*d)))
```

3.832.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(179) = 358$.

Time = 0.30 (sec) , antiderivative size = 1174, normalized size of antiderivative = 6.02

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(c*d^7 + b*d^6*e - (3*a + 5*c)*d^5*e^2 + b*d^4*e^3 + (3*a + 4*c)*d^3*
e^4 - 2*b*d^2*e^5 + (c*d^5*e^2 + b*d^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*d^2*e
^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x^2 + ((2*a + c)*d^4*e^2 - 3*b*d^3*e^3 +
 (a + 2*c)*d^2*e^4 + ((2*a + c)*d^2*e^4 - 3*b*d*e^5 + (a + 2*c)*e^6)*x^2 +
 2*((2*a + c)*d^3*e^3 - 3*b*d^2*e^4 + (a + 2*c)*d*e^5)*x)*sqrt(d^2 - e^2)*
log((d^2*x + d*e + sqrt(d^2 - e^2)*(d*x + e) + (d^2 - e^2 + sqrt(d^2 - e^2
)*d)*sqrt(x^2 - 1))/(e*x + d)) + 2*(c*d^6*e + b*d^5*e^2 - (3*a + 5*c)*d^4*
e^3 + b*d^3*e^4 + (3*a + 4*c)*d^2*e^5 - 2*b*d*e^6)*x + (2*b*d^5*e^2 - (4*a
 + 3*c)*d^4*e^3 - b*d^3*e^4 + (5*a + 3*c)*d^2*e^5 - b*d*e^6 - a*e^7 + (c*d
^5*e^2 + b*d^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*d^2*e^5 + (3*a + 4*c)*d*e^6 -
 2*b*e^7)*x)*sqrt(x^2 - 1))/(d^8*e^2 - 3*d^6*e^4 + 3*d^4*e^6 - d^2*e^8 + (
d^6*e^4 - 3*d^4*e^6 + 3*d^2*e^8 - e^10)*x^2 + 2*(d^7*e^3 - 3*d^5*e^5 + 3*d
^3*e^7 - d*e^9)*x), 1/2*(c*d^7 + b*d^6*e - (3*a + 5*c)*d^5*e^2 + b*d^4*e^3
 + (3*a + 4*c)*d^3*e^4 - 2*b*d^2*e^5 + (c*d^5*e^2 + b*d^4*e^3 - (3*a + 5*c
)*d^3*e^4 + b*d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x^2 - 2*((2*a + c)*d^
4*e^2 - 3*b*d^3*e^3 + (a + 2*c)*d^2*e^4 + ((2*a + c)*d^2*e^4 - 3*b*d*e^5 +
 (a + 2*c)*e^6)*x^2 + 2*((2*a + c)*d^3*e^3 - 3*b*d^2*e^4 + (a + 2*c)*d*e^5
)*x)*sqrt(-d^2 + e^2)*arctan(-(sqrt(-d^2 + e^2)*sqrt(x^2 - 1)*e - sqrt(-d^
2 + e^2)*(e*x + d))/(d^2 - e^2)) + 2*(c*d^6*e + b*d^5*e^2 - (3*a + 5*c)*d^
4*e^3 + b*d^3*e^4 + (3*a + 4*c)*d^2*e^5 - 2*b*d*e^6)*x + (2*b*d^5*e^2 -...
```

3.832.6 Sympy [F]

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx=\int \frac{a+bx+cx^2}{\sqrt{(x-1)\,(x+1)}\,(d+ex)^3}\,dx$$

input
```
integrate((c*x**2+b*x+a)/(e*x+d)**3/(x**2-1)**(1/2),x)
```

output
```
Integral((a + b*x + c*x**2)/(sqrt((x - 1)*(x + 1))*(d + e*x)**3), x)
```

3.832.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((e-d)*(e+d)>0)', see `assume?` f
or more de
```

3.832.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(179) = 358$.

Time = 0.34 (sec) , antiderivative size = 558, normalized size of antiderivative = 2.86

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx = \frac{(2\,ad^2+cd^2-3\,bde+ae^2+2\,ce^2)\arctan\left(-\frac{e\left(x-\sqrt{x^2-1}\right)+d}{\sqrt{-d^2+e^2}}\right)}{(d^4-2\,d^2e^2+e^4)\sqrt{-d^2+e^2}} + \frac{2\,cd^4e\left(x-\sqrt{x^2-1}\right)^3-2\,ad^2e^3\left(x-\sqrt{x^2-1}\right)^3-5\,cd^2e^3\left(x-\sqrt{x^2-1}\right)^3+3\,bde^4\left(x-\sqrt{x^2-1}\right)^3-ae}{}$$

input
```
integrate((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2),x, algorithm="giac")
```

output
```
(2*a*d^2 + c*d^2 - 3*b*d*e + a*e^2 + 2*c*e^2)*arctan(-(e*(x - sqrt(x^2 - 1
)) + d)/sqrt(-d^2 + e^2))/((d^4 - 2*d^2*e^2 + e^4)*sqrt(-d^2 + e^2)) + (2*
c*d^4*e*(x - sqrt(x^2 - 1))^3 - 2*a*d^2*e^3*(x - sqrt(x^2 - 1))^3 - 5*c*d^
2*e^3*(x - sqrt(x^2 - 1))^3 + 3*b*d*e^4*(x - sqrt(x^2 - 1))^3 - a*e^5*(x -
 sqrt(x^2 - 1))^3 + 2*c*d^5*(x - sqrt(x^2 - 1))^2 + 2*b*d^4*e*(x - sqrt(x^
2 - 1))^2 - 6*a*d^3*e^2*(x - sqrt(x^2 - 1))^2 - 7*c*d^3*e^2*(x - sqrt(x^2
- 1))^2 + 5*b*d^2*e^3*(x - sqrt(x^2 - 1))^2 - 3*a*d*e^4*(x - sqrt(x^2 - 1)
)^2 - 4*c*d*e^4*(x - sqrt(x^2 - 1))^2 + 2*b*e^5*(x - sqrt(x^2 - 1))^2 + 2*
c*d^4*e*(x - sqrt(x^2 - 1)) + 4*b*d^3*e^2*(x - sqrt(x^2 - 1)) - 10*a*d^2*e
^3*(x - sqrt(x^2 - 1)) - 11*c*d^2*e^3*(x - sqrt(x^2 - 1)) + 5*b*d*e^4*(x -
 sqrt(x^2 - 1)) + a*e^5*(x - sqrt(x^2 - 1)) + c*d^3*e^2 + b*d^2*e^3 - 3*a*
d*e^4 - 4*c*d*e^4 + 2*b*e^5)/((d^4*e^2 - 2*d^2*e^4 + e^6)*(e*(x - sqrt(x^2
 - 1))^2 + 2*d*(x - sqrt(x^2 - 1)) + e)^2)
```

3.832.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx = \int \frac{c\,x^2+b\,x+a}{\sqrt{x^2-1}\,{\left(d+e\,x\right)}^3}\,dx$$

input
```
int((a + b*x + c*x^2)/((x^2 - 1)^(1/2)*(d + e*x)^3),x)
```

output
```
int((a + b*x + c*x^2)/((x^2 - 1)^(1/2)*(d + e*x)^3), x)
```

3.833 $\int \frac{1+2x^8}{x(1+x^8)^{3/2}}\,dx$

3.833.1 Optimal result

Integrand size = 20, antiderivative size = 28

$$\int \frac{1+2x^8}{x\left(1+x^8\right)^{3/2}}\,dx = -\frac{1}{4\sqrt{1+x^8}}-\frac{1}{4}\text{arctanh}\left(\sqrt{1+x^8}\right)$$

output
```
-1/4*arctanh((x^8+1)^(1/2))-1/4/(x^8+1)^(1/2)
```

3.833.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1+2x^8}{x\left(1+x^8\right)^{3/2}}\,dx = -\frac{1}{4\sqrt{1+x^8}}-\frac{1}{4}\text{arctanh}\left(\sqrt{1+x^8}\right)$$

input
```
Integrate[(1 + 2*x^8)/(x*(1 + x^8)^(3/2)),x]
```

output
```
-1/4*1/Sqrt[1 + x^8] - ArcTanh[Sqrt[1 + x^8]]/4
```

3.833.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {948, 87, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^8+1}{x\left(x^8+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{8}\int \frac{2x^8+1}{x^8\left(x^8+1\right)^{3/2}}dx^8$$

$$\Big\downarrow \text{ 87}$$

$$\frac{1}{8}\left(\int \frac{1}{x^8\sqrt{x^8+1}}dx^8-\frac{2}{\sqrt{x^8+1}}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{8}\left(2\int \frac{1}{x^{16}-1}d\sqrt{x^8+1}-\frac{2}{\sqrt{x^8+1}}\right)$$

$$\Big\downarrow \text{ 220}$$

$$\frac{1}{8}\left(-2\text{arctanh}\left(\sqrt{x^8+1}\right)-\frac{2}{\sqrt{x^8+1}}\right)$$

input
```
Int[(1 + 2*x^8)/(x*(1 + x^8)^(3/2)),x]
```

output
```
(-2/Sqrt[1 + x^8] - 2*ArcTanh[Sqrt[1 + x^8]])/8
```

3.833.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

3.833.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
pseudoelliptic	$-\frac{1}{4\sqrt{x^8+1}}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^8+1}}\right)}{4}$	21
trager	$-\frac{1}{4\sqrt{x^8+1}}-\frac{\ln\left(\frac{\sqrt{x^8+1}+1}{x^4}\right)}{4}$	27
risch	$-\frac{1}{4\sqrt{x^8+1}}+\frac{-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^8+1}}{2}\right)+(-2\ln(2)+8\ln(x))\sqrt{\pi}}{8\sqrt{\pi}}$	47
meijerg	$\frac{-\sqrt{\pi}+\frac{\sqrt{\pi}}{\sqrt{x^8+1}}-\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^8+1}}{2}\right)+\frac{(2-2\ln(2)+8\ln(x))\sqrt{\pi}}{2}}{4\sqrt{\pi}}+\frac{\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{x^8+1}}}{2\sqrt{\pi}}$	77

input
```
int((2*x^8+1)/x/(x^8+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(x^8+1)^(1/2)-1/4*arctanh(1/(x^8+1)^(1/2))
```

3.833.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(20) = 40$.

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

$$\int \frac{1+2x^8}{x\left(1+x^8\right)^{3/2}}\,dx = -\frac{\left(x^8+1\right)\log\left(\sqrt{x^8+1}+1\right) - \left(x^8+1\right)\log\left(\sqrt{x^8+1}-1\right) + 2\sqrt{x^8+1}}{8\left(x^8+1\right)}$$

input
```
integrate((2*x^8+1)/x/(x^8+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/8*((x^8 + 1)*log(sqrt(x^8 + 1) + 1) - (x^8 + 1)*log(sqrt(x^8 + 1) - 1)
+ 2*sqrt(x^8 + 1))/(x^8 + 1)
```

3.833.6 Sympy [A] (verification not implemented)

Time = 5.91 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

$$\int \frac{1+2x^8}{x\left(1+x^8\right)^{3/2}}\,dx = \frac{\log\left(\sqrt{x^8+1}-1\right)}{8} - \frac{\log\left(\sqrt{x^8+1}+1\right)}{8} - \frac{1}{4\sqrt{x^8+1}}$$

input
```
integrate((2*x**8+1)/x/(x**8+1)**(3/2),x)
```

output
```
log(sqrt(x**8 + 1) - 1)/8 - log(sqrt(x**8 + 1) + 1)/8 - 1/(4*sqrt(x**8 + 1
))
```

3.833.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \frac{1+2x^8}{x\left(1+x^8\right)^{3/2}}\,dx = -\frac{1}{4\sqrt{x^8+1}} - \frac{1}{8}\log\left(\sqrt{x^8+1}+1\right) + \frac{1}{8}\log\left(\sqrt{x^8+1}-1\right)$$

input
```
integrate((2*x^8+1)/x/(x^8+1)^(3/2),x, algorithm="maxima")
```

output
```
-1/4/sqrt(x^8 + 1) - 1/8*log(sqrt(x^8 + 1) + 1) + 1/8*log(sqrt(x^8 + 1) -
1)
```

3.833.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \frac{1+2x^8}{x\left(1+x^8\right)^{3/2}}\,dx = -\frac{1}{4\sqrt{x^8+1}} - \frac{1}{8}\log\left(\sqrt{x^8+1}+1\right) + \frac{1}{8}\log\left(\sqrt{x^8+1}-1\right)$$

input
```
integrate((2*x^8+1)/x/(x^8+1)^(3/2),x, algorithm="giac")
```

output
```
-1/4/sqrt(x^8 + 1) - 1/8*log(sqrt(x^8 + 1) + 1) + 1/8*log(sqrt(x^8 + 1) -
1)
```

3.833.9 Mupad [B] (verification not implemented)

Time = 21.79 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \frac{1+2x^8}{x\left(1+x^8\right)^{3/2}}\,dx = -\frac{\operatorname{atanh}\left(\sqrt{x^8+1}\right)}{4} - \frac{1}{4\sqrt{x^8+1}}$$

input
```
int((2*x^8 + 1)/(x*(x^8 + 1)^(3/2)),x)
```

output
```
- atanh((x^8 + 1)^(1/2))/4 - 1/(4*(x^8 + 1)^(1/2))
```

3.834 $\int \frac{\sqrt{1+x^8}(1+2x^8)}{x+2x^9+x^{17}} dx$

3.834.1 Optimal result

Integrand size = 29, antiderivative size = 28

$$\int \frac{\sqrt{1+x^8}(1+2x^8)}{x+2x^9+x^{17}} dx = -\frac{1}{4\sqrt{1+x^8}} - \frac{1}{4}\text{arctanh}\left(\sqrt{1+x^8}\right)$$

output
```
-1/4*arctanh((x^8+1)^(1/2))-1/4/(x^8+1)^(1/2)
```

3.834.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+x^8}(1+2x^8)}{x+2x^9+x^{17}} dx = -\frac{1}{4\sqrt{1+x^8}} - \frac{1}{4}\text{arctanh}\left(\sqrt{1+x^8}\right)$$

input
```
Integrate[(Sqrt[1 + x^8]*(1 + 2*x^8))/(x + 2*x^9 + x^17),x]
```

output
```
-1/4*1/Sqrt[1 + x^8] - ArcTanh[Sqrt[1 + x^8]]/4
```

3.834.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2019, 2026, 948, 87, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^8+1}(2x^8+1)}{x^{17}+2x^9+x}dx$$

$$\Big\downarrow \text{ 2019}$$

$$\int \frac{2x^8+1}{\sqrt{x^8+1}(x^9+x)}dx$$

$$\Big\downarrow \text{ 2026}$$

$$\int \frac{2x^8+1}{x(x^8+1)^{3/2}}dx$$

$$\Big\downarrow \text{ 948}$$

$$\frac{1}{8}\int \frac{2x^8+1}{x^8(x^8+1)^{3/2}}dx^8$$

$$\Big\downarrow \text{ 87}$$

$$\frac{1}{8}\left(\int \frac{1}{x^8\sqrt{x^8+1}}dx^8-\frac{2}{\sqrt{x^8+1}}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{8}\left(2\int \frac{1}{x^{16}-1}d\sqrt{x^8+1}-\frac{2}{\sqrt{x^8+1}}\right)$$

$$\Big\downarrow \text{ 220}$$

$$\frac{1}{8}\left(-2\text{arctanh}\left(\sqrt{x^8+1}\right)-\frac{2}{\sqrt{x^8+1}}\right)$$

input
```
Int[(Sqrt[1 + x^8]*(1 + 2*x^8))/(x + 2*x^9 + x^17),x]
```

output
```
(-2/Sqrt[1 + x^8] - 2*ArcTanh[Sqrt[1 + x^8]])/8
```

3.834. $\int \frac{\sqrt{1+x^8}(1+2x^8)}{x+2x^9+x^{17}}dx$

3.834.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 948
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^
p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ
[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

3.834. $\int \frac{\sqrt{1+x^8}\left(1+2x^8\right)}{x+2x^9+x^{17}}\,dx$

3.834.4 Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
pseudoelliptic	$-\frac{1}{4\sqrt{x^8+1}}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^8+1}}\right)}{4}$	21
trager	$-\frac{1}{4\sqrt{x^8+1}}+\frac{\ln\left(\frac{\sqrt{x^8+1}-1}{x^4}\right)}{4}$	27
risch	$-\frac{1}{4\sqrt{x^8+1}}+\frac{-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^8+1}}{2}\right)+(-2\ln(2)+8\ln(x))\sqrt{\pi}}{8\sqrt{\pi}}$	47

input
```
int((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(x^8+1)^(1/2)-1/4*arctanh(1/(x^8+1)^(1/2))
```

3.834.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(20) = 40.

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

$$\int \frac{\sqrt{1+x^8}(1+2x^8)}{x+2x^9+x^{17}}\,dx = -\frac{(x^8+1)\log\left(\sqrt{x^8+1}+1\right)-(x^8+1)\log\left(\sqrt{x^8+1}-1\right)+2\sqrt{x^8+1}}{8\,(x^8+1)}$$

input
```
integrate((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x),x, algorithm="fricas")
```

output
```
-1/8*((x^8 + 1)*log(sqrt(x^8 + 1) + 1) - (x^8 + 1)*log(sqrt(x^8 + 1) - 1)
+ 2*sqrt(x^8 + 1))/(x^8 + 1)
```

3.834.6 Sympy [F]

$$\int \frac{\sqrt{1+x^8}(1+2x^8)}{x+2x^9+x^{17}}\,dx = \int \frac{2x^8+1}{x\left(x^8+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((2*x**8+1)*(x**8+1)**(1/2)/(x**17+2*x**9+x),x)
```

output
```
Integral((2*x**8 + 1)/(x*(x**8 + 1)**(3/2)), x)
```

3.834.7 Maxima [F]

$$\int \frac{\sqrt{1+x^8}(1+2x^8)}{x+2x^9+x^{17}}\,dx = \int \frac{(2\,x^8+1)\sqrt{x^8+1}}{x^{17}+2\,x^9+x}\,dx$$

input
```
integrate((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x),x, algorithm="maxima")
```

output
```
integrate((2*x^8 + 1)*sqrt(x^8 + 1)/(x^17 + 2*x^9 + x), x)
```

3.834.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{1+x^8}(1+2x^8)}{x+2x^9+x^{17}}\,dx = -\frac{1}{4\sqrt{x^8+1}} - \frac{1}{8}\log\left(\sqrt{x^8+1}+1\right) + \frac{1}{8}\log\left(\sqrt{x^8+1}-1\right)$$

input
```
integrate((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x),x, algorithm="giac")
```

output
```
-1/4/sqrt(x^8 + 1) - 1/8*log(sqrt(x^8 + 1) + 1) + 1/8*log(sqrt(x^8 + 1) -
1)
```

3.834.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x^8}(1+2x^8)}{x+2x^9+x^{17}}\,dx = \int \frac{\sqrt{x^8+1}\,(2\,x^8+1)}{x^{17}+2\,x^9+x}\,dx$$

input
```
int(((x^8 + 1)^(1/2)*(2*x^8 + 1))/(x + 2*x^9 + x^17),x)
```

output
```
int(((x^8 + 1)^(1/2)*(2*x^8 + 1))/(x + 2*x^9 + x^17), x)
```

3.835 $\int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right) dx$

3.835.1 Optimal result

Integrand size = 20, antiderivative size = 22

$$\int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right) dx = x-3x^3-\frac{1}{9}\sqrt{1-9x^2}$$

output
```
x-3*x^3-1/9*(-9*x^2+1)^(1/2)
```

3.835.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right) dx = x-3x^3-\frac{1}{9}\sqrt{1-9x^2}$$

input
```
Integrate[1 - 9*x^2 + x/Sqrt[1 - 9*x^2],x]
```

output
```
x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

3.835.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-9x^2+\frac{x}{\sqrt{1-9x^2}}+1\right) \, dx$$

$$\Big\downarrow \ 2009$$

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

input
```
Int[1 - 9*x^2 + x/Sqrt[1 - 9*x^2],x]
```

output
```
x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

3.835.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.835.4 Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
default	$x-3x^3-\frac{\sqrt{-9x^2+1}}{9}$	19
trager	$-\left(3x^2-1\right)x-\frac{\sqrt{-9x^2+1}}{9}$	23
risch	$-3x^3+x+\frac{9x^2-1}{9\sqrt{-9x^2+1}}$	26

input
```
int(1-9*x^2+x/(-9*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x-3*x^3-1/9*(-9*x^2+1)^(1/2)
```

3.835.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right)\,dx=-3\,x^3+x-\frac{1}{9}\,\sqrt{-9\,x^2+1}$$

input
```
integrate(1-9*x^2+x/(-9*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

3.835.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right)\,dx=-3x^3+x-\frac{\sqrt{1-9x^2}}{9}$$

input
```
integrate(1-9*x**2+x/(-9*x**2+1)**(1/2),x)
```

output
```
-3*x**3 + x - sqrt(1 - 9*x**2)/9
```

3.835.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right)\,dx=-3\,x^3+x-\frac{1}{9}\,\sqrt{-9\,x^2+1}$$

input
```
integrate(1-9*x^2+x/(-9*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

3.835.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right)\,dx=-3\,x^3+x-\frac{1}{9}\,\sqrt{-9\,x^2+1}$$

input
```
integrate(1-9*x^2+x/(-9*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

3.835.9 Mupad [B] (verification not implemented)

Time = 21.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right)\,dx=x-3\,x^3-\frac{\sqrt{\frac{1}{9}-x^2}}{3}$$

input
```
int(x/(1 - 9*x^2)^(1/2) - 9*x^2 + 1,x)
```

output
```
x - 3*x^3 - (1/9 - x^2)^(1/2)/3
```

3.836 $\int \frac{x+\left(1-9x^2\right)^{3/2}}{\sqrt{1-9x^2}}\,dx$

3.836.1 Optimal result

Integrand size = 25, antiderivative size = 22

$$\int \frac{x+\left(1-9x^2\right)^{3/2}}{\sqrt{1-9x^2}}\,dx = x-3x^3-\frac{1}{9}\sqrt{1-9x^2}$$

output
```
x-3*x^3-1/9*(-9*x^2+1)^(1/2)
```

3.836.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x+\left(1-9x^2\right)^{3/2}}{\sqrt{1-9x^2}}\,dx = x-3x^3-\frac{1}{9}\sqrt{1-9x^2}$$

input
```
Integrate[(x + (1 - 9*x^2)^(3/2))/Sqrt[1 - 9*x^2],x]
```

output
```
x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

3.836.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(1-9x^2\right)^{3/2}+x}{\sqrt{1-9x^2}}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-9x^2+\frac{x}{\sqrt{1-9x^2}}+1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

input
```
Int[(x + (1 - 9*x^2)^(3/2))/Sqrt[1 - 9*x^2],x]
```

output
```
x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

3.836.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.836.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
default	$x-3x^3-\frac{\sqrt{-9x^2+1}}{9}$	19
trager	$-(3x^2-1)\,x-\frac{\sqrt{-9x^2+1}}{9}$	23

input
```
int((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x-3*x^3-1/9*(-9*x^2+1)^(1/2)
```

3.836.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x+(1-9x^2)^{3/2}}{\sqrt{1-9x^2}}\,dx=-3\,x^3+x-\frac{1}{9}\,\sqrt{-9\,x^2+1}$$

input
```
integrate((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

3.836.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{x+(1-9x^2)^{3/2}}{\sqrt{1-9x^2}}\,dx=-3x^3+x-\frac{\sqrt{1-9x^2}}{9}$$

input
```
integrate((x+(-9*x**2+1)**(3/2))/(-9*x**2+1)**(1/2),x)
```

output
```
-3*x**3 + x - sqrt(1 - 9*x**2)/9
```

3.836.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x + (1-9x^2)^{3/2}}{\sqrt{1-9x^2}}\,dx = -3\,x^3 + x - \frac{1}{9}\sqrt{-9\,x^2+1}$$

input
```
integrate((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

3.836.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x + (1-9x^2)^{3/2}}{\sqrt{1-9x^2}}\,dx = -3\,x^3 + x - \frac{1}{9}\sqrt{-9\,x^2+1}$$

input
```
integrate((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

3.836.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{x + (1-9x^2)^{3/2}}{\sqrt{1-9x^2}}\,dx = x - 3\,x^3 - \frac{\sqrt{\frac{1}{9}-x^2}}{3}$$

input
```
int((x + (1 - 9*x^2)^(3/2))/(1 - 9*x^2)^(1/2),x)
```

output
```
x - 3*x^3 - (1/9 - x^2)^(1/2)/3
```

3.837 $\int \frac{(-3+2\sqrt{x})(-3\sqrt{x}+x)^{2/3}}{\sqrt{x}} dx$

3.837.1 Optimal result

Integrand size = 28, antiderivative size = 17

$$\int \frac{(-3+2\sqrt{x})(-3\sqrt{x}+x)^{2/3}}{\sqrt{x}} dx = \frac{6}{5}(-3\sqrt{x}+x)^{5/3}$$

output
```
6/5*(x-3*x^(1/2))^(5/3)
```

3.837.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{(-3+2\sqrt{x})(-3\sqrt{x}+x)^{2/3}}{\sqrt{x}} dx = \frac{6}{5}(-3\sqrt{x}+x)^{5/3}$$

input
```
Integrate[((-3 + 2*Sqrt[x])*(-3*Sqrt[x] + x)^(2/3))/Sqrt[x],x]
```

output
```
(6*(-3*Sqrt[x] + x)^(5/3))/5
```

3.837.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1940, 25, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(2\sqrt{x}-3\right)\left(x-3\sqrt{x}\right)^{2/3}}{\sqrt{x}}dx$$

$$\Big\downarrow \text{1940}$$

$$2\int -\left(\left(3-2\sqrt{x}\right)\left(x-3\sqrt{x}\right)^{2/3}\right)d\sqrt{x}$$

$$\Big\downarrow \text{25}$$

$$-2\int \left(3-2\sqrt{x}\right)\left(x-3\sqrt{x}\right)^{2/3}d\sqrt{x}$$

$$\Big\downarrow \text{1104}$$

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

input
```
Int[((-3 + 2*Sqrt[x])*(-3*Sqrt[x] + x)^(2/3))/Sqrt[x],x]
```

output
```
(6*(-3*Sqrt[x] + x)^(5/3))/5
```

3.837.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1940
```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)
*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /;
FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && I
ntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
 1)/n]] && NeQ[n^2, 1]
```

3.837.4 Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{6\left(x-3\sqrt{x}\right)^{\frac{5}{3}}}{5}$	12
default	$\frac{6\left(x-3\sqrt{x}\right)^{\frac{5}{3}}}{5}$	12
meijerg	$-\frac{18\,3^{\frac{2}{3}}\,\mathrm{signum}\left(-1+\frac{\sqrt{x}}{3}\right)^{\frac{2}{3}} x^{\frac{5}{6}}\,{}_2F_1\left(-\frac{2}{3},\frac{5}{3};\frac{8}{3};\frac{\sqrt{x}}{3}\right)}{5\left(-\mathrm{signum}\left(-1+\frac{\sqrt{x}}{3}\right)\right)^{\frac{2}{3}}}+\frac{3\,3^{\frac{2}{3}}\,\mathrm{signum}\left(-1+\frac{\sqrt{x}}{3}\right)^{\frac{2}{3}} x^{\frac{4}{3}}\,{}_2F_1\left(-\frac{2}{3},\frac{8}{3};\frac{11}{3};\frac{\sqrt{x}}{3}\right)}{2\left(-\mathrm{signum}\left(-1+\frac{\sqrt{x}}{3}\right)\right)^{\frac{2}{3}}}$	84

input
```
int((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
6/5*(x-3*x^(1/2))^(5/3)
```

3.837.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{\left(-3+2\sqrt{x}\right)\left(-3\sqrt{x}+x\right)^{2/3}}{\sqrt{x}}\,dx=\frac{6}{5}\left(x-3\sqrt{x}\right)^{\frac{5}{3}}$$

input
```
integrate((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2),x, algorithm="fricas"
)
```

output
```
6/5*(x - 3*sqrt(x))^(5/3)
```

3.837.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(14) = 28.

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.12

$$\int \frac{\left(-3+2\sqrt{x}\right)\left(-3\sqrt{x}+x\right)^{2/3}}{\sqrt{x}}\,dx = -\frac{18\sqrt{x}\left(-3\sqrt{x}+x\right)^{\frac{2}{3}}}{5} + \frac{6x\left(-3\sqrt{x}+x\right)^{\frac{2}{3}}}{5}$$

input
```
integrate((x-3*x**(1/2))**(2/3)*(-3+2*x**(1/2))/x**(1/2),x)
```

output
```
-18*sqrt(x)*(-3*sqrt(x) + x)**(2/3)/5 + 6*x*(-3*sqrt(x) + x)**(2/3)/5
```

3.837.7 Maxima [F]

$$\int \frac{\left(-3+2\sqrt{x}\right)\left(-3\sqrt{x}+x\right)^{2/3}}{\sqrt{x}}\,dx = \int \frac{\left(x-3\sqrt{x}\right)^{\frac{2}{3}}\left(2\sqrt{x}-3\right)}{\sqrt{x}}\,dx$$

input
```
integrate((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((x - 3*sqrt(x))^(2/3)*(2*sqrt(x) - 3)/sqrt(x), x)
```

3.837.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{\left(-3+2\sqrt{x}\right)\left(-3\sqrt{x}+x\right)^{2/3}}{\sqrt{x}}\,dx = \frac{6}{5}\left(x-3\sqrt{x}\right)^{\frac{5}{3}}$$

input
```
integrate((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2),x, algorithm="giac")
```

output
```
6/5*(x - 3*sqrt(x))^(5/3)
```

3.837.9 Mupad [B] (verification not implemented)

Time = 22.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{\left(-3+2\sqrt{x}\right)\left(-3\sqrt{x}+x\right)^{2/3}}{\sqrt{x}}\,dx=\frac{6\left(x-3\sqrt{x}\right)^{5/3}}{5}$$

input
```
int(((x - 3*x^(1/2))^(2/3)*(2*x^(1/2) - 3))/x^(1/2),x)
```

output
```
(6*(x - 3*x^(1/2))^(5/3))/5
```

3.838 $\int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}}\,dx$

3.838.1 Optimal result

Integrand size = 26, antiderivative size = 17

$$\int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}}\,dx=\frac{6}{5}\big(-3\sqrt{x}+x\big)^{5/3}$$

output
```
6/5*(x-3*x^(1/2))^(5/3)
```

3.838.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}}\,dx=\frac{6}{5}\big(-3\sqrt{x}+x\big)^{5/3}$$

input
```
Integrate[(9 - 9*Sqrt[x] + 2*x)/(-3*Sqrt[x] + x)^(1/3),x]
```

output
```
(6*(-3*Sqrt[x] + x)^(5/3))/5
```

3.838.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2440, 2162, 25, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x-9\sqrt{x}+9}{\sqrt[3]{x-3\sqrt{x}}}dx$$

$$\Big\downarrow \text{2440}$$

$$2\int \frac{\sqrt{x}\left(2x-9\sqrt{x}+9\right)}{\sqrt[3]{x-3\sqrt{x}}}d\sqrt{x}$$

$$\Big\downarrow \text{2162}$$

$$2\int -\left(\left(3-2\sqrt{x}\right)\left(x-3\sqrt{x}\right)^{2/3}\right)d\sqrt{x}$$

$$\Big\downarrow \text{25}$$

$$-2\int \left(3-2\sqrt{x}\right)\left(x-3\sqrt{x}\right)^{2/3}d\sqrt{x}$$

$$\Big\downarrow \text{1104}$$

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

input
```
Int[(9 - 9*Sqrt[x] + 2*x)/(-3*Sqrt[x] + x)^(1/3),x]
```

output
```
(6*(-3*Sqrt[x] + x)^(5/3))/5
```

3.838.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 2162
```
Int[(Pq_)*((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Simp[e   Int[(e*x)^(m - 1)*PolynomialQuotient[Pq, b + c*x, x]*(b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{b, c, e, m, p}, x] && PolyQ[Pq, x] && EqQ[Poly
nomialRemainder[Pq, b + c*x, x], 0]
```

rule 2440
```
Int[(Pq_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{d =
 Denominator[n]}, Simp[d   Subst[Int[x^(d - 1)*(SubstFor[x^n, Pq, x] /. x -
> x^(d*n))*(a*x^(d*j) + b*x^(d*n))^p, x], x, x^(1/d)], x]] /; FreeQ[{a, b,
j, n, p}, x] && PolyQ[Pq, x^n] &&  !IntegerQ[p] && NeQ[n, j] && RationalQ[j
, n] && IntegerQ[j/n] && LtQ[-1, n, 1]
```

3.838.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 2.

Time = 1.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 7.35

method	result
meijerg	$\frac{18\,3^{\frac{2}{3}}\left(-\operatorname{signum}\left(-1+\frac{\sqrt{x}}{3}\right)\right)^{\frac{1}{3}} x^{\frac{5}{6}}\,{}_2F_1\left(\frac{1}{3},\frac{5}{3};\frac{8}{3};\frac{\sqrt{x}}{3}\right)}{5\operatorname{signum}\left(-1+\frac{\sqrt{x}}{3}\right)^{\frac{1}{3}}}+\frac{4\,3^{\frac{2}{3}}\left(-\operatorname{signum}\left(-1+\frac{\sqrt{x}}{3}\right)\right)^{\frac{1}{3}} x^{\frac{11}{6}}\,{}_2F_1\left(\frac{1}{3},\frac{11}{3};\frac{14}{3};\frac{\sqrt{x}}{3}\right)}{11\operatorname{signum}\left(-1+\frac{\sqrt{x}}{3}\right)^{\frac{1}{3}}}-\frac{9\,3^{\frac{2}{3}}\left(-\operatorname{signum}\left(-\right.\right.}{4\operatorname{si}}$

input
```
int((9+2*x-9*x^(1/2))/(x-3*x^(1/2))^(1/3),x,method=_RETURNVERBOSE)
```

output
```
18/5*3^(2/3)/signum(-1+1/3*x^(1/2))^(1/3)*(-signum(-1+1/3*x^(1/2)))^(1/3)*
x^(5/6)*hypergeom([1/3,5/3],[8/3],1/3*x^(1/2))+4/11*3^(2/3)/signum(-1+1/3*
x^(1/2))^(1/3)*(-signum(-1+1/3*x^(1/2)))^(1/3)*x^(11/6)*hypergeom([1/3,11/
3],[14/3],1/3*x^(1/2))-9/4*3^(2/3)/signum(-1+1/3*x^(1/2))^(1/3)*(-signum(-
1+1/3*x^(1/2)))^(1/3)*x^(4/3)*hypergeom([1/3,8/3],[11/3],1/3*x^(1/2))
```

3.838.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}}\,dx=\frac{6}{5}\left(x-3\sqrt{x}\right)^{\frac{5}{3}}$$

input
```
integrate((9+2*x-9*x^(1/2))/(x-3*x^(1/2))^(1/3),x, algorithm="fricas")
```

output
```
6/5*(x - 3*sqrt(x))^(5/3)
```

3.838.6 Sympy [F]

$$\int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}}\,dx=\int \frac{-9\sqrt{x}+2x+9}{\sqrt[3]{-3\sqrt{x}+x}}\,dx$$

input
```
integrate((9+2*x-9*x**(1/2))/(x-3*x**(1/2))**(1/3),x)
```

output
```
Integral((-9*sqrt(x) + 2*x + 9)/(-3*sqrt(x) + x)**(1/3), x)
```

3.838.7 Maxima [F]

$$\int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}}\,dx=\int \frac{2\,x-9\,\sqrt{x}+9}{\left(x-3\,\sqrt{x}\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((9+2*x-9*x^(1/2))/(x-3*x^(1/2))^(1/3),x, algorithm="maxima")
```

output
```
integrate((2*x - 9*sqrt(x) + 9)/(x - 3*sqrt(x))^(1/3), x)
```

3.838.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}}\,dx = \frac{6}{5}\left(x-3\sqrt{x}\right)^{\frac{5}{3}}$$

input
```
integrate((9+2*x-9*x^(1/2))/(x-3*x^(1/2))^(1/3),x, algorithm="giac")
```

output
```
6/5*(x - 3*sqrt(x))^(5/3)
```

3.838.9 Mupad [F(-1)]

Timed out.

$$\int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}}\,dx = \int \frac{2\,x-9\,\sqrt{x}+9}{\left(x-3\,\sqrt{x}\right)^{1/3}}\,dx$$

input
```
int((2*x - 9*x^(1/2) + 9)/(x - 3*x^(1/2))^(1/3),x)
```

output
```
int((2*x - 9*x^(1/2) + 9)/(x - 3*x^(1/2))^(1/3), x)
```

3.839 $\int \frac{1}{\sqrt{4-9x^2}}\,dx$

3.839.1 Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{1}{\sqrt{4-9x^2}}\,dx = \frac{1}{3}\arcsin\left(\frac{3x}{2}\right)$$

output
```
1/3*arcsin(3/2*x)
```

3.839.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(10) = 20.

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.30

$$\int \frac{1}{\sqrt{4-9x^2}}\,dx = \frac{2}{3}\arctan\left(\frac{3x}{-2+\sqrt{4-9x^2}}\right)$$

input
```
Integrate[1/Sqrt[4 - 9*x^2],x]
```

output
```
(2*ArcTan[(3*x)/(-2 + Sqrt[4 - 9*x^2])])/3
```

3.839.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{4-9x^2}}\,dx$$
$$\Big\downarrow \text{223}$$
$$\frac{1}{3}\arcsin\left(\frac{3x}{2}\right)$$

input
```
Int[1/Sqrt[4 - 9*x^2],x]
```

output
```
ArcSin[(3*x)/2]/3
```

3.839.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

3.839.4 Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\arcsin\left(\frac{3x}{2}\right)}{3}$	7
meijerg	$\frac{\arcsin\left(\frac{3x}{2}\right)}{3}$	7
pseudoelliptic	$-\frac{\arctan\left(\frac{\sqrt{-9x^2+4}}{3x}\right)}{3}$	18
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-9x^2+4}+3x\right)}{3}$	31

input
```
int(1/(-9*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*arcsin(3/2*x)
```

3.839.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(6) = 12.

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{4-9x^2}}\,dx = -\frac{2}{3}\arctan\left(\frac{\sqrt{-9\,x^2+4}-2}{3\,x}\right)$$

input
```
integrate(1/(-9*x^2+4)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*arctan(1/3*(sqrt(-9*x^2 + 4) - 2)/x)
```

3.839.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{4-9x^2}}\,dx = \frac{\operatorname{asin}\left(\frac{3x}{2}\right)}{3}$$

input
```
integrate(1/(-9*x**2+4)**(1/2),x)
```

output
```
asin(3*x/2)/3
```

3.839.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{4-9x^2}}\,dx = \frac{1}{3}\arcsin\left(\frac{3}{2}\,x\right)$$

input
```
integrate(1/(-9*x^2+4)^(1/2),x, algorithm="maxima")
```

output
```
1/3*arcsin(3/2*x)
```

3.839.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(6) = 12$.

Time = 0.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{4-9x^2}}\,dx = \frac{1}{2}\sqrt{-9\,x^2+4}x + \frac{2}{3}\arcsin\left(\frac{3}{2}\,x\right)$$

input
```
integrate(1/(-9*x^2+4)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-9*x^2 + 4)*x + 2/3*arcsin(3/2*x)
```

3.839.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{4-9x^2}}\,dx = \frac{\operatorname{asin}\left(\frac{3\,x}{2}\right)}{3}$$

input
```
int(1/(4 - 9*x^2)^(1/2),x)
```

output
```
asin((3*x)/2)/3
```

3.840 $\int \frac{1}{\sqrt{2-3x}\sqrt{2+3x}}\,dx$

3.840.1 Optimal result

Integrand size = 19, antiderivative size = 10

$$\int \frac{1}{\sqrt{2-3x}\sqrt{2+3x}}\,dx=\frac{1}{3}\arcsin\left(\frac{3x}{2}\right)$$

output
```
1/3*arcsin(3/2*x)
```

3.840.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(10) = 20.

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.30

$$\int \frac{1}{\sqrt{2-3x}\sqrt{2+3x}}\,dx=\frac{2}{3}\arctan\left(\frac{3x}{-2+\sqrt{4-9x^2}}\right)$$

input
```
Integrate[1/(Sqrt[2 - 3*x]*Sqrt[2 + 3*x]),x]
```

output
```
(2*ArcTan[(3*x)/(-2 + Sqrt[4 - 9*x^2])])/3
```

3.840.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {39, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2-3x}\sqrt{3x+2}}dx$$

$$\Big\downarrow \ 39$$

$$\int \frac{1}{\sqrt{4-9x^2}}dx$$

$$\Big\downarrow \ 223$$

$$\frac{1}{3}\arcsin\left(\frac{3x}{2}\right)$$

input
```
Int[1/(Sqrt[2 - 3*x]*Sqrt[2 + 3*x]),x]
```

output
```
ArcSin[(3*x)/2]/3
```

3.840.3.1 Defintions of rubi rules used

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

3.840.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(6) = 12$.

Time = 1.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 3.40

method	result	size
default	$\frac{\sqrt{(2-3x)(3x+2)}\,\arcsin\left(\frac{3x}{2}\right)}{3\sqrt{2-3x}\,\sqrt{3x+2}}$	34

input
```
int(1/(2-3*x)^(1/2)/(3*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((2-3*x)*(3*x+2))^(1/2)/(2-3*x)^(1/2)/(3*x+2)^(1/2)*arcsin(3/2*x)
```

3.840.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(6) = 12$.

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.50

$$\int \frac{1}{\sqrt{2-3x}\sqrt{2+3x}}\,dx = -\frac{2}{3}\arctan\left(\frac{\sqrt{3\,x+2}\sqrt{-3\,x+2}-2}{3\,x}\right)$$

input
```
integrate(1/(2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*arctan(1/3*(sqrt(3*x + 2)*sqrt(-3*x + 2) - 2)/x)
```

3.840.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 49, normalized size of antiderivative = 4.90

$$\int \frac{1}{\sqrt{2-3x}\sqrt{2+3x}}\,dx = \begin{cases} -\frac{2i\operatorname{acosh}\left(\frac{\sqrt{3}\sqrt{x+\frac{2}{3}}}{2}\right)}{3} & \text{for}\ \left|x+\frac{2}{3}\right| > \frac{4}{3} \\ \frac{2\operatorname{asin}\left(\frac{\sqrt{3}\sqrt{x+\frac{2}{3}}}{2}\right)}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(2-3*x)**(1/2)/(2+3*x)**(1/2),x)
```

output
```
Piecewise((-2*I*acosh(sqrt(3)*sqrt(x + 2/3)/2)/3, Abs(x + 2/3) > 4/3), (2*
asin(sqrt(3)*sqrt(x + 2/3)/2)/3, True))
```

3.840.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{2-3x}\sqrt{2+3x}}\,dx = \frac{1}{3}\arcsin\left(\frac{3}{2}\,x\right)$$

input
```
integrate(1/(2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="maxima")
```

output
```
1/3*arcsin(3/2*x)
```

3.840.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sqrt{2-3x}\sqrt{2+3x}}\,dx = \frac{2}{3}\arcsin\left(\frac{1}{2}\,\sqrt{3\,x+2}\right)$$

input
```
integrate(1/(2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="giac")
```

output
```
2/3*arcsin(1/2*sqrt(3*x + 2))
```

3.840.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 3.20

$$\int \frac{1}{\sqrt{2-3x}\sqrt{2+3x}}\,dx = -\frac{4\,\mathrm{atan}\left(\frac{\sqrt{2}-\sqrt{2-3\,x}}{\sqrt{2}-\sqrt{3\,x+2}}\right)}{3}$$

input
```
int(1/((2 - 3*x)^(1/2)*(3*x + 2)^(1/2)),x)
```

output
```
-(4*atan((2^(1/2) - (2 - 3*x)^(1/2))/(2^(1/2) - (3*x + 2)^(1/2))))/3
```

3.841 $\int \frac{1}{\sqrt{(2-3x)(2+3x)}}\,dx$

3.841.1 Optimal result

Integrand size = 15, antiderivative size = 10

$$\int \frac{1}{\sqrt{(2-3x)(2+3x)}}\,dx = \frac{1}{3}\arcsin\left(\frac{3x}{2}\right)$$

output
```
1/3*arcsin(3/2*x)
```

3.841.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(10) = 20.

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.30

$$\int \frac{1}{\sqrt{(2-3x)(2+3x)}}\,dx = \frac{2}{3}\arctan\left(\frac{3x}{-2+\sqrt{4-9x^2}}\right)$$

input
```
Integrate[1/Sqrt[(2 - 3*x)*(2 + 3*x)],x]
```

output
```
(2*ArcTan[(3*x)/(-2 + Sqrt[4 - 9*x^2])])/3
```

3.841.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2048, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{(2-3x)(3x+2)}}dx$$
$$\Big\downarrow \text{2048}$$
$$\int \frac{1}{\sqrt{4-9x^2}}dx$$
$$\Big\downarrow \text{223}$$
$$\frac{1}{3}\arcsin\left(\frac{3x}{2}\right)$$

input
```
Int[1/Sqrt[(2 - 3*x)*(2 + 3*x)],x]
```

output
```
ArcSin[(3*x)/2]/3
```

3.841.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

3.841.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\arcsin\left(\frac{3x}{2}\right)}{3}$	7
meijerg	$\frac{\arcsin\left(\frac{3x}{2}\right)}{3}$	7
pseudoelliptic	$-\frac{\arctan\left(\frac{\sqrt{-9x^2+4}}{3x}\right)}{3}$	18
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-9x^2+4}+3x\right)}{3}$	30

input
```
int(1/((2-3*x)*(3*x+2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*arcsin(3/2*x)
```

3.841.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(6) = 12.

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{(2-3x)(2+3x)}}\,dx = -\frac{2}{3}\arctan\left(\frac{\sqrt{-9x^2+4}-2}{3x}\right)$$

input
```
integrate(1/((2-3*x)*(2+3*x))^(1/2),x, algorithm="fricas")
```

output
```
-2/3*arctan(1/3*(sqrt(-9*x^2 + 4) - 2)/x)
```

3.841.6 Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{(2-3x)(2+3x)}}\,dx = \frac{\operatorname{asin}\left(\frac{3x}{2}\right)}{3}$$

input
```
integrate(1/((2-3*x)*(2+3*x))**(1/2),x)
```

output
```
asin(3*x/2)/3
```

3.841.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{(2-3x)(2+3x)}}\,dx = \frac{1}{3}\arcsin\left(\frac{3}{2}\,x\right)$$

input
```
integrate(1/((2-3*x)*(2+3*x))^(1/2),x, algorithm="maxima")
```

output
```
1/3*arcsin(3/2*x)
```

3.841.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(6) = 12.

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{(2-3x)(2+3x)}}\,dx = \frac{1}{2}\sqrt{-9\,x^2+4}x + \frac{2}{3}\arcsin\left(\frac{3}{2}\,x\right)$$

input
```
integrate(1/((2-3*x)*(2+3*x))^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-9*x^2 + 4)*x + 2/3*arcsin(3/2*x)
```

3.841.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{(2-3x)(2+3x)}}\,dx = \frac{\mathrm{asin}\left(\frac{3x}{2}\right)}{3}$$

input
```
int(1/(-(3*x - 2)*(3*x + 2))^(1/2),x)
```

output
```
asin((3*x)/2)/3
```

3.842 $\int \frac{1}{\sqrt{15-2x-x^2}}\,dx$

3.842.1 Optimal result

Integrand size = 14, antiderivative size = 12

$$\int \frac{1}{\sqrt{15-2x-x^2}}\,dx=-\arcsin\left(\frac{1}{4}(-1-x)\right)$$

output
```
arcsin(1/4+1/4*x)
```

3.842.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.92

$$\int \frac{1}{\sqrt{15-2x-x^2}}\,dx=-2\arctan\left(\frac{\sqrt{15-2x-x^2}}{5+x}\right)$$

input
```
Integrate[1/Sqrt[15 - 2*x - x^2],x]
```

output
```
-2*ArcTan[Sqrt[15 - 2*x - x^2]/(5 + x)]
```

3.842.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-x^2-2x+15}}dx$$

$$\Big\downarrow \text{1090}$$

$$-\frac{1}{8}\int \frac{1}{\sqrt{1-\frac{1}{64}(-2x-2)^2}}d(-2x-2)$$

$$\Big\downarrow \text{223}$$

$$-\arcsin\left(\frac{1}{8}(-2x-2)\right)$$

input
```
Int[1/Sqrt[15 - 2*x - x^2],x]
```

output
```
-ArcSin[(-2 - 2*x)/8]
```

3.842.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.842.4 Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

method	result	size
default	$\arcsin\left(\frac{1}{4}+\frac{x}{4}\right)$	7
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2-2x+15}-\operatorname{RootOf}\left(\_Z^2+1\right)\right)$	39

input
```
int(1/(-x^2-2*x+15)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(1/4+1/4*x)
```

3.842.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(6) = 12.

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.42

$$\int \frac{1}{\sqrt{15-2x-x^2}}\,dx = -\arctan\left(\frac{\sqrt{-x^2-2\,x+15}(x+1)}{x^2+2\,x-15}\right)$$

input
```
integrate(1/(-x^2-2*x+15)^(1/2),x, algorithm="fricas")
```

output
```
-arctan(sqrt(-x^2 - 2*x + 15)*(x + 1)/(x^2 + 2*x - 15))
```

3.842.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sqrt{15-2x-x^2}}\,dx = \operatorname{asin}\left(\frac{x}{4}+\frac{1}{4}\right)$$

input
```
integrate(1/(-x**2-2*x+15)**(1/2),x)
```

output
```
asin(x/4 + 1/4)
```

3.842.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{15-2x-x^2}}\,dx = -\arcsin\left(-\frac{1}{4}\,x - \frac{1}{4}\right)$$

input
```
integrate(1/(-x^2-2*x+15)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(-1/4*x - 1/4)
```

3.842.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(6) = 12.

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{1}{\sqrt{15-2x-x^2}}\,dx = \frac{1}{2}\,\sqrt{-x^2-2\,x+15}(x+1) + 8\,\arcsin\left(\frac{1}{4}\,x + \frac{1}{4}\right)$$

input
```
integrate(1/(-x^2-2*x+15)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 - 2*x + 15)*(x + 1) + 8*arcsin(1/4*x + 1/4)
```

3.842.9 Mupad [B] (verification not implemented)

Time = 21.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{15-2x-x^2}}\,dx = \mathrm{asin}\left(\frac{x}{4} + \frac{1}{4}\right)$$

input
```
int(1/(15 - x^2 - 2*x)^(1/2),x)
```

output
```
asin(x/4 + 1/4)
```

3.843 $\int \frac{1}{\sqrt{3-x}\sqrt{5+x}}\,dx$

3.843.1 Optimal result

Integrand size = 17, antiderivative size = 12

$$\int \frac{1}{\sqrt{3-x}\sqrt{5+x}}\,dx = -\arcsin\left(\frac{1}{4}(-1-x)\right)$$

output
```
arcsin(1/4+1/4*x)
```

3.843.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 44 vs. 2(12) = 24.

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 3.67

$$\int \frac{1}{\sqrt{3-x}\sqrt{5+x}}\,dx = \frac{2\sqrt{-3+x}\sqrt{5+x}\,\text{arctanh}\left(\frac{\sqrt{5+x}}{\sqrt{-3+x}}\right)}{\sqrt{-((-3+x)(5+x))}}$$

input
```
Integrate[1/(Sqrt[3 - x]*Sqrt[5 + x]),x]
```

output
```
(2*Sqrt[-3 + x]*Sqrt[5 + x]*ArcTanh[Sqrt[5 + x]/Sqrt[-3 + x]])/Sqrt[-((-3
+ x)*(5 + x))]
```

3.843.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {62, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3-x}\sqrt{x+5}}dx$$
$$\Big\downarrow \; 62$$
$$\int \frac{1}{\sqrt{-x^2-2x+15}}dx$$
$$\Big\downarrow \; 1090$$
$$-\frac{1}{8}\int \frac{1}{\sqrt{1-\frac{1}{64}(-2x-2)^2}}d(-2x-2)$$
$$\Big\downarrow \; 223$$
$$-\arcsin\left(\frac{1}{8}(-2x-2)\right)$$

input
```
Int[1/(Sqrt[3 - x]*Sqrt[5 + x]),x]
```

output
```
-ArcSin[(-2 - 2*x)/8]
```

3.843.3.1 Defintions of rubi rules used

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.843.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(6) = 12$.

Time = 1.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.58

method	result	size
default	$\frac{\sqrt{(3-x)(5+x)}\,\arcsin\left(\frac{1}{4}+\frac{x}{4}\right)}{\sqrt{3-x}\,\sqrt{5+x}}$	31

input
```
int(1/(3-x)^(1/2)/(5+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((3-x)*(5+x))^(1/2)/(3-x)^(1/2)/(5+x)^(1/2)*arcsin(1/4+1/4*x)
```

3.843.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(6) = 12$.

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.42

$$\int \frac{1}{\sqrt{3-x}\sqrt{5+x}}\,dx = -\arctan\left(\frac{\sqrt{x+5}(x+1)\sqrt{-x+3}}{x^2+2\,x-15}\right)$$

input
```
integrate(1/(3-x)^(1/2)/(5+x)^(1/2),x, algorithm="fricas")
```

output
```
-arctan(sqrt(x + 5)*(x + 1)*sqrt(-x + 3)/(x^2 + 2*x - 15))
```

3.843.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 39, normalized size of antiderivative = 3.25

$$\int \frac{1}{\sqrt{3-x}\sqrt{5+x}}\,dx = \begin{cases} -2i\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{x+5}}{4}\right) & \text{for } |x+5|>8 \\ 2\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{x+5}}{4}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(1/(3-x)**(1/2)/(5+x)**(1/2),x)
```

output
```
Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 5)/4), Abs(x + 5) > 8), (2*asin(sqr
t(2)*sqrt(x + 5)/4), True))
```

3.843.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{3-x}\sqrt{5+x}}\,dx = -\arcsin\left(-\frac{1}{4}\,x-\frac{1}{4}\right)$$

input
```
integrate(1/(3-x)^(1/2)/(5+x)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(-1/4*x - 1/4)
```

3.843.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. 2(6) = 12.

Time = 0.33 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{3-x}\sqrt{5+x}}\,dx = 2\arcsin\left(\frac{1}{4}\,\sqrt{2}\sqrt{x+5}\right)$$

input
```
integrate(1/(3-x)^(1/2)/(5+x)^(1/2),x, algorithm="giac")
```

output
```
2*arcsin(1/4*sqrt(2)*sqrt(x + 5))
```

3.843.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.50

$$\int \frac{1}{\sqrt{3-x}\sqrt{5+x}}\,dx = 4\,\mathrm{atan}\left(\frac{\sqrt{3}-\sqrt{3-x}}{\sqrt{x+5}-\sqrt{5}}\right)$$

input
```
int(1/((3 - x)^(1/2)*(x + 5)^(1/2)),x)
```

output
```
4*atan((3^(1/2) - (3 - x)^(1/2))/((x + 5)^(1/2) - 5^(1/2)))
```

3.844 $\int \frac{1}{\sqrt{(3-x)(5+x)}}\,dx$

3.844.1 Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{1}{\sqrt{(3-x)(5+x)}}\,dx = -\arcsin\left(\frac{1}{4}(-1-x)\right)$$

output
```
arcsin(1/4+1/4*x)
```

3.844.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 44 vs. $2(12) = 24$.

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 3.67

$$\int \frac{1}{\sqrt{(3-x)(5+x)}}\,dx = \frac{2\sqrt{-3+x}\sqrt{5+x}\operatorname{arctanh}\left(\frac{\sqrt{5+x}}{\sqrt{-3+x}}\right)}{\sqrt{-((-3+x)(5+x))}}$$

input
```
Integrate[1/Sqrt[(3 - x)*(5 + x)],x]
```

output
```
(2*Sqrt[-3 + x]*Sqrt[5 + x]*ArcTanh[Sqrt[5 + x]/Sqrt[-3 + x]])/Sqrt[-((-3
+ x)*(5 + x))]
```

3.844.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2048, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{(3-x)(x+5)}}dx$$

$$\Big\downarrow \text{2048}$$

$$\int \frac{1}{\sqrt{-x^2-2x+15}}dx$$

$$\Big\downarrow \text{1090}$$

$$-\frac{1}{8}\int \frac{1}{\sqrt{1-\frac{1}{64}(-2x-2)^2}}d(-2x-2)$$

$$\Big\downarrow \text{223}$$

$$-\arcsin\left(\frac{1}{8}(-2x-2)\right)$$

input
```
Int[1/Sqrt[(3 - x)*(5 + x)],x]
```

output
```
-ArcSin[(-2 - 2*x)/8]
```

3.844.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

3.844.4 Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

method	result	size
default	$\arcsin\left(\frac{1}{4}+\frac{x}{4}\right)$	7
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2-2x+15}-\operatorname{RootOf}\left(\_Z^2+1\right)\right)$	39

input
```
int(1/((3-x)*(5+x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(1/4+1/4*x)
```

3.844.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(6) = 12.

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.42

$$\int \frac{1}{\sqrt{(3-x)(5+x)}}\,dx=-\arctan\left(\frac{\sqrt{-x^2-2\,x+15}(x+1)}{x^2+2\,x-15}\right)$$

input
```
integrate(1/((3-x)*(5+x))^(1/2),x, algorithm="fricas")
```

output
```
-arctan(sqrt(-x^2 - 2*x + 15)*(x + 1)/(x^2 + 2*x - 15))
```

3.844.6 Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sqrt{(3-x)(5+x)}}\,dx = \operatorname{asin}\left(\frac{x}{4}+\frac{1}{4}\right)$$

input
```
integrate(1/((3-x)*(5+x))**(1/2),x)
```

output
```
asin(x/4 + 1/4)
```

3.844.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{(3-x)(5+x)}}\,dx = -\arcsin\left(-\frac{1}{4}\,x-\frac{1}{4}\right)$$

input
```
integrate(1/((3-x)*(5+x))^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(-1/4*x - 1/4)
```

3.844.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(6) = 12.

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{1}{\sqrt{(3-x)(5+x)}}\,dx = \frac{1}{2}\sqrt{-x^2-2\,x+15}(x+1)+8\,\arcsin\left(\frac{1}{4}\,x+\frac{1}{4}\right)$$

input
```
integrate(1/((3-x)*(5+x))^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 - 2*x + 15)*(x + 1) + 8*arcsin(1/4*x + 1/4)
```

3.844.9 Mupad [B] (verification not implemented)

Time = 21.09 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{(3-x)(5+x)}}\,dx = \operatorname{asin}\left(\frac{x}{4}+\frac{1}{4}\right)$$

input
```
int(1/(-(x - 3)*(x + 5))^(1/2),x)
```

output
```
asin(x/4 + 1/4)
```

3.845 $\int \frac{1}{\sqrt{-15-8x-x^2}}\,dx$

3.845.1 Optimal result

Integrand size = 14, antiderivative size = 4

$$\int \frac{1}{\sqrt{-15-8x-x^2}}\,dx = \arcsin(4+x)$$

output
```
arcsin(4+x)
```

3.845.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(4) = 8.

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 5.75

$$\int \frac{1}{\sqrt{-15-8x-x^2}}\,dx = -2\arctan\left(\frac{\sqrt{-15-8x-x^2}}{5+x}\right)$$

input
```
Integrate[1/Sqrt[-15 - 8*x - x^2],x]
```

output
```
-2*ArcTan[Sqrt[-15 - 8*x - x^2]/(5 + x)]
```

3.845.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 12 vs. 2(4) = 8.

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-x^2-8x-15}}dx$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{1}{2}\int \frac{1}{\sqrt{1-\frac{1}{4}(-2x-8)^2}}d(-2x-8)$$

$$\Big\downarrow \text{ 223}$$

$$-\arcsin\left(\frac{1}{2}(-2x-8)\right)$$

input
```
Int[1/Sqrt[-15 - 8*x - x^2],x]
```

output
```
-ArcSin[(-8 - 2*x)/2]
```

3.845.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.845.4 Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
default	$\arcsin(x+4)$	5
trager	$\operatorname{RootOf}(\_Z^2+1)\ln\left(-\operatorname{RootOf}(\_Z^2+1)\,x+\sqrt{-x^2-8x-15}-4\operatorname{RootOf}(\_Z^2+1)\right)$	39

input
```
int(1/(-x^2-8*x-15)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(x+4)
```

3.845.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(4) = 8.

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 7.25

$$\int \frac{1}{\sqrt{-15-8x-x^2}}\,dx = -\arctan\left(\frac{\sqrt{-x^2-8\,x-15}(x+4)}{x^2+8\,x+15}\right)$$

input
```
integrate(1/(-x^2-8*x-15)^(1/2),x, algorithm="fricas")
```

output
```
-arctan(sqrt(-x^2 - 8*x - 15)*(x + 4)/(x^2 + 8*x + 15))
```

3.845.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{-15-8x-x^2}}\,dx = \operatorname{asin}(x+4)$$

input
```
integrate(1/(-x**2-8*x-15)**(1/2),x)
```

output
```
asin(x + 4)
```

3.845.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{-15-8x-x^2}}\,dx = -\arcsin\left(-x-4\right)$$

input
```
integrate(1/(-x^2-8*x-15)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(-x - 4)
```

3.845.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(4) = 8.

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 6.00

$$\int \frac{1}{\sqrt{-15-8x-x^2}}\,dx = \frac{1}{2}\sqrt{-x^2-8\,x-15}(x+4)+\frac{1}{2}\arcsin\left(x+4\right)$$

input
```
integrate(1/(-x^2-8*x-15)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 - 8*x - 15)*(x + 4) + 1/2*arcsin(x + 4)
```

3.845.9 Mupad [B] (verification not implemented)

Time = 21.15 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-15-8x-x^2}}\,dx = \mathrm{asin}(x+4)$$

input
```
int(1/(- 8*x - x^2 - 15)^(1/2),x)
```

output
```
asin(x + 4)
```

3.846 $\int \frac{1}{\sqrt{-3-x}\sqrt{5+x}} dx$

3.846.1 Optimal result

Integrand size = 17, antiderivative size = 4

$$\int \frac{1}{\sqrt{-3-x}\sqrt{5+x}} dx = \arcsin(4+x)$$

output
```
arcsin(4+x)
```

3.846.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 44 vs. 2(4) = 8.

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 11.00

$$\int \frac{1}{\sqrt{-3-x}\sqrt{5+x}} dx = \frac{2\sqrt{3+x}\sqrt{5+x}\text{arctanh}\left(\frac{\sqrt{5+x}}{\sqrt{3+x}}\right)}{\sqrt{-((3+x)(5+x))}}$$

input
```
Integrate[1/(Sqrt[-3 - x]*Sqrt[5 + x]),x]
```

output
```
(2*Sqrt[3 + x]*Sqrt[5 + x]*ArcTanh[Sqrt[5 + x]/Sqrt[3 + x]])/Sqrt[-((3 + x
)*(5 + x))]
```

3.846.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 12 vs. $2(4) = 8$.

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {62, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-x-3}\sqrt{x+5}}dx$$
$$\Big\downarrow \text{62}$$
$$\int \frac{1}{\sqrt{-x^2-8x-15}}dx$$
$$\Big\downarrow \text{1090}$$
$$-\frac{1}{2}\int \frac{1}{\sqrt{1-\frac{1}{4}(-2x-8)^2}}d(-2x-8)$$
$$\Big\downarrow \text{223}$$
$$-\arcsin\left(\frac{1}{2}(-2x-8)\right)$$

input
```
Int[1/(Sqrt[-3 - x]*Sqrt[5 + x]),x]
```

output
```
-ArcSin[(-8 - 2*x)/2]
```

3.846.3.1 Defintions of rubi rules used

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

3.846.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(4) = 8.

Time = 1.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 7.25

method	result	size
default	$\frac{\sqrt{(-3-x)(5+x)}\,\arcsin(x+4)}{\sqrt{-3-x}\,\sqrt{5+x}}$	29

input
```
int(1/(-3-x)^(1/2)/(5+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((-3-x)*(5+x))^(1/2)/(-3-x)^(1/2)/(5+x)^(1/2)*arcsin(x+4)
```

3.846.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(4) = 8.

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 7.25

$$\int \frac{1}{\sqrt{-3-x}\sqrt{5+x}}\,dx = -\arctan\left(\frac{\sqrt{x+5}(x+4)\sqrt{-x-3}}{x^2+8\,x+15}\right)$$

input
```
integrate(1/(-3-x)^(1/2)/(5+x)^(1/2),x, algorithm="fricas")
```

output
```
-arctan(sqrt(x + 5)*(x + 4)*sqrt(-x - 3)/(x^2 + 8*x + 15))
```

3.846.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 39, normalized size of antiderivative = 9.75

$$\int \frac{1}{\sqrt{-3-x}\sqrt{5+x}}\,dx = \begin{cases} -2i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+5}}{2}\right) & \text{for } |x+5|>2 \\ 2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+5}}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(1/(-3-x)**(1/2)/(5+x)**(1/2),x)
```

output
```
Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 5)/2), Abs(x + 5) > 2), (2*asin(sqr
t(2)*sqrt(x + 5)/2), True))
```

3.846.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{-3-x}\sqrt{5+x}}\,dx = -\arcsin\left(-x-4\right)$$

input
```
integrate(1/(-3-x)^(1/2)/(5+x)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(-x - 4)
```

3.846.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. 2(4) = 8.

Time = 0.43 (sec) , antiderivative size = 13, normalized size of antiderivative = 3.25

$$\int \frac{1}{\sqrt{-3-x}\sqrt{5+x}}\,dx = 2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+5}\right)$$

input
```
integrate(1/(-3-x)^(1/2)/(5+x)^(1/2),x, algorithm="giac")
```

output
```
2*arcsin(1/2*sqrt(2)*sqrt(x + 5))
```

3.846.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 8.25

$$\int \frac{1}{\sqrt{-3-x}\sqrt{5+x}}\,dx = 4\,\mathrm{atan}\left(\frac{-\sqrt{-x-3}+\sqrt{3}\,1\mathrm{i}}{\sqrt{x+5}-\sqrt{5}}\right)$$

input
```
int(1/((- x - 3)^(1/2)*(x + 5)^(1/2)),x)
```

output
```
4*atan((3^(1/2)*1i - (- x - 3)^(1/2))/((x + 5)^(1/2) - 5^(1/2)))
```

3.847 $\int \frac{1}{\sqrt{(-3-x)(5+x)}} dx$

3.847.1 Optimal result

Integrand size = 13, antiderivative size = 4

$$\int \frac{1}{\sqrt{(-3-x)(5+x)}} dx = \arcsin(4+x)$$

output
```
arcsin(4+x)
```

3.847.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 44 vs. 2(4) = 8.

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 11.00

$$\int \frac{1}{\sqrt{(-3-x)(5+x)}} dx = \frac{2\sqrt{3+x}\sqrt{5+x}\text{arctanh}\left(\frac{\sqrt{5+x}}{\sqrt{3+x}}\right)}{\sqrt{-((3+x)(5+x))}}$$

input
```
Integrate[1/Sqrt[(-3 - x)*(5 + x)],x]
```

output
```
(2*Sqrt[3 + x]*Sqrt[5 + x]*ArcTanh[Sqrt[5 + x]/Sqrt[3 + x]])/Sqrt[-((3 + x
)*(5 + x))]
```

3.847.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 12 vs. $2(4) = 8$.

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2048, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{(-x-3)(x+5)}}\,dx$$
$$\Big\downarrow \text{ 2048}$$
$$\int \frac{1}{\sqrt{-x^2-8x-15}}dx$$
$$\Big\downarrow \text{ 1090}$$
$$-\frac{1}{2}\int \frac{1}{\sqrt{1-\frac{1}{4}(-2x-8)^2}}d(-2x-8)$$
$$\Big\downarrow \text{ 223}$$
$$-\arcsin\left(\frac{1}{2}(-2x-8)\right)$$

input
```
Int[1/Sqrt[(-3 - x)*(5 + x)],x]
```

output
```
-ArcSin[(-8 - 2*x)/2]
```

3.847.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

3.847.4 Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
default	$\arcsin\left(x+4\right)$	5
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2-8x-15}-4\operatorname{RootOf}\left(\_Z^2+1\right)\right)$	39

input
```
int(1/((-3-x)*(5+x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(x+4)
```

3.847.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(4) = 8.

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 7.25

$$\int \frac{1}{\sqrt{(-3-x)(5+x)}}\,dx = -\arctan\left(\frac{\sqrt{-x^2-8\,x-15}(x+4)}{x^2+8\,x+15}\right)$$

input
```
integrate(1/((-3-x)*(5+x))^(1/2),x, algorithm="fricas")
```

output
```
-arctan(sqrt(-x^2 - 8*x - 15)*(x + 4)/(x^2 + 8*x + 15))
```

3.847.6 Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{(-3-x)(5+x)}}\,dx = \operatorname{asin}(x+4)$$

input
```
integrate(1/((-3-x)*(5+x))**(1/2),x)
```

output
```
asin(x + 4)
```

3.847.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 8, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{(-3-x)(5+x)}}\,dx = -\arcsin(-x-4)$$

input
```
integrate(1/((-3-x)*(5+x))^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(-x - 4)
```

3.847.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(4) = 8.

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 6.00

$$\int \frac{1}{\sqrt{(-3-x)(5+x)}}\,dx = \frac{1}{2}\sqrt{-x^2-8x-15}(x+4) + \frac{1}{2}\arcsin(x+4)$$

input
```
integrate(1/((-3-x)*(5+x))^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 - 8*x - 15)*(x + 4) + 1/2*arcsin(x + 4)
```

3.847.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{(-3-x)(5+x)}}\,dx = \text{asin}(x+4)$$

input
```
int(1/(-(x + 3)*(x + 5))^(1/2),x)
```

output
```
asin(x + 4)
```

3.848 $\int (1-\sqrt{x})\,dx$

3.848.1 Optimal result

Integrand size = 9, antiderivative size = 11

$$\int (1-\sqrt{x})\,dx = x-\frac{2x^{3/2}}{3}$$

output
```
x-2/3*x^(3/2)
```

3.848.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int (1-\sqrt{x})\,dx = x-\frac{2x^{3/2}}{3}$$

input
```
Integrate[1 - Sqrt[x],x]
```

output
```
x - (2*x^(3/2))/3
```

3.848.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-\sqrt{x}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$x-\frac{2x^{3/2}}{3}$$

input
```
Int[1 - Sqrt[x],x]
```

output
```
x - (2*x^(3/2))/3
```

3.848.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.848.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$x-\frac{2x^{\frac{3}{2}}}{3}$	8
default	$x-\frac{2x^{\frac{3}{2}}}{3}$	8
risch	$x-\frac{2x^{\frac{3}{2}}}{3}$	8
parts	$x-\frac{2x^{\frac{3}{2}}}{3}$	8
trager	$x-1-\frac{2x^{\frac{3}{2}}}{3}$	9

input
```
int(1-x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x-2/3*x^(3/2)
```

3.848.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \left(1-\sqrt{x}\right) dx = -\frac{2}{3} x^{\frac{3}{2}} + x$$

input
```
integrate(1-x^(1/2),x, algorithm="fricas")
```

output
```
-2/3*x^(3/2) + x
```

3.848.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \left(1-\sqrt{x}\right) dx = -\frac{2x^{\frac{3}{2}}}{3} + x$$

input
```
integrate(1-x**(1/2),x)
```

output
```
-2*x**(3/2)/3 + x
```

3.848.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \left(1-\sqrt{x}\right) dx = -\frac{2}{3} x^{\frac{3}{2}} + x$$

input
```
integrate(1-x^(1/2),x, algorithm="maxima")
```

output
```
-2/3*x^(3/2) + x
```

3.848.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \left(1-\sqrt{x}\right)\,dx=-\frac{2}{3}\,x^{\frac{3}{2}}+x$$

input
```
integrate(1-x^(1/2),x, algorithm="giac")
```

output
```
-2/3*x^(3/2) + x
```

3.848.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \left(1-\sqrt{x}\right)\,dx=x-\frac{2\,x^{3/2}}{3}$$

input
```
int(1 - x^(1/2),x)
```

output
```
x - (2*x^(3/2))/3
```

3.849 $\int \frac{1-x}{1+\sqrt{x}} dx$

3.849.1 Optimal result

Integrand size = 15, antiderivative size = 11

$$\int \frac{1-x}{1+\sqrt{x}} dx = x - \frac{2x^{3/2}}{3}$$

output
```
x-2/3*x^(3/2)
```

3.849.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1-x}{1+\sqrt{x}} dx = x - \frac{2x^{3/2}}{3}$$

input
```
Integrate[(1 - x)/(1 + Sqrt[x]),x]
```

output
```
x - (2*x^(3/2))/3
```

3.849.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1386, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x}{\sqrt{x}+1} \, dx$$

$$\Big\downarrow \text{1386}$$

$$\int \left(1-\sqrt{x}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$x-\frac{2x^{3/2}}{3}$$

input
```
Int[(1 - x)/(1 + Sqrt[x]),x]
```

output
```
x - (2*x^(3/2))/3
```

3.849.3.1 Defintions of rubi rules used

rule 1386
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Simp[(-e^2/c)^q   Int[u*(d - e*x^n)^p, x], x] /; FreeQ[{a, c,
d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && EqQ[p + q, 0
] && GtQ[d, 0] && LtQ[c, 0] && GtQ[e^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.849.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$x-\frac{2x^{\frac{3}{2}}}{3}$	8
default	$x-\frac{2x^{\frac{3}{2}}}{3}$	8
trager	$x-1-\frac{2x^{\frac{3}{2}}}{3}$	9
meijerg	$2\sqrt{x}-\frac{\sqrt{x}\,(4x-6\sqrt{x}+12)}{6}$	22

input
```
int((1-x)/(1+x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x-2/3*x^(3/2)
```

3.849.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1-x}{1+\sqrt{x}}\,dx=-\frac{2}{3}\,x^{\frac{3}{2}}+x$$

input
```
integrate((1-x)/(1+x^(1/2)),x, algorithm="fricas")
```

output
```
-2/3*x^(3/2) + x
```

3.849.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1-x}{1+\sqrt{x}}\,dx=-\frac{2x^{\frac{3}{2}}}{3}+x$$

input
```
integrate((1-x)/(1+x**(1/2)),x)
```

output
```
-2*x**(3/2)/3 + x
```

3.849.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1-x}{1+\sqrt{x}}\,dx = -\frac{2}{3}\,x^{\frac{3}{2}} + x$$

input
```
integrate((1-x)/(1+x^(1/2)),x, algorithm="maxima")
```

output
```
-2/3*x^(3/2) + x
```

3.849.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1-x}{1+\sqrt{x}}\,dx = -\frac{2}{3}\,x^{\frac{3}{2}} + x$$

input
```
integrate((1-x)/(1+x^(1/2)),x, algorithm="giac")
```

output
```
-2/3*x^(3/2) + x
```

3.849.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1-x}{1+\sqrt{x}}\,dx = x - \frac{2\,x^{3/2}}{3}$$

input
```
int(-(x - 1)/(x^(1/2) + 1),x)
```

output
```
x - (2*x^(3/2))/3
```

3.850 $\int \sqrt{\frac{1}{1-x^2}}\,dx$

3.850.1 Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \sqrt{\frac{1}{1-x^2}}\,dx = \sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\arcsin(x)$$

output
```
arcsin(x)*(1/(-x^2+1))^(1/2)*(-x^2+1)^(1/2)
```

3.850.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int \sqrt{\frac{1}{1-x^2}}\,dx = -2\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[Sqrt[(1 - x^2)^(-1)],x]
```

output
```
-2*Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

3.850.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2044, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{1}{1-x^2}}dx$$

$$\Big\downarrow \ 2044$$

$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\int \frac{1}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \ 223$$

$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\arcsin(x)$$

input
```
Int[Sqrt[(1 - x^2)^(-1)],x]
```

output
```
Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]
```

3.850.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2044
```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)]   Int[u*(a + b*x^n)^(p*q), x], x
] /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

3.850.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	si
meijerg	$\arcsin\left(x\right)\sqrt{\frac{1}{-x^{2}+1}}\,\sqrt{-x^{2}+1}$	2
default	$\sqrt{-\frac{1}{x^{2}-1}}\,\sqrt{x^{2}-1}\,\ln\left(x+\sqrt{x^{2}-1}\right)$	3
trager	$\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-\frac{1}{x^{2}-1}}\,x^{2}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-\frac{1}{x^{2}-1}}+x\right)$	5

input
```
int((1/(-x^2+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(x)*(1/(-x^2+1))^(1/2)*(-x^2+1)^(1/2)
```

3.850.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \sqrt{\frac{1}{1-x^2}}\,dx = 2\,\arctan\left(\frac{(x^2-1)\sqrt{-\frac{1}{x^2-1}}+1}{x}\right)$$

input
```
integrate((1/(-x^2+1))^(1/2),x, algorithm="fricas")
```

output
```
2*arctan(((x^2 - 1)*sqrt(-1/(x^2 - 1)) + 1)/x)
```

3.850.6 Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.26

$$\int \sqrt{\frac{1}{1-x^2}}\,dx = \begin{cases} \operatorname{asin}(x) & \text{for}\ x > -1 \wedge x < 1 \end{cases}$$

input
```
integrate((1/(-x**2+1))**(1/2),x)
```

output
```
Piecewise((asin(x), (x > -1) & (x < 1)))
```

3.850.7 Maxima [F]

$$\int \sqrt{\frac{1}{1-x^2}}\,dx = \int \sqrt{-\frac{1}{x^2-1}}\,dx$$

input
```
integrate((1/(-x^2+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-1/(x^2 - 1)), x)
```

3.850.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.37

$$\int \sqrt{\frac{1}{1-x^2}}\,dx = -\arcsin{(x)}\operatorname{sgn}\left(x^2-1\right)$$

input
```
integrate((1/(-x^2+1))^(1/2),x, algorithm="giac")
```

output
```
-arcsin(x)*sgn(x^2 - 1)
```

3.850.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{1}{1-x^2}}\,dx = \int \sqrt{-\frac{1}{x^2-1}}\,dx$$

input
```
int((-1/(x^2 - 1))^(1/2),x)
```

output
```
int((-1/(x^2 - 1))^(1/2), x)
```

3.851 $\int \sqrt{\frac{1+x^2}{1-x^4}}\,dx$

3.851.1 Optimal result

Integrand size = 19, antiderivative size = 27

$$\int \sqrt{\frac{1+x^2}{1-x^4}}\,dx = \sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\arcsin(x)$$

output
```
arcsin(x)*(1/(-x^2+1))^(1/2)*(-x^2+1)^(1/2)
```

3.851.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int \sqrt{\frac{1+x^2}{1-x^4}}\,dx = -2\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[Sqrt[(1 + x^2)/(1 - x^4)],x]
```

output
```
-2*Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

3.851.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {13, 2044, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{x^2+1}{1-x^4}}dx$$
$$\Big\downarrow \text{13}$$
$$\int \sqrt{\frac{1}{1-x^2}}dx$$
$$\Big\downarrow \text{2044}$$
$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\int \frac{1}{\sqrt{1-x^2}}dx$$
$$\Big\downarrow \text{223}$$
$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\arcsin(x)$$

input
```
Int[Sqrt[(1 + x^2)/(1 - x^4)],x]
```

output
```
Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]
```

3.851.3.1 Defintions of rubi rules used

rule 13
```
Int[(u_.)*((v_.)*((a_) + (b_.)*(x_)^(n_.))^(mm_.)*((c_) + (d_.)*(x_)^(n2_.)
)^(m_.))^(p_), x_Symbol] :> Int[u*(v*(c^m/a^(2*m))*(a - b*x^n)^m)^p, x] /;
FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2*c + a^2*d, 0] && In
tegersQ[m, mm] && EqQ[m + mm, 0]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2044
```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)]   Int[u*(a + b*x^n)^(p*q), x], x
] /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

3.851.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

method	result	si
default	$\sqrt{-\frac{1}{x^2-1}}\,\sqrt{x^2-1}\,\ln\left(x+\sqrt{x^2-1}\right)$	3
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{1}{x^2-1}}\,x^2+\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{1}{x^2-1}}+x\right)$	5

input
```
int(((x^2+1)/(-x^4+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

3.851.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int\sqrt{\frac{1+x^2}{1-x^4}}\,dx=2\arctan\left(\frac{(x^2-1)\sqrt{-\frac{1}{x^2-1}}+1}{x}\right)$$

input
```
integrate(((x^2+1)/(-x^4+1))^(1/2),x, algorithm="fricas")
```

output
```
2*arctan(((x^2 - 1)*sqrt(-1/(x^2 - 1)) + 1)/x)
```

3.851.6 Sympy [F]

$$\int \sqrt{\frac{1+x^2}{1-x^4}}\,dx = \int \sqrt{\frac{x^2+1}{1-x^4}}\,dx$$

input
```
integrate(((x**2+1)/(-x**4+1))**(1/2),x)
```

output
```
Integral(sqrt((x**2 + 1)/(1 - x**4)), x)
```

3.851.7 Maxima [F]

$$\int \sqrt{\frac{1+x^2}{1-x^4}}\,dx = \int \sqrt{-\frac{x^2+1}{x^4-1}}\,dx$$

input
```
integrate(((x^2+1)/(-x^4+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-(x^2 + 1)/(x^4 - 1)), x)
```

3.851.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.37

$$\int \sqrt{\frac{1+x^2}{1-x^4}}\,dx = -\arcsin(x)\,\mathrm{sgn}\left(x^2-1\right)$$

input
```
integrate(((x^2+1)/(-x^4+1))^(1/2),x, algorithm="giac")
```

output
```
-arcsin(x)*sgn(x^2 - 1)
```

3.851.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{1+x^2}{1-x^4}}\,dx=\int \sqrt{-\frac{x^2+1}{x^4-1}}\,dx$$

input
```
int((-(x^2 + 1)/(x^4 - 1))^(1/2),x)
```

output
```
int((-(x^2 + 1)/(x^4 - 1))^(1/2), x)
```

3.852 $\int \sqrt{\frac{1}{-1+x^2}}\,dx$

3.852.1 Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \sqrt{\frac{1}{-1+x^2}}\,dx=\sqrt{1-x^2}\sqrt{\frac{1}{-1+x^2}}\arcsin(x)$$

output
```
arcsin(x)*(-x^2+1)^(1/2)*(1/(x^2-1))^(1/2)
```

3.852.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 56 vs. $2(25) = 50$.

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.24

$$\int \sqrt{\frac{1}{-1+x^2}}\,dx=\frac{1}{2}\sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\left(-\log\left(1-\frac{x}{\sqrt{-1+x^2}}\right)+\log\left(1+\frac{x}{\sqrt{-1+x^2}}\right)\right)$$

input
```
Integrate[Sqrt[(-1 + x^2)^(-1)],x]
```

output
```
(Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*(-Log[1 - x/Sqrt[-1 + x^2]] + Log[1
+ x/Sqrt[-1 + x^2]]))/2
```

3.852.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2045, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{1}{x^2-1}}dx$$
$$\Big\downarrow \text{2045}$$
$$\sqrt{1-x^2}\sqrt{\frac{1}{x^2-1}}\int \frac{1}{\sqrt{1-x^2}}dx$$
$$\Big\downarrow \text{223}$$
$$\sqrt{1-x^2}\sqrt{\frac{1}{x^2-1}}\arcsin(x)$$

input
```
Int[Sqrt[(-1 + x^2)^(-1)],x]
```

output
```
Sqrt[1 - x^2]*Sqrt[(-1 + x^2)^(-1)]*ArcSin[x]
```

3.852.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.852.4 Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

method	result	size
default	$\sqrt{\frac{1}{x^2-1}}\,\sqrt{x^2-1}\,\ln\left(x+\sqrt{x^2-1}\right)$	28
trager	$\ln\left(\sqrt{\frac{1}{x^2-1}}\,x^2-\sqrt{\frac{1}{x^2-1}}+x\right)$	28
meijerg	$\frac{\sqrt{\frac{1}{x^2-1}}\,\sqrt{x^2-1}\,\sqrt{-\operatorname{signum}(x^2-1)}\,\arcsin(x)}{\sqrt{\operatorname{signum}(x^2-1)}}$	38

input
```
int((1/(x^2-1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

3.852.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int\sqrt{\frac{1}{-1+x^2}}\,dx=-\log\left(-x+\sqrt{x^2-1}\right)$$

input
```
integrate((1/(x^2-1))^(1/2),x, algorithm="fricas")
```

output
```
-log(-x + sqrt(x^2 - 1))
```

3.852.6 Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.60

$$\int\sqrt{\frac{1}{-1+x^2}}\,dx=\begin{cases}\log\left(x+\sqrt{x^2-1}\right) & \text{for } x>-1\wedge x<1\end{cases}$$

input
```
integrate((1/(x**2-1))**(1/2),x)
```

output
```
Piecewise((log(x + sqrt(x**2 - 1)), (x > -1) & (x < 1)))
```

3.852.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \sqrt{\frac{1}{-1+x^2}}\,dx = \log\left(2\,x + 2\,\sqrt{x^2-1}\right)$$

input
```
integrate((1/(x^2-1))^(1/2),x, algorithm="maxima")
```

output
```
log(2*x + 2*sqrt(x^2 - 1))
```

3.852.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \sqrt{\frac{1}{-1+x^2}}\,dx = \frac{1}{2}\,\sqrt{x^2-1}x + \frac{1}{2}\,\log\left(\left|-x+\sqrt{x^2-1}\right|\right)$$

input
```
integrate((1/(x^2-1))^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x^2 - 1)*x + 1/2*log(abs(-x + sqrt(x^2 - 1)))
```

3.852.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{1}{-1+x^2}}\,dx = \int \sqrt{\frac{1}{x^2-1}}\,dx$$

input
```
int((1/(x^2 - 1))^(1/2),x)
```

output
```
int((1/(x^2 - 1))^(1/2), x)
```

3.853 $\int \sqrt{\frac{1+x^2}{-1+x^4}}\,dx$

3.853.1 Optimal result

Integrand size = 17, antiderivative size = 25

$$\int \sqrt{\frac{1+x^2}{-1+x^4}}\,dx=\sqrt{1-x^2}\sqrt{\frac{1}{-1+x^2}}\arcsin(x)$$

output
```
arcsin(x)*(-x^2+1)^(1/2)*(1/(x^2-1))^(1/2)
```

3.853.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 56 vs. 2(25) = 50.

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.24

$$\int \sqrt{\frac{1+x^2}{-1+x^4}}\,dx=\frac{1}{2}\sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\left(-\log\left(1-\frac{x}{\sqrt{-1+x^2}}\right)+\log\left(1+\frac{x}{\sqrt{-1+x^2}}\right)\right)$$

input
```
Integrate[Sqrt[(1 + x^2)/(-1 + x^4)],x]
```

output
```
(Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*(-Log[1 - x/Sqrt[-1 + x^2]] + Log[1
+ x/Sqrt[-1 + x^2]]))/2
```

3.853.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {13, 2044, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{x^2+1}{x^4-1}}dx$$
$$\Big\downarrow \text{13}$$
$$\int \sqrt{-\frac{1}{1-x^2}}dx$$
$$\Big\downarrow \text{2044}$$
$$\sqrt{-\frac{1}{1-x^2}}\sqrt{1-x^2}\int \frac{1}{\sqrt{1-x^2}}dx$$
$$\Big\downarrow \text{223}$$
$$\sqrt{-\frac{1}{1-x^2}}\sqrt{1-x^2}\arcsin(x)$$

input
```
Int[Sqrt[(1 + x^2)/(-1 + x^4)],x]
```

output
```
Sqrt[-(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]
```

3.853.3.1 Defintions of rubi rules used

rule 13
```
Int[(u_.)*((v_.)*((a_) + (b_.)*(x_)^(n_.))^(mm_.)*((c_) + (d_.)*(x_)^(n2_.)
)^(m_.))^(p_), x_Symbol] :> Int[u*(v*(c^m/a^(2*m))*(a - b*x^n)^m)^p, x] /;
FreeQ[{a, b, c, d, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2*c + a^2*d, 0] && In
tegersQ[m, mm] && EqQ[m + mm, 0]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2044
```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)]   Int[u*(a + b*x^n)^(p*q), x], x
] /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

3.853.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

method	result	size
default	$\sqrt{\frac{1}{x^2-1}}\,\sqrt{x^2-1}\,\ln\left(x+\sqrt{x^2-1}\right)$	28
trager	$\ln\left(\sqrt{\frac{1}{x^2-1}}\,x^2-\sqrt{\frac{1}{x^2-1}}+x\right)$	28

input
```
int(((x^2+1)/(x^4-1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

3.853.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \sqrt{\frac{1+x^2}{-1+x^4}}\,dx = -\log\left(-x+\sqrt{x^2-1}\right)$$

input
```
integrate(((x^2+1)/(x^4-1))^(1/2),x, algorithm="fricas")
```

output
```
-log(-x + sqrt(x^2 - 1))
```

3.853.6 Sympy [F]

$$\int \sqrt{\frac{1+x^2}{-1+x^4}}\,dx = \int \sqrt{\frac{x^2+1}{x^4-1}}\,dx$$

input
```
integrate(((x**2+1)/(x**4-1))**(1/2),x)
```

output
```
Integral(sqrt((x**2 + 1)/(x**4 - 1)), x)
```

3.853.7 Maxima [F]

$$\int \sqrt{\frac{1+x^2}{-1+x^4}}\,dx = \int \sqrt{\frac{x^2+1}{x^4-1}}\,dx$$

input
```
integrate(((x^2+1)/(x^4-1))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((x^2 + 1)/(x^4 - 1)), x)
```

3.853.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sqrt{\frac{1+x^2}{-1+x^4}}\,dx = -\log\left(\left|-x+\sqrt{x^2-1}\right|\right)\operatorname{sgn}\left(x^2-1\right)$$

input
```
integrate(((x^2+1)/(x^4-1))^(1/2),x, algorithm="giac")
```

output
```
-log(abs(-x + sqrt(x^2 - 1)))*sgn(x^2 - 1)
```

3.853.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{1+x^2}{-1+x^4}}\,dx = \int \sqrt{\frac{x^2+1}{x^4-1}}\,dx$$

input
```
int(((x^2 + 1)/(x^4 - 1))^(1/2),x)
```

output
```
int(((x^2 + 1)/(x^4 - 1))^(1/2), x)
```

3.854 $\int \frac{1}{\sqrt{1-x}} dx$

3.854.1 Optimal result

Integrand size = 9, antiderivative size = 11

$$\int \frac{1}{\sqrt{1-x}} dx = -2\sqrt{1-x}$$

output
```
-2*(1-x)^(1/2)
```

3.854.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x}} dx = -2\sqrt{1-x}$$

input
```
Integrate[1/Sqrt[1 - x],x]
```

output
```
-2*Sqrt[1 - x]
```

3.854.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-x}}\,dx$$
$$\big\downarrow 17$$
$$-2\sqrt{1-x}$$

input
```
Int[1/Sqrt[1 - x],x]
```

output
```
-2*Sqrt[1 - x]
```

3.854.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

3.854.4 Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
gosper	$-2\sqrt{1-x}$	10
derivativedivides	$-2\sqrt{1-x}$	10
default	$-2\sqrt{1-x}$	10
trager	$-2\sqrt{1-x}$	10
pseudoelliptic	$-2\sqrt{1-x}$	10
risch	$\frac{2x-2}{\sqrt{1-x}}$	13
meijerg	$-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{1-x}}{\sqrt{\pi}}$	24

input
```
int(1/(1-x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(1-x)^(1/2)
```

3.854.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{1-x}}\,dx = -2\,\sqrt{-x+1}$$

input
```
integrate(1/(1-x)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(-x + 1)
```

3.854.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{1-x}}\,dx = -2\sqrt{1-x}$$

input
```
integrate(1/(1-x)**(1/2),x)
```

output
```
-2*sqrt(1 - x)
```

3.854.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{1-x}}\,dx = -2\,\sqrt{-x+1}$$

input
```
integrate(1/(1-x)^(1/2),x, algorithm="maxima")
```

output
```
-2*sqrt(-x + 1)
```

3.854.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{1-x}}\,dx = -2\sqrt{-x+1}$$

input
```
integrate(1/(1-x)^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(-x + 1)
```

3.854.9 Mupad [B] (verification not implemented)

Time = 21.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{1-x}}\,dx = -2\sqrt{1-x}$$

input
```
int(1/(1 - x)^(1/2),x)
```

output
```
-2*(1 - x)^(1/2)
```

3.855 $\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}} \, dx$

3.855.1 Optimal result

Integrand size = 19, antiderivative size = 11

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}} \, dx = -2\sqrt{1-x}$$

output
```
-2*(1-x)^(1/2)
```

3.855.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.82

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}} \, dx = -\frac{2\sqrt{1-x^2}}{\sqrt{1+x}}$$

input
```
Integrate[Sqrt[1 + x]/Sqrt[1 - x^2],x]
```

output
```
(-2*Sqrt[1 - x^2])/Sqrt[1 + x]
```

3.855.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {456, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x+1}}{\sqrt{1-x^2}} \, dx$$
$$\Big\downarrow \text{ 456}$$
$$\int \frac{1}{\sqrt{1-x}}dx$$
$$\Big\downarrow \text{ 17}$$
$$-2\sqrt{1-x}$$

input
```
Int[Sqrt[1 + x]/Sqrt[1 - x^2],x]
```

output
```
-2*Sqrt[1 - x]
```

3.855.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

3.855.4 Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

method	result	size
default	$-\frac{2\sqrt{-x^2+1}}{\sqrt{x+1}}$	17
gosper	$\frac{2(x-1)\sqrt{x+1}}{\sqrt{-x^2+1}}$	20
risch	$\frac{2\sqrt{\frac{-x^2+1}{x+1}}\,\sqrt{x+1}\,(x-1)}{\sqrt{-x^2+1}\,\sqrt{1-x}}$	42

input
```
int((x+1)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-x^2+1)^(1/2)/(x+1)^(1/2)
```

3.855.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}}\,dx = -\frac{2\sqrt{-x^2+1}}{\sqrt{x+1}}$$

input
```
integrate((1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(-x^2 + 1)/sqrt(x + 1)
```

3.855.6 Sympy [F]

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{x+1}}{\sqrt{-(x-1)(x+1)}}\,dx$$

input
```
integrate((1+x)**(1/2)/(-x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(x + 1)/sqrt(-(x - 1)*(x + 1)), x)
```

3.855.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}}\,dx = \frac{2\,(x-1)}{\sqrt{-x+1}}$$

input
```
integrate((1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
2*(x - 1)/sqrt(-x + 1)
```

3.855.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}}\,dx = 2\,\sqrt{2} - 2\,\sqrt{-x+1}$$

input
```
integrate((1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(2) - 2*sqrt(-x + 1)
```

3.855.9 Mupad [B] (verification not implemented)

Time = 21.78 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}}\,dx = -\frac{2\,\sqrt{1-x^2}}{\sqrt{x+1}}$$

input
```
int((x + 1)^(1/2)/(1 - x^2)^(1/2),x)
```

output
```
-(2*(1 - x^2)^(1/2))/(x + 1)^(1/2)
```

3.856 $\int \frac{1}{\sqrt{1+x}}\,dx$

3.856.1 Optimal result

Integrand size = 7, antiderivative size = 9

$$\int \frac{1}{\sqrt{1+x}}\,dx = 2\sqrt{1+x}$$

output
```
2*(1+x)^(1/2)
```

3.856.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x}}\,dx = 2\sqrt{1+x}$$

input
```
Integrate[1/Sqrt[1 + x],x]
```

output
```
2*Sqrt[1 + x]
```

3.856.3 Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x+1}}\,dx$$
$$\Big\downarrow \; 17$$
$$2\sqrt{x+1}$$

input
```
Int[1/Sqrt[1 + x],x]
```

output
```
2*Sqrt[1 + x]
```

3.856.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

3.856.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
gosper	$2\sqrt{x+1}$	8
derivativedivides	$2\sqrt{x+1}$	8
default	$2\sqrt{x+1}$	8
trager	$2\sqrt{x+1}$	8
risch	$2\sqrt{x+1}$	8
meijerg	$\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{x+1}}{\sqrt{\pi}}$	21

input
```
int(1/(x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(x+1)^(1/2)
```

3.856.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{1+x}}\,dx = 2\,\sqrt{x+1}$$

input
```
integrate(1/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(x + 1)
```

3.856.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{1+x}}\,dx = 2\sqrt{x+1}$$

input
```
integrate(1/(1+x)**(1/2),x)
```

output
```
2*sqrt(x + 1)
```

3.856.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{1+x}}\,dx = 2\,\sqrt{x+1}$$

input
```
integrate(1/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(x + 1)
```

3.856.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{1+x}}\,dx = 2\sqrt{x+1}$$

input
```
integrate(1/(1+x)^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(x + 1)
```

3.856.9 Mupad [B] (verification not implemented)

Time = 21.41 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{1+x}}\,dx = 2\sqrt{x+1}$$

input
```
int(1/(x + 1)^(1/2),x)
```

output
```
2*(x + 1)^(1/2)
```

3.857 $\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}}\,dx$

3.857.1 Optimal result

Integrand size = 21, antiderivative size = 9

$$\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}}\,dx = 2\sqrt{1+x}$$

output
```
2*(1+x)^(1/2)
```

3.857.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 22 vs. 2(9) = 18.

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.44

$$\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}}\,dx = \frac{2\sqrt{1-x^2}}{\sqrt{1-x}}$$

input
```
Integrate[Sqrt[1 - x]/Sqrt[1 - x^2],x]
```

output
```
(2*Sqrt[1 - x^2])/Sqrt[1 - x]
```

3.857.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {456, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}} \, dx$$
$$\Big\downarrow \; 456$$
$$\int \frac{1}{\sqrt{x+1}}dx$$
$$\Big\downarrow \; 17$$
$$2\sqrt{x+1}$$

input
```
Int[Sqrt[1 - x]/Sqrt[1 - x^2],x]
```

output
```
2*Sqrt[1 + x]
```

3.857.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

3.857.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(7) = 14.

Time = 1.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.44

method	result	size
gosper	$\frac{2(x+1)\sqrt{1-x}}{\sqrt{-x^2+1}}$	22
default	$-\frac{2\sqrt{1-x}\,\sqrt{-x^2+1}}{x-1}$	24
risch	$-\frac{2\sqrt{\frac{(1-x)\left(-x^2+1\right)}{(x-1)^2}}\,(x-1)\sqrt{x+1}}{\sqrt{1-x}\,\sqrt{-x^2+1}}$	47

input
```
int((1-x)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(x+1)*(1-x)^(1/2)/(-x^2+1)^(1/2)
```

3.857.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.56

$$\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}}\,dx = -\frac{2\sqrt{-x^2+1}\sqrt{-x+1}}{x-1}$$

input
```
integrate((1-x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(-x^2 + 1)*sqrt(-x + 1)/(x - 1)
```

3.857.6 Sympy [F]

$$\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{1-x}}{\sqrt{-(x-1)(x+1)}}\,dx$$

input
```
integrate((1-x)**(1/2)/(-x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(1 - x)/sqrt(-(x - 1)*(x + 1)), x)
```

3.857.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}}\,dx = 2\sqrt{x+1}$$

input
```
integrate((1-x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(x + 1)
```

3.857.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}}\,dx = -2\sqrt{2}+2\sqrt{x+1}$$

input
```
integrate((1-x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(2) + 2*sqrt(x + 1)
```

3.857.9 Mupad [B] (verification not implemented)

Time = 21.50 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}}\,dx = \frac{2\sqrt{1-x^2}}{\sqrt{1-x}}$$

input
```
int((1 - x)^(1/2)/(1 - x^2)^(1/2),x)
```

output
```
(2*(1 - x^2)^(1/2))/(1 - x)^(1/2)
```

3.858 $\int \sqrt{1-x}\,dx$

3.858.1 Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \sqrt{1-x}\,dx=-\frac{2}{3}(1-x)^{3/2}$$

output
```
-2/3*(1-x)^(3/2)
```

3.858.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{1-x}\,dx=-\frac{2}{3}(1-x)^{3/2}$$

input
```
Integrate[Sqrt[1 - x],x]
```

output
```
(-2*(1 - x)^(3/2))/3
```

3.858.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-x}\,dx$$

$$\Big\downarrow \; 17$$

$$-\frac{2}{3}(1-x)^{3/2}$$

input
```
Int[Sqrt[1 - x],x]
```

output
```
(-2*(1 - x)^(3/2))/3
```

3.858.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

3.858.4 Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{2(1-x)^{\frac{3}{2}}}{3}$	10
derivativedivides	$-\frac{2(1-x)^{\frac{3}{2}}}{3}$	10
default	$-\frac{2(1-x)^{\frac{3}{2}}}{3}$	10
pseudoelliptic	$-\frac{2(1-x)^{\frac{3}{2}}}{3}$	10
trager	$\left(\frac{2x}{3}-\frac{2}{3}\right)\sqrt{1-x}$	14
risch	$-\frac{2(x-1)^2}{3\sqrt{1-x}}$	15
meijerg	$\frac{\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\,(-2x+2)\sqrt{1-x}}{3}}{2\sqrt{\pi}}$	29

input
```
int((1-x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(1-x)^(3/2)
```

3.858.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \sqrt{1-x}\,dx = \frac{2}{3}\,(x-1)\sqrt{-x+1}$$

input
```
integrate((1-x)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(x - 1)*sqrt(-x + 1)
```

3.858.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \sqrt{1-x}\,dx = -\frac{2(1-x)^{\frac{3}{2}}}{3}$$

input
```
integrate((1-x)**(1/2),x)
```

output
```
-2*(1 - x)**(3/2)/3
```

3.858.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \sqrt{1-x}\,dx = -\frac{2}{3}\,(-x+1)^{\frac{3}{2}}$$

input
```
integrate((1-x)^(1/2),x, algorithm="maxima")
```

output
```
-2/3*(-x + 1)^(3/2)
```

3.858.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \sqrt{1-x}\,dx = -\frac{2}{3}\,(-x+1)^{\frac{3}{2}}$$

input
```
integrate((1-x)^(1/2),x, algorithm="giac")
```

output
```
-2/3*(-x + 1)^(3/2)
```

3.858.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \sqrt{1-x}\,dx = -\frac{2\,(1-x)^{3/2}}{3}$$

input
```
int((1 - x)^(1/2),x)
```

output
```
-(2*(1 - x)^(3/2))/3
```

3.859 $\int \frac{\sqrt{1-x^2}}{\sqrt{1+x}}\,dx$

3.859.1 Optimal result

Integrand size = 19, antiderivative size = 13

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x}}\,dx = -\frac{2}{3}(1-x)^{3/2}$$

output
```
-2/3*(1-x)^(3/2)
```

3.859.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x}}\,dx = -\frac{2(1-x^2)^{3/2}}{3(1+x)^{3/2}}$$

input
```
Integrate[Sqrt[1 - x^2]/Sqrt[1 + x],x]
```

output
```
(-2*(1 - x^2)^(3/2))/(3*(1 + x)^(3/2))
```

3.859.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {456, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{x+1}} \, dx$$

$$\Big\downarrow \text{456}$$

$$\int \sqrt{1-x}dx$$

$$\Big\downarrow \text{17}$$

$$-\frac{2}{3}(1-x)^{3/2}$$

input
```
Int[Sqrt[1 - x^2]/Sqrt[1 + x],x]
```

output
```
(-2*(1 - x)^(3/2))/3
```

3.859.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !Integ
erQ[n]))
```

3.859.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.

Time = 1.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

method	result	size
gosper	$\frac{2(x-1)\sqrt{-x^2+1}}{3\sqrt{x+1}}$	20
default	$\frac{2(x-1)\sqrt{-x^2+1}}{3\sqrt{x+1}}$	20
risch	$-\frac{2\sqrt{\frac{-x^2+1}{x+1}}\,\sqrt{x+1}\,(x-1)^2}{3\sqrt{-x^2+1}\,\sqrt{1-x}}$	44

input
```
int((-x^2+1)^(1/2)/(x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(x-1)*(-x^2+1)^(1/2)/(x+1)^(1/2)
```

3.859.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x}}\,dx = \frac{2\sqrt{-x^2+1}(x-1)}{3\sqrt{x+1}}$$

input
```
integrate((-x^2+1)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(-x^2 + 1)*(x - 1)/sqrt(x + 1)
```

3.859.6 Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x}}\,dx = \int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{x+1}}\,dx$$

input
```
integrate((-x**2+1)**(1/2)/(1+x)**(1/2),x)
```

output
```
Integral(sqrt(-(x - 1)*(x + 1))/sqrt(x + 1), x)
```

3.859.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x}}\,dx = \frac{2}{3}\,(x-1)\sqrt{-x+1}$$

input
```
integrate((-x^2+1)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(x - 1)*sqrt(-x + 1)
```

3.859.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x}}\,dx = -\frac{2}{3}\,(-x+1)^{\frac{3}{2}} + \frac{4}{3}\,\sqrt{2}$$

input
```
integrate((-x^2+1)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
-2/3*(-x + 1)^(3/2) + 4/3*sqrt(2)
```

3.859.9 Mupad [B] (verification not implemented)

Time = 21.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x}}\,dx = \frac{\left(\frac{2x}{3}-\frac{2}{3}\right)\sqrt{1-x^2}}{\sqrt{x+1}}$$

input
```
int((1 - x^2)^(1/2)/(x + 1)^(1/2),x)
```

output
```
(((2*x)/3 - 2/3)*(1 - x^2)^(1/2))/(x + 1)^(1/2)
```

3.860 $\int \sqrt{1+x}\,dx$

3.860.1 Optimal result

Integrand size = 7, antiderivative size = 11

$$\int \sqrt{1+x}\,dx = \frac{2}{3}(1+x)^{3/2}$$

output
```
2/3*(1+x)^(3/2)
```

3.860.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sqrt{1+x}\,dx = \frac{2}{3}(1+x)^{3/2}$$

input
```
Integrate[Sqrt[1 + x],x]
```

output
```
(2*(1 + x)^(3/2))/3
```

3.860.3 Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x+1}\,dx$$

$$\Big\downarrow \text{17}$$

$$\frac{2}{3}(x+1)^{3/2}$$

input
```
Int[Sqrt[1 + x],x]
```

output
```
(2*(1 + x)^(3/2))/3
```

3.860.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

3.860.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
gosper	$\frac{2(x+1)^{\frac{3}{2}}}{3}$	8
derivativedivides	$\frac{2(x+1)^{\frac{3}{2}}}{3}$	8
default	$\frac{2(x+1)^{\frac{3}{2}}}{3}$	8
risch	$\frac{2(x+1)^{\frac{3}{2}}}{3}$	8
trager	$\left(\frac{2}{3}+\frac{2x}{3}\right)\sqrt{x+1}$	12
meijerg	$-\frac{\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\,(2x+2)\sqrt{x+1}}{3}}{2\sqrt{\pi}}$	27

input
```
int((x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(x+1)^(3/2)
```

3.860.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \sqrt{1+x}\,dx = \frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

input
```
integrate((1+x)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(x + 1)^(3/2)
```

3.860.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \sqrt{1+x}\,dx = \frac{2(x+1)^{\frac{3}{2}}}{3}$$

input
```
integrate((1+x)**(1/2),x)
```

output
```
2*(x + 1)**(3/2)/3
```

3.860.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \sqrt{1+x}\,dx = \frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

input
```
integrate((1+x)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(x + 1)^(3/2)
```

3.860.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \sqrt{1+x}\,dx = \frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

input
```
integrate((1+x)^(1/2),x, algorithm="giac")
```

output
```
2/3*(x + 1)^(3/2)
```

3.860.9 Mupad [B] (verification not implemented)

Time = 21.31 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \sqrt{1+x}\,dx = \frac{2\,(x+1)^{3/2}}{3}$$

input
```
int((x + 1)^(1/2),x)
```

output
```
(2*(x + 1)^(3/2))/3
```

3.861 $\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}} \, dx$

3.861.1 Optimal result

Integrand size = 21, antiderivative size = 11

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}} \, dx = \frac{2}{3}(1+x)^{3/2}$$

output
```
2/3*(1+x)^(3/2)
```

3.861.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 24 vs. 2(11) = 22.

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.18

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}} \, dx = \frac{2(1-x^2)^{3/2}}{3(1-x)^{3/2}}$$

input
```
Integrate[Sqrt[1 - x^2]/Sqrt[1 - x],x]
```

output
```
(2*(1 - x^2)^(3/2))/(3*(1 - x)^(3/2))
```

3.861.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {456, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}}\,dx$$

$$\Big\downarrow \ 456$$

$$\int \sqrt{x+1}dx$$

$$\Big\downarrow \ 17$$

$$\frac{2}{3}(x+1)^{3/2}$$

input
```
Int[Sqrt[1 - x^2]/Sqrt[1 - x],x]
```

output
```
(2*(1 + x)^(3/2))/3
```

3.861.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] && !Integ
erQ[n]))
```

3.861.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(7) = 14$.

Time = 1.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

method	result	size
gosper	$\frac{2(x+1)\sqrt{-x^2+1}}{3\sqrt{1-x}}$	22
default	$-\frac{2\sqrt{-x^2+1}\,\sqrt{1-x}\,(x+1)}{3(x-1)}$	27
risch	$-\frac{2\sqrt{\frac{(1-x)\left(-x^2+1\right)}{(x-1)^2}}\,(x-1)(x+1)^{\frac{3}{2}}}{3\sqrt{1-x}\,\sqrt{-x^2+1}}$	47

input
```
int((-x^2+1)^(1/2)/(1-x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(x+1)*(-x^2+1)^(1/2)/(1-x)^(1/2)
```

3.861.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(7) = 14$.

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.36

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}}\,dx = -\frac{2\sqrt{-x^2+1}(x+1)\sqrt{-x+1}}{3(x-1)}$$

input
```
integrate((-x^2+1)^(1/2)/(1-x)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(-x^2 + 1)*(x + 1)*sqrt(-x + 1)/(x - 1)
```

3.861.6 Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}}\,dx = \int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{1-x}}\,dx$$

input
```
integrate((-x**2+1)**(1/2)/(1-x)**(1/2),x)
```

output
```
Integral(sqrt(-(x - 1)*(x + 1))/sqrt(1 - x), x)
```

3.861.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}}\,dx = \frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

input
```
integrate((-x^2+1)^(1/2)/(1-x)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(x + 1)^(3/2)
```

3.861.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}}\,dx = \frac{2}{3}\,(x+1)^{\frac{3}{2}} - \frac{4}{3}\,\sqrt{2}$$

input
```
integrate((-x^2+1)^(1/2)/(1-x)^(1/2),x, algorithm="giac")
```

output
```
2/3*(x + 1)^(3/2) - 4/3*sqrt(2)
```

3.861.9 Mupad [B] (verification not implemented)

Time = 21.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}}\,dx = \frac{\left(\frac{2\,x}{3}+\frac{2}{3}\right)\,\sqrt{1-x^2}}{\sqrt{1-x}}$$

input
```
int((1 - x^2)^(1/2)/(1 - x)^(1/2),x)
```

output
```
(((2*x)/3 + 2/3)*(1 - x^2)^(1/2))/(1 - x)^(1/2)
```

3.862 $\int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx$

3.862.1 Optimal result

Integrand size = 17, antiderivative size = 35

$$\int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx=\sqrt{1+x}\sqrt{2+3x}-\frac{\operatorname{arcsinh}\left(\sqrt{2+3x}\right)}{\sqrt{3}}$$

output
```
-1/3*arcsinh((2+3*x)^(1/2))*3^(1/2)+(1+x)^(1/2)*(2+3*x)^(1/2)
```

3.862.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx=\sqrt{1+x}\sqrt{2+3x}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2+3x}}{\sqrt{3}\sqrt{1+x}}\right)}{\sqrt{3}}$$

input
```
Integrate[Sqrt[2 + 3*x]/Sqrt[1 + x],x]
```

output
```
Sqrt[1 + x]*Sqrt[2 + 3*x] - ArcTanh[Sqrt[2 + 3*x]/(Sqrt[3]*Sqrt[1 + x])]/S
qrt[3]
```

3.862.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {60, 64, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3x+2}}{\sqrt{x+1}}dx$$

$$\Big\downarrow \ 60$$

$$\sqrt{x+1}\sqrt{3x+2}-\frac{1}{2}\int \frac{1}{\sqrt{x+1}\sqrt{3x+2}}dx$$

$$\Big\downarrow \ 64$$

$$\sqrt{x+1}\sqrt{3x+2}-\frac{1}{3}\int \frac{1}{\sqrt{\frac{1}{3}(3x+2)+\frac{1}{3}}}d\sqrt{3x+2}$$

$$\Big\downarrow \ 222$$

$$\sqrt{x+1}\sqrt{3x+2}-\frac{\text{arcsinh}\left(\sqrt{3x+2}\right)}{\sqrt{3}}$$

input
```
Int[Sqrt[2 + 3*x]/Sqrt[1 + x],x]
```

output
```
Sqrt[1 + x]*Sqrt[2 + 3*x] - ArcSinh[Sqrt[2 + 3*x]]/Sqrt[3]
```

3.862.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 64
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[2/b   Subst[Int[1/Sqrt[c - a*(d/b) + d*(x^2/b)], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[c - a*(d/b), 0] && ( !GtQ[a - c*(b/d), 0]
 || PosQ[b])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

3.862.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(27) = 54$.

Time = 1.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.91

method	result	size
default	$\sqrt{x+1}\,\sqrt{3x+2}-\frac{\sqrt{(x+1)(3x+2)}\,\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^{2}+5x+2}\right)\sqrt{3}}{6\sqrt{3x+2}\,\sqrt{x+1}}$	67
risch	$\sqrt{x+1}\,\sqrt{3x+2}-\frac{\sqrt{(x+1)(3x+2)}\,\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^{2}+5x+2}\right)\sqrt{3}}{6\sqrt{3x+2}\,\sqrt{x+1}}$	67

input
```
int((3*x+2)^(1/2)/(x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x+1)^(1/2)*(3*x+2)^(1/2)-1/6*((x+1)*(3*x+2))^(1/2)/(3*x+2)^(1/2)/(x+1)^(1
/2)*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x+2)^(1/2))*3^(1/2)
```

3.862.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx = \frac{1}{12}\,\sqrt{3}\log\left(-4\,\sqrt{3}(6\,x+5)\sqrt{3\,x+2}\sqrt{x+1}+72\,x^{2}+120\,x+49\right) + \sqrt{3\,x+2}\sqrt{x+1}$$

input
```
integrate((2+3*x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*log(-4*sqrt(3)*(6*x + 5)*sqrt(3*x + 2)*sqrt(x + 1) + 72*x^2 +
 120*x + 49) + sqrt(3*x + 2)*sqrt(x + 1)
```

3.862.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.86

$$\int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx = \begin{cases} \sqrt{x+1}\sqrt{3x+2} - \frac{\sqrt{3}\operatorname{acosh}\left(\sqrt{3}\sqrt{x+1}\right)}{3} & \text{for } |x+1| > \frac{1}{3} \\ \frac{\sqrt{3}i\operatorname{asin}\left(\sqrt{3}\sqrt{x+1}\right)}{3} - \frac{3i(x+1)^{\frac{3}{2}}}{\sqrt{-3x-2}} + \frac{i\sqrt{x+1}}{\sqrt{-3x-2}} & \text{otherwise} \end{cases}$$

input
```
integrate((2+3*x)**(1/2)/(1+x)**(1/2),x)
```

output
```
Piecewise((sqrt(x + 1)*sqrt(3*x + 2) - sqrt(3)*acosh(sqrt(3)*sqrt(x + 1))/
3, Abs(x + 1) > 1/3), (sqrt(3)*I*asin(sqrt(3)*sqrt(x + 1))/3 - 3*I*(x + 1)
**(3/2)/sqrt(-3*x - 2) + I*sqrt(x + 1)/sqrt(-3*x - 2), True))
```

3.862.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx = -\frac{1}{6}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+5x+2}+6x+5\right)+\sqrt{3x^2+5x+2}$$

input
```
integrate((2+3*x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x + 2) + 6*x + 5) + sqrt(3*x^2 +
 5*x + 2)
```

3.862.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx = \frac{1}{3}\sqrt{3}\left(\sqrt{3x+3}\sqrt{3x+2}+\log\left(\sqrt{3x+3}-\sqrt{3x+2}\right)\right)$$

input
```
integrate((2+3*x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*(sqrt(3*x + 3)*sqrt(3*x + 2) + log(sqrt(3*x + 3) - sqrt(3*x +
2)))
```

3.862.9 Mupad [B] (verification not implemented)

Time = 24.15 (sec) , antiderivative size = 172, normalized size of antiderivative = 4.91

$$\int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx = \frac{2\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\left(\sqrt{2}-\sqrt{3\,x+2}\right)}{3\left(\sqrt{x+1}-1\right)}\right)}{3} - \frac{\frac{30\left(\sqrt{2}-\sqrt{3\,x+2}\right)}{\sqrt{x+1}-1}+\frac{10\left(\sqrt{2}-\sqrt{3\,x+2}\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{24\sqrt{2}\left(\sqrt{2}-\sqrt{3\,x+2}\right)^2}{\left(\sqrt{x+1}-1\right)^2}}{\frac{\left(\sqrt{2}-\sqrt{3\,x+2}\right)^4}{\left(\sqrt{x+1}-1\right)^4}-\frac{6\left(\sqrt{2}-\sqrt{3\,x+2}\right)^2}{\left(\sqrt{x+1}-1\right)^2}+9}$$

input
```
int((3*x + 2)^(1/2)/(x + 1)^(1/2),x)
```

output
```
(2*3^(1/2)*atanh((3^(1/2)*(2^(1/2) - (3*x + 2)^(1/2)))/(3*((x + 1)^(1/2) -
 1))))/3 - ((30*(2^(1/2) - (3*x + 2)^(1/2)))/((x + 1)^(1/2) - 1) + (10*(2^
(1/2) - (3*x + 2)^(1/2))^3)/((x + 1)^(1/2) - 1)^3 + (24*2^(1/2)*(2^(1/2) -
 (3*x + 2)^(1/2))^2)/((x + 1)^(1/2) - 1)^2)/((2^(1/2) - (3*x + 2)^(1/2))^4
/((x + 1)^(1/2) - 1)^4 - (6*(2^(1/2) - (3*x + 2)^(1/2))^2)/((x + 1)^(1/2)
- 1)^2 + 9)
```

3.863 $\int \frac{\sqrt{1-x}\sqrt{2+3x}}{\sqrt{1-x^2}}\,dx$

3.863.1 Optimal result

Integrand size = 30, antiderivative size = 35

$$\int \frac{\sqrt{1-x}\sqrt{2+3x}}{\sqrt{1-x^2}}\,dx = \sqrt{1+x}\sqrt{2+3x}-\frac{\operatorname{arcsinh}\left(\sqrt{2+3x}\right)}{\sqrt{3}}$$

output
```
-1/3*arcsinh((2+3*x)^(1/2))*3^(1/2)+(1+x)^(1/2)*(2+3*x)^(1/2)
```

3.863.2 Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{1-x}\sqrt{2+3x}}{\sqrt{1-x^2}}\,dx = \frac{3\sqrt{1+x}(2+3x)-\sqrt{6+9x}\operatorname{arcsinh}\left(\sqrt{2+3x}\right)}{3\sqrt{2+3x}}$$

input
```
Integrate[(Sqrt[1 - x]*Sqrt[2 + 3*x])/Sqrt[1 - x^2],x]
```

output
```
(3*Sqrt[1 + x]*(2 + 3*x) - Sqrt[6 + 9*x]*ArcSinh[Sqrt[2 + 3*x]])/(3*Sqrt[2
 + 3*x])
```

3.863.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {639, 60, 64, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x}\sqrt{3x+2}}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{639}$$

$$\int \frac{\sqrt{3x+2}}{\sqrt{x+1}}dx$$

$$\Big\downarrow \text{60}$$

$$\sqrt{x+1}\sqrt{3x+2}-\frac{1}{2}\int \frac{1}{\sqrt{x+1}\sqrt{3x+2}}dx$$

$$\Big\downarrow \text{64}$$

$$\sqrt{x+1}\sqrt{3x+2}-\frac{1}{3}\int \frac{1}{\sqrt{\frac{1}{3}(3x+2)+\frac{1}{3}}}d\sqrt{3x+2}$$

$$\Big\downarrow \text{222}$$

$$\sqrt{x+1}\sqrt{3x+2}-\frac{\text{arcsinh}\left(\sqrt{3x+2}\right)}{\sqrt{3}}$$

input
```
Int[(Sqrt[1 - x]*Sqrt[2 + 3*x])/Sqrt[1 - x^2],x]
```

output
```
Sqrt[1 + x]*Sqrt[2 + 3*x] - ArcSinh[Sqrt[2 + 3*x]]/Sqrt[3]
```

3.863.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 64
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[2/b   Subst[Int[1/Sqrt[c - a*(d/b) + d*(x^2/b)], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[c - a*(d/b), 0] && ( !GtQ[a - c*(b/d), 0]
 || PosQ[b])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 639
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[(c + d*x)^(m + p)*(e + f*x)^n*(a/c + (b/d)*x)^p,
 x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (I
ntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[m]))
```

3.863.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(27) = 54$.

Time = 1.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.46

method	result	size
default	$\frac{\sqrt{1-x}\,\sqrt{3x+2}\,\sqrt{-x^{2}+1}\,\left(\ln\left(\frac{5\sqrt{3}}{6}+x\sqrt{3}+\sqrt{3x^{2}+5x+2}\right)\sqrt{3}-6\sqrt{3x^{2}+5x+2}\right)}{6(x-1)\sqrt{3x^{2}+5x+2}}$	86
risch	$-\frac{(x+1)\sqrt{3x+2}\,\sqrt{\frac{(1-x)(3x+2)\left(-x^{2}+1\right)}{(x-1)^{2}}}\,(x-1)}{\sqrt{(x+1)(3x+2)}\,\sqrt{1-x}\,\sqrt{-x^{2}+1}}+\frac{\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^{2}+5x+2}\right)\sqrt{3}\,\sqrt{\frac{(1-x)(3x+2)\left(-x^{2}+1\right)}{(x-1)^{2}}}\,(x-1)}{6\sqrt{1-x}\,\sqrt{3x+2}\,\sqrt{-x^{2}+1}}$	149

input
```
int((1-x)^(1/2)*(3*x+2)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(1-x)^(1/2)*(3*x+2)^(1/2)*(-x^2+1)^(1/2)*(ln(5/6*3^(1/2)+x*3^(1/2)+(3*
x^2+5*x+2)^(1/2))*3^(1/2)-6*(3*x^2+5*x+2)^(1/2))/(x-1)/(3*x^2+5*x+2)^(1/2)
```

3.863.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(27) = 54$.

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.74

$$\int \frac{\sqrt{1-x}\sqrt{2+3x}}{\sqrt{1-x^2}}\,dx = \frac{\sqrt{3}(x-1)\log\left(-\frac{72\,x^3+4\,\sqrt{3}\sqrt{-x^2+1}(6\,x+5)\sqrt{3\,x+2}\sqrt{-x+1}+48\,x^2-71\,x-49}{x-1}\right)-12\,\sqrt{-x^2+1}\sqrt{3\,x+2}\sqrt{-x+1}}{12\,(x-1)}$$

input
```
integrate((1-x)^(1/2)*(2+3*x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(sqrt(3)*(x - 1)*log(-(72*x^3 + 4*sqrt(3)*sqrt(-x^2 + 1)*(6*x + 5)*sq
rt(3*x + 2)*sqrt(-x + 1) + 48*x^2 - 71*x - 49)/(x - 1)) - 12*sqrt(-x^2 + 1
)*sqrt(3*x + 2)*sqrt(-x + 1))/(x - 1)
```

3.863.6 Sympy [F]

$$\int \frac{\sqrt{1-x}\sqrt{2+3x}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{1-x}\sqrt{3x+2}}{\sqrt{-(x-1)\,(x+1)}}\,dx$$

input
```
integrate((1-x)**(1/2)*(2+3*x)**(1/2)/(-x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(1 - x)*sqrt(3*x + 2)/sqrt(-(x - 1)*(x + 1)), x)
```

3.863.7 Maxima [F]

$$\int \frac{\sqrt{1-x}\sqrt{2+3x}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{3\,x+2}\sqrt{-x+1}}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate((1-x)^(1/2)*(2+3*x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(3*x + 2)*sqrt(-x + 1)/sqrt(-x^2 + 1), x)
```

3.863.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(27) = 54$.

Time = 0.37 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.97

$$\int \frac{\sqrt{1-x}\sqrt{2+3x}}{\sqrt{1-x^2}}\,dx = -\frac{1}{3}\sqrt{3}\left(\sqrt{5}\sqrt{3}\sqrt{2} + \log\left(\sqrt{3}\sqrt{2} - \sqrt{5}\right)\right) + \frac{1}{3}\sqrt{3}\log\left(\sqrt{3}\sqrt{x+1} - \sqrt{3\,x+2}\right) + \sqrt{3\,x+2}\sqrt{x+1}$$

input
```
integrate((1-x)^(1/2)*(2+3*x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*(sqrt(5)*sqrt(3)*sqrt(2) + log(sqrt(3)*sqrt(2) - sqrt(5))) +
1/3*sqrt(3)*log(sqrt(3)*sqrt(x + 1) - sqrt(3*x + 2)) + sqrt(3*x + 2)*sqrt(
x + 1)
```

3.863.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x}\sqrt{2+3x}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{3\,x+2}\,\sqrt{1-x}}{\sqrt{1-x^2}}\,dx$$

input
```
int(((3*x + 2)^(1/2)*(1 - x)^(1/2))/(1 - x^2)^(1/2),x)
```

output
```
int(((3*x + 2)^(1/2)*(1 - x)^(1/2))/(1 - x^2)^(1/2), x)
```

3.864 $\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x}\,dx$

3.864.1 Optimal result

Integrand size = 20, antiderivative size = 43

$$\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x}\,dx = \frac{4\sqrt{1+x}}{\sqrt{1-x}} - \arcsin(x) - \operatorname{arctanh}\left(\sqrt{1-x}\sqrt{1+x}\right)$$

output
```
-arcsin(x)-arctanh((1-x)^(1/2)*(1+x)^(1/2))+4*(1+x)^(1/2)/(1-x)^(1/2)
```

3.864.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60

$$\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x}\,dx = -\frac{4\sqrt{1-x^2}}{-1+x} + 4\arctan\left(\frac{\sqrt{1+x}}{\sqrt{2}-\sqrt{1-x}}\right) + 2\operatorname{arctanh}\left(\frac{\sqrt{1-x^2}}{-1+x}\right)$$

input
```
Integrate[(1 + x)^(3/2)/((1 - x)^(3/2)*x),x]
```

output
```
(-4*Sqrt[1 - x^2])/(-1 + x) + 4*ArcTan[Sqrt[1 + x]/(Sqrt[2] - Sqrt[1 - x])
] + 2*ArcTanh[Sqrt[1 - x^2]/(-1 + x)]
```

3.864.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {109, 27, 140, 39, 103, 219, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^{3/2}}{(1-x)^{3/2}x}dx$$

$$\Big\downarrow \text{109}$$

$$\frac{4\sqrt{x+1}}{\sqrt{1-x}}-2\int -\frac{\sqrt{1-x}}{2x\sqrt{x+1}}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\sqrt{1-x}}{x\sqrt{x+1}}dx+\frac{4\sqrt{x+1}}{\sqrt{1-x}}$$

$$\Big\downarrow \text{140}$$

$$-\int \frac{1}{\sqrt{1-x}\sqrt{x+1}}dx+\int \frac{1}{\sqrt{1-x}x\sqrt{x+1}}dx+\frac{4\sqrt{x+1}}{\sqrt{1-x}}$$

$$\Big\downarrow \text{39}$$

$$-\int \frac{1}{\sqrt{1-x^2}}dx+\int \frac{1}{\sqrt{1-x}x\sqrt{x+1}}dx+\frac{4\sqrt{x+1}}{\sqrt{1-x}}$$

$$\Big\downarrow \text{103}$$

$$-\int \frac{1}{\sqrt{1-x^2}}dx-\int \frac{1}{1-(1-x)(x+1)}d\left(\sqrt{1-x}\sqrt{x+1}\right)+\frac{4\sqrt{x+1}}{\sqrt{1-x}}$$

$$\Big\downarrow \text{219}$$

$$-\int \frac{1}{\sqrt{1-x^2}}dx-\text{arctanh}\left(\sqrt{1-x}\sqrt{x+1}\right)+\frac{4\sqrt{x+1}}{\sqrt{1-x}}$$

$$\Big\downarrow \text{223}$$

$$-\arcsin(x)-\text{arctanh}\left(\sqrt{1-x}\sqrt{x+1}\right)+\frac{4\sqrt{x+1}}{\sqrt{1-x}}$$

input
```
Int[(1 + x)^(3/2)/((1 - x)^(3/2)*x),x]
```

output
```
(4*Sqrt[1 + x])/Sqrt[1 - x] - ArcSin[x] - ArcTanh[Sqrt[1 - x]*Sqrt[1 + x]]
```

3.864.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.864. $\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x} \, dx$

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

3.864.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

method	result	size
default	$\frac{\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)x-\arcsin(x)x+\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)+\arcsin(x)-4\sqrt{-x^2+1}\right)\sqrt{1-x}\,\sqrt{x+1}}{(x-1)\sqrt{-x^2+1}}$	70
risch	$\frac{4\sqrt{x+1}\,\sqrt{(1-x)(x+1)}}{\sqrt{-(x-1)(x+1)}\,\sqrt{1-x}}-\frac{\left(\arcsin(x)+\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)\right)\sqrt{(1-x)(x+1)}}{\sqrt{1-x}\,\sqrt{x+1}}$	75

input
```
int((x+1)^(3/2)/(1-x)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(-arctanh(1/(-x^2+1)^(1/2))*x-arcsin(x)*x+arctanh(1/(-x^2+1)^(1/2))+arcsin
(x)-4*(-x^2+1)^(1/2))*(1-x)^(1/2)*(x+1)^(1/2)/(x-1)/(-x^2+1)^(1/2)
```

3.864.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(35) = 70.

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.72

$$\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x}\,dx = \frac{2\,(x-1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+(x-1)\log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+4\,x-4\,\sqrt{x+1}\sqrt{-x+}}{x-1}$$

input
```
integrate((1+x)^(3/2)/(1-x)^(3/2)/x,x, algorithm="fricas")
```

output
```
(2*(x - 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + (x - 1)*log((sqrt(x
+ 1)*sqrt(-x + 1) - 1)/x) + 4*x - 4*sqrt(x + 1)*sqrt(-x + 1) - 4)/(x - 1)
```

3.864.6 Sympy [F]

$$\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x}\,dx = \int \frac{(x+1)^{\frac{3}{2}}}{x\,(1-x)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+x)**(3/2)/(1-x)**(3/2)/x,x)
```

output
```
Integral((x + 1)**(3/2)/(x*(1 - x)**(3/2)), x)
```

3.864.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

$$\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x}\,dx = \frac{4\,x}{\sqrt{-x^2+1}} + \frac{4}{\sqrt{-x^2+1}} - \arcsin(x) - \log\left(\frac{2\,\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

input
```
integrate((1+x)^(3/2)/(1-x)^(3/2)/x,x, algorithm="maxima")
```

output
```
4*x/sqrt(-x^2 + 1) + 4/sqrt(-x^2 + 1) - arcsin(x) - log(2*sqrt(-x^2 + 1)/a
bs(x) + 2/abs(x))
```

3.864.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(35) = 70.

Time = 0.38 (sec) , antiderivative size = 163, normalized size of antiderivative = 3.79

$$\begin{aligned}\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x}\,dx &= -\pi - \frac{4\,\sqrt{x+1}\sqrt{-x+1}}{x-1} \\ &\quad - 2\,\arctan\left(\frac{\sqrt{x+1}\left(\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1} - 1\right)}{2\left(\sqrt{2}-\sqrt{-x+1}\right)}\right) \\ &\quad - \log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} + \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}} + 2\right|\right) \\ &\quad + \log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} + \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}} - 2\right|\right)\end{aligned}$$

input
```
integrate((1+x)^(3/2)/(1-x)^(3/2)/x,x, algorithm="giac")
```

output
```
-pi - 4*sqrt(x + 1)*sqrt(-x + 1)/(x - 1) - 2*arctan(1/2*sqrt(x + 1)*((sqrt
(2) - sqrt(-x + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1))) - log(abs(-(s
qrt(2) - sqrt(-x + 1))/sqrt(x + 1) + sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1))
+ 2)) + log(abs(-(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + sqrt(x + 1)/(sqrt(
2) - sqrt(-x + 1)) - 2))
```

3.864.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x} \, dx = \int \frac{(x+1)^{3/2}}{x\,(1-x)^{3/2}} \, dx$$

input
```
int((x + 1)^(3/2)/(x*(1 - x)^(3/2)),x)
```

output
```
int((x + 1)^(3/2)/(x*(1 - x)^(3/2)), x)
```

3.865 $\int \frac{(1+x)^3}{x(1-x^2)^{3/2}}\,dx$

3.865.1 Optimal result

Integrand size = 20, antiderivative size = 35

$$\int \frac{(1+x)^3}{x(1-x^2)^{3/2}}\,dx = \frac{4(1+x)}{\sqrt{1-x^2}} - \arcsin(x) - \operatorname{arctanh}\left(\sqrt{1-x^2}\right)$$

output
```
-arcsin(x)-arctanh((-x^2+1)^(1/2))+4*(1+x)/(-x^2+1)^(1/2)
```

3.865.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.69

$$\int \frac{(1+x)^3}{x(1-x^2)^{3/2}}\,dx = -\frac{4\sqrt{1-x^2}}{-1+x} + 2\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right) - 2\operatorname{arctanh}\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[(1 + x)^3/(x*(1 - x^2)^(3/2)),x]
```

output
```
(-4*Sqrt[1 - x^2])/(-1 + x) + 2*ArcTan[Sqrt[1 - x^2]/(1 + x)] - 2*ArcTanh[
Sqrt[1 - x^2]/(1 + x)]
```

3.865.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {528, 538, 223, 243, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^3}{x\left(1-x^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 528}$$

$$\int \frac{1-x}{x\sqrt{1-x^2}}dx+\frac{4(x+1)}{\sqrt{1-x^2}}$$

$$\Big\downarrow \text{ 538}$$

$$-\int \frac{1}{\sqrt{1-x^2}}dx+\int \frac{1}{x\sqrt{1-x^2}}dx+\frac{4(x+1)}{\sqrt{1-x^2}}$$

$$\Big\downarrow \text{ 223}$$

$$\int \frac{1}{x\sqrt{1-x^2}}dx-\arcsin(x)+\frac{4(x+1)}{\sqrt{1-x^2}}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{1-x^2}}dx^2-\arcsin(x)+\frac{4(x+1)}{\sqrt{1-x^2}}$$

$$\Big\downarrow \text{ 73}$$

$$-\int \frac{1}{1-x^4}d\sqrt{1-x^2}-\arcsin(x)+\frac{4(x+1)}{\sqrt{1-x^2}}$$

$$\Big\downarrow \text{ 219}$$

$$-\arcsin(x)-\operatorname{arctanh}\left(\sqrt{1-x^2}\right)+\frac{4(x+1)}{\sqrt{1-x^2}}$$

input
```
Int[(1 + x)^3/(x*(1 - x^2)^(3/2)),x]
```

output
```
(4*(1 + x))/Sqrt[1 - x^2] - ArcSin[x] - ArcTanh[Sqrt[1 - x^2]]
```

3.865.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 528
```
Int[((x_)^(m_)*((c_) + (d_.)*(x_))^(n_.))/((a_) + (b_.)*(x_)^2)^(3/2), x_Sy
mbol] :> Simp[(-2^(n - 1))*c^(m + n - 2)*((c + d*x)/(b*d^(m - 1)*Sqrt[a + b
*x^2])), x] + Simp[c^2/a   Int[(x^m/Sqrt[a + b*x^2])*ExpandToSum[((c + d*x)
^(n - 1) - (2^(n - 1)*c^(m + n - 1))/(d^m*x^m))/(c - d*x), x], x], x] /; Fr
eeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && EqQ[b*c^2 + a*d^2, 0]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

3.865. $\int \frac{(1+x)^3}{x(1-x^2)^{3/2}}\,dx$

3.865.4 Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result
risch	$\frac{4+4x}{\sqrt{-x^2+1}}-\arcsin\left(x\right)-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)$
default	$\frac{4x}{\sqrt{-x^2+1}}-\arcsin\left(x\right)+\frac{4}{\sqrt{-x^2+1}}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)$
trager	$-\frac{4\sqrt{-x^2+1}}{x-1}+\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2+1}\right)+\ln\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$
meijerg	$\frac{-\sqrt{\pi}+\frac{\sqrt{\pi}}{\sqrt{-x^2+1}}-\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^2+1}}{2}\right)+\frac{(2-2\ln(2)+2\ln(x)+i\pi)\sqrt{\pi}}{2}}{\sqrt{\pi}}+\frac{i\left(-\frac{i\sqrt{\pi}\,x}{\sqrt{-x^2+1}}+i\sqrt{\pi}\,\arcsin(x)\right)}{\sqrt{\pi}}-\frac{3\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{-x^2+1}}\right)}{\sqrt{\pi}}+$

input
```
int((x+1)^3/x/(-x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
4*(x+1)/(-x^2+1)^(1/2)-arcsin(x)-arctanh(1/(-x^2+1)^(1/2))
```

3.865.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(31) = 62.

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.80

$$\int \frac{(1+x)^3}{x\left(1-x^2\right)^{3/2}}\,dx=\frac{2\,(x-1)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+(x-1)\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+4\,x-4\,\sqrt{-x^2+1}-4}{x-1}$$

input
```
integrate((1+x)^3/x/(-x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
(2*(x - 1)*arctan((sqrt(-x^2 + 1) - 1)/x) + (x - 1)*log((sqrt(-x^2 + 1) -
1)/x) + 4*x - 4*sqrt(-x^2 + 1) - 4)/(x - 1)
```

3.865.6 Sympy [F]

$$\int \frac{(1+x)^3}{x(1-x^2)^{3/2}}\,dx = \int \frac{(x+1)^3}{x(-(x-1)(x+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+x)**3/x/(-x**2+1)**(3/2),x)
```

output
```
Integral((x + 1)**3/(x*(-(x - 1)*(x + 1))**(3/2)), x)
```

3.865.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int \frac{(1+x)^3}{x(1-x^2)^{3/2}}\,dx = \frac{4x}{\sqrt{-x^2+1}} + \frac{4}{\sqrt{-x^2+1}} - \arcsin(x) - \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

input
```
integrate((1+x)^3/x/(-x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
4*x/sqrt(-x^2 + 1) + 4/sqrt(-x^2 + 1) - arcsin(x) - log(2*sqrt(-x^2 + 1)/a
bs(x) + 2/abs(x))
```

3.865.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \frac{(1+x)^3}{x(1-x^2)^{3/2}}\,dx = \frac{8}{\frac{\sqrt{-x^2+1}-1}{x}+1} - \arcsin(x) + \log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

input
```
integrate((1+x)^3/x/(-x^2+1)^(3/2),x, algorithm="giac")
```

output
```
8/((sqrt(-x^2 + 1) - 1)/x + 1) - arcsin(x) + log(-(sqrt(-x^2 + 1) - 1)/abs
(x))
```

3.865.9 Mupad [B] (verification not implemented)

Time = 25.78 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(1+x)^3}{x\,(1-x^2)^{3/2}}\,dx = \ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)-\mathrm{asin}(x)-\frac{4\sqrt{1-x^2}}{x-1}$$

input
```
int((x + 1)^3/(x*(1 - x^2)^(3/2)),x)
```

output
```
log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)) - asin(x) - (4*(1 - x^2)^(1/2))/(x
- 1)
```

3.866 $\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}} \, dx$

3.866.1 Optimal result

Integrand size = 23, antiderivative size = 51

$$\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}} \, dx = \frac{4\sqrt{1+ax}}{\sqrt{1-ax}} - \arcsin(ax) - \operatorname{arctanh}\left(\sqrt{1-ax}\sqrt{1+ax}\right)$$

output
```
-arcsin(a*x)-arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))+4*(a*x+1)^(1/2)/(-a*x+1
)^(1/2)
```

3.866.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.63

$$\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}} \, dx = 4\left(\frac{\sqrt{1-a^2x^2}}{1-ax} + \arctan\left(\frac{\sqrt{1+ax}}{\sqrt{2}-\sqrt{1-ax}}\right)\right) + 2\operatorname{arctanh}\left(\frac{\sqrt{1-a^2x^2}}{-1+ax}\right)$$

input
```
Integrate[(1 + a*x)^(3/2)/(x*(1 - a*x)^(3/2)),x]
```

output
```
4*(Sqrt[1 - a^2*x^2]/(1 - a*x) + ArcTan[Sqrt[1 + a*x]/(Sqrt[2] - Sqrt[1 -
a*x])]) + 2*ArcTanh[Sqrt[1 - a^2*x^2]/(-1 + a*x)]
```

3.866.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {109, 27, 140, 39, 103, 221, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+1)^{3/2}}{x(1-ax)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 109}$$

$$\frac{4\sqrt{ax+1}}{\sqrt{1-ax}}-\frac{2\int -\frac{a\sqrt{1-ax}}{2x\sqrt{ax+1}}dx}{a}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{\sqrt{1-ax}}{x\sqrt{ax+1}}dx+\frac{4\sqrt{ax+1}}{\sqrt{1-ax}}$$

$$\Big\downarrow \text{ 140}$$

$$-a\int \frac{1}{\sqrt{1-ax}\sqrt{ax+1}}dx+\int \frac{1}{x\sqrt{1-ax}\sqrt{ax+1}}dx+\frac{4\sqrt{ax+1}}{\sqrt{1-ax}}$$

$$\Big\downarrow \text{ 39}$$

$$-a\int \frac{1}{\sqrt{1-a^2x^2}}dx+\int \frac{1}{x\sqrt{1-ax}\sqrt{ax+1}}dx+\frac{4\sqrt{ax+1}}{\sqrt{1-ax}}$$

$$\Big\downarrow \text{ 103}$$

$$-a\int \frac{1}{\sqrt{1-a^2x^2}}dx-a\int \frac{1}{a-a(1-ax)(ax+1)}d\left(\sqrt{1-ax}\sqrt{ax+1}\right)+\frac{4\sqrt{ax+1}}{\sqrt{1-ax}}$$

$$\Big\downarrow \text{ 221}$$

$$-a\int \frac{1}{\sqrt{1-a^2x^2}}dx-\text{arctanh}\left(\sqrt{1-ax}\sqrt{ax+1}\right)+\frac{4\sqrt{ax+1}}{\sqrt{1-ax}}$$

$$\Big\downarrow \text{ 223}$$

$$-\arcsin(ax)-\text{arctanh}\left(\sqrt{1-ax}\sqrt{ax+1}\right)+\frac{4\sqrt{ax+1}}{\sqrt{1-ax}}$$

input
```
Int[(1 + a*x)^(3/2)/(x*(1 - a*x)^(3/2)),x]
```

output
```
(4*Sqrt[1 + a*x])/Sqrt[1 - a*x] - ArcSin[a*x] - ArcTanh[Sqrt[1 - a*x]*Sqrt
[1 + a*x]]
```

3.866.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 39
```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

3.866. $\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}} \, dx$

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

3.866.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.18 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.63

method	result
default	$\frac{\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\operatorname{csgn}(a)ax-\arctan\left(\frac{\operatorname{csgn}(a)ax}{\sqrt{-(ax+1)(ax-1)}}\right)ax+\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\operatorname{csgn}(a)-4\sqrt{-a^2x^2+1}\operatorname{csgn}(a)+\operatorname{arctan}\right.}{(ax-1)\sqrt{-a^2x^2+1}}$

input
```
int((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-arctanh(1/(-a^2*x^2+1)^(1/2))*csgn(a)*a*x-arctan(csgn(a)*a*x/(-(a*x+1)*(
a*x-1))^(1/2))*a*x+arctanh(1/(-a^2*x^2+1)^(1/2))*csgn(a)-4*(-a^2*x^2+1)^(1
/2)*csgn(a)+arctan(csgn(a)*a*x/(-(a*x+1)*(a*x-1))^(1/2)))*csgn(a)*(-a*x+1)
^(1/2)*(a*x+1)^(1/2)/(a*x-1)/(-a^2*x^2+1)^(1/2)
```

3.866.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(43) = 86.

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.82

$$\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}}\,dx = \frac{4\,ax + 2\,(ax-1)\arctan\left(\frac{\sqrt{ax+1}\sqrt{-ax+1}-1}{ax}\right) + (ax-1)\log\left(\frac{\sqrt{ax+1}\sqrt{-ax+1}-1}{x}\right) - 4\,\sqrt{ax -}}{ax-1}$$

input
```
integrate((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x, algorithm="fricas")
```

output
```
(4*a*x + 2*(a*x - 1)*arctan((sqrt(a*x + 1)*sqrt(-a*x + 1) - 1)/(a*x)) + (a
*x - 1)*log((sqrt(a*x + 1)*sqrt(-a*x + 1) - 1)/x) - 4*sqrt(a*x + 1)*sqrt(-
a*x + 1) - 4)/(a*x - 1)
```

3.866.6 Sympy [F]

$$\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}}\,dx = \int \frac{(ax+1)^{\frac{3}{2}}}{x\,(-ax+1)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*x+1)**(3/2)/x/(-a*x+1)**(3/2),x)
```

output
```
Integral((a*x + 1)**(3/2)/(x*(-a*x + 1)**(3/2)), x)
```

3.866.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.27

$$\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}}\,dx = \frac{4\,ax}{\sqrt{-a^2x^2+1}} + \frac{4}{\sqrt{-a^2x^2+1}} - \arcsin(ax) - \log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

input
```
integrate((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x, algorithm="maxima")
```

output
```
4*a*x/sqrt(-a^2*x^2 + 1) + 4/sqrt(-a^2*x^2 + 1) - arcsin(a*x) - log(2*sqrt
(-a^2*x^2 + 1)/abs(x) + 2/abs(x))
```

3.866.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(43) = 86$.

Time = 0.43 (sec) , antiderivative size = 196, normalized size of antiderivative = 3.84

$$\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}}\,dx = -\frac{\left(\pi + 2\arctan\left(\frac{\sqrt{ax+1}\left(\frac{\left(\sqrt{2}-\sqrt{-ax+1}\right)^2}{ax+1}-1\right)}{2\left(\sqrt{2}-\sqrt{-ax+1}\right)}\right)\right)a + a\log\left(\left|-\frac{\sqrt{2}-\sqrt{-ax+1}}{\sqrt{ax+1}} + \frac{\sqrt{ax+1}}{\sqrt{2}-\sqrt{-ax+1}} + 2\right|\right) - a\log\left(\left|-\frac{\sqrt{2}-\cdots}{\sqrt{\cdots}}\right.\right.}{a}$$

input
```
integrate((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x, algorithm="giac")
```

output
```
-((pi + 2*arctan(1/2*sqrt(a*x + 1)*((sqrt(2) - sqrt(-a*x + 1))^2/(a*x + 1)
 - 1)/(sqrt(2) - sqrt(-a*x + 1))))*a + a*log(abs(-(sqrt(2) - sqrt(-a*x + 1
))/sqrt(a*x + 1) + sqrt(a*x + 1)/(sqrt(2) - sqrt(-a*x + 1)) + 2)) - a*log(
abs(-(sqrt(2) - sqrt(-a*x + 1))/sqrt(a*x + 1) + sqrt(a*x + 1)/(sqrt(2) - s
qrt(-a*x + 1)) - 2)) + 4*sqrt(a*x + 1)*sqrt(-a*x + 1)*a/(a*x - 1))/a
```

3.866.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}}\,dx = \int \frac{(a\,x+1)^{3/2}}{x\,(1-a\,x)^{3/2}}\,dx$$

input
```
int((a*x + 1)^(3/2)/(x*(1 - a*x)^(3/2)),x)
```

output
```
int((a*x + 1)^(3/2)/(x*(1 - a*x)^(3/2)), x)
```

3.867 $\int \frac{(1+ax)^3}{x\left(1-a^2x^2\right)^{3/2}}\,dx$

3.867.1 Optimal result

Integrand size = 25, antiderivative size = 45

$$\int \frac{(1+ax)^3}{x\left(1-a^2x^2\right)^{3/2}}\,dx=\frac{4(1+ax)}{\sqrt{1-a^2x^2}}-\arcsin(ax)-\operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right)$$

output
```
-arcsin(a*x)-arctanh((-a^2*x^2+1)^(1/2))+4*(a*x+1)/(-a^2*x^2+1)^(1/2)
```

3.867.2 Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.58

$$\int \frac{(1+ax)^3}{x\left(1-a^2x^2\right)^{3/2}}\,dx=-\frac{4\sqrt{1-a^2x^2}}{-1+ax}+2\arctan\left(\frac{-1+\sqrt{1-a^2x^2}}{ax}\right)-\log(x)+\log\left(-1+\sqrt{1-a^2x^2}\right)$$

input
```
Integrate[(1 + a*x)^3/(x*(1 - a^2*x^2)^(3/2)),x]
```

output
```
(-4*Sqrt[1 - a^2*x^2])/(-1 + a*x) + 2*ArcTan[(-1 + Sqrt[1 - a^2*x^2])/(a*x
)] - Log[x] + Log[-1 + Sqrt[1 - a^2*x^2]]
```

3.867.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {528, 538, 223, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax+1)^3}{x(1-a^2x^2)^{3/2}}dx$$

$$\Big\downarrow \text{528}$$

$$\int \frac{1-ax}{x\sqrt{1-a^2x^2}}dx+\frac{4(ax+1)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{538}$$

$$-a\int \frac{1}{\sqrt{1-a^2x^2}}dx+\int \frac{1}{x\sqrt{1-a^2x^2}}dx+\frac{4(ax+1)}{\sqrt{1-a^2x^2}}$$

$$\Big\downarrow \text{223}$$

$$\int \frac{1}{x\sqrt{1-a^2x^2}}dx+\frac{4(ax+1)}{\sqrt{1-a^2x^2}}-\arcsin(ax)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{1-a^2x^2}}dx^2+\frac{4(ax+1)}{\sqrt{1-a^2x^2}}-\arcsin(ax)$$

$$\Big\downarrow \text{73}$$

$$-\frac{\int \frac{1}{\frac{1}{a^2}-\frac{x^4}{a^2}}d\sqrt{1-a^2x^2}}{a^2}+\frac{4(ax+1)}{\sqrt{1-a^2x^2}}-\arcsin(ax)$$

$$\Big\downarrow \text{221}$$

$$-\text{arctanh}\left(\sqrt{1-a^2x^2}\right)+\frac{4(ax+1)}{\sqrt{1-a^2x^2}}-\arcsin(ax)$$

input
```
Int[(1 + a*x)^3/(x*(1 - a^2*x^2)^(3/2)),x]
```

output
```
(4*(1 + a*x))/Sqrt[1 - a^2*x^2] - ArcSin[a*x] - ArcTanh[Sqrt[1 - a^2*x^2]]
```

3.867.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 528
```
Int[((x_)^(m_)*((c_) + (d_.)*(x_))^(n_.))/((a_) + (b_.)*(x_)^2)^(3/2), x_Sy
mbol] :> Simp[(-2^(n - 1))*c^(m + n - 2)*((c + d*x)/(b*d^(m - 1)*Sqrt[a + b
*x^2])), x] + Simp[c^2/a   Int[(x^m/Sqrt[a + b*x^2])*ExpandToSum[((c + d*x)
^(n - 1) - (2^(n - 1)*c^(m + n - 1))/(d^m*x^m))/(c - d*x), x], x], x] /; Fr
eeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && EqQ[b*c^2 + a*d^2, 0]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

3.867.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(41) = 82$.

Time = 1.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.20

method	result
default	$\frac{3ax}{\sqrt{-a^2x^2+1}}+\frac{4}{\sqrt{-a^2x^2+1}}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)+a^3\left(\frac{x}{a^2\sqrt{-a^2x^2+1}}-\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{a^2\sqrt{a^2}}\right)$
meijerg	$\frac{-\sqrt{\pi}+\frac{\sqrt{\pi}}{\sqrt{-a^2x^2+1}}-\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-a^2x^2+1}}{2}\right)+\frac{\left(2-2\ln(2)+2\ln(x)+\ln\left(-a^2\right)\right)\sqrt{\pi}}{2}}{\sqrt{\pi}}-\frac{a\left(\frac{\sqrt{\pi}\,x\left(-a^2\right)^{\frac{3}{2}}}{a^2\sqrt{-a^2x^2+1}}-\frac{\sqrt{\pi}\left(-a^2\right)^{\frac{3}{2}}\arcsin(ax)}{a^3}\right)}{\sqrt{\pi}\sqrt{-a^2}}-\frac{3(}{}$

input
```
int((a*x+1)^3/x/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3*a*x/(-a^2*x^2+1)^(1/2)+4/(-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2)
)+a^3*(x/a^2/(-a^2*x^2+1)^(1/2)-1/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a
^2*x^2+1)^(1/2)))
```

3.867.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.82

$$\int \frac{(1+ax)^3}{x\,(1-a^2x^2)^{3/2}}\,dx = \frac{4\,ax+2\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+(ax-1)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-4\sqrt{-a^2x^2+}}{ax-1}$$

input
```
integrate((a*x+1)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")
```

output
```
(4*a*x + 2*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (a*x - 1)*lo
g((sqrt(-a^2*x^2 + 1) - 1)/x) - 4*sqrt(-a^2*x^2 + 1) - 4)/(a*x - 1)
```

3.867.6 Sympy [F]

$$\int \frac{(1+ax)^3}{x\,(1-a^2x^2)^{3/2}}\,dx = \int \frac{(ax+1)^3}{x\,(-(ax-1)\,(ax+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate((a*x+1)**3/x/(-a**2*x**2+1)**(3/2),x)
```

output
```
Integral((a*x + 1)**3/(x*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

3.867.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int \frac{(1+ax)^3}{x(1-a^2x^2)^{3/2}}\,dx = \frac{4\,ax}{\sqrt{-a^2x^2+1}} + \frac{4}{\sqrt{-a^2x^2+1}} - \arcsin(ax) - \log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

input
```
integrate((a*x+1)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")
```

output
```
4*a*x/sqrt(-a^2*x^2 + 1) + 4/sqrt(-a^2*x^2 + 1) - arcsin(a*x) - log(2*sqrt
(-a^2*x^2 + 1)/abs(x) + 2/abs(x))
```

3.867.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(41) = 82.

Time = 0.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.93

$$\int \frac{(1+ax)^3}{x(1-a^2x^2)^{3/2}}\,dx = -\frac{a\arcsin(ax)\operatorname{sgn}(a)}{|a|} - \frac{a\log\left(\frac{\left|-2\sqrt{-a^2x^2+1}|a|-2a\right|}{2a^2|x|}\right)}{|a|} + \frac{8a}{\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x}-1\right)|a|}$$

input
```
integrate((a*x+1)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

output
```
-a*arcsin(a*x)*sgn(a)/abs(a) - a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a)
- 2*a)/(a^2*abs(x)))/abs(a) + 8*a/(((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x
) - 1)*abs(a))
```

3.867.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.82

$$\int \frac{(1+ax)^3}{x\,(1-a^2x^2)^{3/2}}\,dx = \frac{4\,a\,\sqrt{1-a^2\,x^2}}{\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}} - \frac{a\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}} - \mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)$$

input
```
int((a*x + 1)^3/(x*(1 - a^2*x^2)^(3/2)),x)
```

output
```
(4*a*(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2))
 - (a*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2) - atanh((1 - a^2*x^2)^(1/2))
```

3.868 $\int \frac{1}{\sqrt{1-x^2}} \, dx$

3.868.1 Optimal result

Integrand size = 11, antiderivative size = 2

$$\int \frac{1}{\sqrt{1-x^2}} \, dx = \arcsin(x)$$

output
```
arcsin(x)
```

3.868.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 20 vs. 2(2) = 4.

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 10.00

$$\int \frac{1}{\sqrt{1-x^2}} \, dx = -2 \arctan \left(\frac{\sqrt{1-x^2}}{1+x} \right)$$

input
```
Integrate[1/Sqrt[1 - x^2],x]
```

output
```
-2*ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

3.868.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$\Big\downarrow \ 223$$
$$\arcsin(x)$$

input
```
Int[1/Sqrt[1 - x^2],x]
```

output
```
ArcSin[x]
```

3.868.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

3.868.4 Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\arcsin\left(x\right)$	3
meijerg	$\arcsin\left(x\right)$	3
pseudoelliptic	$-\arctan\left(\frac{\sqrt{-x^{2}+1}}{x}\right)$	17
trager	$\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-x^{2}+1}+x\right)$	27

input
```
int(1/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(x)
```

3.868.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(2) = 4.

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 9.00

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = -2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

input
```
integrate(1/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

3.868.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \operatorname{asin}(x)$$

input
```
integrate(1/(-x**2+1)**(1/2),x)
```

output
```
asin(x)
```

3.868.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin(x)$$

input
```
integrate(1/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
arcsin(x)
```

3.868.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(2) = 4.

Time = 0.35 (sec) , antiderivative size = 17, normalized size of antiderivative = 8.50

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \frac{1}{2}\,\sqrt{-x^2+1}x + \frac{1}{2}\,\arcsin\left(x\right)$$

input
```
integrate(1/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

3.868.9 Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \mathrm{asin}(x)$$

input
```
int(1/(1 - x^2)^(1/2),x)
```

output
```
asin(x)
```

3.869 $\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx$

3.869.1 Optimal result

Integrand size = 21, antiderivative size = 2

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx = \arcsin(x)$$

output
```
arcsin(x)
```

3.869.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 32 vs. $2(2) = 4$.

Time = 0.54 (sec) , antiderivative size = 32, normalized size of antiderivative = 16.00

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx = -\arctan\left(\frac{x\sqrt{1+x^2}\sqrt{1-x^4}}{-1+x^4}\right)$$

input
```
Integrate[Sqrt[1 + x^2]/Sqrt[1 - x^4],x]
```

output
```
-ArcTan[(x*Sqrt[1 + x^2]*Sqrt[1 - x^4])/(-1 + x^4)]
```

3.869.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1386, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^4}}dx$$

$$\Big\downarrow \text{ 1386}$$

$$\int \frac{1}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{ 223}$$

$$\arcsin(x)$$

input
```
Int[Sqrt[1 + x^2]/Sqrt[1 - x^4],x]
```

output
```
ArcSin[x]
```

3.869.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1386
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Simp[(-e^2/c)^q   Int[u*(d - e*x^n)^p, x], x] /; FreeQ[{a, c,
d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && EqQ[p + q, 0
] && GtQ[d, 0] && LtQ[c, 0] && GtQ[e^2, 0]
```

3.869.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(2) = 4$.

Time = 1.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 14.50

method	result	size
default	$\frac{\sqrt{-x^4+1}\,\arcsin(x)}{\sqrt{x^2+1}\,\sqrt{-x^2+1}}$	29

input
```
int((x^2+1)^(1/2)/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(x^2+1)^(1/2)*(-x^4+1)^(1/2)/(-x^2+1)^(1/2)*arcsin(x)
```

3.869.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(2) = 4$.

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 13.50

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx = -\arctan\left(\frac{\sqrt{-x^4+1}\sqrt{x^2+1}}{x^3+x}\right)$$

input
```
integrate((x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-arctan(sqrt(-x^4 + 1)*sqrt(x^2 + 1)/(x^3 + x))
```

3.869.6 Sympy [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{-(x-1)(x+1)(x^2+1)}}\,dx$$

input
```
integrate((x**2+1)**(1/2)/(-x**4+1)**(1/2),x)
```

output
```
Integral(sqrt(x**2 + 1)/sqrt(-(x - 1)*(x + 1)*(x**2 + 1)), x)
```

3.869.7 Maxima [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate((x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2 + 1)/sqrt(-x^4 + 1), x)
```

3.869.8 Giac [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate((x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^2 + 1)/sqrt(-x^4 + 1), x)
```

3.869.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{1-x^4}}\,dx$$

input
```
int((x^2 + 1)^(1/2)/(1 - x^4)^(1/2),x)
```

output
```
int((x^2 + 1)^(1/2)/(1 - x^4)^(1/2), x)
```

3.870 $\int \frac{1}{\sqrt{1+x^2}} dx$

3.870.1 Optimal result

Integrand size = 9, antiderivative size = 2

$$\int \frac{1}{\sqrt{1+x^2}} dx = \operatorname{arcsinh}(x)$$

output
```
arcsinh(x)
```

3.870.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 16 vs. 2(2) = 4.

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 8.00

$$\int \frac{1}{\sqrt{1+x^2}} dx = -\log\left(-x+\sqrt{1+x^2}\right)$$

input
```
Integrate[1/Sqrt[1 + x^2],x]
```

output
```
-Log[-x + Sqrt[1 + x^2]]
```

3.870.3 Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+1}}\,dx$$
$$\Big\downarrow \; 222$$
$$\text{arcsinh}(x)$$

input
```
Int[1/Sqrt[1 + x^2],x]
```

output
```
ArcSinh[x]
```

3.870.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

3.870.4 Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.50

method	result	size
default	$\text{arcsinh}\left(x\right)$	3
meijerg	$\text{arcsinh}\left(x\right)$	3
trager	$\ln\left(x+\sqrt{x^{2}+1}\right)$	11
pseudoelliptic	$\text{arctanh}\left(\frac{\sqrt{x^{2}+1}}{x}\right)$	13

input
```
int(1/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsinh(x)
```

3.870.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(2) = 4$.

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 7.00

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = -\log\left(-x+\sqrt{x^2+1}\right)$$

input
```
integrate(1/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-log(-x + sqrt(x^2 + 1))
```

3.870.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \operatorname{asinh}(x)$$

input
```
integrate(1/(x**2+1)**(1/2),x)
```

output
```
asinh(x)
```

3.870.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \operatorname{arsinh}(x)$$

input
```
integrate(1/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(x)
```

3.870.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(2) = 4$.

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 12.50

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \frac{1}{2}\,\sqrt{x^2+1}x - \frac{1}{2}\,\log\left(-x+\sqrt{x^2+1}\right)$$

input
```
integrate(1/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

3.870.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \operatorname{asinh}(x)$$

input
```
int(1/(x^2 + 1)^(1/2),x)
```

output
```
asinh(x)
```

3.871 $\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx$

3.871.1 Optimal result

Integrand size = 23, antiderivative size = 2

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx = \operatorname{arcsinh}(x)$$

output
```
arcsinh(x)
```

3.871.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 42 vs. $2(2) = 4$.

Time = 0.51 (sec) , antiderivative size = 42, normalized size of antiderivative = 21.00

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx = \log\left(1-x^2\right) - \log\left(-x+x^3+\sqrt{1-x^2}\sqrt{1-x^4}\right)$$

input
```
Integrate[Sqrt[1 - x^2]/Sqrt[1 - x^4],x]
```

output
```
Log[1 - x^2] - Log[-x + x^3 + Sqrt[1 - x^2]*Sqrt[1 - x^4]]
```

3.871.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1386, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}} \, dx$$
$$\Big\downarrow \text{ 1386}$$
$$\int \frac{1}{\sqrt{x^2+1}}dx$$
$$\Big\downarrow \text{ 222}$$
$$\text{arcsinh}(x)$$

input
```
Int[Sqrt[1 - x^2]/Sqrt[1 - x^4],x]
```

output
```
ArcSinh[x]
```

3.871.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1386
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Simp[(-e^2/c)^q   Int[u*(d - e*x^n)^p, x], x] /; FreeQ[{a, c,
d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && EqQ[p + q, 0
] && GtQ[d, 0] && LtQ[c, 0] && GtQ[e^2, 0]
```

3.871.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(2) = 4.

Time = 1.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 14.50

method	result	size
default	$\frac{\sqrt{-x^4+1}\,\mathrm{arcsinh}(x)}{\sqrt{-x^2+1}\,\sqrt{x^2+1}}$	29

input
```
int((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-x^2+1)^(1/2)/(x^2+1)^(1/2)*(-x^4+1)^(1/2)*arcsinh(x)
```

3.871.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(2) = 4.

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 40.50

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx = -\frac{1}{2}\log\left(\frac{x^3+\sqrt{-x^4+1}\sqrt{-x^2+1}-x}{x^3-x}\right) + \frac{1}{2}\log\left(-\frac{x^3-\sqrt{-x^4+1}\sqrt{-x^2+1}-x}{x^3-x}\right)$$

input
```
integrate((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*log((x^3 + sqrt(-x^4 + 1)*sqrt(-x^2 + 1) - x)/(x^3 - x)) + 1/2*log(-(
x^3 - sqrt(-x^4 + 1)*sqrt(-x^2 + 1) - x)/(x^3 - x))
```

3.871.6 Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{-(x-1)(x+1)(x^2+1)}}\,dx$$

input
```
integrate((-x**2+1)**(1/2)/(-x**4+1)**(1/2),x)
```

output
```
Integral(sqrt(-(x - 1)*(x + 1))/sqrt(-(x - 1)*(x + 1)*(x**2 + 1)), x)
```

3.871.7 Maxima [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{-x^2+1}}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 + 1)/sqrt(-x^4 + 1), x)
```

3.871.8 Giac [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{-x^2+1}}{\sqrt{-x^4+1}}\,dx$$

input
```
integrate((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^2 + 1)/sqrt(-x^4 + 1), x)
```

3.871.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx$$

input
```
int((1 - x^2)^(1/2)/(1 - x^4)^(1/2),x)
```

output
```
int((1 - x^2)^(1/2)/(1 - x^4)^(1/2), x)
```

3.872 $\int \sqrt{1-x^2}\,dx$

3.872.1 Optimal result

Integrand size = 11, antiderivative size = 23

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}x\sqrt{1-x^2} + \frac{\arcsin(x)}{2}$$

output
```
1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

3.872.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}x\sqrt{1-x^2} - \arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[Sqrt[1 - x^2],x]
```

output
```
(x*Sqrt[1 - x^2])/2 - ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

3.872.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-x^2}\,dx$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx+\frac{1}{2}\sqrt{1-x^2}x$$

$$\Big\downarrow \text{223}$$

$$\frac{\arcsin(x)}{2}+\frac{1}{2}\sqrt{1-x^2}x$$

input
```
Int[Sqrt[1 - x^2],x]
```

output
```
(x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

3.872.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

3.872.4 Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{x\sqrt{-x^{2}+1}}{2}+\frac{\arcsin\left(x\right)}{2}$	18
risch	$-\frac{x\left(x^{2}-1\right)}{2\sqrt{-x^{2}+1}}+\frac{\arcsin\left(x\right)}{2}$	23
pseudoelliptic	$\frac{x\sqrt{-x^{2}+1}}{2}-\frac{\arctan\left(\frac{\sqrt{-x^{2}+1}}{x}\right)}{2}$	30
meijerg	$\frac{i\left(-2i\sqrt{\pi}\,x\sqrt{-x^{2}+1}-2i\sqrt{\pi}\,\arcsin\left(x\right)\right)}{4\sqrt{\pi}}$	32
trager	$\frac{x\sqrt{-x^{2}+1}}{2}+\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-x^{2}+1}+x\right)}{2}$	41

input
```
int((-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(-x^2+1)^(1/2)+1/2*arcsin(x)
```

3.872.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}\,\sqrt{-x^{2}+1}x - \arctan\left(\frac{\sqrt{-x^{2}+1}-1}{x}\right)$$

input
```
integrate((-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-x^2 + 1)*x - arctan((sqrt(-x^2 + 1) - 1)/x)
```

3.872.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \sqrt{1-x^2}\,dx = \frac{x\sqrt{1-x^2}}{2} + \frac{\operatorname{asin}(x)}{2}$$

input
```
integrate((-x**2+1)**(1/2),x)
```

output
```
x*sqrt(1 - x**2)/2 + asin(x)/2
```

3.872.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}\sqrt{-x^2+1}x + \frac{1}{2}\arcsin(x)$$

input
```
integrate((-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

3.872.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}\sqrt{-x^2+1}x + \frac{1}{2}\arcsin(x)$$

input
```
integrate((-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

3.872.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \sqrt{1-x^2}\,dx = \frac{\operatorname{asin}(x)}{2} + \frac{x\sqrt{1-x^2}}{2}$$

input
```
int((1 - x^2)^(1/2),x)
```

output
```
asin(x)/2 + (x*(1 - x^2)^(1/2))/2
```

3.873 $\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}}\,dx$

3.873.1 Optimal result

Integrand size = 21, antiderivative size = 23

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}}\,dx = \frac{1}{2}x\sqrt{1-x^2} + \frac{\arcsin(x)}{2}$$

output
```
1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

3.873.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 50 vs. 2(23) = 46.

Time = 0.61 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.17

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}}\,dx = \frac{1}{2}\left(\frac{x\sqrt{1-x^4}}{\sqrt{1+x^2}} + \arctan\left(\frac{x\sqrt{1+x^2}}{\sqrt{1-x^4}}\right)\right)$$

input
```
Integrate[Sqrt[1 - x^4]/Sqrt[1 + x^2],x]
```

output
```
((x*Sqrt[1 - x^4])/Sqrt[1 + x^2] + ArcTan[(x*Sqrt[1 + x^2])/Sqrt[1 - x^4]]
)/2
```

3.873.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1386, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^4}}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{1386}$$

$$\int \sqrt{1-x^2}dx$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx+\frac{1}{2}\sqrt{1-x^2}x$$

$$\Big\downarrow \text{223}$$

$$\frac{\arcsin(x)}{2}+\frac{1}{2}\sqrt{1-x^2}x$$

input
```
Int[Sqrt[1 - x^4]/Sqrt[1 + x^2],x]
```

output
```
(x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

3.873.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1386
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Simp[(-e^2/c)^q   Int[u*(d - e*x^n)^p, x], x] /; FreeQ[{a, c,
d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && EqQ[p + q, 0
] && GtQ[d, 0] && LtQ[c, 0] && GtQ[e^2, 0]
```

3.873.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(17) = 34$.

Time = 1.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

method	result	size
default	$\frac{\sqrt{-x^4+1}\left(x\sqrt{-x^2+1}+\arcsin(x)\right)}{2\sqrt{x^2+1}\,\sqrt{-x^2+1}}$	42
risch	$-\frac{x(x^2-1)\sqrt{\frac{-x^4+1}{x^2+1}}\,\sqrt{x^2+1}}{2\sqrt{-x^2+1}\,\sqrt{-x^4+1}}+\frac{\arcsin(x)\sqrt{\frac{-x^4+1}{x^2+1}}\,\sqrt{x^2+1}}{2\sqrt{-x^4+1}}$	89

input
```
int((-x^4+1)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-x^4+1)^(1/2)/(x^2+1)^(1/2)*(x*(-x^2+1)^(1/2)+arcsin(x))/(-x^2+1)^(1/
2)
```

3.873.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(17) = 34$.

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.61

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{-x^4+1}\sqrt{x^2+1}x-(x^2+1)\arctan\left(\frac{\sqrt{-x^4+1}\sqrt{x^2+1}}{x^3+x}\right)}{2\,(x^2+1)}$$

input
```
integrate((-x^4+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(-x^4 + 1)*sqrt(x^2 + 1)*x - (x^2 + 1)*arctan(sqrt(-x^4 + 1)*sqrt
(x^2 + 1)/(x^3 + x)))/(x^2 + 1)
```

3.873.6 Sympy [F]

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{-(x-1)(x+1)(x^2+1)}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((-x**4+1)**(1/2)/(x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))/sqrt(x**2 + 1), x)
```

3.873.7 Maxima [F]

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{-x^4+1}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((-x^4+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^4 + 1)/sqrt(x^2 + 1), x)
```

3.873.8 Giac [F]

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{-x^4+1}}{\sqrt{x^2+1}}\,dx$$

input
```
integrate((-x^4+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^4 + 1)/sqrt(x^2 + 1), x)
```

3.873.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}}\,dx=\int \frac{\sqrt{1-x^4}}{\sqrt{x^2+1}}\,dx$$

input
```
int((1 - x^4)^(1/2)/(x^2 + 1)^(1/2),x)
```

output
```
int((1 - x^4)^(1/2)/(x^2 + 1)^(1/2), x)
```

3.874 $\int \sqrt{1+x^2}\,dx$

3.874.1 Optimal result

Integrand size = 9, antiderivative size = 21

$$\int \sqrt{1+x^2}\,dx = \frac{1}{2}x\sqrt{1+x^2}+\frac{\operatorname{arcsinh}(x)}{2}$$

output
```
1/2*arcsinh(x)+1/2*x*(x^2+1)^(1/2)
```

3.874.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

$$\int \sqrt{1+x^2}\,dx = \frac{1}{2}x\sqrt{1+x^2}-\frac{1}{2}\log\left(-x+\sqrt{1+x^2}\right)$$

input
```
Integrate[Sqrt[1 + x^2],x]
```

output
```
(x*Sqrt[1 + x^2])/2 - Log[-x + Sqrt[1 + x^2]]/2
```

3.874.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^2+1}\,dx$$

$$\Big\downarrow \ 211$$

$$\frac{1}{2}\int \frac{1}{\sqrt{x^2+1}}dx+\frac{1}{2}\sqrt{x^2+1}x$$

$$\Big\downarrow \ 222$$

$$\frac{\text{arcsinh}(x)}{2}+\frac{1}{2}\sqrt{x^2+1}x$$

input
```
Int[Sqrt[1 + x^2],x]
```

output
```
(x*Sqrt[1 + x^2])/2 + ArcSinh[x]/2
```

3.874.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

3.874.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{\operatorname{arcsinh}(x)}{2}+\frac{x\sqrt{x^2+1}}{2}$	16
risch	$\frac{\operatorname{arcsinh}(x)}{2}+\frac{x\sqrt{x^2+1}}{2}$	16
trager	$\frac{x\sqrt{x^2+1}}{2}+\frac{\ln\left(x+\sqrt{x^2+1}\right)}{2}$	24
meijerg	$-\frac{-2\sqrt{\pi}\,x\sqrt{x^2+1}-2\sqrt{\pi}\,\operatorname{arcsinh}(x)}{4\sqrt{\pi}}$	27
pseudoelliptic	$\frac{x\sqrt{x^2+1}}{2}+\frac{\ln\left(\frac{x+\sqrt{x^2+1}}{x}\right)}{4}-\frac{\ln\left(\frac{\sqrt{x^2+1}-x}{x}\right)}{4}$	46

input
```
int((x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsinh(x)+1/2*x*(x^2+1)^(1/2)
```

3.874.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \sqrt{1+x^2}\,dx = \frac{1}{2}\sqrt{x^2+1}x - \frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)$$

input
```
integrate((x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

3.874.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \sqrt{1+x^2}\,dx = \frac{x\sqrt{x^2+1}}{2} + \frac{\operatorname{asinh}(x)}{2}$$

input
```
integrate((x**2+1)**(1/2),x)
```

output
```
x*sqrt(x**2 + 1)/2 + asinh(x)/2
```

3.874.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \sqrt{1+x^2}\,dx = \frac{1}{2}\sqrt{x^2+1}x + \frac{1}{2}\,\operatorname{arsinh}(x)$$

input
```
integrate((x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(x^2 + 1)*x + 1/2*arcsinh(x)
```

3.874.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \sqrt{1+x^2}\,dx = \frac{1}{2}\sqrt{x^2+1}x - \frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)$$

input
```
integrate((x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

3.874.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \sqrt{1+x^2}\,dx = \frac{\operatorname{asinh}(x)}{2} + \frac{x\sqrt{x^2+1}}{2}$$

input
```
int((x^2 + 1)^(1/2),x)
```

output
```
asinh(x)/2 + (x*(x^2 + 1)^(1/2))/2
```

3.875 $\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}} \, dx$

3.875.1 Optimal result

Integrand size = 23, antiderivative size = 21

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}} \, dx = \frac{1}{2}x\sqrt{1+x^2} + \frac{\operatorname{arcsinh}(x)}{2}$$

output
```
1/2*arcsinh(x)+1/2*x*(x^2+1)^(1/2)
```

3.875.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 70 vs. 2(21) = 42.

Time = 0.62 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.33

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}} \, dx = \frac{1}{2}\left(\frac{x\sqrt{1-x^4}}{\sqrt{1-x^2}} + \log\left(1-x^2\right) - \log\left(-x+x^3+\sqrt{1-x^2}\sqrt{1-x^4}\right)\right)$$

input
```
Integrate[Sqrt[1 - x^4]/Sqrt[1 - x^2],x]
```

output
```
((x*Sqrt[1 - x^4])/Sqrt[1 - x^2] + Log[1 - x^2] - Log[-x + x^3 + Sqrt[1 -
x^2]*Sqrt[1 - x^4]])/2
```

3.875.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1386, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}dx$$
$$\Big\downarrow \text{1386}$$
$$\int \sqrt{x^2+1}dx$$
$$\Big\downarrow \text{211}$$
$$\frac{1}{2}\int \frac{1}{\sqrt{x^2+1}}dx+\frac{1}{2}\sqrt{x^2+1}x$$
$$\Big\downarrow \text{222}$$
$$\frac{\operatorname{arcsinh}(x)}{2}+\frac{1}{2}\sqrt{x^2+1}x$$

input
```
Int[Sqrt[1 - x^4]/Sqrt[1 - x^2],x]
```

output
```
(x*Sqrt[1 + x^2])/2 + ArcSinh[x]/2
```

3.875.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1386
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Simp[(-e^2/c)^q   Int[u*(d - e*x^n)^p, x], x] /; FreeQ[{a, c,
d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && EqQ[p + q, 0
] && GtQ[d, 0] && LtQ[c, 0] && GtQ[e^2, 0]
```

3.875.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(15) = 30$.

Time = 1.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.24

method	result	size
default	$-\frac{\sqrt{-x^4+1}\,\sqrt{-x^2+1}\,\left(x\sqrt{x^2+1}+\operatorname{arcsinh}(x)\right)}{2(x^2-1)\sqrt{x^2+1}}$	47
risch	$-\frac{x\sqrt{x^2+1}\,\sqrt{\frac{(-x^2+1)(-x^4+1)}{(x^2-1)^2}}\,(x^2-1)}{2\sqrt{-x^4+1}\,\sqrt{-x^2+1}}-\frac{\operatorname{arcsinh}(x)\sqrt{\frac{(-x^2+1)(-x^4+1)}{(x^2-1)^2}}\,(x^2-1)}{2\sqrt{-x^4+1}\,\sqrt{-x^2+1}}$	110

input
```
int((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-x^4+1)^(1/2)*(-x^2+1)^(1/2)*(x*(x^2+1)^(1/2)+arcsinh(x))/(x^2-1)/(x
^2+1)^(1/2)
```

3.875.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(15) = 30$.

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 5.71

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx = -\frac{2\sqrt{-x^4+1}\sqrt{-x^2+1}x+(x^2-1)\log\left(\frac{x^3+\sqrt{-x^4+1}\sqrt{-x^2+1}-x}{x^3-x}\right)-(x^2-1)\log\left(-\frac{x^3-\sqrt{-x^4+1}\sqrt{-x^2+1}-x}{x^3-x}\right)}{4(x^2-1)}$$

input
```
integrate((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(-x^4 + 1)*sqrt(-x^2 + 1)*x + (x^2 - 1)*log((x^3 + sqrt(-x^4 +
 1)*sqrt(-x^2 + 1) - x)/(x^3 - x)) - (x^2 - 1)*log(-(x^3 - sqrt(-x^4 + 1)*
sqrt(-x^2 + 1) - x)/(x^3 - x)))/(x^2 - 1)
```

3.875.6 Sympy [F]

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{-(x-1)(x+1)(x^2+1)}}{\sqrt{-(x-1)(x+1)}}\,dx$$

input
```
integrate((-x**4+1)**(1/2)/(-x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))/sqrt(-(x - 1)*(x + 1)), x)
```

3.875.7 Maxima [F]

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{-x^4+1}}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^4 + 1)/sqrt(-x^2 + 1), x)
```

3.875.8 Giac [F]

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{-x^4+1}}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^4 + 1)/sqrt(-x^2 + 1), x)
```

3.875.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx$$

input
```
int((1 - x^4)^(1/2)/(1 - x^2)^(1/2),x)
```

output
```
int((1 - x^4)^(1/2)/(1 - x^2)^(1/2), x)
```

3.876 $\int \left(\frac{a+b+cx^2}{d}\right)^m dx$

3.876.1 Optimal result

Integrand size = 14, antiderivative size = 49

$$\int \left(\frac{a+b+cx^2}{d}\right)^m dx = \frac{dx\left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^{1+m}\text{Hypergeometric2F1}\left(1,\frac{3}{2}+m,\frac{3}{2},-\frac{cx^2}{a+b}\right)}{a+b}$$

output
```
d*x*((a+b)/d+x^2*c/d)^(1+m)*hypergeom([1, 3/2+m],[3/2],-c*x^2/(a+b))/(a+b)
```

3.876.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \left(\frac{a+b+cx^2}{d}\right)^m dx = x\left(\frac{a+b+cx^2}{d}\right)^m\left(1+\frac{cx^2}{a+b}\right)^{-m}\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},-\frac{cx^2}{a+b}\right)$$

input
```
Integrate[((a + b + c*x^2)/d)^m,x]
```

output
```
(x*((a + b + c*x^2)/d)^m*Hypergeometric2F1[1/2, -m, 3/2, -((c*x^2)/(a + b)
)])/(1 + (c*x^2)/(a + b))^m
```

3.876.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2072, 238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{a+b+cx^2}{d}\right)^m dx$$

$$\Big\downarrow \text{2072}$$

$$\int \left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^m dx$$

$$\Big\downarrow \text{238}$$

$$\left(\frac{cx^2}{a+b}+1\right)^{-m}\left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^m\int \left(\frac{cx^2}{a+b}+1\right)^m dx$$

$$\Big\downarrow \text{237}$$

$$x\left(\frac{cx^2}{a+b}+1\right)^{-m}\left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^m \text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},-\frac{cx^2}{a+b}\right)$$

input
```
Int[((a + b + c*x^2)/d)^m,x]
```

output
```
(x*((a + b)/d + (c*x^2)/d)^m*Hypergeometric2F1[1/2, -m, 3/2, -((c*x^2)/(a
+ b))])/(1 + (c*x^2)/(a + b))^m
```

3.876.3.1 Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

rule 2072
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] && !BinomialMatchQ[u, x]
```

3.876.4 Maple [F]

$$\int \left(\frac{c\,x^2+a+b}{d}\right)^m dx$$

input
```
int(((c*x^2+a+b)/d)^m,x)
```

output
```
int(((c*x^2+a+b)/d)^m,x)
```

3.876.5 Fricas [F]

$$\int \left(\frac{a+b+cx^2}{d}\right)^m dx = \int \left(\frac{cx^2+a+b}{d}\right)^m dx$$

input
```
integrate(((c*x^2+a+b)/d)^m,x, algorithm="fricas")
```

output
```
integral(((c*x^2 + a + b)/d)^m, x)
```

3.876.6 Sympy [F]

$$\int \left(\frac{a+b+cx^2}{d}\right)^m dx = \int \left(\frac{a+b+cx^2}{d}\right)^m dx$$

input
```
integrate(((c*x**2+a+b)/d)**m,x)
```

output
```
Integral(((a + b + c*x**2)/d)**m, x)
```

3.876.7 Maxima [F]

$$\int \left(\frac{a+b+cx^2}{d}\right)^m dx = \int \left(\frac{cx^2+a+b}{d}\right)^m dx$$

input
```
integrate(((c*x^2+a+b)/d)^m,x, algorithm="maxima")
```

output
```
integrate(((c*x^2 + a + b)/d)^m, x)
```

3.876.8 Giac [F]

$$\int \left(\frac{a+b+cx^2}{d}\right)^m dx = \int \left(\frac{cx^2+a+b}{d}\right)^m dx$$

input
```
integrate(((c*x^2+a+b)/d)^m,x, algorithm="giac")
```

output
```
integrate(((c*x^2 + a + b)/d)^m, x)
```

3.876.9 Mupad [B] (verification not implemented)

Time = 23.47 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10

$$\int \left(\frac{a+b+cx^2}{d}\right)^m dx = \frac{x\left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^m {}_2F_1\left(\frac{1}{2},-m;\ \frac{3}{2};\ -\frac{cx^2}{a+b}\right)}{\left(\frac{cx^2}{a+b}+1\right)^m}$$

input
```
int(((a + b + c*x^2)/d)^m,x)
```

output
```
(x*((a + b)/d + (c*x^2)/d)^m*hypergeom([1/2, -m], 3/2, -(c*x^2)/(a + b)))/
((c*x^2)/(a + b) + 1)^m
```

3.877 $\int \frac{1}{x-\sqrt{1+x^2}}\,dx$

3.877.1 Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{1}{x-\sqrt{1+x^2}}\,dx=-\frac{x^2}{2}-\frac{1}{2}x\sqrt{1+x^2}-\frac{\operatorname{arcsinh}(x)}{2}$$

output
```
-1/2*x^2-1/2*arcsinh(x)-1/2*x*(x^2+1)^(1/2)
```

3.877.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int \frac{1}{x-\sqrt{1+x^2}}\,dx=\frac{1}{2}\left(-x\left(x+\sqrt{1+x^2}\right)+\log\left(-x+\sqrt{1+x^2}\right)\right)$$

input
```
Integrate[(x - Sqrt[1 + x^2])^(-1),x]
```

output
```
(-(x*(x + Sqrt[1 + x^2])) + Log[-x + Sqrt[1 + x^2]])/2
```

3.877.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2531, 15, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x-\sqrt{x^2+1}}dx$$

$$\Big\downarrow \ 2531$$

$$-\int \sqrt{x^2+1}dx-\int xdx$$

$$\Big\downarrow \ 15$$

$$-\int \sqrt{x^2+1}dx-\frac{x^2}{2}$$

$$\Big\downarrow \ 211$$

$$-\frac{1}{2}\int \frac{1}{\sqrt{x^2+1}}dx-\frac{x^2}{2}-\frac{1}{2}\sqrt{x^2+1}x$$

$$\Big\downarrow \ 222$$

$$-\frac{\operatorname{arcsinh}(x)}{2}-\frac{x^2}{2}-\frac{1}{2}\sqrt{x^2+1}x$$

input
```
Int[(x - Sqrt[1 + x^2])^(-1),x]
```

output
```
-1/2*x^2 - (x*Sqrt[1 + x^2])/2 - ArcSinh[x]/2
```

3.877.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2531
```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> Simp[-b/(a*d)   Int[u*x^n, x], x] + Simp[1/(a*c)   Int[u*Sqrt[a + b*
x^(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 -
 d^2, 0]
```

3.877.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
default	$-\frac{x^2}{2}-\frac{\operatorname{arcsinh}(x)}{2}-\frac{x\sqrt{x^2+1}}{2}$	21
trager	$-\frac{x^2}{2}-\frac{x\sqrt{x^2+1}}{2}+\frac{\ln\left(x-\sqrt{x^2+1}\right)}{2}$	31

input
```
int(1/(x-(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2-1/2*arcsinh(x)-1/2*x*(x^2+1)^(1/2)
```

3.877.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x-\sqrt{1+x^2}}\,dx = -\frac{1}{2}\,x^2-\frac{1}{2}\,\sqrt{x^2+1}x+\frac{1}{2}\,\log\left(-x+\sqrt{x^2+1}\right)$$

input
```
integrate(1/(x-(x^2+1)^(1/2)),x, algorithm="fricas")
```

output
```
-1/2*x^2 - 1/2*sqrt(x^2 + 1)*x + 1/2*log(-x + sqrt(x^2 + 1))
```

3.877.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(22) = 44$.

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.32

$$\int \frac{1}{x-\sqrt{1+x^2}}\,dx = -\frac{x\operatorname{asinh}(x)}{2x-2\sqrt{x^2+1}}+\frac{\sqrt{x^2+1}\operatorname{asinh}(x)}{2x-2\sqrt{x^2+1}}+\frac{\sqrt{x^2+1}}{2x-2\sqrt{x^2+1}}$$

input
```
integrate(1/(x-(x**2+1)**(1/2)),x)
```

output
```
-x*asinh(x)/(2*x - 2*sqrt(x**2 + 1)) + sqrt(x**2 + 1)*asinh(x)/(2*x - 2*sq
rt(x**2 + 1)) + sqrt(x**2 + 1)/(2*x - 2*sqrt(x**2 + 1))
```

3.877.7 Maxima [F]

$$\int \frac{1}{x-\sqrt{1+x^2}}\,dx = \int \frac{1}{x-\sqrt{x^2+1}}\,dx$$

input
```
integrate(1/(x-(x^2+1)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(x - sqrt(x^2 + 1)), x)
```

3.877.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{x-\sqrt{1+x^2}}\,dx = -\frac{1}{2}\,x^2-\frac{1}{2}\,\sqrt{x^2+1}x+\frac{1}{2}\,\log\left(-x+\sqrt{x^2+1}\right)$$

input
```
integrate(1/(x-(x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
-1/2*x^2 - 1/2*sqrt(x^2 + 1)*x + 1/2*log(-x + sqrt(x^2 + 1))
```

3.877.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \frac{1}{x-\sqrt{1+x^2}}\,dx=-\frac{\operatorname{asinh}(x)}{2}-\frac{x\,\sqrt{x^2+1}}{2}-\frac{x^2}{2}$$

input
```
int(1/(x - (x^2 + 1)^(1/2)),x)
```

output
```
- asinh(x)/2 - (x*(x^2 + 1)^(1/2))/2 - x^2/2
```

3.878 $\int \frac{1}{x-\sqrt{1-x^2}}\,dx$

3.878.1 Optimal result

Integrand size = 17, antiderivative size = 37

$$\int \frac{1}{x-\sqrt{1-x^2}}\,dx = -\frac{\arcsin(x)}{2}-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{1-x^2}}\right)+\frac{1}{4}\log\left(1-2x^2\right)$$

output
```
-1/2*arcsin(x)-1/2*arctanh(x/(-x^2+1)^(1/2))+1/4*ln(-2*x^2+1)
```

3.878.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int \frac{1}{x-\sqrt{1-x^2}}\,dx = \arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)+\frac{1}{2}\log\left(-x+\sqrt{1-x^2}\right)$$

input
```
Integrate[(x - Sqrt[1 - x^2])^(-1),x]
```

output
```
ArcTan[Sqrt[1 - x^2]/(1 + x)] + Log[-x + Sqrt[1 - x^2]]/2
```

3.878.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x-\sqrt{1-x^2}} \, dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{x}{2x^2-1}+\frac{\sqrt{1-x^2}}{2x^2-1}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\arcsin(x)}{2}-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{1-x^2}}\right)+\frac{1}{4}\log\left(1-2x^2\right)$$

input
```
Int[(x - Sqrt[1 - x^2])^(-1),x]
```

output
```
-1/2*ArcSin[x] - ArcTanh[x/Sqrt[1 - x^2]]/2 + Log[1 - 2*x^2]/4
```

3.878.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.878.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(29) = 58$.

Time = 0.30 (sec) , antiderivative size = 197, normalized size of antiderivative = 5.32

method	result
default	$\frac{\ln(2x^2-1)}{4} + \frac{\sqrt{2}\left(\frac{\sqrt{-4\left(x-\frac{\sqrt{2}}{2}\right)^2-4\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}{4} - \frac{\sqrt{2}\arcsin(x)}{4} - \frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(1-\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}\right)\sqrt{2}}{\sqrt{-4\left(x-\frac{\sqrt{2}}{2}\right)^2-4\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}\right)}{4}\right)}{2} - \frac{\sqrt{2}\left(\sqrt{-4\right.}{}$
trager	$\operatorname{RootOf}\left(2\_Z^2+2\_Z+1\right)\ln\left(-\frac{\sqrt{-x^2+1}+x}{2x^2-1}\right) - \ln\left(\frac{-2\operatorname{RootOf}\left(2\_Z^2+2\_Z+1\right)^2x^2-4\operatorname{RootOf}\left(2\_Z^2+2\_Z\right.}{2x^2-}\right.$

input
```
int(1/(x-(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(2*x^2-1)+1/2*2^(1/2)*(1/4*(-4*(x-1/2*2^(1/2))^2-4*(x-1/2*2^(1/2))*2
^(1/2)+2)^(1/2)-1/4*2^(1/2)*arcsin(x)-1/4*2^(1/2)*arctanh((1-(x-1/2*2^(1/2
))*2^(1/2))*2^(1/2)/(-4*(x-1/2*2^(1/2))^2-4*(x-1/2*2^(1/2))*2^(1/2)+2)^(1/
2)))-1/2*2^(1/2)*(1/4*(-4*(x+1/2*2^(1/2))^2+4*(x+1/2*2^(1/2))*2^(1/2)+2)^(
1/2)+1/4*2^(1/2)*arcsin(x)-1/4*2^(1/2)*arctanh(((x+1/2*2^(1/2))*2^(1/2)+1)
*2^(1/2)/(-4*(x+1/2*2^(1/2))^2+4*(x+1/2*2^(1/2))*2^(1/2)+2)^(1/2)))
```

3.878.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(29) = 58$.

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.27

$$\int \frac{1}{x-\sqrt{1-x^2}}\,dx = \arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) + \frac{1}{4}\log\left(2\,x^2-1\right) + \frac{1}{4}\log\left(-\frac{x^2+\sqrt{-x^2+1}(x+1)-x-1}{x^2}\right) - \frac{1}{4}\log\left(-\frac{x^2-\sqrt{-x^2+1}(x-1)+x-1}{x^2}\right)$$

input
```
integrate(1/(x-(-x^2+1)^(1/2)),x, algorithm="fricas")
```

output
```
arctan((sqrt(-x^2 + 1) - 1)/x) + 1/4*log(2*x^2 - 1) + 1/4*log(-(x^2 + sqrt
(-x^2 + 1)*(x + 1) - x - 1)/x^2) - 1/4*log(-(x^2 - sqrt(-x^2 + 1)*(x - 1)
+ x - 1)/x^2)
```

3.878.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.46

$$\int \frac{1}{x-\sqrt{1-x^2}}\,dx = \frac{\log\left(-x+\sqrt{1-x^2}\right)}{2} - \frac{\operatorname{asin}(x)}{2}$$

input
```
integrate(1/(x-(-x**2+1)**(1/2)),x)
```

output
```
log(-x + sqrt(1 - x**2))/2 - asin(x)/2
```

3.878.7 Maxima [F]

$$\int \frac{1}{x-\sqrt{1-x^2}}\,dx = \int \frac{1}{x-\sqrt{-x^2+1}}\,dx$$

input
```
integrate(1/(x-(-x^2+1)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(x - sqrt(-x^2 + 1)), x)
```

3.878.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(29) = 58.

Time = 0.33 (sec) , antiderivative size = 140, normalized size of antiderivative = 3.78

$$\int \frac{1}{x-\sqrt{1-x^2}}\,dx = -\frac{1}{4}\,\pi\mathrm{sgn}(x) - \frac{1}{2}\,\arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right) + \frac{1}{4}\,\log\left(\left|x+\frac{1}{2}\,\sqrt{2}\right|\right) + \frac{1}{4}\,\log\left(\left|x-\frac{1}{2}\,\sqrt{2}\right|\right) - \frac{1}{4}\,\log\left(\left|-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}+2\right|\right) + \frac{1}{4}\,\log\left(\left|-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}-2\right|\right)$$

input
```
integrate(1/(x-(-x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
-1/4*pi*sgn(x) - 1/2*arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(
-x^2 + 1) - 1)) + 1/4*log(abs(x + 1/2*sqrt(2))) + 1/4*log(abs(x - 1/2*sqrt
(2))) - 1/4*log(abs(-x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/x + 2))
 + 1/4*log(abs(-x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/x - 2))
```

3.878.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.84

$$\int \frac{1}{x-\sqrt{1-x^2}}\,dx = \frac{\ln\left(x-\frac{\sqrt{2}}{2}\right)}{4} + \frac{\ln\left(x+\frac{\sqrt{2}}{2}\right)}{4} - \frac{\ln\left(\frac{\sqrt{2}\left(\frac{\sqrt{2}\,x}{2}-1\right)\,1\mathrm{i}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\frac{\sqrt{2}}{2}}\right)}{4} + \frac{\ln\left(\frac{\sqrt{2}\left(\frac{\sqrt{2}\,x}{2}+1\right)\,1\mathrm{i}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\frac{\sqrt{2}}{2}}\right)}{4} - \frac{\mathrm{asin}(x)}{2}$$

input
```
int(1/(x - (1 - x^2)^(1/2)),x)
```

output
```
log(x - 2^(1/2)/2)/4 + log(x + 2^(1/2)/2)/4 - log((2^(1/2)*((2^(1/2)*x)/2
- 1)*1i - (1 - x^2)^(1/2)*1i)/(x - 2^(1/2)/2))/4 + log((2^(1/2)*((2^(1/2)*
x)/2 + 1)*1i + (1 - x^2)^(1/2)*1i)/(x + 2^(1/2)/2))/4 - asin(x)/2
```

3.879 $\int \frac{1}{x-\sqrt{1+2x^2}}\,dx$

3.879.1 Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \frac{1}{x-\sqrt{1+2x^2}}\,dx=-\sqrt{2}\text{arcsinh}\left(\sqrt{2}x\right)+\text{arctanh}\left(\frac{x}{\sqrt{1+2x^2}}\right)-\frac{1}{2}\log\left(1+x^2\right)$$

output
```
arctanh(x/(2*x^2+1)^(1/2))-1/2*ln(x^2+1)-arcsinh(x*2^(1/2))*2^(1/2)
```

3.879.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.70

$$\int \frac{1}{x-\sqrt{1+2x^2}}\,dx=\left(1+\sqrt{2}\right)\log\left(2\left(-1+\sqrt{2}\right)x+\left(-2+\sqrt{2}\right)\sqrt{1+2x^2}\right)-\log\left(-2+\sqrt{2}-2x^2+x\sqrt{2+4x^2}\right)$$

input
```
Integrate[(x - Sqrt[1 + 2*x^2])^(-1),x]
```

output
```
(1 + Sqrt[2])*Log[2*(-1 + Sqrt[2])*x + (-2 + Sqrt[2])*Sqrt[1 + 2*x^2]] - L
og[-2 + Sqrt[2] - 2*x^2 + x*Sqrt[2 + 4*x^2]]
```

3.879.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x-\sqrt{2x^2+1}}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{x}{x^2+1}-\frac{\sqrt{2x^2+1}}{x^2+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\sqrt{2}\text{arcsinh}\left(\sqrt{2}x\right)+\text{arctanh}\left(\frac{x}{\sqrt{2x^2+1}}\right)-\frac{1}{2}\log\left(x^2+1\right)$$

input
```
Int[(x - Sqrt[1 + 2*x^2])^(-1),x]
```

output
```
-(Sqrt[2]*ArcSinh[Sqrt[2]*x]) + ArcTanh[x/Sqrt[1 + 2*x^2]] - Log[1 + x^2]/
2
```

3.879.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.879.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

method	result
default	$\operatorname{arctanh}\left(\frac{x}{\sqrt{2x^2+1}}\right)-\frac{\ln(x^2+1)}{2}-\operatorname{arcsinh}\left(x\sqrt{2}\right)\sqrt{2}$
trager	$\operatorname{RootOf}\left(\_Z^2-2\_Z-1\right)\ln\left(\frac{x+\sqrt{2x^2+1}}{x^2+1}\right)-\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2-2\_Z-1\right)^2x^2+3\operatorname{RootOf}\left(\_Z^2-2\_Z-1\right)\sqrt{2x^2}\right.$

input
```
int(1/(x-(2*x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
arctanh(x/(2*x^2+1)^(1/2))-1/2*ln(x^2+1)-arcsinh(x*2^(1/2))*2^(1/2)
```

3.879.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(32) = 64.

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.25

$$\int \frac{1}{x-\sqrt{1+2x^2}}\,dx = \sqrt{2}\log\left(\sqrt{2}x-\sqrt{2\,x^2+1}\right)-\frac{1}{2}\log\left(x^2+1\right) - \frac{1}{2}\log\left(\frac{2\,x^2-\sqrt{2\,x^2+1}(x+1)+x+1}{x^2}\right) + \frac{1}{2}\log\left(\frac{2\,x^2+\sqrt{2\,x^2+1}(x-1)-x+1}{x^2}\right)$$

input
```
integrate(1/(x-(2*x^2+1)^(1/2)),x, algorithm="fricas")
```

output
```
sqrt(2)*log(sqrt(2)*x - sqrt(2*x^2 + 1)) - 1/2*log(x^2 + 1) - 1/2*log((2*x
^2 - sqrt(2*x^2 + 1)*(x + 1) + x + 1)/x^2) + 1/2*log((2*x^2 + sqrt(2*x^2 +
 1)*(x - 1) - x + 1)/x^2)
```

3.879.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.68

$$\int \frac{1}{x-\sqrt{1+2x^2}}\,dx=-\log\left(-x+\sqrt{2x^2+1}\right)-\sqrt{2}\operatorname{asinh}\left(\sqrt{2}x\right)$$

input
```
integrate(1/(x-(2*x**2+1)**(1/2)),x)
```

output
```
-log(-x + sqrt(2*x**2 + 1)) - sqrt(2)*asinh(sqrt(2)*x)
```

3.879.7 Maxima [F]

$$\int \frac{1}{x-\sqrt{1+2x^2}}\,dx=\int \frac{1}{x-\sqrt{2\,x^2+1}}\,dx$$

input
```
integrate(1/(x-(2*x^2+1)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(x - sqrt(2*x^2 + 1)), x)
```

3.879.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(32) = 64.

Time = 0.38 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.20

$$\int \frac{1}{x-\sqrt{1+2x^2}}\,dx=\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{2\,x^2+1}\right)+\frac{1}{2}\log\left(\left(\sqrt{2}x-\sqrt{2\,x^2+1}\right)^2+2\sqrt{2}+3\right)-\frac{1}{2}\log\left(\left(\sqrt{2}x-\sqrt{2\,x^2+1}\right)^2-2\sqrt{2}+3\right)-\frac{1}{2}\log\left(x^2+1\right)$$

input
```
integrate(1/(x-(2*x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
sqrt(2)*log(-sqrt(2)*x + sqrt(2*x^2 + 1)) + 1/2*log((sqrt(2)*x - sqrt(2*x^
2 + 1))^2 + 2*sqrt(2) + 3) - 1/2*log((sqrt(2)*x - sqrt(2*x^2 + 1))^2 - 2*s
qrt(2) + 3) - 1/2*log(x^2 + 1)
```

3.879.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.42

$$\int \frac{1}{x-\sqrt{1+2x^2}}\,dx = -\ln(x-\mathrm{i}) - \frac{\ln\left(x-\frac{\sqrt{2}\sqrt{x^2+\frac{1}{2}}}{2}+\frac{1}{2}\mathrm{i}\right)}{2} + \frac{\ln\left(x+\frac{\sqrt{2}\sqrt{x^2+\frac{1}{2}}}{2}-\frac{1}{2}\mathrm{i}\right)}{2} - \sqrt{2}\,\mathrm{asinh}\left(\sqrt{2}\,x\right)$$

input
```
int(1/(x - (2*x^2 + 1)^(1/2)),x)
```

output
```
log(x + (2^(1/2)*(x^2 + 1/2)^(1/2))/2 - 1i/2)/2 - log(x - (2^(1/2)*(x^2 +
1/2)^(1/2))/2 + 1i/2)/2 - log(x - 1i) - 2^(1/2)*asinh(2^(1/2)*x)
```

3.880 $\int \frac{2x-x^3+x^2\sqrt{2-x^2}}{-2+2x^2}\,dx$

3.880.1 Optimal result

Integrand size = 34, antiderivative size = 54

$$\int \frac{2x-x^3+x^2\sqrt{2-x^2}}{-2+2x^2}\,dx = -\frac{x^2}{4}+\frac{1}{4}x\sqrt{2-x^2}-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{2-x^2}}\right)+\frac{1}{4}\log\left(1-x^2\right)$$

output
```
-1/4*x^2-1/2*arctanh(x/(-x^2+2)^(1/2))+1/4*ln(-x^2+1)+1/4*x*(-x^2+2)^(1/2)
```

3.880.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.46

$$\int \frac{2x-x^3+x^2\sqrt{2-x^2}}{-2+2x^2}\,dx = \frac{1}{2}\left(\frac{1}{2}x\left(-x+\sqrt{2-x^2}\right)-\log\left(-2+\sqrt{4-2x^2}\right)+\log\left(-2+x^2+\sqrt{4-2x^2}+x\left(-\sqrt{2}+\sqrt{2-x^2}\right)\right)\right)$$

input
```
Integrate[(2*x - x^3 + x^2*Sqrt[2 - x^2])/(-2 + 2*x^2),x]
```

output
```
((x*(-x + Sqrt[2 - x^2]))/2 - Log[-2 + Sqrt[4 - 2*x^2]] + Log[-2 + x^2 + S
qrt[4 - 2*x^2] + x*(-Sqrt[2] + Sqrt[2 - x^2])])/2
```

3.880.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-x^3+\sqrt{2-x^2}x^2+2x}{2x^2-2}dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{\sqrt{2-x^2}x^2}{2(x^2-1)}+\frac{x}{x^2-1}-\frac{x^3}{2(x^2-1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{2-x^2}}\right)-\frac{x^2}{4}+\frac{1}{4}\sqrt{2-x^2}x+\frac{1}{4}\log\left(1-x^2\right)$$

input
```
Int[(2*x - x^3 + x^2*Sqrt[2 - x^2])/(-2 + 2*x^2),x]
```

output
```
-1/4*x^2 + (x*Sqrt[2 - x^2])/4 - ArcTanh[x/Sqrt[2 - x^2]]/2 + Log[1 - x^2]
/4
```

3.880.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.880.4 Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result
trager	$-\frac{x^{2}}{4}+\frac{x\sqrt{-x^{2}+2}}{4}-\frac{\ln\left(-\frac{\sqrt{-x^{2}+2}+x}{(x-1)(x+1)}\right)}{2}$
default	$\frac{x\sqrt{-x^{2}+2}}{4}-\frac{\sqrt{-(x+1)^{2}+2x+3}}{4}+\frac{\operatorname{arctanh}\left(\frac{4+2x}{2\sqrt{-(x+1)^{2}+2x+3}}\right)}{4}+\frac{\sqrt{-(x-1)^{2}-2x+3}}{4}-\frac{\operatorname{arctanh}\left(\frac{4-2x}{2\sqrt{-(x-1)^{2}-2x+3}}\right)}{4}-$

input
```
int((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^2+1/4*x*(-x^2+2)^(1/2)-1/2*ln(-((-x^2+2)^(1/2)+x)/(x-1)/(x+1))
```

3.880.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.24

$$\int \frac{2x-x^3+x^2\sqrt{2-x^2}}{-2+2x^2}\,dx = -\frac{1}{4}\,x^{2}+\frac{1}{4}\,\sqrt{-x^{2}+2}x+\frac{1}{4}\,\log\left(x^{2}-1\right) -\frac{1}{8}\,\log\left(-\frac{\sqrt{-x^{2}+2}x+1}{x^{2}}\right)+\frac{1}{8}\,\log\left(\frac{\sqrt{-x^{2}+2}x-1}{x^{2}}\right)$$

input
```
integrate((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x, algorithm="fricas")
```

output
```
-1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(x^2 - 1) - 1/8*log(-(sqrt(-x^2 +
 2)*x + 1)/x^2) + 1/8*log((sqrt(-x^2 + 2)*x - 1)/x^2)
```

3.880.6 Sympy [F]

$$\int \frac{2x-x^3+x^2\sqrt{2-x^2}}{-2+2x^2}\,dx = -\frac{\int\left(-\frac{2x}{x^2-1}\right)dx+\int\frac{x^3}{x^2-1}\,dx+\int\left(-\frac{x^2\sqrt{2-x^2}}{x^2-1}\right)dx}{2}$$

input
```
integrate((2*x-x**3+x**2*(-x**2+2)**(1/2))/(2*x**2-2),x)
```

output
```
-(Integral(-2*x/(x**2 - 1), x) + Integral(x**3/(x**2 - 1), x) + Integral(-
x**2*sqrt(2 - x**2)/(x**2 - 1), x))/2
```

3.880.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(42) = 84$.

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.74

$$\int \frac{2x-x^3+x^2\sqrt{2-x^2}}{-2+2x^2}\,dx = -\frac{1}{4}\,x^2+\frac{1}{4}\,\sqrt{-x^2+2}x+\frac{1}{4}\,\log\left(x^2-1\right) + \frac{1}{4}\,\log\left(\frac{2\,\sqrt{-x^2+2}}{|2\,x+2|}+\frac{2}{|2\,x+2|}+1\right) - \frac{1}{4}\,\log\left(\frac{2\,\sqrt{-x^2+2}}{|2\,x-2|}+\frac{2}{|2\,x-2|}-1\right)$$

input
```
integrate((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x, algorithm="maxima")
```

output
```
-1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(x^2 - 1) + 1/4*log(2*sqrt(-x^2 +
 2)/abs(2*x + 2) + 2/abs(2*x + 2) + 1) - 1/4*log(2*sqrt(-x^2 + 2)/abs(2*x
- 2) + 2/abs(2*x - 2) - 1)
```

3.880.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(42) = 84$.

Time = 0.46 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.17

$$\int \frac{2x-x^3+x^2\sqrt{2-x^2}}{-2+2x^2}\,dx = -\frac{1}{4}\,x^2+\frac{1}{4}\,\sqrt{-x^2+2}x+\frac{1}{4}\,\log\left(\left|x^2-1\right|\right) - \frac{1}{4}\,\log\left(\left|\frac{x}{\sqrt{2}-\sqrt{-x^2+2}}-\frac{\sqrt{2}-\sqrt{-x^2+2}}{x}+2\right|\right) + \frac{1}{4}\,\log\left(\left|\frac{x}{\sqrt{2}-\sqrt{-x^2+2}}-\frac{\sqrt{2}-\sqrt{-x^2+2}}{x}-2\right|\right)$$

input
```
integrate((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x, algorithm="giac")
```

output
```
-1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(abs(x^2 - 1)) - 1/4*log(abs(x/(s
qrt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-x^2 + 2))/x + 2)) + 1/4*log(ab
s(x/(sqrt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-x^2 + 2))/x - 2))
```

3.880.9 Mupad [B] (verification not implemented)

Time = 21.96 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.59

$$\int \frac{2x - x^3 + x^2\sqrt{2 - x^2}}{-2 + 2x^2}\,dx = \frac{\ln(x-1)}{4} + \frac{\ln(x+1)}{4} - \frac{\ln\left(\frac{-x\,1\mathrm{i} + \sqrt{2-x^2}\,1\mathrm{i} + 2\mathrm{i}}{x-1}\right)}{4} + \frac{\ln\left(\frac{x\,1\mathrm{i} + \sqrt{2-x^2}\,1\mathrm{i} + 2\mathrm{i}}{x+1}\right)}{4} + \frac{x\sqrt{2-x^2}}{4} - \frac{x^2}{4}$$

input
```
int((2*x + x^2*(2 - x^2)^(1/2) - x^3)/(2*x^2 - 2),x)
```

output
```
log(x - 1)/4 + log(x + 1)/4 - log(((2 - x^2)^(1/2)*1i - x*1i + 2i)/(x - 1)
)/4 + log((x*1i + (2 - x^2)^(1/2)*1i + 2i)/(x + 1))/4 + (x*(2 - x^2)^(1/2)
)/4 - x^2/4
```

3.881 $\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx$

3.881.1 Optimal result

Integrand size = 30, antiderivative size = 60

$$\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx=-\frac{x^2}{4}+\frac{1}{4}x\sqrt{2-x^2}-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{2-x^2}}\right)+\frac{1}{4}\log(1-x)+\frac{1}{4}\log(1+x)$$

output
```
-1/4*x^2-1/2*arctanh(x/(-x^2+2)^(1/2))+1/4*ln(1-x)+1/4*ln(1+x)+1/4*x*(-x^2
+2)^(1/2)
```

3.881.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.30

$$\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx=\frac{1}{4}\left(x\left(-x+\sqrt{2-x^2}\right)-2\log\left(-2+\sqrt{4-2x^2}\right)+2\log\left(-2+x^2+\sqrt{4-2x^2}+x\left(-\sqrt{2}+\sqrt{2-x^2}\right)\right)\right)$$

input
```
Integrate[(x*Sqrt[2 - x^2])/(x - Sqrt[2 - x^2]),x]
```

output
```
(x*(-x + Sqrt[2 - x^2]) - 2*Log[-2 + Sqrt[4 - 2*x^2]] + 2*Log[-2 + x^2 + S
qrt[4 - 2*x^2] + x*(-Sqrt[2] + Sqrt[2 - x^2])])/4
```

3.881.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{2-x^2}{4(x-1)}+\frac{2-x^2}{4(x+1)}+\frac{\sqrt{2-x^2}}{2(x^2-1)}+\frac{\sqrt{2-x^2}}{2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{2-x^2}}\right)-\frac{x^2}{4}+\frac{1}{4}\sqrt{2-x^2}x+\frac{1}{4}\log(1-x)+\frac{1}{4}\log(x+1)$$

input
```
Int[(x*Sqrt[2 - x^2])/(x - Sqrt[2 - x^2]),x]
```

output
```
-1/4*x^2 + (x*Sqrt[2 - x^2])/4 - ArcTanh[x/Sqrt[2 - x^2]]/2 + Log[1 - x]/4
 + Log[1 + x]/4
```

3.881.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.881.4 Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

method	result
trager	$-\frac{x^2}{4}+\frac{x\sqrt{-x^2+2}}{4}-\frac{\ln\left(-\frac{\sqrt{-x^2+2}+x}{(x-1)(x+1)}\right)}{2}$
default	$\frac{x\sqrt{-x^2+2}}{4}-\frac{\sqrt{-(x+1)^2+2x+3}}{4}+\frac{\operatorname{arctanh}\left(\frac{4+2x}{2\sqrt{-(x+1)^2+2x+3}}\right)}{4}+\frac{\sqrt{-(x-1)^2-2x+3}}{4}-\frac{\operatorname{arctanh}\left(\frac{4-2x}{2\sqrt{-(x-1)^2-2x+3}}\right)}{4}-$

input
```
int(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^2+1/4*x*(-x^2+2)^(1/2)-1/2*ln(-((-x^2+2)^(1/2)+x)/(x-1)/(x+1))
```

3.881.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12

$$\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx = -\frac{1}{4}x^2+\frac{1}{4}\sqrt{-x^2+2}x+\frac{1}{4}\log\left(x^2-1\right) - \frac{1}{8}\log\left(-\frac{\sqrt{-x^2+2}x+1}{x^2}\right)+\frac{1}{8}\log\left(\frac{\sqrt{-x^2+2}x-1}{x^2}\right)$$

input
```
integrate(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)),x, algorithm="fricas")
```

output
```
-1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(x^2 - 1) - 1/8*log(-(sqrt(-x^2 +
 2)*x + 1)/x^2) + 1/8*log((sqrt(-x^2 + 2)*x - 1)/x^2)
```

3.881.6 Sympy [F]

$$\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx = \int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx$$

input
```
integrate(x*(-x**2+2)**(1/2)/(x-(-x**2+2)**(1/2)),x)
```

output
```
Integral(x*sqrt(2 - x**2)/(x - sqrt(2 - x**2)), x)
```

3.881.7 Maxima [F]

$$\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx = \int \frac{\sqrt{-x^2+2}x}{x-\sqrt{-x^2+2}}\,dx$$

input
```
integrate(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)),x, algorithm="maxima")
```

output
```
-1/2*x^2 - integrate(-x^2/(x - sqrt(-x^2 + 2)), x)
```

3.881.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(46) = 92$.

Time = 0.38 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.95

$$\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx = -\frac{1}{4}x^2 + \frac{1}{4}\sqrt{-x^2+2}x + \frac{1}{4}\log\left(\left|x^2-1\right|\right) - \frac{1}{4}\log\left(\left|\frac{x}{\sqrt{2}-\sqrt{-x^2+2}} - \frac{\sqrt{2}-\sqrt{-x^2+2}}{x} + 2\right|\right) + \frac{1}{4}\log\left(\left|\frac{x}{\sqrt{2}-\sqrt{-x^2+2}} - \frac{\sqrt{2}-\sqrt{-x^2+2}}{x} - 2\right|\right)$$

input
```
integrate(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)),x, algorithm="giac")
```

output
```
-1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(abs(x^2 - 1)) - 1/4*log(abs(x/(s
qrt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-x^2 + 2))/x + 2)) + 1/4*log(ab
s(x/(sqrt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-x^2 + 2))/x - 2))
```

3.881.9 Mupad [B] (verification not implemented)

Time = 21.71 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.43

$$\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx = \frac{\ln(x-1)}{4} + \frac{\ln(x+1)}{4} - \frac{\ln\left(\frac{-x\,1\mathrm{i}+\sqrt{2-x^2}\,1\mathrm{i}+2\mathrm{i}}{x-1}\right)}{4} + \frac{\ln\left(\frac{x\,1\mathrm{i}+\sqrt{2-x^2}\,1\mathrm{i}+2\mathrm{i}}{x+1}\right)}{4} + \frac{x\sqrt{2-x^2}}{4} - \frac{x^2}{4}$$

input
```
int((x*(2 - x^2)^(1/2))/(x - (2 - x^2)^(1/2)),x)
```

output
```
log(x - 1)/4 + log(x + 1)/4 - log(((2 - x^2)^(1/2)*1i - x*1i + 2i)/(x - 1)
)/4 + log((x*1i + (2 - x^2)^(1/2)*1i + 2i)/(x + 1))/4 + (x*(2 - x^2)^(1/2)
)/4 - x^2/4
```

3.881. $\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx$

3.882 $\int \frac{x}{-x+\sqrt{2x-x^2}}\,dx$

3.882.1 Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \frac{x}{-x+\sqrt{2x-x^2}}\,dx=-\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\text{arctanh}\left(\sqrt{2x-x^2}\right)-\frac{1}{2}\log(1-x)$$

output
```
-1/2*x+1/2*arctanh((-x^2+2*x)^(1/2))-1/2*ln(1-x)-1/2*(-x^2+2*x)^(1/2)
```

3.882.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int \frac{x}{-x+\sqrt{2x-x^2}}\,dx=\frac{1}{2}\Big(i\pi-x-\sqrt{-((-2+x)x)}+\log(-2+x) -2\log\left(-2+x+\sqrt{-((-2+x)x)}\right)\Big)$$

input
```
Integrate[x/(-x + Sqrt[2*x - x^2]),x]
```

output
```
(I*Pi - x - Sqrt[-((-2 + x)*x)] + Log[-2 + x] - 2*Log[-2 + x + Sqrt[-((-2
+ x)*x)]])/2
```

3.882.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{2x-x^2}-x}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{\sqrt{2x-x^2}}{2(1-x)}-\frac{1}{2(x-1)}-\frac{1}{2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\text{arctanh}\left(\sqrt{2x-x^2}\right)-\frac{1}{2}\sqrt{2x-x^2}-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

input
```
Int[x/(-x + Sqrt[2*x - x^2]),x]
```

output
```
-1/2*x - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x]/2
```

3.882.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.882.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

method	result	size
trager	$\frac{3}{2}-\frac{x}{2}-\frac{\sqrt{-x^{2}+2x}}{2}-\frac{\ln\left(\sqrt{-x^{2}+2x}-1\right)}{2}$	35
default	$-\frac{x}{2}-\frac{\ln(x-1)}{2}-\frac{\sqrt{-(x-1)^{2}+1}}{2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-(x-1)^{2}+1}}\right)}{2}$	38

input
```
int(x/(-x+(-x^2+2*x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
3/2-1/2*x-1/2*(-x^2+2*x)^(1/2)-1/2*ln((-x^2+2*x)^(1/2)-1)
```

3.882.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int \frac{x}{-x+\sqrt{2x-x^2}}\,dx = -\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{-x^{2}+2\,x}-\frac{1}{2}\,\log\left(x-1\right)+\frac{1}{2}\,\log\left(\frac{x+\sqrt{-x^{2}+2\,x}}{x}\right)-\frac{1}{2}\,\log\left(-\frac{x-\sqrt{-x^{2}+2\,x}}{x}\right)$$

input
```
integrate(x/(-x+(-x^2+2*x)^(1/2)),x, algorithm="fricas")
```

output
```
-1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log((x + sqrt(-x^2 +
2*x))/x) - 1/2*log(-(x - sqrt(-x^2 + 2*x))/x)
```

3.882.6 Sympy [F]

$$\int \frac{x}{-x+\sqrt{2x-x^2}}\,dx = -\int \frac{x}{x-\sqrt{-x^{2}+2x}}\,dx$$

input
```
integrate(x/(-x+(-x**2+2*x)**(1/2)),x)
```

output
```
-Integral(x/(x - sqrt(-x**2 + 2*x)), x)
```

3.882.7 Maxima [F]

$$\int \frac{x}{-x+\sqrt{2x-x^2}}\,dx = \int -\frac{x}{x-\sqrt{-x^2+2\,x}}\,dx$$

input
```
integrate(x/(-x+(-x^2+2*x)^(1/2)),x, algorithm="maxima")
```

output
```
-integrate(x/(x - sqrt(-x^2 + 2*x)), x)
```

3.882.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{x}{-x+\sqrt{2x-x^2}}\,dx = -\frac{1}{2}\,x - \frac{1}{2}\,\sqrt{-x^2+2\,x} - \frac{1}{2}\,\log\left(-\frac{2\,\left(\sqrt{-x^2+2\,x}-1\right)}{|-2\,x+2|}\right) - \frac{1}{2}\,\log\left(|x-1|\right)$$

input
```
integrate(x/(-x+(-x^2+2*x)^(1/2)),x, algorithm="giac")
```

output
```
-1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(-2*(sqrt(-x^2 + 2*x) - 1)/abs(-2*x
 + 2)) - 1/2*log(abs(x - 1))
```

3.882.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{-x+\sqrt{2x-x^2}}\,dx = \int -\frac{x}{x-\sqrt{2\,x-x^2}}\,dx$$

input
```
int(-x/(x - (2*x - x^2)^(1/2)),x)
```

output
```
int(-x/(x - (2*x - x^2)^(1/2)), x)
```

3.883 $\int \frac{x+\sqrt{2x-x^2}}{2-2x}\,dx$

3.883.1 Optimal result

Integrand size = 23, antiderivative size = 51

$$\int \frac{x+\sqrt{2x-x^2}}{2-2x}\,dx=-\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\text{arctanh}\left(\sqrt{2x-x^2}\right)-\frac{1}{2}\log(1-x)$$

output
```
-1/2*x+1/2*arctanh((-x^2+2*x)^(1/2))-1/2*ln(1-x)-1/2*(-x^2+2*x)^(1/2)
```

3.883.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int \frac{x+\sqrt{2x-x^2}}{2-2x}\,dx=\frac{1}{2}\left(i\pi-x-\sqrt{-((-2+x)x)}+\log(-2+x)\right.$$
$$\left.-2\log\left(-2+x+\sqrt{-((-2+x)x)}\right)\right)$$

input
```
Integrate[(x + Sqrt[2*x - x^2])/(2 - 2*x),x]
```

output
```
(I*Pi - x - Sqrt[-((-2 + x)*x)] + Log[-2 + x] - 2*Log[-2 + x + Sqrt[-((-2
+ x)*x)]])/2
```

3.883.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x-x^2}+x}{2-2x} \, dx$$
$$\Big\downarrow \ 7293$$
$$\int \left(\frac{\sqrt{2x-x^2}}{2(1-x)}-\frac{x}{2(x-1)}\right)dx$$
$$\Big\downarrow \ 2009$$
$$\frac{1}{2}\text{arctanh}\left(\sqrt{2x-x^2}\right)-\frac{1}{2}\sqrt{2x-x^2}-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

input
```
Int[(x + Sqrt[2*x - x^2])/(2 - 2*x),x]
```

output
```
-1/2*x - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x]/2
```

3.883.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.883.4 Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

method	result	size
trager	$\frac{3}{2}-\frac{x}{2}-\frac{\sqrt{-x^{2}+2x}}{2}-\frac{\ln\left(\sqrt{-x^{2}+2x}-1\right)}{2}$	35
default	$-\frac{x}{2}-\frac{\ln(x-1)}{2}-\frac{\sqrt{-(x-1)^{2}+1}}{2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-(x-1)^{2}+1}}\right)}{2}$	38

input
```
int((x+(-x^2+2*x)^(1/2))/(-2*x+2),x,method=_RETURNVERBOSE)
```

output
```
3/2-1/2*x-1/2*(-x^2+2*x)^(1/2)-1/2*ln((-x^2+2*x)^(1/2)-1)
```

3.883.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int \frac{x+\sqrt{2x-x^2}}{2-2x}\,dx = -\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{-x^{2}+2\,x}-\frac{1}{2}\,\log\left(x-1\right)+\frac{1}{2}\,\log\left(\frac{x+\sqrt{-x^{2}+2\,x}}{x}\right)-\frac{1}{2}\,\log\left(-\frac{x-\sqrt{-x^{2}+2\,x}}{x}\right)$$

input
```
integrate((x+(-x^2+2*x)^(1/2))/(2-2*x),x, algorithm="fricas")
```

output
```
-1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log((x + sqrt(-x^2 +
2*x))/x) - 1/2*log(-(x - sqrt(-x^2 + 2*x))/x)
```

3.883.6 Sympy [F]

$$\int \frac{x+\sqrt{2x-x^2}}{2-2x}\,dx = -\frac{\int \frac{x}{x-1}\,dx+\int \frac{\sqrt{-x^{2}+2x}}{x-1}\,dx}{2}$$

input
```
integrate((x+(-x**2+2*x)**(1/2))/(2-2*x),x)
```

output
```
-(Integral(x/(x - 1), x) + Integral(sqrt(-x**2 + 2*x)/(x - 1), x))/2
```

3.883.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{x+\sqrt{2x-x^2}}{2-2x}\,dx = -\frac{1}{2}\,x - \frac{1}{2}\,\sqrt{-x^2+2\,x} - \frac{1}{2}\,\log\left(x-1\right) + \frac{1}{2}\,\log\left(\frac{2\,\sqrt{-x^2+2\,x}}{|x-1|} + \frac{2}{|x-1|}\right)$$

input
```
integrate((x+(-x^2+2*x)^(1/2))/(2-2*x),x, algorithm="maxima")
```

output
```
-1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log(2*sqrt(-x^2 + 2*x
)/abs(x - 1) + 2/abs(x - 1))
```

3.883.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{x+\sqrt{2x-x^2}}{2-2x}\,dx = -\frac{1}{2}\,x - \frac{1}{2}\,\sqrt{-x^2+2\,x} - \frac{1}{2}\,\log\left(-\frac{2\,\left(\sqrt{-x^2+2\,x}-1\right)}{|-2\,x+2|}\right) - \frac{1}{2}\,\log\left(|x-1|\right)$$

input
```
integrate((x+(-x^2+2*x)^(1/2))/(2-2*x),x, algorithm="giac")
```

output
```
-1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(-2*(sqrt(-x^2 + 2*x) - 1)/abs(-2*x
 + 2)) - 1/2*log(abs(x - 1))
```

3.883.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x+\sqrt{2x-x^2}}{2-2x}\,dx = \int -\frac{x+\sqrt{2\,x-x^2}}{2\,x-2}\,dx$$

input
```
int(-(x + (2*x - x^2)^(1/2))/(2*x - 2),x)
```

output
```
int(-(x + (2*x - x^2)^(1/2))/(2*x - 2), x)
```

3.884 $\int \frac{\sqrt{2-x}\sqrt{x}+x}{2-2x}\,dx$

3.884.1 Optimal result

Integrand size = 25, antiderivative size = 51

$$\int \frac{\sqrt{2-x}\sqrt{x}+x}{2-2x}\,dx=-\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\operatorname{arctanh}\left(\sqrt{2x-x^2}\right)-\frac{1}{2}\log(1-x)$$

output
```
-1/2*x+1/2*arctanh((-x^2+2*x)^(1/2))-1/2*ln(1-x)-1/2*(-x^2+2*x)^(1/2)
```

3.884.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{2-x}\sqrt{x}+x}{2-2x}\,dx=\frac{1}{2}\Big(i\pi-x-\sqrt{-((-2+x)x)}+\log(-2+x) -2\log\left(-2+x+\sqrt{-((-2+x)x)}\right)\Big)$$

input
```
Integrate[(Sqrt[2 - x]*Sqrt[x] + x)/(2 - 2*x),x]
```

output
```
(I*Pi - x - Sqrt[-((-2 + x)*x)] + Log[-2 + x] - 2*Log[-2 + x + Sqrt[-((-2
+ x)*x)]])/2
```

3.884.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+\sqrt{2-x}\sqrt{x}}{2-2x} \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{\sqrt{2x-x^2}}{2(1-x)}-\frac{x}{2(x-1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\text{arctanh}\left(\sqrt{2x-x^2}\right)-\frac{1}{2}\sqrt{2x-x^2}-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

input
```
Int[(Sqrt[2 - x]*Sqrt[x] + x)/(2 - 2*x),x]
```

output
```
-1/2*x - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x]/2
```

3.884.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.884.4 Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{\sqrt{2-x}\,\sqrt{x}\,\left(\sqrt{-x(x-2)}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x(x-2)}}\right)\right)}{2\sqrt{-x(x-2)}}-\frac{x}{2}-\frac{\ln(x-1)}{2}$	51

input
```
int((x+(2-x)^(1/2)*x^(1/2))/(-2*x+2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2-x)^(1/2)*x^(1/2)/(-x*(x-2))^(1/2)*((-x*(x-2))^(1/2)-arctanh(1/(-x*
(x-2))^(1/2)))-1/2*x-1/2*ln(x-1)
```

3.884.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{2-x}\sqrt{x}+x}{2-2x}\,dx = -\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{x}\sqrt{-x+2}-\frac{1}{2}\,\log\left(x-1\right) + \frac{1}{2}\,\log\left(\frac{x+\sqrt{x}\sqrt{-x+2}}{x}\right)-\frac{1}{2}\,\log\left(-\frac{x-\sqrt{x}\sqrt{-x+2}}{x}\right)$$

input
```
integrate((x+(2-x)^(1/2)*x^(1/2))/(2-2*x),x, algorithm="fricas")
```

output
```
-1/2*x - 1/2*sqrt(x)*sqrt(-x + 2) - 1/2*log(x - 1) + 1/2*log((x + sqrt(x)*
sqrt(-x + 2))/x) - 1/2*log(-(x - sqrt(x)*sqrt(-x + 2))/x)
```

3.884.6 Sympy [F]

$$\int \frac{\sqrt{2-x}\sqrt{x}+x}{2-2x}\,dx = -\frac{\int \frac{x}{x-1}\,dx+\int \frac{\sqrt{x}\sqrt{2-x}}{x-1}\,dx}{2}$$

input
```
integrate((x+(2-x)**(1/2)*x**(1/2))/(2-2*x),x)
```

output
```
-(Integral(x/(x - 1), x) + Integral(sqrt(x)*sqrt(2 - x)/(x - 1), x))/2
```

3.884.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{2-x}\sqrt{x}+x}{2-2x}\,dx = -\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{-x^2+2\,x}-\frac{1}{2}\,\log\left(x-1\right)+\frac{1}{2}\,\log\left(\frac{2\,\sqrt{-x^2+2\,x}}{|x-1|}+\frac{2}{|x-1|}\right)$$

input
```
integrate((x+(2-x)^(1/2)*x^(1/2))/(2-2*x),x, algorithm="maxima")
```

output
```
-1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log(2*sqrt(-x^2 + 2*x
)/abs(x - 1) + 2/abs(x - 1))
```

3.884.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(39) = 78.

Time = 0.35 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.14

$$\int \frac{\sqrt{2-x}\sqrt{x}+x}{2-2x}\,dx = -\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{x}\sqrt{-x+2}-\frac{1}{2}\,\log\left(|x-1|\right)+\frac{1}{2}\,\log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+2}}{\sqrt{x}}+\frac{\sqrt{x}}{\sqrt{2}-\sqrt{-x+2}}+2\right|\right)-\frac{1}{2}\,\log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+2}}{\sqrt{x}}+\frac{\sqrt{x}}{\sqrt{2}-\sqrt{-x+2}}-2\right|\right)$$

input
```
integrate((x+(2-x)^(1/2)*x^(1/2))/(2-2*x),x, algorithm="giac")
```

output
```
-1/2*x - 1/2*sqrt(x)*sqrt(-x + 2) - 1/2*log(abs(x - 1)) + 1/2*log(abs(-(sq
rt(2) - sqrt(-x + 2))/sqrt(x) + sqrt(x)/(sqrt(2) - sqrt(-x + 2)) + 2)) - 1
/2*log(abs(-(sqrt(2) - sqrt(-x + 2))/sqrt(x) + sqrt(x)/(sqrt(2) - sqrt(-x
+ 2)) - 2))
```

3.884.9 Mupad [B] (verification not implemented)

Time = 24.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{2-x}\sqrt{x}+x}{2-2x}\,dx = \operatorname{atanh}\left(\frac{\sqrt{x}\left(\sqrt{2}-\sqrt{2-x}\right)}{x+\sqrt{2}\sqrt{2-x}-2}\right)-\frac{\ln\left(x-1\right)}{2}-\frac{x}{2}-\frac{\sqrt{x}\sqrt{2-x}}{2}$$

input
```
int(-(x + x^(1/2)*(2 - x)^(1/2))/(2*x - 2),x)
```

output
```
atanh((x^(1/2)*(2^(1/2) - (2 - x)^(1/2)))/(x + 2^(1/2)*(2 - x)^(1/2) - 2))
 - log(x - 1)/2 - x/2 - (x^(1/2)*(2 - x)^(1/2))/2
```

3.885 $\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}}\,dx$

3.885.1 Optimal result

Integrand size = 25, antiderivative size = 54

$$\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}}\,dx=-\frac{1}{2}\sqrt{2-x}\sqrt{x}-\frac{x}{2}+\frac{1}{2}\text{arctanh}\left(\sqrt{2-x}\sqrt{x}\right)-\frac{1}{2}\log(1-x)$$

output
```
-1/2*x+1/2*arctanh((2-x)^(1/2)*x^(1/2))-1/2*ln(1-x)-1/2*(2-x)^(1/2)*x^(1/2
)
```

3.885.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}}\,dx=-\frac{x}{2}-\frac{1}{2}\sqrt{-((-2+x)x)}+\log\left(-2+\sqrt{4-2x}\right)\\ -\log\left(-2+\sqrt{4-2x}-\sqrt{2}\sqrt{x}+x+\sqrt{-((-2+x)x)}\right)$$

input
```
Integrate[Sqrt[x]/(Sqrt[2 - x] - Sqrt[x]),x]
```

output
```
-1/2*x - Sqrt[-((-2 + x)*x)]/2 + Log[-2 + Sqrt[4 - 2*x]] - Log[-2 + Sqrt[4
 - 2*x] - Sqrt[2]*Sqrt[x] + x + Sqrt[-((-2 + x)*x)]]
```

3.885.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2035, 25, 2532, 27, 243, 49, 380, 27, 291, 219, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}}\,dx$$

$$\Big\downarrow \text{ 2035}$$

$$2\int -\frac{x}{\sqrt{x}-\sqrt{2-x}}d\sqrt{x}$$

$$\Big\downarrow \text{ 25}$$

$$-2\int \frac{x}{\sqrt{x}-\sqrt{2-x}}d\sqrt{x}$$

$$\Big\downarrow \text{ 2532}$$

$$2\left(\int \frac{x^{3/2}}{2(1-x)}d\sqrt{x}+\int \frac{\sqrt{2-x}x}{2(1-x)}d\sqrt{x}\right)$$

$$\Big\downarrow \text{ 27}$$

$$2\left(\frac{1}{2}\int \frac{x^{3/2}}{1-x}d\sqrt{x}+\frac{1}{2}\int \frac{\sqrt{2-x}x}{1-x}d\sqrt{x}\right)$$

$$\Big\downarrow \text{ 243}$$

$$2\left(\frac{1}{4}\int \frac{x}{1-x}dx+\frac{1}{2}\int \frac{\sqrt{2-x}x}{1-x}d\sqrt{x}\right)$$

$$\Big\downarrow \text{ 49}$$

$$2\left(\frac{1}{4}\int \left(\frac{1}{1-x}-1\right)dx+\frac{1}{2}\int \frac{\sqrt{2-x}x}{1-x}d\sqrt{x}\right)$$

$$\Big\downarrow \text{ 380}$$

$$2\left(\frac{1}{4}\int \left(\frac{1}{1-x}-1\right)dx+\frac{1}{2}\left(\frac{1}{2}\int \frac{2}{(1-x)\sqrt{2-x}}d\sqrt{x}-\frac{1}{2}\sqrt{2-x}\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{ 27}$$

$$2\left(\frac{1}{4}\int \left(\frac{1}{1-x}-1\right)dx+\frac{1}{2}\left(\int \frac{1}{(1-x)\sqrt{2-x}}d\sqrt{x}-\frac{1}{2}\sqrt{2-x}\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{291}$$

$$2\left(\frac{1}{4}\int\left(\frac{1}{1-x}-1\right)dx+\frac{1}{2}\left(\int\frac{1}{1-x}d\frac{\sqrt{x}}{\sqrt{2-x}}-\frac{1}{2}\sqrt{2-x}\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{1}{4}\int\left(\frac{1}{1-x}-1\right)dx+\frac{1}{2}\left(\text{arctanh}\left(\frac{\sqrt{x}}{\sqrt{2-x}}\right)-\frac{1}{2}\sqrt{2-x}\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{1}{2}\left(\text{arctanh}\left(\frac{\sqrt{x}}{\sqrt{2-x}}\right)-\frac{1}{2}\sqrt{2-x}\sqrt{x}\right)+\frac{1}{4}(-x-\log(1-x))\right)$$

input
```
Int[Sqrt[x]/(Sqrt[2 - x] - Sqrt[x]),x]
```

output
```
2*((-1/2*(Sqrt[2 - x]*Sqrt[x]) + ArcTanh[Sqrt[x]/Sqrt[2 - x]])/2 + (-x - L
og[1 - x])/4)
```

3.885.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.885. $\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}}\,dx$

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2532
```
Int[(x_)^(m_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x
_Symbol] :> Simp[-d   Int[x^(m + n)/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x], x]
 + Simp[c   Int[(x^m*Sqrt[a + b*x^(2*n)])/(a*c^2 + (b*c^2 - d^2)*x^(2*n)),
x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[p, 2*n] && NeQ[b*c^2 - d^2, 0
]
```

3.885.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{\sqrt{2-x}\,\sqrt{x}\,\left(\sqrt{-x(x-2)}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x(x-2)}}\right)\right)}{2\sqrt{-x(x-2)}}-\frac{x}{2}-\frac{\ln(x-1)}{2}$	51

input
```
int(x^(1/2)/((2-x)^(1/2)-x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2-x)^(1/2)*x^(1/2)/(-x*(x-2))^(1/2)*((-x*(x-2))^(1/2)-arctanh(1/(-x*
(x-2))^(1/2)))-1/2*x-1/2*ln(x-1)
```

3.885.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}}\,dx = -\frac{1}{2}\,x - \frac{1}{2}\,\sqrt{x}\sqrt{-x+2} - \frac{1}{2}\,\log\left(x-1\right) + \frac{1}{2}\,\log\left(\frac{x+\sqrt{x}\sqrt{-x+2}}{x}\right) - \frac{1}{2}\,\log\left(-\frac{x-\sqrt{x}\sqrt{-x+2}}{x}\right)$$

input
```
integrate(x^(1/2)/((2-x)^(1/2)-x^(1/2)),x, algorithm="fricas")
```

output
```
-1/2*x - 1/2*sqrt(x)*sqrt(-x + 2) - 1/2*log(x - 1) + 1/2*log((x + sqrt(x)*
sqrt(-x + 2))/x) - 1/2*log(-(x - sqrt(x)*sqrt(-x + 2))/x)
```

3.885.6 Sympy [F]

$$\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}}\,dx = \int \frac{\sqrt{x}}{-\sqrt{x}+\sqrt{2-x}}\,dx$$

input
```
integrate(x**(1/2)/((2-x)**(1/2)-x**(1/2)),x)
```

output
```
Integral(sqrt(x)/(-sqrt(x) + sqrt(2 - x)), x)
```

3.885.7 Maxima [F]

$$\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}}\,dx = \int -\frac{\sqrt{x}}{\sqrt{x}-\sqrt{-x+2}}\,dx$$

input
```
integrate(x^(1/2)/((2-x)^(1/2)-x^(1/2)),x, algorithm="maxima")
```

output
```
-integrate(sqrt(x)/(sqrt(x) - sqrt(-x + 2)), x)
```

3.885.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(38) = 76.

Time = 0.38 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.02

$$\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}}\,dx = -\frac{1}{2}\,x - \frac{1}{2}\,\sqrt{x}\sqrt{-x+2} - \frac{1}{2}\,\log\left(|x-1|\right) + \frac{1}{2}\,\log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+2}}{\sqrt{x}} + \frac{\sqrt{x}}{\sqrt{2}-\sqrt{-x+2}} + 2\right|\right) - \frac{1}{2}\,\log\left(\left|-\frac{\sqrt{2}-\sqrt{-x+2}}{\sqrt{x}} + \frac{\sqrt{x}}{\sqrt{2}-\sqrt{-x+2}} - 2\right|\right)$$

input
```
integrate(x^(1/2)/((2-x)^(1/2)-x^(1/2)),x, algorithm="giac")
```

output
```
-1/2*x - 1/2*sqrt(x)*sqrt(-x + 2) - 1/2*log(abs(x - 1)) + 1/2*log(abs(-(sq
rt(2) - sqrt(-x + 2))/sqrt(x) + sqrt(x)/(sqrt(2) - sqrt(-x + 2)) + 2)) - 1
/2*log(abs(-(sqrt(2) - sqrt(-x + 2))/sqrt(x) + sqrt(x)/(sqrt(2) - sqrt(-x
+ 2)) - 2))
```

3.885.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}}\,dx = \operatorname{atanh}\left(\frac{\sqrt{x}\left(\sqrt{2}-\sqrt{2-x}\right)}{x+\sqrt{2}\,\sqrt{2-x}-2}\right) - \frac{\ln\left(x-1\right)}{2} - \frac{x}{2} - \frac{\sqrt{x}\,\sqrt{2-x}}{2}$$

input
```
int(x^(1/2)/((2 - x)^(1/2) - x^(1/2)),x)
```

output
```
atanh((x^(1/2)*(2^(1/2) - (2 - x)^(1/2)))/(x + 2^(1/2)*(2 - x)^(1/2) - 2))
 - log(x - 1)/2 - x/2 - (x^(1/2)*(2 - x)^(1/2))/2
```

3.886 $\int \frac{1}{\left((1+x)\left(-1+x^2\right)\right)^{2/3}} \, dx$

3.886.1 Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{1}{\left((1+x)\left(-1+x^2\right)\right)^{2/3}} \, dx=-\frac{3(1-x^2)}{2\left(-\left((1+x)\left(1-x^2\right)\right)\right)^{2/3}}$$

output
```
-3/2*(-x^2+1)/(-(1+x)*(-x^2+1))^(2/3)
```

3.886.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left((1+x)\left(-1+x^2\right)\right)^{2/3}} \, dx=\frac{3(-1+x)(1+x)}{2\left((-1+x)(1+x)^2\right)^{2/3}}$$

input
```
Integrate[((1 + x)*(-1 + x^2))^(-2/3),x]
```

output
```
(3*(-1 + x)*(1 + x))/(2*((-1 + x)*(1 + x)^2)^(2/3))
```

3.886.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2477, 460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{((x+1)(x^2-1))^{2/3}}dx$$

$$\Big\downarrow \; 2477$$

$$\frac{(x+1)^{2/3}(x^2-1)^{2/3}\int \frac{1}{(x+1)^{2/3}(x^2-1)^{2/3}}dx}{(-((x+1)(1-x^2)))^{2/3}}$$

$$\Big\downarrow \; 460$$

$$\frac{3(x^2-1)}{2(-((x+1)(1-x^2)))^{2/3}}$$

input
```
Int[((1 + x)*(-1 + x^2))^(-2/3),x]
```

output
```
(3*(-1 + x^2))/(2*(-((1 + x)*(1 - x^2)))^(2/3))
```

3.886.3.1 Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

rule 2477
```
Int[(Px_)^(p_), x_Symbol] :> With[{a = Coeff[Px, x, 0], b = Coeff[Px, x, 1]
, c = Coeff[Px, x, 2], d = Coeff[Px, x, 3]}, Simp[Px^p/((c + d*x)^p*(b + d*
x^2)^p)   Int[(c + d*x)^p*(b + d*x^2)^p, x], x] /; EqQ[b*c - a*d, 0]] /; Fr
eeQ[p, x] && PolyQ[Px, x, 3] &&  !IntegerQ[p]
```

3.886.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
gosper	$\frac{3(x-1)(x+1)}{2((x+1)(x^2-1))^{\frac{2}{3}}}$	20
risch	$\frac{3(x+1)(x-1)}{2\left((x+1)^2(x-1)\right)^{\frac{2}{3}}}$	20
trager	$\frac{3(x^3+x^2-x-1)^{\frac{1}{3}}}{2(x+1)}$	21

input
```
int(1/((x+1)*(x^2-1))^(2/3),x,method=_RETURNVERBOSE)
```

output
```
3/2*(x-1)*(x+1)/((x+1)*(x^2-1))^(2/3)
```

3.886.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{1}{((1+x)(-1+x^2))^{2/3}}\,dx = \frac{3(x^3+x^2-x-1)^{\frac{1}{3}}}{2(x+1)}$$

input
```
integrate(1/((1+x)*(x^2-1))^(2/3),x, algorithm="fricas")
```

output
```
3/2*(x^3 + x^2 - x - 1)^(1/3)/(x + 1)
```

3.886.6 Sympy [F]

$$\int \frac{1}{((1+x)(-1+x^2))^{2/3}}\,dx = \int \frac{1}{((x+1)(x^2-1))^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/((1+x)*(x**2-1))**(2/3),x)
```

output
```
Integral(((x + 1)*(x**2 - 1))**(-2/3), x)
```

3.886.7 Maxima [F]

$$\int \frac{1}{\left((1+x)\left(-1+x^2\right)\right)^{2/3}}\,dx=\int \frac{1}{\left((x^2-1)(x+1)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/((1+x)*(x^2-1))^(2/3),x, algorithm="maxima")
```

output
```
integrate(((x^2 - 1)*(x + 1))^(-2/3), x)
```

3.886.8 Giac [F]

$$\int \frac{1}{\left((1+x)\left(-1+x^2\right)\right)^{2/3}}\,dx=\int \frac{1}{\left((x^2-1)(x+1)\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/((1+x)*(x^2-1))^(2/3),x, algorithm="giac")
```

output
```
integrate(((x^2 - 1)*(x + 1))^(-2/3), x)
```

3.886.9 Mupad [B] (verification not implemented)

Time = 23.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{1}{\left((1+x)\left(-1+x^2\right)\right)^{2/3}}\,dx=\frac{3\left((x^2-1)\,(x+1)\right)^{1/3}}{2\,(x+1)}$$

input
```
int(1/((x^2 - 1)*(x + 1))^(2/3),x)
```

output
```
(3*((x^2 - 1)*(x + 1))^(1/3))/(2*(x + 1))
```

3.887 $\int \frac{-1+x^2}{(1+x^2)\sqrt{x(1+x^2)}}\,dx$

3.887.1 Optimal result

Integrand size = 24, antiderivative size = 14

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x(1+x^2)}}\,dx = -\frac{2x}{\sqrt{x(1+x^2)}}$$

output
```
-2*x/(x*(x^2+1))^(1/2)
```

3.887.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x(1+x^2)}}\,dx = -\frac{2x}{\sqrt{x+x^3}}$$

input
```
Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[x*(1 + x^2)]),x]
```

output
```
(-2*x)/Sqrt[x + x^3]
```

3.887.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2467, 25, 356}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-1}{(x^2+1)\sqrt{x(x^2+1)}}dx$$
$$\Big\downarrow \text{2467}$$
$$\frac{\sqrt{x}\sqrt{x^2+1}\int -\frac{1-x^2}{\sqrt{x}(x^2+1)^{3/2}}dx}{\sqrt{x(x^2+1)}}$$
$$\Big\downarrow \text{25}$$
$$-\frac{\sqrt{x}\sqrt{x^2+1}\int \frac{1-x^2}{\sqrt{x}(x^2+1)^{3/2}}dx}{\sqrt{x(x^2+1)}}$$
$$\Big\downarrow \text{356}$$
$$-\frac{2x}{\sqrt{x(x^2+1)}}$$

input
```
Int[(-1 + x^2)/((1 + x^2)*Sqrt[x*(1 + x^2)]),x]
```

output
```
(-2*x)/Sqrt[x*(1 + x^2)]
```

3.887.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 356
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] /;
 FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(m + 1) -
b*c*(m + 2*p + 3), 0] && NeQ[m, -1]
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

3.887.4 Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{2x}{\sqrt{x(x^2+1)}}$	13
default	$-\frac{2x}{\sqrt{x(x^2+1)}}$	13
risch	$-\frac{2x}{\sqrt{x(x^2+1)}}$	13
elliptic	$-\frac{2x}{\sqrt{x(x^2+1)}}$	13
pseudoelliptic	$-\frac{2x}{\sqrt{x(x^2+1)}}$	13
trager	$-\frac{2\sqrt{x^3+x}}{x^2+1}$	17
meijerg	$\frac{2x^{\frac{5}{2}}\,{}_2F_1\left(\frac{5}{4},\frac{3}{2};\frac{9}{4};-x^2\right)}{5}-2\sqrt{x}\,{}_2F_1\left(\frac{1}{4},\frac{3}{2};\frac{5}{4};-x^2\right)$	34

input
```
int((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*x/(x*(x^2+1))^(1/2)
```

3.887.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x(1+x^2)}}\,dx = -\frac{2\sqrt{x^3+x}}{x^2+1}$$

input
```
integrate((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(x^3 + x)/(x^2 + 1)
```

3.887.6 Sympy [F]

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x(1+x^2)}}\,dx = \int \frac{(x-1)(x+1)}{\sqrt{x(x^2+1)}\,(x^2+1)}\,dx$$

input
```
integrate((x**2-1)/(x**2+1)/(x*(x**2+1))**(1/2),x)
```

output
```
Integral((x - 1)*(x + 1)/(sqrt(x*(x**2 + 1))*(x**2 + 1)), x)
```

3.887.7 Maxima [F]

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x(1+x^2)}}\,dx = \int \frac{x^2-1}{\sqrt{(x^2+1)x}(x^2+1)}\,dx$$

input
```
integrate((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate((x^2 - 1)/(sqrt((x^2 + 1)*x)*(x^2 + 1)), x)
```

3.887.8 Giac [F]

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x(1+x^2)}}\,dx = \int \frac{x^2-1}{\sqrt{(x^2+1)x}(x^2+1)}\,dx$$

input
```
integrate((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2),x, algorithm="giac")
```

output
```
integrate((x^2 - 1)/(sqrt((x^2 + 1)*x)*(x^2 + 1)), x)
```

3.887.9 Mupad [B] (verification not implemented)

Time = 22.46 (sec) , antiderivative size = 138, normalized size of antiderivative = 9.86

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x(1+x^2)}}\,dx = -\frac{2x}{\sqrt{x^3+x}} - \frac{\sqrt{1-x\,1i}\,\sqrt{\frac{1}{2}+\frac{x\,1i}{2}}\,\mathrm{E}\left(\operatorname{asin}\left(\sqrt{1-x\,1i}\right)\middle|\frac{1}{2}\right)\,\sqrt{x\,1i}\,2i}{\sqrt{x^3+x}} + \frac{\sqrt{1-x\,1i}\,\sqrt{\frac{1}{2}+\frac{x\,1i}{2}}\,\mathrm{F}\left(\operatorname{asin}\left(\sqrt{1-x\,1i}\right)\middle|\frac{1}{2}\right)\,\sqrt{x\,1i}\,2i}{\sqrt{x^3+x}} - \frac{\sqrt{1-x\,1i}\,\sqrt{1+x\,1i}\,\sqrt{-x\,1i}\,\mathrm{E}\left(\operatorname{asin}\left(\sqrt{-x\,1i}\right)\middle|-1\right)\,1i}{\sqrt{x^3+x}}$$

input
```
int((x^2 - 1)/((x*(x^2 + 1))^(1/2)*(x^2 + 1)),x)
```

output
```
((1 - x*1i)^(1/2)*((x*1i)/2 + 1/2)^(1/2)*ellipticF(asin((1 - x*1i)^(1/2)),
 1/2)*(x*1i)^(1/2)*2i)/(x + x^3)^(1/2) - ((1 - x*1i)^(1/2)*((x*1i)/2 + 1/2
)^(1/2)*ellipticE(asin((1 - x*1i)^(1/2)), 1/2)*(x*1i)^(1/2)*2i)/(x + x^3)^
(1/2) - (2*x)/(x + x^3)^(1/2) - ((1 - x*1i)^(1/2)*(x*1i + 1)^(1/2)*(-x*1i)
^(1/2)*ellipticE(asin((-x*1i)^(1/2)), -1)*1i)/(x + x^3)^(1/2)
```

3.888 $\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx$

3.888.1 Optimal result

Integrand size = 22, antiderivative size = 12

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx = -\frac{2x}{\sqrt{x+x^3}}$$

output
```
-2*x/(x^3+x)^(1/2)
```

3.888.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx = -\frac{2x}{\sqrt{x+x^3}}$$

input
```
Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[x + x^3]),x]
```

output
```
(-2*x)/Sqrt[x + x^3]
```

3.888.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2467, 25, 356}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-1}{(x^2+1)\sqrt{x^3+x}}dx$$

$$\Big\downarrow \text{ 2467}$$

$$\frac{\sqrt{x}\sqrt{x^2+1}\int -\frac{1-x^2}{\sqrt{x}(x^2+1)^{3/2}}dx}{\sqrt{x^3+x}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\sqrt{x}\sqrt{x^2+1}\int \frac{1-x^2}{\sqrt{x}(x^2+1)^{3/2}}dx}{\sqrt{x^3+x}}$$

$$\Big\downarrow \text{ 356}$$

$$-\frac{2x}{\sqrt{x^3+x}}$$

input
```
Int[(-1 + x^2)/((1 + x^2)*Sqrt[x + x^3]),x]
```

output
```
(-2*x)/Sqrt[x + x^3]
```

3.888.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 356
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] /;
 FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(m + 1) -
b*c*(m + 2*p + 3), 0] && NeQ[m, -1]
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

3.888.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$-\frac{2x}{\sqrt{x^{3}+x}}$	11
default	$-\frac{2x}{\sqrt{x\left(x^{2}+1\right)}}$	13
risch	$-\frac{2x}{\sqrt{x\left(x^{2}+1\right)}}$	13
elliptic	$-\frac{2x}{\sqrt{x\left(x^{2}+1\right)}}$	13
pseudoelliptic	$-\frac{2x}{\sqrt{x\left(x^{2}+1\right)}}$	13
trager	$-\frac{2\sqrt{x^{3}+x}}{x^{2}+1}$	17
meijerg	$\frac{2x^{\frac{5}{2}}\,{}_{2}F_{1}\left(\frac{5}{4},\frac{3}{2};\frac{9}{4};-x^{2}\right)}{5}-2\sqrt{x}\,{}_{2}F_{1}\left(\frac{1}{4},\frac{3}{2};\frac{5}{4};-x^{2}\right)$	34

input
```
int((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*x/(x^3+x)^(1/2)
```

3.888.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx=-\frac{2\sqrt{x^3+x}}{x^2+1}$$

input
```
integrate((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(x^3 + x)/(x^2 + 1)
```

3.888.6 Sympy [F]

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx = \int \frac{(x-1)(x+1)}{\sqrt{x(x^2+1)}(x^2+1)}\,dx$$

input
```
integrate((x**2-1)/(x**2+1)/(x**3+x)**(1/2),x)
```

output
```
Integral((x - 1)*(x + 1)/(sqrt(x*(x**2 + 1))*(x**2 + 1)), x)
```

3.888.7 Maxima [F]

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx = \int \frac{x^2-1}{\sqrt{x^3+x}(x^2+1)}\,dx$$

input
```
integrate((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x^2 - 1)/(sqrt(x^3 + x)*(x^2 + 1)), x)
```

3.888.8 Giac [F]

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx = \int \frac{x^2-1}{\sqrt{x^3+x}(x^2+1)}\,dx$$

input
```
integrate((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x, algorithm="giac")
```

output
```
integrate((x^2 - 1)/(sqrt(x^3 + x)*(x^2 + 1)), x)
```

3.888.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx = -\frac{2\,x}{\sqrt{x^3+x}}$$

input
```
int((x^2 - 1)/((x^2 + 1)*(x + x^3)^(1/2)),x)
```

output
```
-(2*x)/(x + x^3)^(1/2)
```

3.889 $\int \frac{\sqrt{\frac{\left(-1+x^2\right)^2}{x\left(1+x^2\right)}}}{1+x^2} \, dx$

3.889.1 Optimal result

Integrand size = 30, antiderivative size = 36

$$\int \frac{\sqrt{\frac{\left(-1+x^2\right)^2}{x\left(1+x^2\right)}}}{1+x^2} \, dx=\frac{2x\sqrt{\frac{\left(1-x^2\right)^2}{x\left(1+x^2\right)}}}{1-x^2}$$

output
```
2*x*((-x^2+1)^2/x/(x^2+1))^(1/2)/(-x^2+1)
```

3.889.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{\frac{\left(-1+x^2\right)^2}{x\left(1+x^2\right)}}}{1+x^2} \, dx=-\frac{2x\sqrt{\frac{\left(-1+x^2\right)^2}{x+x^3}}}{-1+x^2}$$

input
```
Integrate[Sqrt[(-1 + x^2)^2/(x*(1 + x^2))]/(1 + x^2),x]
```

output
```
(-2*x*Sqrt[(-1 + x^2)^2/(x + x^3)])/(-1 + x^2)
```

3.889.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {7269, 25, 356}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{(x^2-1)^2}{x(x^2+1)}}}{x^2+1}\,dx$$

$$\Big\downarrow \ 7269$$

$$-\frac{\sqrt{x}\sqrt{\frac{(1-x^2)^2}{x(x^2+1)}}\sqrt{x^2+1}\int -\frac{1-x^2}{\sqrt{x}(x^2+1)^{3/2}}dx}{1-x^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt{x}\sqrt{\frac{(1-x^2)^2}{x(x^2+1)}}\sqrt{x^2+1}\int \frac{1-x^2}{\sqrt{x}(x^2+1)^{3/2}}dx}{1-x^2}$$

$$\Big\downarrow \ 356$$

$$\frac{2x\sqrt{\frac{(1-x^2)^2}{x(x^2+1)}}}{1-x^2}$$

input
```
Int[Sqrt[(-1 + x^2)^2/(x*(1 + x^2))]/(1 + x^2),x]
```

output
```
(2*x*Sqrt[(1 - x^2)^2/(x*(1 + x^2))])/(1 - x^2)
```

3.889.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 356
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] /;
 FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(m + 1) -
b*c*(m + 2*p + 3), 0] && NeQ[m, -1]
```

rule 7269
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Simp[
a^IntPart[p]*((a*v^m*w^n*z^q)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[
p])*z^(q*FracPart[p])))   Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a
, m, n, p, q}, x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !FreeQ[w, x] &&  !F
reeQ[z, x]
```

3.889.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{2\sqrt{\frac{\left(x^2-1\right)^2}{x\left(x^2+1\right)}}\, x}{x^2-1}$	31
risch	$-\frac{2\sqrt{\frac{\left(x^2-1\right)^2}{x\left(x^2+1\right)}}\, x}{x^2-1}$	31
gosper	$-\frac{2x\sqrt{\frac{\left(x^2-1\right)^2}{x\left(x^2+1\right)}}}{(x-1)(x+1)}$	34
trager	$-\frac{2x\sqrt{-\frac{-x^4+2x^2-1}{x^3+x}}}{x^2-1}$	34

input
```
int(((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-2*((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2-1)*x
```

3.889.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}}{1+x^2}\, dx = -\frac{2\, x\sqrt{\frac{x^4-2\, x^2+1}{x^3+x}}}{x^2-1}$$

input
```
integrate(((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="fricas")
```

output
```
-2*x*sqrt((x^4 - 2*x^2 + 1)/(x^3 + x))/(x^2 - 1)
```

3.889.6 Sympy [F]

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{(x-1)^2(x+1)^2}{x^3+x}}}{x^2+1}\,dx$$

input
```
integrate(((x**2-1)**2/x/(x**2+1))**(1/2)/(x**2+1),x)
```

output
```
Integral(sqrt((x - 1)**2*(x + 1)**2/(x**3 + x))/(x**2 + 1), x)
```

3.889.7 Maxima [F]

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{(x^2-1)^2}{(x^2+1)x}}}{x^2+1}\,dx$$

input
```
integrate(((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="maxima")
```

output
```
integrate(sqrt((x^2 - 1)^2/((x^2 + 1)*x))/(x^2 + 1), x)
```

3.889.8 Giac [F]

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{(x^2-1)^2}{(x^2+1)x}}}{x^2+1}\,dx$$

input
```
integrate(((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="giac")
```

output
```
integrate(sqrt((x^2 - 1)^2/((x^2 + 1)*x))/(x^2 + 1), x)
```

3.889.9 Mupad [B] (verification not implemented)

Time = 22.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}}{1+x^2}\,dx = -\frac{(2\,x^3+2\,x)\,\sqrt{\frac{1}{x^2+1}}\,\sqrt{(x^2-1)^2}\,\sqrt{\frac{1}{x}}}{(x^2-1)\,(x^2+1)}$$

input
```
int(((x^2 - 1)^2/(x*(x^2 + 1)))^(1/2)/(x^2 + 1),x)
```

output
```
-((2*x + 2*x^3)*(1/(x^2 + 1))^(1/2)*((x^2 - 1)^2)^(1/2)*(1/x)^(1/2))/((x^2
 - 1)*(x^2 + 1))
```

3.890 $\int \frac{\sqrt{\frac{\left(-1+x^2\right)^2}{x+x^3}}}{1+x^2}\,dx$

3.890.1 Optimal result

Integrand size = 27, antiderivative size = 33

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{1+x^2}\,dx=\frac{2x\sqrt{\frac{(1-x^2)^2}{x+x^3}}}{1-x^2}$$

output
```
2*x*((-x^2+1)^2/(x^3+x))^(1/2)/(-x^2+1)
```

3.890.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{1+x^2}\,dx=-\frac{2x\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{-1+x^2}$$

input
```
Integrate[Sqrt[(-1 + x^2)^2/(x + x^3)]/(1 + x^2),x]
```

output
```
(-2*x*Sqrt[(-1 + x^2)^2/(x + x^3)])/(-1 + x^2)
```

3.890.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {7270, 25, 2467, 356}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{(x^2-1)^2}{x^3+x}}}{x^2+1}dx$$

$$\Big\downarrow \text{ 7270}$$

$$-\frac{\sqrt{\frac{(1-x^2)^2}{x^3+x}}\sqrt{x^3+x}\int -\frac{1-x^2}{(x^2+1)\sqrt{x^3+x}}dx}{1-x^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt{\frac{(1-x^2)^2}{x^3+x}}\sqrt{x^3+x}\int \frac{1-x^2}{(x^2+1)\sqrt{x^3+x}}dx}{1-x^2}$$

$$\Big\downarrow \text{ 2467}$$

$$\frac{\sqrt{x}\sqrt{x^2+1}\sqrt{\frac{(1-x^2)^2}{x^3+x}}\int \frac{1-x^2}{\sqrt{x}(x^2+1)^{3/2}}dx}{1-x^2}$$

$$\Big\downarrow \text{ 356}$$

$$\frac{2x\sqrt{\frac{(1-x^2)^2}{x^3+x}}}{1-x^2}$$

input
```
Int[Sqrt[(-1 + x^2)^2/(x + x^3)]/(1 + x^2),x]
```

output
```
(2*x*Sqrt[(1 - x^2)^2/(x + x^3)])/(1 - x^2)
```

3.890. $\int \frac{\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{1+x^2}dx$

3.890.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 356
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] /;
 FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(m + 1) -
b*c*(m + 2*p + 3), 0] && NeQ[m, -1]
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

3.890.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{2\sqrt{\frac{\left(x^{2}-1\right)^{2}}{x\left(x^{2}+1\right)}}\, x}{x^{2}-1}$	31
risch	$-\frac{2\sqrt{\frac{\left(x^{2}-1\right)^{2}}{x\left(x^{2}+1\right)}}\, x}{x^{2}-1}$	31
gosper	$-\frac{2x\sqrt{\frac{\left(x^{2}-1\right)^{2}}{x\left(x^{2}+1\right)}}}{\left(x-1\right)\left(x+1\right)}$	34
trager	$-\frac{2x\sqrt{-\frac{-x^{4}+2x^{2}-1}{x^{3}+x}}}{x^{2}-1}$	34

input
```
int(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

3.890. $\int \frac{\sqrt{\frac{\left(-1+x^2\right)^2}{x+x^3}}}{1+x^2}\,dx$

output
```
-2*((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2-1)*x
```

3.890.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{1+x^2}\,dx = -\frac{2\,x\sqrt{\frac{x^4-2\,x^2+1}{x^3+x}}}{x^2-1}$$

input
```
integrate(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1),x, algorithm="fricas")
```

output
```
-2*x*sqrt((x^4 - 2*x^2 + 1)/(x^3 + x))/(x^2 - 1)
```

3.890.6 Sympy [F]

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{(x-1)^2(x+1)^2}{x^3+x}}}{x^2+1}\,dx$$

input
```
integrate(((x**2-1)**2/(x**3+x))**(1/2)/(x**2+1),x)
```

output
```
Integral(sqrt((x - 1)**2*(x + 1)**2/(x**3 + x))/(x**2 + 1), x)
```

3.890.7 Maxima [F]

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{(x^2-1)^2}{x^3+x}}}{x^2+1}\,dx$$

input
```
integrate(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1),x, algorithm="maxima")
```

output
```
integrate(sqrt((x^2 - 1)^2/(x^3 + x))/(x^2 + 1), x)
```

3.890.8 Giac [F]

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{(x^2-1)^2}{x^3+x}}}{x^2+1}\,dx$$

input
```
integrate(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1),x, algorithm="giac")
```

output
```
integrate(sqrt((x^2 - 1)^2/(x^3 + x))/(x^2 + 1), x)
```

3.890.9 Mupad [B] (verification not implemented)

Time = 22.95 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{1+x^2}\,dx = -\frac{\sqrt{\frac{1}{x^3+x}}\,(2\,x^3+2\,x)\,\sqrt{(x^2-1)^2}}{(x^2-1)\,(x^2+1)}$$

input
```
int(((x^2 - 1)^2/(x + x^3))^(1/2)/(x^2 + 1),x)
```

output
```
-((1/(x + x^3))^(1/2)*(2*x + 2*x^3)*((x^2 - 1)^2)^(1/2))/((x^2 - 1)*(x^2 +
 1))
```

3.891 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx$

3.891.1 Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx=\frac{\sqrt{b+ax^2}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{b+ax^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{d}\sqrt{a+\frac{b}{x^2}}\,x}$$

output
```
arctanh(d^(1/2)*(a*x^2+b)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*(a*x^2+b)^(1/2)/x
/a^(1/2)/d^(1/2)/(a+b/x^2)^(1/2)
```

3.891.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx=\frac{\sqrt{b+ax^2}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{b+ax^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{d}\sqrt{a+\frac{b}{x^2}}\,x}$$

input
```
Integrate[1/(Sqrt[a + b/x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b + a*x^2]*ArcTanh[(Sqrt[d]*Sqrt[b + a*x^2])/(Sqrt[a]*Sqrt[c + d*x^2
])])/(Sqrt[a]*Sqrt[d]*Sqrt[a + b/x^2]*x)
```

3.891.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {942, 353, 66, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}dx$$

$$\Big\downarrow \text{ 942}$$

$$\frac{\sqrt{ax^2+b}\int \frac{x}{\sqrt{ax^2+b}\sqrt{dx^2+c}}dx}{x\sqrt{a+\frac{b}{x^2}}}$$

$$\Big\downarrow \text{ 353}$$

$$\frac{\sqrt{ax^2+b}\int \frac{1}{\sqrt{ax^2+b}\sqrt{dx^2+c}}dx^2}{2x\sqrt{a+\frac{b}{x^2}}}$$

$$\Big\downarrow \text{ 66}$$

$$\frac{\sqrt{ax^2+b}\int \frac{1}{a-dx^4}d\frac{\sqrt{ax^2+b}}{\sqrt{dx^2+c}}}{x\sqrt{a+\frac{b}{x^2}}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\sqrt{ax^2+b}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{ax^2+b}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{d}x\sqrt{a+\frac{b}{x^2}}}$$

input
```
Int[1/(Sqrt[a + b/x^2]*Sqrt[c + d*x^2]),x]
```

output
```
(Sqrt[b + a*x^2]*ArcTanh[(Sqrt[d]*Sqrt[b + a*x^2])/(Sqrt[a]*Sqrt[c + d*x^2
])])/(Sqrt[a]*Sqrt[d]*Sqrt[a + b/x^2]*x)
```

3.891.3.1 Defintions of rubi rules used

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 942
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symb
ol] :> Simp[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n)^FracPart
[q])   Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x], x] /; FreeQ[{a, b, c,
 d, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

3.891.4 Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.47

method	result	size
default	$\frac{(a\,x^2+b)\ln\left(\frac{2ad\,x^2+2\sqrt{(a\,x^2+b)(d\,x^2+c)}\,\sqrt{ad}+ac+bd}{2\sqrt{ad}}\right)\sqrt{d\,x^2+c}}{2\sqrt{\frac{a\,x^2+b}{x^2}}\,x\sqrt{ad}\,\sqrt{(a\,x^2+b)(d\,x^2+c)}}$	103

input
```
int(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((a*x^2+b)/x^2)^(1/2)/x*(a*x^2+b)*ln(1/2*(2*a*d*x^2+2*((a*x^2+b)*(d*x^
2+c))^(1/2)*(a*d)^(1/2)+a*c+b*d)/(a*d)^(1/2))*(d*x^2+c)^(1/2)/(a*d)^(1/2)/
((a*x^2+b)*(d*x^2+c))^(1/2)
```

3.891.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.97

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx = \left[\frac{\sqrt{ad}\log\left(8\,a^2d^2x^4+a^2c^2+6\,abcd+b^2d^2+8\,(a^2cd+abd^2)x^2+4\,(2\,adx^3+(ac+bd)x)\sqrt{dx^2+c}\sqrt{ad}\sqrt{\right.}{4\,ad}, -\frac{\sqrt{-ad}\arctan\left(\frac{(2\,adx^3+(ac+bd)x)\sqrt{dx^2+c}\sqrt{-ad}\sqrt{\frac{ax^2+b}{x^2}}}{2\,(a^2d^2x^4+abcd+(a^2cd+abd^2)x^2)}\right)}{2\,ad}\right]$$

input
```
integrate(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*sqrt(a*d)*log(8*a^2*d^2*x^4 + a^2*c^2 + 6*a*b*c*d + b^2*d^2 + 8*(a^2*
c*d + a*b*d^2)*x^2 + 4*(2*a*d*x^3 + (a*c + b*d)*x)*sqrt(d*x^2 + c)*sqrt(a*
d)*sqrt((a*x^2 + b)/x^2))/(a*d), -1/2*sqrt(-a*d)*arctan(1/2*(2*a*d*x^3 + (
a*c + b*d)*x)*sqrt(d*x^2 + c)*sqrt(-a*d)*sqrt((a*x^2 + b)/x^2)/(a^2*d^2*x^
4 + a*b*c*d + (a^2*c*d + a*b*d^2)*x^2))/(a*d)]
```

3.891.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx$$

input
```
integrate(1/(a+b/x**2)**(1/2)/(d*x**2+c)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b/x**2)*sqrt(c + d*x**2)), x)
```

3.891.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{dx^2+c}\sqrt{a+\frac{b}{x^2}}}\,dx$$

input
```
integrate(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(d*x^2 + c)*sqrt(a + b/x^2)), x)
```

3.891.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.31

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx = \frac{a\log\left(\left|-\sqrt{ad}\sqrt{b}+\sqrt{a^2c}\right|\right)\operatorname{sgn}(x)}{\sqrt{ad}|a|} - \frac{a\log\left(\left|-\sqrt{ax^2+b}\sqrt{ad}+\sqrt{a^2c+(ax^2+b)ad-abd}\right|\right)}{\sqrt{ad}|a|\operatorname{sgn}(x)}$$

input
```
integrate(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
a*log(abs(-sqrt(a*d)*sqrt(b) + sqrt(a^2*c)))*sgn(x)/(sqrt(a*d)*abs(a)) - a
*log(abs(-sqrt(a*x^2 + b)*sqrt(a*d) + sqrt(a^2*c + (a*x^2 + b)*a*d - a*b*d
)))/(sqrt(a*d)*abs(a)*sgn(x))
```

3.891.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,\sqrt{d\,x^2+c}}\,dx$$

input
```
int(1/((a + b/x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

output
```
int(1/((a + b/x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.892 $\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)}\,dx$

3.892.1 Optimal result

Integrand size = 28, antiderivative size = 83

$$\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)}\,dx = \frac{2\sqrt{-2x^2+x^4}\arctan\left(\frac{1}{2}\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} - \frac{\sqrt{-2x^2+x^4}\arctan\left(\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}}$$

output
```
2/3*arctan(1/2*(x^2-2)^(1/2))*(x^4-2*x^2)^(1/2)/x/(x^2-2)^(1/2)-1/3*arctan
((x^2-2)^(1/2))*(x^4-2*x^2)^(1/2)/x/(x^2-2)^(1/2)
```

3.892.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)}\,dx = \frac{x\sqrt{-2+x^2}\left(2\arctan\left(\frac{1}{2}\sqrt{-2+x^2}\right)-\arctan\left(\sqrt{-2+x^2}\right)\right)}{3\sqrt{x^2(-2+x^2)}}$$

input
```
Integrate[Sqrt[-2*x^2 + x^4]/((-1 + x^2)*(2 + x^2)),x]
```

output
```
(x*Sqrt[-2 + x^2]*(2*ArcTan[Sqrt[-2 + x^2]/2] - ArcTan[Sqrt[-2 + x^2]]))/(
3*Sqrt[x^2*(-2 + x^2)])
```

3.892.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2467, 25, 435, 94, 73, 216, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^4-2x^2}}{(x^2-1)(x^2+2)}dx$$

$$\Big\downarrow \text{2467}$$

$$\frac{\sqrt{x^4-2x^2}\int -\frac{x\sqrt{x^2-2}}{(1-x^2)(x^2+2)}dx}{x\sqrt{x^2-2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sqrt{x^4-2x^2}\int \frac{x\sqrt{x^2-2}}{(1-x^2)(x^2+2)}dx}{x\sqrt{x^2-2}}$$

$$\Big\downarrow \text{435}$$

$$-\frac{\sqrt{x^4-2x^2}\int \frac{\sqrt{x^2-2}}{(1-x^2)(x^2+2)}dx^2}{2x\sqrt{x^2-2}}$$

$$\Big\downarrow \text{94}$$

$$-\frac{\sqrt{x^4-2x^2}\left(-\frac{1}{3}\int \frac{1}{(1-x^2)\sqrt{x^2-2}}dx^2-\frac{4}{3}\int \frac{1}{\sqrt{x^2-2}(x^2+2)}dx^2\right)}{2x\sqrt{x^2-2}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\sqrt{x^4-2x^2}\left(-\frac{2}{3}\int \frac{1}{-x^4-1}d\sqrt{x^2-2}-\frac{8}{3}\int \frac{1}{x^4+4}d\sqrt{x^2-2}\right)}{2x\sqrt{x^2-2}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\sqrt{x^4-2x^2}\left(-\frac{2}{3}\int \frac{1}{-x^4-1}d\sqrt{x^2-2}-\frac{4}{3}\arctan\left(\frac{\sqrt{x^2-2}}{2}\right)\right)}{2x\sqrt{x^2-2}}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\sqrt{x^4-2x^2}\left(\frac{2}{3}\arctan\left(\sqrt{x^2-2}\right)-\frac{4}{3}\arctan\left(\frac{\sqrt{x^2-2}}{2}\right)\right)}{2x\sqrt{x^2-2}}$$

input
```
Int[Sqrt[-2*x^2 + x^4]/((-1 + x^2)*(2 + x^2)),x]
```

output
```
-1/2*(Sqrt[-2*x^2 + x^4]*((-4*ArcTan[Sqrt[-2 + x^2]/2])/3 + (2*ArcTan[Sqrt
[-2 + x^2]])/3))/(x*Sqrt[-2 + x^2])
```

3.892.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

3.892. $\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)}\,dx$

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

3.892.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.70 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$\frac{i\left(-i\arctan\left(\frac{1}{\sqrt{x^2-2}}\right)+\operatorname{arctanh}\left(\frac{\left(i\sqrt{2}-x\right)\sqrt{2}}{2\sqrt{x^2-2}}\right)+\operatorname{arctanh}\left(\frac{\left(x+i\sqrt{2}\right)\sqrt{2}}{2\sqrt{x^2-2}}\right)\right)}{3}$
default	$\frac{\sqrt{x^4-2x^2}\left(\arctan\left(\frac{x+2}{\sqrt{x^2-2}}\right)-\arctan\left(\frac{x-2}{\sqrt{x^2-2}}\right)+4\arctan\left(\frac{\sqrt{x^2-2}}{2}\right)\right)}{6x\sqrt{x^2-2}}$
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2+1\right)x^7-15\operatorname{RootOf}\left(\_Z^2+1\right)x^5+6\sqrt{x^4-2x^2}\,x^4+24\operatorname{RootOf}\left(\_Z^2+1\right)x^3-16\sqrt{x^4-2x^2}\,x}{\left(x^2+2\right)^2(x-1)(x+1)x}\right.}{6}$

input
```
int((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2),x,method=_RETURNVERBOSE)
```

output
```
1/3*I*(-I*arctan(1/(x^2-2)^(1/2))+arctanh(1/2*(I*2^(1/2)-x)*2^(1/2)/(x^2-2
)^(1/2))+arctanh(1/2*(x+I*2^(1/2))*2^(1/2)/(x^2-2)^(1/2)))
```

3.892.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)}\,dx = -\frac{1}{3}\arctan\left(\frac{\sqrt{x^4-2\,x^2}}{x}\right)+\frac{2}{3}\arctan\left(\frac{\sqrt{x^4-2\,x^2}}{2\,x}\right)$$

input
```
integrate((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2),x, algorithm="fricas")
```

output
```
-1/3*arctan(sqrt(x^4 - 2*x^2)/x) + 2/3*arctan(1/2*sqrt(x^4 - 2*x^2)/x)
```

3.892.6 Sympy [F]

$$\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)}\,dx = \int \frac{\sqrt{x^2(x^2-2)}}{(x-1)(x+1)(x^2+2)}\,dx$$

input
```
integrate((x**4-2*x**2)**(1/2)/(x**2-1)/(x**2+2),x)
```

output
```
Integral(sqrt(x**2*(x**2 - 2))/((x - 1)*(x + 1)*(x**2 + 2)), x)
```

3.892.7 Maxima [F]

$$\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)}\,dx = \int \frac{\sqrt{x^4-2\,x^2}}{(x^2+2)(x^2-1)}\,dx$$

input
```
integrate((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^4 - 2*x^2)/((x^2 + 2)*(x^2 - 1)), x)
```

3.892.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)}\,dx = \frac{1}{3}\left(\arctan\left(i\sqrt{2}\right)-2\arctan\left(\frac{1}{2}i\sqrt{2}\right)\right)\mathrm{sgn}(x) + \frac{2}{3}\arctan\left(\frac{1}{2}\sqrt{x^2-2}\right)\mathrm{sgn}(x) - \frac{1}{3}\arctan\left(\sqrt{x^2-2}\right)\mathrm{sgn}(x)$$

input
```
integrate((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2),x, algorithm="giac")
```

output
```
1/3*(arctan(I*sqrt(2)) - 2*arctan(1/2*I*sqrt(2)))*sgn(x) + 2/3*arctan(1/2*
sqrt(x^2 - 2))*sgn(x) - 1/3*arctan(sqrt(x^2 - 2))*sgn(x)
```

3.892.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)}\,dx = \int \frac{\sqrt{x^4-2\,x^2}}{(x^2-1)\,(x^2+2)}\,dx$$

input
```
int((x^4 - 2*x^2)^(1/2)/((x^2 - 1)*(x^2 + 2)),x)
```

output
```
int((x^4 - 2*x^2)^(1/2)/((x^2 - 1)*(x^2 + 2)), x)
```

3.893 $\int \frac{\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}}{2-x^2} dx$

3.893.1 Optimal result

Integrand size = 25, antiderivative size = 47

$$\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2-x^2} dx = \frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\arctan\left(\sqrt{-2+x^2}\right)}{x\sqrt{-2+x^2}}$$

output
```
(-x^2+1)*arctan((x^2-2)^(1/2))*(1-1/(-x^2+1)^2)^(1/2)/x/(x^2-2)^(1/2)
```

3.893.2 Mathematica [A] (verified)

Time = 5.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2-x^2} dx = -\frac{(-1+x^2)\sqrt{1-\frac{1}{(-1+x^2)^2}}\arctan\left(\sqrt{-2+x^2}\right)}{x\sqrt{-2+x^2}}$$

input
```
Integrate[Sqrt[1 - (-1 + x^2)^(-2)]/(2 - x^2),x]
```

output
```
-(((-1 + x^2)*Sqrt[1 - (-1 + x^2)^(-2)]*ArcTan[Sqrt[-2 + x^2]])/(x*Sqrt[-2
 + x^2]))
```

3.893.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {7273, 25, 2467, 281, 353, 73, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-\frac{1}{(x^2-1)^2}}}{2-x^2}dx$$

$$\Big\downarrow \ 7273$$

$$-\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\int -\frac{\sqrt{(x^2-1)^2-1}}{(1-x^2)(2-x^2)}dx}{\sqrt{(x^2-1)^2-1}}$$

$$\Big\downarrow \ 25$$

$$\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\int \frac{\sqrt{(x^2-1)^2-1}}{(1-x^2)(2-x^2)}dx}{\sqrt{(x^2-1)^2-1}}$$

$$\Big\downarrow \ 2467$$

$$\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\int \frac{x\sqrt{x^2-2}}{(1-x^2)(2-x^2)}dx}{x\sqrt{x^2-2}}$$

$$\Big\downarrow \ 281$$

$$-\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\int \frac{x}{(1-x^2)\sqrt{x^2-2}}dx}{x\sqrt{x^2-2}}$$

$$\Big\downarrow \ 353$$

$$-\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\int \frac{1}{(1-x^2)\sqrt{x^2-2}}dx^2}{2x\sqrt{x^2-2}}$$

$$\Big\downarrow \ 73$$

$$-\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\int \frac{1}{-x^4-1}d\sqrt{x^2-2}}{x\sqrt{x^2-2}}$$

$$\Big\downarrow \ 217$$

3.893. $\int \frac{\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}}{2-x^2}\,dx$

$$\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\arctan\left(\sqrt{x^2-2}\right)}{x\sqrt{x^2-2}}$$

input
```
Int[Sqrt[1 - (-1 + x^2)^(-2)]/(2 - x^2),x]
```

output
```
((1 - x^2)*Sqrt[1 - (1 - x^2)^(-2)]*ArcTan[Sqrt[-2 + x^2]])/(x*Sqrt[-2 + x
^2])
```

3.893.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

3.893. $\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2-x^2}\,dx$

rule 7273
```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p])   Int[u*v^(n*p)*(b + a
/v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bi
nomialQ[v, x] &&  !LinearQ[v, x]
```

3.893.4 Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.34

method	result	size
default	$\frac{\sqrt{\frac{\left(x^{2}-2\right)x^{2}}{\left(x^{2}-1\right)^{2}}}\,\left(x^{2}-1\right)\left(\arctan\left(\frac{x+2}{\sqrt{x^{2}-2}}\right)-\arctan\left(\frac{x-2}{\sqrt{x^{2}-2}}\right)\right)}{2x\sqrt{x^{2}-2}}$	63
trager	$-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{-\operatorname{RootOf}\left(\_Z^{2}+1\right)x^{3}+2x^{2}\sqrt{-\frac{-x^{4}+2x^{2}}{x^{4}-2x^{2}+1}}+3\operatorname{RootOf}\left(\_Z^{2}+1\right)x-2\sqrt{-\frac{-x^{4}+2x^{2}}{x^{4}-2x^{2}+1}}}{(x+1)(x-1)x}\right)}{2}$	106

input
```
int((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((x^2-2)*x^2/(x^2-1)^2)^(1/2)*(x^2-1)*(arctan((x+2)/(x^2-2)^(1/2))-arc
tan((x-2)/(x^2-2)^(1/2)))/x/(x^2-2)^(1/2)
```

3.893.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2-x^2}\,dx = -\arctan\left(\frac{(x^2-1)\sqrt{\frac{x^4-2\,x^2}{x^4-2\,x^2+1}}}{x}\right)$$

input
```
integrate((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x, algorithm="fricas")
```

output
```
-arctan((x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))/x)
```

3.893.6 Sympy [F]

$$\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2-x^2}\,dx = -\int \frac{\sqrt{\frac{x^4}{x^4-2x^2+1}-\frac{2x^2}{x^4-2x^2+1}}}{x^2-2}\,dx$$

input
```
integrate((1-1/(x**2-1)**2)**(1/2)/(-x**2+2),x)
```

output
```
-Integral(sqrt(x**4/(x**4 - 2*x**2 + 1) - 2*x**2/(x**4 - 2*x**2 + 1))/(x**
2 - 2), x)
```

3.893.7 Maxima [F]

$$\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2-x^2}\,dx = \int -\frac{\sqrt{-\frac{1}{(x^2-1)^2}+1}}{x^2-2}\,dx$$

input
```
integrate((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x, algorithm="maxima")
```

output
```
-integrate(sqrt(-1/(x^2 - 1)^2 + 1)/(x^2 - 2), x)
```

3.893.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2-x^2}\,dx = -\arctan\left(\sqrt{x^2-2}\right)\operatorname{sgn}\left(x^3-x\right)$$

input
```
integrate((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x, algorithm="giac")
```

output
```
-arctan(sqrt(x^2 - 2))*sgn(x^3 - x)
```

3.893.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2-x^2}\,dx=\int -\frac{\sqrt{1-\frac{1}{(x^2-1)^2}}}{x^2-2}\,dx$$

input
```
int(-(1 - 1/(x^2 - 1)^2)^(1/2)/(x^2 - 2),x)
```

output
```
int(-(1 - 1/(x^2 - 1)^2)^(1/2)/(x^2 - 2), x)
```

3.894 $\int \frac{\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}}{2+x^2}\,dx$

3.894.1 Optimal result

Integrand size = 29, antiderivative size = 123

$$\int \frac{\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}}{2+x^2}\,dx = -\frac{2(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\arctan\left(\frac{1}{2}\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} + \frac{(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\arctan\left(\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}}$$

output
```
-2/3*(-x^2+1)*arctan(1/2*(x^2-2)^(1/2))*((x^4-2*x^2)/(-x^2+1)^2)^(1/2)/x/(
x^2-2)^(1/2)+1/3*(-x^2+1)*arctan((x^2-2)^(1/2))*((x^4-2*x^2)/(-x^2+1)^2)^(
1/2)/x/(x^2-2)^(1/2)
```

3.894.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}}{2+x^2}\,dx = \frac{\sqrt{\frac{x^2(-2+x^2)}{(-1+x^2)^2}}(-1+x^2)\left(2\arctan\left(\frac{1}{2}\sqrt{-2+x^2}\right)-\arctan\left(\sqrt{-2+x^2}\right)\right)}{3x\sqrt{-2+x^2}}$$

input
```
Integrate[Sqrt[(-2*x^2 + x^4)/(-1 + x^2)^2]/(2 + x^2),x]
```

output
```
(Sqrt[(x^2*(-2 + x^2))/(-1 + x^2)^2]*(-1 + x^2)*(2*ArcTan[Sqrt[-2 + x^2]/2
] - ArcTan[Sqrt[-2 + x^2]]))/(3*x*Sqrt[-2 + x^2])
```

3.894.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.67, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {7270, 25, 2467, 435, 94, 73, 216, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{x^4-2x^2}{(x^2-1)^2}}}{x^2+2} \, dx$$

$$\Big\downarrow \text{7270}$$

$$-\frac{(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\int -\frac{\sqrt{x^4-2x^2}}{(1-x^2)(x^2+2)}dx}{\sqrt{x^4-2x^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\int \frac{\sqrt{x^4-2x^2}}{(1-x^2)(x^2+2)}dx}{\sqrt{x^4-2x^2}}$$

$$\Big\downarrow \text{2467}$$

$$\frac{(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\int \frac{x\sqrt{x^2-2}}{(1-x^2)(x^2+2)}dx}{x\sqrt{x^2-2}}$$

$$\Big\downarrow \text{435}$$

$$\frac{(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\int \frac{\sqrt{x^2-2}}{(1-x^2)(x^2+2)}dx^2}{2x\sqrt{x^2-2}}$$

$$\Big\downarrow \text{94}$$

$$\frac{(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\left(-\frac{1}{3}\int \frac{1}{(1-x^2)\sqrt{x^2-2}}dx^2-\frac{4}{3}\int \frac{1}{\sqrt{x^2-2}(x^2+2)}dx^2\right)}{2x\sqrt{x^2-2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\left(-\frac{2}{3}\int \frac{1}{-x^4-1}d\sqrt{x^2-2}-\frac{8}{3}\int \frac{1}{x^4+4}d\sqrt{x^2-2}\right)}{2x\sqrt{x^2-2}}$$

3.894. $\int \frac{\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}}{2+x^2} \, dx$

$$\Big\downarrow 216$$

$$\frac{(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\left(-\frac{2}{3}\int\frac{1}{-x^4-1}d\sqrt{x^2-2}-\frac{4}{3}\arctan\left(\frac{\sqrt{x^2-2}}{2}\right)\right)}{2x\sqrt{x^2-2}}$$

$$\Big\downarrow 217$$

$$\frac{(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\left(\frac{2}{3}\arctan\left(\sqrt{x^2-2}\right)-\frac{4}{3}\arctan\left(\frac{\sqrt{x^2-2}}{2}\right)\right)}{2x\sqrt{x^2-2}}$$

input
```
Int[Sqrt[(-2*x^2 + x^4)/(-1 + x^2)^2]/(2 + x^2),x]
```

output
```
((1 - x^2)*Sqrt[-((2*x^2 - x^4)/(1 - x^2)^2)]*((-4*ArcTan[Sqrt[-2 + x^2]/2
])/3 + (2*ArcTan[Sqrt[-2 + x^2]])/3))/(2*x*Sqrt[-2 + x^2])
```

3.894.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.894. $\int \frac{\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}}{2+x^2}\,dx$

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

3.894.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.61

method	result
default	$\frac{\sqrt{\frac{\left(x^{2}-2\right) x^{2}}{\left(x^{2}-1\right)^{2}}}\,\left(x^{2}-1\right) \left(\arctan\left(\frac{x+2}{\sqrt{x^{2}-2}}\right)-\arctan\left(\frac{x-2}{\sqrt{x^{2}-2}}\right)+4 \arctan\left(\frac{\sqrt{x^{2}-2}}{2}\right)\right)}{6 x \sqrt{x^{2}-2}}$
trager	$\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right) \ln\left(-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right) x^{7}-6 \sqrt{-\frac{-x^{4}+2 x^{2}}{x^{4}-2 x^{2}+1}}\, x^{6}-15 \operatorname{RootOf}\left(\_Z^{2}+1\right) x^{5}+22 x^{4} \sqrt{-\frac{-x^{4}+2 x^{2}}{x^{4}-2 x^{2}+1}}+24 \operatorname{RootOf}\left(\_Z^{2}+1\right) x}{x \left(x^{2}+2\right)^{2} (x-1)(x+1)}\right.}{6}$

input
```
int(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2),x,method=_RETURNVERBOSE)
```

output
```
1/6*((x^2-2)*x^2/(x^2-1)^2)^(1/2)*(x^2-1)*(arctan((x+2)/(x^2-2)^(1/2))-arc
tan((x-2)/(x^2-2)^(1/2))+4*arctan(1/2*(x^2-2)^(1/2)))/x/(x^2-2)^(1/2)
```

3.894. $\int \frac{\sqrt{\frac{-2x^2+x^4}{\left(-1+x^2\right)^2}}}{2+x^2}\,dx$

3.894.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}}{2+x^2}\,dx = -\frac{1}{3}\arctan\left(\frac{(x^2-1)\sqrt{\frac{x^4-2\,x^2}{x^4-2\,x^2+1}}}{x}\right) + \frac{2}{3}\arctan\left(\frac{(x^2-1)\sqrt{\frac{x^4-2\,x^2}{x^4-2\,x^2+1}}}{2\,x}\right)$$

input
```
integrate(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2),x, algorithm="fricas")
```

output
```
-1/3*arctan((x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))/x) + 2/3*arcta
n(1/2*(x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))/x)
```

3.894.6 Sympy [F]

$$\int \frac{\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}}{2+x^2}\,dx = \int \frac{\sqrt{\frac{x^2(x^2-2)}{x^4-2x^2+1}}}{x^2+2}\,dx$$

input
```
integrate(((x**4-2*x**2)/(x**2-1)**2)**(1/2)/(x**2+2),x)
```

output
```
Integral(sqrt(x**2*(x**2 - 2)/(x**4 - 2*x**2 + 1))/(x**2 + 2), x)
```

3.894.7 Maxima [F]

$$\int \frac{\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}}{2+x^2}\,dx = \int \frac{\sqrt{\frac{x^4-2\,x^2}{(x^2-1)^2}}}{x^2+2}\,dx$$

input
```
integrate(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2),x, algorithm="maxima")
```

output
```
integrate(sqrt((x^4 - 2*x^2)/(x^2 - 1)^2)/(x^2 + 2), x)
```

3.894. $\int \frac{\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}}{2+x^2}\,dx$

3.894.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.32

$$\int \frac{\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}}{2+x^2}\,dx = \frac{2}{3}\arctan\left(\frac{1}{2}\sqrt{x^2-2}\right)\operatorname{sgn}\left(x^3-x\right) - \frac{1}{3}\arctan\left(\sqrt{x^2-2}\right)\operatorname{sgn}\left(x^3-x\right)$$

input
```
integrate(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2),x, algorithm="giac")
```

output
```
2/3*arctan(1/2*sqrt(x^2 - 2))*sgn(x^3 - x) - 1/3*arctan(sqrt(x^2 - 2))*sgn
(x^3 - x)
```

3.894.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}}{2+x^2}\,dx = \int \frac{\sqrt{-\frac{2\,x^2-x^4}{(x^2-1)^2}}}{x^2+2}\,dx$$

input
```
int((-(2*x^2 - x^4)/(x^2 - 1)^2)^(1/2)/(x^2 + 2),x)
```

output
```
int((-(2*x^2 - x^4)/(x^2 - 1)^2)^(1/2)/(x^2 + 2), x)
```

3.895 $\int \left(1+\frac{2x}{1+x^2}\right)^{5/2} dx$

3.895.1 Optimal result

Integrand size = 16, antiderivative size = 133

$$\int \left(1+\frac{2x}{1+x^2}\right)^{5/2} dx = -\frac{4}{3}(1-2x)(1+x)\sqrt{1+\frac{2x}{1+x^2}} - \frac{(1-x)(1+x)^3\sqrt{1+\frac{2x}{1+x^2}}}{3\left(1+x^2\right)} - \frac{(4+3x)\left(1+x^2\right)\sqrt{1+\frac{2x}{1+x^2}}}{1+x} + \frac{5\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\operatorname{arcsinh}(x)}{1+x}$$

output
```
-4/3*(1-2*x)*(1+x)*(1+2*x/(x^2+1))^(1/2)-1/3*(1-x)*(1+x)^3*(1+2*x/(x^2+1))
^(1/2)/(x^2+1)-(4+3*x)*(x^2+1)*(1+2*x/(x^2+1))^(1/2)/(1+x)+5*arcsinh(x)*(x
^2+1)^(1/2)*(1+2*x/(x^2+1))^(1/2)/(1+x)
```

3.895.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.57

$$\int \left(1+\frac{2x}{1+x^2}\right)^{5/2} dx = \frac{(1+x)\left(-17-12x-18x^2-8x^3+3x^4-15(1+x^2)^{3/2}\log\left(-x+\sqrt{1+x^2}\right)\right)}{3\sqrt{\frac{(1+x)^2}{1+x^2}}\left(1+x^2\right)^2}$$

input
```
Integrate[(1 + (2*x)/(1 + x^2))^(5/2),x]
```

output
```
((1 + x)*(-17 - 12*x - 18*x^2 - 8*x^3 + 3*x^4 - 15*(1 + x^2)^(3/2)*Log[-x
+ Sqrt[1 + x^2]]))/(3*Sqrt[(1 + x)^2/(1 + x^2)]*(1 + x^2)^2)
```

3.895.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.86, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {7274, 1298, 27, 495, 27, 684, 27, 676, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{2x}{x^2+1}+1\right)^{5/2} dx$$

$$\Big\downarrow \text{7274}$$

$$\frac{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\int \frac{\left(x^2+2x+1\right)^{5/2}}{\left(x^2+1\right)^{5/2}}dx}{\sqrt{x^2+2x+1}}$$

$$\Big\downarrow \text{1298}$$

$$\frac{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\int \frac{32(x+1)^5}{\left(x^2+1\right)^{5/2}}dx}{32(x+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\int \frac{(x+1)^5}{\left(x^2+1\right)^{5/2}}dx}{x+1}$$

$$\Big\downarrow \text{495}$$

$$\frac{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\left(\frac{1}{3}\int \frac{2(3-x)(x+1)^3}{\left(x^2+1\right)^{3/2}}dx-\frac{(1-x)(x+1)^4}{3\left(x^2+1\right)^{3/2}}\right)}{x+1}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\left(\frac{2}{3}\int \frac{(3-x)(x+1)^3}{\left(x^2+1\right)^{3/2}}dx-\frac{(1-x)(x+1)^4}{3\left(x^2+1\right)^{3/2}}\right)}{x+1}$$

$$\Big\downarrow \text{684}$$

$$\frac{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\left(\frac{2}{3}\left(\int \frac{3(1-3x)(x+1)}{\sqrt{x^2+1}}dx-\frac{2(1-2x)(x+1)^2}{\sqrt{x^2+1}}\right)-\frac{(1-x)(x+1)^4}{3(x^2+1)^{3/2}}\right)}{x+1}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\left(\frac{2}{3}\left(3\int \frac{(1-3x)(x+1)}{\sqrt{x^2+1}}dx-\frac{2(1-2x)(x+1)^2}{\sqrt{x^2+1}}\right)-\frac{(1-x)(x+1)^4}{3(x^2+1)^{3/2}}\right)}{x+1}$$

$$\Big\downarrow \; 676$$

$$\frac{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\left(\frac{2}{3}\left(3\left(\frac{5}{2}\int \frac{1}{\sqrt{x^2+1}}dx-\frac{3}{2}\sqrt{x^2+1}x-2\sqrt{x^2+1}\right)-\frac{2(1-2x)(x+1)^2}{\sqrt{x^2+1}}\right)-\frac{(1-x)(x+1)^4}{3(x^2+1)^{3/2}}\right)}{x+1}$$

$$\Big\downarrow \; 222$$

$$\frac{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\left(\frac{2}{3}\left(3\left(\frac{5\text{arcsinh}(x)}{2}-\frac{3}{2}\sqrt{x^2+1}x-2\sqrt{x^2+1}\right)-\frac{2(1-2x)(x+1)^2}{\sqrt{x^2+1}}\right)-\frac{(1-x)(x+1)^4}{3(x^2+1)^{3/2}}\right)}{x+1}$$

input
```
Int[(1 + (2*x)/(1 + x^2))^(5/2),x]
```

output
```
(Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)]*(-1/3*((1 - x)*(1 + x)^4)/(1 + x^
2)^(3/2) + (2*((-2*(1 - 2*x)*(1 + x)^2)/Sqrt[1 + x^2] + 3*(-2*Sqrt[1 + x^2
] - (3*x*Sqrt[1 + x^2])/2 + (5*ArcSinh[x])/2)))/3))/(1 + x)
```

3.895.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

3.895. $\int \left(1+\frac{2x}{1+x^2}\right)^{5/2} dx$

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 684
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Simp[1/(2*a*c*(p + 1))   Int[
(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^
2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a
, c, d, e, f, g}, x] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2]
 && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3, 0])
```

rule 1298
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p]))   Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a
, b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 7274
```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Simp[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p])   I
nt[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Integ
erQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

3.895.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.47

method	result	size
default	$\frac{\left(\frac{x^2+2x+1}{x^2+1}\right)^{\frac{5}{2}}(x^2+1)\left(15\,\operatorname{arcsinh}(x)(x^2+1)^{\frac{3}{2}}+3x^4-8x^3-18x^2-12x-17\right)}{3(x+1)^5}$	62
risch	$\frac{(3x^4-8x^3-18x^2-12x-17)\sqrt{\frac{(x+1)^2}{x^2+1}}}{3(x^2+1)(x+1)}+\frac{5\,\operatorname{arcsinh}(x)\sqrt{x^2+1}\,\sqrt{\frac{(x+1)^2}{x^2+1}}}{x+1}$	82
trager	$\frac{(3x^4-8x^3-18x^2-12x-17)\sqrt{-\frac{-x^2-2x-1}{x^2+1}}}{3(x^2+1)(x+1)}+5\ln\left(\frac{\sqrt{-\frac{-x^2-2x-1}{x^2+1}}\,x^2+x^2+\sqrt{-\frac{-x^2-2x-1}{x^2+1}}+x}{x+1}\right)$	117

3.895. $\int\left(1+\frac{2x}{1+x^2}\right)^{5/2}dx$

input
```
int((1+2*x/(x^2+1))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((x^2+2*x+1)/(x^2+1))^(5/2)/(x+1)^5*(x^2+1)*(15*arcsinh(x)*(x^2+1)^(3/
2)+3*x^4-8*x^3-18*x^2-12*x-17)
```

3.895.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.88

$$\int \left(1+\frac{2x}{1+x^2}\right)^{5/2} dx =$$

$$-\frac{8\,x^3+8\,x^2+15\,(x^3+x^2+x+1)\log\left(-\frac{x^2-(x^2+1)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+x}{x+1}\right)-(3\,x^4-8\,x^3-18\,x^2-12\,x-17)\sqrt{\frac{x^2+}{x}}}{3\,(x^3+x^2+x+1)}$$

input
```
integrate((1+2*x/(x^2+1))^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(8*x^3 + 8*x^2 + 15*(x^3 + x^2 + x + 1)*log(-(x^2 - (x^2 + 1)*sqrt((x
^2 + 2*x + 1)/(x^2 + 1)) + x)/(x + 1)) - (3*x^4 - 8*x^3 - 18*x^2 - 12*x -
17)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + 8*x + 8)/(x^3 + x^2 + x + 1)
```

3.895.6 Sympy [F]

$$\int \left(1+\frac{2x}{1+x^2}\right)^{5/2} dx = \int \left(\frac{2x}{x^2+1}+1\right)^{\frac{5}{2}} dx$$

input
```
integrate((1+2*x/(x**2+1))**(5/2),x)
```

output
```
Integral((2*x/(x**2 + 1) + 1)**(5/2), x)
```

3.895.7 Maxima [F]

$$\int \left(1+\frac{2x}{1+x^2}\right)^{5/2} dx = \int \left(\frac{2\,x}{x^2+1}+1\right)^{\frac{5}{2}} dx$$

input
```
integrate((1+2*x/(x^2+1))^(5/2),x, algorithm="maxima")
```

output
```
integrate((2*x/(x^2 + 1) + 1)^(5/2), x)
```

3.895.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.65

$$\int \left(1+\frac{2x}{1+x^2}\right)^{5/2} dx = \left(\sqrt{2}+5\,\log\left(\sqrt{2}+1\right)\right)\mathrm{sgn}(x+1) - 5\,\log\left(-x+\sqrt{x^2+1}\right)\mathrm{sgn}(x+1) + \frac{(((3\,x\mathrm{sgn}(x+1)-8\,\mathrm{sgn}(x+1))x-18\,\mathrm{sgn}(x+1))x-12\,\mathrm{sgn}(x+1))x-17\,\mathrm{sgn}(x+1)}{3\,(x^2+1)^{\frac{3}{2}}}$$

input
```
integrate((1+2*x/(x^2+1))^(5/2),x, algorithm="giac")
```

output
```
(sqrt(2) + 5*log(sqrt(2) + 1))*sgn(x + 1) - 5*log(-x + sqrt(x^2 + 1))*sgn(
x + 1) + 1/3*((((3*x*sgn(x + 1) - 8*sgn(x + 1))*x - 18*sgn(x + 1))*x - 12*
sgn(x + 1))*x - 17*sgn(x + 1))/(x^2 + 1)^(3/2)
```

3.895.9 Mupad [F(-1)]

Timed out.

$$\int \left(1+\frac{2x}{1+x^2}\right)^{5/2} dx = \int \left(\frac{2\,x}{x^2+1}+1\right)^{5/2} dx$$

input
```
int(((2*x)/(x^2 + 1) + 1)^(5/2),x)
```

output
```
int(((2*x)/(x^2 + 1) + 1)^(5/2), x)
```

3.896 $\int \left(1+\frac{2x}{1+x^2}\right)^{3/2} dx$

3.896.1 Optimal result

Integrand size = 16, antiderivative size = 90

$$\int \left(1+\frac{2x}{1+x^2}\right)^{3/2} dx = -\left((1-x)(1+x)\sqrt{1+\frac{2x}{1+x^2}}\right) - \frac{x(1+x^2)\sqrt{1+\frac{2x}{1+x^2}}}{1+x} + \frac{3\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\operatorname{arcsinh}(x)}{1+x}$$

output
```
-(1-x)*(1+x)*(1+2*x/(x^2+1))^(1/2)-x*(x^2+1)*(1+2*x/(x^2+1))^(1/2)/(1+x)+3
*arcsinh(x)*(x^2+1)^(1/2)*(1+2*x/(x^2+1))^(1/2)/(1+x)
```

3.896.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int \left(1+\frac{2x}{1+x^2}\right)^{3/2} dx = \frac{\sqrt{\frac{(1+x)^2}{1+x^2}}\left(-1-2x+x^2-3\sqrt{1+x^2}\log\left(-x+\sqrt{1+x^2}\right)\right)}{1+x}$$

input
```
Integrate[(1 + (2*x)/(1 + x^2))^(3/2),x]
```

output
```
(Sqrt[(1 + x)^2/(1 + x^2)]*(-1 - 2*x + x^2 - 3*Sqrt[1 + x^2]*Log[-x + Sqrt
[1 + x^2]]))/(1 + x)
```

3.896.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {7274, 1298, 27, 495, 27, 643, 299, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{2x}{x^2+1}+1\right)^{3/2} dx$$

$$\Big\downarrow \text{7274}$$

$$\frac{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\int \frac{\left(x^2+2x+1\right)^{3/2}}{\left(x^2+1\right)^{3/2}}dx}{\sqrt{x^2+2x+1}}$$

$$\Big\downarrow \text{1298}$$

$$\frac{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\int \frac{8(x+1)^3}{\left(x^2+1\right)^{3/2}}dx}{8(x+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\int \frac{(x+1)^3}{\left(x^2+1\right)^{3/2}}dx}{x+1}$$

$$\Big\downarrow \text{495}$$

$$\frac{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\left(\int \frac{2(1-x)(x+1)}{\sqrt{x^2+1}}dx-\frac{(1-x)(x+1)^2}{\sqrt{x^2+1}}\right)}{x+1}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\left(2\int \frac{(1-x)(x+1)}{\sqrt{x^2+1}}dx-\frac{(1-x)(x+1)^2}{\sqrt{x^2+1}}\right)}{x+1}$$

$$\Big\downarrow \text{643}$$

$$\frac{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\left(2\int \frac{1-x^2}{\sqrt{x^2+1}}dx-\frac{(1-x)(x+1)^2}{\sqrt{x^2+1}}\right)}{x+1}$$

$$\Big\downarrow \text{299}$$

$$\frac{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\left(2\left(\frac{3}{2}\int \frac{1}{\sqrt{x^2+1}}dx-\frac{1}{2}x\sqrt{x^2+1}\right)-\frac{(1-x)(x+1)^2}{\sqrt{x^2+1}}\right)}{x+1}$$

$$\Big\downarrow \text{222}$$

$$\frac{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\left(2\left(\frac{3\text{arcsinh}(x)}{2}-\frac{1}{2}x\sqrt{x^2+1}\right)-\frac{(1-x)(x+1)^2}{\sqrt{x^2+1}}\right)}{x+1}$$

input
```
Int[(1 + (2*x)/(1 + x^2))^(3/2),x]
```

output
```
(Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)]*(-(((1 - x)*(1 + x)^2)/Sqrt[1 + x
^2]) + 2*(-1/2*(x*Sqrt[1 + x^2]) + (3*ArcSinh[x])/2)))/(1 + x)
```

3.896.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 495
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(a*d - b*c*x)*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] -
 Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^(p + 1)*Simp[a*
d^2*(n - 1) - b*c^2*(2*p + 3) - b*c*d*(n + 2*p + 2)*x, x], x], x] /; FreeQ[
{a, b, c, d}, x] && LtQ[p, -1] && GtQ[n, 1] && IntQuadraticQ[a, 0, b, c, d,
 n, p, x]
```

rule 643
```
Int[((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_)
^2)^(p_.), x_Symbol] :> Int[(c*e + d*f*x^2)^m*(a + b*x^2)^p, x] /; FreeQ[{a
, b, c, d, e, f, m, n, p}, x] && EqQ[m, n] && EqQ[d*e + c*f, 0] && (Integer
Q[m] || (GtQ[c, 0] && GtQ[e, 0]))
```

rule 1298
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p]))   Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a
, b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 7274
```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Simp[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p])   I
nt[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Integ
erQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

3.896.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.54

method	result	size
default	$\frac{\left(\frac{x^2+2x+1}{x^2+1}\right)^{\frac{3}{2}}(x^2+1)\left(3\,\text{arcsinh}(x)\sqrt{x^2+1}+x^2-2x-1\right)}{(x+1)^3}$	49
risch	$\frac{(x^2-2x-1)\sqrt{\frac{(x+1)^2}{x^2+1}}}{x+1}+\frac{3\,\text{arcsinh}(x)\sqrt{x^2+1}\,\sqrt{\frac{(x+1)^2}{x^2+1}}}{x+1}$	62
trager	$\frac{(x^2-2x-1)\sqrt{-\frac{-x^2-2x-1}{x^2+1}}}{x+1}+3\ln\left(\frac{\sqrt{-\frac{-x^2-2x-1}{x^2+1}}\,x^2+x^2+\sqrt{-\frac{-x^2-2x-1}{x^2+1}}+x}{x+1}\right)$	97

input
```
int((1+2*x/(x^2+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((x^2+2*x+1)/(x^2+1))^(3/2)/(x+1)^3*(x^2+1)*(3*arcsinh(x)*(x^2+1)^(1/2)+x^
2-2*x-1)
```

3.896.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

$$\int \left(1+\frac{2x}{1+x^2}\right)^{3/2} dx = -\frac{3\,(x+1)\log\left(-\frac{x^2-(x^2+1)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+x}{x+1}\right)-(x^2-2\,x-1)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+2\,x+2}{x+1}$$

input
```
integrate((1+2*x/(x^2+1))^(3/2),x, algorithm="fricas")
```

output
```
-(3*(x + 1)*log(-(x^2 - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x
+ 1)) - (x^2 - 2*x - 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + 2*x + 2)/(x + 1)
```

3.896.6 Sympy [F]

$$\int \left(1+\frac{2x}{1+x^2}\right)^{3/2}\,dx=\int \left(\frac{2x}{x^2+1}+1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((1+2*x/(x**2+1))**(3/2),x)
```

output
```
Integral((2*x/(x**2 + 1) + 1)**(3/2), x)
```

3.896.7 Maxima [F]

$$\int \left(1+\frac{2x}{1+x^2}\right)^{3/2}\,dx=\int \left(\frac{2\,x}{x^2+1}+1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((1+2*x/(x^2+1))^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x/(x^2 + 1) + 1)^(3/2), x)
```

3.896.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int \left(1+\frac{2x}{1+x^2}\right)^{3/2}\,dx=-\left(\sqrt{2}-3\,\log\left(\sqrt{2}+1\right)\right)\mathrm{sgn}(x+1)-3\,\log\left(-x+\sqrt{x^2+1}\right)\mathrm{sgn}(x+1)+\frac{(x\mathrm{sgn}(x+1)-2\,\mathrm{sgn}(x+1))x-\mathrm{sgn}(x+1)}{\sqrt{x^2+1}}$$

input
```
integrate((1+2*x/(x^2+1))^(3/2),x, algorithm="giac")
```

output
```
-(sqrt(2) - 3*log(sqrt(2) + 1))*sgn(x + 1) - 3*log(-x + sqrt(x^2 + 1))*sgn
(x + 1) + ((x*sgn(x + 1) - 2*sgn(x + 1))*x - sgn(x + 1))/sqrt(x^2 + 1)
```

3.896.9 Mupad [F(-1)]

Timed out.

$$\int \left(1+\frac{2x}{1+x^2}\right)^{3/2} dx = \int \left(\frac{2\,x}{x^2+1}+1\right)^{3/2} dx$$

input
```
int(((2*x)/(x^2 + 1) + 1)^(3/2),x)
```

output
```
int(((2*x)/(x^2 + 1) + 1)^(3/2), x)
```

3.897 $\int \sqrt{1+\frac{2x}{1+x^2}}\,dx$

3.897.1 Optimal result

Integrand size = 16, antiderivative size = 61

$$\int \sqrt{1+\frac{2x}{1+x^2}}\,dx=\frac{(1+x^2)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}+\frac{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\operatorname{arcsinh}(x)}{1+x}$$

output
```
(x^2+1)*(1+2*x/(x^2+1))^(1/2)/(1+x)+arcsinh(x)*(x^2+1)^(1/2)*(1+2*x/(x^2+1
))^(1/2)/(1+x)
```

3.897.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \sqrt{1+\frac{2x}{1+x^2}}\,dx=\frac{\sqrt{\frac{(1+x)^2}{1+x^2}}\left(1+x^2-\sqrt{1+x^2}\log\left(-x+\sqrt{1+x^2}\right)\right)}{1+x}$$

input
```
Integrate[Sqrt[1 + (2*x)/(1 + x^2)],x]
```

output
```
(Sqrt[(1 + x)^2/(1 + x^2)]*(1 + x^2 - Sqrt[1 + x^2]*Log[-x + Sqrt[1 + x^2]
]))/(1 + x)
```

3.897.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {7274, 1298, 27, 455, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{2x}{x^2+1}+1}\,dx$$

$$\Big\downarrow \text{7274}$$

$$\frac{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\int \frac{\sqrt{x^2+2x+1}}{\sqrt{x^2+1}}dx}{\sqrt{x^2+2x+1}}$$

$$\Big\downarrow \text{1298}$$

$$\frac{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\int \frac{2(x+1)}{\sqrt{x^2+1}}dx}{2(x+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\int \frac{x+1}{\sqrt{x^2+1}}dx}{x+1}$$

$$\Big\downarrow \text{455}$$

$$\frac{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\left(\int \frac{1}{\sqrt{x^2+1}}dx+\sqrt{x^2+1}\right)}{x+1}$$

$$\Big\downarrow \text{222}$$

$$\frac{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\left(\text{arcsinh}(x)+\sqrt{x^2+1}\right)}{x+1}$$

input
```
Int[Sqrt[1 + (2*x)/(1 + x^2)],x]
```

output
```
(Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)]*(Sqrt[1 + x^2] + ArcSinh[x]))/(1
+ x)
```

3.897.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 1298
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p]))   Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a
, b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 7274
```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Simp[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p])   I
nt[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Integ
erQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

3.897.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{\sqrt{\frac{x^2+2x+1}{x^2+1}}\,\sqrt{x^2+1}\left(\sqrt{x^2+1}+\operatorname{arcsinh}(x)\right)}{x+1}$	42
risch	$\frac{(x^2+1)\sqrt{\frac{(x+1)^2}{x^2+1}}}{x+1}+\frac{\operatorname{arcsinh}(x)\sqrt{x^2+1}\,\sqrt{\frac{(x+1)^2}{x^2+1}}}{x+1}$	58
trager	$\frac{\sqrt{-\frac{-x^2-2x-1}{x^2+1}}\,(x^2+1)}{x+1}+\ln\left(\frac{\sqrt{-\frac{-x^2-2x-1}{x^2+1}}\,x^2+x^2+\sqrt{-\frac{-x^2-2x-1}{x^2+1}}+x}{x+1}\right)$	92

input
```
int((1+2*x/(x^2+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((x^2+2*x+1)/(x^2+1))^(1/2)/(x+1)*(x^2+1)^(1/2)*((x^2+1)^(1/2)+arcsinh(x))
```

3.897.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

$$\int \sqrt{1+\frac{2x}{1+x^2}}\,dx = -\frac{(x+1)\log\left(-\frac{x^2-(x^2+1)\sqrt{\frac{x^2+2x+1}{x^2+1}}+x}{x+1}\right)-(x^2+1)\sqrt{\frac{x^2+2x+1}{x^2+1}}}{x+1}$$

input
```
integrate((1+2*x/(x^2+1))^(1/2),x, algorithm="fricas")
```

output
```
-((x + 1)*log(-(x^2 - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x +
1)) - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)))/(x + 1)
```

3.897.6 Sympy [F]

$$\int \sqrt{1+\frac{2x}{1+x^2}}\,dx = \int \sqrt{\frac{2x}{x^2+1}+1}\,dx$$

input
```
integrate((1+2*x/(x**2+1))**(1/2),x)
```

output
```
Integral(sqrt(2*x/(x**2 + 1) + 1), x)
```

3.897.7 Maxima [F]

$$\int \sqrt{1+\frac{2x}{1+x^2}}\,dx = \int \sqrt{\frac{2\,x}{x^2+1}+1}\,dx$$

input
```
integrate((1+2*x/(x^2+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(2*x/(x^2 + 1) + 1), x)
```

3.897.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \sqrt{1+\frac{2x}{1+x^2}}\,dx = -\left(\sqrt{2}-\log\left(\sqrt{2}+1\right)\right)\operatorname{sgn}(x+1) - \log\left(-x+\sqrt{x^2+1}\right)\operatorname{sgn}(x+1)+\sqrt{x^2+1}\operatorname{sgn}(x+1)$$

input
```
integrate((1+2*x/(x^2+1))^(1/2),x, algorithm="giac")
```

output
```
-(sqrt(2) - log(sqrt(2) + 1))*sgn(x + 1) - log(-x + sqrt(x^2 + 1))*sgn(x +
 1) + sqrt(x^2 + 1)*sgn(x + 1)
```

3.897.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+\frac{2x}{1+x^2}}\,dx = \int \sqrt{\frac{2\,x}{x^2+1}+1}\,dx$$

input
```
int(((2*x)/(x^2 + 1) + 1)^(1/2),x)
```

output
```
int(((2*x)/(x^2 + 1) + 1)^(1/2), x)
```

3.898 $\int \frac{1}{\sqrt{1+\frac{2x}{1+x^2}}}\,dx$

3.898.1 Optimal result

Integrand size = 16, antiderivative size = 109

$$\int \frac{1}{\sqrt{1+\frac{2x}{1+x^2}}}\,dx=\frac{1+x}{\sqrt{1+\frac{2x}{1+x^2}}}-\frac{(1+x)\text{arcsinh}(x)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}-\frac{\sqrt{2}(1+x)\text{arctanh}\left(\frac{1-x}{\sqrt{2}\sqrt{1+x^2}}\right)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

output
```
(1+x)/(1+2*x/(x^2+1))^(1/2)-(1+x)*arcsinh(x)/(x^2+1)^(1/2)/(1+2*x/(x^2+1))
^(1/2)-(1+x)*arctanh(1/2*(1-x)*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)/(x^2+1)^(1/2
)/(1+2*x/(x^2+1))^(1/2)
```

3.898.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{1+\frac{2x}{1+x^2}}}\,dx=\frac{(1+x)\left(\sqrt{1+x^2}+2\sqrt{2}\text{arctanh}\left(\frac{1+x-\sqrt{1+x^2}}{\sqrt{2}}\right)+\log\left(-x+\sqrt{1+x^2}\right)\right)}{\sqrt{\frac{(1+x)^2}{1+x^2}}\sqrt{1+x^2}}$$

input
```
Integrate[1/Sqrt[1 + (2*x)/(1 + x^2)],x]
```

output
```
((1 + x)*(Sqrt[1 + x^2] + 2*Sqrt[2]*ArcTanh[(1 + x - Sqrt[1 + x^2])/Sqrt[2
]] + Log[-x + Sqrt[1 + x^2]]))/(Sqrt[(1 + x)^2/(1 + x^2)]*Sqrt[1 + x^2])
```

3.898.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.65, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {7274, 1298, 27, 493, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{2x}{x^2+1}+1}}dx$$

$$\Big\downarrow \text{ 7274}$$

$$\frac{\sqrt{x^2+2x+1}\int \frac{\sqrt{x^2+1}}{\sqrt{x^2+2x+1}}dx}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

$$\Big\downarrow \text{ 1298}$$

$$\frac{2(x+1)\int \frac{\sqrt{x^2+1}}{2(x+1)}dx}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(x+1)\int \frac{\sqrt{x^2+1}}{x+1}dx}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

$$\Big\downarrow \text{ 493}$$

$$\frac{(x+1)\left(\int \frac{1-x}{(x+1)\sqrt{x^2+1}}dx+\sqrt{x^2+1}\right)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

$$\Big\downarrow \text{ 719}$$

$$\frac{(x+1)\left(-\int \frac{1}{\sqrt{x^2+1}}dx+2\int \frac{1}{(x+1)\sqrt{x^2+1}}dx+\sqrt{x^2+1}\right)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{(x+1)\left(2\int \frac{1}{(x+1)\sqrt{x^2+1}}dx-\text{arcsinh}(x)+\sqrt{x^2+1}\right)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

$$\Big\downarrow \text{ 488}$$

$$\frac{(x+1)\left(-2\int \frac{1}{2-\frac{(1-x)^2}{x^2+1}}d\frac{1-x}{\sqrt{x^2+1}}-\text{arcsinh}(x)+\sqrt{x^2+1}\right)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

$$\Big\downarrow \ 219$$

$$\frac{(x+1)\left(-\text{arcsinh}(x)-\sqrt{2}\text{arctanh}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right)+\sqrt{x^2+1}\right)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

input
```
Int[1/Sqrt[1 + (2*x)/(1 + x^2)],x]
```

output
```
((1 + x)*(Sqrt[1 + x^2] - ArcSinh[x] - Sqrt[2]*ArcTanh[(1 - x)/(Sqrt[2]*Sq
rt[1 + x^2])]))/(Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)])
```

3.898.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

3.898. $\int \frac{1}{\sqrt{1+\frac{2x}{1+x^2}}}\,dx$

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1298
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p]))   Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a
, b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 7274
```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Simp[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p])   I
nt[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Integ
erQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

3.898.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

method	result
risch	$\frac{x+1}{\sqrt{\frac{(x+1)^2}{x^2+1}}}+\frac{\left(-\operatorname{arcsinh}(x)-\sqrt{2}\,\operatorname{arctanh}\left(\frac{(-2x+2)\sqrt{2}}{4\sqrt{(x+1)^2-2x}}\right)\right)(x+1)}{\sqrt{\frac{(x+1)^2}{x^2+1}}\,\sqrt{x^2+1}}$
trager	$\frac{\sqrt{-\frac{-x^2-2x-1}{x^2+1}}\,(x^2+1)}{x+1}-\ln\left(\frac{\sqrt{-\frac{-x^2-2x-1}{x^2+1}}\,x^2+x^2+\sqrt{-\frac{-x^2-2x-1}{x^2+1}}+x}{x+1}\right)+\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{2\sqrt{-\frac{-x^2-2x-}{x^2+1}}}{}\right.$

input
```
int(1/(1+2*x/(x^2+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/((x+1)^2/(x^2+1))^(1/2)*(x+1)+(-arcsinh(x)-2^(1/2)*arctanh(1/4*(-2*x+2)*
2^(1/2)/((x+1)^2-2*x)^(1/2)))/((x+1)^2/(x^2+1))^(1/2)/(x^2+1)^(1/2)*(x+1)
```

3.898.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt{1+\frac{2x}{1+x^2}}}\,dx$$
$$= \frac{\sqrt{2}(x+1)\log\left(-\frac{x^2+\sqrt{2}(x^2-1)+\left(2x^2+\sqrt{2}(x^2+1)+2\right)\sqrt{\frac{x^2+2x+1}{x^2+1}}-1}{x^2+2x+1}\right)+(x+1)\log\left(-\frac{x^2-(x^2+1)\sqrt{\frac{x^2+2x+1}{x^2+1}}+x}{x+1}\right)+(x}{x+1}$$

input
```
integrate(1/(1+2*x/(x^2+1))^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(2)*(x + 1)*log(-(x^2 + sqrt(2)*(x^2 - 1) + (2*x^2 + sqrt(2)*(x^2 + 1
) + 2)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) - 1)/(x^2 + 2*x + 1)) + (x + 1)*log
(-(x^2 - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x + 1)) + (x^2 +
1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)))/(x + 1)
```

3.898.6 Sympy [F]

$$\int \frac{1}{\sqrt{1+\frac{2x}{1+x^2}}}\,dx = \int \frac{1}{\sqrt{\frac{2x}{x^2+1}+1}}\,dx$$

input
```
integrate(1/(1+2*x/(x**2+1))**(1/2),x)
```

output
```
Integral(1/sqrt(2*x/(x**2 + 1) + 1), x)
```

3.898.7 Maxima [F]

$$\int \frac{1}{\sqrt{1+\frac{2x}{1+x^2}}}\,dx = \int \frac{1}{\sqrt{\frac{2\,x}{x^2+1}+1}}\,dx$$

input
```
integrate(1/(1+2*x/(x^2+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*x/(x^2 + 1) + 1), x)
```

3.898.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{1+\frac{2x}{1+x^2}}}\,dx = \frac{\sqrt{2}\log\left(\frac{\left|-2\,x-2\,\sqrt{2}+2\,\sqrt{x^2+1}-2\right|}{\left|-2\,x+2\,\sqrt{2}+2\,\sqrt{x^2+1}-2\right|}\right)}{\operatorname{sgn}(x+1)} + \frac{\log\left(-x+\sqrt{x^2+1}\right)}{\operatorname{sgn}(x+1)} + \frac{\sqrt{x^2+1}}{\operatorname{sgn}(x+1)}$$

input
```
integrate(1/(1+2*x/(x^2+1))^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*log(abs(-2*x - 2*sqrt(2) + 2*sqrt(x^2 + 1) - 2)/abs(-2*x + 2*sqrt(
2) + 2*sqrt(x^2 + 1) - 2))/sgn(x + 1) + log(-x + sqrt(x^2 + 1))/sgn(x + 1)
 + sqrt(x^2 + 1)/sgn(x + 1)
```

3.898.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+\frac{2x}{1+x^2}}}\,dx = \int \frac{1}{\sqrt{\frac{2\,x}{x^2+1}+1}}\,dx$$

input
```
int(1/((2*x)/(x^2 + 1) + 1)^(1/2),x)
```

output
```
int(1/((2*x)/(x^2 + 1) + 1)^(1/2), x)
```

3.899 $\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}}\,dx$

3.899.1 Optimal result

Integrand size = 16, antiderivative size = 144

$$\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}}\,dx=\frac{3(2+x)}{2\sqrt{1+\frac{2x}{1+x^2}}}-\frac{1+x^2}{2(1+x)\sqrt{1+\frac{2x}{1+x^2}}}-\frac{3(1+x)\operatorname{arcsinh}(x)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}-\frac{9(1+x)\operatorname{arctanh}\left(\frac{1-x}{\sqrt{2}\sqrt{1+x^2}}\right)}{2\sqrt{2}\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

output
```
3/2*(2+x)/(1+2*x/(x^2+1))^(1/2)+1/2*(-x^2-1)/(1+x)/(1+2*x/(x^2+1))^(1/2)-3
*(1+x)*arcsinh(x)/(x^2+1)^(1/2)/(1+2*x/(x^2+1))^(1/2)-9/4*(1+x)*arctanh(1/
2*(1-x)*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)/(x^2+1)^(1/2)/(1+2*x/(x^2+1))^(1/2)
```

3.899.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}}\,dx=\frac{(1+x)\left(\sqrt{1+x^2}(5+9x+2x^2)+9\sqrt{2}(1+x)^2\operatorname{arctanh}\left(\frac{1+x-\sqrt{1+x^2}}{\sqrt{2}}\right)+6(1+x)^2\log\right.}{2\left(\frac{(1+x)^2}{1+x^2}\right)^{3/2}(1+x^2)^{3/2}}$$

input
```
Integrate[(1 + (2*x)/(1 + x^2))^(-3/2),x]
```

output
```
((1 + x)*(Sqrt[1 + x^2]*(5 + 9*x + 2*x^2) + 9*Sqrt[2]*(1 + x)^2*ArcTanh[(1
 + x - Sqrt[1 + x^2])/Sqrt[2]] + 6*(1 + x)^2*Log[-x + Sqrt[1 + x^2]]))/(2*
((1 + x)^2/(1 + x^2))^(3/2)*(1 + x^2)^(3/2))
```

3.899.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.72, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {7274, 1298, 27, 492, 590, 25, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{2x}{x^2+1}+1\right)^{3/2}}dx$$

$$\Big\downarrow \text{7274}$$

$$\frac{\sqrt{x^2+2x+1}\int \frac{\left(x^2+1\right)^{3/2}}{\left(x^2+2x+1\right)^{3/2}}dx}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

$$\Big\downarrow \text{1298}$$

$$\frac{8(x+1)\int \frac{\left(x^2+1\right)^{3/2}}{8(x+1)^3}dx}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(x+1)\int \frac{\left(x^2+1\right)^{3/2}}{(x+1)^3}dx}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

$$\Big\downarrow \text{492}$$

$$\frac{(x+1)\left(\frac{3}{2}\int \frac{x\sqrt{x^2+1}}{(x+1)^2}dx-\frac{\left(x^2+1\right)^{3/2}}{2(x+1)^2}\right)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

$$\Big\downarrow \text{590}$$

$$\frac{(x+1)\left(\frac{3}{2}\left(\frac{(x+2)\sqrt{x^2+1}}{x+1}-\int -\frac{1-2x}{(x+1)\sqrt{x^2+1}}dx\right)-\frac{\left(x^2+1\right)^{3/2}}{2(x+1)^2}\right)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

$$\Big\downarrow \text{25}$$

$$\frac{(x+1)\left(\frac{3}{2}\left(\int \frac{1-2x}{(x+1)\sqrt{x^2+1}}dx+\frac{\sqrt{x^2+1}(x+2)}{x+1}\right)-\frac{\left(x^2+1\right)^{3/2}}{2(x+1)^2}\right)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

$$\Big\downarrow \text{719}$$

$$\frac{(x+1)\left(\frac{3}{2}\left(-2\int \frac{1}{\sqrt{x^2+1}}dx+3\int \frac{1}{(x+1)\sqrt{x^2+1}}dx+\frac{\sqrt{x^2+1}(x+2)}{x+1}\right)-\frac{\left(x^2+1\right)^{3/2}}{2(x+1)^2}\right)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

$$\Big\downarrow \text{222}$$

$$\frac{(x+1)\left(\frac{3}{2}\left(3\int \frac{1}{(x+1)\sqrt{x^2+1}}dx-2\text{arcsinh}(x)+\frac{\sqrt{x^2+1}(x+2)}{x+1}\right)-\frac{\left(x^2+1\right)^{3/2}}{2(x+1)^2}\right)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

$$\Big\downarrow \text{488}$$

$$\frac{(x+1)\left(\frac{3}{2}\left(-3\int \frac{1}{2-\frac{(1-x)^2}{x^2+1}}d\frac{1-x}{\sqrt{x^2+1}}-2\text{arcsinh}(x)+\frac{\sqrt{x^2+1}(x+2)}{x+1}\right)-\frac{\left(x^2+1\right)^{3/2}}{2(x+1)^2}\right)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(x+1)\left(\frac{3}{2}\left(-2\text{arcsinh}(x)-\frac{3\text{arctanh}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right)}{\sqrt{2}}+\frac{\sqrt{x^2+1}(x+2)}{x+1}\right)-\frac{\left(x^2+1\right)^{3/2}}{2(x+1)^2}\right)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

input
```
Int[(1 + (2*x)/(1 + x^2))^(-3/2),x]
```

output
```
((1 + x)*(-1/2*(1 + x^2)^(3/2)/(1 + x)^2 + (3*(((2 + x)*Sqrt[1 + x^2])/(1
+ x) - 2*ArcSinh[x] - (3*ArcTanh[(1 - x)/(Sqrt[2]*Sqrt[1 + x^2])])/Sqrt[2]
))/2))/(Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)])
```

3.899. $\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}}\,dx$

3.899.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 492
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 1))), x] - Simp[2*b*(p/(d*(n + 1))
)   Int[x*(c + d*x)^(n + 1)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c,
d, n}, x] && GtQ[p, 0] && (IntegerQ[p] || LtQ[n, -1]) && NeQ[n, -1] &&  !IL
tQ[n + 2*p + 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 590
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-(c + d*x)^(n + 1))*(a + b*x^2)^p*((c*(2*p + 1) - d*(n + 1)*x)/(d^2*(
n + 1)*(n + 2*p + 2))), x] + Simp[2*(p/(d^2*(n + 1)*(n + 2*p + 2)))   Int[(
c + d*x)^(n + 1)*(a + b*x^2)^(p - 1)*(a*d*(n + 1) + b*c*(2*p + 1)*x), x], x
] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && LtQ[n, -1] &&  !ILtQ[n + 2*p +
1, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.899. $\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}}\,dx$

rule 1298
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p]))   Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a
, b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 7274
```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Simp[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p])   I
nt[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Integ
erQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

3.899.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.76

method	result
risch	$\frac{2x^4+9x^3+7x^2+9x+5}{2(x+1)(x^2+1)\sqrt{\frac{(x+1)^2}{x^2+1}}}+\frac{\left(-3\,\mathrm{arcsinh}(x)-\frac{9\sqrt{2}\,\mathrm{arctanh}\left(\frac{(-2x+2)\sqrt{2}}{4\sqrt{(x+1)^2-2x}}\right)}{4}\right)(x+1)}{\sqrt{\frac{(x+1)^2}{x^2+1}}\,\sqrt{x^2+1}}$
trager	$\frac{(x^2+1)(2x^2+9x+5)\sqrt{-\frac{-x^2-2x-1}{x^2+1}}}{2(x+1)^3}+3\ln\left(-\frac{\sqrt{-\frac{-x^2-2x-1}{x^2+1}}\,x^2-x^2+\sqrt{-\frac{-x^2-2x-1}{x^2+1}}-x}{x+1}\right)-\frac{9\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{2\sqrt{-\cdots}}{\cdots}\right.}{\cdots}$ …
default	$-\frac{(x+1)\left(-(x^2+1)^{\frac{5}{2}}x+(x^2+1)^{\frac{3}{2}}x^3+(x^2+1)^{\frac{5}{2}}-(x^2+1)^{\frac{3}{2}}x^2-18\sqrt{2}\,\mathrm{arctanh}\left(\frac{(x-1)\sqrt{2}}{2\sqrt{x^2+1}}\right)x^2-5x(x^2+1)^{\frac{3}{2}}+6\sqrt{x^2+1}\,x^3+24\,\mathrm{arcsinh}\cdots\right.}{8(\cdots}$ …

input
```
int(1/(1+2*x/(x^2+1))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*x^4+9*x^3+7*x^2+9*x+5)/(x+1)/(x^2+1)/((x+1)^2/(x^2+1))^(1/2)+(-3*ar
csinh(x)-9/4*2^(1/2)*arctanh(1/4*(-2*x+2)*2^(1/2)/((x+1)^2-2*x)^(1/2)))/((
x+1)^2/(x^2+1))^(1/2)/(x^2+1)^(1/2)*(x+1)
```

3.899.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.42

$$\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}}\,dx = \frac{10\,x^3+9\,\sqrt{2}(x^3+3\,x^2+3\,x+1)\log\left(-\frac{x^2+\sqrt{2}(x^2-1)+\left(2\,x^2+\sqrt{2}(x^2+1)+2\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}-1}{x^2+2\,x+1}\right)+\cdots}{\cdots}$$

input
```
integrate(1/(1+2*x/(x^2+1))^(3/2),x, algorithm="fricas")
```

output
```
1/4*(10*x^3 + 9*sqrt(2)*(x^3 + 3*x^2 + 3*x + 1)*log(-(x^2 + sqrt(2)*(x^2 -
 1) + (2*x^2 + sqrt(2)*(x^2 + 1) + 2)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) - 1)
/(x^2 + 2*x + 1)) + 30*x^2 + 12*(x^3 + 3*x^2 + 3*x + 1)*log(-(x^2 - (x^2 +
 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x + 1)) + 2*(2*x^4 + 9*x^3 + 7*x
^2 + 9*x + 5)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + 30*x + 10)/(x^3 + 3*x^2 +
3*x + 1)
```

3.899.6 Sympy [F]

$$\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{2x}{x^2+1}+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(1+2*x/(x**2+1))**(3/2),x)
```

output
```
Integral((2*x/(x**2 + 1) + 1)**(-3/2), x)
```

3.899.7 Maxima [F]

$$\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{2\,x}{x^2+1}+1\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(1+2*x/(x^2+1))^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*x/(x^2 + 1) + 1)^(-3/2), x)
```

3.899.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}}\,dx = \frac{9\sqrt{2}\log\left(\frac{\left|-2\,x-2\,\sqrt{2}+2\,\sqrt{x^2+1}-2\right|}{\left|-2\,x+2\,\sqrt{2}+2\,\sqrt{x^2+1}-2\right|}\right)}{4\,\mathrm{sgn}\,(x+1)} + \frac{3\,\log\left(-x+\sqrt{x^2+1}\right)}{\mathrm{sgn}\,(x+1)} + \frac{\sqrt{x^2+1}}{\mathrm{sgn}\,(x+1)} + \frac{7\left(x-\sqrt{x^2+1}\right)^3+5\left(x-\sqrt{x^2+1}\right)^2-13\,x+13\,\sqrt{x^2+1}+5}{\left(\left(x-\sqrt{x^2+1}\right)^2+2\,x-2\,\sqrt{x^2+1}-1\right)^2\mathrm{sgn}\,(x+1)}$$

input
```
integrate(1/(1+2*x/(x^2+1))^(3/2),x, algorithm="giac")
```

output
```
9/4*sqrt(2)*log(abs(-2*x - 2*sqrt(2) + 2*sqrt(x^2 + 1) - 2)/abs(-2*x + 2*s
qrt(2) + 2*sqrt(x^2 + 1) - 2))/sgn(x + 1) + 3*log(-x + sqrt(x^2 + 1))/sgn(
x + 1) + sqrt(x^2 + 1)/sgn(x + 1) + (7*(x - sqrt(x^2 + 1))^3 + 5*(x - sqrt
(x^2 + 1))^2 - 13*x + 13*sqrt(x^2 + 1) + 5)/(((x - sqrt(x^2 + 1))^2 + 2*x
- 2*sqrt(x^2 + 1) - 1)^2*sgn(x + 1))
```

3.899.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{2\,x}{x^2+1}+1\right)^{3/2}}\,dx$$

input
```
int(1/((2*x)/(x^2 + 1) + 1)^(3/2),x)
```

output
```
int(1/((2*x)/(x^2 + 1) + 1)^(3/2), x)
```

3.900 $\int \frac{\sqrt{1+\frac{2x}{1+x^2}}}{1+x^2}\,dx$

3.900.1 Optimal result

Integrand size = 24, antiderivative size = 28

$$\int \frac{\sqrt{1+\frac{2x}{1+x^2}}}{1+x^2}\,dx = -\frac{(1-x)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}$$

output
```
-(1-x)*(1+2*x/(x^2+1))^(1/2)/(1+x)
```

3.900.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{1+\frac{2x}{1+x^2}}}{1+x^2}\,dx = \frac{(-1+x)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}$$

input
```
Integrate[Sqrt[1 + (2*x)/(1 + x^2)]/(1 + x^2),x]
```

output
```
((-1 + x)*Sqrt[1 + (2*x)/(1 + x^2)])/(1 + x)
```

3.900.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {7274, 1298, 27, 453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{2x}{x^2+1}+1}}{x^2+1}dx$$

$$\Big\downarrow \text{7274}$$

$$\frac{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\int \frac{\sqrt{x^2+2x+1}}{(x^2+1)^{3/2}}dx}{\sqrt{x^2+2x+1}}$$

$$\Big\downarrow \text{1298}$$

$$\frac{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\int \frac{2(x+1)}{(x^2+1)^{3/2}}dx}{2(x+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\int \frac{x+1}{(x^2+1)^{3/2}}dx}{x+1}$$

$$\Big\downarrow \text{453}$$

$$-\frac{(1-x)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}$$

input
```
Int[Sqrt[1 + (2*x)/(1 + x^2)]/(1 + x^2),x]
```

output
```
-(((1 - x)*Sqrt[1 + (2*x)/(1 + x^2)])/(1 + x))
```

3.900. $\int \frac{\sqrt{1+\frac{2x}{1+x^2}}}{1+x^2}dx$

3.900.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

rule 1298
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p]))   Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a
, b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

rule 7274
```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Simp[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p])   I
nt[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Integ
erQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

3.900.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{\sqrt{\frac{(x+1)^2}{x^2+1}}\,(x-1)}{x+1}$	25
gosper	$\frac{(x-1)\sqrt{\frac{x^2+2x+1}{x^2+1}}}{x+1}$	28
default	$\frac{(x-1)\sqrt{\frac{x^2+2x+1}{x^2+1}}}{x+1}$	28
trager	$\frac{(x-1)\sqrt{-\frac{-x^2-2x-1}{x^2+1}}}{x+1}$	31

input
```
int((1+2*x/(x^2+1))^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/(x+1)*((x+1)^2/(x^2+1))^(1/2)*(x-1)
```

3.900.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{1+\frac{2x}{1+x^2}}}{1+x^2}\,dx = \frac{(x-1)\sqrt{\frac{x^2+2x+1}{x^2+1}}+x+1}{x+1}$$

input
```
integrate((1+2*x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="fricas")
```

output
```
((x - 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x + 1)/(x + 1)
```

3.900.6 Sympy [F]

$$\int \frac{\sqrt{1+\frac{2x}{1+x^2}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{(x+1)^2}{x^2+1}}}{x^2+1}\,dx$$

input
```
integrate((1+2*x/(x**2+1))**(1/2)/(x**2+1),x)
```

output
```
Integral(sqrt((x + 1)**2/(x**2 + 1))/(x**2 + 1), x)
```

3.900.7 Maxima [F]

$$\int \frac{\sqrt{1+\frac{2x}{1+x^2}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{2x}{x^2+1}+1}}{x^2+1}\,dx$$

input
```
integrate((1+2*x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="maxima")
```

output
```
integrate(sqrt(2*x/(x^2 + 1) + 1)/(x^2 + 1), x)
```

3.900.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{1+\frac{2x}{1+x^2}}}{1+x^2}\,dx = \sqrt{2}\mathrm{sgn}(x+1) + \frac{x\mathrm{sgn}(x+1) - \mathrm{sgn}(x+1)}{\sqrt{x^2+1}}$$

input
```
integrate((1+2*x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="giac")
```

output
```
sqrt(2)*sgn(x + 1) + (x*sgn(x + 1) - sgn(x + 1))/sqrt(x^2 + 1)
```

3.900.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{1+\frac{2x}{1+x^2}}}{1+x^2}\,dx = \frac{\sqrt{\frac{2\,x}{x^2+1}+1}\,(x-1)}{x+1}$$

input
```
int(((2*x)/(x^2 + 1) + 1)^(1/2)/(x^2 + 1),x)
```

output
```
(((2*x)/(x^2 + 1) + 1)^(1/2)*(x - 1))/(x + 1)
```

3.901 $\int \sqrt{x-x^2}F(x)\,dx$

3.901.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \sqrt{x-x^2}F(x)\,dx = \text{Int}\left(\sqrt{x-x^2}F(x),x\right)$$

output
```
CannotIntegrate(F(x)*(-x^2+x)^(1/2),x)
```

3.901.2 Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \sqrt{x-x^2}F(x)\,dx = \int \sqrt{x-x^2}F(x)\,dx$$

input
```
Integrate[Sqrt[x - x^2]*F[x],x]
```

output
```
Integrate[Sqrt[x - x^2]*F[x], x]
```

3.901.3 Rubi [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2467, 7284, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x-x^2}F(x)\,dx$$

$$\Big\downarrow \text{ 2467}$$

$$\frac{\sqrt{x-x^2}\int \sqrt{1-x}\sqrt{x}F(x)dx}{\sqrt{1-x}\sqrt{x}}$$

$$\Big\downarrow \text{ 7284}$$

$$\frac{2\sqrt{x-x^2}\int \sqrt{1-x}xF(x)d\sqrt{x}}{\sqrt{1-x}\sqrt{x}}$$

$$\Big\downarrow \text{ 7299}$$

$$\frac{2\sqrt{x-x^2}\int \sqrt{1-x}xF(x)d\sqrt{x}}{\sqrt{1-x}\sqrt{x}}$$

input
```
Int[Sqrt[x - x^2]*F[x],x]
```

output
```
$Aborted
```

3.901.3.1 Defintions of rubi rules used

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

rule 7284
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.901.4 Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int F(x)\sqrt{-x^2+x}dx$$

input
```
int(F(x)*(-x^2+x)^(1/2),x)
```

output
```
int(F(x)*(-x^2+x)^(1/2),x)
```

3.901.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{x-x^2}F(x)\,dx = \int \sqrt{-x^2+x}F(x)\,dx$$

input
```
integrate(F(x)*(-x^2+x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-x^2 + x)*F(x), x)
```

3.901.6 Sympy [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{x-x^2}F(x)\,dx = \int \sqrt{-x\,(x-1)}F(x)\,dx$$

input
```
integrate(F(x)*(-x**2+x)**(1/2),x)
```

output
```
Integral(sqrt(-x*(x - 1))*F(x), x)
```

3.901.7 Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{x-x^2}F(x)\,dx = \int \sqrt{-x^2+x}F(x)\,dx$$

input
```
integrate(F(x)*(-x^2+x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 + x)*F(x), x)
```

3.901.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{x-x^2}F(x)\,dx = \int \sqrt{-x^2+x}F(x)\,dx$$

input
```
integrate(F(x)*(-x^2+x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^2 + x)*F(x), x)
```

3.901.9 Mupad [N/A]

Not integrable

Time = 20.75 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{x-x^2}F(x)\,dx = \int F(x)\,\sqrt{x-x^2}\,dx$$

input
```
int(F(x)*(x - x^2)^(1/2),x)
```

output
```
int(F(x)*(x - x^2)^(1/2), x)
```

3.902 $\int \frac{F(x)}{\sqrt{x-x^2}}\,dx$

3.902.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{F(x)}{\sqrt{x-x^2}}\,dx = \text{Int}\left(\frac{F(x)}{\sqrt{x-x^2}},x\right)$$

output
```
CannotIntegrate(F(x)/(-x^2+x)^(1/2),x)
```

3.902.2 Mathematica [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{F(x)}{\sqrt{x-x^2}}\,dx = \int \frac{F(x)}{\sqrt{x-x^2}}\,dx$$

input
```
Integrate[F[x]/Sqrt[x - x^2],x]
```

output
```
Integrate[F[x]/Sqrt[x - x^2], x]
```

3.902.3 Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2467, 7284, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F(x)}{\sqrt{x-x^2}}dx$$

$$\Big\downarrow \text{2467}$$

$$\frac{\sqrt{1-x}\sqrt{x}\int \frac{F(x)}{\sqrt{1-x}\sqrt{x}}dx}{\sqrt{x-x^2}}$$

$$\Big\downarrow \text{7284}$$

$$\frac{2\sqrt{1-x}\sqrt{x}\int \frac{F(x)}{\sqrt{1-x}}d\sqrt{x}}{\sqrt{x-x^2}}$$

$$\Big\downarrow \text{7299}$$

$$\frac{2\sqrt{1-x}\sqrt{x}\int \frac{F(x)}{\sqrt{1-x}}d\sqrt{x}}{\sqrt{x-x^2}}$$

input
```
Int[F[x]/Sqrt[x - x^2],x]
```

output
```
$Aborted
```

3.902.3.1 Defintions of rubi rules used

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

rule 7284
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.902.4 Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{F(x)}{\sqrt{-x^2+x}}dx$$

input
```
int(F(x)/(-x^2+x)^(1/2),x)
```

output
```
int(F(x)/(-x^2+x)^(1/2),x)
```

3.902.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{F(x)}{\sqrt{x-x^2}}\,dx = \int \frac{F(x)}{\sqrt{-x^2+x}}\,dx$$

input
```
integrate(F(x)/(-x^2+x)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^2 + x)*F(x)/(x^2 - x), x)
```

3.902.6 Sympy [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{F(x)}{\sqrt{x-x^2}}\,dx = \int \frac{F(x)}{\sqrt{-x\,(x-1)}}\,dx$$

input
```
integrate(F(x)/(-x**2+x)**(1/2),x)
```

output
```
Integral(F(x)/sqrt(-x*(x - 1)), x)
```

3.902.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{F(x)}{\sqrt{x-x^2}}\,dx = \int \frac{F(x)}{\sqrt{-x^2+x}}\,dx$$

input
```
integrate(F(x)/(-x^2+x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(F(x)/sqrt(-x^2 + x), x)
```

3.902.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{F(x)}{\sqrt{x-x^2}}\,dx = \int \frac{F(x)}{\sqrt{-x^2+x}}\,dx$$

input
```
integrate(F(x)/(-x^2+x)^(1/2),x, algorithm="giac")
```

output
```
integrate(F(x)/sqrt(-x^2 + x), x)
```

3.902.9 Mupad [N/A]

Not integrable

Time = 20.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{F(x)}{\sqrt{x-x^2}}\,dx = \int \frac{F(x)}{\sqrt{x-x^2}}\,dx$$

input
```
int(F(x)/(x - x^2)^(1/2),x)
```

output
```
int(F(x)/(x - x^2)^(1/2), x)
```

3.903 $\int \sqrt{1-x}\sqrt{x}F(x)\,dx$

3.903.1 Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \sqrt{1-x}\sqrt{x}F(x)\,dx = \mathrm{Int}\left(\sqrt{x-x^2}F(x),x\right)$$

output
```
CannotIntegrate(F(x)*(-x^2+x)^(1/2),x)
```

3.903.2 Mathematica [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \sqrt{1-x}\sqrt{x}F(x)\,dx = \int \sqrt{1-x}\sqrt{x}F(x)\,dx$$

input
```
Integrate[Sqrt[1 - x]*Sqrt[x]*F[x],x]
```

output
```
Integrate[Sqrt[1 - x]*Sqrt[x]*F[x], x]
```

3.903.3 Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7284, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-x}\sqrt{x}F(x)\,dx$$

$$\Big\downarrow \text{ 7284}$$

$$2\int \sqrt{1-x}xF(x)d\sqrt{x}$$

$$\Big\downarrow \text{ 7299}$$

$$2\int \sqrt{1-x}xF(x)d\sqrt{x}$$

input
```
Int[Sqrt[1 - x]*Sqrt[x]*F[x],x]
```

output
```
$Aborted
```

3.903.3.1 Defintions of rubi rules used

rule 7284
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.903.4 Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int F(x)\sqrt{1-x}\,\sqrt{x}dx$$

input
```
int(F(x)*(1-x)^(1/2)*x^(1/2),x)
```

output
```
int(F(x)*(1-x)^(1/2)*x^(1/2),x)
```

3.903.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \sqrt{1-x}\sqrt{x}F(x)\,dx = \int \sqrt{x}\sqrt{-x+1}F(x)\,dx$$

input
```
integrate(F(x)*(1-x)^(1/2)*x^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x)*sqrt(-x + 1)*F(x), x)
```

3.903.6 Sympy [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \sqrt{1-x}\sqrt{x}F(x)\,dx = \int \sqrt{x}\sqrt{1-x}F(x)\,dx$$

input
```
integrate(F(x)*(1-x)**(1/2)*x**(1/2),x)
```

output
```
Integral(sqrt(x)*sqrt(1 - x)*F(x), x)
```

3.903.7 Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \sqrt{1-x}\sqrt{x}F(x)\,dx = \int \sqrt{x}\sqrt{-x+1}F(x)\;dx$$

input
```
integrate(F(x)*(1-x)^(1/2)*x^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)*sqrt(-x + 1)*F(x), x)
```

3.903.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \sqrt{1-x}\sqrt{x}F(x)\,dx = \int \sqrt{x}\sqrt{-x+1}F(x)\;dx$$

input
```
integrate(F(x)*(1-x)^(1/2)*x^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x)*sqrt(-x + 1)*F(x), x)
```

3.903.9 Mupad [N/A]

Not integrable

Time = 20.41 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \sqrt{1-x}\sqrt{x}F(x)\,dx = \int \sqrt{x}\,F(x)\,\sqrt{1-x}\,dx$$

input
```
int(x^(1/2)*F(x)*(1 - x)^(1/2),x)
```

output
```
int(x^(1/2)*F(x)*(1 - x)^(1/2), x)
```

3.904 $\int \frac{F(x)}{\sqrt{1-x}\sqrt{x}} dx$

3.904.1 Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{F(x)}{\sqrt{1-x}\sqrt{x}} dx = \text{Int}\left(\frac{F(x)}{\sqrt{x-x^2}}, x\right)$$

output
```
CannotIntegrate(F(x)/(-x^2+x)^(1/2),x)
```

3.904.2 Mathematica [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{F(x)}{\sqrt{1-x}\sqrt{x}} dx = \int \frac{F(x)}{\sqrt{1-x}\sqrt{x}} dx$$

input
```
Integrate[F[x]/(Sqrt[1 - x]*Sqrt[x]),x]
```

output
```
Integrate[F[x]/(Sqrt[1 - x]*Sqrt[x]), x]
```

3.904.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7284, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F(x)}{\sqrt{1-x}\sqrt{x}}dx$$
$$\Big\downarrow \text{7284}$$
$$2\int \frac{F(x)}{\sqrt{1-x}}d\sqrt{x}$$
$$\Big\downarrow \text{7299}$$
$$2\int \frac{F(x)}{\sqrt{1-x}}d\sqrt{x}$$

input
```
Int[F[x]/(Sqrt[1 - x]*Sqrt[x]),x]
```

output
```
$Aborted
```

3.904.3.1 Defintions of rubi rules used

rule 7284
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.904.4 Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{F(x)}{\sqrt{1-x}\,\sqrt{x}}dx$$

input
```
int(F(x)/(1-x)^(1/2)/x^(1/2),x)
```

output
```
int(F(x)/(1-x)^(1/2)/x^(1/2),x)
```

3.904.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47

$$\int \frac{F(x)}{\sqrt{1-x}\sqrt{x}}\,dx = \int \frac{F(x)}{\sqrt{x}\sqrt{-x+1}}\,dx$$

input
```
integrate(F(x)/(1-x)^(1/2)/x^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(x)*sqrt(-x + 1)*F(x)/(x^2 - x), x)
```

3.904.6 Sympy [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{F(x)}{\sqrt{1-x}\sqrt{x}}\,dx = \int \frac{F(x)}{\sqrt{x}\sqrt{1-x}}\,dx$$

input
```
integrate(F(x)/(1-x)**(1/2)/x**(1/2),x)
```

output
```
Integral(F(x)/(sqrt(x)*sqrt(1 - x)), x)
```

3.904.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{F(x)}{\sqrt{1-x}\sqrt{x}}\,dx = \int \frac{F(x)}{\sqrt{x}\sqrt{-x+1}}\,dx$$

input
```
integrate(F(x)/(1-x)^(1/2)/x^(1/2),x, algorithm="maxima")
```

output
```
integrate(F(x)/(sqrt(x)*sqrt(-x + 1)), x)
```

3.904.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{F(x)}{\sqrt{1-x}\sqrt{x}}\,dx = \int \frac{F(x)}{\sqrt{x}\sqrt{-x+1}}\,dx$$

input
```
integrate(F(x)/(1-x)^(1/2)/x^(1/2),x, algorithm="giac")
```

output
```
integrate(F(x)/(sqrt(x)*sqrt(-x + 1)), x)
```

3.904.9 Mupad [N/A]

Not integrable

Time = 20.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{F(x)}{\sqrt{1-x}\sqrt{x}}\,dx = \int \frac{F(x)}{\sqrt{x}\,\sqrt{1-x}}\,dx$$

input
```
int(F(x)/(x^(1/2)*(1 - x)^(1/2)),x)
```

output
```
int(F(x)/(x^(1/2)*(1 - x)^(1/2)), x)
```

3.905 $\int F\left(\frac{a+bx}{x}\right) dx$

3.905.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int F\left(\frac{a+bx}{x}\right) dx = \mathrm{Int}\left(F\left(b+\frac{a}{x}\right),x\right)$$

output
```
CannotIntegrate(F(b+a/x),x)
```

3.905.2 Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int F\left(\frac{a+bx}{x}\right) dx = \int F\left(\frac{a+bx}{x}\right) dx$$

input
```
Integrate[F[(a + b*x)/x],x]
```

output
```
Integrate[F[(a + b*x)/x], x]
```

3.905.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7292, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F\left(\frac{a+bx}{x}\right) \, dx$$

$$\Big\downarrow \ 7292$$

$$\int F\left(\frac{a}{x}+b\right) \, dx$$

$$\Big\downarrow \ 7299$$

$$\int F\left(\frac{a}{x}+b\right) \, dx$$

input
```
Int[F[(a + b*x)/x],x]
```

output
```
$Aborted
```

3.905.3.1 Defintions of rubi rules used

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.905.4 Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int F\left(\frac{bx+a}{x}\right)dx$$

input
```
int(F((b*x+a)/x),x)
```

output
```
int(F((b*x+a)/x),x)
```

3.905.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int F\left(\frac{a+bx}{x}\right)dx = \int F\left(\frac{bx+a}{x}\right)dx$$

input
```
integrate(F((b*x+a)/x),x, algorithm="fricas")
```

output
```
integral(F((b*x + a)/x), x)
```

3.905.6 Sympy [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int F\left(\frac{a+bx}{x}\right)dx = \int F\left(\frac{a+bx}{x}\right)dx$$

input
```
integrate(F((b*x+a)/x),x)
```

output
```
Integral(F((a + b*x)/x), x)
```

3.905.7 Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int F\left(\frac{a+bx}{x}\right)\,dx=\int F\left(\frac{bx+a}{x}\right)\,dx$$

input
```
integrate(F((b*x+a)/x),x, algorithm="maxima")
```

output
```
integrate(F((b*x + a)/x), x)
```

3.905.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int F\left(\frac{a+bx}{x}\right)\,dx=\int F\left(\frac{bx+a}{x}\right)\,dx$$

input
```
integrate(F((b*x+a)/x),x, algorithm="giac")
```

output
```
integrate(F((b*x + a)/x), x)
```

3.905.9 Mupad [N/A]

Not integrable

Time = 20.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int F\left(\frac{a+bx}{x}\right)\,dx=\int F\left(\frac{a+b\,x}{x}\right)\,dx$$

input
```
int(F((a + b*x)/x),x)
```

output
```
int(F((a + b*x)/x), x)
```

3.906 $\int F\left(\frac{a+bx^2}{x^2}\right) dx$

3.906.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int F\left(\frac{a+bx^2}{x^2}\right) dx = \text{Int}\left(F\left(b+\frac{a}{x^2}\right),x\right)$$

output
```
CannotIntegrate(F(b+a/x^2),x)
```

3.906.2 Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int F\left(\frac{a+bx^2}{x^2}\right) dx = \int F\left(\frac{a+bx^2}{x^2}\right) dx$$

input
```
Integrate[F[(a + b*x^2)/x^2],x]
```

output
```
Integrate[F[(a + b*x^2)/x^2], x]
```

3.906.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7292, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F\left(\frac{a+bx^2}{x^2}\right) dx$$

$$\Big\downarrow \text{7292}$$

$$\int F\left(\frac{a}{x^2}+b\right) dx$$

$$\Big\downarrow \text{7299}$$

$$\int F\left(\frac{a}{x^2}+b\right) dx$$

input
```
Int[F[(a + b*x^2)/x^2],x]
```

output
```
$Aborted
```

3.906.3.1 Defintions of rubi rules used

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.906.4 Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int F\left(\frac{b\,x^{2}+a}{x^{2}}\right)dx$$

input
```
int(F((b*x^2+a)/x^2),x)
```

output
```
int(F((b*x^2+a)/x^2),x)
```

3.906.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int F\left(\frac{a+bx^2}{x^2}\right)\,dx=\int F\left(\frac{bx^2+a}{x^2}\right)\,dx$$

input
```
integrate(F((b*x^2+a)/x^2),x, algorithm="fricas")
```

output
```
integral(F((b*x^2 + a)/x^2), x)
```

3.906.6 Sympy [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int F\left(\frac{a+bx^2}{x^2}\right)\,dx=\int F\left(\frac{a+bx^2}{x^2}\right)dx$$

input
```
integrate(F((b*x**2+a)/x**2),x)
```

output
```
Integral(F((a + b*x**2)/x**2), x)
```

3.906.7 Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int F\left(\frac{a+bx^2}{x^2}\right)\,dx=\int F\left(\frac{bx^2+a}{x^2}\right)\,dx$$

input
```
integrate(F((b*x^2+a)/x^2),x, algorithm="maxima")
```

output
```
integrate(F((b*x^2 + a)/x^2), x)
```

3.906.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int F\left(\frac{a+bx^2}{x^2}\right)\,dx=\int F\left(\frac{bx^2+a}{x^2}\right)\,dx$$

input
```
integrate(F((b*x^2+a)/x^2),x, algorithm="giac")
```

output
```
integrate(F((b*x^2 + a)/x^2), x)
```

3.906.9 Mupad [N/A]

Not integrable

Time = 20.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int F\left(\frac{a+bx^2}{x^2}\right)\,dx=\int F\left(\frac{b\,x^2+a}{x^2}\right)\,dx$$

input
```
int(F((a + b*x^2)/x^2),x)
```

output
```
int(F((a + b*x^2)/x^2), x)
```

3.907 $\int F\left(\frac{x}{a+bx}\right) dx$

3.907.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int F\left(\frac{x}{a+bx}\right) dx = \text{Int}\left(F\left(\frac{x}{a+bx}\right),x\right)$$

output
```
CannotIntegrate(F(x/(b*x+a)),x)
```

3.907.2 Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int F\left(\frac{x}{a+bx}\right) dx = \int F\left(\frac{x}{a+bx}\right) dx$$

input
```
Integrate[F[x/(a + b*x)],x]
```

output
```
Integrate[F[x/(a + b*x)], x]
```

3.907.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F\left(\frac{x}{a+bx}\right) \, dx$$
$$\Big\downarrow \; 7299$$
$$\int F\left(\frac{x}{a+bx}\right) \, dx$$

input
```
Int[F[x/(a + b*x)],x]
```

output
```
$Aborted
```

3.907.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.907.4 Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int F\left(\frac{x}{bx+a}\right) dx$$

input
```
int(F(x/(b*x+a)),x)
```

output
```
int(F(x/(b*x+a)),x)
```

3.907.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int F\left(\frac{x}{a+bx}\right) dx = \int F\left(\frac{x}{bx+a}\right) dx$$

input
```
integrate(F(x/(b*x+a)),x, algorithm="fricas")
```

output
```
integral(F(x/(b*x + a)), x)
```

3.907.6 Sympy [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int F\left(\frac{x}{a+bx}\right) dx = \int F\left(\frac{x}{a+bx}\right) dx$$

input
```
integrate(F(x/(b*x+a)),x)
```

output
```
Integral(F(x/(a + b*x)), x)
```

3.907.7 Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int F\left(\frac{x}{a+bx}\right) dx = \int F\left(\frac{x}{bx+a}\right) dx$$

input
```
integrate(F(x/(b*x+a)),x, algorithm="maxima")
```

output
```
integrate(F(x/(b*x + a)), x)
```

3.907.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int F\left(\frac{x}{a+bx}\right)\,dx=\int F\left(\frac{x}{bx+a}\right)\,dx$$

input
```
integrate(F(x/(b*x+a)),x, algorithm="giac")
```

output
```
integrate(F(x/(b*x + a)), x)
```

3.907.9 Mupad [N/A]

Not integrable

Time = 20.48 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int F\left(\frac{x}{a+bx}\right)\,dx=\int F\left(\frac{x}{a+b\,x}\right)\,dx$$

input
```
int(F(x/(a + b*x)),x)
```

output
```
int(F(x/(a + b*x)), x)
```

3.908 $\int F\left(\frac{x^2}{a+bx^2}\right) dx$

3.908.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int F\left(\frac{x^2}{a+bx^2}\right) dx = \operatorname{Int}\left(F\left(\frac{x^2}{a+bx^2}\right),x\right)$$

output
```
CannotIntegrate(F(x^2/(b*x^2+a)),x)
```

3.908.2 Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int F\left(\frac{x^2}{a+bx^2}\right) dx = \int F\left(\frac{x^2}{a+bx^2}\right) dx$$

input
```
Integrate[F[x^2/(a + b*x^2)],x]
```

output
```
Integrate[F[x^2/(a + b*x^2)], x]
```

3.908.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F\left(\frac{x^2}{a+bx^2}\right) \, dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int F\left(\frac{x^2}{a+bx^2}\right)dx$$

input
```
Int[F[x^2/(a + b*x^2)],x]
```

output
```
$Aborted
```

3.908.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.908.4 Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int F\left(\frac{x^2}{b\,x^2+a}\right)dx$$

input
```
int(F(x^2/(b*x^2+a)),x)
```

output
```
int(F(x^2/(b*x^2+a)),x)
```

3.908.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int F\left(\frac{x^2}{a+bx^2}\right)\,dx=\int F\left(\frac{x^2}{bx^2+a}\right)\,dx$$

input
```
integrate(F(x^2/(b*x^2+a)),x, algorithm="fricas")
```

output
```
integral(F(x^2/(b*x^2 + a)), x)
```

3.908.6 Sympy [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int F\left(\frac{x^2}{a+bx^2}\right)\,dx=\int F\left(\frac{x^2}{a+bx^2}\right)\,dx$$

input
```
integrate(F(x**2/(b*x**2+a)),x)
```

output
```
Integral(F(x**2/(a + b*x**2)), x)
```

3.908.7 Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int F\left(\frac{x^2}{a+bx^2}\right)\,dx=\int F\left(\frac{x^2}{bx^2+a}\right)\,dx$$

input
```
integrate(F(x^2/(b*x^2+a)),x, algorithm="maxima")
```

output
```
integrate(F(x^2/(b*x^2 + a)), x)
```

3.908.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int F\left(\frac{x^2}{a+bx^2}\right)\,dx = \int F\left(\frac{x^2}{bx^2+a}\right)\,dx$$

input
```
integrate(F(x^2/(b*x^2+a)),x, algorithm="giac")
```

output
```
integrate(F(x^2/(b*x^2 + a)), x)
```

3.908.9 Mupad [N/A]

Not integrable

Time = 21.68 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int F\left(\frac{x^2}{a+bx^2}\right)\,dx = \int F\left(\frac{x^2}{b\,x^2+a}\right)\,dx$$

input
```
int(F(x^2/(a + b*x^2)),x)
```

output
```
int(F(x^2/(a + b*x^2)), x)
```

3.909 $\int F\left(\frac{x^2}{(a+bx)^2}\right)\,dx$

3.909.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int F\left(\frac{x^2}{(a+bx)^2}\right)\,dx=\text{Int}\left(F\left(\frac{x^2}{(a+bx)^2}\right),x\right)$$

output
```
CannotIntegrate(F(x^2/(b*x+a)^2),x)
```

3.909.2 Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int F\left(\frac{x^2}{(a+bx)^2}\right)\,dx=\int F\left(\frac{x^2}{(a+bx)^2}\right)\,dx$$

input
```
Integrate[F[x^2/(a + b*x)^2],x]
```

output
```
Integrate[F[x^2/(a + b*x)^2], x]
```

3.909.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F\left(\frac{x^2}{(a+bx)^2}\right) \, dx$$
$$\Big\downarrow \; 7299$$
$$\int F\left(\frac{x^2}{(a+bx)^2}\right) dx$$

input
```
Int[F[x^2/(a + b*x)^2],x]
```

output
```
$Aborted
```

3.909.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.909.4 Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int F\left(\frac{x^2}{\left(bx+a\right)^2}\right) dx$$

input
```
int(F(x^2/(b*x+a)^2),x)
```

output
```
int(F(x^2/(b*x+a)^2),x)
```

3.909.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.08

$$\int F\left(\frac{x^2}{(a+bx)^2}\right) dx = \int F\left(\frac{x^2}{(bx+a)^2}\right) dx$$

input
```
integrate(F(x^2/(b*x+a)^2),x, algorithm="fricas")
```

output
```
integral(F(x^2/(b^2*x^2 + 2*a*b*x + a^2)), x)
```

3.909.6 Sympy [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int F\left(\frac{x^2}{(a+bx)^2}\right) dx = \int F\left(\frac{x^2}{(a+bx)^2}\right) dx$$

input
```
integrate(F(x**2/(b*x+a)**2),x)
```

output
```
Integral(F(x**2/(a + b*x)**2), x)
```

3.909.7 Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int F\left(\frac{x^2}{(a+bx)^2}\right) dx = \int F\left(\frac{x^2}{(bx+a)^2}\right) dx$$

input
```
integrate(F(x^2/(b*x+a)^2),x, algorithm="maxima")
```

output
```
integrate(F(x^2/(b*x + a)^2), x)
```

3.909.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int F\left(\frac{x^2}{(a+bx)^2}\right)\,dx=\int F\left(\frac{x^2}{(bx+a)^2}\right)\,dx$$

input
```
integrate(F(x^2/(b*x+a)^2),x, algorithm="giac")
```

output
```
integrate(F(x^2/(b*x + a)^2), x)
```

3.909.9 Mupad [N/A]

Not integrable

Time = 21.46 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int F\left(\frac{x^2}{(a+bx)^2}\right)\,dx=\int F\left(\frac{x^2}{(a+b\,x)^2}\right)\,dx$$

input
```
int(F(x^2/(a + b*x)^2),x)
```

output
```
int(F(x^2/(a + b*x)^2), x)
```

3.910 $\int F\left(\frac{x^4}{(a+bx^2)^2}\right) dx$

3.910.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right) dx = \text{Int}\left(F\left(\frac{x^4}{(a+bx^2)^2}\right), x\right)$$

output
```
CannotIntegrate(F(x^4/(b*x^2+a)^2),x)
```

3.910.2 Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right) dx = \int F\left(\frac{x^4}{(a+bx^2)^2}\right) dx$$

input
```
Integrate[F[x^4/(a + b*x^2)^2],x]
```

output
```
Integrate[F[x^4/(a + b*x^2)^2], x]
```

3.910.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right) dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right) dx$$

input
```
Int[F[x^4/(a + b*x^2)^2],x]
```

output
```
$Aborted
```

3.910.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.910.4 Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int F\left(\frac{x^4}{\left(b\,x^2+a\right)^2}\right) dx$$

input
```
int(F(x^4/(b*x^2+a)^2),x)
```

output
```
int(F(x^4/(b*x^2+a)^2),x)
```

3.910.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right)\,dx = \int F\left(\frac{x^4}{(bx^2+a)^2}\right)\,dx$$

input
```
integrate(F(x^4/(b*x^2+a)^2),x, algorithm="fricas")
```

output
```
integral(F(x^4/(b^2*x^4 + 2*a*b*x^2 + a^2)), x)
```

3.910.6 Sympy [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right)\,dx = \int F\left(\frac{x^4}{(a+bx^2)^2}\right)\,dx$$

input
```
integrate(F(x**4/(b*x**2+a)**2),x)
```

output
```
Integral(F(x**4/(a + b*x**2)**2), x)
```

3.910.7 Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right)\,dx = \int F\left(\frac{x^4}{(bx^2+a)^2}\right)\,dx$$

input
```
integrate(F(x^4/(b*x^2+a)^2),x, algorithm="maxima")
```

output
```
integrate(F(x^4/(b*x^2 + a)^2), x)
```

3.910.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right)\,dx=\int F\left(\frac{x^4}{(bx^2+a)^2}\right)\,dx$$

input
```
integrate(F(x^4/(b*x^2+a)^2),x, algorithm="giac")
```

output
```
integrate(F(x^4/(b*x^2 + a)^2), x)
```

3.910.9 Mupad [N/A]

Not integrable

Time = 19.76 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right)\,dx=\int F\left(\frac{x^4}{(b\,x^2+a)^2}\right)\,dx$$

input
```
int(F(x^4/(a + b*x^2)^2),x)
```

output
```
int(F(x^4/(a + b*x^2)^2), x)
```

3.911 $\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx$

3.911.1 Optimal result

Integrand size = 37, antiderivative size = 47

$$\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{bx^2+\sqrt{a+b^2x^4}}}\right)}{\sqrt{2}\sqrt{b}}$$

output
```
1/2*arctanh(x*2^(1/2)*b^(1/2)/(b*x^2+(b^2*x^4+a)^(1/2))^(1/2))*2^(1/2)/b^(
1/2)
```

3.911.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

$$\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx=\frac{\log\left(i\sqrt{b}\left(bx^2+\sqrt{a+b^2x^4}+\sqrt{2}\sqrt{b}x\sqrt{bx^2+\sqrt{a+b^2x^4}}\right)\right)}{\sqrt{2}\sqrt{b}}$$

input
```
Integrate[Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]
```

output
```
Log[I*Sqrt[b]*(b*x^2 + Sqrt[a + b^2*x^4] + Sqrt[2]*Sqrt[b]*x*Sqrt[b*x^2 +
Sqrt[a + b^2*x^4]])]/(Sqrt[2]*Sqrt[b])
```

3.911.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2557, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{a+b^2x^4}+bx^2}}{\sqrt{a+b^2x^4}}dx$$

$$\Big\downarrow \text{2557}$$

$$\int \frac{1}{1-\frac{2bx^2}{\sqrt{a+b^2x^4}+bx^2}}d\frac{x}{\sqrt{\sqrt{a+b^2x^4}+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\sqrt{a+b^2x^4}+bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

input
```
Int[Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]
```

output
```
ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]]/(Sqrt[2]*Sqrt
[b])
```

3.911.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2557
```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Simp[d   Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x
^2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2,
0]
```

3.911.4 Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+\sqrt{b^{2}x^{4}+a}}}{\sqrt{b^{2}x^{4}+a}}dx$$

input
```
int((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)
```

output
```
int((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)
```

3.911.5 Fricas [A] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.87

$$\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx = \left[\frac{\sqrt{2}\log\left(4\,b^{2}x^{4}+4\,\sqrt{b^{2}x^{4}+a}bx^{2}+2\left(\sqrt{2}b^{\frac{3}{2}}x^{3}+\sqrt{2}\sqrt{b^{2}x^{4}+a}\sqrt{b}x\right)\sqrt{bx^{2}+\sqrt{b^{2}x^{4}+a}}+a\right)}{4\,\sqrt{b}}, -\frac{1}{2}\,\sqrt{2}\sqrt{-\frac{1}{b}}\arctan\left(\frac{\sqrt{2}\sqrt{bx^{2}+\sqrt{b^{2}x^{4}+a}}\sqrt{-\frac{1}{b}}}{2\,x}\right)\right]$$

input
```
integrate((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="
fricas")
```

output
```
[1/4*sqrt(2)*log(4*b^2*x^4 + 4*sqrt(b^2*x^4 + a)*b*x^2 + 2*(sqrt(2)*b^(3/2
)*x^3 + sqrt(2)*sqrt(b^2*x^4 + a)*sqrt(b)*x)*sqrt(b*x^2 + sqrt(b^2*x^4 + a
)) + a)/sqrt(b), -1/2*sqrt(2)*sqrt(-1/b)*arctan(1/2*sqrt(2)*sqrt(b*x^2 + s
qrt(b^2*x^4 + a))*sqrt(-1/b)/x)]
```

3.911.6 Sympy [F]

$$\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx = \int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx$$

input
```
integrate((b*x**2+(b**2*x**4+a)**(1/2))**(1/2)/(b**2*x**4+a)**(1/2),x)
```

output
```
Integral(sqrt(b*x**2 + sqrt(a + b**2*x**4))/sqrt(a + b**2*x**4), x)
```

3.911.7 Maxima [F]

$$\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx = \int \frac{\sqrt{bx^2+\sqrt{b^2x^4+a}}}{\sqrt{b^2x^4+a}}\,dx$$

input
```
integrate((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(sqrt(b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)
```

3.911.8 Giac [F]

$$\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx = \int \frac{\sqrt{bx^2+\sqrt{b^2x^4+a}}}{\sqrt{b^2x^4+a}}\,dx$$

input
```
integrate((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="
giac")
```

output
```
integrate(sqrt(b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)
```

3.911.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx=\int \frac{\sqrt{\sqrt{b^2\,x^4+a}+b\,x^2}}{\sqrt{b^2\,x^4+a}}\,dx$$

input
```
int(((a + b^2*x^4)^(1/2) + b*x^2)^(1/2)/(a + b^2*x^4)^(1/2),x)
```

output
```
int(((a + b^2*x^4)^(1/2) + b*x^2)^(1/2)/(a + b^2*x^4)^(1/2), x)
```

3.912 $\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx$

3.912.1 Optimal result

Integrand size = 38, antiderivative size = 48

$$\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx = \frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}\right)}{\sqrt{2}\sqrt{b}}$$

output
```
1/2*arctan(x*2^(1/2)*b^(1/2)/(-b*x^2+(b^2*x^4+a)^(1/2))^(1/2))*2^(1/2)/b^(
1/2)
```

3.912.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx = \frac{i\log\left(-ib^{3/2}x^2+i\sqrt{b}\sqrt{a+b^2x^4}+\sqrt{2}bx\sqrt{-bx^2+\sqrt{a+b^2x^4}}\right)}{\sqrt{2}\sqrt{b}}$$

input
```
Integrate[Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]
```

output
```
(I*Log[(-I)*b^(3/2)*x^2 + I*Sqrt[b]*Sqrt[a + b^2*x^4] + Sqrt[2]*b*x*Sqrt[-
(b*x^2) + Sqrt[a + b^2*x^4]]])/(Sqrt[2]*Sqrt[b])
```

3.912.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2557, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{a+b^2x^4}-bx^2}}{\sqrt{a+b^2x^4}}dx$$

$$\Big\downarrow \ 2557$$

$$\int \frac{1}{\frac{2bx^2}{\sqrt{a+b^2x^4}-bx^2}+1}d\frac{x}{\sqrt{\sqrt{a+b^2x^4}-bx^2}}$$

$$\Big\downarrow \ 216$$

$$\frac{\arctan\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\sqrt{a+b^2x^4}-bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

input
```
Int[Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]
```

output
```
ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]]/(Sqrt[2]*Sq
rt[b])
```

3.912.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2557
```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Simp[d   Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x
^2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2,
0]
```

3.912.4 Maple [F]

$$\int \frac{\sqrt{-b\,x^2+\sqrt{b^2x^4+a}}}{\sqrt{b^2x^4+a}}dx$$

input
```
int((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)
```

output
```
int((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)
```

3.912.5 Fricas [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.04

$$\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx = \left[\frac{1}{4}\sqrt{2}\sqrt{-\frac{1}{b}}\log\left(4\,b^2x^4-4\,\sqrt{b^2x^4+a}bx^2 + 2\left(\sqrt{2}b^2x^3\sqrt{-\frac{1}{b}}-\sqrt{2}\sqrt{b^2x^4+a}bx\sqrt{-\frac{1}{b}}\right)\sqrt{-bx^2+\sqrt{b^2x^4+a}}+a\right), -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-bx^2+\sqrt{b^2x^4+a}}}{2\sqrt{b}x}\right)}{2\sqrt{b}}\right]$$

input
```
integrate((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm=
"fricas")
```

output
```
[1/4*sqrt(2)*sqrt(-1/b)*log(4*b^2*x^4 - 4*sqrt(b^2*x^4 + a)*b*x^2 + 2*(sqr
t(2)*b^2*x^3*sqrt(-1/b) - sqrt(2)*sqrt(b^2*x^4 + a)*b*x*sqrt(-1/b))*sqrt(-
b*x^2 + sqrt(b^2*x^4 + a)) + a), -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-b*x
^2 + sqrt(b^2*x^4 + a))/(sqrt(b)*x))/sqrt(b)]
```

3.912.6 Sympy [F]

$$\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx=\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx$$

input
```
integrate((-b*x**2+(b**2*x**4+a)**(1/2))**(1/2)/(b**2*x**4+a)**(1/2),x)
```

output
```
Integral(sqrt(-b*x**2 + sqrt(a + b**2*x**4))/sqrt(a + b**2*x**4), x)
```

3.912.7 Maxima [F]

$$\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx=\int \frac{\sqrt{-bx^2+\sqrt{b^2x^4+a}}}{\sqrt{b^2x^4+a}}\,dx$$

input
```
integrate((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm=
"maxima")
```

output
```
integrate(sqrt(-b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)
```

3.912.8 Giac [F]

$$\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx=\int \frac{\sqrt{-bx^2+\sqrt{b^2x^4+a}}}{\sqrt{b^2x^4+a}}\,dx$$

input
```
integrate((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm=
"giac")
```

output
```
integrate(sqrt(-b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)
```

3.912.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx=\int \frac{\sqrt{\sqrt{b^2\,x^4+a}-b\,x^2}}{\sqrt{b^2\,x^4+a}}\,dx$$

input
```
int(((a + b^2*x^4)^(1/2) - b*x^2)^(1/2)/(a + b^2*x^4)^(1/2),x)
```

output
```
int(((a + b^2*x^4)^(1/2) - b*x^2)^(1/2)/(a + b^2*x^4)^(1/2), x)
```

3.913 $\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx$

3.913.1 Optimal result

Integrand size = 40, antiderivative size = 169

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx = \frac{\left(\frac{1}{2}-\frac{i}{2}\right)\arctan\left(\frac{\sqrt{3}d+2icx}{\sqrt{2ic^2-\sqrt{3}d^2}\sqrt{\sqrt{3}-2ix^2}}\right)}{\sqrt{2ic^2-\sqrt{3}d^2}} - \frac{\left(\frac{1}{2}+\frac{i}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{3}d-2icx}{\sqrt{2ic^2+\sqrt{3}d^2}\sqrt{\sqrt{3}+2ix^2}}\right)}{\sqrt{2ic^2+\sqrt{3}d^2}}$$

output
```
(1/2-1/2*I)*arctan((2*I*c*x+d*3^(1/2))/(-2*I*x^2+3^(1/2))^(1/2)/(2*I*c^2-d
^2*3^(1/2))^(1/2))/(2*I*c^2-d^2*3^(1/2))^(1/2)-(1/2+1/2*I)*arctanh((-2*I*c
*x+d*3^(1/2))/(2*I*x^2+3^(1/2))^(1/2)/(2*I*c^2+d^2*3^(1/2))^(1/2))/(2*I*c^
2+d^2*3^(1/2))^(1/2)
```

3.913.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.89 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.15

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx = -\frac{\sqrt{-2c^2-\sqrt{4c^4+3d^4}}\arctan\left(\frac{d\sqrt{2x^2+\sqrt{3+4x^4}}}{\sqrt{-2c^2-\sqrt{4c^4+3d^4}}}\right)}{\sqrt{4c^4+3d^4}} + \frac{\sqrt{-2c^2+\sqrt{4c^4+3d^4}}\arctan\left(\frac{d\sqrt{2x^2+\sqrt{3+4x^4}}}{\sqrt{-2c^2+\sqrt{4c^4+3d^4}}}\right)}{\sqrt{4c^4+3d^4}} + c\text{RootSum}\Big[9d^2-24c^2\#1-6d^2\#1^2-8c^2\#1^3 + d^2\#1^4\&, \frac{-3\log\left(2x^2+\sqrt{3+4x^4}+2x\sqrt{2x^2+\sqrt{3+4x^4}}-\#1\right)-\log\left(2x^2+\sqrt{3+4x^4}+2x\sqrt{2x^2+\right.}{-6c^2-3d^2\#1-6c^2\#1^2+d^2\#1^3}$$

input
```
Integrate[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)*Sqrt[3 + 4*x^4]),x]
```

output
```
-((Sqrt[-2*c^2 - Sqrt[4*c^4 + 3*d^4]]*ArcTan[(d*Sqrt[2*x^2 + Sqrt[3 + 4*x^
4]])/Sqrt[-2*c^2 - Sqrt[4*c^4 + 3*d^4]]])/Sqrt[4*c^4 + 3*d^4]) + (Sqrt[-2*
c^2 + Sqrt[4*c^4 + 3*d^4]]*ArcTan[(d*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]])/Sqrt[-
2*c^2 + Sqrt[4*c^4 + 3*d^4]]])/Sqrt[4*c^4 + 3*d^4] + c*RootSum[9*d^2 - 24*
c^2*#1 - 6*d^2*#1^2 - 8*c^2*#1^3 + d^2*#1^4 & , (-3*Log[2*x^2 + Sqrt[3 + 4
*x^4] + 2*x*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]] - #1] - Log[2*x^2 + Sqrt[3 + 4*x
^4] + 2*x*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]] - #1]*#1^2)/(-6*c^2 - 3*d^2*#1 - 6
*c^2*#1^2 + d^2*#1^3) & ]
```

3.913.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2558, 488, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{4x^4+3}+2x^2}}{\sqrt{4x^4+3}(c+dx)}\,dx$$

$$\Big\downarrow \textcolor{blue}{2558}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(c+dx)\sqrt{\sqrt{3}-2ix^2}}dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(c+dx)\sqrt{2ix^2+\sqrt{3}}}dx$$

$$\Big\downarrow \textcolor{blue}{488}$$

$$\left(-\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{-2ic^2+\sqrt{3}d^2-\frac{\left(\sqrt{3}d+2icx\right)^2}{\sqrt{3}-2ix^2}}d\frac{\sqrt{3}d+2icx}{\sqrt{\sqrt{3}-2ix^2}}-\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{2ic^2+\sqrt{3}d^2-\frac{\left(\sqrt{3}d-2icx\right)^2}{2ix^2+\sqrt{3}}}d\frac{\sqrt{3}d-2icx}{\sqrt{2ix^2+\sqrt{3}}}$$

$$\Big\downarrow \textcolor{blue}{217}$$

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\arctan\left(\frac{\sqrt{3}d+2icx}{\sqrt{\sqrt{3}-2ix^2}\sqrt{-\sqrt{3}d^2+2ic^2}}\right)}{\sqrt{-\sqrt{3}d^2+2ic^2}}-\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{2ic^2+\sqrt{3}d^2-\frac{\left(\sqrt{3}d-2icx\right)^2}{2ix^2+\sqrt{3}}}d\frac{\sqrt{3}d-2icx}{\sqrt{2ix^2+\sqrt{3}}}$$

$$\Big\downarrow \textcolor{blue}{219}$$

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\arctan\left(\frac{\sqrt{3}d+2icx}{\sqrt{\sqrt{3}-2ix^2}\sqrt{-\sqrt{3}d^2+2ic^2}}\right)}{\sqrt{-\sqrt{3}d^2+2ic^2}}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\text{arctanh}\left(\frac{\sqrt{3}d-2icx}{\sqrt{\sqrt{3}+2ix^2}\sqrt{\sqrt{3}d^2+2ic^2}}\right)}{\sqrt{\sqrt{3}d^2+2ic^2}}$$

input
```
Int[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)*Sqrt[3 + 4*x^4]),x]
```

output
```
((1/2 - I/2)*ArcTan[(Sqrt[3]*d + (2*I)*c*x)/(Sqrt[(2*I)*c^2 - Sqrt[3]*d^2]
*Sqrt[Sqrt[3] - (2*I)*x^2])])/Sqrt[(2*I)*c^2 - Sqrt[3]*d^2] - ((1/2 + I/2)
*ArcTanh[(Sqrt[3]*d - (2*I)*c*x)/(Sqrt[(2*I)*c^2 + Sqrt[3]*d^2]*Sqrt[Sqrt[
3] + (2*I)*x^2])])/Sqrt[(2*I)*c^2 + Sqrt[3]*d^2]
```

3.913.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.913. $\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx$

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 2558
```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Simp[(1 - I)/2   Int[(c + d*x)
^m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Simp[(1 + I)/2   Int[(c + d*x)^m/Sqrt[
Sqrt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && G
tQ[a, 0]
```

3.913.4 Maple [F]

$$\int \frac{\sqrt{2x^2+\sqrt{4x^4+3}}}{(dx+c)\sqrt{4x^4+3}}dx$$

input
```
int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x)
```

output
```
int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x)
```

3.913.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx = \text{Timed out}$$

input
```
integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x, algorit
hm="fricas")
```

output
```
Timed out
```

3.913.6 Sympy [F]

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx=\int \frac{\sqrt{2x^2+\sqrt{4x^4+3}}}{(c+dx)\sqrt{4x^4+3}}\,dx$$

input
```
integrate((2*x**2+(4*x**4+3)**(1/2))**(1/2)/(d*x+c)/(4*x**4+3)**(1/2),x)
```

output
```
Integral(sqrt(2*x**2 + sqrt(4*x**4 + 3))/((c + d*x)*sqrt(4*x**4 + 3)), x)
```

3.913.7 Maxima [F]

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx=\int \frac{\sqrt{2\,x^2+\sqrt{4\,x^4+3}}}{\sqrt{4\,x^4+3}(dx+c)}\,dx$$

input
```
integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)), x)
```

3.913.8 Giac [F]

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx=\int \frac{\sqrt{2\,x^2+\sqrt{4\,x^4+3}}}{\sqrt{4\,x^4+3}(dx+c)}\,dx$$

input
```
integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x, algorit
hm="giac")
```

output
```
integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)), x)
```

3.913.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx = \int \frac{\sqrt{2\,x^2+\sqrt{4\,x^4+3}}}{\sqrt{4\,x^4+3}\,\left(c+d\,x\right)}\,dx$$

input
```
int((2*x^2 + (4*x^4 + 3)^(1/2))^(1/2)/((4*x^4 + 3)^(1/2)*(c + d*x)),x)
```

output
```
int((2*x^2 + (4*x^4 + 3)^(1/2))^(1/2)/((4*x^4 + 3)^(1/2)*(c + d*x)), x)
```

3.914 $\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx$

3.914.1 Optimal result

Integrand size = 40, antiderivative size = 268

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx = \frac{\left(\frac{1}{2}-\frac{i}{2}\right)d\sqrt{\sqrt{3}-2ix^2}}{\left(2ic^2-\sqrt{3}d^2\right)(c+dx)} - \frac{\left(\frac{1}{2}+\frac{i}{2}\right)d\sqrt{\sqrt{3}+2ix^2}}{\left(2ic^2+\sqrt{3}d^2\right)(c+dx)} + \frac{(1+i)c\arctan\left(\frac{\sqrt{3}d+2icx}{\sqrt{2ic^2-\sqrt{3}d^2}\sqrt{\sqrt{3}-2ix^2}}\right)}{\left(2ic^2-\sqrt{3}d^2\right)^{3/2}} + \frac{(1-i)c\,\text{arctanh}\left(\frac{\sqrt{3}d-2icx}{\sqrt{2ic^2+\sqrt{3}d^2}\sqrt{\sqrt{3}+2ix^2}}\right)}{\left(2ic^2+\sqrt{3}d^2\right)^{3/2}}$$

output
```
(1+I)*c*arctan((2*I*c*x+d*3^(1/2))/(-2*I*x^2+3^(1/2))^(1/2)/(2*I*c^2-d^2*3
^(1/2))^(1/2))/(2*I*c^2-d^2*3^(1/2))^(3/2)+(1-I)*c*arctanh((-2*I*c*x+d*3^(
1/2))/(2*I*x^2+3^(1/2))^(1/2)/(2*I*c^2+d^2*3^(1/2))^(1/2))/(2*I*c^2+d^2*3^
(1/2))^(3/2)+(1/2-1/2*I)*d*(-2*I*x^2+3^(1/2))^(1/2)/(d*x+c)/(2*I*c^2-d^2*3
^(1/2))-(1/2+1/2*I)*d*(2*I*x^2+3^(1/2))^(1/2)/(d*x+c)/(2*I*c^2+d^2*3^(1/2)
)
```

3.914.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 3.42 (sec) , antiderivative size = 1464, normalized size of antiderivative = 5.46

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx$$
$$= -\frac{d\sqrt{2x^2+\sqrt{3+4x^4}}\left(3d^2\left(2x^2+\sqrt{3+4x^4}\right)+2c^2\left(3+8x^4+4x^2\sqrt{3+4x^4}\right)\right)}{(4c^4+3d^4)\,(c+dx)\left(3+8x^4+4x^2\sqrt{3+4x^4}\right)}$$
$$+\frac{8c^5\arctan\left(\frac{d\sqrt{2x^2+\sqrt{3+4x^4}}}{\sqrt{-2c^2-\sqrt{4c^4+3d^4}}}\right)}{(4c^4+3d^4)^{3/2}\sqrt{-2c^2-\sqrt{4c^4+3d^4}}}-\frac{6cd^4\arctan\left(\frac{d\sqrt{2x^2+\sqrt{3+4x^4}}}{\sqrt{-2c^2-\sqrt{4c^4+3d^4}}}\right)}{(4c^4+3d^4)^{3/2}\sqrt{-2c^2-\sqrt{4c^4+3d^4}}}$$
$$+\frac{4c^3\arctan\left(\frac{d\sqrt{2x^2+\sqrt{3+4x^4}}}{\sqrt{-2c^2-\sqrt{4c^4+3d^4}}}\right)}{(4c^4+3d^4)\sqrt{-2c^2-\sqrt{4c^4+3d^4}}}-\frac{8c^5\arctan\left(\frac{d\sqrt{2x^2+\sqrt{3+4x^4}}}{\sqrt{-2c^2+\sqrt{4c^4+3d^4}}}\right)}{(4c^4+3d^4)^{3/2}\sqrt{-2c^2+\sqrt{4c^4+3d^4}}}$$
$$+\frac{6cd^4\arctan\left(\frac{d\sqrt{2x^2+\sqrt{3+4x^4}}}{\sqrt{-2c^2+\sqrt{4c^4+3d^4}}}\right)}{(4c^4+3d^4)^{3/2}\sqrt{-2c^2+\sqrt{4c^4+3d^4}}}+\frac{4c^3\arctan\left(\frac{d\sqrt{2x^2+\sqrt{3+4x^4}}}{\sqrt{-2c^2+\sqrt{4c^4+3d^4}}}\right)}{(4c^4+3d^4)\sqrt{-2c^2+\sqrt{4c^4+3d^4}}}$$
$$-\frac{\text{RootSum}\left[9d^2-24c^2\#1-6d^2\#1^2-8c^2\#1^3+d^2\#1^4\&,\frac{128c^4\log\left(2x^2+\sqrt{3+4x^4}+2x\sqrt{2x^2+\sqrt{3+4x^4}}-\#1\right)+3d^4\log}{\quad}\right.}{\quad}$$
$$+\frac{\text{RootSum}\left[9d^2-24c^2\#1-6d^2\#1^2-8c^2\#1^3+d^2\#1^4\&,\frac{512c^8\log\left(2x^2+\sqrt{3+4x^4}+2x\sqrt{2x^2+\sqrt{3+4x^4}}-\#1\right)+408c^4d}{\quad}\right.}{\quad}$$

input
```
Integrate[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)^2*Sqrt[3 + 4*x^4]),x]
```

output
```
-((d*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]*(3*d^2*(2*x^2 + Sqrt[3 + 4*x^4]) + 2*c^
2*(3 + 8*x^4 + 4*x^2*Sqrt[3 + 4*x^4])))/((4*c^4 + 3*d^4)*(c + d*x)*(3 + 8*
x^4 + 4*x^2*Sqrt[3 + 4*x^4]))) + (8*c^5*ArcTan[(d*Sqrt[2*x^2 + Sqrt[3 + 4*
x^4]])/Sqrt[-2*c^2 - Sqrt[4*c^4 + 3*d^4]]])/((4*c^4 + 3*d^4)^(3/2)*Sqrt[-2
*c^2 - Sqrt[4*c^4 + 3*d^4]]) - (6*c*d^4*ArcTan[(d*Sqrt[2*x^2 + Sqrt[3 + 4*
x^4]])/Sqrt[-2*c^2 - Sqrt[4*c^4 + 3*d^4]]])/((4*c^4 + 3*d^4)^(3/2)*Sqrt[-2
*c^2 - Sqrt[4*c^4 + 3*d^4]]) + (4*c^3*ArcTan[(d*Sqrt[2*x^2 + Sqrt[3 + 4*x^
4]])/Sqrt[-2*c^2 - Sqrt[4*c^4 + 3*d^4]]])/((4*c^4 + 3*d^4)*Sqrt[-2*c^2 - S
qrt[4*c^4 + 3*d^4]]) - (8*c^5*ArcTan[(d*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]])/Sqr
t[-2*c^2 + Sqrt[4*c^4 + 3*d^4]]])/((4*c^4 + 3*d^4)^(3/2)*Sqrt[-2*c^2 + Sqr
t[4*c^4 + 3*d^4]]) + (6*c*d^4*ArcTan[(d*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]])/Sqr
t[-2*c^2 + Sqrt[4*c^4 + 3*d^4]]])/((4*c^4 + 3*d^4)^(3/2)*Sqrt[-2*c^2 + Sqr
t[4*c^4 + 3*d^4]]) + (4*c^3*ArcTan[(d*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]])/Sqrt[
-2*c^2 + Sqrt[4*c^4 + 3*d^4]]])/((4*c^4 + 3*d^4)*Sqrt[-2*c^2 + Sqrt[4*c^4
+ 3*d^4]]) - RootSum[9*d^2 - 24*c^2*#1 - 6*d^2*#1^2 - 8*c^2*#1^3 + d^2*#1^
4 & , (128*c^4*Log[2*x^2 + Sqrt[3 + 4*x^4] + 2*x*Sqrt[2*x^2 + Sqrt[3 + 4*x
^4]] - #1] + 3*d^4*Log[2*x^2 + Sqrt[3 + 4*x^4] + 2*x*Sqrt[2*x^2 + Sqrt[3 +
 4*x^4]] - #1] + 16*c^2*d^2*Log[2*x^2 + Sqrt[3 + 4*x^4] + 2*x*Sqrt[2*x^2 +
 Sqrt[3 + 4*x^4]] - #1]*#1 + d^4*Log[2*x^2 + Sqrt[3 + 4*x^4] + 2*x*Sqrt[2*
x^2 + Sqrt[3 + 4*x^4]] - #1]*#1^2)/(6*c^2 + 3*d^2*#1 + 6*c^2*#1^2 - d^2...
```

3.914.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2558, 491, 488, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{4x^4+3}+2x^2}}{\sqrt{4x^4+3}(c+dx)^2}dx$$

$$\Big\downarrow \text{2558}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(c+dx)^2\sqrt{\sqrt{3}-2ix^2}}dx+\left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(c+dx)^2\sqrt{2ix^2+\sqrt{3}}}dx$$

$$\Big\downarrow \text{491}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{2c\int\frac{1}{(c+dx)\sqrt{\sqrt{3}-2ix^2}}dx}{2c^2+i\sqrt{3}d^2}+\frac{d\sqrt{\sqrt{3}-2ix^2}}{\left(-\sqrt{3}d^2+2ic^2\right)(c+dx)}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{2c\int\frac{1}{(c+dx)\sqrt{2ix^2+\sqrt{3}}}dx}{2c^2-i\sqrt{3}d^2}-\frac{d\sqrt{\sqrt{3}+2ix^2}}{\left(\sqrt{3}d^2+2ic^2\right)(c+dx)}\right)$$

$$\Big\downarrow \text{488}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{d\sqrt{\sqrt{3}-2ix^2}}{\left(-\sqrt{3}d^2+2ic^2\right)(c+dx)}-\frac{2c\int\frac{1}{-2ic^2+\sqrt{3}d^2-\frac{\left(\sqrt{3}d+2icx\right)^2}{\sqrt{3}-2ix^2}}d\frac{\sqrt{3}d+2icx}{\sqrt{\sqrt{3}-2ix^2}}}{2c^2+i\sqrt{3}d^2}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(-\frac{2c\int\frac{1}{2ic^2+\sqrt{3}d^2-\frac{\left(\sqrt{3}d-2icx\right)^2}{2ix^2+\sqrt{3}}}d\frac{\sqrt{3}d-2icx}{\sqrt{2ix^2+\sqrt{3}}}}{2c^2-i\sqrt{3}d^2}-\frac{d\sqrt{\sqrt{3}+2ix^2}}{\left(\sqrt{3}d^2+2ic^2\right)(c+dx)}\right)$$

$$\Big\downarrow \text{217}$$

$$\left(\frac{1}{2}+\frac{i}{2}\right)\left(-\frac{2c\int\frac{1}{2ic^2+\sqrt{3}d^2-\frac{\left(\sqrt{3}d-2icx\right)^2}{2ix^2+\sqrt{3}}}d\frac{\sqrt{3}d-2icx}{\sqrt{2ix^2+\sqrt{3}}}}{2c^2-i\sqrt{3}d^2}-\frac{d\sqrt{\sqrt{3}+2ix^2}}{\left(\sqrt{3}d^2+2ic^2\right)(c+dx)}\right)+\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{2c\arctan\left(\frac{\sqrt{3}d+2icx}{\sqrt{\sqrt{3}-2ix^2}\sqrt{-\sqrt{3}d^2+2ic^2}}\right)}{\sqrt{-\sqrt{3}d^2+2ic^2}\left(2c^2+i\sqrt{3}d^2\right)}+\frac{d\sqrt{\sqrt{3}-2ix^2}}{\left(-\sqrt{3}d^2+2ic^2\right)(c+dx)}\right)$$

$$\Big\downarrow \text{219}$$

$$\left(\frac{1}{2}-\frac{i}{2}\right)\left(\frac{2c\arctan\left(\frac{\sqrt{3}d+2icx}{\sqrt{\sqrt{3}-2ix^2}\sqrt{-\sqrt{3}d^2+2ic^2}}\right)}{\sqrt{-\sqrt{3}d^2+2ic^2}\left(2c^2+i\sqrt{3}d^2\right)}+\frac{d\sqrt{\sqrt{3}-2ix^2}}{\left(-\sqrt{3}d^2+2ic^2\right)(c+dx)}\right)+\left(\frac{1}{2}+\frac{i}{2}\right)\left(-\frac{2c\,\text{arctanh}\left(\frac{\sqrt{3}d-2icx}{\sqrt{\sqrt{3}+2ix^2}\sqrt{\sqrt{3}d^2+2ic^2}}\right)}{\left(2c^2-i\sqrt{3}d^2\right)\sqrt{\sqrt{3}d^2+2ic^2}}-\frac{d\sqrt{\sqrt{3}+2ix^2}}{\left(\sqrt{3}d^2+2ic^2\right)(c+dx)}\right)$$

input
```
Int[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)^2*Sqrt[3 + 4*x^4]),x]
```

3.914. $\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx$

output
```
(1/2 - I/2)*((d*Sqrt[Sqrt[3] - (2*I)*x^2])/(((2*I)*c^2 - Sqrt[3]*d^2)*(c +
 d*x)) + (2*c*ArcTan[(Sqrt[3]*d + (2*I)*c*x)/(Sqrt[(2*I)*c^2 - Sqrt[3]*d^2
]*Sqrt[Sqrt[3] - (2*I)*x^2])])/(Sqrt[(2*I)*c^2 - Sqrt[3]*d^2]*(2*c^2 + I*S
qrt[3]*d^2))) + (1/2 + I/2)*(-((d*Sqrt[Sqrt[3] + (2*I)*x^2])/(((2*I)*c^2 +
 Sqrt[3]*d^2)*(c + d*x))) - (2*c*ArcTanh[(Sqrt[3]*d - (2*I)*c*x)/(Sqrt[(2*
I)*c^2 + Sqrt[3]*d^2]*Sqrt[Sqrt[3] + (2*I)*x^2])])/((2*c^2 - I*Sqrt[3]*d^2
)*Sqrt[(2*I)*c^2 + Sqrt[3]*d^2]))
```

3.914.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 491
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b*(c/(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 3, 0]
```

rule 2558
```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Simp[(1 - I)/2   Int[(c + d*x)
^m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Simp[(1 + I)/2   Int[(c + d*x)^m/Sqrt[
Sqrt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && G
tQ[a, 0]
```

3.914. $\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx$

3.914.4 Maple [F]

$$\int \frac{\sqrt{2x^2+\sqrt{4x^4+3}}}{\left(dx+c\right)^2\sqrt{4x^4+3}}dx$$

input
```
int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x)
```

output
```
int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x)
```

3.914.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx = \text{Timed out}$$

input
```
integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x, algor
ithm="fricas")
```

output
```
Timed out
```

3.914.6 Sympy [F]

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx = \int \frac{\sqrt{2x^2+\sqrt{4x^4+3}}}{\left(c+dx\right)^2\sqrt{4x^4+3}}\,dx$$

input
```
integrate((2*x**2+(4*x**4+3)**(1/2))**(1/2)/(d*x+c)**2/(4*x**4+3)**(1/2),x
)
```

output
```
Integral(sqrt(2*x**2 + sqrt(4*x**4 + 3))/((c + d*x)**2*sqrt(4*x**4 + 3)),
x)
```

3.914.7 Maxima [F]

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx=\int \frac{\sqrt{2\,x^2+\sqrt{4\,x^4+3}}}{\sqrt{4\,x^4+3}(dx+c)^2}\,dx$$

input
```
integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)^2), x)
```

3.914.8 Giac [F]

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx=\int \frac{\sqrt{2\,x^2+\sqrt{4\,x^4+3}}}{\sqrt{4\,x^4+3}(dx+c)^2}\,dx$$

input
```
integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x, algor
ithm="giac")
```

output
```
integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)^2), x)
```

3.914.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx=\int \frac{\sqrt{2\,x^2+\sqrt{4\,x^4+3}}}{\sqrt{4\,x^4+3}\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((2*x^2 + (4*x^4 + 3)^(1/2))^(1/2)/((4*x^4 + 3)^(1/2)*(c + d*x)^2),x)
```

output
```
int((2*x^2 + (4*x^4 + 3)^(1/2))^(1/2)/((4*x^4 + 3)^(1/2)*(c + d*x)^2), x)
```

3.915 $\int \frac{-4+x}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx$

3.915.1 Optimal result

Integrand size = 18, antiderivative size = 41

$$\int \frac{-4+x}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = -30\sqrt[6]{x}+2\sqrt{x}-\frac{6x^{5/6}}{5}+\frac{6x^{7/6}}{7}+30\arctan\left(\sqrt[6]{x}\right)$$

output
```
-30*x^(1/6)-6/5*x^(5/6)+6/7*x^(7/6)+30*arctan(x^(1/6))+2*x^(1/2)
```

3.915.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{-4+x}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = \frac{2}{35}\left(-525\sqrt[6]{x}+35\sqrt{x}-21x^{5/6}+15x^{7/6}\right)+30\arctan\left(\sqrt[6]{x}\right)$$

input
```
Integrate[(-4 + x)/((1 + x^(1/3))*Sqrt[x]),x]
```

output
```
(2*(-525*x^(1/6) + 35*Sqrt[x] - 21*x^(5/6) + 15*x^(7/6)))/35 + 30*ArcTan[x
^(1/6)]
```

3.915.3 Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2035, 25, 2429, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-4}{\left(\sqrt[3]{x}+1\right)\sqrt{x}}dx$$
$$\Big\downarrow \text{2035}$$
$$2\int -\frac{4-x}{\sqrt[3]{x}+1}d\sqrt{x}$$
$$\Big\downarrow \text{25}$$
$$-2\int \frac{4-x}{\sqrt[3]{x}+1}d\sqrt{x}$$
$$\Big\downarrow \text{2429}$$
$$-6\int \frac{x\left(4-x^3\right)}{x+1}d\sqrt[6]{x}$$
$$\Big\downarrow \text{2333}$$
$$-6\int \left(-x^3+x^2-x-\frac{5}{x+1}+5\right)d\sqrt[6]{x}$$
$$\Big\downarrow \text{2009}$$
$$-6\left(-5\arctan\left(\sqrt[6]{x}\right)-\frac{x^{7/2}}{7}+\frac{x^{5/2}}{5}-\frac{x^{3/2}}{3}+5\sqrt[6]{x}\right)$$

input
```
Int[(-4 + x)/((1 + x^(1/3))*Sqrt[x]),x]
```

output
```
-6*(5*x^(1/6) - x^(3/2)/3 + x^(5/2)/5 - x^(7/2)/7 - 5*ArcTan[x^(1/6)])
```

3.915.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2429
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{g = Denominator
[n]}, Simp[g   Subst[Int[x^(g - 1)*(Pq /. x -> x^g)*(a + b*x^(g*n))^p, x],
x, x^(1/g)], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && FractionQ[n]
```

3.915.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$-30x^{\frac{1}{6}}-\frac{6x^{\frac{5}{6}}}{5}+\frac{6x^{\frac{7}{6}}}{7}+30\arctan\left(x^{\frac{1}{6}}\right)+2\sqrt{x}$	28
default	$-30x^{\frac{1}{6}}-\frac{6x^{\frac{5}{6}}}{5}+\frac{6x^{\frac{7}{6}}}{7}+30\arctan\left(x^{\frac{1}{6}}\right)+2\sqrt{x}$	28
meijerg	$-\frac{2x^{\frac{1}{6}}\left(-45x+63x^{\frac{2}{3}}-105x^{\frac{1}{3}}+315\right)}{105}+30\arctan\left(x^{\frac{1}{6}}\right)-24x^{\frac{1}{6}}$	33

input
```
int((x-4)/(1+x^(1/3))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-30*x^(1/6)-6/5*x^(5/6)+6/7*x^(7/6)+30*arctan(x^(1/6))+2*x^(1/2)
```

3.915.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \frac{-4+x}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = \frac{6}{7}\,(x-35)x^{\frac{1}{6}} - \frac{6}{5}\,x^{\frac{5}{6}} + 2\,\sqrt{x} + 30\,\arctan\left(x^{\frac{1}{6}}\right)$$

input
```
integrate((-4+x)/(1+x^(1/3))/x^(1/2),x, algorithm="fricas")
```

output
```
6/7*(x - 35)*x^(1/6) - 6/5*x^(5/6) + 2*sqrt(x) + 30*arctan(x^(1/6))
```

3.915.6 Sympy [A] (verification not implemented)

Time = 3.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{-4+x}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = \frac{6x^{\frac{7}{6}}}{7} - \frac{6x^{\frac{5}{6}}}{5} - 30\sqrt[6]{x} + 2\sqrt{x} + 30\,\mathrm{atan}\left(\sqrt[6]{x}\right)$$

input
```
integrate((-4+x)/(1+x**(1/3))/x**(1/2),x)
```

output
```
6*x**(7/6)/7 - 6*x**(5/6)/5 - 30*x**(1/6) + 2*sqrt(x) + 30*atan(x**(1/6))
```

3.915.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{-4+x}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = \frac{6}{7}\,x^{\frac{7}{6}} - \frac{6}{5}\,x^{\frac{5}{6}} + 2\,\sqrt{x} - 30\,x^{\frac{1}{6}} + 30\,\arctan\left(x^{\frac{1}{6}}\right)$$

input
```
integrate((-4+x)/(1+x^(1/3))/x^(1/2),x, algorithm="maxima")
```

output
```
6/7*x^(7/6) - 6/5*x^(5/6) + 2*sqrt(x) - 30*x^(1/6) + 30*arctan(x^(1/6))
```

3.915.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{-4+x}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = \frac{6}{7}\,x^{\frac{7}{6}} - \frac{6}{5}\,x^{\frac{5}{6}} + 2\sqrt{x} - 30\,x^{\frac{1}{6}} + 30\,\arctan\left(x^{\frac{1}{6}}\right)$$

input
```
integrate((-4+x)/(1+x^(1/3))/x^(1/2),x, algorithm="giac")
```

output
```
6/7*x^(7/6) - 6/5*x^(5/6) + 2*sqrt(x) - 30*x^(1/6) + 30*arctan(x^(1/6))
```

3.915.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{-4+x}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = 30\,\mathrm{atan}\left(x^{1/6}\right) + 2\sqrt{x} - 30\,x^{1/6} - \frac{6\,x^{5/6}}{5} + \frac{6\,x^{7/6}}{7}$$

input
```
int((x - 4)/(x^(1/2)*(x^(1/3) + 1)),x)
```

output
```
30*atan(x^(1/6)) + 2*x^(1/2) - 30*x^(1/6) - (6*x^(5/6))/5 + (6*x^(7/6))/7
```

3.916 $\int \frac{1+\sqrt{x}}{x^{5/6}+x^{7/6}} \, dx$

3.916.1 Optimal result

Integrand size = 21, antiderivative size = 26

$$\int \frac{1+\sqrt{x}}{x^{5/6}+x^{7/6}} \, dx = 3\sqrt[3]{x}+6\arctan\left(\sqrt[6]{x}\right)-3\log\left(1+\sqrt[3]{x}\right)$$

output
```
3*x^(1/3)+6*arctan(x^(1/6))-3*ln(1+x^(1/3))
```

3.916.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1+\sqrt{x}}{x^{5/6}+x^{7/6}} \, dx = 3\sqrt[3]{x}+6\arctan\left(\sqrt[6]{x}\right)-3\log\left(1+\sqrt[3]{x}\right)$$

input
```
Integrate[(1 + Sqrt[x])/(x^(5/6) + x^(7/6)),x]
```

output
```
3*x^(1/3) + 6*ArcTan[x^(1/6)] - 3*Log[1 + x^(1/3)]
```

3.916.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2027, 2035, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}+1}{x^{7/6}+x^{5/6}}dx$$
$$\Big\downarrow \text{2027}$$
$$\int \frac{\sqrt{x}+1}{\left(\sqrt[3]{x}+1\right)x^{5/6}}dx$$
$$\Big\downarrow \text{2035}$$
$$6\int \frac{\sqrt{x}+1}{\sqrt[3]{x}+1}d\sqrt[6]{x}$$
$$\Big\downarrow \text{2341}$$
$$6\int \left(\frac{1-\sqrt[6]{x}}{\sqrt[3]{x}+1}+\sqrt[6]{x}\right)d\sqrt[6]{x}$$
$$\Big\downarrow \text{2009}$$
$$6\left(\arctan\left(\sqrt[6]{x}\right)+\frac{\sqrt[3]{x}}{2}-\frac{1}{2}\log\left(\sqrt[3]{x}+1\right)\right)$$

input
```
Int[(1 + Sqrt[x])/(x^(5/6) + x^(7/6)),x]
```

output
```
6*(x^(1/3)/2 + ArcTan[x^(1/6)] - Log[1 + x^(1/3)]/2)
```

3.916.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] && !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.916.4 Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$3x^{\frac{1}{3}}+6\arctan\left(x^{\frac{1}{6}}\right)-3\ln\left(1+x^{\frac{1}{3}}\right)$$

input
```
int((1+x^(1/2))/(x^(5/6)+x^(7/6)),x)
```

output
```
3*x^(1/3)+6*arctan(x^(1/6))-3*ln(1+x^(1/3))
```

3.916.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{1+\sqrt{x}}{x^{5/6}+x^{7/6}}\,dx = 3\,x^{\frac{1}{3}}+6\,\arctan\left(x^{\frac{1}{6}}\right)-3\,\log\left(x^{\frac{1}{3}}+1\right)$$

input
```
integrate((1+x^(1/2))/(x^(5/6)+x^(7/6)),x, algorithm="fricas")
```

output
```
3*x^(1/3) + 6*arctan(x^(1/6)) - 3*log(x^(1/3) + 1)
```

3.916.6 Sympy [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1+\sqrt{x}}{x^{5/6}+x^{7/6}}\,dx = 3\sqrt[3]{x}-3\log\left(\sqrt[3]{x}+1\right)+6\operatorname{atan}\left(\sqrt[6]{x}\right)$$

input
```
integrate((1+x**(1/2))/(x**(5/6)+x**(7/6)),x)
```

output
```
3*x**(1/3) - 3*log(x**(1/3) + 1) + 6*atan(x**(1/6))
```

3.916.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{1+\sqrt{x}}{x^{5/6}+x^{7/6}}\,dx = 3\,x^{\frac{1}{3}} + 6\,\arctan\left(x^{\frac{1}{6}}\right) - 3\,\log\left(x^{\frac{1}{3}} + 1\right)$$

input
```
integrate((1+x^(1/2))/(x^(5/6)+x^(7/6)),x, algorithm="maxima")
```

output
```
3*x^(1/3) + 6*arctan(x^(1/6)) - 3*log(x^(1/3) + 1)
```

3.916.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{1+\sqrt{x}}{x^{5/6}+x^{7/6}}\,dx = 3\,x^{\frac{1}{3}} + 6\,\arctan\left(x^{\frac{1}{6}}\right) - 3\,\log\left(x^{\frac{1}{3}} + 1\right)$$

input
```
integrate((1+x^(1/2))/(x^(5/6)+x^(7/6)),x, algorithm="giac")
```

output
```
3*x^(1/3) + 6*arctan(x^(1/6)) - 3*log(x^(1/3) + 1)
```

3.916.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1+\sqrt{x}}{x^{5/6}+x^{7/6}}\,dx = 6\,\mathrm{atan}\left(x^{1/6}\right) - 3\,\ln\left(36\,x^{1/3} + 36\right) + 3\,x^{1/3}$$

input
```
int((x^(1/2) + 1)/(x^(5/6) + x^(7/6)),x)
```

output
```
6*atan(x^(1/6)) - 3*log(36*x^(1/3) + 36) + 3*x^(1/3)
```

3.917 $\int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx$

3.917.1 Optimal result

Integrand size = 22, antiderivative size = 42

$$\int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx=6\sqrt[6]{x}-3\sqrt[3]{x}+\frac{3x^{2/3}}{2}-6\arctan\left(\sqrt[6]{x}\right)+3\log\left(1+\sqrt[3]{x}\right)$$

output
```
6*x^(1/6)-3*x^(1/3)+3/2*x^(2/3)-6*arctan(x^(1/6))+3*ln(1+x^(1/3))
```

3.917.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx=\frac{3}{2}\left(4-2\sqrt[6]{x}+\sqrt{x}\right)\sqrt[6]{x}-6\arctan\left(\sqrt[6]{x}\right)+3\log\left(1+\sqrt[3]{x}\right)$$

input
```
Integrate[(1 + Sqrt[x])/((1 + x^(1/3))*Sqrt[x]),x]
```

output
```
(3*(4 - 2*x^(1/6) + Sqrt[x])*x^(1/6))/2 - 6*ArcTan[x^(1/6)] + 3*Log[1 + x^
(1/3)]
```

3.917.3 Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2035, 2429, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}+1}{\left(\sqrt[3]{x}+1\right)\sqrt{x}}dx$$

$$\Big\downarrow \text{2035}$$

$$2\int \frac{\sqrt{x}+1}{\sqrt[3]{x}+1}d\sqrt{x}$$

$$\Big\downarrow \text{2429}$$

$$6\int \frac{x\left(x^{3/2}+1\right)}{x+1}d\sqrt[6]{x}$$

$$\Big\downarrow \text{2333}$$

$$6\int \left(x^{3/2}-\sqrt[6]{x}-\frac{1-\sqrt[6]{x}}{x+1}+1\right)d\sqrt[6]{x}$$

$$\Big\downarrow \text{2009}$$

$$6\left(-\arctan\left(\sqrt[6]{x}\right)+\frac{x^2}{4}-\frac{x}{2}+\sqrt[6]{x}+\frac{1}{2}\log(x+1)\right)$$

input
```
Int[(1 + Sqrt[x])/((1 + x^(1/3))*Sqrt[x]),x]
```

output
```
6*(x^(1/6) - x/2 + x^2/4 - ArcTan[x^(1/6)] + Log[1 + x]/2)
```

3.917.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2429
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{g = Denominator
[n]}, Simp[g   Subst[Int[x^(g - 1)*(Pq /. x -> x^g)*(a + b*x^(g*n))^p, x],
x, x^(1/g)], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && FractionQ[n]
```

3.917.4 Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

method	result	
derivativedivides	$6x^{\frac{1}{6}}-3x^{\frac{1}{3}}+\frac{3x^{\frac{2}{3}}}{2}-6\arctan\left(x^{\frac{1}{6}}\right)+3\ln\left(1+x^{\frac{1}{3}}\right)$	
meijerg	$6x^{\frac{1}{6}}-6\arctan\left(x^{\frac{1}{6}}\right)-\frac{x^{\frac{1}{3}}\left(-3x^{\frac{1}{3}}+6\right)}{2}+3\ln\left(1+x^{\frac{1}{3}}\right)$	
default	$\ln\left(x+1\right)+\frac{3x^{\frac{2}{3}}}{2}-\ln\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}+1\right)+2\ln\left(1+x^{\frac{1}{3}}\right)-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}-6\arctan\left(x^{\frac{1}{6}}\right)$	

input
```
int((1+x^(1/2))/(1+x^(1/3))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
6*x^(1/6)-3*x^(1/3)+3/2*x^(2/3)-6*arctan(x^(1/6))+3*ln(1+x^(1/3))
```

3.917.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx=\frac{3}{2}\,x^{\frac{2}{3}}-3\,x^{\frac{1}{3}}+6\,x^{\frac{1}{6}}-6\,\arctan\left(x^{\frac{1}{6}}\right)+3\,\log\left(x^{\frac{1}{3}}+1\right)$$

input
```
integrate((1+x^(1/2))/(1+x^(1/3))/x^(1/2),x, algorithm="fricas")
```

output
```
3/2*x^(2/3) - 3*x^(1/3) + 6*x^(1/6) - 6*arctan(x^(1/6)) + 3*log(x^(1/3) +
1)
```

3.917.6 Sympy [A] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = 6\sqrt[6]{x} + \frac{3x^{\frac{2}{3}}}{2} - 3\sqrt[3]{x} + 3\log\left(\sqrt[3]{x}+1\right) - 6\,\mathrm{atan}\left(\sqrt[6]{x}\right)$$

input
```
integrate((1+x**(1/2))/(1+x**(1/3))/x**(1/2),x)
```

output
```
6*x**(1/6) + 3*x**(2/3)/2 - 3*x**(1/3) + 3*log(x**(1/3) + 1) - 6*atan(x**(
1/6))
```

3.917.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = \frac{3}{2}\,x^{\frac{2}{3}} - 3\,x^{\frac{1}{3}} + 6\,x^{\frac{1}{6}} - 6\,\arctan\left(x^{\frac{1}{6}}\right) + 3\,\log\left(x^{\frac{1}{3}}+1\right)$$

input
```
integrate((1+x^(1/2))/(1+x^(1/3))/x^(1/2),x, algorithm="maxima")
```

output
```
3/2*x^(2/3) - 3*x^(1/3) + 6*x^(1/6) - 6*arctan(x^(1/6)) + 3*log(x^(1/3) +
1)
```

3.917.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

$$\int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = \frac{3}{2}\,x^{\frac{2}{3}} - 3\,x^{\frac{1}{3}} + 6\,x^{\frac{1}{6}} - 6\,\arctan\left(x^{\frac{1}{6}}\right) + 3\,\log\left(x^{\frac{1}{3}}+1\right)$$

input
```
integrate((1+x^(1/2))/(1+x^(1/3))/x^(1/2),x, algorithm="giac")
```

output
```
3/2*x^(2/3) - 3*x^(1/3) + 6*x^(1/6) - 6*arctan(x^(1/6)) + 3*log(x^(1/3) +
1)
```

3.917.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx = \frac{3\,x^{2/3}}{2}+3\,\ln\left(\left(-6+x^{1/6}\,6\mathrm{i}\right)\,\left(6+x^{1/6}\,6\mathrm{i}\right)\right)-3\,x^{1/3}-6\,\mathrm{atan}\left(x^{1/6}\right)+6\,x^{1/6}$$

input
```
int((x^(1/2) + 1)/(x^(1/2)*(x^(1/3) + 1)),x)
```

output
```
3*log((x^(1/6)*6i - 6)*(x^(1/6)*6i + 6)) - 6*atan(x^(1/6)) - 3*x^(1/3) + (
3*x^(2/3))/2 + 6*x^(1/6)
```

3.918 $\int \frac{\sqrt{2+\frac{b}{x^2}}}{b+2x^2}\,dx$

3.918.1 Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \frac{\sqrt{2+\frac{b}{x^2}}}{b+2x^2}\,dx = -\frac{\operatorname{csch}^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

output
```
-arccsch(x*2^(1/2)/b^(1/2))/b^(1/2)
```

3.918.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 48 vs. 2(20) = 40.

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.40

$$\int \frac{\sqrt{2+\frac{b}{x^2}}}{b+2x^2}\,dx = -\frac{\sqrt{2+\frac{b}{x^2}}\,x\operatorname{arctanh}\left(\frac{\sqrt{b+2x^2}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{b+2x^2}}$$

input
```
Integrate[Sqrt[2 + b/x^2]/(b + 2*x^2),x]
```

output
```
-((Sqrt[2 + b/x^2]*x*ArcTanh[Sqrt[b + 2*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[b + 2
*x^2]))
```

3.918.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {941, 858, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{b}{x^2}+2}}{b+2x^2}dx$$

$$\Big\downarrow \text{ 941}$$

$$\int \frac{1}{x^2\sqrt{\frac{b}{x^2}+2}}dx$$

$$\Big\downarrow \text{ 858}$$

$$-\int \frac{1}{\sqrt{\frac{b}{x^2}+2}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 222}$$

$$-\frac{\text{arcsinh}\left(\frac{\sqrt{b}}{\sqrt{2}x}\right)}{\sqrt{b}}$$

input
```
Int[Sqrt[2 + b/x^2]/(b + 2*x^2),x]
```

output
```
-(ArcSinh[Sqrt[b]/(Sqrt[2]*x)]/Sqrt[b])
```

3.918.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

3.918.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(14) = 28$.

Time = 1.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.50

method	result	size
default	$-\frac{\sqrt{\frac{2x^2+b}{x^2}}\, x \ln\left(\frac{2b+2\sqrt{b}\,\sqrt{2x^2+b}}{x}\right)}{\sqrt{2x^2+b}\,\sqrt{b}}$	50

input
```
int((2+b/x^2)^(1/2)/(2*x^2+b),x,method=_RETURNVERBOSE)
```

output
```
-((2*x^2+b)/x^2)^(1/2)*x/(2*x^2+b)^(1/2)/b^(1/2)*ln(2*(b^(1/2)*(2*x^2+b)^(
1/2)+b)/x)
```

3.918.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(14) = 28$.

Time = 0.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.75

$$\int \frac{\sqrt{2+\frac{b}{x^2}}}{b+2x^2}\, dx = \left[\frac{\log\left(-\frac{x^2-\sqrt{b}x\sqrt{\frac{2\,x^2+b}{x^2}}+b}{x^2}\right)}{2\,\sqrt{b}}, \frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x\sqrt{\frac{2\,x^2+b}{x^2}}}{2\,x^2+b}\right)}{b}\right]$$

input
```
integrate((2+b/x^2)^(1/2)/(2*x^2+b),x, algorithm="fricas")
```

output
```
[1/2*log(-(x^2 - sqrt(b)*x*sqrt((2*x^2 + b)/x^2) + b)/x^2)/sqrt(b), sqrt(-
b)*arctan(sqrt(-b)*x*sqrt((2*x^2 + b)/x^2)/(2*x^2 + b))/b]
```

3.918.6 Sympy [F]

$$\int \frac{\sqrt{2+\frac{b}{x^2}}}{b+2x^2}\,dx = \int \frac{\sqrt{\frac{b}{x^2}+2}}{b+2x^2}\,dx$$

input
```
integrate((2+b/x**2)**(1/2)/(2*x**2+b),x)
```

output
```
Integral(sqrt(b/x**2 + 2)/(b + 2*x**2), x)
```

3.918.7 Maxima [F]

$$\int \frac{\sqrt{2+\frac{b}{x^2}}}{b+2x^2}\,dx = \int \frac{\sqrt{\frac{b}{x^2}+2}}{2\,x^2+b}\,dx$$

input
```
integrate((2+b/x^2)^(1/2)/(2*x^2+b),x, algorithm="maxima")
```

output
```
integrate(sqrt(b/x^2 + 2)/(2*x^2 + b), x)
```

3.918.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(14) = 28$.

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{\sqrt{2+\frac{b}{x^2}}}{b+2x^2}\,dx = \frac{\arctan\left(\frac{\sqrt{2\,x^2+b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}} - \frac{\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)\operatorname{sgn}(x)}{\sqrt{-b}}$$

input
```
integrate((2+b/x^2)^(1/2)/(2*x^2+b),x, algorithm="giac")
```

output
```
arctan(sqrt(2*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - arctan(sqrt(b)/sqrt(-b)
)*sgn(x)/sqrt(-b)
```

3.918.9 Mupad [B] (verification not implemented)

Time = 19.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{2+\frac{b}{x^2}}}{b+2x^2}\,dx = -\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}}{2\,x}\right)}{\sqrt{b}}$$

input
```
int((b/x^2 + 2)^(1/2)/(b + 2*x^2),x)
```

output
```
-asinh((2^(1/2)*b^(1/2))/(2*x))/b^(1/2)
```

3.919 $\int \frac{\sqrt{2-\frac{b}{x^2}}}{-b+2x^2} dx$

3.919.1 Optimal result

Integrand size = 24, antiderivative size = 20

$$\int \frac{\sqrt{2-\frac{b}{x^2}}}{-b+2x^2} dx = -\frac{\csc^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

output
```
-arccsc(x*2^(1/2)/b^(1/2))/b^(1/2)
```

3.919.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 52 vs. 2(20) = 40.

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int \frac{\sqrt{2-\frac{b}{x^2}}}{-b+2x^2} dx = \frac{\sqrt{2-\frac{b}{x^2}}x \arctan\left(\frac{\sqrt{-b+2x^2}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{-b+2x^2}}$$

input
```
Integrate[Sqrt[2 - b/x^2]/(-b + 2*x^2),x]
```

output
```
(Sqrt[2 - b/x^2]*x*ArcTan[Sqrt[-b + 2*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[-b + 2*
x^2])
```

3.919.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {941, 858, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2-\frac{b}{x^2}}}{2x^2-b}dx$$
$$\Big\downarrow \text{ 941}$$
$$\int \frac{1}{x^2\sqrt{2-\frac{b}{x^2}}}dx$$
$$\Big\downarrow \text{ 858}$$
$$-\int \frac{1}{\sqrt{2-\frac{b}{x^2}}}d\frac{1}{x}$$
$$\Big\downarrow \text{ 223}$$
$$-\frac{\arcsin\left(\frac{\sqrt{b}}{\sqrt{2}x}\right)}{\sqrt{b}}$$

input
```
Int[Sqrt[2 - b/x^2]/(-b + 2*x^2),x]
```

output
```
-(ArcSin[Sqrt[b]/(Sqrt[2]*x)]/Sqrt[b])
```

3.919.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

3.919.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(14) = 28$.

Time = 1.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.05

method	result	size
default	$-\frac{\sqrt{-\frac{-2x^2+b}{x^2}}\, x \ln\left(\frac{-2b+2\sqrt{-b}\,\sqrt{2x^2-b}}{x}\right)}{\sqrt{2x^2-b}\,\sqrt{-b}}$	61

input
```
int((2-b/x^2)^(1/2)/(2*x^2-b),x,method=_RETURNVERBOSE)
```

output
```
-(-(-2*x^2+b)/x^2)^(1/2)*x/(2*x^2-b)^(1/2)/(-b)^(1/2)*ln(2*((-b)^(1/2)*(2*
x^2-b)^(1/2)-b)/x)
```

3.919.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(14) = 28$.

Time = 0.35 (sec) , antiderivative size = 84, normalized size of antiderivative = 4.20

$$\int \frac{\sqrt{2-\frac{b}{x^2}}}{-b+2x^2}\,dx = \left[-\frac{\sqrt{-b}\log\left(-\frac{x^2-\sqrt{-b}x\sqrt{\frac{2\,x^2-b}{x^2}}-b}{x^2}\right)}{2\,b}, -\frac{\arctan\left(\frac{\sqrt{b}x\sqrt{\frac{2\,x^2-b}{x^2}}}{2\,x^2-b}\right)}{\sqrt{b}}\right]$$

input
```
integrate((2-b/x^2)^(1/2)/(2*x^2-b),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-b)*log(-(x^2 - sqrt(-b)*x*sqrt((2*x^2 - b)/x^2) - b)/x^2)/b, -
arctan(sqrt(b)*x*sqrt((2*x^2 - b)/x^2)/(2*x^2 - b))/sqrt(b)]
```

3.919.6 Sympy [F]

$$\int \frac{\sqrt{2-\frac{b}{x^2}}}{-b+2x^2}\,dx = \int \frac{\sqrt{-\frac{b}{x^2}+2}}{-b+2x^2}\,dx$$

input
```
integrate((2-b/x**2)**(1/2)/(2*x**2-b),x)
```

output
```
Integral(sqrt(-b/x**2 + 2)/(-b + 2*x**2), x)
```

3.919.7 Maxima [F]

$$\int \frac{\sqrt{2-\frac{b}{x^2}}}{-b+2x^2}\,dx = \int \frac{\sqrt{-\frac{b}{x^2}+2}}{2\,x^2-b}\,dx$$

input
```
integrate((2-b/x^2)^(1/2)/(2*x^2-b),x, algorithm="maxima")
```

output
```
integrate(sqrt(-b/x^2 + 2)/(2*x^2 - b), x)
```

3.919.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(14) = 28$.

Time $= 0.32$ (sec) , antiderivative size $= 40$, normalized size of antiderivative $= 2.00$

$$\int \frac{\sqrt{2-\frac{b}{x^2}}}{-b+2x^2}\,dx = \frac{\arctan\left(\frac{\sqrt{2\,x^2-b}}{\sqrt{b}}\right)\operatorname{sgn}(x)}{\sqrt{b}} - \frac{\arctan\left(\frac{\sqrt{-b}}{\sqrt{b}}\right)\operatorname{sgn}(x)}{\sqrt{b}}$$

input
```
integrate((2-b/x^2)^(1/2)/(2*x^2-b),x, algorithm="giac")
```

output
```
arctan(sqrt(2*x^2 - b)/sqrt(b))*sgn(x)/sqrt(b) - arctan(sqrt(-b)/sqrt(b))*
sgn(x)/sqrt(b)
```

3.919.9 Mupad [B] (verification not implemented)

Time = 19.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{2-\frac{b}{x^2}}}{-b+2x^2}\,dx=-\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{-b}}{2\,x}\right)}{\sqrt{-b}}$$

input
```
int(-(2 - b/x^2)^(1/2)/(b - 2*x^2),x)
```

output
```
-asinh((2^(1/2)*(-b)^(1/2))/(2*x))/(-b)^(1/2)
```

3.920 $\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex}\,dx$

3.920.1 Optimal result

Integrand size = 19, antiderivative size = 121

$$\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex}\,dx=\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{c}{x^2}}}{\sqrt{a}}\right)}{e}-\frac{\sqrt{ad^2+ce^2}\text{arctanh}\left(\frac{ad-\frac{ce}{x}}{\sqrt{ad^2+ce^2}\sqrt{a+\frac{c}{x^2}}}\right)}{de}-\frac{\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}}{\sqrt{a+\frac{c}{x^2}}x}\right)}{d}$$

output
```
arctanh((a+c/x^2)^(1/2)/a^(1/2))*a^(1/2)/e-arctanh(c^(1/2)/x/(a+c/x^2)^(1/
2))*c^(1/2)/d-arctanh((a*d-c*e/x)/(a*d^2+c*e^2)^(1/2)/(a+c/x^2)^(1/2))*(a*
d^2+c*e^2)^(1/2)/d/e
```

3.920.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex}\,dx = -\frac{\sqrt{a+\frac{c}{x^2}}x\left(2\sqrt{-ad^2-ce^2}\arctan\left(\frac{\sqrt{a}(d+ex)-e\sqrt{c+ax^2}}{\sqrt{-ad^2-ce^2}}\right)-2\sqrt{c}e\text{arctanh}\left(\frac{\sqrt{a}x-\sqrt{c+ax^2}}{\sqrt{c}}\right)+\sqrt{a}d\log\left(-\sqrt{a}x\right.\right.}{de\sqrt{c+ax^2}}$$

input
```
Integrate[Sqrt[a + c/x^2]/(d + e*x),x]
```

output
```
-((Sqrt[a + c/x^2]*x*(2*Sqrt[-(a*d^2) - c*e^2]*ArcTan[(Sqrt[a]*(d + e*x) -
 e*Sqrt[c + a*x^2])/Sqrt[-(a*d^2) - c*e^2]] - 2*Sqrt[c]*e*ArcTanh[(Sqrt[a]
*x - Sqrt[c + a*x^2])/Sqrt[c]] + Sqrt[a]*d*Log[-(Sqrt[a]*x) + Sqrt[c + a*x
^2]]))/(d*e*Sqrt[c + a*x^2]))
```

3.920.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {1774, 1803, 606, 243, 73, 221, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex} \, dx$$

$$\Big\downarrow \text{ 1774}$$

$$\int \frac{\sqrt{a+\frac{c}{x^2}}}{x\left(\frac{d}{x}+e\right)}dx$$

$$\Big\downarrow \text{ 1803}$$

$$-\int \frac{\sqrt{a+\frac{c}{x^2}}x}{\frac{d}{x}+e}d\frac{1}{x}$$

$$\Big\downarrow \text{ 606}$$

$$\frac{\int \frac{ad-\frac{ce}{x}}{\sqrt{a+\frac{c}{x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}}{e}-\frac{a\int \frac{x}{\sqrt{a+\frac{c}{x^2}}}d\frac{1}{x}}{e}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\int \frac{ad-\frac{ce}{x}}{\sqrt{a+\frac{c}{x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}}{e}-\frac{a\int \frac{x}{\sqrt{a+\frac{c}{x^2}}}d\frac{1}{x^2}}{2e}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\int \frac{ad-\frac{ce}{x}}{\sqrt{a+\frac{c}{x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}}{e}-\frac{a\int \frac{1}{\frac{\sqrt{a+\frac{c}{x^2}}}{c}-\frac{a}{c}}d\sqrt{a+\frac{c}{x^2}}}{ce}$$

$$\Big\downarrow \; 221$$

$$\frac{\int \frac{ad-\frac{ce}{x}}{\sqrt{a+\frac{c}{x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}}{e}+\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{c}{x^2}}}{\sqrt{a}}\right)}{e}$$

$$\Big\downarrow \; 719$$

$$\frac{\frac{(ad^2+ce^2)\int \frac{1}{\sqrt{a+\frac{c}{x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}}{d}-\frac{ce\int \frac{1}{\sqrt{a+\frac{c}{x^2}}}d\frac{1}{x}}{d}}{e}+\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{c}{x^2}}}{\sqrt{a}}\right)}{e}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{(ad^2+ce^2)\int \frac{1}{\sqrt{a+\frac{c}{x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}}{d}-\frac{ce\int \frac{1}{1-\frac{c}{x^2}}d\frac{1}{\sqrt{a+\frac{c}{x^2}}x}}{d}}{e}+\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{c}{x^2}}}{\sqrt{a}}\right)}{e}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{(ad^2+ce^2)\int \frac{1}{\sqrt{a+\frac{c}{x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}}{d}-\frac{\sqrt{c}e\text{arctanh}\left(\frac{\sqrt{c}}{x\sqrt{a+\frac{c}{x^2}}}\right)}{d}}{e}+\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{c}{x^2}}}{\sqrt{a}}\right)}{e}$$

$$\Big\downarrow \; 488$$

$$\frac{-\frac{(ad^2+ce^2)\int \frac{1}{ad^2+ce^2-\frac{1}{x^2}}d\frac{ad-\frac{ce}{x}}{\sqrt{a+\frac{c}{x^2}}}}{d}-\frac{\sqrt{c}e\text{arctanh}\left(\frac{\sqrt{c}}{x\sqrt{a+\frac{c}{x^2}}}\right)}{d}}{e}+\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{c}{x^2}}}{\sqrt{a}}\right)}{e}$$

$$\Big\downarrow \; 219$$

$$\frac{-\frac{\sqrt{ad^2+ce^2}\text{arctanh}\left(\frac{ad-\frac{ce}{x}}{\sqrt{a+\frac{c}{x^2}}\sqrt{ad^2+ce^2}}\right)}{d}-\frac{\sqrt{c}e\text{arctanh}\left(\frac{\sqrt{c}}{x\sqrt{a+\frac{c}{x^2}}}\right)}{d}}{e}+\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+\frac{c}{x^2}}}{\sqrt{a}}\right)}{e}$$

input
```
Int[Sqrt[a + c/x^2]/(d + e*x),x]
```

output
```
(Sqrt[a]*ArcTanh[Sqrt[a + c/x^2]/Sqrt[a]])/e + (-((Sqrt[a*d^2 + c*e^2]*Arc
Tanh[(a*d - (c*e)/x)/(Sqrt[a*d^2 + c*e^2]*Sqrt[a + c/x^2])])/d) - (Sqrt[c]
*e*ArcTanh[Sqrt[c]/(Sqrt[a + c/x^2]*x)])/d)/e
```

3.920. $\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex}\,dx$

3.920.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 606
```
Int[(((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :
> Simp[a/c   Int[(c + d*x)^(n + 1)*((a + b*x^2)^(p - 1)/x), x], x] - Simp[1
/c   Int[(c + d*x)^n*(a*d - b*c*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && GtQ[p, 0] && ILtQ[n, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.920. $\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex} dx$

rule 1774
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

3.920.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(103) = 206$.

Time = 1.10 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.04

method	result
default	$-\frac{\sqrt{\frac{a x^2+c}{x^2}}\, x \left(\sqrt{c}\sqrt{\frac{a d^2+c e^2}{e^2}}\ln\left(\frac{2c+2\sqrt{c}\sqrt{a x^2+c}}{x}\right)e^2-\sqrt{a}\, d\ln\left(\frac{\sqrt{a x^2+c}\sqrt{a}+a x}{\sqrt{a}}\right)e\sqrt{\frac{a d^2+c e^2}{e^2}}-\ln\left(\frac{2\sqrt{a x^2+c}\sqrt{\frac{a d^2+c e^2}{e^2}}\, e-2adx+\cdots}{e x+d}\right.\right.}{\sqrt{a x^2+c}\, d e^2\sqrt{\frac{a d^2+c e^2}{e^2}}}$

input
```
int((a+c/x^2)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-((a*x^2+c)/x^2)^(1/2)*x*(c^(1/2)*((a*d^2+c*e^2)/e^2)^(1/2)*ln(2*(c^(1/2)*
(a*x^2+c)^(1/2)+c)/x)*e^2-a^(1/2)*d*ln(((a*x^2+c)^(1/2)*a^(1/2)+a*x)/a^(1/
2))*e*((a*d^2+c*e^2)/e^2)^(1/2)-ln(2*((a*x^2+c)^(1/2)*((a*d^2+c*e^2)/e^2)^
(1/2)*e-a*d*x+c*e)/(e*x+d))*a*d^2-ln(2*((a*x^2+c)^(1/2)*((a*d^2+c*e^2)/e^2
)^(1/2)*e-a*d*x+c*e)/(e*x+d))*c*e^2)/(a*x^2+c)^(1/2)/d/e^2/((a*d^2+c*e^2)/
e^2)^(1/2)
```

3.920.5 Fricas [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 1532, normalized size of antiderivative = 12.66

$$\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex}\,dx = \text{Too large to display}$$

input
```
integrate((a+c/x^2)^(1/2)/(e*x+d),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a)*d*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + c)/x^2) - c) +
sqrt(c)*e*log(-(a*x^2 - 2*sqrt(c)*x*sqrt((a*x^2 + c)/x^2) + 2*c)/x^2) + sq
rt(a*d^2 + c*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*c^2*e^2 - (2*a^2*d^2 + a*
c*e^2)*x^2 + 2*(a*d*x^2 - c*e*x)*sqrt(a*d^2 + c*e^2)*sqrt((a*x^2 + c)/x^2)
)/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e), -1/2*(2*sqrt(-a)*d*arctan(sqrt(-a)*x^
2*sqrt((a*x^2 + c)/x^2)/(a*x^2 + c)) - sqrt(c)*e*log(-(a*x^2 - 2*sqrt(c)*x
*sqrt((a*x^2 + c)/x^2) + 2*c)/x^2) - sqrt(a*d^2 + c*e^2)*log((2*a*c*d*e*x
- a*c*d^2 - 2*c^2*e^2 - (2*a^2*d^2 + a*c*e^2)*x^2 + 2*(a*d*x^2 - c*e*x)*sq
rt(a*d^2 + c*e^2)*sqrt((a*x^2 + c)/x^2))/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e)
, 1/2*(sqrt(a)*d*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + c)/x^2) - c) +
 sqrt(c)*e*log(-(a*x^2 - 2*sqrt(c)*x*sqrt((a*x^2 + c)/x^2) + 2*c)/x^2) + 2
*sqrt(-a*d^2 - c*e^2)*arctan((a*d*x^2 - c*e*x)*sqrt(-a*d^2 - c*e^2)*sqrt((
a*x^2 + c)/x^2)/(a*c*d^2 + c^2*e^2 + (a^2*d^2 + a*c*e^2)*x^2)))/(d*e), -1/
2*(2*sqrt(-a)*d*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + c)/x^2)/(a*x^2 + c)) - s
qrt(c)*e*log(-(a*x^2 - 2*sqrt(c)*x*sqrt((a*x^2 + c)/x^2) + 2*c)/x^2) - 2*s
qrt(-a*d^2 - c*e^2)*arctan((a*d*x^2 - c*e*x)*sqrt(-a*d^2 - c*e^2)*sqrt((a*
x^2 + c)/x^2)/(a*c*d^2 + c^2*e^2 + (a^2*d^2 + a*c*e^2)*x^2)))/(d*e), 1/2*(
2*sqrt(-c)*e*arctan(sqrt(-c)*x*sqrt((a*x^2 + c)/x^2)/(a*x^2 + c)) + sqrt(a
)*d*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + c)/x^2) - c) + sqrt(a*d^2 +
 c*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*c^2*e^2 - (2*a^2*d^2 + a*c*e^2)*...
```

3.920.6 Sympy [F]

$$\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex}\,dx = \int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex}\,dx$$

input
```
integrate((a+c/x**2)**(1/2)/(e*x+d),x)
```

output
```
Integral(sqrt(a + c/x**2)/(d + e*x), x)
```

3.920.7 Maxima [F]

$$\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex}\,dx = \int \frac{\sqrt{a+\frac{c}{x^2}}}{ex+d}\,dx$$

input
```
integrate((a+c/x^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + c/x^2)/(e*x + d), x)
```

3.920.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+c/x^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.920.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex}\,dx = \int \frac{\sqrt{a+\frac{c}{x^2}}}{d+e\,x}\,dx$$

input
```
int((a + c/x^2)^(1/2)/(d + e*x),x)
```

output
```
int((a + c/x^2)^(1/2)/(d + e*x), x)
```

3.921 $\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex}\,dx$

3.921.1 Optimal result

Integrand size = 24, antiderivative size = 181

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex}\,dx=\frac{\sqrt{a}\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{e}-\frac{\sqrt{c}\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{d}-\frac{\sqrt{ad^2-e(bd-ce)}\text{arctanh}\left(\frac{2ad-be+\frac{bd-2ce}{x}}{2\sqrt{ad^2-e(bd-ce)}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{de}$$

output
```
arctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/2))*a^(1/2)/e-arctanh(1/2*(
b+2*c/x)/c^(1/2)/(a+c/x^2+b/x)^(1/2))*c^(1/2)/d-arctanh(1/2*(2*a*d-b*e+(b*
d-2*c*e)/x)/(a*d^2-e*(b*d-c*e))^(1/2)/(a+c/x^2+b/x)^(1/2))*(a*d^2-e*(b*d-c
*e))^(1/2)/d/e
```

3.921.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex}\,dx=\frac{x\sqrt{a+\frac{c+bx}{x^2}}\left(-2\sqrt{-ad^2+bde-ce^2}\arctan\left(\frac{\sqrt{a}(d+ex)-e\sqrt{c+x(b+ax)}}{\sqrt{-ad^2+bde-ce^2}}\right)+2\sqrt{c}e\text{arctanh}\left(\frac{\sqrt{a}x-\sqrt{c+x(b+ax)}}{\sqrt{c}}\right)-\sqrt{}\right.}{de\sqrt{c+x(b+ax)}}$$

input
```
Integrate[Sqrt[a + c/x^2 + b/x]/(d + e*x),x]
```

output
```
(x*Sqrt[a + (c + b*x)/x^2]*(-2*Sqrt[-(a*d^2) + b*d*e - c*e^2]*ArcTan[(Sqrt
[a]*(d + e*x) - e*Sqrt[c + x*(b + a*x)])/Sqrt[-(a*d^2) + b*d*e - c*e^2]] +
 2*Sqrt[c]*e*ArcTanh[(Sqrt[a]*x - Sqrt[c + x*(b + a*x)])/Sqrt[c]] - Sqrt[a
]*d*Log[e*(b + 2*a*x - 2*Sqrt[a]*Sqrt[c + x*(b + a*x)])]))/(d*e*Sqrt[c + x
*(b + a*x)])
```

3.921.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1773, 1802, 1270, 1154, 219, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{d+ex}dx$$

$$\Big\downarrow \text{1773}$$

$$\int \frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x\left(\frac{d}{x}+e\right)}dx$$

$$\Big\downarrow \text{1802}$$

$$-\int \frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x}{\frac{d}{x}+e}d\frac{1}{x}$$

$$\Big\downarrow \text{1270}$$

$$\frac{\int \frac{ad-be-\frac{ce}{x}}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}}{e}-\frac{a\int \frac{x}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}}{e}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\int \frac{ad-be-\frac{ce}{x}}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}}{e}+\frac{2a\int \frac{1}{4a-\frac{1}{x^2}}d\frac{2a+\frac{b}{x}}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}}{e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\int \frac{ad-be-\frac{ce}{x}}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}}{e}+\frac{\sqrt{a}\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{e}$$

$\Big\downarrow$ 1269

$$\frac{\frac{\left(ad^2-e(bd-ce)\right)\int \frac{1}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}}{d}-\frac{ce\int \frac{1}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}d\frac{1}{x}}{d}}{e}+\frac{\sqrt{a}\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{e}$$

$\Big\downarrow$ 1092

$$\frac{\frac{\left(ad^2-e(bd-ce)\right)\int \frac{1}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}}{d}-\frac{2ce\int \frac{1}{4c-\frac{1}{x^2}}d\frac{b+\frac{2c}{x}}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}}{d}}{e}+\frac{\sqrt{a}\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{e}$$

$\Big\downarrow$ 219

$$\frac{\frac{\left(ad^2-e(bd-ce)\right)\int \frac{1}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}}{d}-\frac{\sqrt{c}e\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{d}}{e}+\frac{\sqrt{a}\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{e}$$

$\Big\downarrow$ 1154

$$\frac{-\frac{2\left(ad^2-e(bd-ce)\right)\int \frac{1}{4\left(ad^2-e(bd-ce)\right)-\frac{1}{x^2}}d\frac{2ad-be+\frac{bd-2ce}{x}}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}}{d}-\frac{\sqrt{c}e\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{d}}{e}+\frac{\sqrt{a}\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{e}$$

$\Big\downarrow$ 219

$$\frac{-\frac{\sqrt{ad^2-e(bd-ce)}\text{arctanh}\left(\frac{2ad+\frac{bd-2ce}{x}-be}{2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{ad^2-e(bd-ce)}}\right)}{d}-\frac{\sqrt{c}e\text{arctanh}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{d}}{e}+\frac{\sqrt{a}\text{arctanh}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{e}$$

input
```
Int[Sqrt[a + c/x^2 + b/x]/(d + e*x),x]
```

3.921. $\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex}\,dx$

output
```
(Sqrt[a]*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/e + (-((S
qrt[c]*e*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c/x^2 + b/x])])/d) - (S
qrt[a*d^2 - e*(b*d - c*e)]*ArcTanh[(2*a*d - b*e + (b*d - 2*c*e)/x)/(2*Sqrt
[a*d^2 - e*(b*d - c*e)]*Sqrt[a + c/x^2 + b/x])])/d)/e
```

3.921.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1270
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Simp[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g))
Int[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Simp[1/(e*(e*f - d*g))
Int[Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*((a + b*x + c*x^2)^(p
- 1)/(f + g*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[p]
&& GtQ[p, 0]
```

rule 1773
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)
^(n2_.))^(p_.), x_Symbol] :> Int[((e + d*x^n)^q*(a + b*x^n + c*x^(2*n))^p)/
x^(n*q), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[n2, 2*n] && EqQ[mn, -
n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

3.921. $\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex}\,dx$

rule 1802
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1
)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b,
c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

3.921.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(157) = 314$.

Time = 1.06 (sec) , antiderivative size = 397, normalized size of antiderivative = 2.19

method	result
default	$-\frac{\sqrt{\frac{a x^2+bx+c}{x^2}}\, x \left(\sqrt{\frac{a d^2-bde+c e^2}{e^2}}\, \sqrt{a}\, \sqrt{c}\, \ln\left(\frac{2c+bx+2\sqrt{c}\,\sqrt{a x^2+bx+c}}{x}\right) e^2-\sqrt{\frac{a d^2-bde+c e^2}{e^2}}\, \ln\left(\frac{2\sqrt{a x^2+bx+c}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right) ade-a^{\frac{3}{2}}\, \mathrm{l}\right.}{}$

input
```
int((a+c/x^2+b/x)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-((a*x^2+b*x+c)/x^2)^(1/2)*x*(((a*d^2-b*d*e+c*e^2)/e^2)^(1/2)*a^(1/2)*c^(1
/2)*ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*x+c)^(1/2))/x)*e^2-((a*d^2-b*d*e+c*e^2)
/e^2)^(1/2)*ln(1/2*(2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*d*e-
a^(3/2)*ln((2*(a*x^2+b*x+c)^(1/2)*((a*d^2-b*d*e+c*e^2)/e^2)^(1/2)*e-2*a*d*
x+b*e*x-b*d+2*c*e)/(e*x+d))*d^2+a^(1/2)*ln((2*(a*x^2+b*x+c)^(1/2)*((a*d^2-
b*d*e+c*e^2)/e^2)^(1/2)*e-2*a*d*x+b*e*x-b*d+2*c*e)/(e*x+d))*b*d*e-a^(1/2)*
ln((2*(a*x^2+b*x+c)^(1/2)*((a*d^2-b*d*e+c*e^2)/e^2)^(1/2)*e-2*a*d*x+b*e*x-
b*d+2*c*e)/(e*x+d))*c*e^2)/(a*x^2+b*x+c)^(1/2)/d/e^2/a^(1/2)/((a*d^2-b*d*e
+c*e^2)/e^2)^(1/2)
```

3.921.5 Fricas [A] (verification not implemented)

Time = 43.57 (sec) , antiderivative size = 2411, normalized size of antiderivative = 13.32

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex}\, dx = \text{Too large to display}$$

input
```
integrate((a+c/x^2+b/x)^(1/2)/(e*x+d),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a)*d*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)
*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) + sqrt(c)*e*log(-(8*b*c*x + (b^2 + 4
*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))
/x^2) + sqrt(a*d^2 - b*d*e + c*e^2)*log((8*b*c*d*e - 8*c^2*e^2 - (b^2 + 4*
a*c)*d^2 - (8*a^2*d^2 - 8*a*b*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 2*(4*a*b*d^2
+ 4*b*c*e^2 - (3*b^2 + 4*a*c)*d*e)*x + 4*sqrt(a*d^2 - b*d*e + c*e^2)*((2*a
*d - b*e)*x^2 + (b*d - 2*c*e)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(e^2*x^2 + 2
*d*e*x + d^2)))/(d*e), -1/2*(2*sqrt(-a)*d*arctan(1/2*(2*a*x^2 + b*x)*sqrt(
-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) - sqrt(c)*e*log(-
(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt((a*x
^2 + b*x + c)/x^2))/x^2) - sqrt(a*d^2 - b*d*e + c*e^2)*log((8*b*c*d*e - 8*
c^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*a^2*d^2 - 8*a*b*d*e + (b^2 + 4*a*c)*e^2)*
x^2 - 2*(4*a*b*d^2 + 4*b*c*e^2 - (3*b^2 + 4*a*c)*d*e)*x + 4*sqrt(a*d^2 - b
*d*e + c*e^2)*((2*a*d - b*e)*x^2 + (b*d - 2*c*e)*x)*sqrt((a*x^2 + b*x + c)
/x^2))/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e), 1/2*(sqrt(a)*d*log(-8*a^2*x^2 -
8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x
^2)) + sqrt(c)*e*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*
c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2) - 2*sqrt(-a*d^2 + b*d*e - c
*e^2)*arctan(-1/2*sqrt(-a*d^2 + b*d*e - c*e^2)*((2*a*d - b*e)*x^2 + (b*d -
 2*c*e)*x)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*d^2 - b*c*d*e + c^2*e^2 + (...
```

3.921.6 Sympy [F]

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex}\,dx = \int \frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{d+ex}\,dx$$

input
```
integrate((a+c/x**2+b/x)**(1/2)/(e*x+d),x)
```

output
```
Integral(sqrt(a + b/x + c/x**2)/(d + e*x), x)
```

3.921.7 Maxima [F]

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex}\,dx = \int \frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{ex+d}\,dx$$

input
```
integrate((a+c/x^2+b/x)^(1/2)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/x + c/x^2)/(e*x + d), x)
```

3.921.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+c/x^2+b/x)^(1/2)/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.921.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex}\,dx = \int \frac{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{d+e\,x}\,dx$$

input
```
int((a + b/x + c/x^2)^(1/2)/(d + e*x),x)
```

output
```
int((a + b/x + c/x^2)^(1/2)/(d + e*x), x)
```

3.922 $\int \frac{\sqrt[6]{x}+\sqrt[5]{x^3}}{\sqrt{x}}\,dx$

3.922.1 Optimal result

Integrand size = 19, antiderivative size = 26

$$\int \frac{\sqrt[6]{x}+\sqrt[5]{x^3}}{\sqrt{x}}\,dx=\frac{3x^{2/3}}{2}+\frac{10}{11}\sqrt{x}\sqrt[5]{x^3}$$

output
```
3/2*x^(2/3)+10/11*(x^3)^(1/5)*x^(1/2)
```

3.922.2 Mathematica [A] (verified)

Time = 6.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[6]{x}+\sqrt[5]{x^3}}{\sqrt{x}}\,dx=\frac{3x^{2/3}}{2}+\frac{10}{11}\sqrt{x}\sqrt[5]{x^3}$$

input
```
Integrate[(x^(1/6) + (x^3)^(1/5))/Sqrt[x],x]
```

output
```
(3*x^(2/3))/2 + (10*Sqrt[x]*(x^3)^(1/5))/11
```

3.922.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[5]{x^3}+\sqrt[6]{x}}{\sqrt{x}} \, dx$$

$$\Big\downarrow \text{2010}$$

$$\int \left(\frac{\sqrt[5]{x^3}}{\sqrt{x}}+\frac{1}{\sqrt[3]{x}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3x^{2/3}}{2}+\frac{10}{11}\sqrt[5]{x^3}\sqrt{x}$$

input
```
Int[(x^(1/6) + (x^3)^(1/5))/Sqrt[x],x]
```

output
```
(3*x^(2/3))/2 + (10*Sqrt[x]*(x^3)^(1/5))/11
```

3.922.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

3.922.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{3x^{\frac{2}{3}}}{2}+\frac{10\left(x^{3}\right)^{\frac{1}{5}}\sqrt{x}}{11}$	17

input
```
int((x^(1/6)+(x^3)^(1/5))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/2*x^(2/3)+10/11*(x^3)^(1/5)*x^(1/2)
```

3.922.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt[6]{x}+\sqrt[5]{x^3}}{\sqrt{x}}\,dx=\frac{10}{11}\left(x^{3}\right)^{\frac{1}{5}}\sqrt{x}+\frac{3}{2}\,x^{\frac{2}{3}}$$

input
```
integrate((x^(1/6)+(x^3)^(1/5))/x^(1/2),x, algorithm="fricas")
```

output
```
10/11*(x^3)^(1/5)*sqrt(x) + 3/2*x^(2/3)
```

3.922.6 Sympy [A] (verification not implemented)

Time = 63.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt[6]{x}+\sqrt[5]{x^3}}{\sqrt{x}}\,dx=\frac{3x^{\frac{2}{3}}}{2}+\frac{10\sqrt{x}\sqrt[5]{x^{3}}}{11}$$

input
```
integrate((x**(1/6)+(x**3)**(1/5))/x**(1/2),x)
```

output
```
3*x**(2/3)/2 + 10*sqrt(x)*(x**3)**(1/5)/11
```

3.922.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt[6]{x}+\sqrt[5]{x^3}}{\sqrt{x}}\,dx = \frac{10}{11}\left(x^3\right)^{\frac{1}{5}}\sqrt{x}+\frac{3}{2}\,x^{\frac{2}{3}}$$

input
```
integrate((x^(1/6)+(x^3)^(1/5))/x^(1/2),x, algorithm="maxima")
```

output
```
10/11*(x^3)^(1/5)*sqrt(x) + 3/2*x^(2/3)
```

3.922.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt[6]{x}+\sqrt[5]{x^3}}{\sqrt{x}}\,dx = \frac{10}{11}\,x^{\frac{11}{10}}+\frac{3}{2}\,x^{\frac{2}{3}}$$

input
```
integrate((x^(1/6)+(x^3)^(1/5))/x^(1/2),x, algorithm="giac")
```

output
```
10/11*x^(11/10) + 3/2*x^(2/3)
```

3.922.9 Mupad [B] (verification not implemented)

Time = 19.99 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt[6]{x}+\sqrt[5]{x^3}}{\sqrt{x}}\,dx = \frac{10\,\sqrt{x}\,\left(x^3\right)^{1/5}}{11}+\frac{3\,x^{2/3}}{2}$$

input
```
int(((x^3)^(1/5) + x^(1/6))/x^(1/2),x)
```

output
```
(10*x^(1/2)*(x^3)^(1/5))/11 + (3*x^(2/3))/2
```

3.923 $\int \frac{2+x}{\sqrt{4x-x^2}}\,dx$

3.923.1 Optimal result

Integrand size = 17, antiderivative size = 26

$$\int \frac{2+x}{\sqrt{4x-x^2}}\,dx=-\sqrt{4x-x^2}-4\arcsin\left(1-\frac{x}{2}\right)$$

output
```
4*arcsin(-1+1/2*x)-(-x^2+4*x)^(1/2)
```

3.923.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.81

$$\int \frac{2+x}{\sqrt{4x-x^2}}\,dx=\frac{(-4+x)x-8\sqrt{-4+x}\sqrt{x}\log\left(\sqrt{-4+x}-\sqrt{x}\right)}{\sqrt{-((-4+x)x)}}$$

input
```
Integrate[(2 + x)/Sqrt[4*x - x^2],x]
```

output
```
((-4 + x)*x - 8*Sqrt[-4 + x]*Sqrt[x]*Log[Sqrt[-4 + x] - Sqrt[x]])/Sqrt[-((
-4 + x)*x)]
```

3.923.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1160, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+2}{\sqrt{4x-x^2}}dx$$

$$\Big\downarrow \text{ 1160}$$

$$4\int \frac{1}{\sqrt{4x-x^2}}dx-\sqrt{4x-x^2}$$

$$\Big\downarrow \text{ 1090}$$

$$-\int \frac{1}{\sqrt{1-\frac{1}{16}(4-2x)^2}}d(4-2x)-\sqrt{4x-x^2}$$

$$\Big\downarrow \text{ 223}$$

$$-4\arcsin\left(\frac{1}{4}(4-2x)\right)-\sqrt{4x-x^2}$$

input
```
Int[(2 + x)/Sqrt[4*x - x^2],x]
```

output
```
-Sqrt[4*x - x^2] - 4*ArcSin[(4 - 2*x)/4]
```

3.923.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

3.923.4 Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result
default	$4 \arcsin\left(-1+\frac{x}{2}\right)-\sqrt{-x^{2}+4x}$
risch	$\frac{x(x-4)}{\sqrt{-x(x-4)}}+4 \arcsin\left(-1+\frac{x}{2}\right)$
pseudoelliptic	$-\sqrt{-x\left(x-4\right)}-8 \arctan\left(\frac{\sqrt{-x(x-4)}}{x}\right)$
meijerg	$4 \arcsin\left(\frac{\sqrt{x}}{2}\right)+\frac{4i\left(\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{-\frac{x}{4}+1}}{2}-i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{x}}{2}\right)\right)}{\sqrt{\pi}}$
trager	$-\sqrt{-x^{2}+4x}-4\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^{2}+1\right)x-2\,\mathrm{RootOf}\left(\_Z^{2}+1\right)+\sqrt{-}\right.$

input
```
int((x+2)/(-x^2+4*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4*arcsin(-1+1/2*x)-(-x^2+4*x)^(1/2)
```

3.923.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \frac{2+x}{\sqrt{4x-x^2}}\,dx = -\sqrt{-x^{2}+4\,x}-8\,\arctan\left(\frac{\sqrt{-x^{2}+4\,x}}{x}\right)$$

input
```
integrate((2+x)/(-x^2+4*x)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-x^2 + 4*x) - 8*arctan(sqrt(-x^2 + 4*x)/x)
```

3.923.6 Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{2+x}{\sqrt{4x-x^2}}\,dx = -\sqrt{-x^2+4x} + 4\,\mathrm{asin}\left(\frac{x}{2}-1\right)$$

input
```
integrate((2+x)/(-x**2+4*x)**(1/2),x)
```

output
```
-sqrt(-x**2 + 4*x) + 4*asin(x/2 - 1)
```

3.923.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{2+x}{\sqrt{4x-x^2}}\,dx = -\sqrt{-x^2+4\,x} - 4\,\arcsin\left(-\frac{1}{2}\,x+1\right)$$

input
```
integrate((2+x)/(-x^2+4*x)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(-x^2 + 4*x) - 4*arcsin(-1/2*x + 1)
```

3.923.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{2+x}{\sqrt{4x-x^2}}\,dx = -\sqrt{-x^2+4\,x} + 4\,\arcsin\left(\frac{1}{2}\,x-1\right)$$

input
```
integrate((2+x)/(-x^2+4*x)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(-x^2 + 4*x) + 4*arcsin(1/2*x - 1)
```

3.923.9 Mupad [B] (verification not implemented)

Time = 19.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{2+x}{\sqrt{4x-x^2}}\,dx = 4\,\mathrm{asin}\left(\frac{x}{2}-1\right)-\sqrt{4\,x-x^2}$$

input
```
int((x + 2)/(4*x - x^2)^(1/2),x)
```

output
```
4*asin(x/2 - 1) - (4*x - x^2)^(1/2)
```

3.924 $\int \frac{3+x}{\sqrt[3]{6x+x^2}} \, dx$

3.924.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{3+x}{\sqrt[3]{6x+x^2}} \, dx = \frac{3}{4}\left(6x+x^2\right)^{2/3}$$

output
```
3/4*(x^2+6*x)^(2/3)
```

3.924.2 Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{3+x}{\sqrt[3]{6x+x^2}} \, dx = \frac{3}{4}(x(6+x))^{2/3}$$

input
```
Integrate[(3 + x)/(6*x + x^2)^(1/3),x]
```

output
```
(3*(x*(6 + x))^(2/3))/4
```

3.924.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+3}{\sqrt[3]{x^2+6x}} \, dx$$

$$\Big\downarrow \; 1104$$

$$\frac{3}{4}\left(x^2+6x\right)^{2/3}$$

input
```
Int[(3 + x)/(6*x + x^2)^(1/3),x]
```

output
```
(3*(6*x + x^2)^(2/3))/4
```

3.924.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

3.924.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$\frac{3(x(6+x))^{\frac{2}{3}}}{4}$	10
default	$\frac{3(x^2+6x)^{\frac{2}{3}}}{4}$	12
trager	$\frac{3(x^2+6x)^{\frac{2}{3}}}{4}$	12
risch	$\frac{3x(6+x)}{4(x(6+x))^{\frac{1}{3}}}$	14
gosper	$\frac{3x(6+x)}{4(x^2+6x)^{\frac{1}{3}}}$	16
meijerg	$\frac{3\,9^{\frac{1}{3}}2^{\frac{2}{3}}x^{\frac{2}{3}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{x}{6}\right)}{4}+\frac{9^{\frac{1}{3}}2^{\frac{2}{3}}x^{\frac{5}{3}}\,{}_2F_1\left(\frac{1}{3},\frac{5}{3};\frac{8}{3};-\frac{x}{6}\right)}{10}$	42

input
```
int((3+x)/(x^2+6*x)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/4*(x*(6+x))^(2/3)
```

3.924.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{3+x}{\sqrt[3]{6x+x^2}}\,dx = \frac{3}{4}\left(x^2+6\,x\right)^{\frac{2}{3}}$$

input
```
integrate((3+x)/(x^2+6*x)^(1/3),x, algorithm="fricas")
```

output
```
3/4*(x^2 + 6*x)^(2/3)
```

3.924.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{3+x}{\sqrt[3]{6x+x^2}}\,dx = \frac{3(x^2+6x)^{\frac{2}{3}}}{4}$$

input
```
integrate((3+x)/(x**2+6*x)**(1/3),x)
```

output
```
3*(x**2 + 6*x)**(2/3)/4
```

3.924.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{3+x}{\sqrt[3]{6x+x^2}}\,dx = \frac{3}{4}\,(x^2+6\,x)^{\frac{2}{3}}$$

input
```
integrate((3+x)/(x^2+6*x)^(1/3),x, algorithm="maxima")
```

output
```
3/4*(x^2 + 6*x)^(2/3)
```

3.924.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{3+x}{\sqrt[3]{6x+x^2}}\,dx = \frac{3}{4}\,(x^2+6\,x)^{\frac{2}{3}}$$

input
```
integrate((3+x)/(x^2+6*x)^(1/3),x, algorithm="giac")
```

output
```
3/4*(x^2 + 6*x)^(2/3)
```

3.924.9 Mupad [B] (verification not implemented)

Time = 19.90 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int \frac{3+x}{\sqrt[3]{6x+x^2}}\,dx = \frac{3\,(x\,(x+6))^{2/3}}{4}$$

input
```
int((x + 3)/(6*x + x^2)^(1/3),x)
```

output
```
(3*(x*(x + 6))^(2/3))/4
```

3.925 $\int \frac{4+x}{(6x-x^2)^{3/2}}\,dx$

3.925.1 Optimal result

Integrand size = 17, antiderivative size = 22

$$\int \frac{4+x}{(6x-x^2)^{3/2}}\,dx = -\frac{12-7x}{9\sqrt{6x-x^2}}$$

output
```
1/9*(-12+7*x)/(-x^2+6*x)^(1/2)
```

3.925.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{4+x}{(6x-x^2)^{3/2}}\,dx = \frac{-12+7x}{9\sqrt{-((-6+x)x)}}$$

input
```
Integrate[(4 + x)/(6*x - x^2)^(3/2),x]
```

output
```
(-12 + 7*x)/(9*Sqrt[-((-6 + x)*x)])
```

3.925.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+4}{(6x-x^2)^{3/2}} \, dx$$

$$\Big\downarrow \; 1158$$

$$-\frac{12-7x}{9\sqrt{6x-x^2}}$$

input
```
Int[(4 + x)/(6*x - x^2)^(3/2),x]
```

output
```
-1/9*(12 - 7*x)/Sqrt[6*x - x^2]
```

3.925.3.1 Defintions of rubi rules used

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

3.925.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

method	result	size
risch	$\frac{-12+7x}{9\sqrt{-x(-6+x)}}$	16
pseudoelliptic	$\frac{-12+7x}{9\sqrt{-x(-6+x)}}$	16
gosper	$-\frac{x(-6+x)(-12+7x)}{9(-x^2+6x)^{\frac{3}{2}}}$	23
trager	$-\frac{(-12+7x)\sqrt{-x^2+6x}}{9x(-6+x)}$	27
default	$\frac{1}{\sqrt{-x^2+6x}}-\frac{7(-2x+6)}{18\sqrt{-x^2+6x}}$	31
meijerg	$-\frac{2\sqrt{3}\,\sqrt{2}\,(1-\frac{x}{3})}{9\sqrt{x}\,\sqrt{1-\frac{x}{6}}}+\frac{\sqrt{x}\,\sqrt{6}}{18\sqrt{1-\frac{x}{6}}}$	40

input
```
int((x+4)/(-x^2+6*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*(-12+7*x)/(-x*(-6+x))^(1/2)
```

3.925.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{4+x}{(6x-x^2)^{3/2}}\,dx = -\frac{\sqrt{-x^2+6\,x}(7\,x-12)}{9\,(x^2-6\,x)}$$

input
```
integrate((4+x)/(-x^2+6*x)^(3/2),x, algorithm="fricas")
```

output
```
-1/9*sqrt(-x^2 + 6*x)*(7*x - 12)/(x^2 - 6*x)
```

3.925.6 Sympy [F]

$$\int \frac{4+x}{(6x-x^2)^{3/2}}\,dx = \int \frac{x+4}{\left(-x\,(x-6)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((4+x)/(-x**2+6*x)**(3/2),x)
```

output
```
Integral((x + 4)/(-x*(x - 6))**(3/2), x)
```

3.925.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{4+x}{(6x-x^2)^{3/2}}\,dx = \frac{7\,x}{9\,\sqrt{-x^2+6\,x}} - \frac{4}{3\,\sqrt{-x^2+6\,x}}$$

input
```
integrate((4+x)/(-x^2+6*x)^(3/2),x, algorithm="maxima")
```

output
```
7/9*x/sqrt(-x^2 + 6*x) - 4/3/sqrt(-x^2 + 6*x)
```

3.925.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{4+x}{(6x-x^2)^{3/2}}\,dx = -\frac{\sqrt{-x^2+6\,x}(7\,x-12)}{9\,(x^2-6\,x)}$$

input
```
integrate((4+x)/(-x^2+6*x)^(3/2),x, algorithm="giac")
```

output
```
-1/9*sqrt(-x^2 + 6*x)*(7*x - 12)/(x^2 - 6*x)
```

3.925.9 Mupad [B] (verification not implemented)

Time = 19.85 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{4+x}{(6x-x^2)^{3/2}}\,dx = \frac{7\,x-12}{9\,\sqrt{6\,x-x^2}}$$

input
```
int((x + 4)/(6*x - x^2)^(3/2),x)
```

output
```
(7*x - 12)/(9*(6*x - x^2)^(1/2))
```

3.926 $\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx$

3.926.1 Optimal result

Integrand size = 17, antiderivative size = 12

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = \arctan\left(\sqrt{2x+x^2}\right)$$

output
```
arctan((x^2+2*x)^(1/2))
```

3.926.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 41 vs. 2(12) = 24.

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.42

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = -\frac{2\sqrt{x}\sqrt{2+x}\arctan\left(1+x-\sqrt{x}\sqrt{2+x}\right)}{\sqrt{x(2+x)}}$$

input
```
Integrate[1/((1 + x)*Sqrt[2*x + x^2]),x]
```

output
```
(-2*Sqrt[x]*Sqrt[2 + x]*ArcTan[1 + x - Sqrt[x]*Sqrt[2 + x]])/Sqrt[x*(2 + x
)]
```

3.926.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1112, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+1)\sqrt{x^2+2x}}dx$$

$$\Big\downarrow \text{1112}$$

$$4\int \frac{1}{4(x^2+2x)+4}d\sqrt{x^2+2x}$$

$$\Big\downarrow \text{216}$$

$$\arctan\left(\sqrt{x^2+2x}\right)$$

input
```
Int[1/((1 + x)*Sqrt[2*x + x^2]),x]
```

output
```
ArcTan[Sqrt[2*x + x^2]]
```

3.926.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1112
```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Simp[4*c   Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a
+ b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

3.926.4 Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
pseudoelliptic	$\arctan\left(\sqrt{x\,(x+2)}\right)$	9
default	$-\arctan\left(\frac{1}{\sqrt{(x+1)^{2}-1}}\right)$	13
trager	$\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)+\sqrt{x^{2}+2x}}{x+1}\right)$	31

input
```
int(1/(x+1)/(x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan((x*(x+2))^(1/2))
```

3.926.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = 2\,\arctan\left(-x+\sqrt{x^{2}+2\,x}-1\right)$$

input
```
integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="fricas")
```

output
```
2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

3.926.6 Sympy [F]

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = \int \frac{1}{\sqrt{x\,(x+2)}\,(x+1)}\,dx$$

input
```
integrate(1/(1+x)/(x**2+2*x)**(1/2),x)
```

output
```
Integral(1/(sqrt(x*(x + 2))*(x + 1)), x)
```

3.926.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = -\arcsin\left(\frac{1}{|x+1|}\right)$$

input
```
integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(1/abs(x + 1))
```

3.926.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = 2\arctan\left(-x+\sqrt{x^2+2x}-1\right)$$

input
```
integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

3.926.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+2x}\,(x+1)}\,dx$$

input
```
int(1/((2*x + x^2)^(1/2)*(x + 1)),x)
```

output
```
int(1/((2*x + x^2)^(1/2)*(x + 1)), x)
```

3.927 $\int \frac{1}{(1+2x)\sqrt{x+x^2}} dx$

3.927.1 Optimal result

Integrand size = 17, antiderivative size = 12

$$\int \frac{1}{(1+2x)\sqrt{x+x^2}} \, dx = \arctan\left(2\sqrt{x+x^2}\right)$$

output
```
arctan(2*(x^2+x)^(1/2))
```

3.927.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 43 vs. 2(12) = 24.

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.58

$$\int \frac{1}{(1+2x)\sqrt{x+x^2}} \, dx = -\frac{2\sqrt{x}\sqrt{1+x}\arctan\left(1+2x-2\sqrt{x}\sqrt{1+x}\right)}{\sqrt{x(1+x)}}$$

input
```
Integrate[1/((1 + 2*x)*Sqrt[x + x^2]),x]
```

output
```
(-2*Sqrt[x]*Sqrt[1 + x]*ArcTan[1 + 2*x - 2*Sqrt[x]*Sqrt[1 + x]])/Sqrt[x*(1
 + x)]
```

3.927.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1112, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x+1)\sqrt{x^2+x}}dx$$

$$\Big\downarrow \text{1112}$$

$$4\int \frac{1}{8(x^2+x)+2}d\sqrt{x^2+x}$$

$$\Big\downarrow \text{216}$$

$$\arctan\left(2\sqrt{x^2+x}\right)$$

input
```
Int[1/((1 + 2*x)*Sqrt[x + x^2]),x]
```

output
```
ArcTan[2*Sqrt[x + x^2]]
```

3.927.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1112
```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Simp[4*c   Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a
+ b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

3.927.4 Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
pseudoelliptic	$\arctan\left(2\sqrt{(x+1)\,x}\right)$	11
default	$-\arctan\left(\frac{1}{\sqrt{4\left(x+\frac{1}{2}\right)^{2}-1}}\right)$	15
trager	$\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)+2\sqrt{x^{2}+x}}{1+2x}\right)$	33

input
```
int(1/(1+2*x)/(x^2+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan(2*((x+1)*x)^(1/2))
```

3.927.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{(1+2x)\sqrt{x+x^2}}\,dx = 2\,\arctan\left(-2\,x+2\,\sqrt{x^2+x}-1\right)$$

input
```
integrate(1/(1+2*x)/(x^2+x)^(1/2),x, algorithm="fricas")
```

output
```
2*arctan(-2*x + 2*sqrt(x^2 + x) - 1)
```

3.927.6 Sympy [F]

$$\int \frac{1}{(1+2x)\sqrt{x+x^2}}\,dx = \int \frac{1}{\sqrt{x\,(x+1)}\,(2x+1)}\,dx$$

input
```
integrate(1/(1+2*x)/(x**2+x)**(1/2),x)
```

output
```
Integral(1/(sqrt(x*(x + 1))*(2*x + 1)), x)
```

3.927.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1}{(1+2x)\sqrt{x+x^2}}\,dx = -\arcsin\left(\frac{1}{|2\,x+1|}\right)$$

input
```
integrate(1/(1+2*x)/(x^2+x)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(1/abs(2*x + 1))
```

3.927.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{(1+2x)\sqrt{x+x^2}}\,dx = 2\,\arctan\left(-2\,x+2\,\sqrt{x^2+x}-1\right)$$

input
```
integrate(1/(1+2*x)/(x^2+x)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(-2*x + 2*sqrt(x^2 + x) - 1)
```

3.927.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+2x)\sqrt{x+x^2}}\,dx = \int \frac{1}{(2\,x+1)\,\sqrt{x^2+x}}\,dx$$

input
```
int(1/((2*x + 1)*(x + x^2)^(1/2)),x)
```

output
```
int(1/((2*x + 1)*(x + x^2)^(1/2)), x)
```

3.928 $\int \frac{-1+x}{\sqrt{2x-x^2}}\,dx$

3.928.1 Optimal result

Integrand size = 17, antiderivative size = 15

$$\int \frac{-1+x}{\sqrt{2x-x^2}}\,dx=-\sqrt{2x-x^2}$$

output
```
-(-x^2+2*x)^(1/2)
```

3.928.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{-1+x}{\sqrt{2x-x^2}}\,dx=-\sqrt{-((-2+x)x)}$$

input
```
Integrate[(-1 + x)/Sqrt[2*x - x^2],x]
```

output
```
-Sqrt[-((-2 + x)*x)]
```

3.928.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-1}{\sqrt{2x-x^2}}\,dx$$
$$\Big\downarrow \ 1104$$
$$-\sqrt{2x-x^2}$$

input
```
Int[(-1 + x)/Sqrt[2*x - x^2],x]
```

output
```
-Sqrt[2*x - x^2]
```

3.928.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

3.928.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

method	result	size
pseudoelliptic	$-\sqrt{-x\left(x-2\right)}$	11
default	$-\sqrt{-x^{2}+2x}$	14
trager	$-\sqrt{-x^{2}+2x}$	14
risch	$\frac{x\left(x-2\right)}{\sqrt{-x\left(x-2\right)}}$	14
gosper	$\frac{x\left(x-2\right)}{\sqrt{-x^{2}+2x}}$	17
meijerg	$-2\arcsin\left(\frac{\sqrt{2}\,\sqrt{x}}{2}\right)+\frac{2i\left(\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{1-\frac{x}{2}}}{2}-i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{2}\,\sqrt{x}}{2}\right)\right)}{\sqrt{\pi}}$	54

input
```
int((x-1)/(-x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-x*(x-2))^(1/2)
```

3.928.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{-1+x}{\sqrt{2x-x^2}}\,dx = -\sqrt{-x^2+2\,x}$$

input
```
integrate((-1+x)/(-x^2+2*x)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-x^2 + 2*x)
```

3.928.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{-1+x}{\sqrt{2x-x^2}}\,dx = -\sqrt{-x^2+2x}$$

input
```
integrate((-1+x)/(-x**2+2*x)**(1/2),x)
```

output
```
-sqrt(-x**2 + 2*x)
```

3.928.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{-1+x}{\sqrt{2x-x^2}}\,dx = -\sqrt{-x^2+2\,x}$$

input
```
integrate((-1+x)/(-x^2+2*x)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(-x^2 + 2*x)
```

3.928.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{-1+x}{\sqrt{2x-x^2}}\,dx = -\sqrt{-x^2+2\,x}$$

input
```
integrate((-1+x)/(-x^2+2*x)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(-x^2 + 2*x)
```

3.928.9 Mupad [B] (verification not implemented)

Time = 20.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{-1+x}{\sqrt{2x-x^2}}\,dx = -\sqrt{-x\ (x-2)}$$

input
```
int((x - 1)/(2*x - x^2)^(1/2),x)
```

output
```
-(-x*(x - 2))^(1/2)
```

3.929 $\int \frac{\sqrt{x-x^2}}{1+x} \, dx$

3.929.1 Optimal result

Integrand size = 17, antiderivative size = 54

$$\int \frac{\sqrt{x-x^2}}{1+x} \, dx = \sqrt{x-x^2}-\frac{3}{2}\arcsin(1-2x)+\sqrt{2}\arctan\left(\frac{1-3x}{2\sqrt{2}\sqrt{x-x^2}}\right)$$

output
```
3/2*arcsin(-1+2*x)+arctan(1/4*(1-3*x)*2^(1/2)/(-x^2+x)^(1/2))*2^(1/2)+(-x^
2+x)^(1/2)
```

3.929.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.52

$$\int \frac{\sqrt{x-x^2}}{1+x} \, dx = \frac{\sqrt{-((-1+x)x)}\left(\sqrt{-1+x}\sqrt{x}-6\text{arctanh}\left(\frac{\sqrt{-1+x}}{-1+\sqrt{x}}\right)+2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}}{\sqrt{\frac{-1+x}{x}}}\right)\right)}{\sqrt{-1+x}\sqrt{x}}$$

input
```
Integrate[Sqrt[x - x^2]/(1 + x),x]
```

output
```
(Sqrt[-((-1 + x)*x)]*(Sqrt[-1 + x]*Sqrt[x] - 6*ArcTanh[Sqrt[-1 + x]/(-1 +
Sqrt[x])] + 2*Sqrt[2]*ArcTanh[Sqrt[2]/Sqrt[(-1 + x)/x]]))/(Sqrt[-1 + x]*Sq
rt[x])
```

3.929.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1162, 1269, 1090, 223, 1154, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x-x^2}}{x+1} \, dx$$

$$\Big\downarrow \text{1162}$$

$$\sqrt{x-x^2}-\frac{1}{2} \int \frac{1-3 x}{(x+1) \sqrt{x-x^2}}dx$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{2} \left(3 \int \frac{1}{\sqrt{x-x^2}}dx-4 \int \frac{1}{(x+1) \sqrt{x-x^2}}dx\right)+\sqrt{x-x^2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{2} \left(-4 \int \frac{1}{(x+1) \sqrt{x-x^2}}dx-3 \int \frac{1}{\sqrt{1-(1-2 x)^2}}d(1-2 x)\right)+\sqrt{x-x^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{2} \left(-4 \int \frac{1}{(x+1) \sqrt{x-x^2}}dx-3 \arcsin (1-2 x)\right)+\sqrt{x-x^2}$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{2} \left(8 \int \frac{1}{-\frac{(1-3 x)^2}{x-x^2}-8}d\left(-\frac{1-3 x}{\sqrt{x-x^2}}\right)-3 \arcsin (1-2 x)\right)+\sqrt{x-x^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2} \left(2 \sqrt{2} \arctan \left(\frac{1-3 x}{2 \sqrt{2} \sqrt{x-x^2}}\right)-3 \arcsin (1-2 x)\right)+\sqrt{x-x^2}$$

input
```
Int[Sqrt[x - x^2]/(1 + x),x]
```

output
```
Sqrt[x - x^2] + (-3*ArcSin[1 - 2*x] + 2*Sqrt[2]*ArcTan[(1 - 3*x)/(2*Sqrt[2
]*Sqrt[x - x^2])])/2
```

3.929.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

3.929. $\int \frac{\sqrt{x-x^2}}{1+x}\,dx$

3.929.4 Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$2\sqrt{2}\,\arctan\left(\frac{\sqrt{-(x-1)x}\,\sqrt{2}}{2x}\right)+\sqrt{-\left(x-1\right)x}-3\arctan\left(\frac{\sqrt{-(x-1)x}}{x}\right)$
default	$\sqrt{-\left(x+1\right)^{2}+3x+1}+\frac{3\arcsin(2x-1)}{2}-\sqrt{2}\,\arctan\left(\frac{(3x-1)\sqrt{2}}{4\sqrt{-(x+1)^{2}+3x+1}}\right)$
risch	$-\frac{(x-1)x}{\sqrt{-(x-1)x}}+\frac{3\arcsin(2x-1)}{2}-\sqrt{2}\,\arctan\left(\frac{(3x-1)\sqrt{2}}{4\sqrt{-(x+1)^{2}+3x+1}}\right)$
trager	$\sqrt{-x^{2}+x}+\operatorname{RootOf}\left(\_Z^{2}+2\right)\ln\left(\frac{3\operatorname{RootOf}\left(\_Z^{2}+2\right)x+4\sqrt{-x^{2}+x}-\operatorname{RootOf}\left(\_Z^{2}+2\right)}{x+1}\right)+\frac{3\operatorname{RootOf}(}{}$

input
```
int((-x^2+x)^(1/2)/(x+1),x,method=_RETURNVERBOSE)
```

output
```
2*2^(1/2)*arctan(1/2*(-(x-1)*x)^(1/2)/x*2^(1/2))+(-(x-1)*x)^(1/2)-3*arctan
((-(x-1)*x)^(1/2)/x)
```

3.929.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{x-x^2}}{1+x}\,dx = 2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+x}}{2\,x}\right)+\sqrt{-x^2+x}-3\,\arctan\left(\frac{\sqrt{-x^2+x}}{x}\right)$$

input
```
integrate((-x^2+x)^(1/2)/(1+x),x, algorithm="fricas")
```

output
```
2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + x)/x) + sqrt(-x^2 + x) - 3*arctan
(sqrt(-x^2 + x)/x)
```

3.929.6 Sympy [F]

$$\int \frac{\sqrt{x-x^2}}{1+x}\,dx=\int \frac{\sqrt{-x\,(x-1)}}{x+1}\,dx$$

input
```
integrate((-x**2+x)**(1/2)/(1+x),x)
```

output
```
Integral(sqrt(-x*(x - 1))/(x + 1), x)
```

3.929.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{x-x^2}}{1+x}\,dx=-\sqrt{2}\arcsin\left(\frac{3\,x}{|x+1|}-\frac{1}{|x+1|}\right)+\sqrt{-x^2+x}+\frac{3}{2}\arcsin\left(2\,x-1\right)$$

input
```
integrate((-x^2+x)^(1/2)/(1+x),x, algorithm="maxima")
```

output
```
-sqrt(2)*arcsin(3*x/abs(x + 1) - 1/abs(x + 1)) + sqrt(-x^2 + x) + 3/2*arcs
in(2*x - 1)
```

3.929.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{x-x^2}}{1+x}\,dx=2\,\sqrt{2}\arctan\left(\frac{1}{4}\,\sqrt{2}\left(\frac{3\,\left(2\,\sqrt{-x^2+x}-1\right)}{2\,x-1}-1\right)\right)+\sqrt{-x^2+x}+\frac{3}{2}\arcsin\left(2\,x-1\right)$$

input
```
integrate((-x^2+x)^(1/2)/(1+x),x, algorithm="giac")
```

output
```
2*sqrt(2)*arctan(1/4*sqrt(2)*(3*(2*sqrt(-x^2 + x) - 1)/(2*x - 1) - 1)) + s
qrt(-x^2 + x) + 3/2*arcsin(2*x - 1)
```

3.929.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x-x^2}}{1+x}\,dx = \int \frac{\sqrt{x-x^2}}{x+1}\,dx$$

input
```
int((x - x^2)^(1/2)/(x + 1),x)
```

output
```
int((x - x^2)^(1/2)/(x + 1), x)
```

3.930 $\int \sqrt{\sqrt[4]{x}+x}\,dx$

3.930.1 Optimal result

Integrand size = 11, antiderivative size = 59

$$\int \sqrt{\sqrt[4]{x}+x}\,dx=\frac{1}{3}\sqrt[4]{x}\sqrt{\sqrt[4]{x}+x}+\frac{2}{3}x\sqrt{\sqrt[4]{x}+x}-\frac{1}{3}\text{arctanh}\left(\frac{\sqrt{x}}{\sqrt{\sqrt[4]{x}+x}}\right)$$

output
```
-1/3*arctanh(x^(1/2)/(x^(1/4)+x)^(1/2))+1/3*x^(1/4)*(x^(1/4)+x)^(1/2)+2/3*
x*(x^(1/4)+x)^(1/2)
```

3.930.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \sqrt{\sqrt[4]{x}+x}\,dx=\frac{1}{3}\sqrt{\sqrt[4]{x}+x}(\sqrt[4]{x}+2x)-\frac{1}{3}\text{arctanh}\left(\frac{\sqrt{x}}{\sqrt{\sqrt[4]{x}+x}}\right)$$

input
```
Integrate[Sqrt[x^(1/4) + x],x]
```

output
```
(Sqrt[x^(1/4) + x]*(x^(1/4) + 2*x))/3 - ArcTanh[Sqrt[x]/Sqrt[x^(1/4) + x]]
/3
```

3.930.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {1910, 1924, 1930, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x+\sqrt[4]{x}}\,dx$$

$$\Big\downarrow \text{ 1910}$$

$$\frac{1}{4}\int \frac{\sqrt[4]{x}}{\sqrt{x+\sqrt[4]{x}}}dx+\frac{2}{3}\sqrt{x+\sqrt[4]{x}}x$$

$$\Big\downarrow \text{ 1924}$$

$$\int \frac{x}{\sqrt{x+\sqrt[4]{x}}}d\sqrt[4]{x}+\frac{2}{3}\sqrt{x+\sqrt[4]{x}}x$$

$$\Big\downarrow \text{ 1930}$$

$$-\frac{1}{2}\int \frac{\sqrt[4]{x}}{\sqrt{x+\sqrt[4]{x}}}d\sqrt[4]{x}+\frac{2}{3}\sqrt{x+\sqrt[4]{x}}x+\frac{1}{3}\sqrt{x+\sqrt[4]{x}}\sqrt[4]{x}$$

$$\Big\downarrow \text{ 1935}$$

$$-\frac{1}{3}\int \frac{1}{1-\sqrt{x}}d\frac{\sqrt{x}}{\sqrt{x+\sqrt[4]{x}}}+\frac{2}{3}\sqrt{x+\sqrt[4]{x}}x+\frac{1}{3}\sqrt{x+\sqrt[4]{x}}\sqrt[4]{x}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{1}{3}\text{arctanh}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt[4]{x}}}\right)+\frac{2}{3}\sqrt{x+\sqrt[4]{x}}x+\frac{1}{3}\sqrt{x+\sqrt[4]{x}}\sqrt[4]{x}$$

input
```
Int[Sqrt[x^(1/4) + x],x]
```

output
```
(x^(1/4)*Sqrt[x^(1/4) + x])/3 + (2*x*Sqrt[x^(1/4) + x])/3 - ArcTanh[Sqrt[x
]/Sqrt[x^(1/4) + x]]/3
```

3.930.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

3.930.4 Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.64

method	result
meijerg	$-\frac{2\left(-\frac{\sqrt{\pi}\,x^{\frac{3}{8}}\left(6x^{\frac{3}{4}}+3\right)\sqrt{1+x^{\frac{3}{4}}}}{6}+\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(x^{\frac{3}{8}}\right)}{2}\right)}{3\sqrt{\pi}}$
derivativedivides	$\frac{1}{6\left(\frac{\sqrt{x^{\frac{1}{4}}+x}}{\sqrt{x}}-1\right)^{2}}+\frac{1}{\frac{6\sqrt{x^{\frac{1}{4}}+x}}{\sqrt{x}}-6}+\frac{\ln\left(\frac{\sqrt{x^{\frac{1}{4}}+x}}{\sqrt{x}}-1\right)}{6}-\frac{1}{6\left(\frac{\sqrt{x^{\frac{1}{4}}+x}}{\sqrt{x}}+1\right)^{2}}+\frac{1}{\frac{6\sqrt{x^{\frac{1}{4}}+x}}{\sqrt{x}}+6}-\frac{\ln\left(\frac{\sqrt{x^{\frac{1}{4}}+x}}{\sqrt{x}}+1\right)}{6}$
default	$\frac{1}{6\left(\frac{\sqrt{x^{\frac{1}{4}}+x}}{\sqrt{x}}-1\right)^{2}}+\frac{1}{\frac{6\sqrt{x^{\frac{1}{4}}+x}}{\sqrt{x}}-6}+\frac{\ln\left(\frac{\sqrt{x^{\frac{1}{4}}+x}}{\sqrt{x}}-1\right)}{6}-\frac{1}{6\left(\frac{\sqrt{x^{\frac{1}{4}}+x}}{\sqrt{x}}+1\right)^{2}}+\frac{1}{\frac{6\sqrt{x^{\frac{1}{4}}+x}}{\sqrt{x}}+6}-\frac{\ln\left(\frac{\sqrt{x^{\frac{1}{4}}+x}}{\sqrt{x}}+1\right)}{6}$

input
```
int((x^(1/4)+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/Pi^(1/2)*(-1/6*Pi^(1/2)*x^(3/8)*(6*x^(3/4)+3)*(1+x^(3/4))^(1/2)+1/2*P
i^(1/2)*arcsinh(x^(3/8)))
```

3.930.5 Fricas [F(-1)]

Timed out.

$$\int\sqrt{\sqrt[4]{x}+x}\,dx=\text{Timed out}$$

input
```
integrate((x^(1/4)+x)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.930.6 Sympy [F]

$$\int \sqrt{\sqrt[4]{x}+x}\,dx=\int \sqrt{\sqrt[4]{x}+x}\,dx$$

input
```
integrate((x**(1/4)+x)**(1/2),x)
```

output
```
Integral(sqrt(x**(1/4) + x), x)
```

3.930.7 Maxima [F]

$$\int \sqrt{\sqrt[4]{x}+x}\,dx=\int \sqrt{x+x^{\frac{1}{4}}}\,dx$$

input
```
integrate((x^(1/4)+x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x + x^(1/4)), x)
```

3.930.8 Giac [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.76

$$\int \sqrt{\sqrt[4]{x}+x}\,dx=\frac{1}{3}\sqrt{x+x^{\frac{1}{4}}}x^{\frac{1}{4}}\left(2\,x^{\frac{3}{4}}+1\right)-\frac{1}{6}\log\left(\sqrt{\frac{1}{x^{\frac{3}{4}}}+1}+1\right)+\frac{1}{6}\log\left(\left|\sqrt{\frac{1}{x^{\frac{3}{4}}}+1}-1\right|\right)$$

input
```
integrate((x^(1/4)+x)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(x + x^(1/4))*x^(1/4)*(2*x^(3/4) + 1) - 1/6*log(sqrt(1/x^(3/4) + 1
) + 1) + 1/6*log(abs(sqrt(1/x^(3/4) + 1) - 1))
```

3.930.9 Mupad [B] (verification not implemented)

Time = 20.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.46

$$\int \sqrt{\sqrt[4]{x}+x}\,dx = \frac{8\,x\,\sqrt{x+x^{1/4}}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{3}{2};\ \frac{5}{2};\ -x^{3/4}\right)}{9\,\sqrt{x^{3/4}+1}}$$

input
```
int((x + x^(1/4))^(1/2),x)
```

output
```
(8*x*(x + x^(1/4))^(1/2)*hypergeom([-1/2, 3/2], 5/2, -x^(3/4)))/(9*(x^(3/4
) + 1)^(1/2))
```

3.931 $\int \sqrt{x+x^{3/2}}\,dx$

3.931.1 Optimal result

Integrand size = 11, antiderivative size = 59

$$\int \sqrt{x+x^{3/2}}\,dx=\frac{32\left(x+x^{3/2}\right)^{3/2}}{105x^{3/2}}-\frac{16\left(x+x^{3/2}\right)^{3/2}}{35x}+\frac{4\left(x+x^{3/2}\right)^{3/2}}{7\sqrt{x}}$$

output
```
32/105*(x+x^(3/2))^(3/2)/x^(3/2)-16/35*(x+x^(3/2))^(3/2)/x+4/7*(x+x^(3/2))
^(3/2)/x^(1/2)
```

3.931.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int \sqrt{x+x^{3/2}}\,dx=\frac{4\sqrt{x+x^{3/2}}\left(8-4\sqrt{x}+3x+15x^{3/2}\right)}{105\sqrt{x}}$$

input
```
Integrate[Sqrt[x + x^(3/2)],x]
```

output
```
(4*Sqrt[x + x^(3/2)]*(8 - 4*Sqrt[x] + 3*x + 15*x^(3/2)))/(105*Sqrt[x])
```

3.931.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1908, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^{3/2}+x}\,dx$$

$$\Big\downarrow \text{1908}$$

$$\frac{4\left(x^{3/2}+x\right)^{3/2}}{7\sqrt{x}}-\frac{4}{7}\int \frac{\sqrt{x^{3/2}+x}}{\sqrt{x}}dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{4\left(x^{3/2}+x\right)^{3/2}}{7\sqrt{x}}-\frac{4}{7}\left(\frac{4\left(x^{3/2}+x\right)^{3/2}}{5x}-\frac{2}{5}\int \frac{\sqrt{x^{3/2}+x}}{x}dx\right)$$

$$\Big\downarrow \text{1920}$$

$$\frac{4\left(x^{3/2}+x\right)^{3/2}}{7\sqrt{x}}-\frac{4}{7}\left(\frac{4\left(x^{3/2}+x\right)^{3/2}}{5x}-\frac{8\left(x^{3/2}+x\right)^{3/2}}{15x^{3/2}}\right)$$

input
```
Int[Sqrt[x + x^(3/2)],x]
```

output
```
(4*(x + x^(3/2))^(3/2))/(7*Sqrt[x]) - (4*((-8*(x + x^(3/2))^(3/2))/(15*x^(
3/2)) + (4*(x + x^(3/2))^(3/2))/(5*x)))/7
```

3.931.3.1 Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.931.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.47

method	result	size
derivativedivides	$\frac{4\sqrt{x+x^{\frac{3}{2}}}\,(1+\sqrt{x})(15x-12\sqrt{x}+8)}{105\sqrt{x}}$	28
default	$\frac{4\sqrt{x+x^{\frac{3}{2}}}\,(1+\sqrt{x})(15x-12\sqrt{x}+8)}{105\sqrt{x}}$	28
meijerg	$-\frac{\frac{32\sqrt{\pi}}{105}-\frac{4\sqrt{\pi}\,(1+\sqrt{x})^{\frac{3}{2}}(15x-12\sqrt{x}+8)}{105}}{\sqrt{\pi}}$	34

input
```
int((x+x^(3/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/105*(x+x^(3/2))^(1/2)*(1+x^(1/2))*(15*x-12*x^(1/2)+8)/x^(1/2)
```

3.931.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.51

$$\int \sqrt{x+x^{3/2}}\,dx=\frac{4\,\left(15\,x^{2}+(3\,x+8)\sqrt{x}-4\,x\right)\sqrt{x^{\frac{3}{2}}+x}}{105\,x}$$

input
```
integrate((x+x^(3/2))^(1/2),x, algorithm="fricas")
```

output
```
4/105*(15*x^2 + (3*x + 8)*sqrt(x) - 4*x)*sqrt(x^(3/2) + x)/x
```

3.931.6 Sympy [F]

$$\int \sqrt{x+x^{3/2}}\,dx=\int \sqrt{x^{\frac{3}{2}}+x}\,dx$$

input
```
integrate((x+x**(3/2))**(1/2),x)
```

output
```
Integral(sqrt(x**(3/2) + x), x)
```

3.931.7 Maxima [F]

$$\int \sqrt{x+x^{3/2}}\,dx=\int \sqrt{x^{\frac{3}{2}}+x}\,dx$$

input
```
integrate((x+x^(3/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^(3/2) + x), x)
```

3.931.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.56

$$\int \sqrt{x+x^{3/2}}\,dx=\frac{4}{105}\left(15\left(\sqrt{x}+1\right)^{\frac{7}{2}}-42\left(\sqrt{x}+1\right)^{\frac{5}{2}}+35\left(\sqrt{x}+1\right)^{\frac{3}{2}}-8\right)\operatorname{sgn}(x)$$

input
```
integrate((x+x^(3/2))^(1/2),x, algorithm="giac")
```

output
```
4/105*(15*(sqrt(x) + 1)^(7/2) - 42*(sqrt(x) + 1)^(5/2) + 35*(sqrt(x) + 1)^
(3/2) - 8)*sgn(x)
```

3.931.9 Mupad [B] (verification not implemented)

Time = 19.83 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.46

$$\int \sqrt{x+x^{3/2}}\,dx=\frac{2\,x\,\sqrt{x+x^{3/2}}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},3;\ 4;\ -\sqrt{x}\right)}{3\,\sqrt{\sqrt{x}+1}}$$

input
```
int((x + x^(3/2))^(1/2),x)
```

output
```
(2*x*(x + x^(3/2))^(1/2)*hypergeom([-1/2, 3], 4, -x^(1/2)))/(3*(x^(1/2) +
1)^(1/2))
```

3.932 $\int x\sqrt{x+x^{3/2}}\,dx$

3.932.1 Optimal result

Integrand size = 13, antiderivative size = 94

$$\int x\sqrt{x+x^{3/2}}\,dx = -\frac{32}{99}\left(x+x^{3/2}\right)^{3/2}+\frac{512\left(x+x^{3/2}\right)^{3/2}}{3465x^{3/2}} -\frac{256\left(x+x^{3/2}\right)^{3/2}}{1155x}+\frac{64\left(x+x^{3/2}\right)^{3/2}}{231\sqrt{x}}+\frac{4}{11}\sqrt{x}\left(x+x^{3/2}\right)^{3/2}$$

output
```
-32/99*(x+x^(3/2))^(3/2)+512/3465*(x+x^(3/2))^(3/2)/x^(3/2)-256/1155*(x+x^
(3/2))^(3/2)/x+64/231*(x+x^(3/2))^(3/2)/x^(1/2)+4/11*(x+x^(3/2))^(3/2)*x^(
1/2)
```

3.932.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.54

$$\int x\sqrt{x+x^{3/2}}\,dx = \frac{4\sqrt{x+x^{3/2}}\left(128-64\sqrt{x}+48x-40x^{3/2}+35x^2+315x^{5/2}\right)}{3465\sqrt{x}}$$

input
```
Integrate[x*Sqrt[x + x^(3/2)],x]
```

output
```
(4*Sqrt[x + x^(3/2)]*(128 - 64*Sqrt[x] + 48*x - 40*x^(3/2) + 35*x^2 + 315*
x^(5/2)))/(3465*Sqrt[x])
```

3.932.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1922, 1922, 1908, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{x^{3/2}+x}\,dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{4}{11}\sqrt{x}\left(x^{3/2}+x\right)^{3/2}-\frac{8}{11}\int\sqrt{x}\sqrt{x^{3/2}+x}dx$$

$$\Big\downarrow \text{1922}$$

$$\frac{4}{11}\sqrt{x}\left(x^{3/2}+x\right)^{3/2}-\frac{8}{11}\left(\frac{4}{9}\left(x^{3/2}+x\right)^{3/2}-\frac{2}{3}\int\sqrt{x^{3/2}+x}dx\right)$$

$$\Big\downarrow \text{1908}$$

$$\frac{4}{11}\sqrt{x}\left(x^{3/2}+x\right)^{3/2}-\frac{8}{11}\left(\frac{4}{9}\left(x^{3/2}+x\right)^{3/2}-\frac{2}{3}\left(\frac{4\left(x^{3/2}+x\right)^{3/2}}{7\sqrt{x}}-\frac{4}{7}\int\frac{\sqrt{x^{3/2}+x}}{\sqrt{x}}dx\right)\right)$$

$$\Big\downarrow \text{1922}$$

$$\frac{4}{11}\sqrt{x}\left(x^{3/2}+x\right)^{3/2}-\frac{8}{11}\left(\frac{4}{9}\left(x^{3/2}+x\right)^{3/2}-\frac{2}{3}\left(\frac{4\left(x^{3/2}+x\right)^{3/2}}{7\sqrt{x}}-\frac{4}{7}\left(\frac{4\left(x^{3/2}+x\right)^{3/2}}{5x}-\frac{2}{5}\int\frac{\sqrt{x^{3/2}+x}}{x}dx\right)\right)\right)$$

$$\Big\downarrow \text{1920}$$

$$\frac{4}{11}\sqrt{x}\left(x^{3/2}+x\right)^{3/2}-\frac{8}{11}\left(\frac{4}{9}\left(x^{3/2}+x\right)^{3/2}-\frac{2}{3}\left(\frac{4\left(x^{3/2}+x\right)^{3/2}}{7\sqrt{x}}-\frac{4}{7}\left(\frac{4\left(x^{3/2}+x\right)^{3/2}}{5x}-\frac{8\left(x^{3/2}+x\right)^{3/2}}{15x^{3/2}}\right)\right)\right)$$

input
```
Int[x*Sqrt[x + x^(3/2)],x]
```

output
```
(4*Sqrt[x]*(x + x^(3/2))^(3/2))/11 - (8*((4*(x + x^(3/2))^(3/2))/9 - (2*((
4*(x + x^(3/2))^(3/2))/(7*Sqrt[x]) - (4*((-8*(x + x^(3/2))^(3/2))/(15*x^(3
/2)) + (4*(x + x^(3/2))^(3/2))/(5*x)))/7))/3))/11
```

3.932.3.1 Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

3.932.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.40

method	result	size
derivativedivides	$\frac{4\sqrt{x+x^{\frac{3}{2}}}\,(1+\sqrt{x})\left(315x^{2}-280x^{\frac{3}{2}}+240x-192\sqrt{x}+128\right)}{3465\sqrt{x}}$	38
default	$\frac{4\sqrt{x+x^{\frac{3}{2}}}\,(1+\sqrt{x})\left(315x^{2}-280x^{\frac{3}{2}}+240x-192\sqrt{x}+128\right)}{3465\sqrt{x}}$	38
meijerg	$-\frac{\frac{512\sqrt{\pi}}{3465}-\frac{4\sqrt{\pi}\,(1+\sqrt{x})^{\frac{3}{2}}\left(315x^{2}-280x^{\frac{3}{2}}+240x-192\sqrt{x}+128\right)}{3465}}{\sqrt{\pi}}$	44

input
```
int(x*(x+x^(3/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/3465*(x+x^(3/2))^(1/2)*(1+x^(1/2))*(315*x^2-280*x^(3/2)+240*x-192*x^(1/2
)+128)/x^(1/2)
```

3.932.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.43

$$\int x\sqrt{x+x^{3/2}}\,dx = \frac{4\left(315\,x^{3}-40\,x^{2}+\left(35\,x^{2}+48\,x+128\right)\sqrt{x}-64\,x\right)\sqrt{x^{\frac{3}{2}}+x}}{3465\,x}$$

input
```
integrate(x*(x+x^(3/2))^(1/2),x, algorithm="fricas")
```

output
```
4/3465*(315*x^3 - 40*x^2 + (35*x^2 + 48*x + 128)*sqrt(x) - 64*x)*sqrt(x^(3
/2) + x)/x
```

3.932.6 Sympy [F]

$$\int x\sqrt{x+x^{3/2}}\,dx = \int x\sqrt{x^{\frac{3}{2}}+x}\,dx$$

input
```
integrate(x*(x+x**(3/2))**(1/2),x)
```

output
```
Integral(x*sqrt(x**(3/2) + x), x)
```

3.932.7 Maxima [F]

$$\int x\sqrt{x+x^{3/2}}\,dx = \int \sqrt{x^{\frac{3}{2}}+x}x\,dx$$

input
```
integrate(x*(x+x^(3/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^(3/2) + x)*x, x)
```

3.932.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.54

$$\int x\sqrt{x+x^{3/2}}\,dx=\frac{4}{3465}\left(315\left(\sqrt{x}+1\right)^{\frac{11}{2}}-1540\left(\sqrt{x}+1\right)^{\frac{9}{2}}+2970\left(\sqrt{x}+1\right)^{\frac{7}{2}}-2772\left(\sqrt{x}+1\right)^{\frac{5}{2}}+1155\right.$$

input
```
integrate(x*(x+x^(3/2))^(1/2),x, algorithm="giac")
```

output
```
4/3465*(315*(sqrt(x) + 1)^(11/2) - 1540*(sqrt(x) + 1)^(9/2) + 2970*(sqrt(x
) + 1)^(7/2) - 2772*(sqrt(x) + 1)^(5/2) + 1155*(sqrt(x) + 1)^(3/2) - 128)*
sgn(x)
```

3.932.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{x+x^{3/2}}\,dx=\int x\,\sqrt{x+x^{3/2}}\,dx$$

input
```
int(x*(x + x^(3/2))^(1/2),x)
```

output
```
int(x*(x + x^(3/2))^(1/2), x)
```

3.933 $\int (1-x^2)\sqrt{\frac{1}{2-x^2}}\,dx$

3.933.1 Optimal result

Integrand size = 21, antiderivative size = 18

$$\int (1-x^2)\sqrt{\frac{1}{2-x^2}}\,dx = \frac{x}{2\sqrt{\frac{1}{2-x^2}}}$$

output
```
1/2*x/(1/(-x^2+2))^(1/2)
```

3.933.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (1-x^2)\sqrt{\frac{1}{2-x^2}}\,dx = \frac{x}{2\sqrt{\frac{1}{2-x^2}}}$$

input
```
Integrate[(1 - x^2)*Sqrt[(2 - x^2)^(-1)],x]
```

output
```
x/(2*Sqrt[(2 - x^2)^(-1)])
```

3.933.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2044, 297}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-x^2\right) \sqrt{\frac{1}{2-x^2}} \, dx$$

$$\Big\downarrow \; 2044$$

$$\sqrt{\frac{1}{2-x^2}} \sqrt{2-x^2} \int \frac{1-x^2}{\sqrt{2-x^2}} dx$$

$$\Big\downarrow \; 297$$

$$\frac{x}{2 \sqrt{\frac{1}{2-x^2}}}$$

input
```
Int[(1 - x^2)*Sqrt[(2 - x^2)^(-1)],x]
```

output
```
x/(2*Sqrt[(2 - x^2)^(-1)])
```

3.933.3.1 Defintions of rubi rules used

rule 297
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[c*
x*((a + b*x^2)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d
, 0] && EqQ[a*d - b*c*(2*p + 3), 0]
```

rule 2044
```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)]   Int[u*(a + b*x^n)^(p*q), x], x
] /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

3.933.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

method	result	size
gosper	$-\frac{(x^2-2)x\sqrt{-\frac{1}{x^2-2}}}{2}$	20
default	$-\frac{(x^2-2)x\sqrt{-\frac{1}{x^2-2}}}{2}$	20
trager	$-\frac{(x^2-2)x\sqrt{-\frac{1}{x^2-2}}}{2}$	20
risch	$-\frac{(x^2-2)x\sqrt{-\frac{1}{x^2-2}}}{2}$	20
meijerg	$\sqrt{\frac{1}{-x^2+2}}\,\sqrt{-x^2+2}\,\arcsin\left(\frac{x\sqrt{2}}{2}\right)-\frac{i\sqrt{\frac{1}{-x^2+2}}\,\sqrt{-x^2+2}\left(\frac{i\sqrt{\pi}\,x\sqrt{2}\,\sqrt{1-\frac{x^2}{2}}}{2}-i\sqrt{\pi}\,\arcsin\left(\frac{x\sqrt{2}}{2}\right)\right)}{\sqrt{\pi}}$	89

input
```
int((-x^2+1)*(1/(-x^2+2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(x^2-2)*x*(-1/(x^2-2))^(1/2)
```

3.933.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (1-x^2)\sqrt{\frac{1}{2-x^2}}\,dx=-\frac{1}{2}\left(x^3-2\,x\right)\sqrt{-\frac{1}{x^2-2}}$$

input
```
integrate((-x^2+1)*(1/(-x^2+2))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(x^3 - 2*x)*sqrt(-1/(x^2 - 2))
```

3.933.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 0.33 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.61

$$\int \left(1-x^2\right)\sqrt{\frac{1}{2-x^2}}\,dx=-\frac{x^3\sqrt{-\frac{1}{x^2-2}}}{2}+x\sqrt{-\frac{1}{x^2-2}}$$

input
```
integrate((-x**2+1)*(1/(-x**2+2))**(1/2),x)
```

output
```
-x**3*sqrt(-1/(x**2 - 2))/2 + x*sqrt(-1/(x**2 - 2))
```

3.933.7 Maxima [F]

$$\int \left(1-x^2\right)\sqrt{\frac{1}{2-x^2}}\,dx=\int -\left(x^2-1\right)\sqrt{-\frac{1}{x^2-2}}\,dx$$

input
```
integrate((-x^2+1)*(1/(-x^2+2))^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 1)*sqrt(-1/(x^2 - 2)), x)
```

3.933.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \left(1-x^2\right)\sqrt{\frac{1}{2-x^2}}\,dx=-\frac{1}{2}\sqrt{-x^2+2}x\operatorname{sgn}\left(x^2-2\right)$$

input
```
integrate((-x^2+1)*(1/(-x^2+2))^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-x^2 + 2)*x*sgn(x^2 - 2)
```

3.933.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \left(1-x^2\right) \sqrt{\frac{1}{2-x^2}}\, dx = -\frac{x\,\left(x^2-2\right)\,\sqrt{-\frac{1}{x^2-2}}}{2}$$

input
```
int(-(x^2 - 1)*(-1/(x^2 - 2))^(1/2),x)
```

output
```
-(x*(x^2 - 2)*(-1/(x^2 - 2))^(1/2))/2
```

3.934 $\int \sqrt{x^2+x^3-x^4}\,dx$

3.934.1 Optimal result

Integrand size = 16, antiderivative size = 107

$$\int \sqrt{x^2+x^3-x^4}\,dx=-\frac{(1-2x)\sqrt{x^2+x^3-x^4}}{8x}-\frac{(1+x-x^2)\sqrt{x^2+x^3-x^4}}{3x}-\frac{5\sqrt{x^2+x^3-x^4}\arcsin\left(\frac{1-2x}{\sqrt{5}}\right)}{16x\sqrt{1+x-x^2}}$$

output
```
-1/8*(1-2*x)*(-x^4+x^3+x^2)^(1/2)/x-1/3*(-x^2+x+1)*(-x^4+x^3+x^2)^(1/2)/x-
5/16*arcsin(1/5*(1-2*x)*5^(1/2))*(-x^4+x^3+x^2)^(1/2)/x/(-x^2+x+1)^(1/2)
```

3.934.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.77

$$\int \sqrt{x^2+x^3-x^4}\,dx=\frac{\sqrt{x^2+x^3-x^4}\left(2\sqrt{-1-x+x^2}(-11-2x+8x^2)+15\log\left(1-2x+2\sqrt{-1-x+x^2}\right)\right)}{48x\sqrt{-1-x+x^2}}$$

input
```
Integrate[Sqrt[x^2 + x^3 - x^4],x]
```

output
```
(Sqrt[x^2 + x^3 - x^4]*(2*Sqrt[-1 - x + x^2]*(-11 - 2*x + 8*x^2) + 15*Log[
1 - 2*x + 2*Sqrt[-1 - x + x^2]]))/(48*x*Sqrt[-1 - x + x^2])
```

3.934.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1950, 1160, 1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-x^4+x^3+x^2}\,dx$$

$$\Big\downarrow \text{1950}$$

$$\frac{\sqrt{-x^4+x^3+x^2}\int x\sqrt{-x^2+x+1}dx}{x\sqrt{-x^2+x+1}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{\sqrt{-x^4+x^3+x^2}\left(\frac{1}{2}\int \sqrt{-x^2+x+1}dx-\frac{1}{3}\left(-x^2+x+1\right)^{3/2}\right)}{x\sqrt{-x^2+x+1}}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\sqrt{-x^4+x^3+x^2}\left(\frac{1}{2}\left(\frac{5}{8}\int \frac{1}{\sqrt{-x^2+x+1}}dx-\frac{1}{4}(1-2x)\sqrt{-x^2+x+1}\right)-\frac{1}{3}\left(-x^2+x+1\right)^{3/2}\right)}{x\sqrt{-x^2+x+1}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{\sqrt{-x^4+x^3+x^2}\left(\frac{1}{2}\left(-\frac{1}{8}\sqrt{5}\int \frac{1}{\sqrt{1-\frac{1}{5}(1-2x)^2}}d(1-2x)-\frac{1}{4}\sqrt{-x^2+x+1}(1-2x)\right)-\frac{1}{3}\left(-x^2+x+1\right)^{3/2}\right)}{x\sqrt{-x^2+x+1}}$$

$$\Big\downarrow \text{223}$$

$$\frac{\sqrt{-x^4+x^3+x^2}\left(\frac{1}{2}\left(-\frac{5}{8}\arcsin\left(\frac{1-2x}{\sqrt{5}}\right)-\frac{1}{4}\sqrt{-x^2+x+1}(1-2x)\right)-\frac{1}{3}\left(-x^2+x+1\right)^{3/2}\right)}{x\sqrt{-x^2+x+1}}$$

input
```
Int[Sqrt[x^2 + x^3 - x^4],x]
```

output
```
(Sqrt[x^2 + x^3 - x^4]*(-1/3*(1 + x - x^2)^(3/2) + (-1/4*((1 - 2*x)*Sqrt[1
 + x - x^2]) - (5*ArcSin[(1 - 2*x)/Sqrt[5]])/8)/2))/(x*Sqrt[1 + x - x^2])
```

3.934.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1950
```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
 :> Simp[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))])   Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))],
x], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

3.934.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.44

method	result
pseudoelliptic	$\frac{(16x^2-4x-22)\sqrt{-x^2(x^2-x-1)}+15\arcsin\left(\frac{\sqrt{5}\,(2x-1)}{5}\right)x}{48x}$
trager	$\frac{(8x^2-2x-11)\sqrt{-x^4+x^3+x^2}}{24x}+\frac{5\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-2\,\mathrm{RootOf}\left(\_Z^2+1\right)x^2+\mathrm{RootOf}\left(\_Z^2+1\right)x+2\sqrt{-x^4+x^3+x^2}}{x}\right)}{16}$
default	$-\frac{\sqrt{-x^4+x^3+x^2}\left(16(-x^2+x+1)^{\frac{3}{2}}-12x\sqrt{-x^2+x+1}+6\sqrt{-x^2+x+1}-15\arcsin\left(\frac{\sqrt{5}\,(2x-1)}{5}\right)\right)}{48x\sqrt{-x^2+x+1}}$
risch	$\frac{(8x^2-2x-11)\sqrt{-x^2(x^2-x-1)}}{24x}-\frac{5\arcsin\left(\frac{2\sqrt{5}\,\left(x-\frac{1}{2}\right)}{5}\right)\sqrt{-x^2(x^2-x-1)}\,\sqrt{-x^2+x+1}}{16x(x^2-x-1)}$

input
```
int((-x^4+x^3+x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*((16*x^2-4*x-22)*(-x^2*(x^2-x-1))^(1/2)+15*arcsin(1/5*5^(1/2)*(2*x-1)
)*x)/x
```

3.934.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.58

$$\int\sqrt{x^2+x^3-x^4}\,dx$$
$$=-\frac{15\,x\arctan\left(-\frac{x-\sqrt{-x^4+x^3+x^2}}{x^2}\right)-\sqrt{-x^4+x^3+x^2}(8\,x^2-2\,x-11)+11\,x}{24\,x}$$

input
```
integrate((-x^4+x^3+x^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/24*(15*x*arctan(-(x - sqrt(-x^4 + x^3 + x^2))/x^2) - sqrt(-x^4 + x^3 +
x^2)*(8*x^2 - 2*x - 11) + 11*x)/x
```

3.934.6 Sympy [F]

$$\int \sqrt{x^2+x^3-x^4}\,dx = \int \sqrt{-x^4+x^3+x^2}\,dx$$

input
```
integrate((-x**4+x**3+x**2)**(1/2),x)
```

output
```
Integral(sqrt(-x**4 + x**3 + x**2), x)
```

3.934.7 Maxima [F]

$$\int \sqrt{x^2+x^3-x^4}\,dx = \int \sqrt{-x^4+x^3+x^2}\,dx$$

input
```
integrate((-x^4+x^3+x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^4 + x^3 + x^2), x)
```

3.934.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.56

$$\begin{aligned}\int \sqrt{x^2+x^3-x^4}\,dx = {} & \frac{1}{48}\left(15\arcsin\left(\frac{1}{5}\sqrt{5}\right)+22\right)\operatorname{sgn}(x) \\ & +\frac{5}{16}\arcsin\left(\frac{1}{5}\sqrt{5}(2x-1)\right)\operatorname{sgn}(x) \\ & +\frac{1}{24}\left(2\left(4x\operatorname{sgn}(x)-\operatorname{sgn}(x)\right)x-11\operatorname{sgn}(x)\right)\sqrt{-x^2+x+1}\end{aligned}$$

input
```
integrate((-x^4+x^3+x^2)^(1/2),x, algorithm="giac")
```

output
```
1/48*(15*arcsin(1/5*sqrt(5)) + 22)*sgn(x) + 5/16*arcsin(1/5*sqrt(5)*(2*x -
 1))*sgn(x) + 1/24*(2*(4*x*sgn(x) - sgn(x))*x - 11*sgn(x))*sqrt(-x^2 + x +
 1)
```

3.934.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{x^2+x^3-x^4}\,dx=\int \sqrt{-x^4+x^3+x^2}\,dx$$

input
```
int((x^2 + x^3 - x^4)^(1/2),x)
```

output
```
int((x^2 + x^3 - x^4)^(1/2), x)
```

3.935 $\int \frac{1}{\sqrt{(a^2+x^2)^3}}\,dx$

3.935.1 Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \frac{1}{\sqrt{(a^2+x^2)^3}}\,dx = \frac{x(a^2+x^2)}{a^2\sqrt{(a^2+x^2)^3}}$$

output
```
x*(a^2+x^2)/a^2/((a^2+x^2)^3)^(1/2)
```

3.935.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{(a^2+x^2)^3}}\,dx = \frac{x(a^2+x^2)}{a^2\sqrt{(a^2+x^2)^3}}$$

input
```
Integrate[1/Sqrt[(a^2 + x^2)^3],x]
```

output
```
(x*(a^2 + x^2))/(a^2*Sqrt[(a^2 + x^2)^3])
```

3.935.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2045, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{(a^2+x^2)^3}}dx$$

$$\Big\downarrow \ 2045$$

$$\frac{\left(\frac{x^2}{a^2}+1\right)^{3/2}\int \frac{1}{\left(\frac{x^2}{a^2}+1\right)^{3/2}}dx}{\sqrt{(a^2+x^2)^3}}$$

$$\Big\downarrow \ 208$$

$$\frac{x\left(\frac{x^2}{a^2}+1\right)}{\sqrt{(a^2+x^2)^3}}$$

input
```
Int[1/Sqrt[(a^2 + x^2)^3],x]
```

output
```
(x*(1 + x^2/a^2))/Sqrt[(a^2 + x^2)^3]
```

3.935.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2045
```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)]   Int[u*(1 + b*(x^n/a))^(p*q)
, x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

3.935.4 Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{x(a^2+x^2)}{a^2\sqrt{(a^2+x^2)^3}}$	24
default	$\frac{x(a^2+x^2)}{a^2\sqrt{(a^2+x^2)^3}}$	24
risch	$\frac{x(a^2+x^2)}{a^2\sqrt{(a^2+x^2)^3}}$	24
trager	$\frac{x\sqrt{a^6+3a^4x^2+3a^2x^4+x^6}}{a^2(a^2+x^2)^2}$	40

input
```
int(1/((a^2+x^2)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*(a^2+x^2)/a^2/((a^2+x^2)^3)^(1/2)
```

3.935.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(23) = 46.

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.56

$$\int \frac{1}{\sqrt{(a^2+x^2)^3}}\,dx = \frac{a^4+2\,a^2x^2+x^4+\sqrt{a^6+3\,a^4x^2+3\,a^2x^4+x^6}x}{a^6+2\,a^4x^2+a^2x^4}$$

input
```
integrate(1/((a^2+x^2)^3)^(1/2),x, algorithm="fricas")
```

output
```
(a^4 + 2*a^2*x^2 + x^4 + sqrt(a^6 + 3*a^4*x^2 + 3*a^2*x^4 + x^6)*x)/(a^6 +
 2*a^4*x^2 + a^2*x^4)
```

3.935.6 Sympy [F]

$$\int \frac{1}{\sqrt{(a^2+x^2)^3}}\,dx = \int \frac{1}{\sqrt{(a^2+x^2)^3}}\,dx$$

input
```
integrate(1/((a**2+x**2)**3)**(1/2),x)
```

output
```
Integral(1/sqrt((a**2 + x**2)**3), x)
```

3.935.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \frac{1}{\sqrt{(a^2+x^2)^3}}\,dx = \frac{x}{\sqrt{a^2+x^2}a^2}$$

input
```
integrate(1/((a^2+x^2)^3)^(1/2),x, algorithm="maxima")
```

output
```
x/(sqrt(a^2 + x^2)*a^2)
```

3.935.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \frac{1}{\sqrt{(a^2+x^2)^3}}\,dx = \frac{x}{\sqrt{a^2+x^2}a^2}$$

input
```
integrate(1/((a^2+x^2)^3)^(1/2),x, algorithm="giac")
```

output
```
x/(sqrt(a^2 + x^2)*a^2)
```

3.935.9 Mupad [B] (verification not implemented)

Time = 21.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{(a^2+x^2)^3}}\,dx=\frac{x\sqrt{(a^2+x^2)^3}}{a^2\,(a^2+x^2)^2}$$

input
```
int(1/((a^2 + x^2)^3)^(1/2),x)
```

output
```
(x*((a^2 + x^2)^3)^(1/2))/(a^2*(a^2 + x^2)^2)
```

3.936 $\int \frac{\sqrt{x}}{1+\sqrt{x}+x}\,dx$

3.936.1 Optimal result

Integrand size = 16, antiderivative size = 42

$$\int \frac{\sqrt{x}}{1+\sqrt{x}+x}\,dx=2\sqrt{x}-\frac{2\arctan\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)}{\sqrt{3}}-\log\left(1+\sqrt{x}+x\right)$$

output
```
-ln(1+x+x^(1/2))-2/3*arctan(1/3*(1+2*x^(1/2))*3^(1/2))*3^(1/2)+2*x^(1/2)
```

3.936.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}}{1+\sqrt{x}+x}\,dx=2\sqrt{x}-\frac{2\arctan\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)}{\sqrt{3}}-\log\left(1+\sqrt{x}+x\right)$$

input
```
Integrate[Sqrt[x]/(1 + Sqrt[x] + x),x]
```

output
```
2*Sqrt[x] - (2*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3] - Log[1 + Sqrt[x]
+ x]
```

3.936.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1693, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{x+\sqrt{x}+1} \, dx$$
$$\Big\downarrow \text{ 1693}$$
$$2 \int \frac{x}{x+\sqrt{x}+1} d\sqrt{x}$$
$$\Big\downarrow \text{ 1143}$$
$$2 \int \left(1-\frac{\sqrt{x}+1}{x+\sqrt{x}+1}\right) d\sqrt{x}$$
$$\Big\downarrow \text{ 2009}$$
$$2 \left(-\frac{\arctan\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\sqrt{x}-\frac{1}{2}\log\left(x+\sqrt{x}+1\right)\right)$$

input
```
Int[Sqrt[x]/(1 + Sqrt[x] + x),x]
```

output
```
2*(Sqrt[x] - ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]]/Sqrt[3] - Log[1 + Sqrt[x] + x
]/2)
```

3.936.3.1 Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.936.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

method	result
derivativedivides	$-\ln\left(1+x+\sqrt{x}\right)-\frac{2\arctan\left(\frac{(1+2\sqrt{x})\sqrt{3}}{3}\right)\sqrt{3}}{3}+2\sqrt{x}$
default	$-\ln\left(1+x+\sqrt{x}\right)-\frac{2\arctan\left(\frac{(1+2\sqrt{x})\sqrt{3}}{3}\right)\sqrt{3}}{3}+2\sqrt{x}$
trager	$2\sqrt{x}-2\ln\left(1+x+\sqrt{x}\right)\operatorname{RootOf}\left(3\_Z^2+3\_Z+1\right)+2\ln\left(-3\operatorname{RootOf}\left(3\_Z^2+3\_Z\right.\right.$

input
```
int(x^(1/2)/(1+x+x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-ln(1+x+x^(1/2))-2/3*arctan(1/3*(1+2*x^(1/2))*3^(1/2))*3^(1/2)+2*x^(1/2)
```

3.936.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{x}}{1+\sqrt{x}+x}\,dx = -\frac{2}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\sqrt{x}+\frac{1}{3}\sqrt{3}\right)+2\sqrt{x}-\log\left(x+\sqrt{x}+1\right)$$

input
```
integrate(x^(1/2)/(1+x+x^(1/2)),x, algorithm="fricas")
```

output
```
-2/3*sqrt(3)*arctan(2/3*sqrt(3)*sqrt(x) + 1/3*sqrt(3)) + 2*sqrt(x) - log(x
 + sqrt(x) + 1)
```

3.936.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{x}}{1+\sqrt{x}+x}\,dx = 2\sqrt{x} - \log\left(4\sqrt{x}+4x+4\right) - \frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate(x**(1/2)/(1+x+x**(1/2)),x)
```

output
```
2*sqrt(x) - log(4*sqrt(x) + 4*x + 4) - 2*sqrt(3)*atan(2*sqrt(3)*sqrt(x)/3
+ sqrt(3)/3)/3
```

3.936.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{x}}{1+\sqrt{x}+x}\,dx = -\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\sqrt{x}+1)\right) + 2\sqrt{x} - \log\left(x+\sqrt{x}+1\right)$$

input
```
integrate(x^(1/2)/(1+x+x^(1/2)),x, algorithm="maxima")
```

output
```
-2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + 2*sqrt(x) - log(x + sqr
t(x) + 1)
```

3.936.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{x}}{1+\sqrt{x}+x}\,dx = -\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\sqrt{x}+1)\right) + 2\sqrt{x} - \log\left(x+\sqrt{x}+1\right)$$

input
```
integrate(x^(1/2)/(1+x+x^(1/2)),x, algorithm="giac")
```

output
```
-2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + 2*sqrt(x) - log(x + sqr
t(x) + 1)
```

3.936.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{x}}{1+\sqrt{x}+x}\,dx = 2\sqrt{x} - \frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\sqrt{x}}{3}\right)}{3} - \ln\left(x+\sqrt{x}+1\right)$$

input
```
int(x^(1/2)/(x + x^(1/2) + 1),x)
```

output
```
2*x^(1/2) - (2*3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^(1/2))/3))/3 - log(x
+ x^(1/2) + 1)
```

3.937 $\int \frac{x}{1+\sqrt{x}+x} dx$

3.937.1 Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \frac{x}{1+\sqrt{x}+x} dx = -2\sqrt{x}+x+\frac{4\arctan\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)}{\sqrt{3}}$$

output
```
x+4/3*arctan(1/3*(1+2*x^(1/2))*3^(1/2))*3^(1/2)-2*x^(1/2)
```

3.937.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{x}{1+\sqrt{x}+x} dx = -2\sqrt{x}+x+\frac{4\arctan\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Integrate[x/(1 + Sqrt[x] + x),x]
```

output
```
-2*Sqrt[x] + x + (4*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3]
```

3.937.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1693, 1143, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x+\sqrt{x}+1}dx$$
$$\Big\downarrow \text{ 1693}$$
$$2\int \frac{x^{3/2}}{x+\sqrt{x}+1}d\sqrt{x}$$
$$\Big\downarrow \text{ 1143}$$
$$2\int \left(\sqrt{x}+\frac{1}{x+\sqrt{x}+1}-1\right)d\sqrt{x}$$
$$\Big\downarrow \text{ 2009}$$
$$2\left(\frac{2\arctan\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{x}{2}-\sqrt{x}\right)$$

input
```
Int[x/(1 + Sqrt[x] + x),x]
```

output
```
2*(-Sqrt[x] + x/2 + (2*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3])
```

3.937.3.1 Defintions of rubi rules used

rule 1143
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Int[ExpandIntegrand[(d + e*x)^m/(a + b*x + c*x^2), x], x] /; FreeQ[{a,
b, c, d, e}, x] && IGtQ[m, 1]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.937.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$x+\frac{4\arctan\left(\frac{(1+2\sqrt{x})\sqrt{3}}{3}\right)\sqrt{3}}{3}-2\sqrt{x}$	26
default	$x+\frac{4\arctan\left(\frac{(1+2\sqrt{x})\sqrt{3}}{3}\right)\sqrt{3}}{3}-2\sqrt{x}$	26
trager	$x-1-2\sqrt{x}-\frac{2\,\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+3\right)\sqrt{x}-x+1}{\mathrm{RootOf}\left(\_Z^2+3\right)x-\mathrm{RootOf}\left(\_Z^2+3\right)+3x+3}\right)}{3}$	58

input
```
int(x/(1+x+x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x+4/3*arctan(1/3*(1+2*x^(1/2))*3^(1/2))*3^(1/2)-2*x^(1/2)
```

3.937.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{x}{1+\sqrt{x}+x}\,dx=\frac{4}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\sqrt{x}+\frac{1}{3}\sqrt{3}\right)+x-2\sqrt{x}$$

input
```
integrate(x/(1+x+x^(1/2)),x, algorithm="fricas")
```

output
```
4/3*sqrt(3)*arctan(2/3*sqrt(3)*sqrt(x) + 1/3*sqrt(3)) + x - 2*sqrt(x)
```

3.937.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{x}{1+\sqrt{x}+x}\,dx = -2\sqrt{x}+x+\frac{4\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate(x/(1+x+x**(1/2)),x)
```

output
```
-2*sqrt(x) + x + 4*sqrt(3)*atan(2*sqrt(3)*sqrt(x)/3 + sqrt(3)/3)/3
```

3.937.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{x}{1+\sqrt{x}+x}\,dx = \frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\sqrt{x}+1)\right)+x-2\sqrt{x}$$

input
```
integrate(x/(1+x+x^(1/2)),x, algorithm="maxima")
```

output
```
4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + x - 2*sqrt(x)
```

3.937.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{x}{1+\sqrt{x}+x}\,dx = \frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\sqrt{x}+1)\right)+x-2\sqrt{x}$$

input
```
integrate(x/(1+x+x^(1/2)),x, algorithm="giac")
```

output
```
4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + x - 2*sqrt(x)
```

3.937.9 Mupad [B] (verification not implemented)

Time = 21.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{x}{1+\sqrt{x}+x}\,dx = x + \frac{4\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}+\frac{2\sqrt{3}\sqrt{x}}{3}\right)}{3} - 2\sqrt{x}$$

input
```
int(x/(x + x^(1/2) + 1),x)
```

output
```
x + (4*3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^(1/2))/3))/3 - 2*x^(1/2)
```

3.938 $\int \frac{1}{\sqrt{x}(1+\sqrt{x}+x)^{7/2}} dx$

3.938.1 Optimal result

Integrand size = 18, antiderivative size = 76

$$\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}+x\right)^{7/2}}\,dx=\frac{4(1+2\sqrt{x})}{15\left(1+\sqrt{x}+x\right)^{5/2}}+\frac{64(1+2\sqrt{x})}{135\left(1+\sqrt{x}+x\right)^{3/2}}+\frac{512(1+2\sqrt{x})}{405\sqrt{1+\sqrt{x}+x}}$$

output
```
4/15*(1+2*x^(1/2))/(1+x+x^(1/2))^(5/2)+64/135*(1+2*x^(1/2))/(1+x+x^(1/2))^
(3/2)+512/405*(1+2*x^(1/2))/(1+x+x^(1/2))^(1/2)
```

3.938.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}+x\right)^{7/2}}\,dx=\frac{4(1+2\sqrt{x})\left(203+304\sqrt{x}+432x+256x^{3/2}+128x^2\right)}{405\left(1+\sqrt{x}+x\right)^{5/2}}$$

input
```
Integrate[1/(Sqrt[x]*(1 + Sqrt[x] + x)^(7/2)),x]
```

output
```
(4*(1 + 2*Sqrt[x])*(203 + 304*Sqrt[x] + 432*x + 256*x^(3/2) + 128*x^2))/(4
05*(1 + Sqrt[x] + x)^(5/2))
```

3.938.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1690, 1089, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\left(x+\sqrt{x}+1\right)^{7/2}}dx$$

$$\Big\downarrow \ 1690$$

$$2\int \frac{1}{\left(x+\sqrt{x}+1\right)^{7/2}}d\sqrt{x}$$

$$\Big\downarrow \ 1089$$

$$2\left(\frac{16}{15}\int \frac{1}{\left(x+\sqrt{x}+1\right)^{5/2}}d\sqrt{x}+\frac{2(2\sqrt{x}+1)}{15\left(x+\sqrt{x}+1\right)^{5/2}}\right)$$

$$\Big\downarrow \ 1089$$

$$2\left(\frac{16}{15}\left(\frac{8}{9}\int \frac{1}{\left(x+\sqrt{x}+1\right)^{3/2}}d\sqrt{x}+\frac{2(2\sqrt{x}+1)}{9\left(x+\sqrt{x}+1\right)^{3/2}}\right)+\frac{2(2\sqrt{x}+1)}{15\left(x+\sqrt{x}+1\right)^{5/2}}\right)$$

$$\Big\downarrow \ 1088$$

$$2\left(\frac{2(2\sqrt{x}+1)}{15\left(x+\sqrt{x}+1\right)^{5/2}}+\frac{16}{15}\left(\frac{16(2\sqrt{x}+1)}{27\sqrt{x+\sqrt{x}+1}}+\frac{2(2\sqrt{x}+1)}{9\left(x+\sqrt{x}+1\right)^{3/2}}\right)\right)$$

input
```
Int[1/(Sqrt[x]*(1 + Sqrt[x] + x)^(7/2)),x]
```

output
```
2*((2*(1 + 2*Sqrt[x]))/(15*(1 + Sqrt[x] + x)^(5/2)) + (16*((2*(1 + 2*Sqrt[
x]))/(9*(1 + Sqrt[x] + x)^(3/2)) + (16*(1 + 2*Sqrt[x]))/(27*Sqrt[1 + Sqrt[
x] + x])))/15)
```

3.938.3.1 Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1690
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

3.938.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{\frac{4}{15}+\frac{8\sqrt{x}}{15}}{\left(1+x+\sqrt{x}\right)^{\frac{5}{2}}}+\frac{\frac{64}{135}+\frac{128\sqrt{x}}{135}}{\left(1+x+\sqrt{x}\right)^{\frac{3}{2}}}+\frac{\frac{512}{405}+\frac{1024\sqrt{x}}{405}}{\sqrt{1+x+\sqrt{x}}}$	53
default	$\frac{\frac{4}{15}+\frac{8\sqrt{x}}{15}}{\left(1+x+\sqrt{x}\right)^{\frac{5}{2}}}+\frac{\frac{64}{135}+\frac{128\sqrt{x}}{135}}{\left(1+x+\sqrt{x}\right)^{\frac{3}{2}}}+\frac{\frac{512}{405}+\frac{1024\sqrt{x}}{405}}{\sqrt{1+x+\sqrt{x}}}$	53

input
```
int(1/x^(1/2)/(1+x+x^(1/2))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
4/15*(1+2*x^(1/2))/(1+x+x^(1/2))^(5/2)+64/135*(1+2*x^(1/2))/(1+x+x^(1/2))^
(3/2)+512/405*(1+2*x^(1/2))/(1+x+x^(1/2))^(1/2)
```

3.938.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}+x\right)^{7/2}}\,dx =$$
$$-\frac{4\left(128\,x^{5}+272\,x^{4}+455\,x^{3}+232\,x^{2}-\left(256\,x^{5}+736\,x^{4}+1366\,x^{3}+1427\,x^{2}+839\,x+101\right)\sqrt{x}-128\,x-\right.}{405\left(x^{6}+3\,x^{5}+6\,x^{4}+7\,x^{3}+6\,x^{2}+3\,x+1\right)}$$

input
```
integrate(1/x^(1/2)/(1+x+x^(1/2))^(7/2),x, algorithm="fricas")
```

output
```
-4/405*(128*x^5 + 272*x^4 + 455*x^3 + 232*x^2 - (256*x^5 + 736*x^4 + 1366*
x^3 + 1427*x^2 + 839*x + 101)*sqrt(x) - 128*x - 203)*sqrt(x + sqrt(x) + 1)
/(x^6 + 3*x^5 + 6*x^4 + 7*x^3 + 6*x^2 + 3*x + 1)
```

3.938.6 Sympy [F]

$$\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}+x\right)^{7/2}}\,dx = \int \frac{1}{\sqrt{x}\left(\sqrt{x}+x+1\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/x**(1/2)/(1+x+x**(1/2))**(7/2),x)
```

output
```
Integral(1/(sqrt(x)*(sqrt(x) + x + 1)**(7/2)), x)
```

3.938.7 Maxima [F]

$$\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}+x\right)^{7/2}}\,dx = \int \frac{1}{\left(x+\sqrt{x}+1\right)^{\frac{7}{2}}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(1+x+x^(1/2))^(7/2),x, algorithm="maxima")
```

output
```
integrate(1/((x + sqrt(x) + 1)^(7/2)*sqrt(x)), x)
```

3.938.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.59

$$\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}+x\right)^{7/2}}\,dx = \frac{4\left(2\left(8\left(2\left(4\sqrt{x}(2\sqrt{x}+5)+35\right)\sqrt{x}+65\right)\sqrt{x}+355\right)\sqrt{x}+203\right)}{405\left(x+\sqrt{x}+1\right)^{\frac{5}{2}}}$$

input
```
integrate(1/x^(1/2)/(1+x+x^(1/2))^(7/2),x, algorithm="giac")
```

output
```
4/405*(2*(8*(2*(4*sqrt(x)*(2*sqrt(x) + 5) + 35)*sqrt(x) + 65)*sqrt(x) + 35
5)*sqrt(x) + 203)/(x + sqrt(x) + 1)^(5/2)
```

3.938.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}+x\right)^{7/2}}\,dx = \int \frac{1}{\sqrt{x}\left(x+\sqrt{x}+1\right)^{7/2}}\,dx$$

input
```
int(1/(x^(1/2)*(x + x^(1/2) + 1)^(7/2)),x)
```

output
```
int(1/(x^(1/2)*(x + x^(1/2) + 1)^(7/2)), x)
```

3.939 $\int \frac{-1+x}{1+\sqrt{1+x^2}}\,dx$

3.939.1 Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \frac{-1+x}{1+\sqrt{1+x^2}}\,dx=-\frac{1}{x}+\sqrt{1+x^2}+\frac{\sqrt{1+x^2}}{x}-\operatorname{arcsinh}(x)-\log\left(1+\sqrt{1+x^2}\right)$$

output
```
-1/x-arcsinh(x)-ln(1+(x^2+1)^(1/2))+(x^2+1)^(1/2)+(x^2+1)^(1/2)/x
```

3.939.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{-1+x}{1+\sqrt{1+x^2}}\,dx=-\frac{1}{x}+\frac{(1+x)\sqrt{1+x^2}}{x}-4\text{arctanh}\left(1-2x+2\sqrt{1+x^2}\right)$$

input
```
Integrate[(-1 + x)/(1 + Sqrt[1 + x^2]),x]
```

output
```
-x^(-1) + ((1 + x)*Sqrt[1 + x^2])/x - 4*ArcTanh[1 - 2*x + 2*Sqrt[1 + x^2]]
```

3.939.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-1}{\sqrt{x^2+1}+1} \, dx$$
$$\Big\downarrow \text{ 7293}$$
$$\int \left(\frac{x}{\sqrt{x^2+1}+1}-\frac{1}{\sqrt{x^2+1}+1}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\text{arcsinh}(x)+\frac{\sqrt{x^2+1}}{x}+\sqrt{x^2+1}-\log\left(\sqrt{x^2+1}+1\right)-\frac{1}{x}$$

input
```
Int[(-1 + x)/(1 + Sqrt[1 + x^2]),x]
```

output
```
-x^(-1) + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x - ArcSinh[x] - Log[1 + Sqrt[1 +
x^2]]
```

3.939.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.939.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

method	result	size
trager	$\frac{x-1}{x}+\frac{(x+1)\sqrt{x^2+1}}{x}+2\ln\left(-\frac{\sqrt{x^2+1}-1-x}{x}\right)$	43
default	$-\frac{1}{x}+\sqrt{x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)-\ln\left(x\right)+\frac{\left(x^2+1\right)^{\frac{3}{2}}}{x}-x\sqrt{x^2+1}-\operatorname{arcsinh}\left(x\right)$	53
meijerg	$-\frac{x\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},1;\frac{3}{2},2;-x^2\right)}{2}+\frac{-4\sqrt{\pi}+4\sqrt{\pi}\,\sqrt{x^2+1}-4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^2+1}}{2}\right)}{4\sqrt{\pi}}$	58

input
```
int((x-1)/(1+(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
(x-1)/x+(x+1)/x*(x^2+1)^(1/2)+2*ln(-((x^2+1)^(1/2)-1-x)/x)
```

3.939.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.39

$$\int \frac{-1+x}{1+\sqrt{1+x^2}}\,dx$$
$$=\frac{x\log\left(2\,x^2-\sqrt{x^2+1}(2\,x+1)+x+1\right)-x\log\left(x\right)-x\log\left(-x+\sqrt{x^2+1}+1\right)+\sqrt{x^2+1}(x+1)+x-}{x}$$

input
```
integrate((-1+x)/(1+(x^2+1)^(1/2)),x, algorithm="fricas")
```

output
```
(x*log(2*x^2 - sqrt(x^2 + 1)*(2*x + 1) + x + 1) - x*log(x) - x*log(-x + sq
rt(x^2 + 1) + 1) + sqrt(x^2 + 1)*(x + 1) + x - 1)/x
```

3.939.6 Sympy [A] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \frac{-1+x}{1+\sqrt{1+x^2}}\,dx=\frac{x}{\sqrt{x^2+1}}+\sqrt{x^2+1}-\log\left(\sqrt{x^2+1}+1\right)-\operatorname{asinh}\left(x\right)-\frac{1}{x}+\frac{1}{x\sqrt{x^2+1}}$$

input
```
integrate((-1+x)/(1+(x**2+1)**(1/2)),x)
```

output
```
x/sqrt(x**2 + 1) + sqrt(x**2 + 1) - log(sqrt(x**2 + 1) + 1) - asinh(x) - 1
/x + 1/(x*sqrt(x**2 + 1))
```

3.939.7 Maxima [F]

$$\int \frac{-1+x}{1+\sqrt{1+x^2}}\,dx = \int \frac{x-1}{\sqrt{x^2+1}+1}\,dx$$

input
```
integrate((-1+x)/(1+(x^2+1)^(1/2)),x, algorithm="maxima")
```

output
```
1/4*x^2 - 1/2*x - integrate(1/2*(x^3 - x^2)/(x^2 + 2*sqrt(x^2 + 1) + 2), x
)
```

3.939.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.72

$$\int \frac{-1+x}{1+\sqrt{1+x^2}}\,dx = \sqrt{x^2+1} - \frac{2}{\left(x-\sqrt{x^2+1}\right)^2-1} - \frac{1}{x} + \log\left(-x+\sqrt{x^2+1}\right) - \log\left(|x|\right) - \log\left(\left|-x+\sqrt{x^2+1}+1\right|\right) + \log\left(\left|-x+\sqrt{x^2+1}-1\right|\right)$$

input
```
integrate((-1+x)/(1+(x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
sqrt(x^2 + 1) - 2/((x - sqrt(x^2 + 1))^2 - 1) - 1/x + log(-x + sqrt(x^2 +
1)) - log(abs(x)) - log(abs(-x + sqrt(x^2 + 1) + 1)) + log(abs(-x + sqrt(x
^2 + 1) - 1))
```

3.939.9 Mupad [B] (verification not implemented)

Time = 20.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{-1+x}{1+\sqrt{1+x^2}}\,dx = \sqrt{x^2+1}-\ln(x)-\operatorname{asinh}(x)+\frac{\sqrt{x^2+1}}{x}-\frac{1}{x}+\operatorname{atan}\left(\sqrt{x^2+1}\,1\mathrm{i}\right)\,1\mathrm{i}$$

input
```
int((x - 1)/((x^2 + 1)^(1/2) + 1),x)
```

output
```
atan((x^2 + 1)^(1/2)*1i)*1i - asinh(x) - log(x) + (x^2 + 1)^(1/2) + (x^2 +
 1)^(1/2)/x - 1/x
```

3.940 $\int \frac{1}{(1+x)^{2/3}(-1+x^2)^{2/3}}\,dx$

3.940.1 Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \frac{1}{(1+x)^{2/3}\left(-1+x^2\right)^{2/3}}\,dx=\frac{3\sqrt[3]{-1+x^2}}{2(1+x)^{2/3}}$$

output
```
3/2*(x^2-1)^(1/3)/(1+x)^(2/3)
```

3.940.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x)^{2/3}\left(-1+x^2\right)^{2/3}}\,dx=\frac{3\sqrt[3]{-1+x^2}}{2(1+x)^{2/3}}$$

input
```
Integrate[1/((1 + x)^(2/3)*(-1 + x^2)^(2/3)),x]
```

output
```
(3*(-1 + x^2)^(1/3))/(2*(1 + x)^(2/3))
```

3.940.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {460}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+1)^{2/3}\left(x^2-1\right)^{2/3}}\,dx$$
$$\Big\downarrow \text{ 460}$$
$$\frac{3\sqrt[3]{x^2-1}}{2(x+1)^{2/3}}$$

input
```
Int[1/((1 + x)^(2/3)*(-1 + x^2)^(2/3)),x]
```

output
```
(3*(-1 + x^2)^(1/3))/(2*(1 + x)^(2/3))
```

3.940.3.1 Defintions of rubi rules used

rule 460
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-d)*(c + d*x)^n*((a + b*x^2)^(p + 1)/(b*c*n)), x] /; FreeQ[{a, b, c, d, n,
 p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + 2*p + 2, 0]
```

3.940.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

method	result	size
gosper	$\frac{3(x-1)(x+1)^{\frac{1}{3}}}{2(x^2-1)^{\frac{2}{3}}}$	18
risch	$\frac{3(x+1)^{\frac{1}{3}}\left(\frac{(x^2-1)^2}{x+1}\right)^{\frac{1}{3}}(x-1)}{2(x^2-1)^{\frac{2}{3}}\left((x+1)(x-1)^2\right)^{\frac{1}{3}}}$	44

input
```
int(1/(x+1)^(2/3)/(x^2-1)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
3/2*(x-1)*(x+1)^(1/3)/(x^2-1)^(2/3)
```

3.940.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{1}{(1+x)^{2/3}(-1+x^2)^{2/3}}\,dx = \frac{3\,(x^2-1)^{\frac{1}{3}}}{2\,(x+1)^{\frac{2}{3}}}$$

input
```
integrate(1/(1+x)^(2/3)/(x^2-1)^(2/3),x, algorithm="fricas")
```

output
```
3/2*(x^2 - 1)^(1/3)/(x + 1)^(2/3)
```

3.940.6 Sympy [F]

$$\int \frac{1}{(1+x)^{2/3}(-1+x^2)^{2/3}}\,dx = \int \frac{1}{((x-1)\,(x+1))^{\frac{2}{3}}\,(x+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(1+x)**(2/3)/(x**2-1)**(2/3),x)
```

output
```
Integral(1/(((x - 1)*(x + 1))**(2/3)*(x + 1)**(2/3)), x)
```

3.940.7 Maxima [F]

$$\int \frac{1}{(1+x)^{2/3}(-1+x^2)^{2/3}}\,dx = \int \frac{1}{(x^2-1)^{\frac{2}{3}}(x+1)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(1+x)^(2/3)/(x^2-1)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((x^2 - 1)^(2/3)*(x + 1)^(2/3)), x)
```

3.940.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.65

$$\int \frac{1}{(1+x)^{2/3}\left(-1+x^2\right)^{2/3}}\,dx = \frac{3}{2}\left(-\frac{2}{x+1}+1\right)^{\frac{1}{3}}$$

input
```
integrate(1/(1+x)^(2/3)/(x^2-1)^(2/3),x, algorithm="giac")
```

output
```
3/2*(-2/(x + 1) + 1)^(1/3)
```

3.940.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x)^{2/3}\left(-1+x^2\right)^{2/3}}\,dx = \int \frac{1}{\left(x^2-1\right)^{2/3}\left(x+1\right)^{2/3}}\,dx$$

input
```
int(1/((x^2 - 1)^(2/3)*(x + 1)^(2/3)),x)
```

output
```
int(1/((x^2 - 1)^(2/3)*(x + 1)^(2/3)), x)
```

3.941 $\int \left((1-x^6)^{2/3}+\frac{(1-x^6)^{2/3}}{x^6}\right) dx$

3.941.1 Optimal result

Integrand size = 27, antiderivative size = 35

$$\int \left((1-x^6)^{2/3}+\frac{(1-x^6)^{2/3}}{x^6}\right) dx=-\frac{(1-x^6)^{2/3}}{5x^5}+\frac{1}{5}x(1-x^6)^{2/3}$$

output
```
-1/5*(-x^6+1)^(2/3)/x^5+1/5*x*(-x^6+1)^(2/3)
```

3.941.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.51

$$\int \left((1-x^6)^{2/3}+\frac{(1-x^6)^{2/3}}{x^6}\right) dx=-\frac{(1-x^6)^{5/3}}{5x^5}$$

input
```
Integrate[(1 - x^6)^(2/3) + (1 - x^6)^(2/3)/x^6,x]
```

output
```
-1/5*(1 - x^6)^(5/3)/x^5
```

3.941.3 Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 2 in optimal.

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{(1-x^6)^{2/3}}{x^6}+(1-x^6)^{2/3}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$x\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{6},\frac{7}{6},x^6\right)-\frac{\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{2}{3},\frac{1}{6},x^6\right)}{5x^5}$$

input
```
Int[(1 - x^6)^(2/3) + (1 - x^6)^(2/3)/x^6,x]
```

output
```
-1/5*Hypergeometric2F1[-5/6, -2/3, 1/6, x^6]/x^5 + x*Hypergeometric2F1[-2/
3, 1/6, 7/6, x^6]
```

3.941.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.941.4 Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.57

method	result	size
trager	$\frac{(x^6-1)(-x^6+1)^{\frac{2}{3}}}{5x^5}$	20
risch	$-\frac{x^{12}-2x^6+1}{5x^5(-x^6+1)^{\frac{1}{3}}}$	25
meijerg	$x\,{}_2F_1\left(-\frac{2}{3},\frac{1}{6};\frac{7}{6};x^6\right)-\frac{{}_2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};x^6\right)}{5x^5}$	27
gosper	$\frac{(-x^6+1)^{\frac{2}{3}}(x^2-x+1)(x^2+x+1)(x+1)(x-1)}{5x^5}$	35

input
```
int((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/5*(x^6-1)/x^5*(-x^6+1)^(2/3)
```

3.941.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.54

$$\int \left(\left(1-x^6\right)^{2/3}+\frac{\left(1-x^6\right)^{2/3}}{x^6}\right) dx = \frac{(x^6-1)(-x^6+1)^{\frac{2}{3}}}{5\,x^5}$$

input
```
integrate((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x, algorithm="fricas")
```

output
```
1/5*(x^6 - 1)*(-x^6 + 1)^(2/3)/x^5
```

3.941.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.94

$$\int \left(\left(1-x^6\right)^{2/3} + \frac{\left(1-x^6\right)^{2/3}}{x^6}\right) dx = \frac{x\Gamma\left(\frac{1}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{6} \\ \frac{7}{6} \end{matrix} \middle| x^6 e^{2i\pi}\right)}{6\Gamma\left(\frac{7}{6}\right)} + \frac{\Gamma\left(-\frac{5}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{6}, -\frac{2}{3} \\ \frac{1}{6} \end{matrix} \middle| x^6 e^{2i\pi}\right)}{6x^5\Gamma\left(\frac{1}{6}\right)}$$

input
```
integrate((-x**6+1)**(2/3)+(-x**6+1)**(2/3)/x**6,x)
```

output
```
x*gamma(1/6)*hyper((-2/3, 1/6), (7/6,), x**6*exp_polar(2*I*pi))/(6*gamma(7
/6)) + gamma(-5/6)*hyper((-5/6, -2/3), (1/6,), x**6*exp_polar(2*I*pi))/(6*
x**5*gamma(1/6))
```

3.941.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \left((1-x^6)^{2/3}+\frac{(1-x^6)^{2/3}}{x^6}\right)\,dx=\frac{(x^6-1)(x^2+x+1)^{\frac{2}{3}}(-x^2+x-1)^{\frac{2}{3}}(x+1)^{\frac{2}{3}}(x-1)^{\frac{2}{3}}}{5\,x^5}$$

input
```
integrate((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x, algorithm="maxima")
```

output
```
1/5*(x^6 - 1)*(x^2 + x + 1)^(2/3)*(-x^2 + x - 1)^(2/3)*(x + 1)^(2/3)*(x -
1)^(2/3)/x^5
```

3.941.8 Giac [F]

$$\int \left((1-x^6)^{2/3}+\frac{(1-x^6)^{2/3}}{x^6}\right)\,dx=\int (-x^6+1)^{\frac{2}{3}}+\frac{(-x^6+1)^{\frac{2}{3}}}{x^6}\,dx$$

input
```
integrate((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x, algorithm="giac")
```

output
```
integrate((-x^6 + 1)^(2/3) + (-x^6 + 1)^(2/3)/x^6, x)
```

3.941.9 Mupad [B] (verification not implemented)

Time = 19.88 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.40

$$\int \left((1-x^6)^{2/3}+\frac{(1-x^6)^{2/3}}{x^6}\right)\,dx=-\frac{(1-x^6)^{5/3}}{5\,x^5}$$

input
```
int((1 - x^6)^(2/3)/x^6 + (1 - x^6)^(2/3),x)
```

output
```
-(1 - x^6)^(5/3)/(5*x^5)
```

3.942 $\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2(a+bx^n)^{3/2}}\,dx$

3.942.1 Optimal result

Integrand size = 37, antiderivative size = 15

$$\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2\,(a+bx^n)^{3/2}}\,dx=\frac{x^m}{\sqrt{a+bx^n}}$$

output
```
x^m/(a+b*x^n)^(1/2)
```

3.942.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.19 (sec) , antiderivative size = 111, normalized size of antiderivative = 7.40

$$\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2\,(a+bx^n)^{3/2}}\,dx=\frac{x^m\sqrt{1+\frac{bx^n}{a}}\left(2a(m+n)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m}{n},\frac{m+n}{n},-\frac{bx^n}{a}\right)+b(2\cdots\right.}{2a(m+n)\sqrt{a+b\cdots}}$$

input
```
Integrate[(x^(-1 + m)*(2*a*m + b*(2*m - n)*x^n))/(2*(a + b*x^n)^(3/2)),x]
```

output
```
(x^m*Sqrt[1 + (b*x^n)/a]*(2*a*(m + n)*Hypergeometric2F1[3/2, m/n, (m + n)/
n, -((b*x^n)/a)] + b*(2*m - n)*x^n*Hypergeometric2F1[3/2, (m + n)/n, 2 + m
/n, -((b*x^n)/a)]))/(2*a*(m + n)*Sqrt[a + b*x^n])
```

3.942.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {27, 951}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{m-1}(2am+b(2m-n)x^n)}{2\left(a+bx^n\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\int \frac{x^{m-1}(b(2m-n)x^n+2am)}{(bx^n+a)^{3/2}}dx$$

$$\Big\downarrow \text{ 951}$$

$$\frac{x^m}{\sqrt{a+bx^n}}$$

input
```
Int[(x^(-1 + m)*(2*a*m + b*(2*m - n)*x^n))/(2*(a + b*x^n)^(3/2)),x]
```

output
```
x^m/Sqrt[a + b*x^n]
```

3.942.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 951
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

3.942.4 Maple [F]

$$\int \frac{x^{-1+m}(2am+b(2m-n)\,x^{n})}{2\,(a+b\,x^{n})^{\frac{3}{2}}}dx$$

input
```
int(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x)
```

output
```
int(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x)
```

3.942.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{x^{-1+m}(2am+b(2m-n)x^{n})}{2\,(a+bx^{n})^{3/2}}\,dx=\frac{xx^{m-1}}{\sqrt{bx^{n}+a}}$$

input
```
integrate(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x, algorithm=
"fricas")
```

output
```
x*x^(m - 1)/sqrt(b*x^n + a)
```

3.942.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.33 (sec) , antiderivative size = 122, normalized size of antiderivative = 8.13

$$\int \frac{x^{-1+m}(2am+b(2m-n)x^{n})}{2\,(a+bx^{n})^{3/2}}\,dx=\frac{aa^{\frac{m}{n}}a^{-\frac{m}{n}-\frac{3}{2}}mx^{m}\Gamma\left(\frac{m}{n}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{3}{2},\frac{m}{n}\\ \frac{m}{n}+1\end{matrix}\right|\frac{bx^{n}e^{i\pi}}{a}\right)}{n\Gamma\left(\frac{m}{n}+1\right)}+\frac{a^{-\frac{m}{n}-\frac{5}{2}}a^{\frac{m}{n}+1}bx^{m+n}(2m-n)\,\Gamma\left(\frac{m}{n}+1\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{3}{2},\frac{m}{n}+1\\ \frac{m}{n}+2\end{matrix}\right|\frac{bx^{n}e^{i\pi}}{a}\right)}{2n\Gamma\left(\frac{m}{n}+2\right)}$$

input
```
integrate(1/2*x**(-1+m)*(2*a*m+b*(2*m-n)*x**n)/(a+b*x**n)**(3/2),x)
```

output
```
a*a**(m/n)*a**(-m/n - 3/2)*m*x**m*gamma(m/n)*hyper((3/2, m/n), (m/n + 1,),
 b*x**n*exp_polar(I*pi)/a)/(n*gamma(m/n + 1)) + a**(-m/n - 5/2)*a**(m/n +
1)*b*x**(m + n)*(2*m - n)*gamma(m/n + 1)*hyper((3/2, m/n + 1), (m/n + 2,),
 b*x**n*exp_polar(I*pi)/a)/(2*n*gamma(m/n + 2))
```

3.942.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2\left(a+bx^n\right)^{3/2}}\,dx=\frac{x^m}{\sqrt{bx^n+a}}$$

input
```
integrate(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x, algorithm=
"maxima")
```

output
```
x^m/sqrt(b*x^n + a)
```

3.942.8 Giac [F]

$$\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2\left(a+bx^n\right)^{3/2}}\,dx=\int \frac{(b(2\,m-n)x^n+2\,am)x^{m-1}}{2\,(bx^n+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x, algorithm=
"giac")
```

output
```
integrate(1/2*(b*(2*m - n)*x^n + 2*a*m)*x^(m - 1)/(b*x^n + a)^(3/2), x)
```

3.942.9 Mupad [B] (verification not implemented)

Time = 20.46 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2\left(a+bx^n\right)^{3/2}}\,dx=\frac{x^m}{\sqrt{a+b\,x^n}}$$

input
```
int((x^(m - 1)*(2*a*m + b*x^n*(2*m - n)))/(2*(a + b*x^n)^(3/2)),x)
```

output
```
x^m/(a + b*x^n)^(1/2)
```

3.942. $\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2(a+bx^n)^{3/2}}\,dx$

3.943 $\int \frac{x-2x^3}{\sqrt{2+3x}} \, dx$

3.943.1 Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \frac{x-2x^3}{\sqrt{2+3x}} \, dx = -\frac{4}{81}\sqrt{2+3x} - \frac{10}{81}(2+3x)^{3/2} + \frac{8}{135}(2+3x)^{5/2} - \frac{4}{567}(2+3x)^{7/2}$$

output
```
-10/81*(2+3*x)^(3/2)+8/135*(2+3*x)^(5/2)-4/567*(2+3*x)^(7/2)-4/81*(2+3*x)^
(1/2)
```

3.943.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.53

$$\int \frac{x-2x^3}{\sqrt{2+3x}} \, dx = -\frac{2\sqrt{2+3x}(164-123x-216x^2+270x^3)}{2835}$$

input
```
Integrate[(x - 2*x^3)/Sqrt[2 + 3*x],x]
```

output
```
(-2*Sqrt[2 + 3*x]*(164 - 123*x - 216*x^2 + 270*x^3))/2835
```

3.943.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2027, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-2x^3}{\sqrt{3x+2}}dx$$

$$\Big\downarrow \ 2027$$

$$\int \frac{x\left(1-2x^2\right)}{\sqrt{3x+2}}dx$$

$$\Big\downarrow \ 522$$

$$\int \left(-\frac{2}{27}(3x+2)^{5/2}+\frac{4}{9}(3x+2)^{3/2}-\frac{5}{9}\sqrt{3x+2}-\frac{2}{27\sqrt{3x+2}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{4}{567}(3x+2)^{7/2}+\frac{8}{135}(3x+2)^{5/2}-\frac{10}{81}(3x+2)^{3/2}-\frac{4}{81}\sqrt{3x+2}$$

input
```
Int[(x - 2*x^3)/Sqrt[2 + 3*x],x]
```

output
```
(-4*Sqrt[2 + 3*x])/81 - (10*(2 + 3*x)^(3/2))/81 + (8*(2 + 3*x)^(5/2))/135
- (4*(2 + 3*x)^(7/2))/567
```

3.943.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.943.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.45

method	result	size
trager	$\left(-\frac{4}{21}x^{3}+\frac{16}{105}x^{2}+\frac{82}{945}x-\frac{328}{2835}\right)\sqrt{3x+2}$	24
gosper	$-\frac{2\left(270x^{3}-216x^{2}-123x+164\right)\sqrt{3x+2}}{2835}$	25
risch	$-\frac{2\left(270x^{3}-216x^{2}-123x+164\right)\sqrt{3x+2}}{2835}$	25
pseudoelliptic	$-\frac{2\left(270x^{3}-216x^{2}-123x+164\right)\sqrt{3x+2}}{2835}$	25
derivativedivides	$-\frac{10(3x+2)^{\frac{3}{2}}}{81}+\frac{8(3x+2)^{\frac{5}{2}}}{135}-\frac{4(3x+2)^{\frac{7}{2}}}{567}-\frac{4\sqrt{3x+2}}{81}$	38
default	$-\frac{10(3x+2)^{\frac{3}{2}}}{81}+\frac{8(3x+2)^{\frac{5}{2}}}{135}-\frac{4(3x+2)^{\frac{7}{2}}}{567}-\frac{4\sqrt{3x+2}}{81}$	38
meijerg	$-\frac{16\sqrt{2}\left(\frac{32\sqrt{\pi}}{35}-\frac{\sqrt{\pi}\left(-135x^{3}+108x^{2}-96x+128\right)\sqrt{1+\frac{3x}{2}}}{140}\right)}{81\sqrt{\pi}}+\frac{2\sqrt{2}\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(-6x+8\right)\sqrt{1+\frac{3x}{2}}}{6}\right)}{9\sqrt{\pi}}$	74

input
```
int((-2*x^3+x)/(3*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-4/21*x^3+16/105*x^2+82/945*x-328/2835)*(3*x+2)^(1/2)
```

3.943.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.45

$$\int \frac{x-2x^3}{\sqrt{2+3x}}\,dx=-\frac{2}{2835}\left(270\,x^{3}-216\,x^{2}-123\,x+164\right)\sqrt{3\,x+2}$$

input
```
integrate((-2*x^3+x)/(2+3*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/2835*(270*x^3 - 216*x^2 - 123*x + 164)*sqrt(3*x + 2)
```

3.943.6 Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \frac{x-2x^3}{\sqrt{2+3x}}\,dx = -\frac{4(3x+2)^{\frac{7}{2}}}{567}+\frac{8(3x+2)^{\frac{5}{2}}}{135}-\frac{10(3x+2)^{\frac{3}{2}}}{81}-\frac{4\sqrt{3x+2}}{81}$$

input
```
integrate((-2*x**3+x)/(2+3*x)**(1/2),x)
```

output
```
-4*(3*x + 2)**(7/2)/567 + 8*(3*x + 2)**(5/2)/135 - 10*(3*x + 2)**(3/2)/81
- 4*sqrt(3*x + 2)/81
```

3.943.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{x-2x^3}{\sqrt{2+3x}}\,dx = -\frac{4}{567}\,(3\,x+2)^{\frac{7}{2}}+\frac{8}{135}\,(3\,x+2)^{\frac{5}{2}}-\frac{10}{81}\,(3\,x+2)^{\frac{3}{2}}-\frac{4}{81}\,\sqrt{3\,x+2}$$

input
```
integrate((-2*x^3+x)/(2+3*x)^(1/2),x, algorithm="maxima")
```

output
```
-4/567*(3*x + 2)^(7/2) + 8/135*(3*x + 2)^(5/2) - 10/81*(3*x + 2)^(3/2) - 4
/81*sqrt(3*x + 2)
```

3.943.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{x-2x^3}{\sqrt{2+3x}}\,dx = -\frac{4}{567}\,(3\,x+2)^{\frac{7}{2}}+\frac{8}{135}\,(3\,x+2)^{\frac{5}{2}}-\frac{10}{81}\,(3\,x+2)^{\frac{3}{2}}-\frac{4}{81}\,\sqrt{3\,x+2}$$

input
```
integrate((-2*x^3+x)/(2+3*x)^(1/2),x, algorithm="giac")
```

output
```
-4/567*(3*x + 2)^(7/2) + 8/135*(3*x + 2)^(5/2) - 10/81*(3*x + 2)^(3/2) - 4
/81*sqrt(3*x + 2)
```

3.943.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.70

$$\int \frac{x-2x^3}{\sqrt{2+3x}}\,dx = \frac{8\,(3\,x+2)^{5/2}}{135} - \frac{10\,(3\,x+2)^{3/2}}{81} - \frac{4\,\sqrt{3\,x+2}}{81} - \frac{4\,(3\,x+2)^{7/2}}{567}$$

input
```
int((x - 2*x^3)/(3*x + 2)^(1/2),x)
```

output
```
(8*(3*x + 2)^(5/2))/135 - (10*(3*x + 2)^(3/2))/81 - (4*(3*x + 2)^(1/2))/81
 - (4*(3*x + 2)^(7/2))/567
```

3.944 $\int \frac{1}{\sqrt[4]{1+x}+\sqrt{1+x}}\,dx$

3.944.1 Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \frac{1}{\sqrt[4]{1+x}+\sqrt{1+x}}\,dx = -4\sqrt[4]{1+x}+2\sqrt{1+x}+4\log\left(1+\sqrt[4]{1+x}\right)$$

output
```
-4*(1+x)^(1/4)+4*ln(1+(1+x)^(1/4))+2*(1+x)^(1/2)
```

3.944.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[4]{1+x}+\sqrt{1+x}}\,dx = 2\left(-2\sqrt[4]{1+x}+\sqrt{1+x}+2\log\left(1+\sqrt[4]{1+x}\right)\right)$$

input
```
Integrate[((1 + x)^(1/4) + Sqrt[1 + x])^(-1),x]
```

output
```
2*(-2*(1 + x)^(1/4) + Sqrt[1 + x] + 2*Log[1 + (1 + x)^(1/4)])
```

3.944.3 Rubi [A] (warning: unable to verify)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1918, 2027, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x+1}+\sqrt[4]{x+1}}dx$$

$$\Big\downarrow \text{1918}$$

$$\int \frac{1}{\sqrt{x+1}+\sqrt[4]{x+1}}d(x+1)$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{1}{\sqrt[4]{x+1}\left(\sqrt[4]{x+1}+1\right)}d(x+1)$$

$$\Big\downarrow \text{798}$$

$$4\int \frac{\sqrt{x+1}}{x+2}d\sqrt[4]{x+1}$$

$$\Big\downarrow \text{49}$$

$$4\int \left(x+\frac{1}{x+2}\right)d\sqrt[4]{x+1}$$

$$\Big\downarrow \text{2009}$$

$$4\left(-x+\frac{\sqrt{x+1}}{2}+\log(x+2)-1\right)$$

input
```
Int[((1 + x)^(1/4) + Sqrt[1 + x])^(-1),x]
```

output
```
4*(-1 - x + Sqrt[1 + x]/2 + Log[2 + x])
```

3.944.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1918
```
Int[((a_.)*(u_)^(j_.) + (b_.)*(u_)^(n_.))^(p_), x_Symbol] :> Simp[1/Coeffic
ient[u, x, 1]   Subst[Int[(a*x^j + b*x^n)^p, x], x, u], x] /; FreeQ[{a, b,
j, n, p}, x] && LinearQ[u, x] && NeQ[u, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.944.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$-4(x+1)^{\frac{1}{4}}+4\ln\left(1+(x+1)^{\frac{1}{4}}\right)+2\sqrt{x+1}$	26
default	$-4(x+1)^{\frac{1}{4}}+4\ln\left(1+(x+1)^{\frac{1}{4}}\right)+2\sqrt{x+1}$	26

input
```
int(1/((x+1)^(1/4)+(x+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-4*(x+1)^(1/4)+4*ln(1+(x+1)^(1/4))+2*(x+1)^(1/2)
```

3.944.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt[4]{1+x}+\sqrt{1+x}}\,dx = 2\sqrt{x+1} - 4\,(x+1)^{\frac{1}{4}} + 4\,\log\left((x+1)^{\frac{1}{4}}+1\right)$$

input
```
integrate(1/((1+x)^(1/4)+(1+x)^(1/2)),x, algorithm="fricas")
```

output
```
2*sqrt(x + 1) - 4*(x + 1)^(1/4) + 4*log((x + 1)^(1/4) + 1)
```

3.944.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt[4]{1+x}+\sqrt{1+x}}\,dx = -4\sqrt[4]{x+1} + 2\sqrt{x+1} + 4\log\left(\sqrt[4]{x+1}+1\right)$$

input
```
integrate(1/((1+x)**(1/4)+(1+x)**(1/2)),x)
```

output
```
-4*(x + 1)**(1/4) + 2*sqrt(x + 1) + 4*log((x + 1)**(1/4) + 1)
```

3.944.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt[4]{1+x}+\sqrt{1+x}}\,dx = 2\sqrt{x+1} - 4\,(x+1)^{\frac{1}{4}} + 4\,\log\left((x+1)^{\frac{1}{4}}+1\right)$$

input
```
integrate(1/((1+x)^(1/4)+(1+x)^(1/2)),x, algorithm="maxima")
```

output
```
2*sqrt(x + 1) - 4*(x + 1)^(1/4) + 4*log((x + 1)^(1/4) + 1)
```

3.944.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt[4]{1+x}+\sqrt{1+x}}\,dx = 2\sqrt{x+1} - 4\,(x+1)^{\frac{1}{4}} + 4\,\log\left((x+1)^{\frac{1}{4}}+1\right)$$

input
```
integrate(1/((1+x)^(1/4)+(1+x)^(1/2)),x, algorithm="giac")
```

output
```
2*sqrt(x + 1) - 4*(x + 1)^(1/4) + 4*log((x + 1)^(1/4) + 1)
```

3.944.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt[4]{1+x}+\sqrt{1+x}}\,dx = 4\,\ln\left((x+1)^{1/4}+1\right) + 2\sqrt{x+1} - 4\,(x+1)^{1/4}$$

input
```
int(1/((x + 1)^(1/2) + (x + 1)^(1/4)),x)
```

output
```
4*log((x + 1)^(1/4) + 1) + 2*(x + 1)^(1/2) - 4*(x + 1)^(1/4)
```

3.945 $\int \frac{1+2x}{\sqrt{x+x^2}} \, dx$

3.945.1 Optimal result

Integrand size = 15, antiderivative size = 11

$$\int \frac{1+2x}{\sqrt{x+x^2}} \, dx = 2\sqrt{x+x^2}$$

output
```
2*(x^2+x)^(1/2)
```

3.945.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1+2x}{\sqrt{x+x^2}} \, dx = 2\sqrt{x(1+x)}$$

input
```
Integrate[(1 + 2*x)/Sqrt[x + x^2],x]
```

output
```
2*Sqrt[x*(1 + x)]
```

3.945.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+1}{\sqrt{x^2+x}}\,dx$$

$$\Big\downarrow \text{1104}$$

$$2\sqrt{x^2+x}$$

input
```
Int[(1 + 2*x)/Sqrt[x + x^2],x]
```

output
```
2*Sqrt[x + x^2]
```

3.945.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

3.945.4 Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$2\sqrt{x^2+x}$	10
default	$2\sqrt{x^2+x}$	10
trager	$2\sqrt{x^2+x}$	10
pseudoelliptic	$2\sqrt{(x+1)\,x}$	10
gosper	$\frac{2(x+1)x}{\sqrt{x^2+x}}$	14
risch	$\frac{2(x+1)x}{\sqrt{(x+1)x}}$	14
meijerg	$2\,\text{arcsinh}\left(\sqrt{x}\right)+\frac{2\sqrt{\pi}\,\sqrt{x}\,\sqrt{x+1}-2\sqrt{\pi}\,\text{arcsinh}\left(\sqrt{x}\right)}{\sqrt{\pi}}$	35

input
```
int((1+2*x)/(x^2+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(x^2+x)^(1/2)
```

3.945.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1+2x}{\sqrt{x+x^2}}\,dx = 2\sqrt{x^2+x}$$

input
```
integrate((1+2*x)/(x^2+x)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(x^2 + x)
```

3.945.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1+2x}{\sqrt{x+x^2}}\,dx = 2\sqrt{x^2+x}$$

input
```
integrate((1+2*x)/(x**2+x)**(1/2),x)
```

output
```
2*sqrt(x**2 + x)
```

3.945.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1+2x}{\sqrt{x+x^2}}\,dx = 2\sqrt{x^2+x}$$

input
```
integrate((1+2*x)/(x^2+x)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(x^2 + x)
```

3.945.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1+2x}{\sqrt{x+x^2}}\,dx = 2\sqrt{x^2+x}$$

input
```
integrate((1+2*x)/(x^2+x)^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(x^2 + x)
```

3.945.9 Mupad [B] (verification not implemented)

Time = 21.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1+2x}{\sqrt{x+x^2}}\,dx = 2\sqrt{x\,(x+1)}$$

input
```
int((2*x + 1)/(x + x^2)^(1/2),x)
```

output
```
2*(x*(x + 1))^(1/2)
```

3.946 $\int \frac{1}{2\sqrt{x}(1+x)} dx$

3.946.1 Optimal result

Integrand size = 14, antiderivative size = 6

$$\int \frac{1}{2\sqrt{x}(1+x)} dx = \arctan\left(\sqrt{x}\right)$$

output
```
arctan(x^(1/2))
```

3.946.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{2\sqrt{x}(1+x)} dx = \arctan\left(\sqrt{x}\right)$$

input
```
Integrate[1/(2*Sqrt[x]*(1 + x)),x]
```

output
```
ArcTan[Sqrt[x]]
```

3.946.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {27, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2\sqrt{x}(x+1)}dx$$
$$\Big\downarrow \; 27$$
$$\frac{1}{2}\int \frac{1}{\sqrt{x}(x+1)}dx$$
$$\Big\downarrow \; 73$$
$$\int \frac{1}{x+1}d\sqrt{x}$$
$$\Big\downarrow \; 216$$
$$\arctan\left(\sqrt{x}\right)$$

input
```
Int[1/(2*Sqrt[x]*(1 + x)),x]
```

output
```
ArcTan[Sqrt[x]]
```

3.946.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.946.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\arctan\left(\sqrt{x}\right)$	5
default	$\arctan\left(\sqrt{x}\right)$	5
meijerg	$\arctan\left(\sqrt{x}\right)$	5
trager	$\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{x}+x-1}{x+1}\right)}{2}$	30

input
```
int(1/2/(x+1)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan(x^(1/2))
```

3.946.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{1}{2\sqrt{x}(1+x)}\,dx = \arctan\left(\sqrt{x}\right)$$

input
```
integrate(1/2/(1+x)/x^(1/2),x, algorithm="fricas")
```

output
```
arctan(sqrt(x))
```

3.946.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{1}{2\sqrt{x}(1+x)}\,dx = \text{atan}\left(\sqrt{x}\right)$$

input
```
integrate(1/2/(1+x)/x**(1/2),x)
```

output
```
atan(sqrt(x))
```

3.946.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{1}{2\sqrt{x}(1+x)}\,dx = \arctan\left(\sqrt{x}\right)$$

input
```
integrate(1/2/(1+x)/x^(1/2),x, algorithm="maxima")
```

output
```
arctan(sqrt(x))
```

3.946.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{1}{2\sqrt{x}(1+x)}\,dx = \arctan\left(\sqrt{x}\right)$$

input
```
integrate(1/2/(1+x)/x^(1/2),x, algorithm="giac")
```

output
```
arctan(sqrt(x))
```

3.946.9 Mupad [B] (verification not implemented)

Time = 21.16 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.67

$$\int \frac{1}{2\sqrt{x}(1+x)}\,dx = \operatorname{atan}\left(\sqrt{x}\right)$$

input
```
int(1/(2*x^(1/2)*(x + 1)),x)
```

output
```
atan(x^(1/2))
```

3.947 $\int \frac{1}{x\sqrt{6x-x^2}}\,dx$

3.947.1 Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \frac{1}{x\sqrt{6x-x^2}}\,dx = -\frac{\sqrt{6x-x^2}}{3x}$$

output
```
-1/3*(-x^2+6*x)^(1/2)/x
```

3.947.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\sqrt{6x-x^2}}\,dx = \frac{-6+x}{3\sqrt{-((-6+x)x)}}$$

input
```
Integrate[1/(x*Sqrt[6*x - x^2]),x]
```

output
```
(-6 + x)/(3*Sqrt[-((-6 + x)*x)])
```

3.947.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{6x-x^2}}\,dx$$
$$\Big\downarrow \ 1123$$
$$-\frac{\sqrt{6x-x^2}}{3x}$$

input
```
Int[1/(x*Sqrt[6*x - x^2]),x]
```

output
```
-1/3*Sqrt[6*x - x^2]/x
```

3.947.3.1 Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

3.947.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{-6+x}{3\sqrt{-x(-6+x)}}$	14
pseudoelliptic	$-\frac{\sqrt{-x(-6+x)}}{3x}$	14
gosper	$\frac{-6+x}{3\sqrt{-x^2+6x}}$	17
default	$-\frac{\sqrt{-x^2+6x}}{3x}$	17
trager	$-\frac{\sqrt{-x^2+6x}}{3x}$	17
meijerg	$-\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{1-\frac{x}{6}}}{3\sqrt{x}}$	19

input
```
int(1/x/(-x^2+6*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-6+x)/(-x*(-6+x))^(1/2)
```

3.947.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{1}{x\sqrt{6x-x^2}}\,dx = -\frac{\sqrt{-x^2+6\,x}}{3\,x}$$

input
```
integrate(1/x/(-x^2+6*x)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(-x^2 + 6*x)/x
```

3.947.6 Sympy [F]

$$\int \frac{1}{x\sqrt{6x-x^2}}\,dx = \int \frac{1}{x\sqrt{-x\,(x-6)}}\,dx$$

input
```
integrate(1/x/(-x**2+6*x)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(-x*(x - 6))), x)
```

3.947.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{1}{x\sqrt{6x-x^2}}\,dx = -\frac{\sqrt{-x^2+6\,x}}{3\,x}$$

input
```
integrate(1/x/(-x^2+6*x)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*sqrt(-x^2 + 6*x)/x
```

3.947.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{1}{x\sqrt{6x-x^2}}\,dx = \frac{2}{3\left(\frac{\sqrt{-x^2+6\,x}-3}{x-3}-1\right)}$$

input
```
integrate(1/x/(-x^2+6*x)^(1/2),x, algorithm="giac")
```

output
```
2/3/((sqrt(-x^2 + 6*x) - 3)/(x - 3) - 1)
```

3.947.9 Mupad [B] (verification not implemented)

Time = 21.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{1}{x\sqrt{6x-x^2}}\,dx = -\frac{\sqrt{6\,x-x^2}}{3\,x}$$

input
```
int(1/(x*(6*x - x^2)^(1/2)),x)
```

output
```
-(6*x - x^2)^(1/2)/(3*x)
```

3.948 $\int \left(1+\sqrt{x}\right) \sqrt{x} \, dx$

3.948.1 Optimal result

Integrand size = 13, antiderivative size = 17

$$\int \left(1+\sqrt{x}\right) \sqrt{x} \, dx = \frac{2x^{3/2}}{3} + \frac{x^2}{2}$$

output
```
2/3*x^(3/2)+1/2*x^2
```

3.948.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \left(1+\sqrt{x}\right) \sqrt{x} \, dx = \frac{2x^{3/2}}{3} + \frac{x^2}{2}$$

input
```
Integrate[(1 + Sqrt[x])*Sqrt[x],x]
```

output
```
(2*x^(3/2))/3 + x^2/2
```

3.948.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sqrt{x}+1\right)\sqrt{x}\,dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(x+\sqrt{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2x^{3/2}}{3}+\frac{x^2}{2}$$

input
```
Int[(1 + Sqrt[x])*Sqrt[x],x]
```

output
```
(2*x^(3/2))/3 + x^2/2
```

3.948.3.1 Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.948.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3}+\frac{x^2}{2}$	12
default	$\frac{2x^{\frac{3}{2}}}{3}+\frac{x^2}{2}$	12
trager	$\frac{(x-1)(x+1)}{2}+\frac{2x^{\frac{3}{2}}}{3}$	15

input
```
int(x^(1/2)*(1+x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^(3/2)+1/2*x^2
```

3.948.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \left(1+\sqrt{x}\right)\sqrt{x}\,dx=\frac{1}{2}\,x^2+\frac{2}{3}\,x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="fricas")
```

output
```
1/2*x^2 + 2/3*x^(3/2)
```

3.948.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \left(1+\sqrt{x}\right)\sqrt{x}\,dx=\frac{2x^{\frac{3}{2}}}{3}+\frac{x^2}{2}$$

input
```
integrate(x**(1/2)*(1+x**(1/2)),x)
```

output
```
2*x**(3/2)/3 + x**2/2
```

3.948.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(11) = 22.

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \left(1+\sqrt{x}\right)\sqrt{x}\,dx = \frac{1}{2}\left(\sqrt{x}+1\right)^4 - \frac{4}{3}\left(\sqrt{x}+1\right)^3 + \left(\sqrt{x}+1\right)^2$$

input
```
integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="maxima")
```

output
```
1/2*(sqrt(x) + 1)^4 - 4/3*(sqrt(x) + 1)^3 + (sqrt(x) + 1)^2
```

3.948.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \left(1+\sqrt{x}\right)\sqrt{x}\,dx = \frac{1}{2}\,x^2 + \frac{2}{3}\,x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="giac")
```

output
```
1/2*x^2 + 2/3*x^(3/2)
```

3.948.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \left(1+\sqrt{x}\right)\sqrt{x}\,dx = \frac{x^2}{2} + \frac{2\,x^{3/2}}{3}$$

input
```
int(x^(1/2)*(x^(1/2) + 1),x)
```

output
```
x^2/2 + (2*x^(3/2))/3
```

3.949 $\int \frac{1-\sqrt{x}}{\sqrt[3]{x}} dx$

3.949.1 Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{1-\sqrt{x}}{\sqrt[3]{x}} dx = \frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7}$$

output
```
3/2*x^(2/3)-6/7*x^(7/6)
```

3.949.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1-\sqrt{x}}{\sqrt[3]{x}} dx = \frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7}$$

input
```
Integrate[(1 - Sqrt[x])/x^(1/3),x]
```

output
```
(3*x^(2/3))/2 - (6*x^(7/6))/7
```

3.949.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-\sqrt{x}}{\sqrt[3]{x}} \, dx$$

$$\Big\downarrow \text{802}$$

$$\int \left(\frac{1}{\sqrt[3]{x}}-\sqrt[6]{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}$$

input
```
Int[(1 - Sqrt[x])/x^(1/3),x]
```

output
```
(3*x^(2/3))/2 - (6*x^(7/6))/7
```

3.949.3.1 Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.949.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

method	result	size
derivativedivides	$\frac{3x^{\frac{2}{3}}}{2}-\frac{6x^{\frac{7}{6}}}{7}$	12
default	$\frac{3x^{\frac{2}{3}}}{2}-\frac{6x^{\frac{7}{6}}}{7}$	12

input
```
int((1-x^(1/2))/x^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/2*x^(2/3)-6/7*x^(7/6)
```

3.949.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\,dx=-\frac{6}{7}\,x^{\frac{7}{6}}+\frac{3}{2}\,x^{\frac{2}{3}}$$

input
```
integrate((1-x^(1/2))/x^(1/3),x, algorithm="fricas")
```

output
```
-6/7*x^(7/6) + 3/2*x^(2/3)
```

3.949.6 Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\,dx=-\frac{6x^{\frac{7}{6}}}{7}+\frac{3x^{\frac{2}{3}}}{2}$$

input
```
integrate((1-x**(1/2))/x**(1/3),x)
```

output
```
-6*x**(7/6)/7 + 3*x**(2/3)/2
```

3.949.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\,dx = -\frac{6}{7}\,x^{\frac{7}{6}} + \frac{3}{2}\,x^{\frac{2}{3}}$$

input
```
integrate((1-x^(1/2))/x^(1/3),x, algorithm="maxima")
```

output
```
-6/7*x^(7/6) + 3/2*x^(2/3)
```

3.949.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\,dx = -\frac{6}{7}\,x^{\frac{7}{6}} + \frac{3}{2}\,x^{\frac{2}{3}}$$

input
```
integrate((1-x^(1/2))/x^(1/3),x, algorithm="giac")
```

output
```
-6/7*x^(7/6) + 3/2*x^(2/3)
```

3.949.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\,dx = -\frac{3\,x^{2/3}\,\left(4\,\sqrt{x}-7\right)}{14}$$

input
```
int(-(x^(1/2) - 1)/x^(1/3),x)
```

output
```
-(3*x^(2/3)*(4*x^(1/2) - 7))/14
```

3.950 $\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx$

3.950.1 Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx = -6\sqrt[6]{x}+2\sqrt{x}-\frac{6x^{5/6}}{5}+\frac{6x^{7/6}}{7}+6\arctan\left(\sqrt[6]{x}\right)$$

output
```
-6*x^(1/6)-6/5*x^(5/6)+6/7*x^(7/6)+6*arctan(x^(1/6))+2*x^(1/2)
```

3.950.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx = \frac{2}{35}\left(-105\sqrt[6]{x}+35\sqrt{x}-21x^{5/6}+15x^{7/6}\right)+6\arctan\left(\sqrt[6]{x}\right)$$

input
```
Integrate[Sqrt[x]/(1 + x^(1/3)),x]
```

output
```
(2*(-105*x^(1/6) + 35*Sqrt[x] - 21*x^(5/6) + 15*x^(7/6)))/35 + 6*ArcTan[x^
(1/6)]
```

3.950.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {864, 60, 60, 60, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt[3]{x}+1}dx$$

$$\Big\downarrow \text{864}$$

$$3\int \frac{x^{7/6}}{\sqrt[3]{x}+1}d\sqrt[3]{x}$$

$$\Big\downarrow \text{60}$$

$$3\left(\frac{2x^{7/6}}{7}-\int \frac{x^{5/6}}{\sqrt[3]{x}+1}d\sqrt[3]{x}\right)$$

$$\Big\downarrow \text{60}$$

$$3\left(\int \frac{\sqrt{x}}{\sqrt[3]{x}+1}d\sqrt[3]{x}+\frac{2x^{7/6}}{7}-\frac{2x^{5/6}}{5}\right)$$

$$\Big\downarrow \text{60}$$

$$3\left(-\int \frac{\sqrt[6]{x}}{\sqrt[3]{x}+1}d\sqrt[3]{x}+\frac{2x^{7/6}}{7}-\frac{2x^{5/6}}{5}+\frac{2\sqrt{x}}{3}\right)$$

$$\Big\downarrow \text{60}$$

$$3\left(\int \frac{1}{\left(\sqrt[3]{x}+1\right)\sqrt[6]{x}}d\sqrt[3]{x}+\frac{2x^{7/6}}{7}-\frac{2x^{5/6}}{5}+\frac{2\sqrt{x}}{3}-2\sqrt[6]{x}\right)$$

$$\Big\downarrow \text{73}$$

$$3\left(2\int \frac{1}{x^{2/3}+1}d\sqrt[6]{x}+\frac{2x^{7/6}}{7}-\frac{2x^{5/6}}{5}+\frac{2\sqrt{x}}{3}-2\sqrt[6]{x}\right)$$

$$\Big\downarrow \text{216}$$

$$3\left(2\arctan\left(\sqrt[6]{x}\right)+\frac{2x^{7/6}}{7}-\frac{2x^{5/6}}{5}+\frac{2\sqrt{x}}{3}-2\sqrt[6]{x}\right)$$

input
```
Int[Sqrt[x]/(1 + x^(1/3)),x]
```

3.950. $\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx$

output
```
3*(-2*x^(1/6) + (2*Sqrt[x])/3 - (2*x^(5/6))/5 + (2*x^(7/6))/7 + 2*ArcTan[x
^(1/6)])
```

3.950.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 864
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x
^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

3.950.4 Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$-6x^{\frac{1}{6}}-\frac{6x^{\frac{5}{6}}}{5}+\frac{6x^{\frac{7}{6}}}{7}+6\arctan\left(x^{\frac{1}{6}}\right)+2\sqrt{x}$	28
default	$-6x^{\frac{1}{6}}-\frac{6x^{\frac{5}{6}}}{5}+\frac{6x^{\frac{7}{6}}}{7}+6\arctan\left(x^{\frac{1}{6}}\right)+2\sqrt{x}$	28
meijerg	$-\frac{2x^{\frac{1}{6}}\left(-45x+63x^{\frac{2}{3}}-105x^{\frac{1}{3}}+315\right)}{105}+6\arctan\left(x^{\frac{1}{6}}\right)$	28

input
```
int(x^(1/2)/(1+x^(1/3)),x,method=_RETURNVERBOSE)
```

output
```
-6*x^(1/6)-6/5*x^(5/6)+6/7*x^(7/6)+6*arctan(x^(1/6))+2*x^(1/2)
```

3.950.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx = \frac{6}{7}\,(x-7)x^{\frac{1}{6}} - \frac{6}{5}\,x^{\frac{5}{6}} + 2\,\sqrt{x} + 6\,\arctan\left(x^{\frac{1}{6}}\right)$$

input
```
integrate(x^(1/2)/(1+x^(1/3)),x, algorithm="fricas")
```

output
```
6/7*(x - 7)*x^(1/6) - 6/5*x^(5/6) + 2*sqrt(x) + 6*arctan(x^(1/6))
```

3.950.6 Sympy [A] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx = \frac{6x^{\frac{7}{6}}}{7} - \frac{6x^{\frac{5}{6}}}{5} - 6\sqrt[6]{x} + 2\sqrt{x} + 6\,\mathrm{atan}\left(\sqrt[6]{x}\right)$$

input
```
integrate(x**(1/2)/(1+x**(1/3)),x)
```

output
```
6*x**(7/6)/7 - 6*x**(5/6)/5 - 6*x**(1/6) + 2*sqrt(x) + 6*atan(x**(1/6))
```

3.950.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx = \frac{6}{7}\,x^{\frac{7}{6}} - \frac{6}{5}\,x^{\frac{5}{6}} + 2\,\sqrt{x} - 6\,x^{\frac{1}{6}} + 6\,\arctan\left(x^{\frac{1}{6}}\right)$$

input
```
integrate(x^(1/2)/(1+x^(1/3)),x, algorithm="maxima")
```

output
```
6/7*x^(7/6) - 6/5*x^(5/6) + 2*sqrt(x) - 6*x^(1/6) + 6*arctan(x^(1/6))
```

3.950.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx = \frac{6}{7}\,x^{\frac{7}{6}} - \frac{6}{5}\,x^{\frac{5}{6}} + 2\,\sqrt{x} - 6\,x^{\frac{1}{6}} + 6\,\arctan\left(x^{\frac{1}{6}}\right)$$

input
```
integrate(x^(1/2)/(1+x^(1/3)),x, algorithm="giac")
```

output
```
6/7*x^(7/6) - 6/5*x^(5/6) + 2*sqrt(x) - 6*x^(1/6) + 6*arctan(x^(1/6))
```

3.950.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx = 6\,\mathrm{atan}\left(x^{1/6}\right) + 2\,\sqrt{x} - 6\,x^{1/6} - \frac{6\,x^{5/6}}{5} + \frac{6\,x^{7/6}}{7}$$

input
```
int(x^(1/2)/(x^(1/3) + 1),x)
```

output
```
6*atan(x^(1/6)) + 2*x^(1/2) - 6*x^(1/6) - (6*x^(5/6))/5 + (6*x^(7/6))/7
```

3.951 $\int \frac{\sqrt[3]{1+\sqrt{x}}}{x} \, dx$

3.951.1 Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \frac{\sqrt[3]{1+\sqrt{x}}}{x} \, dx = 6\sqrt[3]{1+\sqrt{x}} - 2\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1+\sqrt{x}}}{\sqrt{3}}\right) + 3\log\left(1-\sqrt[3]{1+\sqrt{x}}\right) - \frac{\log(x)}{2}$$

output
```
-1/2*ln(x)+3*ln(1-(1+x^(1/2))^(1/3))-2*arctan(1/3*(1+2*(1+x^(1/2))^(1/3))*
3^(1/2))*3^(1/2)+6*(1+x^(1/2))^(1/3)
```

3.951.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt[3]{1+\sqrt{x}}}{x} \, dx = 6\sqrt[3]{1+\sqrt{x}} - 2\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{1+\sqrt{x}}}{\sqrt{3}}\right) + 2\log\left(-1+\sqrt[3]{1+\sqrt{x}}\right) - \log\left(1+\sqrt[3]{1+\sqrt{x}}+\left(1+\sqrt{x}\right)^{2/3}\right)$$

input
```
Integrate[(1 + Sqrt[x])^(1/3)/x,x]
```

output
```
6*(1 + Sqrt[x])^(1/3) - 2*Sqrt[3]*ArcTan[(1 + 2*(1 + Sqrt[x])^(1/3))/Sqrt[
3]] + 2*Log[-1 + (1 + Sqrt[x])^(1/3)] - Log[1 + (1 + Sqrt[x])^(1/3) + (1 +
 Sqrt[x])^(2/3)]
```

3.951.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {798, 60, 69, 16, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{\sqrt{x}+1}}{x}dx$$

$$\Big\downarrow \text{798}$$

$$2\int \frac{\sqrt[3]{\sqrt{x}+1}}{\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{60}$$

$$2\left(\int \frac{1}{\left(\sqrt{x}+1\right)^{2/3}\sqrt{x}}d\sqrt{x}+3\sqrt[3]{\sqrt{x}+1}\right)$$

$$\Big\downarrow \text{69}$$

$$2\left(-\frac{3}{2}\int \frac{1}{1-\sqrt[3]{\sqrt{x}+1}}d\sqrt[3]{\sqrt{x}+1}-\frac{3}{2}\int \frac{1}{x+\sqrt[3]{\sqrt{x}+1}+1}d\sqrt[3]{\sqrt{x}+1}+3\sqrt[3]{\sqrt{x}+1}-\frac{1}{2}\log\left(\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{16}$$

$$2\left(-\frac{3}{2}\int \frac{1}{x+\sqrt[3]{\sqrt{x}+1}+1}d\sqrt[3]{\sqrt{x}+1}+3\sqrt[3]{\sqrt{x}+1}+\frac{3}{2}\log\left(1-\sqrt[3]{\sqrt{x}+1}\right)-\frac{\log\left(\sqrt{x}\right)}{2}\right)$$

$$\Big\downarrow \text{1083}$$

$$2\left(3\int \frac{1}{-x-3}d\left(2\sqrt[3]{\sqrt{x}+1}+1\right)+3\sqrt[3]{\sqrt{x}+1}+\frac{3}{2}\log\left(1-\sqrt[3]{\sqrt{x}+1}\right)-\frac{\log\left(\sqrt{x}\right)}{2}\right)$$

$$\Big\downarrow \text{217}$$

$$2\left(-\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\sqrt{x}+1}+1}{\sqrt{3}}\right)+3\sqrt[3]{\sqrt{x}+1}+\frac{3}{2}\log\left(1-\sqrt[3]{\sqrt{x}+1}\right)-\frac{\log\left(\sqrt{x}\right)}{2}\right)$$

input
```
Int[(1 + Sqrt[x])^(1/3)/x,x]
```

output
```
2*(3*(1 + Sqrt[x])^(1/3) - Sqrt[3]*ArcTan[(1 + 2*(1 + Sqrt[x])^(1/3))/Sqrt
[3]] + (3*Log[1 - (1 + Sqrt[x])^(1/3)])/2 - Log[Sqrt[x]]/2)
```

3.951.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

3.951. $\int \frac{\sqrt[3]{1+\sqrt{x}}}{x}\,dx$

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.951.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3.

Time = 1.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

method	result
meijerg	$-\frac{2\left(-\Gamma\left(\frac{2}{3}\right)\sqrt{x}\,{}_3F_2\left(\frac{2}{3},1,1;2,2;-\sqrt{x}\right)-3\left(3+\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+\frac{\ln(x)}{2}\right)\Gamma\left(\frac{2}{3}\right)\right)}{3\Gamma\left(\frac{2}{3}\right)}$
derivativedivides	$6\left(1+\sqrt{x}\right)^{\frac{1}{3}}-\ln\left(\left(1+\sqrt{x}\right)^{\frac{2}{3}}+\left(1+\sqrt{x}\right)^{\frac{1}{3}}+1\right)-2\arctan\left(\frac{\left(1+2\left(1+\sqrt{x}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}+$
default	$6\left(1+\sqrt{x}\right)^{\frac{1}{3}}-\ln\left(\left(1+\sqrt{x}\right)^{\frac{2}{3}}+\left(1+\sqrt{x}\right)^{\frac{1}{3}}+1\right)-2\arctan\left(\frac{\left(1+2\left(1+\sqrt{x}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)\sqrt{3}+$

input
```
int((1+x^(1/2))^(1/3)/x,x,method=_RETURNVERBOSE)
```

output
```
-2/3/GAMMA(2/3)*(-GAMMA(2/3)*x^(1/2)*hypergeom([2/3,1,1],[2,2],-x^(1/2))-3
*(3+1/6*Pi*3^(1/2)-3/2*ln(3)+1/2*ln(x))*GAMMA(2/3))
```

3.951.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt[3]{1+\sqrt{x}}}{x}\,dx = -2\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\sqrt{x}+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+6\left(\sqrt{x}+1\right)^{\frac{1}{3}} - \log\left(\left(\sqrt{x}+1\right)^{\frac{2}{3}}+\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)+2\log\left(\left(\sqrt{x}+1\right)^{\frac{1}{3}}-1\right)$$

input
```
integrate((1+x^(1/2))^(1/3)/x,x, algorithm="fricas")
```

output
```
-2*sqrt(3)*arctan(2/3*sqrt(3)*(sqrt(x) + 1)^(1/3) + 1/3*sqrt(3)) + 6*(sqrt
(x) + 1)^(1/3) - log((sqrt(x) + 1)^(2/3) + (sqrt(x) + 1)^(1/3) + 1) + 2*lo
g((sqrt(x) + 1)^(1/3) - 1)
```

3.951.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt[3]{1+\sqrt{x}}}{x}\,dx = -\frac{2\sqrt[6]{x}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{3}, -\frac{1}{3} \\ \frac{2}{3} \end{matrix}\middle| \frac{e^{i\pi}}{\sqrt{x}}\right)}{\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((1+x**(1/2))**(1/3)/x,x)
```

output
```
-2*x**(1/6)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), exp_polar(I*pi)/sqrt(x
))/gamma(2/3)
```

3.951.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt[3]{1+\sqrt{x}}}{x}\,dx = -2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)\right)+6\left(\sqrt{x}+1\right)^{\frac{1}{3}} - \log\left(\left(\sqrt{x}+1\right)^{\frac{2}{3}}+\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)+2\log\left(\left(\sqrt{x}+1\right)^{\frac{1}{3}}-1\right)$$

input
```
integrate((1+x^(1/2))^(1/3)/x,x, algorithm="maxima")
```

output
```
-2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(x) + 1)^(1/3) + 1)) + 6*(sqrt(x) +
1)^(1/3) - log((sqrt(x) + 1)^(2/3) + (sqrt(x) + 1)^(1/3) + 1) + 2*log((sqr
t(x) + 1)^(1/3) - 1)
```

3.951.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{1+\sqrt{x}}}{x}\,dx = -2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\Big(2\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\Big)\right)+6\left(\sqrt{x}+1\right)^{\frac{1}{3}} - \log\left(\left(\sqrt{x}+1\right)^{\frac{2}{3}}+\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)+2\log\left(\left|\left(\sqrt{x}+1\right)^{\frac{1}{3}}-1\right|\right)$$

input
```
integrate((1+x^(1/2))^(1/3)/x,x, algorithm="giac")
```

output
```
-2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(x) + 1)^(1/3) + 1)) + 6*(sqrt(x) +
1)^(1/3) - log((sqrt(x) + 1)^(2/3) + (sqrt(x) + 1)^(1/3) + 1) + 2*log(abs(
(sqrt(x) + 1)^(1/3) - 1))
```

3.951.9 Mupad [B] (verification not implemented)

Time = 21.54 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt[3]{1+\sqrt{x}}}{x}\,dx = 2\ln\left(\left(\sqrt{x}+1\right)^{1/3}-1\right)+6\left(\sqrt{x}+1\right)^{1/3} + \ln\left(\left(\sqrt{x}+1\right)^{1/3}+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)-\ln\left(\left(\sqrt{x}+1\right)^{1/3}+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)$$

input
```
int((x^(1/2) + 1)^(1/3)/x,x)
```

output
```
2*log((x^(1/2) + 1)^(1/3) - 1) + 6*(x^(1/2) + 1)^(1/3) + log((x^(1/2) + 1)
^(1/3) - (3^(1/2)*1i)/2 + 1/2)*(3^(1/2)*1i - 1) - log((3^(1/2)*1i)/2 + (x^
(1/2) + 1)^(1/3) + 1/2)*(3^(1/2)*1i + 1)
```

3.952 $\int (1-\sqrt{x}) \, dx$

3.952.1 Optimal result

Integrand size = 9, antiderivative size = 11

$$\int (1-\sqrt{x}) \, dx = x - \frac{2x^{3/2}}{3}$$

output
```
x-2/3*x^(3/2)
```

3.952.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int (1-\sqrt{x}) \, dx = x - \frac{2x^{3/2}}{3}$$

input
```
Integrate[1 - Sqrt[x],x]
```

output
```
x - (2*x^(3/2))/3
```

3.952.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-\sqrt{x}\right) \, dx$$
$$\Big\downarrow \; 2009$$
$$x-\frac{2 x^{3/2}}{3}$$

input
```
Int[1 - Sqrt[x],x]
```

output
```
x - (2*x^(3/2))/3
```

3.952.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.952.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$x-\frac{2 x^{\frac{3}{2}}}{3}$	8
default	$x-\frac{2 x^{\frac{3}{2}}}{3}$	8
risch	$x-\frac{2 x^{\frac{3}{2}}}{3}$	8
parts	$x-\frac{2 x^{\frac{3}{2}}}{3}$	8
trager	$x-1-\frac{2 x^{\frac{3}{2}}}{3}$	9

input
```
int(1-x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x-2/3*x^(3/2)
```

3.952.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \left(1-\sqrt{x}\right) dx = -\frac{2}{3} x^{\frac{3}{2}} + x$$

input
```
integrate(1-x^(1/2),x, algorithm="fricas")
```

output
```
-2/3*x^(3/2) + x
```

3.952.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \left(1-\sqrt{x}\right) dx = -\frac{2x^{\frac{3}{2}}}{3} + x$$

input
```
integrate(1-x**(1/2),x)
```

output
```
-2*x**(3/2)/3 + x
```

3.952.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \left(1-\sqrt{x}\right) dx = -\frac{2}{3} x^{\frac{3}{2}} + x$$

input
```
integrate(1-x^(1/2),x, algorithm="maxima")
```

output
```
-2/3*x^(3/2) + x
```

3.952.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \left(1-\sqrt{x}\right)\,dx=-\frac{2}{3}\,x^{\frac{3}{2}}+x$$

input
```
integrate(1-x^(1/2),x, algorithm="giac")
```

output
```
-2/3*x^(3/2) + x
```

3.952.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \left(1-\sqrt{x}\right)\,dx=x-\frac{2\,x^{3/2}}{3}$$

input
```
int(1 - x^(1/2),x)
```

output
```
x - (2*x^(3/2))/3
```

3.953 $\int \left(1-\sqrt[4]{x}\right) \, dx$

3.953.1 Optimal result

Integrand size = 9, antiderivative size = 11

$$\int \left(1-\sqrt[4]{x}\right) \, dx=x-\frac{4x^{5/4}}{5}$$

output
```
x-4/5*x^(5/4)
```

3.953.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \left(1-\sqrt[4]{x}\right) \, dx=x-\frac{4x^{5/4}}{5}$$

input
```
Integrate[1 - x^(1/4),x]
```

output
```
x - (4*x^(5/4))/5
```

3.953.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-\sqrt[4]{x}\right) \, dx$$

$$\Big\downarrow \; 2009$$

$$x-\frac{4x^{5/4}}{5}$$

input
```
Int[1 - x^(1/4),x]
```

output
```
x - (4*x^(5/4))/5
```

3.953.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.953.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$x-\frac{4x^{\frac{5}{4}}}{5}$	8
default	$x-\frac{4x^{\frac{5}{4}}}{5}$	8
risch	$x-\frac{4x^{\frac{5}{4}}}{5}$	8
parts	$x-\frac{4x^{\frac{5}{4}}}{5}$	8
trager	$x-1-\frac{4x^{\frac{5}{4}}}{5}$	9

input
```
int(1-x^(1/4),x,method=_RETURNVERBOSE)
```

output
```
x-4/5*x^(5/4)
```

3.953.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \left(1-\sqrt[4]{x}\right)\,dx=-\frac{4}{5}\,x^{\frac{5}{4}}+x$$

input
```
integrate(1-x^(1/4),x, algorithm="fricas")
```

output
```
-4/5*x^(5/4) + x
```

3.953.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \left(1-\sqrt[4]{x}\right)\,dx=-\frac{4x^{\frac{5}{4}}}{5}+x$$

input
```
integrate(1-x**(1/4),x)
```

output
```
-4*x**(5/4)/5 + x
```

3.953.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \left(1-\sqrt[4]{x}\right)\,dx=-\frac{4}{5}\,x^{\frac{5}{4}}+x$$

input
```
integrate(1-x^(1/4),x, algorithm="maxima")
```

output
```
-4/5*x^(5/4) + x
```

3.953.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \left(1-\sqrt[4]{x}\right)\,dx=-\frac{4}{5}\,x^{\frac{5}{4}}+x$$

input
```
integrate(1-x^(1/4),x, algorithm="giac")
```

output
```
-4/5*x^(5/4) + x
```

3.953.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \left(1-\sqrt[4]{x}\right)\,dx=x-\frac{4\,x^{5/4}}{5}$$

input
```
int(1 - x^(1/4),x)
```

output
```
x - (4*x^(5/4))/5
```

3.954 $\int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}} dx$

3.954.1 Optimal result

Integrand size = 19, antiderivative size = 11

$$\int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}} \, dx = x - \frac{4x^{5/4}}{5}$$

output
```
x-4/5*x^(5/4)
```

3.954.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}} \, dx = x - \frac{4x^{5/4}}{5}$$

input
```
Integrate[(1 - Sqrt[x])/(1 + x^(1/4)),x]
```

output
```
x - (4*x^(5/4))/5
```

3.954.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1386, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-\sqrt{x}}{\sqrt[4]{x}+1} \, dx$$

$$\Big\downarrow \text{ 1386}$$

$$\int \left(1-\sqrt[4]{x}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$x-\frac{4x^{5/4}}{5}$$

input
```
Int[(1 - Sqrt[x])/(1 + x^(1/4)),x]
```

output
```
x - (4*x^(5/4))/5
```

3.954.3.1 Defintions of rubi rules used

rule 1386
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Simp[(-e^2/c)^q   Int[u*(d - e*x^n)^p, x], x] /; FreeQ[{a, c,
d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && EqQ[p + q, 0
] && GtQ[d, 0] && LtQ[c, 0] && GtQ[e^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.954.4 Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result
derivativedivides	$x-\frac{4x^{\frac{5}{4}}}{5}$
meijerg	$\frac{x^{\frac{1}{4}}\left(4\sqrt{x}-6x^{\frac{1}{4}}+12\right)}{3}-\frac{x^{\frac{1}{4}}\left(12x-15x^{\frac{3}{4}}+20\sqrt{x}-30x^{\frac{1}{4}}+60\right)}{15}$
default	$-\frac{4x^{\frac{5}{4}}}{5}+x+2\ln\left(1+x^{\frac{1}{4}}\right)-\ln\left(1-x\right)-\ln\left(-1+\sqrt{x}\right)+\ln\left(1+\sqrt{x}\right)+2\ln\left(x^{\frac{1}{4}}-1\right)$

input
```
int((1-x^(1/2))/(1+x^(1/4)),x,method=_RETURNVERBOSE)
```

output
```
x-4/5*x^(5/4)
```

3.954.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}}\,dx=-\frac{4}{5}\,x^{\frac{5}{4}}+x$$

input
```
integrate((1-x^(1/2))/(1+x^(1/4)),x, algorithm="fricas")
```

output
```
-4/5*x^(5/4) + x
```

3.954.6 Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}}\,dx=-\frac{4x^{\frac{5}{4}}}{5}+x$$

input
```
integrate((1-x**(1/2))/(1+x**(1/4)),x)
```

output
```
-4*x**(5/4)/5 + x
```

3.954.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}}\,dx = -\frac{4}{5}\,x^{\frac{5}{4}} + x$$

input
```
integrate((1-x^(1/2))/(1+x^(1/4)),x, algorithm="maxima")
```

output
```
-4/5*x^(5/4) + x
```

3.954.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}}\,dx = -\frac{4}{5}\,x^{\frac{5}{4}} + x$$

input
```
integrate((1-x^(1/2))/(1+x^(1/4)),x, algorithm="giac")
```

output
```
-4/5*x^(5/4) + x
```

3.954.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}}\,dx = x - \frac{4\,x^{5/4}}{5}$$

input
```
int(-(x^(1/2) - 1)/(x^(1/4) + 1),x)
```

output
```
x - (4*x^(5/4))/5
```

3.955 $\int \frac{1}{\sqrt{(a+bx)(c+dx)}}\,dx$

3.955.1 Optimal result

Integrand size = 15, antiderivative size = 61

$$\int \frac{1}{\sqrt{(a+bx)(c+dx)}}\,dx=\frac{\text{arctanh}\left(\frac{bc+ad+2bdx}{2\sqrt{b}\sqrt{d}\sqrt{ac+(bc+ad)x+bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

output
```
arctanh(1/2*(2*b*d*x+a*d+b*c)/b^(1/2)/d^(1/2)/(a*c+(a*d+b*c)*x+b*d*x^2)^(1
/2))/b^(1/2)/d^(1/2)
```

3.955.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

$$\int \frac{1}{\sqrt{(a+bx)(c+dx)}}\,dx=\frac{2\sqrt{a+bx}\sqrt{c+dx}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}$$

input
```
Integrate[1/Sqrt[(a + b*x)*(c + d*x)],x]
```

output
```
(2*Sqrt[a + b*x]*Sqrt[c + d*x]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sq
rt[a + b*x])])/(Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])
```

3.955.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2048, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{(a+bx)(c+dx)}}dx$$

$$\Big\downarrow \text{2048}$$

$$\int \frac{1}{\sqrt{x(ad+bc)+ac+bdx^2}}dx$$

$$\Big\downarrow \text{1092}$$

$$2\int \frac{1}{4bd-\frac{(bc+ad+2bdx)^2}{bdx^2+(bc+ad)x+ac}}d\frac{bc+ad+2bdx}{\sqrt{bdx^2+(bc+ad)x+ac}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}\sqrt{x(ad+bc)+ac+bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

input
```
Int[1/Sqrt[(a + b*x)*(c + d*x)],x]
```

output
```
ArcTanh[(b*c + a*d + 2*b*d*x)/(2*Sqrt[b]*Sqrt[d]*Sqrt[a*c + (b*c + a*d)*x
+ b*d*x^2])]/(Sqrt[b]*Sqrt[d])
```

3.955.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

3.955.4 Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bdx}{\sqrt{bd}}+\sqrt{ac+(ad+bc)x+bd\,x^2}\right)}{\sqrt{bd}}$	49

input
```
int(1/((b*x+a)*(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(a*c+(a*d+b*c)*x+b*d*x^2)^(1/2))/(b
*d)^(1/2)
```

3.955.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 192, normalized size of antiderivative = 3.15

$$\int \frac{1}{\sqrt{(a+bx)(c+dx)}}\,dx$$
$$= \left[\frac{\sqrt{bd}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,\sqrt{bdx^2+ac+(bc+ad)x}(2\,bdx+bc+ad)\sqrt{bd}+8\,(b^2cd+\right.}{2\,bd}\right.$$
$$\left. -\frac{\sqrt{-bd}\arctan\left(\frac{\sqrt{bdx^2+ac+(bc+ad)x}(2\,bdx+bc+ad)\sqrt{-bd}}{2\,(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)}{bd}\right]$$

input
```
integrate(1/((b*x+a)*(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*sqrt(
b*d*x^2 + a*c + (b*c + a*d)*x)*(2*b*d*x + b*c + a*d)*sqrt(b*d) + 8*(b^2*c*
d + a*b*d^2)*x)/(b*d), -sqrt(-b*d)*arctan(1/2*sqrt(b*d*x^2 + a*c + (b*c +
a*d)*x)*(2*b*d*x + b*c + a*d)*sqrt(-b*d)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d
 + a*b*d^2)*x))/(b*d)]
```

3.955.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(58) = 116$.

Time = 1.07 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.66

$$\int \frac{1}{\sqrt{(a+bx)(c+dx)}}\,dx = \begin{cases} \frac{\log\left(ad+bc+2bdx+2\sqrt{bd}\sqrt{ac+bdx^2+x(ad+bc)}\right)}{\sqrt{bd}} & \text{for } bd \neq 0 \wedge ac - \frac{(ad+bc)^2}{4bd} \neq 0 \\ \frac{\left(x-\frac{-ad-bc}{2bd}\right)\log\left(x-\frac{-ad-bc}{2bd}\right)}{\sqrt{bd\left(x-\frac{-ad-bc}{2bd}\right)^2}} & \text{for } bd \neq 0 \\ \frac{2\sqrt{ac+x(ad+bc)}}{ad+bc} & \text{for } ad+bc \neq 0 \\ \frac{x}{\sqrt{ac}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/((b*x+a)*(d*x+c))**(1/2),x)
```

output
```
Piecewise((log(a*d + b*c + 2*b*d*x + 2*sqrt(b*d)*sqrt(a*c + b*d*x**2 + x*(
a*d + b*c)))/sqrt(b*d), Ne(b*d, 0) & Ne(a*c - (a*d + b*c)**2/(4*b*d), 0)),
 ((x - (-a*d - b*c)/(2*b*d))*log(x - (-a*d - b*c)/(2*b*d))/sqrt(b*d*(x - (
-a*d - b*c)/(2*b*d))**2), Ne(b*d, 0)), (2*sqrt(a*c + x*(a*d + b*c))/(a*d +
 b*c), Ne(a*d + b*c, 0)), (x/sqrt(a*c), True))
```

3.955.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{(a+bx)(c+dx)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/((b*x+a)*(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.955.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(49) = 98$.

Time = 0.31 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.02

$$\int \frac{1}{\sqrt{(a+bx)(c+dx)}}\,dx = \frac{1}{4}\sqrt{bdx^2+bcx+adx+ac}\left(2\,x+\frac{bc+ad}{bd}\right) + \frac{(b^2c^2-2\,abcd+a^2d^2)\log\left(\left|-bc-ad-2\,\sqrt{bd}\left(\sqrt{bd}x-\sqrt{bdx^2+bcx+adx+ac}\right)\right|\right)}{8\,\sqrt{bd}bd}$$

input
```
integrate(1/((b*x+a)*(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(b*d*x^2 + b*c*x + a*d*x + a*c)*(2*x + (b*c + a*d)/(b*d)) + 1/8*(b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(abs(-b*c - a*d - 2*sqrt(b*d)*(sqrt(b*d)*
x - sqrt(b*d*x^2 + b*c*x + a*d*x + a*c))))/(sqrt(b*d)*b*d)
```

3.955.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{(a+bx)(c+dx)}}\,dx = \int \frac{1}{\sqrt{(a+b\,x)\,(c+d\,x)}}\,dx$$

input
```
int(1/((a + b*x)*(c + d*x))^(1/2),x)
```

output
```
int(1/((a + b*x)*(c + d*x))^(1/2), x)
```

3.956 $\int \frac{1}{\sqrt{(a+bx)(c-dx)}}\,dx$

3.956.1 Optimal result

Integrand size = 16, antiderivative size = 65

$$\int \frac{1}{\sqrt{(a+bx)(c-dx)}}\,dx = -\frac{\arctan\left(\frac{bc-ad-2bdx}{2\sqrt{b}\sqrt{d}\sqrt{ac+(bc-ad)x-bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

output
```
-arctan(1/2*(-2*b*d*x-a*d+b*c)/b^(1/2)/d^(1/2)/(a*c+(-a*d+b*c)*x-b*d*x^2)^
(1/2))/b^(1/2)/d^(1/2)
```

3.956.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sqrt{(a+bx)(c-dx)}}\,dx = -\frac{2\sqrt{a+bx}\sqrt{c-dx}\arctan\left(\frac{\sqrt{b}\sqrt{c-dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c-dx)}}$$

input
```
Integrate[1/Sqrt[(a + b*x)*(c - d*x)],x]
```

output
```
(-2*Sqrt[a + b*x]*Sqrt[c - d*x]*ArcTan[(Sqrt[b]*Sqrt[c - d*x])/(Sqrt[d]*Sq
rt[a + b*x])])/(Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c - d*x)])
```

3.956.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2048, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{(a+bx)(c-dx)}}dx$$
$$\Big\downarrow \text{2048}$$
$$\int \frac{1}{\sqrt{x(bc-ad)+ac-bdx^2}}dx$$
$$\Big\downarrow \text{1092}$$
$$2\int \frac{1}{-\frac{(bc-ad-2bdx)^2}{-bdx^2+(bc-ad)x+ac}-4bd}d\frac{bc-ad-2bdx}{\sqrt{-bdx^2+(bc-ad)x+ac}}$$
$$\Big\downarrow \text{217}$$
$$-\frac{\arctan\left(\frac{-ad+bc-2bdx}{2\sqrt{b}\sqrt{d}\sqrt{x(bc-ad)+ac-bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

input
```
Int[1/Sqrt[(a + b*x)*(c - d*x)],x]
```

output
```
-(ArcTan[(b*c - a*d - 2*b*d*x)/(2*Sqrt[b]*Sqrt[d]*Sqrt[a*c + (b*c - a*d)*x
 - b*d*x^2])]/(Sqrt[b]*Sqrt[d]))
```

3.956.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

3.956.4 Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{\arctan\left(\frac{\sqrt{bd}\,\left(x-\frac{-ad+bc}{2bd}\right)}{\sqrt{ac+(-ad+bc)x-bd\,x^2}}\right)}{\sqrt{bd}}$	55

input
```
int(1/((b*x+a)*(-d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b*d)^(1/2)*arctan((b*d)^(1/2)*(x-1/2*(-a*d+b*c)/b/d)/(a*c+(-a*d+b*c)*x-
b*d*x^2)^(1/2))
```

3.956.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 202, normalized size of antiderivative = 3.11

$$\int \frac{1}{\sqrt{(a+bx)(c-dx)}}\,dx$$
$$= \left[-\frac{\sqrt{-bd}\log\left(8\,b^2d^2x^2+b^2c^2-6\,abcd+a^2d^2-4\,\sqrt{-bdx^2+ac+(bc-ad)x}(2\,bdx-bc+ad)\sqrt{-bd}-8\right.}{2\,bd}\right.$$
$$\left.-\frac{\sqrt{bd}\arctan\left(\frac{\sqrt{-bdx^2+ac+(bc-ad)x}(2\,bdx-bc+ad)\sqrt{bd}}{2\,(b^2d^2x^2-abcd-(b^2cd-abd^2)x)}\right)}{bd}\right]$$

input
```
integrate(1/((b*x+a)*(-d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 - 6*a*b*c*d + a^2*d^2 - 4*sqr
t(-b*d*x^2 + a*c + (b*c - a*d)*x)*(2*b*d*x - b*c + a*d)*sqrt(-b*d) - 8*(b^
2*c*d - a*b*d^2)*x)/(b*d), -sqrt(b*d)*arctan(1/2*sqrt(-b*d*x^2 + a*c + (b*
c - a*d)*x)*(2*b*d*x - b*c + a*d)*sqrt(b*d)/(b^2*d^2*x^2 - a*b*c*d - (b^2*
c*d - a*b*d^2)*x))/(b*d)]
```

3.956.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(60) = 120$.

Time = 1.13 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.49

$$\int \frac{1}{\sqrt{(a+bx)(c-dx)}}\,dx = \begin{cases} \frac{\log\left(-ad+bc-2bdx+2\sqrt{-bd}\sqrt{ac-bdx^2+x(-ad+bc)}\right)}{\sqrt{-bd}} & \text{for } bd \neq 0 \wedge ac + \frac{(ad-bc)^2}{4bd} \neq 0 \\ \frac{\left(x+\frac{ad-bc}{2bd}\right)\log\left(x+\frac{ad-bc}{2bd}\right)}{\sqrt{-bd\left(x+\frac{ad-bc}{2bd}\right)^2}} & \text{for } bd \neq 0 \\ \frac{2\sqrt{ac+x(-ad+bc)}}{-ad+bc} & \text{for } ad-bc \neq 0 \\ \frac{x}{\sqrt{ac}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/((b*x+a)*(-d*x+c))**(1/2),x)
```

output
```
Piecewise((log(-a*d + b*c - 2*b*d*x + 2*sqrt(-b*d)*sqrt(a*c - b*d*x**2 + x
*(-a*d + b*c)))/sqrt(-b*d), Ne(b*d, 0) & Ne(a*c + (a*d - b*c)**2/(4*b*d),
0)), ((x + (a*d - b*c)/(2*b*d))*log(x + (a*d - b*c)/(2*b*d))/sqrt(-b*d*(x
+ (a*d - b*c)/(2*b*d))**2), Ne(b*d, 0)), (2*sqrt(a*c + x*(-a*d + b*c))/(-a
*d + b*c), Ne(a*d - b*c, 0)), (x/sqrt(a*c), True))
```

3.956.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{(a+bx)(c-dx)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/((b*x+a)*(-d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for m
ore detail
```

3.956.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(53) = 106$.

Time = 0.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.02

$$\int \frac{1}{\sqrt{(a+bx)(c-dx)}}\,dx = \frac{1}{4}\sqrt{-bdx^2+bcx-adx+ac}\left(2x-\frac{bc-ad}{bd}\right) - \frac{(b^2c^2+2abcd+a^2d^2)\log\left(\left|-bc+ad-2\sqrt{-bd}\left(\sqrt{-bd}x-\sqrt{-bdx^2+bcx-adx+ac}\right)\right|\right)}{8\sqrt{-bd}bd}$$

input
```
integrate(1/((b*x+a)*(-d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(-b*d*x^2 + b*c*x - a*d*x + a*c)*(2*x - (b*c - a*d)/(b*d)) - 1/8*(
b^2*c^2 + 2*a*b*c*d + a^2*d^2)*log(abs(-b*c + a*d - 2*sqrt(-b*d)*(sqrt(-b*
d)*x - sqrt(-b*d*x^2 + b*c*x - a*d*x + a*c))))/(sqrt(-b*d)*b*d)
```

3.956.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{(a+bx)(c-dx)}}\,dx = \int \frac{1}{\sqrt{(a+b\,x)\,(c-d\,x)}}\,dx$$

input
```
int(1/((a + b*x)*(c - d*x))^(1/2),x)
```

output
```
int(1/((a + b*x)*(c - d*x))^(1/2), x)
```

3.957 $\int \frac{1}{\sqrt{x}(1-x^2)} \, dx$

3.957.1 Optimal result

Integrand size = 15, antiderivative size = 13

$$\int \frac{1}{\sqrt{x}\,(1-x^2)} \, dx = \arctan\left(\sqrt{x}\right) + \operatorname{arctanh}\left(\sqrt{x}\right)$$

output
```
arctan(x^(1/2))+arctanh(x^(1/2))
```

3.957.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x}\,(1-x^2)} \, dx = \arctan\left(\sqrt{x}\right) + \operatorname{arctanh}\left(\sqrt{x}\right)$$

input
```
Integrate[1/(Sqrt[x]*(1 - x^2)),x]
```

output
```
ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

3.957.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\,(1-x^2)}dx$$

$$\Big\downarrow \text{266}$$

$$2\int \frac{1}{1-x^2}d\sqrt{x}$$

$$\Big\downarrow \text{756}$$

$$2\left(\frac{1}{2}\int \frac{1}{1-x}d\sqrt{x}+\frac{1}{2}\int \frac{1}{x+1}d\sqrt{x}\right)$$

$$\Big\downarrow \text{216}$$

$$2\left(\frac{1}{2}\int \frac{1}{1-x}d\sqrt{x}+\frac{\arctan\left(\sqrt{x}\right)}{2}\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{\arctan\left(\sqrt{x}\right)}{2}+\frac{\text{arctanh}\left(\sqrt{x}\right)}{2}\right)$$

input
```
Int[1/(Sqrt[x]*(1 - x^2)),x]
```

output
```
2*(ArcTan[Sqrt[x]]/2 + ArcTanh[Sqrt[x]]/2)
```

3.957.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

3.957.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\arctan\left(\sqrt{x}\right)+\operatorname{arctanh}\left(\sqrt{x}\right)$	10
default	$\arctan\left(\sqrt{x}\right)+\operatorname{arctanh}\left(\sqrt{x}\right)$	10
meijerg	$-\frac{\sqrt{x}\left(\ln\left(1-\left(x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(x^{2}\right)^{\frac{1}{4}}\right)-2\arctan\left(\left(x^{2}\right)^{\frac{1}{4}}\right)\right)}{2\left(x^{2}\right)^{\frac{1}{4}}}$	40
trager	$\frac{\ln\left(\frac{2\sqrt{x}+1+x}{x-1}\right)}{2}-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)x+2\sqrt{x}-\operatorname{RootOf}\left(\_Z^{2}+1\right)}{x+1}\right)}{2}$	56

input
```
int(1/(-x^2+1)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan(x^(1/2))+arctanh(x^(1/2))
```

3.957.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(9) = 18$.

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{x}\,(1-x^2)}\,dx = \arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\sqrt{x}-1\right)$$

input
```
integrate(1/(-x^2+1)/x^(1/2),x, algorithm="fricas")
```

output
```
arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

3.957.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{x}\,(1-x^2)}\,dx = -\frac{\log\left(\sqrt{x}-1\right)}{2} + \frac{\log\left(\sqrt{x}+1\right)}{2} + \operatorname{atan}\left(\sqrt{x}\right)$$

input
```
integrate(1/(-x**2+1)/x**(1/2),x)
```

output
```
-log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 + atan(sqrt(x))
```

3.957.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(9) = 18$.

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{x}\,(1-x^2)}\,dx = \arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\sqrt{x}-1\right)$$

input
```
integrate(1/(-x^2+1)/x^(1/2),x, algorithm="maxima")
```

output
```
arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

3.957.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(9) = 18$.

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{1}{\sqrt{x}\,(1-x^2)}\,dx = \arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\left|\sqrt{x}-1\right|\right)$$

input
```
integrate(1/(-x^2+1)/x^(1/2),x, algorithm="giac")
```

output
```
arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(abs(sqrt(x) - 1))
```

3.957.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{x}\,(1-x^2)}\,dx = \operatorname{atan}\left(\sqrt{x}\right) + \operatorname{atanh}\left(\sqrt{x}\right)$$

input
```
int(-1/(x^(1/2)*(x^2 - 1)),x)
```

output
```
atan(x^(1/2)) + atanh(x^(1/2))
```

3.958 $\int \frac{\sqrt{x}}{x-x^3} dx$

3.958.1 Optimal result

Integrand size = 15, antiderivative size = 13

$$\int \frac{\sqrt{x}}{x-x^3} \, dx = \arctan\left(\sqrt{x}\right) + \operatorname{arctanh}\left(\sqrt{x}\right)$$

output
```
arctan(x^(1/2))+arctanh(x^(1/2))
```

3.958.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}}{x-x^3} \, dx = \arctan\left(\sqrt{x}\right) + \operatorname{arctanh}\left(\sqrt{x}\right)$$

input
```
Integrate[Sqrt[x]/(x - x^3),x]
```

output
```
ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

3.958.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {9, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{x-x^3}\,dx$$
$$\Big\downarrow \ 9$$
$$\int \frac{1}{\sqrt{x}\,(1-x^2)}dx$$
$$\Big\downarrow \ 266$$
$$2\int \frac{1}{1-x^2}d\sqrt{x}$$
$$\Big\downarrow \ 756$$
$$2\left(\frac{1}{2}\int \frac{1}{1-x}d\sqrt{x}+\frac{1}{2}\int \frac{1}{x+1}d\sqrt{x}\right)$$
$$\Big\downarrow \ 216$$
$$2\left(\frac{1}{2}\int \frac{1}{1-x}d\sqrt{x}+\frac{\arctan\left(\sqrt{x}\right)}{2}\right)$$
$$\Big\downarrow \ 219$$
$$2\left(\frac{\arctan\left(\sqrt{x}\right)}{2}+\frac{\text{arctanh}\left(\sqrt{x}\right)}{2}\right)$$

input
```
Int[Sqrt[x]/(x - x^3),x]
```

output
```
2*(ArcTan[Sqrt[x]]/2 + ArcTanh[Sqrt[x]]/2)
```

3.958.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

3.958.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\arctan\left(\sqrt{x}\right)+\operatorname{arctanh}\left(\sqrt{x}\right)$	10
default	$\arctan\left(\sqrt{x}\right)+\operatorname{arctanh}\left(\sqrt{x}\right)$	10
meijerg	$-\frac{\sqrt{x}\left(\ln\left(1-(x^{2})^{\frac{1}{4}}\right)-\ln\left(1+(x^{2})^{\frac{1}{4}}\right)-2\arctan\left((x^{2})^{\frac{1}{4}}\right)\right)}{2(x^{2})^{\frac{1}{4}}}$	40
trager	$\frac{\ln\left(\frac{2\sqrt{x}+1+x}{x-1}\right)}{2}-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)x+2\sqrt{x}-\operatorname{RootOf}\left(\_Z^{2}+1\right)}{x+1}\right)}{2}$	56

input
```
int(x^(1/2)/(-x^3+x),x,method=_RETURNVERBOSE)
```

output
```
arctan(x^(1/2))+arctanh(x^(1/2))
```

3.958.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(9) = 18.

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{x}}{x-x^3}\,dx=\arctan\left(\sqrt{x}\right)+\frac{1}{2}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\log\left(\sqrt{x}-1\right)$$

input
```
integrate(x^(1/2)/(-x^3+x),x, algorithm="fricas")
```

output
```
arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

3.958.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{x}}{x-x^3}\,dx=-\frac{\log\left(\sqrt{x}-1\right)}{2}+\frac{\log\left(\sqrt{x}+1\right)}{2}+\operatorname{atan}\left(\sqrt{x}\right)$$

input
```
integrate(x**(1/2)/(-x**3+x),x)
```

output
```
-log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 + atan(sqrt(x))
```

3.958.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(9) = 18.

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{x}}{x-x^3}\,dx = \arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\sqrt{x}-1\right)$$

input
```
integrate(x^(1/2)/(-x^3+x),x, algorithm="maxima")
```

output
```
arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

3.958.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(9) = 18.

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{\sqrt{x}}{x-x^3}\,dx = \arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\left|\sqrt{x}-1\right|\right)$$

input
```
integrate(x^(1/2)/(-x^3+x),x, algorithm="giac")
```

output
```
arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(abs(sqrt(x) - 1))
```

3.958.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{x}}{x-x^3}\,dx = \mathrm{atan}\left(\sqrt{x}\right) + \mathrm{atanh}\left(\sqrt{x}\right)$$

input
```
int(x^(1/2)/(x - x^3),x)
```

output
```
atan(x^(1/2)) + atanh(x^(1/2))
```

3.959 $\int \frac{x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx$

3.959.1 Optimal result

Integrand size = 25, antiderivative size = 72

$$\int \frac{x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx = \sqrt{\frac{1}{23}\left(13+8\sqrt{3}\right)}\operatorname{arctanh}\left(\frac{1+\sqrt{3}+2x}{\sqrt{2\left(-2+3\sqrt{3}\right)}}\right)+\frac{1}{2}\log\left(2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2\right)$$

output
```
1/2*ln(2+x^2-3^(1/2)+x*(1+3^(1/2)))+1/23*arctanh((1+2*x+3^(1/2))/(-4+6*3^(
1/2))^(1/2))*(299+184*3^(1/2))^(1/2)
```

3.959.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \frac{x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx = \frac{\left(1+\sqrt{3}\right)\operatorname{arctanh}\left(\frac{1+\sqrt{3}+2x}{\sqrt{-4+6\sqrt{3}}}\right)}{\sqrt{-4+6\sqrt{3}}}+\frac{1}{2}\log\left(2-\sqrt{3}+x+\sqrt{3}x+x^2\right)$$

input
```
Integrate[x/(2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2),x]
```

output
```
((1 + Sqrt[3])*ArcTanh[(1 + Sqrt[3] + 2*x)/Sqrt[-4 + 6*Sqrt[3]]])/Sqrt[-4
+ 6*Sqrt[3]] + Log[2 - Sqrt[3] + x + Sqrt[3]*x + x^2]/2
```

3.959.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^2+(1+\sqrt{3})x-\sqrt{3}+2}dx$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{2}\int \frac{2x+\sqrt{3}+1}{x^2+(1+\sqrt{3})x-\sqrt{3}+2}dx-\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{1}{x^2+(1+\sqrt{3})x-\sqrt{3}+2}dx$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{2}\int \frac{2x+\sqrt{3}+1}{x^2+(1+\sqrt{3})x-\sqrt{3}+2}dx+\left(1+\sqrt{3}\right)\int \frac{1}{-\left(2x+\sqrt{3}+1\right)^2-2\left(2-3\sqrt{3}\right)}d\left(2x+\sqrt{3}+1\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\int \frac{2x+\sqrt{3}+1}{x^2+(1+\sqrt{3})x-\sqrt{3}+2}dx+\frac{(1+\sqrt{3})\,\text{arctanh}\left(\frac{2x+\sqrt{3}+1}{\sqrt{2\left(3\sqrt{3}-2\right)}}\right)}{\sqrt{2\left(3\sqrt{3}-2\right)}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{(1+\sqrt{3})\,\text{arctanh}\left(\frac{2x+\sqrt{3}+1}{\sqrt{2\left(3\sqrt{3}-2\right)}}\right)}{\sqrt{2\left(3\sqrt{3}-2\right)}}+\frac{1}{2}\log\left(x^2+\left(1+\sqrt{3}\right)x-\sqrt{3}+2\right)$$

input
```
Int[x/(2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2),x]
```

output
```
((1 + Sqrt[3])*ArcTanh[(1 + Sqrt[3] + 2*x)/Sqrt[2*(-2 + 3*Sqrt[3])]])/Sqrt
[2*(-2 + 3*Sqrt[3])] + Log[2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2]/2
```

3.959.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

3.959.4 Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\ln\left(x\sqrt{3}+x^{2}-\sqrt{3}+x+2\right)}{2}-\frac{2\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\right)\operatorname{arctanh}\left(\frac{1+2x+\sqrt{3}}{\sqrt{-4+6\sqrt{3}}}\right)}{\sqrt{-4+6\sqrt{3}}}$	58

input
```
int(x/(2+x^2-3^(1/2)+(1+3^(1/2))*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x*3^(1/2)+x^2-3^(1/2)+x+2)-2*(-1/2-1/2*3^(1/2))/(-4+6*3^(1/2))^(1/2
)*arctanh((1+2*x+3^(1/2))/(-4+6*3^(1/2))^(1/2))
```

3.959.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(51) = 102$.

Time = 0.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.82

$$\int \frac{x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx$$
$$= \frac{1}{46}\sqrt{23}\sqrt{8\sqrt{3}+13}\log\left(\frac{23x^4+46x^3+\sqrt{23}(11x^3+24x^2-\sqrt{3}(5x^3+13x^2-6x-4)-4x+5)\sqrt{8}}{x^4+2x^3+2x^2+10x+1}\right.$$
$$+\frac{1}{2}\log\left(x^2+\sqrt{3}(x-1)+x+2\right)$$

input
```
integrate(x/(2+x^2-3^(1/2)+x*(1+3^(1/2))),x, algorithm="fricas")
```

output
```
1/46*sqrt(23)*sqrt(8*sqrt(3) + 13)*log((23*x^4 + 46*x^3 + sqrt(23)*(11*x^3
 + 24*x^2 - sqrt(3)*(5*x^3 + 13*x^2 - 6*x - 4) - 4*x + 5)*sqrt(8*sqrt(3) +
 13) + 23*sqrt(3)*(3*x^2 + 5*x + 4) - 23*x + 138)/(x^4 + 2*x^3 + 2*x^2 + 1
0*x + 1)) + 1/2*log(x^2 + sqrt(3)*(x - 1) + x + 2)
```

3.959.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(58) = 116$.

Time = 0.70 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.81

$$\int \frac{x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx = \left(\frac{\sqrt{5+4\sqrt{3}}}{2\cdot\left(2-3\sqrt{3}\right)}\right.$$
$$\left.+\frac{1}{2}\right)\log\left(x-\frac{5\sqrt{3}}{5+4\sqrt{3}}+\left(\frac{\sqrt{5+4\sqrt{3}}}{2\cdot\left(2-3\sqrt{3}\right)}+\frac{1}{2}\right)\left(\frac{47}{22+13\sqrt{3}}+\frac{33\sqrt{3}}{22+13\sqrt{3}}\right)+\frac{11}{5+4\sqrt{3}}\right)$$
$$+\left(\frac{1}{2}\right.$$
$$\left.-\frac{\sqrt{5+4\sqrt{3}}}{2\cdot\left(2-3\sqrt{3}\right)}\right)\log\left(x-\frac{5\sqrt{3}}{5+4\sqrt{3}}+\frac{11}{5+4\sqrt{3}}+\left(\frac{1}{2}-\frac{\sqrt{5+4\sqrt{3}}}{2\cdot\left(2-3\sqrt{3}\right)}\right)\left(\frac{47}{22+13\sqrt{3}}+\frac{33\sqrt{3}}{22+13\sqrt{3}}\right)\right)$$

input
```
integrate(x/(2+x**2-3**(1/2)+x*(1+3**(1/2))),x)
```

output
```
(sqrt(5 + 4*sqrt(3))/(2*(2 - 3*sqrt(3))) + 1/2)*log(x - 5*sqrt(3)/(5 + 4*s
qrt(3)) + (sqrt(5 + 4*sqrt(3))/(2*(2 - 3*sqrt(3))) + 1/2)*(47/(22 + 13*sqr
t(3)) + 33*sqrt(3)/(22 + 13*sqrt(3))) + 11/(5 + 4*sqrt(3))) + (1/2 - sqrt(
5 + 4*sqrt(3))/(2*(2 - 3*sqrt(3))))*log(x - 5*sqrt(3)/(5 + 4*sqrt(3)) + 11
/(5 + 4*sqrt(3)) + (1/2 - sqrt(5 + 4*sqrt(3))/(2*(2 - 3*sqrt(3))))*(47/(22
 + 13*sqrt(3)) + 33*sqrt(3)/(22 + 13*sqrt(3))))
```

3.959.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

$$\int \frac{x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx = -\frac{\left(\sqrt{3}+1\right)\log\left(\frac{2\,x+\sqrt{3}-\sqrt{6\,\sqrt{3}-4}+1}{2\,x+\sqrt{3}+\sqrt{6\,\sqrt{3}-4}+1}\right)}{2\,\sqrt{6\,\sqrt{3}-4}} + \frac{1}{2}\log\left(x^2+x\left(\sqrt{3}+1\right)-\sqrt{3}+2\right)$$

input
```
integrate(x/(2+x^2-3^(1/2)+x*(1+3^(1/2))),x, algorithm="maxima")
```

output
```
-1/2*(sqrt(3) + 1)*log((2*x + sqrt(3) - sqrt(6*sqrt(3) - 4) + 1)/(2*x + sq
rt(3) + sqrt(6*sqrt(3) - 4) + 1))/sqrt(6*sqrt(3) - 4) + 1/2*log(x^2 + x*(s
qrt(3) + 1) - sqrt(3) + 2)
```

3.959.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11

$$\int \frac{x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx = -\frac{\left(\sqrt{3}+1\right)\log\left(\frac{\left|2\,x+\sqrt{3}-\sqrt{6\,\sqrt{3}-4}+1\right|}{\left|2\,x+\sqrt{3}+\sqrt{6\,\sqrt{3}-4}+1\right|}\right)}{2\,\sqrt{6\,\sqrt{3}-4}} + \frac{1}{2}\log\left(\left|x^2+x\left(\sqrt{3}+1\right)-\sqrt{3}+2\right|\right)$$

input
```
integrate(x/(2+x^2-3^(1/2)+x*(1+3^(1/2))),x, algorithm="giac")
```

output
```
-1/2*(sqrt(3) + 1)*log(abs(2*x + sqrt(3) - sqrt(6*sqrt(3) - 4) + 1)/abs(2*
x + sqrt(3) + sqrt(6*sqrt(3) - 4) + 1))/sqrt(6*sqrt(3) - 4) + 1/2*log(abs(
x^2 + x*(sqrt(3) + 1) - sqrt(3) + 2))
```

3.959.9 Mupad [B] (verification not implemented)

Time = 20.60 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.24

$$\int \frac{x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx = \ln\left(x-\left(\frac{\frac{\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}+\frac{\sqrt{3}\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}+\frac{1}{2}\right)\left(2x+\sqrt{3}+1\right)\right)\left(\frac{\frac{\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}+\frac{\sqrt{3}\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}+\frac{1}{2}\right)-\ln\left(x+\left(\frac{\frac{\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}+\frac{\sqrt{3}\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}-\frac{1}{2}\right)\left(2x+\sqrt{3}+1\right)\right)\left(\frac{\frac{\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}+\frac{\sqrt{3}\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}-\frac{1}{2}\right)$$

input
```
int(x/(x*(3^(1/2) + 1) - 3^(1/2) + x^2 + 2),x)
```

output
```
log(x - ((((3^(1/2) - 1)*(3^(1/2) + 7))^(1/2)/2 + (3^(1/2)*((3^(1/2) - 1)*
(3^(1/2) + 7))^(1/2))/2)/((3^(1/2) - 1)*(3^(1/2) + 7)) + 1/2)*(2*x + 3^(1/
2) + 1))*((((3^(1/2) - 1)*(3^(1/2) + 7))^(1/2)/2 + (3^(1/2)*((3^(1/2) - 1)
*(3^(1/2) + 7))^(1/2))/2)/((3^(1/2) - 1)*(3^(1/2) + 7)) + 1/2) - log(x + (
(((3^(1/2) - 1)*(3^(1/2) + 7))^(1/2)/2 + (3^(1/2)*((3^(1/2) - 1)*(3^(1/2)
+ 7))^(1/2))/2)/((3^(1/2) - 1)*(3^(1/2) + 7)) - 1/2)*(2*x + 3^(1/2) + 1))*
((((3^(1/2) - 1)*(3^(1/2) + 7))^(1/2)/2 + (3^(1/2)*((3^(1/2) - 1)*(3^(1/2)
 + 7))^(1/2))/2)/((3^(1/2) - 1)*(3^(1/2) + 7)) - 1/2)
```

3.959. $\int \frac{x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx$

3.960 $\int \sqrt{x^2+x^3}\,dx$

3.960.1 Optimal result

Integrand size = 11, antiderivative size = 37

$$\int \sqrt{x^2+x^3}\,dx=-\frac{4(x^2+x^3)^{3/2}}{15x^3}+\frac{2(x^2+x^3)^{3/2}}{5x^2}$$

output
```
-4/15*(x^3+x^2)^(3/2)/x^3+2/5*(x^3+x^2)^(3/2)/x^2
```

3.960.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int \sqrt{x^2+x^3}\,dx=\frac{2(x^2(1+x))^{3/2}\,(-2+3x)}{15x^3}$$

input
```
Integrate[Sqrt[x^2 + x^3],x]
```

output
```
(2*(x^2*(1 + x))^(3/2)*(-2 + 3*x))/(15*x^3)
```

3.960.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^3+x^2}\,dx$$

$$\Big\downarrow \text{1908}$$

$$\frac{2\left(x^3+x^2\right)^{3/2}}{5x^2}-\frac{2}{5}\int \frac{\sqrt{x^3+x^2}}{x}dx$$

$$\Big\downarrow \text{1920}$$

$$\frac{2\left(x^3+x^2\right)^{3/2}}{5x^2}-\frac{4\left(x^3+x^2\right)^{3/2}}{15x^3}$$

input
```
Int[Sqrt[x^2 + x^3],x]
```

output
```
(-4*(x^2 + x^3)^(3/2))/(15*x^3) + (2*(x^2 + x^3)^(3/2))/(5*x^2)
```

3.960.3.1 Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

3.960.4 Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

method	result	size
gosper	$\frac{2(x+1)(3x-2)\sqrt{x^3+x^2}}{15x}$	23
default	$\frac{2(x+1)(3x-2)\sqrt{x^3+x^2}}{15x}$	23
trager	$\frac{2(3x^2+x-2)\sqrt{x^3+x^2}}{15x}$	23
risch	$\frac{2\sqrt{x^2(x+1)}\,(3x^2+x-2)}{15x}$	23
meijerg	$-\frac{-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\,(x+1)^{\frac{3}{2}}(2-3x)}{15}}{2\sqrt{\pi}}$	27

input
```
int((x^3+x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(x+1)*(3*x-2)*(x^3+x^2)^(1/2)/x
```

3.960.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

$$\int \sqrt{x^2+x^3}\,dx = \frac{2\sqrt{x^3+x^2}(3x^2+x-2)}{15x}$$

input
```
integrate((x^3+x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/15*sqrt(x^3 + x^2)*(3*x^2 + x - 2)/x
```

3.960.6 Sympy [F]

$$\int \sqrt{x^2+x^3}\,dx = \int \sqrt{x^3+x^2}\,dx$$

input
```
integrate((x**3+x**2)**(1/2),x)
```

output
```
Integral(sqrt(x**3 + x**2), x)
```

3.960.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.41

$$\int \sqrt{x^2 + x^3}\,dx = \frac{2}{15}\left(3\,x^2 + x - 2\right)\sqrt{x + 1}$$

input
```
integrate((x^3+x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(3*x^2 + x - 2)*sqrt(x + 1)
```

3.960.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.30

$$\int \sqrt{x^2 + x^3}\,dx = \frac{2}{15}\left(3\,(x + 1)^{\frac{5}{2}} - 10\,(x + 1)^{\frac{3}{2}} + 15\,\sqrt{x + 1}\right)\mathrm{sgn}(x) + \frac{2}{3}\left((x + 1)^{\frac{3}{2}} - 3\,\sqrt{x + 1}\right)\mathrm{sgn}(x) + \frac{4}{15}\,\mathrm{sgn}(x)$$

input
```
integrate((x^3+x^2)^(1/2),x, algorithm="giac")
```

output
```
2/15*(3*(x + 1)^(5/2) - 10*(x + 1)^(3/2) + 15*sqrt(x + 1))*sgn(x) + 2/3*((
x + 1)^(3/2) - 3*sqrt(x + 1))*sgn(x) + 4/15*sgn(x)
```

3.960.9 Mupad [B] (verification not implemented)

Time = 19.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

$$\int \sqrt{x^2 + x^3}\,dx = \frac{2\,\sqrt{x^3 + x^2}\,\left(3\,x^2 + x - 2\right)}{15\,x}$$

input
```
int((x^2 + x^3)^(1/2),x)
```

output
```
(2*(x^2 + x^3)^(1/2)*(x + 3*x^2 - 2))/(15*x)
```

3.961 $\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx$

3.961.1 Optimal result

Integrand size = 17, antiderivative size = 12

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = \arctan\left(\sqrt{2x+x^2}\right)$$

output
```
arctan((x^2+2*x)^(1/2))
```

3.961.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 41 vs. 2(12) = 24.

Time = 0.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.42

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = -\frac{2\sqrt{x}\sqrt{2+x}\arctan\left(1+x-\sqrt{x}\sqrt{2+x}\right)}{\sqrt{x(2+x)}}$$

input
```
Integrate[1/((1 + x)*Sqrt[2*x + x^2]),x]
```

output
```
(-2*Sqrt[x]*Sqrt[2 + x]*ArcTan[1 + x - Sqrt[x]*Sqrt[2 + x]])/Sqrt[x*(2 + x
)]
```

3.961.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1112, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+1)\sqrt{x^2+2x}}dx$$

$$\Big\downarrow \text{ 1112}$$

$$4\int \frac{1}{4\left(x^2+2x\right)+4}d\sqrt{x^2+2x}$$

$$\Big\downarrow \text{ 216}$$

$$\arctan\left(\sqrt{x^2+2x}\right)$$

input
```
Int[1/((1 + x)*Sqrt[2*x + x^2]),x]
```

output
```
ArcTan[Sqrt[2*x + x^2]]
```

3.961.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1112
```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Simp[4*c   Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a
+ b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

3.961.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
pseudoelliptic	$\arctan\left(\sqrt{x\left(x+2\right)}\right)$	9
default	$-\arctan\left(\frac{1}{\sqrt{\left(x+1\right)^{2}-1}}\right)$	13
trager	$\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)+\sqrt{x^{2}+2x}}{x+1}\right)$	31

input
```
int(1/(x+1)/(x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan((x*(x+2))^(1/2))
```

3.961.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = 2\arctan\left(-x+\sqrt{x^2+2\,x}-1\right)$$

input
```
integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="fricas")
```

output
```
2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

3.961.6 Sympy [F]

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = \int \frac{1}{\sqrt{x\left(x+2\right)}\left(x+1\right)}\,dx$$

input
```
integrate(1/(1+x)/(x**2+2*x)**(1/2),x)
```

output
```
Integral(1/(sqrt(x*(x + 2))*(x + 1)), x)
```

3.961.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = -\arcsin\left(\frac{1}{|x+1|}\right)$$

input
```
integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(1/abs(x + 1))
```

3.961.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = 2\arctan\left(-x+\sqrt{x^2+2\,x}-1\right)$$

input
```
integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

3.961.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+2\,x}\,(x+1)}\,dx$$

input
```
int(1/((2*x + x^2)^(1/2)*(x + 1)),x)
```

output
```
int(1/((2*x + x^2)^(1/2)*(x + 1)), x)
```

3.962 $\int \sqrt{1-\sqrt{x}-x}\sqrt{x}\,dx$

3.962.1 Optimal result

Integrand size = 22, antiderivative size = 95

$$\int \sqrt{1-\sqrt{x}-x}\sqrt{x}\,dx=\frac{9}{32}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x}+\frac{5}{12}\left(1-\sqrt{x}-x\right)^{3/2}-\frac{1}{2}\left(1-\sqrt{x}-x\right)^{3/2}\sqrt{x}+\frac{45}{64}\arcsin\left(\frac{1+2\sqrt{x}}{\sqrt{5}}\right)$$

output
```
45/64*arcsin(1/5*(1+2*x^(1/2))*5^(1/2))+5/12*(1-x-x^(1/2))^(3/2)-1/2*(1-x-
x^(1/2))^(3/2)*x^(1/2)+9/32*(1+2*x^(1/2))*(1-x-x^(1/2))^(1/2)
```

3.962.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.75

$$\int \sqrt{1-\sqrt{x}-x}\sqrt{x}\,dx=\frac{1}{96}\sqrt{1-\sqrt{x}-x}\left(67-34\sqrt{x}+8x+48x^{3/2}\right)+\frac{45}{32}\arctan\left(\frac{\sqrt{x}}{-1+\sqrt{1-\sqrt{x}-x}}\right)$$

input
```
Integrate[Sqrt[1 - Sqrt[x] - x]*Sqrt[x],x]
```

output
```
(Sqrt[1 - Sqrt[x] - x]*(67 - 34*Sqrt[x] + 8*x + 48*x^(3/2)))/96 + (45*ArcT
an[Sqrt[x]/(-1 + Sqrt[1 - Sqrt[x] - x])])/32
```

3.962.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1693, 1166, 27, 1160, 1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-x-\sqrt{x}+1}\sqrt{x}\,dx$$

$$\Big\downarrow \text{1693}$$

$$2\int \sqrt{-x-\sqrt{x}+1}xd\sqrt{x}$$

$$\Big\downarrow \text{1166}$$

$$2\left(-\frac{1}{4}\int -\frac{1}{2}(2-5\sqrt{x})\sqrt{-x-\sqrt{x}+1}d\sqrt{x}-\frac{1}{4}\sqrt{x}(-x-\sqrt{x}+1)^{3/2}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{1}{8}\int (2-5\sqrt{x})\sqrt{-x-\sqrt{x}+1}d\sqrt{x}-\frac{1}{4}(-x-\sqrt{x}+1)^{3/2}\sqrt{x}\right)$$

$$\Big\downarrow \text{1160}$$

$$2\left(\frac{1}{8}\left(\frac{9}{2}\int \sqrt{-x-\sqrt{x}+1}d\sqrt{x}+\frac{5}{3}(-x-\sqrt{x}+1)^{3/2}\right)-\frac{1}{4}(-x-\sqrt{x}+1)^{3/2}\sqrt{x}\right)$$

$$\Big\downarrow \text{1087}$$

$$2\left(\frac{1}{8}\left(\frac{9}{2}\left(\frac{5}{8}\int \frac{1}{\sqrt{-x-\sqrt{x}+1}}d\sqrt{x}+\frac{1}{4}\sqrt{-x-\sqrt{x}+1}(2\sqrt{x}+1)\right)+\frac{5}{3}(-x-\sqrt{x}+1)^{3/2}\right)-\frac{1}{4}(-x-\sqrt{x}+1)$$

$$\Big\downarrow \text{1090}$$

$$2\left(\frac{1}{8}\left(\frac{9}{2}\left(\frac{1}{4}(2\sqrt{x}+1)\sqrt{-x-\sqrt{x}+1}-\frac{1}{8}\sqrt{5}\int \frac{1}{\sqrt{1-\frac{x}{5}}}d(-2\sqrt{x}-1)\right)+\frac{5}{3}(-x-\sqrt{x}+1)^{3/2}\right)-\frac{1}{4}(-x-\sqrt{x}$$

$$\Big\downarrow \text{223}$$

$$2\left(\frac{1}{8}\left(\frac{9}{2}\left(\frac{1}{4}(2\sqrt{x}+1)\sqrt{-x-\sqrt{x}+1}-\frac{5}{8}\arcsin\left(\frac{-2\sqrt{x}-1}{\sqrt{5}}\right)\right)+\frac{5}{3}(-x-\sqrt{x}+1)^{3/2}\right)-\frac{1}{4}(-x-\sqrt{x}+1)^{3/2}$$

input
```
Int[Sqrt[1 - Sqrt[x] - x]*Sqrt[x],x]
```

output
```
2*(-1/4*((1 - Sqrt[x] - x)^(3/2)*Sqrt[x]) + ((5*(1 - Sqrt[x] - x)^(3/2))/3
 + (9*(((1 + 2*Sqrt[x])*Sqrt[1 - Sqrt[x] - x])/4 - (5*ArcSin[(-1 - 2*Sqrt[
x])/Sqrt[5]])/8))/2)/8)
```

3.962.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1693
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p,
x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && IntegerQ
[Simplify[(m + 1)/n]]
```

3.962.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$-\frac{(1-x-\sqrt{x})^{\frac{3}{2}}\sqrt{x}}{2}+\frac{5(1-x-\sqrt{x})^{\frac{3}{2}}}{12}-\frac{9(-2\sqrt{x}-1)\sqrt{1-x-\sqrt{x}}}{32}+\frac{45\arcsin\left(\frac{2\sqrt{5}\left(\sqrt{x}+\frac{1}{2}\right)}{5}\right)}{64}$	67
default	$-\frac{(1-x-\sqrt{x})^{\frac{3}{2}}\sqrt{x}}{2}+\frac{5(1-x-\sqrt{x})^{\frac{3}{2}}}{12}-\frac{9(-2\sqrt{x}-1)\sqrt{1-x-\sqrt{x}}}{32}+\frac{45\arcsin\left(\frac{2\sqrt{5}\left(\sqrt{x}+\frac{1}{2}\right)}{5}\right)}{64}$	67

input
```
int(x^(1/2)*(1-x-x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(1-x-x^(1/2))^(3/2)*x^(1/2)+5/12*(1-x-x^(1/2))^(3/2)-9/32*(-2*x^(1/2)
-1)*(1-x-x^(1/2))^(1/2)+45/64*arcsin(2/5*5^(1/2)*(x^(1/2)+1/2))
```

3.962.5 Fricas [A] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

$$\int \sqrt{1-\sqrt{x}-x}\sqrt{x}\,dx$$
$$=\frac{1}{96}\left(2\left(24x-17\right)\sqrt{x}+8x+67\right)\sqrt{-x-\sqrt{x}+1}$$
$$-\frac{45}{128}\arctan\left(-\frac{\left(8x^2-\left(16x^2-38x+11\right)\sqrt{x}-9x+3\right)\sqrt{-x-\sqrt{x}+1}}{4\left(4x^3-13x^2+7x-1\right)}\right)$$

input
```
integrate(x^(1/2)*(1-x-x^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/96*(2*(24*x - 17)*sqrt(x) + 8*x + 67)*sqrt(-x - sqrt(x) + 1) - 45/128*ar
ctan(-1/4*(8*x^2 - (16*x^2 - 38*x + 11)*sqrt(x) - 9*x + 3)*sqrt(-x - sqrt(
x) + 1)/(4*x^3 - 13*x^2 + 7*x - 1))
```

3.962.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.59

$$\int \sqrt{1-\sqrt{x}-x}\sqrt{x}\,dx = 2\sqrt{-\sqrt{x}-x+1}\left(\frac{x^{\frac{3}{2}}}{4}-\frac{17\sqrt{x}}{96}+\frac{x}{24}+\frac{67}{192}\right)+\frac{45\,\mathrm{asin}\left(\frac{2\sqrt{5}\left(\sqrt{x}+\frac{1}{2}\right)}{5}\right)}{64}$$

input
```
integrate(x**(1/2)*(1-x-x**(1/2))**(1/2),x)
```

output
```
2*sqrt(-sqrt(x) - x + 1)*(x**(3/2)/4 - 17*sqrt(x)/96 + x/24 + 67/192) + 45
*asin(2*sqrt(5)*(sqrt(x) + 1/2)/5)/64
```

3.962.7 Maxima [F]

$$\int \sqrt{1-\sqrt{x}-x}\sqrt{x}\,dx = \int \sqrt{x}\sqrt{-x-\sqrt{x}+1}\,dx$$

input
```
integrate(x^(1/2)*(1-x-x^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)*sqrt(-x - sqrt(x) + 1), x)
```

3.962.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.54

$$\int \sqrt{1-\sqrt{x}-x}\sqrt{x}\,dx = \frac{1}{96}\left(2\left(4\sqrt{x}(6\sqrt{x}+1)-17\right)\sqrt{x}+67\right)\sqrt{-x-\sqrt{x}+1} + \frac{45}{64}\arcsin\left(\frac{1}{5}\sqrt{5}(2\sqrt{x}+1)\right)$$

input
```
integrate(x^(1/2)*(1-x-x^(1/2))^(1/2),x, algorithm="giac")
```

output
```
1/96*(2*(4*sqrt(x)*(6*sqrt(x) + 1) - 17)*sqrt(x) + 67)*sqrt(-x - sqrt(x) +
 1) + 45/64*arcsin(1/5*sqrt(5)*(2*sqrt(x) + 1))
```

3.962.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-\sqrt{x}-x}\sqrt{x}\,dx = \int \sqrt{x}\,\sqrt{1-\sqrt{x}-x}\,dx$$

input
```
int(x^(1/2)*(1 - x^(1/2) - x)^(1/2),x)
```

output
```
int(x^(1/2)*(1 - x^(1/2) - x)^(1/2), x)
```

3.963 $\int \sqrt[3]{1+\sqrt{-3+x}}\,dx$

3.963.1 Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \sqrt[3]{1+\sqrt{-3+x}}\,dx = -\frac{3}{2}\left(1+\sqrt{-3+x}\right)^{4/3}+\frac{6}{7}\left(1+\sqrt{-3+x}\right)^{7/3}$$

output
```
-3/2*(1+(-3+x)^(1/2))^(4/3)+6/7*(1+(-3+x)^(1/2))^(7/3)
```

3.963.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \sqrt[3]{1+\sqrt{-3+x}}\,dx = \frac{3}{14}\left(1+\sqrt{-3+x}\right)^{4/3}\left(-3+4\sqrt{-3+x}\right)$$

input
```
Integrate[(1 + Sqrt[-3 + x])^(1/3),x]
```

output
```
(3*(1 + Sqrt[-3 + x])^(4/3)*(-3 + 4*Sqrt[-3 + x]))/14
```

3.963.3 Rubi [A] (warning: unable to verify)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {239, 774, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{\sqrt{x-3}+1}\,dx$$

$$\Big\downarrow \text{ 239}$$

$$\int \sqrt[3]{\sqrt{x-3}+1}d(x-3)$$

$$\Big\downarrow \text{ 774}$$

$$2\int \sqrt{x-3}\sqrt[3]{x-2}d\sqrt{x-3}$$

$$\Big\downarrow \text{ 53}$$

$$2\int \left((x-2)^{4/3}-\sqrt[3]{x-2}\right)d\sqrt{x-3}$$

$$\Big\downarrow \text{ 2009}$$

$$2\left(\frac{3}{7}(x-2)^{7/3}-\frac{3}{4}(x-2)^{4/3}\right)$$

input
```
Int[(1 + Sqrt[-3 + x])^(1/3),x]
```

output
```
2*((-3*(-2 + x)^(4/3))/4 + (3*(-2 + x)^(7/3))/7)
```

3.963.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.963.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$-\frac{3\left(1+\sqrt{-3+x}\right)^{\frac{4}{3}}}{2}+\frac{6\left(1+\sqrt{-3+x}\right)^{\frac{7}{3}}}{7}$	24
default	$-\frac{3\left(1+\sqrt{-3+x}\right)^{\frac{4}{3}}}{2}+\frac{6\left(1+\sqrt{-3+x}\right)^{\frac{7}{3}}}{7}$	24

input
```
int((1+(-3+x)^(1/2))^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-3/2*(1+(-3+x)^(1/2))^(4/3)+6/7*(1+(-3+x)^(1/2))^(7/3)
```

3.963.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int \sqrt[3]{1+\sqrt{-3+x}}\,dx = \frac{3}{14}\left(4\,x+\sqrt{x-3}-15\right)\left(\sqrt{x-3}+1\right)^{\frac{1}{3}}$$

input
```
integrate((1+(-3+x)^(1/2))^(1/3),x, algorithm="fricas")
```

output
```
3/14*(4*x + sqrt(x - 3) - 15)*(sqrt(x - 3) + 1)^(1/3)
```

3.963.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(29) = 58$.

Time = 0.64 (sec) , antiderivative size = 184, normalized size of antiderivative = 5.26

$$\int \sqrt[3]{1+\sqrt{-3+x}}\,dx = \frac{12(x-3)^{\frac{7}{2}}\sqrt[3]{\sqrt{x-3}+1}}{14(x-3)^{\frac{5}{2}}+14(x-3)^2} - \frac{6(x-3)^{\frac{5}{2}}\sqrt[3]{\sqrt{x-3}+1}}{14(x-3)^{\frac{5}{2}}+14(x-3)^2} + \frac{9(x-3)^{\frac{5}{2}}}{14(x-3)^{\frac{5}{2}}+14(x-3)^2} + \frac{15(x-3)^3\sqrt[3]{\sqrt{x-3}+1}}{14(x-3)^{\frac{5}{2}}+14(x-3)^2} - \frac{9(x-3)^2\sqrt[3]{\sqrt{x-3}+1}}{14(x-3)^{\frac{5}{2}}+14(x-3)^2} + \frac{9(x-3)^2}{14(x-3)^{\frac{5}{2}}+14(x-3)^2}$$

input
```
integrate((1+(-3+x)**(1/2))**(1/3),x)
```

output
```
12*(x - 3)**(7/2)*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 14*(x - 3)
**2) - 6*(x - 3)**(5/2)*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 14*(
x - 3)**2) + 9*(x - 3)**(5/2)/(14*(x - 3)**(5/2) + 14*(x - 3)**2) + 15*(x
- 3)**3*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 14*(x - 3)**2) - 9*(
x - 3)**2*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 14*(x - 3)**2) + 9
*(x - 3)**2/(14*(x - 3)**(5/2) + 14*(x - 3)**2)
```

3.963.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int \sqrt[3]{1+\sqrt{-3+x}}\,dx = \frac{6}{7}\left(\sqrt{x-3}+1\right)^{\frac{7}{3}} - \frac{3}{2}\left(\sqrt{x-3}+1\right)^{\frac{4}{3}}$$

input
```
integrate((1+(-3+x)^(1/2))^(1/3),x, algorithm="maxima")
```

output
```
6/7*(sqrt(x - 3) + 1)^(7/3) - 3/2*(sqrt(x - 3) + 1)^(4/3)
```

3.963.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int \sqrt[3]{1+\sqrt{-3+x}}\,dx = \frac{6}{7}\left(\sqrt{x-3}+1\right)^{\frac{7}{3}} - \frac{3}{2}\left(\sqrt{x-3}+1\right)^{\frac{4}{3}}$$

input
```
integrate((1+(-3+x)^(1/2))^(1/3),x, algorithm="giac")
```

output
```
6/7*(sqrt(x - 3) + 1)^(7/3) - 3/2*(sqrt(x - 3) + 1)^(4/3)
```

3.963.9 Mupad [B] (verification not implemented)

Time = 19.37 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.46

$$\int \sqrt[3]{1+\sqrt{-3+x}}\,dx = (x-3)\ {}_2F_1\left(-\frac{1}{3},2;\ 3;\ -\sqrt{x-3}\right)$$

input
```
int(((x - 3)^(1/2) + 1)^(1/3),x)
```

output
```
(x - 3)*hypergeom([-1/3, 2], 3, -(x - 3)^(1/2))
```

3.964 $\int \frac{1}{\sqrt{3+\sqrt{-1+2x}}}\,dx$

3.964.1 Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{1}{\sqrt{3+\sqrt{-1+2x}}}\,dx = -6\sqrt{3+\sqrt{-1+2x}}+\frac{2}{3}\left(3+\sqrt{-1+2x}\right)^{3/2}$$

output
```
2/3*(3+(-1+2*x)^(1/2))^(3/2)-6*(3+(-1+2*x)^(1/2))^(1/2)
```

3.964.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{3+\sqrt{-1+2x}}}\,dx = \frac{2}{3}\left(-6+\sqrt{-1+2x}\right)\sqrt{3+\sqrt{-1+2x}}$$

input
```
Integrate[1/Sqrt[3 + Sqrt[-1 + 2*x]],x]
```

output
```
(2*(-6 + Sqrt[-1 + 2*x])*Sqrt[3 + Sqrt[-1 + 2*x]])/3
```

3.964.3 Rubi [A] (warning: unable to verify)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {239, 774, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sqrt{2x-1}+3}}dx$$
$$\Big\downarrow \text{239}$$
$$\frac{1}{2}\int \frac{1}{\sqrt{\sqrt{2x-1}+3}}d(2x-1)$$
$$\Big\downarrow \text{774}$$
$$\int \frac{\sqrt{2x-1}}{\sqrt{2x+2}}d\sqrt{2x-1}$$
$$\Big\downarrow \text{53}$$
$$\int \left(\sqrt{2x+2}-\frac{3}{\sqrt{2x+2}}\right)d\sqrt{2x-1}$$
$$\Big\downarrow \text{2009}$$
$$\frac{2}{3}(2x+2)^{3/2}-6\sqrt{2x+2}$$

input
```
Int[1/Sqrt[3 + Sqrt[-1 + 2*x]],x]
```

output
```
-6*Sqrt[2 + 2*x] + (2*(2 + 2*x)^(3/2))/3
```

3.964.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 239
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[1/Coefficient[v, x, 1
]   Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Lin
earQ[v, x] && NeQ[v, x]
```

rule 774
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]},
Simp[k   Subst[Int[x^(k - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; Fre
eQ[{a, b, p}, x] && FractionQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.964.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{2\left(3+\sqrt{2x-1}\right)^{\frac{3}{2}}}{3}-6\sqrt{3+\sqrt{2x-1}}$	28
default	$\frac{2\left(3+\sqrt{2x-1}\right)^{\frac{3}{2}}}{3}-6\sqrt{3+\sqrt{2x-1}}$	28

input
```
int(1/(3+(2*x-1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(3+(2*x-1)^(1/2))^(3/2)-6*(3+(2*x-1)^(1/2))^(1/2)
```

3.964.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

$$\int \frac{1}{\sqrt{3+\sqrt{-1+2x}}}\,dx = \frac{2}{3}\,\sqrt{\sqrt{2\,x-1}+3}\left(\sqrt{2\,x-1}-6\right)$$

input
```
integrate(1/(3+(-1+2*x)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(sqrt(2*x - 1) + 3)*(sqrt(2*x - 1) - 6)
```

3.964.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(31) = 62.

Time = 0.62 (sec) , antiderivative size = 265, normalized size of antiderivative = 7.16

$$\int \frac{1}{\sqrt{3+\sqrt{-1+2x}}}\,dx = -\frac{6\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}\sqrt{\sqrt{2}\sqrt{x-\frac{1}{2}}+3}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}}+\frac{36\sqrt{2}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}}+\frac{4\sqrt{3}\left(x-\frac{1}{2}\right)^{3}\sqrt{\sqrt{2}\sqrt{x-\frac{1}{2}}+3}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}}-\frac{36\sqrt{3}\left(x-\frac{1}{2}\right)^{2}\sqrt{\sqrt{2}\sqrt{x-\frac{1}{2}}+3}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}}+\frac{108\left(x-\frac{1}{2}\right)^{2}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}}$$

input
```
integrate(1/(3+(-1+2*x)**(1/2))**(1/2),x)
```

output
```
-6*sqrt(6)*(x - 1/2)**(5/2)*sqrt(sqrt(2)*sqrt(x - 1/2) + 3)/(3*sqrt(6)*(x
- 1/2)**(5/2) + 9*sqrt(3)*(x - 1/2)**2) + 36*sqrt(2)*(x - 1/2)**(5/2)/(3*s
qrt(6)*(x - 1/2)**(5/2) + 9*sqrt(3)*(x - 1/2)**2) + 4*sqrt(3)*(x - 1/2)**3
*sqrt(sqrt(2)*sqrt(x - 1/2) + 3)/(3*sqrt(6)*(x - 1/2)**(5/2) + 9*sqrt(3)*(
x - 1/2)**2) - 36*sqrt(3)*(x - 1/2)**2*sqrt(sqrt(2)*sqrt(x - 1/2) + 3)/(3*
sqrt(6)*(x - 1/2)**(5/2) + 9*sqrt(3)*(x - 1/2)**2) + 108*(x - 1/2)**2/(3*s
qrt(6)*(x - 1/2)**(5/2) + 9*sqrt(3)*(x - 1/2)**2)
```

3.964.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{3+\sqrt{-1+2x}}}\,dx = \frac{2}{3}\left(\sqrt{2x-1}+3\right)^{\frac{3}{2}}-6\sqrt{\sqrt{2x-1}+3}$$

input
```
integrate(1/(3+(-1+2*x)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
2/3*(sqrt(2*x - 1) + 3)^(3/2) - 6*sqrt(sqrt(2*x - 1) + 3)
```

3.964.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{3+\sqrt{-1+2x}}}\,dx = \frac{2}{3}\left(\sqrt{2\,x-1}+3\right)^{\frac{3}{2}} - 6\sqrt{\sqrt{2\,x-1}+3}$$

input
```
integrate(1/(3+(-1+2*x)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
2/3*(sqrt(2*x - 1) + 3)^(3/2) - 6*sqrt(sqrt(2*x - 1) + 3)
```

3.964.9 Mupad [B] (verification not implemented)

Time = 20.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt{3+\sqrt{-1+2x}}}\,dx = \frac{\sqrt{3}\,(2\,x-1)\,{}_2\mathrm{F}_1\left(\frac{1}{2},2;\;3;\;-\frac{\sqrt{2\,x-1}}{3}\right)}{6}$$

input
```
int(1/((2*x - 1)^(1/2) + 3)^(1/2),x)
```

output
```
(3^(1/2)*(2*x - 1)*hypergeom([1/2, 2], 3, -(2*x - 1)^(1/2)/3))/6
```

3.965 $\int \frac{\sqrt{1-x}}{1+\sqrt{x}} dx$

3.965.1 Optimal result

Integrand size = 19, antiderivative size = 29

$$\int \frac{\sqrt{1-x}}{1+\sqrt{x}} dx = -\left(\left(2-\sqrt{x}\right)\sqrt{1-x}\right)-\arcsin\left(\sqrt{x}\right)$$

output
```
-arcsin(x^(1/2))-(1-x)^(1/2)*(2-x^(1/2))
```

3.965.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{1-x}}{1+\sqrt{x}} dx = \left(-2+\sqrt{x}\right)\sqrt{1-x}+2\arctan\left(\frac{\sqrt{1-x}}{1+\sqrt{x}}\right)$$

input
```
Integrate[Sqrt[1 - x]/(1 + Sqrt[x]),x]
```

output
```
(-2 + Sqrt[x])*Sqrt[1 - x] + 2*ArcTan[Sqrt[1 - x]/(1 + Sqrt[x])]
```

3.965.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1388, 900, 89}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x}}{\sqrt{x}+1}dx$$
$$\Big\downarrow \text{1388}$$
$$\int \frac{\sqrt{1-\sqrt{x}}}{\sqrt{\sqrt{x}+1}}dx$$
$$\Big\downarrow \text{900}$$
$$2\int \frac{\sqrt{1-\sqrt{x}}\sqrt{x}}{\sqrt{\sqrt{x}+1}}d\sqrt{x}$$
$$\Big\downarrow \text{89}$$
$$2\left(-\frac{\arcsin\left(\sqrt{x}\right)}{2}-\frac{1}{2}\sqrt{1-x}(2-\sqrt{x})\right)$$

input
```
Int[Sqrt[1 - x]/(1 + Sqrt[x]),x]
```

output
```
2*(-1/2*((2 - Sqrt[x])*Sqrt[1 - x]) - ArcSin[Sqrt[x]]/2)
```

3.965.3.1 Defintions of rubi rules used

rule 89
```
Int[(((a_.) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)])/Sqrt[(e_) + (f_.)*(x_)],
 x_] :> Simp[Sqrt[c*e]*(b*f*x - 2*(b*e - a*f))*(Sqrt[e^2 - f^2*x^2]/(2*e*f^
2)), x] - Simp[Sqrt[c*e]*(b*e - 2*a*f)*(ArcSin[f*(x/e)]/(2*f^2)), x] /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && GtQ[c, 0] && GtQ[e, 0]
```

rule 900
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(a + b*x^(g*n)
)^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
 && NeQ[b*c - a*d, 0] && FractionQ[n]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

3.965.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(23) = 46$.

Time = 1.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.66

method	result	size
default	$-\frac{\sqrt{1-x}\,\sqrt{x}\,\left(-2\sqrt{-(x-1)x}+\arcsin(2x-1)\right)}{2\sqrt{-(x-1)x}}-2\sqrt{1-x}$	48

input
```
int((1-x)^(1/2)/(1+x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(1-x)^(1/2)*x^(1/2)*(-2*(-(x-1)*x)^(1/2)+arcsin(2*x-1))/(-(x-1)*x)^(1
/2)-2*(1-x)^(1/2)
```

3.965.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{1-x}}{1+\sqrt{x}}\,dx = \sqrt{x}\sqrt{-x+1} - 2\sqrt{-x+1} + \arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

input
```
integrate((1-x)^(1/2)/(1+x^(1/2)),x, algorithm="fricas")
```

output
```
sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) + arctan(sqrt(-x + 1)/sqrt(x))
```

3.965.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.38 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{1-x}}{1+\sqrt{x}}\,dx = i\sqrt{x}\sqrt{x-1} - 2i\sqrt{x-1} + i\operatorname{asinh}\left(\sqrt{x-1}\right)$$

input
```
integrate((1-x)**(1/2)/(1+x**(1/2)),x)
```

output
```
I*sqrt(x)*sqrt(x - 1) - 2*I*sqrt(x - 1) + I*asinh(sqrt(x - 1))
```

3.965.7 Maxima [F]

$$\int \frac{\sqrt{1-x}}{1+\sqrt{x}}\,dx = \int \frac{\sqrt{-x+1}}{\sqrt{x}+1}\,dx$$

input
```
integrate((1-x)^(1/2)/(1+x^(1/2)),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x + 1)/(sqrt(x) + 1), x)
```

3.965.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-x}}{1+\sqrt{x}}\,dx = \sqrt{x}\sqrt{-x+1} - 2\sqrt{-x+1} + \arcsin\left(\sqrt{-x+1}\right)$$

input
```
integrate((1-x)^(1/2)/(1+x^(1/2)),x, algorithm="giac")
```

output
```
sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) + arcsin(sqrt(-x + 1))
```

3.965.9 Mupad [B] (verification not implemented)

Time = 21.87 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{1-x}}{1+\sqrt{x}}\,dx = \sqrt{x}\,\sqrt{1-x} - 2\,\sqrt{1-x} - 2\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt{1-x}-1}\right)$$

input
```
int((1 - x)^(1/2)/(x^(1/2) + 1),x)
```

output
```
x^(1/2)*(1 - x)^(1/2) - 2*(1 - x)^(1/2) - 2*atan(x^(1/2)/((1 - x)^(1/2) -
1))
```

3.966 $\int \frac{\sqrt{1-x}}{1-\sqrt{x}}\,dx$

3.966.1 Optimal result

Integrand size = 21, antiderivative size = 25

$$\int \frac{\sqrt{1-x}}{1-\sqrt{x}}\,dx=-\left(\left(2+\sqrt{x}\right)\sqrt{1-x}\right)+\arcsin\left(\sqrt{x}\right)$$

output
```
arcsin(x^(1/2))-(1-x)^(1/2)*(2+x^(1/2))
```

3.966.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt{1-x}}{1-\sqrt{x}}\,dx=\left(-2-\sqrt{x}\right)\sqrt{1-x}+2\arctan\left(\frac{\sqrt{x}}{-1+\sqrt{1-x}}\right)$$

input
```
Integrate[Sqrt[1 - x]/(1 - Sqrt[x]),x]
```

output
```
(-2 - Sqrt[x])*Sqrt[1 - x] + 2*ArcTan[Sqrt[x]/(-1 + Sqrt[1 - x])]
```

3.966.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1388, 900, 89}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x}}{1-\sqrt{x}}dx$$

$$\Big\downarrow \text{1388}$$

$$\int \frac{\sqrt{\sqrt{x}+1}}{\sqrt{1-\sqrt{x}}}dx$$

$$\Big\downarrow \text{900}$$

$$2\int \frac{\sqrt{\sqrt{x}+1}\sqrt{x}}{\sqrt{1-\sqrt{x}}}d\sqrt{x}$$

$$\Big\downarrow \text{89}$$

$$2\left(\frac{\arcsin\left(\sqrt{x}\right)}{2}-\frac{1}{2}\left(\sqrt{x}+2\right)\sqrt{1-x}\right)$$

input
```
Int[Sqrt[1 - x]/(1 - Sqrt[x]),x]
```

output
```
2*(-1/2*((2 + Sqrt[x])*Sqrt[1 - x]) + ArcSin[Sqrt[x]]/2)
```

3.966.3.1 Defintions of rubi rules used

rule 89
```
Int[(((a_.) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)])/Sqrt[(e_) + (f_.)*(x_)],
 x_] :> Simp[Sqrt[c*e]*(b*f*x - 2*(b*e - a*f))*(Sqrt[e^2 - f^2*x^2]/(2*e*f^
2)), x] - Simp[Sqrt[c*e]*(b*e - 2*a*f)*(ArcSin[f*(x/e)]/(2*f^2)), x] /; Fre
eQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && GtQ[c, 0] && GtQ[e, 0]
```

rule 900
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(a + b*x^(g*n)
)^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
 && NeQ[b*c - a*d, 0] && FractionQ[n]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

3.966.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(19) = 38$.

Time = 1.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.92

method	result	size
default	$-2\sqrt{1-x}+\frac{\sqrt{1-x}\,\sqrt{x}\,\left(-2\sqrt{-(x-1)x}+\arcsin(2x-1)\right)}{2\sqrt{-(x-1)x}}$	48

input
```
int((1-x)^(1/2)/(1-x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-2*(1-x)^(1/2)+1/2*(1-x)^(1/2)*x^(1/2)*(-2*(-(x-1)*x)^(1/2)+arcsin(2*x-1))
/(-(x-1)*x)^(1/2)
```

3.966.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{1-x}}{1-\sqrt{x}}\,dx = -\sqrt{x}\sqrt{-x+1}-2\sqrt{-x+1}-\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

input
```
integrate((1-x)^(1/2)/(1-x^(1/2)),x, algorithm="fricas")
```

output
```
-sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) - arctan(sqrt(-x + 1)/sqrt(x))
```

3.966.6 Sympy [A] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt{1-x}}{1-\sqrt{x}}\,dx = -2\left(\begin{cases} \frac{\sqrt{x}\sqrt{1-x}}{2} + \sqrt{1-x} - \frac{\operatorname{asin}\left(\sqrt{x}\right)}{2} & \text{for}\ \sqrt{x} > -1 \wedge \sqrt{x} < 1 \end{cases}\right)$$

input
```
integrate((1-x)**(1/2)/(1-x**(1/2)),x)
```

output
```
-2*Piecewise((sqrt(x)*sqrt(1 - x)/2 + sqrt(1 - x) - asin(sqrt(x))/2, (sqrt
(x) > -1) & (sqrt(x) < 1)))
```

3.966.7 Maxima [F]

$$\int \frac{\sqrt{1-x}}{1-\sqrt{x}}\,dx = \int -\frac{\sqrt{-x+1}}{\sqrt{x}-1}\,dx$$

input
```
integrate((1-x)^(1/2)/(1-x^(1/2)),x, algorithm="maxima")
```

output
```
-integrate(sqrt(-x + 1)/(sqrt(x) - 1), x)
```

3.966.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{1-x}}{1-\sqrt{x}}\,dx = -\sqrt{x}\sqrt{-x+1} - 2\sqrt{-x+1} - \arcsin\left(\sqrt{-x+1}\right)$$

input
```
integrate((1-x)^(1/2)/(1-x^(1/2)),x, algorithm="giac")
```

output
```
-sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) - arcsin(sqrt(-x + 1))
```

3.966.9 Mupad [B] (verification not implemented)

Time = 22.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\int \frac{\sqrt{1-x}}{1-\sqrt{x}}\,dx = 2\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt{1-x}-1}\right) - 2\sqrt{1-x} - \sqrt{x}\,\sqrt{1-x}$$

input
```
int(-(1 - x)^(1/2)/(x^(1/2) - 1),x)
```

output
```
2*atan(x^(1/2)/((1 - x)^(1/2) - 1)) - 2*(1 - x)^(1/2) - x^(1/2)*(1 - x)^(1
/2)
```

3.967 $\int \frac{x}{x-\sqrt{1+x^2}}\,dx$

3.967.1 Optimal result

Integrand size = 17, antiderivative size = 21

$$\int \frac{x}{x-\sqrt{1+x^2}}\,dx=-\frac{x^3}{3}-\frac{1}{3}\left(1+x^2\right)^{3/2}$$

output
```
-1/3*x^3-1/3*(x^2+1)^(3/2)
```

3.967.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x}{x-\sqrt{1+x^2}}\,dx=-\frac{x^3}{3}-\frac{1}{3}\left(1+x^2\right)^{3/2}$$

input
```
Integrate[x/(x - Sqrt[1 + x^2]),x]
```

output
```
-1/3*x^3 - (1 + x^2)^(3/2)/3
```

3.967.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2531, 15, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x-\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{ 2531}$$

$$-\int x^2dx-\int x\sqrt{x^2+1}dx$$

$$\Big\downarrow \text{ 15}$$

$$-\int x\sqrt{x^2+1}dx-\frac{x^3}{3}$$

$$\Big\downarrow \text{ 241}$$

$$-\frac{x^3}{3}-\frac{1}{3}\left(x^2+1\right)^{3/2}$$

input
```
Int[x/(x - Sqrt[1 + x^2]),x]
```

output
```
-1/3*x^3 - (1 + x^2)^(3/2)/3
```

3.967.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2531
```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> Simp[-b/(a*d)   Int[u*x^n, x], x] + Simp[1/(a*c)   Int[u*Sqrt[a + b*
x^(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 -
 d^2, 0]
```

3.967.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{x^{3}}{3}-\frac{\left(x^{2}+1\right)^{\frac{3}{2}}}{3}$	16
trager	$-\frac{x^{3}}{3}+\left(-\frac{x^{2}}{3}-\frac{1}{3}\right)\sqrt{x^{2}+1}$	22

input
```
int(x/(x-(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^3-1/3*(x^2+1)^(3/2)
```

3.967.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{x}{x-\sqrt{1+x^2}}\,dx = -\frac{1}{3}\,x^{3} - \frac{1}{3}\,{\left(x^{2} + 1\right)}^{\frac{3}{2}}$$

input
```
integrate(x/(x-(x^2+1)^(1/2)),x, algorithm="fricas")
```

output
```
-1/3*x^3 - 1/3*(x^2 + 1)^(3/2)
```

3.967.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(15) = 30.

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.67

$$\int \frac{x}{x-\sqrt{1+x^2}}\,dx = \frac{2x^{2}}{3x-3\sqrt{x^{2}+1}} - \frac{x\sqrt{x^{2}+1}}{3x-3\sqrt{x^{2}+1}} + \frac{1}{3x-3\sqrt{x^{2}+1}}$$

input
```
integrate(x/(x-(x**2+1)**(1/2)),x)
```

output
```
2*x**2/(3*x - 3*sqrt(x**2 + 1)) - x*sqrt(x**2 + 1)/(3*x - 3*sqrt(x**2 + 1)
) + 1/(3*x - 3*sqrt(x**2 + 1))
```

3.967.7 Maxima [F]

$$\int \frac{x}{x-\sqrt{1+x^2}}\,dx = \int \frac{x}{x-\sqrt{x^2+1}}\,dx$$

input
```
integrate(x/(x-(x^2+1)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x/(x - sqrt(x^2 + 1)), x)
```

3.967.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{x}{x-\sqrt{1+x^2}}\,dx = -\frac{1}{3}x^3-\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(x/(x-(x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
-1/3*x^3 - 1/3*(x^2 + 1)^(3/2)
```

3.967.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{x}{x-\sqrt{1+x^2}}\,dx = -\sqrt{x^2+1}\left(\frac{x^2}{3}+\frac{1}{3}\right)-\frac{x^3}{3}$$

input
```
int(x/(x - (x^2 + 1)^(1/2)),x)
```

output
```
- (x^2 + 1)^(1/2)*(x^2/3 + 1/3) - x^3/3
```

3.968 $\int \frac{x}{x-\sqrt{1-x^2}}\,dx$

3.968.1 Optimal result

Integrand size = 19, antiderivative size = 65

$$\int \frac{x}{x-\sqrt{1-x^2}}\,dx=\frac{x}{2}+\frac{\sqrt{1-x^2}}{2}-\frac{\operatorname{arctanh}\left(\sqrt{2}x\right)}{2\sqrt{2}}-\frac{\operatorname{arctanh}\left(\sqrt{2}\sqrt{1-x^2}\right)}{2\sqrt{2}}$$

output
```
1/2*x-1/4*arctanh(x*2^(1/2))*2^(1/2)-1/4*arctanh(2^(1/2)*(-x^2+1)^(1/2))*2
^(1/2)+1/2*(-x^2+1)^(1/2)
```

3.968.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int \frac{x}{x-\sqrt{1-x^2}}\,dx=\frac{1}{2}\left(x+\sqrt{1-x^2}+\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}x}{-1-x+\sqrt{1-x^2}}\right)\right)$$

input
```
Integrate[x/(x - Sqrt[1 - x^2]),x]
```

output
```
(x + Sqrt[1 - x^2] + Sqrt[2]*ArcTanh[(Sqrt[2]*x)/(-1 - x + Sqrt[1 - x^2])]
)/2
```

3.968.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2532, 262, 219, 353, 60, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x-\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{2532}$$

$$-\int \frac{x^2}{1-2x^2}dx-\int \frac{x\sqrt{1-x^2}}{1-2x^2}dx$$

$$\Big\downarrow \text{262}$$

$$-\frac{1}{2}\int \frac{1}{1-2x^2}dx-\int \frac{x\sqrt{1-x^2}}{1-2x^2}dx+\frac{x}{2}$$

$$\Big\downarrow \text{219}$$

$$-\int \frac{x\sqrt{1-x^2}}{1-2x^2}dx-\frac{\text{arctanh}\left(\sqrt{2}x\right)}{2\sqrt{2}}+\frac{x}{2}$$

$$\Big\downarrow \text{353}$$

$$-\frac{1}{2}\int \frac{\sqrt{1-x^2}}{1-2x^2}dx^2-\frac{\text{arctanh}\left(\sqrt{2}x\right)}{2\sqrt{2}}+\frac{x}{2}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\sqrt{1-x^2}-\frac{1}{2}\int \frac{1}{\left(1-2x^2\right)\sqrt{1-x^2}}dx^2\right)-\frac{\text{arctanh}\left(\sqrt{2}x\right)}{2\sqrt{2}}+\frac{x}{2}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\int \frac{1}{2x^4-1}d\sqrt{1-x^2}+\sqrt{1-x^2}\right)-\frac{\text{arctanh}\left(\sqrt{2}x\right)}{2\sqrt{2}}+\frac{x}{2}$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\left(\sqrt{1-x^2}-\frac{\text{arctanh}\left(\sqrt{2}\sqrt{1-x^2}\right)}{\sqrt{2}}\right)-\frac{\text{arctanh}\left(\sqrt{2}x\right)}{2\sqrt{2}}+\frac{x}{2}$$

input
```
Int[x/(x - Sqrt[1 - x^2]),x]
```

output
```
x/2 - ArcTanh[Sqrt[2]*x]/(2*Sqrt[2]) + (Sqrt[1 - x^2] - ArcTanh[Sqrt[2]*Sq
rt[1 - x^2]]/Sqrt[2])/2
```

3.968.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

3.968. $\int \frac{x}{x-\sqrt{1-x^2}}\,dx$

rule 2532
```
Int[(x_)^(m_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x
_Symbol] :> Simp[-d   Int[x^(m + n)/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x], x]
 + Simp[c   Int[(x^m*Sqrt[a + b*x^(2*n)])/(a*c^2 + (b*c^2 - d^2)*x^(2*n)),
x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[p, 2*n] && NeQ[b*c^2 - d^2, 0
]
```

3.968.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

method	result
trager	$\frac{x}{2}+\frac{\sqrt{-x^2+1}}{2}-\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{2\sqrt{-x^2+1}+\operatorname{RootOf}\left(\_Z^2-2\right)}{\operatorname{RootOf}\left(\_Z^2-2\right)x-1}\right)}{4}$
default	$\frac{x}{2}-\frac{\operatorname{arctanh}\left(x\sqrt{2}\right)\sqrt{2}}{4}+\frac{\sqrt{-4\left(x+\frac{\sqrt{2}}{2}\right)^2+4\left(x+\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}{8}-\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\left(\left(x+\frac{\sqrt{2}}{2}\right)\sqrt{2}+1\right)\sqrt{2}}{\sqrt{-4\left(x+\frac{\sqrt{2}}{2}\right)^2+4\left(x+\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}\right)}{8}+\sqrt{-4\left(x-\right.}$

input
```
int(x/(x-(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/2*(-x^2+1)^(1/2)-1/4*RootOf(_Z^2-2)*ln(-(2*(-x^2+1)^(1/2)+RootOf(_
Z^2-2))/(RootOf(_Z^2-2)*x-1))
```

3.968.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(45) = 90.

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.49

$$\int \frac{x}{x-\sqrt{1-x^2}}\,dx = \frac{1}{8}\sqrt{2}\log\left(\frac{6x^2-2\sqrt{2}(2x^2-3)+2\sqrt{-x^2+1}(3\sqrt{2}-4)-9}{2x^2-1}\right) + \frac{1}{8}\sqrt{2}\log\left(\frac{2x^2-2\sqrt{2}x+1}{2x^2-1}\right)+\frac{1}{2}x+\frac{1}{2}\sqrt{-x^2+1}$$

input
```
integrate(x/(x-(-x^2+1)^(1/2)),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*log((6*x^2 - 2*sqrt(2)*(2*x^2 - 3) + 2*sqrt(-x^2 + 1)*(3*sqrt(
2) - 4) - 9)/(2*x^2 - 1)) + 1/8*sqrt(2)*log((2*x^2 - 2*sqrt(2)*x + 1)/(2*x
^2 - 1)) + 1/2*x + 1/2*sqrt(-x^2 + 1)
```

3.968.6 Sympy [F]

$$\int \frac{x}{x-\sqrt{1-x^2}}\,dx=\int \frac{x}{x-\sqrt{1-x^2}}\,dx$$

input
```
integrate(x/(x-(-x**2+1)**(1/2)),x)
```

output
```
Integral(x/(x - sqrt(1 - x**2)), x)
```

3.968.7 Maxima [F]

$$\int \frac{x}{x-\sqrt{1-x^2}}\,dx=\int \frac{x}{x-\sqrt{-x^2+1}}\,dx$$

input
```
integrate(x/(x-(-x^2+1)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x/(x - sqrt(-x^2 + 1)), x)
```

3.968.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(45) = 90.

Time = 0.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.62

$$\int \frac{x}{x-\sqrt{1-x^2}}\,dx=\frac{1}{8}\sqrt{2}\log\left(\frac{\left|4x-2\sqrt{2}\right|}{\left|4x+2\sqrt{2}\right|}\right)-\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-6\right|}{\left|4\sqrt{2}+\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-6\right|}\right)+\frac{1}{2}x+\frac{1}{2}\sqrt{-x^2+1}$$

input
```
integrate(x/(x-(-x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*log(abs(4*x - 2*sqrt(2))/abs(4*x + 2*sqrt(2))) - 1/8*sqrt(2)*l
og(abs(-4*sqrt(2) + 2*(sqrt(-x^2 + 1) - 1)^2/x^2 - 6)/abs(4*sqrt(2) + 2*(s
qrt(-x^2 + 1) - 1)^2/x^2 - 6)) + 1/2*x + 1/2*sqrt(-x^2 + 1)
```

3.968.9 Mupad [B] (verification not implemented)

Time = 21.67 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.95

$$\int \frac{x}{x-\sqrt{1-x^2}}\,dx = \frac{x}{2}-\frac{\sqrt{2}\,\ln\left(\frac{\sqrt{2}\left(\frac{\sqrt{2}\,x}{2}-1\right)\,1\mathrm{i}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\frac{\sqrt{2}}{2}}\right)}{8}-\frac{\sqrt{2}\,\ln\left(\frac{\sqrt{2}\left(\frac{\sqrt{2}\,x}{2}+1\right)\,1\mathrm{i}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\frac{\sqrt{2}}{2}}\right)}{8}+\frac{\sqrt{2}\,\ln\left(x-\frac{\sqrt{2}}{2}\right)}{8}-\frac{\sqrt{2}\,\ln\left(x+\frac{\sqrt{2}}{2}\right)}{8}+\frac{\sqrt{1-x^2}}{2}$$

input
```
int(x/(x - (1 - x^2)^(1/2)),x)
```

output
```
x/2 - (2^(1/2)*log((2^(1/2)*((2^(1/2)*x)/2 - 1)*1i - (1 - x^2)^(1/2)*1i)/(
x - 2^(1/2)/2)))/8 - (2^(1/2)*log((2^(1/2)*((2^(1/2)*x)/2 + 1)*1i + (1 - x
^2)^(1/2)*1i)/(x + 2^(1/2)/2)))/8 + (2^(1/2)*log(x - 2^(1/2)/2))/8 - (2^(1
/2)*log(x + 2^(1/2)/2))/8 + (1 - x^2)^(1/2)/2
```

3.969 $\int \frac{x}{x-\sqrt{1+2x^2}}\,dx$

3.969.1 Optimal result

Integrand size = 19, antiderivative size = 31

$$\int \frac{x}{x-\sqrt{1+2x^2}}\,dx = -x-\sqrt{1+2x^2}+\arctan(x)+\arctan\left(\sqrt{1+2x^2}\right)$$

output
```
-x+arctan(x)+arctan((2*x^2+1)^(1/2))-(2*x^2+1)^(1/2)
```

3.969.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.61

$$\int \frac{x}{x-\sqrt{1+2x^2}}\,dx = -x-\sqrt{1+2x^2}+2\arctan\left(\left(2+\sqrt{2}\right)x-\left(1+\sqrt{2}\right)\sqrt{1+2x^2}\right)$$

input
```
Integrate[x/(x - Sqrt[1 + 2*x^2]),x]
```

output
```
-x - Sqrt[1 + 2*x^2] + 2*ArcTan[(2 + Sqrt[2])*x - (1 + Sqrt[2])*Sqrt[1 + 2
*x^2]]
```

3.969.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2532, 262, 216, 353, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x-\sqrt{2x^2+1}}dx$$

$$\Big\downarrow \text{2532}$$

$$-\int \frac{x^2}{x^2+1}dx-\int \frac{x\sqrt{2x^2+1}}{x^2+1}dx$$

$$\Big\downarrow \text{262}$$

$$\int \frac{1}{x^2+1}dx-\int \frac{x\sqrt{2x^2+1}}{x^2+1}dx-x$$

$$\Big\downarrow \text{216}$$

$$-\int \frac{x\sqrt{2x^2+1}}{x^2+1}dx+\arctan(x)-x$$

$$\Big\downarrow \text{353}$$

$$-\frac{1}{2}\int \frac{\sqrt{2x^2+1}}{x^2+1}dx^2+\arctan(x)-x$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\int \frac{1}{(x^2+1)\sqrt{2x^2+1}}dx^2-2\sqrt{2x^2+1}\right)+\arctan(x)-x$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\int \frac{1}{\frac{x^4}{2}+\frac{1}{2}}d\sqrt{2x^2+1}-2\sqrt{2x^2+1}\right)+\arctan(x)-x$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(2\arctan\left(\sqrt{2x^2+1}\right)-2\sqrt{2x^2+1}\right)+\arctan(x)-x$$

input
```
Int[x/(x - Sqrt[1 + 2*x^2]),x]
```

output
```
-x + ArcTan[x] + (-2*Sqrt[1 + 2*x^2] + 2*ArcTan[Sqrt[1 + 2*x^2]])/2
```

3.969.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2532
```
Int[(x_)^(m_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x
_Symbol] :> Simp[-d   Int[x^(m + n)/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x], x]
 + Simp[c   Int[(x^m*Sqrt[a + b*x^(2*n)])/(a*c^2 + (b*c^2 - d^2)*x^(2*n)),
x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[p, 2*n] && NeQ[b*c^2 - d^2, 0
]
```

3.969. $\int \frac{x}{x-\sqrt{1+2x^2}}\,dx$

3.969.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
default	$-x+\arctan\left(x\right)+\arctan\left(\sqrt{2x^{2}+1}\right)-\sqrt{2x^{2}+1}$	28
trager	$-x-\sqrt{2x^{2}+1}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{\sqrt{2x^{2}+1}+\operatorname{RootOf}\left(\_Z^{2}+1\right)}{\operatorname{RootOf}\left(\_Z^{2}+1\right)x+1}\right)$	53

input
```
int(x/(x-(2*x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-x+arctan(x)+arctan((2*x^2+1)^(1/2))-(2*x^2+1)^(1/2)
```

3.969.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{x}{x-\sqrt{1+2x^2}}\,dx = -x-\sqrt{2\,x^{2}+1}+\arctan\left(x\right)-\arctan\left(-\frac{x^{2}-\sqrt{2\,x^{2}+1}+1}{x^{2}}\right)$$

input
```
integrate(x/(x-(2*x^2+1)^(1/2)),x, algorithm="fricas")
```

output
```
-x - sqrt(2*x^2 + 1) + arctan(x) - arctan(-(x^2 - sqrt(2*x^2 + 1) + 1)/x^2
)
```

3.969.6 Sympy [F]

$$\int \frac{x}{x-\sqrt{1+2x^2}}\,dx = \int \frac{x}{x-\sqrt{2x^{2}+1}}\,dx$$

input
```
integrate(x/(x-(2*x**2+1)**(1/2)),x)
```

output
```
Integral(x/(x - sqrt(2*x**2 + 1)), x)
```

3.969.7 Maxima [F]

$$\int \frac{x}{x-\sqrt{1+2x^2}}\,dx = \int \frac{x}{x-\sqrt{2\,x^2+1}}\,dx$$

input
```
integrate(x/(x-(2*x^2+1)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x/(x - sqrt(2*x^2 + 1)), x)
```

3.969.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(27) = 54$.

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.03

$$\int \frac{x}{x-\sqrt{1+2x^2}}\,dx = -\frac{1}{2}\,\pi - x - \sqrt{2\,x^2+1} + \arctan(x) + \arctan\left(-\frac{\left(\sqrt{2}x-\sqrt{2\,x^2+1}\right)^2+1}{2\left(\sqrt{2}x-\sqrt{2\,x^2+1}\right)}\right)$$

input
```
integrate(x/(x-(2*x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
-1/2*pi - x - sqrt(2*x^2 + 1) + arctan(x) + arctan(-1/2*((sqrt(2)*x - sqrt
(2*x^2 + 1))^2 + 1)/(sqrt(2)*x - sqrt(2*x^2 + 1)))
```

3.969.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.06

$$\int \frac{x}{x-\sqrt{1+2x^2}}\,dx = -x - \sqrt{2}\,\sqrt{x^2+\frac{1}{2}} - \ln(x-\mathrm{i})\,1\mathrm{i} + \frac{\ln\left(x-\frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{2}+\frac{1}{2}\mathrm{i}\right)\,1\mathrm{i}}{2} + \frac{\ln\left(x+\frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{2}-\frac{1}{2}\mathrm{i}\right)\,1\mathrm{i}}{2}$$

input
```
int(x/(x - (2*x^2 + 1)^(1/2)),x)
```

output
```
(log(x - (2^(1/2)*(x^2 + 1/2)^(1/2))/2 + 1i/2)*1i)/2 - log(x - 1i)*1i - x
+ (log(x + (2^(1/2)*(x^2 + 1/2)^(1/2))/2 - 1i/2)*1i)/2 - 2^(1/2)*(x^2 + 1/
2)^(1/2)
```

3.969. $\int \frac{x}{x-\sqrt{1+2x^2}}\,dx$

3.970 $\int \sqrt{x}\sqrt{\sqrt{x}+x}\,dx$

3.970.1 Optimal result

Integrand size = 17, antiderivative size = 82

$$\int \sqrt{x}\sqrt{\sqrt{x}+x}\,dx = \frac{5}{32}\left(1+2\sqrt{x}\right)\sqrt{\sqrt{x}+x}-\frac{5}{12}\left(\sqrt{x}+x\right)^{3/2}+\frac{1}{2}\sqrt{x}\left(\sqrt{x}+x\right)^{3/2}-\frac{5}{32}\text{arctanh}\left(\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)$$

output
```
-5/32*arctanh(x^(1/2)/(x+x^(1/2))^(1/2))-5/12*(x+x^(1/2))^(3/2)+1/2*x^(1/2
)*(x+x^(1/2))^(3/2)+5/32*(1+2*x^(1/2))*(x+x^(1/2))^(1/2)
```

3.970.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.70

$$\int \sqrt{x}\sqrt{\sqrt{x}+x}\,dx = \frac{1}{96}\sqrt{\sqrt{x}+x}\left(15-10\sqrt{x}+8x+48x^{3/2}\right)-\frac{5}{32}\text{arctanh}\left(\frac{\sqrt{\sqrt{x}+x}}{\sqrt{x}}\right)$$

input
```
Integrate[Sqrt[x]*Sqrt[Sqrt[x] + x],x]
```

output
```
(Sqrt[Sqrt[x] + x]*(15 - 10*Sqrt[x] + 8*x + 48*x^(3/2)))/96 - (5*ArcTanh[S
qrt[Sqrt[x] + x]/Sqrt[x]])/32
```

3.970.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {1924, 1134, 1160, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\sqrt{x+\sqrt{x}}\,dx$$

$$\Big\downarrow \text{1924}$$

$$2\int x\sqrt{x+\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{1134}$$

$$2\left(\frac{1}{4}\sqrt{x}\left(x+\sqrt{x}\right)^{3/2}-\frac{5}{8}\int \sqrt{x}\sqrt{x+\sqrt{x}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{1160}$$

$$2\left(\frac{1}{4}\sqrt{x}\left(x+\sqrt{x}\right)^{3/2}-\frac{5}{8}\left(\frac{1}{3}\left(x+\sqrt{x}\right)^{3/2}-\frac{1}{2}\int \sqrt{x+\sqrt{x}}d\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{1087}$$

$$2\left(\frac{1}{4}\sqrt{x}\left(x+\sqrt{x}\right)^{3/2}-\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{8}\int \frac{1}{\sqrt{x+\sqrt{x}}}d\sqrt{x}-\frac{1}{4}\left(2\sqrt{x}+1\right)\sqrt{x+\sqrt{x}}\right)+\frac{1}{3}\left(x+\sqrt{x}\right)^{3/2}\right)\right)$$

$$\Big\downarrow \text{1091}$$

$$2\left(\frac{1}{4}\sqrt{x}\left(x+\sqrt{x}\right)^{3/2}-\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\int \frac{1}{1-x}d\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}-\frac{1}{4}\left(2\sqrt{x}+1\right)\sqrt{x+\sqrt{x}}\right)+\frac{1}{3}\left(x+\sqrt{x}\right)^{3/2}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{1}{4}\sqrt{x}\left(x+\sqrt{x}\right)^{3/2}-\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{4}\text{arctanh}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)-\frac{1}{4}\left(2\sqrt{x}+1\right)\sqrt{x+\sqrt{x}}\right)+\frac{1}{3}\left(x+\sqrt{x}\right)^{3/2}\right)\right)$$

input
```
Int[Sqrt[x]*Sqrt[Sqrt[x] + x],x]
```

output
```
2*((Sqrt[x]*(Sqrt[x] + x)^(3/2))/4 - (5*((Sqrt[x] + x)^(3/2)/3 + (-1/4*((1
 + 2*Sqrt[x])*Sqrt[Sqrt[x] + x]) + ArcTanh[Sqrt[x]/Sqrt[Sqrt[x] + x]]/4)/2
))/8)
```

3.970.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1924
```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp
[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x
], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j
] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1
]
```

3.970. $\int \sqrt{x}\sqrt{\sqrt{x}+x}\,dx$

3.970.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.56

method	result	size
meijerg	$-\frac{-\frac{\sqrt{\pi}\, x^{\frac{1}{4}} \left(336 x^{\frac{3}{2}}+56 x-70 \sqrt{x}+105\right) \sqrt{1+\sqrt{x}}}{672}+\frac{5 \sqrt{\pi}\, \operatorname{arcsinh}\left(x^{\frac{1}{4}}\right)}{32}}{\sqrt{\pi}}$	46
derivativedivides	$\frac{\sqrt{x}\, (x+\sqrt{x})^{\frac{3}{2}}}{2}-\frac{5 (x+\sqrt{x})^{\frac{3}{2}}}{12}+\frac{5 (1+2 \sqrt{x}) \sqrt{x+\sqrt{x}}}{32}-\frac{5 \ln\left(\frac{1}{2}+\sqrt{x}+\sqrt{x+\sqrt{x}}\right)}{64}$	54
default	$\frac{\sqrt{x}\, (x+\sqrt{x})^{\frac{3}{2}}}{2}-\frac{5 (x+\sqrt{x})^{\frac{3}{2}}}{12}+\frac{5 (1+2 \sqrt{x}) \sqrt{x+\sqrt{x}}}{32}-\frac{5 \ln\left(\frac{1}{2}+\sqrt{x}+\sqrt{x+\sqrt{x}}\right)}{64}$	54

input
```
int(x^(1/2)*(x+x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/Pi^(1/2)*(-1/672*Pi^(1/2)*x^(1/4)*(336*x^(3/2)+56*x-70*x^(1/2)+105)*(1+
x^(1/2))^(1/2)+5/32*Pi^(1/2)*arcsinh(x^(1/4)))
```

3.970.5 Fricas [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.66

$$\int \sqrt{x}\sqrt{\sqrt{x}+x}\, dx = \frac{1}{96}\left(2\left(24\, x-5\right)\sqrt{x}+8\, x+15\right)\sqrt{x+\sqrt{x}} + \frac{5}{128}\log\left(4\sqrt{x+\sqrt{x}}(2\sqrt{x}+1)-8\, x-8\sqrt{x}-1\right)$$

input
```
integrate(x^(1/2)*(x+x^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/96*(2*(24*x - 5)*sqrt(x) + 8*x + 15)*sqrt(x + sqrt(x)) + 5/128*log(4*sqr
t(x + sqrt(x))*(2*sqrt(x) + 1) - 8*x - 8*sqrt(x) - 1)
```

3.970.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.71

$$\int \sqrt{x}\sqrt{\sqrt{x}+x}\,dx = 2\sqrt{\sqrt{x}+x}\left(\frac{x^{\frac{3}{2}}}{4} - \frac{5\sqrt{x}}{96} + \frac{x}{24} + \frac{5}{64}\right) - \frac{5\log\left(2\sqrt{x}+2\sqrt{\sqrt{x}+x}+1\right)}{64}$$

input
```
integrate(x**(1/2)*(x+x**(1/2))**(1/2),x)
```

output
```
2*sqrt(sqrt(x) + x)*(x**(3/2)/4 - 5*sqrt(x)/96 + x/24 + 5/64) - 5*log(2*sq
rt(x) + 2*sqrt(sqrt(x) + x) + 1)/64
```

3.970.7 Maxima [F]

$$\int \sqrt{x}\sqrt{\sqrt{x}+x}\,dx = \int \sqrt{x+\sqrt{x}}\sqrt{x}\,dx$$

input
```
integrate(x^(1/2)*(x+x^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x + sqrt(x))*sqrt(x), x)
```

3.970.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.61

$$\int \sqrt{x}\sqrt{\sqrt{x}+x}\,dx = \frac{1}{96}\left(2\left(4\sqrt{x}(6\sqrt{x}+1)-5\right)\sqrt{x}+15\right)\sqrt{x+\sqrt{x}} + \frac{5}{64}\log\left(-2\sqrt{x+\sqrt{x}}+2\sqrt{x}+1\right)$$

input
```
integrate(x^(1/2)*(x+x^(1/2))^(1/2),x, algorithm="giac")
```

output
```
1/96*(2*(4*sqrt(x)*(6*sqrt(x) + 1) - 5)*sqrt(x) + 15)*sqrt(x + sqrt(x)) +
5/64*log(-2*sqrt(x + sqrt(x)) + 2*sqrt(x) + 1)
```

3.970.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\sqrt{\sqrt{x}+x}\,dx = \int \sqrt{x}\,\sqrt{x+\sqrt{x}}\,dx$$

input
```
int(x^(1/2)*(x + x^(1/2))^(1/2),x)
```

output
```
int(x^(1/2)*(x + x^(1/2))^(1/2), x)
```

3.971 $\int \frac{1+\sqrt[3]{x}}{1+\sqrt{x}}\,dx$

3.971.1 Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt{x}}\,dx = -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5} - 2\sqrt{3}\arctan\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right) - 4\log\left(1+\sqrt[6]{x}\right) - \log\left(1-\sqrt[6]{x}+\sqrt[3]{x}\right)$$

output
```
-3*x^(1/3)+6/5*x^(5/6)-4*ln(1+x^(1/6))-ln(1-x^(1/6)+x^(1/3))-2*arctan(1/3*
(1-2*x^(1/6))*3^(1/2))*3^(1/2)+2*x^(1/2)
```

3.971.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt{x}}\,dx = -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5} - 2\sqrt{3}\arctan\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right) - 4\log\left(1+\sqrt[6]{x}\right) - \log\left(1-\sqrt[6]{x}+\sqrt[3]{x}\right)$$

input
```
Integrate[(1 + x^(1/3))/(1 + Sqrt[x]),x]
```

output
```
-3*x^(1/3) + 2*Sqrt[x] + (6*x^(5/6))/5 - 2*Sqrt[3]*ArcTan[(1 - 2*x^(1/6))/
Sqrt[3]] - 4*Log[1 + x^(1/6)] - Log[1 - x^(1/6) + x^(1/3)]
```

3.971.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {7267, 2027, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{x}+1}{\sqrt{x}+1}dx$$

$$\Big\downarrow \text{ 7267}$$

$$6\int \frac{x^{7/6}+x^{5/6}}{\sqrt{x}+1}d\sqrt[6]{x}$$

$$\Big\downarrow \text{ 2027}$$

$$6\int \frac{\left(\sqrt[3]{x}+1\right)x^{5/6}}{\sqrt{x}+1}d\sqrt[6]{x}$$

$$\Big\downarrow \text{ 2426}$$

$$6\int \left(\frac{\sqrt[6]{x}(1-\sqrt[6]{x})}{\sqrt{x}+1}+x^{2/3}+\sqrt[3]{x}-\sqrt[6]{x}\right)d\sqrt[6]{x}$$

$$\Big\downarrow \text{ 2009}$$

$$6\left(-\frac{\arctan\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{x^{5/6}}{5}+\frac{\sqrt{x}}{3}-\frac{\sqrt[3]{x}}{2}-\frac{2}{3}\log\left(\sqrt[6]{x}+1\right)-\frac{1}{6}\log\left(\sqrt[3]{x}-\sqrt[6]{x}+1\right)\right)$$

input
```
Int[(1 + x^(1/3))/(1 + Sqrt[x]),x]
```

output
```
6*(-1/2*x^(1/3) + Sqrt[x]/3 + x^(5/6)/5 - ArcTan[(1 - 2*x^(1/6))/Sqrt[3]]/
Sqrt[3] - (2*Log[1 + x^(1/6)])/3 - Log[1 - x^(1/6) + x^(1/3)]/6)
```

3.971.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.971.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{6x^{\frac{5}{6}}}{5}+2\sqrt{x}-3x^{\frac{1}{3}}-4\ln\left(1+x^{\frac{1}{6}}\right)-\ln\left(1-x^{\frac{1}{6}}+x^{\frac{1}{3}}\right)+2\sqrt{3}\arctan\left(\frac{\left(2x^{\frac{1}{6}}-1\right)\sqrt{3}}{3}\right)$
default	$\frac{6x^{\frac{5}{6}}}{5}+2\sqrt{x}-3x^{\frac{1}{3}}-4\ln\left(1+x^{\frac{1}{6}}\right)-\ln\left(1-x^{\frac{1}{6}}+x^{\frac{1}{3}}\right)+2\sqrt{3}\arctan\left(\frac{\left(2x^{\frac{1}{6}}-1\right)\sqrt{3}}{3}\right)$
meijerg	$2\sqrt{x}-2\ln\left(1+\sqrt{x}\right)-\frac{3x^{\frac{1}{3}}\left(-8\sqrt{x}+20\right)}{20}+2x^{\frac{1}{3}}\left(-\frac{\ln\left(1+x^{\frac{1}{6}}\right)}{x^{\frac{1}{3}}}+\frac{\ln\left(1-x^{\frac{1}{6}}+x^{\frac{1}{3}}\right)}{2x^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\,x}{2-x}\right)}{x^{\frac{1}{3}}}\right.$

input
```
int((1+x^(1/3))/(1+x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
6/5*x^(5/6)+2*x^(1/2)-3*x^(1/3)-4*ln(1+x^(1/6))-ln(1-x^(1/6)+x^(1/3))+2*3^
(1/2)*arctan(1/3*(2*x^(1/6)-1)*3^(1/2))
```

3.971.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt{x}}\,dx = 2\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{6}} - \frac{1}{3}\sqrt{3}\right) + \frac{6}{5}x^{\frac{5}{6}} + 2\sqrt{x} - 3x^{\frac{1}{3}} - \log\left(x^{\frac{1}{3}} - x^{\frac{1}{6}} + 1\right) - 4\log\left(x^{\frac{1}{6}} + 1\right)$$

input
```
integrate((1+x^(1/3))/(1+x^(1/2)),x, algorithm="fricas")
```

output
```
2*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/6) - 1/3*sqrt(3)) + 6/5*x^(5/6) + 2*sqrt
(x) - 3*x^(1/3) - log(x^(1/3) - x^(1/6) + 1) - 4*log(x^(1/6) + 1)
```

3.971.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.48 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.09

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt{x}}\,dx = \frac{16x^{\frac{5}{6}}\Gamma\left(\frac{8}{3}\right)}{5\Gamma\left(\frac{11}{3}\right)} - \frac{8\sqrt[3]{x}\Gamma\left(\frac{8}{3}\right)}{\Gamma\left(\frac{11}{3}\right)} + 2\sqrt{x} - 2\log\left(\sqrt{x}+1\right) - \frac{16e^{-\frac{2i\pi}{3}}\log\left(-\sqrt[6]{x}e^{\frac{i\pi}{3}}+1\right)\Gamma\left(\frac{8}{3}\right)}{3\Gamma\left(\frac{11}{3}\right)} - \frac{16\log\left(-\sqrt[6]{x}e^{i\pi}+1\right)\Gamma\left(\frac{8}{3}\right)}{3\Gamma\left(\frac{11}{3}\right)} - \frac{16e^{\frac{2i\pi}{3}}\log\left(-\sqrt[6]{x}e^{\frac{5i\pi}{3}}+1\right)\Gamma\left(\frac{8}{3}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate((1+x**(1/3))/(1+x**(1/2)),x)
```

output
```
16*x**(5/6)*gamma(8/3)/(5*gamma(11/3)) - 8*x**(1/3)*gamma(8/3)/gamma(11/3)
 + 2*sqrt(x) - 2*log(sqrt(x) + 1) - 16*exp(-2*I*pi/3)*log(-x**(1/6)*exp_po
lar(I*pi/3) + 1)*gamma(8/3)/(3*gamma(11/3)) - 16*log(-x**(1/6)*exp_polar(I
*pi) + 1)*gamma(8/3)/(3*gamma(11/3)) - 16*exp(2*I*pi/3)*log(-x**(1/6)*exp_
polar(5*I*pi/3) + 1)*gamma(8/3)/(3*gamma(11/3))
```

3.971.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.74

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt{x}}\,dx = 2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{6}}-1\right)\right)+\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x} - 3x^{\frac{1}{3}}-\log\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)-4\log\left(x^{\frac{1}{6}}+1\right)$$

input
```
integrate((1+x^(1/3))/(1+x^(1/2)),x, algorithm="maxima")
```

output
```
2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/6) - 1)) + 6/5*x^(5/6) + 2*sqrt(x) -
3*x^(1/3) - log(x^(1/3) - x^(1/6) + 1) - 4*log(x^(1/6) + 1)
```

3.971.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.74

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt{x}}\,dx = 2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{6}}-1\right)\right)+\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x} - 3x^{\frac{1}{3}}-\log\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)-4\log\left(x^{\frac{1}{6}}+1\right)$$

input
```
integrate((1+x^(1/3))/(1+x^(1/2)),x, algorithm="giac")
```

output
```
2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/6) - 1)) + 6/5*x^(5/6) + 2*sqrt(x) -
3*x^(1/3) - log(x^(1/3) - x^(1/6) + 1) - 4*log(x^(1/6) + 1)
```

3.971.9 Mupad [B] (verification not implemented)

Time = 20.54 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.28

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt{x}}\,dx = 2\sqrt{x}+\ln\left(\left(-1+\sqrt{3}\,1\mathrm{i}\right)\left(27+\sqrt{3}\,9\mathrm{i}\right)+36\,x^{1/6} +36\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)-\ln\left(\left(1+\sqrt{3}\,1\mathrm{i}\right)\left(-27+\sqrt{3}\,9\mathrm{i}\right)+36\,x^{1/6} +36\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)-4\ln\left(36\,x^{1/6}+36\right)-3\,x^{1/3}+\frac{6\,x^{5/6}}{5}$$

input
```
int((x^(1/3) + 1)/(x^(1/2) + 1),x)
```

output
```
log((3^(1/2)*1i - 1)*(3^(1/2)*9i + 27) + 36*x^(1/6) + 36)*(3^(1/2)*1i - 1)
 - 4*log(36*x^(1/6) + 36) - log((3^(1/2)*1i + 1)*(3^(1/2)*9i - 27) + 36*x^
(1/6) + 36)*(3^(1/2)*1i + 1) + 2*x^(1/2) - 3*x^(1/3) + (6*x^(5/6))/5
```

3.971. $\int \frac{1+\sqrt[3]{x}}{1+\sqrt{x}}\,dx$

3.972 $\int \frac{1+\sqrt[3]{x}}{1+\sqrt[4]{x}}\,dx$

3.972.1 Optimal result

Integrand size = 17, antiderivative size = 115

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt[4]{x}}\,dx = 12\sqrt[12]{x}+4\sqrt[4]{x}-3\sqrt[3]{x}-2\sqrt{x}+\frac{12x^{7/12}}{7}+\frac{4x^{3/4}}{3}-\frac{6x^{5/6}}{5}+\frac{12x^{13/12}}{13} + 4\sqrt{3}\arctan\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right)-8\log\left(1+\sqrt[12]{x}\right)-2\log\left(1-\sqrt[12]{x}+\sqrt[6]{x}\right)$$

output
```
12*x^(1/12)+4*x^(1/4)-3*x^(1/3)+12/7*x^(7/12)+4/3*x^(3/4)-6/5*x^(5/6)+12/1
3*x^(13/12)-8*ln(1+x^(1/12))-2*ln(1-x^(1/12)+x^(1/6))+4*arctan(1/3*(1-2*x^
(1/12))*3^(1/2))*3^(1/2)-2*x^(1/2)
```

3.972.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt[4]{x}}\,dx = \frac{16380\sqrt[12]{x}+5460\sqrt[4]{x}-4095\sqrt[3]{x}-2730\sqrt{x}+2340x^{7/12}+1820x^{3/4}-1638x^{5/6}+1260x^{13/12}}{1365} + 4\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[12]{x}}{\sqrt{3}}\right)-8\log\left(1+\sqrt[12]{x}\right)-2\log\left(1-\sqrt[12]{x}+\sqrt[6]{x}\right)$$

input
```
Integrate[(1 + x^(1/3))/(1 + x^(1/4)),x]
```

output
```
(16380*x^(1/12) + 5460*x^(1/4) - 4095*x^(1/3) - 2730*Sqrt[x] + 2340*x^(7/1
2) + 1820*x^(3/4) - 1638*x^(5/6) + 1260*x^(13/12))/1365 + 4*Sqrt[3]*ArcTan
[1/Sqrt[3] - (2*x^(1/12))/Sqrt[3]] - 8*Log[1 + x^(1/12)] - 2*Log[1 - x^(1/
12) + x^(1/6)]
```

3.972.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {7267, 2027, 2375, 27, 2426, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{x}+1}{\sqrt[4]{x}+1}dx$$

$$\Big\downarrow \text{ 7267}$$

$$12\int \frac{x^{5/4}+x^{11/12}}{\sqrt[4]{x}+1}d\sqrt[12]{x}$$

$$\Big\downarrow \text{ 2027}$$

$$12\int \frac{\left(\sqrt[3]{x}+1\right)x^{11/12}}{\sqrt[4]{x}+1}d\sqrt[12]{x}$$

$$\Big\downarrow \text{ 2375}$$

$$12\left(\frac{1}{13}\int \frac{13\left(1-\sqrt[12]{x}\right)x^{11/12}}{\sqrt[4]{x}+1}d\sqrt[12]{x}+\frac{x^{13/12}}{13}\right)$$

$$\Big\downarrow \text{ 27}$$

$$12\left(\int \frac{\left(1-\sqrt[12]{x}\right)x^{11/12}}{\sqrt[4]{x}+1}d\sqrt[12]{x}+\frac{x^{13/12}}{13}\right)$$

$$\Big\downarrow \text{ 2426}$$

$$12\left(\int \left(-\frac{\sqrt[6]{x}+1}{\sqrt[4]{x}+1}-x^{3/4}+x^{2/3}+\sqrt{x}-x^{5/12}-\sqrt[4]{x}+\sqrt[6]{x}+1\right)d\sqrt[12]{x}+\frac{x^{13/12}}{13}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$12\left(\frac{\arctan\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{x^{13/12}}{13}-\frac{x^{5/6}}{10}+\frac{x^{3/4}}{9}+\frac{x^{7/12}}{7}-\frac{\sqrt{x}}{6}-\frac{\sqrt[3]{x}}{4}+\frac{\sqrt[4]{x}}{3}+\sqrt[12]{x}-\frac{2}{3}\log\left(\sqrt[12]{x}+1\right)-\frac{1}{6}\log\left(\sqrt[6]{\right.$$

input
```
Int[(1 + x^(1/3))/(1 + x^(1/4)),x]
```

output
```
12*(x^(1/12) + x^(1/4)/3 - x^(1/3)/4 - Sqrt[x]/6 + x^(7/12)/7 + x^(3/4)/9
- x^(5/6)/10 + x^(13/12)/13 + ArcTan[(1 - 2*x^(1/12))/Sqrt[3]]/Sqrt[3] - (
2*Log[1 + x^(1/12)])/3 - Log[1 - x^(1/12) + x^(1/6)]/6)
```

3.972.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2375
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Simp[Pqq*(c*x)^(m + q
- n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x] + Si
mp[1/(b*(m + q + n*p + 1))   Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(P
q - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x]] /
; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (
q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

rule 2426
```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

3.972. $\int \frac{1+\sqrt[3]{x}}{1+\sqrt[4]{x}}\,dx$

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

3.972.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

method	result
derivativedivides	$\frac{12x^{\frac{13}{12}}}{13}-\frac{6x^{\frac{5}{6}}}{5}+\frac{4x^{\frac{3}{4}}}{3}+\frac{12x^{\frac{7}{12}}}{7}-2\sqrt{x}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}+12x^{\frac{1}{12}}-8\ln\left(1+x^{\frac{1}{12}}\right)-2\ln\left(1-x\right.$
default	$\frac{12x^{\frac{13}{12}}}{13}-\frac{6x^{\frac{5}{6}}}{5}+\frac{4x^{\frac{3}{4}}}{3}+\frac{12x^{\frac{7}{12}}}{7}-2\sqrt{x}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}+12x^{\frac{1}{12}}-8\ln\left(1+x^{\frac{1}{12}}\right)-2\ln\left(1-x\right.$
meijerg	$\frac{x^{\frac{1}{4}}\left(4\sqrt{x}-6x^{\frac{1}{4}}+12\right)}{3}-4\ln\left(1+x^{\frac{1}{4}}\right)+\frac{3x^{\frac{1}{12}}\left(560x-728x^{\frac{3}{4}}+1040\sqrt{x}-1820x^{\frac{1}{4}}+7280\right)}{1820}-4x^{\frac{1}{12}}\left(\frac{\ln\left(1+x^{\frac{1}{1}}\right.}{x^{\frac{1}{12}}}\right.$

input
```
int((1+x^(1/3))/(1+x^(1/4)),x,method=_RETURNVERBOSE)
```

output
```
12/13*x^(13/12)-6/5*x^(5/6)+4/3*x^(3/4)+12/7*x^(7/12)-2*x^(1/2)-3*x^(1/3)+
4*x^(1/4)+12*x^(1/12)-8*ln(1+x^(1/12))-2*ln(1-x^(1/12)+x^(1/6))-4*3^(1/2)*
arctan(1/3*(2*x^(1/12)-1)*3^(1/2))
```

3.972.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt[4]{x}}\,dx = -4\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{12}}-\frac{1}{3}\sqrt{3}\right)+\frac{12}{13}(x+13)x^{\frac{1}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}} +\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)-8\log\left(x^{\frac{1}{12}}+1\right)$$

input
```
integrate((1+x^(1/3))/(1+x^(1/4)),x, algorithm="fricas")
```

output
```
-4*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/12) - 1/3*sqrt(3)) + 12/13*(x + 13)*x^(
1/12) - 6/5*x^(5/6) + 4/3*x^(3/4) + 12/7*x^(7/12) - 2*sqrt(x) - 3*x^(1/3)
+ 4*x^(1/4) - 2*log(x^(1/6) - x^(1/12) + 1) - 8*log(x^(1/12) + 1)
```

3.972.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.91 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.92

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt[4]{x}}\,dx = \frac{64x^{\frac{13}{12}}\Gamma\left(\frac{16}{3}\right)}{13\Gamma\left(\frac{19}{3}\right)} + \frac{64x^{\frac{7}{12}}\Gamma\left(\frac{16}{3}\right)}{7\Gamma\left(\frac{19}{3}\right)} + \frac{64\sqrt[12]{x}\Gamma\left(\frac{16}{3}\right)}{\Gamma\left(\frac{19}{3}\right)} - \frac{32x^{\frac{5}{6}}\Gamma\left(\frac{16}{3}\right)}{5\Gamma\left(\frac{19}{3}\right)} + \frac{4x^{\frac{3}{4}}}{3} + 4\sqrt[4]{x} - \frac{16\sqrt[3]{x}\Gamma\left(\frac{16}{3}\right)}{\Gamma\left(\frac{19}{3}\right)} - 2\sqrt{x} - 4\log\left(\sqrt[4]{x}+1\right) + \frac{64e^{-\frac{i\pi}{3}}\log\left(-\sqrt[12]{x}e^{\frac{i\pi}{3}}+1\right)\Gamma\left(\frac{16}{3}\right)}{3\Gamma\left(\frac{19}{3}\right)} - \frac{64\log\left(-\sqrt[12]{x}e^{i\pi}+1\right)\Gamma\left(\frac{16}{3}\right)}{3\Gamma\left(\frac{19}{3}\right)} + \frac{64e^{\frac{i\pi}{3}}\log\left(-\sqrt[12]{x}e^{\frac{5i\pi}{3}}+1\right)\Gamma\left(\frac{16}{3}\right)}{3\Gamma\left(\frac{19}{3}\right)}$$

input
```
integrate((1+x**(1/3))/(1+x**(1/4)),x)
```

output
```
64*x**(13/12)*gamma(16/3)/(13*gamma(19/3)) + 64*x**(7/12)*gamma(16/3)/(7*g
amma(19/3)) + 64*x**(1/12)*gamma(16/3)/gamma(19/3) - 32*x**(5/6)*gamma(16/
3)/(5*gamma(19/3)) + 4*x**(3/4)/3 + 4*x**(1/4) - 16*x**(1/3)*gamma(16/3)/g
amma(19/3) - 2*sqrt(x) - 4*log(x**(1/4) + 1) + 64*exp(-I*pi/3)*log(-x**(1/
12)*exp_polar(I*pi/3) + 1)*gamma(16/3)/(3*gamma(19/3)) - 64*log(-x**(1/12)
*exp_polar(I*pi) + 1)*gamma(16/3)/(3*gamma(19/3)) + 64*exp(I*pi/3)*log(-x*
*(1/12)*exp_polar(5*I*pi/3) + 1)*gamma(16/3)/(3*gamma(19/3))
```

3.972.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt[4]{x}}\,dx = -4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}}-1\right)\right) + \frac{12}{13}x^{\frac{13}{12}} - \frac{6}{5}x^{\frac{5}{6}} + \frac{4}{3}x^{\frac{3}{4}} + \frac{12}{7}x^{\frac{7}{12}} - 2\sqrt{x} - 3x^{\frac{1}{3}} + 4x^{\frac{1}{4}} + 12x^{\frac{1}{12}} - 2\log\left(x^{\frac{1}{6}} - x^{\frac{1}{12}} + 1\right) - 8\log\left(x^{\frac{1}{12}}+1\right)$$

input
```
integrate((1+x^(1/3))/(1+x^(1/4)),x, algorithm="maxima")
```

output
```
-4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 12/13*x^(13/12) - 6/5*x^
(5/6) + 4/3*x^(3/4) + 12/7*x^(7/12) - 2*sqrt(x) - 3*x^(1/3) + 4*x^(1/4) +
12*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) - 8*log(x^(1/12) + 1)
```

3.972.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.70

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt[4]{x}}\,dx = -4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}}-1\right)\right)+\frac{12}{13}x^{\frac{13}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}+\frac{12}{7}x^{\frac{7}{12}} - 2\sqrt{x}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}+12x^{\frac{1}{12}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)-8\log\left(x^{\frac{1}{12}}+1\right)$$

input
```
integrate((1+x^(1/3))/(1+x^(1/4)),x, algorithm="giac")
```

output
```
-4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 12/13*x^(13/12) - 6/5*x^
(5/6) + 4/3*x^(3/4) + 12/7*x^(7/12) - 2*sqrt(x) - 3*x^(1/3) + 4*x^(1/4) +
12*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) - 8*log(x^(1/12) + 1)
```

3.972.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.13

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt[4]{x}}\,dx = 4\,x^{1/4} +\ln\left(\left(-2+\sqrt{3}\,2i\right)\left(54-36\,x^{1/12}+\sqrt{3}\,18i\right)-144\,x^{1/12}+144\right)\left(-2+\sqrt{3}\,2i\right)-\ln\left(\left(2+\sqrt{3}\,2i\right)\left(36\,x^{1/12}-\right.\right.$$

input
```
int((x^(1/3) + 1)/(x^(1/4) + 1),x)
```

output
```
log((3^(1/2)*2i - 2)*(3^(1/2)*18i - 36*x^(1/12) + 54) - 144*x^(1/12) + 144
)*(3^(1/2)*2i - 2) - 8*log(144*x^(1/12) + 144) - log((3^(1/2)*2i + 2)*(3^(
1/2)*18i + 36*x^(1/12) - 54) - 144*x^(1/12) + 144)*(3^(1/2)*2i + 2) - 2*x^
(1/2) - 3*x^(1/3) + 4*x^(1/4) + (4*x^(3/4))/3 - (6*x^(5/6))/5 + 12*x^(1/12
) + (12*x^(7/12))/7 + (12*x^(13/12))/13
```

3.973 $\int \frac{x^2}{-1+x^2+\sqrt{1-x^2}}\,dx$

3.973.1 Optimal result

Integrand size = 22, antiderivative size = 4

$$\int \frac{x^2}{-1+x^2+\sqrt{1-x^2}}\,dx = x+\arcsin(x)$$

output
```
x+arcsin(x)
```

3.973.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 22 vs. 2(4) = 8.

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 5.50

$$\int \frac{x^2}{-1+x^2+\sqrt{1-x^2}}\,dx = x+2\arctan\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

input
```
Integrate[x^2/(-1 + x^2 + Sqrt[1 - x^2]),x]
```

output
```
x + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])]
```

3.973.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2587, 24, 25, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{x^2+\sqrt{1-x^2}-1}dx$$
$$\Big\downarrow \text{ 2587}$$
$$-\int -\frac{1}{\sqrt{1-x^2}}dx-\int -1dx$$
$$\Big\downarrow \text{ 24}$$
$$x-\int -\frac{1}{\sqrt{1-x^2}}dx$$
$$\Big\downarrow \text{ 25}$$
$$\int \frac{1}{\sqrt{1-x^2}}dx+x$$
$$\Big\downarrow \text{ 223}$$
$$\arcsin(x)+x$$

input
```
Int[x^2/(-1 + x^2 + Sqrt[1 - x^2]),x]
```

output
```
x + ArcSin[x]
```

3.973.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2587
```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Simp[c   Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Simp[a*e   Int[
u/((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e
, n}, x] && EqQ[b*c - a*d, 0]
```

3.973.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 7.25

method	result	size
trager	$x+\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-x^{2}+1}+x\right)$	29
default	$x+\frac{\ln(x-1)}{2}-\frac{\ln(x+1)}{2}+\operatorname{arctanh}\left(x\right)+\frac{\sqrt{-\left(x+1\right)^{2}+2x+2}}{2}+\arcsin\left(x\right)-\frac{\sqrt{-\left(x-1\right)^{2}-2x+2}}{2}$	51

input
```
int(x^2/(-1+x^2+(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x+RootOf(_Z^2+1)*ln(RootOf(_Z^2+1)*(-x^2+1)^(1/2)+x)
```

3.973.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(4) = 8.

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 5.00

$$\int\frac{x^2}{-1+x^2+\sqrt{1-x^2}}\,dx=x-2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

input
```
integrate(x^2/(-1+x^2+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

output
```
x - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

3.973.6 Sympy [F]

$$\int \frac{x^2}{-1+x^2+\sqrt{1-x^2}}\,dx=\int \frac{x^2}{x^2+\sqrt{1-x^2}-1}\,dx$$

input
```
integrate(x**2/(-1+x**2+(-x**2+1)**(1/2)),x)
```

output
```
Integral(x**2/(x**2 + sqrt(1 - x**2) - 1), x)
```

3.973.7 Maxima [F]

$$\int \frac{x^2}{-1+x^2+\sqrt{1-x^2}}\,dx=\int \frac{x^2}{x^2+\sqrt{-x^2+1}-1}\,dx$$

input
```
integrate(x^2/(-1+x^2+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(x^2/(x^2 + sqrt(-x^2 + 1) - 1), x)
```

3.973.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{-1+x^2+\sqrt{1-x^2}}\,dx=x+\arcsin(x)$$

input
```
integrate(x^2/(-1+x^2+(-x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
x + arcsin(x)
```

3.973.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{-1+x^2+\sqrt{1-x^2}}\,dx = x + \operatorname{asin}(x)$$

input
```
int(x^2/(x^2 + (1 - x^2)^(1/2) - 1),x)
```

output
```
x + asin(x)
```

3.974 $\int \sqrt{\frac{1+x}{x}}\,dx$

3.974.1 Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \sqrt{\frac{1+x}{x}}\,dx = \sqrt{1+\frac{1}{x}}x + \operatorname{arctanh}\left(\sqrt{1+\frac{1}{x}}\right)$$

output
```
arctanh((1+1/x)^(1/2))+x*(1+1/x)^(1/2)
```

3.974.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \sqrt{\frac{1+x}{x}}\,dx = \sqrt{1+\frac{1}{x}}x + \operatorname{arctanh}\left(\sqrt{1+\frac{1}{x}}\right)$$

input
```
Integrate[Sqrt[(1 + x)/x],x]
```

output
```
Sqrt[1 + x^(-1)]*x + ArcTanh[Sqrt[1 + x^(-1)]]
```

3.974.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {2072, 773, 51, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{x+1}{x}}dx$$
$$\Big\downarrow \text{2072}$$
$$\int \sqrt{\frac{1}{x}+1}dx$$
$$\Big\downarrow \text{773}$$
$$-\int \sqrt{1+\frac{1}{x}}x^2d\frac{1}{x}$$
$$\Big\downarrow \text{51}$$
$$\sqrt{\frac{1}{x}+1}x-\frac{1}{2}\int \frac{x}{\sqrt{1+\frac{1}{x}}}d\frac{1}{x}$$
$$\Big\downarrow \text{73}$$
$$\sqrt{\frac{1}{x}+1}x-\int \frac{1}{\frac{1}{x^2}-1}d\sqrt{1+\frac{1}{x}}$$
$$\Big\downarrow \text{220}$$
$$\text{arctanh}\left(\sqrt{\frac{1}{x}+1}\right)+\sqrt{\frac{1}{x}+1}x$$

input
```
Int[Sqrt[(1 + x)/x],x]
```

output
```
Sqrt[1 + x^(-1)]*x + ArcTanh[Sqrt[1 + x^(-1)]]
```

3.974.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 2072
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

3.974.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(18) = 36$.

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

method	result	size
trager	$x\sqrt{-\frac{-x-1}{x}}-\frac{\ln\left(2x\sqrt{-\frac{-x-1}{x}}-2x-1\right)}{2}$	39
default	$\frac{\sqrt{\frac{x+1}{x}}\,x\left(2\sqrt{x^2+x}+\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)\right)}{2\sqrt{(x+1)x}}$	41
risch	$x\sqrt{\frac{x+1}{x}}+\frac{\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)\sqrt{\frac{x+1}{x}}\,\sqrt{(x+1)x}}{2x+2}$	47

input
```
int(((x+1)/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x*(-(-x-1)/x)^(1/2)-1/2*ln(2*x*(-(-x-1)/x)^(1/2)-2*x-1)
```

3.974.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(18) = 36$.

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \sqrt{\frac{1+x}{x}}\,dx = x\sqrt{\frac{x+1}{x}} + \frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}+1\right) - \frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

input
```
integrate(((1+x)/x)^(1/2),x, algorithm="fricas")
```

output
```
x*sqrt((x + 1)/x) + 1/2*log(sqrt((x + 1)/x) + 1) - 1/2*log(sqrt((x + 1)/x)
 - 1)
```

3.974.6 Sympy [F]

$$\int \sqrt{\frac{1+x}{x}}\,dx = \int \sqrt{\frac{x+1}{x}}\,dx$$

input
```
integrate(((1+x)/x)**(1/2),x)
```

output
```
Integral(sqrt((x + 1)/x), x)
```

3.974.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(18) = 36$.

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.27

$$\int \sqrt{\frac{1+x}{x}}\,dx = \frac{\sqrt{\frac{x+1}{x}}}{\frac{x+1}{x}-1} + \frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}+1\right) - \frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

input
```
integrate(((1+x)/x)^(1/2),x, algorithm="maxima")
```

output
```
sqrt((x + 1)/x)/((x + 1)/x - 1) + 1/2*log(sqrt((x + 1)/x) + 1) - 1/2*log(s
qrt((x + 1)/x) - 1)
```

3.974.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \sqrt{\frac{1+x}{x}}\,dx = -\frac{1}{2}\log\left(\left|-2\,x + 2\,\sqrt{x^2+x} - 1\right|\right)\operatorname{sgn}(x) + \sqrt{x^2+x}\operatorname{sgn}(x)$$

input
```
integrate(((1+x)/x)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x) + sqrt(x^2 + x)*sgn(x)
```

3.974.9 Mupad [B] (verification not implemented)

Time = 21.50 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \sqrt{\frac{1+x}{x}}\,dx = \operatorname{atanh}\left(\sqrt{\frac{1}{x}+1}\right) + x\,\sqrt{\frac{1}{x}+1}$$

input
```
int(((x + 1)/x)^(1/2),x)
```

output
```
atanh((1/x + 1)^(1/2)) + x*(1/x + 1)^(1/2)
```

3.975 $\int \sqrt{\frac{1-x}{x}}\,dx$

3.975.1 Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \sqrt{\frac{1-x}{x}}\,dx = \sqrt{-1+\frac{1}{x}}x - \arctan\left(\sqrt{-1+\frac{1}{x}}\right)$$

output
```
-arctan((-1+1/x)^(1/2))+x*(-1+1/x)^(1/2)
```

3.975.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \sqrt{\frac{1-x}{x}}\,dx = \sqrt{-1+\frac{1}{x}}x - \arctan\left(\sqrt{-1+\frac{1}{x}}\right)$$

input
```
Integrate[Sqrt[(1 - x)/x],x]
```

output
```
Sqrt[-1 + x^(-1)]*x - ArcTan[Sqrt[-1 + x^(-1)]]
```

3.975.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2072, 773, 51, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{1-x}{x}}dx$$
$$\Big\downarrow \text{2072}$$
$$\int \sqrt{\frac{1}{x}-1}dx$$
$$\Big\downarrow \text{773}$$
$$-\int \sqrt{\frac{1}{x}-1}x^2d\frac{1}{x}$$
$$\Big\downarrow \text{51}$$
$$\sqrt{\frac{1}{x}-1}x-\frac{1}{2}\int \frac{x}{\sqrt{\frac{1}{x}-1}}d\frac{1}{x}$$
$$\Big\downarrow \text{73}$$
$$\sqrt{\frac{1}{x}-1}x-\int \frac{1}{1+\frac{1}{x^2}}d\sqrt{\frac{1}{x}-1}$$
$$\Big\downarrow \text{216}$$
$$\sqrt{\frac{1}{x}-1}x-\arctan\left(\sqrt{\frac{1}{x}-1}\right)$$

input
```
Int[Sqrt[(1 - x)/x],x]
```

output
```
Sqrt[-1 + x^(-1)]*x - ArcTan[Sqrt[-1 + x^(-1)]]
```

3.975.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 2072
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

3.975.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

method	result	size
default	$\frac{\sqrt{-\frac{x-1}{x}}\, x \left(2\sqrt{-x^{2}+x}+\arcsin\left(2x-1\right)\right)}{2\sqrt{-\left(x-1\right) x}}$	40
risch	$\sqrt{-\frac{x-1}{x}}\, x-\frac{\arcsin\left(2x-1\right)\sqrt{-\frac{x-1}{x}}\, \sqrt{-\left(x-1\right) x}}{2\left(x-1\right)}$	45
trager	$\sqrt{-\frac{x-1}{x}}\, x+\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right) \ln\left(2\sqrt{-\frac{x-1}{x}}\, x-2\operatorname{RootOf}\left(\_Z^{2}+1\right) x+\operatorname{RootOf}\left(\_Z^{2}+1\right)\right)}{2}$	52

input
```
int(((1-x)/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-(x-1)/x)^(1/2)*x*(2*(-x^2+x)^(1/2)+arcsin(2*x-1))/(-(x-1)*x)^(1/2)
```

3.975.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \sqrt{\frac{1-x}{x}}\,dx = x\sqrt{-\frac{x-1}{x}} - \arctan\left(\sqrt{-\frac{x-1}{x}}\right)$$

input
```
integrate(((1-x)/x)^(1/2),x, algorithm="fricas")
```

output
```
x*sqrt(-(x - 1)/x) - arctan(sqrt(-(x - 1)/x))
```

3.975.6 Sympy [F]

$$\int \sqrt{\frac{1-x}{x}}\,dx = \int \sqrt{\frac{1-x}{x}}\,dx$$

input
```
integrate(((1-x)/x)**(1/2),x)
```

output
```
Integral(sqrt((1 - x)/x), x)
```

3.975.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \sqrt{\frac{1-x}{x}}\,dx = -\frac{\sqrt{-\frac{x-1}{x}}}{\frac{x-1}{x}-1} - \arctan\left(\sqrt{-\frac{x-1}{x}}\right)$$

input
```
integrate(((1-x)/x)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(-(x - 1)/x)/((x - 1)/x - 1) - arctan(sqrt(-(x - 1)/x))
```

3.975.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \sqrt{\frac{1-x}{x}}\, dx = \frac{1}{4}\, \pi \mathrm{sgn}(x) + \frac{1}{2}\, \arcsin\left(2\, x - 1\right) \mathrm{sgn}(x) + \sqrt{-x^2 + x} \mathrm{sgn}(x)$$

input
```
integrate(((1-x)/x)^(1/2),x, algorithm="giac")
```

output
```
1/4*pi*sgn(x) + 1/2*arcsin(2*x - 1)*sgn(x) + sqrt(-x^2 + x)*sgn(x)
```

3.975.9 Mupad [B] (verification not implemented)

Time = 20.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \sqrt{\frac{1-x}{x}}\, dx = x\, \sqrt{\frac{1}{x} - 1} - \mathrm{atan}\left(\sqrt{\frac{1}{x} - 1}\right)$$

input
```
int((-(x - 1)/x)^(1/2),x)
```

output
```
x*(1/x - 1)^(1/2) - atan((1/x - 1)^(1/2))
```

3.976 $\int \sqrt{\frac{-1+x}{x}}\,dx$

3.976.1 Optimal result

Integrand size = 11, antiderivative size = 24

$$\int \sqrt{\frac{-1+x}{x}}\,dx=\sqrt{-1+x}\sqrt{x}-\text{arcsinh}\left(\sqrt{-1+x}\right)$$

output
```
-arcsinh((-1+x)^(1/2))+(-1+x)^(1/2)*x^(1/2)
```

3.976.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int \sqrt{\frac{-1+x}{x}}\,dx=\sqrt{-1+x}\sqrt{x}-2\text{arctanh}\left(\frac{\sqrt{-1+x}}{-1+\sqrt{x}}\right)$$

input
```
Integrate[Sqrt[(-1 + x)/x],x]
```

output
```
Sqrt[-1 + x]*Sqrt[x] - 2*ArcTanh[Sqrt[-1 + x]/(-1 + Sqrt[x])]
```

3.976.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {2072, 773, 51, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{x-1}{x}}dx$$
$$\Big\downarrow \text{2072}$$
$$\int \sqrt{1-\frac{1}{x}}dx$$
$$\Big\downarrow \text{773}$$
$$-\int \sqrt{1-\frac{1}{x}}x^2d\frac{1}{x}$$
$$\Big\downarrow \text{51}$$
$$\frac{1}{2}\int \frac{x}{\sqrt{1-\frac{1}{x}}}d\frac{1}{x}+\sqrt{1-\frac{1}{x}}x$$
$$\Big\downarrow \text{73}$$
$$\sqrt{1-\frac{1}{x}}x-\int \frac{1}{1-\frac{1}{x^2}}d\sqrt{1-\frac{1}{x}}$$
$$\Big\downarrow \text{219}$$
$$\sqrt{1-\frac{1}{x}}x-\text{arctanh}\left(\sqrt{1-\frac{1}{x}}\right)$$

input
```
Int[Sqrt[(-1 + x)/x],x]
```

output
```
Sqrt[1 - x^(-1)]*x - ArcTanh[Sqrt[1 - x^(-1)]]
```

3.976. $\int \sqrt{\frac{-1+x}{x}}\,dx$

3.976.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 2072
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

3.976.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(18) = 36.

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

method	result	size
trager	$\sqrt{-\frac{1-x}{x}}\, x+\frac{\ln\left(2 \sqrt{-\frac{1-x}{x}}\, x-2 x+1\right)}{2}$	39
default	$-\frac{\sqrt{\frac{x-1}{x}}\, x \left(-2 \sqrt{x^{2}-x}+\ln\left(x-\frac{1}{2}+\sqrt{x^{2}-x}\right)\right)}{2 \sqrt{\left(x-1\right) x}}$	45
risch	$x \sqrt{\frac{x-1}{x}}-\frac{\ln\left(x-\frac{1}{2}+\sqrt{x^{2}-x}\right) \sqrt{\frac{x-1}{x}}\, \sqrt{\left(x-1\right) x}}{2 \left(x-1\right)}$	49

3.976. $\int \sqrt{\frac{-1+x}{x}}\, dx$

input
```
int(((x-1)/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(1-x)/x)^(1/2)*x+1/2*ln(2*(-(1-x)/x)^(1/2)*x-2*x+1)
```

3.976.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(18) = 36$.

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \sqrt{\frac{-1+x}{x}}\,dx = x\sqrt{\frac{x-1}{x}} - \frac{1}{2}\log\left(\sqrt{\frac{x-1}{x}}+1\right) + \frac{1}{2}\log\left(\sqrt{\frac{x-1}{x}}-1\right)$$

input
```
integrate(((-1+x)/x)^(1/2),x, algorithm="fricas")
```

output
```
x*sqrt((x - 1)/x) - 1/2*log(sqrt((x - 1)/x) + 1) + 1/2*log(sqrt((x - 1)/x)
 - 1)
```

3.976.6 Sympy [F]

$$\int \sqrt{\frac{-1+x}{x}}\,dx = \int \sqrt{\frac{x-1}{x}}\,dx$$

input
```
integrate(((-1+x)/x)**(1/2),x)
```

output
```
Integral(sqrt((x - 1)/x), x)
```

3.976.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(18) = 36$.

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \sqrt{\frac{-1+x}{x}}\,dx = -\frac{\sqrt{\frac{x-1}{x}}}{\frac{x-1}{x}-1} - \frac{1}{2}\log\left(\sqrt{\frac{x-1}{x}}+1\right) + \frac{1}{2}\log\left(\sqrt{\frac{x-1}{x}}-1\right)$$

input
```
integrate(((-1+x)/x)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt((x - 1)/x)/((x - 1)/x - 1) - 1/2*log(sqrt((x - 1)/x) + 1) + 1/2*log(
sqrt((x - 1)/x) - 1)
```

3.976.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \sqrt{\frac{-1+x}{x}}\,dx = \frac{1}{2}\log\left(\left|-2\,x+2\,\sqrt{x^2-x}+1\right|\right)\mathrm{sgn}(x) + \sqrt{x^2-x}\mathrm{sgn}(x)$$

input
```
integrate(((-1+x)/x)^(1/2),x, algorithm="giac")
```

output
```
1/2*log(abs(-2*x + 2*sqrt(x^2 - x) + 1))*sgn(x) + sqrt(x^2 - x)*sgn(x)
```

3.976.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \sqrt{\frac{-1+x}{x}}\,dx = x\,\sqrt{1-\frac{1}{x}} - \mathrm{atanh}\left(\sqrt{1-\frac{1}{x}}\right)$$

input
```
int(((x - 1)/x)^(1/2),x)
```

output
```
x*(1 - 1/x)^(1/2) - atanh((1 - 1/x)^(1/2))
```

3.977 $\int \frac{\sqrt{\frac{1+x}{x}}}{x} \, dx$

3.977.1 Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \frac{\sqrt{\frac{1+x}{x}}}{x} \, dx = -2\sqrt{1+\frac{1}{x}} + 2\text{arctanh}\left(\sqrt{1+\frac{1}{x}}\right)$$

output
```
2*arctanh((1+1/x)^(1/2))-2*(1+1/x)^(1/2)
```

3.977.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{1+x}{x}}}{x} \, dx = -2\sqrt{1+\frac{1}{x}} + 2\text{arctanh}\left(\sqrt{1+\frac{1}{x}}\right)$$

input
```
Integrate[Sqrt[(1 + x)/x]/x,x]
```

output
```
-2*Sqrt[1 + x^(-1)] + 2*ArcTanh[Sqrt[1 + x^(-1)]]
```

3.977.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2073, 798, 60, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{x+1}{x}}}{x} dx$$

$$\Big\downarrow \text{2073}$$

$$\int \frac{\sqrt{\frac{1}{x}+1}}{x}dx$$

$$\Big\downarrow \text{798}$$

$$-\int \sqrt{1+\frac{1}{x}}xd\frac{1}{x}$$

$$\Big\downarrow \text{60}$$

$$-\int \frac{x}{\sqrt{1+\frac{1}{x}}}d\frac{1}{x}-2\sqrt{\frac{1}{x}+1}$$

$$\Big\downarrow \text{73}$$

$$-2\int \frac{1}{\frac{1}{x^2}-1}d\sqrt{1+\frac{1}{x}}-2\sqrt{\frac{1}{x}+1}$$

$$\Big\downarrow \text{220}$$

$$2\text{arctanh}\left(\sqrt{\frac{1}{x}+1}\right)-2\sqrt{\frac{1}{x}+1}$$

input
```
Int[Sqrt[(1 + x)/x]/x,x]
```

output
```
-2*Sqrt[1 + x^(-1)] + 2*ArcTanh[Sqrt[1 + x^(-1)]]
```

3.977. $\int \frac{\sqrt{\frac{1+x}{x}}}{x} dx$

3.977.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2073
```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && BinomialQ[u, x] &&  !BinomialMatchQ[u, x
]
```

3.977. $\int \frac{\sqrt{\frac{1+x}{x}}}{x} dx$

3.977.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

method	result	size
trager	$-2\sqrt{-\frac{-x-1}{x}}-\ln\left(2x\sqrt{-\frac{-x-1}{x}}-2x-1\right)$	39
risch	$-2\sqrt{\frac{x+1}{x}}+\frac{\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)\sqrt{\frac{x+1}{x}}\,\sqrt{(x+1)x}}{x+1}$	46
default	$-\frac{\sqrt{\frac{x+1}{x}}\left(2\left(x^2+x\right)^{\frac{3}{2}}-2\sqrt{x^2+x}\,x^2-\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)x^2\right)}{x\sqrt{(x+1)x}}$	60

input
```
int(((x+1)/x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2*(-(-x-1)/x)^(1/2)-ln(2*x*(-(-x-1)/x)^(1/2)-2*x-1)
```

3.977.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int\frac{\sqrt{\frac{1+x}{x}}}{x}\,dx=-2\sqrt{\frac{x+1}{x}}+\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

input
```
integrate(((1+x)/x)^(1/2)/x,x, algorithm="fricas")
```

output
```
-2*sqrt((x + 1)/x) + log(sqrt((x + 1)/x) + 1) - log(sqrt((x + 1)/x) - 1)
```

3.977.6 Sympy [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int\frac{\sqrt{\frac{1+x}{x}}}{x}\,dx=-2\sqrt{1+\frac{1}{x}}-\log\left(\sqrt{1+\frac{1}{x}}-1\right)+\log\left(\sqrt{1+\frac{1}{x}}+1\right)$$

input
```
integrate(((1+x)/x)**(1/2)/x,x)
```

output
```
-2*sqrt(1 + 1/x) - log(sqrt(1 + 1/x) - 1) + log(sqrt(1 + 1/x) + 1)
```

3.977.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{\frac{1+x}{x}}}{x}\,dx = -2\sqrt{\frac{x+1}{x}} + \log\left(\sqrt{\frac{x+1}{x}}+1\right) - \log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

input
```
integrate(((1+x)/x)^(1/2)/x,x, algorithm="maxima")
```

output
```
-2*sqrt((x + 1)/x) + log(sqrt((x + 1)/x) + 1) - log(sqrt((x + 1)/x) - 1)
```

3.977.8 Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{\frac{1+x}{x}}}{x}\,dx = -\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)\operatorname{sgn}(x) + \frac{2\operatorname{sgn}(x)}{x-\sqrt{x^2+x}}$$

input
```
integrate(((1+x)/x)^(1/2)/x,x, algorithm="giac")
```

output
```
-log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x) + 2*sgn(x)/(x - sqrt(x^2 + x)
)
```

3.977.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\frac{1+x}{x}}}{x}\,dx = 2\,\operatorname{atanh}\left(\sqrt{\frac{1}{x}+1}\right) - 2\sqrt{\frac{1}{x}+1}$$

input
```
int(((x + 1)/x)^(1/2)/x,x)
```

output
```
2*atanh((1/x + 1)^(1/2)) - 2*(1/x + 1)^(1/2)
```

3.978 $\int \sqrt{\frac{x}{1+x}}\,dx$

3.978.1 Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \sqrt{\frac{x}{1+x}}\,dx = \sqrt{x}\sqrt{1+x} - \operatorname{arcsinh}\left(\sqrt{x}\right)$$

output
```
-arcsinh(x^(1/2))+x^(1/2)*(1+x)^(1/2)
```

3.978.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(22) = 44.

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \sqrt{\frac{x}{1+x}}\,dx = \frac{\sqrt{\frac{x}{1+x}}\left(\sqrt{x}(1+x) + \sqrt{1+x}\log\left(-\sqrt{x}+\sqrt{1+x}\right)\right)}{\sqrt{x}}$$

input
```
Integrate[Sqrt[x/(1 + x)],x]
```

output
```
(Sqrt[x/(1 + x)]*(Sqrt[x]*(1 + x) + Sqrt[1 + x]*Log[-Sqrt[x] + Sqrt[1 + x]
]))/Sqrt[x]
```

3.978.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2050, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{x}{x+1}}dx$$
$$\Big\downarrow \ 2050$$
$$\int \frac{\sqrt{x}}{\sqrt{x+1}}dx$$
$$\Big\downarrow \ 60$$
$$\sqrt{x}\sqrt{x+1}-\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{x+1}}dx$$
$$\Big\downarrow \ 63$$
$$\sqrt{x}\sqrt{x+1}-\int \frac{1}{\sqrt{x+1}}d\sqrt{x}$$
$$\Big\downarrow \ 222$$
$$\sqrt{x}\sqrt{x+1}-\text{arcsinh}\left(\sqrt{x}\right)$$

input
```
Int[Sqrt[x/(1 + x)],x]
```

output
```
Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

3.978.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2050
```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

3.978.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(16) = 32$.

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.05

method	result	size
default	$\frac{\sqrt{\frac{x}{x+1}}\,(x+1)\left(2\sqrt{x^2+x}-\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)\right)}{2\sqrt{(x+1)x}}$	45
risch	$(x+1)\sqrt{\frac{x}{x+1}}-\frac{\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)\sqrt{\frac{x}{x+1}}\,\sqrt{(x+1)x}}{2x}$	47
trager	$2\left(\frac{x}{2}+\frac{1}{2}\right)\sqrt{\frac{x}{x+1}}+\frac{\ln\left(2\sqrt{\frac{x}{x+1}}\,x+2\sqrt{\frac{x}{x+1}}-2x-1\right)}{2}$	49

input
```
int((x/(x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x/(x+1))^(1/2)*(x+1)*(2*(x^2+x)^(1/2)-ln(x+1/2+(x^2+x)^(1/2)))/((x+1)
*x)^(1/2)
```

3.978.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(16) = 32$.

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \sqrt{\frac{x}{1+x}}\,dx = (x+1)\sqrt{\frac{x}{x+1}} - \frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}+1\right) + \frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

input
```
integrate((x/(1+x))^(1/2),x, algorithm="fricas")
```

output
```
(x + 1)*sqrt(x/(x + 1)) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(sqrt(x/(x
 + 1)) - 1)
```

3.978.6 Sympy [F]

$$\int \sqrt{\frac{x}{1+x}}\,dx = \int \sqrt{\frac{x}{x+1}}\,dx$$

input
```
integrate((x/(1+x))**(1/2),x)
```

output
```
Integral(sqrt(x/(x + 1)), x)
```

3.978.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(16) = 32$.

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \sqrt{\frac{x}{1+x}}\,dx = -\frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1} - \frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}+1\right) + \frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

input
```
integrate((x/(1+x))^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(x/(x + 1))/(x/(x + 1) - 1) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(
sqrt(x/(x + 1)) - 1)
```

3.978.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(16) = 32$.

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \sqrt{\frac{x}{1+x}}\,dx = \frac{1}{2}\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)\mathrm{sgn}(x+1)+\sqrt{x^2+x}\,\mathrm{sgn}(x+1)$$

input
```
integrate((x/(1+x))^(1/2),x, algorithm="giac")
```

output
```
1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x + 1) + sqrt(x^2 + x)*sgn(x
+ 1)
```

3.978.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \sqrt{\frac{x}{1+x}}\,dx = -\mathrm{atanh}\left(\sqrt{\frac{x}{x+1}}\right)-\frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1}$$

input
```
int((x/(x + 1))^(1/2),x)
```

output
```
- atanh((x/(x + 1))^(1/2)) - (x/(x + 1))^(1/2)/(x/(x + 1) - 1)
```

3.979 $\int \frac{1}{\sqrt{\frac{-1-x}{x}}}\,dx$

3.979.1 Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \frac{1}{\sqrt{\frac{-1-x}{x}}}\,dx=-x\sqrt{-\frac{1+x}{x}}+\arctan\left(\sqrt{-\frac{1+x}{x}}\right)$$

output
```
arctan(((-1-x)/x)^(1/2))-x*((-1-x)/x)^(1/2)
```

3.979.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.79

$$\int \frac{1}{\sqrt{\frac{-1-x}{x}}}\,dx=\frac{\sqrt{x}(1+x)+\sqrt{1+x}\log\left(-\sqrt{x}+\sqrt{1+x}\right)}{\sqrt{x}\sqrt{-\frac{1+x}{x}}}$$

input
```
Integrate[1/Sqrt[(-1 - x)/x],x]
```

output
```
(Sqrt[x]*(1 + x) + Sqrt[1 + x]*Log[-Sqrt[x] + Sqrt[1 + x]])/(Sqrt[x]*Sqrt[
-((1 + x)/x)])
```

3.979.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2072, 773, 52, 73, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{-x-1}{x}}}dx$$

$$\Big\downarrow \ 2072$$

$$\int \frac{1}{\sqrt{-\frac{1}{x}-1}}dx$$

$$\Big\downarrow \ 773$$

$$-\int \frac{x^2}{\sqrt{-1-\frac{1}{x}}}d\frac{1}{x}$$

$$\Big\downarrow \ 52$$

$$\frac{1}{2}\int \frac{x}{\sqrt{-1-\frac{1}{x}}}d\frac{1}{x}-\sqrt{-\frac{1}{x}-1}x$$

$$\Big\downarrow \ 73$$

$$-\int \frac{1}{-1-\frac{1}{x^2}}d\sqrt{-1-\frac{1}{x}}-\sqrt{-\frac{1}{x}-1}x$$

$$\Big\downarrow \ 217$$

$$\arctan\left(\sqrt{-\frac{1}{x}-1}\right)-\sqrt{-\frac{1}{x}-1}x$$

input
```
Int[1/Sqrt[(-1 - x)/x],x]
```

output
```
-(Sqrt[-1 - x^(-1)]*x) + ArcTan[Sqrt[-1 - x^(-1)]]
```

3.979.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 2072
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

3.979.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.52

method	result	size
default	$\frac{(x+1)\left(2\sqrt{-x^{2}-x}+\arcsin(1+2x)\right)}{2\sqrt{-\frac{x+1}{x}}\,\sqrt{-(x+1)x}}$	44
risch	$\frac{x+1}{\sqrt{-\frac{x+1}{x}}}-\frac{\arcsin(1+2x)\sqrt{-(x+1)x}}{2\sqrt{-\frac{x+1}{x}}\,x}$	45
trager	$-\sqrt{-\frac{x+1}{x}}\,x-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(2\sqrt{-\frac{x+1}{x}}\,x-2\operatorname{RootOf}\left(\_Z^{2}+1\right)x-\operatorname{RootOf}\left(\_Z^{2}+1\right)\right)}{2}$	55

input
```
int(1/((-x-1)/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x+1)*(2*(-x^2-x)^(1/2)+arcsin(1+2*x))/(-(x+1)/x)^(1/2)/(-(x+1)*x)^(1/
2)
```

3.979.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{\frac{-1-x}{x}}}\,dx = -x\sqrt{-\frac{x+1}{x}} + \arctan\left(\sqrt{-\frac{x+1}{x}}\right)$$

input
```
integrate(1/((-1-x)/x)^(1/2),x, algorithm="fricas")
```

output
```
-x*sqrt(-(x + 1)/x) + arctan(sqrt(-(x + 1)/x))
```

3.979.6 Sympy [F]

$$\int \frac{1}{\sqrt{\frac{-1-x}{x}}}\,dx = \int \frac{1}{\sqrt{\frac{-x-1}{x}}}\,dx$$

input
```
integrate(1/((-1-x)/x)**(1/2),x)
```

output
```
Integral(1/sqrt((-x - 1)/x), x)
```

3.979.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{\frac{-1-x}{x}}}\,dx = -\frac{\sqrt{-\frac{x+1}{x}}}{\frac{x+1}{x}-1} + \arctan\left(\sqrt{-\frac{x+1}{x}}\right)$$

input
```
integrate(1/((-1-x)/x)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(-(x + 1)/x)/((x + 1)/x - 1) + arctan(sqrt(-(x + 1)/x))
```

3.979.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{\frac{-1-x}{x}}}\,dx = \frac{1}{4}\,\pi\mathrm{sgn}(x) - \frac{\arcsin\left(2\,x+1\right)}{2\,\mathrm{sgn}\left(x\right)} - \frac{\sqrt{-x^2-x}}{\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/((-1-x)/x)^(1/2),x, algorithm="giac")
```

output
```
1/4*pi*sgn(x) - 1/2*arcsin(2*x + 1)/sgn(x) - sqrt(-x^2 - x)/sgn(x)
```

3.979.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{\frac{-1-x}{x}}}\,dx = \mathrm{atan}\left(\sqrt{-\frac{1}{x}-1}\right) - x\,\sqrt{-\frac{1}{x}-1}$$

input
```
int(1/(-(x + 1)/x)^(1/2),x)
```

output
```
atan((- 1/x - 1)^(1/2)) - x*(- 1/x - 1)^(1/2)
```

3.980 $\int \sqrt{(4-x)x}\,dx$

3.980.1 Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \sqrt{(4-x)x}\,dx=-\frac{1}{2}(2-x)\sqrt{4x-x^2}-2\arcsin\left(1-\frac{x}{2}\right)$$

output
```
2*arcsin(-1+1/2*x)-1/2*(2-x)*(-x^2+4*x)^(1/2)
```

3.980.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.48

$$\int \sqrt{(4-x)x}\,dx=\frac{1}{2}\sqrt{-((-4+x)x)}\left(-2+x-\frac{16\text{arctanh}\left(\frac{\sqrt{-4+x}}{-2+\sqrt{x}}\right)}{\sqrt{-4+x}\sqrt{x}}\right)$$

input
```
Integrate[Sqrt[(4 - x)*x],x]
```

output
```
(Sqrt[-((-4 + x)*x)]*(-2 + x - (16*ArcTanh[Sqrt[-4 + x]/(-2 + Sqrt[x])])/(
Sqrt[-4 + x]*Sqrt[x])))/2
```

3.980.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2048, 1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{(4-x)x}\,dx$$

$$\Big\downarrow \text{ 2048}$$

$$\int \sqrt{4x-x^2}dx$$

$$\Big\downarrow \text{ 1087}$$

$$2\int \frac{1}{\sqrt{4x-x^2}}dx-\frac{1}{2}(2-x)\sqrt{4x-x^2}$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{1}{2}\int \frac{1}{\sqrt{1-\frac{1}{16}(4-2x)^2}}d(4-2x)-\frac{1}{2}\sqrt{4x-x^2}(2-x)$$

$$\Big\downarrow \text{ 223}$$

$$-2\arcsin\left(\frac{1}{4}(4-2x)\right)-\frac{1}{2}\sqrt{4x-x^2}(2-x)$$

input
```
Int[Sqrt[(4 - x)*x],x]
```

output
```
-1/2*((2 - x)*Sqrt[4*x - x^2]) - 2*ArcSin[(4 - 2*x)/4]
```

3.980.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

3.980.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{(x-2)x(x-4)}{2\sqrt{-x(x-4)}}+2\arcsin\left(-1+\frac{x}{2}\right)$
default	$-\frac{(4-2x)\sqrt{-x^2+4x}}{4}+2\arcsin\left(-1+\frac{x}{2}\right)$
pseudoelliptic	$-4\arctan\left(\frac{\sqrt{-x(x-4)}}{x}\right)+\frac{(x-2)\sqrt{-x(x-4)}}{2}$
meijerg	$-\frac{8i\left(-\frac{i\sqrt{\pi}\,\sqrt{x}\,\left(-\frac{3x}{2}+3\right)\sqrt{-\frac{x}{4}+1}}{12}+\frac{i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{x}}{2}\right)}{2}\right)}{\sqrt{\pi}}$
trager	$\left(-1+\frac{x}{2}\right)\sqrt{-x^2+4x}+2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2+4x}+2\right.$

input
```
int(((4-x)*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(x-2)*x*(x-4)/(-x*(x-4))^(1/2)+2*arcsin(-1+1/2*x)
```

3.980.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \sqrt{(4-x)x}\,dx = \frac{1}{2}\sqrt{-x^2+4\,x}(x-2) - 4\arctan\left(\frac{\sqrt{-x^2+4\,x}}{x}\right)$$

input
```
integrate(((4-x)*x)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-x^2 + 4*x)*(x - 2) - 4*arctan(sqrt(-x^2 + 4*x)/x)
```

3.980.6 Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

$$\int \sqrt{(4-x)x}\,dx = \left(\frac{x}{2}-1\right)\sqrt{-x^2+4x} + 2\,\mathrm{asin}\left(\frac{x}{2}-1\right)$$

input
```
integrate(((4-x)*x)**(1/2),x)
```

output
```
(x/2 - 1)*sqrt(-x**2 + 4*x) + 2*asin(x/2 - 1)
```

3.980.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \sqrt{(4-x)x}\,dx = \frac{1}{2}\sqrt{-x^2+4\,x}x - \sqrt{-x^2+4\,x} - 2\arcsin\left(-\frac{1}{2}\,x+1\right)$$

input
```
integrate(((4-x)*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-x^2 + 4*x)*x - sqrt(-x^2 + 4*x) - 2*arcsin(-1/2*x + 1)
```

3.980.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \sqrt{(4-x)x}\,dx = \frac{1}{2}\sqrt{-x^2+4\,x}(x-2) + 2\,\arcsin\left(\frac{1}{2}\,x-1\right)$$

input
```
integrate(((4-x)*x)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 + 4*x)*(x - 2) + 2*arcsin(1/2*x - 1)
```

3.980.9 Mupad [B] (verification not implemented)

Time = 21.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \sqrt{(4-x)x}\,dx = 2\,\mathrm{asin}\left(\frac{x}{2}-1\right) + \left(\frac{x}{2}-1\right)\,\sqrt{4\,x-x^2}$$

input
```
int((-x*(x - 4))^(1/2),x)
```

output
```
2*asin(x/2 - 1) + (x/2 - 1)*(4*x - x^2)^(1/2)
```

3.981 $\int \frac{1}{\sqrt{(1-x)x}}\,dx$

3.981.1 Optimal result

Integrand size = 11, antiderivative size = 8

$$\int \frac{1}{\sqrt{(1-x)x}}\,dx = -\arcsin(1-2x)$$

output
```
arcsin(-1+2*x)
```

3.981.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 40 vs. 2(8) = 16.

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 5.00

$$\int \frac{1}{\sqrt{(1-x)x}}\,dx = -\frac{2\sqrt{-1+x}\sqrt{x}\log\left(\sqrt{-1+x}-\sqrt{x}\right)}{\sqrt{-((-1+x)x)}}$$

input
```
Integrate[1/Sqrt[(1 - x)*x],x]
```

output
```
(-2*Sqrt[-1 + x]*Sqrt[x]*Log[Sqrt[-1 + x] - Sqrt[x]])/Sqrt[-((-1 + x)*x)]
```

3.981.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2048, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{(1-x)x}}dx$$
$$\Big\downarrow \text{ 2048}$$
$$\int \frac{1}{\sqrt{x-x^2}}dx$$
$$\Big\downarrow \text{ 1090}$$
$$-\int \frac{1}{\sqrt{1-(1-2x)^2}}d(1-2x)$$
$$\Big\downarrow \text{ 223}$$
$$-\arcsin(1-2x)$$

input
```
Int[1/Sqrt[(1 - x)*x],x]
```

output
```
-ArcSin[1 - 2*x]
```

3.981.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

3.981.4 Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
default	$\arcsin(2x-1)$	7
meijerg	$2\arcsin(\sqrt{x})$	7
pseudoelliptic	$-2\arctan\left(\frac{\sqrt{-(x-1)x}}{x}\right)$	16
trager	$\mathrm{RootOf}(\_Z^2+1)\ln\left(-2\,\mathrm{RootOf}(\_Z^2+1)\,x+\mathrm{RootOf}(\_Z^2+1)+2\sqrt{-x^2+x}\right)$	36

input
```
int(1/((1-x)*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(2*x-1)
```

3.981.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{(1-x)x}}\,dx = -2\arctan\left(\frac{\sqrt{-x^2+x}}{x}\right)$$

input
```
integrate(1/((1-x)*x)^(1/2),x, algorithm="fricas")
```

output
```
-2*arctan(sqrt(-x^2 + x)/x)
```

3.981.6 Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{(1-x)x}}\,dx = \mathrm{asin}\,(2x-1)$$

input
```
integrate(1/((1-x)*x)**(1/2),x)
```

output
```
asin(2*x - 1)
```

3.981.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{(1-x)x}}\,dx = \arcsin{(2\,x-1)}$$

input
```
integrate(1/((1-x)*x)^(1/2),x, algorithm="maxima")
```

output
```
arcsin(2*x - 1)
```

3.981.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(6) = 12.

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 3.12

$$\int \frac{1}{\sqrt{(1-x)x}}\,dx = \frac{1}{4}\,\sqrt{-x^2+x}(2\,x-1) + \frac{1}{8}\,\arcsin{(2\,x-1)}$$

input
```
integrate(1/((1-x)*x)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(-x^2 + x)*(2*x - 1) + 1/8*arcsin(2*x - 1)
```

3.981.9 Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{(1-x)x}}\,dx = \mathrm{asin}(2\,x-1)$$

input
```
int(1/(-x*(x - 1))^(1/2),x)
```

output
```
asin(2*x - 1)
```

3.982 $\int \frac{x}{(x(2+x))^{3/2}}\,dx$

3.982.1 Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{x}{(x(2+x))^{3/2}}\,dx = \frac{x}{\sqrt{2x+x^2}}$$

output
```
x/(x^2+2*x)^(1/2)
```

3.982.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{(x(2+x))^{3/2}}\,dx = \frac{x}{\sqrt{x(2+x)}}$$

input
```
Integrate[x/(x*(2 + x))^(3/2),x]
```

output
```
x/Sqrt[x*(2 + x)]
```

3.982.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2048, 1124, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x(x+2))^{3/2}}dx$$
$$\Big\downarrow \text{2048}$$
$$\int \frac{x}{(x^2+2x)^{3/2}}dx$$
$$\Big\downarrow \text{1124}$$
$$\int 0dx + \frac{x}{\sqrt{x^2+2x}}$$
$$\Big\downarrow \text{24}$$
$$\frac{x}{\sqrt{x^2+2x}}$$

input
```
Int[x/(x*(2 + x))^(3/2),x]
```

output
```
x/Sqrt[2*x + x^2]
```

3.982.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

3.982.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{x}{\sqrt{x(x+2)}}$	10
pseudoelliptic	$\frac{x}{\sqrt{x(x+2)}}$	10
gosper	$\frac{x^2(x+2)}{(x(x+2))^{\frac{3}{2}}}$	15
trager	$\frac{\sqrt{x^2+2x}}{x+2}$	16
meijerg	$\frac{\sqrt{2}\,\sqrt{x}}{2\sqrt{1+\frac{x}{2}}}$	16
default	$-\frac{1}{\sqrt{x^2+2x}}+\frac{2x+2}{2\sqrt{x^2+2x}}$	29

input
```
int(x/(x*(x+2))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x/(x*(x+2))^(1/2)
```

3.982.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{x}{(x(2+x))^{3/2}}\,dx = \frac{x+\sqrt{x^2+2\,x}+2}{x+2}$$

input
```
integrate(x/(x*(2+x))^(3/2),x, algorithm="fricas")
```

output
```
(x + sqrt(x^2 + 2*x) + 2)/(x + 2)
```

3.982.6 Sympy [F]

$$\int \frac{x}{(x(2+x))^{3/2}}\,dx = \int \frac{x}{(x\,(x+2))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(x*(2+x))**(3/2),x)
```

output
```
Integral(x/(x*(x + 2))**(3/2), x)
```

3.982.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{(x(2+x))^{3/2}}\,dx = \frac{x}{\sqrt{x^2+2\,x}}$$

input
```
integrate(x/(x*(2+x))^(3/2),x, algorithm="maxima")
```

output
```
x/sqrt(x^2 + 2*x)
```

3.982.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{x}{(x(2+x))^{3/2}}\,dx = \frac{2}{x-\sqrt{x^2+2\,x}+2}$$

input
```
integrate(x/(x*(2+x))^(3/2),x, algorithm="giac")
```

output
```
2/(x - sqrt(x^2 + 2*x) + 2)
```

3.982.9 Mupad [B] (verification not implemented)

Time = 22.39 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{x}{(x(2+x))^{3/2}}\,dx = \frac{\sqrt{x\ (x+2)}}{x+2}$$

input
```
int(x/(x*(x + 2))^(3/2),x)
```

output
```
(x*(x + 2))^(1/2)/(x + 2)
```

3.983 $\int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2} dx$

3.983.1 Optimal result

Integrand size = 19, antiderivative size = 22

$$\int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2} dx = \sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\frac{1}{x}}}{\sqrt{2}}\right)$$

output
```
arctanh(1/2*(1+1/x)^(1/2)*2^(1/2))*2^(1/2)
```

3.983.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2} dx = \sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1+x}{x}}}{\sqrt{2}}\right)$$

input
```
Integrate[Sqrt[1 + x^(-1)]/(1 - x^2),x]
```

output
```
Sqrt[2]*ArcTanh[Sqrt[(1 + x)/x]/Sqrt[2]]
```

3.983.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1776, 1388, 946, 25, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{1}{x}+1}}{1-x^2} \, dx$$
$$\Big\downarrow \text{1776}$$
$$\int \frac{\sqrt{\frac{1}{x}+1}}{\left(\frac{1}{x^2}-1\right)x^2}dx$$
$$\Big\downarrow \text{1388}$$
$$\int \frac{1}{\left(\frac{1}{x}-1\right)\sqrt{\frac{1}{x}+1x^2}}dx$$
$$\Big\downarrow \text{946}$$
$$-\int -\frac{1}{\left(1-\frac{1}{x}\right)\sqrt{1+\frac{1}{x}}}d\frac{1}{x}$$
$$\Big\downarrow \text{25}$$
$$\int \frac{1}{\left(1-\frac{1}{x}\right)\sqrt{\frac{1}{x}+1}}d\frac{1}{x}$$
$$\Big\downarrow \text{73}$$
$$2\int \frac{1}{2-\frac{1}{x^2}}d\sqrt{1+\frac{1}{x}}$$
$$\Big\downarrow \text{219}$$
$$\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\frac{1}{x}+1}}{\sqrt{2}}\right)$$

input
```
Int[Sqrt[1 + x^(-1)]/(1 - x^2),x]
```

output
```
Sqrt[2]*ArcTanh[Sqrt[1 + x^(-1)]/Sqrt[2]]
```

3.983. $\int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2} \, dx$

3.983.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 1776
```
Int[((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_S
ymbol] :> Int[((d + e*x^n)^q*(c + a*x^(2*n))^p)/x^(2*n*p), x] /; FreeQ[{a,
c, d, e, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

3.983.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(17) = 34$.

Time = 1.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86

method	result	size
default	$\frac{\sqrt{\frac{x+1}{x}}\, x\sqrt{2}\, \operatorname{arctanh}\left(\frac{(1+3x)\sqrt{2}}{4\sqrt{x^{2}+x}}\right)}{2\sqrt{(x+1)x}}$	41
trager	$-\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right) \ln\left(-\frac{3\operatorname{RootOf}\left(\_Z^{2}-2\right)x-4x\sqrt{-\frac{-x-1}{x}}+\operatorname{RootOf}\left(\_Z^{2}-2\right)}{x-1}\right)}{2}$	48

input
```
int((1+1/x)^(1/2)/(-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*((x+1)/x)^(1/2)*x/((x+1)*x)^(1/2)*2^(1/2)*arctanh(1/4*(1+3*x)*2^(1/2)/
(x^2+x)^(1/2))
```

3.983.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2}\, dx = \frac{1}{2}\sqrt{2}\log\left(-\frac{2\sqrt{2}x\sqrt{\frac{x+1}{x}}+3x+1}{x-1}\right)$$

input
```
integrate((1+1/x)^(1/2)/(-x^2+1),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*log(-(2*sqrt(2)*x*sqrt((x + 1)/x) + 3*x + 1)/(x - 1))
```

3.983.6 Sympy [F]

$$\int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2}\, dx = -\int \frac{\sqrt{1+\frac{1}{x}}}{x^2-1}\, dx$$

input
```
integrate((1+1/x)**(1/2)/(-x**2+1),x)
```

output
```
-Integral(sqrt(1 + 1/x)/(x**2 - 1), x)
```

3.983.7 Maxima [F]

$$\int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2}\,dx = \int -\frac{\sqrt{\frac{1}{x}+1}}{x^2-1}\,dx$$

input
```
integrate((1+1/x)^(1/2)/(-x^2+1),x, algorithm="maxima")
```

output
```
-integrate(sqrt(1/x + 1)/(x^2 - 1), x)
```

3.983.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(17) = 34$.

Time = 0.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.32

$$\int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2}\,dx = \frac{1}{2}\sqrt{2}\log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\operatorname{sgn}(x) - \frac{1}{2}\sqrt{2}\log\left(\frac{\left|-2\,x-2\,\sqrt{2}+2\,\sqrt{x^2+x}+2\right|}{\left|-2\,x+2\,\sqrt{2}+2\,\sqrt{x^2+x}+2\right|}\right)\operatorname{sgn}(x)$$

input
```
integrate((1+1/x)^(1/2)/(-x^2+1),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*log((sqrt(2) - 1)/(sqrt(2) + 1))*sgn(x) - 1/2*sqrt(2)*log(abs(
-2*x - 2*sqrt(2) + 2*sqrt(x^2 + x) + 2)/abs(-2*x + 2*sqrt(2) + 2*sqrt(x^2
+ x) + 2))*sgn(x)
```

3.983.9 Mupad [B] (verification not implemented)

Time = 22.37 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2}\,dx = \sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}+1}}{2}\right)$$

input
```
int(-(1/x + 1)^(1/2)/(x^2 - 1),x)
```

output
```
2^(1/2)*atanh((2^(1/2)*(1/x + 1)^(1/2))/2)
```

3.984 $\int \frac{1}{1+\sqrt{5}-x^2+\sqrt{5}x^2}\,dx$

3.984.1 Optimal result

Integrand size = 23, antiderivative size = 24

$$\int \frac{1}{1+\sqrt{5}-x^2+\sqrt{5}x^2}\,dx=\frac{1}{2}\arctan\left(\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}x\right)$$

output
```
1/2*arctan(x*(1/2*5^(1/2)-1/2))
```

3.984.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{1}{1+\sqrt{5}-x^2+\sqrt{5}x^2}\,dx=\frac{1}{4}i\log\left(1+\sqrt{5}-2ix\right)-\frac{1}{4}i\log\left(1+\sqrt{5}+2ix\right)$$

input
```
Integrate[(1 + Sqrt[5] - x^2 + Sqrt[5]*x^2)^(-1),x]
```

output
```
(I/4)*Log[1 + Sqrt[5] - (2*I)*x] - (I/4)*Log[1 + Sqrt[5] + (2*I)*x]
```

3.984.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {6, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{5}x^2 - x^2 + \sqrt{5} + 1} \, dx$$
$$\Big\downarrow \; 6$$
$$\int \frac{1}{\left(\sqrt{5} - 1\right) x^2 + \sqrt{5} + 1} dx$$
$$\Big\downarrow \; 216$$
$$\frac{1}{2} \arctan\left(\sqrt{\frac{1}{2}\left(3 - \sqrt{5}\right)} x\right)$$

input
```
Int[(1 + Sqrt[5] - x^2 + Sqrt[5]*x^2)^(-1),x]
```

output
```
ArcTan[Sqrt[(3 - Sqrt[5])/2]*x]/2
```

3.984.3.1 Defintions of rubi rules used

rule 6
```
Int[(u_.)*((v_.) + (a_.)*(Fx_) + (b_.)*(Fx_))^(p_.), x_Symbol] :> Int[u*(v
+ (a + b)*Fx)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[Fx, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.984.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 1.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

method	result	size
risch	$\frac{\arctan\left(\frac{2x}{\sqrt{5}+1}\right)}{4}-\frac{\arctan\left(\frac{2x}{-\sqrt{5}-1}\right)}{4}$	30
parallelrisch	$-\frac{i\ln\left(-\frac{i\sqrt{5}}{2}-\frac{i}{2}+x\right)}{4}+\frac{i\ln\left(\frac{i\sqrt{5}}{2}+\frac{i}{2}+x\right)}{4}$	30
default	$\frac{4\arctan\left(\frac{4x}{2\sqrt{5}+2}\right)}{\left(\sqrt{5}-1\right)\left(2\sqrt{5}+2\right)}$	32
meijerg	$\frac{\arctan\left(\frac{x\sqrt{\sqrt{5}-1}}{\sqrt{\sqrt{5}+1}}\right)}{\sqrt{\sqrt{5}+1}\,\sqrt{\sqrt{5}-1}}$	33

input
```
int(1/(1-x^2+5^(1/2)+x^2*5^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/4*arctan(2*x/(5^(1/2)+1))-1/4*arctan(2*x/(-5^(1/2)-1))
```

3.984.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.54

$$\int \frac{1}{1+\sqrt{5}-x^2+\sqrt{5}x^2}\,dx=\frac{1}{2}\arctan\left(\frac{1}{2}\sqrt{5}x-\frac{1}{2}\,x\right)$$

input
```
integrate(1/(1-x^2+5^(1/2)+x^2*5^(1/2)),x, algorithm="fricas")
```

output
```
1/2*arctan(1/2*sqrt(5)*x - 1/2*x)
```

3.984.6 Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1}{1+\sqrt{5}-x^2+\sqrt{5}x^2}\,dx = -\frac{\operatorname{atan}\left(x\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\right)}{2}$$

input
```
integrate(1/(1-x**2+5**(1/2)+x**2*5**(1/2)),x)
```

output
```
-atan(x*(1/2 - sqrt(5)/2))/2
```

3.984.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.46

$$\int \frac{1}{1+\sqrt{5}-x^2+\sqrt{5}x^2}\,dx = \frac{1}{2}\arctan\left(\frac{1}{2}x\left(\sqrt{5}-1\right)\right)$$

input
```
integrate(1/(1-x^2+5^(1/2)+x^2*5^(1/2)),x, algorithm="maxima")
```

output
```
1/2*arctan(1/2*x*(sqrt(5) - 1))
```

3.984.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.54

$$\int \frac{1}{1+\sqrt{5}-x^2+\sqrt{5}x^2}\,dx = \frac{1}{2}\arctan\left(\frac{2x}{\sqrt{5}+1}\right)$$

input
```
integrate(1/(1-x^2+5^(1/2)+x^2*5^(1/2)),x, algorithm="giac")
```

output
```
1/2*arctan(2*x/(sqrt(5) + 1))
```

3.984.9 Mupad [B] (verification not implemented)

Time = 22.71 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.88

$$\int \frac{1}{1+\sqrt{5}-x^2+\sqrt{5}x^2}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x\,\left(\sqrt{5}+1\right)}{4\,\left(\frac{\sqrt{5}}{4}+\frac{1}{4}\right)\,\sqrt{\sqrt{5}+3}}\right)\,\left(\sqrt{5}+1\right)}{4\,\sqrt{\sqrt{5}+3}}$$

input
```
int(1/(5^(1/2) + 5^(1/2)*x^2 - x^2 + 1),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*x*(5^(1/2) + 1))/(4*(5^(1/2)/4 + 1/4)*(5^(1/2) + 3)
^(1/2)))*(5^(1/2) + 1))/(4*(5^(1/2) + 3)^(1/2))
```

3.985 $\int \frac{1}{\sqrt{ax+bx^2}}\,dx$

3.985.1 Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{1}{\sqrt{ax+bx^2}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

output
```
2*arctanh(x*b^(1/2)/(b*x^2+a*x)^(1/2))/b^(1/2)
```

3.985.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.96

$$\int \frac{1}{\sqrt{ax+bx^2}}\,dx = -\frac{2\sqrt{x}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

input
```
Integrate[1/Sqrt[a*x + b*x^2],x]
```

output
```
(-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b
]*Sqrt[x*(a + b*x)])
```

3.985.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax+bx^2}}dx$$

$$\Big\downarrow \text{1091}$$

$$2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

input
```
Int[1/Sqrt[a*x + b*x^2],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

3.985.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

3.985.4 Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
pseudoelliptic	$\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{\sqrt{b}}$	23
default	$\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$	29

input
```
int(1/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^(1/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))
```

3.985.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.21

$$\int \frac{1}{\sqrt{ax+bx^2}}\,dx = \left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)}{b}\right]$$

input
```
integrate(1/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

output
```
[log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(
sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

3.985.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(26) = 52.

Time = 0.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.68

$$\int \frac{1}{\sqrt{ax+bx^2}}\,dx = \begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } b\neq 0 \wedge \frac{a^2}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{for } b\neq 0 \\ \frac{2\sqrt{ax}}{a} & \text{for } a\neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

input
```
integrate(1/(b*x**2+a*x)**(1/2),x)
```

output
```
Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(b, 0)
 & Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2
), Ne(b, 0)), (2*sqrt(a*x)/a, Ne(a, 0)), (zoo*x, True))
```

3.985.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{ax+bx^2}}\,dx = \frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}$$

input
```
integrate(1/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

output
```
log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

3.985.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(22) = 44.

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{ax+bx^2}}\,dx = \frac{1}{4}\,\sqrt{bx^2+ax}\left(2\,x+\frac{a}{b}\right)+\frac{a^2\log\left(\left|2\,\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate(1/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(b*x^2 + a*x)*(2*x + a/b) + 1/8*a^2*log(abs(2*(sqrt(b)*x - sqrt(b*
x^2 + a*x))*sqrt(b) + a))/b^(3/2)
```

3.985.9 Mupad [B] (verification not implemented)

Time = 21.39 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{ax+bx^2}}\,dx = \frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}$$

input
```
int(1/(a*x + b*x^2)^(1/2),x)
```

output
```
log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

3.986 $\int \frac{1}{\sqrt{x(a+bx)}}\,dx$

3.986.1 Optimal result

Integrand size = 11, antiderivative size = 28

$$\int \frac{1}{\sqrt{x(a+bx)}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

output
```
2*arctanh(x*b^(1/2)/(b*x^2+a*x)^(1/2))/b^(1/2)
```

3.986.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.96

$$\int \frac{1}{\sqrt{x(a+bx)}}\,dx = -\frac{2\sqrt{x}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

input
```
Integrate[1/Sqrt[x*(a + b*x)],x]
```

output
```
(-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b
]*Sqrt[x*(a + b*x)])
```

3.986.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2048, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x(a+bx)}}dx$$
$$\Big\downarrow \text{2048}$$
$$\int \frac{1}{\sqrt{ax+bx^2}}dx$$
$$\Big\downarrow \text{1091}$$
$$2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}$$
$$\Big\downarrow \text{219}$$
$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

input
```
Int[1/Sqrt[x*(a + b*x)],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

3.986.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

3.986.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
pseudoelliptic	$\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{x(bx+a)}}{x\sqrt{b}}\right)}{\sqrt{b}}$	23
default	$\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$	29

input
```
int(1/(x*(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/b^(1/2)*arctanh((x*(b*x+a))^(1/2)/x/b^(1/2))
```

3.986.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.21

$$\int \frac{1}{\sqrt{x(a+bx)}}\,dx = \left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)}{b}\right]$$

input
```
integrate(1/(x*(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
[log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(
sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

3.986.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(26) = 52$.

Time = 0.71 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.68

$$\int \frac{1}{\sqrt{x(a+bx)}}\,dx = \begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } b \neq 0 \wedge \frac{a^2}{b} \neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{for } b \neq 0 \\ \frac{2\sqrt{ax}}{a} & \text{for } a \neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

input
```
integrate(1/(x*(b*x+a))**(1/2),x)
```

output
```
Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(b, 0)
 & Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2
), Ne(b, 0)), (2*sqrt(a*x)/a, Ne(a, 0)), (zoo*x, True))
```

3.986.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{x(a+bx)}}\,dx = \frac{\log\left(2\,bx + a + 2\,\sqrt{bx^2 + ax}\sqrt{b}\right)}{\sqrt{b}}$$

input
```
integrate(1/(x*(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

3.986.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(22) = 44$.

Time = 0.47 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{x(a+bx)}}\,dx = \frac{1}{4}\sqrt{bx^2+ax}\left(2\,x+\frac{a}{b}\right)+\frac{a^2\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate(1/(x*(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(b*x^2 + a*x)*(2*x + a/b) + 1/8*a^2*log(abs(2*(sqrt(b)*x - sqrt(b*
x^2 + a*x))*sqrt(b) + a))/b^(3/2)
```

3.986.9 Mupad [B] (verification not implemented)

Time = 21.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{x(a+bx)}}\,dx = \frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}$$

input
```
int(1/(x*(a + b*x))^(1/2),x)
```

output
```
log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

3.987 $\int \frac{1}{\sqrt{\left(b+\frac{a}{x}\right)x^2}}\,dx$

3.987.1 Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{1}{\sqrt{\left(b+\frac{a}{x}\right)x^2}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

output
```
2*arctanh(x*b^(1/2)/(b*x^2+a*x)^(1/2))/b^(1/2)
```

3.987.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.96

$$\int \frac{1}{\sqrt{\left(b+\frac{a}{x}\right)x^2}}\,dx=-\frac{2\sqrt{x}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

input
```
Integrate[1/Sqrt[(b + a/x)*x^2],x]
```

output
```
(-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b
]*Sqrt[x*(a + b*x)])
```

3.987.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2078, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2 \left(\frac{a}{x}+b\right)}} \, dx$$

$$\Big\downarrow \text{ 2078}$$

$$\int \frac{1}{\sqrt{ax+bx^2}}dx$$

$$\Big\downarrow \text{ 1091}$$

$$2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

input
```
Int[1/Sqrt[(b + a/x)*x^2],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

3.987.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

3.987.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$	29

input
```
int(1/((b+a/x)*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

3.987.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.21

$$\int \frac{1}{\sqrt{\left(b+\frac{a}{x}\right)x^2}}\,dx = \left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)}{b}\right]$$

input
```
integrate(1/((b+a/x)*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(
sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

3.987.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(26) = 52$.

Time = 0.66 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.68

$$\int \frac{1}{\sqrt{\left(b+\frac{a}{x}\right)x^2}}\,dx = \begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } b\neq 0 \wedge \frac{a^2}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{for } b\neq 0 \\ \frac{2\sqrt{ax}}{a} & \text{for } a\neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

input
```
integrate(1/((b+a/x)*x**2)**(1/2),x)
```

output
```
Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(b, 0)
 & Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2
), Ne(b, 0)), (2*sqrt(a*x)/a, Ne(a, 0)), (zoo*x, True))
```

3.987.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{\left(b+\frac{a}{x}\right)x^2}}\,dx = \frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}$$

input
```
integrate(1/((b+a/x)*x^2)^(1/2),x, algorithm="maxima")
```

output
```
log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

3.987.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(22) = 44$.

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{\left(b+\frac{a}{x}\right)x^2}}\,dx = \frac{1}{4}\sqrt{bx^2+ax}\left(2\,x+\frac{a}{b}\right)+\frac{a^2\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate(1/((b+a/x)*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(b*x^2 + a*x)*(2*x + a/b) + 1/8*a^2*log(abs(2*(sqrt(b)*x - sqrt(b*
x^2 + a*x))*sqrt(b) + a))/b^(3/2)
```

3.987.9 Mupad [B] (verification not implemented)

Time = 21.69 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\left(b+\frac{a}{x}\right)x^2}}\,dx = \frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}$$

input
```
int(1/(x^2*(b + a/x))^(1/2),x)
```

output
```
log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

3.988 $\int \frac{1}{\sqrt{\left(\frac{a}{x^2}+\frac{b}{x}\right)x^3}}\,dx$

3.988.1 Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{1}{\sqrt{\left(\frac{a}{x^2}+\frac{b}{x}\right)x^3}}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

output
```
2*arctanh(x*b^(1/2)/(b*x^2+a*x)^(1/2))/b^(1/2)
```

3.988.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.96

$$\int \frac{1}{\sqrt{\left(\frac{a}{x^2}+\frac{b}{x}\right)x^3}}\,dx=-\frac{2\sqrt{x}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

input
```
Integrate[1/Sqrt[(a/x^2 + b/x)*x^3],x]
```

output
```
(-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b
]*Sqrt[x*(a + b*x)])
```

3.988.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2078, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^3\left(\frac{a}{x^2}+\frac{b}{x}\right)}}dx$$

$$\Big\downarrow \text{2078}$$

$$\int \frac{1}{\sqrt{ax+bx^2}}dx$$

$$\Big\downarrow \text{1091}$$

$$2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

input
```
Int[1/Sqrt[(a/x^2 + b/x)*x^3],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

3.988.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

3.988.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{\sqrt{b}}$	29

input
```
int(1/((a/x^2+b/x)*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

3.988.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.21

$$\int \frac{1}{\sqrt{\left(\frac{a}{x^2}+\frac{b}{x}\right)x^3}}\,dx = \left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)}{b}\right]$$

input
```
integrate(1/((a/x^2+b/x)*x^3)^(1/2),x, algorithm="fricas")
```

output
```
[log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(
sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

3.988.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(26) = 52.

Time = 0.70 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.68

$$\int \frac{1}{\sqrt{\left(\frac{a}{x^2}+\frac{b}{x}\right)x^3}}\,dx = \begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } b\neq 0 \wedge \frac{a^2}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{for } b\neq 0 \\ \frac{2\sqrt{ax}}{a} & \text{for } a\neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

input
```
integrate(1/((a/x**2+b/x)*x**3)**(1/2),x)
```

output
```
Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(b, 0)
 & Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2
), Ne(b, 0)), (2*sqrt(a*x)/a, Ne(a, 0)), (zoo*x, True))
```

3.988.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{\left(\frac{a}{x^2}+\frac{b}{x}\right)x^3}}\,dx = \frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}$$

input
```
integrate(1/((a/x^2+b/x)*x^3)^(1/2),x, algorithm="maxima")
```

output
```
log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

3.988.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(22) = 44$.

Time = 0.33 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{\left(\frac{a}{x^2}+\frac{b}{x}\right)x^3}}\,dx = \frac{1}{4}\sqrt{bx^2+ax}\left(2x+\frac{a}{b}\right)+\frac{a^2\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate(1/((a/x^2+b/x)*x^3)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(b*x^2 + a*x)*(2*x + a/b) + 1/8*a^2*log(abs(2*(sqrt(b)*x - sqrt(b*
x^2 + a*x))*sqrt(b) + a))/b^(3/2)
```

3.988.9 Mupad [B] (verification not implemented)

Time = 22.67 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\left(\frac{a}{x^2}+\frac{b}{x}\right)x^3}}\,dx = \frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}$$

input
```
int(1/(x^3*(a/x^2 + b/x))^(1/2),x)
```

output
```
log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

3.989 $\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}}\,dx$

3.989.1 Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

output
```
2*arctanh(x*b^(1/2)/(b*x^2+a*x)^(1/2))/b^(1/2)
```

3.989.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.96

$$\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}}\,dx = -\frac{2\sqrt{x}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

input
```
Integrate[1/Sqrt[(a*x^2 + b*x^3)/x],x]
```

output
```
(-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b
]*Sqrt[x*(a + b*x)])
```

3.989.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2078, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}}dx$$
$$\Big\downarrow \; 2078$$
$$\int \frac{1}{\sqrt{ax+bx^2}}dx$$
$$\Big\downarrow \; 1091$$
$$2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}$$
$$\Big\downarrow \; 219$$
$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

input
```
Int[1/Sqrt[(a*x^2 + b*x^3)/x],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

3.989.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

3.989.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$	29

input
```
int(1/((b*x^3+a*x^2)/x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

3.989.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.21

$$\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}}\,dx = \left[\frac{\log\left(2\,bx + a + 2\,\sqrt{bx^2 + ax}\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)}{b}\right]$$

input
```
integrate(1/((b*x^3+a*x^2)/x)^(1/2),x, algorithm="fricas")
```

output
```
[log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(
sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

3.989.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(26) = 52$.

Time = 0.66 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.68

$$\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}}\,dx = \begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } b \neq 0 \wedge \frac{a^2}{b} \neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{for } b \neq 0 \\ \frac{2\sqrt{ax}}{a} & \text{for } a \neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

input
```
integrate(1/((b*x**3+a*x**2)/x)**(1/2),x)
```

output
```
Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(b, 0)
 & Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2
), Ne(b, 0)), (2*sqrt(a*x)/a, Ne(a, 0)), (zoo*x, True))
```

3.989.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}}\,dx = \frac{\log\left(2\,bx + a + 2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}$$

input
```
integrate(1/((b*x^3+a*x^2)/x)^(1/2),x, algorithm="maxima")
```

output
```
log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

3.989.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(22) = 44$.

Time = 0.35 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}}\,dx = \frac{1}{4}\sqrt{bx^2+ax}\left(2\,x+\frac{a}{b}\right)+\frac{a^2\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate(1/((b*x^3+a*x^2)/x)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(b*x^2 + a*x)*(2*x + a/b) + 1/8*a^2*log(abs(2*(sqrt(b)*x - sqrt(b*
x^2 + a*x))*sqrt(b) + a))/b^(3/2)
```

3.989.9 Mupad [B] (verification not implemented)

Time = 22.63 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}}\,dx = \frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}$$

input
```
int(1/((a*x^2 + b*x^3)/x)^(1/2),x)
```

output
```
log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

3.990 $\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}}\,dx$

3.990.1 Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

output
```
2*arctanh(x*b^(1/2)/(b*x^2+a*x)^(1/2))/b^(1/2)
```

3.990.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.96

$$\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}}\,dx = -\frac{2\sqrt{x}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

input
```
Integrate[1/Sqrt[(a*x^3 + b*x^4)/x^2],x]
```

output
```
(-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b
]*Sqrt[x*(a + b*x)])
```

3.990.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2078, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}}dx$$

$$\Big\downarrow \text{ 2078}$$

$$\int \frac{1}{\sqrt{ax+bx^2}}dx$$

$$\Big\downarrow \text{ 1091}$$

$$2\int \frac{1}{1-\frac{bx^2}{bx^2+ax}}d\frac{x}{\sqrt{bx^2+ax}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

input
```
Int[1/Sqrt[(a*x^3 + b*x^4)/x^2],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

3.990.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

3.990.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$	29

input
```
int(1/((b*x^4+a*x^3)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

3.990.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.21

$$\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}}\,dx = \left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)}{b}\right]$$

input
```
integrate(1/((b*x^4+a*x^3)/x^2)^(1/2),x, algorithm="fricas")
```

output
```
[log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(
sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

3.990.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(26) = 52$.

Time = 0.69 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.68

$$\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}}\,dx = \begin{cases} \frac{\log\left(a+2\sqrt{b}\sqrt{ax+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } b \neq 0 \wedge \frac{a^2}{b} \neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{b\left(\frac{a}{2b}+x\right)^2}} & \text{for } b \neq 0 \\ \frac{2\sqrt{ax}}{a} & \text{for } a \neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

input
```
integrate(1/((b*x**4+a*x**3)/x**2)**(1/2),x)
```

output
```
Piecewise((log(a + 2*sqrt(b)*sqrt(a*x + b*x**2) + 2*b*x)/sqrt(b), Ne(b, 0)
 & Ne(a**2/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*(a/(2*b) + x)**2
), Ne(b, 0)), (2*sqrt(a*x)/a, Ne(a, 0)), (zoo*x, True))
```

3.990.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}}\,dx = \frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\sqrt{b}\right)}{\sqrt{b}}$$

input
```
integrate(1/((b*x^4+a*x^3)/x^2)^(1/2),x, algorithm="maxima")
```

output
```
log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

3.990.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(22) = 44$.

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}}\,dx = \frac{1}{4}\sqrt{bx^2+ax}\left(2\,x+\frac{a}{b}\right)+\frac{a^2\log\left(\left|2\left(\sqrt{b}x-\sqrt{bx^2+ax}\right)\sqrt{b}+a\right|\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate(1/((b*x^4+a*x^3)/x^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(b*x^2 + a*x)*(2*x + a/b) + 1/8*a^2*log(abs(2*(sqrt(b)*x - sqrt(b*
x^2 + a*x))*sqrt(b) + a))/b^(3/2)
```

3.990.9 Mupad [B] (verification not implemented)

Time = 21.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}}\,dx = \frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}$$

input
```
int(1/((a*x^3 + b*x^4)/x^2)^(1/2),x)
```

output
```
log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

3.991 $\int \frac{1}{\sqrt{acx+bcx^2}}\,dx$

3.991.1 Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \frac{1}{\sqrt{acx+bcx^2}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

output
```
2*arctanh(x*b^(1/2)*c^(1/2)/(b*c*x^2+a*c*x)^(1/2))/b^(1/2)/c^(1/2)
```

3.991.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.40

$$\int \frac{1}{\sqrt{acx+bcx^2}}\,dx = -\frac{2\sqrt{x}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{cx(a+bx)}}$$

input
```
Integrate[1/Sqrt[a*c*x + b*c*x^2],x]
```

output
```
(-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b
]*Sqrt[c*x*(a + b*x)])
```

3.991.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{acx+bcx^2}}dx$$
$$\Big\downarrow \text{ 1091}$$
$$2\int \frac{1}{1-\frac{bcx^2}{bcx^2+acx}}d\frac{x}{\sqrt{bcx^2+acx}}$$
$$\Big\downarrow \text{ 219}$$
$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

input
```
Int[1/Sqrt[a*c*x + b*c*x^2],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])
```

3.991.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

3.991.4 Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

method	result	size
pseudoelliptic	$\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{cx(bx+a)}}{x\sqrt{bc}}\right)}{\sqrt{bc}}$	28
default	$\frac{\ln\left(\frac{\frac{1}{2}ac+bcx}{\sqrt{bc}}+\sqrt{bc\,x^2+acx}\right)}{\sqrt{bc}}$	37

input
```
int(1/(b*c*x^2+a*c*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(b*c)^(1/2)*arctanh((c*x*(b*x+a))^(1/2)/x/(b*c)^(1/2))
```

3.991.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.18

$$\int \frac{1}{\sqrt{acx+bcx^2}}\,dx = \left[\frac{\sqrt{bc}\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\sqrt{bc}\right)}{bc}, -\frac{2\,\sqrt{-bc}\arctan\left(\frac{\sqrt{bcx^2+acx}\sqrt{-bc}}{bcx}\right)}{bc}\right]$$

input
```
integrate(1/(b*c*x^2+a*c*x)^(1/2),x, algorithm="fricas")
```

output
```
[sqrt(b*c)*log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/(b*c), -
2*sqrt(-b*c)*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/(b*c)]
```

3.991.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(39) = 78.

Time = 0.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.42

$$\int \frac{1}{\sqrt{acx+bcx^2}}\,dx = \begin{cases} \frac{\log\left(ac+2bcx+2\sqrt{bc}\sqrt{acx+bcx^2}\right)}{\sqrt{bc}} & \text{for } bc\neq 0 \wedge \frac{a^2c}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{bc\left(\frac{a}{2b}+x\right)^2}} & \text{for } bc\neq 0 \\ \frac{2\sqrt{acx}}{ac} & \text{for } ac\neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

input
```
integrate(1/(b*c*x**2+a*c*x)**(1/2),x)
```

output
```
Piecewise((log(a*c + 2*b*c*x + 2*sqrt(b*c)*sqrt(a*c*x + b*c*x**2))/sqrt(b*
c), Ne(b*c, 0) & Ne(a**2*c/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*
c*(a/(2*b) + x)**2), Ne(b*c, 0)), (2*sqrt(a*c*x)/(a*c), Ne(a*c, 0)), (zoo*
x, True))
```

3.991.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{acx+bcx^2}}\,dx = \frac{\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\sqrt{bc}\right)}{\sqrt{bc}}$$

input
```
integrate(1/(b*c*x^2+a*c*x)^(1/2),x, algorithm="maxima")
```

output
```
log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/sqrt(b*c)
```

3.991.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(30) = 60$.

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{acx+bcx^2}}\,dx = \frac{a^2c\log\left(\left|-ac-2\sqrt{bc}\left(\sqrt{bc}x-\sqrt{bcx^2+acx}\right)\right|\right)}{8\sqrt{bc}b} + \frac{1}{4}\sqrt{bcx^2+acx}\left(2x+\frac{a}{b}\right)$$

input
```
integrate(1/(b*c*x^2+a*c*x)^(1/2),x, algorithm="giac")
```

output
```
1/8*a^2*c*log(abs(-a*c - 2*sqrt(b*c)*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x))
))/(sqrt(b*c)*b) + 1/4*sqrt(b*c*x^2 + a*c*x)*(2*x + a/b)
```

3.991.9 Mupad [B] (verification not implemented)

Time = 21.40 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{acx+bcx^2}}\,dx = \frac{\ln\left(a\,c+2\,\sqrt{b\,c}\,\sqrt{c\,x\,\left(a+b\,x\right)}+2\,b\,c\,x\right)}{\sqrt{b\,c}}$$

input
```
int(1/(a*c*x + b*c*x^2)^(1/2),x)
```

output
```
log(a*c + 2*(b*c)^(1/2)*(c*x*(a + b*x))^(1/2) + 2*b*c*x)/(b*c)^(1/2)
```

3.992 $\int \frac{1}{\sqrt{c(ax+bx^2)}}\,dx$

3.992.1 Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \frac{1}{\sqrt{c\left(ax+bx^2\right)}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

output
```
2*arctanh(x*b^(1/2)*c^(1/2)/(b*c*x^2+a*c*x)^(1/2))/b^(1/2)/c^(1/2)
```

3.992.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.40

$$\int \frac{1}{\sqrt{c\left(ax+bx^2\right)}}\,dx=-\frac{2\sqrt{x}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{cx(a+bx)}}$$

input
```
Integrate[1/Sqrt[c*(a*x + b*x^2)],x]
```

output
```
(-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b
]*Sqrt[c*x*(a + b*x)])
```

3.992.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2078, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c\,(ax+bx^2)}}\,dx$$
$$\Big\downarrow \text{ 2078}$$
$$\int \frac{1}{\sqrt{acx+bcx^2}}dx$$
$$\Big\downarrow \text{ 1091}$$
$$2\int \frac{1}{1-\frac{bcx^2}{bcx^2+acx}}d\frac{x}{\sqrt{bcx^2+acx}}$$
$$\Big\downarrow \text{ 219}$$
$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

input
```
Int[1/Sqrt[c*(a*x + b*x^2)],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])
```

3.992.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

3.992.4 Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

method	result	size
pseudoelliptic	$\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{cx(bx+a)}}{x\sqrt{bc}}\right)}{\sqrt{bc}}$	28
default	$\frac{\ln\left(\frac{\frac{1}{2}ac+bcx}{\sqrt{bc}}+\sqrt{bc\,x^2+acx}\right)}{\sqrt{bc}}$	37

input
```
int(1/(c*(b*x^2+a*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(b*c)^(1/2)*arctanh((c*x*(b*x+a))^(1/2)/x/(b*c)^(1/2))
```

3.992.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.18

$$\int \frac{1}{\sqrt{c\,(ax+bx^2)}}\,dx = \left[\frac{\sqrt{bc}\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\sqrt{bc}\right)}{bc}, -\frac{2\,\sqrt{-bc}\arctan\left(\frac{\sqrt{bcx^2+acx}\sqrt{-bc}}{bcx}\right)}{bc}\right]$$

input
```
integrate(1/(c*(b*x^2+a*x))^(1/2),x, algorithm="fricas")
```

output
```
[sqrt(b*c)*log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/(b*c), -
2*sqrt(-b*c)*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/(b*c)]
```

3.992.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(39) = 78$.

Time = 0.66 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.42

$$\int \frac{1}{\sqrt{c\,(ax+bx^2)}}\,dx = \begin{cases} \frac{\log\left(ac+2bcx+2\sqrt{bc}\sqrt{acx+bcx^2}\right)}{\sqrt{bc}} & \text{for } bc \neq 0 \wedge \frac{a^2c}{b} \neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{bc\left(\frac{a}{2b}+x\right)^2}} & \text{for } bc \neq 0 \\ \frac{2\sqrt{acx}}{ac} & \text{for } ac \neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

input
```
integrate(1/(c*(b*x**2+a*x))**(1/2),x)
```

output
```
Piecewise((log(a*c + 2*b*c*x + 2*sqrt(b*c)*sqrt(a*c*x + b*c*x**2))/sqrt(b*
c), Ne(b*c, 0) & Ne(a**2*c/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*
c*(a/(2*b) + x)**2), Ne(b*c, 0)), (2*sqrt(a*c*x)/(a*c), Ne(a*c, 0)), (zoo*
x, True))
```

3.992.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{c\,(ax+bx^2)}}\,dx = \frac{\log\left(2\,bcx + ac + 2\,\sqrt{bcx^2 + acx}\sqrt{bc}\right)}{\sqrt{bc}}$$

input
```
integrate(1/(c*(b*x^2+a*x))^(1/2),x, algorithm="maxima")
```

output
```
log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/sqrt(b*c)
```

3.992.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(30) = 60$.

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{c\left(ax+bx^2\right)}}\,dx = \frac{a^2c\log\left(\left|-ac-2\sqrt{bc}\left(\sqrt{bc}x-\sqrt{bcx^2+acx}\right)\right|\right)}{8\sqrt{bc}b} + \frac{1}{4}\sqrt{bcx^2+acx}\left(2x+\frac{a}{b}\right)$$

input
```
integrate(1/(c*(b*x^2+a*x))^(1/2),x, algorithm="giac")
```

output
```
1/8*a^2*c*log(abs(-a*c - 2*sqrt(b*c)*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x))
))/(sqrt(b*c)*b) + 1/4*sqrt(b*c*x^2 + a*c*x)*(2*x + a/b)
```

3.992.9 Mupad [B] (verification not implemented)

Time = 20.97 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{c\left(ax+bx^2\right)}}\,dx = \frac{\ln\left(a\,c+2\sqrt{b\,c}\,\sqrt{c\,x\,\left(a+b\,x\right)}+2\,b\,c\,x\right)}{\sqrt{b\,c}}$$

input
```
int(1/(c*(a*x + b*x^2))^(1/2),x)
```

output
```
log(a*c + 2*(b*c)^(1/2)*(c*x*(a + b*x))^(1/2) + 2*b*c*x)/(b*c)^(1/2)
```

3.993 $\int \frac{1}{\sqrt{cx(a+bx)}}\,dx$

3.993.1 Optimal result

Integrand size = 12, antiderivative size = 40

$$\int \frac{1}{\sqrt{cx(a+bx)}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

output
```
2*arctanh(x*b^(1/2)*c^(1/2)/(b*c*x^2+a*c*x)^(1/2))/b^(1/2)/c^(1/2)
```

3.993.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.40

$$\int \frac{1}{\sqrt{cx(a+bx)}}\,dx = -\frac{2\sqrt{x}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{cx(a+bx)}}$$

input
```
Integrate[1/Sqrt[c*x*(a + b*x)],x]
```

output
```
(-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b
]*Sqrt[c*x*(a + b*x)])
```

3.993.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2048, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cx(a+bx)}}\,dx$$

$$\Big\downarrow \text{ 2048}$$

$$\int \frac{1}{\sqrt{acx+bcx^2}}dx$$

$$\Big\downarrow \text{ 1091}$$

$$2\int \frac{1}{1-\frac{bcx^2}{bcx^2+acx}}d\frac{x}{\sqrt{bcx^2+acx}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{cx}}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

input
```
Int[1/Sqrt[c*x*(a + b*x)],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])
```

3.993.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

3.993.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.70

method	result	size
pseudoelliptic	$\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{cx(bx+a)}}{x\sqrt{bc}}\right)}{\sqrt{bc}}$	28
default	$\frac{\ln\left(\frac{\frac{1}{2}ac+bcx}{\sqrt{bc}}+\sqrt{bc\,x^2+acx}\right)}{\sqrt{bc}}$	37

input
```
int(1/(c*x*(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/(b*c)^(1/2)*arctanh((c*x*(b*x+a))^(1/2)/x/(b*c)^(1/2))
```

3.993.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.18

$$\int \frac{1}{\sqrt{cx(a+bx)}}\,dx = \left[\frac{\sqrt{bc}\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\sqrt{bc}\right)}{bc}, -\frac{2\,\sqrt{-bc}\arctan\left(\frac{\sqrt{bcx^2+acx}\sqrt{-bc}}{bcx}\right)}{bc}\right]$$

input
```
integrate(1/(c*x*(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
[sqrt(b*c)*log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/(b*c), -
2*sqrt(-b*c)*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/(b*c)]
```

3.993.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(39) = 78$.

Time = 0.71 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.42

$$\int \frac{1}{\sqrt{cx(a+bx)}}\,dx = \begin{cases} \frac{\log\left(ac+2bcx+2\sqrt{bc}\sqrt{acx+bcx^2}\right)}{\sqrt{bc}} & \text{for } bc \neq 0 \wedge \frac{a^2c}{b} \neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{bc\left(\frac{a}{2b}+x\right)^2}} & \text{for } bc \neq 0 \\ \frac{2\sqrt{acx}}{ac} & \text{for } ac \neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

input
```
integrate(1/(c*x*(b*x+a))**(1/2),x)
```

output
```
Piecewise((log(a*c + 2*b*c*x + 2*sqrt(b*c)*sqrt(a*c*x + b*c*x**2))/sqrt(b*
c), Ne(b*c, 0) & Ne(a**2*c/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*
c*(a/(2*b) + x)**2), Ne(b*c, 0)), (2*sqrt(a*c*x)/(a*c), Ne(a*c, 0)), (zoo*
x, True))
```

3.993.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{cx(a+bx)}}\,dx = \frac{\log\left(2\,bcx + ac + 2\,\sqrt{bcx^2 + acx}\sqrt{bc}\right)}{\sqrt{bc}}$$

input
```
integrate(1/(c*x*(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/sqrt(b*c)
```

3.993.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(30) = 60.

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{cx(a+bx)}}\,dx = \frac{a^2c\log\left(\left|-ac-2\sqrt{bc}\left(\sqrt{bc}x-\sqrt{bcx^2+acx}\right)\right|\right)}{8\sqrt{bc}b} + \frac{1}{4}\sqrt{bcx^2+acx}\left(2x+\frac{a}{b}\right)$$

input
```
integrate(1/(c*x*(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
1/8*a^2*c*log(abs(-a*c - 2*sqrt(b*c)*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x))
))/(sqrt(b*c)*b) + 1/4*sqrt(b*c*x^2 + a*c*x)*(2*x + a/b)
```

3.993.9 Mupad [B] (verification not implemented)

Time = 22.70 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{cx(a+bx)}}\,dx = \frac{\ln\left(a\,c+2\sqrt{b\,c}\,\sqrt{c\,x\,(a+b\,x)}+2\,b\,c\,x\right)}{\sqrt{b\,c}}$$

input
```
int(1/(c*x*(a + b*x))^(1/2),x)
```

output
```
log(a*c + 2*(b*c)^(1/2)*(c*x*(a + b*x))^(1/2) + 2*b*c*x)/(b*c)^(1/2)
```

3.994 $\int \frac{1}{\sqrt{c\left(b+\frac{a}{x}\right)x^2}}\,dx$

3.994.1 Optimal result

Integrand size = 16, antiderivative size = 40

$$\int \frac{1}{\sqrt{c\left(b+\frac{a}{x}\right)x^2}}\,dx=\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

output
```
2*arctanh(x*b^(1/2)*c^(1/2)/(b*c*x^2+a*c*x)^(1/2))/b^(1/2)/c^(1/2)
```

3.994.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.40

$$\int \frac{1}{\sqrt{c\left(b+\frac{a}{x}\right)x^2}}\,dx=-\frac{2\sqrt{x}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{cx(a+bx)}}$$

input
```
Integrate[1/Sqrt[c*(b + a/x)*x^2],x]
```

output
```
(-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b
]*Sqrt[c*x*(a + b*x)])
```

3.994.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2078, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cx^2\left(\frac{a}{x}+b\right)}}dx$$

$$\Big\downarrow \text{ 2078}$$

$$\int \frac{1}{\sqrt{acx+bcx^2}}dx$$

$$\Big\downarrow \text{ 1091}$$

$$2\int \frac{1}{1-\frac{bcx^2}{bcx^2+acx}}d\frac{x}{\sqrt{bcx^2+acx}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

input
```
Int[1/Sqrt[c*(b + a/x)*x^2],x]
```

output
```
(2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])
```

3.994.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 2078
```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

3.994.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\ln\left(\frac{\frac{1}{2}ac+bcx}{\sqrt{bc}}+\sqrt{bc\,x^{2}+acx}\right)}{\sqrt{bc}}$	37

input
```
int(1/(c*(b+a/x)*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln((1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2)
```

3.994.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.18

$$\int \frac{1}{\sqrt{c\left(b+\frac{a}{x}\right)x^2}}\,dx = \left[\frac{\sqrt{bc}\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\sqrt{bc}\right)}{bc},\ -\frac{2\,\sqrt{-bc}\arctan\left(\frac{\sqrt{bcx^2+acx}\sqrt{-bc}}{bcx}\right)}{bc}\right]$$

input
```
integrate(1/(c*(b+a/x)*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[sqrt(b*c)*log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/(b*c), -
2*sqrt(-b*c)*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/(b*c)]
```

3.994.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(39) = 78$.

Time = 0.69 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.42

$$\int \frac{1}{\sqrt{c\left(b+\frac{a}{x}\right)x^2}}\,dx = \begin{cases} \frac{\log\left(ac+2bcx+2\sqrt{bc}\sqrt{acx+bcx^2}\right)}{\sqrt{bc}} & \text{for } bc\neq 0\wedge \frac{a^2c}{b}\neq 0 \\ \frac{\left(\frac{a}{2b}+x\right)\log\left(\frac{a}{2b}+x\right)}{\sqrt{bc\left(\frac{a}{2b}+x\right)^2}} & \text{for } bc\neq 0 \\ \frac{2\sqrt{acx}}{ac} & \text{for } ac\neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

input
```
integrate(1/(c*(b+a/x)*x**2)**(1/2),x)
```

output
```
Piecewise((log(a*c + 2*b*c*x + 2*sqrt(b*c)*sqrt(a*c*x + b*c*x**2))/sqrt(b*
c), Ne(b*c, 0) & Ne(a**2*c/b, 0)), ((a/(2*b) + x)*log(a/(2*b) + x)/sqrt(b*
c*(a/(2*b) + x)**2), Ne(b*c, 0)), (2*sqrt(a*c*x)/(a*c), Ne(a*c, 0)), (zoo*
x, True))
```

3.994.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{c\left(b+\frac{a}{x}\right)x^2}}\,dx = \frac{\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\sqrt{bc}\right)}{\sqrt{bc}}$$

input
```
integrate(1/(c*(b+a/x)*x^2)^(1/2),x, algorithm="maxima")
```

output
```
log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/sqrt(b*c)
```

3.994.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(30) = 60$.

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{c\left(b+\frac{a}{x}\right)x^2}}\,dx = \frac{a^2c\log\left(\left|-ac-2\sqrt{bc}\left(\sqrt{bc}x-\sqrt{bcx^2+acx}\right)\right|\right)}{8\sqrt{bc}b} + \frac{1}{4}\sqrt{bcx^2+acx}\left(2x+\frac{a}{b}\right)$$

input
```
integrate(1/(c*(b+a/x)*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*a^2*c*log(abs(-a*c - 2*sqrt(b*c)*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x))
))/(sqrt(b*c)*b) + 1/4*sqrt(b*c*x^2 + a*c*x)*(2*x + a/b)
```

3.994.9 Mupad [B] (verification not implemented)

Time = 23.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{c\left(b+\frac{a}{x}\right)x^2}}\,dx = \frac{\ln\left(a\,c+2\sqrt{b\,c}\,\sqrt{c\,x\,(a+b\,x)}+2\,b\,c\,x\right)}{\sqrt{b\,c}}$$

input
```
int(1/(c*x^2*(b + a/x))^(1/2),x)
```

output
```
log(a*c + 2*(b*c)^(1/2)*(c*x*(a + b*x))^(1/2) + 2*b*c*x)/(b*c)^(1/2)
```

3.995 $\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx$

3.995.1 Optimal result

Integrand size = 22, antiderivative size = 63

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx = \frac{1}{4}\left(3x+\sqrt{-1+x^2}\right)\sqrt{1-x^2+x\sqrt{-1+x^2}} + \frac{3\arcsin\left(x-\sqrt{-1+x^2}\right)}{4\sqrt{2}}$$

output
```
3/8*arcsin(x-(x^2-1)^(1/2))*2^(1/2)+1/4*(3*x+(x^2-1)^(1/2))*(1-x^2+x*(x^2-
1)^(1/2))^(1/2)
```

3.995.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.75

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx = \frac{1}{8}\left(\frac{2(-1+x^2)\left(3x+\sqrt{-1+x^2}\right)}{\sqrt{1-x^2+x\sqrt{-1+x^2}}\left(-1+x^2+x\sqrt{-1+x^2}\right)} - 3\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-1+x^2}}{\sqrt{1-x^2+x\sqrt{-1+x^2}}}\right)\right)$$

input
```
Integrate[Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]],x]
```

output
```
((2*(-1 + x^2)*(3*x + Sqrt[-1 + x^2]))/(Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]]*(
-1 + x^2 + x*Sqrt[-1 + x^2])) - 3*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[-1 + x^2])/
Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]]])/8
```

3.995.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-x^2+\sqrt{x^2-1}x+1}\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \sqrt{-x^2+\sqrt{x^2-1}x+1}dx$$

input
```
Int[Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]],x]
```

output
```
$Aborted
```

3.995.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.995.4 Maple [F]

$$\int \sqrt{1-x^2+x\sqrt{x^2-1}}dx$$

input
```
int((1-x^2+x*(x^2-1)^(1/2))^(1/2),x)
```

output
```
int((1-x^2+x*(x^2-1)^(1/2))^(1/2),x)
```

3.995.5 Fricas [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.08

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx = \frac{1}{4}\sqrt{-x^2+\sqrt{x^2-1}x+1}\left(3\,x+\sqrt{x^2-1}\right) + \frac{3}{8}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+\sqrt{x^2-1}x+1}}{2\sqrt{x^2-1}}\right)$$

input
```
integrate((1-x^2+x*(x^2-1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(-x^2 + sqrt(x^2 - 1)*x + 1)*(3*x + sqrt(x^2 - 1)) + 3/8*sqrt(2)*a
rctan(1/2*sqrt(2)*sqrt(-x^2 + sqrt(x^2 - 1)*x + 1)/sqrt(x^2 - 1))
```

3.995.6 Sympy [F]

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx = \int \sqrt{-x^2+x\sqrt{x^2-1}+1}\,dx$$

input
```
integrate((1-x**2+x*(x**2-1)**(1/2))**(1/2),x)
```

output
```
Integral(sqrt(-x**2 + x*sqrt(x**2 - 1) + 1), x)
```

3.995.7 Maxima [F]

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx = \int \sqrt{-x^2+\sqrt{x^2-1}x+1}\,dx$$

input
```
integrate((1-x^2+x*(x^2-1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x^2 + sqrt(x^2 - 1)*x + 1), x)
```

3.995.8 Giac [F]

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx = \int \sqrt{-x^2+\sqrt{x^2-1}x+1}\,dx$$

input
```
integrate((1-x^2+x*(x^2-1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x^2 + sqrt(x^2 - 1)*x + 1), x)
```

3.995.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx=\int \sqrt{x\,\sqrt{x^2-1}-x^2+1}\,dx$$

input
```
int((x*(x^2 - 1)^(1/2) - x^2 + 1)^(1/2),x)
```

output
```
int((x*(x^2 - 1)^(1/2) - x^2 + 1)^(1/2), x)
```

3.996 $\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}\,dx$

3.996.1 Optimal result

Integrand size = 29, antiderivative size = 66

$$\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}\,dx=\frac{1}{2}\left(\sqrt{x}+3\sqrt{1+x}\right)\sqrt{-x+\sqrt{x}\sqrt{1+x}}-\frac{3\arcsin\left(\sqrt{x}-\sqrt{1+x}\right)}{2\sqrt{2}}$$

output
```
-3/4*arcsin(x^(1/2)-(1+x)^(1/2))*2^(1/2)+1/2*(x^(1/2)+3*(1+x)^(1/2))*(-x+x
^(1/2)*(1+x)^(1/2))^(1/2)
```

3.996.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}\,dx=\frac{1}{4}\left(2\left(\sqrt{x}+3\sqrt{1+x}\right)\sqrt{-x+\sqrt{x}\sqrt{1+x}}-3\sqrt{2}\arctan\left(\frac{\sqrt{-2x+2\sqrt{x}\sqrt{1+x}}}{-\sqrt{x}+\sqrt{1+x}}\right)\right)$$

input
```
Integrate[Sqrt[-x + Sqrt[x]*Sqrt[1 + x]]/Sqrt[1 + x],x]
```

output
```
(2*(Sqrt[x] + 3*Sqrt[1 + x])*Sqrt[-x + Sqrt[x]*Sqrt[1 + x]] - 3*Sqrt[2]*Ar
cTan[Sqrt[-2*x + 2*Sqrt[x]*Sqrt[1 + x]]/(-Sqrt[x] + Sqrt[1 + x])])/4
```

3.996.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sqrt{x}\sqrt{x+1}-x}}{\sqrt{x+1}}\,dx$$
$$\Big\downarrow \text{7267}$$
$$2\int \sqrt{\sqrt{x}\sqrt{x+1}-x}d\sqrt{x+1}$$
$$\Big\downarrow \text{7299}$$
$$2\int \sqrt{\sqrt{x}\sqrt{x+1}-x}d\sqrt{x+1}$$

input
```
Int[Sqrt[-x + Sqrt[x]*Sqrt[1 + x]]/Sqrt[1 + x],x]
```

output
```
$Aborted
```

3.996.3.1 Defintions of rubi rules used

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.996.4 Maple [F]

$$\int \frac{\sqrt{-x+\sqrt{x}\,\sqrt{x+1}}}{\sqrt{x+1}}dx$$

input
```
int((-x+x^(1/2)*(x+1)^(1/2))^(1/2)/(x+1)^(1/2),x)
```

output
```
int((-x+x^(1/2)*(x+1)^(1/2))^(1/2)/(x+1)^(1/2),x)
```

3.996.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}\,dx = \text{Timed out}$$

input
```
integrate((-x+x^(1/2)*(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="fricas"
)
```

output
```
Timed out
```

3.996.6 Sympy [F]

$$\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}\,dx = \int \frac{\sqrt{\sqrt{x}\sqrt{x+1}-x}}{\sqrt{x+1}}\,dx$$

input
```
integrate((-x+x**(1/2)*(1+x)**(1/2))**(1/2)/(1+x)**(1/2),x)
```

output
```
Integral(sqrt(sqrt(x)*sqrt(x + 1) - x)/sqrt(x + 1), x)
```

3.996.7 Maxima [F]

$$\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}\,dx = \int \frac{\sqrt{\sqrt{x+1}\sqrt{x}-x}}{\sqrt{x+1}}\,dx$$

input
```
integrate((-x+x^(1/2)*(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(sqrt(x + 1)*sqrt(x) - x)/sqrt(x + 1), x)
```

3.996.8 Giac [F]

$$\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}\,dx = \int \frac{\sqrt{\sqrt{x+1}\sqrt{x}-x}}{\sqrt{x+1}}\,dx$$

input
```
integrate((-x+x^(1/2)*(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sqrt(x + 1)*sqrt(x) - x)/sqrt(x + 1), x)
```

3.996.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}\,dx = \int \frac{\sqrt{\sqrt{x}\,\sqrt{x+1}-x}}{\sqrt{x+1}}\,dx$$

input
```
int((x^(1/2)*(x + 1)^(1/2) - x)^(1/2)/(x + 1)^(1/2),x)
```

output
```
int((x^(1/2)*(x + 1)^(1/2) - x)^(1/2)/(x + 1)^(1/2), x)
```

3.997 $\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx$

3.997.1 Optimal result

Integrand size = 31, antiderivative size = 78

$$\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx = -\sqrt{2\left(1+\sqrt{5}\right)}\arctan\left(\sqrt{-2+\sqrt{5}}\left(x+\sqrt{1+x^2}\right)\right) + \sqrt{2\left(-1+\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{2+\sqrt{5}}\left(x+\sqrt{1+x^2}\right)\right)$$

output
```
arctanh((x+(x^2+1)^(1/2))*(2+5^(1/2))^(1/2))*(-2+2*5^(1/2))^(1/2)-arctan((
x+(x^2+1)^(1/2))*(-2+5^(1/2))^(1/2))*(2+2*5^(1/2))^(1/2)
```

3.997.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.06

$$\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx = -\sqrt{2\left(1+\sqrt{5}\right)}\arctan\left(\sqrt{2+\sqrt{5}}\left(x-\sqrt{1+x^2}\right)\right) - \sqrt{2\left(-1+\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{-2+\sqrt{5}}\left(x-\sqrt{1+x^2}\right)\right)$$

input
```
Integrate[-((x + 2*Sqrt[1 + x^2])/(x + x^3 + Sqrt[1 + x^2])),x]
```

output
```
-(Sqrt[2*(1 + Sqrt[5])]*ArcTan[Sqrt[2 + Sqrt[5]]*(x - Sqrt[1 + x^2])]) - S
qrt[2*(-1 + Sqrt[5])]*ArcTanh[Sqrt[-2 + Sqrt[5]]*(x - Sqrt[1 + x^2])]
```

3.997.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 319 vs. 2(78) = 156.

Time = 0.81 (sec) , antiderivative size = 319, normalized size of antiderivative = 4.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {25, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -\frac{2\sqrt{x^2+1}+x}{x^3+\sqrt{x^2+1}+x}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{x+2\sqrt{x^2+1}}{x^3+x+\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{7293}$$

$$-\int \left(\frac{x}{x^3+x+\sqrt{x^2+1}}+\frac{2\sqrt{x^2+1}}{x^3+x+\sqrt{x^2+1}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\sqrt{\frac{2}{5}\left(\sqrt{5}-1\right)}\arctan\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x^2+1}\right)-\sqrt{\frac{2}{5\left(\sqrt{5}-1\right)}}\arctan\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x^2+1}\right)-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)-2\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\arctan\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)+\sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\text{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x^2+1}\right)-\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\text{arctanh}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x^2+1}\right)+\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\text{arctanh}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)-2\sqrt{\frac{2}{5\left(\sqrt{5}-1\right)}}\text{arctanh}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)$$

input
```
Int[-((x + 2*Sqrt[1 + x^2])/(x + x^3 + Sqrt[1 + x^2])),x]
```

output
```
-2*Sqrt[2/(5*(1 + Sqrt[5]))]*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x] - Sqrt[(1 + S
qrt[5])/10]*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x] - Sqrt[2/(5*(-1 + Sqrt[5]))]*A
rcTan[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[1 + x^2]] - Sqrt[(2*(-1 + Sqrt[5]))/5]*A
rcTan[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[1 + x^2]] - 2*Sqrt[2/(5*(-1 + Sqrt[5]))]
*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x] + Sqrt[(-1 + Sqrt[5])/10]*ArcTanh[Sqrt[
2/(-1 + Sqrt[5])]*x] - Sqrt[2/(5*(1 + Sqrt[5]))]*ArcTanh[Sqrt[2/(1 + Sqrt[
5])]*Sqrt[1 + x^2]] + Sqrt[(2*(1 + Sqrt[5]))/5]*ArcTanh[Sqrt[2/(1 + Sqrt[5
])]*Sqrt[1 + x^2]]
```

3.997.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.997.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.33 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.78

method	result
trager	$-\operatorname{RootOf}\left(\operatorname{RootOf}\left(\_Z^4+\_Z^2-1\right)^2+\_Z^2+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\operatorname{RootOf}\left(\_Z^4+\_Z^2-1\right)^2+\_Z^2+1\right)\operatorname{Ro}}{1+\operatorname{RootO}}\right.$
default	$-\frac{\left(3+\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{5\sqrt{2\sqrt{5}+2}}+\frac{\left(\sqrt{5}-3\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{5\sqrt{2\sqrt{5}-2}}+\frac{2\left(\sqrt{5}+1\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{5\sqrt{2\sqrt{5}+2}}-\frac{2\sqrt{5}\left(\sqrt{5}-1\right)\operatorname{arc}}{5\sqrt{2\sqrt{}}}$

input
```
int((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-RootOf(RootOf(_Z^4+_Z^2-1)^2+_Z^2+1)*ln(-(RootOf(RootOf(_Z^4+_Z^2-1)^2+_Z
^2+1)*RootOf(_Z^4+_Z^2-1)^2-(x^2+1)^(1/2)*RootOf(_Z^4+_Z^2-1)^2-RootOf(Roo
tOf(_Z^4+_Z^2-1)^2+_Z^2+1))/(1+RootOf(RootOf(_Z^4+_Z^2-1)^2+_Z^2+1)*RootOf
(_Z^4+_Z^2-1)^2*x))+RootOf(_Z^4+_Z^2-1)*ln(-((x^2+1)^(1/2)*RootOf(_Z^4+_Z^
2-1)^2+RootOf(_Z^4+_Z^2-1)^3+(x^2+1)^(1/2)+2*RootOf(_Z^4+_Z^2-1))/(RootOf(
_Z^4+_Z^2-1)^3*x+RootOf(_Z^4+_Z^2-1)*x+1))
```

3.997.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(58) = 116$.

Time = 0.29 (sec) , antiderivative size = 434, normalized size of antiderivative = 5.56

$$\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx = \frac{1}{4}\sqrt{2}\sqrt{-\sqrt{5}-1}\log\left(4x^2 - \sqrt{x^2+1}\left(\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{-\sqrt{5}-1}+4x\right) + \left(\sqrt{5}\sqrt{2}x-\sqrt{2}x\right)\sqrt{-\sqrt{5}-1}+4\right) - \frac{1}{4}\sqrt{2}\sqrt{-\sqrt{5}-1}\log\left(4x^2 + \sqrt{x^2+1}\left(\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{-\sqrt{5}-1}-4x\right) - \left(\sqrt{5}\sqrt{2}x-\sqrt{2}x\right)\sqrt{-\sqrt{5}-1}+4\right) - \frac{1}{4}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(4x^2-4\sqrt{x^2+1}x + \left(\sqrt{5}\sqrt{2}x-\sqrt{x^2+1}\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)+\sqrt{2}x\right)\sqrt{\sqrt{5}-1}+4\right) + \frac{1}{4}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(4x^2-4\sqrt{x^2+1}x - \left(\sqrt{5}\sqrt{2}x-\sqrt{x^2+1}\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)+\sqrt{2}x\right)\sqrt{\sqrt{5}-1}+4\right) - \frac{1}{4}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(2x+\sqrt{2}\sqrt{\sqrt{5}-1}\right) + \frac{1}{4}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(2x-\sqrt{2}\sqrt{\sqrt{5}-1}\right) - \frac{1}{4}\sqrt{2}\sqrt{-\sqrt{5}-1}\log\left(2x+\sqrt{2}\sqrt{-\sqrt{5}-1}\right) + \frac{1}{4}\sqrt{2}\sqrt{-\sqrt{5}-1}\log\left(2x-\sqrt{2}\sqrt{-\sqrt{5}-1}\right)$$

input
```
integrate((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x, algorithm="fricas"
)
```

output
```
1/4*sqrt(2)*sqrt(-sqrt(5) - 1)*log(4*x^2 - sqrt(x^2 + 1)*((sqrt(5)*sqrt(2)
 - sqrt(2))*sqrt(-sqrt(5) - 1) + 4*x) + (sqrt(5)*sqrt(2)*x - sqrt(2)*x)*sq
rt(-sqrt(5) - 1) + 4) - 1/4*sqrt(2)*sqrt(-sqrt(5) - 1)*log(4*x^2 + sqrt(x^
2 + 1)*((sqrt(5)*sqrt(2) - sqrt(2))*sqrt(-sqrt(5) - 1) - 4*x) - (sqrt(5)*s
qrt(2)*x - sqrt(2)*x)*sqrt(-sqrt(5) - 1) + 4) - 1/4*sqrt(2)*sqrt(sqrt(5) -
 1)*log(4*x^2 - 4*sqrt(x^2 + 1)*x + (sqrt(5)*sqrt(2)*x - sqrt(x^2 + 1)*(sq
rt(5)*sqrt(2) + sqrt(2)) + sqrt(2)*x)*sqrt(sqrt(5) - 1) + 4) + 1/4*sqrt(2)
*sqrt(sqrt(5) - 1)*log(4*x^2 - 4*sqrt(x^2 + 1)*x - (sqrt(5)*sqrt(2)*x - sq
rt(x^2 + 1)*(sqrt(5)*sqrt(2) + sqrt(2)) + sqrt(2)*x)*sqrt(sqrt(5) - 1) + 4
) - 1/4*sqrt(2)*sqrt(sqrt(5) - 1)*log(2*x + sqrt(2)*sqrt(sqrt(5) - 1)) + 1
/4*sqrt(2)*sqrt(sqrt(5) - 1)*log(2*x - sqrt(2)*sqrt(sqrt(5) - 1)) - 1/4*sq
rt(2)*sqrt(-sqrt(5) - 1)*log(2*x + sqrt(2)*sqrt(-sqrt(5) - 1)) + 1/4*sqrt(
2)*sqrt(-sqrt(5) - 1)*log(2*x - sqrt(2)*sqrt(-sqrt(5) - 1))
```

3.997.6 Sympy [F]

$$\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx = -\int \frac{x}{x^3+x+\sqrt{x^2+1}}\,dx - \int \frac{2\sqrt{x^2+1}}{x^3+x+\sqrt{x^2+1}}\,dx$$

input
```
integrate((-x-2*(x**2+1)**(1/2))/(x+x**3+(x**2+1)**(1/2)),x)
```

output
```
-Integral(x/(x**3 + x + sqrt(x**2 + 1)), x) - Integral(2*sqrt(x**2 + 1)/(x
**3 + x + sqrt(x**2 + 1)), x)
```

3.997.7 Maxima [F]

$$\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx = \int -\frac{x+2\sqrt{x^2+1}}{x^3+x+\sqrt{x^2+1}}\,dx$$

input
```
integrate((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x, algorithm="maxima"
)
```

output
```
-x - 1/2*arctan(x) + integrate(1/2*(2*x^6 + 3*x^4 - x^2 - 1)/(x^6 + 2*x^4
+ 2*x^2 + 2*(x^3 + x)*sqrt(x^2 + 1) + 1), x)
```

3.997.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(58) = 116$.

Time = 0.41 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.79

$$\begin{aligned}
&\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx \\
&= -\frac{1}{2}\sqrt{2\sqrt{5}+2}\arctan\left(-\frac{x-\sqrt{x^2+1}+\frac{1}{x-\sqrt{x^2+1}}}{\sqrt{2\sqrt{5}-2}}\right) \\
&\quad -\frac{1}{2}\sqrt{2\sqrt{5}+2}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) \\
&\quad +\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(-x+\sqrt{x^2+1}+\sqrt{2\sqrt{5}+2}-\frac{1}{x-\sqrt{x^2+1}}\right) \\
&\quad -\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)+\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) \\
&\quad -\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(\left|-x+\sqrt{x^2+1}-\sqrt{2\sqrt{5}+2}-\frac{1}{x-\sqrt{x^2+1}}\right|\right)
\end{aligned}$$

input
```
integrate((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
-1/2*sqrt(2*sqrt(5) + 2)*arctan(-(x - sqrt(x^2 + 1) + 1/(x - sqrt(x^2 + 1)
))/sqrt(2*sqrt(5) - 2)) - 1/2*sqrt(2*sqrt(5) + 2)*arctan(x/sqrt(1/2*sqrt(5
) + 1/2)) + 1/4*sqrt(2*sqrt(5) - 2)*log(-x + sqrt(x^2 + 1) + sqrt(2*sqrt(5
) + 2) - 1/(x - sqrt(x^2 + 1))) - 1/4*sqrt(2*sqrt(5) - 2)*log(abs(x + sqrt
(1/2*sqrt(5) - 1/2))) + 1/4*sqrt(2*sqrt(5) - 2)*log(abs(x - sqrt(1/2*sqrt(
5) - 1/2))) - 1/4*sqrt(2*sqrt(5) - 2)*log(abs(-x + sqrt(x^2 + 1) - sqrt(2*
sqrt(5) + 2) - 1/(x - sqrt(x^2 + 1))))
```

3.997.9 Mupad [B] (verification not implemented)

Time = 20.65 (sec) , antiderivative size = 649, normalized size of antiderivative = 8.32

$$\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx = \frac{\ln\left(x+\frac{\sqrt{2}\sqrt{\sqrt{5}-1}}{2}\right)\left(\frac{\sqrt{5}}{2}-\frac{5}{2}\right)}{2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}} - \frac{\ln\left(x-\frac{\sqrt{2}\sqrt{\sqrt{5}-1}}{2}\right)\left(\frac{\sqrt{5}}{2}-\frac{5}{2}\right)}{2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}}$$
$$+\frac{\ln\left(x-\frac{\sqrt{2}\sqrt{-\sqrt{5}-1}}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{5}{2}\right)}{2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}} - \frac{\ln\left(x+\frac{\sqrt{2}\sqrt{-\sqrt{5}-1}}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{5}{2}\right)}{2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}}$$
$$-\frac{\left(\ln\left(x-\frac{\sqrt{2}\sqrt{\sqrt{5}-1}}{2}\right)-\ln\left(\frac{\sqrt{2}\,x\sqrt{\sqrt{5}-1}}{2}+\frac{\sqrt{2}\sqrt{x^2+1}\sqrt{\sqrt{5}+1}}{2}+1\right)\right)\left(\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+2\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)}{\left(2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{\sqrt{5}}{2}+\frac{1}{2}}}$$
$$-\frac{\left(\ln\left(x+\frac{\sqrt{2}\sqrt{\sqrt{5}-1}}{2}\right)-\ln\left(\frac{\sqrt{2}\sqrt{x^2+1}\sqrt{\sqrt{5}+1}}{2}-\frac{\sqrt{2}\,x\sqrt{\sqrt{5}-1}}{2}+1\right)\right)\left(\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+2\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)}{\left(2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{\sqrt{5}}{2}+\frac{1}{2}}}$$
$$+\frac{\left(\ln\left(\frac{\sqrt{2}\sqrt{x^2+1}\sqrt{1-\sqrt{5}}}{2}-\frac{\sqrt{2}\,x\sqrt{-\sqrt{5}-1}}{2}+1\right)-\ln\left(x+\frac{\sqrt{2}\sqrt{-\sqrt{5}-1}}{2}\right)\right)\left(\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+2\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)}{\left(2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}}$$
$$+\frac{\left(\ln\left(\frac{\sqrt{2}\,x\sqrt{-\sqrt{5}-1}}{2}+\frac{\sqrt{2}\sqrt{x^2+1}\sqrt{1-\sqrt{5}}}{2}+1\right)-\ln\left(x-\frac{\sqrt{2}\sqrt{-\sqrt{5}-1}}{2}\right)\right)\left(\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+2\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)}{\left(2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}}$$

input
```
int(-(x + 2*(x^2 + 1)^(1/2))/(x + (x^2 + 1)^(1/2) + x^3),x)
```

output
```
(log(x + (2^(1/2)*(5^(1/2) - 1)^(1/2))/2)*(5^(1/2)/2 - 5/2))/(2*(5^(1/2)/2
 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2)) - (log(x - (2^(1/2)*(5^(1/2) -
1)^(1/2))/2)*(5^(1/2)/2 - 5/2))/(2*(5^(1/2)/2 - 1/2)^(1/2) + 4*(5^(1/2)/2
- 1/2)^(3/2)) + (log(x - (2^(1/2)*(- 5^(1/2) - 1)^(1/2))/2)*(5^(1/2)/2 + 5
/2))/(2*(- 5^(1/2)/2 - 1/2)^(1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2)) - (log(x
+ (2^(1/2)*(- 5^(1/2) - 1)^(1/2))/2)*(5^(1/2)/2 + 5/2))/(2*(- 5^(1/2)/2 -
1/2)^(1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2)) - ((log(x - (2^(1/2)*(5^(1/2) -
1)^(1/2))/2) - log((2^(1/2)*x*(5^(1/2) - 1)^(1/2))/2 + (2^(1/2)*(x^2 + 1)^
(1/2)*(5^(1/2) + 1)^(1/2))/2 + 1))*((5^(1/2)/2 - 1/2)^(1/2) + 2*(5^(1/2)/2
 - 1/2)^(3/2)))/((2*(5^(1/2)/2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2))*(
5^(1/2)/2 + 1/2)^(1/2)) - ((log(x + (2^(1/2)*(5^(1/2) - 1)^(1/2))/2) - log
((2^(1/2)*(x^2 + 1)^(1/2)*(5^(1/2) + 1)^(1/2))/2 - (2^(1/2)*x*(5^(1/2) - 1
)^(1/2))/2 + 1))*((5^(1/2)/2 - 1/2)^(1/2) + 2*(5^(1/2)/2 - 1/2)^(3/2)))/((
2*(5^(1/2)/2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2))*(5^(1/2)/2 + 1/2)^(
1/2)) + ((log((2^(1/2)*(x^2 + 1)^(1/2)*(1 - 5^(1/2))^(1/2))/2 - (2^(1/2)*x
*(- 5^(1/2) - 1)^(1/2))/2 + 1) - log(x + (2^(1/2)*(- 5^(1/2) - 1)^(1/2))/2
))*((- 5^(1/2)/2 - 1/2)^(1/2) + 2*(- 5^(1/2)/2 - 1/2)^(3/2)))/((2*(- 5^(1/
2)/2 - 1/2)^(1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2))*(1/2 - 5^(1/2)/2)^(1/2))
+ ((log((2^(1/2)*x*(- 5^(1/2) - 1)^(1/2))/2 + (2^(1/2)*(x^2 + 1)^(1/2)*(1
- 5^(1/2))^(1/2))/2 + 1) - log(x - (2^(1/2)*(- 5^(1/2) - 1)^(1/2))/2))*...
```

3.997. $\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx$

3.998 $\int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx$

3.998.1 Optimal result

Integrand size = 25, antiderivative size = 126

$$\int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx = -\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\arctan\left(\frac{2\sqrt{5}-\left(5+\sqrt{5}\right)x}{\sqrt{10\left(1+\sqrt{5}\right)}\sqrt{2+2x+x^2}}\right) - \sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\operatorname{arctanh}\left(\frac{2\sqrt{5}+\left(5-\sqrt{5}\right)x}{\sqrt{10\left(-1+\sqrt{5}\right)}\sqrt{2+2x+x^2}}\right)$$

output
```
-1/2*arctanh((x*(5-5^(1/2))+2*5^(1/2))/(x^2+2*x+2)^(1/2)/(-10+10*5^(1/2))^
(1/2))*(-2+2*5^(1/2))^(1/2)-1/2*arctan((2*5^(1/2)-x*(5+5^(1/2)))/(x^2+2*x+
2)^(1/2)/(10+10*5^(1/2))^(1/2))*(2+2*5^(1/2))^(1/2)
```

3.998.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.77

$$\int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx = \text{RootSum}\Big[8-8\#1 +\#1^4\&, \frac{-\log\left(-x+\sqrt{2+2x+x^2}-\#1\right)-\log\left(-x+\sqrt{2+2x+x^2}-\#1\right)\#1+\log\left(-x+\sqrt{2+2x-\right.}{-2+\#1^3}$$

input
```
Integrate[(1 + 2*x)/((1 + x^2)*Sqrt[2 + 2*x + x^2]),x]
```

output
```
RootSum[8 - 8*#1 + #1^4 & , (-Log[-x + Sqrt[2 + 2*x + x^2] - #1] - Log[-x
+ Sqrt[2 + 2*x + x^2] - #1]*#1 + Log[-x + Sqrt[2 + 2*x + x^2] - #1]*#1^2)/
(-2 + #1^3) & ]
```

3.998.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1369, 25, 1363, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+1}{(x^2+1)\sqrt{x^2+2x+2}}dx$$

$$\Big\downarrow \text{1369}$$

$$\frac{\int -\frac{-2\sqrt{5}x-\sqrt{5}+5}{(x^2+1)\sqrt{x^2+2x+2}}dx}{2\sqrt{5}}-\frac{\int -\frac{2\sqrt{5}x+\sqrt{5}+5}{(x^2+1)\sqrt{x^2+2x+2}}dx}{2\sqrt{5}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2\sqrt{5}x+\sqrt{5}+5}{(x^2+1)\sqrt{x^2+2x+2}}dx}{2\sqrt{5}}-\frac{\int \frac{-2\sqrt{5}x-\sqrt{5}+5}{(x^2+1)\sqrt{x^2+2x+2}}dx}{2\sqrt{5}}$$

$$\Big\downarrow \text{1363}$$

$$2\sqrt{5}\left(1-\sqrt{5}\right)\int \frac{1}{\frac{2\left(\left(5-\sqrt{5}\right)x+2\sqrt{5}\right)^2}{x^2+2x+2}+20\left(1-\sqrt{5}\right)}d\left(-\frac{\left(5-\sqrt{5}\right)x+2\sqrt{5}}{\sqrt{x^2+2x+2}}\right)-2\sqrt{5}\left(1+\sqrt{5}\right)\int \frac{1}{\frac{2\left(2\sqrt{5}-\left(5+\sqrt{5}\right)x\right)^2}{x^2+2x+2}+20\left(1+\sqrt{5}\right)}d\frac{2\sqrt{5}-\left(5+\sqrt{5}\right)x}{\sqrt{x^2+2x+2}}$$

$$\Big\downarrow \text{216}$$

$$2\sqrt{5}\left(1-\sqrt{5}\right)\int \frac{1}{\frac{2\left(\left(5-\sqrt{5}\right)x+2\sqrt{5}\right)^2}{x^2+2x+2}+20\left(1-\sqrt{5}\right)}d\left(-\frac{\left(5-\sqrt{5}\right)x+2\sqrt{5}}{\sqrt{x^2+2x+2}}\right)-\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\arctan\left(\frac{2\sqrt{5}-\left(5+\sqrt{5}\right)x}{\sqrt{10\left(1+\sqrt{5}\right)}\sqrt{x^2+2x+2}}\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{(1-\sqrt{5})\operatorname{arctanh}\left(\frac{(5-\sqrt{5})x+2\sqrt{5}}{\sqrt{10(\sqrt{5}-1)}\sqrt{x^2+2x+2}}\right)}{\sqrt{2(\sqrt{5}-1)}}-\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\arctan\left(\frac{2\sqrt{5}-(5+\sqrt{5})x}{\sqrt{10(1+\sqrt{5})}\sqrt{x^2+2x+2}}\right)$$

input
```
Int[(1 + 2*x)/((1 + x^2)*Sqrt[2 + 2*x + x^2]),x]
```

output
```
-(Sqrt[(1 + Sqrt[5])/2]*ArcTan[(2*Sqrt[5] - (5 + Sqrt[5])*x)/(Sqrt[10*(1 +
 Sqrt[5])]*Sqrt[2 + 2*x + x^2])]) + ((1 - Sqrt[5])*ArcTanh[(2*Sqrt[5] + (5
 - Sqrt[5])*x)/(Sqrt[10*(-1 + Sqrt[5])]*Sqrt[2 + 2*x + x^2])])/Sqrt[2*(-1
+ Sqrt[5])]
```

3.998.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1363
```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Simp[-2*a*g*h   Subst[Int[1/Simp[2*a^2*g*h*c + a
*e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ
[{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

rule 1369
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Simp
[1/(2*q)   Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c
*e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[
Simp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a +
 c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x]
&& NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

3.998.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.67 (sec) , antiderivative size = 430, normalized size of antiderivative = 3.41

method	result
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2+4\,\mathrm{RootOf}\left(16\_Z^4+8\_Z^2+5\right)^2+2\right)\ln\left(\frac{32x\,\mathrm{RootOf}\left(16\_Z^4+8\_Z^2+5\right)^4\,\mathrm{RootOf}\left(\_Z^2+4\,\mathrm{RootOf}\left(16\_Z^4+8\_ \right.\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((1+2*x)/(x^2+1)/(x^2+2*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*RootOf(_Z^2+4*RootOf(16*_Z^4+8*_Z^2+5)^2+2)*ln((32*x*RootOf(16*_Z^4+8
*_Z^2+5)^4*RootOf(_Z^2+4*RootOf(16*_Z^4+8*_Z^2+5)^2+2)+52*RootOf(_Z^2+4*Ro
otOf(16*_Z^4+8*_Z^2+5)^2+2)*RootOf(16*_Z^4+8*_Z^2+5)^2*x+80*RootOf(16*_Z^4
+8*_Z^2+5)^2*RootOf(_Z^2+4*RootOf(16*_Z^4+8*_Z^2+5)^2+2)+96*(x^2+2*x+2)^(1
/2)*RootOf(16*_Z^4+8*_Z^2+5)^2-7*RootOf(_Z^2+4*RootOf(16*_Z^4+8*_Z^2+5)^2+
2)*x-10*RootOf(_Z^2+4*RootOf(16*_Z^4+8*_Z^2+5)^2+2)+38*(x^2+2*x+2)^(1/2))/
(4*x*RootOf(16*_Z^4+8*_Z^2+5)^2+x+2))-RootOf(16*_Z^4+8*_Z^2+5)*ln(-(-32*Ro
otOf(16*_Z^4+8*_Z^2+5)^5*x+20*RootOf(16*_Z^4+8*_Z^2+5)^3*x+48*(x^2+2*x+2)^
(1/2)*RootOf(16*_Z^4+8*_Z^2+5)^2+80*RootOf(16*_Z^4+8*_Z^2+5)^3+25*RootOf(1
6*_Z^4+8*_Z^2+5)*x+5*(x^2+2*x+2)^(1/2)+50*RootOf(16*_Z^4+8*_Z^2+5))/(4*x*R
ootOf(16*_Z^4+8*_Z^2+5)^2+x-2))
```

3.998.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.83

$$\int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx = \frac{1}{2}\sqrt{2i-1}\log\left(-x+i\sqrt{2i-1}+\sqrt{x^2+2x+2}-i\right) - \frac{1}{2}\sqrt{2i-1}\log\left(-x-i\sqrt{2i-1}+\sqrt{x^2+2x+2}-i\right) - \frac{1}{2}\sqrt{-2i-1}\log\left(-x+i\sqrt{-2i-1}+\sqrt{x^2+2x+2}+i\right) + \frac{1}{2}\sqrt{-2i-1}\log\left(-x-i\sqrt{-2i-1}+\sqrt{x^2+2x+2}+i\right)$$

input
```
integrate((1+2*x)/(x^2+1)/(x^2+2*x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2*I - 1)*log(-x + I*sqrt(2*I - 1) + sqrt(x^2 + 2*x + 2) - I) - 1/
2*sqrt(2*I - 1)*log(-x - I*sqrt(2*I - 1) + sqrt(x^2 + 2*x + 2) - I) - 1/2*
sqrt(-2*I - 1)*log(-x + I*sqrt(-2*I - 1) + sqrt(x^2 + 2*x + 2) + I) + 1/2*
sqrt(-2*I - 1)*log(-x - I*sqrt(-2*I - 1) + sqrt(x^2 + 2*x + 2) + I)
```

3.998.6 Sympy [F]

$$\int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx = \int \frac{2x+1}{(x^2+1)\sqrt{x^2+2x+2}}\,dx$$

input
```
integrate((1+2*x)/(x**2+1)/(x**2+2*x+2)**(1/2),x)
```

output
```
Integral((2*x + 1)/((x**2 + 1)*sqrt(x**2 + 2*x + 2)), x)
```

3.998.7 Maxima [F]

$$\int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx = \int \frac{2\,x+1}{\sqrt{x^2+2\,x+2}(x^2+1)}\,dx$$

input
```
integrate((1+2*x)/(x^2+1)/(x^2+2*x+2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((2*x + 1)/(sqrt(x^2 + 2*x + 2)*(x^2 + 1)), x)
```

3.998.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(93) = 186.

Time = 0.38 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.52

$$\int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx$$
$$= \frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(256\left(\sqrt{5}\left(x-\sqrt{x^2+2\,x+2}\right)-2\,x+\sqrt{5}\sqrt{\sqrt{5}-2}+\sqrt{5}+2\sqrt{x^2+2\,x+2}-2\sqrt{\sqrt{5}\right.\right.$$
$$\left.\left.+256\left(\sqrt{5}\left(x-\sqrt{x^2+2\,x+2}\right)-2\,x-\sqrt{5}+2\sqrt{x^2+2\,x+2}+\sqrt{\sqrt{5}-2}+2\right)^2\right)$$
$$-\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(256\left(\sqrt{5}\left(x-\sqrt{x^2+2\,x+2}\right)-2\,x-\sqrt{5}\sqrt{\sqrt{5}-2}+\sqrt{5}+2\sqrt{x^2+2\,x+2}+2\sqrt{\right.\right.$$
$$\left.\left.+256\left(\sqrt{5}\left(x-\sqrt{x^2+2\,x+2}\right)-2\,x-\sqrt{5}+2\sqrt{x^2+2\,x+2}-\sqrt{\sqrt{5}-2}+2\right)^2\right)$$
$$+\frac{\left(\pi+4\arctan\left(\frac{1}{2}\left(x-\sqrt{x^2+2\,x+2}\right)\left(2\sqrt{5}\sqrt{\sqrt{5}-2}+\sqrt{5}+4\sqrt{\sqrt{5}-2}+3\right)+\frac{3}{2}\sqrt{5}\sqrt{\sqrt{5}-2}+\frac{1}{2}\right.\right.}{4\left(\sqrt{5}-1\right)}$$
$$-\frac{\left(\pi+4\arctan\left(-\frac{1}{2}\left(x-\sqrt{x^2+2\,x+2}\right)\left(2\sqrt{5}\sqrt{\sqrt{5}-2}-\sqrt{5}+4\sqrt{\sqrt{5}-2}-3\right)-\frac{3}{2}\sqrt{5}\sqrt{\sqrt{5}-2}+\right.\right.}{4\left(\sqrt{5}-1\right)}$$

input
```
integrate((1+2*x)/(x^2+1)/(x^2+2*x+2)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(2*sqrt(5) - 2)*log(256*(sqrt(5)*(x - sqrt(x^2 + 2*x + 2)) - 2*x +
 sqrt(5)*sqrt(sqrt(5) - 2) + sqrt(5) + 2*sqrt(x^2 + 2*x + 2) - 2*sqrt(sqrt
(5) - 2) - 2)^2 + 256*(sqrt(5)*(x - sqrt(x^2 + 2*x + 2)) - 2*x - sqrt(5) +
 2*sqrt(x^2 + 2*x + 2) + sqrt(sqrt(5) - 2) + 2)^2) - 1/4*sqrt(2*sqrt(5) -
2)*log(256*(sqrt(5)*(x - sqrt(x^2 + 2*x + 2)) - 2*x - sqrt(5)*sqrt(sqrt(5)
 - 2) + sqrt(5) + 2*sqrt(x^2 + 2*x + 2) + 2*sqrt(sqrt(5) - 2) - 2)^2 + 256
*(sqrt(5)*(x - sqrt(x^2 + 2*x + 2)) - 2*x - sqrt(5) + 2*sqrt(x^2 + 2*x + 2
) - sqrt(sqrt(5) - 2) + 2)^2) + 1/4*(pi + 4*arctan(1/2*(x - sqrt(x^2 + 2*x
 + 2))*(2*sqrt(5)*sqrt(sqrt(5) - 2) + sqrt(5) + 4*sqrt(sqrt(5) - 2) + 3) +
 3/2*sqrt(5)*sqrt(sqrt(5) - 2) + 1/2*sqrt(5) + 7/2*sqrt(sqrt(5) - 2) + 3/2
))*sqrt(2*sqrt(5) - 2)/(sqrt(5) - 1) - 1/4*(pi + 4*arctan(-1/2*(x - sqrt(x
^2 + 2*x + 2))*(2*sqrt(5)*sqrt(sqrt(5) - 2) - sqrt(5) + 4*sqrt(sqrt(5) - 2
) - 3) - 3/2*sqrt(5)*sqrt(sqrt(5) - 2) + 1/2*sqrt(5) - 7/2*sqrt(sqrt(5) -
2) + 3/2))*sqrt(2*sqrt(5) - 2)/(sqrt(5) - 1)
```

3.998.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx = \int \frac{2\,x+1}{(x^2+1)\,\sqrt{x^2+2\,x+2}}\,dx$$

input
```
int((2*x + 1)/((x^2 + 1)*(2*x + x^2 + 2)^(1/2)),x)
```

output
```
int((2*x + 1)/((x^2 + 1)*(2*x + x^2 + 2)^(1/2)), x)
```

3.999 $\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx$

3.999.1 Optimal result

Integrand size = 27, antiderivative size = 22

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx=\arctan\left(\frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)$$

output
```
arctan(x/(-x^2+(x^4+1)^(1/2))^(1/2))
```

3.999.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 4.36

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx=i\text{arctanh}\left(\sqrt{2}+\sqrt{2}x^4-ix^3\sqrt{-x^2+\sqrt{1+x^4}}+\frac{\sqrt{1+x^4}\left(-2x^2+i\sqrt{2}x\sqrt{-x^2+\sqrt{1+x^4}}\right)}{\sqrt{2}}\right)$$

input
```
Integrate[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
```

output
```
I*ArcTanh[Sqrt[2] + Sqrt[2]*x^4 - I*x^3*Sqrt[-x^2 + Sqrt[1 + x^4]] + (Sqrt
[1 + x^4]*(-2*x^2 + I*Sqrt[2]*x*Sqrt[-x^2 + Sqrt[1 + x^4]]))/Sqrt[2]]
```

3.999.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2553, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^4+1)\sqrt{\sqrt{x^4+1}-x^2}}dx$$

$$\Big\downarrow \text{2553}$$

$$\int \frac{1}{\frac{x^2}{\sqrt{x^4+1}-x^2}+1}d\frac{x}{\sqrt{\sqrt{x^4+1}-x^2}}$$

$$\Big\downarrow \text{216}$$

$$\arctan\left(\frac{x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)$$

input
```
Int[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
```

output
```
ArcTan[x/Sqrt[-x^2 + Sqrt[1 + x^4]]]
```

3.999.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2553
```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x
_)^(n_.))^(p_.)]), x_Symbol] :> Simp[1/a   Subst[Int[1/(1 - c*x^2), x], x,
x/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && Eq
Q[p, 2/n]
```

3.999.4 Maple [F]

$$\int \frac{1}{\left(x^{4}+1\right) \sqrt{-x^{2}+\sqrt{x^{4}+1}}}dx$$

input
```
int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)
```

output
```
int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)
```

3.999.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(18) = 36$.

Time = 0.63 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.82

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = -\frac{1}{4}\,\arctan\left(\frac{4\left(10\,x^{7}-6\,x^{3}+(7\,x^{5}-x)\sqrt{x^{4}+1}\right)\sqrt{-x^{2}+\sqrt{x^{4}+1}}}{17\,x^{8}-46\,x^{4}+1}\right)$$

input
```
integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-1/4*arctan(4*(10*x^7 - 6*x^3 + (7*x^5 - x)*sqrt(x^4 + 1))*sqrt(-x^2 + sqr
t(x^4 + 1))/(17*x^8 - 46*x^4 + 1))
```

3.999.6 Sympy [F]

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{-x^{2}+\sqrt{x^{4}+1}}\,(x^{4}+1)}\,dx$$

input
```
integrate(1/(x**4+1)/(-x**2+(x**4+1)**(1/2))**(1/2),x)
```

output
```
Integral(1/(sqrt(-x**2 + sqrt(x**4 + 1))*(x**4 + 1)), x)
```

3.999.7 Maxima [F]

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{(x^4+1)\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)
```

3.999.8 Giac [F]

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{(x^4+1)\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

input
```
integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)
```

3.999.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx = \int \frac{1}{\sqrt{\sqrt{x^4+1}-x^2}\,(x^4+1)}\,dx$$

input
```
int(1/(((x^4 + 1)^(1/2) - x^2)^(1/2)*(x^4 + 1)),x)
```

output
```
int(1/(((x^4 + 1)^(1/2) - x^2)^(1/2)*(x^4 + 1)), x)
```

3.1000 $\int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx$

3.1000.1 Optimal result

Integrand size = 33, antiderivative size = 40

$$\int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{cx^2+d\sqrt{a+bx^4}}}\right)}{a\sqrt{c}}$$

output
```
arctanh(x*c^(1/2)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2))/a/c^(1/2)
```

3.1000.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{cx^2+d\sqrt{a+bx^4}}}{\sqrt{c}x}\right)}{a\sqrt{c}}$$

input
```
Integrate[1/((a + b*x^4)*Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]),x]
```

output
```
ArcTanh[Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]/(Sqrt[c]*x)]/(a*Sqrt[c])
```

3.1000.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)\sqrt{d\sqrt{a+bx^4}+cx^2}}dx$$

$$\Big\downarrow \text{2553}$$

$$\frac{\int \frac{1}{1-\frac{cx^2}{cx^2+d\sqrt{bx^4+a}}}d\frac{x}{\sqrt{cx^2+d\sqrt{bx^4+a}}}}{a}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{d\sqrt{a+bx^4}+cx^2}}\right)}{a\sqrt{c}}$$

input
```
Int[1/((a + b*x^4)*Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]),x]
```

output
```
ArcTanh[(Sqrt[c]*x)/Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]]/(a*Sqrt[c])
```

3.1000.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2553
```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x
_)^(n_.))^(p_.)]), x_Symbol] :> Simp[1/a   Subst[Int[1/(1 - c*x^2), x], x,
x/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && Eq
Q[p, 2/n]
```

3.1000.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{4}+a\right)\sqrt{c\,x^{2}+d\sqrt{b\,x^{4}+a}}}dx$$

input
```
int(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x)
```

output
```
int(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x)
```

3.1000.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

3.1000.6 Sympy [F]

$$\int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx = \int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx$$

input
```
integrate(1/(b*x**4+a)/(c*x**2+d*(b*x**4+a)**(1/2))**(1/2),x)
```

output
```
Integral(1/((a + b*x**4)*sqrt(c*x**2 + d*sqrt(a + b*x**4))), x)
```

3.1000.7 Maxima [F]

$$\int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx = \int \frac{1}{(bx^4+a)\sqrt{cx^2+\sqrt{bx^4+a}d}}\,dx$$

input
```
integrate(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="maxima
")
```

output
```
integrate(1/((b*x^4 + a)*sqrt(c*x^2 + sqrt(b*x^4 + a)*d)), x)
```

3.1000.8 Giac [F]

$$\int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx = \int \frac{1}{(bx^4+a)\sqrt{cx^2+\sqrt{bx^4+a}d}}\,dx$$

input
```
integrate(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^4 + a)*sqrt(c*x^2 + sqrt(b*x^4 + a)*d)), x)
```

3.1000.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx = \int \frac{1}{\left(b\,x^4+a\right)\,\sqrt{d\,\sqrt{b\,x^4+a}+c\,x^2}}\,dx$$

input
```
int(1/((a + b*x^4)*(d*(a + b*x^4)^(1/2) + c*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^4)*(d*(a + b*x^4)^(1/2) + c*x^2)^(1/2)), x)
```

3.1001 $\int \frac{1}{(a+bx^4)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx$

3.1001.1 Optimal result

Integrand size = 34, antiderivative size = 41

$$\int \frac{1}{(a+bx^4)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx=\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{-cx^2+d\sqrt{a+bx^4}}}\right)}{a\sqrt{c}}$$

output
```
arctan(x*c^(1/2)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2))/a/c^(1/2)
```

3.1001.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{1}{(a+bx^4)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx=-\frac{\arctan\left(\frac{\sqrt{-cx^2+d\sqrt{a+bx^4}}}{\sqrt{c}x}\right)}{a\sqrt{c}}$$

input
```
Integrate[1/((a + b*x^4)*Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]),x]
```

output
```
-(ArcTan[Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]/(Sqrt[c]*x)]/(a*Sqrt[c]))
```

3.1001.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2553, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^4)\sqrt{d\sqrt{a+bx^4}-cx^2}}dx$$

$$\Big\downarrow \text{2553}$$

$$\frac{\int \frac{1}{\frac{cx^2}{d\sqrt{bx^4+a}-cx^2}+1}d\frac{x}{\sqrt{d\sqrt{bx^4+a}-cx^2}}}{a}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{\sqrt{c}x}{\sqrt{d\sqrt{a+bx^4}-cx^2}}\right)}{a\sqrt{c}}$$

input
```
Int[1/((a + b*x^4)*Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]),x]
```

output
```
ArcTan[(Sqrt[c]*x)/Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]]/(a*Sqrt[c])
```

3.1001.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2553
```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x
_)^(n_.))^(p_.)]), x_Symbol] :> Simp[1/a   Subst[Int[1/(1 - c*x^2), x], x,
x/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && Eq
Q[p, 2/n]
```

3.1001.4 Maple [F]

$$\int \frac{1}{\left(b\,x^{4}+a\right)\sqrt{-c\,x^{2}+d\sqrt{b\,x^{4}+a}}}dx$$

input
```
int(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x)
```

output
```
int(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x)
```

3.1001.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="frica
s")
```

output
```
Timed out
```

3.1001.6 Sympy [F]

$$\int \frac{1}{(a+bx^4)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx = \int \frac{1}{(a+bx^4)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx$$

input
```
integrate(1/(b*x**4+a)/(-c*x**2+d*(b*x**4+a)**(1/2))**(1/2),x)
```

output
```
Integral(1/((a + b*x**4)*sqrt(-c*x**2 + d*sqrt(a + b*x**4))), x)
```

3.1001.7 Maxima [F]

$$\int \frac{1}{(a+bx^4)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx = \int \frac{1}{(bx^4+a)\sqrt{-cx^2+\sqrt{bx^4+a}d}}\,dx$$

input
```
integrate(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(1/((b*x^4 + a)*sqrt(-c*x^2 + sqrt(b*x^4 + a)*d)), x)
```

3.1001.8 Giac [F]

$$\int \frac{1}{(a+bx^4)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx = \int \frac{1}{(bx^4+a)\sqrt{-cx^2+\sqrt{bx^4+a}d}}\,dx$$

input
```
integrate(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="giac"
)
```

output
```
integrate(1/((b*x^4 + a)*sqrt(-c*x^2 + sqrt(b*x^4 + a)*d)), x)
```

3.1001.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^4)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx = \int \frac{1}{(b\,x^4+a)\,\sqrt{d\,\sqrt{b\,x^4+a}-c\,x^2}}\,dx$$

input
```
int(1/((a + b*x^4)*(d*(a + b*x^4)^(1/2) - c*x^2)^(1/2)),x)
```

output
```
int(1/((a + b*x^4)*(d*(a + b*x^4)^(1/2) - c*x^2)^(1/2)), x)
```

3.1002 $\int \frac{x}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx$

3.1002.1 Optimal result

Integrand size = 51, antiderivative size = 184

$$\int \frac{x}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}\right)}{2\sqrt{b}d^2} - \frac{c\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)\sqrt{\frac{a+bd^4\left(\frac{c}{d}+x\right)^4}{\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}d^2\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}$$

output
```
1/2*arctanh(d^2*(c/d+x)^2*b^(1/2)/(a+b*d^4*(c/d+x)^4)^(1/2))/d^2/b^(1/2)-1
/2*c*(cos(2*arctan(b^(1/4)*(d*x+c)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)
*(d*x+c)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*(d*x+c)/a^(1/4))),1/2*2^
(1/2))*(a^(1/2)+d^2*(c/d+x)^2*b^(1/2))*((a+b*d^4*(c/d+x)^4)/(a^(1/2)+d^2*(
c/d+x)^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/d^2/(a+b*d^4*(c/d+x)^4)^(1/2)
```

3.1002.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.36 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.11

$$\int \frac{x}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx$$

$$= \frac{\sqrt[4]{-1}\left(\sqrt[4]{-1}\sqrt[4]{a}-\sqrt[4]{b}(c+dx)\right)^2\sqrt{\frac{(1-i)\left((-1)^{3/4}\sqrt[4]{a}-\sqrt[4]{b}(c+dx)\right)}{\sqrt[4]{-1}\sqrt[4]{a}-\sqrt[4]{b}(c+dx)}}\sqrt{-\frac{i\left(\sqrt[4]{-1}\sqrt[4]{a}+\sqrt[4]{b}(c+dx)\right)}{\sqrt[4]{-1}\sqrt[4]{a}-\sqrt[4]{b}(c+dx)}}\sqrt{-\frac{(1+i)\left((-1)^{3/4}\sqrt[4]{\ldots}\right.}{\sqrt[4]{-1}\sqrt[4]{a}\ldots}}}{\ldots}$$

input
```
Integrate[x/Sqrt[a + b*c^4 + 4*b*c^3*d*x + 6*b*c^2*d^2*x^2 + 4*b*c*d^3*x^3
 + b*d^4*x^4],x]
```

output
```
((-1)^(1/4)*((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))^2*Sqrt[((1 - I)*((-1)
^(3/4)*a^(1/4) - b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*
x))]*Sqrt[((-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1
/4) - b^(1/4)*(c + d*x))]*Sqrt[((-1 - I)*((-1)^(3/4)*a^(1/4) + b^(1/4)*(c
+ d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]*(((-1)^(1/4)*a^(1/4) -
b^(1/4)*c)*EllipticF[ArcSin[Sqrt[((-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*(c +
d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]], -1] - 2*(-1)^(1/4)*a^(1
/4)*EllipticPi[-I, ArcSin[Sqrt[((-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*(c + d*
x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]], -1]))/(a^(1/4)*Sqrt[b]*d^
2*Sqrt[a + b*(c + d*x)^4])
```

3.1002.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2459, 2424, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx$$

$$\Big\downarrow \; 2459$$

$$\int \frac{x}{\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}d\left(\frac{c}{d}+x\right)$$

$$\Big\downarrow \text{2424}$$

$$\int \left(\frac{\frac{c}{d}+x}{\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}-\frac{c}{d\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}\right)d\left(\frac{c}{d}+x\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}\right)}{2\sqrt{b}d^2}-\frac{c\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)\sqrt{\frac{a+bd^4\left(\frac{c}{d}+x\right)^4}{\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}d\left(\frac{c}{d}+x\right)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}d^2\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}$$

input
```
Int[x/Sqrt[a + b*c^4 + 4*b*c^3*d*x + 6*b*c^2*d^2*x^2 + 4*b*c*d^3*x^3 + b*d
^4*x^4],x]
```

output
```
ArcTanh[(Sqrt[b]*d^2*(c/d + x)^2)/Sqrt[a + b*d^4*(c/d + x)^4]]/(2*Sqrt[b]*
d^2) - (c*(Sqrt[a] + Sqrt[b]*d^2*(c/d + x)^2)*Sqrt[(a + b*d^4*(c/d + x)^4)
/(Sqrt[a] + Sqrt[b]*d^2*(c/d + x)^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*d*(c/d
 + x))/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*d^2*Sqrt[a + b*d^4*(c/d + x)^4])
```

3.1002.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2424
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]
```

rule 2459
```
Int[(Pn_)^(p_.)*(Qx_), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1
]/(Expon[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x
 -> x - S, x]^p*ExpandToSum[Qx /. x -> x - S, x], x], x, x + S] /; Binomial
Q[Pn /. x -> x - S, x] || (IntegerQ[Expon[Pn, x]/2] && TrinomialQ[Pn /. x -
> x - S, x])] /; FreeQ[p, x] && PolyQ[Pn, x] && GtQ[Expon[Pn, x], 2] && NeQ
[Coeff[Pn, x, Expon[Pn, x] - 1], 0] && PolyQ[Qx, x] &&  !(MonomialQ[Qx, x]
&& IGtQ[p, 0])
```

3.1002.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.99 (sec) , antiderivative size = 1528, normalized size of antiderivative = 8.30

method	result	size
default	Expression too large to display	1528
elliptic	Expression too large to display	1528

input
```
int(x/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(1/2),
x,method=_RETURNVERBOSE)
```

output
```
2*(-(-I/b*(-a*b^3)^(1/4)-c)/d+(1/b*(-a*b^3)^(1/4)-c)/d)*(((-I/b*(-a*b^3)^(
1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-
a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^
(1/2)*(x-(I/b*(-a*b^3)^(1/4)-c)/d)^2*(((I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b
^3)^(1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)/d)/((-1/b*(-a*b^3)^(1/4)-c)/d-(
1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2)*(((I/b*(-a*b^
3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-a*b^3)^(1/4)-c)/d)/((-I
/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)
/d))^(1/2)/((-I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)/((I/b*(-a*
b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(b*d^4*(x-(1/b*(-a*b^3)^(1/4)-c)
/d)*(x-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-
a*b^3)^(1/4)-c)/d))^(1/2)*((I/b*(-a*b^3)^(1/4)-c)/d*EllipticF((((-I/b*(-a*
b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-
I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c
)/d))^(1/2),(((I/b*(-a*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3)^(1/4)-c)/d)*(-(-I/b*
(-a*b^3)^(1/4)-c)/d+(1/b*(-a*b^3)^(1/4)-c)/d)/((1/b*(-a*b^3)^(1/4)-c)/d-(-
1/b*(-a*b^3)^(1/4)-c)/d)/((I/b*(-a*b^3)^(1/4)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)
/d))^(1/2))+(-(I/b*(-a*b^3)^(1/4)-c)/d+(1/b*(-a*b^3)^(1/4)-c)/d)*EllipticP
i((((-I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(
1/4)-c)/d)/((-I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b...
```

3.1002.5 Fricas [F]

$$\int \frac{x}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = \int \frac{x}{\sqrt{bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4+a}}\,dx$$

input
```
integrate(x/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^
(1/2),x, algorithm="fricas")
```

output
```
integral(x/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x
 + b*c^4 + a), x)
```

3.1002.6 Sympy [F]

$$\int \frac{x}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = \int \frac{x}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx$$

input
```
integrate(x/(b*d**4*x**4+4*b*c*d**3*x**3+6*b*c**2*d**2*x**2+4*b*c**3*d*x+b
*c**4+a)**(1/2),x)
```

output
```
Integral(x/sqrt(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**
3*x**3 + b*d**4*x**4), x)
```

3.1002.7 Maxima [F]

$$\int \frac{x}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = \int \frac{x}{\sqrt{bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4+a}}\,dx$$

input
```
integrate(x/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^
(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x
 + b*c^4 + a), x)
```

3.1002.8 Giac [F]

$$\int \frac{x}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = \int \frac{x}{\sqrt{bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4+a}}\,dx$$

input
```
integrate(x/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^
(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x
 + b*c^4 + a), x)
```

3.1002.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = \int \frac{x}{\sqrt{b\,c^4+4\,b\,c^3\,d\,x+6\,b\,c^2\,d^2\,x^2+4\,b\,c\,d^3\,x^3+b\,d^4\,x^4+a}}\,dx$$

input
```
int(x/(a + b*c^4 + b*d^4*x^4 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + 4*b*c*d^3*x
^3)^(1/2),x)
```

output
```
int(x/(a + b*c^4 + b*d^4*x^4 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + 4*b*c*d^3*x
^3)^(1/2), x)
```

3.1003 $\int \frac{1}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx$

3.1003.1 Optimal result

Integrand size = 49, antiderivative size = 131

$$\int \frac{1}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = \frac{\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)\sqrt{\frac{a+bd^4\left(\frac{c}{d}+x\right)^4}{\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}(c+dx)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}d\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}$$

output
```
1/2*(cos(2*arctan(b^(1/4)*(d*x+c)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*
(d*x+c)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*(d*x+c)/a^(1/4))),1/2*2^(
1/2))*(a^(1/2)+d^2*(c/d+x)^2*b^(1/2))*((a+b*d^4*(c/d+x)^4)/(a^(1/2)+d^2*(c
/d+x)^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/d/(a+b*d^4*(c/d+x)^4)^(1/2)
```

3.1003.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = -\frac{i\sqrt{\frac{a+b(c+dx)^4}{a}}\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}(c+dx)\right),-1\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}d\sqrt{a+b(c+dx)^4}}$$

input
```
Integrate[1/Sqrt[a + b*c^4 + 4*b*c^3*d*x + 6*b*c^2*d^2*x^2 + 4*b*c*d^3*x^3
 + b*d^4*x^4],x]
```

output
```
((-I)*Sqrt[(a + b*(c + d*x)^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqr
t[a]]*(c + d*x)], -1])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*d*Sqrt[a + b*(c + d*x)^4
])
```

3.1003.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.041, Rules used = {2458, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}dx$$

$$\Big\downarrow \text{ 2458}$$

$$\int \frac{1}{\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}d\left(\frac{c}{d}+x\right)$$

$$\Big\downarrow \text{ 761}$$

$$\frac{\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)\sqrt{\frac{a+bd^4\left(\frac{c}{d}+x\right)^4}{\left(\sqrt{a}+\sqrt{b}d^2\left(\frac{c}{d}+x\right)^2\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}d\left(\frac{c}{d}+x\right)}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}d\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}$$

input
```
Int[1/Sqrt[a + b*c^4 + 4*b*c^3*d*x + 6*b*c^2*d^2*x^2 + 4*b*c*d^3*x^3 + b*d
^4*x^4],x]
```

output
```
((Sqrt[a] + Sqrt[b]*d^2*(c/d + x)^2)*Sqrt[(a + b*d^4*(c/d + x)^4)/(Sqrt[a]
 + Sqrt[b]*d^2*(c/d + x)^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*d*(c/d + x))/a^
(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*d*Sqrt[a + b*d^4*(c/d + x)^4])
```

3.1003.3.1 Defintions of rubi rules used

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 2458
```
Int[(Pn_)^(p_.), x_Symbol] :> With[{S = Coeff[Pn, x, Expon[Pn, x] - 1]/(Exp
on[Pn, x]*Coeff[Pn, x, Expon[Pn, x]])}, Subst[Int[ExpandToSum[Pn /. x -> x
- S, x]^p, x], x, x + S] /; BinomialQ[Pn /. x -> x - S, x] || (IntegerQ[Exp
on[Pn, x]/2] && TrinomialQ[Pn /. x -> x - S, x])] /; FreeQ[p, x] && PolyQ[P
n, x] && GtQ[Expon[Pn, x], 2] && NeQ[Coeff[Pn, x, Expon[Pn, x] - 1], 0]
```

3.1003.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.36 (sec) , antiderivative size = 1036, normalized size of antiderivative = 7.91

method	result	size
default	Expression too large to display	1036
elliptic	Expression too large to display	1036

input
```
int(1/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(1/2),
x,method=_RETURNVERBOSE)
```

output
```
2*(-(-I/b*(-a*b^3)^(1/4)-c)/d+(1/b*(-a*b^3)^(1/4)-c)/d)*(((-I/b*(-a*b^3)^(
1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-
a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^
(1/2)*(x-(I/b*(-a*b^3)^(1/4)-c)/d)^2*(((I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b
^3)^(1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)/d)/((-1/b*(-a*b^3)^(1/4)-c)/d-(
1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2)*(((I/b*(-a*b^
3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-a*b^3)^(1/4)-c)/d)/((-I
/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)
/d))^(1/2)/((-I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)/((I/b*(-a*
b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(b*d^4*(x-(1/b*(-a*b^3)^(1/4)-c)
/d)*(x-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-
a*b^3)^(1/4)-c)/d))^(1/2)*EllipticF((((-I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-a*b
^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b^3)^(1/4)-c)/d-(1
/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2),(((I/b*(-a*b^3
)^(1/4)-c)/d-(-1/b*(-a*b^3)^(1/4)-c)/d)*(-(-I/b*(-a*b^3)^(1/4)-c)/d+(1/b*(
-a*b^3)^(1/4)-c)/d)/((1/b*(-a*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3)^(1/4)-c)/d)/(
(I/b*(-a*b^3)^(1/4)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)/d))^(1/2))
```

3.1003.5 Fricas [F]

$$\int \frac{1}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = \int \frac{1}{\sqrt{bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4+a}}\,dx$$

input
```
integrate(1/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^
(1/2),x, algorithm="fricas")
```

output
```
integral(1/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x
+ b*c^4 + a), x)
```

3.1003.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = \int \frac{1}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx$$

input
```
integrate(1/(b*d**4*x**4+4*b*c*d**3*x**3+6*b*c**2*d**2*x**2+4*b*c**3*d*x+b
*c**4+a)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**
3*x**3 + b*d**4*x**4), x)
```

3.1003.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = \int \frac{1}{\sqrt{bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4+a}}\,dx$$

input
```
integrate(1/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^
(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x
 + b*c^4 + a), x)
```

3.1003.8 Giac [F]

$$\int \frac{1}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = \int \frac{1}{\sqrt{bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4+a}}\,dx$$

input
```
integrate(1/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^
(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x
 + b*c^4 + a), x)
```

3.1003.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx$$
$$=\int \frac{1}{\sqrt{b\,c^4+4\,b\,c^3\,d\,x+6\,b\,c^2\,d^2\,x^2+4\,b\,c\,d^3\,x^3+b\,d^4\,x^4+a}}\,dx$$

input
```
int(1/(a + b*c^4 + b*d^4*x^4 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + 4*b*c*d^3*x
^3)^(1/2),x)
```

output
```
int(1/(a + b*c^4 + b*d^4*x^4 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + 4*b*c*d^3*x
^3)^(1/2), x)
```

3.1004 $\int \frac{a-cx^4}{\sqrt{a+bx^2+cx^4}(ad+aex^2+cdx^4)} dx$

3.1004.1 Optimal result

Integrand size = 43, antiderivative size = 54

$$\int \frac{a-cx^4}{\sqrt{a+bx^2+cx^4}(ad+aex^2+cdx^4)} dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{bd-ae}x}{\sqrt{d}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{bd-ae}}$$

output
```
arctanh(x*(-a*e+b*d)^(1/2)/d^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d^(1/2)/(-a*e+b*
d)^(1/2)
```

3.1004.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 12.41 (sec) , antiderivative size = 419, normalized size of antiderivative = 7.76

$$\int \frac{a-cx^4}{\sqrt{a+bx^2+cx^4}(ad+aex^2+cdx^4)} dx = \frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right),\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\operatorname{EllipticPi}\left(\frac{(b}{ae-}\right.\right.}{\sqrt{2}\sqrt{}}$$

input
```
Integrate[(a - c*x^4)/(Sqrt[a + b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),
x]
```

output
```
(I*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1
+ (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(
b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]
)] - EllipticPi[((b + Sqrt[b^2 - 4*a*c])*d)/(a*e - Sqrt[a]*Sqrt[-4*c*d^2 +
 a*e^2]), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[
b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticPi[((b + Sqrt[b^2 - 4*a*c
])*d)/(a*e + Sqrt[a]*Sqrt[-4*c*d^2 + a*e^2]), I*ArcSinh[Sqrt[2]*Sqrt[c/(b
+ Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]
))/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*Sqrt[a + b*x^2 + c*x^4])
```

3.1004.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$, Rules used = {2537, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-cx^4}{\sqrt{a+bx^2+cx^4}\left(ad+aex^2+cdx^4\right)}dx$$

$$\Big\downarrow \text{2537}$$

$$a\int \frac{1}{ad-\frac{a(bd-ae)x^2}{cx^4+bx^2+a}}d\frac{x}{\sqrt{cx^4+bx^2+a}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\text{arctanh}\left(\frac{x\sqrt{bd-ae}}{\sqrt{d}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{bd-ae}}$$

input
```
Int[(a - c*x^4)/(Sqrt[a + b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x]
```

output
```
ArcTanh[(Sqrt[b*d - a*e]*x)/(Sqrt[d]*Sqrt[a + b*x^2 + c*x^4])]/(Sqrt[d]*Sq
rt[b*d - a*e])
```

3.1004.3.1 Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2537
```
Int[((u_)*((A_) + (B_.)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v,
x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Co
eff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Simp[A   Subst[Int[1/(d - (b*d - a*e
)*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0]] /;
 FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]
```

3.1004.4 Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{\arctan\left(\frac{d\sqrt{c\,x^{4}+b\,x^{2}+a}}{x\sqrt{(ae-bd)d}}\right)}{\sqrt{(ae-bd)d}}$	47
elliptic	$-\frac{\arctan\left(\frac{d\sqrt{c\,x^{4}+b\,x^{2}+a}}{x\sqrt{(ae-bd)d}}\right)}{\sqrt{(ae-bd)d}}$	47
pseudoelliptic	$-\frac{\arctan\left(\frac{d\sqrt{c\,x^{4}+b\,x^{2}+a}}{x\sqrt{(ae-bd)d}}\right)}{\sqrt{(ae-bd)d}}$	47

input
```
int((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
-1/((a*e-b*d)*d)^(1/2)*arctan(d*(c*x^4+b*x^2+a)^(1/2)/x/((a*e-b*d)*d)^(1/2
))
```

3.1004.5 Fricas [A] (verification not implemented)

Time = 7.88 (sec) , antiderivative size = 305, normalized size of antiderivative = 5.65

$$\int \frac{a-cx^4}{\sqrt{a+bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx$$
$$= \left[\frac{\log\left(-\frac{c^2d^2x^8+2\,(4\,bcd^2-3\,acde)x^6-(8\,abde-a^2e^2-2\,(4\,b^2+ac)d^2)x^4+a^2d^2+2\,(4\,abd^2-3\,a^2de)x^2+4\,(cdx^5+(2\,bd-ae)x^3+adx)\sqrt{cx^4+bx^2}}{c^2d^2x^8+2\,acdex^6+2\,a^2dex^2+(2\,acd^2+a^2e^2)x^4+a^2d^2}\right.}{4\sqrt{bd^2-ade}}\right.$$
$$\left.-\frac{\sqrt{-bd^2+ade}\arctan\left(\frac{2\sqrt{cx^4+bx^2+a}\sqrt{-bd^2+ade}x}{cdx^4+(2\,bd-ae)x^2+ad}\right)}{2\,(bd^2-ade)}\right]$$

input
```
integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2),x, algori
thm="fricas")
```

output
```
[1/4*log(-(c^2*d^2*x^8 + 2*(4*b*c*d^2 - 3*a*c*d*e)*x^6 - (8*a*b*d*e - a^2*
e^2 - 2*(4*b^2 + a*c)*d^2)*x^4 + a^2*d^2 + 2*(4*a*b*d^2 - 3*a^2*d*e)*x^2 +
 4*(c*d*x^5 + (2*b*d - a*e)*x^3 + a*d*x)*sqrt(c*x^4 + b*x^2 + a)*sqrt(b*d^
2 - a*d*e))/(c^2*d^2*x^8 + 2*a*c*d*e*x^6 + 2*a^2*d*e*x^2 + (2*a*c*d^2 + a^
2*e^2)*x^4 + a^2*d^2))/sqrt(b*d^2 - a*d*e), -1/2*sqrt(-b*d^2 + a*d*e)*arct
an(2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-b*d^2 + a*d*e)*x/(c*d*x^4 + (2*b*d - a*
e)*x^2 + a*d))/(b*d^2 - a*d*e)]
```

3.1004.6 Sympy [F]

$$\int \frac{a-cx^4}{\sqrt{a+bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx$$
$$= -\int \left(-\frac{a}{ad\sqrt{a+bx^2+cx^4}+aex^2\sqrt{a+bx^2+cx^4}+cdx^4\sqrt{a+bx^2+cx^4}}\right)dx$$
$$-\int \frac{cx^4}{ad\sqrt{a+bx^2+cx^4}+aex^2\sqrt{a+bx^2+cx^4}+cdx^4\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((-c*x**4+a)/(c*d*x**4+a*e*x**2+a*d)/(c*x**4+b*x**2+a)**(1/2),x)
```

output
```
-Integral(-a/(a*d*sqrt(a + b*x**2 + c*x**4) + a*e*x**2*sqrt(a + b*x**2 + c
*x**4) + c*d*x**4*sqrt(a + b*x**2 + c*x**4)), x) - Integral(c*x**4/(a*d*sq
rt(a + b*x**2 + c*x**4) + a*e*x**2*sqrt(a + b*x**2 + c*x**4) + c*d*x**4*sq
rt(a + b*x**2 + c*x**4)), x)
```

3.1004.7 Maxima [F]

$$\int \frac{a-cx^4}{\sqrt{a+bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx = \int -\frac{cx^4-a}{(cdx^4+aex^2+ad)\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2),x, algori
thm="maxima")
```

output
```
-integrate((c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 + b*x^2 + a))
, x)
```

3.1004.8 Giac [F]

$$\int \frac{a-cx^4}{\sqrt{a+bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx = \int -\frac{cx^4-a}{(cdx^4+aex^2+ad)\sqrt{cx^4+bx^2+a}}\,dx$$

input
```
integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2),x, algori
thm="giac")
```

output
```
integrate(-(c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 + b*x^2 + a))
, x)
```

3.1004.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a-cx^4}{\sqrt{a+bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx = \int \frac{a-c\,x^4}{(c\,d\,x^4+a\,e\,x^2+a\,d)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((a - c*x^4)/((a*d + a*e*x^2 + c*d*x^4)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((a - c*x^4)/((a*d + a*e*x^2 + c*d*x^4)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.1005 $\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4}(ad+aex^2+cdx^4)}\,dx$

3.1005.1 Optimal result

Integrand size = 44, antiderivative size = 53

$$\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx=\frac{\arctan\left(\frac{\sqrt{bd+ae}\,x}{\sqrt{d}\sqrt{a-bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{bd+ae}}$$

output
```
arctan(x*(a*e+b*d)^(1/2)/d^(1/2)/(c*x^4-b*x^2+a)^(1/2))/d^(1/2)/(a*e+b*d)^
(1/2)
```

3.1005.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 11.91 (sec) , antiderivative size = 416, normalized size of antiderivative = 7.85

$$\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx$$
$$=\frac{i\sqrt{2+\frac{4cx^2}{-b+\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\left(\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{-b+\sqrt{b^2-4ac}}}x\right),\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)-\text{EllipticPi}\left(\ldots\right.\right.}{2\ldots}$$

input
```
Integrate[(a - c*x^4)/(Sqrt[a - b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),
x]
```

output
```
((I/2)*Sqrt[2 + (4*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c])]*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(-b + Sqrt[b^2 -
 4*a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])] - EllipticP
i[((b - Sqrt[b^2 - 4*a*c])*d)/(-(a*e) + Sqrt[a]*Sqrt[-4*c*d^2 + a*e^2]), I
*ArcSinh[Sqrt[2]*Sqrt[c/(-b + Sqrt[b^2 - 4*a*c])]*x], (b - Sqrt[b^2 - 4*a*
c])/(b + Sqrt[b^2 - 4*a*c])] - EllipticPi[((-b + Sqrt[b^2 - 4*a*c])*d)/(a*
e + Sqrt[a]*Sqrt[-4*c*d^2 + a*e^2]), I*ArcSinh[Sqrt[2]*Sqrt[c/(-b + Sqrt[b
^2 - 4*a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])]))/(Sqrt
[c/(-b + Sqrt[b^2 - 4*a*c])]*d*Sqrt[a - b*x^2 + c*x^4])
```

3.1005.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2537, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4}\left(ad+aex^2+cdx^4\right)}dx$$

$$\Big\downarrow \text{2537}$$

$$a\int \frac{1}{\frac{a(bd+ae)x^2}{cx^4-bx^2+a}+ad}d\frac{x}{\sqrt{cx^4-bx^2+a}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{x\sqrt{ae+bd}}{\sqrt{d}\sqrt{a-bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{ae+bd}}$$

input
```
Int[(a - c*x^4)/(Sqrt[a - b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x]
```

output
```
ArcTan[(Sqrt[b*d + a*e]*x)/(Sqrt[d]*Sqrt[a - b*x^2 + c*x^4])]/(Sqrt[d]*Sqr
t[b*d + a*e])
```

3.1005.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2537
```
Int[((u_)*((A_) + (B_.)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v,
x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Co
eff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Simp[A   Subst[Int[1/(d - (b*d - a*e
)*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0]] /;
 FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]
```

3.1005.4 Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{\arctan\left(\frac{d\sqrt{c\,x^{4}-b\,x^{2}+a}}{x\sqrt{(ae+bd)d}}\right)}{\sqrt{(ae+bd)d}}$	46
elliptic	$-\frac{\arctan\left(\frac{d\sqrt{c\,x^{4}-b\,x^{2}+a}}{x\sqrt{(ae+bd)d}}\right)}{\sqrt{(ae+bd)d}}$	46
pseudoelliptic	$-\frac{\arctan\left(\frac{d\sqrt{c\,x^{4}-b\,x^{2}+a}}{x\sqrt{(ae+bd)d}}\right)}{\sqrt{(ae+bd)d}}$	46

input
```
int((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
-1/((a*e+b*d)*d)^(1/2)*arctan(d*(c*x^4-b*x^2+a)^(1/2)/x/((a*e+b*d)*d)^(1/2
))
```

3.1005.5 Fricas [A] (verification not implemented)

Time = 7.66 (sec) , antiderivative size = 304, normalized size of antiderivative = 5.74

$$\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx = \left[-\frac{\sqrt{-bd^2-ade}\log\left(-\frac{c^2d^2x^8-2\,(4\,bcd^2+3\,acde)x^6+(8\,abde+a^2e^2+2\,(4\,b^2+ac)d^2)x^4+a^2d^2-2\,(4\,abd^2+3\,a^2de)x^2+4\,(cdx^5-(2\,bd+a}{c^2d^2x^8+2\,acdex^6+2\,a^2dex^2+(2\,acd^2+a^2e^2)x^4+a^2d^2}\right.}{4\,(bd^2+ade)}\right.$$

input
```
integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2),x, algori
thm="fricas")
```

output
```
[-1/4*sqrt(-b*d^2 - a*d*e)*log(-(c^2*d^2*x^8 - 2*(4*b*c*d^2 + 3*a*c*d*e)*x
^6 + (8*a*b*d*e + a^2*e^2 + 2*(4*b^2 + a*c)*d^2)*x^4 + a^2*d^2 - 2*(4*a*b*
d^2 + 3*a^2*d*e)*x^2 + 4*(c*d*x^5 - (2*b*d + a*e)*x^3 + a*d*x)*sqrt(c*x^4
- b*x^2 + a)*sqrt(-b*d^2 - a*d*e))/(c^2*d^2*x^8 + 2*a*c*d*e*x^6 + 2*a^2*d*
e*x^2 + (2*a*c*d^2 + a^2*e^2)*x^4 + a^2*d^2))/(b*d^2 + a*d*e), 1/2*arctan(
2*sqrt(c*x^4 - b*x^2 + a)*sqrt(b*d^2 + a*d*e)*x/(c*d*x^4 - (2*b*d + a*e)*x
^2 + a*d))/sqrt(b*d^2 + a*d*e)]
```

3.1005.6 Sympy [F]

$$\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx = -\int \left(-\frac{a}{ad\sqrt{a-bx^2+cx^4}+aex^2\sqrt{a-bx^2+cx^4}+cdx^4\sqrt{a-bx^2+cx^4}}\right)dx - \int \frac{cx^4}{ad\sqrt{a-bx^2+cx^4}+aex^2\sqrt{a-bx^2+cx^4}+cdx^4\sqrt{a-bx^2+cx^4}}\,dx$$

input
```
integrate((-c*x**4+a)/(c*d*x**4+a*e*x**2+a*d)/(c*x**4-b*x**2+a)**(1/2),x)
```

output
```
-Integral(-a/(a*d*sqrt(a - b*x**2 + c*x**4) + a*e*x**2*sqrt(a - b*x**2 + c
*x**4) + c*d*x**4*sqrt(a - b*x**2 + c*x**4)), x) - Integral(c*x**4/(a*d*sq
rt(a - b*x**2 + c*x**4) + a*e*x**2*sqrt(a - b*x**2 + c*x**4) + c*d*x**4*sq
rt(a - b*x**2 + c*x**4)), x)
```

3.1005.7 Maxima [F]

$$\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx = \int -\frac{cx^4-a}{(cdx^4+aex^2+ad)\sqrt{cx^4-bx^2+a}}\,dx$$

input
```
integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2),x, algori
thm="maxima")
```

output
```
-integrate((c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 - b*x^2 + a))
, x)
```

3.1005.8 Giac [F]

$$\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx = \int -\frac{cx^4-a}{(cdx^4+aex^2+ad)\sqrt{cx^4-bx^2+a}}\,dx$$

input
```
integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2),x, algori
thm="giac")
```

output
```
integrate(-(c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 - b*x^2 + a))
, x)
```

3.1005.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx = \int \frac{a-c\,x^4}{(c\,d\,x^4+a\,e\,x^2+a\,d)\,\sqrt{c\,x^4-b\,x^2+a}}\,dx$$

input
```
int((a - c*x^4)/((a*d + a*e*x^2 + c*d*x^4)*(a - b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((a - c*x^4)/((a*d + a*e*x^2 + c*d*x^4)*(a - b*x^2 + c*x^4)^(1/2)), x)
```

3.1006 $\int \frac{1}{\sqrt{5-2x+x^2}(8+x^3)}\,dx$

3.1006.1 Optimal result

Integrand size = 20, antiderivative size = 84

$$\int \frac{1}{\sqrt{5-2x+x^2}\,(8+x^3)}\,dx=-\frac{\arctan\left(\frac{1-x}{\sqrt{3}\sqrt{5-2x+x^2}}\right)}{4\sqrt{3}}-\frac{\text{arctanh}\left(\frac{7-3x}{\sqrt{13}\sqrt{5-2x+x^2}}\right)}{12\sqrt{13}}+\frac{1}{12}\text{arctanh}\left(\sqrt{5-2x+x^2}\right)$$

output
```
1/12*arctanh((x^2-2*x+5)^(1/2))-1/12*arctan(1/3*(1-x)*3^(1/2)/(x^2-2*x+5)^
(1/2))*3^(1/2)-1/156*arctanh(1/13*(7-3*x)*13^(1/2)/(x^2-2*x+5)^(1/2))*13^(
1/2)
```

3.1006.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{5-2x+x^2}\,(8+x^3)}\,dx =\frac{1}{156}\left(-13\sqrt{3}\arctan\left(\frac{4-2x+x^2-(-1+x)\sqrt{5-2x+x^2}}{\sqrt{3}}\right)+13\text{arctanh}\left(\sqrt{5-2x+x^2}\right)+2\sqrt{13}\text{arctanh}\left(\frac{2+x-\sqrt{5-2x+x^2}}{\sqrt{13}}\right)\right)$$

input
```
Integrate[1/(Sqrt[5 - 2*x + x^2]*(8 + x^3)),x]
```

output
```
(-13*Sqrt[3]*ArcTan[(4 - 2*x + x^2 - (-1 + x)*Sqrt[5 - 2*x + x^2])/Sqrt[3]
] + 13*ArcTanh[Sqrt[5 - 2*x + x^2]] + 2*Sqrt[13]*ArcTanh[(2 + x - Sqrt[5 -
 2*x + x^2])/Sqrt[13]])/156
```

3.1006.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {2533, 1154, 219, 1358, 27, 1313, 216, 1357, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2-2x+5}\,(x^3+8)}dx$$

$$\Big\downarrow \text{2533}$$

$$\frac{1}{12}\int \frac{1}{(x+2)\sqrt{x^2-2x+5}}dx+\frac{1}{12}\int \frac{4-x}{(x^2-2x+4)\sqrt{x^2-2x+5}}dx$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{12}\int \frac{4-x}{(x^2-2x+4)\sqrt{x^2-2x+5}}dx-\frac{1}{6}\int \frac{1}{52-\frac{4(7-3x)^2}{x^2-2x+5}}d\frac{2(7-3x)}{\sqrt{x^2-2x+5}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{12}\int \frac{4-x}{(x^2-2x+4)\sqrt{x^2-2x+5}}dx-\frac{\text{arctanh}\left(\frac{7-3x}{\sqrt{13}\sqrt{x^2-2x+5}}\right)}{12\sqrt{13}}$$

$$\Big\downarrow \text{1358}$$

$$\frac{1}{12}\left(3\int \frac{1}{(x^2-2x+4)\sqrt{x^2-2x+5}}dx-\frac{1}{2}\int -\frac{2(1-x)}{(x^2-2x+4)\sqrt{x^2-2x+5}}dx\right)-\frac{\text{arctanh}\left(\frac{7-3x}{\sqrt{13}\sqrt{x^2-2x+5}}\right)}{12\sqrt{13}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{12}\left(3\int \frac{1}{(x^2-2x+4)\sqrt{x^2-2x+5}}dx+\int \frac{1-x}{(x^2-2x+4)\sqrt{x^2-2x+5}}dx\right)-\frac{\text{arctanh}\left(\frac{7-3x}{\sqrt{13}\sqrt{x^2-2x+5}}\right)}{12\sqrt{13}}$$

$$\Big\downarrow \text{1313}$$

$$\frac{1}{12}\left(\int \frac{1-x}{(x^2-2x+4)\sqrt{x^2-2x+5}}dx+12\int \frac{1}{\frac{8(1-x)^2}{x^2-2x+5}+24}d\left(-\frac{2(1-x)}{\sqrt{x^2-2x+5}}\right)\right)-\frac{\text{arctanh}\left(\frac{7-3x}{\sqrt{13}\sqrt{x^2-2x+5}}\right)}{12\sqrt{13}}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{12}\left(\int \frac{1-x}{(x^2-2x+4)\sqrt{x^2-2x+5}}dx-\sqrt{3}\arctan\left(\frac{1-x}{\sqrt{3}\sqrt{x^2-2x+5}}\right)\right)-\frac{\text{arctanh}\left(\frac{7-3x}{\sqrt{13}\sqrt{x^2-2x+5}}\right)}{12\sqrt{13}}$$

$$\Big\downarrow \text{1357}$$

$$\frac{1}{12}\left(-2\int \frac{1}{2(x^2-2x+5)-2}d\sqrt{x^2-2x+5}-\sqrt{3}\arctan\left(\frac{1-x}{\sqrt{3}\sqrt{x^2-2x+5}}\right)\right)-\frac{\text{arctanh}\left(\frac{7-3x}{\sqrt{13}\sqrt{x^2-2x+5}}\right)}{12\sqrt{13}}$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{12}\left(\text{arctanh}\left(\sqrt{x^2-2x+5}\right)-\sqrt{3}\arctan\left(\frac{1-x}{\sqrt{3}\sqrt{x^2-2x+5}}\right)\right)-\frac{\text{arctanh}\left(\frac{7-3x}{\sqrt{13}\sqrt{x^2-2x+5}}\right)}{12\sqrt{13}}$$

input
```
Int[1/(Sqrt[5 - 2*x + x^2]*(8 + x^3)),x]
```

output
```
-1/12*ArcTanh[(7 - 3*x)/(Sqrt[13]*Sqrt[5 - 2*x + x^2])]/Sqrt[13] + (-(Sqrt
[3]*ArcTan[(1 - x)/(Sqrt[3]*Sqrt[5 - 2*x + x^2])]) + ArcTanh[Sqrt[5 - 2*x
+ x^2]])/12
```

3.1006. $\int \frac{1}{\sqrt{5-2x+x^2}(8+x^3)}\,dx$

3.1006.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1313
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e
)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f,
0]
```

rule 1357
```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g   Subst[Int[1/(b*d - a*e
- b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g,
h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &
& EqQ[h*e - 2*g*f, 0]
```

rule 1358
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-(h*e - 2*g*f)/(2*f)   Int[1/
((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[h/(2*f)   Int[(e +
 2*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c
, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c
*e - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

rule 2533
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]*((a_) + (b_.)*(x_)^3)), x_Sy
mbol] :> With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, Sim
p[r/(3*a)   Int[1/((r + s*x)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[r/(3*a)
  Int[(2*r - s*x)/((r^2 - r*s*x + s^2*x^2)*Sqrt[d + e*x + f*x^2]), x], x]]
/; FreeQ[{a, b, d, e, f}, x] && PosQ[a/b]
```

3.1006.4 Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.82

method	result
default	$-\frac{\sqrt{13}\,\operatorname{arctanh}\left(\frac{(14-6x)\sqrt{13}}{26\sqrt{(x+2)^2-6x+1}}\right)}{156}+\frac{\operatorname{arctanh}\left(\sqrt{x^2-2x+5}\right)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(2x-2)}{6\sqrt{x^2-2x+5}}\right)}{12}$
trager	$\operatorname{RootOf}\left(144\_Z^2+12\_Z+1\right)\ln\left(\frac{2880\operatorname{RootOf}\left(144\_Z^2+12\_Z+1\right)^2 x+126\operatorname{RootOf}\left(144\_Z^2+12\_Z+1\right)\sqrt{x^2-2}}{12\operatorname{RootOf}}\right.$

input
```
int(1/(x^3+8)/(x^2-2*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/156*13^(1/2)*arctanh(1/26*(14-6*x)*13^(1/2)/((x+2)^2-6*x+1)^(1/2))+1/12
*arctanh((x^2-2*x+5)^(1/2))+1/12*3^(1/2)*arctan(1/6*3^(1/2)/(x^2-2*x+5)^(1
/2)*(2*x-2))
```

3.1006.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(64) = 128$.

Time = 0.26 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.83

$$\int \frac{1}{\sqrt{5-2x+x^2}\,(8+x^3)}\,dx = \frac{1}{12}\,\sqrt{3}\arctan\left(-\frac{1}{3}\,\sqrt{3}(x-2)+\frac{1}{3}\,\sqrt{3}\sqrt{x^2-2\,x+5}\right) - \frac{1}{12}\,\sqrt{3}\arctan\left(-\frac{1}{3}\,\sqrt{3}x+\frac{1}{3}\,\sqrt{3}\sqrt{x^2-2\,x+5}\right) + \frac{1}{156}\,\sqrt{13}\log\left(-\frac{\sqrt{13}(3\,x-7)+\sqrt{x^2-2\,x+5}(3\,\sqrt{13}+13)+9\,x-21}{x+2}\right) + \frac{1}{24}\,\log\left(x^2-\sqrt{x^2-2\,x+5}(x-2)-3\,x+6\right) - \frac{1}{24}\,\log\left(x^2-\sqrt{x^2-2\,x+5}x-x+4\right)$$

input
```
integrate(1/(x^3+8)/(x^2-2*x+5)^(1/2),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*arctan(-1/3*sqrt(3)*(x - 2) + 1/3*sqrt(3)*sqrt(x^2 - 2*x + 5)
) - 1/12*sqrt(3)*arctan(-1/3*sqrt(3)*x + 1/3*sqrt(3)*sqrt(x^2 - 2*x + 5))
+ 1/156*sqrt(13)*log(-(sqrt(13)*(3*x - 7) + sqrt(x^2 - 2*x + 5)*(3*sqrt(13
) + 13) + 9*x - 21)/(x + 2)) + 1/24*log(x^2 - sqrt(x^2 - 2*x + 5)*(x - 2)
- 3*x + 6) - 1/24*log(x^2 - sqrt(x^2 - 2*x + 5)*x - x + 4)
```

3.1006.6 Sympy [F]

$$\int \frac{1}{\sqrt{5-2x+x^2}\,(8+x^3)}\,dx = \int \frac{1}{(x+2)\,(x^2-2x+4)\,\sqrt{x^2-2x+5}}\,dx$$

input
```
integrate(1/(x**3+8)/(x**2-2*x+5)**(1/2),x)
```

output
```
Integral(1/((x + 2)*(x**2 - 2*x + 4)*sqrt(x**2 - 2*x + 5)), x)
```

3.1006.7 Maxima [F]

$$\int \frac{1}{\sqrt{5-2x+x^2}\,(8+x^3)}\,dx = \int \frac{1}{(x^3+8)\sqrt{x^2-2\,x+5}}\,dx$$

input
```
integrate(1/(x^3+8)/(x^2-2*x+5)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 8)*sqrt(x^2 - 2*x + 5)), x)
```

3.1006.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(64) = 128.

Time = 0.34 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.95

$$\begin{aligned}\int \frac{1}{\sqrt{5-2x+x^2}\,(8+x^3)}\,dx &= -\frac{1}{12}\,\sqrt{3}\arctan\left(-\frac{1}{3}\,\sqrt{3}\left(x-\sqrt{x^2-2\,x+5}\right)\right) \\ &\quad + \frac{1}{12}\,\sqrt{3}\arctan\left(-\frac{1}{3}\,\sqrt{3}\left(x-\sqrt{x^2-2\,x+5}-2\right)\right) \\ &\quad + \frac{1}{156}\,\sqrt{13}\log\left(\frac{\left|-2\,x-2\,\sqrt{13}+2\,\sqrt{x^2-2\,x+5}-4\right|}{\left|-2\,x+2\,\sqrt{13}+2\,\sqrt{x^2-2\,x+5}-4\right|}\right) \\ &\quad + \frac{1}{24}\,\log\left(\left(x-\sqrt{x^2-2\,x+5}\right)^2-4\,x+4\,\sqrt{x^2-2\,x+5} \right. \\ &\qquad \left. +7\right) - \frac{1}{24}\,\log\left(\left(x-\sqrt{x^2-2\,x+5}\right)^2+3\right)\end{aligned}$$

input
```
integrate(1/(x^3+8)/(x^2-2*x+5)^(1/2),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 - 2*x + 5))) + 1/12*sqrt(3
)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 - 2*x + 5) - 2)) + 1/156*sqrt(13)*log(
abs(-2*x - 2*sqrt(13) + 2*sqrt(x^2 - 2*x + 5) - 4)/abs(-2*x + 2*sqrt(13) +
 2*sqrt(x^2 - 2*x + 5) - 4)) + 1/24*log((x - sqrt(x^2 - 2*x + 5))^2 - 4*x
+ 4*sqrt(x^2 - 2*x + 5) + 7) - 1/24*log((x - sqrt(x^2 - 2*x + 5))^2 + 3)
```

3.1006.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{5-2x+x^2}\,(8+x^3)}\,dx = \int \frac{1}{(x^3+8)\,\sqrt{x^2-2\,x+5}}\,dx$$

input
```
int(1/((x^3 + 8)*(x^2 - 2*x + 5)^(1/2)),x)
```

output
```
int(1/((x^3 + 8)*(x^2 - 2*x + 5)^(1/2)), x)
```

3.1007 $\int \sqrt{\frac{x^2}{1+x^2}}\,dx$

3.1007.1 Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \sqrt{\frac{x^2}{1+x^2}}\,dx=\frac{\sqrt{x^2}\sqrt{1+x^2}}{x}$$

output
```
(x^2)^(1/2)*(x^2+1)^(1/2)/x
```

3.1007.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \sqrt{\frac{x^2}{1+x^2}}\,dx=\frac{x}{\sqrt{\frac{x^2}{1+x^2}}}$$

input
```
Integrate[Sqrt[x^2/(1 + x^2)],x]
```

output
```
x/Sqrt[x^2/(1 + x^2)]
```

3.1007.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2050, 34, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{x^2}{x^2+1}}dx$$
$$\Big\downarrow \text{2050}$$
$$\int \frac{\sqrt{x^2}}{\sqrt{x^2+1}}dx$$
$$\Big\downarrow \text{34}$$
$$\frac{\sqrt{x^2}\int \frac{x}{\sqrt{x^2+1}}dx}{x}$$
$$\Big\downarrow \text{241}$$
$$\frac{\sqrt{x^2}\sqrt{x^2+1}}{x}$$

input
```
Int[Sqrt[x^2/(1 + x^2)],x]
```

output
```
(Sqrt[x^2]*Sqrt[1 + x^2])/x
```

3.1007.3.1 Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2050
```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

3.1007.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

method	result	size
gosper	$\frac{\left(x^2+1\right)\sqrt{\frac{x^2}{x^2+1}}}{x}$	23
default	$\frac{\left(x^2+1\right)\sqrt{\frac{x^2}{x^2+1}}}{x}$	23
trager	$\frac{\left(x^2+1\right)\sqrt{\frac{x^2}{x^2+1}}}{x}$	23
risch	$\frac{\left(x^2+1\right)\sqrt{\frac{x^2}{x^2+1}}}{x}$	23

input
```
int((x^2/(x^2+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x^2+1)/x*(x^2/(x^2+1))^(1/2)
```

3.1007.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \sqrt{\frac{x^2}{1+x^2}}\,dx = \frac{\left(x^2+1\right)\sqrt{\frac{x^2}{x^2+1}}}{x}$$

input
```
integrate((x^2/(x^2+1))^(1/2),x, algorithm="fricas")
```

output
```
(x^2 + 1)*sqrt(x^2/(x^2 + 1))/x
```

3.1007.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \sqrt{\frac{x^2}{1+x^2}}\,dx = x\sqrt{\frac{x^2}{x^2+1}} + \frac{\sqrt{\frac{x^2}{x^2+1}}}{x}$$

input
```
integrate((x**2/(x**2+1))**(1/2),x)
```

output
```
x*sqrt(x**2/(x**2 + 1)) + sqrt(x**2/(x**2 + 1))/x
```

3.1007.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.35

$$\int \sqrt{\frac{x^2}{1+x^2}}\,dx = \sqrt{x^2+1}$$

input
```
integrate((x^2/(x^2+1))^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x^2 + 1)
```

3.1007.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \sqrt{\frac{x^2}{1+x^2}}\,dx = \sqrt{x^2+1}\,\mathrm{sgn}(x) - \mathrm{sgn}(x)$$

input
```
integrate((x^2/(x^2+1))^(1/2),x, algorithm="giac")
```

output
```
sqrt(x^2 + 1)*sgn(x) - sgn(x)
```

3.1007.9 Mupad [B] (verification not implemented)

Time = 21.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.65

$$\int \sqrt{\frac{x^2}{1+x^2}}\,dx = \frac{\sqrt{x^4+x^2}}{x}$$

input
```
int((x^2/(x^2 + 1))^(1/2),x)
```

output
```
(x^2 + x^4)^(1/2)/x
```

3.1008 $\int \sqrt{\frac{x^n}{1+x^n}}\,dx$

3.1008.1 Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \sqrt{\frac{x^n}{1+x^n}}\,dx = \frac{2x\sqrt{x^n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right),\frac{1}{2}\left(3+\frac{2}{n}\right),-x^n\right)}{2+n}$$

output
```
2*x*hypergeom([1/2, 1/2+1/n],[3/2+1/n],-x^n)*(x^n)^(1/2)/(2+n)
```

3.1008.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \sqrt{\frac{x^n}{1+x^n}}\,dx = \frac{2x\sqrt{x^n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}+\frac{1}{n},\frac{3}{2}+\frac{1}{n},-x^n\right)}{2+n}$$

input
```
Integrate[Sqrt[x^n/(1 + x^n)],x]
```

output
```
(2*x*Sqrt[x^n]*Hypergeometric2F1[1/2, 1/2 + n^(-1), 3/2 + n^(-1), -x^n])/(
2 + n)
```

3.1008.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2050, 34, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{x^n}{x^n+1}}dx$$

$$\Big\downarrow \text{ 2050}$$

$$\int \frac{\sqrt{x^n}}{\sqrt{x^n+1}}dx$$

$$\Big\downarrow \text{ 34}$$

$$x^{-n/2}\sqrt{x^n}\int \frac{x^{n/2}}{\sqrt{x^n+1}}dx$$

$$\Big\downarrow \text{ 888}$$

$$\frac{2x^{\frac{n+2}{2}-\frac{n}{2}}\sqrt{x^n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right),\frac{1}{2}\left(3+\frac{2}{n}\right),-x^n\right)}{n+2}$$

input
```
Int[Sqrt[x^n/(1 + x^n)],x]
```

output
```
(2*x^(-1/2*n + (2 + n)/2)*Sqrt[x^n]*Hypergeometric2F1[1/2, (1 + 2/n)/2, (3
 + 2/n)/2, -x^n])/(2 + n)
```

3.1008.3.1 Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2050
```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

3.1008.4 Maple [F]

$$\int \sqrt{\frac{x^n}{1+x^n}}dx$$

input
```
int((x^n/(1+x^n))^(1/2),x)
```

output
```
int((x^n/(1+x^n))^(1/2),x)
```

3.1008.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{\frac{x^n}{1+x^n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((x^n/(1+x^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.1008.6 Sympy [F]

$$\int \sqrt{\frac{x^n}{1+x^n}}\,dx = \int \sqrt{\frac{x^n}{x^n+1}}\,dx$$

input
```
integrate((x**n/(1+x**n))**(1/2),x)
```

output
```
Integral(sqrt(x**n/(x**n + 1)), x)
```

3.1008.7 Maxima [F]

$$\int \sqrt{\frac{x^n}{1+x^n}}\,dx = \int \sqrt{\frac{x^n}{x^n+1}}\,dx$$

input
```
integrate((x^n/(1+x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^n/(x^n + 1)), x)
```

3.1008.8 Giac [F]

$$\int \sqrt{\frac{x^n}{1+x^n}}\,dx = \int \sqrt{\frac{x^n}{x^n+1}}\,dx$$

input
```
integrate((x^n/(1+x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^n/(x^n + 1)), x)
```

3.1008.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{x^n}{1+x^n}}\,dx = \int \sqrt{\frac{x^n}{x^n+1}}\,dx$$

input
```
int((x^n/(x^n + 1))^(1/2),x)
```

output
```
int((x^n/(x^n + 1))^(1/2), x)
```

3.1009 $\int \frac{ef-efx^2}{(ad+bdx+adx^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$

3.1009.1 Optimal result

Integrand size = 52, antiderivative size = 88

$$\int \frac{ef-efx^2}{(ad+bdx+adx^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx=\frac{ef\arctan\left(\frac{ab+(4a^2+b^2-2ac)x+abx^2}{2a\sqrt{2a-c}\sqrt{a+bx+cx^2+bx^3+ax^4}}\right)}{a\sqrt{2a-c}d}$$

output
```
e*f*arctan(1/2*(a*b+(4*a^2-2*a*c+b^2)*x+a*b*x^2)/a/(2*a-c)^(1/2)/(a*x^4+b*
x^3+c*x^2+b*x+a)^(1/2))/a/d/(2*a-c)^(1/2)
```

3.1009.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 246 vs. 2(88) = 176.

Time = 0.94 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.80

$$\int \frac{ef-efx^2}{(ad+bdx+adx^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx = -\frac{ef\left(-2\sqrt{-2a+c}\arctan\left(\frac{b\left(-\sqrt{-2a+c}x+\sqrt{a+bx+cx^2+bx^3+ax^4}\right)}{2a\sqrt{2a-c}(1+x^2)}\right)+\sqrt{2a-c}\left(2\log\left(-\sqrt{-2a+c}x+\sqrt{a+bx\right.\right.\right.}{}$$

input
```
Integrate[(e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*Sqrt[a + b*x + c*x^2 +
b*x^3 + a*x^4]),x]
```

output
```
-1/2*(e*f*(-2*Sqrt[-2*a + c]*ArcTan[(b*(-(Sqrt[-2*a + c]*x) + Sqrt[a + b*x
 + c*x^2 + b*x^3 + a*x^4]))/(2*a*Sqrt[2*a - c]*(1 + x^2))] + Sqrt[2*a - c]
*(2*Log[-(Sqrt[-2*a + c]*x) + Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]] - Log
[a*b^2*(-1 + x^2)^2 + 8*a^3*(1 + x^2)^2 - 4*a^2*c*(1 + x^2)^2 + b^2*x*(b +
 2*c*x + b*x^2 - 2*Sqrt[-2*a + c]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])])
))/(a*Sqrt[-(-2*a + c)^2]*d)
```

3.1009.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.019$, Rules used = {2507}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ef-efx^2}{\sqrt{ax^4+a+bx^3+bx+cx^2}\,(adx^2+ad+bdx)}dx$$

$$\Big\downarrow \ 2507$$

$$\frac{ef\arctan\left(\frac{x(4a^2-2ac+b^2)+abx^2+ab}{2a\sqrt{2a-c}\sqrt{ax^4+a+bx^3+bx+cx^2}}\right)}{ad\sqrt{2a-c}}$$

input
```
Int[(e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*Sqrt[a + b*x + c*x^2 + b*x^3
+ a*x^4]),x]
```

output
```
(e*f*ArcTan[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b*x^2)/(2*a*Sqrt[2*a - c]*S
qrt[a + b*x + c*x^2 + b*x^3 + a*x^4])])/(a*Sqrt[2*a - c]*d)
```

3.1009.3.1 Defintions of rubi rules used

rule 2507
```
Int[((f_) + (g_.)*(x_)^2)/(((d_) + (e_.)*(x_) + (d_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_) + (c_.)*(x_)^2 + (b_.)*(x_)^3 + (a_.)*(x_)^4]), x_Symbol] :> Simp
[a*(f/(d*Rt[a^2*(2*a - c), 2]))*ArcTan[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b
*x^2)/(2*Rt[a^2*(2*a - c), 2]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])], x] /
; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[b*d - a*e, 0] && EqQ[f + g, 0] &&
PosQ[a^2*(2*a - c)]
```

3.1009.4 Maple [A] (verified)

Time = 3.96 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.05

method	result	size
default	$\frac{ef \ln\left(\frac{2\sqrt{-2a+c}\,\sqrt{a\,x^{4}+b\,x^{3}+c\,x^{2}+bx+a}\,a-4a^{2}x+\left(-b\,x^{2}+2cx-b\right)a-b^{2}x}{a\,x^{2}+bx+a}\right)}{d\sqrt{-2a+c}\,a}$	92
pseudoelliptic	$\frac{ef \ln\left(\frac{2\sqrt{-2a+c}\,\sqrt{a\,x^{4}+b\,x^{3}+c\,x^{2}+bx+a}\,a-4a^{2}x+\left(-b\,x^{2}+2cx-b\right)a-b^{2}x}{a\,x^{2}+bx+a}\right)}{d\sqrt{-2a+c}\,a}$	92
elliptic	Expression too large to display	254498

input
```
int((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
e*f/d/(-2*a+c)^(1/2)*ln((2*(-2*a+c)^(1/2)*(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2)*
a-4*a^2*x+(-b*x^2+2*c*x-b)*a-b^2*x)/(a*x^2+b*x+a))/a
```

3.1009.5 Fricas [A] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 324, normalized size of antiderivative = 3.68

$$\int \frac{ef - efx^2}{(ad + bdx + adx^2)\sqrt{a + bx + cx^2 + bx^3 + ax^4}}\,dx$$
$$= \left[-\frac{\sqrt{-2\,a + c}ef \log\left(\frac{2\,ab^3x^3 + 2\,ab^3x - (8\,a^4 - a^2b^2 - 4\,a^3c)x^4 - 8\,a^4 + a^2b^2 + 4\,a^3c + (16\,a^4 + 10\,a^2b^2 + b^4 + 8\,a^2c^2 - 4\,(6\,a^3 + ab^2)c)x^2 - 4\,(a^2b \cdots}{a^2x^4 + 2\,abx^3 + 2\,abx + (2\,a^2 + b^2)x^2 + a^2}\cdots\right)}{2\,(2\,a^2 - ac)d} - \frac{\sqrt{2\,a - c}ef \arctan\left(\frac{2\,\sqrt{ax^4 + bx^3 + cx^2 + bx + a}\sqrt{2\,a - c}a}{abx^2 + ab + (4\,a^2 + b^2 - 2\,ac)x}\right)}{(2\,a^2 - ac)d} \right]$$

input
```
integrate((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/
2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-2*a + c)*e*f*log((2*a*b^3*x^3 + 2*a*b^3*x - (8*a^4 - a^2*b^2 -
 4*a^3*c)*x^4 - 8*a^4 + a^2*b^2 + 4*a^3*c + (16*a^4 + 10*a^2*b^2 + b^4 + 8
*a^2*c^2 - 4*(6*a^3 + a*b^2)*c)*x^2 - 4*(a^2*b*x^2 + a^2*b + (4*a^3 + a*b^
2 - 2*a^2*c)*x))*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*sqrt(-2*a + c))/(a^2
*x^4 + 2*a*b*x^3 + 2*a*b*x + (2*a^2 + b^2)*x^2 + a^2))/((2*a^2 - a*c)*d),
-sqrt(2*a - c)*e*f*arctan(2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*sqrt(2*a
 - c)*a/(a*b*x^2 + a*b + (4*a^2 + b^2 - 2*a*c)*x))/((2*a^2 - a*c)*d)]
```

3.1009.6 Sympy [F]

$$\int \frac{ef - efx^2}{(ad + bdx + adx^2)\sqrt{a + bx + cx^2 + bx^3 + ax^4}}\,dx =$$

$$-\frac{ef\left(\int \frac{x^2}{ax^2\sqrt{ax^4+a+bx^3+bx+cx^2}+a\sqrt{ax^4+a+bx^3+bx+cx^2}+bx\sqrt{ax^4+a+bx^3+bx+cx^2}}\,dx + \int \left(-\frac{}{ax^2\sqrt{ax^4+a+bx^3+bx+cx^2}+a\sqrt{ax}\right.\right.}{d}$$

input
```
integrate((-e*f*x**2+e*f)/(a*d*x**2+b*d*x+a*d)/(a*x**4+b*x**3+c*x**2+b*x+a
)**(1/2),x)
```

output
```
-e*f*(Integral(x**2/(a*x**2*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2) + a*s
qrt(a*x**4 + a + b*x**3 + b*x + c*x**2) + b*x*sqrt(a*x**4 + a + b*x**3 + b
*x + c*x**2)), x) + Integral(-1/(a*x**2*sqrt(a*x**4 + a + b*x**3 + b*x + c
*x**2) + a*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2) + b*x*sqrt(a*x**4 + a
+ b*x**3 + b*x + c*x**2)), x))/d
```

3.1009.7 Maxima [F]

$$\int \frac{ef - efx^2}{(ad + bdx + adx^2)\sqrt{a + bx + cx^2 + bx^3 + ax^4}}\,dx = \int -\frac{efx^2 - ef}{\sqrt{ax^4 + bx^3 + cx^2 + bx + a}(adx^2 + bdx + ad)}\,dx$$

input
```
integrate((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/
2),x, algorithm="maxima")
```

output
```
-integrate((e*f*x^2 - e*f)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(a*d*x^2
 + b*d*x + a*d)), x)
```

3.1009.8 Giac [F]

$$\int \frac{ef - efx^2}{(ad + bdx + adx^2)\sqrt{a + bx + cx^2 + bx^3 + ax^4}}\,dx$$
$$= \int -\frac{efx^2 - ef}{\sqrt{ax^4 + bx^3 + cx^2 + bx + a}(adx^2 + bdx + ad)}\,dx$$

input
```
integrate((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/
2),x, algorithm="giac")
```

output
```
integrate(-(e*f*x^2 - e*f)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(a*d*x^2
 + b*d*x + a*d)), x)
```

3.1009.9 Mupad [F(-1)]

Timed out.

$$\int \frac{ef - efx^2}{(ad + bdx + adx^2)\sqrt{a + bx + cx^2 + bx^3 + ax^4}}\,dx$$
$$= \int \frac{e\,f - e\,f\,x^2}{(a\,d\,x^2 + b\,d\,x + a\,d)\,\sqrt{a\,x^4 + b\,x^3 + c\,x^2 + b\,x + a}}\,dx$$

input
```
int((e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*(a + b*x + a*x^4 + b*x^3 + c*
x^2)^(1/2)),x)
```

output
```
int((e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*(a + b*x + a*x^4 + b*x^3 + c*
x^2)^(1/2)), x)
```

3.1010 $\int \frac{ef-efx^2}{(-ad+bdx-adx^2)\sqrt{-a+bx+cx^2+bx^3-ax^4}}\,dx$

3.1010.1 Optimal result

Integrand size = 57, antiderivative size = 88

$$\int \frac{ef-efx^2}{(-ad+bdx-adx^2)\sqrt{-a+bx+cx^2+bx^3-ax^4}}\,dx = \frac{ef\,\text{arctanh}\left(\frac{ab-(4a^2+b^2+2ac)x+abx^2}{2a\sqrt{2a+c}\sqrt{-a+bx+cx^2+bx^3-ax^4}}\right)}{a\sqrt{2a+c}\,d}$$

output
```
e*f*arctanh(1/2*(a*b-(4*a^2+2*a*c+b^2)*x+a*b*x^2)/a/(2*a+c)^(1/2)/(-a*x^4+
b*x^3+c*x^2+b*x-a)^(1/2))/a/d/(2*a+c)^(1/2)
```

3.1010.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 213 vs. 2(88) = 176.

Time = 0.92 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.42

$$\int \frac{ef-efx^2}{(-ad+bdx-adx^2)\sqrt{-a+bx+cx^2+bx^3-ax^4}}\,dx = \frac{ef\left(2\text{arctanh}\left(\frac{b\left(x-\frac{\sqrt{x(b+cx+bx^2)-a(1+x^4)}}{\sqrt{2a+c}}\right)}{2a(1+x^2)}\right)+2\log\left(-\sqrt{2a+c}\,x+\sqrt{x(b+cx+bx^2)-a(1+x^4)}\right)-\log\right.}{}$$

input
```
Integrate[(e*f - e*f*x^2)/((-(a*d) + b*d*x - a*d*x^2)*Sqrt[-a + b*x + c*x^
2 + b*x^3 - a*x^4]),x]
```

output
```
(e*f*(2*ArcTanh[(b*(x - Sqrt[x*(b + c*x + b*x^2) - a*(1 + x^4)]/Sqrt[2*a +
 c]))/(2*a*(1 + x^2))] + 2*Log[-(Sqrt[2*a + c]*x) + Sqrt[x*(b + c*x + b*x^
2) - a*(1 + x^4)]] - Log[a*b^2*(-1 + x^2)^2 + 8*a^3*(1 + x^2)^2 + 4*a^2*c*
(1 + x^2)^2 - b^2*x*(b + 2*c*x + b*x^2 - 2*Sqrt[2*a + c]*Sqrt[x*(b + c*x +
 b*x^2) - a*(1 + x^4)])]))/(2*a*Sqrt[2*a + c]*d)
```

3.1010.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.018, Rules used = {2508}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ef-efx^2}{\sqrt{-ax^4-a+bx^3+bx+cx^2}\,(-adx^2-ad+bdx)}dx$$

$$\Big\downarrow \text{2508}$$

$$\frac{ef\,\text{arctanh}\left(\frac{-x(4a^2+2ac+b^2)+abx^2+ab}{2a\sqrt{2a+c}\sqrt{-ax^4-a+bx^3+bx+cx^2}}\right)}{ad\sqrt{2a+c}}$$

input
```
Int[(e*f - e*f*x^2)/((-(a*d) + b*d*x - a*d*x^2)*Sqrt[-a + b*x + c*x^2 + b*
x^3 - a*x^4]),x]
```

output
```
(e*f*ArcTanh[(a*b - (4*a^2 + b^2 + 2*a*c)*x + a*b*x^2)/(2*a*Sqrt[2*a + c]*
Sqrt[-a + b*x + c*x^2 + b*x^3 - a*x^4])])/(a*Sqrt[2*a + c]*d)
```

3.1010.3.1 Defintions of rubi rules used

rule 2508
```
Int[((f_) + (g_.)*(x_)^2)/(((d_) + (e_.)*(x_) + (d_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_) + (c_.)*(x_)^2 + (b_.)*(x_)^3 + (a_.)*(x_)^4]), x_Symbol] :> Simp
[(-a)*(f/(d*Rt[(-a^2)*(2*a - c), 2]))*ArcTanh[(a*b + (4*a^2 + b^2 - 2*a*c)*
x + a*b*x^2)/(2*Rt[(-a^2)*(2*a - c), 2]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^
4])], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[b*d - a*e, 0] && EqQ[f +
 g, 0] && NegQ[a^2*(2*a - c)]
```

3.1010.4 Maple [A] (verified)

Time = 3.88 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.09

method	result	size
default	$-\frac{ef\ln\left(\frac{2\sqrt{2a+c}\,\sqrt{-a\,x^4+b\,x^3+c\,x^2+bx-a}\,a+4a^2x+\left(-b\,x^2+2cx-b\right)a+b^2x}{a\,x^2-bx+a}\right)}{d\sqrt{2a+c}\,a}$	96
pseudoelliptic	$-\frac{ef\ln\left(\frac{2\sqrt{2a+c}\,\sqrt{-a\,x^4+b\,x^3+c\,x^2+bx-a}\,a+4a^2x+\left(-b\,x^2+2cx-b\right)a+b^2x}{a\,x^2-bx+a}\right)}{d\sqrt{2a+c}\,a}$	96
elliptic	Expression too large to display	281960

input
```
int((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(1/2),x
,method=_RETURNVERBOSE)
```

output
```
-e*f/d/(2*a+c)^(1/2)*ln((2*(2*a+c)^(1/2)*(-a*x^4+b*x^3+c*x^2+b*x-a)^(1/2)*
a+4*a^2*x+(-b*x^2+2*c*x-b)*a+b^2*x)/(a*x^2-b*x+a))/a
```

3.1010.5 Fricas [A] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 331, normalized size of antiderivative = 3.76

$$\int \frac{ef-efx^2}{(-ad+bdx-adx^2)\sqrt{-a+bx+cx^2+bx^3-ax^4}}\,dx$$
$$= \left[\frac{\sqrt{2\,a+c}ef\log\left(\frac{2\,ab^3x^3+2\,ab^3x+(8\,a^4-a^2b^2+4\,a^3c)x^4+8\,a^4-a^2b^2+4\,a^3c-(16\,a^4+10\,a^2b^2+b^4+8\,a^2c^2+4\,(6\,a^3+ab^2)c)x^2-4\,(a^2bx^2+\cdots}{a^2x^4-2\,abx^3-2\,abx+(2\,a^2+b^2)x^2+a^2}\right.}{2\,(2\,a^2+ac)d}\right.$$
$$\left. -\frac{\sqrt{-2\,a-c}ef\arctan\left(\frac{2\sqrt{-ax^4+bx^3+cx^2+bx-a}a\sqrt{-2\,a-c}}{abx^2+ab-(4\,a^2+b^2+2\,ac)x}\right)}{(2\,a^2+ac)d}\right]$$

input
```
integrate((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(
1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2*a + c)*e*f*log((2*a*b^3*x^3 + 2*a*b^3*x + (8*a^4 - a^2*b^2 + 4
*a^3*c)*x^4 + 8*a^4 - a^2*b^2 + 4*a^3*c - (16*a^4 + 10*a^2*b^2 + b^4 + 8*a
^2*c^2 + 4*(6*a^3 + a*b^2)*c)*x^2 - 4*(a^2*b*x^2 + a^2*b - (4*a^3 + a*b^2
+ 2*a^2*c)*x)*sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*sqrt(2*a + c))/(a^2*x
^4 - 2*a*b*x^3 - 2*a*b*x + (2*a^2 + b^2)*x^2 + a^2))/((2*a^2 + a*c)*d), -s
qrt(-2*a - c)*e*f*arctan(2*sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*a*sqrt(-
2*a - c)/(a*b*x^2 + a*b - (4*a^2 + b^2 + 2*a*c)*x))/((2*a^2 + a*c)*d)]
```

3.1010.6 Sympy [F]

$$\int \frac{ef-efx^2}{(-ad+bdx-adx^2)\sqrt{-a+bx+cx^2+bx^3-ax^4}}\,dx$$
$$= \frac{ef\left(\int \frac{x^2}{ax^2\sqrt{-ax^4-a+bx^3+bx+cx^2}+a\sqrt{-ax^4-a+bx^3+bx+cx^2}-bx\sqrt{-ax^4-a+bx^3+bx+cx^2}}\,dx+\int\left(-\frac{}{ax^2\sqrt{-ax^4-a+bx^3+bx+cx^2}+a\sqrt{}}\right.\right.}{d}$$

input
```
integrate((-e*f*x**2+e*f)/(-a*d*x**2+b*d*x-a*d)/(-a*x**4+b*x**3+c*x**2+b*x
-a)**(1/2),x)
```

output
```
e*f*(Integral(x**2/(a*x**2*sqrt(-a*x**4 - a + b*x**3 + b*x + c*x**2) + a*s
qrt(-a*x**4 - a + b*x**3 + b*x + c*x**2) - b*x*sqrt(-a*x**4 - a + b*x**3 +
 b*x + c*x**2)), x) + Integral(-1/(a*x**2*sqrt(-a*x**4 - a + b*x**3 + b*x
+ c*x**2) + a*sqrt(-a*x**4 - a + b*x**3 + b*x + c*x**2) - b*x*sqrt(-a*x**4
 - a + b*x**3 + b*x + c*x**2)), x))/d
```

3.1010.7 Maxima [F]

$$\int \frac{ef-efx^2}{(-ad+bdx-adx^2)\sqrt{-a+bx+cx^2+bx^3-ax^4}}\,dx$$
$$= \int \frac{efx^2-ef}{\sqrt{-ax^4+bx^3+cx^2+bx-a}(adx^2-bdx+ad)}\,dx$$

input
```
integrate((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(
1/2),x, algorithm="maxima")
```

output
```
integrate((e*f*x^2 - e*f)/(sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*(a*d*x^2
 - b*d*x + a*d)), x)
```

3.1010.8 Giac [F]

$$\int \frac{ef - efx^2}{(-ad + bdx - adx^2)\sqrt{-a + bx + cx^2 + bx^3 - ax^4}}\,dx = \int \frac{efx^2 - ef}{\sqrt{-ax^4 + bx^3 + cx^2 + bx - a}(adx^2 - bdx + ad)}\,dx$$

input
```
integrate((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(
1/2),x, algorithm="giac")
```

output
```
integrate((e*f*x^2 - e*f)/(sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*(a*d*x^2
 - b*d*x + a*d)), x)
```

3.1010.9 Mupad [F(-1)]

Timed out.

$$\int \frac{ef - efx^2}{(-ad + bdx - adx^2)\sqrt{-a + bx + cx^2 + bx^3 - ax^4}}\,dx = \int -\frac{e\,f - e\,f\,x^2}{(a\,d\,x^2 - b\,d\,x + a\,d)\,\sqrt{-a\,x^4 + b\,x^3 + c\,x^2 + b\,x - a}}\,dx$$

input
```
int(-(e*f - e*f*x^2)/((a*d - b*d*x + a*d*x^2)*(b*x - a - a*x^4 + b*x^3 + c
*x^2)^(1/2)),x)
```

output
```
int(-(e*f - e*f*x^2)/((a*d - b*d*x + a*d*x^2)*(b*x - a - a*x^4 + b*x^3 + c
*x^2)^(1/2)), x)
```

3.1011 $\int \frac{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

3.1011.1 Optimal result

Integrand size = 59, antiderivative size = 46

$$\int \frac{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\frac{\sqrt{2}b\,\text{arcsinh}\left(\frac{ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
b*arcsinh((a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2))/a^(1/2))*2^(1/2)/a^(1/2)
```

3.1011.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 107 vs. 2(46) = 92.

Time = 5.00 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.33

$$\int \frac{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=-\frac{\sqrt{2}b\sqrt{x\left(-ax+b\sqrt{\frac{a(-1+ax^2)}{b^2}}\right)}\sqrt{x\left(ax+b\sqrt{\frac{a(-1+ax^2)}{b^2}}\right)}\arctan\left(\sqrt{2}\sqrt{x\left(-ax+b\sqrt{\frac{a(-1+ax^2)}{b^2}}\right)}\right)}{ax}$$

input
```
Integrate[Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]/(x*Sqrt[-(a/b^2
) + (a^2*x^2)/b^2]),x]
```

output
```
-((Sqrt[2]*b*Sqrt[x*(-(a*x) + b*Sqrt[(a*(-1 + a*x^2))/b^2])]*Sqrt[x*(a*x +
 b*Sqrt[(a*(-1 + a*x^2))/b^2])]*ArcTan[Sqrt[2]*Sqrt[x*(-(a*x) + b*Sqrt[(a*
(-1 + a*x^2))/b^2])]])/(a*x))
```

3.1011.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2555, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{x\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}}dx$$

$$\Big\downarrow \text{2555}$$

$$\frac{\sqrt{2}b\int \frac{1}{\sqrt{\frac{\left(\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}b+ax\right)^2}{a}+1}}d\left(\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}b+ax\right)}{a}$$

$$\Big\downarrow \text{222}$$

$$\frac{\sqrt{2}b\text{arcsinh}\left(\frac{b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Int[Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]/(x*Sqrt[-(a/b^2) + (a
^2*x^2)/b^2]),x]
```

output
```
(Sqrt[2]*b*ArcSinh[(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt
[a]
```

3.1011.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2555
```
Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c
_) + (d_.)*(x_)^2]), x_Symbol] :> Simp[Sqrt[2]*(b/a)   Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]
```

3.1011.4 Maple [F]

$$\int \frac{\sqrt{a\,x^{2}+bx\sqrt{-\frac{a}{b^{2}}+\frac{a^{2}x^{2}}{b^{2}}}}}{x\sqrt{-\frac{a}{b^{2}}+\frac{a^{2}x^{2}}{b^{2}}}}dx$$

input
```
int((a*x^2+b*x*(-a/b^2+a^2/b^2*x^2)^(1/2))^(1/2)/x/(-a/b^2+a^2/b^2*x^2)^(1
/2),x)
```

output
```
int((a*x^2+b*x*(-a/b^2+a^2/b^2*x^2)^(1/2))^(1/2)/x/(-a/b^2+a^2/b^2*x^2)^(1
/2),x)
```

3.1011.5 Fricas [A] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 161, normalized size of antiderivative = 3.50

$$\int \frac{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$
$$= \left[\frac{\sqrt{2}b\log\left(-4\,ax^2-4\,bx\sqrt{\frac{a^2x^2-a}{b^2}}-2\sqrt{ax^2+bx\sqrt{\frac{a^2x^2-a}{b^2}}}\left(\sqrt{2}\sqrt{a}x+\frac{\sqrt{2}b\sqrt{\frac{a^2x^2-a}{b^2}}}{\sqrt{a}}\right)+1\right)}{2\sqrt{a}},\; -\sqrt{2}b\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{ax^2+bx\sqrt{\frac{a^2x^2-a}{b^2}}}\sqrt{-\frac{1}{a}}}{2\,x}\right)\right]$$

input
```
integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b
^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*b*log(-4*a*x^2 - 4*b*x*sqrt((a^2*x^2 - a)/b^2) - 2*sqrt(a*x^2
 + b*x*sqrt((a^2*x^2 - a)/b^2))*(sqrt(2)*sqrt(a)*x + sqrt(2)*b*sqrt((a^2*x
^2 - a)/b^2)/sqrt(a)) + 1)/sqrt(a), -sqrt(2)*b*sqrt(-1/a)*arctan(1/2*sqrt(
2)*sqrt(a*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2))*sqrt(-1/a)/x)]
```

3.1011.6 Sympy [F]

$$\int \frac{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx = \int \frac{\sqrt{x\left(ax+b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\right)}}{x\sqrt{\frac{a(ax^2-1)}{b^2}}}\,dx$$

input
```
integrate((a*x**2+b*x*(-a/b**2+a**2*x**2/b**2)**(1/2))**(1/2)/x/(-a/b**2+a
**2*x**2/b**2)**(1/2),x)
```

output
```
Integral(sqrt(x*(a*x + b*sqrt(a**2*x**2/b**2 - a/b**2)))/(x*sqrt(a*(a*x**2
 - 1)/b**2)), x)
```

3.1011.7 Maxima [F]

$$\int \frac{\sqrt{ax^2 + bx\sqrt{-\frac{a}{b^2} + \frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2} + \frac{a^2x^2}{b^2}}}\,dx = \int \frac{\sqrt{ax^2 + \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}bx}}{\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}x}\,dx$$

input
```
integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b
^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 -
a/b^2)*x), x)
```

3.1011.8 Giac [F]

$$\int \frac{\sqrt{ax^2 + bx\sqrt{-\frac{a}{b^2} + \frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2} + \frac{a^2x^2}{b^2}}}\,dx = \int \frac{\sqrt{ax^2 + \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}bx}}{\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}x}\,dx$$

input
```
integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b
^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 -
a/b^2)*x), x)
```

3.1011.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ax^2 + bx\sqrt{-\frac{a}{b^2} + \frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2} + \frac{a^2x^2}{b^2}}}\,dx = \int \frac{\sqrt{a\,x^2 + b\,x\,\sqrt{\frac{a^2\,x^2}{b^2} - \frac{a}{b^2}}}}{x\,\sqrt{\frac{a^2\,x^2}{b^2} - \frac{a}{b^2}}}\,dx$$

input
```
int((a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(1/2))^(1/2)/(x*((a^2*x^2)/b^2 -
a/b^2)^(1/2)),x)
```

output
```
int((a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(1/2))^(1/2)/(x*((a^2*x^2)/b^2 -
a/b^2)^(1/2)), x)
```

3.1011. $\int \frac{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

3.1012 $\int \frac{\sqrt{-ax^2+bx\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

3.1012.1 Optimal result

Integrand size = 58, antiderivative size = 46

$$\int \frac{\sqrt{-ax^2+bx\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\frac{\sqrt{2}b\arcsin\left(\frac{ax-b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
b*arcsin((a*x-b*(a/b^2+a^2*x^2/b^2)^(1/2))/a^(1/2))*2^(1/2)/a^(1/2)
```

3.1012.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 114 vs. 2(46) = 92.

Time = 4.44 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.48

$$\int \frac{\sqrt{-ax^2+bx\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\frac{\sqrt{2}b\sqrt{x\left(-ax+b\sqrt{\frac{a(1+ax^2)}{b^2}}\right)}\sqrt{ax\left(ax+b\sqrt{\frac{a(1+ax^2)}{b^2}}\right)}\arctan\left(\frac{\sqrt{2}\sqrt{ax\left(ax+b\sqrt{\frac{a(1+ax^2)}{b^2}}\right)}}{\sqrt{a}}\right)}{a^{3/2}x}$$

input
```
Integrate[Sqrt[-(a*x^2) + b*x*Sqrt[a/b^2 + (a^2*x^2)/b^2]]/(x*Sqrt[a/b^2 +
 (a^2*x^2)/b^2]),x]
```

output
```
(Sqrt[2]*b*Sqrt[x*(-(a*x) + b*Sqrt[(a*(1 + a*x^2))/b^2])]*Sqrt[a*x*(a*x +
b*Sqrt[(a*(1 + a*x^2))/b^2])]*ArcTan[(Sqrt[2]*Sqrt[a*x*(a*x + b*Sqrt[(a*(1
 + a*x^2))/b^2])])/Sqrt[a]])/(a^(3/2)*x)
```

3.1012.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2555, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}-ax^2}}{x\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}}dx$$

$$\Big\downarrow \text{2555}$$

$$-\frac{\sqrt{2}b\int \frac{1}{\sqrt{1-\frac{\left(b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}-ax\right)^2}{a}}}d\left(b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}-ax\right)}{a}$$

$$\Big\downarrow \text{223}$$

$$-\frac{\sqrt{2}b\arcsin\left(\frac{b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}-ax}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Int[Sqrt[-(a*x^2) + b*x*Sqrt[a/b^2 + (a^2*x^2)/b^2]]/(x*Sqrt[a/b^2 + (a^2*
x^2)/b^2]),x]
```

output
```
-((Sqrt[2]*b*ArcSin[(-(a*x) + b*Sqrt[a/b^2 + (a^2*x^2)/b^2])/Sqrt[a]])/Sqr
t[a])
```

3.1012.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2555
```
Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c
_) + (d_.)*(x_)^2]), x_Symbol] :> Simp[Sqrt[2]*(b/a)   Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]
```

3.1012.4 Maple [F]

$$\int \frac{\sqrt{-a\,x^{2}+bx\sqrt{\frac{a}{b^{2}}+\frac{a^{2}x^{2}}{b^{2}}}}}{x\sqrt{\frac{a}{b^{2}}+\frac{a^{2}x^{2}}{b^{2}}}}dx$$

input
```
int((-a*x^2+b*x*(a/b^2+a^2/b^2*x^2)^(1/2))^(1/2)/x/(a/b^2+a^2/b^2*x^2)^(1/
2),x)
```

output
```
int((-a*x^2+b*x*(a/b^2+a^2/b^2*x^2)^(1/2))^(1/2)/x/(a/b^2+a^2/b^2*x^2)^(1/
2),x)
```

3.1012.5 Fricas [A] (verification not implemented)

Time = 5.16 (sec) , antiderivative size = 161, normalized size of antiderivative = 3.50

$$\int \frac{\sqrt{-ax^2+bx\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx = \left[\frac{1}{2}\sqrt{2}b\sqrt{-\frac{1}{a}}\log\left(4\,ax^2-4\,bx\sqrt{\frac{a^2x^2+a}{b^2}} + 2\sqrt{-ax^2+bx\sqrt{\frac{a^2x^2+a}{b^2}}}\left(\sqrt{2}ax\sqrt{-\frac{1}{a}}-\sqrt{2}b\sqrt{-\frac{1}{a}}\sqrt{\frac{a^2x^2+a}{b^2}}\right)+1\right), -\frac{\sqrt{2}b\arctan\left(\frac{\sqrt{2}\sqrt{-ax^2+bx\sqrt{\frac{a^2x^2+a}{b^2}}}}{2\sqrt{a}x}\right)}{\sqrt{a}}\right]$$

input
```
integrate((-a*x^2+b*x*(a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*b*sqrt(-1/a)*log(4*a*x^2 - 4*b*x*sqrt((a^2*x^2 + a)/b^2) + 2*
sqrt(-a*x^2 + b*x*sqrt((a^2*x^2 + a)/b^2))*(sqrt(2)*a*x*sqrt(-1/a) - sqrt(
2)*b*sqrt(-1/a)*sqrt((a^2*x^2 + a)/b^2)) + 1), -sqrt(2)*b*arctan(1/2*sqrt(
2)*sqrt(-a*x^2 + b*x*sqrt((a^2*x^2 + a)/b^2))/(sqrt(a)*x))/sqrt(a)]
```

3.1012.6 Sympy [F]

$$\int \frac{\sqrt{-ax^2+bx\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx = \int \frac{\sqrt{-x\left(ax-b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}\right)}}{x\sqrt{\frac{a(ax^2+1)}{b^2}}}\,dx$$

input
```
integrate((-a*x**2+b*x*(a/b**2+a**2*x**2/b**2)**(1/2))**(1/2)/x/(a/b**2+a*
*2*x**2/b**2)**(1/2),x)
```

output
```
Integral(sqrt(-x*(a*x - b*sqrt(a**2*x**2/b**2 + a/b**2)))/(x*sqrt(a*(a*x**
2 + 1)/b**2)), x)
```

3.1012.7 Maxima [F]

$$\int \frac{\sqrt{-ax^2 + bx\sqrt{\frac{a}{b^2} + \frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2} + \frac{a^2x^2}{b^2}}}\,dx = \int \frac{\sqrt{-ax^2 + \sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}bx}}{\sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}x}\,dx$$

input
```
integrate((-a*x^2+b*x*(a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-a*x^2 + sqrt(a^2*x^2/b^2 + a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 +
 a/b^2)*x), x)
```

3.1012.8 Giac [F]

$$\int \frac{\sqrt{-ax^2 + bx\sqrt{\frac{a}{b^2} + \frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2} + \frac{a^2x^2}{b^2}}}\,dx = \int \frac{\sqrt{-ax^2 + \sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}bx}}{\sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}x}\,dx$$

input
```
integrate((-a*x^2+b*x*(a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-a*x^2 + sqrt(a^2*x^2/b^2 + a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 +
 a/b^2)*x), x)
```

3.1012.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-ax^2+bx\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\int \frac{\sqrt{b\,x\,\sqrt{\frac{a}{b^2}+\frac{a^2\,x^2}{b^2}}-a\,x^2}}{x\,\sqrt{\frac{a}{b^2}+\frac{a^2\,x^2}{b^2}}}\,dx$$

input
```
int((b*x*(a/b^2 + (a^2*x^2)/b^2)^(1/2) - a*x^2)^(1/2)/(x*(a/b^2 + (a^2*x^2
)/b^2)^(1/2)),x)
```

output
```
int((b*x*(a/b^2 + (a^2*x^2)/b^2)^(1/2) - a*x^2)^(1/2)/(x*(a/b^2 + (a^2*x^2
)/b^2)^(1/2)), x)
```

3.1013 $\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

3.1013.1 Optimal result

Integrand size = 58, antiderivative size = 46

$$\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\frac{\sqrt{2}b\operatorname{arcsinh}\left(\frac{ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
b*arcsinh((a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2))/a^(1/2))*2^(1/2)/a^(1/2)
```

3.1013.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 107 vs. 2(46) = 92.

Time = 0.01 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.33

$$\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=-\frac{\sqrt{2}b\sqrt{x\left(-ax+b\sqrt{\frac{a(-1+ax^2)}{b^2}}\right)}\sqrt{x\left(ax+b\sqrt{\frac{a(-1+ax^2)}{b^2}}\right)}\arctan\left(\sqrt{2}\sqrt{x\left(-ax+b\sqrt{\frac{a(-1+ax^2)}{b^2}}\right)}\right)}{ax}$$

input
```
Integrate[Sqrt[x*(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])]/(x*Sqrt[-(a/b^2
) + (a^2*x^2)/b^2]),x]
```

output
```
-((Sqrt[2]*b*Sqrt[x*(-(a*x) + b*Sqrt[(a*(-1 + a*x^2))/b^2])]*Sqrt[x*(a*x +
 b*Sqrt[(a*(-1 + a*x^2))/b^2])]*ArcTan[Sqrt[2]*Sqrt[x*(-(a*x) + b*Sqrt[(a*
(-1 + a*x^2))/b^2])]])/(a*x))
```

3.1013.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052, Rules used = {2556, 2555, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x \left(b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax\right)}}{x\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}}dx$$

$$\Big\downarrow \ 2556$$

$$\int \frac{\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax^2}}{x\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}}dx$$

$$\Big\downarrow \ 2555$$

$$\frac{\sqrt{2}b\int \frac{1}{\sqrt{\frac{\left(\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}b+ax\right)^2}{a}+1}}d\left(\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}b+ax\right)}{a}$$

$$\Big\downarrow \ 222$$

$$\frac{\sqrt{2}b\text{arcsinh}\left(\frac{b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Int[Sqrt[x*(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])]/(x*Sqrt[-(a/b^2) + (a
^2*x^2)/b^2]),x]
```

output
```
(Sqrt[2]*b*ArcSinh[(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt
[a]
```

3.1013.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2555
```
Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c
_) + (d_.)*(x_)^2]), x_Symbol] :> Simp[Sqrt[2]*(b/a)   Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]
```

rule 2556
```
Int[Sqrt[(e_.)*(x_)*((a_.)*(x_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2])]/((x_)*S
qrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Int[Sqrt[a*e*x^2 + b*e*x*Sqrt[c + d
*x^2]]/(x*Sqrt[c + d*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^
2*d, 0] && EqQ[b^2*c*e + a, 0]
```

3.1013.4 Maple [F]

$$\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}dx$$

input
```
int((x*(a*x+b*(-a/b^2+a^2/b^2*x^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2/b^2*x^2)^(1
/2),x)
```

output
```
int((x*(a*x+b*(-a/b^2+a^2/b^2*x^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2/b^2*x^2)^(1
/2),x)
```

3.1013.5 Fricas [A] (verification not implemented)

Time = 5.51 (sec) , antiderivative size = 161, normalized size of antiderivative = 3.50

$$\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx = \left[\frac{\sqrt{2}b\log\left(-4\,ax^2-4\,bx\sqrt{\frac{a^2x^2-a}{b^2}}-2\,\sqrt{ax^2+bx\sqrt{\frac{a^2x^2-a}{b^2}}}\left(\sqrt{2}\sqrt{a}x+\frac{\sqrt{2}b\sqrt{\frac{a^2x^2-a}{b^2}}}{\sqrt{a}}\right)+1\right)}{2\sqrt{a}}, -\sqrt{2}b\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{ax^2+bx\sqrt{\frac{a^2x^2-a}{b^2}}}\sqrt{-\frac{1}{a}}}{2\,x}\right)\right]$$

input
```
integrate((x*(a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2*x^2/b
^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*b*log(-4*a*x^2 - 4*b*x*sqrt((a^2*x^2 - a)/b^2) - 2*sqrt(a*x^2
 + b*x*sqrt((a^2*x^2 - a)/b^2))*(sqrt(2)*sqrt(a)*x + sqrt(2)*b*sqrt((a^2*x
^2 - a)/b^2)/sqrt(a)) + 1)/sqrt(a), -sqrt(2)*b*sqrt(-1/a)*arctan(1/2*sqrt(
2)*sqrt(a*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2))*sqrt(-1/a)/x)]
```

3.1013.6 Sympy [F]

$$\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx = \int \frac{\sqrt{x\left(ax+b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\right)}}{x\sqrt{\frac{a(ax^2-1)}{b^2}}}\,dx$$

input
```
integrate((x*(a*x+b*(-a/b**2+a**2*x**2/b**2)**(1/2)))**(1/2)/x/(-a/b**2+a*
*2*x**2/b**2)**(1/2),x)
```

output
```
Integral(sqrt(x*(a*x + b*sqrt(a**2*x**2/b**2 - a/b**2)))/(x*sqrt(a*(a*x**2
 - 1)/b**2)), x)
```

3.1013.7 Maxima [F]

$$\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\int \frac{\sqrt{\left(ax+\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}b\right)x}}{\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}x}\,dx$$

input
```
integrate((x*(a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2*x^2/b
^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((a*x + sqrt(a^2*x^2/b^2 - a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 -
a/b^2)*x), x)
```

3.1013.8 Giac [F]

$$\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\int \frac{\sqrt{\left(ax+\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}b\right)x}}{\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}x}\,dx$$

input
```
integrate((x*(a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2*x^2/b
^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt((a*x + sqrt(a^2*x^2/b^2 - a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 -
a/b^2)*x), x)
```

3.1013.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\int \frac{\sqrt{x\,\left(a\,x+b\,\sqrt{\frac{a^2\,x^2}{b^2}-\frac{a}{b^2}}\right)}}{x\,\sqrt{\frac{a^2\,x^2}{b^2}-\frac{a}{b^2}}}\,dx$$

---

input
```
int((x*(a*x + b*((a^2*x^2)/b^2 - a/b^2)^(1/2)))^(1/2)/(x*((a^2*x^2)/b^2 -
a/b^2)^(1/2)),x)
```

output
```
int((x*(a*x + b*((a^2*x^2)/b^2 - a/b^2)^(1/2)))^(1/2)/(x*((a^2*x^2)/b^2 -
a/b^2)^(1/2)), x)
```

3.1013. $$\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

3.1014 $\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

3.1014.1 Optimal result

Integrand size = 57, antiderivative size = 46

$$\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\frac{\sqrt{2}b\arcsin\left(\frac{ax-b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
b*arcsin((a*x-b*(a/b^2+a^2*x^2/b^2)^(1/2))/a^(1/2))*2^(1/2)/a^(1/2)
```

3.1014.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 114 vs. 2(46) = 92.

Time = 0.01 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.48

$$\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$
$$=\frac{\sqrt{2}b\sqrt{x\left(-ax+b\sqrt{\frac{a(1+ax^2)}{b^2}}\right)}\sqrt{ax\left(ax+b\sqrt{\frac{a(1+ax^2)}{b^2}}\right)}\arctan\left(\frac{\sqrt{2}\sqrt{ax\left(ax+b\sqrt{\frac{a(1+ax^2)}{b^2}}\right)}}{\sqrt{a}}\right)}{a^{3/2}x}$$

input
```
Integrate[Sqrt[x*(-(a*x) + b*Sqrt[a/b^2 + (a^2*x^2)/b^2])]/(x*Sqrt[a/b^2 +
 (a^2*x^2)/b^2]),x]
```

output
```
(Sqrt[2]*b*Sqrt[x*(-(a*x) + b*Sqrt[(a*(1 + a*x^2))/b^2])]*Sqrt[a*x*(a*x +
b*Sqrt[(a*(1 + a*x^2))/b^2])]*ArcTan[(Sqrt[2]*Sqrt[a*x*(a*x + b*Sqrt[(a*(1
 + a*x^2))/b^2])])/Sqrt[a]])/(a^(3/2)*x)
```

3.1014.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2556, 2555, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x\left(b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}-ax\right)}}{x\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}}dx$$

$$\Big\downarrow \text{2556}$$

$$\int \frac{\sqrt{bx\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}-ax^2}}{x\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}}dx$$

$$\Big\downarrow \text{2555}$$

$$-\frac{\sqrt{2}b\int \frac{1}{\sqrt{1-\frac{\left(b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}-ax\right)^2}{a}}}d\left(b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}-ax\right)}{a}$$

$$\Big\downarrow \text{223}$$

$$-\frac{\sqrt{2}b\arcsin\left(\frac{b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}-ax}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Int[Sqrt[x*(-(a*x) + b*Sqrt[a/b^2 + (a^2*x^2)/b^2])]/(x*Sqrt[a/b^2 + (a^2*
x^2)/b^2]),x]
```

output
```
-((Sqrt[2]*b*ArcSin[(-(a*x) + b*Sqrt[a/b^2 + (a^2*x^2)/b^2])/Sqrt[a]])/Sqr
t[a])
```

3.1014.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2555
```
Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c
_) + (d_.)*(x_)^2]), x_Symbol] :> Simp[Sqrt[2]*(b/a)   Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]
```

rule 2556
```
Int[Sqrt[(e_.)*(x_)*((a_.)*(x_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2])]/((x_)*S
qrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Int[Sqrt[a*e*x^2 + b*e*x*Sqrt[c + d
*x^2]]/(x*Sqrt[c + d*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^
2*d, 0] && EqQ[b^2*c*e + a, 0]
```

3.1014.4 Maple [F]

$$\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}dx$$

input
```
int((x*(-a*x+b*(a/b^2+a^2/b^2*x^2)^(1/2)))^(1/2)/x/(a/b^2+a^2/b^2*x^2)^(1/
2),x)
```

output
```
int((x*(-a*x+b*(a/b^2+a^2/b^2*x^2)^(1/2)))^(1/2)/x/(a/b^2+a^2/b^2*x^2)^(1/
2),x)
```

3.1014.5 Fricas [A] (verification not implemented)

Time = 5.11 (sec) , antiderivative size = 161, normalized size of antiderivative = 3.50

$$\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx = \left[\frac{1}{2}\sqrt{2}b\sqrt{-\frac{1}{a}}\log\left(4\,ax^2-4\,bx\sqrt{\frac{a^2x^2+a}{b^2}} + 2\sqrt{-ax^2+bx\sqrt{\frac{a^2x^2+a}{b^2}}}\left(\sqrt{2}ax\sqrt{-\frac{1}{a}}-\sqrt{2}b\sqrt{-\frac{1}{a}}\sqrt{\frac{a^2x^2+a}{b^2}}\right)+1\right), -\frac{\sqrt{2}b\arctan\left(\frac{\sqrt{2}\sqrt{-ax^2+bx\sqrt{\frac{a^2x^2+a}{b^2}}}}{2\sqrt{a}x}\right)}{\sqrt{a}}\right]$$

input
```
integrate((x*(-a*x+b*(a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*b*sqrt(-1/a)*log(4*a*x^2 - 4*b*x*sqrt((a^2*x^2 + a)/b^2) + 2*
sqrt(-a*x^2 + b*x*sqrt((a^2*x^2 + a)/b^2))*(sqrt(2)*a*x*sqrt(-1/a) - sqrt(
2)*b*sqrt(-1/a)*sqrt((a^2*x^2 + a)/b^2)) + 1), -sqrt(2)*b*arctan(1/2*sqrt(
2)*sqrt(-a*x^2 + b*x*sqrt((a^2*x^2 + a)/b^2))/(sqrt(a)*x))/sqrt(a)]
```

3.1014.6 Sympy [F]

$$\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx = \int \frac{\sqrt{-x\left(ax-b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}\right)}}{x\sqrt{\frac{a(ax^2+1)}{b^2}}}\,dx$$

input
```
integrate((x*(-a*x+b*(a/b**2+a**2*x**2/b**2)**(1/2)))**(1/2)/x/(a/b**2+a**
2*x**2/b**2)**(1/2),x)
```

output
```
Integral(sqrt(-x*(a*x - b*sqrt(a**2*x**2/b**2 + a/b**2)))/(x*sqrt(a*(a*x**
2 + 1)/b**2)), x)
```

3.1014.7 Maxima [F]

$$\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\int \frac{\sqrt{-\left(ax-\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}b\right)x}}{\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}x}\,dx$$

input
```
integrate((x*(-a*x+b*(a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-(a*x - sqrt(a^2*x^2/b^2 + a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 +
 a/b^2)*x), x)
```

3.1014.8 Giac [F]

$$\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\int \frac{\sqrt{-\left(ax-\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}b\right)x}}{\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}x}\,dx$$

input
```
integrate((x*(-a*x+b*(a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-(a*x - sqrt(a^2*x^2/b^2 + a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 +
 a/b^2)*x), x)
```

3.1014. $\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

3.1014.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\int \frac{\sqrt{-x\,\left(a\,x-b\,\sqrt{\frac{a}{b^2}+\frac{a^2\,x^2}{b^2}}\right)}}{x\,\sqrt{\frac{a}{b^2}+\frac{a^2\,x^2}{b^2}}}\,dx$$

input
```
int((-x*(a*x - b*(a/b^2 + (a^2*x^2)/b^2)^(1/2)))^(1/2)/(x*(a/b^2 + (a^2*x^
2)/b^2)^(1/2)),x)
```

output
```
int((-x*(a*x - b*(a/b^2 + (a^2*x^2)/b^2)^(1/2)))^(1/2)/(x*(a/b^2 + (a^2*x^
2)/b^2)^(1/2)), x)
```

3.1015 $\int \frac{-\sqrt{-4+x}-4\sqrt{-1+x}+\sqrt{-4+x}x+\sqrt{-1+x}x}{(1+\sqrt{-4+x}+\sqrt{-1+x})(4-5x+x^2)}\,dx$

3.1015.1 Optimal result

Integrand size = 66, antiderivative size = 19

$$\int \frac{-\sqrt{-4+x}-4\sqrt{-1+x}+\sqrt{-4+x}x+\sqrt{-1+x}x}{(1+\sqrt{-4+x}+\sqrt{-1+x})(4-5x+x^2)}\,dx = 2\log\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)$$

output
```
2*ln(1+(-4+x)^(1/2)+(-1+x)^(1/2))
```

3.1015.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{-\sqrt{-4+x}-4\sqrt{-1+x}+\sqrt{-4+x}x+\sqrt{-1+x}x}{(1+\sqrt{-4+x}+\sqrt{-1+x})(4-5x+x^2)}\,dx = 4\text{arctanh}\left(1-\frac{2\sqrt{-4+x}}{3}+\frac{2\sqrt{-1+x}}{3}\right)$$

input
```
Integrate[(-Sqrt[-4 + x] - 4*Sqrt[-1 + x] + Sqrt[-4 + x]*x + Sqrt[-1 + x]*
x)/((1 + Sqrt[-4 + x] + Sqrt[-1 + x])*(4 - 5*x + x^2)),x]
```

output
```
4*ArcTanh[1 - (2*Sqrt[-4 + x])/3 + (2*Sqrt[-1 + x])/3]
```

3.1015.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 66 vs. $2(19) = 38$.

Time = 2.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.47, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x-4}x+\sqrt{x-1}x-\sqrt{x-4}-4\sqrt{x-1}}{\left(\sqrt{x-4}+\sqrt{x-1}+1\right)(x^2-5x+4)}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{\sqrt{x-4}x}{\left(\sqrt{x-4}+\sqrt{x-1}+1\right)(x^2-5x+4)}+\frac{\sqrt{x-1}x}{\left(\sqrt{x-4}+\sqrt{x-1}+1\right)(x^2-5x+4)}-\frac{\sqrt{x-4}}{\left(\sqrt{x-4}+\sqrt{x-1}+1\right)(x\right.$$

$$\Big\downarrow \text{2009}$$

$$\text{arcsinh}\left(\frac{\sqrt{x-4}}{\sqrt{3}}\right)-\text{arctanh}\left(\sqrt{x-4}\right)+\text{arctanh}\left(\frac{\sqrt{x-1}}{2}\right)+\text{arctanh}\left(\frac{\sqrt{x-1}}{2\sqrt{x-4}}\right)+\frac{1}{2}\log(5-x)$$

input
```
Int[(-Sqrt[-4 + x] - 4*Sqrt[-1 + x] + Sqrt[-4 + x]*x + Sqrt[-1 + x]*x)/((1
 + Sqrt[-4 + x] + Sqrt[-1 + x])*(4 - 5*x + x^2)),x]
```

output
```
ArcSinh[Sqrt[-4 + x]/Sqrt[3]] - ArcTanh[Sqrt[-4 + x]] + ArcTanh[Sqrt[-1 +
x]/2] + ArcTanh[Sqrt[-1 + x]/(2*Sqrt[-4 + x])] + Log[5 - x]/2
```

3.1015.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.1015.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(15) = 30.

Time = 1.38 (sec) , antiderivative size = 147, normalized size of antiderivative = 7.74

method	result
default	$\frac{\ln(-5+x)}{2}-\frac{\ln(1+\sqrt{x-4})}{2}+\frac{\ln(-1+\sqrt{x-4})}{2}+\frac{\ln(\sqrt{x-1}+2)}{2}-\frac{\ln(\sqrt{x-1}-2)}{2}+\frac{7\sqrt{x-4}\,\sqrt{x-1}\,\operatorname{arctanh}\left(\frac{-17+5x}{4\sqrt{x^2-5x+4}}\right)}{4\sqrt{x^2-5x+4}}+$

input
```
int((-(x-4)^(1/2)+x*(x-4)^(1/2)-4*(x-1)^(1/2)+x*(x-1)^(1/2))/(x^2-5*x+4)/(
1+(x-4)^(1/2)+(x-1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(-5+x)-1/2*ln(1+(x-4)^(1/2))+1/2*ln(-1+(x-4)^(1/2))+1/2*ln((x-1)^(1/
2)+2)-1/2*ln((x-1)^(1/2)-2)+7/4*(x-4)^(1/2)*(x-1)^(1/2)/(x^2-5*x+4)^(1/2)*
arctanh(1/4*(-17+5*x)/(x^2-5*x+4)^(1/2))+1/4*(x-4)^(1/2)*(x-1)^(1/2)*(2*ln
(-5/2+x+(x^2-5*x+4)^(1/2))-5*arctanh(1/4*(-17+5*x)/(x^2-5*x+4)^(1/2)))/(x^
2-5*x+4)^(1/2)
```

3.1015.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(15) = 30.

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 5.05

$$\begin{aligned}&\int \frac{-\sqrt{-4+x}-4\sqrt{-1+x}+\sqrt{-4+x}x+\sqrt{-1+x}x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)(4-5x+x^2)}\,dx\\&=-\frac{1}{2}\log\left(-(4x-11)\sqrt{x-1}\sqrt{x-4}+4x^2-21x+23\right)\\&\quad+\frac{1}{2}\log\left(\sqrt{x-1}\sqrt{x-4}-x+7\right)+\frac{1}{2}\log(x-5)+\frac{1}{2}\log\left(\sqrt{x-1}+2\right)\\&\quad-\frac{1}{2}\log\left(\sqrt{x-1}-2\right)-\frac{1}{2}\log\left(\sqrt{x-4}+1\right)+\frac{1}{2}\log\left(\sqrt{x-4}-1\right)\end{aligned}$$

input
```
integrate((-(-4+x)^(1/2)+x*(-4+x)^(1/2)-4*(-1+x)^(1/2)+x*(-1+x)^(1/2))/(x^
2-5*x+4)/(1+(-4+x)^(1/2)+(-1+x)^(1/2)),x, algorithm="fricas")
```

output
```
-1/2*log(-(4*x - 11)*sqrt(x - 1)*sqrt(x - 4) + 4*x^2 - 21*x + 23) + 1/2*lo
g(sqrt(x - 1)*sqrt(x - 4) - x + 7) + 1/2*log(x - 5) + 1/2*log(sqrt(x - 1)
+ 2) - 1/2*log(sqrt(x - 1) - 2) - 1/2*log(sqrt(x - 4) + 1) + 1/2*log(sqrt(
x - 4) - 1)
```

3.1015.6 Sympy [A] (verification not implemented)

Time = 8.67 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{-\sqrt{-4+x}-4\sqrt{-1+x}+\sqrt{-4+x}x+\sqrt{-1+x}x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)(4-5x+x^2)}\,dx = 2\log\left(\sqrt{x-4}+\sqrt{x-1}+1\right)$$

input
```
integrate((-(-4+x)**(1/2)+x*(-4+x)**(1/2)-4*(-1+x)**(1/2)+x*(-1+x)**(1/2))
/(x**2-5*x+4)/(1+(-4+x)**(1/2)+(-1+x)**(1/2)),x)
```

output
```
2*log(sqrt(x - 4) + sqrt(x - 1) + 1)
```

3.1015.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(15) = 30.

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 4.95

$$\int \frac{-\sqrt{-4+x}-4\sqrt{-1+x}+\sqrt{-4+x}x+\sqrt{-1+x}x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)(4-5x+x^2)}\,dx$$
$$= \frac{1}{2}\log(x-1) + \frac{1}{2}\log\left(\frac{2x^2+2\left((x-1)\sqrt{x-4}+2x-6\right)\sqrt{x-1}+2(2x-3)\sqrt{x-4}-7x+3}{2\left((x-1)\sqrt{x-4}+2x-6\right)}\right) + \frac{1}{2}\log\left(\frac{(x-1)\sqrt{x-4}+2x-6}{x-1}\right)$$

input
```
integrate((-(-4+x)^(1/2)+x*(-4+x)^(1/2)-4*(-1+x)^(1/2)+x*(-1+x)^(1/2))/(x^
2-5*x+4)/(1+(-4+x)^(1/2)+(-1+x)^(1/2)),x, algorithm="maxima")
```

output
```
1/2*log(x - 1) + 1/2*log(1/2*(2*x^2 + 2*((x - 1)*sqrt(x - 4) + 2*x - 6)*sq
rt(x - 1) + 2*(2*x - 3)*sqrt(x - 4) - 7*x + 3)/((x - 1)*sqrt(x - 4) + 2*x
- 6)) + 1/2*log(((x - 1)*sqrt(x - 4) + 2*x - 6)/(x - 1))
```

3.1015.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(15) = 30$.

Time = 0.34 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.05

$$\int \frac{-\sqrt{-4+x}-4\sqrt{-1+x}+\sqrt{-4+x}x+\sqrt{-1+x}x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)(4-5x+x^2)}\,dx = -\log\left(\sqrt{x-1}-\sqrt{x-4}+1\right)-\log\left(\sqrt{x-1}-\sqrt{x-4}\right) + \log\left(\sqrt{x-1}+2\right)+\log\left(\left|-\sqrt{x-1}+\sqrt{x-4}-3\right|\right)$$

input
```
integrate((-(-4+x)^(1/2)+x*(-4+x)^(1/2)-4*(-1+x)^(1/2)+x*(-1+x)^(1/2))/(x^
2-5*x+4)/(1+(-4+x)^(1/2)+(-1+x)^(1/2)),x, algorithm="giac")
```

output
```
-log(sqrt(x - 1) - sqrt(x - 4) + 1) - log(sqrt(x - 1) - sqrt(x - 4)) + log
(sqrt(x - 1) + 2) + log(abs(-sqrt(x - 1) + sqrt(x - 4) - 3))
```

3.1015.9 Mupad [B] (verification not implemented)

Time = 22.53 (sec) , antiderivative size = 132, normalized size of antiderivative = 6.95

$$\int \frac{-\sqrt{-4+x}-4\sqrt{-1+x}+\sqrt{-4+x}x+\sqrt{-1+x}x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)(4-5x+x^2)}\,dx = \frac{\ln(x-5)}{2}+2\,\mathrm{atanh}\left(\frac{\sqrt{x-1}-\sqrt{3}}{\sqrt{x-4}}\right)+\frac{7\,\mathrm{atanh}\left(\frac{4\left(\sqrt{x-1}-\sqrt{3}\right)}{\left(\frac{\left(\sqrt{x-1}-\sqrt{3}\right)^2}{x-4}+1\right)\sqrt{x-4}}\right)}{2} - \frac{5\,\mathrm{atanh}\left(\frac{194400\left(\sqrt{x-1}-\sqrt{3}\right)}{\left(\frac{48600\left(\sqrt{x-1}-\sqrt{3}\right)^2}{x-4}+48600\right)\sqrt{x-4}}\right)}{2}-\mathrm{atanh}\left(\sqrt{x-4}\right)+\mathrm{atanh}\left(\frac{\sqrt{x-1}}{2}\right)$$

input
```
int((x*(x - 1)^(1/2) + x*(x - 4)^(1/2) - 4*(x - 1)^(1/2) - (x - 4)^(1/2))/
((x^2 - 5*x + 4)*((x - 1)^(1/2) + (x - 4)^(1/2) + 1)),x)
```

output
```
log(x - 5)/2 + 2*atanh(((x - 1)^(1/2) - 3^(1/2))/(x - 4)^(1/2)) + (7*atanh
((4*((x - 1)^(1/2) - 3^(1/2)))/((((x - 1)^(1/2) - 3^(1/2))^2/(x - 4) + 1)*
(x - 4)^(1/2))))/2 - (5*atanh((194400*((x - 1)^(1/2) - 3^(1/2)))/(((48600*
((x - 1)^(1/2) - 3^(1/2))^2)/(x - 4) + 48600)*(x - 4)^(1/2))))/2 - atanh((
x - 4)^(1/2)) + atanh((x - 1)^(1/2)/2)
```

3.1015. $\int \frac{-\sqrt{-4+x}-4\sqrt{-1+x}+\sqrt{-4+x}x+\sqrt{-1+x}x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)(4-5x+x^2)}\,dx$

3.1016 $\int \frac{1}{x(3+3x+x^2)\sqrt[3]{3+3x+3x^2+x^3}}\,dx$

3.1016.1 Optimal result

Integrand size = 31, antiderivative size = 90

$$\int \frac{1}{x(3+3x+x^2)\sqrt[3]{3+3x+3x^2+x^3}}\,dx = -\frac{\arctan\left(\frac{1+\frac{2\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}}{\sqrt{3}}\right)}{3^{5/6}} - \frac{\log\left(1-(1+x)^3\right)}{6\sqrt[3]{3}} + \frac{\log\left(\sqrt[3]{3}(1+x)-\sqrt[3]{2+(1+x)^3}\right)}{2\sqrt[3]{3}}$$

output
```
-1/3*arctan(1/3*(1+2*3^(1/3)*(1+x)/(2+(1+x)^3)^(1/3))*3^(1/2))*3^(1/6)-1/1
8*ln(1-(1+x)^3)*3^(2/3)+1/6*ln(3^(1/3)*(1+x)-(2+(1+x)^3)^(1/3))*3^(2/3)
```

3.1016.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.00

$$\int \frac{1}{x\left(3+3x+x^2\right)\sqrt[3]{3+3x+3x^2+x^3}}\,dx = \frac{\arctan\left(\frac{\sqrt{3}\sqrt[3]{3+3x+3x^2+x^3}}{2\sqrt[3]{3}+2\sqrt[3]{3}x+\sqrt[3]{3+3x+3x^2+x^3}}\right)}{3^{5/6}} + \frac{2\log\left(\sqrt[3]{3}+\sqrt[3]{3}x-\sqrt[3]{3+3x+3x^2+x^3}\right)-\log\left(3^{2/3}+2\,3^{2/3}x+3^{2/3}x^2+\sqrt[3]{3}(1+x)\sqrt[3]{3+3x+3x^2+\right.}{6\sqrt[3]{3}}$$

input
```
Integrate[1/(x*(3 + 3*x + x^2)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

output
```
ArcTan[(Sqrt[3]*(3 + 3*x + 3*x^2 + x^3)^(1/3))/(2*3^(1/3) + 2*3^(1/3)*x +
(3 + 3*x + 3*x^2 + x^3)^(1/3))]/3^(5/6) + (2*Log[3^(1/3) + 3^(1/3)*x - (3
+ 3*x + 3*x^2 + x^3)^(1/3)] - Log[3^(2/3) + 2*3^(2/3)*x + 3^(2/3)*x^2 + 3^
(1/3)*(1 + x)*(3 + 3*x + 3*x^2 + x^3)^(1/3) + (3 + 3*x + 3*x^2 + x^3)^(2/3
)])/(6*3^(1/3))
```

3.1016.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {940, 938, 25, 901}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(x^2+3x+3\right)\sqrt[3]{x^3+3x^2+3x+3}}dx$$

$$\Big\downarrow \ 940$$

$$\int \frac{1}{\left((x+1)^3-1\right)\sqrt[3]{(x+1)^3+2}}dx$$

$$\Big\downarrow \ 938$$

$$\int -\frac{1}{\left(1-(x+1)^3\right)\sqrt[3]{(x+1)^3+2}}d(x+1)$$

$$\Big\downarrow \ 25$$

$$-\int \frac{1}{(1-(x+1)^3)\sqrt[3]{(x+1)^3+2}}d(x+1)$$

$$\Big\downarrow \; 901$$

$$-\frac{\arctan\left(\frac{\frac{2\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1}{\sqrt{3}}\right)}{3^{5/6}}-\frac{\log\left(1-(x+1)^3\right)}{6\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}(x+1)-\sqrt[3]{(x+1)^3+2}\right)}{2\sqrt[3]{3}}$$

input
```
Int[1/(x*(3 + 3*x + x^2)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

output
```
-(ArcTan[(1 + (2*3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3))/Sqrt[3]]/3^(5/6))
 - Log[1 - (1 + x)^3]/(6*3^(1/3)) + Log[3^(1/3)*(1 + x) - (2 + (1 + x)^3)^
(1/3)]/(2*3^(1/3))
```

3.1016.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 901
```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

rule 938
```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Simp[1/Coefficient[u, x, 1]   Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
 x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u
, x]
```

rule 940
```
Int[(u_)^(p_.)*(v_)^(q_.)*(x_)^(m_.), x_Symbol] :> Int[NormalizePseudoBinom
ial[x^(m/p)*u, x]^p*NormalizePseudoBinomial[v, x]^q, x] /; FreeQ[{p, q}, x]
 && IntegersQ[p, m/p] && PseudoBinomialPairQ[x^(m/p)*u, v, x]
```

3.1016. $\int \frac{1}{x(3+3x+x^2)\sqrt[3]{3+3x+3x^2+x^3}}\,dx$

3.1016.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 17.66 (sec) , antiderivative size = 1673, normalized size of antiderivative = 18.59

method	result	size
trager	Expression too large to display	1673

input
```
int(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/9*RootOf(_Z^3-9)*ln(-(34139998872*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z
^3-9)+81*_Z^2)*x^3+102419996616*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9
)+81*_Z^2)*x^2-6984737648*RootOf(_Z^3-9)*x^3-20954212944*RootOf(_Z^3-9)*x^
2-20954212944*RootOf(_Z^3-9)*x+102419996616*RootOf(RootOf(_Z^3-9)^2+9*_Z*R
ootOf(_Z^3-9)+81*_Z^2)*x-15322002984*(x^3+3*x^2+3*x+3)^(2/3)*RootOf(_Z^3-9
)^2*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x+1828928511*Root
Of(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^3-37
4182374*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9
)^3*x^3-8232012228*RootOf(_Z^3-9)+2619276618*RootOf(_Z^3-9)^3*RootOf(RootO
f(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)+40236427242*RootOf(RootOf(_Z^3-9)
^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)+20428536825*RootOf(RootOf(_Z^3-9)^2+9*_Z*R
ootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*(x^3+3*x^2+3*x+3)^(1/3)*x^2+40857073
650*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(_Z^3-9)*RootOf(RootOf(_Z^3-9)^2+9*_Z*Ro
otOf(_Z^3-9)+81*_Z^2)*x+5107334328*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(_Z^3-9)^
2-8512490709*(x^3+3*x^2+3*x+3)^(2/3)*x-8512490709*(x^3+3*x^2+3*x+3)^(2/3)-
15322002984*(x^3+3*x^2+3*x+3)^(2/3)*RootOf(_Z^3-9)^2*RootOf(RootOf(_Z^3-9)
^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)+5107334328*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x
+3)^(1/3)*x^2+10214668656*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(_Z^3-9)^2*x+20428
536825*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(_Z^3-9)*RootOf(RootOf(_Z^3-9)^2+9*_Z
*RootOf(_Z^3-9)+81*_Z^2)-12802499577*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootO...
```

3.1016.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(71) = 142$.

Time = 5.03 (sec) , antiderivative size = 458, normalized size of antiderivative = 5.09

$$\int \frac{1}{x(3+3x+x^2)\sqrt[3]{3+3x+3x^2+x^3}}\,dx = -\frac{1}{54}$$

$$\cdot 3^{\frac{2}{3}} \log\left(\frac{3\cdot 3^{\frac{2}{3}}(7\,x^4+28\,x^3+42\,x^2+30\,x+9)(x^3+3\,x^2+3\,x+3)^{\frac{2}{3}}+3^{\frac{1}{3}}(31\,x^6+186\,x^5+465\,x^4+66\cdots}{x^6+6\,x^5+15\,x^{\cdots}}\right.$$

$$+\frac{1}{27}$$

$$\cdot 3^{\frac{2}{3}} \log\left(\frac{2\cdot 3^{\frac{2}{3}}(x^3+3\,x^2+3\,x)-9\cdot 3^{\frac{1}{3}}(x^3+3\,x^2+3\,x+3)^{\frac{1}{3}}(x^2+2\,x+1)+9\,(x^3+3\,x^2+3\,x+3)^{\frac{2}{3}}(x\cdots}{x^3+3\,x^2+3\,x}\right.$$

$$-\frac{1}{9}$$

$$\cdot 3^{\frac{1}{6}} \arctan\left(\frac{3^{\frac{1}{6}}\left(12\cdot 3^{\frac{2}{3}}(7\,x^7+49\,x^6+147\,x^5+240\,x^4+225\,x^3+117\,x^2+27\,x)(x^3+3\,x^2+3\,x+3)^{\frac{2}{3}}-\cdots\right.}{\cdots}\right.$$

input
```
integrate(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="fricas")
```

output
```
-1/54*3^(2/3)*log((3*3^(2/3)*(7*x^4 + 28*x^3 + 42*x^2 + 30*x + 9)*(x^3 + 3
*x^2 + 3*x + 3)^(2/3) + 3^(1/3)*(31*x^6 + 186*x^5 + 465*x^4 + 666*x^3 + 60
3*x^2 + 324*x + 81) + 9*(5*x^5 + 25*x^4 + 50*x^3 + 54*x^2 + 33*x + 9)*(x^3
 + 3*x^2 + 3*x + 3)^(1/3))/(x^6 + 6*x^5 + 15*x^4 + 18*x^3 + 9*x^2)) + 1/27
*3^(2/3)*log((2*3^(2/3)*(x^3 + 3*x^2 + 3*x) - 9*3^(1/3)*(x^3 + 3*x^2 + 3*x
 + 3)^(1/3)*(x^2 + 2*x + 1) + 9*(x^3 + 3*x^2 + 3*x + 3)^(2/3)*(x + 1))/(x^
3 + 3*x^2 + 3*x)) - 1/9*3^(1/6)*arctan(1/3*3^(1/6)*(12*3^(2/3)*(7*x^7 + 49
*x^6 + 147*x^5 + 240*x^4 + 225*x^3 + 117*x^2 + 27*x)*(x^3 + 3*x^2 + 3*x +
3)^(2/3) - 3^(1/3)*(127*x^9 + 1143*x^8 + 4572*x^7 + 11070*x^6 + 18414*x^5
+ 22032*x^4 + 18900*x^3 + 11178*x^2 + 4131*x + 729) - 18*(31*x^8 + 248*x^7
 + 868*x^6 + 1782*x^5 + 2400*x^4 + 2196*x^3 + 1332*x^2 + 486*x + 81)*(x^3
+ 3*x^2 + 3*x + 3)^(1/3))/(251*x^9 + 2259*x^8 + 9036*x^7 + 21546*x^6 + 343
98*x^5 + 38556*x^4 + 30348*x^3 + 16038*x^2 + 5103*x + 729))
```

3.1016.6 Sympy [F]

$$\int \frac{1}{x\left(3+3x+x^2\right)\sqrt[3]{3+3x+3x^2+x^3}}\,dx=\int \frac{1}{x\left(x^2+3x+3\right)\sqrt[3]{x^3+3x^2+3x+3}}\,dx$$

input
```
integrate(1/x/(x**2+3*x+3)/(x**3+3*x**2+3*x+3)**(1/3),x)
```

output
```
Integral(1/(x*(x**2 + 3*x + 3)*(x**3 + 3*x**2 + 3*x + 3)**(1/3)), x)
```

3.1016.7 Maxima [F]

$$\int \frac{1}{x\left(3+3x+x^2\right)\sqrt[3]{3+3x+3x^2+x^3}}\,dx=\int \frac{1}{\left(x^3+3\,x^2+3\,x+3\right)^{\frac{1}{3}}\left(x^2+3\,x+3\right)x}\,dx$$

input
```
integrate(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^2 + 3*x + 3)*x), x)
```

3.1016.8 Giac [F]

$$\int \frac{1}{x\left(3+3x+x^2\right)\sqrt[3]{3+3x+3x^2+x^3}}\,dx=\int \frac{1}{\left(x^3+3\,x^2+3\,x+3\right)^{\frac{1}{3}}\left(x^2+3\,x+3\right)x}\,dx$$

input
```
integrate(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^2 + 3*x + 3)*x), x)
```

3.1016.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\,(3+3x+x^2)\,\sqrt[3]{3+3x+3x^2+x^3}}\,dx = \int \frac{1}{x\,\left(x^2+3\,x+3\right)\,{\left(x^3+3\,x^2+3\,x+3\right)}^{1/3}}\,dx$$

input
```
int(1/(x*(3*x + x^2 + 3)*(3*x + 3*x^2 + x^3 + 3)^(1/3)),x)
```

output
```
int(1/(x*(3*x + x^2 + 3)*(3*x + 3*x^2 + x^3 + 3)^(1/3)), x)
```

3.1017 $\int \frac{1-x^2}{(1-x+x^2)(1-x^3)^{2/3}}\,dx$

3.1017.1 Optimal result

Integrand size = 29, antiderivative size = 103

$$\int \frac{1-x^2}{(1-x+x^2)(1-x^3)^{2/3}}\,dx=\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(1+2(1-x)^3-x^3\right)}{2\ 2^{2/3}}+\frac{3\log\left(\sqrt[3]{2}(1-x)+\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

output
```
-1/4*ln(1+2*(1-x)^3-x^3)*2^(1/3)+3/4*ln(2^(1/3)*(1-x)+(-x^3+1)^(1/3))*2^(1
/3)+1/2*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(
1/3)
```

3.1017.2 Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.76

$$\int \frac{1-x^2}{(1-x+x^2)(1-x^3)^{2/3}}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}(1-x^3)^{2/3}}{2^{2/3}+2^{2/3}x+2^{2/3}x^2-(1-x^3)^{2/3}}\right)-2\log\left(2^{2/3}+2^{2/3}x+2^{2/3}x^2+2(1-x^3)^{2/3}\right)+\log\left(-\left((1+x\right.\right.}{2\ 2^{2/3}}$$

input
```
Integrate[(1 - x^2)/((1 - x + x^2)*(1 - x^3)^(2/3)),x]
```

output
```
-1/2*(2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(2/3))/(2^(2/3) + 2^(2/3)*x + 2^
(2/3)*x^2 - (1 - x^3)^(2/3))] - 2*Log[2^(2/3) + 2^(2/3)*x + 2^(2/3)*x^2 +
2*(1 - x^3)^(2/3)] + Log[-((1 + x + x^2)*(2^(1/3) + 2^(1/3)*x^2 - (2 - 2*x
^3)^(2/3) + 2*(1 - x^3)^(1/3) + x*(2^(1/3) - 2*(1 - x^3)^(1/3))))])/2^(2/3
)
```

3.1017.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 425 vs. $2(103) = 206$.

Time $= 0.96$ (sec) , antiderivative size $= 425$, normalized size of antiderivative $= 4.13$, number of steps used $= 2$, number of rules used $= 2$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used $= \{2583, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{(x^2-x+1)(1-x^3)^{2/3}}dx$$

$$\Big\downarrow \text{ 2583}$$

$$\int \left(-\frac{x^3}{(1-x^3)^{2/3}(x^3+1)}+\frac{x}{(1-x^3)^{2/3}(x^3+1)}+\frac{1}{(1-x^3)^{2/3}(x^3+1)}-\frac{x^2}{(1-x^3)^{2/3}(x^3+1)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt[3]{2}\arctan\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\arctan\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\arctan\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\arctan\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log(x^3+1)}{3\ 2^{2/3}}+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}+\frac{1}{3}\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)-\frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}\right)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\ 2^{2/3}}$$

input
```
Int[(1 - x^2)/((1 - x + x^2)*(1 - x^3)^(2/3)),x]
```

output
```
(2^(1/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3
] + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[
3]) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]
) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log[
1 + x^3]/(3*2^(2/3)) + Log[2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/(3*2^(2/3))
- Log[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3
)^(1/3)]/(3*2^(2/3)) + (2^(1/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)]
)/3 - Log[2*2^(1/3) + (1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/(1 - x
^3)^(1/3)]/(6*2^(2/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/3)) - Log[
-(2^(1/3)*x) - (1 - x^3)^(1/3)]/(2*2^(2/3))
```

3.1017.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2583
```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Simp[1/c^q   Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b
*x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && Poly
Q[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denomina
tor[p], 3]
```

3.1017.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 10.01 (sec) , antiderivative size = 692, normalized size of antiderivative = 6.72

method	result	size
trager	Expression too large to display	692

input
```
int((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(-(-2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z
^3-2)^4*x+2*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_
Z^2)*RootOf(_Z^3-2)^2-RootOf(_Z^3-2)^2*x^2+4*(-x^3+1)^(1/3)*RootOf(RootOf(
_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x+RootOf(_Z^3-2)^2*x-4*(-x^3+1)^(1/3
)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-RootOf(_Z^3-2)^2)/(x
^2-x+1))*RootOf(_Z^3-2)-ln(-(-2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2
)+4*_Z^2)*RootOf(_Z^3-2)^4*x+2*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z
*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2-RootOf(_Z^3-2)^2*x^2+4*(-x^3+1)^(
1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x+RootOf(_Z^3-2)^
2*x-4*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-R
ootOf(_Z^3-2)^2)/(x^2-x+1))*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*
_Z^2)+RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*ln((-2*RootOf(Ro
otOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^4*x+2*(-x^3+1)^(
2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2+
RootOf(_Z^3-2)^2*x^2+2*(-x^3+1)^(1/3)*RootOf(_Z^3-2)*x+4*(-x^3+1)^(1/3)*Ro
otOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x-3*RootOf(_Z^3-2)^2*x+2
*(-x^3+1)^(2/3)-2*(-x^3+1)^(1/3)*RootOf(_Z^3-2)-4*(-x^3+1)^(1/3)*RootOf(Ro
otOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)+RootOf(_Z^3-2)^2)/(x^2-x+1))
```

3.1017.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(78) = 156$.

Time = 2.92 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.81

$$\int \frac{1-x^2}{(1-x+x^2)\left(1-x^3\right)^{2/3}}\,dx = -\frac{1}{6}$$

$$\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\Big(2\cdot 4^{\frac{2}{3}}(x^5-x^4-3\,x^3+3\,x^2+x-1)(-x^3+1)^{\frac{1}{3}}+4\,(x^4-4\,x^3+5\,x^2-4\,x+1)(-x}{6\,(3\,x^6-9\,x^5+6\,x^4-x^3+6\,x^2-9\,x}\right.$$

$$-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(\frac{2\cdot 4^{\frac{1}{3}}(-x^3+1)^{\frac{2}{3}}(x^2-3\,x+1)-4^{\frac{2}{3}}(x^4-3\,x^2+1)-8\,(-x^3+1)^{\frac{1}{3}}(x^2-x)}{x^4-2\,x^3+3\,x^2-2\,x+1}\right)$$

$$+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(-\frac{4^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}(x-1)-4^{\frac{1}{3}}(x^2-x+1)-2\,(-x^3+1)^{\frac{2}{3}}}{x^2-x+1}\right)$$

input
```
integrate((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x, algorithm="fricas")
```

output
```
-1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(2*4^(2/3)*(x^5 - x^4 - 3*
x^3 + 3*x^2 + x - 1)*(-x^3 + 1)^(1/3) + 4*(x^4 - 4*x^3 + 5*x^2 - 4*x + 1)*
(-x^3 + 1)^(2/3) + 4^(1/3)*(x^6 - 7*x^5 + 10*x^4 - 7*x^3 + 10*x^2 - 7*x +
1))/(3*x^6 - 9*x^5 + 6*x^4 - x^3 + 6*x^2 - 9*x + 3)) - 1/24*4^(2/3)*log((2
*4^(1/3)*(-x^3 + 1)^(2/3)*(x^2 - 3*x + 1) - 4^(2/3)*(x^4 - 3*x^2 + 1) - 8*
(-x^3 + 1)^(1/3)*(x^2 - x))/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)) + 1/12*4^(2/3
)*log(-(4^(2/3)*(-x^3 + 1)^(1/3)*(x - 1) - 4^(1/3)*(x^2 - x + 1) - 2*(-x^3
 + 1)^(2/3))/(x^2 - x + 1))
```

3.1017.6 Sympy [F]

$$\int \frac{1-x^2}{(1-x+x^2)\left(1-x^3\right)^{2/3}}\,dx = -\int \frac{x^2}{x^2\left(1-x^3\right)^{\frac{2}{3}} - x\left(1-x^3\right)^{\frac{2}{3}} + \left(1-x^3\right)^{\frac{2}{3}}}\,dx - \int \left(-\frac{1}{x^2\left(1-x^3\right)^{\frac{2}{3}} - x\left(1-x^3\right)^{\frac{2}{3}} + \left(1-x^3\right)^{\frac{2}{3}}}\right)dx$$

input
```
integrate((-x**2+1)/(x**2-x+1)/(-x**3+1)**(2/3),x)
```

output
```
-Integral(x**2/(x**2*(1 - x**3)**(2/3) - x*(1 - x**3)**(2/3) + (1 - x**3)*
*(2/3)), x) - Integral(-1/(x**2*(1 - x**3)**(2/3) - x*(1 - x**3)**(2/3) +
(1 - x**3)**(2/3)), x)
```

3.1017.7 Maxima [F]

$$\int \frac{1-x^2}{(1-x+x^2)\left(1-x^3\right)^{2/3}}\,dx = \int -\frac{x^2-1}{\left(-x^3+1\right)^{\frac{2}{3}}\left(x^2-x+1\right)}\,dx$$

input
```
integrate((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 1)/((-x^3 + 1)^(2/3)*(x^2 - x + 1)), x)
```

3.1017.8 Giac [F]

$$\int \frac{1-x^2}{(1-x+x^2)(1-x^3)^{2/3}}\,dx = \int -\frac{x^2-1}{(-x^3+1)^{\frac{2}{3}}(x^2-x+1)}\,dx$$

input
```
integrate((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x, algorithm="giac")
```

output
```
integrate(-(x^2 - 1)/((-x^3 + 1)^(2/3)*(x^2 - x + 1)), x)
```

3.1017.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1-x^2}{(1-x+x^2)(1-x^3)^{2/3}}\,dx = -\int \frac{x^2-1}{(1-x^3)^{2/3}(x^2-x+1)}\,dx$$

input
```
int(-(x^2 - 1)/((1 - x^3)^(2/3)*(x^2 - x + 1)),x)
```

output
```
-int((x^2 - 1)/((1 - x^3)^(2/3)*(x^2 - x + 1)), x)
```

3.1018 $\int \frac{x^2}{\sqrt{-1+x^4}(1+x^4)}\,dx$

3.1018.1 Optimal result

Integrand size = 20, antiderivative size = 49

$$\int \frac{x^2}{\sqrt{-1+x^4}\,(1+x^4)}\,dx=-\frac{1}{4}\arctan\left(\frac{1+x^2}{x\sqrt{-1+x^4}}\right)-\frac{1}{4}\text{arctanh}\left(\frac{1-x^2}{x\sqrt{-1+x^4}}\right)$$

output
```
-1/4*arctan((x^2+1)/x/(x^4-1)^(1/2))-1/4*arctanh((-x^2+1)/x/(x^4-1)^(1/2))
```

3.1018.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{\sqrt{-1+x^4}\,(1+x^4)}\,dx=\left(-\frac{1}{8}-\frac{i}{8}\right)\arctan\left(\frac{(1+i)x}{\sqrt{-1+x^4}}\right)+\left(\frac{1}{8}-\frac{i}{8}\right)\arctan\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-1+x^4}}{x}\right)$$

input
```
Integrate[x^2/(Sqrt[-1 + x^4]*(1 + x^4)),x]
```

output
```
(-1/8 - I/8)*ArcTan[((1 + I)*x)/Sqrt[-1 + x^4]] + (1/8 - I/8)*ArcTan[((1/2
 + I/2)*Sqrt[-1 + x^4])/x]
```

3.1018.3 Rubi [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 175, normalized size of antiderivative = 3.57, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {993, 1535, 763, 2213, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{x^4-1}\,(x^4+1)}\,dx$$

$$\Big\downarrow \text{ 993}$$

$$\frac{1}{2}\int \frac{1}{(x^2+i)\sqrt{x^4-1}}dx-\frac{1}{2}\int \frac{1}{(i-x^2)\sqrt{x^4-1}}dx$$

$$\Big\downarrow \text{ 1535}$$

$$\frac{1}{2}\left(-\frac{1}{2}i\int \frac{1}{\sqrt{x^4-1}}dx-\frac{1}{2}i\int \frac{i-x^2}{(x^2+i)\sqrt{x^4-1}}dx\right)+\frac{1}{2}\left(\frac{1}{2}i\int \frac{1}{\sqrt{x^4-1}}dx+\frac{1}{2}i\int \frac{x^2+i}{(i-x^2)\sqrt{x^4-1}}dx\right)$$

$$\Big\downarrow \text{ 763}$$

$$\frac{1}{2}\left(-\frac{1}{2}i\int \frac{i-x^2}{(x^2+i)\sqrt{x^4-1}}dx-\frac{i\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4-1}}\right)+\frac{1}{2}\left(\frac{1}{2}i\int \frac{x^2+i}{(i-x^2)\sqrt{x^4-1}}dx+\frac{i\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4-1}}\right)$$

$$\Big\downarrow \text{ 2213}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{i-\frac{2x^2}{x^4-1}}d\frac{x}{\sqrt{x^4-1}}-\frac{i\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4-1}}\right)+\frac{1}{2}\left(\frac{i\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4-1}}-\frac{1}{2}\int \frac{1}{\frac{2x^2}{x^4-1}+i}d\frac{x}{\sqrt{x^4-1}}\right)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{i-\frac{2x^2}{x^4-1}}d\frac{x}{\sqrt{x^4-1}}-\frac{i\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4-1}}\right)+\frac{1}{2}\left(\frac{i\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4-1}}+\left(\frac{1}{4}+\frac{i}{4}\right)\text{arctanh}\left(\frac{(1+i)x}{\sqrt{x^4-1}}\right)\right)$$

$$\downarrow 219$$

$$\frac{1}{2}\left(\left(-\frac{1}{4}-\frac{i}{4}\right)\arctan\left(\frac{(1+i)x}{\sqrt{x^4-1}}\right)-\frac{i\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4-1}}\right)+\frac{1}{2}\left(\frac{i\sqrt{x^2-1}\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4-1}}+\left(\frac{1}{4}+\frac{i}{4}\right)\text{arctanh}\left(\frac{(1+i)x}{\sqrt{x^4-1}}\right)\right)$$

input
```
Int[x^2/(Sqrt[-1 + x^4]*(1 + x^4)),x]
```

output
```
((-1/4 - I/4)*ArcTan[((1 + I)*x)/Sqrt[-1 + x^4]] - ((I/2)*Sqrt[-1 + x^2]*S
qrt[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/(Sqrt[2]*
Sqrt[-1 + x^4]))/2 + ((1/4 + I/4)*ArcTanh[((1 + I)*x)/Sqrt[-1 + x^4]] + ((
I/2)*Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x
^2]], 1/2])/(Sqrt[2]*Sqrt[-1 + x^4]))/2
```

3.1018.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 763
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]))
*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

rule 993
```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*
b)   Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Simp[s/(2*b)   Int[1/((r
 - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

rule 1535
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
1/(2*d)   Int[1/Sqrt[a + c*x^4], x], x] + Simp[1/(2*d)   Int[(d - e*x^2)/((
d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2
+ a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

rule 2213
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Simp[A   Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^
4]], x] /; FreeQ[{a, c, d, e, A, B}, x] && EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d
 + A*e, 0]
```

3.1018.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.87 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\left(\frac{1}{8}-\frac{i}{8}\right)\left(\ln\left(2\right)+\ln\left(\frac{(1-i)\sqrt{x^4-1}-2ix}{x^2+i}\right)+\arctan\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{x^4-1}}{x}\right)\right)$
default	$\frac{\ln\left(\frac{1+\frac{x^4-1}{2x^2}+\frac{\sqrt{x^4-1}}{x}}{1+\frac{x^4-1}{2x^2}-\frac{\sqrt{x^4-1}}{x}}\right)}{16}+\frac{\arctan\left(\frac{\sqrt{x^4-1}}{x}+1\right)}{8}+\frac{\arctan\left(\frac{\sqrt{x^4-1}}{x}-1\right)}{8}$
elliptic	$\frac{\ln\left(\frac{1+\frac{x^4-1}{2x^2}+\frac{\sqrt{x^4-1}}{x}}{1+\frac{x^4-1}{2x^2}-\frac{\sqrt{x^4-1}}{x}}\right)}{16}+\frac{\arctan\left(\frac{\sqrt{x^4-1}}{x}+1\right)}{8}+\frac{\arctan\left(\frac{\sqrt{x^4-1}}{x}-1\right)}{8}$
trager	$\operatorname{RootOf}\left(32\_Z^2-8\_Z+1\right)\ln\left(-\frac{8\operatorname{RootOf}\left(32\_Z^2-8\_Z+1\right)x+\sqrt{x^4-1}}{8x^2\operatorname{RootOf}\left(32\_Z^2-8\_Z+1\right)-x^2+1}\right)+\frac{\ln\left(\frac{8\operatorname{RootOf}\left(32\_Z^2-8\_\right.}{8x^2\operatorname{RootOf}\left(32\_Z^2-\right.}\right.}{4}$

input
```
int(x^2/(x^4+1)/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(1/8-1/8*I)*(ln(2)+ln(((1-I)*(x^4-1)^(1/2)-2*I*x)/(x^2+I))+arctan((1/2+1/2
*I)*(x^4-1)^(1/2)/x))
```

3.1018.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{x^2}{\sqrt{-1+x^4}\,(1+x^4)}\,dx = \frac{1}{4}\arctan\left(\frac{\sqrt{x^4-1}x}{x^2+1}\right)+\frac{1}{8}\log\left(\frac{x^4+2\,x^2+2\,\sqrt{x^4-1}x-1}{x^4+1}\right)$$

input
```
integrate(x^2/(x^4+1)/(x^4-1)^(1/2),x, algorithm="fricas")
```

output
```
1/4*arctan(sqrt(x^4 - 1)*x/(x^2 + 1)) + 1/8*log((x^4 + 2*x^2 + 2*sqrt(x^4
- 1)*x - 1)/(x^4 + 1))
```

3.1018.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{-1+x^4}\,(1+x^4)}\,dx = \int \frac{x^2}{\sqrt{(x-1)\,(x+1)\,(x^2+1)}\,(x^4+1)}\,dx$$

input
```
integrate(x**2/(x**4+1)/(x**4-1)**(1/2),x)
```

output
```
Integral(x**2/(sqrt((x - 1)*(x + 1)*(x**2 + 1))*(x**4 + 1)), x)
```

3.1018.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{-1+x^4}\,(1+x^4)}\,dx = \int \frac{x^2}{(x^4+1)\sqrt{x^4-1}}\,dx$$

input
```
integrate(x^2/(x^4+1)/(x^4-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/((x^4 + 1)*sqrt(x^4 - 1)), x)
```

3.1018.8 Giac [F]

$$\int \frac{x^2}{\sqrt{-1+x^4}\,(1+x^4)}\,dx = \int \frac{x^2}{(x^4+1)\sqrt{x^4-1}}\,dx$$

input
```
integrate(x^2/(x^4+1)/(x^4-1)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/((x^4 + 1)*sqrt(x^4 - 1)), x)
```

3.1018.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{-1+x^4}\,(1+x^4)}\,dx = \int \frac{x^2}{\sqrt{x^4-1}\,(x^4+1)}\,dx$$

input
```
int(x^2/((x^4 - 1)^(1/2)*(x^4 + 1)),x)
```

output
```
int(x^2/((x^4 - 1)^(1/2)*(x^4 + 1)), x)
```

3.1019 $\int \frac{a-cx^4}{(ae+cdx^2)(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

3.1019.1 Optimal result

Integrand size = 46, antiderivative size = 80

$$\int \frac{a-cx^4}{(ae+cdx^2)(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx=\frac{\arctan\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{e}\sqrt{cd^2-bde+ae^2}}$$

output
```
arctan(x*(a*e^2-b*d*e+c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(c*x^4+b*x^2+a)^(1/2))/
d^(1/2)/e^(1/2)/(a*e^2-b*d*e+c*d^2)^(1/2)
```

3.1019.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 10.57 (sec) , antiderivative size = 383, normalized size of antiderivative = 4.79

$$\int \frac{a-cx^4}{(ae+cdx^2)(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$
$$=\frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right),\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\text{EllipticPi}\left(\frac{(b+\cdots}{\cdots}\right.\right.}{\sqrt{2}\sqrt{\frac{\cdots}{b+\cdots}}\cdots}$$

input
```
Integrate[(a - c*x^4)/((a*e + c*d*x^2)*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]
),x]
```

output
```
(I*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1
+ (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(
b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]
)] - EllipticPi[((b + Sqrt[b^2 - 4*a*c])*d)/(2*a*e), I*ArcSinh[Sqrt[2]*Sqr
t[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4
*a*c])] - EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2
]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^
2 - 4*a*c])]))/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e*Sqrt[a + b*x^2
 + c*x^4])
```

3.1019.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2537, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a-cx^4}{(d+ex^2)\sqrt{a+bx^2+cx^4}\,(ae+cdx^2)}dx$$

$$\Big\downarrow \text{2537}$$

$$a\int \frac{1}{\frac{a(cd^2-bed+ae^2)x^2}{cx^4+bx^2+a}+ade}d\frac{x}{\sqrt{cx^4+bx^2+a}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{e}\sqrt{ae^2-bde+cd^2}}$$

input
```
Int[(a - c*x^4)/((a*e + c*d*x^2)*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

output
```
ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c
*x^4])]/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e + a*e^2])
```

3.1019.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2537
```
Int[((u_)*((A_) + (B_.)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v,
x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Co
eff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Simp[A   Subst[Int[1/(d - (b*d - a*e
)*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0]] /;
 FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]
```

3.1019.4 Maple [A] (verified)

Time = 3.40 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\arctan\left(\frac{de\sqrt{c\,x^{4}+b\,x^{2}+a}}{x\sqrt{\left(a\,e^{2}-bde+c\,d^{2}\right)de}}\right)}{\sqrt{(a\,e^{2}-bde+c\,d^{2})de}}$	66
elliptic	$-\frac{\arctan\left(\frac{de\sqrt{c\,x^{4}+b\,x^{2}+a}}{x\sqrt{\left(a\,e^{2}-bde+c\,d^{2}\right)de}}\right)}{\sqrt{(a\,e^{2}-bde+c\,d^{2})de}}$	66
pseudoelliptic	$-\frac{\arctan\left(\frac{de\sqrt{c\,x^{4}+b\,x^{2}+a}}{x\sqrt{\left(a\,e^{2}-bde+c\,d^{2}\right)de}}\right)}{\sqrt{(a\,e^{2}-bde+c\,d^{2})de}}$	66

input
```
int((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/((a*e^2-b*d*e+c*d^2)*d*e)^(1/2)*arctan(d*e*(c*x^4+b*x^2+a)^(1/2)/x/((a*
e^2-b*d*e+c*d^2)*d*e)^(1/2))
```

3.1019.5 Fricas [A] (verification not implemented)

Time = 28.97 (sec) , antiderivative size = 472, normalized size of antiderivative = 5.90

$$\int \frac{a-cx^4}{(ae+cdx^2)(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$
$$= \left[-\frac{\sqrt{-cd^3e+bd^2e^2-ade^3}\log\left(-\frac{c^2d^2e^2x^8-2\,(3\,c^2d^3e-4\,bcd^2e^2+3\,acde^3)x^6+a^2d^2e^2+(c^2d^4-8\,bcd^3e-8\,abde^3+a^2e^4+4\,(2\,b^2+ac}{c^2d^2e^2x^8+2\,(c^2d^3e+acde^3)x^6+}\right.}{4\,(cd^3e-bd^2e^2+}\right.$$

input
```
integrate((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algo
rithm="fricas")
```

output
```
[-1/4*sqrt(-c*d^3*e + b*d^2*e^2 - a*d*e^3)*log(-(c^2*d^2*e^2*x^8 - 2*(3*c^
2*d^3*e - 4*b*c*d^2*e^2 + 3*a*c*d*e^3)*x^6 + a^2*d^2*e^2 + (c^2*d^4 - 8*b*
c*d^3*e - 8*a*b*d*e^3 + a^2*e^4 + 4*(2*b^2 + a*c)*d^2*e^2)*x^4 - 2*(3*a*c*
d^3*e - 4*a*b*d^2*e^2 + 3*a^2*d*e^3)*x^2 + 4*(c*d*e*x^5 + a*d*e*x - (c*d^2
 - 2*b*d*e + a*e^2)*x^3)*sqrt(-c*d^3*e + b*d^2*e^2 - a*d*e^3)*sqrt(c*x^4 +
 b*x^2 + a))/(c^2*d^2*e^2*x^8 + 2*(c^2*d^3*e + a*c*d*e^3)*x^6 + a^2*d^2*e^
2 + (c^2*d^4 + 4*a*c*d^2*e^2 + a^2*e^4)*x^4 + 2*(a*c*d^3*e + a^2*d*e^3)*x^
2))/(c*d^3*e - b*d^2*e^2 + a*d*e^3), 1/2*arctan(2*sqrt(c*d^3*e - b*d^2*e^2
 + a*d*e^3)*sqrt(c*x^4 + b*x^2 + a)*x/(c*d*e*x^4 + a*d*e - (c*d^2 - 2*b*d*
e + a*e^2)*x^2))/sqrt(c*d^3*e - b*d^2*e^2 + a*d*e^3)]
```

3.1019.6 Sympy [F]

$$\int \frac{a-cx^4}{(ae+cdx^2)(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx =$$
$$-\int\left(-\frac{a}{ade\sqrt{a+bx^2+cx^4}+ae^2x^2\sqrt{a+bx^2+cx^4}+cd^2x^2\sqrt{a+bx^2+cx^4}+cdex^4\sqrt{a+bx^2+cx^4}}\right)d$$
$$-\int\frac{cx^4}{ade\sqrt{a+bx^2+cx^4}+ae^2x^2\sqrt{a+bx^2+cx^4}+cd^2x^2\sqrt{a+bx^2+cx^4}+cdex^4\sqrt{a+bx^2+cx^4}}\,dx$$

input
```
integrate((-c*x**4+a)/(c*d*x**2+a*e)/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x
)
```

output
```
-Integral(-a/(a*d*e*sqrt(a + b*x**2 + c*x**4) + a*e**2*x**2*sqrt(a + b*x**
2 + c*x**4) + c*d**2*x**2*sqrt(a + b*x**2 + c*x**4) + c*d*e*x**4*sqrt(a +
b*x**2 + c*x**4)), x) - Integral(c*x**4/(a*d*e*sqrt(a + b*x**2 + c*x**4) +
 a*e**2*x**2*sqrt(a + b*x**2 + c*x**4) + c*d**2*x**2*sqrt(a + b*x**2 + c*x
**4) + c*d*e*x**4*sqrt(a + b*x**2 + c*x**4)), x)
```

3.1019.7 Maxima [F]

$$\int \frac{a-cx^4}{(ae+cdx^2)(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int -\frac{cx^4-a}{\sqrt{cx^4+bx^2+a}(cdx^2+ae)(ex^2+d)}\,dx$$

input
```
integrate((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algo
rithm="maxima")
```

output
```
-integrate((c*x^4 - a)/(sqrt(c*x^4 + b*x^2 + a)*(c*d*x^2 + a*e)*(e*x^2 + d
)), x)
```

3.1019.8 Giac [F]

$$\int \frac{a-cx^4}{(ae+cdx^2)(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \int -\frac{cx^4-a}{\sqrt{cx^4+bx^2+a}(cdx^2+ae)(ex^2+d)}\,dx$$

input
```
integrate((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algo
rithm="giac")
```

output
```
integrate(-(c*x^4 - a)/(sqrt(c*x^4 + b*x^2 + a)*(c*d*x^2 + a*e)*(e*x^2 + d
)), x)
```

3.1019.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a-cx^4}{(ae+cdx^2)\,(d+ex^2)\,\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{a-c\,x^4}{(e\,x^2+d)\,\left(c\,d\,x^2+a\,e\right)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

input
```
int((a - c*x^4)/((d + e*x^2)*(a*e + c*d*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

output
```
int((a - c*x^4)/((d + e*x^2)*(a*e + c*d*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x
)
```

3.1020 $\int \left(x+\frac{1-x^2}{1+x}\right) \, dx$

3.1020.1 Optimal result

Integrand size = 15, antiderivative size = 1

$$\int \left(x+\frac{1-x^2}{1+x}\right) \, dx=x$$

output
```
x
```

3.1020.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \left(x+\frac{1-x^2}{1+x}\right) \, dx=x$$

input
```
Integrate[x + (1 - x^2)/(1 + x),x]
```

output
```
x
```

3.1020.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{1-x^2}{x+1}+x\right) dx$$
$$\Big\downarrow \ 2009$$
$$x$$

input
```
Int[x + (1 - x^2)/(1 + x),x]
```

output
```
x
```

3.1020.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.1020.4 Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
default	x	2
norman	x	2
risch	x	2

input
```
int(x+(-x^2+1)/(x+1),x,method=_RETURNVERBOSE)
```

output
```
x
```

3.1020.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \left(x+\frac{1-x^2}{1+x}\right) dx = x$$

input
```
integrate(x+(-x^2+1)/(1+x),x, algorithm="fricas")
```

output
```
x
```

3.1020.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 0, normalized size of antiderivative = 0.00

$$\int \left(x+\frac{1-x^2}{1+x}\right) dx = x$$

input
```
integrate(x+(-x**2+1)/(1+x),x)
```

output
```
x
```

3.1020.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \left(x+\frac{1-x^2}{1+x}\right) dx = x$$

input
```
integrate(x+(-x^2+1)/(1+x),x, algorithm="maxima")
```

output
```
x
```

3.1020.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \left(x+\frac{1-x^2}{1+x}\right)\,dx=x$$

input
```
integrate(x+(-x^2+1)/(1+x),x, algorithm="giac")
```

output
```
x
```

3.1020.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \left(x+\frac{1-x^2}{1+x}\right)\,dx=x$$

input
```
int(x - (x^2 - 1)/(x + 1),x)
```

output
```
x
```

3.1021 $\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}} \, dx$

3.1021.1 Optimal result

Integrand size = 17, antiderivative size = 42

$$\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}} \, dx = \arcsin(x)-\frac{\arctan\left(\frac{1+4x\sqrt{1-x^2}}{\sqrt{3}(1-2x^2)}\right)}{\sqrt{3}}$$

output
```
arcsin(x)-1/3*arctan(1/3*(1+4*x*(-x^2+1)^(1/2))*3^(1/2)/(-2*x^2+1))*3^(1/2
)
```

3.1021.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.07

$$\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}} \, dx = -2\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)+\text{RootSum}\Big[1+2\#1+2\#1^2-2\#1^3 +\#1^4\&,\frac{-\log(-1+x)+\log\left(\sqrt{1-x^2}+\#1-x\#1\right)-\log(-1+x)\#1^2+\log\left(\sqrt{1-x^2}+\#1-x\#1\right)\ldots}{1+2\#1-3\#1^2+2\#1^3}$$

input
```
Integrate[(x^(-1) + Sqrt[1 - x^2])^(-1),x]
```

output
```
-2*ArcTan[Sqrt[1 - x^2]/(1 + x)] + RootSum[1 + 2*#1 + 2*#1^2 - 2*#1^3 + #1
^4 & , (-Log[-1 + x] + Log[Sqrt[1 - x^2] + #1 - x*#1] - Log[-1 + x]*#1^2 +
 Log[Sqrt[1 - x^2] + #1 - x*#1]*#1^2)/(1 + 2*#1 - 3*#1^2 + 2*#1^3) & ]
```

3.1021.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.90, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-x^2}+\frac{1}{x}}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{x}{x^4-x^2+1}-\frac{x^2\sqrt{1-x^2}}{x^4-x^2+1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\arcsin(x)-\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{x}{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}$$

input
```
Int[(x^(-1) + Sqrt[1 - x^2])^(-1),x]
```

output
```
ArcSin[x] - ArcTan[(1 - 2*x^2)/Sqrt[3]]/Sqrt[3] - ArcTan[x/(Sqrt[-((I - Sq
rt[3])/(I + Sqrt[3]))]*Sqrt[1 - x^2])]/Sqrt[3] - ArcTan[(Sqrt[-((I - Sqrt[
3])/(I + Sqrt[3]))]*x)/Sqrt[1 - x^2]]/Sqrt[3]
```

3.1021.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.1021.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.39 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.12

method	result
trager	$\operatorname{RootOf}(\_Z^2+1)\ln\left(\operatorname{RootOf}(\_Z^2+1)\sqrt{-x^2+1}+x\right)+\frac{\operatorname{RootOf}(\_Z^2+3)\ln\left(\frac{2\operatorname{RootOf}(\_Z^2+3)x^2+3x}{\operatorname{RootOf}(\_Z}\right.}{3}$
default	$\frac{i\sqrt{3}\ln\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{\left(-1+i\sqrt{3}\right)\left(\sqrt{-x^2+1}-1\right)}{x}-1\right)}{6}-\frac{i\sqrt{3}\ln\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{\left(-1-i\sqrt{3}\right)\left(\sqrt{-x^2+1}-1\right)}{x}-1\right)}{6}+\frac{i\sqrt{3}\ln\left(\left(\sqrt{-x}\right.\right.}{}$

input
```
int(1/(1/x+(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
RootOf(_Z^2+1)*ln(RootOf(_Z^2+1)*(-x^2+1)^(1/2)+x)+1/3*RootOf(_Z^2+3)*ln((
2*RootOf(_Z^2+3)*x^2+3*x*(-x^2+1)^(1/2)-RootOf(_Z^2+3))/(RootOf(_Z^2+3)*x^
2-x^2+2))
```

3.1021.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.74

$$\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right) + \frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(2x^2-1)\sqrt{-x^2+1}}{3(x^3-x)}\right) - 2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

input
```
integrate(1/(1/x+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/3*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x^2 - 1)*sqrt(-x^2 + 1)/(x^3 - x)) - 2*arctan((sqrt(-x^2 + 1) - 1)/x
)
```

3.1021.6 Sympy [F]

$$\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}}\,dx=\int \frac{x}{x\sqrt{1-x^2}+1}\,dx$$

input
```
integrate(1/(1/x+(-x**2+1)**(1/2)),x)
```

output
```
Integral(x/(x*sqrt(1 - x**2) + 1), x)
```

3.1021.7 Maxima [F]

$$\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}}\,dx=\int \frac{1}{\sqrt{-x^2+1}+\frac{1}{x}}\,dx$$

input
```
integrate(1/(1/x+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^2 + 1) + 1/x), x)
```

3.1021.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(37) = 74$.

Time = 0.37 (sec) , antiderivative size = 193, normalized size of antiderivative = 4.60

$$\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}}\,dx$$
$$= \frac{1}{2}\pi\mathrm{sgn}(x) - \frac{1}{6}\sqrt{3}\left(\pi\mathrm{sgn}(x) + 2\arctan\left(-\frac{\sqrt{3}x\left(\frac{\sqrt{-x^2+1}-1}{x} + \frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2} - 1\right)}{3\left(\sqrt{-x^2+1}-1\right)}\right)\right)$$
$$- \frac{1}{6}\sqrt{3}\left(\pi\mathrm{sgn}(x) + 2\arctan\left(\frac{\sqrt{3}x\left(\frac{\sqrt{-x^2+1}-1}{x} - \frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2} + 1\right)}{3\left(\sqrt{-x^2+1}-1\right)}\right)\right)$$
$$+ \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right) + \arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2} - 1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)$$

input
```
integrate(1/(1/x+(-x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
1/2*pi*sgn(x) - 1/6*sqrt(3)*(pi*sgn(x) + 2*arctan(-1/3*sqrt(3)*x*((sqrt(-x
^2 + 1) - 1)/x + (sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))) -
1/6*sqrt(3)*(pi*sgn(x) + 2*arctan(1/3*sqrt(3)*x*((sqrt(-x^2 + 1) - 1)/x -
(sqrt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1))) + 1/3*sqrt(3)*arcta
n(1/3*sqrt(3)*(2*x^2 - 1)) + arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1
)/(sqrt(-x^2 + 1) - 1))
```

3.1021. $\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}}\,dx$

3.1021.9 Mupad [B] (verification not implemented)

Time = 20.26 (sec) , antiderivative size = 549, normalized size of antiderivative = 13.07

$$\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}}\,dx = \operatorname{asin}(x) - \frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)-1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}}-\sqrt{1-x^2}\,1\mathrm{i}}{\frac{\sqrt{3}}{2}-x+\frac{1}{2}\mathrm{i}}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}\left(\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}\right)} + \frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)-1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}\left(-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}\right)} - \frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)+1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}\left(-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}\right)} - \frac{\ln\left(x-\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)}{\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}} - \frac{\ln\left(x+\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)}{\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}} + \frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)+1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}\left(\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}\right)} + \frac{\ln\left(x-\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)}{-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}} + \frac{\ln\left(x+\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)}{-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}}$$

input
```
int(1/(1/x + (1 - x^2)^(1/2)),x)
```

output
```
asin(x) - log((((x*(3^(1/2)/2 + 1i/2) - 1)*1i)/(1 - (3^(1/2)/2 + 1i/2)^2)^
(1/2) - (1 - x^2)^(1/2)*1i)/(3^(1/2)/2 - x + 1i/2))/((1 - (3^(1/2)/2 + 1i/
2)^2)^(1/2)*(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)) + log((((x*(3^(1/2)/2
 - 1i/2) - 1)*1i)/(1 - (3^(1/2)/2 - 1i/2)^2)^(1/2) - (1 - x^2)^(1/2)*1i)/(
x - 3^(1/2)/2 + 1i/2))/((1 - (3^(1/2)/2 - 1i/2)^2)^(1/2)*(4*(3^(1/2)/2 - 1
i/2)^3 - 3^(1/2) + 1i)) - log((((x*(3^(1/2)/2 - 1i/2) + 1)*1i)/(1 - (3^(1/
2)/2 - 1i/2)^2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + 3^(1/2)/2 - 1i/2))/((1 -
(3^(1/2)/2 - 1i/2)^2)^(1/2)*(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i)) - (lo
g(x - 3^(1/2)/2 - 1i/2)*(3^(1/2)/2 + 1i/2))/(3^(1/2) - 4*(3^(1/2)/2 + 1i/2
)^3 + 1i) - (log(x + 3^(1/2)/2 + 1i/2)*(3^(1/2)/2 + 1i/2))/(3^(1/2) - 4*(3
^(1/2)/2 + 1i/2)^3 + 1i) + log((((x*(3^(1/2)/2 + 1i/2) + 1)*1i)/(1 - (3^(1
/2)/2 + 1i/2)^2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + 3^(1/2)/2 + 1i/2))/((1 -
 (3^(1/2)/2 + 1i/2)^2)^(1/2)*(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)) + (l
og(x - 3^(1/2)/2 + 1i/2)*(3^(1/2)/2 - 1i/2))/(4*(3^(1/2)/2 - 1i/2)^3 - 3^(
1/2) + 1i) + (log(x + 3^(1/2)/2 - 1i/2)*(3^(1/2)/2 - 1i/2))/(4*(3^(1/2)/2
- 1i/2)^3 - 3^(1/2) + 1i)
```

3.1021. $\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}}\,dx$

3.1022 $\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}}\,dx$

3.1022.1 Optimal result

Integrand size = 33, antiderivative size = 42

$$\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}}\,dx = \arcsin(x) - \frac{\arctan\left(\frac{1+4x\sqrt{1-x^2}}{\sqrt{3}(1-2x^2)}\right)}{\sqrt{3}}$$

output
```
arcsin(x)-1/3*arctan(1/3*(1+4*x*(-x^2+1)^(1/2))*3^(1/2)/(-2*x^2+1))*3^(1/2
)
```

3.1022.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.07

$$\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}}\,dx = -2\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right) + \text{RootSum}\Big[1+2\#1+2\#1^2-2\#1^3 +\#1^4\&, \frac{-\log(-1+x)+\log\left(\sqrt{1-x^2}+\#1-x\#1\right)-\log(-1+x)\#1^2+\log\left(\sqrt{1-x^2}+\#1-x\#1\right)}{1+2\#1-3\#1^2+2\#1^3}$$

input
```
Integrate[(x*Sqrt[1 - x^2])/(x - x^3 + Sqrt[1 - x^2]),x]
```

output
```
-2*ArcTan[Sqrt[1 - x^2]/(1 + x)] + RootSum[1 + 2*#1 + 2*#1^2 - 2*#1^3 + #1
^4 & , (-Log[-1 + x] + Log[Sqrt[1 - x^2] + #1 - x*#1] - Log[-1 + x]*#1^2 +
 Log[Sqrt[1 - x^2] + #1 - x*#1]*#1^2)/(1 + 2*#1 - 3*#1^2 + 2*#1^3) & ]
```

3.1022.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 149, normalized size of antiderivative = 3.55, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{1-x^2}}{-x^3+\sqrt{1-x^2}+x}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{\sqrt{1-x^2}x^2}{x^4-x^2+1}+\frac{(1-x^2)x^3}{x^4-x^2+1}+\frac{x-1}{2}+\frac{x+1}{2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\arcsin(x)-\frac{\arctan\left(\frac{1-2x^2}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{x}{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{\arctan\left(\frac{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{x^2}{2}+\frac{1}{4}(1-x)^2+\frac{1}{4}(x+1)^2$$

input
```
Int[(x*Sqrt[1 - x^2])/(x - x^3 + Sqrt[1 - x^2]),x]
```

output
```
(1 - x)^2/4 - x^2/2 + (1 + x)^2/4 + ArcSin[x] - ArcTan[(1 - 2*x^2)/Sqrt[3]
]/Sqrt[3] - ArcTan[x/(Sqrt[-((I - Sqrt[3])/(I + Sqrt[3]))]*Sqrt[1 - x^2])]
/Sqrt[3] - ArcTan[(Sqrt[-((I - Sqrt[3])/(I + Sqrt[3]))]*x)/Sqrt[1 - x^2]]/
Sqrt[3]
```

3.1022.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.1022.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.12

method	result
trager	$\operatorname{RootOf}\left(\_Z^2+1\right) \ln\left(\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)+\frac{\operatorname{RootOf}\left(\_Z^2+3\right)\ln\left(\frac{2\operatorname{RootOf}\left(\_Z^2+3\right)x^2+3x\cdots}{\operatorname{RootOf}\left(\_Z\cdots\right.}\right.}{3}$
default	$\frac{\sqrt{3}\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{3}+\left(\frac{1}{4}+\frac{i\sqrt{3}}{12}\right)\ln\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{\left(-1+i\sqrt{3}\right)\left(\sqrt{-x^2+1}-1\right)}{x}-1\right)+\left(-\frac{i\sqrt{3}}{12}+\frac{1}{4}\right)\ln\cdots$

input
```
int(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
RootOf(_Z^2+1)*ln(RootOf(_Z^2+1)*(-x^2+1)^(1/2)+x)+1/3*RootOf(_Z^2+3)*ln((
2*RootOf(_Z^2+3)*x^2+3*x*(-x^2+1)^(1/2)-RootOf(_Z^2+3))/(RootOf(_Z^2+3)*x^
2-x^2+2))
```

3.1022.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.74

$$\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}}\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2-1)\right) + \frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(2x^2-1)\sqrt{-x^2+1}}{3(x^3-x)}\right) - 2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

input
```
integrate(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/3*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x^2 - 1)*sqrt(-x^2 + 1)/(x^3 - x)) - 2*arctan((sqrt(-x^2 + 1) - 1)/x
)
```

3.1022.6 Sympy [F]

$$\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}}\,dx = -\int \frac{x\sqrt{1-x^2}}{x^3-x-\sqrt{1-x^2}}\,dx$$

input
```
integrate(x*(-x**2+1)**(1/2)/(x-x**3+(-x**2+1)**(1/2)),x)
```

output
```
-Integral(x*sqrt(1 - x**2)/(x**3 - x - sqrt(1 - x**2)), x)
```

3.1022.7 Maxima [F]

$$\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}}\,dx = \int -\frac{\sqrt{-x^2+1}x}{x^3-x-\sqrt{-x^2+1}}\,dx$$

input
```
integrate(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

output
```
1/2*x^2 + integrate(-(x^4 - x^2)/(x^3 - x - sqrt(x + 1)*sqrt(-x + 1)), x)
```

3.1022.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(37) = 74.

Time = 0.35 (sec) , antiderivative size = 193, normalized size of antiderivative = 4.60

$$\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}}\,dx$$
$$= \frac{1}{2}\,\pi\mathrm{sgn}(x) - \frac{1}{6}\,\sqrt{3}\left(\pi\mathrm{sgn}(x) + 2\,\arctan\left(-\frac{\sqrt{3}x\left(\frac{\sqrt{-x^2+1}-1}{x} + \frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2} - 1\right)}{3\left(\sqrt{-x^2+1}-1\right)}\right)\right)$$
$$- \frac{1}{6}\,\sqrt{3}\left(\pi\mathrm{sgn}(x) + 2\,\arctan\left(\frac{\sqrt{3}x\left(\frac{\sqrt{-x^2+1}-1}{x} - \frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2} + 1\right)}{3\left(\sqrt{-x^2+1}-1\right)}\right)\right)$$
$$+ \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x^2-1)\right) + \arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2} - 1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)$$

input
```
integrate(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
1/2*pi*sgn(x) - 1/6*sqrt(3)*(pi*sgn(x) + 2*arctan(-1/3*sqrt(3)*x*((sqrt(-x
^2 + 1) - 1)/x + (sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))) -
1/6*sqrt(3)*(pi*sgn(x) + 2*arctan(1/3*sqrt(3)*x*((sqrt(-x^2 + 1) - 1)/x -
(sqrt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1))) + 1/3*sqrt(3)*arcta
n(1/3*sqrt(3)*(2*x^2 - 1)) + arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1
)/(sqrt(-x^2 + 1) - 1))
```

3.1022.9 Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 549, normalized size of antiderivative = 13.07

$$\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}}\,dx = \operatorname{asin}(x) - \frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)-1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}}-\sqrt{1-x^2}\,1\mathrm{i}}{\frac{\sqrt{3}}{2}-x+\frac{1}{2}\mathrm{i}}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}\left(\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}\right)} + \frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)-1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}\left(-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}\right)} - \frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)+1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^2}\left(-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}\right)} - \frac{\ln\left(x-\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)}{\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}} - \frac{\ln\left(x+\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)}{\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}} + \frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)+1\right)1\mathrm{i}}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^2}\left(\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}\right)} + \frac{\ln\left(x-\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)}{-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}} + \frac{\ln\left(x+\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)}{-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\mathrm{i}\right)^3+1\mathrm{i}}$$

input
```
int((x*(1 - x^2)^(1/2))/(x - x^3 + (1 - x^2)^(1/2)),x)
```

output
```
asin(x) - log((((x*(3^(1/2)/2 + 1i/2) - 1)*1i)/(1 - (3^(1/2)/2 + 1i/2)^2)^
(1/2) - (1 - x^2)^(1/2)*1i)/(3^(1/2)/2 - x + 1i/2))/((1 - (3^(1/2)/2 + 1i/
2)^2)^(1/2)*(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)) + log((((x*(3^(1/2)/2
 - 1i/2) - 1)*1i)/(1 - (3^(1/2)/2 - 1i/2)^2)^(1/2) - (1 - x^2)^(1/2)*1i)/(
x - 3^(1/2)/2 + 1i/2))/((1 - (3^(1/2)/2 - 1i/2)^2)^(1/2)*(4*(3^(1/2)/2 - 1
i/2)^3 - 3^(1/2) + 1i)) - log((((x*(3^(1/2)/2 - 1i/2) + 1)*1i)/(1 - (3^(1/
2)/2 - 1i/2)^2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + 3^(1/2)/2 - 1i/2))/((1 -
(3^(1/2)/2 - 1i/2)^2)^(1/2)*(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i)) - (lo
g(x - 3^(1/2)/2 - 1i/2)*(3^(1/2)/2 + 1i/2))/(3^(1/2) - 4*(3^(1/2)/2 + 1i/2
)^3 + 1i) - (log(x + 3^(1/2)/2 + 1i/2)*(3^(1/2)/2 + 1i/2))/(3^(1/2) - 4*(3
^(1/2)/2 + 1i/2)^3 + 1i) + log((((x*(3^(1/2)/2 + 1i/2) + 1)*1i)/(1 - (3^(1
/2)/2 + 1i/2)^2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + 3^(1/2)/2 + 1i/2))/((1 -
 (3^(1/2)/2 + 1i/2)^2)^(1/2)*(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)) + (l
og(x - 3^(1/2)/2 + 1i/2)*(3^(1/2)/2 - 1i/2))/(4*(3^(1/2)/2 - 1i/2)^3 - 3^(
1/2) + 1i) + (log(x + 3^(1/2)/2 - 1i/2)*(3^(1/2)/2 - 1i/2))/(4*(3^(1/2)/2
- 1i/2)^3 - 3^(1/2) + 1i)
```

3.1022. $\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}}\,dx$

3.1023 $\int (1+x+x^2+x^3)^{-n} (1-x^4)^n \, dx$

3.1023.1 Optimal result

Integrand size = 23, antiderivative size = 34

$$\int (1+x+x^2+x^3)^{-n} (1-x^4)^n \, dx = -\frac{(1-x)(1+x+x^2+x^3)^{-n} (1-x^4)^n}{1+n}$$

output
```
-(1-x)*(-x^4+1)^n/(1+n)/((x^3+x^2+x+1)^n)
```

3.1023.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int (1+x+x^2+x^3)^{-n} (1-x^4)^n \, dx = \frac{(-1+x)(1+x+x^2+x^3)^{-n} (1-x^4)^n}{1+n}$$

input
```
Integrate[(1 - x^4)^n/(1 + x + x^2 + x^3)^n,x]
```

output
```
((-1 + x)*(1 - x^4)^n)/((1 + n)*(1 + x + x^2 + x^3)^n)
```

3.1023.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^3+x^2+x+1\right)^{-n} \left(1-x^4\right)^n \, dx$$

$$\Big\downarrow \text{7299}$$

$$\int \left(x^3+x^2+x+1\right)^{-n} \left(1-x^4\right)^n dx$$

input
```
Int[(1 - x^4)^n/(1 + x + x^2 + x^3)^n,x]
```

output
```
$Aborted
```

3.1023.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.1023.4 Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

method	result
gosper	$\frac{(x-1)\left(-x^4+1\right)^n \left(x^3+x^2+x+1\right)^{-n}}{1+n}$
parallelrisch	$-\frac{\left(-x\left(-x^4+1\right)^n n+\left(-x^4+1\right)^n n\right)\left(x^3+x^2+x+1\right)^{-n}}{n(1+n)}$
norman	$\left(\frac{x\,\mathrm{e}^{n\ln\left(-x^4+1\right)}}{1+n}-\frac{\mathrm{e}^{n\ln\left(-x^4+1\right)}}{1+n}\right)\mathrm{e}^{-n\ln\left(x^3+x^2+x+1\right)}$
risch	$\frac{(x-1)(x-1)^n \mathrm{e}^{-\frac{i n\pi\left(-\operatorname{csgn}\left(i(x-1)\left(x^3+x^2+x+1\right)\right)^3-\operatorname{csgn}\left(i(x-1)\left(x^3+x^2+x+1\right)\right)^2 \operatorname{csgn}(i(x-1))-\operatorname{csgn}\left(i(x-1)\left(x^3+x^2+x+1\right)\right)^2 \operatorname{csgn}\left(\right.\right.}{}}}{1+n}$

input
```
int((-x^4+1)^n/((x^3+x^2+x+1)^n),x,method=_RETURNVERBOSE)
```

output
```
(x-1)/(1+n)*(-x^4+1)^n/((x^3+x^2+x+1)^n)
```

3.1023.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \left(1+x+x^2+x^3\right)^{-n} \left(1-x^4\right)^n \, dx = \frac{(-x^4+1)^n(x-1)}{(x^3+x^2+x+1)^n(n+1)}$$

input
```
integrate((-x^4+1)^n/((x^3+x^2+x+1)^n),x, algorithm="fricas")
```

output
```
(-x^4 + 1)^n*(x - 1)/((x^3 + x^2 + x + 1)^n*(n + 1))
```

3.1023.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(24) = 48.

Time = 17.96 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.15

$$\int \left(1+x+x^2+x^3\right)^{-n} \left(1-x^4\right)^n \, dx$$
$$= \begin{cases} \frac{x(1-x^4)^n}{n(x^3+x^2+x+1)^n+(x^3+x^2+x+1)^n} - \frac{(1-x^4)^n}{n(x^3+x^2+x+1)^n+(x^3+x^2+x+1)^n} & \text{for } n \neq -1 \\ -\log(x-1) & \text{otherwise} \end{cases}$$

input
```
integrate((-x**4+1)**n/((x**3+x**2+x+1)**n),x)
```

output
```
Piecewise((x*(1 - x**4)**n/(n*(x**3 + x**2 + x + 1)**n + (x**3 + x**2 + x
+ 1)**n) - (1 - x**4)**n/(n*(x**3 + x**2 + x + 1)**n + (x**3 + x**2 + x +
1)**n), Ne(n, -1)), (-log(x - 1), True))
```

3.1023.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.47

$$\int \left(1+x+x^2+x^3\right)^{-n} \left(1-x^4\right)^n \, dx = \frac{(x-1)(-x+1)^n}{n+1}$$

input
```
integrate((-x^4+1)^n/((x^3+x^2+x+1)^n),x, algorithm="maxima")
```

output
```
(x - 1)*(-x + 1)^n/(n + 1)
```

3.1023.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(31) = 62$.

Time = 0.36 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.38

$$\int \left(1+x+x^2+x^3\right)^{-n} \left(1-x^4\right)^n \, dx = \frac{\frac{xe^{\left(n\log\left(x^3+x^2+x+1\right)+n\log\left(-x+1\right)\right)}}{\left(x^3+x^2+x+1\right)^n} - \frac{e^{\left(n\log\left(x^3+x^2+x+1\right)+n\log\left(-x+1\right)\right)}}{\left(x^3+x^2+x+1\right)^n}}{n+1}$$

input
```
integrate((-x^4+1)^n/((x^3+x^2+x+1)^n),x, algorithm="giac")
```

output
```
(x*e^(n*log(x^3 + x^2 + x + 1) + n*log(-x + 1))/(x^3 + x^2 + x + 1)^n - e^
(n*log(x^3 + x^2 + x + 1) + n*log(-x + 1))/(x^3 + x^2 + x + 1)^n)/(n + 1)
```

3.1023.9 Mupad [B] (verification not implemented)

Time = 19.81 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \left(1+x+x^2+x^3\right)^{-n} \left(1-x^4\right)^n \, dx = \frac{\left(1-x^4\right)^n (x-1)}{(n+1) \left(x^3+x^2+x+1\right)^n}$$

input
```
int((1 - x^4)^n/(x + x^2 + x^3 + 1)^n,x)
```

output
```
((1 - x^4)^n*(x - 1))/((n + 1)*(x + x^2 + x^3 + 1)^n)
```

3.1024 $\int \frac{x}{\sqrt{-44375b^4+576000b^3cx+576000b^2c^2x^2+5308416c^4x^4}}\,dx$

3.1024.1 Optimal result

Integrand size = 38, antiderivative size = 177

$$\int \frac{x}{\sqrt{-44375b^4+576000b^3cx+576000b^2c^2x^2+5308416c^4x^4}}\,dx$$
$$= \frac{\log\left(20738073600000000b^8c^4+5970056970240000000b^6c^6x^2+25831007059968000000b^5c^7x^3+9510507144\right.}{}$$

output
```
1/18432*ln(20738073600000000*b^8*c^4+5970056970240000000*b^6*c^6*x^2+258310
0705996800000*b^5*c^7*x^3+9510507144806400000*b^4*c^8*x^4+21641687369515008
000*b^3*c^9*x^5+32462531054272512000*b^2*c^10*x^6+1495873430980877735296*c^
12*x^8+5308416*(12230590464*c^10*x^6+1990656000*b^2*c^8*x^4+1105920000*b^3
*c^7*x^3+38880000*b^4*c^6*x^2+79200000*b^5*c^5*x+12203125*b^6*c^4)*(530841
6*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(1/2))/c^2
```

3.1024.2 Mathematica [A] (verified)

Time = 5.60 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.03

$$\int \frac{x}{\sqrt{-44375b^4+576000b^3cx+576000b^2c^2x^2+5308416c^4x^4}}\,dx =$$
$$-\frac{\log\left(-203863558717440000000000000b^{13}c^7-586880480402472960000000000000b^{11}c^9x^2-25392913\right.}{}$$

input
```
Integrate[x/Sqrt[-44375*b^4 + 576000*b^3*c*x + 576000*b^2*c^2*x^2 + 530841
6*c^4*x^4],x]
```

output
```
-1/18432*Log[-20386355871744000000000000000*b^13*c^7 - 586880480402472960000000000000*b^11*c^9*x^2 - 2539291318023094272000000000000*b^10*c^10*x^3 - 934920894363048345600000000000*b^9*c^11*x^4 - 21274644351728033464320000000000*b^8*c^12*x^5 - 3191196652759205019648000000000*b^7*c^13*x^6 - 1470503417591441673053798400000000*b^5*c^15*x^8 + Sqrt[-44375*b^4 + 576000*b^3*c*x + 576000*b^2*c^2*x^2 + 5308416*c^4*x^4]*(636806076825600000000000*b^11*c^7 + 4132961129594880000000000*b^10*c^8*x + 2028908190892032000000000*b^9*c^9*x^2 + 5771116631870668800000000*b^8*c^10*x^3 + 1038800993736720384000000000*b^7*c^11*x^4 + 6382393305518410039296000000*b^5*c^13*x^6)]/c^2
```

3.1024.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {2505}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{-44375b^4+576000b^3cx+576000b^2c^2x^2+5308416c^4x^4}}\,dx$$

$$\Big\downarrow \; 2505$$

$$\frac{\log \left(20738073600000000b^8c^4+5970056970240000000b^6c^6x^2+25831007059968000000b^5c^7x^3+9510507144806400\right.}{}$$

input
```
Int[x/Sqrt[-44375*b^4 + 576000*b^3*c*x + 576000*b^2*c^2*x^2 + 5308416*c^4*x^4],x]
```

output
```
Log[20738073600000000*b^8*c^4 + 5970056970240000000*b^6*c^6*x^2 + 25831007059968000000*b^5*c^7*x^3 + 95105071448064000000*b^4*c^8*x^4 + 21641687369515008000*b^3*c^9*x^5 + 324625310542725120000*b^2*c^10*x^6 + 149587343098087735296*c^12*x^8 + 5308416*Sqrt[-44375*b^4 + 576000*b^3*c*x + 576000*b^2*c^2*x^2 + 5308416*c^4*x^4]*(12203125*b^6*c^4 + 79200000*b^5*c^5*x + 38880000*b^4*c^6*x^2 + 1105920000*b^3*c^7*x^3 + 1990656000*b^2*c^8*x^4 + 12230590464*c^10*x^6)]/(18432*c^2)
```

3.1024.3.1 Defintions of rubi rules used

rule 2505
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (e_.)*(x_)^4], x_Symbol] :
> With[{Px = (1/320)*(33*b^2*c + 6*a*c^2 + 40*a^2*e) - (22/5)*a*c*e*x^2 + (
22/15)*b*c*e*x^3 + (1/4)*e*(5*c^2 + 4*a*e)*x^4 + (4/3)*b*e^2*x^5 + 2*c*e^2*
x^6 + e^3*x^8}, Simp[(1/(8*Rt[e, 2]))*Log[Px + (1/(8*Rt[e, 2]*x)   D[Px, x]
)*Sqrt[a + b*x + c*x^2 + e*x^4]], x]] /; FreeQ[{a, b, c, e}, x] && EqQ[71*c
^2 + 100*a*e, 0] && EqQ[1152*c^3 - 125*b^2*e, 0]
```

3.1024.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.54 (sec) , antiderivative size = 1597, normalized size of antiderivative = 9.02

method	result	size
default	Expression too large to display	1597
elliptic	Expression too large to display	1597

input
```
int(x/(5308416*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(1/2),
x,method=_RETURNVERBOSE)
```

output
```
1/1152*(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c-5/48*RootOf(_Z^4+10
*_Z^2+96*_Z-71,index=4)*b/c)*((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*
b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c)*(x-5/48*RootOf(_Z^4+10
*_Z^2+96*_Z-71,index=1)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b
/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*
_Z^2+96*_Z-71,index=2)*b/c))^(1/2)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,in
dex=2)*b/c)^2*((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c-5/48*RootOf
(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,
index=3)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)*b/c-5/48*RootOf(
_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,i
ndex=2)*b/c))^(1/2)*((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c-5/48*
RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*
_Z-71,index=4)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*R
ootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+96*_
Z-71,index=2)*b/c))^(1/2)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-
5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+
96*_Z-71,index=2)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)/(c^4
*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)*(x-5/48*RootOf(_Z^4+10
*_Z^2+96*_Z-71,index=2)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)
*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c))^(1/2)*(5/48*R...
```

3.1024.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.93

$$\int \frac{x}{\sqrt{-44375b^4+576000b^3cx+576000b^2c^2x^2+5308416c^4x^4}}\,dx$$
$$= \frac{\log\left(28179280429056\,c^8x^8+6115295232000\,b^2c^6x^6+4076863488000\,b^3c^5x^5+179159040000\,b^4c^4x^4+486\right.}{}$$

input
```
integrate(x/(5308416*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^
(1/2),x, algorithm="fricas")
```

output
```
1/18432*log(28179280429056*c^8*x^8 + 6115295232000*b^2*c^6*x^6 + 407686348
8000*b^3*c^5*x^5 + 179159040000*b^4*c^4*x^4 + 486604800000*b^5*c^3*x^3 + 1
12464000000*b^6*c^2*x^2 + 3906640625*b^8 + (12230590464*c^6*x^6 + 19906560
00*b^2*c^4*x^4 + 1105920000*b^3*c^3*x^3 + 38880000*b^4*c^2*x^2 + 79200000*
b^5*c*x + 12203125*b^6)*sqrt(5308416*c^4*x^4 + 576000*b^2*c^2*x^2 + 576000
*b^3*c*x - 44375*b^4))/c^2
```

3.1024.6 Sympy [F]

$$\int \frac{x}{\sqrt{-44375b^4 + 576000b^3cx + 576000b^2c^2x^2 + 5308416c^4x^4}}\,dx = \int \frac{x}{\sqrt{-44375b^4 + 576000b^3cx + 576000b^2c^2x^2 + 5308416c^4x^4}}\,dx$$

input
```
integrate(x/(5308416*c**4*x**4+576000*b**2*c**2*x**2+576000*b**3*c*x-44375
*b**4)**(1/2),x)
```

output
```
Integral(x/sqrt(-44375*b**4 + 576000*b**3*c*x + 576000*b**2*c**2*x**2 + 53
08416*c**4*x**4), x)
```

3.1024.7 Maxima [F]

$$\int \frac{x}{\sqrt{-44375b^4 + 576000b^3cx + 576000b^2c^2x^2 + 5308416c^4x^4}}\,dx = \int \frac{x}{\sqrt{5308416\,c^4x^4 + 576000\,b^2c^2x^2 + 576000\,b^3cx - 44375\,b^4}}\,dx$$

input
```
integrate(x/(5308416*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^
(1/2),x, algorithm="maxima")
```

output
```
integrate(x/sqrt(5308416*c^4*x^4 + 576000*b^2*c^2*x^2 + 576000*b^3*c*x - 4
4375*b^4), x)
```

3.1024.8 Giac [F]

$$\int \frac{x}{\sqrt{-44375b^4 + 576000b^3cx + 576000b^2c^2x^2 + 5308416c^4x^4}}\,dx = \int \frac{x}{\sqrt{5308416\,c^4x^4 + 576000\,b^2c^2x^2 + 576000\,b^3cx - 44375\,b^4}}\,dx$$

input
```
integrate(x/(5308416*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^
(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(5308416*c^4*x^4 + 576000*b^2*c^2*x^2 + 576000*b^3*c*x - 4
4375*b^4), x)
```

3.1024.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{-44375b^4 + 576000b^3cx + 576000b^2c^2x^2 + 5308416c^4x^4}} \, dx$$
$$= \int \frac{x}{\sqrt{-44375\, b^4 + 576000\, b^3\, c\, x + 576000\, b^2\, c^2\, x^2 + 5308416\, c^4\, x^4}} \, dx$$

input
```
int(x/(5308416*c^4*x^4 - 44375*b^4 + 576000*b^2*c^2*x^2 + 576000*b^3*c*x)^
(1/2),x)
```

output
```
int(x/(5308416*c^4*x^4 - 44375*b^4 + 576000*b^2*c^2*x^2 + 576000*b^3*c*x)^
(1/2), x)
```

3.1025 $\int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}}\,dx$

3.1025.1 Optimal result

Integrand size = 30, antiderivative size = 100

$$\int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}}\,dx = \frac{1}{16}\log\Big(921+2864x+9280x^2+13440x^3+17024x^4+19456x^5+12288x^6+8192x^7+4096x^8+\sqrt{9+120x+64x^2+64x^3+64x^4}\,(179+444x+744x^2+1280x^3+960x^4+768x^5+512x^6)\Big)$$

output
```
1/16*ln(921+2864*x+9280*x^2+13440*x^3+17024*x^4+19456*x^5+12288*x^6+8192*x
^7+4096*x^8+(512*x^6+768*x^5+960*x^4+1280*x^3+744*x^2+444*x+179)*(64*x^4+6
4*x^3+64*x^2+120*x+9)^(1/2))
```

3.1025.2 Mathematica [A] (verified)

Time = 3.63 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}}\,dx = -\frac{1}{16}\log\Big(-921-2864x-9280x^2-13440x^3-17024x^4-19456x^5-12288x^6-8192x^7-4096x^8+\sqrt{9+120x+64x^2+64x^3+64x^4}\,(179+444x+744x^2+1280x^3+960x^4+768x^5+512x^6)\Big)$$

input
```
Integrate[(1 + 4*x)/Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4],x]
```

output
```
-1/16*Log[-921 - 2864*x - 9280*x^2 - 13440*x^3 - 17024*x^4 - 19456*x^5 - 1
2288*x^6 - 8192*x^7 - 4096*x^8 + Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4
]*(179 + 444*x + 744*x^2 + 1280*x^3 + 960*x^4 + 768*x^5 + 512*x^6)]
```

3.1025.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.39, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2506, 27, 2505}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x+1}{\sqrt{64x^4+64x^3+64x^2+120x+9}}dx$$

$$\Big\downarrow \text{2506}$$

$$4\int \frac{2\left(x+\frac{1}{4}\right)}{\sqrt{256\left(x+\frac{1}{4}\right)^4+160\left(x+\frac{1}{4}\right)^2+384\left(x+\frac{1}{4}\right)-71}}d\left(x+\frac{1}{4}\right)$$

$$\Big\downarrow \text{27}$$

$$8\int \frac{x+\frac{1}{4}}{\sqrt{256\left(x+\frac{1}{4}\right)^4+160\left(x+\frac{1}{4}\right)^2+384\left(x+\frac{1}{4}\right)-71}}d\left(x+\frac{1}{4}\right)$$

$$\Big\downarrow \text{2505}$$

$$\frac{1}{16}\log\left(16777216\left(x+\frac{1}{4}\right)^8+20971520\left(x+\frac{1}{4}\right)^6+33554432\left(x+\frac{1}{4}\right)^5+3538944\left(x+\frac{1}{4}\right)^4+23068672\left(x+\right.\right.$$

input
```
Int[(1 + 4*x)/Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4],x]
```

output
```
Log[2560256 + 12795904*(1/4 + x)^2 + 23068672*(1/4 + x)^3 + 3538944*(1/4 +
 x)^4 + 33554432*(1/4 + x)^5 + 20971520*(1/4 + x)^6 + 16777216*(1/4 + x)^8
 + 256*Sqrt[-71 + 384*(1/4 + x) + 160*(1/4 + x)^2 + 256*(1/4 + x)^4]*(781
+ 2112*(1/4 + x) + 432*(1/4 + x)^2 + 5120*(1/4 + x)^3 + 3840*(1/4 + x)^4 +
 4096*(1/4 + x)^6)]/16
```

3.1025.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2505
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (e_.)*(x_)^4], x_Symbol] :
> With[{Px = (1/320)*(33*b^2*c + 6*a*c^2 + 40*a^2*e) - (22/5)*a*c*e*x^2 + (
22/15)*b*c*e*x^3 + (1/4)*e*(5*c^2 + 4*a*e)*x^4 + (4/3)*b*e^2*x^5 + 2*c*e^2*
x^6 + e^3*x^8}, Simp[(1/(8*Rt[e, 2]))*Log[Px + (1/(8*Rt[e, 2]*x)   D[Px, x]
)*Sqrt[a + b*x + c*x^2 + e*x^4]], x]] /; FreeQ[{a, b, c, e}, x] && EqQ[71*c
^2 + 100*a*e, 0] && EqQ[1152*c^3 - 125*b^2*e, 0]
```

rule 2506
```
Int[((A_) + (B_.)*(x_))/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^
3 + (e_.)*(x_)^4], x_Symbol] :> Simp[B   Subst[Int[x/Sqrt[(-3*d^4 + 16*c*d^
2*e - 64*b*d*e^2 + 256*a*e^3)/(256*e^3) + (d^3 - 4*c*d*e + 8*b*e^2)*(x/(8*e
^2)) - (3*d^2 - 8*c*e)*(x^2/(8*e)) + e*x^4], x], x, d/(4*e) + x], x] /; Fre
eQ[{a, b, c, d, e, A, B}, x] && EqQ[B*d - 4*A*e, 0] && EqQ[d*(141*d^3 - 752
*c*d*e - 400*b*e^2) + 16*e^2*(71*c^2 + 100*a*e), 0] && EqQ[144*(3*d^2 - 8*c
*e)^3 + 125*(d^3 - 4*c*d*e + 8*b*e^2)^2, 0]
```

3.1025.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(96) = 192$.

Time = 2.07 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.28

method	result
trager	$-\frac{\ln\left(4096x^8-512\sqrt{64x^4+64x^3+64x^2+120x+9}\,x^6+8192x^7-768\sqrt{64x^4+64x^3+64x^2+120x+9}\,x^5+12288x^6-960\sqrt{64x^4+64x^3+64\right.}{}$
default	Expression too large to display
elliptic	Expression too large to display

input
```
int((1+4*x)/(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16*ln(4096*x^8-512*(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2)*x^6+8192*x^7-76
8*(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2)*x^5+12288*x^6-960*(64*x^4+64*x^3+64
*x^2+120*x+9)^(1/2)*x^4+19456*x^5-1280*(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2
)*x^3+17024*x^4-744*(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2)*x^2+13440*x^3-444
*x*(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2)+9280*x^2-179*(64*x^4+64*x^3+64*x^2
+120*x+9)^(1/2)+2864*x+921)
```

3.1025.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97

$$\int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}}\,dx = \frac{1}{16}\log\left(-4096\,x^8-8192\,x^7-12288\,x^6-19456\,x^5-17024\,x^4-13440\,x^3-9280\,x^2 - \left(512\,x^6+768\,x^5+960\,x^4+1280\,x^3+744\,x^2+444\,x+179\right)\sqrt{64\,x^4+64\,x^3+64\,x^2+120\,x+9} - 2864\,x-921\right)$$

input
```
integrate((1+4*x)/(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2),x, algorithm="frica
s")
```

output
```
1/16*log(-4096*x^8 - 8192*x^7 - 12288*x^6 - 19456*x^5 - 17024*x^4 - 13440*
x^3 - 9280*x^2 - (512*x^6 + 768*x^5 + 960*x^4 + 1280*x^3 + 744*x^2 + 444*x
 + 179)*sqrt(64*x^4 + 64*x^3 + 64*x^2 + 120*x + 9) - 2864*x - 921)
```

3.1025.6 Sympy [F]

$$\int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}}\,dx = \int \frac{4x+1}{\sqrt{64x^4+64x^3+64x^2+120x+9}}\,dx$$

input
```
integrate((1+4*x)/(64*x**4+64*x**3+64*x**2+120*x+9)**(1/2),x)
```

output
```
Integral((4*x + 1)/sqrt(64*x**4 + 64*x**3 + 64*x**2 + 120*x + 9), x)
```

3.1025.7 Maxima [F]

$$\int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}}\,dx=\int \frac{4\,x+1}{\sqrt{64\,x^4+64\,x^3+64\,x^2+120\,x+9}}\,dx$$

input
```
integrate((1+4*x)/(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((4*x + 1)/sqrt(64*x^4 + 64*x^3 + 64*x^2 + 120*x + 9), x)
```

3.1025.8 Giac [F]

$$\int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}}\,dx=\int \frac{4\,x+1}{\sqrt{64\,x^4+64\,x^3+64\,x^2+120\,x+9}}\,dx$$

input
```
integrate((1+4*x)/(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((4*x + 1)/sqrt(64*x^4 + 64*x^3 + 64*x^2 + 120*x + 9), x)
```

3.1025.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}}\,dx=\int \frac{4\,x+1}{\sqrt{64\,x^4+64\,x^3+64\,x^2+120\,x+9}}\,dx$$

input
```
int((4*x + 1)/(120*x + 64*x^2 + 64*x^3 + 64*x^4 + 9)^(1/2),x)
```

output
```
int((4*x + 1)/(120*x + 64*x^2 + 64*x^3 + 64*x^4 + 9)^(1/2), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
          if is_contains_complex(optimal) then
              if debug then
                      print("both result and optimal complex");
              fi;
              if leaf_count_result<=2*leaf_count_optimal then
                  return "A"," ";
              else
                 return "B",cat("Both result and optimal contain complex but leaf count of
                                 convert(leaf_count_result,string)," vs. $2 (",
                                 convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
              end if
          else #result contains complex but optimal is not
              if debug then
                      print("result contains complex but optimal is not");
              fi;
              return "C","Result contains complex when optimal does not.";
          fi;
      else # result do not contain complex
           # this assumes optimal do not as well. No check is needed here.
          if debug then
                 print("result do not contain complex, this assumes optimal do not as well"
          fi;
          if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                  print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A"," ";
          else
              if debug then
                  print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                 convert(leaf_count_result,string),"$ vs. $2(",
                                 convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
          fi;
      fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])  #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```